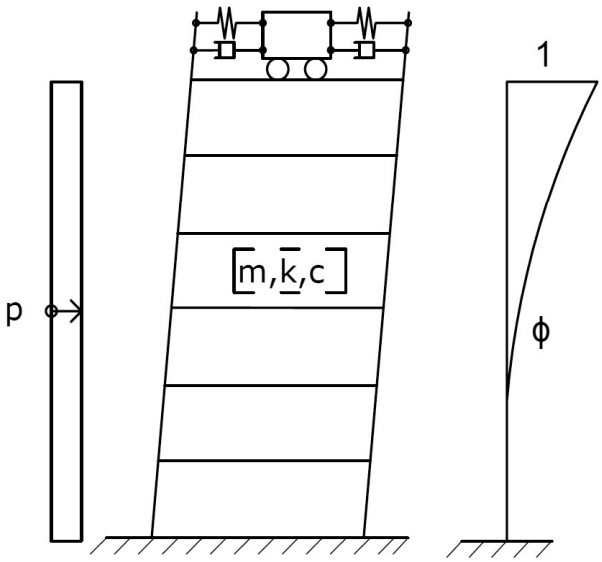
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Dynamic Response in Tall Timber Structures

A parametric study of the dynamic response due to lateral loads in a tall timber structure

Master's thesis in Structural Engineering and Building Technology

Fredrik Ivarsson
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Department of Mechanics and Maritime Sciences
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MASTER'S THESIS IN STRUCTURAL ENGINEERING AND BUILDING TECHNOLOGY

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Master's Thesis 2018:39
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Cover:
A analytical model of a damped structure and the dynamic response due to a lateral load.

Department of Mechanics and Maritime Sciences
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ABSTRACT

Modern building tend to strive towards more slender and lightweight constructions. That is to be more provident with space and materials as well as for aesthetic reasons. The effect of these lightweight slender buildings is an increased sensitivity to lateral loads with regard to the dynamic behaviour of the structure. Since the european union changed to more function based standards, the development of timber and timber products have increased during the past 20 years. It is now both in the interest of and feasible to build taller and larger buildings with the primary load bearing system made of timber. Timber have a relatively low mass compared to other construction materials which can result in larger deformations and discomfort if the dynamic response in the structure is too large.

The purpose of this report is to make a parametric study on how mass, stiffness and damping affect the feasible building height of a tall timber structure with regard to dynamic effects caused by wind. This is performed via simulations and analyses of a planned timber structure above 10 floors. The general design parameters are modified in order to fulfill the acceleration requirements for a structure with an increasing number of floors.

The initial structure is composed of load bearing Cross Laminated Timber (CLT) walls and floors that acts in diaphragm action. A FE-model is used to determine the eigenfrequencies of the structure and the swedish Annex, *EKS 10*, is used to calculate the peak acceleration. The determined eigenfrequency and acceleration curve is compared with the requirements of horizontal acceleration according to *ISO 10137*. If the structure fulfills the requirements, the structure is successively increased with 2 storeys at the time. If the structure does not fulfill the demands, it is improved with mass, stiffness and/or damping in an iteration process until it fulfills the requirements.

The result of this study is divided into a "*general behaviour*" and a "*structural behaviour*" chapter, to make it possible to understand the impact of each individual parameter separately and the combined impact on the structure. The improvements of adding mass and stiffness separately did not result in dramatic improvements of the acceleration. But by combining mass, damping and stiffness, considerable improvements with respect to the dynamic response is achieved and a building height of 26 storeys was feasible. Improvements of mass and damping combined made it possible to fulfill the demands on a 22 storey timber structure. This study conclude that the most feasible solution is to add mass and damping in forms of a concrete top storey (floor and walls) together with a TMD (Tuned Mass Damper) on the top floor.

Key words: Dynamic, Timber, Multi storey structure, Wind load, Acceleration

Dynamisk respons i höga träbyggnader
En parameterstudie av dynamisk respons på grund av lateral last i höga träbyggnader
Examensarbete inom masterprogrammet Structural Engineering and Building Technology
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Institutionen för Mekanik och maritima vetenskaper
Avdelningen för Konstruktionsteknik
Chalmers tekniska högskola

SAMMANFATTNING

Moderna byggnader tenderar mot mer slanka och lätta konstruktioner. Detta är för att vara mer sparsamma med utrymme och material samt av estetiska skäl. Resultatet av dessa lätta slanka byggnader är en ökad känslighet för lateral last med avseende på det dynamiska beteendet hos byggnaden. Sedan Europeiska unionen ändrade till mer funktionsbaserade standarder har utvecklingen av konstruktionsvirke och träprodukter ökat under de senaste 20 åren. Det är nu både intressant och möjligt att bygga högre och större byggnader med primära lastbärande system av trä. Då trä har en relativt låg massa jämfört med andra byggnadsmaterial kan det leda till större deformationer och nedsatt komfort om den dynamiska responsen i byggnaden är för stor.

Syftet med denna rapport är att göra en parameterstudie av hur massa, styvhet och dämpning påverkar byggnadshöjden hos en hög träbyggnad med avseende på dynamiska effekter orsakade av vind. Denna studie genomförs genom simuleringar och analyser av en planerad träbyggnad högre än 10 våningar, där de vanliga dimensionerings parametrar modifieras för att uppfylla accelerations-kraven för en byggnad med ett ökande antal våningar.

Den ursprungliga byggnaden består av lastbärande väggar och golv av Korslaminerat trä (KL-trä) som fungerar i membranverkan. En FE-modell används för att bestämma egenfrekvenserna för byggnaden och den svenska bilagan, *EKS 10,* används för att beräkna den maximala accelerationen. Den bestämda egenfrekvensen och accelerationen jämförs med kraven på horisontell acceleration enligt *ISO 10137*. Om byggnaden uppfyller kraven ökas byggnadshöjden med 2 våningar i taget. Om byggnaden inte uppfyller kraven förbättras den med massa, styvhet och/eller dämpning i en iterationsprocess tills den uppfyller kraven.

Resultatet av denna studie är uppdelat i ett "*generellt beteende*"- och ett "*strukturellt beteende*"- kapitel för att göra det möjligt att förstå effekterna av varje enskild parameter separat samt den kombinerade effekten på byggnaden. Förbättringarna av att lägga till massa och styvhet separat resulterade inte i dramatiska förbättringar av accelerationen. Men genom att kombinera massa, dämpning och styvhet uppnås betydande förbättringar med avseende på det dynamiska beteendet och en byggnadshöjd på 26 våningar var möjlig. Kombinerade förbättringar av massa och dämpning gjorde det möjligt att uppfylla kraven på en 22-vånings träbyggnad. Denna studie visar emellertid att den mest genomförbara lösningen är att lägga till massa och dämpning i form av en betong-våning (golv och väggar) tillsammans med en TMD (Tuned Mass Damper) på högsta våningen.

Nyckelord: Dynamik, Trä, Flervåningshus, Vindlast, Acceleration

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Preface

This report is the final part of the Master programme *Structural engineering and building technology* at Chalmers University of Technology, Sweden. The work is completed during the spring of 2018 at the institution for *Architecture and Civil Engineering* under the department of *Structural Engineering*. The project is cooperated with the company COWI, that also come up with idea of this project.

All tests have been carried out in COWI's office, where necessary materials, software's and workspace have been supplied. We would especially like to thank our supervisor from COWI, Thomas Hallgren for his involvement and guidance along this project. We also want to thanks the Department of Applied Mechanics providing a helpful examiner, Thomas Abrahamsson.

We hope that this report contribute a deeper understanding of dynamics to engineers and the timber industry, and that timber structures will be a competitive method on the market.

Göteborg, June 2018

Joel Sjöholm and Fredrik Ivarsson

Notations

Roman upper case letters

A	cross section area
A_i	area of individual walls
A_{ref}	reference area of structure
B^2	background factor
C_{pe}	pressure coefficient for external pressure
D_s	dynamic magnification factor
E	Young's modulus
F	Kármáns wind energy spectrum
ΣF	total force
F_w	wind force
H	total horizontal load
I	moment of inertia
I_v	turbulence intensity
L	element length
M	mass
R^2	resonance response
S_{bi}	bending stiffness of individual wall
S_{si}	shear stiffness of individual wall
S_T	global torsional stiffness
T_{600}	reference time for basic mean wind velocity
T	global torsional moment
T_a	return period of 5 years
T_n	natural period
T	kinetic energy
$\overline{U}$	complex number of the amplitude
V	potential energy
$\ddot{X}_{max}$	peak acceleration

Roman lower case letters

b	width of the structure
b_i	height/width of wall i depending on direction of wall
c	damping coefficient
c_{cr}	critical damping coefficient
c_f	force coefficient for wind action
c_r	roughness factor
$c_s c_d$	structural factor
c_0	orographic factor
d_i	distance from local to global gravity centre

f_D	damping force
f_n	eigenfrequency
f_0	first natural frequency in a horizontal direction
g	factor dependent on waveform
h	height of the structure
h_i	height/width of wall i depending on direction of wall
h_{ref}	reference height
k	stiffness
k_p	peak factor
k_r	terrain factor
l	height of the structure
l_i	height/width of wall i depending on direction of wall
m	mass of element
$m(s)$	mass per unit length
m_e	equivalent mass
n_1	fundamental eigenfrequency
$n_{1,x}$	the eigenfrequency of the structure
$p(t)$	time dependent forced motion
q_m	mean wind pressure
$q_p(z_e)$	peak velocity pressure
r	frequency ratio
u	displacement
$\dot{u}$	velocity
$\ddot{u}$	acceleration
ν	up-crossing frequency
v_b	basic wind speed with a returning period of 50 years
$v_{b,Ta}$	basic wind speed for a 1 year period
v_m	referens speed of mean wind velocity
$v_m(z)$	mean-wind velocity
z	height of the structure
z_e	reference height for the external pressure
z_0	roughness length

Greek letters

δ_a	aerodynamic damping for the fundamental mode
δ_d	damping due to special devices, e.g. TMD
δ_s	structural damping
ζ	viscous damping factor, a quotation to the critical damping
ρ	density
$\sigma_x(z)$	standard deviation of acceleration
Φ_b	size factor with regard to width of the structure
Φ_h	size factor with regard to height of the structure
Φ_1^2	mode shape (n=1)
ψ	mode shape
ψ''	second derivative of the mode shape
ω_n	eigenfrequency

1. Introduction

There are today a lot of ideas about constructing multi storey timber buildings, and there are currently research projects to implement timber as a main material in these kind of structures. The tallest timber building in Sweden today is Strandparken in Sundbyberg with 8 floors, built by Martinsons entirely in CLT (Cross laminated timber). The tallest timber building in the world is *Treet* in Bergen (Norway), with 14 floors, and there are suggestions to build even higher (*Hellekant, 2016)* .

Gusty wind is a lateral load that creates oscillations in buildings. The dynamic response in a structure can be perceived as a discomfort if the acceleration is too large (ISO 1984). Timber have a relatively low density compared to other construction materials which can result in larger deformations and discomfort (*Reynolds et al., 2015)*. According to the paper *Wind loading on tall buildings* (Mendis et al), the dynamic response is highly affected of both mass and stiffness. It is therefore possible to reduce the response acceleration by increasing these properties. This can however be in conflict with design of the structure. Damping on the contrary contributes to decreased wind response without the need of increased weight or stiffness. (*Mendis et al., 2007*)

1.1. Purpose

The purpose of this report is to make a parametric study on how mass, stiffness and damping affect the building height of a tall timber structure with regard to dynamic effects caused by wind. This is performed via simulations and analyses of a planned timber structure higher than 10 floors. The general design parameters are modified in order to fulfill the acceleration requirements for a structure with an increasing number of floors. For each increase of floors, the design criterias of acceleration are controlled. If they are not fulfilled, the three parameters are modified or added to meet the requirements.

The goal is to achieve a parametric study that can be used as support for designing tall timber structures. Another goal is to clarify the property which is most effective to modify with regard to an increased building height.

The whole study is conducted with regard to the practical applicability of the solutions and the result of the study is weighted with regard to the possibility to realistically implement it in a early design phase.

1.2. Limitations

The main focus of this study is the understanding of the dynamic response of the structure and focus will not be on the theory of dynamics. The calculations will be secondary to the dynamic analysis conducted in the FE-software. This study is based and restricted on a planned structure with load bearing walls in the facade and interior walls. These walls is assumed to only handle horizontal linear dynamic response of acceleration and frequency. The report do not treat design or optimizations of e.g. dampers, building size or placement of walls for maximum stiffness. Angle-acceleration or design-capacities e.g. load bearing, sound, sustainability/resistance and fire resistance will not be treated in this rapport.

- This report is restricted to the result and conditions possible to analyse in the software *FEM-design.*
- The report is based on swedish conditions and regulations.
- The structure is only analysed in serviceability limit state.
- The study is limited to a structure with a specific plan and geometry

1.3. Method

The report is divided into: a *literature study*, a *iteration process of improvements, FEM-tests*, *result* and *analysis* and suggestions of improvements in the *conclusion*.

The literature study consist of an analysis of the relevant standards regarding design of tall buildings with regard to wind induced forces. These standards are the european standard Eurocode *SS-EN 1991-1-4:2005*, as well as the swedish annex *EKS 10*, and the International standards *ISO-6897* and *ISO-10137*. The standards of analysing dynamic behaviour are studied and compared. Other published material on this specific subject such as master theses and articles are used to broaden the knowledge within the area. Simplified hand calculations of tall timber structures are performed for verification and further analysis of the results. The main result from the hand calculations are accelerations which are compared with suggested satisfactory magnitudes from the *ISO* standards. The suggested choices of improvements in the iteration process is based on the literature study and realistic construction methods. A detailed FE-model in the software *FEM-design 17-3D Structures* of the structure is analysed and compared to the hand calculations and the suggested serviceability limits. By changing parameters of material properties in the structure in an iteration process (see *Figure 1.1*) a more appropriate timber structure are developed in order to increase the building height yet still satisfy the requirements.

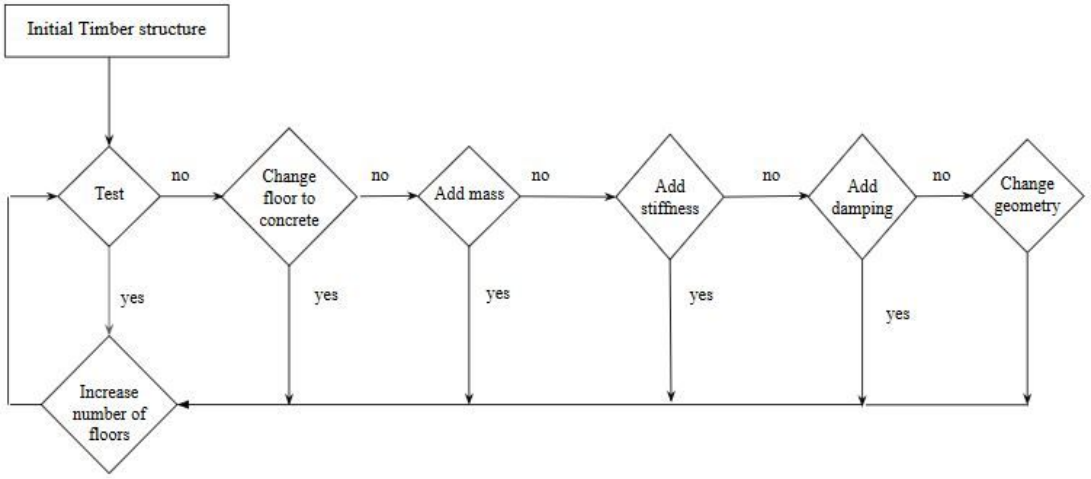


Figure 1.1 Flowchart of iteration process, order of parameter changes to fulfill requirements

2. Background

During the 19th century,the common building height was limited to a few storeys, the existing tall buildings were stocky and designed with over-capacity in accordance with existing knowledge and preferences. The vertical load bearing of these buildings often had infills of granite with high mass. This caused the buildings to be rather insensitive to dynamic effects and have led inhabitants to expect buildings without movements even in high wind conditions. Modern building tend to strive towards more slender and lightweight constructions. This to be more provident with space and materials as well as aesthetic reasons. The effect of these lightweight slender buildings is an increased sensitivity to lateral loads with regard to the dynamic behaviour of the structure. (*ISO ,1984*)

Since the european union changed to more function based standards, the development of timber and timber products have increased during the past 20 years. It is now both in the interest of and possible to build taller and larger buildings with the primary load bearing system made of timber due to the new regulations. (*Svenskt trä, 2018*)

2.1. Tall timber structures

Timber is one of the most eco friendly and longest used building material available. Timber have high strength -to -weight ratio and can transfer stresses in both tension and compression. The Timber´s high strength to weight ratio combined with its good resistance against sound and heat transfer as well as good durability makes it an favourable construction material. The fact that timber easily can be altered and connected to other structural elements increases the benefits of the material. (*Kermani, 2013*)

There are no distinct definition of the term “tall buildings”, instead the definition depends on multiple contexts and is highly subjective. Examples of these contexts could be the surrounding height of buildings or if the building is very slender, see *Figure 2.1*.

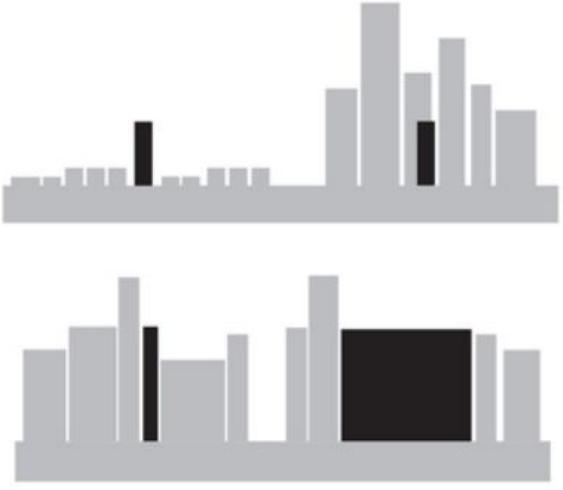


Figure 2.1 Structural contexts of tall buildings (*CTBUH, 2018*)

In high-rise cities, a 14-storey building may not be considered as a tall building, but should have been in e.g. a european suburb where the urban norm is distinctively lower. In the same fashion are a lot of buildings perceived as tall due to their slenderness when they in fact are not that tall. The definition of "tall timber structures" are somewhat unspecified with regard to authorized sources. The Council for Tall Buildings and Urban Habitat (CTBUH) have the following topologies according to the material of the primary vertical and lateral load bearing structure (2015):

- Steel
- Concrete
- Composite
- Mixed structure

A building consisting of a steel frame and a flooring system of concrete elements supported on steel beams is still considered as a steel building by CTBUH, this kind of floor construction is not considered to contribute in the main structural system. This is comparable to the distinction in design sometimes drawn for structures, the primary load bearing system is designed by a structural engineer while the secondary load bearing structure can be contractor designed (*Ctbuh, 2018)*. According to the paper *Proposal for Defining a Tall Timber Building, (Foster* et al., 2016) the definitions used for tall steel and concrete structures can easily be expanded to accommodate timber as building material and the following criterias for a tall timber structure is proposed:

- A single-material tall building is defined as one where the main vertical and lateral structural elements and floor systems are constructed from a single material.
- If a tall building is of timber construction with a floor system of concrete planks or slabs supported on timber beams, it is still considered as a timber building.
- If a tall building is of timber construction with local connections between timber elements formed using another material, it is still considered a timber building.

Further clarifications on the definition on what is a tall timber structure and what is not can be read in *Proposal for Defining a Tall Timber Building.* Worth mentioning is that approximately 85% of the building height of a structure should be of structural timber to be defined as a single-material timber structure. Similar proposals for defining a tall timber structure are conducted by e.g. Emporis (a global provider of building information).

2.2. Sustainability

In order to reduce the usage of non renewable materials with high energy consumption and large carbon dioxide emissions such as concrete and steel, the usage of timber can be increased.

2.2.1. Roadmap 2050

The european council decided in oktober 2009 an objective in order to reduce the emissions of greenhouse gases with 80-95% in the EU. This objective was created in order to fulfill the limitation on a 2 ℃ increase of the temperature until 2050. The primary measure to fulfill this objective is an transformation of the energy system to become more efficient.

There are many ways to increase energy efficiency of the society but the relevant in the civil engineering sector would be to optimise buildings and building materials with regard to energy efficiency, as well as limitation on transportation of materials. Part of this solution would be to increase the usage of timber products as building material. (*Swedish wood, 2018)*

2.2.2. Manufacturing of building materials

Timber products compared to other building materials such as concrete and steel requires little external energy input during the processing of the raw material to finished products. In Sweden, 80 % of the energy consumed by the sawmills is created by the bio waste from their own production. This fact combined with the fact that wood store carbon dioxide makes it far more energy efficient than the two other major building materials. Both steel and concrete requires vast quantities of fossil fuel energy for both production and processing, primary to the heating of the raw material during the processing. This energy consumption leads to significant carbon emissions which can be seen in *Figure 2.2* where the carbon emissions from manufacturing different building materials is compared. (*Swedish wood, 2018)*

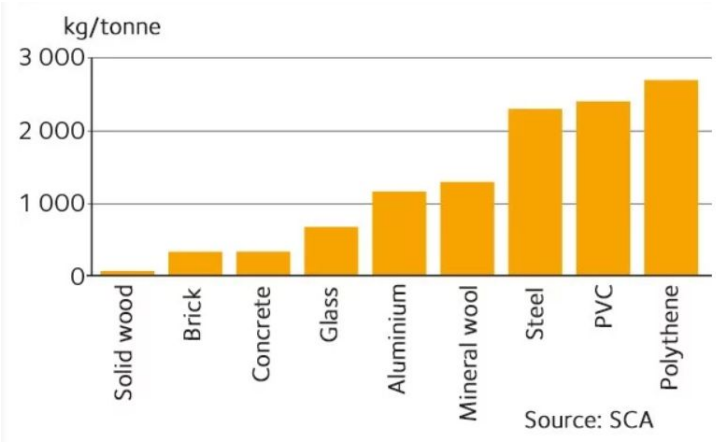


Figure 2.2 Material emissions (*Swedish wood, 2018)*

2.2.3. Timber as building material

Timber is favourable as a substitute to other building material in order to decrease the carbon emissions. This is due to the environmental benefits, combined with the fact that it can replace other building material yet provide the structure the same function and service life.
Another eco friendly aspect of using timber is the opportunity for reuse and recycle of the timber products after the end of its intentional service life. Products of timber may be used with the same purpose in another building if they still fulfill the requirements on the element otherwise the material can be recycled as fuel for energy generation. (*Swedish wood, 2018)*

2.3. Structural systems

The structural system, shape and material properties, are fundamental to efficiently resist vertical and horizontal load. The taller and more slender a structure becomes, the more important does these structural factors gets. With a more complex structure where the building consists of non-regularities in its shape, it is necessary to adjust the structural system to fit the structural demands. It may be necessary to combine different structures to a hybrid structures to fulfill these demands on the building.(*Smith & Coull, 1991*)

2.3.1. Shear wall structure

Shear walls are rigid surface elements that resist forces well in its own plane. (*Swedish wood, 2015*) The action on these kind of structures due to horizontal wind load can be illustrated in *Figure 2.3*.

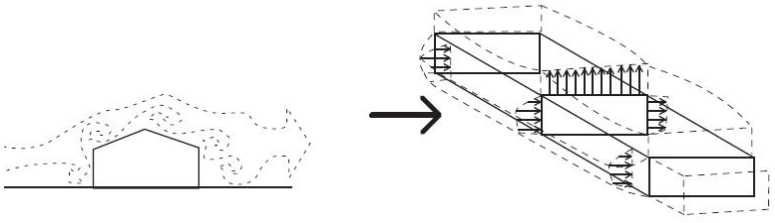


Figure 2.3 Load transfer in shear walls (*Swedish wood, 2015)*

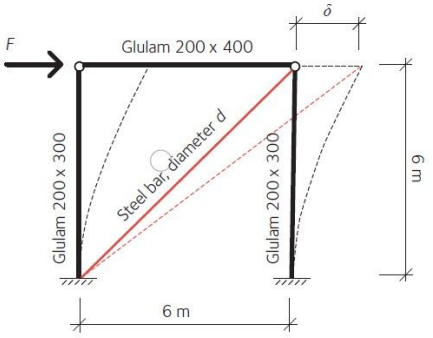
Continuous vertical walls have usually high capacity to carry both gravity and lateral loading and can be seen as a vertical cantilever. To increase the horizontal stiffness, wall elements can be coupled with rigid connections to a composite unit which enables the whole wall to work as one shear wall along the height. A drawback of continuous walls is restricted planning of spaces, this structure is suitable for e.g. residential buildings with permanent and repeating planning on each floor. (*Smith & Coull, 1991*)

2.3.2. Braced frame structure

Beam and column-structures need a bracing-system due to the difficulties to have moment resisting connection especially in timber structures. Bracing systems is efficient to resist lateral load with members with high utilization ratio, acting in fully tension. The lateral restraint is provided by a diagonal member that form a "web" in a vertical truss. This diagonal member acts in tension or compression instead of a web of a beam that transfer shear. A architectural disadvantage with bracing systems is that it can interfere with e.g. windows. (*Smith & Coull, 1991)*

Due to timber's orthotropic behavior with much higher strength in the direction parallel to the grain than transverse to it, it is suited to use in linear structural elements such as trusses. In these types of elements, the material only takes compression and tension in the direction of the grain and by that best utilize the materials benefits. (*Ramage et al., 2017)*

An example of the effect on a structures horizontal stiffness of a bracing truss (in this case, a diagonal steel rod) is presented in *Figure 2.4*. The glulam frame is statically stable without the rod due to the fixed supports of the columns, the horizontal stiffness is however radically increased when the steel rod is added. This can be seen in the table of *Figure 2.4* where the stiffness is expressed for different dimensions of the rod. (*Swedish wood, 2015*)



Diameter	Horizontal stiffness $k = F/\delta$
No diagonal	$k = k_0$
d = 10 mm	$k \approx 7 \cdot k_0$
d = 20 mm	$k \approx 25 \cdot k_0$
d = 30 mm	$k \approx 50 \cdot k_0$

Figure 2.4 Effect of diagonal rod (*Swedish wood, 2015)*

2.3.3. Core structures

This type of structure consists of a central core (or multiple cores), that may be used as elevator or stair shafts. The structural system with a centric core have architectural benefits that allows an open plan and possibilities of having large window areas to inlet natural light. Using a concrete core increases the mass and stiffness, compared to a timber core. A drawback of having only a core with a small effective structural depth is the reduced resistance to lateral load. (*Smith & Coull, 1991*)

3. Material

3.1. Timber

Timber differs in many ways as building material compared to the other traditional building materials such as concrete and steel. Wood is an organic material with varying properties, both due to its orthotropic behaviour and its significant variation in production. Due to the fact that timber is an biological material, the properties of the material cannot be controlled in the same manner as for manmade materials. The grading of timber is based on statistical relationships due the large variation in the raw material. Each log comes from an individual three which have grown over a long period of time with trees growing in different silvicultural environments. The fact that timber is highly hygroscopic influences the behaviour and the implementation of the material further. (*Swedish wood, 2015*)

3.1.1. Moisture and shrinkage

Timber is a hygroscopic and orthotropic material which have different deformation properties in different environments and fiber directions. The strength (in all directions) reduces with increased moisture ratio until the saturation point. The saturation point is when no free water occurs in the cell cavities and the fibers are fully saturated (100% bound water). This saturation point is at approximately 27% moisture content.
Shrinkage is a moisture conditioned effect, the largest effect is in tangential direction and smallest in the fiber direction. Shrinkage starts from the saturation point to fully dry (Burström, 2007). It is important to avoid lager cracks due to dehydration, that affects the load bearing capacity. It is therefore important to avoid both a material with a high moisture content, and a rapid dehydration of the material. In the design phase of a timber structure, it is important to have similar deformation properties in the timber as the composed material. This in order to avoid initial stresses especially when the structure is combined with concrete or steel. (*TräGuiden, 2003*)

3.2. EWP (Engineered wood products)

Sawn timber have a natural limitation on the dimension of its cross section and length due to the size of the log as well as the manufacturing process. According to Svenskt Trä, the maximum depth and length is 245mm and 5,5m respectively. In order to obtain larger dimensions, so called engineered wood products (EWP) can be created, where the final timber product consists of sawn timber boards, veneers, fibres or particles boards and are composite with adhesives. Most of these EWPs such as beams and panels were invented in North America during the 20th century. *Figure 3.1* shows a timeline of the invention of some of the EWPs. The composition of the different materials offer a product which can be manufactured with small diameter trees and logs of lower quality yet still have good structural properties. This is of interest both in the economical view as well for sustainability. (*Swedish wood ,2015*)

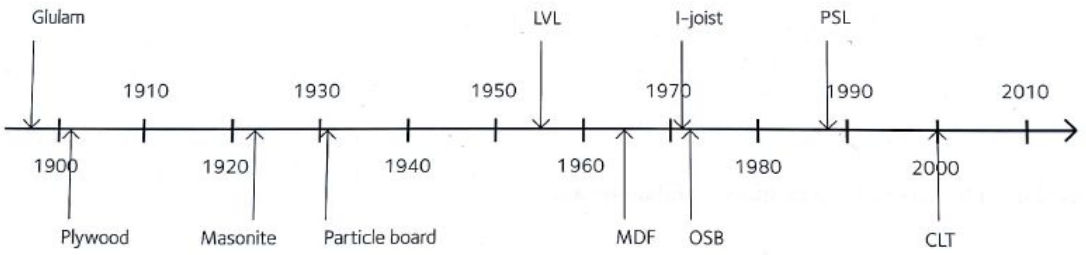


Figure 3.1 Invention of different EWPs (Swedish wood ,2015)

The probability density function of the strength of sawn timber compared to glulam is presented in *Figure 3.2*.

