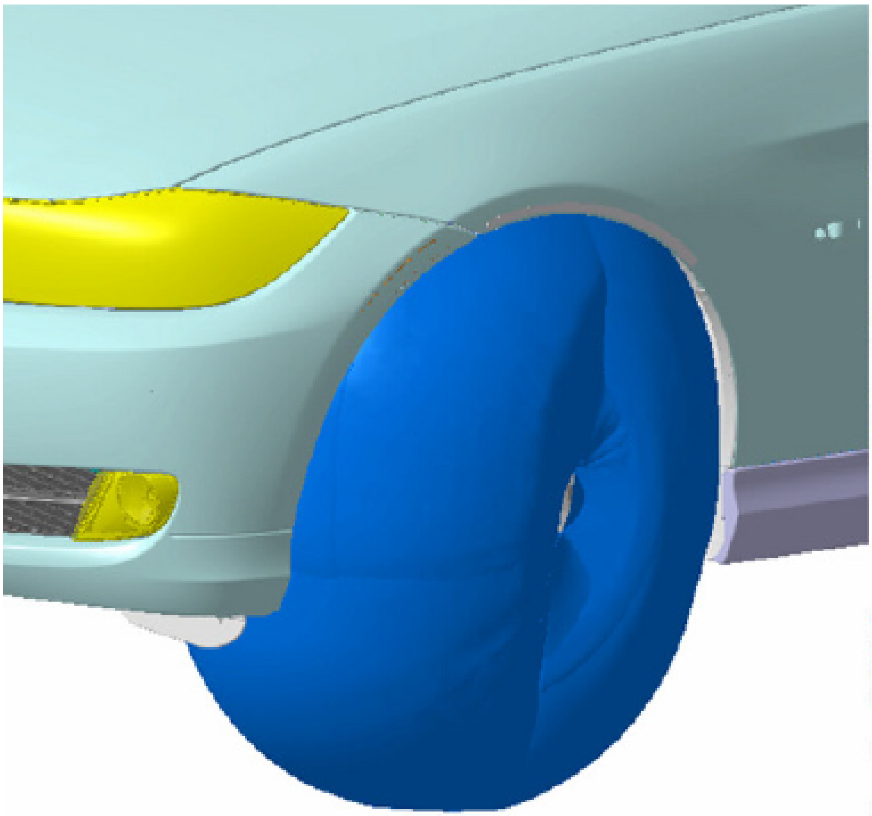
CHALMERS
UNIVERSITY OF TECHNOLOGY



Wheel Envelope Methodology

Master's thesis in Automotive Engineering

KUSHAAL PISHEY

Department of Mechanics and Maritime Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2020

Master's thesis in automotive engineering

Wheel Envelope Methodology

KUSHAAL PISHEY

Department of Mechanics and Maritime Sciences
Division of Vehicle Engineering and Autonomous Systems
Chalmers University of Technology
Gothenburg, Sweden 2020

Wheel Envelope Methodology
Kushaal Pishey

Academic Supervisor & Examiner
Ingemar Johansson
Professor of the Practice, Vehicle Engineering and Autonomous Systems
Chalmers University of Technology
Senior Advisor, Wheel Suspension, CEVT AB

Technical Supervisor
Magnus Nilsson Gatenholm
Concept Leader, Wheel Suspension, CEVT AB

Master's Thesis 2020:08
Department of Mechanics and Maritime Sciences
Division of Vehicle Engineering and Autonomous Systems
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

Cover: Wheel Envelope of left front wheel.
Image Source: Julian Weber, *Automotive Development Processes*, Springer Publication, 2009.

Typeset in LaTeX
Printed by Chalmers Reproservice
Gothenburg, Sweden 2020

Wheel Envelope Methodology
Kushaal Pishey
Department of Mechanics and Maritime Sciences
Division of Vehicle Engineering and Autonomous Systems
Chalmers University of Technology

Abstract

A Wheel envelope is an important aspect of chassis engineering and is developed early in the development process. A Wheel envelope represents the maximum swept volume of a wheel according to the agreed standards and methods.

The aim of this master thesis was to realize the concept of a wheel envelope and develop an analytical method for the wheel envelope development. Knowledge of parameters which will influence the wheel envelope development is required. Knowledge of geometrical tolerances of certain components from the suspension system and the wheel are needed as they contribute to the tolerance stack up. The compliance in the suspension bushings is also studied to understand it's effect on the displacement of wheel center position. A steering strategy is explained based on the driving condition of the vehicle and, details about the tyre sections which include summer tyre, winter tyre and use of snow chains.

For the method, parameters which will influence the envelope design was listed. Geometrical tolerances from components such as control arm, spring, damper were studied and a method to calculate the stack up of tolerances in the tolerance chain was established.

Compliance from the bushings have significant influence on the displacement of wheel center position. A method was established to estimate the deflection in the wheel center position due to compliance of the bushings and forces acting on the wheel in the longitudinal and lateral directions.

Steering strategy is influenced by the type of vehicle for which the envelope is being developed and the driving conditions. Hence, a generic method to create steering strategy by considering the bump travel, rebound travel and steering rack travel was established. Snow Chains are mandatory on certain markets. Therefore, steering strategy for a wheel with a snow chain was created.

Keywords: Wheel Envelope Methodology, Geometric Tolerance, Compliance, Steering Strategy.

Acknowledgements

The completion of this thesis work would not have been possible without thanking some prominent people who have been a part of this thesis.

To start with, I express gratitude to my academic supervisor Professor Ingemar Johansson for his guidance and valuable inputs during the entire thesis work. I am extremely fortunate to have him supervising me at all the instances and helping me understand the intricacies of concepts required for development of wheel envelopes.

I am very thankful to Magnus Nilsson Gatenholm, my technical supervisor at CEVT AB for his invaluable support in the form of sharing technical content and ideas whenever in need. I experienced a pleasant work environment at CEVT for which I would like to thank all the employees.

Finally, I would like to acknowledge my gratitude to my parents and sister for all their love and support during the entire duration of master studies. Also, I would like to thank my friends for supporting me when I was in much need of them.

Thank you very much!

Kushaal Pishey, Gothenburg, June 2020

Dedicated to

My late Grandfather for his tremendous love and support

Nomenclature

OEM	Original Equipment Manufacturer
CAD	Computer Aided Design
ETRTO	European Tyre and Rim Technical Organization
SAE	Society of Automotive Engineers
RSS	Root Sum Square
Jounce	Compression of the Suspension (in mm)
Rebound	Extension of the Suspension (in mm)
δ_i	Angle of rotation of inner front wheel (°)
δ_o	Angle of rotation of outer front wheel (°)
L	Wheelbase (mm)
W	Track width (mm)
S	Tolerance Stack up (mm)
F	Force (N)
k	Stiffness ($\mathrm{N\,mm^{-1}}$)
d	Displacement (mm)

Contents

List of Figures

List of Tables

1 Introduction

1.1 Background

Chassis design is an important aspect of vehicle design for the reason that it is traditionally split up into subsystems such as steering, suspension, brake, wheels, tyres and so on. Of the above-mentioned subsystems, suspension along with wheels are of high priority as they are the primary link between the ground and the vehicle. Also, the packaging of body components adjacent to wheels must be considered. [1]

The selection of the concept of wheel suspension, hard points and the compliance are done to meet the overall attribute targets with regards to vehicle dynamics, ride comfort and NVH. One of the important deliverables in chassis design engineering is the development of wheel envelope for the packaging around the wheel subsystem. The packaging around the wheel composes side members, wheel arches, steering subsystem along with suspension system. Wheel packaging not only affects the turning radius but also the impact performance of the side members. Size of the tyres will affect the wheel packaging; hence the largest selected tyre profile is used for the envelope development.

Under dynamic conditions, the wheel moves vertically due to driving on an uneven road surface and laterally due to cornering forces. In addition the wheels are moving because of compliance when loads are applied and the position because of geometrical tolerances. Therefore, the definitions and strategies are important for the dynamic movement during the development of the wheel envelopes. Hence, all the parameters and their respective tolerances have to be considered for static and dynamic conditions. The wheel envelopes define the total required packaging volume for wheels, suspension and other body parts. Therefore, accurate wheel envelope is required for efficient packaging. [2]

1.2 Aim

The thesis focuses on developing a method which can be used to develop the wheel envelopes with improved accuracy. Static and dynamic parameters, compliance, steering strategy for the front suspension and the process to analyse the tolerances in the system will be studied.

1.3 Deliverables

The deliverables of this thesis are:

- A list of parameters that affects the wheel envelope.
- A study on tolerances of parameters required for geometrical analysis of suspension, steering and wheel combination.
- A method to calculate the total tolerances along with steering strategy which will affect the wheel envelope.
- A review of kinematics and compliance required for wheel envelope.
- Recommendations and create guidelines for the method to develop wheel envelope.
- To present the method in the form of a Master Thesis report.

1.4 Limitations

The limitations of this thesis work are:

- The process to develop wheel envelopes will be general and exact details will be decided in the projects.
- Existing suspension geometries will be used for the study. Some parameters may have to be adjusted to suit other suspension systems.
- Some components are assumed to be stiff to reduce complications during development of the method.
- The Master thesis work is mainly focused on the front suspension.
- Since the focus is on development of method and not the numerical results, the accuracy of the numerical values are limited to a certain extent.

2
Theory

The swept volumes of wheels are extremely important to avoid physical contact between the tyre and any adjacent part when the vehicle is in motion. To study the tyre sweep, movement of wheel center position in longitudinal, lateral and vertical directions due to suspension and steering must be considered. However, to develop a method, individual contribution of certain components of suspension and steering systems needs to be analysed.

2.1 Vehicle Type

The type of vehicle and the segment to which it belongs is to be known beforehand for developing a wheel envelope. Generally available vehicle types on the market are Hatchbacks, Sedans and SUVs to name a few. The above-mentioned vehicle types have a set of space constraints which have to be considered. For instance, the space between the tyre and the wheel arch of a sedan is lesser than the space between the tyre and the wheel arch of a SUV because of different design principles and different driving conditions.

