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Dynamic Analysis of Pedestrian Load Models for Footbridges

A review of current load models and guidelines

Master of Science Thesis in the Master's programme Structural Engineering and Building Technology

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Cover:
Walking pedestrians on a timber deck in Gothenburg harbor.

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ABSTRACT

Modern footbridges are designed slender and with longer spans as a result of technical innovation and more accurate analysis. Due to the low mass of these structures a problem has been discovered regarding uncomfortable vibrations caused by pedestrians. Eurocode does not cover this subject thoroughly and a lot of factors are left to the designer to make reasonable estimates. Following the closure of the London Millennium Bridge and the Solférino footbridge in Paris due to uncomfortable swaying further research on the area of human vibrations have been made. This has resulted in a number of proposed standards and guidelines for the design of footbridges.

The Master's thesis was performed at Reinertsen Sverige AB in Göteborg and aimed to increase the knowledge of how to design footbridges regarding dynamic loads induced by pedestrians. Furthermore the thesis aimed to summarize and explain the current proposed standards and guidelines of how human induced vibrations in pedestrian bridges can be modeled in the design phase.

The analyzed standards propose to model pedestrian induced forces as concentrated or uniformly distributed harmonic loads. Some standards also suggest to model pedestrian loads as Fourier sums. Graphical evaluation and comparison of the acceleration response obtained with the load models in the standards were done valid for all simply supported footbridges in one span. The proposed load models were systematically compared and evaluated with recommended design situations given in Eurocode and ISO 10137.

It was concluded that single pedestrians and groups can be modeled according to ISO 10137. Pedestrian streams relevant for modeling large amounts of pedestrians can be modeled with uniformly distributed load according to other standards. The guidelines and load models in ISO 10137 are not sufficient for this task. Furthermore Eurocode and ISO 10137 needs to be complemented with more specific guidelines to provide a sufficient support during design.

Key words: pedestrian bridge, footbridge, pedestrian induced vibrations, Eurocode, ISO 10137, Sétra, SYNPEX, HIVOSS, JRC, dynamic analysis

Dynamisk analys av lastmodeller för fotgängare på gångbroar
En granskning av nuvarande lastmodeller och riktlinjer

Examensarbete inom mastersprogrammet Structural Engineering and Building Technology

ANDERS MÅRTENSSON
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Avdelningen för konstruktionsteknik

Chalmers tekniska högskola

SAMMANFATTNING

Tack vare teknisk innovation och möjlighet till mer noggrann strukturell analys byggs idag gångbroar slankare och med längre spann än tidigare. Den låga massan hos dessa konstruktioner har gett upphov till ett nytt problem med obekväma vibrationer skapade av fotgängare. Utförliga riktlinjer som beaktar sådana laster finns inte i Eurocode och många faktorer lämnas därför till ingenjören för att göra rimliga uppskattningar. Millennium-bron i London och Solférino-bron i Paris är två gångbroar som fick stängas strax efter öppning på grund av obekväma vibrationer. Detta har gett upphov till forskning inom området och ett antal standarder och riktlinjer för dynamisk dimensionering av gångbroar.

Examensarbetet har utförts på Reinertsen Sverige AB i Göteborg med målet att öka förståelsen och kunskapen om hur gångbroar ska dimensioneras för dynamiska laster på grund av fotgängare. Vidare så syftade examensarbetet till att sammanfatta och förklara dem nuvarande standarderna och riktlinjerna för hur gånglaster kan modelleras vid dimensionering.

De analyserade standarderna föreslår att gånglaster modelleras som punktlaster eller jämnt utbredda laster. Vissa standarder förslår också att modellera gånglaster med Fouriersummor. De erhållna accelerationerna enligt olika standarder har utvärderats grafiskt gällande för alla fritt upplagda gångbroar i ett spann. De föreslagna lastmodellerna har jämförts med varandra och utvärderats enligt rekommenderade situationer för dimensionering givna i Eurocode och ISO 10137.

Examensarbetet visade att individuella fotgängare och grupper kan modelleras enligt ISO 10137. Flöden av fotgängare, relevant för att modellera belastning av stora antal fotgängare, kan beskrivas med hjälp av utbredd last som angivits i andra standarder. Riktlinjerna och lastmodellen definierade i ISO 10137 är inte tillräckliga för att modellera detta. Eurocode och ISO 10137 behöver kompletteras med utförligare riktlinjer för att kunna utgöra ett tillräckligt stöd vid dimensionering av gångbroar.

Nyckelord: gångbroar, gång- och cykelbroar, gånglaster, Eurocode, ISO 10137, Sétra, SYNPEX, HIVOSS, JRC, dynamisk analys

Contents

Preface

The thesis is the final part of the civil engineering program and the masters´s programme Structural Engineering and Building Technology at Chalmers University of Technology. It was carried out from January to June 2014 at Reinertsen Sverige AB in Göteborg in cooperation with the Division of Structural Engineering at Chalmers. Load models for human induced forces on pedestrian bridges have been studied with the aim to increase the knowledge of how to design footbridges regarding dynamic loads induced by pedestrians.

Associate professor Mario Plos at Concrete Structures at Chalmers University of Technology was the examiner and Emanuel Trolin, MSc, at Reinertsen Sverige AB, was the supervisor of this thesis.

The authors thank Mario Plos for his remarks on the thesis and the opposition group Erik Asplund and Daniel Steckmest for their valuable input.

Foremost the authors would like to thank Emanuel Trolin for his support and valuable guidance during the project.

Göteborg June 2014

Anders Mårtensson and Martin Nilsson

Notations

Roman upper case letters

2D	Two-dimensional
A	Cross-sectional area
$\mathbf{C}$	Damping matrix
$D_s(r)$	Steady-state magnification factor
E	Young's modulus
$F(t)$	Time dependent applied load
FE	Finite element
I	Moment of inertia
L	Length
M	Mass matrix
MDOF	Multi degree of freedom
N	Total number of pedestrians
N_{eq}	Equivalent number of pedestrians
P_0	Load amplitude for concentrated loads
K	Stiffness matrix
p	Applied load vector
Q	Static load of a pedestrian
$H(r)$	Frequency response function
SDOF	Single degree of freedom
T_d	Damped period
T_n	Undamped natural period
U	Load amplitude
U_0	Static displacement

Roman lower case letters

a	Acceleration
b	Bridge width
c	Coefficient of viscous damping
c_{cr}	Critical damping coefficient
d	Pedestrian density
f_p	Step frequency of a pedestrian
$f_{p.jogg}$	Step frequency of a jogging pedestrian
f_s	Step frequency of a pedestrian
f_n	Natural frequency

k	Stiffness of a spring
m	Mass
$p(t)$	External force
p_0	Load amplitude for uniformly distributed loads
r	Frequency ratio
u	Displacement
$\boldsymbol{u}$	Displacement
u_0	Initial displacement
$\dot{u}$	First derivative of u with respect to time t, velocity
$\dot{\boldsymbol{u}}$	First derivative of $\boldsymbol{u}$ with respect to time t, velocity vector
$\ddot{u}$	Second derivative of u with respect to time t, acceleration
$\ddot{\boldsymbol{u}}$	Second derivative of $\boldsymbol{u}$ with respect to time t, acceleration vector
v	Velocity
v_0	Initial velocity
v_s	Walking velocity of a pedestrian
t	Time
x	Coordinate
y	Coordinate
z	Coordinate

Greek upper case letters

Ω	Load frequency
$\boldsymbol{\Phi}$	Modal matrix

Greek lower case letters

α	Fourier coefficient
β	Frequency ratio
$\delta(t)$	Dirac delta function
$\boldsymbol{\eta}$	Modal coordinate vector
θ	Angle
λ	Eigenvalue
ξ	Viscous damping factor/ structural damping ratio
ρ	Material density
τ	Normalization factor
ϕ	Phase shift angle
$\boldsymbol{\phi}$	Mode vector
ω_b	Beat frequency

ω_d	Damped natural frequency
ω_n	Undamped natural frequency
ω_l	Load frequency

1 Introduction

This chapter will present background, aim, method, limitations and general layout.

1.1 Background

Modern footbridges have dynamic properties that differ from older more conventional pedestrian bridges. They are built slender and longer as a result of technical innovation and more accurate structural analysis. As a consequence a new problem has been discovered concerning uncomfortable vibrations caused by pedestrian loading.

Two modern bridges that have experienced problems with vibrations because of pedestrian loading are the London Millennium Bridge and the Solferino footbridge in Paris. The two bridges were closed soon after opening due to lateral swaying experienced as uncomfortable by the pedestrians (Sétra, 2006).

In the design of footbridges human induced loads are significant. The dynamic effect of the pedestrian load can cause uncomfortable and excessive vibrations due to its low frequency. Low frequency loads are likely to give rise to resonance in slender and flexible footbridges with low natural frequencies in the same range as the load.

Pedestrian loads are difficult to model due to its characteristics as weight of the pedestrian, walking speed and synchronization amongst pedestrians. Dynamic analysis is not extensively covered in Eurocode which refers to ISO 10137 for further guidance. ISO 10137 includes guidelines for dynamic analysis of footbridges but does not cover the subject thoroughly. The standard leaves a lot of factors for the designer to make reasonable estimates.

Following the closure of the London Millennium Bridge and Solferino footbridge research on the area of human induced vibrations have increased resulting in a number of proposed standards and guidelines for the design of footbridges. The proposed standards and guidelines could complement Eurocode and provide support during a dynamic analysis.

1.2 Aim

The overall aim of the Master Thesis was to increase the knowledge of how to design footbridges regarding dynamic loads induced by pedestrians.

This thesis aimed to summarize and explain the current proposed standards and guidelines of how human induced vibrations in pedestrian bridges can be modeled in the design phase.

The thesis aimed to find a numerical method to be able to compare and evaluate the proposed load models. The evaluation aimed to result in recommendations on how to complement Eurocode and ISO 10137 with accurate methods and load models for a dynamic analysis.

1.3 Method

A literature study was made of available standards, guidelines and research material of how to model pedestrian induced forces and their dynamic actions. A finite element analysis (FE-analysis) of the dynamic response in a simply supported structure with one span was made. The analysis studied how mass and stiffness are related to the acceleration response in a simply supported structure by changing properties during a large number of analyses.

Results from the FE analysis and the increased knowledge about load models from the literature study made it possible to evaluate and compare the considered load models.

The graphical evaluations were made using normalized curves based on the relationship found between bridge characteristics and the acceleration response.

The proposed load models were systematically compared with recommended design situations in ISO 10137. The evaluation resulted in recommendations of how Eurocode and ISO 10137 can be complemented in order to perform an accurate analysis.

The commercial software ADINA (ADINA R & D, 2012) was used for Finite Element (FE) analysis where the results were processed in Excel. The commercial software Matlab was used for simple numerical calculations.

1.4 Limitations

The analyses in the thesis were limited to the study of simply supported structures in one span using two dimensional (2D) analysis.

A literature study was made where the load models considered to be the most applicable and relevant were chosen. The load models used in analysis are recognized by institutions in the field of structural engineering.

The considered structural materials when comparing load models and damping ratios were limited to reinforced concrete, steel and timber.

Vertical and lateral modes of vibrations were treated independently. Torsional and mixed modes were not regarded.

1.5 General layout

Chapter 2 presents basic facts about vibrations in pedestrian bridges and human induced load with the purpose to introduce the reader to the subject. The chapter includes pedestrian forces, how pedestrian can interact and a presentation of two existing bridges that have experienced uncomfortable vibrations due to pedestrians.

Chapter 3 explains basic theory of structural dynamics and aims to provide a support for the reader during the analysis in chapter 5. Chapter 3 includes derivations of the response in simple dynamic systems, dynamic phenomena occurring in structural systems and other relevant aspects.

The literature study of current standards and guidelines regarding dynamic design of pedestrian bridges is presented in chapter 4. The chapter aims to increase the knowledge of how to model pedestrian loads and present applicable methods useful in the design phase.

In chapter 5 an extensive analysis of dynamic behavior in a simply supported beam is presented. The chapter includes used methods and results of how the dynamic response can be normalized.

Based on the results in chapter 5 the load models can be normalized into normalization curves. In chapter 6 all relevant load loads are normalized and graphically presented.

Chapter 7 aimed to evaluate and compare the proposed load models to the load models presented in ISO 10137. The evaluation was done systematically and discussed for the normalized curves and according to design situations recommended by Eurocode and ISO 10137.

The general discussion about how to design pedestrian bridges, proposed standards and the extent of Eurocode and ISO 10137 are presented in chapter 8. The purpose was to present thoughts and opinions for the final conclusions made in chapter 9.

Chapter 9 includes the final conclusions about Eurocode, ISO 10137 and how to design pedestrian bridges induced by pedestrian loads. The purpose was to summarize and present the most relevant results and conclusions in the thesis. Furthermore the chapter includes suggestions for further studies.

2 Pedestrian forces and human interaction

In this section theory about pedestrian induced forces and its effects on pedestrian bridges will be presented in order to familiarize the reader with the subject. Basic facts about human interaction and synchronization will also be presented as well as two examples of footbridges which have had problems with large human induced vibrations

2.1 Pedestrian induced forces

When a pedestrian crosses a bridge a dynamic force is produced which has components in three different directions: vertical, lateral, and longitudinal. Some forms of deliberate loading such as jumping or body swaying can produce forces with different characteristics (S.Živanovic, 2005).

Dynamic forces are described as a function of time and space, periodically repeated with regular time intervals. Dynamic actions are the displacements, velocities, accelerations and energy produced by the vibration source. These actions can often not be predicted in a deterministic way which is why it can be suitable to consider them to be random (ISO, 2008).

The force produced in vertical direction by pedestrians is the one studied the most. The vertical component has the highest magnitude of the three components and has therefore been regarded as the most important (S.Živanovic, 2005). In recent years more detailed studies have been made showing that the lateral force induced by pedestrians also can cause problems regarding the serviceability of footbridges (Pat Dallard, 2001).

2.1.1 Vertical

Vertical ground reaction forces due to walking and running for one foot are presented in Figure 2.1. The first maximum represents the heel hitting the ground and the second maximum the front of the foot pushing of from the ground (Research Fund for Coal and Steel, 2006). The reaction force for running looks different from the walking force because it is a discontinuing contact with the ground (Sétra, 2006).

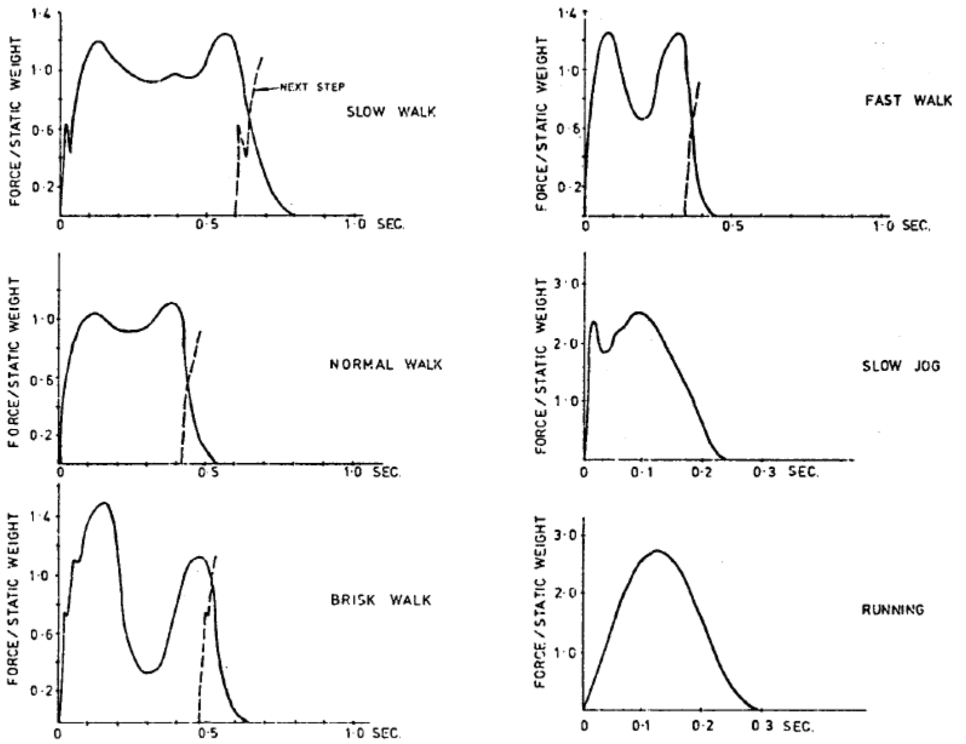


Figure 2.1 *Human induced vertical ground forces over time for different types of activity (Sétra, 2006)*

The actual force for two steps is shown in Figure 2.2. Note that for walking the next step begins before the first has ended which is illustrated with the dotted and dashed lines in the figure.

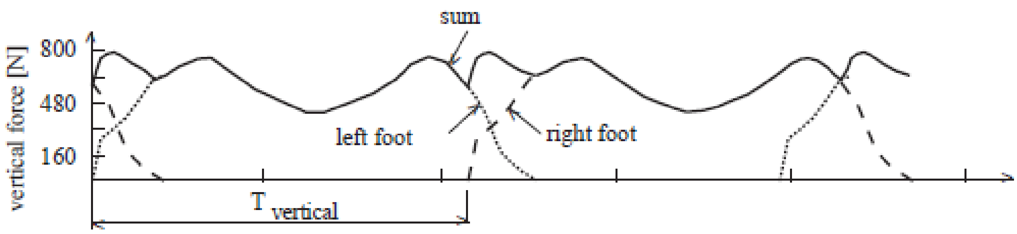


Figure 2.2 *Periodic walking in vertical direction (S.Živanovic, 2005)*

The walking force is a determined by the weight of the pedestrian, its step-length and the walking frequency. The load period for the vertical component is between two consecutive steps. Normal frequency ranges for walking and running have been established through measurements and are shown in Table 2.1.

Table 2.1 Frequency ranges for walking and running (Sétra, 2006)

Designation	Specific features	Frequency range (Hz)
Walking	Continuous contact with the ground	1.6 to 2.4
Running	Discontinuous contact	2 to 3.5

2.1.2 Lateral

The lateral load is created by the pedestrians swaying from side to side giving that the lateral force component is significantly lower than the vertical. The lateral force component is smaller for running than for walking.

The lateral component differs from the vertical where the lateral load period is between two following left or right footsteps. This means that the load period is twice as large as for vertical direction and therefore the lateral load frequency is half of the vertical load frequency (Sétra, 2006) illustrated in Figure 2.3.

According to research done on the London Millennium Bridge which experienced problems with lateral movements the typical frequency of purposeful walking seems to be around 2 Hz. In large groups this rate decreases to 1.4 Hz or lower, resulting in a frequency of the vertical force in the range of 1.2-2.2 Hz. Since the frequency for the lateral loads are half of the vertical it is in the range of 0.6 – 1.1 Hz (Pat Dallard, 2001).

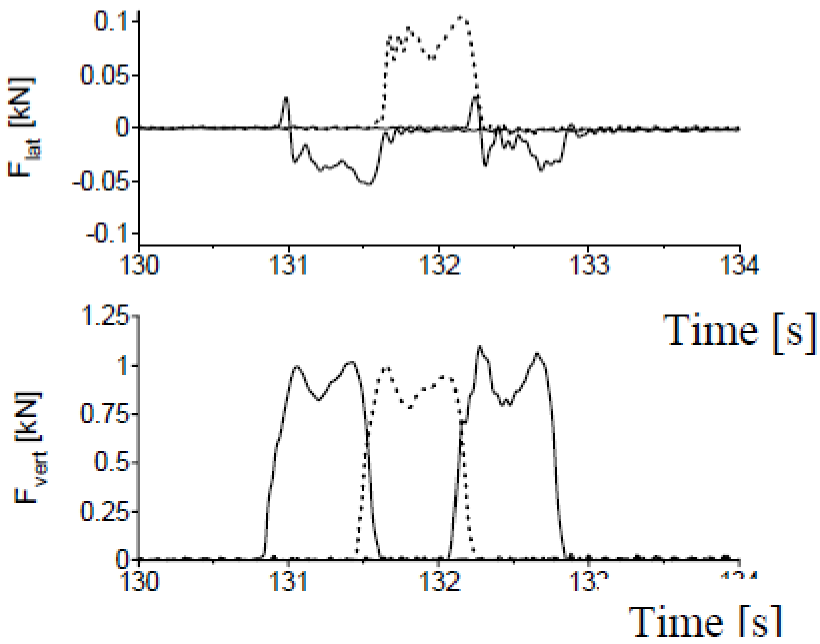


Figure 2.3 Lateral and vertical ground reaction force for three consecutive steps (Research Fund for Coal and Steel, 2006).

2.2 Crowds and interaction

Synchronization occurs between pedestrians in both vertical and lateral direction meaning that pedestrians coordinate their movements with other pedestrians. Pedestrians are more sensitive against vibrations in lateral direction than vertical. In vertical direction pedestrians can compensate with their knees and their balance is better than in lateral direction. The London Millennium Footbridge and Solferino Footbridge in Paris have shown that synchronization in lateral direction can be a problem.

The occurrence of large human induced vibrations in lateral direction is the cause of synchronization within the crowd. For pedestrians it is more comfortable to walk synchronized with the bridges swaying. This instinct to synchronize walking with the bridges movement results in that the pedestrian forces are applied at the resonance frequency of the bridge which increases the movements of the bridge. This phenomenon is called “lock-in” and means that as the amplitude of the bridges motion increases the lateral force added from the pedestrians’ increase. Respectively the degree of synchronization between pedestrians increases with increasing lateral movements (Pat Dallard, 2001). A requirement for lock-in to develop is that the bridge must have lateral natural frequencies that coincide with the frequencies of lateral movements of pedestrians.

2.3 Perception of vibrations

The human perception of vibrations on footbridges is highly subjective and depends on several factors such as personal sensitivity, surroundings of the bridge, bridge type and design, direction of movements, height above ground, exposure time, number of people walking on the bridge and the expectations on the bridge (Christoph Heinemeyer, 2009).

For example vibrations in a more slender lightweight bridge is experienced as less disturbing by pedestrian than if a bridge who appears to be more massive demonstrates the same movements. In the same way vibrations in a bridge that is high above the ground can be experienced as more disturbing (Christoph Heinemeyer, 2009).

Because the perception of vibrations is subjective, comfort limits are stated as ranges to avoid. The limits are often defined as accelerations but can be translated into limits for displacements and speed.

2.4 Footbridges with dynamic problems induced by pedestrians

In this section the London Millennium Bridge and the Solférino footbridge in Paris that have shown large vibrations due to human induced loads are presented. Research has been done based on measurements performed on these two bridges after closing due to large and uncomfortable vibrations.

2.4.1 Solférino footbridge

The Solférino footbridge in Paris is a 106 meters long steel arch bridge with timber deck, spanning over river Seine. The arch is formed from two arches linked by cross-pieces supporting a lower deck (Sétra, 2006). There are two pedestrian walkways as shown in Figure 2.4, one upper and one lower.

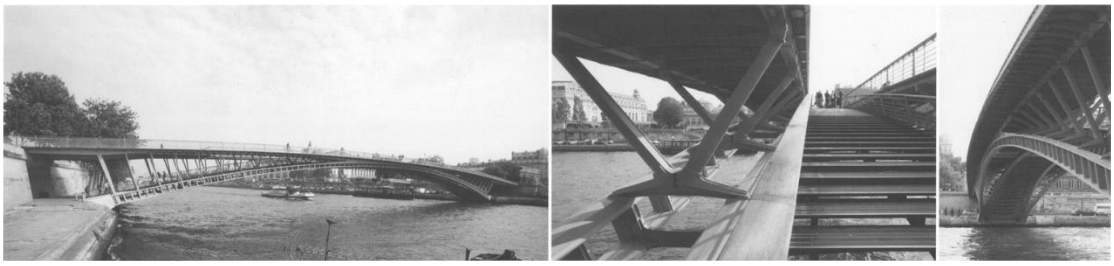


Figure 2.4 Solférino footbridge in Paris (Ursula Baus, 2008).

On the inauguration day in 1999 the bridge had to be closed because of excessive lateral vibrations (Blekherman, 2007). The French road authority, Sétra, published guidelines for dynamic analysis of footbridges partially based on investigations made on the Solferino footbridge. Controlled pedestrian crowd tests where made that indicated the lock-in phenomena (Ingolfsón, 2011).

2.4.2 London Millennium Footbridge

The London Millennium Footbridge is situated in London and spans across the river Thames between St. Paul's Cathedral and the Tate Gallery. The bridge spans over 332 m divided into a north span of 80 m, a central span of 144 m and a southern span of 108 m. It is a shallow suspension bridge, as can be seen in Figure 2.5, where the cables are situated low and sometimes beneath the bridge deck to free the view for the pedestrians. The cable profile sags 2.3 meters in the middle-span which is approximately six times lower than for a more conventional suspension bridge The deck structure is 4 meters wide and is made up of 16 meters long aluminum box sections (P. Dallard, 2001).

Figure 2.5 The London Millennium footbridge. (P. Dallard, 2001)

During the opening day in June 2000 around 80 000 to 100 000 people crossed the bridge. A maximum amount of people at the same time was estimated to 2000 resulting in an approximate density of 1.3 to 1.5 pedestrian per square meter (P. Dallard, 2001). Unexpected large lateral vibrations occurred mainly in the south span of the bridge and the bridge had to be closed to fully investigate the cause.

The frequencies of the movements were around 0.8 Hz which respectively is the first lateral mode of the south span. In the central span movements took place at barely 0.5 Hz and 1.0 Hz, respectively the first and second lateral mode. At the north side movements occurred at a frequency around 1.0 Hz, the first lateral mode of the north span (P. Dallard, 2001).

The vibrations increased as large numbers of pedestrians where walking at the affected span and died down if the number of people reduced or if they stopped walking. Accelerations were so high that people stopped walking and held onto the railings for support. Based on visual estimations the maximum lateral acceleration on the south and north span was between 1.96 m/s^2 and 2.45 m/s^2 (P. Dallard, 2001).

The lateral force created by the pedestrians' synchronization was found to be the cause of the lateral movements. A loading which at that time were not considered in detail in international bridge codes (Pat Dallard, 2001).

The lateral excitations that occurred at the Millennium Bridge are not exclusively dependent on the technical innovation and structural form of the bridge. This kind of behavior could occur for any footbridge with a natural lateral frequency below 1.3 Hz, loaded with a large enough number of pedestrian (Pat Dallard, 2001).

3 Basic dynamics

In this chapter some basic concepts in structural dynamics will be explained which are important for the understanding of and the execution of this Master's thesis

Conclusions and discussions made in the end of the thesis can still be understood without reading this chapter. For the reader not familiar with the subject of dynamics it is recommended in order to understand the analysis performed in the thesis.

Subjects that will be treated are:

- Free vibrations of an undamped and viscously damped single degree of freedom system(SDOF-system) and analytical solution for a SDOF-system
- Multiple degree of freedom systems(MDOF-systems) and how to solve them
- The resonance phenomenon
- How to calculate natural frequencies and modes
- Fourier series
- Dirac delta function
- Monte Carlo simulations
- Euler-Bernoulli beams and how natural frequencies can be calculated analytically

3.1 SDOF system

A system consisting of one single degree of freedom (SDOF) -system is a good way to explain dynamic actions and highly relevant because more complex systems can be transformed into simpler independent SDOF systems.

A SDOF-system can be described with only one parameter for which the system is fully determined. The system must have an elastic component which can store and release energy and mass which can store and release kinetic energy.

In Figure 3.1 an example of a SDOF-system can be seen in the form of a spring-mass-system with a viscous damper, a mass-spring-dashpot model:

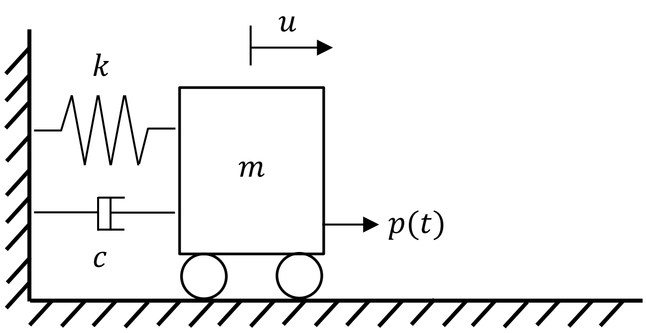


Figure 3.1 Mass-spring-dashpot model

The mass m is a point mass that can only move along the horizontal axis. The displacement on the x-axis from the position where the spring is underformed is described with u. The mass is connected to the fixed connection with a linear spring with the stiffness k. An external force $p(t)$ is acting on the mass and energy is dissipated from the vibrations of the mass through a damping mechanism with the viscous damping coefficient c.

For the derivation of the equation of motion for the SDOF-system Newton's second law is required:

$$\sum F = ma \tag{3-1}$$

Where m is the mass and a is the acceleration of the mass in horizontal direction.

Assuming that the mass is displaced u to the right of the undeformed position of the spring by a force $p(t)$: the spring force and the damping mechanism will act, as shown in the free-body diagram in Figure 3.2, at the left of the mass working against the displacement.

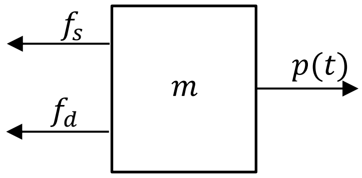


Figure 3.2 Free-body diagram showing forces acting on mass m

Taking this into account Equation (3-1) can be expressed like:

$$p(t) - f_s - f_d = ma \tag{3-2}$$

The acceleration is defined as positive in x-direction and is determined by the second derivative of the displacement meaning that $a = \ddot{u}(t)$. Respectively velocity is given by the first derivative of the displacement $\dot{u}(t)$.

The forces f_s and f_d can be expressed as:

$$f_s = ku$$

$$f_d = c\dot{u}$$

Equation (3-2) can be expressed as:

$$-ku - c\dot{u} + p(t) = m\ddot{u} \tag{3-3}$$

This gives the equation of motion for our damped SDOF-model.

$$m\ddot{u} + c\dot{u} + ku = p(t) \tag{3-4}$$

Equation (3-4) is the fundamental equation in structural dynamics and solving of this equation can be done analytically and numerically.

3.1.1 Free vibration of SDOF-system

The equation of motion for a linear SDOF-system is a second-order ordinary differential equation. In mathematical terms the general solution for this kind of equation consists of a particular solution $u_p(t)$ linked to the forced motion and a complementary solution $u_c(t)$ related to the natural motion of the system. Together these two parts form the total response of the system:

$$u(t) = u_p(t) + u_c(t) \tag{3-5}$$

Specifying initial displacement and initial velocity

$$u(0) = u_0 \text{ and} \tag{3-6}$$

$$\dot{u}(0) = v_0 \tag{3-7}$$

It is convenient to rewrite the equation of motion as stated earlier in Equation (3-4) dividing by the mass m and rewriting to the form:

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = \omega_n^2\frac{p(t)}{k} \tag{3-8}$$

Where ω_n is the undamped natural frequency defined as:

$$\omega_n = \sqrt{\frac{k}{m}} \tag{3-9}$$

Giving

$$\omega_n^2 = \frac{k}{m} \tag{3-10}$$

The factor ξ is a quantity without dimension called the viscous damping factor defined from the critical damping factor c_{cr}:

$$\xi = c/c_{cr} \tag{3-11}$$

$$c_{cr} = 2m\omega_n = 2\sqrt{km} \tag{3-12}$$

Considering only free vibration equation (3-8) becomes:

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = 0 \tag{3-13}$$

For solving this equation a form of the solution is assumed:

$$u(t) = \bar{C}e^{\bar{s}t} \tag{3-14}$$

Equation (3-14) substituted into equation (3-13) gives:

$$(s^2 + 2\xi\omega_n\bar{s} + \omega_n^2)\bar{C}e^{\bar{s}t} = 0 \tag{3-15}$$

For Equation (3-15) to be valid for all times the expression within the parenthesis must be set to:

$$\bar{s}^2 + 2\xi\omega_n\bar{s} + \omega_n^2 = 0 \tag{3-16}$$

Which is called the characteristic equation.

3.1.2 Free vibration of undamped SDOF-system

Equation of motion for an undamped system is written as

$$\ddot{u} + \omega_n^2 u = 0 \tag{3-17}$$

With the corresponding characteristic equation:

$$\bar{s}^2 + \omega_n^2 = 0 \tag{3-18}$$

With roots:

$$\bar{s}_{1,2} = \pm i\omega_n \text{ where } i = \sqrt{-1} \tag{3-19}$$

By substituting these roots into Equation (3-14) the general solution is achieved as

$$u = \bar{C}_1 e^{i\omega_n t} + \bar{C}_2 e^{-i\omega_n t} \tag{3-20}$$

Introducing Euler's equation

$$e^{\pm i\theta} = cos\theta \pm i\ sin\theta \tag{3-21}$$

We get Equation ((3-20) in terms of trigonometric functions

$$u = A_1 \cos \omega_n t + A_2 \sin \omega_n t \tag{3-22}$$

A_1 and A_2 are constants to be determined from initial conditions. Equations (3-6) and (3-22) give

$$\begin{aligned} u(0) = u_0 = A_1, \\ \dot{u}(0) = v_0 = A_2\omega_n \end{aligned} \tag{3-23}$$

Giving

$$u(t) = u_0\ cos\omega_n t + \frac{v_0}{\omega_n} sin\omega_n t \tag{3-24}$$

Which expresses the free vibration response of an undamped SDOF-system. Equation (3-22) is the more general solution which can be used to determine the solution when

$p(t) \neq 0$.

Consider a system that is displaced from its equilibrium position by $u(0)$ and is then released meaning that $v(0) = 0$. This gives the equation for the position of the system like:

$$u(t) = u_0 \cos \omega_n t \tag{3-25}$$

This system has a motion like a simple harmonic motion with the amplitude u_0 and a natural frequency as:

$$f_n = \frac{\omega_n}{2\pi} \quad \text{[Hz]} \tag{3-26}$$

And natural period of:

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} \quad [\mathrm{s}] \tag{3-27}$$

The displacement can be described with Equation (3-24) or by the expression

$$\begin{aligned} u(t) &= U\cos(\omega_n t - \alpha) \\ &= U\cos\omega_n\left(t - \frac{\alpha}{\omega_n}\right) \end{aligned} \tag{3-28}$$

Where the amplitude U and phase angle are

$$U = \sqrt{u_n^2 + \left(\frac{v_0}{\omega_n}\right)^2} \tag{3-29}$$

$$\tan\alpha = \frac{v_0/\omega_n}{u_0} \tag{3-30}$$

3.1.3 Free vibration of viscously damped SDOF-system

Once a system without damping is set in motion it will continue indefinitely but in reality all systems have some kind of damping that will dissipate the energy.

As shown before in Equation (3-13) the expression for motion for free vibration of a system with viscous damping is

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = 0 \tag{3-31}$$

Supposing a solution

$$u(t) = \bar{C}e^{\bar{s}t} \tag{3-32}$$

The characteristic equation is given by

$$\bar{s}^2 + 2\xi\omega_n\bar{s} + \omega_n^2 = 0 \tag{3-33}$$

Where the roots of the equation is

$$\left.\begin{matrix}\bar{s}_1 \\ \bar{s}_2\end{matrix}\right\} = -\xi\omega_n \pm i\,\omega_n\sqrt{\xi^2 - 1} \tag{3-34}$$

The damping factor ξ distinguishes between three different cases of damping, underdamped $0 < \xi < 1$, critically damped $\xi = 1$ and overdamped $\xi > 1$. The three cases are illustrated in Figure 3.3 Displacement decrement over time for three cases of damping. It is only the underdamped case that exhibits oscillating movement with decaying amplitude and this is the most important case for structural dynamics and this thesis.

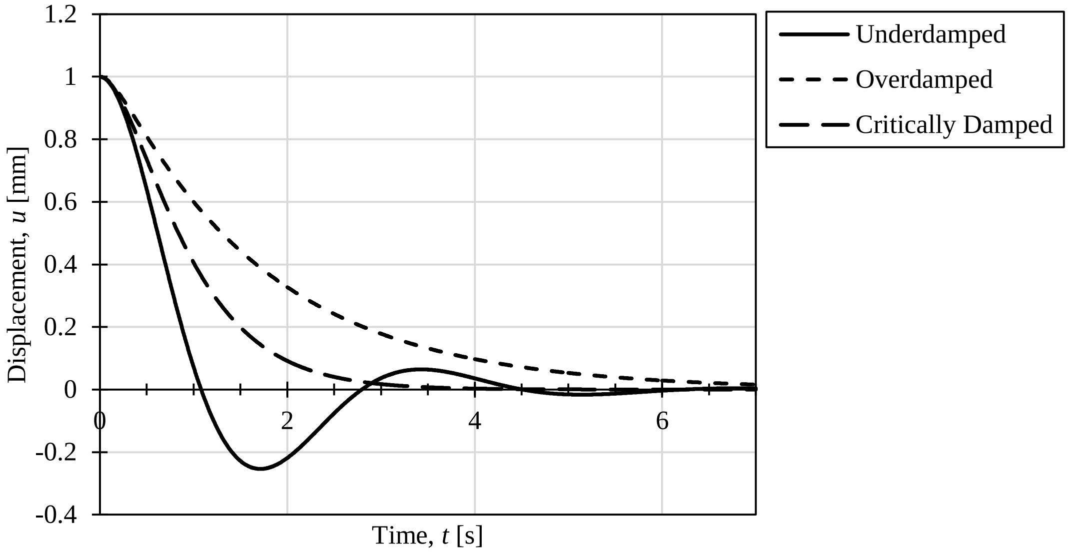


Figure 3.3 Displacement decrement over time for three cases of damping.

3.1.3.1 Underdamped case

For the underdamped case Equation (3-34) can be written as

$$\left.\begin{matrix}\bar{s}_1\\ \bar{s}_2\end{matrix}\right\} = -\xi\omega_n \pm i\,\omega_d \tag{3-35}$$

ω_d is the damped circular natural frequency in (rad/s)

$$\omega_d = \omega_n\sqrt{1-\xi^2} \tag{3-36}$$

With the corresponding damped period T_d

$$T_d = \frac{2\pi}{\omega_d} \quad [\mathrm{s}] \tag{3-37}$$

With help of Euler's formula the general solution can be written as

$$u(t) = e^{-\xi\omega_n t}(A_1\cos\omega_d t + \sin\omega_d t) \tag{3-38}$$

The initial conditions in Equation (3-23) gives A_1 and A_2 resulting in

$$u(t) = e^{-\xi\omega_n t}\left(u_0\cos\ \omega_d t + \frac{v_0 + \xi\omega_n u_0}{\omega_d}\sin\omega_d t\right) \tag{3-39}$$

This equation can also be written as

$$u(t) = Ue^{-\xi\omega_n t}\cos(\omega_d t - \alpha) \tag{3-40}$$

In addition to this it can be shown with the rotating vector technique that the amplitude and phase can be written in the form

$$U = \sqrt{u_0^2 + \left(\frac{v_0 + \xi\omega_n u_0}{\omega_d}\right)^2} \tag{3-41}$$

And

$$\tan\alpha = \frac{v_0 + \xi\omega_n u_0}{\omega_d u_0} \tag{3-42}$$

Regardless to the level of damping the response can be written as

$$u(t) = \frac{v_0}{\omega_d} e^{-\xi\omega_n t} \sin \omega_d t \tag{3-43}$$

Where the difference in damping is expressed in the rate which the motion dies out, meaning the term $e^{-\xi\omega_n t}$. This is illustrated in Figure 3.4 for different levels of damping.

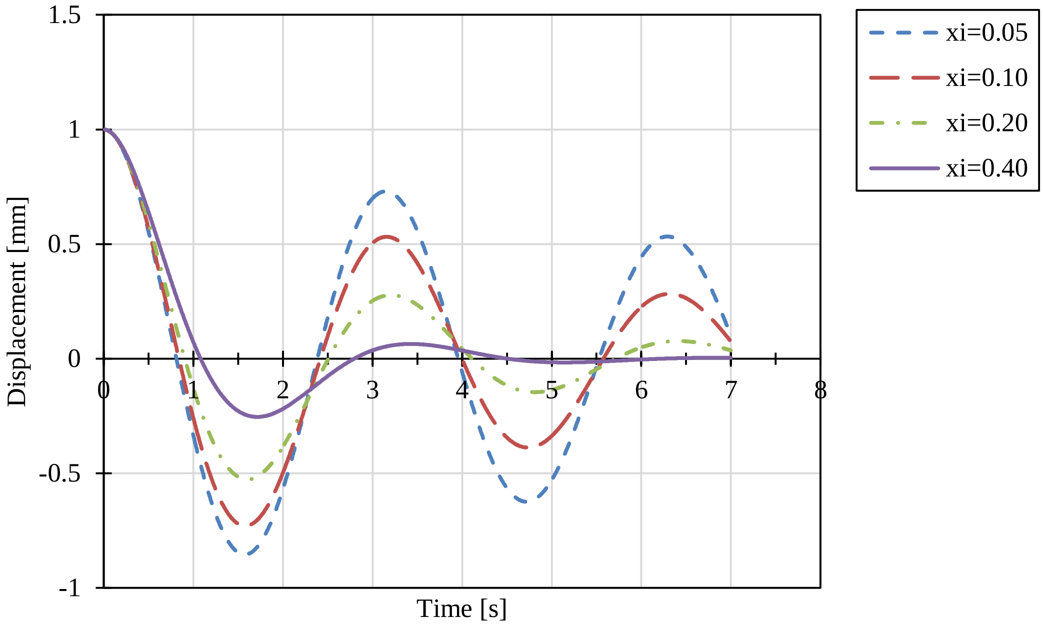


Figure 3.4 Rate of motion for different levels of damping.

3.1.3.2 Critically damped case ($\xi = 1$)

With the damping $\xi = 1$ the characteristic equation Equation (3-16) has one solution

$$\bar{s} = -\omega_n \tag{3-44}$$

The response becomes

$$u(t) = (C_1 + C_2 t)e^{-\omega_n t} \tag{3-45}$$

With the initial conditions taken into account the non-oscillatory response of a critically damped system according to Equation (3-32) becomes

$$u(t) = [u_0 + (v_0 + \omega_n u_0)t]e^{-\omega_n t} \tag{3-46}$$

3.1.3.3 Overdamped case $\xi > 1$

This case of damping results in two negative real roots.

$$\omega^* = \omega_n \sqrt{\xi^2 - 1} \tag{3-47}$$

Which gives that the response of the system can be written as

$$u(t) = e^{-\omega_n t}(C1 \cosh \omega^* t + C_2 \sinh \omega^* t) \tag{3-48}$$

C_1 and C_2 depend on initial conditions which gives Equation (3-49) for the response.

$$u(t) = e^{-\omega_n t}[u_0 \cosh \omega^* t + \frac{v_0 + \xi \omega_n u_0}{\omega^*} \sinh \omega^* t] \tag{3-49}$$

3.1.4 Analytical solution of SDOF

How to solve a SDOF-system analytically differs if the system is damped or not. For more complex systems an analytical solution is often not possible. For a system with a sinusoidal load it is possible to achieve an analytical solution which will be explained in this section.

3.1.4.1 Undamped SDOF system

An undamped SDOF-system as shown in Figure 3.5 subjected to a sinusoidal load can be expressed with the equation of motion in Equation (3-50).

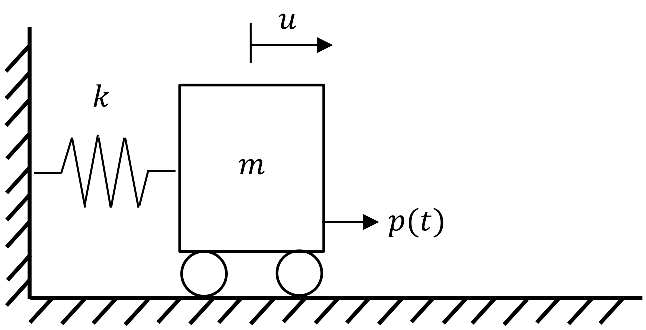


Figure 3.5 Undamped SDOF-system

$$m\ddot{u} + Ku = p_0 \cos(\Omega t) \tag{3-50}$$

As stated before the solution can be divided into a particular and complementary solution:

$$u(t) = u_p(t) + u_c(t) \tag{3-51}$$

To calculate the natural motion u_c when there is no loading Equation (3-50) can be rewritten as:

$$\ddot{u} + \omega_n^2 u = 0 \tag{3-52}$$

With the corresponding characteristic equation:

$$\bar{s}^2 + \omega_n^2 = 0 \tag{3-53}$$

The roots of this characteristic equation are:

$$\bar{s}_{1,2} = \pm i\omega_n \text{ where } \; i = \sqrt{-1} \tag{3-54}$$

When these roots are substituted into the Equation (3-14) the general solution is obtained as:

$$u = \bar{C}_1 e^{i\omega_n t} + \bar{C}_2 e^{-i\omega_n t} \tag{3-55}$$

Introducing Euler's equation:

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$

Gives that Equation (3-51) can be written in terms of trigonometric functions:

$$u_c(t) = A_1\cos(\omega_n t) + A_2 \sin(\omega_n t) \tag{3-56}$$

Were A_1 and A_2 are real constants to be determined from initial conditions. In this case:

$$u(0) = u_0 = A_1$$

And

$$\dot{u}(0) = v_0 = A_2\omega_n$$

Giving an expression for free vibration response of an undamped system:

$$u(t) = u_0\cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t \tag{3-57}$$

Equation (3-52) is the more general equation which will be used to explain the response when the system is subjected to loading $p(t)$ as well.

Equation (3-50) only has even-order derivatives on the left side which means that the forced motion will have the response:

$$u_p(t) = U\cos(\Omega t) \tag{3-58}$$

Where U is the amplitude of the forced motion. The amplitude of the response of the forced motion can be determined by substituting Equation (3-58) into Equation ((3-50) giving:

$$U = \frac{p_0}{k - m\Omega^2} \tag{3-59}$$

Assuming that $k - m\Omega^2 \neq 0$ gives the static displacement, which is the displacement of the mass if the load would be applied statically:

$$U_0 = \frac{p_0}{k} \tag{3-60}$$

This gives that Equation (3-59) can be written

$$\frac{U}{U_0} = \frac{1}{1 - r^2}, \qquad r \neq 1 \tag{3-61}$$

Where r is the frequency ratio between the forcing frequency and the undamped natural frequency ω_n

$$r = \frac{\Omega}{\omega_n} \tag{3-62}$$

The frequency response function $H(r)$ gives the magnitude and sign of the forced motion response as a function of r:

$$H(r) = \frac{U}{U_0} \tag{3-63}$$

Equation (3-58) combined with equation (3-59) gives the steady-state or the forced motion response

$$u_p = \frac{U_0}{1 - r^2} \cos \Omega t, \quad r \neq 1 \tag{3-64}$$

This gives that equation (3-56) combined with (3-61) becomes the total response from the sinusoidal load:

$$u(t) = \frac{U_0}{1 - r^2} \cos \Omega t + A_1 \cos \omega_n t + A_2 \sin \omega_n t \tag{3-65}$$

3.1.4.2 Damped SDOF system

The approach for solving a damped SDOF-system is the same as for an undamped system where the response is divided into natural u_c and forced motion u_p .

For a mass-spring-dashpot model as shown in Figure 3.1 the equation of motion is:

$$m\ddot{u} + c\dot{u} + ku = p_0 \cos \Omega t \tag{3-66}$$

Because of the damping the steady-state, or forced motion, will not be in phase. The excitation will be given by:

$$u_p = U \cos(\Omega t - \alpha) \tag{3-67}$$

U is the steady-state amplitude and α is the phase angle of the steady-state-response, or in other words due to the forced motion

To determine the amplitude and the phase angle rotating vectors are used. For Equation (3-66) the following are stated:

$$\dot{u}_p = -\Omega U \sin(\Omega t - \alpha) \tag{3-68}$$

$$\ddot{u}_p = -\Omega^2 U \cos(\Omega t - \alpha) \tag{3-69}$$

Equations (3-67), (3-68) and (3-69) are substituted into Equation (3-66) giving:

$$-m\Omega^2 U \cos(\Omega t - \alpha) - c\Omega U \sin(\Omega t - \alpha) + kU \cos(\Omega t - \alpha) = \tag{3-70}$$

$$= p_o \cos \Omega t$$

This quation can be described with a force vector polygon since each term in Equation (3-70) represents an action on the mass in consideration. This gives:

$$(kU - m\Omega^2 U)^2 + (c\Omega U)^2 = p_0^2 \tag{3-71}$$

Where:

$$\tan \alpha = \frac{c\Omega}{k - m\Omega^2} \tag{3-72}$$

These equations can therefore be expressed as:

$$D_s(r) \equiv \frac{U(r)}{U} = \frac{1}{[(1 - r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}} \tag{3-73}$$

With:

$$\tan \alpha(r) = \frac{2\xi r}{1 - r^2} \tag{3-74}$$

The total response is given by $u(t) = u_p(t) + u_c(t)$ and with Equations (3-67), (3-73), (3-74) and $u(t) = e^{-\xi \omega_n t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$ for free vibration of viscously damped SDOF system

It can be written as:

$$u(t) = \frac{U_0}{[(1 - r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}} \cos(\Omega \mathrm{t} - \alpha) + \tag{3-75}$$

$$+\mathrm{e}^{-\xi \omega_\mathrm{n} t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

3.2 MDOF system

In a structural system with more than one degree of freedom the SDOF model is not enough to describe its response. For a complex system with multiple degree of freedom (MDOF) a larger system is required. The MDOF-system is based on the SDOF-system but allows multiple degrees of freedom and is written in matrix form.

In Figure 3.6 an example of a multiple degree of freedom system is shown for three masses connected by springs and dampers. The system has three degrees of freedom as the masses are allowed to move in horizontal direction. Mass number two is subjected to external force $p(t)$ acting in horisontal direction.

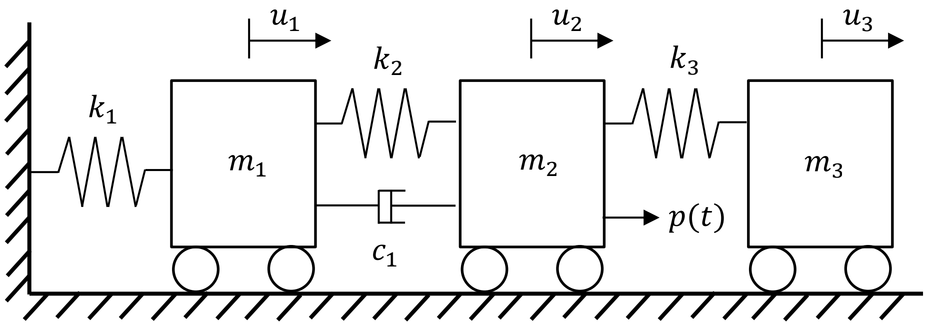


Figure 3.6 An example of MDOF system with three degrees of freedom in horizontal direction.

The equation of motion for a MDOF system is derived by Newton's second law by dividing the system into free-body diagrams for each degree of freedom.

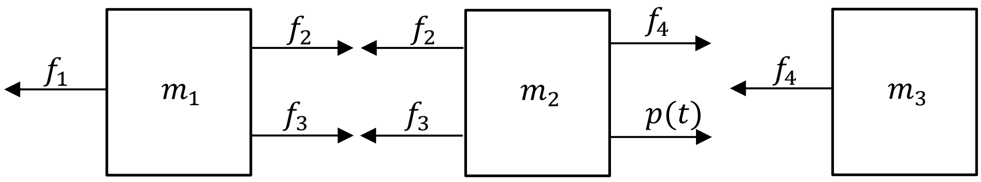


Figure 3.7 Free-body diagram of a MDOF system with three masses

For each free-body diagram the total force F_n is written in equilibrium to the acting forces in equation (3-76) to (3-78).

$$\overset{+}{\rightarrow}\sum F_1 = m_1\ddot{u}_1 = -f_1 + f_2 + f_3 \tag{3-76}$$

$$\overset{+}{\rightarrow}\sum F_2 = m_2\ddot{u}_2 = -f_2 - f_3 + f_4 + p(t) \tag{3-77}$$

$$\overset{+}{\rightarrow}\sum F_3 = m_3\ddot{u}_3 = -f_4 \tag{3-78}$$

The forces f_n are related to the displacement u_n with linear springs and dampers as Equation (3-128) to (3-131).

$$f_1 = k_1(u_1 - 0) \tag{3-79}$$

$$f_2 = k_2(u_2 - u_1) \tag{3-80}$$

$$f_3 = c_1(\dot{u}_2 - \dot{u}_1) \tag{3-81}$$

$$f_4 = k_3(u_3 - u_2) \tag{3-82}$$

Insert the derived forces from Equation (3-79) to (3-82) into the force equilibriums in Equation (3-76) to (3-78) and rearrange.

$$m_1\ddot{u}_1 = -k_1(u_1 - 0) + c_1(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1)$$

$$\Rightarrow \; m_1\ddot{u}_1 + c_1\dot{u}_1 - c_2\dot{u}_2 + (k_1 + k_2)u_1 - k_2u_2 = 0 \tag{3-83}$$

$$m_2\ddot{u}_2 = -k_2(u_2 - u_1) + k_3(u_3 - u_2) + p(t)$$

$$\Rightarrow \; m_2\ddot{u}_2 - c_1\dot{u}_1 + c_1\dot{u}_2 - k_2u_1 + (k_2 + k_3)u_2 - k_3u_3 = p(t) \tag{3-84}$$

$$m_3\ddot{u}_3 = -k_3(u_3 - u_2)$$

$$\Rightarrow \; m_3\ddot{u}_3 - k_3u_2 + k_3u_3 = 0 \tag{3-85}$$

Collect the force equilibriums and insert into the equation of motion in matrix form. The equation of motion on matrix form includes rectangular mass matrix $\mathbf{M}$, damping matrix $\mathbf{C}$ and stiffness matrix $\mathbf{K}$. The matrices are all multiplied by the physical displacement vector $\boldsymbol{u}$ and equal to the load vector $\mathbf{p}$.

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{p} \tag{3-86}$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} c_1 & -c_2 & 0 \\ -c_1 & c_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{bmatrix} + \tag{3-87}$$

$$+\begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ p(t) \\ 0 \end{bmatrix}$$

The mass matrix is a diagonal matrix with the masses inserted in numerical order. The stiffness matrix is not a diagonal matrix as the forces in the springs are dependent on more than one degree of freedom. A system with a stiffness matrix that is not diagonal is said to be a system with stiffness coupling. With coupling in the system it is more difficult to solve but it can be solved with mode superposition explained in section 3.2.1. The damping matrix is similar to the stiffness matrix and is in the most of the cases also coupled.

3.2.1 Mode superposition

Mode superposition is a method to transform a MDOF-system with coupled equations into uncoupled equations. The coupled equation system will generate N equations and N unknows which is a difficult to solve. By uncoupling the system with mode superposition the equations will be uncoupled and independent from each other. The uncoupled equations are independent of each other and can be solved with SDOF-solutions explained in section 3.1.4.

The equation of motion with coupled equations and non-diagonal matrices for a MDOF-system.

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{p}(t) \tag{3-88}$$

To be able to uncouple the equation system the natural frequencies and natural modes has to be known for all N. These are explained in section 3.4 Natural frequencies and natural modes and satisfies Equation (3-89).

$$[\mathbf{K} - \omega_r^2\mathbf{M}]\boldsymbol{\phi}_r = \mathbf{0} \tag{3-89}$$

Where

$$r = 1,2 \ldots N$$

By normalizing the mode vectors it is possible to calculate the modal mass and modal stiffness matrices for each r by Equation (3-90) and (3-91).

$$M_r = \boldsymbol{\phi}_r^T\mathbf{M}\boldsymbol{\phi}_r^T \tag{3-90}$$

$$K_r = \boldsymbol{\phi}_r^T\mathbf{K}\boldsymbol{\phi}_r^T = \omega_r^2 M_r \tag{3-91}$$

The mode vectors for each r can be collected into a modal matrix $\mathbf{\Phi}$ with the mode vectors as columns in the modal matrix.

$$\mathbf{\Phi} = [\boldsymbol{\phi}_1 \quad \boldsymbol{\phi}_2 \quad \cdots \quad \boldsymbol{\phi}_N] \tag{3-92}$$

The modal matrix makes it possible to determine the modal mass and stiffness matrices for the whole system as diagonal matrices using Equation (3-93) and (3-94).

$$\boldsymbol{M} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} \tag{3-93}$$

$$\boldsymbol{K} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} \tag{3-94}$$

The transformation from coupled to uncoupled matrices with diagonal mass and stiffness matrices changes the equation system. Due to the change in system the displacement vectors has to be transformed by Equation (3-95). In mode superposition the coordinate system is expressed in $\boldsymbol{\eta}(t)$ called principle coordinates or modal coordinates**.**

$$\mathbf{u}(t) = \mathbf{\Phi}\boldsymbol{\eta}(t) = \sum_{r=1}^{N} \boldsymbol{\phi}_r\boldsymbol{\eta}_r(t) \tag{3-95}$$

The equation of motion with modal coordinates and uncoupled equations is written in Equation (3-96).

$$\boldsymbol{M\ddot{\eta}} + \boldsymbol{C\dot{\eta}} + \boldsymbol{K\eta} = \boldsymbol{f}(t) \tag{3-96}$$

Where:

$$\boldsymbol{M} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}$$

$$\boldsymbol{C} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}$$

$$\boldsymbol{K} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}$$

$$\boldsymbol{f}(t) = \mathbf{\Phi}^{\mathrm{T}}\boldsymbol{p}(t)$$

It should be noted that mode superposition in many cases does not generate a diagonal damping matrix due to the complexity of damping in a system. If a diagonal damping matrix is not achieved modal damping or Rayleigh damping methods can be used.

3.2.2 Numerical integration

In this chapter numerical integration methods of second order differential equation are presented. Two methods are presented were the main difference is the stability of the solution, they can either be conditionally or unconditionally stable. The stability depends on the time step chosen. If an algorithm only is stable for time step $h \leq h_{cr}$ the algorithm is defined as conditionally stable. A time step $h \geq h_{cr}$ is too large and will give an unstable solution. The critical time step h_{cr} depends on the eigenvalues of the iteration matrix. Unconditionally stable algorithms are stable for all time steps chosen.

The central difference method is the most fundamental numerical solution method in structural dynamics and it is conditionally stable. A common used method is the Newmark-Beta method which is unconditionally stable and implemented in many FE-programs including ADINA (ADINA R & D, 2012).

3.2.3 Central difference method

The central difference method is an approximate numerical solution that is conditionally stable and is based on the definition of the derivate.

Equation of motion with initial boundary conditions.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}(t) \tag{3-97}$$

$$\mathbf{u}(0) = \mathbf{u}_0 \tag{3-98}$$

$$\dot{\mathbf{u}}(0) = \mathbf{v}_0 \tag{3-99}$$

In the derivation of central difference method the displacement in time is defined as equation (3-100.

$$\mathbf{u}(t_n) \equiv \mathbf{u}_n \tag{3-100}$$

The definition of velocity is the derivate of displacement with backward and forward values, equation (3-101.

$$\dot{\mathbf{u}}_n = \frac{\mathbf{u}_{n+1} - \mathbf{u}_{n-1}}{2h} \tag{3-101}$$

From first order derivative in Equation (3-101) the second order derivative of displacement is derived as Equation (3-102).

$$\ddot{\mathbf{u}}_n = \frac{\mathbf{u}_{n+1} - 2\mathbf{u}_n + \mathbf{u}_{n-1}}{h^2} \tag{3-102}$$

The derived equations for acceleration and velocity are inserted into Equation (3-97) and give the discrete governing equation.

$$\left(\frac{1}{h^2}\mathbf{M} + \frac{1}{2h}\mathbf{C}\right)\,\mathbf{u}_{n+1} + \left(\mathbf{K} - \frac{2}{h^2}\mathbf{M}\right)\,\mathbf{u}_n + \tag{3-103}$$

$$+\left(\frac{1}{h^2}\mathbf{M} - \frac{1}{2h}\mathbf{C}\right)\,\mathbf{u}_{n-1} = \mathbf{p}_n$$

Equation (3-103) contains the displacement for all three time steps, $\mathbf{u}_{n-1}$, $\mathbf{u}_n$ and $\mathbf{u}_{n+1}$. By calculating $\mathbf{u}_{n-1}$ in Equation (3-104) only $\mathbf{u}_{n+1}$ remains unknown and can be solved by equation (3-105.

$$\mathbf{u}_{n-1} = \mathbf{u}_0 - h\mathbf{v}_0 + \frac{h^2}{2}\ddot{\mathbf{u}}_0 \tag{3-104}$$

$$\mathbf{u}_{n+1} = \left(\mathbf{p}_n - \left(\mathbf{K} - \frac{2}{h^2}\mathbf{M}\right)\,\mathbf{u}_n - \left(\frac{1}{h^2}\mathbf{M} - \frac{1}{2h}\mathbf{C}\right)\,\mathbf{u}_{n-1}\right) * \tag{3-105}$$

$$* \left(\frac{1}{h^2}\mathbf{M} + \frac{1}{2h}\mathbf{C}\right)^{-1}$$

3.2.4 Newmark integration

Newmark integration, also known as Newton-β integration, is unconditionally stable and does not require any knowledge of the response frequencies in the system. The method is an implicit method using both the present time step, t_n, and next time step, t_{n+1}. Tough it requires that the equation of motion fulfills equation (3-106.

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}\dot{\mathbf{u}}_{n+1} + \mathbf{K}\mathbf{u}_{n+1} \equiv \mathbf{p}(t_{n+1}) \tag{3-106}$$

Initial conditions are the same as for central differential method and given in equation (3-107) and (3-108).

$$\mathbf{u}(0) = \mathbf{u}_0 \tag{3-107}$$

$$\dot{\mathbf{u}}(0) = \mathbf{v}_0 \tag{3-108}$$

The integration method is a generalized method of Taylor series expansion based on the equation (3-109) and (3-110).

$$\dot{\mathbf{u}}_{n+1} = \dot{\mathbf{u}}_n + [(1-\gamma)\,\ddot{\mathbf{u}}_n + \gamma\ddot{\mathbf{u}}_{n+1}]h \tag{3-109}$$

$$\mathbf{u}_{n+1} = \mathbf{u}_n + \dot{\mathbf{u}}_n h + [(1-2\beta)\ddot{\mathbf{u}}_n + 2\beta\ddot{\mathbf{u}}_{n+1}]\frac{h^2}{2} \tag{3-110}$$

To fulfil the criteria of being unconditionally stable Newmark originally proposed values for γ and β in Equation (3-111) and (3-112). This is called the constant acceleration scheme and in Figure 3.8 the principle of acceleration variation is shown.

$$\gamma = \frac{1}{2} \tag{3-111}$$

$$\beta = \frac{1}{4} \tag{3-112}$$

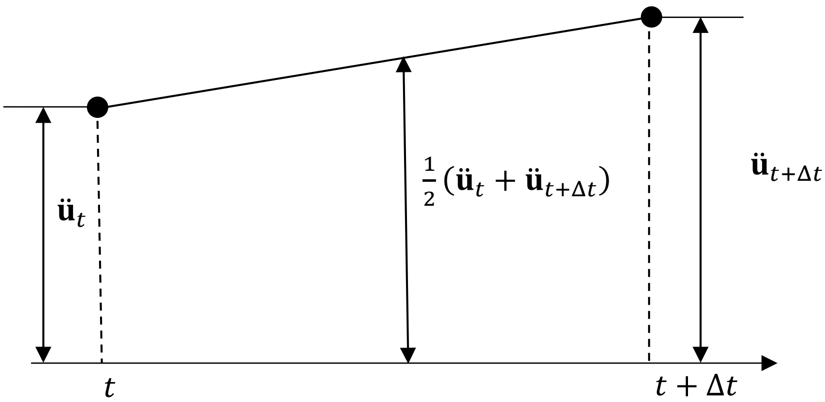


Figure 3.8 The principle of acceleration variation in Newmark's constant acceleration scheme

From initial conditions the initial acceleration is given by Equation (3-113) and initial displacement by Equation (3-114).

$$\ddot{\mathbf{u}}_0 = \mathbf{M}^{-1}(\mathbf{p}(0) - \mathbf{C}\mathbf{v}_0 - \mathbf{K}\mathbf{u}_0) \tag{3-113}$$

$$\mathbf{u}_{-1} = \mathbf{u}_0 - h\mathbf{v}_0 + \frac{h^2}{2}\ddot{\mathbf{u}}_0 \tag{3-114}$$

Insert the two equations above into equation of motion, (3-106).

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}(\dot{\mathbf{u}}_n + [(1-\gamma)\,\ddot{\mathbf{u}}_n + \gamma\ddot{\mathbf{u}}_{n+1}]h) + \tag{3-115}$$

$$+\mathbf{K}\left(\mathbf{u}_n + \dot{\mathbf{u}}_n h + [(1-2\beta)\ddot{\mathbf{u}}_n + 2\beta\ddot{\mathbf{u}}_{n+1}]\frac{h^2}{2}\right) \equiv \mathbf{p}(t_{n+1})$$

The acceleration, $\mathbf{u}_{n+1}$, from equation (3-115 can be solved as $\mathbf{u}_{n-1}$ and $\mathbf{u}_n$ are known.

3.2.4.1 Accuracy of Newmark integration

The Newmark integration is unconditionally stable and independent on the time step length to obtain a stable solution. The time step length will on the other hand affect the computational power needed and the accuracy of the results. In (Bathe, 1996) a study of the accuracy for three integration methods are compared with the exact solution. The results from the study are shown in Figure 3.9 for amplitude decay and period elongation.

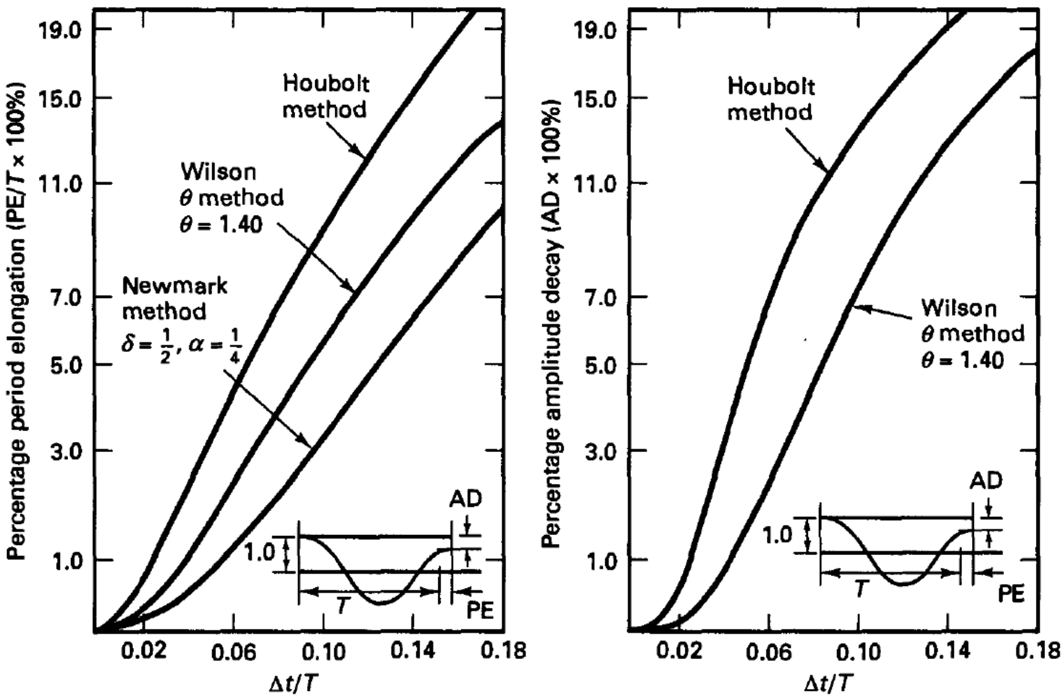


Figure 3.9 Percentage period elongation and amplitude decay for Newmark, Wilson and Houbolt integration methods (Bathe, 1996).

The plot to the left of period elongation and time step length shows that for Newmark integration the deviation is linear proportional to the time step length. A longer time step will give a less accurate solution. The three methods compared gives quite similar results for small time steps but Newmark is the most accurate for larger time steps. The right hand side plot in Figure 3.9 shows the deviation in amplitude and time step length. The time step length in Newmark integration does not affect the resulting amplitude. From this study it can be seen that Newmark integration is the most accurate method regarding the chosen time step length.

3.3 Resonance

One of the most important phenomenon in dynamic is resonance. A system that is in resonance with an exciting force will generate the highest response possible to achieve. Resonance occurs when the exciting forces is harmonic and has the same loading frequency as one of the structures natural frequency. In this chapter two types of resonance phenomena are explained, perfect and beating resonance.

3.3.1 Perfect resonance

Perfect resonance occurs when the exciting force has exactly the same loading frequency, ω_l, as one of the structure´s natural frequency, ω_n. During perfect resonance the displacement and acceleration in the structure will increase linearly over time as shown in Figure 3.10. Resonance is crucial as it will generate the highest response and it can cause discomfort or even failure of a structure.

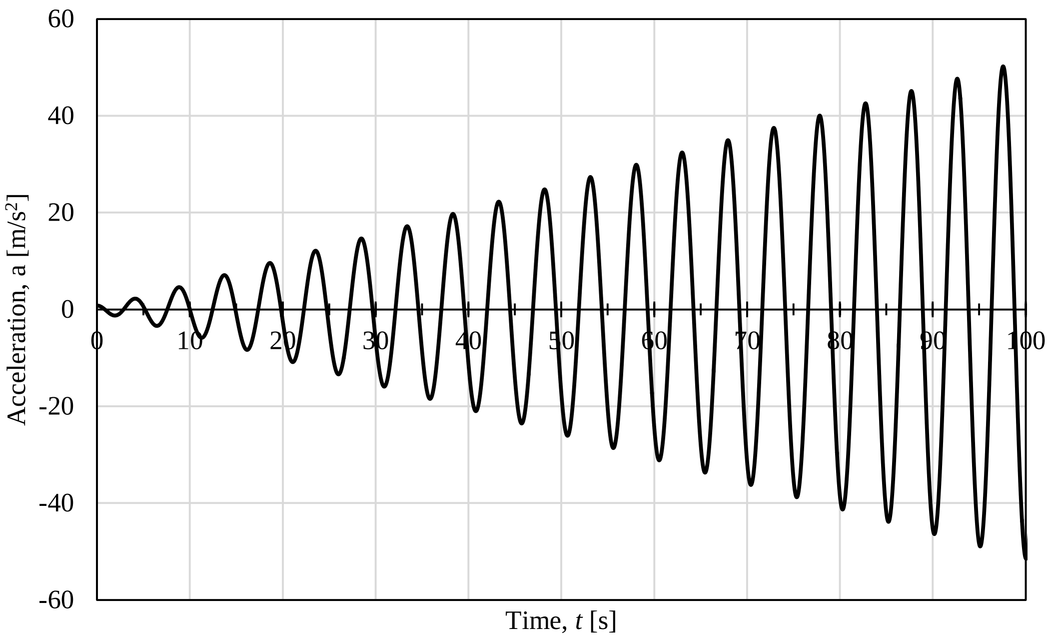


Figure 3.10 Linear increase of the acceleration of a structure in perfect resonance.

The particular solution in Equation (3-116) of an undamped SDOF-system does not allow resonance, as the denominator will be zero equation.

$$u_p = \frac{U_0}{1 - r^2} \cos \Omega_l t \tag{3-116}$$

For an excitation force equal to Equation (3-117) the particular solution for undamped SDOF-system is given in Equation (3-118) and has to be used when $\omega_l = \omega_n$.

$$p(t) = p_0 \cos \omega_n t \tag{3-117}$$

$$u_p(t) = \frac{U_0}{2} \omega_n t \sin \omega_n t \qquad \text{3-118}$$

3.3.2 Beating resonance

The beat phenomenon occurs when a system has closely spaced natural frequencies or when a system is excited with a frequency very close to the systems natural frequency. This results in that the amplitudes of the oscillations are not constant over time but varies (Gaffney, 2002) seen in Figure 3.11.

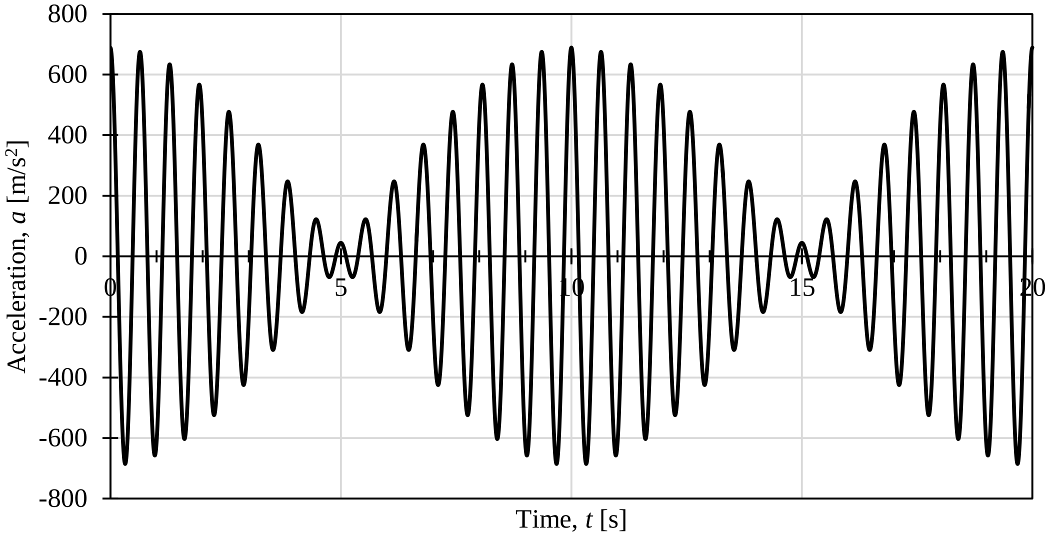


Figure 3.11 Acceleration response in a structure that is excited by an external harmonic force with a frequency close to the natural frequency of the structure.

The beat phenomenon can be explained by free vibration of a spring mass system with two masses shown in Figure 3.12.

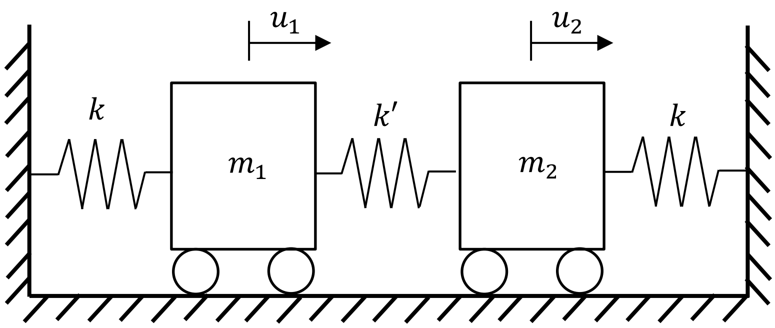


Figure 3.12 Spring mass system

The system has closely spaced natural frequencies if the springs have closely spaced stiffness according to the conditions in Equation ((3-119).

$$k' = k(1 + \delta) \tag{3-119}$$

Where:

δ is close to zero

Free vibration response for a system with two masses can be written as:

$$u_1(t) = \frac{u_0}{2}(\cos \omega_1 t + \cos \omega_2 t) \tag{3-120}$$

$$u_2(t) = \frac{u_0}{2}(\cos \omega_1 t - \cos \omega_2 t) \tag{3-121}$$

The beat frequency ω_b and the average frequency ω_{avg} can be defined by the equations:

$$\omega_b = \omega_2 - \omega_1 \tag{3-122}$$

$$\omega_{avg} = \frac{\omega_2 - \omega_1}{2} \tag{3-123}$$

Resulting in Equations ((3-120) and ((3-121) can be written as:

$$u_1(t) = (u_0 \cos\frac{\omega_b t}{2}) \cos \omega_{avg} t \tag{3-124}$$

$$u_2(t) = (u_0 \sin\frac{\omega_b t}{2}) \sin \omega_{avg} t \tag{3-125}$$

The displacements $u_1(t)$ and $u_2(t)$ can therefore be considered as rapid harmonic movements at frequency ω_{avg} with amplitudes that are varying slowly with $\cos\left(\frac{\omega_b t}{2}\right)$and $\sin(\frac{\omega_b t}{2})$ which means that when the amplitude of one of the masses builds up the other one dies down and vice versa (Roy R. Craig, 2006). This gives rise to the beat phenomenon previously shown in Figure 3.11.

3.3.3 Steady-state

A system excited by a harmonic force will over time settle into equilibrium, a steady state condition. The characteristic of steady state condition is that it will have the same response over time. The response can be either constant or a repeated pattern. By knowing that the system response will not change over time the response of a transient problem can be determined independent of time a variable (Bathe, 1996)

3.3.4 Steady-state magnification factor

At resonance the amplitude is limited only by the damping force. The amplitude for at resonance can be described by the steady-state magnification factor, $D_s(r)$. In Equation (3-126) the factor is given for at resonance for $r = 1$.

$$(D_s)_{r=1} = \frac{1}{2\xi} \tag{3-126}$$

Where:

r	is the nondimensionalized forcing frequency, $r = \Omega/\omega_n$
ξ	structural damping ratio

The magnification factor resonance is plotted in Figure 3.13 for different structural damping ratios. In the figure it can be seen that higher level of damping will generate a lower response amplitude and undamped system will get an infinitely high response at resonance.

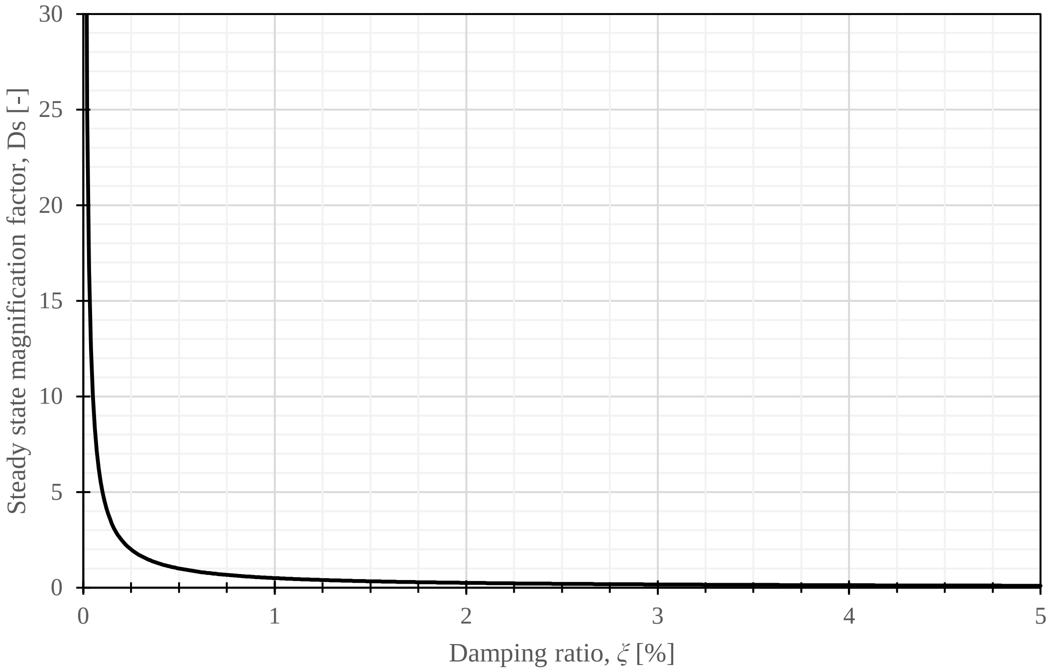


Figure 3.13 Plot of steady state magnification factor and damping ratio

3.3.5 Steady state vs maximum values

The response in a structure varies depending on the frequency of the load if it close to resonance or not explained in section 3.3.1. For a structured being excited by a force it will take some time before steady state is reached. During the analysis it has been discovered that the response in a structure will build up to reach steady state as in FIG! this occurs when the structure and load is in resonance or close to resonance. The steady state response will be the maximum response during its loading time.

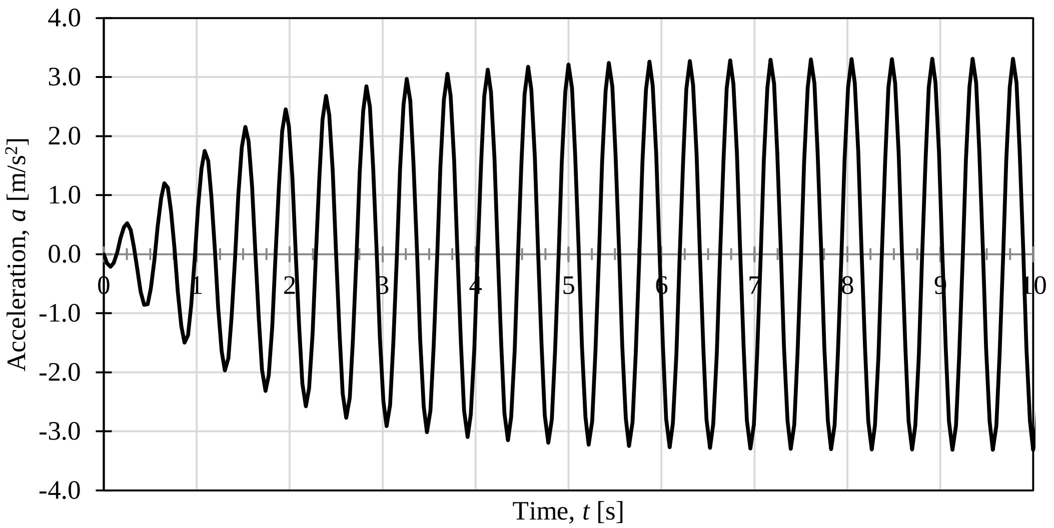


Figure 3.14 Response in a structure in resonance where steady state condition builds up

A different behavior is discovered for structures that are excited by a force which is not in resonance. The response for such a structure is shown in FIG where steady state conditions are reached at 5 seconds. The main differences between the two cases are the initial responses where for the second case the maximum response is reached in the beginning of the loading time. The steady state response is lower than the initial and the response decreases into steady state condition.

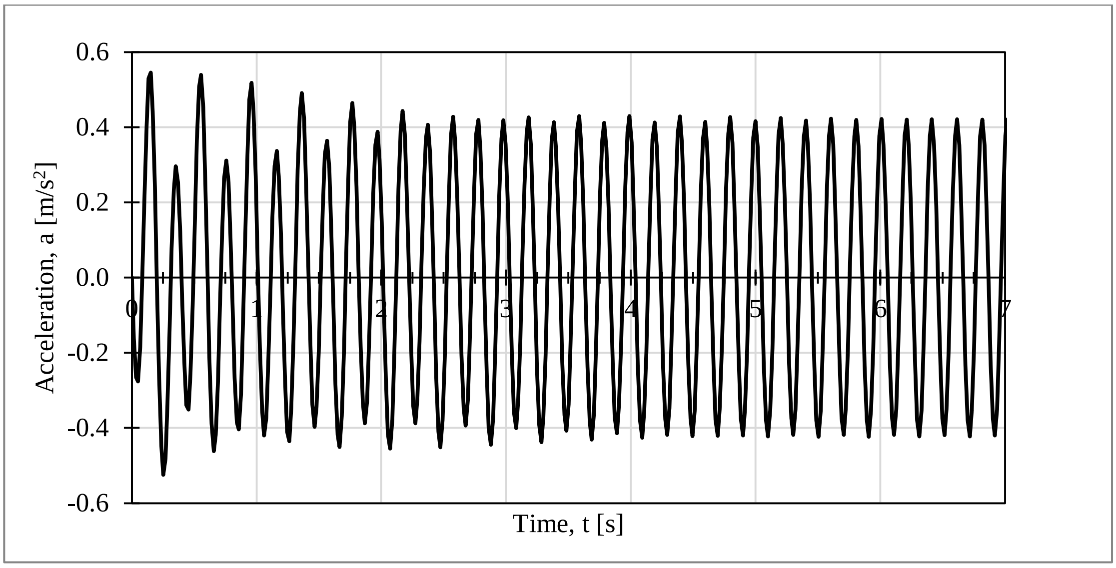


Figure 3.15 Response in a structure not in resonance where the response decreases into steady state conditions

The explanation to this difference in response is resonance. As the second system is not in resonance with the load, the response to the load will counter each other before they tune in to each other's motions. This can generate large responses in the beginning of loading. In the first case resonance occurs from the start and the structure and load will act together without disagreement.

3.4 Natural frequencies and natural modes

The natural frequencies and natural modes are dynamic properties of an element or system. An element that is excited by an external load at the same frequency as its natural frequency will undergo resonance as explained in section 3.3.

The natural modes of a system describe how the system will respond when it vibrates at the natural frequencies. In this chapter the principle of how to obtain the natural frequencies and natural modes for a 2-DOF system are explained. The method is shown in matrix form of a two degree of freedom system and is directly applicable for a MDOF-system.

3.4.1 Natural frequency of a 2-DOF system

The natural frequencies of a 2-DOF for free vibration is an eigenvalue problem and can be solved from the equation of motion. In Equation (3-127) the equation of motion for an undamped 2-DOF system is stated.

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3-127}$$

Assuming that the system will undergo harmonic motion the two solutions to the system is given as:

$$u_1(t) = U_1 \cos(\omega t - \alpha) \tag{3-128}$$

$$u_2(t) = U_2 \cos(\omega t - \alpha) \tag{3-129}$$

Insert the two solutions from Equation (3-128) and (3-129) into the equation of motion gives the algebraic eigenvalue problem in equation (3-130).

$$\left[\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\right] \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3-130}$$

In Equation (3-130) the solution is given for those values of ω that will satisfy the characteristic equation given in Equation (3-131). This is an eigenvalue problem where ω_i^2 is the eigenvalue. For a 2-DOF system the solution is given by two eigenvalues ω_1 and ω_2 where $\omega_1 \leq \omega_2$.

$$\left|\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega_i^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\right| = 0 \tag{3-131}$$

The eigenvalues in equation (3-131 are solved by calculating the determinant of the matrix $\boldsymbol{D}$.

$$\det \begin{bmatrix} k_{11} - \omega^2 m_{11} & k_{12} - \omega^2 m_{12} \\ k_{21} - \omega^2 m_{21} & k_{22} - \omega^2 m_{22} \end{bmatrix} = \det \boldsymbol{D} = 0 \tag{3-132}$$

The determinant of $\boldsymbol{D}$ gives a second order equation equal to zero with the eigenvalues ω_1^2 and ω_2^2 as the solutions. The solution is given as a ratio between stiffness k and mass m.

The natural frequencies are calculated as.

$$f_1 = \frac{\omega_1}{2\pi} \qquad [Hz] \tag{3-133}$$

$$f_2 = \frac{\omega_2}{2\pi} \qquad [Hz] \tag{3-134}$$

3.4.2 Natural modes of a 2-DOF system

The natural modes for a system can be calculated when the natural frequencies are known.

Substitute one of the known eigenvalues from the natural frequencies in Equation (3-133) and (3-134) into equation (3-131) will give the equation system below.

$$\left[\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega_i^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\right] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3-135}$$

The vector containing U_1 and U_2 is unknown and can be expressed as a ratio β with the two amplitudes.

$$\beta_i = \frac{U_1}{U_2} \tag{3-136}$$

The mode shape for mode r is then given in Equation (3-137) where $r = 1{,}2$ for a 2-DOF system. The constant A_i is a normalization factor.

$$\boldsymbol{\phi}_i = \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}_i = A_i \begin{Bmatrix} 1 \\ \beta_i \end{Bmatrix} \tag{3-137}$$

The most efficient way to find the eigenmodes is to set U_1 in equation (3-135) equal to one. This holds for all values of U except if $U_1 = 0$. The equation system in (3-138) can then be solved for ω_i and the eigenvector or mode shape $\boldsymbol{\phi}_i$ is determined.

$$\begin{bmatrix} k_{11} - \omega_i^2 m_{11} & k_{12} - \omega_i^2 m_{12} \\ k_{21} - \omega_i^2 m_{21} & k_{22} - \omega_i^2 m_{22} \end{bmatrix} \begin{bmatrix} 1 \\ U_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3-138}$$

$$\boldsymbol{\phi}_i = \begin{bmatrix} 1 \\ U_i \end{bmatrix} \tag{3-139}$$

The eigenvectors for a 2-DOF system is written as Equation (3-140) and (3-141) below.

$$\boldsymbol{\phi}_1 = \begin{bmatrix} 1 \\ U_1 \end{bmatrix} \tag{3-140}$$

$$\boldsymbol{\phi}_2 = \begin{bmatrix} 1 \\ U_2 \end{bmatrix} \tag{3-141}$$

The mode shapes can be presented graphically as in Figure 3.16.

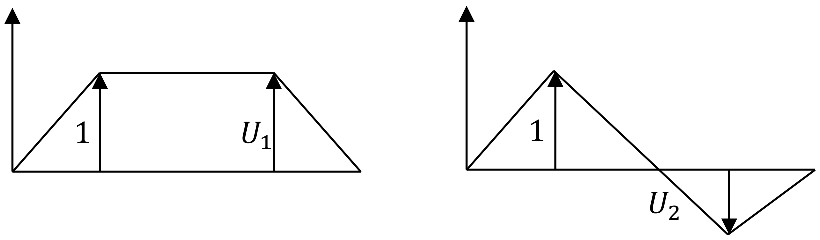


Figure 3.16 Schematic presentation of the mode shapes in a 2-DOF system.

3.5 Fourier series

Forces acting on a structure can in many cases be periodic or can be approximated to be periodic. Tough they are not always continuous or described by different functions in different time intervals. A non-continuous harmonic force can be divided into its harmonic components and described by a Fourier series as a continuous function.

In this chapter the approach and components needed in a Fourier expansion are presented for real Fourier series of a harmonic force. Complex Fourier series are not presented but can be useful in other occasions.

In Figure 3.17 an example of a periodic function over time period T is shown.

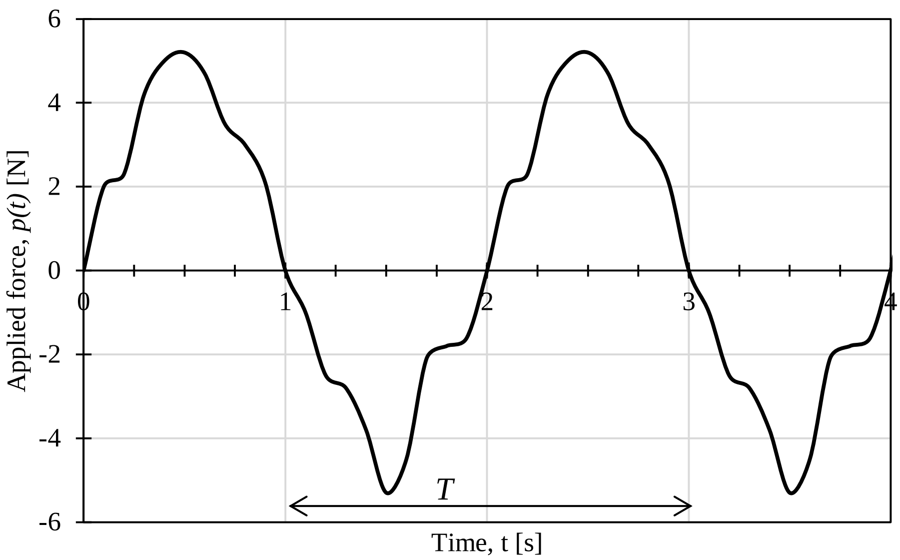


Figure 3.17 Plot of a harmonic force p(t) over time t with period T

A Fourier series expansion of a real periodic function is defined in equation (3-142), where Ω_1 is the fundamental frequency.

$$p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\Omega_1 t + \sum_{n=1}^{\infty} b_n \sin n\Omega_1 t \tag{3-142}$$

$$\Omega_1 = \frac{2\pi}{T_1} \quad [rad/s] \tag{3-143}$$

The coefficients a_0, a_n and b_n are given as in Equation (3-144) to (3-146)

$$a_0 = \frac{1}{T_1}\int_{\tau}^{\tau+T_1} p(t)dt \tag{3-144}$$

$$a_n = \frac{2}{T_1}\int_{\tau}^{\tau+T_1} p(t)\cos n\Omega_1 t\, dt \tag{3-145}$$

$$b_n = \int_{\tau}^{\tau+T_1} p(t)\sin n\Omega_1 t\, dt \tag{3-146}$$

Where:

$$n = 1,2, \ldots$$

It shall be noted that the a-terms are used for even functions and will be equal to zero for all odd functions and vice versa for b-terms which is used for odd functions.

The number of terms, n, used in Fourier expansion affects the accuracy of the expansions ability to describe the original periodic function. The considered number of terms is chosen for individual functions as the terms a and b will decrease for each new n and thereby decrease its affection on the Fourier expansion.

3.6 Dirac delta function

Mathematically speaking the delta function is not a function but a distribution because it is too singular. It can be regarded as an operator that extracts the value of a function at zero

The Dirac delta function $\delta(t)$ is a function that is zero everywhere except zero defined by:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

With

$$\int_{t_1}^{t_2} dt\delta(t) = 1 \tag{3-147}$$

If $0 \in [t_1, t_2]$ it has an infinitely high peak at the origin, $t = 0$, and the function can be seen as a Gaussian limit:

$$\delta(t) = \lim_{\sigma \to 0} \frac{1}{\sqrt{2\pi}\sigma} e^{-t^2/2\sigma^2} \tag{3-148}$$

Or Lorentzian

$$\delta(t) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{t^2 + \epsilon^2} \tag{3-149}$$

The relation

$$\int dtf(t)\delta\,(t) = f(0) \tag{3-150}$$

Is an important property of the Dirac delta function showing that all $\delta(t)$ vanishes everywhere except at $t = 0$. Meaning that is does not matter what values the function $f(t)$ has, except at $t = 0$. From this it can be said that:

$$f(t)\delta(t) = f(0)\delta(t) \tag{3-151}$$

$f(0)$ does not depend on t and can be extracted outside of the integral giving Equation (3-150) as:

$$\int dtf(t)\delta(t - t_0) = f(t_0) \tag{3-152}$$

3.7 Response spectra

A response spectrum is a plot of maximum responses for an SDOF-system to given input versus a system parameter. The response of the system can be for example maximum displacement, acceleration and stress and the system parameter is often the undamped natural frequency.

The purpose of a response spectrum is to provide information to be able to choose one or more system parameters to limit the response of the system when the input is given.

Response Spectra is often used in preliminary design of buildings to secure against earthquake excitation (Roy R. Craig, 2006). It has also been suggested as a way of dealing with pedestrian induced vibrations (Stana Zivanovic, 2010).

In 2013 a research group at the University of Alexandria investigated the response on footbridges of a pedestrian load using Response Spectra Method. The study resulted in a typical Response Spectra plot, Figure 3.18, over the response in N for a range of natural frequencies with varying damping coefficient (El-Sayed Mashaly, Tarek M. Ebrahim, Hamdy Abou-Elfath, Omar A. Ebrahim, 2013).

