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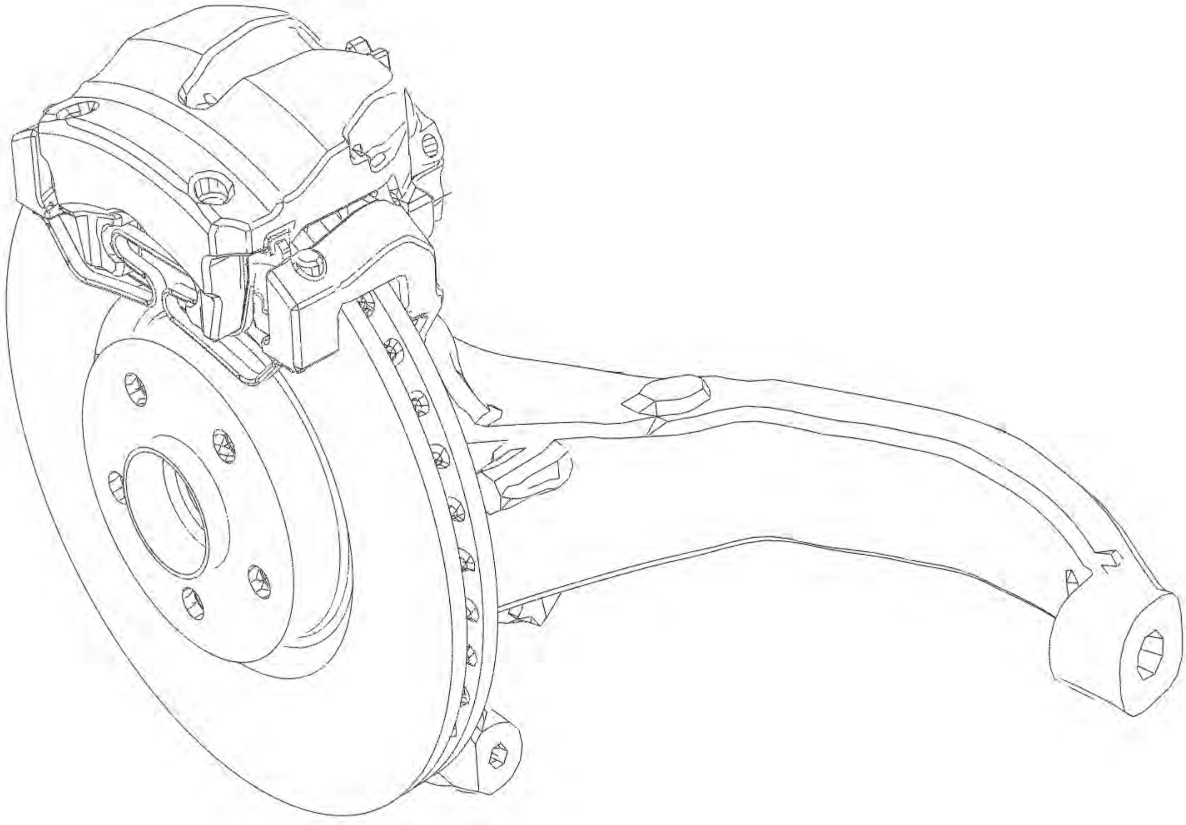

# Brake Squeal Analysis in Time Domain Using ABAQUS

Finite Element Simulation of Brake Squeal for Passenger Cars

*Master's Thesis in Automotive Engineering Programme*

*Mir Arash Keshavarz*

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2017

MASTER’S THESIS IN AUTOMOTIVE ENGINEERING

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## ABSTRACT

One of the main quality concerns of car manufacturers is brake squeal. There are two CAE (Complex Aided Engineering) method for prediction of brake squeal, CEA (Complex Eigenvalue Analysis) and TDA (Transient Dynamic Analysis). However, CEA method is a quite usual tool to evaluate brake squeal propensity despite its intrinsic overestimation of any system instabilities. In contrast, TDA provides a reduced number of predicted instabilities, but it is too time consuming for complete FE models. The purpose of this thesis is the comparison of TDA with CEA to identify brake squeal frequencies. The finite element model of studied brake system consists of a brake disc, and pair of brake pads (friction material and backplate). Using the CEA method, brake squeal frequencies are identified by coalescence modes and calculation of Negative Damping Ratio (NDR) for various pressure and friction coefficient. As a comparison with CEA, TDA is performed by use of dynamic explicit solver to extract the variation of amplitude at a node on the brake disc friction surface over time. The frequency response function of a node on brake disc can be calculated by Discrete Fourier Transform (DFT), the frequencies correspond to the peaks of frequency response function can be considered as a representative of brake squeal frequency.

The comparison of TDA and CEA reveals that even though five modes are introduced by NDR criteria for brake squeal onset, merely two of them are detected by TDA. Taking account of CEA overestimation about unstable modes, it can be concluded that the frequency response derived from the transient response can be a reliable method to represent the system instability. In addition, regarding TDA results brake squeal might happen in an unstable mode even with low value of real part as well as NDR.

Key words: Brake, squeal, complex eigenvalue analysis, transient Dynamic analysis, stability

# CONTENTS

# Preface

This thesis is the final essay of my Master of Science in Automotive Engineering at Chalmers University of Technology, Gothenburg, Sweden. The work was performed at Volvo Cars Corporation, Gothenburg, Sweden, at Driving Dynamics Centre at the department of Braking Analysis and Verification during autumn 2016. It was a great opportunity to learn and update my knowledge in nonlinear vibrations, CAE methods linked to Noise, Vibration and Harshness (NVH) and transient analyses. In addition, it was really interesting to get familiar with Volvo Cars Corporation environment.
I want to appreciate my supervisors Dr. Gaël le Gigan and Dr. Patrick Sabiniaz whom always have been helpful and willing to support me. I appreciate my family members especially my mother to support and encourage me in all stages of my life.

Mir Arash Keshavarz,
Gothenburg, May 2017

# Notations

$\mu_s$ ………………………………….………………...…...static coefficient of friction
$\mu_d$…………………………………..………………....dynamic coefficient of friction
$v_s$…………………………..……………... the speed between sliding pad and disc
$F_f$…………………………..…………….…………...…………friction force
L…………………………..……………………………....normal force on surface
$\theta$……..the slope of line connecting the pivot to the mid-point of a pad's contact area
$\lambda$……………….………...……………………….………….......complex eigenvalue
α………………………...……………………….....real part of complex eigenvalue
β……………………..... ………………....imaginary part of complex eigenvalue
$K$………………………........……………………………….....stiffness matrix
$u_{eq}$……………………….....…………………..……………....equilibrium point
$F_{ext}$……………………………………………………………....external force
$F_{nl}$……………………………………….………....force originated from nonlinearity
$u$…………………………….........……………………….……....displacement vector
$\bar{u}$………………………………........…....displacement relative to equilibrium point
M……………………........…………………………………………......mass matrix
$\ddot{u}$………………………………......………………………............acceleration vector
C………………………………....………………………..................damping matrix
$\dot{u}$……………………………....……………………….................. velocity vector
$A$……………………....………………………………......................eigenvector
$t$……………………....……………………………….................................time
$u_0$……………….………....…………………………………..........initial displacement
$v_0$…………………………………………………………..................initial velocity
$\zeta$……………………………....…………………………......negative damping ratio
$h$……………………………......……………..…….............................time step
$L$……………………………......…………......…......lower triangular matrix
$U$……………………………………………….................upper triangular matrix

# Abbreviation

CAE……..…………………………………………….Computer Aided Engineering
CEA……………………………………………….. Complex Eigenvalue Analysis
TDA……………………………………………….Transient Dynamic Analysis
FRF………………………………………………..Frequency Response Function
DFT…………………………………..……….. Discrete Fourier Transform
NVH……...……………………………...……Noise, Vibration and Harshness
NDR……………………………………………….. Negative Damping Ratio
FE……………………………………………………… …….Finite Element
FEA…………………………………………….………Finite Element Analysis
FEM……………………………………….………….Finite Element Method
GEA …………………………………….……………….. GEnetic Algorithm
MAC……………………………………………….Modal Assurance Criteria
MAI ……………………………………….………..Modal Absorption Index
PSD…………………………......……………………..Power Spectral Density
SCLM ……………………..…..Spectral Criterion based on Linearization Method

# 1 Introduction

## 1.1 Brake squeal in automotive industry

Nowadays, the refinement of vehicle Noise, Vibration and Harshness (NVH) has considerably increased the contribution of brake noise in vehicle design and development process. As a general practice, brake noise is an irritating sound for consumers who may believe that it is symptomatic of a defective brake system and fill a warranty claim, even though the brake is functioning properly [1]. Thus, understanding, prediction and prevention of brake noise and vibration has become an important aspect in brake design and development related to quality processes. Akay noted that, for example, as early as 1930, brake noise appeared in the top-ten noise problems surveys performed by New York City [2].

A wide variety of brake noise and vibration phenomena are described by various terminologies such as brake squeal, creep groan, chatter, brake judder, brake moan, muh, squeak, etc. [1]. Among them, one general term, squeal, is probably the most prevalent, disturbing to both vehicle passengers and environment, and expensive for brake and automotive manufacturers in terms of warranty costs [1]. However, there is no precise definition of brake squeal that has gained complete acceptance, but it is generally agreed that squeal is a sustained, high frequency vibration (above 1 kHz and below 10 kHz) of brake system components during a braking action leads to audible noise to vehicle occupants or passengers [1].

## 1.2 Numerical methods to predict brake squeal

There exist two different methodologies available to predict brake squeal using Finite Element Method (FEM), namely, Complex Eigenvalue Analysis (CEA) and Transient Dynamic Analysis (TDA) [3]. Both methodologies have pros and cons that are discussed in the following chapters. FEM is a widely used numerical method to predict squeal in car brakes because it offers much faster and more cost-efficient solutions than experimental methods, and it can predict squeal noise at early stages of development process before prototyping [3].

The CEA determines the complex eigenvalues by linearization of the equation of motion around equilibrium point. According to the basic stability theory, the positive real parts of the complex eigenvalues indicate the degree of instability of the linear model of a disc brake and are thought to show the likelihood of squeal occurrence [3]. Even though, CEA allows identification of all unstable frequencies in one run at a given operating conditions whereas not all unstable frequencies can be observed in experiments as instability. Thus, it can be understand that CEA overestimates the instabilities. The cause and reason of this over estimation are discussed in the next chapters. On the other hand, TDA is able to predict real unstable frequencies that can be verified by experiments. The drawback of TDA is that it is very time consuming as well as it does not provide any information on unstable mode shapes [3]. The purpose of this project is transient analysis of brake system in commercial FE software as well as the comparison of TDA with CEA.

## 1.3 Brake systems-functions and types

The fundamental functions of a brake system can be summarized as follow [4]

- Reducing the speed of vehicle and if necessary to a requested stationary position (normal braking)
- Prevent unwanted acceleration while travelling downhill
- Maintain the vehicle at stationary condition by parking brake
- Conduct the vehicle to full stop with high deceleration braking (emergency braking)
- Ensure vehicle stability (under and over-steering and maintain tire friction)

The principal type of brake used to generate braking force at the wheels is called friction brake. Friction braking converts the potential and kinetic energy into heat. Friction brakes can be categorized into two types: disc brakes and drum brakes [4]. Figure 1 depicts a disc brake assembly with suspension and part of the subframe. The brake assembly consists mainly of a brake disc, a calliper and a pair of pads with shims and under layer designed to generate a brake torque. During braking, the calliper pushes the pads onto disc by hydraulic piston. The friction force generated at the frictional surfaces between the brake pads and rotating brake disc generates the required braking torque transferred to the wheel/tire to stop the vehicle. Disc brakes are used in all front axle of passenger cars and in some cases can also be found in the rear axle. Despite their higher cost as compared to drum brakes, disc brakes are more robust and have better cooling performance.

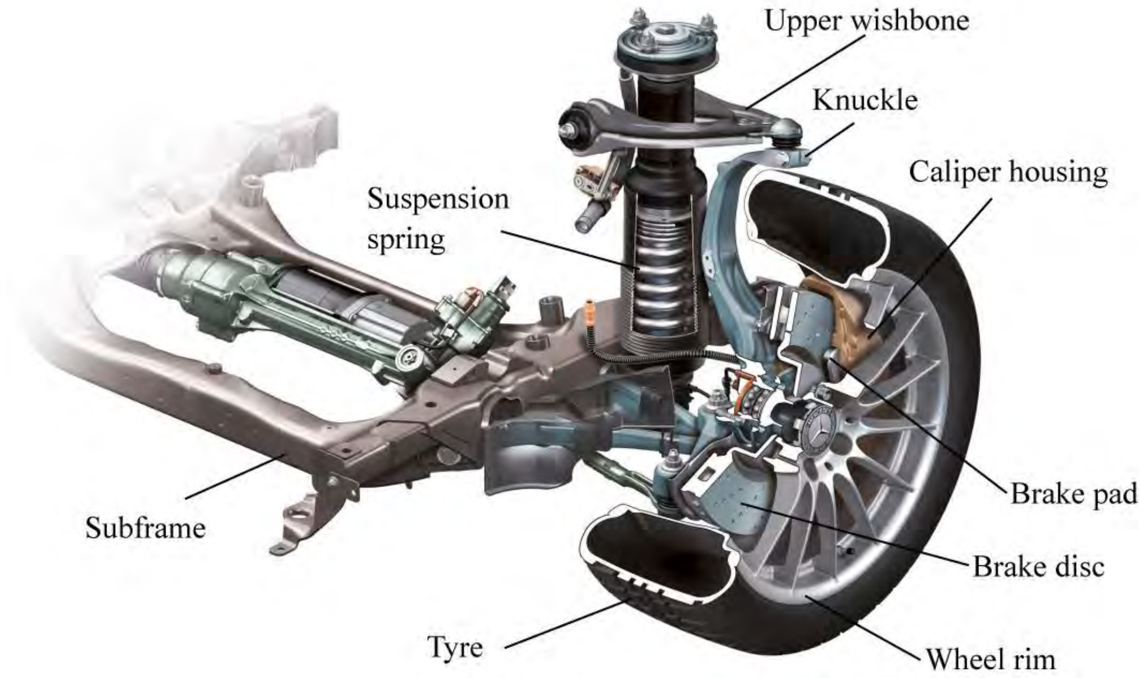


*Figure 1.Suspension assembly with brake system [5].*

Drum brakes, see Figure 2, are radial brakes combining a brake shoe mounted on the stub axle and a rotating brake drum mounted on the axle. Drum brakes are composed of two brake shoes (seldom one only) that are pressed outward against the friction surface of the drum by the action of hydraulic piston during braking. When the braking operation is done, a spring pulls back the brake shoes to ensure a clearance between the surface of drum and brake pad. Drum brakes are less sensitive to external dust and rain as it is a closed component and is cheaper than disc brakes. However, drum brakes suffer from poor cooling characteristics and early cracking.

