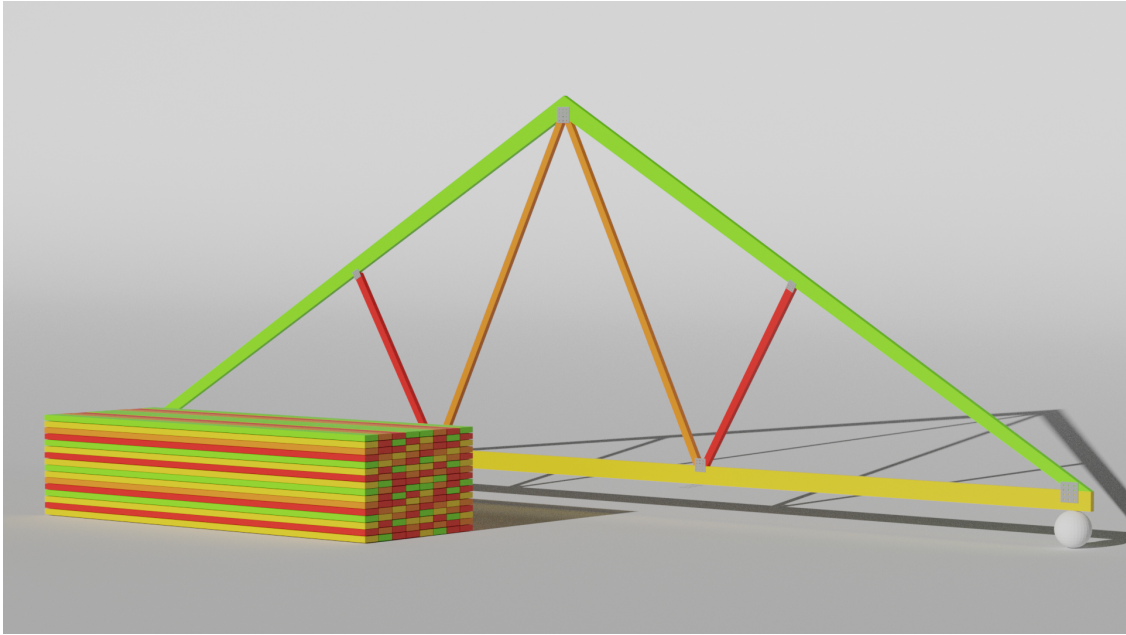




CHALMERS
UNIVERSITY OF TECHNOLOGY



Intelligent Truss Design

Digitalization of the Building Process

Master's thesis in Structural Engineering & Building Technology

Jonathan Pekkala Settlund & Sebastian Jakobsson

DEPARTMENT OF ARCHITECTURE & CIVIL ENGINEERING

CHALMERS UNIVERSITY OF TECHNOLOGY
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MASTER'S THESIS 2022

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JONATHAN PEKKALA SETTTLAND & SEBASTIAN JAKOBSSON

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Master's Thesis 2022
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Cover: A Fink truss with different timber strengths used throughout to enhance the efficiency and utilization ratio of the truss.

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Intelligent Truss Design
Digitalization of the building process
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Abstract

Today, timber is widely used in construction in Sweden, but it is not used to its full potential and strength. Timber is a heterogenous material, but in the construction industry today, assumed to be homogenous. That timber is heterogenous is known, but the reason for it to be assumed homogenous is to simplify the process of grading it. The properties structural timber is assigned in the grading process are their characteristic values, also known as the fifth percentile value. This means that 95% of all timber in each strength class or grade has a value higher than the characteristic one. Another reason for doing this is to minimize the risk of failure. This may result in a potential under utilization if not other, more advanced methods are used. When designing trusses the characteristic value of the timber is used to limit the risk of failure, and not its individual value that can be gathered from the testing carried out during the grading process. If the truss designers can utilize the individual data instead of the characteristic value the trusses can be designed with higher utilization ratio. If the designers know which timber parts are weak and strong, the strong timber will be used where the stresses are the highest, and vice versa with the weaker timber.

In this thesis, softwares such as Grasshopper, Matlab and MiTek Pamir are used to simulate and test whether the risk of failure can be decreased if the truss designers can use individual values instead of the characteristic value. The results show that if the individual values of each timber board instead of the characteristic value are used the risk of failure can be decreased.

Keywords: W-truss, Fink truss, characteristic value, risk of failure, top chord, bottom chord, web

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1

Introduction

In chapter one, prerequisites of the thesis are presented. Firstly, a brief background of the problem is presented, secondly aims, and lastly the method chosen, and limitations.

1.1 Background

Timber used for structural purposes are, within the European standards, divided into strength classes and strength grades. A strength grade is a category that describes the timber's relative quality for construction, and the strength class is a strength grade where the timber has been assigned physical and mechanical properties to be used for structural design computations (Baño, Ridley-Ellis, & Stapel, 2016).

When strength grading timber, it is simplified as a homogenous material and assigned a constant strength along the entirety of the specimen (Hansson & Thelandersson, 2003). The strength the timber is assigned is known as its characteristic value, also known as the 5th percentile, which means that 95% of all specimens have a strength higher than what the assigned grade states (Ridley-Ellis, 2016). This may result in unnecessary high safety margins for timber structures (Hansson & Thelandersson, 2003).

In Europe, three key properties; strength, stiffness and density, are considered when grading timber specimens. Through two European standards, EN408 and EN384, and a reference moisture content of 65% and a temperature of 20°C, the grades are determined. Two sets of grades determining properties exist in the European standards:

- Bending strength, bending stiffness and density (C- and D-classes).
- Tension strength, tension stiffness and density (T-class).

The first set is most commonly used when grading general structural timber, and the second set is more common when grading lamellas used for glued laminated products (Baño, Ridley-Ellis, & Stapel, 2016). The presented thesis will investigate trussed roof rafters, which results in that the first set of properties are of interest. Table 1.1 displays different mechanical and physical properties for soft woods, C-class. The number next to the C shows the characteristic bending strength parallel

1. Introduction

to the grain.

Table 1.1: Physical and mechanical properties of structural timber (Swedish Wood, 2016).

Property	C24	C27	C30	C35	C40
Strength values					
Bending parallel to grain $f_{m,k}$	24	27	30	35	40
Tension parallel to grain $f_{t,0,k}$	14,5	16,5	19	22,5	26
Tension perpendicular to grain $f_{t,90,k}$	0,4	0,4	0,4	0,4	0,4
Compression parallel to grain $f_{c,0,k}$	21	22	24	25	27
Compression perpendicular to grain $f_{c,90,k}$	2,5	2,5	2,7	2,7	2,8
Shear $f_{v,k}$	4,0	4,0	4,0	4,0	4,0
Stiffness value for capacity analysis					
Elastic modulus $E_{0,05}$	7 400	7 700	8 000	8 700	9 400
Stiffness values for deformation calculations, mean values					
Elastic modulus parallel to grain $E_{0,mean}$	11 000	11 500	12 000	13 000	14 000
Elastic modulus perpendicular to grain $E_{90,mean}$	370	380	400	430	470
Shear modulus G_{mean}	690	720	750	810	880
Density					
Density ρ_k ²⁾	350	360	380	390	400
Density ρ_{mean} ³⁾	420	430	460	470	480

The strength of timber cannot be measured non-destructively without rendering the specimen useless in structural purposes. Strength, however, is correlated with other factors and properties that can be measured non-destructively and from these properties the strength of the specimen can be evaluated. These properties and factors are stiffness, density, knots, grain, species and origin.

Stiffness is another property influencing the grading and classification of timber and can be measured non-destructively via mechanical bending or dynamic stiffness (via vibration or measuring the time of flight). Density can, too, be measured non-destructively, the easiest way is to measure the dimensions of the timber and weigh it (Ridley-Ellis, 2016).

As of today when designing roof trusses, the designers are using the tabulated values, see Table 1.1, for structural timber. However, the tabulated value of strength of timber is its characteristic value, which may suggest that the trusses may be stronger than assumed. If the grading and testing data were to be known by the designers, the trusses may be built more efficiently and be optimized in comparison to the methods of today (Jockwer, 2020).

1.2 Aim

The aim of this thesis is to analyze whether the efficiency of roof trusses can be enhanced. If the grading data of the timber is utilized in the designing phase instead of solely the characteristic values of the timber, can the utilization ratio of the truss be bettered? Can the risk of failure be decreased, whilst the efficiency of the structure is increased?

1.3 Method

The thesis is carried out in different steps and stages. Firstly, a literature study is conducted, secondly a design of trusses is carried out in Rhinoceros and Grasshopper, lastly, the design of the truss will be carried out through parametric design, meaning that different parameters will yield different outcome of trusses.

1.3.1 Literature study

The literature study aims to provide knowledge of the grading process of timber. How timber is graded, and why. The study will also cover how the design process of roof trusses is carried out, and how the characteristic properties of timber are implemented when the truss is designed. The information has been gathered from books, scientific articles, conference papers and different studies conducted previously on both grading timber and roof trusses.

1.3.2 Parametric design

When designing the roof truss, the parameters and properties of the timber has a huge impact. This leads to a parametric design where the properties of the timber are altered, which will render in new design of the truss for every simulation. From this parametric study, an intelligent and optimized truss is hopefully designed.

1.4 Limitations

The truss to be designed will be a W-truss. There are many types of trusses that could be analyzed, but due to limited time, solely W-truss will be designed. See Figure 1.1, in this figure the different colours of the members represent different strength properties, i.e., this truss represents a truss that has been designed with regards to the grading data of structural timber. When designing a roof truss, the most common place to find failure is at the fasteners (Höglund, 2022), however though, the fasteners will not be neither designed nor accounted for in failure in this thesis, solely the structural timber members are to be considered.

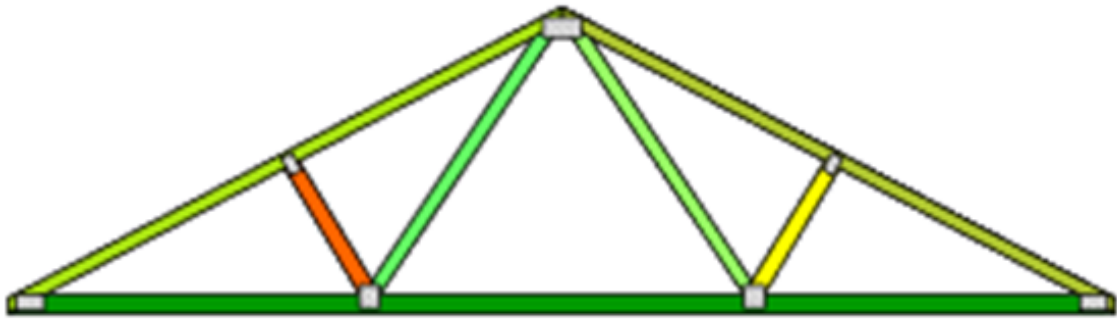


Figure 1.1: W-truss, also known as Fink truss, to be designed in this thesis (Jockwer, 2020).

2

Grading of timber

This chapter aims to provide information and background on how the grading of the timber is carried out as of today. When grading timber, generally there are two main methods, either machine grading or visual grading.

2.1 Visual Grading of Timber

Visual grading, not to be confused with appearance grading, have been used throughout history and is still commonly used until this day. Appearance grading is taking the aesthetics of the timber into account.

In the European standard EN18081-1 there are standards that must be met and followed when visually grading timber. Although there are harmonized standards that must be followed, grading rule for visual strength grading may differ from country to country. The rules in each country have been altered and optimized for the type of wood that is used there.

During visual grading, every specimen of timber is checked for the properties influencing the strength such as stiffness and density, together with knots, slopes of grains, ring widths and reaction wood. All these visual parameters will influence the grade that the specimen is assigned.

Visual characteristics of timber tend not to be the best predictor of properties of timber, and in addition with the safety margin to account for human error, visual strength grading does not make up the best way to estimate the real properties of timber. However, considering that less resources are needed for the visual strength grading it makes a good first route to ensure that more diverse species can be used in construction (Baño, Ridley-Ellis, & Stapel, 2016).

The major limitations of visual grading of timber are the fact that there may be distortions of the timber not on the surface that cannot be accounted for during visual grading. Due to these limitations of not knowing what is going on below the surface, the safety margins with regards to visual grading are higher than machine grading (Baño, Ridley-Ellis, & Stapel, 2016).

2.1.1 Knots in Sawn Timber

There are several types of knots in sawn timber, and all of them have different requirements to meet to be able to pass the visual grading test. In Figure 2.1 knot types are displayed. The criteria for different knots and how they are graded are presented in Table 2.1.

The shape of the knot, if it is intergrown, partially intergrown, encased or loose will affect the grade of the timber, the different shape of knots that may exist in sawn timber are:

- Round or oval knot
- Traversing edge knot
- Traversing or not traversing arris knot
- Spike knot
- Splay knot
- Pin knot



Figure 2.1: Different knot shapes (Swedish Wood, 2016).

Table 2.1: Visual grading criteria of structural timber with regards to knots (Swedish Wood, 2016).

2-sided grading	Feature		Class of the side					
	Maximum permitted on worst meter	2-sided grading ⁴⁾	G2-0	G2-1	G2-2	G2-3	G2-4	
		4-sided grading	G4-0	G4-1	G4-2	G4-3	G4-4	
	Face		Knot size 10 percent of the width + [table value] (mm)					
	Sound intergrown knot		10	20	35	50	⁶⁾	
	Partially intergrown knot / Dead knot		0	10	20	50		
	Encased knot		¹⁾	0	15	40		
	Unsound or loose knot		¹⁾	¹⁾	15	40		
			Knot number					
	Total number		2	4	6	³⁾	⁶⁾	
	Of which encased, unsound and/or loose knots		0	1 ²⁾	2	5		
	Of which spike and/or splay knots		0	4	6	³⁾		
	Edge		Knot size in percent of thickness ⁵⁾ (%)					
	Sound intergrown knot		50	90	100	100	⁶⁾	
	Partially intergrown knot / Dead knot		33	67	75	100		
	Encased knot		¹⁾	33	50	90		
	Unsound or loose knot		¹⁾	¹⁾	50	90		
			Knot number					
	Total number		1	2	4	³⁾	⁶⁾	
	Of which encased, unsound and/or loose knots		0	1 ²⁾	2	3		
	4-sided grading	Feature		Class of the side				
		Maximum permitted on worst meter	2-sided grading ⁴⁾	G2-0	G2-1	G2-2	G2-3	G2-4
			4-sided grading	G4-0	G4-1	G4-2	G4-3	G4-4
		Face		Knot size 10 percent of the width + [table value] (mm)				
Sound intergrown knot		10	20	35	50	⁶⁾		
Partially intergrown knot / Dead knot		0	10	20	50			
Encased knot		¹⁾	0	15	40			
Unsound or loose knot		¹⁾	¹⁾	15	40			
		Knot number						
Total number		2	4	6	³⁾	⁶⁾		
Of which encased, unsound and/or loose knots		0	1 ²⁾	2	5			
Of which spike and/or splay knots		0	4	6	³⁾			
Edge		Knot size in percent of thickness ⁵⁾ (%)						
Sound intergrown knot		50	90	100	100	⁶⁾		
Partially intergrown knot / Dead knot		33	67	75	100			
Encased knot		¹⁾	33	50	90			
Unsound or loose knot		¹⁾	¹⁾	50	90			
		Knot number						
Total number		1	2	4	³⁾	⁶⁾		
Of which encased, unsound and/or loose knots		0	1 ²⁾	2	3			

The number in the grading methods G2 and G4 showcase how many of the sides of the board that has been graded. In Sweden, G4 is most commonly used and has been determined to better predict the properties of the timber (Swedish Wood, 2016). G4-0 is the highest grade and G4-4 the worst. Usually the limit for timber to be used in structural purposes the visual grade must be atleast G4-2, and in Sweden the most commonly used species are spruce and pinewood (Swedish Wood, 2022).

For soft woods, a general correlation between knot ratio and bending strength is presented in Figure 2.2.

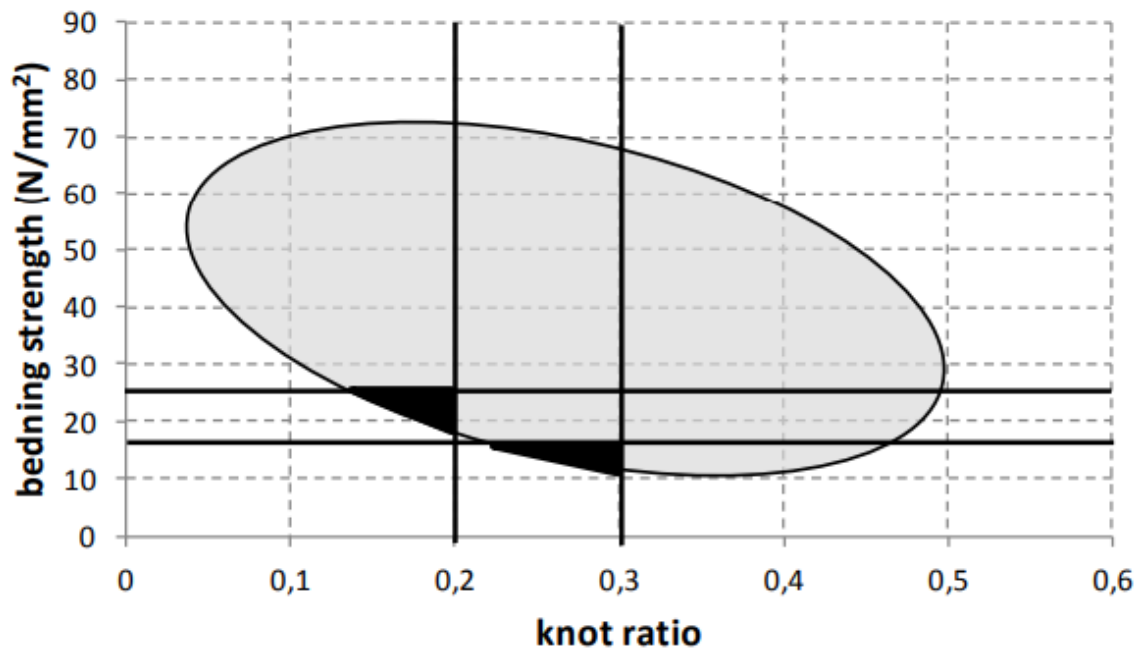


Figure 2.2: Diagram displaying roughly how the knot ratio impacts the bending strength for soft woods (Ravenshorst, 2015).