2.2 Suspension

Wheel envelopes for the front axle and rear axle are different mainly because the front axle of a passenger vehicle consists of a steering system that affects the wheel envelope whereas the rear axle does not usually have a steering system. The type of suspension is important for wheel envelope because of different suspension geometries.

For the front axle, generally used suspension types are MacPherson Suspension and Double Wishbone suspension. Predominantly, MacPherson suspension is used because the system is quite simple and requires less number of linkages which enhances the packaging around the chassis. The MacPherson suspension moves the wheel and suspension mainly through the ball joint which connects the knuckle and the control arm.

Double wishbone suspension consists of two control arms, usually unequal and non-parallel to each other. These control arms are connected to the knuckle through two ball joints. This type of suspension system offers more flexibility due to adjustment in camber gain.

2.3 Parameters Affecting the Wheel Envelope

To develop a working method, the parameters influencing the wheel envelope need to be studied. The parameters can be categorized based on the type of suspension and whether they contribute in static condition or during the suspension kinematics. The parameters are tabulated below.

Static Hard Points	Wheel center position Front bushing of the control arm Rear bushing of the control arm Ball joint Top mount Outer tie rod Inner tie rod Camber angle Toe angle
Suspension Kinematics	Bump travel Rebound travel Tie rod stroke Steering strategy
Compliance	Longitudinal compliance (driving) Longitudinal compliance (braking) Lateral compliance

Table 2.1: Parameters Influencing the Wheel Envelope Development

2.4 List of Hardpoints in suspension

The hardpoints denote the design position of suspension and steering components in the coordinate axes. The general notation for a front MacPherson suspension used in the report is presented in the table and depicted in the figure below.

Hardpoint	Notation
Bush front	PT 3
Bush rear	PT 4
Ball joint	PT 6
Top mount	PT 7
Wheel center position	PT 9
Tie rod outer	PT 12
Tie rod inner	PT 14

Table 2.2: Hardpoint Terminology

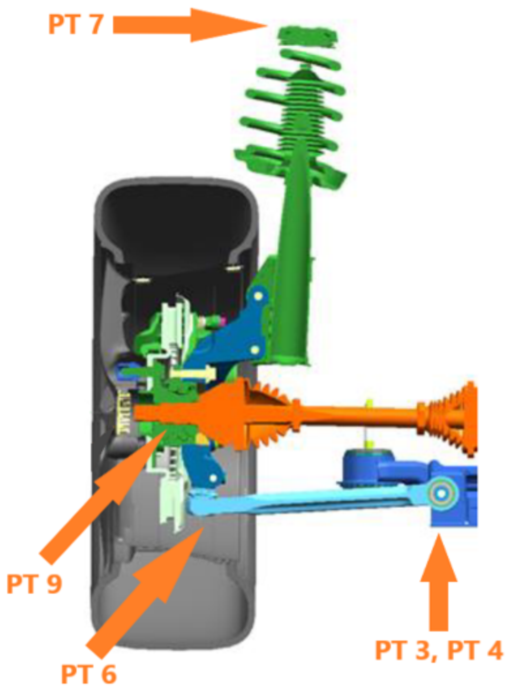


Figure 2.1: Hardpoints with Notation

2.5 Suspension Parameters

Suspension parameters can be classified based on their influence on static and dynamic conditions. When the vehicle is at rest, the design position gives the location of hardpoints for a selected load condition. Hence, they can be categorized under suspension statics. When the vehicle is in motion, a small disturbance from the road surface will influence the location of the hardpoints by displacing them. Hence, they can be categorized under suspension kinematics.

The nominal wheel envelope is generated from the kinematics that is based on the suspension geometry. The total envelope is created from the nominal envelope along with the tolerances and the compliance.

2.5.1 Suspension Statics Hardpoints

2.5.1.1 Rubber Bush

Rubber bushes are ideal in isolating the shock forces coming into the chassis from road disturbances. Two rubber bushes are used between the lower control arm and the subframe mounting.

- **Front bush (PT 3):** This bush is stiff to give required lateral stiffness for the suspension. The radial stiffness, in the lateral direction of the vehicle, is therefore high. It has limited compliance in longitudinal direction.

- **Rear bush (PT 4)**: This bush is the compliance bush that defines longitudinal isolation in the suspension system.

2.5.1.2 Ball joint

The ball joint is placed between the control arm and knuckle. It transfers the lateral and longitudinal loads from wheels also, allows the rotational movement of the knuckle. It is denoted by PT 6 in the list of hardpoints. The compliance in PT 3 and PT 4 will influence the movement of PT 6.

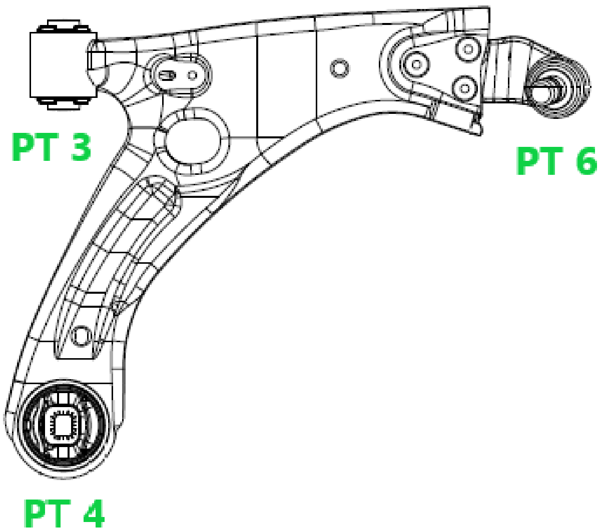


Figure 2.2: Control Arm with Ball Joint and Bushings

2.5.1.3 Top mount

The top mount is where the spring and the damper are fixed to the vehicle body, denoted by PT 7. It is subjected to longitudinal, lateral and vertical forces from the spring and damper coming into the vehicle due to irregularities in the road surface.

2.5.1.4 Wheel Center Position

Wheel center position is a hardpoint which defines the design position of wheel assembly, denoted by PT 9. The movement of wheel center defines the wheel envelopes because the swept volume of the wheel will give an extensive information of wheel position during different driving conditions. Hence, PT 9 directly influences the design of space between the wheel and the wheel arch.

2.5.1.5 Tie rod

The tie rod is an interface between the knuckle and the steering rack. The axial displacement of the steering rack will displace the tie rod which in-turn steers the wheel.

Two hardpoints are considered in the tie rod; the first hardpoint being called as "tie rod inner" denoted by PT 14 and the second hardpoint being called as "tie rod outer" denoted by PT 12. PT 12 and PT 14 provide information regarding the tie rod position when the wheel is steered.

2.5.1.6 Camber angle

Camber angle is defined as the angle of inclination of the wheel with respect to the vertical axis when visualized from the front. Inward leaning of tyre symbolizes negative camber and outward leaning means positive camber. Generally the camber angle is negative for passenger vehicles to enhance the road grip during cornering.

2.5.1.7 Toe angle

Toe angle is defined as the angle of inclination of tyre with respect to the horizontal axis when visualized from the top. Inward leaning of the front wheel symbolizes positive toe (also referred as toe - in) and outward leaning symbolizes negative toe (also referred as toe - out). Usually the static toe angle is positive (toe - in) for passenger vehicles to enhance the straight line stability.

2.5.2 Suspension Kinematics

2.5.2.1 Wheel travel

The displacement of wheel center position in the vertical axis when the vehicle is in motion is defined as Wheel Travel. Further classification of wheel travel can be made by categorizing the direction of wheel travel i.e. Bump travel and Rebound travel.

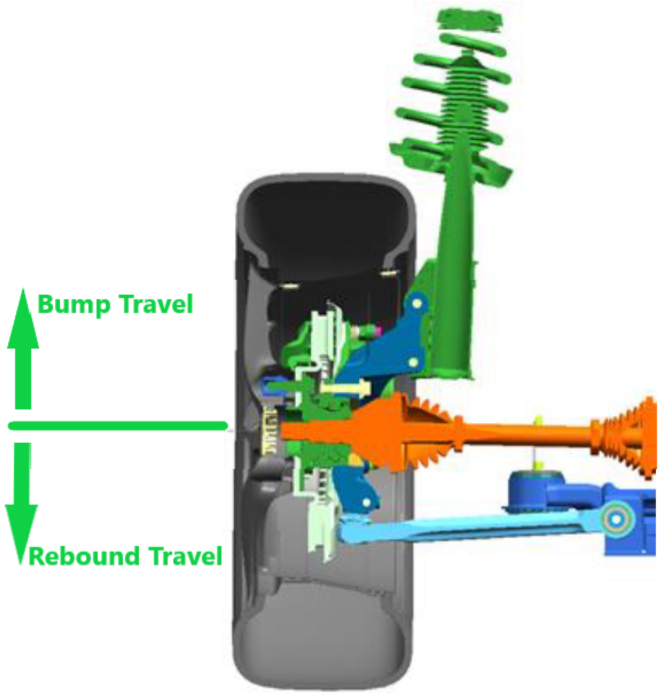


Figure 2.3: Bump Travel and Rebound Travel [3]

- **Bump travel:** The displacement of wheel center position from the design position in upward direction due to driving over road irregularities is defined as Bump travel. Bump travel is sometimes referred to as Jounce. Bump travel is considered to be positive because the the wheel center travels in positive Z direction, towards the chassis, compressing the spring-damper assembly.

- **Rebound travel:** The displacement of wheel center position from the design position in downward direction due to driving over road irregularities is defined as Rebound travel. Rebound travel is considered to be negative because the wheel center travels in negative Z direction, away from the chassis, elongating the spring-damper assembly.

2.5.2.2 Tie rod stroke

Displacement of the tie rod in the Y axis is defined as tie rod stroke. When the suspension system is in motion, tie rod stroke is important to determine the steering angle of the wheels. In the hardpoint list, PT 14 is the tie rod stroke which is defined by geometry in the steering rack.

