CHALMERS
UNIVERSITY OF TECHNOLOGY

Modelling of Dynamic Pressure Conditions in District Heating Systems

Master's thesis in Sustainable Energy Systems

Alexander Lindblom

Department of Energy and Environment
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2015

Modelling of Dynamic Pressure Conditions in District Heating Systems

ALEXANDER LINDBLOM

Supervisors: Olle palmqvist, Stefanìa Òsk Gardarsdòttir

Examiner: Klas Andersson

Department of Energy and Environment
Division of Energy Technology
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2015

Modelling of Dynamic Pressure Conditions in District Heating Systems

ALEXANDER LINDBLOM

Department of Energy and Environment
Division of Energy Technology
Chalmers University of Technology
SE-412 96 Gothenburg
Sweden
Telephone +46 (0)31-772 1000

Department of Energy and Environment
Gothenburg, Sweden 2015

Abstract

Water hammer is a problem that arises in district heating systems when there are abrupt changes in the flow operating conditions. As this often leads to damaged equipment it is of importance to simulate these effects in advance using various simulation tools. This thesis investigates how a general simulation software such as Dymola performs in comparison to the more specialized software available on the market for fluid transient calculations. The finite volume method in Dymola has been compared to the method of characteristics in the PFC simulation software. The overall conclusion is that Dymola may be used in cases where the system to be modelled is not too complex and accurate results are not required.

KEYWORDS:

Water hammer, Dymola, simulation

Acknowledgements

There are several people who have contributed in order to make this project possible. A thanks goes to Olle palmqvist at Solvina for help and guidance with project planning and aim. I would also like to thank Henrik Granberg at Solvina for advice on technical issues in Dymola. I would also like to thank Otto Olsson at Göteborg Energi for supplying all the data that made the project possible. Finally, I would like to thank Stefania Osk Gardarsdottir at Chalmers for giving continuous support and advice during the whole course of the project.

Contents

Nomenclature

λ	Darcy friction factor	[-]
λ_2	Modified friction factor	[-]
f	Fanning friction factor	[-]
a	Wave speed	[m/s]
α	Angle between vertical and horizontal plane	[deg]
P	Static pressure	[Bar]
ρ	Density	[kg/m^3]
x	Coordinate along horizontal axis	[m]
z	Coordinate along verical axis	[m]
A	Cross sectional area of pipe	[m^2]
L	Length of pipe	[m]
H	Manometric pressure head	[mLc]
v	Velocity	[m]
$\dot{V}$	Volumetric flow rate	[m^3/s]
τ	Shear stress	[Pa]
k	Thermal conductance	[W/m,C]
E	Elastic modulus	[GN/m^2]
b	Pipe wall thickness	[m]
C_p	Specific heat capacity	[J/kg,C]
K	Bulk modulus	[Pa]
k_v	Valve coefficient (European standard)	[-]
S	Circumfurance	[m]
$\dot{m}$	Mass flow rate	[kg/s]
m	Mass	[kg]
M	Momentum	[kg m/s]
F	Force	[kg m/s^2]
d	Diameter	[m]
δ	Absolute roughness of pipe	[m]
μ	Dynamic viscosity	[kg/m s]
T_p	Time period of pressure wave	[s]
t	Simulation time	[s]
Φ	Convective flow property [-]	

h	Valve opening	[-]
Re	Reynolds number	[-]
g	Gravitational acceleration	$[m/s^2]$
W'	Ideal work input for a certain hydraulic head	$[J]$
DAE	Differential Algebraic Equation	
ODE	Ordinary Differential Equation	
MOC	Method of Characteristics	
FVM	Finite Volume Method	
$1\,bar = 10.4\,mLc$	Assuming a water temperature of 65 °C	

1 | Introduction

1.1 Background

A District heating system is an effective way of transporting heat around a city. Since many cities in Sweden are dependent on district heating it is of importance that these systems maintain a safe and functional operation.The piping and pumping networks in these systems involve large flow rates and a problem that can occur in the system is water hammer which is mostly caused by sudden valve closure or pump failure. Severe water hammer is damaging for pipes, pumps and valves and the issue needs to be closely examined. A dynamic simulation of the water hammer effects arising from such cases gives valuable information which can be used in system design and assesment of problems and improvement of already existing networks.

The thesis work will be carried out at Solvina AB, a consulting company based in Gothenburg with a lot of experience in dynamic modelling. The Dymola simulation software based on the Modelica programming language will be used for dynamic simulation.

1.2 Objective and Specifications

The primary objective is to determine the capability of the Dymola simulation tool to simulate the water hammer phenomena. Since Dymola is an all round simulation program it will be compared to the most commonly used software on the market. This evaluation will be done in cooperation with Göteborg Energi. Göteborg Energi has provided simulation data from their calculations along with schematics and flow operating conditions of the Gothenburg network.

1.3 Scope and Delimitations

The water hammer concept will be simulated by using simple models that will be compared to the calculations made by Göteborg Energi and a larger model that is more representative of a real district heating system. This larger model is based on the Gothenburg district heating system. No experimental data for external verification of the calculation methods has been gathered. No simulations will be made related to minimizing water hammer effects.

2 | Theory

This chapter provides the theoretical background required for the modelling of fluid transients.

2.1 Water Hammer

Water hammer is a phenomena in the field of fluid transients. When a flowing fluid in a pipe undergoes a sudden velocity deceleration the dynamic pressure is converted into static pressure. This occurs when there is a sudden change in the flow operating conditions such as valve closure or pump failure. It becomes clear that a pipe must be able to withstand not only the static steady state pressure, but also the increase in static pressure due to transient effects. All fluids are to a certain extent compressible and pipes have a specific elasticity depending on the material. If the flow is suddenly stopped at the end of a pipe connected to a valve a pressure wave will propogate backwards through the pipe. Consider a pipe of length 1000 m shown in Figure 2.1.

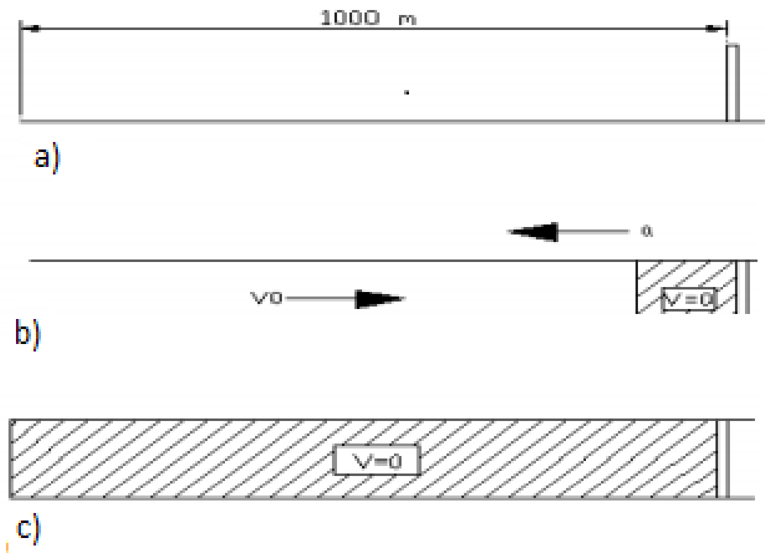

Figure 2.1: a) Before valve closure, b) Suddenly after valve closure, c) one second after valve closure

Since the fluid is slightly compressible, the fluid flow at the valve will stop before the rest of the fluid that will continue with the original velocity v_o until the pressure wave has reached that point along the pipe. The wave velocity a is a function of both the pipe elasticity and the fluid compressibility. If the wave speed is 1000 m/s the wave will reach the end of the pipe after one second and the flow will then have stopped completely. As the fluid has been compressed the pressure is highest close to the valve causing the flow to reverse. The pressure wave will now propagate in the opposite direction. This process will be repeated until the wave has lost all its energy through pipe friction and viscous forces. In some cases, as the pressure wave oscillates the static pressure can drop below the saturation pressure causing cavitation. As the pressure again increases the vapor bubbles implode which can lead to damaging of the pipes.

The time it takes for the wave to travel forward and back through the pipe is the periodic time defined as

$$T_p = \frac{2L}{a} \tag{2.1}$$

If the pipe is completely inelastic and the fluid completely incompressible the wave velocity would approach infinity. The elasticity of the pipe reduces the wave speed to less than the speed of sound in the fluid.[5] If the valve closing time is much shorter than T_p it is referred to as instantaneous valve closure. The opposite case is known as slow valve closure where the valve closing time is much larger than T_p. [6]

2.1.1 One Dimensional Fluid Pipe Flow

Following the previous section on water hammer it becomes evident that equations governing the transformation of fluid states during the transient process are required. During the water hammer process the fluid undergoes a significant change in momentum. At the same time the fluid mass is always conserved regardless of pressure or velocity changes. The following system of equations containing the momentum equation for pipe flow and the continuity equation describe the fluid flow. See appendix A for a derivation.

$$\begin{cases} \underbrace{\frac{\partial(\rho v A)}{\partial t}}_{\mathrm{I}} + \underbrace{\frac{\partial(\rho v^2 A)}{\partial x}}_{\mathrm{II}} = \underbrace{-A\frac{\partial P}{\partial x}}_{\mathrm{III}} - \underbrace{\frac{1}{2}\rho v|v| f S}_{\mathrm{IV}} - \underbrace{A\rho g\frac{\partial z}{\partial x}}_{\mathrm{V}} \\ \underbrace{\frac{\partial(\rho A)}{\partial t}}_{\mathrm{VI}} + \underbrace{\frac{\partial(\rho A v)}{\partial x}}_{\mathrm{VII}} = 0 \end{cases} \tag{2.2}$$

In words the terms are

$$
\begin{cases}
\text{I=accumulation of momentum} \\
\text{II=transport of momentum by convection} \\
\text{III= momentum created by pressure gradient} \\
\text{IV=momentum loss due to friction forces} \\
\text{V=momentum due to potential energy relative to a ground reference level} \\
\text{VI=accumulation of mass} \\
\text{VII=transport of mass along flow direction}
\end{cases}
\tag{2.3}
$$

This system of equations has no analytical solution why numerical methods are required. Moreover, the flow relations are only valid for laminar flow. This is a reasonable assumption for a district heating system were fluid velocities are relatively low.

2.1.2 Calculation Method

In certain special cases simplified relations such as Juokowskis equation for instantaneous valve closure or rigid body theory for slow valve closure can be enough to estimate water hammer effects.[6] These simplified equations would be hard to implement in the Dymola simulation tool even in their range of appliance as they cannot be applied to a dynamic system with initial steady state conditions as is the real case in a district heating system. As the aim of the project is to make dynamic simulations a more sophisticated method such as the method of characteristics or a finite volume method is required. Both techniques will be described in the following sections. Dymola uses the finite volume method in fluid components. The method of characteristics solves the water hammer equations according to a pre-defined algorithm where each time step is dependent on variables at an earlier time, why it is not suitable for Modelica. However, it will be used for model evaluation and comparison between methods.

2.1.3 Finite Volume Method

The finite volume method is one of the most common numerical solution methods in computational fluid dynamics. The first step is to define the computational domain by creating a *grid*. A grid consists of an arbitrary number of *nodes* where physical properties such as temperature and pressure are stored. Surrounding the nodes are the control volume *faces*. When integrating over the control volume boundaries, the values of physical properties at the faces need to be estimated by a suitable approximation known as a *discretization* scheme. The type of scheme will vary depending on application. If the distance between the cell faces is equal to the distance between two adjacent nodes the grid is equidistant.

The idea of the discretization process (integrating and then using a difference scheme) is to yeild a system of equations that can be solved using an iterative procedure. The number of linear equations will equal the number of nodes in the domain. In this way the system becomes coupled since the value of physical properties at a certain node is dependent on all the other nodes. Thus, in a converged solution all equations must be satisfied for the correct solution to be obtained. [7]

2.1.3.1 The Upwind scheme

The upwind discretization scheme is commonly used for convective terms as it is a relatively simple but useful approximation. Consider a grid formation as in figure 2.2

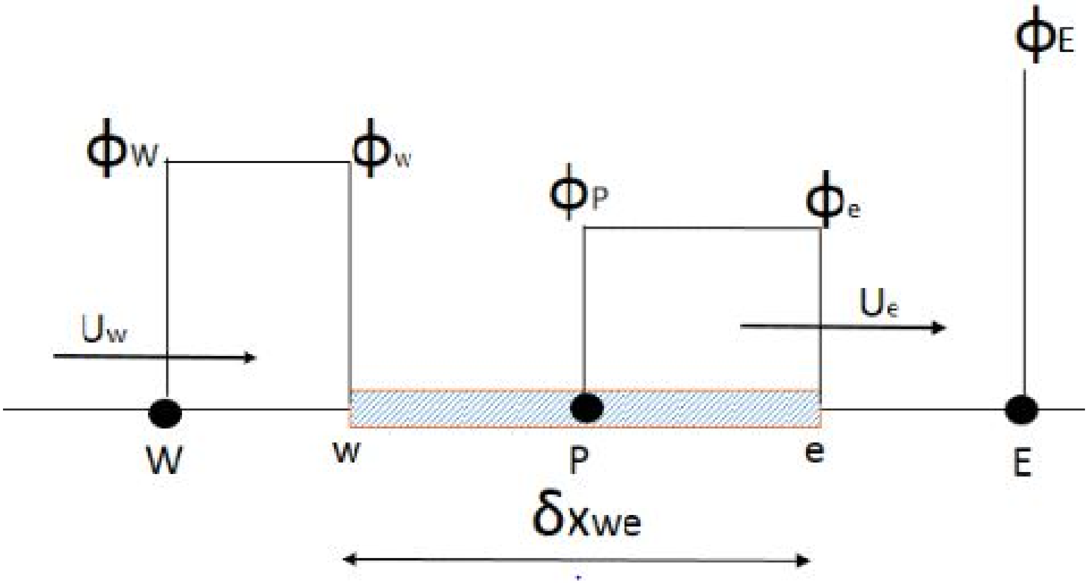

Figure 2.2: Upwind discretization scheme for positive flow direction. Point P is the centre of a control volume with faces e and w. [7]

If the problem were of a diffusive type the transport of a property spreads equally in all directions why central differencing is used in such cases. Convective transport of a flow property Φ is strongly dependent on the direction of flow. The upwind scheme accounts for this by making the following assumptions of values of a property at control volume faces.

$$\begin{cases} \Phi_w = \Phi_W \\ \Phi_e = \Phi_E \end{cases} \tag{2.4}$$

In this way the transportiveness criteria is fulfilled. The drawback is loss of accuracy as the scheme is only first order accurate. If the flow direction was reversed the same priciple will be true in the opposite direction.

