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# Wireless charging using a separate third winding for reactive power supply

Master's thesis in Energy and Environment

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Gothenburg, Sweden 2015

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## Abstract

In this thesis, three different models of wireless charging systems were investigated theoretically and experimentally. The first model that was evaluated was a basic model of a two winding wireless charging system with a capacitor connected in series with each of the coils. An auxiliary winding was added on top of the transmitter coil in order to provide the reactive power and two different topologies of three winding wireless transfer systems were compared to a basic model. The first three winding model had a capacitor connected in series in the receiver loop and in the auxiliary loop, while the last model of a wireless charging system had a capacitor in every loop. In order to predict the behaviour of each wireless charging system, a simulation in Matlab/Simulink was made, as well as the Bode plot for the transfer function of each system. After some practical measurements, it was found that the best overall efficiency was for a basic two winding model, which was 64%. The three winding model with two capacitors had 57% efficiency, while the model with three capacitors had 61% efficiency. These values are rather low compared to the usual values of the efficiency for a wireless charging system for low power, and the main reason is that there were high copper losses, because Litz wires were not used in the experiment.

Index terms: wireless charging; third winding; mutual inductance; reactive power; resonant frequency

## Acknowledgements

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Ivan Šalković

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## Nomenclature

| Symbol | Explanation | Unit |
|---|---|---|
| $V$ | Voltage | V |
| $I$ | Current | A |
| $P$ | Power | W |
| $L$ | Inductance | H |
| $C$ | Capacitance | F |
| $R$ | Resistance | Ω |
| $M$ | Mutual inductance | H |
| $N$ | Number of turns | – |
| $k$ | Coupling coefficient | – |
| $\omega$ | Angular frequency | Rad/s |

# 1 Introduction

## 1.1 Problem background

In the last few decades, the world is facing the consequences of climate changes which are mainly caused by increased $CO_2$ emissions. On a global level, the biggest share in producing $CO_2$ emissions are fossil-fuel power plants and transport sector. To reduce the $CO_2$ emissions from power plants, more wind and solar installed capacities are needed. Inspired by the unstable market of oil and the goals that are set for environmental protection in [1], electric vehicles have become a very important subject of research in most EU countries.

Advantages of vehicles that use internal combustion engine compared to electric vehicles, are greater range, shorter refueling time and highly developed network of gas stations [2]. So far, the charging stations for electric vehicles are mostly using one of the four available connectors [3] that is connected to the power grid with a cable. In order to alleviate the problem of an universal connector for the chargers and achieve greater user safety, a wireless power transfer can be used as a new method to charge the battery of an electric vehicle.

One of the reasons why to choose wireless charging technique instead of a usual conductive charger is that it can be fully autonomous, which means, as soon as the electric vehicle is properly parked over a transmitter coil, a charging process may begin without any human touch. This new charging technique provides a new experience to the driver and according to [4] it is even possible to transfer 100 kW power at 85% transmission efficiency across a 20 cm air gap between the road and the OLEV bus underbody that operates in Gumi, South Korea.

Another advantage of wireless charging is that it has a wide range of output power, so it can also be used as a technique to charge the batteries of mobile phones, TVs, computers and medical devices. To learn how the wireless charging principle works, it is advisable to start with the lower output power and then apply the acquired knowledge to a higher power level which is needed for batteries of electric vehicles.

## 1.2 Previous work

A high power inductive charger is available for use at the Department of Energy and Environment at Chalmers University of Technology which was previously made by 3rd year students [5] and it was used for charging the batteries of an electric go-cart over an air gap. According to [6], the coils of a basic two winding model were evaluated theoretically and by doing FEM analysis, as well as the system with three coils. For the system with an auxiliary coil, it was found that the best position for the auxiliary winding is to be close to the transmitter coil, therefore this premise will be used in this master thesis.

## 1.3 Purpose

The purpose of this Master's thesis is to experimentally verify a wireless charging concept using an auxiliary winding for the reactive power supply. The overall efficiency, amount of copper as well as the losses in the power electronic devices in a three winding system will be compared to a conventional system with two coils.

## 1.4 Layout

The work on this thesis begins with making a model of a two winding wireless charging system with two capacitors in Matlab/Simulink and after that two models with third winding will be made. In the first model there will be capacitors in the auxiliary and in the receiver circuit and the second model will contain capacitors in all three loops. For these three models, phasor analysis will be made, as well as a parametric comparison of currents and transfer functions. Before starting with the laboratory work, a transfer function of each wireless charging system will be determined in order to predict the behaviour of the system.

The main part of this thesis are measurements in the laboratory. Firstly, the self-inductance and the mutual inductance of the coils that are given in [7] and [8] will be measured for different air gap and different misalignment. Also, a script that was previously made in Matlab will be used for calculating the currents in every loop. At the end, the losses in the power semiconductor devices, amount of copper, copper losses and the overall efficiency for each system will be calculated, in order to make a comparison for all three systems.

# 2 Theory

In this chapter, theoretical background will be described that was used in this master thesis work.

## 2.1 Electromagnetism

Faraday's law of electromagnetic induction is considered to be one of the most important discoveries in the history of science which led to the rapid development of electrical engineering as it is the working principle of electric motors, generators and transformers. The Maxwell-Faraday equation, which is a generalization of Faraday's law, can be defined as

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{2.1}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic field and $\nabla \times$ is the curl of a vector field. Equation (2.1) tells us that a time-varying magnetic field is a source of an electric field and vice versa. Faraday's law of electromagnetic induction can be also given as

$$\varepsilon = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{2.2}$$

where $\varepsilon$ is the electromotive force (emf) and $\Phi$ is the magnetic flux. Equation (2.2) shows that emf is equal to the negative of the rate of change of the magnetic flux[9]. An expression for the magnetic flux can be written in the form of a surface integral

$$\Phi = \int_S \boldsymbol{B} \cdot \boldsymbol{n} \, \mathrm{d}S \tag{2.3}$$

where d$S$ is an infinitesimal part of the surface given in the Fig. 2.1. Vector $\boldsymbol{n}$ is a unit normal and $C$ is the contour around the given surface $S$.

