

# CHALMERS



## Calculation method for powering a tramway network

*Master of Science Thesis in Electric Power Engineering*

JAKOB EDSTRAND

Department of Energy and Environment  
*Division of Electric Power Engineering*  
CHALMERS UNIVERSITY OF TECHNOLOGY  
Göteborg, Sweden 2012



MASTER OF SCIENCE THESIS

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Cover:  
Rectifying station 7801 in Göteborg. Photo: Jakob Edstrand.

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## Abstract

Estimating power demand of present and future tramway sections is of importance in order to not overload the electrical equipment supplying the tramway network. This thesis has investigated power demand, current shapes and overload situations in the tramway network of Göteborg. The result is a calculation software where input in form of amount of trams, speed of the trams, passenger loading and track slope outputs station load level and risk of overload.

The developed calculation method has been proven to give good results when comparing the simulation results to measured data. The simplified model developed in this thesis may be used to simulate the load level on rectifying stations when several trams run simultaneously on the same section. The simulations has shown that the simplified model produce accurate results which only differs towards measurement results by 5-10 % in RMS current. The result may be used to see if the tram traffic may be expanded without increasing the risk of short term overload in the rectifying stations.

The conclusion of this thesis is that the load level of some of the rectifying stations in Göteborg is too high. The circuit breakers in these stations are tripped several times per week due to short term overload originated from two or more trams starting simultaneously on the same section. The network of rectifying stations for the tramway system in Göteborg needs to be expanded to cope with the increasing number of passengers and new trams with higher current demand.

Keywords: Calculation method, Tram, Tramway, Power demand, Current, Rectifying station, Göteborg, Simulation software, Tractive effort



# Preface

This master thesis was carried out during March to September 2012 at Vectura Consulting AB, division Railway West and at Chalmers University of Technology, department of Energy and Environment, division of Electric Power Engineering.

I would like to thank everyone who has helped me with this thesis and I would like to direct a special thanks to following people:

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- Lennart Englund at Trafikkontoret in Göteborg for access and support of the local ITS KomFram.



Göteborg, September 2012  
Jakob Edstrand



# Glossary

Brief glossary of some words associated with this thesis (Swedish translation).

<b>A/D-converter</b>	Analogue-to-digital converter, device for converting analogue signals to digital for further processing (A/D-omvandlare)
<b>AC</b>	Alternating Current, current which alternates periodically with a specific frequency (Växelström)
<b>Catenary</b>	Metallic conductor suspended above the rail tracks to transfer current to rail vehicles via a pantograph mounted on the roof (Kontaktledning)
<b>Circuit breaker</b>	Device designed to safely break normal load and fault current (Effektströmbrytare)
<b>Commuter rail</b>	Passenger railway primarily for commuting between city centres and surrounding suburbs, running on regular train tracks (Pendeltåg)
<b>Disconnecter</b>	Device used for safe disconnection of a circuit for maintenance etc. (Frånskiljare)
<b>Grade</b>	Slope in per cent defined as height divided by distance (Lutning)
<b>DC</b>	Direct Current, unidirectional flow of current (Likström)
<b>Göteborgs Spårvägar</b>	Tram operator in Göteborg (Göteborgs Spårvägar)
<b>IGBT</b>	Insulated Gate Bipolar Transistor, transistor commonly used in power electronics (IGBT)
<b>Jerk</b>	Derivative of acceleration against time (Ryck)
<b>KomFram</b>	Intelligent Transportation System for the public transportation in Göteborg (KomFram)
<b>Light rail</b>	Trams operating mostly on traffic separated tracks, with fewer stops and higher speed (Snabbspårväg)
<b>Matlab</b>	Computing software and programming language from MathWorks used for calculation, analysis and visualization of data (MATLAB)

<b>Mixed traffic tracks</b>	Tramway tracks in streets mixed with other traffic (Gatuspår)
<b>N-1 criteria</b>	Design criteria for electrical grids to always give full operation in a network even with one broken link (N-1 kriterium)
<b>Pantograph</b>	Spring loaded arm mounted on the roof of railway vehicles which connects the vehicle to the catenary and conducts current (Strömvtagare)
<b>Protective relay</b>	Device used for fault indication and circuit breaker tripping in electrical power networks (Reläskydd)
<b>Rapid transit</b>	Umbrella term for tram, subway, commuter rail etc. (Snabbkollektivtrafik)
<b>Regenerative braking</b>	Braking of a vehicle using an electric motor as a generator and feeding electric energy back to the source (Regenerativ bromsning)
<b>RMS</b>	Root Mean Square, effective value of sine wave signals (Effektivvärde)
<b>Sirio</b>	The latest delivered tram type in Göteborg, made by the Italian company AnsaldoBreda (M32)
<b>Subway</b>	Passenger railway running in tunnels or on street level but always completely separated from other traffic (Tunnelbana)
<b>Surge arrester</b>	Device that divert dangerous voltages from lightning strikes etc. to ground (Ventilavledare)
<b>Switch-disconnector</b>	Disconnecter which also may break normal load current (Lastfrånskiljare)
<b>Third rail</b>	Metallic conductor on the ground next to the rail tracks used to transfer electric current to rail vehicles using a contact shoe mounted on the vehicle (Strömskena)
<b>Tractive Effort</b>	Propulsion force from a motor (Dragkraft)
<b>Traffic separated tracks</b>	Tramway tracks completely separated from other traffic (Särskild banvall)
<b>Tram</b>	Railbound vehicle for passenger transportation (Spårvagn)
<b>Tramway</b>	Passenger railway running mostly in street level mixed with other traffic (Spårväg)

# List of notations

Table of notations used in this thesis.

$a$	Acceleration [ $m/s^2$ ]
$A_{catenary}$	Cross-sectional area of catenary [ $mm^2$ ]
$d$	Distance [ $m$ ]
$F$	Force [ $N$ ]
$g$	Gravity of Earth [ $9.81\ m/s^2$ ]
$G$	Grade [%]
$I$	Current [ $A$ ]
$l_{catenary}$	Length of catenary section [ $m$ ]
$m$	Mass [ $kg$ ]
$M$	Mass [ $ton$ ]
$P$	Power [ $W$ ]
$\hat{P}$	Normalized peak power [ $W$ ]
$P_{loss}$	Power loss [ $W$ ]
$R_{catenary}$	Resistance in the catenary [ $\Omega$ ]
$R_{curve}$	Curve resistance [ $lb_f$ ]
$R_{grade}$	Grade resistance [ $lb_f$ ]
$R_{rolling}$	Rolling resistance [ $lb_f$ ]
$R_{total}$	Total resistance [ $lb_f$ ]
$\rho_{copper}$	Electrical resistivity of copper [ $1.68 * 10^{-8}\ \Omega \cdot m$ ]
$S$	Velocity [ $km/h$ ]
$t$	Time [ $s$ ]
$U$	Voltage [ $V$ ]
$\hat{U}$	Peak value of voltage [ $V$ ]
$U_{drop}$	Voltage drop [ $V$ ]
$v$	Velocity [ $m/s$ ]
$V$	Velocity [ $mph$ ]
$w$	Weight per axle [ $ton$ ]
$W$	Energy [ $J$ ]



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# 1 Introduction

It is always important to correctly dimension electrical equipment, both from safety and economical points of view. Over dimensioned networks are unnecessary expensive to build and under dimensioned components are more likely to become overloaded which may trip circuit breakers and create power outages. Electrical tramway as a passenger transportation system is well over 100 years old and a well balanced power network is as important today as it was a century ago [1].

## 1.1 Background

The first tramway line in Göteborg was opened in 1879. The city with about 70 000 inhabitants could now travel with horse drawn carriages through the central parts of the town. The population of Göteborg steadily increased to about 125 000 inhabitants in the year 1900 and the traffic with the slow and small horse carriages saw its end. The first electrical tramway line opened in 1902 and in only a few years the entire tramway network was electrified. Electrical tramway networks became a big success and by the end of 1910, electrically driven trams were running on the streets of the 10 largest cities in Sweden [1].

The tramway network in Göteborg was powered from a centrally placed power station with three coal-fired steam-boilers which produced a power of 275 kW each. The voltage level from the generators in the power station was 600 V DC and the current was transferred to the trams via catenary, with the track as return conductor. A buffer battery of 444 Ah was installed in the power station to even out the load of the steam-boilers and to help out at power outages and during power peaks [1].

The tramway network increased fast in size and the traffic was intensified which lead to that the power station quickly became insufficient. The power station was modernized several times and in 1910, the tramway was connected to the newly built hydroelectric power station in Trollhättan, about 75 km north of Göteborg. The old steam powered power station in Göteborg was now serving as a backup which alone could serve the entire tram network during power outages. In 1930 the total output power from the power station was 21 000 kW [1]. The power station in Rosenlund has not been powering the trams for many years now. The electricity is instead coming from a well distributed network of rectifying stations.

The number of passengers travelling with trams in Göteborg has steadily in-

creased with about 5 % annually during the last few years. The rate of increasing passengers is expected to continue in the same pace for the coming years, especially due to the planned introduction of congestion taxes in 2013 [3]. More than 118 million trips were made with trams in Göteborg in 2011 [2].

The increase of passengers also increases the electrical loading of the rectifying stations feeding electricity to the tramway. Another reason for the increased electrical loading is the newest tram type Sirio which was introduced in Göteborg in 2005. The power demand for any vehicle driven by an engine increases with speed, weight and acceleration. The Sirio tram is no exception as it is heavier, faster and quicker during accelerations, compared to the older trams in Göteborg. Other electrical facilities onboard such as heating, air conditioning and electrical doors also strongly contributes to the increasing power demand. The Sirio tram has shown to draw considerably more current than existing trams in Göteborg. The higher power demand from the tram has introduced an overloading problem to the tramway network [27].

## 1.2 Aim

There is a need to easily and relatively fast be able to perform power requirement calculations in different sections of a complex tramway network. By simulating present and future traffic, there are possibilities to find overload situations before they occur. The purpose of this thesis is to facilitate a simple way of calculating the actual required power of an arbitrary section of a tramway network.

## 1.3 Objective

The objectives of this thesis are to

- Measure and record voltage and current in some existing rectifying stations for the tramway in Göteborg.
- Form a theoretical model which fits the recorded data and gives a way to predict the shape of the rectifying station output current during an acceleration.
- Create a software which utilizes the theoretical model created to easily investigate tram frequenting possibilities in an arbitrary tramway section and to foresee possible overloading scenarios.

## 1.4 Scope

The thesis project is performed in the city of Göteborg and data collecting is only carried out in rectifying stations in Göteborg. No rectifying stations in other tramway cities are investigated in this thesis because of practical reasons and shortness of time. No measurements of voltage and current are performed on individual trams, as the interesting part to investigate is the rectifying stations. The current measurement is performed on the negative cable returning the current from the track. By measuring the returning current instead of the output current, a negligible measurement error is introduced. Due to the limited time of this thesis project, the data is collected during the spring and summer of 2012. Possible seasonal variations in power demand due to climate is not measurable although not believed to be of significance.

Modern trams use regenerative braking which feeds back electric energy to the catenary thus reducing the load on the rectifying stations. Regenerative braking will not be considered in the calculation method. This gives a conservative estimation of the load level in the rectifying stations as regenerative braking normally helps supplying the load.

The calculation software created in the project is only presented in this report with general functional descriptions and screen shots. No source code is published due to secrecy reasons.

## 1.5 Methodology

The first stage of this project was to collect data for sufficient time to analyse and draw conclusions that corresponds to reality with a reasonable margin of error. Output voltage and current was recorded in different rectifying stations throughout Göteborg and speed measurements were performed on different tram types operated by Göteborgs Spårvägar. To determine which tram that corresponds to certain recorded data, the voltage and current was recorded along with time that later was compared with the local ITS system KomFram. This system stores all movements of all trams for 18 months and makes it possible to see which unique tram that originates a certain current peak in the measurement data. The theoretical model of the tram power demand was based on literature studies and analysis of the recorded measurement data.

## 1.6 Outline

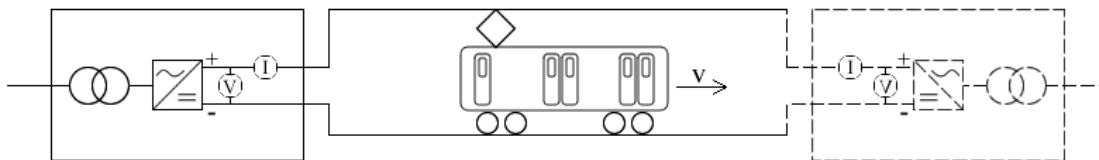
The following pages of this thesis report are divided into the main chapters theory, measurements, analysis, calculation method and conclusion. The theory chapter contains all necessary theory needed to read and understand the rest of the report. The measurements chapter describes how the measurements have been performed and the equipment used. The analysis chapter mostly contains graphical plots from recorded data which have been particularly investigated. The calculation method chapter contains specific results from the theoretical model and a brief description of the software. Finally, the conclusion chapter summarizes the report and recommends some possible future work in this field.

## 2 Theory

This chapter contains the underlying theory which has been used for this thesis. The theory part is quite extensive as the intent of this report is that it should be easy to read and understand the contents.

### 2.1 Transferring energy to railroad vehicles

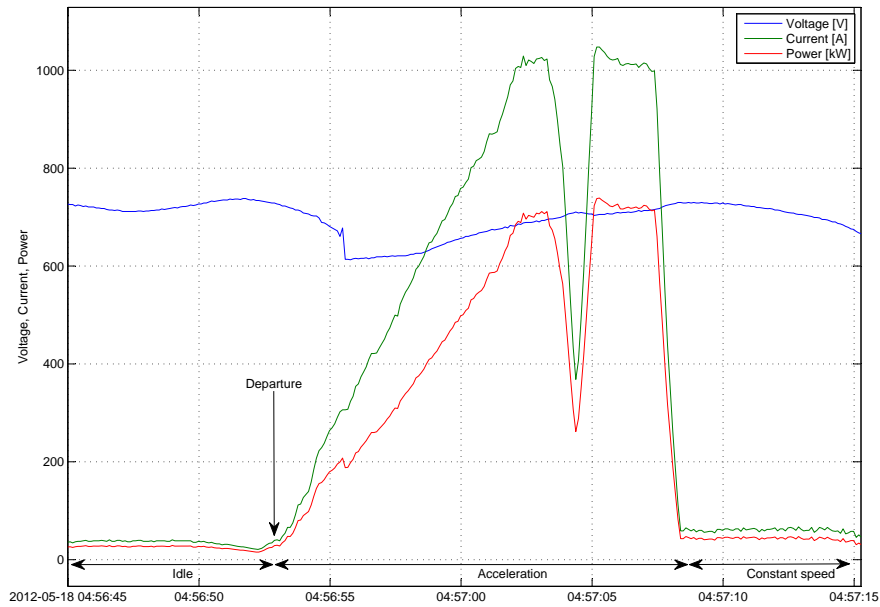
Transferring energy to electrical railroad vehicles may be performed with a few different techniques. The oldest and most commonly occurring method is to use overhead catenary made of copper as phase conductor and the track as return conductor. This method is used in both tram and train electrification all over the world. The advantage is that the dangerous voltage is high up in the air, on a safe distance, and the track voltage level is low enough to be considered safe. This way, it is safe for pedestrians and cars to cross a tramway track without the risk of electrocution. The basic principle of supplying the catenary with DC voltage is shown in Figure 2.1.1.



**Figure 2.1.1:** The power is transferred to the catenary from one or two rectifying stations.

An alternative method is to use a third rail with a relatively high voltage level running parallel to the track. This is often used in subway networks as their tracks always are entirely separated from other traffic. The advantage of using the third-rail technique is that the cross-sectional area of the conductor may be a lot larger than when using overhead catenary. This means that a larger current may be transferred without having a major voltage drop over the conductor, something that is necessary in subway traffic.

A typical shape of the output current from a rectifying station is shown in Figure 2.1.2. The figure shows output voltage, current and power from a rectifying station which feeds a single fed section, as a M32 tram accelerates. In the beginning, the tram is standing still thus only drawing idle current to supply the on board equipment. At the departure from the tram stop, the current increases rapidly as the speed of the tram increases. After the acceleration, the current drops down to a lower value again as the tram continues in constant speed. This is a typical behaviour in tramway networks as the highest current consumption occurs during acceleration and the current consumption during constant speed is low [19].



**Figure 2.1.2:** Voltage, current and power delivered from a rectifying station to a M32 tram on a single fed section during an acceleration.

### 2.1.1 Standardized voltage levels

Regardless of how the electricity is transferred to the vehicle, there are many different voltage levels that may be used. Historically, voltage levels from 50 V up to 50 kV has been used to propel railroad vehicles around the world. Today, there are six voltage levels that are standardized according to IEC 60850 [4] and these are also the most common ones used in all railway, tramway and subway systems, see Table 2.1.1. These voltage levels are the nominal voltages in the system and

the standard also contains maximum and minimum levels that the nominal voltage level is allowed to vary between.

**Table 2.1.1:** Standardized nominal voltage levels in railroad systems [4]

DC				AC	
600 V	750 V	1500 V	3000 V	15 kV, 16.7 Hz	25 kV, 50 Hz

A relatively low DC voltage has both advantages and drawbacks compared to a higher AC voltage. The reasons for choosing either DC or AC have a lot to do with loss reduction, equipment cost, laws and regulations. DC voltage networks are in many ways simpler to design and momentary power may always be calculated as the product of voltage and current. Some drawbacks of DC voltage networks are more stress to circuit breakers compared to AC circuit breakers and difficulties of transferring power a long distance without introducing too large voltage drops. AC voltage may easily be transformed to another voltage level by the use of a transformer, something that must be performed by power electronics in DC systems. AC current are also simpler to break for a circuit breaker as the current alternates and the circuit may be opened without flash-overs.

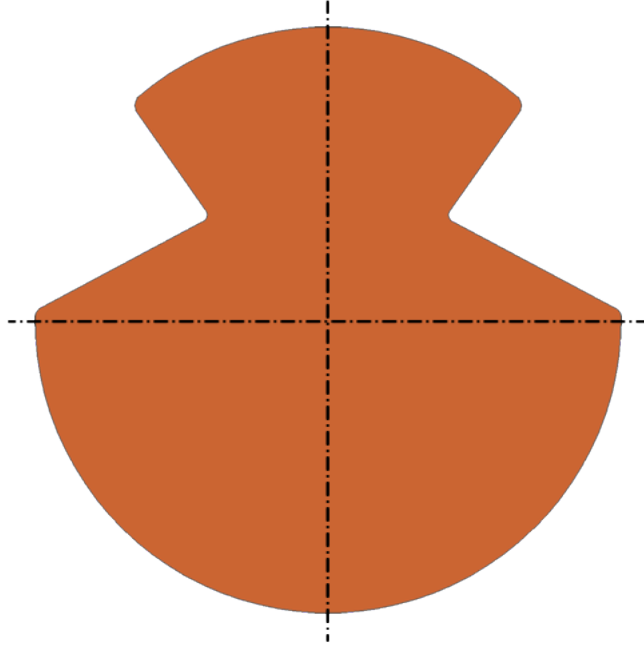
Safety distances increases with increasing voltages. A voltage level below 1000 V AC or 1500 V DC is considered to be low voltage [5]. Another reason for choosing a relatively low voltage for power transmission is that the trams do not need to have any large transformer on board, since the motors may be run directly on the catenary voltage.

The major drawback of choosing a low voltage is that the current increase compared to a high voltage, for the same power being transmitted. The voltage level in the tramway network in Göteborg was originally 600 V and it is today raised to the more common standard voltage 750 V. From a loss point of view, the 25 % increase in voltage gives a 25 % decrease in current for the same power being transmitted. The conduction losses are depending on the current according to

$$P_{loss} = I^2 R_{catenary} \quad (2.1)$$

Equation 2.1 shows that the copper losses increase with the current squared and a lower current is thereby decreasing the conduction losses accordingly. A high current is also negative from a voltage drop point of view. The voltage drop over a single fed copper catenary depends on how far the tram is from the feeding point and may be calculated as

$$U_{drop} = IR_{catenary} = I \frac{\rho_{copper} l_{catenary}}{A_{catenary}} \quad (2.2)$$



**Figure 2.1.3:** Cross-section of catenary wire. The cut in the wire is for fastening clips which in turn are held up by suspension wires and fixed links.

where  $\rho_{copper}$  is the electrical resistivity of copper,  $l_{distance}$  is the length of the catenary between the feeding point and the tram and  $A_{catenary}$  is the cross-sectional area of the catenary.

### 2.1.2 Conducting the current to the trams

To reduce the voltage drop and copper losses in the catenary, the current must be kept low. One way of utilizing this is to evenly distribute the rectifying stations along the tracks and make sure that all stations contribute to the load. The distance between the rectifying stations in Göteborg is about 1-2 km [6]. Another way to reduce the voltage drop and copper losses is to increase the cross-sectional area of the catenary and thereby reduce the resistance in the copper wire. Catenary wires are normally made of 100 % pure copper and a cross-section of a normal catenary is shown in Figure 2.1.3. The cut in the wire is for fastening clips which in turn are held up by suspension wires and fixed links.

Historically, the cross-sectional area of the catenary in Göteborg was 65 mm<sup>2</sup> [1]. With higher currents being transferred today, the area has been increased to 100 mm<sup>2</sup> on traffic separated tracks and 80 mm<sup>2</sup> on mixed traffic tracks. There are a suggestion in Göteborg to replace all the 80 mm<sup>2</sup> and 100 mm<sup>2</sup> catenary

with  $120 \text{ mm}^2$ . This would decrease the resistance and also give possibilities to raise the maximum allowable speed to  $80 \text{ km/h}$ . It is possible to go a lot faster even on a smaller catenary area, but the larger catenary area would have a greater mechanical strength as well as lower risk of melting due to high currents [7].

Subway systems are more common to have a third rail for powering the trains instead of overhead catenary. Even though a third rail is made of steel instead of copper, the much larger cross-sectional area may transfer much more current without being too hot or give too high voltage drop. Subway trains are generally run in train sets which are  $150\text{--}300 \text{ m}$  long and thereby much heavier than a typical  $30 \text{ meter}$  long tram. The energy needed to accelerate a subway train is therefore much higher compared to the typical tram and since subway trains are propelled on the same voltage levels as trams, the needed current also becomes much higher compared to a tram. This is impractical to achieve with catenary as the cross-sectional area would need to be much larger.

### 2.1.3 Higher current demand on modern trams

With faster and heavier trams, the current consumption has increased throughout the years. The technology development has also brought several other power consumers on board such as heating, air condition, lights, ticket machines and electrical doors. But the main power consumer is still the motors, especially during acceleration. During the introducing of the latest tram type in Göteborg, Sirio, measurements in the tram showed that the peak power during acceleration could be as high as three times the nominal motor power. This led to circuit breakers tripping in the rectifying stations and the following power outages stopped the traffic. Today, the motor current is being limited by software in the tram computer but a further reduction of motor current is needed as the number of Sirio trams in service has increased [7].

## 2.2 Rectifying stations

All DC powered tramway and subway networks need to convert the grid AC voltage to a lower DC voltage. In most cities, electrical power is distributed with an AC voltage level of  $10\text{--}20 \text{ kV RMS}$  [8]. This is a voltage level that is well balanced between losses and small physical size of the equipment needed to transform the voltage down to the customer level. A higher voltage would reduce the current and thereby the losses would decrease but the physical size of the components would be much larger. Rectifying stations in tram and subway networks running through cities are normally connected to these distribution voltage networks. This

is mostly due to cost efficiency since the infrastructure already exists and the reasonably small physical size needed for the rectifying stations.

