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A Simplified Agent Based Model of a Sinks and Faucets Resource Economy, Using a blind commitment market

Master's Thesis in Complex Adaptive Systems

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CHALMERS UNIVERSITY OF TECHNOLOGY
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Master's Thesis 2014:1

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Printed in Sweden by Reproservice, Chalmers Tekniska Hogskola, 2013

Abstract

This masters thesis seeks to model a simple financial system by using an agent based model. It seeks to do this by constructing an as simple as possible market and agent dynamic. To do this the blind commitment market is introduced and the general model constructed around this. For this model a number of questions are then posed concerning the model's stability and effectiveness at modelling the core market dynamics. Furthermore an analytical analog is created in order to analyse the equilibrium dynamics. From the analytical representation parallels are drawn to the agent based model. The agent based model is constructed in such a way that agents are encouraged to interact in order to meet all demands. The arising dependancy network of agents is then analysed using a graphical representation, which shows how much the different agents depend on each other. The general dynamics are also examined as the three main parameters of the model are varied.

It is shown that the bind commitment market upholds supply and demand characteristics, and that the agent based model functions as an economy with cooperating agents, however these agents are not making any profit. Furtheromre it is shown that the analytical model is stable for all possible parameters of the allowed parameter space in the sense that feedback processes steer the supply and demand dynamics to a stable fix point, an equilibrium price. The agent based model also gravitates towards an equilibrium price, however minority game dynamics prevent the system from reaching, or staying, at the equilibrium price for long. The minority game dynamics fling the system away from the stable point, causing the system to restart its trajectory towards the equilibrium price, aided by the feedback processes. Lastly the three main parameters of the model are examined with respect to how much they influence the stability of prices inside the model.

Acknowledgements

To whoever has managed to get a hold on a copy of my masters thesis I thank you for setting aside the time to have a read through of an otherwise, to my knowledge, undocumented model. I'd especially like to thank my supervisor Kristian Lindgren, for taking on the role supervisor, simply for the thrill of what I would figure out concerning this new model. Furthermore I would like to thank those people around me whom i have pestered with questions concerning my endeavour. In no particular order I'd like to thank Simon Larsson, for helping me with the *Mathematica* analysis, Rasmus Einarsson, for getting my head straight on the stability analysis, Marcus Johansson, for helping me with the spelling, Staffan Nilsson, for giving me someone to test my theories on and last but not least Marie Doverbo for being ever so supportive throughout this project. There are many more people whom I have interacted with in one way or another during this thesis, your support and interest in my thesis has been duly noted and I am humbled by it. Those whom I have studies with over the years and those of us who spent the summer of 2013 at Chalmers simply because it was fun. Thank you, the time we spent together has been marvellous.

Anton Dalsmo, Gothenburg 5/11/13

Contents

1

Introduction

This project aims to examine networks of depending, interacting agents in a simple economy. To do this we first outline a simple economic model, which is done below. From this model we then formulate the specific questions which we seek to answer.

1.1 Simple financial model outline

Consider a simple economy of resource sinks and faucets. By this we mean that there exists a source, a faucet, of one or more resources representing an inflow of those goods into the economy. Conversely a resource sink is an outflow of one or more resources from the economy. This can be analogous to the flow of resources across a country's, or the EU-region's, border. The resource faucets represent imports and domestic production of resources, and resource sinks represent exports and domestic consumption.

Now that we have models representing the boundary of a region, consider the effect of industry and business inside said region. We may view a automobile factory as a conversion of steal, rubber and other such resources into automobiles ready for consumption. The same holds for a mobile phone producer or a consultancy firm, which turns man hours into information and project results. We can thus conclude that the internal workings of a country convert resources of one type into resources of another type. Such a process may involve just one step or many concurrent steps, increasing the complexity and dependancy of the final output on the initial goods.

We can consider the inflow, outflow and conversion processes available in a region as the basis for an economy. Into this basis we introduce profit seeking agents. Agents introduced into this economy, acting to maximise profits, will seek to meet the demands of the sinks by acquiring resources from the faucets. Simply put, the agents act as market makers between the sinks and the faucets through the conversion processes, and gather their profits from the difference in price.

Lastly we acknowledge that for the internal economy to function there needs to be a market where sinks, faucets and agents can exchange resources. Such a market may function in many different ways from open outcry to other market making mechanics. The important part is that a market's primary function is to facilitate the exchange of resources, and in this process determine the market price of said resources.

In conclusion we seek to construct a simple financial model consisting of sinks, faucets, conversion processes, agents and a market.

1.2 Aim of study

The general aim of this master's thesis was to construct and study a economic system in the simplest possible manner. Many economic models have advanced supply and demand functions[4] as well as complex market structures and thus is is required that the agents of the model have substantial intelligence [3]. Such intelligence can be modelled through the use of neural networks or some other machine learning paradigm[5]. This leads to an enormous amount of model parameters which have to be tuned in order for the model to be analysed. The main design paradigm behind this master's thesis was to reduce the number of parameters needed, in order to not make the system more complex than it needed bee.

This was achieved by firstly deciding that agents would be no more than greedy profit seeking agents with no more than the most basic of memory. Thus effectively removing all the parameters associated with machine learning such as particle swarm optimisation parameters or number of training iterations.

Secondly it was decided that the boundaries of the economy where to be fixed, i.e. the supply of any good from the boundary will kept constant even though the demand for that good would vary. Boundaries with varying supply and demand are usually considered by the following train of thought.

Consider five peasants wishing to buy pieces of pastry. If pastry is priced at six pence a piece two of the five peasants will buy one piece of pastry each, giving the demand at that price point of two pieces of pastry at six pence a piece. However if the pastry where priced at two pence a piece, even the poorest peasants would be willing to buy, yielding the demand at that price point of five pieces of pastry at two pence a peace. There are several models relating the price of a good to the demand or supply of that same good. However even the simplest of these incur the model with extra parameters, which we avoid by fixing the boundary demand and supply.

Lastly it was decided that the market was to be as simple as possible. Usually a market may function by agents placing bids for resources on a market, and those bids are met buy sell orders on the same market. However this requires that the agents place their bids in an intelligent manner, which requires thinking and an evaluation of how much they require a specific resource. This intelligent behaviour would incur to many parameters. We shall thus construct a market mechanism that does not require intelligent agents or parameters.

Keeping in mind theses restrictions on model complexity we still sought a model that gives rise to complex behaviour. The model was thus designed to give rise to a system of agents, where all individual agents depend (to some degree) on all other agents. This was achieved by having some good sinks which do not have a corresponding good faucet, which means that a conversion process needs to be used in order to meet the sink demand. By having several goods linked in value chains it was meant to give rise to depending agents.

1.3 Problem formulation

Since the focus of this project can be divided into two stages we shall do the same with our problem formulation. The first part concerns the basic workings of the financial model and it is detailed below.

1. Is it possible to create a financial model under the constraints specified in section 1.2, which gives normal supply and demand characteristics as well as a functioning market?

2. Does the financial model yield profitable agents?

3. Does the price of all goods in the financial model converge to some market equilibrium?

4. How does the specified model depend on the economic boundary conditions, i.e. sink and faucet strengths, as well as the exact nature of the available conversion processes?

When these questions have been thoroughly discussed we will attempt to answer the following, more insightful questions.

5. Does the model give rise to niche markets occupied by only a small number of agents?

6. Does the model give rise to a network of agents that depend on one another for their individual profits?

7. How resilient are these networks of depending agents to the removal of one or more processes in the network?

8. What is the topological equivalence of such networks and other networks?

9. What conclusions can we draw from the perspective of game-theory, specifically concerning minority games?

1.3.1 Restrictions

Now that we have defined the aims of this project we shall discuss what this project shall not cover. Since the aim is to find a model with as few parameters as possible this thesis will not cover agent models with any sort of intelligence. This rules out all forms of agents involving neural networks and strategy models. This also rules out all forms of price prediction and price filtering. The only data to be made available to the agents will be the previous price, and nothing else. This makes the agents susceptible to price noise, inuring them to act on a particular good being overpriced for one iteration. This was considered an acceptable shortcoming in keeping the project scope constrained.

Furthermore the project would not dabble in lots of different variants of available conversion processes. If one decides to analyse many different structures of the available processes, this opens a can of worms which may not be closed.

Lastly the invented dynamics, such as the market mechanism, were to be investigated with the aim of understanding what dynamics they yielded, and not to be adapted to suit preconceptions of how a market should function.

2

Theory

This chapter covers the relevant parts of game theory and general market dynamics necessary to fully understand this master's thesis.

2.1 Game theory and the minority game

We shall start by examining the Minority game, exemplified with the El Farol bar problem. The El Farol bar problem is based on a bar in Santa Fe, New Mexico [1]. For the inhabitants in the surrounding area the bar presents a problem, each Thursday a music event is held, and the inhabitants must independently decide if they shall attend that Thursday or not. However the bar is quite small and easily becomes overcrowded. So much so, that if more than 60% of the population decides to attend an event the bar will become overcrowded, in which case they would have had a better time if they stayed at home. However if less than 60% of the population attends the event, they will have a better time than if they stayed at home. This poses a problem, whether to attend or not to attend.

The El Farol bar problem was described by Brian Arthur in 1994 [1]. In his first paper on the problem he creates a community of 100 agents; theses agents are given strategies which describes how to handle the bar problem given a set history of the crowdedness of the bar. The agents then choose the strategy which is most profitable to follow for every successive Thursday. This results in the bar attendance rate converging quickly to around 60%, however it never settles down.

The El Farol bar problem is an example of a minority game, in which agents are asked to chose one of two alternatives. The minority is then rewarded and the majority is penalised. It should be noted that minority in this case refers to the less crowded group, with respect to its maximum capacity, and not in the general sense. An every day students example of a minority game would be choosing a computer hall for the day. At Chalmers there are several different computer halls of different sizes and students must chose which one to use, with the aim of finding the least crowded one. A further example of a minority game is the game posed by Yi-Cheng Zhang and Damien Challet [6] in their 2005 paper on Interacting Agents in Financial Markets. This game was much simpler than the El Farole bar problem, it features an odd number of agents choosing between two alternatives, rewarding the alternative which the minority chose.

2.2 The basic market

A market is a place where buyers and sellers meet to exchange goods and currency. There are several methods to facilitate this. The most famous being open outcry, still used by the New York Stock Exchange as stated by its wikipedia page [7]. In open outcry buyers and sellers participate in an open auction where they continuously announce at what price they are willing to buy or sell a specific good. At this point an open auction takes place until a price is determined.

Open outcry will find the market price, given a set of buyers and sellers each with their own maximum or minimum price as well as how much they wish to buy or sell of a specific good. The market price is the price at which the largest amount of good can change hands. This is expressed in Figure 3.2 in which the supply curve depicts the price quantity relation presented buy the sellers, and the demand curve depicts the price quantity relation presented buy the buyers. The intersection of theses curves determine the market price. If demand increases the price goes up, this corresponds to the demand curve being shifted to the right thus increasing the equilibrium price. If the demand decreases, the demand curve shifts to the left and the price decreases. The same dynamics affect the supply curve in a similar way.

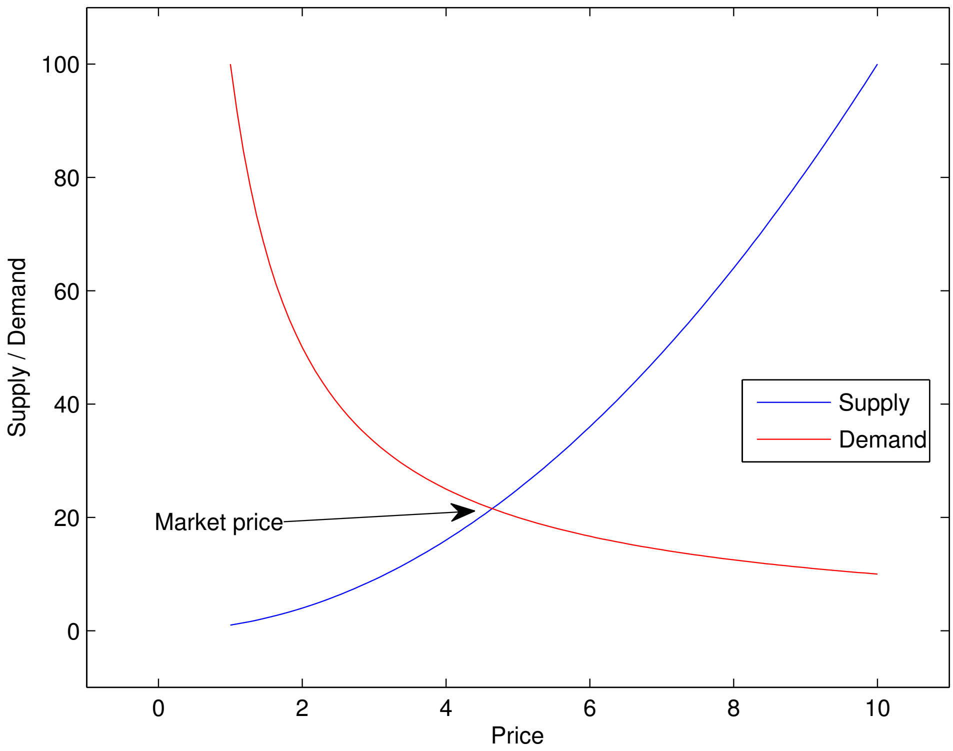

Figure 2.1: Figure depicting supply and demand curves with the market equilibrium price marked.

Open outcry is very common but there are other market mechanisms which should be noted. One mechanism is where both buyers and sellers place their orders with a market maker. The market maker then matches these orders and tries to fulfil as many as possible. This gives the same mechanism as open outcry but has the added value that the market maker may make a profit on the spread of the orders. The spread of the orders is the difference in accept price between the buy and sell orders. However many market making functions are often required to facilitate as many orders as possible. Thus market makers have to make their profit by fulfilling more sell orders in a period of low market price, and fulfilling more buy orders in a period of high market price. In effect matching buyers and sellers over time and not just in the given time point, as the open outcry market.

Lastly we note that markets have negative feedback, in that if a price for a particular good rises this will incentivise producers to produce more of that specific good, thus increasing supply and bringing the price down again.This is because the profit margin of producing such a good increases, thereby incentivising more supply.

2.3 Conversions of goods into other goods

A typical large company is usually based on one core process. Shell use steel and man hours to pump crude oil from the ground and refine it into gasoline. Samsung uses silicone and heat to make microchips. Apple uses microchips and magic to produce a product with an image. All these core processes are examples of converting one or more resources into other resources.

We can say that for a given amount of steel and rubber Ford may produce a set amount of cars. There thus exists a set amount of steel and rubber needed to produce one car. For example 3 pieces of rubber and seven pieces of steal may be needed to produce one car. Which means that a car is 30% rubber and 70% steel. In an economy of only steal, rubber and cars we may say that the Ford process has the following input and output vectors

$$\text{input vector} = [0.3, 0.7, 0], \quad \text{output vector} = [0, 0, 1] \tag{2.1}$$

One can argue that such a representation indicates that 30 % of a unit of rubber and 70 % of a unit of steel combined to make one unit of car, which is not what we intended when we stated that 3 pieces of rubber and 7 pieces of steel where combined to one car. However who is to say that one car is the functional unit of cars, the functional unit of cars may well be 1/10th of a car. In which case the above input output vector would be in the units [pieces of rubber,pieces of steal, $\frac{1}{10}$piece of car]. The case may well be that we need 300 pieces of rubber, in which case the functional units would be [100pieces of rubber,piece of steel, $\frac{1}{10}$piece of car] but still keeping the same input output vectors.

As we can see by allowing the functional units of each good to scale with those actually needed by the process we can define any process in the above manner. By scaling all of the processes in such a manner that the sum of the input vector, as well as the sum of the output vector, becomes unity we assure that no process creates or destroys resources. If we where to allow output vectors with the total sum larger than unity this could allow a system of several such processes in combination to create a faucet of all of the involved resources, something which we do not want.

