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Design, analysis and construction of hydrostatic masonry bridges

An assessment of load capacity using the Discrete Element Method and physical scale models

Master's thesis in Structural Engineering and Building Technology

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
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Cover: Physical scale model of a hydrostatic masonry shell bridge.

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Abstract

Masonry is a building technique extensively used throughout history in different applications. However, structures in concrete have become common practice during the last century. Even though masonry bridges cover a large portion of existing bridges, and are a sustainable choice because of their adaptability and long service life, sufficient methods to assess and design masonry bridges are not clear.

In this master thesis, the collapse load of masonry bridges designed as hydrostatic shells is investigated. The methods used are hand calculations, load tests on physical scale models and computational analysis using the Discrete Element Method (DEM). To validate the computational method an analysis is performed on a simple masonry arch as it is a known case. To validate the computational method, an analysis is performed on a simple masonry arch, a well-documented case, thereby obtaining useful coefficients for the unproven concept of the hydrostatic bridge.

The form finding process is done in *Rhinoceros3D* and *Grasshopper*, and the geometry created is used to model both the Discrete Element Model and the physical model.

The hand calculations provided highly conservative results, as the resulting collapse load from the DEM analysis is 66 times larger and from the physical load test six times larger. The computational and physical models behave similarly when loaded, and both indicate that the critical loading position is in the midspan of the bridge. The physical model was loaded with 13.9 kg before collapse. By applying dimensional scaling, this result translates to 46.9 tonnes for a full-scale bridge. This indicates that the bridge concept is viable at full-scale. The result from the DEM-analysis is qualitatively good but over-estimates the collapse load.

Further work includes adding features to the computational model to provide more reliable results and investigating different geometries for the hydrostatic bridge.

Keywords: masonry bridges, form finding, discrete element method, masonry shells, physical bridge models, hydrostatic shell.

Design, analys och konstruktion av hydrostatiska murverksbroar
En bedömning av lastkapacitet med hjälp av den diskreta elementmetoden och fysiska skalmodeller

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Sammanfattning

Murning är en byggteknik som har använts i olika tillämpningar genom historien. Under det senaste århundradet har dock andra material, såsom betong blivit vanligare. Trots att murade broar utgör en stor del av det befintliga beståndet av broar och är ett hållbart val på grund av deras anpassningsförmåga och långa livslängd, är metoderna för att beräkna och designa murade broar inte särskilt väldefinierade.

I denna masteruppsats undersöks kollapslasten på murade broar designade som hydrostatiska skal. De metoder som används är handberäkningar, belastningstester på fysiska skalmodeller och datorbaserad analys med hjälp av Diskreta Element-Metoden (DEM). För att validera den datorbaserade metoden utförs en analys på ett enkelt stenvalv, ett väl dokumenterat fall, vilket ger koe□cienter som används för det oprövade konceptet med den hydrostatiska bron.

Formfindningsprocessen utförs i *Rhinoceros3D* och *Grasshopper*, och den skapade geometrin används för att modellera både den diskreta elementmodellen och den fysiska modellen.

Handberäkningarna gav mycket konservativa resultat, där kollapslasten från DEM är 66 gånger större och från det fysiska belastningstestet sex gånger större. De datorbaserade och fysiska modellerna beter sig likartat vid belastning och båda indikerar att den kritiska belastningspositionen är vid brospannets mitt. Den fysiska modellen belastades med 13,9 kg före kollaps. Genom att tillämpa dimensionsskalning översätts detta resultat till 46,9 ton för en fullskalig bro. Detta indikerar att brokonceptet är genomförbart i full skala. Resultatet från DEM-analysen är kvalitativt bra men överskattar kollapslasten.

Ytterligare arbete inkluderar att lägga till funktioner i den datorbaserade modellen för att ge mer pålitliga resultat och att undersöka olika geometriska former för den hydrostatiska bron.

Nyckelord: murverksbroar, form finding, diskreta elementmetoden, murverksskal, broskalmodeller, hydrostatiskt skal.

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A special thank you to Prof. Karl-Gunnar Olsson and others involved in the Architecture and Engineering program for sparking an interest in architecture, construction and computational design. Which has continued to grow and develop during our years at Chalmers University of Technology.

And finally, a big thank you to all the friends made along the way.

Jacob Forsberg and Emil Svedjer, Gothenburg, June 2024

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

CAE	Complete Abaqus Environment
CD-algorithm	Central Di□erence algorithm
DEM	Discrete Element Method
DOF	Degree of Freedom
FEM	Finite Element Method
GH	Grasshopper (plug-in to Rhino3D)
LC	Load Combination
ODB	Output Database

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1
Introduction

Throughout history, masonry structures has been an important part of the built environment. Masonry structures such as the Notre Dame in Paris, Ponte di Rialto and St Paul's Cathedral are monuments of their respective city, both valued for their historical signi cance and architectural qualities. Today, the use of masonry has declined since other building techniques have been developed, and the building industry has evolved.

This thesis explores the analysis and construction of masonry structures, speci cally arches and vaults. It examines what we can learn from historical structures and how this knowledge can be combined with modern techniques. The main focus is applying these insights to the design of form-found hydrostatic shell masonry bridges.

1.1 Background

Masonry is a building technique that has been in use for extensive parts of building history [43]. Ranging from simple stone walls to bridges to gothic cathedrals, masonry displays a large diversity in both architectural style and structural concepts. Despite its limitation in mainly being able to carry compression, masonry has throughout history been used in numerous innovative and creative applications.

It is estimated that around 40% of the existing road bridges in Great Britain are masonry bridges, and that a large number of them have been in use for over 100 years [36]. One well-known example is the Maidenhead Railway bridge, Figure 1.1, which was constructed in 1839 and is still in use today. Even though it has been standing for almost 200 years, and carrying larger loads compared to what it was initially designed for. This is in contrast to more recent steel and concrete bridges, of which many have required repairing and some even being replaced after only a couple of decades [33]. Production costs of masonry bridges have historically been high and have therefore been replaced with bridges of other materials. However, due to many of these bridges needing early refurbishments, maintenance costs have already passed the initial production costs after around 20 years of service. In the modern building industry, digital techniques for production are available for use which can decrease the cost of initial investment of a masonry bridge [24]. It could therefore be argued that masonry bridges not only have a longer service life than most concrete or steel structures, but also with good maintenance and refurbishments can adapt to di erent and increased utilization. Together with low maintenance costs, the total

expense for the full service life can also be lower than most bridges built today.

Figure 1.1: Maidenhead Railway Bridge designed and constructed by Isambard Kingdom Brunel. Photo by Stephen Daglish licensed under CC BY-SA2.0

In terms of sustainability, masonry bridges o er a few advantages compared to other materials, such as reinforced concrete. A typical masonry bridge is built as shown in Figure 1.2. It is observed that a large part of the bridge contains ll material that is placed atop the stone arch. This material can be sourced from excavations needed for construction or nearby construction sites. Additionally, the long service life and adaptability contribute to sustainability as refurbishments can be performed instead of demolishing and building new bridges.

Figure 1.2: Typical stone arch bridge and terminology. Redrawn from Commonwealth of Pennsylvania [10].

The proven durability, along with new digital techniques for analysis and fabrication, could be the start of a renaissance of masonry and stone work [16]. However, the most common computational methods in use today do not capture the mechanical behavior of masonry structures. This is due to masonry having very di erent characteristics compared to structural assemblies in timber and reinforced concrete, for which the theory and methods are mainly developed. To continue building with masonry and restore existing structures, a di erent type of methodology for analysis of masonry bridges must be developed. This thesis suggests a combination of physical models and computational analysis, to both develop new types of masonry structures as well as addressing their load capacities.

1.1.1 Analysis of Masonry Structures

The procedure for analysing masonry structures has varied throughout history. During medieval and ancient times, masonry was considered a question of geometric proportions. If the geometry was deemed correct, the structure satis ed the criterion for sti ness, strength and most importantly, stability. Questions of load capacity had not yet been posed and masons relied on passed-down knowledge for sizing of elements. It was not until the 17th century when Galileo sought to establish a function for the load capacity of transversely loaded beams, and in the 1800s when Navier formulated stress criterion for the working state of a structure that these questions were formulated. Hence, the problem of structures went from being solely geometrical to analysing stresses and forces in the building elements [26]. This way of thinking has continued to the modern day, where the most common calculation methods are based on elasticity with stress and strain.

When analysing masonry on an architectural scale, questions of internal stresses are often redundant in the architectural scale. This is due to a very high compressive strength of the material, often combined with large dimensions of the masonry blocks, resulting in low compressive stresses. The resistance against tensile forces is on the contrary very low, and therefore the design must translate applied forces to compression in the elements. Thus, the structural problem is almost purely geometrical [25].

Working with masonry therefore requires a di erent way of thinking, a combination of modern methods together with application of the same geometrical knowledge as the medieval masons. With this combination, new concepts can be created. In the article The construction of new masonry bridges inspired by Paul Séjourné (2021), Emil Adiels and Chris J.K. Williams [10] present and explore the idea of constructing a masonry bridge in the form of a hydrostatic shell. By taking inspiration from hydrostatic arch bridges made by Paul Séjourné [40], a form nding method was derived for the geometry of the shell structure. The work by Adiels and Williams is used as a basis for the design of the masonry bridge covered in this thesis.

1.2 Aim

The aim encompasses two parts. The first is to assess and test methods of analysing masonry structures, specifically shells and arches, and how they can be applied today using computational methods as well as traditional scale model tests.
The second part is to form a better understanding of the load capacity at full scale, for the proposed hydrostatic shell bridges. More specifically to examine the effect of concentrated loads on the bridge deck.

Thesis Questions:

- What is the effect of concentrated live loads on hydrostatic shell masonry bridges, and how does it relate to beam and arch theory?
- How do different analysis methods compare to each other when calculating the maximum load capacity? And what is the collapse load for a hydrostatic masonry bridge based on the different methods?
- Is a hydrostatic shell bridge a viable concept that can be realised in full scale?

1.3 Objective

The research in this thesis will be a necessary step in the process of finding plausible calculation procedures for masonry shell bridges. Determination of the location for critical loading is of importance to achieve this. By using digital, analytical and physical methods, the system can be evaluated in multiple ways to assess the load-bearing capabilities.

