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Balancing Braking Performance and Lateral Traction on Soft Ground in Off-Road Heavy Vehicles

Degree project report in Systems, control and mechatronics, MSc

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Heavy Vehicles
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Abstract

Off-road heavy vehicles operate in various terrains and industries such as agriculture, mining and on construction sites. The widely used braking system ABS is designed for paved road driving and experiments testing it in off-road scenarios have resulted in longer braking distances compared to braking with ABS disabled. Research in off-road conditions is mainly focused on traction control and lateral motion rather than braking.

A physics based tire model has been implemented and analyzed along with an automatic braking controller tailored for off-road conditions. The model describes the acquired forces from a tire when it sinks into soft terrain and includes parameters with values specific for different terrain types such as clay and sand. According to this model, the braking force is monotonically increasing as braking slip increases, thus finding the maximum braking force when the tire is locked and the slip value is -100% . This differs from paved road driving which is why ABS aims to regulate slip between -30% and -10% . As a slip of -100% significantly sacrifices the ability to steer the vehicle in the lateral direction, the objective of the project is to brake the vehicle with the greatest possible force while not losing more than 40% of lateral traction relative to its peak value.

An $\mathcal{H}_\infty$ controller has been designed to brake the vehicle by tracking a desired slip value. Test cases were simulated where the vehicle was decelerated from 30 km/h in various conditions such as varying terrain type and load. Based on the weights in the $\mathcal{H}_\infty$ configuration, decoupled PI-controllers were tuned to compare the centralized $\mathcal{H}_\infty$ controller against individual slip controllers acting on each tire axis. It was found that the centralized controller exhibits superior slip tracking performance in most test cases compared to the decoupled controllers. The $\mathcal{H}_\infty$ controller can effectively track the slip value it was linearized around, namely -24% for front tires and -20% for rear tires. It was able to track this slip reference on different terrain types, including terrain shifts during the braking action, while also being heavily laden with $20 \cdot 10^3\text{ kg}$. In addition, slip tracking was also successful when the slip reference was set to -90% for all tires which reduced the braking distance to approximately 9 m in comparison to roughly 12 m , depending on terrain type and extra load.

The implemented tire model gives insight about why ABS can be ineffective on soft terrain as it regulates slip around -20% where longitudinal braking force is sub-optimal. With slip references of around -24% for front tires and -20% for rear tires, the $\mathcal{H}_\infty$ does not bring obvious advantages over the ABS. However, the fact that the developed controller showed effective slip tracking performance in various

scenarios, including the more aggressive slip reference of -90% , enables the possibility to expand the $\mathcal{H}_\infty$ controller to control more dimensions. Further research is encouraged to include lateral dynamics into the control configuration such that a tradeoff between trying to maximize longitudinal braking force while keeping lateral traction is balanced by weights in the $\mathcal{H}_\infty$ control configuration.

Keywords: tire model, off-road, braking system, robust control, $\mathcal{H}_\infty$, autonomous driving.

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List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

ABS	Anti-lock Braking System
BW	Bekker Wong, referring to the tire model they have influenced
LFT	Linear Fractional Transformation
LTI	Linear Time Invariant system
PI	Proportional–integral controller
PID	Proportional–integral–derivative controller

Nomenclature

Below is the nomenclature of indices, sets, parameters, and variables that have been used throughout this thesis.

Indices

0	Linearization point, exception: z_0, q_0 .
k	Generic integer index for vector elements and discrete-time indexing.
i	Generic tire axle.
f	Front tire axle.
$r1$	Second tire axle. Leading tire axle of the two rear tire axles.
$r2$	Third tire axle. Trailing tire axle of the two rear tire axles.
r	Refers to both axle $r1$ and $r2$ in cases where their dynamics are equal. Only the terrain type under these tire axles distinguishes their dynamics.
x	Longitudinal direction.
y	Lateral direction.
z	Vertical direction.
$\max$	Maximum value.
$\min$	Minimum value.

Sets

$\mathcal{A}$	Set of tire axle indices.
$\mathcal{B}$	Set of slip angles.
$\mathcal{W}_f$	Set of increasing vertical loads for the front tires.
$\mathcal{W}_r$	Set of decreasing vertical loads for the rear tires.

Parameters

b	Tire width.
c	Terrain cohesion.
g	Gravitational acceleration.
h	Vertical distance from terrain surface to vehicle center of mass.
J	Tire moment of inertia around the tire's axis of rotation.
k_c	Pressure-sinkage parameter.
k_ϕ	Pressure-sinkage parameter.
k_x, k_y	Shear deformation modulus.
L_i	Longitudinal distance from vehicle center of mass to tire axle i .
L	Loss function for tire model calibration.
m	Tire mass.
M	Vehicle mass.
n	Pressure-sinkage parameter.
q	Terrain parameter vector.
q_m	Terrain parameter vector subject to the multipass effect.
q_s	Terrain parameter vector not subject to the multipass effect.
q_e	Expanded terrain parameter vector including p_m and p_s .
q_0	Initial guess of terrain parameter values for tire model calibration.
r	Tire radius.
S	Signal selection matrix in the control configuration.
w_L	Weight used in tire model calibration.
ϕ	Angle of internal friction.
ζ	Damping factor.
ρ	Terrain density.
φ	Ground swell angle in bulldozing resistance.

Variables

a	Longitudinal acceleration.
a_a	Achievable longitudinal deceleration.
d	Longitudinal distance.
F_b	Total bulldozing resistance.

F_i	Resulting force in direction i .
F_t	Tractive longitudinal force.
j	Imaginary unit, $j^2 = -1$.
j_i	Shear displacement in direction i .
R_b	Bulldozing resistance at certain contact point.
R_c	Compaction resistance.
r_κ	Slip reference.
$\tilde{r}_\kappa$	Slip reference after filtering.
s	Complex Laplace frequency.
T	Torque.
u	Control input signal.
$\tilde{u}$	Control input signal used for nonlinear plant after potential termination of braking system.
u_Δ	Model uncertainty term, output from Δ .
v_i	Translational velocity in direction i .
v_j	Slip velocity.
v	Longitudinal velocity is denoted in this way in the results section. Otherwise it is denoted as v_x .
w	Exogenous input.
w_s	Decision weights for simplified tire model.
W_i	Vertical load applied to tire axle i .
X_c	Destructive angle in bulldozing resistance.
x	State vector.
y_Δ	Model uncertainty term, input to Δ .
z_0	Sinkage from terrain surface to bottom of tire.
z	Performance output.
z_e	Slip error performance output.
z_u	Input control signal performance output.
α	Tire angular position.
α'	Angle of bulldozing blade.
β	Slip angle.
γ	Upper limit of $\mathcal{H}_\infty$ norm.
η	Generic input signal to the feedback controller in $\mathcal{H}_\infty$ configuration.
θ	Angle of a contact point between tire and terrain.
θ_f	Entry angle.

θ_m	Angle where normal stress is maximized.
θ_r	Exit angle.
κ	Longitudinal slip.
$\tilde{\kappa}$	Longitudinal slip subject to model uncertainty.
λ	Eigenvalues.
σ	Singular value.
σ_n	Normal stress.
τ	Shear stress.
ω	Tire angular velocity.
Ω	Imaginary part of the complex Laplace frequency.

Transfer functions

G_{nom}	Model of vehicle driving on nominal terrain.
G_{unc}	Uncertain model of vehicle driving on uncertain terrain.
K	Generic feedback controller.
K_{PI}	PI feedback controller.
N	Nominal closed loop system.
P	Generalized plant.
W_e	Scalar slip error weight.
W_E	Expansion of W_e to all tire axles.
W_r	Scalar reference signal filter.
W_R	Expansion of W_r to all tire axles.
W_u	Scalar control signal weight.
W_U	Expansion of W_u to all tire axles.
Δ	Uncertainty block.

Notations

$\dot{x}$	Time derivative of a variable x .
$\delta \cdot$	Prefix for the succeeding variable denoting that it is considered in the error coordinate frame.

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1

Introduction

1.1 Background

Off-road heavy vehicles operate in various terrains throughout different industries such as mining, agriculture and construction sites. These terrains have characteristics different to paved roads while also being mutually diverse. In contrast to paved roads, the interaction between tires and off-road terrain differ in terms of resistance and traction as tires may sink into the ground [1]. As a result, braking in these scenarios exhibit different dynamics compared to braking on paved roads, where the latter has been extensively studied in literature [1, 2, 3] while the former has not.

The motivation for exploring braking control is increased by the progress being made in the autonomous driving sector. As the need for robust braking performance is not restricted to autonomous vehicles, reliable braking performance is critical for predictable vehicle behavior. Some benefits of autonomous heavy vehicles have been presented for the mining industry where personnel operate in hazardous environments with life threatening risks such as tunnel collapses and landslides. They also face gradual health issues that may develop over time with examples such as respiratory and skin diseases following long term exposure to dust and chemicals. Autonomous vehicles is a promising solution for these challenges as reported in case studies from mining sites where productivity increased by 18 % and 20 % respectively as well as a decrease of safety incidents of 40 % and 35 % from different sites following decrease in human error [4].

In order to understand braking performance in different terrains, Anti-lock Braking Systems (ABS) provides a fundamental framework as it conceptualizes the relationship between slip and braking force while also being extensively used in the automotive industry. ABS has resulted in several benefits with regards to braking performance when included in cars driving on certain surfaces [5]. Specifically, the benefits with ABS include reduction of braking distance, improved stability, steering and average deceleration in braking maneuvers when driving on ice such as lane change, as well as improved lateral stability and ability to stay within the lane when braking in curves. When the braking torque applied to a wheel is great enough, the wheel locks and slides on the road. Slip describes the relationship between the angular velocity of the tire and its translational velocity. In case of wheel lock at high braking torque, the braking slip value reaches an extreme value of -100% , which is outside the range where maximum braking effect can be generated, which is between -30% and -10% for paved roads. The ABS therefore aims at keeping

the slip in this interval during full braking effort [1, 6].

1.2 Purpose

While braking systems on paved roads such as ABS are well established, research regarding development of braking systems tailored for off-road conditions is currently lacking. Studies focusing on off-road conditions address longitudinal traction control [7, 8] as well as lateral motion control [9, 10, 11] where less attention has been given to braking performance. Experiments have, however, been conducted to test the efficiency of ABS in off-road environments. A study [12] tested the braking performance of nine vehicles on different surfaces doing various maneuvers such as turning and driving straight ahead. Stopping with ABS enabled resulted in the shortest stopping distance on most surfaces for most vehicles compared to braking without ABS. When driving on a straight line, the stopping distance was reduced by 6.9 % on average for all surfaces with ABS enabled. The stopping distance on loose gravel, however, was the only exception to this trend as all nine test vehicles had a longer braking distance with ABS enabled compared to tests when ABS was disabled. Specifically, enabling ABS when having fully laden vehicles on loose gravel resulted in 24.6 % longer braking distance when the vehicles were fully laden and 30.0 % when they were lightly laden. Another study also reported longer braking distances with ABS enabled in certain terrain compared to the case when ABS was disabled [13]. This phenomenon was experienced in terrains where material is built up in front of a locked wheel. Locked wheels may cause more material to assemble in front of the tire compared to a tire regulated by ABS with slippage around -20% . Locked wheels may also dig further into deformable terrain compared to rolling wheels [14], which may also be a reason for shorter braking distances without ABS as the tire may encounter greater terrain resistance as it digs deeper into soft terrain.

This motivates investigation of a reliable braking system able to ensure stability and robust vehicle behavior in off-road terrain. The purpose of this thesis is therefore to investigate how braking systems can be designed to minimize braking distance while also minimizing motion in lateral and yaw dimensions as losing lateral control may cause safety issues. Due to the complexity of such a system and the current state of the research in the field, this thesis focuses on longitudinal braking.

1.3 Objective

The research questions to be explored are as follows:

- How can off-road heavy duty vehicle dynamics be accurately modelled for a subset of surface types while being simple enough to allow real time computation and synthesis of model based controllers? Depending on real world measurement data, how to calibrate off-road vehicle models for proper simulation environment, especially tyre models?
- How can a model based controller for autonomously braking the vehicle be designed from the developed model, and how do different design choices affect

stability, robustness and braking performance in off-road conditions? How to select braking and other driving performance weightings for robust controller design (off-road handling quality models)? Can we select performance penalties to ensure proper closed loop braking performance?

- To what extent can the developed controller maintain stable and robust braking performance and suppress wheel slip deviations while having some uncertain parameters in the model as well as other time varying parameters such as friction coefficient and load?
- Run a multitude of simulated off-road scenario and analyze closed loop braking performance characteristics. Can we give robust performance guarantees on braking distance? How tight/loose are those bounds?
- If time permits, an optional route to explore is if a controller can be developed for braking the vehicle like described above while also suppressing motion deviations in lateral and yaw dimensions.

1.4 Limitations

As models for tire dynamics in deformable terrain are established [1, 15], the modelling part of the project will mainly ensure a valid implementation of these models alongside presenting them in a structured manner with minor analysis. The tire models differ, however, based on e.g. the inflation pressure of the tire, splitting the models in two modes of operation. This thesis is restricted to handle the rigid tire mode where the rim of the tire remains circular [1]. The torque applied to the wheels are a central component in these models. Available measured vehicle data include torque measurements from the transmission line which are subject to losses before torque is transmitted to the wheels. As these losses may require thorough investigation, the developed tire models will be validated against simulated data. This method is supported by related work which have used the physics simulation software Project Chrono [7, 9].

Although longitudinal motion is the main research topic, some consideration needs to be done with regards to lateral motion to have practical relevance. In order to keep a focused scope, some dynamics are neglected from the model. These include suspension between vehicle and wheels which are omitted in favour of a rigid attachment. This likely alters the vertical load applied to a tire compared to a suspended tire. On deformable terrain, this will probably model the depth at which a tire sinks into the ground in a less realistic manner, which probably alters the resulting forces. Further simplifications are made in the braking system where ideal actuators are assumed with ability to realize the desired torque without delay. In reality, these have inertia which prevents immediate response. In this thesis, this is therefore compensated for, to some extent, by slower changes in the desired braking torque.

The performance of the controller is to be evaluated based on its ability to control the derived vehicle model. Testing the braking system with hardware is therefore not to be done. A single braking system is desired for different off-road terrain conditions such as loam and clay. It is, however, necessary to restrict the number

of terrain types to keep the scope feasible. The model and corresponding braking system is therefore going to be designed for a subset of terrain types. A multitude of simulations will be performed to analyze the performance of the braking system in various scenarios. These include laden and unladen vehicle along with varying terrain types.

2

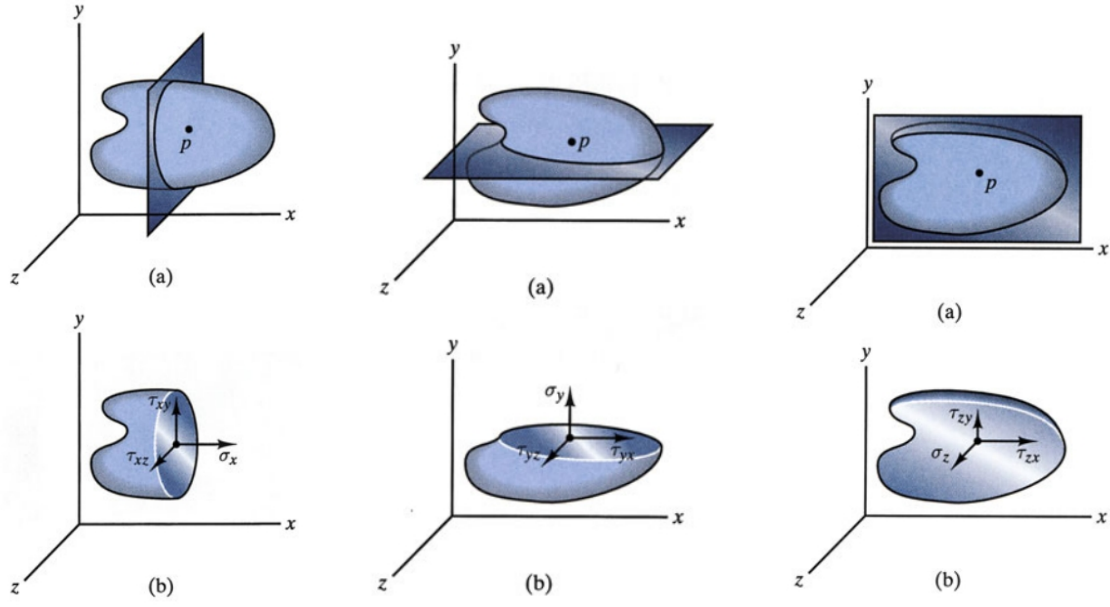
Theory

2.1 Off-road tire modelling

2.1.1 Mechanical stress

To fully understand the concepts described below, it is important to introduce the physical quantity called mechanical stress. This is the force per unit area acting on a point in a material relative to a reference frame [16]. There are two categories of stress defined by their direction relative to a plane intersecting the point of concern in the material. These are normal stress, directed perpendicularly to the plane, and shear stress, directed tangentially to the plane. Consider the point p in the blue object illustrated in Fig. 2.1a and the plane intersecting p , perpendicular to the x axis. The stress normal to this plane in the x -direction is the normal stress at p , denoted $\sigma_{n,x}$. The component in the y direction in p tangential to the plane is the shear stress component τ_{xy} . Equivalently, the tangential component in p , in the z -direction is the shear stress τ_{xz} .

If the plane instead lies in the xz -plane as in Fig. 2.1b, the normal stress relative to this plane is denoted $\sigma_{n,y}$ and the tangential components in p in the x and z -directions are τ_{yx} and τ_{yz} , respectively. For completeness, the normal component in p , relative to the xy -plane is $\sigma_{n,z}$ and the shear stress components are τ_{zx} and τ_{zy} , with directions along the x and y -axes, respectively. This is illustrated in Fig. 2.1c.



(a) yz -plane.

(b) xz -plane.

(c) xy -plane.

Figure 2.1: Normal and shear stresses in different planes [16].

These stress components in p , relative to the given coordinate system, define the state of stress at the point p and can be compactly represented in the stress tensor

$$\Sigma = \begin{bmatrix} \sigma_{n,x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{n,y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{n,z} \end{bmatrix}, \quad (2.1)$$

where the properties

$$\tau_{xy} = \tau_{yx}, \quad (2.2a)$$

$$\tau_{xz} = \tau_{zx}, \quad (2.2b)$$

$$\tau_{yz} = \tau_{zy}, \quad (2.2c)$$

hold. These components are also shown on the cubic object in Fig. 2.2, which faces are normal to the coordinate axes x, y, z . For any given point in a continuum material that is under stress, there exist at least three principal planes [17]. In these planes, there is no stress acting in its tangential direction and the only acting stress vector is perpendicular to the orientation of the principal plane. The principal stresses, σ_p , are the eigenvalues of the stress tensor,

$$\sigma_p = \lambda(\Sigma) \in \mathbb{R}^3. \quad (2.3)$$

There is a close connection between stress and pressure. In fact, the hydrostatic pressure, σ_h , can be derived by taking the average of the principal stresses,

$$\sigma_h = \frac{\sigma_{p,1} + \sigma_{p,2} + \sigma_{p,3}}{3} \in \mathbb{R}. \quad (2.4)$$

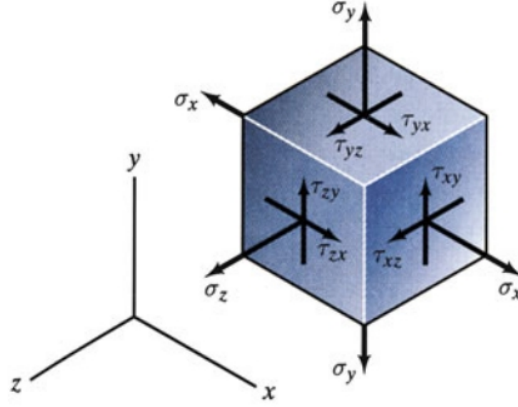


Figure 2.2: Illustration of stress tensor [16]. The illustrated stresses $\sigma_x, \sigma_y, \sigma_z$ represent the normal stresses denoted $\sigma_{n,x}, \sigma_{n,y}, \sigma_{n,z}$.

2.1.2 Pressure and sinkage relationship

When a vehicle is operating in terrain, the ground may be soft enough for the terrain to deform when pressure is applied. This has major impact in off-road tire dynamics as the tire will most likely sink into the ground when driving in off-road conditions. Bekker proposed an equation describing the relationship between sinkage and pressure under the assumption that the terrain is considered homogeneous [1]. This is formulated as

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi \right) z^n, \quad (2.5)$$

describing the applied pressure σ_n for a sinkage height z where k_c, k_ϕ, n are terrain dependent parameters and b is the smaller dimension of the plate applying the pressure, i.e. the width of a rectangle, or the radius of a circle.

2.1.3 Shear strength

For the given application, the cohesiveness of the ground is an important property of the terrain and gives information about when the surface of the terrain fails for a given combination of normal pressure and shear stress [1]. This happens when the applied shear stress exceeds the shear strength of the soil. At this point, the terrain loses cohesiveness and structure, starts to deform, slide and exhibits a plastic flow. Different modes of failure appear for different states of stress according to the Mohr circles shown in Fig. 2.4 where c, ϕ are the terrain specific parameters cohesion and angle of internal friction (or angle of internal shearing resistance). Cohesion c of a material is how strong the bonds between particles in a material naturally are,

without external impact. However, some materials hold together only when normal stress is applied. The shear strength of saturated clay, for example, does not depend on normal stress while the shear strength of dry sand increases when the normal stress increases [15].

Failure of the soil happens when the maximum shear strength is exceeded according to the Mohr-Coloumb failure criterion

$$\tau_{\max} = c + \sigma_n \tan(\phi), \quad (2.6)$$

where σ_n is the normal stress. To evaluate this property, a direct shear-strength test is performed where different pressures are applied to a material. If a terrain sample experiences different states of stress, one may construct a Mohr circle for each mode of failure. Consider a material under stress in two dimensions as illustrated in Fig. 2.3a. The state of stress in this illustration can be described by the stresses $\sigma_x, \tau_{xy}, \sigma_y$. Mohr's circle provides a graphical method [16] for finding the state of stress $\sigma_{x'}, \tau_{x'y'}, \sigma_{y'}$ after the reference frame is rotated by θ according to Fig. 2.3b. Several Mohr circles can be constructed in order to find the maximum shear strength according to Fig. 2.4. If a straight line is drawn that intersects the top of the circles, as depicted, the line can be described as the maximum shear strength (2.6).

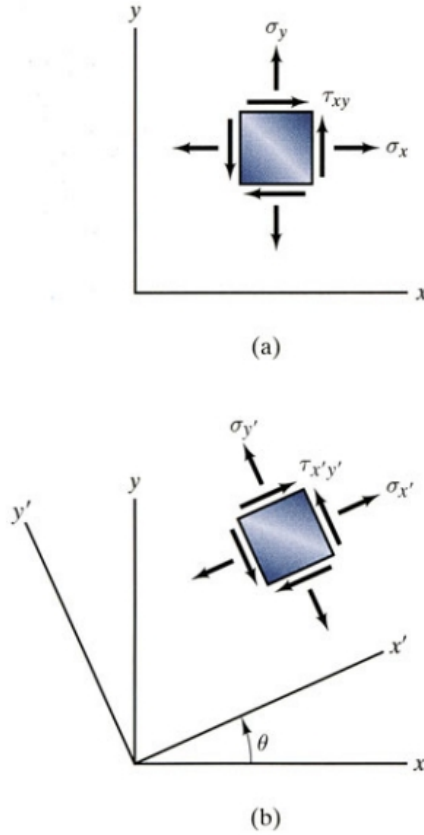


Figure 2.3: States of stress of a material before (a) and after (b) the reference frame is rotated by the angle θ [16].

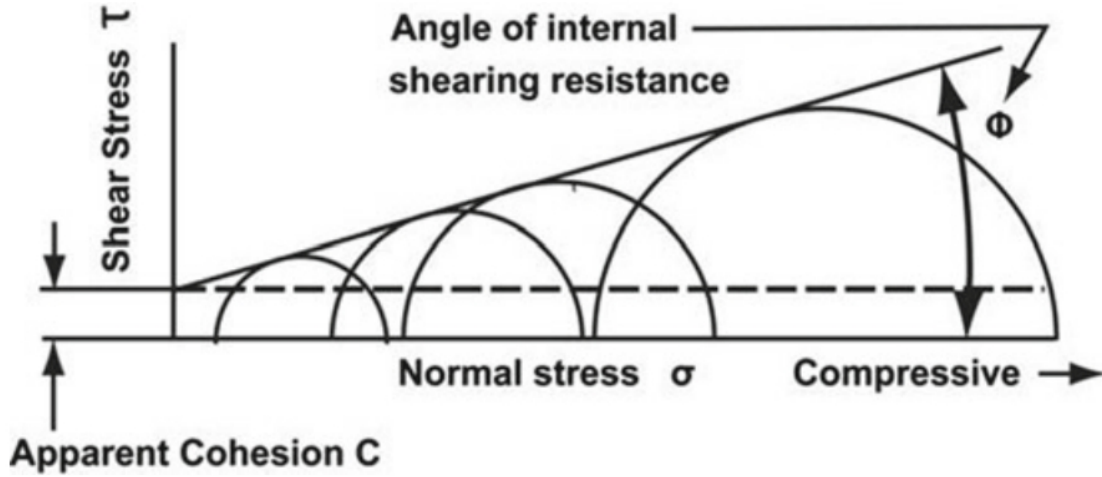


Figure 2.4: Illustration of maximum shear strength from Mohr circles [1].

2.1.4 Longitudinal and vertical forces

The coordinate system of a single tire is shown in Fig. 2.5 where the forward direction x is called the longitudinal dimension, the sideways direction y is called the lateral dimension and z is the vertical direction. Slip is a relationship between the longitudinal velocity v_x of a wheel and its angular velocity ω . It may be defined in several ways [1, 3, 6, 18], one of which is

$$\kappa = \frac{r\omega - v_x}{\max(r\omega, v_x)}, \quad (2.7)$$

where $\kappa > 0$ cause acceleration and $\kappa < 0$ results in deceleration. When a tire is rolling in deformable terrain in off-road conditions, the terrain is compressed and deformed permanently. The terrain underneath a tire exhibits flow patterns according to Fig. 2.6 for different states of horizontal, translational, velocity v_x and rotational velocity ω . These patterns are dependent on many factors, including vertical load and tire slip, and usually consists of two zones. In one zone, the soil moves along ω which is usually backwards and in the other zone, the soil follows v_x , in the tire's longitudinal direction. This latter zone forms a wedge in front of the tire that is being pushed forward and behaves like a bulldozing blade. This effect is called the bulldozing effect [1].

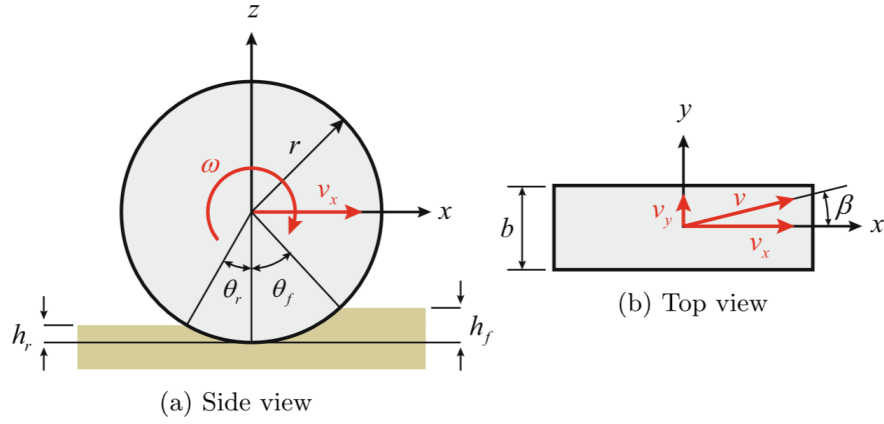


Figure 2.5: Coordinate frame of a single tire [19]. The front sinkage h_f is referred to as z_0 and the rear sinkage h_r is referred to as z_{0r} .

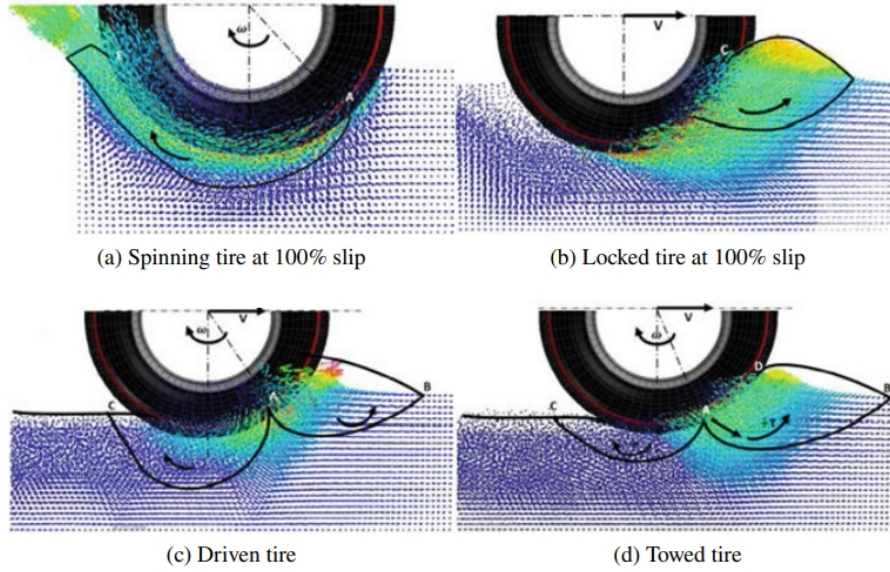


Figure 2.6: Flow patterns underneath a moving tire at different slip ratios [1]. The longitudinal velocity v in the figure is denoted v_x .