2.1.2 Other Natural Features Influencing Visual Grading

There are other natural features of structural timber that will influence the grade a timber will be assigned, these may be:

- Bark or resin pocket
- Resin wood
- Compression wood
- Grain structure
- Pitch

A bark pocket is defined as "*bark that is partly or wholly enclosed in the wood*" (Swedish wood, 2016). A resin pocket is a pocket that either contains or has contained resin creating a hollow in the timber (Swedish Wood, 2016). Both bark pocket and resin pocket are displayed in figures 2.3 and 2.4, respectively.

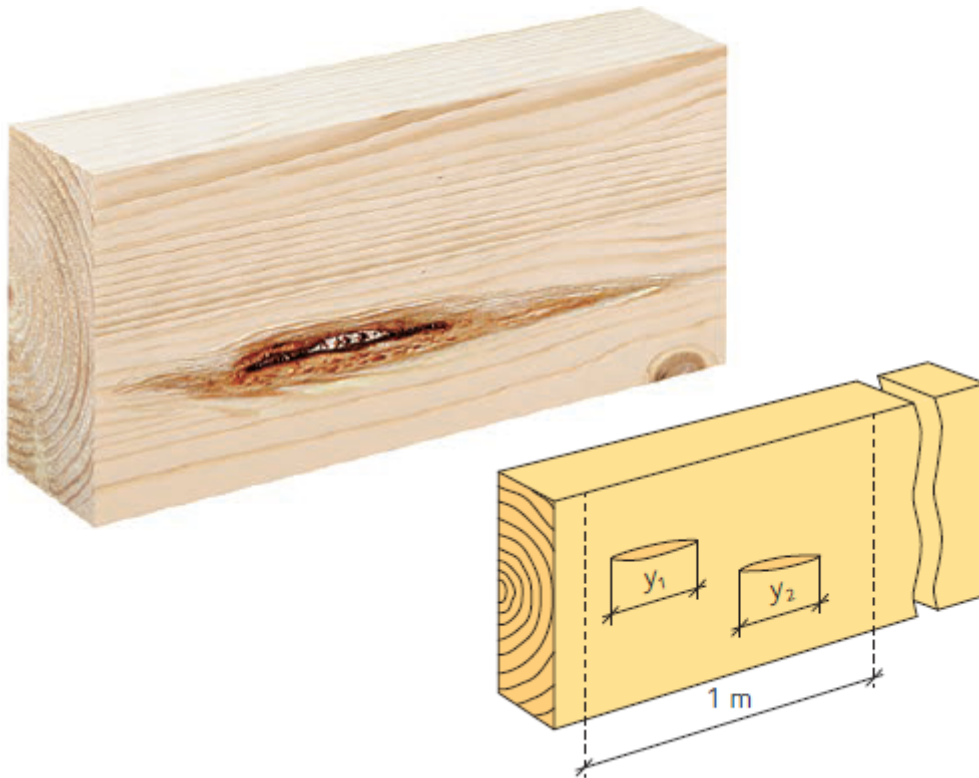


Figure 2.3: Bark pocket (Swedish Wood, 2016).

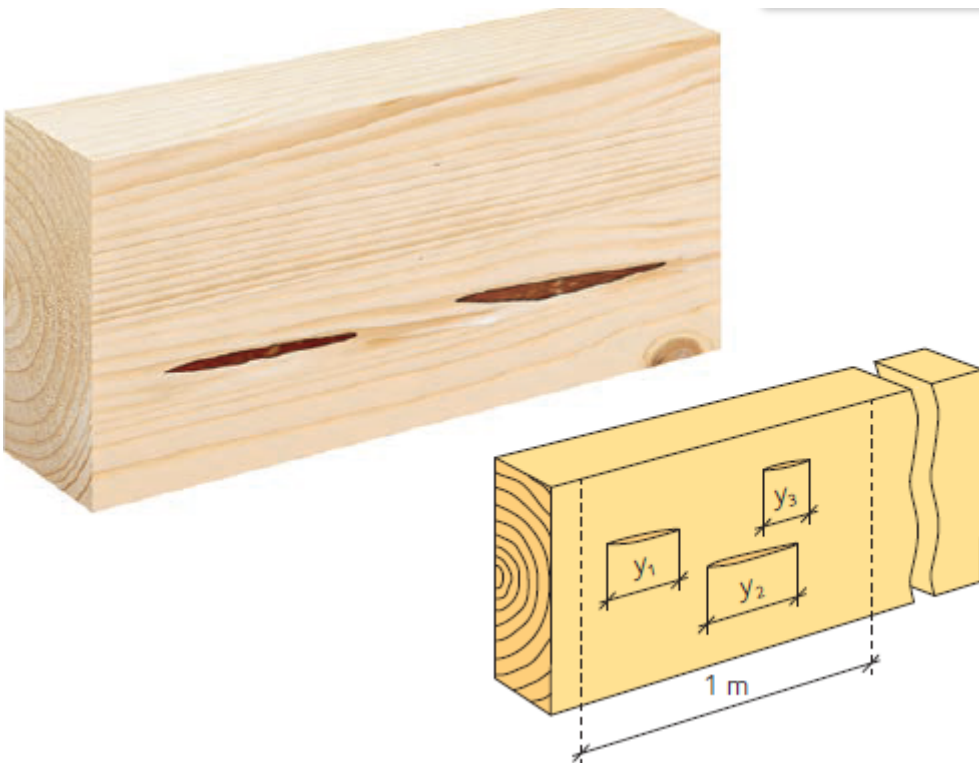


Figure 2.4: Resin pocket (Swedish Wood, 2016).

The different criteria with regards to, bark or resin pockets, resin or compression

2. Grading of timber

wood, grain structure, pitch and rottenness are displayed in Table 2.2.

Table 2.2: Visual grading criteria of structural timber with regards to some other natural features, including rottenness (Swedish Wood, 2016).

Feature		Class of the side				
Maximum permitted	2-sided grading ⁴⁾	G2-0	G2-1	G2-2	G2-3	G2-4
	4-sided grading	G4-0	G4-1	G4-2	G4-3	G4-4
Bark pocket						
Number on worst meter		¹⁾	2	2	4	³⁾
Total length (mm)		¹⁾	100	200	300	³⁾
Resin pocket						
Number on worst meter		2	4	4	4	³⁾
Total length (mm)		75	100	200	300	³⁾
Resin wood						
Percent of the area of the side (%)		¹⁾	10	30	50	³⁾
Reaction wood (Compression wood)						
Percent of the area of the side (%)		¹⁾	10	30	50	³⁾
Strong, abnormal grain structure		¹⁾	¹⁾	⁵⁾	⁵⁾	⁵⁾
Soft rot		¹⁾	¹⁾	¹⁾	¹⁾	²⁾
Blue stain and dote						
Deep blue or dote, percent of the area of the side (%)		¹⁾	¹⁾	10	50	³⁾
Surface blue, percent of the area of the side (%)		¹⁾	¹⁾	20	100	³⁾
Insect attack, diameter < 2 mm						
Percent of the area of the side (%)		¹⁾	¹⁾	¹⁾	15	⁵⁾
Pith		¹⁾	⁵⁾	⁵⁾	⁵⁾	⁵⁾

2.2 Machine Grading of Timber

Machine, or mechanical, grading of timber consist of two methods, machine controlled, and output controlled. Machine controlled is based on predetermined machine settings, and output controlled is carried out by continuous testing of outputs. Similarly, to visual strength grading, the machine carries out non-destructive tests on every specimen. The indicating properties that the machines asses are assumed to be a better predictor of strength than the properties that can be measured during a visual strength grading. The machine grading can also be carried out faster than visual grading and most commonly with a better accuracy, both thanks to x-rays in the machines where they detect knots that are not on visible on the surface, and there is less risk of human errors (Baño, Ridley-Ellis, & Stapel, 2016).

The modern production of structural timber needs to be carried out speedily, reliably, and cheaply, which results in that machine grading is a viable option (Deublein, Steiger, & Köhler, 2010).

2.2.1 Machine Control Grading

The machine control grading systems rely on predetermined settings coming from a previously conducted destructive test. The settings used are different for different species, and different for timber with different origin. Even if the species are the same, different settings must be used for the machine control if the origin of the timber is different (Baño, Ridley-Ellis, & Stapel, 2016).

There are different types of test that can be conducted to measure certain properties that are of interest. To estimate the Modulus of Elasticity, henceforth MoE, a three point bending test can be conducted in a so called Cook-Bolinder Grading machine. The timber boards are exposed to a preset deflection, and the mean force required to reach the preset deflection is measured at every 100mm. The minimum force required to reach the deflection will be the forced used when estimating the MoE for the entire board (Johansson, Brunding & Gruber, 1992). Since timber is seldom straight, this must be compensated for in the Cook-Bolinder machine, and this is done by rotating the timber 180° and performing the test again, the timber is assigned the minimum value of MoE (Boström, 1994). In Figure 2.5 a sketch of a three point bending test that is used in the Cook Bolinder grading machine is shown.

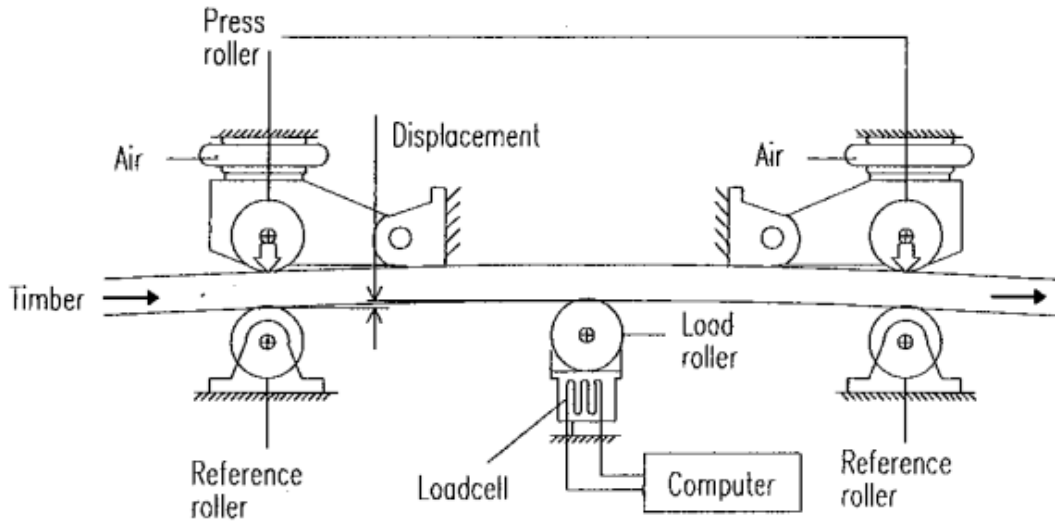


Figure 2.5: A three point bending test, as performed in the Cook-Bolinder machine (Boström, 1994)

The MoE is calculated as:

$$MoE = \frac{F * L^3}{\delta * 48 * I}$$

Where

F = force used to reach preset deflection

L = length between supports

δ = preset deflection

I = moment of inertia

The most common machine in Sweden for timber grading, in 1992, was the Computermatic machine. Instead of a constant deflection, as used in the Cook-Bolinder machine, a constant bending stress is applied to the timber. This bending stress causes a deflection which is measured every 152mm. The largest deflection will be used when calculating the MoE, and the MoE is calculated the same way as in the Cook-Bolinder grading (Johansson, Brunding & Gruber, 1992). A simple sketch of how the Computermatic machine works is provided in Figure 2.6.

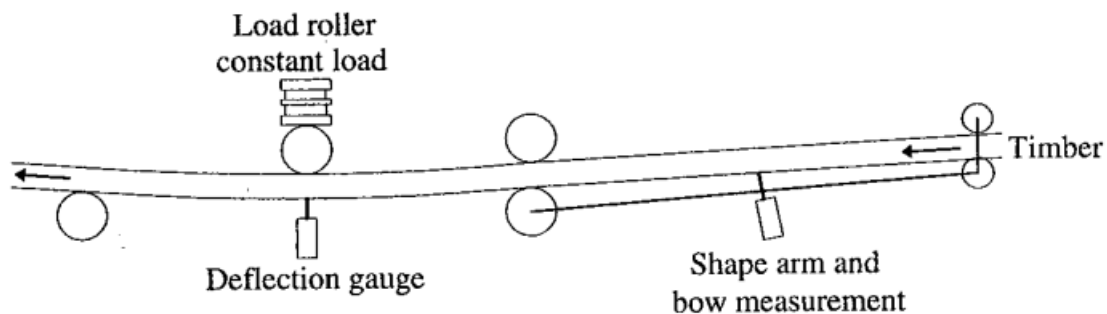


Figure 2.6: Principle of the computermatic machine (Boström, 1994).

Thirdly, there are a machine named Finnograder that does not work like the previously mentioned Cook-Bolinder and Computermatic. Finnograder works without touching the timber. The machine measures knots, both size and location, slope of grain, density and also the moisture content. The moisture content, henceforth MC, is measured to more adjust the density and the timber's specified MC. These measurements are made every 10mm of the board, and along the entirety of it, the measurements stop circa 50mm from the edges.

The density is measured via gamma-rays and the result shows mass of wood and water. The MC is measured via microwaves. The grain has displays a property so that the dielectric constant is bigger in the grain direction than perpendicular to it, making it possible to assess the slope of the grain with the help of polarized microwaves.

Knots can be measured via velocity of radiation. For wider timber boards upto 15 sensors can be used to detect knots. The measurements and readings of knots are weighted and summed up.

From the measurements of knots, slope of grain, density and MC, the bending strength can be estimated, and that is carried out by using an empirically developed equation (Boström, 1994):

$$f = (C_1 + C_2 * \rho + C_3 * \rho^2 + C_4 * \rho^3) * e^{f(\rho, KVS, KVD, SFG, SFK, u)}$$

Where:

f = Characteristic bending strength

C_n = Empirically derived constants

ρ = Density

KVS = Sum of knots

KVD = Difference between knot values at both halves

SFG = Slope of grain in knot-free areas

SFK = Slope of grain at knots

u = MC

After the timber has been graded, either via machine or visually, it is batched together with timber that is assigned the same strength class, according to the table presented in Section 1. However, considering that the strength value of timber is its characteristic value, there are potentially large difference in strength between timber boards within the same strength class.

2.2.2 Output Control Grading

This method requires that the producer analyzes the outputs and adjust the settings of the machine continuously. This method is not commonly used, so the procedures explained in the European Standard EN14081-1 is not well developed (Baño, Ridley-Ellis, & Stapel, 2016).

2.3 Strength of Timber

Timber shows great variability of its properties and characteristics, even within the same species with the same origin. Due to this variability of properties within timber, strength related properties are assigned a characteristic value, commonly known as the 5th percentile value (Porteous & Kermani, 2013). Considering that timber is assigned a strength based on its characteristic value, this means that 95% of timber of that strength grade exhibit a strength greater than the assigned value (Ridley-Ellis, 2016). The strength of timber follows a log-normal distribution, which is depicted in Figure 2.7.

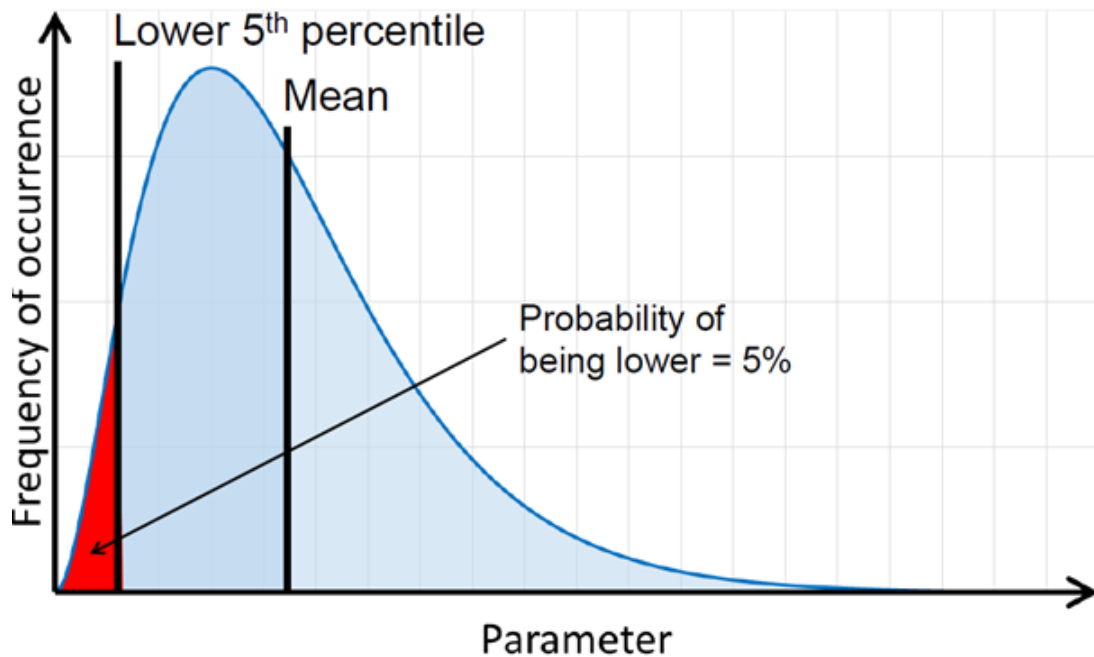


Figure 2.7: Log-normal distribution of strength of timber (Ridley-Ellis, 2016)

According to a study conducted by Alpo-Ranta Maunus and Mikael Fonselius (2001), when testing 589 specimens of ungraded spruce the mean bending strength was estimated as 45.2 MPa with a coefficient of variation at 25%. The same number of boards were then visually graded and 367 of the boards were graded as C24, and then tested. The C24 specimens exhibited a mean bending strength of 49.4 MPa with a coefficient of variation of 20%. The characteristic value of these boards were 33.1 MPa, roughly 38% greater than the strength class would suggest. After this, all 589 specimens would again go through a machine grading to see if any would be graded as C30. According to the machine grading, 496 of the boards were strong enough to be graded as C30, with a characteristic value of 30.5 MPa, a mean value of 47.5 MPa and a coefficient of variation of 22%. The results from this study would suggest that machine grading of timber can be carried out with greater accuracy, and suggest that a large number of specimens within each grade will exhibit strengths greater than what the strength class states.