It is also of interest to evaluate if the concept of building a hydrostatic bridge is realistic at full scale. To answer this question, the bridge must be proved viable as a concept through testing and analysis. Possible construction methods should also be considered.

1.4 Method

The work flow is presented in Figure 1.3 . Starting with a literature review, which includes structural theory of masonry and studies previous works concerning physical models, analytical- and computational methods. The literature review is followed by three separate but parallel parts: I. Hand calculations, II. Computational analysis and III. Physical model testing, all of which are discussed further later in this chapter. The fifth and final part of the project is to compare the results from the three different methods to draw conclusions of the load capacity for the proposed bridges.

Figure 1.3: Flowchart description of work method.

Parts I. and II. starts with an exploration phase in the form of an analysis of an arch. This step involves evaluating the proposed method for calculating the load capacity of the bridge, by applying the method on a simple arch structure. The masonry arch is a known concept that has been studied and constructed for a long time, meaning there are a lot of data and theories available that can be used to evaluate the validity of this study. The exploration phase is important in developing the method in order to make it applicable and reliable for the hydrostatic bridge.

1.4.1 Hand calculations

Hand calculations are necessary for developing an understanding of the hydrostatic shell bridge. The calculation methods in use are mainly simpli cations that can be used to approximate the load capacity in an early stage. The hand calculations will provide implications if the results are reasonable. As a rst test, the bridge is simpli ed to a 2D arch bridge, in order for the hand calculation methods to be applicable.

1.4.2 Computational analysis

To evaluate the capacity of the bridge in detail, the computer software Abaqus is used. Analysis is performed using the Discrete Element Method (DEM). The geometry will be created using Rhinoceros3D (Rhino) and Grasshopper (GH). Rhino is a 3D-modeling software for creating geometry, and GH is a plug-in that uses visual programming to generate geometry [38].

Firstly, an analysis of a common arch will be performed in the software Abaqus. As the arch is a proven concept, with known and well studied structural behaviour, this analysis provides information on the robustness of the chosen method. Secondly, the bridge geometry will be analysed in Abaqus as well based on the evaluation from the first analysis. The results are being compared to the test data from the hand calculations and physical tests.

The computational analysis is also important to determine the critical loading position. This could technically be done by physical load testing as well. However, since constructing a scale model is time-consuming, computational analysis will be efficient for this purpose.

1.4.3 Physical test

Since the structural behaviour of masonry shells are complex, physical models are necessary to evaluate and understand the response of the system during loading. The tests are performed on scale models, which are created using a script in GH and Rhino that generates the form-found bridge as well as production drawings. This geometry is used to 3D-print the parts to construct the scale model.

An increasing concentrated load will be applied during the test at the expected critical point. As the load increases, deformation and crack initiation are observed. Eventually, when the bridge collapses, the applied load is noted.

All load tests are performed on previously constructed scale models. An evaluation is done after each test to improve the method for the next load test.

1.4.4 Comparison of results

All results from the used methods, described in 1.4.1 - 1.4.3, are compared with each other. This in order to examine the coherence of the solution results, and make an approximation for the load capacity of a full scale bridge. As the load tests are performed on scale models, the results from this specific method can be scaled to full scale.

1.5 Limitations

The form nding technique for deriving the shape of the bridge assumes the ll material to act as a hydrostatic force on the blocks as described in section 2.1.3. Assuming a hydrostatic pressure for the ll is a simpli cation, since frictional forces between the ll material and the blocks are not considered.

Furthermore, the purpose of this thesis does not involve optimizing the shape in the form nding stage. Options for initial cross-sections of the bridge are endless. However, for the purpose of this thesis, two di erent cross-sections will be chosen and used throughout all the individual assessments.

All tests are performed at model scale. This means that the dimensions of the computational model are the same as for the physical model. This is done to facilitate comparison of the results.

The hydrostatic bridge is modelled as a dry-stacked masonry structure. Therefore, the interface between the blocks will be modelled as a frictional interaction between the voussoirs without any mortar in the connection.

2

Theory

This chapter covers the theory and state of the art of masonry construction, followed by theory about the three analysis methods used. Section 2.1 presents the theory of different types of masonry structures. Section 2.2 covers the history of using physical models. Section 2.3 introduces analytical methods for solving the collapse load of masonry arches. Section 2.4 presents theory about modelling strategies for masonry structures as well as information about the Discrete Element Method. Finally, section 2.5 presents how DEM can be implemented in Abaqus.

2.1 Masonry Structures

Masonry is a material most commonly composed of an assembly of blocks and mortar, which binds the blocks together [30]. However, masonry can also be dry-stacked, meaning no usage of mortar. There are three important general assumptions made about masonry structures, which are presented by Heyman as [26]:

1. Masonry has no tensile strength
2. Stresses are so low that masonry effectively has an unlimited compressive stress
3. Sliding failure does not occur

The material of the blocks may vary. It could be either tile work or cut stone. However, the general properties stated above are assumed to be true regardless of the material.

2.1.1 Arches

An arch is a construction that ideally is only intended to carry the load in compression. While this was well-known by master builders in medieval times, it was not until Robert Hooke stated the hanging chain principle in 1675 that modern analysis of arches was developed [26]. Hooke stated that a hanging chain influenced by its own self-weight will form the ideal shape, a catenary, to carry compression if it is inverted. This inverted line is called the thrust line and is seen in Figure 2.1. The theory was put into practice by Sir Christopher Wren for the dome of Saint Paul's cathedral. Comparing the span-to-thickness ratio of this dome and other domes at the time, it can be seen that Wren could successfully construct thinner domes out of bricks by designing according to Hooke's hanging chain rule [5].

Figure 2.1: The hanging chain principle explaining the concept of the thrust line. Redrawn from Poleni [37]

Even though this is the ideal shape to carry compression forces in an arch, where the self weight is the dominant load. There are numerous other con gurations that are possible as well. The rule is that to ensure stability, the thrust line must be contained within the boundaries of the arch.

However, if the arch deforms slightly, a unique position of the thrust line can be calculated due to hinge formations. Small movements of the abutments can be considered to change the span length as illustrated in Fig 2.2. Since the arch is made of rigid blocks, tension cracks will appear in the mortar connection which forces the thrust line to follow the uncracked part. If the arch is dry-stacked, the blocks will lose connection.

In the positions where the thrust line is close to the edge of the arch, hinges are formed. This is due to the stress distribution changing in the interaction between the blocks, with tension close to the edges. The system is transferred to a statically determinate system called the*three pin arch* simply by small movements of the abutments. However, if more than three hinges form the structure becomes a mechanism and become unstable (see Section 2.3.1).

Figure 2.2: a) Statically indeterminate arch. b) Statically determinate arch under self weight due to movement of the abutments [26]

2.1.2 Shells

There are many types of di erent structural elements, each requires its own de nition. For example, a beam could be described as a long straight line compared to the dimensions of its cross-section that works primarily in bending. An arch, in this case, would be described in a similar way but with a curved line instead and carrying forces in compression. Describing a shell similarly would be a curved surface with a thickness perpendicular to said surface. The surface, in turn, could be described as a net of di erent curved lines, or per the previous de nition of arches, a system of hanging chains [11].

Using this de nition, the hanging chain theory can be applied to shells where to goal is attain a shape that works primarily through membrane action. Consequently, the geometry of a shell generates an in nite number of solutions to a speci c problem since the thrust line can take an unlimited amount of paths in the structure as explained in 2.1.1.

The construction of speci cally masonry shells has been a common practice for a long time, where a frequent example is masonry vaults in churches. These are called membrane shells which are funicular structures meant to only carry loads in either tension or compression. Since masonry is only able to carry compression according to Heyman's de nition, it as a suitable material choice for such a structure.

Masonry is an assembly of high-density material, which entails the self-weight being dominant. Meaning live loads will have a smaller relative impact on the structure [11]. Therefore, it is a suitable construction material for bridges. Masonry arch bridges could be considered to be shell structures, with retaining walls holding back

the ll material. For traditional arch bridges, the spandrel walls are straight vertical masonry walls (see Figure 1.2. This is not optimal as the ll material will add lateral pressure on the retainers, and a straight wall is not the optimal shape to withstand lateral pressure [10].

2.1.3 Hydrostatic Shells

To explain the concept of hydrostatic shells, one can imagine a cloth lying on water. The cloth is being pulled down, beneath the surface, using ropes that are attached to the cloth, the ropes can be seen as geodesic curves. Geodesics are curves on the surface that has zero geodesic curvature, and as consequence follows the (locally) shortest path between two surface points [42]. As the cloth is pulled further down, a pressure will form, acting perpendicular to the cloth. The pressure will vary in magnitude depending on the depth. The greatest pressure is where the depth is at the maximum. As the magnitude of the pressure is proportional to the depth or height of the ll, so must the curvature be as well.

The same problem is described by Euler in Figure 2.3, except for the depth being measured horizontally. Therefore, the gure can be rotated 90 degrees and solved numerically to obtain the needed shape of the hydrostatic arch [10].

Figure 2.3: Euler's elastica [15], which has the same shape as an hydrostatic arch.

Using this logic, the form nding of the bridge geometry is made, where equilibrium equations for each point decide where the adjacent point should be placed. The vectors used and calculated for the form nding method are shown in Figure 2.4

Figure 2.4: Vectors used for form nding of hydrostatic bridge. Redrawn from [7].

To start the form nding process, a cross-section must be assumed. From that, the same cross-section is o set along the bridge's length axis to initiate the form nding process. The displayed vectors in Figure 2.4 can be derived from the assumptions made, as vectors a and c are set between already given points. To nd the next point in the system (2.1) and (2.2) are used [10]. As the equations satisfy equilibrium in all points, the shell will carry compression in all blocks. This entails the full structure being in compression, thus eliminating the issue with the spandrel walls as in a typical masonry bridge.

$$q = \frac{gz}{4T} \quad c \tag{2.1}$$

$$b = \frac{(1 \quad q \quad q)a + 2(a \quad q)q \quad 2a \quad q}{1 + q \quad q} \tag{2.2}$$

Where q is the load vector and T is the force density.