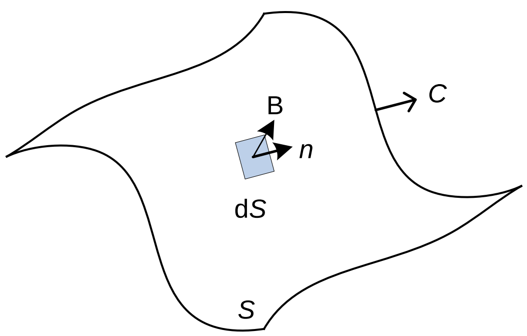


*Figure 2.1: Surface S bounded by the closed contour C*

### 2.1.1 Self-inductance and mutual inductance

Fig. 2.2 presents a coil with $N$ turns that carries current $I$ in the counterclockwise direction [10]. If the current is time-varying, then according to Faraday's law, a self-induced electromotive force will occur to oppose the change in the current and it can be expressed as

$$\varepsilon = -N \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{2.4}$$

The self-induced electromotive force can also be determined as

$$\varepsilon = -L \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \tag{2.5}$$

where $L$ is the self-inductance.

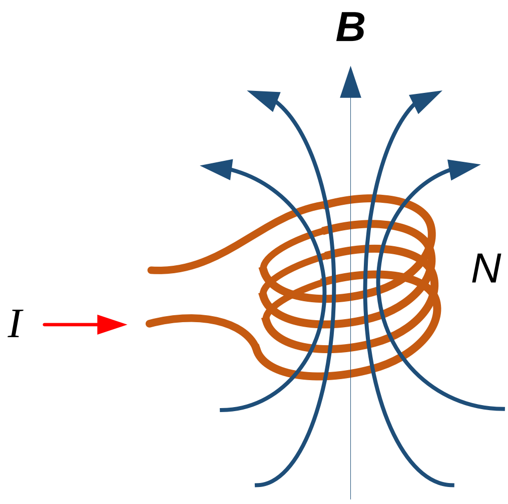


*Figure 2.2: Magnetic field lines through a coil with N turns*

Combining (2.4) and (2.5), an expression for the self-inductance of a coil with $N$ turns can be given as

$$L = N\frac{\Phi}{I} \tag{2.6}$$

In the case of two coils that are close to each other, some of the magnetic field lines will go through the second coil so its magnetic flux through one turn can be given as

$$\Phi_{21} = \int_{S_2} \boldsymbol{B_1} \cdot \boldsymbol{n}\, \mathrm{d}S_2 \tag{2.7}$$

where $\Phi_{21}$ represents the magnetic flux in the second coil that was produced by a time-varying current in the first coil, as shown in Fig. 2.1.

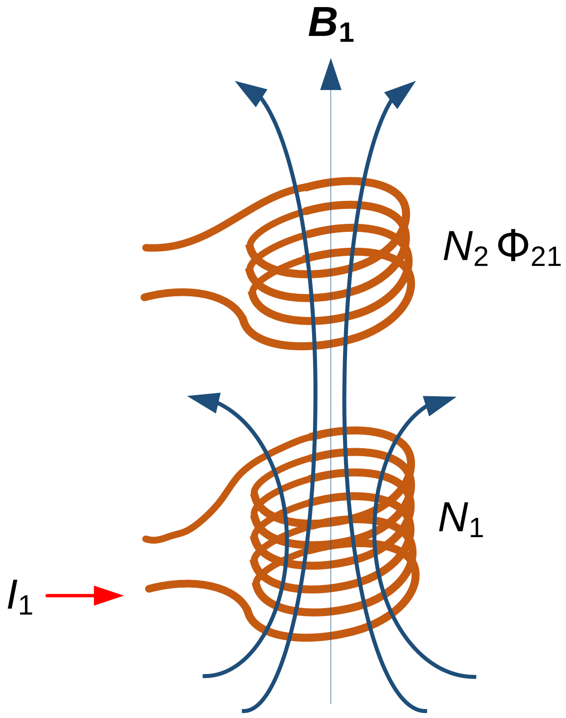


*Figure 2.3: The current in the first coil creates the magnetic flux in the second coil*

Now, according to Faraday's law, an induced electromotive force $\varepsilon_{21}$ in the second coil will be equal to the negative of the rate of change of the magnetic flux times the number of turns in the second coil and it can be written as

$$\varepsilon_{21} = -N_2 \frac{\mathrm{d}\Phi_{21}}{\mathrm{d}t} \tag{2.8}$$

The time-varying magnetic flux in the second coil is proportional to the time-varying current in the first coil, so the induced electromotive force $\varepsilon_{21}$ can be given as

$$\varepsilon_{21} = -M_{21} \frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{2.9}$$

where $M_{21}$ is called the mutual inductance and by equating (2.8) and (2.9) it can be determined as

$$M_{21} = \frac{N_2 \cdot \Phi_{21}}{I_1} \tag{2.10}$$

In the same way, we can determine the magnetic flux $\Phi_{12}$ in the first coil if there is a time-varying current in the second coil, as shown in Fig. 2.4.

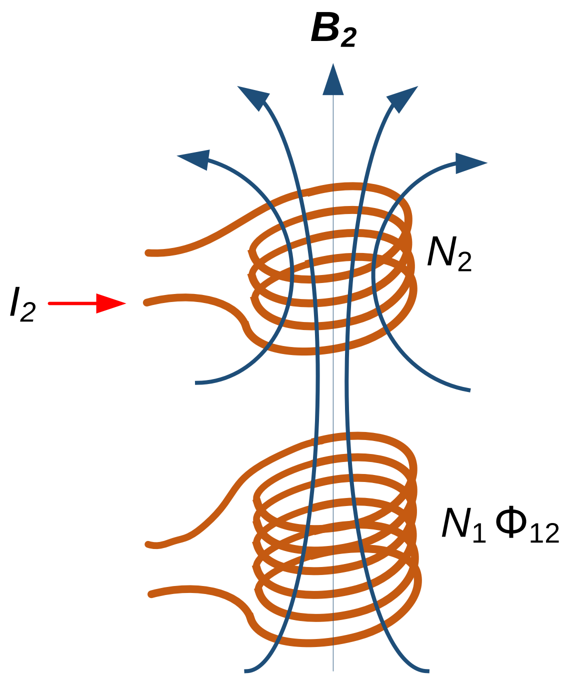


*Figure 2.4: The current in the second coil creates the magnetic flux in the first coil*