As a tire is rolling on soft ground, it compacts the soil in front of it and forms an entry angle θ_f at the point of the rim where the tire comes in contact with the ground, as depicted in Fig. 2.7.

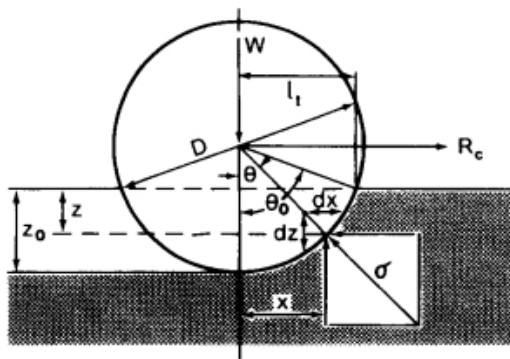


Figure 2.7: Rigid single tire single sinking into the ground [15]. The entry angle in front of the wheel θ_0 is referred to as θ_f .

As the wheel moves forward, its slip velocity v_j at a point on its peripheral is the difference between the theoretical speed on the wheel peripheral, $r\omega$, and the tangential component of the translational velocity v_x [1, 15], and its magnitude can be expressed as

$$\begin{aligned} v_j &= r\omega - v_x \cos(\theta) \\ &= r\omega - r\omega ((1 - \kappa) \cos(\theta)) \\ &= r\omega (1 - (1 - \kappa) \cos(\theta)), \end{aligned} \quad (2.8)$$

where the angle θ is shown in Fig. 2.7. The shear displacement along the interface between tire and terrain is the integral of v_j over time, and has a close form solution [15], expressed as

$$\begin{aligned} j_x(\theta) &= \int_0^t v_j dt \\ &= \int_\theta^{\theta_f} r\omega (1 - (1-s)\cos(\theta)) \frac{d\theta}{\omega} \\ &= r((\theta_f - \theta) - (1-\kappa)(\sin(\theta_f) - \sin(\theta))). \end{aligned} \quad (2.9)$$

The shear stress at a contact point, defined by θ , is described by

$$\begin{aligned}\tau_x(\theta) &= \tau_{\max} \left(1 - e^{-j_x(\theta)/k_x}\right) \\ &= (c + \sigma_n(\theta) \tan(\phi)) \left(1 - e^{-j_x(\theta)/k_x}\right)\end{aligned}\tag{2.10}$$

where k_x is a terrain parameter called shear deformation modulus. As the wheel moves horizontally across the ground with an entry angle θ_f , every point at an angle θ along the rim experiences a radial normal stress $\sigma_n(\theta)$ whose horizontal component will form a resistive force against the tire's horizontal movement. This is called the compaction force

$$R_c = rb \int_0^{\theta_f} \sigma_n(\theta) \sin(\theta) d\theta, \quad (2.11)$$

following the notion that the tire is compressing the ground in front of it in order to move forward. The normal stress σ_n thus causes resistance to the horizontal motion while the tangential stress (2.10), on the other hand, contributes to traction force according to

$$F_t = rb \int_0^{\theta_f} \tau_x(\theta) \cos(\theta) d\theta, \quad (2.12)$$

where the horizontal component of the shear stress contributes to the tractive force. Consequently, the resulting force in the longitudinal dimension can be described as

$$F_x = F_t - R_c = rb \int_0^{\theta_f} (\tau_x(\theta) \cos(\theta) - \sigma_n(\theta) \sin(\theta)) d\theta, \quad (2.13)$$

When the tire is in a state of significant sinkage, the bulldozing resistance needs to be considered [1]. In many traction models, this is not the case where compaction resistance is the only present resistive force acting on the tire [20, 7, 9, 6, 10].

As for the vertical translational motion, both normal and shear stress contribute to upward force F_z which can be described as

$$F_z = rb \int_0^{\theta_f} (\sigma_n(\theta) \cos(\theta) + \tau_x(\theta) \sin(\theta)) d\theta, \quad (2.14)$$

and the relationship between vertical sinkage z_0 , as illustrated in Fig. 2.7, and entry angle θ_f can be described as

$$\theta_f = \arccos\left(1 - \frac{z_0}{r}\right). \quad (2.15)$$

2.1.5 Lateral force

When cornering, the notion of slip angle is important. This angle is shown in Fig. 2.5 as β and is the angle between the tire's horizontal velocity v and its longitudinal axis [19, 1]. The shear displacement in the lateral direction depends on both the longitudinal slip κ and the slip angle β [1] as

$$j_y(\theta) = r(1 - \kappa)(\theta_f - \theta) \tan(\beta). \quad (2.16)$$

The lateral shear stress is equivalent to (2.10), namely

$$\tau_y(\theta) = (c + \sigma_n(\theta) \tan(\phi)) \left(1 - e^{-j_y(\theta)/k_y}\right), \quad (2.17)$$

where $k_y = k_x$ [6]. Lateral shear stress is the reason for the generated lateral force after integrating over the tire area which is in contact with the ground, which can be seen later. When including lateral motion one, also needs to consider the exit angle θ_r , shown in Fig. 2.5, which can be calculated as

$$\theta_r = \arccos\left(1 - \frac{z_{0r}}{r}\right). \quad (2.18)$$

This exit angle extends the area of contact between tire and ground and therefore also affects longitudinal and lateral motion. The equation of motion therefore becomes

$$F_x = rb \int_{\theta_r}^{\theta_f} (\tau_x(\theta) \cos(\theta) - \sigma_n(\theta) \sin(\theta)) d\theta, \quad (2.19a)$$

$$F_y = rb \int_{\theta_r}^{\theta_f} \tau_y(\theta) d\theta, \quad (2.19b)$$

$$F_z = rb \int_{\theta_r}^{\theta_f} (\sigma_n(\theta) \cos(\theta) + \tau_x(\theta) \sin(\theta)) d\theta, \quad (2.19c)$$

for three degrees of freedom. However, in the lateral case where $\theta_r \neq 0$, the normal stress σ_n is calculated differently in the front and rear region of the tire [1]. More specifically, the normal stress distribution depends on whether the current point, θ , on the peripheral of the tire, is in front of, or behind, an angle θ_m , which is the angle where the normal stress is greatest. Taking this into consideration, the normal stress distribution can be calculated as

$$\sigma_n(\theta) = \begin{cases} \left(\frac{k_c}{b} + k_\phi\right) r^n (\cos(\theta) - \cos(\theta_f))^n, & \theta_m \leq \theta \leq \theta_f, \\ \left(\frac{k_c}{b} + k_\phi\right) r^n \left(\cos\left(\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r}(\theta_f - \theta_m)\right) - \cos(\theta_f)\right)^n, & \theta_r \leq \theta \leq \theta_m. \end{cases} \quad (2.20)$$

2.1.5.1 Bulldozing resistance

When a tire is moving forward with a slip angle $\beta \neq 0$, the side of the tire comes in contact with, possibly untouched, terrain. This terrain applies force to the tire sideways, resulting in the bulldozing effect appearing in the lateral direction [1]. Fig. 2.8 shows the Hegedus model that estimates this resistance by pushing a plate rather than terrain. This forms a swollen soil with angle φ relative to the horizontal ground and a destructive angle

$$X_c = 45^\circ - \frac{\varphi}{2}. \quad (2.21)$$

For a blade of unit width, the bulldozing resistive force can be described as

$$R_b = \frac{\cot(X_c) + \tan(X_c + \varphi)}{1 - \tan(\alpha') \tan(X_c + \varphi)} \cdot \left(h_b c + \frac{1}{2} \rho h^2 \left(\cot(X_c) - \tan(\alpha') + \frac{(\cot(X_c) - \tan(\alpha'))^2}{\tan(\alpha') + \cot(\varphi)} \right) \right), \quad (2.22)$$

where α' is the angle between the plate and a vertical line, ρ is the terrain density and the height h_b can be seen in Fig. 2.8. Integrating over the part of the tire side that is in contact with terrain yields the total resistive bulldozing force for one tire as

$$F_b = \int_{\theta_r}^{\theta_f} R_b(h_b) d\theta = \int_{\theta_r}^{\theta_f} R_b(r - h_b(\theta) \cos(\theta)) d\theta. \quad (2.23)$$

Since the bulldozing resistance is caused by longitudinal motion and $\beta \neq 0$, only the component of F_b acting perpendicular to the tire side contributes to the lateral force caused by bulldozing. Therefore the total lateral force acting on the tire can be expressed as

$$F_{y_{tot}} = F_b \sin(\beta) + F_y. \quad (2.24)$$

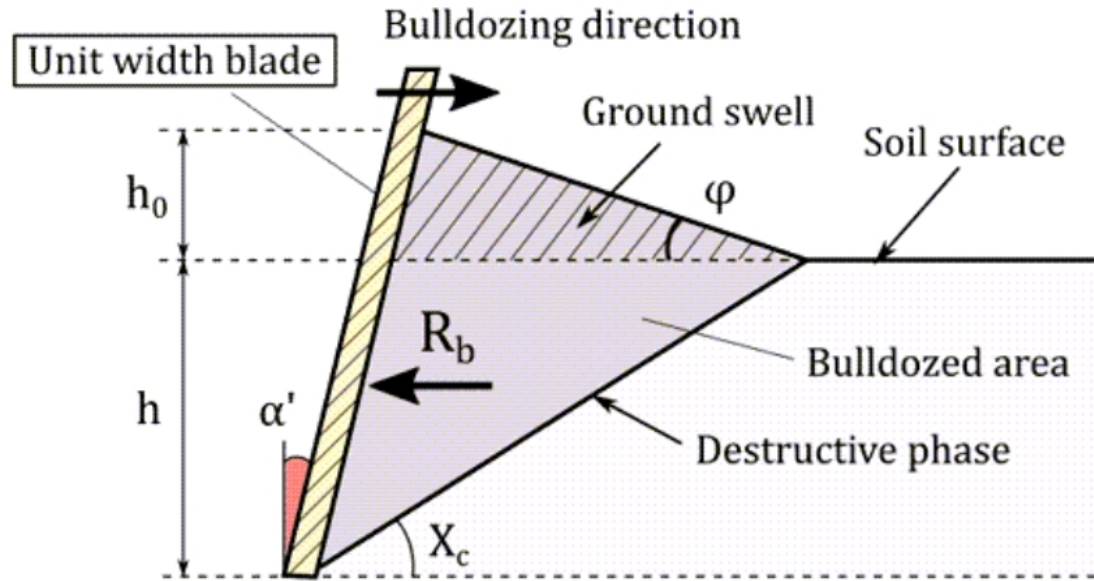


Figure 2.8: Illustration of the Hegedus model for calculation of the bulldozing resistive force [20]. The height h of the bulldozed area is referred to as h_b .

2.1.6 Terrain specific parameters

The type of terrain the tire is driving in can be described by the parameters mentioned above. Four types of terrain have been chosen for testing of the tire model and braking system, and these are defined by the parameter values given in Tab. 2.1. Notice that the terrain density ρ is only needed for bulldozing resistance.

Table 2.1: Terrain parameters. The density values ρ are referenced to [20], the remaining terrain parameters to [6] and some parameter units to [21].

Soil type	n [—]	c [Pa]	ϕ [rad]	k_c [N/m ⁿ⁺¹]	k_ϕ [N/m ⁿ⁺²]	k_x, k_y [m]	ρ [kg/m ³]
Loam	0.90	22 670	0.383972	73 678	5 786 800	0.015	1 330
Sandy loam	0.70	1 700	0.506145	5 300	1 515 000	0.025	1 600
Clay	0.50	4 140	0.226893	13 200	692 200	0.010	1 660
Sand	1.08	200	0.471239	1 000	1 528 600	0.024	1 600

2.1.7 Multipass effect

When having consecutive wheels, the first wheel deforms the terrain, thus affecting the terrain for the succeeding wheels. This is called the multipass effect and is an important aspect to consider in off-road environments on soft and deformable ground. The rolling resistance is a force that depend on a multitude of factors, including tire construction and terrain roughness. If this force is normalized by the vertical load of the tire, the coefficient of rolling resistance is obtained. The impact of the multipass effect was tested by Volvo Group Trucks Technology on gravelly soil with the test vehicle illustrated in Fig. 2.9 which has one front tire axle and three rear tire axles. The result of the test is presented in Fig. 2.10 for both laden and unladen vehicle. Fig. 2.10a shows the rolling resistance coefficient for each consecutive wheel axle with 1 being the front axle. This test also includes the contact area between the soil and tire in Fig. 2.10b. The result indicate greater rolling resistance when the truck was laden compared to unladen. Another observation from this test is that the front tires have significantly greater contact area and rolling resistance coefficient compared to the rear tires. One explanation for this is that the front tires face untouched soil as it progresses while the rear tires are rolling on compacted terrain. It can also be observed that the difference between the first and second tire axle is significantly greater than the mutual difference between the rear tire axles which may be explained by the notion that the front tires are compacting untouched soil while the the rear tires moves over more or less compacted soil.

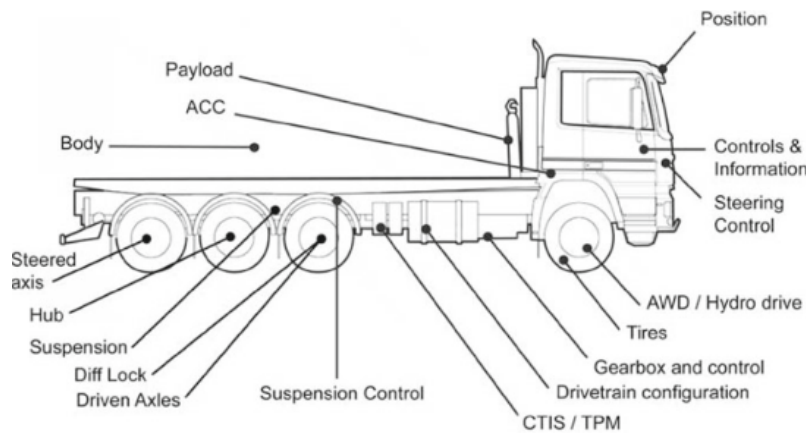


Figure 2.9: Vehicle used for testing the multipass effect with one front axle and three rear axles [1].

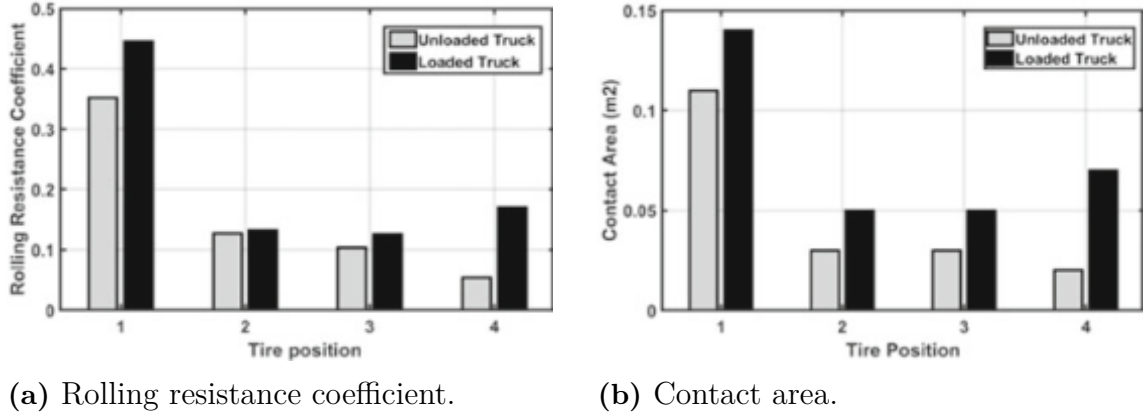


Figure 2.10: Results from testing the multipass effect [1].

2.2 $\mathcal{H}_\infty$ Robust Control

2.2.1 Singular values and $\mathcal{H}_\infty$ norm as prerequisites

Consider the complex valued matrix $P(s)$ where s is the complex valued frequency. The singular values of P are the positive square roots of the eigenvalues of $P^H(j\Omega)P(j\Omega)$ [22], namely

$$\sigma_k(P(j\Omega)) = \sqrt{\lambda_k(P^H(j\Omega)P(j\Omega))}, \quad (2.25)$$

where λ are the eigenvalues and

$$P^H(j\Omega) = P^T(-j\Omega). \quad (2.26)$$

The maximum and minimum gain, in terms of the 2 norm, for any input direction is equal to the maximum and minimum singular values of P respectively. Denoting the maximum and minimum singular values as $\sigma_{\max}$ and $\sigma_{\min}$ respectively, and the input vector to P as u , this can be formulated as

$$\sigma_{\min}(P(j\Omega)) \leq \frac{\|P(j\Omega)u\|_2}{\|u\|_2} \leq \sigma_{\max}(P(j\Omega)), \quad (2.27)$$

where $\|u\|_2$ is the 2-norm of the vector u , formally defined as

$$\|u\|_2 = \sqrt{\sum_k |u_k|^2}. \quad (2.28)$$

Building on this, it is appropriate to present the $\mathcal{H}_\infty$ norm which is a fundamental metric of the controller that is to be synthesized. It is defined as the maximum value of the maximum singular value of the frequency response [22], formally

$$\|P(s)\|_\infty \triangleq \max_{\Omega} \sigma_{\max}(P(j\Omega)). \quad (2.29)$$

2.2.2 Robustness

Model based controllers use information about the system in the process of developing the controller. Therefore, a model of the plant is used in order to give the controller knowledge about the system it is to control. This gives the controller the advantage of basing its control actions on the system it is supposed to operate on, possibly resulting in more careful control input signals. More likely than not, this model is not completely accurate and this mismatch is referred to as model uncertainty [22]. If the control system is insensitive to model uncertainty it is said to be robust. In $\mathcal{H}_\infty$ control design, the aim is to design a controller with ability to handle even the worst case uncertainty, which is a desirable feature for safety critical systems as braking. It is therefore necessary to define an uncertainty set that describes the model uncertainties. Robustness may then be analyzed by checking the stability of all the plants in the the uncertainty set and, if they are stable, also check if it is possible to meet performance specifications for all plants in the uncertainty set. These criteria are called robust stability (RS) and robust performance (RP) respectively. However, this framework considers linear models and since the model to be controlled (2.13) is not linear, the performance of the controller when applied to the nonlinear model will be analyzed by evaluating the closed loop system in different scenarios. Model uncertainty may be categorized into one of the following classes:

- Parametric/structured uncertainty
- Unstructured uncertainty

In parametric uncertainty descriptions, the model structure is known but some parameter values are uncertain. This is not the case for unstructured uncertainty where the uncertainty set may be derived in various ways.

2.2.3 Closed loop architecture

Fig. 2.11 presents the nominal closed loop system architecture of the robust control framework where P is the generalized plant, and K is the controller. The generalized plant P includes weights and performance specifications used for controller synthesis and tuning. The plant takes exogenous inputs w and control signals u and outputs performance outputs z and η which may take several forms, e.g. measurements or differences between states and references. Given the weights in P , the performance output z is calculated and defined by the user based on desired control objectives when excited by the exogenous inputs w which can include references, process disturbances and measurement noise. The system in Fig. 2.11 can be written as

$$\begin{bmatrix} z \\ \eta \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}, \quad (2.30a)$$

$$u = K \eta, \quad (2.30b)$$

from which the closed loop $z = N w$ can be derived [22] from P and K by calculating the lower linear fractional transformation (LFT) as

$$N = \mathcal{F}_l(P, K) \triangleq P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \quad (2.31)$$

As P include an LTI system defined by the common matrices A, B, C, D , the system in Fig. 2.11 can also be written in a compact way as

$$\begin{bmatrix} \dot{x} \\ z \\ \eta \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}. \quad (2.32)$$

Equivalently, the uncertainty matrix Δ can be included into the closed loop system as depicted in Fig. 2.12 and the resulting transfer function from w to z can be calculated using the upper LFT as

$$\mathcal{F}_u(N, \Delta) \triangleq N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1}N_{12}. \quad (2.33)$$

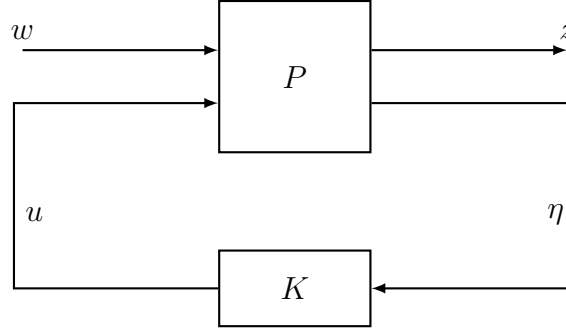


Figure 2.11: Closed loop architecture of nominal plant model, PK -structure [22].

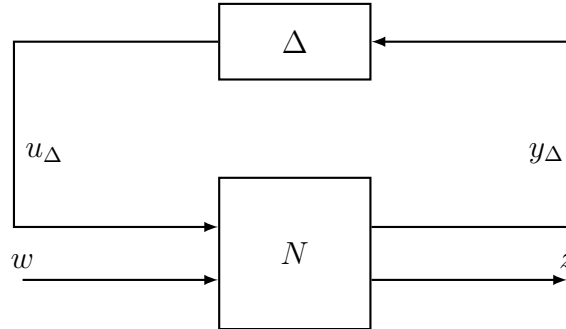


Figure 2.12: $N\Delta$ -structure of the closed loop system architecture [22].

Unstructured model uncertainties can be described in variable ways as depicted in Fig. 2.13 where the uncertainty block Δ_x is accompanied by a weight w_x . Common for these representations is that the uncertainty matrix has a bounded infinity norm as

$$\|\Delta\|_\infty \leq 1. \quad (2.34)$$

Finally, the general architecture for control synthesis is depicted in Fig. 2.14.

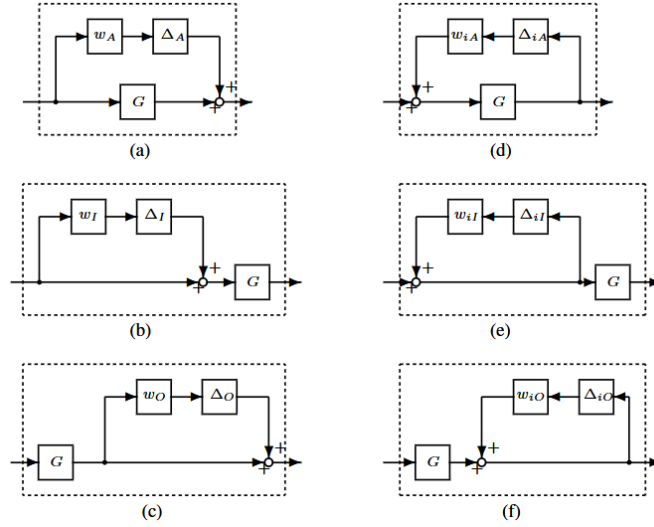


Figure 2.13: Different architectures of unstructured model uncertainties [22]. G represents the nominal plant model, Δ_x is the uncertainty matrix accompanied by a weight w_x . The dashed lines embrace the uncertain model which includes the nominal plant model and uncertainty. a) Additive uncertainty, b) multiplicative input uncertainty, c) multiplicative output uncertainty, d) inverse additive uncertainty, e) inverse multiplicative input uncertainty, f) inverse multiplicative output uncertainty.

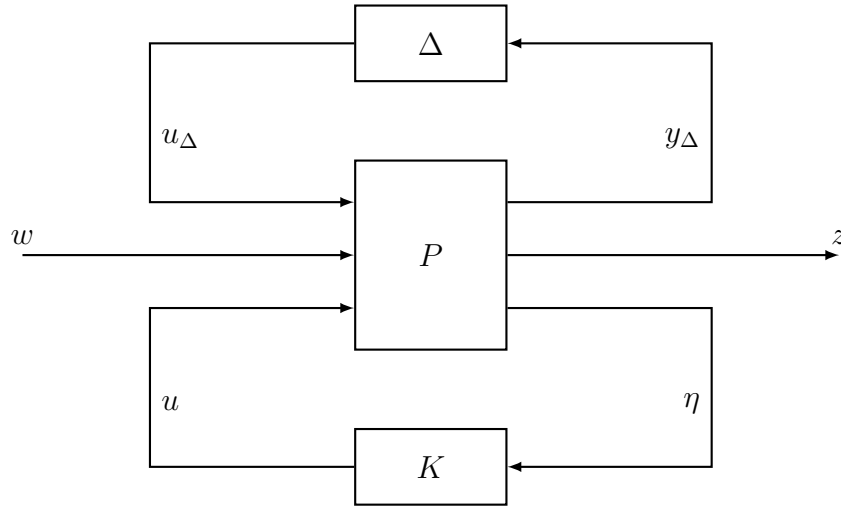


Figure 2.14: $PK\Delta$ - structure, the general architecture used for controller synthesis [22].

2.2.4 $\mathcal{H}_\infty$ controller

Recall the nominal closed loop in Fig. 2.11 and the lower LFT in (2.31). Given the control configuration in the nominal closed loop, the default $\mathcal{H}_\infty$ optimal control problem seeks to find the stabilizing controllers K that minimize

$$\|N\|_\infty = \|\mathcal{F}_l(P, K)\|_\infty = \max_{\Omega} \sigma_{\max}(\mathcal{F}_l(P, K)(j\Omega)) = \max_{\Omega} \sigma_{\max}(N), \quad (2.35)$$

and is therefore a min-max problem. One interpretation of this is that the peak of the maximum singular value of N is to be minimized [22]. In practice, however, it often suffices to find a suboptimal $\mathcal{H}_\infty$ controller rather than the optimal one. Let

$$\gamma_{\min} = \min_K \|\mathcal{F}_l(P, K)\|_\infty = \min_K \|N\|_\infty = \min_K \max_{\Omega} \sigma_{\max}(N), \quad (2.36)$$

then the suboptimal control problem is to find all stabilizing controllers K such that

$$\|N\|_\infty = \|\mathcal{F}_l(P, K)\|_\infty < \gamma, \quad (2.37)$$

where

$$\gamma_{\min} < \gamma. \quad (2.38)$$

A general algorithm to find such controllers does this by iteratively reducing γ . The process of synthesizing $\mathcal{H}_\infty$ typically make some assumptions [22]. The most common assumptions can be formulated using the notations from (2.32) as:

- (A, B_2, C_2) is detectable and stabilizable. This is a requirement in order to find stabilizing controllers.
- D_{12} and D_{21} have full rank. This is a sufficient criterium to ensure proper and realizable controllers.
- The matrix $\begin{bmatrix} A - j\Omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank $\forall \Omega$. This prevents cancellation of poles and zeros on the imaginary axis by the controller which would cause instability of the closed loop.
- The matrix $\begin{bmatrix} A - j\Omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall \Omega$. This serves the same purpose as the previous assumption.
- $D_{11} = D_{22} = 0$. This is not required but simplifies the $\mathcal{H}_\infty$ controller synthesis.

3

Methods

3.1 Single wheel state space model

Inspired by related research [7, 6, 19], an off-road single tire model may be validated against the open source physics simulation software Project Chrono. The soil parameters in Tab. 2.1 are used also in this software along with additional parameters for stiffness and damping. Since no information about reasonable values for the latter ones was found, these were manually tuned to reflect realistic vertical dynamics by reducing vertical oscillations. The following states were included in the single tire model to be evaluated

$$x(t) = \begin{bmatrix} d(t) \\ \alpha(t) \\ z_0(t) \\ v_x(t) \\ \omega(t) \\ v_z(t) \end{bmatrix}, \quad (3.1)$$

where d is covered longitudinal distance, α is the accumulative angle of the wheel, z_0 is the vertical sinkage from the ground surface to the lowest point of the tire. The remaining states, v_x, ω, v_z are time derivatives of $x(t)_{1-3}$ respectively. As all states are time varying, explicitly denoting this as e.g. $x(t)$ is hereafter excluded for simplicity and replaced by the simpler notation x . The dynamics of the single tire is thus described by

$$\dot{x} = \begin{bmatrix} v_x \\ \omega \\ v_z \\ \dot{v}_x \\ \dot{\omega} \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ \omega \\ v_z \\ \frac{F_x}{m} \\ \frac{T - F_t \cdot r}{J} \\ \frac{F_z - m g}{m} \end{bmatrix}, \quad (3.2)$$

where J is the inertia of the wheel around its rotational axis. A damping viscous

friction parameter is neglected as the goal thus far is adequate gross model behavior, hence leaving such fine tuning for further exploration. The tire parameter values used are presented in Tab. 3.1. The comparison is performed by applying a predefined torque to the input variable T in the state space model (3.2) while also applying the same torque to the tire in Chrono, resulting in the plots shown in Fig. 3.1 - 3.4 for each terrain type respectively. In these figures and throughout the thesis, the model in (2.19) is referred to as "BW" following two of its founders, Bekker and Wong. For all terrain types, the longitudinal velocity v_x and angular velocity ω from the BW model follow the validation data from Chrono well. This is reflected in the integrated states, longitudinal position d and angular position α , where the positions from the BW model remain close to the positions from Chrono. Regarding the vertical direction, oscillations occur for the velocities v_z in the BW model which propagates to sinkage z_0 . This is not seen in the data from Chrono and remains a topic to research further. Nevertheless, this does not seem to have significant impact on v_x and ω . It is also noticed that the slip amplitudes from both BW and Chrono increase when velocities, translational or angular, approach zero. This leads to small denominators in the slip calculation (2.7) causing slip amplitudes to increase.

