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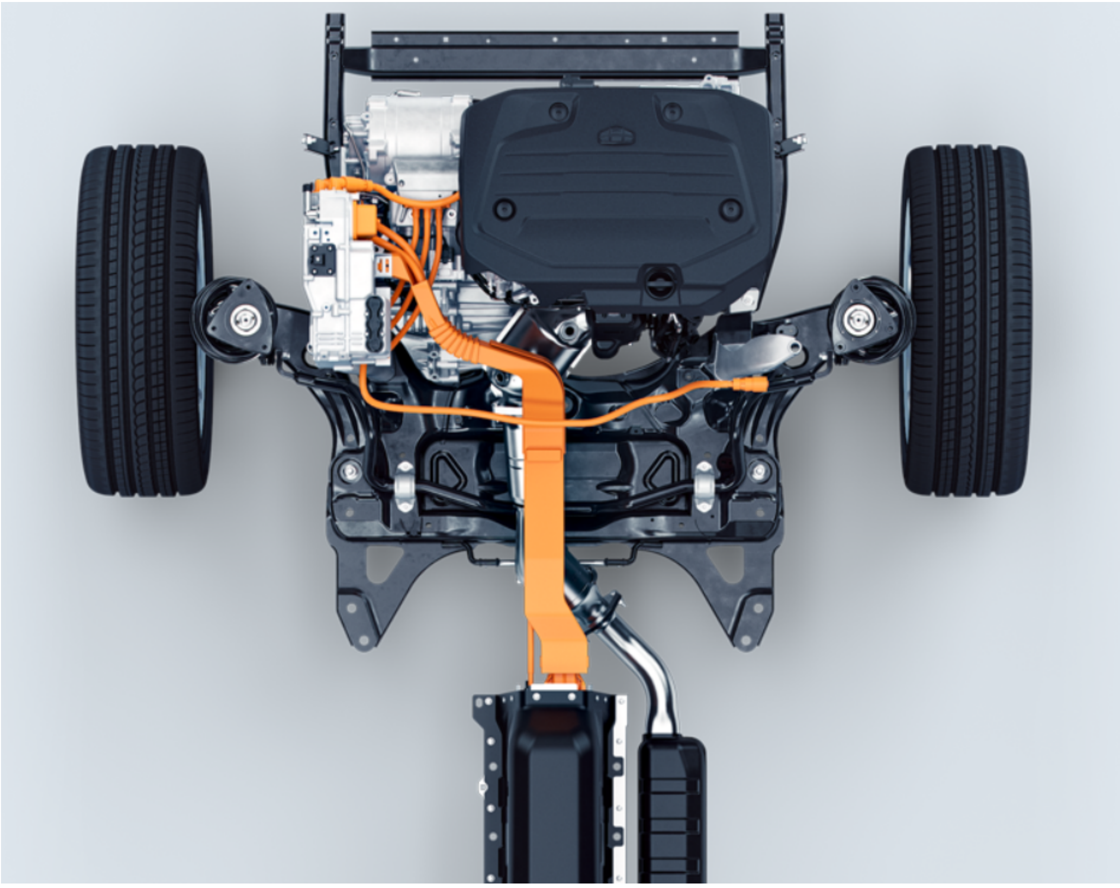

MATLAB Model For Simulation Of Powertrain Customer Collective

Master's Thesis In Automotive Engineering

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CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2020

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Abstract

With the increasing impact on global climate change and the contribution of combustion vehicle emissions to this problem, it is time to look for alternative transmission technologies that promote the use of cleaner propulsion systems using electric motors or reduced usage of combustion engines with the help of hybrid transmission. This thesis is mainly aimed at studying the operation of transmission systems involving both electric motor and combustion engine, analyse typical customer cycles and obtain dimensioning data required for transmission design.

A systematic approach for determining the duty values of an automotive transmission, for a defined vehicle and driver profile helps a long way in analysing the durability of the said transmission system. The study aims at developing a MATLAB tool to accept inputs in the form of velocity profiles, driver usage profile and a well defined vehicle case to determine the duty cycles of the transmission components. This duty cycle gives an overview of the damage taken by the components in the form of torque and rotational speeds. The data generated may also be used as a dimensioning factor for the components such as gears, differential and driveshafts. The tool primarily focuses on Hybrid Electric Vehicles(HEV) and Battery Electric Vehicles(BEV), with the flexibility of analysing a pure combustion based transmission system. Primary focus of these models is obtaining duty cycles of transmission components like the gearbox, differential and driveshaft. The tool can also be used to study the effects of various damage mechanisms, gear shifting patterns, combustion engine - electric motor sizing, and the effects of inertia on the duty values of transmission components.

Keywords: Transmission, Design, Dimensioning, Customer Cycles, Duty Values, HEV, BEV, Damage Values, Inertia.

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Glen Rodrigues, Gothenburg, September 2020

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Introduction

1.1 Background

In the early stages of automotive development the sizing of various components that can be used in a powertrain system are quite uncertain. It is generally dependent on several factors like the type of vehicle, usage pattern expected, terrain that it would be used on etc. Hence, it is very essential to formulate a typical user population distribution that can be used to analyse the damage taken by various powertrain components based on the vehicle usage. This user population/customer population distribution can then be used to simulate different driver behaviour. The data would then be processed to analyse the damage taken by various powertrain components based on individual driver usage style. This data typically called as Duty Cycle can then be used to estimate how individual components are subjected to different levels of cyclic loading, analyse component fatigue and predict service life. The type of loading that is considered for this study is the cyclic variations of torque transmitted by the components. [1]

1.2 Aim

- The aim of this thesis is to improve an exiting MATLAB based script/-tool at CEVT AB to obtain customer collectives for different powertrain systems(conventional, hybrid and electric) for a well defined vehicle case and driver profile.
- The tool would accept inputs in the form of vehicle parameters, driving cycle (Velocity vs Time Profiles) and customer usage pattern. The output generated would be used as dimensioning data required for transmission design.
- Implement models for controlling different hybrid modes of powertrain to understand impact on duty values.
- Cooperation with integration and system design departments at CEVT AB to carry out relevant simulation cases.
- The scripts and the tool will be built around the study previously conducted by Nikitas Sidiropoulos [1] and documentation received from CEVT AB.

1.3 Limitations

- The thesis is limited to determining the duty values of various transmission components based on the torque transmitted by them. This is primarily done by determining the revolutions of components such as gears, differential and driveshaft at each load/torque level. Further analysis or fatigue analysis is not possible since the component data such as material, dimensions, manufacturer specifications etc. is not available at the pilot stage of any vehicle development.
- The torque transmitted by the driveshafts from the differential to the wheels is assumed to be half of the torque input received at the differential. Torque vectoring is not included within the scope of this thesis.
- The tool uses a binning algorithm to collect the revolutions or usage time of the components at various torque and RPM levels. In certain cases the algorithm may collect the data based on a rounded off value of torque or RPM in the format, Torque Nm(Y axis) - Wheel RPM(X axis) - Revolutions (Z axis). For eg: Revolutions of the driveshaft occurring at 1226 Nm torque will be assigned to 1250Nm torque bin and the revolutions occurring at 1224Nm will be assigned to 1220Nm torque bin since the bin sizes are set apart by a step size of 50Nm.
- The effects of temperature, pressure and other environmental factors on the damage values are not considered in this analysis.

1.4 Methodology - Overview

The overall methodology implemented for executing the thesis is explained in stages below. These stages are later elaborated in section 3 of this report.

1. **Knowledge Transfer**
   The first stage of the thesis involved acquiring in-depth knowledge about the duty value analysis currently being performed at CEVT Powertrain Integration department. This stage included obtaining documentation and exisiting MATLAB scripts from CEVT. This was then followed by a literature study which included going through publications and other scholarly articles with respect to duty cycle analysis of transmission components.
2. **Implementation Of Powertrain Models**
   This stage included implementation of the required models i.e. conventional, hybrid and electric powertrain models and their control in the programming environment.
3. **Input Data Processing**
   The input data in the form of vehicle parameters, customer test cycles are then processed by the MATLAB model and the required output data is then stored in matrices assigned with a customer index for easier identification.
4. **Post Processing** The stored data from the previous stage is then analysed statistically and converted into the required useful information in the form of Duty Values, Torque Vs RPM maps and other usage statistics such as ICE usage or EM usage, percentage usage of each gear etc.

2
Theory

The following section includes basic definitions and theory about Operating Parameters, Gears, Gearboxes, Different Powertrains, Duty Value Analysis and Rain-flow Algorithm.

2.1 Definitions

2.1.1 Transmission

The term transmission can be defined as one unit within the driveline of the vehicle, often the main gearbox, or as a general term for number of units.[3]

2.1.2 Driveline

Driveline can be defined as the assembly(s) between the output of the engine or the electric motor and the road wheel hubs.[3]

2.1.3 Powertrain

The driveline and the prime mover (engine/electric motor) together can be referred to as a Powertrain.[3]

2.1.4 Regenerative Braking or Recuperation

Regenerative braking or recuperation is the process through which the kinetic energy of the vehicle is recovered during braking or decceleration. This kinetic energy is otherwise lost as heat at the mechanical brakes. The energy recovered maybe stored as kinetic energy using a mechanical flywheel or as electric energy using a generator and battery.

2.1.5 Boosting

The term boosting in hybrid powertrains can be referred to a stage when the output of an ICE is supported or enhanced by an electric motor or a mechanical flywheel to improve the net total power output at the wheels. The mechanical flywheel releases the kinetic energy stored during regeneration, whereas the electric motor maybe powered by the electrical energy stored during regeneration or by a separate battery.

2.1.6 Gearbox

Gearboxes are elements that transform the mechanical power provided by a power source at a certain speed ω_1 and torque T_1 to a different speed ω_2 and torque T_2 level. [4]

$$\omega_1 = \gamma * \omega_2, T_2 = \gamma * T_1 \tag{2.1}$$

where γ is the gear ratio.

2.1.6.1 Classification Of Gearboxes

In simple terms gearboxes can be classified into 2 main categories,
Manual Gearboxes Manual Gearboxes used to be the most commonly used type of gearboxes due to its low cost and high efficiency. This gearbox is generally characterised by the presence of a friction clutch, often used to disconnect the prime mover and the rest of the transmission, while stopping or changing gears. Selection of the desired gears and operation of the clutch is performed manually by the driver and hence the name.
Automatic Gearboxes Automatic Gearboxes are slightly expensive in comparison to manual gearboxes depending on the level of automation. The presence of a clutch pedal is replaced by automated shift control units that changes the gear based on the torque or speed demand. These gearboxes are characterised by the presence of a torque converter as a replacement to friction clutches.

2.1.7 Gear Step or Step Ratio

Consider a gearbox that comprises of 3 gears with ratios $\gamma_1, \gamma_2, \gamma_3$ respectively, the ratio step φ between each consecutive pair is called Step Ratio and can then be calculated as,

$$\varphi_{1-2} = \frac{\gamma_1}{\gamma_2} \tag{2.2}$$

$$\varphi_{2-3} = \frac{\gamma_2}{\gamma_3} \tag{2.3}$$

2.1.8 Ratio Coverage

Ratio Coverage refers to the total span coverage of the gears. For the same example considered in 2.1.7 it can be calculated as the ratio of the largest gear divided by the ratio of the smallest gear,

$$\varphi_{tot} = \frac{\gamma_1}{\gamma_3} \tag{2.4}$$

2.2 Importance of Transmission Systems

"The transmission takes the power from the engine to the wheels, in doing so actually makes the vehicle usable", Happian Smith[3]. In certain cases the torque output from the prime mover(engine or the electric motor) alone will not be sufficient enough to overcome the resistive forces acting on the vehicle. Hence, the power output from the prime mover needs to be amplified or reduced based on the actual driving conditions of the vehicle. It also facilitates easy disconnecting of the prime mover from the rest of the system when the vehicle is being stopped, this also facilitates easy operation in stop and go traffic situations. [3]

The main objective of a gearbox is to enable optimal usage of the engine or the electric motor and adjust its performance based on the road conditions and driver's requirements. The engine/motor torque and speed can be amplified or sometimes reduced by choosing a range of gear ratios thus making the vehicle usable over a wide load range.[6]

In order to move, the vehicle needs to overcome the resistive forces acting on it. The gearbox aids the prime mover (engine/electric motor) to overcome the resistive forces acting against the vehicle motion. Road resistance can be expressed as tractive resistance(kN). The propelling thrust at the tyre to road interface required to overcome this resistance is known as tractive effect(kN).[6]

The resistive forces primarily acting on vehicle motion can be categorised as,

- **Rolling Resistance** Rolling resistance is generally due to the deformation of the tire at the tire-road interface [6] and can be expressed as,

$$F_{roll} = C_r * M * g \tag{2.5}$$

  where C_r is the coefficient of rolling resistance
  M is the mass of the vehicle in kg
  g is the acceleration due to gravity m/s^2
- **Aerodynamic Resistance** Aerodynamic resistance is the resitive force acting on the vehicle when it is moving through air. This is sometimes referred to as Aerodynamic Drag. This force is mainly due to the turbulence created around the vehicles body.[6]

$$F_{air} = \frac{1}{2} * \rho * C_d * A * v^2 \tag{2.6}$$

  where ρ is the density of air in $kg/m{-}3$ C_d is the coefficient of discharge
  A is the frontal area of the vehicle body in m^2
  v is the velocity of the vehicle in m/s
- **Resistance Due To Gradient** Gradient resistance is the resistive force that the vehicle needs to overcome while travelling on a road incline. It is dependent on the vehicle mass and the slope of the road and can be expressed as,

$$F_{grad} = M * g * sin(\theta) \tag{2.7}$$

where M is the mass of the vehicle in kg
g is the accelaration due to gravity in m/s^2
θ is the slope angle in radians

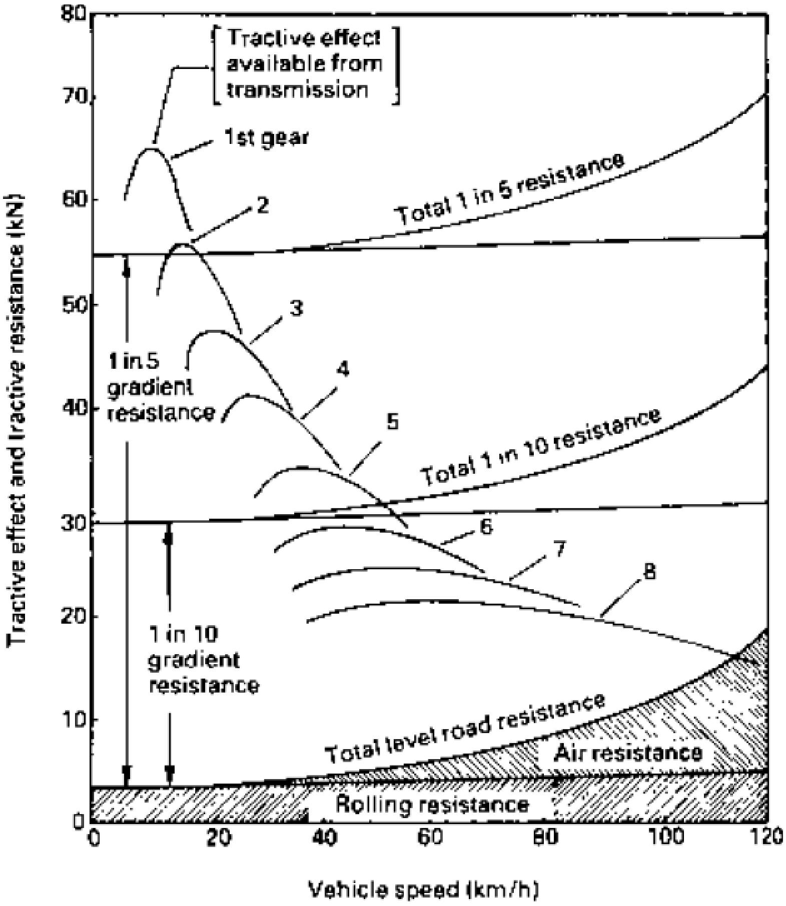

Figure 2.1: Tractive resistance and performance chart for a 8 speed transmission[6]

Figure 2.1 shows how a gearbox helps in overcoming various resistive forces acting on the vehicle. A simple glance at the figure shows that Rolling resistance is fairly constant and is not dependent on the vehicle speed, whereas Aerodynamic resistance increases at a rate equivalent to the square of vehicle's velocity. Gradient force is again independent of vehicle's speed, but is directly proportional to the mass of the vehicle and the gradient angle or the road slope.