2.6 Ackermann Criteria

Ackermann criteria is important aspect for steering geometry due to the fact that the two front wheels are steered to different angles because of the steering geometry. The center lines of inner front wheel (left wheel) and the outer front wheel (right wheel) are separated by a certain distance called Track width (W). The inner front wheel rotates at an angle (δ_i) and the outer front wheel rotates at an angle (δ_o). The steering angle of the wheels need to be different because of different positions the front wheels have to turn the vehicle. These wheel rotations influence the wheel envelopes in terms of swept volume of the wheel. Ackermann condition can be determined from the kinematic analysis of suspension [4].

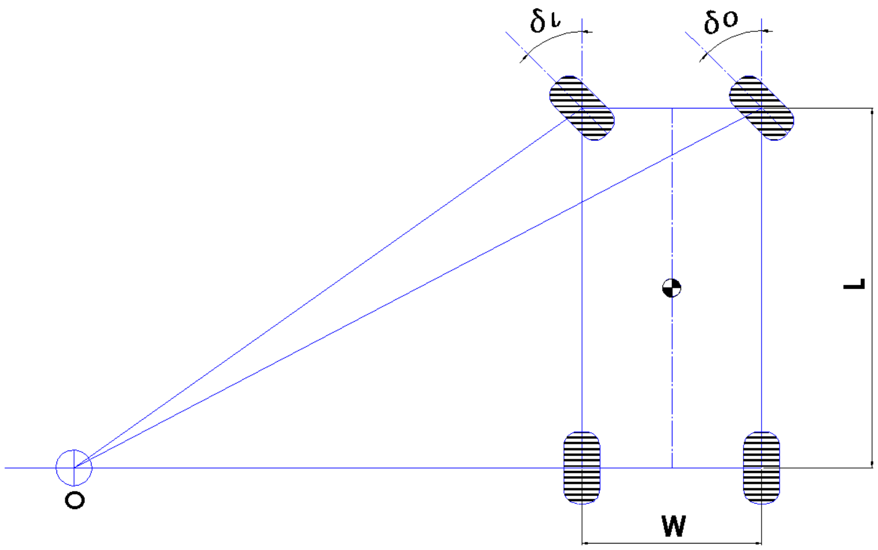


Figure 2.4: Ackermann Geometry for Front Axle Steering Leftwards

In the figure, O represents the point of intersection where all the rotation axes of the wheels meet. The equation to calculate the Ackermann geometry is given by:

$$\frac{1}{tan(\delta_o)} = \frac{1}{tan(\delta_i)} + \frac{W}{L} \tag{2.1}$$

2.7 Wheel

The wheel is commonly referred to combination a tyre and a rim and they are off-the-shelf products, selected directly from suppliers. Since a large number of tyre and rim combinations can be possible, the largest possible wheel size in the vehicle specification is chosen to design the wheel envelope. Understanding of tyre profile is paramount in development of accurate wheel envelope. The largest possible tyre and rim in the vehicle specification are selected based on the tyre designation as illustrated.

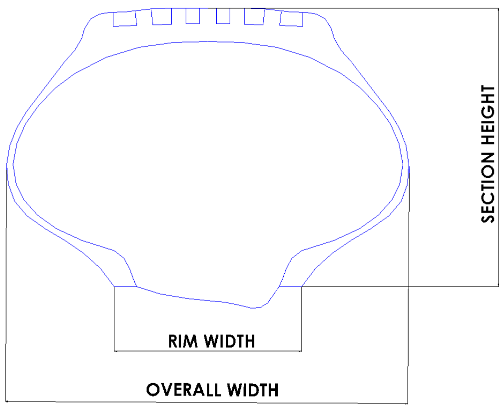


Figure 2.5: Section Width of Tyre

Figure 2.6: Tyre Profile

Image Source: *Merityre Specialists.* [5]

The markings on the tyre is explained as tabulated below.

255	Nominal section width (mm)
55	Nominal sidewall height to nominal section width (%)
R	Radial construction
16	Rim diameter (inch)

Table 2.3: Markings on the Tyre

2.8 Snow Chain

Snow chain is a legal requirement for many markets and need to be considered for the wheel envelope method. Snow chains are used to provide traction to the wheels when driving on low friction road surface. The chains are made of metal links which provide grip to the tyre when it starts to slip. Snow chains have to be considered for wheel envelope because it adds to the tyre section. Sometimes a snow socks may be used on which is quite similar to chains. These socks are thinner than the chains however, for development of an accurate wheel envelope these accessories must be taken into account.

Figure 2.7: Snow Chain and Snow Socks (Single sided) [6]

2.9 Geometric Tolerance Analysis

Geometric tolerance is important for wheel envelopes because the produced parts cannot be accurately manufactured and positioned according to the nominal dimensions. These tolerances are small when a single component is considered. However, for wheel envelopes, the stack up of tolerances of various components will influence the swept volume of the wheel. The strategy for establishing the total tolerances is important for the wheel envelope method.

Determination of tolerance stack up in the early design phase will enable the design engineer to quantify the maximum required stack up of tolerances and develop the envelope accordingly. Tolerance stack up can be determined by 2 methods: Arithmetic tolerance method and Statistical tolerance method.

- ***Arithmetic Tolerance:*** Arithmetic Tolerance is also known as **Straight Tolerance**. In this method, the largest possible tolerance stack up is calculated by considering the nominal dimension and the maximum tolerance on a given part. Similarly, the smallest possible tolerance stack up is calculated by considering the nominal dimension and the minimum tolerance on a given part.

- ***Statistical Tolerance:*** In this method, tolerance values are taken from tolerance specification. A normal distribution for these tolerance values would represent the deviation from the nominal. Normal distribution curve is the best representation of distribution of tolerances. In the figure, it can be seen that the tolerances predominantly fall between $\pm 1\sigma$ and the variation of tolerances is between $\pm 3\sigma$. Root Sum Square Method (RSS Method) is an example of statistical tolerance [7].

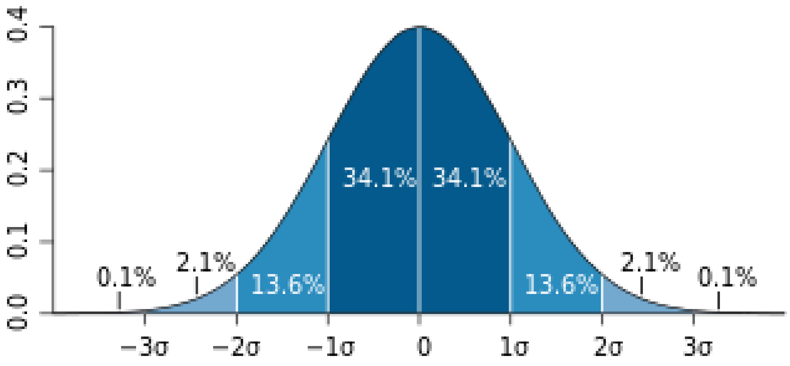


Figure 2.8: Normal Distribution of Tolerances [8]

In this thesis work, Root Sum Square Method (RSS Method) is used to determine the tolerance stack up in X, Y and Z directions respectively. The expression to calculate tolerance stack up using RSS method is:

$$S = \sqrt{\sum_{i=1}^{n}(n_1^2 + n_2^2 + ... + n_n^2)} \tag{2.2}$$

Where,
S = Tolerance stack up (in mm)
n_1, n_2,..., n_n = Individual tolerance from the components (in mm)

The results obtained by using equation 2.2 is inclusive of the spread of the tolerance values according to $\pm 3\sigma$ method.

2.9.1 Spring

The Spring is designed with a purpose of isolating the vertical disturbances coming into the vehicle and thereby enhance the ride quality and occupant comfort. Also, springs play a pivotal role in maintaining the contact between the wheel and the road surface. The spring displaces only along the vertical axis; hence, the positional tolerance along the Z axis is important.

2.9.2 Damper

Damper is a device used to absorb the shock forces coming onto the springs when the vehicle is in motion. The damper absorbs the shock impulses in the form of kinetic energy and dissipates it in the form of heat thereby providing the necessary damping of the wheel. Damper strut consists of a spring seat where the spring can be connected. Since the damper moves along the vertical axis when loaded, its important that the positional tolerance of spring seat is considered. The damper controls the bump and rebound travel and there are tolerances that will effect the wheel envelope.

2.9.3 Subframe Tolerance and Installation Tolerance

MacPherson suspension consists of one control arm which is connected to the knuckle through a ball joint and, to a sub-frame through two bushes (PT 3 and PT 4). During assembly, the local tolerance present in the control arm for proper positioning must be considered. Also, installation tolerance in the subframe will give the positional tolerance. These tolerance values can be determined from the 2D drawing of a control arm.

2.9.4 Tyre

Tyres are compounds of rubber which are manufactured in moulds. Surface tolerance exists because of the manufacturing process; also considering the wear of the moulds. During these processes it is quite difficult to maintain the nominal dimension and manufacture with extreme accuracy. Hence, the supplier suggests certain tolerance values for the tyre profile.

Maximum tolerances in the tyres are described as Maximum in service and, used for standard dimensions and tolerances for tyres as specified by ETRTO. All tyres with a certain size have to be within these standard dimensions and tolerances.Maximum in service dimensions and tolerances are therefore larger than each specific tyre. However, these tolerances vary according to the chosen tyre dimensions.

2.10 Compliance Analysis

Compliance in a component is due to displacement under load. When a loaded component undergoes certain displacement, the geometry of the component also changes. Suspension bushings, springs are some of the known components which can displace under load and thereby induce compliance in the system. stiffness in the bushings is selected to achieve the expected compliance. Compliance analysis is important aspect for wheel envelopes and can be categorized as:

- ***Longitudinal Compliance:*** Longitudinal compliance is important for the ride comfort and developed to suit the vehicle. It is introduced because of the longitudinal forces encountered during driving and braking. However, the braking forces are higher than the driving force. Also, longitudinal compliance can be induced in the suspension when the vehicle drives over irregular road surface.

- ***Lateral Compliance:*** Lateral compliance is introduced because of the lateral forces a vehicle will experience while driving in a corner. Both suspension and steering subsystems will be influenced from these forces.