The strength in sawn timber is limited by irregularities e.g. twigs in its cross-section. EWP in the other hand can more easily be quality controlled and the combination with larger sections (Weibull-effect) increases the strength and decreases the spread compared to sawn timber. (*TräGuiden, 2017*)

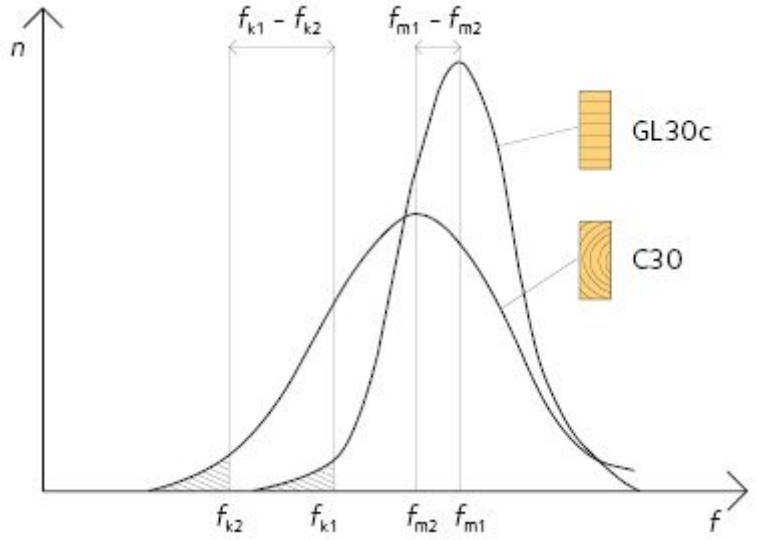


Figure 3.2: *Probability curve of sawn timber (C30) and EWP (GL30c), strength (f) and spread (f_1 - f_2) of tested products (n) (TräGuiden, 2017)*

3.2.1. CLT (Cross-laminated timber)

CLT is an EWP that is manufactured of sawn timber lamina which is glued together in layers and then assembled perpendicular to each other, see *Figure 3.3*. This allows the thickness of the material to be as thick as 500 mm, depending on the number of layers. The procedure of manufacturing CLT also permits sizes of elements as large as 3x24m, this allows CLT to be used as load bearing vertical walls as well as floor diaphragms. *(Swedish wood, 2015*)

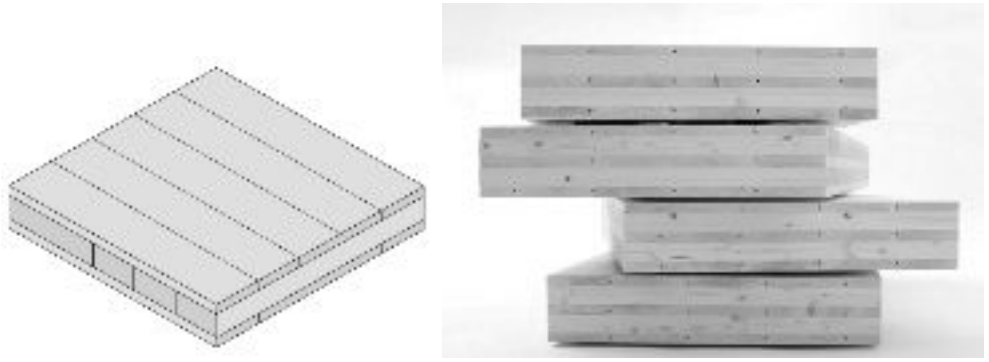


Figure 3.3 CLT with multiple layers glued perpendicular to each other (Swedish wood ,2015) (*Martinsson, 2018)*

The panels of CLT comes to the working site ready to assemble with installations and perforations finished from the factory, see *Figure 3.4*. Due to CLTs broad applicability as well as the ease on the production site, CLT have grown in the timber industry during the last two decades and is of interest as an material alternative to concrete. *(Swedish wood, 2015*)



Figure 3.4 Assembling of CLT-elements, a wall diaphragm with prepared window opening (*Alter, 2018*)

4. Dynamics

A dynamic load is one whose magnitude, direction or point of application varies with time and needs to be considered, i.e. the load cannot be simplified to a static load without severe approximation error. The resulting time-varying displacement and stresses constitute the dynamic response. If the load is known the load is prescribed and the analysis is called deterministic analysis. If the time history of the loading is unknown or only known in statically sense, the loading is said to be random. Of equal importance as the time varying load in a structural dynamics problem is acceleration. This acceleration give rise to a inertia force, if this have a significant contribution to the deflection, a dynamic investigation is required. This dynamic investigation can be described in a flow-chart (see *Figure 4.1*) where the main steps are: *design, analysis* and *testing.* (*Craig, 2006*)

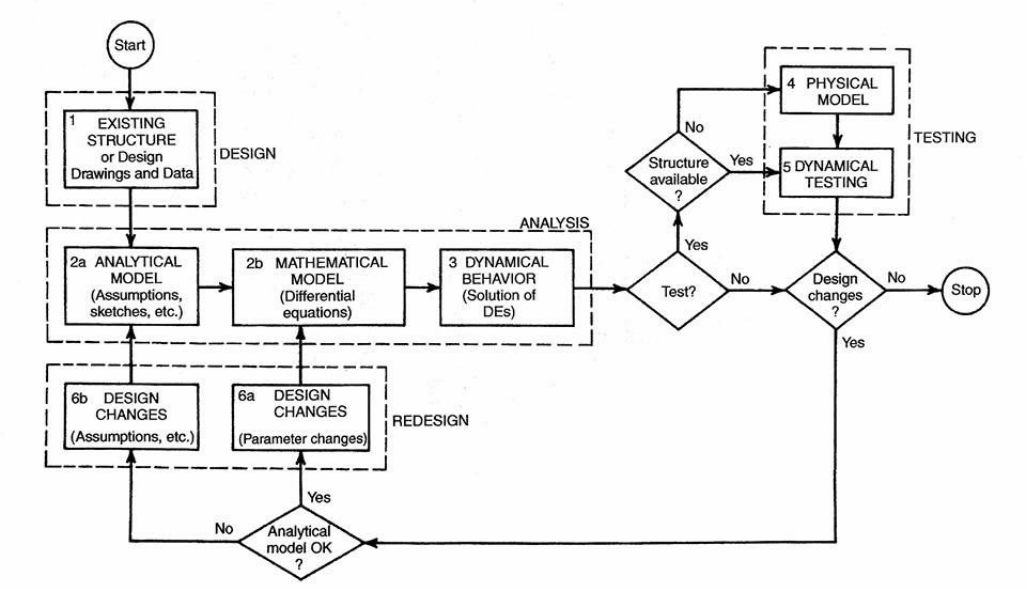


Figure 4.1 Steps in a dynamical investigation (Craig, 2006)

This *Analytical model* consist of:

1. A list of the simplifying assumptions made in reducing the real system to the analytical model
2. Drawings that depict the the analytical model
3. A list of the design parameters (i.e., size, materials, etc,)

To create an analytical part three fundamental equation must be fulfilled:

1. Newton's laws (or equivalent energy principles) must be satisfied
2. The force deformation behaviour elastic elements and force velocity for damping elements must be characterized
3. The kinematics of deformation must be incorporated

4.1. Eigenfrequency

A frequency is a oscillation per time (cycle/s) with the unit [Hz] where the time for one cycle is a period, with the unit [s] (*Craig, 2006*). The eigenfrequency describes the specific frequency at which the system oscillates when released from an offset. *Figure 4.2* describes how a undamped system is offset in position 1, with the magnitude u_0 and released. The mass passes the equilibrium position, 2, ($u(T/4) = 0$) and continues to position 3 ($u(T/2) = -u_0$) where it turns and return to its initial position, i.e $u(T) = u_0$.

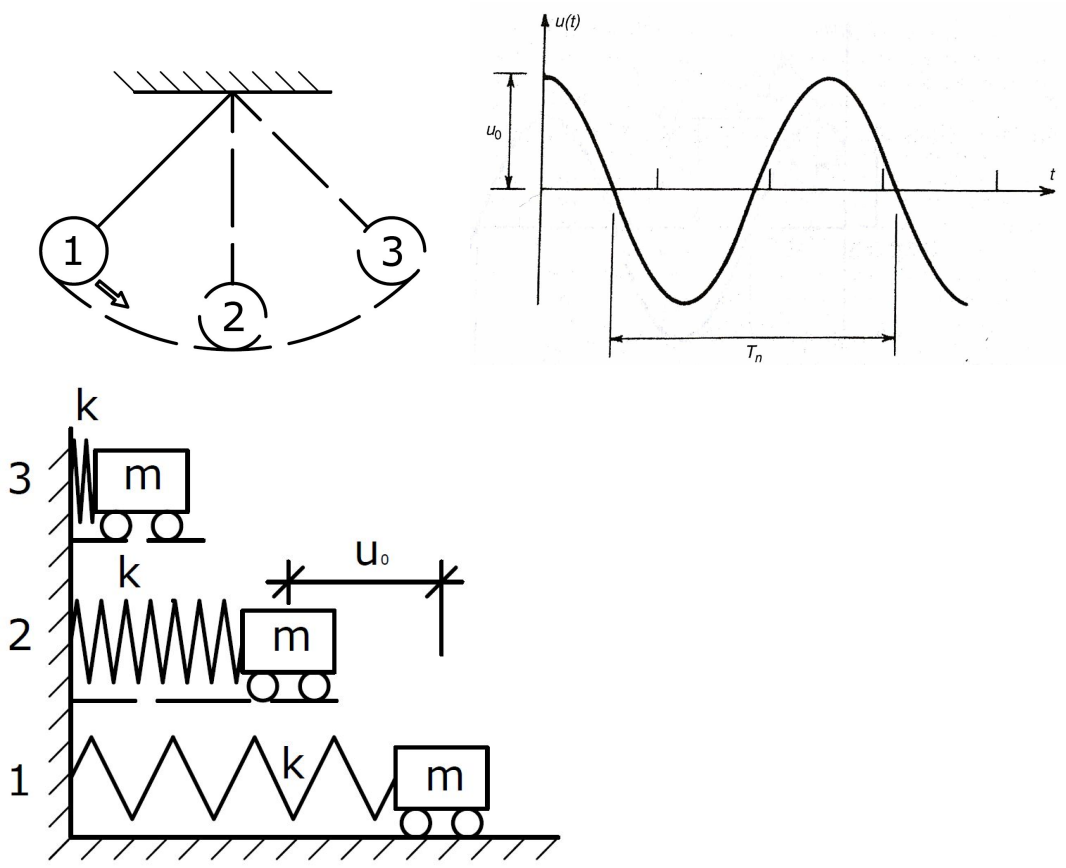


Figure 4.2 Model of two undamped SDOF-system (pendulum and mass-spring)

The eigenfrequency can be determined from the basic example in in *Figure 4.2* by the expression for an undamped SDOF-system (single degree of freedom):

$$f_n = \frac{\omega_n}{2\pi} \quad \text{(Eq.1)}$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(Eq.2)}$$

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} \quad \text{(Eq.3)}$$

where:

f_n	is the eigenfrequency [Hz]
ω_n	is the eigenfrequency [rad/s]
T_n	is the natural period [s]
k	is stiffness [N/m]
m	is mass of element [kg]

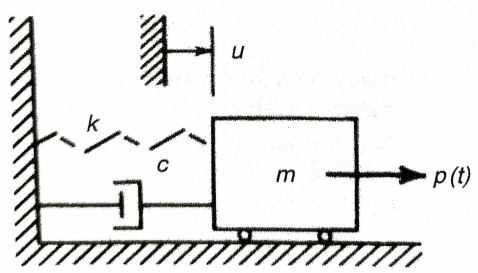


Figure 4.3 A SDOF-system that include the foundations of dynamics (Craig, 2006)

A damped SDOF-system with a forced motion as can be seen in *Figure 4.3* can be expressed by the following equations, see chapter 4.6 *Damping* for further information about damping:

$$\zeta = \frac{c}{2\sqrt{k \cdot m}} \quad \text{(Eq.4)}$$

where:

ζ	is the relative viscous damping factor [%]
c	is damping coefficient [Ns/m]
$2\sqrt{k \cdot m}$	is the critical damping factor

The *second-order differential equation of motion* can be written as:

$$m\ddot{u} + c\dot{u} + ku = p(t) \qquad \text{(Eq.5)}$$

By combining (Eq.4) and (Eq.5), the equation of motion can be reformulated into (Eq.6):

$$m\ddot{u} + 2\zeta\sqrt{km}\,\dot{u} + ku = p(t) \qquad \text{(Eq.6)}$$

where:

$\ddot{u}$	is acceleration [m/s^2]
$\dot{u}$	is velocity [m/s]
u	is displacement [m]
$p(t)$	is a time dependent forcing function

For sinusoidal loads, a general solution is to assume (Eq.7) :

$$u(t) = \overline{U}e^{\bar{s}t} \qquad \text{(Eq.7)}$$

where:

$\bar{}$	indicate complex numbers
$\overline{U}$	is a complex number of the amplitude
$\bar{s}$	is $\pm i\omega_n$, $i = \sqrt{-1}$

The equation of motion can now be rewritten to (Eq.8):

$$(-\omega^2 m + (i\omega)2\xi\sqrt{km} + k) \cdot \overline{C}e^{i\omega t} = \overline{p}e^{i\omega t} \qquad \text{(Eq.8)}$$

Simplified to:

$$(-\omega^2 m + (i\omega)2\xi\sqrt{km} + k) \cdot \overline{C} = \overline{p}$$

(*Craig, 2006*)

4.2. Acceleration

Acceleration, $\ddot{u}(t)$ is determined by the second derivative of the displacement, $u(t)$. The acceleration is therefore directly proportional to the displacement and always directed towards the equilibrium position, see *Figure 4.4*.

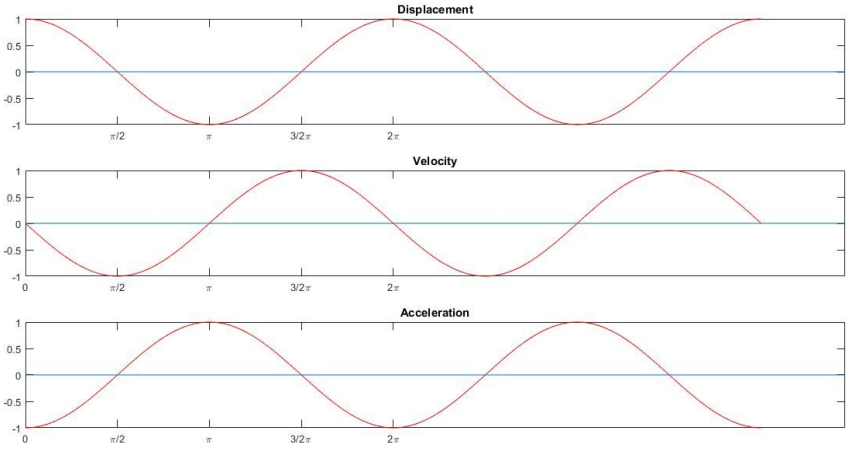


Figure 4.4 Two periods of oscillation, describing the displacement, velocity and acceleration versus time

To obtain the the mathematical terms of the mass-spring system in *Figure 4.2*, Newton's Second Law can be applied according to (Eq.9). (*Craig, 2006*)

$$\Sigma F = m\ddot{u} \tag{Eq.9}$$

where:

ΣF is the total force acting on the undamped system and thus

$$\Sigma F = p(t) - ku \tag{Eq.10}$$

Combining (Eq.9) and (Eq.10) results in:

$$m\ddot{u} + ku = p(t) \tag{Eq.11}$$

In order to maintain dynamic equilibrium in the system when reducing the mass, the acceleration must increase, according to (Eq.9).

4.3. Resonance

In a case where a system is exitated by a forced frequency , Ω, e.g. a periodic wind force acting at that frequency, it is very important to avoid the same eigenfrequency of the structure, ω_n. If the forced motion frequency and eigenfrequency is the same, ($r = 1$) from (Eq.12), the amplitude of the response will continue to grow without bounds. In other words it will lead to a collapse of the structure (*Craig, 2006*). *Figure 4.5* describes how the amplitude increases due to resonance during the first 3 periods.

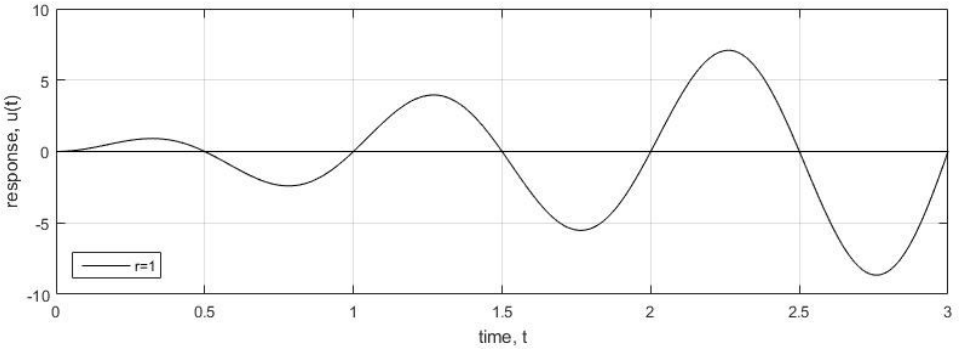


Figure 4.5 Amplitude increase due to resonance, r = 1

Figure 4.6 and (Eq.13) describes how the dynamic magnification factor, D_s, and the response amplitude , U, tend to infinity when the frequency ratio r in (Eq.12) for an undamped system tend towards the value 1. (*Craig, 2006*)

$$r = \frac{\Omega}{\omega_n} \tag{Eq.12}$$

$$D_s(r) \equiv \left|\frac{U}{U_0}\right| = \left|\frac{1}{1-r^2}\right| \tag{Eq.13}$$

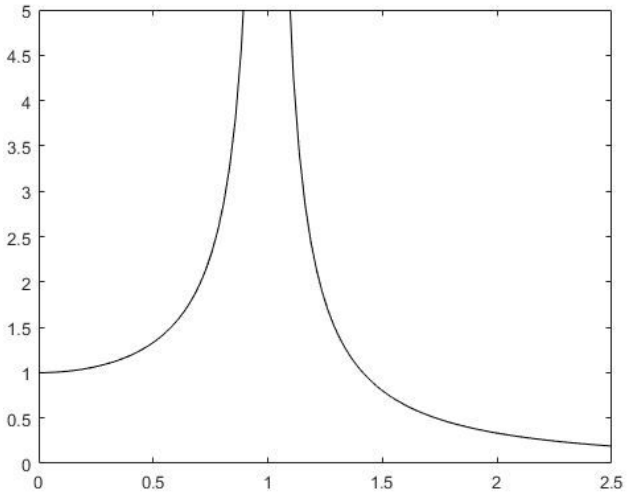


Figure 4.6 Dynamic magnification factor D versus frequency ratio r.

4.4. MDOF

The previous described theory of dynamics is based on a SDOF-system, that have one eigenfrequency. A structural system is a so called MDOF-systems (multi degree of freedom), a system with n DOFs that have n numbers of eigenfrequencies. A vibrating system with a natural frequency also have a natural mode or mode shape which describes how the system respond, i.e. an eigenvector (in mathematical terms), see *Figure 4.9*. Each eigenfrequency has its own mode shape, that governs how the system moves with that specific frequency. It is unnecessary to compute every eigenfrequency and mode shape for a MDOF systems with thousand or millions of degree of freedom. It may be enough to only compute the first and fundamental (or the first few) eigenfrequency of the system (*Craig, 2006*). A vibrating model can be described as a SDOF or a MDOF, they are both based on the same formula but a MDOF system is described in matrices and vectors instead of a scalar equation. *Figure 4.7* illustrate a 4 story building or a MDOF-system with 4 degrees of freedom, this result in 4 eigenfrequencies. The corresponding equation of motion is:

$$M\ddot{u} + C\dot{u} + Ku = p(t) \qquad \text{(Eq.14)}$$

Where M,C and K are the system's mass, damping and stiffness matrices.

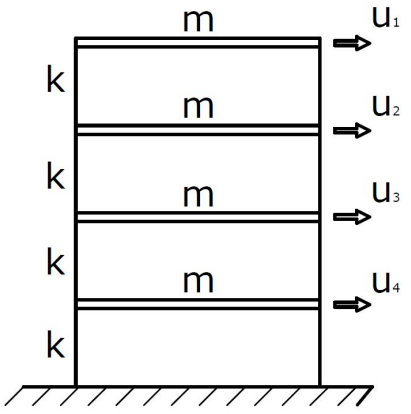


Figure 4.7 A multi story building, represented by an undamped MDOF system

4.5. Mass

Mass have a significant role in dynamics and highly affects the eigenfrequency. Parameters to modify in order to increase mass is density, cross section area and volume of elements, which is described for a beam and a rod in (Eq.15). *Figure 4.8* describe how a 2-DOF system oscillates as a flexible pendulum with a mode shape for the uniform beam and top mass (*M*). (Eq.15) and (Eq.16) defines the rod and beam element parameters for mass and stiffness along the element and for the top mass. *Figure 4.8* can be seen as a local element in a structure or a condensated building with a heavy top floor. (*Craig, 2006*)

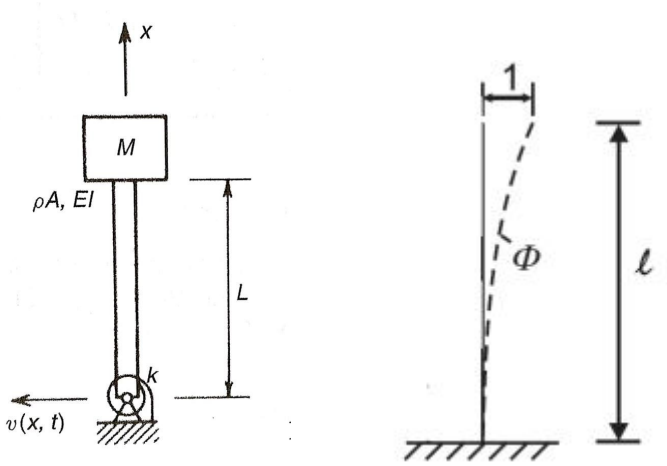


Figure 4.8 Rotation of a 2-DOF system with its mode shape (Craig, 2006)

For rod elements:	For beam elements:	
$m_u = \int_0^L \rho A(\psi_u)^2 dx$	$m_v = \int_0^L \rho A(\psi_v)^2 dx + M$	(Eq.15)
$k_u = \int_0^L EA(\psi_u'')^2 dx$	$k_v = \int_0^L EI(\psi_v'')^2 dx$	(Eq.16)

where on the element:

A	is cross area [m^2]
ρ	is density [kg/m^3]
E	is Young's modulus [N/m^2]
I	is moment of inertia [m^4]
L	is the element length [m]
ψ	is an assumed mode shape, see (Eq.17)
ψ''	is the second derivative of the mode shape
M	is a mass [kg]

The mode shape describes the form/shape of the oscillating system and have significant importance to the calculations of m and k . The mode shape factorize the parameters depending on the placement along the element ($\psi = 0$ while $x = 0$ and $\psi = 1$ while $x = L$), a mass M in the top have largest impact where $\psi = 1$.

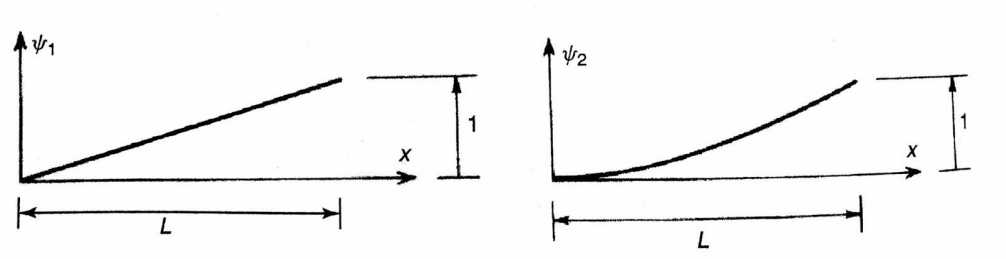


Figure 4.9 Mode shape 1 and 2 of an element with length L (Craig, 2006)

$$\psi_1 = \frac{x}{L} \qquad \psi_2 = \left(\frac{x}{L}\right)^2 \qquad \text{(Eq.17)}$$

4.5.1. Equivalent mass

A building has a distribution of mass, e.g. a heavy floor on a specific elevation. In the first bending mode shape (see *Figure 4.10*) of a structure, a mass which is placed higher up in the structure do have larger impact on the equivalent mass (*Craig, 2006*). The equivalent mass, m_e of the fundamental mode is given by the expression :

$$m_e = \frac{\int_0^l m(s)\cdot\Phi_1^2(s)ds}{\int_0^l \Phi_1^2(s)ds} \qquad \text{(Eq.18)}$$

(*SS-EN 1991-1-4, 2005*)

where:

$m(s)$	is the mass per unit length
Φ_1	is the mode shape of the first mode
l	is the height of the structure

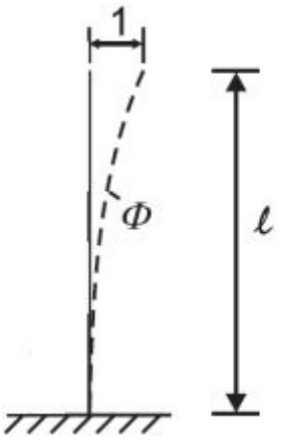


Figure 4.10 Modeshape

The mode shape Φ for a slender timber structure can according to *EN 1991-1-4* be assumed to be:

$$\Phi(x)_1 = (\frac{x}{l})^{1.5} \quad \text{(Eq.19)}$$

According to *EN 1991-1-4* for cantilever buildings with varying mass distribution e.g. flooring, m_e may also (in addition to (Eq.18)) be approximated by the average value of *m* over the upper third of the structure (SS-*EN 1991-1-4, 2005*).

4.6. Damping

If a system is totally undamped and the system is set into a free vibrated motion, the system should continue to vibrate forever. In reality, all systems have some damping, energy dissipate from the vibrating structure e.g. to heat due to friction. The exact natural damping in a structure is usually unrealistically hard to determine before the building is constructed. However, there are many suggestions to determine the damping e.g. according to *EN 1991-1-4*. The magnitude of the damping is characterized by a damping factor, ζ, (Eq.20) that is an dimensionless ratio to the critical damping. (*Craig, 2006)*

$$\zeta = \frac{c}{c_{cr}} \quad \text{(Eq.20)}$$

where:

c is damping coefficient

c_{cr} is critical damping coefficient

$$c_{cr} = 2m\omega_n = 2\sqrt{km} \quad \text{(Eq.21)}$$

Linear viscous damping is the simplest form to handle damping, where the damping force (f_D) is directly proportional to the velocity ($\dot{u}$), (Eq.22) (*Craig, 2006*).

$$f_D = -c\dot{u} \quad \text{(Eq.22)}$$

There are three cases of viscous-damping:

Underdamped ($\zeta < 1$)

This is the most important case, the free vibration slowly decreases in a harmonic motion.

Critically damped ($\zeta = 1$)

Only one solution, the vibration disappear instantly with no oscillations.

Overdamped ($\zeta > 1$)

Similar to critical damping with no oscillations, faster decay.

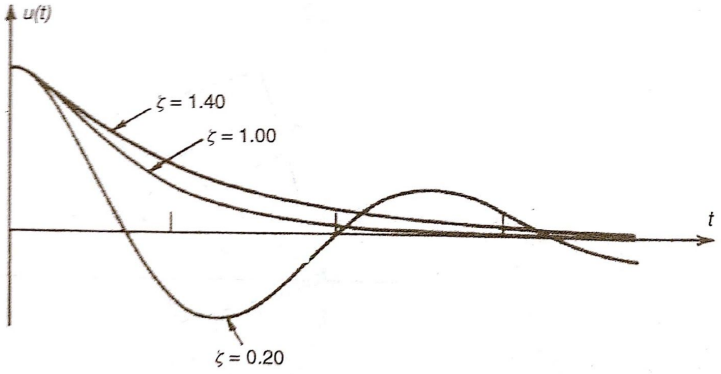


Figure 4.11 System response of different viscous damping (Craig, 2006)

A method to determine the damping factor is by the logarithmic decrement of the damping i.e. how much the amplitude have decreased for one period according to (Eq.23). *Figure 4.12* describes how the amplitude decreases from $u(p)$ to $u(q)$ for one cycle due to damping in a system. This is meaningful for underdamped systems where small damping ($\zeta < 20\%$) can be approximated according to (Eq.24). (*Craig, 2006*)

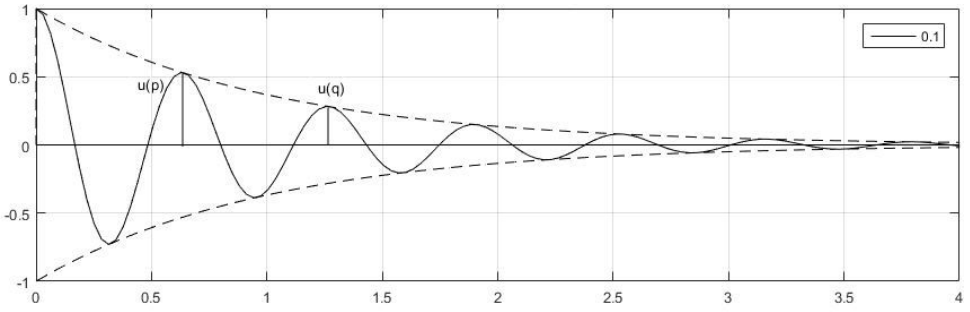


Figure 4.12Amplitude reduction due to damping

$$\delta = ln\frac{u(p)}{u(q)} \quad \text{(Eq.23}$$

$$\delta \approx 2\pi \cdot \zeta \quad \text{(Eq.24)}$$

Figure 4.13 describes how the amplitude decreases during the first 3 seconds of different viscously underdamped systems, where 0.4 fade first followed by 0.2 and 0.1, the undamped system (0.0) continues infinitely

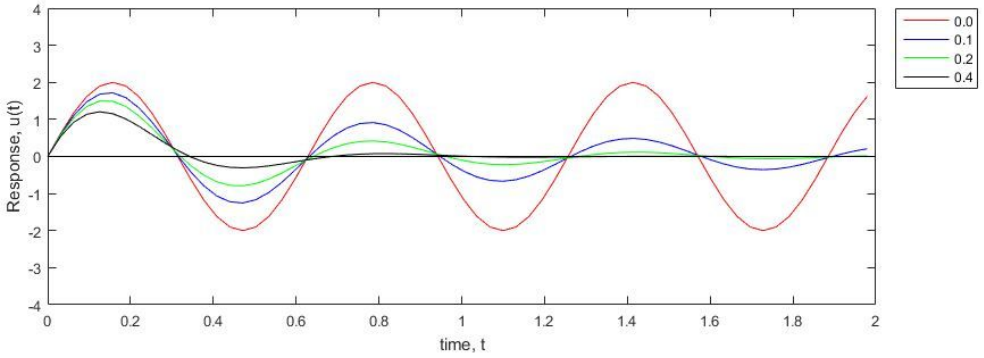


Figure 4.13 Amplitude reduction due to introduction of damping in a free vibrating system

The resonance response or magnification factor, *D,* due to the frequency ratio, r, is described in *Figure 4.14* and (Eq.25) with the effect of introducing a viscous damper to a system. This so called *frequency response* is one of the most important factors of dynamics in order to avoid resonance with large amplitudes (*Craig, 2006*). A frequency ratio with reduced magnification factor, result in a reduced amplitude and acceleration see *Figure 4.13*.

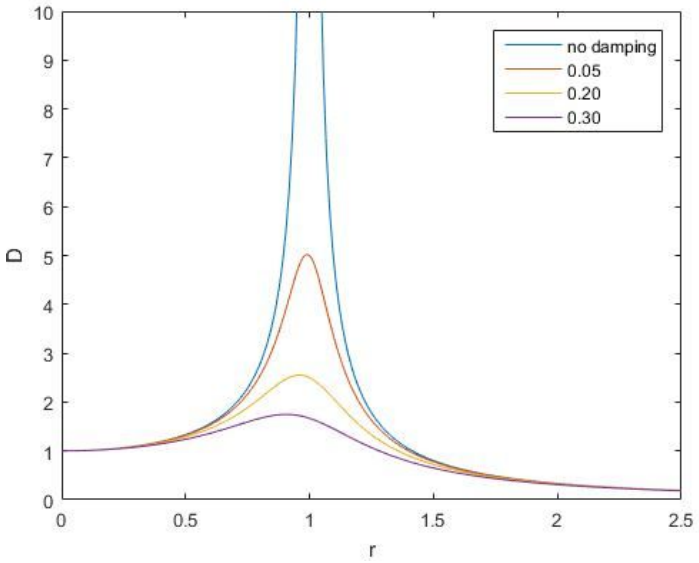


Figure 4.14 Magnification factors - frequency ratio with different damping factors

$$D_s(r) = \frac{1}{[(1-r^2)^2+(2\zeta r)^2]^{1/2}} \qquad \text{(Eq.25)}$$

4.6.1. Damping according to SS-EN 1991-1-4

The total logarithmic decrement of damping (δ) for a fundamental bending mode is estimated according to *EN 1991-1-4* :

$$\delta = \delta_s + \delta_a + \delta_d \quad \text{(Eq.26)}$$

where:

- δ_s is the structural damping
- δ_a is the aerodynamic damping for the fundamental mode
- δ_d is the damping due to special devices, e.g. TMD

The structural damping coefficient for timber varies and have according to *EN 1991-1-4* no typical value. According to a swedish National Annex Boverkets manual a recommended damping factor (ζ) for timber structures is 1.0-1.5%, depending on if the structure are without or with mechanical fasteners. By using (Eq.24), the logarithmic decrement for structural damping is 0.06 - 0.09. In this project it has been assumed that δ_s= 0.09 due to mechanical fasteners.

The aerodynamic damping is estimated by the expression according to *EN 1991-1-4*:

$$\delta_a = \frac{c_f \cdot \rho \cdot b \cdot v_m(z_s)}{2 \cdot n_1 \cdot m_e} \quad \text{(Eq.27)}$$

where:

- c_f is the force coefficient for wind action, pressure coefficient $c_{pe,10}$ is used
- ρ is air density, 1.25 [kg/m^3]
- v_m is reference speed of mean wind velocity [m/s]
- n_1 is fundamental eigenfrequency [Hz]
- m_e is equivalent mass [kg/m]
- b is the width of the building

(*SS-EN 1991-1-4, 2005*)

4.6.2. Equivalent Damping

The definition of damping is the removal of energy from a oscillating system. The energy reduction result in an amplitude decay (for free vibrations), and the oscillation finally disappear. The damping described above has been based on linear viscous damping where the damping force is proportional to the velocity. Damping in real structures are more complex and there are several other mechanisms that removes energy and act as dampers. It may therefore be more appropriate to define similar peak response of the frequency response function as an equivalent damping, that is in proportion to the same frequency response of a system with linear viscous damping.

The frequency response in *Figure 4.15* for a linear viscous damped system and a *structural damped* system expressed in equivalent damping have the same max-amplitude ,"*". When theses amplitudes match, it is possible to express the equivalent damping factor of the complex mechanism equal to the known damping factor of linear viscous damping. (*Craig, 2006*)

Figure 4.15 show how the pendulum (local system) counteract the buildings (global) movement with the same frequency but in opposite direction. The corresponding frequency response plot describes how an added vibration absorber reduce the magnification factor to two small peaks on each side of resonance. The two peaks is the first and second eigenfrequency of the 2DOF-system. (*Craig, 2006*). The frequency positions of the peak have no major impact on the human perception. A human only perceive the amplitude motion, not if the frequency ratio, *r* varies from *1.0* to *0.9*. The structural damper in *Figure 4.15* is called a TMD (Tuned Mass Damper) that is an commercial damping system.