The time-varying current in the second coil produces magnetic field lines that will go through first coil so the induced electromotive force in the first coil can be given as

$$\varepsilon_{12} = -N_1 \frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t} \tag{2.11}$$

where $N_1$ is the number of turns of the first coil and $\Phi_{12}$ is the magnetic flux in the first coil created by the current in the second coil. Magnetic flux $\Phi_{12}$ is proportional to the time-varying current in the second coil and the induced electromotive force $\varepsilon_{12}$ can be determined again as

$$\varepsilon_{12} = -M_{12} \frac{\mathrm{d}I_2}{\mathrm{d}t} \tag{2.12}$$

where $M_{12}$ is called the mutual inductance and by equating (2.11) and (2.12) it can be determined as

$$M_{12} = \frac{N_1 \cdot \Phi_{12}}{I_2} \tag{2.13}$$

By using (2.10) and (2.13) it can be shown that the mutual inductance $M_{21}$ is equal to the mutual inductance $M_{12}$

$$M_{21} = M_{12} = M \tag{2.14}$$

The mutual inductance $M$ can be written in the terms of $L_1$ and $L_2$ as

$$M = k \cdot \sqrt{L_1 \cdot L_2}, \qquad 0 \le k \le 1 \tag{2.15}$$

where $k$ is the coupling coefficient. In case when $k = 1$, all magnetic flux produces by a time-varying current in one coil, will go through the second coil.

### 2.1.2 The *RLC* series circuit

An example of a *RLC* series circuit is shown in Fig. 2.5. It contains an inductor $L$, a resistor $R$, a capacitor $C$ and an AC voltage source which can be determined as

$$v(t) = V_0 \sin(\omega t) \tag{2.16}$$

where $V_0$ is the amplitude of the voltage source and $\omega$ is equal to $2\pi f$.

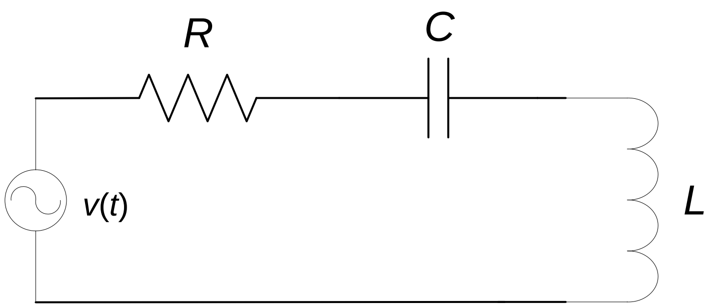


*Figure 2.5: The RLC series circuit*

According to [11], Kirchhoff's second law applied to the series RLC circuit can be given as

$$v(t) = v_R(t) + v_L(t) + v_C(t) \tag{2.17}$$

where $v_R(t)$, $v_L(t)$ and $v_C(t)$ are the voltages across the resistor, the inductor and the capacitor, respectively.

Equation (2.17) can also be written as

$$V_0 \sin(\omega t) = I \cdot R + L \frac{\mathrm{d}I}{\mathrm{d}t} + \frac{Q}{C} \tag{2.18}$$

where $Q$ is an electric charge in the capacitor $C$.

The current $I$ in the series RLC circuit can be determined as

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{2.19}$$

By utilizing (2.19), the differential equation given in (2.18) can be expressed as

$$\frac{V_0}{L} \sin(\omega t) = \frac{\mathrm{d}^2 Q}{\mathrm{d}t^2} + \frac{R}{L} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{L \cdot C} \tag{2.20}$$

Equation (2.20) can be given in another form as

$$\frac{V_0}{L} \sin(\omega t) = \frac{\mathrm{d}^2 Q}{\mathrm{d}t^2} + 2\delta \cdot \frac{\mathrm{d}Q}{\mathrm{d}t} + \omega_0^2 \cdot Q \tag{2.21}$$

where $\omega_0$ is the resonant angular frequency and $\delta$ is the attenuation factor. They can be determined as

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}} = 2\pi \cdot f_0 \tag{2.22}$$

$$\delta = \frac{R}{2 \cdot L} \tag{2.23}$$

Another important term of the series *RLC* circuit is the quality factor *Q* which can be described as a ratio of the stored energy in the inductor and in the capacitor to the energy that is dissipated in the resistor. The quality factor Q of the series RLC circuit can be given as

$$Q = \frac{\omega_0}{2\delta} = \frac{\omega_0 \cdot L}{R} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}} \tag{2.24}$$

## 2.2 Resonant conversion

In order to accomplish wireless charging, a resonant dc-dc converter is required which is shown in Fig. 2.6. A resonant converter contains a dc voltage source, full-bridge inverter, resonant tank, rectifier with low-pass filter and a load resistance.

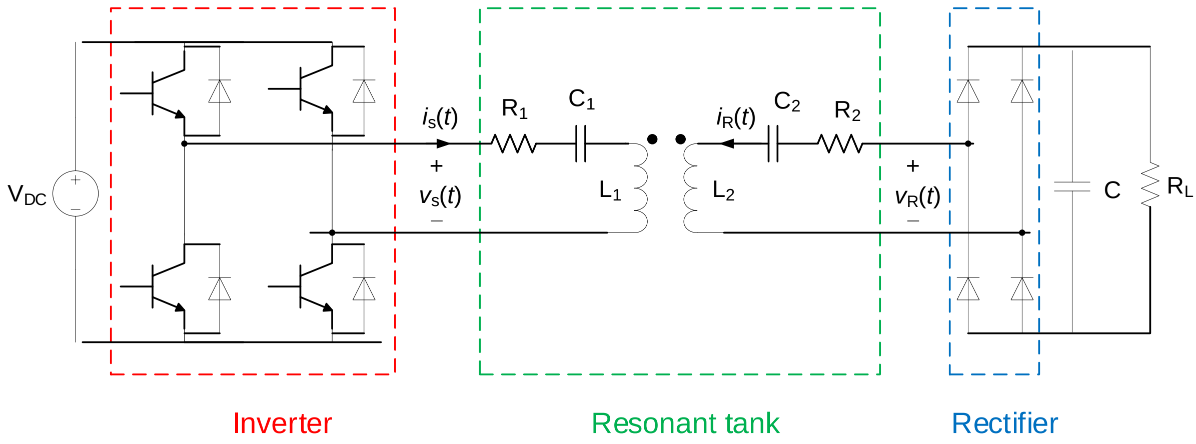


*Figure 2.6: Resonant dc-dc converter*

The inverter shown in Fig. 2.6 produces a square wave voltage output $v_s(t)$ and its spectrum contains fundamental plus the odd harmonics which can be written as