Thus a process is defined by an input and an output vector scaled in such a way that the process does neither create nore destroy resources, it only converts them from one form to another. This means that processes are mass conserving, and do not produce waste. They may, however, produce some goods which can be considered waste by the market, meaning that they are not valuable in any sense, but that is for the market to decide.

Since it has been shown that the functional unit of any good may vary without impacting the underlying process we may also consider this train of thought for inflows of goods into the system. If there is an inflow of 200 bails of straw to the market or 200 pieces of straw, depending on the functional unit, we say that there is a source of that straw into the market. However in this case it is not important if the functional unit is bails of straw or pieces of straw since it matters only in relation to the conversion processes. Thus the actual value of the inflow, be it 200 or 10^{10} bails of straw is irrelevant, the only thing of relevance is the existence of an inflow or not.

3

The agent based model

Below we describe the agent based model used in this thesis. This is done by first describing the market model and then building on with economic boundary values and agents.

3.1 Market

In our model we seek only to model the most fundamental function of a market, the supply and demand characteristic. We do this by modifying the minority game into a market making functionality which we will see yeilds a simple market.

Firstly we must stop thinking in supply and demand curves, and their intersection, the market price. Since they are usually created from agents with a set price at which they wish to buy or sell, as well as a volume which they wish to buy or sell. This incurs extra thinking on the part of the market actor in order to set this price, which demands extra parameters, which we have deemed undesirable.

Thus consider a different market, a market in which agents acting on the market know not how much they wish to buy of a good and at what maximum price, but instead only know how much they wish to spend. If agents have a previous price at market they still have the option to think in terms of how much quantity of good they wish to buy, and at what cost, they are just uncertain if the market price they used for this calculation will be the actual market price.

We can construct such a market by having agents issue buy or sell orders to a market maker. Buy orders are in the form of "I wish to buy salt at a market value of 500 dollars" and sell orders are of the form "I wish to sell 200 buckets of salt". The market maker may then calculate the total supply and total demand on the market which gives

$$\text{market price} = \frac{\sum D}{\sum S}, \tag{3.1}$$

where D and S are demand in dollars and supply in goods respectively. We may also note that the market price follows market dynamics, i.e. if supply goes down, prises go up and so on. This market approximation holds if all buyers and all sellers are willing to accept the market price.

We shall call this market type a blind commitment market. Because market actors blindly commit resources or money to the market in hopes of getting the price they projected. When the market price has been determined goods and money can change hands, this process is graphically described in Figure 3.1.

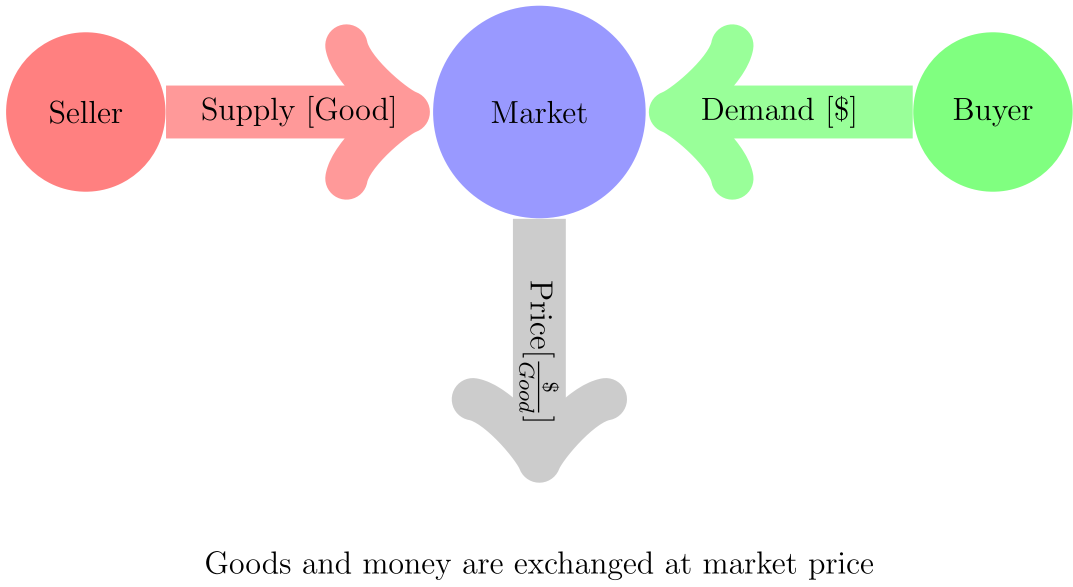

Figure 3.1: Graphical depiction of a simple market.

We may now write out the blind commitment market in pseudo-code and iron out the specifics. Below follows general market dynamics in pseudo code, for a market containing only one good.

1. Market opens
2. Buyers commit money to buying a specific resource. How much money each buyer commits is decided by the buyer.
3. Sellers commit an amount of a specific good to be sold. How much they commit is up to each individual seller.
4. The total market demand and supply for a specific good is calculated.
5. The market price is calculated from the total supply and demand using the following paradigm.

- If the total supply and demand are both greater than zero then the price is determined by
$$\text{Price} = \frac{\text{Total demand}}{\text{Total supply}} \tag{3.2}$$
- If the total supply is zero the the price is determined by
$$\text{Price} = (1+\epsilon)\text{Previous price} \tag{3.3}$$
- If the total demand is zero the the price is determined by
$$\text{Price} = (1-\epsilon)\text{Previous price} \tag{3.4}$$
- If there is no demand and no supply then the price is unchanged.

6. If both supply and demand are nonzero, goods and money are redistributed according to the market price.
   - Sellers receive money proportional to how much good they committed.
   $$\text{Received money} = \text{Committed good} \cdot \text{Market price} \tag{3.5}$$
   - Buyers receive good proportional to how much money they committed.
   $$\text{Received good} = \frac{\text{Committed money}}{\text{Market price}} \tag{3.6}$$
7. If at least one of supply ore demand is zero then goods and money are refunded to the seller and buyers respectively.
8. Market price stored for future reference.
9. Market closes

The pseudo code above describes a market which only handles a single good, however it can easily be extended to a market which handles multiple goods. ϵ is called the price drift. It is introduced in order to deal with situations in which ether supply or demand is not present. If one were to apply equation (3.1) in these instances it would lead to infinite market price or zero market price. However since no goods were exchanged at this price we argue that it was never an actual market price, since market price requires that goods be exchanged. Thus we have to figure out another way of calculating the market price. This is why a drift is applied to the price, to show that there was an unmet demand or undesired supply.

3.2 Boundaries of a sinks and faucets economy

Now consider the economic boundaries of a country or region as discussed before in section 1.1. There exists inflow and outflow of goods to a specific region, the region which we wish to model. We could model this with boundary supply and demand curves as in as in [4]. However soft boundaries require a parameter to set the softness of the supply or demand curve, in keeping with the design philosophy that superfluous parameters should be avoided, let us consider a different type of economic boundary. What if the boundary where to function in the same way as previously described for the blind commitment market?

This would mean that a resource faucet would commit a fix amount of good to the market, and receive compensation at market price. Conversely a resource sink would commit to the market a fix amount of money to buy a specific good, thus posing as a demand in dollars for that good, and receive goods in accordance with the market price. In this way we could pose a fixed exterior supply and demand for all resources on the market.

The strength of the faucets are characterised by how much good flows from it into the market. Goods faucets thus have a characteristic supply in amount of good. The strength of the sinks is characterised by how much consuming power it introduces to the market. Goods sinks thus have a characteristic demand in dollars. This is visualised in Figure 3.2

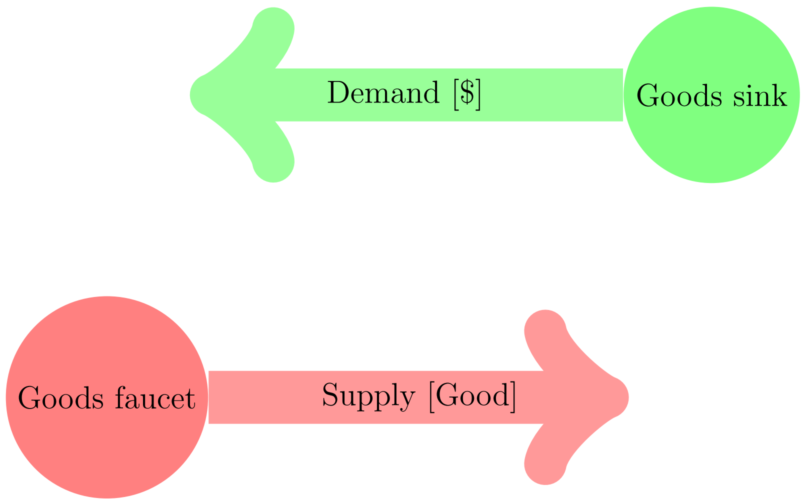

Figure 3.2: Graphical representation of sinks and faucets, with their respective demand and supply.

3.3 Introducing agents into our economy

Let us now introduce into this economy greedy profit maximising agents. By greedy we mean short sighted and with simple guiding rules. Let us start by describing the attributes and workings of an agent. Agents are aware of all available processes and they are aware of the previous market price for all resources. From this information agents decide what process they are going to run in order to make the most profit. To facilitate this they keep track of their own money, as well as how much of each good they have in their possession. A graphical representation of an agent is given in Figure 3.3.

Figure 3.3: Graphical representation of an agent and its traits.

Figure 3.3 depicts all the information an agent possesses, however it does not depict its thought process, this will be described below. An agent has only one decision to make, and that is which conversion process to run. If an agent knows what process it wishes to run, working out which resources to buy, and in what quantity, follows from this knowledge. Without guessing or estimating an agent may only assume that the previous market price will be the current market price, –a reasonable approximation. The question of which process to run then simply becomes a question of which process p is the most profitable. This is given by

$$p = \text{argmax}_{i \in \mathcal{N}}((\text{CPO}_i E_i - \text{CPI}_i) \cdot P), \tag{3.7}$$

where CPI_i and CPO_i is the i:th's conversion process input and output vector as described in 2.3. Furthermore E_i is the agents efficiency parameter for each individual process, a value from 0 to 1. This means that while processes are defined as mass conserving in section 2.3, when an agent implements a process it becomes non-mass conserving. P is the current market price vector of goods and $\mathcal{N}$ denotes the possible process indexes. In equation (3.7) $\cdot$ denotes the inner product, every other operation is element wise. By equation (3.7) each agent evaluates, by themselves, which process is most profitable according to their own process efficiencies.

Now that the agents know which process they wish to run consider the case when they have some money and no resources. They must now acquire the resources they need, in the proportions they need, in order to run their process. Given that the market price stays the same an agent can easily post this demand to the market. Using this reasoning an agents may calculate how much money it wishes to commit to buying each resource by

$$D_i = \frac{C_1 M_i}{\mathrm{CPI}_p \cdot P} \mathrm{CPI}_p P, \tag{3.8}$$

where M_i is the amount of money the i:th agent has and D_i is then the i:th agents demand vector. C_1 is a parameter specifying how much of their total money an agent will spend during each iteration. Equation (3.8) lets the agent calculate how much it is willing to spend and divide that by the total cost of one unit of the conversion process. For a discussion on why this is used, even when the agent has stock in a resource which they are planing to buy, see 3.5.1.

Now that the agent (hopefully) has acquired the resources it was hoping for, it is time to convert them using the process p. However first we must determine at what volume V the agent can run the process. This is defined by

$$V = \min\left(\frac{\text{agent Supply}}{\mathrm{CPI}_p}\right). \tag{3.9}$$

Now that we have calculated the maximum volume at which the agent can run the process it is time to alter the agents stock of goods as if the agent has run the process. This is done by

$$\text{Agent supply after process} = \text{Agent supply before process} + V(\mathrm{CPO}_p E_i - \mathrm{CPI}_p). \tag{3.10}$$

As we can see equation (3.10) simply deducts the resources needed to run the process and adds the resources gained from the process to the agents stock.

By now the agent is hopefully looking at a profit, however, they still need to sell their goods on the open market in order to make a profit. If an agent were to sell all their goods at once it would expose the agent to the current market price only, running the risk of the current market price not suiting their needs. However if an agent only sells part of their stock each iteration this risk is minimised, in effect allowing the agent to sell at a weighted average of the current, and following, iterations market price. How much of their current stock to commit to market can thus be calculated with

$$\text{Supply to market} = (\text{Agent Supply}) C_2, \tag{3.11}$$

where C_2 is a new parameter and the fraction of an agents goods which it intends to sell, of course the agent's stock is deducted for the goods committed to market.

3.4 Constructing a simple agent based economy simulation

Now we have defined both how the boundaries of our economy work, how these boundaries interact with the agents through a market, and how these agents use available conversion processes to make money. Now it is time to combine it all together into a functioning economy model. As all the individual functionalities of agents, markets, conversions and boundaries have already been discussed we need only to tie the strings together. This is done below, using pseudo code.

Consider an economy with a given number of sinks and faucets. These sinks and faucets have a fix demand or supply which they each iteration impose on the market. Furthermore there exists a fix amount of conversion processes. Likewise there exists a fix amount of agents with nothing more than starting capital and a randomly selected process which they intend to run if nothing changes. We may now iterate the model forward in time, and thus investigate the properties prescribed in the problem formulation 1.3.

For each successive iteration every agent has a probability of reevaluating what process they are going to run. This probability is denoted σ and is the third and final parameter introduced in our model. When agents have, possibly, re-evaluated their production choice they post their demands to the market. At the same time all agents post the fraction of the previous resources they have gathered which they wish to sell to the market. By having both selling and buying agents on the same market we allow agents to sell resources to each other. This enables them to depend upon each other and create successive value chains in order to meet the demands of all sinks.

Now that all agents have posted both their buy and sell orders to the market it is time for the sinks and faucets to do the same. In this way buyers, sellers, sinks and faucets are present on the same market, and compete on equal terms for the same resources.

Since the market has received orders from all market actors it can now calculate the market prices and redistribute the goods and money accordingly, after which the market is reset.

Lastly the agents run their desired processes and convert their stock of resources to another set of resources. This last step completes a full iteration and the process starts over with a new iteration. An iteration can be described with a simple bullet list, as follows.

- Agents have the chance to reevaluate which process they wish to run with the probability σ (as in 3.4)
- Agents post demand to market in accordance with C_1 (3.8)
- Agents post supply to market in accordance with C_2 (3.11)
- Sinks and faucets post their demand to market
- Market computes market price of resources and redistributes accordingly
- Agents run their respective processes
- The new market price is fed to the agents to be used in the next iteration

This process encapsulates a flow of resources, money and information which may be visualised, this has been done in Figure 3.4. In this figure we see that faucets relay a supply of goods into the market. This supply is then spun around a couple of times by the agents until it is in the form defined by the sink which in presenting a demand may incur an outflow of resources from the market. Likewise the sink imposes a capital inflow in the market by buying resources from the market in exchange for capital. This capital is then spun around a couple of times until it is sucked out of the system by the resource faucet which exchanges it for the resources it introduces into the market. Thus we can see that the general flow of resources is from left to right, and the general flow of capital is from right to left. This is visualised in Figure 3.4 by the topmost arrows in a lighter shade of green and red.

In Figure 3.4 we may also view the process of feedback present in the model. When agents have posted their demand and supply to the market, and the market has computed the market price and redistributed the resources, agents are made aware of the new market price which may incur the agents to change their behaviour, altering how much they intend to spend on a certain resource and so forth. This process constitutes the main negative feedback present in the model and we shall call it agent switching inducing feedback.

The other main cause of feedback in the model is that if an agent has chosen a process which is not profitable this agent will lose money over several iterations. Thus the agents wealth declines meaning that the agent presents a smaller demand on the market which reduces prices and in turn increases its profitability. This mechanism functions in a similar way for profitable agents, gradually reducing their profit margin to zero. We call this break even feedback.