However, as the form nding process follows the principles of Euler's elastica, the solution will be sensitive. With increasing depth, the curvature of the arch will increase as they are proportional. This will inevitably result in such large curvature that it will form a loop, as can be seen in Figure 2.3. Therefore, if increasing the span of the arch, either the cross-section must be changed or the force density increased, alternatively form a multi-span bridge which Williams and Adiels present in [10].

The form nding technique is based on geodesic coordinates, which consists of coordinate curves following geodesics in one direction and its orthogonal trajectories in the other direction with a constant spacing. Two characteristics of the geodesic coordinates are especially useful in this case: The geodesic curves and the orthogonal trajectories intersect at a right angle, which is useful for the building blocks. And, as the geodesics follow the surface, they will also follow the force path of equilibrium as the form is created from equilibrium equations [10].

2.2 Physical Models

For large parts of history, scale models have been used to describe both the aesthetics of a building as well as understanding and test the structural system. Scale models can vary from simple soap lm models describing minimal surfaces to block models of arches to Gaudi's or Heinz Isler's famous hanging chain models of complex buildings. When making a physical model of masonry, block models are the most common type to use.

In the book Physical Models: Their historical and current use in civil and building engineering design [28], Huerta describes the history of block models. Huerta begins by stating that scale models of stone structures date back to ancient Egypt. The usage of scale models of masonry has been present all through history, where for

example block models were built when constructing St Paul's cathedral in the 1690s, as this was one of the first projects where Hooke's hanging chain theory was applied.

Block models have been used to prove theories formulated for masonry structures. One famous example was performed during the early 18th century by Augustin Danyzy. This was the first systematic test performed to explain the collapse modes of masonry arches. Danyzy's intention was to test Couplet's theories: Masonry has no tensile strength, infinite compressive strength and failure occurs due to the forming of hinges instead of sliding between the bricks [27]. He performed tests on scale models of arches, which would confirm Couplet's theories. In turn, also demonstrate that scale models could be used to explain the theory behind masonry structures. Further tests were performed by various engineers during the 19th century. Most of them were performed to establish the theory of the thrust line being contained within the arch, where the most prominent were done by William Henry Barlow in 1846. Barlow's models would prove that an arch could contain multiple lines of thrust, depending on the load case. The models were constructed with convex sides, shown in Figure 2.5, and would therefore change the geometry of the arch depending on the load case and subsequently, the shape of the thrust line [28].

Figure 2.5: Drawing of Barlow's arch scale model [28]

The construction of scale models of masonry is an effective way to predict the behaviour of the full-scale structure. This is due to masonry being a problem of stability and geometry, rather than material strength as with for example concrete structures. Historical experiments have proven formulated theories, such as the theory of the thrust line in an arch and what type of failure mode is prominent. Since the hydrostatic bridge is an unproven concept, smaller scale models can be used to test the collapse load and better understand the failure mode. Failure loads for block models are also scalable, meaning the load test can be translated to a larger bridge through dimensional analysis.

2.2.1 Dimensional analysis

Working with physical models for a structural problem is relevant if the results can be scaled to a larger dimension. This is not always simple as stresses do not vary linearly with increasing size. However, as Heyman describes, it is assumed that stresses are relatively low in masonry structures and the risk of failure being induced by stresses is unlikely [26]. Because of this, the problem is purely geometrical and results from a scale model can be directly translated into the full scale.

To scale the results given from a smaller model, dimensional analysis must be applied. This involves expressing all variables as fundamental dimensions. For this case, the dimensions are force (F) and length (L). These dimensions can be multiplied and divided in order to derive other dimensions. The used variables for a masonry bridge are displayed in table 2.1 together with its fundamental dimensions [29]. However, scaling of the ll material could be troublesome as cohesion is dependent on scale. If dry sand is used, it could be argued that it is cohesionless. Meaning that the ll is only frictional and therefore contributes to greater friction if scaled, resulting in a conservative solution. Tables 2.1 and 2.2 are taken from [9].

Variables	Dimensions
Density times gravity, g	$F=L^3$
Span, S	L
Collapse load, P	F
Compressive strength,	$F=L^2$
Young's modulus, E	$F=L^2$

Table 2.1: Variables for a masonry bridge and its dimensions

Using the variables presented in Table 2.1, non-dimensional groups can be constructed and used for scaling the results, the groups in Table 2.2 are extracted.

Non-dimensional groups	Variables
Independent Group 1	$P= gS^3$
Independent Group 2	$P= S^2$
Independent Group 3	=E

Table 2.2: Non-dimensional groups for a masonry bridge

$$\left(\frac{P}{gS^3}\right)_{\text{full scale}} = \left(\frac{P}{gS^3}\right)_{\text{model}} \tag{2.3}$$

The independent groups for a scale model can be compared with a full-scale bridge. From (2.3) the resulting collapse load can be obtained by altering the span length. If the span increases by a factor of 10, the collapse load is increased by 1000.

2.3 Analytical Methods

Analysing a masonry structure is different from many types of ordinary engineering analyses. This is due to the problem mainly being a question of stability rather than stresses, with unlimited positions of the thrust line. However, there are methods for calculating the capacity of masonry structures, which are presented below. For hand calculations, these are primarily suited for arches. The bridge covered in this report is not an arch but could be simplified to a 2D arch, and using these hand calculation methods could indicate an approximation of load capacity.

2.3.1 Limit state analysis

The limit state theory, as explained by Heyman [26], is an application of plastic hinge theory on masonry structures. Consider the arch in Figure 2.6(a) loaded by a concentrated force P. This load will alter the internal thrust line to the one seen in Figure 2.6(b). As the load P increases and reaches its critical value P_{cr}, the thrust can barely be contained in the domain of the arch. In the position where the thrust line is close to the boundary seen in Figure 2.6(c), the arch forms hinges perpendicular to the stress field. This four hinged arch in Figure 2.6(d) is unlike the three hinged arch not a stable structure. This corresponds to the failure mechanism of an arch loaded by a concentrated force.

Figure 2.6: Collapse mechanism of an arch in uenced by a concentrated force [26].

Now, the question is, where is the most critical position for a point load to act on? Heyman illustrates this by considering a geometrical factor of safety [25]. The critical position can be obtained by moving a constant concentrated live load along the length of the arch and evaluating how much the thickness can be reduced and still contain the thrust line. The di erence between the real thickness of the arch and the minimum thickness when four hinges have formed gives the geometrical safety factor.

Figure 2.7 shows the general result for the geometric safety factor for di erent positions of the concentrated live load. The exact shape of the curve depends on the relation of magnitude between the self-weight and the live load, but generally, two minimum points are found in the two quarter points of the bridge. This indicates that the critical loading position is in the quarter span of the arch.

Figure 2.7: Geometrical safety factor for an arch. [25]

2.3.2 Heyman's plastic method

Jaques Heyman describes in The Masonry Arch [25] a possible method of finding the critical point load of an arch with land fill. This method should only be used as a quick approximation as the method of collapse is assumed to be a specific shape of the thrust line when placing a point load at one-fourth of the total length of the arch. The method also assumes a flat surface on top of the fill, with no strength from the fill itself, the load P is not distributed in the fill but instead acts directly on the arch. The fill and masonry are assumed to have the same unit weight

Figure 2.8: Physical quantities needed for prediction of load capacity of concentrated load P. [25]

Figure 2.8 details the needed dimensions to perform the calculation of P. However, the drawing is not representative of the shape of the arch, as this may vary in shape and thickness. Therefore, the normalized values, giving a measurement for the shape, , giving a measure of the depth and , giving a measure of the vertical thickness, are used in order to apply the static equilibrium equation 2.4.

$$P = 16\frac{W_2 x_2 f \quad + (1 \quad 1=4k) \quad g \quad (W_1 x_1 + 1=4W_2) f (1 \quad a) \quad (1 + 1=4k) \quad g}{(3 \quad 2\ ) \quad (2 + k)} \tag{2.4}$$

It should be noted that certain compositions will cause the denominator to be equal to zero, and therefore result in an in nite high load P.

Eq. (2.4) can be reduced to only contain the parameters, and , by assuming $k = 1$. This will entail the vertical thickness at the abutment being equal to the thickness at $1=4l$, implicating a lower bound solution as k has a small impact on the value of P. Thus reducing to (2.5). Where p is a value that is dependant on , and , and can be retrieved from Appendix A.1.1, where interpolation is needed depending on the values of , and .

$$p = \frac{P}{\frac{1}{6} l h_c}) \quad P = \frac{p\, l h_c}{6} \tag{2.5}$$

2.4 Computational analysis

Di erent computational methods can be applied to assess a masonry structure in greater detail. A common method to use when analysing structures in general is the Finite Element Method (FEM). However, there are alternatives and alterations to this method that could be useful as well. The following sections will present the theory behind these methods and the advantages and disadvantages of applying them to this speci c problem.

2.4.1 Modelling strategies

Modelling masonry computationally can be done in a few di erent ways. As the structure can be described as a composite material and a combination of building parts acting together, di erent strategies have been assessed and developed depending on the aim of the analysis. These strategies are described by Godio et al. together with their bene ts and drawbacks in [21] as follows:

1. Detailed micro-modelling : Mortar and bricks are modelled separately with material properties, and interactions between the materials are included in the analysis.
   - ˆ Favourable when approximating cracks and smaller deformations
   - ˆ Computationally heavy due to its high detail level

2. Simpli ed micro-modelling : Each brick is modelled as a continuous element, interactions between the blocks are considered, without consideration of the behaviour of the mortar.
   - ˆ Favourable when stability and failure loads are of interest

- Will not describe the crack pattern between the bricks, although it will display where the crack initiation appears

3. Macro-modelling : The structure is modelled as one continuous element, ignoring interactions and is instead treated as a composite material.
   - Favourable when analysing large-scale models, shortest computation time
   - Detail level not adequate for more precise studies

All modelling strategies are useful for different applications. However, it should be carefully considered what detail level to use depending on the problem and intended outcome of the analysis, as this affects the computational time. This will entail the creation of a highly efficient model.