From equations 2.5,2.6 and 2.6 it becomes clear that the minimum traction force required to overcome the resistive forces is given by,

$$F_{TracMin} = F_{roll} + F_{air} + F_{grad} \tag{2.8}$$

But the vehicle needs to have some power in excess, for it to be able to accelerate or climb steep roads. From Newton's Second Law Of Motion we have,

$$F_{acc} = M * a \tag{2.9}$$

Where F is the force in N (Newton)
M is the mass of the body in kg
a is the acceleration of the body in m/s^2
Hence the actual traction force required for a vehicle to accelerate would be,

$$F_{Trac} = F_{roll} + F_{air} + F_{grad} + F_{acc} \tag{2.10}$$

$$Torque_{req} = F_{Trac} * R_{wheel} \tag{2.11}$$

$$Acceleration = \frac{F_{Trac}}{M} \tag{2.12}$$

where R_{wheel} is the wheel radius in m

2.3 Inertia Effect On Transmission

Newton's First Law Of Motion: Every body continues to remain in a state of rest or uniform motion unless acted upon by an external force to change it's state.
Inertia is the resistance of a body to it's motion. In transmission systems, it is important to consider the effects of inertia, since it acts as a resistive component, decreasing the net torque transferred to the wheels. This resistive component results in inertia torque, which acts in a direction opposite to that of the driving torque. From equation 2.11 we can then deduce that the net torque requested from the engine or the electric motor would be the sum of $Torque_{req}$ and the Inertial Torque [7] .

$$ActualTorqueRequested = Torque_{req} + J * \ddot{\theta} \tag{2.13}$$

where J is the inertia of the rotating component in kgm^2
$\ddot{\theta}$ is the angular acceleration of the rotating component in rad/s^2

2.4 Type of Powertrains

Powertrains can be generally classified into three types,

2.4.1 Conventional ICE Powertrain

Conventional ICE Powertrains are characterised by use of an Internal Combustion Engine as a prime mover. Fuel energy is converted into mechanical work by the combustion engine, which is then supplied to a gearbox, where the output of the engine maybe amplified or reduced by selecting the right gear ratio based on the driving conditions. The following block diagram can be used to represent a simple ICE based Powertrain.

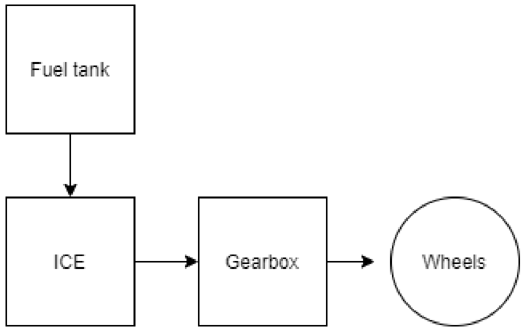

Figure 2.2: Simple ICE Powertrain indicating energy flow from fuel tank to wheels

2.4.2 Hybrid Powertrain

Hybrid Powertrain are characterised by use of one of more prime mover, generally a combination of an ICE and Electric Motor. Based on the placement of the electric motor and the usage, the system can be further classified as Series Hybrid Powertrain and Parallel Hybrid Powertrain. In series hybrids, the ICE is used to run a generator which in turn powers another electric motor that drives the vehicle. In case of Parallel Hybrids, the output of the ICE and the electric motor maybe combined or used individually to drive the vehicle. In this section we concentrate more on Parallel hybrids as Series Hybrid system is out of scope for this thesis. The following block diagram maybe used to represent a simple Parallel Hybrid Powertrain.

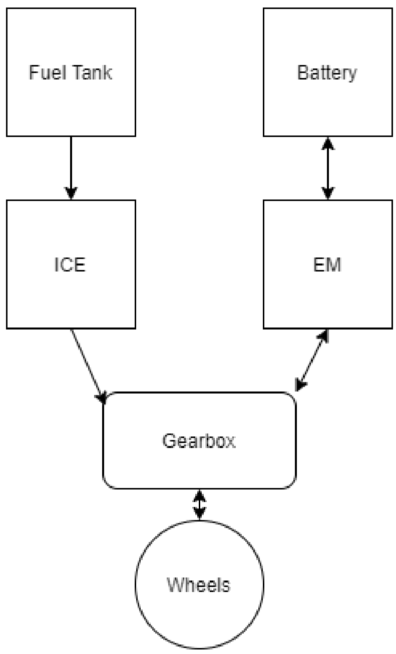

Figure 2.3: Simple Parallel Hybrid Powertrain indicating energy flow from dual sources to wheels

2.4.3 Electric Powertrain

A pure electric powertrain is characterised by the use of an electric motor as the sole prime mover. The system converts electric energy stored in the battery to mechanical work. There is no use of combustion engines in this type of powertrain, resulting in zero emissions. Another interesting characteristic of an electric powertrain is the capability to regenerate a part of the power during braking.

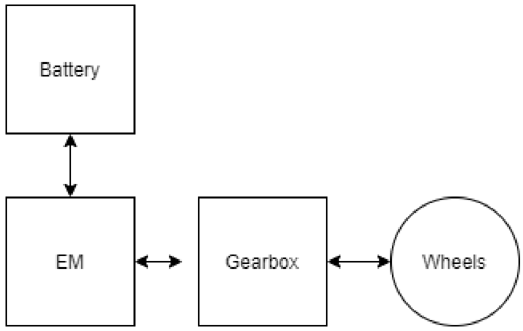

Figure 2.4: Simple EV Powertrain indicating energy flow from battery to wheels

2.5 Duty Value Analysis

"The term Duty Value is used as a measure of accumulated load" [1]. The transmission components are more commonly subjected to cyclic loading in the form of positve and negative torque. This results in torsional stresses or twisting forces being induced on the material. On the long run, due to this cyclic loading and unloading, the transmission components may undergo fatigue failure depending on the intensity of the loading and the cyclic variations.

It is very important to identify the fatigue loading mechanism of transmission elements as they become a governing factor for dimensioning. Some common transmission components and their fatigue mechanisms are discussed below,

- Shafts undergo torsional loading and fracture when fatigued.
- Gears are subjected to local pressure, due to contact between the two gear teeth, this pressure is generally termed as Hertz contact pressure or Hertz contact Stress and results in pitting damage at the tooth surface. Pitting is basically a surface fatigue mechanism that occurs when the contact load exceeds the materials surface strength. They are also subjected to bending loads at the base of the gear tooth which may cause fracture when fatigued. The second case is very much similar to a beam being subjected to bending load.[8]

Hence, it becomes necessary to analyse the durability requirement of the transmission components during the R&D phase of vehicle development. The Duty Values provide an overview of the accumulated load during a components life cycle, making it easier for design engineer to select right materials and dimension the components to ensure optimal life.

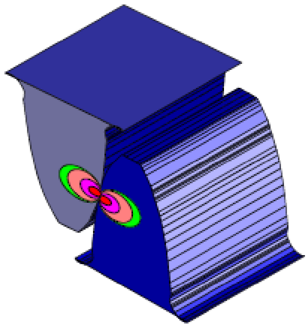

Figure 2.5: Contact Pressure[8]

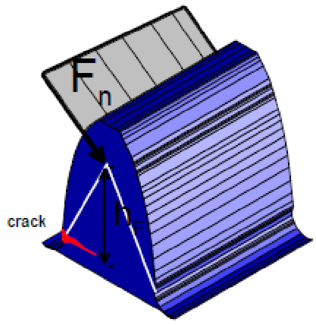

Figure 2.6: Tooth Bending [8]

2.5.1 Customer Population Distribution

In order to determine the expected service life of any transmission component, it is required to know the target customer type, driving pattern, terrain and the loads that the vehicle maybe subjected to. Service Life or Duty value analysis thus starts with defining the driving patterns, terrain and load distribution, generally termed as Customer Population Distribution in this thesis.

Section 2.5.1 is primarily based on the conference paper presented by Nikitas Sidiropoulos [1] and change request documents obtained from CEVT AB [2].

2.5.1.1 Driver Type

Drivers can be generalised into three main categories,

1. **Aggressive Driver** An aggressive driver can be termed as someone who is accelerating rapidly and driving at higher speeds than most of the drivers, using the vehicle closer to its limits, thus causing higher damage to the components and reducing their service life.
2. **Moderate Driver** A moderate driver is someone who drives reasonably, knowing the vehicle's limits and does not push it beyond it's performance boundaries, thus preventing unwanted damage to the transmission components. Also termed as 'Normal Driver'.
3. **Mild Driver** A mild driver can be termed as a person who is extra careful and usually drives slow. Drivers belonging to this category rarely cause any damage.

On a scale of 1-100, with 100 being the most aggressive, the driving behaviour distribution can be considered as per the below scale.

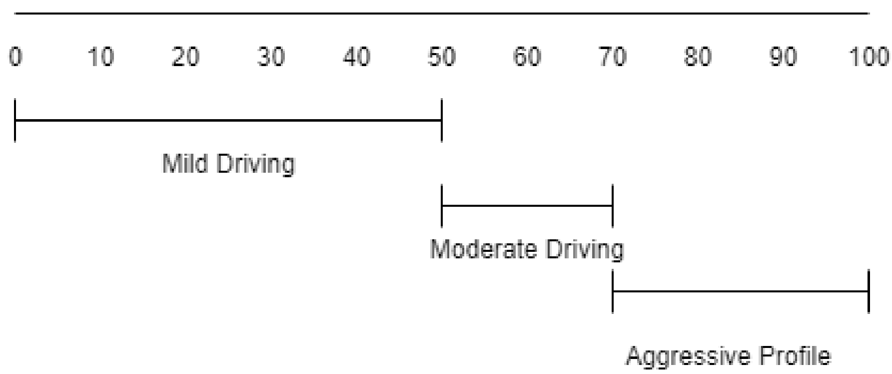

Figure 2.7: Driver Type Distribution

2.5.1.2 Driver Profiles

Driver behaviour cannot be expected to stay the same. Sometimes, an aggressive driver may drive moderately or even the other way around would be possible, based on the traffic conditions. Hence, we need to consider a combination of these behaviours and categorise them into driving profiles. Driver profiles can be again generalised into three categories, these profiles are generally combinations of one or more driving behaviours.

1. **Aggressive Profile** Aggressive profile is the combination of Aggressive-Moderate-Mild driving behaviour in the ratio 4:5:1 which basically indicates that the vehicle would be driven aggressively over 40% of it's service life.
2. **Moderate Profile** Moderate profile is the combination of Aggressive-Moderate-Mild driving behaviour in the ratio 1:7:2, indicating that the vehicle will be used moderately for up to 70% of it's service life.
3. **Mild Profile** Mild profile is the combination of Aggressive-Moderate-Mild driving behaviour in the ratio 0.5:2:7.5, indicating that the vehicle would be used very conservatively over 75% of it's lifetime.

2.5.1.3 Driver Profile Distribution

A driver population distribution is then chosen to assign a percentage distribution over each driving profile. For R&D purposes the design targets are usually customers belonging to Moderate Driving Profile. These statistics are customisable based on the type of the vehicle being designed and region under consideration. In our specific case we concentrate more on passenger cars. Figure 2.8 shows the basic driver type distribution considered for the analysis.

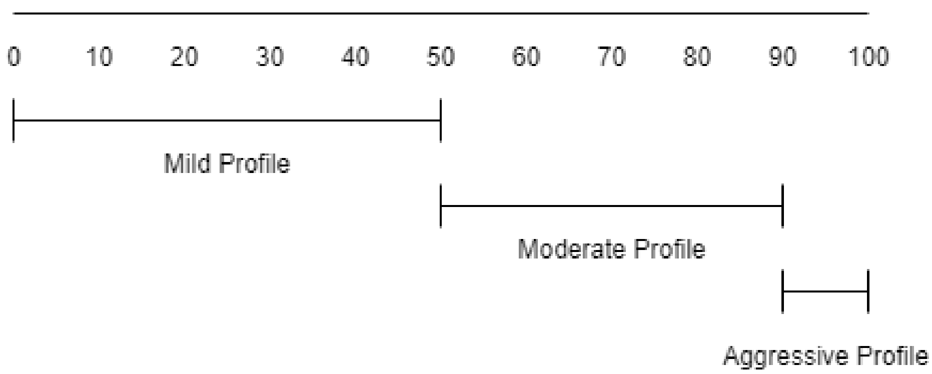

Figure 2.8: Driver Profile Distribution

2.5.1.4 Driving Environment And Driver Profile Combinations

As discussed in section 2.5.1 for durability analysis of the components, it is required that we know the possible terrain and the driving profile combinations to ensure that the testing is as close as possible to real usage conditions. This is done by analysing the target region of sale, and the type of terrains available, where the driver would possibly drive the vehicle. The driver environment combinations are then created using the driving cycles that mimic these terrains and a scaling factor would then be assigned to simulate a typical driving scenario. The test cycles are generally classified as field cycles and test track parts.

Based on the above conditions a typical driving environment and driving profile combination is created as shown below,

Environments

1. City Driving Cycle
2. Driving Cycle to simulate driving on gravel
3. Rural Driving Cycle
4. Mountain Driving Cycle
5. Highway Driving Cycle
6. Autobahn Driving to simulate high speed driving
7. Driving Cycle to simulate hill start
8. Combinations of driving cycles to simulate handling, comfort, country driving etc. with trailer

The first 5 cycles are referred to as field cycles and the latter 3 are the test track parts.

2.5.2 Rainflow Counting Algorithm

Rainflow Counting carries the key term "rainflow" due to its comparison with how the rain falling on the pagoda roof, flows through its edges. The technique is primarily used to count the number of closed cycles or load reversals that the component of interest is subjected to.[11]

According to ASTM E 1049-85[10] and Metal fatigue Analysis Handbook [11] a

"Three-Point Counting Technique" is generally preferred as a standard rainflow counting method. A three point method is called so due to the fact that this technique considers three consecutive cycles or load reversals at any given instance for cycle counting. The method is excellently explained in the Metal fatigue Analysis Handbook [11].

Three-Point Rainflow Counting can be performed as explained below,

- Consider a standing cycle and a hanging cycle hysteresis loop as shown in the figures below, Now before we can proceed further, we need to understand

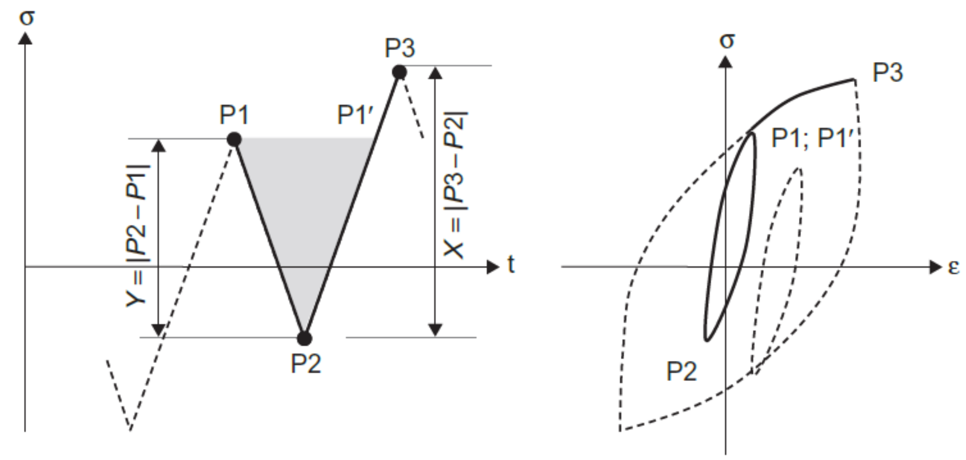

Figure 2.9: Hanging Cycle[11]

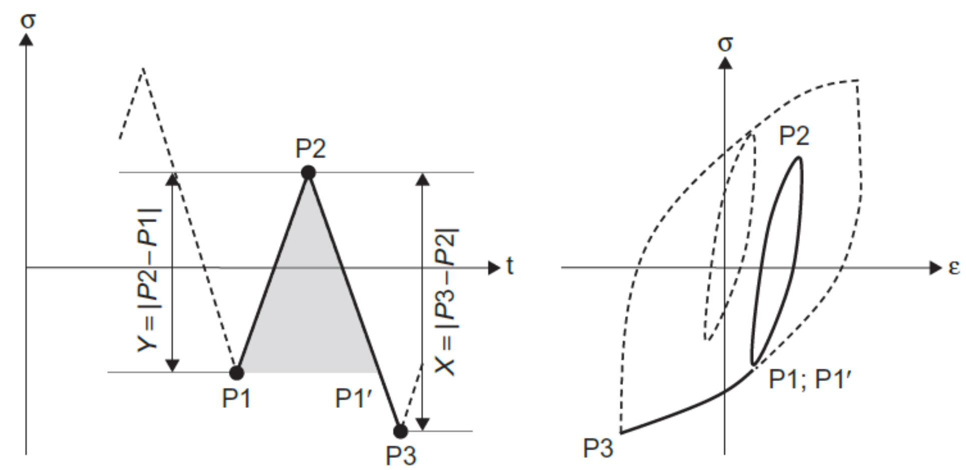

Figure 2.10: Standing Cycle[11]

what a hanging cycle and a standing cycle refer to in fatigue loading. Fatigue loading occurs when a material is loaded cyclically, resulting in a stress-strain response in the form of a hysteresis loop. A load cycle will include at least 2 load reversals or load switches. For hanging cycles, the major chunk of this hysteresis loop is concentrated in the first quadrant of the stress-strain hysteresis diagram and generally occurs when the load cycles are positive or added. Similarly in case of standing cycles major portion of the hysteresis loop will be concentrated towards the third quadrant and occurs when loading is removed [16]. This is illustrated in figure 2.11,

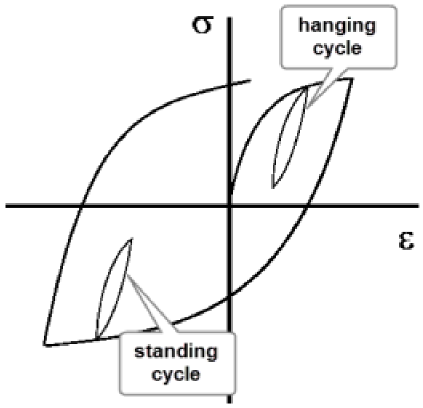

Figure 2.11: Hysteresis Loop [16]

- As shown in figure 2.9 and 2.10 P_1, P_2, P_3 are the three peak values under consideration. Let $X = |P_3 - P_2|$ and $Y = |P_2 - P_1|$. According to ASTM E 1049-85[10] the path $P_1 - P_2 - P_1^1$ can be counted as one cycle if $X \geq Y$ and the cycle count is ignored if $X < Y$ [11] and we then move on to the next consecutive point say P_4 if available. In both our examples we can see that $X \geq Y$ and hence the cycle count is 1. A more complex example is available on Metal fatigue Analysis Handbook [11] as well as on MATLAB rainflow documentation[12].