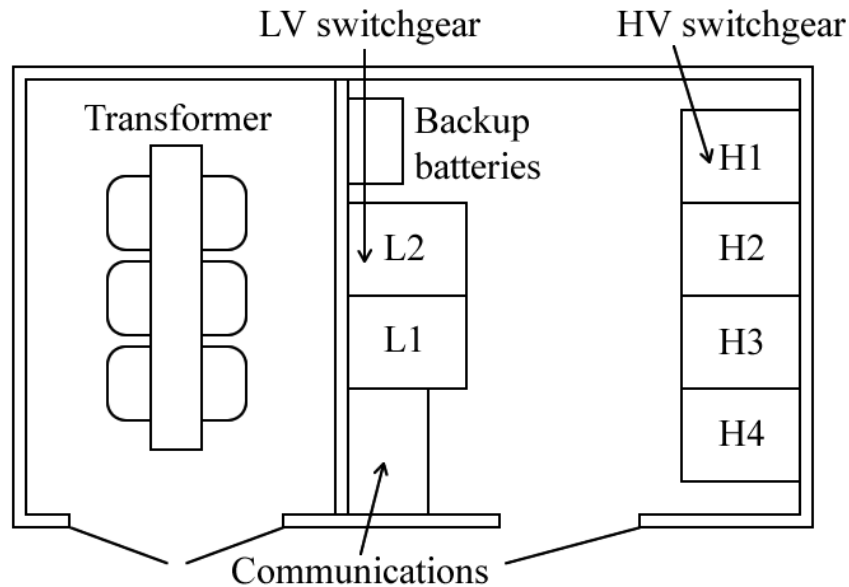
### 2.2.1 Rectifying stations in Göteborg

In Göteborg there are currently 61 rectifying stations serving about 80 km of double tracks. This gives an average track length of 1.31 km that each rectifying station serves. The actual distance between the stations differ between a few hundred meters in the central parts of the city to about 2 km further out. The network is currently divided into 74 electrically isolated sections which are fed from one (single fed) or two (parallel fed) connection points. There are 51 parallel fed and 23 single fed sections in the network excluding the sections around the tram depots [6].

Parallel fed sections have several advantages compared to single fed sections. The most obvious is that if one of the feeding modules would trip for any reason, it is mostly possible to run the section from the other station, at least for a short time. The voltage level during a fault in one of the stations could though drop well below the lowest allowed voltage especially in the end close to the faulted feeder. Another reason for using parallel feeds is that the total load will be shared between the two stations thus loading each transformer and rectifier less, which in the end will lead to a longer life span of the electrical equipment. By using parallel feeding, the distance between the stations may also be increased without introducing a too high voltage drop in the catenary.

Single fed sections are used where it is considered to be enough with one feed. Some tracks are divided into shorter sections which are single fed and these do always have the possibility to be fed from an adjacent section if the feed would break down. The most common use for single fed sections is in the end of a track where one section commonly feed a turning loop and a track distance of about 1 km. By this design, the last rectifying station on the track may be used to power two adjacent sections and there is no need for an extra station in the end close to the turning loop.

The 61 rectifying stations in Göteborg are made by several different manufacturers and the age span of the stations reaches roughly from 1960 until today. The stations are generally built on the same principle but small variations occur. Old stations are continuously exchanged for new ones as the components are worn out and there are seldom spare parts to find for the oldest stations [7]. A typical rectifying station in Göteborg consists of a high voltage AC switchgear, one transformer, a low voltage switchgear with one or two rectifiers and two DC modules with circuit breakers. A typical layout of the building is illustrated in Figure 2.2.1 where H1-H4 is the high voltage switchgear with incoming high voltage cables and L1 and L2 is the low voltage switchgear where feeding cables to the catenary is connected. There are also a few stations with a double setup, with two transform-



**Figure 2.2.1:** Typical layout of a rectifying station in Göteborg. H1-H4 is the high voltage switchgear with incoming high voltage cables and L1-L2 is the low voltage switchgear where the feeding cables to the catenary is connected.

ers and four DC modules. Figure 2.2.2 shows a photograph from station 7836 with a double setup consisting of two separate high voltage switchgears on the right hand side and one combined low voltage switchgear on the left hand side [6].

Power to the station is delivered from the local distribution network with a nominal voltage of 10.5 kV AC (3-phase, 50 Hz). The voltage is usually called 10 kV, but the actual RMS-value of the voltage is 10.5 kV. The high voltage distribution network is built in loops of district distribution stations, rectifying stations and other customers fed directly with 10 kV. The loop design implies that each station must have two incoming connections from similar stations in the vicinity. This redundant way of distributing electricity is made to comply with the N-1 criteria, which mean that if one link to a distribution station is interrupted, the disrupted station may be fed from the next station in the loop. A one-line diagram over a typical rectifying station is found in Figure 2.2.3 where H1-H4 is the same high voltage switchgear as in Figure 2.2.1 and the rectifier L and the two DC feeding modules L1 and L2 are located in the low voltage switchgear in the same figure. Transformer UT1 is a measurement transformer located in the high voltage switchgear and transformer T is the power transformer located in a separate room of the station. Circuit breakers are installed in L1+, L2+ and H4 switch-disconnectors are installed in H1 and H2. Ampere-meters are installed in

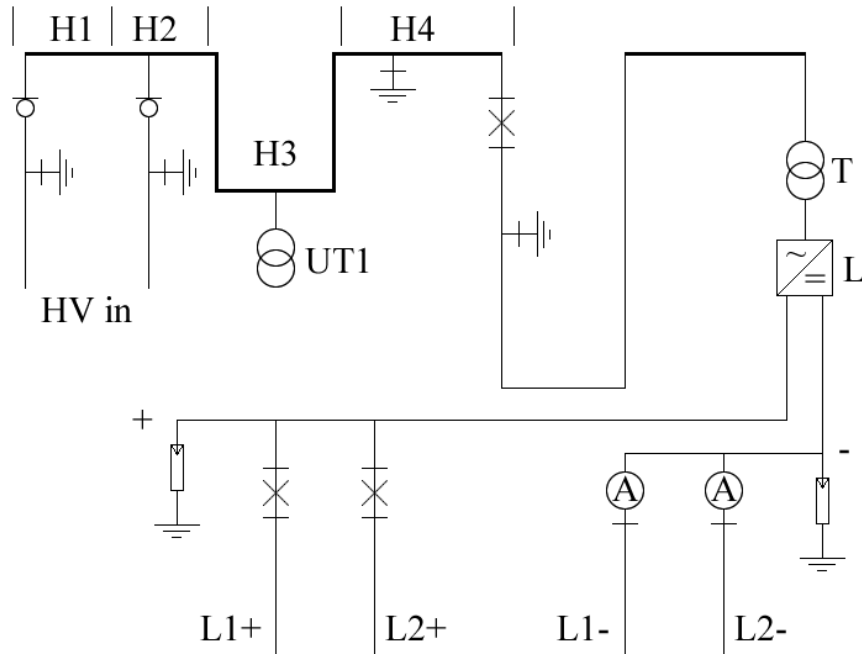


**Figure 2.2.2:** Photograph from a rectifying station. The red painted equipment on the right are the high voltage switchgears and the low voltage switchgear is located to the left.

series with the minus cables and surge arresters are installed on both sides of the rectifier.

### High voltage switchgear

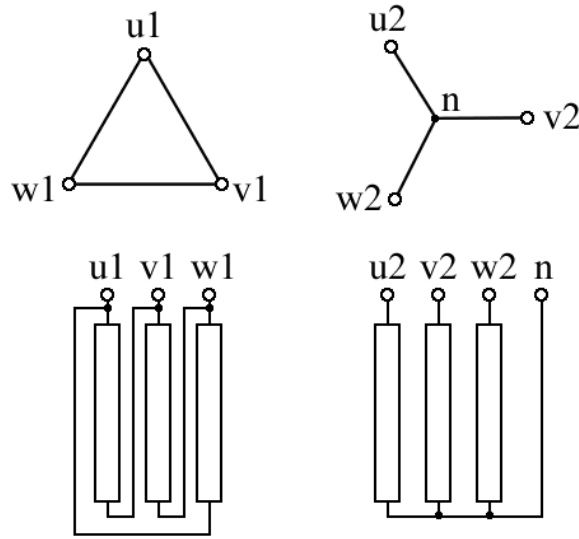
Some new stations are to be built during the coming years in Göteborg and they are being constructed according to a design document that is established [9]. According to this design document, the high voltage switchgear should consist of four modules as shown in Figure 2.2.3; two modules are the incoming 10 kV-connections from nearby distribution stations (H1 and H2), one module is for measuring the power drawn (H3) and one is the outgoing module connecting to the high voltage side of the transformer (H4). The incoming 10 kV-modules should be equipped with switch-disconnectors which are able to disrupt normal load current and act as a disconnector during maintenance work etc. The incoming high voltage power cables are supplied from the local energy company and the dimension of each cable is 3x240/35 mm<sup>2</sup> aluminium. The measuring module should be equipped with current and voltage transformers for measuring the drawn power. The outgoing module should have a circuit breaker to protect the transformer from overloading and a disconnector for maintenance work etc [9].



**Figure 2.2.3:** One-line diagram over a typical rectifying station. H1-H4 is the high voltage switchgear seen in Figure 2.2.1 and L, L1 and L2 is located in the low voltage switchgear.

## Power transformer

The power transformer in a new built rectifying station in Göteborg should have a rated power of 1250 kVA. The transformers in the existing rectifying stations may have different ratings. Typical values of rated power are 1000, 1050, 1125 and 1250 kVA. The reason for choosing 1250 kVA as today's standard rating is that it is a transformer with a good power-to-price ratio. This is because it is an off-the-shelf product that is very common in distribution networks around the world. The rating is also sufficient for the needs of a standard rectifying station as the power demand is very peak-like and the transformer risks only to be overloaded for very short time periods. The high voltage side of the transformer is connected to the fourth module of the high voltage switchgear. The transformer ratio is  $10.5 \pm 2 \times 2.5 \% / 0.530 \text{ kV}$  which means that the RMS voltage on the low voltage side is 530 V. This gives a peak value of  $\hat{U} = \sqrt{2} * 530 = 749.5 \text{ V}$ . Since the voltage historically was 600 V and has been raised in steps throughout the years, power transformers with different ratios still occur. Typical voltages of the low voltage side which are still in operation are 500 and 510 V. The transformer should be connected as DY11 – primary side is delta connected, secondary side is



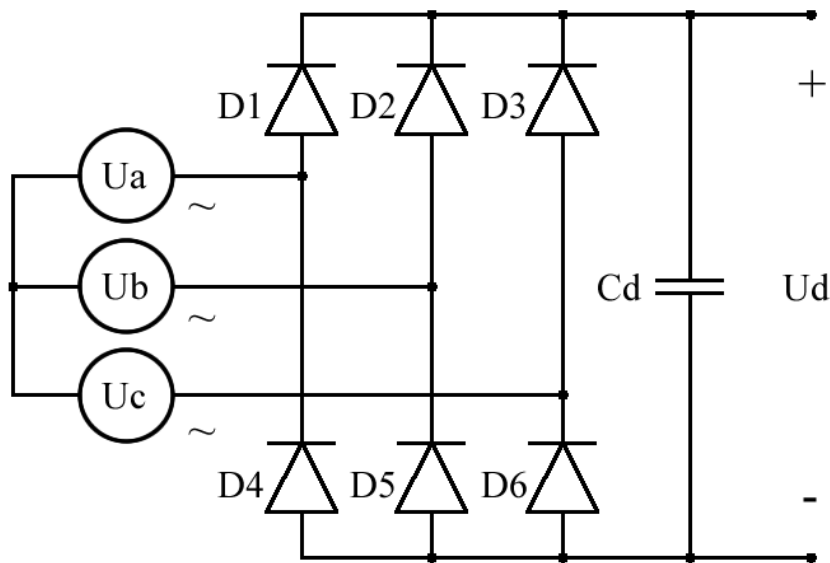
**Figure 2.2.4:** Transformer connection in DY11

we connected and the secondary side leads the primary side by 30 degrees, see Figure 2.2.4 [10].

## Rectifier

The rectifier in the station should be a 6-pulse diode rectifier, also known as a 3-phase full wave rectifier. A basic circuit of the rectifier is shown in Figure 2.2.5. A 3-phase rectifier functions the same way as a single phase rectifier, where all negative half-cycles of a sine wave signal are “mirrored” in the zero-line and added to the positive half-cycle. The major difference in using a 3-phase rectifier is that there are six half-pulses shifted by 60 degrees being rectified during one 360 degree cycle. The voltage over the three legs is then added to each other and the resulting output is shown in Figure 2.2.6. The output DC voltage from a 3-phase rectifier contains a lot less harmonics compared to single phase rectifiers and the voltage ripple is very low. In some rectifiers, there is also a capacitor mounted in parallel with the output voltage to further reduce the DC voltage ripple.

Since the rectifier is made of diodes which are passive components, the current is one way directed. This means that when a modern tram brakes on the section, the line voltage will increase but the rectifier will not feed energy back to the distribution network. There is also no energy storage in the station which means that the braking energy has to be used momentarily by another tram on the



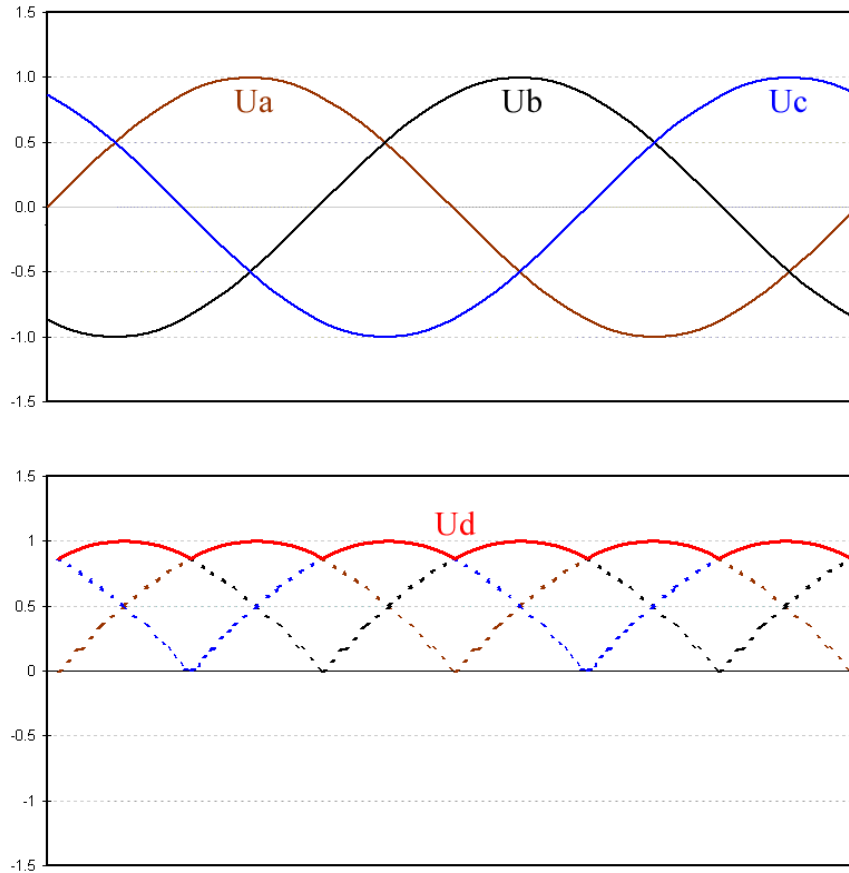
**Figure 2.2.5:** Circuit diagram showing the principle of a three-phase rectifier

section, otherwise it will be burned off in the internal braking resistors on the tram instead. There are rectifying stations available today that use a rectifier and a DC/AC inverter in parallel to be able to feed back the braking energy to the grid, but these are not in use in Göteborg [11].

### DC feeding modules

The feeding modules is the equipment denoted L1 and L2 in Figure 2.2.1 and Figure 2.2.3. The modules should contain a withdrawable circuit breaker which is able to break currents up to 100 kA. The circuit breaker should be able to be operated locally or from the traffic control center. It should also have automatic controls for disconnection and reconnection depending on the type of fault occurring. The combined relay protection should be set for 1.5 kA for each module and it should consist of

- Momentary over-current protection with adjustable level 2-4 kA.
- Over-current protection
- Over-current protection with  $di/dt$ -measuring
- Overload protection with adjustable time constant  $\tau = 2-15$  minutes



**Figure 2.2.6:** Input and output voltage over the rectifier

- Earth fault protection on plus cable and control of isolation strength between shields
- Voltage control towards line

The feeding module should also contain an ampere meter and two counters for counting the number of trips performed by the circuit breaker [9].

If a circuit breaker would trip from of *normal* over-current, there should be a reconnection attempt made after 12 seconds. This must only be performed if the breaker was tripped because of over-current, not any other reason. If the breaker should trip because of any other reason, the breaker must be blocked for automated reconnection and it has to be reset manually, either locally or from the operations central [9]. Figure 2.2.7 shows a photograph of three feeding modules denoted L2-L4 and a rectifier denoted V2.

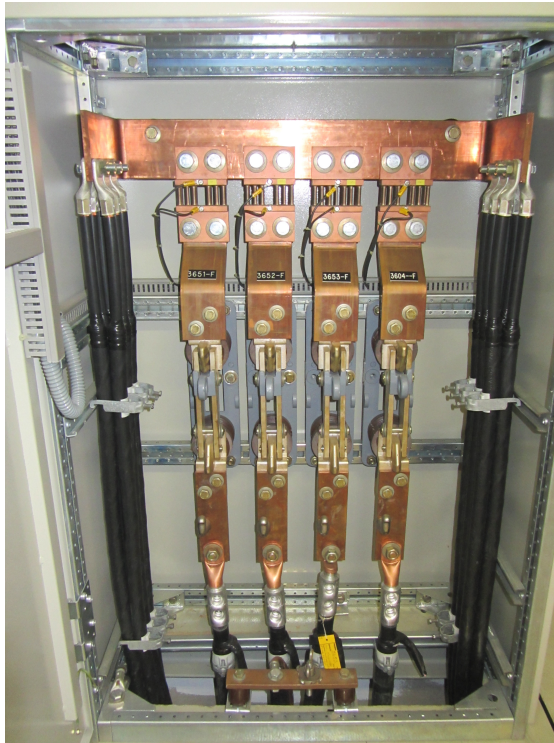


**Figure 2.2.7:** Feeding modules and rectifier

The outgoing cable to the catenary is connected either via the feeding module or a separate plus module. The dimension of this cable should be  $1 \times 500/120/30 \text{ mm}^2$  meaning that the current carrying conductor should be of  $500 \text{ mm}^2$  aluminium, and the two outer layers are shields. Earlier, there has only been one shield and it has been connected to ground potential in the connection point of the catenary but left unconnected in the rectifying station. This is because the different grounding potentials in the two systems and if they are connected it would create a short circuit. The idea with the double shielded cable is that one shield should be connected to ground in the catenary connection point and the other shield should be connected in the rectifying station, but they should not be connected to each other. This safety measure will ensure that if the cable ever would be cut off by for instance an excavator, the short circuit created will always go towards ground and not the dangerous way through the excavator [9].

### Minus module

The minus module is where the minus cable from the tracks is connected to create a closed circuit. The so called minus cable transfers the same amount of current as the feeding cable and the dimensions should therefore be the same as for the plus cable. The minus side voltage is close to zero volts, but not negative as the name might imply. This module is often combined for all incoming minus cables



**Figure 2.2.8:** Minus module in a rectifying station. The incoming cables from below the surface are the minus cables from the track. The grey levers in the middle of the photo is the disconnectors and the current shunts are shown just below the common busbar.

to the station and there is a current shunt and a disconnector for each cable [9]. Figure 2.2.8 shows a photograph of a minus module where the incoming minus cables are in the bottom, the disconnectors are the grey levers in the middle and the current shunts are on top, close to the common minus busbar. The multiple cables on the sides are connections between the minus module and the rectifier.

## Protection

To further protect the rectifying station against over-voltages, there should be a surge arrester installed both on the plus side of the rectifier and in the minus module. This is to protect the electronics against dangerous over-voltages from lightning strokes etc. To protect the station from arc flash-overs, there should also be an arc guard installed that breaks the circuit breaker in module four of the high voltage switchgear (H4) [9].

## Backup and communication

The station is normally fed with a separate service line of 400 V for heating, light and remote controls etc. Since a lot of the equipment in the station is lethal if it is handled the wrong way or if there is a fault, it is crucial to always be able to break the current and block the breaker for reconnection. These functions as well as the communication with the operations central is therefore backed up by a battery unit of 48 V.

## 2.3 Trams

There are currently four different types of trams frequenting the tracks of Göteborg: M28, M29, M31 and M32. The development of new trams has led to an increasing power demand as they become heavier and get more powerful motors. There are a lot of differences in how the motors are designed and controlled but despite all differences, they still run on the same tracks and with the same power supply. A comparison between the different tram types is found in Table 2.3.1 where the most interesting information is the weight and nominal motor power which most affects how much power the different tram types need to accelerate [12, 13, 14].

**Table 2.3.1:** Current tram types in Göteborg. Photos: Wikipedia Commons.

	M28	M29	M31	M32
				
Built by	ASEA/ASJ	Häggglunds	ASEA/MGB	AnsaldoBreda
Trams ordered	70	60	80	65
Number series	701-770	801-860	300-380	401-465
Delivered	1965-1967	1969-1972	1998-2002	2004-2012
Weight	16.8 ton	17.0 ton	34.5 ton	38.9 ton
Length	15.1 m	15.1 m	30.7 m	29.6 m
Sitting capacity	38	36	81	82
Standing capacity	78	82	109	104
Nominal power	176 kW	200 kW	300 kW	424 kW
Maximum speed	60 km/h	60 km/h	70 km/h	80 km/h

### 2.3.1 M28

The M28 was built to run in right-hand traffic that was introduced in Sweden on September 3<sup>rd</sup>, 1967. The model looks a lot like its predecessor M25, built by Hägglunds 1958-1962, but was built by ASEA (electronics) and ASJ (mechanics). The vehicle is non low-floor and it is possible to connect multiple M25, M28 and M29 with a coupler to be run by one driver. Today, it is common to see a train consisting of one M29 followed by one M28 in Göteborg [15].

The DC motors in the M28 are directly driven on the voltage of the catenary and the speed is controlled with a speed pedal and a brake pedal. To limit the starting current, several resistors are connected in series with the motors during start and they are being disconnected one by one during acceleration. In run mode, the resistors are disconnected but to reach the top speed, field weakening of the motors is performed by connecting the resistors in parallel with the field windings.

The M28 tram has three different ways of braking; electromechanical braking, mechanical friction braking acting on the wheels and electromechanical friction braking pressing a brake block against the track. The most commonly used brake is the electromechanical braking which uses the motors as generators and burns of the excess kinetic energy in the same resistors used for decreasing the starting current. The mechanical friction brakes are used for completely stopping the tram, usually in low speeds and the electromechanical friction brakes are used for emergency braking only [15].

### 2.3.2 M29

The M29 was built by Hägglunds which also built the similar looking M25 earlier. The tram is by looks and mechanics a lot like the M25 but the electronic system is updated. For instance, the M29 has a static converter for 48 V but the M25 has a rotating converter. The motors of the M29 were also designed to be run in higher speeds with fewer stops compared to the M25. The control of the DC motors is the same as in M28 and the three braking systems are alike [15].

### 2.3.3 M31

The M31 was originally called M21 and was built between 1984 and 1991. The design was a two-part non low-floor vehicle which was joined together with possibility to walk between the two parts. The model was rebuilt between 1998 and 2002 when a middle part with low floor was inserted between the two original parts. The tram is propelled by thyristor-controlled DC motors placed in the front and rear bogies and the driver uses a joystick to give a reference speed to the motor controller.

The M31 has three types of brakes; electromechanical regenerative braking using the motors as generators, mechanical braking using brakes directly acting on the wheels and track braking using electromechanical braking shoes. The most commonly used brake is the regenerative braking which may be used in both high and low speeds with a smooth and constant deceleration. When using regenerative braking, the motors are used as generators and the current is reversed into the catenary. Some of the braking energy is used for the tram's own need such as cabin light and onboard computers. If no other tram on the same section need the energy momentarily, the voltage of the catenary will be raised to a maximum of 900 V at which the energy will be burned off in the internal braking resistances instead [15].