2.1.3.2 The Staggered Grid

When solving general transport equations of flow properties usually the standard collocated grid formation in figure 2.2 is sufficient. However, this causes a fundamental problem when solving the momentum equation. If the velocity and pressure are stored at the same nodal points a non uniform pressure feild can act as a uniform pressure field when calculating the pressure gradient. This leads to an unphysical solution of the momentum equation. A common solution to this problem is the staggered grid formation shown in figure 2.3.

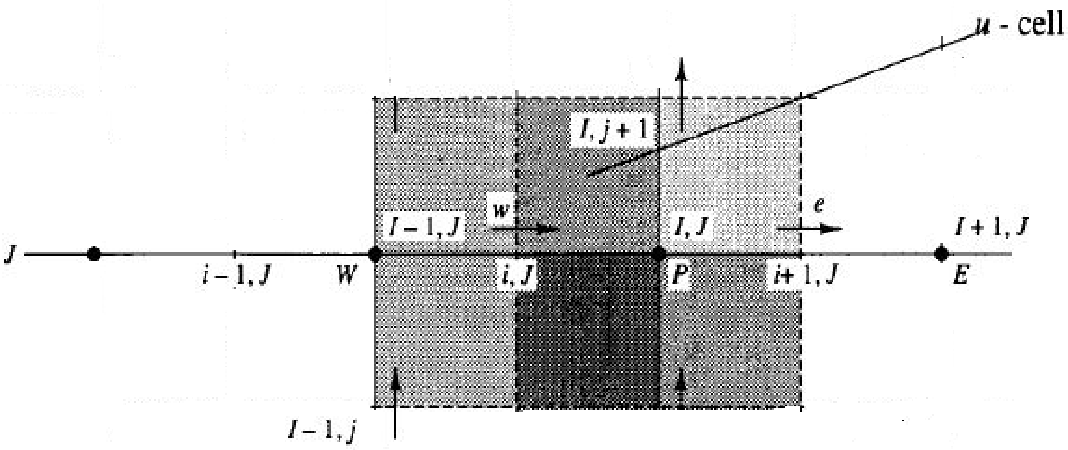

Figure 2.3: Staggered grid in one dimension. [7]

A new set of notations are introduced in order to correctly define the location of nodes and faces. Since only one-dimensional flow is considered the J coordinate is constant. The pressure and other scalar variables are stored in the scalar control volume which centre is denoted by capital letters. See point (I, J) in figure 2.3 where the pressure node is located. The velocities are stored in the u-control volume that is shifted half a control volume to the left of the P-control volume. The center of the u-control volume is located at point (i, J) in figure 2.3. The pressure nodes coincide with the control volume boundaries of the u-control volume.

All terms in the momentum equation are integrated over u-control volumes and the continuity equation is integrated over scalar control volumes. [7]

2.1.3.3 Solution of Equations

The Dymola simulation program can only handle ODE's with respect to time. However, the momentum and continuity equations are partial differential equations in space and time. This requires pre treatment of the equations by discretizing over a spatial control volume.

Consider a pipe of length L with constant cross sectional area divided into n control volumes along an equidistant grid spacing $\Delta x = (I, J) - (I - 1, J)$

Using staggered grid notations to integrate the continuity equation over the scalar control volume gives

$$\int_{i,J}^{i+1,J} \frac{\partial (\rho A)}{\partial t} dx + \int_{i,J}^{i+1,J} \frac{\partial (\rho A v)}{\partial x}\, dx = 0 \tag{2.5}$$

Assuming that the control volume boundaries are constant the integral and derivate are interchanged as

$$\frac{d}{dt} \int_{i,J}^{i+1,J} \rho A \, dx + \int_{i,J}^{i+1,J} \frac{\partial (\rho A v)}{\partial x} dx = 0 \tag{2.6}$$

Assuming an average density over the scalar control volume gives

$$\frac{d}{dt} A \rho_{I,J} \Delta x + \rho_{i+1,J} v_{i+1,J} A - \rho_{i,J} v_{i,J} A = 0 \tag{2.7}$$

The velocities at the scalar control volume faces don't require interpolation as they coincide with the velocity nodes in the u-control volume. Note that v in the equations above is equivalent to u in figure 2.3.

In a similiar way the momentum balance is integrated over the u-control volume.

$$\int\limits_{I-1,J}^{I,J} \frac{\partial\left(\rho v A\right)}{\partial t} dx + \int\limits_{I-1,J}^{I,J} \frac{\partial\left(\rho v^2 A\right)}{\partial x}\, dx = -\int\limits_{I-1,J}^{I,J} A\frac{\partial p}{\partial x} dx - \int\limits_{I-1,J}^{I,J} \frac{1}{2}\rho v|v| f S dx - \int\limits_{I-1,J}^{I,J} A\rho g\frac{\partial z}{\partial x}\, dx \tag{2.8}$$

Interchanging the integral and derivative for the accumulation term as in equation 2.6 the resulting terms after integration are

$$\frac{d}{dt}A\rho_{i,J}v_{i,J}\Delta x + \left(\rho v^2 A\right)_{I,J} - \left(\rho v^2 A\right)_{I-1,J} = -A\left(P_{I,J} - P_{I-1,J}\right) - \frac{1}{2}\rho_{i,J}v_{i,J}|v_{i,J}| f S - A\rho_{i,J}g\,\Delta z \tag{2.9}$$

The velocities at cell faces (I, J) and $(I-1, J)$ are estimated by the upwind scheme.

$$\frac{d}{dt}A\rho_{i,J}v_{i,J}\Delta x + \rho_{I,J}v_{i,J}^2 A - \rho_{I-1,J}v_{i-1,J}^2 A = -A\left(P_{I,J} - P_{I-1,J}\right) - \frac{1}{2}\rho_{i,J}v_{i,J}|v_{i,J}| f S - A\rho_{i,J}g\,\Delta z \tag{2.10}$$

Note that the pressures at faces (I, J) and $(I-1, J)$ are nodal point values in the scalar control volume. Equations 2.7 and 2.10 are two coupled equations for determining the correct velocity and pressure field since both equations are a function of the velocity. The system contains non-linear differential equations with respect to time. Integration over a time step is required, followed by iteration until a converged solution is obtained. The integration method *dassl* by Petzold(1982) was used for solving the equations and is breifly described in the next section. [8, 9]

2.1.3.4 Dassl

Dassl is one of the most universal algorithms for solving systems of DAE's of type index zero and index one. The index of a DAE is the amount of differentiations needed to turn the system into an ODE.

Consider a DAE on the general form with given initial conditions.

$$\begin{cases} F(t, y, y') = 0 \\ y(t_0) = y_0 \\ y'(t_0) = y'_0 \end{cases} \tag{2.11}$$

F, y and y' are assumed to be N-dimensional vectors. The derivative in 2.11 is replaced by a difference approximation known as the implicit Euler equation. This gives

$$F\left(t_{n+1}, y_{n+1}, \frac{y_{n+1}-y_n}{t_{n+1}-t_n}\right) = 0 \tag{2.12}$$

The algorithm can be extended beyond first order backward differencing to order k, which ranges from one to five. The order varies with the behaviour of the solution. The resulting non-linear system is solved using Newton's method. In order to start the solution *dassl* needs to solve an initialization problem of the form

$$F(t, y, y') = 0, t = t_0 \tag{2.13}$$

The transient behavior of fluids in the Dymola simulatiuons are always initiated in steady state which provides the required conditions for initialization. [10]

2.1.4 Method of Characteristics

The method of characteristics is a mathematical method for solving partial differential equations. It is widely used in a lot of commercial software specializing in fluid flow simulations. The idea is to transform the original equations into a set of ODE's valid along certain characteristic lines in the $x-t$ domain. Equation system 2.2 is expressed on a slightly different form.The convective transport term in the momentum equation is assumed negligible as is true for liquids with low compressibility. The static pressure is expressed as a manometric head. This gives

$$\begin{cases} \frac{\partial\left(\rho v^2 A\right)}{\partial x} = 0 \\ \frac{\partial p}{\partial x} = \rho g\left(\frac{\partial H}{\partial x} - \frac{\partial z}{\partial x}\right) = \rho g\left(\frac{\partial H}{\partial x} - \sin\alpha\right) \end{cases} \tag{2.14}$$

Dividing the momentum equation in 2.2 by ρA combined with the above assumptions the following equation is obtained.

$$g\frac{\partial H}{\partial x} + \frac{\partial v}{\partial t} + \frac{fv|v|}{2d} = 0 \tag{2.15}$$

The treatment of the continuity equation in this case is more extensive. Instead of integrating over a control volume the equation is expanded and expressed as a function of the wave speed.

$$\frac{a^2}{g}\frac{\partial v}{\partial x} + \frac{\partial H}{\partial t} = 0 \tag{2.16}$$

Equations 2.15 and 2.16 form a system of partial differential equations. Applying the The method yeilds close to exact solutions. A small error arises due to the required linearization of the friction term between two time steps.[5] See Appendix A for a more detailed description.

2.1.5 Valves

The pressure drop over a valve is a function of the valve coefficient and the volumetric flow rate.

$$\Delta P = \left(\frac{\dot{V}}{k_v}\right)^2 \tag{2.17}$$

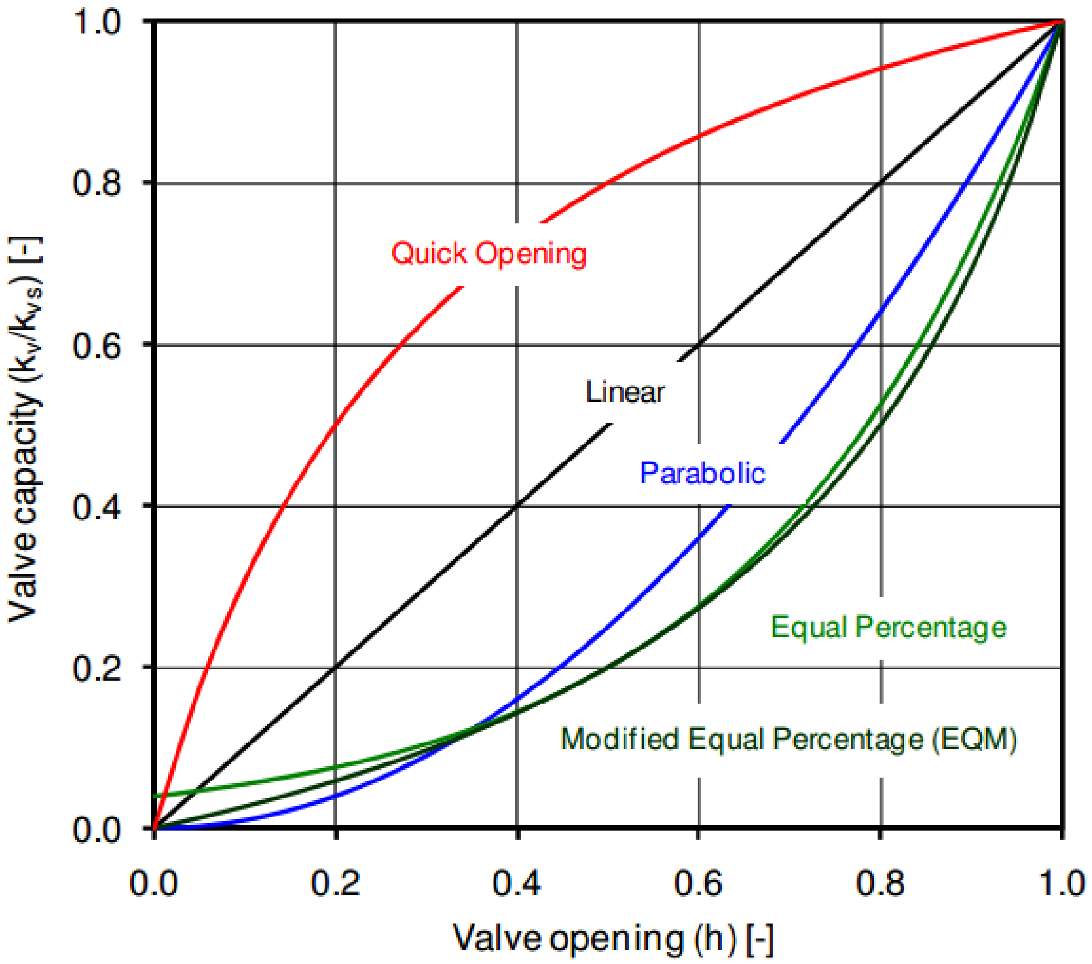

Figure 2.4: Various definitions of the valve characteristic

The standard unit of k_v is m^3/h although it can be seen from the above equation that this is not strictly dimensionally correct. If $k_v = 1\ m^3/h$ it implies that the flow through the valve is 1 m^3/h for a pressure drop of 1 bar.