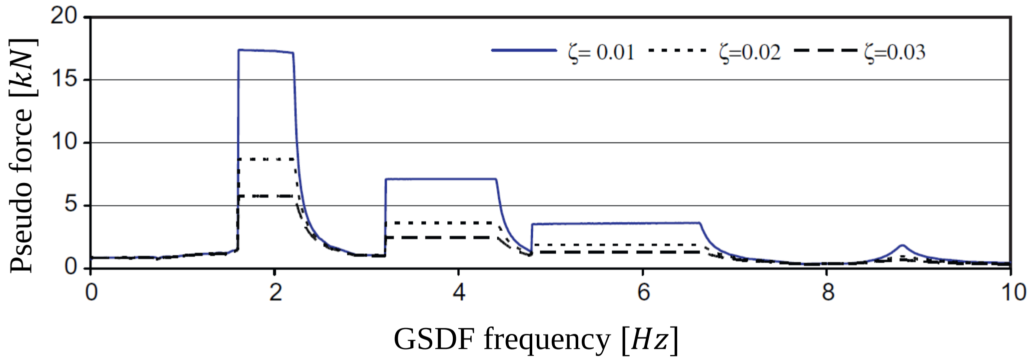


Figure 3.18 Plotted results of a Response Spectra study of pedestrian loading on footbridges

3.8 Monte Carlo simulations

Monte Carlo simulations are based on a method where many analyses with random input variables generates a large amount of random results. The method is applied on mathematical models simulating real systems that are too complicated to solve analytically.

The output data is analyzed using statistical methods to determine the performance of the system (Thomopoulos, 2013).

3.9 Euler-Bernoulli beams

In dynamics and mechanics the Euler-Bernoulli beam theory is widely used for describing the behavior of a beam element. A beam element undergoing transverse vibration the Euler-Bernoulli beam equation is written as Equation (3-153).

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right) + \rho A\frac{\partial^2 v}{\partial t^2} = p_y(x,t) \tag{3-153}$$

$$0 < x < L$$

3.9.1 Analytical solution of natural frequencies

The natural frequencies of a beam element for transverse vibrations can be calculated analytically with Euler-Bernoulli beam equation.

The natural frequencies are calculated when the element is not subjected to an external load which reduces equation (3-153) into equation (3-154).

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right)+\rho A\frac{\partial^2 v}{\partial t^2}=0 \tag{3-154}$$

Assume harmonic motion with solution according to equation (3-155.

$$v(x,t)=V(x)\cos(\omega t-\alpha) \tag{3-155}$$

Obtain the eigenvalue equation by inserting equation (3-155) into beam equation (3-154).

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 V(x)}{\partial x^2}\right)-\rho A V(x)\omega^2=0 \tag{3-156}$$

For a uniform beam the eigenvalue Equation (3-156) can be reduced to a more close form by introducing the eigenvalue λ.

$$\frac{d^4V(x)}{dx^4}-\lambda^4V(x)=0 \tag{3-157}$$

Where:

$$\lambda^4=\omega^2\frac{\rho A}{EI} \tag{3-158}$$

Equation (3-157) is only dependent of x and since the beam equation is of fourth order the general solution to the differential equation is written as.

$$V(x)=C_1\sinh\lambda x+C_2\cosh\lambda x+C_3\sin\lambda x+C_4\cos\lambda x \tag{3-159}$$

Equation (3-159) includes five constants C_1 to C_4 and the eigenvalue λ which can be decided by boundary conditions. In this case a simply supported beam is studied with boundary conditions as below.

$$V(0)=0$$

$$\frac{d^2V(0)}{dx^2}=0$$

$$V(0)=0$$

$$\frac{d^2V(0)}{dx^2}=0$$

Evaluating the boundary conditions at $x=0$.

$$C_2+C_4=0 \tag{3-160}$$

$$\lambda^2\,(C_2-C_4)=0 \tag{3-161}$$

To fulfil the conditions in equation (3-160) and (3-161), C_2 has to be equal to C_4 and equal to zero.

$$C_2 = C_4 = 0 \tag{3-162}$$

The remaining constants are evaluated for boundary conditions at $x = L$ and give.

$$C_1 \sinh \lambda L + C_3 \sin \lambda L = 0 \tag{3-163}$$

$$\lambda^2 \, (C_1 \sinh \lambda L - C_3 \sin \lambda L) = 0 \tag{3-164}$$

By calculating the determinant of the equation system without the constants the non-trivial solutions can be obtained.

$$\begin{vmatrix} \sinh \lambda L & \sin \lambda L \\ \lambda^2 \sinh \lambda L & -\lambda^2 \sinh \lambda L \end{vmatrix} = 0 \tag{3-165}$$

The determinant gives the final equation (3-166.

$$\sinh \lambda L \sin \lambda L = 0 \tag{3-166}$$

To fulfil the conditions in equation (3-166), λL has to be equal to zero since $\sinh \lambda L$ can only be zero for that specific value. Or $\sin \lambda L$ has to be zero which is satisfied for

$$\lambda_r = r\pi/L$$

Where:

$$r = 1,2..N$$

The eigenvalues for a beam element are given from equation (3-167).

$$\lambda_r = \frac{r\pi}{L} \tag{3-167}$$

Where:

$$r = 1,2 \ldots N$$

The natural frequencies can be obtained by inserting the derived eigenvalues from equation (3-167) in equation (3-158). Finally the eigenfrequency for mode r is calculated as in equation (3-169).

$$\lambda_r^4 = \omega^2 \frac{\rho A}{EI} \tag{3-168}$$

$$\omega_r = \left(\frac{r\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \tag{3-169}$$

The eigenfunctions or mode shapes are given by substituting the eigenvalues into equation (3-163) which results in:

$$C_1 = 0$$

And therefore

$$C_1 = C_2 = C_4 = 0 \tag{3-170}$$

The mode shapes are the remaining of equation (3-159 as stated in Equation (3-171).

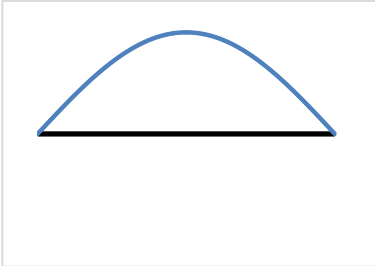
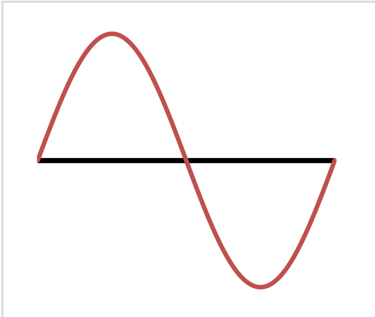
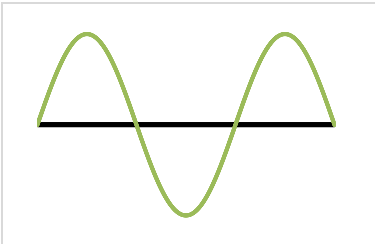
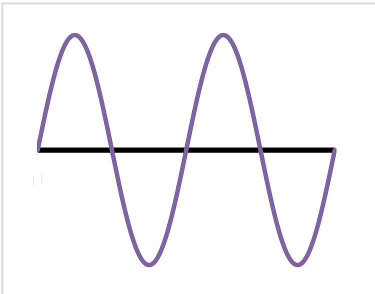
$$V_r(x) = C_3 \sin \lambda_r x \tag{3-171}$$

The constant C_3 is arbitrary but with normalization it is chosen to be equal to one. Inserting the eigenvalue from equation (3-169 into the mode shape the final equation becomes.

$$\phi_r(x) = \sin \frac{r\pi x}{L} \tag{3-172}$$

In Table 3.1 the first four eigenfrequencies and mode shapes are listed and illustrated.

Table 3.1 The first four eigenfrequencies and mode shapes of a simply supported beam

Mode number, r	Eigen frequency for mode r $[rad/s]$	Mode shape for mode, r	
1	$\omega_1 = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\phi_r(x) = \sin\frac{\pi x}{L}$	
2	$\omega_2 = \frac{4\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\phi_r(x) = \sin\frac{2\pi x}{L}$	
3	$\omega_3 = \frac{9\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\phi_r(x) = \sin\frac{3\pi x}{L}$	
4	$\omega_4 = \frac{16\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\phi_r(x) = \sin\frac{4\pi x}{L}$	

4 Standards, regulations and guidelines

In this chapter available standards and guidelines are presented and explained. The considered standards and guidelines are all regarded as applicable and important in the the design of pedestrian bridges due to human induced vibrations.

The presented standards and guidline are:

- Eurocode
- ISO 10137:2008
- UK National Annex to Eurocode
- Sétra - Assessment of vibrational behavior of footbridges under pedestrian loading
- HIVOSS – Human induced vibrations of steel structures
- SYNPEX - Advanced load models for synchronous pedestrian excitation and optimized design guideline for steel footbridges
- JRC - Design of Lightweight Footbridges for Human Induced Vibrations

Finally the Response Spectra method will be presented which is a numerical method recommended in HIVOSS, SYNPEX and JRC.

4.1 Eurocode

Eurocode does not include an exclusive and complete section considering the design of pedestrian bridges due to human induced forces. In this section extracts from several relevant sections in Eurocode are presented.

In Eurocode 1991-2: section 5.7 regarding pedestrian loads on bridges it is stated that depending on the dynamic characteristics of the bridge, the natural frequencies of the structure should be calculated to determine if resonance phenomenon can occur. Eurocode states that resonance can occur for walking, running, jumping and dancing pedestrians. The natural frequencies should be calculated for vertical, lateral and longitudinal directions using an appropriate model of the structure (CEN, 2010).

In Eurocode 1990/A1:2005: section A2.4.3 regarding verifications of comfort in pedestrian bridges it is stated that bridges with a natural frequency lower than 5 Hz in vertical direction or 2.5 Hz for lateral and torsional should be verified against the comfort criteria given in Section 4.1.1. Eurocode enlightens four important load cases to be considered in design given below.

- One person traversing the structure while another one stands in mid-span, acting as the receiver
- Presence of a group with 8 to 15 pedestrians
- Presence of pedestrians streams significantly more than 15 pedestrians
- Festive and choreographic events

(CEN, 2006).

If the natural frequencies of the structure coincide with the interval presented in equation (4-1) the dynamic force from pedestrians should be taken into account in limit state verifications. The standard states that an appropriate load model of pedestrian loads should depend on number of pedestrian and external circumstances to fulfil comfort criteria (CEN, 2010).

Suggested interval for walking frequency:

$$1 \leq f_p \leq 3 \text{ Hz} \quad (4\text{-}1)$$

Suggested frequency for jogging:

$$f_{p.jogg} = 3 \text{ Hz} \quad (4\text{-}2)$$

(CEN, 2010).

The paragraph refers to National Annex for possible pedestrian load models and comfort criteria. The UK National Annex EN 1991-2 (2003) is an annex presenting load models and guidance about pedestrians loading on bridge structures.

Eurocode EN 1990 Annex 1 recommends methods and rules for actions on buildings including a section concerning vibrations in buildings. It states that for buildings with a natural frequency lower than an appropriate value a more refined analysis of the dynamic response should be performed. In this section of Eurocode it refers to the standard ISO 10137 for further guidance (CEN, 2010).

Eurocode 1993-2:2006 concerning steel structures informs that footbridges and cycle-bridges with high vibrations could cause discomfort. The standard recommends to design the bridge with an appropriate natural frequency or with dampers to avoid unpleasant vibrations (CEN, 2009).

4.1.1 Comfort criteria

Eurocode SS-EN 1990/A1:2005 states that comfort criteria should be defined as maximum acceptable accelerations on any part of the bridge. Eurocode provides acceleration limits for vertical and lateral vibrations including limits during exceptional crowded conditions given in Table 4.1. Additional criteria can be given in national annexes. (CEN, 2006)

Table 4.1 Maximum acceptable accelerations recommended by Eurocode

Load case	Acceleration limit [m/s^2]
Vertical vibration	0.7
Lateral vibration	0.2
Exceptional crowd conditions	0.4

4.1.2 Damping

Structural damping ratios to be used for bridges are recommended in Eurocode 1991-2 and given in Table 4.2. The ratios are functions of bridge length with higher damping ratios for shorter bridges (CEN, 2010).

Table 4.2 Structural damping ratio for bridges recommended by Eurocode.

Bridge Type	ξ Lower limit of percentage of critical damping [%]	
	Span L < 20 m	Span L ≥ 20 m
Steel and Composite	$0.5 + 0.125(20 - L)$	0.5
Prestressed concrete	$1.0 + 0.07(20 - L)$	1.0
Filler beam and reinforced concrete	$\xi = 1.5 + 0.07(20 - L)$	$\xi = 1.5$

Table 4.2 is complemented by damping ratios for timber bridges by Eurocode 1995-2:2004. The recommended values are given in Table 4.3 (CEN, 2004).

Table 4.3 Damping ratios for timber bridges according to Eurocode

Type of timber structure	Damping ratio, [%]
Structures without Mechanical Joints	1.0
Structures with Mechanical Joints	1.5

4.1.3 Load model for timber bridges

Eurocode 1995-2:2004, standard for timber structures, presents rules to be applied on timber bridges excited by pedestrian forces. The rules are applicable on bridges designed as simply supported beams or truss systems (CEN, 2004).

4.1.3.1 Vibrations caused by a single pedestrian

The vertical and horizontal vibrations caused by a single pedestrian crossing the bridge is gives by Equation (4-3) and (4-4) for vertical and Equation (4-5).

$$a_{vert,1} = \frac{200}{M\xi} \qquad \text{for } f_{ver} \leq 2.5 \text{ Hz} \tag{4-3}$$

$$a_{vert,1} = \frac{100}{M\xi} \qquad \text{for } 2.5 < f_{ver} \leq 5.0 \text{ Hz} \tag{4-4}$$

$$a_{lat,1} = \frac{50}{M\xi} \qquad \text{for } 0.5 \leq f_{lat} \leq 2.5 \text{ Hz} \tag{4-5}$$

Where:

M total mass of the bridge, [kg]

ξ damping ratio according to Table 4.3

f_{ver} fundamental natural frequency of the bridge in vertical direction

f_{lat} fundamental natural frequency of the bridge in lateral direction

4.1.3.2 Vibrations caused by several pedestrian

The vertical and lateral vibrations for several pedestrian crossing the bridge is given by Equation (4-6) and (4-7).

$$a_{vert,n} = 0.23 a_{vert,1} n\, k_{vert} \tag{4-6}$$

$$a_{lat,n} = 0.18 \alpha_{lat,1}\, n\, k_{lat} \tag{4-7}$$

Where:

n	number of pedestrians according to Table 4.4
k_{vert}	coefficient according to Figure 4.1
k_{lat}	coefficient according to Figure 4.2
$a_{vert,1}$	vertical acceleration caused by a single pedestrian given by Equation (4-3) or (4-4)
$a_{lat,1}$	lateral acceleration caused by a single pedestrian given by Equation (4-5)

The number of pedestrians are given in Table 4.4.

Table 4.4 Number of pedestrians loading the bridge deck

Group of pedestrians	$n = 13$
Continuous stream of pedestrians	$n = 0.6\, A$

Where:

A	Total bridge deck area, [m^2]

The vertical coefficient k_{ver} and lateral coefficient k_{lat} are given in Figure 4.1and Figure 4.2 below.

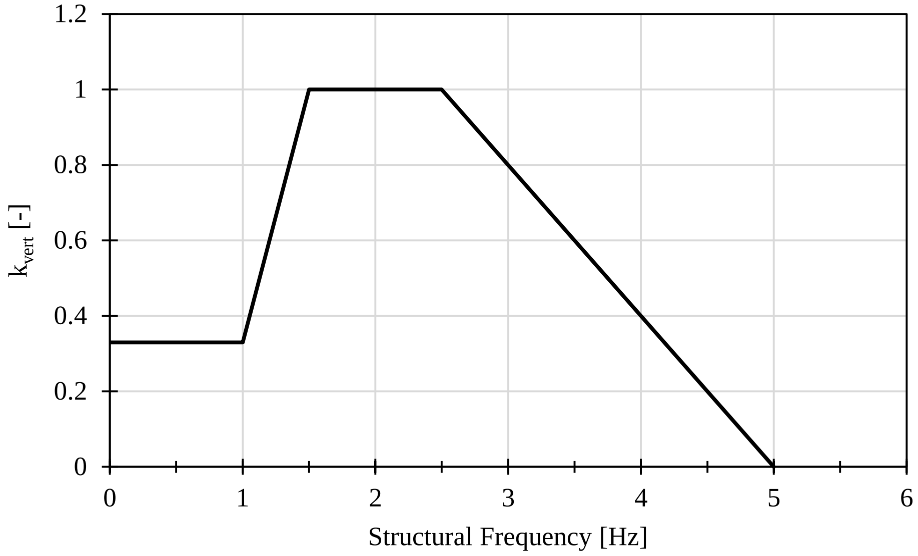


Figure 4.1 *Vertical coefficient k_{ver} for vertical acceleration calculation caused by several pedestrians*

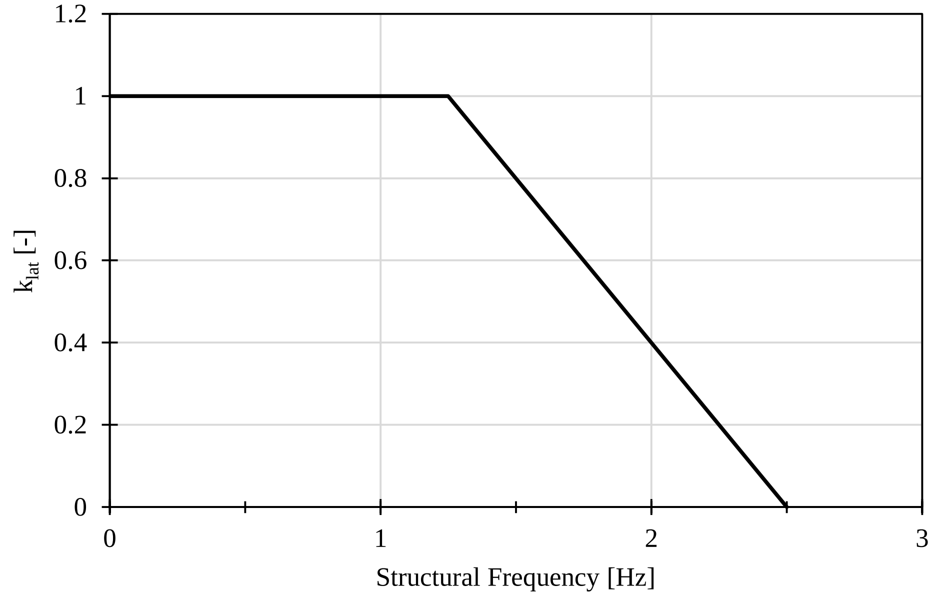


Figure 4.2 *Lateral coefficient k_{lat} for lateral acceleration calculation caused by several pedestrians*

4.2 SS-ISO 10137:2008

ISO 10137 is an international standard presenting bases for design of structures regarding serviceability of buildings and walkways for vibration. Eurocode refers to the standard for further guidance about pedestrian induced vibrations.

ISO 10137 provides basic rules for the design of pedestrian bridges and buildings with regard to vibrations in serviceability limit state. It is an international standard which applies in Sweden. It provides limits for displacements, velocity or acceleration for serviceability limit state. These limits are usually combined with ranges for frequencies and other parameters.

Aspects that are regarded in the standard to provide sufficient design and evaluation criteria:

- Difference in tolerance against vibrations amongst human occupants because of cultural, regional or economic factors
- Building contents sensibility to vibrations and changes in use and occupancy
- Dynamic loadings that are not explicitly regarded in this standard
- Use of materials whose dynamic characteristics change over time
- Limitations of analysis because of the complexity of the structure or the loading
- Consequences of unsatisfactory performance for social or economic factors

(ISO, 2008)

4.2.1 Design guidelines for walkways

The design situations that are to be regarded according to ISO 10137 depend on the pedestrian traffic that will affect the walkway during its life time. The following situations are recommended to consider:

- One person traversing the structure while another one stands in mid-span, acting as the receiver
- A flow of pedestrians, for example in a group of 8 to 15 people, that depends on the length and the width of the walkway
- The possibility of streams of pedestrians significantly larger than 15 people
- Festive or choreographic events that are relevant

4.2.2 Comfort criteria

The comfort criteria are based on acceleration limits given in the standard. For walkways over roads and waterways the level of vibrations should not exceed the proposed acceleration limits. The limits are calculated by multiplying the base curve in Figure 4.3 with 60 for vertical vibrations. Lateral acceleration limits are given by multiplying the curve in Figure 4.4 with the factor 30.

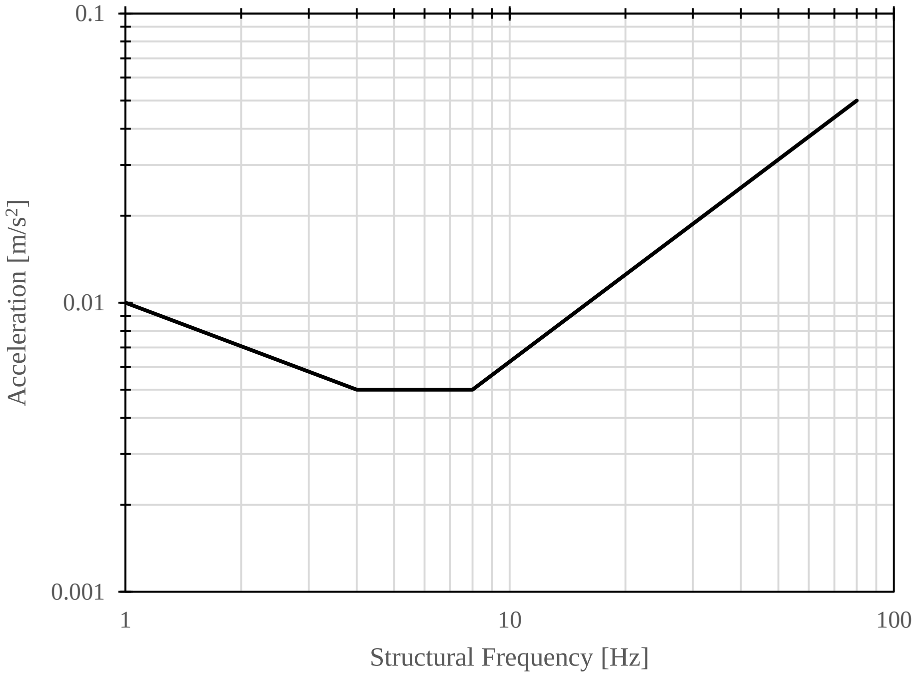


Figure 4.3 Base curve for acceleration limits in vertical direction

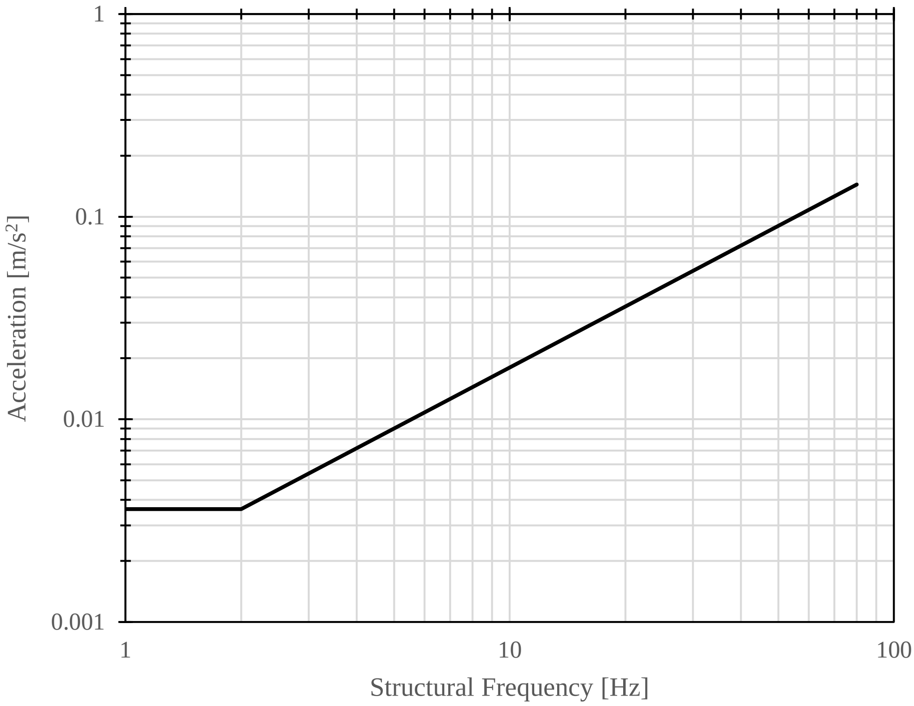


Figure 4.4 Base curve for acceleration limits in lateral direction.

4.2.3 Dynamic load model for single pedestrians

Persons traversing structures can be described as an action that varies with time and position as the person walks over the structure.

The vertical and lateral action from one pedestrian can be described as Fourier series. The vertical load model is given in Equation (4-8) and the lateral load model in Equation (4-9) (ISO, 2008).

$$F_{ver}\ (t) = Q\left(1 + \sum_{n=1}^{k} \alpha_{n,ver} \sin\left(2\pi f t + \phi_{n,ver}\right)\right) \tag{4-8}$$

$$F_{lat}\ (t) = Q\left(1 + \sum_{n=1}^{k} \alpha_{n,lat} \sin\left(2\pi f t + \phi_{n,lat}\right)\right) \tag{4-9}$$

Where:

$\alpha_{n,ver}$	Fourier coefficient corresponding to the n:th harmonic in vertical direction according to Table 4.5, [-]
$\alpha_{n,lat}$	Fourier coefficient corresponding to the n:th harmonic in horizontal direction according to Table 4.5, [-]
Q	static load of the participating person, [N]
f	step frequency. Note that for lateral vibrations, f is one half of the rate of walking or running, [Hz]
$\Phi_{n,ver}$	phase angle of the n:th harmonic in vertical direction according to section 4.2.3.1, [deg]
$\Phi_{n,lat}$	phase angle of the n:th harmonic in lateral direction according to section 4.2.3.1, [deg]
n	number of considered harmonics, [-]
k	number of harmonics that characterize the forcing function in the frequency range of interest. The number of harmonics k that is needed for an accurate model depends on the complexity of the load and its time history, [-].

4.2.3.1 Fourier coefficient and phase angles

The dynamic forces for running and walking are described with Fourier series as in equation (4-8) and (4-9). Fourier coefficient, α, is given in

Table 4.5 for continuous series of steps with different walking frequencies (ISO, 2008).

Table 4.5 Numerical coefficient for pedestrian load models

Activity	Harmonic number	Common range of forcing frequency, nf [Hz]	Numerical coefficient for vertical direction, $\alpha_{n,v}$ [-]	Numerical coefficient for lateral direction, $\alpha_{n,h}$ [-]
Walking	1	1.2 to 2.4	0.37(f - 1.0)	0.1
	2	2.4 to 4.8	0.1	
	3	3.6 to 7.2	0.06	
	4*	4.8 to 9.6	0.06	
	5*	6.0 to 12.0	0.06	
Running	1	2 to 4	1.4	0.2
	2	4 to 8	0.4	
	3	6 to 12	0.1	

*These harmonics are not relevant for human perception of vibration and can be neglected.

The phase angle Φ can be chosen to $\frac{\pi}{2}$ for a conservative approach.

4.2.4 Dynamic actions due to groups of participants

The dynamic response of a group of people will primarily depend on three aspects:

- Weight of participants
- Density of people per unit floor area
- Degree of coordination

The dynamic response will be reduced due to not perfect coordination of the people in a group. The reduced response for uncoordinated movements can approximately be calculated by multiplying the load from a single pedestrian with the coordination factor $C(N)$ in Equation (4-11) (ISO, 2008).

For uncoordinated movement of a group of people the coordination factor is given in Equation (4-10).

$$F(t)_N = F(t)\,C(N) \tag{4-10}$$

Where:

$$C(N) = \frac{\sqrt{N}}{N} \tag{4-11}$$

N number of participants in a group.

4.2.5 Structural damping

ISO 10137 recommends structural damping ratios suitable for pedestrian bridges. The ratios are given for steel, concrete and bridges with both steel and concrete in Table 4.6.

Table 4.6 Structural damping ratios recommended by ISO 10137

Type of structure	Damping ratio ξ % of critical
Steel with asphalt or expoxy surfacing	0.5
Composite steel/concrete	0.6
Prestressed and reinforced concrete	0.8

4.3 UK National Annex

Eurocode refers to national annexes for further guidance regarding design of footbridges for pedestrian loads. The UK National Annex (UK-NA) is an annex to EN 1991-2 (2003) including further guidelines and load models for pedestrian induced forces.

In UK-NA the pedestrian induced forces in vertical and lateral directions are treated. It gives recommendations of categorizing brigdes into bridge classes based on their location. The bridge classes correspond to the expected pedestrian loading. Two different load models are proposed to be used in design regarding vertical vibrations.

The first load model simulates a single pedestrian or a group of pedestrian moving across the bridge span as a concentrated force. The second load model simulates crowds of pedestrians as a uniformly distributed load applied over the entire bridge deck, adapted to the considered mode shape (British Standards Institute, 2008).

Recommendations about the design limits to fulfil SLS criteria are based on maximum vertical acceleration. The maximum vertical acceleration is calculated with the proposed load models and has to be lower than the design acceleration limit for both analyzed models. The design limit is a combined value of factors concerning the users' expectations about bridge vibration.

The lateral stability is assessed with a method using a ratio between mass and damping that indicates if lateral response can be expected.

The annex does not consider exceptional loads as deliberate pedestrian synchronization, vandal loading and mass gathering i.e. marathons and demonstrations.

4.3.1 Bridge classes and expected pedestrian traffic

Bridge classes are defined in UK-NA from class A to D according to Table 4.7. The bridges are categorized in bridge classes by their location and expected usage. All bridges should be categorized into a bridge class which gives recommended group sizes and crowd densities for pedestrians and joggers. The recommended values in Table 4.7 for each bridge class should cover the intended usage of the bridge.

Table 4.7 Bridge classes with corresponding group sizes and crowd densities

Bridge Class	**Bridge usage**	**Group size** (walking)	**Group size** (jogging)	**Crowd density, ρ** (persons/m^2) (walking)
A	Rural locations seldom used and in sparsely populated areas.	N = 2	N = 0	0
B	Suburban locations likely to experience slight variations in pedestrian loading intensity on an occasional basis.	N = 4	N = 1	0.4
C	Urban routes subject to significant variation in daily usage (e.g. structures serving access to offices and schools).	N = 8	N = 2	0.8
D	Primary access to major public assembly facilities such as sports stadia or major public transportation facilities.	N = 16	N = 4	1.5

4.3.2 Vertical response calculations

The maximum vertical acceleration should be determined by applying the load models at the most unfavorable location. The vertical response should include effects from other modes than the fundamental, including torsional if required, to obtain the maximum response. If the modes are not well separated a more complex analysis with combined modes should be applied. The maximum acceleration response calculated for both models has to be lower than the defined limits to fulfill comfort criteria (British Standards Institute, 2008).

4.3.3 Load model for single pedestrians and groups

The maximum vertical acceleration should be obtained by calculating the response from a vertical pulsating force moving along the span. The force applied is a concentrated force simulating a single pedestrian or a group of pedestrians moving at constant speed.

The force $F(N)$ is time and location dependent

$$F(N,t) = F_0 k(f_v)\sqrt{1+\gamma(N-1)}\,\sin(2\pi f_v t) \tag{4-12}$$

Where:

N	number of pedestrians according Table 4.7, [-]
F_0	reference amplitude according to Table 4.8, [N]
f_v	natural frequency of the considered vertical mode, [Hz]
$k(f_v)$	combined factor given in Figure 4.6 considering realistic pedestrian population, harmonic response and weighting of pedestrian sensitivity to vibration, [-]
t	elapsed time, [s]
γ	reduction factor to allow unsynchronization of pedestrian given in Figure 4.7, [-]

(British Standards Institute, 2008)

4.3.3.1 Effective span length

Effective span length is related to the mode shape of the considered mode and is calculated by using the geometry of the mode shape according to Figure 4.5. It is always conservative to use the entire span length instead of the effective.

$$S_{eff} = \frac{Area_A + Area_B}{0.634\,\gamma_{max}} \tag{4-13}$$

$$\lambda = 0.634\frac{S_{eff}}{S} \tag{4-14}$$

Where:

S_{eff}	effective span length given in Equation (4-13), [m]
S	span length, [m]

(British Standards Institute, 2008)

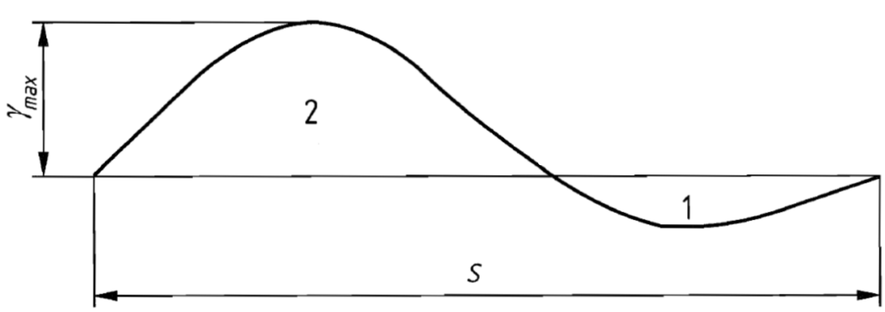


Figure 4.5 Illustration of effective span length. 1 indicates area B and 2 indicates area A

4.3.3.2 Static reference load and pedestrian crossing speed

Recommended values according to UK-NA for reference load and pedestrian crossing speed are given in Table 4.8.

Table 4.8 Reference load and crossing speed for walking and running

Load parameters	Walking	Jogging
Reference load, F_0[N]	280	910
Pedestrian crossing speed, v_t [m/s]	1.7	3

4.3.3.3 Design factor k(f$_v$) for structural frequencies

The factor $k(f_v)$ is described by the two curves in Figure 4.6 representing walking and jogging actions. The factor is a combined factor taken into account three effects.

- A more realistic pedestrian population
- Harmonic response
- Relative weighting of pedestrian sensitivity to response

The factor k(f_v) is a function of the considered mode frequency of the structure on the x-axis, peaking around 2 Hz taking into account the fact that most people walk with a frequency around 2 Hz. The second peak of the function around 3.6 Hz takes the second harmonic into account.

(British Standards Institute, 2008)

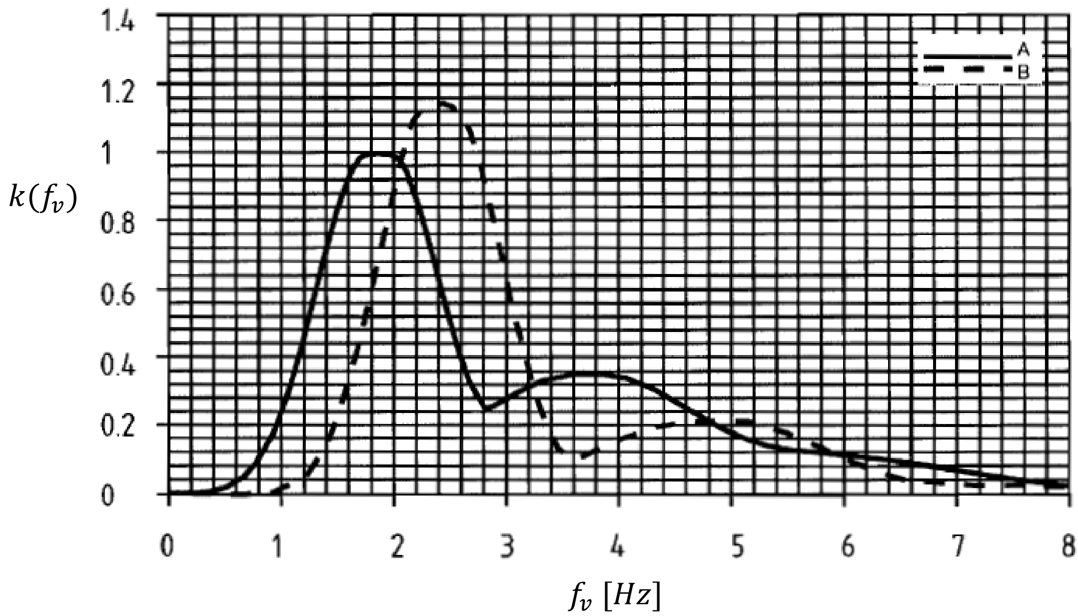


Figure 4.6 Relationship between $k(f_v)$ and mode frequencies f_v. Curve A represents walking pedestrian and curve B for jogging pedestrians.

4.3.3.4 Reduction factor γ

The reduction factor γ for unsynchronized combination of pedestrian actions is a function between structural damping, Equation (4-15) and effective span length given in Figure 4.7. The figure can be used for the concentrated and the uniformly distributed load models. The reduction factor γ for concentrated load models is dependent of the span length with individual curves (British Standards Institute, 2008).

$$\delta = 2\pi\ \xi \tag{4-15}$$

Where:

ξ structural damping ratio, [Hz]

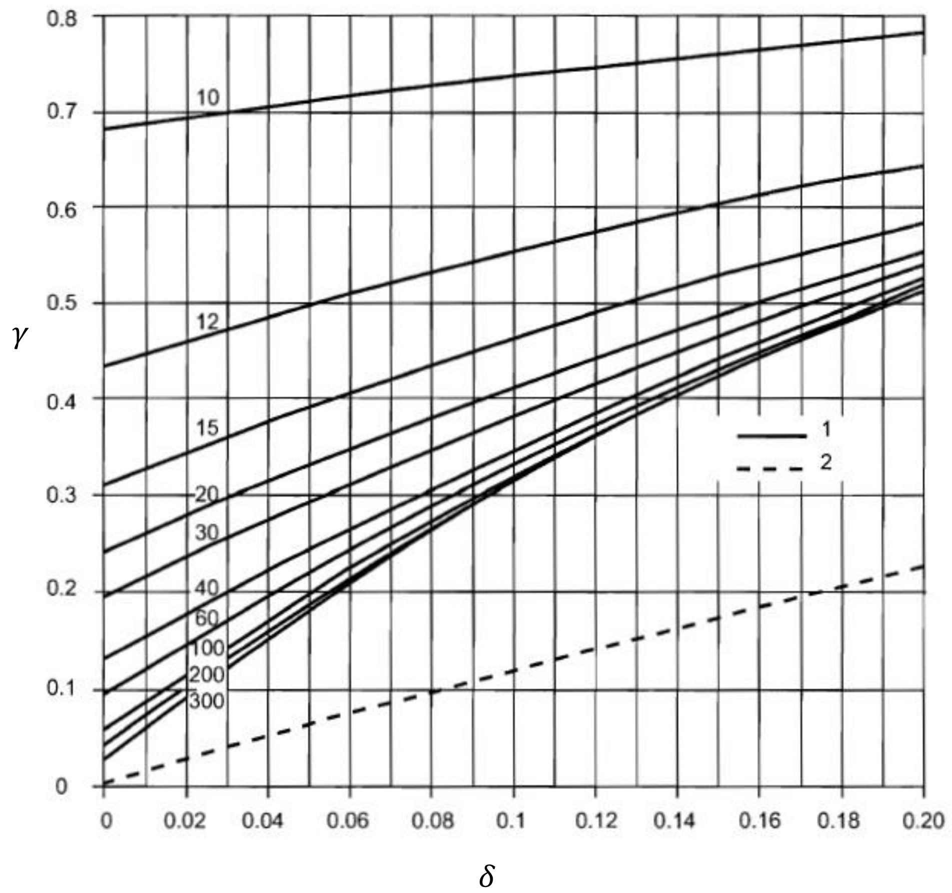


Figure 4.7 Reduction factor γ, to allow for unsynchronized combination of pedestrian actions within groups and crowds. The factor is a function of structural damping δ. Curve 1 is used for pedestrian groups and curve 2 for crowd loading.

Note: All curves represent the variation of the reduction factor with structural damping for the value of effective span, S_{eff}, given.

4.3.4 Steady-state modelling of pedestrian crowds

In crowded situations the maximum vertical acceleration can be determined by a pulsating distributed load applied over the span according to Equation (4-16) defined in UK-NA. The direction of the loading is adapted to the considered mode shape in the most unfavorable way. The load is applied for sufficient time until steady state conditions are achieved.

$$w(N,t) = 1{,}8\left(\frac{F_0}{A}\right)k(f_v)\sqrt{\gamma\frac{N}{\lambda}}\ \sin(2\pi f_v t)\quad [N/m^2] \tag{4-16}$$

Where:

N	number of pedestrians, equation (4-17)
F_0	reference amplitude given in Table 4.8, [N]
A	bridge deck area, [m^2]
$k(f_v)$	combined factor given in Figure 4.6 taking into account realistic pedestrian population, harmonic response and weighting of pedestrian sensitivity to vibrations, [-]
γ	reduction factor to allow for unsynchronization of pedestrians given in Figure 4.7, [-]
λ	reduction factor for effective number of pedestrians given in section 4.3.3.1, [-]
f_v	natural frequency of the considered vertical mode, [Hz]

4.3.4.1 Number of pedestrians

The number of pedestrians of a crowded bridge is based on the bridge class and bridge deck area as seen in equation (4-17).

$$N = \rho A = \rho S b \tag{4-17}$$

Where:

ρ	crowd density according to bridge class, ped/m^2
S	span length, m
b	width of the bridge deck exposed to pedestrian loading
λ	Reduction factor for the effective number of pedestrians when loading from only part of the span contributes to the considered mode, Equation (4-18).

Reduction factor λ is given in Equation (4-18) as a function of span length.

$$\lambda = 0.634\left(\frac{S_{eff}}{S}\right) \tag{4-18}$$

4.3.5 Damping

The UK National Annex refers to Eurocode 1991-2 table 6.6 for appropriate damping values.

Table 4.9 Damping ratios recommended by the UK National Annex

Bridge Type	ξ Lower limit of percentage of critical damping [%]	
	Span $L < 20m$	Span $L \geq 20m$
Steel and Composite	0.5 + 0.125 (20 - L)	0.5
Pre-stressed concrete	1.0 + 0.07 (20 - L)	1.0
Filler beam and reinforced concrete	1.5 + 0.07 (20 - L)	1.5
Timber structures without Mechanical Joints	1.0	
Timber structures with Mechanical Joints	1.5	

4.3.6 Recommended acceleration limits in SLS

The service limit state recommendations are based on the maximum vertical acceleration solved from the appropriate load case applied in the most unfavorable way. To fulfill the requirements in SLS the maximum vertical acceleration should be less than the design acceleration limits. The design limits is a maximum acceleration limit taking four factors into account based on location, pedestrian perception and bridge height. Exceptions from the recommended values are allowed for some type of bridges in remote locations if a suitable risk assessment is done for the individual project.

Maximum acceleration limit:

$$a_{limit} = 1.0 k_1 k_2 k_3 k_4 \; [m/s^2] \qquad (4\text{-}19)$$

Where:

$$0.5 \leq a_{limit} \leq 2.0 \; [m/s^2] \qquad (4\text{-}20)$$

k_1 site usage factor given in Table 4.10

k_2 route redundancy factor given in Table 4.11

k_3 height of the structure given in Table 4.12

k_4 exposure factor given in equation (4-21 and (4-22

The site usage factor depends on the bridge location which corresponds to certain demands and expectations in comfort (British Standards Institute, 2008).

Table 4.10 Recommended values for site usage factor, k_1

Bridge function	$\boldsymbol{k_1}$
Primary route for hospitals or other high sensitivity routes	0.6
Primary route for school	0.8
Primary routes for sport stadia or other high usage routes	0.8
Major urban centers	1.0
Suburban crossings	1.3
Rural environments	1.6

The recommended route redundancy factor is based on how important the bridge is, if there are other alternative bridges or routes as an option for the user.

Table 4.11 Recommended values for route redundancy factor, k_2

Route redundancy	$\boldsymbol{k_2}$
Sole means of access	0.7
Primary route	1.0
Alternative routes readily available	1.3

The structure height factor depends on the bridge height meaning the height between the ground surface and the bridge deck.

Table 4.12 Recommended values for structure height factor, k_3

Bridge height [m]	$\boldsymbol{k_3}$
Greater than 8 m	0.7
4 m to 8 m	1.0
Less than 4 m	1.1

The recommended value of exposure factor k_4, defined by equations (4-21 and (4-22, is primarily suggested to be equal to 1. For individual projects the value can be adjusted to consider other aspect that can affect the users' perception of vibrations. Such aspects can be the parapets design, quality of the walking surface or other aspects. The value must be within the interval defined by equation (4-22.

Recommended value for exposure factor k_4:

$$k_4 = 1.0 \tag{4-21}$$

Recommended limits for exposure factor k_4:

$$0.8 \leq k_4 \leq 1.2 \tag{4-22}$$

4.3.7 Lateral vibrations

The UK Annex defines a frequency limit, equation (4-23) for which lateral vibration could occur and which therefore should be avoided. If the structure has a lateral frequency below the suggested limit a method is established to demonstrate that no unstable lateral vibrations will occur during crowd loading. The method is based on a relationship between mass of the bridge, mass of the pedestrians and structural damping. The relationship between the parameters is called pedestrian mass damping parameter, D, defined in equation (4-24). The parameter is to be compared with a given curve, Figure 4.8 indicating if unstable response is to be expected.

Lateral frequency limit:

$$f_{limit} = 1.5\ Hz \tag{4-23}$$

If at least one of the natural lateral frequencies is below the limit the pedestrian mass damping parameter has to be calculated as:

$$D = \frac{m_{bridge}\ \xi}{m_{pedestrian}} \tag{4-24}$$

Where:

m_{bridge}	mass of the bridge, [kg/m]
$m_{pedestrian}$	mass of the pedestrians, [kg/m]
ξ	structural damping ratio, [Hz]

The pedestrian mass given by the bridge class and corresponding crowd density assuming one person weight is equal to70 kg.

The structural damping ratio is given by recommendations explained in section 4.3.5.

The lateral stability of the structure is assessed by inserting the pedestrian mass damping parameter and the considered lateral frequency of the structure into Figure 4.8. The structure is considered stable if the inserted value falls above the given curve, otherwise unstable conditions are expected.

The curve in Figure 4.8 is based on measured data between 0.5 and 1.1 Hz. The extension of the curve is based on a theoretical model and should be used with caution.

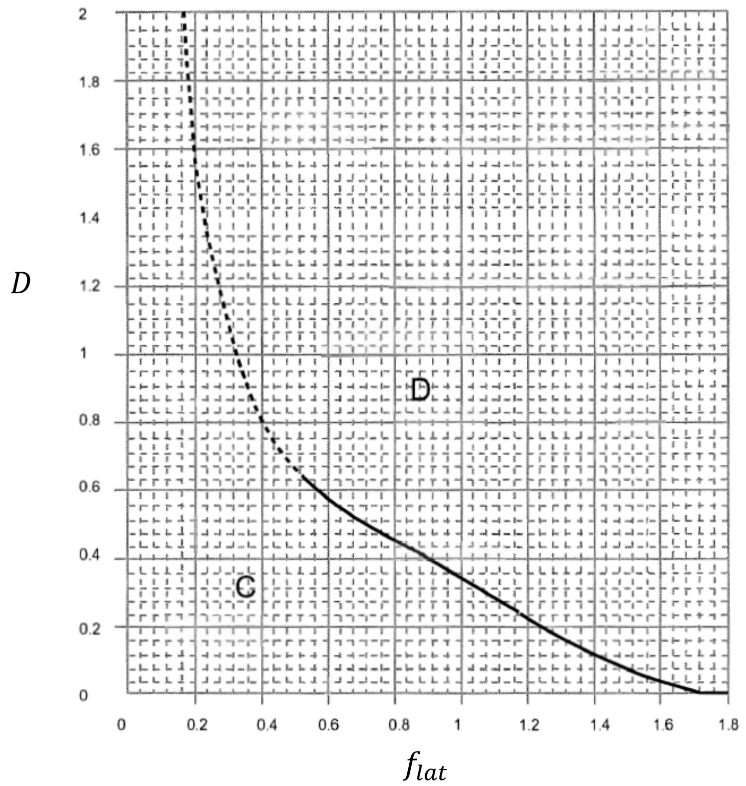


Figure 4.8 Curve for assessing lateral stability as a function of lateral natural frequency flat and pedestrian mass parameter D. Zone C indicates unstable and zone D indicates stable conditions.

4.4 Sétra – Assessment of vibrational behavior of footbridges under pedestrian loading

The report is conducted and published by Sétra (Service d'études techniques des routes et autoroutes), the Technical Department for Transport, Roads and Bridges Engineering and Road Safety in France. The report will be refered to as Sétra in the master thesis report. The purpose of Sétra is to summarize the current knowledge about dynamical behavior of footbridges due to pedestrian loading. The report provides guidelines for design of footbridges regarding dynamical effects with recommendations and a proposed design methodology. The methodology is based on tests and measurements performed on the Solferino footbridge in Paris and experimental tests on platforms in laboratory (Sétra, 2006).

4.4.1 Step frequencies

Sétra suggest estimated step frequencies for walking and jogging to be regarded in the design. The frequencies are defined in ranges and given in Table 4.13 below.

Table 4.13 Step frequency ranges for walking and jogging

Activity	**Step frequency range, [Hz]**
Walking	$1.6 \leq f_s \leq 2.4$
Jogging	$2.0 \leq f_s \leq 3.5$

4.4.2 Load model for a single pedestrian

Pedestrian induced loading can be modeled as a dynamic concentrated force that is a function of time and position of the pedestrian. The load of a human walking or running over a structure can therefore be described with the product of a time, $F(t)$, and space component, $\delta(x - vt)$. The space component is described by a Dirac operator where x is the pedestrian's relation to the centerline of the walkway and v is the speed of the pedestrian traversing the structure according to Equation (4-25) (Sétra, 2006).

$$P(x,t) = F(t)\,\delta(x - vt) \tag{4-25}$$

The guideline proposes a periodic function, $F(t)$, as a Fourier series.

$$F(t) = G_0 + G_1 \sin(2\pi f_m t) + \sum_{i=2}^{n} G_i \sin(2\pi i f_m t - \varphi_i) \tag{4-26}$$

Where:

G_o	Static force from the weight of the pedestrian, 700 N
G_1	First harmonic amplitude, Table 4.14
G_i	i-th harmonic amplitude, Table 4.14
f_m	Walking frequency, [Hz]
φ_i	Phase angle of the i-th harmonic, Table 4.14
n	Number of harmonics used in the calculation

The weight of one person is a mean value of 700 N adopted as G_o. The Fourier coefficients are calculated from a mean frequency of 2 Hz and implemented to the harmonic amplitudes in Table 4.14. Coefficients for i greater than 3 are not considered because they are smaller than 0.1 and gives no significant contribution.

Table 4.14 Harmonic amplitude and phase angle

Number of harmonic, i	Harmonic amplitude [-]	Phase angle [deg]
1	$G_1 = 0.4G_0$	-
2	$G_2 = 0.1G_0$	$\varphi_2 = \pi/2$
3	$G_3 = 0.1G_0$	$\varphi = \pi/2$

In practice Sétra recommends to limit the Fourier sum to the first harmonic. The load model in Equation (4-26) can be adapted to consider vertical action, Equation (4-27), lateral action in Equation (4-28) and longitudinal action in Equation (4-29).

Vertical

$$F_{ver}(t) = G_0 + 0.4G_0\sin(2\pi f_m t) \tag{4-27}$$

Lateral

$$F_{lat}(t) = 0.05G_0\sin\left(2\pi\left(\frac{f_m}{2}\right)t\right) \tag{4-28}$$

Longitudinal

$$F_{long}(t) = 0.02G_0\sin(2\pi f_m t) \tag{4-29}$$

4.4.3 Analysis methodology

Sétra proposes a methodology for how to design a footbridge regarding dynamic analysis. Judging the expected amount of traffic a footbridge class and a desired level of comfort is defined. The calculated natural frequency of the bridge together with the footbridge class results in a dynamic load case that is defined to represent different effects of pedestrian traffic.

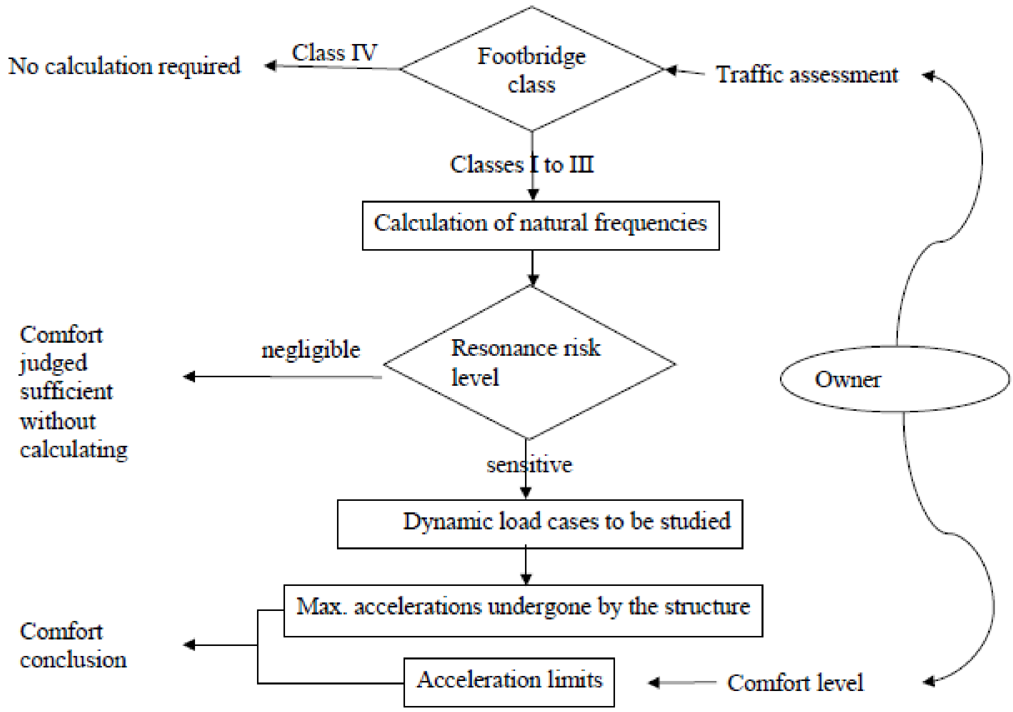


Figure 4.9 Methodology chart

4.4.4 Required dynamic calculations in design for pedestrian loading

The required calculations for design of footbridges with regard to pedestrians streams are depending on bridge classes and natural frequency ranges illustrated in Table 4.15. The bridge classes are explained in section 4.4.5. The frequency ranges need are presented and explained in section 4.4.7. From the table below it is possible to see what type of load that is required in the design. The load cases are explained thoroughly in section 4.4.8.

Table 4.15 Load cases to select based on bridge class and frequency range

<table>
<tr><td rowspan="2">Traffic</td><td rowspan="2">Class</td><td colspan="3">Natural frequency range</td></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Sparse</td><td>III</td><td rowspan="2">Case 1</td><td>Nil</td><td>Nil</td></tr>
<tr><td>Dense</td><td>II</td><td>Case 1</td><td>Case 3</td></tr>
<tr><td>Very dense</td><td>I</td><td>Case 2</td><td>Case 2</td><td>Case 3</td></tr>
</table>

Load cases I to III in Table 4.15 are defined as:

Case 1 Sparse and dense crowd

Case 2 Very dense crowd

Case 3 Crowd complement

4.4.5 Footbridge class

Sétra defines four footbridge classes based on location and usage according to Table 4.16.

Table 4.16 Footbridge classes

Footbridge class	Description
Class IV	Seldom used footbridge, built to link sparsely populated areas or to ensure continuity of the pedestrian footpath in motorway or express lane areas.
Class III	Footbridge for standard use that may occasionally be crossed by large groups of people but that will never be loaded throughout its bearing area
Class II	Urban footbridge linking populated areas subjected to heavy traffic and that may occasionally be loaded throughout its bearing area
Class I	Urban footbridge linking up high pedestrian density areas subjected to very heavy traffic (for instance, nearby presence of a rail or underground station) or that is frequently used by dense crowds

Footbridges in Class IV may not necessarily require a check of the dynamic response but for lightweight and slender footbridges it is advised to choose at least Class III to be conservative. Sétra informs further that a light footbridge can present high accelerations without resonance.

4.4.6 Comfort levels

Three comfort levels are defined in the Sétra guidelines according to Table 4.17. The comfort levels are related to the desired level of vibration judged to be acceptable and defines the allowed acceleration ranges.

Table 4.17 Comfort levels

Comfort level	Description
Maximum comfort	Accelerations undergone by the structure are practically imperceptible to the users
Average comfort	Accelerations undergone by the structure are merely perceptible to the users
Minimum comfort	Under loading configurations that seldom occur, accelerations undergone by the structure are perceived by the users, but do not become intolerable.

If the risk of resonance is deemed negligible after calculating the bridge’s natural frequency the comfort level is automatically met.

4.4.6.1 Acceleration ranges associated with comfort levels

Acceleration limits are stated in ranges due to the uncertainty and subjectivity of the comfort concept. In Table 4.18 and Table 4.19 ranges for accelerations in vertical, longitudinal and lateral direction are stated. Range 1, 2 and 3 correspond to the maximum, average and minimum comfort levels stated in section 4.4.6. Range 4 corresponds to uncomfortable non-acceptable levels of acceleration. The acceleration in lateral direction is in any case limited to 0.10 m/s^2 to avoid lock-in effect.

Table 4.18 Acceleration ranges for vertical vibrations $[m/s^2]$

Acceleration ranges	0	0.5	1	2.5
Range 1	Max			
Range 2		Mean		
Range 3			Min	
Range 4				

Table 4.19 Acceleration ranges for horizontal vibrations $[m/s^2]$

Acceleration ranges	0	0.1	0.15	0.3	0.8
Range 1	Max				
Range 2			Mean		
Range 3				Min	
Range 4					

4.4.7 Frequency range classification

The natural frequencies of bridges are divided into frequency ranges where the ranges correspond to the risk of resonance due to pedestrian induced forces. The ranges are illustrated in Table 4.20 for vertical and longitudinal frequencies and in Table 4.21 for lateral frequencies.

Range 1: maximum risk of resonance

Range 2: medium risk of resonance

Range 3: low risk of resonance

Range 4: negligible risk of resonance

Table 4.20 Frequency ranges of vertical and longitudinal vibrations

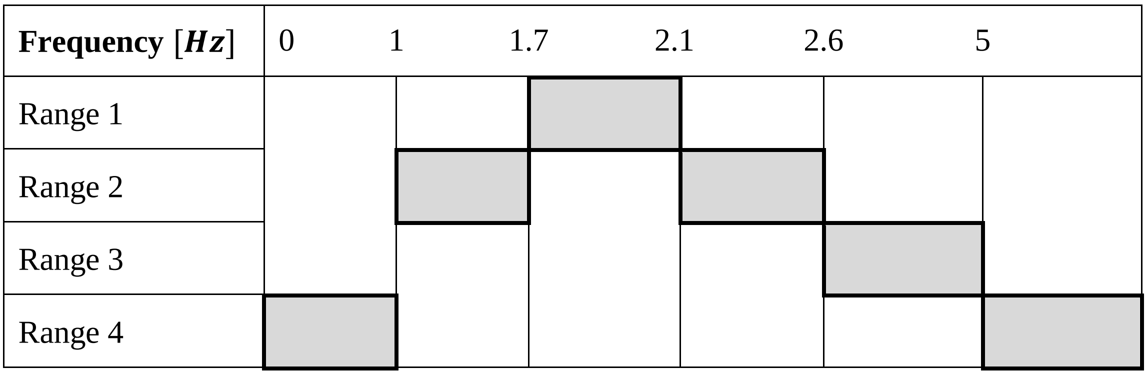
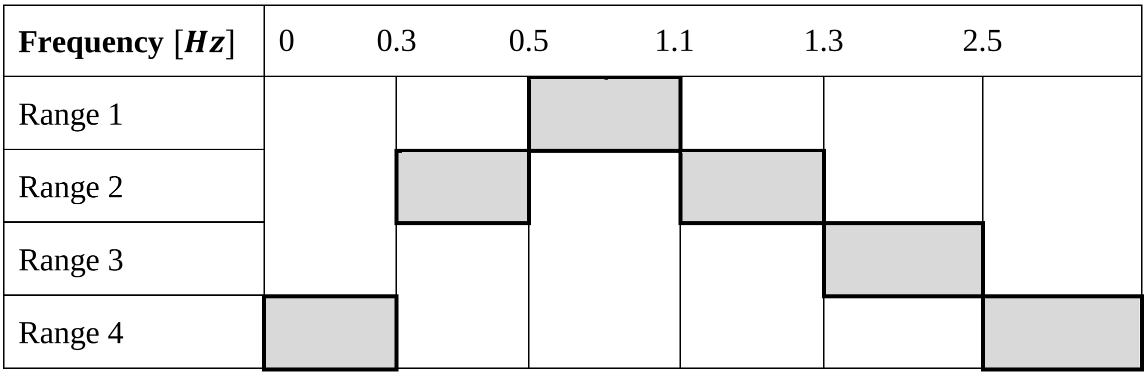


Table 4.21 Frequency range of lateral vibrations



The required calculations depending on bridge classes and natural frequency ranges of the bridge are explained in section 4.4.4.

4.4.8 Load cases

The load cases are to be applied for each relevant vertical, longitudinal and transversal mode at risk. Adjusting the frequency of the load to the natural frequency concerned and applied until steady-state conditions are reached.

The load does not include the static load of the pedestrians which has to be added to the total mass of the footbridge.

4.4.8.1 Load case 1 - sparse and dense crowd

Load case I applies for structures with frequency range 1 and bridge class II and III as well as structures with frequency range 2 and bridge class II.

Table 4.22 Density of pedestrian crowd according to bridge class II and III for load case 1

Class	Density of crowd
III	0,5 pedestrians/m^2
II	0,8 pedestrians/m^2

Number of pedestrians on the bridge deck area based on uniformly distribution.

$$N = S\, d \tag{4-30}$$

Where:

N	Number of pedestrians, [-]
S	Total area of the bridge deck, [m^2]
d	pedestrian density of the crowd according to Table 4.22, [ped/m^2]

The equivalent number of pedestrians represents a group of people with random walking frequencies with an equivalent number of pedestrians at same frequency and in phase:

$$N_{eq} = 10{,}8\,\sqrt{\xi\, N} \tag{4-31}$$

Where:

ξ Critical damping ratio, [%]

The applied load is modified with the modification factor ψ to take into account the probability of resonance. The factor is equal to one if the natural frequency of the bridge is close to normal walking frequencies and zero for unlikely walking frequencies.

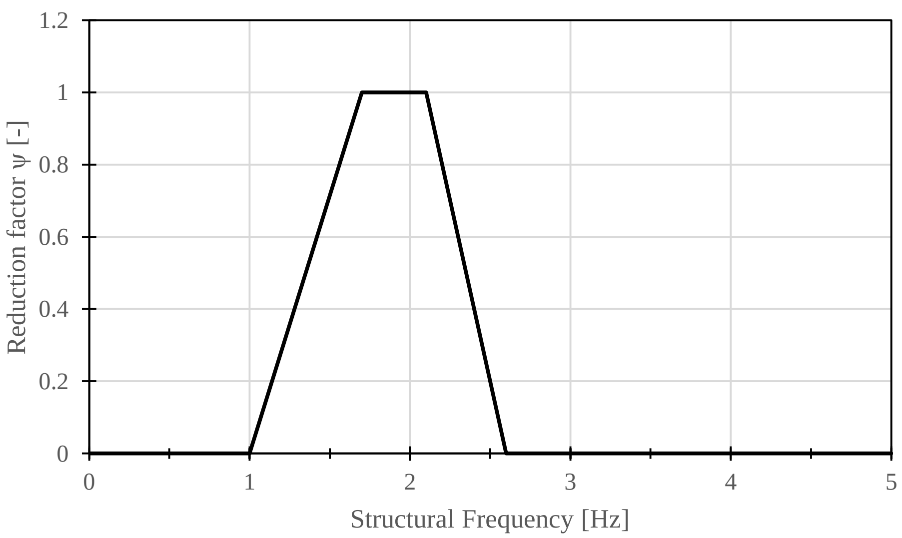


Figure 4.10 Modification factor ψ in the case of walking for vertical and longitudinal vibrations.

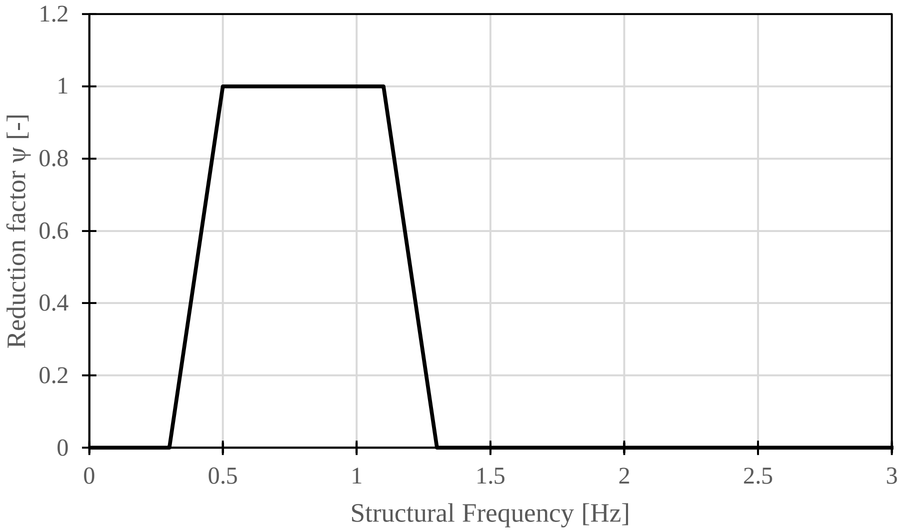


Figure 4.11 Modification factor ψ in the case of walking for lateral vibrations.

The load models for load case 1 are given as:

Vertical direction

$$F_{ver}(t) = d\ 280\cos(2\pi f_v t)\ 10{,}8\sqrt{\xi/n}\,\psi \quad [N/m^2] \tag{4-32}$$

Longitudinal direction

$$F_{long}(t) = d\ 140\cos(2\pi f_v t)\ 10{,}8\sqrt{\xi/n}\,\psi \quad [N/m^2] \tag{4-33}$$

Lateral direction

$$F_{lat}(t) = d\ 35\cos(2\pi f_t t)\ 10{,}8\sqrt{\xi/n}\,\psi \quad [N/m^2] \tag{4-34}$$

4.4.8.2 Load case 2 - very dense crowd

Load case 2 is to be applied for class I footbridges giving pedestrian density equal to 1 pedestrian/m^2.

The equivalent number of pedestrians is calculated as:

$$N_{eq} = 1.85\sqrt{n} \quad [-] \tag{4-35}$$

The load models for load case 2 are given as:

Vertical direction

$$F_{ver}(t) = 1.0\ (280N)\cos(2\pi f_v t)\ 1.85\sqrt{1/n}\,\psi \quad [N/m^2] \tag{4-36}$$

Longitudinal direction

$$F_{long}(t) = 1.0\ (140N)\cos(2\pi f_v t)\ 1.85\sqrt{1/n}\,\psi \quad [N/m^2] \tag{4-37}$$

Lateral direction

$$F_{lat}(t) = 1.0\ (35N)\cos(2\pi f_v t)\ 1.85\sqrt{1/n}\ \psi \quad [N/m^2] \tag{4-38}$$

The value for the modification factor ψ is given in Figure 4.10 and Figure 4.11.

4.4.8.3 Load case 3 - effect of the second harmonic of the crowd

Load case 3 is applied on footbridges of class I and II considering second harmonics. The density and equivalent number of pedestrians are taken according to Table 4.23

Table 4.23 Density and equivalent number of pedestrians for load case 3.

Class	Equivalent number of pedestrians, N_{eq} [-]	Density, $d[ped/m^2]$
I	$1.85\sqrt{n}$	1.0
II	$10{,}8\sqrt{\xi\ N}$	0.8

The load models for load case 3 are given as:

Vertical direction

$$F_{ver}(t) = d(70N)\cos(2\pi f_v t)\ N_{eq}\ \psi \quad [N/m^2] \tag{4-39}$$

Longitudinal direction

$$F_{long}(t) = d(35N)\cos(2\pi f_v t)\ N_{eq}\psi \quad [N/m^2] \tag{4-40}$$

Lateral direction

$$F_{lat}(t) = d\ (7N)\cos(2\pi f_v t)\ N_{eq}\psi \quad [N/m^2] \tag{4-41}$$

The modification factor ψ is given by Figure 4.12 and Figure 4.13 depending on the natural frequency of the considered mode:

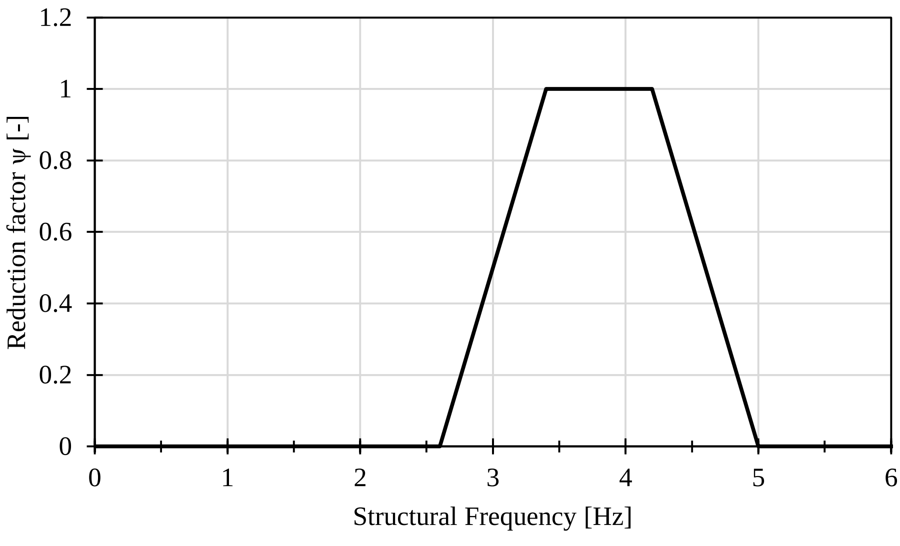


Figure 4.12 Modification factor ψ for vertical vibrations for load case 3.

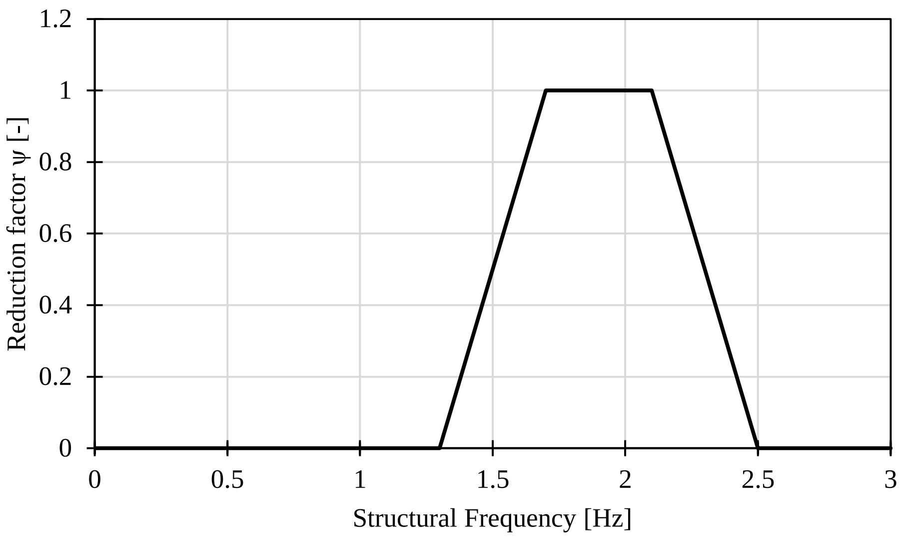


Figure 4.13 Modification factor ψ for lateral vibrations for load case 3.

4.4.9 Structural damping

Following values for structural damping in Table 4.24 are considered during calculations (Sétra, 2006).

Table 4.24 Damping ratios for different materials.

Type	Critical damping ratio
Reinforced concrete	1.3%
Pre-stressed concrete	1%
Mixed	0.6%
Steel	0.4%
Timber	1%

If the construction combines different materials the damping ratio can be taken as an average of the ratios of the different materials combined, weighted by their contribution to the overall rigidity

$$\xi_{mode\ i} = \frac{\sum_{material\ m} \xi_m k_{m,i}}{\sum_{material\ m} k_{m,i}} \tag{4-42}$$

$k_{m,i}$ is the contribution of material m to the overall rigidity in mode i

The determination of $k_{m,i}$ is difficult to determine, so for traditional footbridges with a section that doesn't vary that much the following formula can be used:

$$\xi_{mode\ i} = \frac{\sum_{material\ m} \xi_m EI_m}{\sum_{material\ m} EI_m} \tag{4-43}$$

Where EI is the contribution of the material m to the overall rigidity.

4.5 HIVOSS – Human induced vibrations of steel structures

HIVOSS is short for Human induced vibrations of steel structures, it is based on the results from SYNPEX. The report provides guidelines for design of footbridges regarding dynamical effects with recommendations and a proposed design methodology. The guideline presents uniformly distributed load model for simulating pedestrian streams and alternative methods to regard pedestrian induced vibrations in footbridges (Research Fund for Coal and Steel, 2007). The alternative methods are Response Spectra and a numerical SDOF-solution.

4.5.1 Critical frequency ranges

The natural frequencies of the structure can be determined in many ways. HIVOSS recommends considering the pedestrian mass when determining the natural frequencies but only if the pedestrian mass is larger than 5 % of the structural modal mass.

The guideline suggests critical ranges of natural frequencies for vertical, longitudinal and lateral directions. If the natural frequency of the structure falls in the critical interval a dynamic analysis should be done. For vertical and longitudinal vibrations one interval is defined for the first harmonic, equation (4-44), and one for the second harmonic, equation (4-45). Lateral vibrations are not affected by the second harmonic and only one interval is defined in equation (4-46).

Critical interval for vertical and longitudinal vibrations

$$1.25 \le f_{ver} \le 2.3\ [Hz] \tag{4-44}$$

$$2.5 \le f_{ver.2nd} \le 4.6\ [Hz] \tag{4-45}$$

Critical interval for lateral vibrations

$$0.5 \le f_{lat} \le 1.2\ [Hz] \tag{4-46}$$

4.5.2 Structural damping

The standard recommends minimum and average damping values to be used in serviceability limit state, Table 4.25.

Table 4.25 Minimum and average damping values for serviceability limit state

Construction type	Minimum ξ [%]	Average ξ [%]
Reinforced concrete	0.8	1.3
Prestressed concrete	0.50	1.0
Composite steel-concrete	0.30	0.60
Steel, welded joints	0.20	0.40
Steel, bolted joints	1.0	1.5
Reinforced elastomers	0.70	1.0

Light footbridges excited by intentional loading can undergo large vibrations and leads to higher damping ratios. Damping ratios for large vibrations are given in Table 4.26.

Table 4.26 Damping ratios for structures with large vibrations

Construction type	Damping ratio ξ [%]
Reinforced concrete	5.0
Prestressed concrete	2.0
Steel, welded joints	2.0
Steel, bolted joints	4.0
Reinforced elastomers	7.0

4.5.3 Traffic classes

HIVOSS defines five traffic classes in Table 4.27 with corresponding pedestrian densities. Exceptional loading during formations, processions or matching soldiers are not treated in the standard but need additional consideration.

Table 4.27 Traffic classes with corresponding pedestrian densities.

Traffic class	Density, d [ped/m^2]	Description	Characteristics
TC 1	Group of 15 pedestrians $d = 15\, ped/BL$	Very weak traffic	B – width of bridge deck L – length of bridge deck
TC 2	0.2	Weak traffic	Comfortable and free walk Overtaking is possible Single pedestrians can freely choose pace
TC 3	0.5	Dense traffic	Still unrestricted walking Overtaking can intermittently be inhibited
TC 4	1.0	Very dense traffic	Freedom of movement is restricted Obstructed walking Overtaking is no longer possible
TC 5	1.5	Exceptionally dense traffic	Unpleasant walking Crowding begins One can no longer freely choose pace

4.5.4 Comfort classes and lock-in

The degree of comfort is represented by acceleration limits in vertical and lateral direction. Four comfort classes are defined in the standard and presented in Table 4.28. The highest demands are set to comfort class 1 with lowest allowed acceleration.

Table 4.28 Comfort classes with acceleration limits

Comfort class	Degree of comfort	Vertical a_{limit} $[m/s^2]$	Lateral a_{limit} $[m/s^2]$
CL 1	Maximum	< 0.50	< 0.10
CL 2	Medium	0.50 – 1.00	0.10 – 0.30
CL 3	Minimum	1.00 – 2.50	0.30 – 0.80
CL 4	Unacceptable discomfort	> 2.50	> 0.80

4.5.4.1 Lock-in

Check of lateral lock-in can be performed with two different approaches. The first determines a trigger number of pedestrians and in the second a trigger lateral acceleration is defined (Research Fund for Coal and Steel, 2007).

The triggering number of pedestrians for lateral lock-in can is calculated by equation (4-47).

$$N_L = \frac{8\pi\ \xi\ m^* f}{k} \quad [-] \tag{4-47}$$

Where:

N_L	trigger number of pedestrian for lock-in phenomena, $[-]$
ξ	structural damping ratio according to Table 4.25 and Table 4.26, [-]
m^*	modal mass for the considered mode, [kg]
f	natural frequency for the considered mode, [Hz]
k	constant given in Equation (4-48)

The constant k is derived from experiments on the Millenium Bridge valid in the span 0.50 - 1.0 Hz.

$$k = 300 \quad [Ns/m] \tag{4-48}$$

Instead of describing when lock-in will occur as a trigger value of the number of pedestrian on the bridge has been shown in test that lock-in can be related to lateral acceleration. Lock in can ocurr for the trigger amplitude interval given in equation (4-49).

$$a_{lock-in} = \text{ 0.1 to 0.15 m/s}^2 \tag{4-49}$$

4.5.5 Load model for pedestrian streams

The harmonic load model defined in HIVOSS is a uniformly distributed load over the bridge deck. The load model should be applied according to the mode shapes in the most unfavorable way to obtain maximum acceleration. In order to reach maximum acceleration the walking frequency should be set equal to one of the natural frequencies of the structure in the critical ranges defined. The load model varies in amplitude depending on loading direction and traffic class. In equation (4-50 the basic appearance on the harmonic load model is given.

$$p(t) = P\cos(2\pi f_s t)\ n'\psi\ [N/m^2] \tag{4-50}$$

Where:

P	force component of one single pedestrian with walking frequency f_s, $[N]$
f_s	walking frequency, $[Hz]$
n'	equivalent number of pedestrians on the bridge deck S, $[-]$
S	bridge deck area, $[m^2]$
ψ	modification factor given in section 4.5.5.2

4.5.5.1 Static load amplitude *P*

The load amplitude, P, depends on the considered direction of loading and given in Table 4.29.

Table 4.29 Static load P for varying loading directions

Loading direction	Vertical	Longitudinal	Lateral
Static force, P [N]	280	140	35

4.5.5.2 Modification factor ψ

Modification factor ψ for vertical and longitudinal direction is given in Figure 4.14 and for lateral direction given in Figure 4.15. The factor taking into account the probability that the walking frequency and natural frequency of the structure will fall in the critical range.

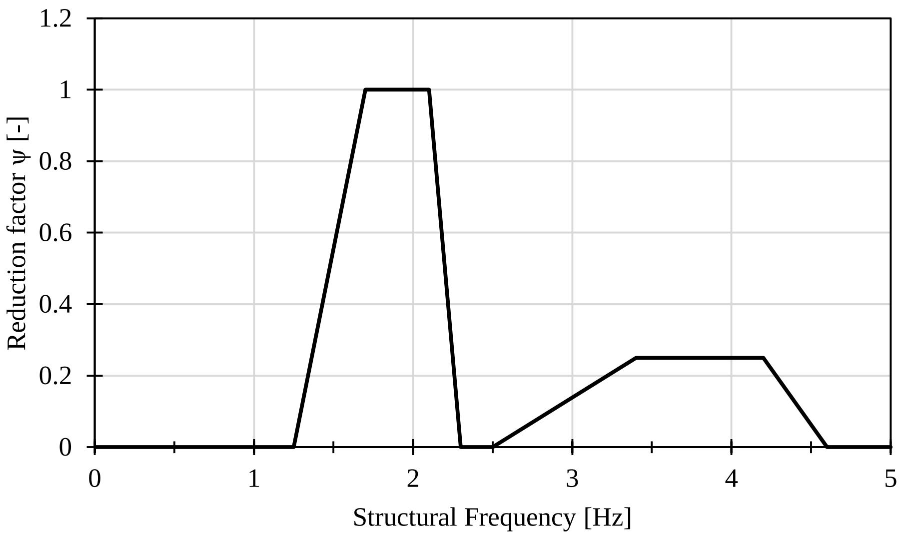


Figure 4.14 Modification factor ψ for vertical and longitudinal loading direction

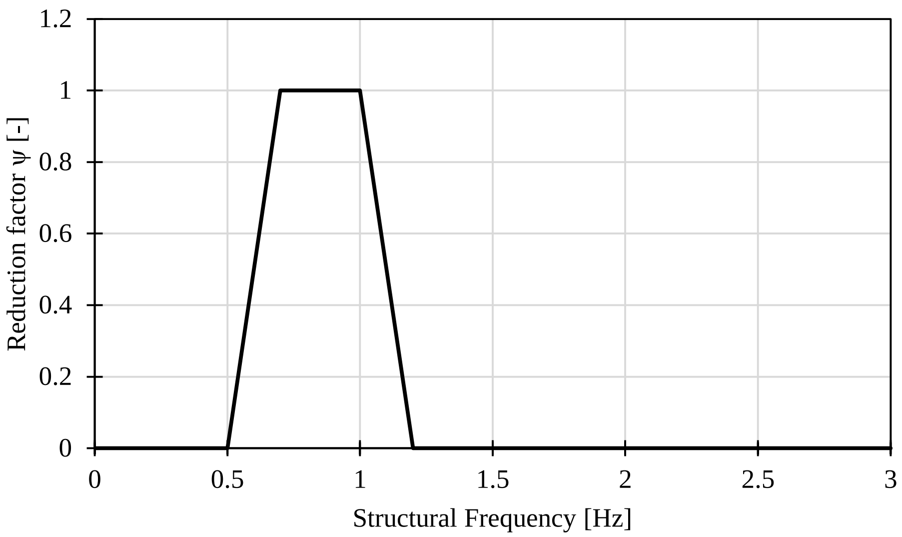


Figure 4.15 Modification factor ψ for lateral loading direction

4.5.5.3 Equivalent number of pedestrians

The equivalent number of pedestrians is defined in HIVOSS depending on traffic class and calculated according to Equation (4-51) and (4-52). The number is a function of pedestrian density, structural damping and bridge deck area.

$$n' = \frac{10.8\sqrt{\xi n}}{S} \quad \text{For d} < 1.0 \text{ ped/m}^2 \tag{4-51}$$

$$n' = \frac{1.85\sqrt{n}}{S} \quad \text{For d} \geq 1.0 \text{ ped/m}^2 \tag{4-52}$$

Where:

ξ	structural damping ratio, [-]
n	total number of pedestrian on the bridge deck area according to Equation (4-53)
S	bridge deck area, [m^2]

Total number of pedestrians on the bridge deck.

$$n = Sd \tag{4-53}$$

Where:

S	total bridge deck area, [m^2]
d	pedestrian density according to Table 4.27, [ped/m^2]

4.5.6 SDOF-solution

HIVOSS proposes an alternative and numerical method of assessing the vibrations in a footbridge due to pedestrian loading. The method is based on derivation from a single degree of freedom system, SDOF-system.

A structure can be transformed into several different spring mass oscillators, where each spring mass represents a natural frequency of the structure. A combination of these can describe arbitrary oscillations of the structure.

For each natural frequency of the bridge an equivalent SDOF-system is used to calculate the maximum acceleration due to dynamic loading.

Maximum acceleration is calculated by:

$$a_{max} = \frac{p^*}{m^*}\frac{\pi}{\delta} = \frac{p^*}{m^*}\frac{1}{2\xi} \tag{4-54}$$

Where

p^*	generalized load
m^*	generalized modal mass
ξ	structural damping ratio
δ	logarithmic decrement of damping

The generalized load and mass are given in equation the generalized load is based on the applied load $p^* \sin(2\pi f t)$.

$$m^* = \int_{L_D} \mu\big(\phi(x)\big)^2 \, dx \tag{4-55}$$

$$p^* \sin(2\pi f t) = \int_{L_D} p(x)\phi(x)\, dx \sin(2\pi f t) \tag{4-56}$$

In Table 4.30 the generalized load and mass are given of a simply supported beam for the first three mode shapes.

Table 4.30 Generalized load and mass with tuning time

Mode number, n	Generalized mass, m^*	Generalized distributed load, p^*	Generalized moving load, p^*	Tuning time, t_{max}
1	$\frac{1}{2}\mu L$	$\frac{2}{\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{v}$
2	$\frac{1}{2}\mu L$	$\frac{1}{\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{2v}$
3	$\frac{1}{2}\mu L$	$\frac{2}{3\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{3v}$

Where:

$p(x)$	distributed load, $[kN/m^2]$
P_{mov}	moving load, $[kN]$

L length of the beam, $[m]$

n number of mode, $[-]$

μ mass distribution per length, $[kg/m]$

v velocity of moving load, $[m/s]$

4.5.7 Response Spectra method

HIVOSS propose the Response Spectra method to calculate the maximum acceleration in a bridge structure. The method is a numerical method explained in section 4.8.

4.6 SYNPEX - Advanced load models for synchronous pedestrian excitation and optimized design guideline for steel footbridges

The report Advanced load models for synchronous pedestrian excitation and optimized design guideline for steel footbridges (SYNPEX) is published by the Research Fund of Coal and Steel in 2008. The purpose of the report was to develop advanced load models for synchronous pedestrian excitation with an optimized design guideline for steel footbridges. The report includes thorough information about current knowledge of pedestrian induced vibrations with several mathematical models for simulating the load. Also existing footbridges have been studied where vibrations has been measured. The report proposes different ways of calculating the vibrations in the guideline with suitable traffic classes and comfort criteria (Research Fund for Coal and Steel, 2006).

4.6.1 Critical step frequencies

SYNPEX suggests critical step frequency intervals to be considered in dynamic design. The frequency ranges corresponds to normal step frequencies by pedestrians which can cause excessive vibrations in footbridges. The critical step frequencies defines critical natural frequencies for footbridges for vertical and lateral natural frequencies.

Critical natural frequencies in vertical direction:

$$1.3 \leq f_i \leq 2.3 \text{ Hz} \tag{4-57}$$

Critical natural frequencies in lateral direction:

$$0.50 \leq f_i \leq 1.2 \text{ Hz} \tag{4-58}$$

The interval recommended for the concentrated load model is given in (4-59) for vertical and in (4-60) for lateral loading directions.

Critical step frequencies in vertical direction applied for concentrated load model.

$$1.25 \leq f_i \leq 2.3 \text{ Hz} \tag{4-59}$$

$$0.625 \leq f_i \leq 1.15 \text{ Hz} \tag{4-60}$$

4.6.2 Traffic classes

SYNPEX suggests five traffic classes predicting the pedestrian densities. For an individual project the class should be chosen with regard to the bridges location. In Table 4.31 the traffic classes are presented with respective pedestrian density.

Table 4.31 Table of traffic classes with pedestrian densities according to SYNPEX

Traffic class	Density, d $[ped/m^2]$	Description
TC 1	Group of 15 pedestrians	Very weak traffic
TC 2	0.2	Weak traffic
TC 3	0.5	Dense traffic
TC 4	1.0	Very dense traffic
TC 5	1.5	Exceptional dense traffic

The characteristics for each traffic class is defined below.

TC 1 Pedestrian density is given by dividing 15 pedestrians by the bridge deck area

TC 2 Comfortable and free walking

Overtaking is possible

Single pedestrian can freely choose pace

TC 3 Significantly dense traffic

Unrestricted walking

Overtaking can intermittently inhibited

TC 4 Freedom of movement is restricted

Uncomfortable situation, obstructed walking

Overtaking is no longer possible

TC 5 Very dense traffic and unpleasant walking

Crowding begins

One can no longer freely choose pace

4.6.3 Comfort classes

SYNPEX recommends four comfort classes defined by acceleration limits. The limits are given for vertical and lateral acceleration with reference to the Sétra standard. In Table 4.32 the comfort classes with limits and accelerations are given.

Table 4.32 Comfort classes with acceleration limits in vertical and lateral directions

Comfort class	Degree of comfort	Vertical acceleration limit, a_{limit} $[m/s^2]$	Lateral acceleration limit, a_{limit} $[m/s^2]$
CL 1	Maximum	< 0.50	< 0.10
CL 2	Medium	0.50 – 1.00	0.10 – 0.30
CL 3	Minimum	1.00 – 2.50	0.30 – 0.80
CL 4	Unacceptable discomfort	> 2.50	> 0.80

Lateral vibrations are affected by the lock-in phenomena which according to the report can occur if both the step frequency and lateral acceleration fulfill the criteria in Equation (4-61) and (4-62).

Step frequency

$$0.8 < \frac{f_{s.m}}{2} < 1.2 \qquad (4\text{-}61)$$

Lateral acceleration

$$a_{max.lateral} > 0.1\ [m/s^2] \qquad (4\text{-}62)$$

4.6.4 Damping

SYNPEX includes structural damping ratios with reference to Setrá and Eurocode. The report enlightens that the value of damping is of great importance in predicting the amplitude of acceleration though it is difficult to decide the proper damping ratio in the design phase. In Table 4.33 the damping ratios from Setrá guideline are presented as minimum and average damping ratios. Eurocode suggest damping ratios related to the span length which are given in Table 4.34.

Table 4.33 Structural damping ratios for different construction material proposed by SYNPEX with reference to Setrá.

Construction type	Minimum ξ [%]	Average ξ [%]
Reinforced concrete	0.80	1.3
Pre-stressed concrete	0.5	1.0
Composite steel-concrete	0.30	0.60
Steel	0.20	0.40
Timber	1.50	3.0

Table 4.34 Structural damping ratios for different construction material proposed by SYNPEX with reference to Eurocode.

Construction Type	Average ξ [%]	
	$L < 20m$	$L \geq 20m$
Steel and Composite	$\xi = 0.5 + 0.125(20 - L)$	$\xi = 0.5$
Prestressed concrete	$\xi = 1.0 + 0.07(20 - L)$	$\xi = 1.0$
Filler beam and reinforced concrete	$\xi = 1.5 + 0.07(20 - L)$	$\xi = 1.5$
Timber	Without mechanical joints	1.0
	With mechanical joints	1.5

4.6.5 Load model for a single pedestrian

SYNPEX proposes two load models for simulating a single pedestrian walking over the bridge. The models are based on Fourier series and given for vertical and lateral vibrations. The loads should be applied as concentrated loads moving over the bridge span with a constant velocity, v_s.

The load model for vertical vibration is given in Equation (4-63) and for lateral in Equation (4-64).

$$F_{p.ver}(t) = G\left[1 + \sum_{i=1}^{n} \alpha_{i,vert} \sin(2\pi i f_s t - \phi_i)\right] \tag{4-63}$$

$$F_{p.lat}(t) = G \sum_{i=1}^{n} \alpha_{i,lat} \sin(\pi i f_s t - \phi_i) \tag{4-64}$$

Where:

G	Static weight of one person, [N]
n	number of harmonic, [-]
α_i	Fourier coefficient of the considered loading direction and number of harmonic given in Table 4.35, [-]
f_s	step frequency, [Hz]
t	time, [s]
ϕ_i	phase shift of the harmonic given in Table 4.36, [deg]

The Fourier coefficients α are given in Table 4.35. The coefficients are constant for the first three harmonics in lateral direction. In vertical direction are the coefficients functions of the step frequency for the first three harmonics.

Table 4.35 Fourier coefficients α for the vertical and lateral load model.

Harmonic number, i [-]	Fourier coefficient vertical model, $\alpha_{i.ver}$ [-]	Fourier coefficient lateral model, $\alpha_{i.lat}$ [-]
1	$\alpha_{1.ver} = 0.0115f_s^2 + 0.2803f_s - 0.2902$	0.1
2	$\alpha_{2.ver} = 0.0669f_s^2 + 0.1067f_s - 0.0417$	0.1
3	$\alpha_{3.ver} = 0.0247f_s^2 + 0.1149f_s - 0.1518$	0.1

The phase angles for the first three harmonics are given in Table 4.36. The angels are constant in lateral direction and functions of the step frequency in vertical direction.

Table 4.36 Phase shifts ϕ for vertical and lateral load model.

Harmonic number, i [-]	Phase shift vertical model, $\phi_{i.ver}$ [°]	Phase shift lateral model, $\phi_{i.lat}$ [°]
1	$\phi_{1.ver} = 0$	0
2	$\phi_{2.ver} = -99.76f_s^2 + 478.93f_s - 387.8$	$\frac{\pi}{2} = 180$
3	If f_s < 2.0 Hz $\phi_{3.ver} = -150.88f_s^3 + 819.65f_s^2 - 1431.35f_s + 811.93$ If $f_s \geq$ 2.0 Hz $\phi_{3.ver} = 813.12f_s^2 - 5357.6f_s^2 + 11726f_s - 8505.9$	$\frac{\pi}{2} = 180$

The pedestrian crossing velocity is given by Equation (4-65) as a function of the step frequency. The function is empirically determined from measurements with step frequencies between 1.3 and 1.8 Hz.

$$v_s = 1.271\ f_s - 1 \qquad (4\text{-}65)$$

4.6.6 Load model for pedestrian streams

SYNPEX defines a uniformly distributed load model to simulate pedestrian streams. The model is applicable in vertical, lateral and longitudinal direction on loading. The governing equation is given in Equation (4-66). The load model should be applied in resonance for one of the structures natural frequencies until steady state conditions are achieved.

$$p(t) = G\cos(2\pi f t)\ n'\psi \qquad (4\text{-}66)$$

Where:

G	static load due to a single pedestrian
f	considered natural frequency
t	time, [s]
n'	equivalent number of pedestrians for synchronization
ψ	reduction coefficient considering the probability of the walking frequency to approach the bridge´s natural frequency

The static load of one pedestrian, G, varies for different loading directions given in *Table 4.37*.

Table 4.37 Static load G for different loading directions.

Loading direction	Vertical	Lateral	Longitudinal
Static load, G [N]	280	35	140

The reduction factor, ψ, is related to the considered natural frequency of the structure according to Figure 4.16 for vertical and for lateral vibrations.

