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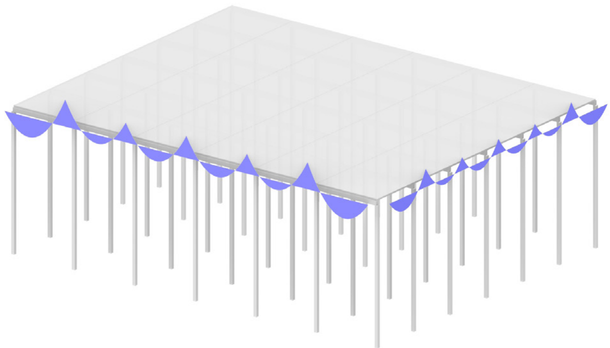

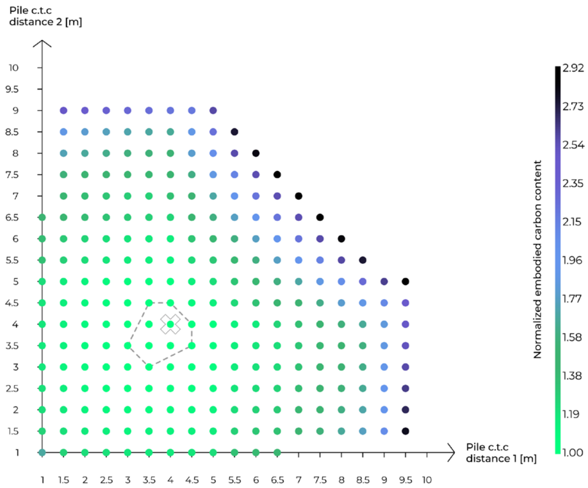


# Optimization of piled foundations

## An optimization study to reduce embodied carbon using parametric design tools

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING

Chalmers University of Technology
Gothenburg, Sweden 2023
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Cover: An illustration of a piled foundation with the corresponding slab- and grade beam moment distribution together with an optimal pile center-to-center distance graph.

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## Abstract

Today, approximately 11% of the global carbon dioxide emissions are connected to the construction industry, including manufacturing of materials. Due to a continuing global population growth, these values are expected to increase beyond the already high emission levels.

A way to reduce construction emissions is to opt for materials with low embodied carbon to replace conventional materials like steel and concrete. However, for building foundations, material strength and durability are critical factors and the freedom of selecting alternative materials is therefore more restricted. Instead, engineers need to strive to optimize the foundation structure to minimize the carbon emissions. Because of the great portion of material used in foundations, a large decrease of the total carbon emissions for a large variety of structures could be expected by material optimizing the foundation alone.

This thesis investigates on material optimization of piled foundations by utilizing computational tools and optimization algorithms, with the purpose to guide structural engineers to create more $CO_2$-efficient structures. More specifically, it explores how various structural parameters influence the need for material in a piled foundation.

The main results, also summarized in a guideline, includes suggestions on optimal pile center-to-center distances and slab thicknesses for different imposed loads and foundation types. The results also includes comparisons between one-way and two-way foundation slabs, concrete and steel piles, concrete classes as well as a comparison to common practice in the industry.

The thesis concludes that there are possibilities within the design process for engineers to significantly decrease the embodied carbon content of piled foundations. The most important aspects are to reduce the slab thickness, select the pile center-to-center distances to fully utilize the slab, followed by designing the piles accordingly. The carbon optimized design shows potential to save up to 52% of embodied carbon compared to mean values from common practice in the industry.

Keywords: optimization, evolutionary algorithms, computational tools, parametric design, sustainability, embodied carbon, piled foundation, design guideline, life cycle analysis, grasshopper

# Acknowledgements

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We are very grateful for our supervisor Carl Hoff for his invaluable guidance and encouragement throughout the project. We also want to give a special thank you to the people at Sweco Structures, for welcoming us and supplying us with necessary material and expertise. We would like to thank as well our supervisor and examiner Associate Professor Ignasi Fernandez, for his very helpful support and feedback during the thesis.

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Anna Stigenberg & Johan Lindqvist
Gothenburg, June 2023

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# 1

# Introduction

## 1.1 Background

Today, approximately 40% of the global carbon dioxide emissions are connected to the building sector. The emissions produced for the operation of buildings is estimated to 29% and the construction industry emissions, including manufacturing of materials, is estimated to 11%, see Figure 1.1.

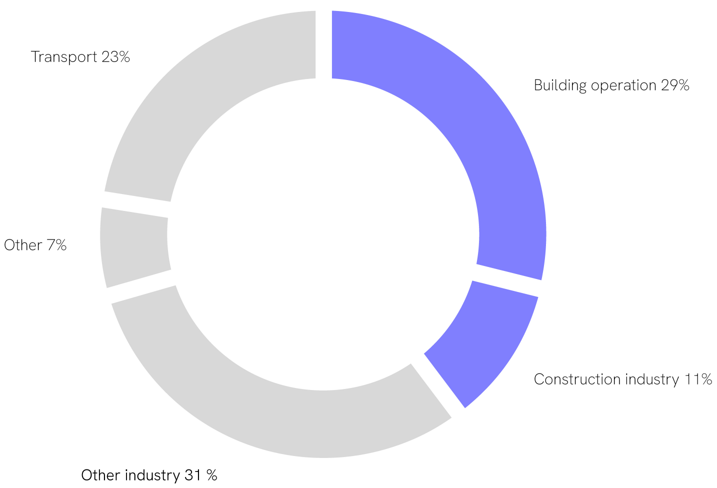



**Figure 1.1:** Building sector global emission share, 2018 (International Energy Agency, 2019).

Due to a continuing global population growth, as illustrated in Figure 1.2, these values are expected to increase. The global floor area has recently increased with more than 2.6% per year (Hart et al., 2021) and if continued in the same rate, the global floor area will be more than doubled before year 2050 compared to year 2010 when the measurements started.

At that same year, the European Union as one of the signatories for the Paris Agreement, has set the goal of being a carbon-neutral continent (Broer Rutger et al., 2022). To meet these ambitious goals, great reductions in carbon emissions from the building sector needs to be made, both on the operational side and the construction side.

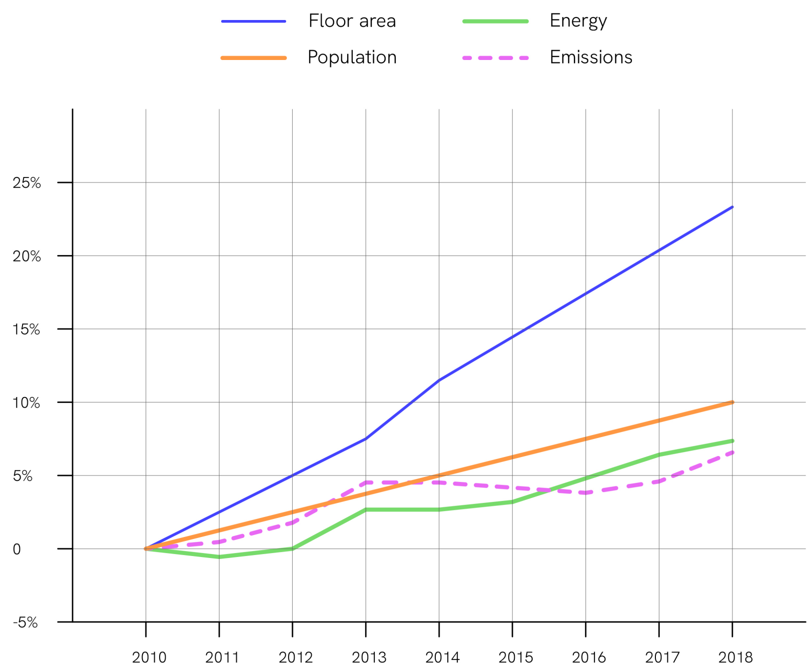



**Figure 1.2:** Changes in floor area, population and building sector energy and emissions globally 2010-2018 (International Energy Agency, 2019).

For structural engineers, one way trying to reduce the emissions has been to select the structural system with the minimum environmental impact. Timber frames are a popular alternative with the median embodied carbon equivalent of 200 $kgCO_2e/m^2$ compared to concrete or steel frames with the corresponding value of 350 - 380 $kgCO_2e/m^2$ (De Wolf et al., 2021). This implies that the carbon emissions of the frame of a building could be reduced by approximately 45% by using timber. However, building with timber could require additional materials for building functions and requirements on for example sound vibration and fire protection, making the emission savings potentially smaller.

On the contrary, the freedom of selecting materials when designing the foundation of a building is much more restricted. Other materials discovered today with smaller embodied carbon content cannot replace reinforced concrete or steel due to its strength and durability properties. Instead, engineers need to strive to optimize the foundation structure to minimize the carbon emissions. Because of the great portion of material used in foundations, a large decrease of the total carbon emissions for a large variety of structures could be achieved by optimizing the foundation design and minimizing its material consumption.

The use of reinforced concrete or steel piling is a widely and historically known practice for structural foundations. In cases where the soil conditions are poor, relative to the weight of the structure, piles can be used to support the structure on the bedrock or in deeper soil providing greater resistance. For certain larger buildings or bad soil conditions, piled foundations are necessary. However, piling is expensive in terms of production and installation emissions, as a single structure can require several hundreds or thousands of piles.

When comparing the relative carbon emissions from a heavy foundation project, including installation, operation, materials and transport, the major portion is coming from manufacturing the materials as these emissions can make up to 80% of the total project emissions (European Federation of Foundation Contractors, 2022), see Figure 1.3. This is because steel and cement (the binding material in concrete) are both very carbon intensive to produce and consequently, a large portion of the carbon dioxide emissions for a structure with a piled foundation is due to the material of the piled foundation alone.

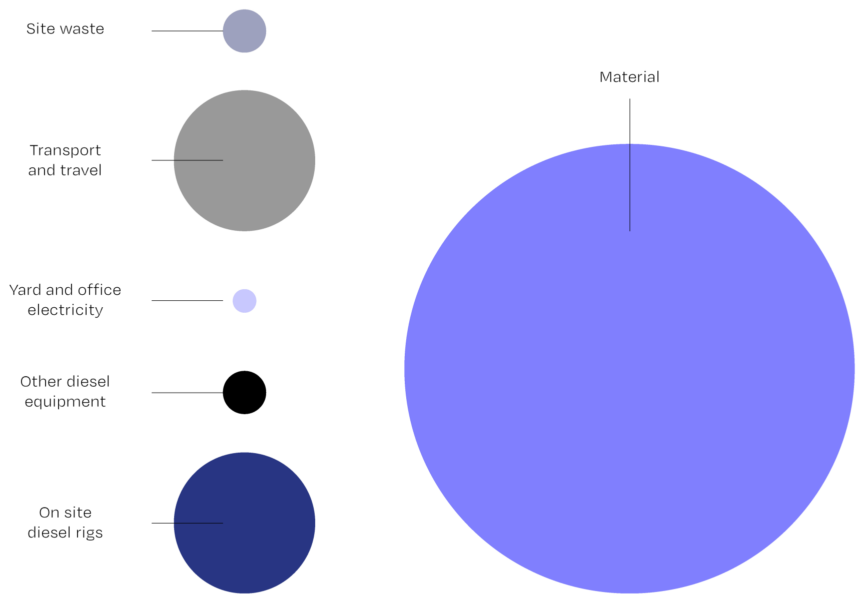



**Figure 1.3:** Relative size of a geotechnical company's carbon emissions on a heavy foundation project (Keller, 2022).

## 1.2 Aim

This thesis will research on material optimization of piled foundations by utilizing computational tools, with the purpose to guide structural engineers to create more $CO_2$ efficient structures.

The thesis will investigate how different structural parameters influence the need for material in piles and foundation slabs for piled foundations. Further, the different combinations of design parameters that represents an optimized design will be investigated. Consequently, the aim of this thesis is to evaluate in detail relevant design choices and study how those can be combined to find an optimal design regarding material use and especially embodied carbon content.

The expected outcome from the thesis is a thorough description of the previously mentioned design choices indicating the impact of such choices regarding the carbon dioxide emissions of piled foundations, as well as a guideline for design based on different initial project conditions. It is also expected a comparison of how much the $CO_2$ emissions can be reduced by using the $CO_2$-optimized design approach compared to common practice in the industry.

## 1.3 Objectives

The objectives of the thesis are:

- To gather information of relevant subjects by conducting a literature review.
- To develop a case study with predefined assumptions on soil profile, loads and other relevant properties to be able to generalize and compare the results.
- To create an automatic optimization tool in Grasshopper for Rhino 3D by scripting in the programming language *C#* and utilizing two different optimization engines. Decisions must be made regarding what parameters to include and how simplified the calculation model will be.
- To create a design guideline by summarizing and drawing conclusions from gathered results from the optimization tool.
- To create a form and interview structural engineers regarding their preliminary design choices to evaluate how much embodied carbon that possibly can be saved by utilizing the developed tool.

## 1.4 Methodology

Initially, a literature review is conducted to gather information on the subject. The literature review will primarily include research on parametric modelling, optimization theory, calculation procedures for piled foundations and life cycle analysis of materials (LCA).

In order to find an optimal combination of structural parameters, as well as to compare the effect of individual parameters, a large series of calculation iterations are needed. To be able to compute such a large amount of iterations, an automatic calculation tool will be developed. The calculation tool will for each set of input parameters calculate the material need and, via an evolutionary based optimization engine or an iteration engine, the inputs will be altered and the results iterated until a minimum value of embodied carbon content is found.

The parameters are differentiated between those defined by the limitations of a project, set parameters (such as ground conditions), and those adaptable by the designer, variable parameters (such as number of piles). The tool will be implemented in Grasshopper for Rhino 3D where custom code will be written using *C#* components. Stock components will be used to create geometry, in order to gain a visual understanding of the procedure. The plug-ins Wallacei and Colibri will be used as optimization and iteration engines.

A simplified version of the tool will initially be set up to test the functionality. Thereafter, work will be done continuously on developing the tool, including more parameters and detailing the calculations.

The tool will be used on a series of different initial conditions, i.e. set parameters, and structural typologies. The results, i.e. combinations of variable parameters with the corresponding embodied carbon content, will be enveloped and used to create a guideline represented by tables and graphs. The guideline will aid structural engineers to select the $CO_2$-optimal structural scheme in early stages of design without having to use the tool itself. An exploration of how individual parameters affect the overall material need will also be conducted.

Furthermore, a form will be sent out to interview structural engineers regarding their design approach for the developed case study, to be able to address how much the embodied carbon content can be reduced using the developed carbon driven design approach.

The design tool will include considerations to buildabilty, meaning intent to increase simplicity in construction as well as to save time and cost. This is done by following common industry practices, and ensures credible comparisons.

## 1.5 Scope and limitations

The scope of the thesis will include:

- $CO_2$-driven optimization of one-way and two-way reinforced concrete foundation slabs supported on piles. The optimization considers LCA stages A1-A3.

- The piles considered are low-displacement piles with hollow circular steel sections and displacement piles with square concrete sections.

- The soil profiles investigated consists of a layer of cohesive soil with an alternating depth to bedrock to explore different site specific scenarios.

- The set parameters in the optimization are:
  - Slab type
  - Foundation slab area
  - Pile type and bearing mechanism
  - Soil profile
  - Pile buckling length

  - Imposed loads
  - Grade beam shear reinforcement dimension
  - Angle of concrete compression strut
  - Crack width limitation

- The variable parameters in the optimization are:
  - Pile center-to-center distances in two transversal directions
  - Foundation slab thickness
  - Grade beam width and height
  - Slab- and grade beam concrete class
  - Grade beam bending reinforcement dimensions
  - Foundation slab reinforcement mesh dimensions

The limitations of the thesis are:

- The results are limited to areas with cohesive soils based on available ground investigation reports.

- The results are based on hand calculations and the current standards in Sweden. No finite element analyses are included.

- The time limit of the thesis requires motivated simplifications of the calculations.

- Due to the uncertain and fluctuating material and installation prices, cost analyses are excluded.

- Construction time and buildability will be considered but not prioritized.

- Assessment of each design's performance is based solely on the total embodied carbon content in the design. Considerations to negative effects in the local environment is neglected.

- The optimization will be done on a foundation floor that is column free, subjected to three simplified load cases, to generalize the results and to neglect the potential impact of columns that varies from project to project.

# 2
# Theory

In this chapter, the theory covering the topics investigated in the thesis are presented.

## 2.1 Parametric modelling

Parametric modelling is a way of working for structural engineers to more readily be able to explore a large variety of design options in order to optimize and create more efficient structural solutions. The workflow is based on parameterizing calculations or geometry inputs by specifying key parameters that can be altered within specified limits. The limited parameters and their corresponding combinations represents the design space of all possible solutions and by automating the procedure of combining parameters, the exploration may reveal high-performing solutions that could have been overlooked when using more time-consuming traditional methods.

The selected key parameters of a structure may be variables of a continuous range or discrete integers and could, for example, represent:

- The column grid spacing in two different transversal directions in an office building limited by feasible span lengths of the elements used.
- The cross-sectional dimensions of a concrete beam.
- The radius of an arch.
- The number of panels in a truss.

When automating the process of testing a parameter or a combination of parameters, the performance of the structure could be measured for each iteration as a performance metric, to be compared with the performances of all the combinations within the design space.

Typical performance metrics within structural engineering could, for example, be:

- Material volume and embodied carbon content of a structural system.
- Crack widths in a concrete beam.

- Reaction forces of an arch.
- Deflections of a truss.

Today, typical engineering workflows include parametric modelling to some extent with the use of spreadsheets where cells, as parameters, can be linked and updated by the user later in the calculation stages. However, with architects increasingly engaging tools like Grasshopper for Rhino 3D and Dynamo for Revit, creating models parametrically and directly in building design, there is a major benefit for structural engineers to do the same since these kinds of tools can connect directly to analysis softwares to produce performance metrics (Fang & Mueller, 2021). Figure 2.1 illustrates an example of parametric modelling where the design of a roof truss is evaluated based on its multi-criteria performance.

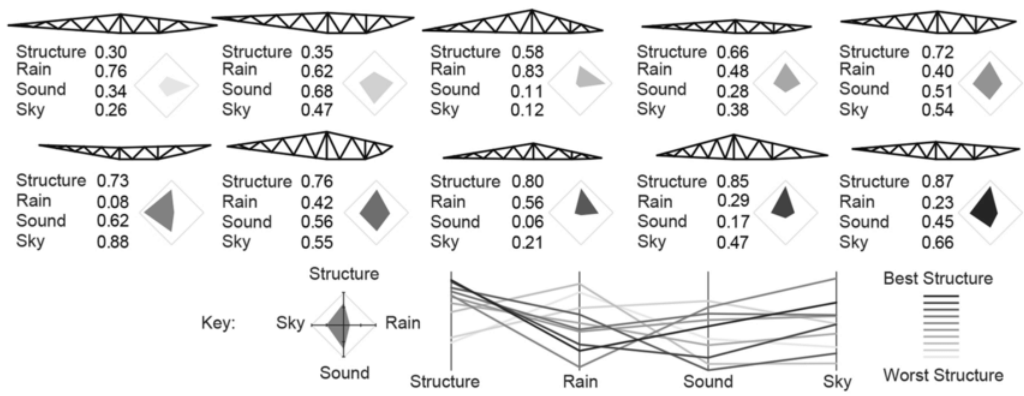



**Figure 2.1:** Parametric modelling of the shape of a roof truss. The overall performance is divided between the metrics structure, rain, sound and sky (Abdullah & Hassanpour, 2020).

Another advantage of utilizing parametric tools is that they offer opportunities to easily automate the procedure of testing parameter combinations by incorporating optimization engines that finds the best sets of parameters for specified performance metrics, further called objectives, of the optimization. The engines can include multiple objectives and can be further read about in Section 2.2.

## 2.2 Evolutionary optimization

The term optimization refers to finding the best possible solution to a problem defined by a given set of limitations or constraints (Coello Coello, 2006). Optimization can be done for a single objective, for example the cost of a design, or for multiple objectives, for example the deflections and the embodied carbon of a bridge.

When optimizing for a single objective, the aim is to find the best possible solution, called *the global optimum*. However, in most cases when optimization is carried out, there exists multiple objectives that are often conflicting. When two or more objectives exists, we call it *multi-objective optimization* which also requires different mathematical and algorithmic tools compared to optimization problems with only one objective.

To solve both single objective and multi-objective optimization problems, *evolutionary algorithms* which is a type of *metaheuristic algorithm*, classified in Figure 2.2, can be used and have become more common recently due to its advantages compared to other similar techniques (Coello Coello, 2006).

Evolutionary multi-objective optimization (EMO) is a type of evolutionary algorithm used in optimization processes. The aim of the process is to find a solution, defined by a unique combination of input variables (genes), that balances a set of objectives (Zitzler et al., n.d.).

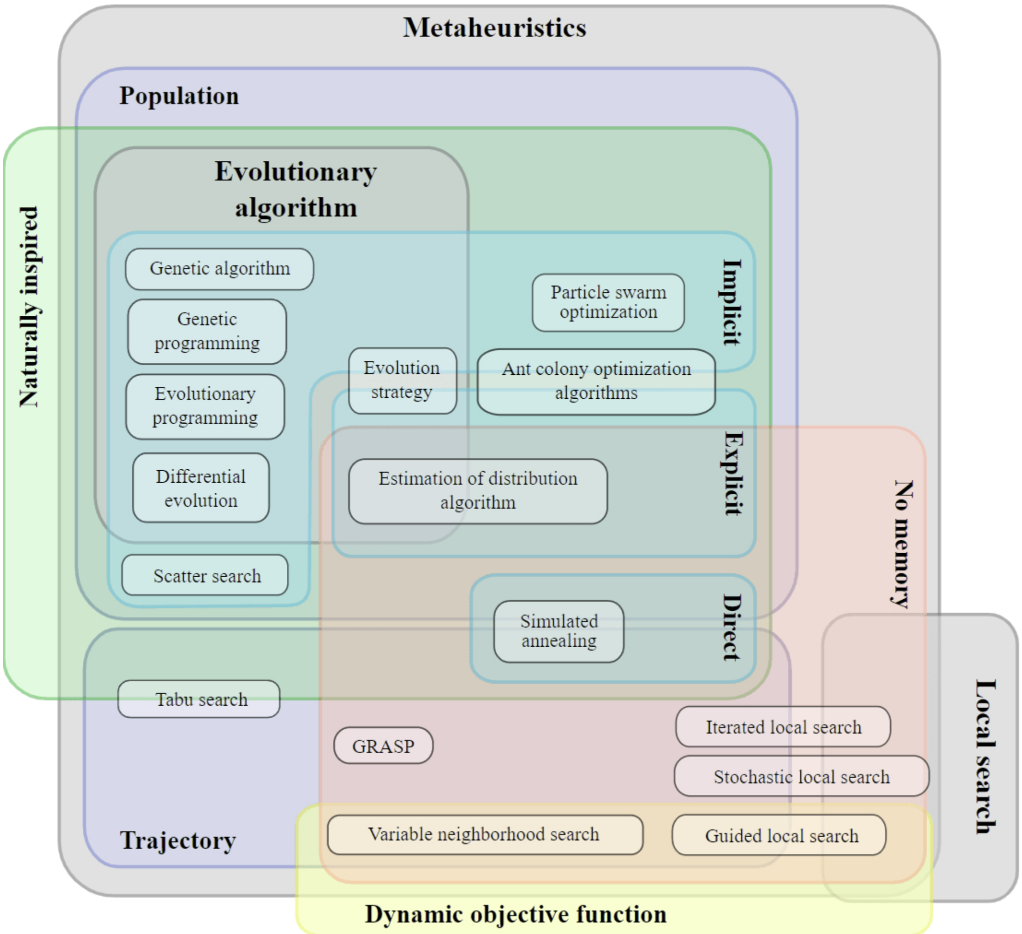



**Figure 2.2:** Classification of metaheuristic algorithms (Dréo, 2007).

An evolutionary algorithm mimics the process of natural evolution, where the populations of solutions are iterated by genetic operations. These genetic operations considers both selection and variation processes. Selection operations represent the process of evolution in which the population evolves through reproduction of current favourable candidates, so called natural selection. Variation operations instead represent the appearance of new candidates, through mutation and recombination.

In a multi-objective optimization problem, optimal solutions are referred to as *Pareto solutions*. There can exists multiple Pareto solutions in a single optimization problem. A Pareto solution is defined as a solution for which the increase of fitness for one objective causes a decrease of fitness of another.

Search mechanisms such as EMO are used in order to avoid too large data sets, requiring heavy computational power. Evolutionary multi-objective algorithms are proven as particularly robust and powerful search mechanisms, especially for problems including multiple conflicting objectives (Zitzler et al., n.d.).

### 2.2.1 Wallacei

Wallacei is an evolutionary multi-objective optimization engine developed for Grasshopper in Rhinoceros 3D. The main component of Wallacei is shown in Figure 2.3. The engine can be used to solve multi-objective optimization problems defined in Grasshopper. Wallacei makes the user in control of the optimization by visualizing the process and letting the user select and highlight desired solutions. Furthermore, the user is able to modify the evolutionary algorithm, by changing certain parameters such as mutation probability, population size and crossover probability. The necessary inputs for Wallacei are the objectives and the set of genes, or parameters, that defines the problem (Maki et al., 2022).

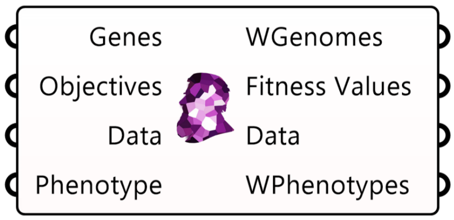



**Figure 2.3:** Wallacei optimization engine component in Grasshopper for Rhino 3D.

## 2.3 Foundation slab

A foundation slab is typically used to distribute the loads acting on the ground floor of a building. The slab may be designed to transfer the loads with a one-way or a two-way action. Additionally, the slab structure is often combined with other structural elements that connects- and transfers the loads acting on the superstructure to the substructure, typically via columns or walls to foundation pads, piles or beams. In contrast to other structural elements in a building, where the limitations of selecting materials is less restrictive, foundations are most often designed with reinforced concrete due to its strength and durability properties.

When selecting the structural scheme of a superstructure, consideration for the foundation design must be taken into account. If the design of the superstructure and the substructure is carried out in isolation, undesired internal stress changes may occur within the structure and especially within the foundation slab (Fleming et al., 2009).

### 2.3.1 Slab types and load transfer mechanisms

Two common foundation slab types for a piled foundation are:

- Continuous one-way slabs spanning over continuous beams, that in turn are spanning over the piles. The load is transferred in one direction to the transversal spanning grade beams.

- Continuous two-way slabs with pile caps that spans directly over the piles. The load is transferred in two perpendicular directions directly to the piles.

#### 2.3.1.1 One-way slab with grade beams

A one-way slab is supported on two edges, normally the edges closest to each other, and is designed with the purpose of transferring the load in only one direction, see Figure 2.4. Hence, the reinforcement is placed mainly in the span direction and the secondary transversal reinforcement is kept to a minimum. The slab may be designed as simply supported or continuous based on the boundary conditions and it is often designed without shear reinforcement. The supports of the slab, in this case simply supported or continuous grade beams, are transferring the loads to their respective supports, the piles. The beams are designed according to traditional methods with longitudinal bending reinforcement and with transversal shear reinforcement.

The one-way slab requires exact placements of the piles where any displacements during installation are undesired and troublesome since the grade beams needs to be cast straight. Different piling methods allow for different tolerances on placement where some methods are more beneficial than others. The achievable tolerances are related to the variations in the soil profile, obstacles in the ground, inclination of soil layers and accuracy of setting up the installation equipment and operator error (Fleming et al., 2009).

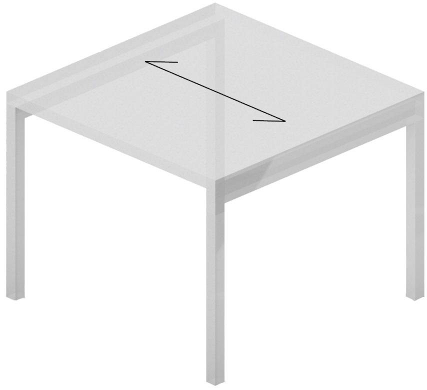


**Figure 2.4:** A one-way slab with grade beam on piles.

#### 2.3.1.2 Two-way slab with pile caps

A two-way slab is normally supported on four corner points, and is designed with the purpose of transferring the load in two transversal directions, see Figure 2.5. Hence, the reinforcement is placed in both directions where the amount is dependent on the moment distribution further described in Section 3.5.5.1. The slab may be designed as simply supported or continuous over the supports based on the boundary conditions and it is often designed without shear reinforcement for buildability reasons. Instead, drop panels or pile caps are often utilized to resist the shear peaks over the supports. The supports of the slab, in this case the piles, are transferring the loads to depths where the required resistance can be mobilized alternatively all the way down to bedrock.

Compared to a one-way slab, this method requires less accuracy of the pile placements where the pile caps often are designed accounting for a certain displacement tolerance during installation.

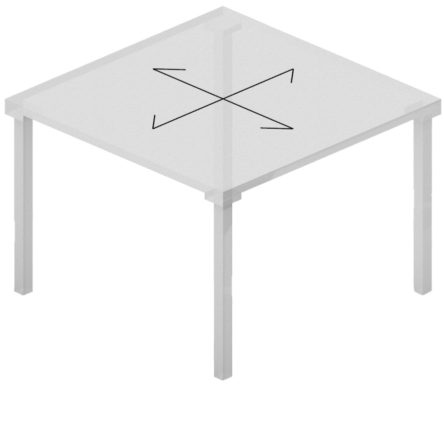


**Figure 2.5:** A two-way slab with pile caps on piles.

## 2.4 Piles

To use piles in construction is a widely and historically known practice. It is believed that the first piles were used about 4000 years ago by people living close to lake shores where food, water and transport were close to hand (Fleming et al., 2009). Today, piles are used in cases where the soil conditions are poor, relative to the weight of the structure, to support the structure on the bedrock or in deeper soil providing greater resistance.

### 2.4.1 Pile types and load transfer mechanisms

There are many different types of piles used in practice today. The industry is constantly evolving with new pile types and installation methods frequently entering the market to meet the stricter environmental requirements and commercial expectations. Generally, piles can be categorized by their installation method, 'driven' or 'bored', or with a more accurate categorization of 'displacement' or 'non-displacement' piles. These categorizations covers the majority of the different piles available on the market, however, the rapid development of pile types and installation methods which are a combination of the above mentioned makes it difficult to categorize them all. A high level summary of the categories of piles is illustrated in Figure 2.6.

The most common pile type and installation method used in Sweden today is driven solid concrete piles. In 2018, the pile type and method represented 60% of the total length of installed piles (Hercules Grundläggning, 2018).

The capacity of a pile or a pile group can be divided into the geotechnical capacity (GEO), referring to the capacity of the surrounding soil for individual piles and for the total pile group, and the structural capacity (STR), referring to the capacity of the pile itself.

#### 2.4.1.1 Displacement piles

A displacement pile is generally a steel or a precast concrete pile, driven into the ground. When the tip of the pile is being pushed into the ground, the soil is displaced mainly in the radial direction and slightly in the vertical direction. The displacement of the soil is sometimes a drawback for the method, however when a pile is driven into frictional soils, an effect of compaction occurs which could be beneficial. Steel displacement piles are often referred to as 'low displacement piles', as these have the advantage of reducing the soil movements while being driven due to the smaller cross sectional area. Low displacement piles are recommended to use if the piles needs to be driven deep through frictional soil, if clay heave might be a problem or if the piles in a pile group needs to be positioned close to each other (Fleming et al., 2009).

#### 2.4.1.2 Non-displacement piles

A non-displacement pile is generally bored with very small to no soil displacements. These can be of steel or cast in place concrete. With this method, the potential drawbacks of the displacement piles are eliminated as well as the beneficial compaction. A drawback of bored concrete piles is the production of spoil, as the soil is excavated from the ground before casting, and the method is thus not recommended in areas with contaminated soil (Fleming et al., 2009).

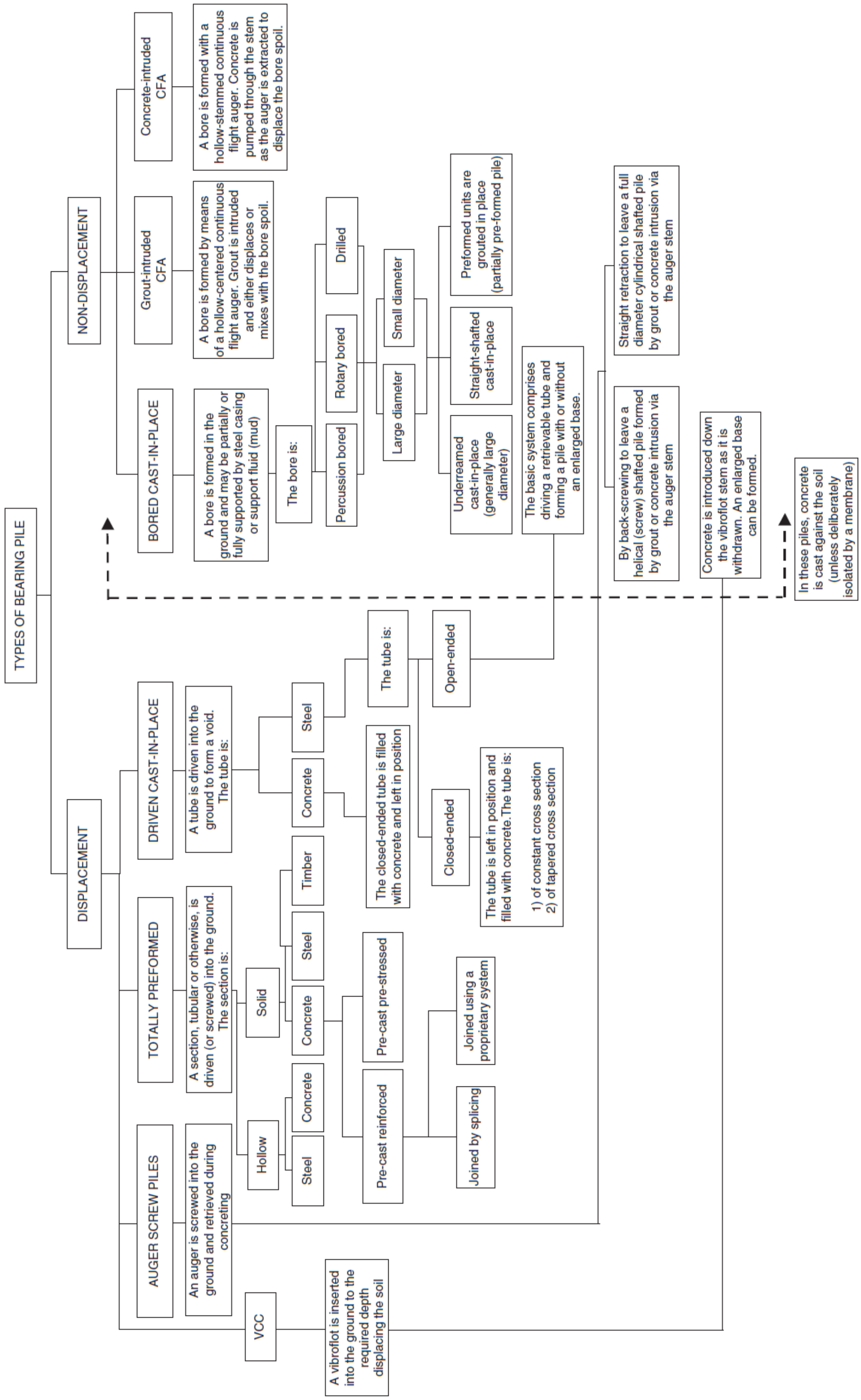



**Figure 2.6:** Categorization of pile types (Fleming et al., 2009).

### 2.4.2 Structural capacity

When verifying the capacity of the pile itself, all the parts of the pile needs to be included. That are the main part of the pile, eventual connections and details. The calculations should, apart from the axial loads, include an eccentricity of the load effects, initial imperfections and transversal loads. Additionally, alteration of material properties as an effect of the chosen installation method should be included. Depending on the load effects present, the pile needs to have sufficient capacity in compression, tension and bending as well as to not exceed limitations of crack widths. If dynamic loads are governing, the pile also needs fulfil the requirements on fatigue (Hercules Grundläggning, 2018).

The structural capacity and behaviour of the pile is dependent on the support of the surrounding soil. Typically, pile buckling and crack width limitation are governing when dimensioning a pile. Consequently, the average soil resistance within the top elastic buckling region of the pile is critical for the capacity. The elastic buckling length is estimated through calculations in the dimensioning procedure, and it is dependent on the geometry and material properties of the pile as well as the soil shear resistance along the pile length (Pålkommissionen, 1998).

When calculating the load effects on a pile, consideration must be taken to potential negative friction. Negative friction is the effect, in cases when a pile settles, where the above ground reconsolidates causing an added load on the pile and pushes the pile downwards. This added load should be combined with other loads acting on the pile, except for the variable loads as they will momentarily reduce the negative friction effect. The required structural capacity for piles subjected to negative friction is not determined in a section at the top of the pile, otherwise typical, but instead at the point of the neutral layer. The neutral layer is the point along the pile where the relative displacements between the pile and soil is zero, which is where the negative friction will have caused the greatest effect, typically at a depth of two thirds of the pile length (Per Eriksson et al., n.d.).

### 2.4.3 Geotechnical capacity

The load transfer mechanisms from a pile to the ground can be divided into 'end bearing' and 'shaft bearing'. The total capacity of a pile is the sum of the mechanisms, however when one of the mechanisms is much greater than the other, a pile can be categorized as either an 'end bearing pile' or a 'shaft bearing pile', both visualized in Figure 2.7. The ground conditions where the pile is installed determines the bearing types possible (Fleming et al., 2009).