Table 3.1: Tire parameters values. Radius r , tire width b , mass m and moment of inertia around the tire's rotational axis, J .

Parameter	r [m]	b [m]	m [kg]	J [kg m ²]
Value	0.9	0.5	500	202.5

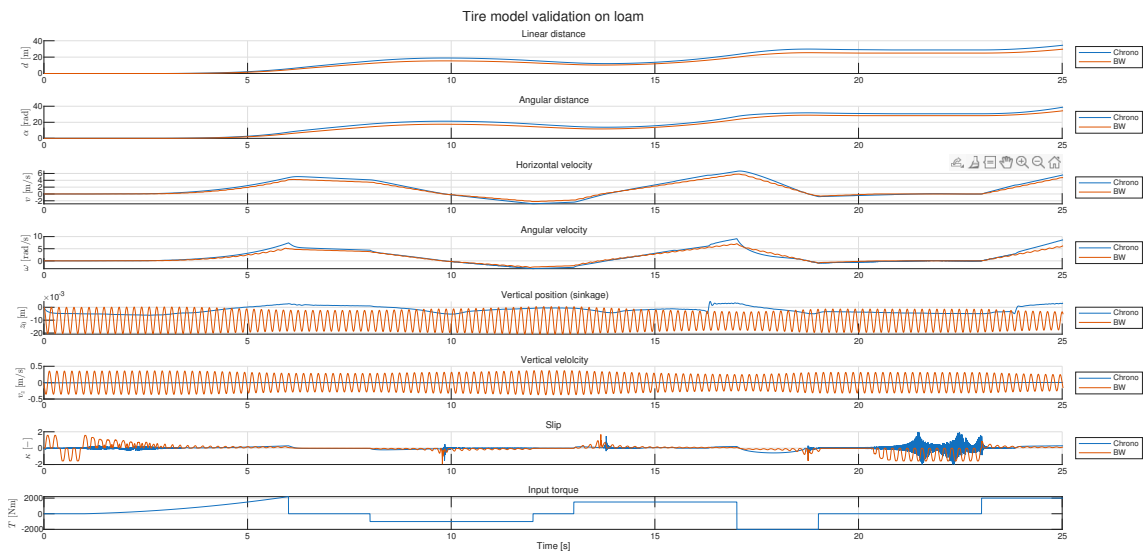


Figure 3.1: Validation of BW model against Project Chrono on loam.

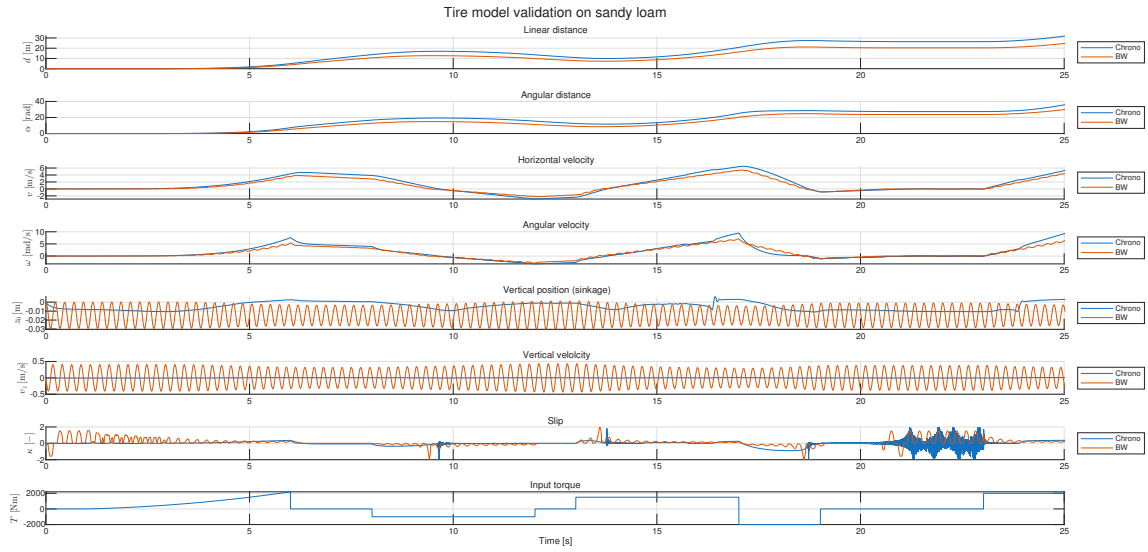


Figure 3.2: Validation of BW model against Project Chrono on sandy loam.

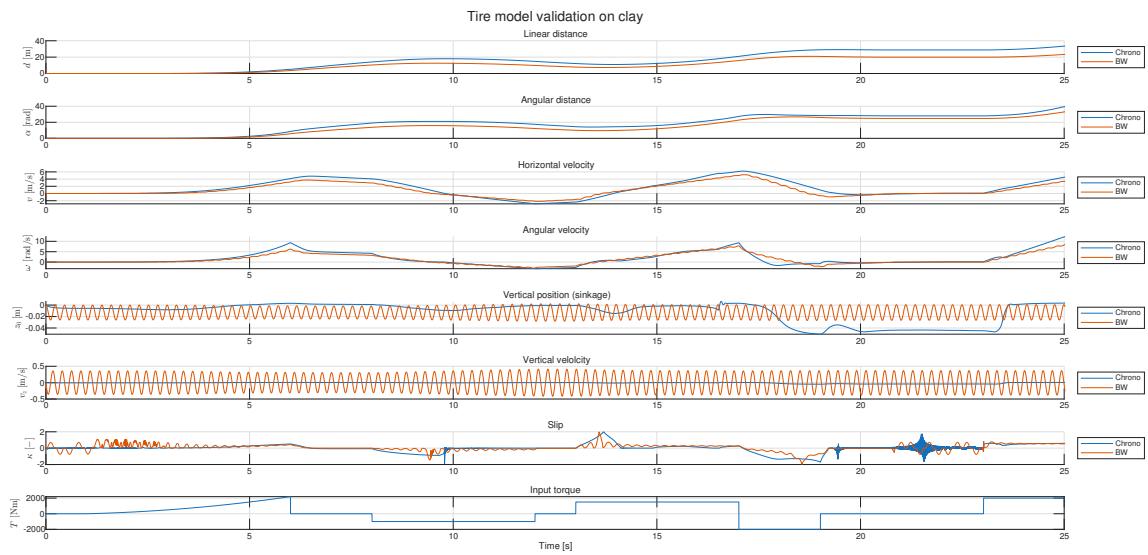


Figure 3.3: Validation of BW model against Project Chrono on clay.

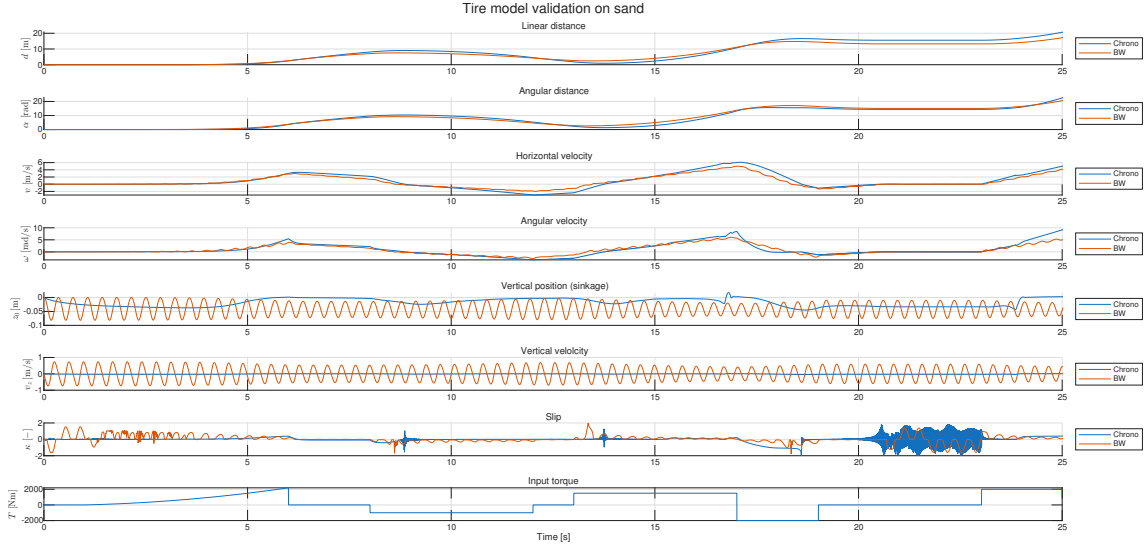


Figure 3.4: Validation of BW model against Project Chrono on sand.

Since (2.8) describes the magnitude of the slip velocity, there remains uncertainty about how to handle the direction in (2.9) and (2.10). The magnitude of shear stress follows the procedure in [18] and the direction is implemented as the direction of the shear displacement (2.9). Therefore, the shear stress was implemented as

$$\tau_x(\theta) = \tau_{\max} \left(1 - e^{-|j_x|/k_x} \right) \cdot \text{sign}(j_x). \quad (3.3)$$

Another possibility may be to use the direction of the slip velocity v_j in (2.8), but since the shear displacement j_x has a closed form expression, independent of v_j , the slip velocity v_j is not explicitly modelled and therefore the direction of shear displacement j_x was used. This may result in a delayed response when directions change. But, as seen in Fig. 3.1 - 3.4, the longitudinal dynamics track the validation data well. It was noticed when simulating the model that the use of dynamic time steps for integrators like Matlab's ode45 often required significant processing time. Using absolute values and sign functions as done in (3.3) likely introduces discontinuities which hinder the dynamic time step integrators from converging at certain time steps. Therefore, a fixed time step simulation was used for the rest of the project, namely the fourth order Runge Kutta method. Moreover, this thesis follows the mentioned common procedure of neglecting the bulldozing resistance in the longitudinal direction. As the BW model behavior appears to be similar to its counterpart in Chrono, this simplification seems to be a reasonable choice.

3.2 Tire model calibration

Although the BW model behavior is similar to the validation data acquired from Chrono, there is still room for improvement. One possibility to calibrate the model to correspond more closely to the validation data is by optimizing the terrain parameters in Tab 2.1. Since oscillations occurred for v_z in the BW model, the terrain parameters were extended with a damping factor ζ such that

$$\dot{x}_6 = \dot{v}_z = \frac{F_z - m g - \zeta v_z}{m}. \quad (3.4)$$

The extended terrain parameter vector can then be formulated as

$$q = \begin{bmatrix} n \\ c \\ \phi \\ k_c \\ k_\phi \\ k_x \\ \zeta \end{bmatrix} \in \mathbb{R}^7, \quad (3.5)$$

which constitute the decision variables in an attempt to calibrate the tire model to fit the validation data better, which is given by

$$x^{\text{meas}} = \begin{bmatrix} d^{\text{meas}} \\ \alpha^{\text{meas}} \\ z_0^{\text{meas}} \\ v_x^{\text{meas}} \\ \omega^{\text{meas}} \\ v_z^{\text{meas}} \end{bmatrix}. \quad (3.6)$$

The cost function for this operation is described by

$$L(q) = \sum_{i=1}^{N_s} \begin{bmatrix} (d_i - d_i^{\text{meas}})^2 \\ (\alpha_i - \alpha_i^{\text{meas}})^2 \\ (z_{0,i} - z_{0,i}^{\text{meas}})^2 \\ (v_{x,i} - v_{x,i}^{\text{meas}})^2 \\ (\omega_i - \omega_i^{\text{meas}})^2 \\ (v_{z,i} - v_{z,i}^{\text{meas}})^2 \end{bmatrix}^T w_L, \quad (3.7)$$

where w_L are weights for the different states and N_s the number of samples. The weights w_L are selected to prioritize fitting the velocities over positions since the positions are purely integrated states from the velocities. Therefore, w_L is selected as

$$w_L = [1 \quad 1 \quad 1 \quad 10 \quad 10 \quad 10]^T. \quad (3.8)$$

By analyzing the longitudinal distance d in the drivecycles, one notices that the tire is passing the same points of terrain multiple times. In reality, the tire deforms the soil in these positions such that the soil characteristics are changed. This multipass effect is yet not considered in the model but since the longitudinal position d is tracked in the tire model, it is possible to know the positions where the tire has already compacted the terrain. One could therefore keep a separate set of terrain

parameters for untouched terrain and already compacted terrain and expand the terrain parameter vector in (3.5) from $\mathbb{R}^7$ to $\mathbb{R}^{14}$, as

$$q_e = \begin{bmatrix} q_s \in \mathbb{R}^7 \\ q_m \in \mathbb{R}^7 \end{bmatrix} \in \mathbb{R}^{14}, \quad (3.9)$$

where q_s is the terrain parameter vector corresponding to the initial passing of the terrain (singlepass) and q_m is the terrain parameter vector for the multipass case. The cost function in this case is the same as before (3.7) but the number of decision variables is doubled from 7 to 14. The curvefitting is executed by minimizing the cost as

$$\begin{aligned} \min_{q_e} \quad & L(q_e) \\ \text{s.t.} \quad & 0 \leq q_e \leq 5 \begin{bmatrix} q_0 \\ q_0 \end{bmatrix}, \end{aligned} \quad (3.10)$$

where q_0 is the initial guess given by the terrain parameter values in Tab. 2.1 for each terrain type respectively. The resulting parameter values are presented in Tab. 3.2 - 3.5.

Table 3.2: Loam soil parameters: singlepass and multipass curvefitting.

Parameter	Default	Singlepass fitted	Multipass fitted
n [—]	0.90	0.88	0.75
c [Pa]	22 670	19 170	41 886
ϕ [rad]	0.383972	0.305323	0.455664
k_c [N/m ⁿ⁺¹]	73 678	101 312	76 993
k_ϕ [N/m ⁿ⁺²]	5 786 800	6 030 166	6 049 120
k_x [m]	0.015	0.030	0.006
ζ [N · kg · s/m]	0	4 869	4 974

Table 3.3: Sandy loam soil parameters: singlepass and multipass curvefitting.

Parameter	Default	Singlepass fitted	Multipass fitted
n [—]	0.70	0.64	0.58
c [Pa]	1 700	1 360	4 559
ϕ [rad]	0.506145	0.410589	0.662528
k_c [N/m ⁿ⁺¹]	5 300	7 658	1 378
k_ϕ [N/m ⁿ⁺²]	1 515 000	1 635 850	1 695 015
k_x [m]	0.025	0.029	0.018
ζ [N · kg · s/m]	0	5 689	5 850

Table 3.4: Clay soil parameters: singlepass and multipass curvefitting.

Parameter	Default	Singlepass fitted	Multipass fitted
n [—]	0.50	0.44	0.47
c [Pa]	4 140	3 991	3 799
ϕ [rad]	0.226893	0.225335	0.297762
k_c [N/m ⁿ⁺¹]	13 200	13 554	13 574
k_ϕ [N/m ⁿ⁺²]	692 200	718 430	770 258
k_x [m]	0.010	0.014	0.008
ζ [N · kg · s/m]	0	5 079	4 571

Table 3.5: Sand soil parameters: singlepass and multipass curvefitting.

Parameter	Default	Singlepass fitted	Multipass fitted
n [—]	1.08	1.04	0.93
c [Pa]	200	183	187
ϕ [rad]	0.471239	0.408483	0.492648
k_c [N/m ⁿ⁺¹]	1 000	1 068	1 082
k_ϕ [N/m ⁿ⁺²]	1 528 600	1 502 262	1 721 731
k_x [m]	0.024	0.040	0.018
ζ [N · kg · s/m]	0	4 763	6 087

The results when simulating the calibrated model are shown in Fig. 3.5 - 3.8 where the original BW model in (2.19) is also present along with the validation data from Chrono and the calibrated model "BW fitted". By including the damper ζ , the sinkage z_0 often finds an equilibrium sinkage where the tire stays vertically until a large enough disturbance occurs, after which a new sinkage equilibrium is found. Although the vertical velocity v_z is not oscillating anymore, its trajectory is not following the data from Chrono, which is obvious when comparing the sinkage z_0 from both models. Further research is therefore needed to understand the vertical dynamics. Better results were seen in the longitudinal and angular dimensions where the calibrated model follows the trajectories from Chrono more closely, although further improvement is still possible. As the decision variable in (3.10) contains many parameters, better curvefitting may be achieved by trying different initial guesses q_0 .

3. Methods

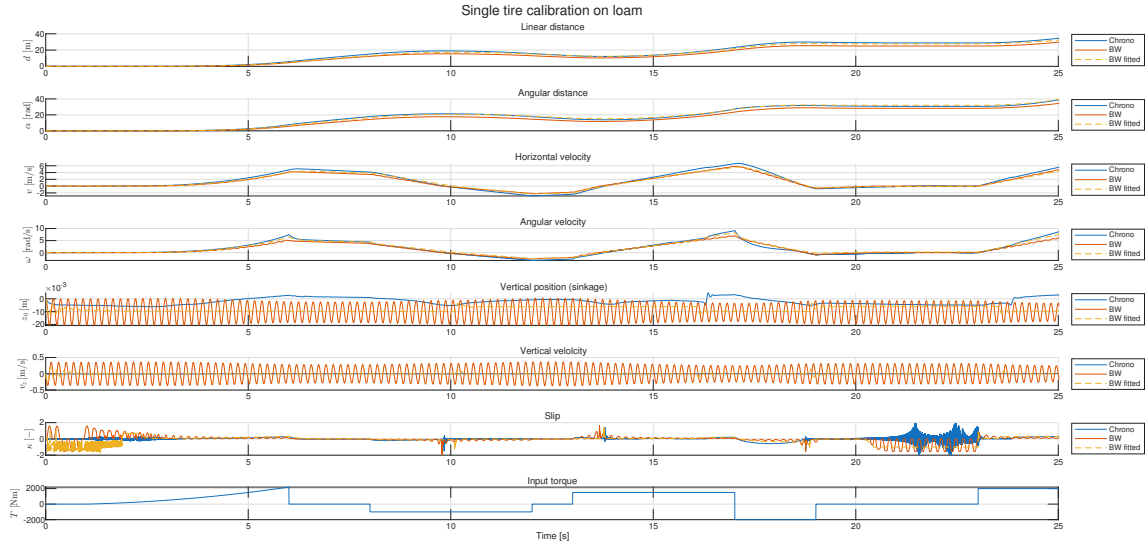


Figure 3.5: Calibration of BW model on loam by using least-squares fitting to simulated data in Project Chrono. Comparison with uncalibrated BW model is also included as reference.

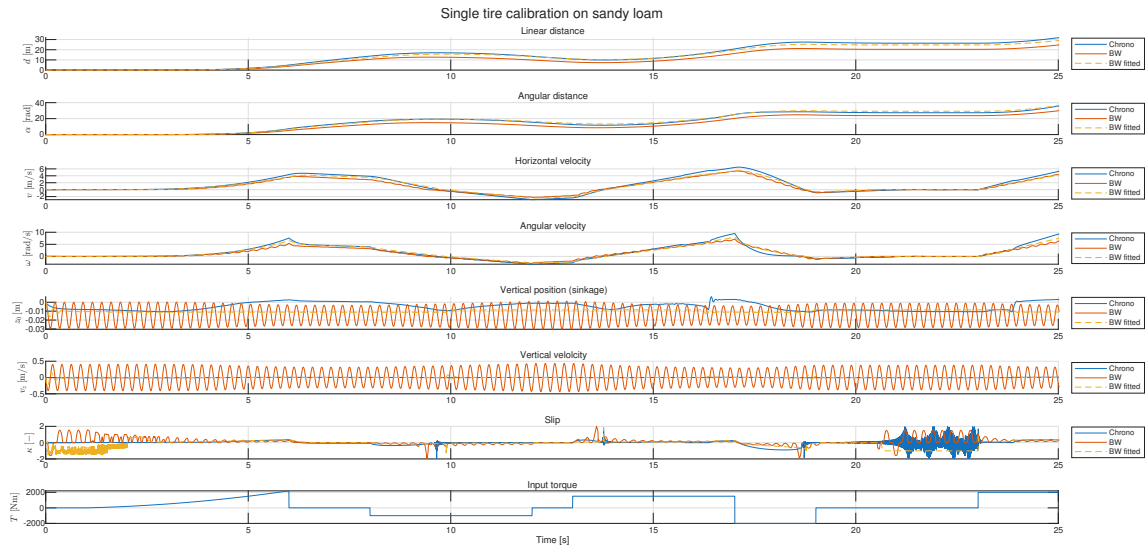


Figure 3.6: Calibration of BW model on sandy loam by using least-squares fitting to simulated data in Project Chrono. Comparison with uncalibrated BW model is also included as reference.

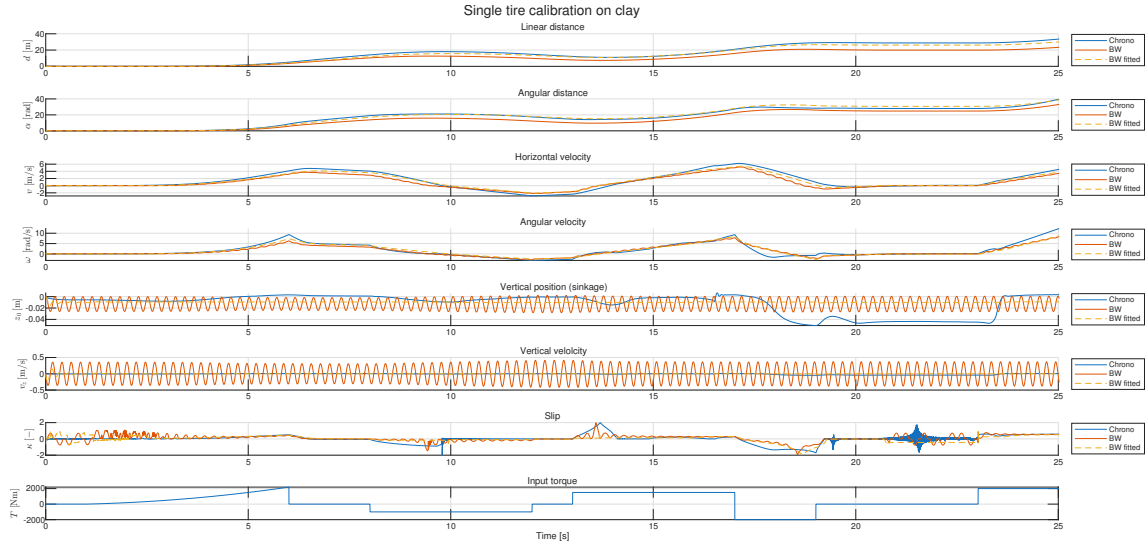


Figure 3.7: Calibration of BW model on clay by using least-squares fitting to simulated data in Project Chrono. Comparison with uncalibrated BW model is also included as reference.

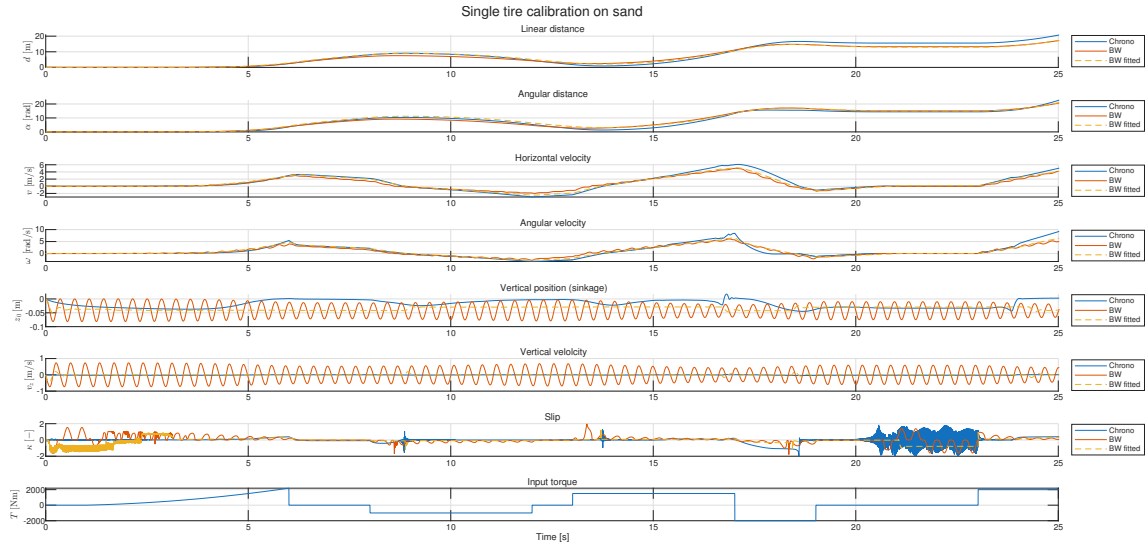


Figure 3.8: Calibration of BW model on sand by using least-squares fitting to simulated data in Project Chrono. Comparison with uncalibrated BW model is also included as reference.

3.3 Tire model considerations

As several slip definitions exists in the literature [1, 6, 18] one needs to be cautious about which one to use along with the shear deformation formulation (2.9). The chosen combination of slip definition (2.7) and shear deformation (2.9) was therefore chosen according to [6, 18] where the slip definition was implemented as

$$\kappa = \frac{r\omega - v_x}{\max(|r\omega|, |v_x|, \epsilon)}, \quad \epsilon = 0.01, \quad (3.11)$$

in order to handle both longitudinal velocity directions and remain numerically stable through ϵ . Even though not explicitly mentioned, a slip definition using only positive slip values even when braking is possibly suggested in [1]. As the tire model has been successfully validated, further analysis of which slip definition to use in off-road conditions is left for further exploration.

As mentioned in the scope of the project, the model is restricted to handle the case when the tire is considered as rigid and completely circular. This assumption is based on, for example, the inflation pressure [1] of the tire. The tire used in Chrono for validation purposes was also chosen as rigid. In reality, the tires are likely not completely circular even under the circumstances when the model considers it to be. It can therefore be interesting to further validate the model against tires which are not completely rigid as the performed validation process compares a rigid tire to another rigid tire which is probably not the case in real world settings.

3.4 Vehicle model

3.4.1 Transient load transfer

The vehicle to be analyzed is shown in Fig. 3.9 and its parameters are given in Tab. 3.6. During acceleration and braking, the vehicle experiences a pitching moment around its center of mass which affects the vertical load on the wheel axles. When braking, vertical load is transferred from the rear axles to the front axle which may have significant impact on the braking performance [1]. The vehicle is modelled as a rigid body under influence of horizontal acceleration a with a rear bogie axle sharing the vertical load equally among the rear axles. By denoting the height from the ground to the center of mass as h and the vehicle mass as M , the following system of equations describes the vertical equilibrium, equilibrium around center of mass and equilibrium around the bogie axle

$$\begin{cases} W_f + W_{r1} + W_{r2} - Mg = 0 \\ -W_f L_f + W_{r1} L_{r1} + W_{r2} (L_{r1} + L_{r2}) - Mah = 0 \\ -W_{r1} \frac{L_{r2} - L_{r1}}{2} + W_{r2} \frac{L_{r2} - L_{r1}}{2} = 0 \end{cases}, \quad (3.12)$$

where W_f , W_{r1} and W_{r2} are the vertical loads on the front, and rear axles corresponding to their distances from the center of mass L_f , L_{r1} and L_{r2} . Solving for the vertical loads using the simplified notation for the rear loads

$$W_r = W_{r1} = W_{r2}, \quad (3.13)$$

yields

$$\begin{cases} W_r = \frac{MgL_f + Mah}{L_{r1} + L_{r2} + 2L_f} \\ W_f = Mg - 2W_r \end{cases} \quad (3.14)$$

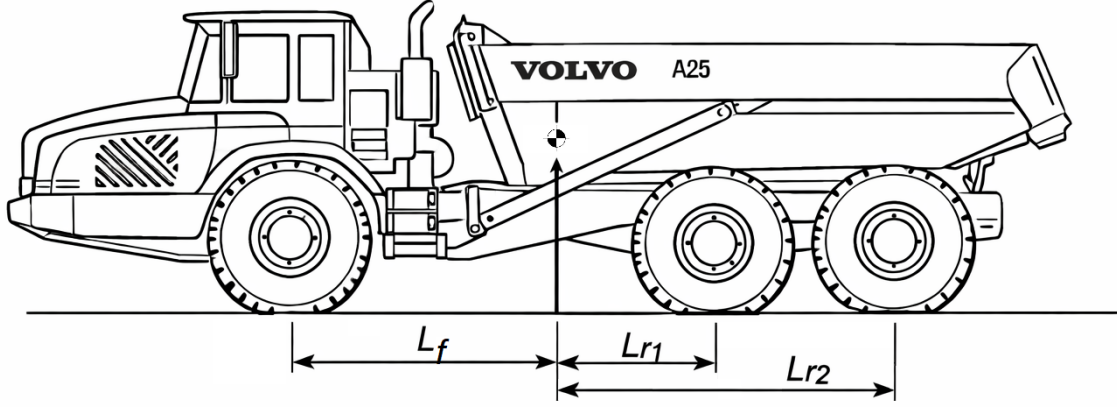


Figure 3.9: Schematic illustration of the vehicle to be analyzed. It has 1 front wheel axle and two rear wheel axles. The image is initially generated by ChatGPT and thereafter modified manually. L_f is the distance from center of mass to front wheel axle, L_{r1} is the distance from center of mass to the first rear wheel axle and L_{r2} is the distance from center of mass to the second rear wheel axle.