Another study with 50 specimens of spruce from Sweden, states that the timber exhibited a mean bending strength of 62.5 MPa with a standard deviation of 9.6 MPa and a local MoE of 16 100 MPa and a standard deviation of 2000 MPa (Ravenshorst, 2015). These specimens have not been graded prior to testing, neither visually nor with machine.

A third study conducted in Italy on 278 specimens of spruce measured a mean bending strength of 41.5 MPa, with a coefficient of variation of 31%, mean MoE was measured to 10500 MPa with a coefficient of variation of 24% (Nocetti, Bacher, Brunetti, Crivellaro, G. Van de Kuile, 2010).

3

Roof Trusses

In this chapter a brief description of trusses will be presented, different types of trusses, and the main truss treated in this thesis. The design phase will be outlined.

Trusses are assemblies of members, in which the members respond in either pure axial compression or tension. The top and bottom members in a truss are called chords, and the members in between the chords are called web members. Web members are usually separated by their response of the forces, web members in tension are called ties, and web members in compression are called struts.

If it is an ideal truss, members meet at joints or nodes, which are most commonly idealized as hinges or pins, incapable of transmitting any bending moment in between members. However, ideal trusses are not realistic, since in an ideal truss, loads are only applied at the nodes or joints, whilst in reality the load is most commonly applied along the chords, causing a combination of axial stress, bending moments and shear (Timber frame engineering council, 2020).

Different types of trusses and their respective names are presented in Figure 3.1.

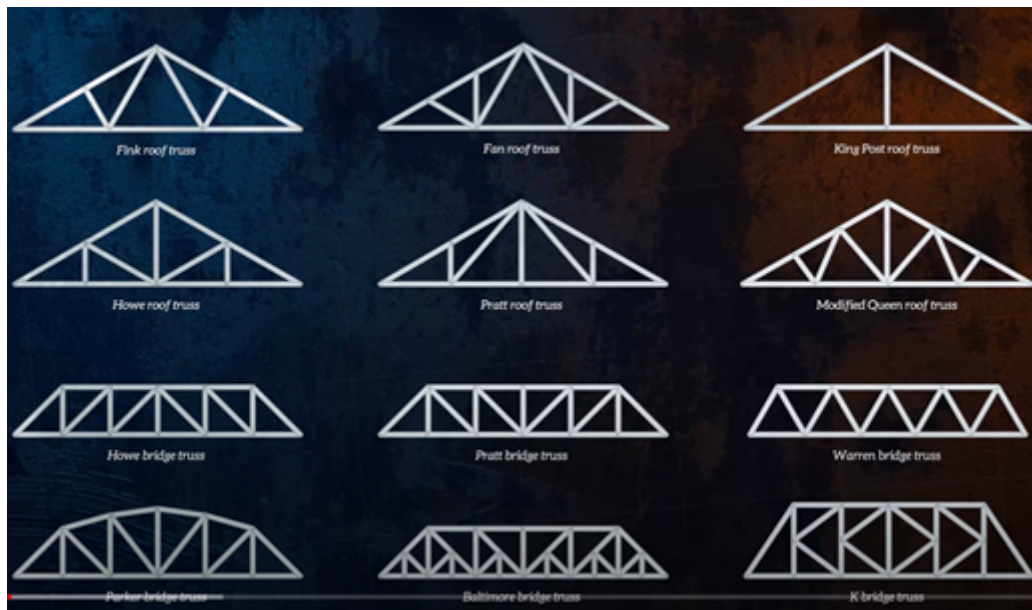


Figure 3.1: Different types of roof trusses (The Efficient Engineer, 2021).

These types of trusses are mainly used in houses with a roof inclination of 14° - 30° .

The shape of the truss is to make the forces acting in the truss to be mainly normal forces (Svenska Takstolsföreningen Svenskt Trä, 2021).

There are other types of trusses than the one presented above, and these are mainly framework. These are more commonly used if the area above the bottom chord of the truss is going to be used, such as in a house with one and a half storeys (Svenska Takstolsföreningen Svenskt Trä, 2021).

3.1 Designing of Roof Trusses

As of today, production of structural timber and design of roof trusses are disconnected and have little to no contact. This results in that the designers need to use the characteristic strength of the timber when designing the trusses. See Figure 3.2 for clarification of how today's production process is carried out.

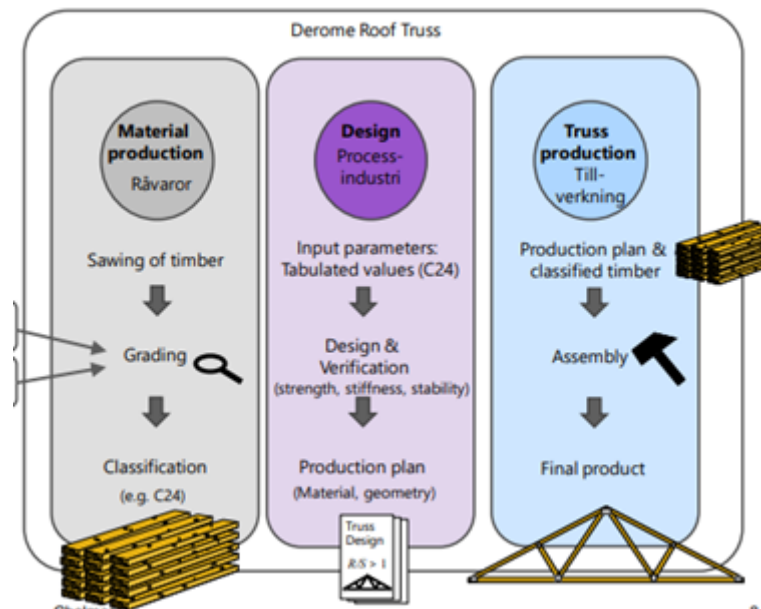


Figure 3.2: Process of manufacturing a roof truss, as of today (Jockwer, 2020).

In Derome, a manufacturer of roof trusses, the designer designs the truss based on what type of building that is going to be erected, size of the building, location of the building, etc. All of these parameters influence how the truss will be designed. Further on, the design may differ whether the truss will be covered or possible to be seen by the occupants of the building. When the designer is finished, the drawings of the truss are sent to the production team that will cut and assemble the parts needed for the truss. In Derome, they have a large machine into which the saw plan can be input so that all the boards and members used for the truss are cut directly and labelled accordingly.

The assembly team will be guided by laser attached to the roof of the assembly hall, so that the truss members are put out correctly, with the right distance and angle,

before attaching the members to each other. When the truss is laid out perfectly, the steel plates are added and attached to the timber with the help of a hydraulic pressure machine that ensures that the steel plates are fastened firmly. When one side of the truss is fastened, the truss is flipped 180° so the members are attached to each other on both sides of the truss. The process in Derome is similar to how the process is displayed in Figure 3.3.

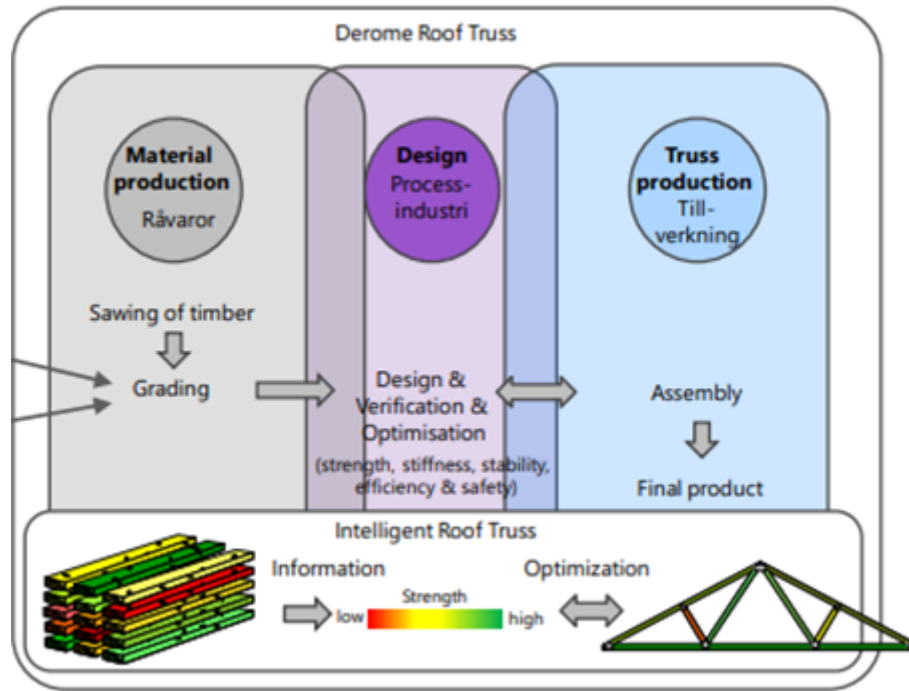


Figure 3.3: Improved process of roof truss manufacturing (Jockwer, 2020)

3.2 Optimizing Roof Truss

Optimizing the shape of trusses is crucial to minimize the amount of material used while also ensuring a high safety factor of the structure. It is necessary to analyse angles, impact of angles and shapes of members in the structure, and this can be done with many different methods. Ultimately, each individual member needs to be strong enough to resist the stresses it is subjected to in its specific position (Kaur, Singh Bansal, & Sanjeev, 2016).

Another way of optimizing trusses is by analysing specific members and comparing them to a given data set of timber strengths and qualities. By taking grading data into consideration when placing members in the truss, it is possible to keep structural redundancy at an acceptable level while also ensuring a high safety factor.

These two ways of optimizing the structure, with regards to truss shape and materials, can be used in tandem to further optimize structures beyond the possibilities of the methods when used separately.

4

Loads on Roof Trusses

Roof trusses are structural members carrying load and ensures that the load is distributed to the walls or columns, which are the vertical load carrying members.

Main loads acting on roof trusses are its self-weight, roof cover, including potential purlins and sheeting, snow load and wind load. The self-weight of the roof truss may be neglected, this is because that the truss is assumed to be able to carry itself, and the self-weight of the truss is small in comparison to the loads coming from the roof cover, snow, and wind.

4.1 Self-weight

The self-weight of the roof structure is dependent on both the dimensions and class of timber that makes up the truss, and the outer roof structure. The self-weight of the roof truss is classified as a permanent load.

To calculate the self-weight of the truss used in this thesis, the volume of the timber must be computed. To see how this is carried out see Appendix A. The density of the timber structure is assumed to be its characteristic value (fifth percentile) and will be log-normally distributed, similarly to how the characteristic value of the timber strength is distributed.

For this thesis, an assumed self-weight of the roof structure, not including the truss, is assumed to be:

$$g_k = 0.8 \frac{kN}{m^2}$$

This is a tabulated value from Träguiden (2017) for wood wool panels, insulation and roofing felt. An assumption that has been made for simplicity for this thesis is that is no ceiling attached to the bottom of the truss, nor any ventilation or other necessary installations.

4.2 Snow Load

The snow load is dependent on the location of the structure, which decides the snow load on the ground. The snow load is estimated as an occurrence of once every fifty years. The snow load on the roof top is both dependent on the snow zone, and the roof angle, where a steeper angle results in smaller snow loads on the roof top, and vice versa (Boverket, 2019). Figure 4.1 displays the division of snow zones of the

southern half of Sweden. This thesis will focus on a structure located in Gothenburg, which has a characteristic snow load of $1.5 \frac{kN}{m^2}$.

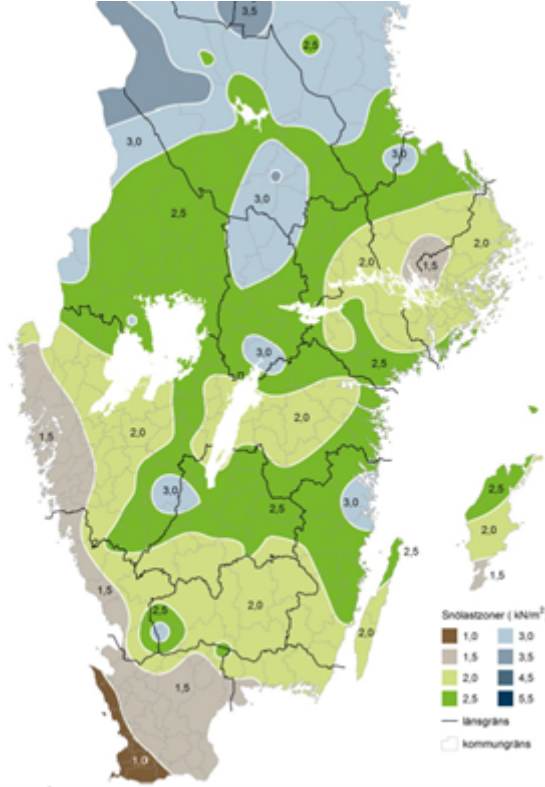


Figure 4.1: Snow zones of roughly half of Sweden (Boverket, 2019)

The snow load is not a permanent load, but a variable one. This results in that the snow load should be designed, similarly to the strength of the timber, as a normal distribution, where the characteristic snow load, that is estimated to appear once every fifty years, is assumed to be the 98th percentile value.

Characteristic snow load of Gothenburg:

$$s_k = 1.5 \frac{kN}{m^2}$$

Roof angle:

$$\alpha = 30^\circ$$

To translate the characteristic snow load, a form factor dependent on the roof angle is used, see Figure 4.2 for how it translates depending on the slope of the roof. For a saddle roof there are two different form factors, the reason for this is that one side is assumed to be on the leeward side of the house, meaning that the snow do not slide of as easily. Form factor for the side exposed to the wind:

$$\mu = \begin{cases} 0.8 & \text{if } \alpha > 30^\circ \\ 0.8 * \frac{60 - \alpha}{30} & \text{if } 30 < \alpha < 60 \\ 0 & \text{if } \alpha \geq 60 \end{cases}$$

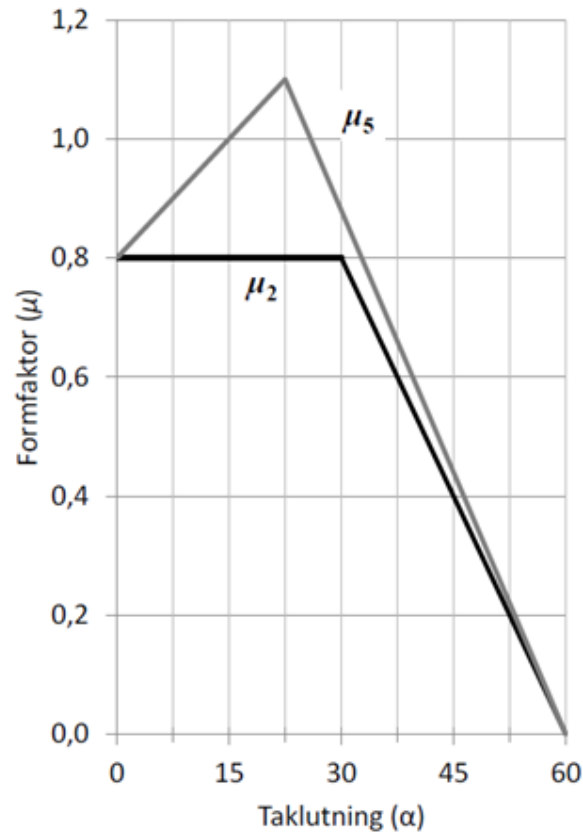


Figure 4.2: Coefficient to translate snow load on the ground to the roof top, depending on the roof angle (Boverket, 2019).

Snow load on wind exposed side:

$$s_k = 1.5 \frac{kN}{m^2}$$

$$\mu_2 = 0.8$$

$$S_{wind} = s_k * \mu_2 = 1.2 \frac{kN}{m^2}$$

Snow load on leeward side:

$$s_k = 1.5 \frac{kN}{m^2}$$

$$\mu_2 = 0.88$$

$$S_{leeward} = s_k * \mu_2 = 1.32 \frac{kN}{m^2}$$

Characteristic snow load per roof truss:

Distance between roof truss:

$$cc = 1.2m$$

A cc measurement of 1.2 meter is standard practice in Sweden (Björklund, 2022).
Windy side:

$$q_{snow_{wind}} = S_{wind} * cc = 1.44 \frac{kN}{m^2}$$

Leeward side:

$$q_{snow_{leeward}} = S_{leeward} * cc = 1.584 \frac{kN}{m^2}$$

In Figure 4.4 it shows how the snow load will differ with varying roof inclination, one can see that the most snow load appears on the roof at an angle of 22.5°, on the leeward side.

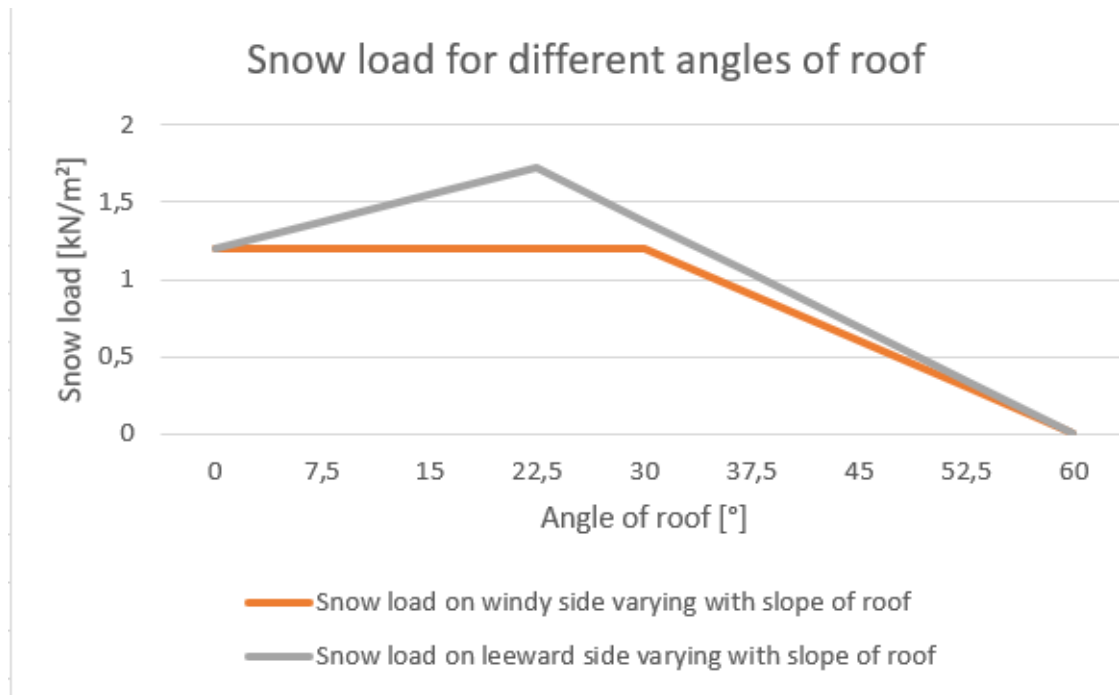


Figure 4.3: Varying snow load with varying inclination of roof (Source: Authors).