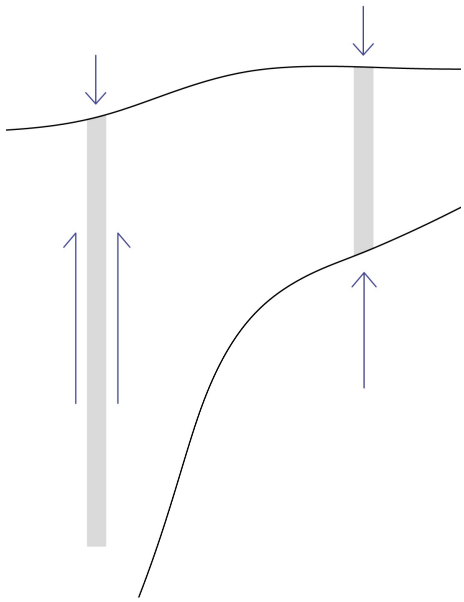


**Figure 2.7:** Shaft and end resistance of a pile.

The total geotechnical capacity of a pile, Q, under axial load is calculated as (Fleming et al., 2009):

$$Q = Q_b + Q_s = A_b \cdot q_b + A_s \cdot \overline{\tau_s} \tag{2.1}$$

where:

| | |
|---|---|
| $A_b$ | is the area of the pile base |
| $q_b$ | is the end bearing pressure |
| $A_s$ | is the area of the pile shaft |
| $\overline{\tau_s}$ | is the average shear stress along the pile shaft |

The axial capacity can be calculated either for individual piles or for a pile group. For shaft bearing piles placed close together, the soil between the piles may move with the piles, thus acting as one large pile. This is referred to as block failure. The axial capacity must therefore be calculated for both the single pile and the pile group. For end bearing piles, the axial capacity of the group is the sum of the axial capacities for the single piles (Fleming et al., 2009).

Steel piles typically have a smaller circumference than concrete piles due to the greater structural strength of the steel. Consequently, shaft bearing piles made of steel are required of large lengths to compensate for the smaller pile shaft area. To avoid this, a common method is to attach thin elements with a large area to the end of the pile. These can be referred to as 'wings', and can reduce the required length of the pile (Shojaei et al., 2021).

To verify the geotechnical capacity, it is common practice to first calculate and then test the piles on site to be able to maximize the capacity. In principle, it is only possible to verify the geotechnical capacity of a pile through calculations solely for shaft bearing piles in cohesive soils (clay). When dimensioning piles in frictional soils through calculations without testing, the safety factors are increasing and the design becomes unnecessary material intensive (Hercules Grundläggning, 2018).

Regarding the geotechnical serviceability limit state, foundations with shaft bearing piles must be designed to limit non-uniform settlements of the piles. The settlements of piles is largely dependant on the overconsolidation ratio, OCR, of the soil and non-uniform settlements can cause considerable redistributions of stresses in the overbearing structure. Contrarily, uniform settlements cause little risk to the structural capacity and is rather a potential practical concern (Statens Geotekniska Institut (SGI), 1993).

Additionally, when estimating the settlement behaviour of the piles, concern must be taken to the added load due to negative friction, as earlier discussed in Section 2.4.2. In practice, negative friction is considered in a length portion of the pile where the soil is expected to settle at least 5 mm more than the pile (Per Eriksson et al., n.d.). This length is typically provided in a geotechnical supporting document and often varies between projects.

#### 2.4.3.1 End bearing piles

The axial capacity of an end bearing pile primarily consists of end resistance and any contributing shear stress at the pile shaft is neglected. The total resistance from this type is therefore determined by the properties of the bedrock or of the firm soil at the pile toe. Verification of the axial capacity of these piles is difficult to predict through calculations and is typically rather done by dynamic testing methods on site. By using these methods, the pile is driven into the ground until sufficient axial capacity is acquired. However, it is possible to roughly estimate the maximum end bearing capacity a given pile type can possibly acquire through calculations. Following the Swedish Commission on Pile Research, the following formula can be used (Alheid et al., 2014):

$$R_{d,max} = \frac{F_{stuk} \cdot k_1 \cdot k_2}{\gamma_t \cdot \gamma_{Rd} \cdot \xi_5} \tag{2.2}$$

where:

| | |
|---|---|
| $F_{stuk}$ | is the pile effective area times the characteristic strength |
| $k_1$, $k_2$ | are factors considering the installation and testing methods of the pile |
| $\gamma_t$ | is a bearing capacity partial factor |
| $\gamma_{Rd}$ | is a model factor for dynamic testing |
| $\xi_5$ | is a correlation coefficient regarding the number of tested piles |

Settlements for end bearing piles are often small and therefore non-governing in the dimensioning of the pile. The dimensioning method of end bearing piles assumes that there exists no decrease of soil stiffness below the pile toe (Statens Geotekniska Institut (SGI), 1993).

#### 2.4.3.2 Cohesion piles (shaft bearing piles)

Cohesion piles are defined as piles installed in cohesive soils which are characterized by a small grain size, where clay is a typical example. The axial resistance is achieved by shear forces in the interface between the pile shaft and the surrounding soil, and the piles are therefore characterized as shaft bearing. There is typically no firm soil layer present in the ground. The shear resistance of cohesive piles is most commonly calculated using the *$\alpha$-method*, where the factor $\alpha$ describes the relation between the maximum possible shear resistance mobilized and the soil undrained shear strength. The factor is normally set to 1.0 for so called normally consolidated soils, further described in Section 2.4.4, and can decrease to 0.4 for overconsolidated soils. The total axial resistance is then calculated as (Per Eriksson et al., n.d.):

$$R = \int_L \alpha_d \cdot \theta_d \cdot c_{ud} \cdot dz + N_p \cdot A \cdot c_{ud} \tag{2.3}$$

where:

| | |
|---|---|
| $\theta$ | is the pile circumference |
| $c_u$ | is the undrained soil shear strength |
| A | is the pile cross sectional area |
| $N_p$ | is the factor for end bearing resistance, normally set to 9 |
| L | is the length of pile |

Piles installed in cohesive soil are at greater risk for large long-term settlements (Per Eriksson et al., n.d.). Design of cohesive piles therefore needs to include an estimation of expected settlements across the foundation. The most common method used in practice for the calculation of settlements is called *the analogue method*. It is based on a fictive foundation reaching two thirds of the total pile depth from where additional soil stresses are calculated using a 2:1 method. The calculation of the settlements is then done as a summation of the soil stress and effective compression modulus along the depth of the soil (Per Eriksson et al., n.d.).

#### 2.4.3.3 Friction piles

Friction piles are defined as piles installed in frictional soil. Frictional soils differ from cohesive soils by having, on average, a larger grain size. Hence, frictional soils are lacking the cohesive behaviour between the soil grains. For a friction pile, the total capacity is divided between the shaft and the base resistance where the contributions from each mechanism will be more even compared to a cohesion pile.

During installation of piles in frictional soil, large disturbances occur and the geotechnical properties of the soil changes. The properties of the soil are derived from the undisturbed soil, and consequently, the final bearing capacity of the soil after the installation of piles is difficult to predict. Verification of the resistance through testing is therefore necessary.

### 2.4.4 Soil profile

When dimensioning the geotechnical capacity, GEO, of a pile foundation in safety class 2 or 3, certain information regarding the properties of the soil is required, depending on the level of detail for the calculations. The properties of the soil may vary along the depth, which is often described by a soil profile developed from testing on site.

Primarily, resistance from the soil is determined by the soil undrained shear strength, abbreviated $c_u$. The soil shear strength typically increases with the soil depth. Characteristic values for $c_u$ can be derived from testing of the ground and are corrected using a factor determined by the yield limit of the soil, $w_L$. The undrained shear strength is dimensioning primarily for normal to slightly overconsolidated soils and the drained shear strength must be regarded for overconsolidated soils (Statens Geotekniska Institut (SGI), 2007).

The overconsolidation ratio, OCR, of a soil describes the loading history of the soil. It is defined as the ratio between maximum imposed stress on the soil in the past, also referred to as the preconsolidation stress, and the current imposed soil pressure. The overconsolidation ratio is along with the shear strength an important factor for dimensioning cohesion piles (El-Reedy, 2012). Soils with OCRs around 1.0 are called normal consolidated soils and with OCRs larger than that, up to approximately 1.5, they are called slightly overconsolidated soils (Statens Geotekniska Institut (SGI), 2007).

Overconsolidation ratios can often be related to the type of a soil. Most of the natural soil types are normally consolidated and clay is typically overconsolidated. Underconsolidated soils are very uncommon (Statens Geotekniska Institut (SGI), 2007).

Additional properties of the soil are necessary for some calculations, most commonly when estimating settlements. Additional common soil properties include (Implementeringskommission för Europastandarder inom geoteknik., 2008):

- Friction angle $\phi$
- Pile end bearing-pressure $q_b$
- Density $\gamma$
- Density index (relative density) in frictional soil $I_D$
- Compression modulus of clay $M_L$
- Groundwater table

In some cases, the following information could also be useful:

- Modulus of deformation (Young's modulus $E$, shear modulus $G$)
- Clay sensitivity
- Yield stress $\sigma$

When dimensioning the structural capacity (STR) of a pile foundation, the following information on the soil properties are needed (Implementeringskommission för Europastandarder inom geoteknik., 2008):

- Soil resistance against pile buckling
- Exposure class

The soil resistance against pile buckling is calculated as the average shear resistance of the soil at a depth corresponding to the buckling length of the pile. This is typically close to 5 m (Hercules Grundläggning, 2020).

## 2.5 Embodied carbon and LCA

The embodied carbon value of a building is defined as the total amount of greenhouse gas emissions emitted during its whole life cycle. The value includes emissions caused by the material manufacturing, transportation and construction as well as maintenance and demolition. Embodied carbon does not consider emissions caused by the energy consumption of a building, referred to instead as operational carbon (Carbon Cure, n.d.).

The concept of embodied carbon was founded in the recent years as a method to compare the climate footprint of different structural designs. Research is ongoing to create a simple calculation method unified for all types of structures to ensure fair comparisons of designs which in turn creates an increased understanding of carbon emissions (Orr et al., 2020).

Life cycle analysis, or LCA, methods are used to track and quantify emissions through the life cycle of a product or process. For buildings, the life cycle is typically categorized in stages according to Figure 2.8. Note that the figure also includes operational carbon, marked in green. Analyses show that the vast majority of a building's embodied carbon is caused by the material production, stages A1-A3, including harvesting and transportation of the raw material as well as the product manufacturing. Consequently, embodied carbon assessments are most important in the very early stages of design (Orr et al., 2020).

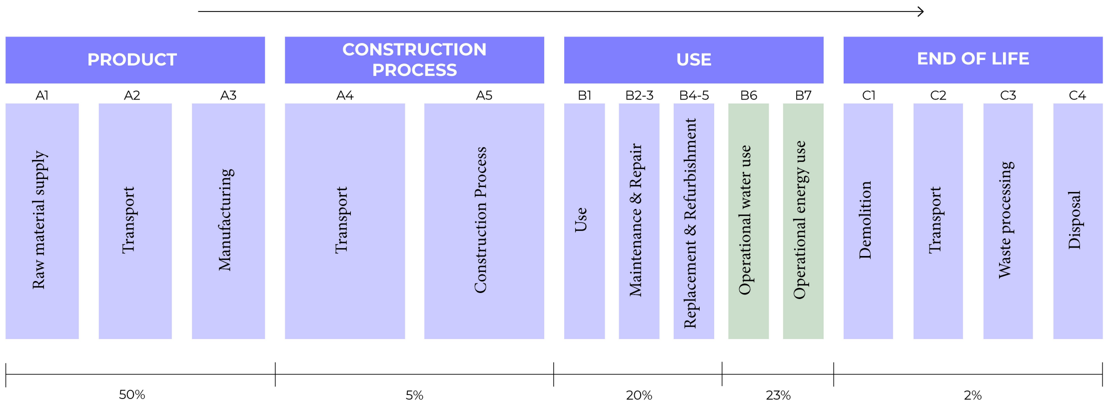



**Figure 2.8:** LCA stages (Orr et al., 2020).

Fundamentally, the embodied carbon value is calculated as the relevant material quantity times the embodied carbon factor of the specific material. The embodied carbon factor is an estimation of the total embodied $CO_2$ per weight or volume material. The accuracy of the estimations for both values increase throughout the design process. As the majority of the embodied carbon comes from the material production, it is a valid estimation in preliminary design to only consider carbon factors from stages A1-A3.

Values for embodied carbon factors can be found as typical material specific values. However, product manufacturers often provide environmental product declarations, EPDs, with figures of global warming potentials, GWPs, for their products (Orr et al., 2020). GWP is an embodied carbon factor, measured as $CO_2$-eq/kg material, which considers all potential greenhouse gases and their relative potency compared to $CO_2$ (Eurostat, 2014).

For concrete, the main cause of carbon emissions is the production of cement, more specifically during the burning of raw materials to clinker. Typically, higher strength concretes have a higher cement content. Research to find more $CO_2$ efficient concrete primarily investigates the possibility to replace some of the cement with other materials. Today, concrete mixes often contains cement replacers such as fly ash, silica fume or ground granulated blast-furnace slag that emits less carbon during production than typical Portland cement (The Concrete Centre, n.d.).

# 3

# Optimization tool

This chapter describes the structure of the optimization tool, the involved parameters and the calculation methods used.

In the design procedure, consideration is taken to follow common industry practices where concerns regarding buildability limits the design choices and number of individual possible structural designs. Buildability refers to increasing ease of construction, thereby also limiting production time and cost. Consideration to follow common industry practices is done to ensure credible comparisons.

## 3.1 Tool structure

The optimization tool is built entirely in the Grasshopper for Rhino3D environment. The tool starts with a set of parameters that can be altered within specified limits set by the user. The parameters are then fed into an automated dimensioning workflow, which employs calculation methods to generate a design for the foundation based on the input parameters. Additionally, the dimensioning workflow identifies if the design is valid or not, based on utilization ratios.

Calculations are based on hand calculation methods, following Eurocode, and are most often written as *C#* code in the program. The methods used are described in Sections 3.5 to 3.7. For full details of the calculations, see Appendix B.

For each design solution, i.e. combination of parameters, the tool computes a total embodied carbon value, defined as a $CO_2$-equivalent, or $CO_2e$, per square meter of floor area. Thereafter, the input parameters, embodied carbon value and validation check is fed to an optimization or iteration engine. The engine operates on the input variables and iterates through multiple solutions in the process to find a valid solution with the lowest possible embodied carbon value.

The input parameters are categorized in two groups, set parameters and variable parameters. The set parameters *defines the problem* and does not change during the optimization process. A set parameter could be, for example, the choice of a steel or a concrete pile. The variable parameters *defines a solution* to the problem and changes in the process by the optimization engine. The height of a slab is, for example, a variable parameter. The full lists of parameters are given in Section 3.8.

Figure 3.1 illustrates the schematic structure of the optimization tool. The large blue square represents the portion of the tool composing the automatic dimensioning workflow and the green squares represents the variable parameters and how the optimization engine operates on them.

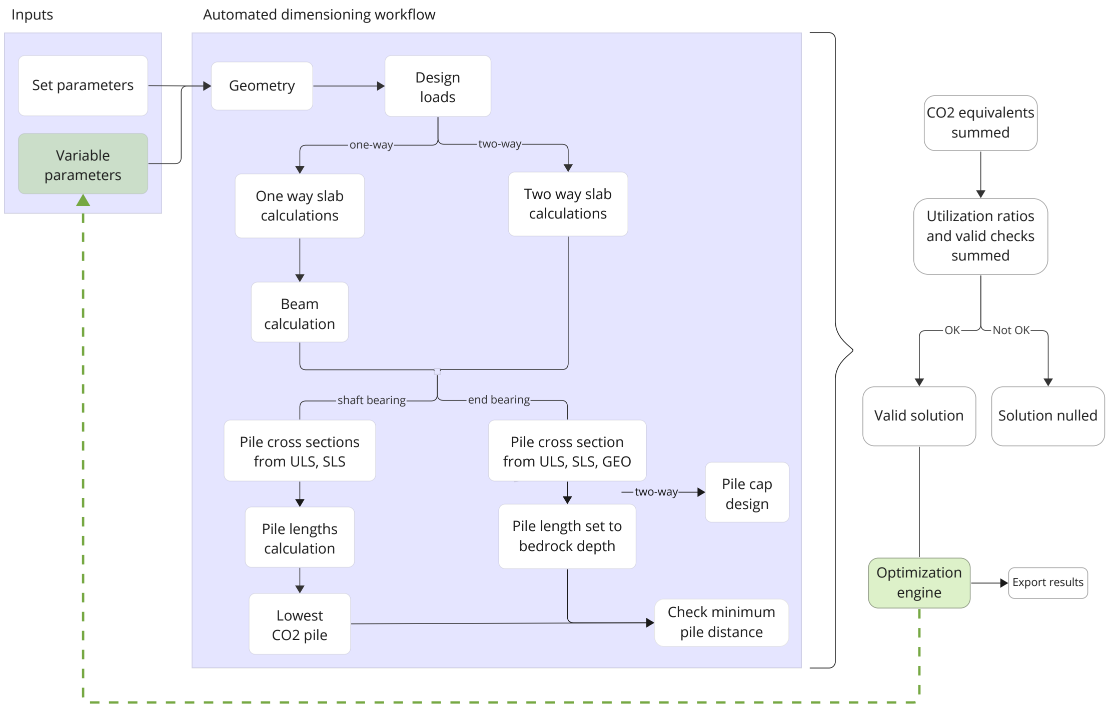



**Figure 3.1:** Schematic structure of the optimization tool.

### 3.1.1 Optimization engines

Depending on the type of information being sought, two distinct optimization engines are employed for the design analysis, Wallacei and Colibri.

Wallacei is an evolutionary multi-objective optimization engine as described in Section 2.2.1. Although Wallacei is identified as a multi-objective engine, it can also run optimization problems with only one objective. In this case, the set parameters define the problem to be solved by Wallacei. The variable parameters represents the genes for the population and the objective is the embodied carbon value. Being an evolutionary engine, Wallacei neglects solutions with a low objective fitness, and instead reproduces variations of more favourable solutions. The benefit with Wallacei as an optimization engine is how quickly it finds the optimal solution without having to compute all possible combinations of parameters. This is especially useful when the user wants to find the best solution only with a limited amount of time.

Colibri is an iteration engine plug-in also developed for Grasshopper. The engine takes all the variable input parameters and iterates through all possible combinations as inputs to the automatic dimensioning workflow. The tool computes a result for each iteration and, when having a large set of input parameters, this process becomes very time consuming compared to when using Wallacei. However, for research purposes where not only the best but where all the different combinations are of interest, this iteration engine is useful. To limit the computational time for this procedure, step increments and outer limits for the variable input parameters have to be set by the user to limit the amount of combinations. The step increments are set to comply with industry standards and for buildability purposes where, for example, dimensions of the different structural elements follow the dimensions used in practice today, see Section 3.8.2.

## 3.2 Case study

To be able to compare the results of how different structural parameters affects the embodied carbon content of the structure, the optimization tool is applied on a case study. The building type chosen for the case study is a large single story facility with a fully open plan and a long-way spanning roof, see Figure 3.2. Plausible functions of the facility could be an industrial storage facility, a sports arena or concert hall, a parking garage or similar. The reason of the choice is to disregard the effects of placement of load-bearing elements on the foundation, such as walls and columns, typically limiting the choice of pile placement.

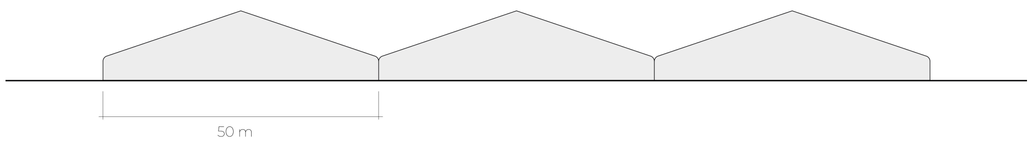



**Figure 3.2:** Elevation of the building used for the case study.

To be able to compare the results in the unit $kgCO_2e/m^2$, a convergence study is performed to make sure that a sufficiently large floor area is considered, to avoid the variations in the results when looking at a too small foundation slab. The variations occur due to the perimeter row of piles. Each perimeter pile supports a slab in only one direction, resulting in a higher embodied carbon per square meter slab in the edge spans. As the total slab area decreases, the ratio of perimeter piles to inner piles increases, leading to a greater embodied carbon value per square meter slab, see Figure 3.3. Since different pile center-to-center distances is one of the main objectives to study, a larger foundation slab is chosen to reduce the impact of this size effect.

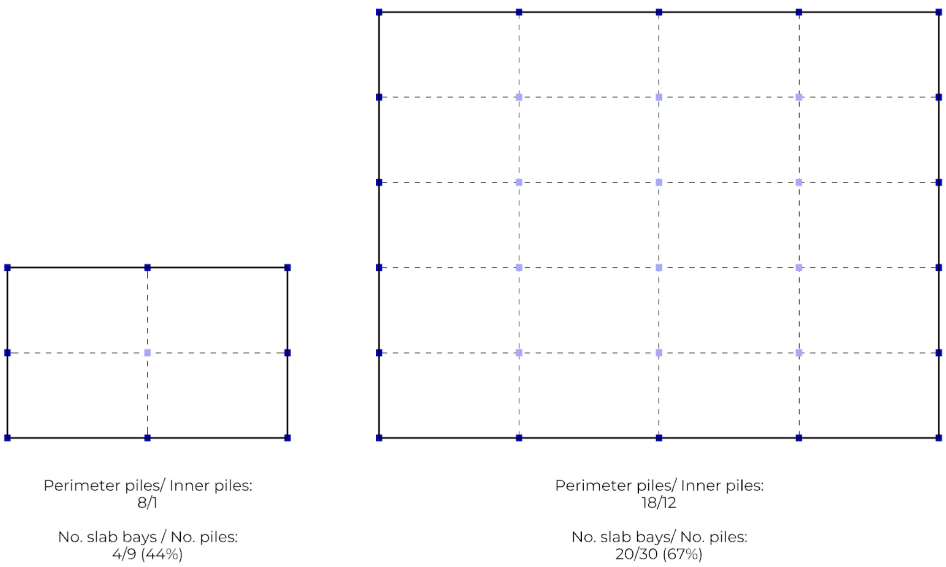



**Figure 3.3:** Effect of increased slab size on number of piles to number of slab bays ratio.

The convergence study investigates, for the square foundation floor, different floor side lengths until the carbon content per square meter is converging. Based on the convergence study illustrated in Figure 3.4, the optimization tool assumes a floor area of 150x150 m, where the results are considered to have converged.

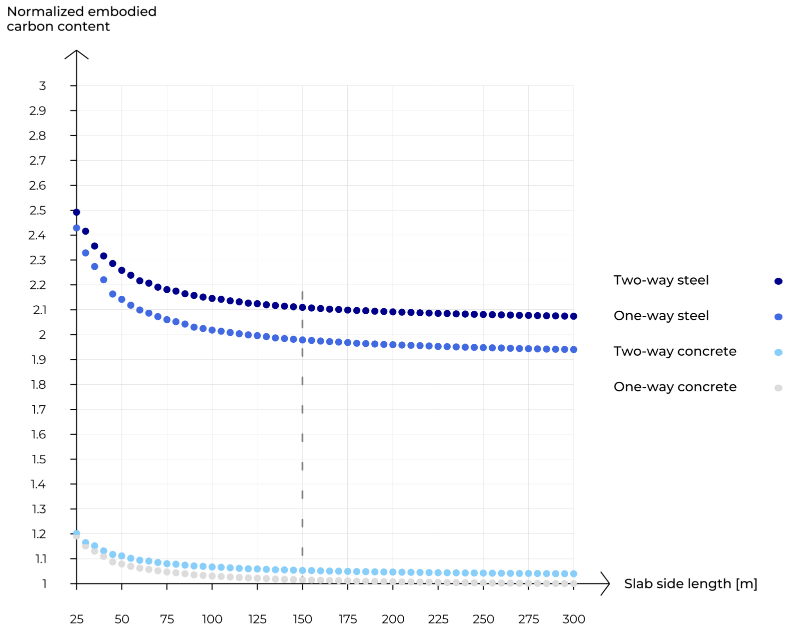



**Figure 3.4:** Convergence study between normalized embodied carbon content and slab side length of the square foundation for a one-way and two-way foundation with concrete or steel piles.

## 3.3 Material and soil properties

The material and soil properties assigned in the optimization tool are according to the Eurocode, national standards and common practice in Sweden.

### 3.3.1 Concrete

The concrete classes used when dimensioning the foundation slab, grade beams and pile caps are ranging from C20/25 to C60/75. The concrete class used in the piles by the manufacturer is C50/60 (Hercules Grundläggning, 2018).

**Table 3.1:** Strength- and deformation properties for concrete (European Committee for Standardisation, 2004).

<table>
<tr><th></th><th colspan="9">Strength classes for concrete</th></tr>
<tr><th>Class</th><th>20/25</th><th>25/30</th><th>30/37</th><th>35/45</th><th>40/50</th><th>45/55</th><th>50/60</th><th>55/67</th><th>60/75</th></tr>
<tr><td>$f_{ck}$ [MPa]</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td><td>45</td><td>50</td><td>55</td><td>60</td></tr>
<tr><td>$f_{cm}$ [MPa]</td><td>28</td><td>33</td><td>38</td><td>43</td><td>48</td><td>53</td><td>58</td><td>63</td><td>68</td></tr>
<tr><td>$f_{ctm}$ [MPa]</td><td>2.2</td><td>2.6</td><td>2.9</td><td>3.2</td><td>3.5</td><td>3.8</td><td>4.1</td><td>4.2</td><td>4.4</td></tr>
<tr><td>$f_{ctk}$ [MPa]</td><td>1.5</td><td>1.8</td><td>2.0</td><td>2.2</td><td>2.5</td><td>2.7</td><td>2.9</td><td>3.0</td><td>3.1</td></tr>
<tr><td>$E_{cm}$ [GPa]</td><td>30</td><td>31</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td></tr>
<tr><td>$\varepsilon_{cu}$ [‰]</td><td colspan="7">3.5</td><td>3.1</td><td>2.9</td></tr>
<tr><td>$\rho_c$ [kg/m$^3$]</td><td colspan="9">2350</td></tr>
</table>

### 3.3.2 Reinforcing steel

The reinforcing steel type used when dimensioning the foundation slab, grade beams, pile caps and piles are K500C-T and K500AB-W. The reinforcing steel properties are shown in Table 3.2 and the dimensions used are shown in Tables 3.3 to 3.5.

**Table 3.2:** Reinforcing steel properties (European Committee for Standardisation, 2004).

| Type | Characteristic yield strength $f_{yk}$ | Elastic modulus $E_s$ | Density $\rho$ |
|---|---|---|---|
| K500C-T | 500 MPa | 200 GPa | 7850 kg |
| K500AB-W | 500 MPa | 200 GPa | 7850 kg |

**Table 3.3:** Reinforcement mesh dimensions used in the foundation slab, steel type K500AB-W (Celsa Steel Service AB, 2023).

| Foundation slab | |
|---|---|
| **Mesh type** | **Dimensions [mm]** |
| FS6100 | 100x100x6 |
| FS6150 | 150x150x6 |
| FS7100 | 100x100x7 |
| FS7150 | 150x150x7 |
| FS8100 | 100x100x8 |
| FS8150 | 150x150x8 |
| FS9100 | 100x100x9 |
| FS9150 | 150x150x9 |
| FS10100 | 100x100x10 |
| FS10150 | 150x150x10 |
| FS12100 | 100x100x12 |
| FS12150 | 150x150x12 |

**Table 3.4:** Reinforcement bar dimensions used in the grade beams, steel type K500C-T (Celsa Steel Service AB, 2023).

| Grade beam | |
|---|---|
| **Type** | **Dimensions $\phi$ [mm]** |
| Longitudinal reinforcement | 8, 10, 12, 16, 20, 25, 32, 40 |
| Transversal reinforcement | 12 |

**Table 3.5:** Reinforcement bar dimensions used in the concrete piles, steel type K500C-T (Hercules Grundläggning, 2018).

| Concrete pile | |
|---|---|
| **Type** | **Dimensions $\phi$ [mm]** |
| Longitudinal reinforcement | 12, 16 |
| Transversal reinforcement | 5 |

### 3.3.3 Steel

The steel type used in the steel piles, by the manufacturer, is S460MH (SSAB, 2022).

**Table 3.6:** Steel pile properties (European Committee for Standardisation, 2008).

| **Type** | **Characteristic yield strength $f_{yk}$** | **Elastic modulus $E_s$** | **Density $\rho$** |
|---|---|---|---|
| S460M | 460 MPa | 210 GPa | 7850 kg |

### 3.3.4 Soil profile

The geotechnical capacity is considered in the tool only in the ultimate limit state, ULS. Therefore, the necessary information regarding the properties of the soil profile for this calculation are the:

- Dimensioning undrained shear strength variation along the soil depth, $c_{ud}$
- Average overconsolidation ratio of the soil, OCR

Data for the soil profile for the studies is collected from the Swedish Geotechnical Institute (SGI). Chosen geographic area and soil type is Gothenburg, Sweden, with cohesive soil i.e. clay. The motivation for the chosen area is the presence of great depths with cohesive clay, providing necessary information for dimensioning long and shaft bearing piles. It is also common practice to construct foundations with shaft bearing piles in this area. The soil properties will also be used for calculations of the structural capacity and behaviour of end bearing piles.

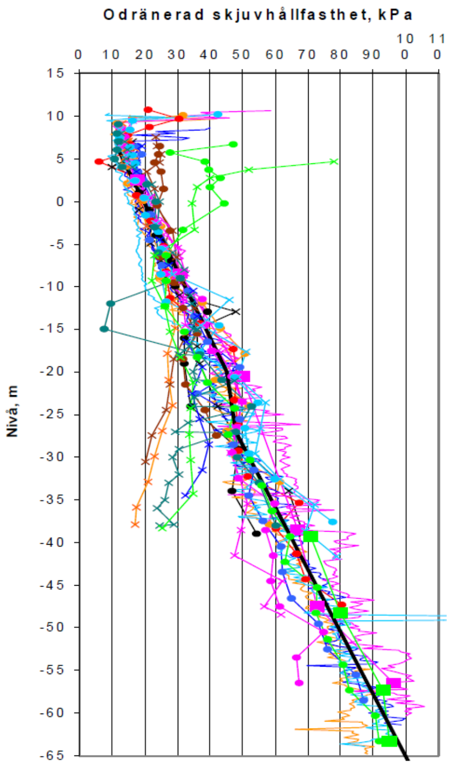



**Figure 3.5:** Soil undrained shear resistance.

Figure 3.5 illustrates results of undrained shear resistance from 8 vane shear tests and 4 cone penetrometer tests of deep clay at different locations in the Gothenburg area. The black line illustrates average values. The horizontal axis shows the derived values for undrained shear resistance [kPa] and the vertical axis shows the depth below ground [m] (Statens Geotekniska Institut (SGI), 2007).

In the optimization routine, a linear approximation of the results is used for the undrained shear resistance at different soil depths:

$$c_{uk}(d) = 20 + 1.2 \cdot d \tag{3.1}$$

where:

| | |
|---|---|
| $c_{uk}$ | is the undrained shear strength [kPa] |
| $d$ | is the ground depth [m] |

The derived values for the undrained shear resistance is thereafter corrected to dimensioning values using the safety factor $\gamma_M = 1.5$ (Trafikverket., 2011). Furthermore, the soil in this area is slightly overconsolidated, with overconsolidation ratios close to 1.1.

### 3.3.5 Global warming potential values

Ideally, values of global warming potentials for different materials should be chosen according to the environmental product declaration, the EPD, of the specific product manufacturer. Additionally, values should be chosen including all stages of the life cycle analysis, the LCA. As this thesis aims to provide aid for structural engineers in early design stages, the GWP values considering the production stages A1-A3 are used only, to calculate the final embodied carbon value. This is a valid estimation due to the large proportion of the total embodied carbon these stages account for.

In this thesis, GWP values for concrete are collected from the Swedish national board of housing's climate database (Boverket, 2023). The database contains values of GWP for building materials based on the existing set of EPDs from Swedish manufacturers. The average resulting value is referred to as the typical value. However, the national board of housing suggests using typical values with a 25% increase, to ensure conservative calculations of embodied carbon in designs. Additionally, the recommendation for conservative values motivates manufacturers to provide product specific data. As one objective for this thesis is to compare results of embodied carbon with results from existing structures, the typical values without the 25% increase are used.

Additionally, this thesis assumes usage of concrete mixes with cement replacements for all designs. Therefore, GWP values are collected from the database for so called 'climate-improved' concretes. The carbon emissions for climate-improved concretes are generally decreased by 25% compared to non-cement replacement concrete, according to the database.

For the design of the foundation slab, reinforcement layouts are limited to provided reinforcement meshes from the steel manufacturer Celsa, typically used in Sweden and further described in Section 3.6. Specific GWP data for Celsas reinforcement meshes is provided in Table 3.10. Celsa is world leading regarding sustainable steel production, with low carbon equivalents emitted per kilogram steel produced (Celsa Steel Service AB, 2023). This can be seen by comparing Celsa's GWP values to those provided by the Swedish national board of housing, see Table 3.9.

The piles are chosen according to provided data of structural capacity from Swedish pile manufacturers, described further in Section 3.7. The corresponding GWP values used to estimate the embodied carbon for piles are therefore collected from EPDs provided by the specific pile manufacturer. The GWP for the piles are, in contrast to other GWP values, assigned in the unit of $kgCO_2e/m$ instead of $kgCO_2e/kg$. See Tables 3.11 and 3.12 below for the collected pile GWP values.

**Table 3.7:** Global warming potential of concrete in foundation elements A1-A3, typical values, not used in the tool (Boverket, 2023).

| **Concrete** | | **Comments** |
|---|---|---|
| **Class** | **GWP [kg$CO_2$e/kg]** | |
| C20/25 | 0.097 | Ready-mix concrete, buildings |
| C25/30 | 0.103 | |
| C30/37 | 0.116 | |
| C35/45 | 0.130 | |
| C40/50 | 0.140 | |
| C45/55 | 0.151 | |
| C50/60 | 0.163 | |
| C55/67 | 0.176 | |
| C60/75 | 0.184 | |

**Table 3.8:** Global warming potential of concrete in foundation elements A1-A3, climate-improved typical values, used in the tool (Boverket, 2023).

| **Concrete** | | **Comments** |
|---|---|---|
| **Class** | **GWP [kg$CO_2$e/kg]** | |
| C20/25 | 0.073 | Ready-mix concrete, buildings, climate-improved |
| C25/30 | 0.077 | |
| C30/37 | 0.087 | |
| C35/45 | 0.098 | |
| C40/50 | 0.105 | |
| C45/55 | 0.114 | |
| C50/60 | 0.122 | |
| C55/67 | 0.132 | |
| C60/75 | 0.138 | |

**Table 3.9:** Global warming potential of reinforcement steel A1-A3, typical value, not used in the tool (Boverket, 2023).

| **Reinforcement steel** | | **Comments** |
|---|---|---|
| **Type** | **GWP [kg$CO_2$e/kg]** | |
| K500C-T | 0.596 | 100% scrap based, excl. alloy |
| K500AB-W | 0.596 | |

**Table 3.10:** Global warming potential of reinforcement steel A1-A3, according to manufacturer, used in the tool (Celsa Steel Service AB, 2021).

| **Reinforcement steel** | | **Comments** |
|---|---|---|
| **Type** | **GWP [kg$CO_2$e/kg]** | |
| K500C-T | 0.398 | 100% scrap based |
| K500AB-W | 0.398 | |

**Table 3.11:** Global warming potential of concrete piles A1-A3, according to manufacturer, used in the tool (Hercules Grundläggning, 2020).

| **Concrete pile** | |
|---|---|
| **Pile type** | **GWP [kg$CO_2$e/m]** |
| HP 235-0412 | 24.0 |
| HP 235-0416 | 24.0 |
| HP 270-0812 | 33.9 |
| HP 270-0816 | 37.1 |
| HP 350-0816 | 57.8 |

**Table 3.12:** Global warming potential of steel piles A1-A3, according to manufacturer, used in the tool (SSAB, 2022).

| **Steel pile** | |
|---|---|
| **Pile type** | **GWP [kg$CO_2$e/m]** |
| RR75x6.3 | 25.6 |
| RR90x6.3 | 30.3 |
| RR115x6.3 | 39.8 |
| RR115x8.0 | 49.8 |
| RR140x8.0 | 61.6 |
| RR140x10.0 | 75.8 |
| RR170x10.0 | 92.4 |
| RR170x12.5 | 113.8 |
| RR220x10.0 | 122.3 |
| RR220x12.5 | 151.0 |

## 3.4 Durability

To ensure that the designed structure is durable through its entire service life, meeting the requirements on strength and serviceability without excessive maintenance, the environmental conditions and protection of the structure needs to be considered. The foundation structure in the thesis is designed for a lifetime of 100 years and the measures taken to protect it is presented below.

### 3.4.1 Exposure class

The exposure class describes the environmental conditions for which the structure is being designed. In addition to the mechanical actions on the structure, it can also be exposed to physical and chemical attacks. The exposure class used when designing the piles and foundation is presented below.

**Table 3.13:** Exposure class for designing foundation and piles (European Committee for Standardisation, 2004).

| **Corrosion induced by carbonation** | | |
|---|---|---|
| **Class** | **Description of the environment** | **Informative examples** |
| XC2 | Wet, rarely dry | Concrete surfaces subject to long-term water contact (many foundations) |

### 3.4.2 Crack width allowance

The maximum potential crack width is limited due to durability as well as appearance concerns. For the exposure class XC2, the crack width limitation is set to 0.3 mm according to the Eurocode (European Committee for Standardisation, 2004).

### 3.4.3 Concrete cover

To protect the reinforcing steel inside the concrete structure from chemical and physical attacks, a minimum cover between the reinforcement surface and the concrete surface is required. The required concrete cover is based on the exposure class, the properties of the reinforcing steel and if extra protection measures are being used. The nominal cover is calculated as (European Committee for Standardisation, 2004):

$$c_{nom} = c_{min} + \Delta c_{dev} \tag{3.2}$$

where:

$$c_{min} = max[c_{min,b}; c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add}; 10\ mm] \tag{3.3}$$

and where:

| | |
|---|---|
| $c_{min,b}$ | is the minimum cover due to bond requirement |
| $c_{min,dur}$ | is the minimum cover due to environmental conditions |
| $\Delta c_{dur,\gamma}$ | is an additive safety element |
| $\Delta c_{dur,st}$ | is the reduction of minimum cover for use of stainless steel |
| $\Delta c_{dur,add}$ | is the reduction of minimum cover for use of additional protection |
| $\Delta c_{dev}$ | is the addition to allow for deviation |

### 3.4.4 Steel pile corrosion allowance

When dimensioning a foundation with hollow steel piles, additional thickness of steel needs to be added on the inside and outside of the piles to protect them from corrosion and to maintain the intended structural capacity. The additional thickness required, for the considered exposure class and service life time, on the inside could be assumed to be 1 mm and on the outside to be 2 mm (Bengtsson et al., 2000).