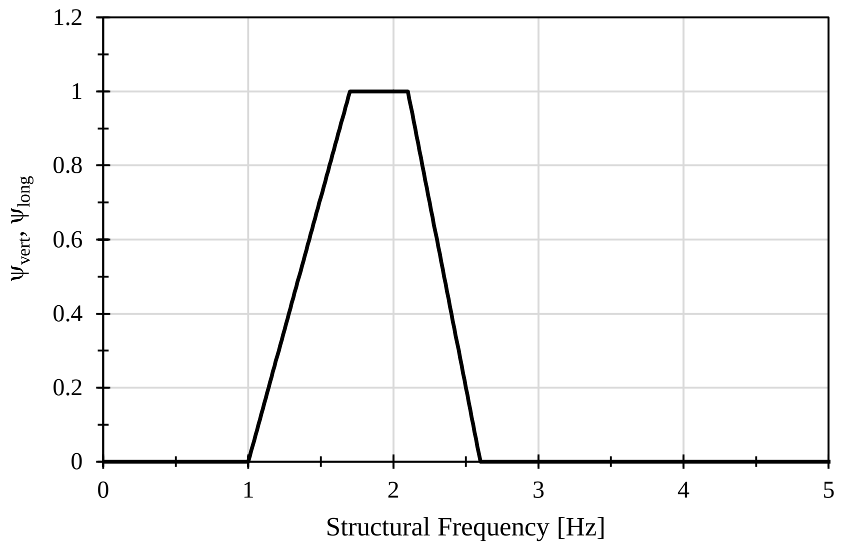


Figure 4.16 Reduction factor ψ as a function of structural frequency for vertical and longitudinal vibrations

The reduction factor ψ for lateral loading is given as a function of the lateral natural frequencies in Figure 4.17.

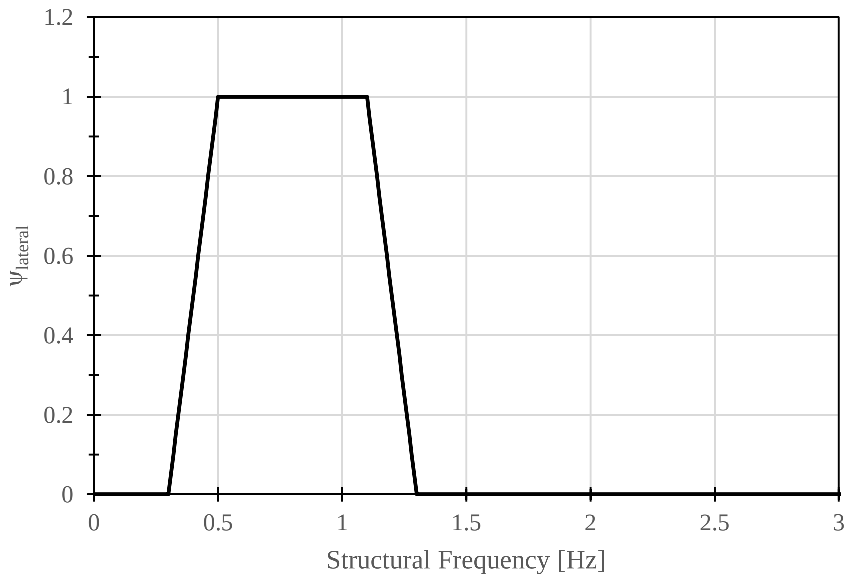


Figure 4.17 Reduction factor ψ as a function of structural frequency for lateral vibrations.

4.6.6.1 Equivalent number of pedestrians

The equivalent number of pedestrians n' included in the load model is given by the equations below. Two equations are given depending on the pedestrian density considered. Equation (4-67) applies for low densities and Equation (4-68) for higher densities as given below.

$$n' = \frac{10.8\sqrt{\xi n}}{S}$$ For d < 1.0 ped/m^2 (4-67)

$$n' = 1.85\sqrt{n}$$ For d ≥ 1.0 ped/m^2 (4-68)

Where

ξ	structural damping ratio, [%]
n	total number of pedestrian on the bridge deck area according to equation (4-69), [-]
S	bridge deck area, [m^2]

Total number of pedestrians on the bridge deck.

$$n = Sd \quad [-] \tag{4-69}$$

Where:

S	total bridge deck area, [m^2]
d	pedestrian density according to Table 4.31, [ped/m^2]

4.6.7 The Response Spectra method

SYNPEX suggest the Response Spectra method as a suitable way of determine the vibrations. The method is described in section 4.8.

4.7 JRC - Design of Lightweight Footbridges for Human Induced Vibrations

This report is based on the European research projects SYNPEX (Advanced Load Models for Synchronous Pedestrian Excitation and Optimized Design Guidelines for Steel footbridges) and HIVOSS (Human induced vibrations of steel structures).

4.7.1 Critical ranges of natural frequencies

Critical ranges for natural frequencies susceptible of excitation from pedestrian forces are stated in Table 4.38 (Christoph Heinemeyer, 2009).

Table 4.38 Critical ranges for natural frequencies in different directions.

Direction	Frequency range
Vertical	$1.25 \leq f_i \leq 2.3$ Hz
Longitudinal	$1.25 \leq f_i \leq 2.3$ Hz
Lateral	$0.5 \leq f_i \leq 1.2$ Hz

Footbridges with a natural frequency in vertical or longitudinal direction within the interval given in Equation (4-70) could also be excited by second harmonics of the pedestrian load.

$$2.5 \leq f_i \leq 4.6 \text{ Hz} \tag{4-70}$$

Though this is very unlikely the critical range for vertical and longitudinal vibrations in this case is expanded to the interval in Equation (4-71).

$$1.25 \leq f_i \leq 4.6 \text{ Hz} \tag{4-71}$$

4.7.2 Traffic classes

Five traffic classes estimate the density of pedestrians which is governing in the design (Christoph Heinemeyer, 2009).

Table 4.39 Recommended traffic classes to estimate the pedestrian density on the bridge

Traffic Class	Density, d [ped/m^2]	Description
TC 1	Group of 15 ped	Very weak traffic
TC 2	0.2	Weak traffic
TC 3	0.5	Dense traffic
TC 4	1.0	Very dense traffic
TC 5	1.5	Exceptional dense traffic

Characteristics of traffic classes:

TC 1 Density is calculated by dividing the number of pedestrian by the bridge deck area.

TC 2 Comfortable and free walking, overtaking is possible, single pedestrians can freely choose pace

TC 3 Unrestricted walking, overtaking can intermittently be inhibited

TC 4 Freedom of movement is restricted, uncomfortable walking, obstructed walking, overtaking is not possible.

TC 5 Unpleasant walking, crowding begins, one cannot freely choose pace.

4.7.3 Comfort classes and lateral lock-in

The pedestrian comfort criteria are categorized according to JRC into four comfort classes with different demands on the experienced degree of comfort. The degree of comfort is determined by acceleration limitations in vertical and horizontal direction. Maximum comfort is reached in CL 1 with low acceptance of acceleration.

The comfort classes do not consider lock-in for horizontal vibrations.

Table 4.40 Definition of comfort classes with corresponding acceleration limits

Comfort level	**Degree of comfort**	**Acceleration level vertical a_{limit} $[m/s^2]$**	**Acceleration level horizontal a_{limit} $[m/s^2]$**
CL 1	Maximum	< 0.50	< 0.10
CL 2	Medium	0.50 - 1.00	0.10 - 0.30
CL 3	Minimum	1.00 - 2.50	0.30 - 0.80
CL 4	Unacceptable discomfort	> 2.50	> 0.80

4.7.3.1 Lateral lock-in

The risk of lateral lock-in on the bridge can be assessed by calculating a trigger number of pedestrians. By calculateing the trigger number in Equation (4-72) it is possible to estimate the number of pedestrians on the bridge it requires for lock-in to occur.

$$N_L = \frac{8\pi\ \xi\ m^* f}{k} \quad [-] \tag{4-72}$$

Where:

ξ	structural damping ratio, [%]
m^*	modal mass for the considered mode, [kg]
f	natural frequency for the considered mode, [Hz]
k	constant given by Equation (4-73), [Ns/m]

The constant k is derived from experiments on the London Millenium Bridge valid in the interval 0.5 - 1.0 Hz.

$$k = 300\ [Ns/m] \tag{4-73}$$

Alternatively of calculating a trigger number of pedestrians the lock-in phenomena is related to the lateral acceleration. If the lateral acceleration is in the range in Equation (4-74) there will be a risk of lock-in to occur.

$$a_{lock-in} = 0.1 \text{ to } 0.15 \text{ m/s}^2 \tag{4-74}$$

4.7.4 Damping

Average damping ratios presented in Table 4.41 are recommended for design in service limits state (Christoph Heinemeyer, 2009).

Table 4.41 Damping ratios to be used in SLS-design

Construction type	Minimum ξ [%]	Average ξ [%]
Reinforced concrete	0.8	1.3
Prestressed concrete	0.5	1.0
Composite steel-concrete	0.3	0.6
Steel	0.2	0.4
Timber	1.0	1.5
Stress-ribbon	0.7	1.0

4.7.5 Load model for pedestrian streams

JRC recommends using a uniformly distributed harmonic load adapted to the considered mode shape. The applied load should have the same frequency as the natural frequency of the considered footbridge

Uniformly distributed harmonic load model:

$$p(t) = P\cos(2\pi f_s t)\ n'\psi\ [N/m^2] \tag{4-75}$$

Where:

P	static load from a single pedestrian, [N]
f_s	step frequency, [Hz]
n'	equivalent number of pedestrian, [-]
S	bridge deck area, [m^2]
ψ	reduction factor considering the probability that the step frequency and the natural frequency will coincide, [-]

4.7.5.1 Static load amplitude of a single pedestrian

The force amplitude depends on considered direction of analysis according to Table 4.42

Table 4.42 Force amplitude depending on considered direction of analysis.

Traffic class	P [N]		
	Vertical	Longitudinal	Lateral
TC 1 – TC 5	280	140	35

4.7.5.2 Equivalent number of pedestrians

Equivalent number of pedestrian, n', depends on traffic class due to different degree of synchronization among pedestrians as seen in Table 4.43.

Table 4.43 Equivalent number of pedestrians due to traffic class.

Traffic class	Euqivalent number of pedestrians, n'
TC 1 – TC 3	$n' = \frac{10.8\sqrt{\xi\, n}}{S}$
TC 4- TC 5	$n' = \frac{1.85\sqrt{n}}{S}$

Where:

ξ structural damping, [%]

S bridge deck area, [m^2]

n number of pedestrians, [-]

Number of pedestrians on the bridge deck is calculated according to traffic class with:

$$n = S\,\rho \tag{4-76}$$

Where:

ρ pedestrian density, [ped/m^2]

4.7.5.3 Modification factor ψ

The applied load is modified with the reduction factor ψ to take into account the probability of resonance.

Reduction factor, ψ, considers the probability that the step frequency and the natural frequency of the bridge will coincide. The factor is equal to one if the natural frequency of the bridge is close to normal walking frequencies and zero for unlikely walking frequencies. As shown in Figure 4.18, for vertical and longitudinal direction, and Figure 4.19 for lateral direction. When second harmonic is considered the dotted line should be used

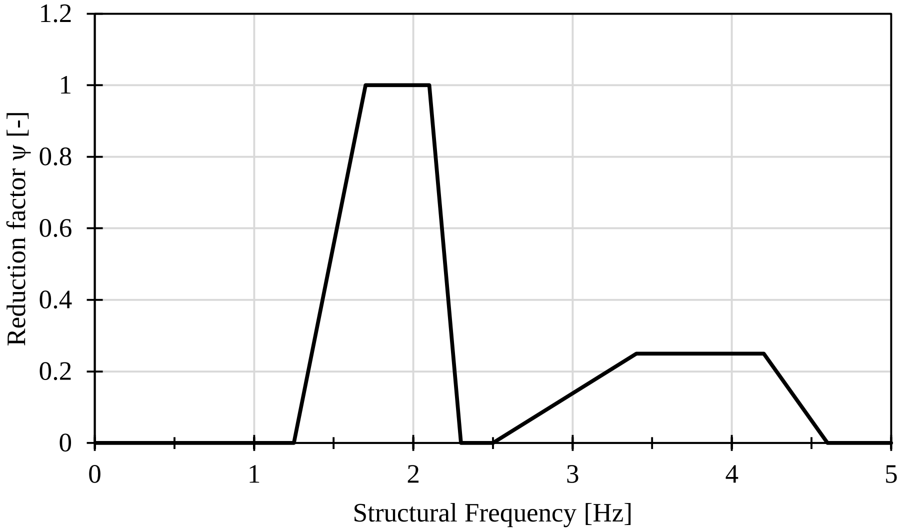


Figure 4.18 Reduction factor, ψ, in vertical and longitudinal direction.

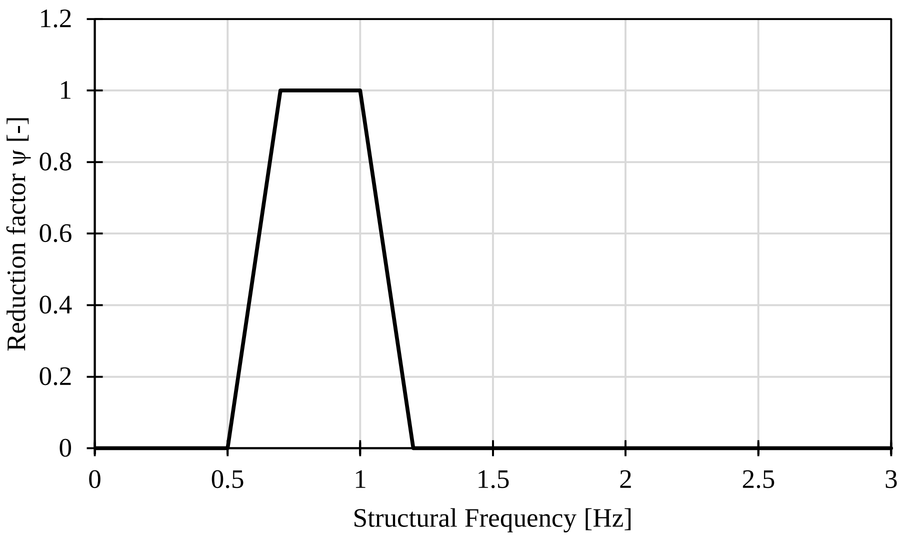


Figure 4.19 Reduction factor, ψ, in lateral direction.

4.7.6 Load model for jogging

As an appendix to the guideline there is a proposed load model for jogging where the velocity of the jogging person affects the dynamic force. The load should be applied with the same frequency as the natural frequency of the considered bridge.

The load should be applied as a point load moving along the span at a specified velocity.

$$p(t, v) = P\cos(2\pi f_s t)\ n'\psi \tag{4-77}$$

Where:

P	static load from a single jogger
P	force amplitude from a single jogger with step frequency f_s
f_s	step frequency
n'	equivalent number of joggers
S	bridge deck area
ψ	reduction factor considering the probability that the step frequency and the natural frequency will coincide
v	running velocity

The static load of a single jogger is defined for vertical direction as.

$$P = 1250\ N$$

The equivalent number of joggers is equal to the total number of joggers on the bridge.

$$n' = n$$

Where:

n total number of joggers

The velocity of the running person is equal to the velocity that the concentrated force should be applied with when moving along the span.

$$v = 3\ m/s$$

The reduction factor, ψ, in Figure 4.20 considers the probability that the step frequency and the natural frequency of the bridge will coincide

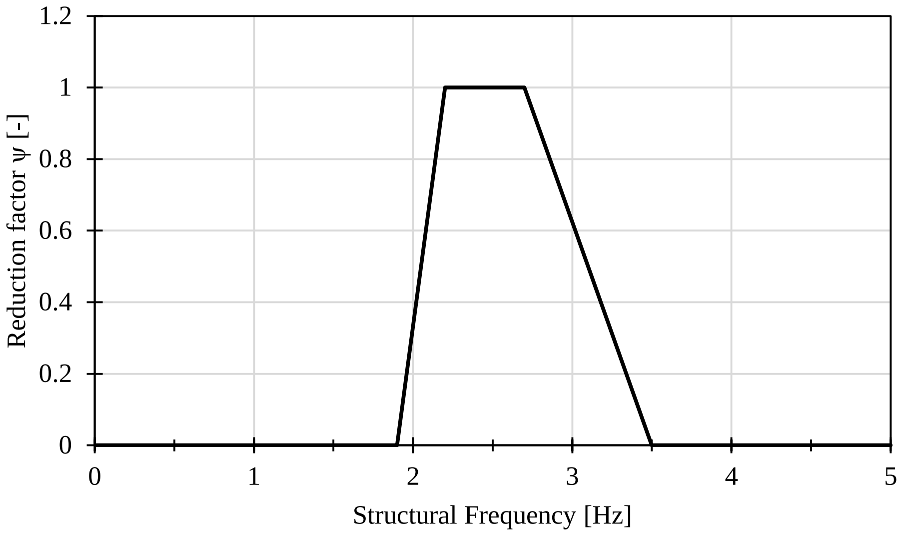


Figure 4.20. Reduction factor, ψ, for jogging.

4.7.7 SDOF solution

HIVOSS propose an alternative and numerical method of assessing the vibrations in a footbridge due to pedestrian loading which is presented in JRC. The method is based on derivation from a single degree of freedom system, SDOF-system.

A structure can be transformed into several different spring mass oscillators, where each spring mass represents a natural frequency of the structure. A combination of these can describe arbitrary oscillations of the structure.

For each natural frequency of the bridge an equivalent SDOF-system is used to calculate the maximum acceleration due to dynamic loading.

Maximum acceleration is calculated by Equation (4-78).

$$a_{max} = \frac{p^*}{m^*}\frac{\pi}{\delta} = \frac{p^*}{m^*}\frac{1}{2\xi} \tag{4-78}$$

Where

p^*	generalized load, [kN]
m^*	generalized modal mass, [kg]
ξ	structural damping ratio, [-]
δ	logarithmic decrement of damping, [-]

The generalized mass and load are given in Equation (4-79) and (4-80). The generalized load is based on the applied load $p^* \sin(2\pi f t)$.

$$m^* = \int_{L_D} \mu\big(\phi(x)\big)^2 \, dx \quad [kg] \tag{4-79}$$

$$p^* \sin(2\pi f t) = \int_{L_D} p(x)\phi(x)\, dx \sin(2\pi f t) \quad [kN] \tag{4-80}$$

In Table 4.44 the generalized load and mass are given of a simply supported beam for the first three mode shapes.

Table 4.44. Generalized load and mass with tuning time

Mode number, n	Generalized mass, m^*	Generalized distributed load, p^*	Generalized moving load, p^*	Tuning time, t_{max}
1	$\frac{1}{2}\mu L$	$\frac{2}{\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{v}$
2	$\frac{1}{2}\mu L$	$\frac{1}{\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{2v}$
3	$\frac{1}{2}\mu L$	$\frac{2}{3\pi}p(x)L$	$\frac{2}{\pi}P_{mov}$	$\frac{L}{3v}$

Where

$p(x)$	distributed load, $[kN/m^2]$
P_{mov}	moving load, $[kN]$
L	length of the beam, $[m]$
n	number of mode, $[-]$
μ	mass distribution per length, $[kg/m]$
v	velocity of moving load, $[m/s]$

4.7.8 Simplified numerical model for required modal mass estimations

Alternatively a simplified approach is derived where the required modal mass for the structure can be calculated based on the acceleration limits, Equation (4-81). The calculation is done separately for vertical and lateral directions. To avoid vibration phenomena due to pedestrian walking the condition of modal mass and acceleration limit should be fulfilled.

$$m_i^* \geq \frac{\sqrt{n}\,(k_1\xi^{k_2} + 1.65k_3\xi^{k_4})}{a_{limit}} \tag{4-81}$$

Where:

m_i^*	modal mass of the considered mode, [kg]
n	total number of pedestrians, [-]
ξ	structural damping ratio, [%]
k_1 to k_4	constants according to given in Table 4.45 and Table 4.46.

The constants k_1 to k_4 are different in vertical and torsional and lateral directions given in Table 4.45 and Table 4.46.

Table 4.45 Constants k_1 to k_4 for vertical and torsional modal mass

$d\ [P/m^2]$	k_1	k_2	k_3	k_4
≤ 0.5	0.7603		0.050	
1.0	0.5700	0.468	0.040	0.675
1.5	0.4000		0.035	

Table 4.46 Constants k_1 to k_4 for lateral modal mass

$d\ [P/m^2]$	k_1	k_2	k_3	k_4
≤ 0.5				
1.0	0.1205	0.45	0.012	0.6405
1.5				

4.8 Response Spectra

The guidelines JRC, HIVOSS and SYNPEX refer to the Response Spectra method as a possible method for dynamic analysis of lightweight footbridges. The method presented in the guidelines is the same for all three guidelines with the compiled information given in this chapter.

The method is adopted from wind engineering where the engineers use it to predict the response of wind gusts on a swaying structures. In similar to wind gusts the pedestrian loading on footbridges is a stochastic type of loading. In footbridge design the maximum peak acceleration was chosen as the design parameter. Due to stochastic loading the peak acceleration is related to the standard deviation derived from Monte Carlo simulations. The constants given in the Response Spectra method are based on stochastic loading of bridges with varying length and width each loaded 5000 times by randomly selected pedestrian streams. The random selection of loading is statistically based with random selection of pedestrians' weight, step frequency, lateral footfall, start position and moment of first step (Christoph Heinemeyer, 2009).

4.8.1 Load model for pedestrian streams according to JRC, HIVOSS and SYNPEX

The response spectra method aims to in a simplified way describe the stochastic loading (Christoph Heinemeyer, 2009). The method covers vertical and lateral vibrations and is based on five assumptions:

- The bridge structure is in resonance with the loading, i.e. the mean step frequency of the pedestrian stream is equal to one of the structure's natural frequencies.
- The mass is uniformly distributed over the bridge span
- The mode shapes are sinusoidal
- No modal coupling exists
- Linear-elastic structural behavior

(Research Fund for Coal and Steel, 2006)

Maximum acceleration, $a_{max,d}$, in Equation (4-82) of the structure including stochastic loading. The peak factor $k_{a,d}$ transforms the deviation of response to a maximum characteristic design value of the acceleration response. To be coherent with Eurocode a 95-procentile value is used in SLS design which is equal to $k_{a.95\%}$ tabulated in the guideline (Christoph Heinemeyer, 2009).

$$a_{max,d} = k_{a,d}\sigma_a \tag{4-82}$$

Where:

$k_{a,d}$	peak factor equal to $k_{a.95\%}$ in SLS design according to Table 4.47 and Table 4.48.
σ_a	standard deviation of acceleration response, Equation (4-83)

The standard deviation considers the stochastic loading of pedestrians calculated in Equation (4-83).

$$\sigma_a^2 = k_1 \xi^{k_2} \frac{C\ \sigma_F^2}{m{*}_i^2} \tag{4-83}$$

Where:

k_1	factor including considered frequency, Equation (4-85).
k_2	factor including considered frequency, Equation (4-86).
f_i	considered frequency, [Hz]
ξ	structural damping ratio according to the considered guideline, [-]
C	constant describing maximum load in spectrum, Table 4.47 and Table 4.48
σ_F^2	variance of loading according to Equation (4-84)
m^*	modal mass of the considered mode, [kg]

The variance of loading in square dependent on a constant and total number of pedestrians is given by Equation (4-84).

$$\sigma_F^2 = k_F n \tag{4-84}$$

Where

k_F constant in Table 4.47 and Table 4.48, [kN^2]

n total number of pedestrians of the bridge, [-]

The two factors k_1 and k_2 includes the considered frequency and tabulated constants that are dependent on the considered directions. Values for a and b are given in Table 4.47 and Table 4.48

$$k_1 = a_1 f_i^2 + a_2 f_i + a_3 \tag{4-85}$$

$$k_2 = b_1 f_i^2 + b_2 f_i + b_3 \tag{4-86}$$

Table 4.47 Constants for vertical accelerations

$d\ [P/m\text{^}2\ ]$	k_F	C	a_1	a_2	a_3	b_1	b_2	b_3	$k_{a.95\%}$
≤ 0.5	$1.20 \cdot 10^{-2}$	2.95	-0.07	0.60	0.075	0.003	-0.040	-1.000	3.92
1.0	$7.00 \cdot 10^{-3}$	3.70	-0.07	0.56	0.084	0.004	-0.045	-1.000	3.80
1.5	$3.34 \cdot 10^{-3}$	5.10	-0.08	0.50	0.085	0.005	-0.060	-1.005	3.74

Table 4.48: Constants for lateral accelerations

d [P/m²]	k_F	C	a_1	a_2	a_3	b_1	b_2	b_3	$k_{a95\%}$
≤ 0.5	$2.85 \cdot 10^{-4}$	6.8	-0.08	0.50	0.085	0.005	-0.06	-1.005	3.77
1.0		7.9	-0.08	0.44	0.096	0.007	-0.071	-1.000	3.73
1.5		12.6	-0.07	0.31	0.120	0.009	-0.094	-1.020	3.63

5 Analysis of acceleration response for a simply supported beam

In this chapter an analysis is presented studying how mass, stiffness and dynamic properties of a simply supported beam affects the acceleration response during dynamic loading.

Firstly an analysis of the acceleration response for a beam with varying density and stiffness is presented. The beam with varying properties is analyzed for concentrated and uniformly distributed harmonic loads.

Furthermore the dependence of the acceleration response due to load amplitude, bridge length and structural damping is analyzed.

The aim of the analysis is to provide a foundation for an evaluation and comparison of the studied load models.

The results from each analysis is discussed separately in each section with a final discussion and summarization of the results in the end of this chapter.

5.1 Study of acceleration response due to concentrated load

The guidelines presented in chapter 4 recommend different concentrated loads to model pedestrian forces. In this section the acceleration response due to a stationary concentrated load is studied. Some guidelines propose a moving point load to simulate pedestrians induced forces. A simplification has been made to only study stationary loads in order to compare the different guidelines in a sufficient way.

5.1.1 Method

The concentrated force will be applied for a cross-section with varying mass and stiffness. The load will be applied until steady-state conditions are reached. The maximum acceleration at steady-state at the mid node is of interest. The beam with varying properties will be analyzed in the FE-software ADINA.

5.1.2 Input data

Applied load in the analysis and considered bridge properties are explained in this chapter.

5.1.2.1 Applied load – concentrated force

In the analysis a concentrated sinusoidal load, $p(t)$ in Equation (5-1) is applied in the middle of the span. The load will be applied in vertical direction with constant load amplitude equal to 700 N throughout the analysis.

$$p(t) = 700\sin(2\pi f_l t) \quad [N] \tag{5-1}$$

The load frequency f_l will be varied in the frequency range defined in Equation (5-2). The load will be varied for all frequencies in the span with steps of 0.05 Hz resulting in 101 different walking frequencies within the interval.

$$1 < f_l < 6\ Hz \tag{5-2}$$

5.1.2.2 Bridge properties

The beam used in the analysis has been varied for five different values of stiffness and five different densities presented in Table 5.1.

Table 5.1 Varying mass and stiffness used in the analysis

Nr	E-modulus, E [GPa]	Density, ρ [kg/m^3]
1	140	4000
2	170	5667
3	200	7333
4	230	9000
5	260	10667

The properties are combined into 25 unique beam configurations all with different natural frequencies. The properties are chosen so that the natural frequencies will vary between 1 and 4 Hz. This range of natural frequencies has been chosen so that resonance will occur when the beams are excited to the sinusoidal load with load frequencies in the interval 1 to 6 Hz.

The different combinations of mass and stiffness are named according to the system EIiMj where i is the number of the E-modulus according to Table 5.1 and j respectively is the mass (density) according to Table 5.1. For example beam EI2M3 has the E-modulus 170 GPa and the density 7333 kg/m^3.

The beam is analyzed for a 15 m long span with a box cross section. The length and cross-section remains constant through all analysis. The cross-sectional dimensions are given in Table 5.2.

Table 5.2 Geometric constants of the cross-section

Width, b	0.4 [m]
Height, h	0.16 [m]
Thickness, t	0.04 [m]

The beam cross-section is illustrated in Figure 5.1.

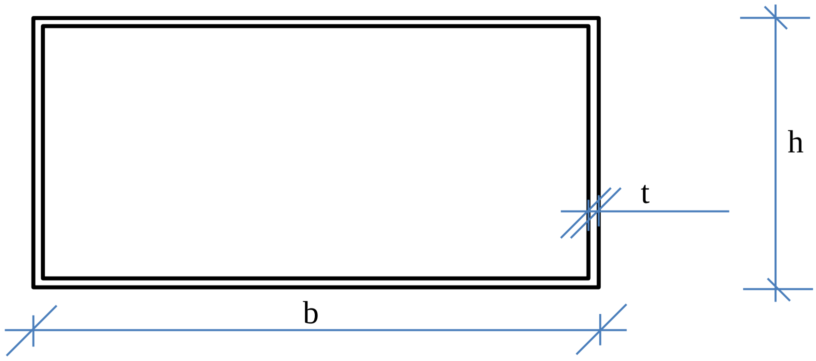


Figure 5.1 Beam cross-section

5.1.3 Results

Results are achieved as maximum acceleration responses at steady state by analysis in the FE-software ADINA. The results are imported into Excel where curves are plotted showing the response varying over several load frequencies for different beams with varying stiffness and mass.

5.1.3.1 Varying of mass over constant stiffness

The maximum acceleration response in the mid node of the beam when the mass is varied over constant stiffness is shown in Figure 5.2 to Figure 5.6. The figures show the relation between applied load frequency and acceleration response. It can be seen that acceleration response is dependent on the mass of the bridge. The beam with the lowest mass exhibits the highest accelerations and the beam with the highest mass has the lowest acceleration.

The curves are displaced on the x-axis depending on the relation between the load frequency and the natural frequency of the beam. The beams with the highest mass results in the lowest natural frequency and have its maximum in resonance for low frequencies. The beams with the highest responses are given for those beams with low mass consequenctly they will have high natural frequency and their maximum responses in resonance for high frequency loads.

Note that in the plots for different stiffness the acceleration response is approximately the same for the different masses independent of the stiffness. Giving that the acceleration response is dependent on the mass of the bridge.

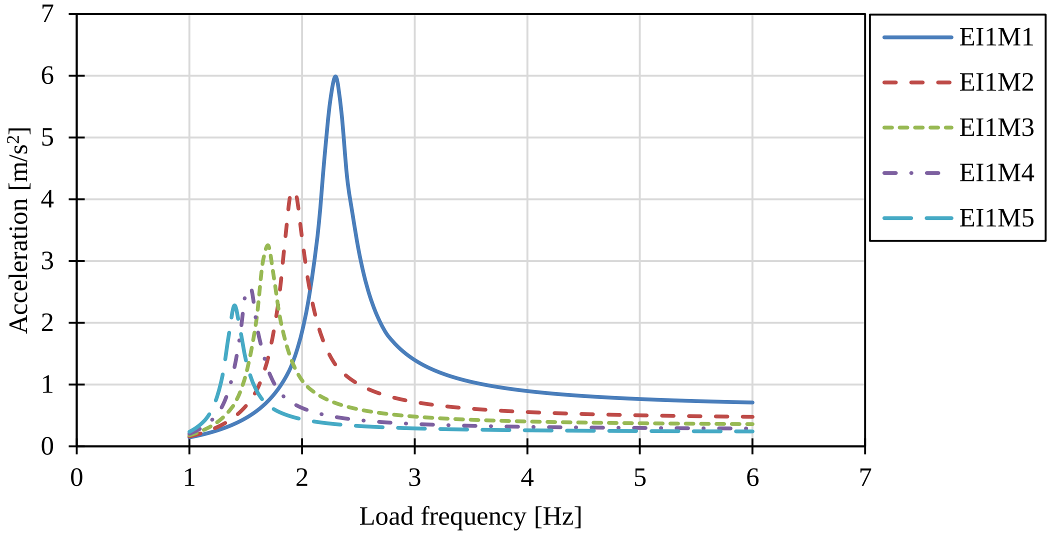


Figure 5.2 Acceleration response for EI1 with varying mass.

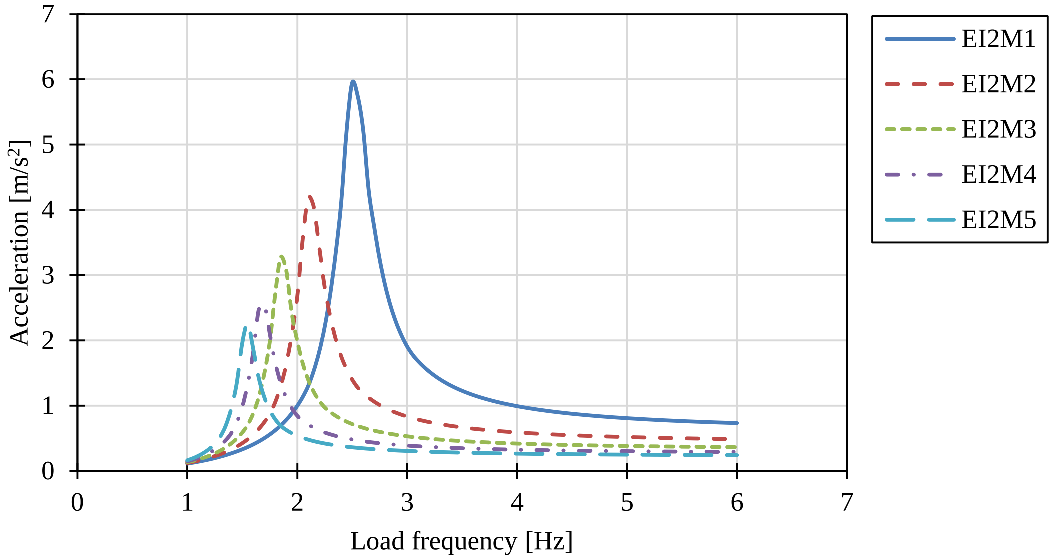


Figure 5.3 Acceleration response for EI2 with varying mass.

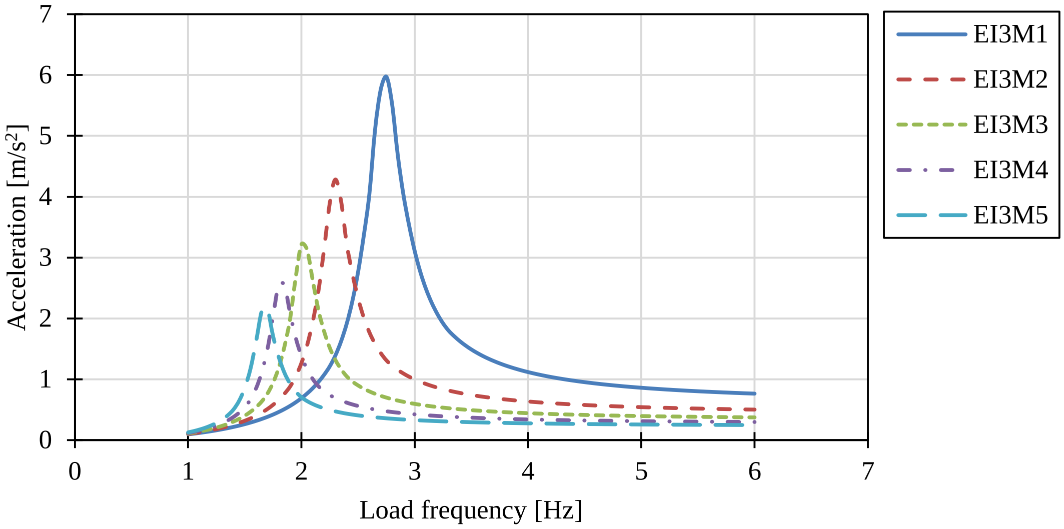


Figure 5.4 Acceleration response for EI3 with varying mass

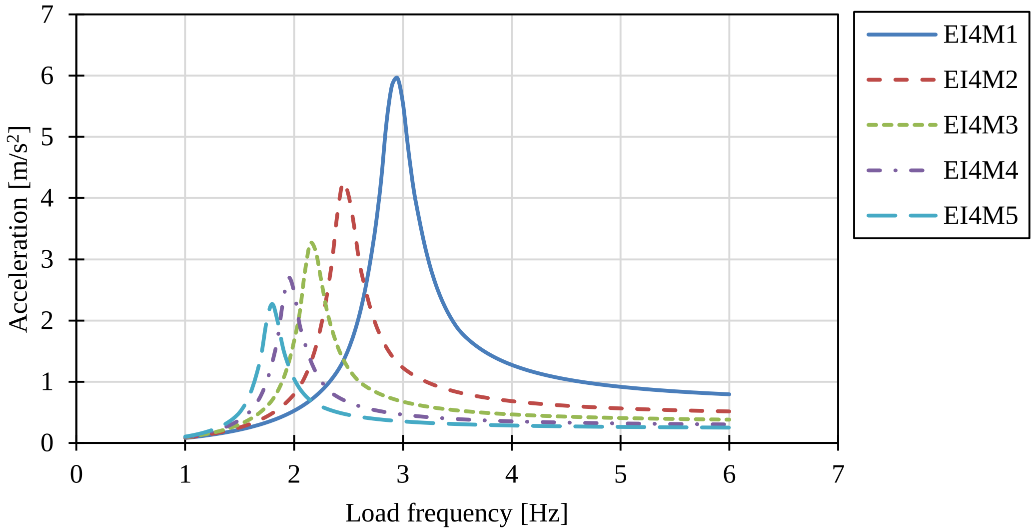


Figure 5.5 Acceleration response for EI4 with varying mass

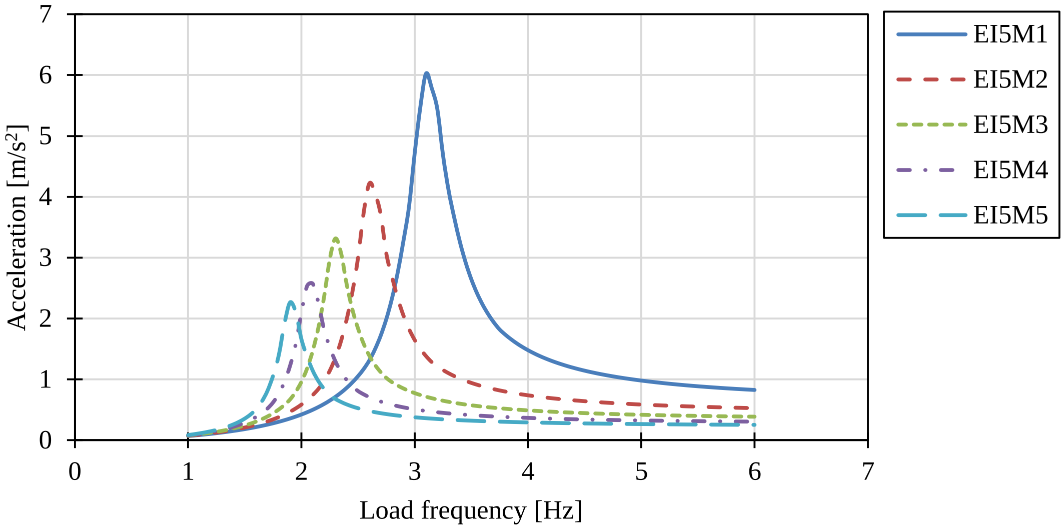


Figure 5.6 Acceleration response for EI5 with varying mass

5.1.3.2 Varying of stiffness over constant mass

The acceleration response when the stiffness is varied over constant mass is shown in Figure 5.7 to Figure 5.11. Each figure is plotted for five beam configurations with varying stiffnesses and constant mass. These plots show, as stated about varying mass and constant stiffness, that the acceleration response is dependent on the mass of the bridge. In the plots it can be seen that the acceleration response is the same for beams with the same mass and that the size of the maximum acceleration changes between the plots for different masses. The curves are displaced on the x-axis dependent on the relation between load frequency and the natural frequency of the beam. The maximum acceleration for a beam configuration occurs when the beam and load frequency are in resonance.

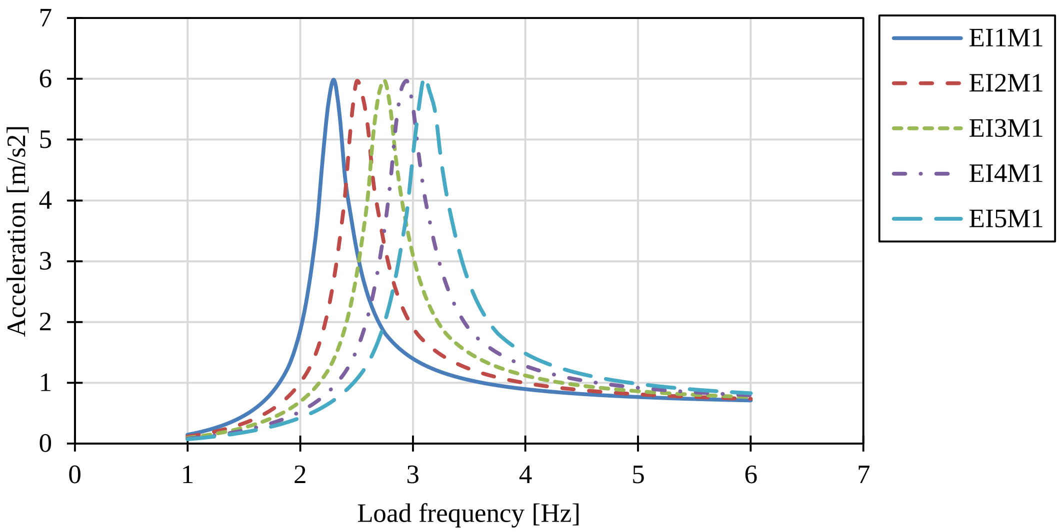


Figure 5.7 Acceleration response for M1 with varying stiffness.

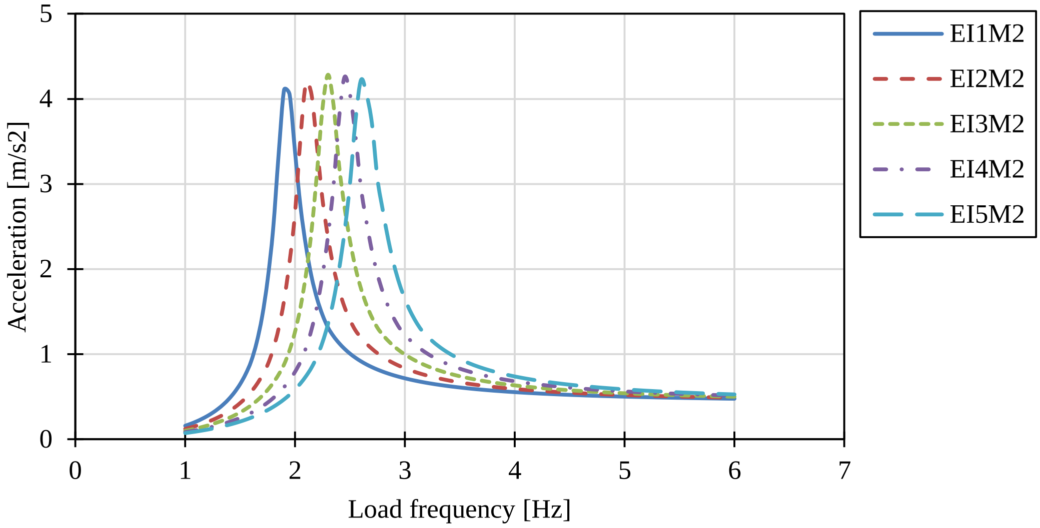


Figure 5.8 *Acceleration response for M2 with varying stiffness.*

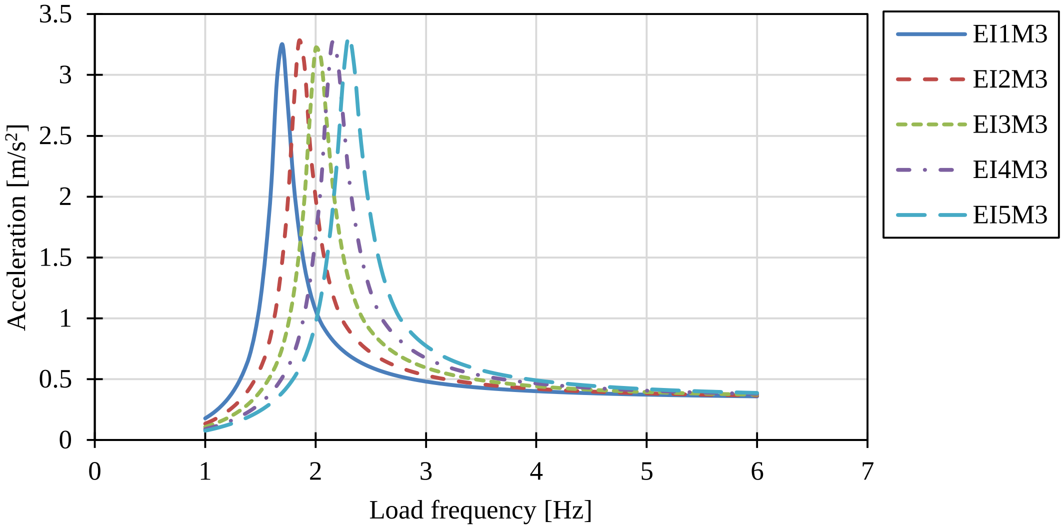


Figure 5.9 *Acceleration response for M3 with varying stiffness.*

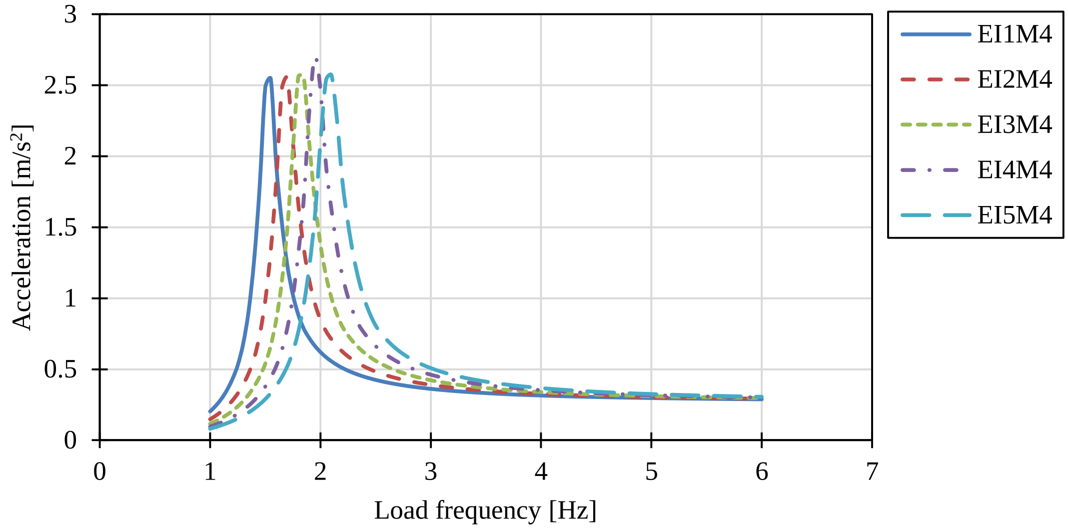


Figure 5.10 Acceleration response for M4 with varying stiffness.

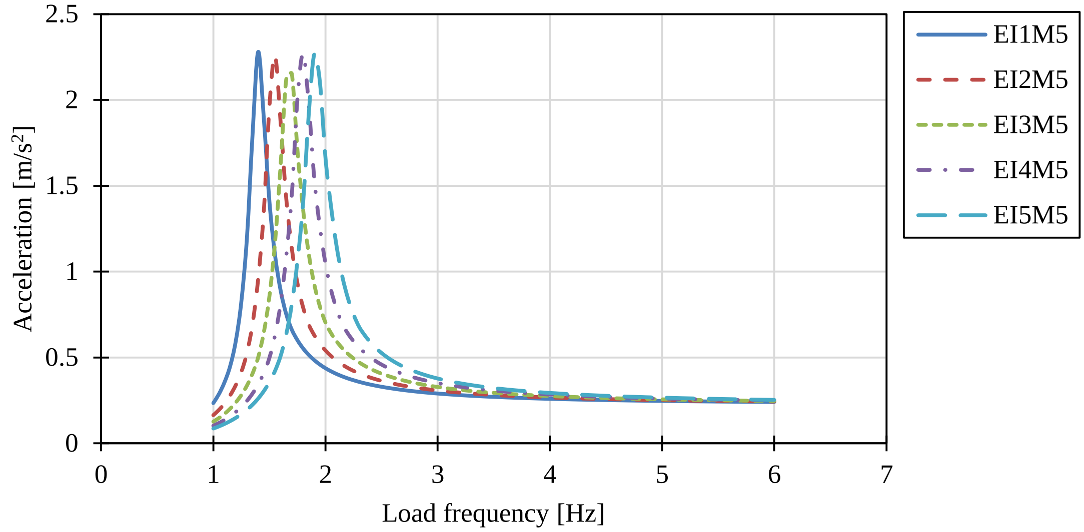


Figure 5.11 Acceleration response for M5 with varying stiffness.

5.1.3.3 Conclusion

It is clear that the size of the acceleration response depends on the mass of the beam. The graphs displacement along the load frequency axis depends on the frequency of the load in comparison with the natural frequency of the beam. This means that the results can be normalized by choosing appropriate factors for normalization.

5.1.4 Normalization

The results can be normalized into one curve making the results independent of the applied load frequency and the mass of the beam. In this way the responses due to different load models can be evaluated independently from which beam configuration that has been used in the analysis.

In Figure 5.12 a plot is shown of acceleration response for different masses with constant stiffness. Note that the response increases for beam sections with smaller mass. The beam section with the smallest mass EI5M1 exhibits the largest accelerations and the section with the highest mass EI5M5 has the smallest accelerations.

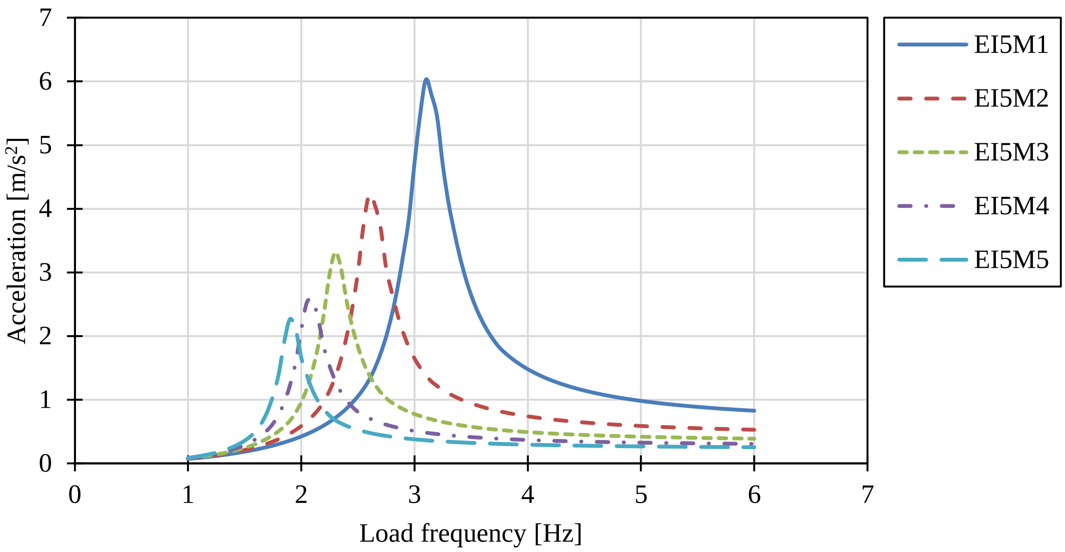


Figure 5.12 Plot of acceleration response for different masses with constant stiffness.

In Figure 5.13 a plot of acceleration response for different stiffnesses with constant mass is shown. Note that the size of the acceleration response is approximately the same, where the small difference depends on how well the natural frequency has coincided with the load frequency in the FE-analysis. It can be seen that the graphs are displaced on the load frequency axis. The beam with the highest stiffness, resulting in the highest natural frequency, is situated furthest to the right and the beam with the lowest stiffness is situated furthest to the left due to its low natural frequency.

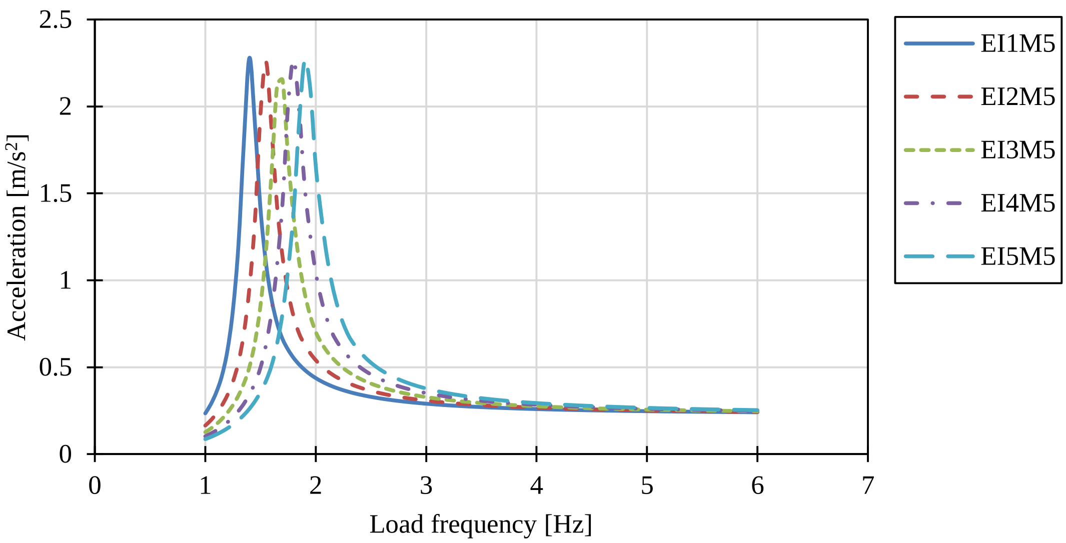


Figure 5.13 Plot of acceleration response for different stiffness's with constant mass.

5.1.4.1 X-axis

Considering the relation between acceleration response and the resonance phenomena the x-axis is chosen to be normalized with a factor β according to Equation (5-3). The factor β is the frequency ratio between load frequency, f_l, and natural frequency, f_n.

$$\beta = \frac{f_l}{f_n} \quad [-] \tag{5-3}$$

Where:

f_l load frequency, [Hz]

f_n natural frequency of the beam, [Hz]

This displaces all curves so that they have their peak value at the same point along the x-axis as illustrated in Figure 5.14.

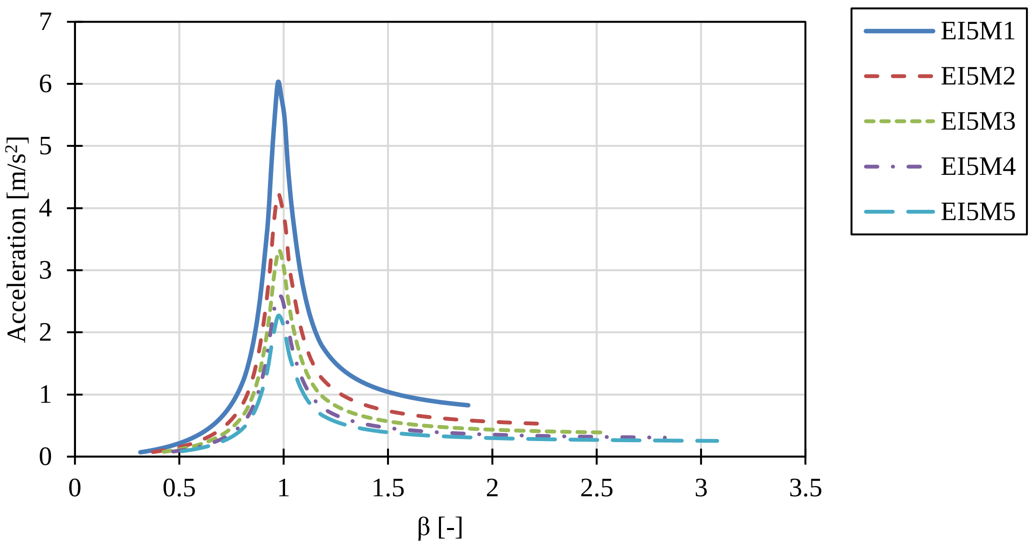


Figure 5.14 Plot of acceleration response with normalized x-axis.

5.1.4.2 Y-axis

The y-axis has to be normalized regarding mass to make the axis independent of varying cross-section properties. The normalization factor called τ is given in Equation (5-4) with the unit [kg/s^2].

$$\tau = a\,\rho\,A \quad [\mathrm{N/m}] \tag{5-4}$$

Where:

a acceleration response, [m/s^2]

ρ density, [kg/m^3]

A area of the cross-section, [m^2]

This displaces all curves so their peak value has the same magnitude according to Figure 5.15.

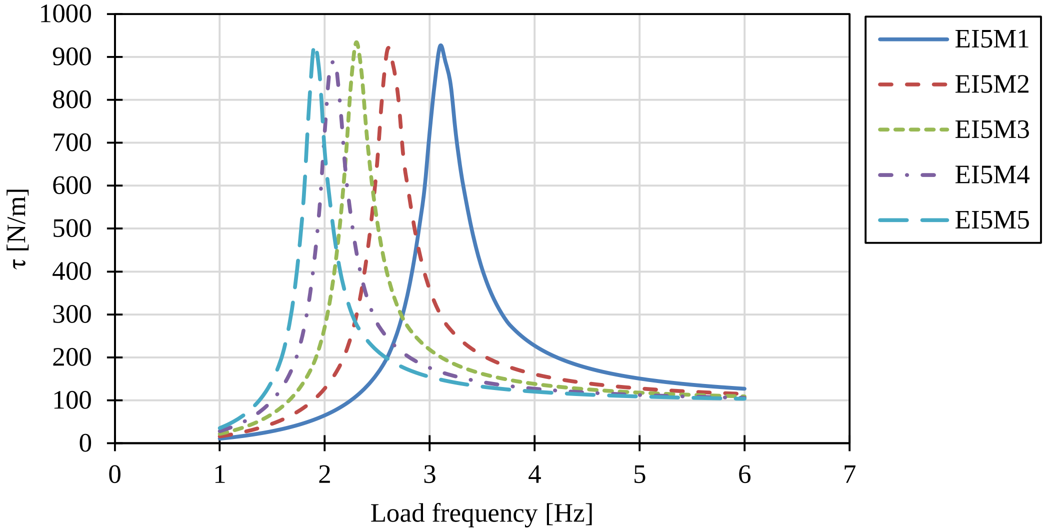


Figure 5.15 Plot of acceleration response with normalized y-axis.

5.1.5 Normalization and design curves

In previous sections it has been shown that the acceleration curves can be normalized separately for the x- and y-axis. By applying the normalizing of the curves for both axes design curves for acceleration response can be created. In Figure 5.16 the normalized curve over both axes is shown with β on the x-axis and τ on the y-axis.

Note that all graphs will peak at $\beta = 1$ where the load frequency and the natural frequency is equal meaning that resonance occurs. The maximum response occurs at the same magnitude for all beams independent of mass.

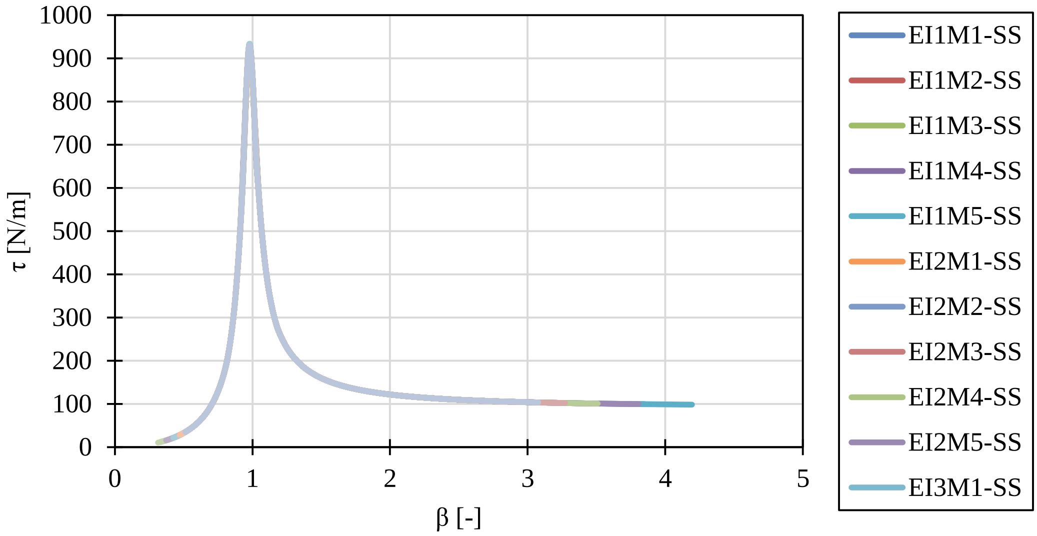


Figure 5.16 Normalized response curve over x and y-axis for steady state response

The normalized curves from the analysis are almost equal with very small deviations. The peak value where $\beta \approx 1$ differs due to the resonance response. Some beam configurations are more close to resonance than others and therefore the response will change. By a closer look it can be determined that beam configuration EI5M3 has the highest response. To be conservative this curve is used as the design curve for further studies as it gives the highest peak value. The variation in responses is very small and is explained by how close to perfect resonance the load frequency is. In the normalized curves it was shown that EI5M3 was the beam configuration closest to perfect resonance. The normalized response curve for beam configuration EI5M3 is shown in Figure 5.17.

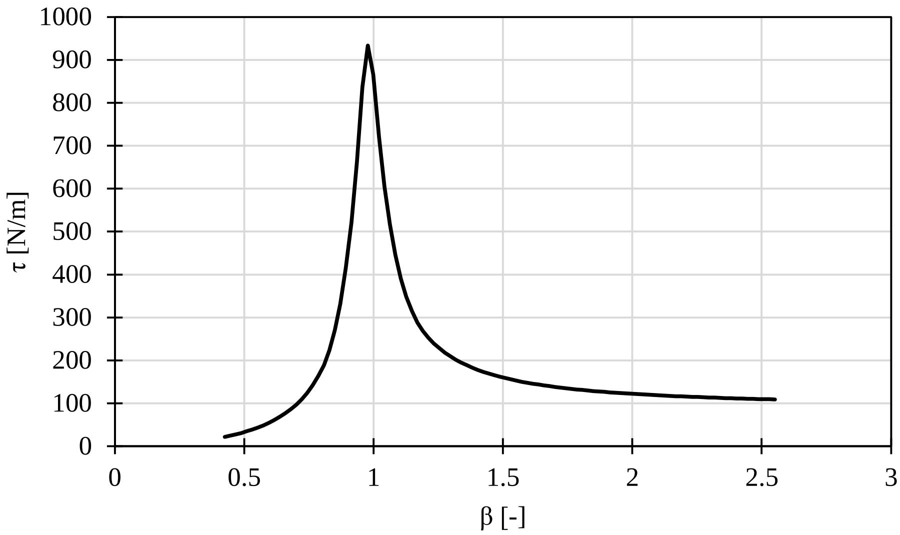


Figure 5.17 Plot of normalized response curve for beam configuration EI5M3

5.2 Study of acceleration response due to distributed load

In the guidelines presented in section 4 distributed loads to model pedestrian forces are proposed and need to be analyzed. In this section an analysis of the response of a uniformly distributed load will be evaluated and compared to the results given in previous section regarding concentrated loads.

5.2.1 Method

The acceleration response is dependent on how well the load frequency fits with the natural frequency of the bridge. This means that the acceleration response for distributed and point load will have the same appearance as they both have a sinusoidal load shape. Therefore the relation between point load and distributed load for $\beta = 1$ can be used to extract a normalized curve for distributed load from the corresponding curve for point load.

The analysis is run for one beam cross-section, EI5M3, giving the maximum acceleration response in steady-state.

5.2.2 Input data

This section presents the applied load and beam properties.

5.2.2.1 Applied force

A uniformly distributed load will be analyzed according to Equation (5-5). The load amplitude is adapted so that the total applied load is equal as for the concentrated load analyzed in section 5.1. The load will be applied in resonance with the beam with f_l equal to 2.30 Hz.

$$p(t) = \frac{700}{15}\sin(2\pi f_{load}t) \quad [N/m] \tag{5-5}$$

5.2.2.2 Beam cross-section

The beam cross-section is the same as used in section 5.1 with dimensions according to Table 5.3 shown in Figure 5.18.

Table 5.3 Geometric constants of the cross-section

Width, b	0.4 m
Height, h	0.16 m
Thickness, t	0.04 m

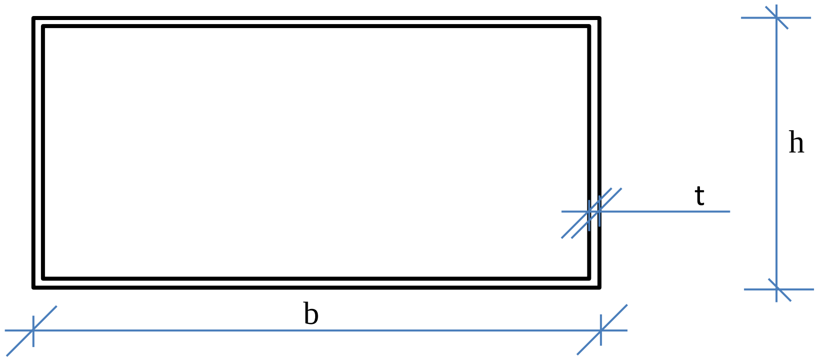


Figure 5.18. Beam cross-section for distributed load

Beam cross-section EI5M3 has the E-modulus 260 GPa and the density 7333 kg/m^3.

5.2.3 Results

The maximum acceleration response for the uniformly distributed load can be seen in Table 5.4. In the table is a comparison also presented with the maximum response for the concentrated load and the uniformly load.

Table 5.4 Results for and relation between acceleration response for point load and distributed load for EI5M3 in resonance.

Level of damping [%]	Max acceleration point load [m/s^2]	Max acceleration distributed load [m/s^2]	Relation acceleration [point load/distributed load]
5	3.22	2.11	1.526

5.2.4 Conclusions

Distributed load results in a lower acceleration response which is logical due to that the same load, total applied N, is distributed over the entire length instead of placed in the middle giving the worst case scenario.

With this relationship the normalized graphs for distributed load can be calculated for the corresponding level of damping from existing graphs for the response due to point loads, as shown in Figure 5.19. The relationship 1.526 has been used to extract the plot for distributed load from the already existing point load for 5 % damping.

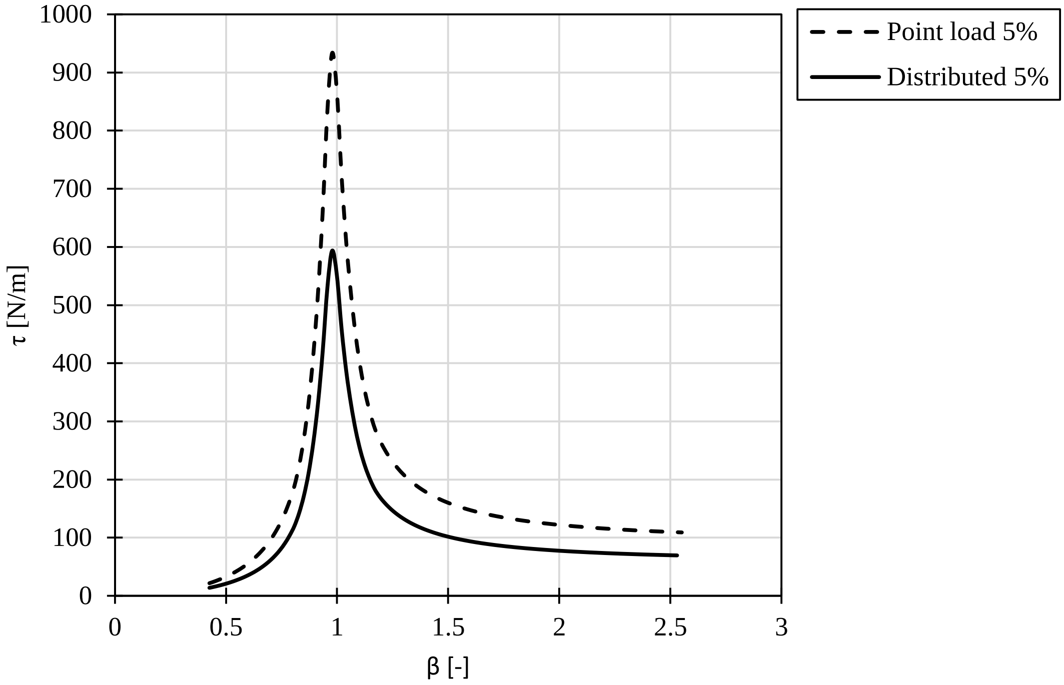


Figure 5.19 Normalized plots for point load and distributed load for 5% damping.

5.3 Force Amplitude study

In this section is an analysis of the relation between load amplitude and acceleration response for a sinusoidal load presented. The analysis is done by numerical calculations of a SDOF-system excited by different load amplitudes.

Pedestrian loads are harmonic and described in the load models presented in section 4 by sinusoidal forces with varying force amplitudes, P_0. The unique factor when comparing different load models is often the force amplitude. By investigating how the force amplitude affects the response, in this case the maximum acceleration, in a structure it is possible to establish an efficient and accurate way of evaluating the models.

5.3.1 Method

The analysis is done analytically to solve the acceleration response of a SDOF-system. The SDOF-system is given in Figure 5.20 which is excited by the harmonic load $p(t)$. By exciting the system with different load amplitudes a relation between load amplitude and acceleration reponse can be found. The analytical solution for various load amplitudes calculated and plotted in Matlab.

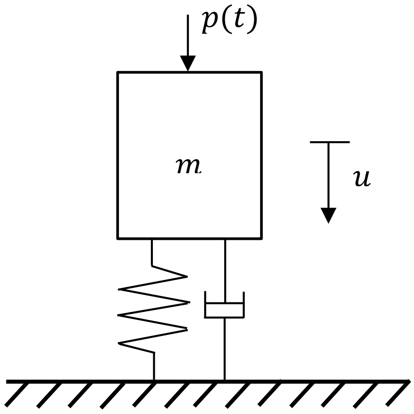


Figure 5.20 Damped SDOF-system excited by the force $p(t)$.

5.3.2 Input data

The applied dynamic load $p(t)$ is given in equation (5-6) with the force amplitude P_0.

$$p(t) = P_0 \cos(2\pi t) \tag{5-6}$$

The input data used throughout this study is given in Table 5.5

Table 5.5 Input data for force amplitude study

Damping ratio, ξ	5.0 %
Natural frequency, ω_n	20 Hz
Load frequency, Ω	10 Hz
Time interval	$0\ s \leq t\ \leq 10\ s$
Time step	0.005 s

The force amplitudes that are varied in the analysis are presented in Table 5.6.

Table 5.6 Varying force amplitudes used in the analysis

Force amplitude, P_0 $[N]$
1
10
40
80
120
160
200

5.3.3 Analytical solution of acceleration response

The acceleration is solved analytically for the SDOF-system by the steady state solution of a damped system. The displacement is given by Equation (5-7) introduced in section 3.1.4.2.

The analytical solution for displacement of a damped SDOF-system in steady state is given by Equation (5-7).

$$u(t) = U\cos(\Omega t - \alpha) + e^{-\xi\omega_n t}(A_1 cos\omega_d t + A_2 sin\omega_d t) \tag{5-7}$$

Where:

$$U = \frac{U_0}{[(1-r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}}$$

The acceleration given in Equation (5-8) is derived by differentiating Equation (5-7) twice over time, t.

$$u''(t) = -\Omega^2 U\cos(\Omega t - \alpha) - \tag{5-8}$$

$$-e^{-\xi\omega_n t}(A_2\omega_d - A_1\xi\omega_n)(\xi\omega_n\cos(\omega_d t) + \omega_d\sin(\omega_d t)) +$$

$$+e^{-\xi\omega_n t}(A_1\omega_d + A_2\xi\omega_n)(\xi\omega_n\sin(\omega_d t) - \omega_d\cos(\omega_d t))$$

5.3.4 Results

The acceleration response over time is plotted in Figure 5.21 for the varying force amplitudes.

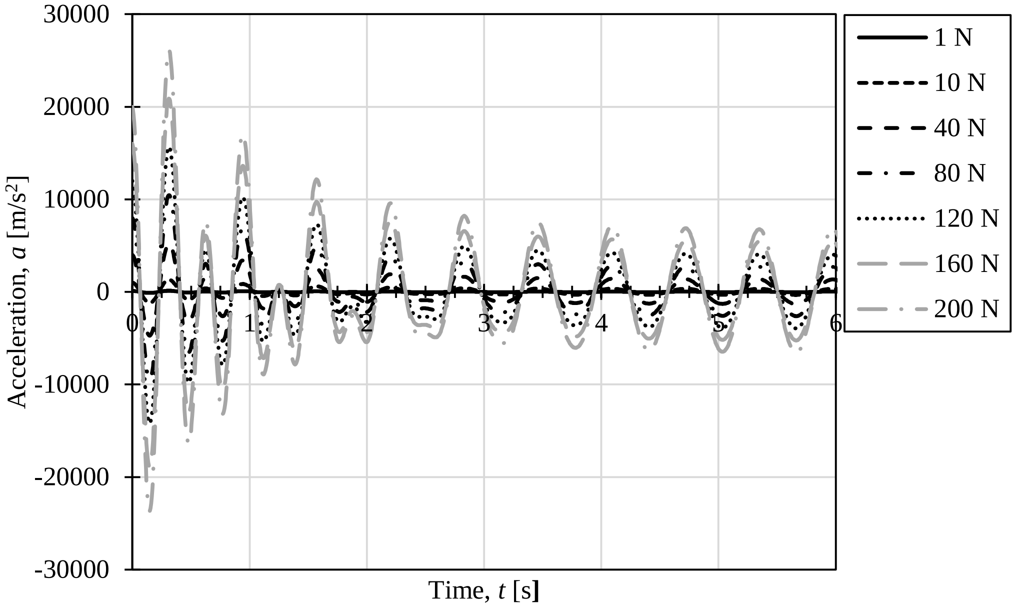


Figure 5.21 Acceleration over time for different force amplitudes

Larger accelerations occur in the beginning due to the initial impulse loading and are disregarded. The maximum accelerations in steady state, $a_{max.ss}$, according to Figure 5.21 are presented in Table 5.7.

Table 5.7 Maximum acceleration at steady state for various force amplitudes

Analyze number	Force amplitude, P_0 $[N]$	Maximum acceleration, $a_{max.ss}$ [m/s^2]
1	1	32.21
2	10	322.1
3	40	1288
4	80	2577
5	120	3865
6	160	5153
7	200	6442

5.3.5 Conclusions

The acceleration response in a damped SDOF-system for steady-state is linear proportional to the force amplitude as seen in Figure 5.22. The relationship makes it is possible to calculate the acceleration response of a sinusoidal load for any given force amplitude.

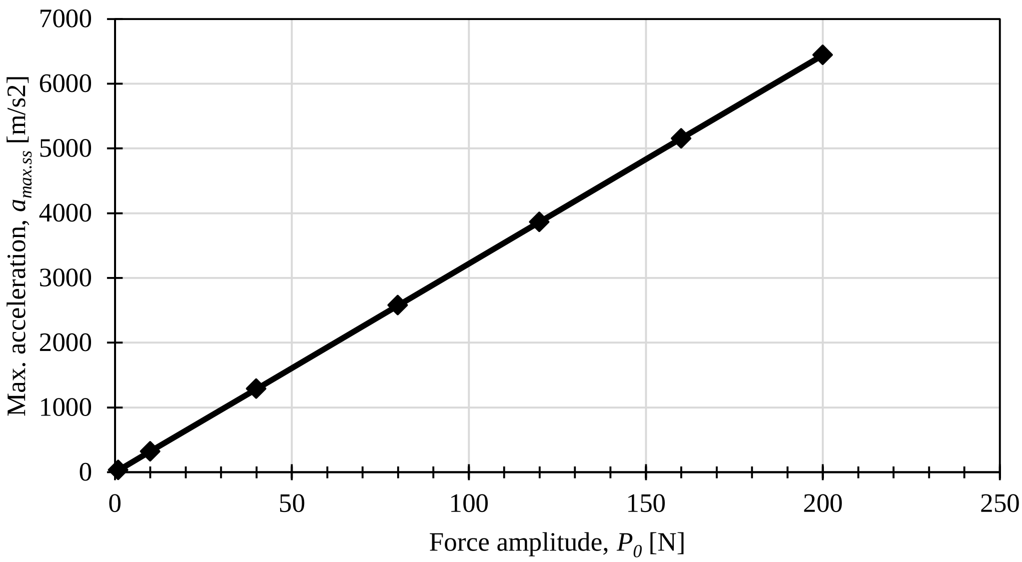


Figure 5.22 Plot of maximum acceleration in steady-state and force amplitude

5.3.6 Normalization

The acceleration response is linear proportional to the force amplitude which means that the previous design curve seen in Figure 5.17 for concentrated load can be normalized according to the load amplitude.

Where the previous τ is according to (5-9):

$$\tau = a\ \rho\ A \quad [kg/s^2] \tag{5-9}$$

Giving a new τ according to Equation (5-13) regarding the load amplitude

$$\tau = \frac{a\rho A}{P_0} \quad [1/m] \tag{5-10}$$

Where the load amplitude, P_0, in this case is equal to 700 N, giving the design curve independent of load amplitude as presented in Figure 5.23.

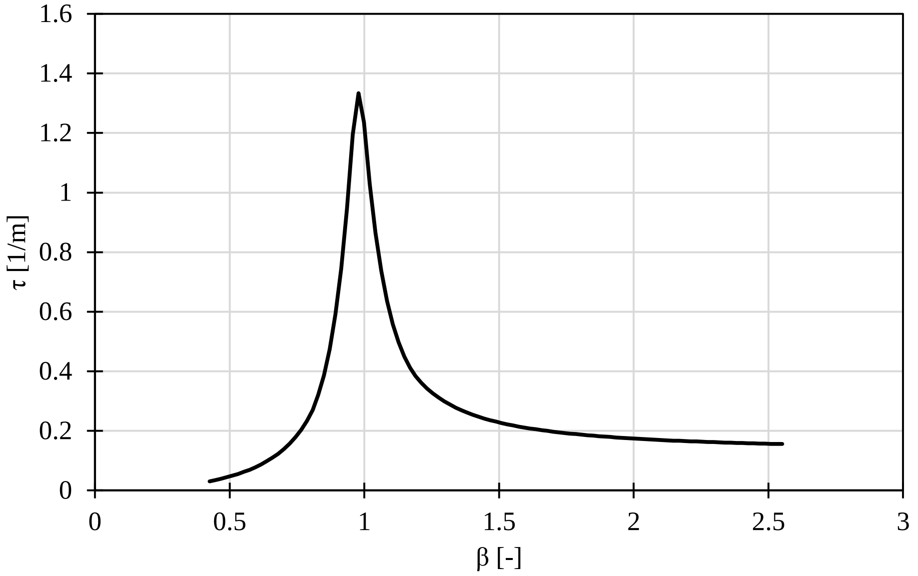


Figure 5.23 Normalized acceleration response for concentrated load independent of force amplitude

The normalized curve for uniformliy distributed load in Figure 5.19 is divided by the total applied load as for concentrated load.

The new τ for uniformly distributed load is given in Equation (5-12) where P_0 is equal to 700 N which is the total applied load.

$$\tau = \frac{a\rho A}{P_0} \quad [1/\text{m}] \tag{5-11}$$

The normalized response curve for uniformly distributed load is given in Figure 5.24.

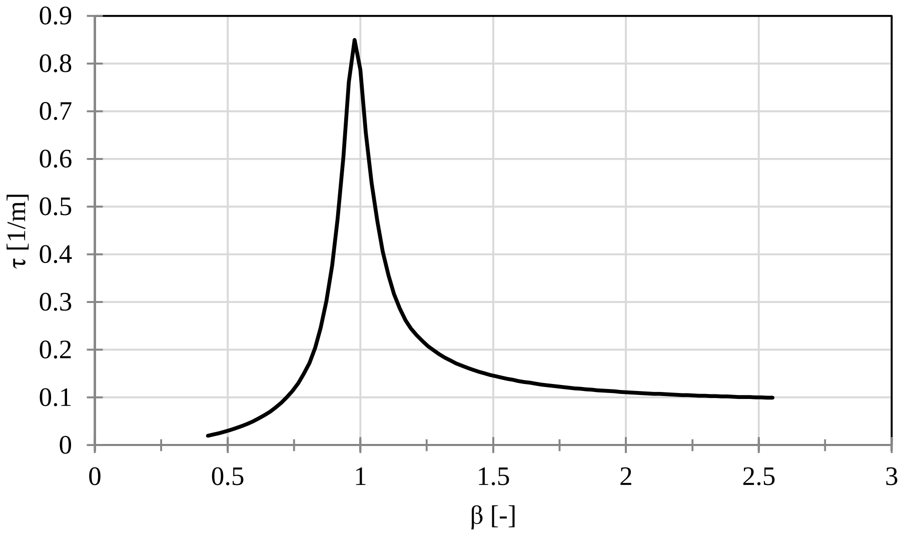


Figure 5.24 Normalized acceleration response for uniformly distributed load independent on load amplitude.

The relationship stated in section 5.2 between distributed load and point load is valid for different load amplitudes as well. This can be derived from that the force amplitude study is made for an SDOF-system. For analysis of distributed load a sum of SDOF-systems can be used which will give the same results as for one singular.

This gives normalized curves for 5 % damping ratio according to Figure 5.25

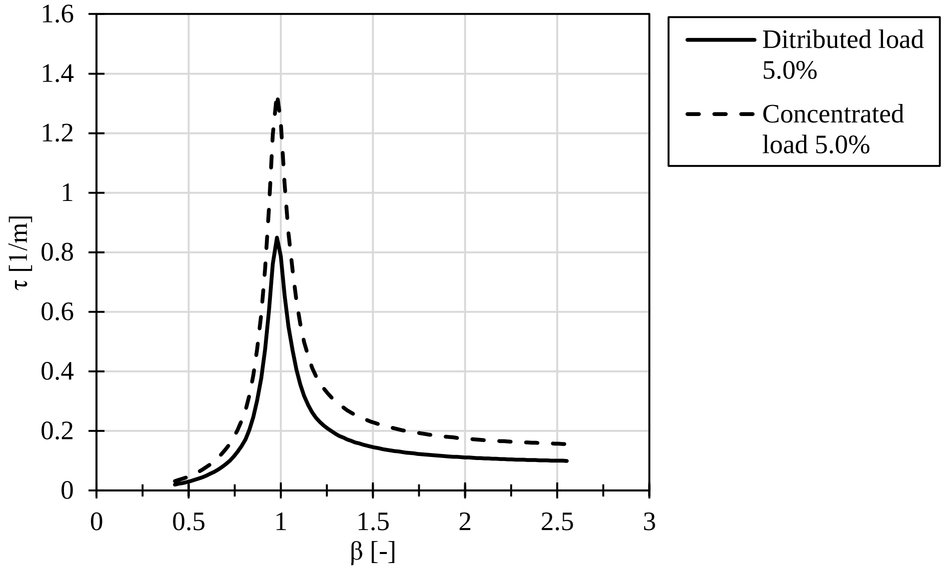


Figure 5.25 Normalized curves regarding load amplitude for distributed load and point load for 5 % damping.

It is clear though that the distributed load depends on the length of the bridge, where the total applied load changes with changing length. This is studied further in the next section.

5.4 Acceleration response due to varying length

In this chapter the acceleration response due to varying bridge length for a simply supported structure is analyzed for distributed load and point load.

5.4.1 Method

A static load representing one pedestrian is taken as the load amplitude, P_0, equal to 700 N. For the load case with distributed load this point load is redistributed over the length of the bridge as presented in section 5.2. The same load is used for all bridge lengths according to Equation (5-12) giving the same relationship between mass and total load for all bridge lengths. The load amplitude is divided by the span length equal 15 m.

$$p(t) = \frac{700}{15}\sin(2\pi f_l t) = 46.67\sin(2\pi f_l t) \quad [N/m] \tag{5-12}$$

Where:

$$p_0 = 46.67 \quad \text{[N/m]}$$

The acceleration response is calculated at resonance for different lengths due to this load.

5.4.2 Input data

The analyzed lengths are 15, 20, 30, 35 and 40 meters and are analyzed for a damping level of 5% with the cross-section EI5M3 which results in the largest response. The analysis is done in the FEM-program ADINA.

5.4.3 Results

The acceleration responses are presented separately for the analysis done for distributed and concentrated loads.

5.4.3.1 Distributed load

Acceleration response and input values due to varying length with distributed load are presented in Table 5.8.

Table 5.8 Acceleration response due to varying length for distributed load.

Bridge length, L [m]	15m	20m	30m	35m	40m
Maximum acceleration in steady-state, a_{max} [m/s^2]	2.11	2.09	2.09	2.07	2.11
Natural frequency, f_n [Hz]	2.35	1.32	0.59	0.43	0.33
Load frequency, f_l [Hz]	2.35	1.32	0.59	0.43	0.33
Density, ρ [kg/m^3]	7333.33	7333.33	7333.33	7333.33	7333.33
Area, A [m^2]	0.0384	0.0384	0.0384	0.0384	0.0384
Tau steady-state, τ_{ss}	594	589	589	583	594
Tau without amplitude, $\tau_{amp,1}$	12.73	12.62	12.62	12.49	12.73

Tau for steady-state is calculated according to Equation (5-13) as the maximum acceleration multiplied with the density and area of the cross-section.

$$\tau_{ss} = a_{max}\rho A \tag{5-13}$$

τ_{ss} is divided with the load amplitude for distributed load from with size 46.67 N/m from Equation (5-12) according to Equation (5-14).

$$\tau_{amp,1} = \frac{a_{max}\rho A}{p_0} \tag{5-14}$$

Where p_0 has the unit N/m

It should be noted that the acceleration response and $\tau_{amp,1}$ both have the same value for all lengths with the same applied load. This gives that the acceleration response is independent of the bridge length and solely dependent on the relationship between the total applied load and the mass of the bridge as stated in section 5.1. This gives that the acceleration response for distributed load per meter can be related to mass per meter according Equation (5-15)

$$a = \frac{\tau_{amp,1}p_0}{\rho A} \tag{5-15}$$

5.4.3.2 Point load

Acceleration response and analysis values due to varying length with a concentrated harmonic load with force amplitude, P_0, equal to 700 N placed in the middle of the span are presented in Table 5.9.

Table 5.9 Acceleration responses due to varying length for point load 700 N.

Bridge length, L [m]	15m	20m	30m	35m	40m
Maximum acceleration in steady-state, a_{max} [m/s^2]	3.22	2.42	1.60	1.39	1.21
Natural frequency, f_n [Hz]	2.35	1.32	0.59	0.43	0.33
Load frequency, f_l [Hz]	2.35	1.32	0.59	0.43	0.33
Density, ρ [kg/m^3]	7333.33	7333.33	7333.33	7333.33	7333.33
Area, A [m^2]	0.0384	0.0384	0.0384	0.0384	0.0384
Tau steady-state, τ_{ss}	907	680	449	392	340
Tau without amplitude, $\tau_{amp,1}$	19.43	19.43	19.24	19.60	19.43

Tau for steady-state is calculated according (5-16) to as the maximum acceleration multiplied with the density and area of the cross-section.

$$\tau_{ss} = a_{max}\rho A \tag{5-16}$$

τ_{ss} is, as for distributed load, divided with the total applied load and multiplied with the length of the bridge to get the total mass of the bridge and the same unit as for distributed loading according to Equation (5-17).

$$\tau_{amp,1} = \frac{a_{max}\rho AL}{P_0} = \frac{a_{max}\rho AL}{700} \tag{5-17}$$

Where P_o has the unit N

It should be noted that $\tau_{amp,1}$ in this case has approximately the same value meaning that the response can be normalized depending on bridge length for point load. It shows that the stated relationship between total applied load and total mass is valid for point loading when the length is varied.

5.4.4 Conclusion

From the acceleration response and the calculated $\tau_{amp,1}$ for distributed and concentrated load it can be seen that the earlier stated relationship between mass and load amplitude is valid. The length of the bridge does not affect the acceleration response in other regard than that the total mass of the bridge is changed which can be taking into regard by adding to the already existing normalization factor. The principles are shown and explained in the next section.

5.4.5 Normalization with regard to length

The normalization presented in section 5.1.4 should be complemented with the factor for varying length and force amplitude as shown in section: 5.3, Force Amplitude study, changing the earlier normalization graphs. This gives that τ becomes:

5.4.5.1 For point load

$$\tau = \frac{a\rho AL}{P_0}\ [-] \tag{5-18}$$

Where a is calculated by:

$$a = \frac{\tau P_0}{A\rho L}\ [m/s^2] \tag{5-19}$$

Where P_0 has the unit N.

Note that $A\rho L$ is the total mass of the bridge giving that the relationship between the amplitude of the force τP_0 and the mass of the bridge $A\rho L$ determines the size of the acceleration as presented earlier in section 5.1.4.

5.4.5.2 For distributed load

$$\tau = \frac{a\rho A}{p_0} \quad [-] \tag{5-20}$$

Where a is calculated by:

$$a = \frac{\tau p_0}{A\rho} \quad [m/s^2] \tag{5-21}$$

Where p_0 has the unit N/m.

5.4.5.3 Normalized curves with regard to bridge length

This gives new normalization curves with regard to force amplitude and bridge-length according to Figure 5.26.

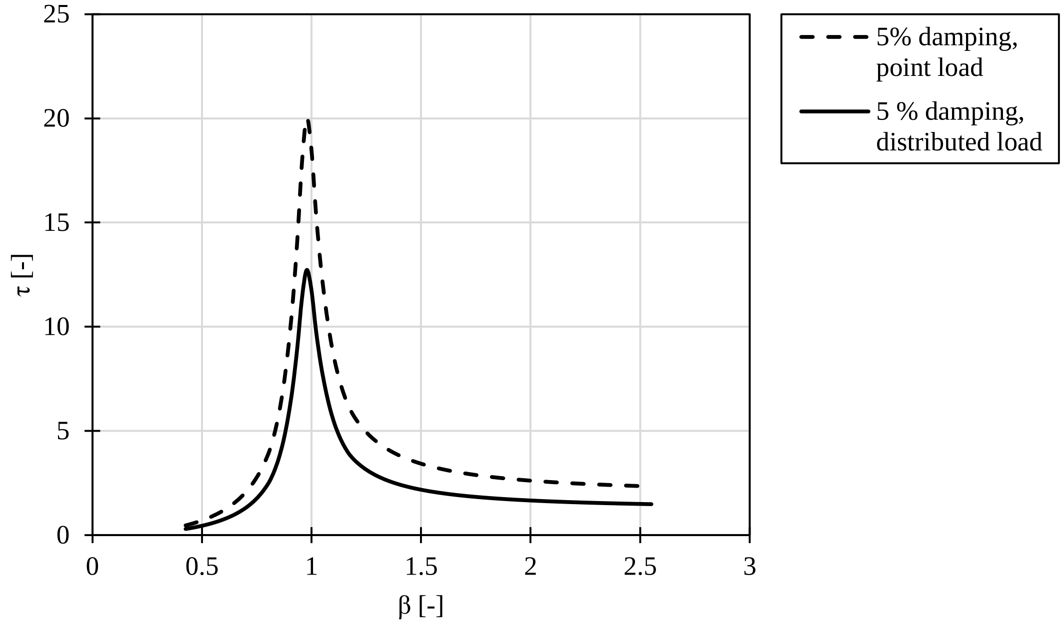


Figure 5.26 Normalized curves for acceleration response with regard to force amplitude and length of the bridge

5.5 Analysis of damping dependence

The damping ratio affects the maximum acceleration of a system and is of great importance when investigating load models. In this chapter a study of the response of the simply supported beam for various damping ratios is presented.

5.5.1 Method

The study is made for a simply supported beam with fixed stiffness and material properties but various damping ratios.

The beam is subjected to a sinusoidal concentrated force in the mid node and a distributed load. It is analyzed in ADINA for maximum acceleration at steady state. The damping ratio is implemented in ADINA as modal damping for all possible modes. All damping ratios are not analyzed for both concentrated and distributed load only those which are relevant to consider.

5.5.2 Input data

The input data presented in this section is the geometrical properties of the beam, applied load and considered damping ratios

5.5.2.1 Geometric properties

The analyzed beam has constant stiffness and density representing the configuration with the highest response, EI5M3. The geometric properties are given in Table 5.10.

Table 5.10 Geometric input data for study of damping ratio

Length, L	15 m
Width, b	0.160 m
Height, h	0.400 m
Thickness, t	0.040 m
Density, ρ	7333 kg/m^3
Modulus of elasticity, E	260 GPa
Natural frequency, f_n	2.35 Hz

5.5.2.2 Applied load

The beam is subjected to a concentrated load and a distributed load separately according to Equation (5-22) and (5-23).

$$P(t) = P_0 \sin(2\pi f t) \quad [N] \tag{5-22}$$

$$p(t) = \frac{P_0}{L} \sin(2\pi f t) \quad [N/m] \tag{5-23}$$

The loads are applied close to resonance to excite the beam in maximum vibration with loading frequency and load amplitude given below. It has been shown in the studies that a load frequency close to resonance gives the highest response. In this analysis the load frequency 2.30 Hz has shown to give the higest response even though the natural frequency is 2.35 Hz.

$$f = 2.30\ Hz$$

$$P_0 = 700\ N$$

5.5.2.3 Damping parameters

The damping ratios used in this study are presented in Table 5.11. The analyzed damping ratios in the study are chosen according to the standards for reinforced concrete, steel and timber bridges.

Table 5.11 Damping ratios used in the study according to standards for reinforced concrete, steel and timber bridges

Standard	Construction material	Damping ratio, ξ [%]
Reference beam	-	5.00
ISO 10137	Reinforced concrete	0.80
	Steel	0.50
	Timber	1.0
UK National Annex	Reinforced concrete	1.50
	Steel	0.50
	Timber	1.0
Sétra	Reinforced concrete	1.30
	Steel	0.40
	Timber	1.0
JRC	Reinforced concrete	1.30
	Steel	0.40
	Timber	1.50
HIVOSS	Reinforced concrete	1.30
	Steel	0.40
	Timber	1.50
SYNPEX	Reinforced concrete	1.30
	Steel	0.40
	Timber	3.00

5.5.3 Results

The results with maximum acceleration response for the considered damping ratios are presented in the following two sections for concentrated and uniformly distributed loads separately.

5.5.3.1 Results – Concentrated load

The maximum acceleration calculated in ADINA for the concentrated load for given damping ratios are presented in Table 5.12.

Table 5.12 Damping ratios with respective maximum acceleration for concentrated force

Damping ratio, ξ [%]	Maximum acceleration, a_{max} [m/s^2]
0.40	37.9
0.50	31.6
0.60	26.7
0.80	20.3
1.00	16.4
1.13	14.5
1.30	12.7
1.35	12.2
1.50	11.0
1.85	8.93
5.00	3.22

5.5.3.2 Results – distributed load

This load is analyzed for the different levels of damping proposed in the load models presented in Section 4, Standards, regulations and guidelines giving values as presented in Table 5.13.

Table 5.13 Acceleration response for different levels of damping for distributed load.