2.4.2 Discrete Element Method for masonry structures

The Discrete Element Method (DEM) refers to a computational analysis method where elements are modelled as individual blocks [39]. DEM is not commonly used in structural analysis, where FEM is standard, but application of DEM is widespread in other fields. Generally it is used to model systems of discrete elements undergoing large displacements. An integral part of the analysis method concerns the interaction between elements, which is why it is especially useful for masonry structures.

The theory was developed by Peter A. Cundall in 1971 with the intent to analyse rock slopes as discontinuous systems [13]. He made use of an explicit solution algorithm instead of an implicit matrix iteration method which is used in FEM. This method is more general since it can guarantee numerical convergence even for nonlinear systems or when deformations are large. The algorithm builds on the equations of motion and how they change over time [2]. Performing DE-analysis does not necessarily mean that an explicit algorithm has to be used. Tests have been performed to use an implicit algorithm, but explicit algorithms are still considered the standard method[31].

Figure 2.9: Flow chart describing the procedure for an implicit and explicit routine

2.4.2.1 Kinematic algorithm

The kinematic state of the system is solved with an explicit central difference integration rule. The algorithm advances with the acceleration from the current time increment and velocity from the previous time mid-increment ($\dot{u}_{(i-\frac{1}{2})}$ and $\ddot{u}_{(i)}$), where i is the increment number. Although other integration schemes exist, the Central Difference algorithm (CD-algorithm) is most commonly used due to its simplicity and accuracy [17].

The equilibrium conditions for a dynamic system are given by Newton's second law:

$$M\ddot{u} = f \tag{2.6}$$

Where M is the systems mass matrix, $\ddot{u}$ is the acceleration vector and f is the applied force vector. The acceleration for a given increment number i can be computed by considering the force vector as the difference between applied forces F and internal forces I:

$$\ddot{u}^N_{(i)} = (M^{N:})^{-1}(F_{(i)} - I_{(i)}) \tag{2.7}$$

Where the superscript N refers to a single degree of freedom. The velocities are calculated at the midincrement between $t_{(i)}$ and $t_{(i+1)}$, further discussion of the importance in step-size is presented in 2.4.2.2:

$$\dot{u}^N_{(i+\frac{1}{2})} = \dot{u}^N_{(i-\frac{1}{2})} + \frac{t_{(i+1)} - t_{(i)}}{2} \ddot{u}^N_{(i)} \tag{2.8}$$

Since the midincrement velocity depends on the velocity state in the previous mid-increment, it must be obtained for the initial state:

$$\dot{u}^N_{(+\frac{1}{2})} = \dot{u}^N_{(0)} + \frac{t_{(1)}}{2} \ddot{u}^N_{(0)}$$

$$\dot{u}^N_{(-\frac{1}{2})} = \dot{u}^N_{(0)} - \frac{t_{(0)}}{2} \ddot{u}^N_{(0)}$$

Now the displacements for the next time step can be computed. This is the final step of the algorithm before it proceeds to the next time increment.

$$u^N_{(i+1)} = u^N_{(i)} + t_{(i+1)} \dot{u}^N_{(i+\frac{1}{2})} \tag{2.9}$$

Efficiency of algorithm:

The strength of the CD-integration scheme is that the displacements can be obtained without iteration of a matrix system. Although it usually requires smaller time steps to ensure numerical stability than an implicit routine, it is of relatively small computational cost since it does not require the system's stiffness matrix. Faster computation is also ensured by the use of a lumped mass matrix, which only contains non-zero values along the diagonal and can be inverted easily.

2.4.2.2 Time step for convergence

The solution procedure is dynamic as the kinematic relations change over time. Unlike an implicit routine, the convergence of the result relies heavily on the size of the time step. Convergence is ensured by using a time increment that is smaller than the critical time step t_{crit}. This value depends on the systems maximum eigenfrequency ω_{max}

$$t_{crit} \quad \frac{2}{\omega_{max}} \tag{2.10}$$

Generally, the size of the time increments is very small for dynamic explicit analyses. However, there is no equation system to solve, which means that analysis time is fast and not too computationally heavy.

Quasi-static analysis:

Discrete Element analysis (DE-analysis) is performed with a dynamic explicit algorithm. However, this thesis aims to analyse a masonry bridge's response against static concentrated forces. A widely endorsed solution to perform static analysis

in a dynamic procedure is to perform a quasi-static analysis [31]. To avoid non-desired dynamic responses from inertial forces, viscous damping is introduced. This effectively prevents vibratory movements from dominating the response and preventing the system from reaching a steady-state solution [23]. One damping model that can be used is Rayleigh damping which consists of a mass-proportional and stiffness-proportional part. The damping of mode k is obtained by the damping ratio $_k$:

$$_k = \frac{1}{2!_k} \; _M + \frac{!_k}{2} \; _K \tag{2.11}$$

where $_M$ and $_K$ are constants defining the magnitude of the damping proportion. The calibration of these parameters affects both the result and the size of the stable time increment which can be seen in (2.12). The mass-proportional part can cause an overestimation of the result in terms of capacity [22], but gives a much smaller reduction for the stable time increment. Stiffness proportional damping reduces the time step significantly but has less effect on the reliability of the result [1].

$$t \quad \frac{2}{!_{max}} \left(\sqrt{1 + \; ^2_{max}} \quad _{max} \right) \tag{2.12}$$

The recommendation is to keep the damping value low to avoid unwanted responses in the collapse mechanism or overestimating the collapse load. This will, however, cause oscillations in the results since it means the higher modes will be underdamped [32]. A drawback of Rayleigh damping is that it is defined in the material data, which requires the model to be made of deformable bodies.

A more general way to apply damping is proportional to the rate of relative motion. This damping is applied in the interface between interacting surfaces both in the normal and tangential direction opposite the velocity. The damping force f_{vd} is given by (2.13)

$$f_{vd} = \; _0 A v^{el}_{rel} \tag{2.13}$$

v^{el}_{rel} is the rate of relative motion between two interaction surfaces with nodal area A. With this approach, a large amount of damping is introduced to achieve convergence in the result.

2.4.2.3 Contact definition

One important aspect of discrete element modelling is the interaction between the individual elements. The process of detecting and defining contact is essential for the global stability of the system. The representation of contact was proposed by Cundall in the paper Formulation of a Three-dimensional Distinct Element Model [14].

Initially, Cundall proposed a general contact algorithm, meaning that all possible connections between the two elements vertices v_i, edges e_i and faces f_i were tested. The total number of possible contact combinations N would then be:

$$N = (v_a + e_a + f_a)(v_b + e_b + f_b)$$

This requires a lot of unnecessary checks and is computationally heavy to run. For example, contact between two cubes requires a total of 676 checks. A new concept of a common plane proved to be more e cient. This can be explained by letting the two elements move towards each other with a solid plate held up between them. When the elements make contact, the plate will be stuck at a certain angle. This corresponds to the sliding plane between the blocks.

Initially, a quick overview is performed where neighbouring blocks are detected according to an algorithm developed by A. Munjiza [35]. This is done by creating a boundary box around every element, as displayed in Figure 2.10. This corresponds to the smallest possible box that can contain the entire shape. The boxes are mapped in a grid system called the cell space. If two elements' boundary boxes are within the same cell, they are considered neighbouring and tested for interaction by the common plane method [18].

Figure 2.10: Munjizas detection algorithm: Areas marked in darker grey are occupied by two or more bodies

When neighbouring blocks have been detected, the position of the common plane is established in the middle point between the two blocks' centre points as displayed in Figure 2.11. With the centre points of the elements called A and B, and the index i taking the value 1 to 3 to denote the components of a vector, the centre point of the common plane C is:

$$C_i = \frac{A_i + B_i}{2} \tag{2.14}$$

The unit normal vector de nes the rotation of the plane and is given by:

$$n_i = Z_i / z \tag{2.15}$$

Where Z_i is the distance between the two element centroids, $Z_i = B_i - A_i$ and $z = Z_i Z_i$

Figure 2.11: Visualization of the common plane between two bodies

The objective of the contact algorithm is to apply transformations to the common plane to maximize the gap between the elements if they are not in contact, or minimize the overlap if they are. This is done by translation and rotation of the plane based on the closest vertex. With this contact model, the number of contact checks N is reduced to:

$$N = v_a + v_b$$

2.5 DEM modelling in Abaqus

The software Abaqus provides two different analysis methods, Abaqus/Standard and Abaqus/Explicit. The difference is that Abaqus/Standard uses the implicit routine that guarantees the stability of the analysis solution, while Abaqus/Explicit uses the dynamic explicit procedure as described in 2.4.2.

For this report, the explicit routine has been used. While the implicit routine would guarantee unconditionally stable results, there are reasons why explicit analysis is more suitable in this case:

- An analysis of a masonry structure puts heavy emphasis on contact interaction in the system rather than the stress state of the individual blocks. This is

because the system is assumed to collapse due to a mechanism forming in the joints between blocks. Abaqus/Explicit provides a stronger contact formulation than Abaqus/Standard [41]

- As Abaqus/Standard employs a matrix iteration method, it depends on the amount of degrees of freedom (DOFs) in the system. Abaqus/Explicit solves the kinematic state independently for each node. Thus, the computational cost of an increase in model complexity is linear for an explicit method and quadratic for an implicit method. This is explained by Godio et al. in [20], where a DEM modelling strategy for masonry wall strips was built in Abaqus and validated on laboratory tests [19].
- The non-linear effects in the system can cause Abaqus/Standard to have problems converging.

2.5.1 Element interaction

The modelling of the interaction is a crucial part of the analysis that plays an integral part in the result, as the system is assumed to fail due to hinges forming in the joints.

Interaction definition:
Connection between surfaces in Abaqus is handled as a pair of a main- and a secondary-surface. The difference between these is that the secondary surface is restricted from penetrating the main surface, whereas the opposite is not restricted. The creation of these can be made in two different ways.[1]

General contact: The general contact algorithm automatically selects contact pairs if the elements are making contact according to Cundall's contact formulation (presented in Section 2.4.2.3). Contact checks are performed throughout the analysis as the individual elements move. The drawback is that main- and secondary surfaces cannot be individually selected.

Surface to surface-contact: In this setup, the surfaces are selected by the user. This allows for specification regarding main- and secondary surfaces and specification of what type of connection to use.