2.5.3 Wöhler Curve, Basquin's Law Of Fatigue and Miner Palmgren Damage Accumulation Rule

In order to understand the concept of fatigue loading cycles, we discuss certain parameters in this section that will help us in deducing the relationship between the number of loading cycles and the load amplitude.

Consider the following figure,

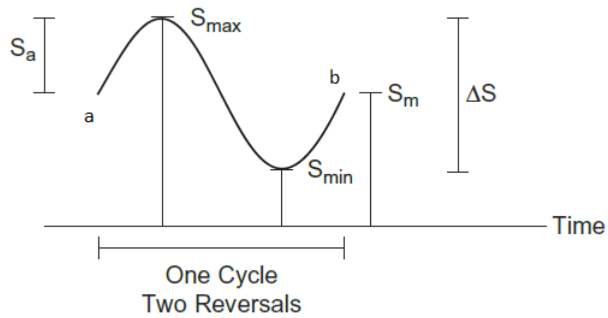

Figure 2.12: Stress Cycle [13]

Here S_a refers to the stress amplitude and is given by $\frac{S_{max}-S_{min}}{2}$
S_{max} is the maximum stress that the element is subjected to by the load cycle
S_{min} is the minimum stress that the element is subjected to by the stress cycle
S_m is the mean stress and can be obtained as $\frac{S_{max}+S_{min}}{2}$
ΔS can be calculated as $S_{max} - S_{min}$

Since, Mr.August Wöhler was the first ever engineer to perform a fatigue study the constant amplitude S-N curves are named after him.[13]

The constant amplitude S-N curve or Stress-Cycle curve is generally plotted as a straight line in log-log co-ordinate and represents High Cycle Fatigue(HCF) regime with plastic deformation. The figure 2.13 shows a typical constant amplitude S-N curve for steel. The straight line segment represents HCF whereas the horizontal part refers to the fatigue limit. From the figure we can deduce that,

$$k = \frac{-1}{b} \tag{2.14}$$

where b is the slope
k is the slope factor
The relation between 2 points S_1, N_1 and S_2, N_2 can be established with the slope factor and this also means that if the reference point S_1, N_1 and the slope factor k or slope b is known then it is possible to determine S_2, N_2,

$$\frac{N_2}{N_1} = (\frac{S_1}{S_2})^k = (\frac{S_1}{S_2})^{\frac{-1}{b}} \tag{2.15}$$

The equations can be then generalised as,

$$N * S_a^k = A \tag{2.16a}$$

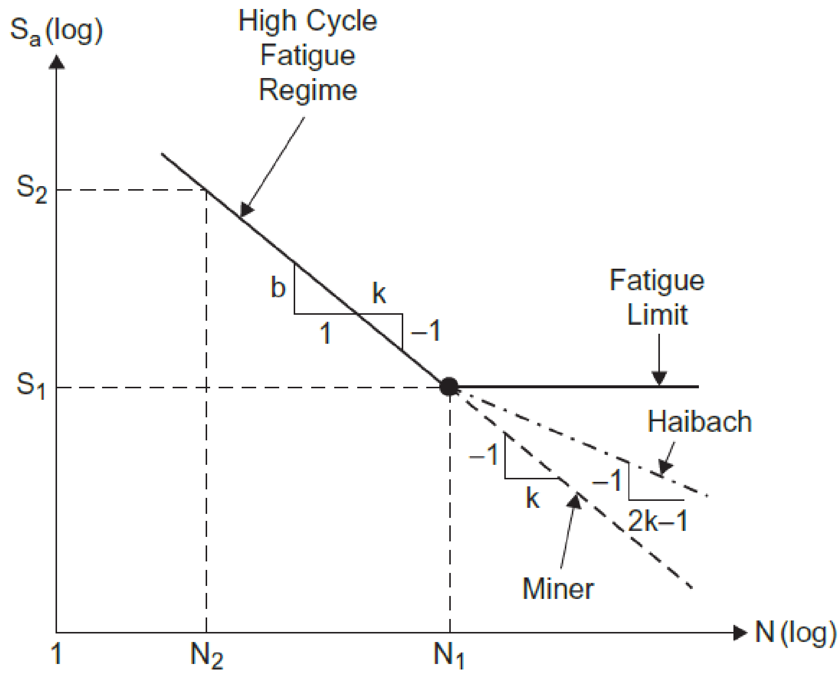

Figure 2.13: S-N Curve For Steel Component [13]

$$S_a = S_f^{'}(2N)^b \tag{2.16b}$$

where A is the fatigue parameter
$S_f^{'}$ is the fatigue strength co-efficient indicating fatigue strength of the material at one load reversal.[13]

"*Basquin's law of fatigue states that the lifetime of the system has a power-law dependence on the external load amplitude, $t_f \sim \sigma_0^{-\alpha}$, where the exponent α has a strong material dependence*".[9]

As seen before, a typical S-N curve can be used as a reference during design process,. This would help us in knowing the endurance limit(S_e) of the material and thus keep a check on the load reversals that the component is subjected to. If the load levels(S_r) are maintained within the endurance limit then theoretically the material will last for indefinite amount of load cycles. This however would result in higher material usage and hence higher costs. Instead, if the load limits are slightly above the endurance limit we could still have a well designed component with slightly less material usage and finite life. This is generally a favorable design target.

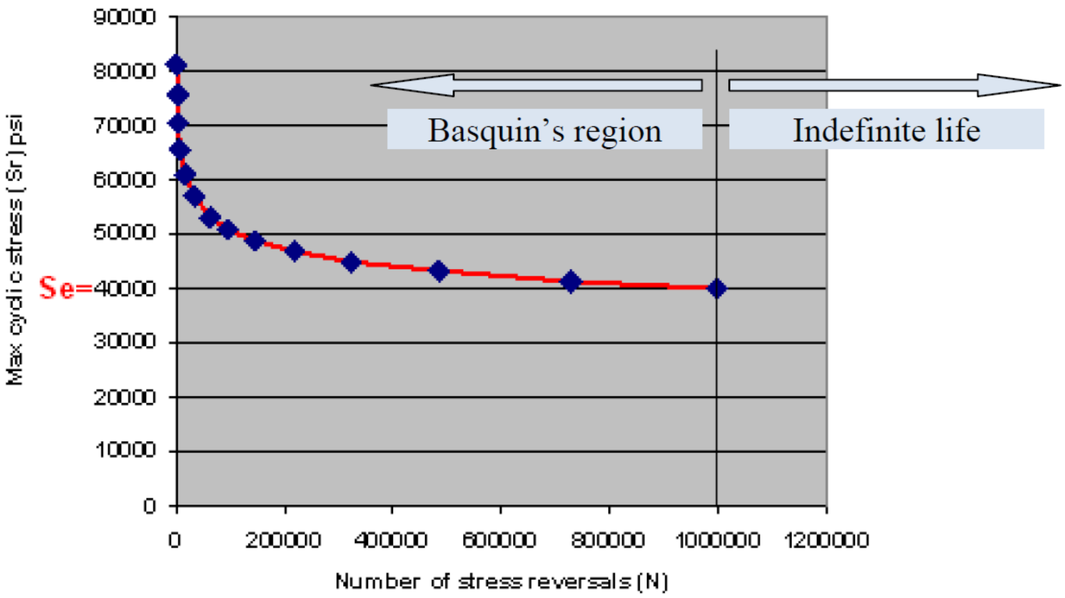

Figure 2.14: S-N Curve [14]

We have Basquin's Equation,

$$S_r = AN^B \tag{2.17}$$

This equation is valid for Basquin's region where $N_{cycles} < 10^6$
S_r is the cyclic stress
A & B are material constants[14]
The equation can also be re-written as,

$$N = (\frac{S_r}{A})^{\frac{1}{B}} \tag{2.18}$$

Thus, when the reversing load or stress value is known, it is possible to determine the number of cycles after which the material will fail [14].

Most of the load amplitudes acting on transmission components are generally variable in nature. Rainflow cycle counting helps eliminate this irregularity by converting the signals into constant amplitude signals that consists of mean stress and cycle counts for each load window. As a result the material can be considered to be subjected to segmented loading histories where each history corresponds to a constant amplitude loading cycle.

Let n_i be the number of cycles at i^{th} load level and let N_{Fi} be the number of cycle to failure at the i^{th} load, then for i = 1:n, the life of the component can be estimated by Palmgren-Miner's Equation[14],

$$\sum_{i=1}^{n} \frac{n_i}{N_{Fi}} = 1 \tag{2.19}$$

if the sum reaches unity or exceeds it, the material is said to have failed due to fatigue.[13]

Equations 2.16a and 2.19 can then be combined to obtain Duty Value as,

$$DV_L = \sum_{k=1}^{m} *n_k * L_k^{We} \tag{2.20}$$

where DV_L is the Duty Value in $[Nm^{We}]$
m is the total number of torque classifications in rainflow matrix
n_k number of load torque cycles in class k
L_k reversed torque amplitude of class k
We is the Wöhler Exponent[1]

3

Methodology

The Duty Value Analysis Tool was completely developed with MATLAB R2019b and further testing was done with MATLAB R2020a. The tool processes the input data in several stages before the duty values and the dimensioning customers can be determined. The following section will explain the various stages involved in obtaining the duty values for any given transmission system.

3.1 Process Flow Diagram

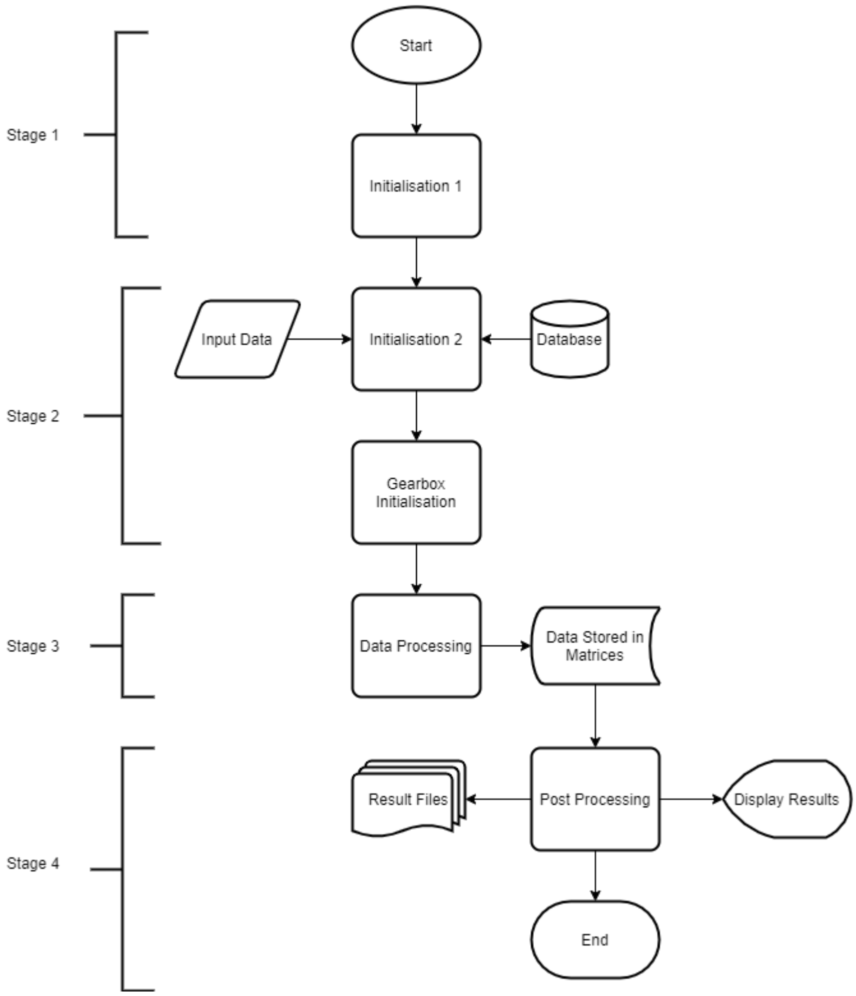

Figure 3.1: Process Flow Overview

3.1.1 Stage 1

Stage 1 indicated by the section "Start" and "Initialisation 1" in figure 3.1 basically involves initialising UI components and creating data placeholders required by the tool. Data placeholders are empty variables in MATLAB workspace and will be replaced by actual data during the later stages of the duty value analysis. This is a critical stage since the tool also performs a system check on the code files to verify the integrity of the files and checks if all the necessary toolboxes are installed. In the first run, the system always triggers the installation of accompanying toolboxes from the internet. The entire process is automated for ease of use.

3.1.2 Stage 2

3.1.2.1 Input Data

The input data is basically a "well defined" vehicle case on which the analysis will be performed. The accuracy of the input data is very critical as it decides the accuracy of the results generated, hence increasing the reliability of the tool. Three major types of Input Data Files are possible depending on the type of the powertrain being analysed.

Basic data that is required irrespective of the type of powertrain

- Mass of the vehicle in kg.
- Mass of the trailer in kg (if required).
- Radius of the wheel.
- Co-efficient of rolling resistance.
- Co-efficient of aerodynamic discharge.
- Frontal area of the vehicle in m^2.
- Wöhler exponent which indicates the type of damage being analysed.
- Customer usage profile indicating percentage usage of each of the 8 driving environments explained in section 2.5.1.4.
- Gearbox information such as number of gears, gear ratios, final drive, torque limit on each gear and RPM limits on each gear to trigger gear up-shift and down-shifts.
- Throttle Position Map which indicates percentage increase in torque output as a function of throttle position.
- Moment of inertia of transmission components.

In addition to the above some more data will be required based on the type of powertrain being analysed,

ICE Powertrain

- RPM-Torque map for the engine.
- Fuel type (Petrol, Diesel) and consumption map.
- Torque converter map (if the transmission is automatic).
- Simplified moment of inertia values for gearbox. (if the transmission is automatic then moment of inertia values for torque converter (turbine and pump) needs to be provided).

Electric Powertrain

- RPM-Torque map for the electric motor.
- Battery information such as initial SOC, upper SOC limit, capacity in Wh etc.

Hybrid Powertrain

- RPM-Torque map for the engine.
- Fuel type (Petrol, Diesel) and consumption map.
- Torque converter map (if the transmission is automatic).
- Simplified moment of inertia values for gearbox. (if the transmission is automatic then moment of inertia values for torque converter (turbine and pump) needs to be provided).
- RPM-Torque map for the electric motor.
- Battery information such as initial SOC, upper SOC limit, capacity in Wh etc.

3.1.2.2 Initialisation 2

On reading the input data the user is prompted to select a simulation mode,

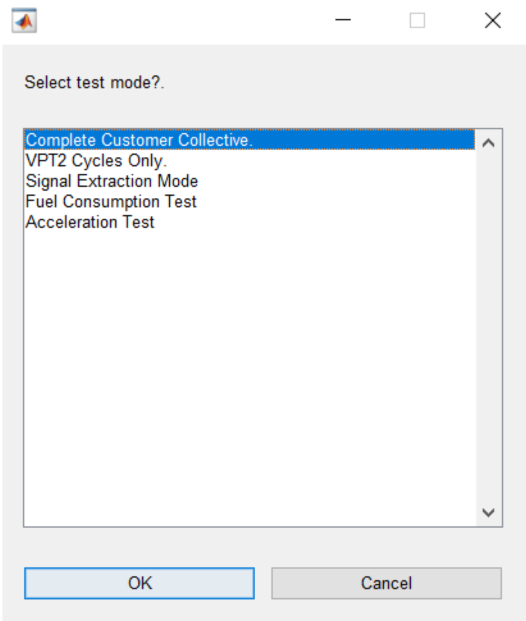

Figure 3.2: Test Mode Selection

- "Complete Customer Collective" mode is used to analyse duty values for an entire customer-environment combination i.e the vehicle is tested across both the field parts as well as the track parts.
- "VPT2 Cycles Only" mode is used to analyse duty values for customer-track environment combination only i.e the vehicle is tested only on track parts.
- "Signal Extraction Mode" is used to extract the time signals generated as part of the vehicle simulations and includes information such as wheel torque, motor/engine torque, vehicle speed, gear being used and so on, as a function of time.

- "Fuel Consumption Test" option is only applicable for ICE based and Hybrid Powertrains and can be used to measure the fuel consumption of the vehicle on WLTP and NEDC cycles.
- "Acceleration Test" can be used to measure the acceleration performance of the vehicle, in this mode throttle position is set to 100% and the road is considered to be perfectly flat i.e 0% gradient.

Once a selection is made, the system accesses the required files from the database.

3.1.2.3 Database

The database acts as a repository and stores all the files required to make the tool run as smoothly as possible. This includes,

- MATLAB Code Files.
- Logs generated during simulations (This also helps in de-bugging the tool).
- Efficiency maps for different gearboxes, Electric Motors etc.

3.1.2.4 Gearbox Initialisation

Based on the vehicle case definition, the system then selects the appropriate gearbox model from the database, this also includes the efficiency maps indicating the transmission efficiency at each operating point. This part of the stage calculates the gearstep between each consecutive gear and also assigns the RPM limits required to indicate an up-shift or a down-shift based on the engine speed and torque demand.