$$v_s(t) = \frac{4 \cdot V_{DC}}{\pi} \sum_{n=1,3,5,...} \frac{1}{n} \sin(n \cdot \omega_s \cdot t) \tag{2.25}$$

where $V_{DC}$ is the input voltage in the primary circuit and the switching frequency $f_s$ or the frequency of the fundamental component is equal to

$$f_s = \frac{\omega_s}{2\pi} \tag{2.26}$$

According to [12], the resonant frequency of the resonant tank network shown in Fig. 2.6 is usually tuned to the switching frequency $f_s$, so the current in the resonant tank $i_s(t)$ as well as the load voltage $v_R(t)$ and the load current $i_R(t)$ have approximately sinusoidal waveforms of frequency $f_s$ with negligible harmonics. In order to control the magnitude of the $i_s(t)$, $v_R(t)$ and $i_R(t)$, the switching frequency $f_s$ can be changed either closer to or further from the resonant frequency $f_0$.

The fundamental component of the inverter's square wave voltage output $v_{s1}(t)$ can be given as

$$v_{s1}(t) = \frac{4 \cdot V_{DC}}{\pi} \sin(\omega_s \cdot t) = V_{s1} \cdot \sin(\omega_s \cdot t) \tag{2.27}$$

and both the fundamental component $v_{s1}(t)$ as well as the square wave voltage output $v_s(t)$ are shown in Fig. 2.7 where the peak amplitude of the fundamental component is $\frac{4}{\pi}$ times the input dc voltage $V_{DC}$.

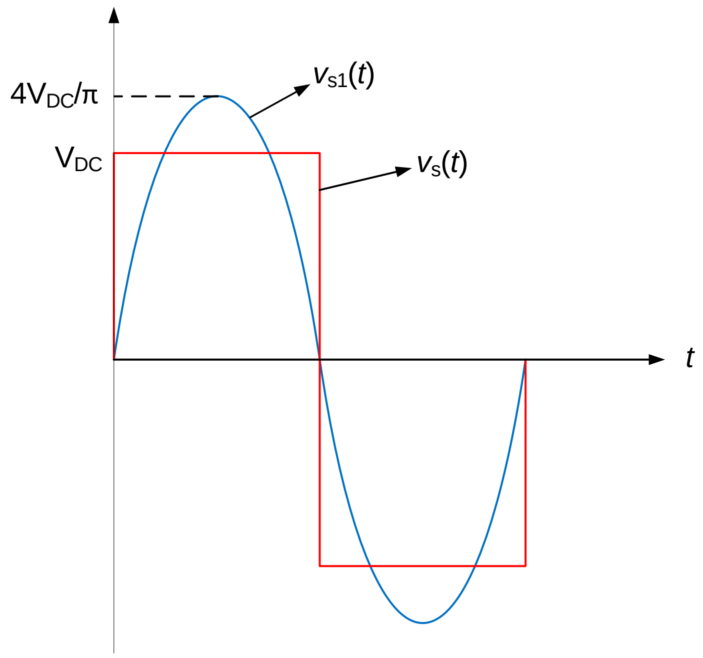


*Figure 2.7: Inverter square wave output voltage and its fundamental component*

The last part of a resonant dc-dc converter is a diode bridge rectifier whose purpose is to rectify the approximately sinusoidal output load current $i_R(t)$. After rectifying, this current is filtered by a large capacitor $C$ which gives small peak to peak ripple dc voltage, so we can expect that the current through the load resistance $R_L$ will be constant.

## 2.3 Losses in power electronic devices

Power electronic components that will be used in the case set-up are MOSFETs and diodes. In order to find the total losses of power electronic devices, detailed information from datasheets about the components will be provided later.

### 2.3.1 MOSFET losses

A MOSFET is a voltage controlled device and it is shown in Fig. 2.8. After applying a gate-to-source voltage $v_{GS}$, the MOSFET is turned on and can be modelled as a resistor $R_{DS,on}$, because a voltage between the drain and the source $v_{DS}$ is linearly proportional to the drain current $i_D$ and it can be written as

$$v_{DS} = R_{DS,on} \cdot i_D \tag{2.28}$$

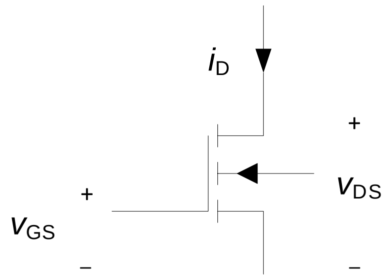


*Figure 2.8: MOSFET (N-channel)*

The conduction losses can be represented as

$$P_{cond} = R_{DS,on} \cdot I_{rms}^2 \tag{2.29}$$

where $i_{D,rms}$ is the rms value of the drain current $i_D$ that is shown in Fig. 2.9. The expression for the current $i_{D,rms}$ can be given as

$$i_{D,rms} = \frac{1}{2} i_{D,pk} \tag{2.30}$$

where $i_{D,pk}$ is the peak value of the drain current.