The relationship between the valve coefficient and the opening h of the valve is known as the valve characteristic. See Figure 2.4. The k_{vs} value is the valve coefficient for a completely open valve.

Assume that at some nominal opening h the pressure drop and flow through the valve are known. Based on this operating point equation 2.17 then determines the k_v value for the opening. The flow in the system can then be determined for the real operating case with another value of ΔP in the same equation since k_v is unchanged as long as the valve opening position remains the same. [11]

2.1.6 Pipes

Pipes are the central component in a district heating system. The governing flow equations have been derived in section 2.1.1. The pressure drop due to friction is an important part of the momentum balance and is described in the following section.

2.1.6.1 Pressure Drop

The pressure drop over a pipe is dependent on several variables. One of the most determining factors is the state of flow with regard to turbulent or laminar conditions. For a flow in a circular pipe the following relations hold.

$$\begin{cases} Re = \dfrac{|v|d\rho}{\mu} = \dfrac{4\dot{m}}{\pi d\mu} \\ \dot{m} = \rho v A \\ A = \dfrac{\pi d^2}{4} \\ D = \dfrac{d}{\delta} \end{cases} \tag{2.18}$$

The solution to the 3-dimensional Navier-Stokes equation for laminar steady flow with constant pressure gradient, viscosity and density yeilds the Hagen-Poiseuille relation for the wall friction factor.

$$\lambda = \frac{64}{Re} \tag{2.19}$$

Generally λ is a function of both the Reynolds number and the relative roughness. The exact relationship between these variables can be found in a standard moody chart.

However in equation 2.19 the Reynolds number is zero when there is no mass flow. This leads to numerical difficulties in a simulation program. The solution to this is to define a modified friction coefficient as

$$\lambda_2 = \lambda Re^2 \tag{2.20}$$

Figure 2.5 shows the relationship between the flow and pipe parameters in this case.

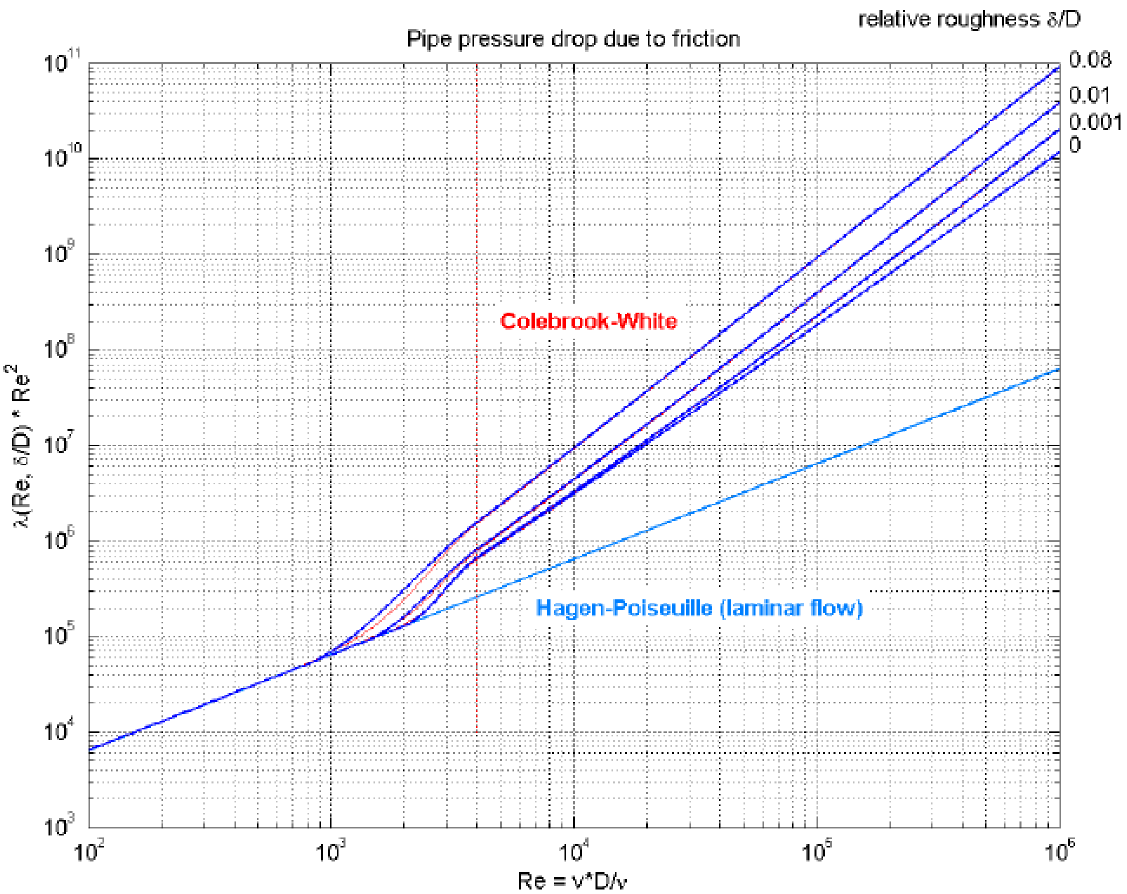

Figure 2.5: Correlations based on a modified friction factor λ_2

2.1.6.2 Laminar Flow

The pressure drop over a circular pipe is

$$\Delta P = \lambda(Re, d)\frac{L}{2d}\rho v|v| \tag{2.21}$$

By definition of the modified wall friction factor and assuming laminar flow where $Re \leq 2000$ the pressure drop is. [9]

$$\Delta P = 128\dot{m}\frac{\mu L}{\pi d^4 \rho} \tag{2.22}$$

λ_2 is estimated as a cubic polynomial between $Re_1(\delta/d)$ and $Re_2 = 4000$. [14]

2.1.7 Pumps

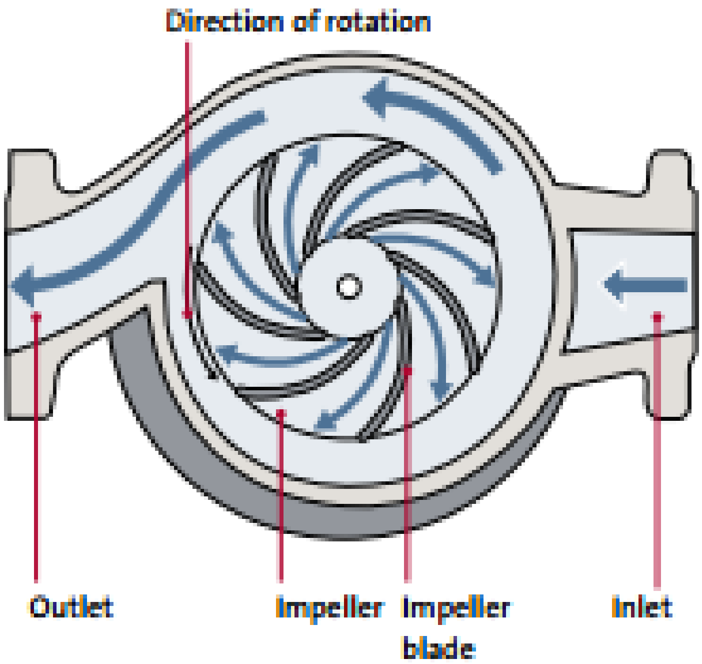

Figure 2.6: Centrifugal pump with impeller and diffuser at the exit. [15]

The most common type of pumps in district heating networks are centrifugal pumps which are in turbomachinery also referred to as radial-flow machines since the fluid enters axially and leaves in the radial direction. The main components of a centrifugal pump are a rotating impeller followed by a diffuser at the oulet. Fluid flows into the impeller eye on the suction side where the pressure is lower than the atmospheric pressure, as seen in figure 2.6. The impeller does work on the fluid by whirling it outward to increase the angular momentum. Both velocity and static pressure are increased during this process. As the flow eneters the diffuser with an increased flow radius the fluid velocity is converted into static pressure.

The pressure increase of the pump is often measured as the hydraulic head defined as

$$H = \frac{p}{\rho g} \tag{2.23}$$

3 | Method

3.1 Modelling Tools

3.1.1 Modelica

Modelica is a modeling language used for describing physical systems, created by the Modelica Association.[1] The language is object-oriented and handles dynamical systems. There are two different approaches to modelling physical systems, causal and acausal. In a causal system the model equations are evaluated according to a pre-defined order by a certain algorithm. Modelica is an example of an acausal or equation based system. In short, Modelica code has to be translated into an executable C-code and then linked to a numerical integration algorithm known as a solver. When the equations are manipulated by the compiler the resulting equation system is solved simultaneously by the solver. It is therefore enough to specify which equations are to be solved without specifying how or in what order. In order to initiate a Modelica simulation initial conditions need to be specified. This can be done either by assigning start values to individual variables or as certain initiation equations only valid at the initial time. [2]

Modelica aims to provide a language for model development over a wide range of engineering domains, which are divided into libraries. Components are built using a hierarchical structure and areas of application are for example fluids, mechanics or electrics. [2] The Modelica standard library offers a wide range of components programmed with the Modelica Modeling language. [3, 4]

3.1.2 Dymola

Dymola stands for *Dynamic Modeling Laboratory* and is a modeling and simulation tool from *Dassault Systèmes* based on the Modelica language. The software utilizes a graphical user interface to display components. Components can easily be found in the Modelica standard library for various applications. There are two modes, simulation and modeling. The simulation environment contains results of calculated variables and plotted figures, along with the solution algorithms.

3.2 General Procedure

The starting point for the project was to gather information and understand the theory behind the water hammer concept. At the same time, the basics of Dymola and Modelica were studied. The Governing equations of fluid transients were analyzed in combination with practical examples. In order to put theory into practice simulations were made in Dymola utilizing the finite volume method.

Studying the Modelica language and component interactions has been a large part of the project work. Chapter 4 gives a breif explanation of the components in the Dymola models and gives the details of the three models that have been simulated.

3.3 Model Verification

In order to observe that the models are functioning and simulate the water hammer concept several test models have been built. The transient flow situations analyzed are valve closure and pump failure. An example of each case is described in chapter 4 and the results of the simulations in chapter 5.

An effort to verify the Modelica model has been done in cooperation with Göteborg Energi who use the method of characteristics for transient flow calculations. The method has proven to give accurate results for water hammer calculations and is therefore considered to be the best reference point for comparison. The accuracy of the Dymola models depending on the number of control volumes in the pipes will be tested by a sensitivity analysis. The results of the sensitivity analysis will then be compared to the method of characteristics. All the tests will be run for a simple model of valve closure. Moreover, all results from the method of characteristics have been simulated at Göteborg Energi.

3.4 District Heating System of Gothenburg

The District heating system in Gothenburg consists of over a 1000 miles of underground pipes and supplies heat to the city as well as some neighbouring municipalities. The energy is supplied mostly by waste heat from industries and heat pumps at certain distribution points in the city. The area to be modelled is outside of the main city where the network connects to the Ale/Älvängen community. The case to be modelled is a pump failiure occuring along a main pipe of the network. A comparison between the FVM (Finite Volume Method) and MOC (Method of Characteristics) will not be made in this case as it becomes more difficult to recreate the same initial operating conditions. However, a sensitivity analysis of a varying number of nodes will be done. This model is considerably more complex than the previous ones but is also much more representative of a real district heating system. It will be used to test how the Modelica model performs in a more complex scenario and compared to the more basic case of pump failiure. The model has been simplified to an extent compared to the real system of the Göteborg network. See section 4.2.3.1

4 | Model Design

4.1 Modelica Fluid Package

This section gives a description of the assumptions and settings behind the components used for simulation in the models.

4.1.1 Incompressible Valve

Valves are a central component for flow regulation of a fluid in a pipe network. Dymola offers a range of different valve types. The component *Valve Incompressible* has been used in the models as it applies to almost incompressible fluids such as liquid water. The valve sizing is given according to the $IEC534/ISAS.75$ standards. [9] The component has a wide range of specifications for different flow conditions.

The Valves in the model are connected to a ramp function generator that controls the valve opening. An offset value determines the initial state of the valve opening ranging from 0 to 1. The time it takes to close or open the valve to a new position is then specified. Pressure drop and flow are determined by the equations in section 2.1.5.