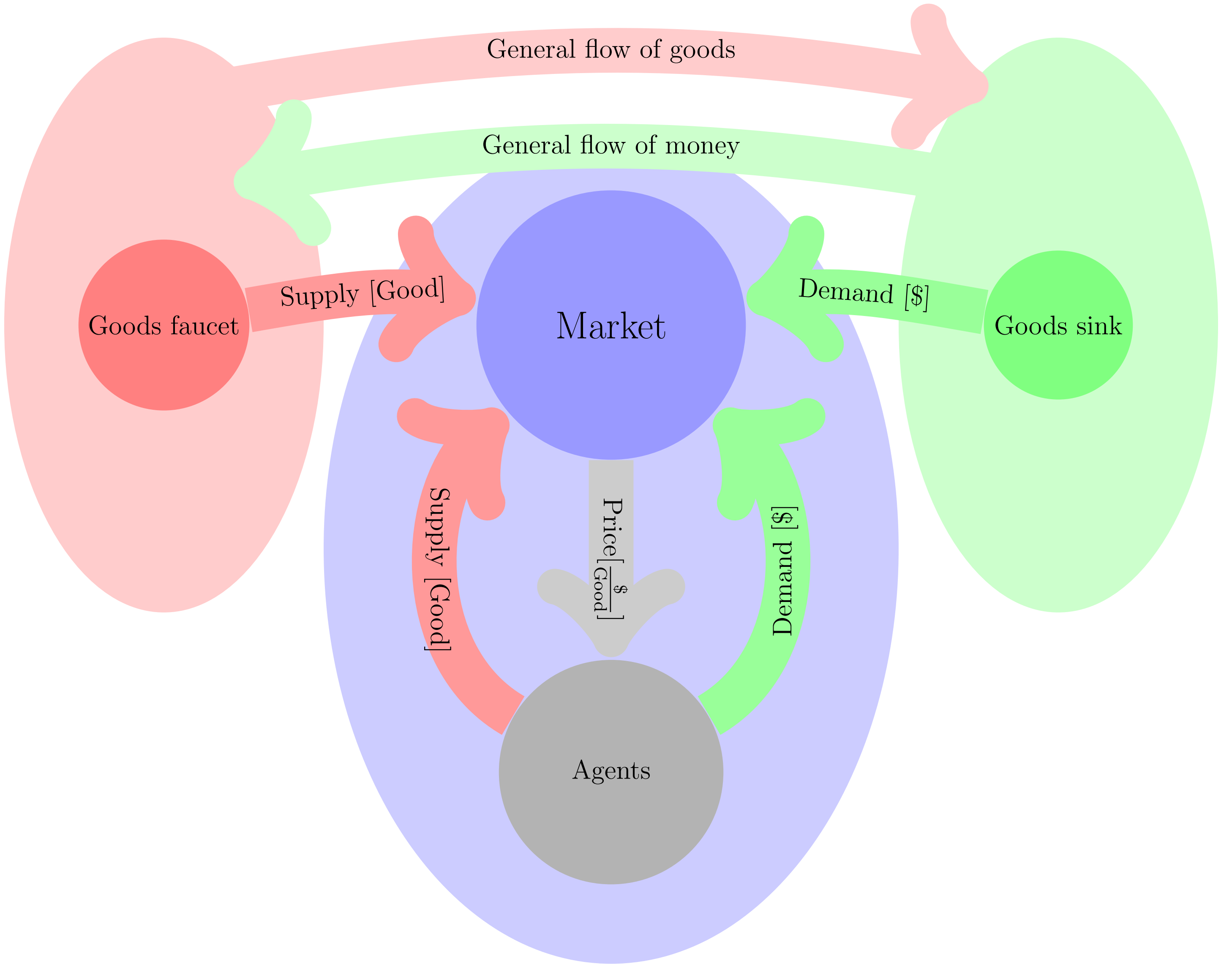

Figure 3.4: Graphical representation of a simple agent based sinks and faucets market economy system with a single market.

3.5 Finer points

To conclude our discussion of the model we need to discuss some of the non-possibilities present in the model as well as elaborate on why some of the financial simplifications are reasonable.

3.5.1 Agents selling and buying the same good

It is poignant to note that in the description of agents acquiring goods it was assumed that the agents had no stock of good to begin with, and one may make the argument that an agent may wish to use part of this stock to run a process. However this would complicate the model even more, so let us entertain the thought of not including such intelligence for the agents.

Assume an agent possesses half of the stock it needs to run a process. This means that by the current paradigm the agent will commit a portion of this stock to market and at the same time commit to buying the same stock using its capital. This will result in the agents selling some goods to itself at market price. This action does not loose the agent any money, it simply reduces the force with which it exploits its current process choice.

This shortcoming on behalf of the agent is considered acceptable in keeping the model simple. It would of cource be possible to create a similar agent buying demand mechanism which took its stock of goods into account, however this was considered outside of the project scope.

3.5.2 Using profits from goods committed to market as payment for other goods

Consider an agent which is short on capital but heavy on goods. The agent has committed some of its goods to market as well as some capital towards buying goods for its desired process. However since the agent has goods committed to market, standing to make a profit, one could argue that the agent would wish to obtain goods for this capital as well. A theoretical order to the market maker would thus be:

- Sell goods A, B and C
- Buy good D and E in equal proportions for the money received, as well as a supplementary $ 500.

For the market this represents a demand in goods D and E as well as supply in goods A, B, C and $. Now consider a normal order, where agents may not use the money previously earned to buy goods:

- Sell goods A, B and C
- Buy good D and E in equal proportions for $500.

This represents a demand of goods D, E and \$ as well as a supply of A, B, C and \$. In the normal case we note that there is a both a supply and demand for money, however in the proposed case there is no demand for money. This means that if all agents posed this type of order there would be no demand for money at all. This would make money worthless (since nobody wants it) and thus the market maker can not solve the equation it needs to solve in order to determine the market price, because the market price for everything would be infinite.

One should note that if we enable agents to trade one good for other goods the above formulation is possible, since money is just another form of good. However this would imply a barter economy, an economy in which goods are exchanged for other goods with no form of currency involved. While this type of economy is possible to construct with the same market making mechanism it was deemed to be outside the scope of this project, and thus not pursued.

3.5.3 General structure of available conversions, sinks and faucets

Previously we have mentioned that the economy possesses several different processes and that these processes are linked in such a way that it is possible to meet the demands of sinks in all resources even though there are only faucets in a couple of resources. This is in order to model value chains and increase the agents dependancies on each other. However we have not specified exactly how this is implemented.

This was done by grouping resources into tiers, where each tier has a set amount of good types associated to it. If we have the resources grouped into tiers we can group the processes as being from one tier to another or from one to the same tier. Of course this does not cover all types of conversions the system could encompass, however this restriction gives a good framework for modelling a value chain.

For our value chain we have chosen to have 4 resource tiers with 2 resources in each tier. Furthermore there exist resource sinks for all resources at a strength of \$ 1000 but only resource faucets for tier I resources of 100 units. Between the tiers there exists a value chain facilitated by 10 conversions with random proportions between resources but still mass conserving. Lastly there exist 5 conversions from each tier onto itself, also with random proportions and still mass conserving. This system of conversions, sinks and faucets is visualised in Figure 3.5.

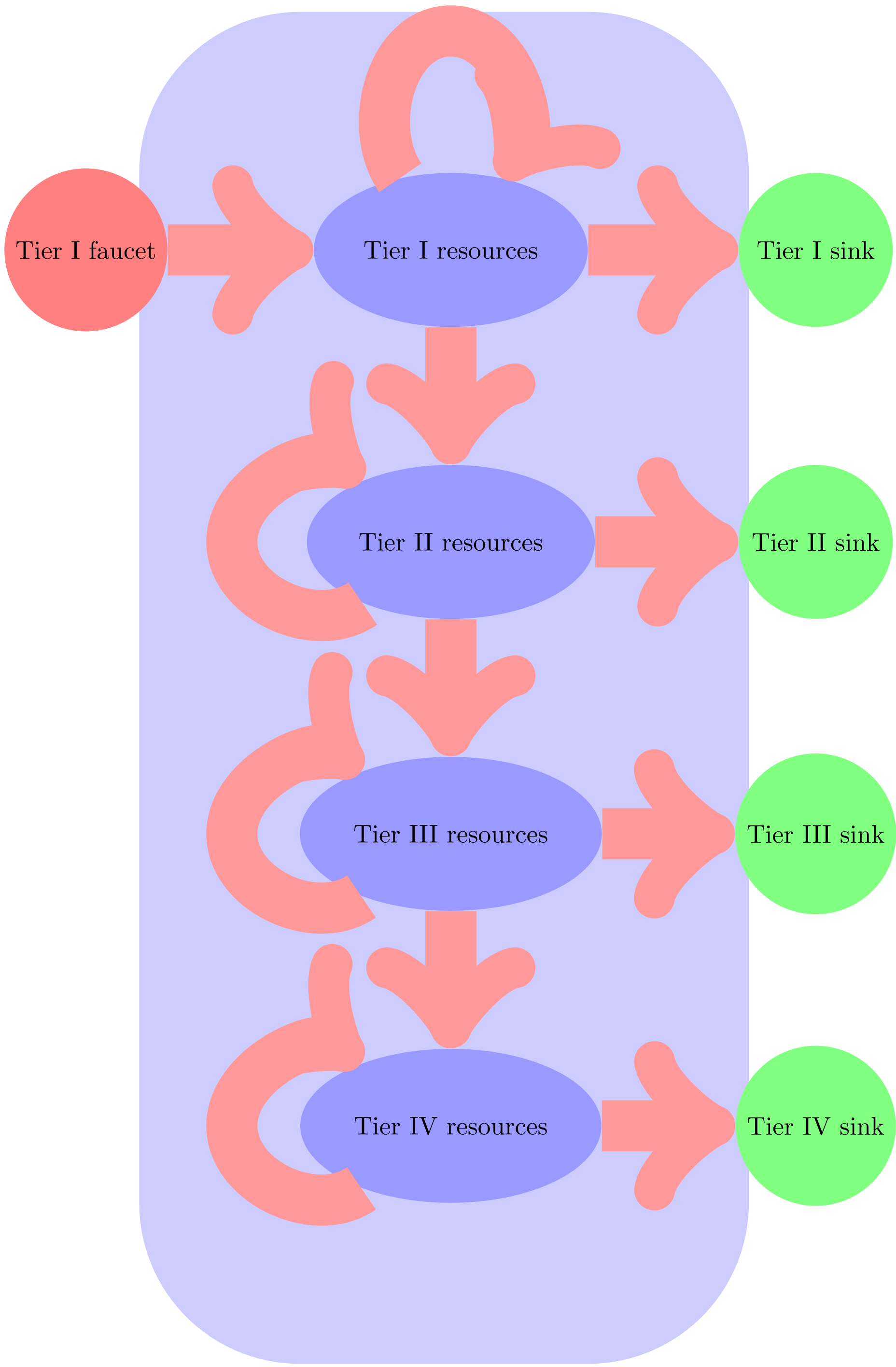

Figure 3.5: Graphical representation of the flow of goods throughout the economy. It is poignant to note that each flow of good has a corresponding flow of money in the opposite direction.

4

Analytical model

The structure of the available conversions offers the possibility of modelling the system analytically. Consider the flow of demand and supply posted by the agents to the market as state variables of the system. Since the system has four tiers this means that we get eight state variables, four representing demand for resources(D_{1-4}) and four representing supply of resources(S_{1-4}). Now focus specifically on the flow of supply. For a given tier agents convert a proportion of this resource into resources of the next tier, and another proportion of these resources into a different proportion of the same tiers resources. We define these proportions as $\lambda \in [0,1[$ and $\nu \in [0,1[$ respectively. If the agents were 100 percent efficient we would know that the proportions have to sum to unity, $\lambda + \nu = 1$, however since this is not the case we know that this is actually an inequality.

Since the agents compete for resources with the sinks when buying on the market we know that the amount of resources that they receive is proportional to the demand posed for the same tier. This may be be formulated as

$$\text{Supply received}_t = S_{i,t}\frac{D_{i,t}}{D_{i,t} + D_{i,R}}, \tag{4.1}$$

where $D_{i,R}$ is the competing demand posed by the sink and t denotes the time step. The supply received is then divided and reconverted into other resources by the processes signified by λ and ν. This means that we can define the supply at each tier through the following iterative equation:

$$S_{i,t+1} = \lambda S_{i-1,t}\frac{D_{i-1,t}}{D_{i-1,t} + D_{i-1,R}} + \nu S_{i,t}\frac{D_{i,t}}{D_{i,t} + D_{i,R}} + S_{i,R}. \tag{4.2}$$

All that remains is to define equations for the five demands and we will have described the system. We know that each and every flow of goods has an opposite flow of money, or demand. By this we can define an analog demand equation to equation (4.2) namely equation (4.3).

$$D_{i,t+1} = \gamma D_{i+1,t}\frac{S_{i+1,t}}{S_{i+1,t} + S_{i+1,R}} + \eta D_{i,t}\frac{S_{i,t}}{S_{i,t} + S_{i,R}} + D_{i,R} \tag{4.3}$$

In equation (4.3) γ and η correspond to λ and ν respectively, and $D_{i,R}$ inflow of demand presented by the sink. This of course means that $\gamma + \eta \leq 1$ and that $\gamma \in [0,1[$ and $\eta \in [0,1[$.

Let us now iron out the specifics for our system, this yields the equation system (4.4).

$$
\begin{aligned}
S_{1,t+1} &= & & \nu S_{1,t}\frac{D_{1,t}}{D_{1,t}+D_{1,R}} &+& S_{1,R} && (4.4a)\\
S_{2,t+1} &= \lambda S_{1,t}\frac{D_{1,t}}{D_{1,t}+D_{1,R}} &+& \nu S_{2,t}\frac{D_{2,t}}{D_{2,t}+D_{2,R}} &+& S_{2,R} && (4.4b)\\
S_{3,t+1} &= \lambda S_{2,t}\frac{D_{2,t}}{D_{2,t}+D_{2,R}} &+& \nu S_{3,t}\frac{D_{3,t}}{D_{3,t}+D_{3,R}} &+& S_{3,R} && (4.4c)\\
S_{4,t+1} &= \lambda S_{3,t}\frac{D_{3,t}}{D_{3,t}+D_{3,R}} &+& \nu S_{4,t}\frac{D_{4,t}}{D_{3,t}+D_{4,R}} &+& S_{4,R} && (4.4d)\\
D_{1,t+1} &= \gamma D_{2,t}\frac{S_{2,t}}{S_{2,t}+S_{2,R}} &+& \eta D_{1,t}\frac{S_{1,t}}{S_{1,t}+S_{1,R}} &+& D_{1,R} && (4.4e)\\
D_{2,t+1} &= \gamma D_{3,t}\frac{S_{3,t}}{S_{3,t}+S_{3,R}} &+& \eta D_{2,t}\frac{S_{2,t}}{S_{2,t}+S_{2,R}} &+& D_{2,R} && (4.4f)\\
D_{3,t+1} &= \gamma D_{4,t}\frac{S_{4,t}}{S_{4,t}+S_{4,R}} &+& \eta D_{3,t}\frac{S_{3,t}}{S_{3,t}+S_{3,R}} &+& D_{3,R} && (4.4g)\\
D_{4,t+1} &= & & \eta D_{4,t}\frac{S_{4,t}}{S_{4,t}+S_{4,R}} &+& D_{4,R} && (4.4h)
\end{aligned}
$$

Equations (4.4) is thus the difference equations equivalence of the system in Figure 3.5. By setting $D_{i,R} = 2 * 1000$ for all i and $S_{i,R} = 0$ for $i \in [2,4]$ and $S_{1,R} = 2 * 100$ we create a difference system mimicking the flow of resources in our agent based model. The free parameters λ, γ, ν and η are affected by the efficiencies and capital of each agent involved in that specific tiers processes. The values λ, γ, ν and η varies for each iteration of the agent based model, since agents switch production processes, we do however know that they always stay within the [0,1[domain. This means that if the equation system (4.4) always has a stable fixed point inside this domain, thus we know that at least some of the underlying dynamics of the agent based model lead to the system converging to a stable fix point.