## 3.5 Loads and load combinations

In order to dimension the foundation structure, the imposed (variable) loads that acts on the elements needs to be assumed and the corresponding load effects, including approximations of self-weight, needs to be calculated.

### 3.5.1 Imposed load

When including the imposed loads in the tool, the category of intended use of the structure needs to be defined. In the Eurocode, a range of categories are defined with corresponding characteristic values of distributed load. Depending on the intended use, the input loads may be changed where every category and load may be examined. In this study, multiple load types corresponding to different plausible purposes of the facility type, as discussed in Section 3.2, will be tested. Load categories and corresponding imposed loads are chosen according to Tables 3.14 and 3.15.

**Table 3.14:** Categories of use A-E (European Committee for Standardisation, 2009).

| Category | Specific use | Example |
|---|---|---|
| A | Areas for domestic and residential activities | Rooms in residential buildings and houses; bedrooms and wards in hospitals; bedrooms in hotels and hostel kitchens and toilets. |
| B | Office areas | |
| C | Areas where people may congregate | C1: Areas with tables etc. e.g. areas in schools, cafés, restaurants, dining halls, reading rooms, receptions.<br>C2: Areas with fixed seats, e.g. areas in churches, theatres or cinemas, conference rooms, lecture halls, assembly halls, waiting rooms, railway waiting rooms.<br>C3: Areas without obstacles for moving people, e.g. areas in museums, exhibition rooms, access areas in public and administration buildings, hotels, hospitals, railway station forecourts.<br>C4: Areas with possible physical activities e.g. dance halls, gymnastic rooms, stages.<br>C5: Areas susceptible to large crowds, e.g. in buildings for public events like concert halls, sports halls including stands, terraces and access areas and railway platforms. |
| D | Shopping areas | D1: Areas in general retail shops.<br>D2: Areas in department stores. |
| E1 | Areas susceptible to accumulation of goods, including access areas | Areas for storage use including storage of books and other documents. |
| E2 | Industrial use | - |

**Table 3.15:** Imposed loads for category A-E (European Committee for Standardisation, 2009).

| **Categories of loaded areas** | $q_k$ **[kN/m²]** | $Q_k$ **[kN]** |
|---|---|---|
| Category A | | |
| - Floors | 1.5 to 2.0 | 2.0 to 3.0 |
| - Stairs | 2.0 to 4.0 | 2.0 to 4.0 |
| - Balconies | 2.5 to 4.0 | 2.0 to 3.0 |
| Category B | 2.0 to 3.0 | 1.5 to 4.5 |
| Category C | | |
| - C1 | 2.0 to 3.0 | 3.0 to 4.0 |
| - C2 | 3.0 to 4.0 | 2.5 to 7.0 |
| - C3 | 3.0 to 5.0 | 4.0 to 7.0 |
| - C4 | 4.5 to 5.0 | 3.5 to 7.0 |
| - C5 | 5.0 to 7.5 | 3.5 to 4.5 |
| Category D | | |
| - D1 | 4.0 to 5.0 | 3.5 to 7.0 |
| - D2 | 4.0 to 5.0 | 3.5 to 7.0 |
| Category E1 | 7.5 | 7.0 |
| Category E2 | For intended use | |

### 3.5.2 Self-weight

The self-weight of the foundation elements, that are the foundation slab, grade beam and pile cap, are approximated by using the geometry inputs set in the beginning of the tool. The volume of each concrete element is multiplied with the reinforced concrete density $\rho_{rc} = 2500$ kg/m$^3$. This density is used, compared to the standard concrete density $\rho_c = 2350$ kg/m$^3$, as the elements will be reinforced with normal amounts to be able to approximate the final self-weight without knowing the reinforcement amount in beforehand, according to the Eurocode (European Committee for Standardisation, 2009). However, when calculating the final embodied carbon values for each solution, the exact material properties and densities are being used, see Chapter 3.3.

The self-weight of the piles are considered small in relation to the rest of the structure and is therefore neglected.

### 3.5.3 Load cases

To comply with the intent of creating an optimization tool in early stages of design, only three load cases will be considered. The three load cases are chosen from common elementary cases and are the cases expected to cause the greatest stresses in the foundation elements. The permanent load, or self-weight, is evenly distributed across all spans in all load cases, with the variable load distributed evenly in certain spans according to Figure 3.6. In each load case, the load is distributed similarly in both directions.

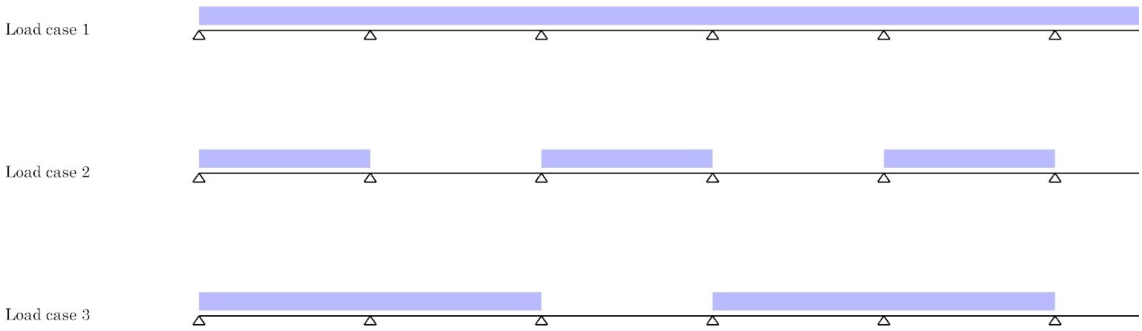



**Figure 3.6:** Variable load distribution in each load case.

### 3.5.4 Load combinations

With only one, partly- and evenly distributed, imposed load assumed for the structure and with the self-weight being calculated, one load combination for ULS and one load combination for SLS are necessary.

The load combination used for ULS, with both loads seen as unfavourable, is (European Committee for Standardisation, 2010):

$$E_{d,ULS} = E(\gamma_G \cdot G + \gamma_Q \cdot Q) \tag{3.4}$$

where:

| | |
|---|---|
| $G$ | is the self-weight of the structure |
| $Q$ | is the evenly or partly evenly distributed imposed load |
| $\gamma_G$ | is 1.35 |
| $\gamma_Q$ | is 1.50 |

The *quasi-permanent* load combination, used for long-term effects in SLS, with both loads seen as unfavourable, is (European Committee for Standardisation, 2010):

$$E_{d,SLS,q} = E(G + \psi_2 \cdot Q) \tag{3.5}$$

where:

| | |
|---|---|
| $G$ | is the self-weight of the structure |
| $Q$ | is the evenly or partly evenly distributed imposed load |
| $\psi_2$ | is 0.8 |

### 3.5.5 Moment- and shear force distribution

In the stage of dimensioning the structure in the ultimate limit state, the moment- and shear force distribution for the selected loads, load cases and load combinations, across the foundation slab and grade beam, may be assumed for an uncracked section according to the theory of linear elasticity (Al-Emrani et al., 2013).

With the flexural rigidity EI being known and constant for the uncracked sections (without accounting for the contributions of the reinforcement) and with predefined span lengths and applied distributed loads, both the moment- and the shear force distribution along the one-way spanning slab and continuous grade beam may be calculated with the *Three Moment Equation* (Gavin, 2009). The principle is based on the *Moment-Area Theorem*, assuming continuity over the supports, and is converted into matrix form and implemented into code in the optimization tool. Figures 3.7 and 3.8 illustrates resulting force distributions.

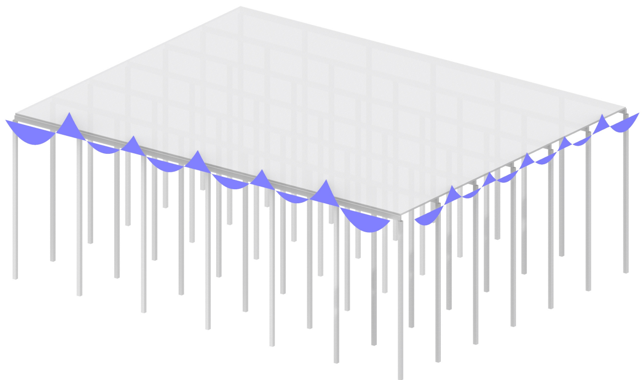


**Figure 3.7:** Moment distribution of continuous slab and beam according to the *Three Moment Equation*.

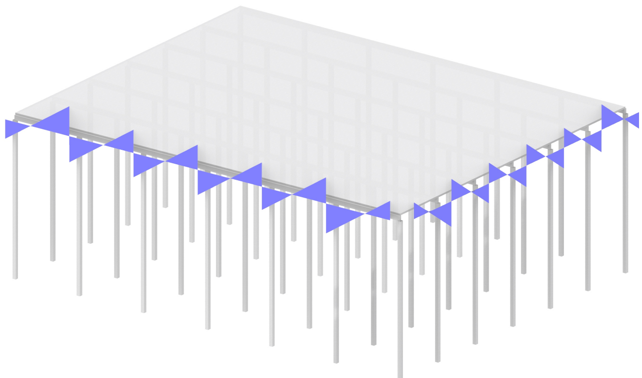


**Figure 3.8:** Shear distribution of continuous slab and beam according to the *Three Moment Equation*.

The same method is used to determine the moment- and shear distributions for the serviceability limit state load combination. For both ULS and SLS, the maximum moment over the support is reduced with an amount depending on the width of the support and the support reaction force, according to Eurocode (European Committee for Standardisation, 2004). It can, however, not be reduced by more than 10%.

#### 3.5.5.1 Strip method

For determining the moment distribution in the two-way spanning slab, the *Strip Method* by Hillerborg is used. The basic principles of the strip method is to divide a slab with reinforcement in two transversal directions into strips, parallel to the longitudinal reinforcement using load dividing lines. The load is thereafter partitioned to each strip and the strip is then treated as a one-way spanning beam. Typically, the load distribution in each strip is determined using a lower bound plastic approach (Engström, 2011).

In this case, the two-way slab is continuous and simply supported on piles. The load dividing lines are identified as the lines between the piles where the shear force in the slab along a strip is zero. In this case, load can be assumed to be distributed evenly between two inner piles, ratio 50-50, and with a ratio of 60-40 between an inner pile and an edge pile, see Figure 3.9. A critical strip is thereafter identified in each direction, marked in blue, along the outer row of inner piles. These two strips are wider than the others and will therefore carry more load. The critical section, subjected to the greatest loads, will be the section where these two strips overlap. The design loads, dimensioning for the entire slab, are determined as the maximum experienced loads from the different load cases in this critical section.

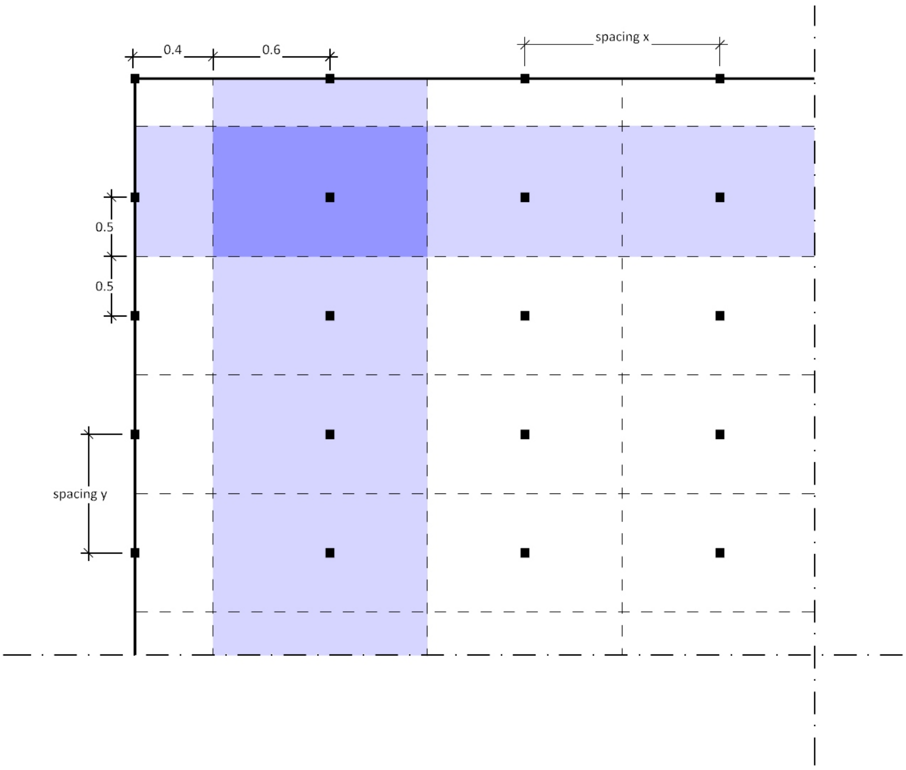



**Figure 3.9:** Critical strips and critical section in a two-way slab.

After identifying the critical strips, the moment- and shear force distribution in each strip is determined using the theory of linear elasticity and calculated using the *Three Moment Equation*, as described previously. For pile supported slabs or sections, the entire load must be carried by a single strip in any direction (Engström, 2011).

## 3.6 Slab dimensioning

Two different types of slabs are being examined in the optimization tool. The first type is a one-way spanning slab that spans between grade beams, that in turn spans between the piles. The second type is a two-way spanning slab with pile caps that spans directly between the piles.

The reinforcement design for the one-way and two-way spanning slab is determined by available reinforcement meshes from the steel manufacturer Celsa, a commonly used manufacturer with sustainable production methods (Celsa Steel Service AB, 2023). One reinforcement mesh is chosen for the top reinforcement and one for the bottom reinforcement, both placed across the entire slab. Multiple variations of mesh sizes are evaluated in the tool.

Calculations assume that the slab is supported by the piles only where any additional support due to contact between ground and slab is neglected in the dimensioning process.

### 3.6.1 One-way slab with grade beams

The dimensioning procedure for the one-way spanning slab and grade beam is based on the same theory. The slab is treated as a continuous wide beam and does not contain any shear reinforcement. The beam is designed as a continuous T-beam over the piles, thus taking into account the effective flange contribution from the slab, see Figure 3.10. The beam is designed with shear reinforcement when necessary. Both elements have one cross sectional design in a support section and one in a field section, for the corresponding moment and shear force in each section.

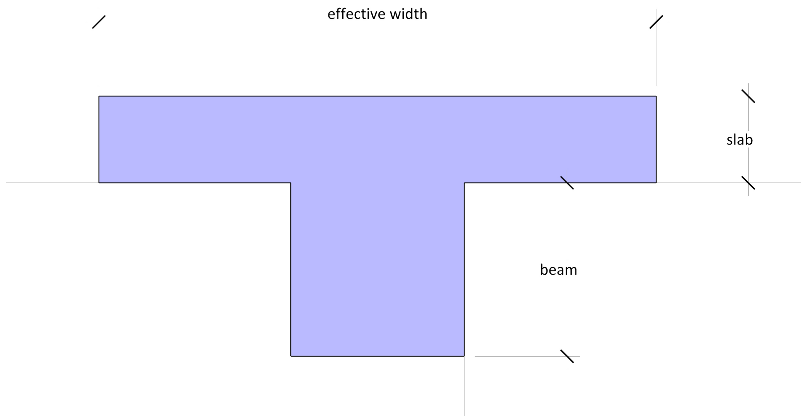



**Figure 3.10:** Grade beam section and effective flange contribution.

#### 3.6.1.1 Ultimate Limit State

In ULS, the one-way slab and grade beam are designed with the elastic moment distribution without plastic redistribution. Both elements are designed to fail in bending with a capacity according to the ULS combination, and the required shear capacity is thereafter verified. Furthermore, the required moment resistance in ULS determines the reinforcement amount in the beam.

The required tensile reinforcement in the grade beam is calculated based on the geometry of the cross section, the concrete strength, the steel strength and the corresponding load effect. The calculations are based on a simplified approach, including using a rectangular stress block, provided by the Swedish Concrete Association (Svenska Betongföreningen, 2020). Using this method, the reinforcement amount is adapted to the response of concrete in compression in ULS. Consequently, multiple valid cross sections acquire a moment utilization ratio close to 100%. The required tensile reinforcement is calculated based on the following equations:

$$m = \frac{M_N}{bd^2\eta f_{cd}} \qquad \text{Relative moment} \tag{3.6a}$$

$$\omega = 1 - \sqrt{1-2m} \qquad \text{Required mechanical reinforcement} \tag{3.6b}$$

$$A_s = \frac{M_N}{d(1-\omega/2)f_{yd}} \qquad \text{Required reinforcement area} \tag{3.6c}$$

where:

| | |
|---|---|
| $M_N$ | is the design moment |
| $b$ | is the cross section width |
| $d$ | is the tensile reinforcement lever arm |
| $\eta$ | is the effective rectangular stress block strength factor |
| $f_{cd}$ | is the design value of concrete compressive strength |
| $f_{yd}$ | is the design yield strength of reinforcement |

For the slab, the reinforcement amounts are determined by chosen reinforcement meshes in the top and bottom of the slab. Thereafter, moment resistances are calculated following the same simplified methodology. This simplified approach neglects the positive effect of the compressive reinforcement (Svenska Betongföreningen, 2020).

Furthermore, the slab is designed to resist the dimensioning shear force without any shear reinforcement for buildability purposes. In the grade beam, if shear reinforcement is needed, the number of links, or stirrups, per section is determined based on the number of tensile reinforcement bars, and the spacing of each group of links is calculated to withstand the dimensioning shear force for the considered section. The shear reinforcement layout along the length of the grade beam is divided into three sections to comply with buildability purposes and also to reduce the amount of reinforcement.

#### 3.6.1.2 Serviceability Limit State

In SLS, both the slab and grade beam are designed to not exceed limits for crack widths. Crack widths are calculated for sustained loading, using the quasi-permanent load combination. The limitation is set to 0.3 mm, see section 3.4.2. In the automated dimensioning workflow, the slab is checked to not exceed the limitation with the chosen cross section and reinforcement mesh. For the grade beam, if necessary, reinforcement bars are added to what was previous calculated for the ULS capacity. The maximum number of reinforcement bars is limited by the geometry of the beam.

To account for the long-term effect of creep, a creep coefficient is calculated for each section, which decreases the stiffness of the element and consequently affects the calculation of the crack width. The creep effect is determined partly by the relative humidity and the exposure class, i.e. the surrounding environment of the element.

Any deflection and shrinkage calculations are neglected in the dimensioning procedure.

#### 3.6.1.3 Reinforcement design one-way slab

Figure 3.11 illustrates the concept of reinforcement design in the one-way slab and grade beam. The reinforcement consists of two meshes in the slab, one upper and one lower, as well as longitudinal and transversal reinforcement in the grade beam.

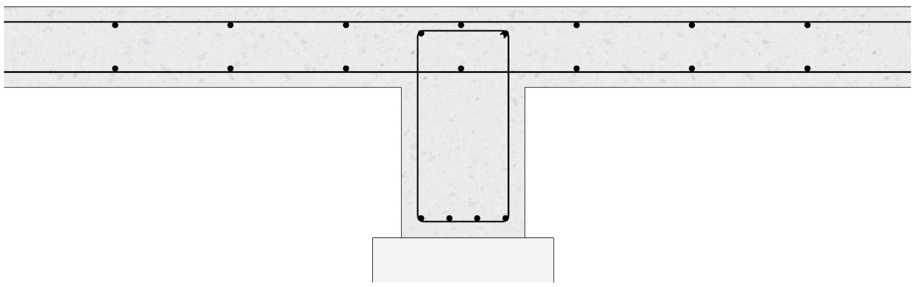


**Figure 3.11:** Detail of the one-way slab and grade beam reinforcement.

### 3.6.2 Two-way slab with pile caps

The two-way slab is designed similarly to the one-way spanning slab. One top and one bottom reinforcement mesh is chosen, and the corresponding moment utilization ratios, in both directions of the slab, are calculated using the simplified method mentioned in Section 3.6.1.1.

With the two-way slab being designed, as for the one-way slab, without shear reinforcement for buildability purposes, the thickness required to resist the shear peaks over the pile supports would become unnecessary large. To reduce the thickness of the slab and to increase the resistance against punching shear over the piles, pile caps are being used.

The two-way slab is also designed to not exceed limits for crack widths, as for the one-way spanning slab. Similarly, any deflection and shrinkage calculations are neglected in the dimensioning procedure.

#### 3.6.2.1 Pile cap

To avoid punching shear and to reduce the thickness of the slab, a pile cap is placed on top of the pile. The width of the pile cap is determined as the sum of the pile width, a 100 mm pile installation tolerance, space for shear links and cover thickness. The pile cap height is determined in the calculations for punching shear resistance. Included in the height is a 100 mm overlap of the pile and pile cap.

The punching shear resistance is calculated based on the strut-and-tie method. This method assumes a spread of stresses from the top of the pile with a 45 degree angle, creating a pressure cone, see Figure 3.12. The punching shear resistance is thereafter based on the width of the pressure cone at the level of the bottom surface of the slab, referred to here as the effective width. The height from the top of pile to the bottom slab, noted $x$ in the figure, is calculated to fit the required effective width.

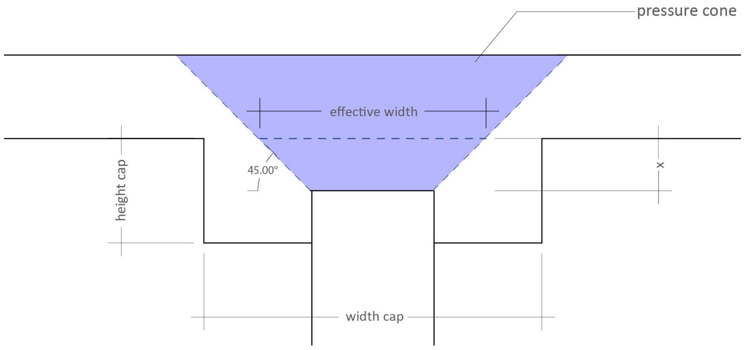



**Figure 3.12:** Elevation view: pile cap and pressure cone for punching shear calculation for the two-way slab.

From the effective width of the pressure cone at the base of the slab, longitudinal reinforcement in the slab within a control region of 2 times the reinforcement lever arm $d$ is considered to contribute to the punching shear resistance, see Figure 3.13.

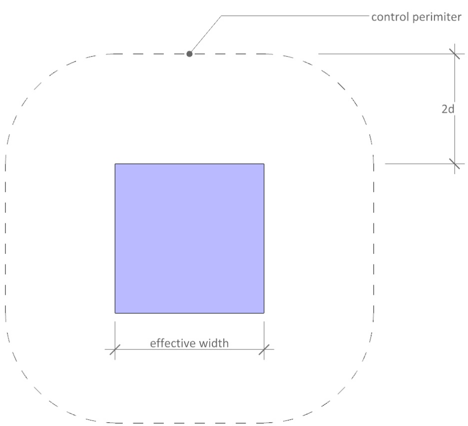



**Figure 3.13:** Plan view: control perimeter.

#### 3.6.2.2 Reinforcement design two-way slab

In addition to the bending reinforcement meshes in the slab, as earlier discussed, additional reinforcement is added to the pile cap following common practice. The positive contribution is, however, neglected in the calculation of the punching shear resistance. A total of 4 C-links are added as additional shear reinforcement and are enclosed by N-links, where the number depends on the height of the cap. Figure 3.14 illustrates the concept of the reinforcement design.

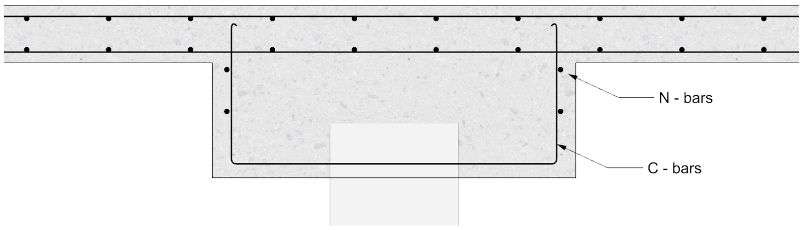



**Figure 3.14:** Detail of the two-way slab and pile cap reinforcement.

### 3.6.3 Limitations of design

- Cross sectional designs must not exceed utilization ratios in bending and shear above 100% and must meet the requirements of maximum crack width.

- The spacing of the reinforcement bars (part of the mesh) in the slab must not exceed the maximum values set by Eurocode:
  - For the reinforcement in the principal direction, this is set to the smaller value of 3 times the slab height or 400 mm.
  - For the non-principal direction, in the case of the one-way spanning slab, this is set to the smaller value of 3.5 times the slab height or 450 mm.

- In the grade beam, a minimum reinforcement amount is calculated as a function of the area of the tensile zone in ULS.

- The height of the slab and the height and width of the beam is allowed to vary in steps of 5 mm to comply with common practice and buildability.

- The ratio between tensile reinforcement area to concrete area must not exceed 4%.

- For the grade beam, the minimum number of tensile reinforcement bars is set to two.

- The height of slab and the width of beam must be large enough to fit needed reinforcement bars with required spacing and concrete cover.

## 3.7 Pile dimensioning

The pile design is done by selecting pile types from tables provided by pile manufacturers. To be able to select the appropriate pile in the optimization tool, the tables require the pile head load in ULS, SLS as well as the soil shear resistance. The capacity of the piles in the tables are calculated in accordance with the Eurocode. Chosen pile manufacturers are *Hercules Grundläggning AB* for concrete piles, and *Svenskt Stål AB (SSAB)* for steel piles, two of the most common manufacturers in Sweden.

The optimization tool is tested on both cohesion shaft bearing piles as well as on end bearing piles. Due to the great uncertainty in calculating the geotechnical capacity of piles in frictional soils, this is excluded from the thesis.

The pile length, for piles installed in cohesive soil, is determined by the necessary geotechnical capacity of the soil in ULS. The length of end bearing piles is for simplicity in this thesis set to the length of the distance to bedrock.

### 3.7.1 Structural capacity

The structural capacity of the piles is determined by the manufacturer and stated in pile capacity tables. The available pile types for the tool to select from are shown in Tables 3.16 and 3.17, ordered in ascending structural capacity.

In ULS, calculations of bearing capacity comprises checks on stresses and buckling of the pile element, as well as capacity checks of the pile joints. In SLS, calculations comprises checks of stresses and, for concrete piles, crack widths. The dimensioning section is chosen within the top buckling length region of the pile. Additionally, the soil shear resistance used to dimension the pile is calculated as the average soil shear resistance within the top buckling length. The buckling length for any pile is set to an approximate value of 4.5 m, following recommendations from Hercules Grundläggning AB (Hercules Grundläggning, 2018) and the Swedish Commission on Pile Research (“Pålkommissionen”, n.d.).

Effect of negative friction is neglected as this effect is dependent on the length of the section of the pile where settlements of the soil relative to the pile is expected to reach above 5 mm. This can vary greatly dependent on the ground conditions and is therefore neglected to avoid uncertain approximations. Additionally, the greatest effect of the negative friction does not occur in the top buckling region of the pile, which is typically dimensioning despite the added load effect at the neutral layer (Hercules Grundläggning, 2018).

All piles are assumed to be fully surrounded by soil. Furthermore, the dimensioning calculations for concrete piles are based on the following assumptions (Hercules Grundläggning, 2018):

- Crack width limit: 0.40 mm for concrete cover 25 mm and 0.15 mm for 45 mm
- Operational lifetime: 100 years
- Long-term loading: 100% in ULS, 100% in SLS
- Reinforcement class: B500B
- Concrete class: C50/60

The following concrete piles are available in the optimization tool, ordered in ascending structural capacity:

**Table 3.16:** Available concrete cross section designs (Hercules Grundläggning, 2018).

| **Concrete pile** | | | |
|---|---|---|---|
| **Width [mm]** | **No. bars** | **Bar diameter [mm]** | **Concrete cover [mm]** |
| 235 | 4 | 12 | 25 |
| 235 | 4 | 16 | 25 |
| 270 | 8 | 12 | 25 |
| 270 | 8 | 16 | 25 |
| 350 | 8 | 16 | 45 |

For steel piles, a common driven steel pile is chosen. The pile types are selected according to the capacity tables that are based on the following assumptions:

- Corrosion allowance: 2 mm exterior, 1 mm interior
- Operational lifetime: 100 years
- Geotechnical category: 2
- Long term loading: 85% in ULS, 100% in SLS
- Steel quality: S460MH
- Expected straightness in loose to firm soil: bucking length / 300

The following steel piles are available in the optimization tool, ordered in ascending structural capacity:

**Table 3.17:** Available steel cross section designs (SSAB, 2022).

| **Steel pile** | |
|---|---|
| **Diameter [mm]** | **Thickness [mm]** |
| 75 | 6.3 |
| 90 | 6.3 |
| 115 | 6.3 |
| 115 | 8 |
| 140 | 8 |
| 170 | 10 |
| 170 | 12.5 |
| 220 | 10 |
| 220 | 12.5 |

### 3.7.2 Geotechnical capacity

The geotechnical capacity of cohesion piles, determining the required pile length, is calculated according to the method described in Section 2.4.3. In the optimization tool, all pile types able to resist, structurally, the dimensioning load are chosen and computed a required length for. Thereafter, the pile design with the lowest embodied carbon value is chosen for the foundation design.

The geotechnical capacity of end bearing piles is only verified by calculations of the maximal possible bearing capacity for a given pile type, as described in 2.4.3.1. The length of the end bearing piles is set equal to the depth to bedrock.

The geotechnical behaviour in SLS is neglected as a single soil profile is considered for the entire project and all the pile locations. Consequently, there is no possibility for non-uniform settlements for shaft bearing piles to occur.

### 3.7.3 Limitations of design

Other than limitations regarding the geotechnical and structural capacity and behavior, the center-to-center distance between individual piles must not subceed minimum values according to the Swedish Geotechnical Institute and the Swedish Transport Administration. The minimum distance between two parallel piles is set according to Table 3.18 where $D$ is the pile diameter [m] and $B$ is the pile width [m].

**Table 3.18:** Minimum distance between individual piles (Statens Geotekniska Institut (SGI), 1993), (Swedish Transport Administration, 2004).

| **Pile length [m]** | **End bearing** | | **Shaft bearing** | |
|---|---|---|---|---|
| | circular | square | circular | square |
| $< 10$ | 3D | 3.4B | 4D | 4.4B |
| 10-25 | 4D | 4.5B | 5D | 5.6B |
| $> 25$ | 5D | 5.6B | 6D | 6.8B |

In practice, the most slender steel pile available, with diameter 75 mm, have the spacing limit of 255 mm to 450 mm depending on the pile type and length. Similar, for the most slender concrete pile with width 235 mm, the limit is 800 mm to 1600 mm.

### 3.7.4 Common practice considerations

The method chosen, following information provided by pile manufacturers, is in line with the common practice regarding pile design in Sweden. Additionally, chosen pile manufacturers are both leading companies on the Swedish market.

## 3.8 Parameter properties

The inputs for the optimization tool are defined as parameters, earlier discussed in Section 3.1. The parameters are categorized between set and variable. The set parameters defines the problem and are fixed throughout the optimization process. The variable parameters defines a solution to the problem and changes throughout the process by the optimization engine.

The input parameters are the basis for the optimization process where all possible combinations of parameters represents the design space. A larger set of parameters and increments equals to a larger design space and consequently requires more computational power and time.

To compute the total number of combinations possible defined by the parameters, the number of increments per variable parameter are multiplied with each other as below:

$$N_{tot} = N_1 \cdot N_2 \cdot ... \cdot N_{i-1} \cdot N_i \tag{3.7}$$

where:

| | |
|---|---|
| $N_{tot}$ | is the total number of possible combinations |
| $N_i$ | is the number of increments for variable parameter $i$ |
| $i$ | is the total number of variable parameters |

To adapt to this fact, the range and the incrementation of the variable parameters in the analyses are selected to fit the optimization engine used, to generate fair and comparable results while keeping the computational time to a minimum. The range and incrementation of the parameters are also selected with industry standards and buildability in mind, for example, the height of the slab incrementation is 25 mm when using Colibri.

The following figures 3.15 and 3.16 illustrates the variable parameters for the one-way and two-way slab foundation optimization problems. The available set and variable parameters are summarized in Table 3.19. Further, tables are provided for each optimization engine used, summarizing the parameter sets for each foundation type, comprising of set parameters with their value as well as variable parameters and their available range and incrementation.

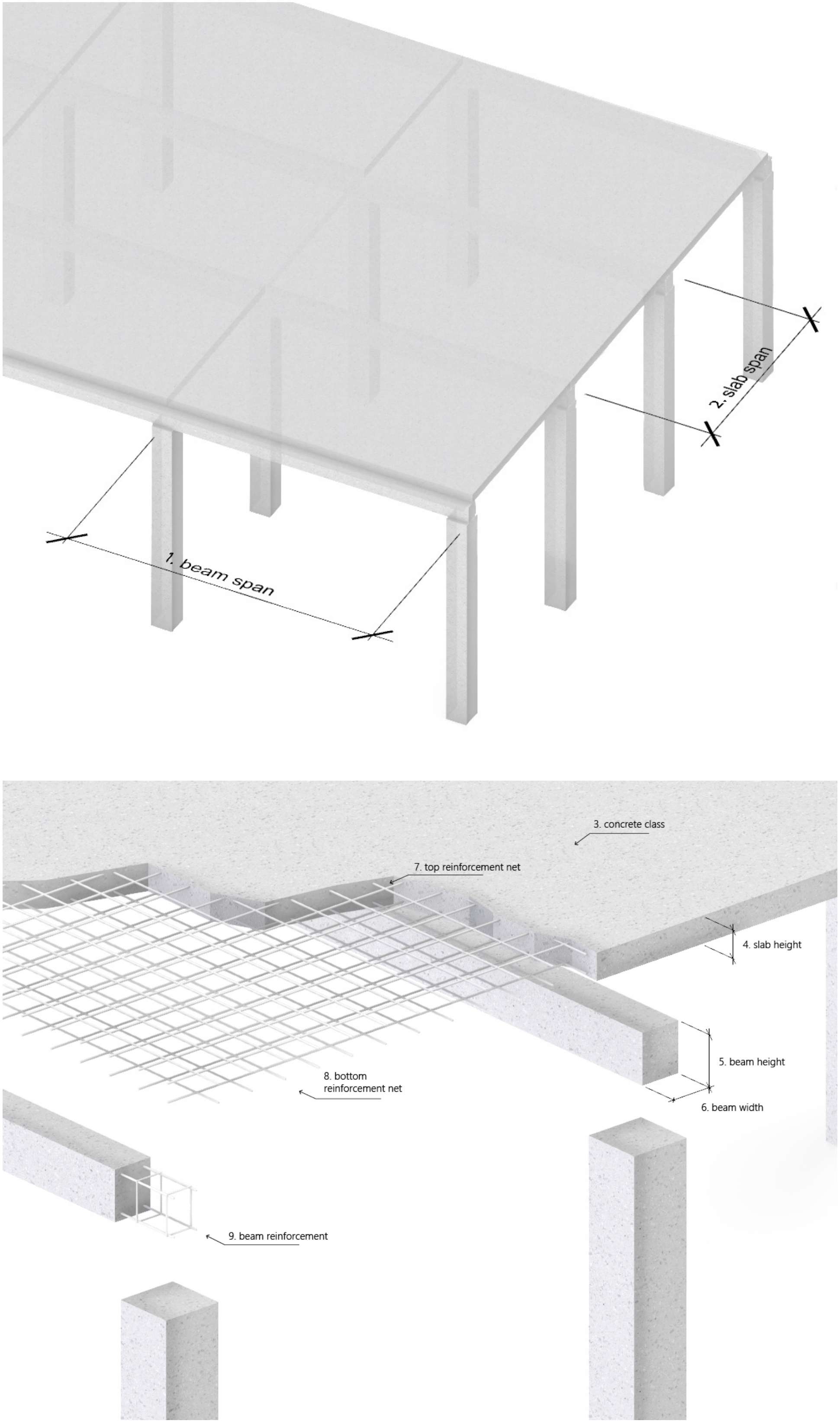



**Figure 3.15:** Variable parameters for the design of the one-way foundation slab.

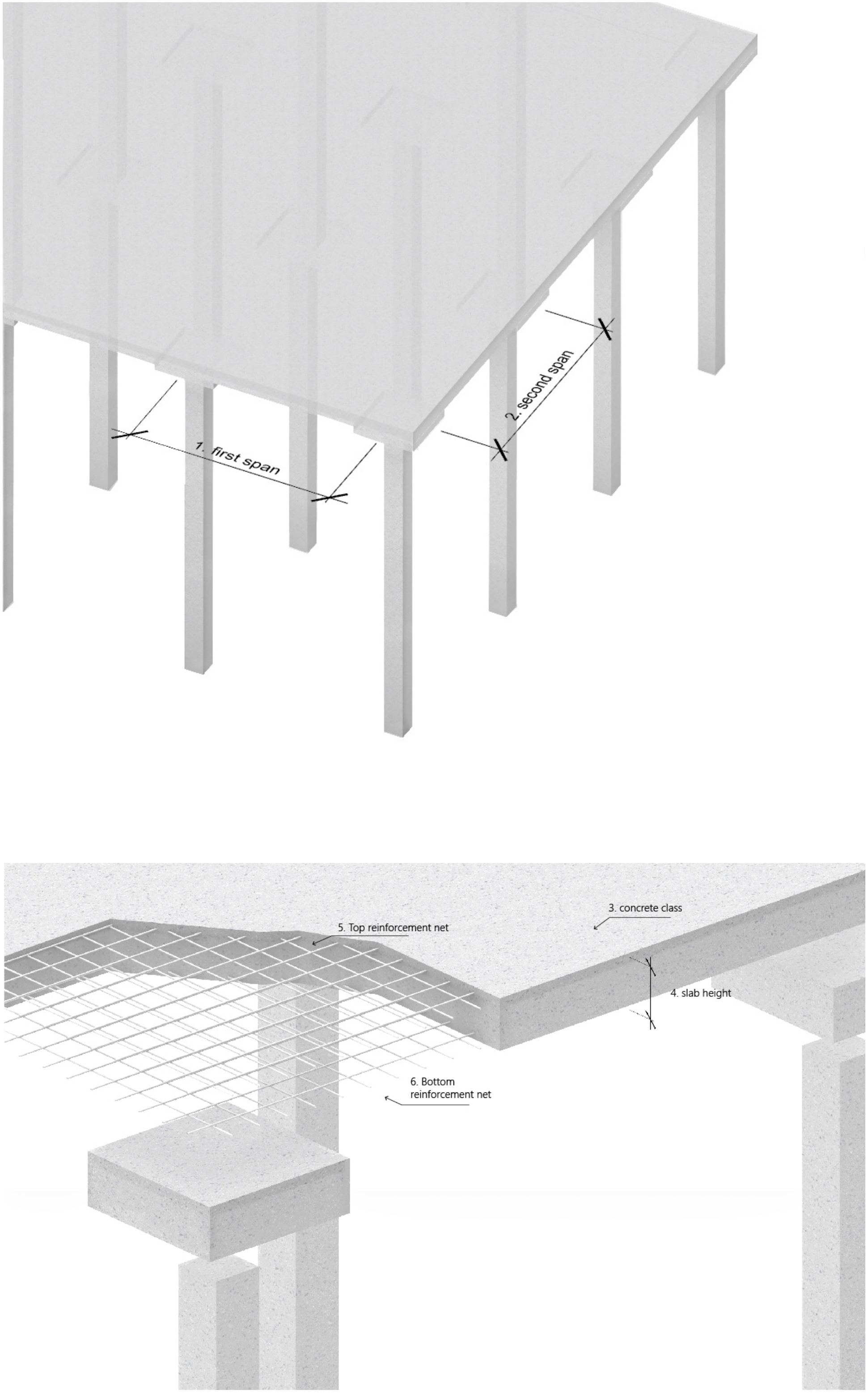



**Figure 3.16:** Variable parameters for the design of the two-way foundation slab.

**Table 3.19:** Summary of set- and variable parameters.

| | Set parameter | Unit | | Variable parameter | Unit |
|---|---|---|---|---|---|
| s.1 | Concrete/steel piles | - | v.1 | CC - distance 1 (slab span) | m |
| s.2 | One-way/two-way slab | - | v.2 | CC - distance 2 (beam span) | m |
| s.3 | Shaft bearing/end bearing pile | - | v.3 | Concrete class | - |
| s.4 | Imposed load | Pa | v.4 | Slab height | mm |
| s.5 | Pile buckling length | m | v.5 | Beam width | mm |
| s.6 | Min. soil shear resistance | Pa | v.6 | Beam height | mm |
| s.7 | Soil shear resistance increase | Pa/m | v.7 | Slab top reinforcement mesh | - |
| s.8 | Soil overconsolidation ratio | - | v.8 | Slab bottom reinforcement mesh | - |
| s.9 | Depth to bedrock | m | v.9 | Beam reinforcement diameter | mm |

### 3.8.1 Wallacei incrementation

With Wallacei as the optimization engine (based on evolutionary algorithms, see Section 2.2) the parameter ranges and incrementations are allowed to be large and detailed without requiring too much computational power. The parameters are, therefore, only adapted to comply with industry standards and buildability purposes. Available set and variable parameters with corresponding values or increments are presented in Table 3.20 and Table 3.21 for the one-way and two-way slab.