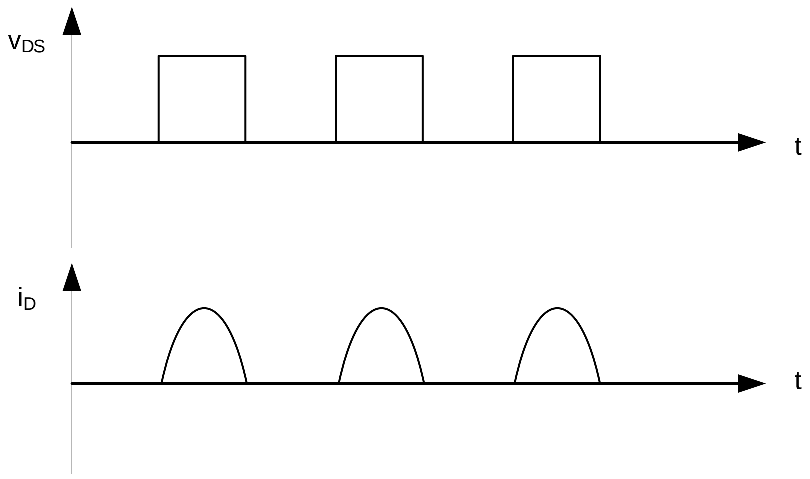


*Figure 2.9: Drain to source voltage and the drain current*

Therefore, the conduction losses of the MOSFET can be given as

$$P_{\mathrm{cond}} = R_{\mathrm{DS,on}} \cdot i_{\mathrm{rms}}^2 = \frac{1}{4} \cdot R_{\mathrm{DS,on}} \cdot i_{\mathrm{D,pk}}^2 \tag{2.31}$$

And the total conduction losses of the full-bridge inverter can be defined as

$$P_{\mathrm{cond,total}} = 4 \cdot P_{\mathrm{cond}} = 4 \cdot \frac{1}{4} \cdot R_{\mathrm{DS,on}} \cdot i_{\mathrm{D,pk}}^2 = R_{\mathrm{DS,on}} \cdot i_{\mathrm{D,pk}}^2 \tag{2.32}$$

The switching losses of the MOSFET can be described as the sum of the energy loss during the on and the off time multiplied by the switching frequency $f_{\mathrm{s}}$ and that can be given as

$$P_{\mathrm{switch}} = \left(E_{\mathrm{on}} + E_{\mathrm{off}}\right) \cdot f_{\mathrm{s}} \tag{2.33}$$

According to [13], the energy loss during the time when MOSFET is turned on and when it is turned off can be determined as

$$E_{\mathrm{on}} = \frac{v_{\mathrm{DS}} \cdot i_{\mathrm{D,pk}} \cdot T_{\mathrm{on}}}{2} \tag{2.34}$$

$$E_{\mathrm{off}} = \frac{v_{\mathrm{DS}} \cdot i_{\mathrm{D,pk}} \cdot T_{\mathrm{off}}}{2} \tag{2.35}$$

where $T_{\mathrm{on}}$ and $T_{\mathrm{off}}$ represent the time that is needed to fully turn on and turn off the MOSFET. The total switching losses are multiplied by four and can be written as

$$P_{\mathrm{switch}} = 4 \cdot \left( \frac{v_{\mathrm{DS}} \cdot i_{\mathrm{D,pk}} \cdot T_{\mathrm{on}}}{2} + \frac{v_{\mathrm{DS}} \cdot i_{\mathrm{D,pk}} \cdot T_{\mathrm{off}}}{2} \right) \cdot f_{\mathrm{s}} = 2 \cdot v_{\mathrm{DS}} \cdot i_{\mathrm{D,pk}} \cdot f_{\mathrm{s}} \cdot \left(T_{\mathrm{on}} + T_{\mathrm{off}}\right)$$

### 2.3.2 Diode losses

A diode is an uncontrollable power electronic device and it is shown in Fig. 2.10.

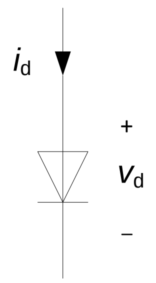


*Figure 2.10: Diode*

The conduction losses in the diode can be given as

$$P_{\text{cond}} = v_{\text{D0}} \cdot i_{\text{D,av}} + R_{\text{D}} \cdot i_{\text{D,rms}}^2 \tag{2.36}$$

where $v_{\text{D0}}$ is the constant voltage drop, $i_{\text{D,av}}$ is the average value of the diode current $i_{\text{D}}$, $R_{\text{D}}$ is the resistance of the diode and $i_{\text{D,rms}}$ is the rms value of the current $i_{\text{D}}$ shown in Fig. 2.11.

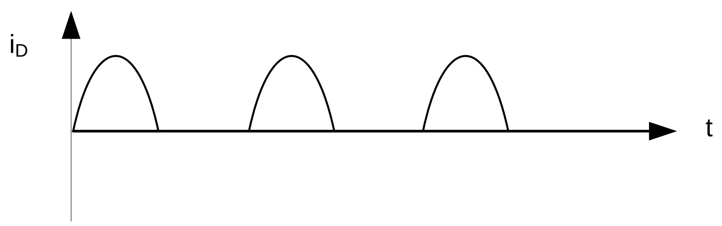


*Figure 2.11: The diode current*

The average value of the current $i_{\text{D}}$ is equal to

$$i_{\text{D,av}} = \frac{i_{\text{D,pk}}}{\pi} \tag{2.37}$$

while the rms value can be determined as

$$i_{\text{D,rms}} = \frac{1}{2} \cdot i_{\text{D,pk}} \tag{2.38}$$

The total conduction losses of the full-bridge rectifier are multiplied by four and can be given as

$$P_{\text{cond,total}} = 4 \cdot \left( v_{D0} \cdot i_{D,av} + R_D \cdot i_{D,rms}^2 \right)$$

$$P_{\text{cond,total}} = 4 \cdot \left( v_{D0} \frac{i_{D,pk}}{\pi} + R_D \cdot \frac{1}{4} \cdot i_{D,pk}^2 \right)$$

$$P_{\text{cond,total}} = 4 \cdot v_{D0} \frac{i_{D,pk}}{\pi} + R_D \cdot i_{D,pk}^2$$

# 3 Currents in the transmitter, receiver and auxiliary circuit

This chapter provides three different theoretical models of wireless charging systems. The first model that will be analyzed is a basic model with two coils and two capacitors. An additional winding will be added to this system, so the second model will contain three coils and capacitors in the auxiliary loop and in the receiver loop. The last model will have a capacitor connected in series with each coil.

## 3.1 Model with two coils and two capacitors

Fig. 3.1 shows a model of a two winding wireless charging system with capacitors in the transmitter and in the receiver coil.