Table 3.6: Vehicle parameter values. The parameters of the tires are those given in Tab. 3.1. The distances L_f, L_{r1}, L_{r2} are explained in Fig. 3.9. The height h is the vertical distance from the ground to the center of mass and M is the vehicle mass.

Parameter	L_f [m]	L_{r1} [m]	L_{r2} [m]	M [kg]	h [m]
Value	3.0	1.0	2.0	$20 \cdot 10^3$	1.5

The vertical loads W_f, W_r and acceleration a are, however, mutually dependent and for simulation purposes the previous acceleration sample was used although one may consider solving for a solution of the acceleration in every iteration of the simulation. In the previous attempt to fit a damping coefficient to accurately model the vertical position z_0 of the single wheel, an equilibrium was found as seen in Fig. 3.5 - 3.8. As the damping coefficient ζ was calibrated for a single tire producing less vertical load onto the ground, it is uncertain how this damping translates to the vehicle model where a significant increase in vertical load will be applied from each tire onto the ground. Given this uncertainty and the notion that the single tire often settled at an equilibrium sinkage, it was decided that an equilibrium sinkage for each tire was to be solved for in each iteration of the vehicle model simulation given its dynamic vertical load $W_i, i \in \{f, r\} = \mathcal{A}$ by solving

$$F_{z,i}(z_0) - \frac{W_i}{2} = 0, \quad (3.15)$$

for z_0 according to (2.14), (2.15) since the vertical loads on an axle, W_i , is distributed over two tires.

3.4.2 Vehicle state space model

The following states are used to represent the vehicle state space model

$$x = \begin{bmatrix} d \\ \alpha_f \\ \alpha_{r1} \\ \alpha_{r2} \\ v \\ \omega_f \\ \omega_{r1} \\ \omega_{r2} \end{bmatrix}, \quad (3.16)$$

with dynamics given by

$$\dot{x} = \begin{bmatrix} v \\ \omega_f \\ \omega_{r1} \\ \omega_{r2} \\ \frac{2(F_{x,f} + F_{x,r1} + F_{x,r2})}{M} \\ \frac{2T_f - 2F_{t,f} \cdot r}{2J} \\ \frac{2T_{r1} - 2F_{t,r1} \cdot r}{2J} \\ \frac{2T_{r2} - 2F_{t,r2} \cdot r}{2J} \end{bmatrix} = \begin{bmatrix} v \\ \omega_f \\ \omega_{r1} \\ \omega_{r2} \\ \frac{2(F_{x,f} + F_{x,r1} + F_{x,r2})}{M} \\ \frac{T_f - F_{t,f} \cdot r}{J} \\ \frac{T_{r1} - F_{t,r1} \cdot r}{J} \\ \frac{T_{r2} - F_{t,r2} \cdot r}{J} \end{bmatrix}, \quad (3.17)$$

where $F_{x,i}$ is the resulting horizontal force from one tire on axle $i \in \{f, r1, r2\}$, $F_{t,i}$ is the traction force generated from shear stress as in (2.12) and T_i is the input torque applied to a wheel on axle i . As mentioned previously, the vertical load applied to each wheel axle is calculated with (3.14) using the translational acceleration $a = \dot{x}_5$ from the previous sample. From this, the equilibrium entry angle θ_f is solved in every iteration by numerically solving (3.15). Since the vertical load is equally split among the rear axes, the only thing distinguishing axle $r1$ and $r2$ is the terrain underneath them. For example, whether one wheel enters a previously non-compacted terrain point while the other does not and is subject to the multipass effect. The model keeps track of its longitudinal position d and since the distances to each axle L_f, L_{r1}, L_{r2} are known, it is possible for the model to know if a certain tire is approaching a soil area previously compacted by itself or not by also utilizing the longitudinal direction v_x . In a braking scenario, the vehicle is moving forward

and both rear axles enter terrain previously compacted by the front tires. Both rear axles will therefore use the same multipass soil parameters q_m and their dynamics are hence equal in that case.

3.5 Trajectory planning

When driving on paved roads, there is usually a slip between -30% and -10% where the braking force is maximized and is the reason why the objective of ABS is to keep the slip in this interval. This relationship can be seen in Fig. 3.10 where the peak tractive force, for accelerating and braking, happens when slip is around ± 0.1 .

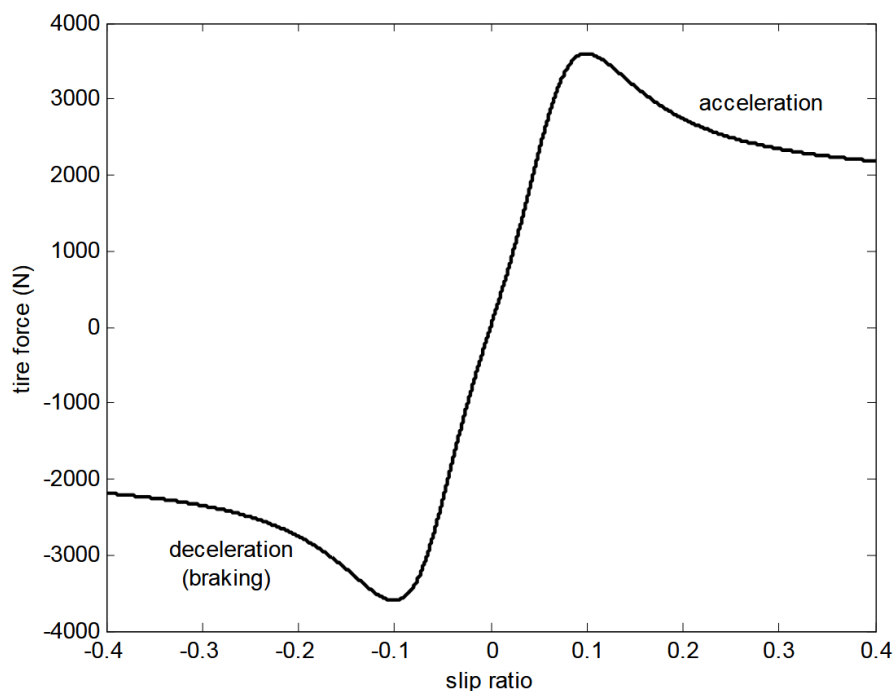


Figure 3.10: Force-slip curve on paved road [2]. The tractive effort reaches a peak value when the slip is approximately ± 0.1 .

In off-road conditions the peak tractive effort often, however, appears at the extreme slip values [15, 6]. This is the case for the model in (2.19) which can be seen in Fig. 3.11 where the longitudinal force F_x is plotted over slip κ for the different terrain types. As can be seen, the force curve is monotonically increasing for increasing slip. A controller with the purpose of minimizing the braking distance will therefore likely lock the wheels where $\kappa = -1$, or even spin the wheels backwards if possible. This action may, however, compromise the controllability of the vehicle in lateral and yaw dimensions as the vehicle may start to skid. Compromising lateral traction resulting in skid may be dangerous and pursuing such a braking scenario is not desirable. It is therefore of interest to consider the lateral dynamics of the vehicle to some extent. As a rule of thumb, the lateral traction must not decrease by more than 50 % from its maximum value. This may therefore be a target for a controller with the purpose of

minimizing braking distance while not sacrificing lateral traction too much, leaving the possibility to a driver or external system to control the lateral motion of the vehicle. There is, however, a risk in closed loop systems when tracking a reference signal, to overshoot the reference and in this case, therefore, risk compromising lateral traction by more than 50 %. A more conservative aim is therefore to minimize braking distance while not losing lateral traction by more than 40 % from its peak value. The force slip curves in Fig. 3.11 can be a reason why the use of ABS in off-road conditions sometimes results in longer braking distances. In this thesis, the slip is defined as negative when braking. Given that definition, ABS regulates aims at keeping the slip in the interval $\kappa \in [-0.3, -0.1]$ where the braking force $-F_x$ is not maximized for any of the tested terrain types. It is also worth noting that the force-slip curves does not cross the origin. One reason for that is that the compaction resistance R_c is included in the braking force which biases the braking F_x to become more negative.

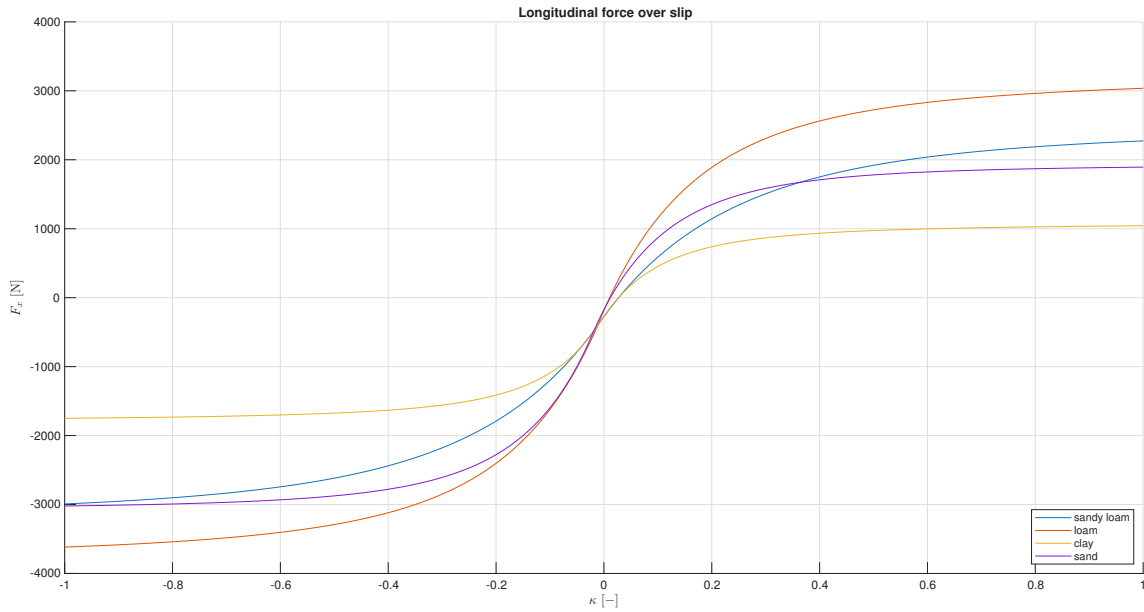


Figure 3.11: Force-slip curves when driving on different terrains.

Recall from (2.19) that the exit angle θ_r is zero for purely longitudinal motion but not necessarily zero when including lateral motion. Thus, it is necessary to find the exit angle θ_r and the angle θ_m where normal stress is maximized in order to explore the lateral dynamics of the tire model. The exit angle may very well depend on both the longitudinal slip κ and slip angle β . Nevertheless, the only estimation [23] found for the exit angle is only dependent on the longitudinal slip as

$$\theta_r = -0.01 \theta_f \kappa, \quad (3.18)$$

which will serve as exit angle estimation while finding θ_m from (2.20) is done numerically. As the terrain is ambiguous to the direction of shear stress in the tangential plane of the tire peripheral, only the absolute shear displacement matters in the pur-

suit of the total shear displacement. Thus, the total shear displacement is calculated as

$$\dot{j}_{\text{tot}} = \sqrt{\dot{j}_x^2 + \dot{j}_y^2}, \quad (3.19)$$

and the total shear stress as

$$\tau_{\text{tot}} = \tau_{\text{max}} \left(1 - e^{-\dot{j}_{\text{tot}}/k}\right). \quad (3.20)$$

Following this, the stress in longitudinal and lateral direction may be treated as proportional contributions to the total shear displacement τ_{tot} as

$$\tau_x = \tau_{\text{tot}} \frac{\dot{j}_x}{\dot{j}_{\text{tot}}}, \quad (3.21a)$$

$$\tau_y = \tau_{\text{tot}} \frac{\dot{j}_y}{\dot{j}_{\text{tot}}}. \quad (3.21b)$$

It is now possible to illustrate the possible combinations of longitudinal and lateral forces. In Fig. 3.12 it can be seen that lateral force F_y decrease as F_t becomes more negative which is the case when braking. As decreasing F_t is caused by decreasing slip κ , there exists a slip value for $\kappa < 0$ where F_y drops by 40 % from its peak value which is to be found. Lateral bulldozing resistance increase the lateral force as in (2.24) but since the value of F_b is uncertain while controllability is a desired property, only the traction generated by the longitudinal slip, F_y , is considered here. A brute force approach for finding the greatest possible deceleration while not sacrificing lateral traction more than 40 % from its peak value is described in Alg. 1.

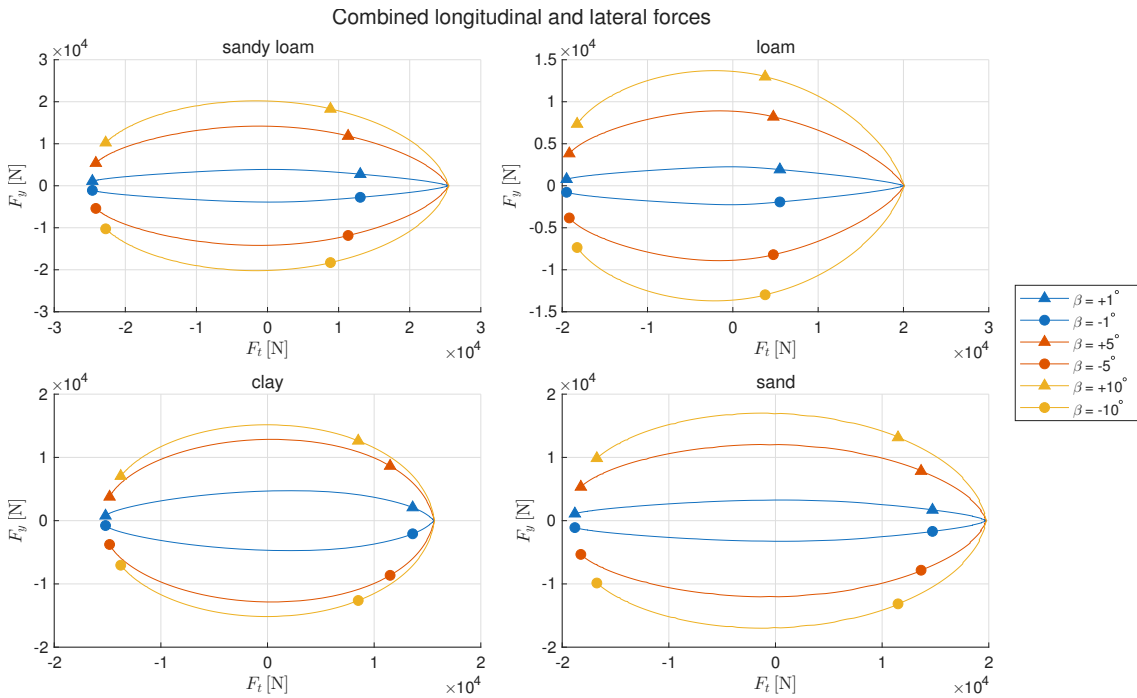


Figure 3.12: Combined longitudinal and lateral forces.

Algorithm 1 Maximum feasible braking deceleration

```

1: for trial decelerations  $a_t \in [0, \dots, -g]$  do
2:   Compute axle loads  $W_f(a_t), W_r(a_t)$ 
3:   Find admissible  $F_{x,f}(W_f), F_{x,r}(W_r)$  such that
      
$$|F_{y,i}(\kappa, \beta)| \geq 0.6 |F_{y,i,\max}(\kappa, \beta)| \quad \forall \kappa \in [-1, 1], \beta \in \mathcal{B}, i \in \mathcal{A}$$

4:   Compute achievable deceleration
      
$$a_a = \frac{2F_{x,f} + 4F_{x,r}}{m}$$

5:   if  $a_a > a_t$  then
6:     break, deceleration  $a_a$  not feasible
7:   else
8:     cache  $a_a$  as current best achievable deceleration
9:   end if
10: end for
11: return  $a_a$ 

```

The magnitudes of the lateral forces F_y are considered as the the slip angle β may be in either direction given that the the set of slip angles tested is

$$\mathcal{B} = \{\pm 2^\circ, \pm 4^\circ, \pm 6^\circ, \pm 8^\circ, \pm 10^\circ\}. \quad (3.22)$$

As mentioned previously, in this scenario when the vehicle is braking, the dynamics of the rear axle tires are equal. Alg. 1 utilizes this notion where rear axle tire forces are simply denoted $F_{x,r}$ and equal for all four rear tires. The result from this algorithm is presented in Tab. 3.7 for each type of soil. The maximum possible decelerations for all types of terrain are similar between 4 m/s^2 and 5 m/s^2 . The front tires contribute with roughly three times greater braking force compared to the rear tires following increased vertical load due to the induced pitch while braking. This also means that the rear tires experience less vertical load compared to an equilibrium state with zero deceleration. It is also noticeable that lateral traction is compromised earlier on clay, in terms of longitudinal slip κ , compared to the other terrain types. This is a limiting factor since a terrain agnostic braking system needs to consider this limiting case where slip may not be allowed to drop to more than roughly -0.24 on the front tires and -0.20 on the rear tires. This may result in sub-optimal braking performance on terrain types other than the limiting one, which is clay.

Table 3.7: Minimum achievable braking acceleration and corresponding slip and traction forces for different terrain types.

Soil	min a_a [m/s ²]	Front κ_f [-]	Rear κ_r [-]	$F_{x,f}$ [N]	$F_{x,r}$ [N]
Sandy loam	-4.992	-0.3265	-0.4490	$-3.07 \cdot 10^4$	$-8.96 \cdot 10^3$
Loam	-4.056	-0.3878	-0.5918	$-2.32 \cdot 10^4$	$-8.41 \cdot 10^3$
Clay	-4.205	-0.2449	-0.2041	$-2.72 \cdot 10^4$	$-7.32 \cdot 10^3$
Sand	-4.783	-0.4694	-0.4490	$-2.91 \cdot 10^4$	$-8.76 \cdot 10^3$

3.6 Simplified tire model

For model based controllers, the dynamic model of the system is of great importance. As no closed form solutions to the tire model integrals (2.19) have been found [18], dealing with these integrals to grant a controller knowledge of the system dynamics is a topic under exploration [6]. This thesis presents a simple model to replace the integrals in the tire model. When inspecting the force-slip curves in Fig. 3.11 it is evident that the shape of these curves are similar to the shapes of the sigmoid and tangens hyperbolicus functions which are defined as

$$\text{sig}(x) = \frac{1}{1 + \exp^{-x}}, \quad (3.23a)$$

$$\tanh(x) = \frac{\exp^x - \exp^{-x}}{\exp^x + \exp^{-x}}. \quad (3.23b)$$

The horizontal force F_x consists of two terms, namely traction force and compaction resistance as in (2.13). When inspecting the force-slip curves in Fig. 3.11 it is noticeable that they do not cross the origin which may be because compaction resistance biases F_x towards a resistive force. In F_x , it is only the tractive force F_t , that is slip dependent while compaction resistance R_c is not slip dependent. Both are, however, sinkage dependent through normal stress σ_n and therefore relies on the vertical load. The candidate simplified model is therefore split into two terms where one term represents F_t and the other represents R_c . Therefore, the sigmoid shaped functions in (3.23) remain as candidate models for simplifying F_t while R_c is simplified as an affine function of vertical load. As it is desired that the simplified model for F_t is a function of both slip and vertical load, curvefitting of the candidate functions (3.23) is done against F_t for a subset of vertical loads resulting in different sinkages z_0 . When braking, vertical load is transferred from the rear tires to the front tires following the induced pitch from the deceleration as in (3.14). Let W_f, W_r be the static loads on the front and rear tire axles respectively when the vehicle is not accelerating. When braking, greater vertical load is experienced on the front tires why the set of vertical loads used for curvefitting the front tire models was chosen to $\mathcal{W}_f = \{W_f, 1.2 W_f, 1.4 W_f, 1.6 W_f\}$. Equivalently, as vertical load decreases on the rear tire axles, the set of vertical loads used for that model was chosen to $\mathcal{W}_r = \{W_r, 0.8 W_r, 0.6 W_r, 0.4 W_r\}$. The front tires use calibrated terrain parameters for singlepass while the rear tires use calibrated terrain parameters for

3. Methods

the multipass case. This since the front tires compacts the terrain for the rear tires, assuming the vehicle is driving in untouched terrain. As mentioned, an affine function of slip was used to simplify R_c . But for F_t , the weight vector $w_s \in \mathbb{R}^4$ were included in the candidate functions (3.23) as

$$\text{sig}_{w_s}(\kappa) = W (w_{s1} \text{sig}(w_{s2} \kappa + w_{s3}) + w_{s4}) \quad \forall W \in \mathcal{W}_i, \quad (3.24a)$$

$$\tanh_{w_s}(\kappa) = W (w_{s1} \tanh(w_{s2} \kappa + w_{s3}) + w_{s4}) \quad \forall W \in \mathcal{W}_i, \quad (3.24b)$$

for each tire axle $i \in \mathcal{A}$. Curvefitting was performed for each candidate function (3.24) with regards to the optimization variable w_s in the least square sense. The result of this optimization along with the affine curvefitting can be seen in Fig. 3.13, 3.14 for the front and rear tires respectively.

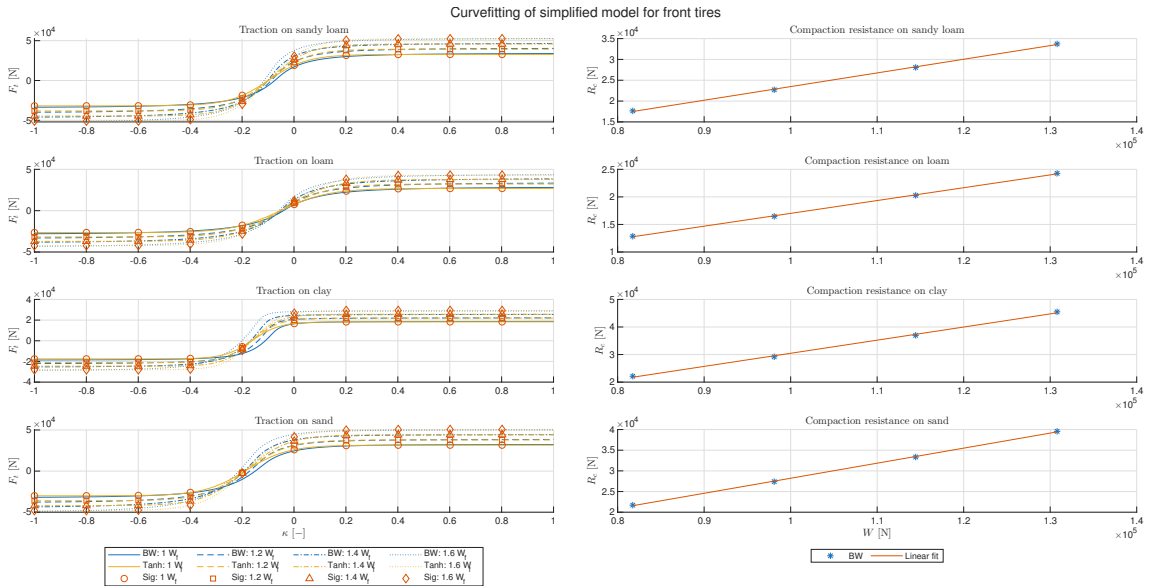


Figure 3.13: Curvfitting of simplified tire models to match the BW model better for the front tires. The left column shows the traction force F_t and the right column shows the compaction resistance R_c on the different terrain types.

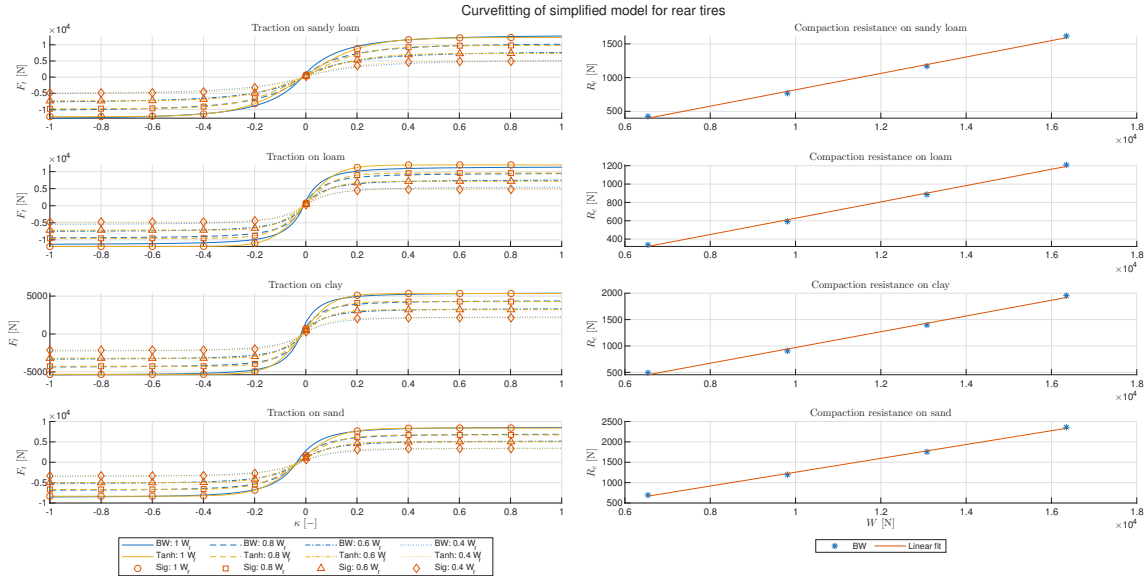


Figure 3.14: Curvefitting of simplified tire models to match the BW model better for the rear tires. The left column shows the traction force F_t and the right column shows the compaction resistance R_c on the different terrain types.

In the left column in i Fig. 3.13 and 3.14, where the goal is to find a simplification for F_t , both candidate functions in (3.24) exhibit seemingly similar result. This is reflected in the average root mean square error which is practically indistinguishable between the candidates. The sigmoid function was selected to use as a simplified tire model for the design of the model based controller as it performed equivalently as the tangens hyperbolicus function but may bring advantages over the tangens hyperbolicus function in terms of simplicity as it is to be linearized when designing the controller.