**Table 3.20:** Variable- and set (blue) parameters for the one-way foundation in the optimization tool.

**One-way foundation parameters**

| Parameter | Range / Values (increment size) | Unit | No. increments |
|---|---|---|---|
| Pile CC distance 1 / slab span | 1.0 to 12.0 (0.1) | m | 110 |
| Pile CC distance 2 / grade beam span | 1.0 to 12.0 (0.1) | m | 110 |
| Slab thickness | 0.05 to 0.5 (0.005) | m | 100 |
| Grade beam width | 0.05 to 0.4 (0.005) | m | 70 |
| Grade beam height | 0.02 to 0.52 (0.005) | m | 100 |
| Slab reinforcement mesh top | 100x100x6 | mm | 12 |
| | 150x150x6 | mm | |
| | 100x100x7 | mm | |
| | 150x150x7 | mm | |
| | 100x100x8 | mm | |
| | 150x150x8 | mm | |
| | 100x100x9 | mm | |
| | 150x150x9 | mm | |
| | 100x100x10 | mm | |
| | 150x150x10 | mm | |
| | 100x100x12 | mm | |
| | 150x150x12 | mm | |
| Slab reinforcement mesh bottom | As mesh top | | 12 |
| Grade beam reinforcement bar | 8, 10, 12, 16, 20, 25, 32, 40 | mm | 8 |
| Concrete class | C20/25 to C60/75 | - | 9 |
| Imposed load | 7.5 | kPa | 1 |
| Grade beam shear reinforcement | 12 | mm | 1 |
| Compression strut angle $cot\phi$ | 1.0 | - | 1 |
| Pile buckling length | 4.5 | m | 1 |
| Min. soil shear resistance | 20.0 | kPa | 1 |
| Soil shear resistance increase per meter | 1.2 | kPa | 1 |
| Soil overconsolidation ratio | 1.1 | - | 1 |
| Depth to bedrock | $\infty$ or 15 | m | 1 |
| **No. of possible combinations** | | | 4.2E+14 |

**Table 3.21:** Variable- and set (blue) parameters for the two-way foundation in the optimization tool.

| **Two-way foundation parameters** | | | |
|---|---|---|---|
| **Parameter** | **Range / Values (increment size)** | **Unit** | **No. increments** |
| Pile CC distance 1 / slab span | 1.0 to 12.0 (0.1) | m | 110 |
| Pile CC distance 2 / grade beam span | 1.0 to 12.0 (0.1) | m | 110 |
| Slab thickness | 0.05 to 0.5 (0.005) | m | 100 |
| Slab reinforcement mesh top | 100x100x6<br>150x150x6<br>100x100x7<br>150x150x7<br>100x100x8<br>150x150x8<br>100x100x9<br>150x150x9<br>100x100x10<br>150x150x10<br>100x100x12<br>150x150x12 | mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm | 12 |
| Slab reinforcement mesh bottom | As mesh top | | 12 |
| Concrete class | C20/25 to C60/75 | - | 9 |
| Imposed load | 7.5 | kPa | 1 |
| Pile buckling length | 4.5 | m | 1 |
| Min. soil shear resistance | 20.0 | kPa | 1 |
| Soil shear resistance increase per meter | 1.2 | kPa | 1 |
| Soil overconsolidation ratio | 1.1 | - | 1 |
| Depth to bedrock | $\infty$ or 15 | m | 1 |
| **No. of possible combinations** | | | 1.1E+9 |

### 3.8.2 Colibri incrementation

With Colibri as the optimization engine, iterating through all possible input combinations (see Section 3.1.1), the parameter ranges and incrementations needs to be limited to avoid requiring too much computational power and time. When setting up the increments, parameter ranges or values to select from, the most important parameters (where the largest difference in the results are expected to occur) are prioritized, hence being given more increments to select from.

The parameters are, therefore, adapted to comply with industry standards and buildability purposes whilst allowing the analyses to be computed within the time limit of the thesis. Available set and variable parameters with corresponding values or increments are presented in Table 3.22 and Table 3.23 for the one-way and two-way slab.

**Table 3.22:** Variable- and set (blue) parameters for the one-way foundation in the optimization tool.

| One-way foundation parameters | | | |
|---|---|---|---|
| **Parameter** | **Range / Values (increment size)** | **Unit** | **No. increments** |
| Pile CC distance 1 / slab span | 1.0 to 10.0 (0.5) | m | 19 |
| Pile CC distance 2 / grade beam span | 1.0 to 10.0 (0.5) | m | 19 |
| Slab thickness | 0.125 to 0.6 (0.025) | m | 20 |
| Grade beam width & height | 0.15 to 1.0 (0.05) | m | 18 |
| Slab reinforcement mesh top | 100x100x10<br>150x150x10<br>100x100x12<br>150x150x12 | mm<br>mm<br>mm<br>mm | 4 |
| Slab reinforcement mesh bottom | 100x100x10<br>150x150x10<br>100x100x12<br>150x150x12 | mm<br>mm<br>mm<br>mm | 4 |
| Grade beam reinforcement bar | 16 | mm | 1 |
| Concrete class | C20/25 | - | 1 |
| Imposed load | 7.5 | kPa | 1 |
| Grade beam shear reinforcement | 12 | mm | 1 |
| Compression strut angle $cot\phi$ | 1.0 | - | 1 |
| Pile buckling length | 4.5 | m | 1 |
| Min. soil shear resistance | 20.0 | kPa | 1 |
| Soil shear resistance increase per meter | 1.2 | kPa | 1 |
| Soil overconsolidation ratio | 1.1 | - | 1 |
| Depth to bedrock | $\infty$ or 15 | m | 1 |
| **No. of possible combinations** | | | 2.1E+6 |

**Table 3.23:** Variable- and set (blue) parameters for the two-way shaft bearing pile foundation in the optimization tool.

| Two-way foundation parameters | | | |
|---|---|---|---|
| **Parameter** | **Range / Values (increment size)** | **Unit** | **No. increments** |
| Pile CC distance 1 | 1.0 to 10.0 (0.5) | m | 19 |
| Pile CC distance 2 | 1.0 to 10.0 (0.5) | m | 19 |
| Slab thickness | 0.125 to 0.6 (0.025) | m | 20 |
| Slab reinforcement mesh top | 100x100x6<br>150x150x6<br>100x100x7<br>150x150x7<br>100x100x8<br>150x150x8<br>100x100x9<br>150x150x9<br>100x100x10<br>150x150x10<br>100x100x12<br>150x150x12 | mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm<br>mm | 12 |
| Slab reinforcement mesh bottom | As mesh top | | 12 |
| Concrete class | C20/25 | - | 1 |
| Imposed load | 7.5 | kPa | 1 |
| Pile buckling length | 4.5 | m | 1 |
| Min. soil shear resistance | 20.0 | kPa | 1 |
| Soil shear resistance increase per meter | 1.2 | kPa | 1 |
| Soil overconsolidation ratio | 1.1 | - | 1 |
| Depth to bedrock | $\infty$ or 15 | m | 1 |
| **No. of possible combinations** | | | 1.0E+6 |

# 4
# Results and discussion

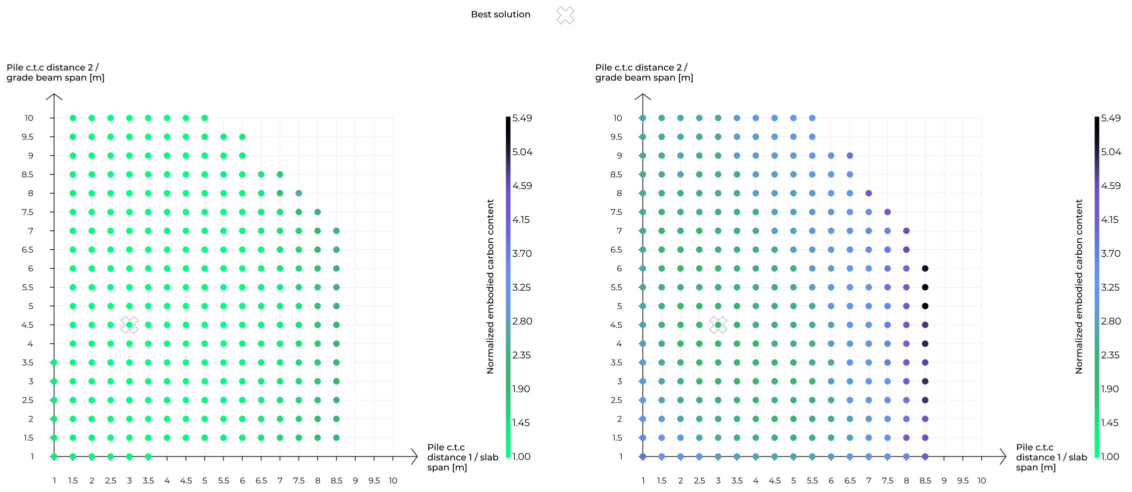
In this chapter the results of the optimization exploration will be presented and discussed.

To be able to compare the results between the graphs for different set parameter combinations and foundation types distinctively, normalizations are done both globally and locally. The global normalization ranges between 1.00 to 5.49, where 1.00 represents the minimum embodied carbon value of all foundation types and combinations while 5.49 represents the corresponding maximum. The limits are set to fit the results obtained and to be able to spot differences, comparable in percent, when studying the graphs produced. This normalization allows for clear comparisons on the global scale to compare which foundation type and parameter combination that performs the best.

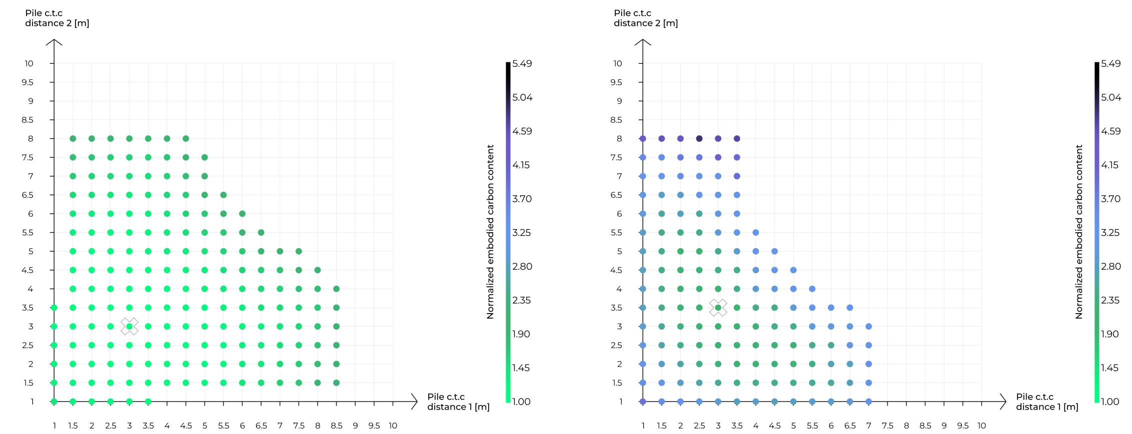
The local normalizations are made in a similar way, ranging from 1.00 to the corresponding maximum. However, the minimum- and maximum values represents in these cases the individual analyses minimum and maximum values only, to decrease the normalization span and to increase readability when looking at the details on a local scale.

## 4.1 Pile center-to-center distance and its effect on embodied carbon

When comparing the influence of different pile center-to-center distances on the embodied carbon, to find the optimal distance for the considered foundation types, Colibri is used as the iteration optimization engine. The inputs for the analyses, motivated and described in Chapter 3, are also shown in Table 3.22 and Table 3.23. The range of center-to-center distances examined is set to 1-10 m, to limit computational time whilst considering a wide range of feasible design options. The solutions with the lowest embodied carbon value for each combination of pile center-to-center distances for each foundation type are illustrated in Figure 4.1 and Figure 4.2, in a global respectively local scale.




(a) One-way slab with concrete and steel piles, respectively.

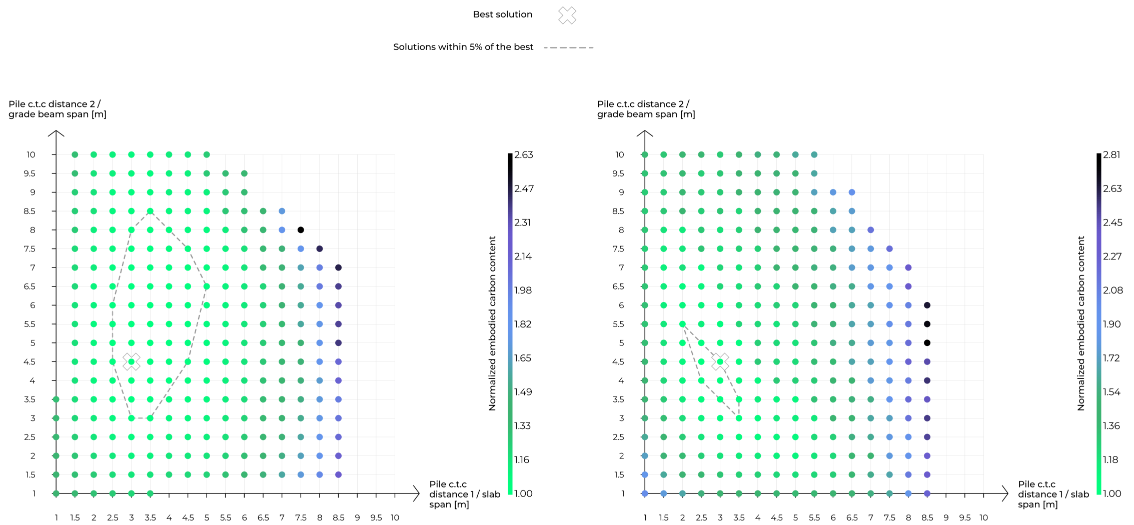



(b) Two-way slab with concrete and steel piles, respectively.

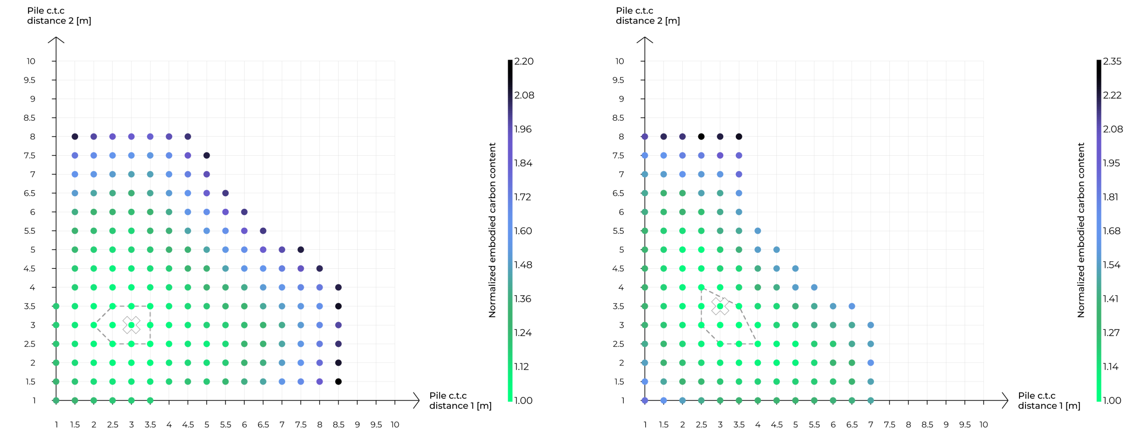
**Figure 4.1:** Global comparison of embodied carbon for different shaft bearing pile center-to-center distances. The best solution for respective foundation type is marked with a cross.

In the global comparison of the different slab and pile types above, there is a distinct difference of the performance of the steel versus concrete piles. All steel pile designs emits a greater amount of carbon than the corresponding concrete pile designs. Comparing the one-way versus two-way slab option for the same type of pile, the optimal solutions results in very similar values of embodied carbon. The results for the optimal center-to-center distance differs however, where the results suggests a longer beam than slab span for the case of a one-way slab.

It should be noted that consideration to error in installation is not considered in the one-way slab design. In cases where the pile is displaced, additional width can be added to the beam along its length or in certain areas. This proved, however, to have minimal impact on the embodied carbon.




(a) One-way slab with concrete and steel piles, respectively.




(b) Two-way slab with concrete and steel piles, respectively.

**Figure 4.2:** Local comparison of embodied carbon for different shaft bearing pile center-to-center distances. The best solution is marked with a cross and the spread of the solutions within 5% is marked with a dashed line.

On the local scale, it is observed from the graphs that the variations of solutions for all cases follows a clear pattern and that the embodied carbon value decreases towards a point in the graph. This suggests the existence of a single Pareto solution for all of the design types. However, the area of optimal solutions, encircled in the graphs and representing the solutions within 5% of the total embodied carbon of the best solution, is much larger for the one-way slab with concrete piles. As observed previously in Figure 4.1, the optimal solutions for the one-way and two-way slab with the same pile type have very similar values of embodied carbon. Consequently, having a one-way slab with concrete piles, it is possible to maintain these low values while choosing from a larger domain of span lengths compared to the two-way option.

### 4.1.1 Slab thickness of the optimal pile center-to-center distances

With the pile center-to-center distance being closely connected to the thickness of the slab when dimensioning the foundation, the optimal combinations of pile center-to-center distances together with their respective slab thickness are studied. The results for each foundation type, in their respective local normalization scale, are presented in Figures 4.3 to 4.6.

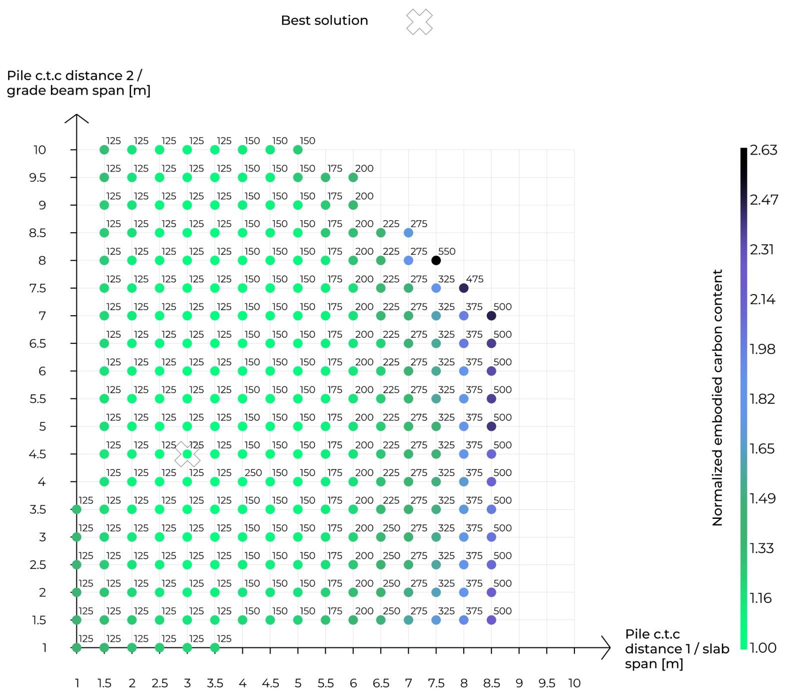



**Figure 4.3:** Embodied carbon and slab thickness [mm] of different pile center-to-center distances. The best solution is marked with a cross. Pile type: shaft bearing concrete, slab type: one-way.

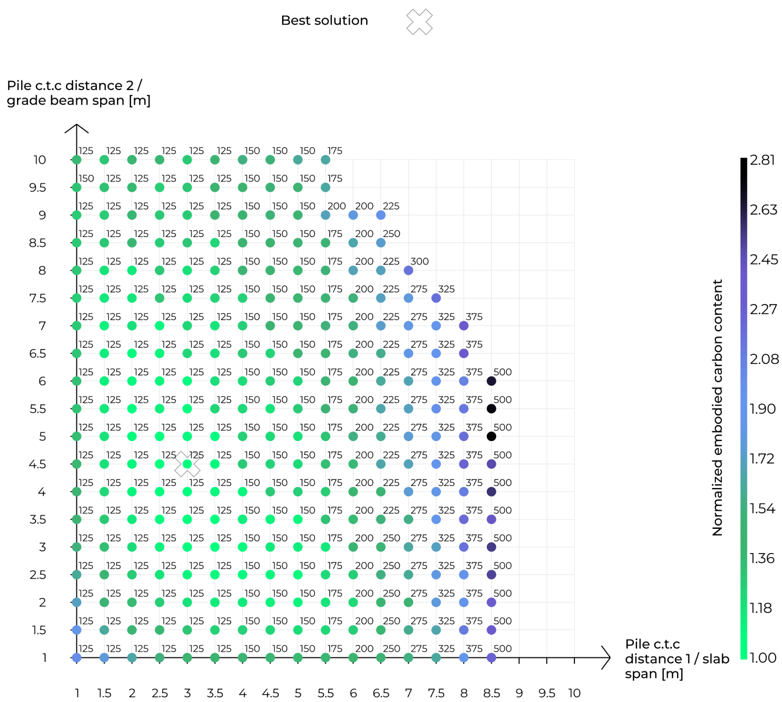



**Figure 4.4:** Embodied carbon and slab thickness [mm] of different pile center-to-center distances. The best solution is marked with a cross. Pile type: shaft bearing steel, slab type: one-way.

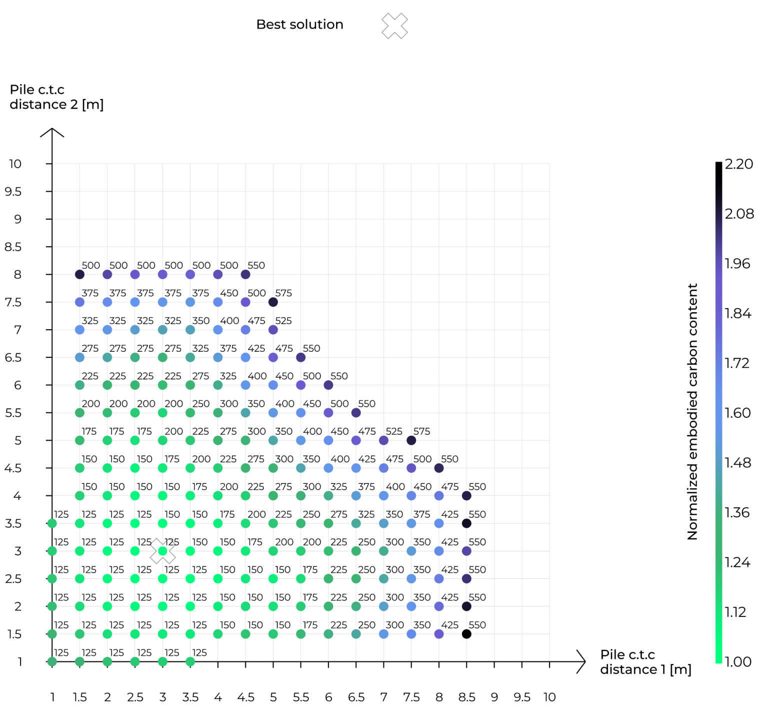



**Figure 4.5:** Embodied carbon and slab thickness [mm] of different pile center-to-center distances. The best solution is marked with a cross. Pile type: shaft bearing concrete, slab type: two-way.

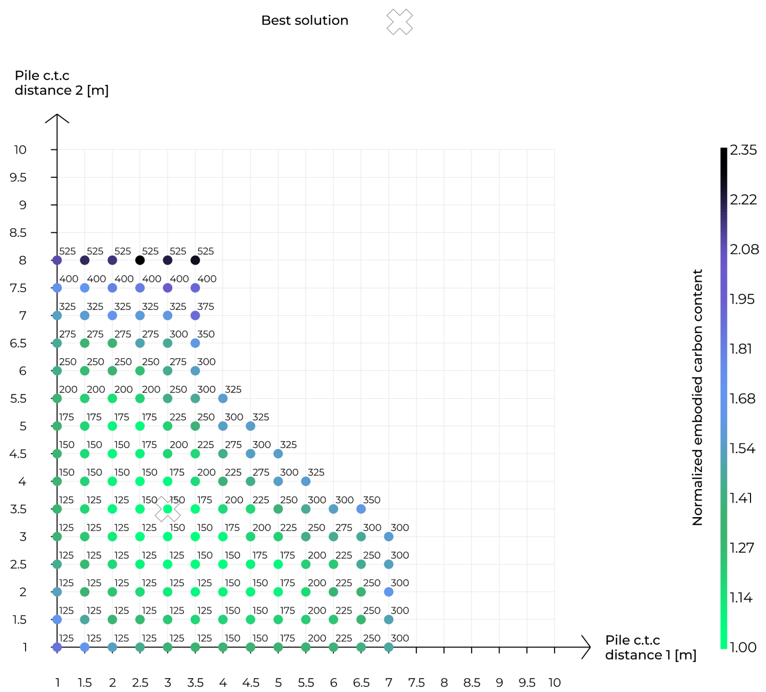



**Figure 4.6:** Embodied carbon and slab thickness [mm] of different pile center-to-center distances. The best solution is marked with a cross. Pile type: shaft bearing steel, slab type: two-way.

All foundation types suggests optimal design solutions with relatively thin slabs, which is expected considering the suggested short spans, relative to the available range. The optimal slab thicknesses are close to the lower limit set, which considers the space needed for reinforcement, required reinforcement spacing and concrete cover.

The lower limit of slab thickness is 105-115 mm, depending on the reinforcement mesh selected. However, to comply with common practice considerations and to reduce computational time, the analyses were performed with a slab thickness incrementation of 25 mm, starting from the thinnest slab of 125 mm.

This provides an explanation to the large area of solutions for the two-way slabs with short spans that have the same 125 mm slab thickness. For the one-way slabs, the thin slab is maintained for increased beam lengths. The results indicate that the ideal design should strive towards achieving the thinnest possible slab, and then select the span length to fully utilize the slab's capacity.

### 4.1.2 Slab-to-piles embodied carbon ratio of the optimal center-to-center distances

As an alternative approach to analyse the optimal design, this evaluation studies the relationship between the total embodied carbon in the foundation versus the ratio between the embodied carbon in the slab and piles. Figure 4.7 shows an average relationship between slab-to-piles embodied carbon ratio and the total embodied in the foundation, for four different foundation types.

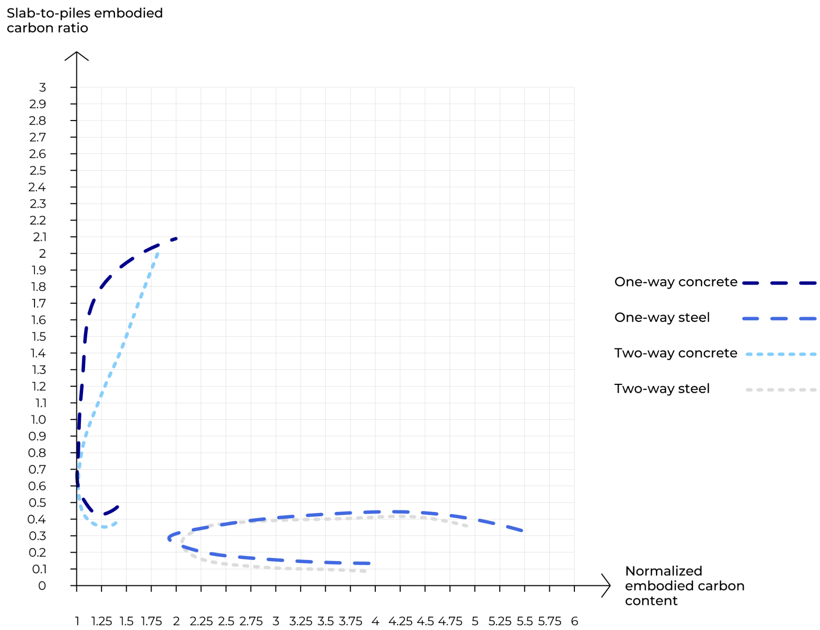



**Figure 4.7:** Slab-to-piles embodied carbon ratio [-] versus total embodied carbon of the optimal pile center-to-center distances. Pile type: shaft bearing.

It is observed in the graph that for each case, there exists an extreme point with a minimum value for the total embodied carbon. This proves the existence of a single Pareto solution in the optimization problem. The point, as well as the sensitivity to variations of slab-to-piles ratio, differ between steel and concrete piles. This is explained by the fact that steel piles emits more carbon per volume unit than concrete.

In the case of a one-way slab with concrete piles, it is shown that a larger range of ratios can be considered while still achieving a low embodied carbon result. This finding aligns with the observations drawn in Figure 4.2 (a). A potential explanation for this phenomenon is that a concrete grade beam bears a resemblance to a concrete pile in terms of embodied carbon. This similarity suggests that as the grade beam span increases, the increased embodied carbon in the beam, due to its necessary size increase, corresponds to the decreased amount of embodied carbon in the piles. Consequently, the two factors balance each other out, making the embodied carbon value independent of their ratio. However, this flexibility is not observed for the case of steel piles.

## 4.2 Embodied carbon with concrete or steel piles

The results shown in both Figure 4.1 and Figure 4.7 indicate that foundations with concrete piles are more carbon-efficient compared to those with steel piles in the case study. Additional analyses are conducted to investigate this effect in the case for end bearing piles, as well as to further investigate the case for shaft bearing piles with another optimization engine.

The analyses are, instead of using the iteration optimization engine Colibri, now conducted using the evolutionary optimization engine Wallacei to find the optimal solution for each foundation type. The analyses evaluates one-way slabs only and the results are presented in Figure 4.8. The results are normalized between 0.0 and 1.0, with 1.0 corresponding to the greatest embodied carbon value for any of the four foundation types and 0.0 represents an embodied carbon value of zero.

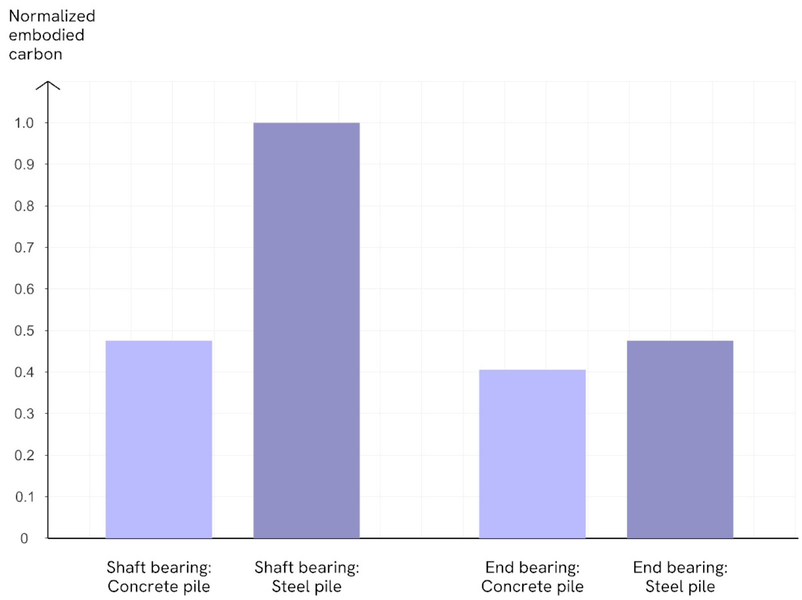



**Figure 4.8:** Embodied carbon comparison between optimized solutions with concrete or steel piles. Values are normalized between 0.0 and 1.0.

The results show, once again, that foundations using steel piles have a higher total embodied carbon value than those using concrete piles, regardless of whether shaft or end bearing piles are being used. However, the difference between the steel and concrete option is relatively small in the case of end bearing piles. The general increase of embodied carbon for steel piles can be explained by the fact that steel piles have a higher embodied carbon value per meter than a concrete pile with the same structural capacity. Moreover, when using steel piles for shaft bearing foundations, the smaller circumference of steel piles, relative the concrete pile with similar structural capacity, leads to longer piles and consequently a higher embodied carbon value. However, this analysis only regards cylindrical steel piles where any additions of wings or other elements that may increase the shaft bearing capacity is not considered.

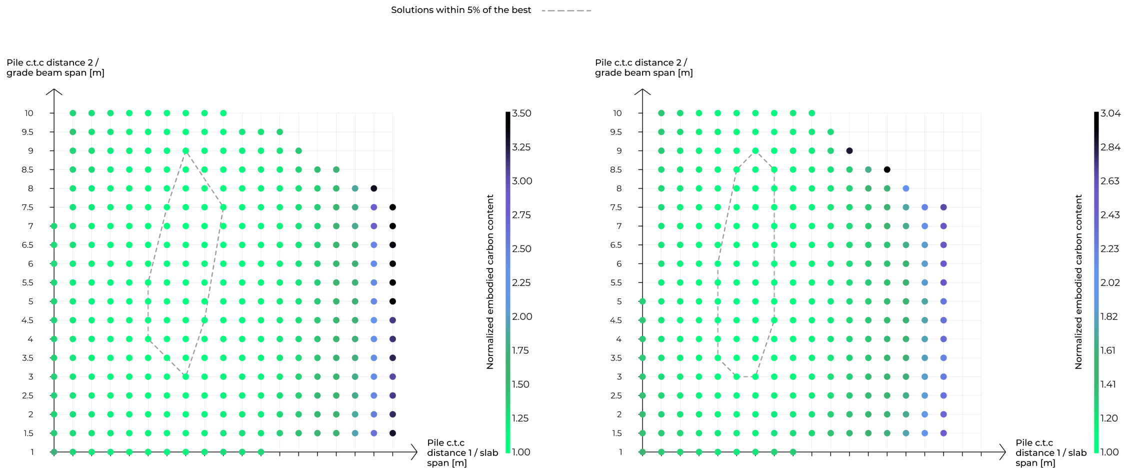
From the results so far, it can be concluded that concrete piles are the preferred choice for piled foundations from a climate standpoint in the case study. However, in situations where disturbance to the soil must be minimized, or room for installation is limited, steel piles are advantageous due to their classification as low-displacement piles and with the possibility to be drilled, i.e. being non-displacement piles. Concrete non-displacement piles have the same advantage but are, however, not preferred in cases of polluted soil. Therefore, in some scenarios, steel piles might be necessary.

Additionally, low or non-displacement piles have the advantage of limiting the pile displacement during installation. This is beneficial in specific cases, such as for one-way spanning slabs with grade beams. However, whenever feasible, concrete piles should be prioritized as the preferred option.