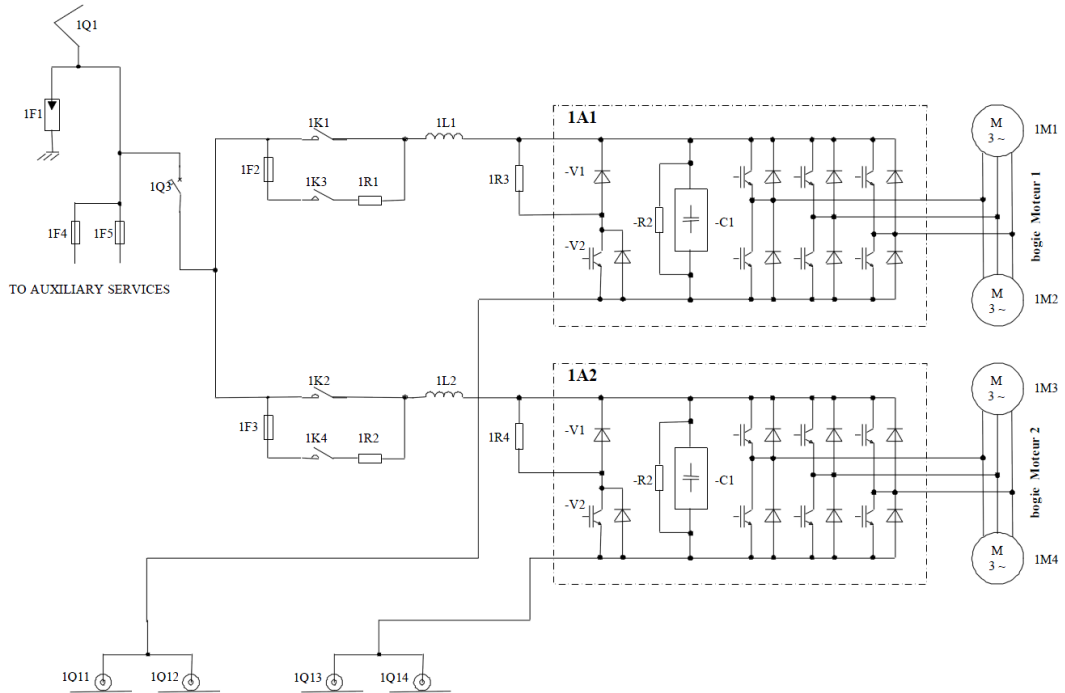
The thyristor-based control of the motors has led to very rough control of the braking voltage and unwanted spike-like behaviour of the catenary voltage is commonly occurring. The thyristor control computer is old and spare parts are hard to come by today. Therefore, the trams are being rebuilt with new control electronics based on IGBT's instead of thyristors. Tests have shown considerably better braking voltages with the new electronics and all trams of this model are scheduled to be rebuilt. The difference is noticeable in the measurements in Chapter 4 [15].

### 2.3.4 M32

The M32 is the latest tram delivered in Göteborg, originally intended as a replacement for the M28 and M29 trams. The first series of 40 M32 trams was intended to be delivered in 2005 but the last tram in this series was not delivered until 2011. The last trams in the second series of 25 are expected to be delivered during fall 2012.

The technology in M32 is in many ways different from the previous trams in Göteborg. The catenary voltage of 750 V DC is inverted to 380 V AC, which supplies the four 3-phase induction motors by varying the frequency. Other converters on the tram converts the catenary voltage to 24 V DC and 380/220 V AC, 50 Hz, to supply the rest of the electrical equipment [16]. The driver of the tram controls the speed with a joystick which gives a reference speed to the inverter controlling the motors. The motor controller then varies the frequency of the AC voltage feeding the motors and the tram is increasing or decreasing the speed. The electrical setup of the M32 tram is shown in Figure 2.3.1 where the motors and the motor control thyristors are seen to the right in the diagram [16].

There are many advantages with 3-phase induction motors compared to DC motors. They generally need less maintenance, they are more robust, they are relatively cheap, they have higher efficiency and they are produced in large scales [17]. The difficulty with induction motors is that they need to be supplied with AC voltage in order to control them. Since the catenary contains DC voltage, it needs



**Figure 2.3.1:** The electrical system of a M32 tram [16]. The four 3-phase induction motors and the motor controller are seen to the right in the diagram.

to be inverted by power electronics, something that is much easier to come by today compared to a few decades ago [17, 18].

The IGBT's in the motor inverter are also able to feed braking energy back to the catenary, similarly to the rebuilt M31 electronics. The high switching frequency of the IGBT's compared to the frequency of the motors creates a smooth voltage of almost 900 V DC during electromechanical braking. This has also been seen in the measurements in Chapter 4.

The M32 tram has shown to be problematic when it comes to high starting current, much higher than the specified current. As the tram was new in Göteborg, tests showed that the starting current could be as high as 1700 A during acceleration. This led to an increasing number of circuit breaker trips in the rectifying stations and there is currently a test with one M32 tram having its motor current limited to 1300 A in the motor controller. If the test falls well out, all M32 trams will probably be equipped with a motor current limit to reduce the load on the rectifying stations [7].

## 2.4 Tractive Effort

The possibility of a vehicle to increase its speed may be written in many different ways. One of the more common ones is to use Newton's second law and denote the force as tractive effort

$$F = ma \quad (2.3)$$

The benefit of using tractive effort instead of for instance power, horse power or torque is that it is easy to compare it with the resistive forces and get the net force acting on the vehicle. Tractive effort is inversely proportional to the speed of the vehicle and may be calculated with the formula

$$F = \frac{P}{v} \quad (2.4)$$

where  $P$  is the output power of the motors and  $v$  is the speed of the vehicle. Since tractive effort is limited by the available power at a given speed, it would mean that the tractive effort would be infinitely high at a start from standstill. However, the power in an electric motor is not constant for any rotational speed. At low speeds, the motor power is also limited by the maximum torque as

$$T \propto I \rightarrow T_{max} \propto I_{max} \quad (2.5)$$

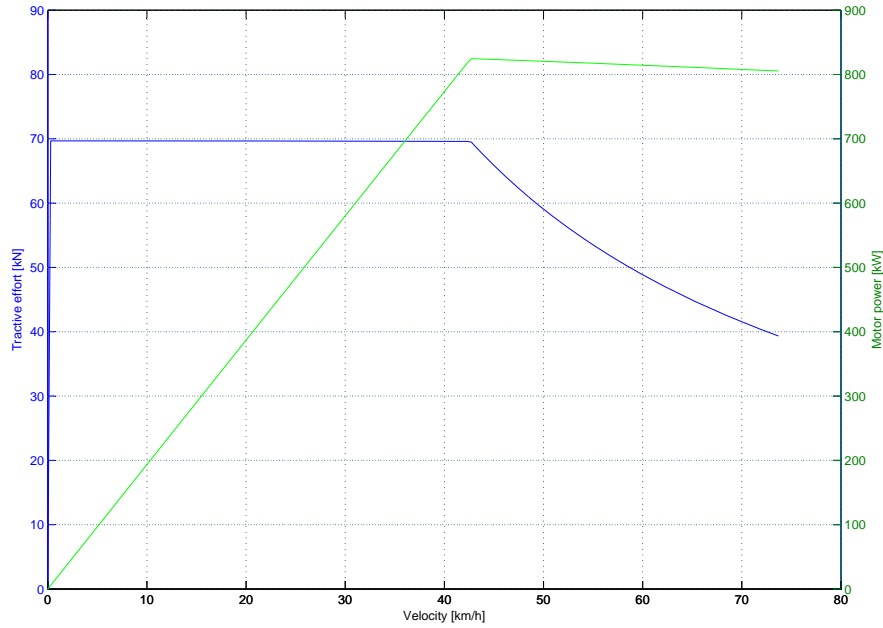
where  $T$  is the motor torque and  $I$  is the motor current. The maximum torque is limited by the maximum current,  $I_{max}$ , and the motor power is limited by the torque according to

$$P = \omega T \rightarrow P_{max} = \omega T_{max} \propto \omega I_{max} \quad (2.6)$$

where  $\omega$  is the angular frequency of the motor. This means that the tractive effort is limited by the available motor power.

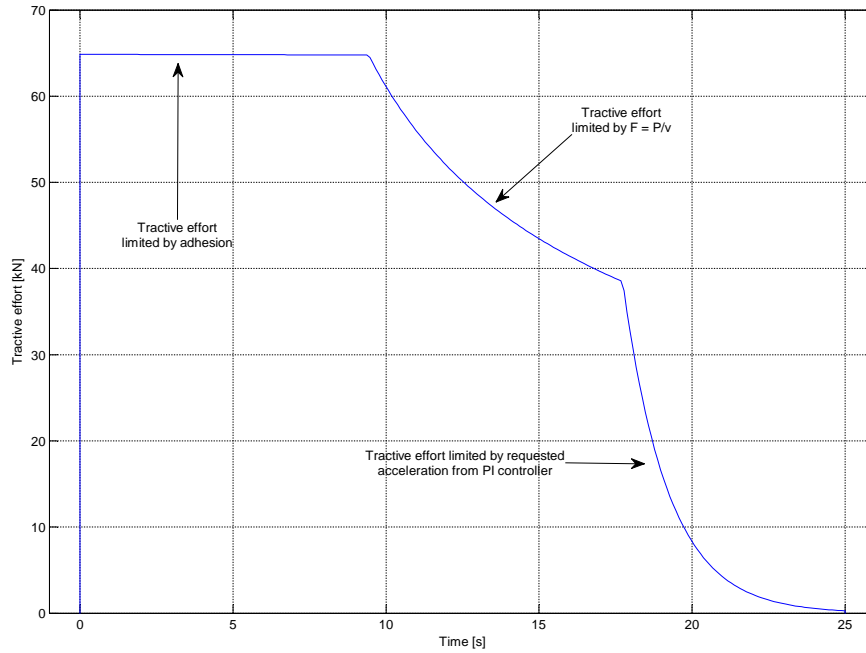
The tractive effort is also limited by the adhesion between the wheels and the track when the tram starts from standstill. To increase the adhesion when starting in a slope or when the track is slippery due to rain or leaves, trams have built in anti-spin control and automated sand spreaders in front of the traction wheels [16].

An example of available tractive effort and motor power in a tram is shown as a function of speed in Figure 2.4.1 and as a function of time during an acceleration from 0 to 60 km/h in Figure 2.4.2. The flat profile in the tractive effort in the figures is the limit due to adhesion and the next part is where the available motor power is limited. In Figure 2.4.2, the acceleration is determined from a simulated speed controller and at about 17 seconds, the control signal (acceleration) decreases. This is due to the fact that the target speed is almost reached and the tram will only need to overcome the resistive forces to keep the same speed.



**Figure 2.4.1:** Example of tractive effort and motor power as a function of speed. The flat profile of the curve is the limited due to adhesion and the inversely decreasing curve is due to the decreasing amount of power available from the motors.

To reduce jerk and to give the passengers a pleasant ride, acceleration is normally limited with software in the tram speed control. Common limitations for acceleration is about 10–15 % of earth gravity or  $1.0\text{--}1.5\text{ m/s}^2$  [20]. As can be noted from (2.3), this limitation will also limit the maximum tractive effort and it will contribute to the flat part of the tractive effort in Figure 2.4.1, in the same way as adhesion does. The decreasing part of the tractive effort curves is due to the limit depending on available power. Since this part is decreasing with increasing speed, the available effort will eventually reach a point which corresponds to maximum possible speed. This is the point where the available tractive effort equals the resistive forces acting on the vehicle. This is why tractive effort curves play important roles in railroad freight. Basically, one can from these curves calculate how heavy load a locomotive is able to pull and in what speed. For trams and subways, the speed is relatively low and the available power at these low speeds is always higher than the resistance. The maximum speed is therefore limited by other factors than the resistive force.



**Figure 2.4.2:** Example of tractive effort as a function of time during an acceleration. The flat profile is the limit due to adhesion and the inversely decreasing curve is due to the decreasing amount of power available from the motors.

## 2.5 Resistive effort

The resistive forces acting on a vehicle may be divided into starting resistance, rolling resistance, grade resistance and curve resistance.

### 2.5.1 Starting resistance

The starting resistance is more or less only used in railroad freight where the locomotive must have enough tractive effort to overcome the starting resistance in order to move the train. In rapid transit systems such as tramway, subway and commuter rail, acceleration is more important than top speed. This means that the starting resistance is practically never a problem to overcome since the available starting power is relatively high. A popular way to determine the maximum starting weight for a vehicle is to look at the power-to-weight ratio. In railroad freight, a ratio of 1 [HP/ton] is a common value while in rapid transit systems the ratio may be as high as 5-10 [HP/ton] [19]. In this thesis project, the starting resistance has been neglected because of the high power-to-weight ratio.

## 2.5.2 Rolling resistance

The rolling resistance plays an important role in the total resistive effort as it increases rapidly with the speed of the train. In railroad freight, the rolling resistance determines the theoretical maximum speed a train may reach which occurs when the available traction force equals the resistive force subjected to the train. This is also the case for rapid transit systems although their maximum speed is commonly limited by other factors before this limit is reached. Still, it is the largest part of the resistive force. Rolling resistance may be written on the form

$$R_{rolling} = Aw + Bv + Cv^2 \quad (2.7)$$

where  $R_{rolling}$  is the rolling resistance and the  $A$ -,  $B$ - and  $C$ -parts are constants multiplied with weight ( $w$ ), speed ( $v$ ) and speed squared. The  $A$ -constant is mostly journal and bearing resistances (wheel axle), the  $B$ -constant is mostly the flange resistance (wheels) and the  $C$ -constant is the air drag resistance (frontal area and coefficient of drag) [19].

## 2.5.3 Grade resistance

The formula for rolling resistance is only valid on flat ground unless the grade resistance is added. The grade resistance is not depending on the speed of the vehicle but very much on the weight. The basic form for grade resistance is

$$R_{grade} = 20mgG \quad (2.8)$$

where  $R_{grade}$  is the resistive force in pounds-force,  $m$  is the weight in tonnes and  $G$  is the grade in percentage slope. The constant 20 is derived from earth gravity and the wish of expressing the force in pounds-force. The grade is positive for increasing resistance in uphill slopes and negative for decreasing resistance in downhill slopes and is measured by dividing height difference by length.

## 2.5.4 Curve resistance

Curve resistance is the resistance that is required to guide the wheels through a curve. Curve resistance is considered to be equivalent to 4 % up grade per degree of curvature, thus not depending on speed through the curve. This is however an estimation and at very low speeds, the curve resistance is considered to be closer to 5 % up grade per degree of curvature [21]. By assuming that the curve resistance is equivalent to 4 % up grade per degree of curvature, the resistance can be expressed as

$$R_{curve} = 0.04R_{grade} * D \quad (2.9)$$

where  $R_{curve}$  is the resistive force in pounds-force,  $R_{grade}$  is the grade resistance force and  $D$  is the degree of curvature. Because of curves mostly having an altering degree through a curve, it might be hard to estimate the resistive force. It is therefore customary to compensate for horizontal curves by including the curvature in the grade resistance[21].

Rail lubricators may be used to reduce the noise through curves and it is also reducing the curve resistance by as much as 50 % [22]. This is used on several places in Göteborg where the curve radius is narrow. The curve resistance is however very small compared to rolling and grade resistance and therefore it has not been considered in the theoretical calculations in this thesis.

### 2.5.5 Total resistive effort

In railroad transportation, there are some similar formulas used to calculate the total resistance and one formula that has been guiding for a very long time is known as Davis formula. Already in 1926, the American W.J. David, Jr. presented his formula in the paper “The Tractive Resistance of Electric Locomotives and Cars” which was printed in General Electric Review (October 1926). This is known as the first empirical formula for computing rolling resistance and it was found to give representative results by several investigators [21, 22].

Although the formula is quite old, the basic principles in railroad transportation are still the same. Due to better equipment on the train cars – such as journal bearings and roller bearings – some coefficients have been altered throughout the years and therefore the formula is still valid. The American Railway Engineering Association (AREA) has altered Davis original formula to better reflect onto the rolling stock that is available today [23]. The resulting formula that is valid for most kind of trains is

$$R_{total} = (0.6wn + 20n) + bwnV + KV^2 + 20wnG \quad (2.10)$$

where  $R_{total}$  is the total rolling resistance in pounds-force,  $w$  is weight per axle in tonnes,  $n$  is number of axles,  $b$  is the coefficient of moving friction,  $V$  is velocity in miles per hour,  $K$  is a lumped coefficient for aerodynamic resistance and  $G$  is the grade in percentage (positive for uphill slopes and negative for downhill slopes). The coefficient for rolling friction is difficult to determine and according to various sources it may vary between 0.001 to 0.01 thus giving very different results. In the original Davis formula, the coefficient varies between 0.03 to 0.045 for different train cars and locomotives. The aerodynamic resistance coefficient is another coefficient that is hard to estimate since it is very depending on how streamlined the design of the vehicle is and how large it is. It is possible to divide this coefficient into its three parts, but mostly it is more convenient to just use

a standard value of  $K$  instead. For railroad vehicles,  $K$  ranges from about 0.1 to 0.3 [24]. AnsaldoBreda has their own version of the Davis formula to calculate the rolling resistances of their tram Sirio (called M32 in Göteborg) [16]. The formula is

$$R_{total} = Mg(0.1G + A + \frac{S^2}{B}) \quad (2.11)$$

where  $R_{total}$  is the total resistance in Newton,  $M$  is the mass in tonnes,  $G$  is the grade in percentage,  $A$  is a constant equal to 2.5,  $S$  is the vehicle speed in kilometer per hour and  $B$  is a constant equal to 850. It is not stated how the coefficients  $A$  and  $B$  are derived in the technical description, but it is likely to believe that they originate from the conversion of the units from imperial to metric system.

To see if both formulas gave similar results, some sample values were inserted into both formulas. For a flat track, weight of 45 tonnes, 12 axles (the M32 has split axles), speed of 60 km/h, coefficient of moving friction of 0.01 and an aerodynamic resistance coefficient of 0.29, the resulting resistance from the applied AnsaldoBreda formula was 2.98 kN. The more general AREA formula gave the result 3.04 kN, which is only a two per cent difference. Similar results were encountered when introducing a gradient or changing the velocity or mass of the vehicle. Because of the more general characteristics of the AREA formula, this was chosen to be the base in the resistive effort calculations in this thesis project.

## 2.6 Resulting effort

According to Newton's first law, a body has constant velocity if the net force acting on the body is zero

$$F_{net} = \Sigma F = 0 \rightarrow \frac{dv}{dt} = 0 \quad (2.12)$$

This general formula is applicable on all moving bodies and it may therefore be used for trams. As the measurements in Chapter 4 shows, a natural drive cycle for a tram starts with a fast, almost constant acceleration up to a desired velocity. After the velocity is reached, the force needed to keep the tram in constant speed equals the resistive forces acting on the tram. This speed-maintaining force is typically only 5 % of the force needed to accelerate the tram, thus giving very non homogeneous power consumption from the rectifying stations. The resulting effort that acts on the vehicle may be calculated with a small rewriting of Newton's second law (2.3)

$$F_{net} = F_{acc} - R_{total} = ma \rightarrow F_{acc} = ma + R_{total} \quad (2.13)$$

where  $F_{acc}$  is the total tractive effort needed from the motors to overcome the resistive forces and accelerate the tram. When the tram brakes, the force  $F_{acc}$  will be negative.

## 2.7 Power calculations

When calculating the total power transmission capability needed in a building or in an area of buildings, it is very uneconomic to assume that every house or apartment maximize their power consumption at once. Instead, a merged power level may be reached by calculating the normalized power demand from statistical energy use and multiplying with the number of apartments or houses and a security factor to manage power demand peaks. This is possible because all habitants of an area never peak their power demand at once. To reach a normalized power demand, one may use Velanders formula [25]

$$\hat{P} = k_1 W + k_2 \sqrt{W} \quad (2.14)$$

where the result  $\hat{P}$  is the normalized power needed,  $W$  is the annual energy use and  $k_1$  and  $k_2$  are constants which are statistically determined and differ depending on the type of load.

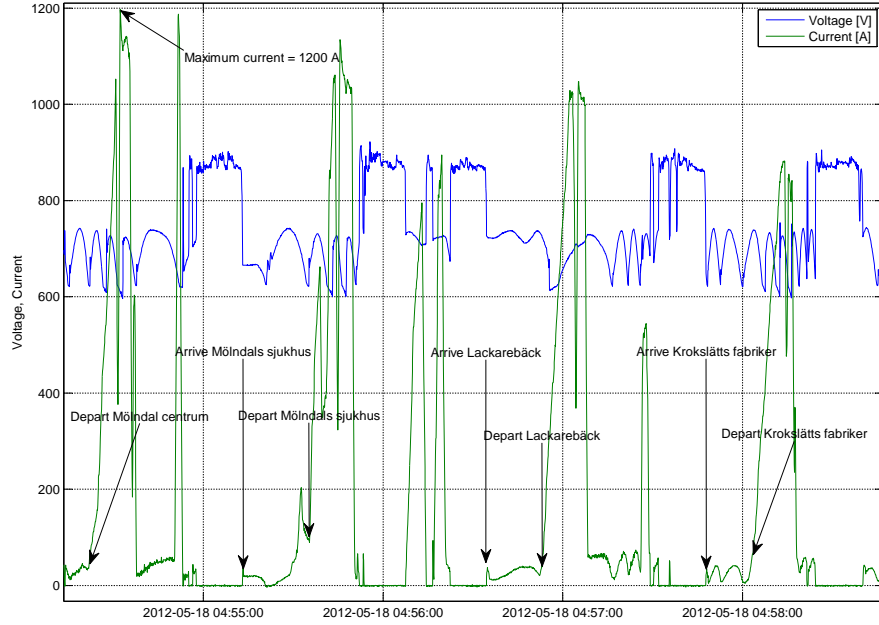
When it comes to powering a tramway network, the power demand is not at all as well distributed as the power demand of a residential area or an industry where multiple consumers even out the load. Figure 2.7.1 shows measurements of voltage and current in a rectifying station in Göteborg as *one* tram travel a distance of about 2.4 km. Almost all current is used during the accelerations which makes it more or less impossible to use Velanders formula for a similar result as with a residential area.

To see what the results with Velanders formula would be, one would need to find good constants to use for tramways. As no such constants were found, these calculations are based on the constants used for railroad,  $k_1 = 0.000230$  and  $k_2 = 0.3160$  [25]. The total energy used by the entire tramway network in Göteborg 2010 was 56 197 kWh or about 4.18 kWh per train-kilometer [26]. The resulting power level would be

$$\hat{P}_{total} = 0.000230 * 56.197 * 10^6 + 0.3160 * \sqrt{56.197 * 10^6} = 15\,294\,kW \quad (2.15)$$

If the power would be evenly distributed along the 61 rectifying stations, the result would be

$$\hat{P}_{station} = \frac{15\,294}{61} = 250\,kW \quad (2.16)$$



**Figure 2.7.1:** Voltage and current from station 7809

This result is not applicable on the stations, but more on the network feeding the stations, if it is part of a larger residential network. If one instead suppose that all rectifying stations shares the total annual energy equally, the result would be

$$\hat{P}_{total} = 0.000230 * \frac{56.197 * 10^6}{61} + 0.3160 * \sqrt{\frac{56.197 * 10^6}{61}} = 515 \text{ kW} \quad (2.17)$$

Because of the peak-like current demand from the trams, Velanders formula with railroad constants is not applicable. As a comparative figure, the transformer in a new-built rectifying station in Göteborg should have the capacity of 1250 kVA [9]. The measurements performed have also showed that power demand from *one* tram could momentarily be as high as 825 kW. To use Velanders formula for tramway applications, one must develop better constants which considers the quick acceleration sequences.