Damping ratio, ξ [%]	Maximum acceleration, a_{max} [m/s^2]
0.40	24.6
0.50	20.1
0.60	16.9
1.00	10.3
1.13	9.25
1.30	8.03
1.35	7.77
1.50	7.00
1.85	5.69
3.00	3.51
5.00	2.11

5.5.4 Conclusions – damping dependence

In this section conclusions from the results of damping dependence analysis are presented. The conclusions are divided into three parts for concentrated load, distributed load and a comparison between the two loading types.

5.5.4.1 Point load

The relationship between the structural damping ratio and acceleration response for a concentrated load given in Table 5.11 is plotted in Figure 5.27. The acceleration is exponential decreasing for higher damping ratios which is reasonable according to the theory presented in section 3.3.4. The acceleration response cannot easily be calculated for any given damping ratio.

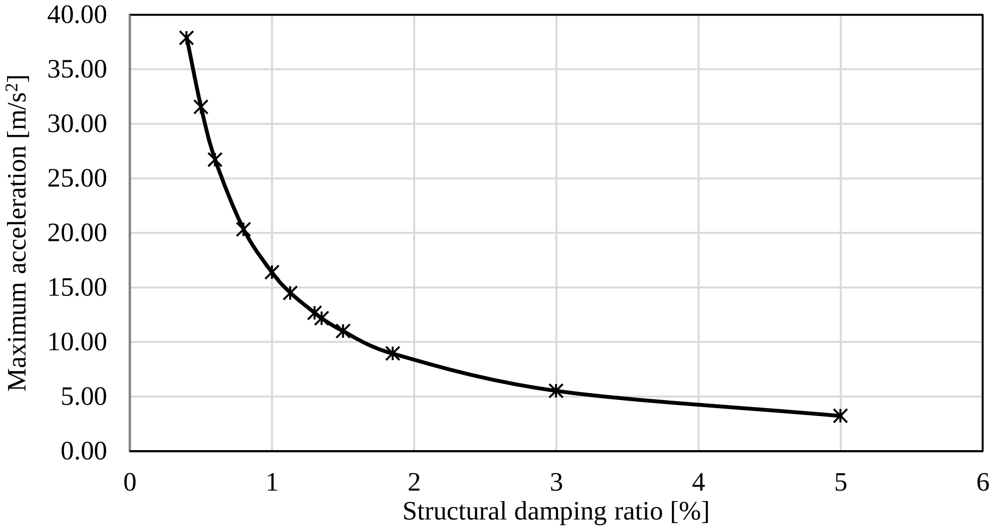


Figure 5.27 Acceleration response due to different levels of structural damping for point load

5.5.4.2 Distributed load

A plot of the acceleration response for different levels of damping is shown Figure 5.28 where it can be seen that the relationship is complex, as shown for the acceleration response due to concentrated loads. The acceleration response cannot easily be calculated for any given level of damping. The plot has the same shape as for the concentrated load which is reasonable.

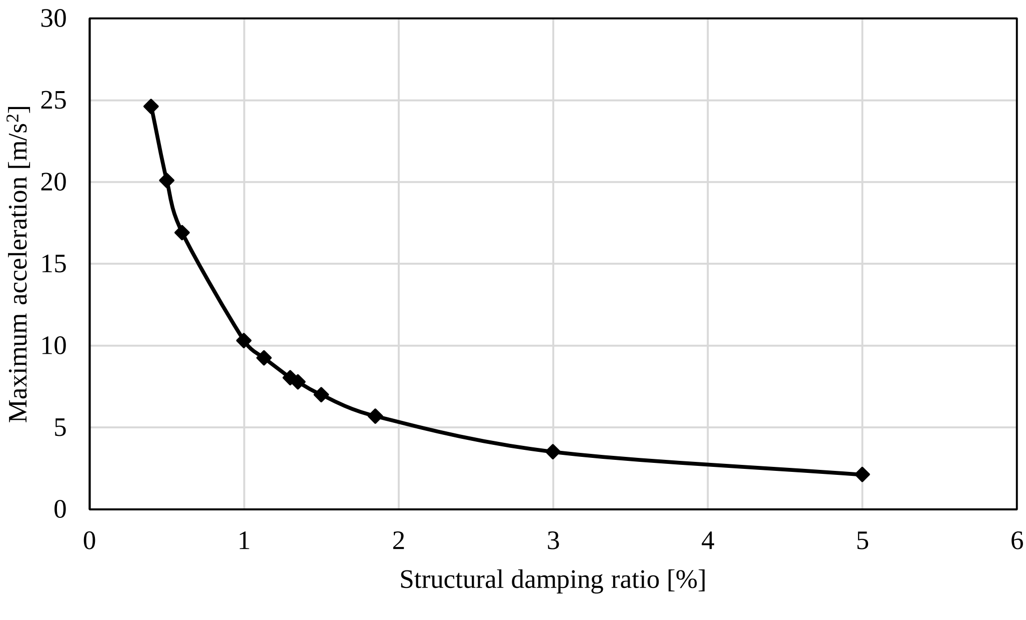


Figure 5.28 Plot of maximum acceleration for different levels of damping for distributed load

5.5.4.3 Comparison concentrated and distributed loads

The acceleration response for concentrated and distributed load for different damping ratios is related according to Table 5.14. In the table the acceleration response for the two types of loading is presented and the ratio between them for each studied damping ratio. The ratio is given in column four and it can be seen that is constant at approximately 1.56 for all damping ratios. As the sum of the applied load is equal for concentrated and distributed load it is shown that a concentrated load gives about 1.56 times higher acceleration than applying the same load as distributed. This is a reasonable conclusion considering that the concentrated load is applied at the worst position as creates a bigger impact. As the relation is constant for all damping ratios the results can be seen as reliable and verified.

Table 5.14 Table of acceleration response for point load and distributed load.

Damping ratio, ξ [%]	Max acceleration point load [m/s^2]	Max acceleration distributed load [m/s^2]	Relation [acc. concent./acc. dist.]
0.40	37.87	24.63	1.538
0.50	31.6	20.1	1.57
0.60	26.69	16.94	1.576
1.13	14.49	9.25	1.566
1.30	12.66	8.03	1.577
1.35	12.16	7.77	1.565
1.50	11.0	7.00	1.57
1.85	8.93	5.69	1.569
5.00	3.22	2.11	1.526

5.5.5 Normalization according to level of damping

As shown in section 5.4.4 the acceleration response due to different levels of damping is complex and therefore not suitable for normalization. To be able to implement the damping ratio in the normalization curves each curve has to be derived empirically as τ is unique for each damping ratio.

As known from analysis in previous chapter the normalization factor τ is calculated according to Equation (5-24).

$$\tau = \frac{a\rho AL}{P_0} \tag{5-24}$$

The normalization factor can now be calculated for all studied damping ratios with the given input data in Table 5.15.

Table 5.15 Normalization factor τ for different damping ratios

Damping ratio, ξ [%]	Normalization factor τ [-]	
	Concentrated load	Distributed load
0.40	228.5	148.6
0.50	190.4	121.2
0.60	161.1	102.2
0.80	122.6	-
1.00	98.8	62.04
1.13	86.98	55.82
1.30	76.41	48.44
1.35	73.38	46.87
1.50	66.35	42.24
1.85	53.88	34.31
5.00	20.01	12.73

A normalization curve for each damping ratio is created by using the curve for 5% damping as a reference curve. The relation between normalization factor for the considered damping ratio and the reference normalization factor by Equation (5-25)

$$\mathrm{T}_{ratio} = \frac{\tau_i}{\tau_{5\%}} \tag{5-25}$$

Where:

T_{ratio}	normalization factor ratio
τ_i	normalization factor for the considered damping ratio i
$\tau_{5\%}$	reference normalization factor 5% damping

The acceleration ratio for each damping ratio is presented in tables below. Table 5.16 contains the ratios for concentrated loads and Table 5.17 for distributed loads.

Table 5.16 Acceleration response and acceleration ratio for concentrated load according to studied damping ratios and reference damping ratio 5%

Damping ratio, ξ [%]	τ_i, [??]	$\tau_{5\%}$, [??]	T_{ratio}, [-]
0.4	228.5	20.01	11.4
0.5	190.4	20.01	9.52
0.6	161.1	20.01	8.02
0.8	122.6	20.01	6.10
1.13	86.98	20.01	4.35
1.3	76.41	20.01	3.82
1.35	73.38	20.01	3.67
1.85	53.88	20.01	2.69
5	20.01	20.01	1.00

Table 5.17. Acceleration response and acceleration ratio for distributed load according to studied damping ratios and reference damping ratio 5%

Damping ratio, ξ [%]	τ_i, [??]	$\tau_{5\%}$, [??]	T_{ratio}, [-]
0.40	148.6	12.73	11.7
0.60	102.2	12.73	8.03
1.00	62.04	12.73	4.87
1.13	55.82	12.73	4.38
1.30	48.44	12.73	3.81
1.35	46.87	12.73	3.68
1.50	42.24	12.73	3.32
1.85	34.31	12.73	2.70
5.00	12.73	12.73	1.00

The plots of normalization curves for different levels of damping are shown in Figure 5.29and Figure 5.30 for point load and distributed load respectively. These plots are extracted from the reference curve for 5 % damping which is multiplied with the relationship between the maximum acceleration for each damping and the reference acceleration for 5 % damping.

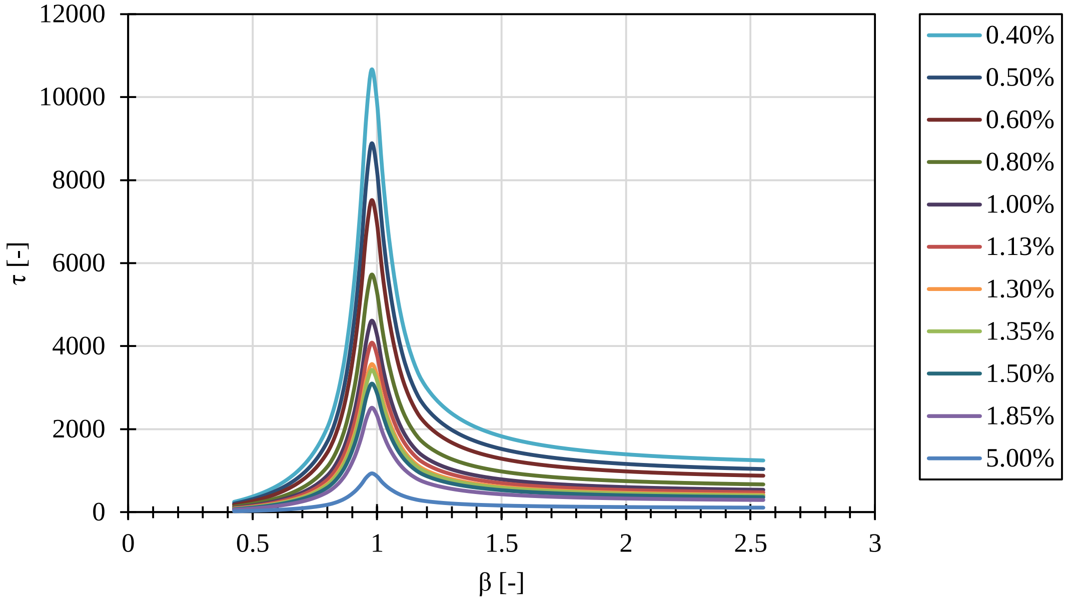


Figure 5.29 Normalization curve with varying damping ratio for concentrated load

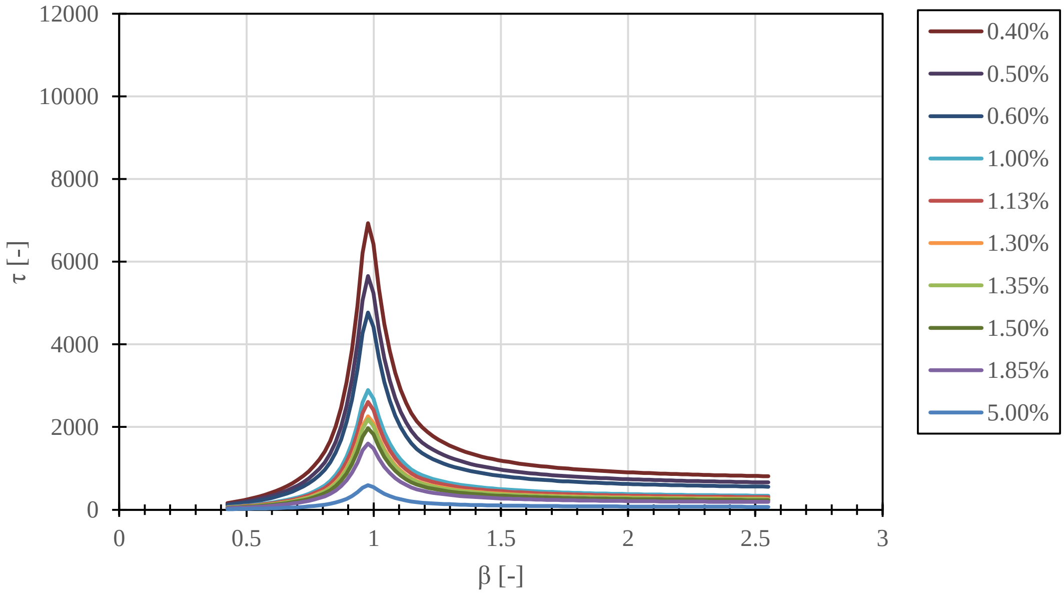


Figure 5.30 Normalization curve with varying damping ratio for distributed load

5.6 General conclusions on analysis and outcome for further studies

In the first analysis of a simply supported beam with varying properties it is shown that the maximum acceleration response is obtained at resonance. By varying mass and stiffness it is shown that the acceleration response in resonance is independent on stiffness and solely dependent on mass. The response in the beam can be normalized according to mass and natural frequency independent on beam properties.

The load amplitude is direct proportional to the acceleration response in a system. In the analysis regarding load amplitude it is shown that the acceleration response in a system excited by a sinusoidal load is linear proportional to the applied load amplitude. It means that the acceleration response in a system excited by a sinusoidal load with any given load amplitude can be calculated.

In the analysis it is shown for varying length that the acceleration response for a simply supported beam is independent on span length. The acceleration response is solely dependent on the ratio between applied force and mass of the excited system.

The acceleration response due to a concentrated load is dependent on the ratio between total applied load in N and total mass in kg independent on span length. The obtained acceleration response for a uniformly distributed load is shown to be the ratio between applied load per meter, N/m, and mass per meter, kg/m independent on span length.

The acceleration response in a system is highly dependent on the structural damping ratio in the system. In the analysis a relationship between acceleration response and damping ratio can be seen but due to its complexity specific normalization factors are derived.

The normalization factors τ are derived for specific damping ratios and represent the acceleration response in a system excited by a sinusoidal load with load amplitude equal to one. The normalization factor τ is dependent on the type of loading with different values for concentrated and uniformly distributed load. The factor is derived individually for all considered damping ratios.

The final result from the analyzes is that the acceleration response at resonance, for a simply supported beam in one span excited by a sinusoidal load, can be calculated with given structural damping, load amplitude and mass of the beam.

The acceleration response of a concentrated load model is calculated by equation (5-26) and by equation (5-27) for uniformly distributed load models.

$$a = \frac{\tau P_0}{A\rho L} \quad [m/s^2] \tag{5-26}$$

$$a = \frac{\tau p_0}{A\rho} \quad [m/s^2] \tag{5-27}$$

6 Normalization of load models

The normalization presented in Chapter 5 can be used to evaluate the studied guidelines independent of force amplitude, bridge length and varying cross-section for a simply supported beam. The possibility to calculate the response for any simply supported beam for arbitrary load amplitude is used for comparison of the studied guidelines.

The normalization is made by division of the load models into two parts, load amplitude and the normalization factor τ. The acceleration response due to a sinusoidal load is represented by the normalization factor τ. The load amplitudes, P_0 for concentrated load and p_0 for distributed load, are derived for every guideline dependent on the proposed load for a pedestrian and empirical factors defined in the corresponding guideline. This results in an evaluation of the guidelines with regard to their individual factors.

Uniformly distributed load models will be analyzed independent on bridge deck area to be consistent when considering different standards and will have the unit N/m. This gives that the models are comparable for arbitrary bridge geometry and cross-sectional properties. Point load are independent on bridge geometry and will have the unit N.

The results from the normalization will be presented in plots of acceleration response dependent on structural frequency for each studied guideline. The results will be discussed to provide a basis for further discussion and conclusions.

The normalization curves are useful to calculate the acceleration response in any type of bridge in one span. By reading τP_0 from the normalized acceleration response curve of the concentrated load model the acceleration response can be calculated by equation (5-26). The acceleration response for uniformly distributed load models is given by τp_0 from the normalization curve and equation (5-27).

6.1 ISO 10137

The ISO 10137 standard gives recommendations for which design situations to consider and provides a load model describing the pedestrian vertical and lateral force as Fourier series according to Equation (4-8) and (4-9) in section 4.2. The load should be applied to achieve the worst case scenario, for example at the center of a simply supported span of uniform mass.

For an accurate analysis the force should be applied moving across the structure but no walking speed is presented in ISO 10137. In the analysis the load will be simplified as a stationary load in the middle of the span to be able to normalize the load according to the method presented in chapter 5.

6.1.1 Effect of phase shift and Fourier coefficients

The load is applied as a concentrated load situated in the middle of the span with a loading frequency equal to the natural frequency of the structure. The load is applied until steady-state is achieved which is the worst case scenario.

The vertical and lateral force for pedestrians in ISO 10137 are modelled with a Fourier series as described in section 4.2 meaning that the force varies due to how many Fourier coefficients and sums that are considered.

In Figure 6.1 and Figure 6.2 the sinusoidal force due to different number of Fourier coefficients and sums, not in phase and in phase respectively are shown. Note that he force is oscillating around 700 N which is the static load of one pedestrian.

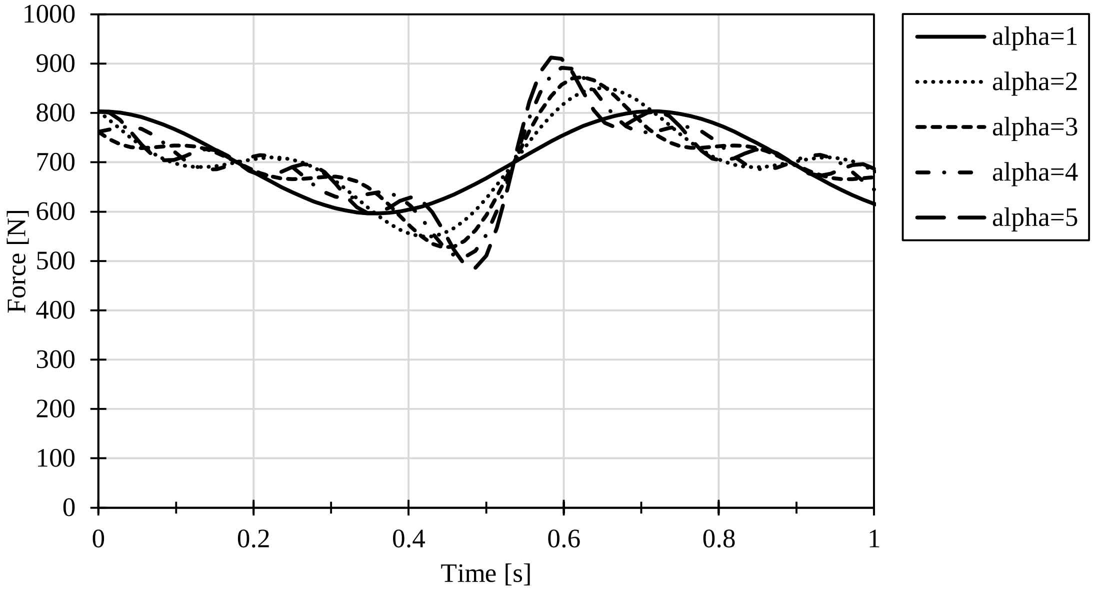


Figure 6.1 Force due to different amount of phase shifts not in phase.

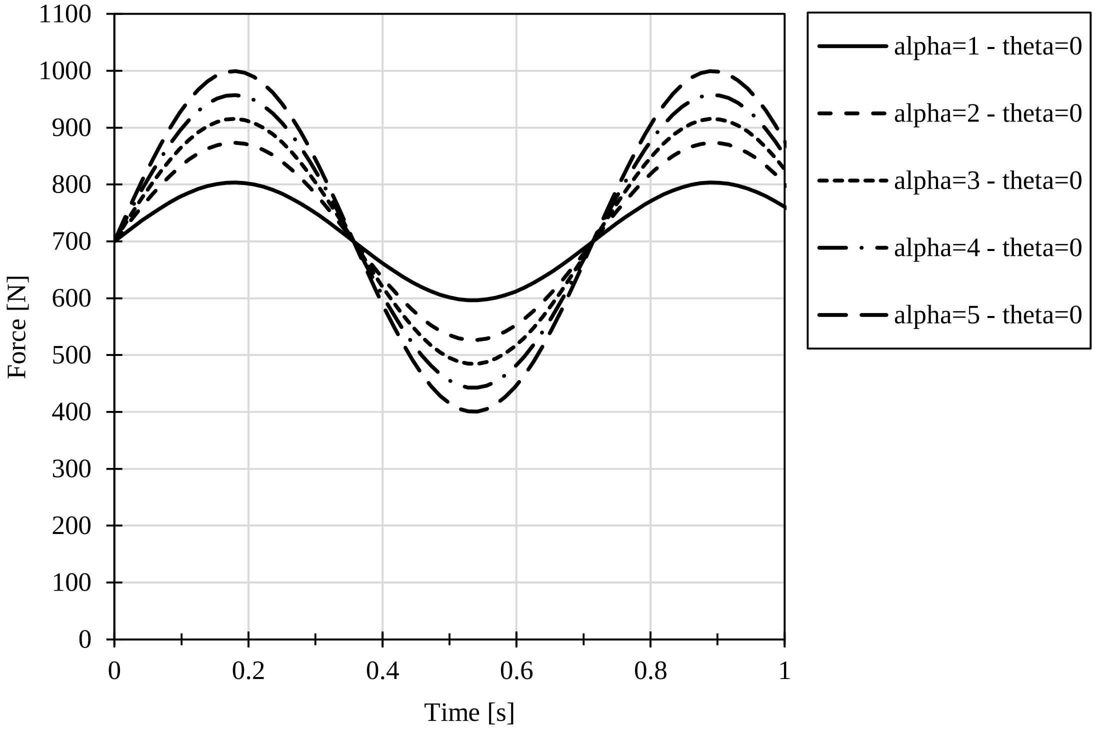


Figure 6.2 Force due to different amount of phase shifts in phase.

By neglecting phase shift the load amplitude will increase and perfect resonance can occur due to the appearance of the load.

The maximum force amplitudes due to different sums and phase angles in phase and not in phase are shown in Figure 6.3. It can be seen that the amplitude for the sums in phase are slightly higher which also is illustrated in Figure 6.2 and Figure 6.1.

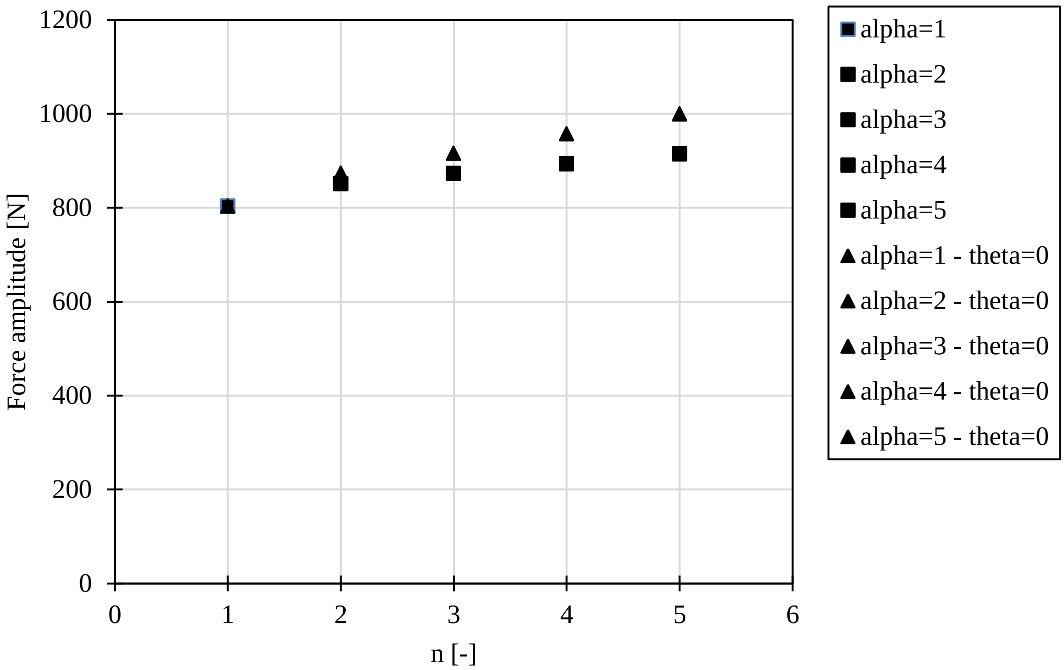


Figure 6.3 *Maximum force amplitude due to different amount of phase angles, in phase and not in phase.*

6.1.2 Normalized acceleration response due to concentrated load

The fact that the force varies due to how many sums and Fourier coefficients that are considered mean that the acceleration response also varies. Applying Fourier sums in phase and not in phase gives different load history and acceleration response. A simplification is made to look at sums in phase without the defined phase shift in ISO 10137. This gives that calculations can be made numerically for different amounts of sums. The static load does not affect the acceleration response and is neglected which gives Equation (6-1) from Equation (4-8) in section 4.2.

$$F_{ver}\ (t) = Q\sum_{n=1}^{k}\alpha_{n,ver}\sin\left(2\pi ft+\phi_{n,ver}\right) = Qsin(2\pi ft)\sum_{n=1}^{k}\alpha_{n,ver} \qquad (6\text{-}1)$$

Introducing normalization factor τ defined as the acceleration response due to a sinusoidal concentrated load empirically derived in chapter 5 and the load amplitude P_0, shown in Equation (6-2):

$$F_v(t) = Q\sin(2\pi f t)\sum_{n=1}^{k}\alpha_{n,ver} \tag{6-2}$$

Where:

$$P_0 = Q\sum_{n=1}^{k}\alpha_{n,ver}$$

Equation (6-1) is calculated regarding different amounts of sums and presented in Figure 6.4 for vertical response without phase shift. The values are calculated with normalization factor τ corresponding to a damping ratio of 5.0 %.

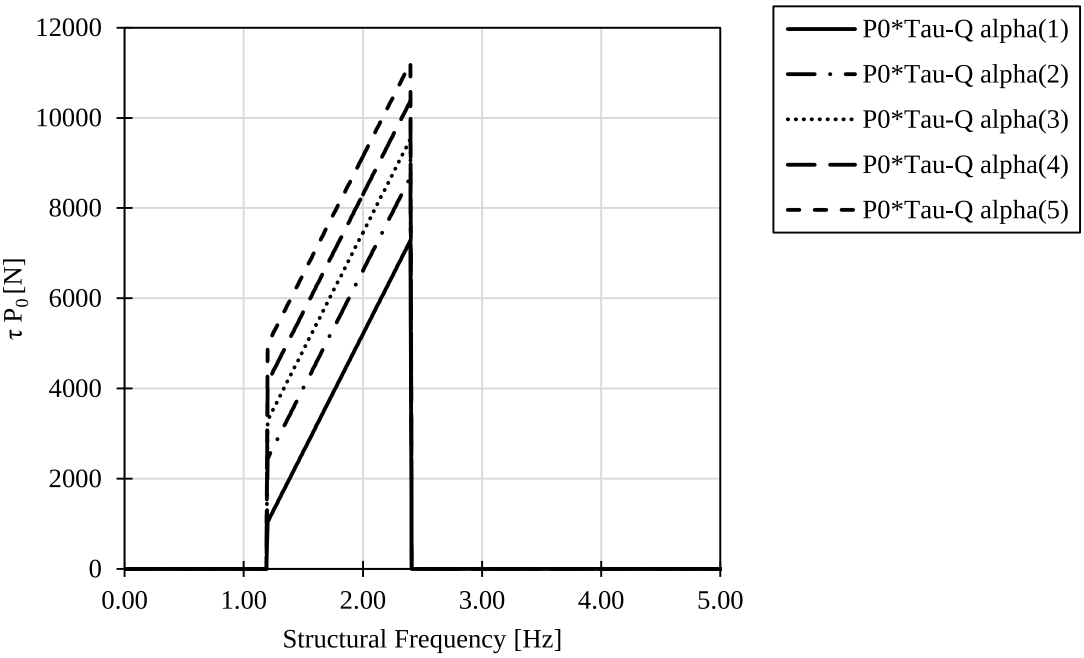


Figure 6.4 Vertical acceleration response due to different amounts of Fourier coefficients for SS-ISO 10137:2008 with 5 % damping.

A comparison of the acceleration response due to applying the sum of forces in phase and not in phase can be seen in Figure 6.5. The curves plotted in Figure 6.4 are compared to three analyses that have been calculated in ADINA. Three beams with varying natural frequencies have been analyzed when applying the Fourier sum with phase shift resulting in maximum accelerations. The Fourier sum has been applied for one, two and three sums. These accelerations give τP_0 for the three Fourier sums which are plotted in Figure 6.5 and can be compared with the numerically derived curves without phase shift.

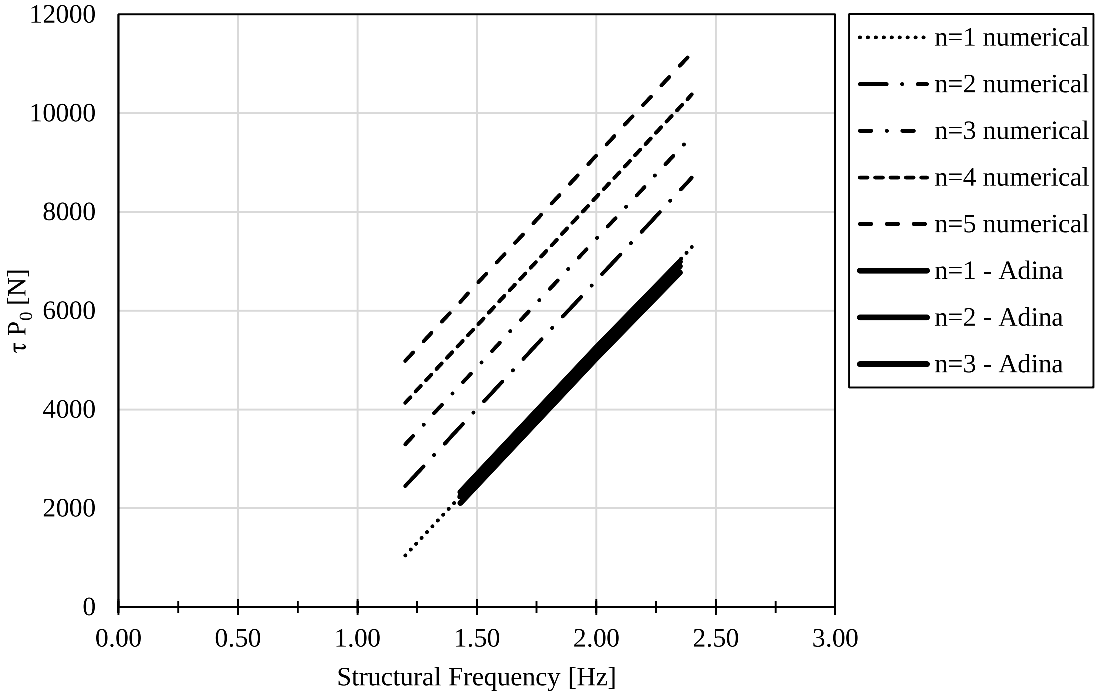


Figure 6.5 Response due to applying ISO 10137 Fourier-force in phase vs not in phase.

It can be seen that the force due to applying the Fourier sums with phase shift are much lower than for applying without and have approximately the same value as the first Fourier sum for all three sums. Note that the calculated values from ADINA calculated with phase shift has a slight curve in comparison with linear curves from hand calculations. This curve can be explained with that the phase shift always is constant which makes the load hit more or less the resonance frequency of the regarded cross-section.

From the results shown in Figure 6.5 it can be concluded that the ISO 10137 load can be simplified to regard only one Fourier sum without phase shift in the analysis, resulting in that the normalization is done according to Equation (6-3)

$$F_{ver}(t) = Q(1 + \alpha_{1,\mathrm{ver}}\sin(2\pi f t)\ )\, N_{eq} \tag{6-3}$$

Where the static load does not affect the acceleration response which gives that Equation (6-3) the load amplitude is identified in Equation (6-4).

$$F_{ver}(t) = Q\ \alpha_{1,\mathrm{ver}}\sin(2\pi f t)\ \ N_{eq} \tag{6-4}$$

Where

$$P_0 = Q\alpha_{1,\mathrm{ver}}N_{eq}$$

Values for the normalization factor τ for concentrated load models for considered damping ratios are empirically derived in chapter 5 and presented in Table 6.1.

Table 6.1 *Values for the normalization factor according to material and corresponding damping.*

Input	Reference	Concrete	Steel	Timber
Damping ratio [%]	5.0	0.80	0.50	1.0
τ [-]	20.01	122.6	190.4	98.8

6.1.2.1 Normalized curves in vertical direction

The vertical response due to varying amount of pedestrians from 1 to 15 people as stated in the standard can be seen in Figure 6.6 and Figure 6.10 for vertical and lateral response respectively for 5.0 % damping. The values are calculated for a Fourier sum with one Fourier coefficient.

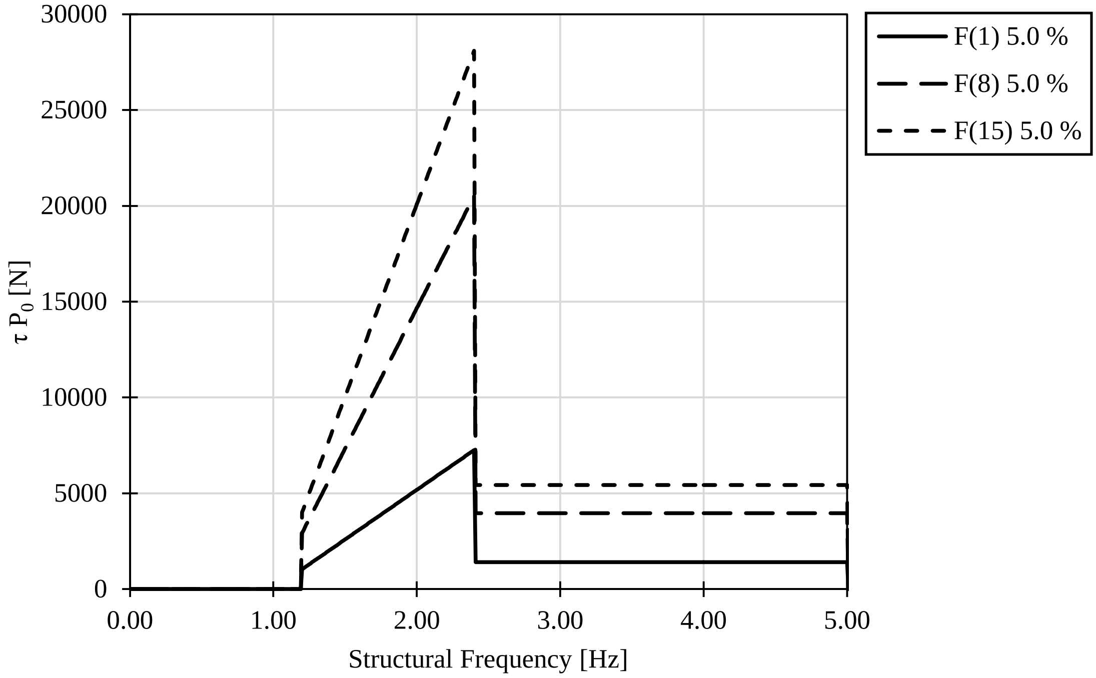


Figure 6.6 *Vertical acceleration response due to varying amount of pedestrians for SS-ISO 10137:2008 with 5.0 % damping.*

Normalized curves for ISO 10137 in vertical direction for concrete with damping ratio 0.8 % can be seen in Figure 6.7

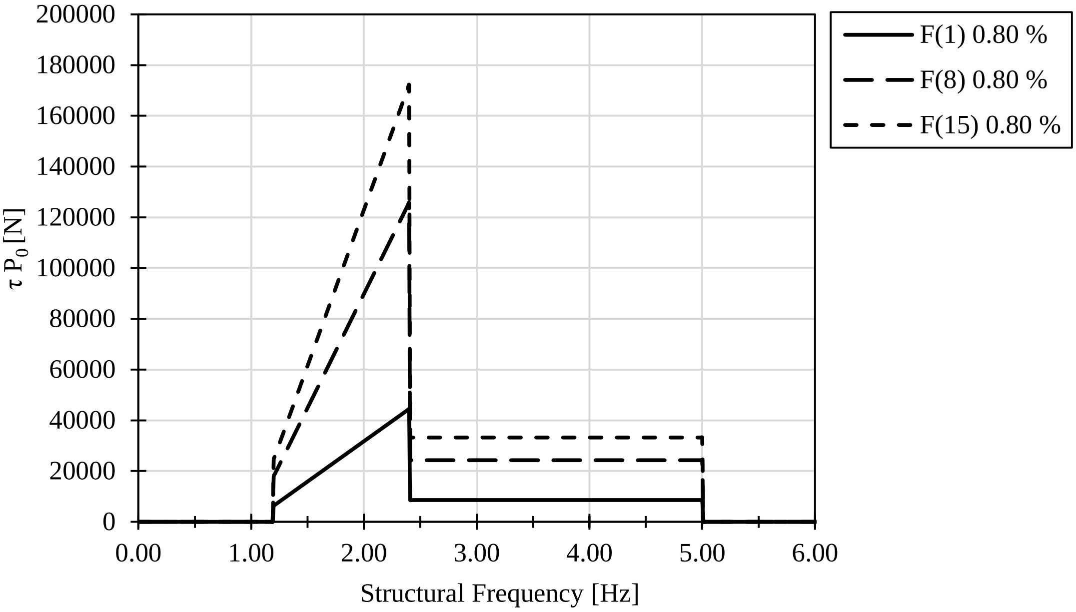


Figure 6.7 Normalized acceleration response for concrete in vertical direction according to ISO 10137 with 0.80 % damping ratio.

Normalized curves for ISO 10137 in vertical direction for steel with damping ratio 0.5 % can be seen in Figure 6.8

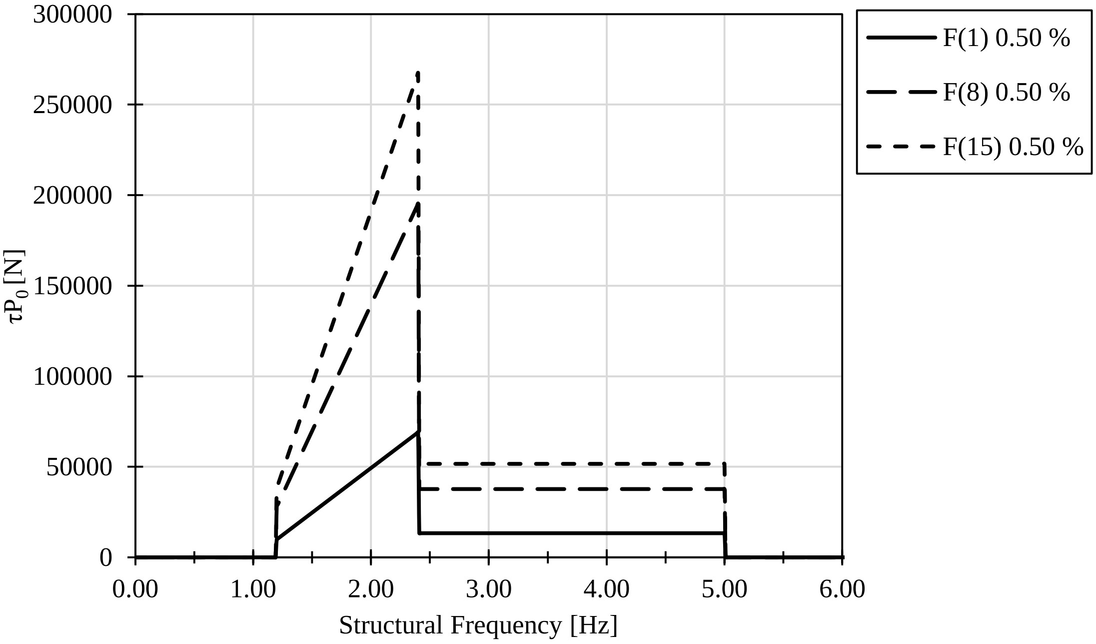


Figure 6.8 Vertical acceleration response for steel due to varying amount of pedestrians for SS-ISO 10137:2008 with 0.50 % damping ratio.

Normalized curves for ISO 10137 in vertical direction for Timber with damping ratio 1.0 % can be seen in Figure 6.7.The damping ratio for timber has been chosen for bridges with mechanical joints to 1.0 %

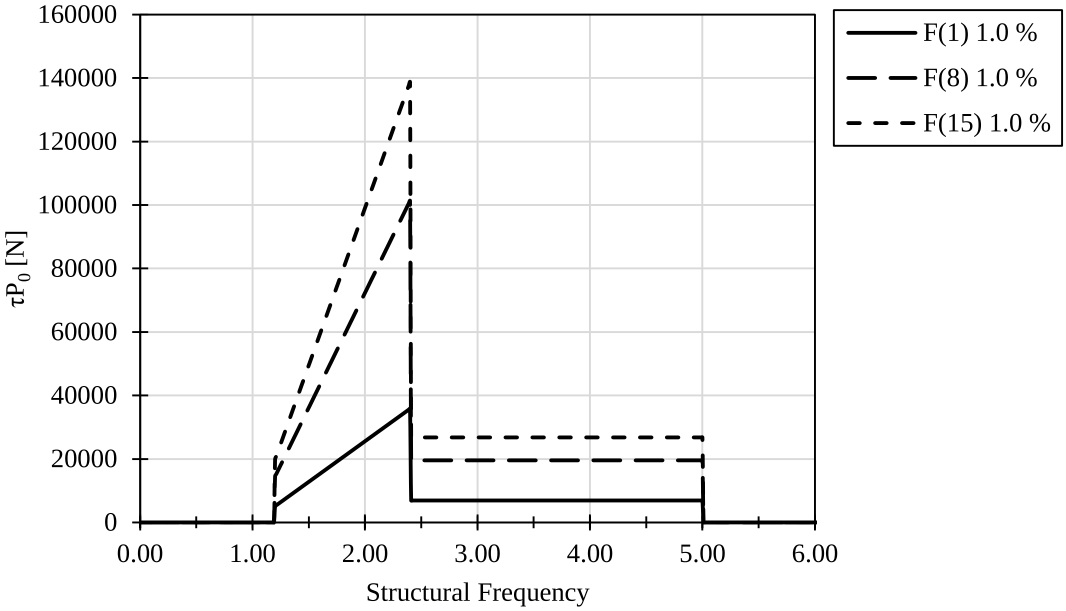


Figure 6.9 *Normalized acceleration response for timber in vertical direction according to ISO 10137 with 1.0 % damping ratio.*

6.1.2.2 Normalized curves in lateral direction

Normalized curves for ISO 10137 in lateral direction with reference damping ratio 0.80 % can be seen in Figure 6.10.

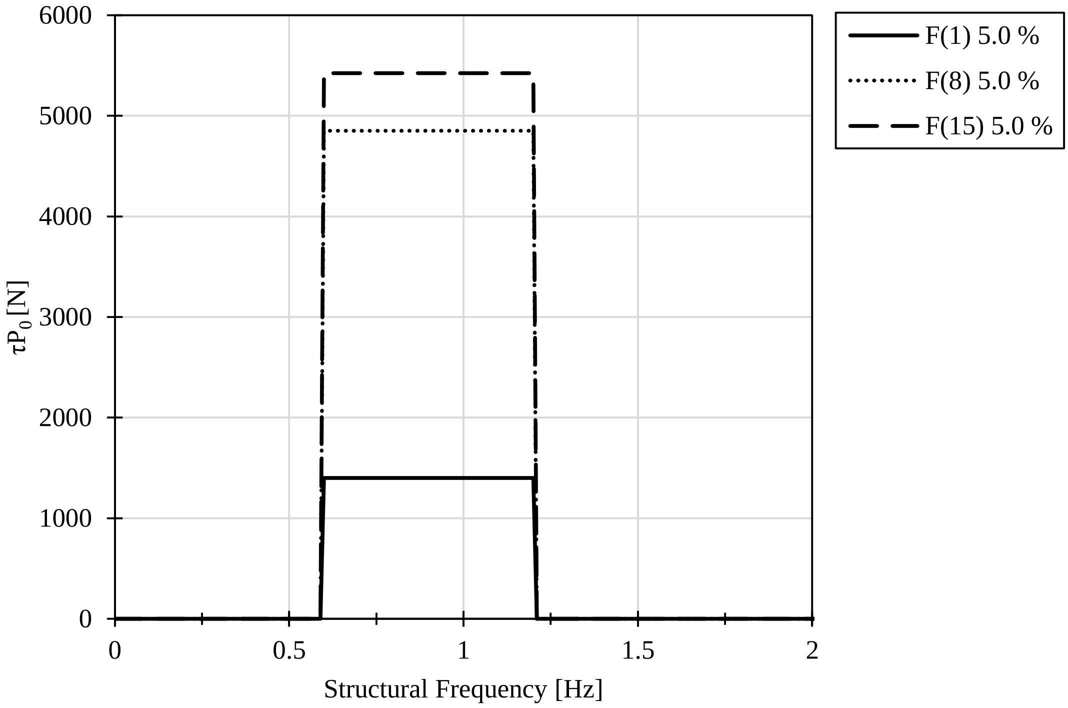


Figure 6.10 *Lateral response due to varying amount of pedestrians for ISO 10137 with 5.0 % damping*

Normalized curves for ISO 10137 in lateral direction for concrete with damping ratio 0.80 % can be seen in Figure 6.11

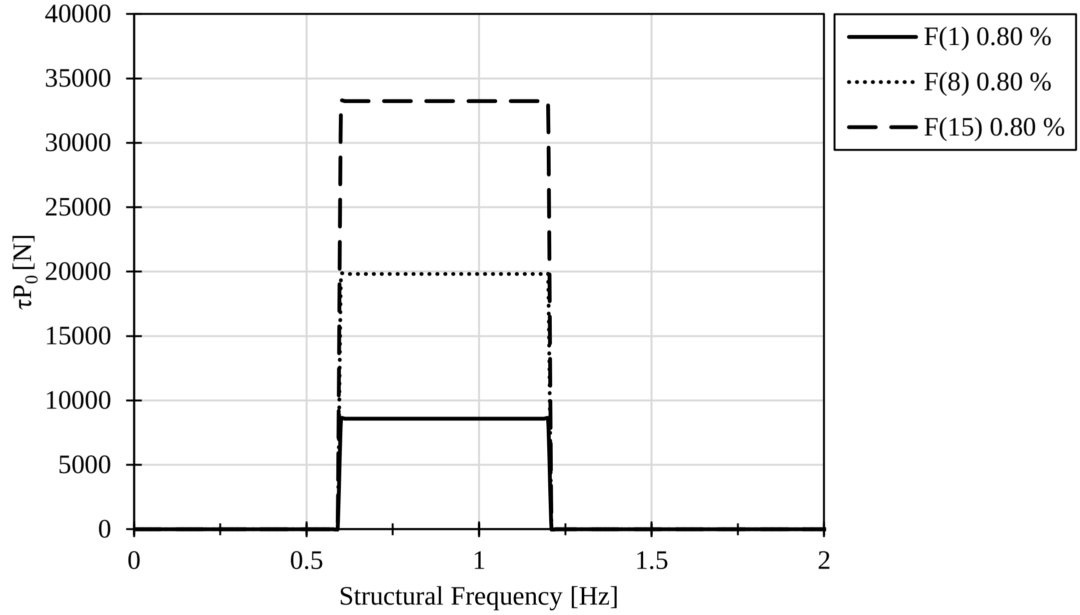


Figure 6.11 Normalized acceleration response for concrete in lateral direction according to ISO 10137 with 0.80 % damping ratio.

Normalized curves for ISO 10137 in lateral direction for steel with damping ratio 0.50 % can be seen in Figure 6.12

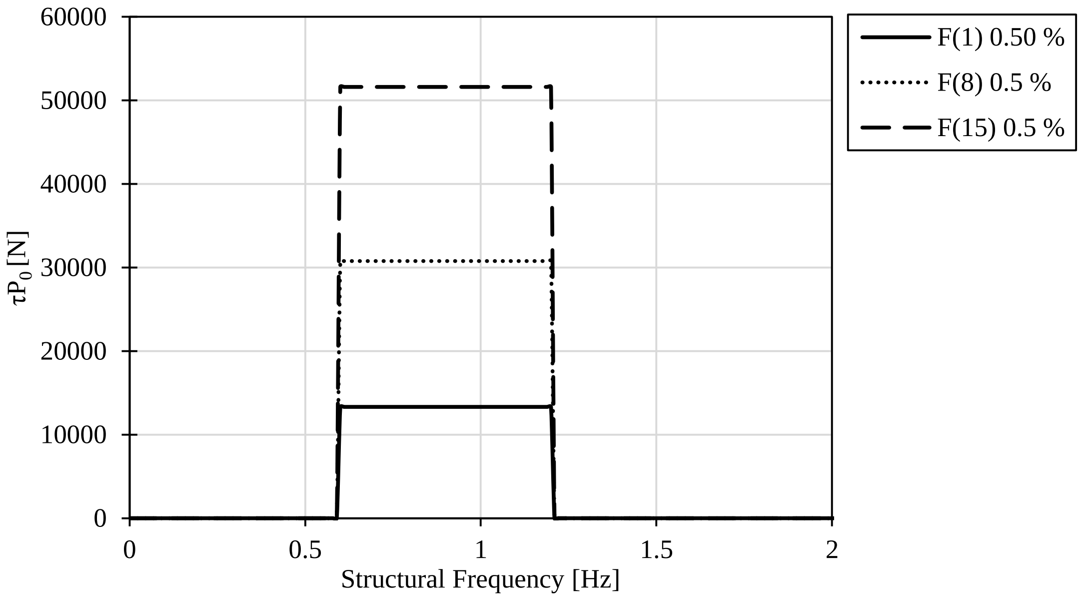


Figure 6.12 Normalized acceleration response for steel in lateral direction according to ISO 10137 with 0.50 % damping ratio.

Normalized curves for ISO 10137 in lateral direction for timber with damping ratio 1.0 % can be seen in Figure 6.13

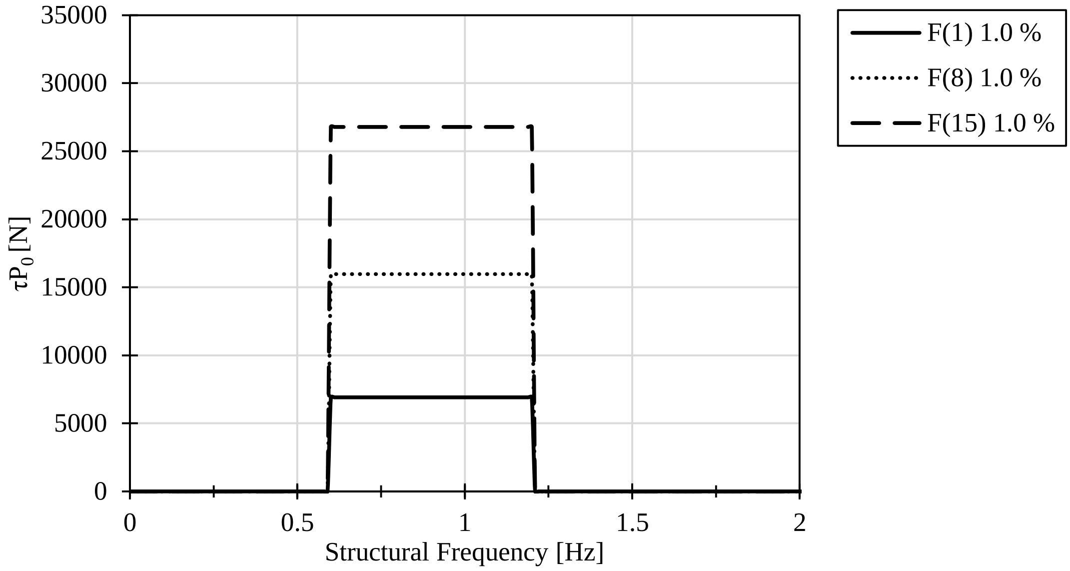


Figure 6.13 Normalized acceleration response for timber in lateral direction according to ISO 10137 with 1.0 % damping ratio.

6.1.3 Comments and discussion

Eurocode states that the natural frequency of a footbridge should be over 5.0 Hz and that if lower a dynamic analysis should be made. The ISO 10137 standard considers structural frequencies from 0 to 5.0 Hz. The first harmony is defined between 1.2 and 2.4 Hz and the second harmony has a defined interval from 2.4 to 5.0 Hz.

In vertical direction the ISO 10137 standard has an increasing response for structural frequencies from 1.2 up to 2.4 Hz which differs from other standards that considers intervals of frequencies with the same large response. These intervals are defined from which step frequencies that occur the most and therefore should be the structural frequencies combined with the greatest risk, something that ISO 10137 do not take into account.

The lateral frequencies regarded are taken as half of the vertical giving 0.60 to 1.2 Hz. The response due to the lateral force constant for the whole interval and the second harmony is not regarded to give rise to any acceleration.

Modelling the pedestrian load with a Fourier sum aims to create a more exact model of the pedestrian load closer to the actual pedestrian force from consecutive footsteps. The fact that the acceleration response does not differ when applying the load with the first, second and third Fourier sum according to the standard gives that it might be unnecessary to model the pedestrian load more accurate than with a sinusoidal load.

6.2 UK National Annex

The UK National Annex to Eurocode proposes two load models that have to be considered when designing for pedestrians induced vibrations. The two models presented in section 4.3.3 and 4.3.4 should be applied for the chosen traffic density as a concentrated load and a uniformly distributed load.

In this chapter the applications of the load models are presented together with normalizations curves.

The UK National Annex proposes five traffic classes defined as pedestrian groups and pedestrian densities given in Table 4.7. Pedestrian groups are used for the concentrated load model and pedestrian densities should be applied in the uniform load model. The traffic classes are presented in Table 4.7.

6.2.1.1 Structural damping ratios

UK National Annex does not present any suitable damping ratios but damping ratios given in Eurocode are implemented to the annex see section 4.3.5. The damping ratios chosen in normalization are given in Table 6.2.

Table 6.2 Damping ratios applicable on the load models in UK National Annex for the reference material and for concrete, steel and timber.

Material	Reference	Concrete	Steel	Timber
Damping ratio, ξ [%]	5.0%	1.5%	0.50%	1.0%

6.2.2 Point load

The UK National annex to Eurocode proposes a concentrated load, Equation (4-12), moving along the span at constant speed. The method will simulate a single pedestrian or a group of pedestrians crossing the bridge as explained in section 4.3.3.

6.2.2.1 Load history

The applied concentrated load is a sinusoidal load and will act on the beam with the load history given in Figure 6.14. The maximum force amplitude is decided by the factors in the load model and will vary depending on loading direction, structural frequency, structural damping ratio and number on pedestrians. The load history in Figure 6.14 is plotted with input data presented in Table 6.3.

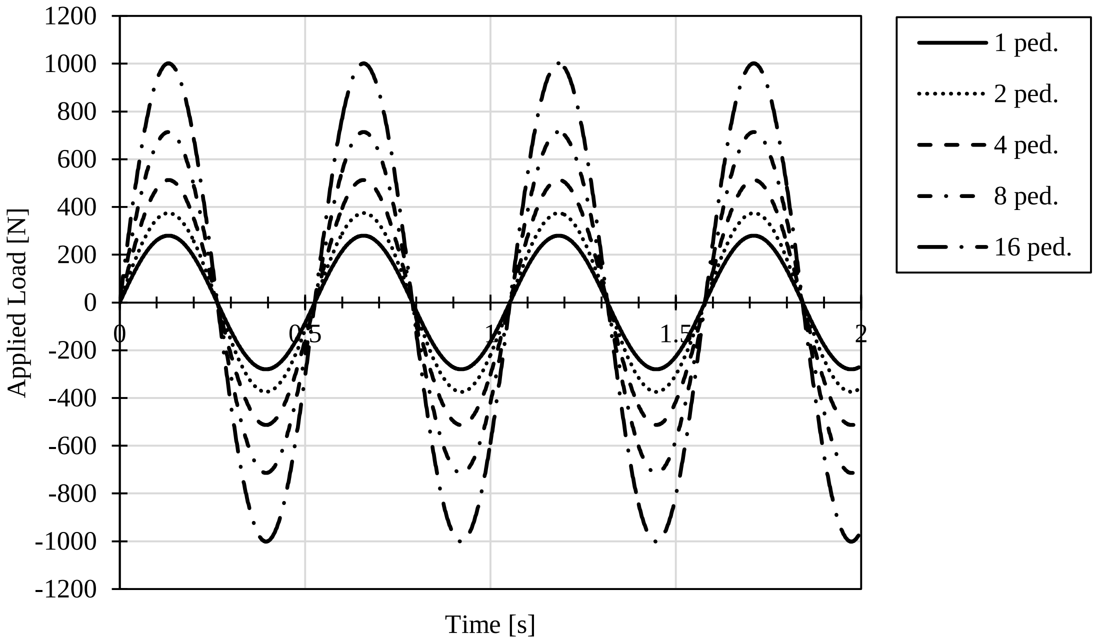


Figure 6.14 Load history for concentrated load applied for 1, 2, 4, 8 and 16 pedestrians.

Table 6.3 Input data used in the plot of loading history for the concentrated load model.

Load model parameter	Given constant value
F_0	280 N
$k(1.9Hz)$	1.0
ξ	5.0%
γ	0.786
f_v	1.9 Hz

6.2.2.2 Normalization and simplifications

The load model is normalized according to the derived method in section: 5.6 according to Equation (6-5). The load model is presented in equation below where the load amplitude P_0 is identified. The normalization factor τ is empirically derived in chapter representing the acceleration response.

$$F(N) = F_0 k(f_v)\sqrt{1+\gamma(N-1)}\sin(2\pi f_v t) \quad [N] \tag{6-5}$$

Where:

$$P_0 = F_0 k(f_v)\sqrt{1+\gamma(N-1)} \quad [N]$$

In the standard the load is supposed to be applied as a moving load along the span with a given constant velocity. A simplification has been made in the normalization to consider the concentrated load as stationary in the middle of the beam. Meaning that the load acts in the worst position and is applied until steady state conditions are achieved.

6.2.2.3 Input data for normalization plot

The normalization is done for 5.0% damping ratio and according to the recommended damping ratios for concrete, steel and timber. The input data used in normalization for each damping case are given in Table 6.4.

Table 6.4 Input given for the reference material and the construction materials concrete, steel and timber for the concentrated load model.

Input	Reference	Concrete	Steel	Timber
Damping ratio [%]	5.0	1.5	0.50	1.0
F_0 [N]	280 N	280 N	280 N	280 N
γ [-]	0.786	0.736	0.700	0.721
τ [-]	20.01	66.35	190.4	98.8

6.2.2.4 Normalization plots for concentrated load model

The design curves are plotted for a single pedestrian and the defined traffic classes according to Table 4.7.

The design curve for the concentrated load model is plotted in Figure 6.15 for 5.0% structural damping ratio and according to the defined traffic classes.

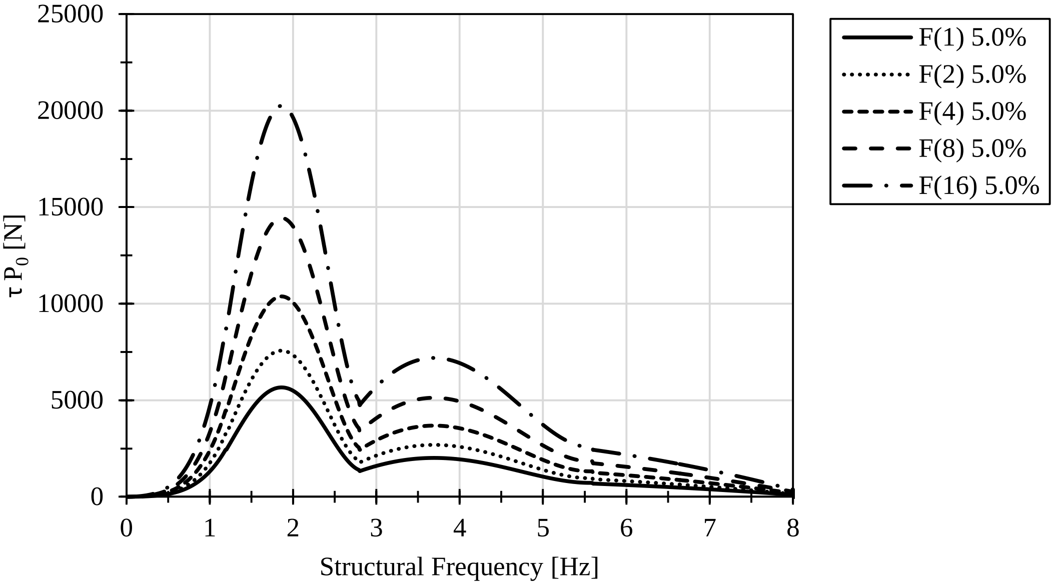


Figure 6.15 Design curve for UK concentrated load with 5.0% structural damping ratio for single pedestrian and according to defined traffic classes.

The design curve for the concentrated load model is plotted in Figure 6.16 for concrete bridges with 1.5% structural damping ratio and according to the defined traffic classes.

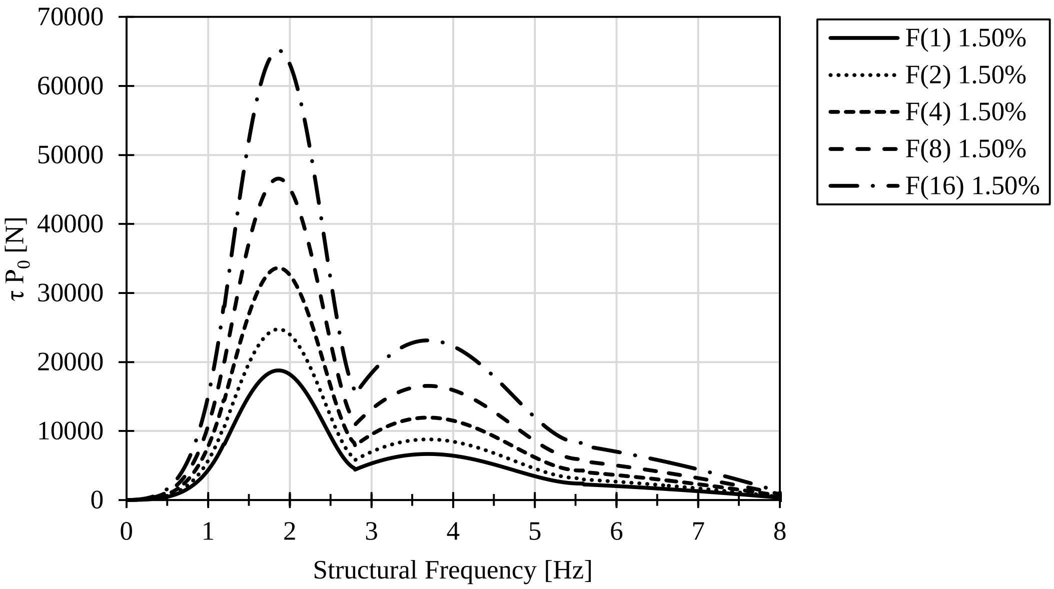


Figure 6.16 Design curve for UK concentrated load for concrete bridges with 1.5% structural damping ratio for single pedestrian and according to defined traffic classes.

The design curve for the concentrated load model is plotted in Figure 6.17 for steel bridges with 0.50% structural damping ratio and according to the defined traffic classes.

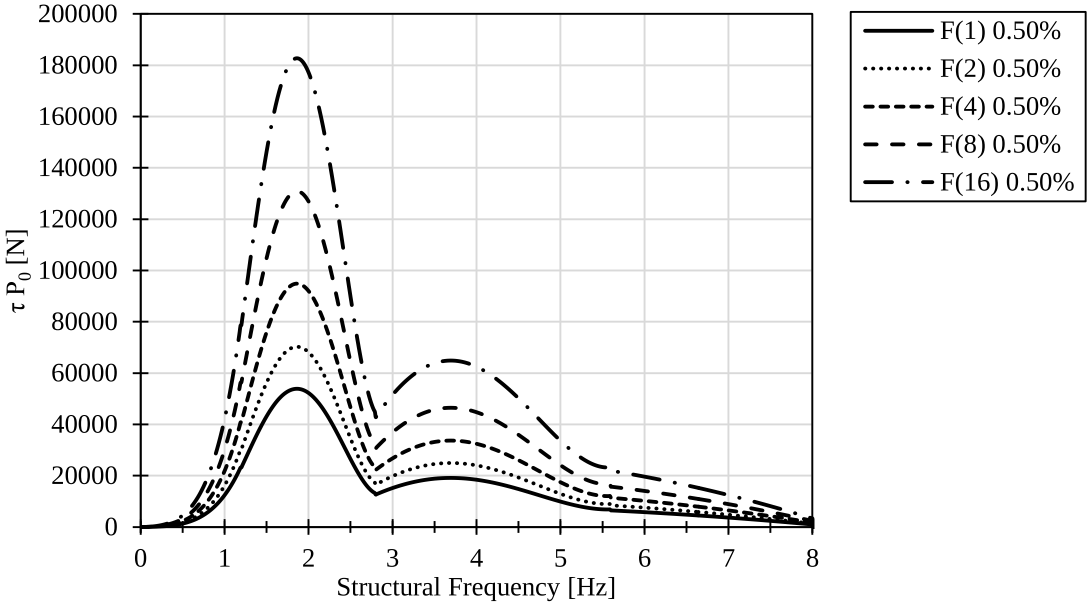


Figure 6.17 Design curve for UK concentrated load for steel bridges with 0.50% structural damping ratio for single pedestrian and according to defined traffic classes.

The design curve for the concentrated load model is plotted in Figure 6.18 for timber bridges with 1.0% structural damping ratio and according to the defined traffic classes.

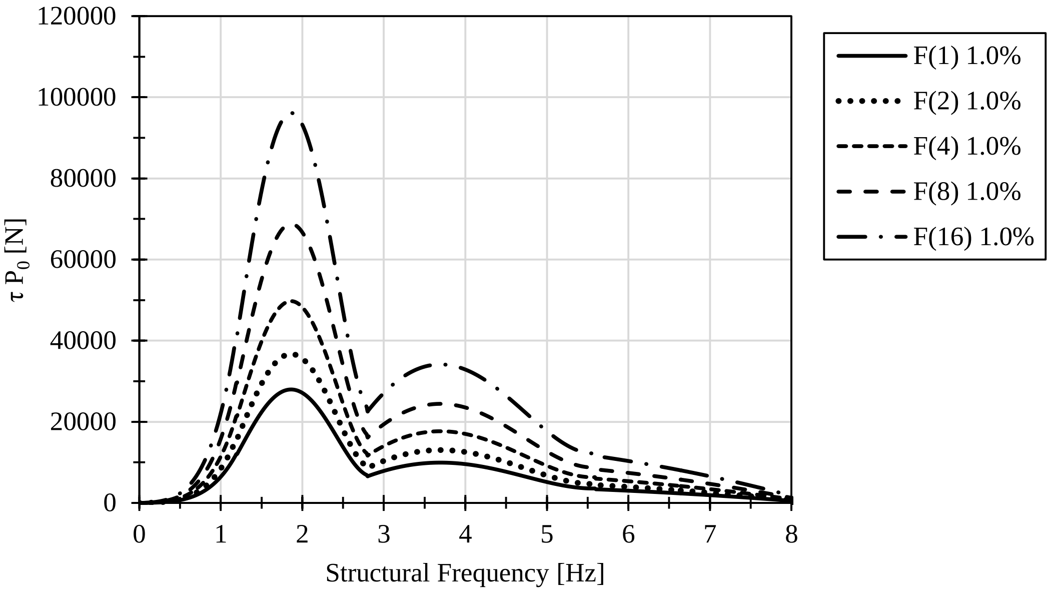


Figure 6.18 Design curve for UK concentrated load for timber bridges with 1.0% structural damping ratio for single pedestrian and according to defined traffic classes.

In Figure 6.19 is traffic class TC 3 plotted for the reference material and the structural materials for a comparison of magnitude between the different damping ratios.

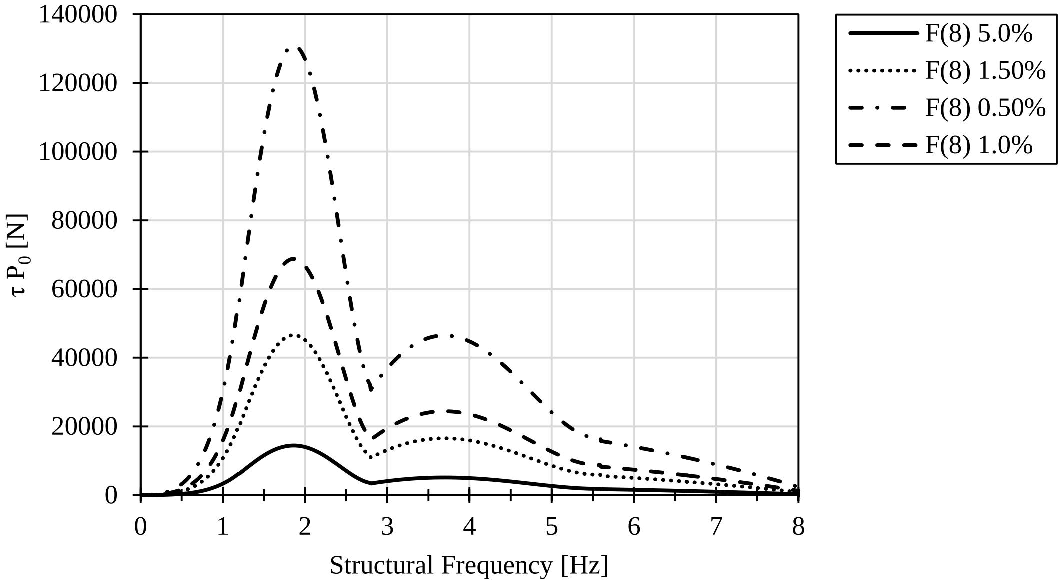


Figure 6.19 Comparison of normalized curve for varying damping ratios plotted for traffic class TC 3.

6.2.3 Uniformly distributed

The uniformly distributed load model in the UK National Annex to Eurocode is given in Equation (4-16). The load simulates a pedestrian stream with the number of pedestrians defined as pedestrian densities, d, for respective traffic class. The load model is normalized according to the derived method in section 5.6. The load amplitude p_0 is identified in Equation (6-6).

$$w = 1.8\left(\frac{F_0}{A}\right) k(f_v)\sqrt{\gamma N/\lambda}\,\sin(2\pi f_v t) \quad [N/m^2] \tag{6-6}$$

Where:

$$p_0 = 1.8\left(\frac{F_0}{A}\right) k(f_v)\sqrt{\gamma N/\lambda} \quad [N/m^2]$$

6.2.3.1 Relation between load amplitude and bridge geometry

The load model generates a load in N/m^2, in normalization curves the load should be given in N/m to be consistent among the models. It is done by multiplying the load amplitude with the bridge width as in Equation (6-7).

$$p_0 = 1.8\left(\frac{F_0}{A}\right) k(f_v)\sqrt{\gamma N/\lambda}\; b \quad [N/m] \tag{6-7}$$

The equation can be simplified into Equation (6-8) and a geometry factor ω can be extracted.

$$p_0 = 1.8\left(\frac{F_0}{bL}\right) k(f_v)\sqrt{\frac{\gamma\, dbL}{\lambda}} b = 1.8F_0 k(f_v)\sqrt{\frac{\gamma d}{\lambda}}\,\frac{\sqrt{bL}b}{bL} = \tag{6-8}$$

$$= 1.8F_0 k(f_v)\sqrt{\frac{\gamma d}{\lambda}}\ \omega = p_0\ \omega \quad [N/m]$$

Where:

$$p_0 = 1.8F_0 k(f_v)\sqrt{\frac{\gamma d}{\lambda}} \quad [N/m]$$

$$\omega = \sqrt{\frac{b}{L}} \quad [-]$$

The force amplitude p_0 is now given in N/m and independent on the bridge deck area. It means that the normalization curves can be plotted for and arbitrary bridge geometry independent on bridge deck area. For a specific area the normalized curves has to be multiplied with the geometry factor ω.

The relationship between load amplitude and geometry is shown in the figures below. In Figure 6.20 is the load amplitude in, N/m, from Equation (6-6) plotted against bridge width with constant length equal to 1 m for an arbitrary traffic class with arbitrary structural properties.

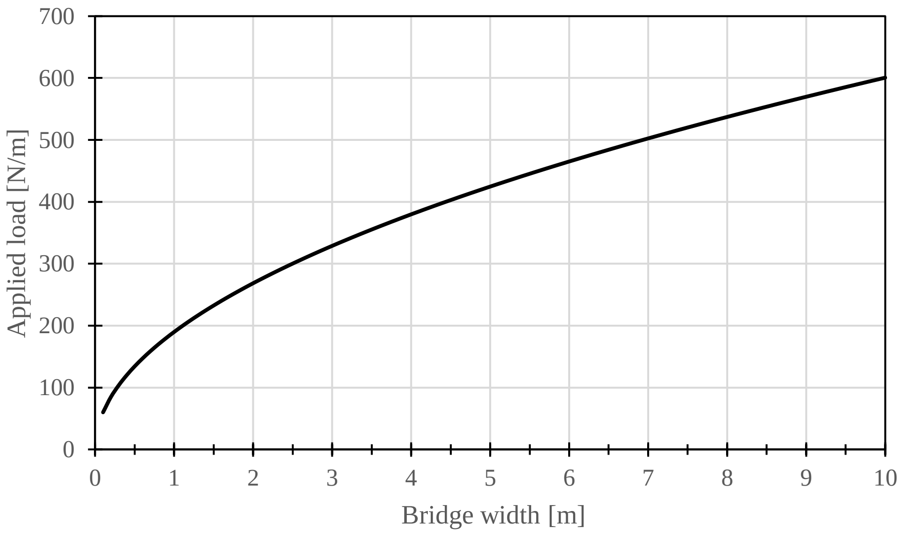


Figure 6.20 Plot of the relation between load amplitude and width of the bridge.

In Figure 6.21 the load amplitude, N/m, from Equation (6-6) is plotted against varying bridge length with constant bridge width equal to 1 m.

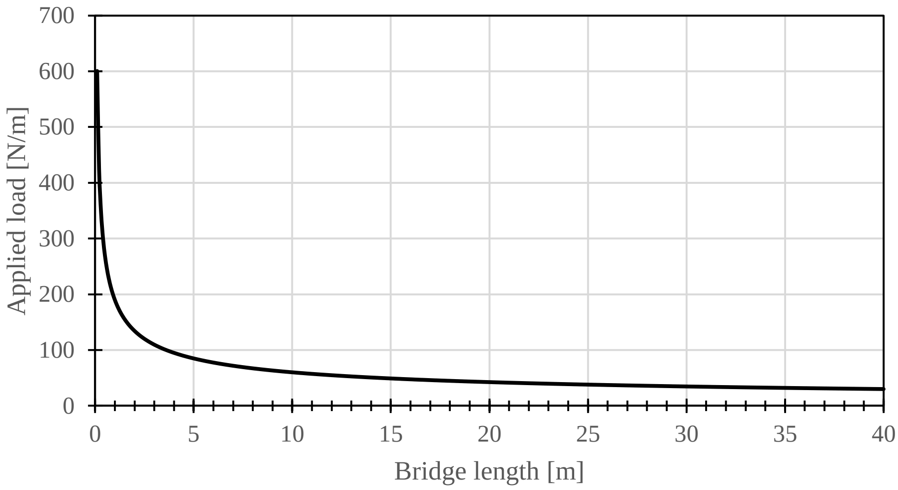


Figure 6.21 Plot of the relation between load amplitude and bridge length

6.2.3.2 Input data for normalization plots

Normalization curves are plotted in the figures below for a reference damping ratio of 5.0% and for construction materials concrete, steel and timber. Includes the input data used in the normalization plots for each material.

Table 6.5 Input data used in the normalization plots for UK NA uniformly distributed load model

Input data	Reference material	Concrete	Steel	Timber
Damping ratio, [%]	5.0	1.5	0.5	1.0
F_0 [N]	280	280	280	280
A [m^2]	1	1	1	1
γ [-]	0.2250	0.114	0.0429	0.07857
λ [-]	0.634	0.634	0.634	0.634
τ [-]	12.74	42.24	122.1	62.04

6.2.3.3 Normalization plots for uniformly distributed load model

The normalization curve for reference damping ratio 5.0% is plotted in Figure 6.22 with the four defined traffic classes.

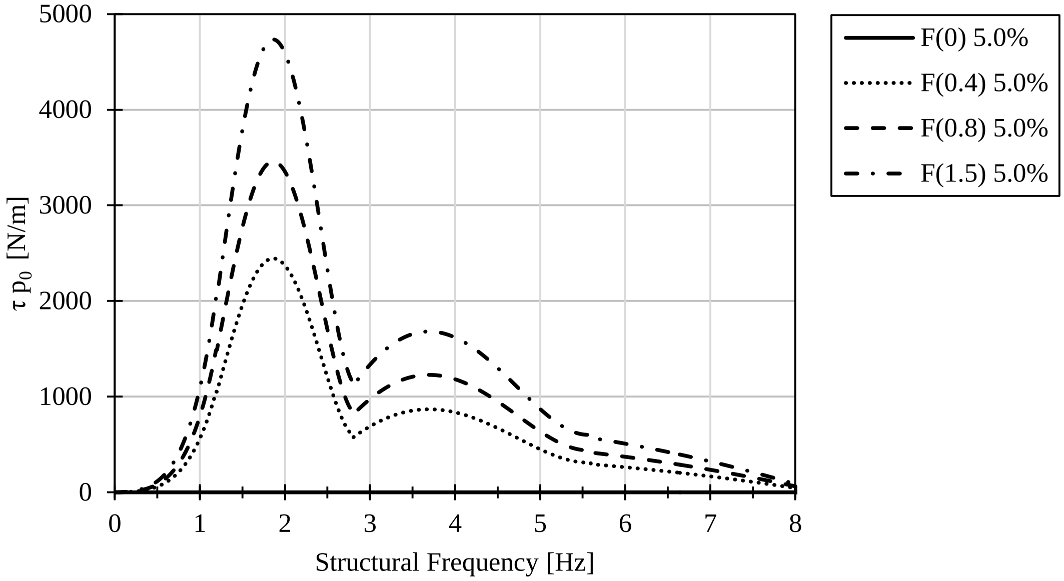


Figure 6.22 Normalization curve for UK NA uniformly distributed load model with reference damping ratio 5.0%.

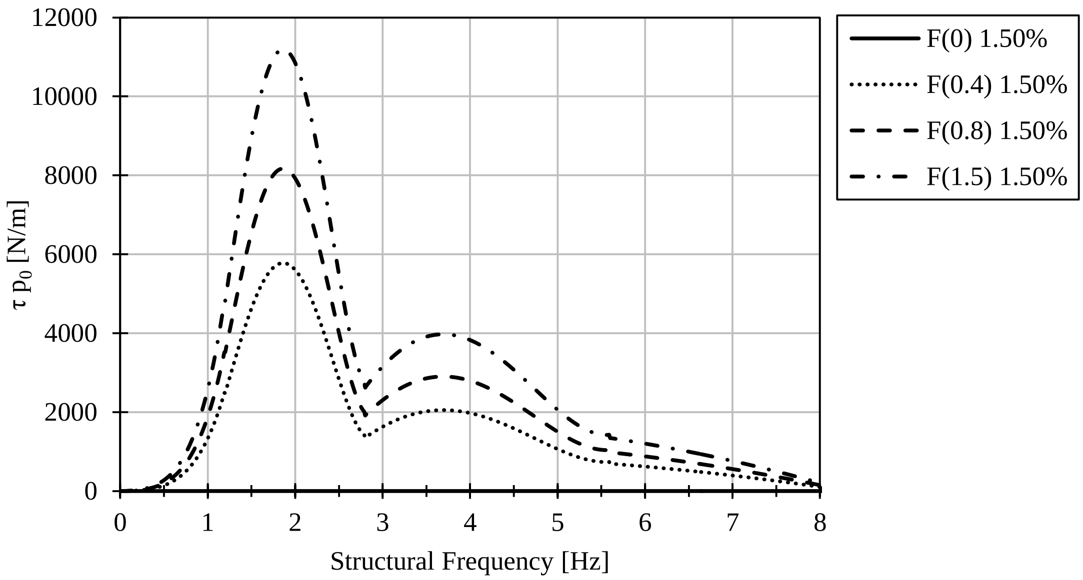


Figure 6.23 Normalization curve for UK NA uniformly distributed load model with concrete structural damping ratio 1.50%

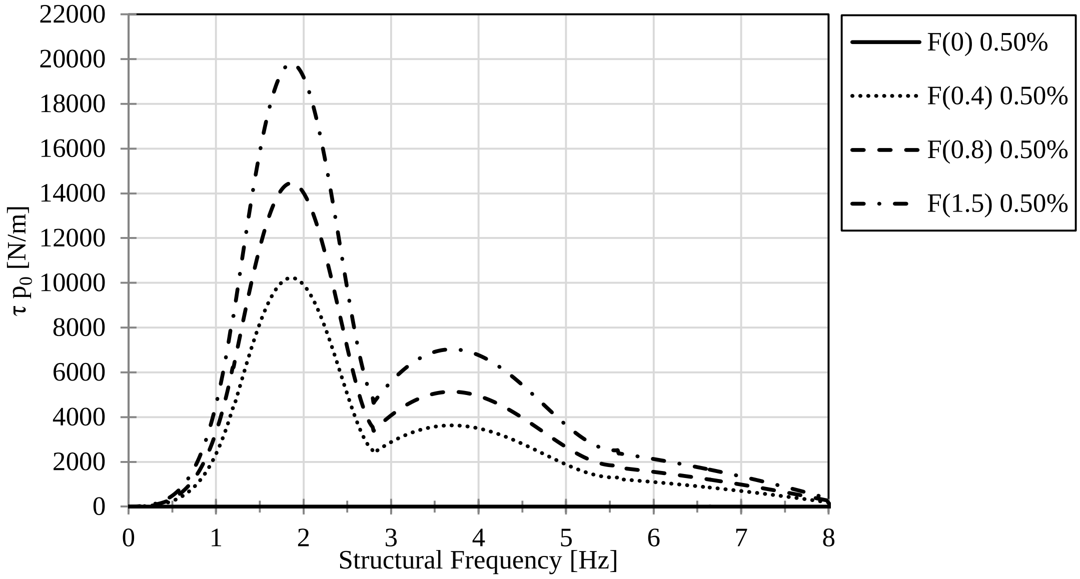


Figure 6.24 Normalization curve for UK NA uniformly distributed load model with steel structural damping ratio 0.50%

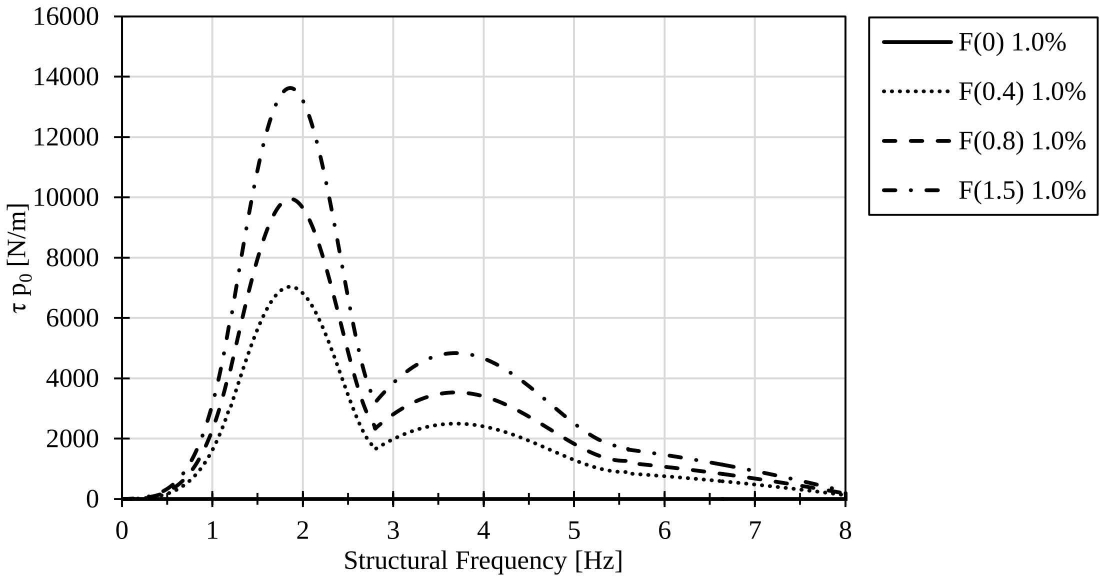


Figure 6.25 Normalization curve for UK NA uniformly distributed load model with timber structural damping ratio 1.0%

In Figure 6.26 are the normalization curves for varying damping ratios compared where all curves are plotted for traffic class TC 3.

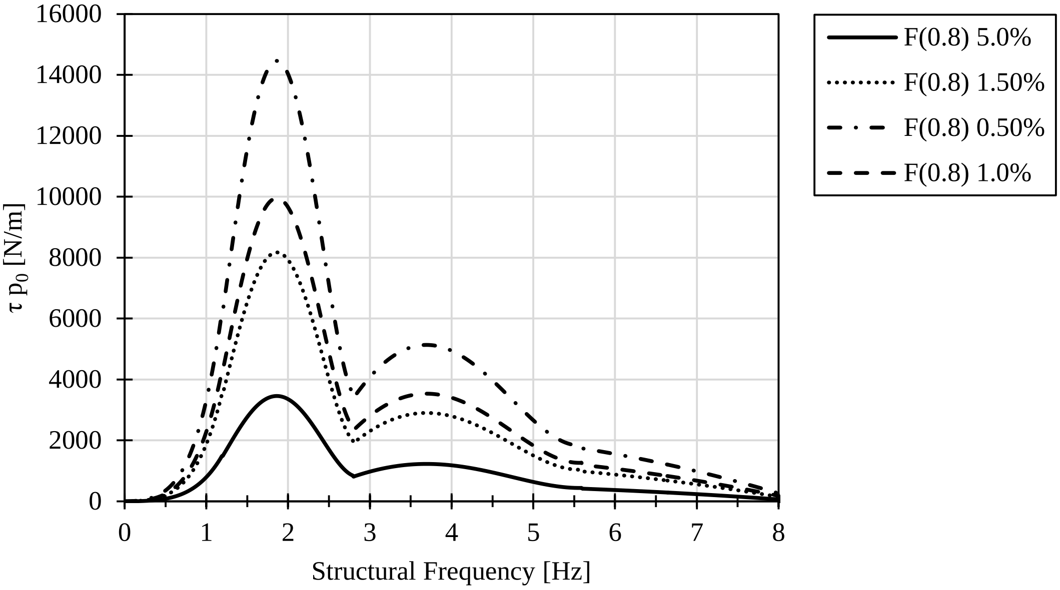


Figure 6.26 Comparison of normalization curves for varying damping ratios plotted with traffic class TC 3.

6.2.4 Comments and discussion

The UK National Annex to Eurocode has well developed load models for how a dynamic analysis should be performed. The standard includes both a concentrated and uniformly distributed load model for vertical vibrations. On the other hand no model is given for lateral vibrations which is in the guideline considered in a simplified way. For vertical vibrations both models has to be analysed in the design to find the highest possible acceleration.

The models are similar to each other with the same design coefficient $k(f_v)$ which can be seen as the governing factor of load amplitude variation over the frequencies. The coefficient is maximum between 1.8 and 2.0 Hz which generates the highest load amplitude for those frequencies. It is a reasonable assumption as it coincides with normal walking frequencies found in literature. The coefficient decreases into above 2 Hz to about 30% of its maximum to increase again at a second peak at 3.7 Hz. The second peak takes the second harmonic into account occurring at the double walking frequency. From 3.7 Hz the coefficient decreases to finally be zero at 8 Hz. The coefficient covers a wide range of frequencies, even higher than Eurocode suggest. Eurocode has set the 5 Hz structural frequency limit to determine which bridges that should be analysed dynamically. An adjustment could be suitable to limit the design coefficient at 5 Hz to be zero for all frequencies above the limit.

In the design process the standard request that both the concentrated and the uniform load model is analysed to find the maximum acceleration for the considered traffic class. This is unique among the standards and by doing this it covers the variety of bridge sizes. Smaller bridges will undergo the largest vibrations with the concentrated load as the pedestrian frequency will be high. Larger bridges will be designed according to the uniform load simulating pedestrian streams. It is reasonable and a logical way of analysing a bridge as the concentrated load model considers pedestrian groups which always is relevant. People will walk in groups no matter the size of the bridge but for a small bridge the stream of pedestrian cannot be too high as the bridge area is small.

The standard recommends four traffic classes categorised by the location of the bridge. The maximum pedestrian density given is 1.5 ped/m^2 which is a very crowded condition. Though the standard states that densities higher than 1.0 ped/m^2 are not required as the vertical action will decrease. Because a too crowded bridge will slow down the forward motion of the pedestrians. In this case the pedestrian density for traffic class TC 4 is not valid and a pedestrian density of 1.0 ped/m^2 should be applied, although TC 4 is valid for pedestrian groups.

The UK National Annex refers to Eurocode for suitable structural damping ratios. The damping ratios for concrete, reinforced concrete and steel are functions of the span length for bridges up to 20 m. Shorter bridges will have higher damping ratios and the minimum is applied for bridges longer than 20 m. Damping ratios for timber bridges are constant for all span lengths but higher for structures without mechanical joints. Damping ratios are hard to predict in the design stage as it is a result of the overall behaviour of the structure including all elements attached and material. It is always conservative to choose a lower damping ratio as the acceleration response will be higher for low damping ratios. In the load models the damping ratio is considered in the reduction factor γ. For concentrated loads the reduction factor is a function of the span length, shorter spans will generate higher reduction factor which leads to higher load amplitude. The uniform load model is independent on span length when choosing reduction factor. It is conservative to choose a high value of γ for both load models.

The normalized curve for concentrated load model is directly applicable on any bridge size as the load amplitude is independent on the bridge deck area. Although the reduction factor is dependent on the span length, which for an accurate analysis should be considered, it is en empirical factor and does not follow a simple relationship. The uniform load model on the other hand is highly dependent on bridge deck area. As seen Figure 6.20 and Figure 6.21of the normalized curves the bridge size is related to the load amplitude by the square root of bridge width divided by its length. Considering a bridge with normal proportions where the length does not exceed its width the maximum load amplitude is given for square bridges where the length is equal to the width. In other words the normalized curves will always be reduced as it is based on one square meter.

6.3 Sétra

Sétra propose two different load models, one concentrated load model and one uniformly distributed load model. The concentrated load model simulates a single pedestrian crossing the bridge and the distributed load model simulates pedestrian streams. Both models are applicable for vertical, longitudinal and lateral directions.

In this chapter normalized load models will be derived into normalized acceleration response curves for vertical and lateral directions.

6.3.1 Normalization of concentrated load model

Sétra propose a concentrated load moving along the span to simulate a single pedestrian crossing a bridge. The load is applicable in vertical, longitudinal and lateral directions. The load is based on Fourier series up to fourth order in vertical direction and first order for lateral and longitudinal directions. The standard suggests limiting the series to first order in practice.

6.3.1.1 Load history

The load history in vertical, longitudinal and lateral loading direction are plotted in Figure 6.27 for the first harmonic.

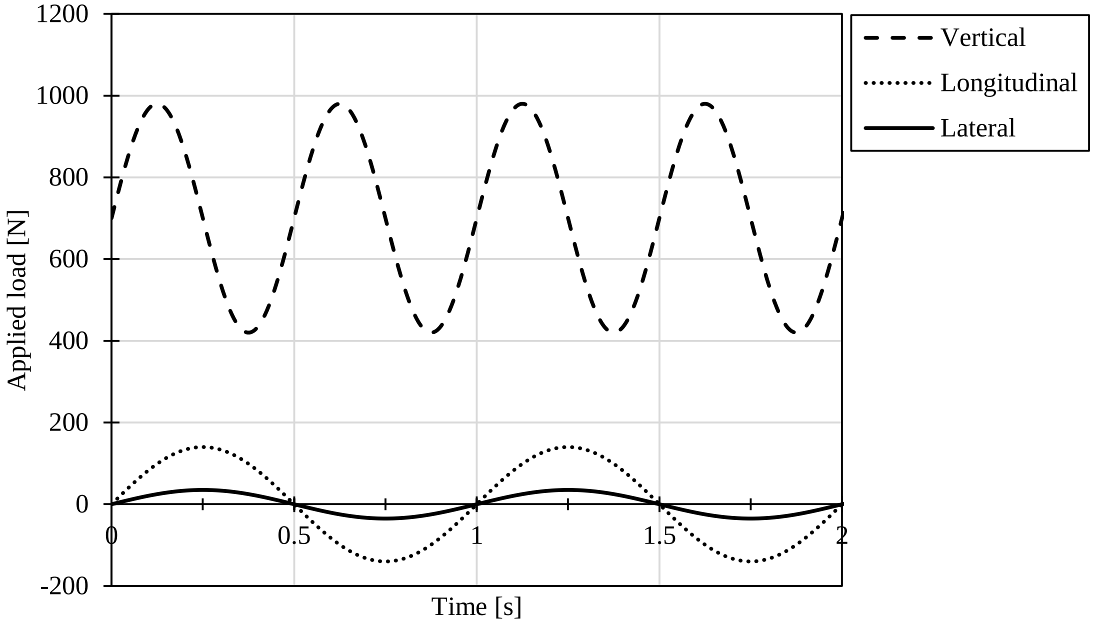


Figure 6.27 Load history for the concentrated load model in vertical, longitudinal and lateral loading directions.

The vertical load model oscillates around 700 N which is the static load of a single pedestrian. The lateral and longitudinal models oscillate at 0 N as no static force is applied in those directions.