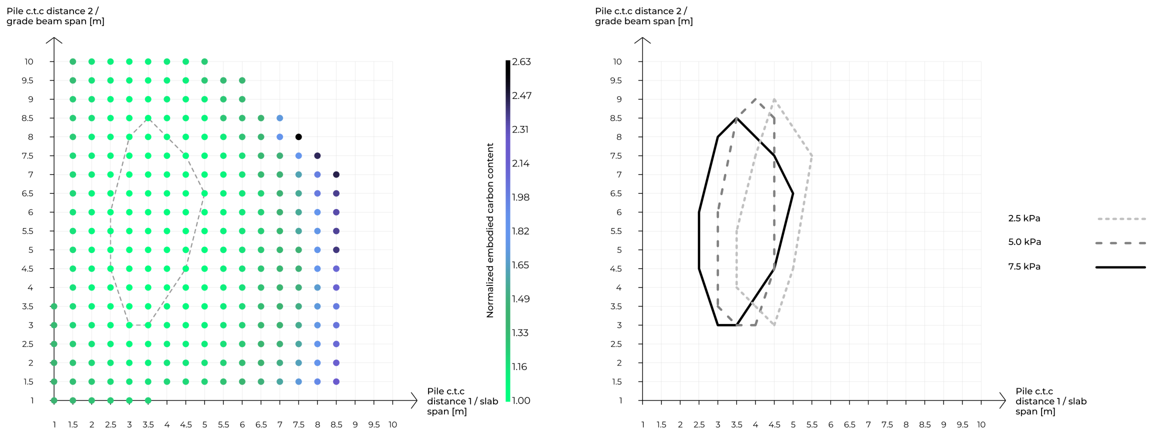
Given the aim of this thesis, which is to identify carbon optimized designs of piled foundations, additional analyses and discussions will focus on the utilization of concrete piles.

## 4.3 Embodied carbon with varying imposed load

The impact of varying imposed loads, representing different facility purposes, on the optimal design is also being investigated. Results from different foundation types are presented in Figures 4.9 to 4.12. For each foundation type, the optimal pile center-to-center distances are studied, depending on the imposed load, in their respective local normalization scale. Imposed loads of 2.5, 5.0 and 7.5 kPa are examined, corresponding to load categories B, C or D and E1 in the Eurocode, see Table 3.15. Additionally, a graph is included for each foundation type, summarizing the range of optimal center-to-center distances for each load.



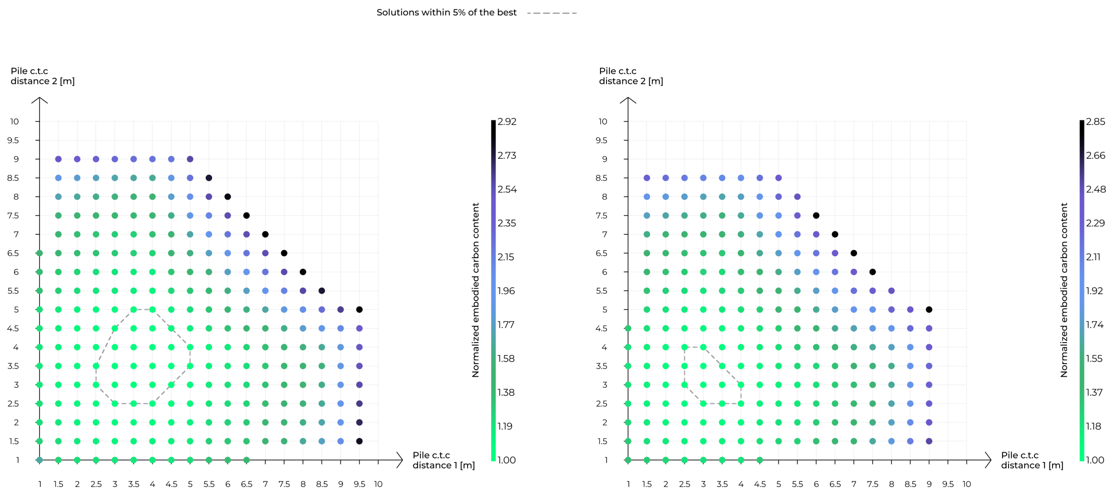

(a) 2.5 kPa respectively 5.0 kPa load.



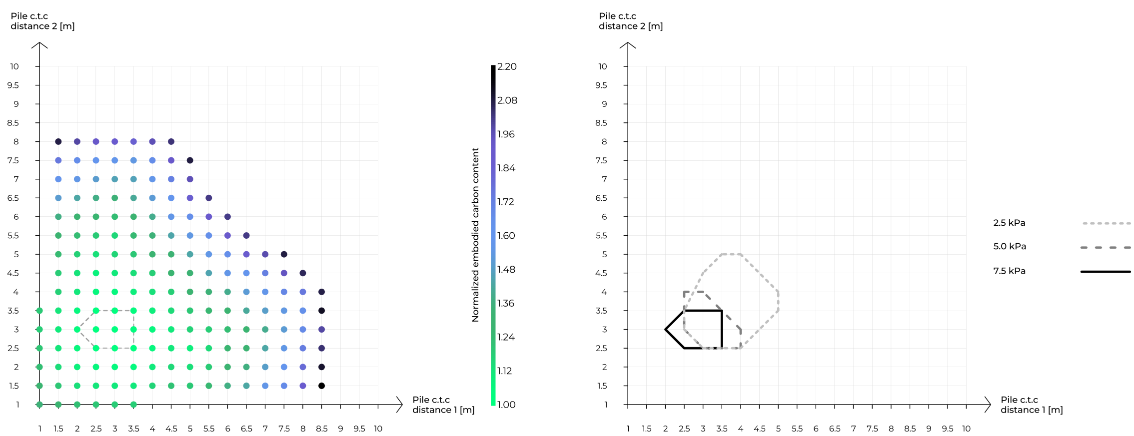

(b) 7.5 kPa load respectively the c.t.c distance spread of the solutions within 5% of the best solution.

**Figure 4.9:** Load influence on the optimal pile c.t.c distance. The spread of the solutions within 5% of the best is marked with a dashed line. Pile type: shaft bearing concrete, slab type: one-way.

For the one-way slab with shaft bearing piles, the results shown in Figure 4.9 suggests that the optimized range of beam spans is large, between 3.0 to 8.5 or 9.0 m, and almost similar for all imposed loads examined. The slab span should, however, be decreased as more load is applied to minimize the slab thickness as much as possible, in accordance with the discussion in Section 4.1.1.



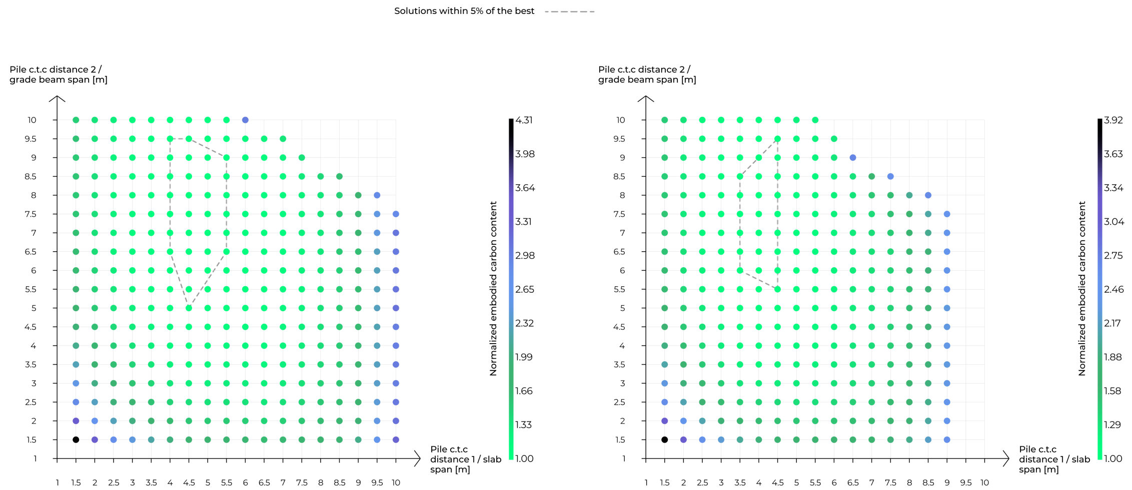

(a) 2.5 kPa respectively 5.0 kPa load.



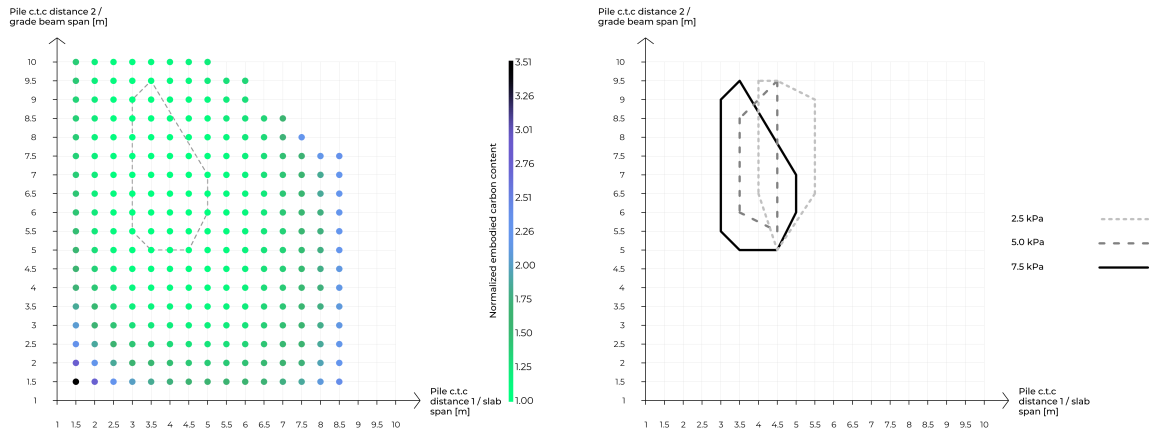

(b) 7.5 kPa load respectively the c.t.c distance spread of the solutions within 5% of the best solution.

**Figure 4.10:** Load influence on the optimal pile c.t.c distance. The spread of the solutions within 5% of the best is marked with a dashed line. Pile type: shaft bearing concrete, slab type: two-way.

In Figure 4.10, the results for the two-way slab with shaft bearing piles shows that the optimal pile center-to-center distances decreases more uniformly in both directions as the load increases. Also, the range (or area) of optimal solutions decreases as the load increases to a more concentrated area of solutions.



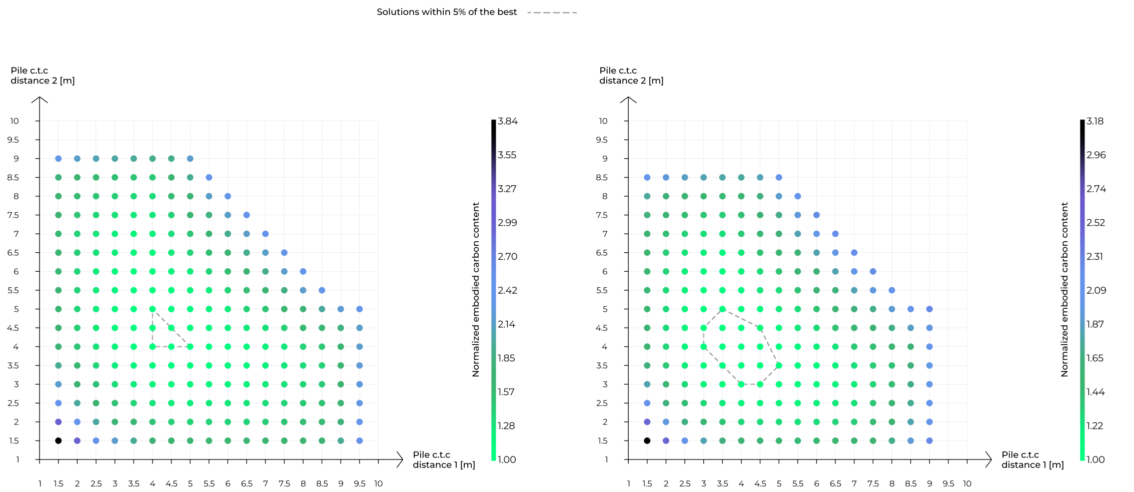

(a) 2.5 kPa respectively 5.0 kPa load.



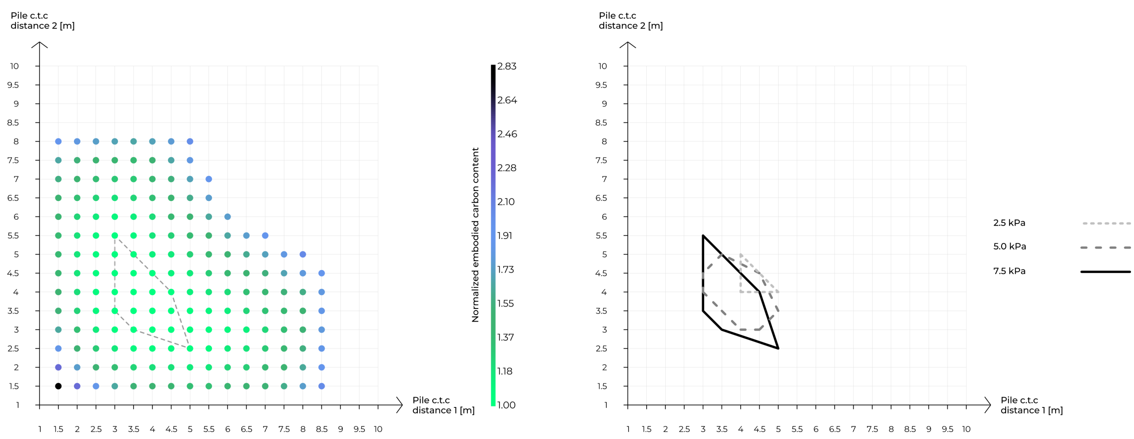

(b) 7.5 kPa load respectively the c.t.c distance spread of the solutions within 5% of the best solution.

**Figure 4.11:** Load influence on the optimal pile c.t.c distance. The spread of the solutions within 5% of the best is marked with a dashed line. Pile type: end bearing concrete, slab type: one-way.

For the one-way slab, now end bearing and shown in Figure 4.11, the results suggests similar optimized solutions as the shaft bearing option for the slab span. However, the beam span is instead recommended to be longer, covering only the upper half of the domain of the optimized solutions for the shaft bearing option. This difference, for the end bearing option, occurs due to the fixed depth to bedrock where different depths likely will affect the optimized grade beam span lengths.




(a) 2.5 kPa respectively 5.0 kPa load.




(b) 7.5 kPa load respectively the c.t.c distance spread of the solutions within 5% of the best solution.

**Figure 4.12:** Load influence on the optimal pile c.t.c distance. The spread of the solutions within 5% of the best is marked with a dashed line. Pile type: end bearing concrete, slab type: two-way.

In Figure 4.12, the two-way end bearing slab shows a similar behavior as the shaft bearing option during load increase. However, instead of decreasing the range (or area) of optimal pile center-to-center distances when increasing the load, the end bearing option instead increases the range.

When comparing the graphs of different foundation types with different imposed loads, it is observed generally that as the load increases, the optimal center-to-center distance between the piles decreases. With an increasing load, the slab thickness must increase. To compensate for this, the span length decreases to limit the necessary thickness increase, which is proved to be a good strategy to minimize the embodied carbon, discussed in Section 4.1.1.

## 4.4 Optimal center-to-center distance effect of varying ground conditions

Separate studies are conducted to examine the sensitivity of the results to the assumed ground conditions. The analyses are performed on a two-way spanning slab with concrete piles subjected to an imposed load of 7.5 kPa. Both end bearing and shaft bearing piles are investigated.

For shaft bearing piles, the effect of varying undrained soil shear resistance is investigated. Alterations are done to the linear soil shear resistance increase along the soil depth, originally set to 1.2 kPa/m. The results indicates that with a lower soil shear resistance, the length of the piles increases and thus the total pile volume. A longer pile can carry more load than two short piles with the same total length, because of the increasing shear resistance along the depth. Consequently, to compensate for the increased pile volume, the optimal span length increases. However, it is observed that the recommended span length changes only slightly compared to significant variations in the soil shear resistance. A change of the linear soil shear resistance increase by 33%, to 0.8 or 1.6 kPa/m, results in an approximate 100 mm change in the recommended center-to-center distance. Variations of the soil shear resistance increase in these magnitudes are therefore deemed negligible for the optimal pile center-to-center distances.

For end bearing piles, the effect of varying depth to bedrock is investigated. Different depths of 5, 10, and 20 meters are examined in addition to the original case study of 15 meters. Similarly to the case for the shaft bearing piles, results indicate that with an increase in pile length, the recommended span length increases, and vice versa. The results reveal an approximate 250 mm increase in recommended span length for each 5 meter increment in depth to bedrock.

## 4.5 Embodied carbon effect of concrete class

To investigate the effect of using different concrete classes in the foundation design, multiple Wallacei analyses are conducted, one for each concrete class and foundation type. Investigated are both a one-way and a two-way slab with shaft bearing concrete piles. Results from the analyses are presented in Figure 4.13.

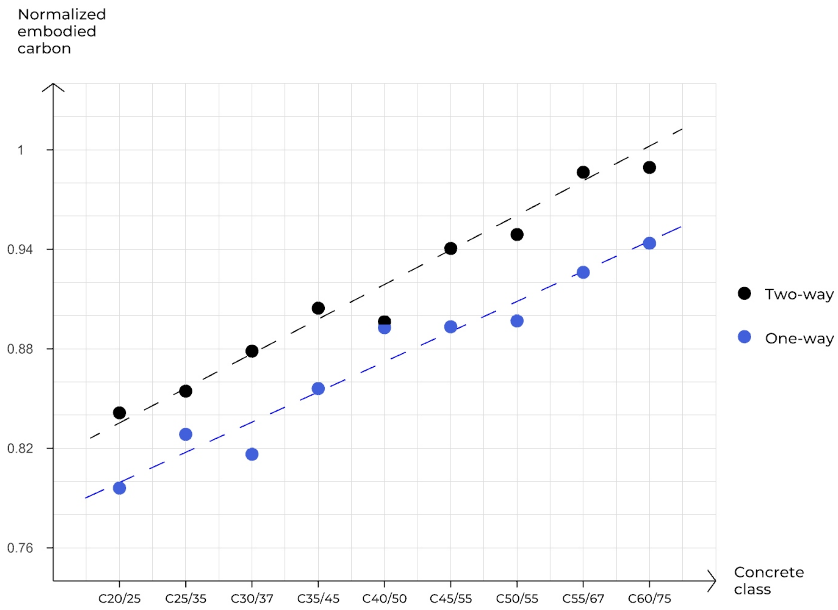



**Figure 4.13:** Effect of concrete class for the embodied carbon.

It is observed in the graph that concrete classes with a lower characteristic strength generally allow for design solutions with lower embodied carbon. Consequently, using lower strength concrete results in a lower total cement content, despite the total increase of concrete volume. As the concrete class C20/25 results in the lowest embodied carbon content for both the one-way and two-way foundation, the analyses regarding center-to-center distances between piles and presented in Sections 4.1 to 4.4, are based on the use of C20/25 concrete.

## 4.6 Asking structural engineers

In addition to meet the thesis objectives of providing recommendations of design to reduce the carbon footprint of piled foundations, it is of interest to evaluate the potential impact of these results. The results presented allow for a comparison of different design choices within the case study, however does not give an indication of how much embodied carbon that potentially could be saved by following the recommendations in practice. To acquire an indication of how foundation designs have been commonly made during the last years, a form was sent out to structural engineer employees at Sweco Sweden where approximately 300 engineers had access.

The form asked the engineers to create an approximate piled foundation design for the same building type, site and loads used in the case study, based on intuition and previous experience from similar projects. They were informed to make basic or no calculations. The engineers were then asked to make selections regarding the following to be used in their design:

- One-way or two-way spanning slab
- Concrete or steel piles
- Center-to-center distances between piles in both directions
- Slab thickness
- Concrete class
- Reinforcement mesh diameter and spacing

The engineers were also requested to provide information regarding their experience in the topic, in terms of number of projects experienced. They were not asked specifically to design with sustainability in mind, although the topic of the thesis was presented with the form.

A total of 14 engineers, with varied experience, answered the form. Based on the responses, a two-way spanning slab with concrete piles is most common for design of piled foundations, as shown in Figure 4.14. Additionally, the majority chose concrete class C30/37 and either a 100x100x10mm or a 150x150x12mm reinforcement mesh.

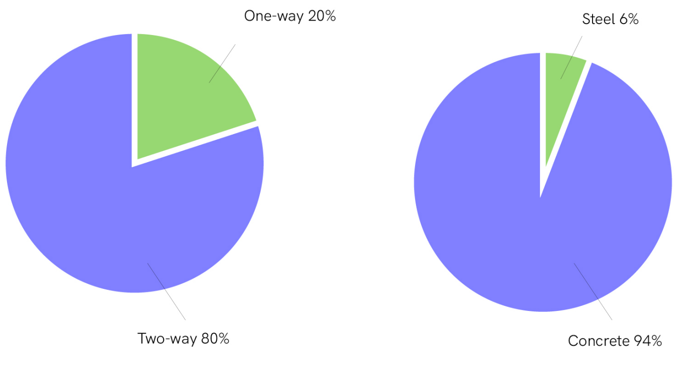



**Figure 4.14:** Division of responses on slab type and pile material.

Following, in Table 4.1, are the results from the form with the engineers' preferred choices with their respective level of experience in the topic. With a large majority selecting a two-way slab as the preferred option, the results of these were computed with the tool and compared to the optimal solution in Figure 4.15.

**Table 4.1:** Choice of spans and slab thickness from form, one-way slab in grey.

| CC-distance 1 (slab span) [m] | CC-distance 2 (beam span) [m] | Slab thickness [mm] | Project exp. |
|---|---|---|---|
| 6 | 5 | 250 | 1-3 |
| 7 | 7 | 400 | 10 + |
| 6 | 6 | 300 | 10 + |
| 6 | 4.5 | 250 | 10 + |
| 7 | 7 | 250 | 7-10 |
| 6.8 | 6.8 | 250 | 10 + |
| 4 | 3 | 250 | 4-6 |
| 3 | 3 | 250 | 10 + |
| 4 | 4 | 250 | 1-3 |
| 5 | 4 | 250 | 7-10 |
| 4 | 4 | 500 | 4-6 |
| 8 | 8 | 200 | 4-6 |
| 6 | 3 | 250 | 4-6 |
| 6 | 6 | 250 | 1-3 |

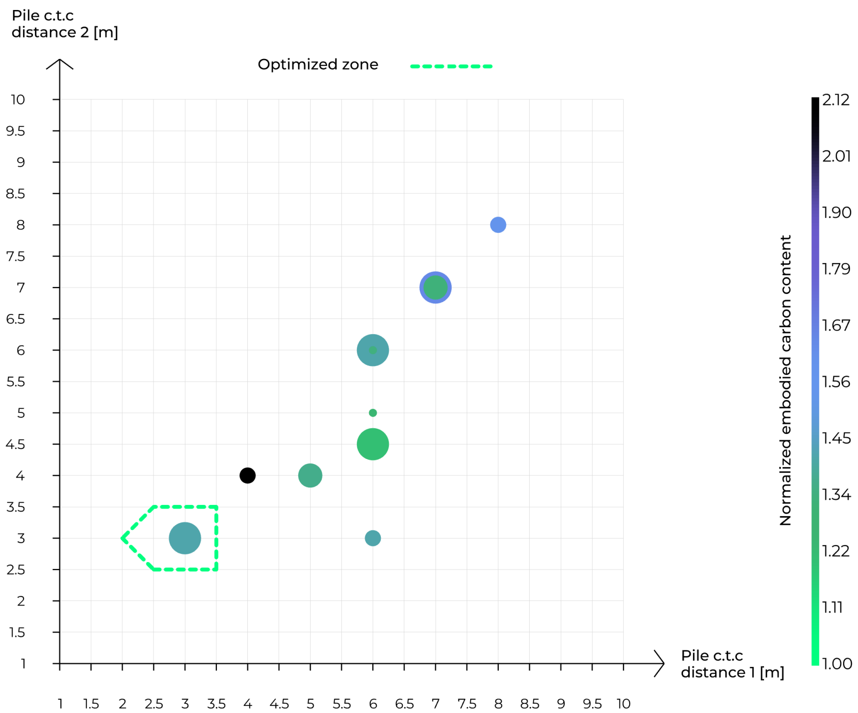



**Figure 4.15:** Engineers' choice of c.t.c distance compared to the optimized solution zone. A larger dot size indicates a more experienced person's answer. Pile type: shaft bearing concrete, slab type: two-way.

The results from the form shows that 60% of the engineers chose a c.t.c distance of 6 meters or more in one direction. Additionally, 55% of those responses had a high degree of experience with 7 or more experienced projects. As seen in Figure 4.15, the embodied carbon content of the suggested designs from the form ranges from a 20% to a 112% increase compared to the suggested optimal solution, with a mean value of 52%. These findings indicate that the results from the research in the thesis recommend engineers to decrease the typical span length, also thereby decreasing the slab thickness, in designs of piled foundation to reduce the total embodied carbon.

Not included in this thesis, however most likely a consideration of highly experienced engineers, is the estimation of project cost. Generally, reducing the amount of material in a structure leads to decreased material cost, making a material-optimized design advantageous from both a financial and environmental perspective. However, in the case of piled foundations, the installation of each pile requires additional time and labour. Consequently, it is reasonable to assume that an increased number of piles would raise the overall project cost. This possibility provides a plausible explanation for the design suggestions put forth by the experienced engineers.

## 4.7 Design guideline

The results and findings from the research is summarized into a design guideline. The guideline is created with the aim to provide engineers with an indication in early design stages of how to design piled foundations for low embodied carbon. It includes an introduction to the subject, a section of general design recommendations and tables with carbon optimized center-to-center distances and slab thicknesses. The recommendations should serve as an initial estimation of design, or as a design to strive towards. Additionally, information is provided of the case study investigated, the impact of variations in ground conditions and a description of the difference in results of one-way and two-way slabs.

The general recommendations include opting for a low concrete class with cement replacements, to go for concrete piles as well as minimizing slab thickness. Figure 4.16 shows the guideline in its entirety, and a full scale version is attached in Appendix A.


**Figure 4.16:** Design guideline.

The tables included in the guideline presents recommendations of pile center-to-center distances and slab thicknesses for a set of different foundation types and loads, shown in Figure 4.17. The tables include one-way and two-way slabs with end bearing or shaft bearing concrete piles in cohesive soil, and three different values of imposed load. Recommended design choices are based on results from both previously presented Colibri analyses, as well as detailed analyses using Wallacei.

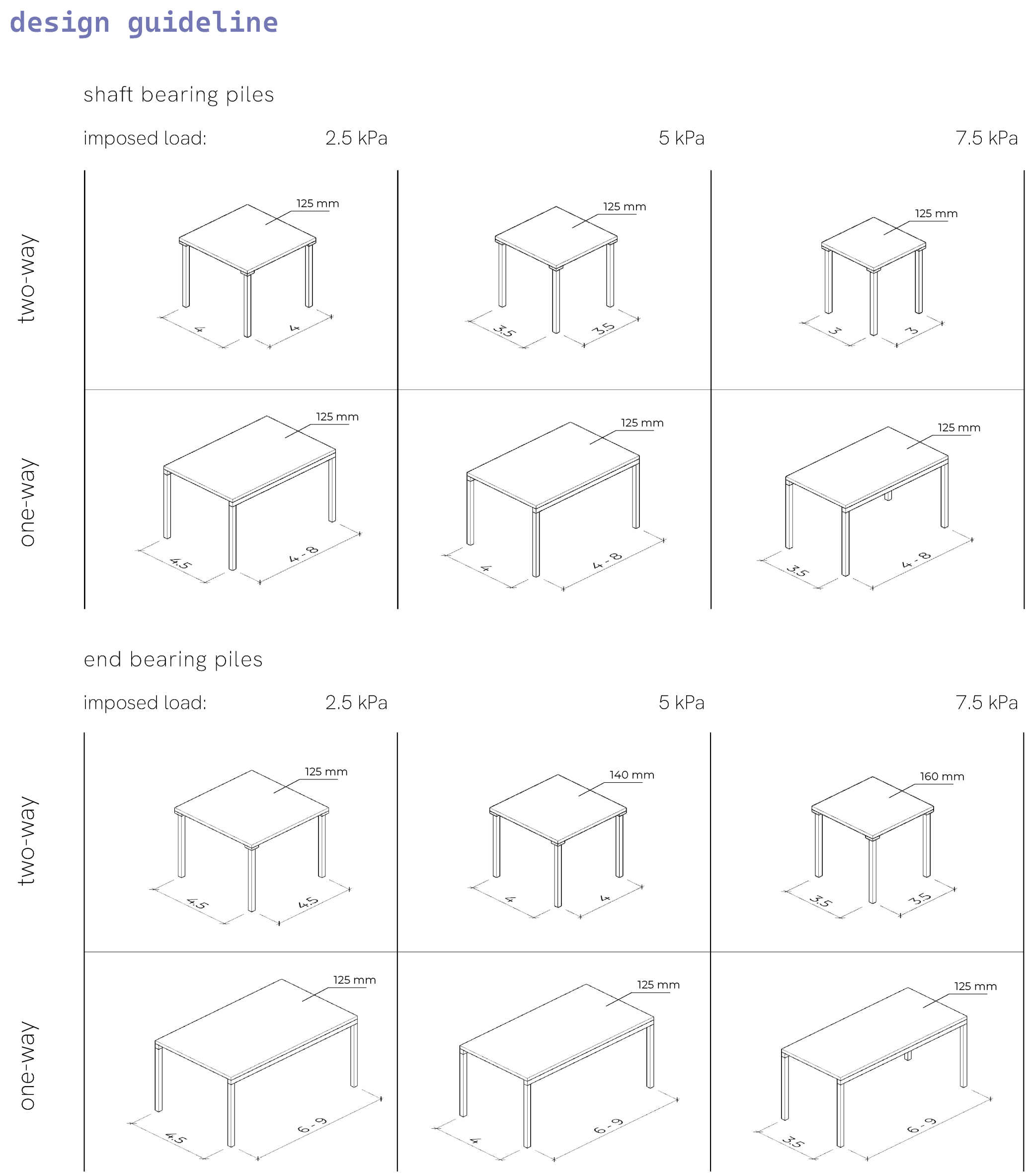



**Figure 4.17:** Design guide of recommended c.t.c distances and slab thicknesses.

# 5 Conclusion

The findings in the thesis concludes that there are possibilities within the design process for engineers to significantly decrease the embodied carbon content of piled foundations. The foremost finding is that an optimal design approach is to opt for the thinnest feasible slab. Afterward, the span length should be selected to fully utilize the chosen slab's load-bearing capacity, followed by designing the piles accordingly. This results in an optimal choice of span lengths for the foundation slab, depending on the applied loading. Furthermore, in the case of a one-way spanning foundation slab, the slab can remain thin even as the grade beam span increases. Subsequently, a larger range of spans is recommended for a one-way slab with concrete piles. The choice between a one-way or two-way slab is not significant from a sustainability standpoint if the corresponding recommendations are followed. Consequently, the one-way slab is advantageous as it offers more design freedom while maintaining a low embodied carbon value.

The results in the thesis suggest a general higher embodied carbon content in foundations with steel piles compared to concrete piles, especially in the case for shaft bearing foundations in cohesive soil. It is important to note, however, that the efficiency of shaft bearing steel piles can be increased, thus reducing the aforementioned difference. Nevertheless, the thesis provides conclusive recommendations to opt for concrete piles whenever feasible to limit the total embodied carbon content.

The analysis results have been summarized into a design guideline, which includes general recommendations and specific design recommendations for pile center-to-center distances and slab thicknesses for different scenarios. The general recommendations suggest opting for a low concrete class, using cement replacers, choosing concrete over steel, and minimizing slab thickness.

By following the recommendations presented in the design guideline regarding span length and slab thickness, assuming the use of concrete piles, the total embodied carbon content can be significantly reduced. Based on a qualitative study, engineers experienced in foundation design suggested designs that varied from a 20% to 112% increase of the embodied carbon compared to the recommended design concluded from the research, with a mean value of 52%. Based on this study, the findings in the thesis recommends engineers to decrease the slab thickness and pile spacing in future piled foundation designs, and thereby reducing the embodied carbon.

The use of parametric modelling, combined with optimization and iteration engines, has been demonstrated as a powerful approach in the thesis for evaluating numerous structural design options. Without these tools, conducting a comparative study on the climate impact of different design choices at this scale would not have been feasible. In conclusion, the application and advancement of these tools have the potential to make a significant impact on the building industry's journey towards achieving climate neutrality.

## 5.1 Future research

For future research, it would be interesting and useful to develop the optimization tool to be project-specific. The aim of the thesis was to create a general guideline for designs to provide to structural engineers. Therefore, the knowledge acquired is accessible to everyone, not just limited to engineers with knowledge of parametric design tools. However, this approach limits the accuracy of the results as it investigates a uniform and simplified structure, as well as assumes certain site and project conditions. Therefore, it would be interesting to develop the tool to be more project-specific as well as user-friendly. Possible additional tool features include the addition of point loads or line loads and the consideration of varying ground conditions across the site.

Furthermore, evaluating the production costs of different designs would be a valuable addition to the tool. As mentioned briefly, the project cost is likely a significant concern for engineers during the design process. Therefore, an estimation of material, manufacturing and installation costs would be a useful inclusion in the tool, allowing for economic evaluation alongside embodied carbon considerations. This addition would transform the optimization problem into a multi-objective one, where production cost becomes an additional objective. Additionally, another potential objective could be to achieve a specific favorable center-to-center distance based on both these factors.

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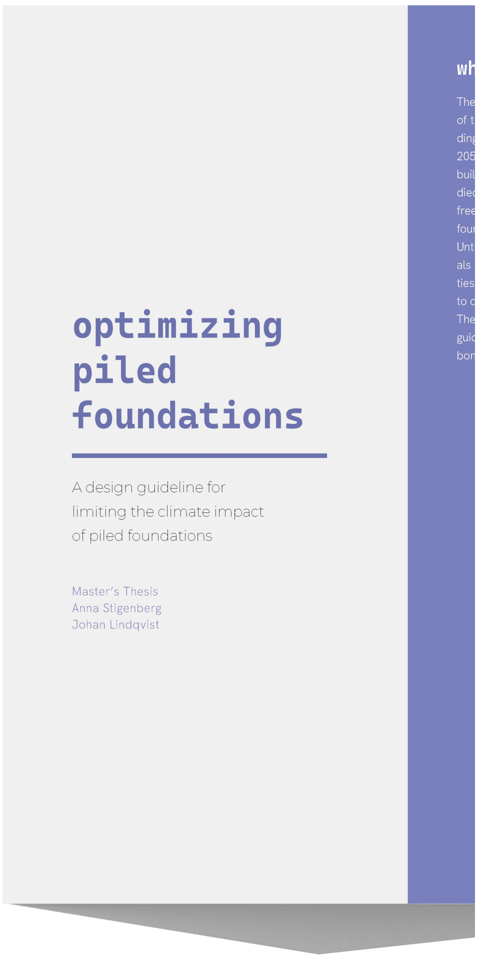
# A

# Design guideline



## why optimize foundations?

The construction industry is responsible for 11% of the world's carbon emissions and must, according to the Paris Agreement, become net zero by 2050. One method to reduce carbon emissions in buildings is to opt for materials with lower embodied carbon content, such as timber. However, the freedom of selecting materials when designing the foundation of a building is much more restricted. Until the invention of more carbon-neutral materials with the same strength and durability properties as steel and concrete, engineers must strive to optimize the material use in foundation designs. The recommendations given in this folder serve to guide engineers in designing more embodied carbon-efficient piled foundations.

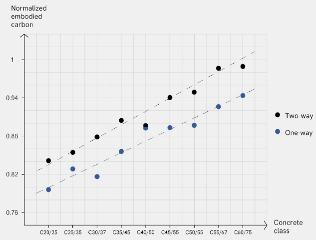
## what is embodied carbon?

The embodied carbon value for a building is defined as the total amount of greenhouse gas emissions emitted throughout the life cycle of its construction, maintenance, and demolition. It serves as a measurement to compare the climate footprint of different structures. Embodied carbon is measured in $CO_2$ equivalents per cubic meter of material, taking into account all potential greenhouse gases and their relative potency compared to CO2. The majority of embodied carbon is emitted during material production and manufacturing stages, specifically denoted as A1-A3, which are the stages considered in this guideline.

## general recommendations

### 01. Choose a low concrete class

The results demonstrate that concrete classes with lower characteristic strength generally enable designs with a slightly reduced embodied carbon, leading to potential reductions of up to 15%. Lower strength concrete facilitates designs with a lower total cement content, despite the necessary increase in concrete volume. Consequently, engineers should consider opting for the lowest acceptable concrete class in foundation designs.

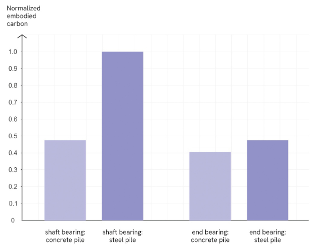



### 02. Use cement replacers

The primary source of carbon emissions in concrete production is the production of cement. By partially substituting cement with waste materials like fly ash or slag, it is possible to reduce the cement content and subsequently also the embodied carbon. This substitution can lead to a reduction in carbon emissions of up to 25% without compromising the strength of the concrete. Ongoing research is continuously exploring methods to further decrease cement content and achieve sustainable concrete production.

## 03. Choose concrete over steel

Analyses indicate that, from a climate standpoint, concrete piles are the preferred choice for piled foundations, regardless of whether shaft or end bearing piles are used. Steel piles have a greater embodied carbon value per meter than a concrete pile with the same structural capacity, along with a smaller circumference. This explains the greater difference observed in the case of shaft bearing piles, as seen in the graph below. However, in some cases, opting for steel piles might be necessary due to potential benefits during installation, and there are special steel piles that results in lower embodied carbon. Nonetheless, concrete piles should be preffered whenever feasible.




## 04. Minimize slab thickness

A key finding from the reasearch, is the importance of a thin foundation slab. The results indicate that the ideal design should strive towards achieving the thinnest possible slab, and then select the span length to fully utilize the slab's capacity. Recommendations provided in the guide all propose a slab thickness below 200 mm.

### how to use the guide

The design guideline is created with the purpose to provide engineers with an indication in early design stages of how to design for low embodied carbon. It should serve as an initial estimate of design, or as a design to strive towards. Recommendations of slab spans and slab thickness is provided for various loads and foundation types.