3.1.3 Stage 3

3.1.3.1 Data Processing

The data processor forms the major core of the tool. Based on the vehicle model definition in the input file and data received from Stage 1 and Stage 2, the processor identifies the type of the powertrain being tested and accordingly triggers the required control algorithm.

ICE Powertrain

Steps

1. Read velocity and slope information from Driving Cycle.
2. Determine the traction torque required at the wheel as explained in section 2.2. Requested acceleration is also calculated during this stage.
3. It is assumed that the vehicle always starts moving with first gear activated.
4. Determine the engine speed and torque required from the engine, based on the total ratio currently active between the engine and the wheels(Gear Ratio of the active gear * Final drive ratio).
5. Determine the torque required to overcome inertia of the gearbox and in case of automatic transmission, the inertia of the torque converter pump and turbine.
6. The total torque required from the engine will then be the sum of the torques obtained from step 4 and step 5.

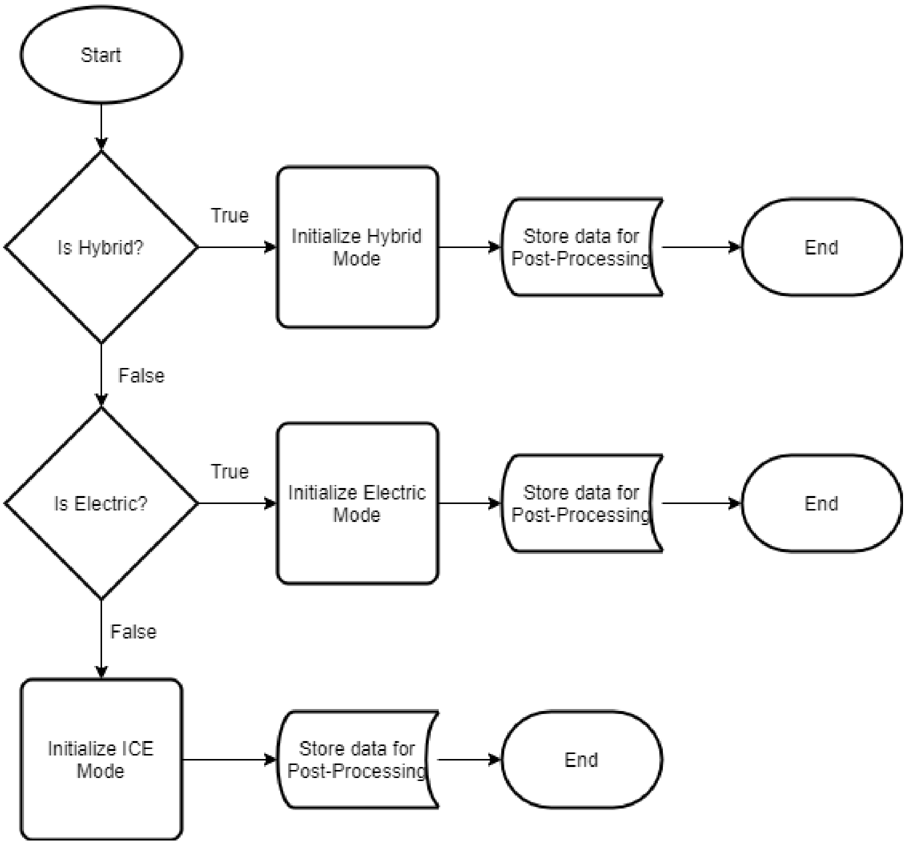

Figure 3.3: Data Processor

7. Check if the torque and engine speed requirements are within the engine's limits. If yes, then verify if the torque supplied from the engine is within the torque transfer capability of the current gear. If both these conditions satisfy then set the engine torque and speed to required limits. If either of these conditions are not satisfied, then the engine torque and the speed will be set to the limiting value, which may either be the limit torque or the engine RPM limit.
8. Fuel used by the combustion engine is determined based on the engines fuel map and stored for later determination of fuel economy.
9. The system then determines the required throttle and updates the same. The throttle is varied in a controlled manner to prevent sudden increase or decrease in throttle position and response.
10. Gear Shifter algorithm automatically shifts the gears based on the torque being requested at the wheel and the torque available at the engine. It also takes into consideration the torque transfer limit of each gear, and the engine RPMs defined in the input file at which a up-shift or down-shift maybe desired. To prevent nervous shifting and to simulate better stability of the gear shifter, a delay is considered between each consecutive gear shift.
11. The generated response signals are then stored in a matrix for post processing and the variables are updated for the next time step .
12. The system now moves on to the next operating point on the driving cycle

and process repeats.

EV Powertrain

Steps

1. Read velocity and slope information from Driving Cycle.
2. Determine the traction torque required at the wheel as explained in section 2.2. Requested acceleration is also calculated during this stage.
3. It is assumed that the vehicle always starts moving with first gear active.
4. Determine the speed and torque required from the electric motor based on the total ratio currently active between the engine and the wheels(Gear Ratio of the active gear * Final drive ratio).
5. Determine the torque required to overcome inertia of the gearbox.
6. The total torque required from the engine will then be the sum of the torques obtained from step 4 and step 5.
7. Check if the SOC available at the battery is above the minimum limit and then verify if the torque and motor speed requirements are within the motor's limits. If yes, then verify if the torque supplied from the motor is within the torque transfer capability of the current gear. If all these conditions satisfy, then set the motor torque and speed to required operating point. If either of these conditions are not satisfied then the motor torque and the speed will be set to the limiting value, which may either be the limit torque or the motor RPM limit.
8. if the torque requested at the wheel is negative then the vehicle is considered to be in regenerative mode and the battery will be charged during this stage. If the torque requested is positive , then the vehicle is in drive mode and is drawing power from the battery. The battery will then be discharged and the SOC will be updated accordingly.
9. The system then determines the required throttle and updates the same. The throttle is varied in a controlled manner to prevent sudden increase or decrease in throttle position and response.
10. Gear Shifter algorithm automatically shifts the gears based on the torque being requested at the wheel and the torque available at the motor. It also takes into consideration the torque transfer limit of each gear, and the motor RPMs defined in the input file at which a up-shift or down-shift maybe desired. To prevent nervous shifting and to simulate better stability of the gear shifter a delay is considered between each consecutive gear shift.This does not apply to a single speed transmission systems.
11. The generated response signals are then stored in a matrix for post processing and the variables are updated for the next time step .
12. The system now moves on to the next operating point on the driving cycle and process repeats.

Hybrid Powertrain

Steps

1. Read velocity and slope information from Driving Cycle
2. Determine the traction torque required at the wheel as explained in section 2.2. Requested acceleration is also calculated during this stage.

3. In case of hybrid powertrains it is highly possible that the vehicle with a Dual Clutch Transmission might either start moving with first gear or second gear activated based on the drive mode.
4. Determine the speed and torque required from the prime mover based on the total ratio currently active between the engine and the wheels(Gear Ratio of the active gear * Final drive ratio).
5. Determine the torque required to overcome inertia of the gearbox.
6. The total torque required from the engine will then be the sum of the torques obtained from step 4 and step 5.
7. Check if the SOC available at the battery is above the minimum limit and then verify if the torque and motor speed requirements are within the motor's limits. If yes, then verify if the torque supplied from the motor is within the torque transfer capability of the current gear. If all three conditions satisfy, then set the motor torque and speed to required limits and update the system to operate in electric mode. If SOC is above the required limit and the torque and speed requested are over the motors limit, then the torque and the speed requested from the motor will be set to the limiting value of the motor, which may either be the limit torque or the motor RPM limit. The difference in torque required will be then used as torque demand from the combustion engine and the system will be updated to operate in Hybrid mode.
8. if the battery SOC is below the limit, then the vehicle cannot be operated in electric mode, in this case the vehicle will try to draw the required torque and power from the combustion engine and at the same moment, if any excess power is available. the battery will be charged and the vehicle will then be operated in ICE mode.
9. if the torque requested at the wheel is negative, then the vehicle is considered to be in regenerative mode and the battery will be charged during this stage. If the torque requested is positive , the vehicle is said to be in drive mode. If the vehicle is in electric drive mode, it is then drawing power from the battery. The battery will then be discharged and the SOC will be updated accordingly. In hybrid mode, the fuel used by the combustion engine is determined based on the engines fuel map and stored separately.
10. The system then determines the required throttle and updates the same. The throttle is varied in a controlled manner to prevent sudden increase or decrease in throttle position and response.
11. Gear Shifter algorithm automatically shifts the gears based on the torque being requested at the wheel and the torque available at the motor. It also takes into consideration the torque transfer limit of each gear, and the motor RPMs defined in the input file at which a up-shift or down-shift maybe desired. To prevent nervous shifting and to simulate better stability of the gear shifter a delay is considered between each consecutive gear shift.
12. The generated response signals are then stored in a matrix for post processing and the variables are updated for the next time step .
13. The system now moves on to the next operating point on the driving cycle and process repeats.

3.1.3.2 Data Storage In Matrices

The data obtained from data processing as explained in 3.1.3.1 is then collated into matrices. The data is obtained from the vehicle models operated in ICE/EV/Hybrid mode. The data collected and the significance of it is explained below,

- Engine speed collective in speed vs torque intervals, stores the cycles when engine is operated and can help in engine sizing or tuning.
- Velocity matrix stores the distance covered in each velocity window. The matrix also stores the velocity collective in vehicle speed vs gearbox input torque intervals when the vehicle is in drive mode.
- The system also stores the distance covered and the fuel consumed(if any) by the vehicle in each driving cycle.
- Matrix that stores the number of up-shifts and downshifts to each gear.
- An array that stores time vs wheel torque output signal for each cycle. Similar data is stored for the torque transfer between the differential and the wheels. This is used as an input for rainflow counting.
- Electric motor collective in torque vs motor speed intervals, stores the time for which the motor was operated within each time interval. This can be used to analyse motor usage and can help with motor sizing or tuning.
- Input torque collective in speed vs torque intervals, stores the cycles when gearbox is supplied with an input torque from the prime mover.
- Output torque collective in speed vs torque intervals, stores the cycles when torque is transmitted by the differential to the wheels.
- Combined matrix or Mode matrix stores the percentage usage of the vehicle in each mode. The color coding is applicable for certain hybrid transmission

EM GEAR / ICE GEAR	2	4	6	R
1	A			
2	B			
3	C	D		
4		E		
5		F	G	
6			H	
7			I	
R				J

Figure 3.4: Operation Mode Hybrid [Dual Clutch Transmission]

systems with Dual Clutch Transmission. The electric motor is connected to the shaft carrying even gears. The blocks coded in Green are permitted combinations of gears in Hybrid Mode, generally indicated by data stored diagonally in the matrix. The combinations indicated in yellow are theoretically possible, but not practical. The combinations in red are physically impossible. Data stored horizontally corresponds to gear usage in EV mode and data stored vertically corresponds to gear usage in ICE Mode.

- Rainflow matrix is generated using three point technique explained in section 2.5.2 and the MATLAB function *dat2rfm*[17], it accepts inputs in the form of time series of torque signals.

3.1.4 Stage 4

The final stage of simulation is Post Processing. This stage is very critical since the data collected in Stage 3 is processed and converted into the required duty values and other valuable information based on the statistical analysis of customer population distribution.

3.1.4.1 Customer Population Distribution

A matrix with 20 customer requirement combinations is then created by assigning usage percentage of each of the driving environments as described in section 2.5.1.4. These are primarily based on surveys conducted to determine the customer usage in each environment type and distance driven [1][19]. The resultant is then a customer matrix with 20 rows indicating customer usage profiles and 8 columns assigning driver environment types to each customer. The matrix on the right which is a $20 * 1$ matrix is used to specify the frequency of occurrence of each of these 20 combinations[2].

$$\begin{bmatrix}
C1E1 & C1E2 & C1E3 & C1E4 & C1E5 & C1E6 & C1E7 & C1E8 \\
C2E1 & C2E2 & C2E3 & C2E4 & C2E5 & C2E6 & C2E7 & C2E8 \\
C3E1 & C3E2 & C3E3 & C3E4 & C3E5 & C3E6 & C3E7 & C3E8 \\
C4E1 & C4E2 & C4E3 & C4E4 & C4E5 & C4E6 & C4E7 & C4E8 \\
C5E1 & C5E2 & C5E3 & C5E4 & C5E5 & C5E6 & C5E7 & C5E8 \\
C6E1 & C6E2 & C6E3 & C6E4 & C6E5 & C6E6 & C6E7 & C6E8 \\
C7E1 & C7E2 & C7E3 & C7E4 & C7E5 & C7E6 & C7E7 & C7E8 \\
C8E1 & C8E2 & C8E3 & C8E4 & C8E5 & C8E6 & C8E7 & C8E8 \\
C9E1 & C9E2 & C9E3 & C9E4 & C9E5 & C9E6 & C9E7 & C9E8 \\
C10E1 & C10E2 & C10E3 & C10E4 & C10E5 & C10E6 & C10E7 & C10E8 \\
C11E1 & C11E2 & C11E3 & C11E4 & C11E5 & C11E6 & C11E7 & C11E8 \\
C12E1 & C12E2 & C12E3 & C12E4 & C12E5 & C12E6 & C12E7 & C12E8 \\
C13E1 & C13E2 & C13E3 & C13E4 & C13E5 & C13E6 & C13E7 & C13E8 \\
C14E1 & C14E2 & C14E3 & C14E4 & C14E5 & C14E6 & C14E7 & C14E8 \\
C15E1 & C15E2 & C15E3 & C15E4 & C15E5 & C15E6 & C15E7 & C15E8 \\
C16E1 & C16E2 & C16E3 & C16E4 & C16E5 & C16E6 & C16E7 & C16E8 \\
C17E1 & C17E2 & C17E3 & C17E4 & C17E5 & C17E6 & C17E7 & C17E8 \\
C18E1 & C18E2 & C18E3 & C18E4 & C18E5 & C18E6 & C18E7 & C18E8 \\
C19E1 & C19E2 & C19E3 & C19E4 & C19E5 & C19E6 & C19E7 & C19E8 \\
C20E1 & C20E2 & C20E3 & C20E4 & C20E5 & C20E6 & C20E7 & C20E8
\end{bmatrix}
\begin{bmatrix}
F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \\ F7 \\ F8 \\ F9 \\ F10 \\ F11 \\ F12 \\ F13 \\ F14 \\ F15 \\ F16 \\ F17 \\ F18 \\ F19 \\ F20
\end{bmatrix}$$

In order to simulate a specific customer requirement, the last row of the $20 * 8$ matrix can then be replaced with the required customer usage profile. Customer usage profile basically indicates the typical vehicle usage in each environment type over a duration of its service life. In section 2.5.1.2 we have seen that there are 3 driving profiles and the combination of 20 customers, frequency of occurrence and these 3 driving profiles will result in 60 customer combinations with different driving styles and usage patterns.

3.1.4.2 Customer Scaling

To begin with , we first need to determine the frequency of occurrence of each driving behaviour, which is quite different from frequency of occurrence of a customer-environment combination. In sections 2.5.1.2 and 2.5.1.3 we have seen that an aggressive driver is characterised by a frequency distribution of 40:50:10, moderate driver is characterised by a distribution of 10:70:20 and mild behaviour is characterised by a distribution of 5:20:75. Then, according to section 2.5.1.3 the three behaviours are then characterised by their frequency of occurrence and are distributed over the entire customer population as 10:40;50, implying that about 10% of the drivers will drive aggressively, 40% of the drivers will use the car moderately and the remaining 50% will be assigned to mild profile.

The overall frequency of occurrence would then be the combination of the frequency of occurrence of each profile (1*3 matrix) and the frequency of occurrence of each each driver distribution (20*1 matrix) resulting in a total of 60 combinations (1*60 matrix). This is then followed by re-scaling the customer matrix (20*8 matrix) according to the various driving behaviours, such as aggressive, moderate and mild. It is important to note that the driver behaviour scaling is only applied to field environments. This will then result in a 60*18 customer matrix, where each set of 20 rows correspond to a specific driving behaviour (20*3 sets) and 18 columns as 5 field cycles are scaled with 3 driving profiles resulting in 15 columns and the remaining 3 columns correspond to track parts.

The next step would be to consider the total life expectancy of the vehicle in kilometres. This value is generally specified in the simulation settings, in Stage 1.

$$Customer\_Scaling = (\frac{Life\_Expected}{Distance\_Covered}) * (\frac{(Scaled\_Customer\_Matrix)}{100}) \tag{3.1}$$

Where $Life\_Expected$ is the life expectancy of the components in kms
$Distance\_Covered$ is the distance covered by the vehicle in each of the 18 scaled environment combinations in kms
$Scaled\_Customer\_Matrix$ is the 60*18 matrix obtained from the previous step.

The size of the resultant scaling matrix will be the same as the 60*18 customer matrix. This matrix would then be used to scale the output signals obtained from the vehicle models in Stage 3 (3.1.3.2) and 60 scaled copies of the same would be created.

Duty Values are then calculated using the input torque and output torque matrices using equation 2.20. Wöhler exponents are chosen based on the type of damage being analysed. For pitting damage in gears a Wöhler exponent of 3.33 is used and for bending loads a Wöhler exponent of 6 is used [2]. The duty values are calculated for each gear and sorted according to the frequency of occurrence in an ascending order. Accumulated damage is calculated using the Palmgren-Miner rule explained in section 2.5.3 and MATLAB function *cmat2dam*[18]. The 2 functions have been adapted from WAFO Toolbox [15].