4.3 Wind Load

Wind load is assumed to be acting on the roof top, and the load is assumed to be average velocity pressure during a ten minute period once every fifty years (Boverket, 2019). The peak wind velocity in Gothenburg is set to 25 m/s, see Figure 4.4 for the different peak wind velocities for different parts of Sweden.

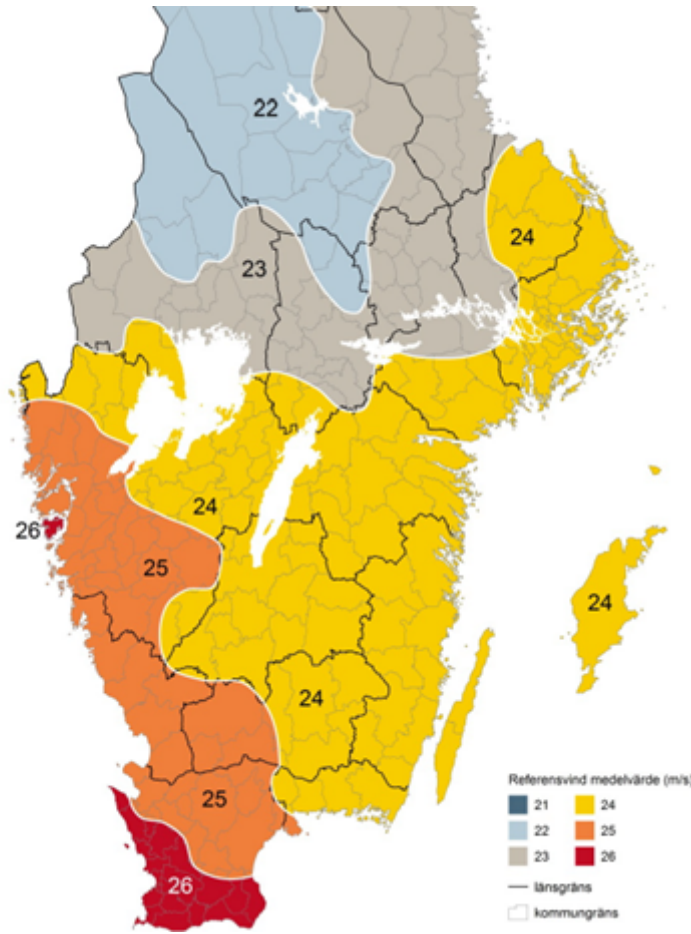


Figure 4.4: Reference wind speed in different parts of Sweden (Boverket, 2019)

Reference wind speed in Gothenburg:

$$v_b = 25 \frac{m}{s}$$

Density of air:

$$\rho_{air} = 1.25 \frac{kg}{m^3}$$

Reference mean velocity wind pressure:

$$q_b = \frac{1}{2} * \rho_{air} * v_b^2$$

Assume terrain category III and building height of 10m:

$$k_r = 0.22, z_0 = 0.3, z = 10m$$

Terrain and building height factor:

$$c_e = 1.783$$

Peak Velocity pressure:

$$q_p = c_e * q_b$$

External pressure coefficient:

$$c_{pe10} = 0.7$$

External pressure acting due to wind:

$$w_e = q_p * c_{pe10} = 0.488 \frac{kN}{m^2}$$

The wind load is a variable load that is assumed to be acting on the structure every fifty years. This results in a normal distribution with the load caused by the peak velocity assumed to be the 98th percentile value of the wind load (Boverket, 2019).

For this thesis the wind is assumed to solely act in pressure, no suction occurs, and there is no internal pressure. Both suction and internal pressure will cause uplift and for these forces, the fasteners between truss and roof cover must be checked.

4.4 Horizontal Loads and Bracing

For this thesis, horizontal bracing has not been checked and the trusses are assumed to be sufficiently braced. The trusses are assumed to be braced by the roof cover and that is assumed to be sufficient. However, normally bracing must be checked and trusses can be braced together, see Figure 4.5 for an example of such horizontal stabilizing method.



Figure 4.5: Horizontal stabilizing truss for a roof truss, to prevent too large horizontal displacement and failure in that direction (Svenska Takstolsföreningen & Swedish Wood, 2021).

5

Software

For the purpose of this thesis, different software have been used to produce models and analyze trusses. Main software utilized have been Rhinoceros 3D and MiTek Pamir. Both are introduced in this chapter.

5.1 Rhinoceros

Rhinoceros is a 3D CAD modeling software that is widely used in general design, but that can also be used to design buildings and structural systems. In this report, Rhino will be used to construct and analyse a fink truss. Seen in Figure 5.1, the model displays elements with their respective cross-sections, applied loads, and support conditions.

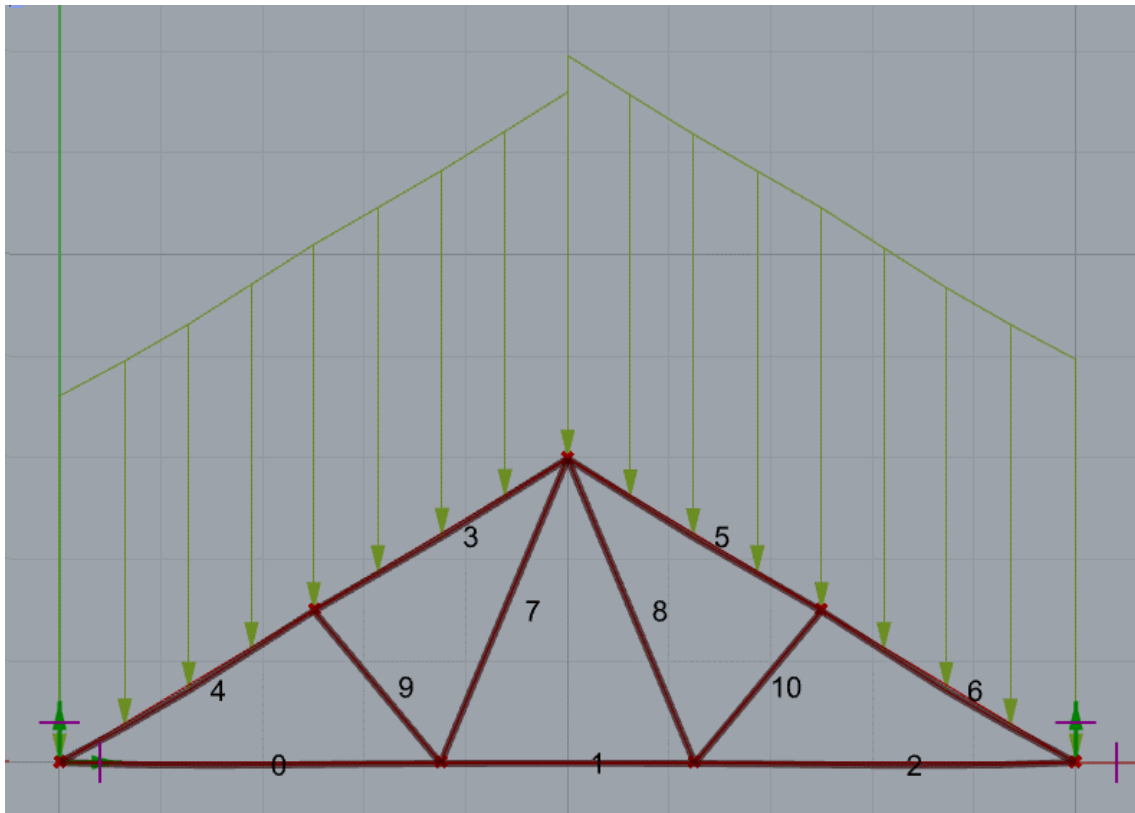


Figure 5.1: Truss model designed in Rhino, with element numbering and applied loads.

A truss is constructed by defining the relevant points and their position, and then connecting these with lines. This forms a basic structure that can be used for analysis.

5.1.1 Grasshopper

Grasshopper is an add-on to Rhinoceros that allows the user to through a node-based approach define geometry and handle data extracted from the model. An example of the node structure can be seen in Figure 5.2. However, each node in Figure 5.2 represents multiple nodes, and is only a simplified representation.

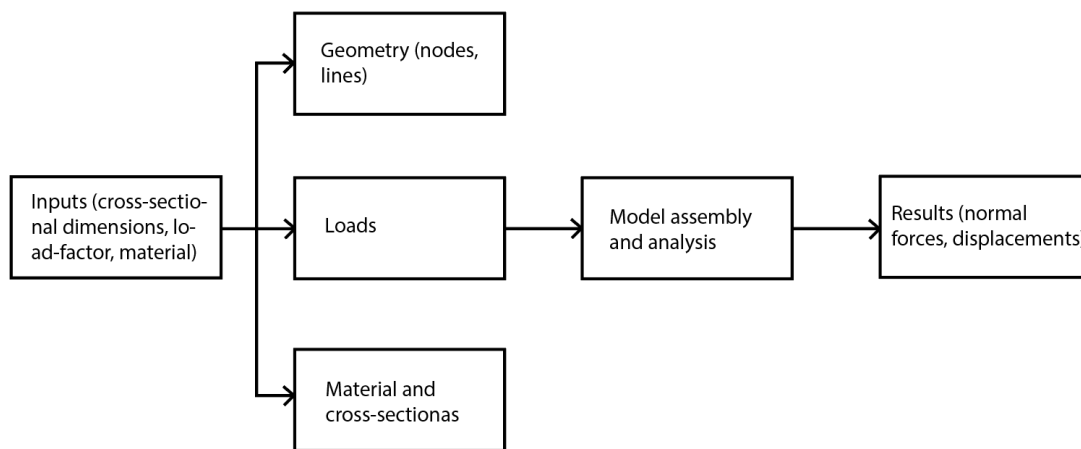


Figure 5.2: Node script structure used in Grasshopper

5.1.2 Karamba

Karamba3D is a Grasshopper-based add-on to Rhinoceros that is able to perform a wide range of different structural analysis. In this report, Karamba3D is used to perform structural analysis relevant to a link truss, such as normal forces and stresses.

5.2 MiTek Pamir

MiTek is a manufacturer of steel plates for fastening of roof trusses. MiTek has simultaneously developed a software named Pamir for designing purposes of structural members of houses, including slabs, walls, trusses, etc. MiTek Pamir performs calculations and displays stresses, utilization ratios and displacements for the designers' chosen load case (Holmlund, 2022). Pamir has timber properties added, so the designers need only choose which strength class of timber to use. MiTek Pamir has built-in load case scenarios that the designer can choose from, meaning that the designers of roof trusses need only know the magnitude of the loads acting on the

roof truss.

When opening MiTek Pamir, one needs to either open an existing project or start a new one. When opening a new project, the shape of the building needs to be set, see Figure 5.3 see for the shape of a building on to which trusses can be placed.

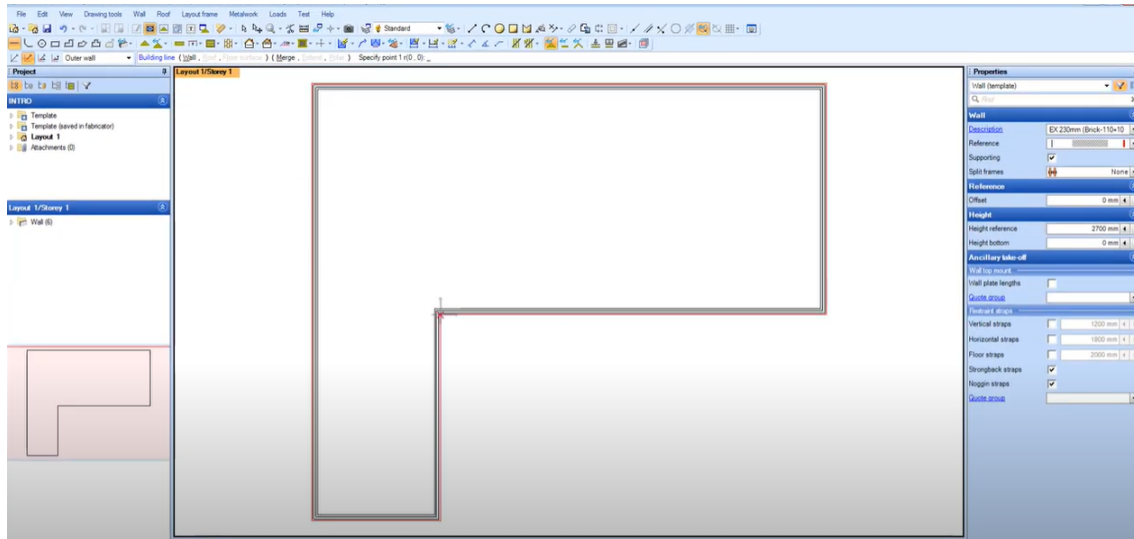


Figure 5.3: Shape of a building onto which the roof trusses will be placed (MiTek Industries South Africa, 2019).

When the shape of the building is set, including the height which can be altered on the right hand side, which can be seen in figure 5.3, the designer can start to add the roof trusses to the building. This can be done by using the property "Frame" and then marking the entire building onto which the trusses should be placed. The result looks as Figure 5.4.

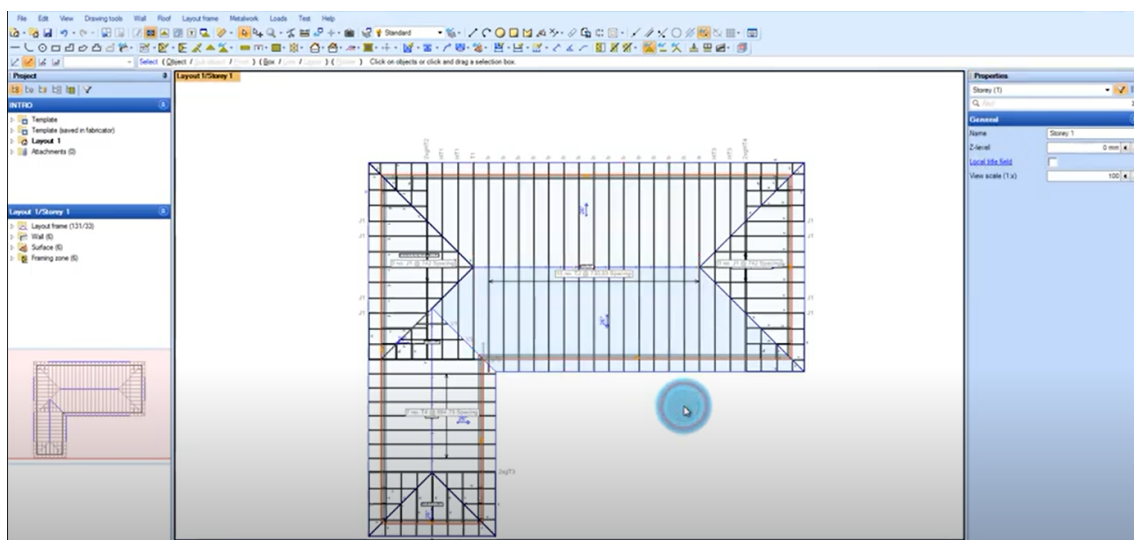


Figure 5.4: Trusses added to the building (MiTek Industries South Africa, 2019).

After the trusses have been added to the building, they can be designed in the truss design property of the software. MiTek Pamir firstly suggests a truss, and then it is the designers' job to add, remove and alter the truss to try to optimize it and make it better. Derome's truss designers use MiTek Pamir, and according to Johan Björklund (2022), the designers at Derome usually disregard the trusses suggested by the software completely and starts from scratch.

See Figure 5.5 which is a fink roof truss designed in Pamir, where the percentages displayed is the estimated utilization ratio of the member of the roof truss, whether it is a steel plate or timber member.

Pamir is used by several Swedish roof truss designers and engineers, promoted during courses carried out by the association for Swedish roof truss designers (Holmlund, 2022).

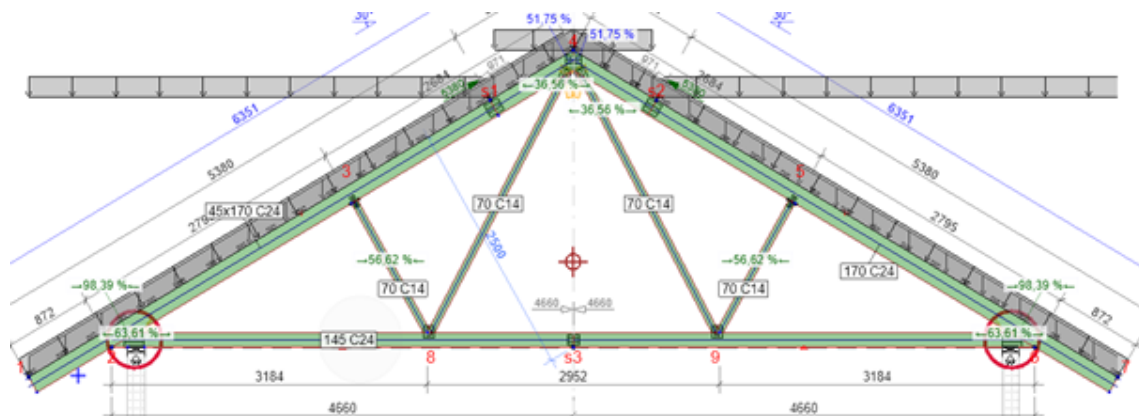


Figure 5.5: Fink-truss designed in MiTek Pamir.