### what is analysed?

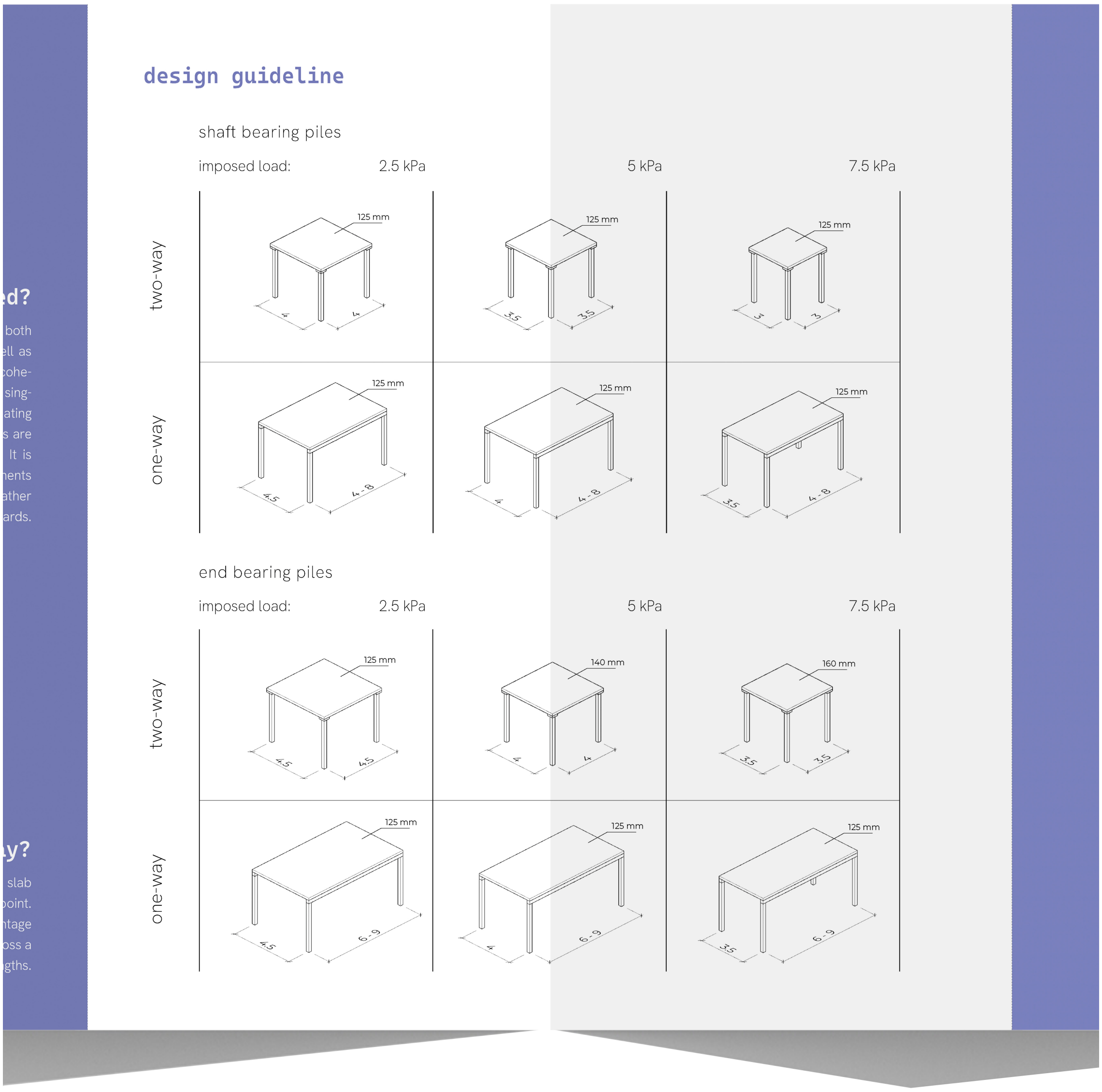
Investigated for the design guideline are both one-way and two-way spanning slabs, as well as both end bearing and shaft bearing piles in cohesive soil. The case study focuses on a large, single-story facility with wide roof spans, eliminating the presence of point loads. Optimal designs are examined for various imposed load values. It is important to note that the design of the elements themselves is not aimed at optimization but rather follows common industry dimensioning standards.

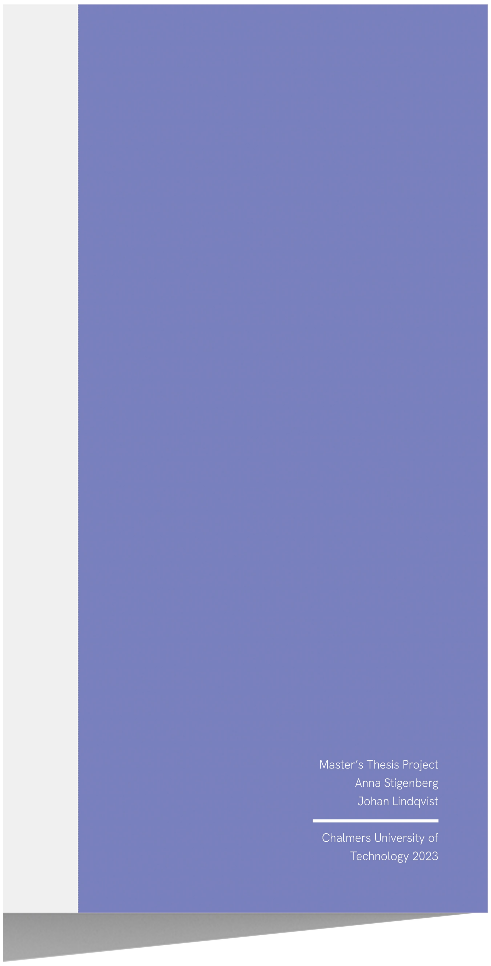
### other ground conditions

For the analysis of shaft bearing piled foundations, a deep clay area is assumed with an undrained shear strength profile of 20 + 1.2 times the soil depth [kPa], characteristic value. Changing the increase value by 33% proved to have minimal affect on the recommended design choices. For end bearing piles, a depth of 15 meters to bedrock is assumed. The analyses show that an increase in depth by 10 meters results in an increase of one meter in the recommeded slab span length.

### one-way or two-way?

The choice between an one-way or two-way slab is not significant from a sustainability standpoint. However, the one-way slab offers the advantage of mainting a low embodied carbon value across a wide range of beam span lengths.


design guideline
shaft bearing piles
imposed load:
2.5 kPa
5 kPa
7.5 kPa
two-way
one-way
125 mm
end bearing piles
imposed load:
2.5 kPa
5 kPa
7.5 kPa
two-way
one-way
125 mm
140 mm
160 mm


Master's Thesis Project
Anna Stigenberg
Johan Lindqvist
Chalmers University of
Technology 2023

# B
# Design calculations

# B.1 One-way slab

## One-way slab

- Designed according to theory of elasticity (linear elastic) without plastic redistribution

- Designed for bending, shear, reinforcement anchorage and crack width

### 1. Inputs

#### 1.1 Geometry

$w_{slab} := 5\ \mathrm{m}$ *slab width*

$h_{slab} := 0{,}135\ \mathrm{m}$ *slab height*

$L_{slab} := 3\ \mathrm{m}$ *slab span*

$\phi_{top} := 0{,}01\ \mathrm{m}$ *top reinforcement diameter*

$s_{top} := 0{,}15\ \mathrm{m}$ *top reinforcement spacing*

$\phi_{bottom} := 0{,}007\ \mathrm{m}$ *bottom reinforcement diameter*

$s_{bottom} := 0{,}1\ \mathrm{m}$ *bottom reinforcement spacing*

#### 1.2 Material

1.2.1 Concrete

**EN-1992-1-1 Table 3.1**

$f_{ck} := 20000000\ \mathrm{Pa}$ *concrete compression strength*

$f_{cm} := f_{ck} + 8\ \mathrm{MPa} = 28\ \mathrm{MPa}$ *concrete mean compression strength*

*concrete mean axial tensile strength*

$$\begin{array}{l} \text{if } f_{ck} \le 50\ \mathrm{MPa} \qquad\qquad = 2{,}2104\ \mathrm{MPa} \\ \quad f_{ctm} := 0{,}3 \cdot \left(\dfrac{f_{ck}}{\mathrm{MPa}}\right)^{\frac{2}{3}} \mathrm{MPa} \\ \text{else} \\ \quad f_{ctm} := 2{,}12 \cdot \ln\left(1 + \left(\dfrac{\dfrac{f_{cm}}{\mathrm{MPa}}}{10}\right)\right) \mathrm{MPa} \end{array}$$

$f_{ctk} := 0{,}7 \cdot f_{ctm} = 1{,}5473\ \mathrm{MPa}$ *concrete tensile strength*

$$E_{cm} := 22 \cdot \left(\frac{\left(\dfrac{f_{cm}}{\mathrm{MPa}}\right)}{10}\right)^{0{,}3} \mathrm{GPa} = 29{,}962\ \mathrm{GPa}$$ *concrete modulus of elasticity*

$$\varepsilon_{cu} = \begin{cases} \text{if } f_{ck} \le 50\ \text{MPa} \\ \quad \varepsilon_{cu} := 3{,}5 \cdot 10^{-3} \\ \text{else} \\ \quad \varepsilon_{cu} := 2{,}6 + 35 \cdot \left( \dfrac{\left( 90 - \dfrac{f_{ck}}{\text{MPa}} \right)}{100} \right)^4 \end{cases} = 0{,}0035$$

concrete ultimate strain

$\rho_c := 23500\ \dfrac{\text{N}}{\text{m}^3}$

concrete density

### 1.2.2 Reinforcement steel K500CT

**EN-1992-1-1 Chapter 3.2**

$f_{yk} := 500\ \text{MPa}$

reinforcement tensile strength

$E_s := 200\ \text{GPa}$

reinforcement modulus of elasticity

### 1.2.3 Design compressive and tensile strengths

**EN-1992-1-1 Chapter 3.1.6**

$\gamma_{c,ULS} := 1{,}5 \qquad \gamma_{c,SLS} := 1$

concrete safety factor (2.4.2.4)

$\gamma_{s,ULS} := 1{,}15 \qquad \gamma_{s,SLS} := 1$

reinforcement safety factor (2.4.2.4)

$\alpha_{cc} := 1{,}0$

coefficient taking account of long term effects on the compressive strength

$\alpha_{ct} := 1{,}0$

coefficient taking account of long term effects on the tensile strength

$$f_{cd} := \frac{\alpha_{cc} \cdot f_{ck}}{\gamma_{c,ULS}} = 13{,}3333\ \text{MPa}$$

(3.15)

$$f_{ctd} := \frac{\alpha_{ct} \cdot f_{ctk}}{\gamma_{c,ULS}} = 1{,}0315\ \text{MPa}$$

(3.16)

$$f_{yd} := \frac{f_{yk}}{\gamma_{s,ULS}} = 434{,}7826\ \text{MPa}$$

(Fig. 3.8)

$$\varepsilon_{yd} := \frac{f_{yd}}{E_s} = 0{,}0022$$

steel ultimate strain

## 1.3 Loads

*Calculated seperately. Support moment reduced with regard to the width of the section.*

$M_{max,supp,ULS} := 73513{,}9225562547\ \text{N m}$

max moment in support in ULS

$M_{max,field,ULS} := 66350{,}542185252\ \text{N m}$

max moment in field in ULS

$V_{max,ULS} := 145774{,}253724539\ \text{N}$

max shear force in ULS

$M_{max,supp,SLS} := 43251{,}3908189458\ \text{N m}$

max moment in support in SLS

$M_{max,field,SLS} := 38664{,}1619868879\ \text{N m}$

max moment in field in SLS

## 1.4 Concrete cover

**EN-1992-1-1 Chapter 4.4**

Exposure class XC2

*minimum concrete cover with respect to durability*

$$\begin{array}{l} \text{if } f_{ck} \geq 35\ \text{MPa} \quad = 25\ \text{mm} \\ \quad c_{min,dur} := 20\ \text{mm} \\ \text{else} \\ \quad c_{min,dur} := 25\ \text{mm} \end{array}$$

$$c_{min,b} := \max\left(\begin{bmatrix} \phi_{top} & \phi_{bottom} \end{bmatrix}\right) = 10\ \text{mm}$$

*minimum concrete cover with respect to anchorage*

$$\Delta c_{dev} := 10\ \text{mm}$$

*extra for deviatons*

$$c_{min,main} := \max\left(\begin{bmatrix} c_{min,b} & c_{min,dur} \end{bmatrix}\right) + \Delta c_{dev} = 35\ \text{mm}$$

*minimum concrete cover main reinforcement*

Dimensioning bending reinforcement with rectangular stress block

**EN-1992-1-1 Chapter 3.1.7**

$$d_{supp} := h_{slab} - c_{min,main} - \frac{\phi_{top}}{2} = 95\ \text{mm}$$

*internal lever arm tensile reinforcement support section*

$$d_{supp,1} := c_{min,main} + \frac{\phi_{bottom}}{2} = 0,0385\ \text{m}$$

*internal lever arm compression reinforcement support section*

$$d_{field} := h_{slab} - c_{min,main} - \frac{\phi_{bottom}}{2} = 96,5\ \text{mm}$$

*internal lever arm tensile reinforcement field section*

$$d_{field,1} := c_{min,main} + \frac{\phi_{top}}{2} = 0,04\ \text{m}$$

*internal lever arm compression reinforcement field section*

*effective stress block height factor*

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 0,8 \\ \quad \lambda := 0,8 \\ \text{else} \\ \quad \lambda := 0,8 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{400} \end{array}$$

*effective stress block strength factor*

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 1 \\ \quad \eta := 1,0 \\ \text{else} \\ \quad \eta := 1,0 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{200} \end{array}$$

## 1.5 Reinforcement areas

$$n_{s,top} := \frac{w_{slab}}{s_{top}} = 33,3333 \qquad n_{s,bottom} := \frac{w_{slab}}{s_{bottom}} = 50$$

*number of bars possible (top and bottom main reinforcement)*

$$n_{s,top} := 33 \qquad n_{s,bottom} := 50$$

*adjusted to lower possible value*

$$A_{s,top} := \left(\frac{\phi_{top}}{2}\right)^2 \cdot \pi \cdot n_{s,top} \qquad A_{s,bottom} := \left(\frac{\phi_{bottom}}{2}\right)^2 \cdot \pi \cdot n_{s,bottom}$$

*reinforcement area*

X

Secondary reinforcement

$$n_{s,2nd,top} := \frac{L_{slab}}{s_{top}} = 20$$

*number of bars possible, secondary reinforcement top layer*

$$n_{s,2nd,top} := 20$$

$$A_{s,2nd,top} := \left(\frac{\phi_{top}}{2}\right)^2 \cdot \pi \cdot n_{s,2nd,top}$$

$$n_{s,2nd,bottom} := \frac{L_{slab}}{s_{bottom}} = 30$$

*number of bars possible, secondary reinforcement bottom layer*

$$n_{s,2nd,bottom} := 30$$

$$A_{s,2nd,bottom} := \left(\frac{\phi_{bottom}}{2}\right)^2 \cdot \pi \cdot n_{s,2nd,bottom}$$

### 1.6 Check minimum reinforcement

**EN-1992-1-1 Chapter 9.3.1.1 (3)**

```
if  s_top > min([400 mm  h_slab·3]) = "OK"
  "NOT OK"
else
  "OK"
```

```
if  s_bottom > min([400 mm  h_slab·3]) = "OK"
  "NOT OK"
else
  "OK"
```

## 2. Moment resistance

### 2.1 Support section

**Uncracked section**

$$W_{supp} := \frac{w_{slab} \cdot h_{slab}^{2}}{6} = 0,0152\ m^3$$

$$M_{Rd1\_supp} := W_{supp} \cdot f_{cd} = 202,5\ kN\ m$$

*bending moment resistance uncracked section*

**Cracked section**

**Svenska Betongföreningens "Handbok till Eurocode"' Vol. 1 Chapter X4.2.6.**

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,top} \cdot f_{yd}}{w_{slab} \cdot d_{supp} \cdot \eta \cdot f_{cd}} = 0,1779$$

*mechanical reinforcment*

$$x := \omega \cdot \frac{d_{supp}}{\lambda} = 0,0211\ m$$

*height of pressure zone*

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0,4935$$

*balanced reinforcement amount*

$$\boxed{\begin{array}{l} \text{if } \omega > \omega_{bal} \\ \quad \sigma_{s2} := E_s \cdot \dfrac{d_{supp} - x}{x} \cdot \varepsilon_{cu} \\ \text{else} \\ \quad \sigma_{s2} := f_{yd} \end{array}}$$

*recalculate reinforcement tensile stress if necessary*

$$\omega := \frac{A_{s,top} \cdot \sigma_{s2}}{w_{slab} \cdot d_{supp} \cdot \eta \cdot f_{cd}} = 0,1779$$

$$x := \omega \cdot \frac{d_{supp}}{\lambda} = 0,0211\ m$$

$$F_{cc} := \eta \cdot f_{cd} \cdot w_{slab} \cdot \lambda \cdot x \qquad z_{cc} := d_{supp} - \frac{\lambda}{2} \cdot x$$

*force in concrete*

$$M_{Rd2,supp} := F_{cc} \cdot z_{cc} = 97,5293\ kN\ m$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,supp,ULS}}{W_{supp}} = 4,8404 \text{ MPa}$$ *concrete tensile stress*

$$\begin{array}{|l|}\hline \text{if } \sigma_c < f_{ctm} \qquad = 97,5293 \text{ kN m} \\ \quad M_{Rd,supp} := M_{Rd1\_supp} \\ \text{else} \\ \quad M_{Rd,supp} := M_{Rd2,supp} \\ \hline \end{array}$$

$$\eta_{M,supp} := \frac{M_{max,supp,ULS}}{M_{Rd,supp}} = 0,7538$$ *utilization ratio*

## 2.2 Field section

**Uncracked section**

$$W_{field} := \frac{w_{slab} \cdot h_{slab}^{\ 2}}{6} = 0,0152 \text{ m}^3$$

$$M_{Rd1\_field} := W_{field} \cdot f_{cd} = 202,5 \text{ kN m}$$

**Cracked section**

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,bottom} \cdot f_{yd}}{w_{slab} \cdot d_{field} \cdot \eta \cdot f_{cd}} = 0,13$$

$$x := \omega \cdot \frac{d_{field}}{\lambda} = 0,0157 \text{ m}$$

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0,4935$$

$$\begin{array}{|l|}\hline \text{if } \omega > \omega_{bal} \\ \quad \sigma_{s2} := E_s \cdot \dfrac{d_{field} - x}{x} \cdot \varepsilon_{cu} \\ \quad \omega := \dfrac{A_{s,field} \cdot \sigma_{s2}}{w_{slab} \cdot d_{field} \cdot \eta \cdot f_{cd}} \\ \quad x := \omega \cdot \dfrac{d_{field}}{\lambda} \\ \hline \end{array}$$

$$M_{Rd2,field} := \eta \cdot f_{cd} \cdot w_{slab} \cdot \lambda \cdot x \cdot \left( d_{field} - \frac{\lambda}{2} \cdot x \right) = 75,4843 \text{ kN m}$$ *bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,field,ULS}}{W_{field}} = 4,3688\ \text{MPa}$$

$$\begin{array}{l} \text{if } \sigma_c < f_{ctm} \qquad = 75,4843\ \text{kN m} \\ \quad M_{Rd,field} := M_{Rd1\_field} \\ \text{else} \\ \quad M_{Rd,field} := M_{Rd2,field} \end{array}$$

$$\eta_{M,field} := \frac{M_{max,field,ULS}}{M_{Rd,field}} = 0,879$$

*utilization ratio*

### 2.4 Placement of reinforcement bars

**BBK 3.9.6**

$c_{min,main} = 35\ \text{mm}$

*minimum cover for main bars (as earlier)*

$d_{min,main,I} := 2 \cdot \max\left(\begin{bmatrix} \phi_{top} & \phi_{bottom} \end{bmatrix}\right) = 20\ \text{mm}$

*minimum distance between main bars in one layer*

$d_{min,main,II} := 1,5 \cdot \max\left(\begin{bmatrix} \phi_{top} & \phi_{bottom} \end{bmatrix}\right) = 15\ \text{mm}$

*minimum distance between main bars between layers*

## 3. Shear capacity

**EN-1992-1-1 Chapter 6.2**

$$C_{Rd,c} := \frac{0,18}{\gamma_{c,ULS}} = 0,12$$ *(6.2.2)*

$$k := \min\left(\left[1 + \sqrt{\frac{200\ \mathrm{mm}}{d_{supp}}}\ \ 2\right]\right) = 2$$ *(6.2.2)*

$$\rho_1 := \min\left(\left[\frac{\left(n_{s,top}\right)\cdot\left(\frac{\phi_{top}}{2}\right)^2\cdot\pi}{w_{slab}\cdot d_{supp}}\ \ 0,02\right]\right) = 0,0055$$ *(6.2.2)*

$$V_{Rd,c} := \left(C_{Rd,c}\cdot k\cdot\left(100\cdot\rho_1\cdot\frac{f_{ck}}{\mathrm{MPa}}\right)^{\frac{1}{3}}\right)\cdot w_{slab}\cdot 1000\cdot d_{supp}\cdot 1000\ \frac{\mathrm{N}}{\mathrm{m}^2} = 252,8628\ \mathrm{kN}$$

$$v_{min} := 0,035\cdot k^{\frac{3}{2}}\cdot\left(\frac{f_{ck}}{\mathrm{MPa}}\right)^{\frac{1}{2}} = 0,4427$$

$$V_{Rd,c,min} := v_{min}\cdot w_{slab}\cdot 1000\cdot d_{supp}\cdot 1000\ \frac{\mathrm{N}}{\mathrm{m}^2} = 210,2915\ \mathrm{kN}$$

$$V_{Rd,c} := \max\left(\left[V_{Rd,c}\ \ V_{Rd,c,min}\right]\right) = 252,8628\ \mathrm{kN}$$ *shear capacity without reinforcement*

$$\eta_V := \frac{V_{max,ULS}}{V_{Rd,c}} = 0,5765$$ *utilization ratio*

## 5. Crack widths

**EN-1992-1-1 Chapter 7.3.4**

$w_{max} := 0,3\ \mathrm{mm}$ — *maximum crack width for exposure class XC2*

$k_1 := 0,8$ — *factor concerning reinforcement adhesion properties, 0.8 for non plain bars*

$k_2 := 0,5$ — *factor concerning strain distribution, 0.5 for bending*

$k_3 := 3,4$ — *recommended value*

$k_4 := 0,425$ — *recommended value*

$k_t := 0,4$ — *long term loading*

$c := c_{min,main} = 35\ \mathrm{mm}$

$f_{ct,eff} := f_{ctm}$

### 5.1 Creep coefficient

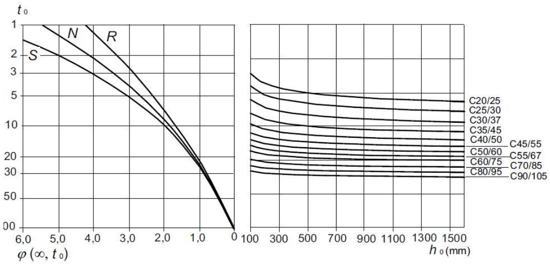
**EN-1992-1-1 Annex B**

$A_c := w_{slab} \cdot h_{slab}$ — *area*

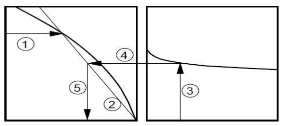
$u := 2 \cdot w_{slab}$ — *circumference exposed to drying*

$$h_0 := \frac{2 \cdot \dfrac{A_c}{u}}{\mathrm{mm}} = 135$$

*ficitve height in mm*




b) utomhusförhållanden – RH = 80%




$t_0 := 28$ $\quad t := 365 \cdot 70$ — *assumed time for loading and life time*

$RH := 80$ — *assumed relative humidity (outdoors)*

$$\alpha_1 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,7} = 1,1691$$

*factors considering the concrete strength*

$$\alpha_2 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,2} = 1,0456$$

$$\alpha_3 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,5} = 1,118$$

$$\begin{aligned}&\text{if } f_{cm} \le 35\,\text{MPa} \quad = 549{,}6197\\ &\quad \beta_H := \min\left(\left[1{,}5\cdot\left(1+(0{,}012\cdot RH)^{18}\right)\cdot h_0 + 250 \;\; 1500\right]\right)\\ &\text{else}\\ &\quad \beta_H := \min\left(\left[1{,}5\cdot\left(1+(0{,}012\cdot RH)^{18}\right)\cdot h_0 + 250\cdot\alpha_3 \;\; 1500\cdot\alpha_3\right]\right)\end{aligned}$$

*factor considering the relative humidity and equivalent thickness*

$$\beta_{c,t,t0} := \left(\frac{t-t_0}{\beta_H + t - t_0}\right)^{0,3} = 0{,}9936$$

*factor considering creep development after loading*

$$\beta_{t0} := \frac{1}{0{,}1 + t_0^{\;0,2}} = 0{,}4884$$

*factor considering the concrete age at loading*

$$\beta_{fcm} := \frac{16{,}8}{\sqrt{\dfrac{f_{cm}}{\text{MPa}}}} = 3{,}1749$$

*factor considering the concrete strength*

$$\alpha_1 := \left(\frac{35\,\text{MPa}}{f_{cm}}\right)^{0,7} = 1{,}1691$$

*factors considering the concrete strength*

$$\alpha_2 := \left(\frac{35\,\text{MPa}}{f_{cm}}\right)^{0,2} = 1{,}0456$$

$$\alpha_3 := \left(\frac{35\,\text{MPa}}{f_{cm}}\right)^{0,5} = 1{,}118$$

$$\begin{aligned}&\text{if } f_{cm} \le 35\,\text{MPa} \quad = 1{,}3899\\ &\quad \varphi_{RH} := 1 + \left(\frac{1-\dfrac{RH}{100}}{0{,}1\cdot h_0^{\frac{1}{3}}}\right)\\ &\text{else}\\ &\quad \varphi_{RH} := \left(1 + \left(\frac{1-\dfrac{RH}{100}}{0{,}1\cdot h_0^{\frac{1}{3}}}\right)\cdot\alpha_1\right)\cdot\alpha_2\end{aligned}$$

*factor considering relative humidity*

$$\varphi_0 := \varphi_{RH}\cdot\beta_{fcm}\cdot\beta_{t0} = 2{,}1554$$

*nominal creep factor*

$$\varphi_{t,t0} := \varphi_0\cdot\beta_{c,t,t0} = 2{,}1416$$

*final creep factor*

$$\varphi_0 := \varphi_{t,t0} = 2{,}1416$$

### 5.2 Support section

Steel tension in cracked section, long term loading

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 2 Chapter A.8**

$$\rho\alpha_e := \frac{(1+\varphi_0)\cdot E_s}{E_{cm}} \cdot \frac{A_{s,top}}{w_{slab}\cdot d_{supp}} = 0{,}1144$$

$$x_{SLS} := d_{supp}\cdot\rho\alpha_e\cdot\left(\sqrt{1+\frac{2}{\rho\alpha_e}}-1\right) = 0{,}0359\ \mathrm{m}$$

*height of pressure zone*

$$\sigma_{s,supp} := \frac{M_{max,supp,SLS}}{A_{s,top}\cdot\left(d_{supp}-\frac{x_{SLS}}{3}\right)} = 200{,}9421\ \mathrm{MPa}$$

*steel tension*

Check of concrete compression

$$\sigma_{c,supp} := \frac{2\cdot M_{max,supp,SLS}}{w_{slab}\cdot x_{SLS}\cdot\left(d_{supp}-\frac{x_{SLS}}{3}\right)} = 5{,}8096\ \mathrm{MPa}$$

*concrete compression*

$$\text{if } \sigma_{c,supp} > 0{,}45\cdot f_{ck}$$

$$\varphi := \varphi_0\cdot\exp\left(1{,}5\cdot\left(\frac{\sigma_{c,supp}}{f_{ck}}-0{,}45\right)\right)$$

$$\rho\alpha_e := \frac{(1+\varphi)\cdot E_s}{E_{cm}}\cdot\frac{A_{s,top}}{w_{slab}\cdot d_{supp}}$$

$$x_{SLS} := d_{supp}\cdot\rho\alpha_e\cdot\left(\sqrt{1+\frac{2}{\rho\alpha_e}}-1\right)$$

$$\sigma_{s,supp} := \frac{M_{max,supp,SLS}}{A_{s,top}\cdot\left(d_{supp}-\frac{x_{SLS}}{3}\right)}$$

*limit for linear creep*

Calculate final crack width

**EN-1992-1-1 Chapter 7.3.4**

$$h_{c,eff} := \min\left(\left[2{,}5\cdot(h_{slab}-d_{supp})\quad \frac{(h_{slab}-x_{SLS})}{3}\quad \frac{h_{slab}}{2}\right]\right) = 33{,}0473\ \mathrm{mm}$$

$$A_{c,eff} := w_{slab}\cdot h_{c,eff} = 1{,}6524\cdot 10^5\ \mathrm{mm}^2$$

$$\rho_{p,eff} := \frac{A_{s,top}}{A_{c,eff}} = 0{,}0157$$

$$s_{r,max} := k_3\cdot c + \frac{k_1\cdot k_2\cdot k_4\cdot\phi_{top}}{\rho_{p,eff}} = 227{,}3804\ \mathrm{mm}$$

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,supp}-k_t\cdot\frac{f_{ct,eff}}{\rho_{p,eff}}\cdot\left(1+\frac{E_s}{E_{cm}}\cdot\rho_{p,eff}\right)}{E_s} = 0{,}0007$$

$$w_{k,supp} := s_{r,max} \cdot \left( \max \left( \left[ \left[ \varepsilon_{sm\_cm} \quad 0,6 \cdot \frac{\sigma_{s,supp}}{E_s} \right] \right] \right) \right) = 0,1577 \text{ mm}$$

*crack width*

```
if  w_k,supp < w_max = "OK!"
  "OK!"
else
  "NOT OK!"
```

### 5.3 Field section

Steel tension in cracked section, long term loading

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 2 Chapter A.8**

$$\rho\alpha_e := \frac{(1+\varphi_0) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,bottom}}{w_{slab} \cdot d_{field}} = 0,0836$$

$$x_{SLS} := d_{field} \cdot \rho\alpha_e \cdot \left( \sqrt{1 + \frac{2}{\rho\alpha_e}} - 1 \right) = 0,0322 \text{ m}$$

*height of pressure zone*

$$\sigma_{s,field} := \frac{M_{max,field,SLS}}{A_{s,bottom} \cdot \left( d_{field} - \frac{x_{SLS}}{3} \right)} = 234,2911 \text{ MPa}$$

*steel tension*

Check of concrete compression

$$\sigma_{c,field} := \frac{2 \cdot M_{max,field,SLS}}{w_{slab} \cdot x_{SLS} \cdot \left( d_{field} - \frac{x_{SLS}}{3} \right)} = 5,5981 \text{ MPa}$$

*concrete compression*

*limit for linear creep*

$$\text{if } \sigma_{c,field} > 0,45 \cdot f_{ck}$$

$$\varphi := \varphi_0 \cdot \exp\left( 1,5 \cdot \left( \frac{\sigma_{c,field}}{f_{ck}} - 0,45 \right) \right)$$

$$\rho\alpha_e := \frac{(1+\varphi) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,bottom}}{w_{slab} \cdot d_{field}}$$

$$x_{SLS} := d_{field} \cdot \rho\alpha_e \cdot \left( \sqrt{1 + \frac{2}{\rho\alpha_e}} - 1 \right)$$

$$\sigma_{s,field} := \frac{M_{max,field,SLS}}{A_{s,bottom} \cdot \left( d_{field} - \frac{x_{SLS}}{3} \right)}$$

Calculate final crack width

**EN-1992-1-1 Chapter 7.3.4**

$$h_{c,eff} := \min \left( \left[ \left[ 2,5 \cdot (h_{slab} - d_{supp}) \quad \frac{(h_{slab} - x_{SLS})}{3} \quad \frac{h_{slab}}{2} \right] \right] \right) = 34,2624 \text{ mm}$$

$$A_{c,eff} := w_{slab} \cdot h_{c,eff} = 1,7131 \cdot 10^5 \text{ mm}^2$$

$$\rho_{p,eff} := \frac{A_{s,bottom}}{A_{c,eff}} = 0,0112$$

$$s_{r,max} := k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{bottom}}{\rho_{p,eff}} = 224,9446 \text{ mm}$$

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,field} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0,0007$$

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0,6 \cdot \frac{\sigma_{s,field}}{E_s}\right]\right)\right) = 0,1683 \text{ mm}$$

*crack width*

```
if  w_k,supp < w_max    = "OK!"
  "OK!"
else
  "NOT OK!"
```

## 6. Volume

$$V_{steel} := \left(A_{s,top} + A_{s,bottom}\right) \cdot L_{slab} + \left(A_{s,2nd,top} + A_{s,2nd,bottom}\right) \cdot w_{slab} = 0,0272 \text{ m}^3$$

$$V_{concrete} := h_{slab} \cdot w_{slab} \cdot L_{slab} - V_{steel} = 1,9978 \text{ m}^3$$

# B.2 Grade beam

## Grade beam

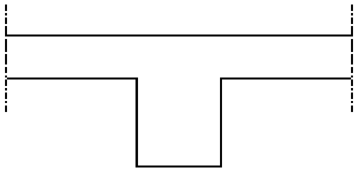
Designed as a continous beam according to Eurocode 2 (Svenska Betongföreningens handbok Vol. 2)

- Designed according to theory of elasticity (linear elastic) without plastic redistribution

- Designed for bending, shear, reinforcement anchorage and crack width

### 1. Inputs

#### 1.1 Geometry



$b_{beam} := 0,235\ \mathrm{m}$ *beam width*

$h_{beam} := 0,36\ \mathrm{m}$ *beam height*

$h_{slab} := 0,135\ \mathrm{m}$ *slab height*

$L_{beam} := 5\ \mathrm{m}$ *span length*

$cc_{beam} := 3\ \mathrm{m}$ *cc distance between beams*

$\phi_{main} := 0,02\ \mathrm{m}$ *main reinforcement diameter*

$\phi_{link} := 0,012\ \mathrm{m}$ *shear reinforcement diameter*

#### 1.2 Material

1.2.1 Concrete

**EN-1992-1-1 Table 3.1**

$f_{ck} := 20000000\ \mathrm{Pa}$ *concrete compression strength*

$f_{cm} := f_{ck} + 8\ \mathrm{MPa} = 28\ \mathrm{MPa}$ *concrete mean compression strength*

*concrete mean axial tensile strength*

$$\begin{array}{l} \text{if } f_{ck} \le 50\ \mathrm{MPa} \qquad = 2,2104\ \mathrm{MPa} \\ \quad f_{ctm} := 0,3 \cdot \left(\dfrac{f_{ck}}{\mathrm{MPa}}\right)^{\frac{2}{3}} \mathrm{MPa} \\ \text{else} \\ \quad f_{ctm} := 2,12 \cdot \ln\left(1 + \left(\dfrac{\frac{f_{cm}}{\mathrm{MPa}}}{10}\right)\right) \mathrm{MPa} \end{array}$$

$f_{ctk} := 0,7 \cdot f_{ctm} = 1,5473\ \mathrm{MPa}$ *concrete tensile strength*

$$E_{cm} := 22 \cdot \left(\dfrac{\left(\frac{f_{cm}}{\mathrm{MPa}}\right)}{10}\right)^{0,3} \mathrm{GPa} = 29,962\ \mathrm{GPa}$$ *concrete modulus of elasticity*

$$\begin{array}{|l|}\hline \text{if } f_{ck} \leq 50\ \text{MPa} \qquad = 0{,}0035 \\ \quad \varepsilon_{cu} := 3{,}5 \cdot 10^{-3} \\ \text{else} \\ \quad \varepsilon_{cu} := 2{,}6 + 35 \cdot \left( \dfrac{\left( 90 - \dfrac{f_{ck}}{\text{MPa}} \right)}{100} \right)^4 \\ \hline \end{array}$$

concrete ultimate strain

$$\rho_c := 23500\ \frac{\text{N}}{\text{m}^3}$$

concrete density

## 1.2.2 Reinforcement steel K500CT

**EN-1992-1-1 Chapter 3.2**

$f_{yk} := 500\ \text{MPa}$ — *reinforcement tensile strength*

$E_s := 200\ \text{GPa}$ — *reinforcement modulus of elasticity*

## 1.2.3 Design compressive and tensile strengths

**EN-1992-1-1 Chapter 3.1.6**

$\gamma_{c,ULS} := 1{,}5 \qquad \gamma_{c,SLS} := 1$ — *concrete safety factor (2.4.2.4)*

$\gamma_{s,ULS} := 1{,}15 \qquad \gamma_{s,SLS} := 1$ — *reinforcement safety factor (2.4.2.4)*

$\alpha_{cc} := 1{,}0$ — *coefficient taking account of long term effects on the compressive strength*

$\alpha_{ct} := 1{,}0$ — *coefficient taking account of long term effects on the tensile strength*

$$f_{cd} := \frac{\alpha_{cc} \cdot f_{ck}}{\gamma_{c,ULS}} = 13{,}3333\ \text{MPa}$$

*(3.15)*

$$f_{ctd} := \frac{\alpha_{ct} \cdot f_{ctk}}{\gamma_{c,ULS}} = 1{,}0315\ \text{MPa}$$

*(3.16)*

$$f_{yd} := \frac{f_{yk}}{\gamma_{s,ULS}} = 434{,}7826\ \text{MPa}$$

*(Fig. 3.8)*

$$\varepsilon_{yd} := \frac{f_{yd}}{E_s} = 0{,}0022$$

*steel ultimate strain*

$$\begin{array}{|l|}\hline \text{if } f_{ck} \leq 50\ \text{MPa} \qquad = 0{,}002 \\ \quad \varepsilon_{c2} := 2 \cdot 10^{-3} \\ \text{else} \\ \quad \varepsilon_{c2} := 2 + 0{,}085 \cdot \left( \left( \dfrac{f_{ck}}{\text{MPa}} \right) - 50 \right)^{0{,}53} \\ \hline \end{array}$$

### 1.3 Loads

Calculated previous to script. Support moments and shear forces are reduced.