6.3.1.2 Normalization and simplifications

The vertical and lateral load models explained in section 4.4.2 are given in Equation (6-9) and (6-10) where the load amplitude P_0 is identified. The load is simplified to be stationary at the middle of the span until steady state conditions are reached. The static load G_0 in the vertical model will not contribute to any vibrations and is neglected. The normalization factor τ is empirically derived in chapter 5 for the considered damping ratios. The model gives a load in N which is independent on the bridge geometry.

$$F_{ver}(t) = G_0 + G_1 \sin(2\pi f_m t) = G_1 \sin(2\pi f_m t) \quad [N] \tag{6-9}$$

$$F_{lat}(t) = 0.05 G_0 \sin\left(2\pi \frac{f_m}{2} t\right) \quad [N] \tag{6-10}$$

Where:

$P_0 = G_1$ In vertical direction

$P_0 = 0.05 G_0$ In lateral direction

6.3.1.3 Input data in normalization plots

The normalized acceleration response is calculated for 5.0% damping ratio and damping ratios recommended by Sétra for concrete, steel and timber structures. The damping ratios with corresponding normalization factors are given in Table 6.6.

Table 6.6 Considered damping ratios with corresponding normalization factors τ for concentrated load models.

Material	Reference	Concrete	Steel	Timber
Damping ratio [%]	5.0	1.3	0.40	1.0
Normalization factor τ [-]	20.01	76.41	228.5	98.80

6.3.1.4 Normalization plots in vertical direction

The normalized acceleration response in vertical direction of loading is calculated and plotted in Figure 6.28. The figure includes the curves for 5.0% damping ratio and the damping ratios according to concrete, steel and timber structures.

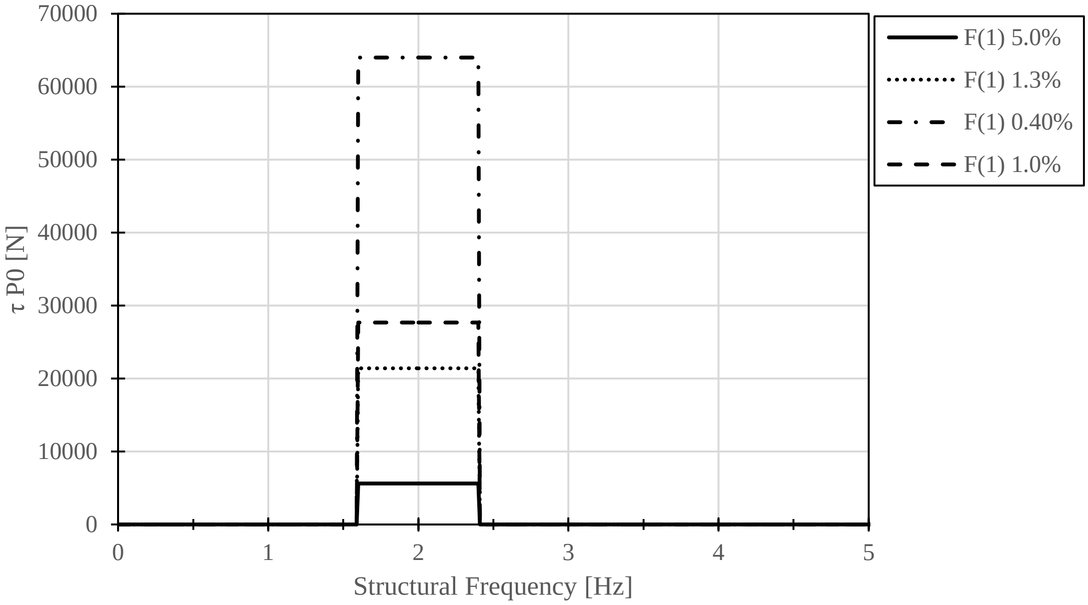


Figure 6.28 Normalized acceleration response for concentrated load model in vertical direction calculated for the four considered damping ratios.

6.3.1.5 Normalization plots in lateral direction

The normalized acceleration response in lateral direction of loading is calculated and plotted inFigure 6.29. The figure includes the curves for 5.0% damping ratio and the damping ratios according to concrete, steel and timber structures.

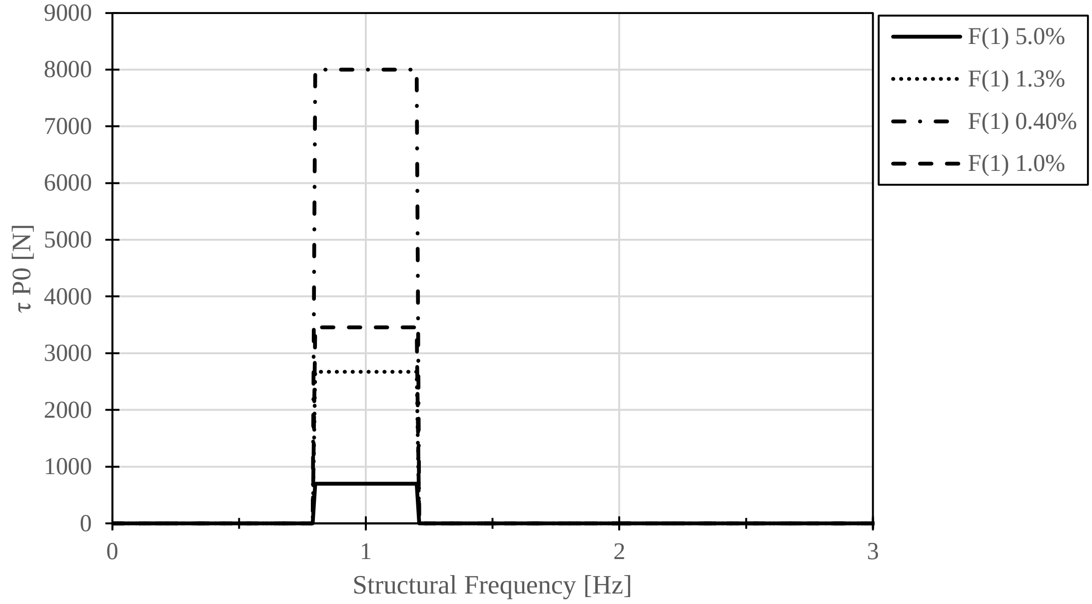


Figure 6.29 Normalized acceleration response for the concentrated load model in lateral direction plotted for four considered damping ratios.

6.3.2 Normalization of distributed load

The methodology presented in Sétra is based on footbridge classification as a function of the expected level of traffic. The load model proposed in Sétra is a distributed harmonic load according to equation (6-11) representing different pedestrian densities as presented in section: 4.4.8. The load is to be applied until steady-state conditions are reached with frequencies corresponding to natural frequencies of the regarded bridge.

$$F_v(t) = d\; P \cos(2\pi f_v t)\, N_{eq}\psi \quad [N/m^2] \tag{6-11}$$

The proposed pedestrian densities and equivalent number of pedestrians according to Sétra that are to be used in the normalization are presented in Table 6.7

Table 6.7 Proposed pedestrian densities and corresponding equivalent number of pedestrians according to Sétra.

Density, $d[pedestrians/m^2]$	Equivalent number of pedestrians, N_{eq}
1.0	$N_{eq} = 1.85\sqrt{1/n}$
0.8	$N_{eq} = 10{,}8\,\sqrt{\xi/n}$
0.5	$N_{eq} = 10{,}8\,\sqrt{\xi/n}$

The load model is normalized according to the derivations in chapter 5. The load model is given in Equation (6-12) where the load amplitude p0 is identified.

$$F_v(t) = d\ P \cos(2\pi f_v t)\ N_{eq}\psi \quad [N/m^2] \tag{6-12}$$

Where:

$$p_0 = dPN_{eq}\psi$$

In Table 6.8 values for the normalization factor τ are given for different materials.

Table 6.8 Values for normalization factor τ for distributed load for different materials with corresponding damping.

Material	Reference	Concrete	Steel	Timber
τ- uniform	12.74	48.45	148.6	62.04
Damping ratio [%]	5	1.3	0.4	1.0

6.3.2.1 Relationship between load amplitude and bridge geometry

The amplitude p_0 is dependent on factors regarding equivalent number of pedestrians, relevant step frequencies for walking and the proposed static load for a pedestrian.

The load amplitude p_0 generates a load in N/m^2. In normalization the load should be applied in N/m which is done by multiplying the load model with the bridge width according to Equation (6-13).

$$p_0 = dPN_{eq}\psi\ b \quad [N/m] \tag{6-13}$$

The equation can be simplified by inserting the equivalent number of pedestrians for respective traffic class in Equation (6-14) and (6-15). The factors including bridge geometry can be extracted to a geometry factor ω which will make the load amplitude independent of bridge geometry.

$$p_0 = d\ P\psi\ 10.8\sqrt{\frac{\xi}{d\ bL}}\ b = d\ P\ \psi\ 10.8\sqrt{\frac{\xi}{d}}\sqrt{\frac{b}{L}} = p_0\omega \tag{6-14}$$

For Class II and III

$$p_0 = d\ P\ \psi 1.85\sqrt{\frac{1}{d\ bL}}\ b = d\ P\ \psi\ 1.85\sqrt{\frac{1}{d}}\sqrt{\frac{b}{L}} = p_0\omega \tag{6-15}$$

For Class I

Where:

$$p_0 = d\, P\, \psi\, 10.8 \sqrt{\frac{\xi}{d}} \quad [N/m]$$

For Class II and III

$$p_0 = d\, P\, \psi\, 1.85 \sqrt{\frac{1}{d}} \quad [N/m]$$

For Class I

$$\omega = \sqrt{\frac{b}{L}} \quad [-]$$

By introducing the geometry factor ω the normalization curves can be plotted independent on bridge geometry. To obtain the curves for an arbitrary geometry the curve has to be multiplied with the geometry factor ω.

The relationship between load amplitude and the geometry factor can be seen in figures below. In Figure 6.30 is the load amplitude plotted for a bridge with varying bridge width and constant length equal to 1 m.

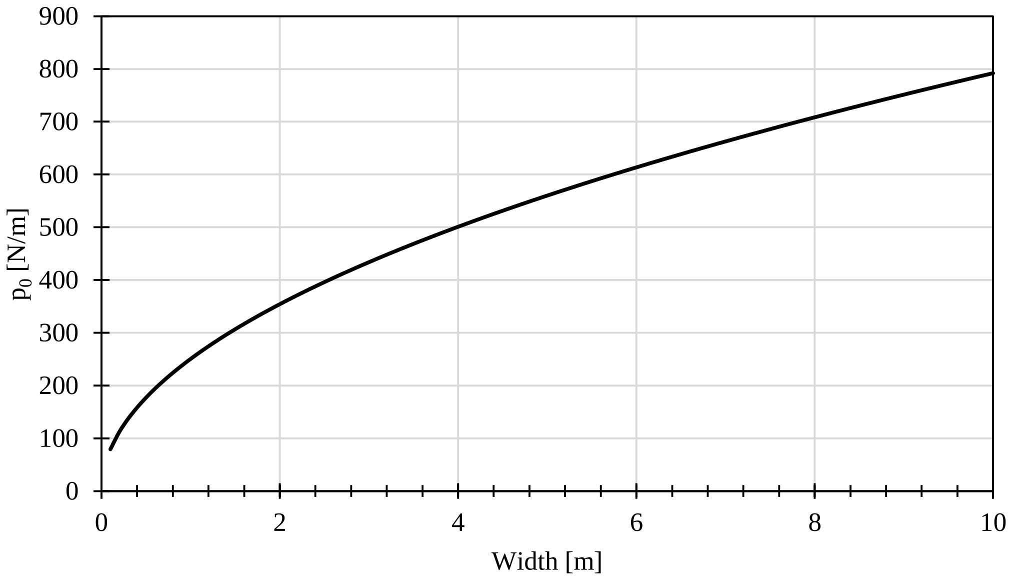


Figure 6.30 Load amplitude for varying width, constant length 1 m.

In Figure 6.31 is the load amplitude plotted for a bridge with varying bridge length and constant width equal to 1 m.

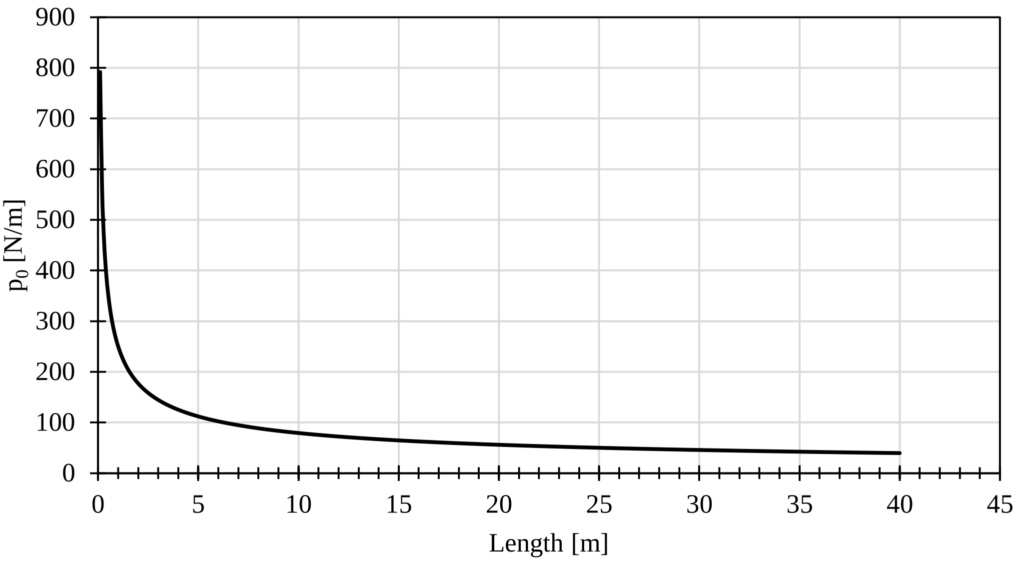


Figure 6.31 Load amplitude for varying length, constant width 1 m.

6.3.2.2 Normalized response according to Sétra in vertical direction

Input variables for the different proposed densities in vertical direction are given in Table 6.9.

Table 6.9 Input variables for proposed densities in vertical direction according to Sétra.

Input variables	Class I	Class II	Class III
Pedestrian force [N]	280	280	280
Density [pedestrians/m^2]	1.0	0.8	0.5
n_{eq} [-]	$1.85\sqrt{1/n}$	$10.8\sqrt{\xi/n}$	$10.8\sqrt{\xi/n}$

The normalized curves for the proposed pedestrian densities in Sétra with a structural damping of 5 % are shown in Figure 6.32 calculated for a 1 m wide bridge.

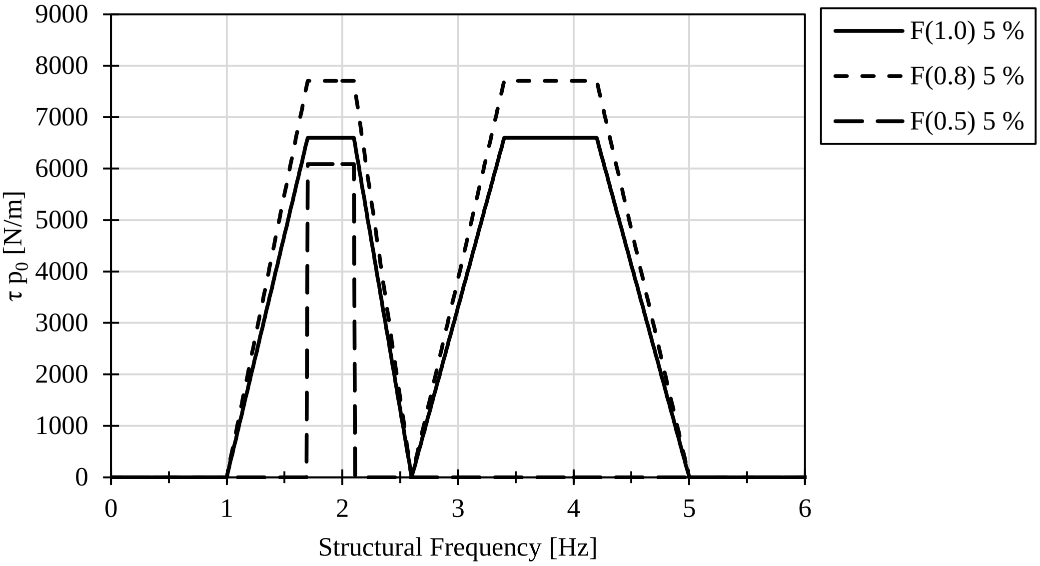


Figure 6.32 Normalized curves in vertical direction for proposed pedestrian densities according to Sétra for 5.0 % damping, established for a 1m² bridge

The normalized curves for concrete, steel and timber are shown in Figure 6.33 to Figure 6.35 for the different proposed densities. The curves are calculated for a 1 m² bridge.

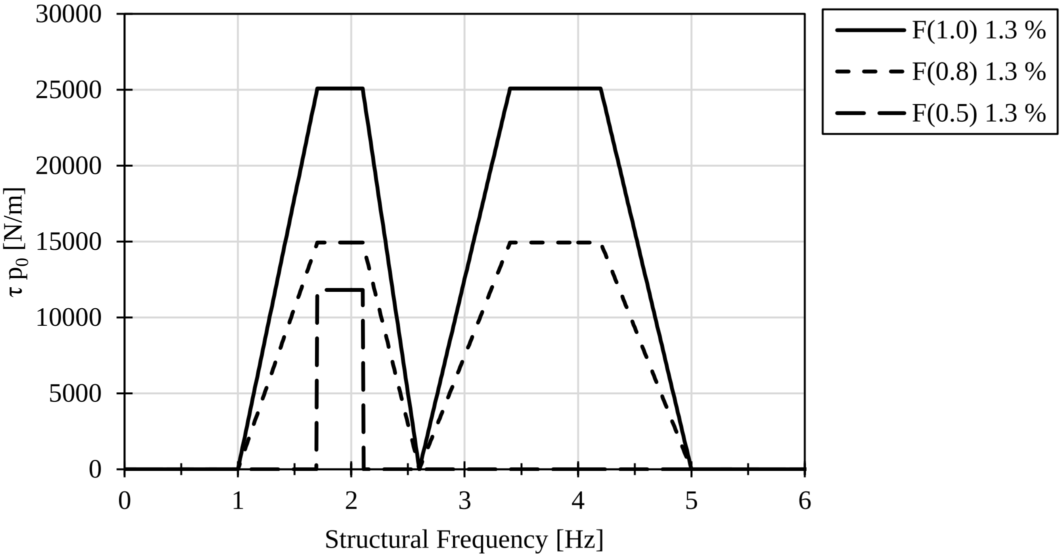


Figure 6.33 Normalized curves according to bridge class for concrete with 1.3 % damping, established for a 1m² bridge.

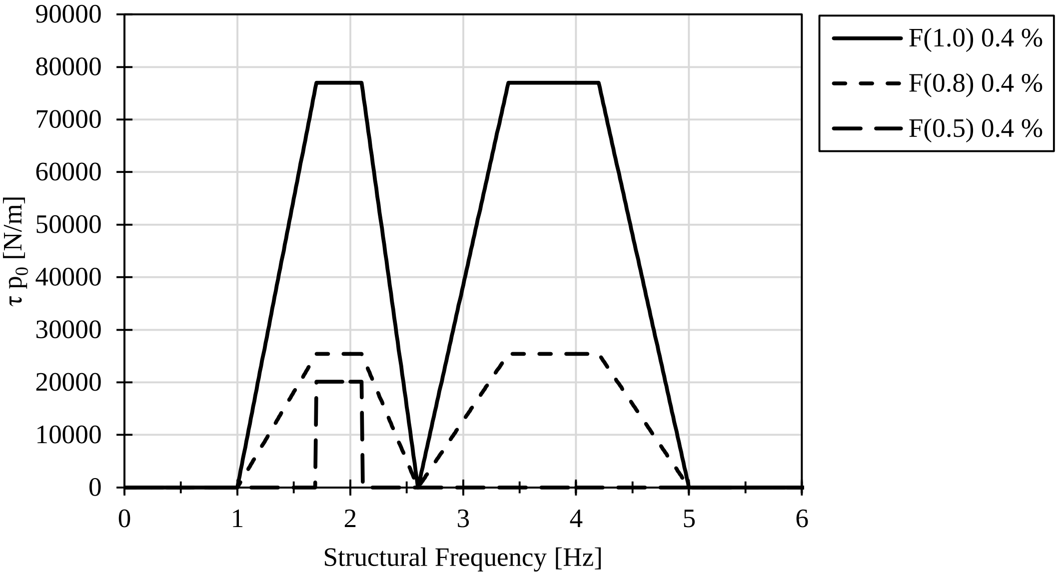


Figure 6.34 Normalized curves according to bridge class for steel with 0.40% damping, established for a 1m^2 bridge.

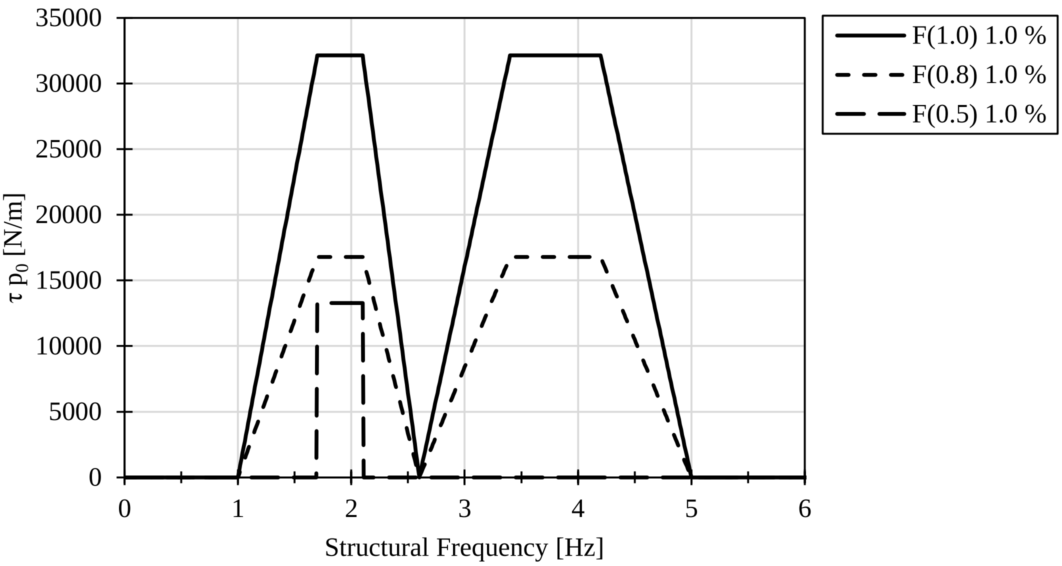


Figure 6.35 Normalized curves according to bridge class for timber with 1.0% damping, established for a 1m^2 bridge

In Figure 7.36 the normalized curves for a density of 0.8 ped/m^2 are shown for different materials presented in Table 6.8.

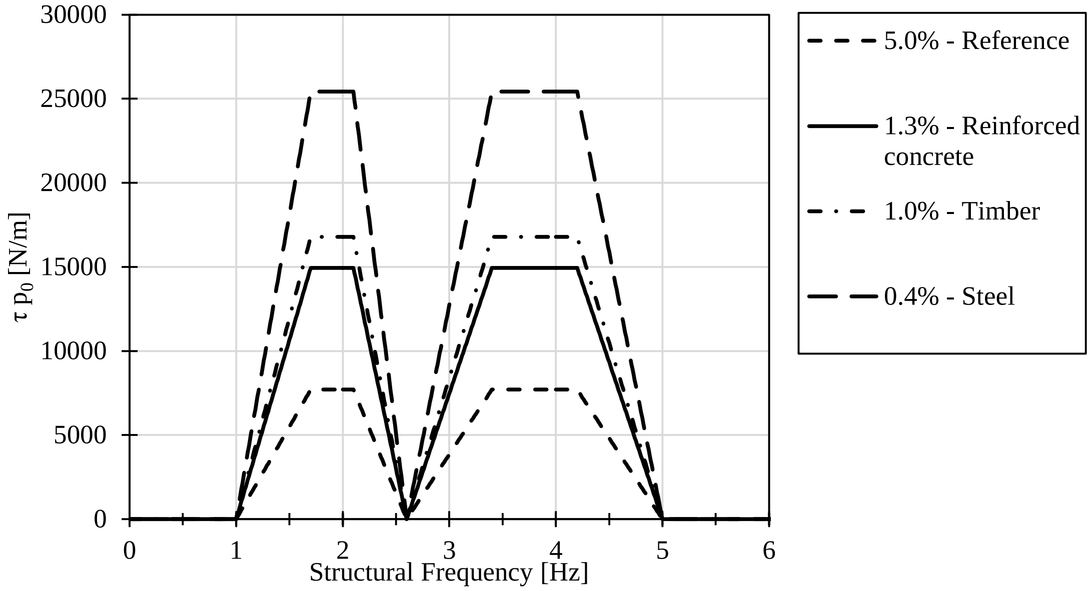


Figure 6.36 Normalized curves for pedestrian density 0.8 ped/m^2 for different materials.

6.3.2.3 Normalized response according to Sétra in lateral direction

The same load model is used in lateral direction but with different load amplitude. Input variables for the different proposed densities are given in Table 6.10.

Table 6.10 Input variables for traffic classes in lateral direction

Bridge class	Class I	Class II	Class III
Pedestrian force [N]	35	35	35
Density [pedestrians/m^2]	1.0	0.8	0.5
n_{eq} [-]	$1.85\sqrt{1/n}$	$10.8\sqrt{\xi/n}$	$10.8\sqrt{\xi/n}$

In Figure 6.37 the normalized curves for different proposed pedestrian densities with 5 % structural damping in lateral direction can be seen.

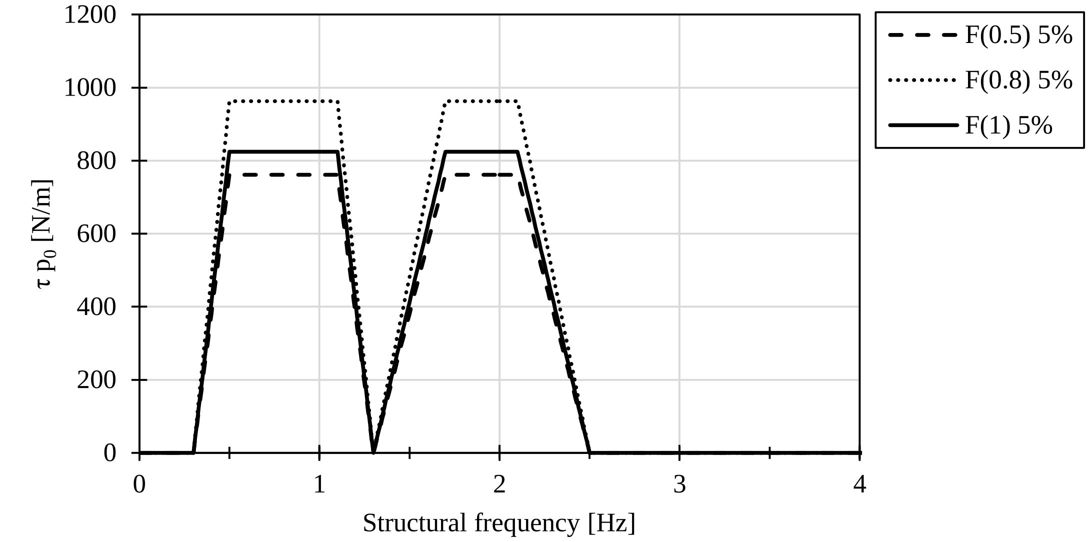


Figure 6.37 Normalized curves for proposed pedestrian densities in lateral direction with 5 % damping.

In Figure 6.38 the normalized curves for different proposed densities for concrete can be seen.

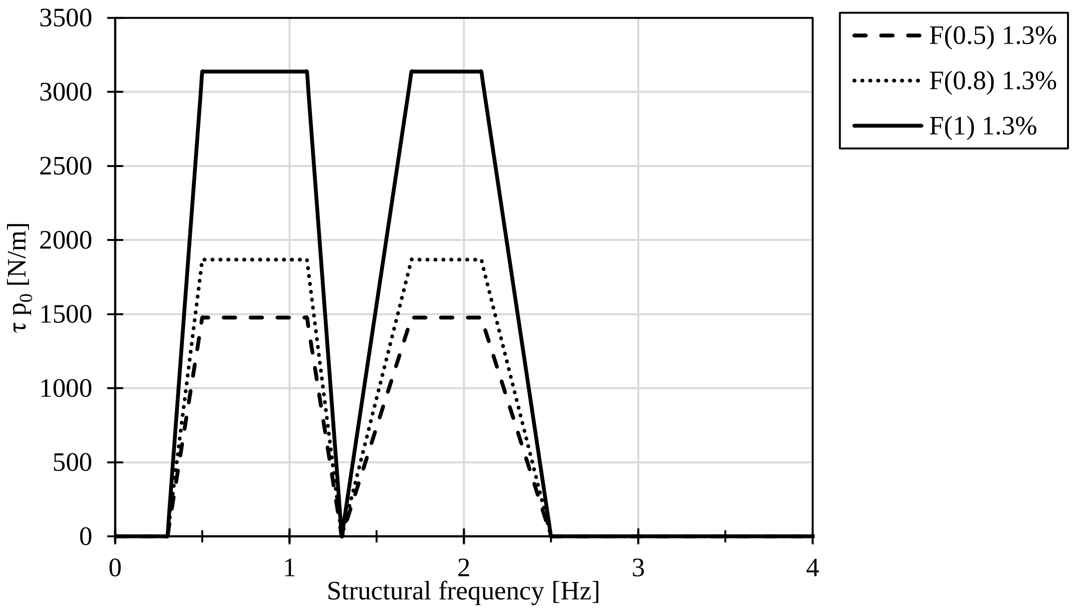


Figure 6.38 Normalized curves for proposed densities in lateral direction for concrete with 1.3 % damping.

In Figure 6.39 normalized curves for the different proposed densities in lateral direction for steel with 0.4 % damping can be seen.

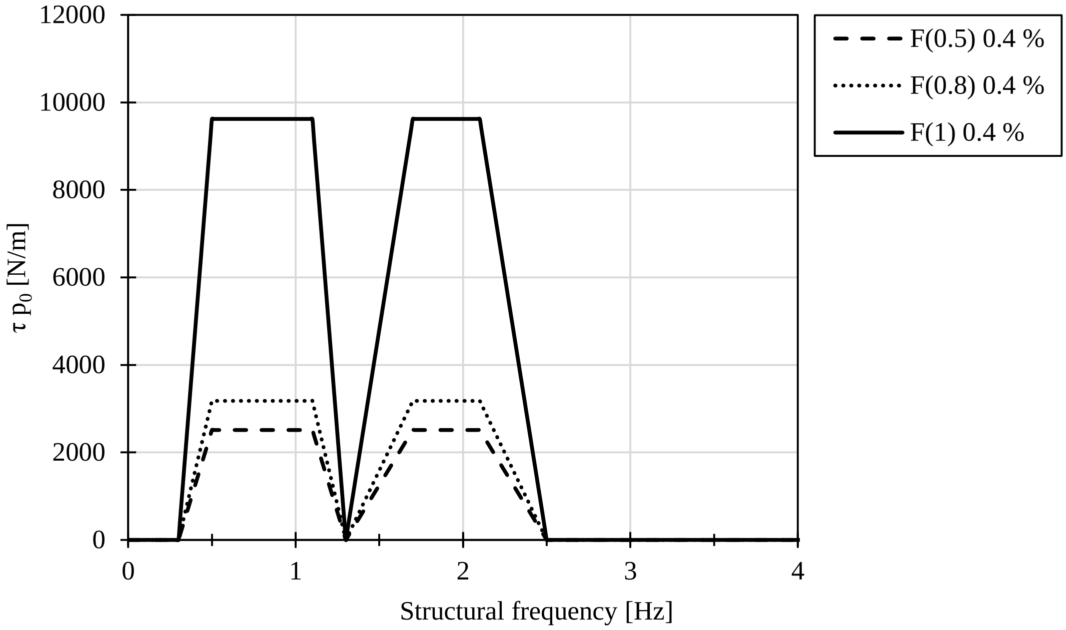


Figure 6.39 Normalized curves for proposed densities in lateral direction for steel with 0.4 % damping.

In Figure 6.40 normalized curves for the different proposed densities in lateral direction for timber with 1.0 % damping cam be seen.

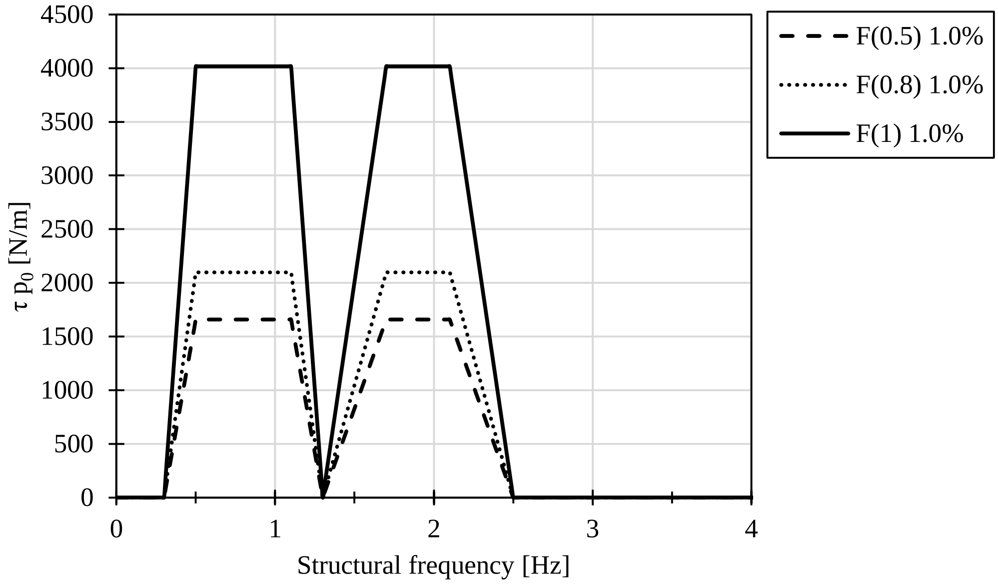


Figure 6.40 Normalized curves for proposed densities in lateral direction for timber with 1.0 % damping

In Figure 6.41 the normalized curves for a density of $0.8\ pedestrians/m^2$ are shown for different materials and corresponding damping.

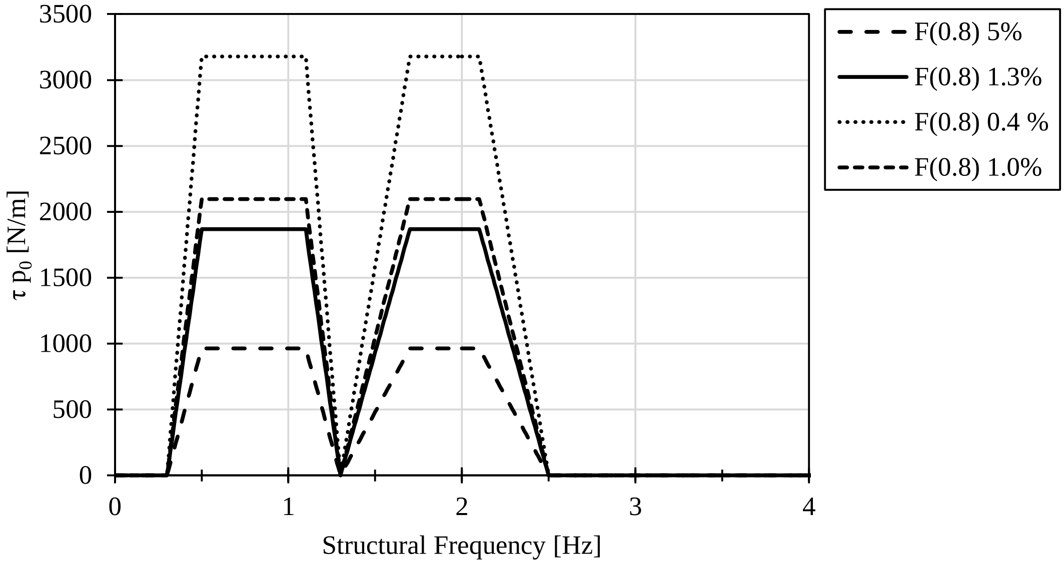


Figure 6.41 Normalized curves for 0.8 pedestrians/m²for different materials.

6.3.3 Comments and discussion

In this section comments are made about the acceleration response obtained with the Sétra standard, what factors it depend on and what conclusions that can be draw from it.

6.3.3.1 Range and weighing of step frequencies

Sétra considers frequencies from 0 to 5 Hz for vertical direction and 0 to 2.5 Hz for lateral. These ranges seem reasonable because normal step frequencies for pedestrians stated in the literature coincide.

Furthermore Sétra weighs the step frequencies between 1.7 and 2.1 Hz as the most relevant step frequencies in vertical direction which can be seen in the normalized curves giving the largest response. These values are regarded as the most likely to occur which is taken into account by the modification factor ψ. The second harmony is regarded to give the same response as the first which can be regarded as conservative when other standards weighs the second harmonies lower than the first.

In lateral direction the same load model is used but with a lower load amplitude meaning that it will give rise to smaller accelerations. The interval of step frequencies giving the biggest response is estimated to be 0.5 to 1.1 Hz. For lateral vibrations the acceleration response from the second harmony is also valued to be as high as for the first harmony.

6.3.3.2 Empirical factors, soft aspects

The equivalent number of pedestrians, n_{eq}, regards the synchronization within groups for different densities. For the higher density of 1.0 ped/m^2 the structural damping is not included as it is for the lower densities.

This affects the response so that for higher levels of damping the density 0.8 for Class II results in higher acceleration response than for Class I with a higher pedestrian density. This can be seen by comparing Figure 6.32 and Figure 6.33 Where Class II gives the highest response for 5 % damping and Class I results in the highest for 1.3 % damping for concrete.

6.3.3.3 Max/min values

It can be seen that the level of damping proposed for the different materials affect the acceleration response. For high levels of damping, as can be seen for the reference damping 5%, the pedestrian density 0.8 pedestrians/m^2 results in the highest response because if its n_{eq} number that takes into account structural damping. The difference between the highest response for density 1.0 for Class I and the lowest for density 0.5 for Class III increases as the damping decreases.

For steel the attained τp_0-value is 3.85 times higher for Class I than for Class III. This relationship decreases to 1.3 for 5% damping.

6.4 SYNPEX

SYNPEX propose several methods to determine the maximum acceleration in a footbridge induced by pedestrians. In this chapter two types of loading are considered, concentrated loads and uniformly distributed. Two concentrated loads are proposed to model the vertical and lateral action based on Fourier series which should be applied as moving over the span. A uniform load model is suggested to simulate pedestrian streams which can be adapted to consider vertical, lateral and longitudinal loading. The models are complemented with acceleration limits and traffic classes for pedestrian streams.

In this chapter the load models are briefly presented and normalized according to the methods presented in section 5.6. For a complete presentation of the models see section 4.6.

The guideline suggest damping ratios for concrete, steel and timber bridges which are in common for concentrated and uniform load models. The chosen ratios for normalization are given in and thoroughly described in section 4.6.4.

Table 6.11 Structural damping ratios for used in normalization for concrete, steel and timber recommended by SYNPEX.

Material	Reference	Concrete	Steel	Timber
Damping ratio, ξ [%]	5.0	1.3	0.40	3.0

6.4.1 Concentrated load

SYNPEX propose two concentrated loads defined by a Fourier series to simulate a single pedestrian walking over the bridge, Equation (4-63) and (4-64).

6.4.1.1 Fourier coefficients, phase angels and load history

The vertical Fourier coefficients α are calculated by the equations in Table 4.35 and plotted in Figure 6.42 for the suggested walking frequency interval. It can be seen in the plot that α_2 is highest and α_3 is lowest for all considered frequencies.

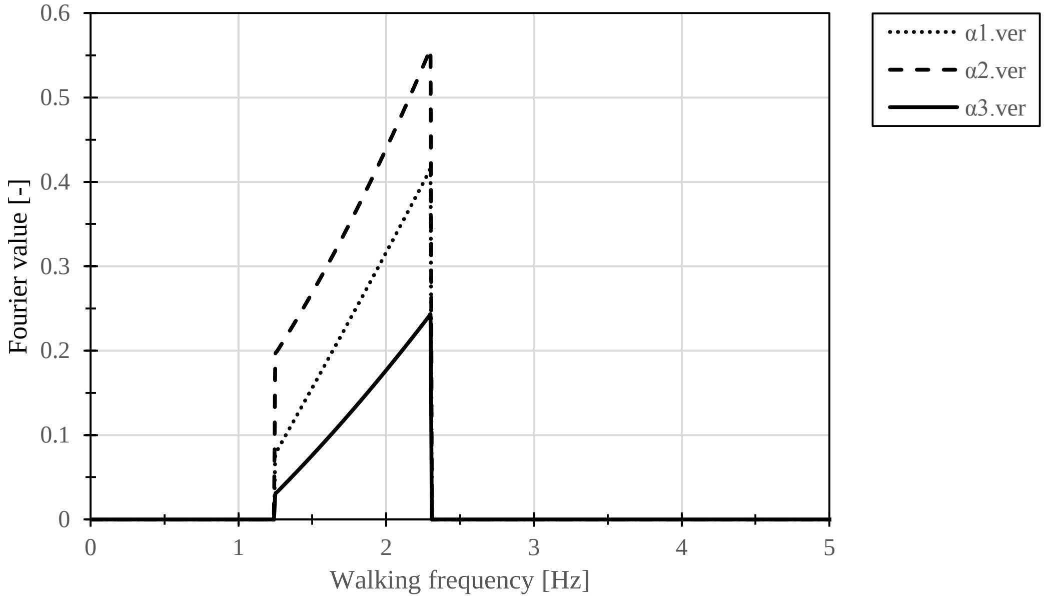


Figure 6.42 Plot of vertical Fourier coefficient, α, and walking frequency according to SYNPEX.

The phase shift for vertical model is calculated by the equations in Table 4.36 as functions of the walking frequency. The lateral phase shifts are independent on walking frequency and given in Table 4.36. Note that the phase shifts are given in degrees angle.

The phase angles for vertical load model are plotted in for the suggested walking frequencies between 1.25 and 2.30 Hz.

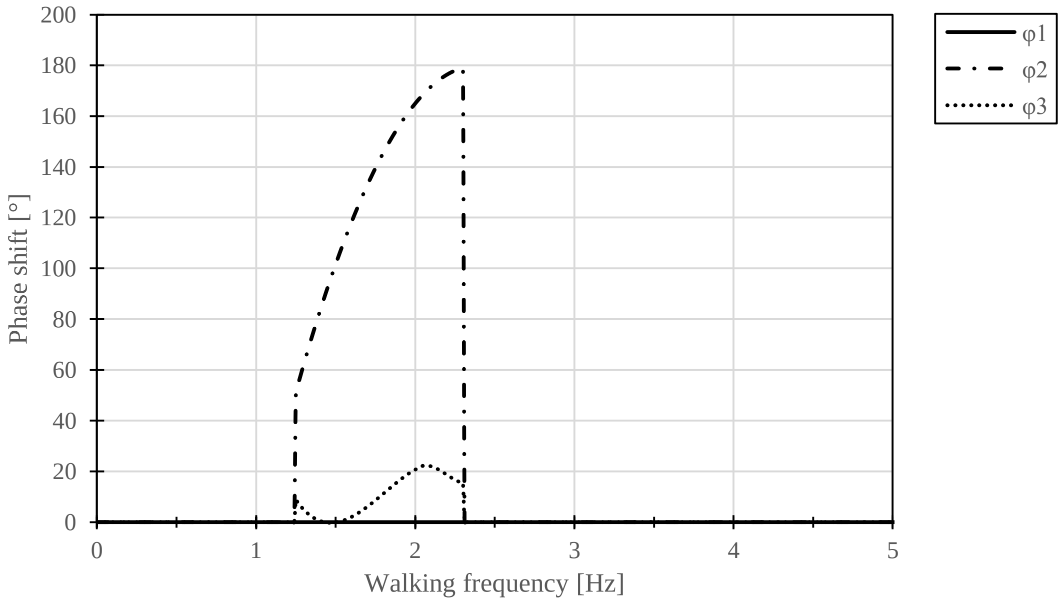


Figure 6.43 Phase shift angels for vertical load model plotted for first, second and third harmonics.

An example of the loading history for vertical action is given in Figure 6.44. The load history is plotted by Equation (4-63) for 2 Hz walking frequency for the first three Fourier sums.

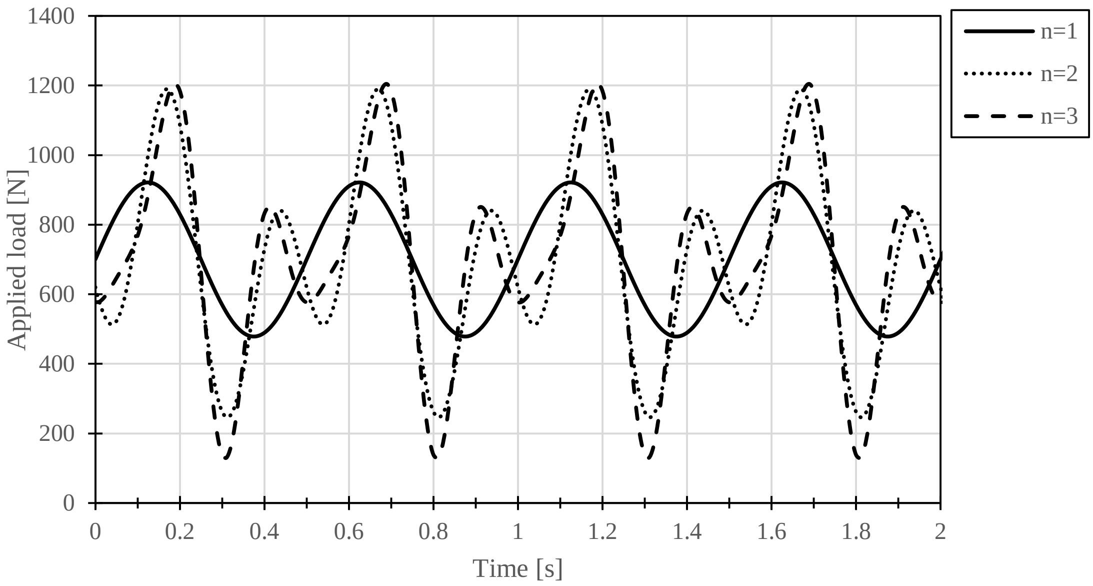


Figure 6.44 Load history of the vertical load model for 1, 2 and 3 harmonics including phase shits.

The load history shows how the load acts over time for first, second and third harmonic. The load oscillates at 700 N which is the static load of one pedestrian. It is clearly seen that when the second harmonic is considered the load amplitude increases well above the curve for the first harmonic.

An example of the load history of the lateral load model is plotted by Equation (4-64) in Figure 6.45. The load history is plotted for walking frequency 2 Hz for one, two and three Fourier sums.

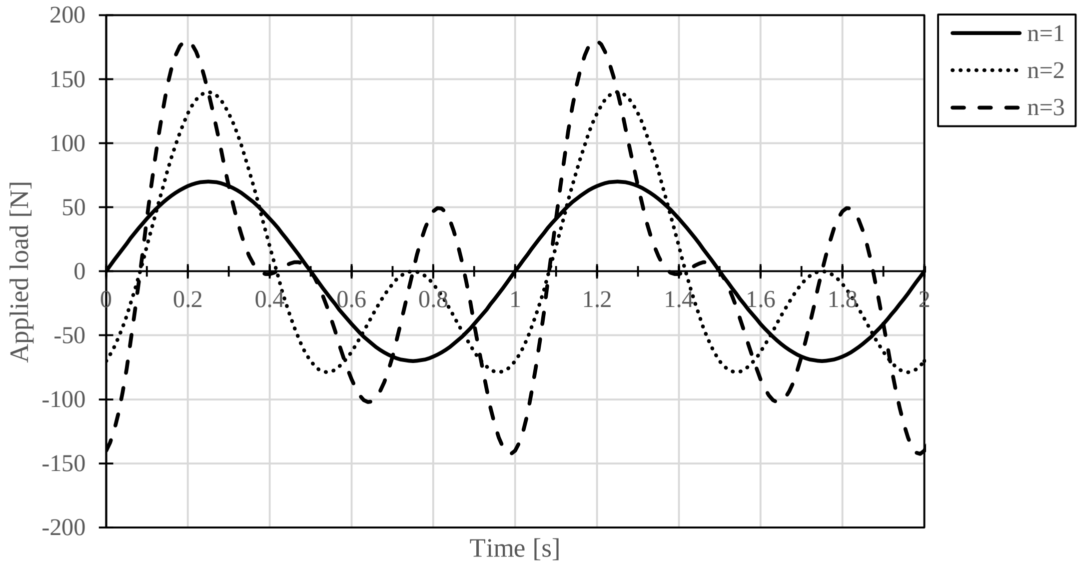


Figure 6.45. Load history of the lateral load model for 1, 2, and 3 harmonics.

In the load history is can be seen how the number of harmonics considered affects the load amplitude. The highest amplitude is given when all three harmonics are considered.

6.4.1.2 Damping and normalization factor τ

The normalization is based on damping ratios that are referred in SYNPEX as average damping ratios. In Table 6.12 the chosen damping ratios are presented with respective normalization factor τ used for concentrated loads.

Table 6.12 Damping ratios and normalization factors for the reference bridge and construction materials concrete, steel and timber.

Material	Reference	Concrete	Steel	Timber
Damping ratio, ξ [%]	5.0	1.3	0.40	3.0
Norm factor, τ [-]	20.01	76.41	228.5	33.32

6.4.1.3 Normalization and simplifications

The concentrated load models proposed by SYNPEX can be normalized based on the analysis done in section 5.6. Two simplifications are made, firstly only to normalize the load models for the first harmonic. This will lead to lower load amplitude than considering two harmonics which can be seen in Figure 6.44 and Figure 6.45. Secondly the loads should be applied as a moving load over the span with a constant velocity. The load models are simplified to be considered as stationary at the middle of the bridge. The loads will be applied until steady state conditions are reached which will generate larger amplitude than from a moving load. The vertical load model as stated in the guideline is given in Equation (6-16) for the first harmonic.

$$F_{vert}(t) = G[1 + \alpha_1 \sin(2\pi f_s t)] \tag{6-16}$$

The vertical load can be divided into two parts, static and dynamic. The static part will not contribute to any vibrations in the bridge and can therefore be neglected. The load is divided according to Equation (6-17) where the terms multiplied with sinus are dynamic.

$$F_{vert}(t) = G[1 + \alpha_1 \sin(2\pi f_s t)] = G + G\alpha_1 \sin(2\pi f_s t) \tag{6-17}$$

As the static load will not contribute to the acceleration response it is neglected and the final vertical load model is given in Equation (6-18) where the load amplitude P_0 is identified.

$$F_{vert}(t) = G\alpha_1 \sin(2\pi f_s t) \ \ [N] \tag{6-18}$$

Where:

$$P_0 = G\alpha_1$$

The lateral load model does not include a static load as the vertical and will be normalized according to Equation (6-19) where the load amplitude P_0 is identified for the first harmonic.

$$F_{lat}(t) = G\alpha_{1.lat}\sin(\pi f_s t) \quad [N] \tag{6-19}$$

Where:

$$P_0 = G\alpha_{1.lat} \quad [N]$$

6.4.1.4 Normalization plots – vertical direction

The normalization curve for a single pedestrian is plotted for the reference damping ratio of 5.0% and with damping ratios defined by SYNPEX for concrete, steel and timber structures. The normalization factor τ is given in Table 6.12.

The normalization curve for the concentrated load model of a single pedestrian is plotted in Figure 6.46 for the reference damping ratio of 5.0%.

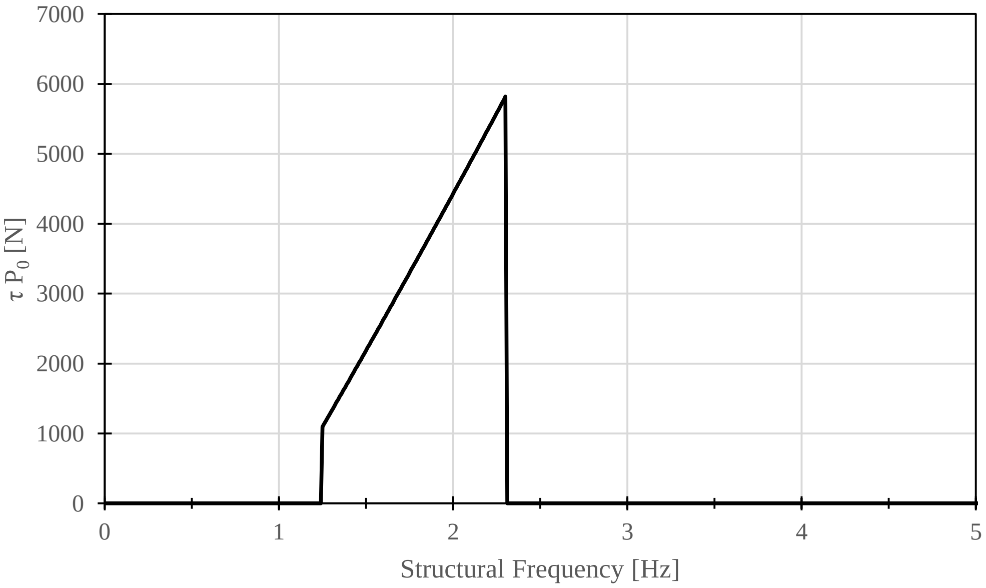


Figure 6.46 Normalization curve for a single pedestrian according to SYNPEX with structural damping ratio 5.0%

The normalization curve for vertical loading direction with damping ratio 1.3% according to concrete structures is plotted in Figure 6.47.

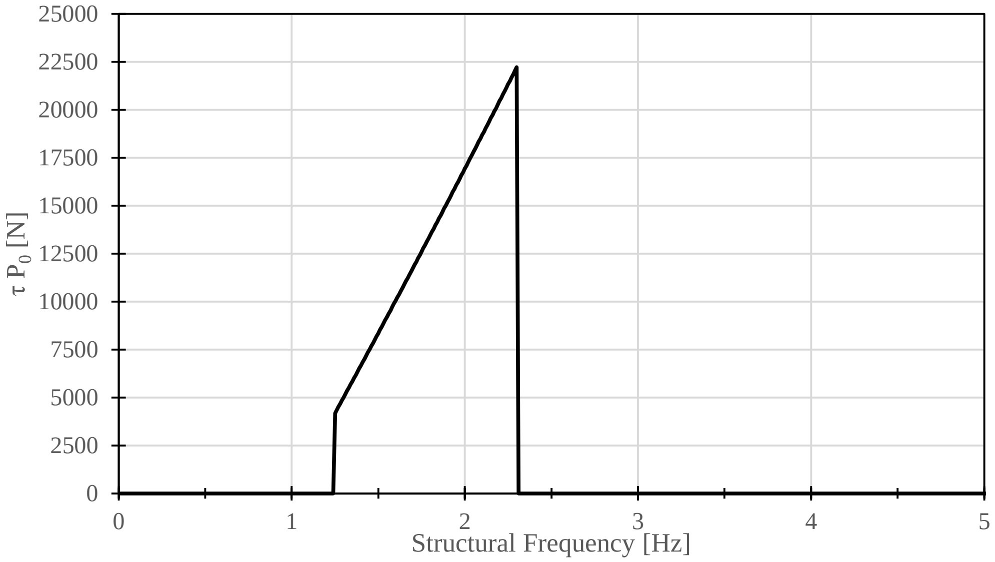


Figure 6.47 Normalization curve for a single pedestrian according to SYNPEX for concrete bridges with structural damping ratio 1.3%

The normalization curve for vertical loading direction with damping ratio 0.40% according to steel structures is plotted in Figure 6.48.

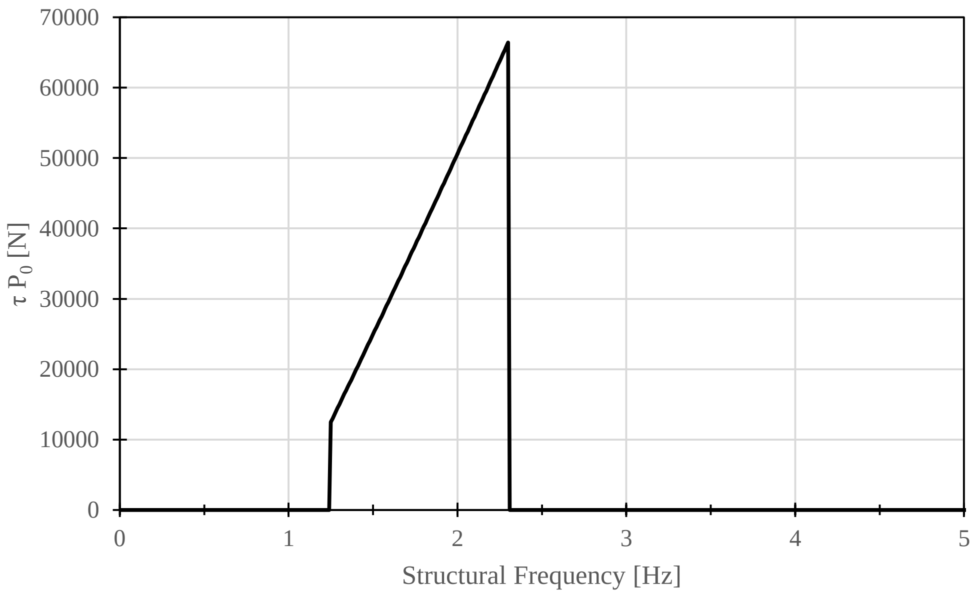


Figure 6.48 Normalization curve for a single pedestrian according to SYNPEX for steel bridges with structural damping ratio 0.4%

The normalization curve for vertical loading direction with damping ratio 3.0% according to timber structures is plotted in Figure 6.49.

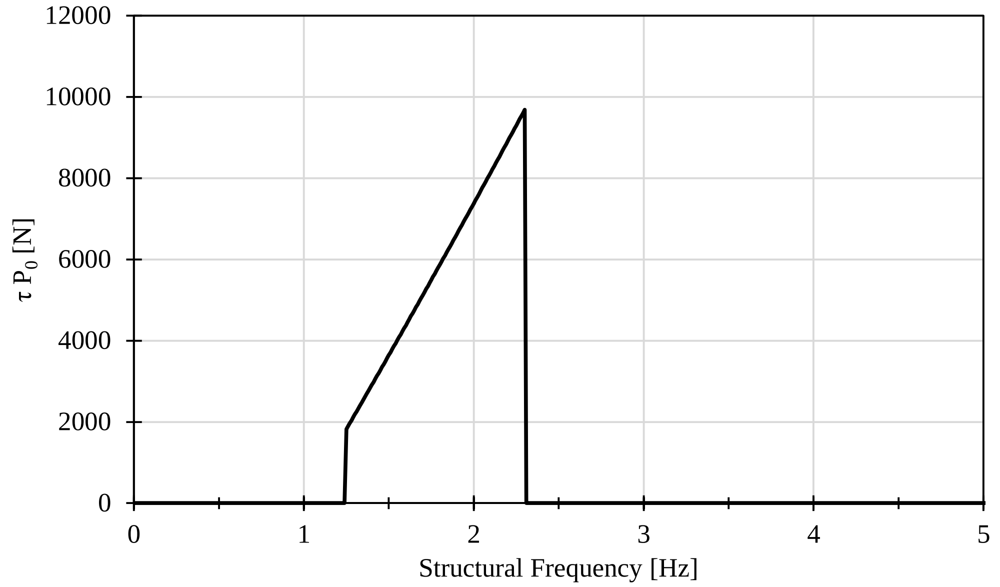


Figure 6.49 Normalization curve for a single pedestrian according to SYNPEX for timber bridges with structural damping ratio 3.0%.

In Figure 6.50 the reference bridge and the three materials are compared in the same plot for a single pedestrian causing vertical vibrations.

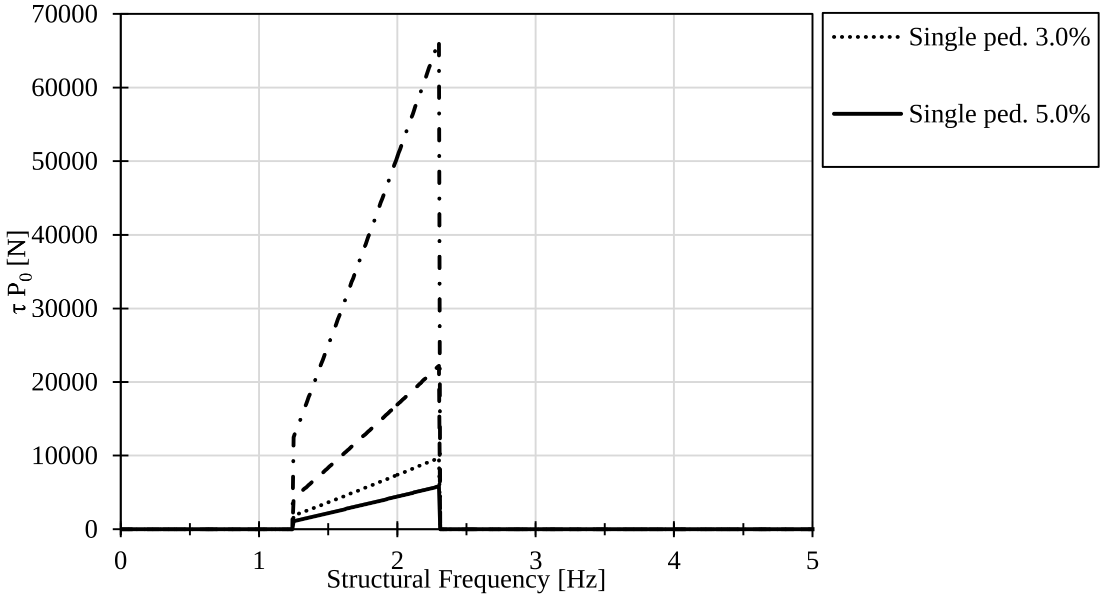


Figure 6.50 Normalization curve for a single pedestrian according to SYNPEX with structural damping ratios for the reference bridge and concrete, steel and timber.

6.4.1.5 Normalization of SYNPEX concentrated model - lateral action

The normalization curves are plotted for a reference damping of 5.0% and for damping ratios representing concrete, steel and timber bridges according to SYNPEX. The damping ratios and normalization factor τ are given in Table 6.12.

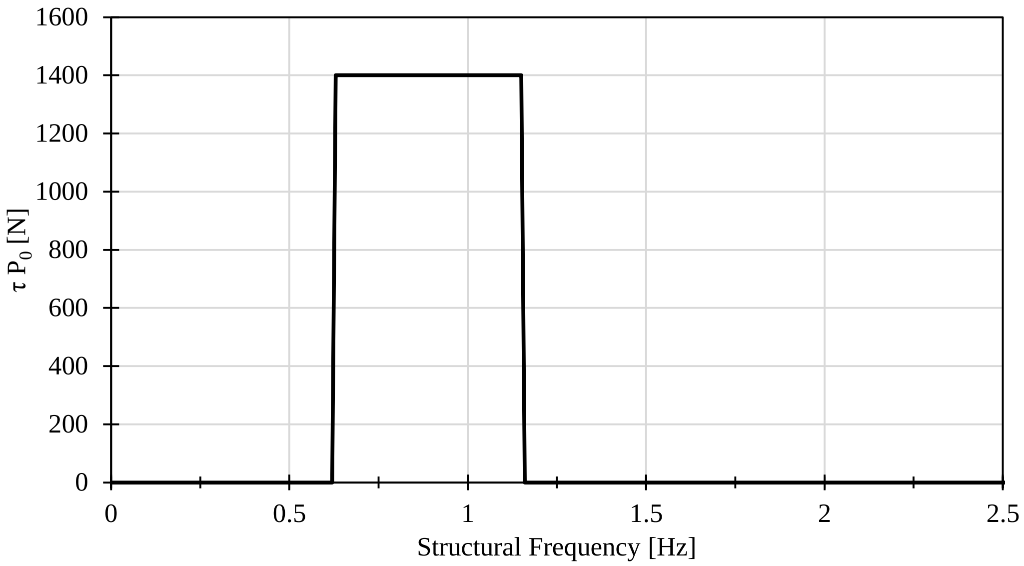


Figure 6.51 Normalization curve for a single pedestrian according to SYNPEX for the reference bridge with structural damping ratio 5.0%.

The normalization curve for lateral loading direction with damping ratio 1.3% according to concrete structures is plotted in Figure 6.52.

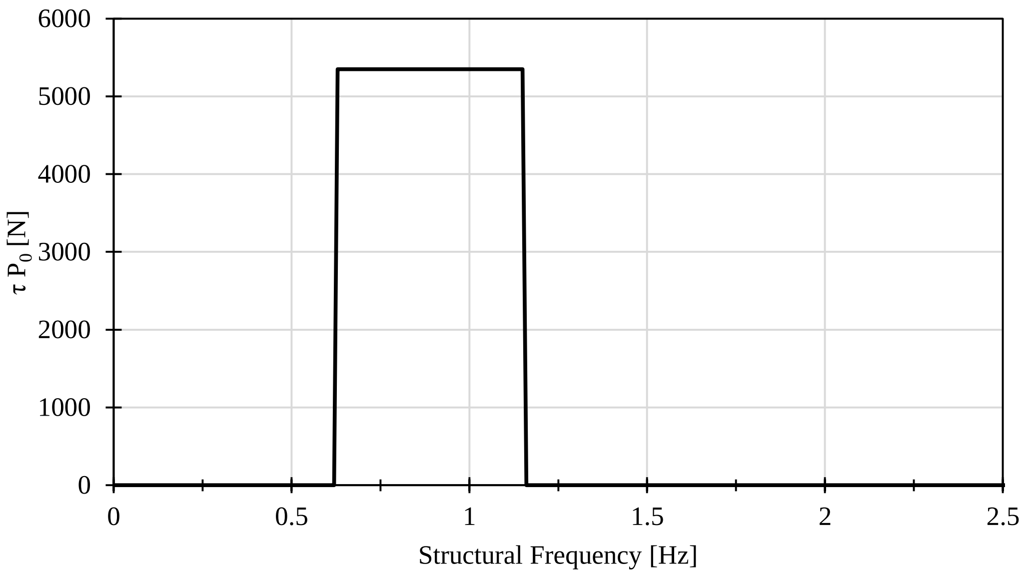


Figure 6.52 Normalization curve for a single pedestrian according to SYNPEX for a concrete structure with structural damping ratio 1.3%.

The normalization curve for lateral loading direction with damping ratio 3.0% according to steel structures is plotted in Figure 6.53.

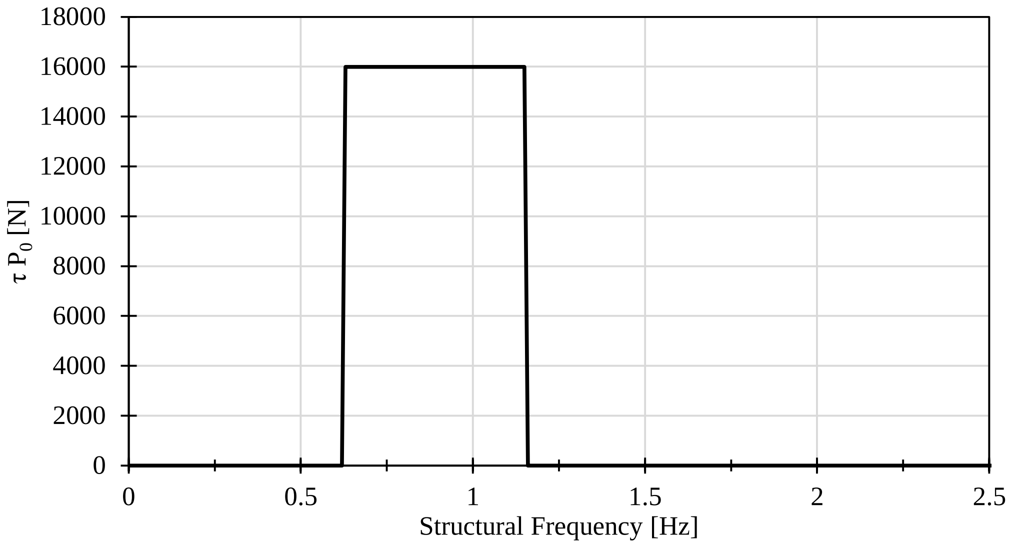


Figure 6.53 Normalization curve for a single pedestrian according to SYNPEX for a concrete bridge with structural damping ratio 0.40%.

The normalization curve for lateral loading direction with damping ratio 3.0% according to timber structures is plotted in Figure 6.54.

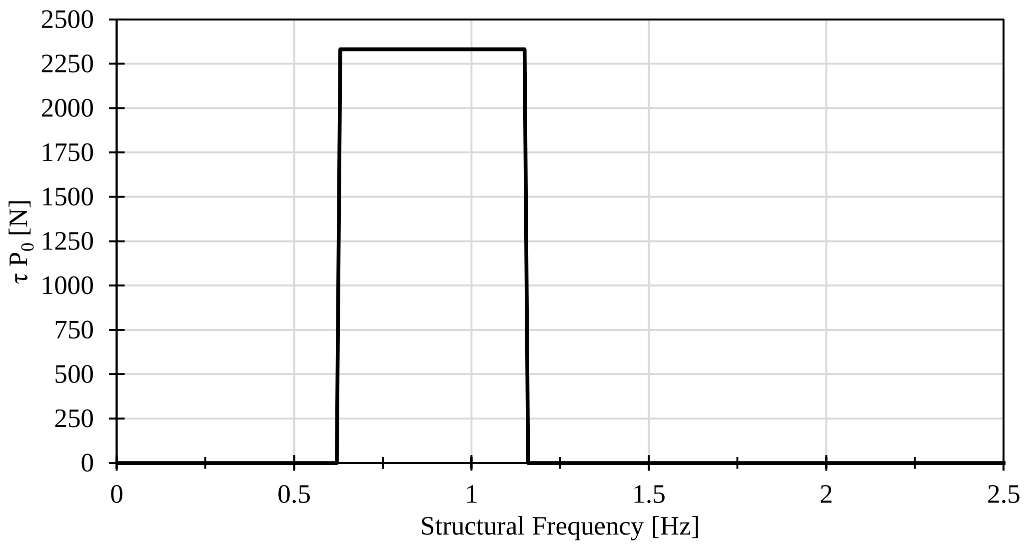


Figure 6.54 Normalization curve for a single pedestrian according to SYNPEX for a timber bridge with structural damping ratio 3.0%.

In Figure 6.55 are the normalization curves of lateral loading for different damping ratios compared.

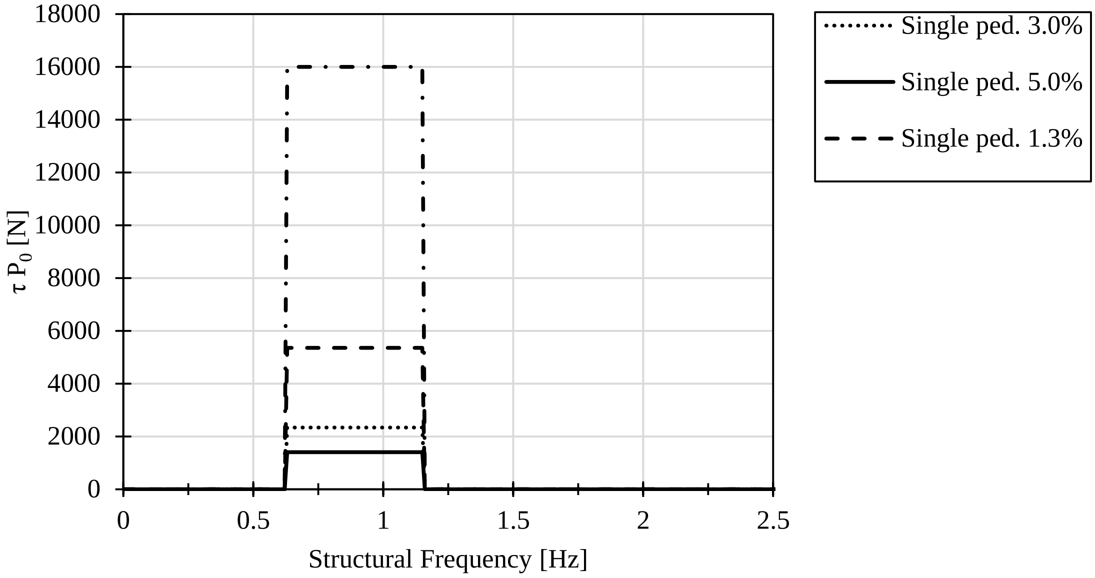


Figure 6.55 Normalization curve for a single pedestrian according to SYNPEX with structural damping ratios for the reference bridge and concrete, steel and timber

6.4.2 Uniformly distributed load

Pedestrian streams are suggested by SYNPEX to be modeled as a uniformly distributed load model over the bridge deck. The load model can be normalized according to chapter 5 and is given Equation (6-20) where the load amplitude p_0 is indetified. The force amplitude is dependent on loading direction, equivalent number of pedestrians and a design coefficient ψ which is a function of the bridge natural frequency. The normalization factor is dependent of the structural damping ratio. The normalization in Equation (6-20) applies for vertical, lateral and longitudinal loading directions.

$$p(t) = G\cos(2\pi f_s t)\, n'\psi = P_0\tau \quad [N/m^2] \tag{6-20}$$

Where:

$$P_0 = G_v n'\psi \quad [N/m^2]$$

The load model is defined in vertical, lateral and longitudinal direction where the load amplitude G and design coefficient ψ varies between the loading directions.

The equivalent number of pedestrians n' is calculated depending on traffic class according to Equation (4-67) and (4-68). Five traffic classes are defined in SYNPEX and given in 4.6.2 and are considered in the normalization curves.

6.4.2.1 Relationship between load amplitude and bridge geometry

The force amplitude, p_0, generates an applied load in N/m^2. In normalization the load should be given in N/m and is therefore multiplied with the bridge width, b, according to Equation (6-21).

$$p_0 = G\ 10.8\ n'\psi\ b \quad [N/m] \tag{6-21}$$

By inserting equivalent number of pedestrians gives Equation (6-22) for TC 2 - 3 and Equation (6-23) for TC 4 -5.

For TC 2 - 3

$$p_0 = G\ 10.8 \frac{\sqrt{\xi n}}{S} \psi\ b \quad [N/m] \tag{6-22}$$

For TC 4 - 5

$$p_0 = G\ 1.85 \frac{\sqrt{1.0\ n}}{S} \psi\ b \quad [N/m] \tag{6-23}$$

The equations can be simplified into Equation (6-24) and (6-25) as n and S includes the width.

For TC 2 - 3

$$p_0 = G\ 10.8 \frac{\sqrt{\xi dbL}}{bL} \psi\ b = \\ = G\ 10.8 \sqrt{\xi d}\ \psi \sqrt{\frac{b}{L}} \quad [N/m] \tag{6-24}$$

For TC 4 - 5

$$p_0 = G\ 1.85 \frac{\sqrt{dbL}}{bL}\ \psi\ b = G\ 1.85 \sqrt{d}\ \psi \sqrt{\frac{b}{L}} \quad [N/m] \tag{6-25}$$

In normalization the curve has to be independent on bridge geometry which is done by extracting the terms including length and width according to Equation (6-26) and (6-27). The extracted term is unit less and called geometry factor ω.

For TC 2 - 3

$$p_0 = G\ 10.8 \sqrt{\xi d}\ \psi \sqrt{\frac{b}{L}} = G\ 10.8 \sqrt{\xi d}\ \psi\ \omega \quad [N/m] \tag{6-26}$$

For TC 4 - 5

$$p_0 = G\ 1.85 \sqrt{d}\ \psi \sqrt{\frac{b}{L}} = G\ 1.85 \sqrt{d}\ \psi\ \omega \quad [N/m] \tag{6-27}$$

Where:

$$\omega = \sqrt{\frac{b}{L}} \quad [-]$$

The relation between load amplitude and bridge geometry can be seen in the figures below. In Figure 6.56 is the amplitude plotted for various lengths with constant width equal to 1 m and in Figure 6.57 for various widths with constant length equal to 1 m. The geometry factor ω will convert the load amplitude for arbitrary bridge geometry.

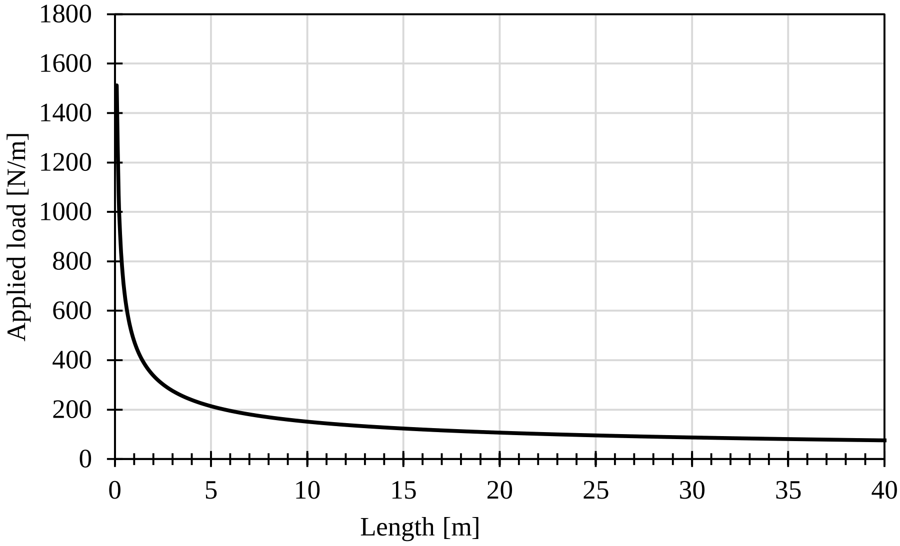


Figure 6.56 Relationship between load amplitude and bridge length for a constant width equal to 1 m.

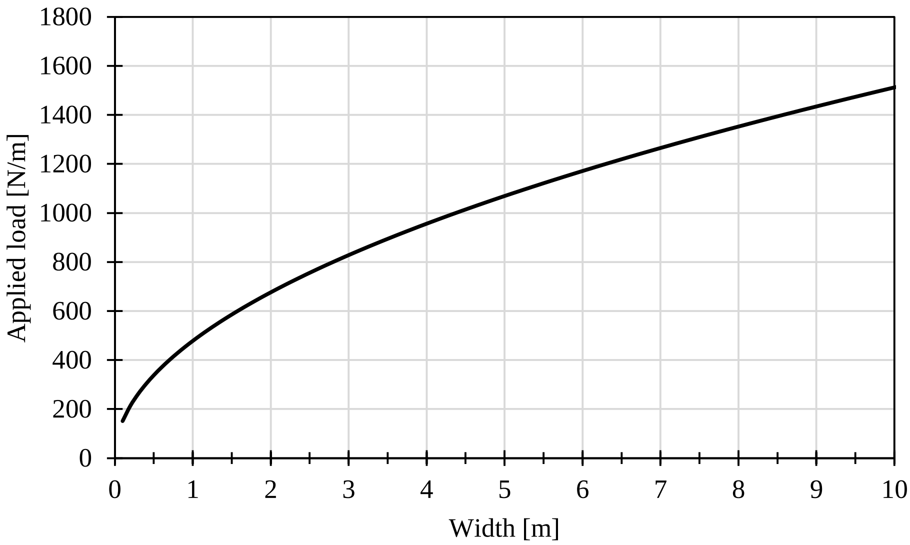


Figure 6.57 Relationship between load amplitude and bridge width for a constant length equal to 1 m.

Traffic class 1 is different from the other classes as it always considers 15 pedestrians independent on bridge area. The geometry factor can be derived in the same way, to be independent on bridge width according Equation (6-28).

$$p_0 = G\ 10.8 \frac{\sqrt{\frac{\xi N}{bL}\ bL}}{bL}\ \psi\ b = G\ 10.8\ \sqrt{N}\ \psi \frac{1}{L} = p_0\ \omega_{TC1} \quad [N/m] \tag{6-28}$$

Where:

$$p_0 = G\ 10.8\ \sqrt{N}\ \psi \quad [N]$$

$$\omega_{TC1} = \frac{1}{L} \quad [1/m\,]$$

The normalization is plotted for load amplitude, p_0, multiplied by the normalization factor, τ. For arbitrary bridge geometry the curve has to be multiplied by the geometry factor ω with the considered length and width.

6.4.2.2 Structural damping ratio and normalization factor τ

SYNPEX suggest several damping ratios for the same materials. In normalization are the damping ratios referred to as average ratios and given in Table 6.13 with corresponding normalization factor for uniformly distributed loads.

Table 6.13 Static load, damping ratios and normalization factors for the reference bridge and construction materials concrete, steel and timber used in the normalization for the vertical load model.

Material	Reference	Concrete	Steel	Timber
Damping ratio, ξ [%]	5.0	1.3	0.40	3.0
Norm factor, τ [-]	12.73	48.44	148.6	21.21

6.4.2.3 Normalized acceleration response for uniform load model – vertical action

The normalization curves for traffic class 1 are plotted in Figure 6.58 for the reference structural damping ratio of 5.0% and according to damping ratios defined for concrete, steel and timber bridges.

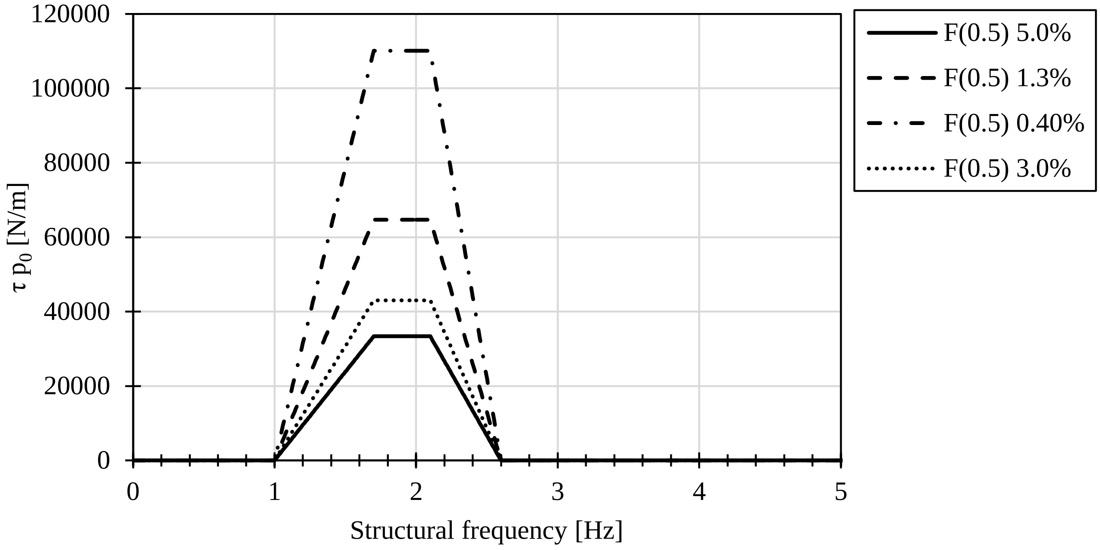


Figure 6.58 Normalization curves for the vertical load model calculated for TC 1 with structural damping ratios according to the reference beam and concrete, steel and timber bridges.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 5 % are plotted in Figure 6.59.

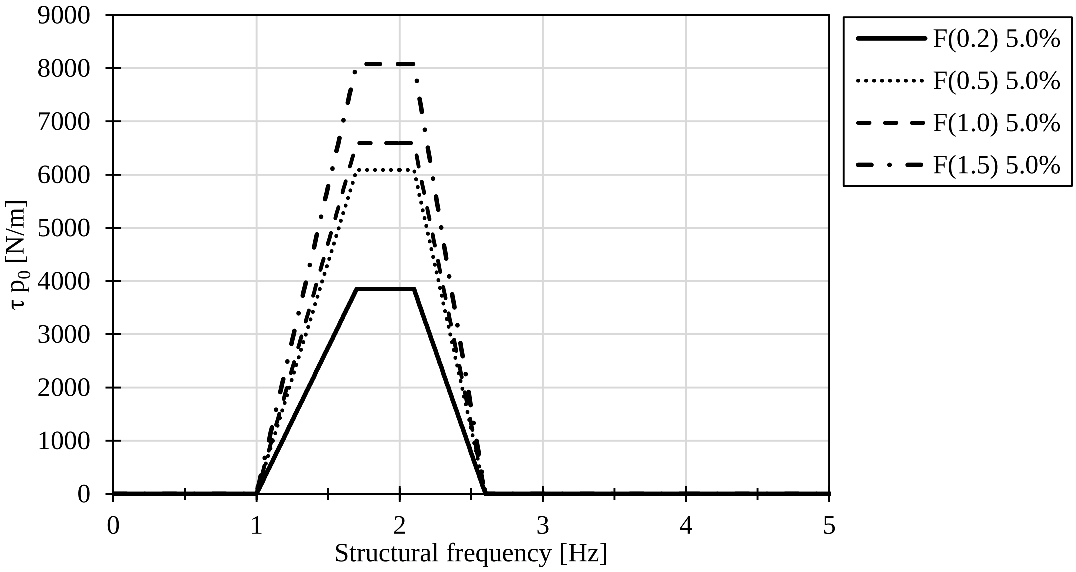


Figure 6.59 Normalization curve for the vertical load model with structural damping ratio 5.0% according to the reference beam plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 1.3% according to concrete bridges are plotted in Figure 6.60.

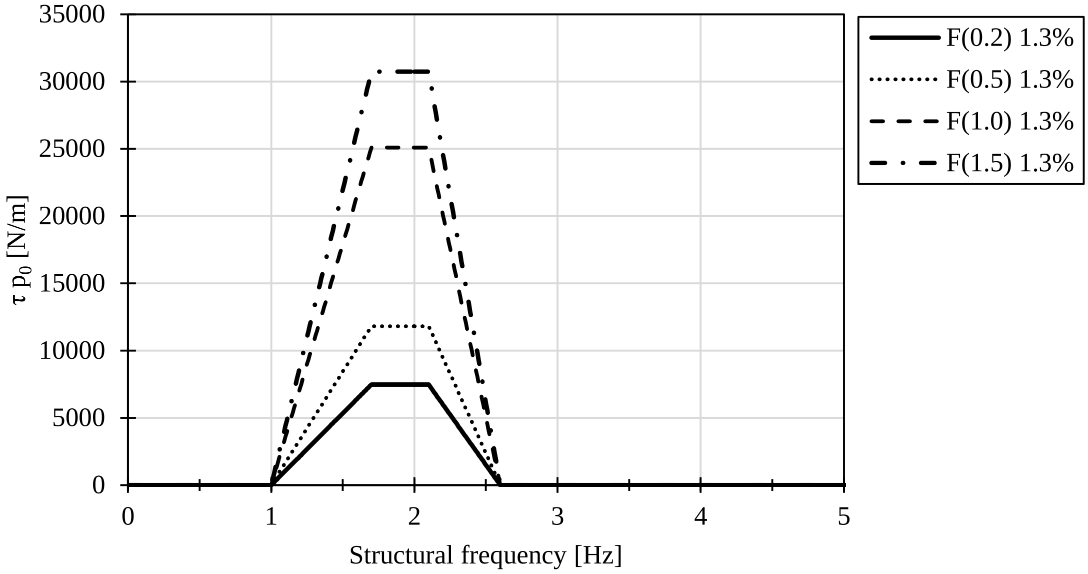


Figure 6.60 Normalization curve for the vertical load model with structural damping ratio 1.3% according to concrete bridges plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 0.40% according to steel bridges are plotted in Figure 6.61.

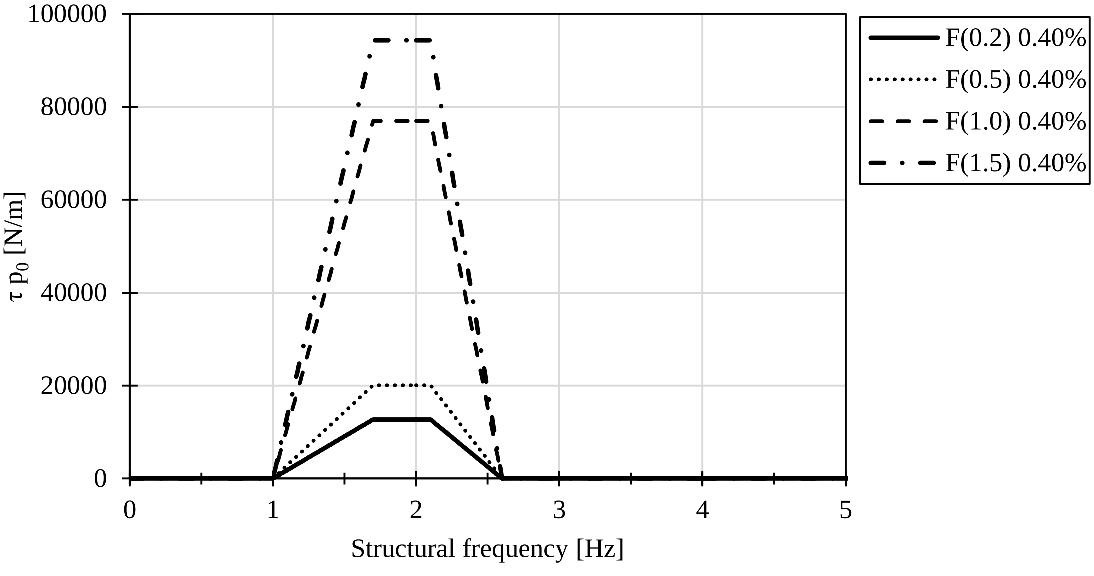


Figure 6.61 Normalization curve for the vertical load model with structural damping ratio 0.4% according to steel bridges plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 3.0% according to timber bridges are plotted in Figure 6.62.

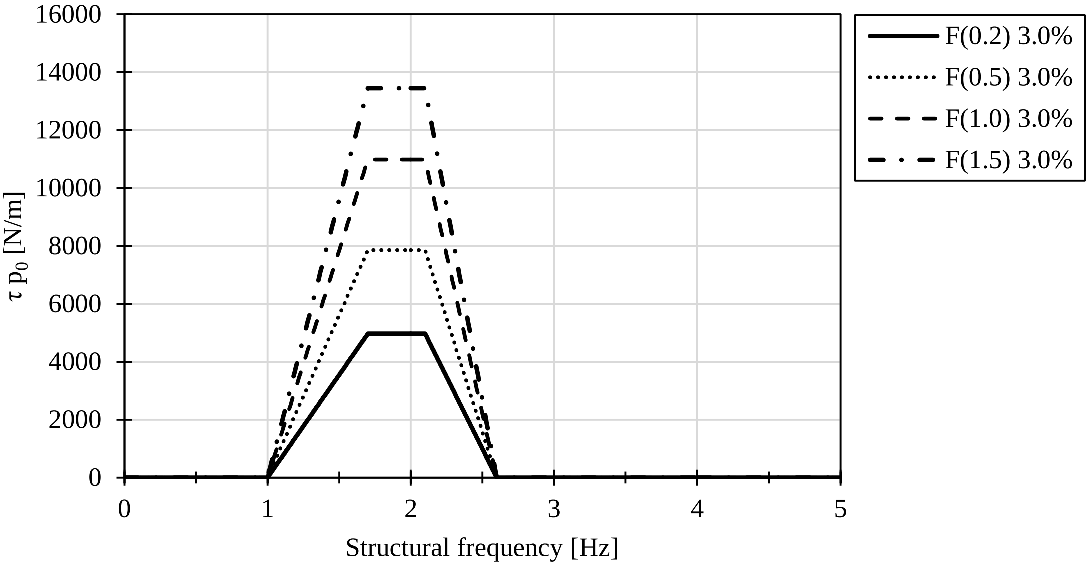


Figure 6.62 Normalization curve for the vertical load model with structural damping ratio 3.0% according to timber bridges plotted for TC 2 to TC5.

A comparison of the different damping ratios plotted in previous figures is shown in Figure 6.63 for traffic class TC 2 to TC 5.

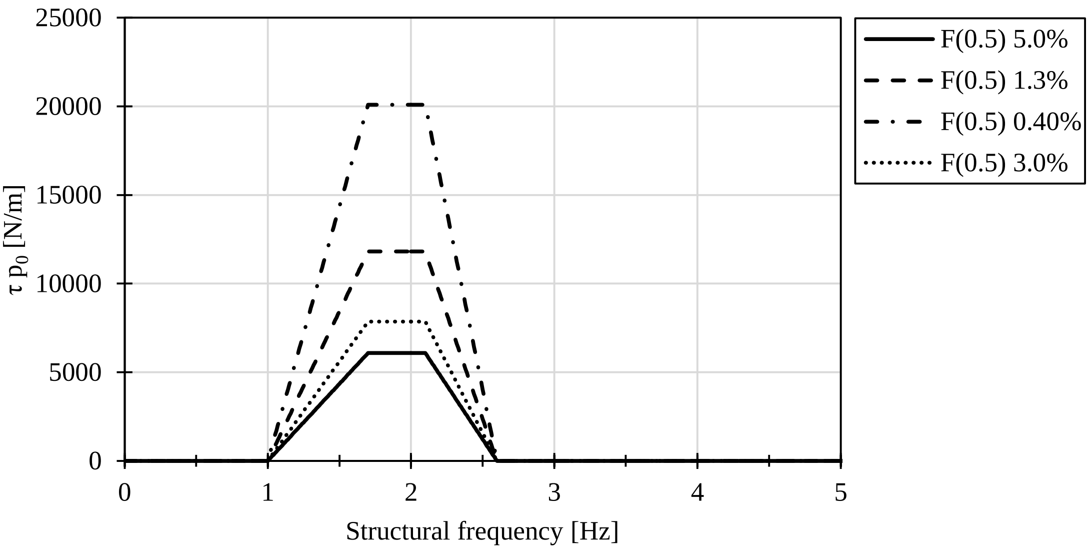


Figure 6.63 Comparison of the normalization curves for the vertical load model calculated for various damping ratios and plotted for TC 3.

6.4.2.4 Normalized acceleration response for uniform load model – lateral action

The normalization curves for traffic class 1 are plotted in Figure 6.58 for the reference structural damping ratio of 5.0% and according to damping ratios defined for concrete, steel and timber bridges.