## 3 Measurements

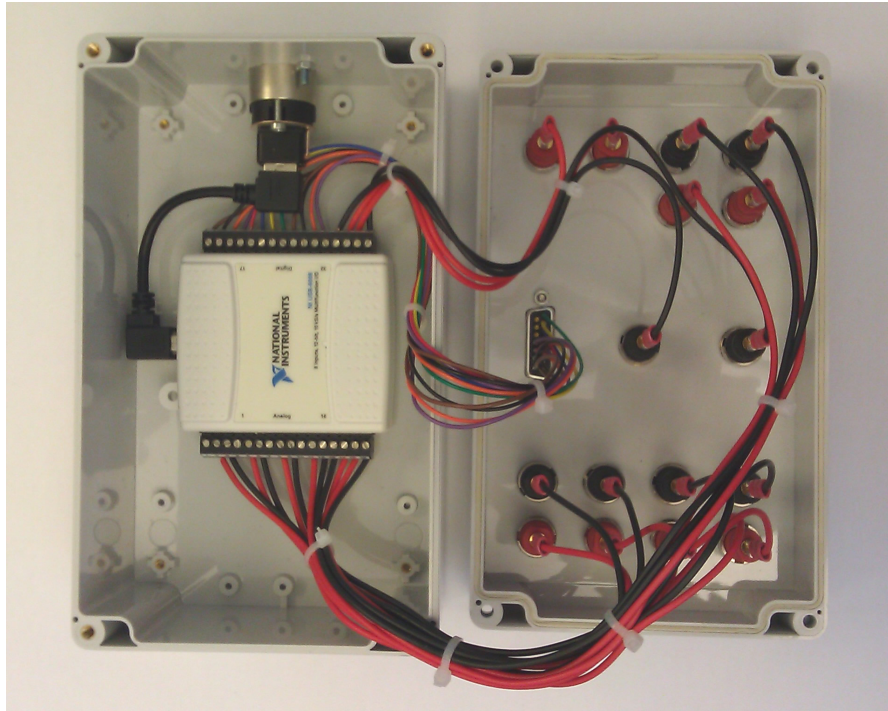
Since one of the objectives of this thesis project is facilitate calculations of the load level in new and existing rectifying stations, it was decided that the power measurements should take place in rectifying stations and not in actual trams. Measuring directly in the rectifying stations gives more measurement data from several different trams and different types of trams compared to measuring in specific trams. It also requires a smaller effort and data could be acquired during a longer time than if it were to be acquired in a tram. The interesting quantities to study are DC output current and voltage from the rectifying stations. From these measurements, the output power can be calculated as  $P = UI$  and from this, the load level of the station can be determined.

### 3.1 Measurement equipment

To acquire and save data for later analysis, some kind of A/D-converter and storage device is needed. Since double fed sections were to be measured, a double setup of equipment was needed and to get most accurate data, it is always best to use similar equipment in both setups. Research engineer Magnus Ellsén at Chalmers University of Technology recommended a small but powerful USB-powered A/D-converter with eight analogue input channels, that would be sufficient to use for this kind of project. Connected to a standard laptop, this converter may sample and save voltage levels of  $\pm 10$  V in frequencies up to several kilohertz. To measure the voltage, a voltage probe was needed to reduce the voltage down to a voltage level that the A/D-converter could handle. To measure the current, a current probe with low output voltage was needed for the same reason.

#### 3.1.1 A/D-converter

The A/D-converter used was a DAQ (Data acquisition device) from National Instruments called NI USB 6008. This converter has eight single ended inputs which also may be used as four differential inputs for better resolution. The sample frequency is up to 10 kHz and the digital resolution is 12 bits with differential measuring. See Appendix A for a data sheet of the converter. The converter was built-in in a plastic box and 4 mm safety plugs were mounted on the lid of the box for a safe and robust handling of the equipment, see Figure 3.1.1.



**Figure 3.1.1:** The inside of the measurement box

### 3.1.2 Voltage probe

The voltage probe used was a LeCroy AP032 with an attenuation ratio of 200:1 and a measuring range of  $\pm 1400$  V DC. The bandwidth of the probe is 25 MHz and it may be used in both AC and DC applications. The probe was powered from the local 230 V-grid through a DC/DC-converter and the connection to the 750 V-side of the rectifier was protected with a 1 A fuse in the station. See Appendix B for a data sheet of the probe.

### 3.1.3 Current probe

The current probe used was a LEM PR2000 with an output of 1 mV/A and maximum measurable current of 2 kA. The probe is built on a Hall effect sensor that can sense both DC and AC currents up to 10 kHz. The drawback of the probe is that it is powered by a 9 V PP3-battery which lasts about 100 hours (with a lithium battery) before a battery change is needed. See Appendix C for a data sheet of the probe.

### 3.1.4 Computer with measurement software

Two standard laptops of the same model were used to run the software needed for the data acquisition. The software is called LabVIEW SignalExpress and it was delivered with the A/D-converters. The software is simple to use and a small program for data sampling and storing was written. To reduce the risk of data loss, the program was set to create a new file for every hour so that there would be lower risk of losing all recorded data if one file should become corrupted. Each file also saved date and time for the start of the file so that the data could be synchronised in the analysis afterwards. The date and time was taken from the internal clock of the computers and these were manually synchronised before each measurement. The data was interpreted directly during the recording and multiplied with the corresponding attenuation ratio so that the recorded data was saved in its physical significance.

### 3.1.5 Measurement accuracy

When measuring an analogue signal with digital equipment, the signal should be sampled with a high enough frequency to be able to recreate the analogue signal. The sampling frequency must be set considering the frequency content of the measured signal, but a too high sampling frequency will give unnecessary amounts of data making it more time consuming to analyse afterwards. This is a factor to consider as the voltage in the system is DC and the current normally changes slower than the voltage. The sampling frequency was set to 10 Hz as this was considered to be fast enough for capturing the changes in the signals but still not creating too large files for the data processing. The size of the acquired data was limited to approximately 1.5 MB per logged hour with this frequency. The resolution of the A/D-converter is 12 bits which is equal to 2.048 A and 2.048 V respectively in the ranges used during the measurements.

To connect the voltage probe in the rectifying stations without needing to disconnect them from the network, a possibility is to connect the voltage probe in parallel over the existing analogue voltage meter on the rectifier. This existing voltage meter is protected with a fuse and when it is removed, connection of the voltage probe may be performed in a secure manner. The data sheet of the voltage probe shows that the accuracy of the selected attenuation is  $\pm 2\%$ .

The current probe is isolated and it should be clamped around an isolated conductor to reduce the risk of electric shock. To further remove the risk of electric shock, the current is measured on the minus side of the rectifier as this voltage should be close to zero. The plus and minus cable to the catenary and the tracks should be 500 mm<sup>2</sup> aluminium with one or two shields connected to ground. When measuring the current, the current probe must be clamped around only the center

conductor, so that the magnetic field from the current is not disturbed by the grounded shields. The accuracy of the instrument is according to the data sheet  $\pm 1$  % of the reading and  $\pm 0.5$  A offset. There is also an offset error of  $\pm 1.5$  % if the conductor not is placed in the center of the probe.

All accuracies put together, it still gives a reasonably good signal and since the measured magnitudes vary a lot from tram to tram, the largest differences is not depending on measurement errors but differences in the loading of the trams, how fast they accelerate etc.

## 3.2 Performing the measurements

To find interesting stations to perform the measurements in, a map of all sections and rectifying stations was analysed [6]. The local energy company, Göteborg Energi, is the company which is responsible for the operation of the rectifying stations. Each week, they provide a fault log which contains all occurred faults during the week [27]. These fault logs contains all events in both the high voltage and the low voltage sides in all rectifying stations in Göteborg. A quick analysis of these fault logs showed an abnormally high number of circuit breaker trips due to over-current in feeding modules in some stations. According to the fault logs, single fed sections feeding turning loops in the end of the track are more likely to trip due to over-current. Station 7809 (Mölndal, southern part of Göteborg) had about four trips due to over-current per week, and station 7829 (Länsmansgården, northern part of Göteborg) had about two trips due to over-current per week. Apart from these stations, there was also interest from Göteborg Energi to perform measurements in some specific stations due to previous faults and problems.

Several stations were marked as interesting in terms of many trams passing, long distance to feed or abnormal tripping frequency. To get good reference measurements for the calculation method, measurements including both single and double fed sections, straight track and slope was needed. To get most usable data out of the measurements, measurements was decided to be performed in station 7809 (Mölndal, high speed, high load level, straight track, single fed section), 7829 (Länsmansgården, high speed, high load level, slight slope, single fed section), 7836 (Vasagatan, low speed, steep slope, parallel fed sections) and simultaneous measurements on the double fed section between station 7810 and 7831 (on the island Hisingen, high speed, straight track, double fed section).

As it turned out, the measurement in station 7831 failed and therefore, a new double fed section was needed. Stations 7858 and 7863 parallel feed a section which has a steep slope and a lot of traffic and it was deemed suitable. A brief description of each station and the measurements performed follows below.

### 3.2.1 First measurement - station 7836

Station 7836 is located centrally in Göteborg and it is a large station with four DC feeding modules, feeding four different parallel fed sections. There are two transformers of 1085 kVA each which transform the voltage to 510 V (720 V peak value) and they supply one rectifier each. There are four incoming 10 kV cables which feed the two transformers in two separated high voltage loops. Both rectifiers are connected to the same positive and negative rails that supply all four feeding modules. This way, the transformers and rectifiers share the load under normal conditions but if one side would trip for any reason, all four modules may be fed from the remaining side.

This was the first measurement and the gathered data was the output DC current from two feeding modules and the common positive voltage over the rectifiers. The two investigated feeding modules supply current to the two tram lines frequenting each section with seven trams per hour. This first measurement was merely a test run to see how well the equipment worked together and if the data would give any good results. In total, 70 hours of data was recorded and no circuit breakers opened in this station during the measurements.

### 3.2.2 Second measurement - station 7809

Station 7809 is located in Mölndal and it is the southernmost rectifying station in the tramway network of Göteborg. There is one transformer of 1000 kVA with two secondary windings and the voltage on the secondary side is 510 V. There are two rectifiers which supply one DC feeding module each and the circuit breaker is placed before the rectifier, thus breaking the AC current instead of the DC current. In normal operation, there is one module feeding the catenary and the other one is used as a spare.

This catenary section about 2.8 km long and it is a parallel fed section, fed from station 7808 in the northern end and from station 7809 placed approximately in the middle of the section. As station 7809 is closest to the southern end of the section, the biggest part of the total current on the section is supplied from 7809 and it may be seen as a single fed section from station 7809 and further south.

The reason for measuring in this station is that there recently has been doubled traffic as two lines run along the track instead of one. This has led to an increase in power demand and the station is being tripped due to over-current 3-4 times per week. In total, 144 hours of data was recorded during this measurement but the battery in the current probe ran out after about 110 hours, so the actual usable data was about 110 hours. During this measurement, the circuit breaker opened no less than three times due to overload.

### 3.2.3 Third measurement - stations 7810 and 7831

Station 7810 and 7831 parallel feed a section of 1.5 km double track on the island Hisingen in northern Göteborg. The section is frequented by three tram lines and the reason for this measurement was to get a good reading of an average parallel fed distance. However, a problem occurred when the current probe could not reach around the minus cable in station 7831, because it had different dimensions. Because of this, the measurement in 7831 was cancelled and the measurement continued with only measurements in station 7810. The result from station 7810 was analysed, but the gathered data was insufficient to draw any conclusions for the entire power demand on the section. In total, about 72 hours of data was recorded in station 7810 and no circuit breaker was opened in the station during this time.

### 3.2.4 Fourth measurement - station 7829

Station 7829 is located in the northern part of Hisingen and it is feeding two sections; one parallel fed and one single fed section with a double turning loop. The interesting section is the single fed section in the end of the tracks and it was chosen for the same reasons as station 7809 in Mölndal; the circuit breaker of the single fed section is tripped about two times per week due to over-current. The traffic on the section is maintained by two tram lines, each frequenting the section with up to 7 trams per hour.

During the connection of the equipment it was discovered that the two minus cables going to the rectifier was parallel connected in the minus cabinet close to the tracks. This made it impossible to measure only the current in one of the sections, and the result from this measurement was therefore the current from both feeding modules in the station. Nevertheless, the results from this measurement was still interesting as to see how the total load in the station looked like. In total, about 71 hours of data was recorded and the circuit breaker for the single fed section opened once due to overload during the measurement.

### 3.2.5 Fifth measurement - stations 7858 and 7863

This measurement was the last measurement of voltage and current and it was performed to compensate for the lost data of the parallel section in the third measurement. Station 7858 and 7863 parallel feed a distance of about 800 m just outside the most central parts of the town and the section is frequented by four tram lines. This makes a tram appear on this section once per minute on an average. The section also includes a steep slope of 40 ‰ and a connection point where the tracks divide into two different directions. This measurement gave many

interesting results, both from a parallel feed and a slope point of view. In total, 71 hours of common data was recorded in both stations. No circuit breaker opened in any of the stations during the measurement.

### **3.2.6 Speed measurements**

As the results from the measurements were analysed, it was clear that there were big differences in the power usage during different operating conditions. Since the data was acquired in the rectifying stations rather than in the actual trams, no speed data was recorded. To get an approximated value of the speed corresponding to the recorded power, and to better be able to fit the data to the theoretical model, speed was recorded in various tram types on the measured distances. The speed was measured using a GPS logging software in an Android cellular phone.



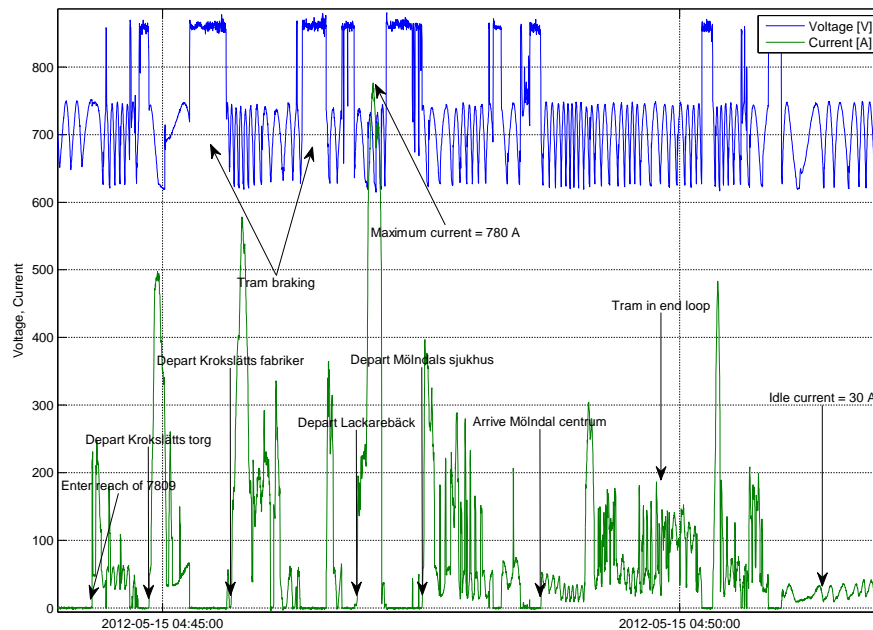
## 4 Analysis of measurements

The measurements of voltage and current were recorded into several txt-files, each containing 36 000 rows or exactly one hour with 10 rows per second. In total, 465 hours of raw data was recorded during the measurements. To handle the data quickly and efficient, a MATLAB-script was written which automatically reads all files from one measurement and stores them together with a time vector in one MATLAB data file. From the MATLAB files, the voltage and current was plotted against time for a first visual overview of the data. The power was also plotted as the product of voltage and current and the consumed energy was calculated with numerical integration of the power.

To help in the analysis of the plotted data, access was granted to the ITS KomFram which contains real-time information of all vehicles in the public transportation system in Göteborg. With this system, all movements of the vehicles are saved for 18 months and data may be extracted to see which tram that passed a certain switch or tram stop at a given time. KomFram was utilized in this project by looking at what exact time a current peak appeared in the measurement data. By comparing that specific time to occurrences on the section in KomFram, a tram identification number could be extracted thus revealing what tram type that caused the current peak.

### 4.1 Data from the second measurement, station 7809 in Mölndal

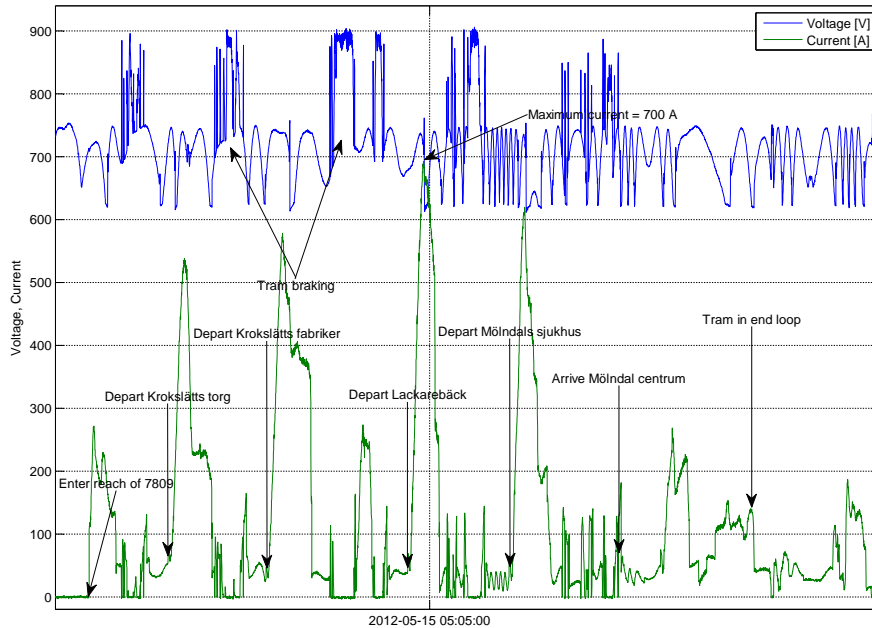
By comparing the times from the recorded data with the times for a tram arriving at and departing from a tram stop, a complete journey could be recreated. An example is shown in Figure 4.1.1, where a M31 tram runs southbound towards Mölndal an early morning. The reason for choosing an early morning is that it is easiest to evaluate the current when there is only one tram on the section. The plot shows recorded voltage and current for the single M31 tram on the section. From the plot, it is possible to see that the investigated tram is one of the M31 trams which have new control electronics described in Chapter 2.3.3. This is easiest seen as the tram brakes and the voltage is raised to almost 900 V. The M31 trams with new control electronics has a smooth and even braking voltage compared to the trams which have the old control electronics, compare with Figure 4.1.2.



**Figure 4.1.1:** Plot of voltage and current in station 7809 as a M31 tram with new control electronics travels from Krokslätt to Mölndal Centrum.

The current shape in Figure 4.1.1 is very typical for rail bound vehicles, with high amplitude spikes during acceleration and much less current during constant speed. The accelerations may be seen in the plot as when the tram departs from and arrives at the different stops, marked with arrows. The current during constant speed is harder to find in the plot. Around 04:50, the tram is in the end loop and the current needed is around 100-200 A. In the end of the plot, the tram stands still and the idle current is about 30 A. The electromechanical regenerative braking is shown clearly as the voltage is raised to almost 900 V where it is electrically controlled. Some of the braking current is used by the internal systems of the tram but since there is no other tram on the section, the rest of the energy is burned off in the braking resistances on the roof. As the voltage of the catenary is raised to 900 V during braking, the rectifier is saturated and no current is fed from the rectifying station during this time. The maximal current being fed from the station during this run is 780 A and the peak power is 525 kW or 175 % of the nominal power of the M31 tram, see Table 2.3.1.

Similar analyses have been made on all recorded data to find typical power demands of the different tram types and some worst-case scenarios. Figure 4.1.2 shows a similar situation with an other M31 tram. The tram travels the same distance as the tram in Figure 4.1.1 and the current shapes are very alike in the

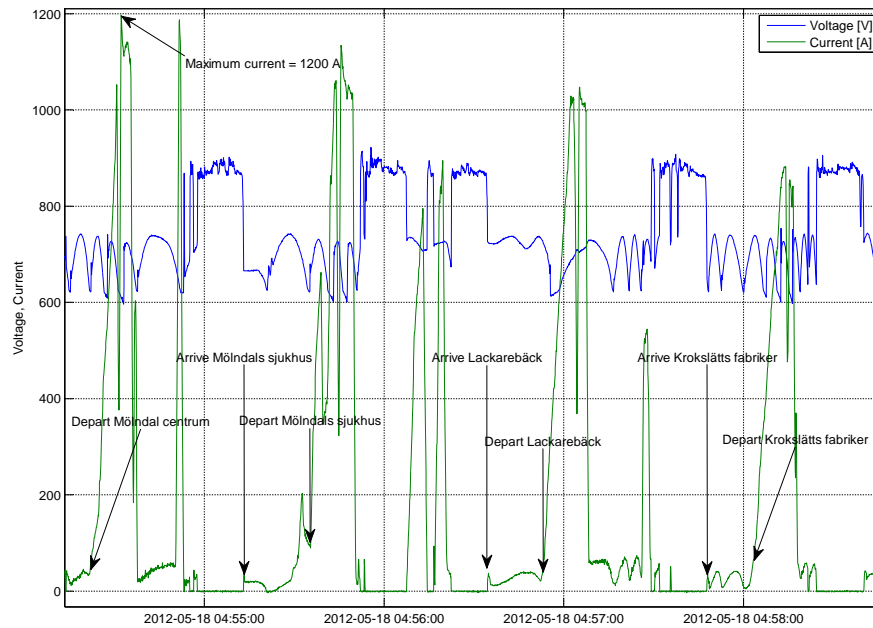


**Figure 4.1.2:** Plot of voltage and current in station 7809 as a M31 tram with old control electronics travels from Krokslätt to Mölndal Centrum.

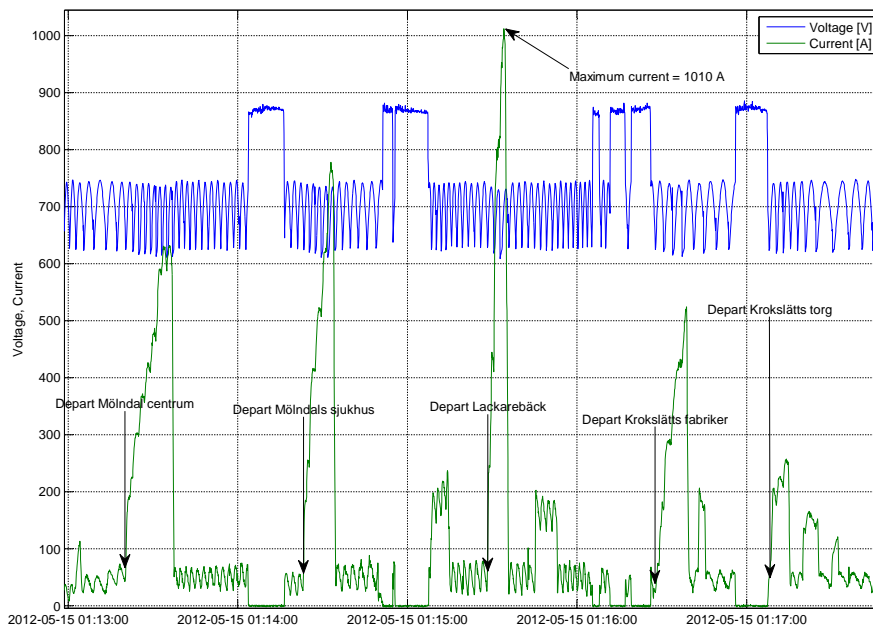
two plots. There is however one particular difference which reveals that there is old control electronics in the M31 tram in Figure 4.1.2. The voltage during the braking sessions is more rough compared to Figure 4.1.1, and this is the result from the old control electronics based on thyristors compared to the new control electronics based on IGBT's.

The newest tram type in Göteborg, M32, is known to consume more power than the previous tram types do. Figure 4.1.3 shows a M32 tram running on the same section as the M31 trams in the two previous figures, but in the reverse direction. The recording is made an early morning and the shape of the current looks a lot like the previous two figures. A general difference between the M31 trams and M32 trams is that the amplitudes of the current peaks are considerably higher for M32 trams. The maximum current recorded in this figure is 1200 A and the magnitudes are often peaking over 1000 A during accelerations. The largest power peak discovered during the measurements was 825 kW or 195 % of the nominal power of the M32 tram, see Table 2.3.1.