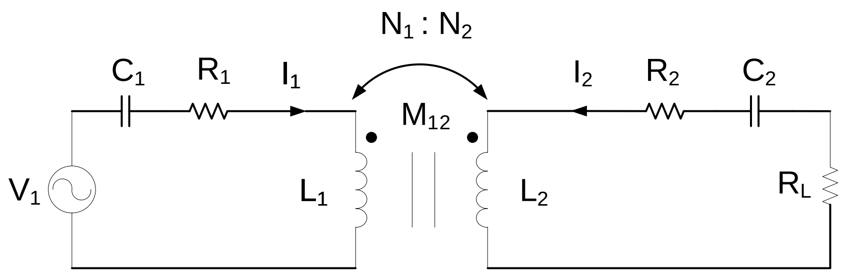


*Figure 3.1: Wireless charging system with capacitors in both coils*

Kirchhoff's second law applied to both loops gives

$$V_1 = R_1 \cdot I_1 + j\omega\left(L_1 \cdot I_1 + M_{12} \cdot I_2\right) + I_1 \cdot \frac{1}{j\omega \cdot C_1} \tag{3.1}$$

$$0 = R_2 \cdot I_2 + j\omega \cdot \left(L_2 \cdot I_2 + M_{12} \cdot I_1\right) + I_2 \cdot \frac{1}{j\omega \cdot C_2} + R_L \cdot I_2 \tag{3.2}$$

where $L_1$ and $L_2$ represent the inductance of the coils, $I_1$ and $I_2$ are the currents through the primary and the secondary circuit and $M_{12}$ represents the mutual inductance. A capacitor $C_1$ is connected in series with the transmitter coil and a load resistance $R_L$ and a capacitor $C_2$ are connected in series with the receiver coil. The input voltage source $V_1$ is represented as a peak value of the fundamental component of the inverter's output square wave.

Equation (3.2) can be written again as

$$0 = \left( R_2 + R_\mathrm{L} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right) \cdot I_2 + j\omega \cdot M_{12} \cdot I_1 \tag{3.3}$$

After utilizing (3.3), the expression for the current in the transmitter loop can be determined as

$$I_1 = \frac{-I_2 \cdot \left( R_2 + R_\mathrm{L} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right)}{j\omega \cdot M_{12}} \tag{3.4}$$

Equation (3.1) can be written again as

$$V_1 = I_1 \cdot \left( R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1} \right) + j\omega \cdot M_{12} \cdot I_2 \tag{3.5}$$

The expression for the current $I_1$ given in (3.4) can be inserted into (3.5), so the expression for the input voltage in the transmitter coil can be determined as

$$V_1 = \frac{-I_2 \cdot \left( R_2 + R_\mathrm{L} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right)}{j\omega \cdot M_{12}} \cdot \left( R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1} \right) + j\omega \cdot M_{12} \cdot I_2 \tag{3.6}$$

From (3.6), the expression for the current $I_2$ in the receiver coil can be expressed as

$$I_2 = \frac{j\omega \cdot M_{12} \cdot V_1}{\left( j\omega \cdot M_{12} \right)^2 - \left( R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1} \right) \cdot \left( R_2 + R_\mathrm{L} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right)} \tag{3.7}$$

## 3.2 Model with three coils and two capacitors

Fig. 3.2 presents a model of a three winding wireless charging system with capacitors connected in series with the receiver and the auxiliary coil.

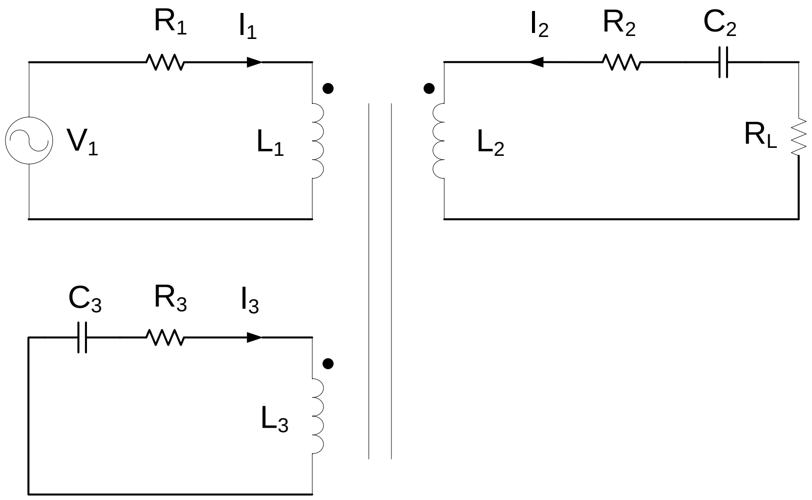


*Figure 3.2: Wireless charging system with capacitors in the auxiliary loop and in the receiver loop*

Kirchhoff's second law applied to three loops gives

$$V_1 = R_1 \cdot I_1 + j\omega\left(L_1 \cdot I_1 + M_{12} \cdot I_2 + M_{13} \cdot I_3\right) \tag{3.8}$$

$$0 = R_2 \cdot I_2 + j\omega \cdot \left(L_2 \cdot I_2 + M_{12} \cdot I_1 + M_{23} \cdot I_3\right) + I_2 \cdot \frac{1}{j\omega \cdot C_2} + R_L \cdot I_2 \tag{3.9}$$

$$0 = R_3 \cdot I_3 + j\omega \cdot \left(L_3 \cdot I_3 + M_{13} \cdot I_1 + M_{23} \cdot I_2\right) + I_3 \cdot \frac{1}{j\omega \cdot C_3} \tag{3.10}$$

where $R_1$, $R_2$ and $R_3$ are the resistances of the coils, $L_1$, $L_2$ and $L_3$ represent the inductance of the coils and $M_{12}$, $M_{13}$ and $M_{23}$ represent the mutual inductances. The currents through the transmitter, receiver and the auxiliary loop are $I_1$, $I_2$ and $I_3$ and the input voltage source is $V_1$.