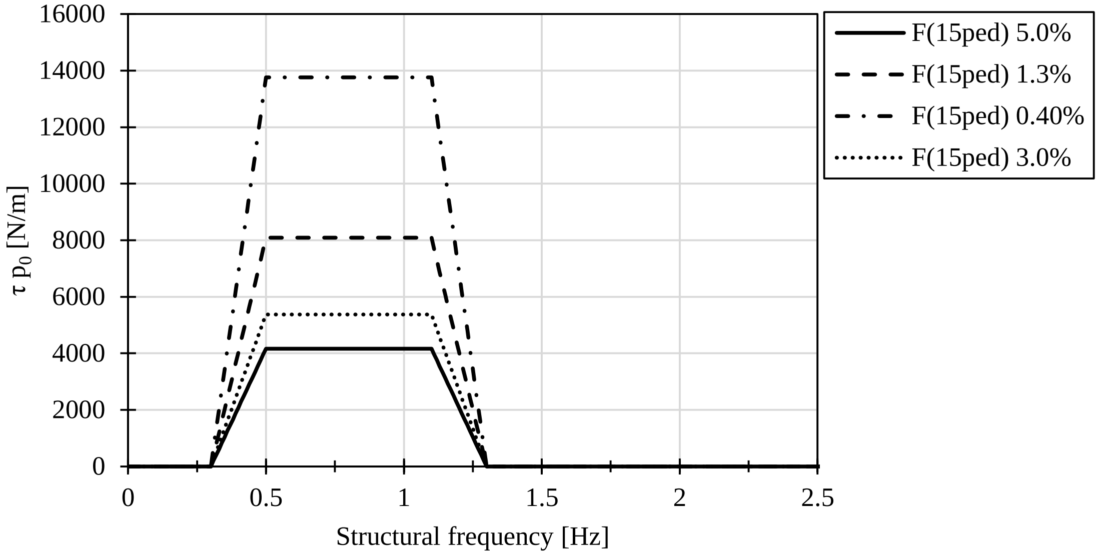


Figure 6.64 Normalization curves for the lateral load model calculated for TC 1 with structural damping ratios according to the reference beam and concrete, steel and timber bridges.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 5.0% are plotted in Figure 6.65.

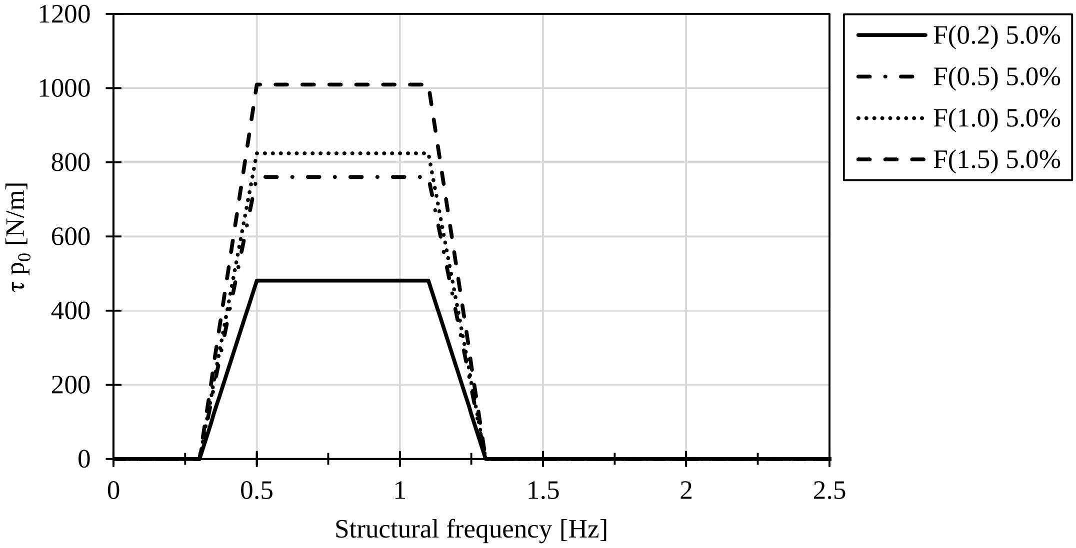


Figure 6.65 Normalization curve for the lateral load model with structural damping 5.0% according to the reference beam plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 1.3% according to concrete bridges are plotted in Figure 6.66.

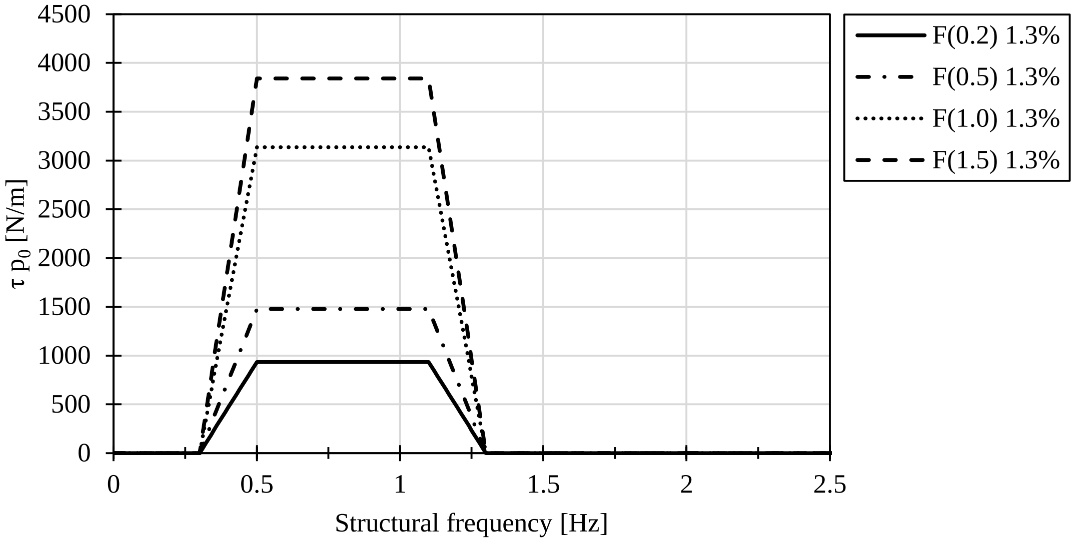


Figure 6.66 Normalization curve for the lateral load model with structural damping ratio 1.3% according to concrete bridges plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 0.4% according to steel bridges are plotted in Figure 6.67.

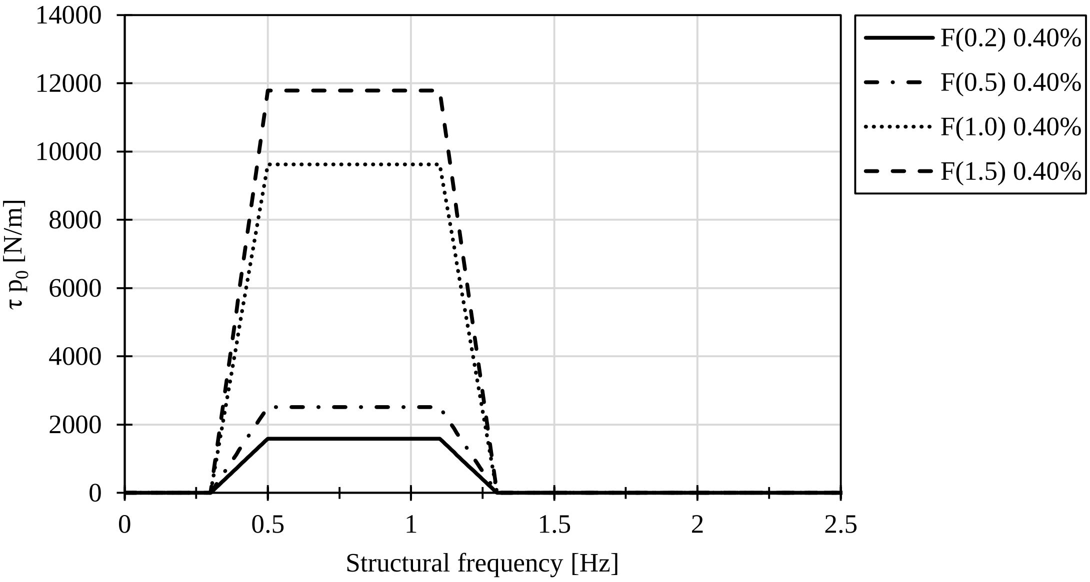


Figure 6.67 Normalization curve for the lateral load model with structural damping ratio 0.4% according to steel bridges plotted for TC 2 to TC5.

Normalized curves for the defined traffic classes TC 2 to TC 5 with a structural damping of 3.0% according to timber bridges are plotted in Figure 6.68.

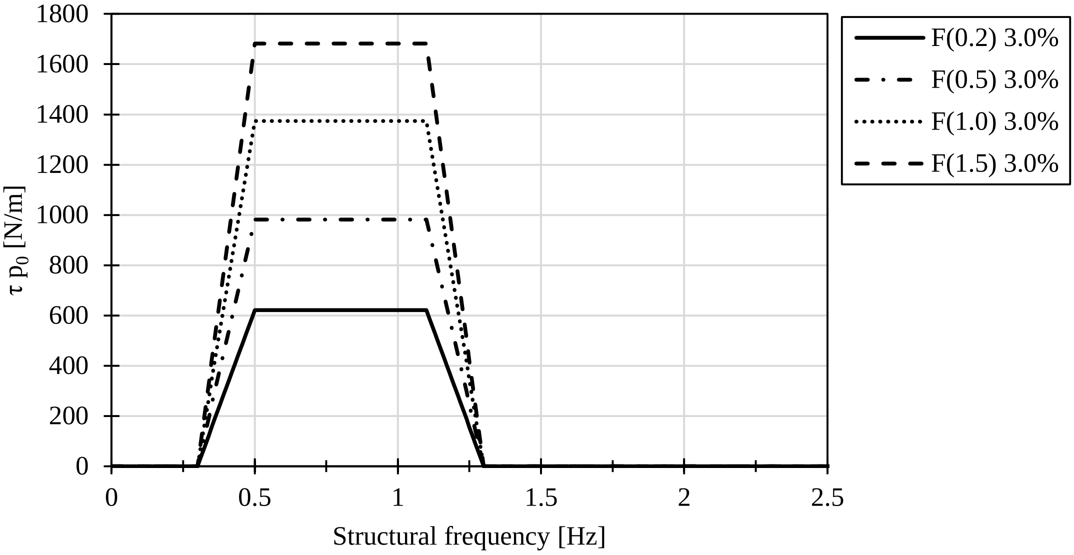


Figure 6.68 Normalization curve for the lateral load model with structural damping ratio 3.0% according to timber bridges plotted for TC 2 to TC5.

A comparison of the different damping ratios plotted in previous figures is shown in Figure 6.69 for traffic class TC 2 to TC 5.

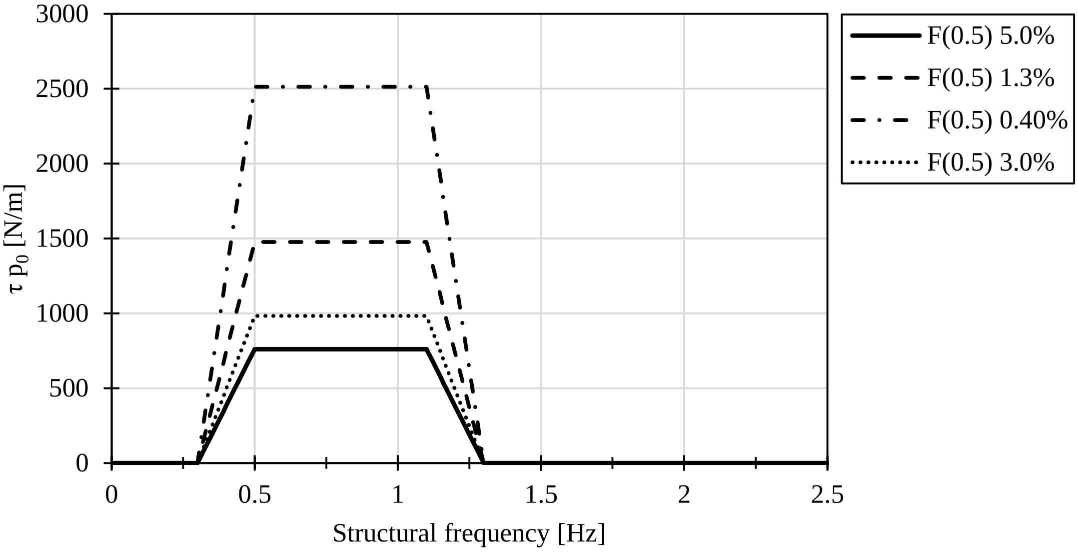


Figure 6.69 Comparison of the normalization curves for the lateral load model calculated for various damping ratios and plotted for TC 3.

6.4.3 Comments and discussion

SYNPEX recommends several methods to analyse and calculated the maximum response in a footbridge induced by pedestrians. The load models treated in the normalization are well developed and well defined leaving no gaps in the description of application. The concentrated loads cover both vertical and lateral direction of loading and should be applied as moving over the span with a given velocity. The velocity recommended in the guideline is a function of the step frequency which is reasonable according to literature. The uniformly distributed load model simulating pedestrian streams is applicable in vertical, lateral and longitudinal direction with different magnitude.

6.4.3.1 Critical frequencies

The step frequency interval for concentrated loads is defined to be within 1.25 and 2.30 Hz. The second harmonic is the double step frequency between 2.50 and 4.60 Hz. The interval is comparable with other intervals found in literature. For uniformly distributed loads the interval is defined to be within 1.0 and 2.6 Hz covering a longer span of step frequencies.

In lateral direction of loading the frequency interval is determined to be within 0.625 and 1.15 Hz for the concentrated load model defined as half the vertical frequency interval. The uniformly distributed load model is defined between 0.30 and 1.3 Hz which is wider than for concentrated loads. The upper limit of the interval is half of the vertical upper limit but the lower limit is extended to be lower than 0.50 Hz. With an interval covering as low as 0.30 Hz would imply a walking frequency as low as 0.60 Hz.

SYNPEX is not consistent about normal walking frequencies as an overall suggestion the vertical interval should be between 1.30 and 2.30 Hz and between 0.50 and 1.20 Hz in lateral direction.

6.4.3.2 Load amplitude – concentrated load model

The concentrated load models are Fourier series with coefficients and phase angles well presented in the report. In vertical direction the coefficients and phase angles are functions of the step frequency based on measurements and methods presented in the report. The functions for vertical action are of third order which can make them complex to use but should give accurate results based on the derivations done by SYNPEX. Higher step frequencies will result in higher Fourier coefficient and therefore higher load amplitude. It should be noted when comparing the magnitude of the coefficients that the second harmonic always will contribute more than the first as the coefficient is higher for all step frequencies. Tough it is complex to make a general conclusion as the phase angle also affects the load amplitude and resonance.

The effect of considered harmonics on load amplitude can be seen in the plot of load history. In the plot it is seen that a load model only considering the first harmonic will generate significantly lower load amplitude than when the harmonics are considered. Three harmonics will give approximately the same load amplitude as for two. Though the difference in load amplitude is not direct proportional to the response as the degree of resonance affect the response. The degree of resonance will decrease as the number of harmonics regarded increase due to phase shifts. I.e. a load with low amplitude and without phase shift can create a higher response than one with high amplitude and phase shifts.

In normalization the second harmonic is considered by only applying the second harmony of the load model. This generates a higher response as the second Fourier coefficient is higher than the first. Is should be treated with caution as the load should be applied with two harmonics when analysing the response in the second harmonic frequency interval.

The lateral coefficients and phase angles are constant and independent on step frequency. The model does not include a static load of the pedestrian which is reasonable as the weight of the pedestrian only acts in vertical direction. As a result the load amplitude is oscillating at zero Newton with the load amplitude increasing with the number of considered harmonics.

The load amplitude for the concentrated load model is independent on the bridge deck area, the same load is applied for all geometries.

6.4.3.3 Load amplitude – uniformly distributed load model

The uniformly distributed load model is applicable for vertical, lateral and longitudinal direction by changing load amplitude and reduction coefficient. The reduction coefficient ψ takes the probability of resonance between the structure and normal step frequencies into account based on the defined frequency intervals. The factor has its maximum, equal to one, between 1.70 and 2.10 Hz for vertical action. In vertical direction it covers the most common walking frequencies and also not so common with reduced magnitude. The factor is zero under 1.0 and above 2.6 Hz which are reasonable assumptions. SYNPEX is neglecting the second harmonic for the vertical load model as the reduction factor is zero. In the report it is stated that no vertical vibration due to the second harmonic has occurred in reality and therefore it is not considered as a problem.

In lateral direction the reduction factor is equal to one between 0.50 and 1.1 Hz and zero below 0.30 Hz and above 1.3 Hz. Normally the lateral frequency interval is half of the vertical interval limits. In this case the interval for lateral action is extended down to 0.30 Hz and covers a broader range of frequencies with less magnitude.

The uniform load model is dependent on the bridge deck area with decreasing load amplitude with increasing span length. With increasing width the load only increases with the square root of the width considering the applied load in N/m. The fact that the load is decreasing per square meter for an increase in area does not make the model consistent. This means that a longer bridge will have lower load per square meter area than a shorter for the same width but still the governing input is the pedestrian density is defined in ped/m^2 evenly over the bridge deck. The varying load amplitude is in normalization adjusted by the derived geometry factor ω.

6.4.3.4 Traffic classes

In SYNPEX five traffic classes for pedestrian streams are defined by characteristics of how the traffic is experienced. The lowest class always considers 15 pedestrians and the maximum is given as a pedestrian density of 1.5 ped/m^2. The traffic classes are illustrated in photos in the report and according to literature the values seems reasonable. No recommendations regarding traffic class based on the bridge location is given.

6.4.3.5 Equivalent number of pedestrians

The guideline does not propose an equivalent number of pedestrians applicable on the concentrated load model and therefore the concentrated load can only be used for simulating a single pedestrian. Several persons can be simulated by one load per pedestrian but it is inefficient and inaccurate as the degree of synchronisation cannot be considered.

SYNPEX shows in the report how the expression for equivalent number of pedestrians is derived for pedestrian streams. A Monte Carlo simulation has been done to investigate how pedestrians synchronize and how it is affected by the structural damping. Traffic class 4 and 5 are not affected by the structural damping which will influence the load amplitude. By comparing the load amplitude between traffic classes and damping ratio it can be seen that the difference in load amplitude between traffic class 3 and 4 is lower for high damped structures and larger difference for structures with low damping.

6.4.3.6 Damping

SYNPEX refers to two different sources Eurocode and Sétra for suitable damping ratios. Eurocode has defined the ratios as functions of the span length and Sétra as maximum and average values. The difference in definition makes the ratios diverge i.e. for a footbridge with 15 m span the damping ratio for steel is defined between 0.2% and 1.25% and timber 1.0% and 3.0%. The report does not decide which of the values to be correct or how they have been determined. As the damping affects both the load amplitude and the structural response a less correct damping ratio will have a large impact on the acceleration response. In the normalization average damping ratios are used.

6.5 JRC and HIVOSS

JRC and HIVOSS are presenting the same load models with the same input value and are considered as equal. This section is presenting the normalized acceleration response for both standards but refers to the section describing JRC.

JRC proposes a load model as a distributed harmonic load adapted to the mode shape of the structure according to section 4.7.5. The harmonic load according to Equation (4-75) should have the same frequency as the natural frequency of the considered bridge and be distributed over the bridge deck until steady-state is reached.

The guideline is based on classification of footbridges due to the expected traffic level according to 4.7. The proposed densities are according to Table 4.39 where Traffic class 1 differs from the others defined as 15 pedestrians distributed evenly over the bridge.

6.5.1 Normalization of distributed load

The load model can be normalized according to the derivations in chapter 5. The load model is given in Equation (6-29) where the load amplitude p0 is identified.

$$p(t) = P\cos(2\pi f_s t)\ n'\psi \quad [\mathrm{N/m^2}] \tag{6-29}$$

Where:

$$p_0 = Pn'\psi \quad [N/m^2]$$

Input variables for different materials according to JRC and values for normalization factor τ are presented in Table 6.14.

Table 6.14 Input variables for different materials according to JRC.

	Reference	Concrete	Steel	Timber
Damping ratio [%]	5	1.3	0.4	1.5
τ- uniform	12.74	48.45	148.6	42.24
Po [N]	280	280	280	280

Equivalent number of pedestrian, n', depends on traffic class due to different degree of synchronization among pedestrians as seen in Table 6.15.

Table 6.15 Equivalent number of pedestrians due to traffic class.

Traffic class	Equivalent number of pedestrians, n'
TC 1 – TC 3	$n' = \dfrac{10.8\sqrt{\xi\, n}}{S}$
TC 4- TC 5	$n' = \dfrac{1.85\sqrt{n}}{S}$

6.5.1.1 Relationship between load amplitude and bridge geometry

The load amplitude p_0 is dependent on factors regarding equivalent number of pedestrians, relevant step frequencies for walking and the proposed static load for a pedestrian. The load amplitude, p_0, generates an applied load in N/m^2. In normalization the load should be given in N/m and is therefore multiplied with the bridge width, b, according to Equation (6-30).

$$p_0 = P\, n'\psi\, b \quad [N/m] \tag{6-30}$$

By inserting equivalent number of pedestrians gives Equation (6-31) for TC 2 - 3 and Equation (6-32) for TC 4 -5.

$$p_0 = G\, \frac{10.8\sqrt{\xi\, n}}{S}\psi\, b \quad [N/m] \qquad \text{For TC 2 - 3} \tag{6-31}$$

$$p_0 = G\, \frac{1.85\sqrt{n}}{S}\psi\, b \quad [N/m] \qquad \text{For TC 4 - 5} \tag{6-32}$$

The equations can be simplified into Equation (6-33) and (6-34) as n and S includes the width.

$$p_0 = P\ 10.8 \frac{10.8\sqrt{\xi dbL}}{bL} \psi\ b = \qquad \text{For TC 2 - 3} \quad (6\text{-}33)$$

$$= P\ 10.8\sqrt{\xi d}\ \psi \sqrt{\frac{b}{L}} \quad [N/m]$$

$$p_0 = P\ \frac{1.85\sqrt{dbL}}{bL}\ \psi\ b = P\ 1.85\sqrt{d}\ \psi \sqrt{\frac{b}{L}} \quad [N/m] \qquad \text{For TC 4 - 5} \quad (6\text{-}34)$$

In normalization the curve has to be independent on bridge geometry which is done by extracting the terms including length and width according to Equation (6-35) and (6-36). The extracted term is unit less and called geometry factor ω.

$$p_0 = P\ 10.8\sqrt{\xi d}\ \psi \sqrt{\frac{b}{L}} = P\ 10.8\sqrt{\xi d}\ \psi\ \omega \quad [N/m] \qquad \text{For TC 2 - 3} \quad (6\text{-}35)$$

$$p_0 = P\ 1.85\sqrt{d}\ \psi \sqrt{\frac{b}{L}} = P\ 1.85\sqrt{d}\ \psi\ \omega \quad [N/m] \qquad \text{For TC 4 - 5} \quad (6\text{-}36)$$

Where:

$$\omega = \sqrt{\frac{b}{L}} \quad [-]$$

The relation between load amplitude and bridge geometry can be seen in the figures below. In Figure 6.70 the amplitude is plotted for various lengths with constant width equal to 1 m and in for various widths with constant length equal to 1 m. The geometry factor ω will convert the load amplitude for arbitrary bridge geometry.

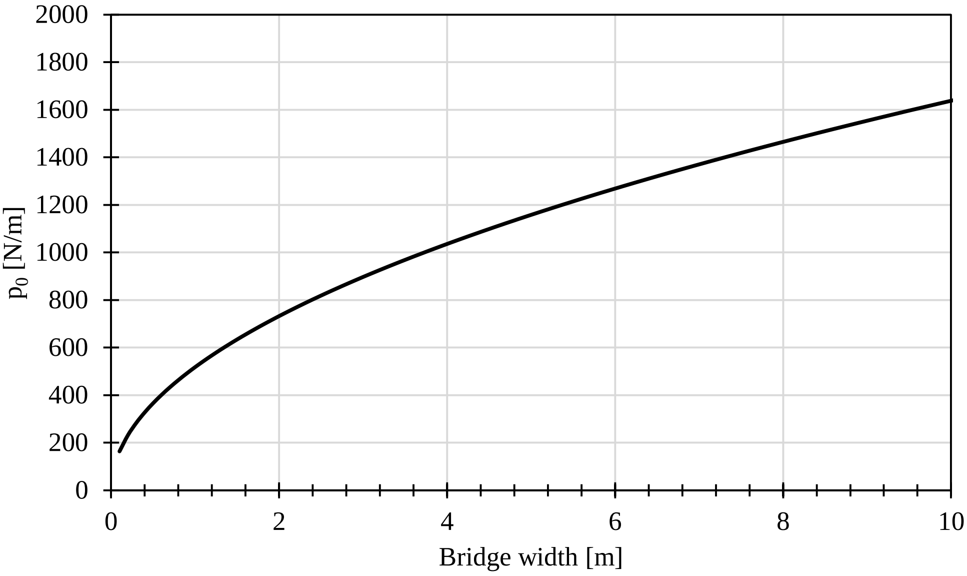


Figure 6.70 Plot of load amplitude p_0 [N/m] for varying width with constant length 1 m.

Figure 6.71 shows a plot of load amplitude for varying length with constant width of 1m.

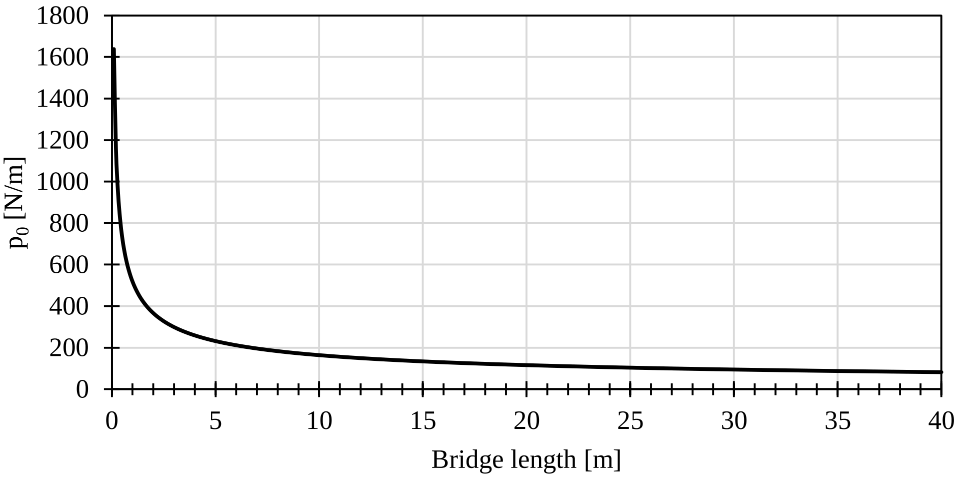


Figure 6.71 Plot of load amplitude p_0 [N/m] for varying length with constant width 1 m.

Traffic class 1 is different from the other classes as it always considers 15 pedestrians independent on bridge area. The geometry factor can be derived in the same way, to be independent on bridge width according Equation (6-37).

$$p_0 = P\ 10.8 \frac{\sqrt{\frac{\xi N}{bL}\ bL}}{bL}\ \psi\ b = P\ 10.8 \sqrt{N}\ \psi \frac{1}{L} = p_0\ \omega_{TC1} \quad [N/m] \tag{6-37}$$

Where:

$$p_0 = P\ 10.8\sqrt{N}\,\psi \quad [N]$$

$$\omega_{TC1} = \frac{1}{L} \quad [1/m\,]$$

This can be seen in Figure 6.72 where the load amplitude is plotted for varying length with constant width of 1m.

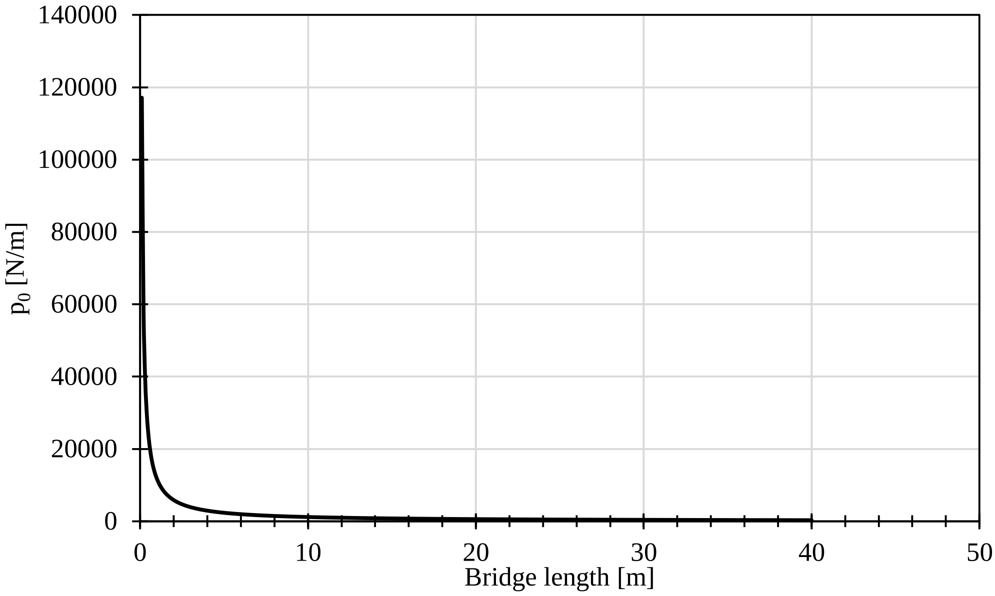


Figure 6.72 Plot of load amplitude for varying length with constant width of 1 m

6.5.1.2 Normalized response according to JRC in vertical direction

Input variables for different materials are shown in Table 6.16.

Table 6.16 Input variables for different materials according to JRC.

	Reference	Concrete	Steel	Timber
Damping ratio [%]	5	1.3	0.4	1.5
τ- uniform	12.74	48.45	148.6	42.24
Po [N]	280	280	280	280

The normalized curves for the proposed pedestrian densities, except TC1, with a structural damping of 5.0 % are shown in Figure 6.73.

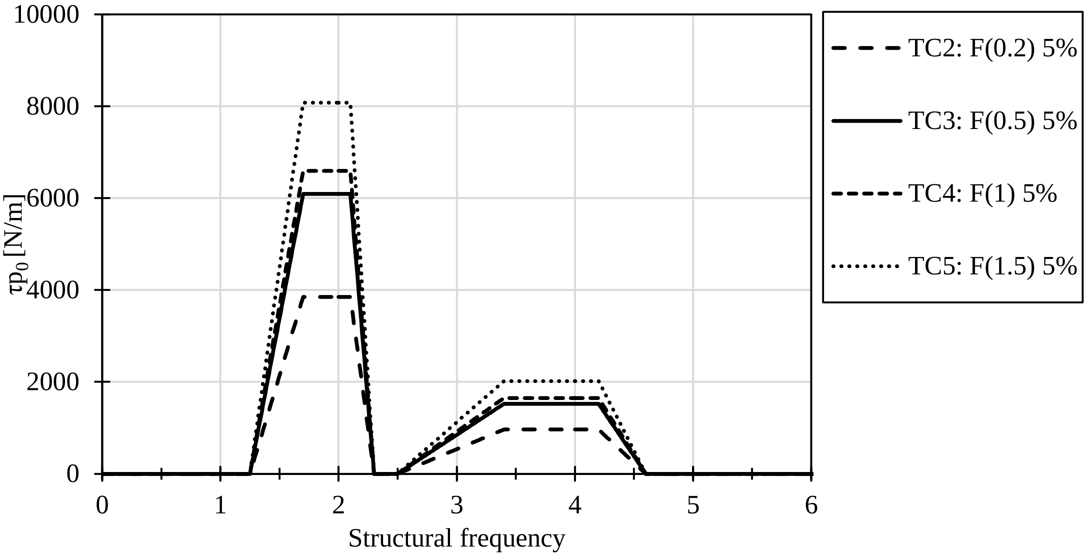


Figure 6.73 Normalized curves for TC2-TC5 with 5 % damping, established for a 15 m^2 bridge.

In Figure 6.74 the normalized curves are plotted which shows the difference between Traffic class 1 and the other densities. In this case 15 pedestrians are distributed over a very small bridge which gives rise to a big response in comparison with the other densities.

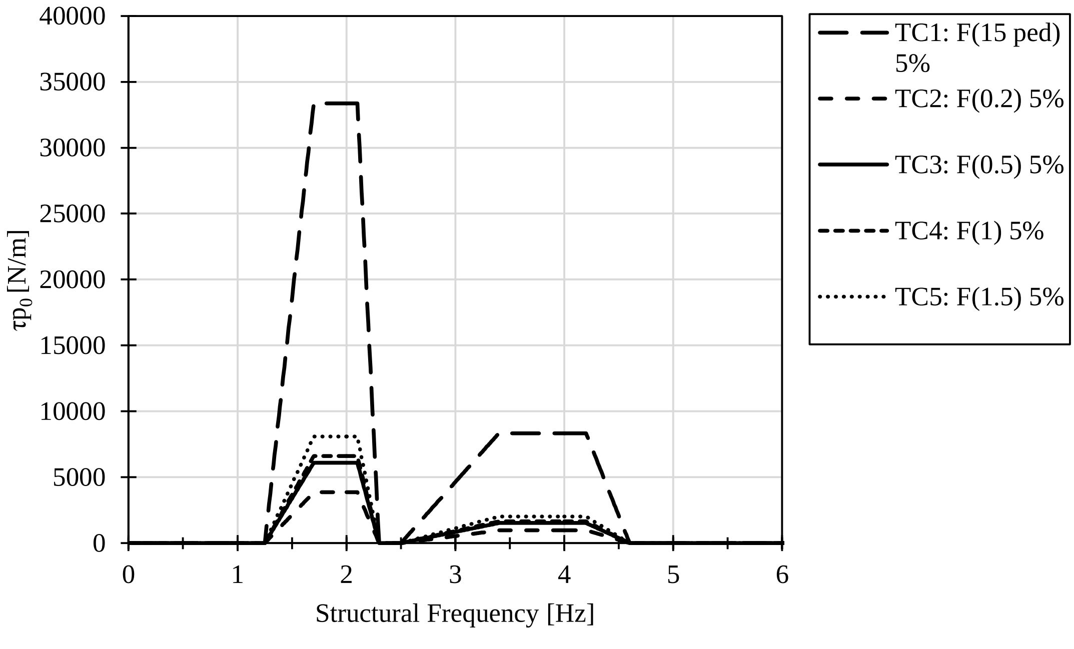


Figure 6.74 Normalized curves for proposed pedestrian densities, 5 % damping, 1m^2 bridge

In Figure 6.75 the normalized curves for TC1 for three different lengths are plotted for a 1 m wide bridge. It can be seen that the accelerations due to TC1 for a small bridge will be big.

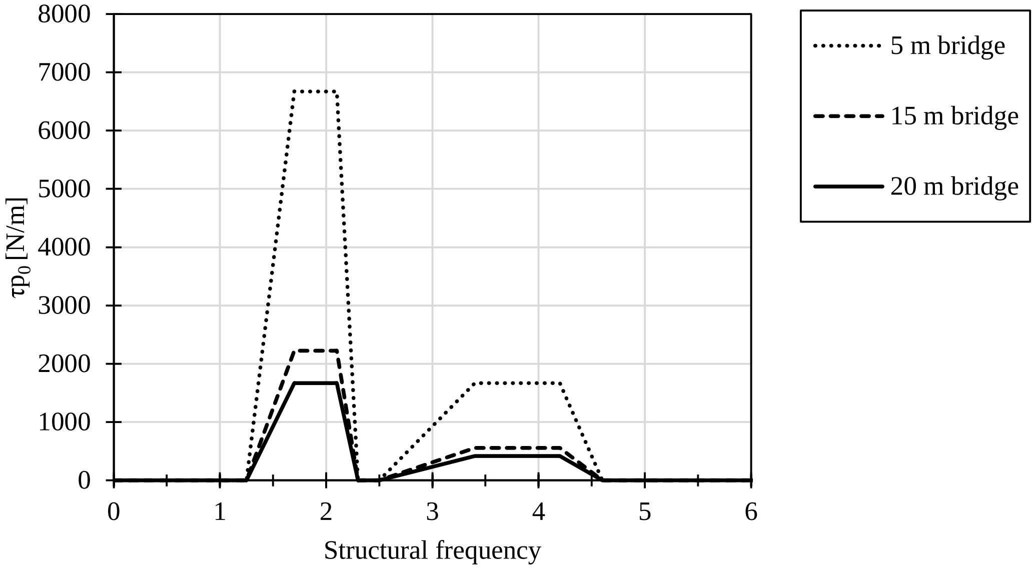


Figure 6.75 Normalized curves for TC1 for different lengths with 1 m wide bridge, 5 % damping.

In Figure 6.76 normalized curves for the proposed densities except TC1 are shown for concrete with 1.3 % damping. Note that TC4 and TC5, with densities 1.0 and 1.5 ped/m^2 respectively, do not take structural damping into account in the equivalent number of pedestrians. This gives big difference in response compared to TC2 and TC3.

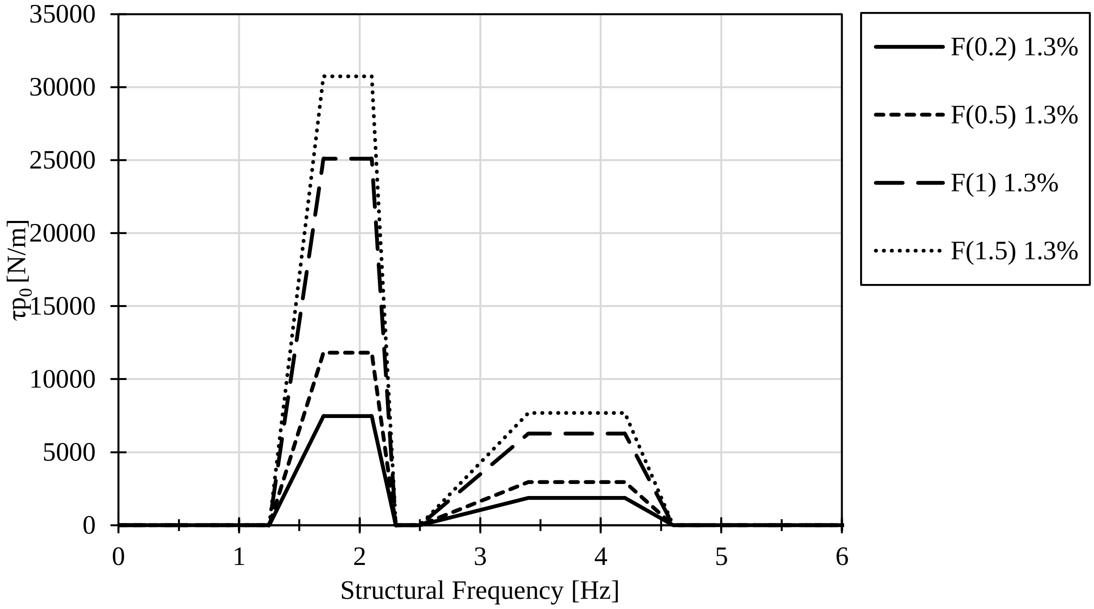


Figure 6.76 Normalized curves in vertical direction for concrete 1.3 % damping

In Figure 6.77 normalized curves for the proposed densities except TC1 are shown for steel with 0.4 % damping. Note that the difference in response between TC2, TC3 and TC4, TC5 increases for the lower damping proposed for steel in comparison with the curve for concrete.

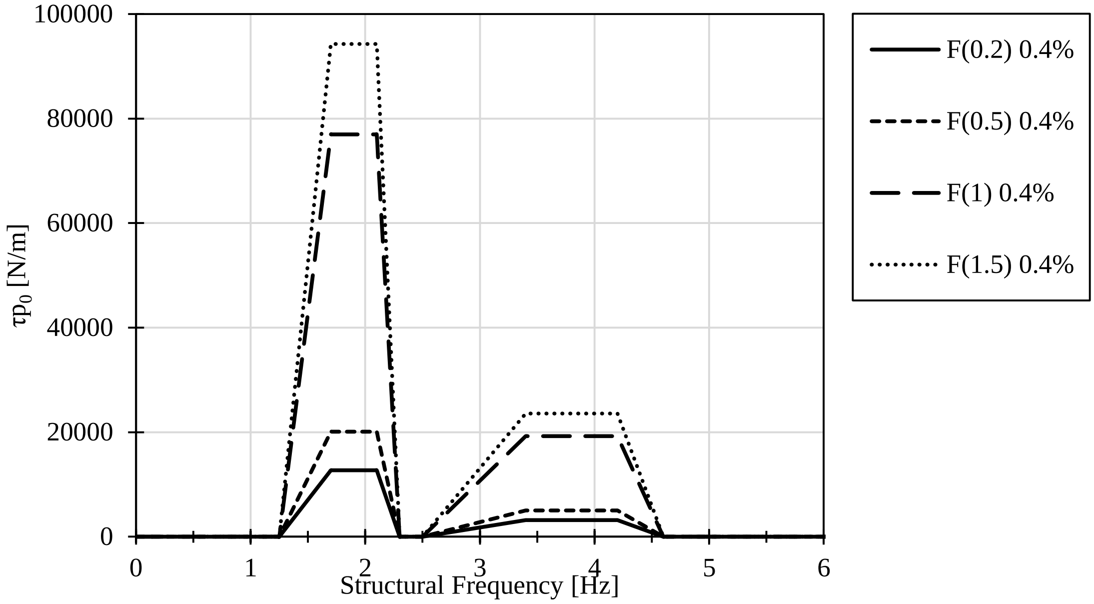


Figure 6.77 Normalized curves in vertical direction for steel, 0.40 % damping.

In Figure 6.78 normalized curves for the proposed densities except TC 1 are shown for timber with 1.5 % damping.

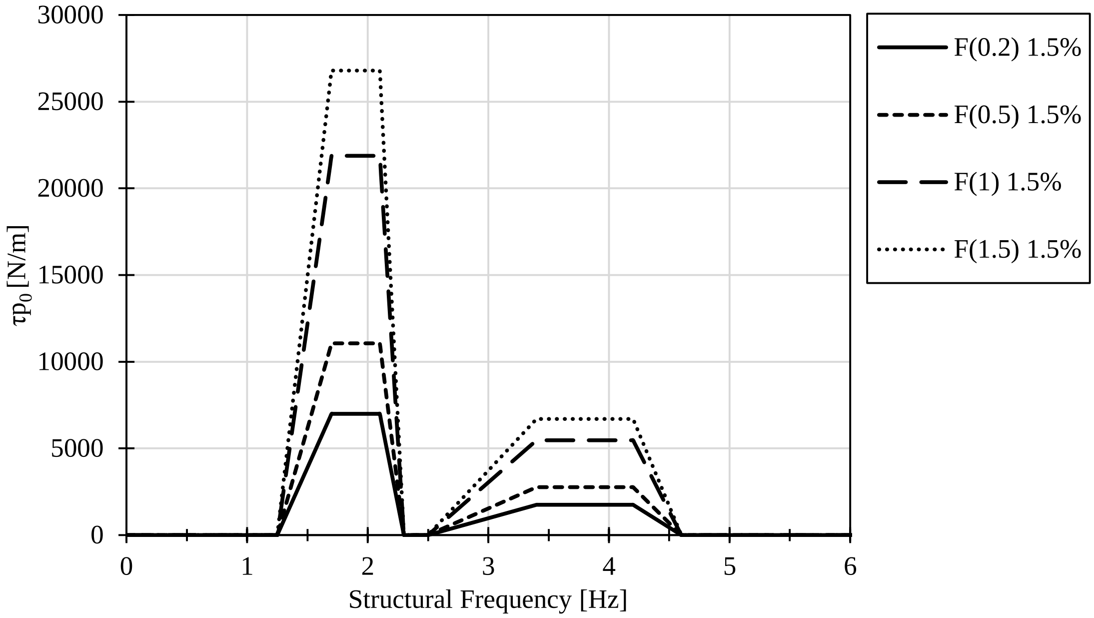


Figure 6.78 Normalized curves in vertical direction for timber, 1.5 % damping.

In Figure 6.79 the normalized curves for TC1 for different materials are plotted with structural damping according to Table 6.16.

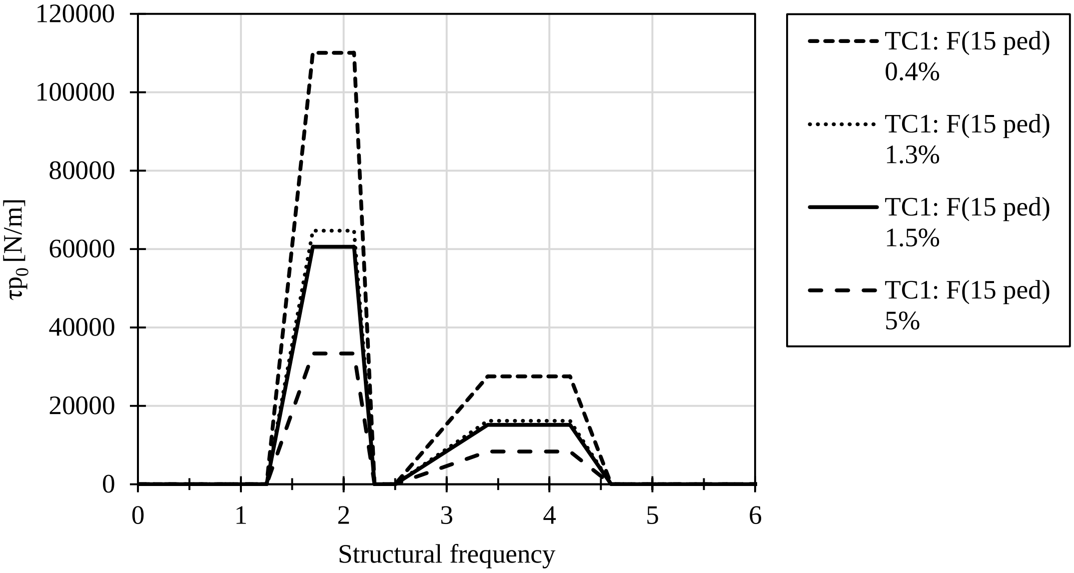


Figure 6.79 Normalized curves for TC1 for different material and corresponding structural damping.

6.5.1.3 Normalized response according to JRC in lateral direction

The same load models as for vertical direction is used for lateral direction. The difference is the load amplitude p_0 and the reduction coefficient ψ. In Table 6.17 input variables for the normalization in lateral direction can be seen.

Table 6.17 Input variables for different materials in lateral direction according to JRC.

	Reference	Concrete	Steel	Timber
Damping ratio [%]	5	1.3	0.4	1.5
τ- uniform	12.74	48.45	148.6	42.24
Po [N]	35	35	35	35

In Figure 6.80 the normalized curves for TC2 to TC5 in lateral direction for 5 % damping can be seen.

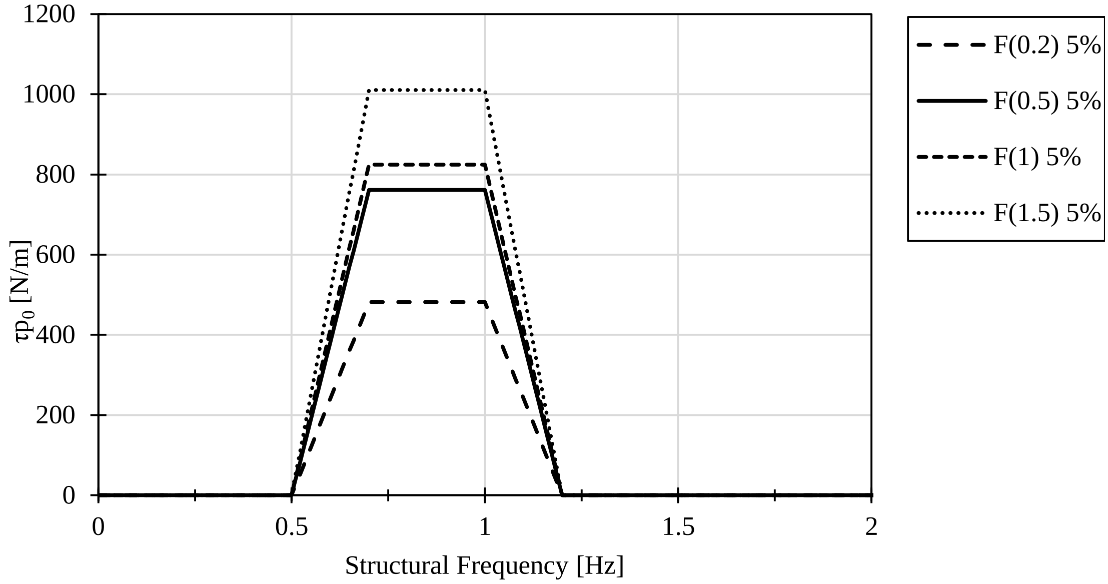


Figure 6.80 Normalized curves in lateral direction for TC2-TC5 for 5 % damping.

In the normalized curves for TC1 to TC5 can be seen which shows the difference for TC1 in comparison with the other traffic classes for a 1 m^2 bridge.

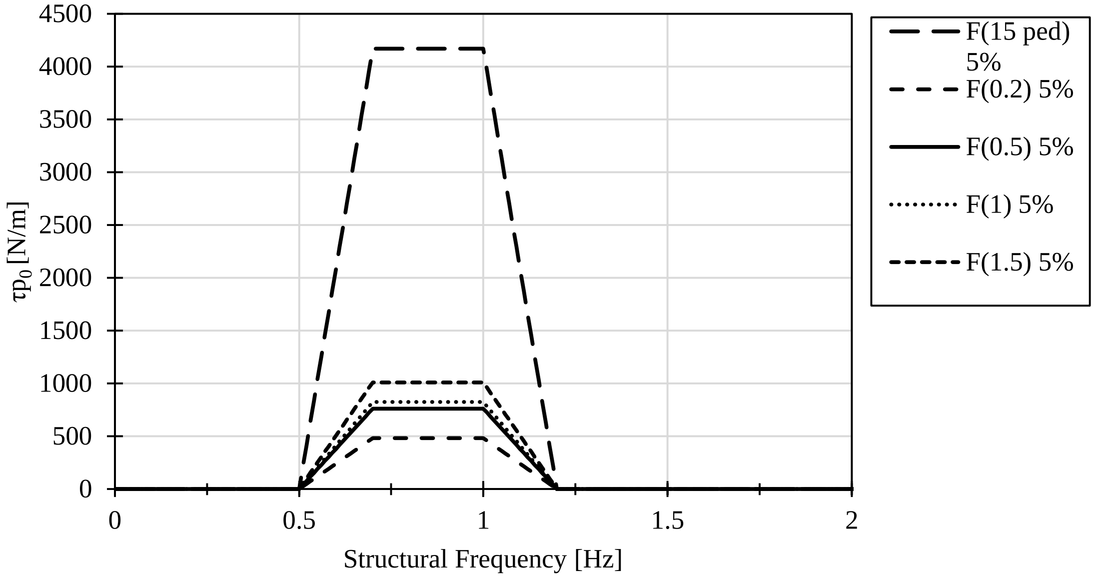


Figure 6.81 Normalized curves in lateral direction for TC1-TC5 for 5 % damping.

In Figure 6.75 the normalized curves for TC1 for three different lengths are plotted for a 1 m wide bridge with 5 % damping. It can be seen that the accelerations due to TC1 for a small bridge will be big.

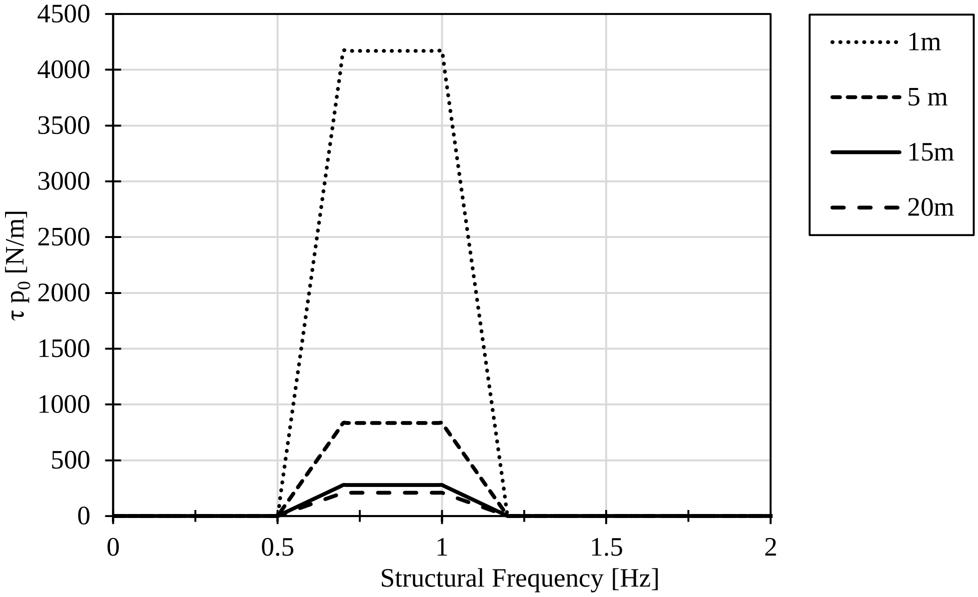


Figure 6.82 Normalized curves in lateral direction for TC1 for different lengths.

In Figure 6.83 Normalized curves in lateral direction for TC2-TC5 for concrete with 1.3 % damping. In Figure 6.83 the normalized curves in lateral direction for TC2 to TC5 for concrete can be seen.

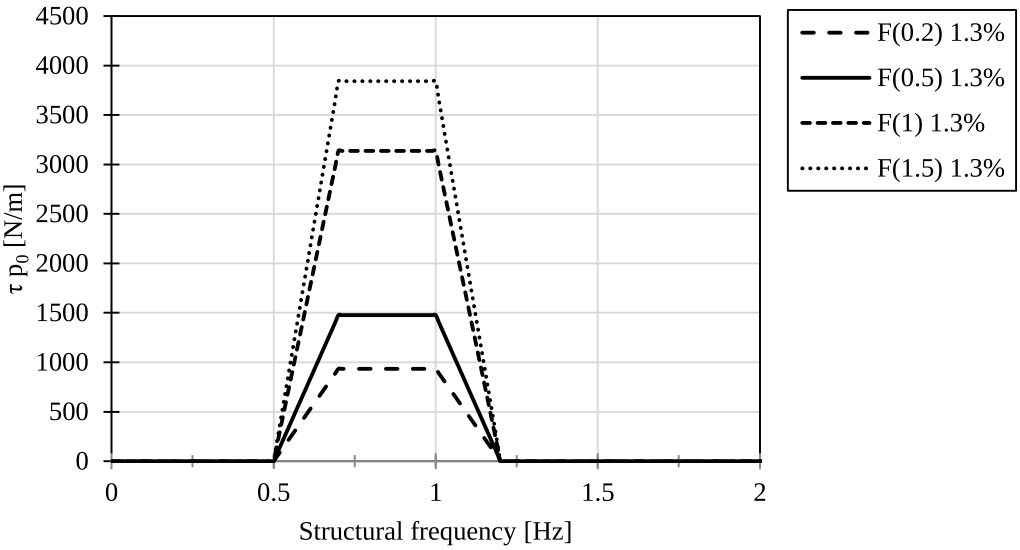


Figure 6.83 Normalized curves in lateral direction for TC2-TC5 for concrete with 1.3 % damping.

In Figure 6.84 the normalized curves in lateral direction for TC2-TC5 for steel can be seen.

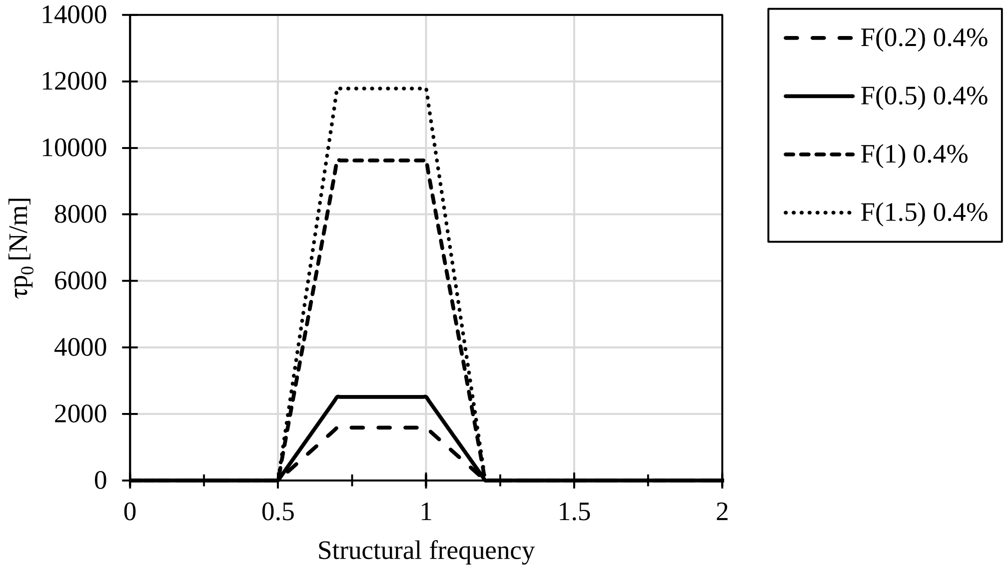


Figure 6.84 Normalized curves in lateral direction for TC5-TC5 for steel with 0.4 % damping.

In Figure 6.85 the normalized curves for TC2 to TC5 for timber with 1.5 % damping can be seen.

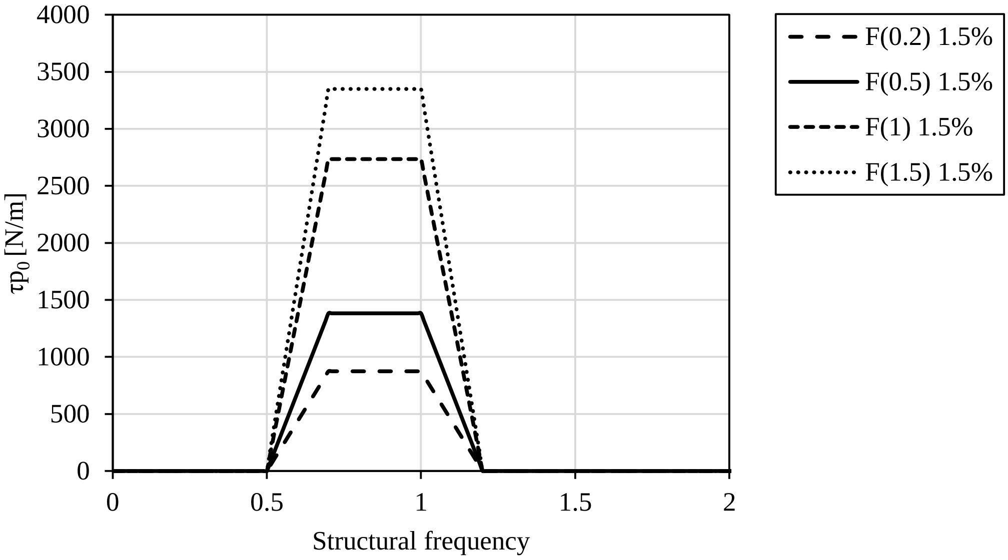


Figure 6.85 Normalized curves in lateral direction for TC2-TC5 for timber with 1.5 % damping.

In Figure 6.86 normalized curves for TC1 for different materials with corresponding damping and reference damping can be seen.

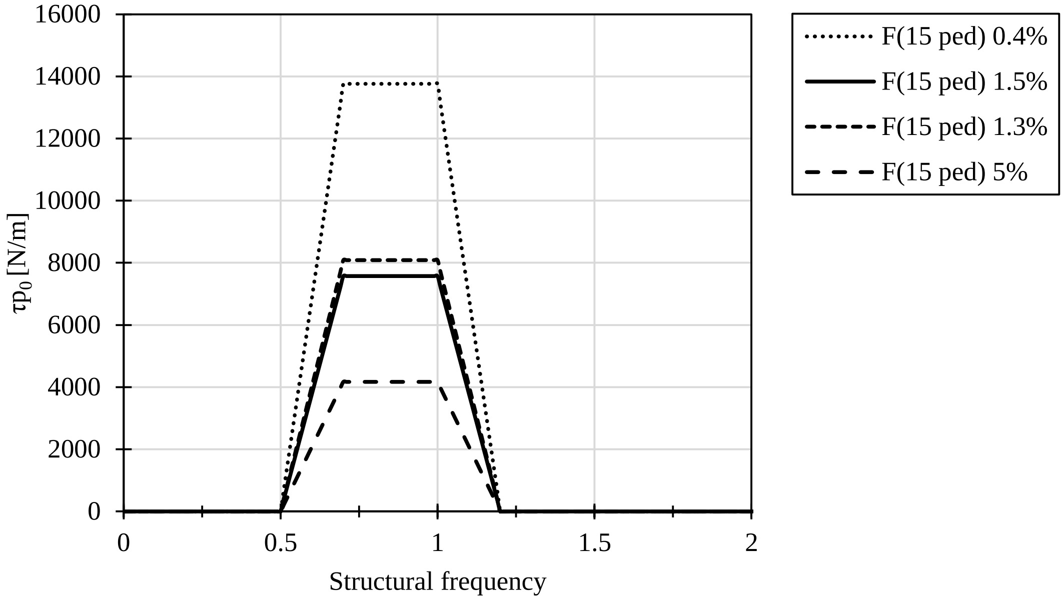


Figure 6.86 Normalized curves for TC1 for different materials with corresponding damping.

6.5.2 Comments and discussion

In this section discussions and comments regarding the normalized curves are presented.

6.5.2.1 Traffic classes

JRC proposes five traffic classes based on expected loading and accepted comfort demands during the bridge's lifetime. The classes are defined by characteristics describing the walking experience.

The lowest class, TC1, is based on a constant number of 15 pedestrians evenly distributed over the loaded area in addition to density defined loads as ped/m^2. Disregarding the expected traffic level it is logical to assume that there will be a smaller group of pedestrians crossing the bridge during its lifetime which makes TC1 reasonable in addition to the other traffic classes. Traffic class 2 to 5 are given as pedestrian densities with the maximum value at 1.5 ped/m^2.

6.5.2.2 Empirical factors, soft aspects

The equivalent number of pedestrians, n', considers the fact that pedestrians synchronize differently depending on the amount of people present at the bridge. For the traffic classes with lower density, TC1 to TC3, the structural damping is considered to affect the synchronization but not for TC4 and TC5 with higher densities. An explanation of this can be that for larger densities walking is obstructed and therefore the dynamic effect of the load decreases whilst the synchronization between pedestrians increase, giving that the structural damping has a lower importance.

6.5.2.3 Max/min values

TC4 and TC5 results in bigger accelerations than TC2 and TC3 due to their higher pedestrian density and their defined equivalent number of pedestrians n'. The difference in n' between traffic classes gives that the difference in acceleration response between the traffic classes are bigger for lower values of structural damping which can be seen in Figure 6.76 to Figure 6.78.

6.5.2.4 Range of frequencies

JRC considers step frequencies between 1.25 to 2.3 Hz in vertical direction for the first harmonic and frequencies between 2.5 and 4.6 Hz for the second harmonic. In lateral direction the considered frequencies are between 0.50 and 1.2 Hz. JRC has based their frequency intervals on empirical investigation. Most common is to define the lateral frequency interval as half of the vertical interval. JRC has chosen to increase the lower limit for the interval to 0.50 instead of 0.60 which would be the case for half the frequency. The second harmonic is not considered for lateral vibrations.

In vertical direction the structural frequencies that give the biggest responses are 1.7 to 2.1 Hz. This is done through the reduction coefficient ψ which can be seen in the normalized curves where these frequencies give rise to the largest response. This is reasonable because these are the step frequencies mentioned in literature as relevant.

The reduction coefficient ψ weighs accelerations due to the second harmony to be lower than for the first. This seems reasonable when other standards weigh the response from the second harmony lower as well. In the literature the effect of second harmonies are considered to be much lower than for the first which can be seen in most guidelines for dynamic analysis.

In lateral direction structural frequencies between 0.7 and 1.0 Hz are weighed to results in the largest accelerations which can be seen for the reduction coefficient ψ for lateral direction. The second harmony is not considered relevant in lateral direction.

7 Results and comments

In this chapter discussions on the analysis done in chapter 5 and the normalization in section 5.6 will be made in order to evaluate what Eurocode and ISO 10137 need to be complemented with to be able to make an accurate and sufficient analysis of expected vibrations in lightweight footbridges. The comparison is systematically done for each required design situations defined by ISO 10137.

ISO 10137 suggests the following design situations according to section 4.2.1:

- One person traversing the structure while another one stands in mid-span, acting as the receiver
- A flow of pedestrians, for example in a group of 8 to 15 people, that depends on the length and the width of the walkway
- The possibility of streams of pedestrians significantly larger than 15 people
- Festive or choreographic events that are relevant

All of these design situations are not covered in ISO 10137 and guidelines for design need to be found elsewhere. Furthermore the guidelines leave a lot of factors to the designer to make reasonable estimates, for example how big the flow of people should be and how it should depend on the geometry of the walkway.

The design situations proposed in ISO 10137 will be compared with guidelines for similar situations as recommended in the studied guidelines. Design situations that are not covered in ISO 10137 will be evaluated with other studied guidelines that are applicable.

ISO 10137 suggest a point load modelled as a Fourier series and an equivalent number of pedestrians. This can be used to cover the two first proposed design situations where a single pedestrian can be modelled with a Fourier sum and a group of people can be modelled with the help of an equivalent number of pedestrians. The last two design situations are not covered in ISO 10137. Pedestrian streams can be modelled with a uniformly distributed load for different pedestrian densities according to other standards as shown in chapter 4. Festive and choreographic events are not completely presented in any guideline and will only be discussed based on compiled knowledge.

The normalized acceleration response is compared for 5.0% structural damping which is referred to as the reference beam or reference damping. In this way the load models will be compared independent on damping ratio. The standards propose different structural damping ratios for the same construction materials. The comparison includes a study of how the normalized acceleration response varies for damping ratios according to reinforced concrete, steel and timber bridges.

Vertical and lateral load models are compared for all considered standards and guidelines when recommended.

7.1 Comparison between concentrated loads and ISO 10137

In this section the first design situation will be presented simulating a single pedestrian crossing a bridge. It is recommended in ISO 10137 to be modeled as a concentrated load moving across the span. Similar load models are given in UK-NA, SYNPEX and Sétra which are compared with ISO 10137 in this section. All models are simplified to be stationary at the middle of them bridge. The models will be compared and discussed regarding acceleration response, critical frequency ranges, and structural damping ratios.

Considered load models in this section are:

- ISO 10137
- UK-NA
- SYNPEX
- Sétra

7.1.1 UK National Annex in comparison to ISO 10137

In this section the obtained response with the load model according to UK-NA will be compared to the obtained response with ISO 10137. This will be done for a reference damping ratio of 5.0 % and for the materials concrete, steel and timber with corresponding damping.

7.1.1.1 Reference damping – 5.0%

The normalized curves for UK point load and ISO 10137 for one single pedestrian are plotted together in Figure 7.1 for 5.0 % damping.

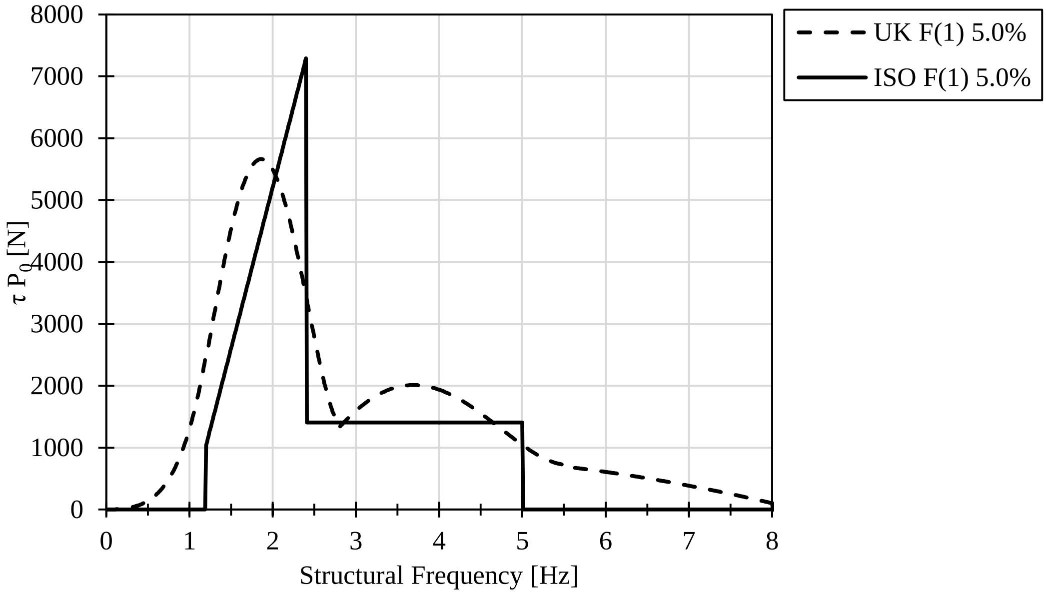


Figure 7.1. Normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 with 5 % damping.

It can be seen that ISO 10137 results in a larger maximum value than UK-NA for the same level of damping but that the maximum values are obtained for different structural frequencies. ISO 10137 weighs walking frequencies differently than UK-NA where the acceleration response increases from 1.20 up to 2.4 Hz whereas UK-NA exhibits the largest accelerations as a peak value at 1.8 Hz. It can be seen that larger acceleration response is obtained with UK-NA in comparison with ISO 10137 for lower frequencies below approximately 2 Hz.

UK-NA weighs the effect due to second harmonies to be larger than ISO 10137 with a peak at approximately 3.7 Hz. The maximum value due to second harmonics for UK-NA is approximately 0.25% larger than the value obtained with ISO 10137

7.1.1.2 Concrete

In Figure 7.2 normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 for concrete is plotted.

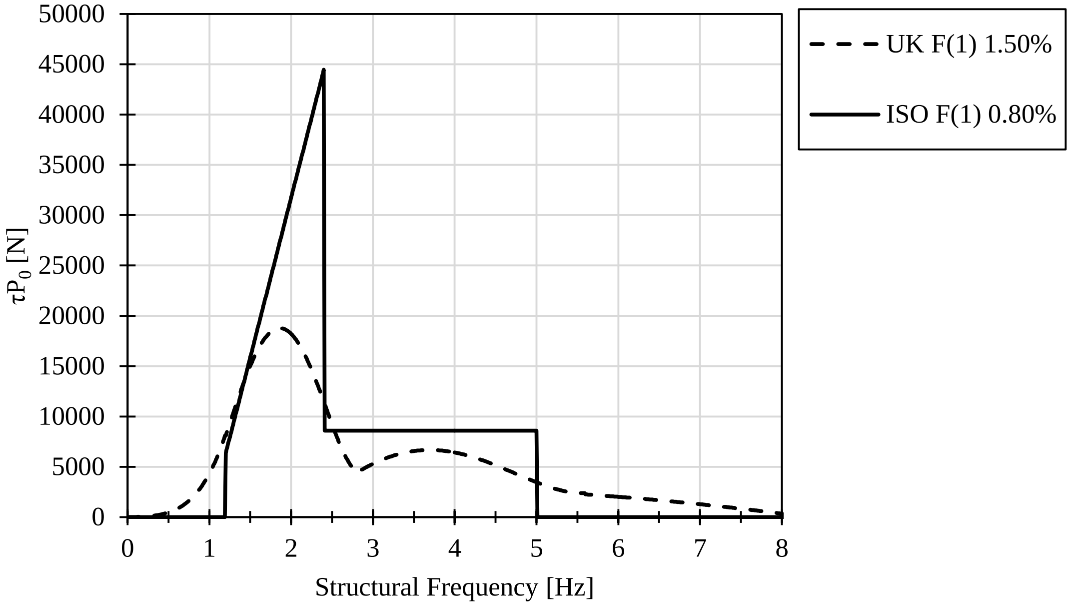


Figure 7.2. Normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 with corresponding damping for concrete.

For concrete the proposed damping ratio differs between UK-NA and ISO 10137. This affects the obtained accelerations so that ISO 10137 results in larger values than UK-NA for almost all structural frequencies, except one point at approximately 2.4 Hz where the acceleration response is marginally smaller than UK-NA. The normalization factor τ is different for the two damping ratios which is the factor affecting the results as the structural damping is not included in the load models. For a damping ratio of 1.5 % τ has the value 66.35 and for 0.80% τ has the value 122.6.

7.1.1.3 Steel

In Figure 7.3 normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 for steel is plotted.

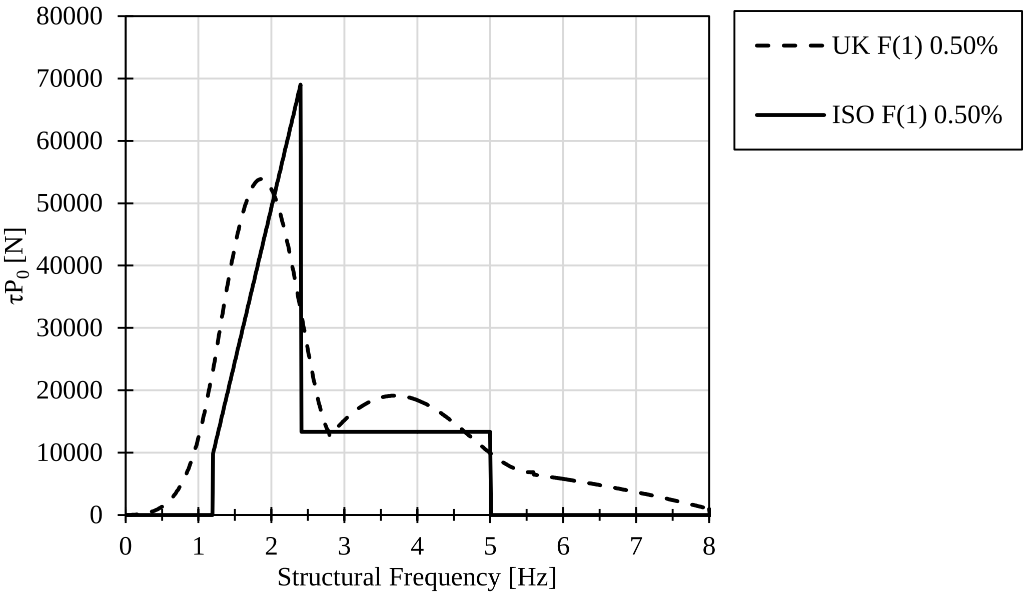


Figure 7.3. Normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 with corresponding damping for steel.

7.1.1.4 Timber

In Figure 7.4 normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 for timber is plotted.

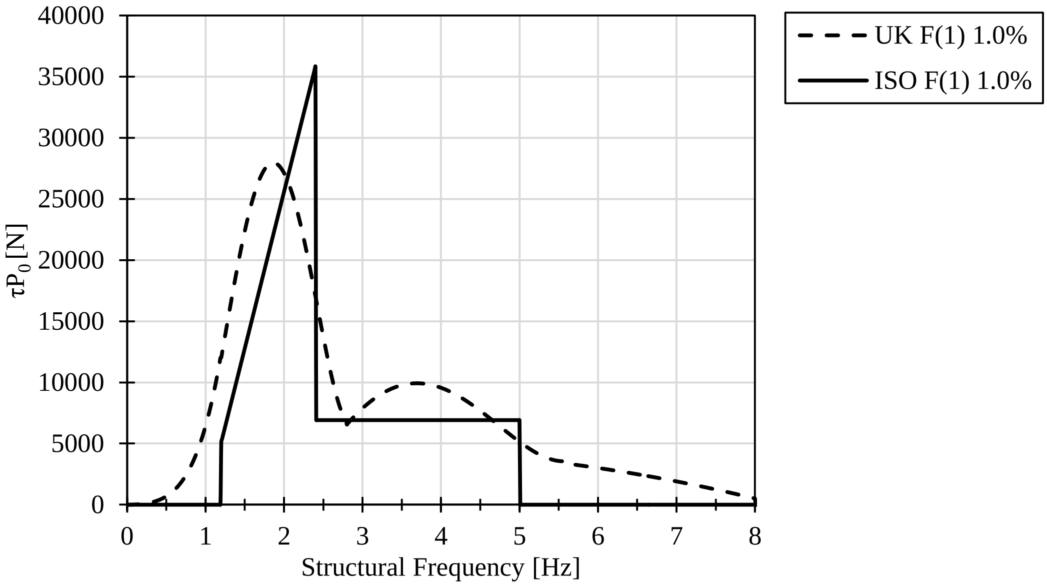


Figure 7.4. Normalized curves for concentrated load for 1 pedestrian according to UK and ISO 10137 with corresponding damping for timber.

For materials with the same damping as for steel and timber the relationship between the normalized curves for UK-NA and ISO 10137 is the same. The factors determining the difference are load amplitude and the weighing of frequencies as discussed about the normalized curves for a reference damping ratio of 5%.

7.1.2 SYNPEX in comparison to ISO 10137

SYNPEX and ISO 10137 presents load models for simulating a single pedestrian modeled as a concentrated load moving over the span. ISO 10137 recommends a load model that is adjustable to both vertical and lateral loading. SYNPEX recommends two different load models one for vertical and one for lateral loading. All models are defined as Fourier series and compared in the following chapters.

SYNPEX recommends applying the concentrated load as moving over the span in the same way as ISO 10137. The models from both standards have been simplified as to stationary which makes them comparable though the real acceleration response will be lower.

7.1.2.1 Reference damping – 5.0%

The normalized acceleration response of the concentrated load model for vertical direction propose by SYNPEX for a single pedestrian is plotted with the corresponding curve from the ISO 10137 standard in Figure 7.5 with 5.0% damping ratio.

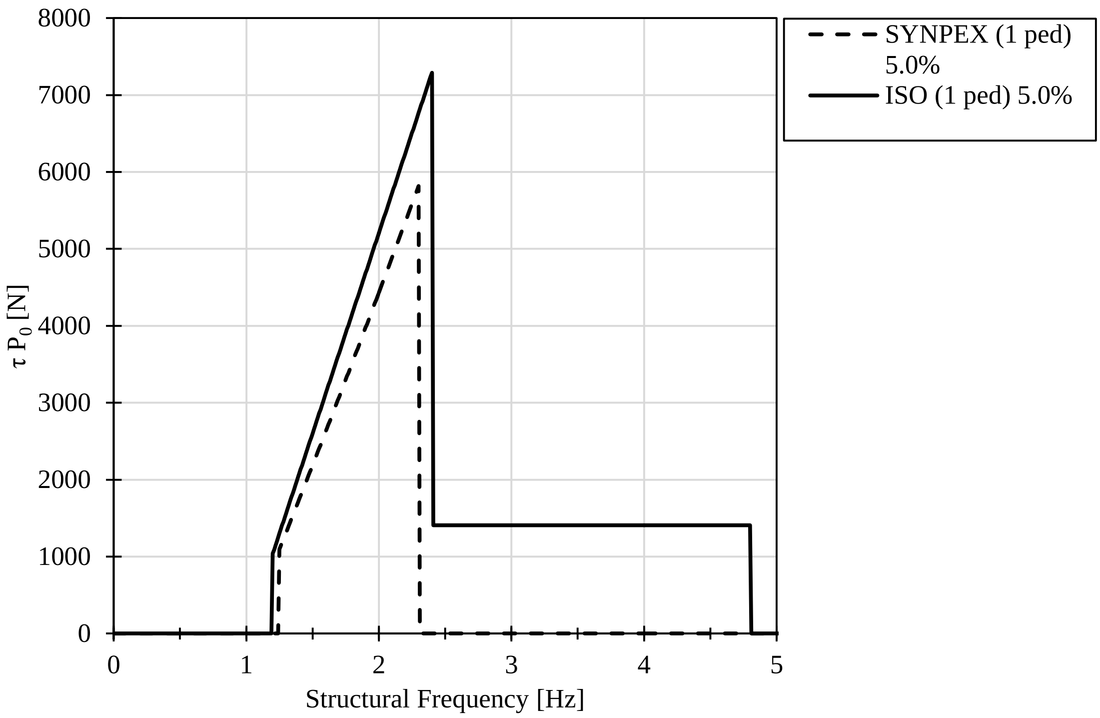


Figure 7.5 Comparison of normalized acceleration response curves of SYNPEX and ISO 10137 single pedestrians with 5.0% damping ratio.

The curves are compared for the same damping ratio which makes the response only dependent on Fourier coefficients and frequency intervals from the two standards. The curves have the same shape for the first harmonic with similar inclination and frequency intervals. The ISO 10137 curve has a higher maximum than SYNPEX approximately 25% higher generating higher load amplitude at 2.5 Hz. The acceleration response from the second harmonic is much higher for SYNPEX than ISO 10137.

The second harmony is not considered in SYNPEX based on the theory in the standard. ISO 10137 takes the second harmony into account with about 20% of the maximum value for the first harmony. In the following comparisons between ISO 10137 and SYNPEX only the first harmony will be considered.

The considered frequency intervals are approximately the same in the standards though ISO 10137 has a slightly wider range for the first harmonic. The frequency range for the second harmonic begins where the first ends and leaves no gap.

The lateral load models are for damping ratio 5.0% compared with their normalized acceleration response curves in Figure 7.6.

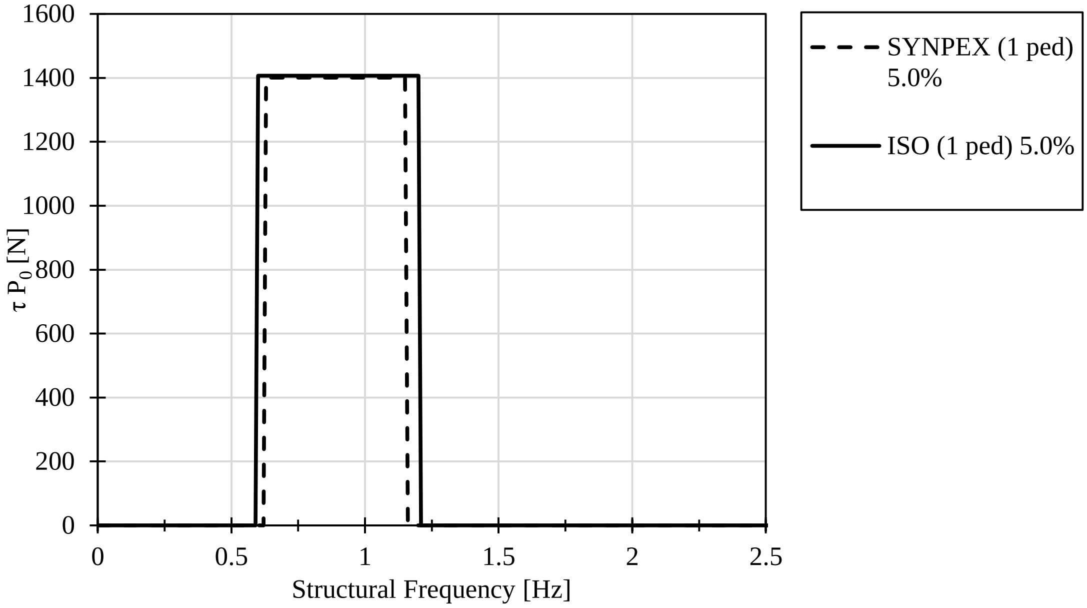


Figure 7.6 Comparison of normalized acceleration response curves in lateral direction of SYNPEX and ISO 10137 single pedestrians with damping ratio 5.0%.

The curves in Figure 7.6 are very similar with the same maximum for their defined frequency interval. Both curves are constant in the interval as the Fourier coefficients are constant and independent on step frequency. The frequency interval defined by SYNPEX as half the step frequency of the vertical action is shorter than for ISO 10137. Though SYNPEX is inconsistent when it comes to critical frequencies, an overall recommendation in the standard is between 0.50 and 1.2 Hz which not is applied here.

7.1.2.2 Concrete

In Figure 7.7 is the two models compared for vertical loading with damping ratios recommended by the standards for concrete structures. ISO 10137 recommends a damping ratio of 0.80% and SYNPEX 1.3%.

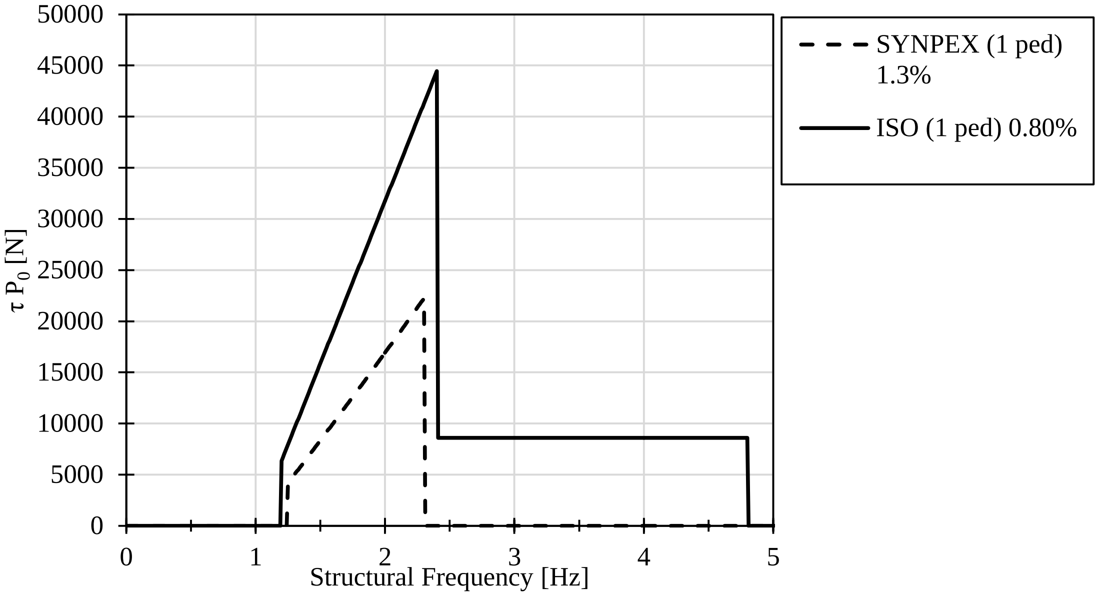


Figure 7.7 *Comparison of normalized acceleration response curves of SYNPEX and ISO 10137 single pedestrians with damping ratio according to concrete structures.*

By applying the loads for vertical direction on a concrete structure the acceleration will approximately twice as high for the ISO 10137 load model than SYNPEX load model. The difference between the two curves is the normalization factor τ due to the difference in damping ratio as the load models are independent on damping ratios.

In are the normalized acceleration response plotted for damping ratios according to concrete structures.

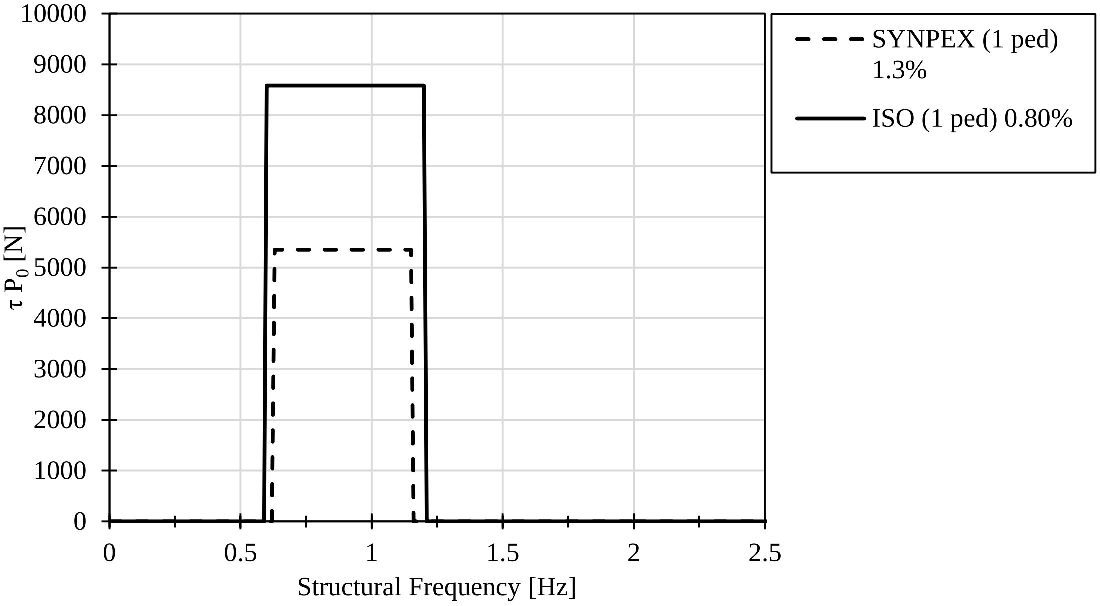


Figure 7.8 *Comparison of normalized acceleration response curves in lateral direction of SYNPEX and ISO 10137 single pedestrians with damping ratio according to concrete structures.*

In lateral direction the acceleration will almost be twice as high by applying the load model given in ISO 10137 than the model in SYNPEX. As the curves have equal maximum value for the same damping ratio only the normalization factor separates the curves.

In the plot of both vertical and lateral direction the damping is the separating factor. In SYNPEX is the damping ratio recommended to be 1.3% which corresponds to a normalization factor equal to 76.41. ISO 10137 recommends 0.80% damping corresponding to a normalization factor of 122.6. The normalization factor is almost twice as high which governing factor in the divergence between the two standards.

7.1.2.3 Steel

In Figure 7.9 the normalized acceleration response for vertical direction is compared for structures with damping ratios according to steel. SYNPEX suggest the damping ratio to be 0.40% and ISO 10137 0.50%.

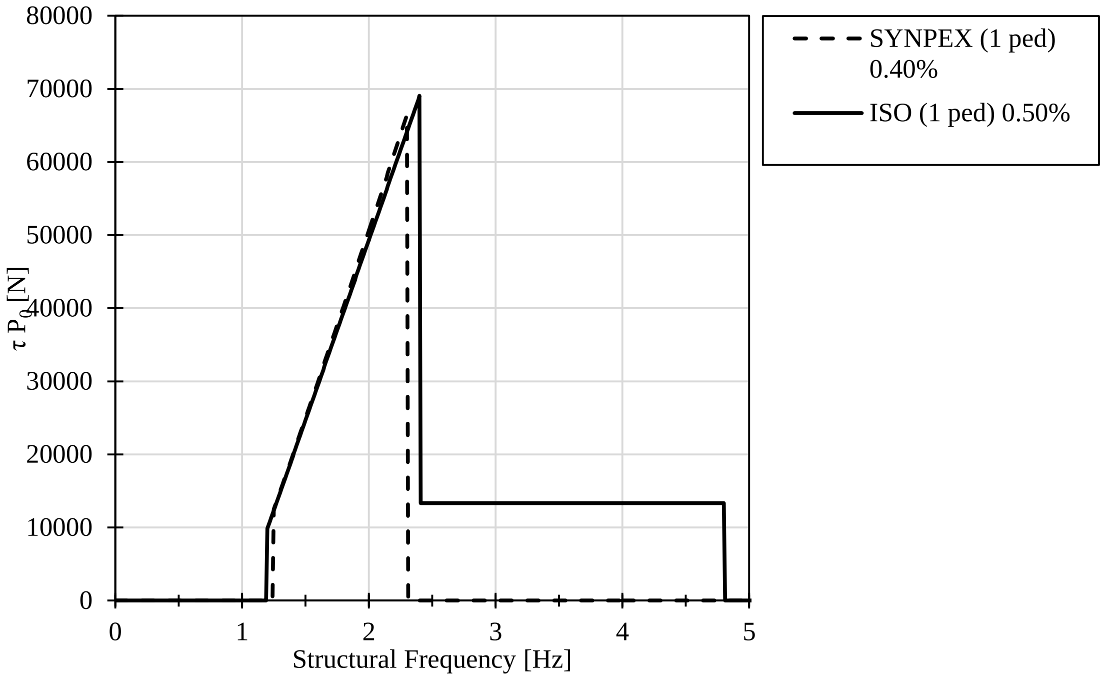


Figure 7.9 *Comparison of normalized acceleration response curves of SYNPEX and ISO 10137 single pedestrians with damping ratio according to steel structures.*

The curves in Figure 7.9 are on the same level for the first harmonic. The frequency interval for ISO 10137 has a higher upper limit which makes the maximum higher than for SYNPEX. In the interval where SYNPEX in defined the curves are the same. SYNPEX has a lower damping ratio than ISO 10137 which makes the normalization factor τ higher than for ISO 10137 which compensates for the difference shown in Figure 7.5 where the same damping is used.

In Figure 7.10 are the normalized acceleration response plotted for lateral loading with damping ratios according to steel structures.

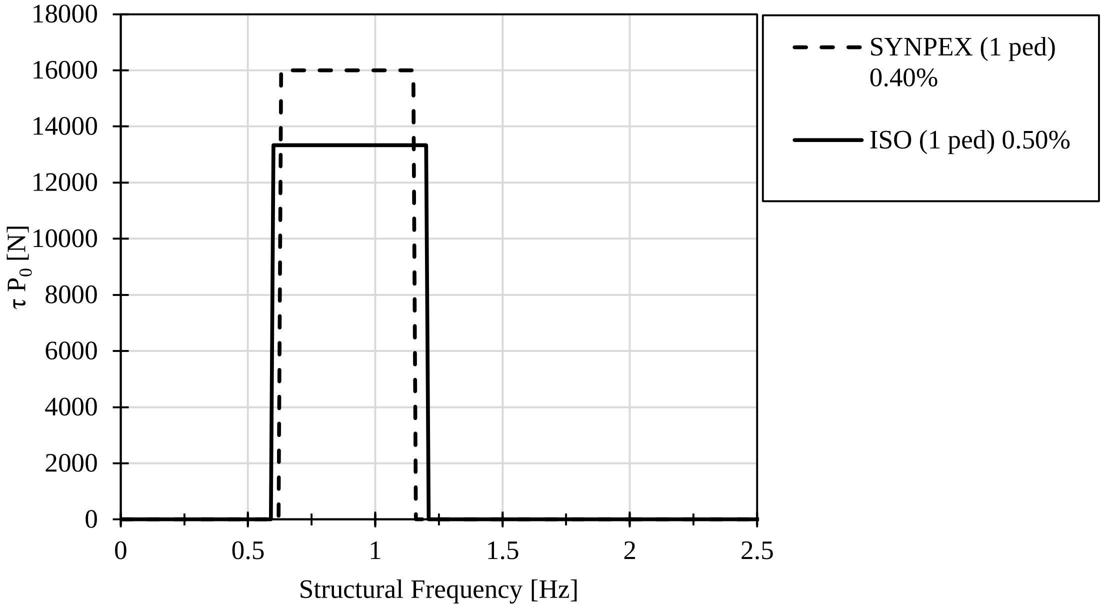


Figure 7.10 Comparison of normalized acceleration response curves in lateral direction of SYNPEX and ISO 10137 single pedestrians with damping ratio according to steel structures.

In lateral direction SYNPEX has a higher maximum value than ISO 10137 due to the difference in damping ratio.

SYNPEX recommends a slightly lower damping ratio than ISO 10137 for steel structures which makes the SYNPEX curve increase compared to ISO 10137 curves. The damping ratios are similar and low which is reasonable according to literature. As seen for lateral loading even small changes in damping ratio can have large impact. Though for vertical direction the damping ratio compensated for SYNPEX to increase to the same level as ISO 10137.

7.1.2.4 Timber

In Figure 7.11 the normalized acceleration responses are plotted for damping ratios according to timber structures.