4.1.2 Dynamic Pipe

The transient pipe behavior has been modelled using the Dymola component *Dynamic Pipe*. The pipe is discretized over an equidistant grid of one dimensional control volumes using the upwind scheme and staggered grid as described in section 2.2.2. A wide range of input specifications and assumptions are available. The initial boundary conditions are implemented by knowing the steady state conditions before a time dependent process starts. Other parameters such as diameter, pipe length and wall roughness are all specified in the component menu. The flow model *detailed pipe characteristic* has been used to calculate the pressure drop from wall friction according to the correlations in the theory section. The component is set to allow flow reversal as this naturally occurs for an oscillating pressure wave. Pipes are assumed to be inelastic. The effects of water hammer due to cavitation in the pipes will not be considered as this would complicate calculations significantly.

4.1.3 Connectors

All components in the Modelica fluid package are equipt with ports at the begining and end of the component in order to make it possible to connect to other components, thereby creating a flow network. The standard connectors use certian approximations that can reduce model accuracy. The mass balances are always exact but the momentum balances are only exact if connecting pipes are of equal diameter. This causes an error in the magnitude of the dynamic pressure but will be ignored in the simulations.

4.1.4 Controlled Pump

The Centrifugal pump described in the section 2.1.7 is represented in the model by the component *Controlled Pump*. The pump operating point is defined by the head, volumetric flowrate and rotational speed. Every pump has a certain point of maximum efficiency. For a certain rotational speed, the pump hydraulic characteristic defines the relationship between the head increase and the flow. In the modelica models the pressure head is set by controlling the rotational speed.[16]

4.1.5 Fluid Properties

The Fluid data has been taken from the International Association for the Properties of Water and Steam (IAPWS). The IF97 standard has been used in this case which is implmented in the Dymola fluid library. [9]

4.2 Modelica Models

The assumptions and settings in the Modelica models built and simulated in Dymola are presented here.A total of the models have been simulated. These include a simple case of valve closure and pump failure along with a pump failiure in a more complex system.

4.2.1 Valve Closure

As mentioned earlier, valve closure is one of the most common causes of water hammer. The test model used to simulate a valve closure event is shown in figure 4.1

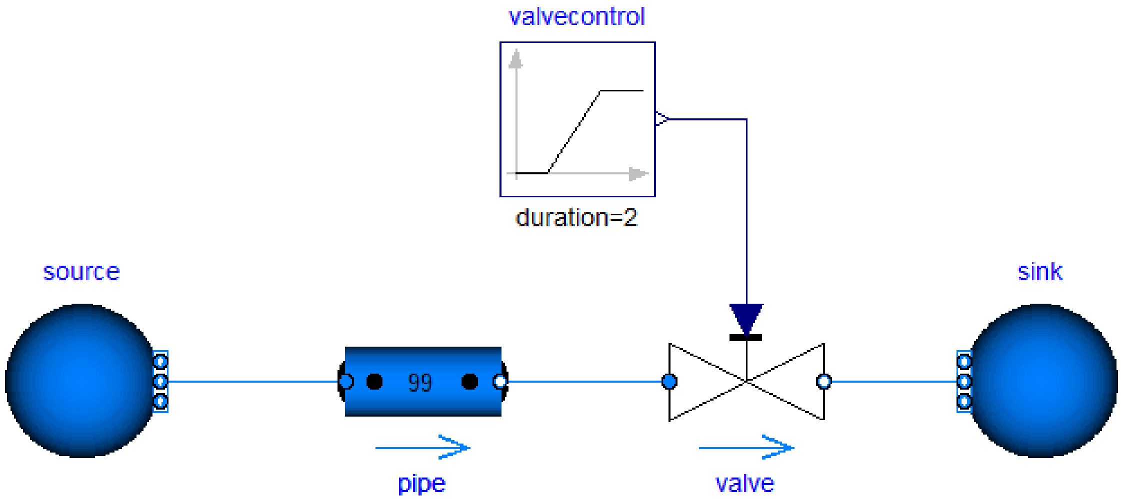

Figure 4.1: Experimental model for simulating sudden valve closure.

The various components are configured with values that determine the initial flow operating case. The difference in pressure between the source and sink drives the flow in the system. The pressure continously falls in the pipes until it reaches the end of the pipe. The pressure at the sink is set to zero in the Göteborg Energi model. In modelica it is only numerically possible to set the flow rate very close to zero resulting in a small error of the initial flow operating conditions between the two calculatuon methods. The model is initialized in steady state and the valve is assumed to be fully open before being closed to 20 % of its original opening during 2 seconds. The model has been simulated for cases of 5, 9, 17 and 99 nodes in the pipe. The pipe is assumed to be completely inelastic in the case of the FVM and consist of steel for the MOC. Specific model details are given in table 4.1

Table 4.1: Valve closure model specifications

Pipe Ref nr	d	Nominal mass flow/ΔP	Nominal h	Pressure mLc	L
source	-	-	-	20	-
sink	-	-	-	1	-
pipe	0.4	-	-	-	2000
valve	-	115/1	1	-	-

4.2.2 Pump Failure

Apart from valve closure, pump failiure will produce similiar transient effects on the fluid flow. The phenomena has been simulated in the system shown in figure 4.2

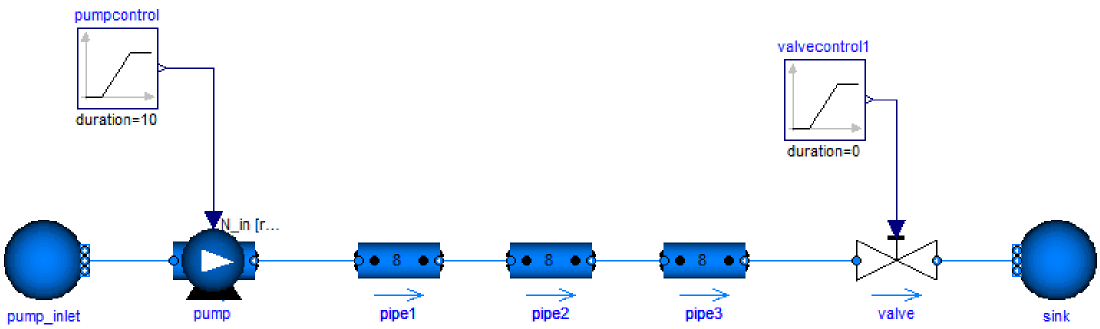

Figure 4.2: Experimental model for simulating pump failiure.

The model consists of three long pipes in series so that the system shares similarities with the larger model of the Gothenburg network where the effects of pump failiure are to be investigated. The pump receives a flow at low pressure and raises it to a higher pressure level depending on the rotational speed. A nominal rotational speed has been set as default. The pump characteristic is determined by a quadratic extrapolation function from three specified operating points of pressure increase and flow rate. The initial flow situation is otherwise similiar to the case of valve closure. The model is run for 5 seconds in steady state. The pump then suddenly stops during 10 seconds as the rotational speed becomes zero. Table 4.2 shows the specific model settings.

Table 4.2: model specifications for the case of simple pump failiure.

parameters	d	Nominal $\dot{m}/\Delta P$	$h_{nominal}$	$P_{initial}$	L	N/N_{nom}
pump inlet	-	-	-	0.5	-	-
sink	-	-	-	1	-	-
pipe 1	0.4	-	-	18	5000	-
pipe 2	0.4	-	-	18	5000	-
pipe 3	0.4	-	-	18	5000	-
valve 1	-	115/1	1	-	-	-
pump	-	-	-	-	-	2000/1500

4.2.3 Model of Gothenburg Network

Figure 4.3 shows the flow path through the different locations along the network. The loads in the system giving heat to surrounding areas are denoted SB. Along the main line are also subsystems containing pumps and boilers to control temperature and pressure which are represented by the red squares in figure 4.3.

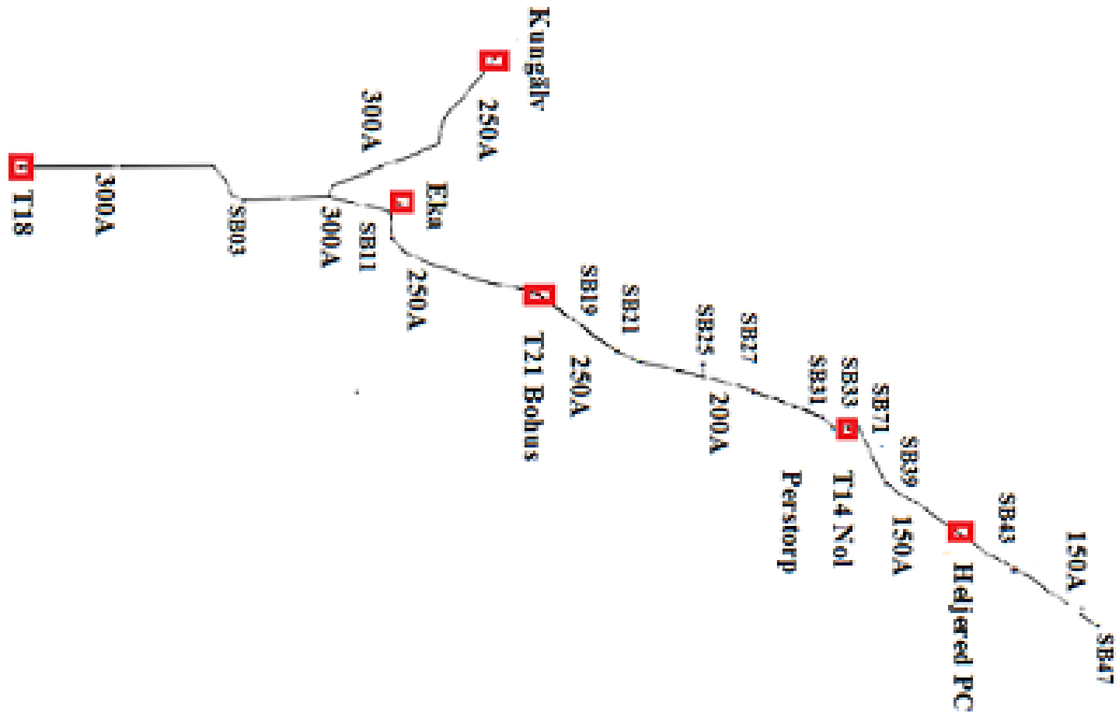

Figure 4.3: Flow schematic of the district heating network including loads and subsystems.

The flow in the system is a closed loop. As heat is exchanged at the loads the flow connects to a return system for reheat going back to pump T18. The somewhat simplified system that has been simulated in modelica only takes into account a return system with one load going back to pump T18. See figure 4.4

The boundary conditions for pressure in the system have been chosen to be as representative as possible of the real system. The pump design pressure is 16 bar as in the real district heating system of Göteborg Energi. The simulation procedure is similar to the preceding case of pump failure. The machine is shut down during 7 seconds. The red lines indicate the forward system and these pipes are denoted with an F. The return pipes are blue indicating that heat has been transferred to a surrounding area and are denoted with an R. The results of pressure distribution and sensitivity analysis are presented in chapter 5.

4.2.3.1 Modelling Assumptions

As the amount of components in the system increases the model becomes more unstable and the calculations more sensitive. It is therefore important to make reasonable assumptions in order to reduce complexity and at the same time maintain an acceptable level of accuracy. Heat losses in pipes are assumed to be negligible as the main focus is on system pressure. The curvature of the pipes has also been ignored as it is hard to get exact data and the pressure loss in the bends is negligible compared to the pressure loss due to pipe friction. Further simplifications involve the modelling of only one branch of the return flow system. As the boundary conditions are not the same as in the real network the flow rates will also differ. The tolerance for the residual in the iteration process is 10^{-4} for the simple cases of pump failiure and valve closure and 10^{-3} for the Gothenburg network.