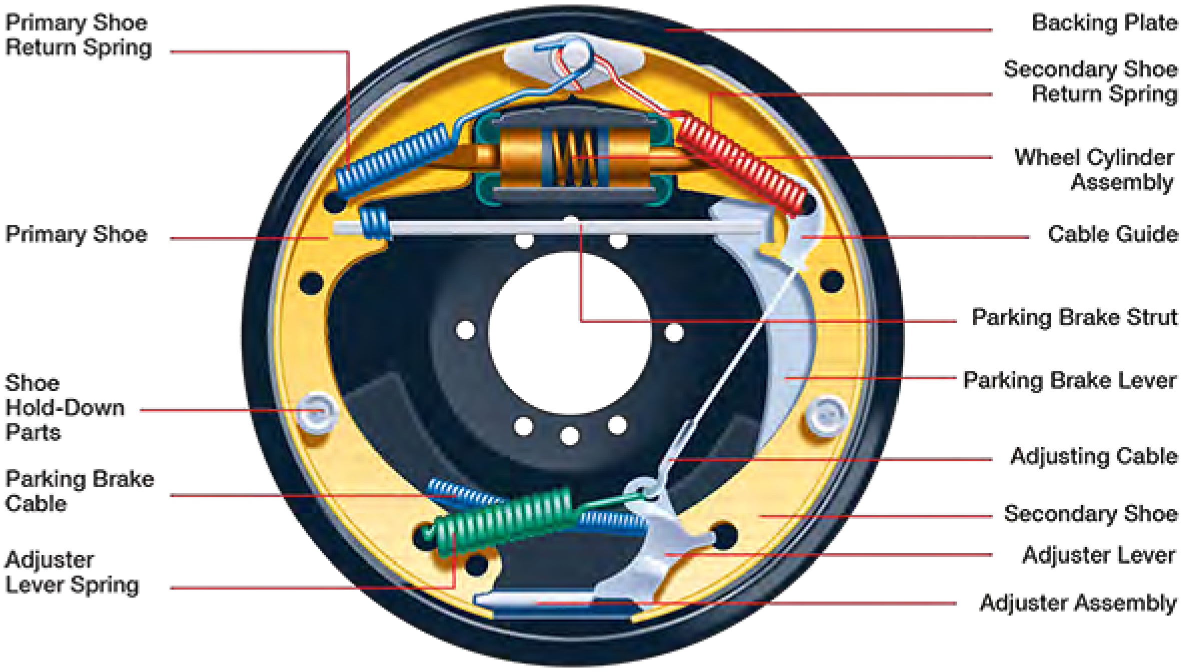


*Figure 2. The drum brake components. [6]*

## 1.4 Disc brake components in brake squeal

The "simplified FE model" of a brake system, see Figure 3, is composed of brake disc, a pair of brake pads. Here, the brake pads include the backplate but no shim neither friction material under layer is modelled. The backplates is made of steel, see *Figure 4*, to push the pad onto the brake disc and generate the requested friction torque. The normal force and frictional torque, i e tangential friction force, between the brake pads and brake disc may excite the brake disc to vibrate. The friction force makes the problem as nonlinear vibration so that the linear vibration theory is not able to model the squeal problem properly. The equation of motion and mathematical modeling of nonlinear vibration of simplified brake system will be discussed in the following chapters.

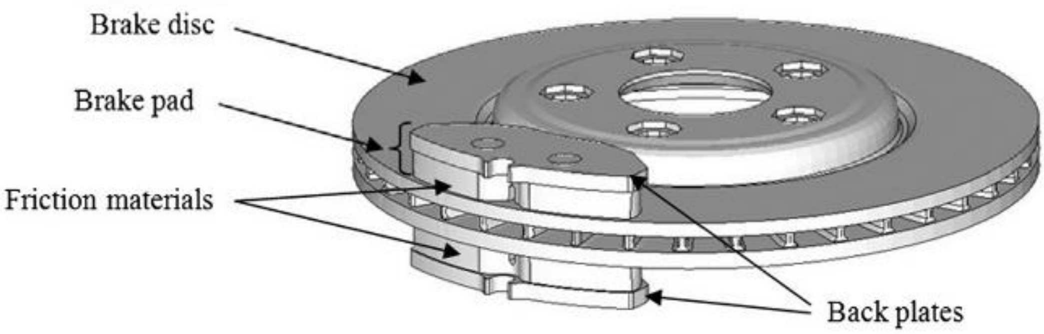


*Figure 3. A simplified model of disc brake system. [7]*

Figure 4, shows the multi-layer pad and backplate structure in detail. The shim function is damping of vibration propensity that is modeled in complete FE model.

The slot and chamfer are introduced to control the brake squeal propensity. For this project, the pad material is considered as anisotropic in FE model.

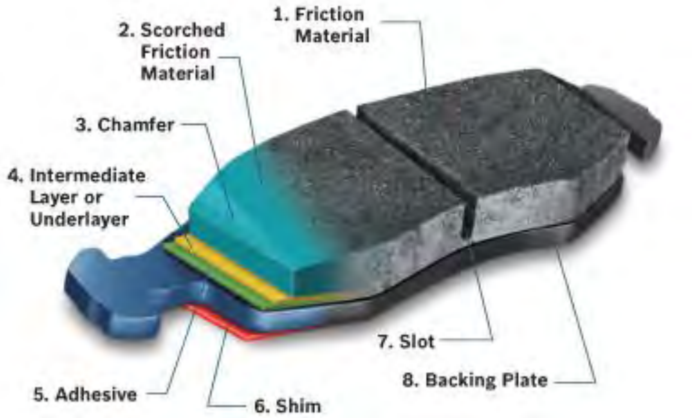

*Figure 4. Brake pad composition. [8]*

## 1.5 Brake noises and vibrations

There are various kinds of brake noises and vibrations with numerous terminologies to nominate the phenomena in the literature that are probably inconsistent. One of the terminologies is shown in Figure 5. It can be seen that noises and vibrations can be classified into groups that are based on frequency range. Some types of brake noise and vibration can be summarized as follow:

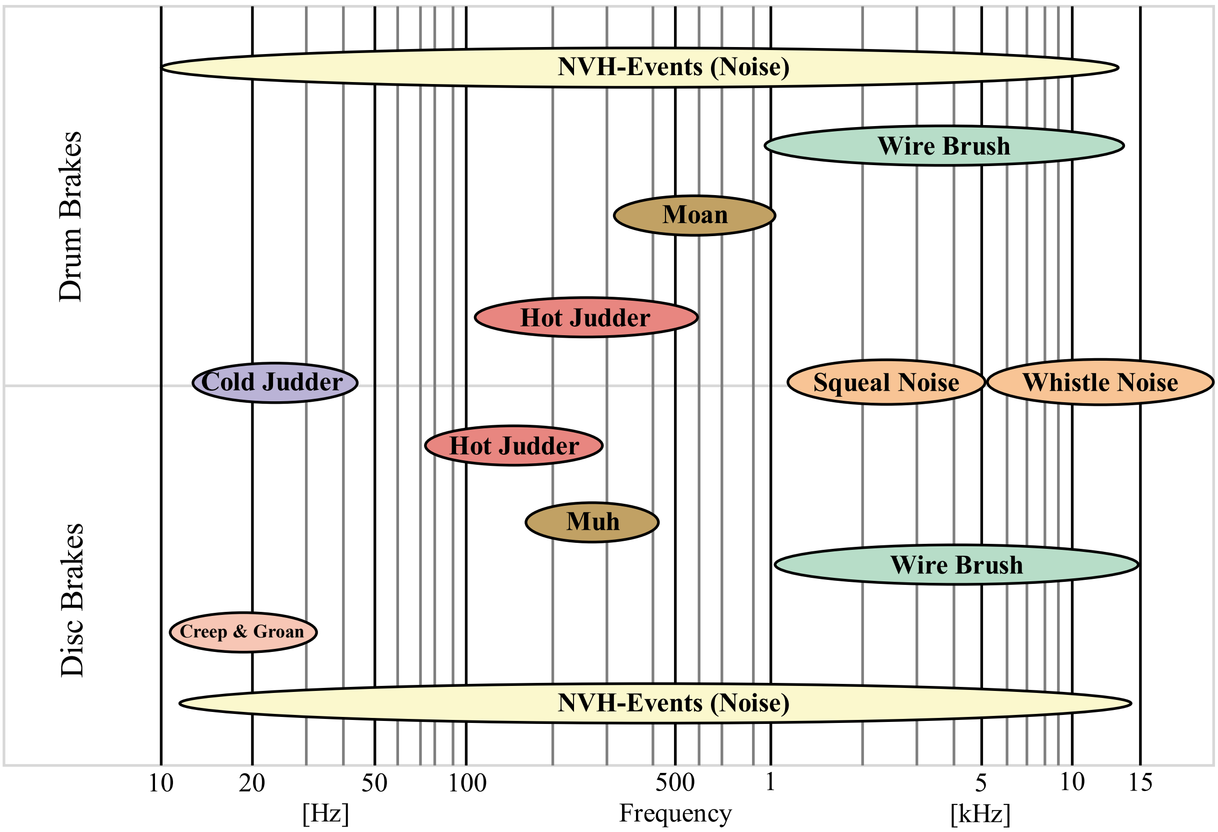

*Figure 5. Classification of brake noise and vibrations during braking.*

- Squeal [9] is a high frequency noise (1-15 kHz) in which the sound is emitted from vibrating brake disc. Squeal will happen at vehicle speed less than 50 km/h in rare few cases but mainly under 10 km/h.
- Wire Brush [9] is a kind of high frequency noise. The excitation mechanism is the same as squeal but with the modulations of sound wave.
- Judder [9] is a high speed vibration problem, and can usually be felt in the brake pedal. It is the result of friction variation due to either Disc Thickness Variation (DTV caused by uneven wear) or by variations of coefficient of friction at disc surface.
- Muh [9] is another high speed phenomenon with frequency below 500 Hz. A vibration in the brake parts transferred to some larger surface such as door panel or window.
- Groan [9] is a low frequency noise (less than 100 Hz) originated from stick-slip behaviour in the brake and make resonance in drive shaft and suspension.

## 1.6 Brake squeal: roots and causes

As mentioned above, brake squeal is a type of friction induced vibration. The brake system may not lead to squeal until at least one of the following phenomenon excited by friction force. The phenomenon that generate brake squeal are as follows:

- **Stick-slip**

  Disc-brake squeal has the characteristic of a frictional vibration which can be induced by a frictional pair having either a static coefficient of friction $\mu_s$ higher than the dynamic coefficient ($\mu_d$), or the negative slope of dynamic coefficient with respect to speed ($\frac{d\mu_d}{dv_s} < 0$, $v_s$ is the speed between sliding brake pads and brake disc) [1]. Indeed, $\frac{d\mu_d}{dv_s} < 0$ theory has not received much attention in recent years [1].

- **Sprag-slip**

  Brake squeal occur in such a position in which the frictional force is increased much above the value it would have in a perfectly rigid system. In other words, the Sprag-slip can be illustrated as in Figure 6. The friction force can be derived as $F_f = \frac{\mu_d L}{1-\mu_d.\tan(\theta)}$ in which L, $\mu_d$ and angle $\theta$ are respectively, normal force, dynamic friction coefficient and the slope of the line connecting the pivot point $P$ to the mid-point of a pad's contact area. According to Sprag-slip theory, if $\theta \to tan^{-1}\left(\frac{1}{\mu_d}\right)$, then $F_f$ will be a large value that represent the squeal condition.

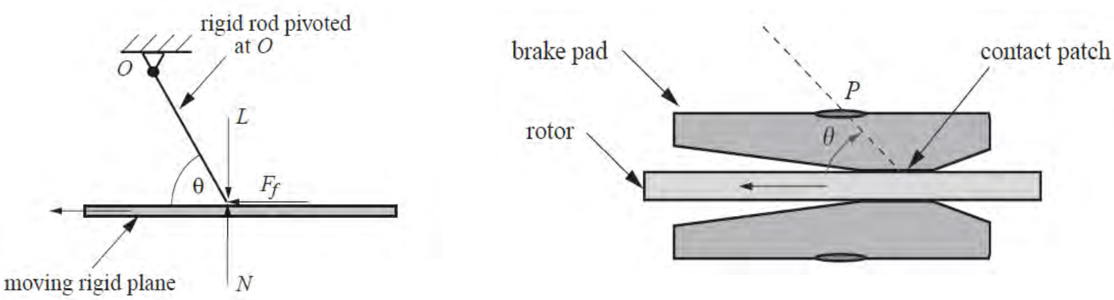


*Figure 6. Schematic model of disc and pad in Sprag-Slip theory [1].*

- **Dynamic instability (flutter type)**

  The premise of this theory is that the squeal happens when the complex eigenvalue correspond to a pair of modes, has conjugate real part and equal imaginary part (two modes happen at the same frequency). This phenomenon is called coalescence, in which two eigenfrequencies - one eigenfrequency with positive real part cause the instability and the other eigenfrequency with negative real part is stable - has coincidence with each other and excite the system to unstable condition. For this thesis, this theory is applied to detect brake squeal frequencies for CEA method.

- **Hammering**

  This mechanism corresponds to imperfection and uneven surface of pad material during the rotation generate the periodic impacts on disc and excite it to instability.

As mentioned above, there are various mechanisms that contribute to the brake squeal onset. In addition, the identification of dominant mechanism might be a complex problem. As more test evidence and analysis results become available, it seems quite obvious that none of the above mechanisms alone can provide a complete explanation of the squeal phenomenon [10]. In some cases, Sprag-slip may seem more proper, but in others, dynamic instability may be a better explanation [10]. Consequently, as discussed in the following section, many studies tried to predict brake squeal by transient and nonlinear methods. However, there are still some issues to work on.

## 1.7 State-of-the-art

Various theories and methods have been formulated in order to simulate brake squeal using Finite Element Method (FEM). At first, it is beneficial to understand the main parameters and mechanisms responsible for brake squeal such as pressure, friction coefficient and component stiffness. For example, Baillet et. al [11] studied the effect of simplified brake system parameters that could contribute to occurrence of instability. The simplified FE model consists of a sliding beam and stationary pad in laboratory code, PLAST3 in explicit dynamic formulation, was implemented. The various values for model parameters such as Young's modulus of pad, beam speed, pad dimensions, pressure, and friction coefficient are investigated in time domain and frequency domain. Other researchers such as Liu [12], tried to gain better understandings about brake squeal by implementing a detailed FE model in which the interaction of friction components are modelled with more accuracy. For instance, Liu P. [12] studied brake squeal using FE model consisting of a brake disc and a pair of brake pads in the frequency domain using CEA method. In addition, Liu P. [12] investigated the effects of the stiffness of the brake disc and the stiffness of the brake backplate. The results show that increasing the coefficient of friction and pad stiffness and decreasing the disc stiffness result in high tendency to squeal. The complex mode shapes showed that the backplate bending mode around 12 kHz, see Figure 7, was the dominant mode for squealing while the disc out of plate bending mode was negligible.