$M_{max,supp,ULS} := 146613,253283657\ N\ m$ — *max ULS moment over support*

$M_{max,field,ULS} := 115144,300881863\ N\ m$ — *max ULS moment in field*

$V_{max,ULS} := 101838,858272077\ N$ — *max ULS shear force in critical section*

$M_{max,supp,SLS} := 87616,136179262\ N\ m$ — *max SLS moment over support*

$M_{max,field,SLS} := 68810,2782005166\ N\ m$ — *max SLS moment in field*

$l_{0,supp} := 2,5\ m$ — *length of support section*

$l_{0,field} := 3,85\ m$ — *length of field section*

### 1.4 Concrete cover

**EN-1992-1-1 Chapter 4.4**

Exposure class XC2

$$\begin{array}{l} \text{if } f_{ck} \geq 35\ \text{MPa} \quad = 25\ \text{mm} \\ \quad c_{min,dur} := 20\ \text{mm} \\ \text{else} \\ \quad c_{min,dur} := 25\ \text{mm} \end{array}$$

*minimum concrete cover with respect to durability*

$$c_{min,b} := \phi_{main} = 20\ \text{mm}$$

*minimum concrete cover with respect to anchorage*

$$\Delta c_{dev} := 10\ \text{mm}$$

*extra for deviatons*

$$c_{min,main} := \max\left(\left[\ c_{min,b}\ \ c_{min,dur}\ \right]\right) + \Delta c_{dev} = 35\ \text{mm}$$

*minimum concrete cover main reinforcement*

$$c_{min,link} := c_{min,main} - \phi_{link} = 23\ \text{mm}$$

*minimum concrete cover shear reinforcement*

$$\begin{array}{l} \text{if } c_{min,link} \geq c_{min,dur} + \Delta c_{dev} = \text{"NOT OK!"} \\ \quad \text{"OK!"} \\ \text{else} \\ \quad \text{"NOT OK!"} \end{array}$$

*concrete cover check shear reinforcement*

$$\begin{array}{l} \text{while } c_{min,link} < c_{min,dur} + \Delta c_{dev} \\ \quad c_{min,link} := c_{min,link} + 1\ \text{mm} \end{array}$$

*iterate until sufficient cover for shear links*

$$\begin{array}{l} \text{if } c_{min,link} \geq c_{min,dur} + \Delta c_{dev} = \text{"OK!"} \\ \quad \text{"OK!"} \\ \text{else} \\ \quad \text{"NOT OK!"} \end{array}$$

*concrete cover check shear reinforcement*

$$c_{min,main} := c_{min,link} + \phi_{link} = 47\ \text{mm}$$

*minimum concrete cover main reinforcement*

$$c_{min,link} = 35\ \text{mm}$$

*minimum concrete cover shear reinforcement*

### 1.5 Rectangular stress block

**EN-1992-1-1 Chapter 3.1.7**

$$d := \left(h_{beam} + h_{slab}\right) - c_{min,main} - \frac{\phi_{main}}{2} = 438\ \text{mm}$$

*internal lever arm (from compressed edge to main reinforcement)*

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 0,8 \\ \quad \lambda := 0,8 \\ \text{else} \\ \quad \lambda := 0,8 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{400} \end{array}$$

*effective stress block height factor*

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 1 \\ \quad \eta := 1,0 \\ \text{else} \\ \quad \eta := 1,0 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{200} \end{array}$$

*effective  stress block strength factor*

## 2. Reinforcement design and moment capacity

### 2.1 Support section

#### 2.1.1 Contributing flange width

EN-1992-1-1 Chapter 5.3.2.1

$$b_i := \frac{cc_{beam} - b_{beam}}{2}$$

$$b_{i,eff} := 0,2 \cdot b_i + 0,1 \cdot l_{0,supp} = 0,5265 \text{ m}$$ *(5.7)*

$$\boxed{\begin{array}{l} \text{if } b_{i,eff} > 0,2 \cdot l_{0,supp} \\ \quad b_{i,eff} := 0,2 \cdot l_{0,supp} \end{array}}$$ *(5.7a)*

$$\boxed{\begin{array}{l} \text{if } b_{i,eff} > b_i \\ \quad b_{i,eff} := b_i \end{array}}$$ *(5.7b)*

$$b_{eff,supp} := 2 \cdot b_{i,eff} + b_{beam} = 1,235 \text{ m}$$

$$\boxed{\begin{array}{l} \text{if } b_{eff,supp} > cc_{beam} \\ \quad b_{eff,supp} := cc_{beam} \end{array}}$$

$$x_{tp,supp} := \frac{b_{eff,supp} \cdot \frac{h_{slab}^{\,2}}{2} + b_{beam} \cdot h_{beam} \cdot \left(h_{slab} + \frac{h_{beam}}{2}\right)}{b_{eff,supp} \cdot h_{slab} + b_{beam} \cdot h_{beam}} = 0,1508 \text{ m}$$

*center of gravity uncracked cross section, calculated from compression side*

#### 2.1.2 Calculate needed reinforcement amount

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 1 Chapter X4.2.2.2**

$$m := \frac{M_{max,supp,ULS}}{b_{beam} \cdot d^2 \cdot \eta \cdot f_{cd}} = 0,2439$$

*relative moment without axial force*

$$\omega := 1 - \sqrt{1 - 2 \cdot m} = 0,2843$$

*required mechanical reinforcement amount*

$$A_{s,supp} := \frac{M_{max,supp,ULS}}{d \cdot \left(1 - \frac{\omega}{2}\right) \cdot f_{yd}} = 897,4734 \text{ mm}^2$$

*required reinforcement*

$$n := \max\left(\left[\left[\frac{A_{s,supp}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} \quad 2\right]\right]\right) = 2,8567$$

*number of main bars required*

*chosen number of bars*

$$n_{supp} := 3$$

$$A_{s,supp} := n_{supp} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 942,4778 \text{ mm}^2$$

*area selected*

**2.1.3 Minimum reinforcement** **EN-1992-1-1 Chapter 7.3**

Web part

$k_c := 1$ *for pure tension*

$$\begin{array}{l} \text{if } (h_{beam} + h_{slab}) < 200\ \text{mm} = 0{,}5 \\ \quad k := 0{,}9 \\ \text{else} \\ \quad k := 0{,}5 \end{array}$$

*coeffecient compensating for unequal stresses*

$$f_{ct,eff} := f_{ctm} = 2{,}2104\ \text{MPa}$$

$$A_{ct} := x_{tp,supp} \cdot b_{beam} = 0{,}0354\ \text{m}^2$$

*height of tension zone of concrete section*

$$\sigma_s := f_{yk} = 5 \cdot 10^8\ \text{Pa}$$

*maximum stress permitted in reinforcement, may be set to fyk*

$$A_{s,min} := \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s} = 78{,}3393\ \text{mm}^2$$

*eq. 7.1*

$$n_{min,web} := \frac{A_{s,min}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} = 0{,}2494$$

*number of main bars required*

Flange part

$$A_{ct} := h_{slab} \cdot \frac{(b_{eff,supp} - b_{beam})}{2} = 0{,}0675\ \text{m}^2$$

$$A_{s,min} := \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s} = 149{,}2033\ \text{mm}^2$$

$$n_{min,flange} := \frac{A_{s,min}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} = 0{,}4749$$

$$n_{min} := n_{min,web} + 2 \cdot n_{min,flange} = 1{,}1992$$

$$n_{min} := 2$$

*chosen number of bars*

$$A_{s,min} := n_{min} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 628{,}3185\ \text{mm}^2$$

**EN-1992-1-1 Chapter 9.2**

$$A_{s,min,g} := \min\left(\left[\left[ 0{,}26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_{beam} \cdot d \quad 0{,}0013 \cdot b_{beam} \cdot d \right]\right]\right) = 0{,}0001\ \text{m}^2$$

*general minimum reinforcement*

$$A_{s,supp} := \max\left(\left[ A_{s,supp} \quad A_{s,min} \quad A_{s,min,g} \right]\right)$$

*choosing if minimum area reinforcement or bending moment reinforcement is governing*

$$n_{supp} := \frac{A_{s,supp}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} = 3$$

#### 2.1.4 Calculating moment resistance

**Uncracked section**

Moment of inertia

$$I_{1,supp} := \frac{b_{eff,supp} \cdot h_{slab}^{3}}{12} + b_{eff,supp} \cdot h_{slab} \cdot \left(x_{tp,supp} - \frac{h_{slab}}{2}\right)^{2} + = 0,0032\ \mathrm{m}^{4}$$
$$+ \frac{b_{beam} \cdot h_{beam}^{3}}{12} + b_{beam} \cdot h_{beam} \cdot \left(h_{slab} + \frac{b_{beam}}{2} - x_{tp,supp}\right)^{2}$$

$$W_{supp} := \frac{I_{1,supp}}{h_{beam} + h_{slab} - x_{tp,supp}} = 0,0093\ \mathrm{m}^{3}$$

$$M_{Rd1\_supp} := W_{supp} \cdot f_{cd} = 123,9216\ \mathrm{kN\ m}$$

*bending moment resistance uncracked section*

**Cracked section**

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 1 Chapter X4.2.6.**

Recalculate ω

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,supp} \cdot f_{yd}}{b_{beam} \cdot d \cdot \eta \cdot f_{cd}} = 0,2986$$

$$x := \omega \cdot \frac{d}{\lambda} = 0,1635\ \mathrm{m}$$

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0,4935$$

$$\boxed{\begin{array}{l} \text{if } \ \omega > \omega_{bal} \\ \quad \sigma_{s2} := E_s \cdot \dfrac{d - x}{x} \cdot \varepsilon_{cu} \\ \quad \omega := \dfrac{A_{s,supp} \cdot \sigma_{s2}}{b_{beam} \cdot d \cdot \eta \cdot f_{cd}} \\ \quad x := \omega \cdot \dfrac{d}{\lambda} \end{array}}$$

$$M_{Rd2,supp} := \eta \cdot f_{cd} \cdot b_{beam} \cdot \lambda \cdot x \cdot \left(d - \frac{\lambda}{2} \cdot x\right) = 152,6858\ \mathrm{kN\ m}$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,supp,ULS}}{W_{supp}} = 15,7748\ \mathrm{MPa}$$

*concrete tensile stress*

$$\begin{aligned}&\text{if } \sigma_c < f_{ctm} \\ &\quad M_{Rd,supp} := M_{Rd1\_supp} \\ &\text{else} \\ &\quad M_{Rd,supp} := M_{Rd2,supp}\end{aligned} = 152{,}6858\ \text{kN m}$$

$$\eta_{M,supp} := \frac{M_{max,supp,ULS}}{M_{Rd,supp}} = 0{,}9602$$

*utilization ratio*

## 2.2 Field section

### 2.2.1 Contributing flange width

**EN-1992-1-1 Chapter 5.3.2.1**

$$b_i := \frac{cc_{beam} - b_{beam}}{2} = 1{,}3825\ \mathrm{m}$$

*(5.7)*

$$b_{i,eff} := 0{,}2 \cdot b_i + 0{,}1 \cdot l_{0,field} = 0{,}6615\ \mathrm{m}$$

*(5.7a)*

$$\boxed{\begin{aligned}&\text{if } b_{i,eff} > 0{,}2 \cdot l_{0,field}\\ &\quad b_{i,eff} := 0{,}2 \cdot l_{0,field}\end{aligned}}$$

*(5.7b)*

$$\boxed{\begin{aligned}&\text{if } b_{i,eff} > b_i\\ &\quad b_{i,eff} := b_i\end{aligned}}$$

$$b_{eff,field} := 2 \cdot b_{i,eff} + b_{beam} = 1{,}558\ \mathrm{m}$$

$$x_{tp,field} := \frac{b_{eff,field} \cdot \dfrac{h_{slab}^{\,2}}{2} + b_{beam} \cdot h_{beam} \cdot \left(h_{slab} + \dfrac{h_{beam}}{2}\right)}{b_{eff,field} \cdot h_{slab} + b_{beam} \cdot h_{beam}} = 0{,}1385\ \mathrm{m}$$

*center of gravity uncracked cross section, calculated from tension side*

### 2.2.2 Required reinforcement amount

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 1 Chapter X4.2.3.3 a)**

$$m := \frac{M_{max,field,ULS}}{b_{eff,field} \cdot d^2 \cdot \eta \cdot f_{cd}} = 0{,}0289$$

*relative moment without axial force*

$$\omega := 1 - \sqrt{1 - 2 \cdot m} = 0{,}0293$$

*required mechanical reinforcement amount*

Checking if height of pressure zone is greater than the flange height and recalculating

$$F_{cf} := \left(b_{eff,field} - b_{beam}\right) \cdot h_{slab} \cdot \eta \cdot f_{cd}$$

*(X4-27)*

$$z_{cf} := d - h_{slab}$$

*(X4-27)*

$$\boxed{\begin{aligned}&\text{if } \omega \cdot d > h_{slab}\\ &\quad m := \frac{M_{max,field,ULS} - F_{cf} \cdot z_{cf}}{b_{beam} \cdot d^2 \cdot \eta \cdot f_{cd}}\\ &\quad \omega := 1 - \sqrt{1 - 2 \cdot m}\end{aligned}}$$

*check height of pressure zone*

*(X4-30)*

*(X4-31)*

$$A_{s,field} := \frac{M_{max,field,ULS}}{d \cdot \left(1 - \dfrac{\omega}{2}\right) \cdot f_{yd}} = 613{,}6357\ \mathrm{mm}^2$$

*required reinforcement*

$$n_{field} := \max\left(\left\lceil \frac{A_{s,field}}{\left(\dfrac{\phi_{main}}{2}\right)^2 \cdot \pi} \right\rceil, 2\right) = 2$$

*number of main bars required*

$$n_{field} := 2$$

*chosen number of bars*

$$A_{s,field} := n_{field} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 628,3185\ mm^2$$

*area selected*

**2.2.3 Minimum reinforcement**

**EN-1992-1-1 Chapter 7.3**

$$k_c := 1$$

*for pure tension*

$$\boxed{\begin{array}{l} \text{if } \left(h_{beam} + h_{slab}\right) < 200\ mm = 0,5 \\ \quad k := 0,9 \\ \text{else} \\ \quad k := 0,5 \end{array}}$$

*coeffecient compensating for unequal stresses*

$$f_{ct,eff} := f_{ctm} = 2,2104\ MPa$$

$$A_{ct} := \left(h_{slab} + h_{beam} - x_{tp,field}\right) \cdot b_{beam} = 0,0838\ m^2$$

*height of tension zone of concrete section*

$$\sigma_s := f_{yk} = 5 \cdot 10^8\ Pa$$

*maximum stress permitted in reinforcement, may be set to fyk*

$$A_{s,min} := \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s} = 185,1861\ mm^2$$

*eq. 7.1*

$$n_{min} := \frac{A_{s,min}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} = 0,5895$$

*number of main bars required*

$$n_{min} := 2$$

*chosen number of bars*

$$A_{s,min} := n_{min} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 628,3185\ mm^2$$

$$A_{s,min,g} := \min\left(\left[0,26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_{beam} \cdot d \quad 0,0013 \cdot b_{beam} \cdot d\right]\right) = 0,0001\ m^2$$

**EN-1992-1-1 Chapter 9.2**

*general minimum reinforcement*

$$A_{s,field} := \max\left(\left[A_{s,field} \quad A_{s,min} \quad A_{s,min,g}\right]\right)$$

*choosing if minimum area reinforcement or bending moment reinforcement is governing*

$$n_{field} := \frac{A_{s,field}}{\left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi} = 2$$

#### 2.2.4 Moment resistance

**Uncracked section**

*Moment of intertia*

$$I_{1,field} := \frac{b_{eff,field} \cdot h_{slab}^{3}}{12} + b_{eff,field} \cdot h_{slab} \cdot \left(x_{tp,field} - \frac{h_{slab}}{2}\right)^2 + = 0,0034\ m^4$$
$$+ \frac{b_{beam} \cdot h_{beam}^{3}}{12} + b_{beam} \cdot h_{beam} \cdot \left(h_{slab} + \frac{b_{beam}}{2} - x_{tp,field}\right)^2$$

$$W_{field} := \frac{I_{1,field}}{h_{beam} + h_{slab} - x_{tp,field}} = 0,0095\ m^3$$

$$M_{Rd1\_field} := W_{field} \cdot f_{cd} = 126,8911\ kN\ m$$

**Cracked section**

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 1 Chapter X4.2.6**

Recalculate ω

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,field} \cdot f_{yd}}{b_{eff,field} \cdot d \cdot \eta \cdot f_{cd}} = 0,03$$

$$\boxed{\begin{array}{l} \text{if } \ \omega \cdot d > h_{slab} \\ \quad m := \dfrac{M_{max,field,ULS} - F_{cf} \cdot z_{cf}}{b_{beam} \cdot d^2 \cdot \eta \cdot f_{cd}} \\ \quad \omega := \dfrac{A_{s,field} \cdot f_{yd}}{b_{eff,field} \cdot d \cdot \eta \cdot f_{cd}} \end{array}}$$

$$x := \omega \cdot \frac{d}{\lambda} = 0,0164\ m$$

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0,4935$$

$$\omega_{bal} := \min\left(\left[\ \omega_{bal} \quad \lambda \cdot \frac{\varepsilon_{c2} + 2 \cdot \dfrac{h_{slab}}{2 \cdot d} \cdot \varepsilon_{yd}}{\varepsilon_{cu} + \varepsilon_{yd}}\ \right]\right) = 0,3765$$

$$
\begin{array}{l}
\text{if } \omega > \omega_{bal} \\
\quad \sigma_{s2} := E_s \cdot \dfrac{d - x}{x} \cdot \varepsilon_{cu} \\
\quad \omega := \dfrac{A_{s,field} \cdot \sigma_{s2}}{b_{eff,field} \cdot d \cdot \eta \cdot f_{cd}} \\
\quad \text{if } \omega \cdot d > h_{slab} \\
\quad\quad m := \dfrac{M_{max,field,ULS} - F_{cf} \cdot z_{cf}}{b_{beam} \cdot d^2 \cdot \eta \cdot f_{cd}} \\
\quad\quad \omega := \dfrac{A_{s,field} \cdot \sigma_{s2}}{b_{eff,field} \cdot d \cdot \eta \cdot f_{cd}} \\
\quad x := \omega \cdot \dfrac{d}{\lambda}
\end{array}
$$

$$
\begin{array}{l}
\text{if } x \cdot \lambda < h_{slab} \qquad\qquad\qquad\qquad = 117{,}8574 \ \mathrm{kN\ m} \\
\quad M_{Rd2,field} := \eta \cdot f_{cd} \cdot b_{eff,field} \cdot \lambda \cdot x \cdot \left(d - \dfrac{\lambda}{2} \cdot x\right) \\
\text{else} \\
\quad F_{cw} := b_{beam} \cdot \lambda \cdot x \cdot \eta \cdot f_{cd} \\
\quad z_{cw} := d - \lambda \cdot \dfrac{x}{2} \\
\quad M_{Rd2,field} := \dfrac{F_{cw} \cdot z_{cw} + F_{cf} \cdot z_{cf}}{F_{cw} + F_{cf}}
\end{array}
$$

*(X4-26)*

*(X4-26)*

*(X4-33)*

$$\sigma_c := \frac{M_{max,field,ULS}}{W_{field}} = 12{,}099 \ \mathrm{MPa}$$

*concrete tensile stress*

$$
\begin{array}{l}
\text{if } \sigma_c < f_{ctm} \qquad\qquad = 117{,}8574 \ \mathrm{kN\ m} \\
\quad M_{Rd,field} := M_{Rd1\_field} \\
\text{else} \\
\quad M_{Rd,field} := M_{Rd2,field}
\end{array}
$$

$$\eta_{M,field} := \frac{M_{max,field,ULS}}{M_{Rd,field}} = 0{,}977$$

*utilization ratio*

### 2.3 Minimum reinforcement spacing

**BBK 3.9.6**

$$c_{min,main} = 47 \ \mathrm{mm}$$

*minimum cover for main bars (as earlier)*

$$d_{min,main,I} := 2 \cdot \phi_{main} = 40 \ \mathrm{mm}$$

*minimum distance between main bars in one layer*

$$d_{min,main,II} := 1{,}5 \cdot \phi_{main} = 30 \ \mathrm{mm}$$

*minimum distance between main bars between layers*

## 4. Crack width

**EN-1992-1-1 Chapter 7.3.4**

$w_{max} := 0,3\ mm$ — *maximum crack width for exposure class XC2*

$k_1 := 0,8$ — *factor concerning reinforcement adhesion properties, 0.8 for non pllain bars*

$k_2 := 0,5$ — *factor concerning straindistribution, 0.5 for bending*

$k_3 := 3,4$ — *recommended value*

$k_4 := 0,425$ — *recommended value*

$k_t := 0,4$ — *long term loading*

$c := c_{min,main} = 47\ mm$

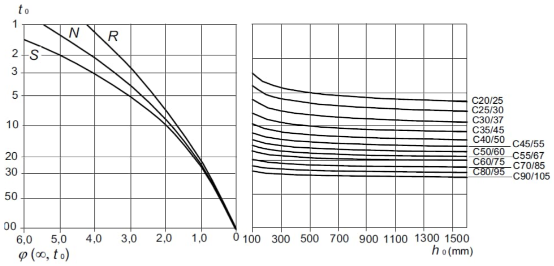
### 4.1 Creep coefficient

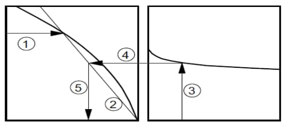
**EN-1992-1-1 Annex B.**

$A_c := b_{eff,supp} \cdot h_{slab} + b_{beam} \cdot h_{beam}$ — *area*

$u := 2 \cdot b_{eff,supp} + 2 \cdot h_{beam}$ — *circumference exposed to drying*

$$h_0 := \frac{2 \cdot \frac{A_c}{u}}{mm} = 157,5705$$ *ficitve height in mm*




b) utomhusförhållanden – RH = 80%

$t_0 := 28$ $\quad t := 365 \cdot 70$ — *assumed time for loading and life time*

$RH := 80$ — *assumed relative humidity (outdoors)*

$$\alpha_1 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0,7} = 1,1691$$ *factors considering the concrete strength*

$$\alpha_2 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0,2} = 1,0456$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0,5} = 1,118$$

$$\begin{aligned}&\text{if } f_{cm} \le 35\ \text{MPa} \qquad = 599{,}7128\\&\quad \beta_H := \min\left(\left[1{,}5\cdot\left(1+(0{,}012\cdot RH)^{18}\right)\cdot h_0 + 250\ \ 1500\right]\right)\\&\text{else}\\&\quad \beta_H := \min\left(\left[1{,}5\cdot\left(1+(0{,}012\cdot RH)^{18}\right)\cdot h_0 + 250\cdot\alpha_3\ \ 1500\cdot\alpha_3\right]\right)\end{aligned}$$

*factor considering the relative humidity and equivalent thickness*

$$\beta_{c,t,t0} := \left(\frac{t-t_0}{\beta_H + t - t_0}\right)^{0{,}3} = 0{,}9931$$

*factor considering creep development after loading*

$$\beta_{t0} := \frac{1}{0{,}1 + t_0^{\ 0{,}2}} = 0{,}4884$$

*factor considering the concrete age at loading*

$$\beta_{fcm} := \frac{16{,}8}{\sqrt{\dfrac{f_{cm}}{\text{MPa}}}} = 3{,}1749$$

*factor considering the concrete strength*

$$\alpha_1 := \left(\frac{35\ \text{MPa}}{f_{cm}}\right)^{0{,}7} = 1{,}1691$$

*factors considering the concrete strength*

$$\alpha_2 := \left(\frac{35\ \text{MPa}}{f_{cm}}\right)^{0{,}2} = 1{,}0456$$

$$\alpha_3 := \left(\frac{35\ \text{MPa}}{f_{cm}}\right)^{0{,}5} = 1{,}118$$

$$\begin{aligned}&\text{if } f_{cm} \le 35\ \text{MPa} \qquad = 1{,}3703\\&\quad \varphi_{RH} := 1 + \left(\frac{1-\dfrac{RH}{100}}{0{,}1\cdot h_0^{\frac{1}{3}}}\right)\\&\text{else}\\&\quad \varphi_{RH} := \left(1 + \left(\frac{1-\dfrac{RH}{100}}{0{,}1\cdot h_0^{\frac{1}{3}}}\right)\cdot\alpha_1\right)\cdot\alpha_2\end{aligned}$$

*factor considering relative humidity*

$\varphi_0 := \varphi_{RH} \cdot \beta_{fcm} \cdot \beta_{t0} = 2{,}125$ *nominal creep factor*

$\varphi_{t,t0} := \varphi_0 \cdot \beta_{c,t,t0} = 2{,}1103$ *final creep factor*

$\varphi_0 := \varphi_{t,t0} = 2{,}1103$ *final creep factor*

### 4.2 Updated reinforcment areas

$n_{supp} := 4$ *number of bars top*

$n_{field} := 3$ *number of bars bottom*

$$A_{s,supp} := n_{supp} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 0{,}0013\ m^2$$

$$A_{s,field} := n_{field} \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi = 0{,}0009\ m^2$$

### 4.3 Support section

Steel tension in cracked section, long term loading

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 2 Chapter A.8**

$$\rho\alpha_e := \frac{(1+\varphi_0) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,supp}}{b_{beam} \cdot d} = 0{,}2535$$

$$x := d \cdot \rho\alpha_e \cdot \left(\sqrt{1+\frac{2}{\rho\alpha_e}} - 1\right) = 0{,}22\ m$$ *height of pressure zone*

$$h_{c,eff} := \min\left(\left[2{,}5 \cdot (h_{beam} + h_{slab} - d) \quad \frac{(h_{beam} + h_{slab} - x)}{3} \quad \frac{(h_{beam} + h_{slab})}{2}\right]\right) = 91{,}6645\ mm$$

$$A_{c,eff} := b_{eff,supp} \cdot h_{c,eff} = 1{,}1321 \cdot 10^5\ mm^2$$ *area around tensile reinforcement*

$$\rho_{p,eff} := \frac{A_{s,supp}}{A_{c,eff}} = 0{,}0111$$

$$s_{r,max} := k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{main}}{\rho_{p,eff}} = 466{,}0929\ mm$$ *maximum distance between cracks*

Check of concrete compression

$$\sigma_{c,supp} := \frac{2 \cdot M_{max,supp,SLS}}{b_{beam} \cdot x \cdot \left(d - \frac{x}{3}\right)} = 9{,}2943\ MPa$$ *concrete compression*

$$\text{if } \sigma_{c,supp} > 0,45 \cdot f_{ck}$$
$$\varphi := \varphi_0 \cdot \exp\left(1,5 \cdot \left(\frac{\sigma_{c,supp}}{f_{ck}} - 0,45\right)\right)$$
$$\rho\alpha_e := \frac{(1+\varphi) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,supp}}{b_{beam} \cdot d}$$
$$x := d \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right)$$

*limit for linear creep*

$$\sigma_{s,supp} := \frac{M_{max,supp,SLS}}{A_{s,supp} \cdot \left(d - \frac{x}{3}\right)} = 191,3893 \text{ MPa}$$

*steel tension*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,supp} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0,0005$$

**EN-1992-1-1 Chapter 7.3.4**

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0,6 \cdot \frac{\sigma_{s,supp}}{E_s}\right]\right)\right) = 0,2676 \text{ mm}$$

*crack width*

```
if  w_k,supp < w_max = "OK!"
  "OK!"
else
  "NOT OK!"
```

## 4.2 Field section

Steel tension in cracked section, long term loading

$$\rho\alpha_e := \frac{(1+\varphi_0) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,field}}{b_{eff,field} \cdot d} = 0,0287$$

$$x := d \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right) = 0,0931 \text{ m}$$

*height of pressure zone*

$$h_{c,eff} := \min\left(\left[2,5 \cdot (h_{beam} + h_{slab} - d) \quad \frac{(h_{beam} + h_{slab} - x)}{3} \quad \frac{(h_{beam} + h_{slab})}{2}\right]\right) = 133,9735 \text{ mm}$$

$$A_{c,eff} := b_{beam} \cdot h_{c,eff} = 31483,7666 \text{ mm}^2$$

$$\rho_{p,eff} := \frac{A_{s,field}}{A_{c,eff}} = 0,0299$$

$$s_{r,max} := k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{main}}{\rho_{p,eff}} = 273,3781 \text{ mm}$$

*maximum distance between cracks*

Check of concrete compression

$$\sigma_{c,field} := \frac{2 \cdot M_{max,field,SLS}}{b_{eff,field} \cdot x \cdot \left(d - \frac{x}{3}\right)} = 2,3318\ \text{MPa}$$

*concrete compression*

$$\text{if}\ \ \sigma_{c,field} > 0,45 \cdot f_{ck}$$

$$\varphi := \varphi_0 \cdot \exp\left(1,5 \cdot \left(\frac{\sigma_{c,field}}{f_{ck}} - 0,45\right)\right)$$

$$\rho\alpha_e := \frac{(1+\varphi) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,field}}{b_{eff,field} \cdot d}$$

$$x := d \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right)$$

$$\sigma_{s,field} := \frac{M_{max,field,SLS}}{A_{s,field} \cdot \left(d - \frac{x}{3}\right)}$$

*limit for linear creep*

$$\sigma_{s,field} := \frac{M_{max,field,SLS}}{A_{s,field} \cdot \left(d - \frac{x}{3}\right)} = 179,3974\ \text{MPa}$$

*steel tension*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,field} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0,0007$$

**EN-1992-1-1 Chapter 7.3.4**

$$w_{k,field} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm}\ \ 0,6 \cdot \frac{\sigma_{s,field}}{E_s}\right]\right)\right) = 0,1968\ \text{mm}$$

*crack width*

```
if  w_k,field < w_max  = "OK!"
  "OK!"
else
  "NOT OK!"
```

## 5. Shear

EN-1992-1-1 Chapter 6.2

### 5.1 Without shear reinforcement

$C_{Rd,c} := \frac{0,18}{\gamma_{c,ULS}} = 0,12$ (6.2.2)

$k := \min\left(\left[1 + \sqrt{\frac{0,2\ \text{m}}{d}} \quad 2\right]\right) = 1,6757$ (6.2.2)

$\rho_1 := \min\left(\left[\frac{\min\left(\left[n_{supp} \quad n_{field}\right]\right) \cdot \left(\frac{\phi_{main}}{2}\right)^2 \cdot \pi}{b_{beam} \cdot d} \quad 0,02\right]\right) = 0,0092$ (6.2.2)

$V_{Rd,c} := \left(C_{Rd,c} \cdot k \cdot \left(100 \cdot \rho_1 \cdot \frac{f_{ck}}{\text{MPa}}\right)^{\frac{1}{3}}\right) \cdot b_{beam} \cdot 1000 \cdot d \cdot 1000\ \frac{\text{N}}{\text{m}^2} = 54,5568\ \text{kN}$

$v_{min} := 0,035 \cdot k^{\frac{3}{2}} \cdot \left(\frac{f_{ck}}{\text{MPa}}\right)^{\frac{1}{2}} = 0,3395$

$V_{Rd,c,min} := v_{min} \cdot b_{beam} \cdot 1000 \cdot d \cdot 1000\ \frac{\text{N}}{\text{m}^2} = 34,9489\ \text{kN}$

$\boxed{V_{Rd,c} := \max\left(\left[V_{Rd,c} \quad V_{Rd,c,min}\right]\right) = 54,5568\ \text{kN}}$

*shear capacity without reinforcement*

### 5.2 Dimension shear reinforcement

$z := 0,9 \cdot d = 394,2\ \text{mm}$

*inner lever arm EN-1992-1-1 Chapter 6.2.3 (1)*

$cot\theta := 1$

*can be chosen between 1 to 2,5, higher values give less reinforcement*

$tan\theta := \frac{1}{cot\theta} = 1$

$\alpha_{cw} := 1$

*no pre-stress*

$v := 0,6 \cdot \left(1 - \frac{\frac{f_{ck}}{\text{MPa}}}{250}\right) = 0,552$

$v_1 := v$

$V_{Rd,max} := \frac{\alpha_{cw} \cdot b_{beam} \cdot z \cdot v_1 \cdot f_{cd}}{cot\theta + tan\theta} = 340,9042\ \text{kN}$

$$\rho_{w,min} := 0,08 \cdot \frac{\sqrt{\frac{f_{ck}}{\mathrm{MPa}}}}{\frac{f_{yk}}{\mathrm{MPa}}} = 0,0007$$

*minimum shear reinforcement EN-1992-1-1 Chapter 9.2.2 (5)*

$$A_{sw,min} := \rho_{w,min} \cdot b_{beam} = 0,1682\ \mathrm{mm}$$

*Asw,min/s = minimum shear reinforcement per m* $\frac{\mathrm{mm}^2}{\mathrm{m}}$

$$A_{s,l} := \left(\frac{\phi_{link}}{2}\right)^2 \cdot \pi \cdot 2 = 226,1947\ \mathrm{mm}^2$$

*Asw,req/s = required shear reinforcement per m* $\frac{\mathrm{mm}^2}{\mathrm{m}}$

$$s_l := 0,3285\ \mathrm{m}$$

*area of shear link, (2 links per section)*

$$V_{Rd,s} := \frac{A_{s,l}}{s_l} \cdot z \cdot f_{yd} \cdot \cot\theta = 118,0146\ \mathrm{kN}$$

$$V_{Rd,s} := \min\left(\left[\, V_{Rd,s} \quad V_{Rd,max} \,\right]\right) = 118,0146\ \mathrm{kN}$$

$$\eta_V := \frac{V_{max,ULS}}{V_{Rd,s}} = 0,8629$$

# B.3 Two-way slab and pile cap

## Two-way slab

- Designed according to theory of elasticity (linear elastic) without plastic redistribution

- Designed for bending, punching shear and crack width

### 1. Inputs

#### 1.1 Geometry

$w_{slab,x} := 3\ \mathrm{m}$ — *slab width x direction*

$w_{slab,y} := 3\ \mathrm{m}$ — *slab width y direction*

$h_{slab} := 0{,}12\ \mathrm{m}$ — *slab height*

$c_{pile} := 0{,}235\ \mathrm{m}$ — *pile side length (diameter for circular piles)*

$\phi_{top} := 0{,}009\ \mathrm{m}$ $\quad s_{top} := 0{,}1\ \mathrm{m}$ — *mesh reinforcement properties top*

$\phi_{bot} := 0{,}008\ \mathrm{m}$ $\quad s_{bot} := 0{,}1\ \mathrm{m}$ — *mesh reinforcement properties bot*

$w_{strip,x} := w_{slab,x} \cdot 1{,}1 = 3{,}3\ \mathrm{m}$ — *mid strip width, x*

$w_{strip,y} := w_{slab,y} \cdot 1{,}1 = 3{,}3\ \mathrm{m}$ — *mid strip width, y*

$\phi := \begin{bmatrix} \phi_{top} & \phi_{top} & \phi_{bot} & \phi_{bot} \end{bmatrix}$ $\qquad \begin{bmatrix} supp_x & supp_y & field_x & field_y \end{bmatrix}$

$s := \begin{bmatrix} s_{top} & s_{top} & s_{bot} & s_{bot} \end{bmatrix}$

$w := \begin{bmatrix} w_{strip,x} & w_{strip,y} & w_{strip,x} & w_{strip,y} \end{bmatrix}$

#### 1.2 Material

Concrete (EN-1992-1-1 Table 3.1)

$f_{ck} := 20000000\ \mathrm{Pa}$ — *concrete compression strength*

$f_{cm} := f_{ck} + 8\ \mathrm{MPa} = 28\ \mathrm{MPa}$ — *concrete mean compression strength*

$$\begin{array}{l} \text{if } f_{ck} \le 50\ \mathrm{MPa} \qquad = 2{,}2104\ \mathrm{MPa} \\ \quad f_{ctm} := 0{,}3 \cdot \left(\dfrac{f_{ck}}{\mathrm{MPa}}\right)^{\frac{2}{3}} \mathrm{MPa} \\ \text{else} \\ \quad f_{ctm} := 2{,}12 \cdot \ln\left(1 + \left(\dfrac{\dfrac{f_{cm}}{\mathrm{MPa}}}{10}\right)\right) \mathrm{MPa} \end{array}$$

*concrete mean axial tensile strength*

$f_{ctk} := 0,7 \cdot f_{ctm} = 1,5473\ \text{MPa}$ *concrete tensile strength*

$E_{cm} := 22 \cdot \left( \dfrac{\left( \dfrac{f_{cm}}{\text{MPa}} \right)}{10} \right)^{0,3} \text{GPa} = 29,962\ \text{GPa}$ *concrete modulus of elasticity*

$$\begin{array}{l} \text{if } f_{ck} \le 50\ \text{MPa} \quad\quad = 0,0035 \\ \quad \varepsilon_{cu} := 3,5 \cdot 10^{-3} \\ \text{else} \\ \quad \varepsilon_{cu} := 2,6 + 35 \cdot \left( \dfrac{\left( 90 - \dfrac{f_{ck}}{\text{MPa}} \right)}{100} \right)^{4} \end{array}$$

*concrete ultimate strain*

$\rho_c := 23500\ \dfrac{\text{N}}{\text{m}^3}$ *concrete density*

Reinforcement steel K500CT (EN-1992-1-1 Chapter 3.2)

$f_{yk} := 500\ \text{MPa}$ *reinforcement tensile strength*

$E_s := 200\ \text{GPa}$ *reinforcement modulus of elasticity*

Design compressive and tensile strengths (EN-1992-1-1 Chapter 3.1.6)

$\gamma_{c,ULS} := 1,5$ $\quad \gamma_{c,SLS} := 1$ *concrete safety factor (2.4.2.4)*

$\gamma_{s,ULS} := 1,15$ $\quad \gamma_{s,SLS} := 1$ *reinforcement safety factor (2.4.2.4)*

$\alpha_{cc} := 1,0$ *coefficient taking account of long term effects on the compressive strength*

$\alpha_{ct} := 1,0$ *coefficient taking account of long term effects on the tensile strength*

$f_{cd} := \dfrac{\alpha_{cc} \cdot f_{ck}}{\gamma_{c,ULS}} = 13,3333\ \text{MPa}$ *(3.15)*

$f_{ctd} := \dfrac{\alpha_{ct} \cdot f_{ctk}}{\gamma_{c,ULS}} = 1,0315\ \text{MPa}$ *(3.16)*

$f_{yd} := \dfrac{f_{yk}}{\gamma_{s,ULS}} = 434,7826\ \text{MPa}$ *(Fig. 3.8)*

$\varepsilon_{yd} := \dfrac{f_{yd}}{E_s} = 0,0022$ *steel ultimate strain*

## 1.3 Loads

Loads calculated previous to script. Support moments reduced with regard to width of support.