4.1 Iterating and stability analysis

Iterating the equations in (4.4) can be written as the map $[S_{1-4,t+1}, D_{1-4,t+1}] = F[S_{1-4,t}, D_{1-4,t}]$ for an initial condition. This has been done with $\lambda = \gamma = \nu = \eta = \frac{9}{20}$ in Figure 4.1.

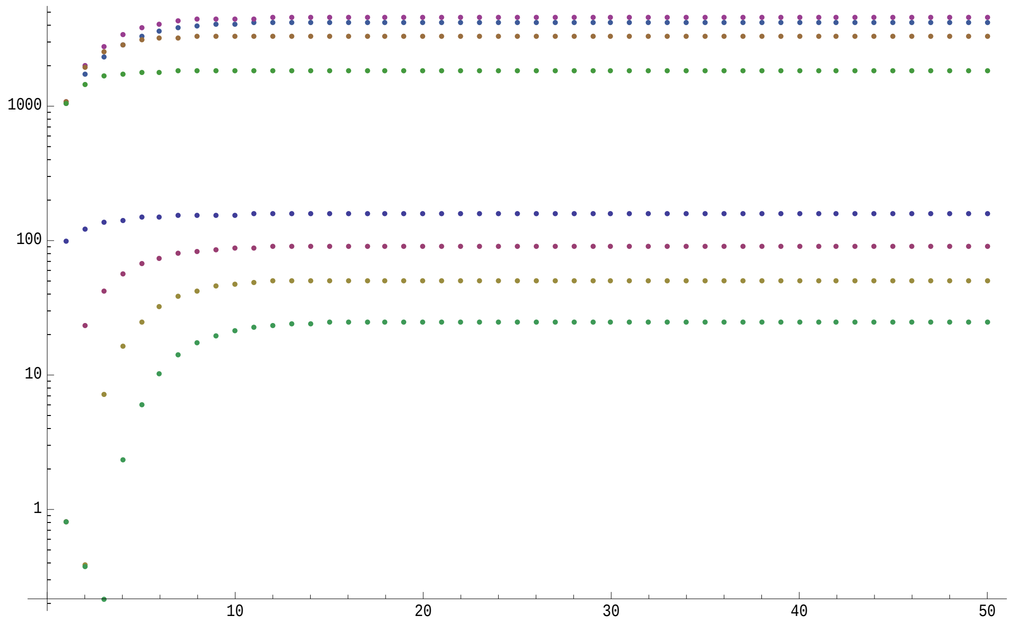

Figure 4.1: Figure depicting the supply and demand progression of the iterated map as defined by equations (4.4) with initial conditions $S_{i,0} = 10$, $D_{i,0} = 100$ and $\lambda = \gamma = \nu = \eta = \frac{9}{20}$.

As we can see the system quickly converges to a stable state, and does not deviate from it. We know that the system has a fixed point iff $[S_{1-4,t}, D_{1-4,t}] = F[S_{1-4,t}, D_{1-4,t}]$ for positive values of S_{1-4} and D_{1-4}. This was investigated using the *Mathematica* solve function for the algebraic equation system (4.4). It was found that there is always a positive fixed point for the system for all combinations of λ, γ, ν and η. In order to determine if the fix point is a stable fix point we must examine the Jacobian of F and determine that all its eigenvalues, in absolute value, are less than one at the fix point [2]. This was also done in *Mathematica*, however computing the eigenvalues algebraically was, after extensive investigation, determined to computationally intensive. Instead the maximum eigenvalue was searched for numerically, using NMaximize. The result of this search for the ten eigenvalues is stated in table 4.1.

absolute Eigenvaule	Domain point			
1.0	$\lambda = 1.0$	$\gamma = 1.0$	$\sigma = 0.000750163$	$\alpha = 1.0$
1.0	$\lambda = 1.0$	$\gamma = 1.0$	$\sigma = 0.998388$	$\alpha = 0.698399$
1.0	$\lambda = 1.0$	$\gamma = 0.999782$	$\sigma = 0.245852$	$\alpha = 0.988721$
1.0	$\lambda = 0.908019$	$\gamma = 1.0$	$\sigma = 0.171524$	$\alpha = 0.94187$
1.0	$\lambda = 0.908019$	$\gamma = 1.0$	$\sigma = 0.171524$	$\alpha = 0.94187$
1.0	$\lambda = 0.908019$	$\gamma = 1.0$	$\sigma = 0.171524$	$\alpha = 0.94187$
1.0	$\lambda = 1.0$	$\gamma = 1.0$	$\sigma = 0.0806639$	$\alpha = 964177$
1.0	$\lambda = 1.0$	$\gamma = 0.997256$	$\sigma = 0.00565015$	$\alpha = 0.305871$

Table 4.1: Table of the largest attained absolute value of each eignevalue of the Jacobian at the systems fixed point, with the corresponding parameter space point at which is was attained.

As we can see none of the eigenvalues achieve their maximum on the interior of parameter domain, and while several eigenvalues achieve the value of one on the boundary of the domain, this is considered unattainable since the boundary is not included in the domain. Thus we conclude that none of the eigenvalues of the Jacobian of F at the fix point are equal to or larger than one. This means that there always exists a stable fix point for our algebraic system. We thus conclude that the underlying dynamics of our agent based model give rise to a stable fix point defining a price equilibrium. For a the complete Mathematica code see Appendix A.

Lastly it is poignant to note that at the fix point the demand and supply for resources may not increase nor decrease, since it is a fix point. If this holds for the agent based case that means that the agents as a whole may not make any money, they many only become richer or poorer in relation to each other. But their total money may not change, since the demand present on the market is a linear function of the agents money and C_1.

5

Implementations

The agent based model was implemented both in MATLAB and C++ because two implementations served two different purposes. The MATLAB implementation was aimed at simulating the system many times, varying the parameters C_1, C_2 , σ and collecting aggregated results. In contrast the C++ model was aimed at visualising the system throughout successive iterations. Both implementations are outlined blow.

5.1 Matlab implementation

Since the MATLAB implementation focused on aggregating results from several instances it was written with a stand alone function for iterating the system given different parameters. This function then saved the entire result to disk, which meant that several computers could be used to simulate multiple systems independently. This was complemented with a result compiler function which read an entire directory of results and compiled the relevant statistics as averages over all simulations. Since MATLAB is optimised for matrix equations all formulations where rewritten in order to utilised this to the fullest extent.

5.2 C++ implementation

Since part of this project was to visualise the flow of resources inside the economy, with the aim of visualising the dependancy networks of different processes, the agent based model was implemented in C++. This was done for two reasons, firstly MATLAB is not able to update visualisations of this caliber fast enough. Secondly the added computational speed of C++ was needed in order to compute the underlying model as well as the simulated annealing needed for the visualisations. Furthermore the added strength of C++ being object oriented made it an easy choice.

The aim of the C++ implementation was to be able to visualise all relevant flows of goods and money within the economy from source, through processes, to sink. In order to do this one first needs a functioning object oriented implementation of the model. This implementation needs to contain market agents as well as sinks and faucets, and be possible to iterate forward in time in a simple way. Secondly one needs to monitor the model and extract the relevant statistics, how much do the agents running process 5 depend on the agents running process 19? Such characteristics were calculated between all processes and stored for every iteration of the model.

Lastly one has to visualise all this data. This was done by letting every process be a node in 3D space and by letting the connections which one wishes to visualise be attracting forces between these nodes. By applying a Coulomb force between every node, and every other node, it is possible to use simulated annealing to organise the system, and thus visualise it in a simple manner.

6

Simulations and results

The results section is divided into several parts, the first part is a section that depicts one typical simulation chosen in order to showcase specific aspects of the model. The second part details data collected from multiple simulations where the parameters C_1, C_2, and σ are systematically varied in order to thoroughly examine the solution space. Finally a typical dependancy networks are examined using the tool written in C++, focusing on the flow of resources inside the economy.

6.1 A typical economy

For this experiment the agent based model was configured as perviously described in 3.3, with $\sigma = 0.01$, $C_1 = 0.5$, and $C_2 = 0.5$. All agents were initiated with \$100 of money and no goods in stock. They were also given a specific start process and randomised efficiencies for all processes between 0 and 100%. Finally the first 1000 iterations of the model were discarded as this period is considered to be ”burn in”, necessary for the simulation and get rid of initial transients.

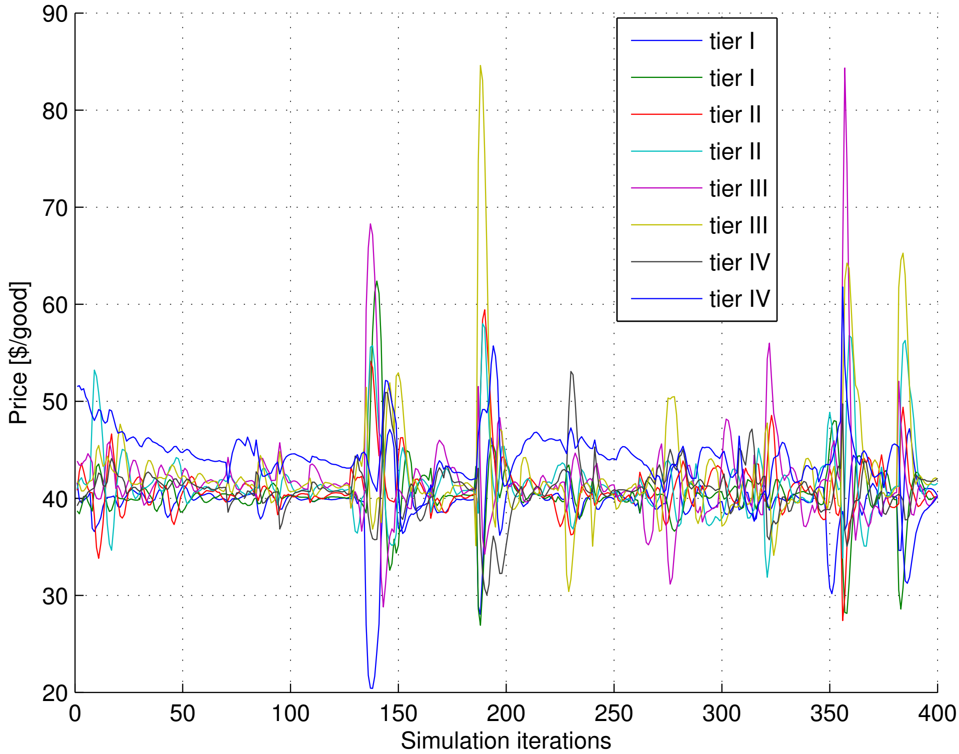

Figure 6.1: Figure depicting the price of each good for every time step following the burn in.

Figure 6.1 showcases the price progression of the first 400 iterations after the burn in period. As we can see the price has converged to around $40 per good. However the price never fully stabilises with periods of increasing calm followed by abrupt turbulence in the market, which slowly dies down before new turbulence arises yet again.

Since it has ben shown that for every configuration of agents, the corresponding analytical model has a fixed point we know that the same dynamics act in the agents based model. Thus we know that the price will converge to this equilibrium, however during this process agents are allowed to reevaluate which process they wish to run. As the price of each good approaches its equilibrium every process becomes equally profitable, leading to agents switching process more haphazardly. If a cash heavy agent switches process during this point, they remove significant consuming power from one process and deposit it in another. This significantly changes the demand and supply for all resources in the general system, and this is what causes the repeated sudden turbulence in the market. However this new agent configuration still has a stable fix point in the underlying dynamics, which means that the process of convergence starts over.

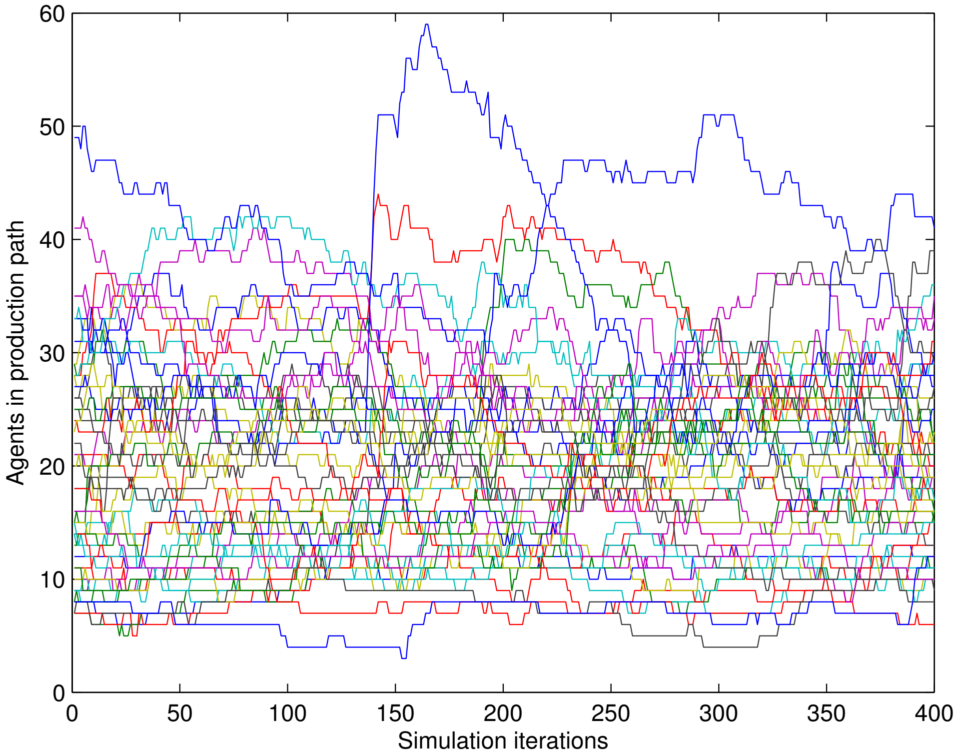

Figure 6.2: Figure depicting the number of agents engaged in each process at every time step following the burn in.

In Figure 6.2 note that there are always agents occupying each and every process at all times. Furthermore we may note that the number of agents in each process varies with both low frequency and high frequency components. This means that even though some agents find a particular process more profitable, some are still leaving that same process for another, in their eyes, more profitable process. This is because agents have a different efficiencies for each process.

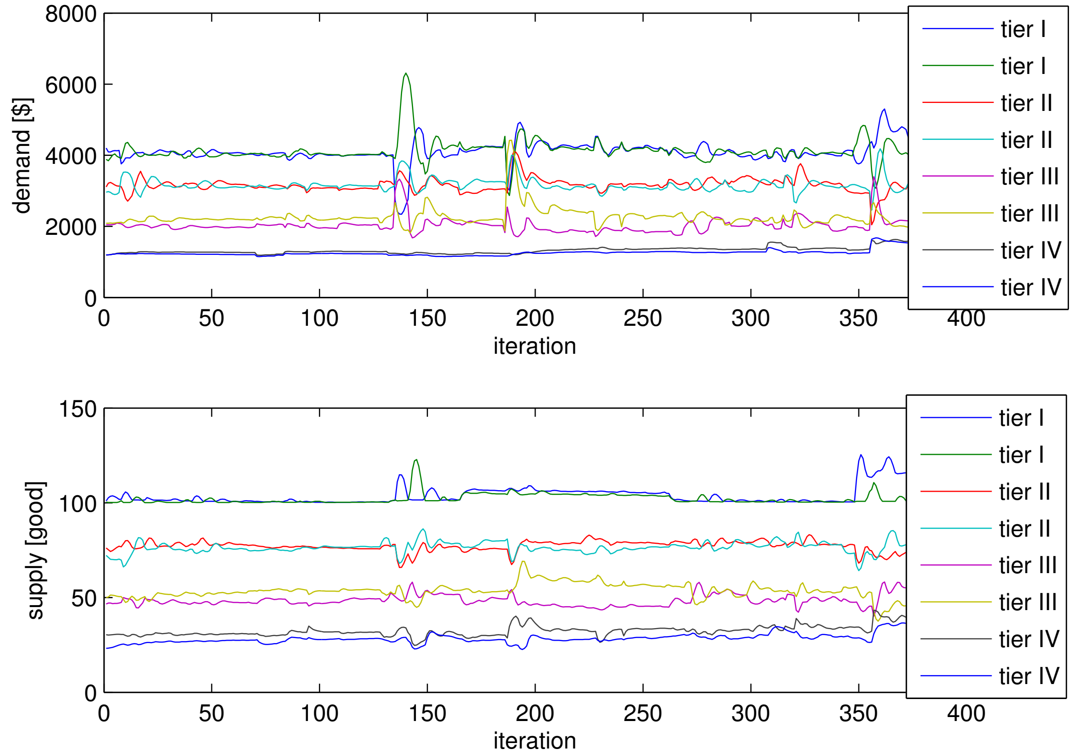

Figure 6.3: Figure depicting the total supply and demand of each resource at every time step following the burn in.