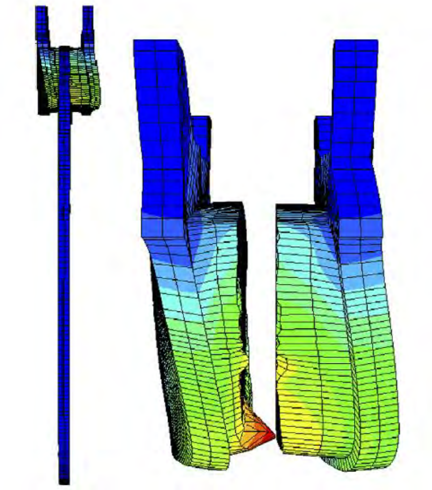

*Figure 7. Brake pad bending mode, i e-the dominant mode for brake squeal [12]*

Other studies [13, 16, 20] compared CEA and TDA method to examine their capabilities in brake squeal prediction in which it is observed that CEA method produces an over prediction of the unstable eigenfrequencies as compared to physical testing [13]. Massi et. al [13] compared the CEA method with TDA and experimental analysis to identify brake squeal instabilities. The FE model implemented for CEA is depicted in Figure 8. It is a "simplified model" where the brake pad and backplate are modelled as a cube and a beam, respectively. The unstable modes are identified with CEA and TDA for various values of friction coefficient and pad stiffness, concluding that increasing friction coefficient and pad stiffness increase brake squeal propensity. The comparison of TDA and CEA results, revealed the over prediction of CEA so that both methods detected instability at 3.5 kHz, but CEA predict the instability in a wider range of pad stiffness than TDA.

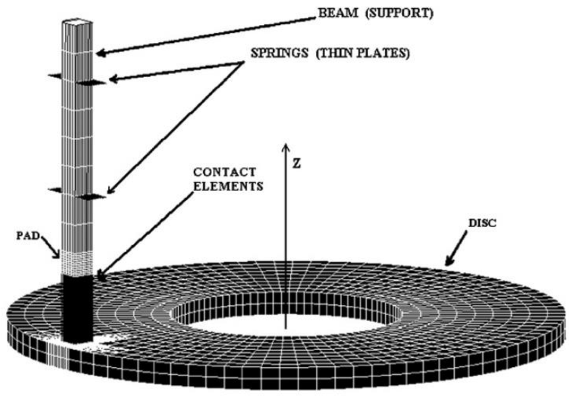


*Figure 8: The simplified brake model for CEA and TDA methods [13]*

It is a well-known fact that a high friction coefficient increases the probability of squeal [14], so the measurement of critical value of friction coefficient can be used as a criteria for the prediction of squeal [14]. Huang et. al. [14], introduced a new method to predict the onset of brake squeal, called as 'reduced-order characteristic equation method' to study the coupling mode and estimate the critical value of friction coefficient. Using modal expansion method (this method approximates the response of a system by the summation of its mode shapes), the possibility of instability is studied in different types of couplings mode such as curve crossing, curve away and curve toward in which the instability is likely to take place in curve toward mode. Then, a reduced-order characteristic equation method utilized the coupled eigenvalues in 'toward mode' to estimate the critical value of friction coefficient. In comparison with CEA, the reduced order method estimate only critical friction coefficient with good accuracy.

Grange et. al. [15], introduced a new method, so called, Spectral Criterion based on Linearization Method (SCLM) for the calculation of eigenvalues of brake squeal through linearization of nonlinear response such that the Power Spectral Density (PSD) should be the same as nonlinear model. SCLM method implemented on an explicit 2D FE model of pad and beam. The SCLM results were capable of identifying modal parameters such as eigenvalues and mode shapes that are closely correlated with CEA.

Regarding that the contact forces between brake disc and pads are a function of the nodes displacement, the friction and normal forces make the brake squeal as a nonlinear problem. Thus, linear methods such as CEA cannot be able to correctly predict the unstable modes because the linearized function around the equilibrium point is not valid during transient condition. As a result, one of the drawbacks of CEA is that it cannot model nonlinear transient vibration accurately. Few papers based on FEM consider the transient nonlinear behaviours of brake systems subjected to multi-instabilities. Sinou [16] identified the two unstable frequency ($f_1$ and $f_2$ in Figure 9) and their harmonics ($\pm n.f_1 \pm m.f2$ - $n, m=1,2,3\dots$) for various friction coefficient using transient response and wavelet spectrum method (Figure 10). Sinou [16] investigated the contribution of harmonics (Figure 10-right) in non-linear transient and stationary condition for disc brake system using a finite element model of brake disc and pads in which the contact elements between disc and pad modelled the friction force as a cubic function to approximate the first and third order of experimental brake pad compression curves. Furthermore, considering Figure 9-left, it is notable that the coalescence modes (A and B or C and D) join each other to make one equal frequency ($f_1$ and $f_2$) at bifurcation point. In addition, Figure 9-right, shows that real part of coalesce mode (A and B or C and D) has the same absolute amount but in apposite sign after the bifurcation points. In this project, the coalescence modes are also derived to predict brake squeal characteristics in the next chapters.

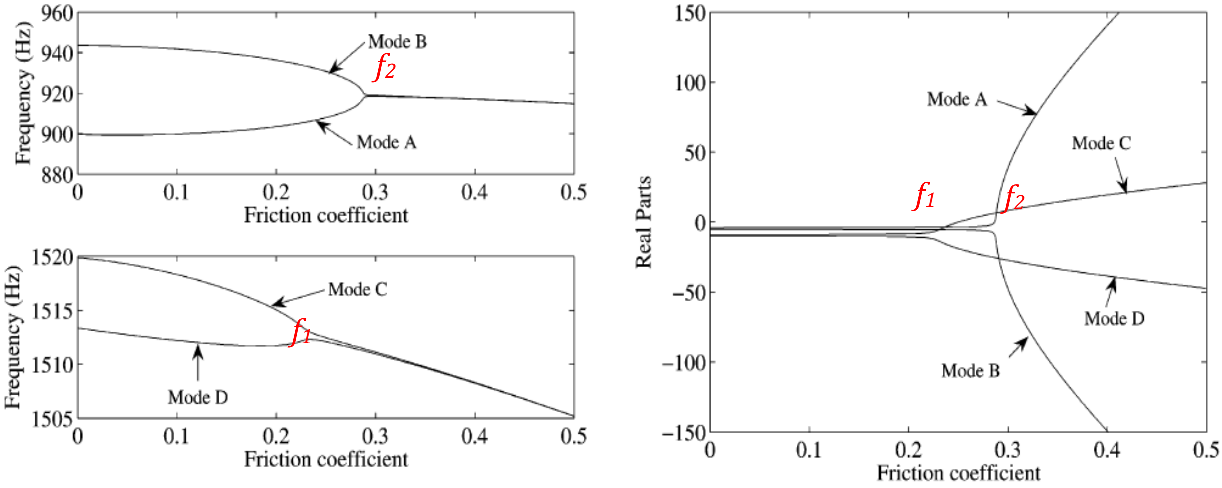


*Figure 9: Bifurcation plot for the brake system at f1 and f2 (left) and real parts (right) [16].*

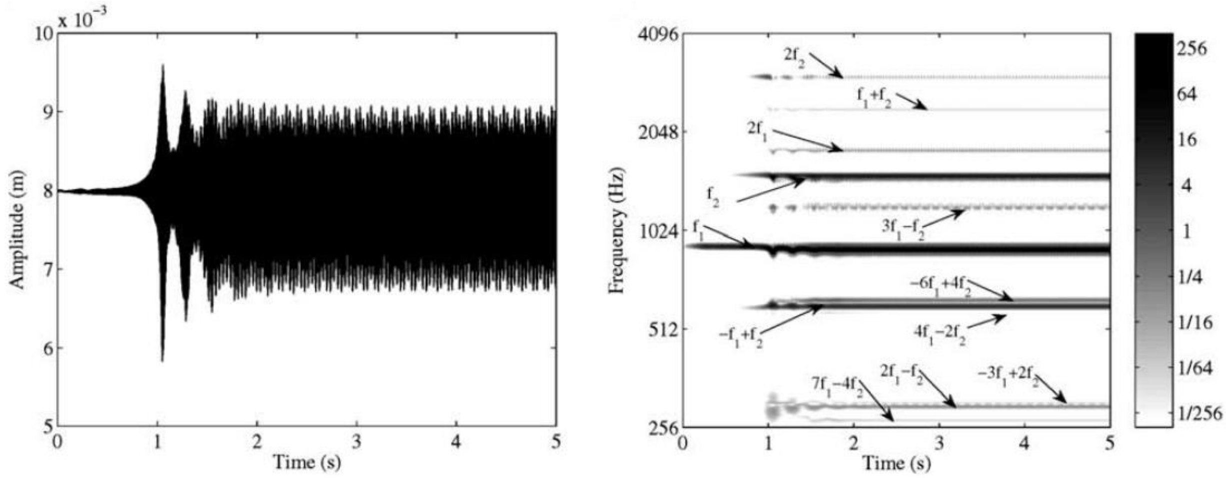


*Figure 10: Transient nonlinear response of the brake system: time history (left) and its corresponding wavelet power spectrum (right) [16].*

In order to control brake squeal, passive methods such as eigenvalue placement (avoidance of resonance frequencies or setting the eigenvalues with negative real part), are the easiest way. The symmetry of disc can cause multiple eigenfrequencies, which leads to instability and squeal, so the asymmetric design of the brake disc can be implemented as a tool for splitting the symmetric modes of the brake disc and prevent squeal [17]. In order to use this method, Spelsberg [17] studied the mathematical relation between the symmetry of disc and its multiple eigenvalues. It was shown that for every elastic body (especially a disc) having an angle of less than $\pi$ has at least one double eigenfrequency, which might excite to self-excitation vibration and squeal. Using finite element model of a brake disc, some structural modifications such as vary the brake disc thickness, add mass and making holes, are proposed to disrupt disc symmetry. The experimental result from scanning laser vibrometer shows that the symmetric brake disc started to squeal almost immediately whereas brake squeal was not found (or harder to produce) in asymmetrical brake disc [17]. Scanning laser vibrometer [17] is an experimental method to record vibration of any structure in 3-Dimension (3D) by use of Doppler effect [17].

Although so many researches on brake squeal have been conducted through CEA approaches, only a few investigate the brake squeal problem with uncertainties. Since, uncertainty always exists in reality, parameter uncertainties have to be introduced into the model of brake systems for obtaining more reliable results [18]. Lu et. al. [18] studied the optimization of brake performance through considering some uncertainties in design variables such as Young modulus, brake disc thickness and friction coefficient at the contact interface. Since there is no sufficient information on the mentioned parameters, each parameter is limited to a specified interval. Lu et. al. [18] proposed a method called as reliability based optimization to optimize the brake squeal propensity using GEnetic Algorithm (GEA). The dominant unstable modes are extracted using FEM and CEA. The objective function of optimization was the backplate thickness in which the reliability index— defined as a function of damping ratio, which should be greater than one to ensure stability — is considered as constraint. The results show that brake squeal propensity can be reduced by using stiffer back plates. The proposed approach shows potentialities in improving the stability of the vehicle disc brake system.

Although, it is usual to study brake squeal exclusively in time domain by TDA or frequency domain by CEA, other studies, such as [19], solved the problem by combining implicit and explicit method. Esgandari M. et. al. [19], investigated the effectiveness of hybrid co-simulation implicit/explicit FE model which combines frequency domain analysis and time domain analysis to predict unstable modes. The implicit model consists of a brake disc, a pair of brake pads, hub, carrier and calliper that is solved in frequency domain. The explicit model includes only a brake disc, the hub and brake pad friction materials which is solved in time domain. The pad nodes are defined as common region for transformation of boundary condition and loading from implicit model to explicit model. The hybrid model correlated by test results with Modal Assurance Criteria (MAC). The results show that unstable frequencies derived from hybrid model was identical to test results. In addition, one of the advantages of hybrid method is that it is capable of predicting instabilities in time domain in a shorter computation time compared to full explicit model.

However, the magnitude of positive real part is an unreliable indicator of squeal, TDA has rarely been applied owing to high computational cost [20]. For this purpose, Oberst [20], investigated the nonlinear behaviour of a pad-on-disc model as a simple FE model of a brake system, consisting of a rotating disc and stationary brake pad using CEA method as well as TDA. As it is shown in Figure 11 to Figure 13, the transient analysis is done through the time series and spectrogram of displacements and phase space plot for pad, disc and the contact patch of disc and pad for various values of friction coefficient to identify the dominant resonance and displacement of unstable modes.

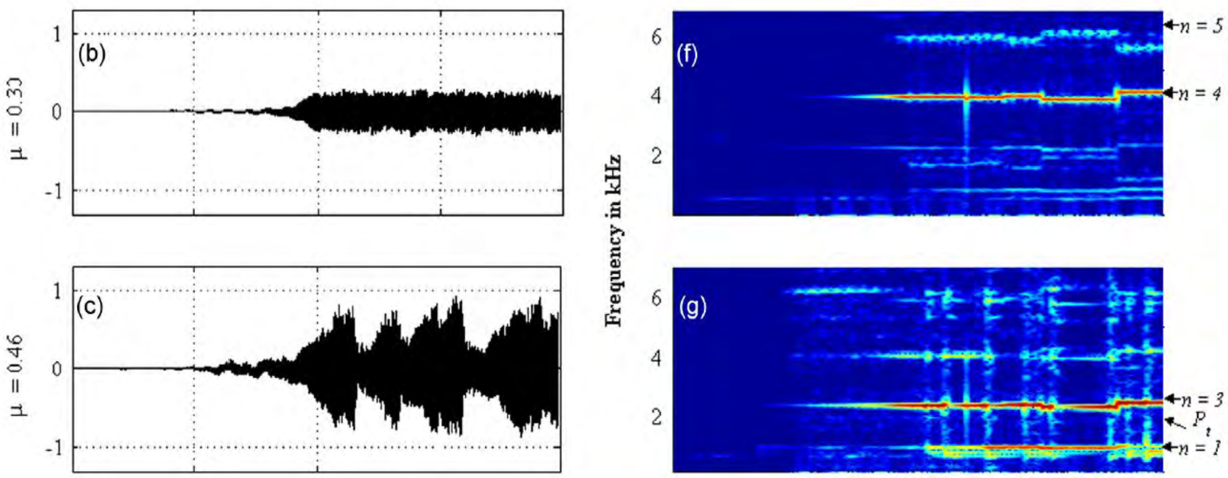


*Figure 11:Time series (left) and spectrograms (right) of Uz for μ=0.3, 0.46 [20].*

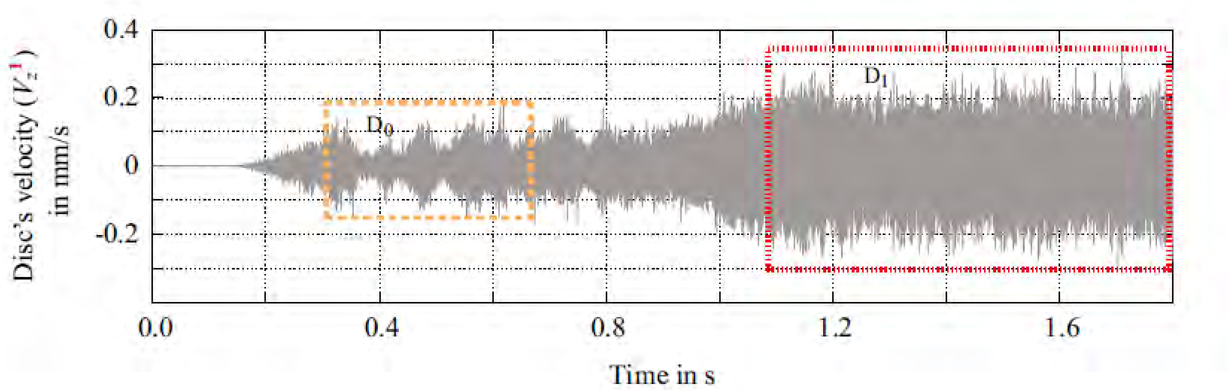


*Figure 12:Time series of out-of-plane velocity of the brake disc in D0 and D1 dynamic regimes [20].*