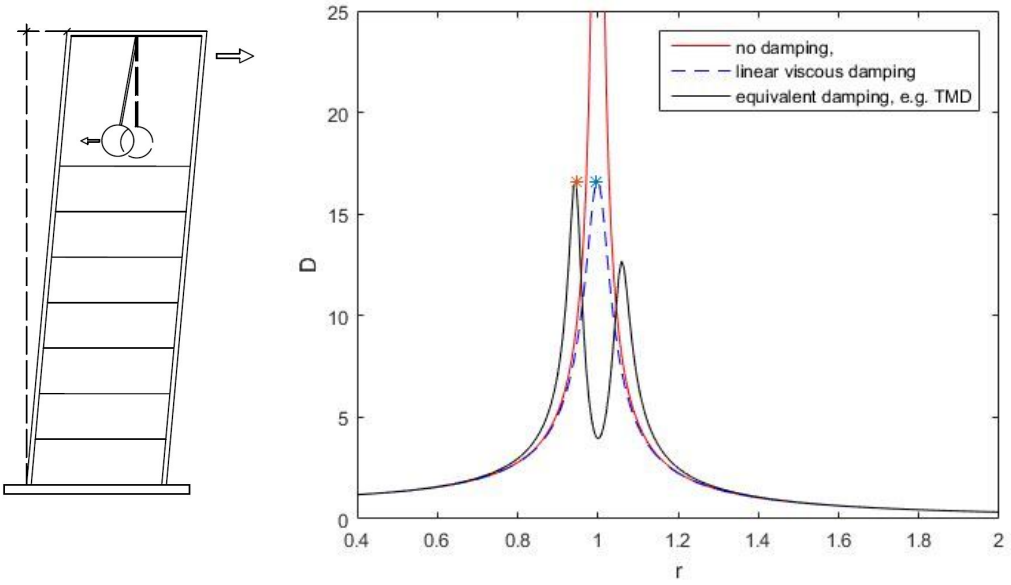


Figure 4.15 Structural damper, TMD, and the frequency response

4.6.3. Commercial damping systems

Additional damping is an efficient solution to minimize the problem of vibrations in tall buildings. The fact that damping characteristics of a structure is hard to estimate before the building is complete may result in a need for additional damping of external devices. These devices are easy to model to estimate the damping and is considered to be a significant part of reducing dynamic behaviour when designing tall buildings. Today, there are numerous damping systems available on the market and the first experiment on damping in buildings dates back to the 1950s. The first commercial installation was in the *World Trade Centre* in 1969. The method was thereafter installed in numerous building during the 1980s, both in existing building such as *John Hancock Tower* in Boston and in new constructions such as *Yokohama Landmark Tower* in Yokohama. (*Lago et al., 2018*)

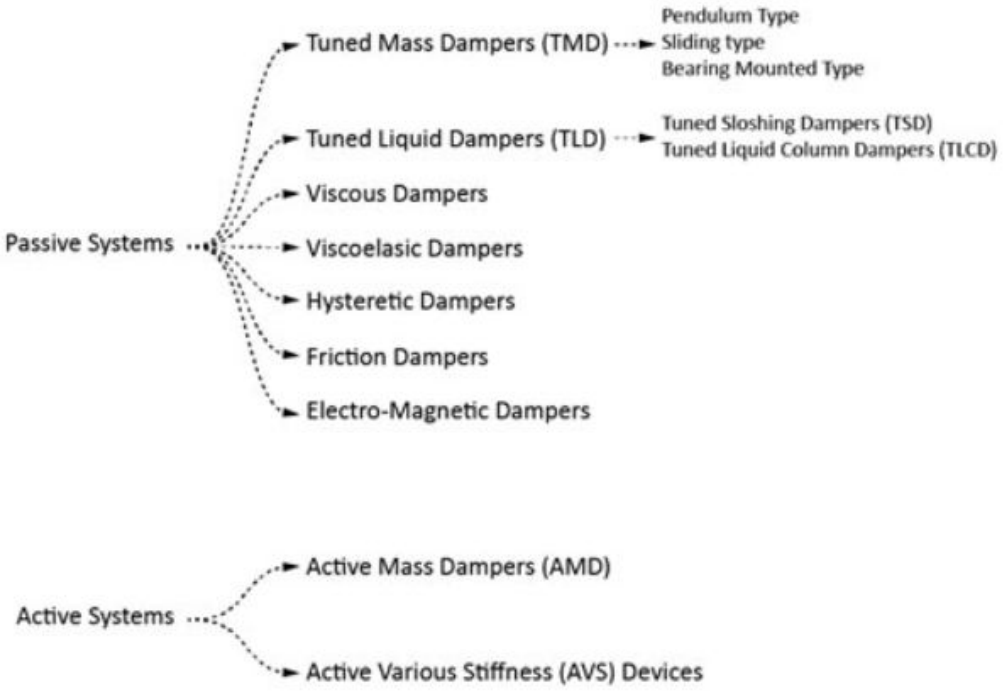


Figure 4.16 Damping systems (Lago et al., 2018)

Additive damping systems can be divided in two main categories,passive and active system as can be seen in *Figure 4.16* . The main difference between them is that the active systems change their properties based on load case and the passive system have constant properties. The more simple passive system is the most economical and reliable yet less effective system. This is used in a greater extent compared to the active. The additional damping provided by a passive system can be up to 3 to 4% while the active may provide additional damping that can be 10% or more. However the active system can reach a cost of 2% of the total building cost while the passive is below half that cost. Due to the extensive cost of an active damping system, it is not economically defendable in the kind of structures analysed in this report and will not be treated as an solution. (*Lago et al., 2018*)

The passive system can further be divided in subcategories as can be seen in *Figure 4.16*, the focus will be held upon the TMD in the this report. A TMD consists of a sprung mass which is allowed to move out of phase in horizontal or vertical direction with the fundamental period that matches that of the building, see *Figure 4.17*. (*Lago et al., 2018*)

4.6.4. Tuned Mass damper (TMD)

A TMD is a device that reduce the dynamic response of a structure and the amplitude of the oscillations for a specific frequency (*Flow engineering, 2018*). TMD systems for a horizontal oscillations can be divided into two subcategories: pendulum and spring coupled mass, see *Figure 4.17*. The pendulum is an effective design, due to the possibilities to adjust the length of the pendulum and tune the response after it is constructed. The disadvantage of a pendulum is the bulky design that occupies several floors, see *Figure 4.17*. The number of floors is too few in this case to use a pendulum, therefore a sliding spring coupled TMD is more suitable which only occupies a room on one floor. A sliding TMD acts in a similar as a pendulum but the mass is instead coupled to the structure via springs. (*Connor, 2014)*

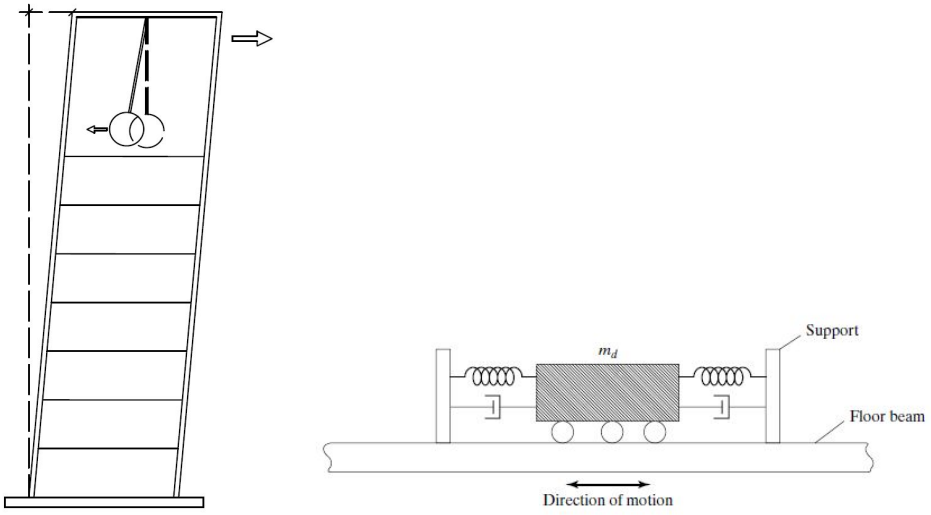


Figure 4.17 Schematic pendulum TMD and a sliding spring coupled TMD (*Connor et al., 2014)*

The eigenfrequency of the TDM is tuned to be similar to the fundamental eigenfrequency of the structure and the motion of the mass counteract the structure which is used to dampen out the vibrations for a specific mode. The couteraction of the TMD reduces the total vibration energy of the building compared to a viscous damper were energy dissipate from the system (*Connor, 2014)*. The amount of equivalent damping which the TMD add is according to the technical information from *Flow engineering* mostly dependant on:

- Location of the TMD in the structure: -The TMD should be located where the amplitude of the vibration is largest
- Mass ratio: -The ratio between the total mass of the structure and the added mass.
- Frequency ratio: - The ratio between the frequency of the structure and that of the added mass -spring system. (*Flow engineering, 2018*)

To determine the response of a TMD, following steps and equations are used. To establish a simplified analytical model, the structural mass is converted to an equivalent mass according to (Eq.28) that generate a condensated 2-DOF mass-spring system, see *Figure 4.18*.

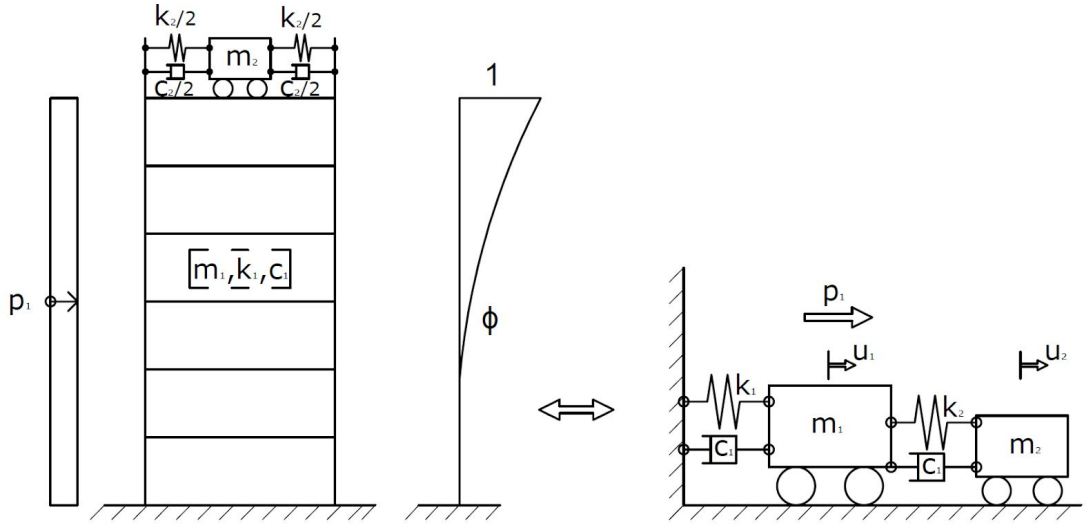


Figure 4.18 A damped structure is converted to a analytical model

$$m_1 = \int_0^l m \cdot \Phi^2 dx \qquad \text{(Eq.28)}$$

where:

m_1 is equivalent mass of the structure

m_2 is mass of damper

Φ is the mode shape, see (Eq.19)

The eigenfrequency of the structure and the damper is assumed to be equal:

$$\omega_n = \omega_d \qquad \text{(Eq.29)}$$

The stiffness of the springs can be calculated from a rewriting of (Eq.2) to (Eq.30).

$$k_{1,2} = \omega_{n,d}^2 \cdot m_{1,2} \qquad \text{(Eq.30)}$$

A desired damping factor and frequency response for linear viscous damping is prescribed to the SDOF-system in order to fulfill the acceleration requirements (lower single peak in *Figure 4.19*). The mass of the TMD is determined so that the maximum peak value of the frequency response of the TMD have the same as the linear viscous damped SDOF system (the levels of the double peak and single peak are equal, see *Figure 4.19*).

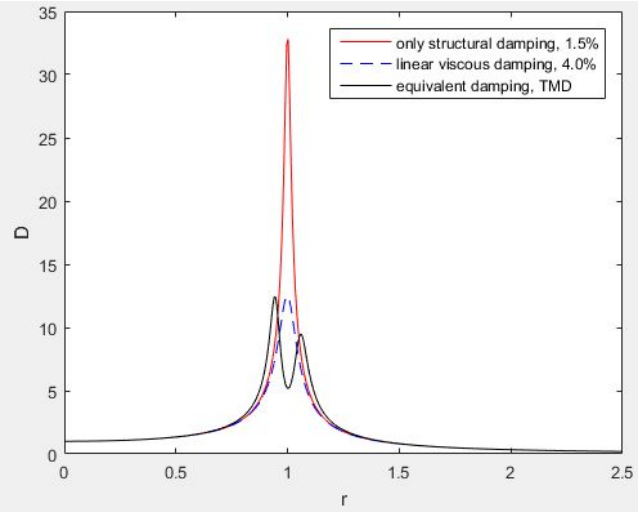


Figure 4.19 Reduced magnification factor due to a TMD

The amplitude response of the structure with a TMD (double peaks) is determined from the equation of motion for a 2DOF-system in (Eq.31). This equation is reformulated via the general solution to (Eq.32), where the amplitude is solved for a specific mass on the TMD. See Appendix C for calculated TMD used in this report.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ 0 \end{bmatrix} \quad \text{(Eq.31)}$$

Equation of motion in matrix form:

$$M\ddot{u}+C\dot{u}+Ku=p(t)$$

Assuming the general solution:

$$(K + Ci\omega - M\omega^2)\overline{U} = \overline{p} \quad \text{(Eq.32)}$$

Simplify equation:

$$K + Ci\omega - M\omega^2 = Z$$

Determine the complex amplitude:

$$\overline{U} = \left|Z^{-1}\right| \cdot \overline{p}$$

Determine the magnification factor, *D*, of the structure:

$$D = \frac{\overline{U}_1}{u_{st}} \quad \text{(Eq.33)}$$

$$u_{st} = \frac{p_1}{k_1} \quad \text{(Eq.34)}$$

Figure 4.20 describes how the TMD mass affect the vibration reduction. It is seen that a small damping increase may require a large mass increase of the TMD.

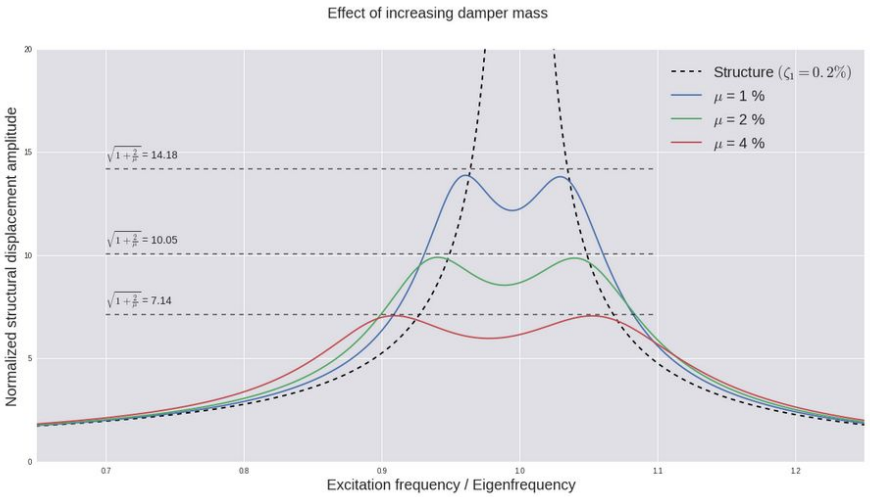


Figure 4.20 Effect of different mass of the TMD (*Flow engineering, 2018*)

Figure 4.21 describes how the choice of eigenfrequency in the TMD affect the magnification factor, it also shows that the amplitude is lowest when the two peaks are made equal. The amplitude response due to the TMD in *Figure 4.19* have a similar response as f=105% in *Figure 4.21*. It is possible to optimize the eigenfrequency of the TMD in order to equalize the two peaks which result in a decreased amplitude response i.e. larger equivalent damping factor.

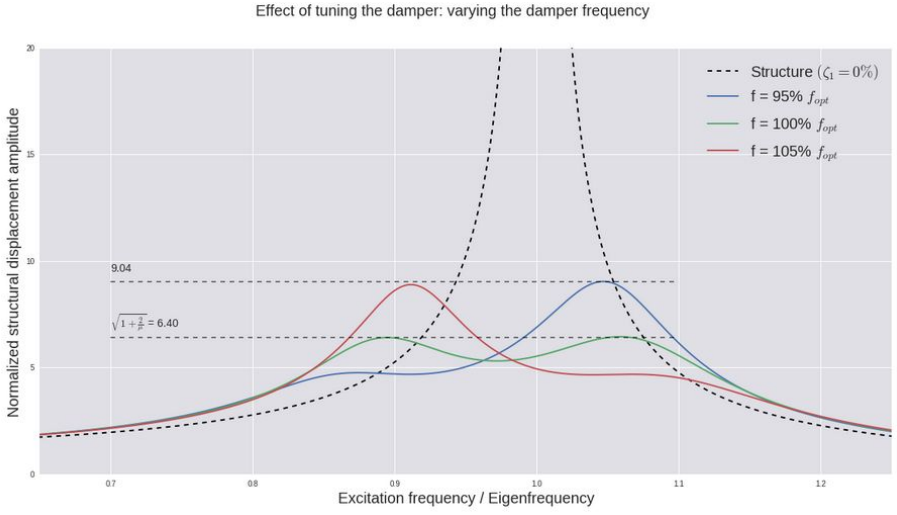


Figure 4.21 Optimizing of the TMD response (*Flow engineering, 2018*)

This choices and optimizations result in a general design of a TMD with larger impact at a lower added mass.

4.6.5. Self Mass Damper (SMD)is the simplest form to handle damping

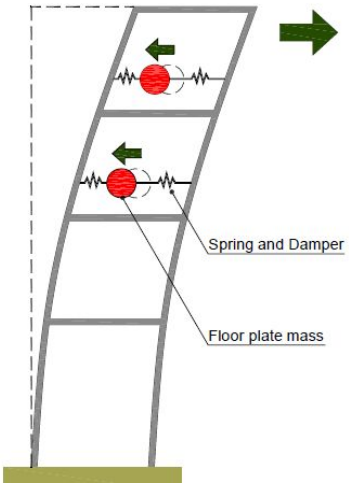


Figure 4.22 Principle sketch of the SMD-system (*Kidokoro, 2008)*

This is achieved by e.g. disconnecting the upper floors from the structure via rubber bearings. This principle is presented in the article *Self Mass Damper (SMD): Seismic Control System Inspired by the Pendulum Movement of an Antique Clock* by Ryota Kidokoro. The article is based on the Nicolas G. Hayek Center, Tokyo where the principle of making use of existing mass in form of concrete floors is applied. This generate the opportunity to deal with seismic loads without the need of additional mass in the structure. The SMD systems are located at floor 9,10,12,13 where parts of the concrete slab is disconnected from the structure to allow movements, see *Figure 4.23*. (*Kidokoro, 2008)*

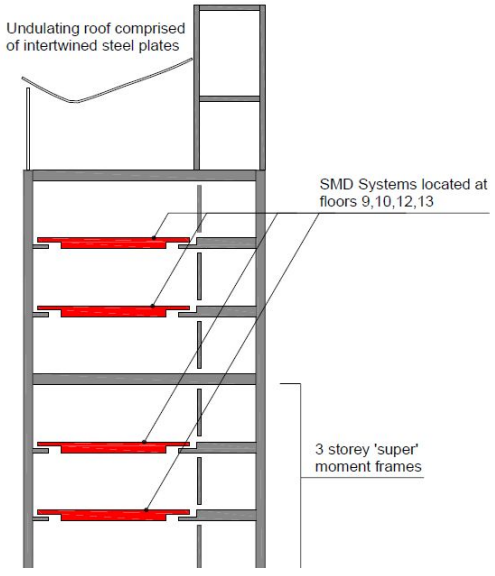


Figure 4.23 Position of SMD-floors (*Kidokoro, 2008)*

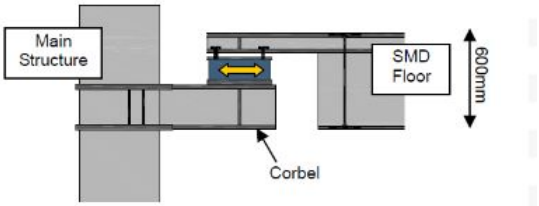


Figure 4.24 Principle details of the suspension of the SMD-floors (*Kidokoro, 2008)*

The disconnection from the main structure is conducted via a combination of sliders and rubber bearing with high damping, the principle of these details can be seen in *Figure 4.24*. Each connection is tuned via applied stiffness to maximize the increase the equivalent global damping.

Extensive quantities of mass needs to be mobilised in order to deal with large seismic forces in a effective way. The damper begins to have notable effect at about 5% of the total building mass and increases in effect with further increased mass. The acquired mass of the damper in Nicolas G. Hayek Center is approximately 10% of the total building mass, this is achieved by the 4 concrete floors with a weight of 100 tons each. This SMD system reduces the seismic forces in the structure with up to 37%. According to *Kidokoro* does the seismic response in a building require far more mass than the low energy response of wind. (*Kidokoro, 2008)*

5. Wind load

When designing low to medium rise buildings, wind load is generally treated as quasi-static loading. A quasi-static assumption can easily lead to underestimation of the load effect on tall buildings and does not take phenomena that affect the dynamic response into account such as acceleration, damping and structural stiffness, (*Mendis, 2007*).
A complete description of wind speed can be defined from meteorological records. Also atmospheric boundary layers, turbulence and variation of wind speed with height and aerodynamic forces contribute to the structural response. All this affects combined are practically infeasible to simulate as a time dependent wind load. (*Smith & Coull, 1991*)

5.1. Meteorological wind

Wind can be defined as movements of air relative to the surface of the earth, where the main driving forces are pressure differences and forces produced by the earth's rotation. There can also be local origins of wind such are local heating and terrain which can cause local wind effects, (Holmes, 2001). The wind itself consists of many different eddies of varying diameter and speed which travels along a main stream of air over the earth's surface. The many different flow situations of wind phenomena are very complex due to the interaction of the wind with other structures. The wind effect can be described as one static component, the mean wind, and one dynamic component which describes the turbulence of the wind. (*Mendis, 2007*)

5.2. Direction of wind load

Slender and tall buildings are more sensitive to dynamic effects due to wind load. There are various phenomena causing dynamic effects on structures due to wind, such as buffeting, galloping, flutter and vortex shedding. Where turbulence buffeting are related to dynamic response in the direction of the wind the transverse response is primary related to vortex shedding and galloping. Instability is primary associated with flutter which is a motion consisting of bending and torsion. The wind phenomena acting on a building is complex as well as the behavior of the flow pattern created around a building, which is affected by distortion of the mean flow, flow separation and formation of vortices and wakes. This arises substantial variation in the wind pressure acting on the surface of a building and consequently large localised dynamic loads acts on the structural system as well as the facade. As a result from this fact the building tends to vibrate in linear and torsional modes, see *Figure 5.1.* (*Mendis, 2007*)

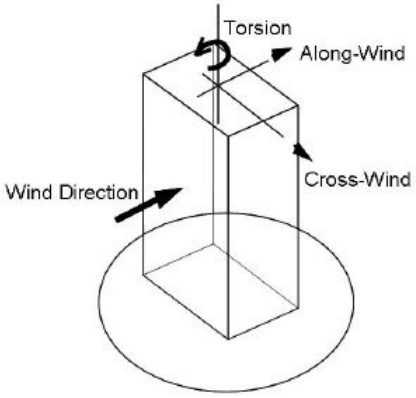


Figure 5.1 Wind direction (*Mendis, 2007*)

5.2.1. Along wind loading

As mentioned, the wind load on a structures can be divided in one static and one dynamic component, the same principle can be applied on along wind loading. The dynamic component is related to the irregular combination of gusts and eddies of different diameter. The natural frequency of structures are often higher than the frequency of the eddies with bigger diameter which entails that there are no dynamic effects of the larger eddies and they can be treated in the same fashion as the mean wind. On the other hand, smaller eddies occur more often and can conform with the structures natural frequency, causing an substantial dynamic load effect on the structure. (*Mendis, 2007*)

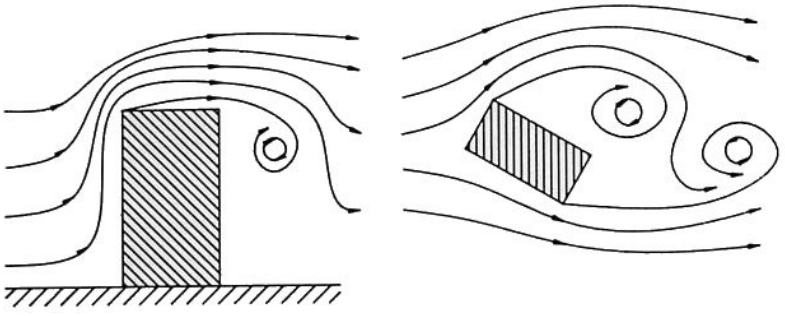


Figure 5.2 Formation of eddies (*Mendis, 2007*)

5.2.2. Cross wind loading

Slender structures with low structural damping are especially sensitive to dynamic motions perpendicular to the wind direction. Actions of crosswind on tall buildings can be divided into three main phenomena were vortex shedding is the most common action due to crosswind. Due to the fact that most tall buildings does not have streamlined design the structure causes the wind to separate from the facade of the structure. This causes effects such as the critical velocity effect where the structure gets exposed to periodic cross loading due to the shed vortices. Displacement with large magnitude can be the result if the frequency of the vortices correspond to the natural frequency of the structure. (Mendis, 2007)

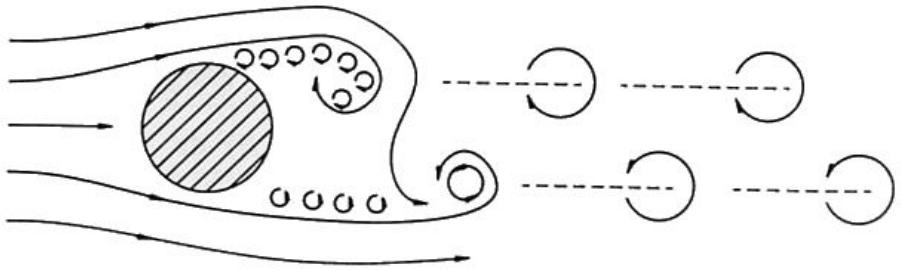


Figure 5.3 Formation of vortex of a bluff object (*Mendis, 2007*)

The other two main phenomena are "the incident turbulence mechanism" and higher derivatives of crosswind displacement such as galloping and flutter. Both phenomena are highly dependant on turbulence. (*Mendis, 2007*)

5.3. Wind load in standards

The wind action in EN 1991-1-4 is simplified as: " set of pressures or forces whose effects are equivalent to the extreme effects of the turbulent wind" and are classified as variable fixed actions. This is accomplished by calculating an equivalent static wind force. (*SS-EN 1991-1-4, 2005*)

The wind pressure acting on external surfaces, W_e :

$$W_e = q_p(z_e) \cdot C_{pe} \quad \text{(Eq.35)}$$

where:

$q_p(z_e)$	is the peak velocity pressure
z_e	is the reference height for the external pressure
C_{pe}	is the pressure coefficient for the external pressure

The wind force F_w acting on a structure:

$$F_w = c_s c_d \cdot c_f \cdot q_p(Z_e) \cdot A_{ref} \quad \text{(Eq.36)}$$

where:

$c_s c_d$	is the structural factor
c_f	is the force coefficient for the structure
A_{ref}	is the reference area of the structure

The part of the standard that treats dynamics covers only response due to "along wind turbulence in resonance with along wind vibrations of a fundamental flexural mode shape with constant sign". In order to investigate the serviceability of a structure, the maximum displacement and the standard deviation of the characteristic acceleration at a certain height Z in the direction of the wind should be controlled. (*SS-EN 1991-1-4, 2005*)

6. Human response

Motions can be perceived with various senses e.g. visually or with vestibular organs. This can lead to a feeling of illness. The human response of motions is psychological and highly individual, a general studie of the human response is shown in *Figure 7.1* where different levels of human response is presented (*Mendis, 2007*). The otolith organs in the vestibular system is sensitive of detecting accelerations (*Johann et al., 2015*), linear acceleration is therefore a critical design parameter and chosen to evaluate the comfort criteria. Internal organs and the whole body perceive a discomfort when it attains resonance, i.e. when the oscillations of the structure correspond to the eigenfrequency of the organ. The first eigenfrequency of the structure is used to determine the acceleration of the building. International ISO-standards ISO 6897 (0,063-1 Hz) and ISO 10137 (0.06-5 Hz) predict a comfort dependence of frequency and acceleration to evaluate the comfort criteria.

LEVEL	ACCELERATION (m / sec^2)	EFFECT
1	< 0.05	Humans cannot perceive motion
2	0.05 - 0.1	a) Sensitive people can perceive motion; b) hanging objects may move slightly
3	0.1 - 0.25	a) Majority of people will perceive motion; b) level of motion may affect desk work: c) long - term exposure may produce motion sickness
4	0.25 - 0.4	a) Desk work becomes difficult or almost impossible; b) ambulation still possible
5	0.4 - 0.5	a) People strongly perceive motion; b) difficult to walk naturally; c) standing people may lose balance.
6	0.5 - 0.6	Most people cannot tolerate motion and are unable to walk naturally
7	0.6 - 0.7	People cannot walk or tolerate motion.
8	> 0.85	Objects begin to fall and people may be injured

Figure 7.2 General human perspective levels (*Mendis, 2007*)

7. Standards

According to the planning and building regulation in Sweden there are the following demands on buildings and structures:

A building should be designed and managed such that the effects which the building is likely to be exposed for during its construction and service life does not lead to:

1. rapture or partial rapture of the structure,
2. unacceptable large deformations,
3. harm on other parts of the building, its installations or fixed equipment as a result of large deformation in the load bearing structure or
4. harm that is unproportional to the event that caused the harm.