Equation (3.8) can be written as

$$V_1 = I_1 \cdot \left(R_1 + j\omega L_1\right) + j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right) \tag{3.11}$$

so, an expression for the current $I_1$ in the transmitter circuit can be given as

$$I_1 = \frac{V_1 - j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{R_1 + j\omega L_1} \tag{3.12}$$

After inserting (3.12) into (3.10), the procedure for finding the expression for the current $I_3$ in the auxiliary circuit can be given as

$$0 = R_3 \cdot I_3 + j\omega \cdot \left( L_3 \cdot I_3 + M_{13} \cdot \left( \frac{V_1 - j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{R_1 + j\omega L_1} \right) + M_{23} \cdot I_2 \right) + I_3 \cdot \frac{1}{j\omega \cdot C_3}$$

$$0 = I_3 \cdot \left( R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} \right) + j\omega \cdot M_{23} \cdot I_2 + \frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega L_1} + \omega^2 \cdot \frac{M_{13}^2 \cdot I_3 + M_{12} \cdot M_{13} \cdot I_2}{R_1 + j\omega L_1}$$

$$-\frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega L_1} = I_3 \cdot \left( R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega L_1} \right) + I_2 \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega L_1} \right)$$

The final expression for the auxiliary current $I_3$ can be determined as

$$I_3 = \frac{-\dfrac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega L_1} - I_2 \cdot \left( j\omega \cdot M_{23} + \dfrac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega L_1} \right)}{R_3 + j\omega \cdot L_3 + \dfrac{1}{j\omega \cdot C_3} + \dfrac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega L_1}} \tag{3.13}$$

The same procedure will be applied to (3.9) in order to determine the current $I_2$ in the secondary circuit. First, the expression for the current $I_1$ in the primary circuit given in (3.12) is inserted into (3.9) and it can be written as

$$0 = \left(R_2 + R_\mathrm{L}\right) \cdot I_2 + j\omega \cdot \left(L_2 \cdot I_2 + M_{12} \cdot I_1 + M_{23} \cdot I_3\right) + I_2 \cdot \frac{1}{j\omega \cdot C_2}$$

$$0 = \left(R_2 + R_\mathrm{L}\right) \cdot I_2 + j\omega \cdot \left( L_2 \cdot I_2 + M_{12} \cdot \left[ \frac{V_1 - j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{R_1 + j\omega L_1} \right] + M_{23} \cdot I_3 \right) + I_2 \cdot \frac{1}{j\omega \cdot C_2}$$

$$\begin{aligned} \frac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega L_1} = I_2 \cdot \left[ R_2 + R_\mathrm{L} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} + \frac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega L_1} \right] + \\ I_3 \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega L_1} \right) \end{aligned} \tag{3.14}$$

The next step is to insert the expression for the auxiliary current given in (3.13) into (3.14) and it can be written as

$$\frac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega L_1} = I_2 \cdot \left[ R_2 + R_L + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} + \frac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega L_1} \right] +$$
$$\left[ \frac{-\frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega L_1} - I_2 \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega L_1} \right)}{R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega L_1}} \right] \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega L_1} \right)$$

Finally, the current $I_2$ in the receiver circuit can be expressed as

$$I_2 = \frac{\frac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega L_1} + \frac{\left( \frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega L_1} \right) \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega L_1} \right)}{R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega L_1}}}{\left( R_2 + R_L + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} + \frac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega L_1} \right) - \frac{\left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega L_1} \right)^2}{\left( R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega L_1} \right)}} \tag{3.15}$$

## 3.3 Model with three coils and three capacitors

Fig. 3.3 presents a model of a three winding wireless charging system with capacitors $C_1$, $C_2$ and $C_3$ connected in series with each coil.

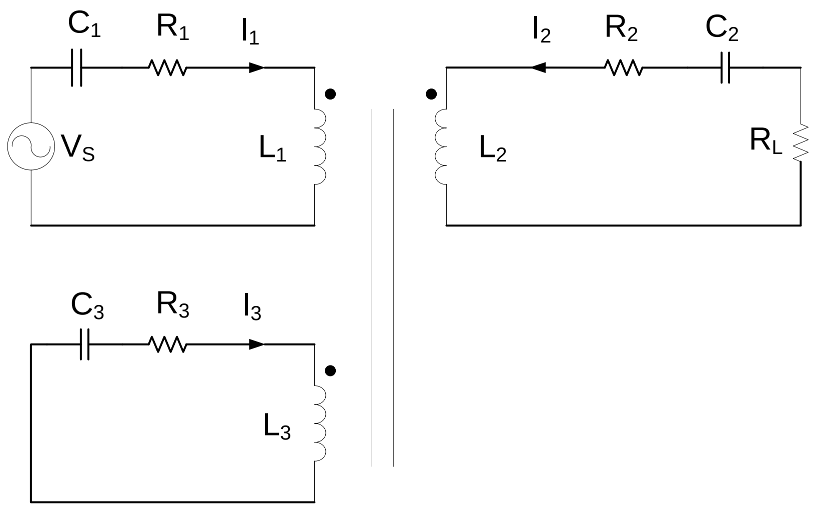


*Figure 3.3: Wireless charging system with capacitors in each circuit*

Kirchhoff's second law applied to three loops gives:

$$V_1 = R_1 \cdot I_1 + j\omega\left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right) + j\omega \cdot L_1 \cdot I_1 + I_1 \cdot \frac{1}{j\omega \cdot C_1} \tag{3.16}$$

$$0 = \left(R_2 + R_L\right) \cdot I_2 + j\omega\left(M_{12} \cdot I_1 + M_{23} \cdot I_3\right) + j\omega \cdot L_2 \cdot I_2 + I_2 \cdot \frac{1}{j\omega \cdot C_2} \tag{3.17}$$

$$0 = R_3 \cdot I_3 + j\omega \cdot \left(M_{13} \cdot I_1 + M_{23} \cdot I_2\right) + j\omega \cdot L_3 \cdot I_3 + I_3 \cdot \frac{1}{j\omega \cdot C_3} \tag{3.18}$$

where $L_1$, $L_2$ and $L_3$ are the inductance of the coils, $R_1$, $R_2$ and $R_3$ are the resistances of the coils and $M_{12}$, $M_{13}$ and $M_{23}$ represent the mutual inductances. The currents through the transmitter, receiver and the auxiliary circuit are $I_1$, $I_2$ and $I_3$. The input voltage source is $V_1$ and the load resistance is represented as $R_L$.