Interaction properties:
To simulate the connection between the voussoirs, the interaction is modelled as a definition of tangential and normal behaviour between surfaces.

Normal behaviour: Normal behaviour between surfaces can be defined using 'hard' or 'soft' contact properties. A hard constraint means that Abaqus enforces a zero-penetration method, meaning that the nodes on the secondary surface are prevented from penetrating the main surface. The interaction can transmit any compression between elements as presented in Figure 2.12. However, the surface separates if the contact pressure is zero, simulating the low tensile strength of the joint.

Figure 2.12: Default pressure-overclosure relationship redrawn from Abaqus User's Manual[1]

Tangential behaviour: The tangential behaviour of a connection is related to the normal behaviour according to the Coulomb friction model. It means that the shear stress $_{crit}$ in the interface depends on the current contact pressure p. The relationship is de ned in (2.16)

$$_{crit} = p \tag{2.16}$$

The friction coe cient can be de ned based on the relative slip in the connection or directly as a static and kinematic friction coe cient. In the case of a masonry structure, it is assumed to be no slip in the interface, which means that the friction model can simply be described by the relationship in (2.16) and displayed in Figure 2.13.

Figure 2.13: The Coulomb friction model redrawn from Abaqus User's Manual[1]

Damping: As explained in section 2.4.2, damping is a crucial component when working with a quasi-static analysis in Abaqus. Abaqus/Explicit allows the user to set a damping coefficient which is associated with the rate of relative motion between surfaces [1]. The value is used in both tangential and normal directions opposite the direction of the velocity. The magnitude of the damping is difficult to obtain numerically. Godio et al. suggest a trial and error method of analysis to make sure that the kinematic energy is less than 5-10% of the internal energy [20].

2.5.2 Load application

The choice of load application method has a large effect on how the system responds. This can be performed as either a load-controlled or displacement/velocity-controlled application. In terms of computational speed, the load-control method is preferred. However, a displacement/velocity-controlled approach can capture the post-peak behaviour after the collapse is initiated [39].

The conditions in which the load is applied are of larger importance for quasi-static analysis than in static analysis. As the application is in a dynamic time-step and the structural response depends on the kinematics of the system, the loads must be applied so they do not induce oscillations in the system [20]. In Abaqus the user can control the load applications utilizing amplitude curves. Most importantly, changes in accelerations from load applications should be kept at a minimum during the time step. The smooth step amplitude curve shown in Figure 2.14 applies the loads without abrupt changes in the start and end of loading.

Figure 2.14: Smooth amplitude loading curve used in Abaqus

2.5.3 Establish a time step

The explicit algorithm requires that the system is solved in a dynamic time step. Unlike in static analysis where time is arbitrary, a dynamic time step displays the kinematic response at a given time. The size of the stable time increment is based on (2.17) but requires de ning in Abaqus/Explicit.[1]

Automatic time incrementation: Abaqus automatically selects the time step based on estimates of the system's dilation wave speed and element characteristic length (2.17). Initially, it estimates the time step based on the highest element frequency in the system. However, such an estimation is conservative and can be better approximated by calculating the system's maximum frequency. The switch between element-by-element estimation and global estimation is handled automatically depending on computational cost.

$$\Delta t = \min \frac{L^e}{c_d}; \qquad c_d = \sqrt{\frac{E}{\rho}} \tag{2.17}$$

Where the time step Δt is given as the smallest value for the mesh element characteristic length L^e over the material dilation wave speed c_d. This is an approximation of the stable time increment which is proven to be a conservative estimate[39]. The dilation wave speed is dependent on the sti ness E and density ρ of the material and can be easily controlled in Abaqus

User-de ned time incrementation: The user selects the time incrementation manually. Either by using the initial element-by-element time step consistently throughout the analysis or by a xed value. If a xed value is used, careful consideration must be made not to go higher than the stable time increment to ensure valid results.

Mass scaling: To improve convergence speed in the result, Abaqus/Explicit provides the option of mass scaling in the time step module. The program arti cially increases the mass density of the material in order to increase the time step size. By looking at (2.17) it can be seen that a mass increase of 100 will render a minimum time step increase of 10. This will shorten the computational time but should be used with caution. As the physics of the simulation is changed, it can introduce oscillations in the result if not used properly.

Mass scaling is usually added as a xed value on the entire model for a quasi-static analysis. In Abaqus/Explicit this can be de ned as an arti cial multiplication factor for the masses. The user can also de ne a desired minimum stable time increment and let the program select the required scale factor. It can also be applied to a selection of the model when, for instance, di erent parts have di erent sti ness values or mesh sizes.

3

Method

The method was brie y presented in 1.4, using Figure 1.3. This chapter presents the method in greater detail. It also describes how the method evolved during the thesis work, as tests were performed to prove the reliability of the chosen method. This chapter begins by describing the analysed hydrostatic bidges in 3.1, followed by the separate parts displayed in Figure 1.3. With 3.2 describing hand calculations, 3.3 describing the load tests performed on the scale models and lastly 3.4 explaining the implementation of the computational model.

3.1 Bridge types

Analysis was performed on two di erent types of hydrostatic shell bridges. The di erence between the bridge types is the initial cross-section chosen for the form nding process. One with a semi-circular cross-section, which is referred to as the circle bridge or circle cross-section bridge henceforth. The second type has a catenary cross-section and is referred to as the catenary bridge or the catenary cross-section bridge.

Figure 3.1 shows the initial cross section at the middle of the bridge, with its dimensions. Figure 3.2 displays the catenary bridge in section with important dimensions noted.

Figure 3.1: Catenary cross-section of the bridge [mm]

Figure 3.2: Elevation of the catenary bridge [mm]

Figure 3.3 shows the initial cross-section, the cross-section at the middle of the bridge, with its dimensions. Figure 3.4 displays the circle bridge in section with important dimensions noted.

Figure 3.3: Circle cross-section of the bridge [mm]

Figure 3.4: Elevation of the circle bridge [mm]

3.2 Hand calculations

Hand calculations were performed in order to approximate the load capacity, as well as develop an understanding of the structure as it is an unproven concept.

The hydrostatic bridges require detailed calculations to obtain reliable results. Therefore an intricate analysis was performed using DEM, which is further described in section 3.4. However, the problem could be simpli ed to perform approximate hand calculations of the collapse load. By assuming that the bridge acts as an arch, calculations using the theory and equations presented in section 2.3.2 were performed. In Figure 3.5 and Figure 3.6 the dimensions of the simpli ed geometry for the circle and catenary bridge are displayed. The simpli ed geometry was obtained by drawing the bridge in section, as a 2D-projection, with the section cut being in the middle of the bridge width. Equation (2.4) was used for the calculation of the collapse load.

Figure 3.5: Dimensions of simpli ed circle cross-section bridge for hand calculations [mm]. Width of the bridge 139 mm.

Figure 3.6: Dimensions of simpli ed catenary cross-section bridge for hand calculations [mm]. Width of bridge 138 mm.

The result from Heyman's method is a critical line load acting along the cross section of the bridge. Thus, to obtain a critical point load, the value for the critical line load is multiplied with the bridge's width. As the hydrostatic bridge's cross section vary along the width, this is a simpli cation as shown in Figure 3.7.

Figure 3.7: 3D-visualization of the simpli ed geometry for the hand calculations performed. With a line load acting perpendicular to the bridge span in the quarter point.

3.3 Physical test

Three physical tests were performed on scale models of the proposed bridges. The method for load testing these bridges was developed during the work of this thesis.

The bridges that were tested were all constructed using the same process and technique as shown in section 4.2. The bridge type for the physical tests was the catenary bridge. They were assembled with blocks that were compressed between two supports. This without any mortar in the interface, thus eliminating the possibility of tensile strength in the connections. Small pins were added in the connection between elements to simplify the assembly of the model. The parts are printed in gypsum which has a high compression strength. Considering this, the model was very similar to a real masonry construction.

3.3.1 Construction of Load-testing-rig

To enable the preliminary test, a loading rig was built, which included the following parts:

1. Wooden frame: A frame that was mounted around the cross-section of the bridge, with its centre of gravity beneath the cross-section to ensure stability. In the bottom of the frame, a bucket was hung, and sand was poured into it to increase the load.
2. Scale: A scale was placed between the frame and the bucket, which measured the weight of the applied load simultaneously as it was applied.
3. Rulers: Paper with millimetre lines was placed behind and in the bridge to measure deformation during the test.

The setup for the test, together with the bridge, is shown in Figure 3.8

Figure 3.8: Setup for first physical load test.

The method was constructed with mainly two goals in mind, finding the critical load and measuring the deformations of the bridge. By measuring the deformations, a load and displacement graph could in theory be constructed and compared with the analysis performed in Abaqus.

3.3.2 First load test

The following section describes the first load test and how it was performed, and important takeaways to enable a more accurate method for the coming load tests. The test was conducted on the catenary model produced during a workshop for students attending the bachelor's program Architecture and Engineering at Chalmers University of Technology. As the critical load position was unknown, the load was placed at the quarter point of the span for the first test, as this is the critical load position for arches as explained in Section 2.1.1. A video showing the first load test is found in [8].

3.3.2.1 Evaluation of Method

Large clamps were attached to the supports to ensure that the bridge was compressed between the abutments. This resulted in the supports being deflected inwards as the bridge started to deform. This could have increased the total load capacity. For future tests, it was decided that the clamps had to be less tensioned to avoid excessive compression forces in the bridge.

The deformations of the bridge before collapse were very small, and therefore di - cult to extract from the footage taken during the test. The rulers that were attached close to the bridge were also hard to read. This resulted in the ambition of plotting a force-displacement diagram being impossible to achieve. However, it was decided to keep the rulers for future tests, as this gave an implication of the movement of the bridge.

3.3.3 Second load test

After the rst load test was performed, the bridge was fully rebuilt without producing any new material. The second time, the bridge had a similar number of imperfections as previously. This was due to some building elements being skewed from the start, mainly the wooden supports and the formwork, which were unsymmetrical. However, this was preferable as it gave similar conditions for loading the bridge, therefore making the results comparable.