In MacPherson suspension, forces act on the contact point of tyre and induce compliance in the bushings through a ball joint. Hence the displacement of bushings will in-turn introduce some amount of movements in the ball joint and wheel center.

2.10.1 Mechanical Snubbing

Elastomeric components like rubber bushings are used in automotive applications to sustain the overloading condition or to limit the displacement of the component and avoid contact with other parts. This concept of curtailing displacement and withstanding the excessive loading is called Snubbing. The figure below is a representation of force displacement curve for elastomers [9].

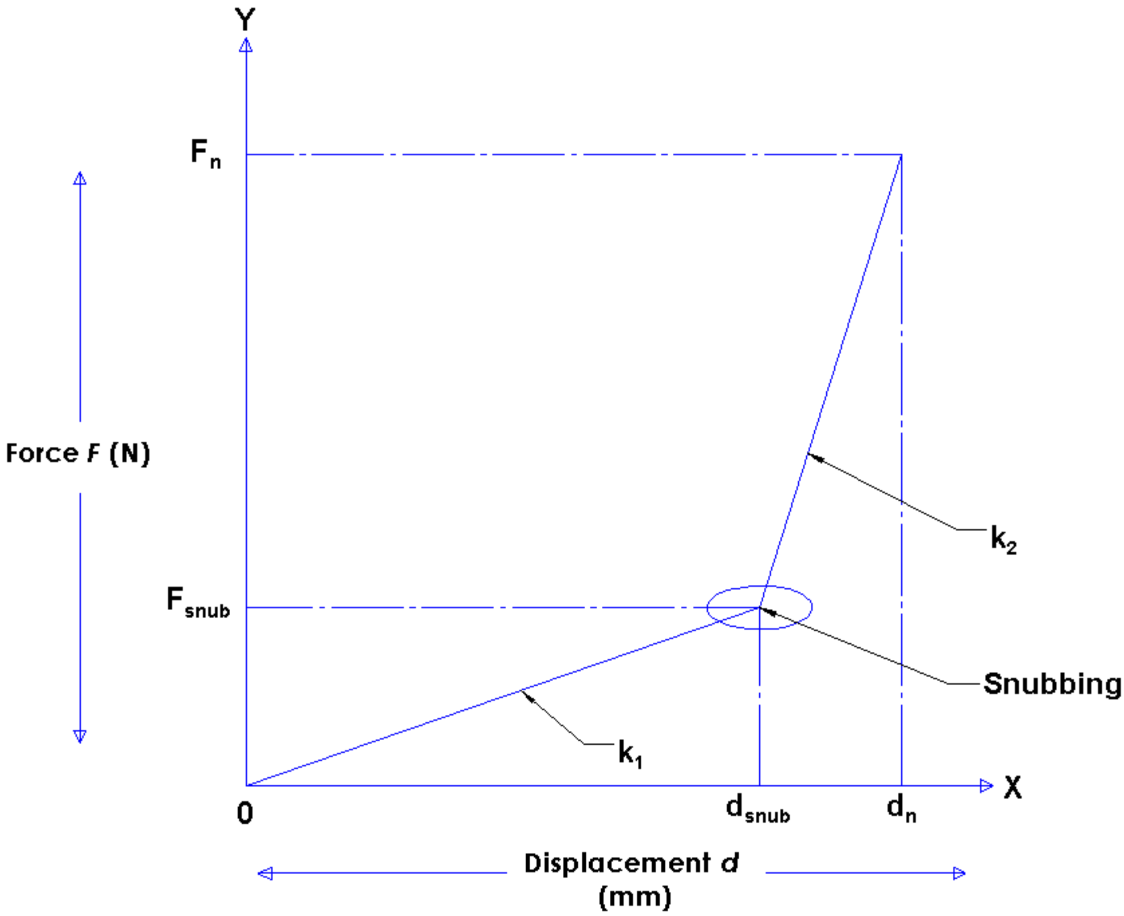


Figure 2.9: Snubbing in Elastomers

In the XY axis, **(0,0)** to **($\mathbf{d}_{snub}$,$\mathbf{F}_{snub}$)** represents the linear stiffness region and **($\mathbf{d}_{snub}$,$\mathbf{F}_{snub}$)** to **($\mathbf{d}_n$,$\mathbf{F}_n$)** represents the non-linear stiffness region. Hence, two different stiffness **($\mathbf{k}_1$)** and **($\mathbf{k}_2$)** are denoted. Bushing stiffness can be determined from bushing specifications.

The displacement in the linear region can be calculated by using the conventional force-displacement formula:

$$d = \frac{F}{k_1} \quad ; \quad \forall \ (0 \leq d \leq d_{snub}) \tag{2.3}$$

The displacement in the non-linear region can be calculated by:

$$d = \left(\frac{F - F_{snub}}{k_2} \right) + d_{snub} \quad ; \quad \forall \ (d_{snub} \leq d \leq d_n) \tag{2.4}$$

With equations 2.3 and 2.4, the displacements can be determined.

2.11 Steering Strategy

A vehicle may never be driven with a 100% rack travel and a 100% bump travel of wheel. Hence, a strategy is developed which helps in understanding and defining the relationship between steering rack travel required in the lateral axis for a certain wheel travel in the vertical axis. Steering strategy is based on the driving condition and the type of the vehicle (Sedan, SUV, Hatchback etc). Hence, each type of vehicle requires a unique steering strategy. The figure appended below is a depiction of generally represented steering strategy.

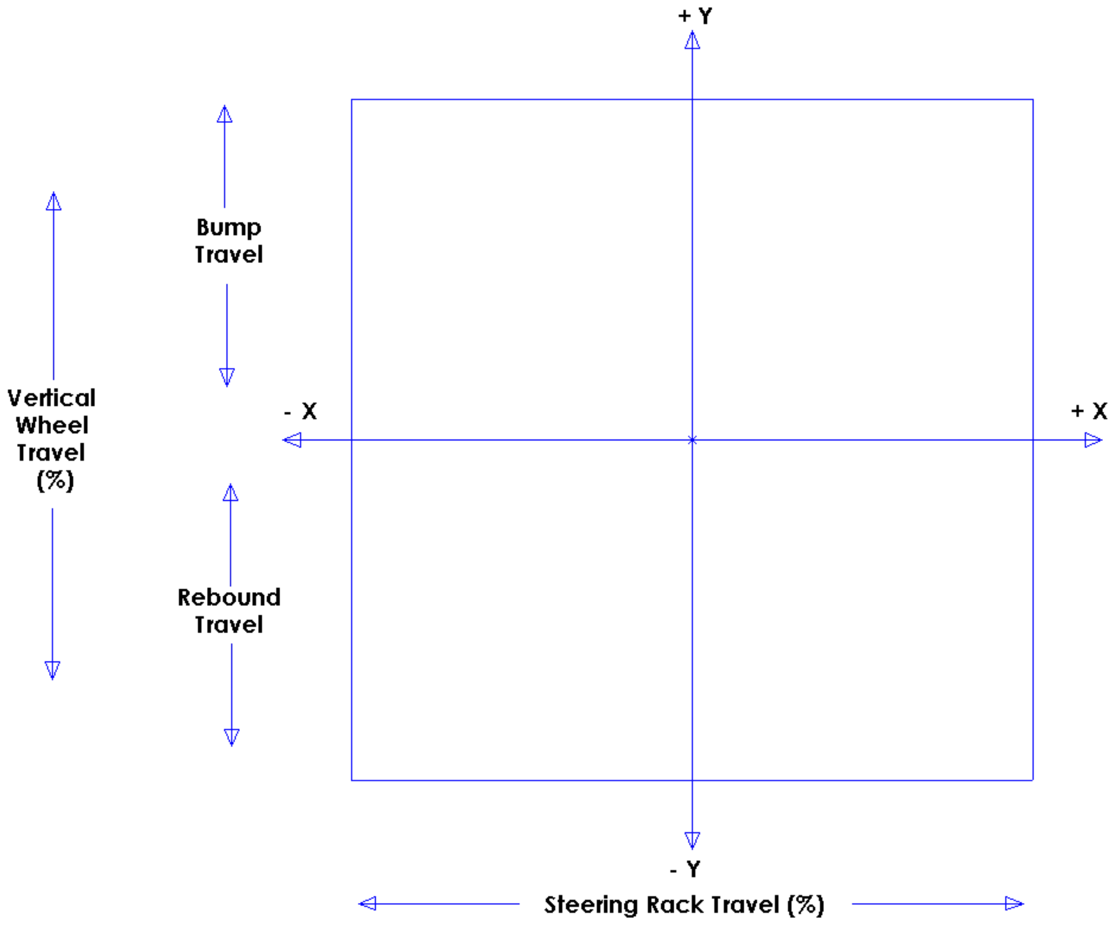


Figure 2.10: Representation of Maximum Wheel Travel and Maximum Steering Travel

The principle for a steering strategy is according to the figure 2.10. It is not possible to have a 100% of rack travel and a 100% of bump travel of the wheel at the same time. Hence, a steering strategy is created which depicts the practical scenario.