The joint in the bottom right corner from Figure 5.5 is presented in Figure 5.6. It shows the utilization ratio of the top chord, bottom chord and the steel plate. One can tell that the steel plate has a utilization ratio that is too large.

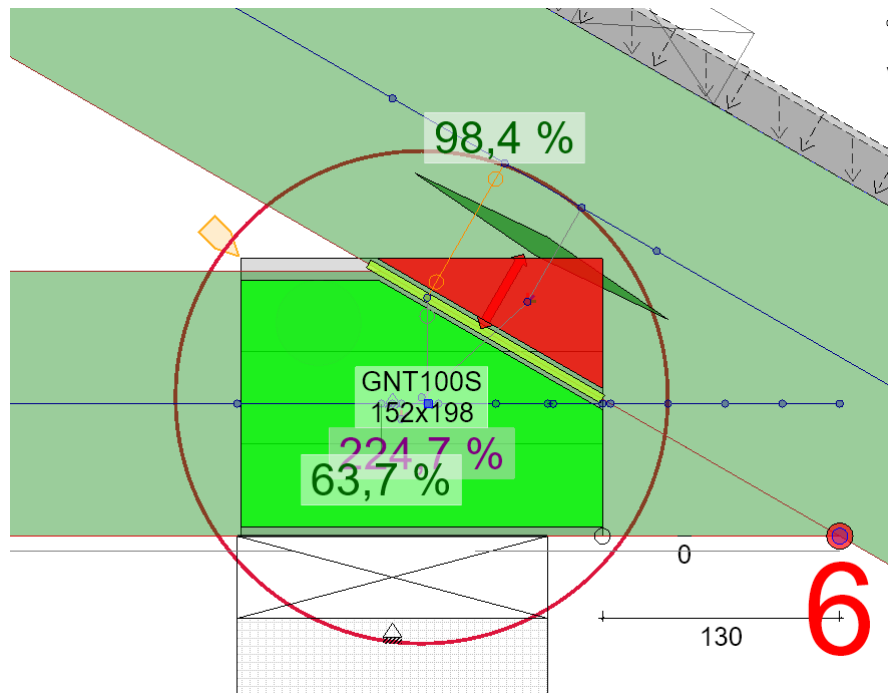


Figure 5.6: Utilization ratio of steel plate in the bottom right corner.

Advantages of using Pamir is mainly its simplicity and user friendly interface, where the designer can move around the members to alter the design, and material properties are programmed into the software, which means that the designer simply chooses the grade to use for which member.

Similarly to the timber properties, load combinations are existing in the software. I.e., the designer needs to know the magnitude of the load, but the programme itself does the combination calculations. As the designer, one can choose to utilize the wind or the snow load as the main load. It is possible to perform analyses both in ULS and SLS.

6

Method

6.1 Assumptions

In order to construct a model for which a structural analysis can be performed, a wide range of assumptions need to be made. These assumptions are made in part to reduce the complexity of the analysis, but also to take the limitations of the software into consideration.

The truss is supported on pinned supports, allowing rotation around the y-axis, while restricting vertical and horizontal movement in the support nodes. Support conditions used in Rhino can be seen in Figure 6.1, with arrows representing locked translations, circles representing locked rotation.

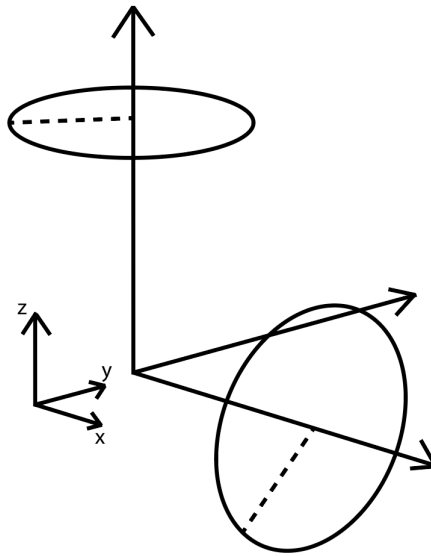


Figure 6.1: Support conditions used in Rhino.

Connections between members are semi-rigid in Rhino, allowing some rotation and deformation from the non-loaded position. While elements 0, 1, and 2 (and 3, 4 & 5, 6) would in reality be a singular member spanning the full length of the truss, Rhino does not allow for this to be the case. Instead, the semi-rigid connections between members (in nodes) is assumed to be enough to accurately represent the behaviour of a singular member.

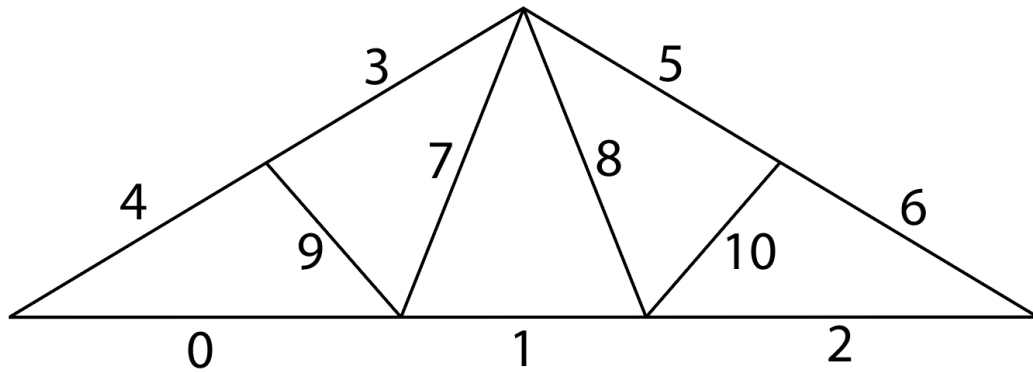


Figure 6.2: Numbering of truss elements.

The Rhino model is constructed by defining nodes and lines, to which material and cross-sections are assigned. This causes the geometry of members to intersect, unlike a real-life model. This is assumed to not cause any difference in results, as each member is able to utilize its full capacity. See Figure 6.3 for clarification.

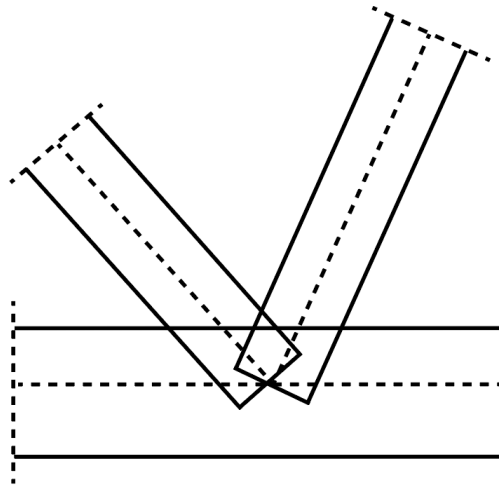


Figure 6.3: Intersection between members 0 (and 1), 7, and 9.

6.2 Variables

The Rhino model is designed such that the height and width of the truss are fully adjustable, but for the sake of analysis, a height of 3 meters and a width of 10 meters will be used.

Each member (lower chord consisting of elements 0, 1, and 2) is assigned a material available from a material node in Karamba3D. The range of materials include C14

to C50, but only C14, C24, and C30 will be relevant for this analysis. A specific density is also assigned and accounted for in the load calculation, depending on which material that is used.

A load-factor is used to adjust the applied load. This is simply a value multiplied with the ultimate limit state load combination. For the sake of this analysis, a scale ranging from 0 to 5 is used.

Each member has rectangular cross-sections, with adjustable dimensions. Width and height of the cross-section is determined by standard measurements.

6.3 Material

The material used for the Monte Carlo simulation is generated from a normal distribution for each C14, C24, and C30. These samples of each material is generated such that 95% of the values are above 14, 24, and 30 MPa respectively. This is different from real-life, where the distribution is more similar to a log-normal distribution, as mentioned in section 2.3. They are assumed to have mean values of 18, 30, and 38 MPa. Each material sample consists of 10,000 data points. See Figure 6.4 for a visual representation of the distributions.

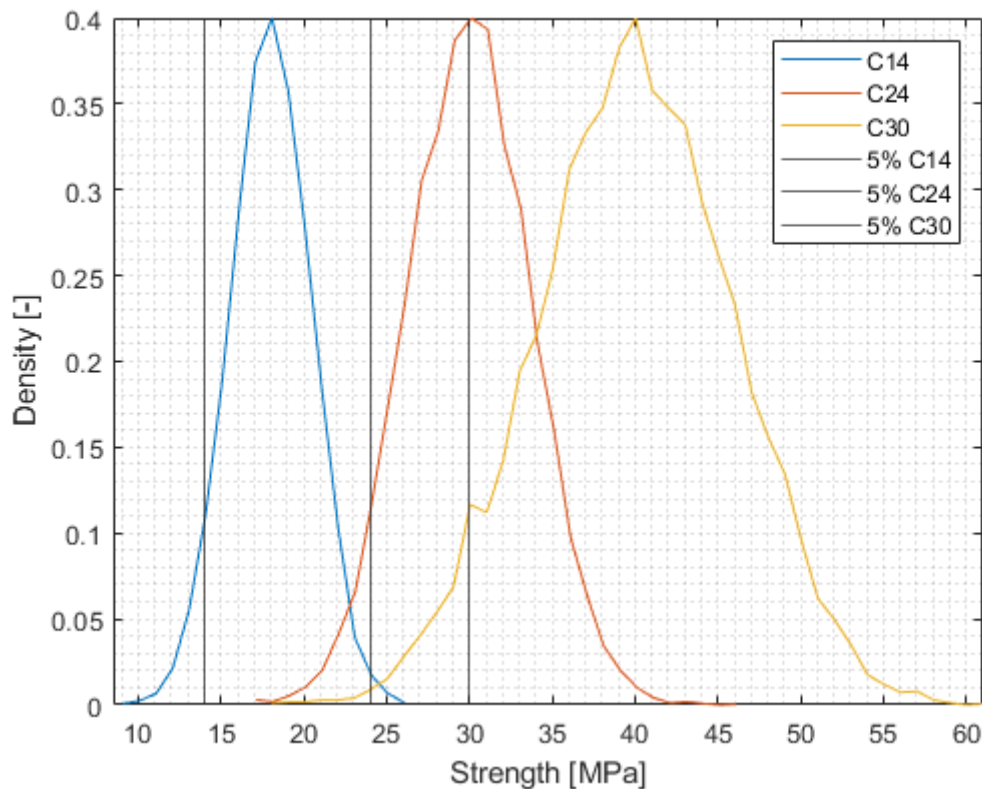


Figure 6.4: Distributions for C14, C24, and C30 and their 5th percentile. Visualized similarly to a normal distribution.

6.4 Frequency of load

The distribution used for the frequency and magnitude of the applied load is a modified normal distribution generated in a similar manner to the materials in section 6.3. The original distribution was generated with 1.0, the ULS load, in the 98th percentile, which resulted in some negative values within the distribution. These negative values were replaced with some positive, more extreme values (2.0-5.0) with decreasing frequency of occurrence. This allowed the ULS load (1.0) to remain in the 98th percentile, while introducing greater loads than what was generated in the original normal distribution.

A better, more ideal way of performing this analysis would be to use either a log-normal distribution, or a gamma distribution. This would give a more realistic distribution of live loads and their distribution, including that of more extreme loads beyond the 98th percentile. However, for the sake of simplicity, this modified distribution is used instead.

See Figure 6.5 for the distribution, and Table 6.1 for the corresponding loads used in the structural analysis in Rhino, where "Load factor [-]" 1.0 represents the ULS load.

Load factor [-]	Load [kN/m]
0.5	2.005
1.0	4.01054
1.5	6.03987
2.0	8.02108
3.0	12.0316
4.0	16.0422
5.0	20.5295

Table 6.1: Load factors and corresponding loads in kN/m.

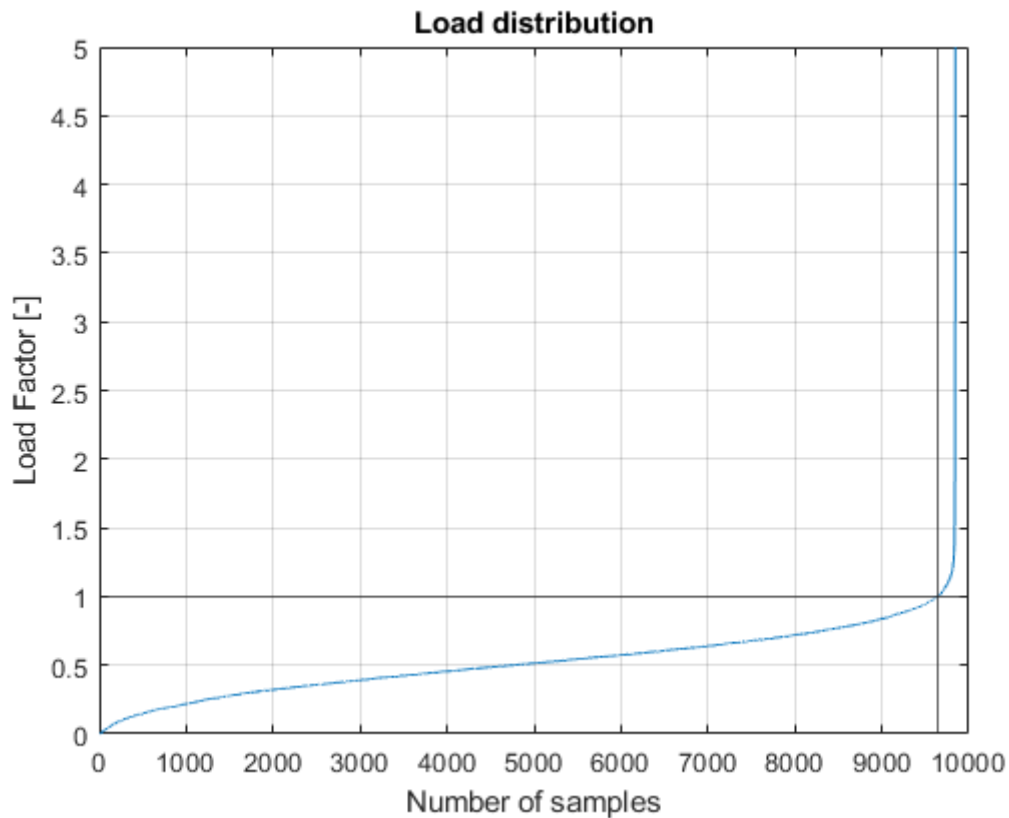


Figure 6.5: Modified normal distribution of the load, with ULS (1.0) being the 98th-percentile.

6.5 Monte Carlo simulation

The calculated stresses from Rhino are copied into an Excel sheet, which are then imported into MatLab.

Each member is randomly assigned a material strength from the normal-distributions in Figure 6.5. The assigned material strength is then compared to the normal

stresses in each member. If the normal stress in a given member exceeds the material strength, the member has failed and a 1 is stored in a vector in the position of the element in question. At the end of the simulation, a total tally of failures in each element is saved. This total number of failures is then expressed as the ratio between failures and iterations performed. As the number of performed iterations increase, the number of cases where failure occurs also increases proportionally. The Monte Carlo simulation is performed with 100,000 iterations.

Flow diagram of MatLab code can be seen in Figure 6.6. Full script can be found in Appendix B.

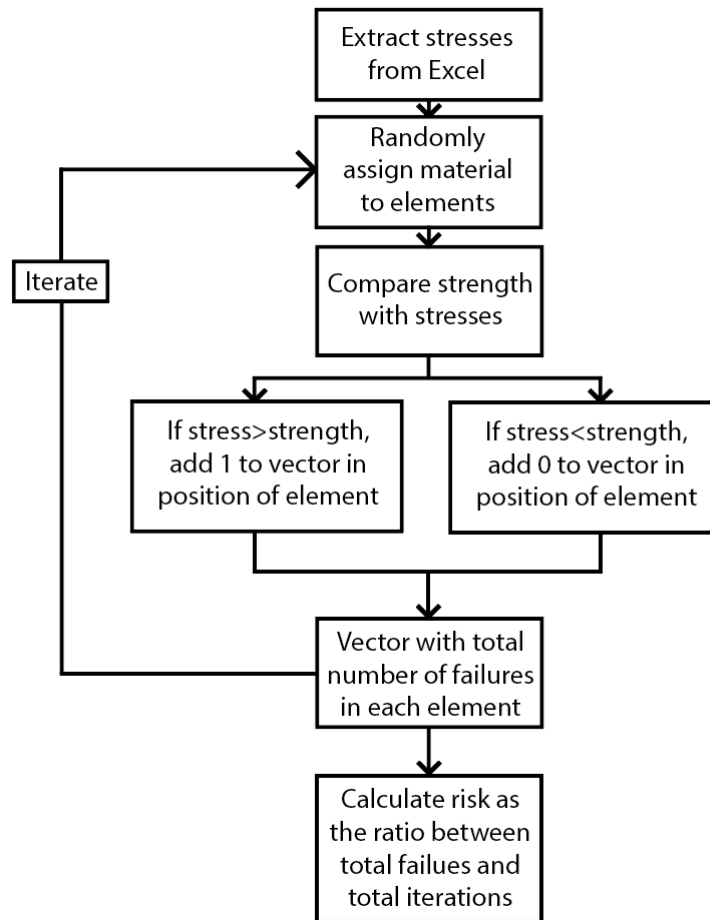


Figure 6.6: Flow diagram of MatLab Monte Carlo simulation.

6.6 Optimization using Monte Carlo

The Monte Carlo simulation is also performed with optimized placement of members, instead of random placement. First, 11 material strengths are generated from the distributions in 6.4, which are then sorted from weakest to strongest. These materials are then assigned to elements according to the stresses they are subjected to. The weakest material is assigned to the least stressed element, the strongest

material is assigned to the most stresses element, and so on.

Flow diagram of MatLab Code can be seen in Figure 6.7. Full script can be found in appendix B.