3.3.3.1 Test setup

The test was performed in the same manner as the rst load test, using the same loading rig and the same method for measuring deformations as explained in 3.3.1. However, some adjustments were made to ensure the stability and precision of the results. One more clamp was added to the supports and larger blocks than previously used were added between the clamps and supports. This was done to provide a more even pressure distribution and therefore decrease the horizontal deformation of the abutments during loading, as this was considered a source of error in the previous test. More cameras and angles were also added in order to capture the deformations and collapse of the bridge.

The loading point was moved to 0.30 m from the support, meaning in between the midpoint and the quarter point. The continuous ow of added weight was altered as the use of sand was not su cient for the collapse load. Thus, weights of 1 kilogram each were placed in the bucket before the sand was poured in. The weight had a collated weight of 10 kilograms.

3.3.4 Third load test

The catenary model was rebuilt once again for the third and nal load test. Using the same building parts as previously, the model had similar imperfections for the third test making the results comparable with previous tests. The gypsum blocks were inspected for deformations or dents since they had been loaded to failure two times before. However, the material displayed no considerable imperfections that would skew the results.

3.3.4.1 Test setup

The load point was moved to the middle of the span to facilitate comparison with the results given by the different load positions in the DE-model. The test was performed using the same method and setup as the second load test.

3.4 Computational analysis

This section covers the method used for creating the computational model and how the analysis was performed. The software for performing analysis was Abaqus and the numerical method was the discrete element method.

3.4.1 Construction of DEM-model

The following section describes the method for creating the computational model of the bridge, both geometry and preparation for conducting the DEM analysis.

The system was analysed using an explicit routine rather than an implicit one, with multiple sources claiming it to be a more reliable method for masonry contact problems [41][39][20]. The analysis was performed in two different time steps corresponding to the application of self-weight and concentrated load respectively. Both of these steps were of type Dynamic Explicit.

3.4.1.1 Defining geometry

The voussoirs for the hydrostatic bridge were modelled in Rhino/GH, a parametric 3D-modelling software that facilitates geometry creation in the early stages. While Abaqus CAE, the software used for analysis, also has a built-in option to define geometry. However, it is more complicated and time-consuming for larger more complex geometries as each element must be defined individually. Instead, the Rhino/GH geometry was exported as individual parts into Abaqus CAE as a STEP-file (.stp) as this was proven to be the most efficient method for complex geometries.

Figure 3.9: Work ow for exporting parts to Abaqus

With the geometry de ned in Rhino/GH , performing analyses of di erent bridge types is less time-consuming. A completely new design can be created by only changing the shape of the cross-section in the middle of the span.

3.4.1.2 Automated tasks

Abaqus CAE requires the application of needed properties for each part. Because of the large number of blocks, individual assignment of properties would be time-consuming. Therefore some parts of the modelling were facilitated by writing a Python-script for Abaqus

Deciding what parts to simplify by scripting depends on how general the problem is. For example, the material properties of the bricks will always be the same, regardless of the geometry, allowing for automation. Parts that were unique for every bridge type were solved directly in theCAE instead. Full documentation of what tasks were automated is presented in section 4.3.5 and Appendix A.3.

3.4.2 Veri cation of analysis model

There are many parameters that are important to the general result of an explicit analysis of a masonry structure. An analysis of an arch was performed to develop a functional modelling system. This veri cation was both an assessment of the work-ow that was used for the real bridge as well as a sensitivity study to de ne the interaction properties.

3.4.2.1 De ning the arch model

The arch studied was a simple half-circular masonry arch shown in Figure 3.10 It was made up of 25 individual voussoirs, which were made in Rhino/GH

Figure 3.10: Arch used for Veri cation of DEM model

The geometry was exported to Abaqus CAE as deformable bodies, and section assignment and meshing were done by using a Python script. The material used was granite, with the following properties:

Parameter	Value	Unit
Young's modulus	70000	MPa
Poisson's ratio	0:3	
Density	$2{:}75 \quad 10^{\ 9}$	$kg{=}m^3$

Table 3.1: Properties of granite used in Abaqus[12]

As for de ning the boundary conditions, the surface of the lower voussoirs was assumed to be fully xed to the abutments. To simplify the de nition, the surface was kinematically coupled to one reference point in the middle of the surface, where the boundary condition was applied.

The general contact de nition was used for interaction between blocks as an individual assignment of main- and secondary surfaces was not important in this case. The result had to be veri ed to ensure that signi cant overlap between elements did not occur. The interaction properties are de ned as follows:

- Normal behaviour: "Hard contact". Default constraint enforcement method
- Tangential behaviour: Penalty formulation. Find a suitable friction coe cient that prevents sliding failure. Assumed to be larger than 1
- Damping: Find a damping coe cient that minimizes oscillations in the response without preventing collapse. A suitable number is found by trial and error.

The setup was tested for two different load combinations. Both include a first step with a gravity load applied in a smooth step over 6 seconds. The load cases were:

1. Load case 1: Concentrated point load applied in mid-span
2. Load case 2: Concentrated point load applied in the quarter span

3.4.2.2 Load Case 1

For the first case, a concentrated force was added in the vertical direction on the keystone of the arch. Similarly to the boundary conditions, the force was added on one reference point that was kinematically coupled to the entire extrados of the keystone. The load was applied in a smooth step over a duration of 6 seconds up to a maximum value of $350 kN$, which means the total analysis time was 12 seconds.

The contact between elements was visualized in the Output database file (ODB) from the Abaqus simulation. Figure 3.11 and 3.12 shows the magnitude of the opening between interacting blocks. A negative value indicates that the elements are overlapping. The figures show that the overlap is at a maximum of approximately 0.12 mm. This was considered to be a sufficiently small value that did not affect the result of the simulation, due to real voussoirs not having perfectly smooth surfaces.

Figure 3.11: LC1: Contact openings after gravity load: t = 6s

Figure 3.12: LC1: Contact openings at collapse stage: t = 10.4s

The system must be in equilibrium with the applied forces until it collapses. The arch was supported on two boundaries which must take all the load that was applied. in Figure 3.13 the reaction forces are shown with the applied load superimposed. The system is shown to be in equilibrium until collapse was initiated around 10.4 seconds into the analysis.

Figure 3.13: LC1: Vertical reaction force and applied load

Another way of assessing the reliability of the result is to ensure that the kinetic energy is kept low during the analysis. Preferably it should be less than 10% of the internal energy [20]. Figure 3.14 shows the internal and kinetic energy during the simulation. It can be seen that the kinetic energy was very low compared to the internal energy until it eventually increased rapidly at around 10.4s. This was another veri cation that collapse was initiated around this time.

Figure 3.14: LC1: Internal (ALLIE) and kinetic (ALLKE) energy

The energy balance for the output data is further explained in [3]. The collapse load can be retrieved from the applied load at time 10.4s as:

$$P_{cr} = 305\,kN$$

3.4.2.3 Load Case 2

In the second case, the arch was loaded at the quarter point of the span. Theoretically, this should collapse under a lower load than the first load case as explained in 2.3.1. The same load of 350kN was applied over 6s in a smooth amplitude. Figure 3.15 and 3.16 highlight areas where the blocks were in contact with each other. The formation and positions of the hinges closely resemble the theoretical hinge pattern shown in Figure 2.6.

Figure 3.15: LC2: initialization of collapse: t = 9.9s

Figure 3.16: LC2: Collapse mode at t = 10.5s

Figure 3.17 shows the distribution of the vertical reaction force in the left and right abutments respectively. Unlike the rst load case, where the reaction forces in the abutments were equal due to symmetrical loading, the left abutment in this case takes the majority of the concentrated force.

Figure 3.17: LC2: Vertical reaction forces in the left and right abutment

The kinetic energy was still just a fraction of the internal energy as in LC1. However, Figure 3.18 shows that the drastic increase in kinetic energy happened earlier at around 9.9s which indicates that collapse was initiated at a smaller force than for an arch loaded in the midpoint.

Figure 3.18: LC2: Internal (ALLIE) and kinetic (ALLKE) energy

The collapse load can be retrieved from the applied load at time 9.9s as:

$$P_{cr} = 268\,\text{kN}$$

3.4.2.4 E ect of Mass Scaling

The e ect of mass scaling was evaluated to visualize its impact on the result. For this study, load case 2 was analysed with three di erent values for the mass scaling (0, 10, 100). Figure 3.19 shows how the kinetic energy in the system was a ected by the applied load.

Figure 3.19: E ect of mass scaling on the kinetic energy

The kinetic energy displays a more gradual increase for lower load values when mass scaling is used. Ultimately, the peak in energy appears at approximately the same

time. Therefore, mass scaling does not alter the failure load if it is used carefully so it does not increase the stable time increment above the critical time step.

The e ect on computational time from mass scaling is signi cant. Thus, it is a choice for the user to balance the stability of the result with computational cost.

3.4.2.5 Evaluation of method

This study was mainly done to establish what parameters to use to achieve a quasi-static response for the analysis of the hydrostatic shell bridge. However, it also served as an introduction to general analysis procedures of masonry constructions and how they relate to the assumptions of masonry as a material. Some validation for the models' reliability can be stated when comparing the results with the properties of masonry in section 2.1.

1. The blocks display very small or no deformations during the analysis even though deformable bodies are used. This captures the "in nite" strength of the voussoirs that is assumed.
2. The tensile strength of the mortar is non-existent, meaning that hinges are formed when parts of the interface experience tension. This is eventually what leads the system to failure.
3. The frictional model employed is su cient to prevent shear failure from happening. If the blocks were allowed to slide relative to each other, it could change the entire failure mechanism and would not represent an accurate masonry response according to Heyman's postulates (2.1).

By testing and altering various interaction properties, usable numbers could be retrieved when the predicted failure mode occurred. To achieve these results, the following interaction properties were used:

Friction coe cient	5
Damping coe cient	0.4

Table 3.2: Interaction properties used in Abaqus for the simpli ed arch

The results from both load cases show that the decisions taken to achieve a quasi-static solution work as they are intended. In both cases, the kinematic energy is kept low during the entire analysis until collapse eventually takes place. For the load application, both a force-driven- and a velocity-driven approach were tested. By using a prescribed velocity, the system is displaced at a constant rate so that no accelerations are induced. However, this e ectively prevents the system from collapsing into a state of free fall when failure eventually occurs. As one intention of the modelling is to accurately represent the failure mode of the hydrostatic bridge, the use of a prescribed force works better in this application. Some important considerations must be taken into account to achieve a trust-worthy quasi-static response with this consideration:

- The damping model used is proportional to the relative motion of the individual blocks. This effectively dampens the relative motions of the blocks to reduce oscillations from the accelerations.
- The load is applied in one smooth step in the analysis. This keeps the acceleration to a minimum at all points.
- The effect of mass scaling was tested to visualize the effect it has on the kinematic state of the structure. It should be used in a responsible way to ensure that the physics of the system is not altered.