3.7 Controller design

The $\mathcal{H}_\infty$ controller setup is defined for a linear nominal plant $G_{\text{nom}}(s)$. As the sandy loam terrain type reasonably includes characteristics similar to the terrain types sand and loam, one may argue that sandy loam remains close these terrain types in terrain parameter space. This analysis is however not done but the sandy loam terrain type was chosen for the nominal plant based on this logic with the terrain type names. Recall that a simplified model was developed for the tire model which will be the model that the control design is based on. The system (3.17) is not linear in the states and inputs and therefore needs to be linearized. For simplicity, the integration states x_{1-4} are dropped since positions are not needed for slip control. This leaves the velocity states x_{5-8} with simplified forces F_x, F_t and R_c to base the controller on. As one slip controller is desired to work in all terrain types, there is a need to define a common goal for the controller based on the acquired maximum decelerations in Tab. 3.7. The chosen strategy is to choose the most conservative slip based on Tab. 3.7 to not risk losing too much lateral traction. Therefore, the slip values from clay are chosen as the slip values to track

when braking the vehicle, i.e. $\kappa_f = -0.2449$, $\kappa_r = -0.2041$. The rear tire slip values are duplicated for both rear axles as the dynamics for these are equal as they both will experience similar multipass effect when driving forward while being subject to equal vertical load following the bogey axle setup. The rear tire slip target κ_r is therefore duplicated to use for the first and second rear axles. This results in the slip operating point

$$\kappa_0 = \begin{bmatrix} \kappa_{0,f} \\ \kappa_{0,r} \\ \kappa_{0,r} \end{bmatrix} = \begin{bmatrix} \kappa_f \\ \kappa_{r1} \\ \kappa_{r2} \end{bmatrix} = \begin{bmatrix} -0.2449 \\ -0.2041 \\ -0.2041 \end{bmatrix}, \quad (3.25)$$

which will also serve as the premiss when linearizing the model around longitudinal and angular velocities. As these slip values are derived from driving on clay, and not the nominal plant which is sandy loam, the corresponding target acceleration needs to be found from these slips on the nominal plant. Using the values in (3.25) and the equations (3.14), (3.15), this was found to be

$$a_0 = -3.3153 \text{ m/s}^2, \quad (3.26)$$

with corresponding traction

$$F_{t0} = \begin{bmatrix} -1.0762 \cdot 10^4 \\ -7.2417 \cdot 10^3 \\ -7.2417 \cdot 10^3 \end{bmatrix} \text{ N}, \quad (3.27)$$

for each tire axle. As the test case is to brake the vehicle from $30 \text{ km/h} \approx 8.3 \text{ m/s}$, the linearization point for longitudinal velocity v_x was selected to

$$v_0 = 7 \text{ m/s}, \quad (3.28)$$

in an attempt to create a stabilizing controller for the initial phase of braking and then, possibly, allowing for gain scheduling as v_x decreases during braking. Using the slip calculation for the braking case in (2.7) as

$$\kappa_b(v_x, \omega) = \frac{r\omega}{v_x} - 1, \quad (3.29)$$

gives the operating point for angular velocity

$$\omega_0 = \begin{bmatrix} \omega_{0,f} \\ \omega_{0,r1} \\ \omega_{0,r2} \end{bmatrix} = \begin{bmatrix} 5.8730 \\ 6.1903 \\ 6.1903 \end{bmatrix} \text{ rad/s}. \quad (3.30)$$

The baseline torque can be found from (3.17), (3.25), (3.26), (3.27), (3.29) as

$$u_{0,i} = T_{0,i} = \dot{\omega}_i \cdot J + F_{t0,i} \cdot r = (1 + \kappa_{0,i}) \frac{a_0}{r} \cdot J + F_{t0,i} \cdot r, \quad i \in \mathcal{A}, \quad (3.31)$$

by treating slip as constant around this operating point. This yields the input torque operation point

$$u_0 = T_0 = \begin{bmatrix} T_{0,f} \\ T_{0,r1} \\ T_{0,r2} \end{bmatrix} = \begin{bmatrix} -1.0249 \\ -0.7111 \\ -0.7111 \end{bmatrix} \cdot 10^4 \text{ Nm}. \quad (3.32)$$

The state dynamics in $G_{\text{nom}}(s)$ is hence derived by linearizing $\dot{x}_{5-8}$ around the state and input operating points v_0, ω_0 and u_0 . Often u_0 is the input required to keep a system in an equilibrium $\dot{x} = 0$. In this case, however, the system is linearized around velocities resulting in negative slip, $\kappa < 0$, corresponding to a braking trajectory where states are not in an equilibrium point but are decreasing. As the braking slip definition for a single tire (3.29) is not linear in the states, this also needs to be linearized in order to be included in the $\mathcal{H}_\infty$ control framework. For this, first order Taylor expansion is used as

$$\begin{aligned} \kappa(v_x, \omega) &\approx \kappa(v_0, \omega_0) + \frac{\partial \kappa(v_0, \omega_0)}{\partial v_x} (v_x - v_0) + \frac{\partial \kappa(v_0, \omega_0)}{\partial \omega} (\omega - \omega_0) \\ &= \kappa(v_0, \omega_0) + \frac{-r\omega_0}{v_0^2} (v_x - v_0) + \frac{r}{v_0} (\omega - \omega_0), \end{aligned} \quad (3.33)$$

and by defining the error coordinates as

$$\delta\kappa = \kappa(v_x, \omega) - \kappa(v_0, \omega_0) = \kappa - \kappa_0, \quad (3.34a)$$

$$\delta v_x = v_x - v_0, \quad (3.34b)$$

$$\delta\omega = \omega - \omega_0, \quad (3.34c)$$

$$\delta u = u - u_0, \quad (3.34d)$$

the slip deviation $\delta\kappa$ can be expressed as linear in the state deviation variable δx as

$$\begin{aligned} \delta\kappa &\approx \begin{bmatrix} \frac{-r\omega_{0,f}}{v_0^2} \delta v_x + \frac{r}{v_0} \delta\omega_f \\ \frac{-r\omega_{0,r1}}{v_0^2} \delta v_x + \frac{r}{v_0} \delta\omega_{r1} \\ \frac{-r\omega_{0,r2}}{v_0^2} \delta v_x + \frac{r}{v_0} \delta\omega_{r2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-r\omega_{0,f}}{v_0^2} & \frac{r}{v_0} & 0 & 0 \\ \frac{-r\omega_{0,r1}}{v_0^2} & 0 & \frac{r}{v_0} & 0 \\ \frac{-r\omega_{0,r2}}{v_0^2} & 0 & 0 & \frac{r}{v_0} \end{bmatrix} \begin{bmatrix} \delta v_x \\ \delta\omega_f \\ \delta\omega_{r1} \\ \delta\omega_{r2} \end{bmatrix} \\ &= C_\kappa \begin{bmatrix} \delta v_x \\ \delta\omega_f \\ \delta\omega_{r1} \\ \delta\omega_{r2} \end{bmatrix} = C_\kappa \delta x. \end{aligned} \quad (3.35)$$

The nominal plant $G_{\text{nom}}(s)$ is designed to measure its states as well as the slip errors by utilizing (3.35) such that

$$C = \begin{bmatrix} I_{4 \times 4} \\ C_\kappa \end{bmatrix} \in \mathbb{R}^{7 \times 4}. \quad (3.36)$$

Hence, the error coordinates of states and slips are obtained from the nominal plant according to

$$\delta y = \begin{bmatrix} \delta v_x \\ \delta v_f \\ \delta \omega_f \\ \delta \omega_{r1} \\ \delta \omega_{r2} \\ \delta \kappa_f \\ \delta \kappa_{r1} \\ \delta \kappa_{r2} \end{bmatrix} = G_{\text{nom}}(s) \delta u. \quad (3.37)$$

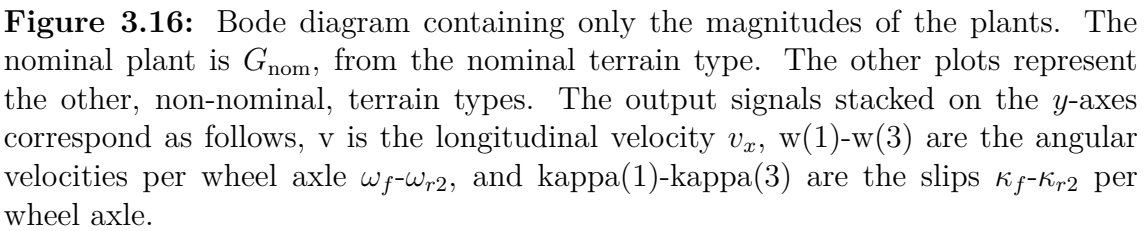
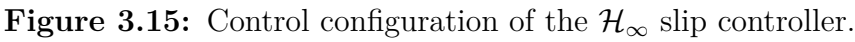
Following the same linearization procedure also for the non-nominal plants, which are the terrain types loam, sand and clay, a set of linear systems is obtained. By passing these four plants to the Matlab function *ucover*, the weight $W_\Delta(s)$ associated with the uncertainty Δ is returned. The uncertain model is then defined as

$$G_{\text{unc}}(s) = G_{\text{nom}}(s) + \Delta W_\Delta(s), \quad (3.38)$$

and the magnitude bode diagram and sigma value plot of the different plants representing different terrain types are shown in Fig. 3.16 and 3.17 respectively. The control configuration is presented in Fig. 3.15 where $\delta \tilde{y}$ represents the measurements from the uncertain model, i.e. the nominal measurements in error coordinates, δy , with additive uncertainties δu_Δ . The matrix S is not explicitly implemented but represents that only a subset of the elements in $\delta \tilde{y}$ are selected for the following summation block, namely the derived slip errors $\delta \tilde{\kappa} \in \mathbb{R}^3$. Therefore, S represents the matrix

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3.39)$$

which selects the last 3 elements of $\delta \tilde{y}$. Hence $\delta \tilde{\kappa}$ are the uncertain, approximated, slip values from the Taylor expansion (3.33) in the linearized coordinate frame.



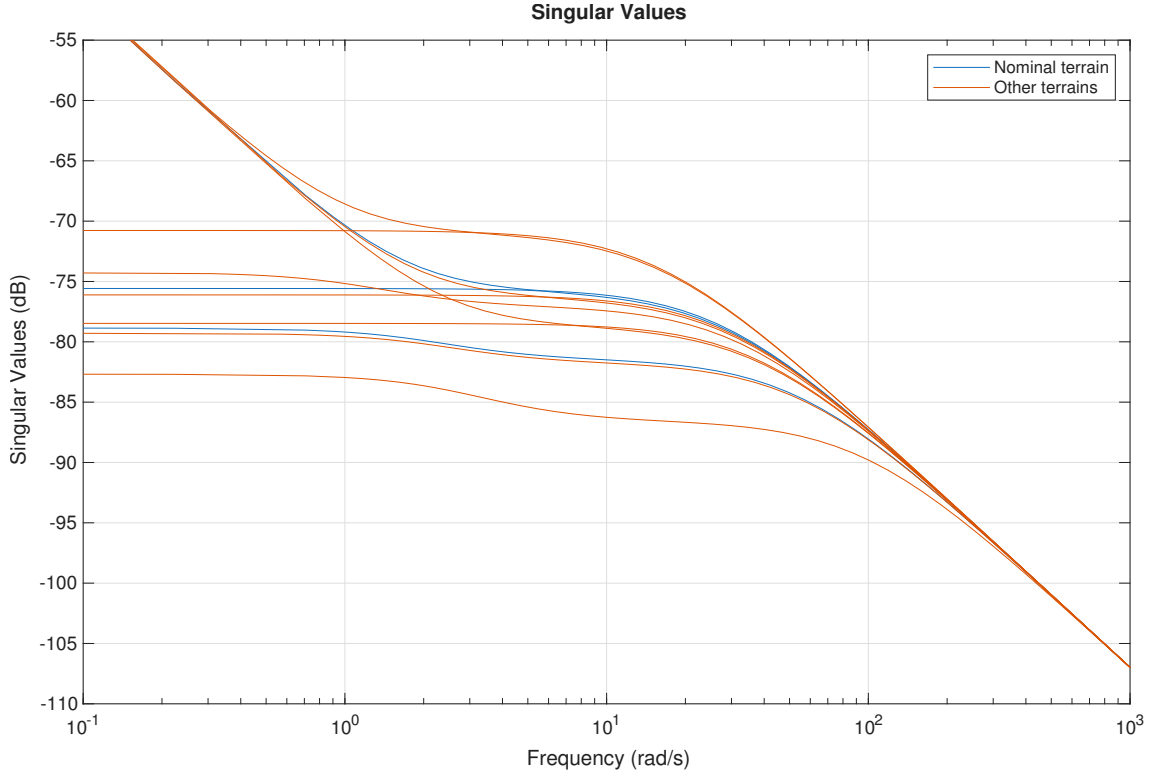


Figure 3.17: Singular value plot of the nominal plant, and the non-nominal plants from the other terrain types.

In the control configuration, $r_\kappa \in \mathbb{R}^3$ represents the slip reference values. These are filtered through $W_R(s)$ to represent a smoothly demanded slip, therefore avoiding unrealistic slip commands. This also partly compensates for the lack of actuator dynamics as it avoids aggressive reference jumps. The resulting reference signal is denoted $\tilde{r}_\kappa$ and is expressed in absolute values rather than relative to the slip linearization point κ_0 . As mentioned, the term κ_0 does not represent an equilibrium point, $\dot{x} \neq 0$, but the slip reference value causing the vehicle to brake with the desired slip, $\dot{x} < 0$. Since the uncertainty formulation adds uncertain dynamics to the nominal measured slip κ , the error coordinates in (3.34) also apply to the uncertain slip measurements $\tilde{\kappa}$ as

$$\delta\tilde{\kappa} = \tilde{\kappa} - \kappa_0. \quad (3.40)$$

By using deviation variables for slip, $\delta\tilde{\kappa}$, and absolute values for reference, $\tilde{r}_\kappa$, the linearization point gets cancelled in the error computation, resulting in

$$e = \tilde{\kappa} - \tilde{r}_\kappa. \quad (3.41)$$

Each element in the reference signal vector is filtered with the scalar transfer function

$$W_r(s) = \frac{1}{0.25s + 1}, \quad (3.42)$$

representing a low pass filter with a time constant of 250 ms and its magnitude bode diagram is shown in Fig. 3.18. This weight is applied to all elements in $r_\kappa \in \mathbb{R}^3$ through

$$W_R(s) = \begin{bmatrix} W_r(s) & 0 & 0 \\ 0 & W_r(s) & 0 \\ 0 & 0 & W_r(s) \end{bmatrix} = W_r(s) \cdot I_{3 \times 3}. \quad (3.43)$$

Similar to the bode diagram of the plant, only the magnitude of the bode diagrams are of interest in this framework, and not the phase. Hence, for all bode diagrams, only the magnitudes are considered although they may be referred to as bode diagrams rather than magnitude bode diagrams.

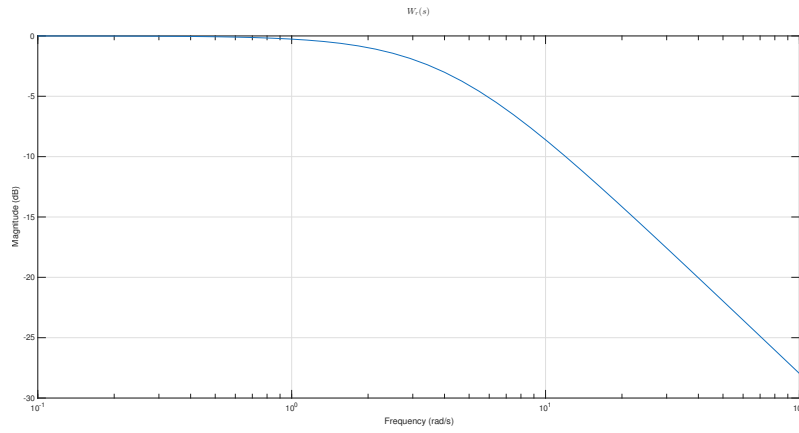


Figure 3.18: Magnitude bode diagram of $W_r(s)$.

As mentioned, the weight W_Δ is returned by Matlab in order to represent the uncertain plants and has the value

$$W_\Delta(s) = \frac{5.254 \cdot 10^{-9} s + 0.00136}{s + 9.272}. \quad (3.44)$$

The tuning of the controller is mainly done by adjusting the weights $W_e(s)$ and $W_u(s)$ which resulted in

$$W_e(s) = \frac{0.1 s + 1}{s + 0.1}, \quad (3.45)$$

and

$$W_u(s) = \frac{-7.4 \cdot 10^{-5} s - 7.515 \cdot 10^{-4}}{s + 0.2031}. \quad (3.46)$$

As for the reference, these scalar transfer functions apply to each element in their respective input vectors as

$$W_E(s) = W_e(s) \cdot I_{3 \times 3}, \quad (3.47a)$$

$$W_U(s) = W_u(s) \cdot I_{3 \times 3}, \quad (3.47b)$$

which can be seen in the control configuration in Fig. 3.15. The bode diagrams for (3.45) and (3.46) are shown in Fig. 3.19 and 3.20 respectively. The performance outputs from these weights are denoted z_e and z_u , and needs to be properly balanced through their corresponding weights in order to attain adequate closed loop performance. As seen in (3.25) and (3.32), the input signals are far greater than the slip values at the linearization point and needs to be addressed accordingly. Therefore, the gain from $W_u(s)$ is roughly -48 dB for low frequencies and -83 dB for high frequencies while corresponding gains from $W_e(s)$ are 20 dB and -20 dB respectively. For example, at 1 rad/s, the front tire torque performance output at the linearization point is

$$\begin{aligned} z_{u,f} &= |W_u(j \cdot 1)| u_{0,f} \approx (-63 \text{ dB}) \cdot (-1.0249 \cdot 10^4 \text{ Nm}) \\ &\approx 7.0795 \cdot 10^{-4} \cdot (-1.0249 \cdot 10^4 \text{ Nm}) \approx -7 \text{ Nm}. \end{aligned} \quad (3.48)$$

Assume the front tire slip error is equal to the front tire slip linearization point, $\kappa_{0,f}$, then this performance output would be

$$z_{e,f} = |W_e(j \cdot 1)| \kappa_{0,f} \approx 0 \text{ dB} \cdot (-0.2449) = -0.2449, \quad (3.49)$$

at 1 rad/s, which is in roughly the same range as the input penalty in (3.48).

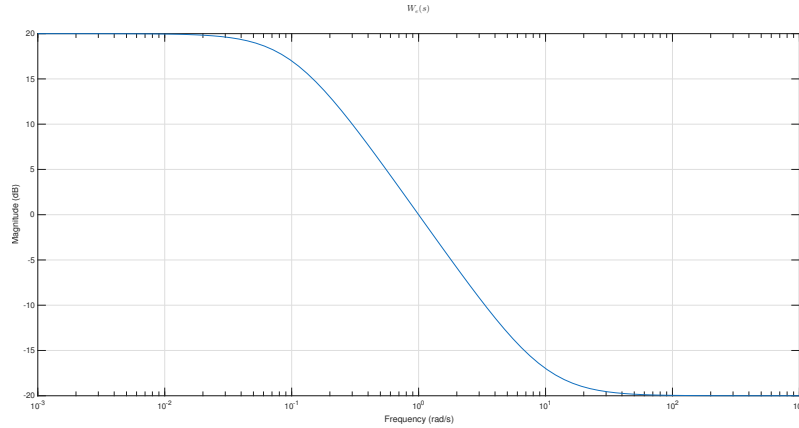


Figure 3.19: Magnitude bode diagram of $W_e(s)$.

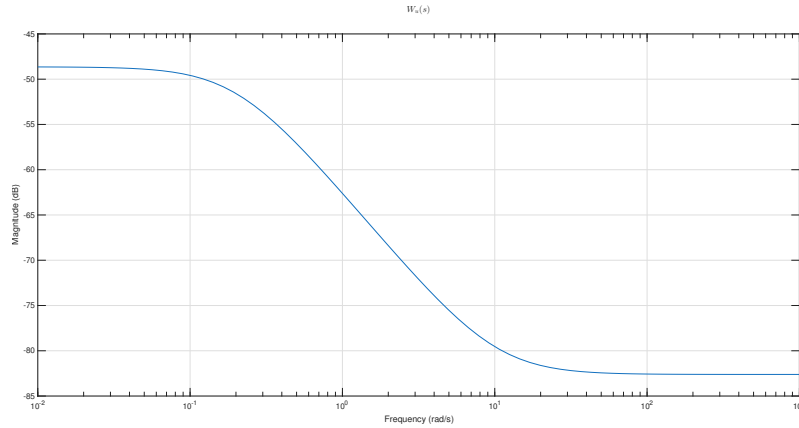


Figure 3.20: Magnitude bode diagram of $W_u(s)$.

The slip error weight $W_e(s)$ penalizes low frequent deviations from the slip reference more than deviations of higher frequencies. Slip may change rapidly following, for example, irregularities in the terrain. Real actuators may have inertia and slower dynamics compared to such rapid slip errors deviations, making it impossible to account for high frequent slip errors. Such errors may also have other, unbiased, causes such as zero mean measurement noise and tracking such high frequent slip deviations is therefore not meaningful if the low frequent content of the slip signal follows the slip reference. Lower frequent slip errors are, however, penalized more according to $W_e(s)$ since it is desired to attenuate low frequent slip deviations such as steady state errors. The input weight $W_u(s)$ is used to balance the ambition of reducing slip errors with moderate control effort in hope of finding a smooth braking force trajectory. As $W_e(s)$ and $W_u(s)$ have a similar shape where low frequencies are penalized more than high frequencies they may appear to counteract each other. Instead, they are creating a balanced trade-off between control effort and slip error attenuation over the entire bandwidth. High frequent input signals are penalized less than low frequent input signals. This allows the controller to adjust its control actions to possible high frequent slip errors. These deviations, as mentioned, are less penalized than steady state errors but cannot be disregarded either. Greater DC gain in $W_u(s)$ is used to counteract excessive and perhaps unnecessary actuator effort over longer time periods. The bode plot in Fig. 3.16 and sigma plots in Fig. 3.17 give insight to which frequencies are considered low compared to high. The response from the plant decrease as input frequencies increase beyond roughly 10 rad/s which is therefore considered a limit that can categorize frequencies as low compared to high. This also means that the plant has inherent low pass properties since its response diminishes as input frequencies grow beyond this limit.

3.8 Controller Comparison

Instead of using a model based approach where all tires are included in one vehicle model, slip tracking can also be performed locally per wheel axle. In order to compare the $\mathcal{H}_\infty$ controller against such decoupled controllers, PI controllers were automatically tuned based on the control configuration and its weights in Fig. 3.15.

This is why it is occasionally referred to as a PI- $\mathcal{H}_\infty$ controller. The automatic tuning was done using the Matlab function *hinfstruct*, where the PI regulators control one wheel axle each. These controllers can be described as one diagonal matrix as

$$K_{\text{PI}}(s) = \begin{bmatrix} \frac{-2.11 \cdot 10^4 s - 6.58 \cdot 10^4}{s} & 0 & 0 \\ 0 & \frac{-1543 s - 7.474 \cdot 10^4}{s} & 0 \\ 0 & 0 & \frac{-2603 s - 6.593 \cdot 10^4}{s} \end{bmatrix}. \quad (3.50)$$

Initially, PID-controllers were tested but as they resulted in unstable closed loop behavior, PI-controllers were used instead with stable closed loop results. The expression for the $\mathcal{H}_\infty$ controller is excluded due to its large size.

4

Results

Several simulations of the closed loop system have been performed. The closed loop contains the nonlinear vehicle model (3.17) and a slip controller which can be selected in a modular fashion. When velocities decrease in magnitude, slip calculations become unstable since velocities enter in the denominator of the slip calculation (2.7). This can be seen in single tire simulations in Fig. 3.1 - 3.4 and 3.5 - 3.8 where slip oscillates with large amplitudes as velocity magnitudes become low. This is especially a risk when considering that velocity measurements and estimations are subject to measurement noise and, possibly, other uncertainties. At low velocities, one could therefore consider switching the control strategy to regulate the angular velocity of the wheels directly. The incentive of keeping lateral traction diminishes as velocities become small since the vehicle loses momentum as speed decreases. The risk of safety incidents from losing lateral traction at low velocities therefore diminishes which motivates controlling angular velocities directly, which may not consider lateral traction. The performance of the slip controllers is therefore to be evaluated based on its ability to brake the vehicle down to 1.0 m/s when the control strategy can be switched. In the following tests, the control signals are therefore set to zero when longitudinal velocity decreases below 1.0 m/s, allowing the vehicle to slowly approach standstill through compaction resistance as another control strategy is not implemented. The closed loop system used for testing the controllers on the nonlinear plant with the logic described is displayed in Fig. 4.1 where $\tilde{u}$ is the input torque used on the vehicle model which is set to zero if the vehicle longitudinal velocity drops below 1.0 m/s. The slip controller acts in the error coordinate frame why the baseline control signal u_0 is added retroactively.

The initial translational velocity is chosen to 30 km/h $\approx$ 8.3 m/s with angular velocities corresponding to zero slip. The control action begins immediately and braking distance up to the point when $v_x < 1$ m/s is displayed in the simulations in the first subplot corresponding to the covered vehicle distance during the simulation as the distance d is initialized as $d = 0$ m. Henceforth, the longitudinal velocity v_x is referred to as v as only the longitudinal direction is considered. Other quantities shown on the simulation plots are given by their titles. The markers $f, r1, r2$ correspond to the consecutive tire axles shown on the vehicle in Fig. 3.9 where f corresponds to the first tire axle while $r1, r2$ corresponds to the leading and trailing rear axles respectively. As the vehicle only moves in the longitudinal direction, the angular velocities on a given tire axle are considered equal and, thus, represents the angular velocity of both wheels on that axle.

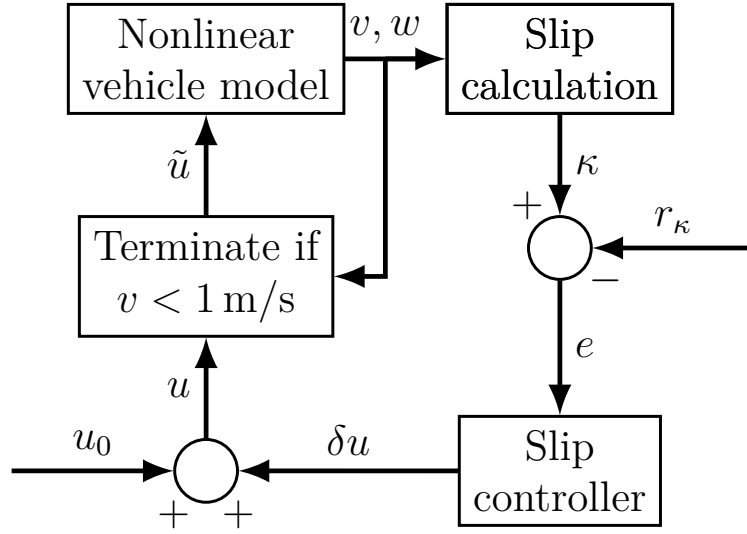
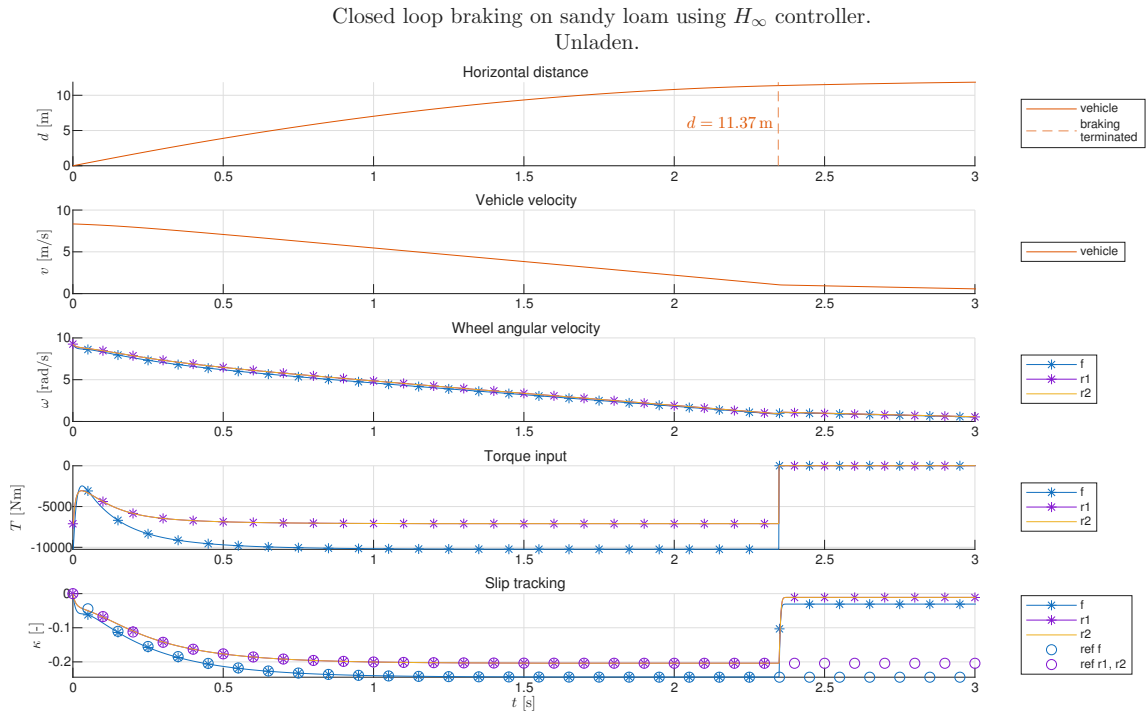
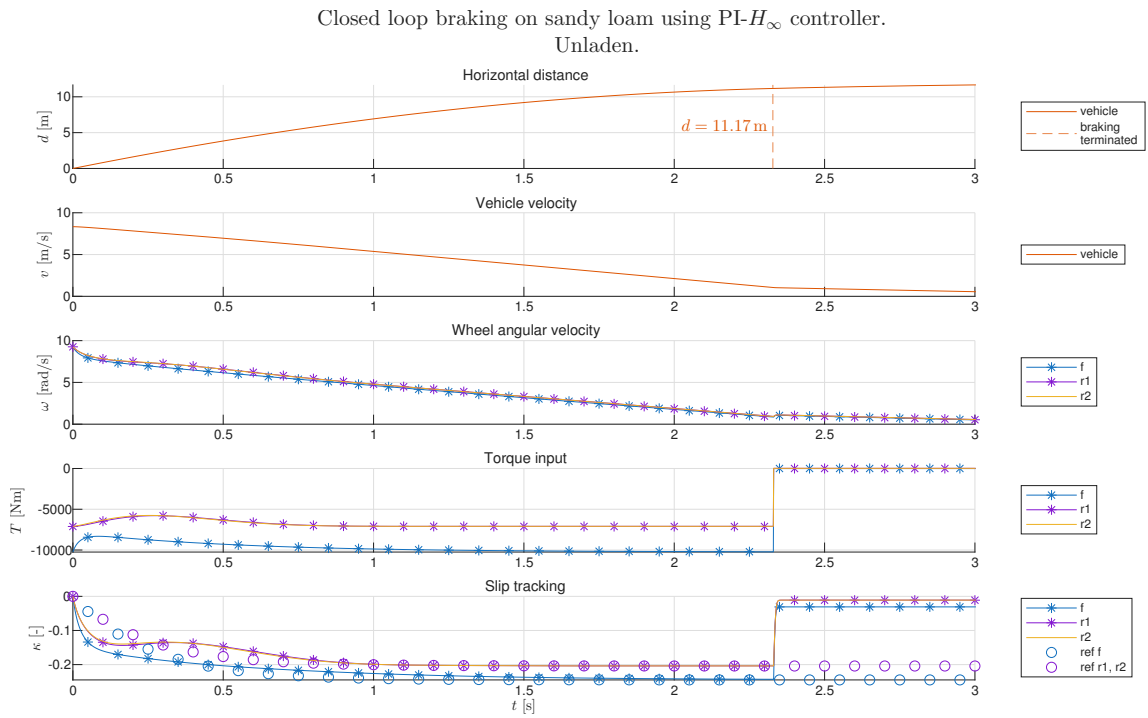


Figure 4.1: Closed loop system with nonlinear vehicle model.