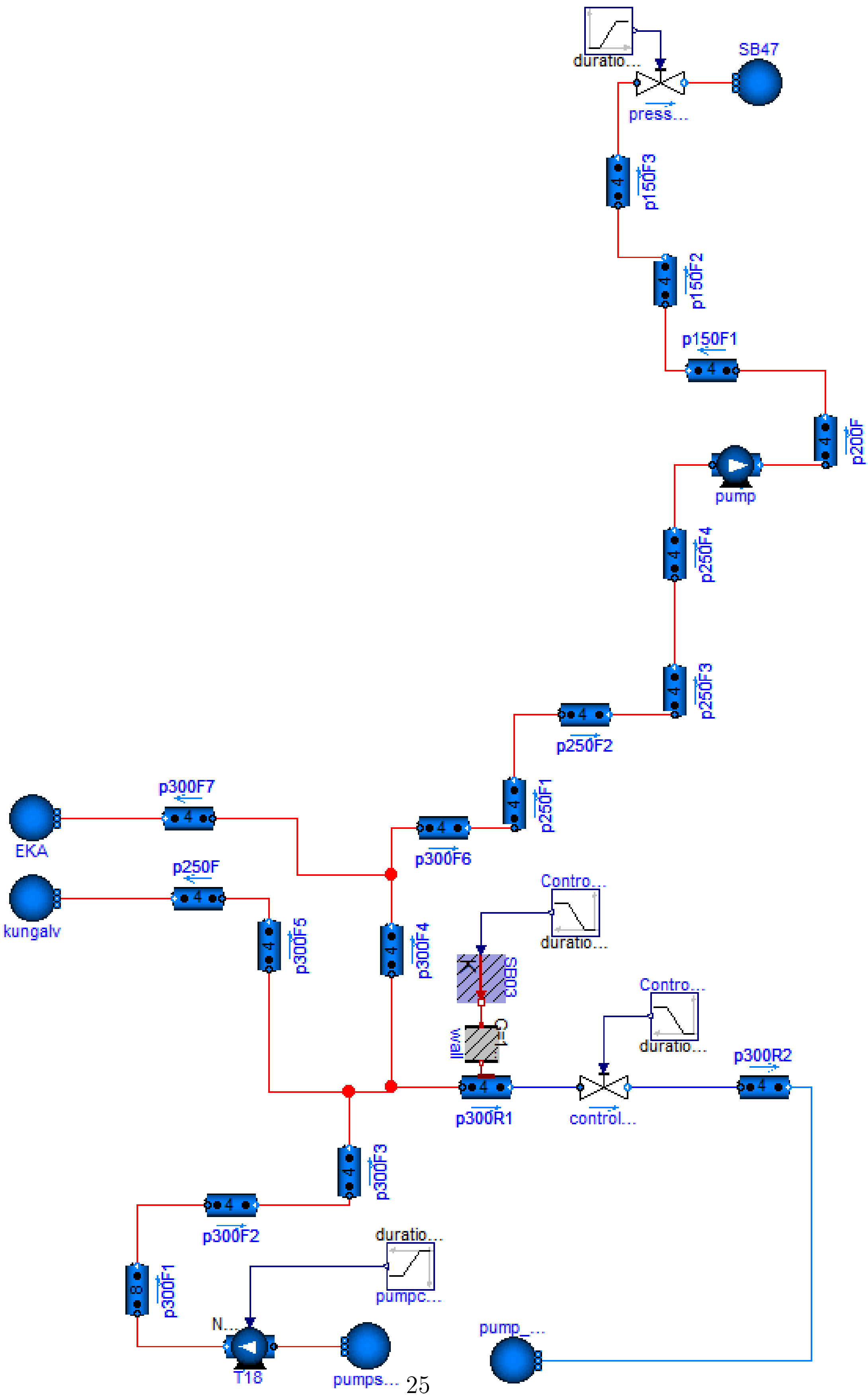

Figure 4.4: Flow schematic of Göteborg Energi in Dymola with a single return flow.

5 | Results

This chapter provides the results from the models described in chapter 4. All results from the finite volume method have been calculated in Dymola but some of the data has been reconstructed in Matlab to provide better visualization of the water hammer concept. In a real case district heating system pump failure is the most common cause of water hammer since technical problems with these machines will occur. However, water hammer due to valve closure can be avoided easier by not closing the valve too quickly.

5.1 Simple Valve Closure

The results presented in this section are from simulation of the valve closure case discussed in section 4.2.1. The results will be presented by showing pressure distribution over pipe length and pressure at pipe cross-sections as a function of time. All time scales are in seconds.

5.1.1 Method of Characteristics

As the method of characteristics is considered to give the most accurate results it will be used as a basis for comparing models. Figure 5.1 shows the propagation of a pressure wave resulting from valve closure, based on simulations performed by Göteborg Energi.

5.1.2 Finite Volume Method

Figures 5.3, 5.2 5.4 and 5.5 show the results from the Finite Volume Method with a different number of nodes along the pipe providing the sensitivity analysis.

The pressure distribution at a certain cross section of the pipe as a function of time will be shown for simulations with a varying amount of nodal points. See figure 5.6

In order to clarify the difference between the different Finte Volume Method cases see figure 5.7

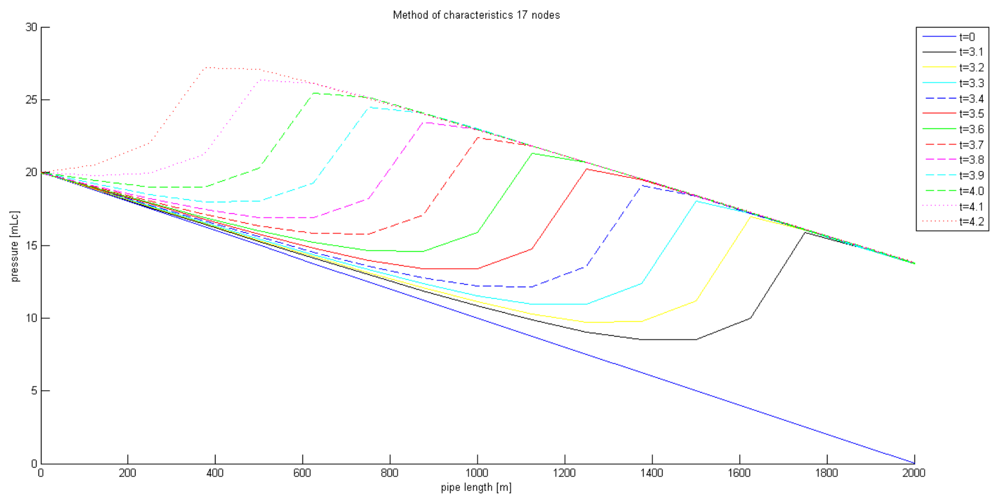

Figure 5.1: Pressure distribution along the pipe calculated by the method of characteristics.

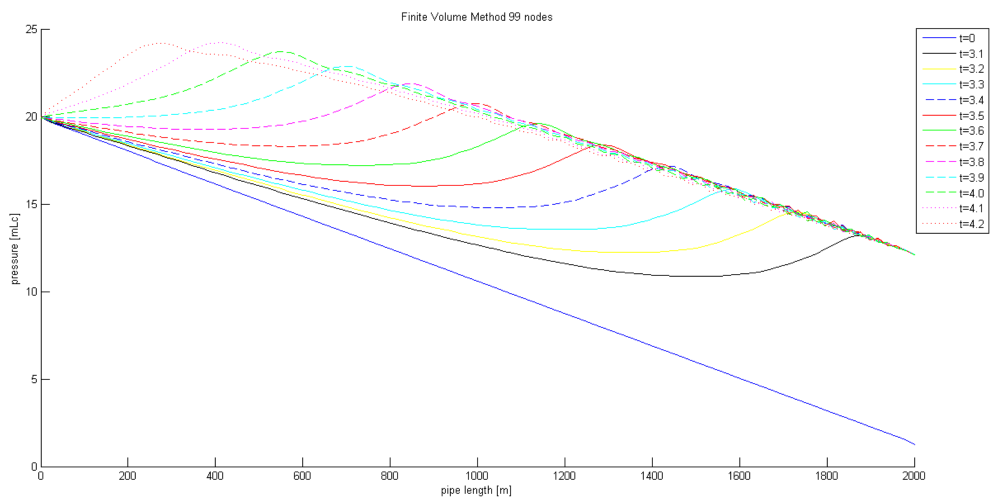

Figure 5.2: Pressure distribution along the pipe calculated by the Finite Volume Method with 99 nodes.

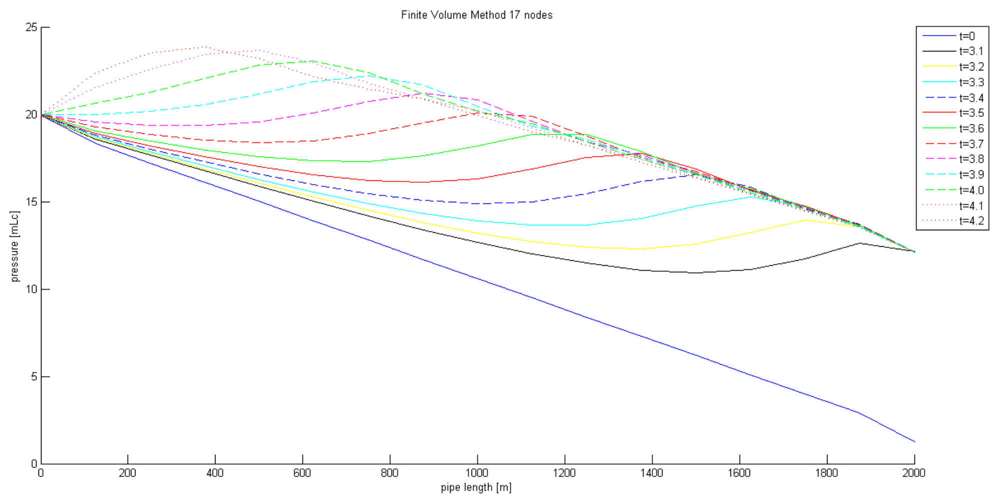

Figure 5.3: Pressure distribution along the pipe calculated by the Finite Volume Method with 17 nodes.

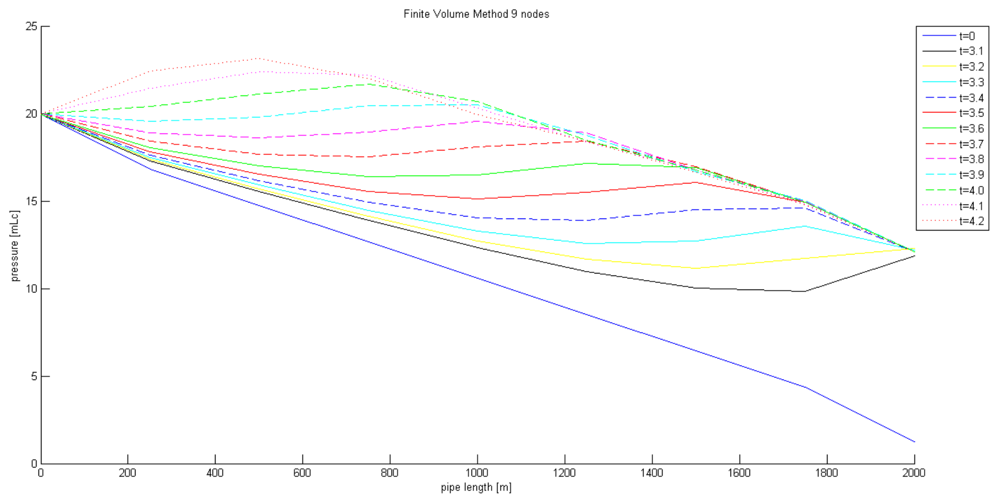

Figure 5.4: Pressure distribution along the pipe calculated by the Finite Volume Method with 9 nodes.

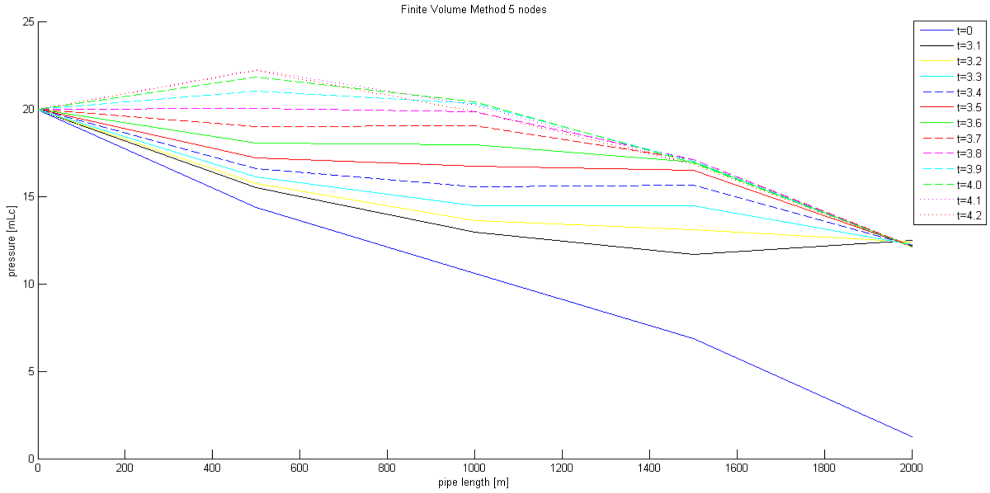

Figure 5.5: Pressure distribution along the pipe calculated by the Finite Volume Method with 5 nodes.

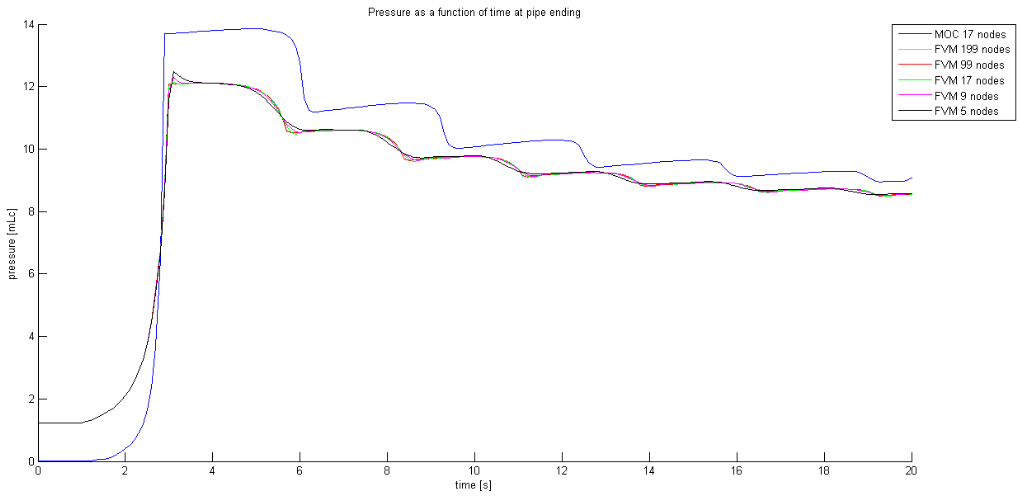

Figure 5.6: Variation of pressure with time at the end of the pipe.