Figure 6.3 depicts the total supply and demand present on the market at every time step after the burn in period. As we can see the demand for tier IV resources has the lowest variance, with an average just above \$1000. This is due to the ever present demand from the sink for tier IV resources at \$1000. If we move down in through, the resource tiers we see that the total demand increases with decreasing tier level. This is because tier IV demand is moved down to tier III demand through the conversion process converting tier III resources to tier IV resources which explains why there exists a demand for tier III just above \$2000. This reasoning holds for tier II and I as well, which explains the demands of \$3000 and \$4000 respectively.

Furthermore we see that the same dynamics apply for the supply of each tier of goods, but in a different manner. If we consider that demand for each good is the same we know prices for all goods are basically the same, this means that the sinks ought to buy equal amounts of goods from the system each iteration. However since a supply of tier IV good has to be met by a supply in tier III during some previous iteration one easily deduces that as tier number decreases the supply has to be the cumulative sum of the higher tier supply, which is what we see in Figure 6.3.

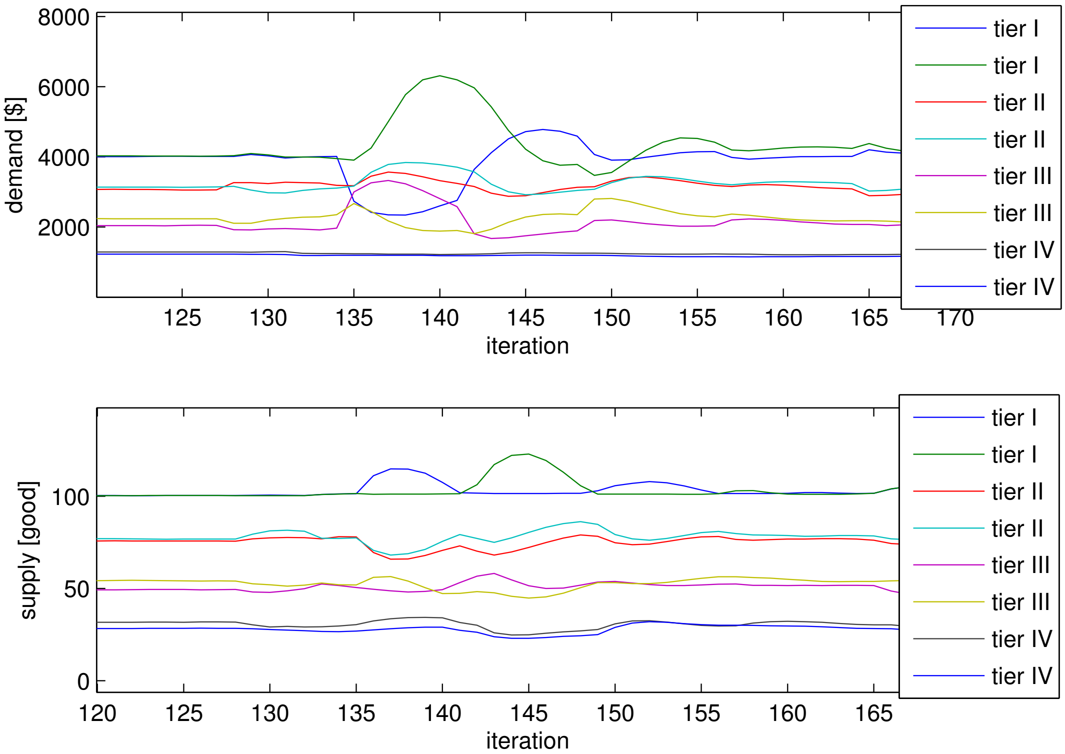

Figure 6.4: Enhanced view of a particular section supply and demand depicting the supply and demand characteristics.

Further examining Figure 6.3 yields Figure 6.4, wherein we have focused on a specific time slot of the time series. We see that the demand of one tier I resource decreases dramatically between iteration 134 and 135, this is accompanied by the demand of one tier III increasing equally dramatically. This change in demand represent one or more cash heavy agents changing the process they wish to run, thus changing the demands for the resources they trade in. This move results in the remaining agents having trouble estimating the price and thus placing non optimal orders to the market. This results in agents not being able to utilise all resources they have bought from the market, which is why the supply of tier I resources increases above the level supplied by the resource faucet. Examining the supply of tier I resources for the iterations after 135 we note that the supply alternates between being 100 and being about 10% more, with decreasing amplitude. This is because the system is trying to balance itself, progressing back to equilibrium, alternating which of the resources it is constrained in, and thus creating a surplus of the other. This surplus is created because if the agents do not manage to acquire resources in the proportion they wish, they sell the unused resources during the following iteration effectively moving resources from one iteration to another.

Since agents acting on the tier I market are having trouble acquiring resources in the correct proportion this means that the production of tier II resources is negatively affected, meaning that supply is reduced.

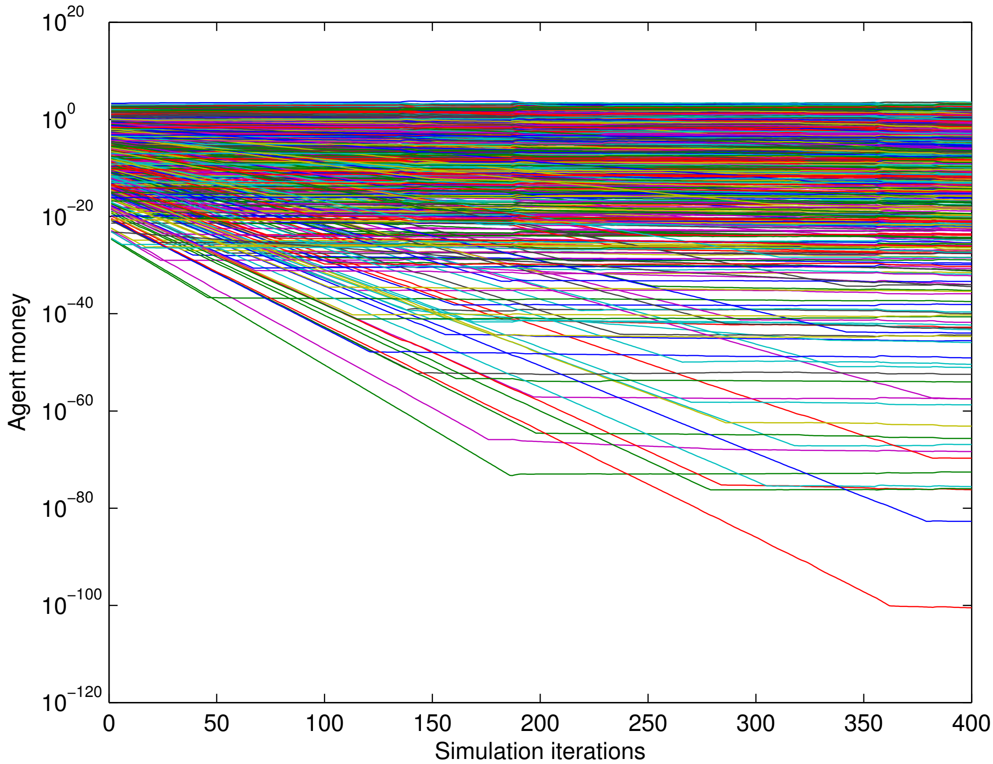

Figure 6.5: Figure depicting every agents money at every time step following the burn in.

From Figure 6.5 we se that there are a lot of agents which simply do not make money. This is because all agents are not apt to function on the present market. This is possibly due to their efficiency being too low in key processes, however there is no mechanism for removing such agents from the model, and thus they stand out in this graph.

Furthermore we see that even though the economy is functioning none of the agents are making a significant amount of money. This is because of the minority game dynamics of the market. If an agent finds a profitable niche in the market it may make a profit for a couple of iterations. However since the agent now has a higher net worth this means that the agent will present a higher demand towards the market, since it always reinvests a proportion of its money in goods. The agent thus increases the price of goods it needs to run the process it has chosen. This self induced increase in price results in a reduction of the agents profitability, and thus it is only a matter of time before the profitability of the niche is reduced to zero.

Of course other agents may also find the specific niche profitable, which means that this process of reduced profitability is even faster. This notion that agents have trouble turning a profit, that is accumulating money, is further visualised in Figure 6.6. In Figure 6.6 we see that the total outflow of money through the resource faucet matches that of the inflow from the resource sink. This means that even though the economy is functioning, with respect to meeting all the demands of the sinks and faucets, the agents have collectively removed their own profit margins.

The reduction of profitability in a specific niche and agents flowing into a profitable niche are examples of the break even and agents switching feedback's respectively. We can thus conclude the feedbacks proposed in 3.4 are present in the model.

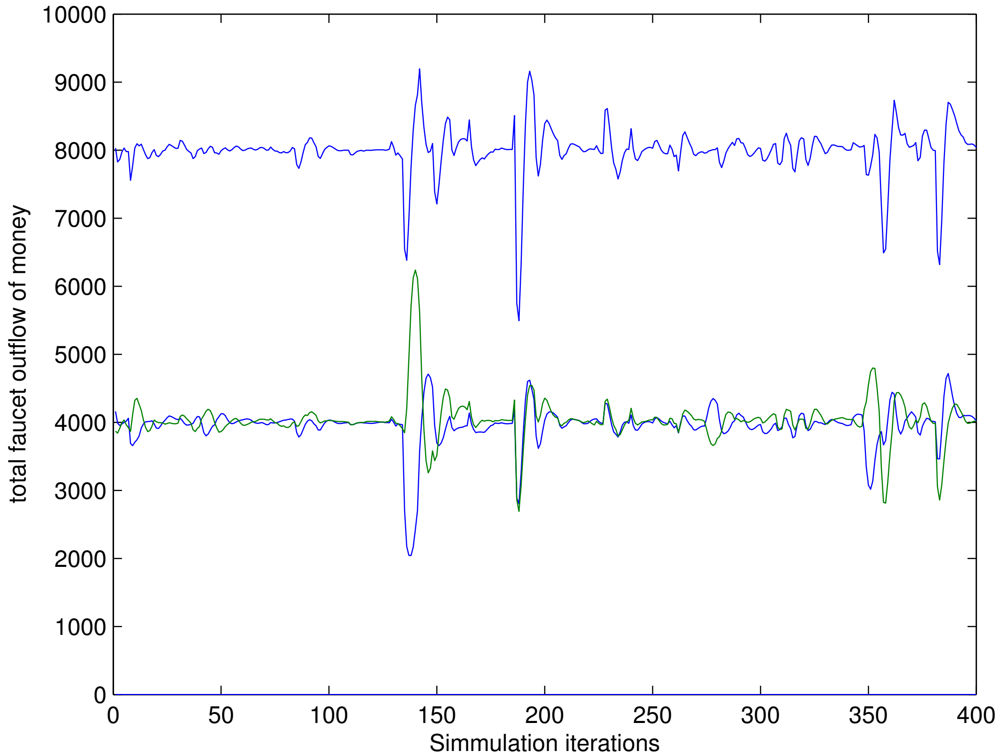

Figure 6.6: Figure depicting the total outflow of money at the goods faucet. The bottom two lines are the outflows of money for each tier one resource, whereas the top most line if the sum of these two.

6.2 Examining the parameter space

In order to analyse the complete parameter space of the model multiple simulations were run successively varying C_1, C_2, and σ. Since the available conversions are random numbers 41 simulations were run for each datapoint. Data was collected for 3000 iterations, after 1000 iterations of burn in.

Several quantities where measured, of which four will be presented here. The three first are the δ of price, supply, and demand, where δ is the standard deviation, of the the time series τ normalised by its mean, as stated in equation (6.1). A low δ represents a stable time series, in this case a stable price or supply time series. Higher values of δ correspond to a higher variation of the time series from iteration to iteration.

$$\delta = \text{std}\left(\frac{\tau}{\text{mean}(\tau)}\right) \tag{6.1}$$

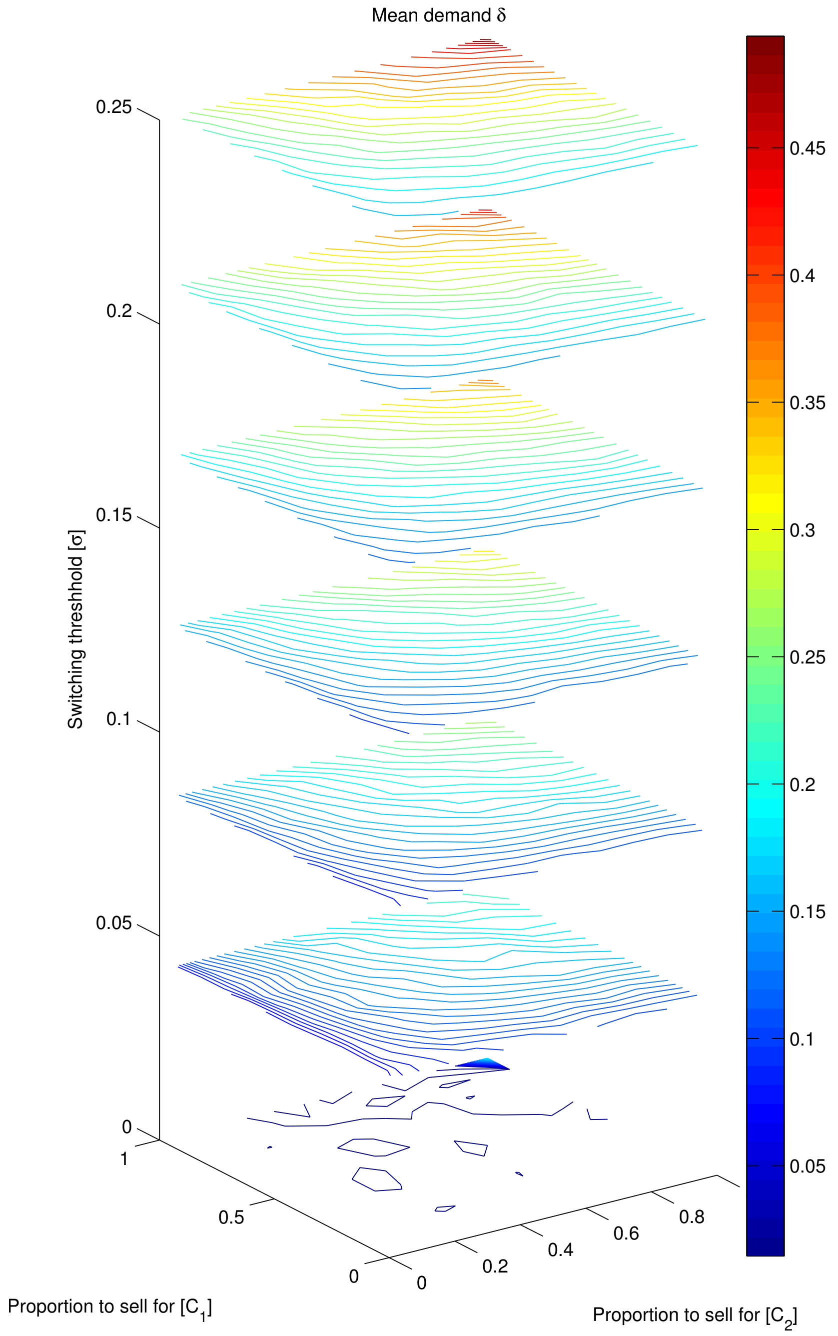

Figure 6.7: Figure depicting the contour planes of demand δ for different σ, agents switching, over varied C_1 and C_2.