Figure 4.1.4 shows another M32 tram running the same distance as the tram in Figure 4.1.3. The recording is made late on a tuesday night and it is most likely the last tour for the day for this tram driver. The shape of the current plot is similar to Figure 4.1.3 but the magnitudes are somewhat lower. In the



**Figure 4.1.3:** Plot of voltage and current in station 7809 as a M32 tram travels from Mölndal Centrum to Krokslätt.



**Figure 4.1.4:** Plot of voltage and current in station 7809 as a M32 tram travels from Mölndal Centrum to Krokslätt.

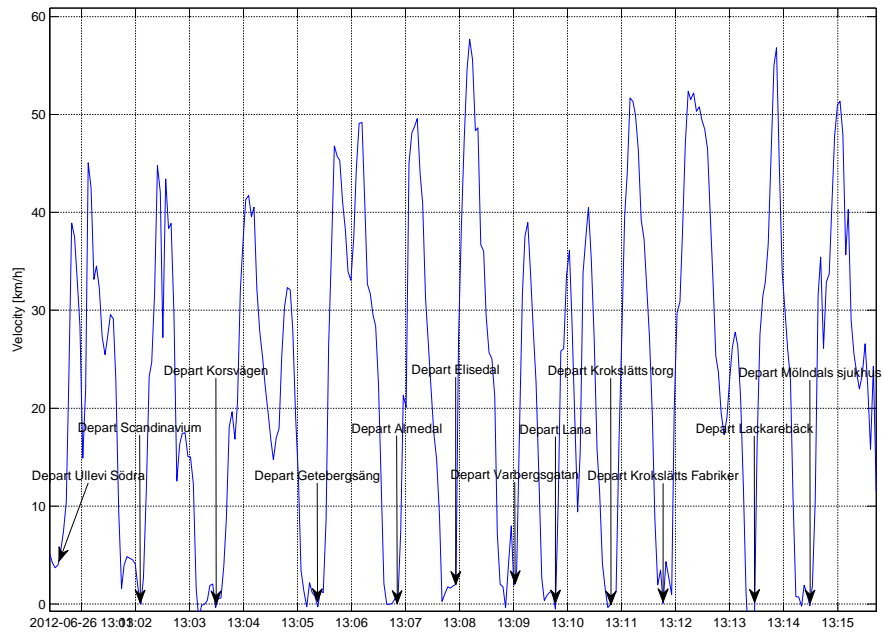
beginning of Figure 4.1.4, just before departing from Mölndal Centrum, the idle current is found to be around 60 A. By comparing the two M32 plots, one may draw some interesting conclusions. Table 4.1.1 shows acceleration times, total time and average speed in the two plots.

**Table 4.1.1:** Acceleration times, total time and average speed exported from Figure 4.1.3 and Figure 4.1.4. The abbreviations used for the tram stops in the table are Mölndal Centrum (MC), Mölndals Sjukhus (MS) and Lackarebäck (L).

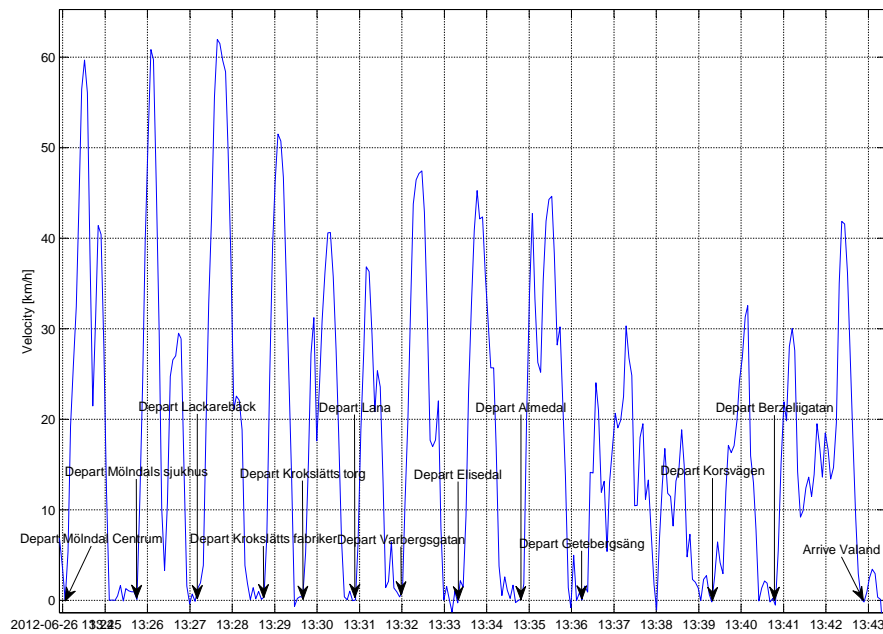
Figure	Acc. MC	Acc. MS	Acc. L	Total time	Avg. speed
4.1.3	16 s	15 s	16 s	3 min, 41 s	30.1 km/h
4.1.4	17 s	11 s	9 s	3 min, 7 s	35.6 km/h

The total time and average speed in the table is measured between departing from Mölndal Centrum and departing from Kroksläatts Fabriker and the total distance between these tram stops is about 1850 m. By looking at the current amplitude in the two figures, the tram in Figure 4.1.4 seems to be run somewhat slower and less aggressive. However, the average speed in Figure 4.1.4 is somewhat higher than in Figure 4.1.3 thus implying that it would have been run faster and more aggressively. The answer to this dilemma is found by looking at how long time the trams stop at each tram stop. In Figure 4.1.3, it is seen that the tram stops for about 15-20 seconds at each stop whereas the tram in Figure 4.1.4 only slows down and then starts accelerating again without stopping completely at any stop. This is also the reason for the much shorter acceleration times from Mölndals Sjukhus and Lackarebäck for the tram in Figure 4.1.4. If the stop times are subtracted from the total time in Figure 4.1.3, the average speed for that tram is increased to 39.4 km/h.

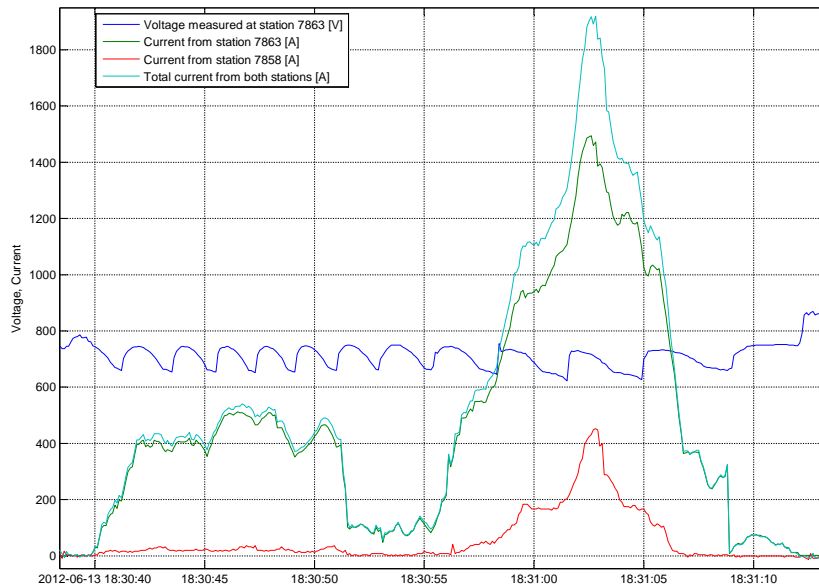
During all the measurements in the rectifying stations, no speed of the trams was measured. To get a hint of how fast the trams travel on the measured distances, some separate speed measurements were performed by travelling with different trams on the distances and logging the speed along the travelled distance. Figure 4.1.5 and Figure 4.1.6 shows the measured speed with a M31 tram and a M32 tram travelling along the measured distances. The figures might be hard to interpret if one not knows the distances between the stops and if the tracks are in mixed or separated traffic. The major conclusions to draw from these measurements are that when there are short distances and mixed traffic, the top speed tends to be around 30-40 km/h and when there are longer distances and separated tracks, the top speed tends to be 50-60 km/h. The speed log from the M32 tram also shows slightly higher speeds than the speed log from the M31 tram.



**Figure 4.1.5:** Speed plot of a M31 tram travelling from Ullevi Södra to Mölndal centrum.



**Figure 4.1.6:** Speed plot of a M32 tram travelling from Mölndal centrum to Valand.

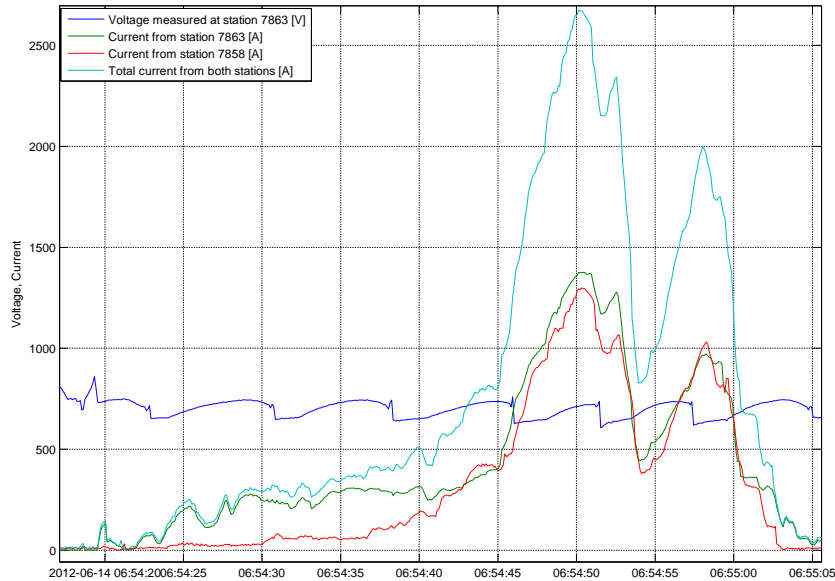


**Figure 4.2.1:** Plot of voltage and currents from stations 7858 and 7863 showing that parallel fed sections not always share the load evenly. This is however not an overload situation as the individual stations never exceeds 1500 A.

## 4.2 Data from the fifth measurement, stations 7858 and 7863 around Redbergsplatsen and Olskrokstorget

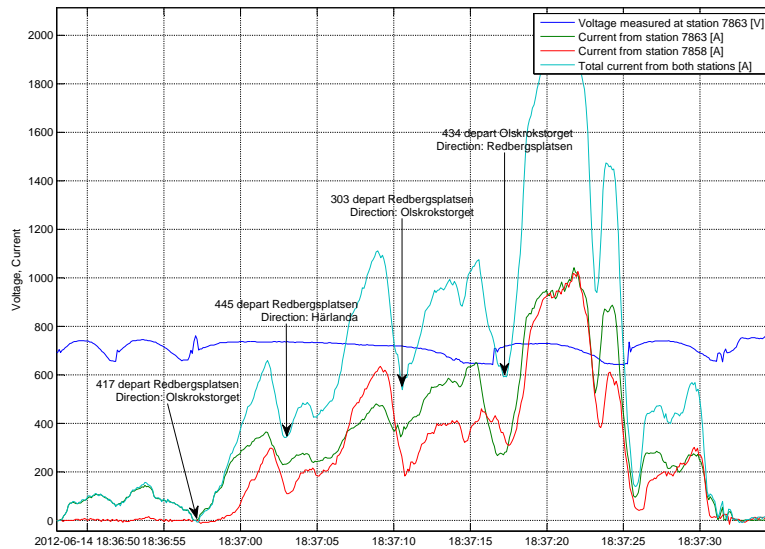
Parallel fed sections are interesting to analyse as the two rectifying stations feeding the section may be different in choice of components, age and wear. As the age span of the stations reach from 1960's until today, their components are varying in both design and wear. The DC output voltage may vary depending on the transformer ratio and the load may not be shared equal between two stations. The distance between the stations are also important as the voltage drop over the catenary increases linearly with the distance from the load. As a tram run along the section, it gives varying impedances for the two voltage sources. Since the current always takes the path with lowest resistance, the major part of the load will be taken by the closest station. If the two stations have slightly different output voltages, the station with the highest voltage is also more likely to take the largest part of the load.

Figure 4.2.1 and Figure 4.2.2 shows two different situations of the section which is parallel fed from station 7863 and 7858. The plots show current from each station, total current from both stations and voltage from station 7863. Since

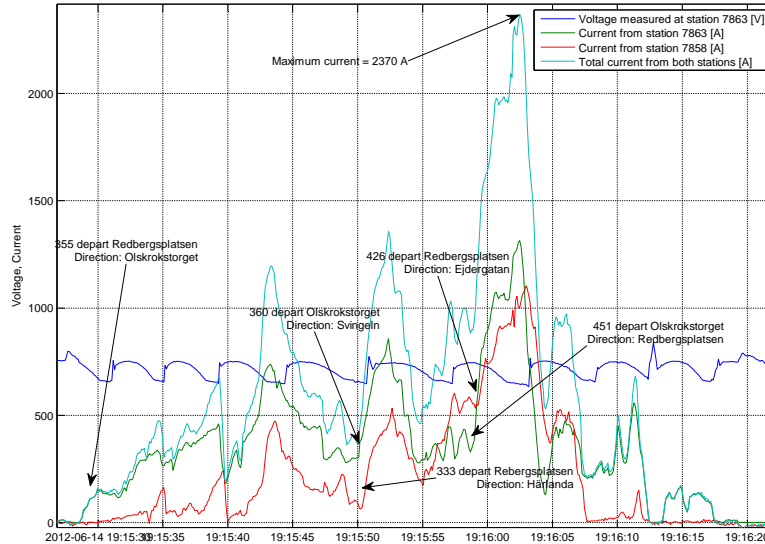


**Figure 4.2.2:** Plot of voltage and currents from stations 7858 and 7863 showing a better load sharing situation compared to Figure 4.2.1. The currents in both stations never exceeds their individual limits of 1500 A, and together, they supply almost 1.9 MW of power to the trams on the section.

the catenary section is parallel fed, the voltage looks alike in both measurements. This makes it unnecessary to show the voltage in both stations. The plots show a normal operating scenario where several trams run on the section simultaneously. The difference in load sharing may depend on several different reasons. Throughout the entire analysis of this measurement, it has been very uncommon to see that station 7858 has taken more load than station 7863. Even as the trams pass directly outside station 7858, station 7863 has taken most of the load. During the first 15 seconds of Figure 4.2.1, station 7858 takes almost no load at all while station 7863 takes the biggest part of it. As the load rises, station 7858 increases its part of the load but overall, station 7863 takes the biggest part of the load. The major difficulty for sharing the load seems to be associated with the age of the two stations and possibly, the output DC voltage of station 7863 is slightly higher than in station 7858. Figure 4.2.3 and Figure 4.2.4 shows two more examples of how the load varies as several trams on the same section runs simultaneously. In both figures, station 7858 and 7863 share the total load well and the current never exceeds the individual limits of 1500 A in the rectifying stations. Individual trams have been localized in these two figures and the trams are distributed evenly over the section during both plots.



**Figure 4.2.3:** Plot of voltage and currents from stations 7858 and 7863 showing another situation where the load is shared good between the two stations. Different trams on the section have been localized and their position and direction is shown in the figure.



**Figure 4.2.4:** Plot of voltage and currents from stations 7858 and 7863 showing a situation where the load is high but never exceeds the limits in the rectifying stations. Different trams on the section have been localized and their position and direction is shown in the figure.

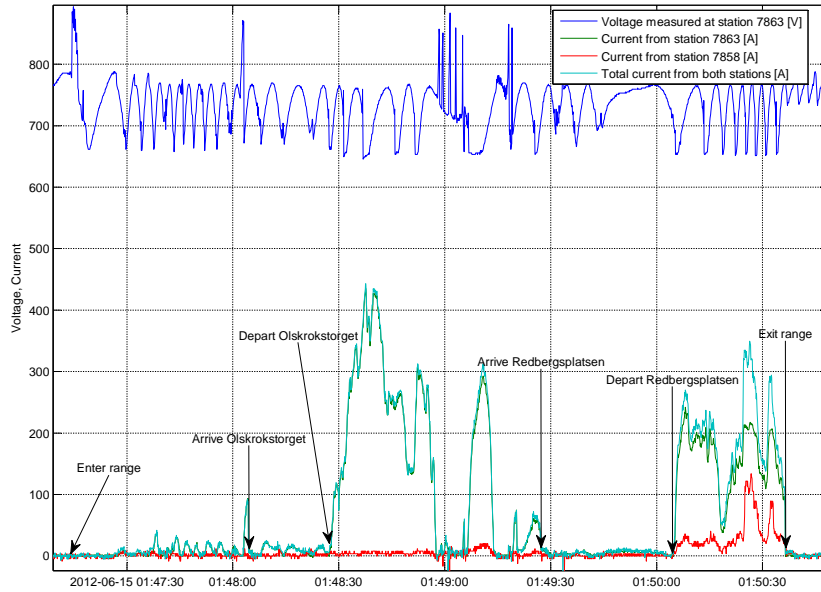
### 4.3 Effect on power consumption while driving in a slope

As mentioned in Chapter 2.5, vehicles going up a slope need considerably more tractive effort than when it runs on a straight track. The same principle is valid for going down a slope where much less effort is needed. The following six figures are extracted from the recorded data of measurement 5, with the catenary section fed by station 7858 and 7863. The section includes a steep slope of 40 ‰ which starts at the stop Olskrokstorget and continues about 300 m before decreasing a bit when the tracks approaches the stop Redbergsplatsen. There is also a gradient of 30 ‰ which is about 150 m long as the trams travel from Redbergsplatsen towards H rlanda. The next figures shows plots from the slope when different tram types travel both up and down the section.

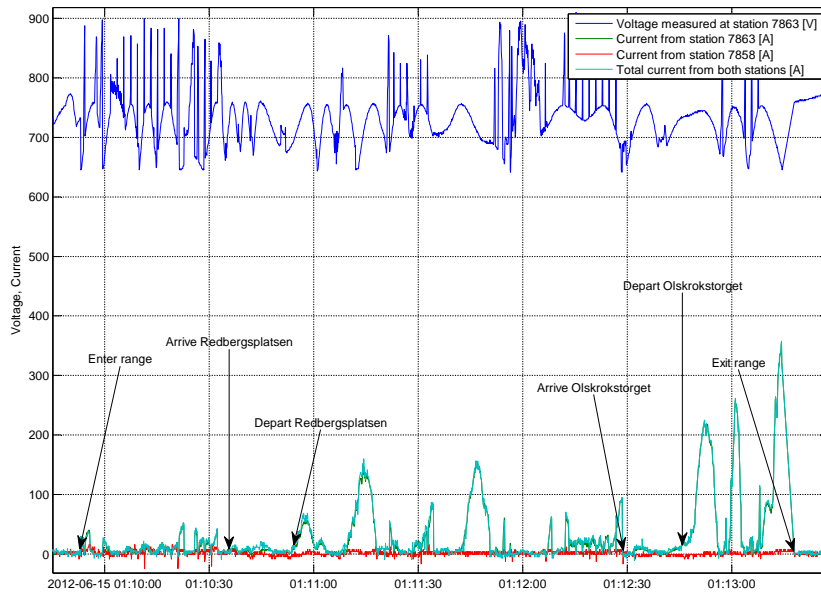
Figure 4.3.1 shows a M31 tram travelling up the slopes. The current is not abnormally high, but it draws current during a longer time span compared to running on a flat track. During the acceleration as the tram starts from Olskrokstorget, it draws current during for about 30 seconds compared to about 15 seconds which is common on a flat track. By integrating the power during the travelling from Olskrokstorget to Redbergsplatsen, the total amount of energy needed for the climb is 5.5 MJ. Much of the energy is stored as potential energy in the tram. This energy will help the tram accelerate when it travels down the slope and much less energy will be needed for travelling in the opposite direction.

The measurements from station 7809 showed that a typical energy consumption during an acceleration on a flat track is about 6.0 MJ or about the same amount that is needed to travel up this slope. The reason for the energy being higher on a flat track is that the top speed on the flat track is about 60 km/h and in the slope about 40 km/h. Figure 4.3.2 shows another M31 tram, but in the reverse direction. It is travelling from Redbergsplatsen to Olskrokstorget down the steep slope and there is not much current needed here as gravity helps increase the speed. The total energy needed for the descent is 1.0 MJ or about one fifth of the energy needed going up the slope. The typical voltage spikes of a braking M31 tram also appears during the descent which indicates that some of the stored potential energy is burned off in the braking resistors on the tram.

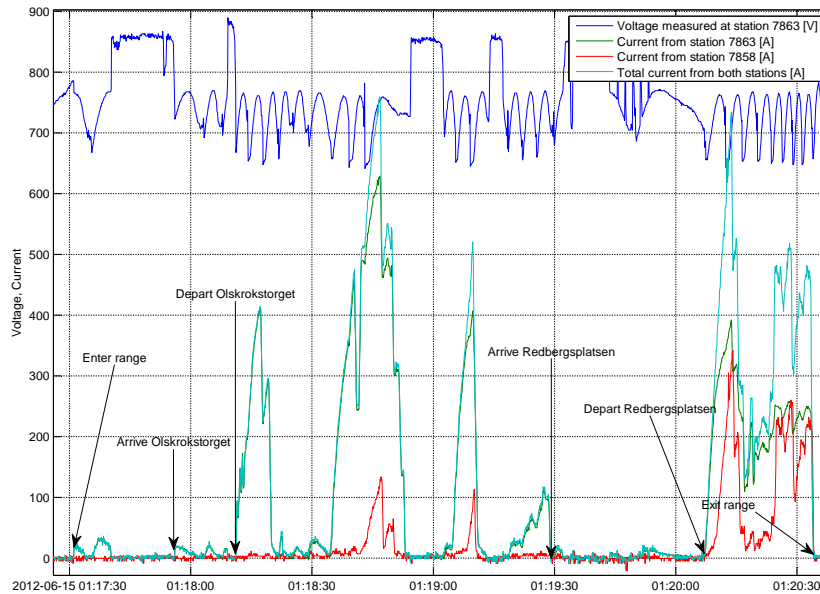
Figure 4.3.3 shows a M32 tram travelling up the same slope as in the previous figures. Compared to Figure 4.3.1, the current amplitude is higher, peaking at about 750 A. It takes about 80 seconds for the tram to travel from Olskrokstorget to Redbergsplatsen, compared to about 60 seconds for the M31 tram in Figure 4.3.1. The reason for this difference is seen in Figure 4.3.3 where the current is divided into three current spikes with a few seconds in between. This is probably because that the tram needed to slow down for traffic lights twice before reaching Red-



**Figure 4.3.1:** Plot of voltage and currents from stations 7858 and 7863 as a M31 tram travel up the slope between Olskrokstorget and Redbergsplatsen. The total amount of energy needed is about 5.5 MJ.



**Figure 4.3.2:** Plot of voltage and currents from stations 7858 and 7863 as a M31 tram goes down the slope between Redbergsplatsen and Olskrokstorget. The total amount of energy needed is about 1.0 MJ.



**Figure 4.3.3:** Plot of voltage and currents from stations 7858 and 7863 as a M32 tram travel up the slope between Olskrokstorget and Redbergsplatsen. The total amount of energy needed is about 5.5 MJ.

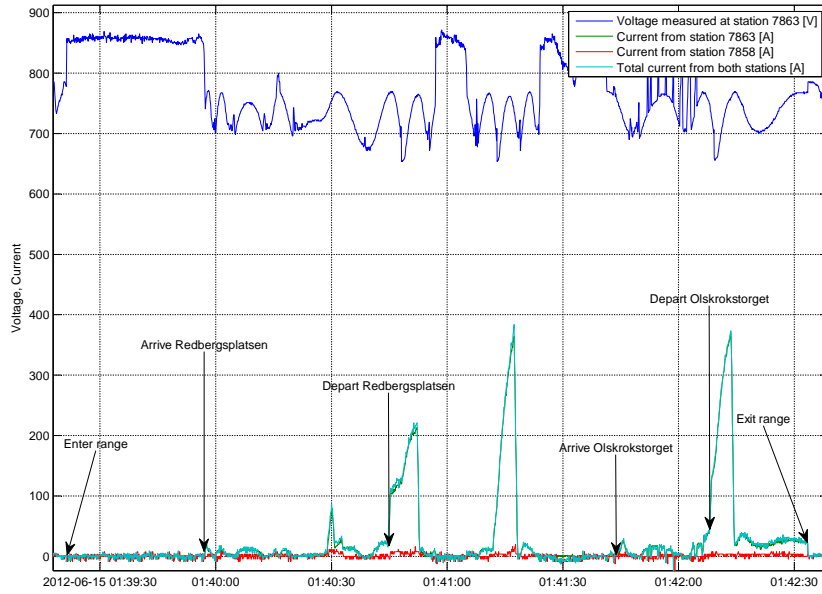
bergsplatsen. Even though the journey took longer time than for the M31 tram, the energy needed to climb the slope was equal to the energy for the M31 tram, in total 5.5 MJ.

Figure 4.3.4 shows the same M32 tram as in Figure 4.3.3, but this time it is going down the slope between Redbergsplatsen and Olskrokstorget. The figure resembles Figure 4.3.2 with low current amplitudes and narrow current spikes. The total energy needed for the descent to Olskrokstorget is only about 0.9 MJ or one sixth of the energy needed to up the slope.