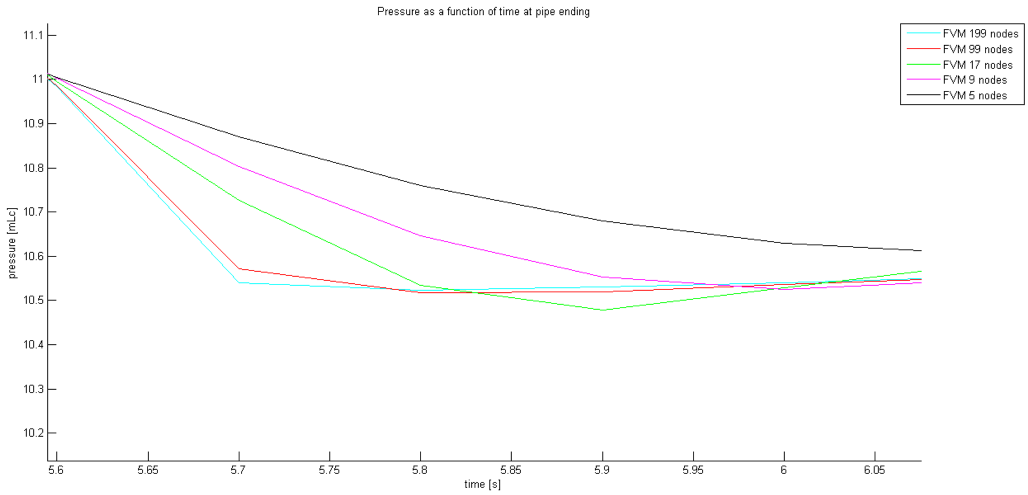

Figure 5.7: Variation of pressure with time at the end of the pipe.

5.2 Simple Pump Failiure

Recall the model discussed in section 4.2.2. To make the results of the simulation as representative as possible the pressure distribution will be shown as a function of both time and pipe length. Figure 5.8 shows the distribution of pressure along the whole pipe distance for different points in time in pipe 1.

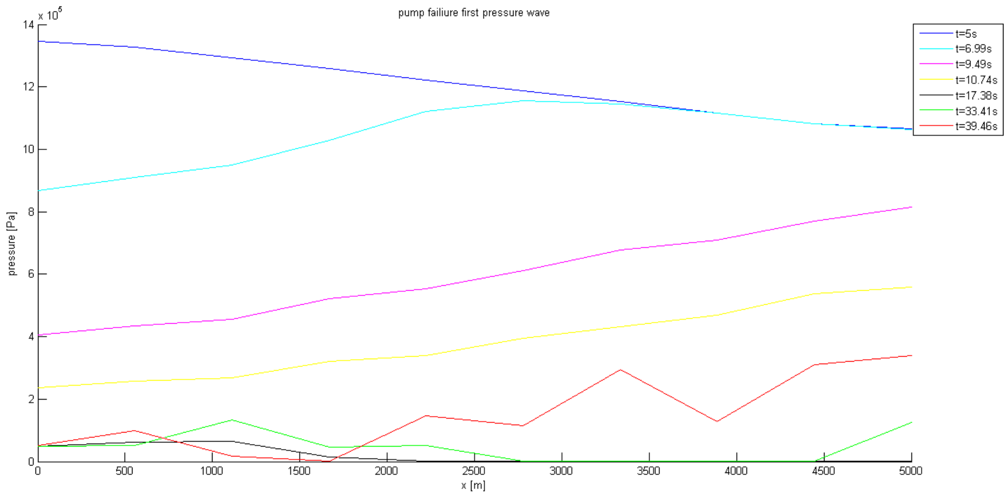

Figure 5.8: Pressure distribution in steel pipe 1 for different time periods after a pump failiure.

Figure 5.8 represents the whole process of pump failiure. The pump is suddenly shut down after 5 seconds. Before the event the only reason for lower pressure at the end of the pipe is frictional losses. As time progresses, it can be seen that the pressure starts to fall in the pipe as there is no longer a driving force. This causes the fluid to deccelerate. The change in velocity is then converted to pressure energy causing the pressure to rise again and spred backwards through the pipe. See the green and red lines in figure 5.8

To illustrate that the process continues in an oscillating manner as in the case of valve closure, see figure 5.9. Figure 5.9 shows the oscillating behaviour of the pressure after pump failiure at the end of pipe 1, pipe 2 and in the middle of pipe 3. Figure 5.8 and 5.9 provide two different perspectives of the process for the first 70 seconds.

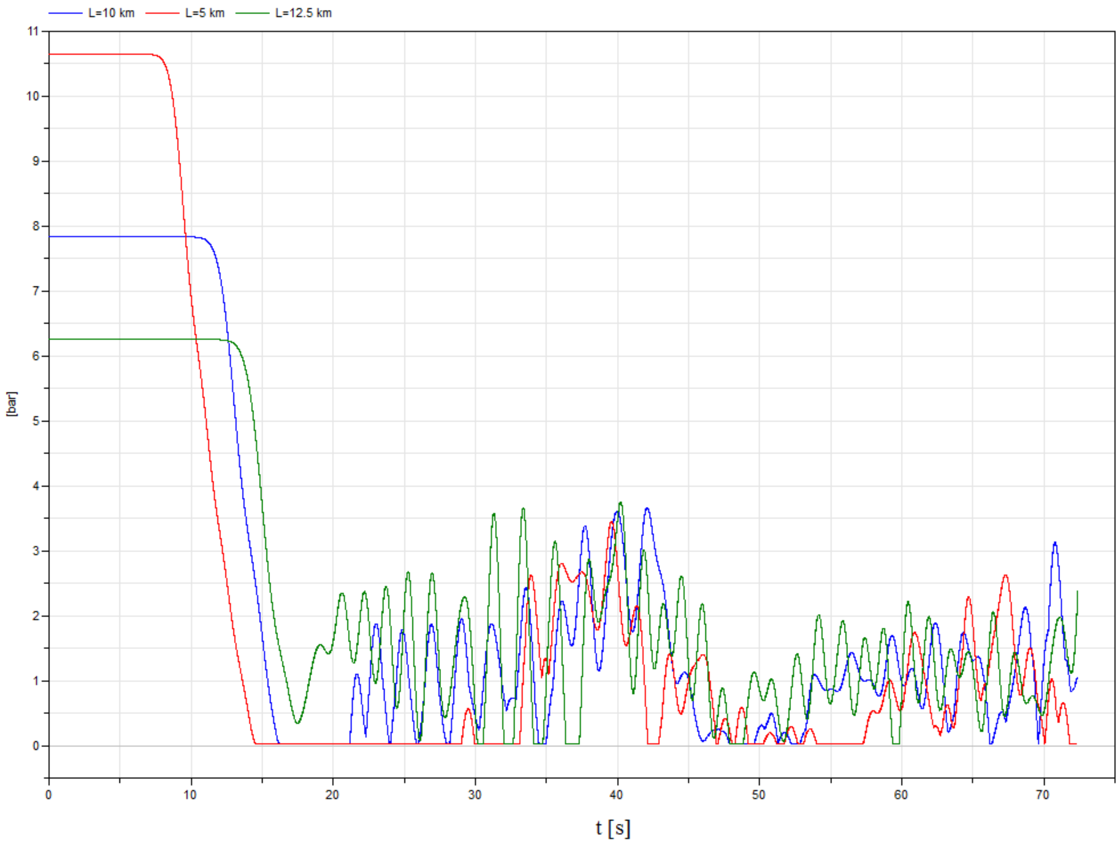

Figure 5.9: Pressure as a function of time for specific pipe cross sections.

5.3 Gothenburg Network

The simulations of the model presented in section 4.2.3 are presented below. Figures 5.10 and 5.11 show the pressure distribution in pipe P300F1(see fig 4.4) at the end and middle of the pipe for a varying number of nodes. Two points along the pipe have been chosen for the evaluation to confirm that the results at a certain point are not a coincidence. Figure 5.12 shows the same analysis but for a lower total pressure exerted by the pump for a setting of a lower rotational speed.

Figure 5.13 gives a representation of the effects of a pressure wave at different areas of the system.

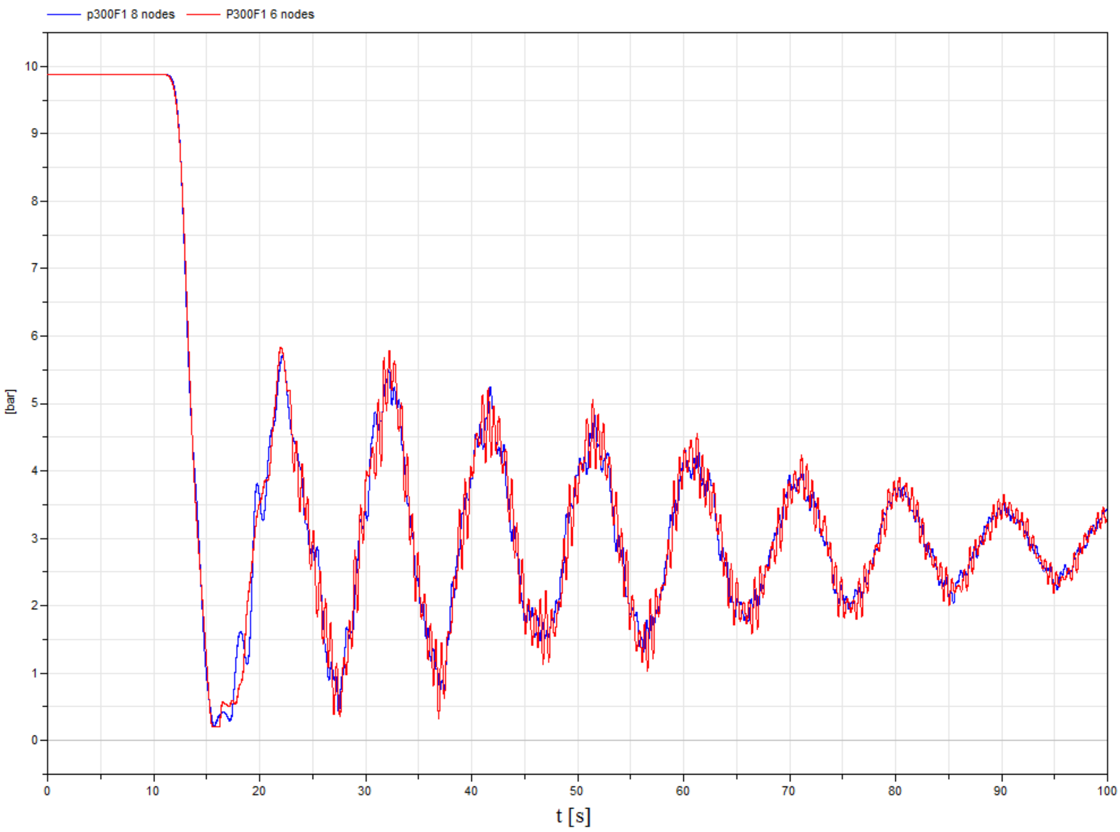

Figure 5.10: Pressure at the end of pipe P300F1 for 6 and 8 nodes.