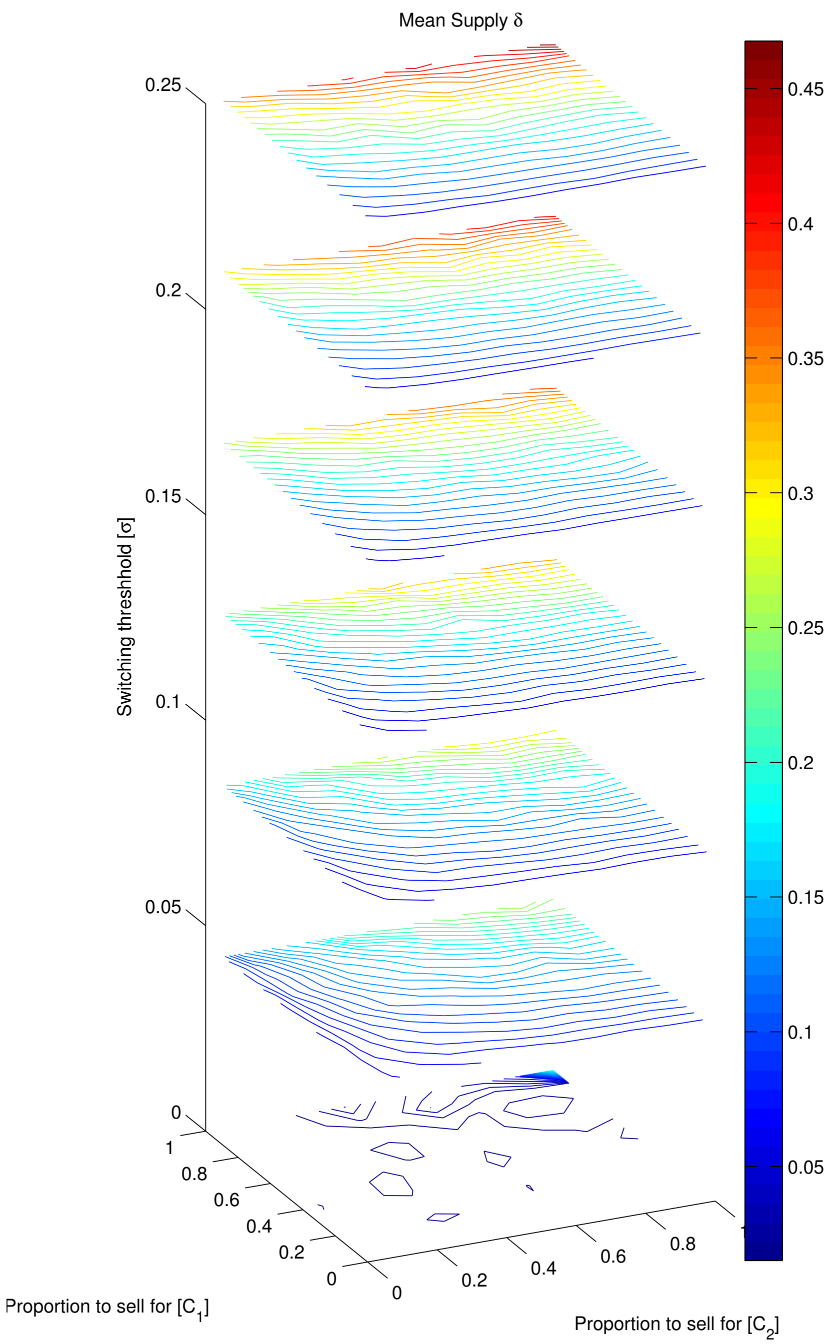

Figure 6.8: Figure depicting the contour planes of supply δ for different σ, agents switching, over varied C_1 and C_2.

Figure 6.7 and 6.8 depict the average δ over 41 iterations of supply and demand for all resources. As we can be seen δ decreases as C_1 and C_2 decreases for both supply and demand, in all levels of σ. This is because reducing the proportion of money agents spend each iteration as well as how much they sell helps to alleviate effects of fluctuations in demand and supply. This is due to $C_1$and C_2 acting as low pass filters on the signals of supply and demand presented by the agents towards the market. Instead of sampling only the current market price when selling goods, one samples a weighted average of the coming prices in such a way that high frequency fluctuations are reduced. Because of the overall feedback mechanisms of the model this lowers the δ of supply and demand.

Furthermore it is shown in Figures 6.7 and 6.8 that reducing the number of agents allowed to switch processes each iteration reduces the average δ. This is because if no agents are allowed to change process the only dynamic present is the break even feedback which drives all processes towards zero profit. In doing so the dynamic stabilises the supply and demand present in the system. This is of course exemplified by the drastic reduction of volatility as the number of agents allowed to switch is reduced to zero. This validates the results from the sample run, that agents switching processes is the main cause of supply and demand fluctuation.

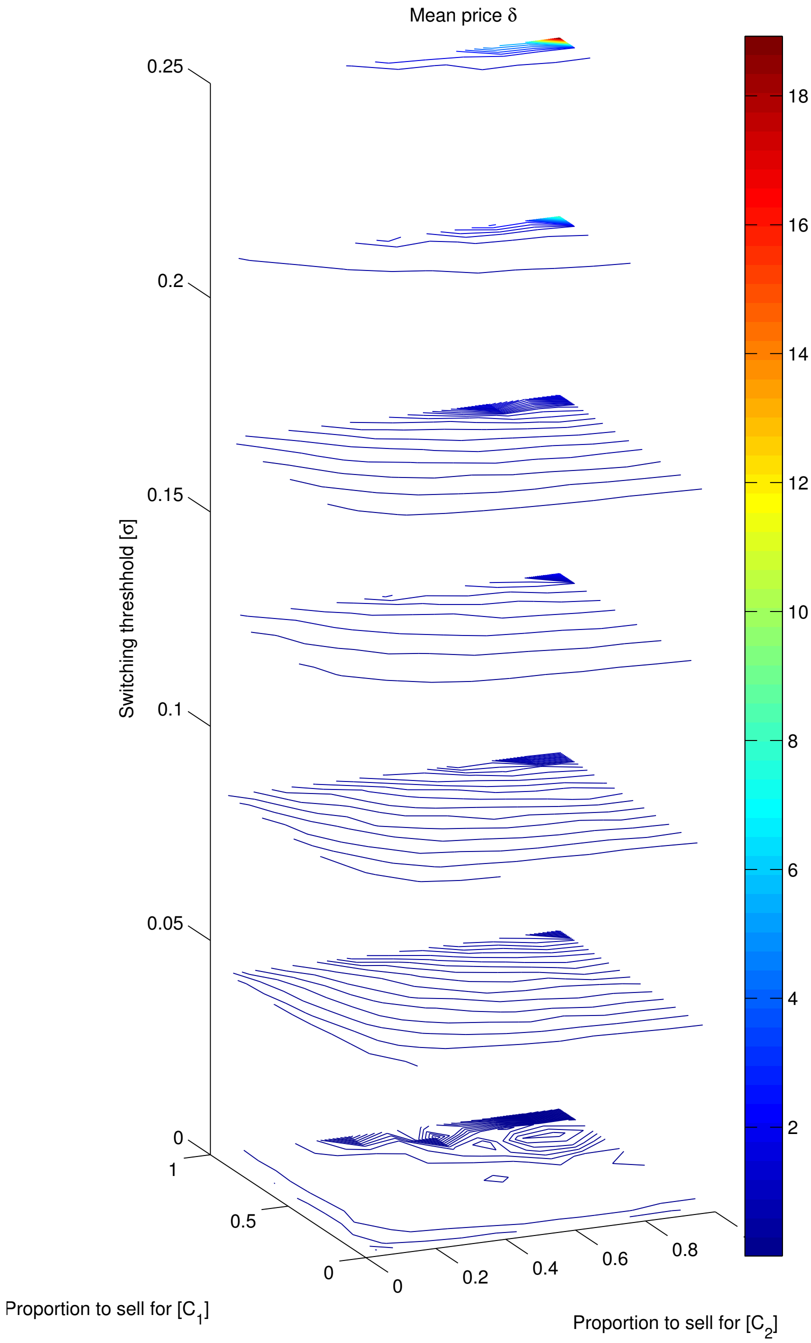

Figure 6.9: Figure depicting the contour planes of price δ for different σ, agents switching, over varied C_1 and C_2.

The reduction in average δ for supply and demand is mimicked in the δ of price. Average price δ has been plotted in Figure 6.9 wherein we can se that reduction in δ follows a reduction in C_1, C_2, or σ, due to the fact that price is calculated from supply and demand.

With lower δ agents are given a more stable environment to act in, this leads to an increase in the overall efficiency of the system as measured by average, average efficiency of agents depicted in Figure 6.10. In Figure 6.10 we se that as C_1, C_2, or σ decreases the average efficiency slowly increases, disregarding $\sigma = 0$ where the average efficiency is 0.5, since no agents are allowed to switch processes. We can thus conclude that more stable markets lead to agents sticking to the process they are most arpt at running. This is probably mostly due to low price fluctuations, which otherwise may induce the greedy agents to change to a process which in the short term is really profitable, even if said agents are not efficient at running that particular process.

The correlation between lower δ and average efficiency is further visualised in Figure 6.11 where the measured average efficiencies are plotted against recorded mean supply δ. As we can see higher efficiency corresponds well with lower δ. Furthermore we note that the fragmentation of the data into eight different arms at low supply δ is due to the sample rate of the domain, which indicates that the correlation between mean efficiency and supply δ also depends on other parameters.

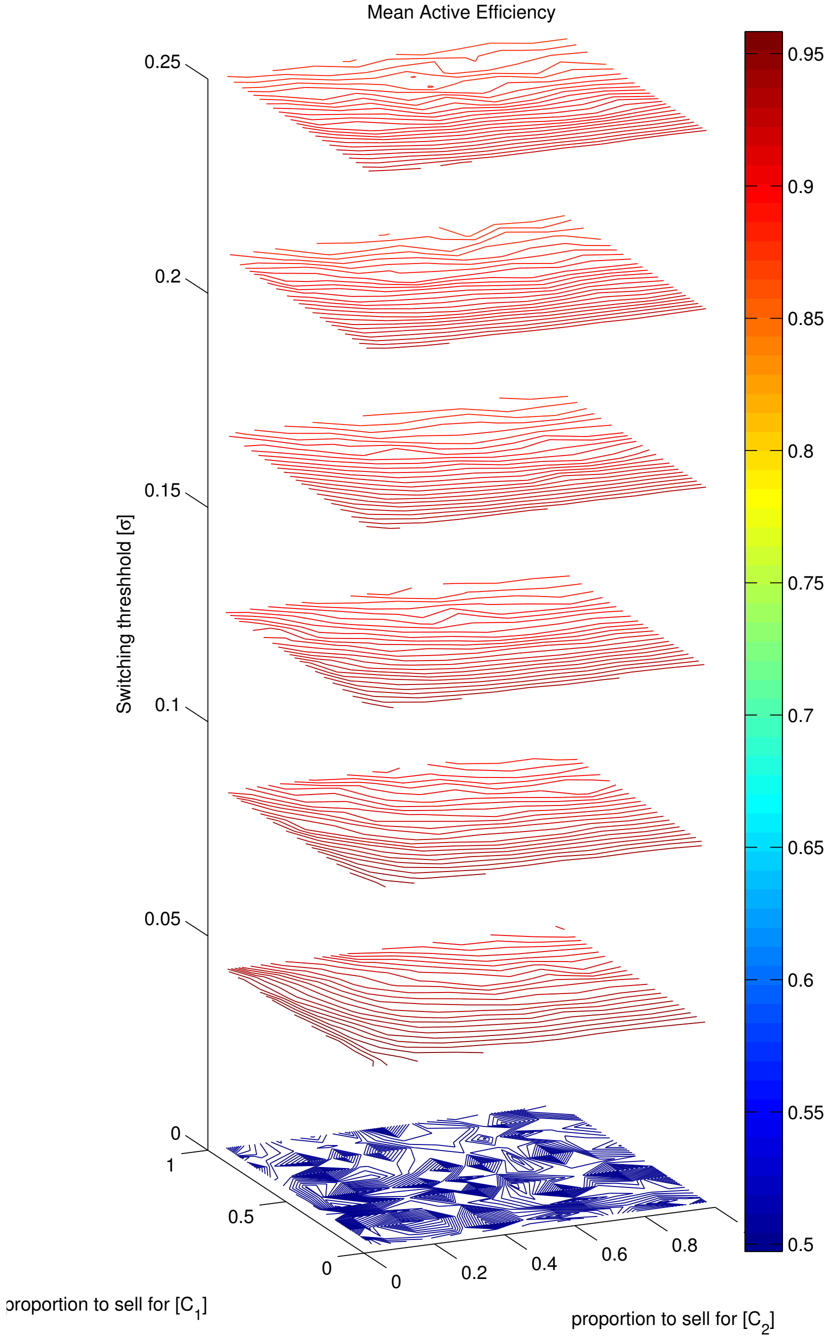

Figure 6.10: Figure depicting the contour planes of average, average efficiency for different σ, agents switching, over varied C_1 and C_2.

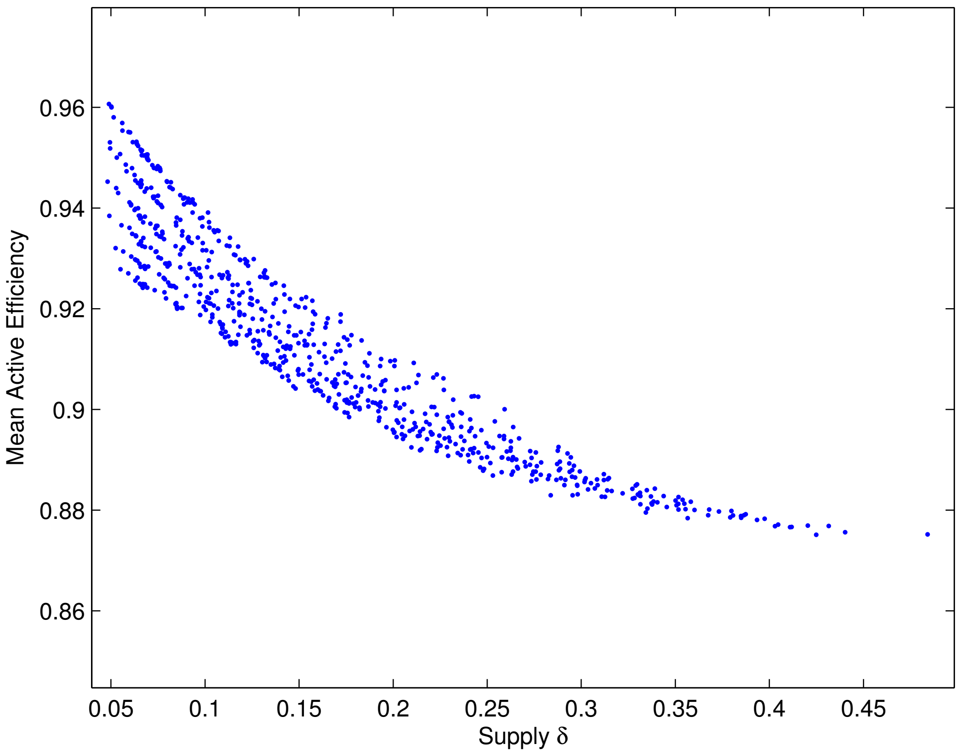

Figure 6.11: Figure depicting the average efficiency versus supply δ, for all collected datapoints except those where $\sigma = 0$.