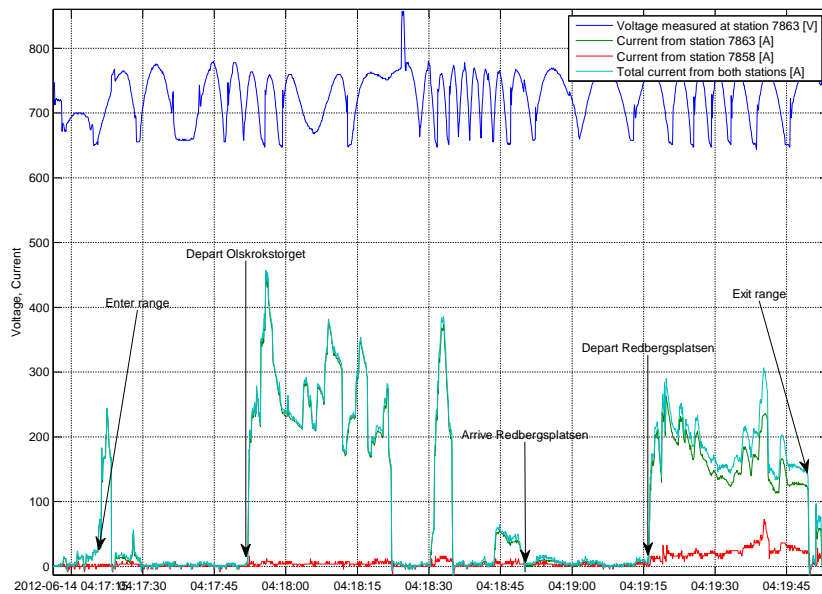
Figure 4.3.5 shows a set of one M29 and one M28 travelling up the slope between Olskrokstorget and Redbergsplatsen. The current is not especially high and it peaks just above 400 A. Just like the M31 and M32 trams, it takes longer time to accelerate and in total it takes almost 60 seconds to reach Redbergsplatsen. The total amount of energy needed for the train to travel up the slope is about 5.4 MJ which is about the same as for the M31 and the M32 trams.

Figure 4.3.6 shows the same set of trams as in Figure 4.3.5 but now the train is travelling down the slope from Redbergsplatsen to Olskrokstorget. Not much energy is needed to accelerate and in total it needs about 1.1 MJ or one fifth of energy needed to run up the slope.

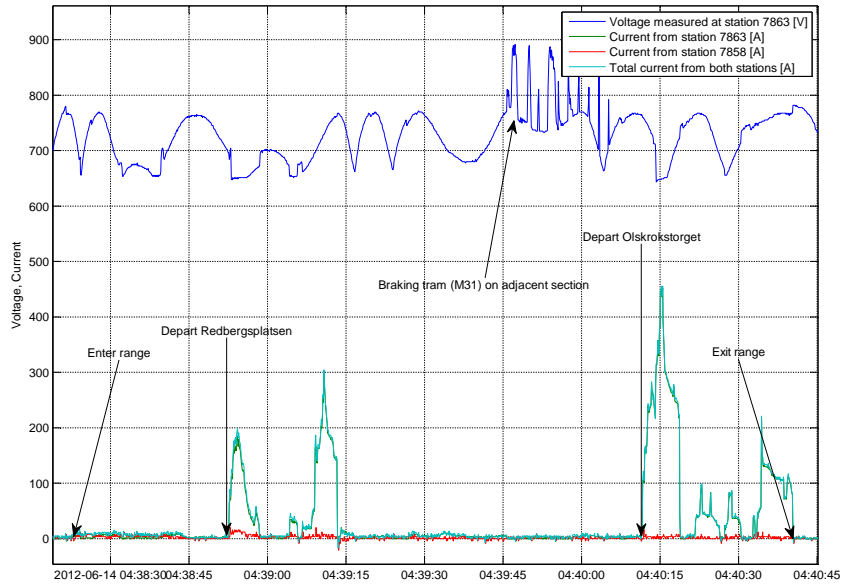
As the six plots have shown, a lot more energy is needed when a tram travel uphill than downhill. As one may have noticed, the energy is almost the same for



**Figure 4.3.4:** Plot of voltage and currents from stations 7858 and 7863 as a M32 tram goes down the slope between Redbergsplatsen and Olskrokstorget. The total amount of energy needed is about 0.9 MJ.



**Figure 4.3.5:** Plot of voltage and currents from stations 7858 and 7863 as a train consisting of one M28 and one M29 travel up the slope between Olskrokstorget and Redbergsplatsen. The total amount of energy needed is about 5.4 MJ.



**Figure 4.3.6:** Plot of voltage and currents from stations 7858 and 7863 as a train consisting of one M28 and one M29 goes down the slope between Redbergsplatsen and Olskrokstorget. The total amount of energy needed is about 1.1 MJ.

all the tram types and this is basically because that the mass of the trams are similar. The M32 weighs about 38.9 tonnes, the M31 weighs about 34.5 tonnes and a train with one M28 and one M29 weighs about 33.8 tonnes, see Table 2.3.1. The newer tram types have more efficient motors, but the major part of the energy consumption depends on the weight and the difference in height between the two tram stops. An interesting note is that the energy also depends on the top speed of the tram during an ascend. If the trams were to travel in 60 km/h up the slope, as they do on flat track, about 50 % more energy would be needed.

## 4.4 Circuit breaker action from measurement 2 and measurement 4

The measurements in station 7809 and 7829 were mainly performed to try to record opening of circuit breakers due to over-current in a DC feeding module. Three occasions were recorded in measurement 2 in station 7809 and one occasion was found in measurement 4 in station 7829. All four occasions were analysed to see the reasons for the circuit breaker opening, see Figures 4.4.1 through 4.4.4.

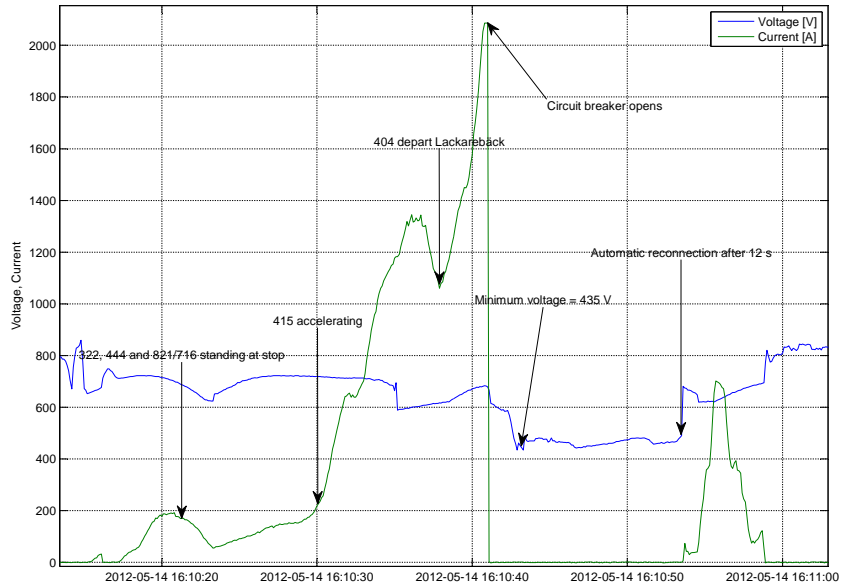
Figure 4.4.1 shows the current and voltage supplied from station 7809 during an over-current situation. At the overload moment there are six trams on the section: one M28, one M29, one M31 and three M32 trams. Most of them are standing still at a stop or in the turning loop and the circuit breaker is overloaded by two M32 trams accelerating within eight seconds of each other. The over-current part of the relay protection is set to open above a current of 1500 A but the current reaches over 2000 A before opening. The settings of the relay protection for this particular station has not been studied thoroughly but the plot shows a behaviour which is expected from this kind of relay protection.

During the overload situation, the section is fed from station 7808, about 1.4 km north of station 7809. The voltage drop over the catenary is large and the lowest recorded voltage at station 7809 is 435 V. The two M32 trams moving on the section might have been standing still during this time since their electric equipment is designed to stop working if the voltage level drops below 500 V [16]. The breaker re-closes automatically after 12 seconds, just like it is set to do.

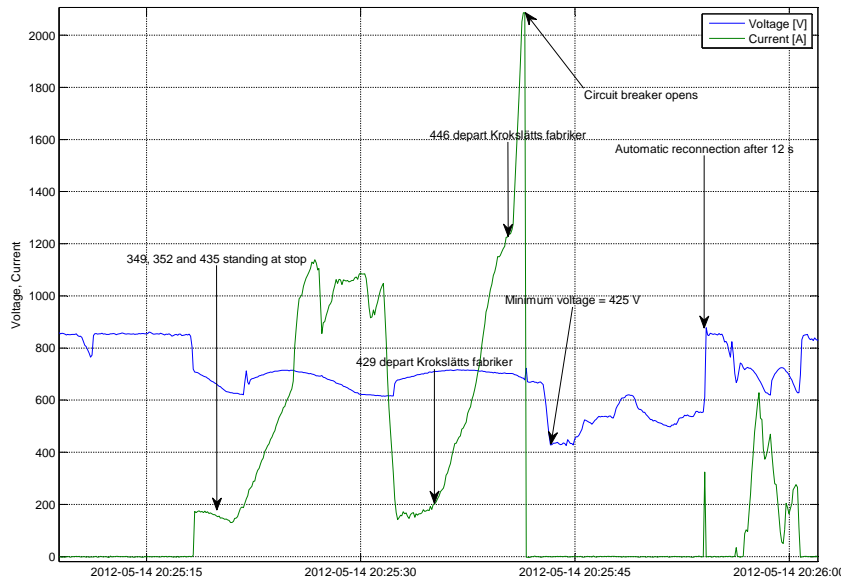
Figure 4.4.2 shows an over-current situation similar to the one in Figure 4.4.1 and at the moment there are two M31 and three M32 trams on the section. Two M32 trams start within six seconds of each other and the current demand increases rapidly. The relay protection trips the circuit breaker after only 0.5 seconds of over-current and during the fault, the voltage goes as low as 425 V. The breaker re-closes automatically after 12 seconds, just like it is set to do.

Figure 4.4.3 shows an over-current situation with one M28, one M29 and three M32 trams in motion at the same time. The current exceeds the limit of 1500 A during four seconds before the breaker opens and the peak current being reached is unknown as the measurement equipment hits its limit at approximately 2100 A. The circuit breaker re-closes automatically after 12 seconds but at the same time there is an unknown tram braking which raises the voltage. The minimum voltage level recorded is 460 V.

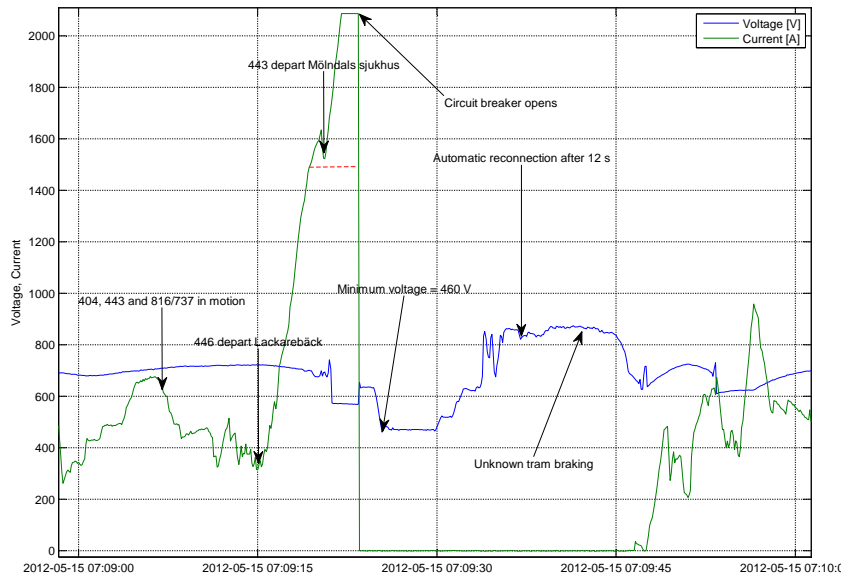
Figure 4.4.4 shows the total current from both feeding modules in station 7829 (from measurement 4) during an overload situation. Several trams are in motion on the single fed section when the circuit breaker opens. The total current at the fault is about 2350 A in the two modules and when one of the breakers open, the



**Figure 4.4.1:** Plot of voltage and current from station 7809 as the circuit breaker in one of the feeding modules is tripped. Two M32 trams accelerating within eight seconds creates the overload situation.



**Figure 4.4.2:** Plot of voltage and current from station 7809 as the circuit breaker in one of the feeding modules is tripped. Two M32 trams accelerating within six seconds creates an overload situation.

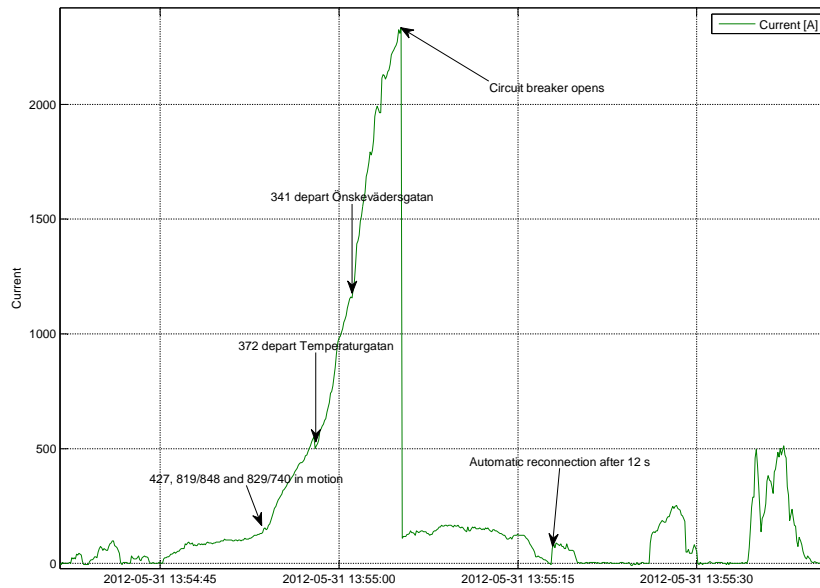


**Figure 4.4.3:** Plot of voltage and current from station 7809 as the circuit breaker in one of the feeding modules is tripped. Several trams in motion creates the overload situation.

remaining current is about 150 A, which means that the current in the overloaded feeder should have been about 2200 A. The voltage measurement was unfortunately not recorded but according to the fault report [27], the breaker was re-closed after 12 seconds.

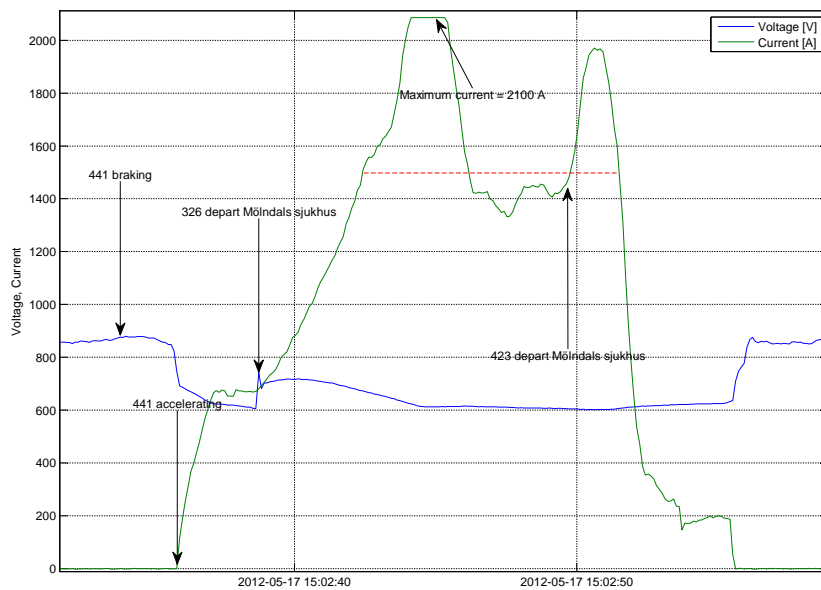
There are several occasions found in the recorded data where the limit in the feeders are hit but the circuit breaker never was tripped. Figures 4.4.5 through 4.4.7 show three situations when the current is well above the limit of 1500 A for several seconds without opening the circuit breaker. It is difficult to say why the relay protection circuit did not trip the breaker in the following three figures and it may seem a bit random that the breaker opens during some over current situations but not in similar cases. The relay protection circuit is built-up by several different parts, and the fault report only states that it was a "normal" over current situation which tripped the breakers in all four recorded cases shown in Figures 4.4.1 - 4.4.4.

Figure 4.4.5 shows one M31 and two M32 trams in motion on the southernmost section in Mölndal. The current exceeds the limit of 1500 A for about four seconds before it drops just below the limit for another four seconds and then exceeds the limit again for another two seconds. The breaker never opens despite the current peaking at over 2100 A. Figure 4.4.6 shows two M32 trams starting almost simultaneously on the same section. The current exceeds the limit of 1500 A for almost five seconds but the breaker never opens. Figure 4.4.7 shows a mix of three

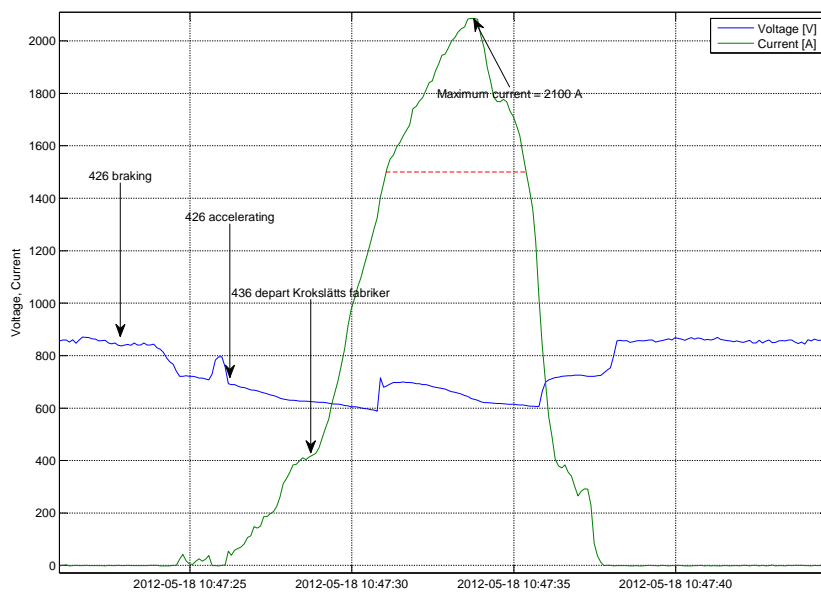


**Figure 4.4.4:** Plot of total current from station 7829 as the circuit breaker in one of the feeding modules is tripped.

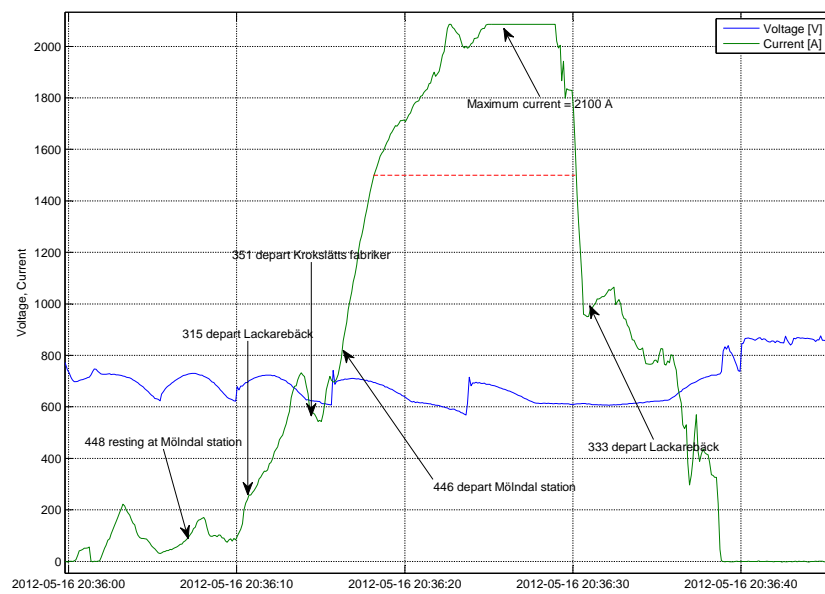
M31 trams and two M32 trams which together makes the current exceed the limit of 1500 A for about 12 seconds. This is the worst over current case recorded during the measurements when the breaker never opened.



**Figure 4.4.5:** Plot of voltage and current from station 7809 during an over current situation. The breaker was never tripped.



**Figure 4.4.6:** Plot of voltage and current from station 7809 during an over current situation. The breaker was never tripped.



**Figure 4.4.7:** Plot of voltage and current from station 7809 during an over current situation. The breaker was never tripped.

## 5 Calculation method

Calculating the exact power demand for one tram is not an easy task to do. There are many factors that play an important role and small adjustments may have large impacts on the total amount of power needed. The problem when calculating the power is that the main components – force, acceleration and speed – depend on each other. The resulting effort from the motors needed to overcome the resistance and accelerate the tram is described in Chapter 2.6 as

$$F_{acc} = ma + R_{total} \quad (5.1)$$

where the acceleration force is depending on mass, acceleration and the total resistive forces. The resistive forces is in turn depending on speed according to

$$R_{total} = (0.6wn + 20n) + bwnV + KV^2 + 20wnG \quad (5.2)$$

Equation 5.2 is described thoroughly in Chapter 2.5.5. The used calculation method approximates the power drawn from the catenary by the tram with the mechanical output power from the motors, i.e. it is assumed that the electrical system of the tram is loss-less. The mechanical output power of the motors can be calculated with (2.4) as

$$P = vF_{acc} \quad (5.3)$$

The acceleration force is set by the speed controller of the tram and the available acceleration force is also depending on the speed of the vehicle. The acceleration can be found by rewriting (5.1) as

$$a = \frac{F_{acc} - R_{total}}{m} \quad (5.4)$$

and the speed of the vehicle may be determined by integrating the acceleration

$$v = \int a \, dt \quad (5.5)$$

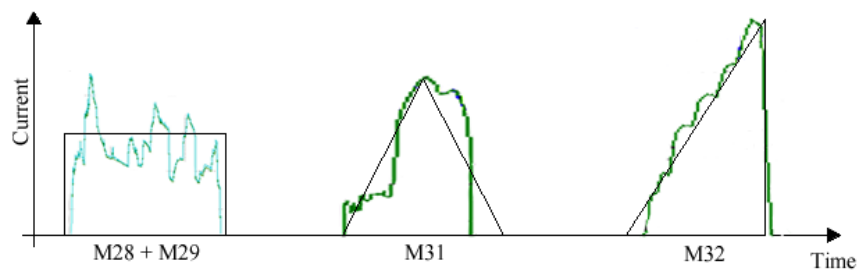
Although the three main variables in the equations depend on each other, a unique solution may be solved by numerical simulation. By assuming that the voltage of the catenary is constant, the current consumption of the tram can be determined as

$$I = \frac{P}{U} \quad (5.6)$$

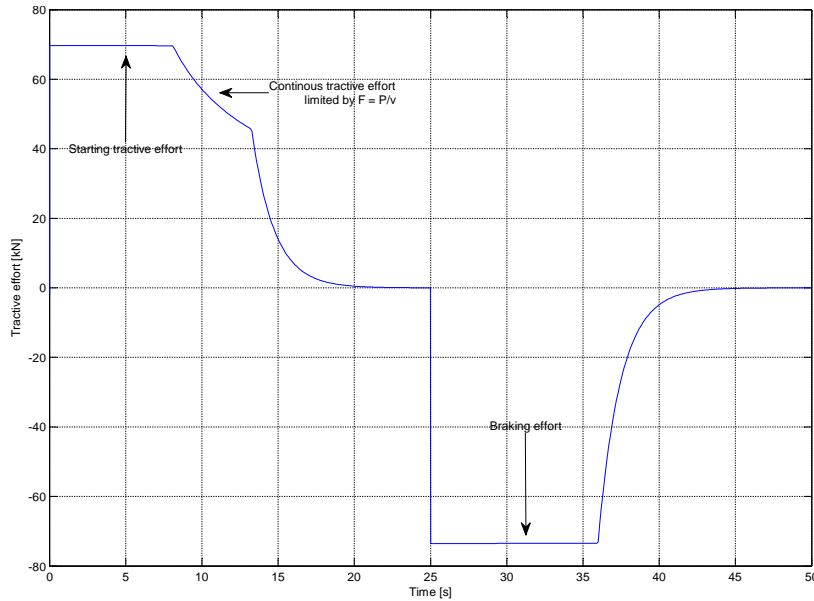
## 5.1 Tractive force, energy and current need during an acceleration

The current drawn by a tram may be divided into three different phases: acceleration, constant speed and idling. As described in Chapter 2.6, the largest energy consumption takes place during the acceleration phase and these peaks are the major reason for short term overloading of the rectifying stations. The current need during constant speed and idle phases are not unimportant by any means, but they are simpler to estimate with a fairly accurate result.