#### 1.3.1 Shear

$V_{ULS,1} := 182899,325569766\ \mathrm{N}$ — *max shear force uniform loadcase*

$V_{ULS,2} := 111136,229012017\ \mathrm{N}$ — *max shear force non uniform loadcase*

#### 1.3.2 Moment x - direction

$M_{max,supp,x,ULS} := 47089,3582438946\ \mathrm{N\ m}$ — *max moment in support section in x direction in ULS*

$M_{max,field,x,ULS} := 42634,3541122892\ \mathrm{N\ m}$ — *max moment in field section in x direction in ULS*

$M_{max,supp,x,SLS} := 27486,7841307016\ \mathrm{N\ m}$ — *max moment in support section in x direction in SLS*

$M_{max,field,x,SLS} := 24658,0913451829\ \mathrm{N\ m}$ — *max moment in field section in x direction in SLS*

#### 1.3.3 Moment y - direction

$M_{max,supp,y,ULS} := 47089,3582438946\ \mathrm{N\ m}$ — *max moment in support section in y direction in ULS*

$M_{max,field,y,ULS} := 42634,3541122892\ \mathrm{N\ m}$ — *max moment in field section in y direction in ULS*

$M_{max,supp,y,SLS} := 27486,7841307016\ \mathrm{N\ m}$ — *max moment in support section in y direction in SLS*

$M_{max,field,y,SLS} := 24658,0913451829\ \mathrm{N\ m}$ — *max moment in field section in y direction in SLS*

### 1.4 Concrete cover

**EN-1992-1-1 Chapter 4.4**

Exposure class XC2

$$\begin{array}{l} \text{if } f_{ck} \geq 35\ \text{MPa} \quad = 25\ \text{mm} \\ \quad c_{min,dur} := 20\ \text{mm} \\ \text{else} \\ \quad c_{min,dur} := 25\ \text{mm} \end{array}$$

*minimum concrete cover with respect to durability*

**Top**

$$c_{min,b,top} := \phi_{top} = 9\ \text{mm}$$

*minimum concrete cover with respect to anchorage*

$$\Delta c_{dev} := 10\ \text{mm}$$

*extra for deviatons*

$$c_{min,top} := \max\left(\begin{bmatrix} c_{min,b,top} & c_{min,dur} \end{bmatrix}\right) + \Delta c_{dev} = 35\ \text{mm}$$

*minimum concrete cover main reinforcement*

**Bottom**

$$c_{min,b,bot} := \phi_{bot} = 8\ \text{mm}$$

*minimum concrete cover with respect to anchorage*

$$c_{min,bot} := \max\left(\begin{bmatrix} c_{min,b,bot} & c_{min,dur} \end{bmatrix}\right) + \Delta c_{dev} = 35\ \text{mm}$$

*minimum concrete cover main reinforcement*

### 1.5 Reinforcement lever arm

$$d_{supp,x} := h_{slab} - c_{min,top} - \frac{\phi_{top}}{2} = 80,5\ \text{mm}$$

*reinforcement in the x direction is always on top of the reinforcement in the y direction*

$$d_{supp,y} := d_{supp,x} - \phi_{top} = 71,5\ \text{mm}$$

*internal lever arm (from compressed edge to main reinforcement)*

$$d_{field,x} := h_{slab} - c_{min,bot} - \frac{3 \cdot \phi_{bot}}{2} = 73\ \text{mm}$$

$$d_{field,y} := d_{field,x} + \phi_{bot} = 81\ \text{mm}$$

### 1.6 Rectangular stress block factors

**EN-1992-1-1 Chapter 3.1.7**

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 0,8 \\ \quad \lambda := 0,8 \\ \text{else} \\ \quad \lambda := 0,8 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{400} \end{array}$$

*effective stress block height factor*

$$\begin{array}{l} \text{if } f_{ck} \leq 50\ \text{MPa} \quad = 1 \\ \quad \eta := 1,0 \\ \text{else} \\ \quad \eta := 1,0 - \dfrac{\left(\dfrac{f_{ck}}{\text{MPa}} - 50\right)}{200} \end{array}$$

*effective  stress block strength factor*

## 3. Moment resistance mid strip

### 3.1 Support section

#### 3.1.1 X - direction

**Uncracked section**

$$W_x := \frac{w_{strip,x} \cdot h_{slab}^{\ 2}}{6} = 0{,}0079\ m^3$$

$$M_{Rd1,x} := W_x \cdot f_{cd} = 105{,}6\ kN\ m$$

*bending moment resistance uncracked section*

**Svenska Betongföreningens "Handbok till Eurocode" Vol. 1 Chapter X4.2.6.**

**Cracked section**

$$A_{s,supp,x} := \left(\frac{\phi_{top}}{2}\right)^2 \cdot \pi \cdot \frac{w_{strip,x}}{s_{top}} = 2099{,}3693\ mm^2$$

*main tensile reinforcement amount*

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,supp,x} \cdot f_{yd}}{w_{strip,x} \cdot d_{supp,x} \cdot \eta \cdot f_{cd}} = 0{,}2577$$

*mechanical reinforcment*

$$x := \omega \cdot \frac{d_{supp,x}}{\lambda} = 0{,}0259\ m$$

*height of pressure zone*

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0{,}4935$$

*balanced reinforcement amount*

$$\begin{array}{l} \text{if } \omega > \omega_{bal} \\ \quad \sigma_{s2} := E_s \cdot \dfrac{d_{supp,x} - x_{sls}}{x_{sls}} \cdot \varepsilon_{cu} \\ \quad \omega := \dfrac{A_{s,supp,x} \cdot \sigma_{s2}}{w_{strip,x} \cdot d_{supp,x} \cdot \eta \cdot f_{cd}} \\ \quad x_{sls} := \omega \cdot \dfrac{d_{supp,x}}{\lambda} \end{array}$$

*recalculate reinforcement tensile stress if necessary*

$$M_{Rd2,supp,x} := \eta \cdot f_{cd} \cdot w_{strip,x} \cdot \lambda \cdot x \cdot \left(d_{supp,x} - \frac{\lambda}{2} \cdot x\right) = 64{,}0103\ kN\ m$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,supp,x,ULS}}{W_x} = 5{,}9456\ MPa$$

*force in concrete*

$$\begin{array}{l} \text{if } \sigma_c < f_{ctm} \qquad = 64{,}0103\ kN\ m \\ \quad M_{Rd,supp,x} := M_{Rd1,x} \\ \text{else} \\ \quad M_{Rd,supp,x} := M_{Rd2,supp,x} \end{array}$$

$$\eta_{M,supp,x} := \frac{M_{max,supp,x,ULS}}{M_{Rd,supp,x}} = 0{,}7357$$

*utilization ratio*

#### 3.1.1 Y - direction

**Uncracked section**

$$W_y := \frac{w_{strip,y} \cdot h_{slab}^2}{6} = 0,0079\ m^3$$

$$M_{Rd1,y} := W_y \cdot f_{cd} = 105,6\ kN\ m$$

*bending moment resistance uncracked section*

**Cracked section**

$$A_{s,supp,y} := \left(\frac{\phi_{top}}{2}\right)^2 \cdot \pi \cdot \frac{w_{strip,y}}{s_{top}} = 2099,3693\ mm^2$$

*main tensile reinforcement amount*

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,supp,y} \cdot f_{yd}}{w_{strip,y} \cdot d_{supp,y} \cdot \eta \cdot f_{cd}} = 0,2901$$

*mechanical reinforcment*

$$x := \omega \cdot \frac{d_{supp,y}}{\lambda} = 0,0259\ m$$

*height of pressure zone*

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0,4935$$

*balanced reinforcement amount*

*recalculate reinforcement tensile stress if necessary*

$$\text{if } \omega > \omega_{bal}$$
$$\sigma_{s2} := E_s \cdot \frac{d_{supp,y} - x_{sls}}{x_{sls}} \cdot \varepsilon_{cu}$$
$$\omega := \frac{A_{s,supp,y} \cdot \sigma_{s2}}{w_{strip,y} \cdot d_{supp,y} \cdot \eta \cdot f_{cd}}$$
$$x_{sls} := \omega \cdot \frac{d_{supp,y}}{\lambda}$$

$$M_{Rd2,supp,y} := \eta \cdot f_{cd} \cdot w_{strip,y} \cdot \lambda \cdot x \cdot \left(d_{supp,y} - \frac{\lambda}{2} \cdot x\right) = 55,7954\ kN\ m$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,supp,y,ULS}}{W_y} = 5,9456\ MPa$$

*force in concrete*

$$\text{if } \sigma_c < f_{ctm} \qquad = 55,7954\ kN\ m$$
$$M_{Rd,supp,y} := M_{Rd1\_y}$$
$$\text{else}$$
$$M_{Rd,supp,y} := M_{Rd2,supp,y}$$

$$\eta_{M,field} := \frac{M_{max,supp,y,ULS}}{M_{Rd,supp,y}} = 0,844$$

*utilization ratio*

## 3.2 Field section

### 3.2.1 X - direction

**Cracked section**

$$A_{s,field,x} := \left(\frac{\phi_{bot}}{2}\right)^2 \cdot \pi \cdot \frac{w_{strip,x}}{s_{bot}} = 1658{,}7609\ mm^2$$

*main tensile reinforcement amount*

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,field,x} \cdot f_{yd}}{w_{strip,x} \cdot d_{field,x} \cdot \eta \cdot f_{cd}} = 0{,}2245$$

*mechanical reinforcment*

$$x := \omega \cdot \frac{d_{field,x}}{\lambda} = 0{,}0205\ m$$

*height of pressure zone*

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0{,}4935$$

*balanced reinforcement amount*

*recalculate reinforcement tensile stress if necessary*

$$\text{if } \omega > \omega_{bal}$$
$$\sigma_{s2} := E_s \cdot \frac{d_{field,x} - x_{sls}}{x_{sls}} \cdot \varepsilon_{cu}$$
$$\omega := \frac{A_{s,field,x} \cdot \sigma_{s2}}{w_{strip,x} \cdot d_{field,x} \cdot \eta \cdot f_{cd}}$$
$$x_{sls} := \omega \cdot \frac{d_{field,x}}{\lambda}$$

$$M_{Rd2,field,x} := \eta \cdot f_{cd} \cdot w_{strip,x} \cdot \lambda \cdot x \cdot \left(d_{field,x} - \frac{\lambda}{2} \cdot x\right) = 46{,}7371\ kN\ m$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,field,x,ULS}}{W_x} = 5{,}3831\ MPa$$

*force in concrete*

$$\text{if } \sigma_c < f_{ctm} \quad = 46{,}7371\ kN\ m$$
$$M_{Rd,field,x} := M_{Rd1,x}$$
$$\text{else}$$
$$M_{Rd,field,x} := M_{Rd2,field,x}$$

$$\eta_{M,field,x} := \frac{M_{max,field,x,ULS}}{M_{Rd,field,x}} = 0{,}9122$$

*utilization ratio*

### 3.1.1 Y - direction

**Cracked section**

$$A_{s,field,y} := \left(\frac{\phi_{bot}}{2}\right)^2 \cdot \pi \cdot \frac{w_{strip,y}}{s_{bot}} = 1658{,}7609\ mm^2$$

*main tensile reinforcement amount*

Assume that the reinforcement is yielding

$$\omega := \frac{A_{s,field,y} \cdot f_{yd}}{w_{strip,y} \cdot d_{field,y} \cdot \eta \cdot f_{cd}} = 0{,}2024$$

*mechanical reinforcment*

$$x := \omega \cdot \frac{d_{field,y}}{\lambda} = 0{,}0205\ m$$

*height of pressure zone*

Calculate balanced reinforcement amount

$$\omega_{bal} := \lambda \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0{,}4935$$

*balanced reinforcement amount*

*recalculate reinforcement tensile stress if necessary*

$$\boxed{\begin{array}{l} \text{if } \ \omega > \omega_{bal} \\ \quad \sigma_{s2} := E_s \cdot \dfrac{d_{field,y} - x_{sls}}{x_{sls}} \cdot \varepsilon_{cu} \\ \quad \omega := \dfrac{A_{s,field,y} \cdot \sigma_{s2}}{w_{strip,y} \cdot d_{field,y} \cdot \eta \cdot f_{cd}} \\ \quad x_{sls} := \omega \cdot \dfrac{d_{field,y}}{\lambda} \end{array}}$$

$$M_{Rd2,field,y} := \eta \cdot f_{cd} \cdot w_{strip,y} \cdot \lambda \cdot x \cdot \left(d_{field,y} - \frac{\lambda}{2} \cdot x\right) = 52{,}5067\ kN\ m$$

*bending moment resistance cracked section*

$$\sigma_c := \frac{M_{max,field,y,ULS}}{W_y} = 5{,}3831\ MPa$$

*force in concrete*

$$\boxed{\begin{array}{l} \text{if } \ \sigma_c < f_{ctm} \\ \quad M_{Rd,field,y} := M_{Rd1,y} \\ \text{else} \\ \quad M_{Rd,field,y} := M_{Rd2,field,y} \end{array}} = 52{,}5067\ kN\ m$$

$$\eta_{M,field,y} := \frac{M_{max,field,y,ULS}}{M_{Rd,field,y}} = 0{,}812$$

*utilization ratio*

$$A_s := \left[ A_{s,supp,x}\ \ A_{s,supp,y}\ \ A_{s,field,x}\ \ A_{s,field,y} \right]$$

## 4. Placing of reinforcement

### 4.1 Maximum spacing bars

**BBK 3.9.6**

$d_{min,main,I} := \phi_{top} + \phi_{bot} = 17\ mm$

*minimum distance between main bars in one layer*

$d_{min,main,II} := 1,5 \cdot \frac{\left(\phi_{top} + \phi_{bot}\right)}{2} = 12,75\ mm$

*minimum distance between main bars between layers*

## 5. Crack widths

### 5.1 Creep coefficients

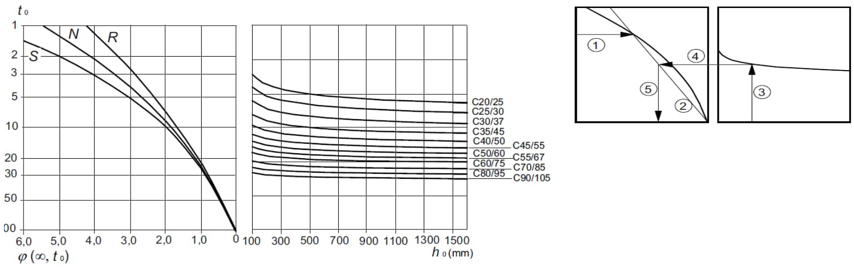
**EN-1992-1-1 Annex B**

$A_{c,x} := w_{strip,x} \cdot h_{slab}$ $\quad A_{c,y} := w_{strip,y} \cdot h_{slab}$ *area*

$u_x := 2 \cdot w_{strip,x}$ $\quad u_y := 2 \cdot w_{strip,y}$ *circumference exposed to drying*

$$h_{0,x} := \frac{2 \cdot \frac{A_{c,x}}{u_x}}{\mathrm{mm}} = 120 \qquad h_{0,y} := \frac{2 \cdot \frac{A_{c,y}}{u_y}}{\mathrm{mm}} = 120$$

*ficitve height in mm*




b) utomhusförhållanden – RH = 80%

$t_0 := 28$ $\quad t := 365 \cdot 70$ *assumed time for loading and life time*

$RH := 80$ *assumed relative humidity (outdoors)*

$$\alpha_1 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,7} = 1,1691$$

*factors considering the concrete strength*

$$\alpha_2 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,2} = 1,0456$$

$$\alpha_3 := \left(\frac{35\ \mathrm{MPa}}{f_{cm}}\right)^{0,5} = 1,118$$

$$\beta_{t0} := \frac{1}{0,1 + t_0^{\ 0,2}} = 0,4884$$

*factor considering the concrete age at loading*

$$\beta_{fcm} := \frac{16,8}{\sqrt{\frac{f_{cm}}{\mathrm{MPa}}}} = 3,1749$$

*factor considering the concrete strength*

$h_0 := [\, h_{0,x} \;\; h_{0,y} \,]$

$\varphi_0 := [\, 0 \;\; 0 \,]$

for $i \in [1..2]$

  if $f_{cm} \le 35\ \mathrm{MPa}$

  $\beta_H := \min\left(\left[\, 1{,}5 \cdot \left(1 + (0{,}012 \cdot RH)^{18}\right) \cdot h_{0_i} + 250 \;\; 1500 \,\right]\right)$

  *factor considering the relative humidity and equivalent thickness*

  else

  $\beta_H := \min\left(\left[\, 1{,}5 \cdot \left(1 + (0{,}012 \cdot RH)^{18}\right) \cdot h_{0_i} + 250 \cdot \alpha_3 \;\; 1500 \cdot \alpha_3 \,\right]\right)$

  $\beta_{c,t,t0} := \left(\dfrac{t - t_0}{\beta_H + t - t_0}\right)^{0{,}3}$

  *factor considering creep development after loading*

  if $f_{cm} \le 35\ \mathrm{MPa}$

  $\varphi_{RH} := 1 + \left(\dfrac{1 - \dfrac{RH}{100}}{0{,}1 \cdot \left(h_{0_i}\right)^{\frac{1}{3}}}\right)$

  *factor considering relative humidity*

  else

  $\varphi_{RH} := \left(1 + \left(\dfrac{1 - \dfrac{RH}{100}}{0{,}1 \cdot \left(h_{0_i}\right)^{\frac{1}{3}}}\right) \cdot \alpha_1\right) \cdot \alpha_2$

  $\varphi_{0_i} := \varphi_{RH} \cdot \beta_{fcm} \cdot \beta_{t0} \cdot \beta_{c,t,t0}$

  *final creep factor*

$\varphi_0 = [\, 2{,}1665 \;\; 2{,}1665 \,]$

L

## 5.2 Steel stress

Svenska Betongföreningens "Handbok till Eurocode" Vol. 2 Chapter A.8

### 5.2.1 Support section x

$$\rho\alpha_e := \frac{\left(1+\varphi_{0_1}\right)\cdot E_s}{E_{cm}} \cdot \frac{A_{s,supp,x}}{w_{strip,x}\cdot d_{supp,x}} = 0{,}167$$

$$x_{supp,x} := d_{supp,x}\cdot\rho\alpha_e\cdot\left(\sqrt{1+\frac{2}{\rho\alpha_e}}-1\right) = 0{,}035\ \mathrm{m}$$

*height of pressure zone*

$$\sigma_{s,supp,x} := \frac{M_{max,supp,x,SLS}}{A_{s,supp,x}\cdot\left(d_{supp,x}-\frac{x_{supp,x}}{3}\right)} = 1{,}902\cdot 10^{8}\ \mathrm{Pa}$$

*steel tension*

$$\sigma_c := \frac{2\cdot M_{max,supp,x,SLS}}{w_{strip,x}\cdot x_{supp,x}\cdot\left(d_{supp,x}-\frac{x_{supp,x}}{3}\right)} = 6{,}917\cdot 10^{6}\ \mathrm{Pa}$$

*concrete compression*

$$\text{if } \sigma_c > 0{,}45\cdot f_{ck}$$

$$\varphi := \varphi_{0_1}\cdot\exp\left(1{,}5\cdot\left(\frac{\sigma_c}{f_{ck}}-0{,}45\right)\right)$$

$$\rho\alpha_e := \frac{(1+\varphi)\cdot E_s}{E_{cm}}\cdot\frac{A_{s,supp,x}}{w_{strip,x}\cdot d_{supp,x}}$$

$$x_{supp,x} := d_{supp,x}\cdot\rho\alpha_e\cdot\left(\sqrt{1+\frac{2}{\rho\alpha_e}}-1\right)$$

$$\sigma_{s,supp,x} := \frac{M_{max,supp,x,SLS}}{A_{s,supp,x}\cdot\left(d_{supp,x}-\frac{x_{supp,x}}{3}\right)}$$

*limit for linear creep*

### 5.2.2 Support section y

$$\rho\alpha_e := \frac{\left(1+\varphi_{0_2}\right)\cdot E_s}{E_{cm}} \cdot \frac{A_{s,supp,y}}{w_{strip,y}\cdot d_{supp,y}} = 0{,}1881$$

$$x_{supp,y} := d_{supp,y}\cdot\rho\alpha_e\cdot\left(\sqrt{1+\frac{2}{\rho\alpha_e}}-1\right) = 0{,}0324\ \mathrm{m}$$

*height of pressure zone*

$$\sigma_{s,supp,y} := \frac{M_{max,supp,y,SLS}}{A_{s,supp,y}\cdot\left(d_{supp,y}-\frac{x_{supp,y}}{3}\right)} = 2{,}1572\cdot 10^{8}\ \mathrm{Pa}$$

*steel tension*

$$\sigma_c := \frac{2\cdot M_{max,supp,y,SLS}}{w_{strip,y}\cdot x_{supp,y}\cdot\left(d_{supp,y}-\frac{x_{supp,y}}{3}\right)} = 8{,}4663\cdot 10^{6}\ \mathrm{Pa}$$

*concrete compression*

$$\text{if } \sigma_c > 0{,}45 \cdot f_{ck}$$
$$\varphi := \varphi_{0_2} \cdot \exp\left(1{,}5 \cdot \left(\frac{\sigma_c}{f_{ck}} - 0{,}45\right)\right)$$
$$\rho\alpha_e := \frac{(1+\varphi) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,supp,y}}{w_{strip,y} \cdot d_{supp,y}}$$
$$x_{supp,y} := d_{supp,y} \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right)$$
$$\sigma_{s,supp,y} := \frac{M_{max,supp,y,SLS}}{A_{s,supp,y} \cdot \left(d_{supp,y} - \frac{x_{supp,y}}{3}\right)}$$

*limit for linear creep*

**5.2.3 Field section x**

$$\rho\alpha_e := \frac{\left(1 + \varphi_{0_1}\right) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,field,x}}{w_{strip,x} \cdot d_{field,x}} = 0{,}1455$$

$$x_{field,x} := d_{field,x} \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right) = 0{,}0302\ \text{m}$$

*height of pressure zone*

$$\sigma_{s,field,x} := \frac{M_{max,field,x,SLS}}{A_{s,field,x} \cdot \left(d_{field,x} - \frac{x_{field,x}}{3}\right)} = 2{,}3617 \cdot 10^{8}\ \text{Pa}$$

*steel tension*

$$\sigma_c := \frac{2 \cdot M_{max,field,x,SLS}}{w_{strip,x} \cdot x_{field,x} \cdot \left(d_{field,x} - \frac{x_{field,x}}{3}\right)} = 7{,}8699 \cdot 10^{6}\ \text{Pa}$$

*concrete compression*

$$\text{if } \sigma_c > 0{,}45 \cdot f_{ck}$$
$$\varphi := \varphi_{0_1} \cdot \exp\left(1{,}5 \cdot \left(\frac{\sigma_c}{f_{ck}} - 0{,}45\right)\right)$$
$$\rho\alpha_e := \frac{(1+\varphi) \cdot E_s}{E_{cm}} \cdot \frac{A_{s,field,x}}{w_{strip,x} \cdot d_{field,x}}$$
$$x_{field,x} := d_{field,x} \cdot \rho\alpha_e \cdot \left(\sqrt{1 + \frac{2}{\rho\alpha_e}} - 1\right)$$
$$\sigma_{s,field,x} := \frac{M_{max,field,x,SLS}}{A_{s,field,x} \cdot \left(d_{field,x} - \frac{x_{field,x}}{3}\right)}$$

*limit for linear creep*

### 5.2.3 Field section y

$$\rho\alpha_e := \frac{\left(1+\varphi_{0\ 2}\right)\cdot E_s}{E_{cm}} \cdot \frac{A_{s,field,y}}{w_{strip,y} \cdot d_{field,y}} = 0{,}1312$$

*height of pressure zone*

$$x_{field,y} := d_{field,y} \cdot \rho\alpha_e \cdot \left(\sqrt{1+\frac{2}{\rho\alpha_e}} - 1\right) = 0{,}0322 \text{ m}$$

*steel tension*

$$\sigma_{s,field,y} := \frac{M_{max,field,y,SLS}}{A_{s,field,y} \cdot \left(d_{field,y} - \frac{x_{field,y}}{3}\right)} = 2{,}1156 \cdot 10^{8} \text{ Pa}$$

*concrete compression*

$$\sigma_c := \frac{2 \cdot M_{max,field,y,SLS}}{w_{strip,y} \cdot x_{field,y} \cdot \left(d_{field,y} - \frac{x_{field,y}}{3}\right)} = 6{,}6047 \cdot 10^{6} \text{ Pa}$$

*limit for linear creep*

if $\sigma_c > 0{,}45 \cdot f_{ck}$

$$\varphi := \varphi_{0\ 2} \cdot \exp\left(1{,}5 \cdot \left(\frac{\sigma_c}{f_{ck}} - 0{,}45\right)\right)$$

$$\rho\alpha_e := \frac{(1+\varphi)\cdot E_s}{E_{cm}} \cdot \frac{A_{s,field,y}}{w_{strip,y} \cdot d_{field,y}}$$

$$x_{field,y} := d_{field,y} \cdot \rho\alpha_e \cdot \left(\sqrt{1+\frac{2}{\rho\alpha_e}} - 1\right)$$

$$\sigma_{s,field,y} := \frac{M_{max,field,y,SLS}}{A_{s,field,y} \cdot \left(d_{field,y} - \frac{x_{field,y}}{3}\right)}$$

## 5.3 Crack width

**EN-1992-1-1 Chapter 7.3.4**

Inputs

$w_{max} := 0,3\ mm$ — *maximum crack width for exposure class XC2*

$k_1 := 0,8$ — *factor concerning reinforcement adhesion properties, 0.8 for non pllain bars*

$k_2 := 0,5$ — *factor concerning straindistribution, 0.5 for bending*

$k_3 := 3,4$ — *recommended value*

$k_4 := 0,425$ — *recommended value*

$k_t := 0,4$ — *long term loading*

$c_{top} := c_{min,top} = 35\ mm$

$c_{bot} := c_{min,bot} = 35\ mm$

$f_{ct,eff} := f_{ctm}$

#### 5.3.1 Support section x direction

$$h_{c,eff} := \min\left(\left[2,5 \cdot \left(h_{slab} - d_{supp,x}\right) \quad \frac{\left(h_{slab} - x_{supp,x}\right)}{3} \quad \frac{h_{slab}}{2}\right]\right) = 28,338\ mm$$

$$A_{c,eff} := w_{strip,x} \cdot h_{c,eff} = 93515,2353\ mm^2$$

$$\rho_{p,eff} := \frac{A_{s,supp,x}}{A_{c,eff}} = 0,0224$$

$$s_{r,max} := k_3 \cdot c_{top} + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{top}}{\rho_{p,eff}} = 187,153\ mm$$ *maximum distance between cracks*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,supp,x} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0,0007$$

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0,6 \cdot \frac{\sigma_{s,supp,x}}{E_s}\right]\right)\right) = 0,1356\ mm$$ *crack width*

```
if  w_k,supp < w_max = "OK!"
   "OK!"
else
   "NOT OK!"
```

#### 5.3.2 Support section y direction

$$h_{c,eff} := \min\left(\left[2,5 \cdot \left(h_{slab} - d_{supp,y}\right) \quad \frac{\left(h_{slab} - x_{supp,y}\right)}{3} \quad \frac{h_{slab}}{2}\right]\right) = 29,1935\ mm$$

$$A_{c,eff} := w_{strip,y} \cdot h_{c,eff} = 96338,66\ mm^2$$

$$\rho_{p,eff} := \frac{A_{s,supp,y}}{A_{c,eff}} = 0,0218$$

$$s_{r,max} := k_3 \cdot c_{top} + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{top}}{\rho_{p,eff}} = 189,2107\ mm$$ *maximum distance between cracks*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,supp,y} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0,0008$$

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0,6 \cdot \frac{\sigma_{s,supp,y}}{E_s}\right]\right)\right) = 0,1601\ mm$$ *crack width*

```
if  w_k,supp < w_max = "OK!"
   "OK!"
else
   "NOT OK!"
```

### 5.3.3 Field section x direction

$$h_{c,eff} := \min\left(\left[2{,}5 \cdot \left(h_{slab} - d_{field,x}\right) \quad \frac{\left(h_{slab} - x_{field,x}\right)}{3} \quad \frac{h_{slab}}{2}\right]\right) = 29{,}9438\ mm$$

$$A_{c,eff} := w_{strip,x} \cdot h_{c,eff} = 98814{,}6633\ mm^2$$

$$\rho_{p,eff} := \frac{A_{s,field,x}}{A_{c,eff}} = 0{,}0168$$

$$s_{r,max} := k_3 \cdot c_{bot} + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{bot}}{\rho_{p,eff}} = 200{,}0171\ mm$$

*maximum distance between cracks*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,field,x} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0{,}0009$$

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0{,}6 \cdot \frac{\sigma_{s,field,x}}{E_s}\right]\right)\right) = 0{,}1776\ mm$$

*crack width*

```
if  w_k,supp < w_max = "OK!"
  "OK!"
else
  "NOT OK!"
```

### 5.3.3 Field section y direction

$$h_{c,eff} := \min\left(\left[2{,}5 \cdot \left(h_{slab} - d_{field,y}\right) \quad \frac{\left(h_{slab} - x_{field,y}\right)}{3} \quad \frac{h_{slab}}{2}\right]\right) = 29{,}2662\ mm$$

$$A_{c,eff} := w_{strip,y} \cdot h_{c,eff} = 96578{,}413\ mm^2$$

$$\rho_{p,eff} := \frac{A_{s,field,y}}{A_{c,eff}} = 0{,}0172$$

$$s_{r,max} := k_3 \cdot c_{bot} + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi_{bot}}{\rho_{p,eff}} = 198{,}1836\ mm$$

*maximum distance between cracks*

$$\varepsilon_{sm\_cm} := \frac{\sigma_{s,field,y} - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{p,eff}\right)}{E_s} = 0{,}0008$$

$$w_{k,supp} := s_{r,max} \cdot \left(\max\left(\left[\varepsilon_{sm\_cm} \quad 0{,}6 \cdot \frac{\sigma_{s,field,y}}{E_s}\right]\right)\right) = 0{,}1528\ mm$$

*crack width*

```
if  w_k,supp < w_max = "OK!"
  "OK!"
else
  "NOT OK!"
```

## 6. Pile cap design

**EN-1992-1-1 Chapter 6.4**

*- Width determined by installation tolerances*

*- Height determined to ensure sufficient punching shear resistance*

### 6.1 Geometry

Width:

$c_{pile} = 0{,}235\ \text{m}$ — *width pile*

$h_{slab} = 0{,}12\ \text{m}$ — *height slab*

$c_{tol} := 100\ \text{mm}$ — *pile tolerance in plan*

$c_{bot} = 0{,}035\ \text{m}$

$w_{cap} := c_{pile} + 2 \cdot c_{tol} + 2 \cdot c_{bot} = 0{,}505\ \text{m}$ — *width pile cap*

Effective width and control perimeter:

$$d_{eff} := \frac{d_{supp,y} + d_{supp,x}}{2} = 76\ \text{mm}$$ *effective slab height*

$w_{pile,eff} := 0{,}509\ \text{m}$ *solved in script* — *effective width of pile at bottom slab*

$u_l := w_{pile,eff} \cdot 4 + 2 \cdot d_{eff} \cdot 2 \cdot \pi = 2{,}991\ \text{m}$ — *control perimeter*

Height:

$h_{cap,cover} := 100\ \text{mm}$ — *height from top of pile to bottom cap*

### 6.2 Punching shear

$\beta := 1{,}15$ — eccentricity factor (6.4.3 (6))

$V_{ULS} := \max\left(\left[\,V_{ULS,1}\quad \beta \cdot V_{ULS,2}\,\right]\right)$ — dimensioning shear force

$$\rho_x := \frac{\pi \cdot \phi_{top}^{\ 2}}{4 \cdot d_{supp,x} \cdot s_{top}} = 0{,}0079$$

$$\rho_y := \frac{\pi \cdot \phi_{top}^{\ 2}}{4 \cdot d_{supp,y} \cdot s_{top}} = 0{,}0089$$

$$\rho := \min\left(\left[\,\sqrt{\rho_x \cdot \rho_y}\quad 0{,}02\,\right]\right) = 0{,}0084$$ reinforcement

$$k := 1 + \sqrt{\frac{200\ \text{mm}}{d_{eff}}} = 2{,}6222$$

$$v_{min} := 0{,}035 \cdot k^{\frac{3}{2}} \sqrt{\frac{f_{ck}}{\text{MPa}}}\ \text{MPa} = 0{,}6646\ \text{MPa}$$ minimum punching resistance (Chapter 6.2.2)

$$C_{Rd,c} := \frac{0,18}{\gamma_{c,ULS}} = 0,12$$ (6.4.4 (1))

$$v_{Rd,c} := C_{Rd,c} \cdot k \cdot \left(100 \cdot \rho \cdot \frac{f_{ck}}{\text{MPa}}\right)^{\left(\frac{1}{3}\right)} \text{MPa} = 0,8054 \text{ MPa}$$ (6.47)

$$V_{Rd,c} := v_{Rd,c} \cdot d_{eff} \cdot u_l = 183,0925 \text{ kN}$$ punching shear resistance

$$\eta_{punching} := \frac{V_{ULS}}{V_{Rd,c}} = 0,9989$$ utilization ratio

## B.4 Shaft bearing piles

# Shaft bearing piles

Calculated according to the Swedish Commission on Pile Research: Cohesion piles

Inputs

Geometry

$A_s := 0,055225\ \mathrm{m}^2$ *pile area*

$\theta := 0,94\ \mathrm{m}$ *pile circumferance*

$L := 25\ \mathrm{m}$ *length pile*

$cc_x := 3\ \mathrm{m}$ *cc distance between piles x axis*

$cc_y := 5\ \mathrm{m}$ *cc distance between piles y axis*

$n_x := 51$ *number of piles in x direction*

$n_y := 31$ *number of piles in y direction*

$N := n_x \cdot n_y$ *total number of piles*

Loads

$Q_{ULS} := 321067,375015155\ \mathrm{N}$ *load per pile*

$Q_{SLS} := 700000\ \mathrm{N}$

Soil and ground properties

$c_{u,min} := 13333,3333333333333333333333333333\ \mathrm{Pa}$

$c_{u,inc} := 1200\ \mathrm{Pa}$

Mean shear resistance

$$c_{u,max} := \frac{L}{\mathrm{m}} \cdot c_{u,inc} + c_{u,min}$$

$$c_{u,mean} := \frac{c_{u,max} + c_{u,min}}{2} = 28333,3333\ \mathrm{Pa}$$

Soil effective modulus

$M_1 := 700\ \mathrm{kPa}$ *(Distributed over soil depth similar to c.u)*

$M_2 := 900\ \mathrm{kPa}$

$M_3 := 1500\ \mathrm{kPa}$

$M_k := \begin{bmatrix} M_1 & M_2 & M_3 \end{bmatrix}$

$\gamma_m := 1,5$

$$M_d := \frac{M_k}{\gamma_m}$$

Additional soil properties:

$\gamma_{soil} := 16\ \frac{\mathrm{kN}}{\mathrm{m}^3}$ *soil density*

$OCR := 1,1$ *over consolidation ratio*

$N_p := 9$ *ground resistance capacity for normal piles*

**ULS**

## Partial safety factors

Factors for caracteristic capacity

$\gamma_{Rd} := 1,1$ *modelling factor (BFS 2015.6 EKS10 Table I-4)*

$n := 5$ *number of geotechnic test*

```
if n > 5
  n := 6
```

$$\xi := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1,4 & 1,35 & 1,33 & 1,31 & 1,29 & 1,27 \end{bmatrix}$$

$\xi_3 := \xi_{2\,n} = 1,29$ *correlation coefficient considering number of geotechnical studies (assuming 5)*

Factors for dimensioning capacity

$\gamma_s := 1,2$ *partial coefficient for compression piles (BFS 2015.6 EKS10 Table I-7)*

## Calculation of adhesion factor

$\alpha_{okorr} := 1,0$

$\kappa_\phi := 0,9$ *factor considering pile diameter*

$\kappa_f := 1,0$ *factor considering pile form*
*calculations restricted to constant width piles*

$\kappa_T := 1$ *factor considering time after installation*

$k_t := 0,9$ *factor considering durance of loads*

$\alpha_x := [1,25 ; 1,26 .. 2,5]$ $\alpha_y := [0,5 ; 0,6 .. 1,0]$

$o := OCR$

```
if o < 2,5
  if o < 1,25
    κ_OCR := 1,0
  else
    κ_OCR := linterp(α_x ; α_y ; 0,5)
else
  κ_OCR := 0,5
```

$\alpha := \alpha_{okorr} \cdot \kappa_\phi \cdot \kappa_f \cdot \kappa_{OCR} \cdot \kappa_T = 0,9$

### Final single pile resistance

$$R_{single,shaft} := \alpha \cdot \theta \cdot c_{u,mean} \cdot L$$

$$R_{single,end} := N_p \cdot A_s \cdot c_{u,mean}$$

$$R_{single} := k_t \cdot \left( \left( R_{single,shaft} + R_{single,end} \right) \cdot \frac{1}{\gamma_{Rd} \cdot \xi_3 \cdot \gamma_s} \right) = 324{,}1714 \text{ kN}$$

Relative contribution from end of pile:

$$\frac{R_{single,end}}{R_{single}} = 4{,}3441 \text{ \%}$$

### Group effects

$S_1 := cc_x \cdot n_x$ *pile group width*

$S_2 := cc_y \cdot n_y$ *pile group depth*

$$R_{group} := \frac{2 \cdot \left( S_1 + S_2 \right) \cdot L \cdot c_{u,mean} + N_p \cdot S_1 \cdot S_2 \cdot c_{u,mean}}{\gamma_{Rd} \cdot \xi_3 \cdot \gamma_s} = 3{,}8076 \cdot 10^6 \text{ kN}$$

### Final resistance

$$R_M := \left[ N \cdot R_{single} \quad R_{group} \right]$$

$$R := \min\left( R_M \right) = 5{,}1252 \cdot 10^5 \text{ kN}$$

### Utilization rates

$$\eta_{ULS} := \frac{Q_{ULS} \cdot N}{R} = 99{,}0425 \text{ \%}$$

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