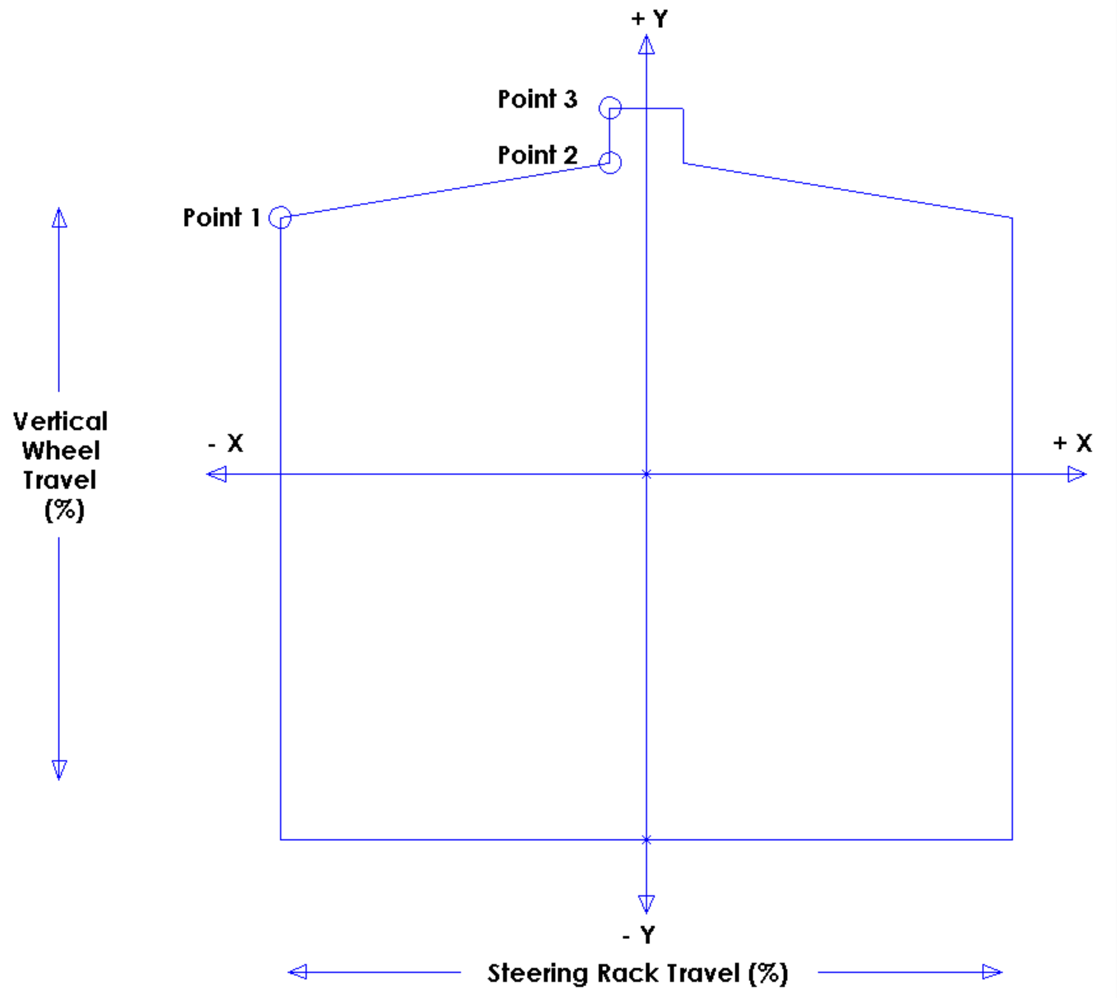


Figure 2.11: Ideal Representation of Steering Strategy

Steering strategy changes according to the vehicle type which implies that a company has to create it's own strategy either by physically driving the vehicle or through computer simulations. Hence, the steering strategy presented in this thesis work is generic in nature. Some companies experiment by driving a vehicle which consists of clay formations in the wheels houses to see how much the wheels are moving.

In the figure, certain points are marked with numbers which are important for the creation of steering strategy. They are elaborated as follows:

- ***Point 1:*** At this point, the steering rack is allowed to travel 100% while a maximum bump travel needs to be determined to suit the type of vehicle and the expected driving condition.

- ***Point 2:*** At this point, the steering rack travel is reduced whereas the vertical wheel travel is to be determined to suit the vehicle type and driving condition.

- ***Point 3:*** At this point, full wheel travel is possible. This condition may be present only when the vehicle is driving straight ahead.

Snow chains may be equipped while driving on snowy/slippery surfaces. The speed of the vehicle will be reduced. Considering these points, an appropriate percentage of rack travel and wheel travel can be estimated for steering strategy with a snow chain.

3
Methods

The thesis work emphasises on development of a wheel envelope methodology. The primary aspects required to improve the accuracy of the method are:

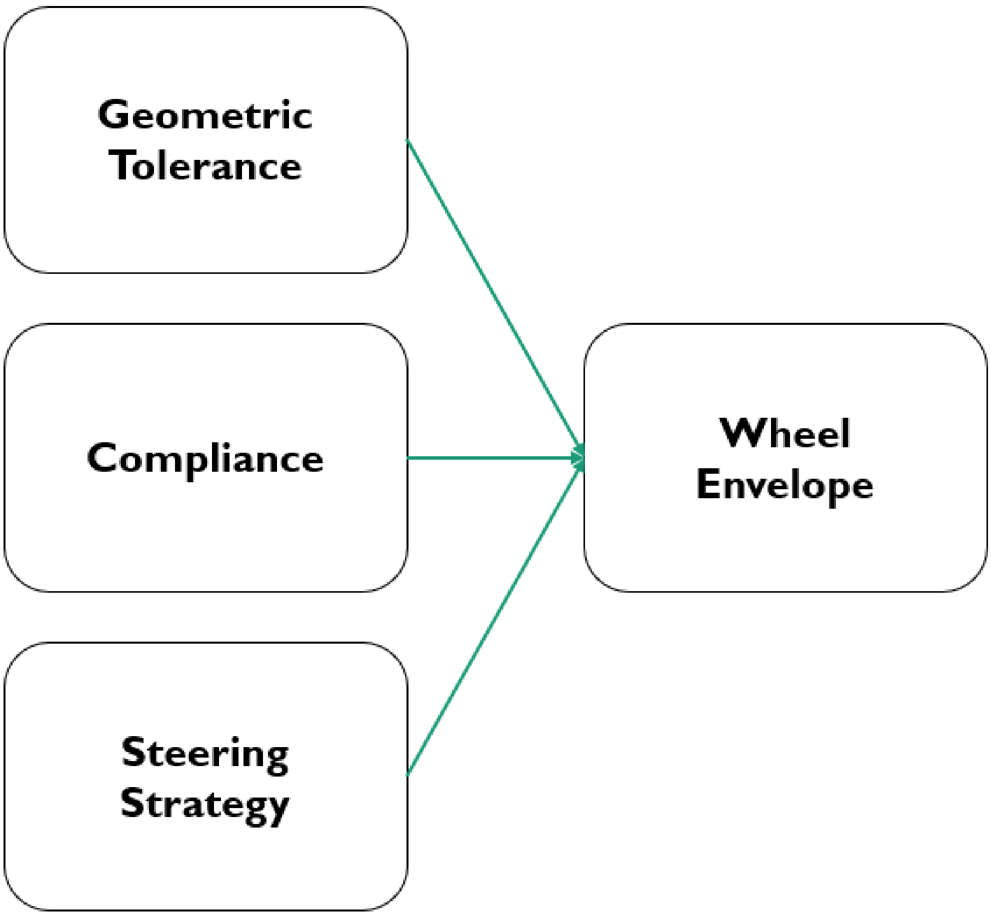


Figure 3.1: Wheel Envelope Methodology

3.1 Geometric Tolerance

Individual tolerances from the components of suspension, steering and the wheel contribute to the tolerance stack up in the wheel envelope. Each of these components have different tolerances and the approach to determine them are discussed in detail.

3.1.1 Spring

The primary tolerances from the spring which affect the wheel envelope method are the spring load at design length with the stiffness of the spring.

At design length, the spring is loaded with the design load (Kerb+2). Design load has some tolerance based on type of the vehicle and its principal design. Variation in design load will cause variation in the vehicle height i.e. heavier vehicles are closer to the ground and vice versa. Spring rate is a parameter which represents the stiffness chosen to suit the vehicle based on its characteristics.

The spring stiffness and design load at the design length have tolerances which have to be converted into the total tolerance from the spring. Different type of vehicles have different design load tolerance and different stiffness. Therefore, they must be investigated to determine the largest geometrical tolerance. The kerb weight, design load and the spring rate can be obtained from the 2D drawing. The table below contains the details of the above-mentioned parameters used in this project work.

Kerb Weight (kg)	995 - 1033
Design Load (N)	4935 ± 140
Spring Rate ($\mathrm{N\,mm^{-1}}$)	28 ± 3%

Table 3.1: Geometrical Tolerance from Spring

3.1.1.1 Tolerance calculation

- **Contribution from weight class:**
  - Difference between the kerb weight for the springs on all 4 wheels (in kg):

$$1033 - 995 = 38 \tag{3.1}$$

  - Difference between the kerb weight for spring per wheel (in kg):

$$\frac{38}{4} = 9.5 \tag{3.2}$$

  - Converting kerb weight difference per wheel from kg to N:

$$9.5 * 9.81 \approx \pm 93\,\mathrm{N} \tag{3.3}$$

  - From Hooke's law,

$$\delta = \frac{F}{k} \tag{3.4}$$

    where:

    - k = Spring rate (in $\mathrm{N\,mm^{-1}}$)
    - F = Force (in N)
    - δ = displacement (in mm)

$$\delta_1 = \frac{\pm 93}{28} \approx \pm 3\,\mathrm{mm} \tag{3.5}$$

- **Contribution from design load:**
  From table 3.1, the tolerance for design load is $\pm 140\,\mathrm{N}$. Substituting this tolerance value for F in equation 3.4, the tolerance contribution can be calculated as shown below:

$$\delta_2 = \frac{\pm 140}{28} = \pm 5\,\mathrm{mm} \tag{3.6}$$

  The total tolerance from the spring in Z direction is the sum of tolerance contribution from weight class and tolerance contribution from design load.

$$\delta = \delta_1 + \delta_2$$

$$\delta = (\pm 3\,\mathrm{mm}) + (\pm 5\,\mathrm{mm})$$

$$\delta = \pm 8\,\mathrm{mm} \tag{3.7}$$

Spring stiffness has a tolerance of 3% which is small and may/may not be considered for the tolerance calculation.

3.1.2 Damper

Damper and Spring are connected by means of a spring seat on which the spring rests. The position of the spring seat contributes a value of $\pm 1.5\,\mathrm{mm}$ to the tolerance stack up in Z direction.