The measurements showed that the different tram types in Göteborg have slightly different current consumption curves, which originates from how the motor controller is built. The measurements have shown that the amplitude and shape of a current curve during acceleration makes it easy to see which tram type that where accelerating. Figure 5.1.1 shows a simple sketch of how the different current shapes look like during acceleration. The figure shows that a M31 tram looks like an isosceles triangle and a M32 tram is similar to a right triangle. For both these tram types, the current is linearly depending on the speed which gives a peak at the time when the acceleration is complete. For a double set of either two M28 trams, two M29 trams or one of each, the shape is estimated by a rectangle. The reason that the current shape look like a rectangle is that the resistance based control



**Figure 5.1.1:** Current shape sketches (black) for different tram types during an acceleration sequence. Coloured curves are typical recorded shapes.

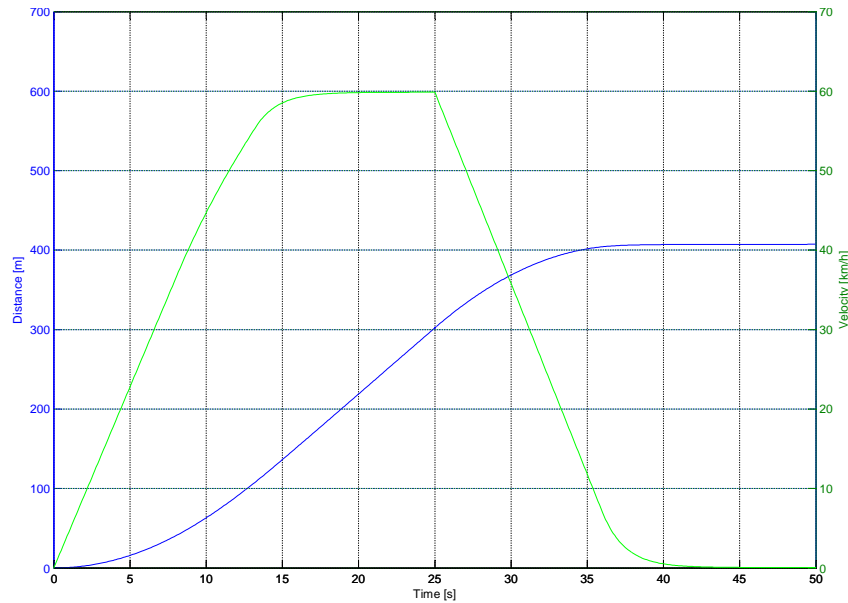


**Figure 5.1.2:** Simulation of tractive effort for a M32 tram.

system of the trams decrease the peak current and give a somewhat constant current during an acceleration.

### 5.1.1 Simulink model

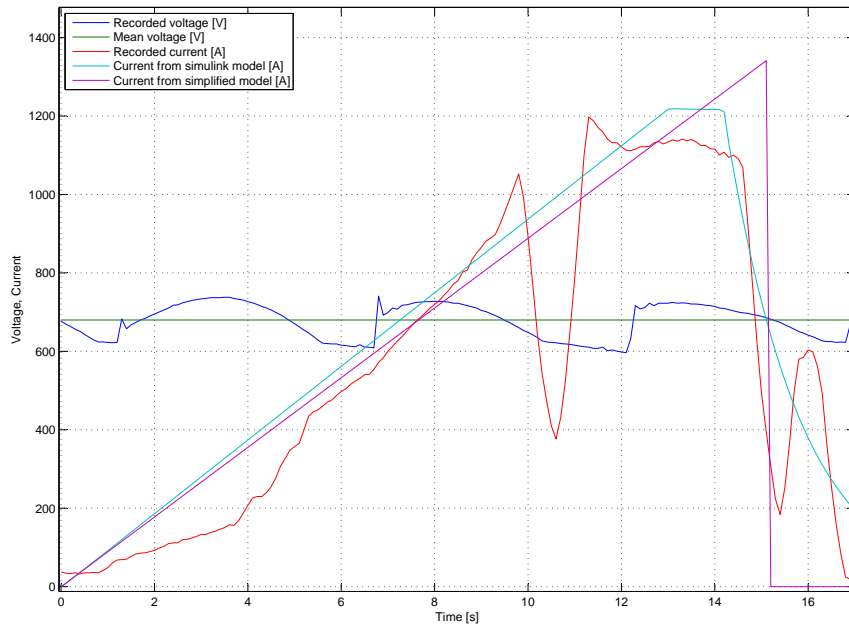
To achieve a solution which outputs force, energy, power and current, a simulation environment utilizing the described formulas was setup in MATLAB Simulink. A block diagram of the model file may be found in Appendix D. The simulation uses a speed control built on a simple PI-controller to simulate the desired tractive effort from the driver to the motors. To show the model and compare it to measurements, a simulation was run where there first is a speed-step to increase the speed from standstill to 60 km/h and after 25 seconds a counter-step is set to brake the tram to standstill. The needed inputs to the simulation is the different parts of the resistive forces and limits for maximum acceleration, maximum tractive force and maximum power. Since the tram with highest power demand is in some sense most interesting, the values for a M32 tram were inserted into the model. Figure 5.1.2 shows the tractive effort in kN during the run and Figure 5.1.3 shows speed and distance covered during the run. The result from these figures is as expected and very similar to figures from the technical specification for the M32 tram [16]. The simulation also outputs power and current curves and the total amount of energy needed during one acceleration. These figures and results are presented in Figures 5.1.4, 5.1.5 and Table 5.1.1.



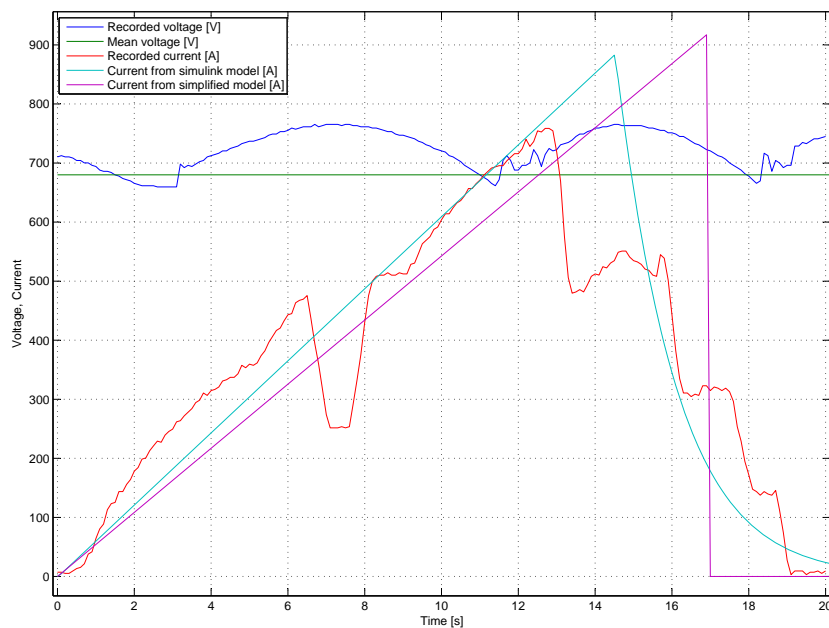
**Figure 5.1.3:** Speed and distance plot of a M32 tram simulation.

### 5.1.2 Simplified model

To be able to calculate the power and current demand on a section with several trams in movement simultaneously, it is necessary to simplify the simulink model. The simplified model is based on the same physical equations mentioned earlier in this chapter, but with some simplifications which removes the need of running the simulation with numerical iterations in computer software. The major difference is that the force is considered to be constant for the entire acceleration, which also gives a constant acceleration and linearly increasing speed. These approximations are built on the observation that the current increases almost linearly during the first part of an acceleration, see Figure 2.1.2. This leads to a somewhat higher current peak and must therefore be compensated by reducing the acceleration somewhat to compensate for the fact that the acceleration is limited by the available power during higher speed. Figure 5.1.4 shows a comparison of a recorded current peak during an acceleration with a M32 tram on a flat track and the corresponding current peak output from the Simulink model and the simplified model. The Simulink model fits the recorded data quite well, both in magnitude and shape, and the simplified model has a somewhat higher amplitude. Figure 5.1.5 shows a similar situation as in Figure 5.1.4 but the recorded data comes from a M32 tram running up a 40 ‰ slope. Both the current from the Simulink model and the simplified model have slightly higher amplitudes than the recorded data, and the shapes of the simulated curves are somewhat deviating from the recorded data.



**Figure 5.1.4:** Voltage and current plot comparing the recorded current versus the current output from the Simulink model and the simplified model. The recorded data originates from a M32 tram acceleration on a flat track.



**Figure 5.1.5:** Voltage and current plot comparing the recorded current versus the current output from the Simulink model and the simplified model. The recorded data originates from a M32 tram acceleration in a slope of about 40 %.

**Table 5.1.1:** Energy usage from measured data, Simulink model and simplified model.

Slope	Recorded data	Simulink model	Simplified model
0 ‰	6.52 MJ	6.87 MJ	6.77 MJ
40 ‰	5.48 MJ	5.27 MJ	5.30 MJ

To determine the accuracy of the two models, the total amount of energy used during the acceleration may also be compared. The energy from the recorded data is found from integrating the product of voltage and current:

$$W = \int P = \int UI \quad (5.7)$$

The energy from the Simulink model is generated automatically during the simulation and the energy from the simplified model is also found from integrating the product of voltage and current. The voltage is here estimated as the average voltage from the measurements and is equal to 680 V. The result of the comparison is found in Table 5.1.1. The table shows that both models are accurate enough and the acceleration sequence is therefore hereafter modelled with the simplified model. One may also notice that the energy use climbing the slope is lower than the energy use during the acceleration on the flat track. This is because the speed on the flat track is about 50 % higher compared to the speed during the climb.

## 5.2 Resistive forces and current need during constant speed

As trams travel in constant speed, the resistive forces from air drag and track are the ones that need to be overcome. As these forces are constant at constant speed and constant slope, they may be estimated by a constant current. The resistive forces on a flat track are relatively low in speeds up to 100 km/h but it increase fast if the slope increases. By analysing the current shapes of the recorded data, it was found that the driving cycles of the trams mostly consists of accelerations and decelerations. After an acceleration, it is common that the tram runs on only idling current since the speed barely decreases by the low resistive forces. The effect of this is that constant speed current may be approximated with only idling current. A problem with this approximation is that when a tram enter a slope, the current needed to compensate for the increased resistance will not be accounted for. This may be compensated for by raising the idling current or by introducing one extra acceleration when simulating a section with a slope.

### 5.3 Current need during idling

As trams stand still at stops and in turning loops, the systems onboard the tram draws an idle current which may be seen as constant. The measurements showed that the idle current from a set of two M28/M29 trams is about 10 A, a M31 tram draws about 30 A in idling current and a M32 tram draws about 60 A in idling current. To run a simulation with other tram types, the idle current must be measured, taken from the specification or estimated by comparing to known values. Even though the idle current may seem as a small quantity in contrast to the acceleration current, it must be taken into consideration, especially as it is not uncommon for several trams to stand at a turning loop at the same time.

### 5.4 Simulating several trams on the same section

When calculating the power demand for a section today, one is often talking about the number of trams passing the section during a specified time period. For a heavily frequented section, a tram may pass every 30 seconds during peak hours and for a less frequented section it may pass one tram every 300 seconds. This is vital information for the design of the electrical network and it is one of the most dimensioning factors. Since the service life of the electrical equipment is several decades, one must also take future tramway frequenting into consideration and provide a solution with a good margin to not limit the requested traffic on the section.

With input values such as the mix of different trams, number of passengers on each tram, number of stops on the section and tram frequenting, a simple result for power demand estimation may be found. The problem with this simple setup is that the trams are not always exactly on time. The insecurity factor is very important as the number of trams on the investigated section may be heavily over frequented due to previous stops and delays. One may either design a system which will withstand normal load but may be overloaded during peaks or one may introduce an insecurity factor to keep the load on acceptable levels at all times.

As the power demand for trams arises mostly during accelerations, it is important to know how many tram stops and traffic lights there are on the investigated section. Traffic lights often have a priority for trams and therefore there must also be a likeliness-for-stop factor multiplied with the number of traffic lights. In Göteborg, there are a total of 132 tram stops which today are frequented by an average of 2.6 tram lines per stop. The total number of sections is 74 which gives an average of 1.78 stops per section. The actual number of stops per section differs between zero and six [6]. With all this data known, it is possible to estimate how many accelerations that will occur on a section during a given time period. To-

gether with the rest of the parameters, such as passenger count, speed, slope and rolling resistance it is possible to estimate how much energy that each acceleration requires and thereby, total energy per time unit may be found. RMS and mean power may be found from the total energy, but the really interesting part is to find how often a station will be overloaded due to superimposed currents caused by simultaneous accelerations by different trams.

During all four circuit breaker openings occurring in the recorded data, the cause has been simultaneous accelerations by at least two trams. Therefore, it is of great importance to find a reliable model which calculates the risk of simultaneous starts under given conditions. To investigate how often simultaneous starts will occur, one may use different statistical distributions to see if any known model fit the recorded data. With only one tram on the section, the accelerations are by definition never simultaneous, but as soon as more than one tram appears on the section, the risk of simultaneous accelerations increases. Under normal service conditions, there may be as many as five or six trams travelling on the same section at once and after a standstill, there may be several more trams on queue.

To simulate how the load on a rectifying station is varying throughout the day, a MATLAB-script was created which simulates the total load on a section during one hour. The general inputs to the script is:

- Section data divided in up to four different subsections with different slopes and lengths.
- The number of tram stops and traffic lights and a likeliness-for-stop factor.
- The total number of trams passing the section during one hour.
- The mixture of different tram types and at which speed and acceleration rate they are run.

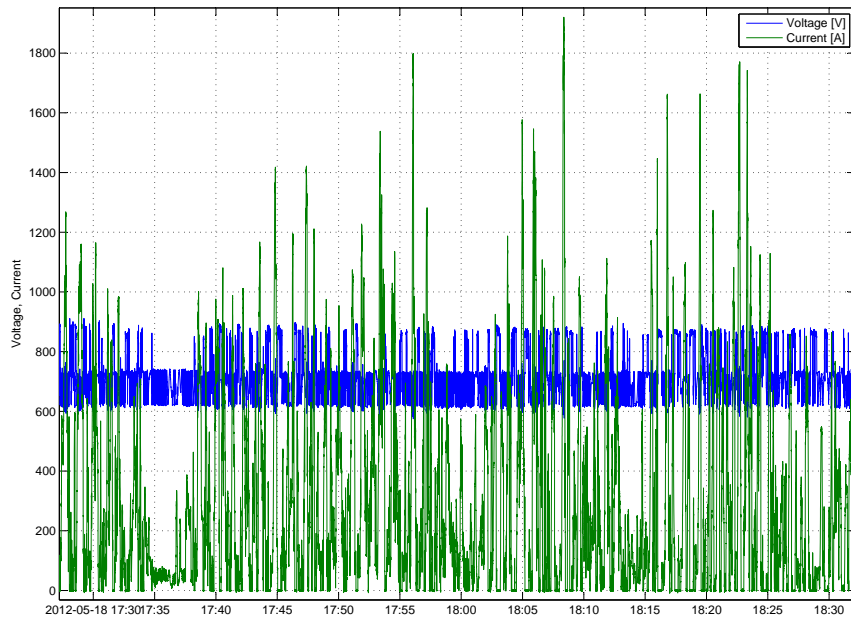
The script creates an array which contains the section profile and the different slopes on the different parts of the section. It continues by calculating the total time needed to run the total distance and the mean time between each acceleration. A normal distribution with standard deviation of 10 is then run onto the time to simulate how different drivers have different driving approaches and thereby the time between accelerations is differentiated for a more realistic setup.

An outer for-loop is used to create the calculated number of trams on the section during one hour and assign them correct properties such as tram model, total weight, acceleration rate and maximum speed. They are also given different start times which are selected randomly during the simulation run time. The reason for using a random distribution for the start times is that this often is the case as the trams seldom follows the time table entirely, and a more realistic load level of the rectifying stations is achieved.

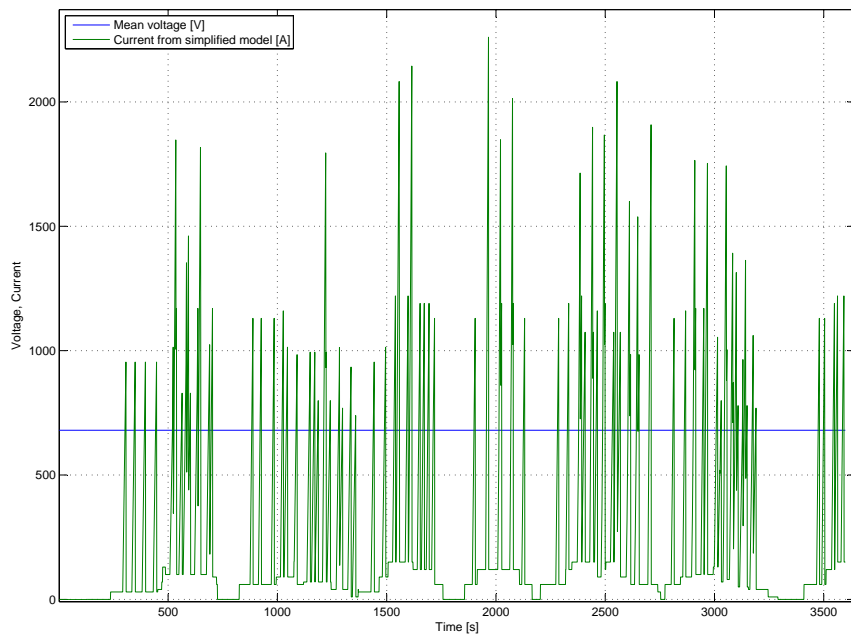
For each tram, an inner for-loop defines how each tram is run on the section, adding idling current and one acceleration sequence for each stop. All trams are simulated to stop at each tram stop, but the location of traffic related stops may be different for each tram. The acceleration current is created for each unique stop with the previously developed simplified acceleration model. Since the slope may vary on the investigated section, it is randomly selected from the slope array created earlier to simulate that traffic related stops may occur on different parts of the section.

The result of this simulation is the RMS current during one hour and a plot which shows the current peaks and the load level on the rectifying stations on the section. Figure 5.4.1 shows the measured current and voltage during one hour on the southernmost section in Mölndal. This data comes from the second measurement and the section is parallel fed, but the location of station 7809 makes it take the biggest part of the load. The RMS current during this hour is 450 A. Figure 5.4.2 shows the output from the simulation which is set to emulate the current demand in Figure 5.4.1. The current peaks is much alike the recorded data, both in frequency and amplitude and the RMS current during this hour is 458 A. Figure 5.4.3 shows a comparison between the measured and simulated data, sorted from highest to lowest. As the plot shows, the simulation follows the measured data well along the highest current peaks. As the current levels falls down to lower current levels, the simulation lies somewhat below the measured data. This may be explained by the use of the simplified model, which looks like a triangle. The comparison of RMS current between the simulated and measured currents also shows that the simulation current much resembles the measured current.

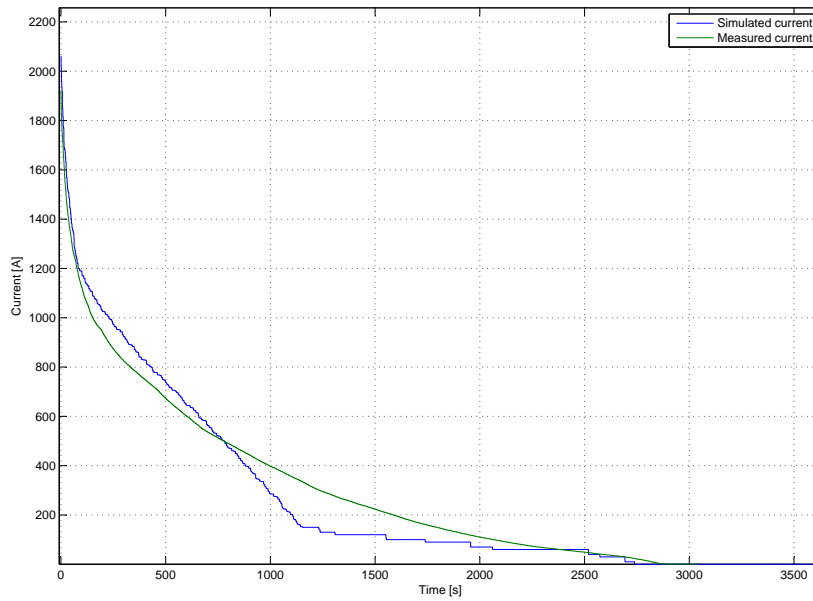
Figure 5.4.4 shows the measured voltage and total current during one hour from stations 7863 and 7858 (see Chapter 3.2.5 for more information). The RMS current during this hour is 320 A and the plot shows the total load on both stations. Figure 5.4.5 shows the simulated current situation on the same section. The current in these two plots are quite similar in both amplitude and frequency and the RMS current from the simulation is 335 A. Figure 5.4.6 shows a comparison between the measured and simulated data, sorted from highest to lowest. As in the previous case, the simulated current follows the measured current quite well down to about 500 A where the simulated current lies below the measured current curve. The RMS level for the whole hour is somewhat higher in the simulated curve than the measured. Overall, the figures in this chapter shows that the simulation software is accurate both on flat track and when introducing a gradient.



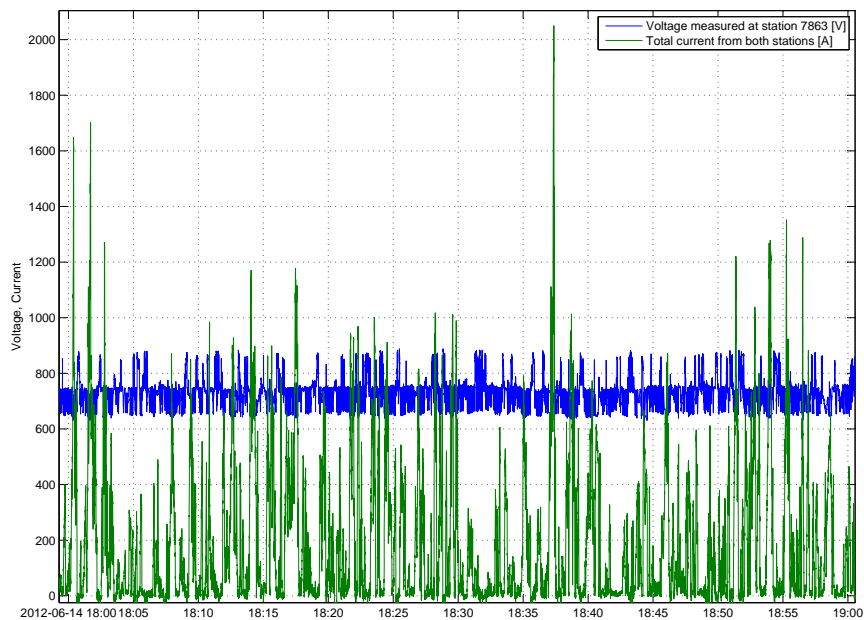
**Figure 5.4.1:** Voltage and current plot during one hour of measurements in station 7809. The RMS current is 450 A.