3.4.3 Load cases for bridge analysis

A few different load cases and designs were analysed in order to answer the posed questions. The first question to be answered was where the critical load point was located, hence enabling drawing conclusions for one of the thesis questions, is the structure a beam or an arch?
The load cases were set up according to table 3.3 and performed on two different geometries, a catenary and a circle cross-section. Since the bridges are symmetric, the results could be mirrored to cover the full length of the bridge.

Load points [mm]	200	300	325	350	375	400
Relative to span	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{1}{2}$

Table 3.3: Load points for parametric study using Abaqus, distance from support [mm]

4

Modelling of the Hydrostatic Bridges

After the arch model was created, analysed and discussed, the work on the bridges was initiated. Based on findings from the verification model, the same work process and specifics, such as material parameters, were utilized.

This chapter aims to show the entire work flow for the digital and physical fabrication method for the hydrostatic bridge. From form finding to finished analysis.

4.1 Geometry generation

The form finding process is based on the theory presented in section 2.1.3. Only the modelling of the bridge with a catenary cross-section is presented in this chapter. However, the steps presented are done in the same way for both types of bridges. The basis of the geometry design was made in the computer software Rhino together with the plug-in Grasshopper(GH). The software facilitates the design process by using a parametric approach to 3D modelling meaning that it is ideal for an iterative design process.

The numerical method for form finding of a hydrostatic shell follows a process by Adiels and Williams in the paper The construction of new masonry bridges inspired by Paul Séjourné [10]. The process is initiated by a chosen cross-section in the mid-span of the bridge, and the form is iterated to its final shape using a Python-scripting component in GH. The script is shown in App. A.2. The input needed for the Python-component is a number of points along a discretized curve as well as parameters for the fill density and geometrical values defining the final length of the bridge. The points extracted from the curve are used to calculate the vectors that defines the geodesic curves of the form-found surface. The geodesic curves, and the orthogonal trajectories, are used to draw a surface grid as displayed in Figure 4.1. The last step of the process is to create individual bricks from the grid with a given thickness.

Figure 4.1: Generation of geometry in Rhino/GH

4.2 Construction of the Physical model

The physical models used for testing were first built during workshops during a course in parametric tools for Architecture and Engineering students at Chalmers University of Technology. A video showing the construction is found in [6]. The bridges were constructed of mainly four building parts and were produced accordingly:
Numbering according to Figure 4.2

1. Abutments: Made from wood and cut in the correct angles using a CNC router
2. Bottom plate: Made from a thick plywood board with some bracing for added stiffness beneath. The plywood was also cut using a CNC router for the placement of the abutments.
3. Gypsum blocks: Printed using a gypsum 3D-printer (Projet 660 Pro), Figure 4.2 displaying the placement when printing
4. Formwork: Made from styrofoam and cut to the shape of the form found bridge using a CNC router

Figure 4.2: All building parts for physical model

When constructing the model, the abutments were placed on the bottom plate together with the formwork placed alongside the abutments. The building blocks were marked with a letter and a number during the printing, to display the placement of each block. The gypsum was then placed in the formwork accordingly. When all

blocks were laid out, clamps were used to push the abutments together as sand was poured into the bridge, which created the needed self-weight as well as horizontal support. When the abutments were screwed into the bottom plate, the formwork could be removed together with the clamps as the bridge was stable. The weight of the bridge was 2.49 kg gypsum blocks and 7.34 kg ll material.

The 3D-printed gypsum blocks had two modeling details that were added to simplify the assembly process. Firstly, the blocks have small pins on one side and indents on the other, this to make sure that the blocks are tted properly, see Figure 4.3. Secondly, the blocks closest to the support has a rounded edge facing the abutments, to ensure a well tted joint.

Figure 4.3: Special features of the 3D-printed blocks

4.2.1 Rebuilding the model

After each test, all parts of the bridge were collected and rebuilt using the same process as the rst build, see Figure 4.4. This enabled multiple tests to be performed on each bridge, without having to produce new material.

Figure 4.4: Left: All the blocks sorted according to the numbering system. Right: Assembly process in the formwork

4.3 DEM

The DE-model of the bridge was modelled in the same scale as the physical model to facilitate a comparison of results. Consequently, the DE-model can be seen as a digital twin of the scale model with the same material properties.

The geometry was simpli ed for the DE-analysis compared to the physical model. The blocks did not include the pins that were used to simplify construction of the model. The ll material was not modelled as this increases computational time signi cantly.

4.3.1 De ning Properties

The study of the simple arch in section 3.4.2 was the basis for the design of the hydrostatic bridge. The di erence from the veri cation model was that the bridge was constructed with gypsum blocks which has other material properties. The material density was calculated by weighing a set of pieces from the physical model and dividing the value with the volume given in Grasshopper, and Young's modulus was given by the manufacturer of the 3D-printer[4]. The material properties used for analysis are presented in Table 4.1.

Parameter	Value	Unit
Young's modulus,E	9450	MPa
Poisson's ratio,	0:3	
Block density, $_{block}$	$1{:}48 \quad 10^{\ 9}$	tonne=mm^3
Sand density, $_{sand}$	$1{:}60 \quad 10^{\ 9}$	tonne=mm^3
Mesh seed size	10	mm

Table 4.1: Material properties used for the bridge analysis in Abaqus

The interaction properties were de ned in a Python-script, attached in Appendix A.3 as a frictional contact. The tangential behaviour was modelled as a penalty formulation with a friction coe cient of 5. A hard contact formulation with a default enforcement method was used for the normal behaviour. Damping was added with a coe cient of 0.4. The interaction was modelled with a general contact algorithm since the speci cation of main- and secondary surfaces was not of importance.

4.3.2 Loads

Loading was performed in two di erent analysis steps in a dynamic explicit procedure. First, the system's self-weight was added for 6 seconds. In the second step, the concentrated load is applied for 6 seconds. All the forces were applied with a smooth amplitude to decrease unwanted dynamic responses.

4.3.2.1 Self Weight

For the self-weight of the bridge, two di erent loads must be considered. Since the ll material is not included in the DEM-model, the weight from the sand is introduced as a pressure load on the inside of the shell. The load is expressed as an analytical eld in Abaquswith the formula:

$$p = {}_{sand} gz$$

where $_{sand}$ is the density of the ll material and z is the depth of the ll at a given point. With this function, the pressure p from the sand will increase with increasing depth of the ll as shown in Figure 4.5.

Figure 4.5: Sand pressure modelled in Abaqus

The self-weight from the voussoirs is de ned as a gravity load. Abaqus automatically sets the magnitude based on the volume and density of the bricks.

4.3.2.2 Applied Load

The concentrated point load on the structure needs to be modelled to include the e ect of the ll material. Namely, the dispersion of the force through the sand. A load model is created to mimic the e ects of load distribution through the ll material.

An assumption was made that the forces would spread out in a cone-formed shape from the application point down to the shell as seen in Figure 4.6. With this approach, most of the load will be put directly under the load application point with a gradual decrease towards the edges of the cone. Houlsby et al. suggest that an appropriate value for the load spread angle in granular ll is $= \tan^{1}(0{:}5) \quad 45°$ [34]. Which was selected for this analysis.

Figure 4.6: Assumed load distribution in the ll material

To achieve this loading condition, the load was modelled in an analytical eld expression. With the use of a locally de ned spherical coordinate system in the loading point (Figure 4.7), the distribution can be de ned to spread in the sand at a given angle' . The expression gives a relation of magnitude between the loading vectors.

Figure 4.7: Spherical coordinate system de ned in the load point

$$y = a\cos(b\varphi')$$

Where b defines the period of the function. The function is positive in the range $-45° < \varphi' < 45°$ for $b = 2$. By including a minimum value statement in the expression, the expression was generalized to only include positive vectors. The final expression was:

$$y = \max(0; a\cos(2\varphi')) \tag{4.1}$$

Where a defines the magnitude of the applied force. Depending on where the point load is applied, the total loaded area differs depending on the bridge depth. Therefore, the load magnitude should be considered as a scaling value of the vectors. To get the total applied force at a given time, all contributions from the vectors were added to the resulting ODB-file.

With 4.1 as a analytical field expression, Abaqus automatically creates the given load vectors based on the position of a spherical coordinate system. The load model is displayed in Figure 4.8.

Figure 4.8: Load dispersion model in Abaqus for a concentrated force in the quarter point

4.3.3 Boundary Conditions

To properly de ne the support conditions of the bridge, the model has to represent the real support condition for the physical model accurately. This was achieved by including a surface (the black surfaces in Figure 4.9), in the Abaqus model on which the end stones were resting. These surfaces represent the abutments in the physical model and were locked in all directions (rotation and translation). The blocks directly in contact with the surface have the same contact properties with the abutments, as they have to all other adjacent blocks.

Figure 4.9: Boundary conditions for the hydrostatic bridge

With this model, the bricks are held in place by frictional shear forces but can still rotate and form hinges at the boundaries as the general assumptions of masonry state.

4.3.4 Analysis procedure

The importance of mass scaling in the analysis is evident in this case. As the complexity of the block geometry combined with the large number of elements decreases the Stable time incrementsigni cantly, making analysis almost impossible to perform in a reasonable time. With a mass scaling factor of 10000, and running the analysis on four processors, a simulation takes around 2h 15min hours. Where the gravity step takes apporixmately 45 minutes and the load application step 1.5 hours.

Analysis for di erent load points was made more e cient by running a restart analysis. This means that the rst analysis step, the gravity step, corresponding to the application of self-weight was only performed once per bridge. The subsequent loading step was then analysed based on the data from the gravity step without the need to run it again, e ectively saving 45 minutes of analysis time. With this

method, di erent loading positions and magnitudes were analysed more quickly.