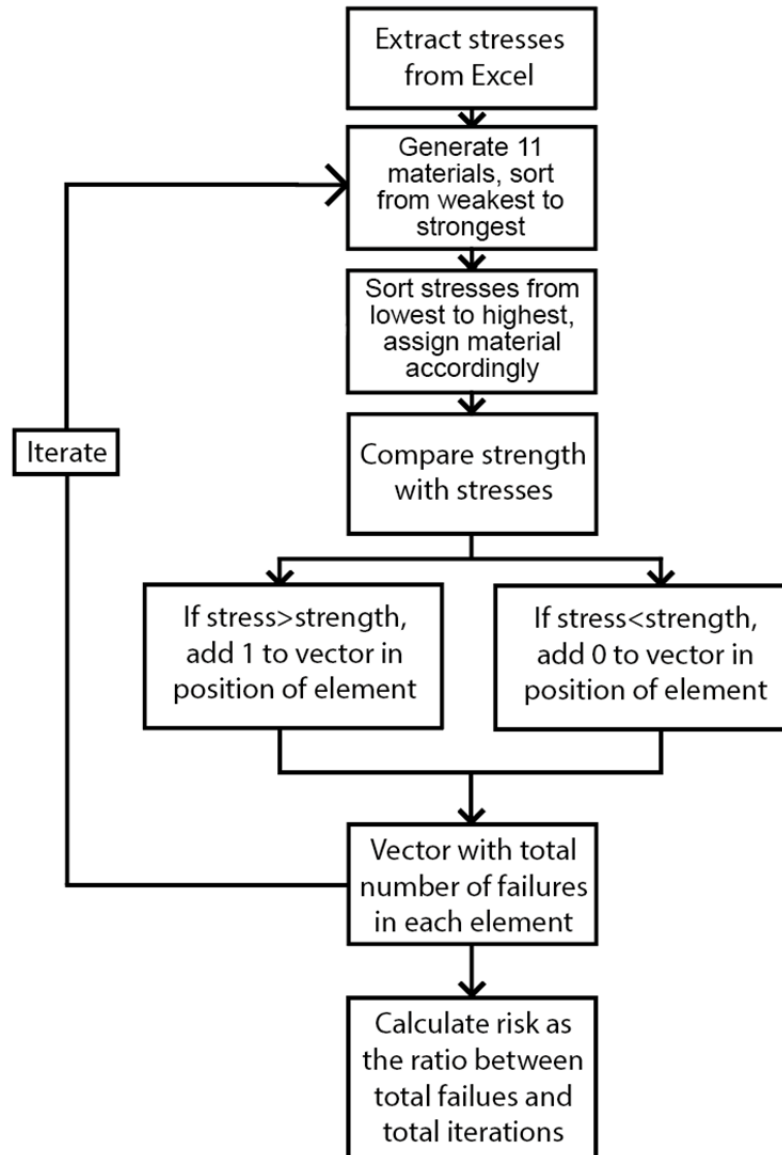


Figure 6.7: Flow diagram of MatLab Monte Carlo simulation with optimized placement of material.

7

Results

Presented in this chapter are the results from the Monte Carlo simulation performed in MatLab, with the respective cross-sectional dimensions for each version of the truss. Deflections of the bottom chord and volume of used material are also presented. A convergence study is also performed to verify how the result from the Monte Carlo simulation is dependent on the number of iterations performed.

Element specific normal stresses are presented in Appendix C.

7.1 Initial iteration

The first test is performed with the same dimensions for all members, which will be refined in further tests.

The inputs that vary with each iteration include cross-sectional areas of the members, and their specific assigned material, which in this case is C14, C24, and C30. Dimensions are presented in Table 7.1.

Table 7.1: Dimensions of members used in the first iteration.

Member(s)	Height [mm]	Width [mm]
0,1,2	220	45
3,4,5,6	220	45
7,8	220	45
9,10	220	45

7.1.1 Monte Carlo results

The results from the Monte Carlo simulation are presented in Table 7.2-7.4 for C14, C24, and C30. The load factors are as defined by the normal distribution in Figure 6.5. The second column "Risk (mat.)" presents the risk of failure in the material, more specifically the risk of failure in the most stressed member, #6 in Figure 6.2. Complete frequency of material failure is presentend in Appendix C. The third column "Risk (comb.)" presents the the combined risk of failure of material failure multiplied with the frequency of the specific load factor.

Red text indicates a risk of failure greater than 1/100,000.

Table 7.2: Risk of failure with C14.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0.01504	3.06E-06
4.0	0.33651	3.42E-05
5.0	0.90968	9.1E-05

Table 7.3: Risk of failure with C24.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0	0
4.0	0.00011	1.12E-08
5.0	0.00812	8.12E-07

Table 7.4: Risk of failure with C30.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0	0
4.0	0	0
5.0	0.000137	1.37E-07

7.2 Second iteration

The second test reduces the dimensions of the top and bottom chords, specifically elements 0-2 and 3-6, and the dimensions of the inner members, as these do not experience high stresses.. Dimensions are presented in Table 7.5.

Table 7.5: Dimensions of members used in the second iteration.

Member(s)	Height [mm]	Width [mm]
0,1,2	145	45
3,4,5,6	145	45
7,8	70	45
9,10	70	45

7.2.1 Monte Carlo results

Results from Monte Carlo simulation can be seen in tables 7.6-7.8. Red text indicates a risk of failure greater than 1/100,000.

Table 7.6: Risk of failure with C14.

Load factor	Risk (mat.)	Risk (comb.)
0.5	0	0
1.0	0	0
1.5	0.00024	2.00E-07
2.0	0.01473	6.00E-06
3.0	0.68262	1.39E-04
4.0	0.9994	1.02E-04
5.0	1.0	1.0E-04

Table 7.7: Risk of failure with C24.

Load factor	Risk (mat.)	Risk (comb.)
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0.00207	4.20E-07
4.0	0.11474	1.17E-05
5.0	0.7003	7.0E-05

Table 7.8: Risk of failure with C30.

Load factor	Risk (mat.)	Risk (comb.)
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0.00046	9.8E-08
4.0	0.00688	6.99E-07
5.0	0.09047	9.05E-06

7.3 Final iteration

The final test further reduces the dimensions of the top and bottom chord down to the size of the inner members. Dimensions are presented in Table 7.9.

Table 7.9: Dimensions of members used in the third iteration.

Member(s)	Height [mm]	Width [mm]
0,1,2	70	45
3,4,5,6	70	45
7,8	70	45
9,10	70	45

7.3.1 Monte Carlo results

Results from Monte Carlo simulation can be seen in tables 7.10-7.12. Red text indicates a risk of failure greater than 1/100,000.

Table 7.10: Risk of failure with C14.

Load factor	Risk (mat.)	Risk (comb.)
0.5	0	0
1.0	0.02407	5.6E-04
1.5	0.78906	6.41E-04
2.0	1.00	4.06E-04
3.0	1.00	2.06E-04
4.0	1.00	1.02E-04
5.0	1.00	1.00E-04

Table 7.11: Risk of failure with C24.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0.00356	2.90E-06
2.0	0.17243	7.01E-05
3.0	0.99534	2.02E-04
4.0	1.00	1.02E-04
5.0	1.00	1.00E-04

Table 7.12: Risk of failure with C30.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0.00073	6.00E-07
2.0	0.01211	4.90E-06
3.0	0.47924	9.74E-04
4.0	0.98806	1.00E-04
5.0	1.00	1.00E-04

7.4 Verification of Pamir model

The same Rhino calculations and Monte Carlo simulation is also performed for the dimensions and material used in MiTek Pamir.

Results from Monte Carlo simulation can be seen in Table 7.14. Red text indicates a risk of failure greater than 1/100,000.

Table 7.13: Dimensions of members used in the Pamir model.

Member(s)	Height [mm]	Width [mm]
0,1,2	145	45
3,4,5,6	170	45
7,8	70	45
9,10	70	45

Table 7.14: Risk of failure with C24 for the Pamir model.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0	0
4.0	0.0115	1.17E-06
5.0	0.22567	2.27E-05

7.5 Optimized placement

A Monte Carlo simulation and analysis with optimized placement of material is also performed, according to the procedure detailed in section 6.6. The analysis is performed with dimensions according to section 7.2.

7.5.1 Monte Carlo results

Results from Monte Carlo simulation can be seen in tables 7.15-7.17. Red text indicates a risk of failure greater than 1/100,000.

Table 7.15: Risk of failure with C14.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0.01666	3.40E-06
4.0	0.99948	1.02E-04
5.0	1.00	1.00E-04

Table 7.16: Risk of failure with C24.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0	0
4.0	0	0
5.0	0.01984	1.98E-06

Table 7.17: Risk of failure with C30.

Load factor	Risk (mat.)	Risk (comb.)]
0.5	0	0
1.0	0	0
1.5	0	0
2.0	0	0
3.0	0	0
4.0	0	0
5.0	0	0

7.6 Convergence

A convergence study is performed to verify the variation of results depending on the number of Monte Carlo iterations. The value chosen for the convergence study is the risk of material failure in iteration 1, with material C24, at 4.0 load factor. This specific combination of dimensions, material, and load was selected for the convergence study because the value was close enough to zero to generate zero-valued results for a low number of iterations, thus giving larger relative fluctuations in result. A visual representation of the convergence is presented in Figure 7.1, with the x-axis scaled logarithmically.

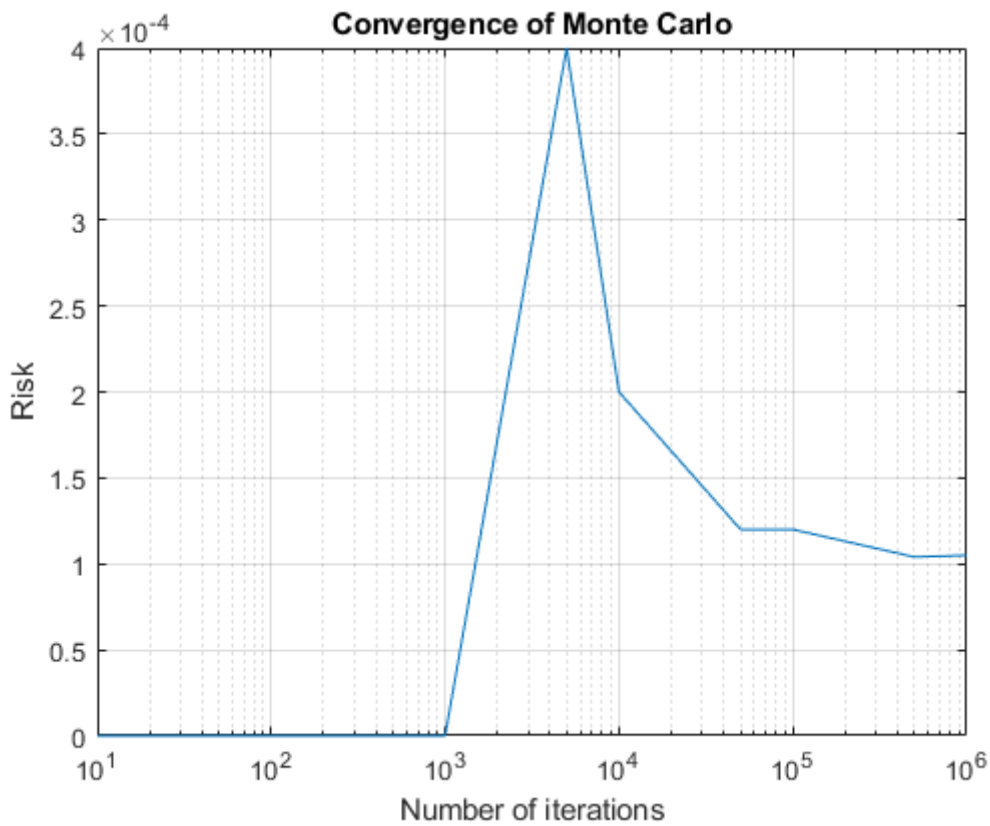


Figure 7.1: Convergence of Monte Carlo simulation, Risk of failure in first iteration with different number of Monte Carlo iterations.

7.7 Other observations

Other than the risk of failure with different materials, the largest displacements of the bottom chord and the total volume of used material was also documented for each of the truss versions. Bottom chord displacements are presented in Table 8.15, volume of used material in Table 8.16.

Table 7.18: Displacements of the bottom chord.

Test #	Displacement	L/x
1	0.009402	L/1064
2	0.015425	L/648
3	0.029371	L/340
Pamir	0.013713	L/729

Table 7.19: Total amount of material used.

Test	Volume of material [m ³]
1	0.317464
2	0.17412
3	0.101011
Pamir	0.18724

8

Discussion

This chapter aims to discuss the results. The benefits and downsides of altering dimensions or the grade will be discussed. Possibility for further studies within this area will, too, be brought up.

8.1 Different iterations

The results gathered from the simulations are considered reasonable. Normal stresses increase as the cross-sectional dimensions of each member decreases, and combined with weaker material, increases the risk of failure. Any other result would indicate that something is incorrectly defined in the structural- or statistical model, but this is not the case. For each of the simulation that has been performed in this thesis, the number of iterations are 10^5 . If the number of iterations performed in this thesis would be higher, the number of failures would too be higher, however, it is estimated that 100 000 iterations are sufficient to produce reliable results with regards to risk of failure.

8.1.1 Iteration 1

In the first iteration the size of the members are 45x220, which is the largest dimensions used by Derome (Björklund, 2022). One can from Tables 7.2, 7.3 and 7.4, presented in Section 7.1, see that as the strength of the timber increases, the risk of failure decreases, which is to be expected. The risk of failure for the smallest dimension is zero for the ultimate limit state, which leads to the conclusion that the truss members can be smaller than to enhance the efficiency of the truss.

8.1.2 Iteration 2

For the second iteration the dimension of the members were smaller, 45x145 for the top and bottom chords, and 45x70 for the web. 45x145 and 45x70 are two of the standard dimensions to use when designing a timber truss. For this dimensions the risk of failure at ultimate limit state is zero for all strength grades. For C14 to reach the threshold value of 1 in 100 000, which is estimated to be the risk of failure for trusses designed with guidelines of today, the load must be increased to two times the ultimate limit state. For C24, the load must be increased to four times the load, and for C30 the load has to be increased five times. These numbers speak to that the truss is still over dimensioned with timber of the size 45*145.

8.1.3 Iteration 3

For the smallest size used in this analysis, 45*70, to reach a risk of failure greater than 1 in 100 000, the ultimate limit state is sufficient. For C24, 1.5 times the load will cause a risk of failure greater than that. For timber of C30, the load needs to be close to three times larger than the ultimate limit state load. The failure that occurs for the truss with the smallest dimensions occur in the top chord close to the joint where it is fastened to the bottom chord. The risk of this failure can be mitigated by using a stronger material in that member. If the top chord would instead be of C24 or C30, the risk of failure would be decreased, this conclusion can be drawn from the fact that for C24, the load needed to be greater than the ultimate limit state load to be greater than zero.

8.1.4 Optimization of placements of timber members

When a batch of eleven boards are randomly selected from a graded board and placed in the truss according to their strength the risk of failure is decreased significantly for trusses with strength class of C24 and C30.

The risk of failure when optimizing a truss solely made up of timber of strength class C24, the risk of failure is zero for four times the design load, whilst being zero for three times the design load prior to optimizing. For C30, the risk of failure is zero for upto five times the design load, which indicates that it is not necessary to solely use C30 timber throughout the entire truss.

The reason to why there is little to no difference for the truss made of C14 timber may be due to that the batch of timber boards randomly selected from the strength distribution may not differ from the timber used in the first iteration. Randomness may play a part. Randomness may similarly play a role as to why the differences are larger for C24 and C30. Another factor that will influence the outcome are the different standard deviations used for the normal distributions assumed for the strength distributions of the strength classes.

It is possible that the standard deviation for the different materials are the same in real-life, giving a similar distribution around the mean for all classes. This would result in similar decrease in risk of failure for all materials.

8.1.5 Altering dimensions or grade?

From the results presented one can see that the dimensions of the members have a large impact on the risk of failure, more so than the grade of the timber. The stresses negligibly changed when altering the strength of the timber, but when the dimensions of the member are altered the stresses are significantly increased. If the cross-sectional area of the member are halved the stress in said member will be doubled. The most efficient way to enhance the structural soundness and efficiency of the truss is to alter both the dimensions and the strength of the timber.

8.2 Pros and cons of using the same grade and/or dimensions

To use the same dimensions and strength of timber throughout the entire truss is not necessary, and not the most efficient way to do it. The largest stresses occur in the top chord, causing it to be in need of greater strength and/or size than the other members. Similarly, the members with the smallest stresses are the web members, so to most efficiently design a truss, the top chord should be stronger than the web members, and the bottom chord should be somewhere in between these two.

Although, the truss will be more efficient if designing each member separately and cater its dimensions and grade to the stress acting in that member, this may not be the best way to do it practically. When using several different timber grades the increase of failure may be enhanced due to errors in assembling. In Derome the trusses are assembled by hand and human factors and errors may influence the integrity of the truss. Even though that the trusses will be more structurally efficient if some of the members are of lower strength class, the production may take longer due to the fact that the people assembling the truss must ensure that the right strength class of timber is at the right place.

In this case, the largest stresses are in the bottom right end of the top chord, element #6 in Figure 5.1, which may lead to it being in need of being of a higher grade than its mirrored counter part. This could lead to mix ups and errors assembling, or that it takes longer time to assemble the trusses.

8.3 Failure of the truss

However, throughout the analysis discussed above, deflection has not been taken into account. The largest deflections occur in the bottom chord, which may lead to the conclusion that it should be stiffer than other parts of the truss. Usually for trusses a maximum deflection value of $L/400$ is set as a limit in the field, usually displacements of joints are disregarded (Träguiden, 2017). For this thesis, solely the displacements of joints have been examined, and these are roughly $L/1000$, which makes it hard to analyse whether the truss will have too large of displacement and deflections.

The steel plates have been assumed to not fail throughout this analysis, but the steel plates are the most common places for the trusses to fail. See figure 5.4, where the utilization ratio of the bottom right is presented, it reaches a value of almost 225%, meaning that it will fail. In the analysis carried out in Rhinoceros and Matlab, the failure for the truss occurs in the top chord, close to the right hand steel plate. The result from MiTek Pamir, strenghtens the notion that it is reasonable for failure to occur here, and that it is normally the steel plate that will fail in the truss.