3.1.4.3 Result File Generation and Display

On successful execution of previous processes, the system generates a result document in ".xls" format. The result document contains about 26 sheets and each sheet contains results generated over the entire simulation process. The sheet descriptions and the data contained in each sheet is explained below,

- Sheet 1 contains generic information such as the test cycles included in the simulation and a short description of result files.
- Sheet 2 contains the overall summary of the simulation performed with the Duty Values and Palmgren-Miner damage values.
- Sheet 3 contains the output torque collective for the 90^{th} percentile customer stored as revolutions at each torque level measured at the differential.
- Sheet 4 contains the output torque collective for 90^{th} percentile customer stored in a 3D matrix as torque classes on y-axis , wheel RPM classes on x-axis and collected revolutions at the differential on z-axis.
- Sheet 5 contains the output torque collective for the 59^{th} percentile or the dimensioning customer stored as revolutions at each torque level measured at the differential.
- Sheet 6 contains the output torque collective for 59^{th} percentile or the dimensioning customer stored in a 3D matrix as torque classes on y-axis , wheel RPM classes on x-axis and collected revolutions at the differential on z-axis.
- Sheet 7 contains the input torque collective for 90^{th} percentile customer stored in a 3D matrix as torque classes on y-axis , wheel RPM classes on x-axis and collected revolutions at the driveshaft on z-axis.
- Sheet 8 contains the output torque collective for 59^{th} percentile or the dimensioning customer stored in a 3D matrix as torque classes on y-axis , wheel RPM classes on x-axis and collected revolutions at the driveshaft on z-axis.
- Sheet 9 contains the output torque collective for the 59^{th} percentile or the dimensioning customer stored as revolutions at each torque level measured at the driveshaft.
- Sheet 10 contains the output torque collective for the 90^{th} percentile customer stored as revolutions at each torque level measured at the driveshaft.
- Sheet 11 contains the output power measured at the differential stored in a 3D matrix as torque classes on y-axis , wheel RPM classes on x-axis and power output at the differential on z-axis.
- Sheet 12 contains velocity collective and stores distance(Column 2) and time(Column3:end) in velocity vs torque intervals.
- Sheet 13 contains RPM collectives and stores engine speed collective in engine speed and torque intervals.
- Sheet 14 contains the combined matrix that stores the distance/percentage usage information of gears in each mode. Diagonal data represents usage in

hybrid mode.

- Sheet 15 contains the electric motor collective that stores time information for EM usage in torque and motor speed intervals.
- Sheet 16 stores the rainflow matrix obtained for the 59^{th} percentile customer with the cycle counting performed using the torque output at the differential.
- Sheet 17 stores the rainflow matrix obtained for the 90^{th} percentile customer with the cycle counting performed using the torque output at the differential.
- Sheet18 stores the rainflow matrix obtained for the 59^{th} percentile customer with the cycle counting performed using the torque output at the driveshaft.
- Sheet 19 stores the rainflow matrix obtained for the 90^{th} percentile customer with the cycle counting performed using the torque output at the driveshaft.
- Sheet 20 stores the mean torque matrix derived from the 59^{th} percentile customer's rainflow matrix measured at the driveshaft.
- Sheet 21 stores the mean torque matrix derived from the 90^{th} percentile customer's rainflow matrix measured at the driveshaft.
- Sheet 22 stores the mean torque matrix derived from the 59^{th} percentile customer's rainflow matrix measured at the differential.
- Sheet 23 stores the mean torque matrix derived from the 90^{th} percentile customer's rainflow matrix measured at the differential.
- Sheet 24 stores the description of each sheet of the result file.
- Sheet 25 stores the revolutions at each load level and the duty values for the 59^{th} percentile customer and uses the rainflow matrix from Sheet 16 as an input.
- Sheet 26 stores the revolutions at each load level and the duty values for the 90^{th} percentile customer and uses the rainflow matrix from Sheet 17 as an input.

In section 2.5.1, we have studied what a customer population distribution is and how it is used for scaling potential design customers. From figure 2.7 on a scale of 1-100 with 100 being the most aggressive driving behaviour, 90^{th} percentile customer is used to study the customer with aggressive driving patterns. At CEVT 59^{th} percentile customer is used to represent an average driving behaviour and this falls within the moderate behaviour category [2]. Driveshaft here refers to the components transferring the torque from the differential to the wheels.

For converting the rainflow matrix to mean torque matrix we calculate the mean torque and torque range of each cell/position in the rainflow matrix. These values are then written to a new empty matrix.

Next step is to get the size of the mean-range matrix by checking maximum and minimum of the mean torque matrix and the torque range matrix. Now each cell in the mean-range matrix is defined and the final step is to go through each cell in mean-range matrix and check which positions in the rainflow matrix fulfills the conditions of that cell (mean torque and torque range) and then move the values from those cells in rainflow to a correct position within mean-range.

The results generated by the tool act as a dimensioning criteria and help in selection of materials or component sizing , since the approximate loading histories of these components is now known.

4
Results

In the following section various results obtained from the simulation tool are discussed.

4.1 Comparison Between Simulations And Actual Car Measurement

RLDA or Road Load Data Acquisition measurements were performed on a Volvo XC40 with an 8 speed automatic transmission. CAN signals were logged and torque data was extracted by mounting torque sensors on the drive shaft.

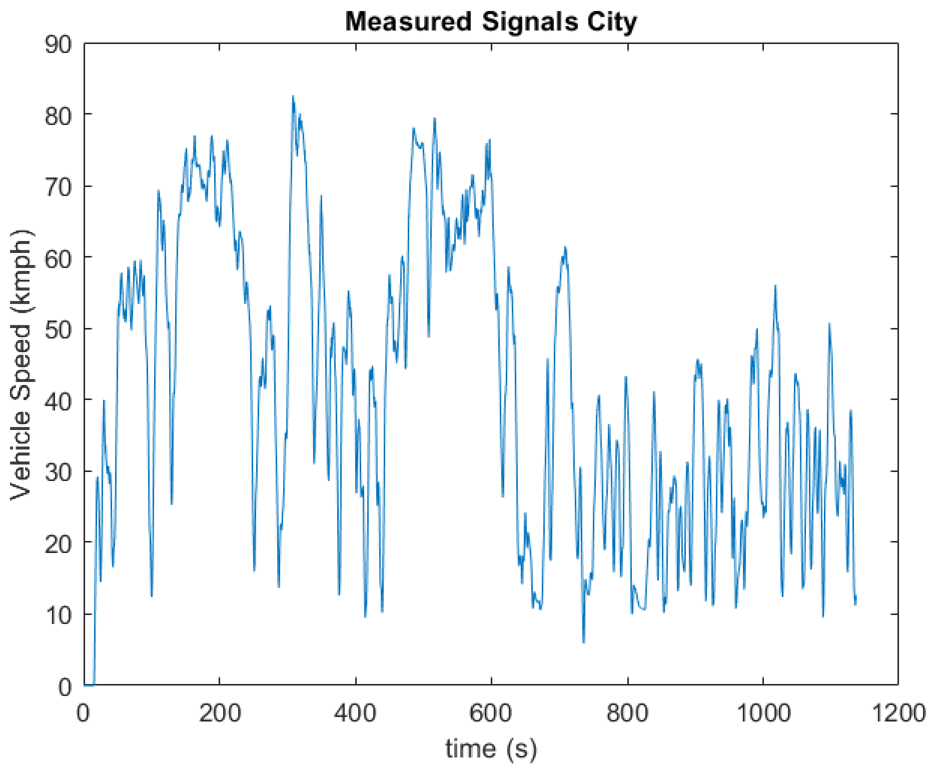

Figure 4.1: Measured Signal for City Driving Cycle

Similar setup was created in the simulation tool to replicate the tests that were performed on the car as part of the manual measurements. The prime focus of this experiment was to check the comparability of the simulation results with that of the manual measurements.

Table 4.1: Volvo XC40 Base Specifications.

Vehicle	
Brand	Volvo
Model	XC40
Drivetrain	AWD
Curb Weight	2078.5 kg
Trailer	2000 kg
Wheel Radius	0.31 m
Transmission	AWF22
Engine	VEP4

Based on the manual measurements performed , the extracted speed time history was used as an input to the simulation tool. The measured signals for a city cycle are represented in the figure 4.1. Figure 4.2 shows the comparison between the RPM

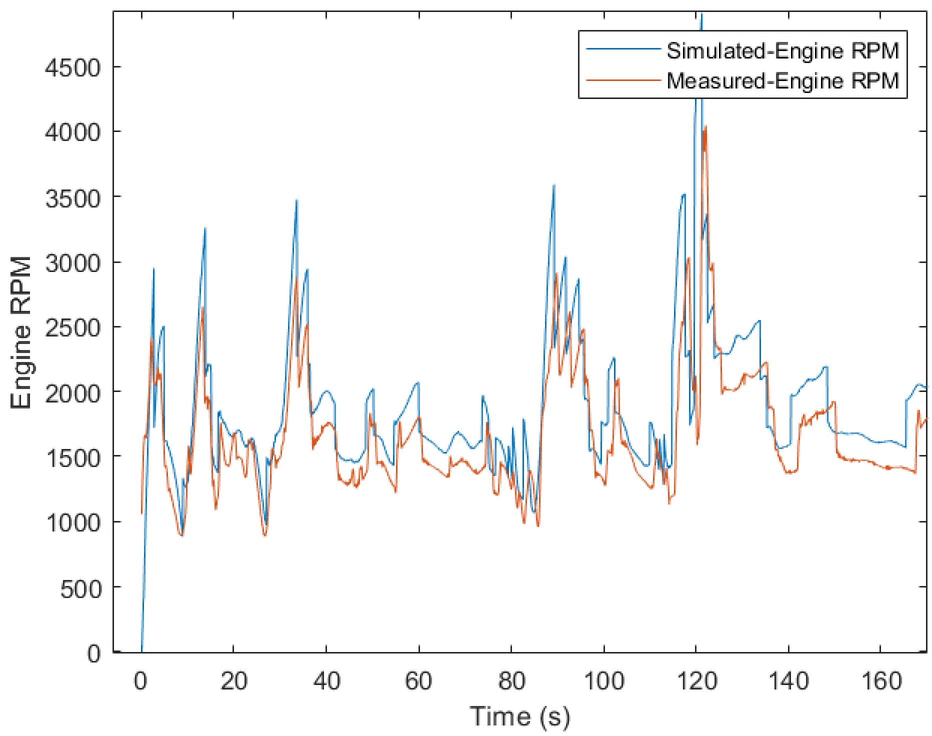

Figure 4.2: Simulated Engine RPM Vs Measured Engine RPM

measured in the vehicle against the Engine RPM simulated in the tool. The simulated signal is slightly exaggerated when compared to the signal measured due to time stepping performed in the simulation and the effects from the torque converter modelled in the simulation tool. On careful observation it can be noticed that both the signals follow a very similar pattern over the total time duration of simulation. The peaks observed in the simulated signals are due to the gear shifts and inertial response from the powertrain control.

The following figure shows the comparison between the measured gear shifts against the gear shifts of the simulated powertrain. The signals are found to be almost in agreement owing to the fact that the gear shift points specified in the input file were approximated to match the performance of the actual vehicle. the simulation model prevents rapid up-shift and down-shifts to main shift stability. This is done by choosing a time delay between shifts to simulate drive comfort.

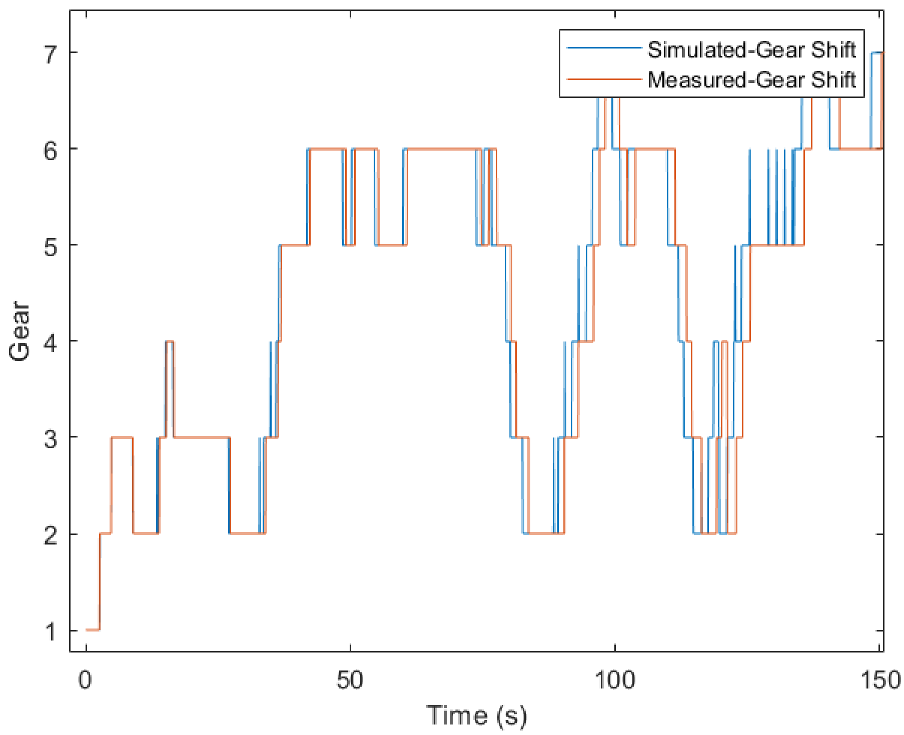

Figure 4.3: Simulated Gear Shifts Vs Measured Gear Shifts

This was then followed by comparing measured wheel torque output from the car with the signals from the simulated model as shown in figure 4.4. Negative wheel torque was not considered in case of the ICE model, since regen is not applicable. Simulated torque signals follow a similar pattern to that of the measured signals however certain exaggerated peaks appear due to the approximation of the gear shift schedules and effect of inertia torque. The results can be further improved by improving the data provided in the input files. The simulations are majorly dependent on the data provided in the input files and need to be as realistic as possible.

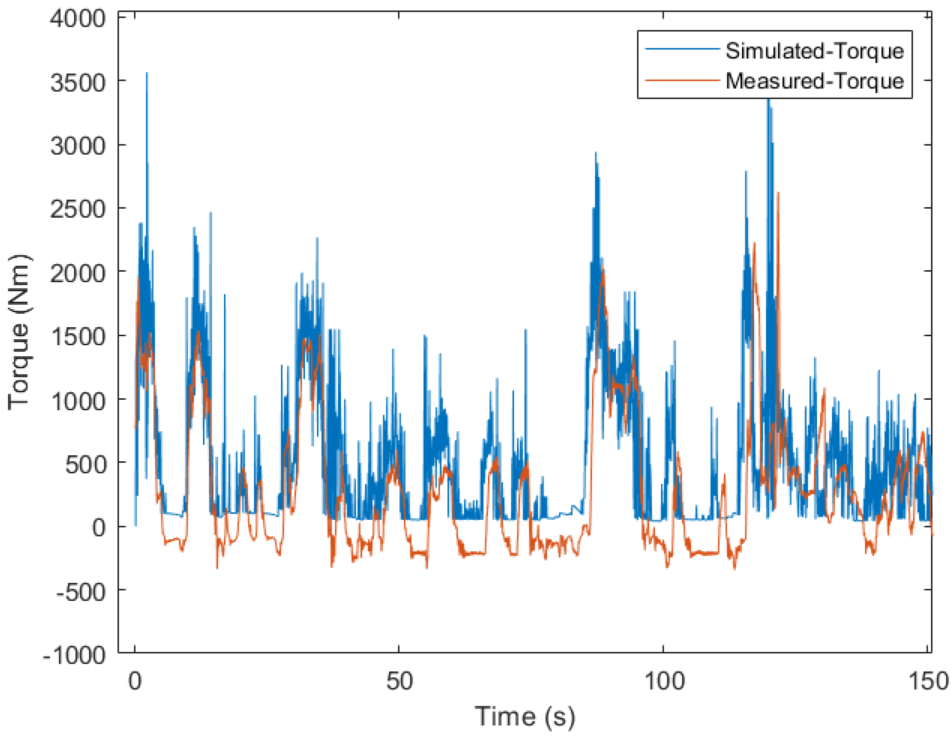

Figure 4.4: Simulated Torque Output Vs Measured Torque Output

It is very important to note that the comparison was made between the signals calculated by the simulation tool and the signals recorded from a car driven by a human driver. It is highly impossible to replicate exact human behavior and can also be said that no two drivers will have the same driving behavior. Our goal here is to develop a model that can help us approximate average response from the vehicle, to determine the damage values on transmission components, that represent real time usage.

4.2 Duty Value Analysis Of Single Speed BEV

The simulation was performed on a Battery Electric Vehicle model with single speed transmission, to study the load histories and determine the duty values of various components such as the gear and the differential. Effects of gearbox inertia was included in the simulations and the overall efficiency of the transmission was set to 97% [2]. Electric motor efficiency was considered at each operating point based on the efficiency map provided by the manufacturer for the said motor [2].