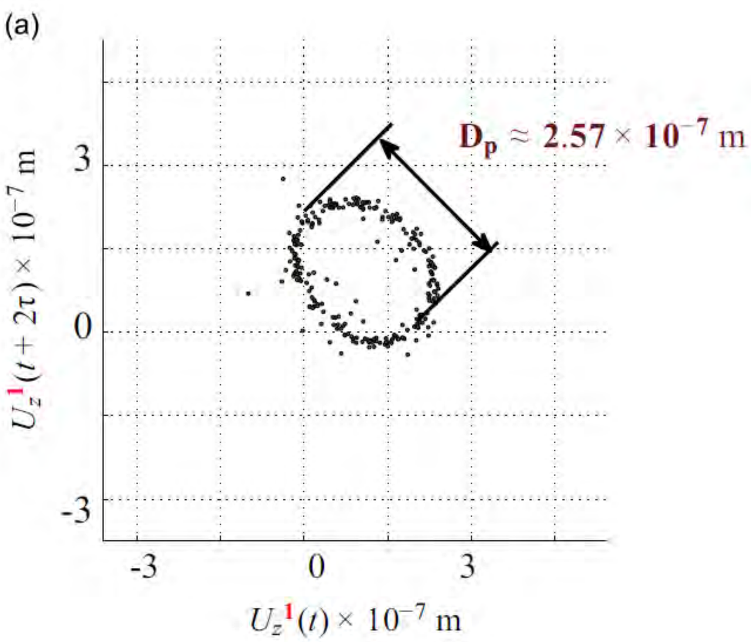


*Figure 13:Phase-space of out-of-plane displacement of the brake disc in z direction for the regime D0 [20].*

The comparison of CEA with TDA reveals that unstable modes in CEA may not be unstable in TDA. For instance, CEA cannot detect in-plane brake pad vibrations that may excite a dominant out-of-plane disc mode which is probable to generate squeal [20]. It is shown that the negative dissipated energy at frequencies of brake pad modes obtained by frequency response analysis of model subjected to harmonic contact pressure, can be used as complementary tool for CEA to predict brake squeal [20]. In addition, the nonlinear interaction of brake pad and disc are classified based on chaos types (intermittency, weak torus, turbulent), dynamic regime (steady state, periodic and quasi periodic) [20].
Regarding that TDA needs large computational efforts and the results of CEA suffer from an over prediction, a more reliable index is needed to be implemented along with CEA outputs and energy calculations, to identify the unstable modes. As a result, Brunetti et. al. [21], presented a new stability index called Modal Absorption Index (MAI), for prediction of unstable modes by using CEA results. As it can be seen in Figure 14, a lump mass model-connected by damper and springs- consist of one or more modules such that each module is composed of two mass for modeling. One mass in each module is in frictional contact with rigid sliders such that the contact formulation can model four different states which are sliding modes, reverse sliding, sticking and detachment. The MAI is defined as the ratio of total energy variations to time period of oscillation. The total energy variations are equal to the sum of contact exchanged power and dissipated power for each of complex and conjugate modes. The main advantage of the MAI is the possibility to evaluate the capability of each unstable mode to absorb energy from the contact interface. The results of new method are verified by CEA and TDA analysis for only two and three modules.

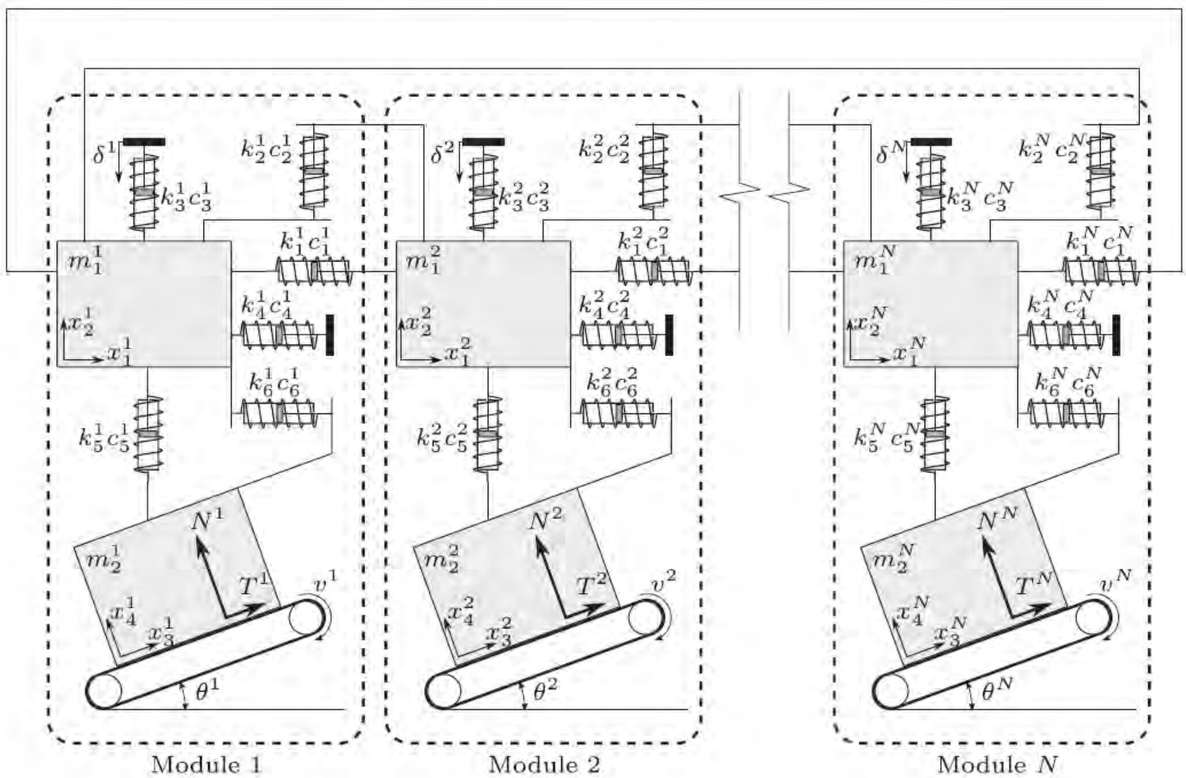


*Figure 14: Lumped spring-mass model with N modules [21].*

The present state-of-the-art survey shows that our understanding of the brake squeal is still insufficient. Moreover, there are several parameters such as pressure, friction and loss of contact that make brake squeal a significantly nonlinear problem. Since braking is basically time dependent, the steady state methods such as CEA cannot predict instability accurately but provide hints on possible instable modes. For example, the study done by Sinou [16] and Oberst [20] reveal over estimation of CEA method. In addition, other studies such as Grange [15] and Brunetti [21] proposed new methods for brake squeal prediction that can be used in specific condition and need more development to be applicable for detailed FE model. For instance, Grange [15] used a laboratory 2D explicit FE code, PLAST2, for pad and beam. Furthermore, Brunetti [21] implemented a spring-lumped mass model that needs development to work with commercial finite element codes. Esgandari [19] introduced a hybrid explicit/implicit FE model that is not as time consuming as pure explicit model and it can be used for detailed FE model. Moreover, Haung [14] reduced order method is able to investigate the sensitivity of eigenvalues to model parameters (i. e. coefficient of friction and lining stiffness) as well as performing design optimization. Although TDA can provide a better insight into brake squeal prediction, CEA can be useful in cases that the over prediction of results are sufficient to have an insight about brake squeal occurrence.

# 2 Mathematical formulation of brake vibration

## 2.1 Complex Eigenvalue Analysis (CEA) and Stability Condition

The classical method for instability prediction in squeal analysis can be done by calculation of unstable eigenfrequencies of linearized equation of motion around the equilibrium point. Thus, the first step is to find the equilibrium point ($u_0$) which can be done using Eq. (1).

$$K.u_0 = F_{ext} + F_{nl} \tag{1}$$

Where $F_{ext}$ is the external force such as the pressure on back plates and $F_{nl}$ is the nonlinear force at the contact interface of disc and pad.

Having the equilibrium point, the perturbation around the equilibrium point can be defined as follows.

$$u = u_{eq} + \bar{u} \tag{2}$$

Using Eq. (2), the classical equation of motion for vibration of a brake system with friction between pad and disc can be written as Eq. (3).

$$M \cdot \ddot{\bar{u}} + C \cdot \dot{\bar{u}} + K \cdot \bar{u} = F_{nl} + F_{ext} \tag{3}$$

Where *M, C* and *K* are the mass, damping and stiffness matrix, respectively.

Regarding that $F_{\mathrm{nl}} = K_{nl}.\bar{u}$, the equation of motion can be written as

$$M \cdot \ddot{\bar{u}} + C \cdot \dot{\bar{u}} + (K - K_{nl}) \cdot \bar{u} = F_{ext} \tag{4}$$

In Eq. (4), $K_{\mathrm{nl}}$ represents the nonlinear stiffness generated by friction force between pad and disc. Considering $\bar{u}(t) = A.e^{\lambda t}$ and $F_{ext} = 0$, the closed form of Eq. (4) can be written as

$$(M \cdot \lambda^2 + C \cdot \lambda + K - K_{\mathrm{nl}}) \cdot A = 0 \tag{5}$$

Where $\lambda$ is the complex eigenvalue and $A$ is the corresponding eigenvector. In fact, complex eigenvalue analysis is solving Eq. (5). The eigenvalues, can be expressed as $\lambda = \alpha \pm j\omega$, where $\sqrt{j} = -1$ , in which $\alpha$ is the real part of complex eigenvalue, nominate as $Re(\lambda)$, represents the stability of the system, and $\omega$ is the imaginary part of complex eigenvalue, nominated as $Im(\lambda)$, represents the mode frequency. The Eq. (6) expresses the generalized displacement of the system, $\bar{u}$.

$$\bar{u}(t) = exp(\alpha t) \cdot (\mathrm{A}_1 cos\, \omega t \pm \mathrm{A}_2\, sin\, \omega t) \tag{6}$$

Where $\mathrm{A}_1$ and $\mathrm{A}_2$ are constants calculated by initial conditions (e.g. $\bar{u}(0) = u_0$ and $\dot{\bar{u}}(0) = v_0$).

According to Eq. (6), if $\alpha$ is negative, $\bar{u}$ decreases exponentially with time and the system will be stable. Otherwise, when $\alpha$ is positive, $\bar{u}$ increases with time and the system is then unstable. Consequently, the unstable and stable region can be illustrated as depicted in Figure 15. It is essential to have all eigenvalues on the left hand side, to have a stable system. Although, all of the eigenvalues with positive real part represent the instability but the squeal may not happen in all of them.

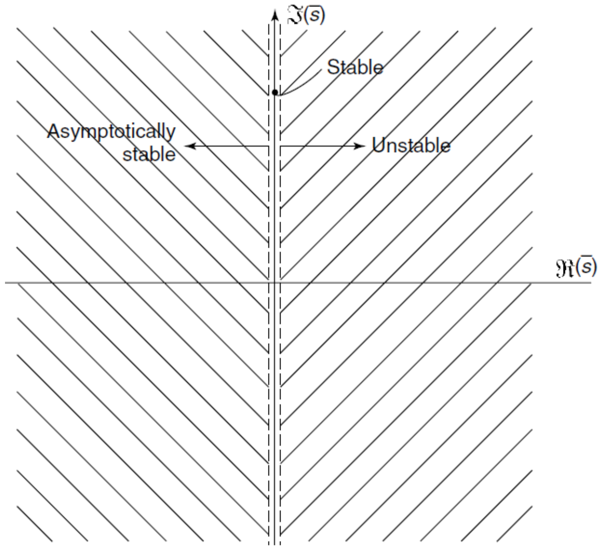


*Figure 15. Stability region in the complex plane [22]*

## 2.2 Negative Damping Ratio (NDR)

The complex eigenvalue corresponding to the k^th order of the complex mode can be expressed as Eq. (7)

$$\lambda_k = \alpha_k + j\omega_k \tag{7}$$

Where $\alpha_k$ is real part and $\omega_k$ is the imaginary part of the complex eigenvalue.
In theory, the modes with positive real part ($\alpha_k$) are unstable. However, a more realistic assessment is that only when the positive real part ($\alpha_k$) of the eigenvalue reaches a certain value, brake squeal is likely to occur [23]. Therefore, the potential of brake system to become unstable can be assessed by a so-called Negative Damping Ratio (NDR) that is defined as Eq. (8) [23]:

$$\zeta = \frac{-2\alpha_k}{|\omega_k|} \tag{8}$$

According to past experience, the eigenfrequencies corresponding to damping ratio lower than - 0.01 are considered as unstable [23]. It implies that the eigenfrequencies with low value of real part are not able to excite the brake system to make instability. Note that results presented later take –NDR and there the displayed results are with positive values only.

## 2.3 Transient Dynamic Analysis (TDA)

In this project, the transient analysis is performed using 'Dynamic, Explicit' in Abaqus/Explicit which uses central difference method as time integration. The differential equation for structural dynamics can be considered as

$$M \cdot \ddot{u}(t) + C \cdot \dot{u}(t) + K \cdot u(t) = P(t) \tag{9}$$

Such second order differential equation needs two initial conditions in order to be solved in a non-generic state. Let denote $u(0) = u_0$ and $\dot{u}(0) = v_0$ as initial conditions.
Considering $u(t_n) = u_n$, the foundation of central difference algorithm can be written as

$$\dot{u}_n = \frac{u_{n+1} - u_{n-1}}{2h} \tag{10}$$

In which $h$ is the time step. The Eq. (10) can be interpreted as the derivative at time $t$ is approximated as the slope of the line passing through the function at $t_{n-1}$ and $t_{n+1}$.

Using the Tylor series for the $u_{n+1}$ and $u_{n-1}$, we have

$$u_{n+1} = u_n + h.\dot{u}_n + \frac{h^2}{2}.\ddot{u}_n \tag{11}$$

$$u_{n-1} = u_n - h.\dot{u}_n + \frac{h^2}{2}.\ddot{u}_n \tag{12}$$

By adding Eq. (11) to Eq. (12) , $\ddot{u}_n$ can be derived as Eq. (13)

$$\ddot{u}_n = \frac{u_{n+1} - 2u_n + u_{n-1}}{h^2} \tag{13}$$

By substituting the first and second derivative in Eq. (9), the discrete governing equation can be written as

$$\left(\frac{1}{h^2}M + \frac{1}{2h}C\right).u_{n+1} = p_n - \left(K - \frac{2}{h^2}M\right).u_n - \left(\frac{1}{h^2}M - \frac{1}{2h}C\right)u_{n-1} \tag{14}$$

The central difference procedure for solving Eq. (14) can be shown in Table (1).

Central difference method is conditionally stable provided that the step size $h$ is smaller than the critical step size that can be calculated as below:

$l_{min}$= minimum element length

$$h_{cr} \leq \frac{l_{min}}{c} \quad c = \sqrt{\frac{E}{\rho}} \tag{15}$$

As a general rule, in explicit transient analyses, the small value for $h$ leads to the very time consuming calculation that limit generating detailed FE models in terms of time, file volume and software capability for handling very large result files. Therefore, there is a trade-off between the time step ($h$) and total simulation duration that should be managed in a correct way.