The eurocodes combined with the national regulation *EKS* are used to fulfill these demands. Were the Swedish annex, *EKS*, and *EN 1991-1-4* are enforcement regulations to the demands on the mechanical resistance, stability and durability on structures within Sweden. (*Boverket, 2017*)

7.1. ISO 6897

ISO 6897 is a guideline for evaluate the response of structures exposed of low frequency horizontal movements (0,063-1 Hz). It gives satisfactory magnitudes that are based on wind storms with a returning period of 5 years during the worst 10 minutes, see *Figure 7.3*. The recommendation of ISO 6897 is to have less than 12% of the occupancy sense a discomfort during a 5 years period. The suggested satisfactory acceleration magnitudes for storms with 1 year return period can be calculated as 0,72 times those for a 5 years period. This would represent less than 2% of the occupancy sense a discomfort due to acceleration. There are two different approaches to analyze the acceleration of the building, peak acceleration and r.m.s (root mean square) acceleration. Peak acceleration is based on that the preceiver only remember the largest cycle of a 20-60 min period. R.m.s acceleration on the contrary based on that the occupancy perceive the history of cycles and intensity over the same period. (*ISO, 1984*)

$$Peak = rms \cdot g \tag{Eq.37}$$

where:

g is a factor dependent on waveform, see k_p (Eq.41)

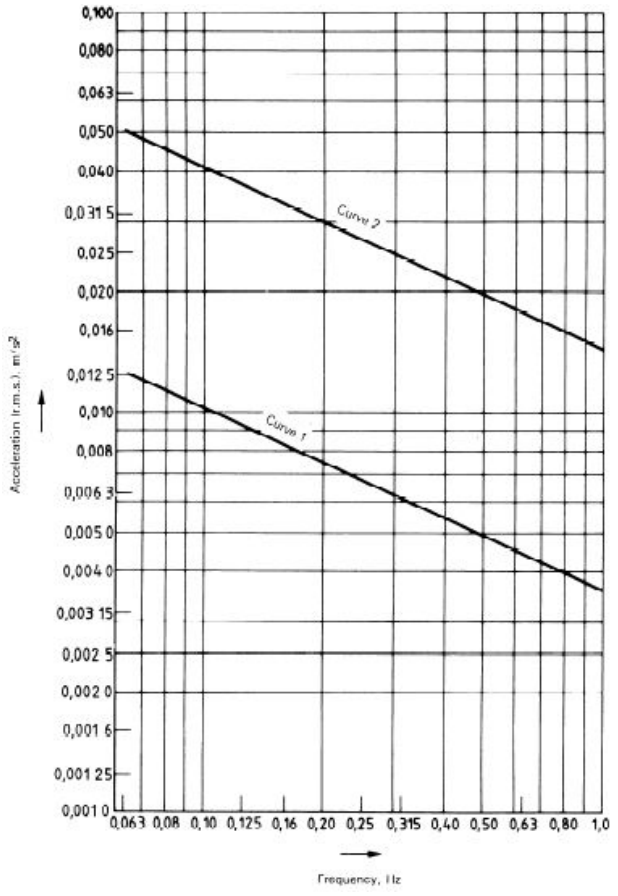


Figure 7.3: A suggested satisfactory magnitudes of horizontal motion of buildings of a five years return period. Average (curve 2) and lower threshold (curve 1) (*ISO, 1984*)

7.2. ISO 10137

An evaluation curve for horizontal motions due to wind induced vibrations is presented in *Figure 7.4*. The graph represent peak acceleration for wind induced vibration of a 1 year return period. The calculated acceleration should not exceed the evaluation curve at the same natural frequency. The lower curve is a suggested satisfactory acceleration for residence, where 90% of the occupancy probably perceive the motion. The upper curve represent offices, which is suggested to be at a level 3/2 above that residence curve. The recommendations presented in this standards are for serviceability (accelerations in the scope of 0.06-5 Hz), safety and occurrence of resonance is not treated. The requirements of the ISO-standard should be evaluated based on the presented calculations of acceleration in *EN 1991-1-4*, combinated with the national annex *EKS 10*. (*ISO, 2007*)

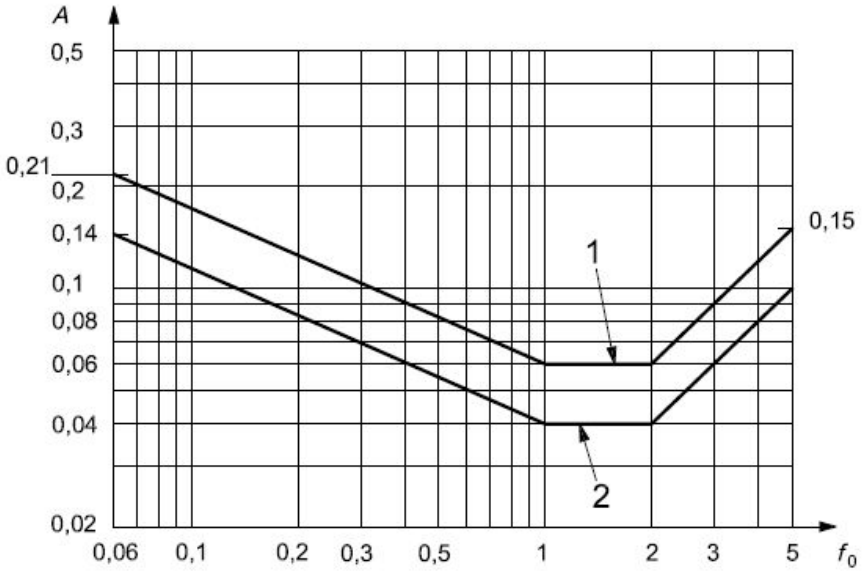


Figure 7.4 Evaluation curve for wind induced vibrations. Cuve 1 (offices), curve 2 (residence), (*ISO, 2007*)

where:

A is the peak acceleration [m/s^2]

f_0 is the first natural frequency in a horizontal direction [Hz]

7.3. Eurocode

According to *SS-EN 1990* are there several requirements in the serviceability limit state on a structure, among them is it stated that the control of vibrations should be evaluated with regard to the discomfort of people as well as limitation of the functional effectiveness of the structure. It is in the annex *A1.4.4 vibrations* stated that if the natural frequency of the structure is lower than the appropriate value, a more detailed analysis including damping should be performed according to *EN 1991-1-4* and *ISO 10137*. In *EN 1991-1-4*, Annex F *Dynamic characteristics of structures* it is expressed how the calculation procedure of dynamic behaviour assumes that "structures possess linear elastic behaviour and classical normal modes". The dynamic structural properties are consequently described by:

- Natural frequency
- Modal shapes
- Equivalent masses
- logarithmic decrement of damping

And the fundamental dynamic properties can be evaluated in approximate terms, using simplified analytical, semi-empirical or empirical equations, which is presented in the following chapters. (*SS-EN 1990, 2004*), (*SS-EN 1991-1-4, 2005*)

7.3.1. Frequency

The fundamental flexural frequency n_1 of multi storey buildings with a height larger than 50m can be estimated by the expression according to *EN 1991-1-4*:

$$n_1 = \frac{46}{h} \text{ [Hz]} \qquad \text{(Eq.38)}$$

where:

h is the height [m]

46 is based on a trendline from empirical frequency values [m/s]

7.4. EKS 10 (Swedish annex)

EKS is the national regulations to Eurocode and it is based on national conditions such as climate, geology and way of living. The latest revision of the national regulations is *EKS 10* which began to apply on 1 january 2016. (*Boverket, 2017*)

7.4.1. Referens wind loads in Sweden

A map of Sweden from the national standards is used to assign the wind reference speed at the location of the construction and with that assign the static wind load (see *Figure 7.5*). This reference wind speed is a mean wind speed measured at a height of 10m from the ground with and return period of 50 years. (*Boverket, 2015*)

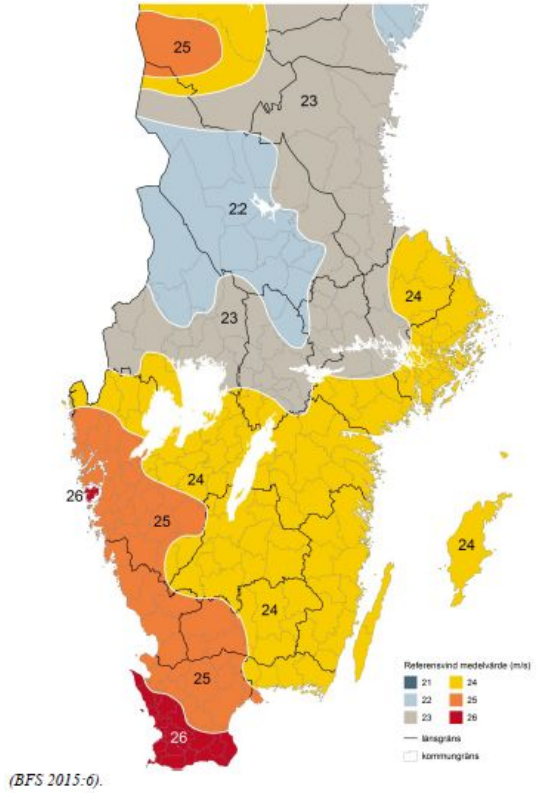


Figure 7.5 Mean reference wind speed in southern part of Sweden (*Boverket, 2015*)

7.4.2. Acceleration

The calculation of the peak acceleration of the structure, including the following equations below are calculated according to *EKS 10* (6.3.1 (1)) if nothing else is specified. This acceleration is compared to suggested ISO-standard for a specific frequency. See Appendix A for complete calculations.

Peak acceleration, $\ddot{X}_{max}(z)$:

$$\ddot{X}_{max}(z) = 0,72 \cdot k_p \sigma_x(z) \quad \text{(Eq.39)}$$

Where 0,72 is a suggested reduction factor according ISO 6897 that convert a 5 year return period to a 1 year return period (see chapter 7.1, *ISO 6897*).

Standard deviation of acceleration, σ_x(z):

$$\sigma_x(z) = \frac{3I_v(h)\, R\, q_m(h)\, b\, c_f\, \Phi_{1,x}(z_{n-1})}{m} \quad \text{(Eq.40)}$$

Peak factor, k_p:

$$k_p = \sqrt{2 \cdot ln(\nu T_{600})} + \frac{0,6}{\sqrt{2 \cdot ln(\nu T_{600})}} \quad \text{(Eq.41)}$$

where:

T_{600} is the time where the basic mean wind velocity is measured, 600s

ν is the up-crossing frequency:

$$\nu = n_{1,x} \cdot \frac{R}{\sqrt{B^2 + R^2}} \quad \text{(Eq.42)}$$

$n_{1,x}$ is the eigenfrequency of the structure, determined using *FEM Design*

The background factor B^2 allowing for the lack of full correlation of the pressure on the structure surface may be calculated, according to *BSV 97* (3:22e):

$$B^2 = exp\left[-0,05 \cdot \left(\frac{h}{h_{ref}}\right) + \left(1 - \frac{b}{h}\right) \cdot \left(0,04 + 0,01 \cdot \left(\frac{h}{h_{ref}}\right)\right)\right] \quad \text{(Eq.43)}$$

where:

b is width of the structure

h is height of the structure

h_{ref} is 10 m, a recommended value according to *BSV 97*

The resonance response (allowing for turbulence in resonance), the damping (δ_x) is described in chapter 4.6.1:

$$R^2 = \frac{2\pi \cdot F \cdot \Phi_b \cdot \Phi_h}{\delta_s + \delta_a} \quad \text{(Eq.44)}$$

Non dimensional (Kármán) wind energy spectrum, *BSV 97* (fig 3:22d) and non dimensional frequency:

$$F = \frac{4 \cdot y_c}{(1+70{,}8y_c^2)^{\frac{5}{6}}} \quad \text{(Eq.45)}$$

$$y_c = \frac{150 n_1}{v_m} \quad \text{(Eq.46)}$$

Size factor with regard to width and height of the structure:

$$\Phi_b = \frac{1}{1+\frac{3{,}2 n_1 b}{v_m}} \quad \text{(Eq.47)}$$

$$\Phi_h = \frac{1}{1+\frac{2 n_1 h}{v_m}} \quad \text{(Eq.48)}$$

The turbulence intensity, according to *EN 1991-1-4* (4.7):

$$I_v = \frac{1}{c_0 \cdot ln(\frac{z}{z_0})} \quad \text{(Eq.49)}$$

where:

- c_0 is a orography factor
- z is the height of the structure
- z_0 is the roughness length

The fundamental flexural mode of buildings:

$$\Phi_{1,x}(z) = (\frac{z}{h})^{\zeta} \quad \text{(Eq.50)}$$

Where a recommended value for ζ for slender cantilever buildings according to *EN 1991-1-4* (F.13) is 1.5.

By assuming that no activity will take place on the roof (storey *n*), the acceleration calculation takes place on the highest located apartment (i.e. *n-1*). (Eq.50) is therefore rewritten as:

$$\Phi_{1,x}(z_{n-1}) = \left(\frac{z_{n-1}}{h}\right)^{1,5} \quad \text{(Eq.51)}$$

The mean wind velocity $v_m(z)$ at the height z above the terrain, *EN 1991-1-4* (4.3)

$$v_m(z) = c_0(z) \cdot c_r(z) \cdot v_{b,Ta} \quad \text{(Eq.52)}$$

where:

$v_{b,Ta}$ is basic wind speed for a 1 year period
c_0 is the orography factor, taken as 1,0
c_r is roughness factor, according to *EN 1991-1-4* (4.4)

$$c_r = k_r \cdot ln\left(\frac{z}{z_0}\right) \quad \text{(Eq.53)}$$

k_r is terrain factor depending on the roughness length z_0, according *EN 1991-1-4* (4.5)

$$k_r = 0,19\left(\frac{z_0}{z_{0,II}}\right)^{0,07} \quad \text{(Eq.54)}$$

The mean wind pressure can be calculated:

$$q_m = \tfrac{1}{2} \cdot \rho \cdot v_m^2 \quad \text{(Eq.55)}$$

where:

ρ is the density of air

The requirements of acceleration needs to be fulfilled according to *ISO 10137*, this acceleration is based on a one year return period. According to *ISO 6897* (ch.3, NOTES 3) should the suggested satisfactory acceleration of a one year period be 0,72 times those for a five-year period.
A wind speed with a five year return can be calculated:

$$v_{b,Ta} = 0,75 v_b \cdot \sqrt{(1 - 0,2 ln(-ln(1 - \tfrac{1}{T_a})))} \quad \text{(Eq.56)}$$

where:

v_b is basic wind speed with a returning period of 50 years, see *Figure 7.5*)
T_a is the return period of 5 years

8. Iteration process

The study of the dynamic response for the structure follows an iteration process, described in the flowchart in *Figure 8.1*. A initial structure of CLT is tested with regard to dynamic response, the number of floors is increased successively until the demands associated to vibration are no longer fulfilled. Then the first improvement, change of floor to concrete, is tested until it no longer fulfills the demands, and so on. The flowchart and the iteration process have been arranged with regard to the practical applicability of the solutions as well as its efficiency to mitigate the dynamic response of the structure. Each improvement in the flowchart is weighted with regard to its ability to be practically implemented in the structure, where the most realistic procedure is tested first.

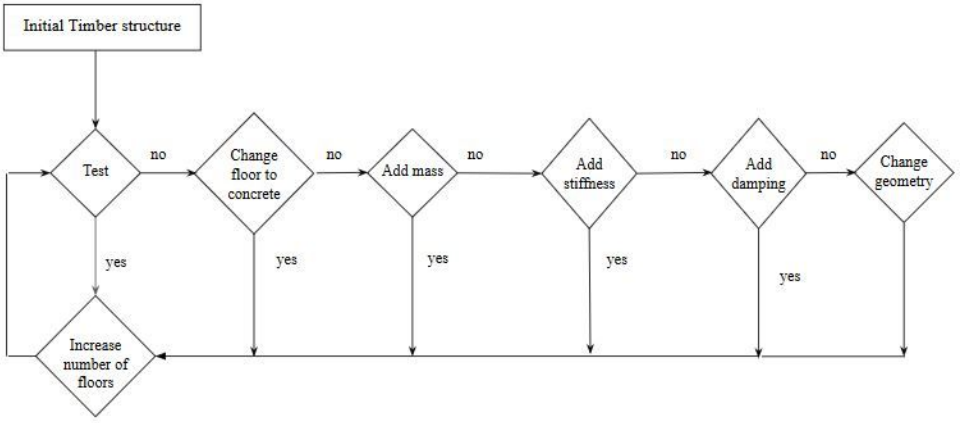


Figure 8.1 Flowchart of iteration process, order of parameter changes to fulfill requirements

8.1. Mass

Adding mass is an efficient way to modify the dynamic response and decrease the acceleration. Mass placed in the top (or in the upper third of the building) have large impact on the equivalent mass and strong impact on the dynamic response. Concrete is a suitable material to increase mass efficiently. When changing the material of elements from timber to concrete, an increased mass is achieved without loss of functionality. By only add mass (dead load) with no further pursue, both space, sustainability and economics of the building would be severely affected to the negative. Replacing floor elements is an efficient method to achieve effective mass increment, adding mass (e.g. wall elements) is the next step in the iteration process. Adding mass with no further purpose than only dead load will not be treated in this iteration process. *Table 8.1* demonstrates the mass difference using alternative materials as flooring.

Table 8.1 Weight of adequate floor material

Product	Element weight [kg/m^2]
CLT (230mm)	92
HD/f 120/27 F155	457
Solid concrete (300mm)	750

8.1.1. HD-f

HD-f (hollow core) elements is a standard prefabricated product that is fast to install without the need to cast in situ. HD-f is a mass reduced product and a “relatively” light structural element, which in this study is a disadvantage. HD-f is included and tested in this project due to its efficient building method and commonness on the market. HD-f is used as long as the element achieve the requested equivalent mass.

8.1.2. Solid concrete

Solid concrete is an effective alternative to increase mass and can both be prefabricated or cast in situ. Prefabricated solid concrete elements are similar to HD-f but generates more mass. They can however be harder to assemble due to the increased mass compared to the HD-f. Cast in situ is possible but not treated into this report due to casting difficulties on the top of the structure compared to an assembling of prefabricated products.

8.1.3. Concrete core

The core is an addition to the existing shear walls and may therefore be an improvement of the dynamic response due to the increase of both equivalent mass and stiffness. A cast in situ core is appropriate due to continuity and higher stiffness than prefabricated elements. The usage of a stabilizing concrete core is a well established method in order to handle the lateral loads. The method have many practical advantages such as: localization of mass and stiffness to where it is needed the most (such as elevator shaft), symmetry and centralization of the core to the rotation centre of the building reduces the rotational forces (see chapter *9.6.1 Model for verification)* and a practical building process with logical workflow on site. A proposal of the building process is to first cast the core and than assemble the timber elements.

8.2. Stiffness

The structure is often adapted to a plan of the building which makes it difficult to make larger changes. Material stiffness, described as the Young's modulus are a material specific parameter which cannot be increased without an exchange of the original material in the case of CLT. In order to increase the stiffness of a building, the dimensions of the structural elements e.g wall thickness needs to be increased or alternatively improved geometry or added bracing. Stiffer structural elements can be added to the cross section of the building, these could be a type of bracing trusses or fixed columns in the facade.

8.2.1. Geometry

A tall building structure can be seen as a fixed cantilever beam, a magnified or improved cross-section increases the global moment of inertia i.e. increases the stiffness. However, it can be of great difficulty to change the plan of the building with regard to increased cross section of the building. Often is the size of the building regulated by detailed development plans and other public regulations.

Also it can be of great architectural disadvantage to use a building with deep cross section when it comes to the requirements of natural light in the residential areas. Therefore the effect of geometry modification will only be analyzed, but will not be a part of a presented solution.

8.2.2. Truss

A way to stiffen the structure is to add a bracing truss in the facade, i.e. improve the moment of inertia of the structure. A structure made of CLT elements primary resists lateral forces via shear walls that resist forces well in its own plane. When adding a bracing truss in the facade some of the horizontal force are diverted to the stiffer trusses in the exterior of the building due to the increased moment of inertia and added stiffness. (*Swedish wood, 2015*)
Three different bracing truss systems in the facade of the structure (*T1, T2, T3)* are analysed with regard to the dynamic response of the structure. These have different advantages with regard to architectural and structural aspects. The two truss systems to the left in *Figure 8.2* have some architectural benefits, each floor have spaces without bracing elements in the facade yet still an increased stiffness. *T1* is braced in the center along the facade, based on the theory that maximum shear acts in the gravity center of a structure. The truss in *T2* are located in the outer part along the facade, and giv increased lever arm to the bending in the structure. The truss to the right, *T3,* have greater depth of the truss which entails a continuous deep web i.e. large lever arm, the facade are though inferior with regard to architectural aspects.

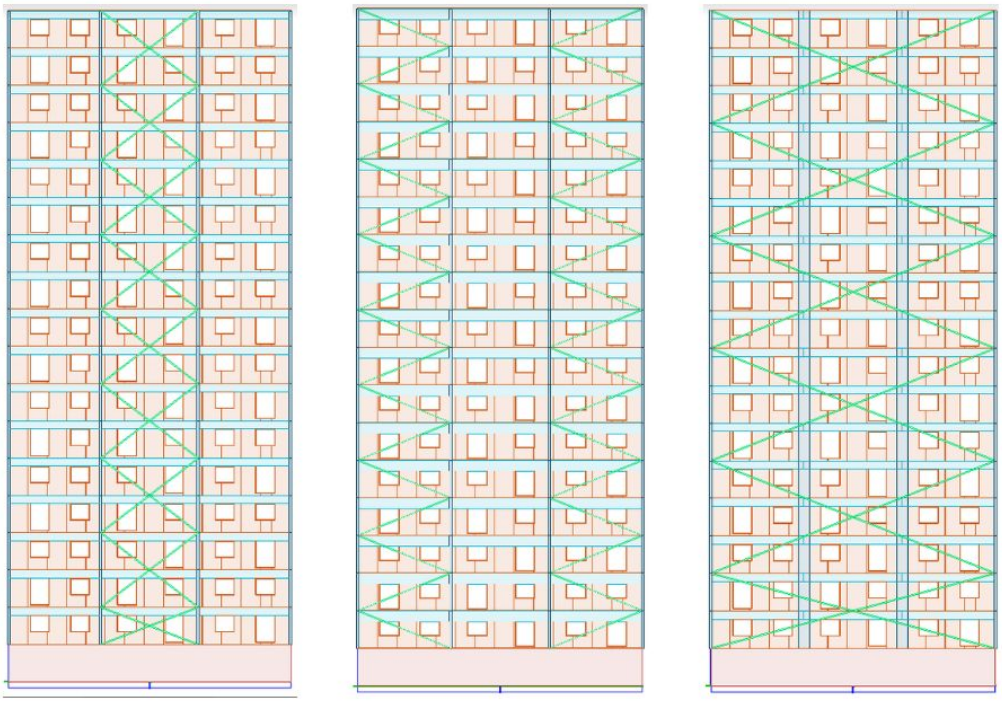


Figure 8.2 T*russ systems in the facade, T1, T2 and T3*

The proposed cross-section dimensions of the GL36c trusses is roughly estimated to:
Beam/horizontal elements: 215x1305mm
Column/vertical elements: 215x855mm
Oblique strut/tie elements: 215x90mm

8.2.3. Wall thickness

An increased wall thickness increases the cross-section, A, which results in an increased mass and stiffness. An increased thickness is an uncomplicated improvement but interfere with the plan and decreases the living space.

8.3. Damping

A damper is evaluated to fit the structure and provide the required damping factor. A spring coupled TMD is used in this study and the estimated design of this damper is adapted to this type of structure. The TMD should be located where the amplitude of the deflection is largest, i.e. in the top of the structure. The frequency ratio between the TMD and the structure should be equal to 1 for a maximum counter movement. The mass of the damper affect the mass ratio and the impact of the damping. The maximum allowed mass of the damper is chosen to 10 tons with respect of practical applicability. In order to anchor the damper properly to the structure, the TMD is always coupled to added mass in the form of concrete floors. A proposed solution to achieve a taller building height of the building is to combine only the damper with increased mass-increase and neglect the stiffness improvements. In this way, the two different solutions complement each other yet the cost and practical applicability is taken into consideration.

9. FEM-design

FEM (Finite element method) is a numerical approach that approximately solves differential equations that are too complicated to solve analytically. A model can be subdivided to smaller elements (finite elements) in a mesh, where more simple approximations are made over each element. These approximations are then patched together for a solution of the entire model. (*Ottosen & Petersson, 2007*)

The software that is used in this project is *FEM-Design 17 - 3D Structures* that is a Strusoft software system who provide structural modeling of 2D and 3D systems. Materials and structural elements can be analyzed manually or related to Eurocode 2, Eurocode 3 (Timber), Eurocode 5 and Eurocode 7. The modeling and analysing of the program is divided into the following steps: *Structure, Loads, Finite elements, Analysis.* (*StruSoft AB, 2018*)

9.1. Structure

The model is based on proposed plan, this plan will be repeated to the requested numbers of floors. The plan of the proposed 22x22m building consists of a staircase and elevator core in the center of the building and four apartments, see *Figure 9.1*. The model will include the fundamental structural system that have the main influence in order to give reasonable eigenfrequencies. The model consider windows, doors and similar perforation that affect the stiffness on the structure, see *Figure 9.2*. The first storey of the structure will be cast in-situ concrete to fulfill the requirements of a sustainable fundament, i.e. a stiff and moisture resistance fundament.

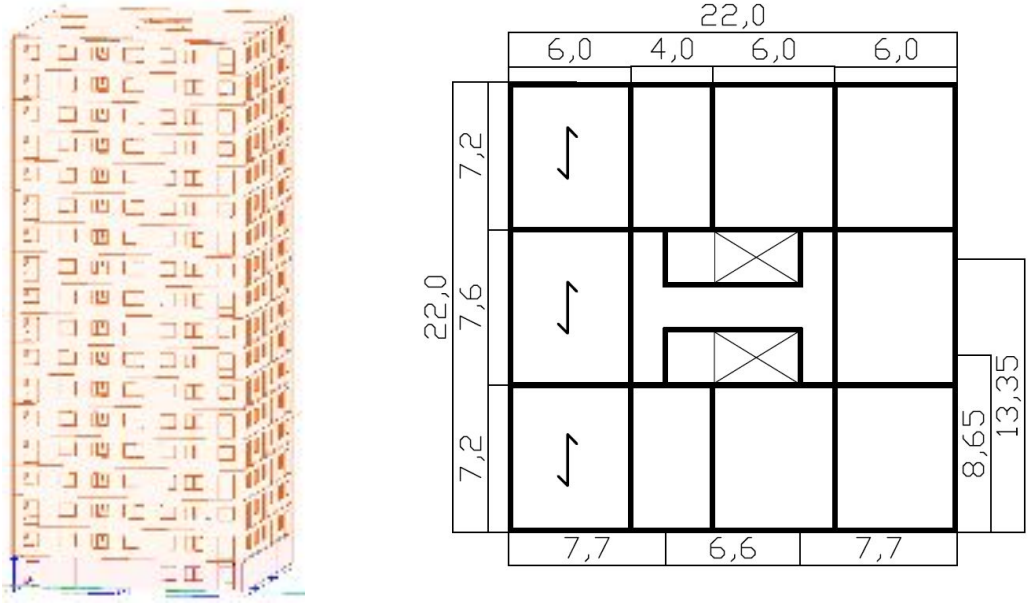


Figure 9.1 The proposed structure and plan with 18 storeys

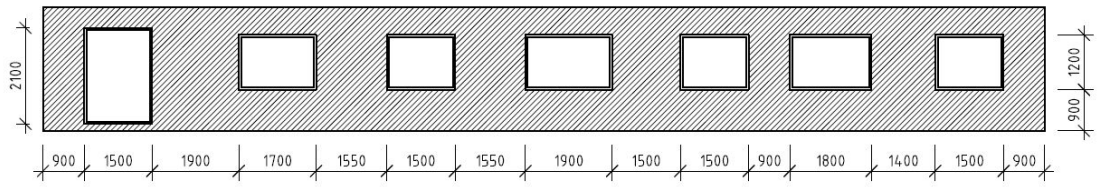


Figure 9.2 Location of Windows

9.1.1. Floor thickness

Assuming a floor thickness of approximately 500mm, based on a standard product for residential separating floors from Martinssons (*Figure 9.3*). The height of each ceiling is set to 2.4m, normal ceiling height in residential buildings, that result in a storey height of 2.9m. Using the lowest accepted ceiling height consequently result in higher equivalent mass and more floors on same building height.

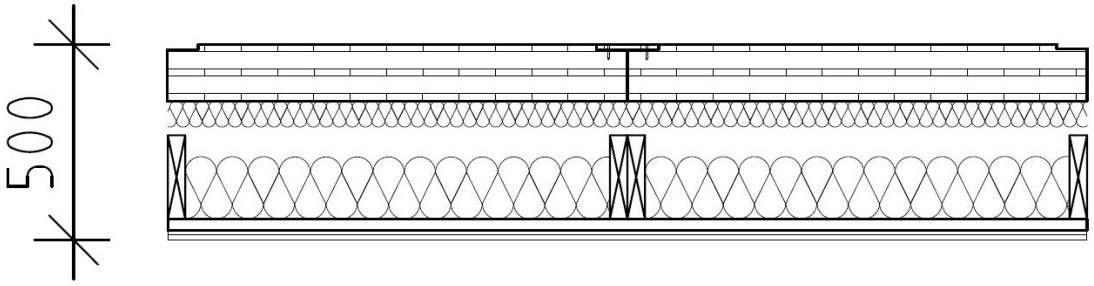
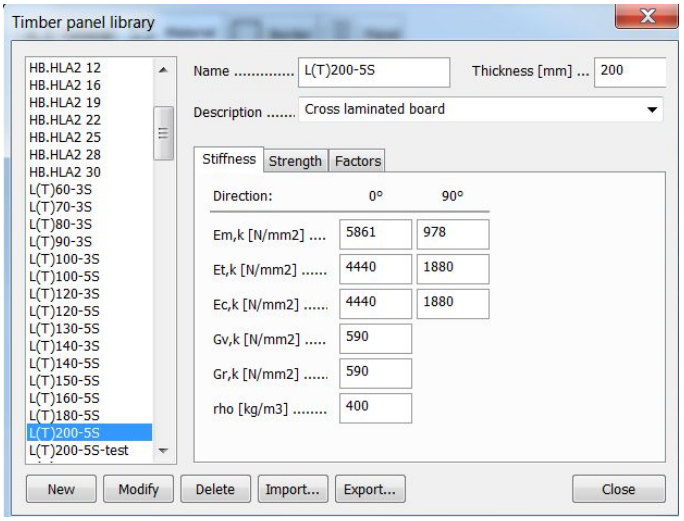


Figure 9.3 A cross section of floor element inspired by Martinssons floor product.

9.1.2. Materials

The choice of material affects the dynamic response on the structure due to mass and material stiffness. *FEM-Design* takes into account the orthotropic behaviour of timber using local material coordinates and applies appropriate material properties (e.g. Young's modulus) for the loded direction. The material data in *FEM-Design* is synchronized with actual products. Data have been compared with Martinsson and choices of element sizes (e.g. thinesses of walls and floors) is based on available products. See in *Figure 9.4*. data and choices from *FEM-Design* and material properties (50% fractile) from Martinssons CLT products used in this report (200, 230 and 300mm).



Tjocklek [mm] 2)	E-modul, böjning kring i-axeln, $E_{mi,50}$				E-modul, drag // i-axeln $E_{ti,50}$			E-modul, tryck // i-axeln $E_{ci,50}$			Skjuvmodul, $G_{ij,50}$		
L / T 3)	Z Styv	X Vek	Y_1 Skivv.styv	Y_2 Skivv.vek	X Styv	Z Vek	Y Tvärdrag	X Styv	Z Vek	Y ⊥ Fibrerna	XY Böj.styv	YZ Böj.vek	XZ Böj.skiv
60-3s	10 606	764	7 457	3 913	7 457	3 913	370				117	96	690
70-3s	10 152	892	6 384	3 211	6 384	3 211	310				95	108	582
80-3s	10 832	474	8 308	2 028	2 028	2 028	335				148	87	627
90-3s	10 601	616	7 410	2 580	7 410	2 580	323	Samma värden som för drag.			117	96	606
100-3s	10 601	616	7 410	2 580	7 410	2 580	323				117	96	606
120-3s	10 601	616	7 410	2 580	7 410	2 580	323				117	96	606
140-3s	10 601	616	7 410	2 580	7 410	2 580	323				117	96	606
100-5s	8 760	1 749	6 692	3 022	6 692	3 022	314				100	104	590
120-5s	7 859	2 304	5 615	3 685	5 615	3 685	300				83	119	565
130-5s	9 451	1 324	7 686	2 410	7 686	2 410	327				125	93	613
140-5s	7 106	2 767	4 846	4 159	4 846	4 159	290				73	133	547
150-5s	8 760	1 749	6 692	3 022	6 692	3 022	314	Samma värden som för drag.			100	104	590
160-5s	9 822	1 095	8 308	2 028	8 308	2 028	335				148	87	627
180-5s	7 859	2 304	5 615	3 685	5 615	3 685	300				83	119	565
200-5s	8 760	1 749	6 692	3 022	6 692	3 022	314				100	104	590
230-5s	8 760	1 749	6 692	3 022	6 692	3 022	314				100	104	590
210-7s	7 891	2 284	6 384	3 211	6 384	3 211	310				95	108	582
240-7s	9 401	1 354	8 308	2 028	8 308	2 028	335	Samma värden som för drag.			148	87	627
270-7s	8 740	1 761	7 410	2 580	7 410	2 580	323				117	96	606
280-7s	7 891	2 284	6 384	3 211	6 384	3 211	310				95	108	582

Figure 9.4 Material dialog box from FEM-Design and material properties from CLT-producer "Martinssons"

9.1.3. Support and connections

The support condition is modeled as a hinged line-supports along all basement walls. The connections between wall-floor is assumed to be hinged where the floor is simply supported in three spans (7.2m/span) see *Figure 9.1*. The floor-floor connection in each span and wall-wall connections are assumed and modeled as a composite *Figure 9.5*.

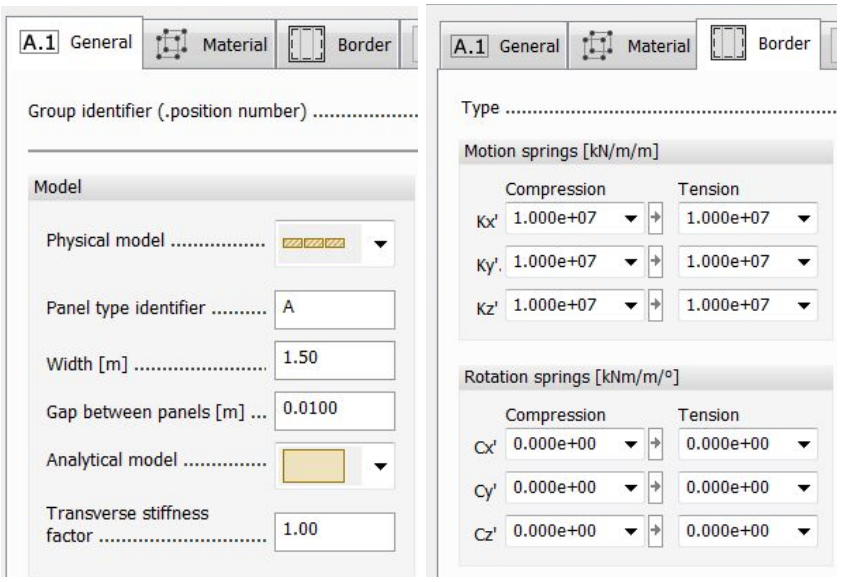


Figure 9.5 Connection dialog boxes from FEM-Design

9.2. Loads

The wind load is simplified as a line load which acts on each floor, using the same input as in *EKS 10,* see load-application in *Figure 9.6*. Beside wind load, live load have been added as an surface load to represent some activity in the building, this result in favorable response due to "higher mass". The added live load is 30% of Eurocodes recommended live load value of residents (2,0 kN/m^2) which equals 0.6kN/m^2.