8.4 Assumptions and simplifications

For trusses the most common failure to occur is failure in the steel plates, however, for this analysis the plates are assumed to not fail and that only the timber members can fail. It is not realistic to assume no failure in the steel plates and this should be taken into consideration when analyzing the results gathered.

By using MiTek Pamir, one can design both the timber members and the steel plates, this software could be used further to minimize the risk of failure both in steel plates and in the timber members. It is possible to change material dimensions and classes of each member separately which can be carried out to get more precise results. However, in MiTek Pamir the timber materials are programmed into it and it is using the characteristic values and these are not possible to alter which would be needed to use a normal distribution of the timber properties.

8.5 Further studies

Further studies can be conducted where the joints and steel plates of the truss are analyzed. The steel plates are the most common failure to occur in a real truss, however, in this analysis the risk of failure in the plates have been disregarded.

In Rhinoceros, different type of angles were tested for the top chord, but not accurately. In this thesis, the top chord is set to have an angle of 30° , after the testing of at which angle the maximum stresses in the truss are the smallest. Exactly 30° may not be correct since the analysis regarding the angles were not precise. In other words, there are potential to enhance the efficiency of the truss further by investigating more accurately at what angle the top chord should be attached to the bottom chord. A similar analysis should be carried out with the web members to ensure that all angles of the members have been designed optimally.

This thesis has solely investigated a Fink truss, but there are several other types of trusses that can be investigated. Other trusses may be more intricate and be more difficult to analyze, but this can be beneficial for further studies to look at. Not all roof tops use Fink trusses, hence there are need of analyses regarding other shapes of trusses. For instance, in Sweden when constructing houses with one and a half floors, the timber roof truss may be a framework instead and these trusses can be analyzed further. For different trusses, a Monte Carlo simulation similar to the one used in this report could also be used, as it only takes the normal stresses of individual members into consideration.

9

Conclusion

This chapter will provide conclusions with regard to the aims and results of the thesis.

The aim of the thesis was to investigate whether timber trusses can be optimized if the data from the testing and grading can be used when designing trusses. This aim has been achieved, as we have shown that it is possible to reduce the risk of failure using optimized and intentional placement of materials.

There are steps to take towards greater efficiency with regards to designing and constructing timber trusses, which can be seen from the results in Section 7.

Timber is not a homogenous material, and if the grading data can be utilized in the designing process the trusses can be constructed more efficiently with a lower risk of failure. Although the structural integrity of the truss can be assumed to be of greater quality when designing according to the testing data, than if solely the tabulated strength classes are used, there may be a higher risk of human errors which may lead to the need of more meticulous controls of the trusses prior to mounting them.

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A

Appendix 1: MathCAD calculations

Wind and snow load of a roof truss

Geometry

Length of roof truss:

$$L_{\text{truss}} := 10\text{m}$$

Distance between chord and web members:

$$L_1 := \frac{L}{3}$$

Angle of top chord:

$$\alpha := 30\text{deg}$$

Height of roof truss:

$$H_{\text{truss}} := \tan(\alpha) \cdot \frac{L}{2} = 2.887\text{m}$$

Height where outer web member attaches to top chord:

$$H_1 := \frac{H}{3}$$

Length of top chord:

$$d := \sqrt{\left(\frac{L}{2}\right)^2 + H^2} = 5.774\text{m}$$

Dimensions of members:

$$b := 45\text{mm} \quad h := 220\text{mm}$$

Assumed centre-to-centre distance between roof trusses:

$$cc := 1200\text{mm}$$

Properties of timber

$$f_{\text{mk}} := 24\text{MPa}$$

Bending strength parallel to grain:

$$\text{Class} := \left(\begin{array}{l} \text{"C14"} \quad \text{if } f_{\text{mk}} \leq 14\text{MPa} \\ \text{"C16"} \quad \text{if } 14\text{MPa} < f_{\text{mk}} \leq 16\text{MPa} \\ \text{"C18"} \quad \text{if } 16\text{MPa} < f_{\text{mk}} \leq 18\text{MPa} \\ \text{"C20"} \quad \text{if } 18\text{MPa} < f_{\text{mk}} \leq 20\text{MPa} \\ \text{"C22"} \quad \text{if } 20\text{MPa} < f_{\text{mk}} \leq 22\text{MPa} \\ \text{"C24"} \quad \text{if } 22\text{MPa} < f_{\text{mk}} \leq 24\text{MPa} \\ \text{"C27"} \quad \text{if } 24\text{MPa} < f_{\text{mk}} \leq 27\text{MPa} \\ \text{"C30"} \quad \text{if } 27\text{MPa} < f_{\text{mk}} \leq 30\text{MPa} \\ \text{"C35"} \quad \text{if } 30\text{MPa} < f_{\text{mk}} \leq 35\text{MPa} \\ \text{"C40"} \quad \text{if } 35\text{MPa} < f_{\text{mk}} \leq 40\text{MPa} \end{array} \right) = \text{"C24"}$$

Density of timber:

$$\rho_k := \begin{cases} 290 \frac{\text{kg}}{\text{m}^3} & \text{if } f_{mk} \leq 14\text{MPa} \\ 310 \frac{\text{kg}}{\text{m}^3} & \text{if } 14\text{MPa} < f_{mk} \leq 16\text{MPa} \\ 320 \frac{\text{kg}}{\text{m}^3} & \text{if } 16\text{MPa} < f_{mk} \leq 18\text{MPa} \\ 330 \frac{\text{kg}}{\text{m}^3} & \text{if } 18\text{MPa} < f_{mk} \leq 20\text{MPa} \\ 340 \frac{\text{kg}}{\text{m}^3} & \text{if } 20\text{MPa} < f_{mk} \leq 22\text{MPa} \\ 350 \frac{\text{kg}}{\text{m}^3} & \text{if } 22\text{MPa} < f_{mk} \leq 24\text{MPa} \\ 360 \frac{\text{kg}}{\text{m}^3} & \text{if } 24\text{MPa} < f_{mk} \leq 27\text{MPa} \\ 380 \frac{\text{kg}}{\text{m}^3} & \text{if } 27\text{MPa} < f_{mk} \leq 30\text{MPa} \\ 390 \frac{\text{kg}}{\text{m}^3} & \text{if } 30\text{MPa} < f_{mk} \leq 35\text{MPa} \\ 400 \frac{\text{kg}}{\text{m}^3} & \text{if } 35\text{MPa} < f_{mk} \leq 40\text{MPa} \end{cases} = 350 \cdot \frac{\text{kg}}{\text{m}^3}$$

Self-weight of roof

Volume of roof truss:

$$\underline{V} := L \cdot b \cdot h + \frac{L}{2} \cdot \cos(\alpha) \cdot b \cdot h \cdot 2 + 2 \cdot \sqrt{\left[\left(\frac{L}{2} - L_1\right)^2 + H^2\right]} \cdot b \cdot h \dots = 0.289 \cdot \text{m}^3$$

$$+ 2 \cdot \sqrt{\left[\left(L_1 - \frac{H_1}{\tan(\alpha)}\right)^2 + H_1^2\right]} \cdot b \cdot h$$

Self-weight of roof truss:

$$g_{k,\text{truss}} := \frac{(V \cdot \rho_k \cdot g)}{L} = 0.099 \cdot \frac{\text{kN}}{\text{m}}$$

Self-weight of roof truss assumed negligible.

Assumed self-weight of the roof, from www.traguiden.se:

$$g_{k,\text{rooftop}} := 0.8 \frac{\text{kN}}{\text{m}^2}$$

Self-weight of roof on each truss:

$$\underline{g_{k,\text{rooftop}}} := g_{k,\text{rooftop}} \cdot cc = 0.96 \cdot \frac{\text{kN}}{\text{m}}$$

Self-weight of roof structure:

$$g_k := g_{k,\text{truss}} + g_{k,\text{rooftop}} = 1.059 \cdot \frac{\text{kN}}{\text{m}}$$

Snow load

Gothenburg:

$$s_k := 1.5 \frac{\text{kN}}{\text{m}^2}$$

Angle of pitch of roof:

$$\alpha = 30 \cdot \text{deg}$$

Factor regarding angle of roof:

$$\mu_2 := \begin{cases} 0.8 & \text{if } 0 \leq \alpha \leq 30 \text{deg} \\ 0.8 \cdot \frac{60 - \alpha}{30} & \text{if } 30 < \alpha < 60 \\ 0 & \text{if } \alpha \geq 60 \end{cases} = 0.8$$

$$\mu_5 := 0.87$$

Snow load on roof top:

$$S_2 := s_k \cdot \mu_2 = 1.2 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$S_5 := s_k \cdot \mu_5 = 1.305 \cdot \frac{\text{kN}}{\text{m}^2}$$

Snow load on each roof truss:

$$q_{k,\text{snow},2} := S_2 \cdot cc = 1.44 \cdot \frac{\text{kN}}{\text{m}}$$

$$q_{k,\text{snow},5} := S_5 \cdot cc = 1.566 \cdot \frac{\text{kN}}{\text{m}}$$

Wind load

Reference wind speed in Gothenburg:

$$v_b := 25 \frac{\text{m}}{\text{s}}$$

Density of air:

$$\rho_{\text{air}} := 1.25 \frac{\text{kg}}{\text{m}^3}$$

Reference mean velocity wind pressure:

$$q_b := \frac{1}{2} \cdot \rho_{\text{air}} \cdot v_b^2 = 390.625 \text{ Pa}$$

Assume terrain III and building height of 10m.

$$k_r := 0.22 \quad z_0 := 0.3 \quad z := 10$$

Terrain and building height factor:

$$c_e(z, z_0) := \left(k_r \cdot \ln\left(\frac{z}{z_0}\right) \right)^2 \cdot \left(1 + \frac{7}{\ln\left(\frac{z}{z_0}\right)} \right)$$

$$c_e(10, 0.3) = 1.783$$

Peak velocity pressure:

$$q_p := c_e(10, 0.3) \cdot q_b = 696.541 \text{ Pa}$$

External pressure coefficient:

$$c_{pe,10} := \begin{cases} 0 & \text{if } \alpha < 5\text{deg} \\ 0 + 0.2 \cdot \frac{\alpha - 5}{15 - 5} & \text{if } 5\text{deg} \leq \alpha < 15\text{deg} \\ 0.2 + 0.5 \cdot \frac{\alpha - 15}{30 - 15} & \text{if } 15\text{deg} \leq \alpha < 30\text{deg} \\ 0.7 & \text{if } 30\text{deg} \leq \alpha < 60\text{deg} \\ 0.7 + 0.1 \cdot \frac{\alpha - 60}{75 - 60} & \text{if } 60\text{deg} \leq \alpha < 75\text{deg} \\ 0.8 & \text{if } \alpha \geq 75\text{deg} \end{cases} = 0.7$$

External pressure acting on roof due to wind:

$$w_e := q_p \cdot c_{pe,10} \cdot c_e = 0.585 \cdot \frac{\text{kN}}{\text{m}}$$

Ultimate limit state loads:

Snow as the main load:

$$q_{d,\text{snow},2} := q_{k,\text{snow},2} \cdot 1.5 + w_e + g_k \cdot 1.35 = 4.175 \cdot \frac{\text{kN}}{\text{m}}$$

$$q_{d,\text{snow},5} := q_{k,\text{snow},5} \cdot 1.5 + w_e + g_k \cdot 1.35 = 4.364 \cdot \frac{\text{kN}}{\text{m}}$$

Wind as the main load:

$$q_{d,\text{wind},2} := q_{k,\text{snow},2} + w_e \cdot 1.5 + g_k \cdot 1.35 = 3.747 \cdot \frac{\text{kN}}{\text{m}}$$

$$q_{d,\text{wind},5} := q_{k,\text{snow},5} + w_e \cdot 1.5 + g_k \cdot 1.35 = 3.873 \cdot \frac{\text{kN}}{\text{m}}$$

Snow and wind load as main load respectively, largest load become the designing case:

$$q_d := \begin{cases} q_{d,\text{snow},5} & \text{if } q_{d,\text{snow},5} \geq q_{d,\text{wind},5} \\ q_{d,\text{wind},5} & \text{if } q_{d,\text{snow},5} < q_{d,\text{wind},5} \end{cases} = 4.364 \cdot \frac{\text{kN}}{\text{m}}$$

B

Appendix 2: MatLab scripts

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```
%-----%
% Master's thesis: Intelligent truss design           %
% Data extraction & Monte Carlo simulation           %
% Sebastian Jakobsson & Jonathan Pekkala Settland 2022 %
% Chalmers University of Technology                 %
%-----%
```

Data extraction from Excel file

```
clc
clear

k = 17; % Different
        sheets in the Excel-file used to store data. Each sheet holds different
        information. % Sheet

        6,7,8, and 17 stores stresses from Rhino.
data = readmatrix('iterations.xlsx','UseExcel',k,'Sheet',k); % Extract
        stresses from data sheet

C = zeros(11,1); % Empty
    vector

applied_load = data(1,:);

for j=1:width(data) % Organize
    data from data sheet
    A = data(3:24,j);
    for i=1:length(C)
        A2 = A(2*i-1);
        B2 = A(2*i);
        if abs(A2) > abs(B2)
            C(i)=A2;
        else
            C(i)=B2;
        end
    end
    Stress(:,j) = abs(C); % Maximum
    stresses in each element
end
```

```

Stress_1 = Stress; % Defines a
matrix of stressed with all load elements. Below are specific elements.
% Stress_1 = Stress(1:3,:); % 0 1 2, lower
% Stress_1 = Stress(4:7,:); % 3 4 5 6, upper
% Stress_1 = Stress(8:9,:); % 7 8, inner
% Stress_1 = Stress(10:11,:); % 10 11, inner

```

Material strengths

```

b = 11; % 10 = C14, 11 = C24, 12 = C30
        % Different values of 'b' gives a different sample of material
        % strengths, depending on their respective normal distributions

y = readmatrix('iterations.xlsx','UseExcel',b,'Sheet',b); % Pre-determined
normal distributed material, see MatLab-script 'Generate_Material'.
y = y(3:length(y),1);

```

Monte Carlo simulation

```

for k = 1:width(Stress_1)
    Stress_new = Stress_1(:,k);
    Total = zeros(length(Stress_new),1);
    n=100000; % Using 100,000 iterations
    for j=1:n
        Tot = zeros(length(Stress_new),1);
        for i=1:length(Stress_new)
            p = y(randi(length(y),1)); % Extract a random value,
strength, from the material
            p_save(i) = p;
            Element(i) = i;
            if Stress_new(i)>p % If the stress is greater than
the material strength, save a '1'
                Tot(i) = Tot(i)+1; % otherwise, continue
            else
                continue
            end
        end
        Total = Total + Tot; % Total number of failures
    end
    Risk_member = Total/sum(Total); % Risk of failure in individual
member
    [M,I] = max(Total); % Maximum number of failures,
with index of element
    Risk = M/n;
    if Risk == 0 % If the risk of failure is
equal to 0, display
        Z = applied_load(k);
        disp(['No risk of failure with load factor ' num2str(Z)])
    else % Else, display
        Z = applied_load(k);
        disp([num2str(Risk) ' risk of failure with load factor ' num2str(Z)])
    end
    freq(k) = Risk;
end

```

end

No risk of failure with load factor 0.5
No risk of failure with load factor 1
No risk of failure with load factor 1.5056
No risk of failure with load factor 2
No risk of failure with load factor 3
0.01169 risk of failure with load factor 4
0.22791 risk of failure with load factor 5

Total risk, material failure combined with load frequency

```
load = readmatrix('read_this.xlsx','UseExcel',1,'Sheet',1); % Load normal
distribution of the load from Excel

J = [0.5 1 1.5056 2 3 4 5]; % Load factors to
be used
B = zeros(1,length(J));

for i=1:length(J) % Find index of
load-factors specified in matrix 'J'
    k = find(abs(load-J(i)) < 0.0001);
    if i < length(J)
        B(i) = mean(k/length(load)); % If more than
one value apply, take mean
    else
        B(i) = 0.9999; % For last,
assign risk of 0.0001
    end
end
risk_load = 1-B; % Risk of load
occurring

Total_risk = risk_load.*freq % Total risk of
failure, combination of material failure and laod frequency

Total_risk =

1.0e-04 *

0 0 0 0 0 0.0119 0.2279
```

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Optimized Monte Carlo Simulation	2
Total risk, material failure combined with load frequency	3

```
%-----%  
% Master's thesis: Intelligent truss design           %  
% Data extraction & Optimized Monte Carlo simulation %  
% Sebastian Jakobsson & Jonathan Pekkala Settlund 2022 %  
% Chalmers University of Technology                  %  
%-----%
```

Data extraction from Excel file

```
clc  
clear  
  
k = 8; % Different  
       sheets in the Excel-file used to store data. Each sheet holds different  
       information. % Sheet  
  
       6,7,8, and 17 stores stresses from Rhino.  
data = readmatrix('iterations.xlsx','UseExcel',k,'Sheet',k); % Extract  
       stresses from data sheet  
  
C = zeros(11,1); % Empty  
    vector  
  
applied_load = data(1,:);  
  
for j=1:width(data) % Organize  
    data from data sheet  
    A = data(3:24,j);  
    for i=1:length(C)  
        A2 = A(2*i-1);  
        B2 = A(2*i);  
        if abs(A2) > abs(B2)  
            C(i)=A2;  
        else  
            C(i)=B2;  
        end  
    end  
    Stress(:,j) = abs(C); % Maximum  
    stresses in each element  
end  
  
Stress_1 = Stress;
```

Optimized Monte Carlo Simulation

```
clc

b = 12; % 10 = C14, 11 = C24, 12 = C30
        % Different values of 'b' gives a different sample of material
        % strengths, depending on their respective normal distributions

y = readmatrix('iterations.xlsx','UseExcel',b,'Sheet',b); % Pre-determined
normal distributed material, see MatLab-script 'Generate_Material'.
y = y(3:length(y),1);
stress_sort = sort(Stress_1); % Sort stresses from lowest
to highest.
Total = zeros(1,length(stress_sort));

n = 100000;

for k=1:width(Stress_1)
    Total = zeros(length(Stress_1),1);
    stress_comp = stress_sort(:,k);
    Tot = zeros(length(stress_sort),1);

    for j=1:n
        y_gen = y(randi(length(y),1,11)); % Generate 11 random values
        from distribution.