*Table 1. The central difference algorithm for explicit analysis*

| | |
|---|---|
| Step 0 | • Input mass (M), damping (C), stiffness (*K)*<br>• Calculate LU factorization [24] of $M$ such that *M=LU*<br>Note that LU factorization of matrix M means to define it as *M=LU* in which *L*: lower triangular matrix *U*:upper triangular matrix<br>• Input initial conditions $u_0$, $v_0$ and step size (h)<br>• Calculate the initial acceleration from Eq. *(16)*<br>$\ddot{u}_0 = M^{-1}.[p(0) - C \cdot \dot{u}_0 - K \cdot u_0]$ (16)<br>• Calculate LU factorization of $\frac{1}{h^2}M + \frac{1}{2h}C$ such that $\frac{1}{h^2}M + \frac{1}{2h}C = LU$ in which L: lower triangular matrix U:upper triangular matrix<br>• Calculate the starting displacement from Taylor series (Eq. (17))<br>$u_{-1} = u_0 - h.\dot{u}_0 + \frac{h^2}{2}\ddot{u}_0$ (17*)* |
| Step 1 | Loop for each time step, n=1……n |
| Step 2 | Calculate the right hand side (Eq. (18)) of the iteration<br>$RHS_n = p_n - \left(K - \frac{2}{h^2}M\right)u_n - \left(\frac{1}{h^2}M - \frac{1}{2h}C\right)u_{n-1}$ (18) |
| Step 3 | Solve for displacement at the next time step<br>$\left(\frac{1}{h^2}M + \frac{1}{2h}C\right).u_{n+1} = RHS_n$ (19) |
| Step 4 | Evaluate the set of velocity and acceleration from Eq. (10) and Eq. (13) |
| Step 5 | Set $n \rightarrow n + 1$ and continue to the next step |

## 2.4 Discrete Fourier Transform (DFT)

The Discrete Fourier Transform (DFT) converts the time domain signal to frequency domain. The transient dynamic analysis gives the results as a discrete values in the time intervals ($\Delta t$). Suppose that the signal is sampled at $N$ points in total time period $T$, the sampled times can be defined as $t_m = m.\Delta t$ ($m = 0,1, \ldots, N-1$). Thus, the corresponding frequency response function $u(f_n)$ at each frequency ($f_n$) can be derived as

$u(f_n) = \sum_{m=0}^{N-1} u(t_m)\exp(-j2\pi mn/N)$ *n=0, 2,….N-1* (20)

Indeed, $u(f_n)$ in Eq. (20) represent the frequency response function. At first, the explicit integration calculates the time response, $u(t_m)$, of brake in each time step. Then, DFT, Eq. (20), converts the time response to frequency response function (FRF). Consequently, the frequencies correspond to peaks of frequency response represent the frequencies in which the brake squeal is probable to happen. It is clear that TDA gives the candidate frequencies that are likely to generate brake squeal. So, it cannot give any information about the unstable mode shapes that can be useful for modal refinement.

## 2.5 Advantages and disadvantages of CEA and TDA

The advantage of CEA is that it gives mode shape of unstable eigenvalues. In other words, it gives unstable modes that can be used for vibration refinement in terms of geometrical revisions in brake system.

As mentioned earlier, CEA uses the linearized equation of motion around the equilibrium point, thus it can be considered as a limitation of this method because it is valid only at the vicinity of the equilibrium point. Furthermore, it is a steady state method that is not able to model the transient behavior and nonlinearities of squeal phenomena. In addition, it uses some assumptions such as a constant contact area between disc and pads and linear friction law. In fact, these assumptions are not compatible with reality.

As mentioned earlier, the transient analysis gives only the frequencies that may contribute to brake squeal together with the "dominating" displacement field. It does not give any information about the unstable mode shapes at a given frequency. Therefore, having knowledge about the brake squeal mode shape, is essential to do structural modification and prevent brake squeal as well. On the other hand, due to the time dependency of moving load on disc, it is better to use TDA for modeling the transient variables of model such as displacement, velocity and acceleration. The main drawback of the transient analysis is mainly long computational time and having convergence for each iteration. Time-domain simulations require huge data storage in case of using small time step. In this case, it is hard to handle it by commercial software such as ABAQUS. The computational time and data storage will be a serious problem, particularly, when the FRF up to high frequencies is desirable so that the time steps should be very small in the central difference integration to give the Discrete Fourier Transform (DFT) in high frequencies with reasonable accuracy.

# 3 Finite element modeling

## 3.1 FE mesh, material property and boundary conditions

The finite element model of this project consists of three FE models of a 17 inch Volvo Car brake system: two models (“complete” and “simplified model”) for CEA and a third model (simplified model only) for TDA. The first CEA model consists of the full brake assembly without suspension, see Figure 16 (left). The details of modeling of complete model are beyond the scope of this project. The second CEA model consists of so called, “simplified model” with the brake disc, a pair of brake pads together with their backplate, see Figure 16 (right). The unique TDA model is identical to the “simplified model” used in CEA.

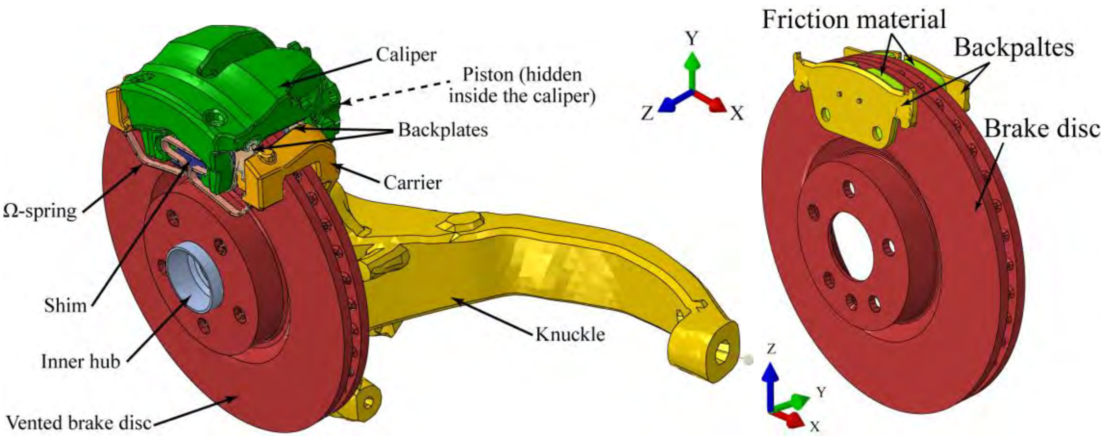


*Figure 16. Complete (left) and simplified (right) FE model of brake system for CEA*

The boundary conditions, see Table 2, for the simplified model, are adjusted according to the disc rotation around positive Y-axis in which the rotation push the back plate to forward direction so the translational DOF of edges that have contact with other parts-see Figure 17- in normal direction relative to each surface will be zero. For example, Figure 17 displays that the rotation pushes the backplate to move in x-direction. So, A10 and A6 go ahead in x-direction and touch the caliper edges. Thus, the DOF of A10 and A6 are zero in x-direction. The boundary condition for other surfaces- see Table 2- are determined similarly.

*Table 2. Boundary condition for inner and outer backplate for the simplified FE model.*

| Face | Boundary condition |
|---|---|
| A1,A2,A3,A4, A8, A9 | Uz=0 |
| A5,A6,A7,A10 | Ux=0 |

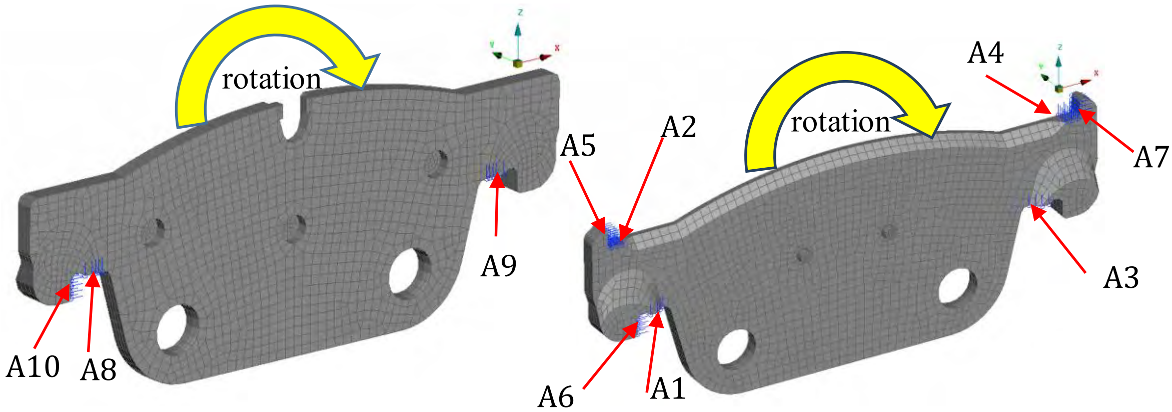


*Figure 17. Boundary condition for inner backplate (left) and outer backplate (right) for the simplified FE model.*

Table 3 shows the material properties of brake pads, disc and back plate. The disc and backplate are isotropic gray iron and steel respectively. The pad material is anisotropic, Figure 18 shows anisotropy of the brake pads is identical in 1 (x-direction) and 2 direction (y-direction) which mean that anisotropy plays a role in the brake pad thickness.

*Table 3: Anisotropic material properties of the brake pad.*

| | E1 [MPa] | E2 [MPa] | E3 [MPa] | ν12 | ν13 | ν23 | Density [kg/mm3] |
|---|---|---|---|---|---|---|---|
| Pad-inner | 11212.5 | 11212.5 | 2317.5 | 0.1 | 0.57 | 0.57 | 2.751E-6 |
| Pad-outer | 10863 | 10863 | 2482 | 0.12 | 0.51 | 0.51 | 2.659E-6 |
| Disc | 111613 | 111613 | 111613 | 0.25 | 0.25 | 0.25 | 7.2E-6 |
| Back plate | 206800 | 206800 | 206800 | 0.29 | 0.29 | 0.29 | 7.82E-6 |

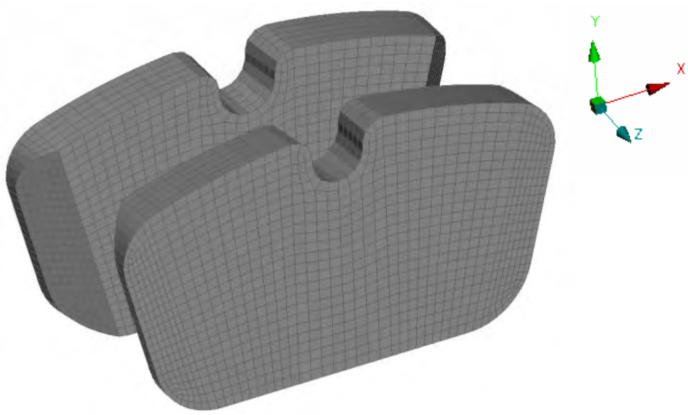


*Figure 18: The orientation of anisotropic pad material*

## 3.2 Simplified FE model for CEA

The FE model created in ANSA consists of 76886 hexa-elements and 1286 penta-elements which are linear. The average mesh size is about 3 mm with a maximum of 4 mm and a minimum of 1.4 mm. Compared to the complete brake assembly model where the hydraulic pressure is applied onto the piston, the pressure is here directly applied onto the two backplates of the brake pads. The friction interface is modelled by introducing contact surfaces between the brake disc and brake pad friction materials. Two types of contacts are used in the present model which are tie contact

and contact pair. The tie contact is introduced for the full attachment of backplate to the outer and inner brake pad nodes. The contact pair is used to model the friction interface between the brake pads and the brake disc which introduces nonlinearity in the model. The classical Coulomb law is applied for friction modeling.
To perform the complex eigenvalue analysis in Simulia-ABAQUS/standard, four main steps are defined as 1) pressure step, 2) disc rotation, 3) modal analysis and 4) complex eigenvalue analysis, respectively. Note that the pressure and rotation steps represent Eq. (1) in which linearization of equation of motion for deriving the equilibrium point will be done.

- Pressure step

This load step is a quasi-static step in which the pressure is a linear function of time, see Figure 19 (left). The pressure, green arrows in Figure 19 (right), is applied to both inner and outer back plates by the piston (on the inner side) and the calliper (on the outer side). In fact, the normal force between pad and disc will be generated in this step.

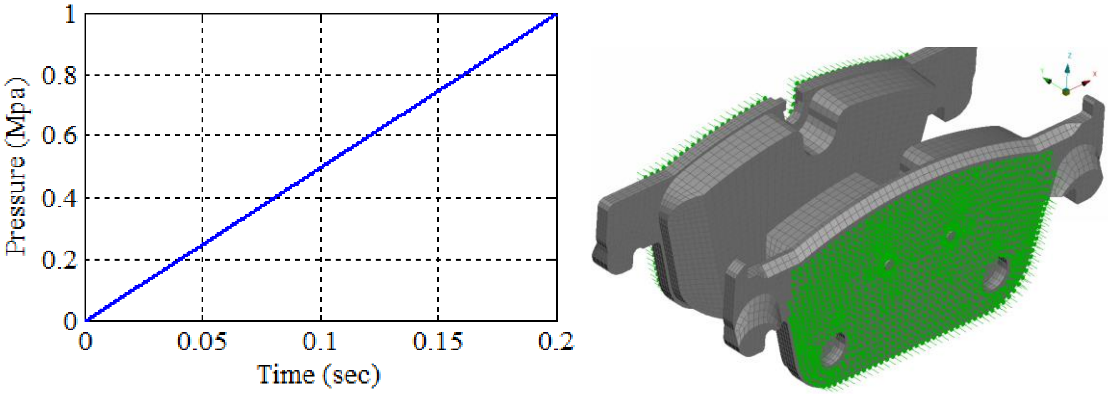


*Figure 19: Pressure variation (left) and surface of pressure application symbolized by green arrows (right) in pressure step.*

- Disc rotation step

This step is a quasi-static load step in which the displacement and rotation of the nodes are calculated by STATIC card in Abaqus/Standard. As indicated in Figure 20 (left) the angular velocity varies with time. Figure 20 (right) illustrates the application of the angular velocity in Y-direction (1 rad/s) which is exerted to all nodes on the brake disc using the MOTION card in Abaqus/Standard. In order to avoid convergence problems, the ramping-up of the rotation is nonlinear. This load step can be interpreted as the generation of friction force between pad and disc using the normal force that has already created in the pressure step.