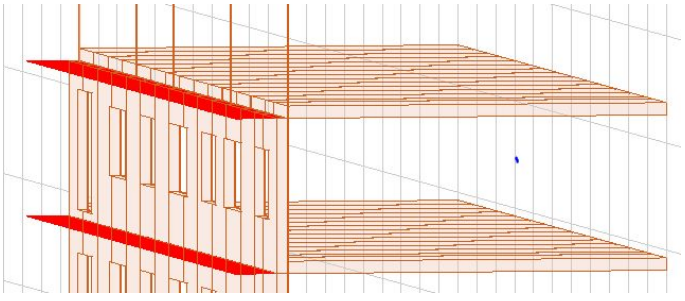


Figure 9.7 Live load (2kN/m^2) acting on the floor and wind load in y-direction.

Structural mass (dead load) is not included by default to the calculations, but must be included to generate the dynamic calculations.

9.3. Linear Dynamic

According to Strusoft's theory manual, if the structure is unloaded, Q = 0, the stiffness and mass matrices result in an eigenvalue problem, see (Eq.57).

$$[K - \omega^2 M]\,\varphi = 0 \qquad \text{(Eq.57)}$$

where:

φ is the vibration shape/mode

If the load Q is varying it result in a displacement varying with time. The external loads are treated and added to inertia loading (d'Alembert's theorem) that result in the basic equation, (Eq.58), in which damping is ignored.

$$Ku = Q - M\ddot{u} \qquad \text{(Eq.58)}$$

Wind load according to *EN 1991-1-4* is a horizontal quasi-static force and the result from FEM design is free oscillations. The calculation according to (Eq.57) gives eigenfrequencies from the problem $det[\boldsymbol{K} - \omega^2\mathbf{M}] = 0$. $\boldsymbol{K}$ and $\boldsymbol{M}$ are (n x n) matrices (*n* numbers of degrees-of-freedoms) that result in *n* eigenfrequencies where the lower frequencies require less energy than a high frequence. Wind load has an energy content in the low rage of the frequency spectrum and therefore only the lower eigenfrequency modes contribute significantly to the structural response. To decrease the calculation time, only the two first eigenfrequencies are calculated. There are the lowest eigenfrequency in x and y-direction separately (the wind can blow in any direction).

FEM-Design do not include damping in the calculations and therefore it needs to be considered in separate hand calculations. Adding damping changes the response of the structure and should, in a more exact calculation be included in one complex calculation (where the frequency and acceleration is determined) over a time domain. This is above the capacity of the software and outside the scope of *EN 1991-1-4* and will not be treated in this report.

9.4. Finite elements

FE methods subdivides the structure into finite elements that creates a mesh of the structure which the calculations is based on. A finer mesh generate in general better approximations, but require more computer time. The mesh in this model is in general lage(1m x 1m) to insure fast calculations. The size of the elements cannot be too large so it affect the accuracy of the calculations. The linear dynamic calculations of eigenfrequencies is based on mass and stiffnesses, that need to be represented well in each element to not deteriorate accuracy. *Figure 9.8* shows a 18 store structure with an average 1m element mesh. The calculation time of two first eigenfrequencies was approximately 2min.

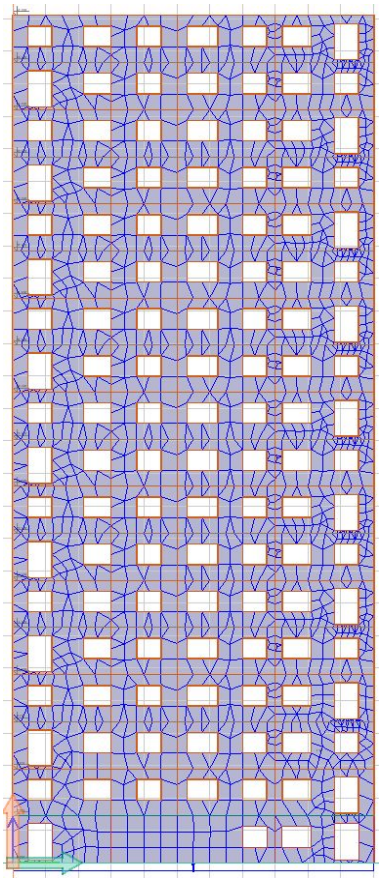


Figure 9.9 A meshed facade of an 18 storey structure

9.5. Post processing in FEM-Design

The results from *FEM-Design* can be presented both in a graphical view as well as in tables of individual node and element result. The result of global reaction forces can be obtained as resultants or line forces along the line supports in the foundation as can be seen in *Figure 9.10* where the graphical illustration is presented.

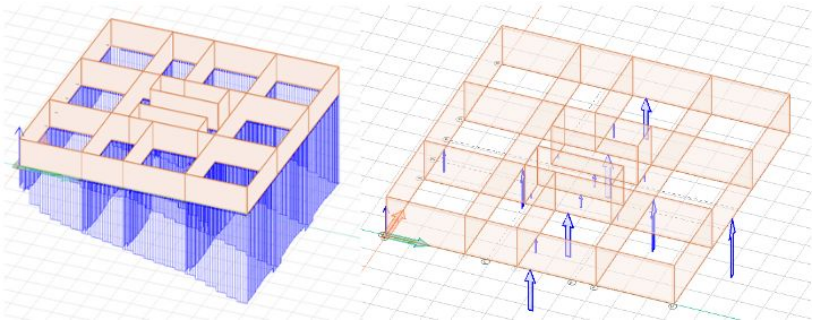


Figure 9.10 Line support forces and resultants

For each individual wall, detailed results can be obtained. Where the force distribution along the wall length can be visualised both as a linearized or more accurate graph.

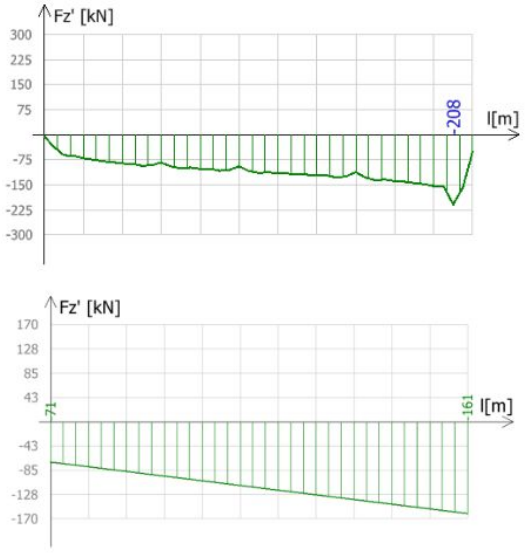


Figure 9.11 Linearization

The results which will be used in the verification of the model will be based on the linearized result as can be seen in the lower graph of *Figure 9.11* in order to correlate with the hand calculations.

9.6. Verification of the FE-model

In order to verify that the model conducted in *FEM-Design* corresponds to reality and to the calculation procedure in *EN 1991-1-4*, the horizontal and vertical equilibrium of the load bearing walls in the transversal direction are compared to simplified hand calculations. A simple comparison of floor mass from the program is also conducted to verify that no further model errors occur. This comparison is based on the mean value of level masses obtained in *FEM-Design* for a typical CLT storey and hand quantification of the same case.

The hand calculations are based on *SS-EN-1991-1* (actions on structures) and *Distribution of Horizontal load on Bracing elements*. The calculations are based on the fact that all the bracing elements are oriented in x- and y-directions and that the stiffness does not vary along the height. The stiffness of each wall is expressed as a function of its cross section where members which act primary in shear, like shear panels in timber buildings can be represented by the shear stiffness only. All bracing members have the same thickness and the shear stiffness will be directly proportional to the depth of the wall section and can be expressed as the sections depth. More generally, the stiffness of each bracing wall depend on both the shear and bending stiffness and can be expressed as:

$$\frac{1}{S_i} = \frac{1}{S_{si}} + \frac{1}{S_{bi}} \qquad \text{(Eq.59)}$$

where:

S_{si} Shear stiffness of wall *i*

S_{bi} Bending stiffness of wall *i*

And can be expressed as:

$$S_{si} = k_s \cdot \frac{E_i \cdot A_i}{l_i} \qquad \text{(Eq.60)}$$

$$S_{bi} = k_b \cdot \frac{E_i \cdot I_i}{l_i^3} \qquad \text{(Eq.61)}$$

When consider only shear stiffness of the bracing timber walls. Thus the only factor varying between the different walls are the depth of the section. Young's modulus, wall thickness and wall height are all represented by the same parameters and can be reduced of the expression. The same procedure applies for the bending stiffness which is represented by the area moment of inertia.
The bending stiffness however, is affected by the orientation of the wall. The expressions used for shear and bending stiffness in these calculations are:

$$S_{si} = l_i \qquad \text{(Eq.62)}$$

$$S_{b.i} = \frac{b_i \cdot h_i^3}{12} + (A_i \cdot d_i^2) \qquad \text{(Eq.63)}$$

where:

l_i Height/width of wall i depending on direction of wall

b_i Height/width of wall i depending on direction of wall

h_i Height/width of wall i depending on direction of wall

A_i Area of each individual wall

d_i Distance from local to global gravity centre

The total horizontal force acting on a bracing member can in x- and y-direction seprately be expressed as:

$$H_{xi} = H\frac{S_{s.xi}}{\Sigma S_{s.xi}} + T\frac{S_{b.xi} \times y_i}{S_T} \quad \text{(Eq.64)}$$

$$H_{yi} = H\frac{S_{s.yi}}{\Sigma S_{s.yi}} + T\frac{S_{b.yi} \times x_i}{S_T} \quad \text{(Eq.65)}$$

where:

H Total horizontal load

S_T Global torsional stiffness

T Global torsional moment

$$T = H \cdot e \quad \text{(Eq.66)}$$

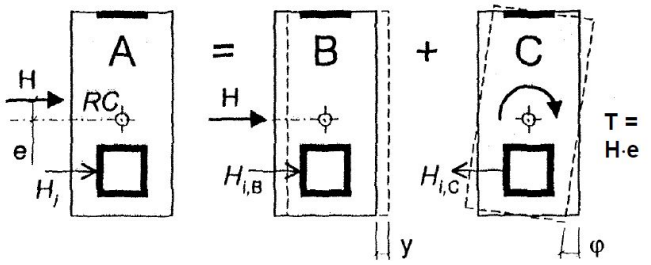


Figure 9.12 Eccentric horizontal load (Engström)

The first term of each equation corresponds to the translation of each wall and the second term corresponds to horizontal contribution from the rotation, see *Figure 9.12*. The stiffness of each contribution is dependant on the type of behavior according to: (Eq.60) and (Eq.61). The individual contribution from the rotation is dependant of the torsional stiffness:

$$S_T = \Sigma(S_{b.xi} \cdot y_i^2) + \Sigma(S_{b.yi} \cdot x_i^2) \quad \text{(Eq.67)}$$

9.6.1. Model for verification

The model which is used to verify the calculations from the software is a simplified 20 storey high timber building. The structure is of the same type as the original model. However, some simplifications are used in order to compare the result with simplified hand calculations. These are:

- No windows in the load bearing walls
- All storeys made of timber
- Floor acting as diaphragm
- Loads are applied without partial coefficients

The reason to neglect the windows in the load bearing facade is the fact that there is a large number of windows at each floor, dividing the shear walls in narrow vertical elements. Comparison with the result from the software proved that it was difficult to estimate the contributing wall area. The contributing area was somewhere greater than the case when neglecting the wall area above and below windows but less than the case when neglecting window area, see *Figure 9.13* .

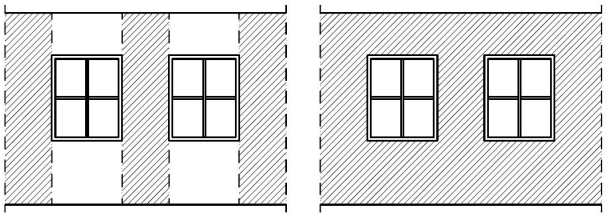


Figure 9.13 Effective contributing wall area

In order to use the calculation procedure in "Distribution of Horizontal load on Bracing elements" the stiffness of each wall is assumed to be constant along the height of the wall. This would not be the case with a basement of cast in situ concrete. The floor is assumed to distribute the forces acting on the facade to the shear walls without deforming itself and thus act as a rigid diaphragm (see *Figure 9.14*). This is due to the fact that a semi rigid diaphragm (see *Figure 9.14*) is assumed to distribute the load as a continuous beam which is highly indeterminate (Design of timber structures; p207). In order to simplify the calculations further, the partial coefficients of the loads are neglected.

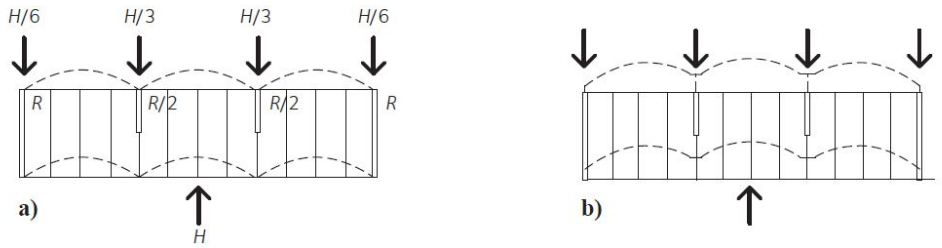


Figure 9.14 Rigid diaphragm VS Non rigid diaphragm (*Swedish wood, 2015)*

Due to symmetry of the walls around the global rotation centre of the building in X-direction the contribution from the torsional moment will be zero, see (Eq.66). This can be seen in *Figure 9.15* where wall *1*, *7* and *13* have their counterpart in wall *3*, *8* and *10* and no additional horizontal force due to rotation will arise. In Y-direction however, wall *15* and *16* are slightly offset from the centrum line of the section. This causes the rotation centre to move slightly to the left and by that create a torsional contribution to the horizontal equilibrium.

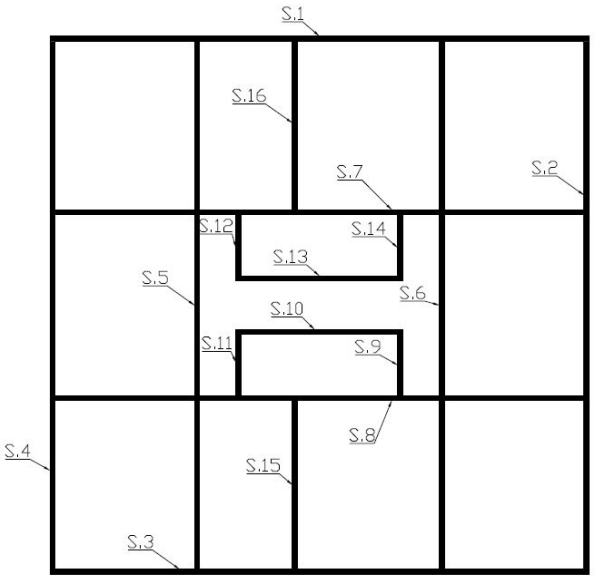


Figure 9.15 Plan and Wall ID for each individual wall

9.6.2. Result from verification

Table 9.2 and *Table 9.3* present the comparison of maximum values of horizontal and vertical forces as well as compressive stress, obtained both in *FEM-Design* and hand calculations. The forces is represented by the global reaction forces in vertical and horizontal direction and the compressive stress is related to the vertical reaction force. *Table 9.1* shows the comparison of the level masses obtained in *FEM-Design* and hand calculations.

Table 9.1 Level mass comparison

	FEM-Design	Hand calculations	Difference
Total level mass [tons]	113.44	113.14	0.265%

The difference of 0.265% is minimal and the comparison verify that no model error occurs, e.g. double elements or similar.

The difference in result are small enough to prove the model to be reliable. The stiffness in the hand calculations of wall 1,3,7,8 with the same orientation and length, (see *Figure 9.15*) are equal, which result in a horizontal force of the same magnitude for wind in X-direction acting on these walls.
The result from the software corresponds well for wall *7* and *8* but wall *1* and *3* have a slightly smaller value. This is probably due to redistribution of forces due to stiffness variation of floor diaphragms and facade walls. The hand calculations are based on the assumption that the magnitude of force acting on each individual wall is directly proportional to its stiffness. By that it neglects the stiffness of the load distributing panels in walls and floors, see *Figure 9.14*.

The result of the compressive stress is based on a simplified linearized result of the reaction force along the length of each wall, see *Figure 9.11*. The maximum value for each wall is divided by the cross section area of a one meter long segment of the wall to represent the compressive stress within the wall. This generalisation of the result from the software gives a larger difference in *Table 9.2* compared to the other comparisons.

Table 9.2 Wind in X-direction

	FEM-Design		Hand calculations		
	Value	Wall ID	Value	Wall ID	Difference
Maximum Horizontal force [kN]	358	7,8	340	1,3,7,8	5%
Maximum compressive stress [MPa]	0.81	7,8	1.0	7,8	19%
Maximum reaction force [kN]	3425	2	3229	2	6%

For *Table 9.3* with wind direction in positive Y-direction, the most loaded walls differ from the result in *Table 9.2* primary due to the rotational contribution related to the asymmetry of the walls. The reason of the difference in the wall ID of the most loaded wall in horizontal and vertical direction between *FEM-Design* and Hand calculations is the same as for wind in X-direction. The floor diaphragm is assumed to be stiff in the hand calculations while *FEM-Design* accounts for the actual stiffness of the floor diaphragm. This result in a larger load acting on the interior walls in *FEM-Design* (*5, 6*) see *Figure 9.15* compared to wall *2* and *4*. Accordingly, the general distribution of horizontal load acting on each individual wall dislocates in order to maintain equilibrium. This slightly affects the vertical equilibrium which can be seen in the comparison of reaction force. The compressive stress and reaction force in *Table 9.3* have a deviation of less than five percent.

Table 9.3 Wind in Y-direction

	FEM-Design		Hand calculations		
	Value	Wall ID	Value	Wall ID	Difference
Maximum Horizontal load [kN]	342.7	6	310.4	4	9.4%
Maximum compressive stress [MPa]	0.835	16	0.868	16	3.8%
Maximum reaction force [kN]	3503	1	3587	7	2.3%

Generally these values presented in *Table 9.1*, *Table 9.2* and *Table 9.3* do correspond well and prove the model to be reliable enough to proceed with further calculations. For a more detailed comparison between the result from Hand calculations and *FEM-Design,* see Appendix D

9.6.3. Element size

A convergence study of a 10 storey structure with different element sizes can be seen in *Figure 9.16* and described in *Table 9.4*. A larger structure would have the same behaviour but takes longer to calculate. A convergence study is made to ensure a fast calculations with minimized errors. The calculation time decreases rappidly up to an element size of 0.75m and the decreasing have almost stopped at an element size of 1.0m. It is sufficient to use an element size of 1.0m where the error is sufficiently small, that requires less time consuming calculations.

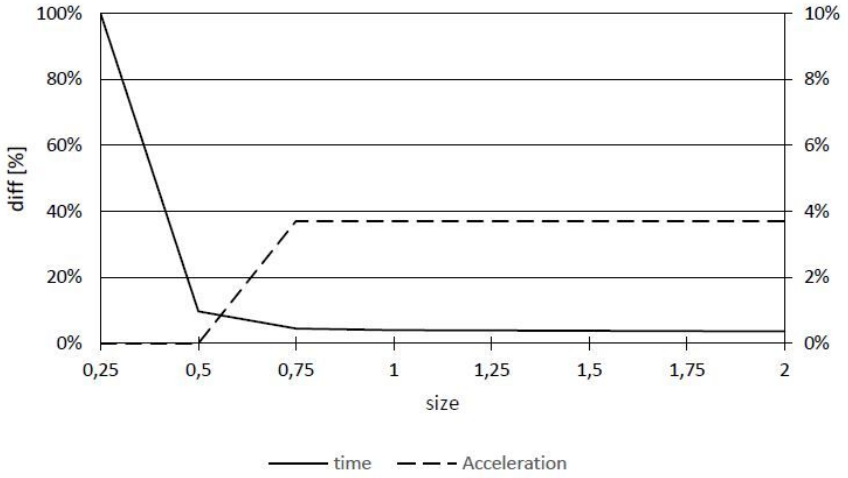


Figure 9.16 Convergence, time reduction y-left and acceleration diff., y-right

Table 9.4 Convergence

Element size [m]	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Time
0.25	36.72	3.203	0.027	20:12
0.5	36.73	3.223	0.027	01:58
0.75	36.70	3.243	0.026	00:54
1.0	36.79	3.258	0.026	00:48
2.0	36.76	3.276	0.026	00:44

10. Result

In the following chapter, the results of the parametric study is presented. The results are separated in a more general behaviour chapter and a more specific solution chapter.
The *general behavior* chapter present the effect on acceleration of the different parameters separately such as mass,stiffness and damping and how these are practically increased. The more solution oriented *structural behaviour* chapter present the combination of the effect of the different parameters in order to achieve an increased building height. To mitigate the risk of misleading information it is worth to clarify that graphs are presented in the following chapter with both logarithmic and linear scaling depending on the intention of the graph. Generally are the graphs where a comparison with the acceleration demand from the ISO-standard 10137 logarithmic due to the scaling of the demand.
Figure 10.1 shows the dynamic response in a 10-20 storey timber structure with no mass, stiffness or damping improvements and the trend line of increased acceleration due to a increased building height. Where 10 storeys (*CLT_10*) is in the lower right corner and 20 storeys (*CLT_20*) in the top left. This structure allow a building height up to 12 storey that fulfils the requirements. An increase of the building height reduces the frequency and increases the acceleration in a linear log-log behaviour. Structural adjustments have been done to fulfill the requirements according ISO 10137 and the proportion to ISO 10137 ≤ 1.

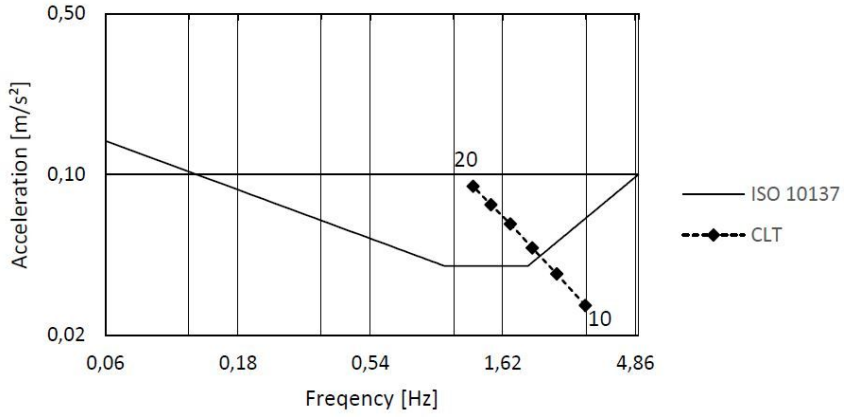


Figure 10.1 Linear trend of increased acceleration and increased number of floors.

Table 10.1 Result of initial CLT-structures

Model	m_e [ton/m]	Freq.[Hz]	Acc. [m/s^2]	Proportion to ISO 10137
CLT_10	36.72	3.211	0.027	42 %
CLT_12	37.05	2.533	0.037	73 %
CLT_14	37.30	2.066	0.048	117 %
CLT_16	37.50	1.725	0.061	152 %
CLT_18	37.65	1.467	0.074	186 %
CLT_20	37.78	1.265	0.089	222 %

10.1. General behaviour

To make changes with most impact, it is necessary to understand the behaviour and the response of the structure. A theoretical response-test in *Figure 10.2* shows the impact of changing mass (*m*) and stiffness (*k*). This test only include structural material, to be able to weight the importance of the material parameters (wind and live load is not included). It is possible to see that stiffness have larger influence than mass, a doubled stiffness (*2k/m*) reduces the acceleration more than a doubled mass (*k/2m*) from the initial values of timber (*k/m*). A structure with twice the stiffness and mass (*2k/2m*) almost result in a "pure" acceleration decrease (i.e. no change in frequency), but a slight increasing in frequency indicate a bit higher importance of stiffness than mass.

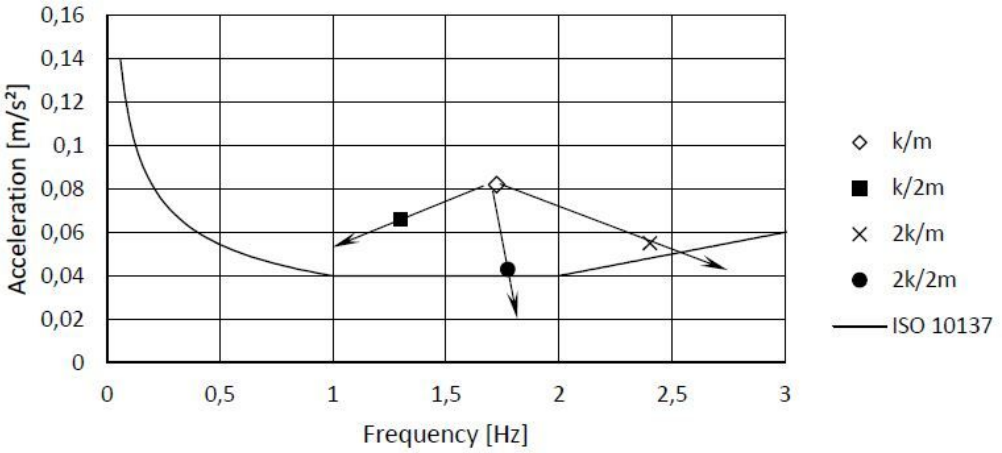


Figure 10.2 Theoretical behaviour by changing material parameters in timber

10.1.1. Equivalent mass

Increasing mass reduces the acceleration according to *Figure 10.2*, this increase can be done in two ways:

- Material changes, e.g. concrete sections/increase ρ or *A*
- Mass placement, i.e. mode shape scaling

Increase mass:

Figure 10.3 describes a mass-acceleration relationship with 3 different frequencies that are in the scope of timber structures. The Acceleration decreases rapidly up to a mass of 40 ton/m, but the requirements are not fulfilled until the mass reaches 83 ton/m for a frequency of 1.3Hz. The requirements of 0.80Hz. is fulfilled with an equivalent mass of 142 ton/m, i.e. a lower frequency needs more equivalent mass.

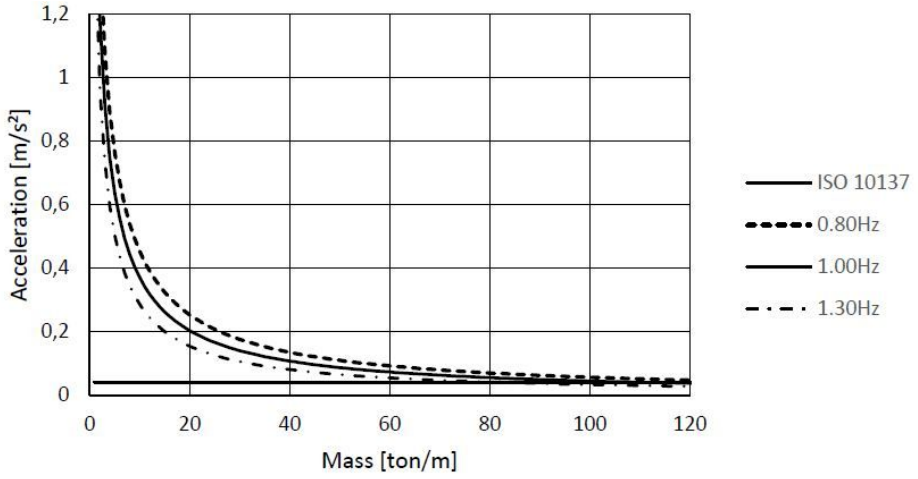


Figure 10.3 Acceleration decrease due to mass increment for 3 different frequencies

Figure 10.4 shows in similarity to *Figure 10.3* how the acceleration decrease by increased frequency, for the same mass. A higher mass (81.2 and 112.3ton/m) with the same frequency have lower acceleration (than 37.5ton/m). The test result from *FEM-Design* with the same mass is marked in *Figure 10.4*. As mentioned in earlier in general behavior and shown in *Figure 10.2*, by increasing the equivalent mass, the frequency and acceleration decreases, see *Table 10.2*.

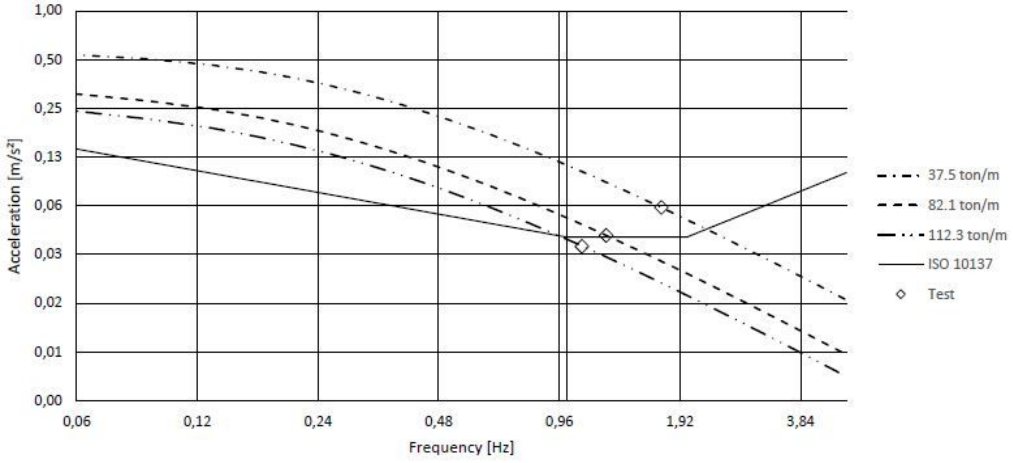


Figure 10.4 Acceleration decreases due to mass and frequency increment.

Table 10.2 Test result from FEM-Design of a structure with same stiffness

Mass [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
37.5	1.725	0.061	152%
82.1	1.256	0.041	102%
112.3	1.093	0.035	88%

Adding material that affect mass but not the structural stiffness will decrease the acceleration and frequency in a logarithmic linear behaviour, e.g. *k/m* to *k/2m*. With two known responses, this linearity makes it possible to interpolate/extrapolate the mass to an optimal utilization in proportion to ISO 10137. *Figure 10.5*. shows two buildings in CLT and one with added glulam truss in the facade. The result in *Figure 10.5* from *CLT_n* (n-numbers of floors) cannot be used as reference data for e.g. *Truss_n* or another building height *CLT_n+1,* due to that they have other stiffness as well. *CLT_n+1* have two known responses, over and under the requirements, and it is therefore possible to interpolate a needed mass with 100% in proportion to ISO 10137, this mass can be used in order to design the thickness of the concrete slab. *Truss_n*, with added glulam truss system have higher frequency that indicate a higher stiffness and can therefore not be compared with *CLT_n*.

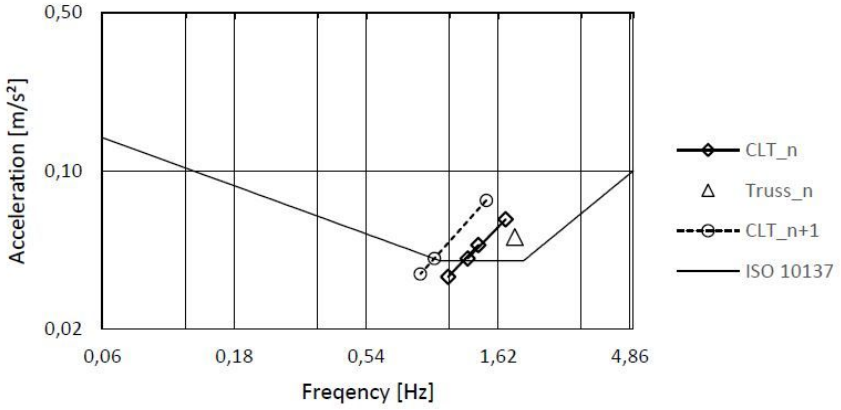


Figure 10.5 Interpolating a needed mass to fulfill requirements

Mode shape:

By examine the mode shape $\Phi_1^2(s)$ in *Figure 10.6*, it is possible to identify the principle of mass placement. The mode shape vary in value from 0 at the supported ground floor up to 1 (top of the building). The timber building acts as a cantilever beam and the value of the mode shape is multiplied with the mass on that level, to obtain equivalent mass see Appendix B. This motivates the strong reducing effect on the dynamic response of mass placed far from the support (in the top of the building). This also motivates the approximation of the average value of *m* over the upper third (x >0.7) of the structure from *EN 1991-1-4*.

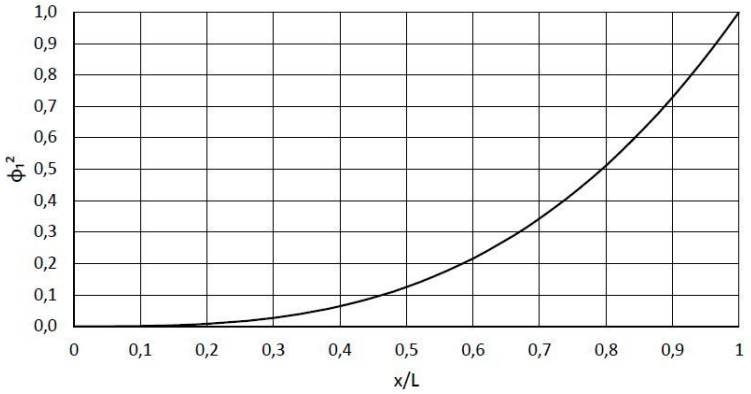


Figure 10.6 Mode shape, mass influence

10.1.2. Stiffness

An increased number of storeys reduces the global stiffness, due to the fact that the structure acts as a more slender/less stiff beam. This result in a reduced frequency and increased acceleration (see trend in *Figure 10.1*). Increasing the stiffness according *Figure 10.2* result in an increased frequency and decreased acceleration, i.e. counteract the response from a floor increase.