6.3 Dependancy networks of agents through the C++ visualisations

In order to showcase the C++ visualisation of the dependance network a sample case has been documented. The parameters are the same as for the sample run previously described in 6.1. The results are visualised in Figure 6.12, 6.13, and 6.14. In these figures we see a snapshot of the 3D representation of the resource flow network. Every node is a market actor, or group of market actors. The sinks and faucets are represented by one node each (nodes 51 and 52), furthermore all agents involved in a particular process are given a node. This means that the $10 * 3 + 5 * 4 = 50$ nodes are process nodes, where 4 tiers gives 5*4 processes from one tier to the same and 3 times 10 processes between tiers.

Connections between nodes represent flow of goods. This means that a connection from A to B represents the total value of goods sold from A to B in dollars. Since all agents may trade in all goods and conduct business with each other we get 52^2

connections, which is arguably quite impractical to visualise. Thus Figures 6.12, 6.13, and 6.14 only visualise those flow which are no less than 5% of the largest flow. This gives us a good insight into the dependancy network at three distinctively different time steps. By distinctly different we mean time steps which are separated by at least one significant shift in the available supply and demand, and its subsequent convergence back to the stability point described by the analytical model.

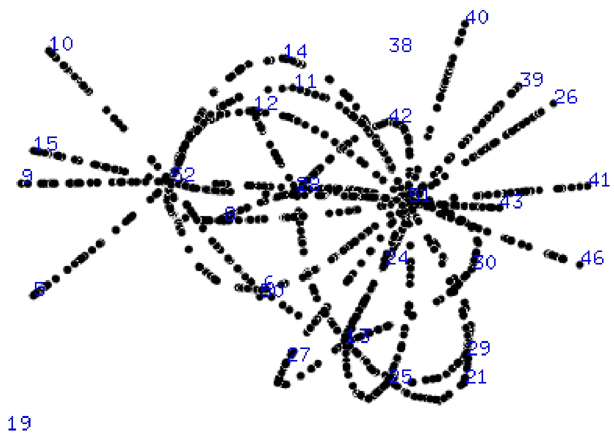

Figure 6.12: Figure depicting the flow of resources between the different groups of market actors for one iteration of a sample simulation. All resource flows which are shown are at least 5% of the largest flow, when counted in dollars. Nodes 51 and 52 are the sink and faucet respectively.

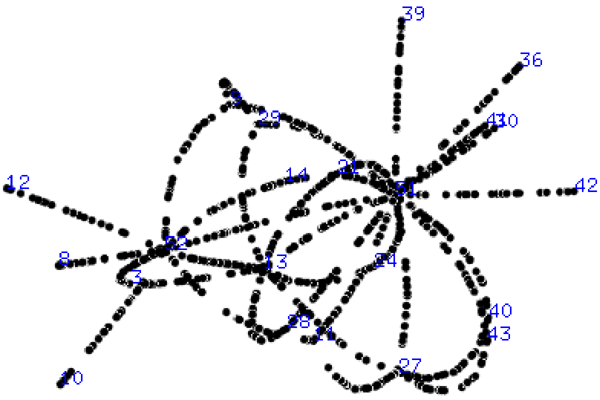

Figure 6.13: Figure depicting the flow of resources between the different groups of market actors for one iteration of a sample simulation. All resource flows which are shown are at least 5% of the largest flow, when counted in dollars. Nodes 51 and 52 are the sink and faucet respectively.

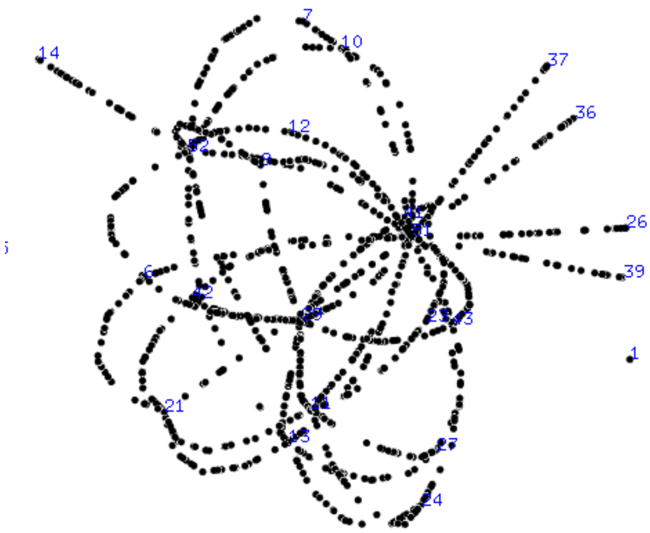

Figure 6.14: Figure depicting the flow of resources between the different groups of market actors for one iteration of a sample simulation. All resource flows which are shown are at least 5% of the largest flow, when counted in dollars. Nodes 51 and 52 are the sink and faucet respectively.

As we can see from Figures 6.12, 6.13, and 6.14 there are several connections between sink and faucet, indicating that the flow of resources go through several processes. We may also note that several processes seem like dead ends, with free floating ends. Theses are the processes to which the connecting edges are lower than the 5% visualisation limit.

Furthermore we see that the visualised processes make up one network and not several disjoint clusters. This picture is the same for a wide range of the lower visualisation limits, however if one asks to visualise the network with a visualisation limit of 30% the network may sometimes be disjoint. This shows that all major resource follows are connected to every other resource flow with equal or greater flows. Lastly we may note that the visualised networks are different, referring to involved nodes, when separated by a significant shift in the supply and demand characteristics.

7

Conclusions

In conclusion we note that both the feedback processes, agent switching inducing feedback and break even feedback have combined to create a model which progresses towards a market equilibrium. This in combination with the minority dynamics of the market price setting algorithm have yielded price progressions which converge towards this market equilibrium, only to spontaneously take a step back when one or more cash heavy agents switch processes.

It has been shown, in Section 6.1, that the prescribed model exhibits normal supply and demand characteristics, answering question 1 of the problem formulation. However as was shown in 6.1 agents do not make any significant profits once the model has stabilised, which answers question 2, which was further backed up by the stability analysis of Section 4 wherein the stable state involves a constant demand present on the market. This leads us to the question of if this model converges to a market equilibrium as posed in question 3. As we have seen, in both Section 4 and 6.1, strong feedback mechanisms push the system to a price equilibrium, however due to the minority game dynamics the system never fully reaches it. This also showcases what we can say with a game theoretic perspective on this model. Showing that the system behaves just as in the case of the El Farol bar problem where feedback moves the attending/not attending ratio towards 60% but never settling. Lastly it has been shown, through visualisation in Section 6, that all processes connected with major flows are connected, via the same network of flows, to all other agents involved in the major flows. This means that agents rely on theses processes for input resources and output demand, in essense relying on neighbouring agents in the network. Meaning that all major players rely on each other. However if the system changes, through its supply ore demand, the system quickly adapts through the feedback processes.

In short, the pursued model amply models a simple market, with minimal parameters and extremely simple agents. The model has a stable price point which it is drawn towards, however it may at any point unexpectedly detract from this path, causing a new convergence process to start.

8

Discussion

It should be noted that the analysis of this new model and market system is not as deep as the model merits. There are several instances where the dynamics merit further investigation, however time constraints made this endeavour impossible.

One such endeavour which aught to be undertaken is the solidification of the connection between the analytical model and the agent based model. If the number of agents allowed to switch each iteration is set to zero, the agent based model becomes an analytical model where each process has a set amount of money and from this makes a profit or loss depending on the market state. However this analytical model possesses far more state variables then the four supply and demand variables used in Section 4, making it a far better analytical model of the agent based system than the analytical model presented in Section 4. Preliminary numerical excursions into the dynamics of such a model show that the model does converge to a stable price dynamic. However this price dynamic is not a fix point in the general sense but can be viewed as a strange attractor in a nondimensionlised supply demand plane. However further research is needed on this in order to draw any conclusions.

Lastly no analysis of the models independence, relating to conversion processes, has been presented. Specifically relating to the fact that the conversion processes where randomised for every simulation. In general the analysis concerning functional unit relates this matter, indicating that these randomisations do not interfere with the general behaviour of the model. However a high standard deviation in the recorded value of δ in Section 6.2 would indicate the contrary, which means that is merits further investigation.

8.1 Undiscussed parts of the problem formulation

The avid reader may note that some questions from the problem formulation have been ignored in the conclusions section. However we shall detail some speculation or give a reason for why these questions are left unanswered.

Firstly the question of weather the model gives rise to niches, question 5, was left unanswered because no method of detecting these niches was found. If we define a niche as a small section of the market occupied by a small number of agents, fulfilling a supply to a main market which could otherwise not be meet, then it would be possible to detect such a niche by removing it, monitoring if it is filled by agents from the main market. However deciding what constitutes the main market and what constitutes a niche was simply too hard to define in the network of interacting agents.

Secondly question 7 referred to the resilience of the network to additions or withdrawals of the available processes, and while the C++ model was written with this functionality in mind it was never fully implemented, due to time constraints. However one might speculate that since the market in the model adapts well to price changes and fluctuations in supply and demand, one could argue that removing or adding one process would not upset the system in a spectacular way, only move the equilibrium price point. This could be further backed up by arguments made for the stability of the analytical model, and its relation to the agent based one, in that it does depend on the specifics of the processes only that at least one exists between tiers. This means that as long as the system has processes connecting the tiers the system will not crash.

Lastly the question of topological equivalence, question 8, was not discused because it was found that the topology of the flow network was heavily influenced by the topology of the available resources. Thus the structure dependancy network for the agents was dictated by a force other than the agents, meaning that no further effort was put on investigating this question.

Bibliography

[1] W. Brian Arthur, "Inductive Reasoning and Bounded Rationality, " *American Economic Review (Papers and Proceedings)*, 84,406-411. (1994)

[2] Ramm, Alexander G. Hoang, Nguyen S. *Dynamical Systems Method and Applications: Theoretical Developments and Numerical Examples.*Wiley, Hoboken, NJ, USA 2012.

[3] R.G. Palmer, W. Brian Arthur, John H. Holland, Blake LeBaron, Paul Tayler,"Artificial economic life: a simple model of a stockmarket,"*Physica D: Nonlinear Phenomena.* Volume 75, Issues 1-3, Pages 264-274. (1994)

[4] David K. Bryngelsson, Kristian Lindgren, "Why large-scale bioenergy production on marginal land is unfeasible: A conceptual partial equilibrium analysis," *Energy Policy* Volume 55, April 2013, Pages 454-466. (2013)

[5] Christophe Deissenberg, Sander van der Hoog, Herbert Dawid," EURACE: A massively parallel agent-based model of the European economy," *Applied Mathematics and Computation*Volume 204, Issue 2, 15 October 2008, Pages 541-552. (2008)

[6] D. Challet, M. Marsili, Y.-C. Zhang, Minority Games: Interacting Agents in Financial Markets, *Oxford University Press*, Oxford (2005)

[7] Wikipedia (2013) Open outcry `www.en.wikipedia.org/wiki/open_outcry` (October 31, 2013)

A

Mathematica code for analytical model stability analysis

The Mathematica code used in the analytical model has been appended below.

```
JacobianMatrix[f_List?VectorQ, x_List] :=
 Outer[D, f, x] /; Equal @@ (Dimensions /@ {f, x})
JacobianDeterminant[f_List?VectorQ, x_List] :=
 Det[JacobianMatrix[f, x]] /; Equal @@ (Dimensions /@ {f, x})

F[S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4] := {λ S1 Dem1/(Dem1 + 1000) + 100,
  σ S1 Dem1/(Dem1 + 1000) + λ S2 Dem2/(Dem2 + 1000),
  σ S2 Dem2/(Dem2 + 1000) + λ S3 Dem3/(Dem3 + 1000),
  σ S3 Dem3/(Dem3 + 1000) + λ S4 Dem4/(Dem4 + 1000),

  α Dem2  + γ Dem1 S1/(S1 + 100) + 1000,
  α Dem3  + γ Dem2 + 1000,
  α Dem4  + γ Dem3 + 1000,
  γ Dem4 + 1000}
Print["   ------ -- avalible somutions -- -----"]
sol = FullSimplify[
  Solve[{S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4} == F[S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4],
   {S, S1, S2, S3, S4, Dem, Dem1, Dem2, Dem3, Dem4}]]
Print["---- Domain where solutions are always positive -----"]
Print["-- Domain for 1 --"]
point = {S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4} /. Part[sol, 1];
thePlace = {};
For[i = 1, i <= Length[point], i++,
 thePlace = Join[thePlace, {Part[point, i] > 0}]]
thePlace = Join[thePlace,
   {0 < λ < 1, 0 < σ < 1, 0 < γ < 1, 0 < α < 1}];
Reduce[thePlace, {λ, σ, γ, α}, Reals]
Print["-- domain for 2 --"]
point = {S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4} /. Part[sol, 2];
thePlace = {};
For[i = 1, i <= Length[point], i++,
 thePlace = Join[thePlace, {Part[point, i] > 0}]]
thePlace = Join[thePlace,
   {0 < λ < 1, 0 < σ < 1, 0 < γ < 1, 0 < α < 1}];
Reduce[thePlace, {λ, σ, γ, α}, Reals]
Print["----- sol1 keep away points --------"]
denom1 = Denominator[{S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4} /. Part[sol, 1]];
For[i = 1, i <= Length[denom1], i++,
 Print[Solve[{Part[denom1, i] == 0, 0 < λ < 1,
    0 < σ < 1, 0 < γ < 1, 0 < α < 1}, {λ, σ, γ, α}]]]
Print["----- sol2 keep away points------ "]
denom2 = Denominator[{S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4} /. Part[sol, 2]];
For[i = 1, i <= Length[denom2], i++, Print[Solve[
   {Part[denom2, i] == 0, 0 < λ < 1, 0 < σ < 1, 0 < γ < 1, 0 < α < 1}, {λ, σ, γ, α}]]]
(*chose a point to run alog on*)
```

```
Print["----- compute eigenvlues of jacobian -----"]
JACK = JacobianMatrix[F[S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4],
   {S1, S2, S3, S4, Dem1, Dem2, Dem3, Dem4}] ;
eigenVals = FullSimplify[Eigenvalues[JACK]]
thePoint = Part[sol, 2];
Print["----- max val of eig at the point 2-----"]
For[i = 1, i <= Length[eigenVals], i++,
 hamster = Abs[Part[eigenVals, i] /. thePoint];
 (*Reduce[{Abs[Part[eigenVals,i]/.thePoint] <1,0<λ<1,0<σ<1,0<γ<1,0<α<1},{λ,σ,γ,α}]*)
 Print[NMaximize[{hamster,
    1 > λ > 0  && 1 > γ > 0 && 1 > σ > 0 && 1 > α > 0}, {λ, γ, σ, α}]]
 (*Print[FindInstance[{hamster>1 &&1>λ>0  &&1>γ>0 && 1>σ>0 && 1>α>0},
   {λ,γ,σ,α},Reals]]*)
]
```