3.1.3 Top Mount

The top mount is a mounting point for the suspension system which means that the component is part of assembly. Hence, some internal tolerance is already present in the mount for facilitating the assembly. Apart from that, a positional tolerance also needs to be considered. The internal tolerance is approximately $\pm 0.5\,\mathrm{mm}$ and the positional tolerance is approximately $\pm 3\,\mathrm{mm}$. Hence, the total contribution from the top mount to the tolerance stack up is approximately $\pm 3.5\,\mathrm{mm}$ in Z direction.

3.1.4 Control Arm

The control arm is a connection between the wheel and the sub-frame of the chassis. Therefore, some amount of internal tolerance is present in the component to facilitate the assembly. The contribution from the control arm to the tolerance stack up is approximately $\pm 0.5\,\mathrm{mm}$ in X and Y direction.

3.1.5 Installation Tolerances

The estimated installation tolerance from the sub-frame is approximately $\pm 3\,\mathrm{mm}$ in X and Y directions. The tolerances in the bushings and control arm add up to the mentioned value.

3.1.6 Tyre

The tyre size chosen in this project was 235/55 R 18. From the passenger car tyre manual by ETRTO, for the chosen tyre size, the actual section width is 235 mm and the design section width is 245 mm. The tyre's section width is influenced by the width of the rim upon which it is mounted. The tyre can be mounted on a rim which may be narrower or wider than the measuring rim. To accommodate these variations, the actual section width of a new tyre is usually 4% smaller than the design section width. The design overall diameter is 715 mm.

The tolerance is given by maximum-in-service overall width and maximum-in-service overall diameter which are 255 mm and 725 mm respectively. It can be observed that the tolerance for section width and overall diameter is ±10 mm each.

3.1.7 Snow Chain

SAE Class S snow chain is used on vehicles with restricted wheel well clearance. Generally, the thickness of a snow chain is 9 mm. The snow chains are mounted over the tyres and therefore consumes some space whenever used.

3.2 Compliance

3.2.1 Longitudinal Compliance

Longitudinal compliance in the bushings arise due to longitudinal forces (braking force or driving force) acting on the wheel center position. Since the bushings and the wheel center are connected through a knuckle, ball joint and a control arm, displacement at the ball joint is determined initially. Displacement in the ball joint is later translated into the wheel center position.

For longitudinal compliance, the influence of braking force is higher than the driving force. The front bush (PT 3) offers high stiffness in the Y direction; hence, the low displacement. Whereas it has lower stiffness in the X direction. The stiffness of the rear bush (PT 4) in the Y direction is very low which enables the bush to displace in the Y direction. The displacement in the bushings will induce displacement in the wheel center through the ball joint.

Assumption: The control arm is rigidly connected between the ball joint and the rubber bushings.

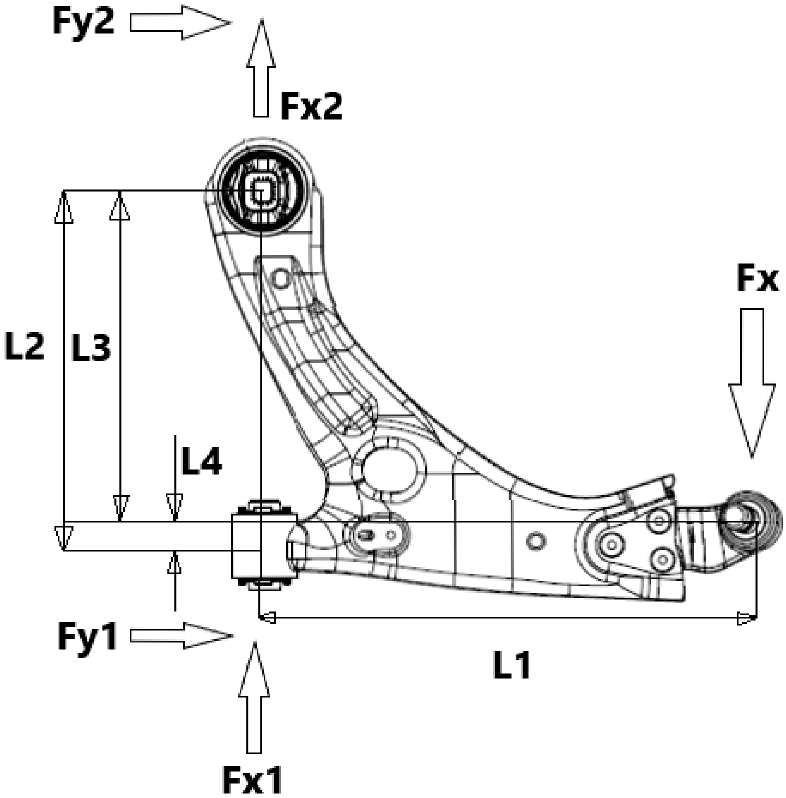


Figure 3.2: Longitudinal Compliance

The symbols used in the figure 3.2 can be explained as:

Fx	Longitudinal Force acting on PT 6 (N)
Fx_1	Longitudinal Force acting on PT 3 (N)
Fy_1	Lateral Force acting on PT 3 (N)
Fx_2	Longitudinal Force acting on PT 4 (N)
Fy_2	Lateral Force acting on PT 4 (N)
L_1	Horizontal distance between centre points of PT 6 and PT 3 (mm)
L_2	Vertical distance between centre points of PT 3 and PT 4 (mm)
L_3	Vertical distance between centre points of PT 6 and PT 4 (mm)
L_4	Vertical distance between centre points of PT 6 and PT 3 (mm)

Table 3.2: Definition of Symbols in Figure 3.2

A free body diagram can be drawn to represent the forces and moment acting on the control arm. In figure 3.2, it can be observed that an offset distance L_4 exists between the ball joint (PT 6) and the front bush (PT 3) such that the sum of L_3 and L_4 is equal to L_2.

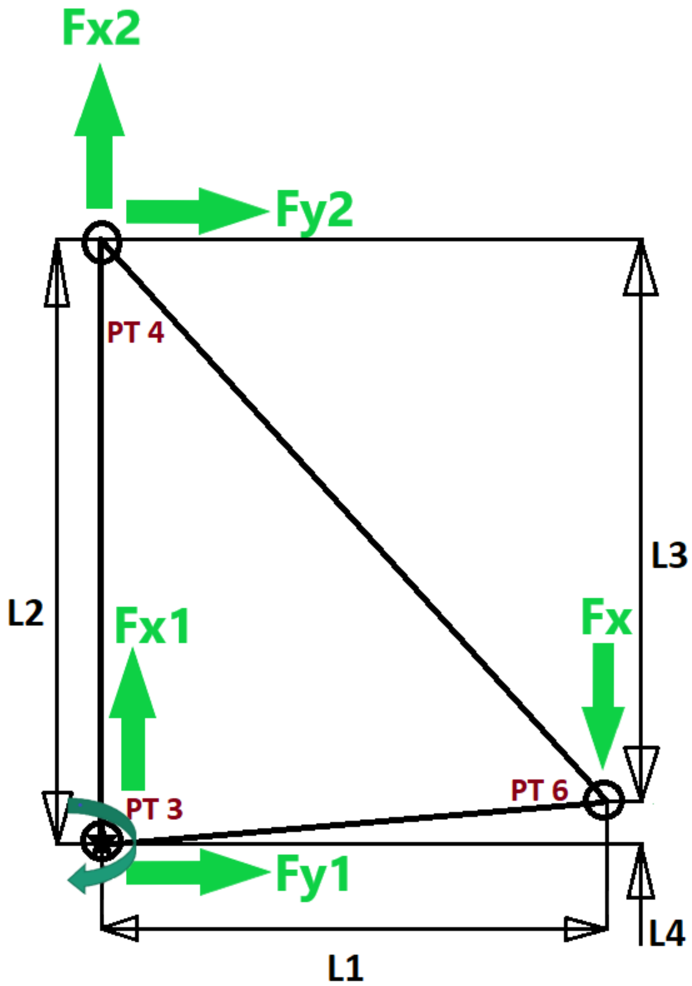


Figure 3.3: Free Body Diagram for Longitudinal Compliance

The equilibrium equations can be written as:

- Summation of forces along X axis:

$$\Sigma Fx = 0$$

$$Fx = Fx_1 + Fx_2 \tag{3.8}$$

Longitudinal force Fx acts on the bushings PT 3 and PT 4. Hence, Fx splits into Fx_1 and Fx_2.

Assumption: Fx_1 and Fx_2 are equal in magnitude which implies that:

$$Fx_1 = Fx_2. \tag{3.9}$$

- Summation of forces along Y axis:

$$\Sigma Fy = 0$$

$$Fy = Fy_1 + Fy_2 \tag{3.10}$$

Since lateral force Fy acting on the system is considered to be zero:

$$Fy_1 = -Fy_2 \tag{3.11}$$

- Moment across PT 3

$$(Fx * L_1) + (Fy_2 * L_2) = 0$$

Fx is the net longitudinal force acting on PT 6 but, Fy_2 is unknown. Hence, Fy_2 can be determined by rearranging the equation.

$$(Fy_2 * L_2) = -(Fx * L_1)$$

$$Fy_2 = -Fx * \frac{L_1}{L_2} \tag{3.12}$$

The equations derived above aids in determining the force across the bushings. The displacement in the bushings will induce displacement in the ball joint. To calculate the displacement in the bushings, the stiffness of the bushing must be determined.