10.1.2.1. Geometry

This project have not focused to optimize the plan or size of a structure (due to infinite alternatives) and have only scaled the existing floor plan. An increase of size, result in a more stubby/less slender structure which increases the stiffness and the equivalent mass. A total stiffness can not be calculated in *FEM Design,* but the result in *Figure 10.7* indicate a acceleration decrease due to stiffness and mass increase. While the acceleration decrease, the frequency have a small increase, this patten indicate a "pure" acceleration decrease (described earlier in chapter 10.1 *General behaviour*). The equivalent mass is increased by 50%, see *Table 10.3*, due to pure acceleration behaviour the stiffness can be assumed to increases with 50%. A scaling of the geometry is not a part of the solution of mitigating the dynamic response of the analysed structure due its limitations described in chapter *8.2.1 Geometry.*

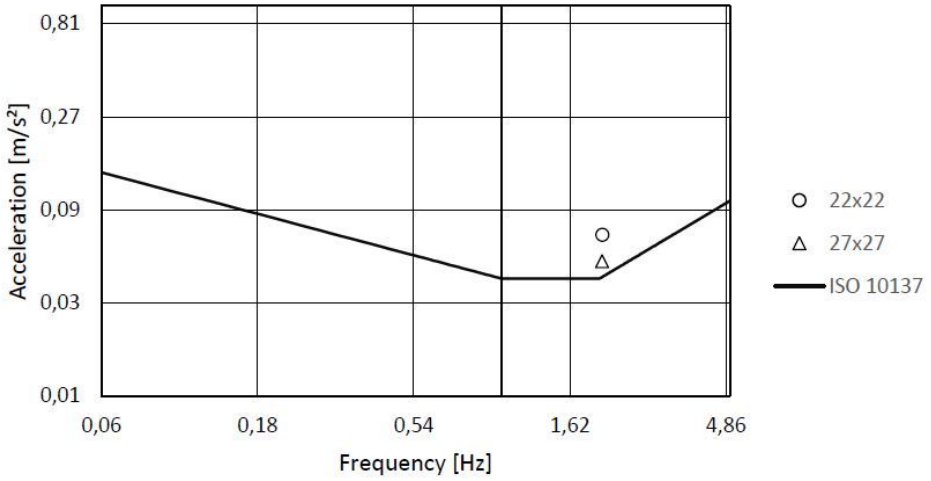


Figure 10.7 Response of size scaling of the structure

Table 10.3 Response of size scaling of the structure

b x h [m]	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
22x22	27.77	2.037	0.067	164%
27x27	42.07	2.034	0.049	120%
diff. [-]	*51.5%*	*0.1%*	*26.9%*	*26.9%*

10.1.2.2. Bracing

A 18 storey structure are tested with the 3 different bracing systems described in chapter 8.2.2 Truss. The result from the 3 different bracing systems showed not a large disparity (3%), and clearly stiffened the structure (40% reduction of acceleration),see *Figure 10.8* and *Table 10.4*. *T1* is a bracing in the center of the facade and therefore with architectural benefits. It was the stiffest analysed truss and therefore the used bracing in the iteration process.

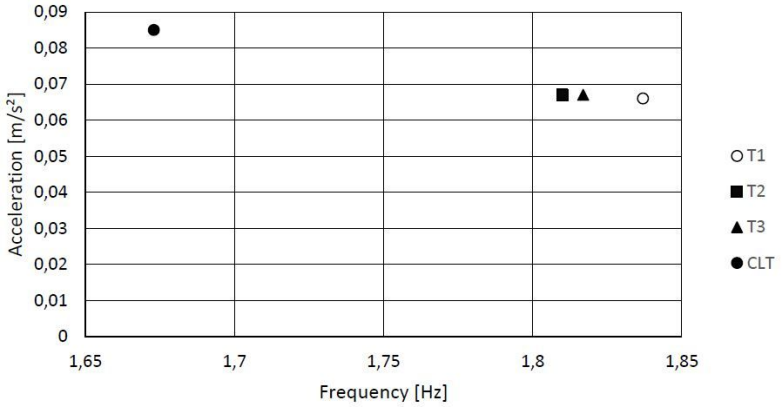


Figure 10.8 Dynamic response of 3 different braced structures

Table 10.4 Utilization of bracing systems/added trusses.

Model	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
CLT_18	27.93	1.673	0.083	213%
T1	32.54	1.837	0.066	165%
T2	32.56	1.810	0.067	168%
T3	32.62	1.817	0.067	168%

10.1.2.3. Wall Thickness

Wall thickness increases in the similar way as the geometry both stiffness and equivalent mass. *Figure 10.9* and *Table 10.5* demonstrate an improvement of the dynamic response of a 18 storey structure with increased wall thickness between 200mm up to 300mm. The test also include the cases where only the facade walls have a thickness of 300mm as well as the case where only the inner walls have a thickness of 300mm, and the remaining walls have have a thickness of 200mm. The frequency increase from 1.673 Hz to 1.811 Hz when the wall thickness is increased from 200mm to 300mm while the acceleration decreases with 26%.

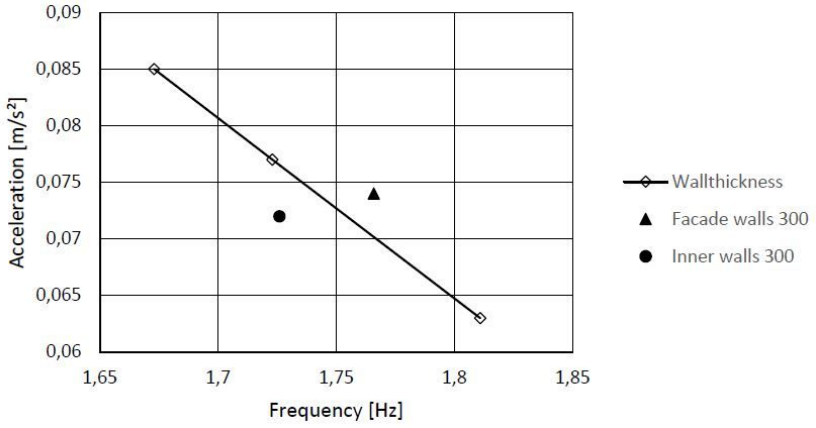


Figure 10.9 Response of increased wall thickness

Table 10.5 Response of increased wall thickness

Wall thickness [mm]	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
200	27.93	1.673	0.083	213%
230	29.92	1.723	0.077	193%
300	34.56	1.811	0.063	158%
Facade walls 300	30.34	1.766	0.074	185%
Inner walls 300	32.15	1.726	0.072	180%

10.1.3. Damping

Adding damping is an effective method to reduce the acceleration due to dynamic response, *Figure 10.10* shows how the acceleration is reduced proportional to the damping factor. The structural damping coefficient is chosen to 1.5% according to *EN 1991-1-4*. A study of high efficient passive damping of 5.5% and 3.0% are also included. The response shows generally a similar behaviour of acceleration with different damping coefficients.

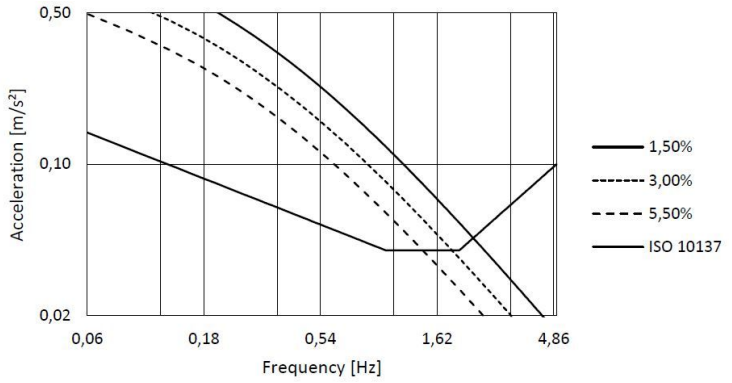


Figure 10.10 Frequency-acceleration response for 3 different damping levels

Figure 10.11 shows that the largest acceleration reduction is in the first percent of added damping and then begins to level out. *Table 10.6* describes that the acceleration reduces with approximately 41% with added damping, from structural damping (1.5%) to a total damping of 4.0% for all 3 different type of structures. The acceleration reduction is largest for light structures with low frequencies (0.8Hz, 37 ton), see difference of acceleration in *Table 10.6* .

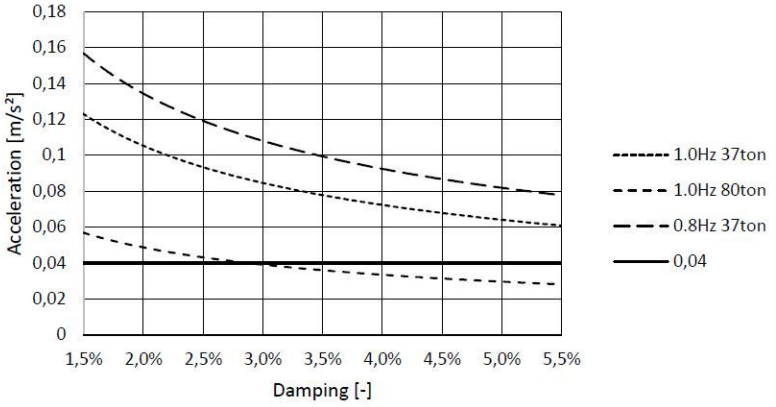


Figure 10.11 Response due to added damping with 3 expected structural masses and frequencies

Table 10.6 Acceleration reduction due to TMD, damping factor 1.5% - 4.0%.

	1.0Hz, 37 ton/m	1.0Hz, 80 ton/m	0.8Hz, 37 ton/m
Diff [m/s^2]	0.0508	0.0235	0.0646
diff [-]	*41.2%*	*41.2%*	*41.1%*

Figure 10.12 describes how the frequency response for a non damped timber structure is reduced to less than ⅜ (*38%*) of the magnification factor due to an added TMD with a total

damping factor of 4.0%. The TMD is optimized to make the double peaks equal, this by reducing and displacing the eigenfreqency of the TMD. By optimizing the frequency, the equivalent damping can be increased from 4.0% to 5.0% without any mass increase. This makes it possible to increase the equivalent damping factor for the same mass, or reduce the mass yet still achieve the same damping factor.

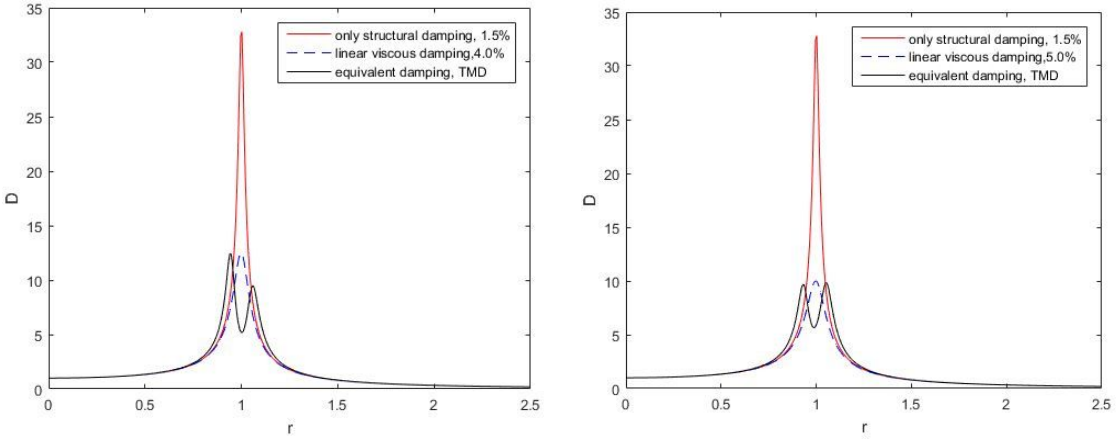


Figure 10.12 Amplitude reduction due to added TMD with equivalent damping factor of 4.0% and 5.0% with optimal frequency

The mass of the damper as mentioned before effects the equivalent damping, *Figure 10.13* describes how the mass of a optimized TMD strives towards unreasonable dimensions for high damping factors.

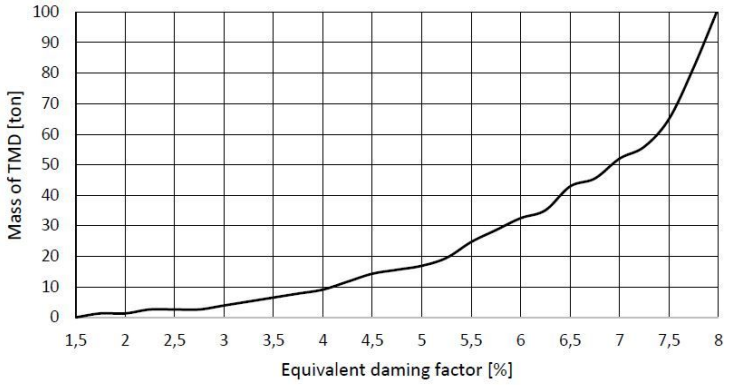


Figure 10.13 Needed mass to fulfill the damping factor

10.1.4. Eurocode

The simplified calculation method of the eigenfrequency according to *EN 1991-1-4* (Eq.38), n=46/h is compared to test result of the frequency with the same building height, varying from 10 to 20 storeys in *Figure 10.14*. The test result have a much higher eigenfrequency but a less steep trend line, which result that these will merge for a lower frequency i.e. a higher structure. *Figure 10.14* describes how the difference between test result and *EN 1991-1-4* decreases while the number of storeys increases.

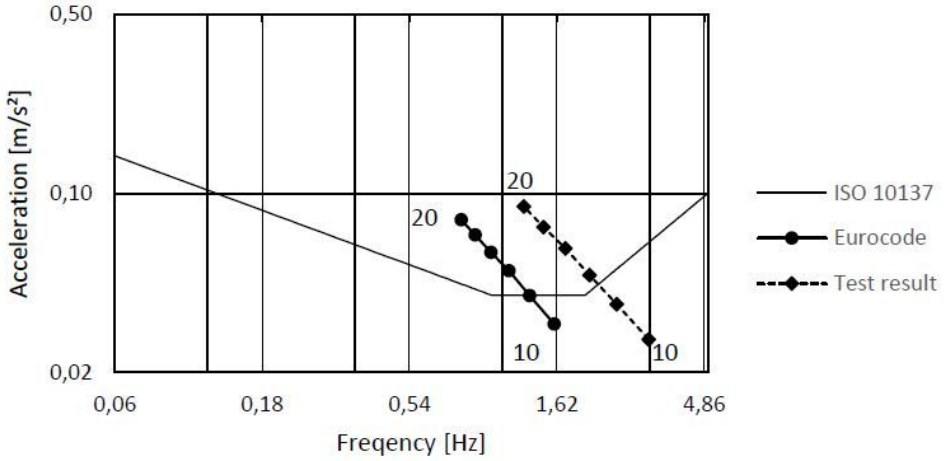


Figure 10.14 Comparison of test result and simplified hand calculations according to EN 1991-1-4

Table 10.7 Difference/dissimilarities from test result to EN 1991-1-4

n-storeys	diff. frequency
10	50.6%
12	47.8%
14	45.2%
16	42.6%
18	39.9%
20	37.3%

10.1.5. Acceleration adjustments

By assuming no activity on the top floor/roof, the measurements have been done at one floor below (*n-1*), according (Eq.51). This assumption have less importance on a high rise building than a low or medium rise building due to the lower ratio of the floor height to total height. *Figure 10.15* compare the impact of determine the acceleration one floor below of a high- and low rise buildings. For a low rise building (10 storeys) the reduction is 15% and for a high rise the reduction is only 7%. This reduction is thought of great importance if the acceleration is slightly above the requirements.

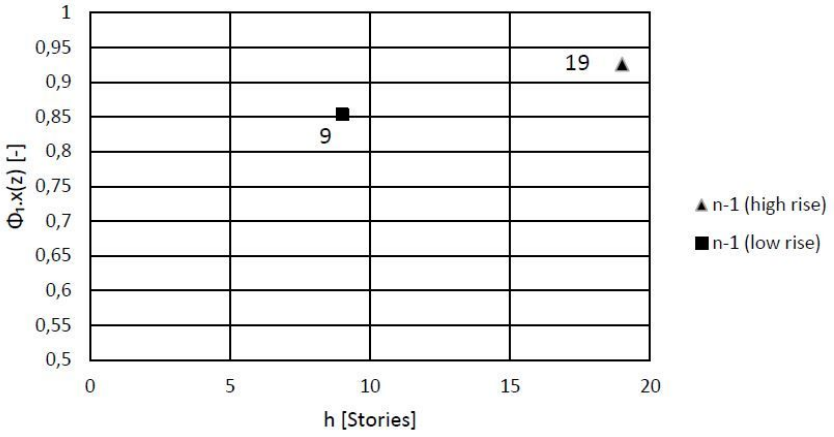


Figure 10.15 Acceleration reduction on floor n-1 due to fundamental flexural mode ($\Phi_{1,x}(z_{n-1})$) on a high rise (20 storey) and low rise (10 storey) building.

The effect of calculating the acceleration one floor below the roof (n-1) is displayed in *Table 10.8*, where the impact decreases with increased building height. The limits for a office building is presented in bracket.

Table 10.8 Variation of acceleration on roof and top floor for different structures

	CLT_14	CLT_20
acceleration **(n)** [m/s^2]	0.054	0.096
acceleration **(n-1)** [m/s^2]	0.048	0.089
diff. [-]	11.1%	7.3%
Limit, residential (office) [m/s^2]	0.041 (0.062)	0.04 (0.06)

10.2. Structural behaviour

This chapter present the combination of the effects of the different parameters in order to achieve an increased building height, each solution is tested until the proportion to ISO 10137 is less than 1 of the structure. Thereafter the next solution according to the flowchart (described in chapter *8. Iteration process*) is introduced and added to the improved structure. The examined structure fulfill the requirements for a 12 storey structure without improvements, the iteration process therefore begins at a initial building height of 14 floors. *Table 10.9* present the acceleration of the structure in proportion to ISO 10137 with different solutions, e.g. HD-f on the top floor or damping.

Table 10.9 Acceleration of the structure in proportion to ISO 10137 of proposed solutions.

Structure	10	12	14	16	18	20	26
CLT	42%	73%	117%	152%	186%	222%	319%
HD-f 120/27	-	-	100%	116%	-	-	-
Solid Concrete	-	-	-	100%	-	-	-
Truss	-	-	-	-	98%	-	-
Wall thickness	-	-	-	-	-	99%	-
Damper	-	-	-	-	-	-	99%

Table 10.10 describes how many floors that is possible to achieve with only one parameter changed or added, and with all parameters combined. 12 storeys fulfill the requirements with no need of improvements. Changing mass and stiffness result in small, similar improvements, adding a damper result in the best individual response with 6 storey increase of the building height. A combination of all 3 parameters results in 14 storey increase of the building height.

Table 10.10 Achievable numbers of storeys with improved parameters, individual and combined

Parameter	Number of storeys	Proportion to ISO 10137
Stiffness [k]	14	90%
Mass [m]	16	100%
Damping [c]	18	100%
Combined [k+m+c]	26	99%

10.2.1. HD-f

The first improvement of the 14 story structure is adding mass in form of HD-f 120/27 F155. The test show that it is enough to add HD-f floors only on the top floor on a 14 story building to manage the acceleration requirements, the resulting acceleration of the structure is equal to the requirements. The added mass from HD-f floor is though not enough for a 16 storey building, HD-f on floor 16 and 15 resulted in 116% acceleration in the structure in proportion to ISO 10137 and solid concrete is therefore used instead.

10.2.2. Solid concrete

In order to increase the mass on each floor and not replace more timber, solid concrete floors are used for 16 storey buildings and upwards. It is sufficient to use a 300mm thick concrete floor on level 15 and 16 to fulfill the requirements. The thickness is interpolated for an optimized mass, the resulting acceleration due to the added mass is equal to the requirements of ISO 10137 for 16 storeys.

The acceleration demands of a building height of 18 storeys was not fulfilled with 300mm solid concrete floors on both floor 18 and 17. In order to minimize the replacement of timber, in combination with practical on site demands, no further timber floors were replaced. Instead, the material of the walls on floor 17 are changed to concrete in order to have a sufficient impact on the equivalent mass, i.e 300mm concrete floor on 18 and 17 together with 200mm concrete walls on floor 17 (the acceleration proportion to ISO 10137 is 117%). This material change of the walls resulted in an increase of the mass with a factor 6,25, see (Eq.68) .

$$\frac{\rho_{Concrete}}{\rho_{CLT}} = \frac{2500}{400} = 6,25 \qquad \text{(Eq.68)}$$

With the gathering of concrete elements at level 17 and 18, different subcontractors on the working site at the same time can be avoided, yet an increased equivalent mass is reached. This solution are nevertheless not sufficient for a 18 storey building, no further timber floors is replaced and concrete core as a solutions is instead evaluated.

10.2.3. Concrete core

A concrete core is a well established solution, used to stabilize the structure of a building. The principle are investigated in order to increase the mass and stiffness of the structure. The implementation of a concrete core to the structure gave no major effect in the dynamic response. This is due to the influence of the modal mass, i.e the extra mass in the lower ⅔ of the building have low impact on the equivalent mass. The core is in the center of the building that result in a small lever arm compared to the facade elements that transfer load and stiffen the building much more. Comparing the improvements of acceleration in proportion to ISO 10137, from a plane CLT structure, a concrete core and added mass in the top (CLT+m) in *Table 10.11* show that added mass have larger impact than the concrete core. Due to the low impact on the dynamic response of the concrete core compared to the added mass of solid concrete, the solution of using a concrete core is not further used in the iteration process.

Table 10.11 Improvements of a 18 storey structure

Model	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
CLT	37.65	1.467	0.074	186%
CLT+m	91.29	1.034	0.047	117%
Concrete core	47.12	1.537	0.057	142%

10.2.4. Stiffness

In order to increase the building height further without replacing more timber elements, the parameter *stiffness* is modified. This is made with the implementation of glulam bracing and increasing wall thickness, the plan and geometry is however not changed.
The most effective glulam truss (*T1*), described in chapter *10.1.4,* is added in the center of the alongside of the facade to increase the stiffness in the structure, see *Figure 10.16*.

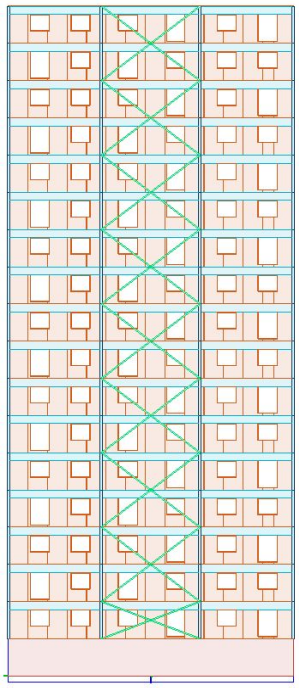


Figure 10.16 T1, the stiffest bracing that was tested

The relationship of increased frequency and decrease of acceleration comparing *CLT* and *T1* is possible to see in *Figure 10.17,* this indicates a structural stiffening due to the bracing. Adding a truss along the facade only made it possible to increase the height with 2 floors to 18 storeys with an acceleration utilization of 99% in proportion to ISO 10137. The wall thickness for all walls is thereafter increased from 200mm to 300mm, this result in an utilization of 93% for 18 floors.

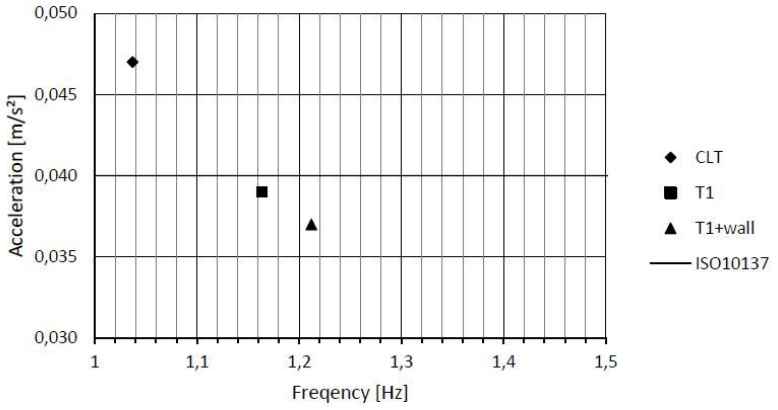


Figure 10.17 Improvements of adding bracing and wall thickness

The same structure is tested for 20 storeys and the acceleration is equal to the requirments. No further stiffness improvements is evaluated in this report, damping is instead evaluated.

10.2.5. Damping

Adding a external damping in form of a TMD (in addition to structural damping and aerodynamic damping) result in further remarkable improvements for the 20 story building with added mass and bracing. A added TMD with a mass of 7.1 *ton* and a total equivalent damping of 3.5% result in large acceleration reduction, even a building height of 26 storeys maintain the acceleration demands, see *Table 10.12*. *Figure 10.18* describes how the acceleration is reduced for the 26 story building with and without a TMD according to EKS 10.

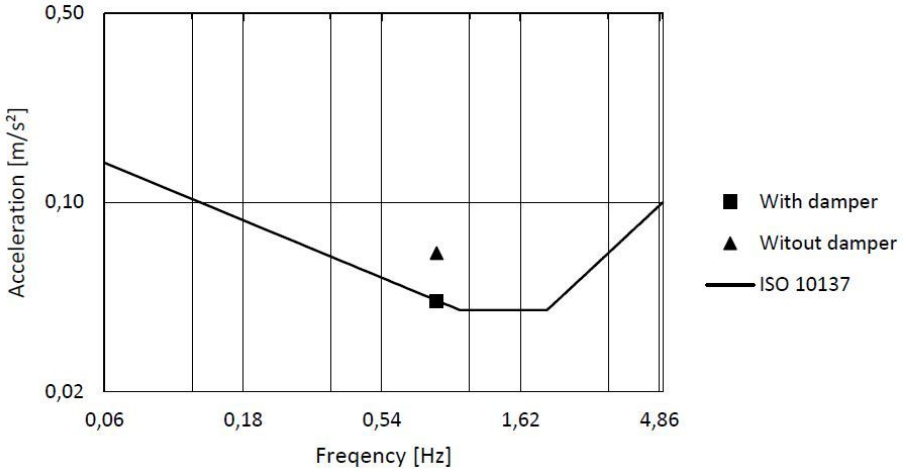


Figure 10.18 Influence of adding a damper on a 26 story building

Damping is also evaluated in combination with added mass (the added stiffness is removed). It is possible to achieve 22 storeys with a acceleration equal to the requirements, which is close to 26 storeys and is therefore a proposed solution. The needed mass of the TMD is 7.3 ton and a internal damping factor of 3.6% is required. Adding a TMD to the initial structure entirely in CLT results in a increase of 6 storeys and is the best individual improvement. This structure have no improved mass or stiffness, the TMD therefore requires the highest mass and internal damping factor to weigh up the missing mass and stiffness improvements.
A higher internal damping factor requires more optimization of the frequency, see *f* in *Table 10.12*. A damping factor of 5.0% requires 3.3% reduction of the frequency compared to a damping factor of 3.5% and 1% reduction of the frequency.

Table 10.12 Influence of adding damper

Model	m_e [ton/m]	m_{TMD} [ton]	ζ_{eq} [-]	ζ_{TMD} [-]	f [-]	Freq. [Hz]	Proportion to ISO 10137
26 storeys [m+k+c]	87.18	7.1	3.5%	3.5%	99%	0.833	99%
22 storeys [m+c]	83.88	7.3	3.7%	3.6%	99.8%	0.801	100%
18 storeys [c]	37.65	9.8	5.1%	5.0%	96.7%	1.467	100%

In *Table 10.13* are all different solutions that meet the requirements from the iteration process summarized. The maximum numbers of floors is 26 storeys with all parameters combined, the initial CLT structure is improved with a solid concrete top story, added truss along the facade, wall thickness is increased with 50% (from 200 to 300mm in all walls) and a TMD in the top. A proposed solution where only damping in combination with added mass (solid concrete top storey) is also presented in the bottom of the table.

Table 10.13 Summarizing of all different solutions

Structure	Number of floors	m_e [ton/m]	Freq. [Hz]	Acc. [m/s^2]	Proportion to ISO 10137
CLT [-]	12	37.05	2.533	0.037	73%
HD-f 120/27 [*m*]	14	54.13	1.783	0.040	100%
Solid Concrete [*m*]	16	83.20	1.276	0.040	100%
Truss [*m*+*k*]	18	95.91	1.164	0.039	98%
Wall thickness [*m*+*k*]	20	96.55	1.151	0.040	100%
Damper [*m*+*k*+*c*]	26	87.18	0.833	0.043	99%
Proposed [*m*+*c*]	22	83.88	0.801	0.044	100%

11. Analysis

This chapter discusses the results from the simulations as well as the plausibility of these in order to decrease the dynamic response of the structure. The proposed and analysed solutions in this study have been adapted to be practically feasible, it is of course impossible to generalise to this extent due to the vast diversity in different projects and buildings. The general assumption made have been that a timber building of this magnitude, today is considered as a role model and inspiration in order to increase the usage of timber and timber products in tall buildings. Based on this fact have the economy partly been breached in some cases and have not been widely investigated. The plausibility of the different solutions is based on reason and experience of professionals in the field.

11.1. Mass

Increasing the equivalent mass via additional mass and mass placement was the fundamental idea of this study, in order to mitigate the dynamic response of a timber structure. It have via simulations and analyses shown to be appropriate method when the dynamic response of the unmodified building is relatively close to the requirements. Considering the practical applicability, adding mass in the top of the structure result in the most effective impact and minimize the mixture of materials through the building process. However adding only mass is not the complete solution in order to significantly increase the building height.

11.1.1. HD-f

The implementation of mass placement increases the building height with 2 storeys up to 14 storeys with the use of HD-f elements, this is in the context not sufficient to be a effective solution in order to increase the building height of tall timber structures alone. However, the method can easily be adopted to the existing building without large changes of the structure. The broad variety of standard products, selection of supplier and the assembling efficiency is beneficial. This can favour the use of HD-f in a large variety of structures as long as the requested equivalent mass is achieved and not too many timber floors are replaced.

11.1.2. Solid concrete

Solid concrete is the most efficient method of mass placement. When changing the material of the top floor to solid concrete, the building height can be increased with 2 further storeys to a height of 16 storeys. This would in fact be the tallest timber structure in the world if it had been built, which demonstrates the great impact of the mode shape an the equivalent mass. To place a mass as large as the one obtained with solid concrete elements in the top of the structure affects the dynamic behavior considerably. Having a top floor that is as heavy as possible without severely affecting strength and stability result in maximized equivalent mass and efficiency, which enables less modifications and the possibility to preserve more timber.

The usage of concrete wall elements on the uppermost storey instead of changing the material on more of the two floors at the top enables the separating of timber and concrete elements. The risk of interfering contractors on the working site are minimized, yet the equivalent mass is increased significantly. The solution with the concrete top storey is used further in this study due to its practical applicability in combination with its effect on the equivalent mass.

11.1.3. Concrete core

Replacing the stairwell/elevator shaft to a concrete core is not that effective and therefore not further investigated. The concrete core have a low acceleration reduction (low mass and stiffness addition) compared to a mass addition of a solid concrete floor in the top. The concrete core was, due to its stiffness and mass properties assumed to influence the structures dynamic response more significantly than it showed in the analysis of the whole structure. This fact indicates that the outdated assumptions of a structure needs a stabilizing concrete core can be reevaluated, which as a result can support the usage of stabilizing CLT cores.

11.2. Stiffness

Change of stiffness compared to change of mass is a lager procedure and a more difficult operation, both in architectural, structural and economically perspectives. Generally, it is hard to increase the stiffness with notable effect with regard to acceleration when a buildings structure and material is determined. Material properties such as Young's modulus for a EWP may be difficult to increase with a significant importance. It is therefore more favourably to change/add parameters of the structure, such as *Plan view, Wall thickness and/or Bracings.*

11.2.1. Geometry

The changing of the plan in a global scale have not been considered as a part of the solution when constructing tall timber structures in this report. The plan is assumed to be more or less determined before the choice of material and type of structure, and is therefore not possible to change significant. The depth of a building with central core is limited by several practical demands e.g. requirements of daylight, adaptation to surrounding buildings, e.t.c. However it can be of interest to investigate the effect of the plan size with regard to the dynamic behaviour.

When changing the depth of the building from 22 to 27 meters, both equivalent mass and stiffness increased. This resulted in a decrease with *31%* of the acceleration which is considerably in the context. Worth mentioning is also the fact that the corresponding area increase of the plan is approximately *50%*. This area increase is quite substantial and perhaps not so realistic or relevant in the design phase of a building. The area increase needed to provide enough mass and stiffness in order to manage the dynamic demands is to large in comparison to the initial building. This supports the initial decision that the change of area is not considered as a solution of the dynamic response of tall timber structures.