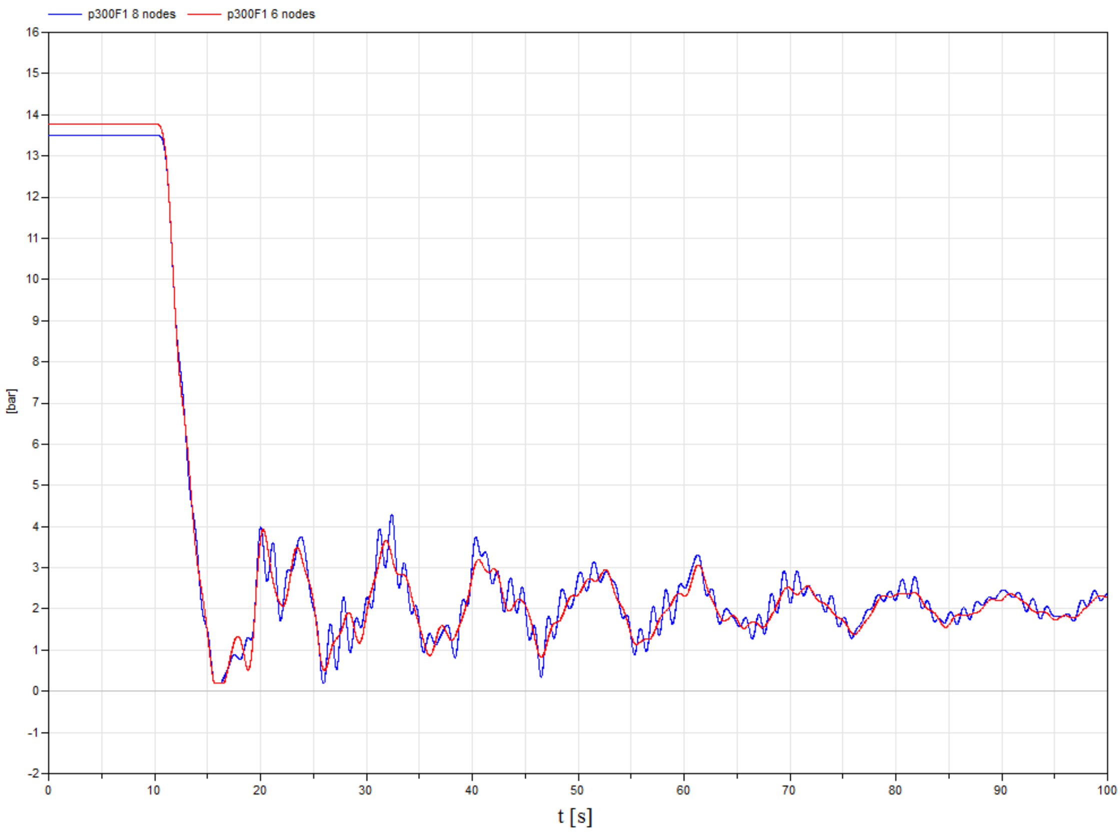

Figure 5.11: Pressure at the middle of pipe P300F1 for 6 and 8 nodes.

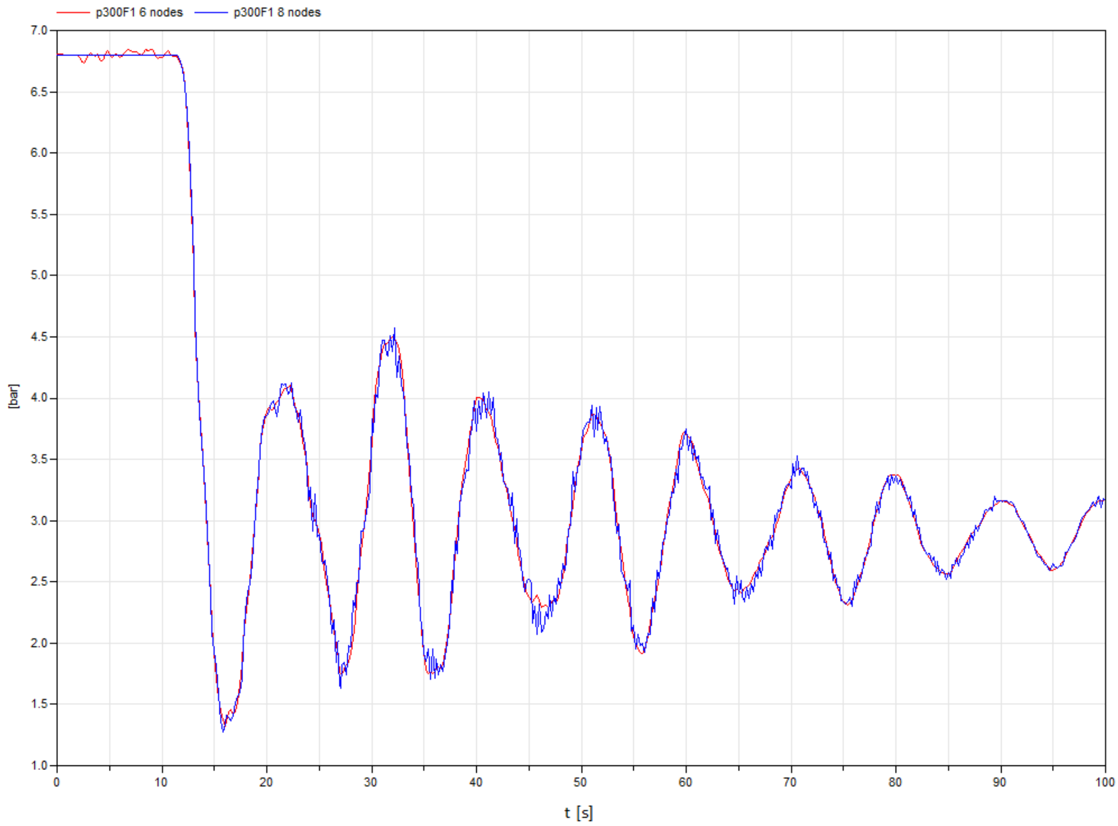

Figure 5.12: Pressure at the end of pipe P300F1 for 6 and 8 nodes with lower total pressure.

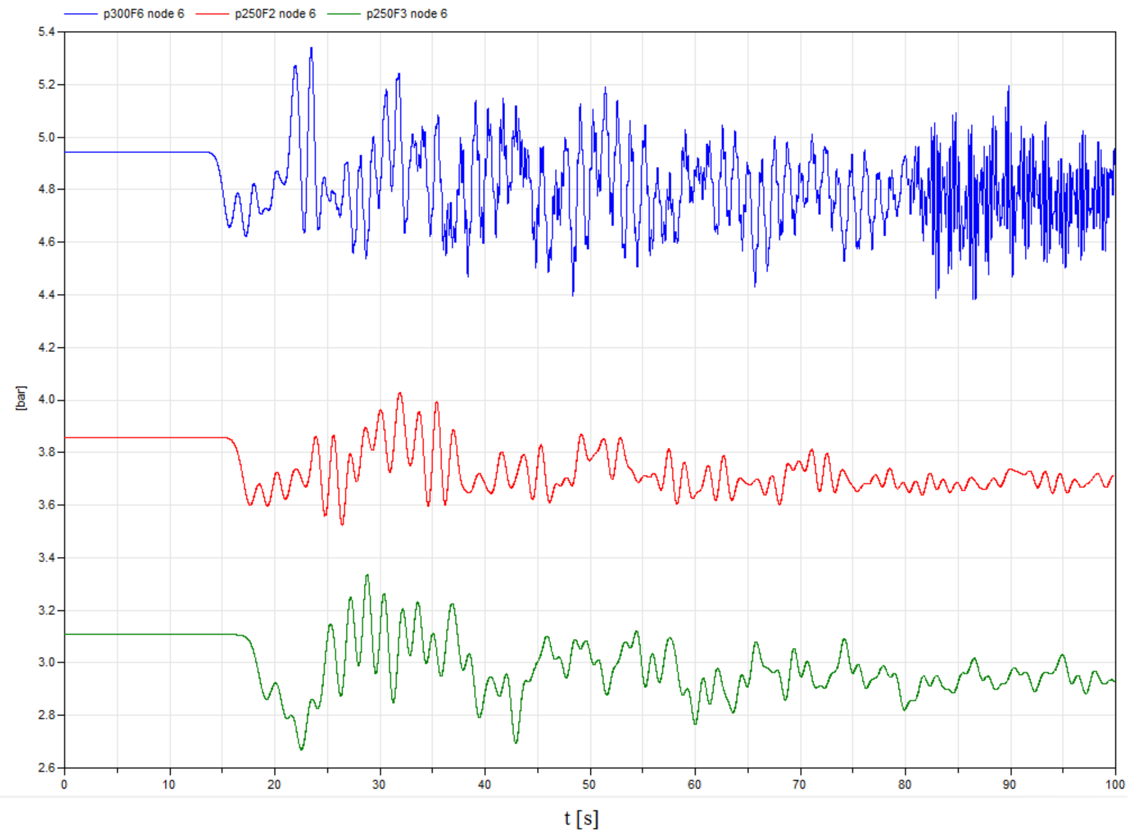

Figure 5.13: Pressures at the end of pipes P300F6, P250F2 and p250F3.

6 | Analysis of Results

6.1 Valve Closure

Considering the results from the simulations of valve closure in chapter 5 several important observations can be made. The results confirm that both calculation methods are applicable for solving the water hammer problem. When using the finite volume method with an equal number of nodes as the method of characteristics the pressure distribution follows the same trend. However, there is also a clearly observable difference. The accuracy of the finite volume method is poorer and declines even further as the number of nodes decreases. When using only 5 nodes as in figure 5.5 the finite volume method is not a good estimate as the pattern differs significantly from the MOC and the FVM case of 17 nodes.

In theory, when increasing the number of nodes the results from the FVM should approach those of the MOC. Recall figures 5.7 and 5.6. The general pattern of both calculation methods are the same but offset with respect to both time and absolute value. The reason for the offset in time is most likely due to the difference in wave speed between the two models. Recall that the MOC assumes the pipe to be elastic and the FVM considers the pipe to be inelastic. The wave speed will therefore be lower for the MOC. As the number of nodes in the FVM increases to 99 nodes the absolute value does not approach that of the MOC by much compared to the case of 17 nodes. On the other hand there is a clear difference in absolute value between the cases of 5 and 17 nodes. When using an increasing number of nodes the curvature of the graph is significantly different from using fewer nodes and more similiar to the MOC. This difference is seen most clearly for the highest resolution case of 99 nodes. As no sensitivity analysis has been done on the MOC it is hard to say how large the difference would be if the MOC case was simulated using 99 nodes compared to 17 nodes. Moreover, figure 5.7 shows the expected oscillating behavior of a pressure wave resulting from valve closure. The figure shows how the pressure varies at the end of the pipe as a function of time. It can be seen that an increase to 199 nodes in the FVM still doesn't give an absolute value that is close to that of the MOC. However, there is a large difference in curvature of the graphs. An increase in number of nodes from 5 to 9 and from 99 to 199 is in relative terms an increase by double for both cases. The difference in pressure is a lot larger between 5 and 9 nodes than between 99 and 199 nodes. This means that the relationship between number of nodes and accuracy is not linear and increasing the number of nodes when the amount of nodes is already relatively large will cost in terms of computational power compared to a small gain in accuracy.

It is difficult to estimate how the small difference in the steady state operating conditions influences the difference in presure distribution why it is not possible to make any assumptions on how much it has affected the results. The difference in the steady state case can clearly be seen from all the figures in the case of valve closure since the pressure at the end of the pipe is not zero before the start of each simulation.

6.2 Pump Failure

Comparing the different simulations of pump failiure the overall trend is the same in both the simpler system and more complex one of Göteborg Energi. Figure 5.13 indicates that when moving further away from pump T18 which is the source of water hammer in this case, the effect and magnitude of the pressure waves decrease. This is to be expected as energy and momentum are lost due to friction.

The sensitivity analysis in figures 5.10 and 5.11 imply that a small number of nodes in a large system will yield results with poor accuracy and uncertainty. When changing from 6 to 8 nodes the pressure can differ by almost 1 bar which is a considerable amount if a detailed analysis is to be done. Two points along the pipe where chosen to confirm that the sensitivity analysis was not just a coincidence at a certain point. Increasing the relative difference in the amount of nodes in a pipe has a larger effect on the the more complex model than the simple one when it comes to difference in pressure distribution. This can be seen by comparing figures 5.10-5.12 with figure 5.6. Note that the pressure units are different. Consider figure 5.12. The pump has been shut down at exactly the same rate but the design pressure in the system is lower. The difference between the 6 and 8 node simulation is somewhat smaller for this case. It is hard from this result to conclude that a lower total pressure decreases the difference in pressure distribution when simulating with different amounts of nodes. The reason for this is the effect of other boundary conditions. As the total design pressure from the pump decreases so does the relative gradient compared to the fixed pressure at for example Kungälv and Eka. This will lead to a lower flow velocity in the system which in turn implies a lower change in momentum during pump failure.

As the number of nodes in a Dymola simulation increases so does the computational time due to an increase in complexity and amount of equations to be solved. This leads to a trade off between accuracy and computational time. However, the biggest problem with a large and complex system is the initialization of the model. This means finding the initial solution based on the given boundary conditions and is an iterative procedure. If this crucial process fails no simulations of the model can be made at all. When using more than 8 nodes in the gothenburg network the initialization failed. It is possible to guess new starting values to make the initialization possible but this may become very tedious and difficult. A secondary problem in Dymola are the sharp gradients that occur when simulating instantaneous valve closure or pump failure. Often the calculations fail in these cases, probably because of divergence of the solution. In the Gothenburg network, if the pump was to shut down in less than 7 seconds the simulation would fail. The gradient problem can be improved somewhat by increasing the number of time steps in the integrator.

7 | Conclusion

A big advantage of the MOC is that no iteration is required. This method is therefore more stable for larger systems. Recall that the exact momentum equation is simplified for the MOC. This simplified equation is then solved with a good level of accuracy and the simplification in itself is a very good approximation for incompressible fluids. The FVM on the other hand solves the exact momentum equation but a lot of accuracy is lost after discretization and using a small amount of nodes. When taking all these matters into consideration the overall conclusion is that Dymola is suitable for making fairly rough estimations of the water hammer process in systems that are not too complex.