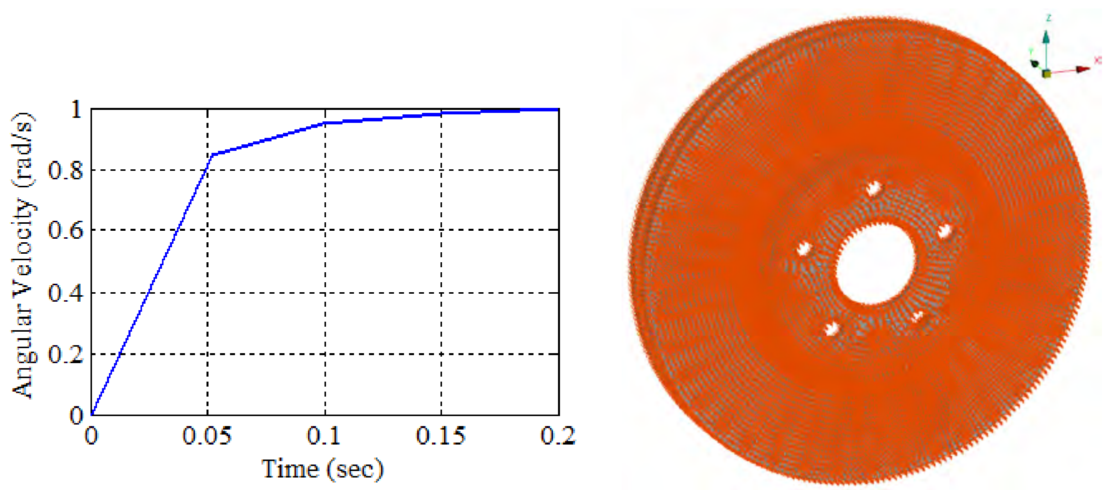


*Figure 20. Angular velocity of disc (left) and applied motion around Y-axis symbolized by orange arrows (right) in disc rotation step*

The pressure and disc rotation vary with respect to time in the mentioned load steps. The solution should be converged in each time step. The friction makes the problem highly nonlinear that may cause some problems in convergence. Thus, Due to avoid convergence problem, the pressure load step and disc rotation load step are performed separately.

- Real eigenvalue step

In this step, ABAQUS/standard solver uses the normal pressure and friction force calculated in the previous load steps and performs modal analysis. CEA linearizes the equation of motion around equilibrium point and finds the equilibrium point. Then, the real part of eigenvalues (natural frequencies) and related mode shapes of model derived through Lanczos method up to 10 kHz.

- Complex eigenvalue step

By use of the linearization method around equilibrium point, ABAQUS/standard solver calculates the complex part of eigenvalues and complex mode shapes in this step.

## 3.3 Complete FE model of brake assembly for CEA

The focus of the present work is not the modeling of the complete brake assembly neither its calibration against experimental testing. However, a short description is given in the following.

This complete brake assembly is modelled in commercial FE software and consists of 327071 solid elements (303873 tetra and 15524 hexa elements) and 4314 shell elements. All elements are quadratic. As depicted in Figure 16 (left), the brake assembly includes 1) brake disc, 2) inner and outer hub, 3) modified hub bearing (two torus instead of bearing balls), 4) hub ring, 5) two brake pads (lining friction material, backplates, adhesive shim, rubber shim and steel shim), 6) carrier, 7) caliper, 8) Ω-spring, 9) piston, 10) piston fastening clip, 11) knuckle and 12) ball bracket joints. The wheel suspension was chosen to not be modeled in this study.

The FE model is solved in the same way as the simplified FE model with modification of the boundary condition to account for the new components and mounting on the wheel suspension.

## 3.4 Simplified FE model for TDA

For this thesis, Transient Dynamic Analysis (TDA) that solves the equation of motion in time domain is used to identify the brake squeal. TDA can be implemented by DYNAMIC-EXPLICIT card in ABAQUS Standard in time domain. As a general rule, the time domain behaviour of braking condition can be considered in two phase, firstly transient phase in which the disc rotation and pressure vary linearly and secondly, steady state phase inwhich the disc rotation and pressure are constant. For this case study, the time response of simplified brake model is derived in time domain (0-0.42 s). Regarding that the brake squeal will happen in the steady state phase, the time period for transient phase is considered very short (0-0.02s). Therefore, the steady state load condition with constant the pressure and rotation will take place in time period (0.02-0.42 s). In this case study, the braking procedure is divided into three steps that can be described as below:

Load step 1: In the first load step, the disc rotation vary linearly up to 2 rad/s in time period (0-0.01 s) and pressure is considered as zero

Load step 2: The disc rotation is constant (2 rad/s) but pressure vary linearly up to 1.2 (Mpa) in time period (0.01-0.02 s). It is notable that the pressure is applied with a time delay of 0.01 (s). Regardless of constant rotation at this load step, the combination of rotation at the previous load step and pressure at this load step can represent the transient loading condition.

Load step 3: The disc rotation and pressure are constant in time period (0.02-0.42 s) and in addition an impulsive force, as depicted in Figure 21, is applied to one node on the brake disc for a very short time period. In fact, this impulsive force is applied to disc to excite the out of plane bending modes of disc.

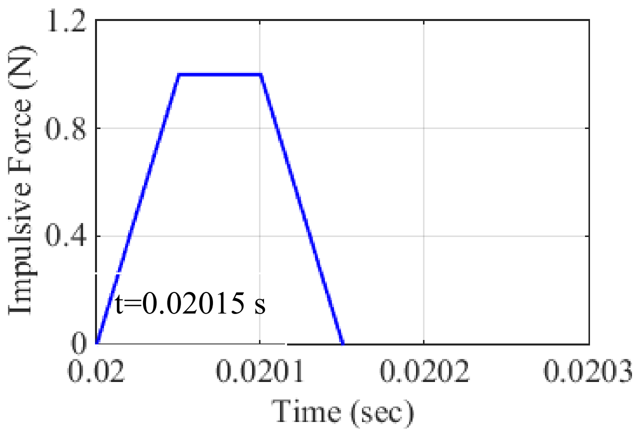


*Figure 21: Impulsive force applied on brake disc*

The superposition of the mentioned load steps can be shown as Figure 22 in which the disc rotation and pressure are stabilized in 2 (rad/s) and 1.2 (Mpa).

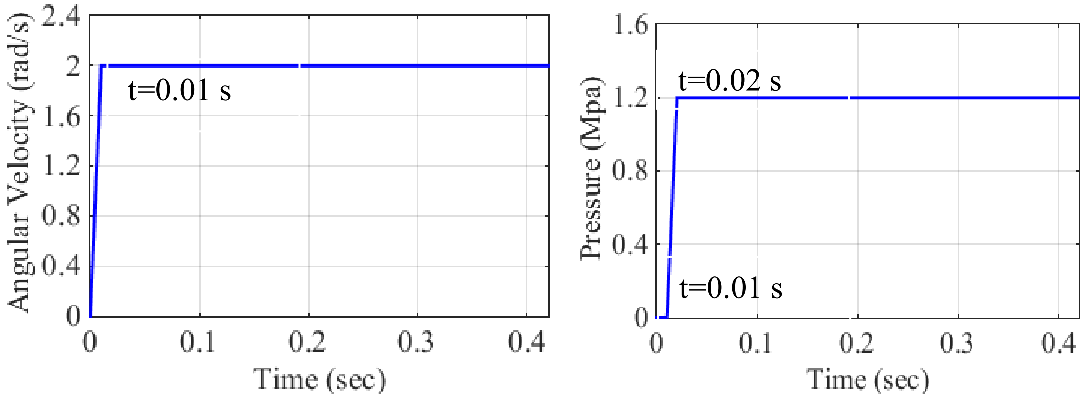


*Figure 22. The angular velocity (left) and pressure (right) in TDA analysis.*

The FE model is basically the same as the simplified CEA model. However, the method of applying the rotation of the brake disc was changed from MOTION in CEA to initial condition at the centre of disc (Figure 23) in TDA. As depicted in Figure 23, the initial condition is applied to the central point of disc in Y-axis and the nodes around the disc centre are coupled to central point of disc with MPC (Multi Point Constraint) coupling. The position of impulsive force in Y-direction is illustrated in Figure 23 as well. The boundary condition and material properties are the same as complex eigenvalue analysis.

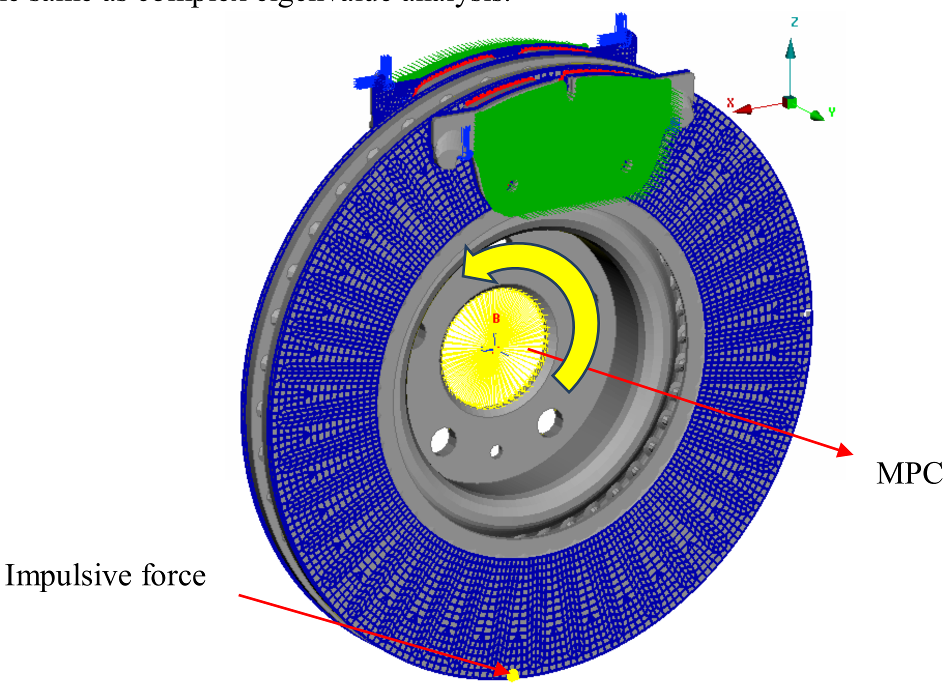


*Figure 23: The simplified brake model for TDA.*

## 3.5 Limitation of TDA

As a general rule, the time integration of equation of motion in transient explicit analysis takes a longer time than CEA method. In order to compare the TDA results with CEA, the time response should be converted to frequency response by using DFT (Discreet Fourier Transform). In signal processing problems, the signal aliasing is defined as phenomenon in which the reconstructed signal is completely different from the original signal. Thus, the sampling signal should be at least twice the maximum signal frequency based on the Nyquist criteria. Otherwise, the sampled signal and consequently the derived FRF (Frequency Response Function) are not accurate enough to be comparable to the CEA unstable frequencies. According to Nyquist criteria, having the frequency response range up to a certain $f$, the sample time without aliasing should be $1/(2 \times f)$. As a result, the 'TIME INTERVAL' in OUTPUT, HISTORY card in ABAQUS/Standard should be used to avoid signal aliasing in DFT process.

The time step for integration steps is taken by ABAQUS/Standard solver automatically which is around $\Delta t = 6\ \mathrm{e}^{-8}$. The sample time step for writing the results in the output file is done with $\Delta t = 5\ \mathrm{e}^{-5}$ that corresponds to the frequency range up to 10 000 Hz. The output file was adjusted to save only external nodes results for display purposes. It then creates only 52 GB output file. The simulation time was around 72 hours.

It is notable that this TDA model does not contain all brake components for example, calliper, piston and knuckle and all solid elements are linear. Thus, if TDA is done with complete brake model, it will take a very long time (maybe some months) for doing the simulation and the given results file will be too large to handle in ABAQUS/viewer.

# 4 Results

## 4.1 CEA results for “simplified model”

### 4.1.1 Eigenfrequencies

The complex eigenvalue analysis for $P$ = 10 bar and friction coefficient of 0.7 are shown in Table 4 for only the first six unstable modes. The first instability originated from the out-of-plane bending mode of the brake disc takes place at 1 931 Hz. In comparison with other modes, the disc out-of-plane bending mode (at 1 931 Hz) has higher positive real part ($\alpha = 98.9$). Although there are a lot of eigenvalues with positive real part, the real parts of most of them are very close to zero. So, it is necessary to have a criteria such as NDR (Negative Damping Ratio) that can filter the unstable modes with low value of positive real part.

*Table 4.The first six unstable mode of brake system in complex eigenvalue analysis. The bold font shows a possible strong instability.*

| Mode No. | Natural frequency (Hz) | Real part | Mode shape |
|---|---|---|---|
| 1 | 1931 | **98.9** | Disc out of plane bending |
| 2 | 2642 | 4.67E-12 | Back plate bending |
| 3 | 2999 | 2.4E-11 | Back plate bending |
| 4 | 3379 | 1.77E-11 | Back plate bending |
| 5 | 4223 | 2.6E-11 | Back plate bending |
| 6 | 4682 | 1.36E-11 | Disc out of plane bending and back plate bending |

As mentioned in Section 2.2, in order to have effective instability to excite the brake system to make a squeal, the NDR (Negative Damping Ratio) should be higher than a certain value. In other words, the ratio of real part of eigenvalue to imaginary part should be more than 0.01 [23]. Hence, the NDR is calculated for various pressure and friction values in the following section.

### 4.1.2 Negative Damping Ratio (NDR) for simplified FE model

Since the instability depends on various parameters such as pressure, friction and geometrical parameters, it is beneficial to calculate the NDR for various values of friction and pressure. Figure 24, shows a plot of NDR versus frequency for pressures 3, 6, 9 and 12 bars and frictions coefficient of 0.4, 0.5, 0.6 and 0.7. In fact, the variation of pressure and friction changes the stiffness term in Eq. (5) that result in various NDR values. As mentioned in Section 2.2, the modes that have NDR higher than 0.01 are likely to squeal. According to this criteria, the first frequency in which the squeal may happen is 1 931 Hz. It can be realized that the first instability at 1 931 Hz correspond to the case (I) P = 3 bar, $\mu = 0.7$ or case (II) P = 12 bar, $\mu = 0.7$. It implies that the high value of friction will increase the squeal probability.

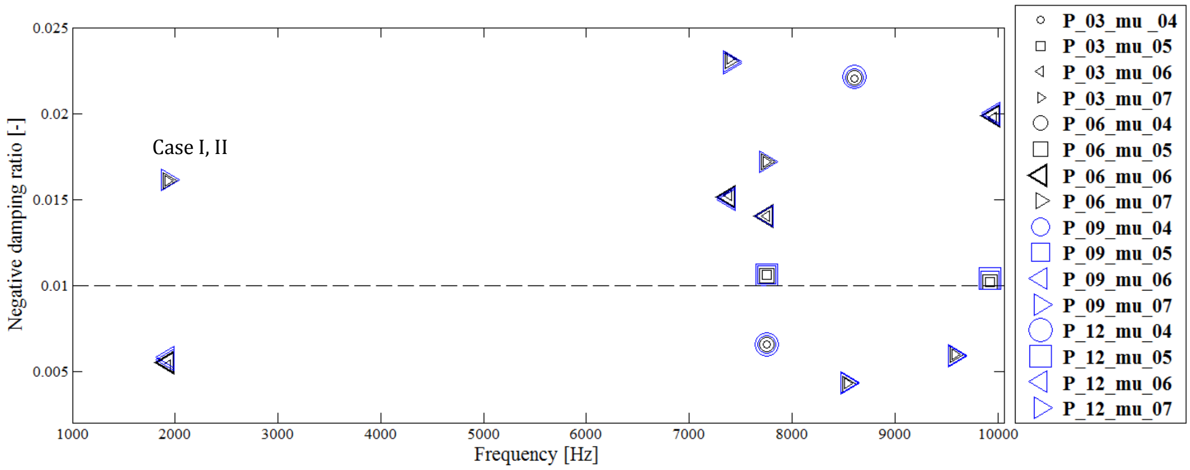


*Figure 24: Negative damping ratio for various amounts of pressure and friction*

According to Figure 24, all of the unstable modes for $P$ = 12 bar and $\mu$ = 0.7 can be presented in *Table 5*.