4.1 Nominal plant

As an initial test, the closed loop system is tested on the nominal plant which means braking on sandy loam terrain with no extra load added to the vehicle. The slip reference is set to κ_0 , as in (3.25), and the resulting simulations from the two controllers are shown in Fig. 4.2. The last plot in Fig. 4.2a shows that the $\mathcal{H}_\infty$ controller tracks the slip reference well. It also shows a subtle transition phase initially which may be a consequence of the initial state values connected to the $\mathcal{H}_\infty$ controller. The PI controller in Fig. 4.2b is able to track the slip reference after roughly 1.0 s. The slip tracking shows an initial overshoot in the negative direction which highlights the drawback of the PI controller of not having a dampening term that a PID controller has. The fact that the PI controller results in more negative slip in the initial phase compared to the $\mathcal{H}_\infty$ controller is a reasonable explanation for achieving a shorter braking distance compared to the $\mathcal{H}_\infty$ controller, namely 11.17 m for the PI controller and 11.37 m for the $\mathcal{H}_\infty$ controller.

(a) H_∞ controller.

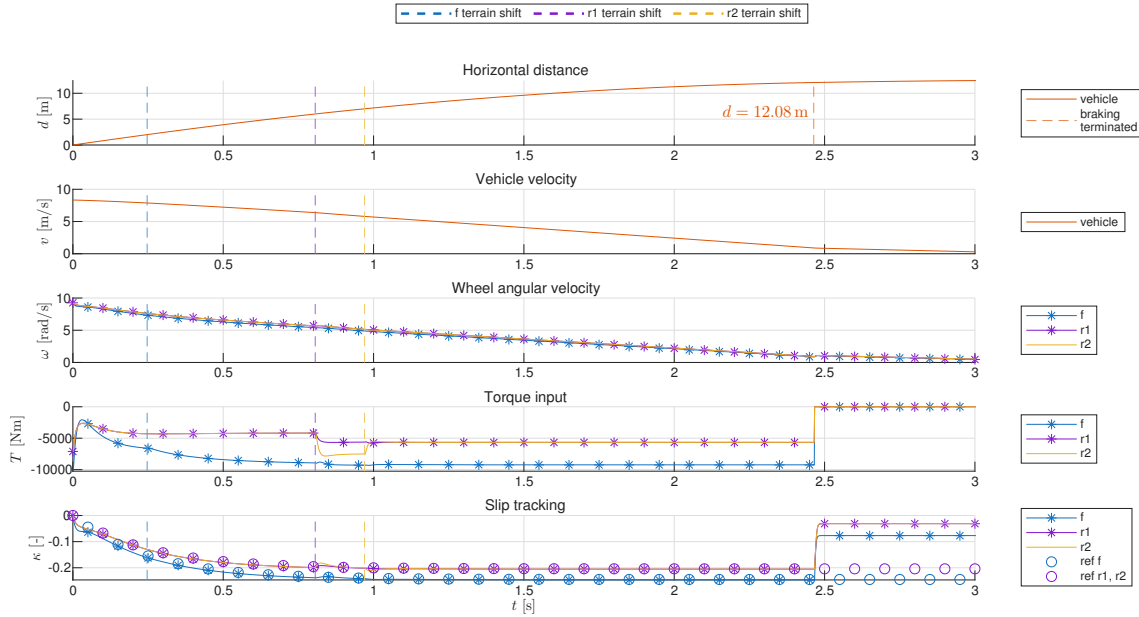
(b) PI controller.

Figure 4.2: Closed loop simulation using the nonlinear and nominal plant with different controllers.

4.2 Terrain shift during braking

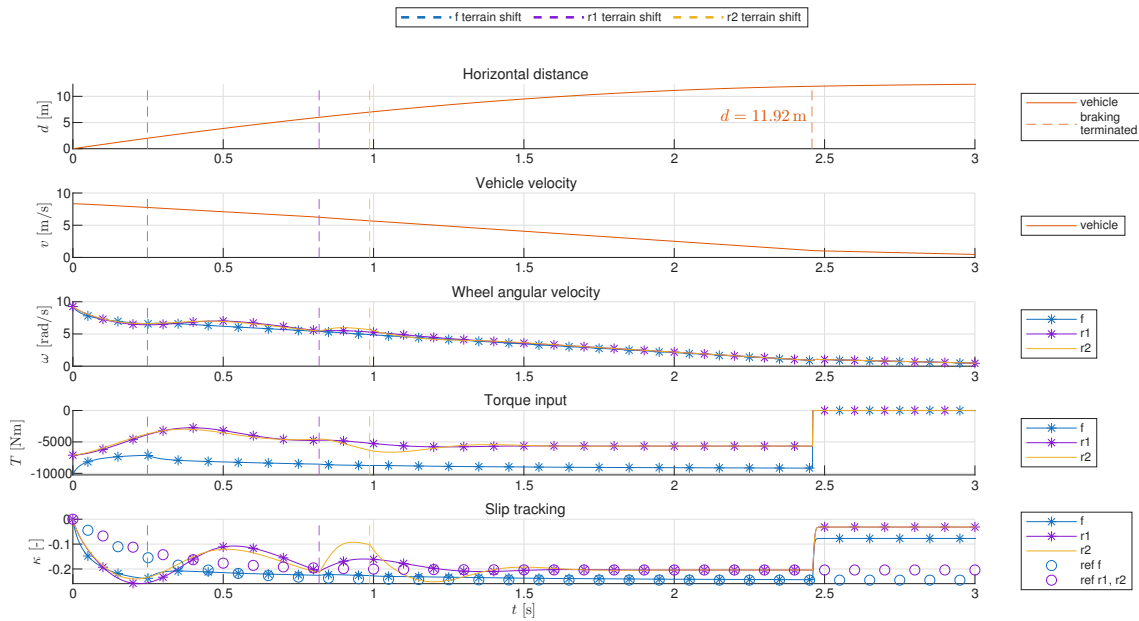
In these simulations, the controllers are tested on different terrain types. The vehicle initially brakes on one terrain type and then enters another terrain type during the braking action. This sudden shift is indicated by the dashed vertical lines where the front tire axle, f , enters the new terrain first, followed by the leading and trailing rear tire axles, $r1$ and $r2$. As seen in Fig. 4.3 - 4.6, the $\mathcal{H}_\infty$ controller tracks the slip references well and handles the terrain transitions well for all tested combinations. The PI controller, on the other hand, shows different behavior in Fig. 4.3b, 4.4b compared to Fig. 4.5b, 4.6b. In the former cases, it struggles to converge to adequate slip tracking when the initial terrain type is clay. In Fig. 4.5b, however, the vehicle enters clay as secondary terrain type and manages to track slip well in that case. When designing the $\mathcal{H}_\infty$ controller, clay was the most difficult terrain to handle as it probably has less similarities with the nominal terrain type sandy loam compared to the other non-nominal terrain types which are sand and loam. Given these figures, this behavior may apply to the PI controller as well which was tuned with the same weights as the $\mathcal{H}_\infty$ controller. Regarding the other terrain types, the PI controller handles them well in those scenarios.

Closed loop braking on clay followed by sand using H_∞ controller.
Unladen.



(a) H_∞ controller.

Closed loop braking on clay followed by sand using PI- H_∞ controller.
Unladen.

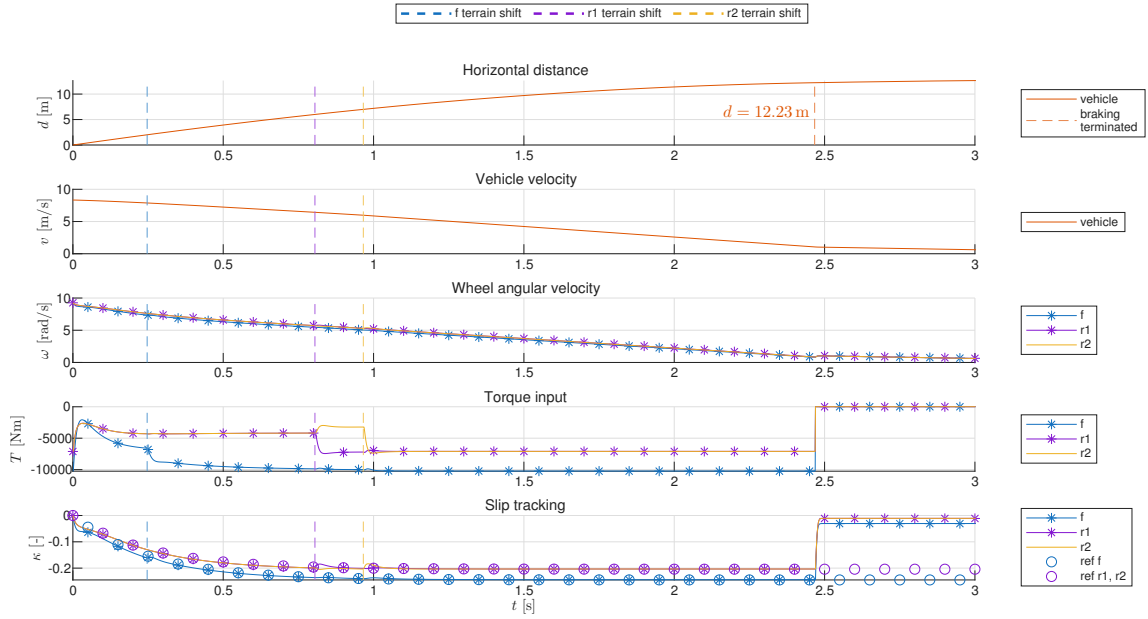


(b) PI controller.

Figure 4.3: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on clay and then each tire axle consecutively enters the new terrain type sand, indicated by dashed vertical lines.

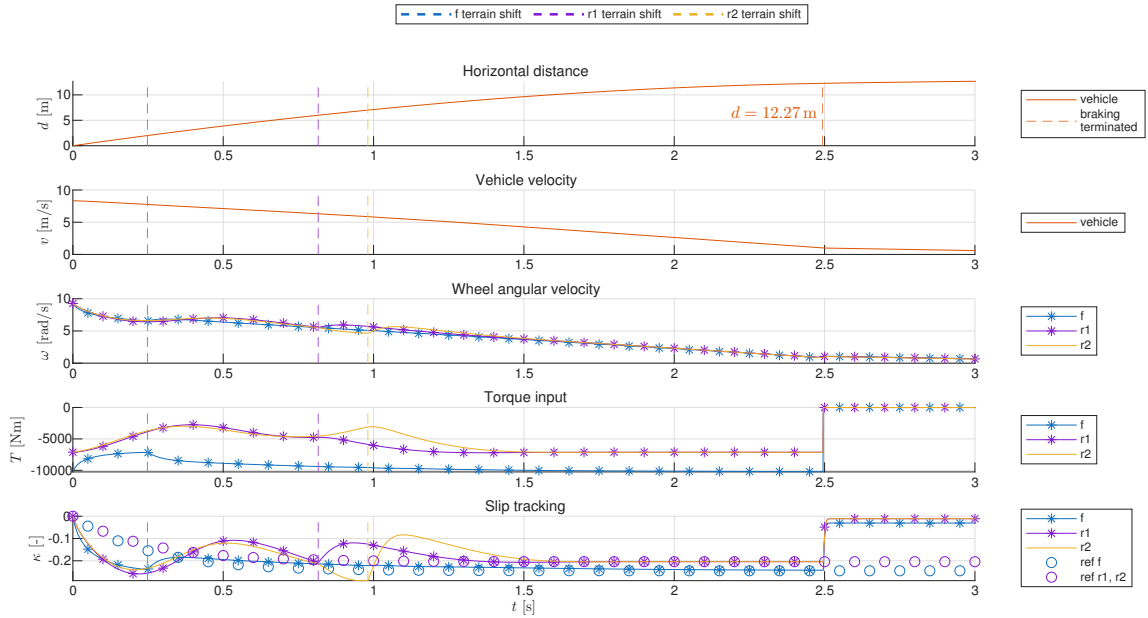
4. Results

Closed loop braking on clay followed by sandy loam using H_∞ controller.
Unladen.



(a) H_∞ controller.

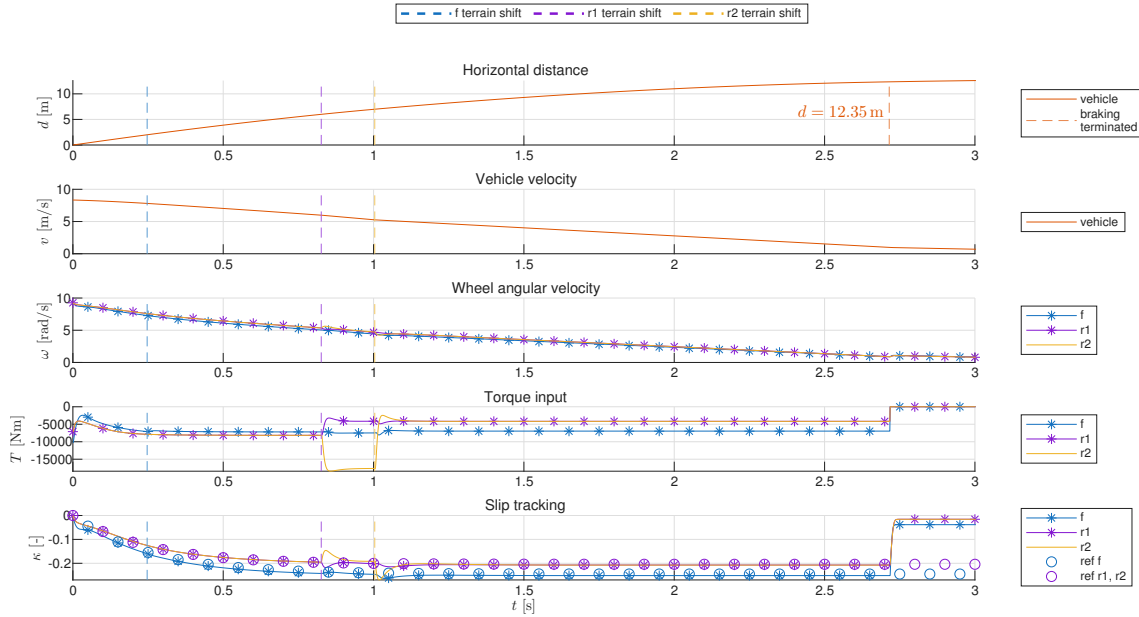
Closed loop braking on clay followed by sandy loam using PI- H_∞ controller.
Unladen.



(b) PI controller.

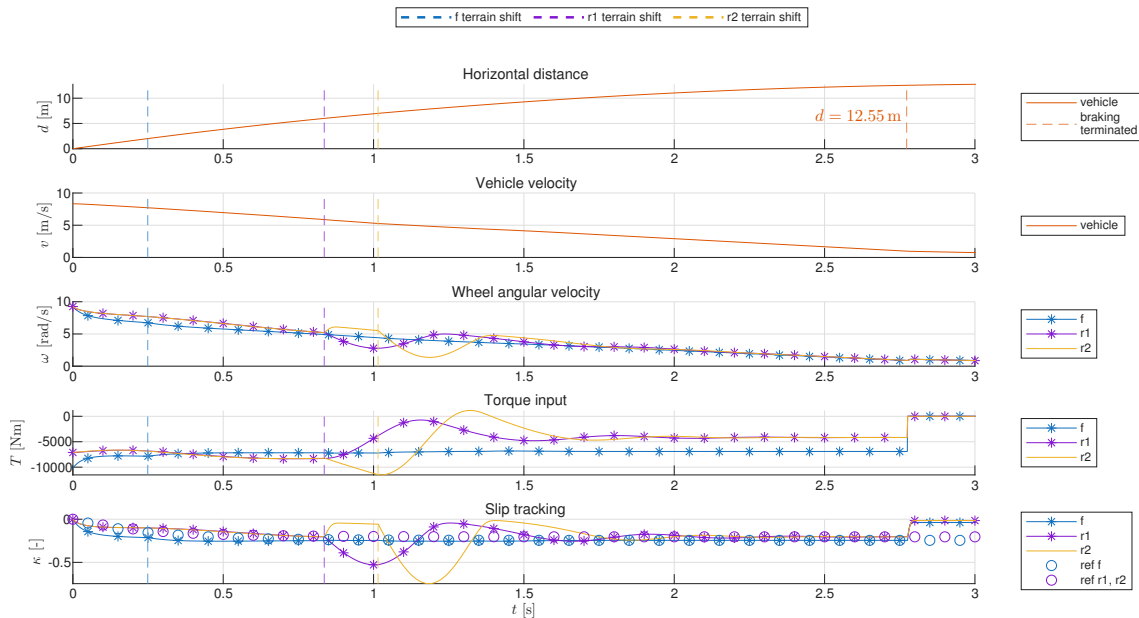
Figure 4.4: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on clay and then each tire axle consecutively enters the new terrain type sandy loam, indicated by dashed vertical lines.

Closed loop braking on loam followed by clay using H_∞ controller.
Unladen.



(a) H_∞ controller.

Closed loop braking on loam followed by clay using PI- H_∞ controller.
Unladen.

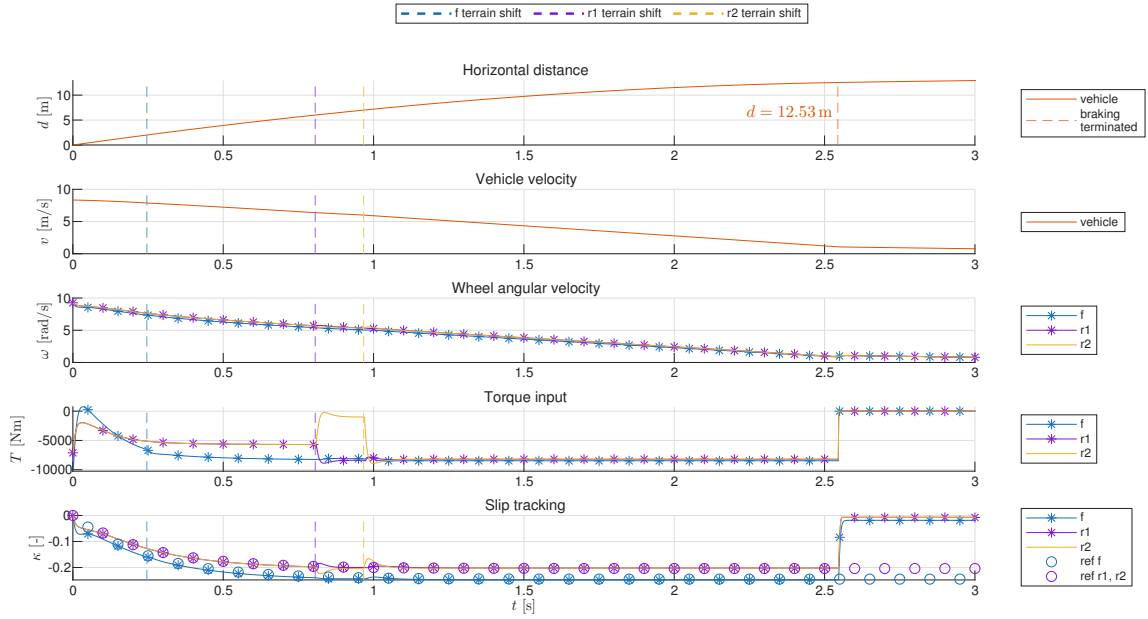


(b) PI controller.

Figure 4.5: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type clay, indicated by dashed vertical lines.

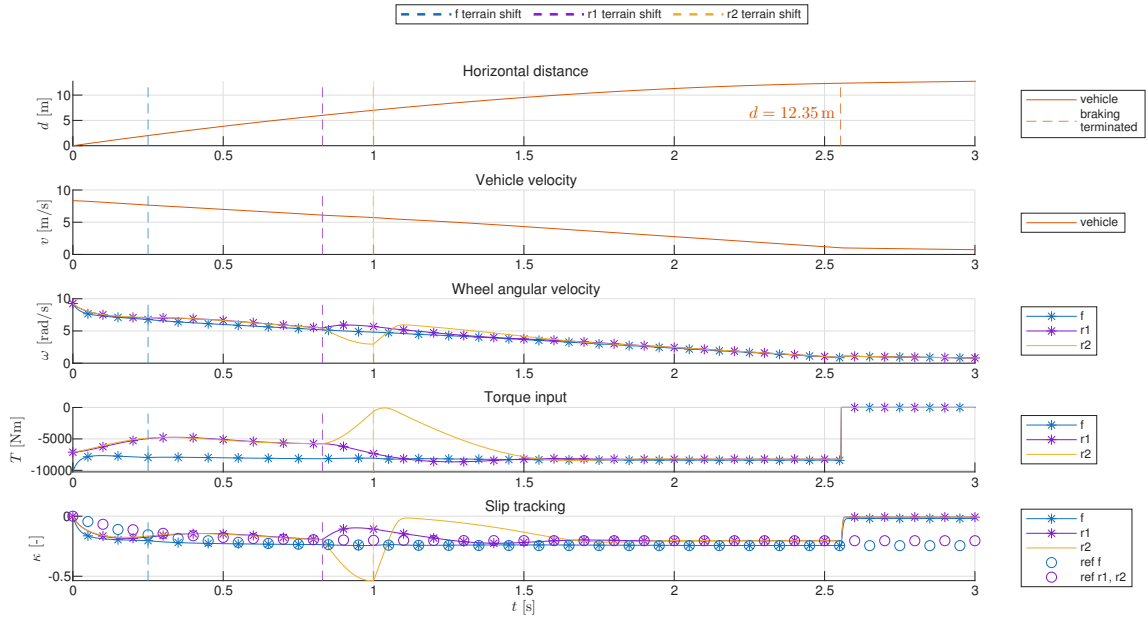
4. Results

Closed loop braking on sand followed by loam using H_∞ controller.
Unladen.



(a) H_∞ controller.

Closed loop braking on sand followed by loam using PI- H_∞ controller.
Unladen.



(b) PI controller.

Figure 4.6: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on sand and then each tire axle consecutively enters the new terrain type loam, indicated by dashed vertical lines.

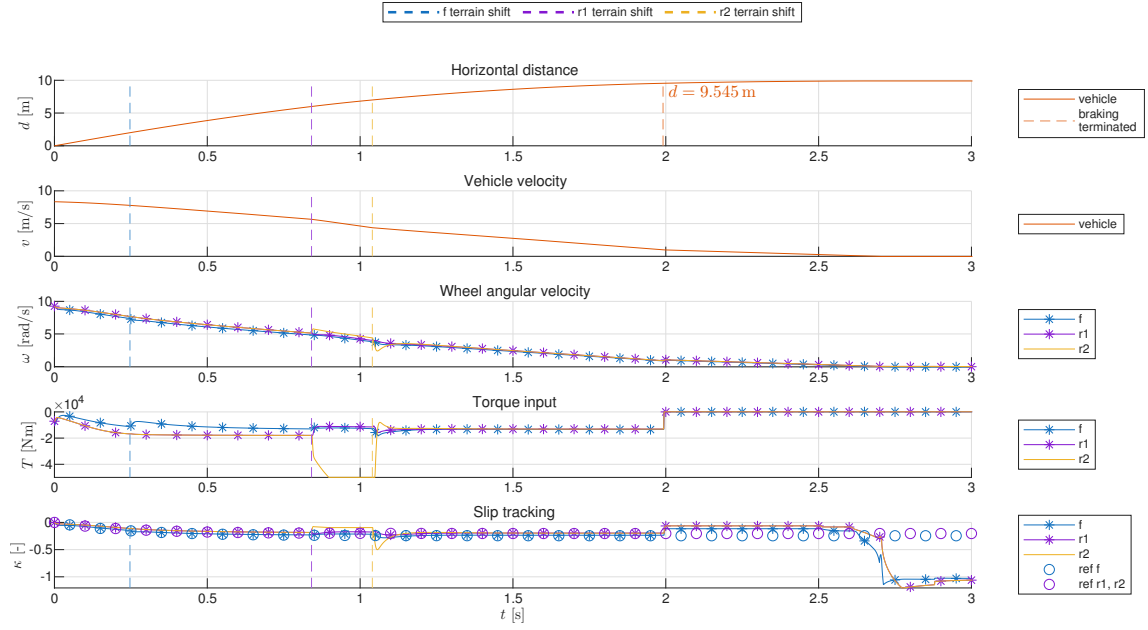
4.3 Extra load

In this scenario, extra load is added to the vehicle's cargo bed and vertical load on the tire axes are adjusted accordingly. The mass of the payload is equal to the vehicle mass, namely $20 \cdot 10^3$ kg. The test cases will continue to include terrain shifts in order to test the closed loop system in a more compact manner. The results of the tests with a laden truck and terrain shifts are shown in Fig. 4.7 - 4.9. Even with a heavily laden truck, the $\mathcal{H}_\infty$ controller tracks the slip reference very well, even during terrain shifts. The PI controller is not as effective and exhibits a steady state error even for the nominal terrain type as sequel terrain type, which can be seen in Fig. 4.8b. Since the PI controller produces a more positive slip on average compared to the $\mathcal{H}_\infty$ controller, it results in a longer braking distance throughout all these tests when comparing to the $\mathcal{H}_\infty$ controller.

On certain terrain types, as in Fig. 4.7a, 4.8a, the braking distance is reduced significantly compared to the average stopping distance with an unladen truck. Although the vehicle has greater momentum when being laden, the braking distance may be decreased due to the extra sinkage of the tires when the vehicle is laden compared to unladen. This results in greater interaction area between tire and terrain which results in both greater braking traction F_t and compaction resistance R_c . Note, however, that the shorter braking distance when being laden is not confirmed as these distances are compared when braking on different terrain types.

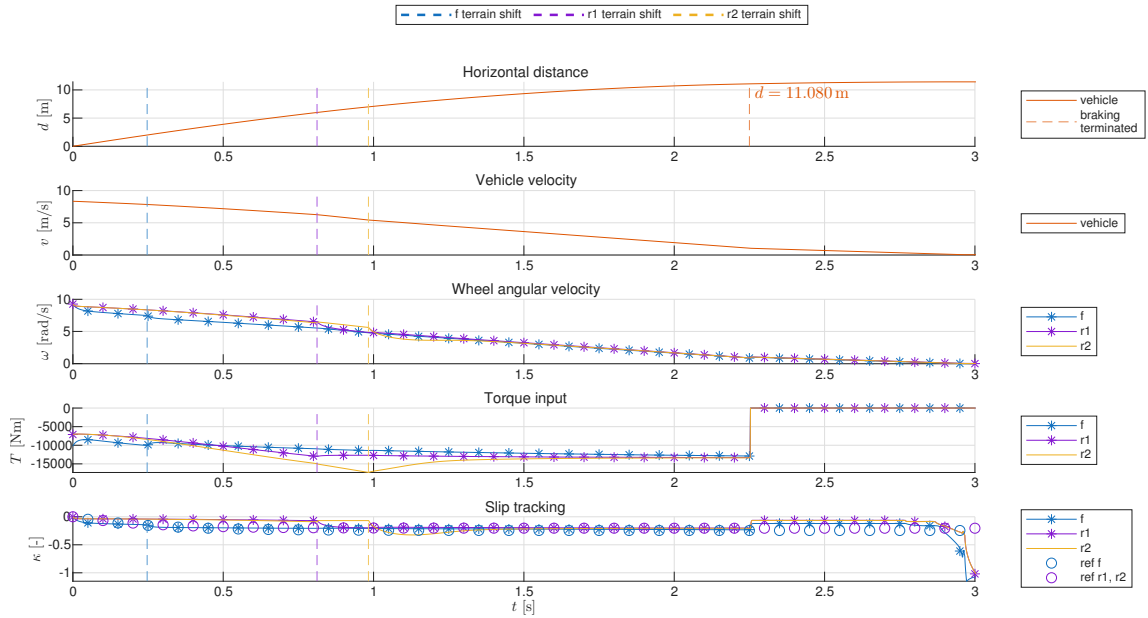
4. Results

Closed loop braking on loam followed by sand using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

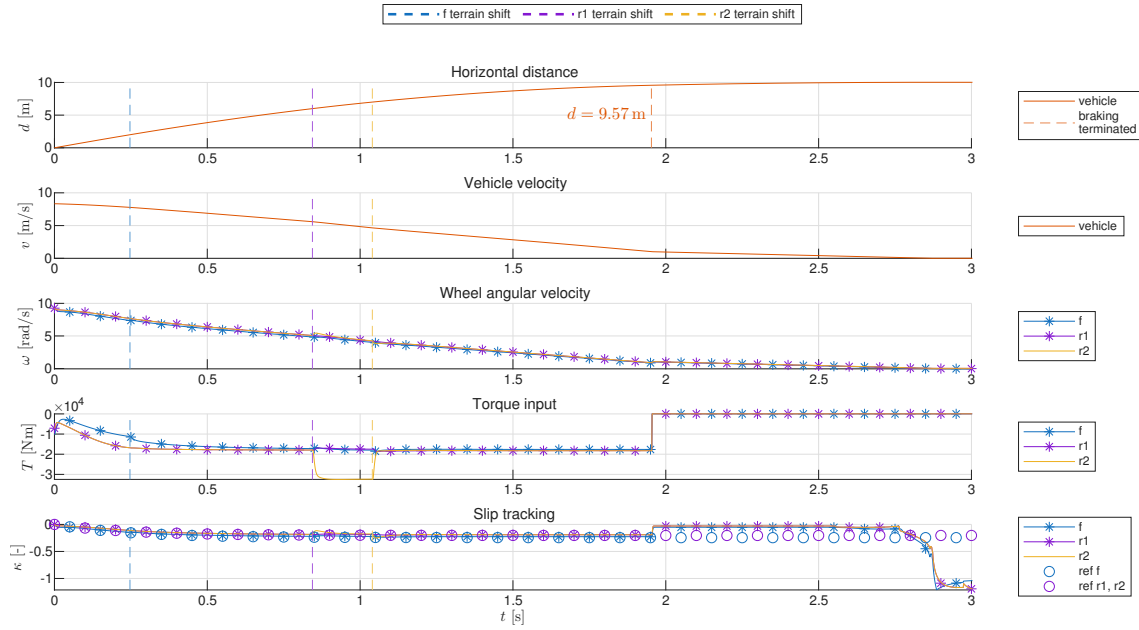
Closed loop braking on loam followed by sand using PI- H_∞ controller.
Laden with 20 tons.