11.2.2. Bracing

The effect of added a glulam bracing in the facade was not that remarkably, a total height of 14 stories is obtained with only the implementation of bracing. This is a increase of 2 stories from the initial CLT structure and the least effective modification of the three parameters studied.

The implementation of glulam trusses in the facade correlates well with the scope of timber structures and is beneficial in the definition process of the structure. No timber material is removed in the benefit of other building materials such as concrete, instead, more timber material added and this in the lateral load bearing structure. This is of course advantageously with regard to the classification of timber buildings but is contradictory with the environmental aspects of optimization and effective usage of building materials. It can be question if the goal of more eco friendly buildings primary of timber justifies the non effective usage of the material.

A glulam truss along the facade affects the placing and distribution of windows to a large extent, and by that the functions on the rooms on the interior side. This aspect makes the glulam bracing system unfavourable to introduce when the plan is set and the windows are placed. The introduction of the glulam bracing should be done early in the design phase so that the architect can account for the interference with windows and strive to use the glulam as a aesthetic aspect instead.

11.2.3. Wall thickness

The increase of wall thickness affects the building in the same fashion as the increase of geometry, with both increase mass and stiffness. The response of the structure is linear with decreasing acceleration and increasing frequency when the wall thickness is increased, i.e stiffness increase more than mass. A total reduction of the acceleration with *26%* due to a wall thickness increase with *50%* (from 200mm to 300mm) is not sufficient as a solution on its own for a 18 storey building. This low acceleration reduction with a large material contribution may not be economically or environmentally defensible. The increase of wall thickness interferes with the plan of the building, but it does not affect the intended function of the plan such as a glulam truss with large dimensions would. Therefore it is more favourable to increase the wall thickness in a later design phase. The drawbacks of an increased wall thickness is a ineffective and careless material consumption than glulam bracing, this also increases building costs yet still provides less space in the building.

11.3. Damping

The implementation of damping on the structure have shown to be advantageous with regard to the mitigating of the dynamic response. The damper have largest influence on the amplitude for a equivalent daming up to 4-5%, above this damping factor does the effect decrease and at the same time does the mass of the TMD strives to unreasonable dimensions. The mass limit (10 tons) of the TMD is chosen in this study to be in proportion and reasonable dimensions for the structural together with a size that is economically and environmentally defensible. Adding a damper to the initial structure results in a increase of 6 storeys to a total building height of 18 storeys, and is the best individual improvement. The mass of the TMD is 9.8 ton, 0.2 ton below the limit and have a viscous damping of 5.5%. Optimizing and tuning of the damper in order to equal the peaks (described in chapter 4.6.4) have large impact on the mass of the damper. An optimized TMD can result in large amplitude reduction with a small mass of the damper. This theoretic optimization is very sensitive and is difficult to achieve in practice, the impact of a TMD in practice may therefore not have the same effect. It is difficult to estimate if the viscous damping factor of 5.5% is feasible to achieve for a TMD, due to lack of information from distributor.

The use of dampers are not the most common solution in buildings but is still widely used in especially tall and slender buildings. The building height in this report is not that outstanding with regard to the majority of high rise buildings, but it is very tall with regard to other timber buildings. The use of a passive damping system (TMD) is perhaps motivated by this fact, both with regard to the cost (estimated to circa 1% of the building cost) and the practical applicability. A further benefit of damping is its ability to change the structural response after the building is assembled and the real response is known, and added as an external component.

The decreasing mass of the TMD with an increased number of storeys, which can be obtained in *Table 10.12* can be interpreted as contradictory, a higher building would have had higher acceleration and consequently larger mass of the TMD. The reason behind this result is the influence of the added mass and stiffness in the 22 and 26 storey high buildings. These parameters affect the buildings dynamic response such that the needed mass of the TMD is reduced compared to the initial structure.

The use of a self mass damper (SMD)-system would also perhaps be suitable as a damping system in a timber structure with a applied concrete floor in the top of the structure. In this way is both the mass is utilized as a dynamic parameter to increase the equivalent mass, and as a part of an effective damping system without the need of additional mass. The principle of the SMD is however not extensively used or analysed and further investigations of the system as well as its applicability in combination with tall timber structures should be made. Alternative damping systems could be the tuned liquid damper (TLD) with water tanks on the roof structure with the same eigenfrequency as the structure generating a passive damping effect. The water tanks can also be part of a solar panel system to provide water heating to the building.

11.4. Requirements

The simplified hand calculation (Eq.38) according *EN 1991-1-4* of frequency, 46/h, do not correspond to result of the tested building. The hand calculations is rough approximations for multi story structures higher than 50m, (Eq.38) is not suitable for a timber structure with this height. Investigations are needed for a more appropriate approximation so it can be used as a control of calculated frequency or a rough check of the acceleration requirements.

The requirements of residential buildings are rather strict, it may therefore be more convenient to have offices in the top of the building due to lower requirements. A realistic interpretation is also to do the calculations at the top floor (z_{n-1}) and not on the roof, that reduce the acceleration considerably. This is though not a recommended solution, but a method to fulfill the requirements.

11.5. General analysis

It is possible to build a multistory building mainly in timber, but it may be hard to fulfill the acceleration requirements with only the current timber products for taller buildings.

The effect of combining the three modified parameters, mass, stiffness and damping is remarkable, a total increase of 14 storey is achieved and a total building height of 26 storeys fulfils the acceleration demands. However, this solution is considered to be to hard to implement practically as well as it would be hard to make it economically defendable. The different modifications on the structure can interfere with the scope of the building as well as the aesthetics and economy. The added stiffness contributes least with regard to increased building height, yet interfere with the architectural aspects, the environmental aspects and the economic aspects. It is of interest to build sustainable and using material efficiently, an excessive use of timber in order to increase the stiffness of the structure is therefore not defensible. Using two structural systems and increase the CLT thickness to increase the height of the building with a few storeys is considered as inefficient and leads to an overuse of timber. Stiffness is nevertheless one of the most important factors in dynamics, it is therefor of high importance to design an effective plan and choose suitable materials that creates a structure with high moment of inertia and modulus of elasticity in the early design phase. Due to the disadvantage of the stiffness adjustments, a solution with combined *mass* and *damping* is suggested. These improvements increases the building height to 22 storeys with regard to the acceleration demands. The fact that an increase of the building height of 10 storeys is achieved in combination with its practical benefits makes this solution more feasible than the solution mentioned before. The combination of mass and damping is advantageous due to the cooperation of the two systems. The mass in form of concrete floor on the top contribute to the equivalent mass while it also acts as a rigid support for the damper. In this way is a rigid base for the TMD achieved which would be hard to manage with a CLT floor. The mass of the TMD in the 22 storey building is only 7.1 ton, which entails realistic dimension of the device in this type of structure. It is possible to increase the storey height further if the mass and damping on the TMD can be increased with regard to space and weight.

The building in this report have both a concrete basement and a concrete top floor. The basement of concrete is used in order to secure a rigid base for the structure as well as mitigating moisture leaks from the ground. The top floor in concrete is used in order to insure maximum increase of equivalent mass, yet still supply the structure a purpose in form of a floor or walls. It can be discussed how many actual "timber floors" the building have with regard to the definition of timber buildings. When only using concrete slabs supported on timber walls, the definition of a "timber building" according to *CTBUH* is fulfilled. However when walls of concrete is added as well, the definition: "*A single-material tall building is defined as one where the main vertical and lateral structural elements and floor systems are constructed from a single material*" is challenged. The placement of the interfering concrete elements is though located to the top of the structure in order to stay close to the demands from *CTBUH*. This minimize the spread of concrete elements on each floor yet still increase the equivalent mass in the most efficient way.

It also simplifies the work of manufacturers of timber elements which installs the elements on site. A minimized mix of different construction workers under the construction time avoids interference and inefficiency. Timber building or not, it may be more efficient and sustainable, defined in Roadmap 2050, to use the most suitable material where it is the most efficient, than to fulfill the demands of timber building certifications. This means that the structure fulfills the dynamic requirements regarding acceleration with the use of timber products as the primary material.

The recommended solution of this study is to use the most suitable material that result in efficient impact on the dynamic response. A combination of mass placement and increased damping of a timber structure would allow a building height competitive with prefabricated concrete. In this way would producers and actors in the timber occupation have the opportunity to act in the multi storey structure domain for a more sustainable future in an urbanizing society.

12. Sources of error

In order to accomplish a study of this kind, some simplifications and generalisations have to be made. Some of the larger sources of error due to these simplifications follows here.

- The damper in this study have not been tested together with the structure in the FEM-calculations, it is instead added to the hand calculations of the acceleration according to *EKS 10*. The frequency and equivalent mass is therefore not changed when damping is introduced, only the acceleration is decreased. A more detailed FE-model including damping may be more accurate than a simplified 2-DOF system with a frequency response equivalent to a viscous damping factor.
- The hand calculated acceleration according to the Swedish annex *EKS 10* is simplified to the actual response of the real structure. It could be a more accurate result of the eigenfrequency and acceleration if the used software can simulate acceleration, damping and a randomized time dependent forced motion. This is more difficult and should have been more time consuming.
- The tested bracing systems, e.g. truss and concrete core is not the most optimal. A more optimal structure could affect the response more and adding stiffness could therefore be a more recommended solution.
- The equivalent damping factor of the TMD can be overestimated in the simplified calculations. The optimization of the damper is highly sensitive to small adjustments, therefore it can be difficult to achieve this precise optimization in reality.

13. Conclusion

A summary of the result from this study is presented in *Table 13.1* the conclusions based on this result are:

- The parameter ***mass*** increased the building height from 12 to 16 storeys with regard to the acceleration demands. This fact in combination with its practical applicability makes it the most suitable improvement parameter of the building on its own.
- The parameter ***stiffness*** increased the building height from 12 to 14 storeys with regard to the acceleration demands. The hardship of implementation of increased stiffness in the design phase in combination with its low effect on the demands makes it the least suitable improvement parameter of the building.
- The parameter ***damping*** increased the building height from 12 to 18 storeys with regard to the acceleration demands. This makes damping to the parameter with the highest individual effect on the building height. The implementation of a damper in buildings is however not the most conventional solution, this makes damping to a prominent improvement with the need of further studies of effect and implementation in tall timber buildings.
- When combining the three parameters of improvement, a building height of 26 storeys was achieved with regard to the acceleration demands. This is an increase of 14 storeys compared to the initial building. However, this solution is considered to be to hard to implement practically as well as it would be hard to make it economically defensible.
- A solution with combined *mass* and *damping* improvements is instead suggested. This increases the building height to 22 storeys with regard to the acceleration demands. The fact that an increase of the building height of 10 storeys is achieved in combination with its practical benefits makes this solution feasible. The combination of mass and damping is advantageous due to the cooperation of the two systems. The mass in form of concrete floor on the top contribute to the equivalent mass while it also acts as a rigid support for the damper.

Table 13.1 Summarization of result

Parameter	Number of storeys	Proportion to ISO 10137
Stiffness [k]	14	90%
Mass [m]	16	100%
Damping [c]	18	100%
Combined [k+m+c]	26	99%
Suggested [m+c]	22	100%

14. Further studies

This report have only analysed the dynamic response and horizontal acceleration due to wind induced motions on a structure with a specific geometry and plan. There are several other aspects that needs to be considered to design a multi story timber structure. Further studies are proposed to be considered before a design of a 22 story timber building is realized:

- Analysis of implementation possibilities of dampers in tall timber structures, suitable for timber structures e.g. TMD, SMD
- Analysis of the "realistic effect" of additional damping in tall timber structures
- Control of remaining demands in ULS and SLS of the structure e.g. load bearing capacity

15. References

Alter, Lloyd. (2018) No Timber Towers Happening in America as Concrete Industry Blocks Tall Wood from International Building Code. *TreeHugger*. www.treehugger.com/sustainable-product-design/no-timber-towers-happening-america-concrete-industry-blocks-tall-wood-international-building-code.html. (2018-02-03)

Boverket (1997). *Boverkets handbok om snö- och vindlast,* (*BSV 97*). Utgåva 2.

Boverket. (2015). *Boverkets föreskrifter om ändring i verkets föreskrifter och allmänna råd (2011:10) om tillämpning av europeiska konstruktionsstandarder (eurokoder)*, (*EKS 10*) Sverige.

Boverket. (2016) *Om Boverkets Konstruktionsregler, EKS.* www.boverket.se/sv/byggande/regler-for-byggande/om-boverkets-konstruktionsregler-eks/. (2018-02-23)

Burström, Per Gunnar. (2007) *Byggnadsmaterial.* Studentlitteratur

Connor, J., Laflamme, S. (2014) *Structural motion engineering.* Springer

Craig,R. R., Kurdila, A. J. (2006) *Fundamentals of structural dynamics.* Wiley

Ctbuh (no date) *CTBUH - Search*, *Council on Tall Buildings and Urban Habitat*. http://www.ctbuh.org/AboutCTBUH/SearchCTBUHWebsite/tabid/37/language/en-US/Default.aspx?q=definition#gsc.tab=0&gsc.q=definition&gsc.page=1. (2018-04-23)

Engström, B. (no date) *Distribution of horizontal load on bracing elements.* Chalmers University of Technology

Flow engineering.(no date) *Technical information and products* ftp://ceres.udc.es/ITS_Caminos/Optativas/Calculo_Dinamico_Estructuras/teoria/sismos/TMD/technical_information.pdf (2018-04-15)

Flow Engineering (no date) *Tuned Mass Dampers.* https://flow-engineering.com/tuned-mass-dampers/ (2018-05-02)

Foster, Robert M., et al. (2016) Proposal for Defining a Tall Timber Building. *Journal of Structural Engineering*, vol. 142, no. 12, 2016, p. 02516001., doi:10.1061/(asce)st.1943-541x.0001615. (2018-04-23)

Hellekant, Johan. (2016) Ny Trend: Smarta Höghus i Trä – Uppåt 20 Våningar. *SvD* www.svd.se/ny-trend-smarta-hoghus-i-tra--uppat-20-vaningar#sida-8. (2018-02-05)

Holmes D.J. (2001) *Wind Loading of Structures*. Spon Press

ISO 10137, (2007). SS-ISO 10137:2008. *Bases for Design of Structures - Serviceability of Buildings and Walkways against Vibrations*. Stockholm: SIS Förlag AB

ISO 6987, (1984). *Guidelines for the evaluation of the response of occupants of fixed structures,especially buildings and off-shore structures, to low-frequency horizontal motion (0,063 to 1 Hz)*.Switzerland: International Organization for Standardization

Johann, Ferrareto A., et al. (2015) Wind-Induced Motion on Tall Buildings: A Comfort Criteria Overview. *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 142, 2015, pp. 26–42. DOI:10.1016/j.jweia.2015.03.001.

Kermani, Abdy. *Structural Timber Design*. Knovel, 2007.

Kidokoro,R. (2008) Self Mass Damper (SMD): Seismic Control System Inspired by the Pendulum Movement of an Antique Clock. | *The 14th World Conference on Earthquake Engineering;* October 12-17, 2008, Beijing, China

Lago, A., Wood, A., Trabucco, D. (2018) *Damping Technologies for Tall Buildings: New Trends in Comfort and Safety.* Elsevier. https://www.elsevier.com/physical-sciences/engineering-and-technology/article-selections/damping-technologies-for-tall-buildings-new-trends-in-comfort-and-safety (2018-04-10)

Martinssons. (no date) *KL-Trä* www.martinsons.se/byggprodukter/kl-tra. (2018-03-12)

Mendis, P., Ngo, T., Haritos, N., Hira, A., Samali, B. & Cheung, J. (2007) *Wind Loading on Tall Buildings*. EJSE Special Issue: Loading on Structures.

Ottosen, Niels Saabye., and Hans Petersson. (2007) *Introduction to the Finite Element Method*. Prentice Hall.

Ramage, Michael, et al. (2016) Super Tall Timber: Design Research for the next Generation of Natural Structure. *The Journal of Architecture*, vol. 22, no. 1, Feb. 2017, pp. 104–122., doi:10.1080/13602365.2016.1276094.

Reynolds, Thomas, et al. (2015) Ambient Vibration Tests of a Cross-Laminated Timber Building. *Proceedings of the Institution of Civil Engineers - Construction Materials*, vol. 168, no. 3, 2015, pp. 121–131., doi:10.1680/coma.14.00047.

SS-EN 1991-1-4 (2005). Eurokod 1: Laster på bärverk – Del 1-4: Allmänna laster – Vindlast. Stockholm: SIS Förlag AB

SS-EN 1990 (2002). Eurokod - Grundläggande dimensioneringsregler för bärverk. Stockholm: SIS Förlag AB

Stafford Smith, B. & Coull, A. (1991). Tall building structures: analysis and design. New York: Wiley

StruSoft AB. (2018) *FEM-Design*. www.strusoft.com/products/fem-design. (2018-01-24)

Svenskt Trä. (no date) *Trä Är Ett Hållbart Byggmaterial*. www.svenskttra.se/om-tra/att-valja-tra/tra-och-miljo/tra-ar-ett-hallbart-byggmaterial/. (2018-02-17)

Swedish wood. (2015). *Design of timber structures*. Volume 1. Stockholm: Swedish wood.

TräGuiden (no date) *Pelar-Balkstommar - Generellt*. www.traguiden.se/planering/planera-ett-trabygge/byggsystem/byggnader-med-stor-spannvidd/pelar-balkstommar---generellt/. (2018-03-02)

Appendix A

Dynamic response

According to EKS and EC 1991-1-4

2018-02-15
Joel Sjöholm
Fredrik Ivarsson

The calculation methods assume linear elastic structures and normal modes,
The dynamic properties is described by:

- eigenfrequens
- modeshape
- eqvivalent mass
- logaritmic decrement for damping

Input

$n := 26$ Numbers of floors

$cc := 2.9\ m$ Ceiling height $h := n \cdot cc = 75.4\ m$ Building height

$b := 22\ m$ Width of building (depth is same as width)

$n_1 := 0.872\ Hz$ First eigenfrequency

$n_{i.y} := 0.884\ Hz$ First eigenfrequency, other direction

$v_b := 25\ \frac{m}{s}$ Wind speed in Gothenburg
(EKS 10 fig. C-4)

$y := 1$ Wind return period, "1" (ISO 10137) or "5" (ISO 6897) years

Terraintype: 3

$z_0 := 0.3\ m$ Roughness length

$c_0(z) := 1.0$

$z_{0.11} := 0.05\ m$ EC1991-1-4 (Tab.4.1)

$\delta_d := 0\% \cdot 2 \cdot \pi$ TMD, added damping

$m_e := 38.18 \cdot 10^3\ \frac{kg}{m}$ Equivalent mass

$z := h$ Assuming that the building is on flat ground/no hill

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Constans from EKS

$\rho := 1.25 \ \frac{kg}{m^3}$ Density air

$h_{ref} := 10 \ m$

Wall, EC 1991-1-4 (tab.7.1), D+(-E)

$$c_{pe.10} := 1.3 + \frac{\left(\frac{h}{b}\right) - 1}{5 - 1} \cdot (1.5 - 1.3) = 1.421$$

$c_f := c_{pe.10}$

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Wind-load

$$I_v(z) := \frac{1}{c_0(z) \cdot \ln\left(\frac{z}{z_0}\right)}$$ Turbulence intensity, EC 1991-1-4 (4.7)

$$k_r := 0.19 \cdot \left(\frac{z_0}{z_{0.11}}\right)^{0.07} = 0.215$$ EC 1991-1-4 (eq.4.5)

$T_e := 5$ Number of years, wind return period.
5 years acc. ISO 6897

$$v_{b.Ta} := 0.75 \cdot v_b \cdot \sqrt{\left(1 - 0.2 \cdot \ln\left(-\ln\left(1 - \frac{1}{T_e}\right)\right)\right)} = 21.378 \ \frac{m}{s}$$

$$c_r := k_r \cdot \ln\left(\frac{z}{z_0}\right)$$ EC 1991-1-4 (4.4)

$$v_m(h) := c_0(z) \cdot c_r \cdot v_{b.Ta}$$ EC 1991-1-4 (4.3)

$$v_m := v_m(h) = 25.449 \ \frac{m}{s}$$

$$q_m(h) := \frac{1}{2} \cdot \rho \cdot {v_m}^2 = 404.776 \ Pa$$ Mean pressure

Eigenfrequencies

$$uc := Hz \cdot m$$

$$n_{1.EC} := \frac{46}{h} \cdot uc$$ EC 1991-1-4 (F.2)

$$n_{1.x} := \begin{cases} n_1 & \text{if } n_1 > 0 \\ n_{1.EC} & \text{else} \end{cases} = 0.872 \ \frac{1}{s}$$

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Maximum acceleration

According to EKS 6.3.1(1)

$uc := m$

$$y_c := \frac{150 \cdot n_{1.x}}{v_m} \cdot uc = 5.14$$ [m]

$$F := \frac{4 \cdot y_c}{\left(1 + 70.8 \cdot {y_c}^2\right)^{\frac{5}{6}}}$$

$$\Phi_b := \frac{1}{1 + \frac{3.2 \cdot n_{1.x} \cdot b}{v_m}}$$

$$\Phi_h := \frac{1}{1 + \frac{2 \cdot n_{1.x} \cdot h}{v_m}}$$

$\delta_s := 1.5\% \cdot 2\ \pi = 0.094$

Recomended mechanical damping for timber structures with mechanical fastners, converted to logaritmic decrement

$$\delta_a := \frac{c_f \cdot \rho \cdot b \cdot v_m}{2 \cdot n_{1.x} \cdot m_e} = 0.015$$

Aerodynamic damping cofficient
EC 1991-1-4 (F.18)

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$$B := \sqrt{\exp\left(-0.05 \cdot \left(\frac{h}{h_{ref}}\right) + \left(1 - \frac{b}{h}\right) \cdot \left(0.04 + 0.01 \cdot \left(\frac{h}{h_{ref}}\right)\right)\right)} = 0.863$$

$$R := \sqrt{\left(\frac{2 \cdot \pi \cdot F \cdot \Phi_b \cdot \Phi_h}{\delta_s + \delta_a + \delta_d}\right)} = 0.325$$

$$v := n_{1.x} \cdot \frac{R}{\sqrt{B^2 + R^2}} = 0.307 \ \frac{1}{s}$$

$$\Phi_{1.x}(h) := \left(\frac{(z - cc)}{h}\right)^{1.5}$$

acceleration one floor below roof (last inhabitaed floor)

$$\sigma_{a.x}(z) := \frac{3 \cdot I_v(h) \cdot R \cdot q_m(h) \cdot b \cdot c_f \cdot \Phi_{1.x}(h)}{m_e} = 0.055 \ \frac{m}{s^2}$$

$$\sigma_{a.x} := \sigma_{a.x}(z)$$

$$T := 600 \ s$$

$$k_p := \text{if} \left(\sqrt{2 \cdot \ln(v \cdot T)} + \frac{0.6}{\sqrt{2 \cdot \ln(v \cdot T)}}\right) > 3 \ \left\| \sqrt{2 \cdot \ln(v \cdot T)} + \frac{0.6}{\sqrt{2 \cdot \ln(v \cdot T)}} \right. \ \text{else} \ \left\| 3 \right. = 3.416$$

$$x_{a.max}(z) := \text{if } y = 1 \ \left\| 0.72 \cdot k_p \cdot \sigma_{a.x} \right. \ \text{else if } y = 5 \ \left\| k_p \cdot \sigma_{a.x} \right. = 0.1355 \ \frac{m}{s^2}$$

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ISO 10137 requirements

log-log interpolation:

$$V_x := \begin{bmatrix} 0.06 \\ 1 \\ 2 \\ 5 \end{bmatrix} \qquad V_y := \begin{bmatrix} 0.14 \\ 0.04 \\ 0.04 \\ 0.1 \end{bmatrix}$$

$$k := 0 .. 2$$

$$\Delta V_{x_k} := \text{for } j \in 0 .. 2 \left\| \log\left(V_{x_{k+1}}\right) - \log\left(V_{x_k}\right) \right| = \begin{bmatrix} 1.222 \\ 0.301 \\ 0.398 \end{bmatrix}$$

$$\Delta V_{y_k} := \text{for } j \in 0 .. 2 \left\| \log\left(V_{y_{k+1}}\right) - \log\left(V_{y_k}\right) \right| = \begin{bmatrix} -0.544 \\ 0 \\ 0.398 \end{bmatrix}$$

$$m_{xy_k} := \left\| \frac{\Delta V_{y_k}}{\Delta V_{x_k}} \right| = \begin{bmatrix} -0.445 \\ 0 \\ 1 \end{bmatrix}$$

$$k_{xy_k} := \text{for } j \in 0 .. 2 \left\| \frac{V_{y_k}}{V_{x_k}^{m_{xy_k}}} \right| = \begin{bmatrix} 0.04 \\ 0.04 \\ 0.02 \end{bmatrix}$$

$$uc := s$$

$$n_{1.x} := n_{1.x} \cdot uc = 0.872$$

$$uc := \frac{s^2}{m}$$

$$x_{a.max} := x_{a.max}(z) \cdot uc$$

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$$x_{max} := \begin{cases} \text{if } n_{1.x} < 1 \\ \quad \left\| k_{xy_0} \cdot n_{1.x}^{\,m_{xy_0}} \right. \\ \text{else if } 1 \le n_{1.x} \le 2 \\ \quad \left\| k_{xy_1} \cdot n_{1.x}^{\,m_{xy_1}} \right. \\ \text{else if } n_{1.x} > 2 \\ \quad \left\| k_{xy_2} \cdot n_{1.x}^{\,m_{xy_2}} \right. \\ \text{else} \\ \quad \left\| \text{“Not in the scope of ISO 10137”} \right. \end{cases} = 0.043$$

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Plott

$i := 0 .. 123$

$$x := 0.06, 0.1 .. 5 = \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ \vdots \\ 123 \end{matrix} \begin{bmatrix} 0.06 \\ 0.1 \\ 0.14 \\ 0.18 \\ 0.22 \\ 0.26 \\ 0.3 \\ 0.34 \\ 0.38 \\ 0.42 \\ 0.46 \\ 0.5 \\ \vdots \end{bmatrix}$$

$$y_i := \left\| \begin{array}{l} \text{for } j \in 0 .. 123 \\ \quad \left\| \begin{array}{l} \text{if } x_i < 1 \\ \quad \left\| k_{xy_0} \cdot x_i^{m_{xy_0}} \right. \\ \text{else if } 1 \le x_i \le 2 \\ \quad \left\| k_{xy_1} \cdot x_i^{m_{xy_1}} \right. \\ \text{else if } x_i > 2 \\ \quad \left\| k_{xy_2} \cdot x_i^{m_{xy_2}} \right. \end{array} \right. \end{array} \right. = \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ \vdots \\ 123 \end{matrix} \begin{bmatrix} 0.14 \\ 0.112 \\ 0.096 \\ 0.086 \\ 0.078 \\ 0.073 \\ 0.068 \\ 0.065 \\ 0.062 \\ 0.059 \\ 0.057 \\ 0.054 \\ \vdots \end{bmatrix}$$

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Risk of Vortex shedding or Galloping

$v_m := c_0(z) \cdot c_r \cdot v_b = 29.76 \ \frac{m}{s}$

Vortex shedding

$St := 0.12$ Strouhal number, EC 1991-1-4, fig (E1)

$v_{crit.i} := \frac{b \cdot n_{i.y}}{St} = 162.067 \ \frac{m}{s}$ EC 1991-1-4 (E.2)

$Vortex_shedding := \text{if } v_{crit.i} > 1.25 \cdot v_m$
$\quad \text{“No risk”}$
else
$\quad \text{“Risk”}$

Galloping

$Sc := \frac{2 \cdot \delta_s \cdot m_e}{\rho \cdot b^2}$ Scruton number, EC 1991-1-4 E. 1.3.3 (1)

$a_G := 1.2$ EC 1991-1-4, Tab. E.7

$v_{CG} := \frac{2 \cdot Sc}{a_G} \cdot n_{i.y} \cdot b$ EC 1991-1-4 (E18)

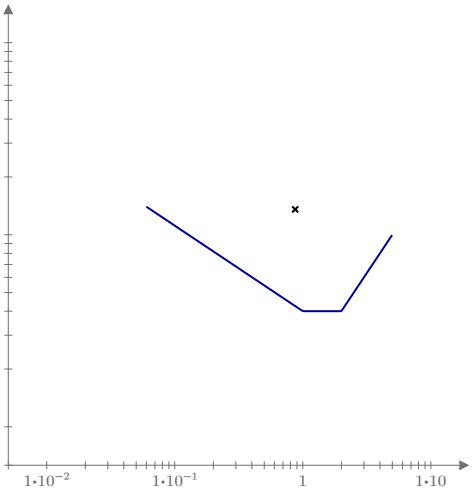
$Galloping := \text{if } v_{CG} > 1.25 \cdot v_m$
$\quad \text{“No risk”}$
else
$\quad \text{“Risk”}$

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Result

Number of floors	Height	Eigenfrequency
$n = 26$	$h = 75.4\ m$	$n_{1.x} = 0.872$

Acceleration	ISO 10137	
$x_{a.max} = 0.136$	$x_{max} = 0.043$	$\frac{x_{a.max}}{x_{max}} = 319\%$



$Vortex_shedding =$ "No risk"

$\frac{v_{crit.i}}{1.25 \cdot v_m} = 4.357$

$Galloping =$ "No risk"

$\frac{v_{CG}}{1.25 \cdot v_m} = 10.365$

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Appendix B

	20		
Floor	mass [ton]	Φ_1^2	$m*\Phi_1^2$
0	133,238	0,000	0,00
1	425,726	0,000	0,05
2	113,282	0,001	0,11
3	113,478	0,003	0,38
4	113,57	0,008	0,91
5	113,518	0,016	1,77
6	113,258	0,027	3,06
7	113,478	0,043	4,87
8	113,401	0,064	7,26
9	113,514	0,091	10,34
10	113,285	0,125	14,16
11	113,648	0,166	18,91
12	113,288	0,216	24,47
13	113,517	0,275	31,17
14	113,907	0,343	39,07
15	113,285	0,422	47,79
16	113,648	0,512	58,19
17	113,288	0,614	69,57
18	113,517	0,729	82,75
19	113,907	0,857	97,66
20	88,9535	1,000	88,95
		m_e:	37,62