The bushing stiffness values and that the bushing can undergo are tabulated as follows:

Axis	Bushing	$k_{nominal}(\mathrm{N\,mm^{-1}})$
X	Front Bush	1300
	Rear Bush	1620
Y	Front Bush	20000
	Rear Bush	240

Table 3.3: Bushing Parameters

The derivation of formula to calculate the total bushing stiffness in the x direction is:

$$\frac{1}{kx_{total}} = \frac{1}{kx_1} + \frac{1}{kx_2} \tag{3.13}$$

Taking LCM:

$$\frac{1}{kx_{total}} = \frac{kx_2 + kx_1}{kx_1 * kx_2}$$

Taking inverse on both LHS & RHS:

$$kx_{total} = \frac{kx_1 * kx_2}{kx_1 + kx_2} \tag{3.14}$$

Using the equation 3.14 and substituting the stiffness values in the equation, the total bushing stiffness in the X direction can be calculated.

$$kx_{total} = \frac{1300 * 1620}{1300 + 1620}$$

The total stiffness of the individual bushes PT 3 and PT 4 in X direction is:

$$kx_{total} = 721\,\mathrm{N\,mm^{-1}}$$

Total displacement in the ball joint (PT 6) can be calculated using the equation:

$$Dx = dx + dy(\mathrm{mm}) \tag{3.15}$$

3.2.2 Lateral Compliance

Lateral compliance in the bushings arise due to lateral forces acting on the wheel center position. The front bush (PT 3) offers high radial stiffness due to which the displacement is low. However, the radial stiffness of the rear bush (PT 4) is very low which enables the bush to displace in the Y direction. The displacement in the bushings will induce displacement in the wheel center position. Hence, lateral compliance is required to understand the influence of the forces on the system.

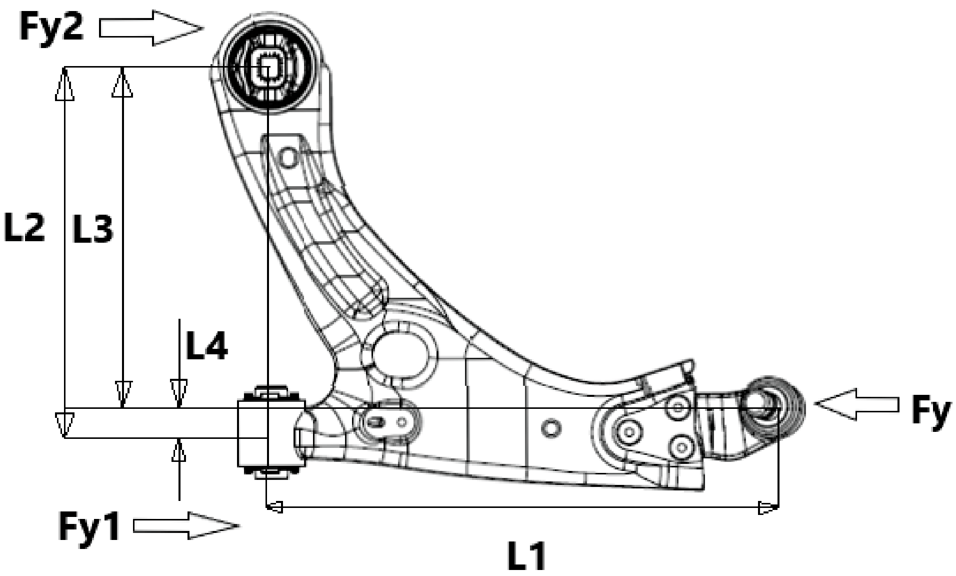


Figure 3.4: Lateral Compliance

The symbols used in the figure 3.2 can be explained as:

Fy	Lateral Force acting on PT 6 (N)
Fy_1	Lateral Force acting on PT 3 (N)
Fy_2	Lateral Force acting on PT 4 (N)
L_1	Horizontal distance between centre points of PT 6 and PT 3 (mm)
L_2	Vertical distance between centre points of PT 3 and PT 4 (mm)
L_3	Vertical distance between centre points of PT 6 and PT 4 (mm)
L_4	Vertical distance between centre points of PT 6 and PT 3 (mm)

Table 3.4: Definition of Symbols in Figure 3.4

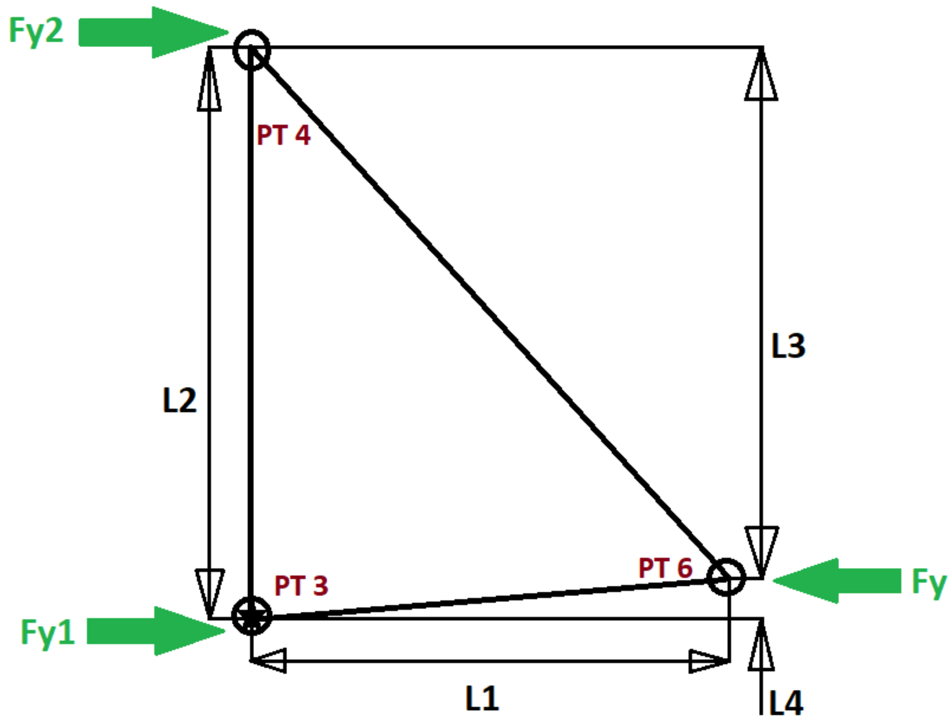


Figure 3.5: Free Body Diagram for Lateral Compliance

The equilibrium equations can be written as:

- Summation of forces along Y axis:

$$\Sigma Fy = 0$$

$$Fy = Fy_1 + Fy_2 \tag{3.16}$$

- Calculation of Fy_1 and Fy_2

  The force in the front bush (PT 3) and rear bush (PT 4) are different because of the control arm's geometry. From the figure 3.5, it can be observed that there exists an offset in vertical distance between the ball joint (PT 6) and front bush (PT 3). The force acting on PT 3 can be denoted as Fy_1 and the force acting on PT 4 can be denoted as Fy_2. Hence, by taking the ratio of vertical distances between these points, Fy_1 and Fy_2 can be obtained as shown below.

  – Force acting on PT 3:

$$Fy_1 = Fy * \frac{L_3}{L_2} \tag{3.17}$$

  – Force acting on PT 4:

$$Fy_2 = Fy * \frac{L_4}{L_2} \tag{3.18}$$

3.3 Steering Strategy

Parameters for creating a steering strategy are wheel travel in vertical direction and rack travel in lateral direction. Hence, knowledge of bump travel, rebound travel and rack travel is necessary. Since these values change for each vehicle type based on the design and driving condition, the values are converted into percentage. This provides flexibility for using the same method on various vehicle types. The table below contains the parameters and the converted percentage values and the general method for creating steering strategy is depicted in the picture below.

Parameter	Value & SI Unit	Axis
Bump travel	100 %	+ Z
Rebound travel	100 %	- Z
Rack travel	100 %	± Y

Table 3.5: Design Parameters for Steering Strategy

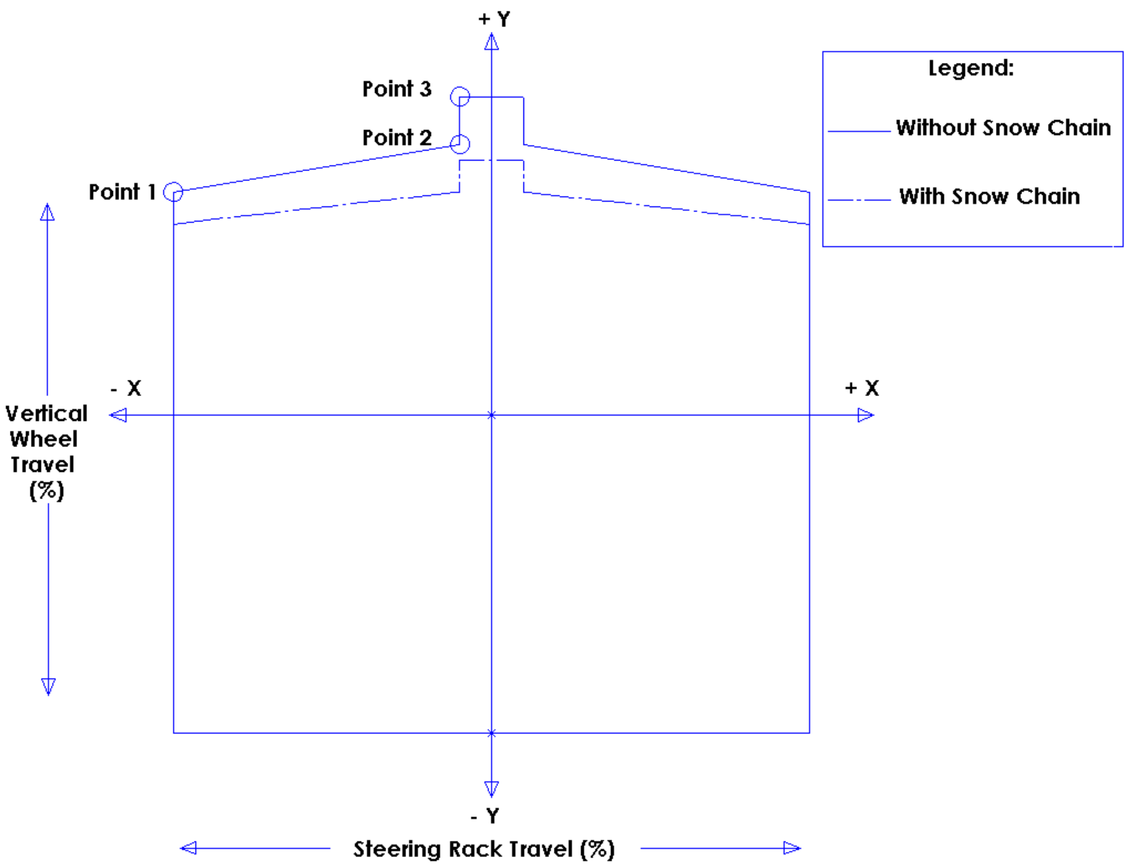


Figure 3.6: Description of Steering Strategy

The steering strategy presented here consists a plot with snow chain and without snow chain. The points 1, 2 and 3 are to be determined based on the wheel travel of the vehicle. Hence, the strategy must be created by considering the type of the vehicle and the driving condition.