(b) PI controller.

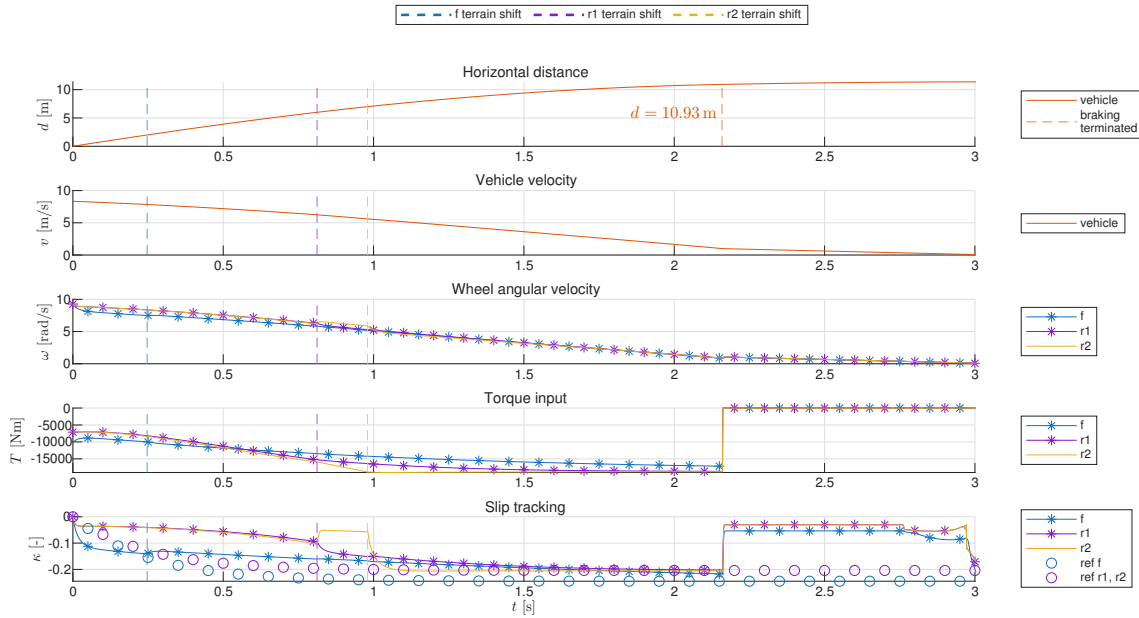
Figure 4.7: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type, sand, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load.

Closed loop braking on loam followed by sandy loam using H_∞ controller.
Laden with 20 tons.



(a) H_∞ controller.

Closed loop braking on loam followed by sandy loam using PI- H_∞ controller.
Laden with 20 tons.

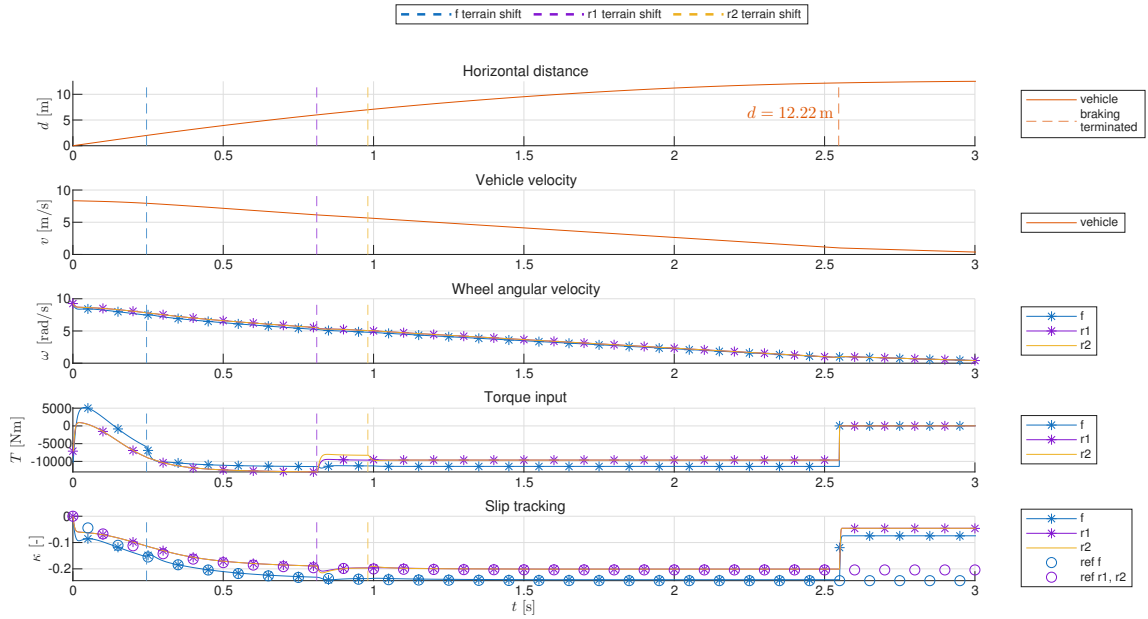


(b) PI controller.

Figure 4.8: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type, sandy loam, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load.

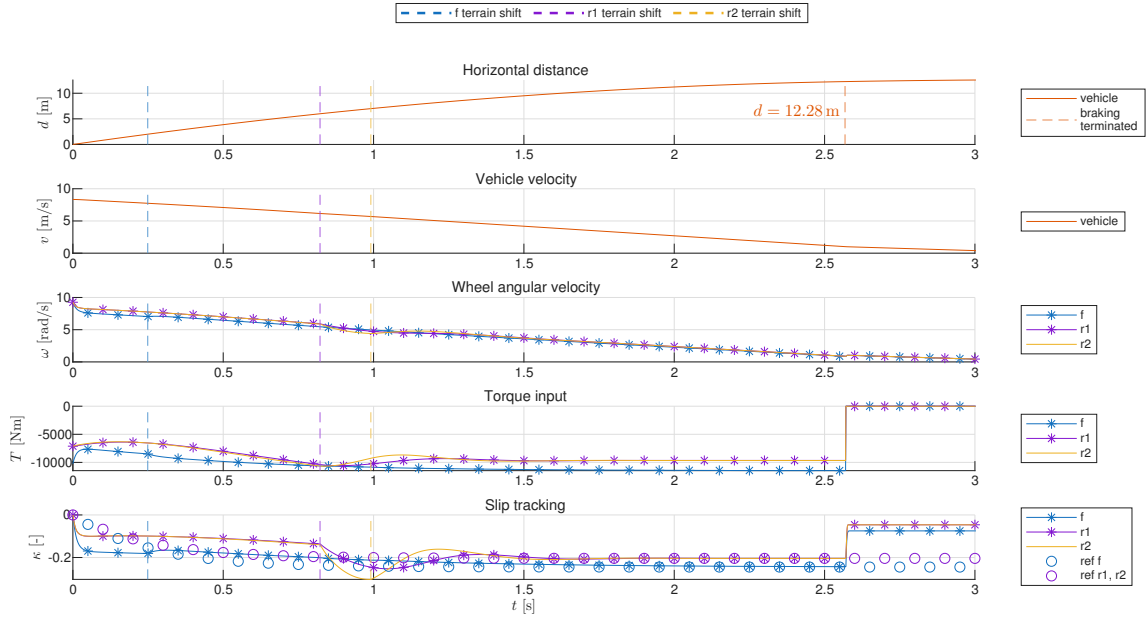
4. Results

Closed loop braking on sand followed by clay using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

Closed loop braking on sand followed by clay using PI- H_∞ controller.
Laden with 20 tons.



(b) PI controller.

Figure 4.9: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on sand and then each tire axle consecutively enters the new terrain type, clay, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load.

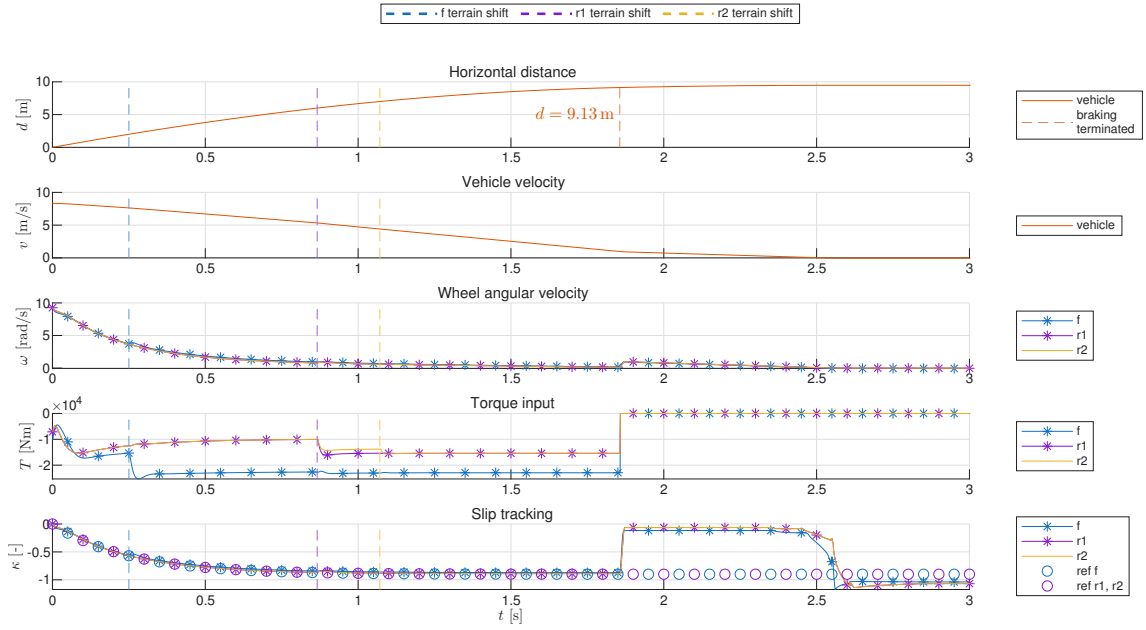
4.4 Different slip references

As discussed, the controller is linearized around the slip operating point in (3.25). Here, a more aggressive slip reference is tested, namely -0.9 for all wheels. A well behaving closed loop even for this slip reference can be valuable in cases when it is desirable to sacrifice lateral traction in favour of reducing the longitudinal braking distance. The resulting simulations are shown in Fig. 4.10 - 4.13 where the vehicle is still laden with $20 \cdot 10^3$ kg and experiences terrain shifts. For the first two scenarios, the slip references approach -0.9 from the beginning while the latter two experiments initially use the default slip reference (3.25) and later updates the reference to -0.9 . As also these cases are handled very well by the $\mathcal{H}_\infty$ controller, it is appropriate to ask why the controller can handle scenarios outside the uncertainty region which intention is to include only the non-nominal terrain types. Firstly, since the vehicle model does not have slip but velocities as states, slip is estimated algebraically from the states. Consequently, the controller does not have information about the slip dynamics $\dot{\kappa}$. Therefore, the control signals δu from the controller may rely more on the slip errors than the state dynamics. This is similar to a PID controller that relies solely on a specified error signal for computing the control signal. Secondly, each torque input directly impacts the angular acceleration of a specific tire. Therefore, one may argue that the system is weakly coupled since each input signal has significant influence over the angular velocity of the corresponding tire. This can also be seen in the bode diagram in Fig. 3.16 where the gain from each input signal to the slip of each tire follows a diagonal pattern with greater gain along the diagonal diagrams compared to the off-diagonal couplings.

The PI controller manages to track the slip reference for the front tire better than for the rear tires in these scenarios as the front tires exhibit ok slip tracking ability in all scenarios. The rear axles do not track the more aggressive slip reference of -0.9 in any case except for the last experiment in Fig. 4.13b where the leading rear axle tracks the reference well while the slip amplitude of the trailing rear axle seems to decay around the reference. Interestingly, this results in significantly longer braking distance compared to the $\mathcal{H}_\infty$ controller in Fig. 4.13a suggesting that an oscillating slip around the reference slip may suffer more in terms of braking distance from having a less negative slip compared to what it gains when having more negative slip.

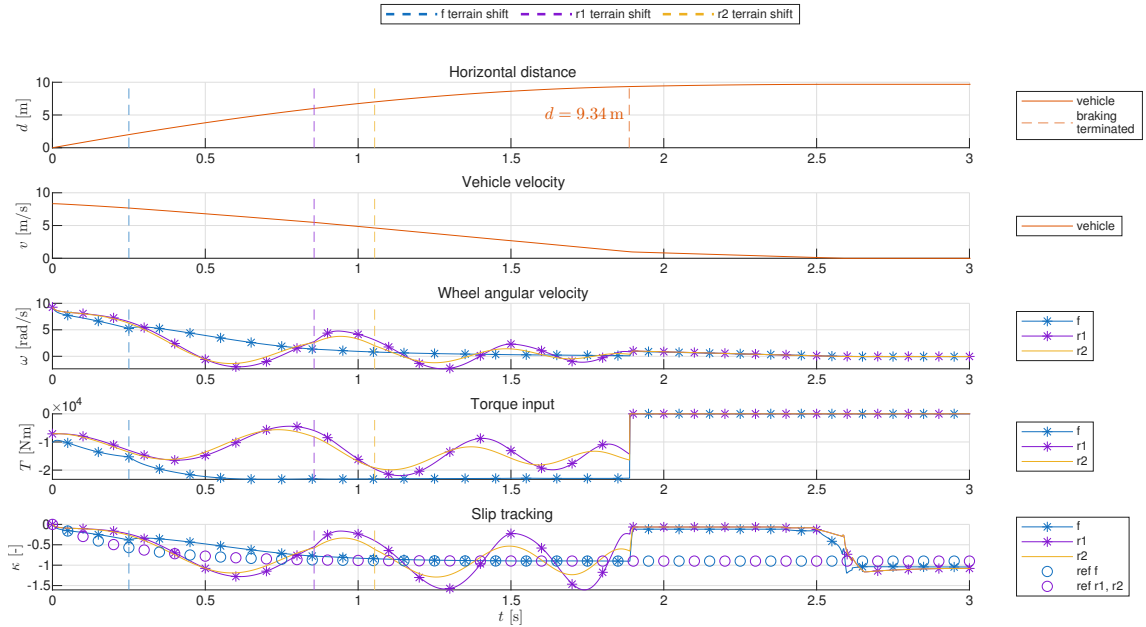
4. Results

Closed loop braking on clay followed by sand using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

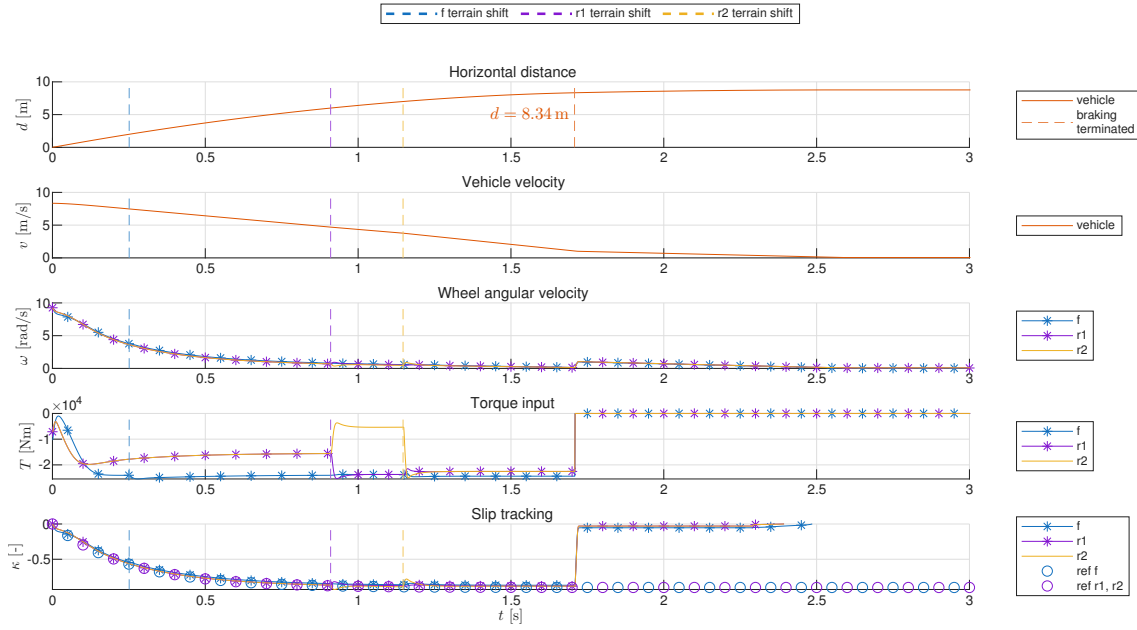
Closed loop braking on clay followed by sand using PI- H_∞ controller.
Laden with 20 tons.



(b) PI controller.

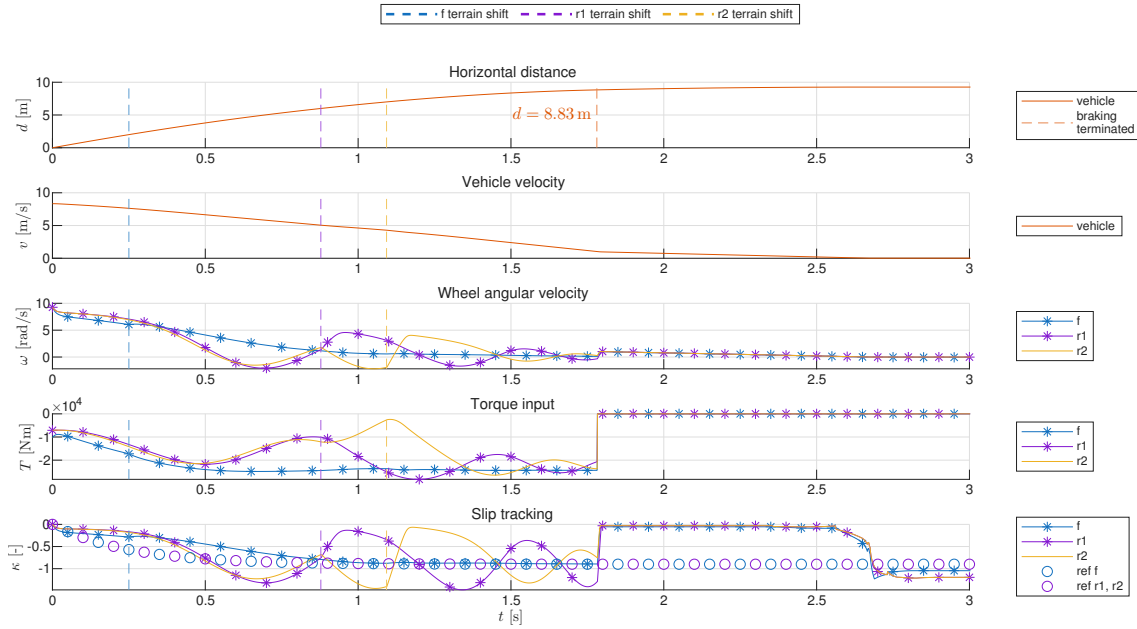
Figure 4.10: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on clay and then each tire axle consecutively enters the new terrain type, sand, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load and the reference slip is more aggressive compared to the linearization point, namely at -0.9 for all tires.

Closed loop braking on sand followed by sandy loam using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

Closed loop braking on sand followed by sandy loam using PI- H_∞ controller.
Laden with 20 tons.

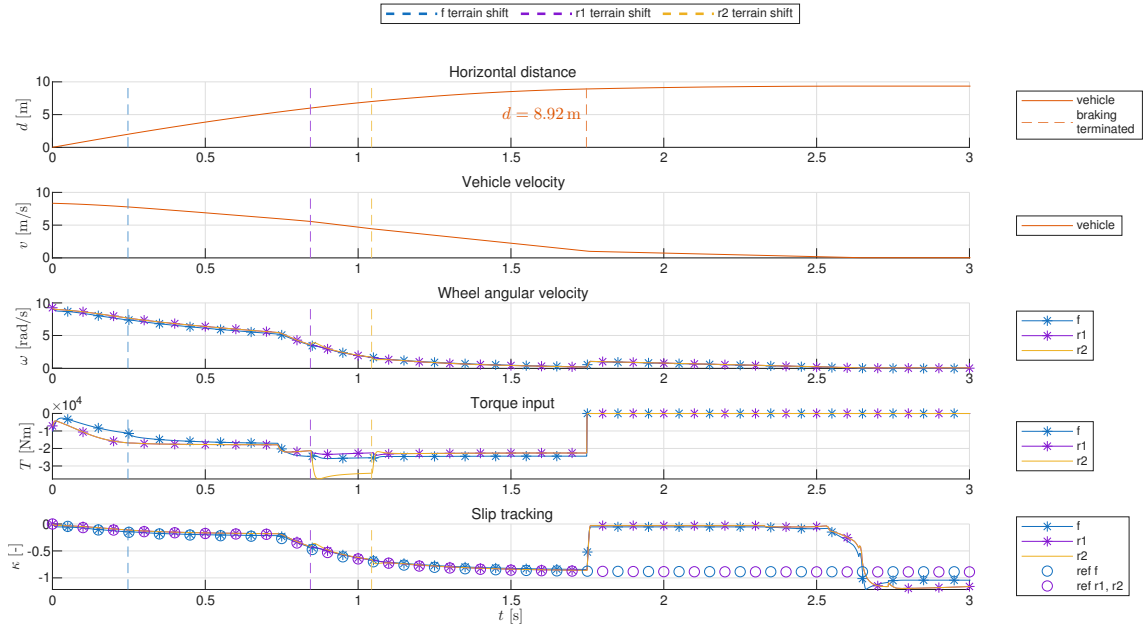


(b) PI controller.

Figure 4.11: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on sand and then each tire axle consecutively enters the new terrain type, sandy loam, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load and the reference slip is more aggressive compared to the linearization point, namely at -0.9 for all tires.

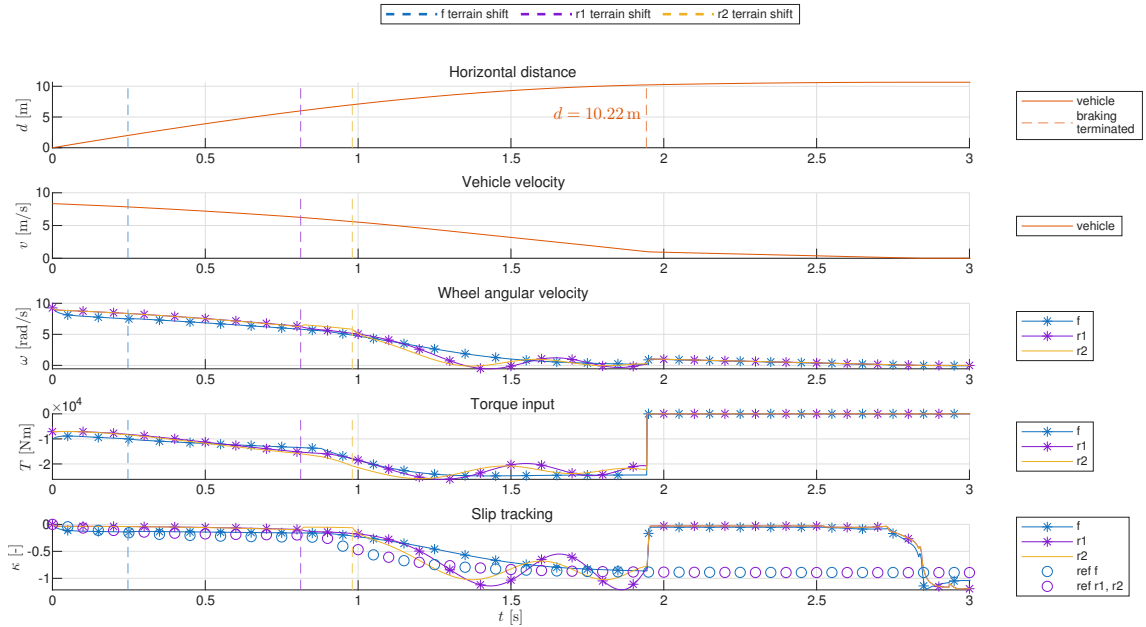
4. Results

Closed loop braking on loam followed by sandy loam using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

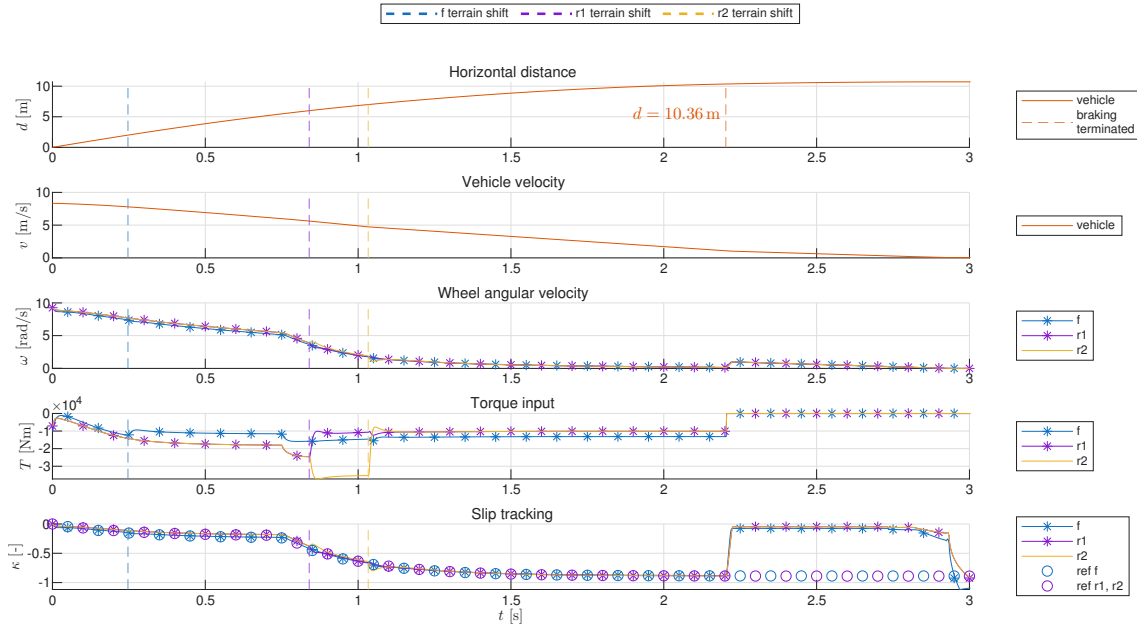
Closed loop braking on loam followed by sandy loam using $PI-H_\infty$ controller.
Laden with 20 tons.



(b) PI controller.