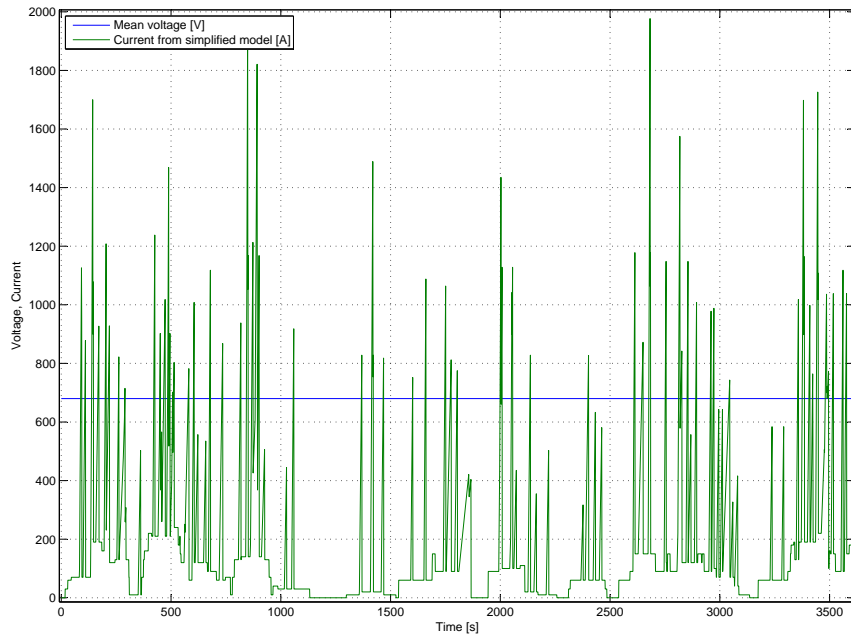
**Figure 5.4.2:** Output from the simulation script, simulating the same section as measured in Figure 5.4.1. The RMS current is 458 A.



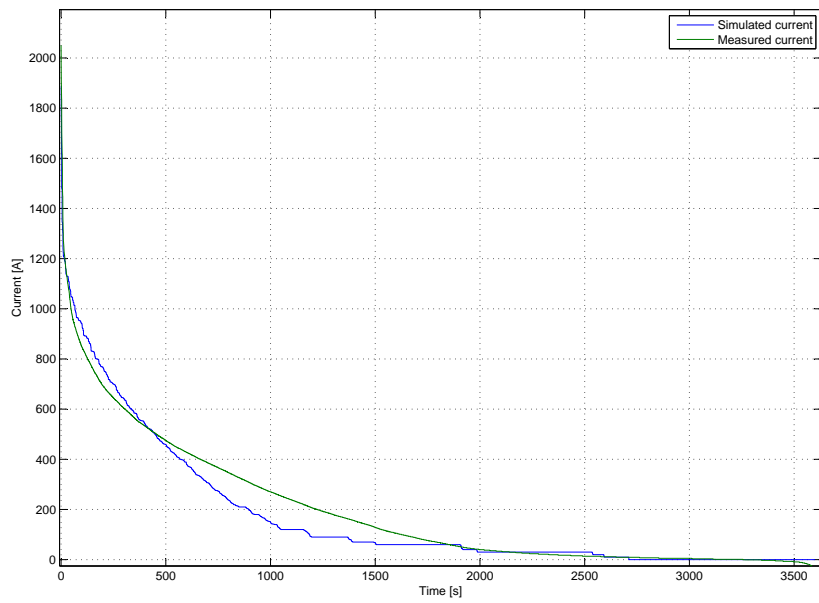
**Figure 5.4.3:** Simulated and measured current levels sorted from highest to lowest during one hour. The current levels are the same as in Figure 5.4.1 and Figure 5.4.2.



**Figure 5.4.4:** Voltage and current plot during one hour of measurements in stations 7863 and 7858. The RMS current is 320 A.



**Figure 5.4.5:** Output from the simulation script, simulating the same section as measured in Figure 5.4.4. The RMS current is 335 A.



**Figure 5.4.6:** Simulated and measured current levels sorted from highest to lowest during one hour. The current levels are the same as in Figure 5.4.4 and Figure 5.4.5.

## 5.5 Software for tramway current demand calculations

One of the objectives of this thesis was to create a user-friendly, simple-to-use software for calculating the load of rectifying stations and risk of overloading them under certain conditions. This software has been coded in PHP and HTML because of the easiness to create a good graphical user interface and store data in a database. The software is actually only a interface making it easier to utilize the Simplified current model and the MATLAB-script described in the previous chapter. The name of the software is BAMSE which is short for "Beräkning, Analys och Simulering av Spårvagnars Effektbehov" or "Calculation, Analysis and Simulation of Trams need for Power" translated into English.

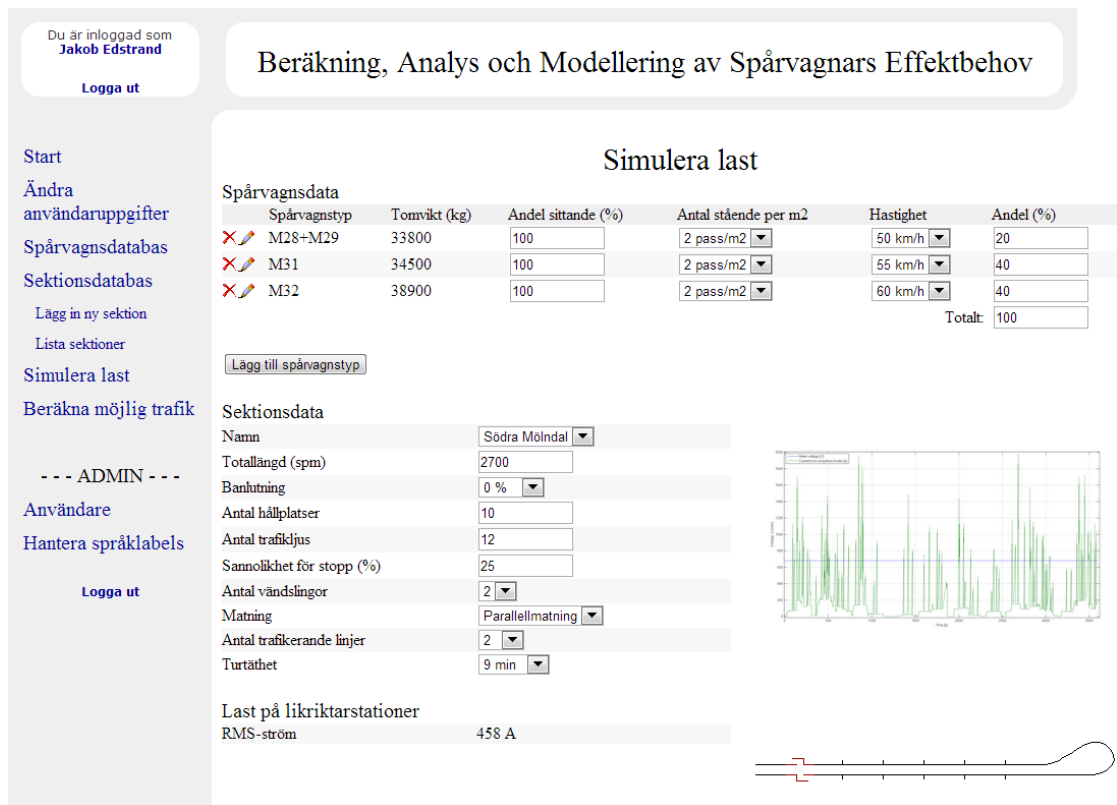
The software consists of:

- One page for inserting and modifying tram data such as length, weight, passenger capacity and idle current which is saved into the database.
- One page for inserting and modifying tramway section data such as track length, single or parallel feed, number of stops and traffic lights and it is saved into the database.
- One page for calculating the electrical load by combining a mix of tram types frequenting a section with a given speed, frequency and number of passengers.

A screen shot of the software is shown in Figure 5.5.1. This figure shows the calculation page where the user inputs tram data such as different tram types, their loading and maximum speed on the section and which mix of trams that is simulated. The user then chooses which section to investigate, which will load from the database, and changes data if necessary. The output is a current and voltage plot similar to Figure 5.4.2 and the calculated RMS-value for a one hour period.

To interpret the output, one must have good knowledge of how the relay protection is set up in the rectifying stations feeding the investigated section. A higher RMS value means higher risk of overloading the rectifying stations. From the measurements, there has been no definite relation between RMS-value and amplitude of current peaks. Therefore, one must also analyse the current plot output from the simulation to see how frequent current peaks over a set limit occurs. This will give a hint to the risk of over current situations occurring in the rectifying stations.

A typical rectifying station in Göteborg has the "normal" over current limit set to 1500 A and the instant over current limit set to 2500 A. The thermal limit



**Figure 5.5.1:** Screen shot of the interface for the simplified model

is depending on cable length, but mostly it is set around 1000 A for about one minute. The RMS current that a rectifying station may handle without opening the circuit breaker should always be lower than the thermal limit in the station. The measurements in Göteborg has however shown that the normal over current limit is the most common reason for circuit breaker opening. In all four cases of recorded circuit breaker openings (see Chapter 4.4) the reason for opening the circuit breaker was normal over current caused by two M32 trams accelerating within 10 seconds of each other. If there would be a current limiter installed on board the trams, these problems would probably decrease.

## 6 Conclusion

This thesis has focused on creating a simplified calculation method for powering a tramway network and verifying that the method produces useful and accurate results by comparing the simulation output with measurement data from real rectifying stations. The result of this work is a software that utilize the developed calculation method to fast and easy determine if a tramway section is correctly dimensioned for the tram traffic or if the electrical equipment risks to be overloaded.

The basis in the created software is a theoretical model of the energy need of an accelerating tram and the current shape that the rectifying station is subjected to. The tractive and resistive efforts which form the basis of the theoretical model have been derived from well established physical theorems and reliable formulas from the train industry, which are proven to give reliable results.

The two models of a tram acceleration has proven to match well with the recorded data. Both the Simulink model and the simplified model have given results which are satisfactory in both current amplitude and total energy during the acceleration. Both models deviate 4-5 % in total energy during an acceleration and under 10 % in maximum current amplitude, compared to measurements.

The results from the section simulations has shown to emulate the recorded data well. The results from the simulated sections has shown that the simplified model works well for simulating the superimposed currents from several trams travelling simultaneously on the same section. RMS currents from these simulations has shown only 3-5 % deviations from a sample hour from the measured data. The comparison between the measured and simulated currents has shown that the used model well matches the measured data, both on flat track and when simulating a slope.

The measurements has shown that one may not conclude that two rectifying stations parallel feeding a tramway section always split the load evenly. This could potentially be a problem and must be considered when new rectifying stations are being constructed.

The measurements performed in this thesis has recorded four circuit breaker trips due to temporary overload in the rectifying stations. All four of these circuit breaker trips were due to two or more M32 trams starting on the same section within a few seconds. The developed software outputs a plot showing the load level on the rectifying stations and the risk of simultaneous accelerations leading to overload situations. With experience and knowledge about the relay protection

settings in the rectifying stations, the output from the developed software may be used to foresee the risk of short term overload in the station.

The analysis of the fault reports from the rectifying stations in Göteborg has shown that some rectifying stations are very high loaded and short term overload in these stations are common. The network of rectifying stations for the tramway system in Göteborg must be expanded to cope with the increasing number of passengers and new trams with higher current demand.

## 6.1 Future work

This thesis has investigated how contemporary and future problems with power supplies in tramway sections may be detected and fixed in advance. The developed software will come to great use in future consulting assignments where new tracks are built, current tram frequenting is increased or the speed is raised. It will also be helpful for calculating possibilities of temporary traffic re-routing during maintenance work etc. The software has also much potential when it comes to increased functionality and improvement of the mathematical expressions. One such function could be to include short circuit calculations to find a maximum allowable length of a catenary section that a rectifying station may serve without risking that a circuit breaker fails to detect a short circuit.

The result from the simulation run in the software is not easy to interpret for a person who is unfamiliar with how the relay protection is set up in the rectifying stations. To further help the user analyse the result from the simulation, the program could be expanded to include the same calculations as the relay protection in the rectifying station performs. By including calculations for thermal, momentary and short term overload current, the result of the simulation would be interpretable for more users.

This thesis has not taken into consideration that some tram types are able to feed back braking energy into the catenary which may be used to propel other trams on the same section. It would be interesting to investigate how much of this braking energy that actually is used for powering other trams and if it would have any effect to install rectifying stations with some kind of energy storage units.

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# Appendix A: NI-6008 datasheet

## Low-Cost, Bus-Powered Multifunction DAQ for USB – 12- or 14-Bit, up to 48 kS/s, 8 Analog Inputs

### Specifications

Typical at 25 °C unless otherwise noted.

#### Analog Input

##### Absolute accuracy, single-ended

Range	Typical at 25 °C (mV)	Maximum (0 to 55 °C) (mV)
±10	14.7	138

##### Absolute accuracy at full scale, differential<sup>1</sup>

Range	Typical at 25 °C (mV)	Maximum (0 to 55 °C) (mV)
±20	14.7	138
±10	7.73	84.8
±5	4.28	58.4
±4	3.59	53.1
±2.5	2.56	45.1
±2	2.21	42.5
±1.25	1.70	38.9
±1	1.53	37.5

Number of channels..... 8 single-ended/4 differential  
Type of ADC..... Successive approximation

##### ADC resolution (bits)

Module	Differential	Single-Ended
USB-6008	12	11
USB-6009	14	13

##### Maximum sampling rate (system dependent)

Module	Maximum Sampling Rate (kS/s)
USB-6008	10
USB-6009	48

Input range, single-ended..... ±10 V  
Input range, differential..... ±20, ±10, ±5, ±4, ±2.5, ±2, ±1.25, ±1 V  
Maximum working voltage..... ±10 V  
Overvoltage protection..... ±35 V  
FIFO buffer size..... 512 B  
Timing resolution..... 41.67 ns (24 MHz timebase)  
Timing accuracy..... 100 ppm of actual sample rate  
Input impedance..... 144 kΩ  
Trigger source..... Software or external digital trigger  
System noise..... 5 m V<sub>rms</sub> (±10 V range)

#### Analog Output

Absolute accuracy (no load)..... 7 mV typical, 36.4 mV maximum at full scale  
Number of channels..... 2  
Type of DAC..... Successive approximation  
DAC resolution..... 12 bits  
Maximum update rate..... 150 Hz, software-timed

Output range..... 0 to +5 V  
Output impedance..... 50 Ω  
Output current drive..... 5 mA  
Power-on state..... 0 V  
Slew rate..... 1 V/μs  
Short-circuit current..... 50 mA

#### Digital I/O

Number of channels..... 12 total  
8 (P0.<0..7>)  
4 (P1.<0..3>)  
Direction control..... Each channel individually programmable as input or output  
Output driver type  
USB-6008..... Open-drain  
USB-6009..... Each channel individually programmable as push-pull or open-drain  
Compatibility..... CMOS, TTL, LVTTL  
Internal pull-up resistor..... 4.7 kΩ to +5 V  
Power-on state..... Input (high impedance)  
Absolute maximum voltage range..... -0.5 to +5.8 V

#### Digital logic levels

Level	Min	Max	Units
Input low voltage	0.3	0.8	V
Input high voltage	2.0	5.8	V
Input leakage current	—	50	μA
Output low voltage (I = 8.5 mA)	—	0.8	V
Output high voltage (push-pull, I = 8.5 mA)	2.0	3.5	V
Output high voltage (open-drain, I = 0.6 mA, nominal)	2.0	5.0	V
Output high voltage (open-drain, I = 8.5 mA, with external pull-up resistor)	2.0	—	V

#### Counter

Number of counters..... 1  
Resolution..... 32 bits  
Counter measurements..... Edge counting (falling edge)  
Pull-up resistor..... 4.7 kΩ to 5 V  
Maximum input frequency..... 5 MHz  
Minimum high pulse width..... 100 ns  
Minimum low pulse width..... 100 ns  
Input high voltage..... 2.0 V  
Input low voltage..... 0.8 V

#### Power available at I/O connector

+5 V output (200 mA maximum)..... +5 V typical  
+4.85 V minimum  
+2.5 V output (1 mA maximum)..... +2.5 V typical  
+2.5 V output accuracy..... 0.25% max  
Voltage reference temperature drift... 50 ppm/°C max

<sup>1</sup>Input voltages may not exceed the working voltage range.

BUY ONLINE at [ni.com](http://ni.com) or CALL 800 813 3693 (U.S.)

Low-Cost, Bus-Powered Multifunction DAQ for USB – 12- or 14-Bit, up to 48 kS/s, 8 Analog Inputs

Physical Characteristics

If you need to clean the module, wipe it with a dry towel.

Dimensions (without connectors) .....	6.35 by 8.51 by 2.31 cm (2.50 by 3.35 by 0.91 in.)
Dimensions (with connectors) .....	8.18 by 8.51 by 2.31 cm (3.22 by 3.35 by 0.91 in.)
Weight (without connectors) .....	59 g (2.1 oz)
Weight (with connectors) .....	84 g (3 oz)
I/O connectors .....	USB series B receptacle (2) 16-position (screw-terminal) plug headers
Screw-terminal wiring .....	16 to 28 AWG
Screw-terminal torque .....	0.22 to 0.25 N•m (2.0 to 2.2 lb•in.)

Power Requirement

USB (4.10 to 5.25 VDC) .....	80 mA typical 500 mA maximum
USB suspend .....	300 µA typical 500 µA maximum

Environmental

The USB-6008 and USB-6009 are intended for indoor use only.

Operating environment	
Ambient temperature range .....	0 to 55 °C (tested in accordance with IEC-60068-2-1 and IEC-60068-2-2)
Relative humidity range .....	10 to 90%, noncondensing (tested in accordance with IEC-60068-2-56)
Storage environment	
Ambient temperature range .....	-40 to 85 °C (tested in accordance with IEC-60068-2-1 and IEC-60068-2-2)
Relative humidity range .....	5 to 90%, noncondensing (tested in accordance with IEC-60068-2-56)
Maximum altitude .....	2,000 m (at 25 °C ambient temperature)
Pollution degree .....	2

Safety and Compliance

Safety

This product is designed to meet the requirements of the following standards of safety for electrical equipment for measurement, control, and laboratory use:

- IEC 61010-1, EN 61010-1
- UL 61010-1, CSA 61010-1

**Note:** For UL and other safety certifications, refer to the product label or visit [ni.com/certification](http://ni.com/certification), search by model number or product line, and click the appropriate link in the Certification column.

Electromagnetic Compatibility

This product is designed to meet the requirements of the following standards of EMC for electrical equipment for measurement, control, and laboratory use:

- EN 61326 EMC requirements; Minimum Immunity
- EN 55011 Emissions; Group 1, Class A
- CE, C-Tick, ICES, and FCC Part 15 Emissions; Class A

**Note:** For EMC compliance, operate this device according to product documentation.

CE Compliance

This product meets the essential requirements of applicable European Directives, as amended for CE marking, as follows:

- 2006/95/EC; Low-Voltage Directive (safety)
- 2004/108/EC; Electromagnetic Compatibility Directive (EMC)

**Note:** Refer to the Declaration of Conformity (DoC) for this product for any additional regulatory compliance information. To obtain the DoC for this product, visit [ni.com/certification](http://ni.com/certification), search by model number or product line, and click the appropriate link in the Certification column.

Waste Electrical and Electronic Equipment (WEEE)

**EU Customers:** At the end of their life cycle, all products must be sent to a WEEE recycling center. For more information about WEEE recycling centers and National Instruments WEEE initiatives, visit [ni.com/environment/weee.htm](http://ni.com/environment/weee.htm).

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compliance, go to [sl.solutions.ni.com/doc/rohs\\_china](http://sl.solutions.ni.com/doc/rohs_china).)

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# Appendix B: AP032 datasheet

## II. Specifications

### GENERAL CHARACTERISTICS

	AP031	AP032
Bandwidth	25 MHz	
Rise Time	14 ns	
Attenuation	1:10 / 1:100	1:20 / 1:200
Atten. Accuracy	±2 %	
Input Resistance	4 M $\Phi$	
Input Capacitance	10 pF each side to ground	
Input Configuration	Differential	
Input Voltage:		
Max. Differential	<b>1:100 Range</b> $\pm 700$ V ( DC + peak AC ) or 500 V r.m.s.	<b>1:200 Range</b> $\pm 1400$ V ( DC + peak AC ) or 1000 V r.m.s.
	<b>1:10 Range</b> $\pm 70$ V ( DC + peak AC ) or 50Vrms	<b>1:20 Range</b> $\pm 700$ V ( DC + peak AC ) or 100Vrms
Max. Common Mode	$\pm 700$ V ( DC + peak AC ) or 500 V r.m.s.	$\pm 1400$ V ( DC + peak AC ) or 1000 V r.m.s.
Max. Absolute	$\pm 1400$ V ( DC + peak AC ) or 1000 V r.m.s.	
CMRR:		
50 Hz	- 86 dB	- 80 dB
20 kHz	- 66 dB	- 60 dB
200 kHz	- 56 dB	- 50 dB
Output Offset (Typical)	<±5 mV	
Output Noise (Typical)	1.5 to 2 mV typical	



# Appendix C: PR2000 datasheet



## Current Probe Model PR2000

The PR2000 current probe is based on Hall Effect technology for use in measurement of both DC and AC current. The PR2000 may be used in conjunction with multimeters, recorders and other suitable equipment for accurate non intrusive current measurement.



### Electrical Characteristics

Current Range .....	2000 A AC <sub>RMS</sub> or DC
Measuring Range .....	± 2800 A
Output Sensitivity .....	1 mV/A
Accuracy (at +25°C) .....	± 1% of reading ± 500 mA
Resolution .....	± 100 mA
Load Impedance .....	> 10 k Ohms and ≤ 100 pF
Conductor Position Sensitivity .....	± 1.5% relative to centre reading
Frequency Range..... (small signal) .....	DC to 10 kHz (- 1 dB)
Temperature Coefficient .....	± 0.1% of reading / °C
Power Supply .....	9 V Alkaline, MN1604/PP3
.....	75 Hours, low battery indicator
Working Voltage (see Safety Standards section) .....	600 V AC <sub>RMS</sub> or DC

### General Characteristics

Maximum Conductor Size .....	50 mm diameter
Output Connection .....	4mm safety plugs
Output Zero .....	Manual adjust via thumbwheel
Cable Length .....	1.5 meters
Operating Temperature Range .....	0 to +50 °C
Storage Temperature Range (with battery removed) .....	-20 to +85 °C
Operating Humidity .....	15% to 85% (non condensing)
Weight .....	570 g

LEM Probes

Website : [www.lem.com](http://www.lem.com)  
Email : [probe@lem.com](mailto:probe@lem.com)

### Safety Standards

BSEN61010-1: 1993 and Amendment A2: July 1995  
BSEN61010-2-032: 1995  
BSEN61010-2-031: 1995

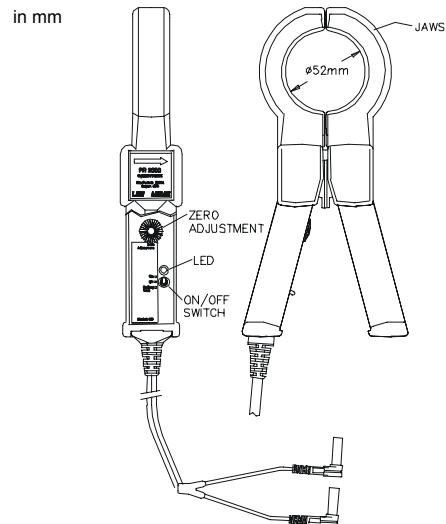
600 V<sub>RMS</sub>, Category III, Pollution Degree 2

Use of the probe on **uninsulated conductors** is limited to 600 V AC<sub>RMS</sub> or DC and frequencies below 1 kHz.

### EMC Standards

EN61326 :1998

### Dimensions



PR2000\_DS\_E\_010703\_1

Specifications subject to change without notice

LEM Probes

Website : [www.lem.com](http://www.lem.com)  
Email : [probe@lem.com](mailto:probe@lem.com)

# Appendix D: Simulink model