------ -- avalible somutions -- -----

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$
\begin{aligned}
&\{\{S_1 \to (50 (\gamma - \gamma^4 - (1+\alpha)(1+\alpha^2)\lambda + (3+\alpha(2+\alpha))\gamma\lambda + \gamma^3(3+\lambda) - \gamma^2(3+(3+\alpha)\lambda) + \\
&\quad \sqrt{(((-1+\gamma)^3(\gamma-\lambda) + \alpha^3(-2+\lambda) - \alpha^2(-1+\gamma)(-2+\lambda) + \alpha(-1+\gamma)^2(-2+\lambda))^2 +} \\
&\quad\quad \overline{8(\alpha^2+(-1+\gamma)^2)(1+\alpha-\gamma)(-1+\gamma)^3(-2+\gamma+\lambda))})) / \\
&\quad (\alpha^3(-1+\lambda) + \alpha(-1+\gamma)^2(-1+\lambda) - (-1+\gamma)^3(-2+\gamma+\lambda) + \alpha^2(-1+\gamma+\lambda-\gamma\lambda)), \\
&S_2 \to -(50(\alpha+\alpha^2-(-2+\gamma)(-1+\gamma)^2-\alpha\gamma) \\
&\quad (\alpha^3(2-3\lambda) + \alpha^2(-1+\gamma)(-2+3\lambda) - \alpha(-1+\gamma)^2(-2+3\lambda) + (-1+\gamma)^3(-4+\gamma+3\lambda) + \\
&\quad \sqrt{(((-1+\gamma)^3(\gamma-\lambda) + \alpha^3(-2+\lambda) - \alpha^2(-1+\gamma)(-2+\lambda) + \alpha(-1+\gamma)^2(-2+\lambda))^2 +} \\
&\quad\quad \overline{8(\alpha^2+(-1+\gamma)^2)(1+\alpha-\gamma)(-1+\gamma)^3(-2+\gamma+\lambda))}\,\sigma) / \\
&\quad (\lambda(\alpha^5(-1+\lambda)^2 - 2\alpha^4(-1+\gamma)(-1+\lambda)^2 + 2\alpha(-1+\gamma)^4(-1+\lambda)(-2+\gamma+\lambda) - \\
&\quad (-1+\gamma)^5(-2+\gamma+\lambda)^2 + \alpha^3(-1+\gamma)^2(-1+\lambda)(-4+\gamma+3\lambda) - \\
&\quad \alpha^2(-1+\gamma)^3(-1+\lambda)(-5+2\gamma+3\lambda))), \\
&S_3 \to (50(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\gamma)(2+\alpha-3\gamma+\gamma^2)(\alpha^3(2-3\lambda) + \alpha^2(-1+\gamma)(-2+3\lambda) - \\
&\quad \alpha(-1+\gamma)^2(-2+3\lambda) + (-1+\gamma)^3(-4+\gamma+3\lambda) + \\
&\quad \sqrt{(((-1+\gamma)^3(\gamma-\lambda) + \alpha^3(-2+\lambda) - \alpha^2(-1+\gamma)(-2+\lambda) + \alpha(-1+\gamma)^2(-2+\lambda))^2 +} \\
&\quad\quad \overline{8(\alpha^2+(-1+\gamma)^2)(1+\alpha-\gamma)(-1+\gamma)^3(-2+\gamma+\lambda))}\,\sigma^2) / \\
&\quad (\lambda(\alpha^6(-1+\lambda)^3 - 3\alpha(-1+\gamma)^5(-1+\lambda)(-2+\gamma+\lambda)^2 + (-1+\gamma)^6(-2+\gamma+\lambda)^3 - \\
&\quad \alpha^5(-1+\gamma)(-1+\lambda)^2(-4+\gamma+3\lambda) + \alpha^2(-1+\gamma)^4(-1+\lambda)(-2+\gamma+\lambda)(-7+2\gamma+5\lambda) + \\
&\quad \alpha^4(-1+\gamma)^2(-1+\lambda)^2(-8+3\gamma+5\lambda) - \\
&\quad \alpha^3(-1+\gamma)^3(-1+\lambda)(13+\gamma^2+6(-3+\lambda)\lambda+\gamma(-8+6\lambda)))), \\
&S_4 \to (50(1+\alpha-\gamma)(-2+\gamma)(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\gamma) \\
&\quad (\alpha^3(2-3\lambda) + \alpha^2(-1+\gamma)(-2+3\lambda) - \alpha(-1+\gamma)^2(-2+3\lambda) + (-1+\gamma)^3(-4+\gamma+3\lambda) + \\
&\quad \sqrt{(((-1+\gamma)^3(\gamma-\lambda) + \alpha^3(-2+\lambda) - \alpha^2(-1+\gamma)(-2+\lambda) + \alpha(-1+\gamma)^2(-2+\lambda))^2 +} \\
&\quad\quad \overline{8(\alpha^2+(-1+\gamma)^2)(1+\alpha-\gamma)(-1+\gamma)^3(-2+\gamma+\lambda))}\,\sigma^3) / \\
&\quad (\lambda(-2+\gamma+\lambda)(\alpha-\alpha\lambda+(-1+\gamma)(-2+\gamma+\lambda))(-\alpha^3(-1+\lambda)+\alpha^2(-1+\gamma)(-1+\lambda) - \\
&\quad \alpha(-1+\gamma)^2(-1+\lambda) + (-1+\gamma)^3(-2+\gamma+\lambda)) \\
&\quad (\alpha^2(-1+\lambda) + (-1+\gamma)^2(-2+\gamma+\lambda) + \alpha(-1+\gamma+\lambda-\gamma\lambda))), \\
&Dem_1 \to \frac{1}{(-1+\gamma)^3(-2+\gamma+\lambda)}\, 500\, (-(-1+\gamma)^3(\gamma-\lambda) - \alpha^3(-2+\lambda) + \\
&\quad \alpha^2(-1+\gamma)(-2+\lambda) - \alpha(-1+\gamma)^2(-2+\lambda) -
\end{aligned}
$$

$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big),$$

$$\mathrm{Dem}_2\to-\frac{1000\ \big(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\,\gamma\big)}{(-1+\gamma)^3},\ \mathrm{Dem}_3\to\frac{1000\ (1+\alpha-\gamma)}{(-1+\gamma)^2},$$

$$\mathrm{Dem}_4\to$$
$$-\frac{1000}{-1+\gamma}\Big\},$$

$$\Big\{S_1\to\Big(50\ \big(\gamma-\gamma^4-(1+\alpha)\ \big(1+\alpha^2\big)\ \lambda+(3+\alpha\ (2+\alpha))\ \gamma\,\lambda+\gamma^3\ (3+\lambda)-\gamma^2\ (3+(3+\alpha)\ \lambda)-$$
$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big)\Big)\Big/$$
$$\big(\alpha^3\ (-1+\lambda)+\alpha\ (-1+\gamma)^2\ (-1+\lambda)-(-1+\gamma)^3\ (-2+\gamma+\lambda)+\alpha^2\ (-1+\gamma+\lambda-\gamma\,\lambda)\big),$$

$$S_2\to\Big(50\ \big(\alpha+\alpha^2-(-2+\gamma)\ (-1+\gamma)^2-\alpha\,\gamma\big)$$
$$\Big(\alpha^3\ (-2+3\,\lambda)-\alpha^2\ (-1+\gamma)\ (-2+3\,\lambda)+\alpha\ (-1+\gamma)^2\ (-2+3\,\lambda)-(-1+\gamma)^3\ (-4+\gamma+3\,\lambda)+$$
$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big)\ \sigma\Big)\Big/$$
$$\Big(\lambda\ \big(\alpha^5\ (-1+\lambda)^2-2\,\alpha^4\ (-1+\gamma)\ (-1+\lambda)^2+2\,\alpha\ (-1+\gamma)^4\ (-1+\lambda)\ (-2+\gamma+\lambda)-$$
$$(-1+\gamma)^5\ (-2+\gamma+\lambda)^2+\alpha^3\ (-1+\gamma)^2\ (-1+\lambda)\ (-4+\gamma+3\,\lambda)-$$
$$\alpha^2\ (-1+\gamma)^3\ (-1+\lambda)\ (-5+2\,\gamma+3\,\lambda)\big)\Big),$$

$$S_3\to-\Big(50\ \big(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\,\gamma\big)\ \big(2+\alpha-3\,\gamma+\gamma^2\big)$$
$$\Big(\alpha^3\ (-2+3\,\lambda)-\alpha^2\ (-1+\gamma)\ (-2+3\,\lambda)+\alpha\ (-1+\gamma)^2\ (-2+3\,\lambda)-(-1+\gamma)^3\ (-4+\gamma+3\,\lambda)+$$
$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big)\ \sigma^2\Big)\Big/$$
$$\Big(\lambda\ \big(\alpha^6\ (-1+\lambda)^3-3\,\alpha\ (-1+\gamma)^5\ (-1+\lambda)\ (-2+\gamma+\lambda)^2+(-1+\gamma)^6\ (-2+\gamma+\lambda)^3-\alpha^5\ (-1+\gamma)$$
$$(-1+\lambda)^2\ (-4+\gamma+3\,\lambda)+\alpha^2\ (-1+\gamma)^4\ (-1+\lambda)\ (-2+\gamma+\lambda)\ (-7+2\,\gamma+5\,\lambda)+\alpha^4\ (-1+\gamma)^2$$
$$(-1+\lambda)^2\ (-8+3\,\gamma+5\,\lambda)-\alpha^3\ (-1+\gamma)^3\ (-1+\lambda)\ \big(13+\gamma^2+6\ (-3+\lambda)\ \lambda+\gamma\ (-8+6\,\lambda)\big)\big)\Big),$$

$$S_4\to\Big(50\ (1+\alpha-\gamma)\ (-2+\gamma)\ \big(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\,\gamma\big)\ \Big(\alpha^3\ (2-3\,\lambda)+\alpha^2\ (-1+\gamma)\ (-2+3\,\lambda)-$$
$$\alpha\ (-1+\gamma)^2\ (-2+3\,\lambda)+(-1+\gamma)^3\ (-4+\gamma+3\,\lambda)-$$
$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big)\ \sigma^3\Big)\Big/$$
$$\Big(\lambda\ (-2+\gamma+\lambda)\ (\alpha-\alpha\,\lambda+(-1+\gamma)\ (-2+\gamma+\lambda))\ \big(-\alpha^3\ (-1+\lambda)+\alpha^2\ (-1+\gamma)\ (-1+\lambda)-$$
$$\alpha\ (-1+\gamma)^2\ (-1+\lambda)+(-1+\gamma)^3\ (-2+\gamma+\lambda)\big)$$
$$\big(\alpha^2\ (-1+\lambda)+(-1+\gamma)^2\ (-2+\gamma+\lambda)+\alpha\ (-1+\gamma+\lambda-\gamma\,\lambda)\big)\Big),$$

$$\mathrm{Dem}_1\to\frac{1}{(-1+\gamma)^3\ (-2+\gamma+\lambda)}\ 500\ \Big(-(-1+\gamma)^3\ (\gamma-\lambda)-\alpha^3\ (-2+\lambda)+$$
$$\alpha^2\ (-1+\gamma)\ (-2+\lambda)-\alpha\ (-1+\gamma)^2\ (-2+\lambda)+$$
$$\sqrt{\Big(\big((-1+\gamma)^3\ (\gamma-\lambda)+\alpha^3\ (-2+\lambda)-\alpha^2\ (-1+\gamma)\ (-2+\lambda)+\alpha\ (-1+\gamma)^2\ (-2+\lambda)\big)^2+}$$
$$8\ \big(\alpha^2+(-1+\gamma)^2\big)\ (1+\alpha-\gamma)\ (-1+\gamma)^3\ (-2+\gamma+\lambda)\Big)\Big),$$

$$\mathrm{Dem}_2\to-\frac{1000\ \big(\alpha+\alpha^2+(-1+\gamma)^2-\alpha\,\gamma\big)}{(-1+\gamma)^3},\ \mathrm{Dem}_3\to\frac{1000\ (1+\alpha-\gamma)}{(-1+\gamma)^2},$$

$$\mathrm{Dem}_4\to$$
$$-\frac{1000}{-1+\gamma}\Big\}\Big\}$$

---- Domain where solutions are always positive -----

```
-- Domain for 1 --
False
-- domain for 2 --
0 < λ < 1 && 0 < σ < 1 && 0 < γ < 1 && 0 < α < 1
----- sol1 keep away points --------
{}
{}
{}
{}
{}
{}
{}
{}
----- sol2 keep away points------
{}
{}
{}
{}
{}
{}
{}
{}
----- compute eigenvlues of jacobian -----
```

$$\left\{\frac{\lambda\,\mathrm{Dem}_4}{1000+\mathrm{Dem}_4},\ \frac{\lambda\,\mathrm{Dem}_3}{1000+\mathrm{Dem}_3},\ \frac{\lambda\,\mathrm{Dem}_2}{1000+\mathrm{Dem}_2},\ \gamma,\ \gamma,\ \gamma,\ \frac{1}{2}\left(\frac{\lambda\,\mathrm{Dem}_1}{1000+\mathrm{Dem}_1}+\frac{1}{(100+\mathrm{S}_1)^2}\Big(\gamma\,\mathrm{S}_1\,(100+\mathrm{S}_1)-\left(\sqrt{\left((1000+\mathrm{Dem}_1)^2\,(1000+\mathrm{Dem}_2)^4\,(1000+\mathrm{Dem}_3)^4\,(1000+\mathrm{Dem}_4)^4\,(100+\mathrm{S}_1)^2\,\left(10\,000\,\lambda^2\,\mathrm{Dem}_1^2+200\,\lambda\,\mathrm{Dem}_1\,(1000\,\gamma+(-\gamma+\lambda)\,\mathrm{Dem}_1)\,\mathrm{S}_1+(1000\,\gamma+(\gamma-\lambda)\,\mathrm{Dem}_1)^2\,\mathrm{S}_1^2\right)\right)}\right)\Big/\left((1000+\mathrm{Dem}_1)^2\,(1000+\mathrm{Dem}_2)^2\,(1000+\mathrm{Dem}_3)^2\,(1000+\mathrm{Dem}_4)^2\right)\Big)\right),\right.$$

$$\left.\frac{1}{2}\left(\frac{\lambda\,\mathrm{Dem}_1}{1000+\mathrm{Dem}_1}+\frac{1}{(100+\mathrm{S}_1)^2}\Big(\gamma\,\mathrm{S}_1\,(100+\mathrm{S}_1)+\left(\sqrt{\left((1000+\mathrm{Dem}_1)^2\,(1000+\mathrm{Dem}_2)^4\,(1000+\mathrm{Dem}_3)^4\,(1000+\mathrm{Dem}_4)^4\,(100+\mathrm{S}_1)^2\,\left(10\,000\,\lambda^2\,\mathrm{Dem}_1^2+200\,\lambda\,\mathrm{Dem}_1\,(1000\,\gamma+(-\gamma+\lambda)\,\mathrm{Dem}_1)\,\mathrm{S}_1+(1000\,\gamma+(\gamma-\lambda)\,\mathrm{Dem}_1)^2\,\mathrm{S}_1^2\right)\right)}\right)\Big/\left((1000+\mathrm{Dem}_1)^2\,(1000+\mathrm{Dem}_2)^2\,(1000+\mathrm{Dem}_3)^2\,(1000+\mathrm{Dem}_4)^2\right)\Big)\right)\right\}$$

```
----- max val of eig at the point 2-----
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

Infinity::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

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General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

Infinity::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

NMaximize::nnum : The function value Indeterminate is not a number at $\{\alpha, \gamma, \lambda, \sigma\}$ = {1., 1., 1., 0.}. ≫

```
{1., {α → 1., γ → 1., λ → 1., σ → 0.000750163}}
{1., {λ → 1., γ → 1., σ → 0.998388, α → 0.698399}}
```

Infinity::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

General::stop : Further output of Infinity::indet will be suppressed during this calculation. ≫

NMaximize::nnum :
The function value Indeterminate is not a number at $\{\alpha, \gamma, \lambda, \sigma\}$ = {0.990675, 1., 1., 0.175947}. ≫

```
{1., {α → 0.988721, γ → 0.999782, λ → 1., σ → 0.245852}}
{1., {λ → 0.908019, γ → 1., σ → 0.171524, α → 0.94187}}
{1., {λ → 0.908019, γ → 1., σ → 0.171524, α → 0.94187}}
{1., {λ → 0.908019, γ → 1., σ → 0.171524, α → 0.94187}}
{1., {λ → 1., γ → 1., σ → 0.0806639, α → 0.964177}}
{1., {λ → 1., γ → 0.997256, σ → 0.00565015, α → 0.305871}}
```