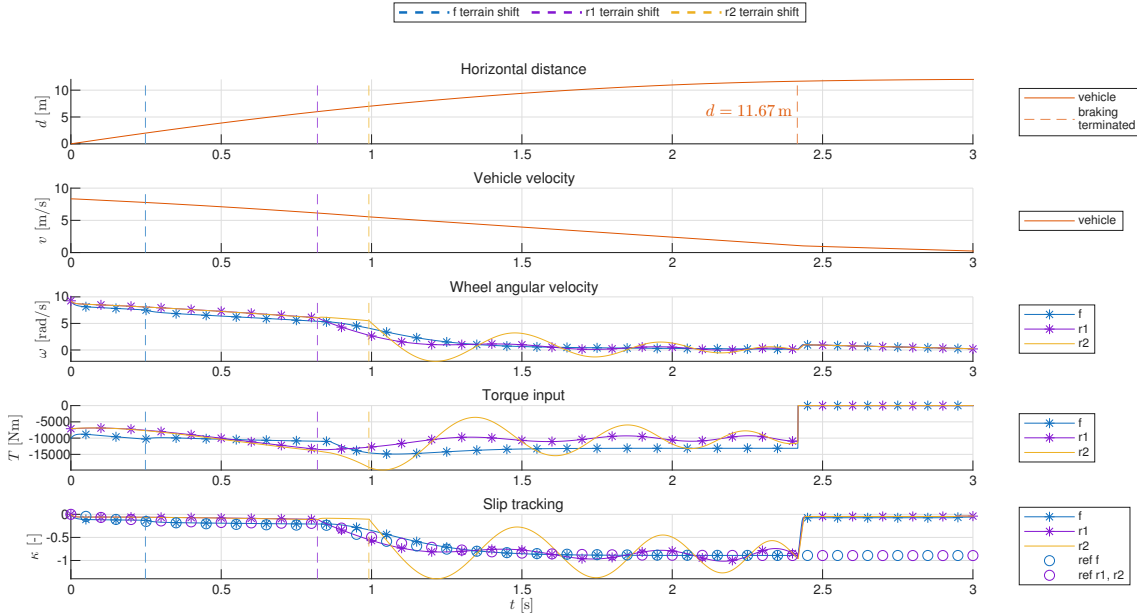
Figure 4.12: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type, sandy loam, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load and the reference slip is changed when the vehicle reaches 6 m/s to a more aggressive one compared to the linearization point, namely at -0.9 for all tires.

Closed loop braking on sandy loam followed by clay using H_∞ controller.
Laden with 20 tons.



(a) $\mathcal{H}_\infty$ controller.

Closed loop braking on sandy loam followed by clay using PI- H_∞ controller.
Laden with 20 tons.



(b) PI controller.

Figure 4.13: Closed loop simulation using the nonlinear plant with different controllers. The vehicle starts driving on sandy loam and then each tire axle consecutively enters the new terrain type, clay, indicated by the dashed vertical lines. The vehicle carries $20 \cdot 10^3$ kg extra load and the reference slip is changed when the vehicle reaches 6 m/s to a more aggressive one compared to the linearization point, namely at -0.9 for all tires.

4.5 Measurement noise and actuator dynamics

Lastly, additional tests were performed to examine the limits of the $\mathcal{H}_\infty$ controller. Zero mean gaussian measurement noise was added to the states v, ω with standard deviation 0.1 m/s and 0.5 rad/s for v and ω respectively. These plots are shown in Fig. A.1 - A.2 in the appendix. The $\mathcal{H}_\infty$ controller outputs very noisy control signals when measurement noise is included and no low pass filter is used for the measurements. However, the slip tracking is decent for the first 1.5s when they start to deviate in both cases. The ability of the closed loop system to track the slip reference initially can to some extent be described by the bode diagram in Fig. 3.16 and sigma plot in Fig. 3.17 where it can be seen that the response from the plants to input signals decrease for higher frequent input signals. For the PI-controller, however, the noise in the control signals have far smaller amplitude for the rear tires compared to the front tires and it manages to track the slip reference decently in both cases.

Further, actuator dynamics equal to $W_R(s)$ in (3.43) was included in the closed loop, but not in the control configuration in Fig. 3.15. These results are also included in the appendix and can be seen in Fig. A.3 - A.4. The control dynamics is included after the possible termination of braking control in Fig. 4.1. This means that $\tilde{u}$ is subject to actuator dynamics and is the reason why the control effort is not immediately terminated when $v < 1$ m/s in the cases including actuator dynamics.

As seen in Fig. A.3a - A.4a, the $\mathcal{H}_\infty$ controller is not able to track the slip reference after inclusion of actuator dynamics. The modelled actuator delay may cause the response time of the actuators to be too slow for adequate slip tracking. This could, however, possibly be resolved if actuator dynamics is included in the control configuration. This may require retuning of the weights in the control configuration. The PI-controller seemingly tracks the slip references better compared to the $\mathcal{H}_\infty$ controller although its closed loop behavior leaves room for improvement as well. This can be seen in Fig. A.3b - A.4b where the front tire follows the slip reference more closely than the rear tires when actuator dynamics is included.

5

Conclusion

A tire model suitable for off-road conditions is well established in the literature. This model has been implemented and validated against a trusted software, called Project Chrono, with adequate accuracy. Relevant vehicle data was not available and validation was therefore only done by comparing the model to simulated single tire data from Project Chrono. The model was further calibrated towards the simulated data by adjusting its terrain parameters which resulted in increased model accuracy. As the model contains integrals without closed form solutions [18], a simplified tire model was developed by fitting a sigmoid function to the tire model. By using this simplified model, a model based controller could be designed which performed beyond expectation. Slip tracking was maintained within the given uncertainty for different terrain types, as well as outside the modelled uncertainty with heavy extra load and slip references not at the linearization point.

Based on the tire model, it was discovered that the longitudinal force is monotonically increasing as slip increases from -1 to 1 . The maximum braking force is therefore achieved when slip is -1 , i.e. locking the wheels or perhaps even spinning the wheels in reverse direction. Therefore, the evaluation of the stated research questions needs to be addressed according to the updated objective to not sacrifice lateral traction more than 40 % from its peak value as loosing too much lateral traction may compromise safety.

The proposed $\mathcal{H}_\infty$ controller, with configuration and weights in Fig. 3.15, manages to track the desired slip reference beyond expectations. It was not only robust enough to track the slip reference in all terrains including terrain shifts during braking, but could also handle heavy load as well as an aggressive slip reference far beyond the linearization point. Following the example of performance output values in (3.48) and (3.49), it can be argued that the controller may prioritize attenuating control signals over slip errors. This turned out not to be an issue based on the presented slip tracking ability of the controller. As the results were satisfactory, it is possible to conclude that the proposed weights and performance outputs in the control configuration shown in Fig. 3.15 can be selected to ensure proper closed loop performance using the $\mathcal{H}_\infty$ controller.

The off-road tire model was found not to include a specific friction coefficient but, instead, a set of other terrain specific parameters. The braking system was tested in various settings where these parameters were changed from one set to another during braking to simulate the vehicle leaving one terrain and entering another one. The controller was able to handle this transition very well even in the case when

being heavily laden and also for slip references beyond the linearization point. In real world settings, the values of these parameters likely change slightly even within the same terrain type over a distance. For example, the vehicle might first drive on dry clay which becomes more wet during the along the track. The fact that the controller could track the reference slip in case of sudden terrain changes from e.g. sand to clay, it is reasonable to assume that real world variations within one terrain type such as dry and wet clay can also be handled by the controller.

Cases where the vehicle was both unladen, as in the case of the nominal plant, and heavily laden were tested. Also these scenarios were handled well in the closed loop. It is however worth mentioning that all test cases were subject to the same initial velocities which also reflect the linearization point for the controller. Since the initial longitudinal velocity was chosen to approximately 8.3 m/s, the controller was linearized around 7 m/s to facilitate closed loop stability in the initial braking phase and, if necessary, allow gain scheduling as the longitudinal velocity decreases. The successful closed loop simulations are therefore limited to this initial velocity and further research is needed in order to include other scenarios. On the other hand, as the vehicle velocity and tire angular velocities decrease during braking, the closed loop maintained stable behavior as the velocities decreased from their linearization values. In case of successful slip tracking, however, the slip remains close to its linearization point as only the velocities depart from their linearization points. This is under the assumption that the slip reference is equal to the slip linearization point. Nevertheless, a single linearized $\mathcal{H}_\infty$ controller was able to handle the complete braking trajectory down to 1 m/s without gain scheduling.

Since the control system could maintain adequate slip tracking during terrain shifts while being heavily laden, further tests were performed to analyze the limitations of the closed loop systems. Firstly, reference slip values beyond the linearization point were tested, namely at -0.9 for all wheels. The controller was able to track these as well, including cases when slip references was changed from the linearization point to -0.9 during the braking action. Secondly, addition of measurement noise and actuator dynamics were included without addressing these neither by filtering nor in the control configuration. With only measurement noise added, the controller could track the slip reference only for a limited time but not when velocities became too low. As the actuator was simplified to ideal, the control signals in this simulation are, however, unrealistic as they are not dampened by any inertia. Thus, actuator dynamics were included in the closed loop but not in the control configuration which resulted in an unstable closed loop system.

The $\mathcal{H}_\infty$ controller was compared to decoupled PI controllers for each wheel. The $\mathcal{H}_\infty$ controller exhibited better slip tracking performance compared to the decoupled controller which can partly be explained by the lack of damping in the PI controllers, which is used in PID controllers. Using PID controllers with the given tuning setup, however, showed unstable closed loop behavior as they were automatically tuned. The tuning was performed by analyzing closed loop behavior using the $\mathcal{H}_\infty$ controller which also contributes to the better performance of the $\mathcal{H}_\infty$ controller compared to the PI-controller. The robust performance guarantees, as requested in the research questions, are therefore to be analyzed based on the performance of

the $\mathcal{H}_\infty$ controller rather than the PI-controller. The longest braking distance down to 1.0 m/s among the test cases, excluding inclusion of measurement noise and actuator dynamics, is 12.53 m with the $\mathcal{H}_\infty$ controller. This therefore serves a worst case braking distance given the initial velocity and set of test cases. For an unladen truck and slip references at the linearization point, where sufficient lateral traction is maintained, the results indicate that the braking distance down to 1.0 m/s for various terrains is between 11.0 m/s and 13.0 m.

5.1 Future work

Although the controller shows ability to track references of -0.9 , the $\mathcal{H}_\infty$ controller was designed to track slip references around -0.2 . This is similar to the ABS which regulates slip between -0.3 and -0.1 . One may then wonder what the benefit of the developed $\mathcal{H}_\infty$ is compared to ABS. This study has shown potential of an $\mathcal{H}_\infty$ controller to track various slip references in order to brake a heavy vehicle in various conditions. As it is a MIMO controller, it may be expanded to include more dimensions. This can be the inclusion of lateral dynamics into the control configuration which could allow the controller to attenuate lateral motion while trying to maximize the longitudinal braking force. The trade-off between these ambitions can be designed by carefully selecting the weights in the $\mathcal{H}_\infty$ control configuration. Another possibility is to track a desired slip angle β while braking to attain desired braking behavior while turning. The developed braking system is therefore not to be seen as a replacement for ABS but rather as a proof of concept for centralized braking controllers in off-road conditions. It demonstrates the potential of the approach and serves as a foundation for future research to develop more sophisticated braking systems.

In order to expand the system with lateral dynamics, several topics need to be considered. This is especially the case if the goal is to brake the vehicle when turning as a rolling motion will appear. Consequently, vertical load will increase for the outer tires and decrease for the inner tires. This will make the sinkage differ between inner and outer tires and therefore effect both longitudinal force as well as lateral traction. More demand may be put on the outer tires compared to the inner tires and the ability of the braking system to produce the demanded torque needs to be analyzed. Also, when turning on soft ground, when the tires sink into the terrain, the truck may be subject to greater rolling motion compared to when turning on asphalt, potentially causing the vehicle to tilt to the extent where there is a risk for the cargo to fall over. Additionally, as vertical load increase for the front tires when braking and also increase for the outer tires when turning, extensive vertical load may be experienced by the outer front tire in when braking and turning simultaneously. This affects the tire dynamics of the front tire and also puts high demands on the hardware such as axles and suspension system to withstand great forces. This may restrict the feasible braking torques while turning while the vertical load shifts towards the outer tires possibly alters the optimal combination of braking torques to be demanded.

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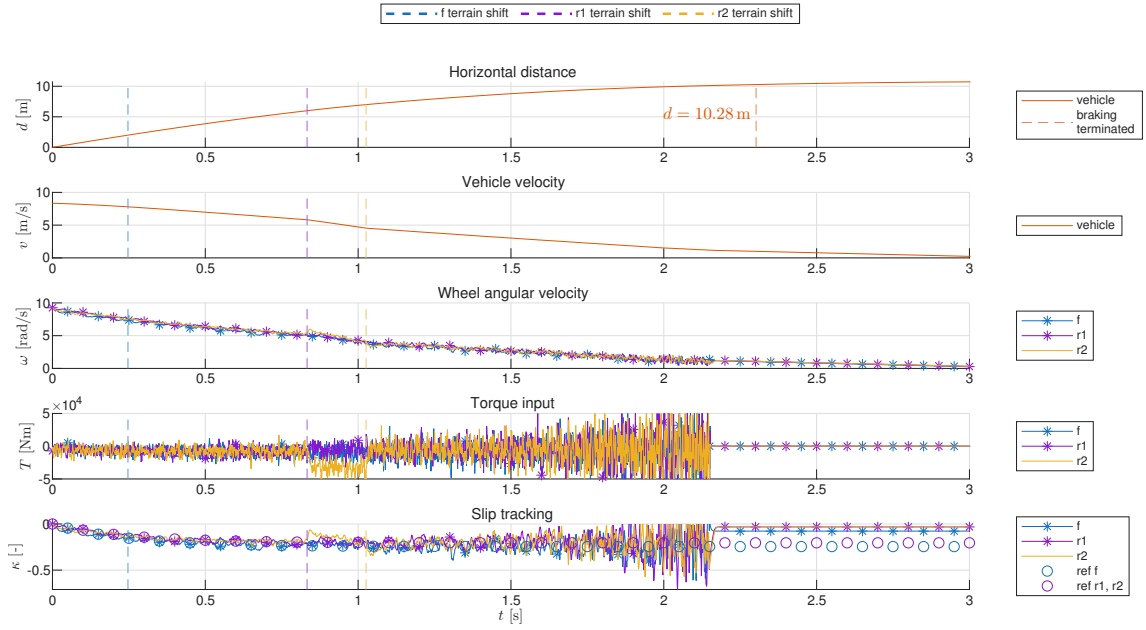
A

Appendix 1

A.1 Results with measurement noise

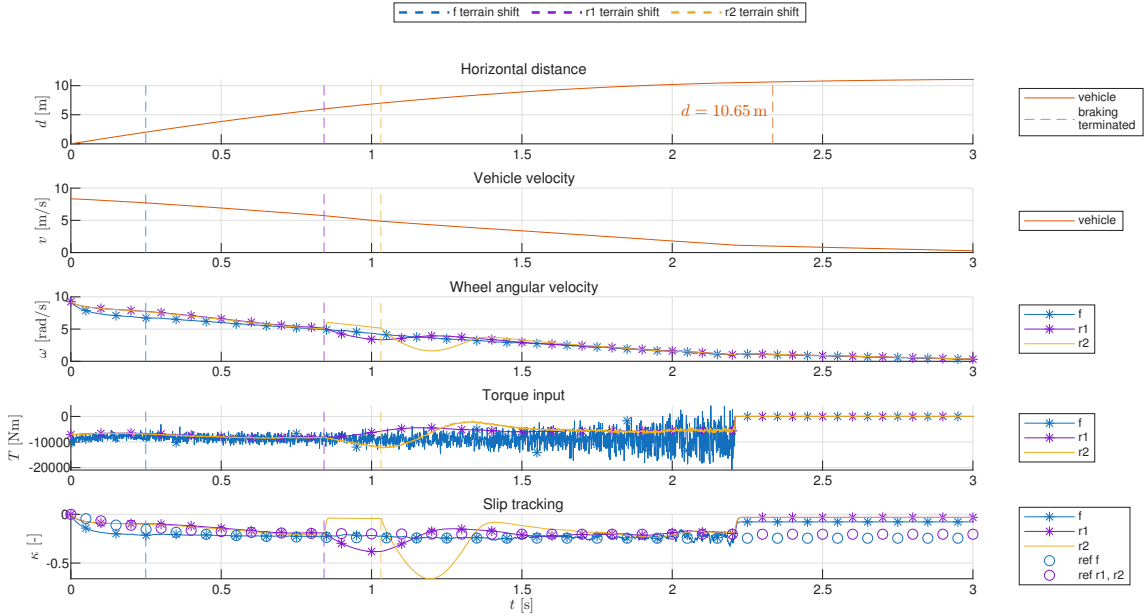
A. Appendix 1

Closed loop braking on loam followed by sand using H_∞ controller.
Unladen. Measurement noise included.



(a) H_∞ controller.

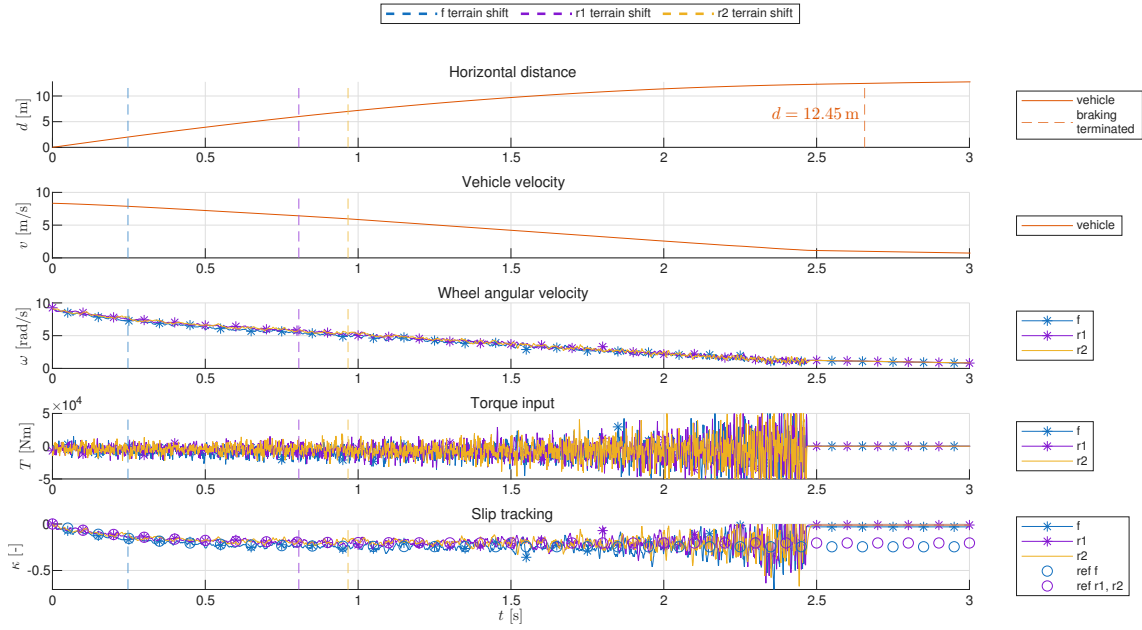
Closed loop braking on loam followed by sand using PI- H_∞ controller.
Unladen. Measurement noise included.



(b) PI controller.

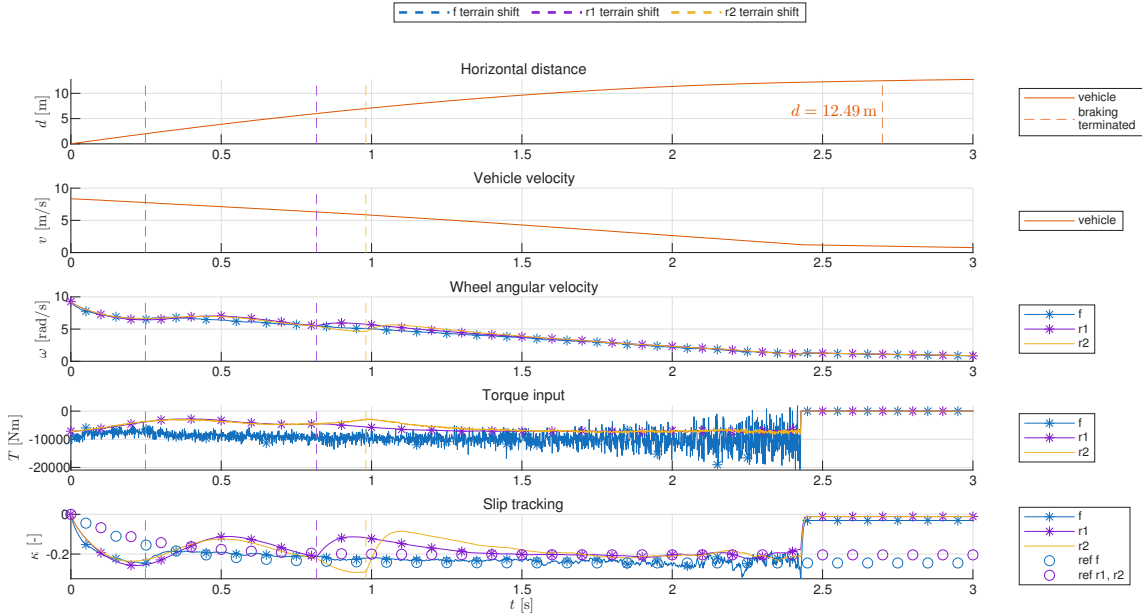
Figure A.1: Closed loop simulation using different controllers and nonlinear plant. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type, sand, indicated by the dashed vertical lines. The vehicle is unladen and measurement noise is added to v and ω .

Closed loop braking on clay followed by sandy loam using H_∞ controller.
Unladen. Measurement noise included.



(a) H_∞ controller.

Closed loop braking on clay followed by sandy loam using PI- H_∞ controller.
Unladen. Measurement noise included.

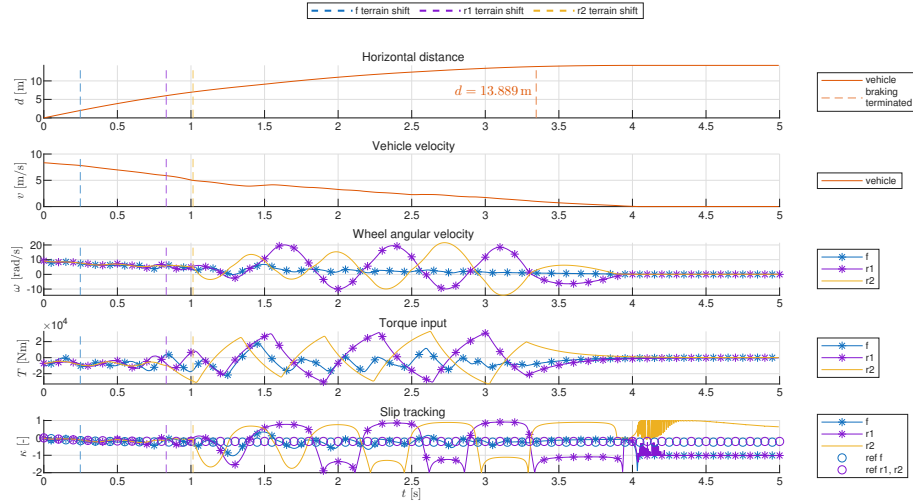


(b) PI controller.

Figure A.2: Closed loop simulation using different controllers and nonlinear plant. The vehicle starts driving on clay and then each tire axle consecutively enters the new terrain type, sandy loam, indicated by the dashed vertical lines. The vehicle is unladen and measurement noise is added to v and ω .

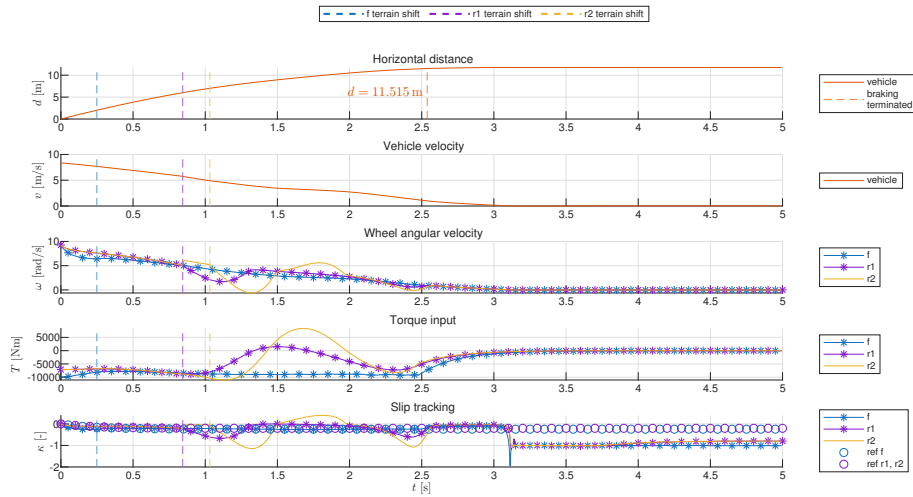
A.2 Results with measurement noise and actuator dynamics

Closed loop braking on loam followed by sand using H_∞ controller.
Unladen. Measurement noise included. Actuator dynamics included.



(a) H_∞ controller.

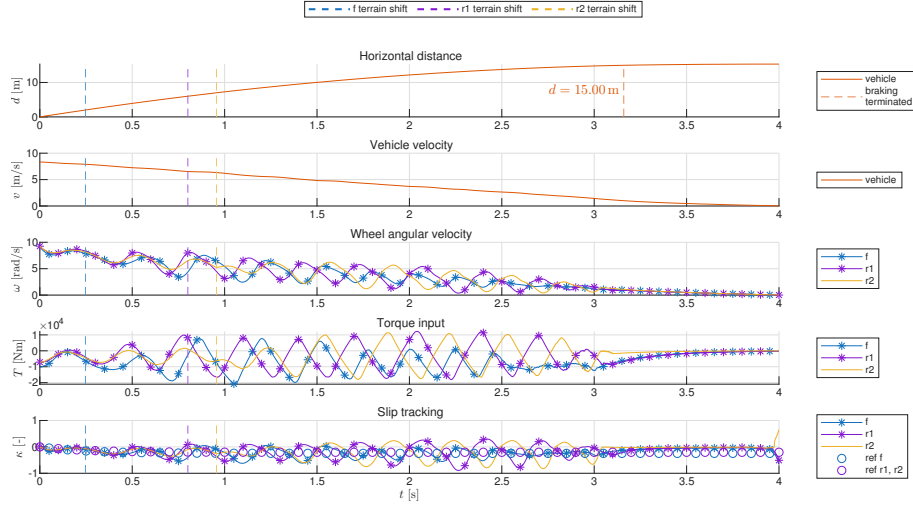
Closed loop braking on loam followed by sand using PI- H_∞ controller.
Unladen. Measurement noise included. Actuator dynamics included.



(b) PI controller.

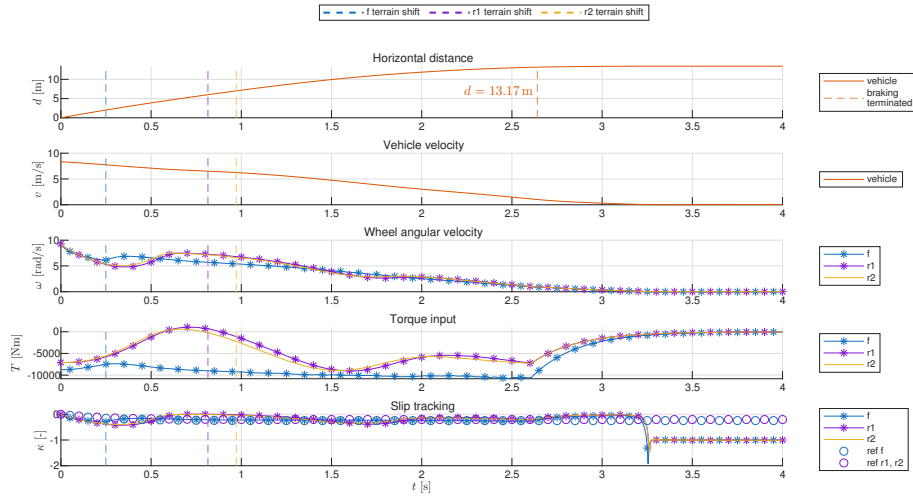
Figure A.3: Closed loop simulation using different controllers and nonlinear plant. The vehicle starts driving on loam and then each tire axle consecutively enters the new terrain type, sand, indicated by the dashed vertical lines. The vehicle is unladen and measurement noise is added to v and ω . Actuator dynamics is added in the closed loop but not included in the H_∞ configuration.

Closed loop braking on clay followed by sandy loam using H_∞ controller.
Unladen. Measurement noise included. Actuator dynamics included.



(a) H_∞ controller.

Closed loop braking on clay followed by sandy loam using PI- H_∞ controller.
Unladen. Measurement noise included. Actuator dynamics included.



(b) PI controller.

Figure A.4: Closed loop simulation using different controllers and nonlinear plant. The vehicle starts driving on clay and then each tire axle consecutively enters the new terrain type, sandy loam, indicated by the dashed vertical lines. The vehicle is unladen and measurement noise is added to v and ω . Actuator dynamics is added in the closed loop but not included in the H_∞ configuration.

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