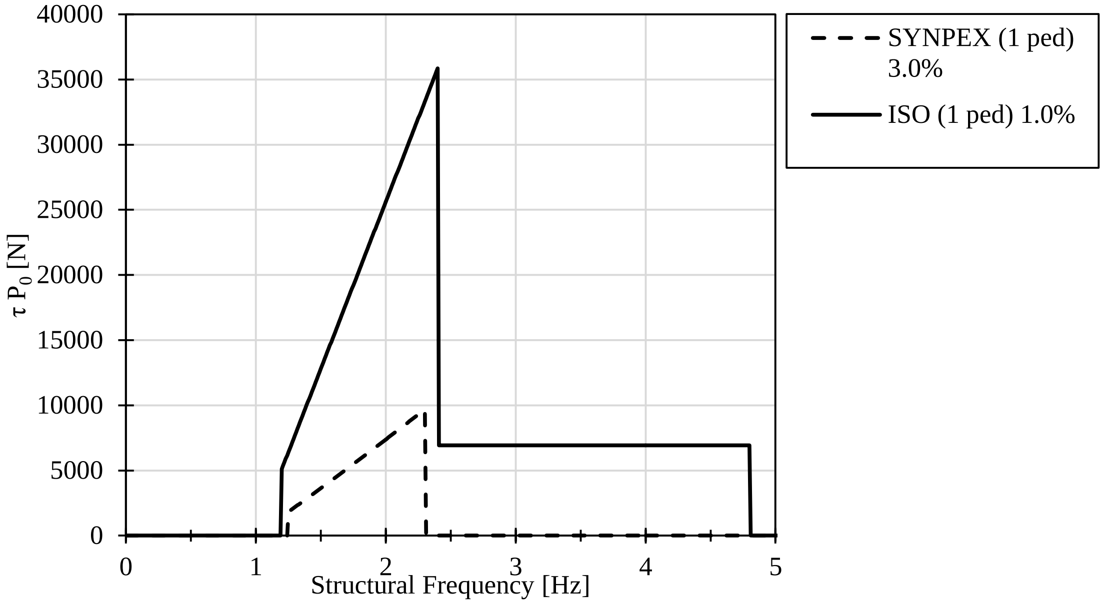


Figure 7.11 Comparison of normalized acceleration response curves in vertical direction of SYNPEX and ISO 10137 single pedestrians with damping ratio according to timber structures.

The large difference in damping ratio highly affects the normalized acceleration response curves for timber. The maximum value is over three times higher for ISO 10137 than SYNPEX.

In Figure 7.12 the normalized acceleration response curves are plotted with damping ratios according to timber structures.

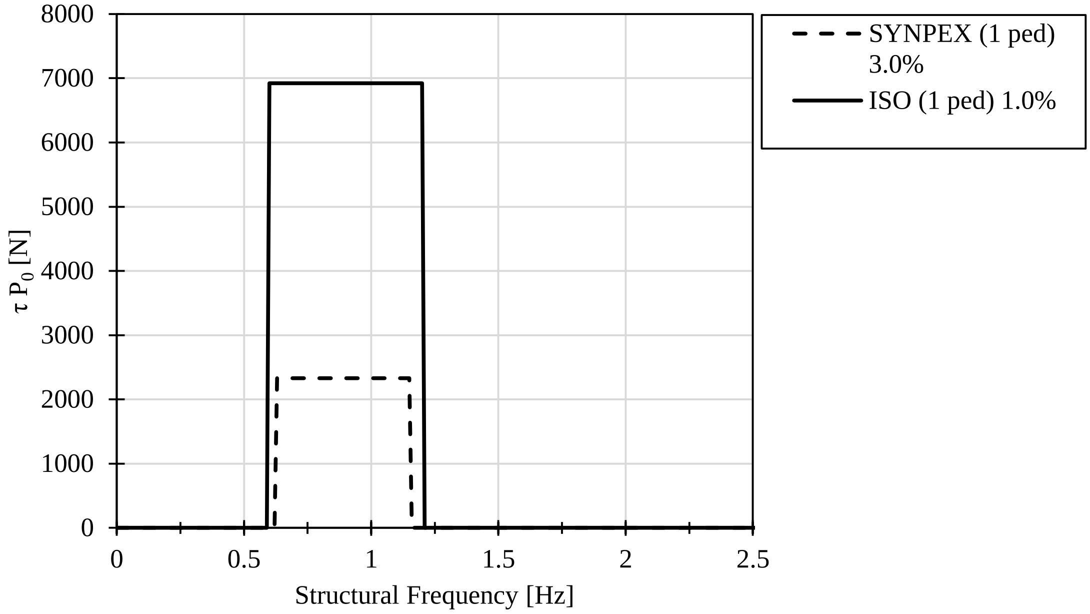


Figure 7.12 Comparison of normalized acceleration response curves in lateral direction of SYNPEX and ISO 10137 single pedestrians with damping ratio according to timber structures.

The normalized acceleration response curves diverge for timber structures with three times higher damping in SYNPEX than ISO 10137.

The damping ratios recommended by two standards diverge the most for timber structures. SYNPEX recommend three times higher damping ratio, 3.0% in comparison to ISO 10137 recommendation of 1.0% damping. The large difference in damping ratios affects the normalization factor τ where 3.0% damping corresponds to τ equal to 33.3 and 1.0% to 98.8. The damping ratio for SYNPEX can be regarded as too high, no other standard recommends values over 1.5%. A too high damping ratio means that the acceleration response will decrease, it is always conservative to assign a lower damping ratio.

7.1.3 Sétra in comparison to ISO 10137

Sétra and ISO 10137 propose moving concentrated load models to simulate a single pedestrian crossing the bridge. The model in Sétra is applicable in vertical, lateral and longitudinal direction of loading and the load model in ISO 10137 can be applied for vertical and lateral directions. The models are all based on Fourier series which in this comparison on the first Fourier sum is regarded.

In this chapter is the normalized acceleration response from the load models proposed in the standards compared for vertical and lateral loading. The responses are compared for a reference damping of 5.0% and by the standards recommended damping ratios according to concrete, steel and timber bridges.

7.1.3.1 Reference damping – 5.0%

The models are compared for the same reference damping ratio of 5.0% firstly in vertical and secondly in lateral direction. The differences between the acceleration responses will be results of Fourier coefficients and frequency intervals.

In Figure 7.13 is the normalized acceleration response in vertical direction plotted for Sétra and ISO 10137.

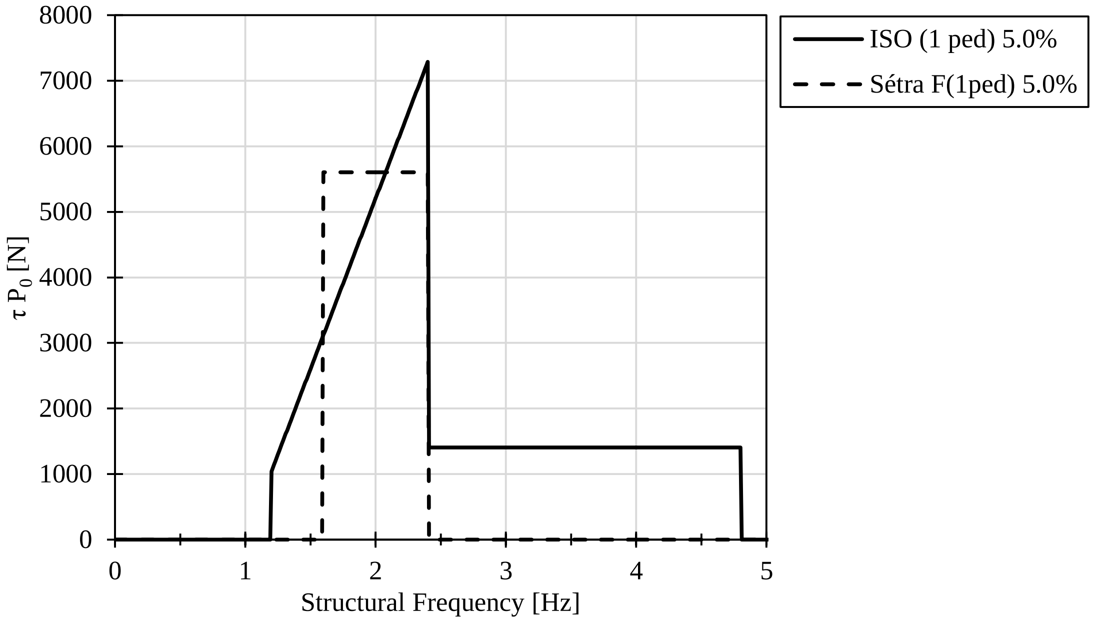


Figure 7.13 Normalized acceleration response of a single pedestrian with vertical load models according to Sétra and ISO 10137 and 5.0% damping.

As seen in the comparison above the load model given in ISO 10137 is defined for a wider range of frequencies. The lower limit is in ISO 10137 set to 1.2 Hz compared to 1.6 Hz in Sétra. The upper limit for the first harmony is equal between the models and set to 2.4 Hz. ISO 10137 is defined for the second harmony but not Sétra. It is recommended in Sétra for practical reasons to only consider the first Fourier sum which means that the second harmony is neglected automatically.

The acceleration response in Sétra is constant for the critical frequencies compared to ISO 10137 which is linear increasing. The response is higher for Sétra between 1.6 and 2.1 Hz. ISO 10137 has the highest maximum response at 2.4 Hz.

In Figure 7.14 is the normalized acceleration response in lateral direction plotted for Sétra and ISO 10137.

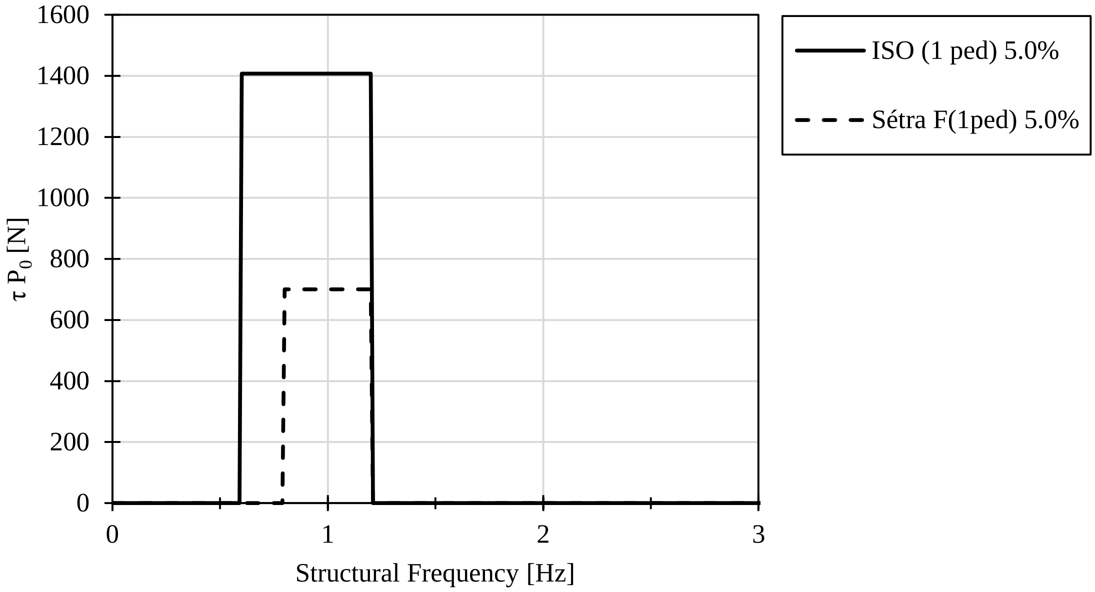


Figure 7.14 Normalized acceleration response of a single pedestrian with lateral load models according to Sétra and ISO 10137 and 5.0% damping.

In the comparison above of lateral acceleration response it can be seen that ISO 10137 is defined for a wider frequency range than Sétra. The critical frequency range for Sétra is quite small only covering frequencies between 0.80 and 1.2 Hz. The acceleration response is constant for both models in their defined frequency ranges with approximately twice as high response with ISO 10137.

7.1.3.2 Concrete

In Figure 7.15 is the normalized acceleration response in vertical direction plotted for Sétra and ISO 10137 with damping ratios recommended for concrete structures.

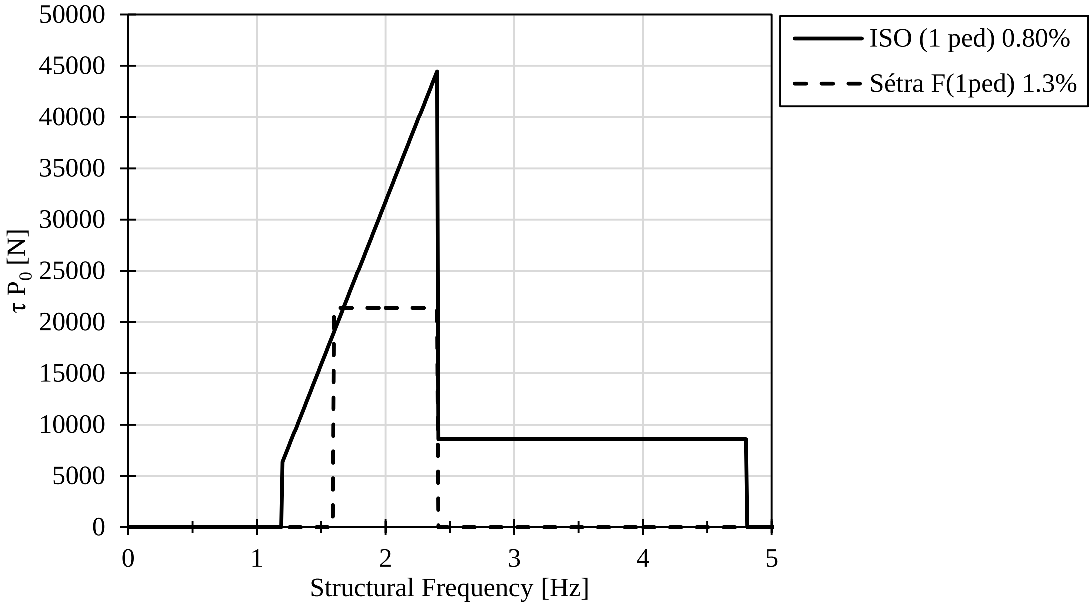


Figure 7.15 Normalized acceleration response of a single pedestrian with vertical load models according to Sétra and ISO 10137 for concrete bridges

The acceleration response for concrete structures is higher for ISO 10137 for all defined frequencies. This is explained by the differences in recommended damping ratios where ISO 10137 has a lower ratio which gives a higher response.

In Figure 7.16 is the normalized acceleration response in lateral direction plotted for Sétra and ISO 10137 with damping ratios recommended for concrete structures.

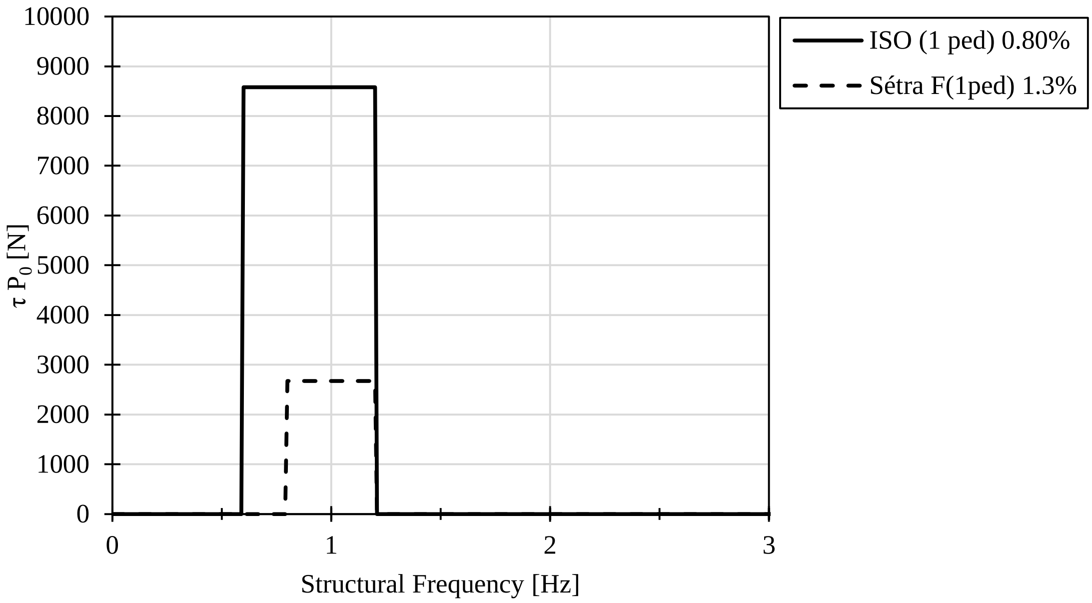


Figure 7.16 Normalized acceleration response of a single pedestrian with lateral load models according to Sétra and ISO 10137 for concrete bridges

As seen in the figure above the acceleration response is much higher for ISO 10137 compared to Sétra. The differences in damping ratio increase the gap between the models.

7.1.3.3 Steel

In Figure 7.17 is the normalized acceleration response in vertical direction plotted for Sétra and ISO 10137 with damping ratios recommended for steel structures.

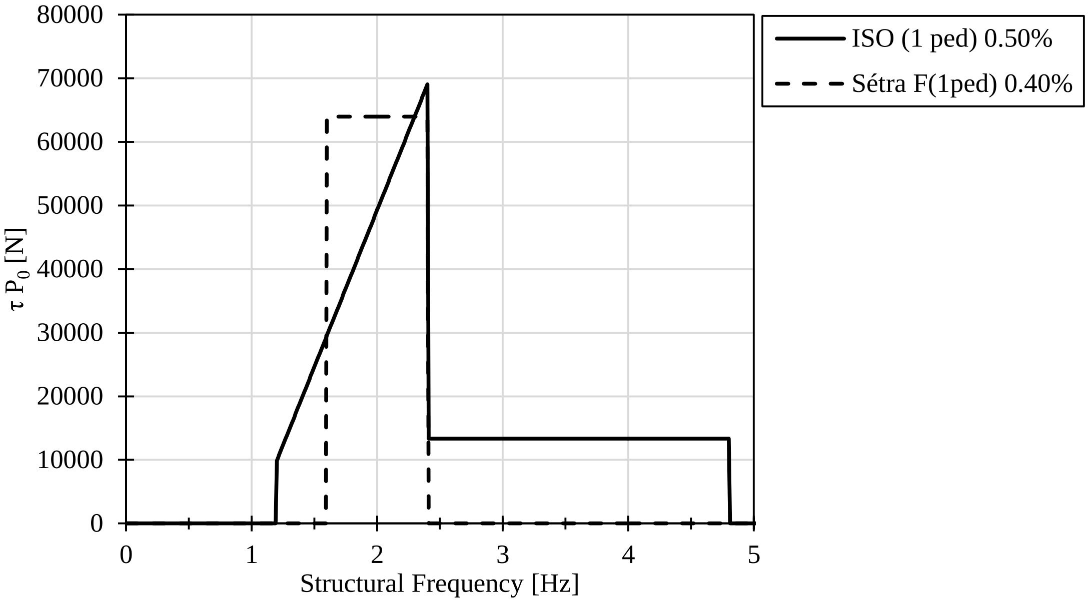


Figure 7.17 Normalized acceleration response of a single pedestrian with vertical load models according to Sétra and ISO 10137 for steel bridges.

The damping ratio for steel structures is similar with 0.50% damping for ISO 10137 and 0.40% for Sétra. The maximum response in vertical direction is approximately the same and Sétra has higher response than ISO 10137 for the most common step frequencies just below 2.0 Hz.

In Figure 7.18 is the normalized acceleration response in lateral direction plotted for Sétra and ISO 10137 with damping ratios recommended for steel structures.

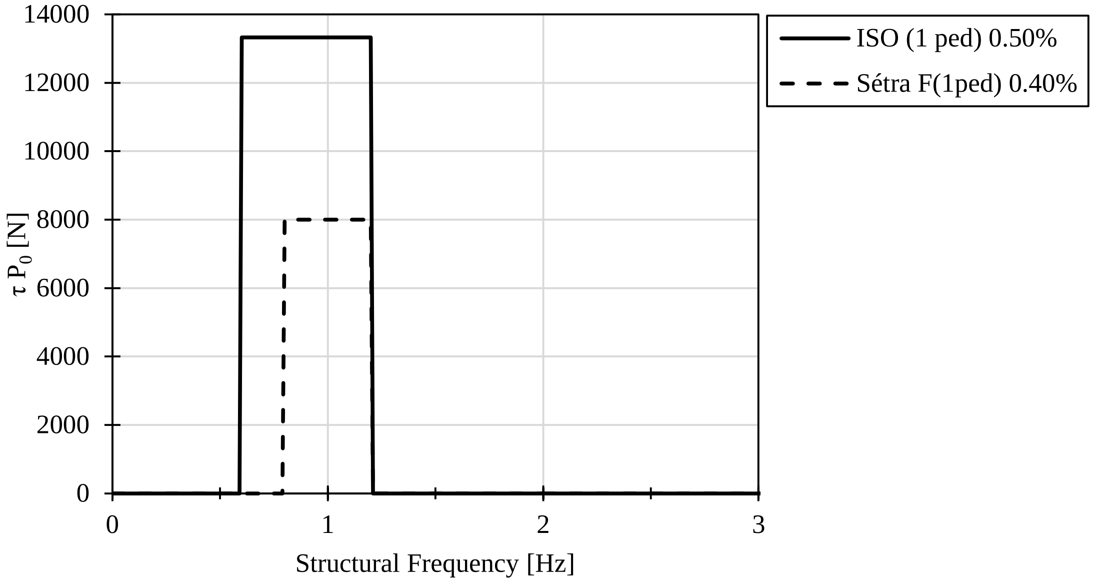


Figure 7.18 Normalized acceleration response of a single pedestrian with lateral load models according to Sétra and ISO 10137 for steel bridges.

The lateral acceleration response is still higher for ISO 10137 even though the gap has decreased due to the differences in damping ratio. The difference in response is still significant and about 50%.

7.1.3.4 Timber

In Figure 7.19 is the normalized acceleration response in vertical direction plotted for Sétra and ISO 10137 with damping ratios recommended for timber structures.

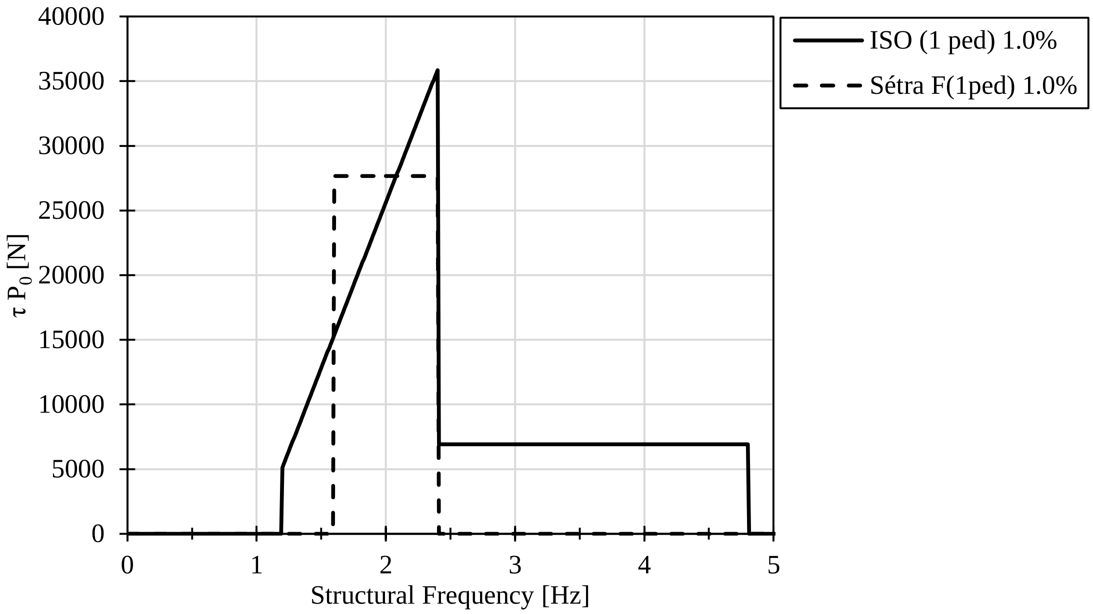


Figure 7.19 Normalized acceleration response of a single pedestrian with vertical load models according to Sétra and ISO 10137 for timber bridges.

The damping ratio for timber structures is recommended by both standards to 1.0%. This means that the relation between the responses is the same as for the comparison of 5.0% damping which applies for vertical and lateral direction.

In Figure 7.20 is the normalized acceleration response in vertical direction plotted for Sétra and ISO 10137 with damping ratios recommended for timber structures.

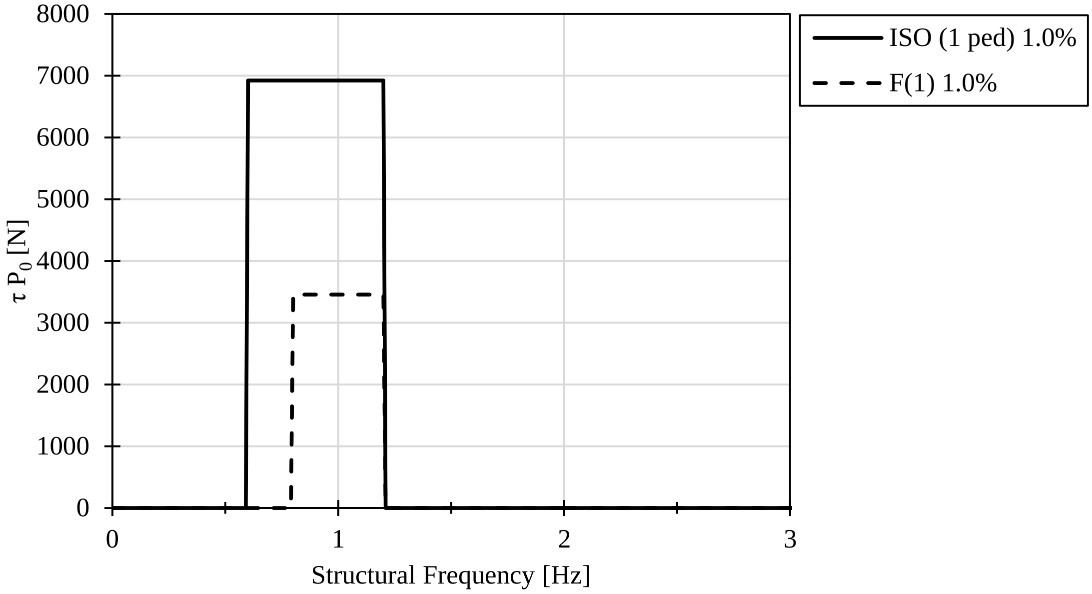


Figure 7.20 Normalized acceleration response of a single pedestrian with lateral load models according to Sétra and ISO 10137 for timber bridges.

7.2 Comparison between groups of pedestrian

Groups of pedestrians can be modelled according to ISO 10137 with an equivalent number of pedestrians and the load model defined for one single pedestrian. In this section a comparison between groups of people modelled with ISO 10137 and UK-NA is done which are the two standards defining groups with the help of an equivalent number of pedestrians. The concentrated load models which do not include an applicable equivalent number of pedestrian are not considered. The synchronization among pedestrians has to be considered otherwise too high and unrealistic results are given.

The load models will be compared for 8 and 15 pedestrians according to the second design situation in ISO 10137. The discussion regards primarily normalized acceleration response and equivalent number of pedestrians.

The considered standards in this section are:

- ISO 10137
- UK-NA

7.2.1 UK National Annex in comparison with ISO 10137

In this section acceleration response for group sizes of 8 and 15 people are compared for UK-NA and ISO 10137 for a reference damping of 5 % and for the materials steel, timber and concrete with corresponding damping.

7.2.1.1 Normalized curves for reference damping, steel and timber

The normalized curves for groups of people for UK-NA point load and ISO 10137 are plotted together in Figure 7.21 for 5 % damping for group sizes of 8 and 15 people as the interval is defined in ISO 10137.

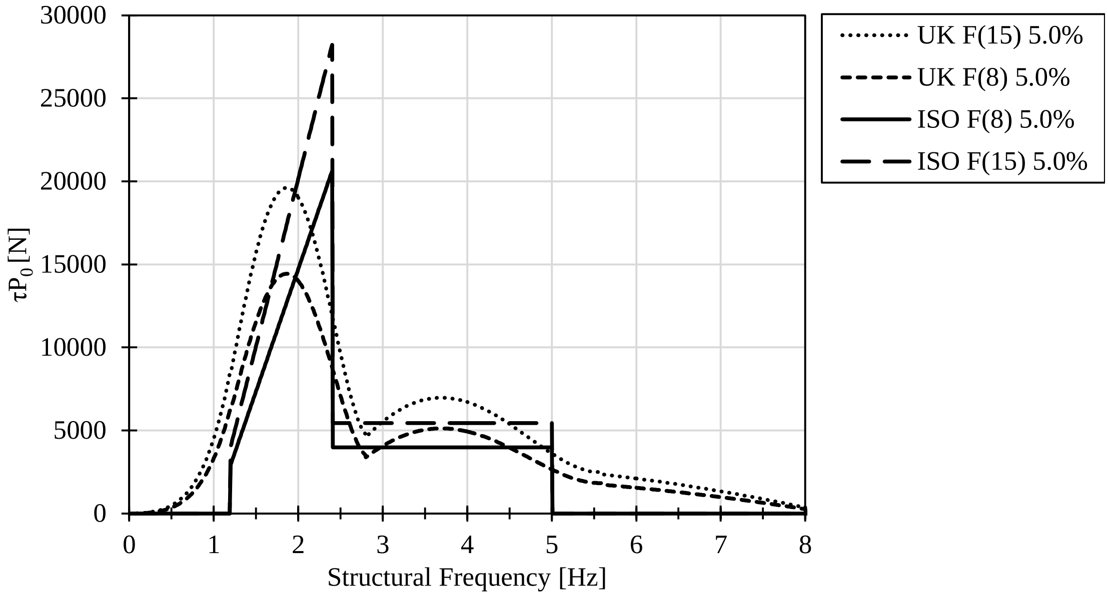


Figure 7.21 Normalized curves for concentrated load for several pedestrians according to UK-NA and ISO 10137 with 5 % damping.

It can be seen that ISO 10137 results in a higher maximum value than UK-NA for the same level of damping but that higher acceleration response is obtained with UK-NA for lower frequencies below approximately 1.8 Hz. ISO 10137 weighs walking frequencies differently than UK-NA where the acceleration response increases up to 2.4 Hz whereas UK-NA exhibits the largest accelerations at 1.8 Hz. UK-NA weighs the frequencies based on which are the most occurring and therefore the most relevant to consider in design. ISO 10137 seems to weigh the structural frequencies from that a pedestrian load of 2.4 Hz would result in the highest response. It should be noted that ISO 10137 does not consider structural frequencies below 1.2 Hz to be in any danger of vibrations due to pedestrian loading.

UK-NA weighs the second harmonics higher than ISO 10137 and results in higher acceleration response with a peak value at approximately 3.7 Hz. ISO 10137 has the same value for frequencies between 2.4 and 5 Hz which can be seen gives bigger accelerations response around 5 Hz in comparison with UK-NA.

In Figure 7.22 normalized curves for groups of pedestrians for UK-NA and ISO 10137 are plotted for steel with 0.50 % damping.

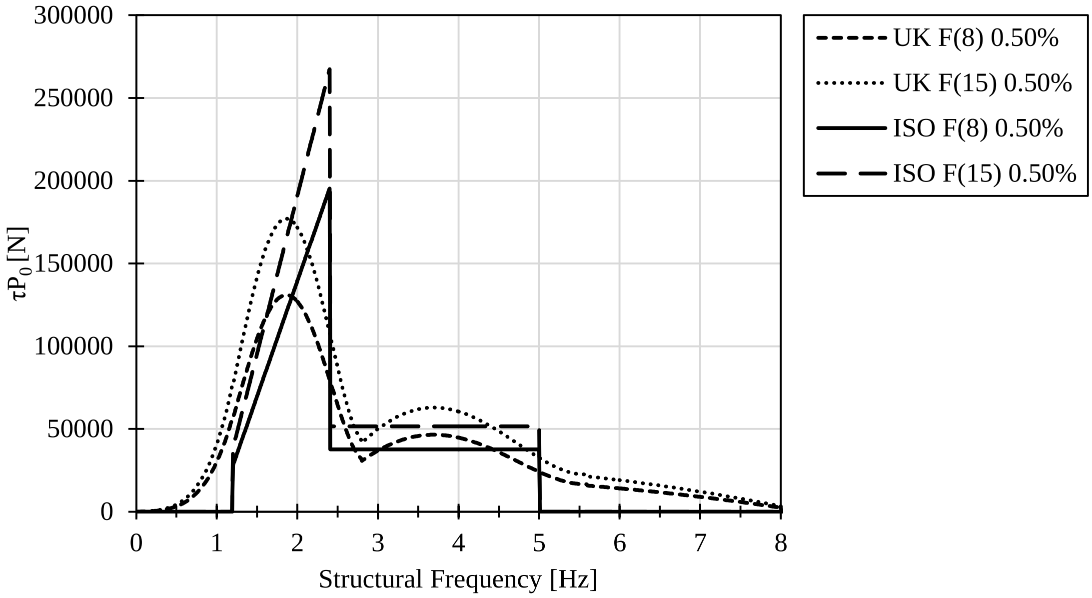


Figure 7.22 Normalized curves for groups of pedestrians for UK-NA and ISO 10137 for steel with 0.5 % damping.

Because the proposed damping ratio is the same for UK-NA and ISO 10137 the relationship is the same as for the normalized curve for 5 % damping.

In Figure 7.23 normalized curves for groups of pedestrians for UK-NA and ISO 10137 are plotted for timber 1.0 % damping ratio.

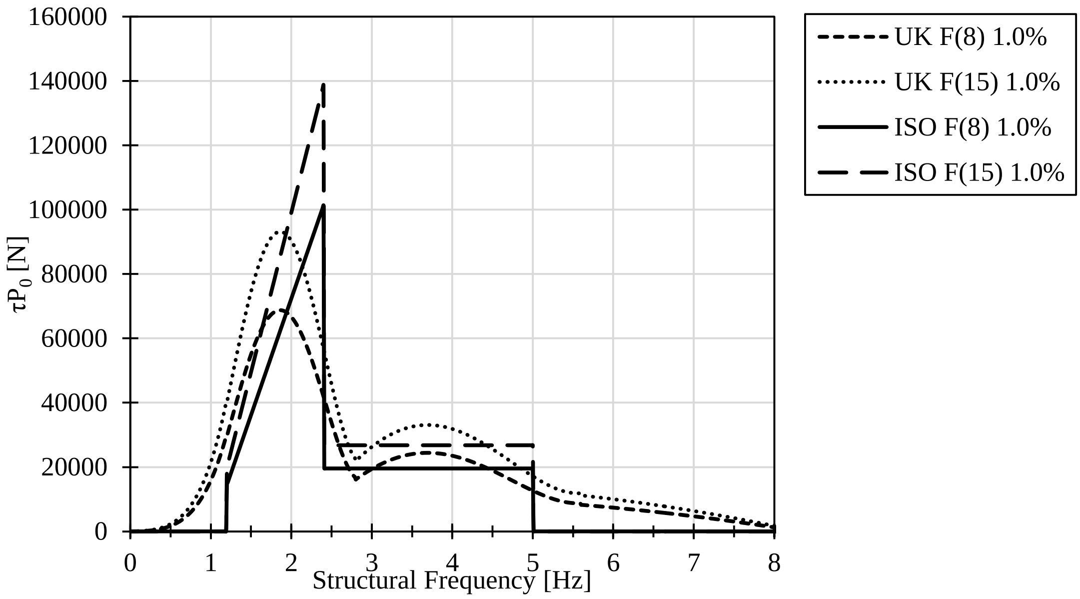


Figure 7.23 Normalized curves for groups of pedestrians for UK-NA and ISO 10137 for timber with 1.0 % damping.

Due to that the proposed damping ratio is the same for timber for UK-NA and ISO 10137 the relationship between the normalized acceleration responses is the same as discussed for a damping ratio of 5 % showed in Figure 7.21.

7.2.1.2 Concrete

In Figure 7.22 normalized curves for groups of pedestrians for UK-NA and ISO 10137 are plotted for concrete with 1.5 and 0.80 % damping ratio respectively.

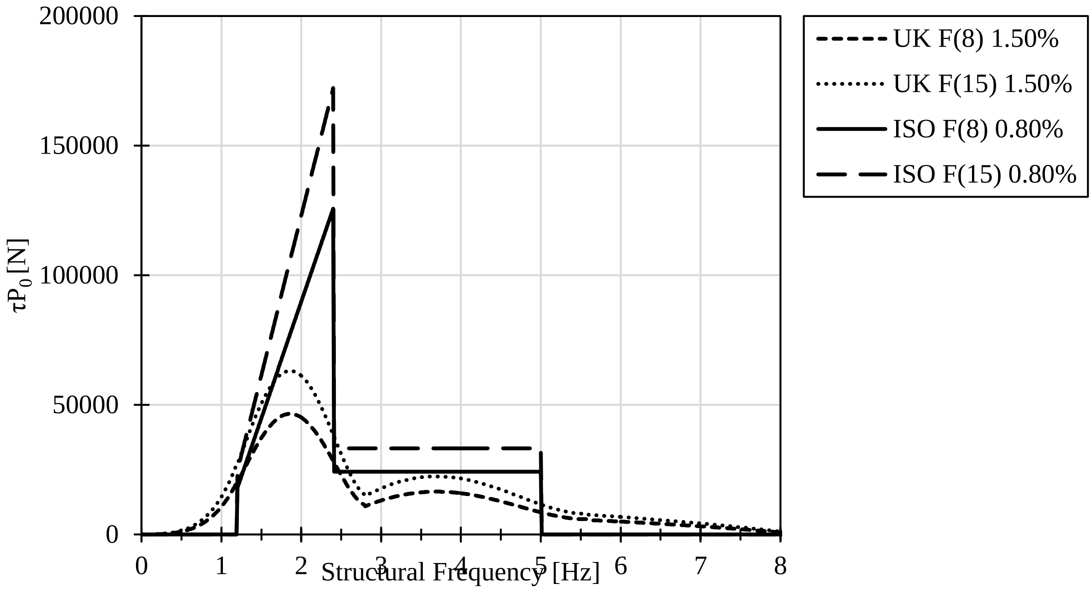


Figure 7.24 Normalized curves for groups of pedestrians for UK-NA and ISO 10137 for concrete with 1.5 and 0.80 % damping.

The difference in proposed damping ratios between the standards affects the obtained acceleration response so that the acceleration response for ISO 10137 is larger or the same for all frequencies above 1.2 Hz in comparison with UK-NA.

The values obtained with ISO 10137 are larger due to its lower damping ratio which means that τ for ISO 10137 is 122.6 where τ for UK is 66.35 corresponding to a damping ratio of 1.5 %. The damping is not included in the equivalent number of pedestrian which describes the size of the group and therefore the difference in damping affects the response through the corresponding τ-value.

7.3 Distributed loads describing streams of pedestrians

The third design situation in ISO 10137 is to analyze the acceleration response of a pedestrian stream crossing the bridge. Based on the literature study in chapter 4 pedestrians stream are preferably modeled as uniformly distributed loads applied on the entire bridge deck. The load case or a method for this analyze is not covered in ISO 10137 and is therefore in this chapter compared for the load models recommended by other standards and guidelines.

The load models considered in this section are:

- UK-NA
- Sétra
- SYNPEX
- JRC
- HIVOSS

All load models except UK-NA includes load models for vertical and lateral vibrations. UK-NA covers only vertical vibrations and is therefore not included in the lateral comparison. JRC and HIVOSS are equally defined for uniformly distributed load and will therefore be considered with the same curve.

The load models are applied with pedestrian densities according to traffic classes based on location or expected traffic. The comparison in this section is systematically done for traffic classes corresponding to the traffic situation given below. The way of comparing the models based on traffic situation makes it possible to compare how the models are regarding expected traffic.

The considered locations and corresponding expected loading are:

- Group of 15 pedestrians
- Suburban location
- Urban location – normal use
- Urban location – crowded
- Exceptionally dense traffic

The standards which do not include the defined load cases above will not be included in the specific comparison.

7.3.1 Group of 15 pedestrians

The lowest traffic class for pedestrian streams in some of the standards is defined as a pedestrian group of 15 pedestrians evenly distributed over the bridge deck. The load case is dependent on the bridge geometry which means that independent on bridge size it should fulfill the requirements of 15 pedestrians on the bridge simultaneously.

Three standards have defined their lowest traffic class as a group of 15 pedestrians, SYNPEX, JRC and HIVOSS. SYNPEX is the oldest of the three where JRC and HIVOSS are based on the recommendations in SYNPEX. JRC and HIVOSS recommend the same load models, frequency interval and damping ratios and will be regarded as equal in the comparison.

The normalized acceleration response curves for load models for a group of 15 pedestrians are calculated independent on bridge geometry. The curves have to be multiplied with the geometry factor ω_{TC1} to be considered for arbitrary geometry. The factor is equal for the considered standards which make them comparable.

$$\omega_{TC1} = \frac{1}{L} \quad [m]$$

The damping ratio is regarded by the normalization factor τ for different damping ratios. The damping ratio is also included in the load models in the derivation of equivalent number of pedestrians.

$$n_{eq} = 10.8\sqrt{\xi N} \quad [-]$$

7.3.1.1 Reference damping – 5.0%

In Figure 7.25 the normalized acceleration response curves for vertical loading are plotted for JRC/HIVOSS and SYNPEX with 5.0% structural damping ratio.

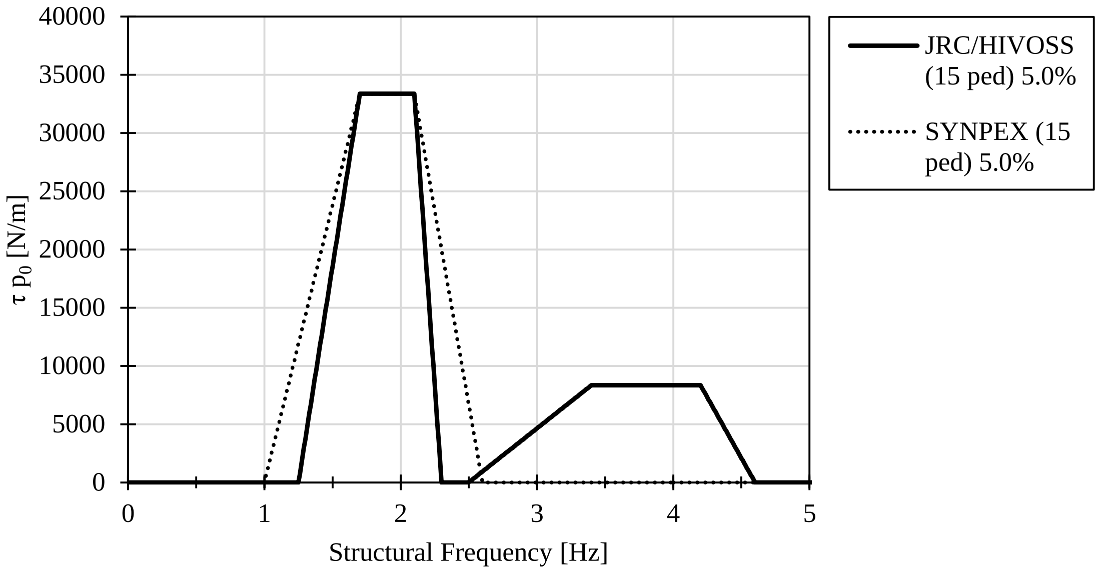


Figure 7.25 Normalized acceleration response for the vertical load models recommended by SYNPEX, JRC and HIVOSS, plotted for 5.0% structural damping ratio.

The normalized acceleration response curves in figure above are plotted for the same damping ratio which makes the differences between them to only dependent on the load model and definitions in the standards.

The two load models have the same maximum in the first harmonic at the same frequency interval between 1.75 and 2.10 Hz. SYNPEX on the other hand considers a wider frequency range for the first harmonic between 1.0 and 2.6 Hz. JRC/HIVOSS has a defined the frequency interval between 1.25 and 1.3 Hz which generates that the slope of the curve is steeper from maximum to zero. The difference in frequency interval implies that there will be large differences in acceleration response in the intervals 1.0 and 1.75 Hz as well as between 2.10 and 2.60 Hz where SYNPEX covers a greater area in the chart. Though the differences are for the not so common step frequencies the acceleration response is significantly larger for SYNPEX.

The second harmonic is not considered in SYNPEX based on the assumption that the phenomena never have happened in reality. JRC/HIVOSS has chosen to weigh the second harmonic as 25% of the first harmonic. JRC/HIVOSS is based on SYNPEX which is an older standard. During the development of JRC/HIVOSS they choose, despite the overall impression from literature, to consider the second harmonic even though an actual case never occurred in reality.

In Figure 7.26 the normalized acceleration responses in lateral direction are plotted for the load models presented in JRC/HIVOSS and SYNPEX for 5.0% damping.

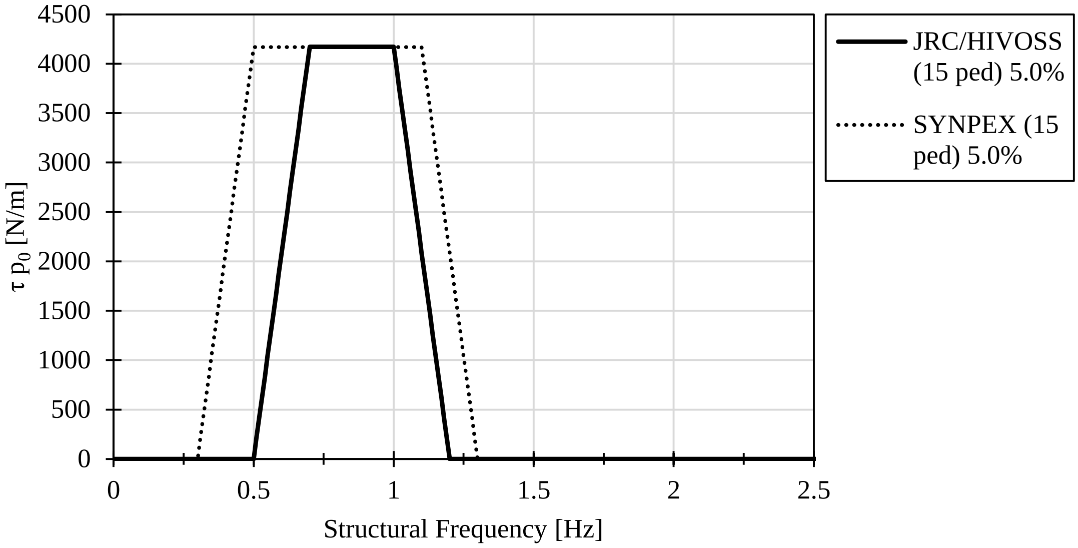


Figure 7.26 Normalized acceleration response for the lateral load models recommended by SYNPEX, JRC and HIVOSS, plotted for 5.0% structural damping ratio.

The normalized acceleration responses from the two standards have the same maximum value when considering the same structural damping ratio. SYNPEX has defined a wider frequency range both for the maximum value and the overall considered frequencies. As the lateral frequencies are based on the vertical frequencies it is reasonable that SYNPEX has wider ranges for both vertical and lateral direction than JRC/HIVOSS. SYNPEX defines the worst frequencies between 0.50 and 1.10 Hz which equally wide as the total range defined by JRC/HIVOSS. This means that i.e. 0.50 Hz and 1.10 Hz SYNPEX weigh the frequency at maximum and JRC/HIVOSS as and not considered at all. The great difference can have large impact on the design of footbridges as lateral vibrations around 0.50 Hz has occurred even in the most studied case, London Millennium Bridge.

7.3.1.2 Concrete

In Figure 7.27 the normalized acceleration response curve in vertical direction are plotted for damping ratios according to concrete structures. The same damping ratio 1.3% is recommended in SYNPEX and JRC/HIVOSS.

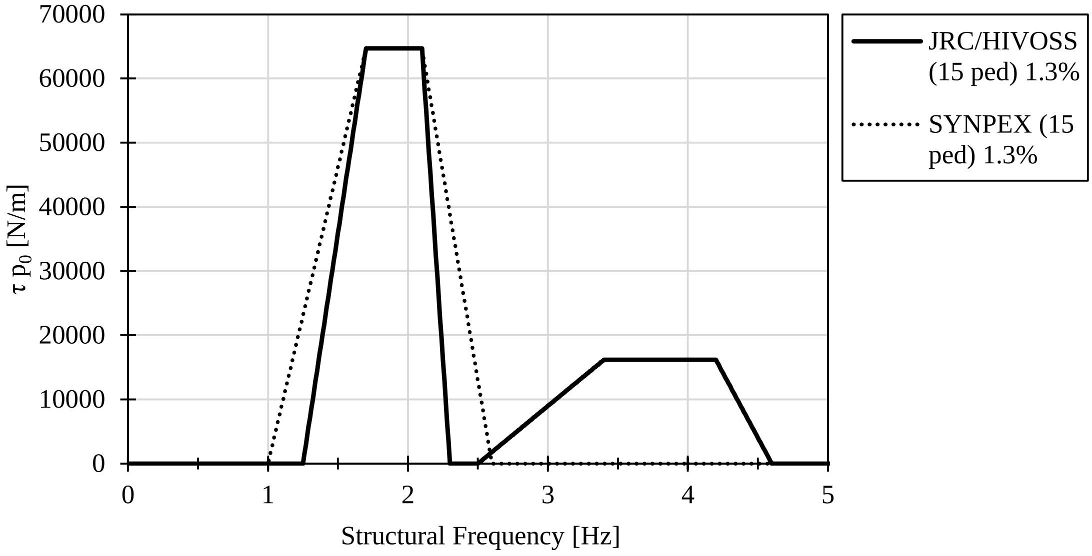


Figure 7.27 Normalized acceleration response for the vertical load models recommended by SYNPEX, JRC and HIVOSS, plotted for concrete structural damping ratios.

The normalized acceleration response plots have the same maximum and same relation as for 5.0% damping because of the fact that the standards applies the same damping 1.3% for concrete. When the same damping ratio is chosen the same normalization factor τ applies and the relation between the curves remains as for 5.0%.

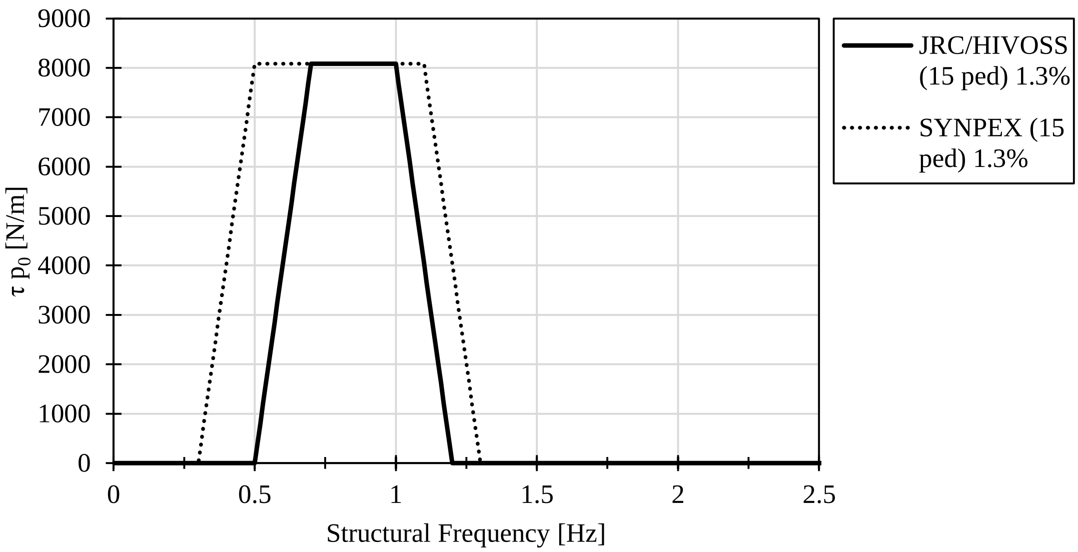


Figure 7.28. Normalized acceleration response for the lateral load models recommended by SYNPEX, JRC and HIVOSS, plotted for concrete structural damping ratios.

The relation between the lateral normalized acceleration responses for the standards is the same as for 5.0% damping.

The recommended damping ratio 1.3% for concrete seems reasonable as it is similar to the ratios given in literature. Eurocode suggest a higher ratio of 1.5% for reinforced concrete which is higher but on the other hand ISO 10137 suggest a lower ratio of 0.80%.

7.3.1.3 Steel

In Figure 7.29 the normalized acceleration response curves are plotted for vertical load model with damping ratios according to steel structures. SYNPEX and JRC/HIVOSS recommends the same ratio of 0.40%.

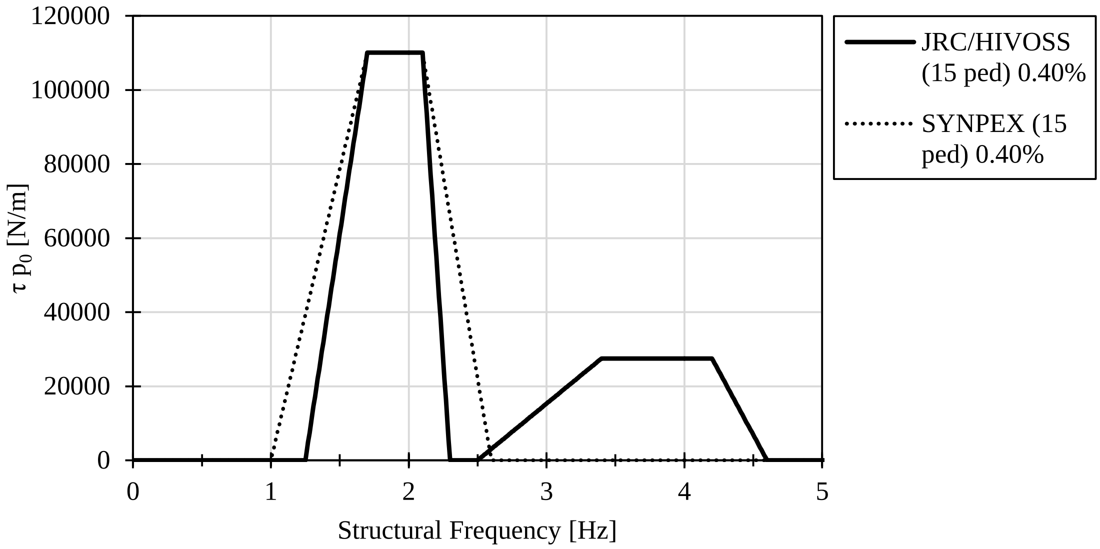


Figure 7.29 Normalized acceleration response for the vertical load models recommended by SYNPEX, JRC and HIVOSS, plotted for steel structural damping ratios.

The same damping ratios are applied in the standards which does not affect the relation between the normalized curves which is same as for 5.0% damping. The load models have the same maximum values with the main differences in the defined frequency intervals.

In Figure 7.30 are the normalized acceleration response plotted for the lateral load models. The same situation applied for the lateral load models as for vertical, it mainly the frequency interval that separates the models.

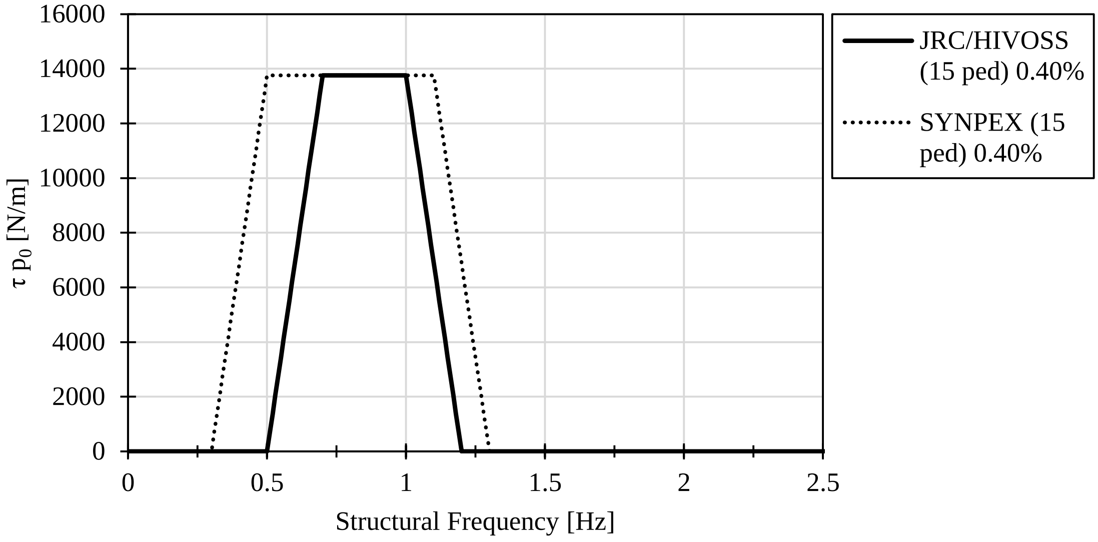


Figure 7.30 Normalized acceleration response for the lateral load models recommended by SYNPEX, JRC and HIVOSS, plotted for steel structural damping ratios.

7.3.1.4 Timber

The structural damping ratio for timber structures differs between the standards. In SYNPEX 3.0% is recommended and in JRC/HIVOSS 1.5% is recommended.

In Figure 7.31 the normalized acceleration responses for vertical loading are plotted with damping ratios according to timber structures.

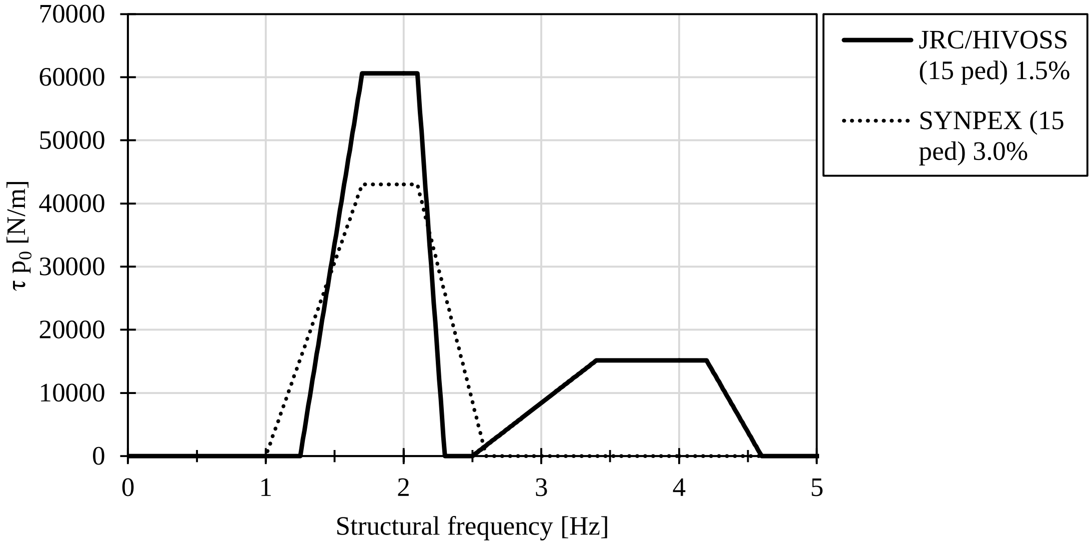


Figure 7.31 Normalized acceleration response for the vertical load models recommended by SYNPEX, JRC and HIVOSS, plotted timber structural damping ratios.

The maximum vertical response is given for the load model stated in JRC/HIVOSS about 40% higher than SYNPEX. The difference is a result of the damping ratios where recommends SYNPEX twice as high ratio than JRC/HIVOSS which means that the normalization factor τ is half as high. Due to the expression for equivalent number of pedestrians where the damping ratio is included affects the difference to decrease to 30%. The higher damping the higher number of equivalent pedestrians and higher normalized acceleration response.

In Figure 7.32 are the normalized acceleration responses in lateral direction plotted for the standards.

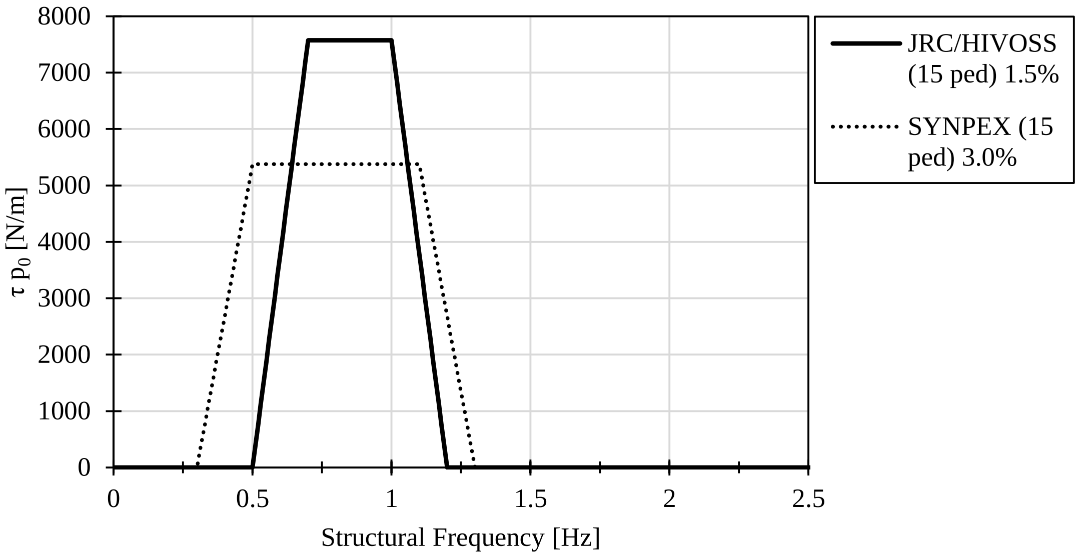


Figure 7.32 Normalized acceleration response for the lateral load models recommended by SYNPEX, JRC and HIVOSS, plotted for timber structural damping ratios.

The maximum lateral response is higher for JRC/HIVOSS due to the difference in damping ratio. The difference is about 40% and the same explanation applies for lateral loading as for vertical because of the similarities in load model.

7.3.2 Suburban location

"Suburban location" is a traffic class considering locations with weak traffic where slight variation in pedestrian loading and intensity occurs. An example can be a footbridge over a motorway linking two populated areas.

SYNPEX, JRC and HIVOSS propose a density of 0.2 ped/m^2 defined as weak traffic. Sétra proposes a slightly higher pedestrian density of 0.5 ped/m^2 for Class III which fits well with the definition of the traffic class "Suburban location". UK-NA recommends a pedestrian density of 0.4 ped/m^2 for Bridge class B defined for suburban locations.

Considered load models with corresponding traffic class:

SYNPEX	TC2
HIVOSS	TC2
JRC	TC2
UK	B
Sétra	Class III

7.3.2.1 Reference damping – 5.0%

In Figure 7.33 and Figure 7.34 the normalized acceleration response for the considered standards in vertical and lateral direction can be seen established for a damping ratio of 5 %.

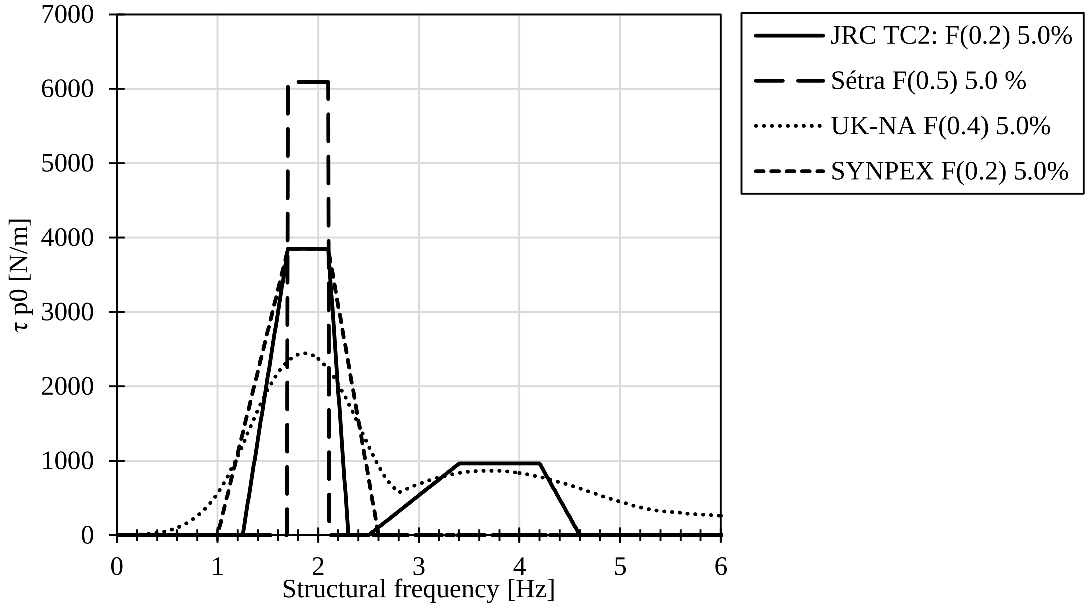


Figure 7.33 Normalized acceleration response in vertical direction describing suburban location for a reference damping ratio of 5.0 %.

Sétra is defined with a higher pedestrian density than the other standards and results in the highest accelerations but for a short interval of frequencies. JRC and SYNPEX which are defined with the same pedestrian density results in the same maximum value but JRC consider a shorter range of frequencies for the first harmony of structural frequencies. JRC considers second harmonics as well which SYNPEX considers as irrelevant. UK-NA results in the lowest accelerations where the maximum peak is approximately half of Sétra. Though the response according to UK-NA gives the smallest accelerations it considers the widest range of frequencies.

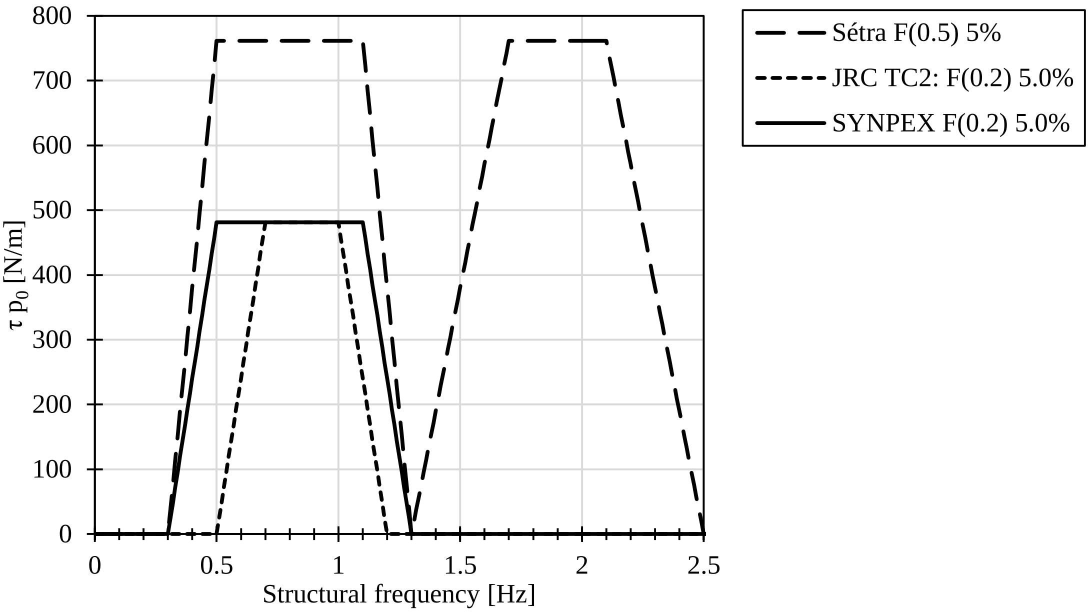


Figure 7.34 Normalized acceleration response in lateral direction describing suburban location for a reference damping ratio of 5.0 %.

Sétra results in the highest accelerations also in lateral direction. JRC and SYNPEX results in the same maximum value but SYNPEX consider a smaller interval of frequencies. Sétra is the only standard that considers the second harmony of structural frequencies in lateral direction where SYNPEX and JRC consider them to be irrelevant. UK-NA does not propose any load for lateral direction and is therefore not considered.

7.3.2.2 Concrete

In Figure 7.35 and Figure 7.36 the normalized acceleration response for the considered standards for concrete with corresponding damping ratios in vertical and lateral direction can be seen.

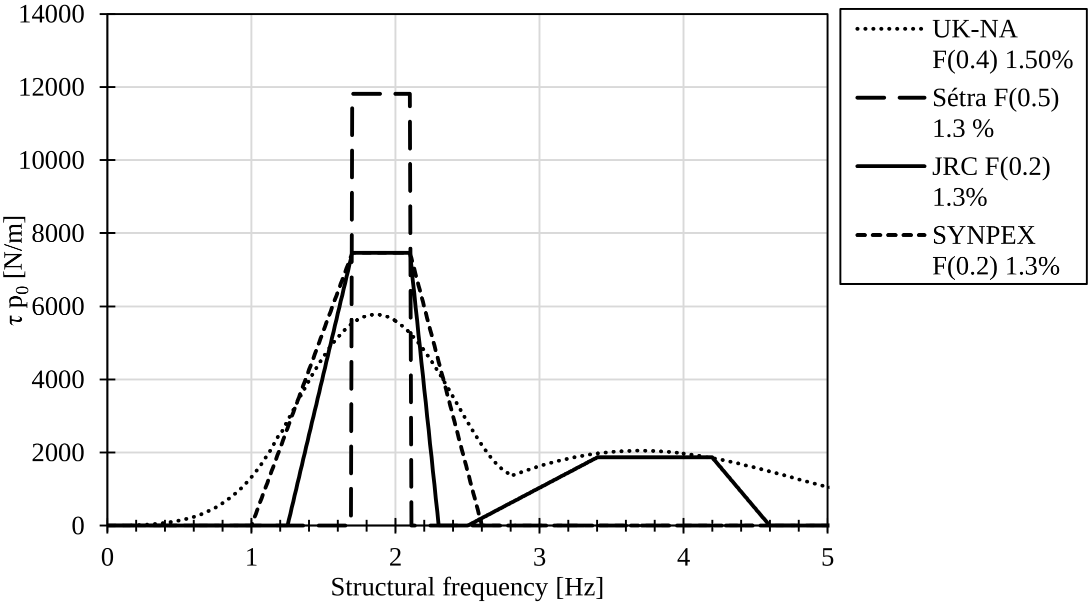


Figure 7.35 Normalized acceleration response in vertical direction describing suburban location for concrete with corresponding damping.

Sétra, JRC and SYNPEX propose the same damping ratio of 1.3 % for concrete where UK-NA proposes a higher value of 1.5 %. Sétra results in the highest accelerations and JRC and SYNPEX are defined with the same damping ratio and pedestrian density and results in the same maximum value. UK-NA results in the lowest where the maximum value obtained with UK-NA is significantly lower than the one obtained with Sétra. In this case the accelerations for the second harmony are bigger due to UK-NA than for JRC which differs from the relationship obtained for a damping ratio of 5 %. This is due to that the damping ratio is considered in different ways in JRC and UK-NA.

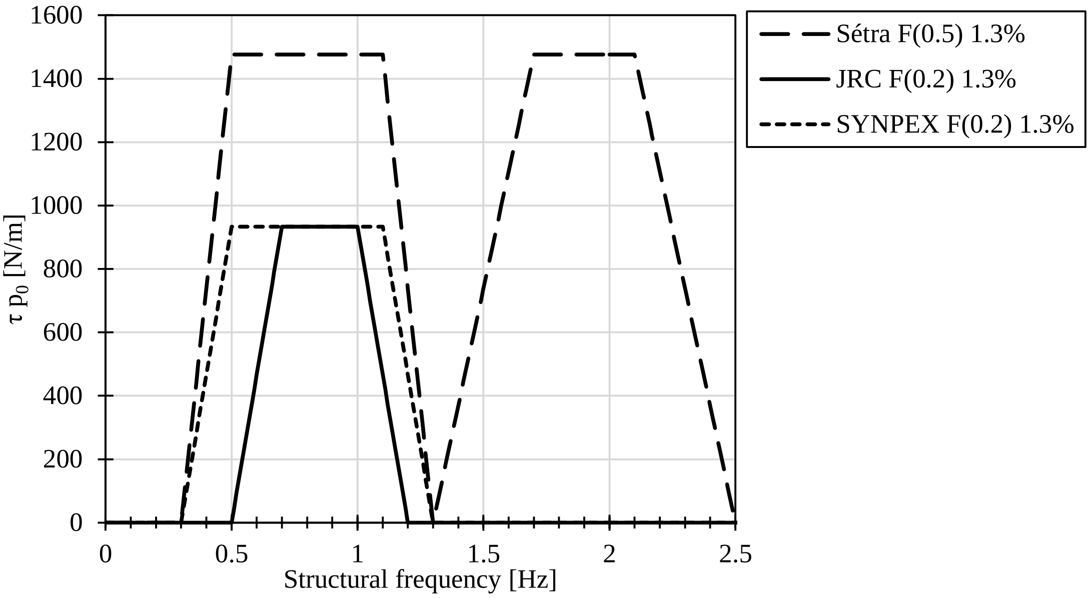


Figure 7.36 Normalized acceleration response in lateral direction describing suburban location for concrete with corresponding damping.

Sétra, JRC and SYNPEX are defined with the same damping for concrete. Sétra exhibits significantly larger response than JRC and SYNPEX which results in the same maximum value.

7.3.2.3 Steel

In Figure 7.37 and Figure 7.38 the normalized acceleration response for the considered standards for steel with corresponding damping ratios in vertical and lateral direction can be seen.

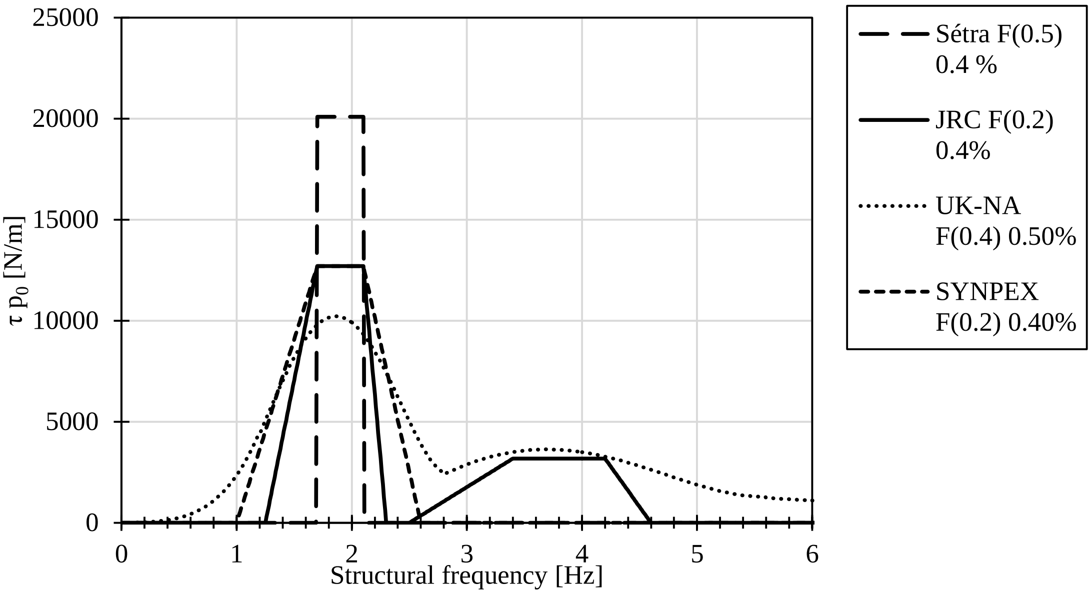


Figure 7.37 Normalized acceleration response in vertical describing suburban location for steel with corresponding damping.

Sétra, JRC and SYNPEX propose the same damping ratio of 0.40 % for steel where UK-NA proposes a slightly higher value of 0.50 %. Sétra results in the highest accelerations and JRC and SYNPEX results in the same maximum value. UK-NA results in the lowest acceleration response for the first harmony but larger than JRC for the second harmonies of structural frequencies.

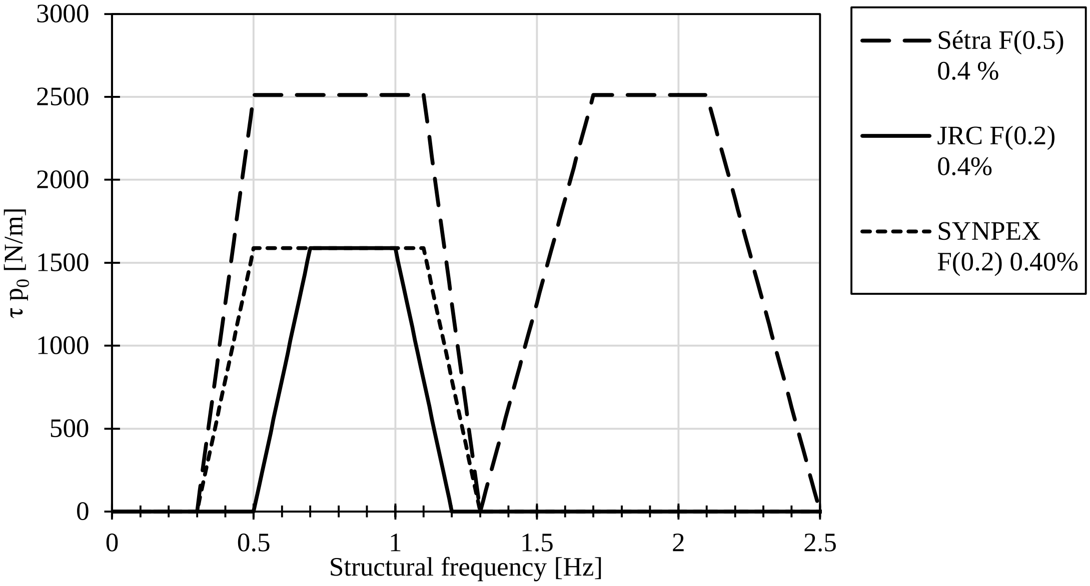


Figure 7.38 Normalized acceleration response in lateral direction describing suburban location for steel with corresponding damping.

The same damping is proposed for Sétra, JRC and SYNPEX. Sétra results in much larger response as shown for other materials with corresponding damping.

7.3.2.4 Timber

In Figure 7.39 and Figure 7.40 the normalized acceleration response for the considered standards for timber with corresponding damping ratios in vertical and lateral direction can be seen.

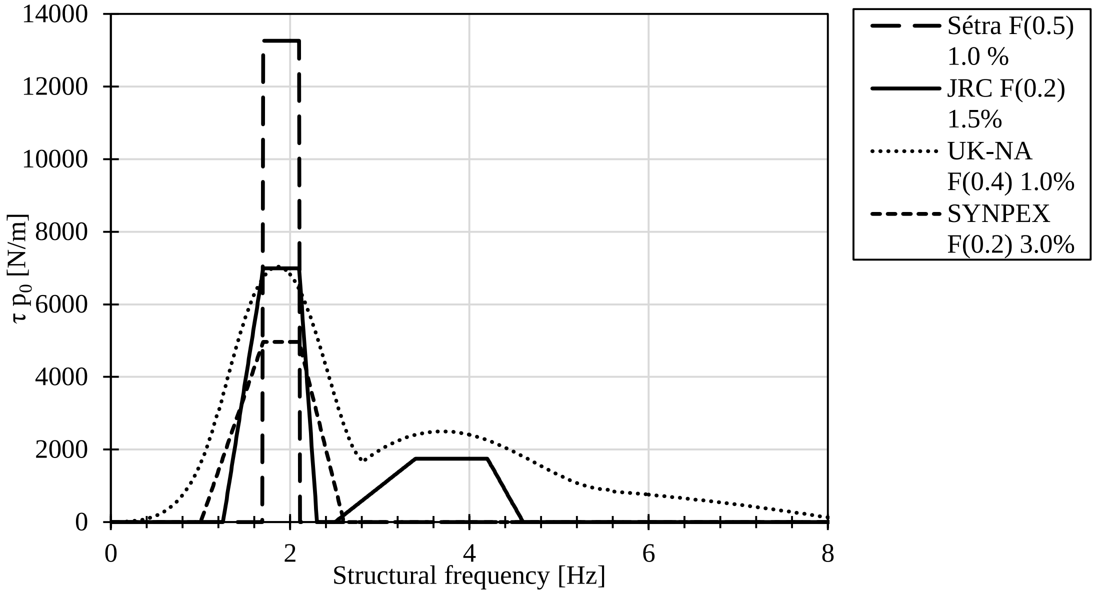


Figure 7.39 Normalized acceleration response in vertical direction describing suburban location for timber with corresponding damping.

Different damping ratios for timber are proposed in the standards. Sétra results in the highest maximum value for a damping ratio of 1.0 %. For timber the acceleration response obtained with JRC is larger than for SYNPEX due to its lower damping ratio of 1.5 % compared to the damping ratio of 3.0 % proposed for SYNPEX. UK-NA with a damping ratio of 1.0 % and a higher pedestrian density of 0.4 ped/m^2 compared to JRC results in approximately the same maximum value as for JRC for the first harmony of structural frequencies. Disregarding the maximum value UK-NA results in higher accelerations for all structural frequencies compared to JRC and SYNPEX including second harmonics.

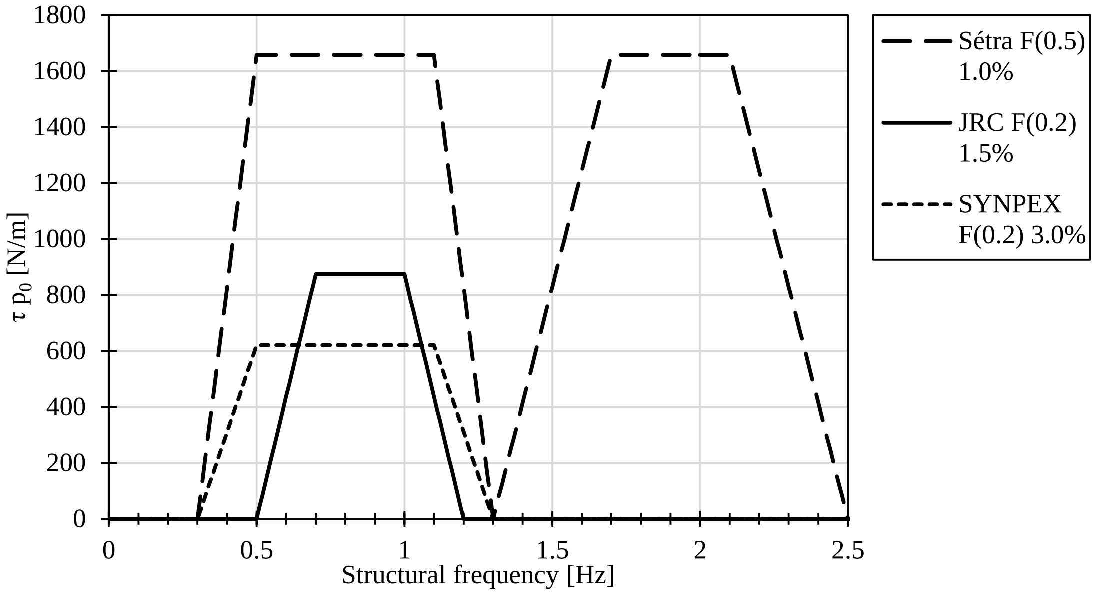


Figure 7.40 Normalized acceleration response in lateral direction for different standards describing suburban location for timber with corresponding damping.

The maximum acceleration response obtained with JRC for a damping ratio of 1.5 % is larger than for SYNPEX with a corresponding damping ratio of 3.0 %. Sétra results in the highest accelerations for all structural frequencies with a proposed damping ratio of 1.0 % for timber.

7.3.3 Urban location – normal use

The traffic class “Urban location – normal use” is defined as a location in a city linking populated area with heavy traffic. The traffic may vary significant over the day and could occasionally be loaded throughout its loading area. In UK-NA the situation corresponds to bridge class C with 0.8 ped/m^2 which is the second most crowded condition. In Sétra the description relates to footbridge class II which also is the second most crowded with 0.8 ped/m^2. SYNPEX, JRC and HIVOSS have the same traffic classes and the situation is considered as “dense traffic” equal to 0.5 ped/m^2. Due to the different classes the normalized acceleration response will be compared for various pedestrian densities.

7.3.3.1 Reference damping – 5.0%

In Figure 7.41 is the normalized acceleration response in vertical loading direction plotted for the reference damping 5.0%.

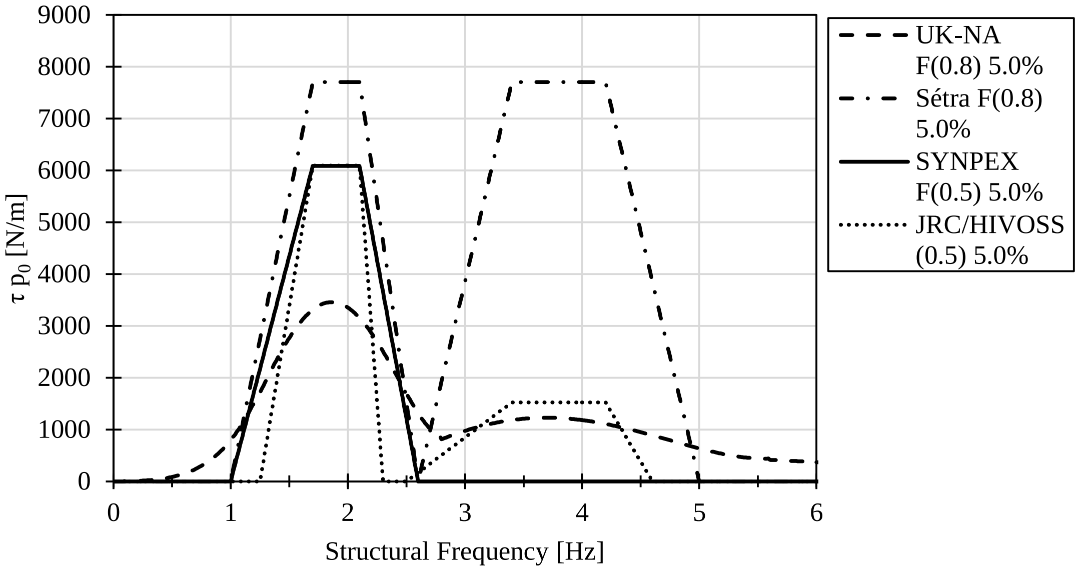


Figure 7.41 Comparison of the normalized acceleration response in vertical direction for damping ratio equal to 5.0%.

In the figure above is the acceleration response for the load models calculated with the same damping ratio and normalization factor τ. The difference in response is determined by the pedestrian density, the load model itself and the critical frequency intervals defined in the standards.

The maximum response for the first harmony is given by the load model in Sétra. The response is some higher than for SYNPEX and JRC/HIVOSS. The three standards mentioned are based on each other with very similar load models and Sétra gives the highest maximum because of a denser pedestrian density. UK-NA and Sétra has the same pedestrian density but UK-NA has a maximum less than 50% of Sétras maximum. The difference is big which leads to significant variations in the design.

Sétra has the highest maximum for the second harmony as well with the same magnitude as the first. SYNPEX does not consider the second harmony and is zero for all frequencies greater than 2.6 Hz. JRC/HIVOSS and UK-NA has approximately the same maximum for the second harmony. JRC/HIVOSS weighs the second harmony as 25% of the first and UK-NA about a third of the first harmony.

Overall has Sétra the highest response considering all frequencies with the highest maximums and widest frequency ranges.

In Figure 7.42 is the normalized acceleration response in lateral direction plotted for 5.0% damping ratio.

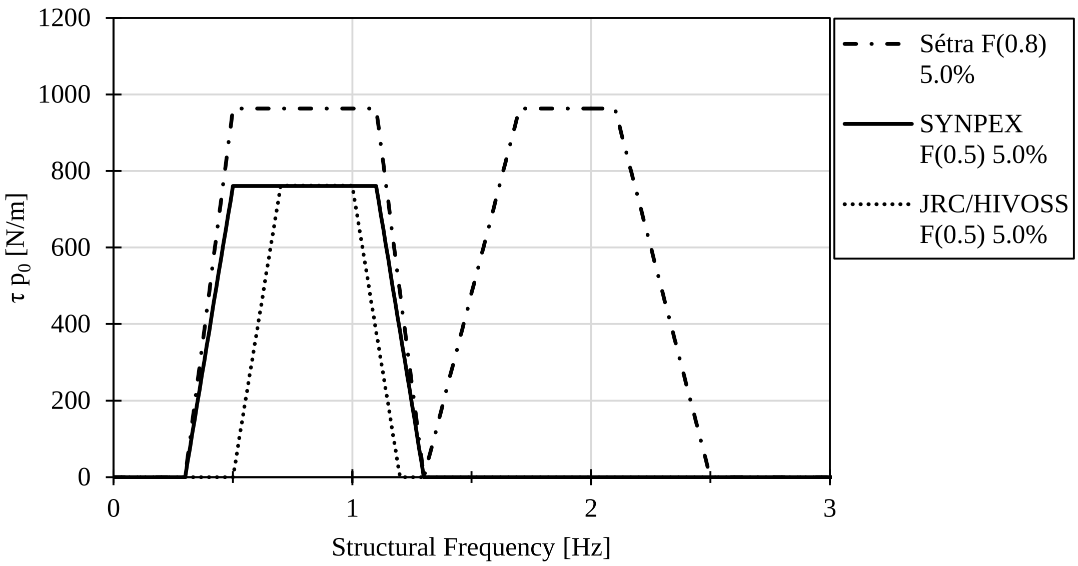


Figure 7.42 Comparison of the normalized acceleration response in lateral direction for damping ratio equal to 5.0%

In lateral direction is the normalized acceleration response from Sétra, SYNPEX and JRC/HIVOSS compared. UK-NA has not defined a load model for lateral direction and is therefore not included in the plot.

Sétra has the highest maximum for the first harmony due to a higher pedestrian density. SYNPEX and JRC/HIVOSS has the same maximum for the first harmony slightly lower than Sétra. The critical frequency interval for Sétra and SYNPEX is the same both for the maximum values and the total range for first harmony. JRC/HIVOSS does not consider frequencies lower than 0.50 Hz compared to the other standards minimum at 0.30 Hz.

The second harmony is only considered by Sétra for lateral vibrations with the same maximum as for its first harmony.

7.3.3.2 Concrete

The damping ratio for concrete bridges varies among the standards. The highest damping ratio is given by UK-NA at 1.5% which is given for recommended ratios in Eurocode. Sétra, SYNPEX and JRC/HIVOSS has all the same recommended ratio equal to 1.3%, slightly lower than Eurocode but not decisive.

In Figure 7.43the normalized acceleration response in vertical direction is plotted for damping ratios according to concrete structures.

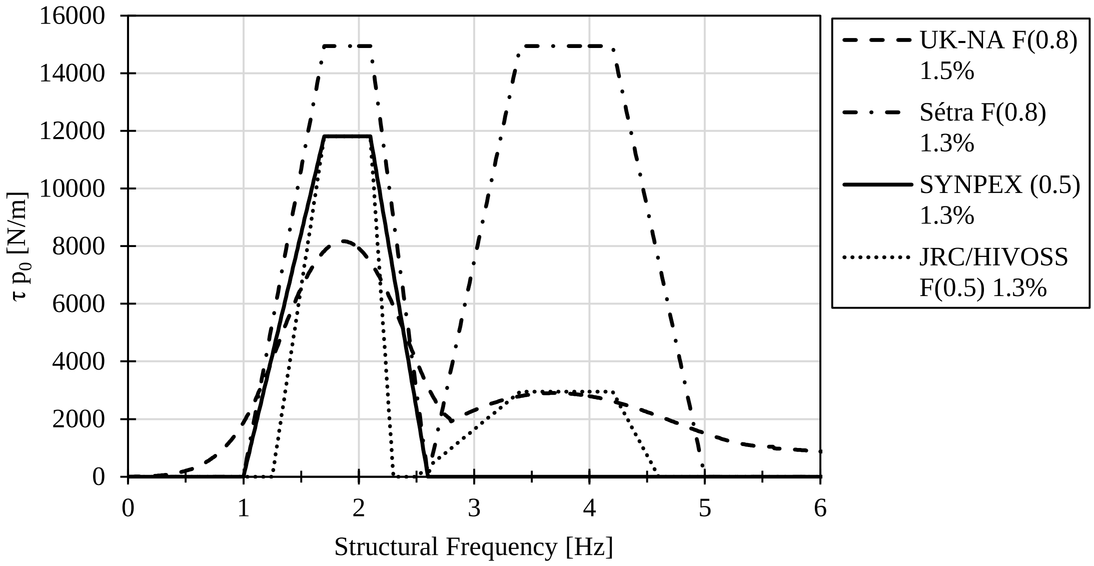


Figure 7.43 Comparison of the normalized acceleration response in vertical direction for damping ratios according to concrete structures.

The damping ratios for Sétra, SYNPEX and JRC/HIVOSS are all the same meaning that the relation between the responses has not changed for the comparison of 5.0 % damping ratio. This is because the models are defined in the same way. The gap between the maximum responses of UK-NA and Sétra has decreased compared to the plot with 5.0% damping. Which means that UK-NAs response for lower damping ratios will increase more in comparison to the other models.

For the second harmony have JRC/HIVOSS and UK-NA same response but much lower than Sétra.

In Figure 7.44 is the normalized acceleration response in lateral direction plotted for damping ratios according to concrete structures.

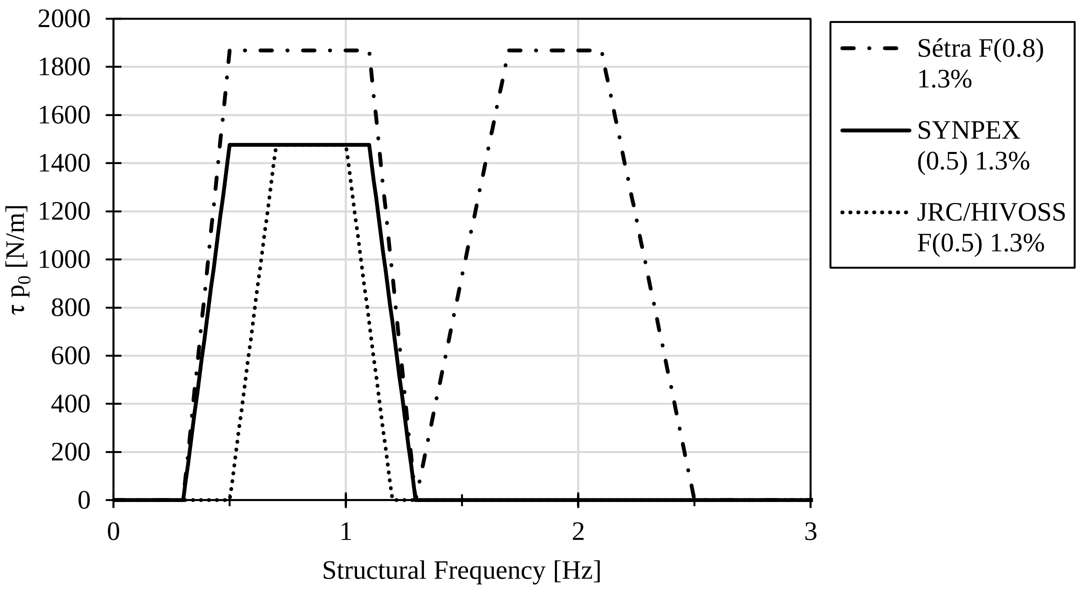


Figure 7.44 Comparison of the normalized acceleration response in lateral direction for damping ratios according to concrete structures.