        for i=1:length(stress_sort)
            strength = y_gen;
            strength_sort = sort(y_gen); % Sort materials from
            weakest to strongest.
            if strength_sort(i) < stress_comp(i) % Compare material strength
            to stresses.
                Tot(i) = Tot(i)+1;
            else
                continue
            end
        end
    end
    [M,I] = max(Tot);
    Risk = M/n;
    if Risk == 0 % If the risk of failure is
    equal to 0, display
        Z = applied_load(k);
        disp(['No risk of failure with load factor ' num2str(Z)])
    else % Else, display
        Z = applied_load(k);
        disp([num2str(Risk) ' risk of failure with load factor ' num2str(Z)])
    end
    freq(k) = Risk;
end
```

No risk of failure with load factor 0.5
 No risk of failure with load factor 1
 No risk of failure with load factor 1.5056
 No risk of failure with load factor 2
 0.00042 risk of failure with load factor 3
 0.95808 risk of failure with load factor 4
 1 risk of failure with load factor 5

Total risk, material failure combined with load frequency

```

load = readmatrix('read_this.xlsx','UseExcel',1,'Sheet',1); % Load normal
distribution of the load from Excel

J = [0.5 1 1.5056 2 3 4 5]; % Load factors to
be used
B = zeros(1,length(J));

for i=1:length(J) % Find index of
load-factors specified in matrix 'J'
    k = find(abs(load-J(i)) < 0.0001);
    if i < length(J)
        B(i) = mean(k/length(load)); % If more than
one value apply, take mean
    else
        B(i) = 0.9999; % For last,
assign risk of 0.0001
    end
end
risk_load = 1-B; % Risk of load
occurring

Total_risk = risk_load.*freq % Total risk of
failure, combination of material failure and laod frequency

Total_risk =

    1.0e-04 *

         0         0         0         0    0.0009    0.9732    1.0000
  
```

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```
%-----%
% Master's thesis: Intelligent truss design          %
% Generate normal distributed material with 5th-percentile %
% Sebastian Jakobsson & Jonathan Pekkala Settlund 2022 %
% Chalmers University of Technology                  %
%-----%
```

Generate normal-distributed C14/C24/C30-material

```
clc
clear

limit = 0.05;           % Desirable percentile below 24 MPa
pres = 0.00001;         % Precision for condition
n = 10000;
b = [18 30 38];         % Mean of strength (18, 30, 38 for C14,
    C24, C30 respectively)
c = [14 24 30];
a = linspace(0,10,n);

for k = 1:length(c)
    for j=1:length(a)
        y = a(j).*randn(n,1) + b(k); % Normal-distributed value. Function
        'randn' gives normal-distributed value between 0 and 1.
        y1=sort(y);                % Sort vector in ascending order.

        for i=1:length(y1)
            if y1(i)<=c(k)           % 14, 24, 30, depending on which
material is desirable
                T(i)=1;
            else
                T(i)=0;
            end
        end
        if abs(sum(T)/n-limit) < pres % if 5th-percentile is true with
precision, break
            break
        end
    end
    disp([num2str(sum(T)/n*100) '% under ' num2str(c(k)) 'MPa'])
    figure(1)
    plot(1:n,y1);
    hold on
end

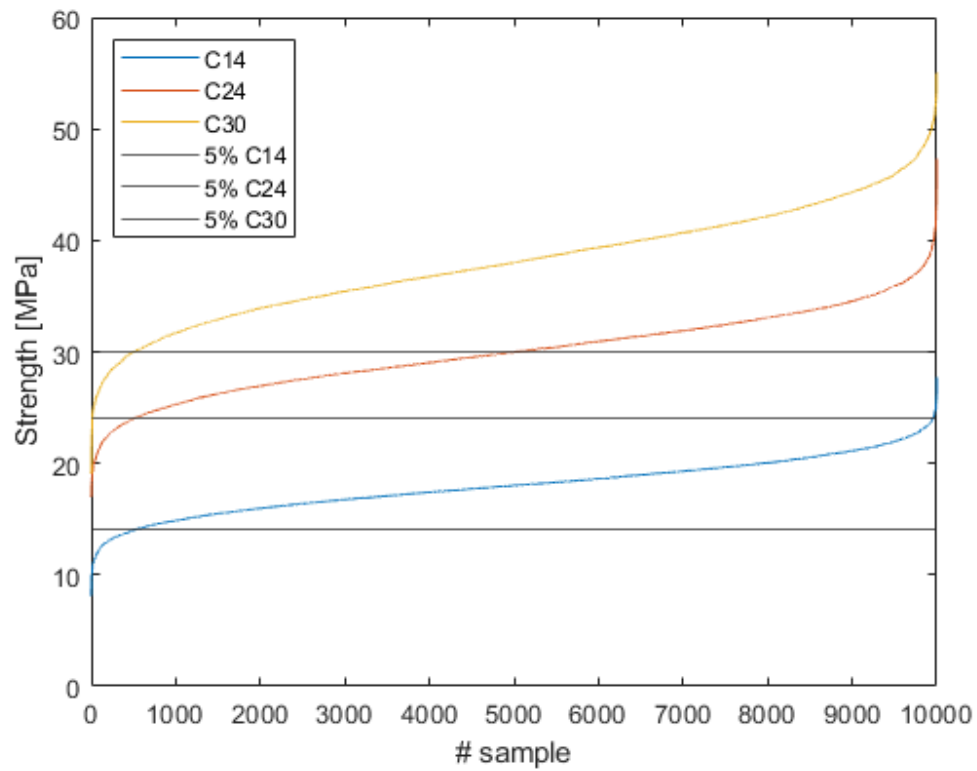
yline(14)
yline(24)
yline(30)
ylabel('Strength [MPa]')
xlabel('# sample')
```

```
legend('C14','C24','C30','5% C14','5% C24','5% C30','Location','NorthWest')  
xlim([0 10000])
```

5% under 14MPa

5% under 24MPa

5% under 30MPa

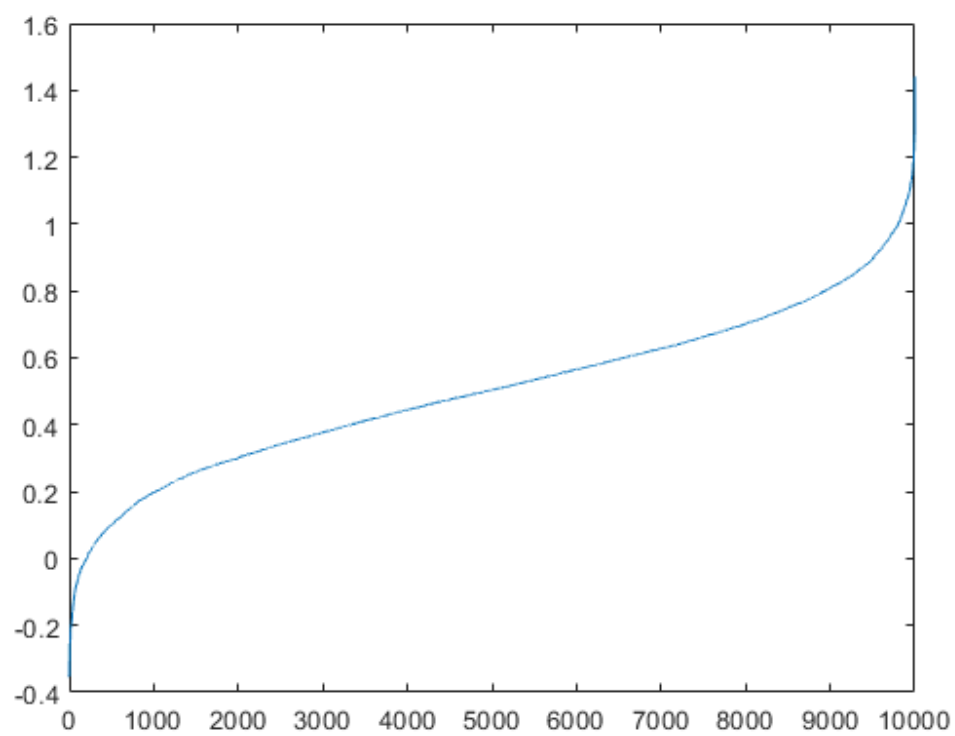


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```
%-----%  
% Master's thesis: Intelligent truss design %  
% Generate normal distributed load factor, ULS at 98th percentile %  
% Sebastian Jakobsson & Jonathan Pekkala Settland 2022 %  
% Chalmers University of Technology %  
%-----%
```

Generate normal-distributed load factor

```
clc  
clear  
  
limit = 0.98; % Desirable percentile below 1.0 load factor  
    (ULS)  
var = 0.0001; % Precision for condition  
n = 10000;  
b = 0.5; % Mean of load factor  
a = linspace(0,10,n);  
  
for j=1:length(a)  
    y = a(j).*randn(n,1) + b; % Normal-distributed value. Function 'randn'  
    gives normal-distributed value between 0 and 1.  
    y1=sort(y); % Sort vector in ascending order.  
    for i=1:length(y1)  
        if y1(i)<=1 % 1 to get 98th percentile  
            T(i)=1;  
        else  
            T(i)=0;  
        end  
    end  
    if abs(sum(T)/n-limit) < var % if 98th-percentile is true with precision,  
        break  
    end  
end  
  
figure(1)  
plot(1:n,y1);
```



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C

Appendix 3: Element stresses Rhino & Material failure in each member

Iteration 1	0.5	1.0	1.5056	2.0	3.0	4.0	5.0
Element #							
0	1.627031	3.254063	4.900619	6.508126	9.762189	13.016252	16.270315
	1.627031	3.254063	4.900619	6.508126	9.762189	13.016252	16.270315
1	1.100048	2.200096	3.313345	4.400192	6.600288	8.800384	11.000481
	1.100048	2.200096	3.313345	4.400192	6.600288	8.800384	11.000481
2	1.686521	3.373042	5.079801	6.746084	10.119126	13.492168	16.865209
	1.686521	3.373042	5.079801	6.746084	10.119126	13.492168	16.865209
3	-1.324646	-2.649292	-3.989833	-5.298584	-7.947875	-10.597167	-13.246459
	-1.634779	-3.269558	-4.923954	-6.539115	-9.808673	-13.07823	-16.347788
4	-2.040092	-4.080184	-6.144757	-8.160368	-12.240552	-16.320736	-20.400919
	-1.729959	-3.459918	-5.210637	-6.919836	-10.379754	-13.839673	-17.299591
5	-1.336589	-2.673178	-4.025807	-5.346357	-8.019535	-10.692713	-13.365891
	-1.677735	-3.355471	-5.053339	-6.710941	-10.066412	-13.421883	-16.777353
6	-1.782024	-3.564048	-5.367456	-7.128095	-10.692143	-14.256191	-17.820239
	-2.12317	-4.24634	-6.394988	-8.49268	-12.73902	-16.98536	-21.2317
7	0.457189	0.914377	1.377052	1.828754	2.743132	3.657509	4.571886
	0.457189	0.914377	1.377052	1.828754	2.743132	3.657509	4.571886
8	0.50782	1.01564	1.529553	2.031279	3.046919	4.062558	5.078198
	0.50782	1.01564	1.529553	2.031279	3.046919	4.062558	5.078198
9	0.50782	1.01564	1.529553	2.031279	3.046919	4.062558	5.078198
	-0.537596	-1.075191	-1.619238	-2.150382	-3.225574	-4.300765	-5.375956
10	-0.595858	-1.191716	-1.794725	-2.383432	-3.575149	-4.766865	-5.958581
	-0.595858	-1.191716	-1.794725	-2.383432	-3.575149	-4.766865	-5.958581

Iteration 2	0.5	1.0	1.5056	2.0	3.0	4.0	5.0
Element #							
0	2.449094	4.898188	7.376671	9.796376	14.694565	19.592753	24.490941
	2.449094	4.898188	7.376671	9.796376	14.694565	19.592753	24.490941
1	1.647568	3.295136	4.962475	6.590273	9.885409	13.180545	16.475681
	1.647568	3.295136	4.962475	6.590273	9.885409	13.180545	16.475681
2	2.539845	5.079689	7.650012	10.159379	15.239068	20.318757	25.398446
	2.539845	5.079689	7.650012	10.159379	15.239068	20.318757	25.398446
3	-2.005191	-4.010382	-6.039635	-8.020763	-12.031145	-16.041526	-20.051908
	-2.469403	-4.938806	-7.437842	-9.877612	-14.816418	-19.755224	-24.69403
4	-3.065904	-6.131807	-9.234502	-12.263614	-18.395421	-24.527229	-30.659036
	-2.601691	-5.203383	-7.836294	-10.406766	-15.610148	-20.813531	-26.016914
5	-2.023788	-4.047575	-6.095649	-8.095151	-12.142726	-16.190302	-20.237877
	-2.534421	-5.068842	-7.633676	-10.137685	-15.206527	-20.275369	-25.344211
6	-2.680871	-5.361743	-8.074784	-10.723485	-16.085228	-21.44697	-26.808713
	-3.191505	-6.383009	-9.612812	-12.766019	-19.149028	-25.532037	-31.915047
7	1.464402	2.928804	4.410779	5.857608	8.786411	11.715215	14.644019
	1.464402	2.928804	4.410779	5.857608	8.786411	11.715215	14.644019
8	1.631178	3.262357	4.913109	6.524713	9.78707	13.049426	16.311783
	1.631178	3.262357	4.913109	6.524713	9.78707	13.049426	16.311783
9	1.631178	3.262357	4.913109	6.524713	9.78707	13.049426	16.311783
	-1.697025	-3.394049	-5.111438	-6.788098	-10.182148	-13.576197	-16.970246
10	-1.887534	-3.775068	-5.685253	-7.550137	-11.325205	-15.100274	-18.875342
	-1.887534	-3.775068	-5.685253	-7.550137	-11.325205	-15.100274	-18.875342

Iteration 3 Element #	0.5	1.0	1.5056	2.0	3.0	4.0	5.0
0	5.093868	10.187737	15.342731	20.375473	30.56321	40.750946	50.938683
	5.093868	10.187737	15.342731	20.375473	30.56321	40.750946	50.938683
1	3.380123	6.760247	10.180931	13.520493	20.28074	27.040986	33.801233
	3.380123	6.760247	10.180931	13.520493	20.28074	27.040986	33.801233
2	5.280987	10.561974	15.906333	21.123948	31.685922	42.247896	52.80987
	5.280987	10.561974	15.906333	21.123948	31.685922	42.247896	52.80987
3	-4.129636	-8.259271	-12.438463	-16.518542	-24.777814	-33.037085	-41.296356
	-5.084526	-10.169052	-15.314592	-20.338103	-30.507155	-40.676207	-50.845259
4	-6.35872	-12.717441	-19.152466	-25.434881	-38.152322	-50.869763	-63.587203
	-5.40383	-10.80766	-16.276336	-21.61532	-32.422981	-43.230641	-54.038301
5	-4.169876	-8.339753	-12.559668	-16.679506	-25.019259	-33.359012	-41.698765
	-5.220256	-10.440511	-15.72341	-20.881023	-31.321534	-41.762046	-52.202557
6	-5.567768	-11.135536	-16.770117	-22.271071	-33.406607	-44.542143	-55.677678
	-6.618147	-13.236294	-19.933859	-26.472588	-39.708883	-52.945177	-66.181471
7	1.469159	2.938317	4.425105	5.876634	8.814951	11.753268	14.691585
	1.469159	2.938317	4.425105	5.876634	8.814951	11.753268	14.691585
8	1.631253	3.262505	4.913333	6.52501	9.787515	13.05002	16.312526
	1.631253	3.262505	4.913333	6.52501	9.787515	13.05002	16.312526
9	1.631253	3.262505	4.913333	6.52501	9.787515	13.05002	16.312526
	-1.725328	-3.450657	-5.196689	-6.901313	-10.35197	-13.802626	-17.253283
10	-1.907953	-3.815906	-5.746755	-7.631813	-11.447719	-15.263626	-19.079532
	-1.907953	-3.815906	-5.746755	-7.631813	-11.447719	-15.263626	-19.079532

Pamir Element #	0.5	1.0	1.5056	2.0	3.0	4.0	5.0
0	2.450387	4.900774	7.380565	9.801548	14.702321	19.603095	24.503869
	2.450387	4.900774	7.380565	9.801548	14.702321	19.603095	24.503869
1	1.649983	3.299965	4.969748	6.59993	9.899896	13.199861	16.499826
	1.649983	3.299965	4.969748	6.59993	9.899896	13.199861	16.499826
2	2.541393	5.082785	7.654675	10.165571	15.248356	20.331141	25.413927
	2.541393	5.082785	7.654675	10.165571	15.248356	20.331141	25.413927
3	-1.709421	-3.418842	-5.148776	-6.837684	-10.256525	-13.675367	-17.094209
	-2.105861	-4.211722	-6.342854	-8.423444	-12.635167	-16.846889	-21.058611
4	-2.614107	-5.228214	-7.873691	-10.456428	-15.684643	-20.912857	-26.141071
	-2.217667	-4.435334	-6.679613	-8.870668	-13.306001	-17.741335	-22.176669
5	-1.72492	-3.449839	-5.195458	-6.899679	-10.349518	-13.799358	-17.249197
	-2.161004	-4.322008	-6.508944	-8.644016	-12.966024	-17.288032	-21.61004
6	-2.285087	-4.570174	-6.882682	-9.140348	-13.710522	-18.280697	-22.850871
	-2.721171	-5.442343	-8.196168	-10.884685	-16.327028	-21.76937	-27.211713
7	1.456676	2.913352	4.387509	5.826705	8.740057	11.65341	14.566762
	1.456676	2.913352	4.387509	5.826705	8.740057	11.65341	14.566762
8	1.622555	3.24511	4.887135	6.490219	9.735329	12.980438	16.225548
	1.622555	3.24511	4.887135	6.490219	9.735329	12.980438	16.225548
9	1.622555	3.24511	4.887135	6.490219	9.735329	12.980438	16.225548
	-1.702356	-3.404711	-5.127495	-6.809422	-10.214133	-13.618844	-17.023556

10 -1.894963 -3.789926 -5.707629 -7.579853 -11.369779 -15.159706 -18.949632
-1.894963 -3.789926 -5.707629 -7.579853 -11.369779 -15.159706 -18.949632

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