$\Phi_1^2=((n_i/n_{20})^{1,5})^2$

$m_e=\sum(m*\Phi_1^2)/((\sum\Phi_1^2)*2,9)$

Appendix C

```
clc
clear fig
clear all
close all
%Input
f1=0.801; %[Hz]frequency of the structure
m1=1462; %[ton] mass of the structure
zeta_s=1.5; %[%] damping factor of timber structure
p1=10; %[kN] force
my=0.001; %[%]start value of mass ratio of TMD-sturucter
zeta_eq=3.7;%[%]equivalent damping
zeta_d=3.5; %[%]damping in TMD
f_opt=0.988;
%Calculations
n=400; %number of points
r=linspace(0,2.5,n);%domain of frequency ratio
w1=2*pi*f1; %[rad/s]
omega=r*w1; %frequency of forced motion
zeta_s=zeta_s/100;
zeta_eq=zeta_eq/100;
zeta_d=zeta_d/100;
k1=m1*w1.^2; %[kN/m]sfiffness of the structure
u_st=p1/k1; %static deflection of the primary system
%frequency response of only structural damping
for i=1:n;
Z1(i)=k1+omega(i)*2*zeta_s*sqrt(k1*m1)*1i-m1*omega(i).^2;
U1(i)=p1/Z1(i);
D1(i)=abs(U1(i))/u_st;
end
%frequency response of viscous damping
for i=1:n;
Z2(i)=k1+omega(i)*2*zeta_eq*sqrt(k1*m1)*1i-m1*omega(i).^2;
U2(i)=inv(Z2(i))*p1;
D2(i)=abs(U2(i))/u_st;
end
%frequency response of TMD
limit=max(D2);%The seeked magnification
%with a determined viscous damping factor
Dmax=max(D1);
count=0;
%find m2
while Dmax>limit
m2=my*m1;%mass of TMD due to ratio my
k2=f_opt*m2*w1.^2; %[kN/m] stiffness of spring to TMD
c1=zeta_s*2*sqrt(k1*m1); %structural damping coefficent
c2=zeta_d*2*sqrt(k2*m2);%TMD damping coefficent
%matirx form
M=[m1 0;0 m2];
K=[k1+k2 -k2;-k2 k2];
C=[c1+c2 -c2;-c2 c2];
```

```
P=[p1;0];
for i=1:n
Z3=[K+C*omega(i)*1i-M*omega(i).^2];
U3=abs((inv(Z3))*P);
D3(i)=U3(1)/u_st;
end
Dmax=max(D3);
count=count+1;
if count>1000, break, end
my=my+0.001;
end
my=m2/m1;
D_ratio_1=max(D3)/max(D1);%magnification reduction from no damping
D_ratio_2=max(D3)/max(D2);%diff from viscous damping
%%Ans.
count %numbers of iterations
m2 %mass of TMD
my %mass ratio
D_ratio_1%magnification reduction from no damping
D_ratio_2%diff from viscous damping
figure(1)
clf
plot(r,D1,'r',r,D2,'b--',r,D3,'k')
set(gcf,'color','w');
legend('only structural damping, 1.5%','linear viscous
 damping,4.0%','equivalent damping, TMD')
xlabel('r')
ylabel('D')
```

```
count =

     6


m2 =

    8.7720


my =

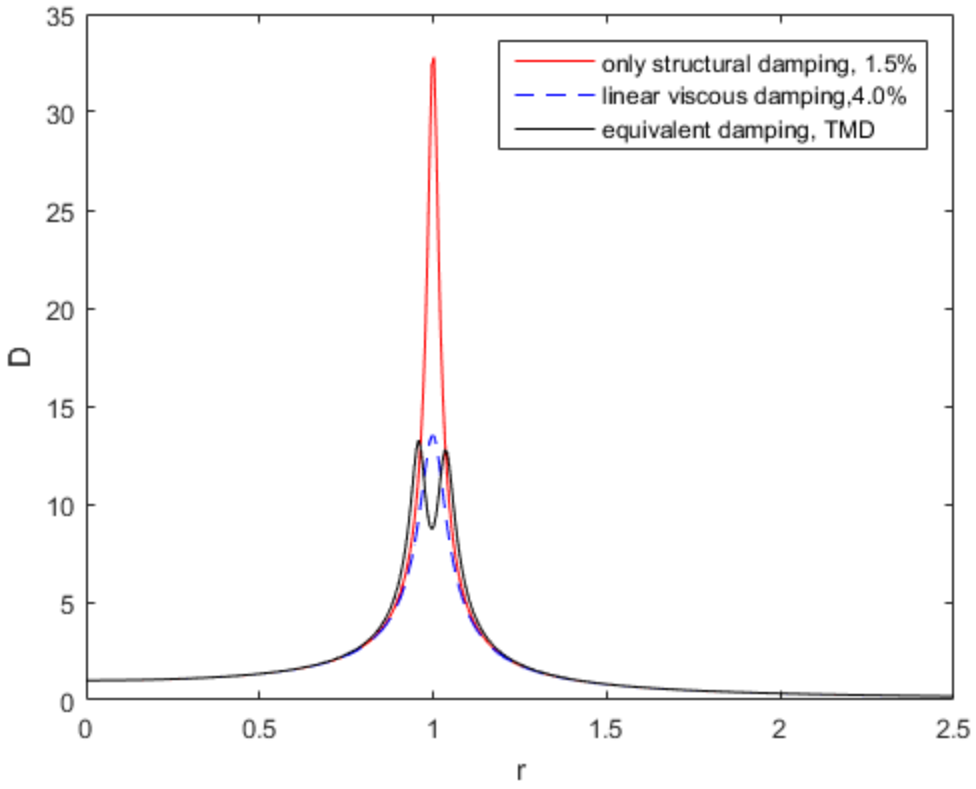
    0.0060


D_ratio_1 =

    0.4041


D_ratio_2 =

    0.9821
```



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Appendix D

Verification of FE-model

Solid CLT Structure

2018-05-24
Joel Sjöholm
Fredrik Ivarsson

Input

Constants building:

$n := 20$	Number of floors
$h_1 := 2.9 \cdot m$	Height of each floor
$h_{tot} := n \cdot h_1 = 58\ m$	Total building height
$d_1 := 22 \cdot m$	Width, x and y

Wind load

Constants wind load:

$v_b := 25\ \frac{m}{s}$ Wind speed gothenburg

$z_0 := 0.3\ m$ Terrain type 3

$z_{min} := 5\ m$

Minimum height

$z_{0.11} := 0.05\ m$

$k_r := 0.19 \cdot \left(\frac{z_0}{z_{0.11}}\right)^{0.07} = 0.215$ Terrain factor

$c_0 := 1$

$\rho_{air} := 1.25\ \frac{kg}{m^3}$ Density air

$q_b := \frac{\rho_{air} \cdot {v_b}^2}{2} = 390.625\ Pa$

Reference mean velocity pressure

$hd := \frac{h_{tot}}{d_1} = 2.636$ height/depth-ratio

$c_{pe} := 1.3 + \frac{hd - 1}{5 - 1} \cdot (1.5 - 1.3) = 1.382$ Pressure coefficient

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$$level := 20..1 = \begin{bmatrix} 20 \\ 19 \\ 18 \\ 17 \\ 16 \\ 15 \\ 14 \\ 13 \\ 12 \\ 11 \\ 10 \\ 9 \\ \vdots \end{bmatrix} \qquad i := 0..19$$

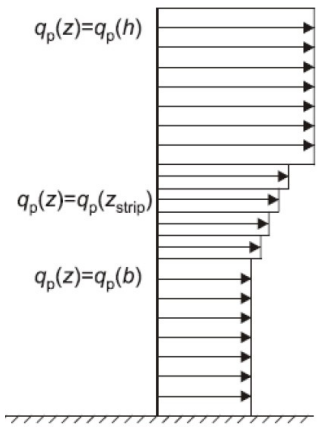
Vector of storeys

Vector of storey heigt

$$h_i := level_i \cdot h_1 = \begin{bmatrix} 58 \\ 55.1 \\ 52.2 \\ 49.3 \\ 46.4 \\ 43.5 \\ 40.6 \\ 37.7 \\ 34.8 \\ 31.9 \\ 29 \\ 26.1 \\ \vdots \end{bmatrix} m$$

Vector of reference height

$$z_{e_i} := \begin{array}{l} \text{if } h_i \le d_1 \\ \quad \| \; d_1 \\ \text{else if } h_i \ge (h_{tot} - d_1) \\ \quad \| \; h_{tot} \\ \text{else} \\ \quad \| \; h_i \end{array} = \begin{bmatrix} \vdots \\ 58 \\ 58 \\ 58 \\ 58 \\ 58 \\ 34.8 \\ 31.9 \\ 29 \\ 26.1 \\ 23.2 \\ 22 \\ 22 \\ \vdots \end{bmatrix} m$$



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Turbulence intensity

$$I_{v_i} := \frac{1}{c_0 \cdot \ln\left(\frac{z_{e_i}}{z_0}\right)} = \begin{bmatrix} \vdots \\ 0.21 \\ 0.214 \\ 0.219 \\ 0.224 \\ 0.23 \\ 0.233 \\ 0.233 \\ 0.233 \\ 0.233 \\ 0.233 \\ 0.233 \\ 0.233 \\ \vdots \end{bmatrix}$$

Exposure factor

$$c_{e_i} := \left(1 + 6 \cdot I_{v_i}\right) \cdot \left(k_r \cdot \ln\left(\frac{z_{e_i}}{z_0}\right) \cdot c_0\right)^2 = \begin{bmatrix} 2.751 \\ 2.751 \\ 2.751 \\ 2.751 \\ 2.751 \\ 2.751 \\ 2.751 \\ 2.751 \\ 2.372 \\ 2.309 \\ 2.242 \\ 2.168 \\ \vdots \end{bmatrix}$$

Peak velocity pressure

$$q_{p.z_i} := c_{e_i} \cdot q_b = \begin{bmatrix} 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 1.075 \\ 0.926 \\ 0.902 \\ 0.876 \\ 0.847 \\ \vdots \end{bmatrix} \ \textit{kPa}$$

Wind pressure

$$w_{e.z_i} := q_{p.z_i} \cdot c_{pe}$$

$$Q_{w.z_i} := \begin{cases} \text{if } i = 0 \\ \quad w_{e.z_i} \cdot \frac{h_1}{2} \cdot d_1 \\ \text{else} \\ \quad \left(w_{e.z_{i-1}} + w_{e.z_i}\right) \cdot \frac{h_1}{2} \cdot d_1 \end{cases} = \begin{bmatrix} 47.371 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 88.205 \\ 80.597 \\ 78.365 \\ 75.939 \\ \vdots \end{bmatrix} \ \textit{kN}$$

Total horizontal wind load on each floor

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Vertical load

Cross laminated timber:

$\rho_{clt} := 400 \ \frac{kg}{m^3}$ Density CLT

$g_{k.clt} := \rho_{clt} \cdot g = 3.923 \ \frac{kN}{m^3}$ Self weight CLT

Concrete:

$\rho_{c45} := 2500 \ \frac{kg}{m^3}$ Density CLT

$g_{k.c45} := \rho_{c45} \cdot g = 24.517 \ \frac{kN}{m^3}$ Self weight concrete

Building parameters:

$A_{floor} := d_1{}^2 = 484 \ m^2$ Area of each floor level

$t_{floor} := 0.23 \ m$ Thickness floor structure

$t_{wall} := 0.2 \ m$ Thickness wall structure

$l_{wall} := 213.2 \ m$ Total wall length of each floor

Wall areas:

$A_1 := 44.49 \ m^2$
$A_2 := 55.62 \ m^2$
$A_3 := 35.64 \ m^2$
$A_4 := A_3$
$A_5 := A_2$
$A_6 := A_1$

$A_a := 45.9 \ m^2$
$A_b := 52.47 \ m^2$
$A_c := 7.1 \ m^2$
$A_d := 38.88 \ m^2$
$A_e := 12.96 \ m^2$
$A_f := A_b$
$A_g := A_a$

Contributing wall area without windows

$A_{wall} := A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_a + A_b + A_c + A_d + A_e + A_f + A_g = 527.18 \ m^2$

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Self weights:

$G_{k.wall} := g_{k.clt} \cdot A_{wall} \cdot t_{wall} = 413.59\ kN$ Total dead load of walls on one floor level

$g_{k.wall} := \frac{G_{k.wall}}{l_{wall}} = 1.94\ \frac{kN}{m}$

$g_{k.wall.in} := h_1 \cdot t_{wall} \cdot g_{k.clt} = 2.275\ \frac{kN}{m}$

$G_{k.floor} := g_{k.clt} \cdot A_{floor} \cdot t_{floor} = 436.671\ kN$ Total dead load of one floor

$g_{k.floor} := \frac{G_{k.floor}}{A_{floor}} = 0.902\ \frac{kN}{m^2}$

$G_{k.slab} := g_{k.c45} \cdot A_{floor} \cdot t_{floor} = \left(2.729 \cdot 10^3\right)\ kN$ Total dead load of one concrete slab

$g_{k.slab} := g_{k.c45} \cdot t_{floor} = 5.639\ \frac{kN}{m^2}$

$G_{k.wall.c45} := g_{k.c45} \cdot A_{wall} \cdot t_{wall} = \left(2.585 \cdot 10^3\right)\ kN$ Total dead load of concrete walls

$g_{k.wall.c45} := g_{k.c45} \cdot t_{wall} \cdot h_1 = 14.22\ \frac{kN}{m}$

$q_{k.live} := 0.3 \cdot 2\ \frac{kN}{m^2}$ Live load residencial

$Q_{k.live} := q_{k.live} \cdot A_{floor} = 290.4\ kN$ Total live load of one floor

$$G_{k.z_i} := \begin{cases} G_{k.floor} + \frac{G_{k.wall}}{2} + Q_{k.live} & \text{if } i = 0 \\ \frac{G_{k.wall}}{2} & \text{else if } i = 19 \\ G_{k.floor} + G_{k.wall} + Q_{k.live} & \text{else} \end{cases} = \begin{bmatrix} \vdots \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 1.141 \cdot 10^3 \\ 206.795 \end{bmatrix} kN$$ Total dead load

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Unintended inclination

$\theta_0 := \frac{1}{200}$ $M := 40$ Number of vertical elements

$$\alpha_n := \text{if}\left(\frac{2}{\sqrt{h_{tot} \cdot \frac{1}{m}}} > \frac{2}{3}, \frac{2}{\sqrt{h_{tot}}}, \frac{2}{3}\right) = 0.667 \qquad \alpha_m := \sqrt{0.5 \cdot \left(1 + \frac{1}{M}\right)} = 0.716$$

$\theta_1 := \theta_0 \cdot \alpha_n \cdot \alpha_m = 0.002$

$$H_{k_i} := G_{k.z_i} \cdot \theta_1 = \begin{bmatrix} \vdots \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 2.722 \\ 0.493 \end{bmatrix} \; kN$$

Horisontal load on each floor due to unintended inclination

Reaction forces

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Reaction forces

Horizontal equilibrium

$j := 0 .. 15$ See chapter 9.6.1 *Model for verification*

Coordinates G.C in (x.y)

$$coord := \begin{bmatrix} 11 & 21.9 \\ 21.9 & 11 \\ 11 & 0.1 \\ 0.1 & 11 \\ 6 & 11 \\ 16 & 11 \\ 11 & 14.8 \\ 11 & 7.2 \\ 14.3 & 8.65 \\ 11 & 9.9 \\ 7.7 & 8.65 \\ 7.7 & 13.35 \\ & \vdots \end{bmatrix} m$$

Length of each wall in (x,y)

$$length := \begin{bmatrix} 22 & 0.2 \\ 0.2 & 22 \\ 22 & 0.2 \\ 0.2 & 22 \\ 0.2 & 22 \\ 0.2 & 22 \\ 22 & 0.2 \\ 22 & 0.2 \\ 0.2 & 2.8 \\ 6.8 & 0.2 \\ 0.2 & 2.8 \\ 0.2 & 2.8 \\ & \vdots \end{bmatrix} m$$

Shear stiffness

$S_{j,0} := length_{j,0}$ $S_{j,1} := length_{j,1}$

$a_i S_{yi_j} := coord_{j,0} \cdot S_{j,1}$ $b_i S_{xi_j} := coord_{j,1} \cdot S_{j,0}$

Rotation center:

$$R_{c_{0,0}} := \frac{\sum a_i S_{yi}}{\sum S^{\langle 1 \rangle}}$$ $$R_{c_{0,1}} := \frac{\sum b_i S_{xi}}{\sum S^{\langle 0 \rangle}}$$ $R_c = \begin{bmatrix} 10.873 & 11 \end{bmatrix} m$

$coord_{new_{j,0}} := coord_{j,0} - R_{c_{0,0}}$ $coord_{new_{j,1}} := coord_{j,1} - R_{c_{0,1}}$

$S_{xi} y_{i_j} := S_{j,0} \cdot coord_{new_{j,1}}^{2}$ $S_{yi} x_{i_j} := S_{j,1} \cdot coord_{new_{j,0}}^{2}$

$S_T := \sum S_{yi} x_i + \sum S_{xi} y_i = \left(1.237 \cdot 10^4\right) m^3$ Global torsional stiffnes

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Load distribution

$$H_i := Q_{w.z} = \begin{bmatrix} 47.371 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 88.205 \\ 80.597 \\ 78.365 \\ 75.939 \\ \vdots \end{bmatrix} \ kN$$

Total horisontal load, both x and y direction

$$H_{tot} := \sum H_i = \left(1.602 \cdot 10^3\right) \ kN$$

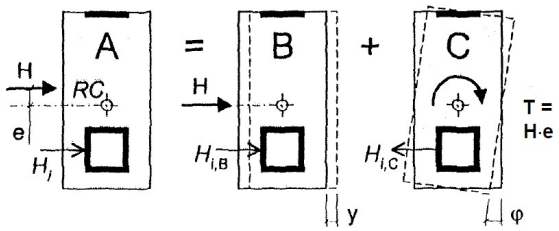
$$H_i := Q_{w.z} + H_k$$

If second order effects are included

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Wind in y-direction (eccentricity)

$T = H \cdot e$



$H_{tot.y} := H_{tot}$

$H_{tot.x} := 0\ \boldsymbol{N}$

$e := R_{c_{0,0}} - \frac{d_1}{2} = -12.696\ \boldsymbol{cm}$ Eccentricity

$T := H_{tot} \cdot e = -203.393\ \boldsymbol{kN \cdot m}$ Torsional moment

$Hy_{j,0} := H_{tot.x} \cdot \frac{S_{j,0}}{\sum S^{\langle 0 \rangle}} + T \cdot \frac{S_{j,0} \cdot coord_{new_{j,1}}}{S_T}$ Horizontal load on each individual wall in x-direction

$Hy_{j,1} := H_{tot} \cdot \frac{S_{j,1}}{\sum S^{\langle 1 \rangle}} + T \cdot \frac{S_{j,1} \cdot coord_{new_{j,0}}}{S_T}$ Horizontal load on each individual wall in y-direction

$$Hy = \begin{bmatrix} -3.943 & 2.786 \\ -5.842 \cdot 10^{-18} & 302.493 \\ 3.943 & 2.786 \\ -5.842 \cdot 10^{-18} & 310.38 \\ -5.842 \cdot 10^{-18} & 308.246 \\ -5.842 \cdot 10^{-18} & 304.628 \\ -1.375 & 2.786 \\ 1.375 & 2.786 \\ 0.008 & 38.849 \\ 0.123 & 2.786 \\ 0.008 & 39.153 \\ -0.008 & 39.153 \\ & \vdots \end{bmatrix} \boldsymbol{kN}$$

Horizontal load on each individual wall in both x and y direction

$cont_y := \sum Hy^{\langle 1 \rangle} = \left(1.602 \cdot 10^3\right)\ \boldsymbol{kN}$ Control of total horizontal load

$cont_x := \sum Hy^{\langle 0 \rangle} = -2.725 \cdot 10^{-15}\ \boldsymbol{kN}$ Control of total horizontal load

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Wind in x-direction (No eccentricity)

$H_{tot.y} := 0 \cdot N$ $\quad e := 0\ m$

$H_{tot.x} := H_{tot}$ $\quad T := H_{tot} \cdot e = 0\ kN \cdot m$ No torisonal moment

$$Hx_{j,0} := H_{tot.x} \cdot \frac{S_{j,0}}{\sum S^{\langle 0 \rangle}} + T \cdot \frac{S_{j,0} \cdot coord_{new_{j,1}}}{S_T}$$

Horizontal load on each individual wall in X-direction

$$Hx_{j,1} := H_{tot.y} \cdot \frac{S_{j,1}}{\sum S^{\langle 1 \rangle}} + T \cdot \frac{S_{j,1} \cdot coord_{new_{j,0}}}{S_T}$$

Horizontal load on each individual wall in Y-direction

$$Hx = \begin{bmatrix} 340.208 & 0 \\ 3.093 & 0 \\ 340.208 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ 340.208 & 0 \\ 340.208 & 0 \\ 3.093 & 0 \\ 105.155 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ \vdots \end{bmatrix} kN$$

Horizontal load on each individual wall in both x and y direction

$cont_y := \sum Hx^{\langle 1 \rangle} = 0\ kN$ Control of total horizontal load

$cont_x := \sum Hx^{\langle 0 \rangle} = \left(1.602 \cdot 10^3\right)\ kN$ Control of total horizontal load

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Vertical equilibrium

Wall id, contributing floor area and contributing wall length, actual wall length

$$A_{con} := \begin{bmatrix} 1 & 39.6 & 22 & 22 \\ 2 & 33 & 22 & 22 \\ 3 & 39.6 & 22 & 22 \\ 4 & 33 & 22 & 22 \\ 5 & 54.07 & 22 & 22 \\ 6 & 54.07 & 22 & 22 \\ 7 & 72.06 & 22 & 22 \\ 8 & 72.06 & 22 & 22 \\ 9 & 3.745 & 2.8 & 2.8 \\ 10 & 13.75 & 6.8 & 6.8 \\ 11 & 3.745 & 2.8 & 2.8 \\ 12 & 3.745 & 2.8 & 2.8 \\ & & & \vdots \end{bmatrix}$$

$$pos := \begin{array}{c} \text{x} \quad \text{x} \quad \text{y} \quad \text{y} \\ \begin{bmatrix} 0 & 22 & 21.8 & 22 \\ 21.8 & 22 & 0 & 22 \\ 0 & 22 & 0 & 0.2 \\ 0 & 0.2 & 0 & 22 \\ 5.9 & 6.1 & 0 & 22 \\ 15.9 & 16.1 & 0 & 22 \\ 0 & 22 & 14.7 & 14.9 \\ 0 & 22 & 7.1 & 7.3 \\ 14.2 & 14.4 & 7.3 & 10.1 \\ 7.6 & 14.4 & 9.9 & 10.1 \\ 7.6 & 7.8 & 7.3 & 10.1 \\ 7.6 & 7.8 & 12.1 & 14.7 \\ 7.6 & 14.4 & 12.1 & 12.3 \\ 14.2 & 14.4 & 12.1 & 14.7 \\ 9.9 & 10.1 & 0 & 7.2 \\ 9.9 & 10.1 & 14.9 & 22 \end{bmatrix} \end{array} \ m$$

Global tilting moment

$M_{i_i} := H_{i_i} \cdot h_i$ Contribution from each individual storey

$M_{tot} := \sum M_i = \left(5.086 \cdot 10^4\right)\ kN \cdot m$ Global moment

Vertical load on each storey

$$g_{k.i_j} := \frac{A_{con_{j,1}} \cdot m^2 \cdot \left(n \cdot \left(g_{k.floor} + q_{k.live}\right)\right) + A_{con_{j,2}} \cdot m \cdot \left(g_{k.wall.in} \cdot n\right)}{A_{con_{j,3}} \cdot m}$$

$G_{k.i_j} := g_{k.i_j} \cdot A_{con_{j,3}} \cdot m$

$G_{k.tot} := \sum G_{k.i} = \left(2.41 \cdot 10^4\right)\ kN$ Total vertical load

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Bending stiffness in X and Y

$$S_{b_{j,0}} := \frac{length_{j,1} \cdot length_{j,0}^{3}}{12} + \left(length_{j,1} \cdot length_{j,0} \cdot \left(\left| \left(R_{c_{0,0}} - coord_{j,0} \right) \right| \right)^{2} \right) \qquad \text{X}$$

$$S_{b_{j,1}} := \frac{length_{j,0} \cdot length_{j,1}^{3}}{12} + \left(length_{j,1} \cdot length_{j,0} \cdot \left(\left| \left(R_{c_{0,1}} - coord_{j,1} \right) \right| \right)^{2} \right) \qquad \text{Y}$$

Moment in X and Y direction:

$M_{tot.x} := M_{tot}$ Total moment X-direction

$M_{tot.y} := M_{tot}$ Total moment Y-direction

$$M_{j,0} := M_{tot.x} \cdot \frac{S_{b_{j,0}}}{\sum S_b^{\langle 0 \rangle}}$$

Moment on each individual wall in X-direction

$$M_{j,1} := M_{tot.y} \cdot \frac{S_{b_{j,1}}}{\sum S_b^{\langle 1 \rangle}}$$

Moment on each individual wall in Y-direction

$$M = \begin{bmatrix} 4.485 \cdot 10^{3} & 1.283 \cdot 10^{4} \\ 1.352 \cdot 10^{4} & 4.355 \cdot 10^{3} \\ 4.485 \cdot 10^{3} & 1.283 \cdot 10^{4} \\ 1.29 \cdot 10^{4} & 4.355 \cdot 10^{3} \\ 2.64 \cdot 10^{3} & 4.355 \cdot 10^{3} \\ 2.922 \cdot 10^{3} & 4.355 \cdot 10^{3} \\ 4.485 \cdot 10^{3} & 1.56 \cdot 10^{3} \\ 4.485 \cdot 10^{3} & 1.56 \cdot 10^{3} \\ 166.205 & 84.877 \\ 132.955 & 40.497 \\ 142.495 & 84.877 \\ 142.495 & 84.877 \\ & \vdots \end{bmatrix} kN \cdot m$$

Moment on each individual wall

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Stresses due to moment in X and Y direction

$$\sigma_{M_{j,0}} := \frac{M_{j,0}}{S_{b_{j,0}}} \cdot \left(R_{c_{0,0}} - pos_{j,1} \right) \qquad \sigma_{M_{j,1}} := \frac{M_{j,1}}{S_{b_{j,1}}} \cdot \left(R_{c_{0,1}} - pos_{j,3} \right)$$

$$\sigma_M = \begin{bmatrix} -0.281 & -0.27 \\ -0.281 & -0.27 \\ -0.281 & 0.265 \\ 0.27 & -0.27 \\ 0.121 & -0.27 \\ -0.132 & -0.27 \\ -0.281 & -0.096 \\ -0.281 & 0.091 \\ -0.089 & 0.022 \\ -0.089 & 0.022 \\ 0.078 & 0.022 \\ 0.078 & -0.091 \\ \vdots & \end{bmatrix} MPa$$

Stresses due to normal force

$$\sigma_N := -\frac{g_{k.i}}{t_{wall}} = \begin{bmatrix} -0.498 \\ -0.453 \\ -0.498 \\ -0.453 \\ -0.597 \\ -0.597 \\ -0.72 \\ -0.72 \\ -0.428 \\ -0.531 \\ -0.428 \\ -0.428 \\ \vdots \end{bmatrix} MPa$$

Addition of stresses and sorting after direction as well as dimension of wall

Left side positive in x,y

$$\sigma_{x_{j,0}} := \text{if } length_{j,0} > 0.2\ m \left\| -\sigma_{M_{j,0}} + \sigma_{N_j} \right\| \text{else} \left\| \text{if } coord_{j,0} > R_{c_{0,0}} \left\| 0 \right\| \text{else} \left\| \left| \sigma_{M_{j,0}} \right| + \sigma_{N_j} \right\| \right\|$$

Right side pos x,y

$$\sigma_{x_{j,1}} := \text{if } length_{j,0} > 0.2\ m \left\| \sigma_{M_{j,0}} + \sigma_{N_j} \right\| \text{else} \left\| \text{if } coord_{j,0} > R_{c_{0,0}} \left\| \sigma_{M_{j,0}} + \sigma_{N_j} \right\| \text{else} \left\| 0 \right\| \right\|$$

$$\sigma_{y_{j,0}} := \text{if } length_{j,1} > 0.2\ m \left\| -\sigma_{M_{j,1}} + \sigma_{N_j} \right\| \text{else} \left\| \text{if } coord_{j,1} > R_{c_{0,1}} \left\| 0 \right\| \text{else} \left\| \left| \sigma_{M_{j,1}} \right| + \sigma_{N_j} \right\| \right\|$$

$$\sigma_{y_{j,1}} := \text{if } length_{j,1} > 0.2\ m \left\| \sigma_{M_{j,1}} + \sigma_{N_j} \right\| \text{else} \left\| \text{if } coord_{j,1} > R_{c_{0,1}} \left\| \sigma_{M_{j,1}} + \sigma_{N_j} \right\| \text{else} \left\| 0 \right\| \right\|$$

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Stresses in each individual wall, wind in X-direction

$$\sigma_x = \begin{bmatrix} -0.217 & -0.779 \\ 0 & -0.734 \\ -0.217 & -0.779 \\ -0.183 & 0 \\ -0.476 & 0 \\ 0 & -0.729 \\ -0.438 & -1.001 \\ -0.438 & -1.001 \\ 0 & -0.518 \\ -0.442 & -0.62 \\ -0.351 & 0 \\ -0.351 & 0 \\ & \vdots \end{bmatrix} \boldsymbol{MPa}$$

Stresses in each individual wall, wind in X-direction

$$\sigma_y = \begin{bmatrix} 0 & -0.768 \\ -0.183 & -0.723 \\ -0.233 & 0 \\ -0.183 & -0.723 \\ -0.327 & -0.867 \\ -0.327 & -0.867 \\ 0 & -0.815 \\ -0.629 & 0 \\ -0.451 & -0.406 \\ -0.509 & 0 \\ -0.451 & -0.406 \\ -0.338 & -0.519 \\ & \vdots \end{bmatrix} \boldsymbol{MPa}$$

Reaction forces

$c := 0 \,..\, 1$

$f_{x_{j,c}} := \overrightarrow{\sigma_{x_{j,c}}}\, t_{wall}$

$f_{y_{j,c}} := \overrightarrow{\sigma_{y_{j,c}}}\, t_{wall}$

$$F_{x_j} := \text{if } \left(\sigma_{x_{j,0}} \neq 0 \wedge \sigma_{x_{j,1}} \neq 0\right) \left\| \frac{\sigma_{x_{j,0}} + \sigma_{x_{j,1}}}{2} \cdot \max\left(length_{j,0}, length_{j,1}\right) t_{wall} \right. \text{else} \left\| \min\left(\sigma_{x_{j,0}}, \sigma_{x_{j,1}}\right) \cdot \max\left(length_{j,0}, length_{j,1}\right) t_{wall} \right.$$

Different depandant on shape of stress, triangular/ rectangular (direction od wall)

$$F_{y_j} := \text{if } \left(\sigma_{y_{j,0}} \neq 0 \wedge \sigma_{y_{j,1}} \neq 0\right) \left\| \frac{\sigma_{y_{j,0}} + \sigma_{y_{j,1}}}{2} \cdot \max\left(length_{j,0}, length_{j,1}\right) t_{wall} \right. \text{else} \left\| \min\left(\sigma_{y_{j,0}}, \sigma_{y_{j,1}}\right) \cdot \max\left(length_{j,0}, length_{j,1}\right) t_{wall} \right.$$

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RESULT

$$H_i = \begin{bmatrix} 47.371 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 94.741 \\ 88.205 \\ 80.597 \\ 78.365 \\ 75.939 \\ \vdots \end{bmatrix} \ kN$$

Load on each floor level

$$R_c = \begin{bmatrix} 10.873 & 11 \end{bmatrix} \ m$$

Rotation centre

Horizontal load in X and Y direction of each wall with wind load in X-direction

$$Hx = \begin{bmatrix} 340.208 & 0 \\ 3.093 & 0 \\ 340.208 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ 340.208 & 0 \\ 340.208 & 0 \\ 3.093 & 0 \\ 105.155 & 0 \\ 3.093 & 0 \\ 3.093 & 0 \\ & \vdots \end{bmatrix} \ kN$$

Horizontal load in x and Y direction of each wall with wind load in Y-direction

$$Hy = \begin{bmatrix} -3.943 & 2.786 \\ -5.842 \cdot 10^{-18} & 302.493 \\ 3.943 & 2.786 \\ -5.842 \cdot 10^{-18} & 310.38 \\ -5.842 \cdot 10^{-18} & 308.246 \\ -5.842 \cdot 10^{-18} & 304.628 \\ -1.375 & 2.786 \\ 1.375 & 2.786 \\ 0.008 & 38.849 \\ 0.123 & 2.786 \\ 0.008 & 39.153 \\ -0.008 & 39.153 \\ & \vdots \end{bmatrix} \ kN$$

$$\max\left(Hx^{\langle 0 \rangle}\right) = 340.208 \ kN$$

$$\max\left(Hy^{\langle 1 \rangle}\right) = 310.38 \ kN$$

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Maximum Compression / Tension **stress** in each wall,wind load in X-direction

$$\sigma_{x.min_j} := \min\left(\sigma_{x_{j,0}}, \sigma_{x_{j,1}}\right) = \begin{bmatrix} -0.779 \\ -0.734 \\ -0.779 \\ -0.183 \\ -0.476 \\ -0.729 \\ -1.001 \\ -1.001 \\ -0.518 \\ -0.62 \\ -0.351 \\ -0.351 \\ \vdots \end{bmatrix} \textit{MPa} \qquad \min\left(\sigma_{x.min}^{\langle 0 \rangle}\right) = -1.001 \textit{ MPa}$$

Maximum Compression / Tension **stress** in each wall,wind load in Y-direction

$$\sigma_{y.min_j} := \min\left(\sigma_{y_{j,0}}, \sigma_{y_{j,1}}\right) = \begin{bmatrix} \vdots \\ -0.867 \\ -0.867 \\ -0.815 \\ -0.629 \\ -0.451 \\ -0.509 \\ -0.451 \\ -0.519 \\ -0.563 \\ -0.519 \\ -0.691 \\ -0.868 \end{bmatrix} \textit{MPa} \qquad \min\left(\sigma_{y.min}^{\langle 0 \rangle}\right) = -0.868 \textit{ MPa}$$

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Maximum Compressive / Tensioned **reaction force** in each wall,wind load in X-direction

$$F_x = \begin{bmatrix} -2.191 \cdot 10^3 \\ -3.229 \cdot 10^3 \\ -2.191 \cdot 10^3 \\ -806.053 \\ -2.095 \cdot 10^3 \\ -3.207 \cdot 10^3 \\ -3.166 \cdot 10^3 \\ -3.166 \cdot 10^3 \\ -289.824 \\ -722.528 \\ -196.445 \\ -196.445 \\ \vdots \end{bmatrix} \ kN$$

$$\min\left(F_x^{\langle 0 \rangle}\right) = -3.229 \cdot 10^3 \ kN$$

Maximum Compressive / Tensioned **reaction force** in each wall,wind load in Y-direction

$$F_y = \begin{bmatrix} -3.379 \cdot 10^3 \\ -1.993 \cdot 10^3 \\ -1.025 \cdot 10^3 \\ -1.993 \cdot 10^3 \\ -2.626 \cdot 10^3 \\ -2.626 \cdot 10^3 \\ -3.587 \cdot 10^3 \\ -2.767 \cdot 10^3 \\ -239.924 \\ -692.488 \\ -239.924 \\ -239.924 \\ \vdots \end{bmatrix} \ kN$$

$$\min\left(F_y^{\langle 0 \rangle}\right) = -3.587 \cdot 10^3 \ kN$$

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