The compared models in figure above are all calculated for the same damping ratio and the relation between them is unchanged compared to the comparison for 5.0% damping.

7.3.3.3 Steel

The recommended damping ratios for steel structures are low which generates a high acceleration response. The same structural damping ratios is recommended in Sétra, SYNPEX and JRC/HIVOSS equal to 0.40%. The UK-NA recommends the damping ratio to 0.50% which is lightly higher than the other standards.

In Figure 7.45 is the normalized acceleration response in vertical direction compared for the considered standards with damping ratios according to steel structures.

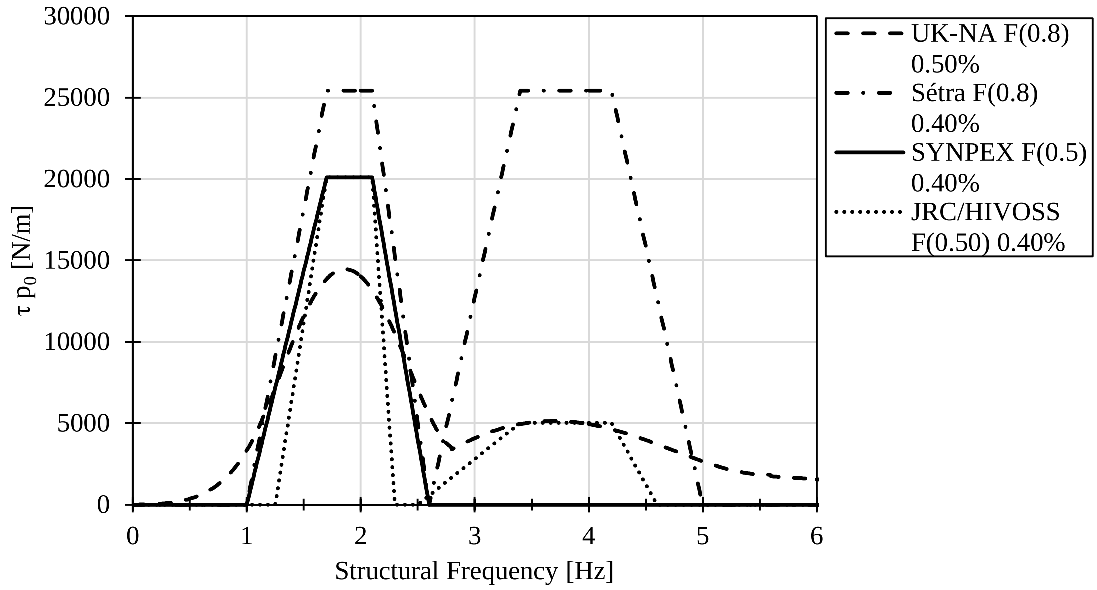


Figure 7.45 Comparison of the normalized acceleration response in vertical direction for damping ratios according to steel structures.

In the comparison above it can be seen that Sétra has the maximum value for the first and second harmony. As the damping ratios between Sétra, SYNPEX and JRC/HIVOSS is the same the relation between them is the same as for 5.0% damping. UK-NA has a lower response for the first harmony about 55% of the maximum response in Sétra. JRC/HIVOSS and UK-NA has the same response for the second harmony and SYNPEX is equal to zero.

In Figure 7.46 is the normalized acceleration response in lateral direction compared for damping ratios according to steel structures.

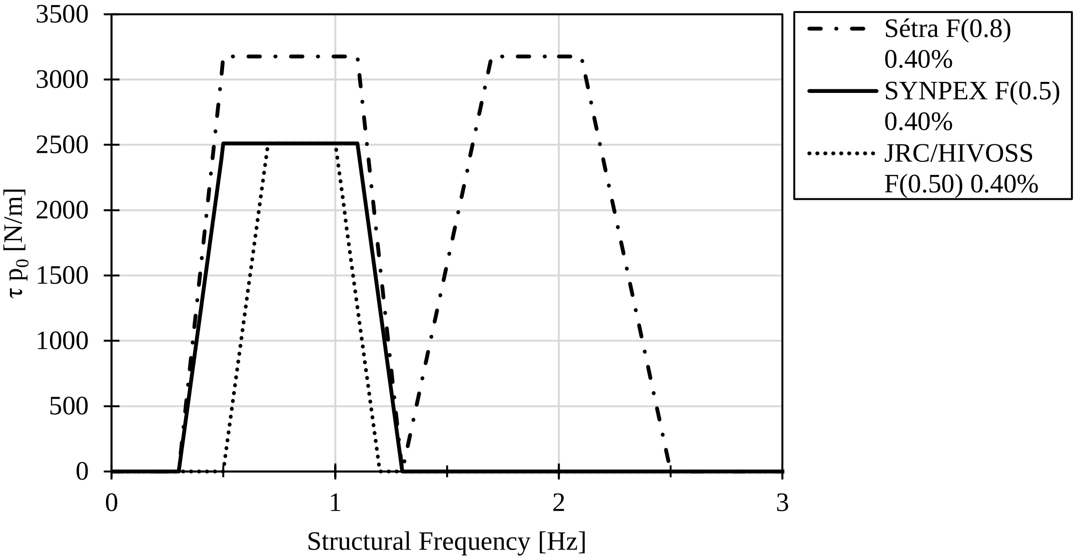


Figure 7.46 Comparison of the normalized acceleration response in lateral direction for damping ratios according to steel structures.

The same damping ratio applies for the three compared standards. The relation between the acceleration responses is unchanged compared to the comparison with 5.0% damping.

7.3.3.4 Timber

The recommended damping ratio for timber bridges varies among the standards with large differences. Highest damping ratio is recommended in SYNPEX equal to 3.0%. The damping ratio seems high and will be unconservative as the acceleration response decreases with increasing damping ratio. In JRC/HIVOSS is 1.5% damping recommended and comparable to the ratio recommended by UK-NA at 1.0%. The large scatter of recommended ratios affects the responses and creates uncertainties for the designer to assign a proper ratio.

In Figure 7.47 is the normalized acceleration response in vertical direction compared for damping ratios according to timber structures.

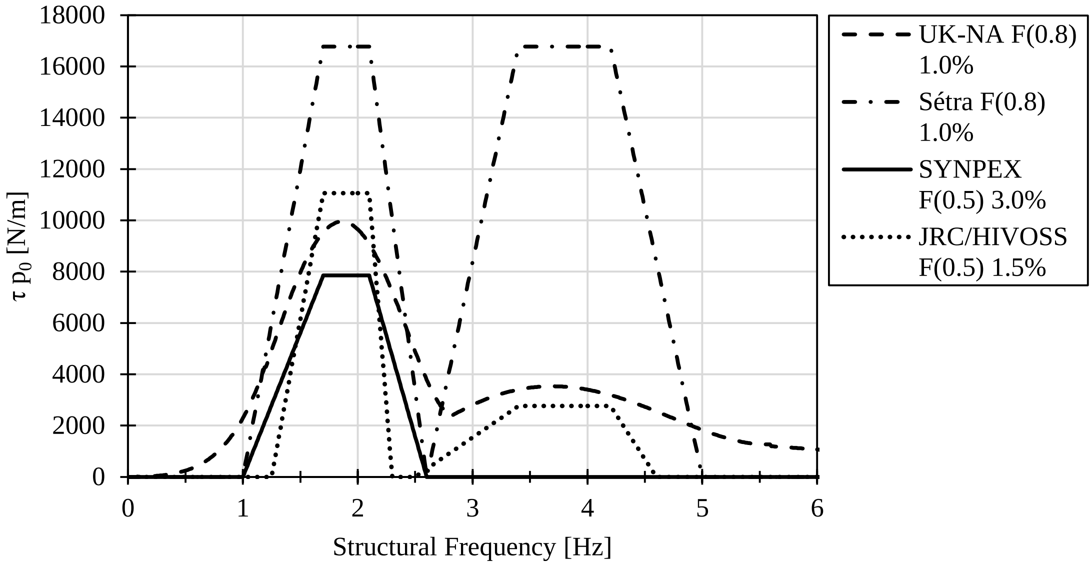


Figure 7.47 Comparison of the normalized acceleration response in vertical direction for timber bridges.

In the comparison above Sétra has clearly the highest maximum in response for the first and second harmony. The response between the other standards is more equal where SYNPEX has the lowest response due to the high damping ratio assigned. UK-NA and JRC/HIVOSS have similar response for both the first and second harmony. A higher damping ratio is applied in JRC/HIVOSS but gives still a slightly higher response for the first harmony though UK-NA has a higher response for the second harmony.

In Figure 7.48 is the normalized acceleration response in lateral direction compared for damping ratios according to timber bridges.

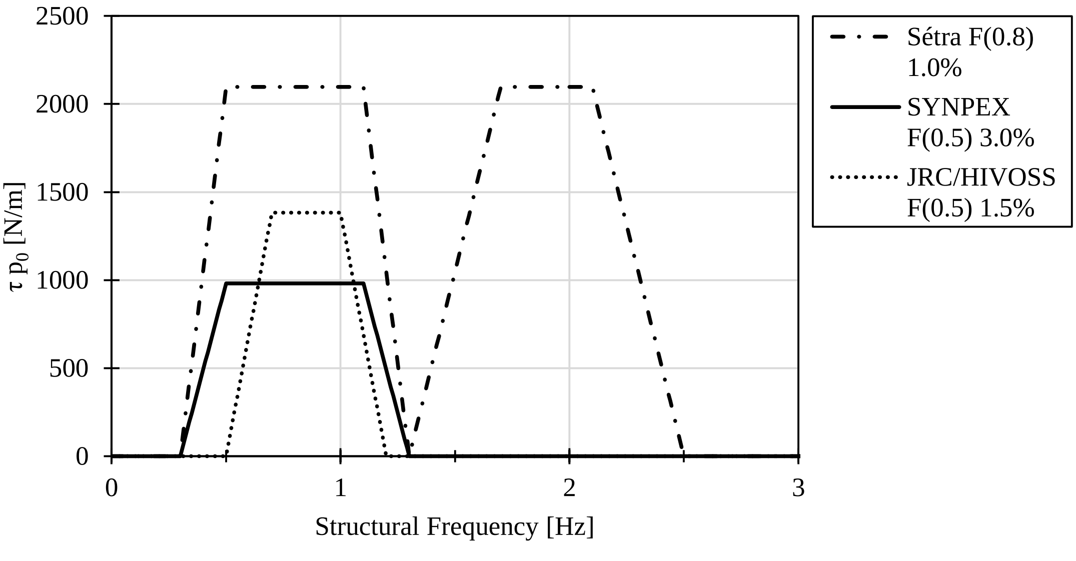


Figure 7.48 Comparison of the normalized acceleration response in lateral direction for timber bridges.

The lateral response varies among the compared standards in the figure above. SYNPEX has the lowest response at less than 50% of the maximum response by Sétra. The low response in SYNPEX is explained by the high damping ratio applied. Sétra which considers the highest pedestrian density has also the lowest applied damping ratio has the absolute highest acceleration response. JRC/HIVOSS has a lower damping ratio than SYNPEX which generates a higher response.

7.3.4 Urban location – crowded

The traffic class “Urban location – crowded” is a traffic class considering primary route at important facility in a city such as a metro station or close to a sports stadium. The condition is in SYNPEX, JRC and HIVOSS regarded as a very dense traffic according to traffic class 4 with 1.0 ped/m^2. In UK-NA traffic class D applies with 1.0 ped/m^2 as the maximum considered density in the standard. In Sétra class I applies which suits well to the description and corresponds to 1.0 ped/m^2. This means that all standards will be compared for the same pedestrian density.

SYNPEX	TC4
HIVOSS	TC4
JRC	TC4
UK	D (1.0 ped/m^2)
Setra	Class I

7.3.4.1 Reference damping – 5.0%

The standards are in figure above all compared with the same damping ratio and pedestrian density. The difference between the acceleration responses is dependent on the load model and critical frequency intervals defined.

In Figure 7.49 is the normalized acceleration response in vertical direction plotted for 5.0% structural damping ratio.

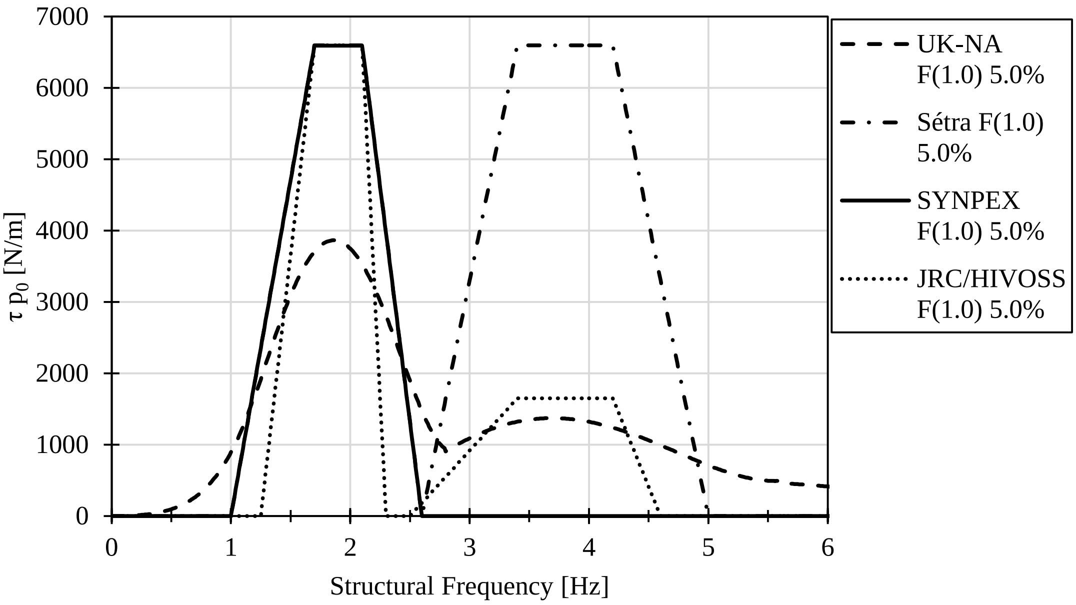


Figure 7.49 Comparison of the normalized acceleration response in vertical direction for a reference damping ratio of 5.0 %.

In the lowest interval for the first harmony Sétra, SYNPEX and JRC/HIVOSS has all the maximum value. The models are based on each other which make them similar and therefore generate the same acceleration response. The differences between them are the frequency interval where JRC/HIVOSS has the shortest range of frequencies and Sétra together with SYNPEX are defined for a wider range. Though at the maximum response they are all defined for the same frequencies. The UK-NA response is significantly lower than the other standards almost half as high value. The load model in UK-NA is different from the others where it takes into account the damping ratio and equivalent number of pedestrians in a different manner. The frequency interval for the first harmony is approximately the same for all standards.

For the second harmony Sétra reaches the highest response at the same level as its first harmony. JRC/HIVOSS and UK-NA has approximately the same response where they peak at the same frequency interval. SYNPEX does not consider the second harmony and is zero for all frequencies greater than 2.5 Hz.

In Figure 7.50 is the normalized acceleration response in lateral direction plotted for 5.0% damping ratio.

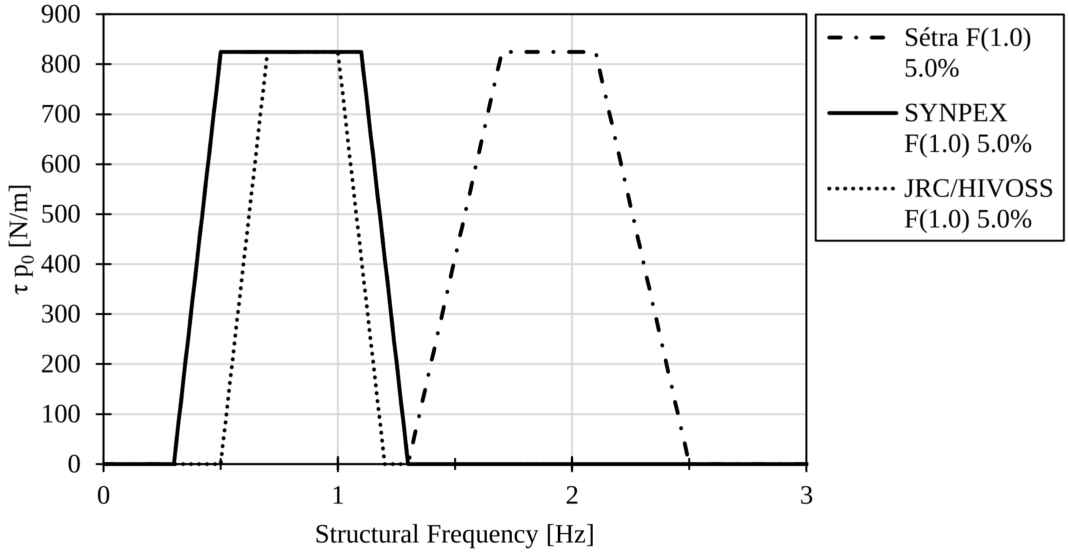


Figure 7.50 Comparison of the normalized acceleration response in lateral direction for a reference damping ratio of 5 %

The UK-NA does not recommend any load model for lateral vibrations. The response from the compared curves is similar as the load models are based on each other. SYNPEX and Sétra have the widest range of considered frequencies for the first harmony where only JRC/HIVOSS is zero for frequencies below 0.5 Hz. The maximum response for the first harmony is the same for all models but only Sétra considers the second harmony where it has weighted it as equal to the first.

7.3.4.2 Concrete

In Figure 7.51 is the normalized acceleration response plotted with damping ratios according to concrete structures.

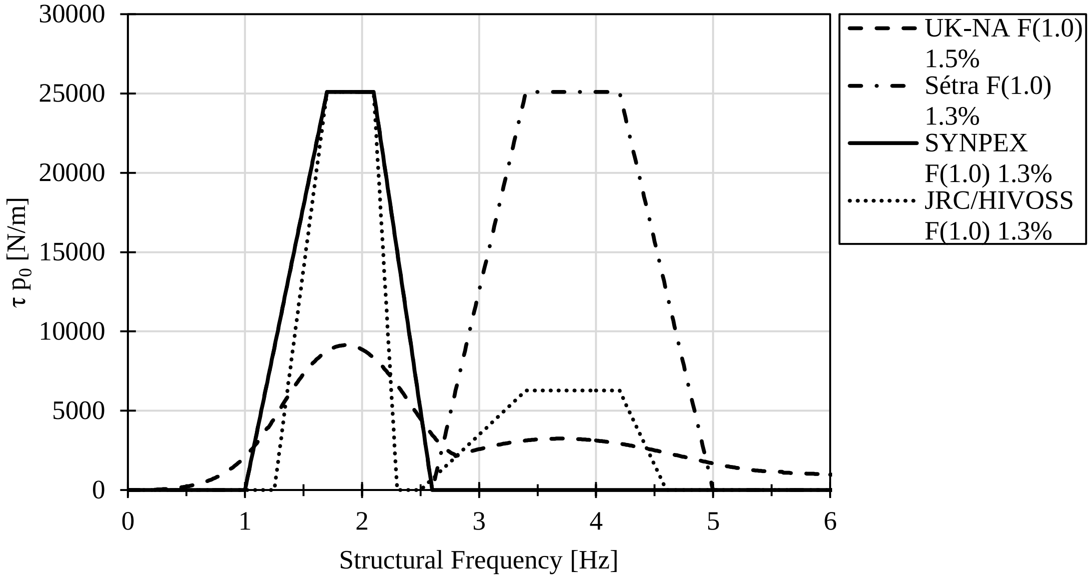


Figure 7.51 Comparison of the normalized acceleration response in vertical direction for reinforced concrete with corresponding damping.

The damping ratios for concrete are in Sétra, SYNPEX and JRC/HIVOSS recommended to 1.3% and in UK-NA to 1.5%. This means that a lower normalization factor τ is applied for the UK-NA load model giving a lower response. In Sétra, SYNPEX and HIVOSS is the damping ratio considered in the same manner which will give no difference in the response. The UK-NA load model has the lowest value when comparing them for the same damping ratio and becomes even lower for concrete. Mainly because of the higher damping ratio with lower τ but the fact that the model also includes the damping ratio could compensate for a higher ratio which not seems to be the case.

In Figure 7.52 is the normalized acceleration response in lateral direction plotted for damping ratios according to concrete structures.

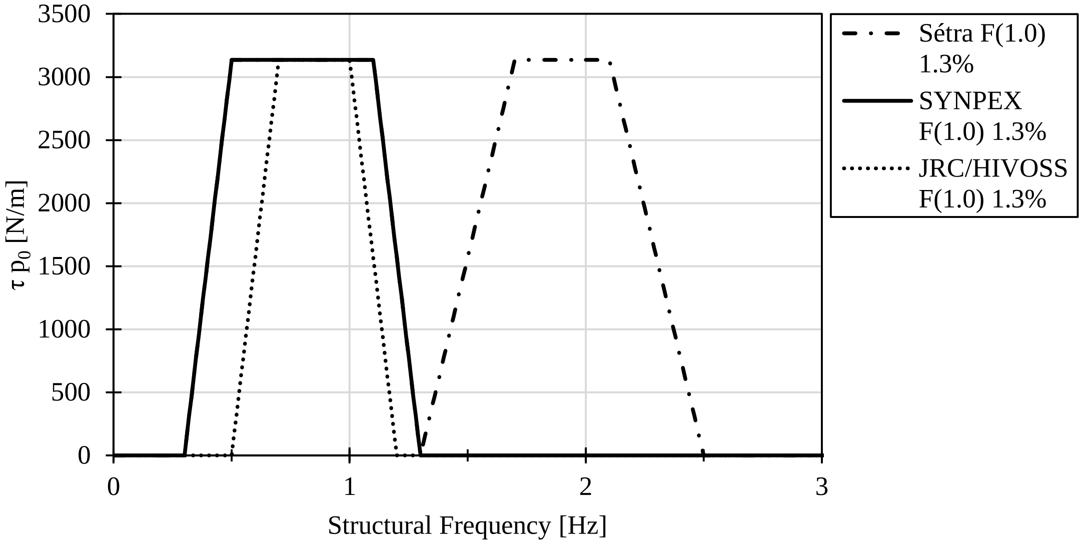


Figure 7.52 Comparison of the normalized acceleration response in lateral direction for reinforced concrete with corresponding damping.

The damping ratio is the same for all compared standards which means that the relation between them will be the same for concrete structures.

7.3.4.3 Steel

In Figure 7.53 is the normalized acceleration response plotted with damping ratios according to steel structures.

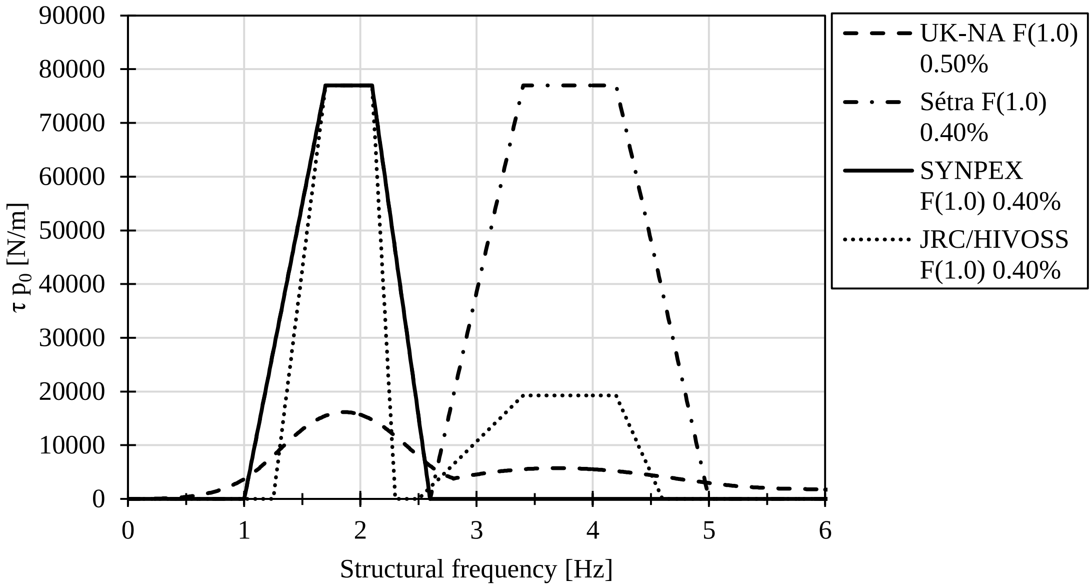


Figure 7.53 Comparison of the normalized acceleration response in vertical direction for steel with corresponding damping.

The damping ratios for steel structures are Sétra, SYNPEX and JRC/HIVOSS is equal to 0.40% and in UK-NA set to 0.50%. The damping ratios are all low and approximately the same which makes them reasonable. UK-NA has a slightly higher damping ratio than the other standards which also was the case for concrete structures. In the figure it can clearly be seen that UK-NA load model has a much lower acceleration response about 20% of the maximum from the other models.

In Figure 7.54 is the normalized acceleration response for lateral vibrations plotted with damping ratios according to steel structures.

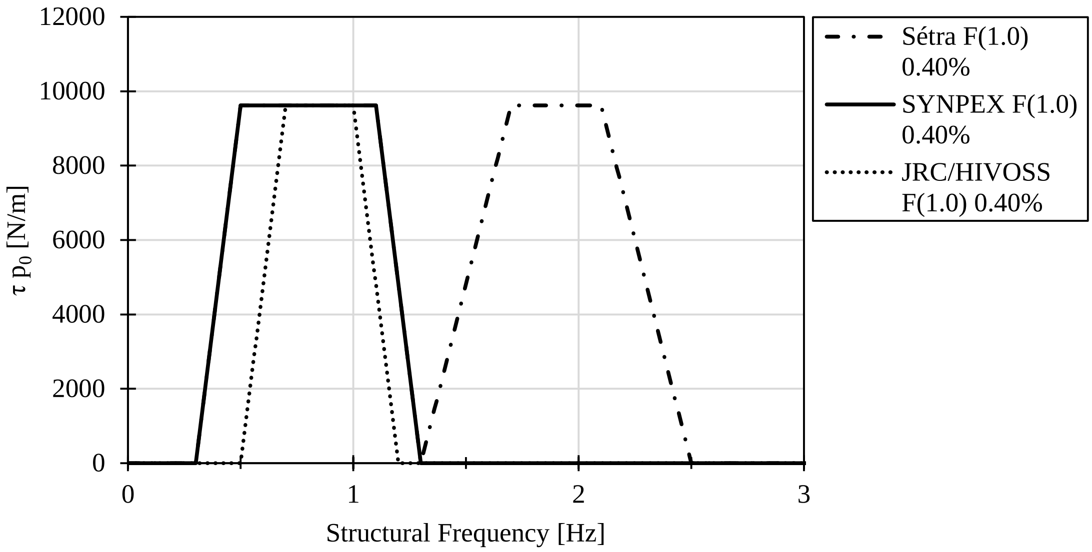


Figure 7.54 Comparison of the normalized acceleration response in lateral direction for steel with corresponding damping.

The same damping ratio is applied in the compared load models giving no difference in the relation between them.

7.3.4.4 Timber

The damping ratios for timber structures vary among the standards where SYNPEX has the highest damping ratio of 3.0%. UK-NA and Sétra recommends 1.0% and JRC/HIVOSS recommends 1.5%.

In Figure 7.55 is the normalized acceleration response in vertical direction plotted for damping ratios according to timber.

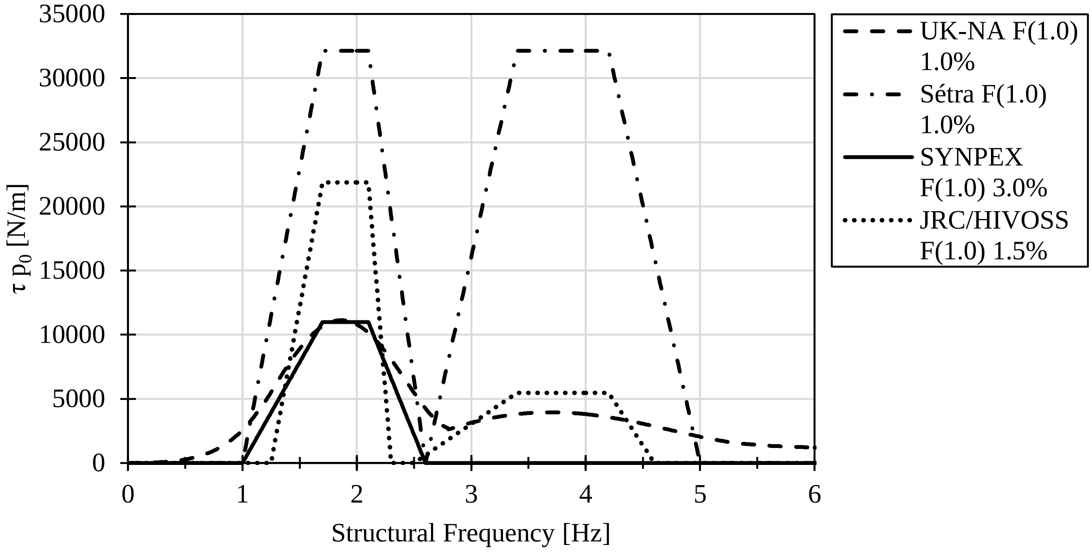


Figure 7.55 Comparison of the normalized acceleration response in vertical direction for timber with corresponding damping.

Due to the varying damping ratios the models generates different response. The maximum response is given for the load model in Sétra which also has the lowest daping ratio. UK-NA and SYNPEX has different recommendations about the damping ratio but ends up with the same response. The damping ratio is three times as high for SYNPEX in comparison to UK-NA. there is a big difference between the highest and lowest response as SYNPEX is about one third of the response in Sétra. The two models are defined in a similar way but the damping ratios affects the response drastically.

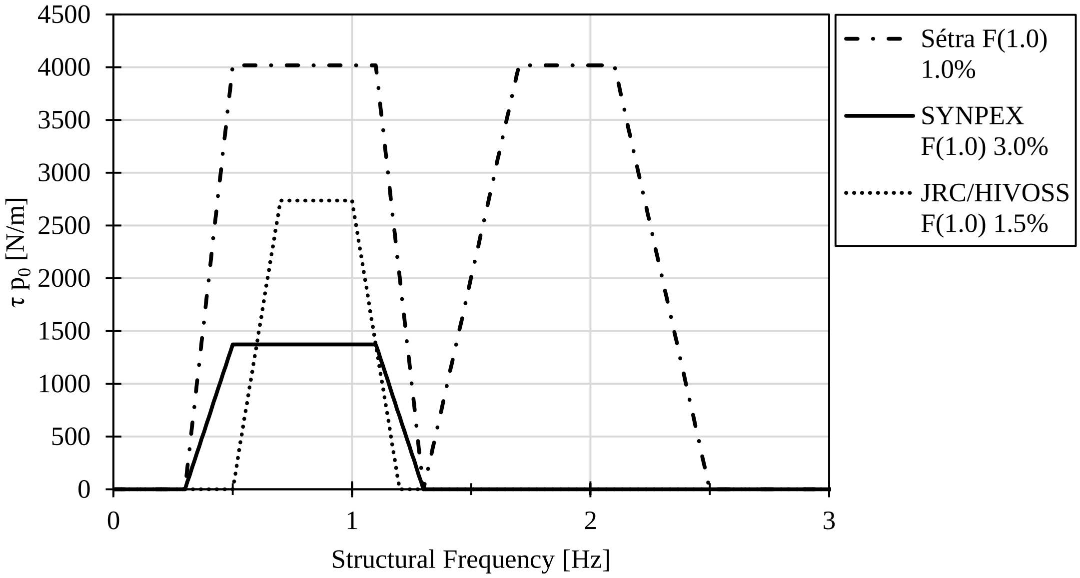


Figure 7.56 Comparison of the normalized acceleration response in lateral direction for timber with corresponding damping.

In lateral direction of loading is the relation between the load models similar to the vertical loading. The highest response is given for Sétra and lowest for SYNPEX. JRC/HIVOSS has a recommended ratio of 1.5% in between of the others and will therefore have a response higher than SYNPEX and lower than Sétra.

7.3.5 Exceptional dense traffic

The traffic situation “Exceptional dense traffic” is a load case that very seldom will occur during the bridge´s life time. It could happen during a special occasion for example during the opening ceremony of the bridge or other special occasion in a nearby location to the bridge. The traffic situation corresponds to the worst traffic classes defined in the standards. In SYNPEX, JRC and HIVOSS it is referred to traffic class 5 which is described as exceptional dense traffic with a pedestrian density equal to 1.5 ped/m^2. In UK-NA is the highest traffic situation regarded as bridge class D but with the definition given in the standard does not qualify as an exceptional dense traffic situation. Sétra does not either provide a suitable traffic class for exceptional dense traffic as the highest traffic class presented corresponds to pedestrian traffic at a train station.

In this chapter will traffic class TC5 for SYNPEX, JRC and HIVOSS be compared by the normalized acceleration response curve. Due to the fact that JRC and HIVOSS recommends the same load model and damping ratio only two curves will be compared.

7.3.5.1 Reference damping – 5.0%

The normalized acceleration response is calculated for the considered standards with the same structural damping ratio 5.0%.

In Figure 7.57 is the normalized acceleration response in vertical direction compared for 5.0% damping ratio.

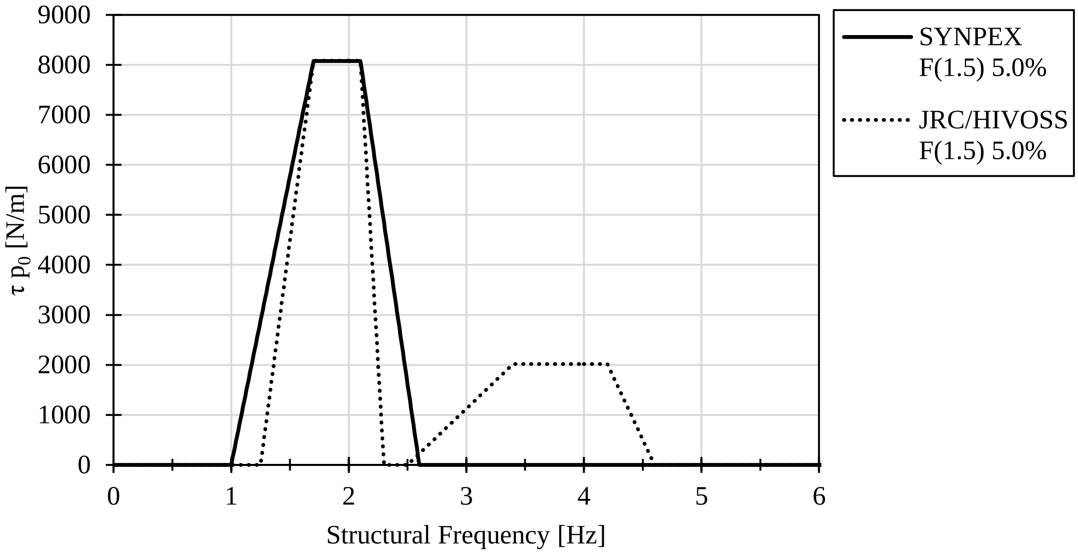


Figure 7.57 Comparison of the normalized acceleration response in vertical direction for 5.0% damping ratio.

In the comparison over the responses in figure above are all load models plotted with the same damping ratio. The responses is dependent on the load models and the by the standards defined critical frequency intervals. The load models in SYNPEX and

JRC/HIVOSS are based on each other which makes them similar. The maximum response for the first interval is equal between the models but SYNPEX is defined for a wider frequency range. SYNPEX does not consider the second harmony and is equal to zero. JRC/HIVOSS weigh the second harmony as 25% of the value in the first harmony.

In is the normalized acceleration response in lateral direction plotted for the considered standards with 5.0% damping.

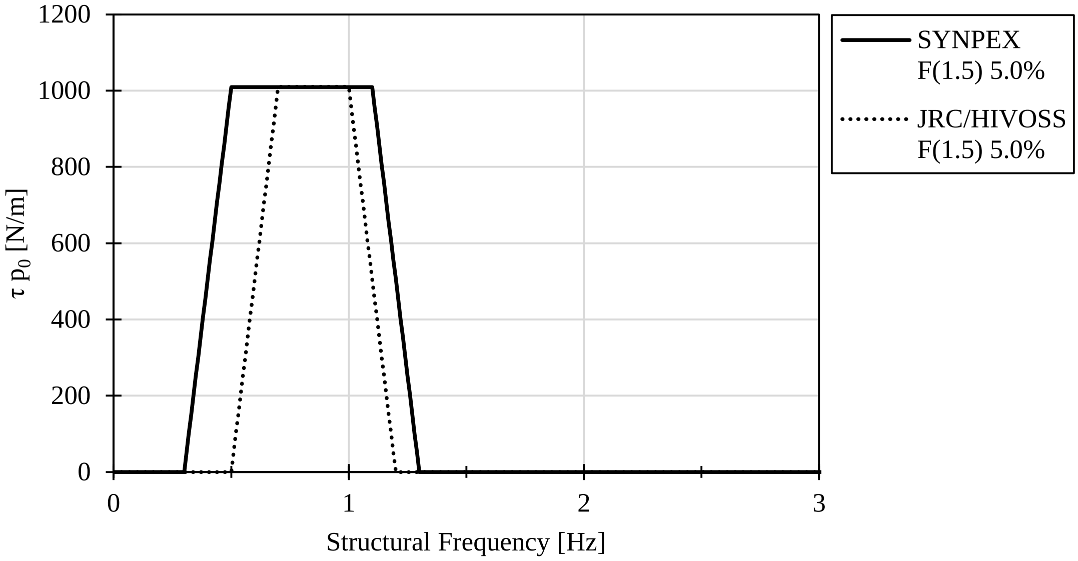


Figure 7.58 Comparison of the normalized acceleration response in vertical direction for 5.0% damping ratio.

In lateral direction are the responses from the two load models the same for the first harmony. SYNPEX has a wider frequency interval than JRC/HIVOSS and considers frequencies lower than 0.50 Hz. None of the standards is defined for the second harmony.

7.3.5.2 Concrete

The recommended damping ratio for concrete structures is in SYNPEX and JRC/HIVOSS set to 1.3%.

In Figure 7.59 the normalized acceleration response in vertical direction is plotted for the considered standards with damping ratios according to concrete structures.

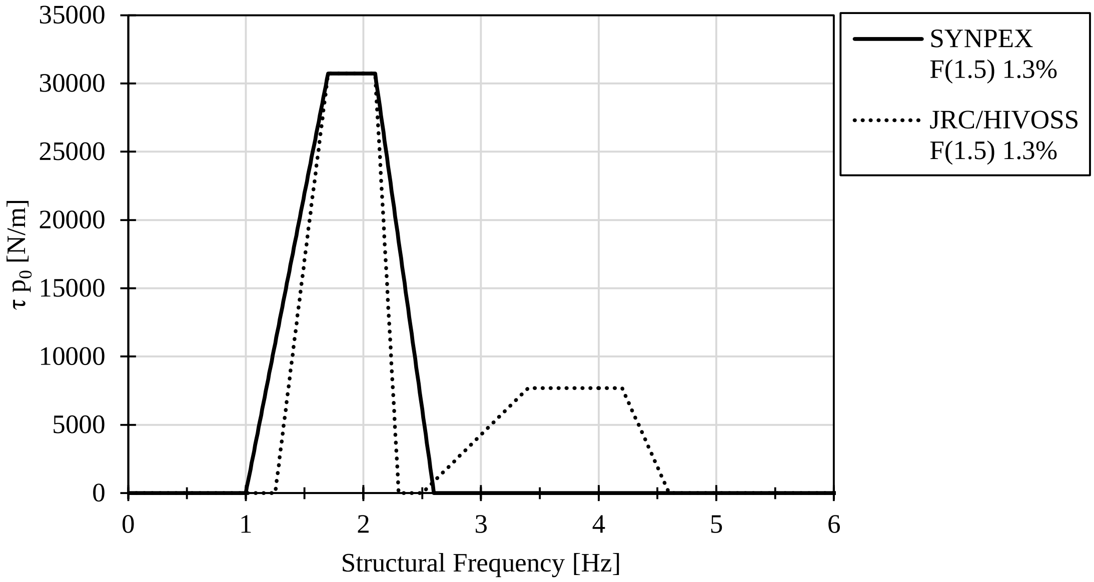


Figure 7.59 Comparison of the normalized acceleration response in vertical direction for damping ratios according to concrete structures.

As the damping ratio is equal in both standards the relation between the responses remains compared to the comparison of 5.0% damping. In is the normalized acceleration response plotted for lateral direction. The same conclusion applies for lateral direction of loading with no difference in the relation between the curves.

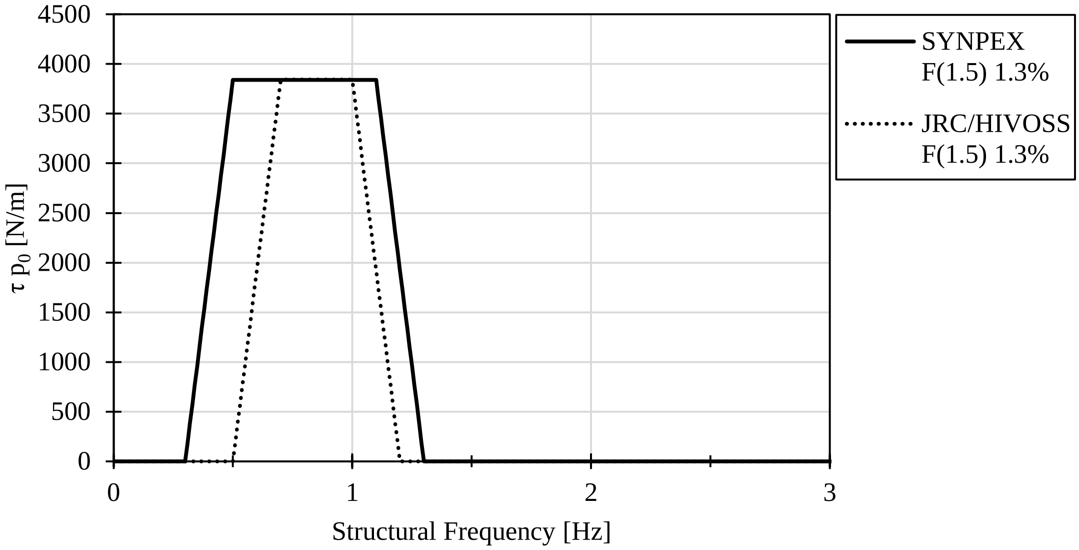


Figure 7.60 Comparison of the normalized acceleration response in lateral direction for damping ratios according to concrete structures.

7.3.5.3 Steel

The damping ratios recommend by the considered standards for steel structures is 1.5% for both SYNPEX and JRC/HIVOSS.

In Figure 7.61 is the normalized acceleration response plotted for the considered standards with damping according to steel structures.

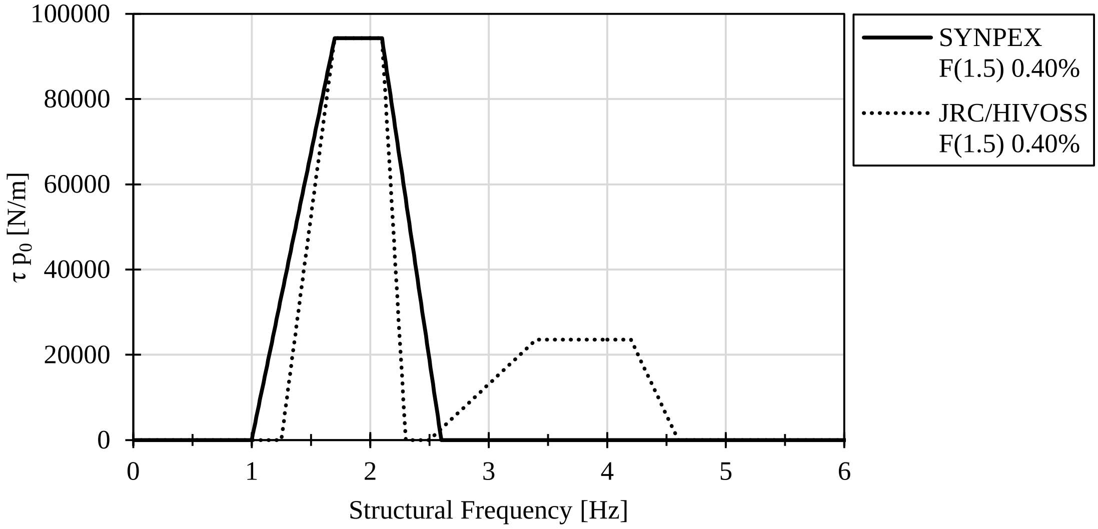


Figure 7.61 Comparison of the normalized acceleration response in vertical direction for damping ratios according to steel structures.

The damping ratio is equal for SYNPEX and JRC/HIVOSS which means that the responses are related to each other in the same way as in the comparison made for 5.0% damping. The same applies for the lateral response which is plotted in Figure 7.62.

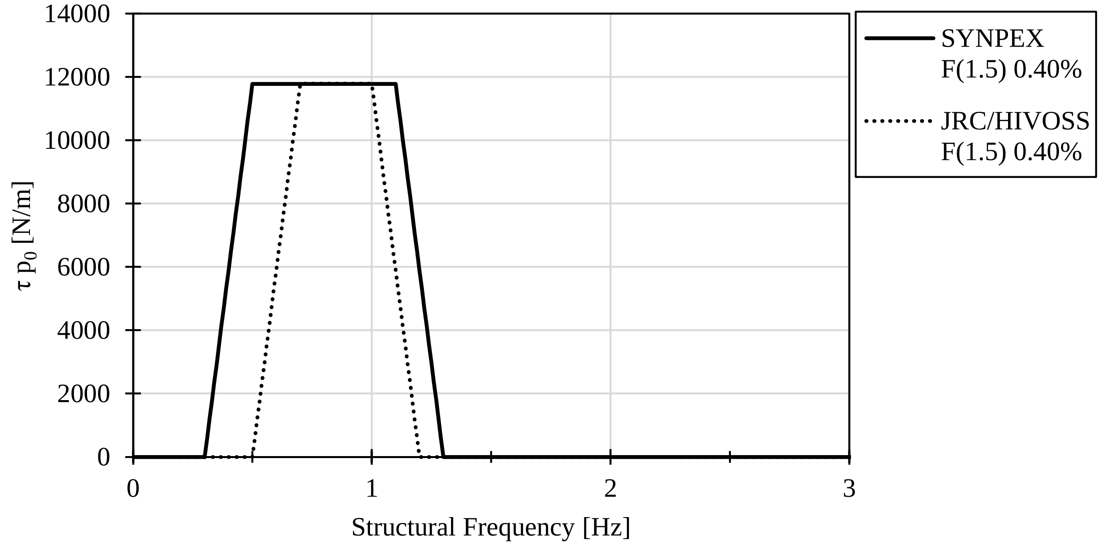


Figure 7.62 Comparison of the normalized acceleration response in lateral direction for damping ratios according to steel structures

7.3.5.4 Timber

The damping ratios recommended by the standards for timber bridges vary where SYNPEX recommends 3.0% and JRC/HIVOSS 1.5%.

In Figure 7.63 the normalized acceleration response in vertical direction is plotted for damping ratios according to timber bridges.

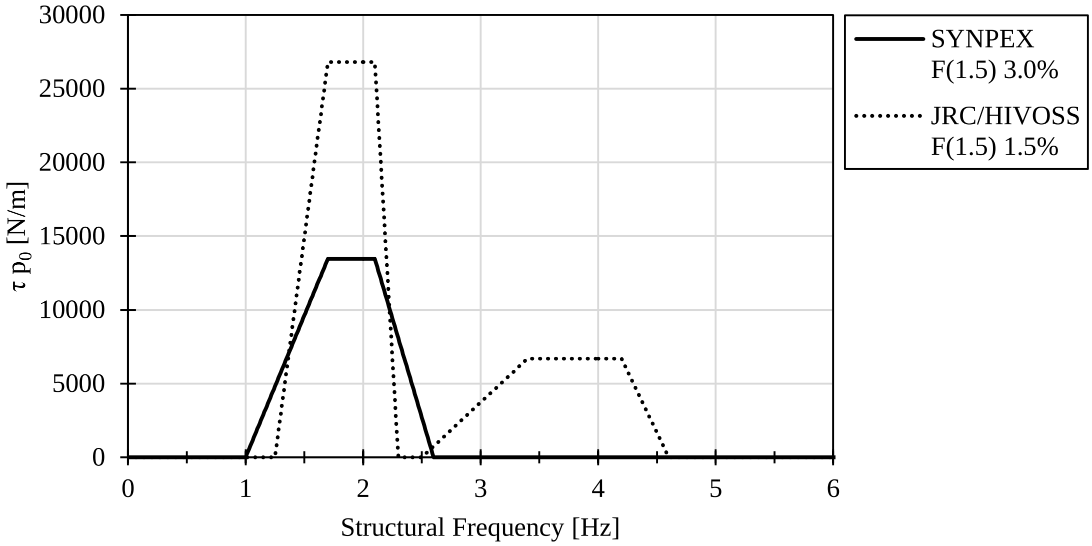


Figure 7.63 Comparison of the normalized acceleration response in vertical direction for damping ratios according to timber structures.

In Figure 7.63 above it is shown that JRC/HIVOSS has approximately twice as high response for the first harmony. The different in response is due to different damping ratios recommended. SYNPEX recommends 3.0% damping ratio which seems to an over estimation according to other standards given a too low response.

In Figure 7.64 the normalized acceleration response in lateral direction is plotted with damping according to timber.

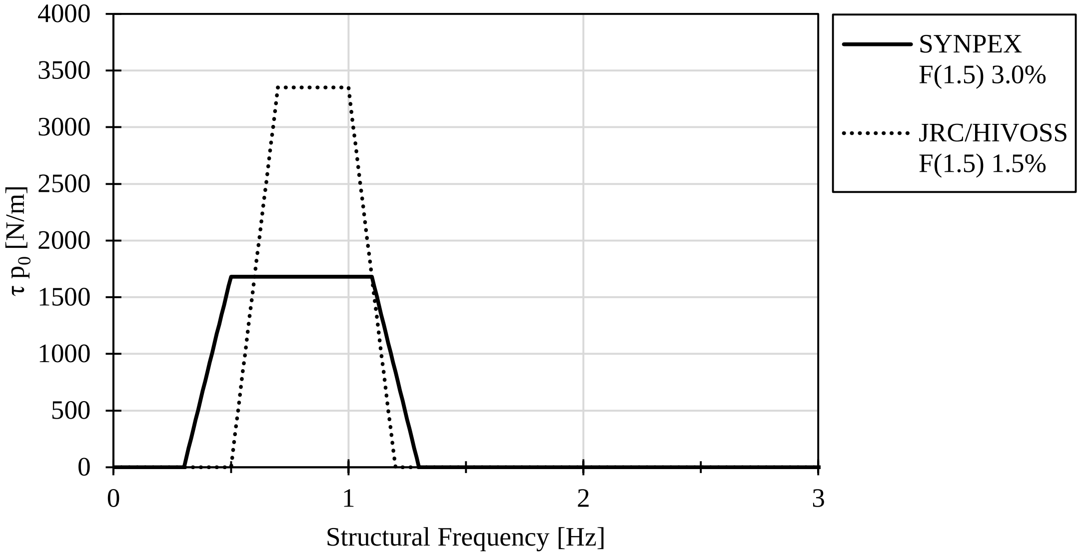


Figure 7.64 Comparison of the normalized acceleration response in lateral direction for damping ratios according to timber structures.

The response in lateral direction is twice as high for JRC/HIVOSS compared to SYNPEX due to different damping ratios.

7.4 Festive and choreographic events

That a footbridge should be designed with regard to festive and choreographic events is mentioned in the literature but is not included in the load models proposed for walking pedestrians.

In ISO 10137 there are guidelines that refer to swaying, rhythmical jumping and similar activities performed by groups of participants where a high level of coordination between the participants is assumed. Examples of design values for crowd densities are proposed for example coordinated jumping and other activities at a stationary location. This could be applied for bridges if similar events are expected to take place on the bridge itself.

The load has the same sinusoidal shape as defined for walking but with different coordination factors for high, medium and low degree of coordination adapted to group sizes equal to or larger than 50 people.

The load should be applied with given coordination factor but it is not mentioned if the load should be applied as a concentrated load with an amplitude describing a group of people or a distributed load, even though values for pedestrian densities are given.

The other considered standards do not propose any pedestrian density or load model regarding festive events which leads to the conclusion that load models adapted for buildings rather than pedestrian bridges according to Eurocode or ISO 10137 for dynamic actions could be used in this case where the load no longer resembles a walking load.

8 Discussion

In this chapter relevant aspects regarding load models and guidelines are presented and discussed.

8.1 Frequency intervals

Eurocode states that all bridges with at least one natural frequency below 5 Hz in vertical direction and 2.5 Hz in lateral and torsional directions have to be analyzed dynamically. The analysis should be done to determine the risk of resonance and excessive vibrations due to pedestrians induced forces. Eurocode recommends frequency ranges of normal step frequencies for which resonance can occur. The range is in vertical direction defined between 1.0 and 3.0 Hz. This corresponds to an interval between 0.50 and 1.5 Hz in lateral direction. A recommended frequency for jogging is set to 3.0 Hz.

The given frequency intervals does not cover all frequencies below 5 Hz. Bridges with vertical natural frequency below 1.0 Hz and above 3.0 Hz are not considered. As well as for bridges with lateral natural frequencies below 0.50 Hz and above 2.5 Hz. Resonance between the applied pedestrian load and the bridge structure will only occur for the frequencies in the specified ranges. The frequencies outside the intervals will not be considered in the load models and no dynamic analysis is needed. Eurocode does not mention that resonance can occur for the second harmony of the applied load. The second harmony will double the critical frequency ranges to consider all frequencies below 5 Hz and up to 6 Hz. There is a risk of mistakes to be made by the engineer as Eurocode lacks information about the second harmony. The engineer could, by only the information in Eurocode, suppose that a dynamic analysis is not required for bridges with natural frequencies below 5 Hz and outside the critical ranges. Even though resonance and excessive vibrations could occur for the second harmony. Eurocode refers to ISO 10137 for further guidance where different critical frequency ranges are defined. The critical frequency ranges given in Eurocode are reasonable though the upper limit of 3.0 Hz can be regarded as too high. Compared to research and other standards step frequencies above 2.5 Hz are rare.

The step frequencies suggested in the standards are similar but there are differences. ISO 10137 suggest a vertical step frequency range between 1.2 and 2.4 Hz. The range is smaller than in Eurocode and should correspond better to the actual step frequencies. The studied standards suggest approximately the same range as in ISO 10137 but the ranges defined as step frequencies does not always correspond to the critical ranges considered in design.

Sétra suggest a step frequency between 1.6 and 2.4 to be applied for the single pedestrian load model and frequencies between 1.0 and 2.6 Hz are regarded for pedestrian streams. The same applies for the load models in SYNPEX where different ranges are considered depending on the type of load model. The UK-NA is characterized by covering all frequencies between 0 and 8 Hz in the load models. It is not consistent to Eurocode by defining step frequencies up to 8 Hz as bridges with natural frequencies above 5 Hz are not required to be analyzed. Walking frequencies up to 8 Hz are not realistic even when considering the second harmonic.

The load models simulating a single pedestrian are compared with ISO 10137 in section 0 where the considered frequency ranges are shown graphically. The most critical frequencies defined by UK-NA are between 1.2 and 2.6 Hz which is comparable to ISO 10137. SYNPEX has a shorter interval between 1.25 and 2.3 Hz still covering the most critical frequencies. Sétra on the other hand is defined between 1.6 and 2.4 Hz. In vertical direction the interval can be accepted but it would correspond to a too high frequency interval in lateral direction. All concentrated load models except SYNPEX are defined for the second harmony where the double frequency applies.

The load models simulating pedestrian streams as uniformly distributed loads consider a wider range of frequencies. The models are all compared in section 7.3 where the frequency ranges are seen for the first and second harmony in vertical and lateral directions. SYNPEX has the widest frequency range regarded in design for the first vertical harmony between 1.0 and 2.6 Hz for all traffic classes. Sétra applies the same range but a shorter interval is considered for low density crowds only covering 1.7 to 2.1 Hz. An explanation why a shorter range is of interest for low density crowds, which is unique for Sétra, is not found. The standard weighs the frequencies differently in the load models and is directly related to load amplitude. The maximum load amplitude is given between 1.7 and 2.1 Hz for all standards. The range is reasonable according to normal step frequencies and by the fact that the standards agree. The second harmony in vertical direction is considered in UK-NA, Sétra, JRC and HIVOSS for all traffic classes. SYNPEX does not consider the second harmony and Sétra applies the second harmony for higher pedestrian densities. According to literature a second harmony resonance phenomena has never occurred in reality but still some of the standards enhance its importance.

Low frequencies are considered in the uniform load models for lateral vibrations where SYNPEX and Sétra are defined for a minimum frequency of 0.30 Hz. The minimum limit is far lower than half the considered vertical step frequency used in design. JRC and HIVOSS are defined to the minimum limit 0.50 Hz. This gives a great difference between the models as SYNPEX and Sétra weigh 0.50 Hz with maximum importance compared to JRC and HIVOSS which are set to zero. According to the knowledge gained by the London Millennium Bridge frequencies as low as at least 0.50 Hz should be regarded to avoid future mistakes. The second harmony is only considered by Sétra in lateral direction.

The compared standards have similar or equally defined step frequencies in their load models for vertical and lateral action of the first harmony. The defined step frequencies in ISO 10137 are relevant whereas the defined upper limit in Eurocode of 3 Hz seems too high according to literature. The frequency range defined in UK-NA can with reference to Eurocode be shortened to consider at maximum 5 Hz. The second harmony is considered in all standards except Eurocode and SYNPEX. The importance of excessive vibrations due to the second harmony diverges in literature. According to theory it seems reasonable to occur but the fact that it never happened in reality remains. An explanation why no known real case have occurred can be that other modes are excited instead. The second harmonic is theoretical and in reality torsional and mixed modes could be excited instead. The second harmonic response is interesting and can be studied further both because of its theoretical effect and if it in fact can or will occur in reality.

8.2 Effect of pedestrian mass on dynamic properties

The amount of pedestrians situated at a footbridge that is vibrating affects the damping and natural frequency of the structure. The pedestrians mass is added to the mass of the bridge and they are moving together.

JRC, HIVOSS and SYNPEX recommend that when calculating the natural frequencies of a bridge the mass of pedestrians should be taken into account. For pedestrian streams this is relevant when the added mass due to the pedestrians can be large in comparison with the weight of the bridge. The increase in mass due to pedestrians results in a lower natural frequency. This is important to pay attention to as it changes the conditions of the analysis resulting in that the risk of resonance could increase or decrease. A limit is defined in JRC, HIVOSS and SYNPEX that the mass of pedestrians should be regarded when calculating the natural frequency of the structure if the modal mass of the pedestrians is more than 5 % of the modal mass of the bridge. This limit is based on Monte Carlo simulations and should be a reasonable estimate.

Sétra recommends incorporating the mass of each pedestrian within the mass of the footbridge when analyzing with the uniformly distributed load. This is because that the static load of the pedestrian has no influence on the acceleration disregarding the fact that added mass changes the behavior of the structure. The authors think it is reasonable and in line with other recommendations found in literature.

Other regarded aspects in the literature show that the mass of the pedestrians affect the dynamic properties of footbridges. The risk of unstable lateral response is regarded with a damping mass parameter in UK-NA. A relation between the bridge mass in combination with structural damping and the mass of the pedestrians is compared with test measurements for lateral frequencies of footbridges. This indicates when unstable lateral responses are to be expected. This relationship seems reasonable as it considers the relationship between pedestrian and bridge mass which is strengthened by the analysis in section 5. The relationship between the load amplitude and the mass of the bridge is concluded to be the factor that affects the obtained accelerations which is comparable to the relationship proposed in UK-NA.

ISO 10137 mentions that the presence of pedestrians may change footbridges dynamic properties and that “some form of reliability analysis my need to be carried out”. A method for performing this reliability analysis is not presented. It is also mentioned that people will add to the overall level of damping but that damping in general is complicated to calculate. The recommendation is made that data from measurements should be used. The effect of pedestrian mass on structural damping is an aspect that should be investigated further. Field measurements on existing structures could be a good way to gather more information about this subject.

To conclude it can be said that pedestrian mass is mentioned in the literature as relevant to consider but that the connection with structural damping and natural frequencies is complex. Regarding structural damping there are a lot of uncertainties considering other factors in addition to the effect of pedestrian mass which makes this hard to assess. For accurate calculations of structural dynamic behavior the pedestrians mass should be included. The effects of pedestrian mass on the damping ratio and natural frequencies are not included in this master´s thesis.

8.3 Damping

Structural damping has a significant effect on the acceleration response of a structure and is therefore important to consider correctly. A problem is that the damping ratio is complex to calculate and involves a lot of uncertainty. It is affected by many factors as for example the structure of the bridge, if the experienced vibrations are large or small or the choice of material. Reliable values can be obtained through measurements and earlier experience but this is based on that there are available values from earlier similar bridges.

Choosing an appropriate damping ratio is a problem when defining FE-models in the purpose of an accurate analysis. As it can be seen in section 5.5 the damping ratio affects the obtained acceleration response significantly. If the damping ratio is not accurate the results from the FE-analysis will not be reliable. It can be better to choose a lower value of damping for design and to consider the calculated values as indications on the real behavior of the structure. If accelerations are calculated that are near the defined comfort limits it is recommended to take measures adding extra damping to be sure to avoid uncomfortable vibrations.

The defined damping ratios for different materials differ between the studied guidelines. The damping ratio for reinforced concrete is defined as 0.80 % in ISO 10137 and 1.5 % for UK-NA. Timber differs between a damping ratio of 3.0 % for SYNPEX and 1.0 % according to UK-NA and ISO 10137. This results in that the obtained acceleration response with different standards differs for a bridge built with the same material.

Eurocode define damping ratios dependent on material and if the span length is shorter or longer than 20 meters. The damping ratio according to Eurocode is larger for a shorter span beneath 20 m and increases as the span decreases. For span lengths over 20 m the damping ratio is constant. The proposed values are larger than the ones presented in ISO 10137 where for example the damping for reinforced concrete is defined as 0.8 % and the value in Eurocode is defined as 1.5 % for spans larger than 20 m.

This result in questions regarding which damping ratio that should be used in design. A conservative approach is to take the lower value defined in ISO 10137 giving rise to a larger acceleration response. For example regarding reinforced concrete other standards propose a damping ratio closer to 1.5 % as for example Sétra that gives a value of 1.3 % and UK-NA which gives a value of 1.5 % as well. This implies that the damping ratios defined in ISO 10137 might be conservative.

It should also be noted that damping ratios are dependent on the structure of the bridge. For example non-structural elements such as railings can add to the overall damping of the structure in a way that can be hard to predict. Damping ratios defined from the type of structure in addition to choice of material would be useful in design.

The authors recommend a conservative approach choosing low damping ratios. ISO 10137 is the most conservative of the studied standards and can be a good alternative.

8.4 Load models

Concentrated and uniformly distributed loads are discussed separately in the following two sections.

8.4.1 Concentrated load models for single pedestrians and groups of pedestrians

The humans induced vibrations of a single pedestrian is simulated by a concentrated force. The proposed models in ISO 10137, SYNPEX and Sétra are based on Fourier series considering up to five Fourier sums. The UK-NA proposes a model different from the others but similar to the uniform load proposed in the standard.

Common for all models is that it should be applied as a moving load over the span. The idea is to in an accurate way simulate a pedestrian crossing the bridge. The walking velocities are given both as constant values independent on step frequency and as functions of the frequency. ISO 10137 does not recommend a walking velocity as the other standards do. It is reasonable to consider the step frequency in the walking velocity as they are highly related. To model the load as moving increases the complexity of application in a FE-software or simple hand calculations. The structural response will be dependent on the bridge length and walking velocity as the loading time is related to the time it takes for the load to cross the bridge. The loading time is probably not sufficiently long to reach steady state. It is even questionable if steady state conditions can be achieved as the load does not act at the worst position during the entire loading time.

A simplification has been made in this thesis to only analyze the concentrated loads as stationary. The loads have been applied until steady state was reached which gave higher acceleration responses than from a moving load. Since all concentrated load models should be applied as moving they are all comparable on the same basis even though the acceleration responses cannot be compared to reality and to the recommended acceleration limits.

The difference in maximum response between a moving and a stationary load has not been studied in this thesis but is of high interest. Interesting aspects could be how the bridge length affects the response and if the load models can be simplified as stationary but with a modification factor. The simplest way is to apply the loads in a FE software as stationary and for an engineer it can be of interest to know how the response is related to the way of load application.

ISO 10137 proposes a concentrated load model applicable in vertical and lateral direction. It includes up to the fifth harmonic in vertical direction and only the first in lateral direction. It is recommended in the standard to consider a maximum of three harmonics as the higher harmonics are not perceptible by humans. In the normalized acceleration response only the first Fourier sum is considered. A study has been made in section 6.1.1 of how the number of considered harmonics affects the response in a structure. It shows that the first sum gives the governing response. The additional sums will not increase the vibration and can be neglected. Based on the study only the first Fourier sum is used in normalization. Concerning the second harmony it has been shown that only the second Fourier sum affects the structural response. Meaning that the second Fourier sum can be used individually without considering the first sum which has been used in normalization.

The load model for vertical loading in ISO 10137 is defined between 1.2 and 2.4 Hz for the first harmony and between 2.4 and 4.8 Hz for the second harmony. In lateral direction the frequency range is defined between 0.60 and 1.2 Hz. ISO 10137 gives the highest acceleration response for 2.4 Hz.

The load model in UK-NA is defined for all frequencies from 0 to 8 Hz as seen in Figure 7.1 which presents the comparison between ISO 10137 and UK-NA for 5.0% damping. Both standards consider the first and second harmony and have a similar shape over the frequencies. UK-NA has higher response for frequencies below 2 Hz but ISO 10137 has higher response between 2.0 and 2.4 Hz and the highest maximum response. The curves are parallel between 1.2 Hz and up to 2.0 Hz where UK-NA has a higher response for all frequencies. ISO 10137 has constant response for the second harmony lower than UK-NA. UK-NA weighs 3.8 Hz the highest for the second harmony which is reasonable according to common step frequencies. Overall the response is similar for the two load models even though they diverge in response at the most common walking frequencies just below 2 Hz. ISO 10137 weighs 2.4 Hz as the most important differently than UK-NA. The chosen frequency is not the most common step frequency but could give high dynamic effects as high frequency means fast walking speed close to running and generates higher load amplitude. A walking frequency at 2.4 Hz means fast walking which results in a high force when the foot hits the ground. The assumption seems reasonable for single pedestrians. UK-NA generates a much lower acceleration response than ISO 10137 for 2.4 Hz which can give great differences in design.

SYNPEX propose a concentrated load model applicable for a single pedestrian in vertical and lateral directions. The load model is a Fourier series with coefficients and phase angles based on complex functions of the step frequency. SYNPEX does not consider the second harmony in their load models. In vertical direction it is defined from 1.25 to 2.3 Hz and in lateral direction between 0.30 and 1.3 Hz. In Figure 7.5 is the model compared with the load model proposed by ISO 10137 for 5.0% damping. The models are almost identical for the first harmony with increasing response for higher step frequencies. ISO 10137 generates higher response for all frequencies defined but the curves are close to parallel with small differences in response. The critical frequency range is slightly shorter for SYNPEX and it is not defined for the second harmony. In lateral direction compared in Figure 7.6 it can be seen that the two load models gives the same response. Both curves are constant and equal at the maximum value. Even in lateral direction SYNPEX is defined for a slightly shorter frequency range.

Sétra proposes a concentrated load model based on Fourier series that is adjustable to consider vertical, longitudinal and lateral loading. Different from the other standards, Sétra recommends the engineer to only consider the first Fourier sum of practical reasons. The first Fourier sum of the vertical load model is compared to the vertical model in ISO 10137 in Figure 7.13. Because of the recommendations in Sétra the main difference between the responses in the two load models is due to the critical frequency ranges. Sétra is defined for a short interval in vertical direction between 1.6 and 2.4 Hz. This gives big differences in response for frequencies below 1.6 Hz which are common step frequencies. Although Sétra gives the highest response for frequencies between 1.6 and 2.1 Hz which are the most common step frequencies. ISO 10137 reaches the highest maximum response due to weighing of high step frequencies where Sétra is constant in the defined range. The response in lateral direction diverges between the models as seen in Figure 7.14. ISO 10137 gives twice as high response and is defined for a significantly wider frequency range. Sétra considers step frequencies from 0.80 to 1.2 Hz which is short in comparison to other standards. It clearly does not cover all important frequencies that can cause excessive lateral vibrations referring to research done on the London Millennium Bridge.

The concentrated load models are primarily used to simulate a single pedestrian but can also be used for group of pedestrians. For an accurate and realistic load model the concentrated load need to be multiplied with an equivalent number of pedestrians. Multiplying the load by the total number of pedestrians in a group gives too high load amplitude as it implies that the pedestrians all walk with perfect synchronization. The equivalent number of pedestrians takes the probability of synchronization among the pedestrians into account and should always be used. ISO 10137 and UK-NA gives recommendations about the equivalent number of pedestrians. In Figure 7.21 are groups of 8 and 15 pedestrians compared between the models with 5.0% structural damping ratio.

It was earlier shown that ISO 10137 has higher response than UK-NA for frequencies above 2 Hz maximum for a single pedestrian. Though it has a lower response for frequencies below 2 Hz and for the second harmony for the same load case. The relation between the responses remains constant even for groups of pedestrians shown in Figure 7.21. This implies that the two models have a similar expression for the equivalent number of pedestrians for high damping. The model in UK-NA includes the damping ratio in the load model which can affect the response in a more complex way for other damping ratios. In Figure 7.22 shows the same comparison but for a low damping ratio equal to 0.50%. The relation between the curves remains constant even for low damping. It can therefore be concluded that ISO 10137 and UK-NA are similar in acceleration response for single pedestrians and group of pedestrians for all damping ratios. Still there are differences in the acceleration response for specific frequencies especially at 2.4 Hz.

The overall impression from the considered load models is that ISO 10137 shows acceleration response that generally is comparable and of the same magnitude as for SYNPEX, UK-NA, and Sétra. Though there are big differences in response for specific frequencies. The load application is similar among the models but ISO 10137 is not considered as the most complete.

SYNPEX proposes the load model which is most equal to ISO 10137. The models show approximately the same response in both vertical and lateral direction of loading though SYNPEX do not consider the second harmony. Both models weigh high step frequencies of most importance. SYNPEX refers to ISO 10137 in their report but it is not clear if SYNPEX has chosen to follow the assumptions in ISO 10137. It is hard to predict what the correct answer is without measured data from field measurements. SYNPEX has the far most complex Fourier coefficient and phase angles. They are calculated by complicated functions resulting in similar coefficients given by ISO 10137.

The load model proposed by UK-NA is regarded as the most complete model. The model includes factors not regarded in the other models concerning synchronization in groups of pedestrians, span length, structural damping and a design factor for weighing the importance of the frequencies. It is at the same time the only model which is not based on Fourier series. Different from ISO 10137 and SYNPEX it weighs 1.9 Hz as the most important frequency which is reasonable for groups according to common step frequencies. The model is not applicable in lateral direction which had been desirable.

Sétra propose a model which is simple to apply and recommends the engineer to use only the first Fourier sum. In vertical direction it gives accurate results compared to ISO 10137 and the other standards. The model generates constant response for its defined frequencies. In lateral direction it differs from the other standards with a lower response.

The greatest difference between the load models is the acceleration response at 2.4 Hz where ISO 10137 gives the highest response. It can be a reasonable estimate regarding a single pedestrian as high step frequency gives high step forces. Though for a group of pedestrian the probability is low that all pedestrians are walking at the same high frequency generating the highest response. In UK-NA the probability of likely step frequencies is considered which gives the highest response just below 2 Hz. It makes the model applicable for groups but could give too low response for a single pedestrian.

To conclude ISO 10137 recommends a good load model for a single pedestrian that is applicable in vertical and lateral direction. Due to its simplicity defined as a Fourier sum without design factors the model is by the authors of the thesis considered to be too simple to model pedestrian groups. The authors think that the model recommended by UK-NA is the most complete and well defined for pedestrian groups even though lateral vibrations are not covered.

8.4.2 Pedestrian crowds simulated as uniformly distributed loads

During crowded conditions the pedestrian loading can be modeled as a uniformly distributed load applied over the entire bridge deck. It is the most convenient way of modeling the pedestrian induced forces caused by more than a single pedestrian. SYNPEX is the oldest of the considered standards that recommends this type of loading. Sétra, JRC and HIVOSS are all referring to SYNPEX but have made their own interpretation and adjustments to the model. All models can be modified to consider vertical, lateral and longitudinal loading directions. UK-NA proposes a uniform load model different from the other standards which only applies for vertical vibrations. ISO 10137 does no propose a uniform load model or any model to simulate pedestrian streams even though it recommends the designer to perform such an analysis.

In section 7.3 the models are compared graphically where the relation between acceleration responses and the structural frequency of the bridge is shown. The acceleration response for the same damping ratios varies due to pedestrian density, critical frequency ranges, equivalent number of pedestrians and individual factors defined in the standards.

In Figure 7.49 the uniform load models are compared for the same damping ratio 5.0% and pedestrian density equal to 1.0 ped/m^2. It can clearly be seen that SYNPEX, Sétra, JRC and HIVOSS has the same maximum acceleration response for the first harmony. The models are similar and do not deviate for the first harmony. The parameters affecting the acceleration response are equal.

The maximum response by the UK-NA load model, in Figure 7.49, is about 60% of the others maximum response. The model is completely different but should simulate the same situation which is confusing. For this specific load case with 1.0 ped/m^2, some main differences between the models are identified. SYNPEX, Sétra, JRC and HIVOSS do not consider the structural damping ratio in the applied load as they do for lower pedestrian densities. The standards state that the damping does not affect the synchronization among the pedestrians if the bridge is crowded. The load model in UK-NA always considers the damping ratio in the reduction factor γ. The factor is increasing with increased structural damping and varies between 0 and the square root of 0.20. Figure 7.41 shows the acceleration response for 5.0% damping ratio and pedestrian density 0.8 ped/m^2 for Sétra and UK-NA. The relation between the two curves remains as UK-NA has about half as high acceleration response. This means that UK-NA gives about half as high response independent on pedestrian density for high damping ratios. For low damping ratios the relation between the responses from Sétra and UK-NA can be seen in Figure 7.55. The response is plotted for 1.0% damping ratio and 1.0 ped/m^2 where Sétra has an acceleration response three times as high. It can be concluded that UK-NA gives a low acceleration response compared to the other standards with increasing difference for low damping ratios.

The critical frequencies and the weighing of their importance are crucial factors when comparing the acceleration responses. In Figure 7.49 the models are compared with the same input data and the differences in frequency ranges can be seen. UK-NA covers all frequencies from 0 to 8 Hz with varying importance. SYNPEX and Sétra consider a wider span than JRC and HIVOSS. The models are defined for the same frequency interval resulting in the maximum response. The difference in critical frequency range gives the largest impact for the frequencies in the border line between the curves. I.e. at 1.25 Hz where JRC and HIVOSS are zero SYNPEX and Sétra have reached approximately a third of their maximum. An even greater difference is identified at 2.3 Hz where SYNPEX and Sétra reach over 50% of their maximum compared to zero for JRC and HIVOSS.

The second harmony is considered by Sétra, JRC, HIVOSS and UK-NA but not by SYNPEX. Sétra has the highest acceleration response for the second harmony in all analyzed cases presented in section 7.3. Sétra weighs the second harmony equal to the first as no other standards do. JRC and HIVOSS weigh the second harmonic as 25% of the maximum of the first harmony. UK-NA weighs the second harmony higher than JRC and HIVOSS at 38% of the maximum in the first harmony. As a consequence the difference between JRC, HIVOSS and UK-NA is lower for the second harmonic compared to the first harmonic.

In lateral direction the compared load models SYNPEX, Sétra, JRC and HIVOSS all have the same load amplitude for the same pedestrian density and damping ratio. The difference between the models is the considered frequencies. Sétra and SYNPEX are defined between 0.30 and 1.3 Hz for the first harmonic. The maximum response is given between 0.50 and 1.1 Hz. The range is wide and represents frequencies even lower than the normal step frequencies presented in literature. JRC and HIVOSS are defined between 0.50 and 1.2 Hz with maximum response between 0.70 and 1.0 Hz. The range is narrower than in SYNPEX and Sétra but still covers the most critical frequencies. As learnt by the London Millennium Bridge frequencies at 0.5 Hz can cause excessive vibrations and discomfort and should be treated with caution. A wide frequency range that covers frequencies well below 0.5 Hz can be recommended for a safe and reliable design.

The overall impression regarding the most preferable load model is that none of them fulfill all desired requirements. The UK-NA is easy to use and complete as it covers all important factors affecting the vibrations induced by pedestrian streams. It weighs the first and second harmonics in a reasonable way but it generates lower acceleration response for all considered load cases and damping ratios. This leaves a lot of unanswered questions that cannot be evaluated without further investigations of real loads on existing bridges. The model covers only vertical direction of loading which is insufficient for a complete analysis. The other standards provide load models that cover all directions of loading and all important factors for simulating pedestrian streams. JRC and HIVOSS consider the second harmonic in a reasonable way as 25% of the response for the first harmonic. Though the models have too small frequency intervals for lateral loading as they do not cover frequencies below 0.5 Hz. SYNPEX and Sétra cover low lateral frequencies in a preferable way but are not to be recommended for vertical direction. SYNPEX is incomplete as it does not consider the second harmonic. Sétra on the other hand considers both the first and second harmonic with same magnitude which is questionable. The standard defines a shorter range of frequencies dependent on traffic class in comparison to the other load models. This seems unreasonable as it results in that relevant frequencies are not regarded.

The fact remains that Eurocode and ISO 10137 needs to be complemented to model pedestrian streams. For a conservative approach the authors of this thesis would recommend the load model presented in Sétra. The model generates the highest acceleration responses for the considered traffic classes in both vertical and lateral direction. It is the most conservative load model regarding second harmonics, possibly too conservative. Though it should be used with caution for low traffic classes as the considered frequency interval is narrower than for higher traffic classes. The authors recommend considering the frequency interval defined for high traffic classes even for low traffic classes in order to make an accurate analysis.

An alternative solution to complement ISO 10137 with a model of pedestrian streams is to apply the concentrated load model given in the standard as a uniformly distributed load. It has not been studied in this thesis but is recommended for further investigations. The authors think it possible to adapt the concentrated load model to simulate pedestrian crowds. The load can in a FE-analysis be applied over the span recalculated to consider pedestrian densities and multiplied with the equivalent number of pedestrians recommended in the standard. The authors believe that the load application will simulate an evenly distributed crowd in the same way as the other models do. The acceleration response in comparison to other models is currently unknown and is of great interest to investigate in further studies if it can be a suitable solution with accurate results.

8.5 Acceleration limits

Sétra, SYNPEX, HIVOSS and JRC states acceleration limits according to a maximum, medium and minimum level. In vertical direction the maximum level of comfort is defined as smaller than 0.50 m/s^2 and the minimum level as an interval from 1.0 to 2.5 m/s^2 where accelerations above 2.5 m/s^2 are defined as unacceptable. The fact that these limits are similar point to that a consensus exists and that they can be regarded as reasonable.

UK-NA proposes an acceleration limit of 2.0 m/s^2 which in comparison with Sétra, SYNPEX, HIVOSS and JRC corresponds to a minimum level of comfort.

ISO 10137 defines acceleration limits calculated from a base curve for vertical and lateral direction. In vertical direction this results in an acceleration limit defined as decreasing from 0.60 m/s^2 for 1 Hz to 0.30 m/s^2 for a structural frequency of 5 Hz. Compared with the limits defined in the other regarded standards this corresponds to a medium and maximum level of comfort. This implies that the limits in ISO 10137 for vertical direction are conservatively defined.

The research done on the London Millennium bridge and Solférino footbridge shows that lateral "lock-in" is a phenomenon that must be regarded during design. Sétra that is based on research done on the Solférino footbridge limits accelerations in lateral direction to 0.10 m/s^2 in any case to avoid "lock-in".

UK-NA states that if there are no lateral modes below 1.5 Hz it can be assumed that unstable lateral response will not occur. For lateral modes below 1.5 Hz a method is proposed where the mass per unit length of the bridge in combination with the structural damping is divided with the mass of pedestrians per unit length. This relationship is compared with a stability boundary established through measurements to estimate if unstable lateral movements will occur. A limitation with this method is that the stability curve only is valid for lateral frequencies from 0.5 to 1.1 Hz and extracted theoretically beyond these frequencies. This curve should therefore be used with caution outside the valid range.

In lateral direction JRC, HIVOSS and SYNPEX defines acceleration limits as maximum and minimum level of comfort. Maximum level is defined as smaller than 0.10 m/s^2 and the minimum level as an interval from 0.30 to 0.80 m/s^2 where accelerations above 0.80 m/s^2 are defined as unacceptable.

JRC and HIVOSS define a critical number of pedestrians and a range of frequencies when "lock-in" is likely to occur. The critical number is based on a relationship between the modal mass, natural frequency, the structural damping and a constant that is derived from the measurement on the London Millennium Bridge. The range for when "lock-in" is likely to occur is defined as 0.10 m/s^2 to 0.15 m/s^2 in JRC and HIVOSS which is close to the defined limit 0.1m/s^2 to avoid according to Sétra. SYNPEX defines an acceleration limit of 0.1 m/s^2 for which lateral "lock-in" can be expected if a certain relationship between the step frequencies of the pedestrians and the natural frequency of the bridge also is met.

IS0 10137 defines acceleration limits in lateral direction but does not consider lateral "lock-in". Based on the literature and the other considered guidelines it seems relevant for this to be added to the limits in ISO 10137.

The lateral limits defined in ISO 10137 are constant from 1 to 2 Hz and increasing for higher frequencies. This results in a limit of 0.108 m/s^2 for 1 Hz to 2 Hz which increases linearly for frequencies above 2 Hz resulting in 0.27 m/s^2 for 5 Hz. These limits are similar to other standards.

Eurocode defines maximum acceleration limits for vertical and lateral direction to 0.70 m/s^2 and 0.20 m/s^2 respectively. The vertical limit is acceptable if compared with the given values in other standards. In lateral direction it is larger than the limit for lateral "lock-in". The vertical limit is low defined concerning all type of bridges in all locations. It would have been preferable to define ranges of acceptable accelerations. Pedestrians experience vibrations differently and different demands are therefore relevant due to location and situations. The limit in lateral direction is reasonable but a lower limit of 0.10 m/s^2 is recommended to avoid "lock-in".

8.6 Bridges of different material

It is clear that the mass of a bridge in comparison with the applied load is the decisive factor for the acceleration response. This means that the same bridge designed with different materials will result in different acceleration response as different materials have different structural properties.

The materials studied in this thesis are reinforced concrete, steel and timber. Steel and timber are strong materials in comparison to their weight and should therefore be more likely to exhibit large vibrations than a corresponding bridge in concrete which is heavier and demands a larger cross-section. This is not studied extensively in this thesis and is a subject for further studies. A comparison with a reference bridge designed with different materials for the same conditions can be made in order to examine this thoroughly. A case study of a bridge with a natural frequency of for example 2 Hz designed with different materials could be done examining the obtained accelerations.

8.7 User-friendliness

The standards and guidelines present their recommendations with different methods and to different extents. This affects how easy they are to apply for an engineer during design.

ISO 10137 suggest a load model to describe the pedestrian force of a single pedestrian or a group of pedestrians but does not give any recommendations on how the force should be applied. It does not give any clear recommendations on how big the groups should be and defines that streams of pedestrians should be analyzed but no model for pedestrian streams is given. When designing a footbridge for dynamic loading due to pedestrians there are a lot of factors and information missing in ISO 10137.

SYNPEX gives a lot of recommendations of methods and load models but does not point out the most relevant or accurate. This makes it hard to use during design when it is not clear which load models and limits to consider as relevant.

UK-NA is easy to follow but does not explain all empirical factors. It is unclear what the factors account for and this gives rise to uncertainties in the design phase.

All standards defining a load model for a single pedestrian state that the load should be applied as a moving load across the bridge. A moving load is significantly more complex to apply in FE-software or in hand calculations than a stationary load. The obtained accelerations depend on bridge span and velocity as this decides the time the load takes to cross the structure. It is very likely that the loading time is not sufficient for steady-state to occur or that it is even possible. The load do not act at the most critical position along the span and the fact that the load moves could just as easily counteract the resonance phenomenon as to create it.

An uncertainty regarding load models for single pedestrians is the number of Fourier coefficients and harmonies included in Fourier sums which are suggested as load models in ISO 10137 and SYNPEX. Regarding the Fourier coefficients defined in ISO 10137 the obtained accelerations seem to be dependent only on the first term in the sum while the others are working against each other. The Fourier coefficients presented in SYNPEX are very complex and takes effort to implement in design. Considering two or three coefficients results in larger amplitude compared to regarding one. It is not sure that this larger amplitude results in larger acceleration response. The shape of the first Fourier term gives rise to the resonance phenomenon whereas the load resulting from two or three terms could just as easily counteract each other resulting in a lower acceleration response than for one term. This can be seen in Figure 6.44.

Traffic classes based on expected traffic or locations with corresponding pedestrian densities are considered in the guidelines. This is very helpful for a designer if the pedestrian loading is not well defined by the client. In addition pedestrian streams are modeled with a uniformly distributed load with different size corresponding to the density expected on the bridge. This is easy to implement during design in comparison to modeling pedestrian streams with single pedestrians or groups which is complex and time consuming. Guidelines for which steps to take during design are also suggested which is helpful.

The guidelines presented in Sétra are hard to follow and “traffic classes” and “load cases” are confused with each other which makes is hard to use. JRC and HIVOSS provide relevant information with well-defined traffic classes.

8.8 Design situations given in Eurocode and ISO 10137

The proposed design situations in Eurocode and ISO 10137 seem reasonable as they include situations likely to occur. Situations where a single pedestrian, groups of pedestrians and streams of pedestrians traverse a structure are recommended to consider.

All of these design situations are not covered in Eurocode and ISO 10137 and guidelines for design need to be found elsewhere. Furthermore the guidelines leave a lot of factors to the designer to make reasonable estimates, for example how big the pedestrian streams should be and how it should depend on the geometry of the walkway which is not defined. Recommendations on how much pedestrian traffic that can be expected for certain locations are not presented as are done in other guidelines which could be helpful.

The design situation considering a single pedestrian proposes that one pedestrian should be standing in the middle of the span while another traverses the structure. This might be unnecessary to model as the pedestrian standing at mid span only adds a static load without dynamic contribution and with no significant effect to the bridge structure. This design situation could take into account that the pedestrian in the middle would experience any vibrations more disturbing than the pedestrian traversing the bridge. The authors think that Eurocode and ISO 10137 suggests to measure the acceleration in the middle of the span while a pedestrian is crossing the bridge rather than adding the static load of an extra pedestrian in the middle of the span. However this is not evident from the standards.

The other standards studied in this thesis provide guidelines on how pedestrian streams should be modeled and what densities of pedestrians that can be expected. This can work as a complement to the design situations proposed in Eurocode and ISO 10137 where the load from pedestrian streams is not defined. The more detailed guidelines and additional load models in these standards together with ISO 10137 can be used to cover the design situations proposed in Eurocode and ISO 10137.

JRC and HIVOSS propose only distributed load to models the pedestrians load as no single pedestrian load models are defined. It seems as the load from single pedestrians are neglected. The authors think it is relevant to define a concentrated load model for single pedestrians in order to model groups of pedestrians to cover the design situation in Eurocode and ISO 10137. This is relevant for footbridges as a group of pedestrians can give rise to higher acceleration than to model streams with low pedestrian densities.

9 Concluding remarks

The conclusions from this master´s thesis and suggestion for further studies are presented in this chapter.

9.1 Conclusions

In this Master´s Thesis current standards and guidelines have been presented, introduced and explained concerning how to design pedestrian footbridges regarding human induced vibrations. Several complete and accurate load models, methods and recommendations were found and presented which has increased the knowledge of dynamic design of footbridges.

A numerical method has been derived to be able to compare and evaluate proposed load models in a sufficiently accurate way valid for a simply supported structure with arbitrary length, width, cross-section and density. It has been shown that the acceleration response depends on the ratio between load amplitude and mass of the bridge. A normalization factor dependent on the structural damping was derived in order to describe the acceleration response. The normalization factor and the relationship between load amplitude and structural mass results in that all load models can be analyzed and compared solely dependent on individual properties such as empirical factors and defined load amplitude.

The literature study of current standards and the derived evaluation method has been used to compare the proposed load models with recommendations in Eurocode, according to section 4.1, and ISO 10137 (ISO, 2008). It was concluded that Eurocode and ISO 10137 need to be complemented with more specific guidelines to provide a sufficient support during design. Guidelines for an accurate and sufficient dynamic analysis can be found in the proposed standards.

The defined design situations in Eurocode and ISO 10137 are judged as relevant and appropriate for design of pedestrian bridges. However the design situations proposed in Eurocode and ISO 10137 are not covered by the two standards and guidance for a thorough design need to be found elsewhere.

ISO 10137 suggests a concentrated load, applicable in vertical and lateral direction, based on a Fourier sum sufficient to cover the first two design situations corresponding to a single and to a group of pedestrians. Alternative concentrated load models can be found in SYNPEX (Research Fund for Coal and Steel, 2006), Sétra (Sétra, 2006) and UK-NA (British Standards Institute, 2008) resulting in responses with same magnitude but different for specific structural frequencies. UK-NA proposes a different concentrated load model as a simple harmonic load and is regarded as the most complete, including additional parameters not regarded in the other models, though it does not cover lateral vibrations. ISO 10137 is recommended for single pedestrians and UK-NA for group of pedestrians.

No instructions are given in Eurocode or ISO 10137 of how to model pedestrian streams. The other standards studied propose to model pedestrian streams with uniformly distributed load which seems accurate and applicable. Modeling a stream of pedestrians with concentrated loads representing single pedestrians is time consuming. A distributed load should be defined in both vertical and lateral direction taking into account second harmonies preferably with a lower acceleration response in comparison to the first. A distributed load could advantageously be defined according to traffic classes representing a location or expected loading as shown in the standards. Structural frequencies most likely to be excited by pedestrian walking frequencies should be regarded as the most relevant. The range for lateral frequencies should include frequencies below 0.5 Hz to take the “lock-in” phenomenon into account. These aspects can be found in the studied standards but none of them fulfill all desired criteria. Sétra recommends the most conservative load models and can be used for safe side calculations though it should be used with caution regarding relevant structural frequencies. Class III bridges in Sétra is recommended to be analyzed for all frequency ranges.

It can be concluded that the standards and design procedures are dependent on several factors in addition to the load models.

The defined step frequencies likely to occur affect which structural frequencies that are probable to be excited to resonance. ISO 10137 weighs 2.4 Hz as the structural frequency resulting in the largest response which is a frequency that does not occur in the literature as the most likely step frequency. The weighing of frequencies according to ISO 10137 is not optimized for groups but is applicable for single pedestrians. A weighing similar to UK-NA with a peak in acceleration response for the most occurring step frequencies just below 2 Hz is reasonable. The approach according to JRC, HIVOSS, Sétra and SYNPEX is also reasonable giving the same acceleration response for an interval of relevant step frequencies.

The acceleration limits defined in Eurocode and ISO 10137 for vertical direction are conservative whereas the lateral acceleration limits are comparable to other standards. Eurocode and IS0 10137 do not consider lateral “lock-in” which is a phenomenon that must be regarded during design. Acceleration limits could preferably be stated as ranges instead of an upper limit as defined in Eurocode. This seems reasonable because the perception of vibrations is subjective and that the accepted level of vibrations varies depending on the location and the structure of the bridge. An upper limit valid for all bridges can generate too high demands. The ranges could be defined from the accepted level of comfort dependent on location and the expected vibrational behavior. Good examples are shown in Sétra, JRC, HIVOSS and SYNPEX.

The choice of structural damping ratio is complex and depends not only on the material of the bridge but alos on the structure. For an accurate analysis the pedestrian mass should be included in the damping properties of the structure. This is mentioned in the literature but without precise guidelines and is a subject for further studies. The defined damping ratios differ between the standards but have similar magnitude. Because of this the damping values in Eurocode can be taken as valid. To conclude it is judged better to choose a lower damping ratio for a conservative approach; ISO 10137 recommends the most conservative damping ratios.

9.2 Suggestions for further studies

The load application of the concentrated load models have been simplified to be applied as stationary loads instead of loads moving across the span as recommended by the standards. In this thesis it is not analyzed how this simplification affects the acceleration response though it is obviously smaller for a moving load. It is clear that the acceleration response is dependent on loading time and position of loading. A suggestion for further studies is to find a relationship between the acceleration response due to moving and stationary loads. Such a relationship could help the designer to simplify the load application to a stationary load generating a comparable response to a moving load.

In the standards studied the pedestrian mass is mentioned to affect the structural properties of the footbridge. The subject is not extensively presented and the connection between structural damping and natural frequencies is complex. For accurate calculations the pedestrian mass should be included as it affects both the acceleration response and probability of resonance due to change in natural frequencies. A study could include when the effects of pedestrian mass should be considered and to what extent it affects the design.

The recommendations concerning acceleration response due to the second harmonics differ between the standards. It is mentioned that vibrations in existing bridges never have occurred due to second harmonics of the pedestrian load. Theoretically this should be relevant to take into account; why it is an interesting subject for a further study of whether it can occur in reality.

The analysis and the normalization curves are limited to simply supported bridges in one span, excited by their first mode. It is of interest to examine the possibility of creating similar normalization curves for bridges with several spans and different boundary conditions.

A further study can also be made regarding pedestrian bridges of different materials and how they respond to relevant walking frequencies. A comparison of a reference bridge with different materials for the same conditions can be made in order to examine this thoroughly. Footbridges designed in different materials have different structural properties with corresponding natural frequencies which should affect the acceleration response. The normalization curves could be useful to evaluate which cross-sections with corresponding material that are susceptible to exhibit excessive vibrations.

A study of how the concentrated load defined in ISO 10137 can be applied as a uniformly distributed load to simulate pedestrian streams is of great interest. The concentrated load model can be recalculated to represent pedestrian crowds evenly distributed by the factor for equivalent number of pedestrians. The acceleration response in comparison with other uniformly distributed load models is not studied in this master´s thesis.

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