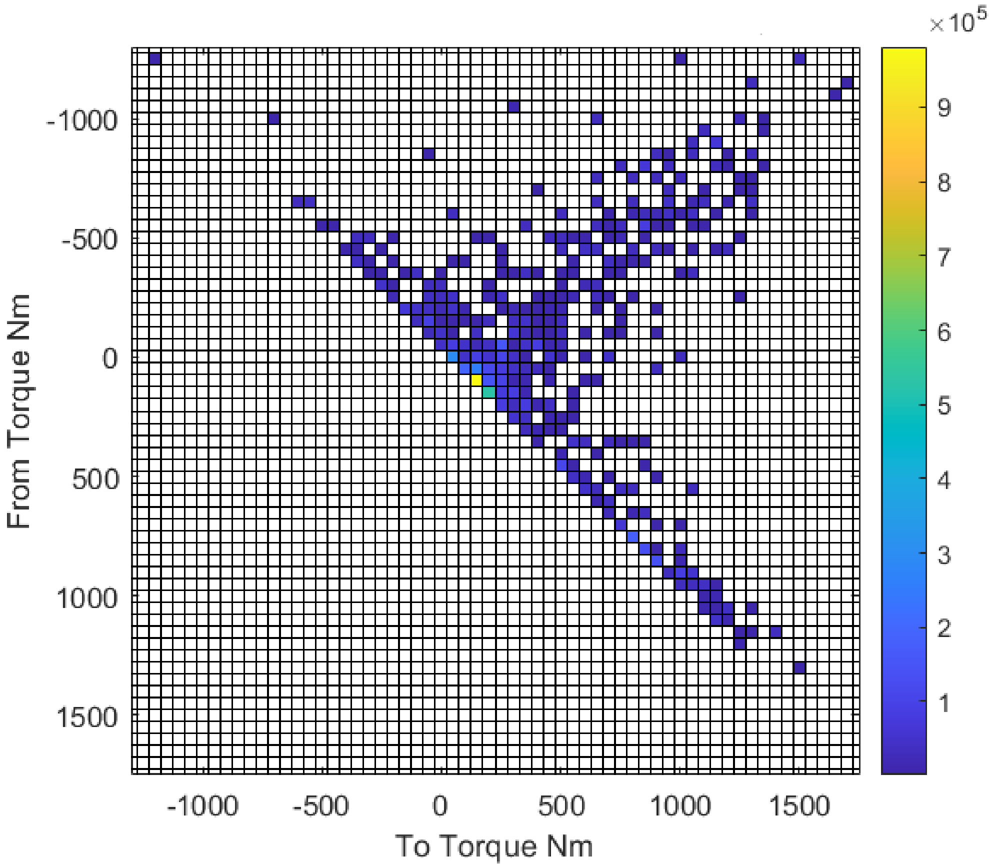

Figure 4.5: Rainflow Cycle Visualisation For Dimensioning Customer Calculated At The EV Driveshaft

Figure 4.5 visualises the rainflow counts obtained from the output torque signals calculated at the driveshaft. For this particular simulation, driveshaft torque was assumed to be half of the torque output at the differential. The visualisation of the rainflow matrix is done by means of a pseudocolor plot. A pseudocolor plot is generally used to visualise a matrix and the colors assigned to each block is based on the magnitude of the value occupying the block [20]. Figure 4.5 represents the cycles occurring at various load torque intervals. The terms "From Torque" and "to Torque" are used to indicate the start and the end of each load reversal. The simulation was performed for a life expectancy value of 8,00,000 kms. In case of single speed transmissions a single gear is subjected to all the load reversals throughout the vehicles service life. By determining the number of cycles expected at each load level, the data can then be used for dimensioning the gears so that the failure of the component at the load levels with higher frequencies can be prevented.

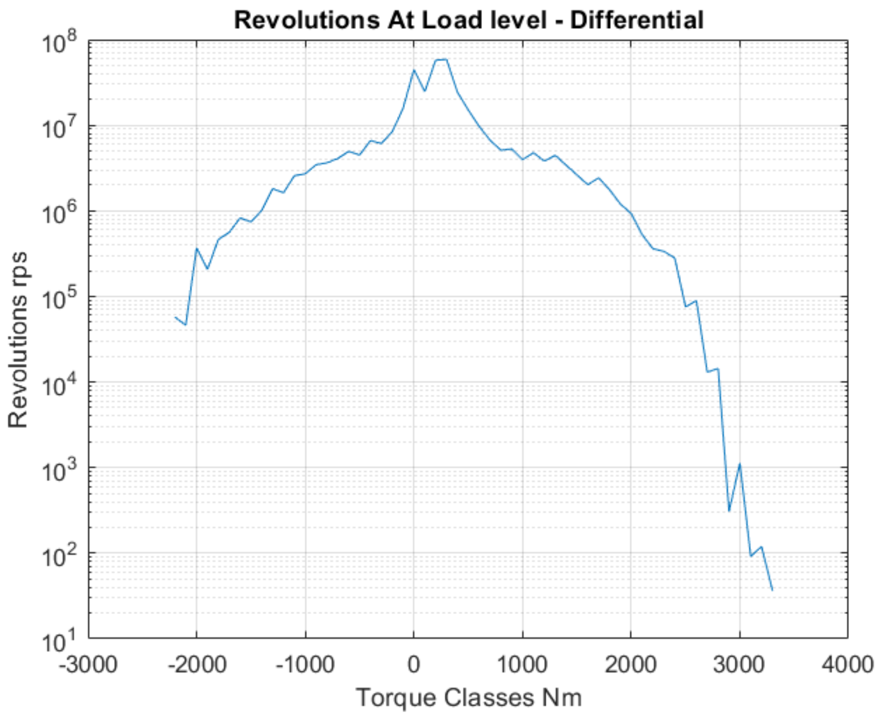

Figure 4.6: Revolutions At Load Level Calculated At The EV Differential

Figure 4.6 can be used to study the load cycles that the differential is subjected to at various load torque levels. In general terms we know that the unit of frequency is Hertz and is defined as the number of cycles occurring per second. The same concept is applied in Duty Value analysis by considering the frequency in terms of the component revolutions occurring per second. The data then provides information on the load repetitions occurring at each load level. For example in this particular simulation it was observed that $5.89 * 10^7$ cycles occur at a load of 300Nm. Then according to Palmgren-Miner Damage accumulation rule explained in section 2.5.3 and from equation 2.19 the material selected for the differential should at least have a N_{Fi} value of greater than $5.89 * 10^7$ cycles to ensure finite fatigue life at 300Nm.

Figure 4.7: Revolutions At Load Level Calculated At The EV Transmission Input

The revolutions at load level collective for the input gear is shown in figure 4.7. It was observed that the transmission input was subjected to about $7.91 * 10^8$ load cycles at 20Nm. Then according to Palmgren-Miner Damage accumulation rule and equation 2.19 the material selected for the differential should at least have a N_{Fi} value of greater than $7.91 * 10^8$ cycles to ensure finite fatigue life at 20Nm. The duty values are then calculated using equation 2.20 and are found to be $4.26 * 10^{21} Nm^{We}$ at the input parts and $8.05 * 10^{26} Nm^{We}$ at the output parts with We=6.

4.3 Duty Value Analysis Of A 3 Speed Hybrid Transmission

A simulation was performed on a hybrid vehicle with 3 speed planetary gearbox. The gearbox parameters were pre-calculated and fed as a simplified model in the input file. The overall performance of the vehicle in all three modes (EV, ICE, Hybrid) is considered here to determine the total duty values of the transmission setup.

Figure 4.8 represents the rainflow counts calculated by using the torque output

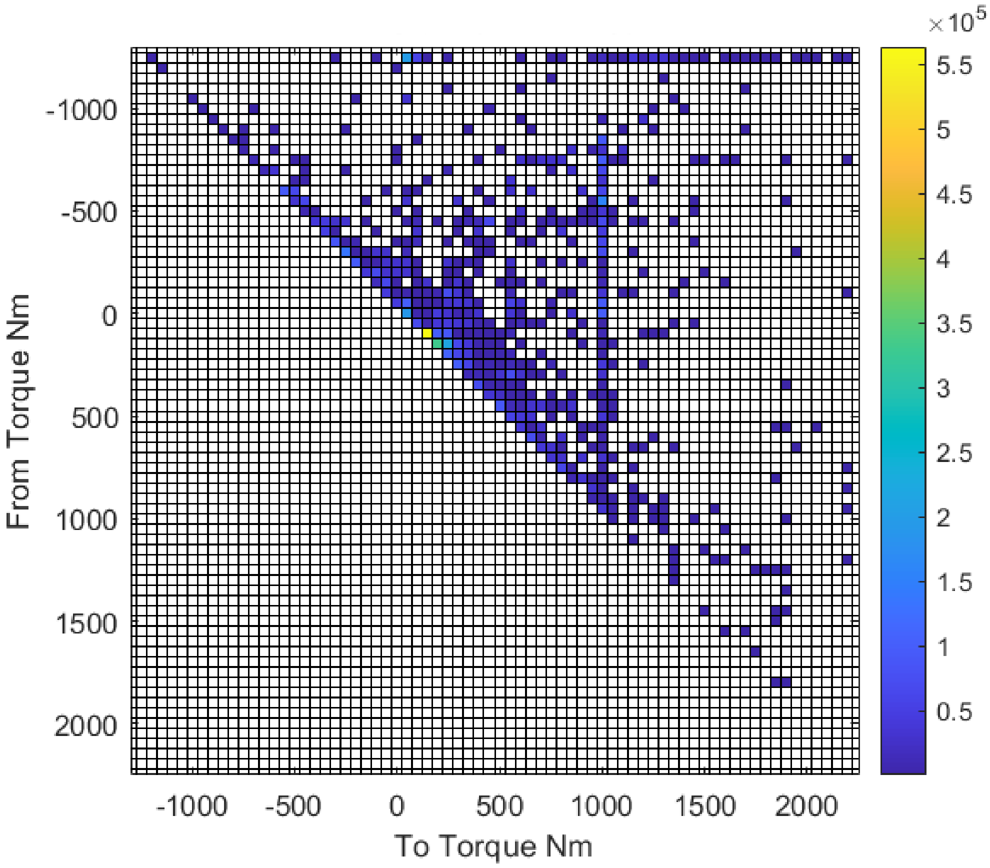

Figure 4.8: Rainflow Cycle Visualisation For Dimensioning Customer Calculated At The HEV Driveshaft

signals at the driveshaft. As discussed in section 4.2 the driveshaft torque here was considered to be half of the torque output available at the differential. The simulation was performed for a life expectancy of 3,50,000 kms which is standard for Duty Value simulations [2]. As explained in section 4.2 a pseudocolor plot is used to visualise the rainflow matrix obtained from the simulation. The colorbar on the right is used as a reference scale to indicate the number or revolutions/cycles occurring at each load level indicated by a specific color within the pseudocolor plot. The From Torque-To Torque plot then provides useful information such as the number of cycles occurring within each load window. This data can then be used to select a suitable material and dimensions for the driveshaft to prevent fatigue failure in the operating zones or load windows.

For any hybrid vehicle, we need to consider the loading histories for the transmis-

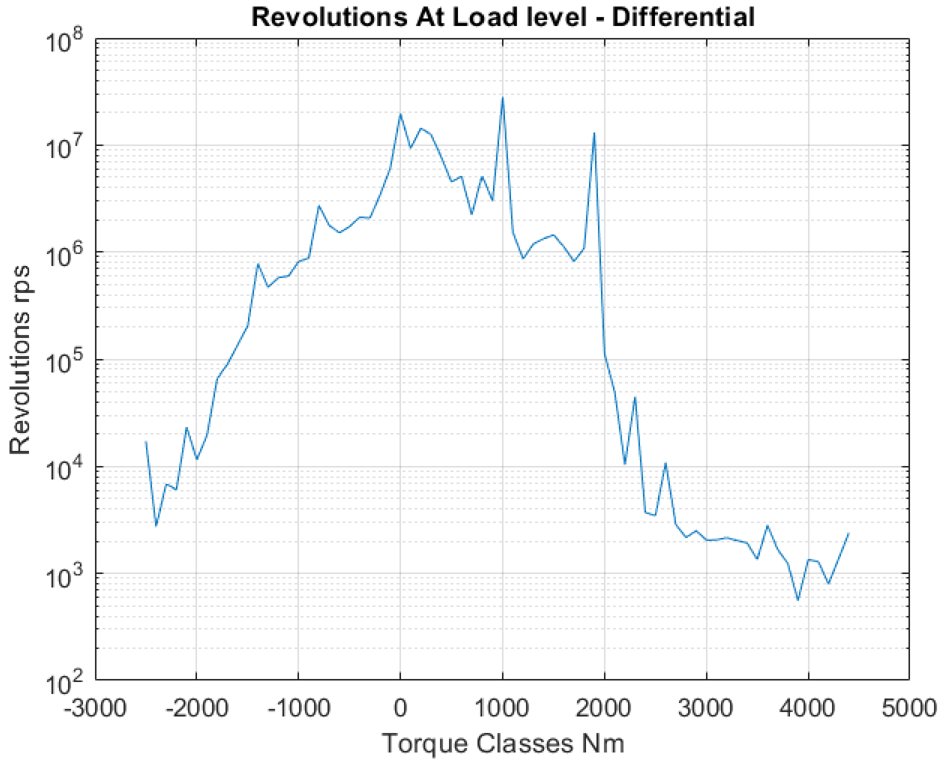

Figure 4.9: Revolution At The Load Level Calculated At The HEV Differential

sion components in both the positive torque domain as well as the negative torque domain, due to the vehicle's capability to regenerate some of the power during braking. In drive mode or positive torque mode it was observed that the differential was subjected to about $2.80 * 10^7$ cycles at a load class of 1000Nm. Then according to Palmgren-Miner Damage Accumulation rule 2.5.3 and equation 2.19 the component or the material selected should at-least be able to withstand more than $2.80 * 10^7$ load cycles at 1000Nm for finite life. In regen mode or in negative torque domain we can consider the load cycles occurring at -800Nm torque level. About $2.72 * 10^6$ loading cycles was noted at -800Nm torque class. Hence for finite life the material selected must also support at least $2.72 * 10^6$ cycles at -800Nm for finite fatigue life. Similarly the load cycles occurring at each load class needs to be considered and the design engineer needs to then ensure that material selected can withstand these load cycles occurring at every specific load case.

Now moving on to the input section of the transmission, the load collectives at each individual gear was analysed in a similar fashion to that of the differential. For the first gear it was observed that the number of cycles recorded peaked with a value of $1.13 * 10^8$ cycles at a load of 210Nm.

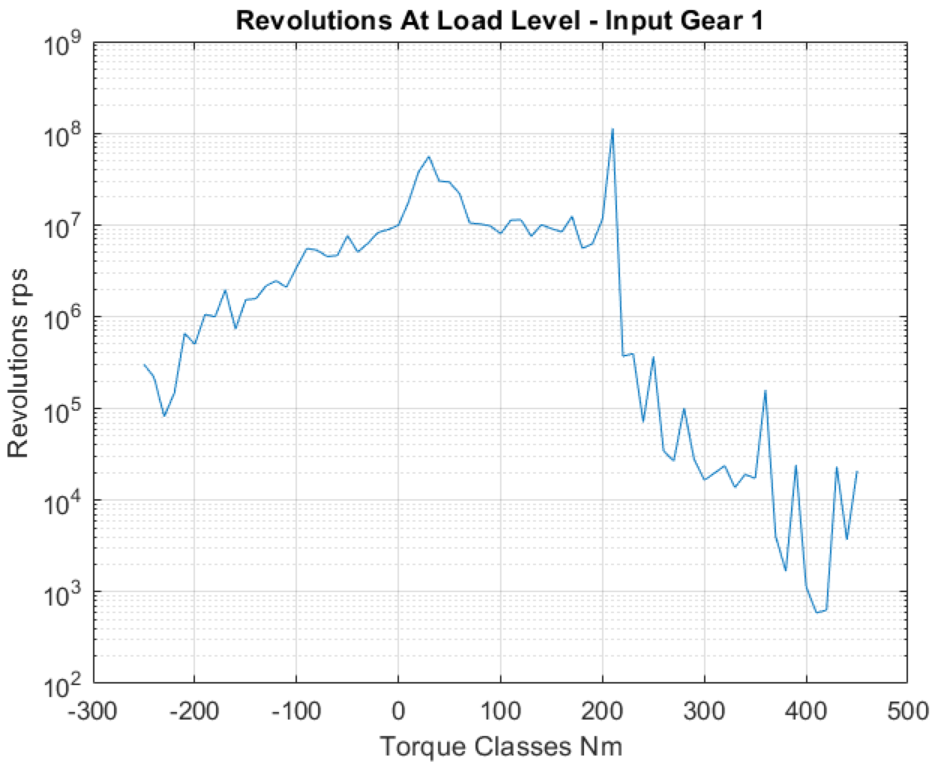

Figure 4.10: Revolutions At Load Level Calculated AT The HEV Transmission Input (Gear 1)