8 | Further Work

In this project reducing water hammer effects during simulation has not been considered. Therefore, a future project could investigate the capability of using Dymola for this purpose. As Dymola is already sensitive to complex systems it may be the case that such an investigation is not feasible. However, efforts could be made to improve initialization and thereby create more stability in the calculations which is the major concern for the program to perform well when simulating water hammer.

Bibliography

[1] *Modelica Association*. Mar. 19, 2015. URL: `http://www.modelica.org`.

[2] Modelica language specification 3.3. URL: `http://www.Modelica.org`.

[3] S. E. Mattsson et al. "Initialization of Hybrid Differential-Algebraic Equations in Modelica 2". In: *Proceedings of the 2nd International Modelica Conference*. Oberpfaffenhofen, Germany, Mar. 18–19, 2002, pp. 9–15.

[4] *Dymola - User Manual Volume 1*. 15th ed. Dassault Systemes AB. Sept. 2013.

[5] G.Larsson. *Reglerdynamik, trychållning och tryckslag i stora rörsystem*. Institutionen för Energiteknik, Chalmers Tekniska Högskola., 2003.

[6] G. Rasmusson. *Analys av tryckslag i rörsystem*. Karlstads universitet, June 27, 2006.

[7] H.K Veersteg, W.Malalasekera. *An Introduction To Computational Fluid Dynamics, 2nd edition*. Pearson Education Limited, 2007. ISBN: 9780131274983.

[8] H.Elmqvist. *Finite Volume Modelica package for Modelica.Fluid*. Aug. 27, 2004.

[9] *User's Guide to the Dymola Fluid package, 2015.*

[10] Brenan K.E, Campbell S.L,Petzold L.R. *Numerical Solution To Initial Value Problems in Differential-Algebraic Equations*. North-Holland,New York, 1989. ISBN: 0898713536.

[11] Anders Truschel. *Hydronic Heating And Cooling Systems, Lecture notes*. Feb. 27, 2014.

[12] Colebrook, F. *Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws*. Inst Civ. Eng no.4, pp 14-25, 1939.

[13] Swamee P.K, Jain A.K. *Explicit equations for pipe-flow problems.* Proc. ASCE, J.Hydraul Div, 102 (HY5), pp. 657-664, 1976.

[14] Samoilenko, L.A. *Investigation of the Hydraulic Resistance of Pipelines in the Zone of Transition from Laminar into Turbulent Motion.* Thesis (Cand. of Technical Science), Leningrad, 1968.

[15] *Crane Enginnering.* Feb. 12, 2015. URL: `http://www.craneengineering.net`.

[16] Dixon S.L, Hall C.A. *Fluid Mechanics and Thermodynamics of Turbomachinery, 7th edition.* Elsevier Inc, 2014. ISBN: 9780124159549.

A | Appendix A

A.1 Method of Characteristics

This sections provides a more in detail explenation of how the MOC solves the combined momentum and continuity problem.

The system of partial differential equations to be solved is

$$\begin{cases} L_1 = g\dfrac{\partial H}{\partial x} + \dfrac{\partial v}{\partial t} + \dfrac{fv|v|}{2d} = 0 \\ L_2 = \dfrac{a^2}{g}\dfrac{\partial v}{\partial x} + \dfrac{\partial H}{\partial t} = 0 \end{cases} \tag{A.1}$$

The addition of the above equations is assumed to be a function of an unknown multiplier q.

$$L = L_1 + L_2 = q\left(\frac{\partial H}{\partial x}\frac{g}{q} + \frac{\partial H}{\partial t}\right) + \left(\frac{\partial v}{\partial x}q\frac{a^2}{g} + \frac{\partial v}{\partial t}\right) + \frac{fv|v|}{2d} = 0 \tag{A.2}$$

The differentials of H and v are written as

$$\begin{cases} \dfrac{dH}{dt} = \dfrac{\partial H}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial H}{\partial t} \\ \dfrac{dv}{dt} = \dfrac{\partial v}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial v}{\partial t} \end{cases} \tag{A.3}$$

By combining equations A.2 and A.3 it can be shown that if

$$\frac{dx}{dt} = \frac{g}{q} = \frac{qa^2}{g} \quad , \tag{A.4}$$

Then equation A.2 will become an ODE of the form

$$q\frac{dH}{dt} + \frac{dv}{dt} + \frac{fv|v|}{2d} = 0 \tag{A.5}$$

According to A.4 q has two solutions

$$q = \pm \frac{g}{a} \tag{A.6}$$

Inserting both values of q into equation A.4 gives $\frac{dx}{dt} = \pm a$.

After inserting q in A.5 the characteristic line functions are obtained.

$$C^{+}: \begin{cases} \frac{g}{a}\frac{dH}{dt} + \frac{dv}{dt} + \frac{fv|v|}{2d} = 0 \\ \frac{dx}{dt} = +a \end{cases} \tag{A.7}$$

$$C^{-}: \begin{cases} -\frac{g}{a}\frac{dH}{dt} + \frac{dv}{dt} + \frac{fv|v|}{2d} = 0 \\ \frac{dx}{dt} = -a \end{cases} \tag{A.8}$$

Figure A.1 shows an arbitrary point P in the $x - t$ plane located at the centre of points A and B along a pipe divided into N sections. The properties of pressure and flow rate at point P are a function of these properties at points A and B. The connecting functions are the characteristic lines C^{+} and C^{-} which represent the wave velocity. Thus, values at the current time step are calculated from values assumed to be known at a previous time step.

Integrating equation A.7 between A and P and equation A.8 between B and P gives the following relationship for the characteristic line functions.

$$\begin{cases} C^{+}: H_P = H_A - B(\dot{V}_P - \dot{V}_A) - R|\dot{V}_A| \\ C^{-}: H_P = H_B + B(\dot{V}_P - \dot{V}_A) + R|\dot{V}_B| \quad , \end{cases} \tag{A.9}$$

Where $B = \frac{a}{gA}$ and $R = \frac{f\Delta x}{2gdA^2}$.

On a more general form equation system A.9 can be expressed as

$$\begin{cases} C^{+}: H(x,t) = C_P - B_P\dot{V}(x,t) \\ C^{-}: H(x,t) = C_m + B_m\dot{V}(x,t) \quad , \end{cases} \tag{A.10}$$

Where

$$\begin{cases} C_P = H(t-\Delta t, x-\Delta x) + B\dot{V}(t-\Delta t, x-\Delta x) \\ B_P = B + R|\dot{V}(t-\Delta t, x-\Delta x)| \\ C_m = H(t-\Delta t, x+\Delta x) - B\dot{V}(t-\Delta t, x+\Delta x) \\ B_m = B + R|\dot{V}(t-\Delta t, x+\Delta x)| \end{cases} \tag{A.11}$$

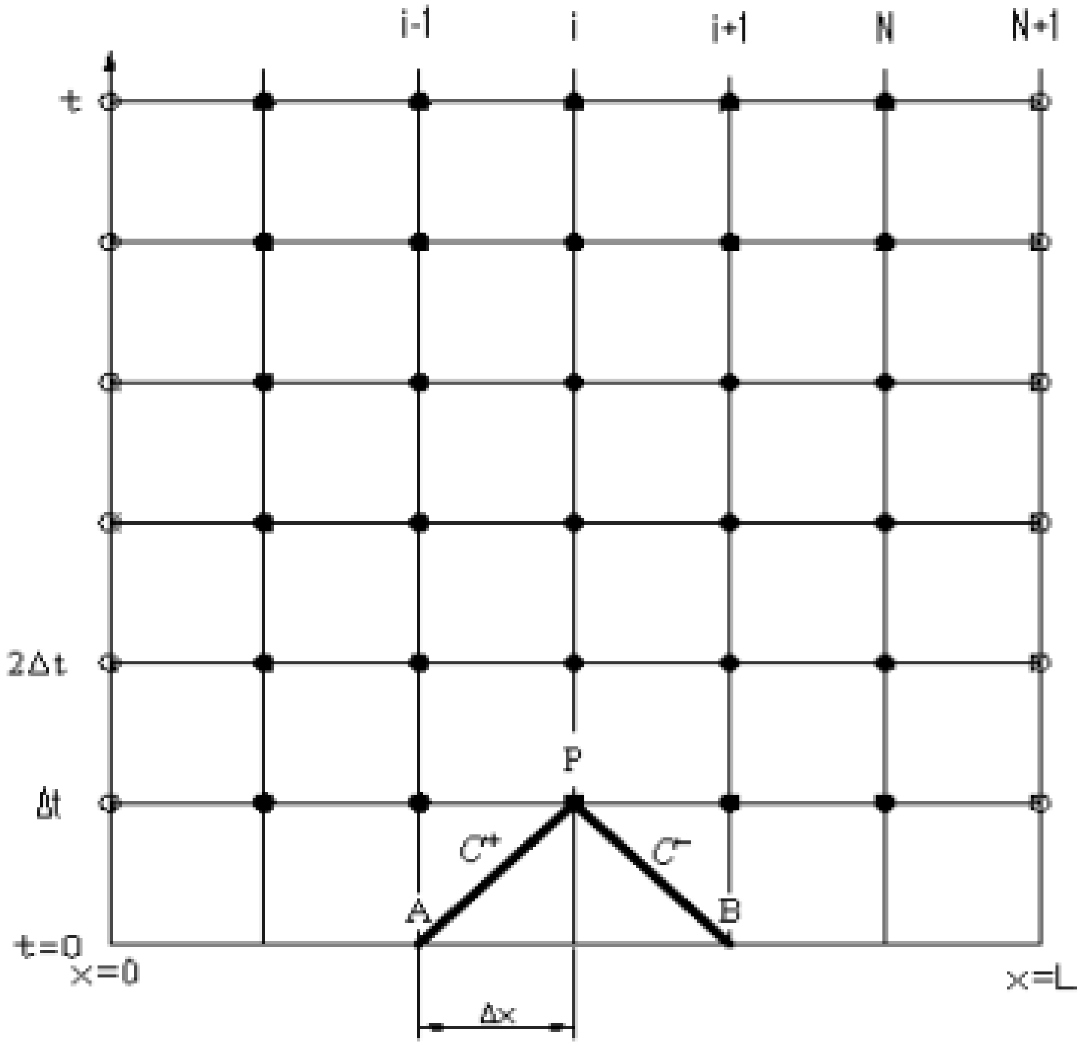

Figure A.1: Characteristic lines in the $x - t$ plane relating points A,B and P. [6]

Combining the equations in A.10 gives an expression for the pressure head.

$$H(x,t) = \frac{C_P B_m + C_m B_P}{B_p + B_m} \tag{A.12}$$

The flow rate $\dot{V}(x,t)$ is determined from any of the equations in A.10. The initial Values $\dot{V}(x,0)$ and $H(x,0)$ are determined along the x-axis. The calculation then proceeds by determining $\dot{V}(x,t)$ and $H(x,t)$ for each time step. Observe that at the begining and end of the pipe C^+ or C^- do not exist. This means that extra relationships are needed to define the values of $\dot{V}(0,t)$, $H(0,t)$, $\dot{V}(L,t)$ and $H(L,t)$ for a solution to be possible. [6]

B | Appendix B

B.1 Specific Model Data of Gothenburg Network

```
C    Pipe Performance Table(5)
C    -------------------------
C    Pipe    Diam   Rough  Kval   E-mod  Thick  Wall Heat
Design P  Comment
C    Ref-Nr  Di mm  Ks mm  W/m,C  GN/m2   mm     MJ/m3,C
bar       (text)
       50A     54.5  0.10   0.2115   210   2.9     3.60
16
       65A     70.3  0.10   0.2395   210   2.9     3.60
16
       80A     82.5  0.10   0.2505   210   3.2     3.60
16
      100A    107.1  0.10   0.2605   210   3.6     3.60
16
      125A    132.5  0.10   0.2560   210   3.6     3.60
16
      175S    174.8  0.10   0.2560   210   3.6     3.60
16
      150A    160.3  0.10   0.2835   210   4.0     3.60
16
      212S    211.5  0.10   0.2835   210   4.0     3.60
16
      175A    182.0  0.10   0.2835   210   4.0     3.60
16
      200A    210.1  0.10   0.3645   210   4.5     3.60
16
      225A    237.0  0.10   0.3645   210   4.5     3.60
16
      250A    263.0  0.10   0.3565   210   5.0     3.60
16
      277S    277.2  0.10   0.3565   210   5.0     3.60
16
      300A    312.7  0.10   0.4095   210   5.6     3.60
16
      350A    344.4  0.10   0.5135   210   5.6     3.60
16
      400A    393.8  0.10   0.5480   210   6.3     3.60
16
      404S    403.9  0.10   0.5480   210   6.3     3.60
16
      450A    455.0  0.10   1.096    210   6.3     3.60
16
      520S    519.6  0.10   1.09     210  13.0     3.60
16
      500A    495.4  0.10   0.5345   210   6.3     3.60
16
      503S    502.6  0.10   0.5345   210   6.3     3.60
16
      518S    517.8  0.10   0.5345   210   6.3     3.60
16
      600A    595.8  0.10   0.6565   210   7.1     3.60
16
      654S    654.0  0.10   0.6565   210   7.1     3.60
16
      786S    786.2  0.10   1.3      210  14.0     3.60
16
      613S    613.0  0.10   0.6565   210   7.1     3.60
```