4.3.5 Automated tasks

When building the model of the bridge in Abaqus, the Python script used in Section 3.4.1 was updated and more functionalities were added in order to minimize modelling time. As this geometry is more complex, compared to the simple arch, the process of for example selecting the surfaces of the surfaces facing the ll material became more time-consuming. Therefore, the following features were included in the script.

- De ning material properties and section assignment
- Mesh the parts
- Assemble the model
- De ning interaction properties between the parts
- De ning time steps
- Creating a surface for the intrados to apply hydrostatic load
- De ning a spherical coordinate system at the point load
- De ning needed amplitudes and analytical elds
- De ning all loads: gravity, sand pressure and point load

The full process of creating the model was as follows:

1. Export STEP- le from Rhino
2. Export a CSV- le from GH containing the coordinates of the midpoint of the surface facing the ll material of each masonry unit (a separate script for this is found in A.4 which was implemented in GH)
3. Import STEP- le in Abaqus
4. De ne needed inputs in the Python script (these include e.g. material properties and size of loads, see A.3 for full script)
5. Run Python script in Abaqus
6. Manually add the rest of the properties in the CAE. Eg. BC:s, output data and de ne the restart model.

The Python scripts used for automating the model creation is shown in full in Appendix A.3 and Appendix A.4.

5
Results

The following chapter presents the results of all calculations. Section 5.1 presents the results from the hand calculations, section 5.2 shows the results from all the physical load tests and section 5.3 presents the results for the Discrete Element Method. Lastly, the results are analysed using dimensional analysis in section 5.4.

5.1 Hand calculations

Using the equations presented in section 2.3, the results presented in Table 5.1 were calculated. This by simplifying the geometry to a simple arch bridge. The result from this method is a critical line load F_{cr} acting along the width of the bridge, see Figure 3.7. To get the critical concentrated force P_{cr}, the result was multiplied by the bridge width. The calculated bridges were the circle and the catenary cross-section. The load was placed at 1/4 of the span length (the critical load point for arches) for both bridges. The calculations in full for both bridges are found in Appendix A.1.

Cross-section	Load F_{cr} [kg=m]	Load P_{cr} [kg]
Circle	28.0	4.4
Catenary	33.8	4.5

Table 5.1: Approximated collapse load using hand calculations

5.2 Physical test

In total, three test were performed on the bridge with a catenary cross section. The results from all three tests are compared and presented in 5.2.4.

5.2.1 Load placed at quarter point

The scale model was loaded in the quarter point, 0.2 m from the support. It was loaded with a continuous flow of added weight until it collapsed at a total load of 28.9 kilograms. Figure 5.1 displays the curvature of the bridge at 20 kilograms of loading.

Figure 5.1: Load test of catenary with the load placed at quarter point, loaded with 20 kilograms.

Figure 5.2 displays the deformed shape of the bridge, where the black dashed line is the same as in Figure 5.1, and the red line is the outline of the deformed bridge. It is noted that the loaded side of the bridge has de ected downwards and the unloaded side upwards. Above the bridge, an exaggerated curve is shown to further display the deformation.

Figure 5.2: Load test of catenary with the load placed at the quarter point, deformation at approximately 27 kilograms. Red line: outlines of the bridge, black dashed: initial shape.

The moment of collapse is shown in Figure 5.3. The outline is similar to gure 5.2. The rupture is present directly under the load, with a bulging shape, its movement is noted with the red curve in the gure.

Figure 5.3: Load test of catenary with the load placed at quarter point, collapse at 28.9 kilograms.

5.2.2 Load placed between quarter and mid point

During the second load test, the scale model was loaded between the quarter point and the midpoint, 0.30 m from the support. With the middle being at 0.4 m, thus the load was placed at 3/8 of the total span. It was loaded with a continuous ow of added weight until it collapsed at a total load of 22.9 kilograms. Figure 5.4 displays the undeformed bridge, loaded only by the frame of 1 kg.

Figure 5.4: Load test of catenary with the load placed at 3/8 point, loaded with 1 kilogram.

Figure 5.5 displays the deformed shape of the bridge, where the black dashed line is the same as in Figure 5.4, and the red line is the outline of the deformed bridge.

Figure 5.5: Load test of catenary with the load placed at the 3/8 point, deformation at approximately 20 kilograms. Red line: outlines of the bridge, black dashed: initial shape.

The moment of collapse is shown in Figure 5.6. The outline is similar to gure 5.5. The rupture is present directly under the load, with a small bulging shape, its movement is noted with the red curve in the gure.

Figure 5.6: Load test of catenary with the load placed at 3/8 point, collapse at 22.9 kilograms.

Figure 5.7 and 5.8 display the initial shape of the bridge and the deformed bridge respectively, captured at an angle. It is noted that the bulging shape is more prominent than observed in Figure 5.5.

Figure 5.7: Load test of catenary with the load placed at 3/8 point, loaded with 1 kilogram. Angled perspective.

Figure 5.8: Load test of catenary with the load placed at 3/8 point, deformation at approximately 20 kilograms. Angled perspective.

5.2.3 Load placed at mid point

For the final load test performed on the catenary cross-section, the scale model was loaded in the mid point. It was loaded with a continuous flow of added weight until it collapsed at a total load of 13.9 kilograms. Figure 5.9 displays the undeformed bridge, loaded only by the frame of 1 kg.

Figure 5.9: Load test of catenary with the load placed at midpoint, loaded with 1 kilogram.

Figure 5.10 displays the deformed shape of the bridge, where the black dashed line is the same as in Figure 5.9, and the red line is the outline of the deformed bridge. It is noted that the joints between the blocks in the middle have started to open.

Figure 5.10: Load test of catenary with the load placed at the midpoint, deformation at approximately 10 kilograms. Red line: outlines of the bridge, black dashed: initial shape.

The moment of collapse is shown in Figure 5.11. The outline is similar to gure 5.10. The rupture is present directly under the load, with a bulging shape, its movement is noted with the red curve in the gure.

Figure 5.11: Load test of catenary with the load placed at midpoint, collapse at 13.9 kilograms.

5.2.4 Combined results

Table 5.2 and Figure 5.12 shows the result from all the load tests based on the loading position. There is a 70% di erence in load capacity between the strongest and weakest loading position.

Catenary	1/4 point	3/8 point	1/2 point
Mass [kg]	28.9	22.9	13.9
Force [kN]	0.28	0.23	0.14

Table 5.2: Comparison of results from load tests of physical models

Figure 5.12: Collapse load for the analysed loading positions

5.3 Computational Analysis

This section presents the result of the Discrete Element Analysis for the different types of bridges tested.

5.3.1 Catenary bridge

The hydrostatic bridge concept with a catenary cross section was analysed by applying a concentrated force in the points presented in Table. 3.3. Figure 5.13 shows the system's kinetic energy as a function of the applied load. The critical load value is obtained by analysing at which load magnitude the kinetic energy drastically increases.

Figure 5.13: Catenary: Force-Kinetic Energy graph for different loading positions defined from the support

Figure 5.14 displays the difference in collapse load for all the analysed loading positions along the bridge.

Figure 5.14: Catenary: Collapse load for all the analysed loading positions

Figure 5.15 and 5.16 shows the vertical displacement of the bridge when the load is placed in the middle of the span and at the quarter point respectively. In gure 5.16 a positive vertical displacement can be observed at the unloaded side of the bridge. For loading positions closer to the middle, this e ect is less apparent. Deformation gures for all the loading positions are shown in Appendix A.5.

Figure 5.15: Catenary: Elevation of vertical displacement [mm] for a load positioned in the middle of the span (y = 400 mm). Force: 1.83 kN.

Figure 5.16: Catenary: Elevation and top view of vertical displacement [mm] for a load positioned in the quarter point of the span (y = 200 mm). Force: 2.97 kN

The horizontal displacement for the same load cases is shown in Figure 5.17 and 5.18.

Figure 5.17: Catenary: Top view of horizontal displacement [mm] for a load positioned in the middle of the span (y = 400 mm). Force: 1.83 kN. Scale factor: X=5

Figure 5.18: Catenary: Top view of horizontal displacement [mm] for a load positioned in the quarter point of the span (y = 200 mm). Force: 2.97 kN. Scale factor: X=5

During load application, the thrust lines carrying compression to the abutments must adapt to changes in displacement. Figure 5.19 and 5.20 show the contact pressure in the system at the time when the collapse is initiated for a load placed in the middle and the quarter point respectively. Areas in black display contact pressure lower than 1 MPa.

Figure 5.19: Catenary: Top view of contact pressure between the voussoirs of the bridge when loaded in the middle [MPa]

Figure 5.20: Catenary: Top view of contact pressure between the voussoirs of the bridge when loaded in the quarter point [MPa]

The obtained collapse load are obtained by interpolation of the applied load for the time increment before and after collapse is initiated. The result for all the analysed points are shown in table. 5.3

Load application point [mm]	200	300	325	350	375	400
Collapse load [kN]	2.92	1.99	1.74	1.90	1.55	1.80

Table 5.3: Catenary: Collapse load for all the analysed points

5.3.2 Circle bridge

The circle bridge was analysed in the same way as the catenary bridge, with loading positions according to Table 3.3. The resulting collapse load, presented in Figure 5.21 is obtained by observing a sudden increase of kinematic energy at a given load magnitude.

Figure 5.21: Circle: Force-Kinetic Energy graph for di erent loading positions de ned from the support

Figure 5.22 shows the collapse load based on the loading position along the bridge length.

Figure 5.22: Circle: Collapse load for all the analysed loading positions

The deformation in vertical direction for a load placed in the middle and the quarter point are shown in Figure 5.23 and 5.24. Deformation gures for all the loading positions are shown in Appendix A.5.

Figure 5.23: Circle: Elevation of vertical displacement [mm] for a load positioned in the middle of the span (y = 400 mm). Force: 1.46 kN

Figure 5.24: Circle: Elevation and top view of vertical displacement [mm] for a load positioned in the quarter point of the span (y = 200 mm). Force: 2.67 kN

The corresponding horizontal displacements at the same time step are shown in