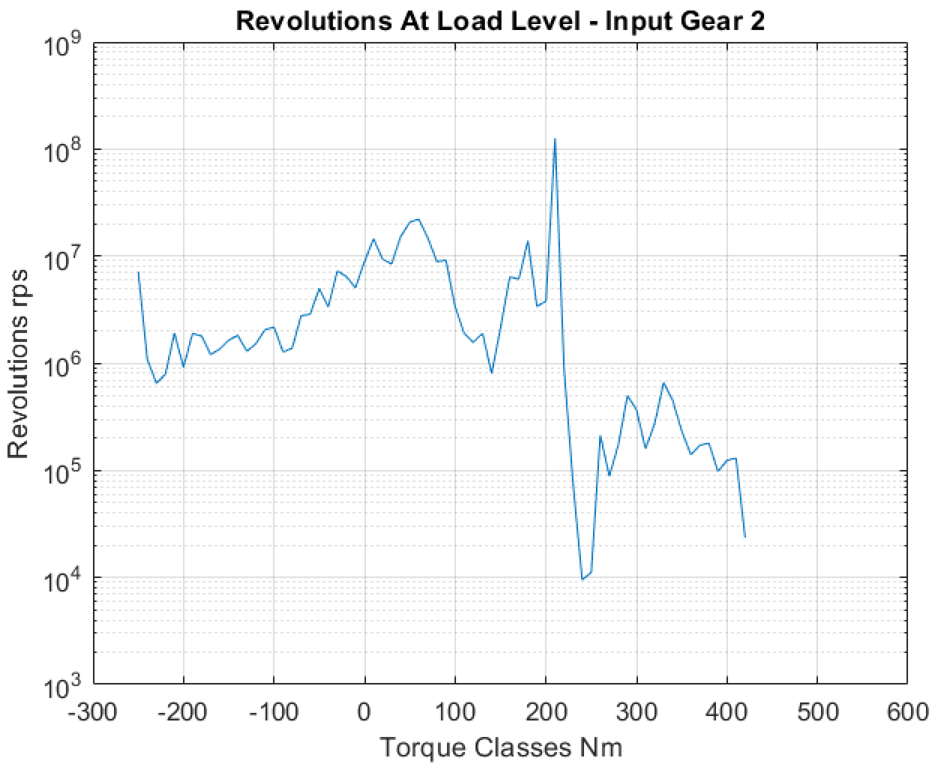

Figure 4.11: Revolutions At Load Level Calculated AT The HEV Transmission Input (Gear 2)

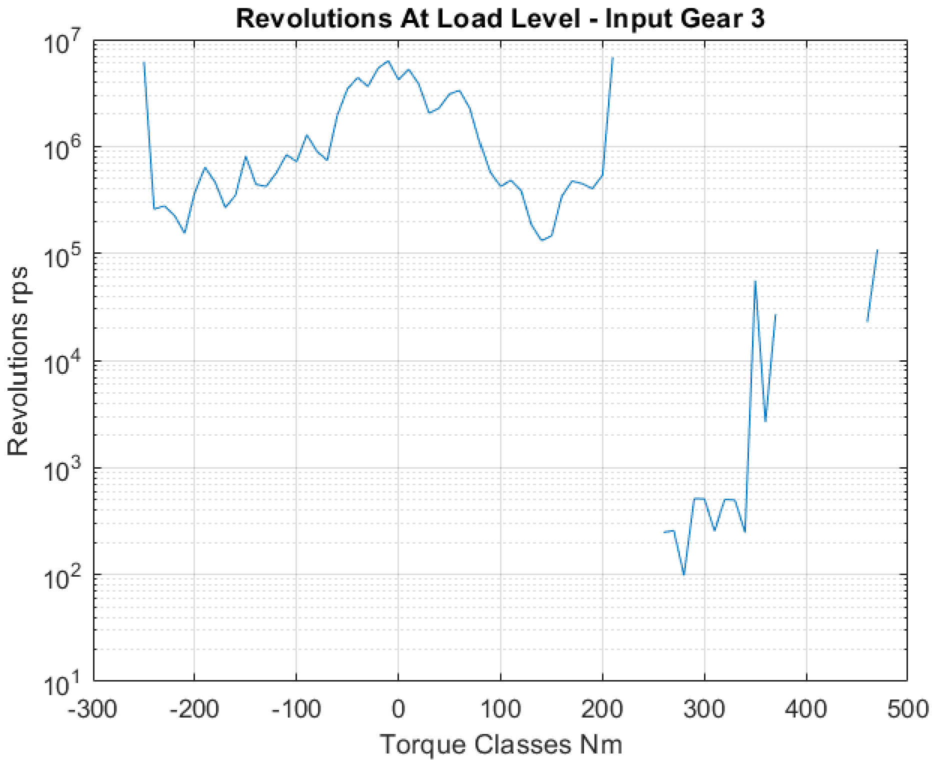

Figure 4.12: Revolutions At Load Level Calculated AT The HEV Transmission Input (Gear 3)

Similarly for the second gear it was observed that the gear was subjected to $1.26 * 10^8$ load cycles at 210Nm, but the third gear was subjected to $6.31 * 10^6$ cycles at -10Nm and $6.82 * 10^6$ cycles at 210Nm, indicating that the gear was also used more in regenerative mode. For material selection and gear design, one needs to ensure that the meshing gears are made of the same material or at least have similar hardness. If materials of different hardness levels are used, it is possible that the gear with higher hardness may cause surface damage to the comparatively softer mating gear. The materials selected need to satisfy all the load cycle demands to ensure longevity of the components.

Table 4.2: Duty values For 3 Speed HEV With We = 3.33.

Duty Values (Nm^{We})	
Gear 1	$9.26 * 10^{15}$
Gear 2	$1.30 * 10^{16}$
Gear 3	$1.40 * 10^{15}$
Differential	$1.94 * 10^{18}$

4.4 Duty Value Analysis Of A 7 Speed Dual Clutch Hybrid Transmission (DCT)

A simulation was performed on a hybrid vehicle with 7 speed Dual Clutch Transmission. A DCT is generally characterised by the presence of two supporting shafts carrying even gears and odd gears separately, controlled with two different clutches. In our current setup, both the even gears as well as the odd gears receive torque and speed input from combustion engine. However, the electric motor is only connected to the even shaft, resulting in the even gears being subjected to higher loads in comparison with odd gears. This, typically would result in the even gears having slightly higher duty values. The gear ratios of the even gears and odd gears have been simplified further and a single array of gear ratios representing a standard-/ordinary gearbox was created. The same was then used in the input file. The simulation was run for a life expectancy of 3,50,000kms.

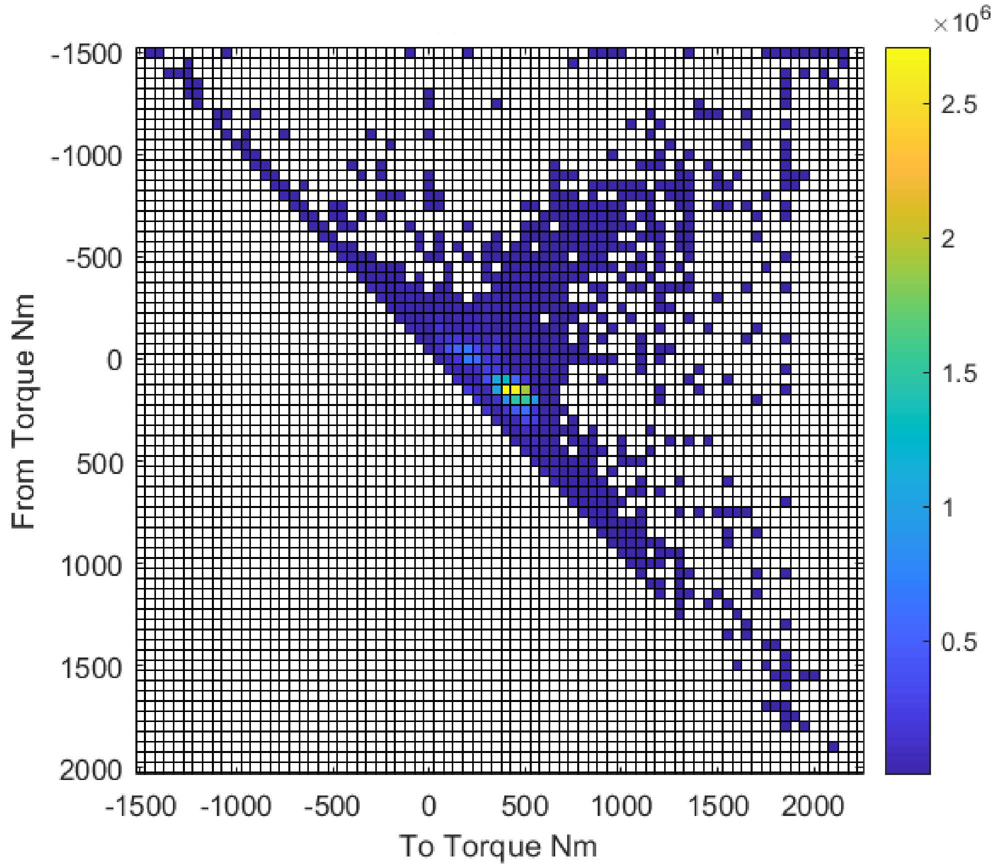

Figure 4.13: Rainflow Cycle Visualisation For Dimensioning Customer Calculated at The 7DCT HEV Driveshaft

The rainflow counts were extracted from the output torque signal calculated at the vehicle's driveshaft. The driveshaft torque was considered to be half of the torque output available at the differential for simplifying the analysis. It was observed that the differential was subjected to a wide range of loads ranging from -1500Nm to 2150Nm. The same is visualised in a pseudocolor plot with each color indicating the intensity or cycles sustained within each loading interval. The from-to plot indicates the start and end of each loading interval. A colorbar is then used to indicate a scale denoting the cyclic distribution. High number of cycles occurring at higher load amplitude will generally cause more damage than the cycles occurring at lower load amplitude.

Figure 4.14: Revolution At The Load Level Calculated At The DCT HEV Differential

Figure 4.14 represents the revolutions at each load level calculated at the differential. Here, a differential is used to indicate the final useful torque available before it is distributed to the wheels for traction. It was observed that the differential was subjected to $2.07 * 10^7$ load cycles at 300Nm. Then according Palmgren-Miner damage accumulation rule 2.5.3 and equation 2.19 the material selected should be able to withstand at least $2.07 * 10^7$ cycles at 300Nm to ensure good service life at 300Nm load. Similarly the cycles or revolutions at each load level are considered and the material will have finite life, if the number of cycles at each load level are lesser than the number of cycles for fatigue failure at the same load level. The resulting ratios are then summed and if the resultant is less than 1, then the material is said to have finite life and if the sum exceeds 1, then the material or the component will undergo fatigue failure. Similar analysis was then carried out for each of the 7 gears.

As discussed earlier for the current gearbox under investigation we need to have a slightly different approach since the even numbered gears and odd numbered gears are separated and subjected to different loading histories. Gear 1 being an odd gear, transmits the torque from the combustion engine to the final drive. Hence, only positive load amplitudes are being considered for this particular gear. The following figure shows the cycles simulated at each load level for gear 1 calculated at the transmission input.

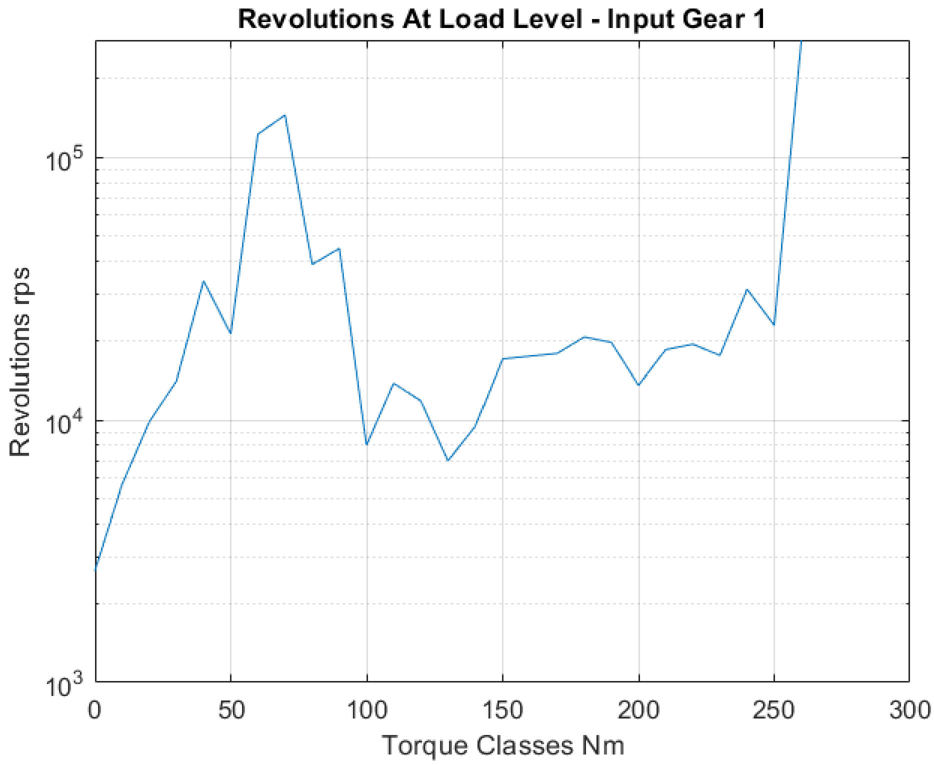

Figure 4.15: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 1)

It was observed that the loading histories for first gear indicated two significant peaks at 260Nm with a cycle count of $2.79 * 10^5$ cycles and $1.45 * 10^5$ cycles at 70Nm load. The loads are then analysed at each load level using the Palmgren-Miner rule and duty values are then calculated using equation 2.20. It is important to pay special attention to high number of cycles occurring at higher load points as these are generally the cause for fatigue failure of the component.

The second gear being an even gear, it was subjected to loads from both the combustion engine as well as the electric motor. Due to instant torque capability of the electric motor , special attention was paid to the components being operated in pure electric mode. It is also possible that the gear would be subjected to loads from both the combustion engine as well as the electric motor when operated in hybrid mode. The following figure 4.16 then represents the revolutions at load level collective, calculated at the input for second gear. The second gear was subjected to loads ranging from -250Nm to 370Nm.

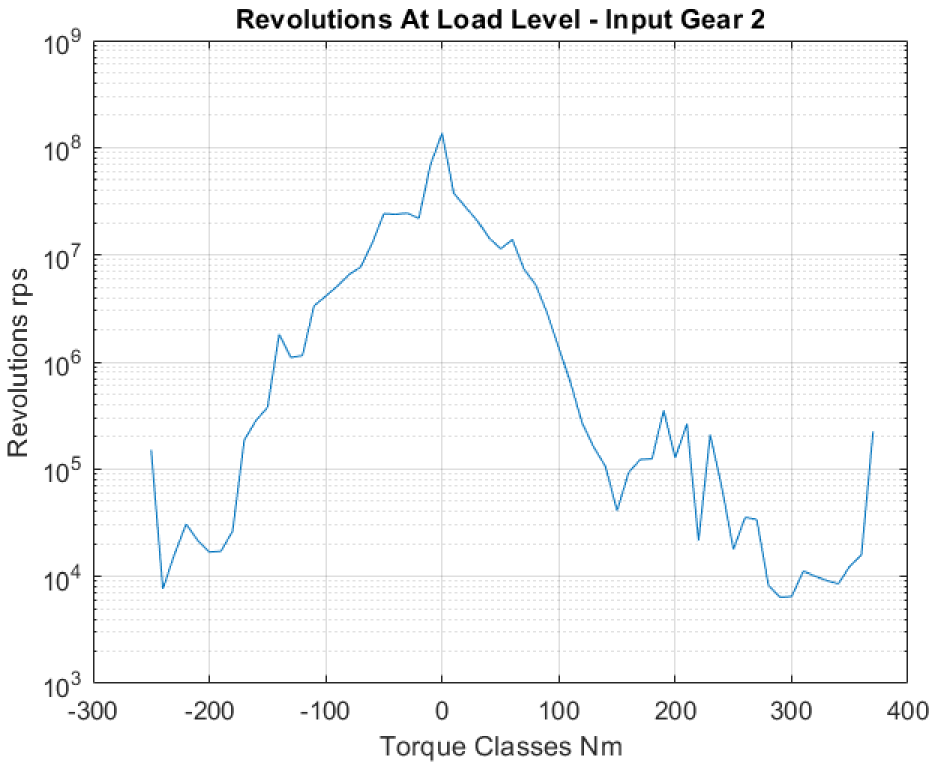

Figure 4.16: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 2)

The second gear is then subjected to loads in both positive and negative domain due to the presence of an electric motor and its capability to regenerate power during braking. The collective can then be evaluated to study the accumulated damage and duty values using equations 2.19 and 2.20. A high cycle count of $1.36 * 10^8$ at 0Nm indicated gear idle and does not have a major impact on duty life. A cycle count of $2.46 * 10^7$ at -30Nm was observed in regen mode and a count of $3.79 * 10^7$ was observed at 10Nm load in drive mode.

Now coming to the load histories of the third gear, visualised in figure 4.17, high cycles($2.38 * 10^6$ cycles) were noted at about 260Nm load. The third gear being an odd numbered gear, received input from the combustion engine and was subjected to positive loads only. The data then was processed in a similar manner compared to previous cases and the duty values and accumulated damage values were determined.

Figure 4.17: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 3)

The fourth gear being an even numbered gear was analysed similar to the second gear. The loading histories are presented both in positive as well as negative torque domain.

The first glance at the collective indicates the gear usage in a wide load band ranging between -250 to 450Nm. The peak occurs at 0Nm with $8.03 * 10^6$ cycles and indicates gear idle . These cycles will not cause a major impact on the duty life of the gear. The next peak reading appeared at 260Nm with $3.74 * 10^6$. It becomes critical to analyse this particular reading, since the load is quite high and the component is subjected to multiple cycles of the same. As discussed earlier high load cycles at higher load points are the major reason for fatigue failure. The same is applicable to the loading histories in both the positive as well as the negative torque domain. A cycle count of $1.23 * 10^6$ was noted at -10Nm indicating high regen within this torque class.

The recorded data was then analysed for the fifth gear. This being an odd numbered gear, the loading histories were recorded only in the positive torque domain. A peak cycle count of $3.01 * 10^7$ was observed at a load class of 260Nm which indicates high usage in drive mode. The load histories are then analysed by determining the accumulated load and calculating the duty values.