*Table 5. Unstable mode shapes with $P$ = 12 bar and $\mu$ = 0.7(The highlighted frequencies have NDR higher than critical value)*

| Mode | Natural frequency (Hz) | Mode shape | Max. NDR |
|---|---|---|---|
| 1 | **1931** | Out of plane bending mode of disc | **0.016** |
| 2 | **7387** | back plate bending coupled with out of plane mode of disc | **0.023** |
| 3 | **7754** | back plate torsion coupled with out of plane mode of disc | **0.017** |
| 4 | 8538 | Local mode of back plate torsion | 0.004 |
| 5 | **8552** | back plate bending coupled with out of plane mode of disc | **0.022** |
| 6 | 9584 | Local mode of back plate bending | 0.005 |
| 7 | **9962** | back plate torsion coupled with out of plane mode of disc | **0.020** |

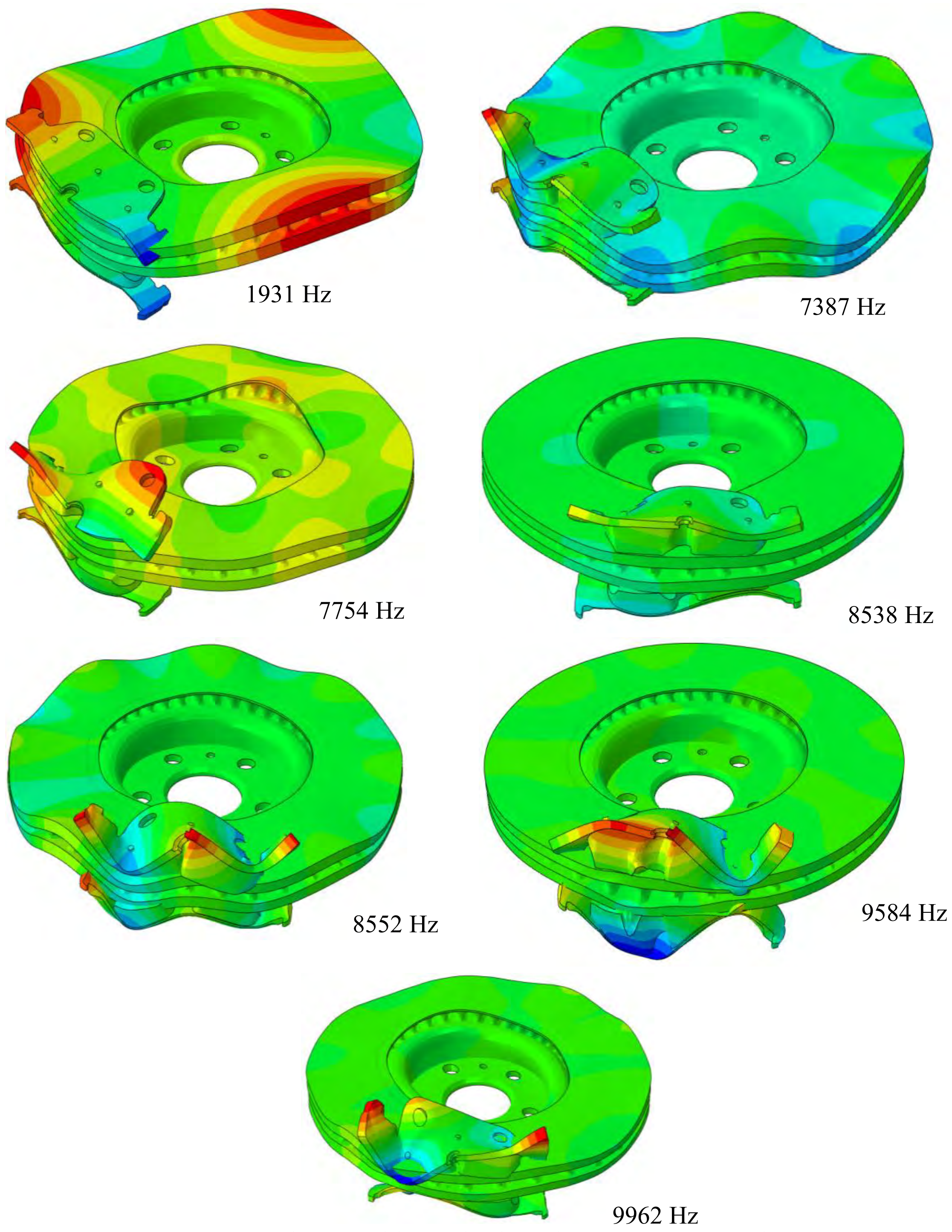

*Figure 25.The unstable modes based on NDR up to 10 kHz*

Figure 25 illustrates that regardless of first instability at 1 931 Hz, the other instabilities correspond to the coupled mode of disc and backplate. Regarding that the backplate structure has high stiffness and consequently high natural frequency, so the coupling of brake disc and backplate take place in relatively high frequency range. Figure 25 shows that coupling of out of plane bending mode of brake disc with backplate bending/torsion mode can make unstable modes with NDR higher than 0.01. As a result, it is probable to have squeal when the backplate bending or torsion modes excites out-of-plane bending mode of the brake disc.

The friction coefficient is one of the key parameters in brake squeal that constantly varies under braking as it is a function of brake disc temperature, brake pressure and brake disc rotation. As mentioned earlier, one of the mechanisms of brake squeal is mode coupling. In fact, coupling of modes occur when a pair of modes that have equal conjugate value for the real part. The imaginary parts of eigenvalues (that represent the mode frequency) join each other at coalescence point and their frequency become identical. Thus, for a certain value of friction coefficient, one mode with negative real part will be stable and another mode (with positive real part) will be unstable. While the friction coefficient reaches a specific value, the stable pair of mode may excite the unstable one and consequently this leads to squeal. It can be seen that for a certain value of friction, two curves intersect and join each other. The points A, B, C, D, E, F and G are called as coalescence frequency. It is notable that the frequencies correspond to A, B, C, D, E, F and G that are presented in Figure 26 to Figure 28 are the same as unstable modes that are presented in Table 5. So, it implies that the modes with NDR higher than 0.01 generate coalescence modes and make instability.

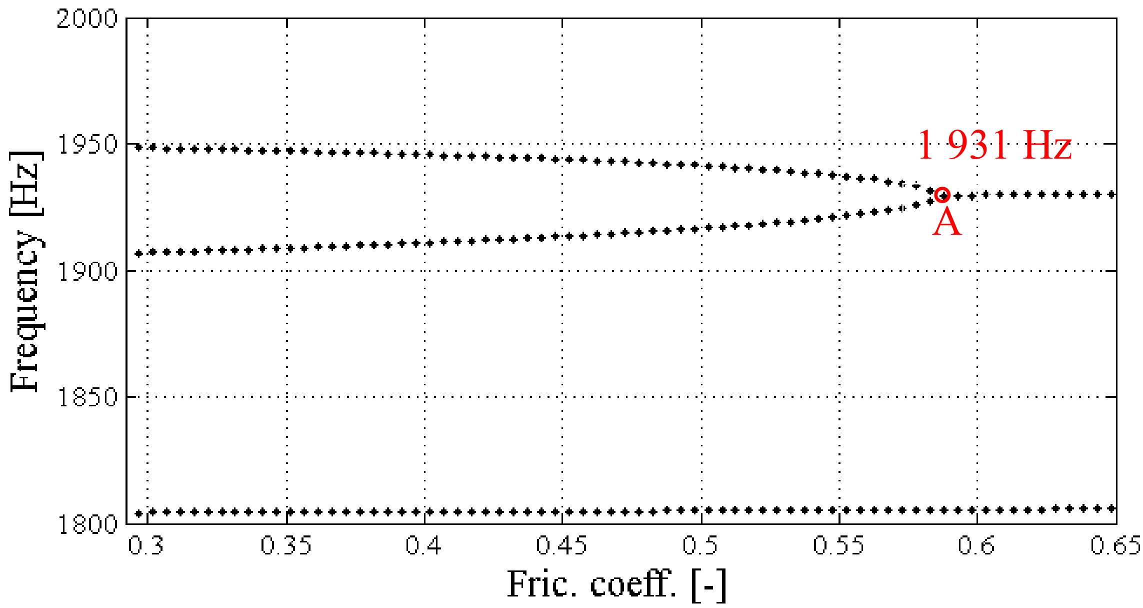


*Figure 26. The imaginary part of eigenfrequency at 1 931 Hz*

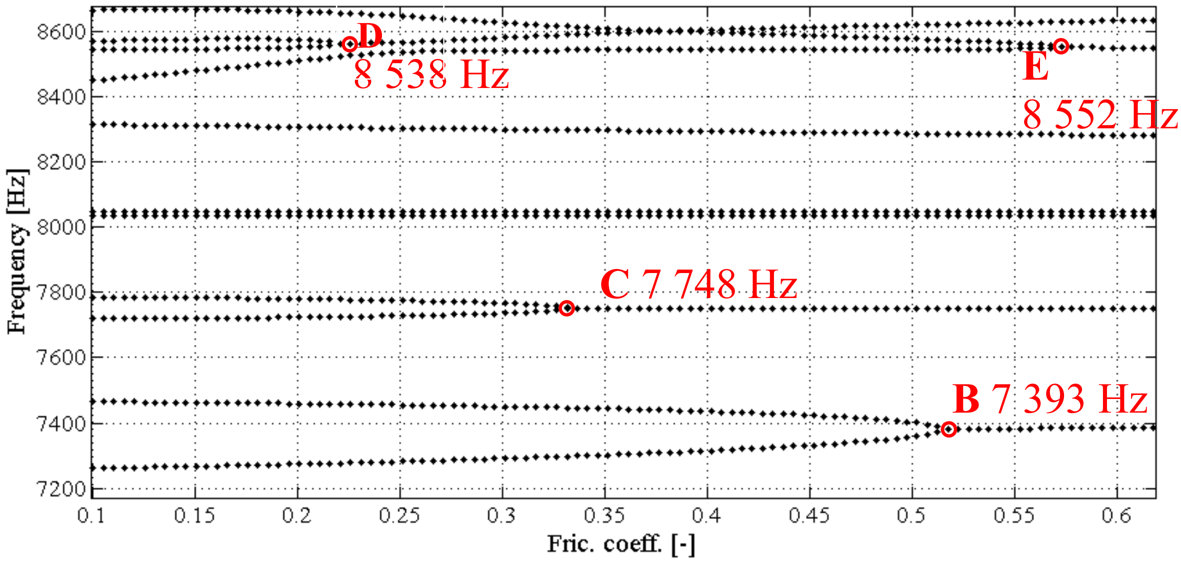


*Figure 27. The imaginary part of eigenfrequency at 7393, 7748, 8538 and 8552 Hz*

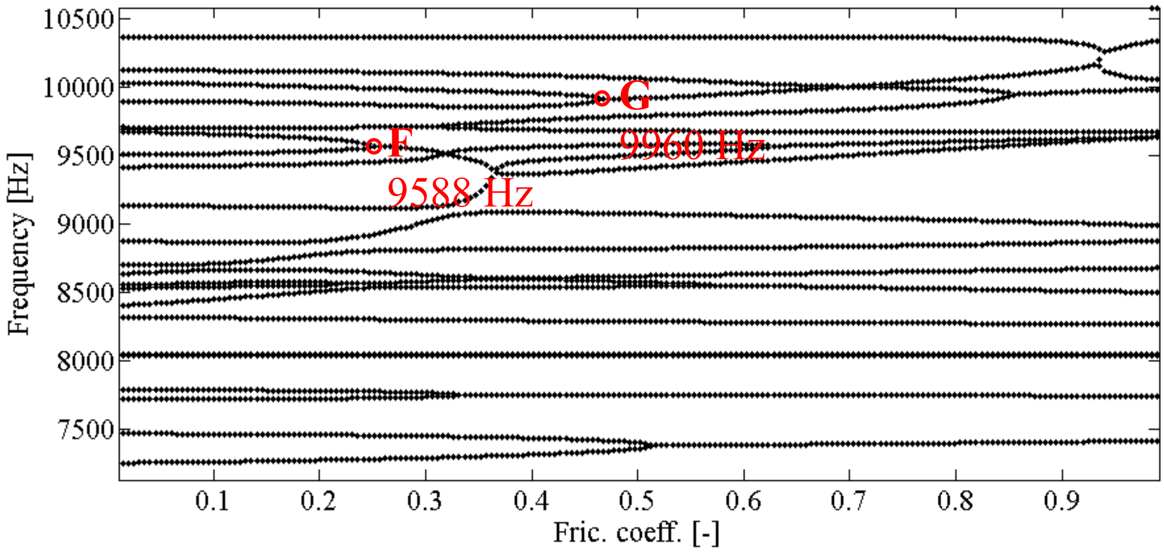


*Figure 28: The imaginary part of eigenfrequency at 9 588 and 9960 Hz*

Figure 29, shows real part of eigenfrequency versus friction coefficient in which the real parts curve split into two branches near the critical friction coefficient (also called as bifurcation-$\mu_{critical}$). Before the bifurcation points, the real part of eigenvalues are zero and the eigenvalues are purely imaginary with different frequency. But, after the bifurcation point, one branch goes for positive real part of eigenvalues and become unstable. The other branch goes toward the negative real part and become stable. Consequently, the stable mode will excite unstable mode and vice versa. This clearly shows that increasing the friction coefficient will increase the force and moments at the disc and pad interface which results in excitation of disc by backplate vibration. By the time, the frequency of two modes takes the same value (modes coalescence will happen), the mode which is unstable will then be excited by stable mode and vice versa (energy exchange), and consequently the brake squeal occurs.

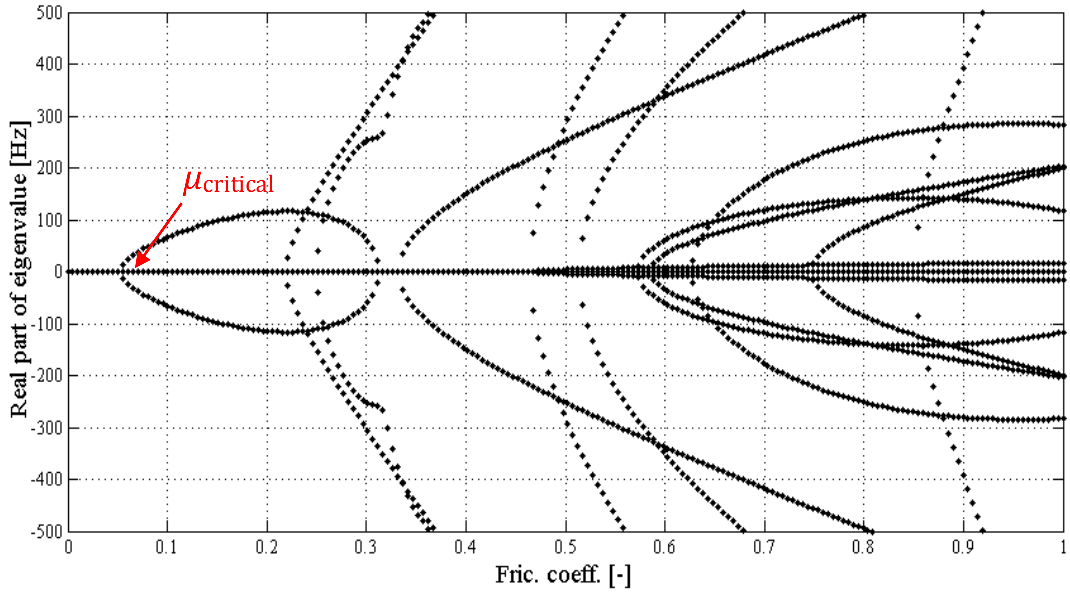


*Figure 29. Real part of eigenvalues versus friction coefficient.*

## 4.2 NDR of complete FE model

The experimental results from complete vehicle testing show that the observed squeal instabilities are around 1 700 Hz and 3 400 Hz when driving forward under “warm” condition brake disc assembly, i e above 100 °C. Therefore, the calibrated complete FE model of brake system, Figure 16 (left), is studied to find out the mentioned frequencies in simulation results. The details of modeling of complete model are presented in [25].