When snow chains are equipped, the speed of the vehicle is reduced; also, wheel travel in +Y direction is reduced. Therefore the reduction in steering strategy is observed. For different type of vehicles, the steering strategy also changes. For instance, a SUV usually has more wheel travel for a certain rack travel when compared to a sedan. Hence the strategy has to be created as the vehicle type changes.

4

Results

4.1 Geometric Tolerance

Component	Axis	Tolerance
PT 6	X	± 3 mm
	Y	± 3 mm
Control Arm	X	± 0.5 mm
	Y	± 0.5 mm
Spring	Z	± 3.3 mm
	Z	± 5 mm
Damper	Z	± 1.5 mm
Top mount	Z	± 0.5 mm
	Z	± 3 mm

Table 4.1: Geometrical Tolerance Contribution from Components

From the Theory, Method and table 4.1, the geometric tolerance in the X, Y and Z axes can be calculated as follows.

- ***Geometric Tolerance in X axis:*** The contributors of tolerance stack up in X axis are the ball joint and the control arm. Therefore stack up of tolerance in X axis is:

$$S_x = \sqrt{(\pm 3)^2 + (\pm 0.5)^2}$$
$$S_x = \pm 3.04\,\text{mm}$$

- ***Geometric Tolerance in Y axis:*** The contributors of tolerance stack up in Y axis are the ball joint and the control arm. Therefore stack up of tolerance in Y axis is:

$$S_y = \sqrt{(\pm 3)^2 + (\pm 0.5)^2}$$
$$S_y = \pm 3.04\,\text{mm}$$

- ***Geometric Tolerance in Z axis:*** The tolerances in the Z axis contribute to vehicle height and the contributors are the spring, damper and the top mount. Therefore, stack up of tolerance in Z axis is:

$$S_z = \sqrt{(\pm 3.33)^2 + (\pm 5)^2 + (\pm 1.5)^2 + (\pm 3)^2 + (\pm 0.5)^2}$$

$$S_z = 6.90\,\text{mm}$$

4.2 Compliance

4.2.1 Longitudinal Compliance

A method was established to determine the displacement in the ball joint due to the compliance of the bushings. The figure below depicts the calculation method to determine the compliance. Assuming that the maximum longitudinal force is 10 kN, the displacement in the ball joint is approximately 14.52 mm. The sum displacements of PT 3 and PT 4 in X direction and the displacement of PT 4 in Y direction are considered for calculating the displacement at PT 6.

PT 6	PT3	PT4	PT3	PT4								PT6
N	N	N	N	N	mm	mm	mm	N/mm	N/mm	mm	mm	mm
Fx	Fx1	Fx2	Fy1	Fy2	L1	L2	L1/L2	Kx_total	Ky	dx	dy	Dx
500	250	250	-694	694	397	286	1.39	721	800	0.35	0.87	1.21
1000	500	500	-1388	1388	397	286	1.39	721	800	0.69	1.74	2.43
1500	750	750	-2082	2082	397	286	1.39	721	800	1.04	2.60	3.64
2000	1000	1000	-2776	2776	397	286	1.39	721	800	1.39	3.47	4.86
2500	1250	1250	-3470	3470	397	286	1.39	721	800	1.73	4.34	6.07
3000	1500	1500	-4164	4164	397	286	1.39	721	800	2.08	5.21	7.29
3500	1750	1750	-4858	4858	397	286	1.39	721	800	2.43	6.07	8.50
4000	2000	2000	-5552	5552	397	286	1.39	721	5000	2.77	6.55	9.32
4500	2250	2250	-6247	6247	397	286	1.39	721	5000	3.12	6.69	9.81
5000	2500	2500	-6941	6941	397	286	1.39	721	5000	3.47	6.83	10.29
6000	3000	3000	-8329	8329	397	286	1.39	721	5000	4.16	7.11	11.27
7000	3500	3500	-9717	9717	397	286	1.39	721	5000	4.85	7.38	12.24
8000	4000	4000	-11105	11105	397	286	1.39	1000	5000	5.30	7.66	12.96
9000	4500	4500	-12493	12493	397	286	1.39	1000	5000	5.80	7.94	13.74
10000	5000	5000	-13881	13881	397	286	1.39	1000	5000	6.30	8.22	14.52

Figure 4.1: Calculation of Longitudinal Compliance

4.2.2 Lateral Compliance

A method was established to determine the displacement in PT 3 and PT 4 due to compliance of the bushings. The figure below depicts the calculation method to determine the lateral compliance. Due to the geometry of control arm, magnitude forces acting on PT 3 are higher than the magnitude of forces on PT 4. Hence, the lateral compliance can be assumed to be approximately equal to the radial compliance of PT 3 in this specific scenario.

PT 6	PT3	PT4							PT3	PT4	PT3	PT4
N	N	N	mm	mm	mm	mm	Ratio	Ratio	N/mm	N/mm	mm	mm
Fy	Fy1	Fy2	L1	L2	L3	L4	L3/L2	L4/L2	Ky1	Ky2	Dy1	Dy2
500	470.28	29.72	397	286	269	17	0.94	0.06	17500	800	0.03	0.04
1000	940.56	59.44	397	286	269	17	0.94	0.06	17500	800	0.05	0.07
1500	1410.84	89.16	397	286	269	17	0.94	0.06	17500	800	0.08	0.11
2000	1881.12	118.88	397	286	269	17	0.94	0.06	17500	800	0.11	0.15
2500	2351.40	148.60	397	286	269	17	0.94	0.06	17500	800	0.13	0.19
3000	2821.68	178.32	397	286	269	17	0.94	0.06	17500	800	0.16	0.22
3500	3291.96	208.04	397	286	269	17	0.94	0.06	17500	800	0.19	0.26
4000	3762.24	237.76	397	286	269	17	0.94	0.06	17500	800	0.21	0.30
4500	4232.52	267.48	397	286	269	17	0.94	0.06	17500	800	0.24	0.33
5000	4702.80	297.20	397	286	269	17	0.94	0.06	17500	800	0.27	0.37
6000	5643.36	356.64	397	286	269	17	0.94	0.06	17500	800	0.32	0.45
7000	6583.92	416.08	397	286	269	17	0.94	0.06	40000	800	0.39	0.52
8000	7524.48	475.52	397	286	269	17	0.94	0.06	40000	800	0.41	0.59
9000	8465.03	534.97	397	286	269	17	0.94	0.06	40000	800	0.44	0.67
10000	9405.59	594.41	397	286	269	17	0.94	0.06	40000	800	0.46	0.74

Figure 4.2: Calculation of Lateral Compliance

5
Conclusion

The following conclusions can be made from the work carried out in this project.

- The parameters influencing the wheel envelope were listed and detailed study on how they would affect the development of a method for developing a wheel envelope was conducted.

- A study on geometrical tolerance was conducted by considering the geometry of different components from the steering system, suspension system and the wheel. The tolerance of the components were determined and their contribution to the stack up was determined. A method was established to compute the tolerance stack up.

- Compliance in the bushings was studied and its influence on the wheel center position was analysed. A method to calculate the longitudinal compliance and lateral compliance was proposed by considering the forces acting on the system, geometry of the components and the stiffness of the rubber bushings. It can be inferred that major contribution to wheel envelope is from the longitudinal compliance and minor contribution is from the lateral compliance due to the level of compliance.

- A method to create the Steering strategy was established. The steering strategy is to be created based on type of the vehicle and the driving condition. Steering strategy changes when snow chains are equipped. Since the speed of the vehicle is reduced, the steering strategy also reduces.

6
Future Work

One of the limitation of this project work was the in-depth study of the tolerance for various components; however the focus was on the method and its establishment. Hence, detailed study of tolerance chain will be beneficial for improving the accuracy of the wheel envelope.

This project work focused on the developing a wheel envelope method for a system consisting of MacPherson suspension in the front. It would be interesting to develop a method for other type of suspension systems available today.

The project work focused on developing a wheel envelope method for the front wheel. Hence, a method to compute the geometrical tolerance stack up, the compliance and the steering strategy for the MacPherson suspension on the front wheel was developed. The wheel envelope method for the rear wheels needs to be developed.

The entire focus of this thesis work was on developing a method. Using this method and exact details of the vehicle, an accurate wheel envelope can be created.

Bibliography

[1] *Bosch Automotive Handbook, 9th edition.* Contents: Chassis Systems, Robert Bosch GmbH, 2014, ISBN: 978-1-119-03294-6.

[2] Henrik Andersson. *Simulation and Validation of Tire Deformation under Certain Load Cases*. ISSN: 1402 - 1617, 2006.

[3] Ingemar Johansson, *Lecture Notes, Suspension System*, 2018.

[4] Bengt Jacobson et.al, *Vehicle Dynamics Compendium*, 2018.

[5] Merityre Specialists, `https://www.merityre.co.uk/tyres/introduction-to-tyre-markings`

[6] Snowchains and Snowsocks technical information,
`https://www.snowchains.com/car-snow-chains/thule-konig-zip-ultra-9mm`
`https://www.snowchains.com/snow-socks/fit-and-go-suv-sox`

[7] Romeo Bardini, Torsten Bertram, Christian Hörsken, O. Lange and Marc Torlo. *Vehicle Modeling and Simulation in the Duisburg Mechatronics Laboratory*, 1999.

[8] Standard Distribution of Tolerances,
`https://en.wikipedia.org/wiki/Standard_deviation`

[9] Sudhir Kaul. *Multi-Degree-of-Freedom Modelling of Mechanical Snubbing Systems*, Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference 2011, DETC2011-47144.