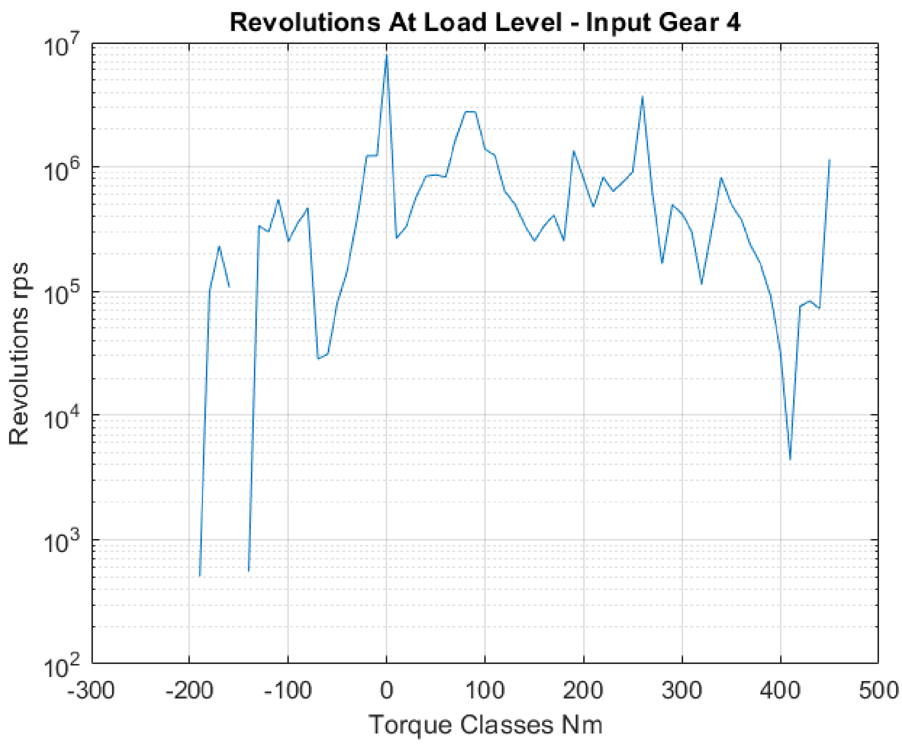

Figure 4.18: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 4)

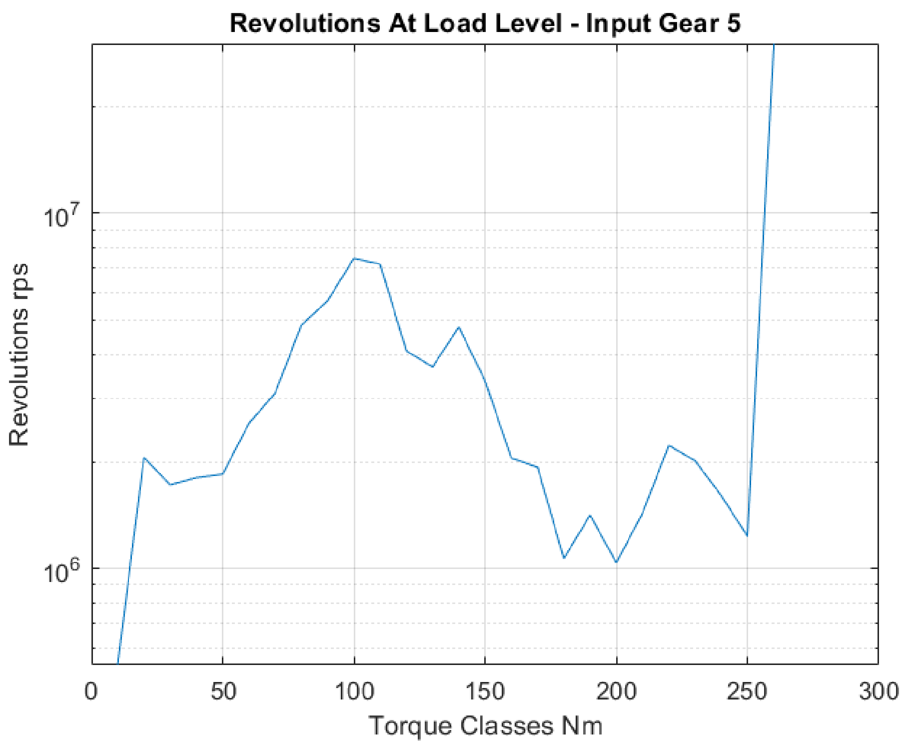

Figure 4.19: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 5)

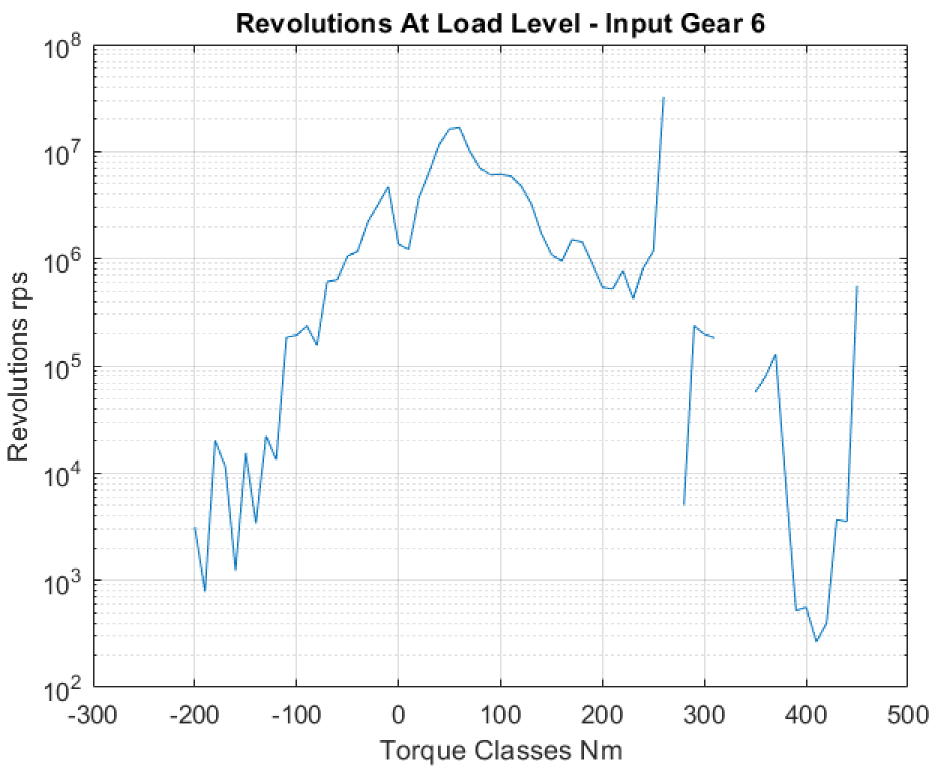

Figure 4.20: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 6)

This was then followed by RAL (Revolutions At Load Level) analysis of the data collected at the sixth gear. The gear was subjected to input loads ranging from -250Nm to 450Nm, however the load repetitions beyond 260Nm are lower in comparison with the load cycles appearing before 260Nm and are not visible in the RAL graph. Peak recording of $3.25 * 10^7$ cycles appears at 260Nm indicating usage in drive mode, whereas a cycle count of $4.71 * 10^6$ appears at -10Nm indicates usage in regen mode. A second peak appears in drive mode with $1.68 * 10^7$ cycles appearing at 60Nm. However the occurrence of $3.25 * 10^7$ cycles at 260Nm would be more dangerous owing to the fact that those cycles occur at a much higher load.

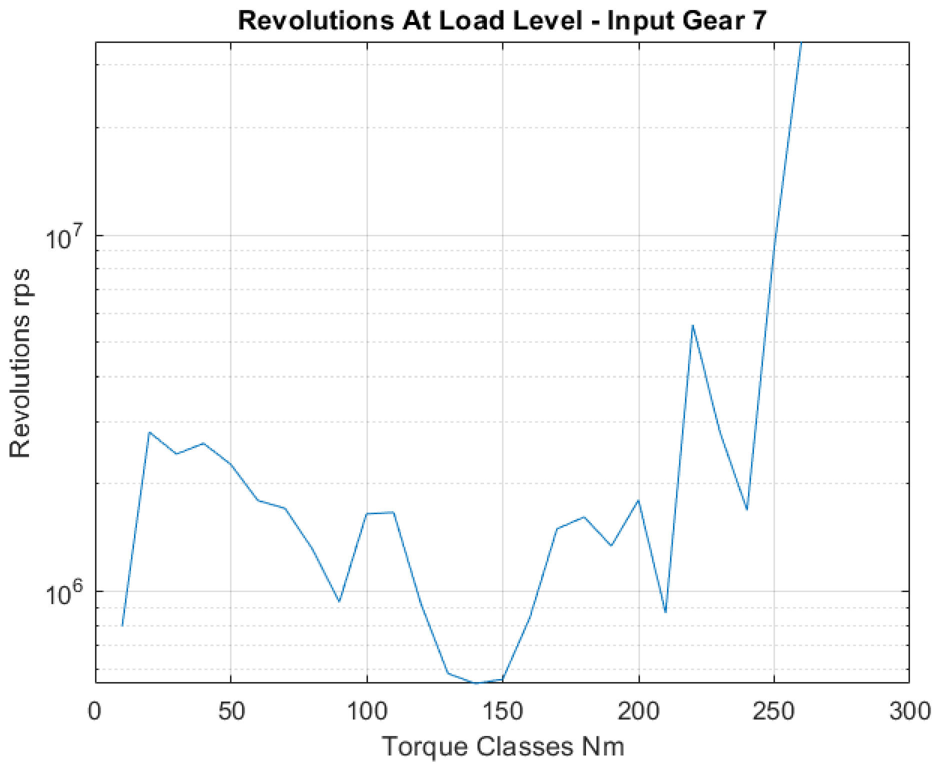

Figure 4.21: Revolutions At Load Level Calculated AT The DCT HEV Transmission Input (Gear 7)

Finally, analysing the RAL collective of the seventh gear, it was observed that the gear was subjected to $3.50 * 10^7$ load cycles at 260Nm. Comparing the RAL collectives from first gear to the seventh a common observation was that the gears were subjected to higher cycles at around 260Nm of load, specifically in odd gears. This is mainly due to the torque transfer limits assigned to each gear and the limitations set in input file.

The duty values were then compiled for each individual gear and the differential using equation 2.20 and are tabulated below. The test was primarily carried out with We = 6 emphasizing bending loads.

Table 4.3: Duty values For 7 Speed HEV.

Duty Values (Nm^{We})	
Gear 1	$1.04 * 10^{20}$
Gear 2	$9.32 * 10^{20}$
Gear 3	$1.30 * 10^{21}$
Gear 4	$1.93 * 10^{22}$
Gear 5	$1.09 * 10^{22}$
Gear 6	$1.67 * 10^{22}$
Gear 7	$1.48 * 10^{22}$
Differential	$3.62 * 10^{26}$

4.5 Impact Of Gearbox Efficiencies On Duty Values

Transmission efficiency plays a vital role in determining the duty values of the components and helps in further reducing the inaccuracies in the calculation steps. In general terms, if a component is operated at a lower efficiency point than expected, the component then would have to work harder to overcome the loss of useful work transfer. The gearbox efficiency map is generally stored in the tool's database and will be in the form of a 3D matrix with the efficiencies specified for each torque-speed operating interval.

Each individual gear has its own efficiency map . The efficiencies for the first gear varied between 35% to 97% depending on the operating speed and torque received. The efficiency map for the first gear is shown below.

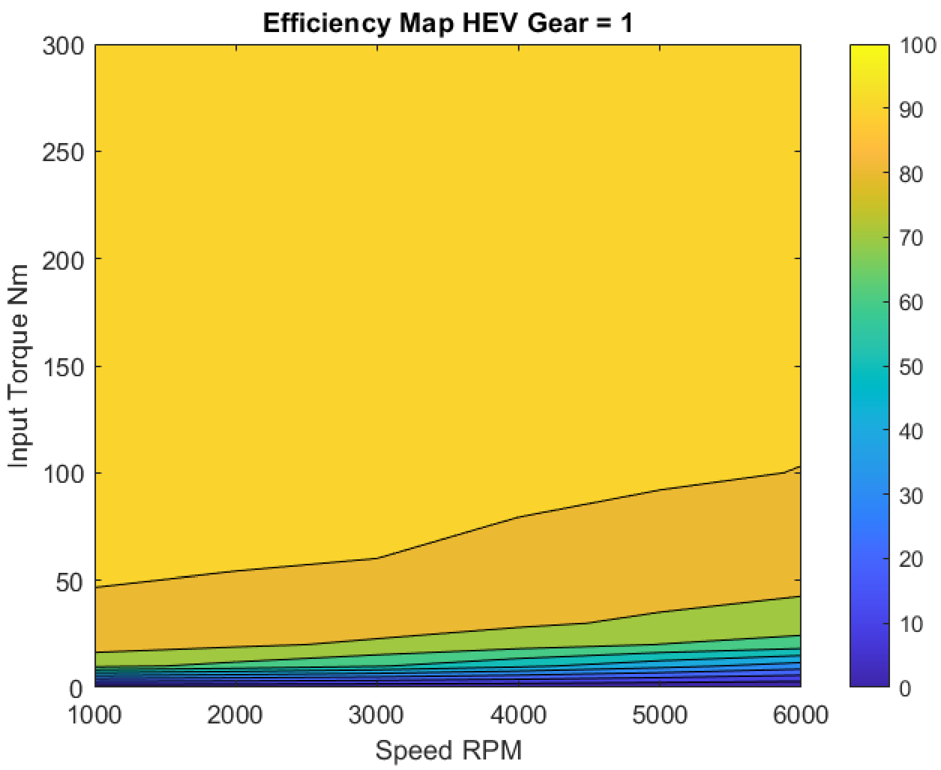

Figure 4.22: Gearbox Efficiency Map HEV Gear 1

For simplicity of the analysis, we will now consider the 3 speed hybrid transmission model, similar to the one discussed in section 4.3. The simulation was conducted with "Efficiency Maps = 'On'"(Active Tracking Of Operating Points) and "Efficiency Maps = 'Off'(Set to 1 or 100%).

It was observed that the percentage increase in duty values was in the range of 30-35% when the system included the gearbox efficiencies. This change is clearly due to the fact that the system senses that the gearbox is not functioning at its optimum and requests higher torque from the prime mover in order to satisfy the demand at the wheels. In simple terms the system must compensate for the losses incurred during torque transmission.

Table 4.4: Duty values comparison (Efficiency).

Duty Values (Nm^{We})			
We	Efficiency Off	Efficiency On	Percentage Increase
3.33	$1.46 * 10^{18}$	$1.92 * 10^{18}$	31.5
6	$6.57 * 10^{26}$	$8.91 * 10^{26}$	31.61
7	$1.38 * 10^{30}$	$1.8 * 10^{30}$	30.43

4.6 Impact Of Inertia On Duty Values

Component inertia is one of the important aspects to be considered during duty value analysis. Since, the analysis is majorly considered on torque loads, the impact of component inertia and its effect on torque transfer plays a vital role in determining the durability of the components.

Table 4.5: Duty values comparison (Inertia).

Duty Values (Nm^{We})			
We	Inertia Off	Inertia On	Percentage Increase
3.33	$1.83 * 10^{18}$	$1.92 * 10^{18}$	4.92
6	$8.51 * 10^{26}$	$8.91 * 10^{26}$	4.70
7	$1.72 * 10^{30}$	$1.8 * 10^{30}$	4.65

The study was conducted on a 3 speed HEV with an automatic transmission. Vehicle specifications or the input file used was similar to the one discussed in section 4.3. Detailed duty value analysis of the 3 speed HEV model is discussed in section 4.3.

As shown in table 4.5, it was observed that there was a considerable amount of increase in duty values, when inertia of the gearbox and the torque converter was considered in the simulation. This increase is due to the fact that the transmission components need to be supplied with slightly higher amount of torque than that requested at the wheel, so as to overcome the inertia of the components to prevent transmission losses.

5
Conclusion And Future Work

5.1 Conclusion

As discussed earlier in section 1.1, in the early stages of any vehicle development, it is difficult to know the exact sizing of transmission components or the materials that can be used to design these components with. Manual testing or verification of the components is destructive in nature, meaning the components cannot be re-used if damaged during the test phase. Manual testing or physical testing of the component costs a lot of money and time. The whole idea behind the concept of computer simulations is to avoid manual testing as much as possible or prevent premature designs, not knowing the nature of loading that these components maybe subjected to. The tool tries to bridge this gap by providing near realistic loading histories of each of these components, hence helping the design team to select the right material and size the components accordingly.

The simulation tool analyses the vehicle model over a desired life expectancy range (distance in kms) and provides an insight on how each component will be loaded during the course of its service life. On the other hand it is also possible to study how the vehicle model behaves in each of the driving environments. Factors like acceleration capability, top speed, fuel consumption, electric range etc can also be studied with these models.

The tool enables the users to perform simulations that can be easily set up by simply providing the vehicle specifications. The tool can also be used to quickly analyse a particular transmission component and test different variants of the same.

5.2 Future Work

As part of future work on this project the following options could be considered,

- Further enhancement of the scripts to reduce run-time on slower computers.
- Inclusion of factors such as temperature, pressure, quality of lubricating oil etc and their effects on duty values.
- Further inclusion of more transmission components such as bearing, washers etc. maybe possible.
- Torque vectoring to determine the exact damage cause to the driveshafts.
- Possibilities to include material test data to enable the system to automatically detect fatigue failure.
- Possibility to convert the MATLAB scripts into a standalone application.

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