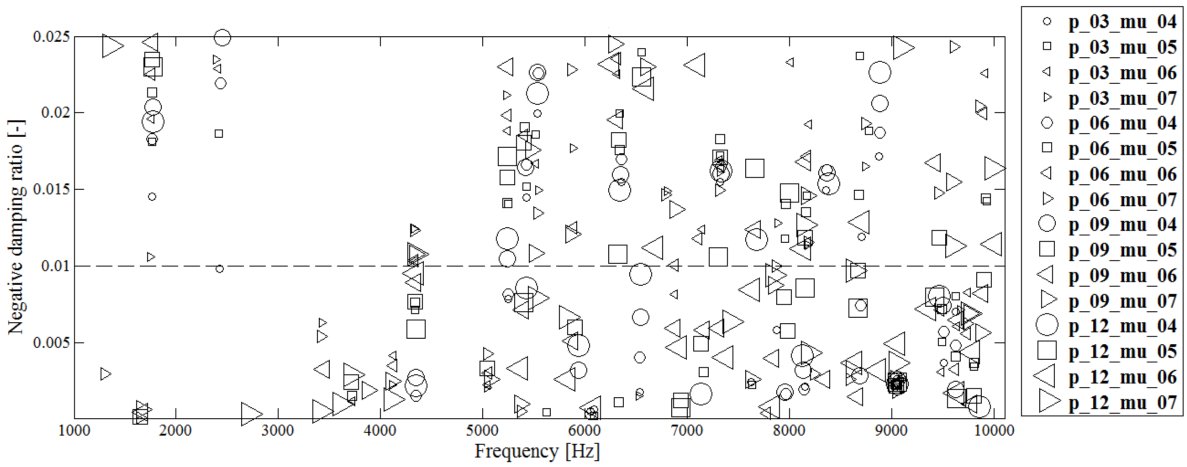


*Figure 30: NDR for complete FE model up to 10 kHz*

Compared to the experimental results, Figure 30 shows that CEA can predict the first instability (1766 Hz) so that NDR values related to 1766 Hz is higher than 0.01, but no instability with NDR higher than 0.01 can be seen at 3400 Hz. Although there are some instability around that frequency, their NDR is just as high as 0.006 that is much lower than the critical value. As mentioned earlier, this matter can be regarded as an evidence to prove that CEA along with critical value of NDR=0.01 cannot be a reliable method to predict brake squeal. So, it is useful to investigate the capability of TDA in the prediction of brake squeal. It should be noted that Figure 30 is generated for frequency up to 10 000 Hz. However, the model is assumed to accurately predict squeal up to about 5 000 Hz for the given mesh size considered in the study.

## 4.3 Results of TDA for simplified model

Using DYNAMIC-EXPLICIT in ABAQUS/Standard, the velocity-time response of all nodes are calculated. Then, the Discrete Fourier Transform (DFT) is used to derive the frequency response function of some nodes. It is noticed that the frequency response of the nodes may be different because each node can have different displacement in several modes. So, the derived frequency response depends on the position of node on disc. As an example, Figure 31 shows the velocity response of node on the friction surface of the brake disc for a time span of 0.2 s.

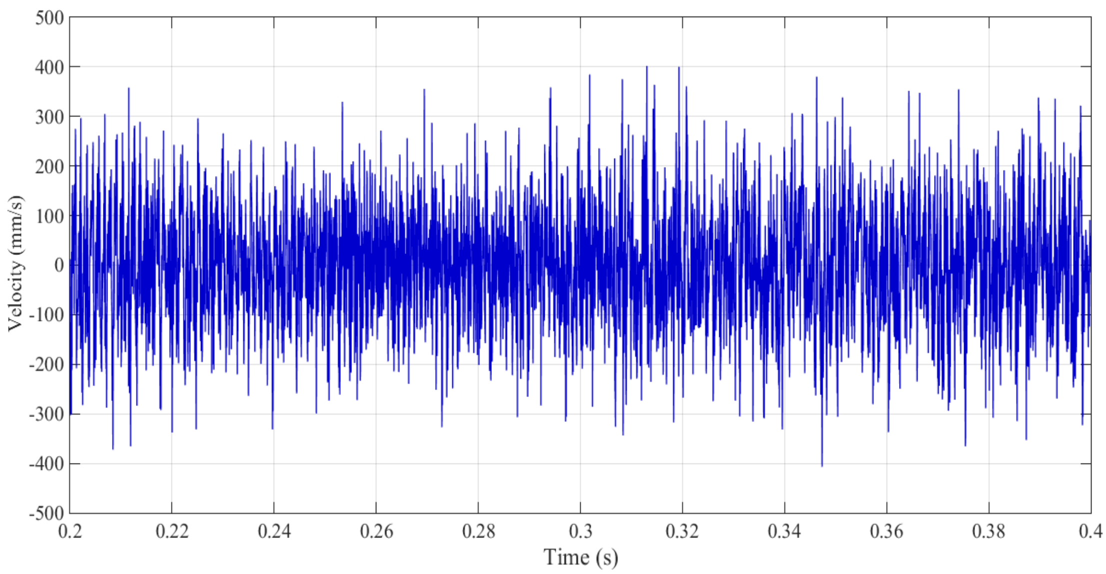


*Figure 31. Velocity of a node on the brake disc hat friction surface.*

In order to compare TDA with CEA, the time domain response should be converted into frequency response in which the peak frequencies correspond to the natural frequencies of the brake assembly. The peaks that have high amplitude are related to unstable frequencies, so they can be considered as candidates for brake squeal onset.

For this thesis, the frequency response of some nodes on the various part of disc are investigated, one of the mentioned frequency responses that can be comparable to CEA results with respect to brake squeal frequencies is shown in Figure 32. It can be seen that there is a peak at 1 972 Hz in frequency response that corresponds to the first instability mode (at 1 931 Hz) in CEA. In fact, this mode is the first out of plane bending mode of disc. In the vehicle, the observed frequency is at about 1 700 Hz.

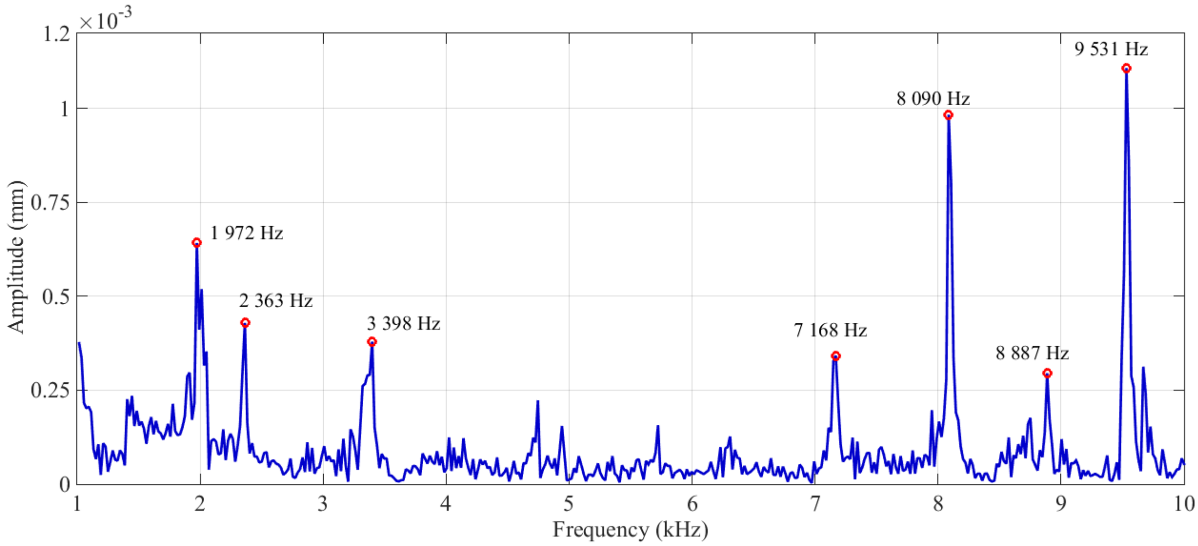


*Figure 32: DFT of a node on the brake disc hat friction surface with highlighted squealing peak frequencies.*

Figure 32 shows that the peaks at about 2 360 Hz, 3 400 Hz, 7 170 Hz and 8 890 Hz have relatively low amplitudes in comparison with other peaks. Some of the peaks cannot be seen in the NDR plot for simple model due to low level of NDR. This incompatibility shows that the complex eigenvalue analysis and real part of eigenvalue cannot predict instabilities accurately.

Table 6 summarizes the unstable modes for NDR values higher than critical value. It can be seen that the coalescence frequency at 8538 Hz and 9584 Hz correspond to NDR values lower than 0.01 but observed anyway in TDA. In addition, considering that NDR values higher than 0.01 as a criterion for brake squeal, Table 6 shows that CEA introduced five unstable modes but only two of them are predicted by the TDA.

On the other hand, considering the coalescence frequency as a criteria for brake squeal, four out of seven frequencies are extracted by TDA.

*Table 6. Comparison of TDA with CEA (the frequencies with NDR lower than 0.01 are highlighted)*

| Unstable mode | Squeal frequency-TDA (Hz) | Squeal frequency (Coalescence frequency)-CEA (Hz) | Max NDR |
|---|---|---|---|
| 1 | 1972 | 1931 | 0.016 |
| 2 | 7168 | 7393 | 0.023 |
| 3 | N/A | 7748 | 0.017 |
| 4 | 8887 | **8538** | **0.004** |
| 5 | N/A | 8605 | 0.022 |
| 6 | 9531 | **9584** | **0.005** |
| 7 | N/A | 9962 | 0.02 |

With regard to the mentioned criteria for brake squeal detection using NDR and coalescence frequencies, the over prediction of CEA is obvious. For example, Table 6 introduces an instability at 7 748 Hz but no such instability is observed in TDA. Therefore, it can be concluded that having the positive real part of eigenvalues as well as NDR higher than 0.01 are the essential condition for squeal, but it is not sufficient condition for brake squeal prediction in CEA.
The comparison of TDA results in Table 6 and Figure 33 shows that the mode shapes at 7 168 Hz and 8 887 Hz could correspond to the out of plane bending mode of brake disc and backplate at 7393 Hz and 8538 Hz in CEA simulation. This implies that the brake squeal is the consequence of coalescence of backplate and brake disc mode shapes.

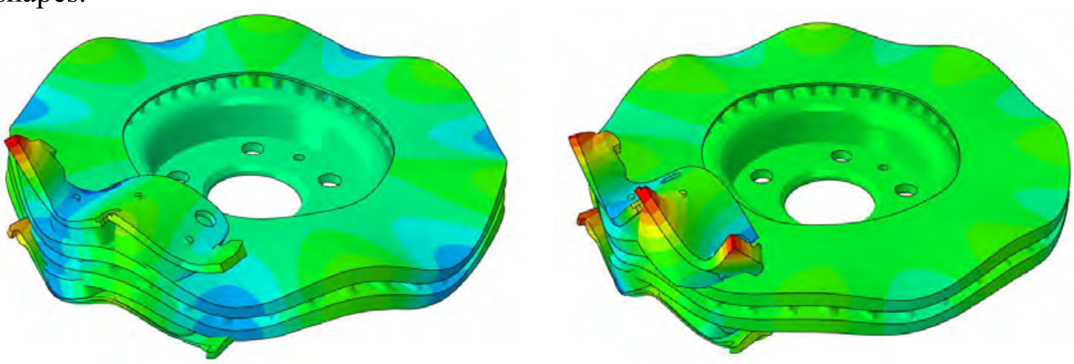

*Figure 33: Out of plane bending mode of brake disc and pad at 7393 Hz (left) and 8538 Hz (right)*

# 5 Concluding and remarks

Although the TDA analysis gives a better overview about the brake transient behaviour, it has some drawbacks that confine the utility of this approach to just simple and "small" FE models. For example, the duration of simulation and data storage are the main problems of this approach so that we cannot use this method for complete FE model as well as the FE models that contains a large amount of solid elements. This results in a limited application of this method to simplified FEM models. On the other hand, the CEA is not as problematic as TDA, even though it over estimate the unstable modes but it can be run in a short time. This item can be a main advantage of this model particularly in complete FE model of brake system. However, testing will be required to identify problematic frequencies and propose design modification to uncouple the modes. Consequently, it is essential to make some fundamental revisions to improve TDA to make it suitable for complete FE models.

# 6 Future work

Despite great progress in understanding and numerical simulation of brake squeal, there is still considerable progress to provide to achieve efficient numerical approaches for squeal prediction [26]. One of the most important drawbacks that needs to be investigated is the capability to reduce computational time and data storage [26]. Model reduction is a method that aims at reducing the DOF (Degree Of Freedom) of equation of large and complex system, can be introduced as good methodology for brake squeal problem. The main aim of all reduced finite elements methods is to condense the DOF of system through making a relationship between internal nodes and interface nodes that leads to reduce the computational time and data storage problems. One of the reduced FE method is Craig and Bampton (C&B) reduction method [26]. In fact, this method expresses the internal DOF of pad and disc as a function of generalized DOF and boundary DOF. Another reduction method, is Double Modal Synthesis (DMS) that is more powerful than C&B method to predict the bifurcation frequency [26].

There are some novel methods for solving the nonlinear differential equations that can be implemented for nonlinear dynamic problems especially brake squeal prediction. The Constrained Harmonic Balance Method (CHBM) [27], are introduced to calculate the bifurcation frequency and response of brake system (velocity versus displacement). This approach uses the Fourier series for approximating stationary nonlinear responses of self-excited systems subjected to flutter instabilities. Compared to TDA, this method can perform the computation in less time and data storage due to the fact that it condenses the equations to only nonlinear DOFs (Degree Of Freedom). The main problem of CHBM and DMS is that these methods are implemented for FE model containing only disc and pad, so further development are required to make use of this method for complete FE models.

Some researchers such as [19] used the co-simulation methods for transient analysis. In fact, co-simulation method uses explicit solver for part of model and implicit solver for other parts. Then, one part can be selected as an interface region (for example, brake pads [19]) to exchange data between explicit and implicit parts. The main advantage of this method is that it uses explicit method for only part of model, not for whole model. As a result, the calculation time can be reduced significantly.

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