The expression for the current $I_1$ in the transmitter circuit can be determined from (3.16) and it can be given as

$$I_1 = \frac{V_1 - j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} \tag{3.19}$$

In order to find the expression for the current $I_3$ in the auxiliary circuit, (3.19) is inserted into (3.18) and it can be written as

$$0 = \left(R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3}\right) \cdot I_3 + j\omega \cdot \left(M_{13} \cdot I_1 + M_{23} \cdot I_2\right)$$

$$0 = \left(R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3}\right) \cdot I_3 + j\omega \cdot \left(M_{13} \cdot \left(\frac{V_1 - j\omega \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right) + M_{23} \cdot I_2\right)$$

$$0 = \left(R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3}\right) \cdot I_3 + j\omega \cdot M_{23} \cdot I_2 + \frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} + \omega^2 \cdot \frac{M_{13}^2 \cdot I_3 + M_{12} \cdot M_{13} \cdot I_2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}$$

$$-\frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} = I_3 \cdot \left(R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right) +$$

$$I_2 \cdot \left(j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)$$

The final expression for the current $I_3$ in the auxiliary loop can be determined as

$$I_3 = \frac{-\dfrac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} - I_2 \cdot \left(j\omega \cdot M_{23} + \dfrac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)}{\left(R_3 + j\omega \cdot L_3 + \dfrac{1}{j\omega \cdot C_3} + \dfrac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)} \tag{3.20}$$

The same procedure will be applied to the expression given in (3.17) in order to determine the current $I_2$ in the secondary circuit. First, the expression for the current $I_1$ in the primary circuit that was given in (3.19) is inserted into (3.17) and it can be written as

$$0 = \left( R_2 + R_{\mathrm{L}} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right) \cdot I_2 + j\omega \left( M_{12} \cdot I_1 + M_{23} \cdot I_3 \right)$$

$$0 = \left( R_2 + R_{\mathrm{L}} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} \right) \cdot I_2 + j\omega \cdot \left( M_{12} \cdot \left[ \frac{V_1 - j\omega \cdot \left( M_{12} \cdot I_2 + M_{13} \cdot I_3 \right)}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right] + M_{23} \cdot I_3 \right)$$

$$\begin{aligned} \frac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} = I_2 \cdot \left[ R_2 + R_{\mathrm{L}} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} + \frac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right] + \\ I_3 \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right) \end{aligned} \quad (3.21)$$

The next step is to insert the expression for the current $I_3$ that was previously given in (3.20) into (3.21) and it can be determined as

$$\begin{aligned} \frac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} = I_2 \cdot \left[ R_2 + R_{\mathrm{L}} + j\omega \cdot L_2 + \frac{1}{j\omega \cdot C_2} + \frac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right] + \\ \frac{-\frac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} - I_2 \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right)}{\left( R_3 + j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} + \frac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right)} \cdot \left( j\omega \cdot M_{23} + \frac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega \cdot L_1 + \frac{1}{j\omega \cdot C_1}} \right) \end{aligned}$$

The final expression for the current $I_2$ in the receiver circuit can be given as

$$I_2 = \frac{\dfrac{-j\omega \cdot M_{12} \cdot V_1}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} + \dfrac{\left(\dfrac{j\omega \cdot M_{13} \cdot V_1}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right) \cdot \left(j\omega \cdot M_{23} + \dfrac{\omega^2 \cdot M_{13} \cdot M_{12}}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)}{\left(R_3 + j\omega \cdot L_3 + \dfrac{1}{j\omega \cdot C_3} + \dfrac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)}}{R_2 + R_L + j\omega \cdot L_2 + \dfrac{1}{j\omega \cdot C_2} + \dfrac{\omega^2 \cdot M_{12}^2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}} - \dfrac{\left(j\omega \cdot M_{23} + \dfrac{\omega^2 \cdot M_{12} \cdot M_{13}}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}\right)^2}{R_3 + j\omega \cdot L_3 + \dfrac{1}{j\omega \cdot C_3} + \dfrac{\omega^2 \cdot M_{13}^2}{R_1 + j\omega \cdot L_1 + \dfrac{1}{j\omega \cdot C_1}}}}$$

# 4 Transfer functions

## 4.1 Transfer function of a two winding model with two capacitors

A model of a two winding wireless charging system with two capacitors is presented in Fig. 4.1. The sinusoidal voltage source $v_{s1}(t)$ represents the fundamental component of the inverter's output square wave. A source impedance $Z_S$ consists of a capacitor $C_1$ and a source resistance $R_S$ which will be neglected in this analysis. An output impedance $Z_2$ consists of a capacitor $C_2$ and a load resistance $R_L$.

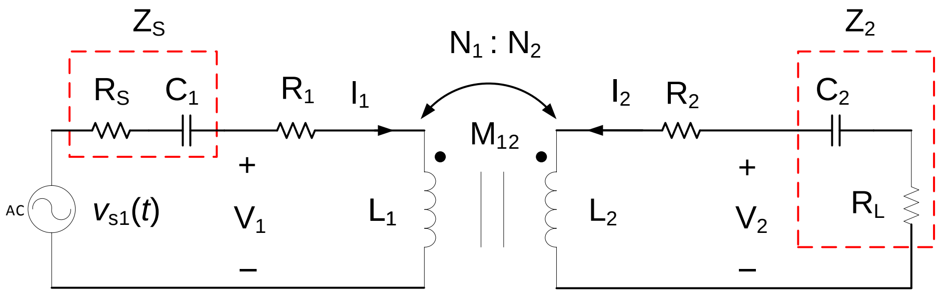


*Figure 4.1: Two winding model with two capacitors*

Kirchhoff's second law applied to both inductors gives

$$V_1 = s \cdot L_1 \cdot I_1 + s \cdot M_{12} \cdot I_2 + R_1 \cdot I_1 \tag{4.1}$$

$$V_2 = s \cdot L_2 \cdot I_2 + s \cdot M_{12} \cdot I_1 + R_2 \cdot I_2 \tag{4.2}$$

where $V_1$ and $V_2$ are the voltages across the inductors, $R_1$ and $R_2$ are the coil's resistances, $L_1$ and $L_2$ represent the inductance of the coils, $I_1$ and $I_2$ are the currents through the primary and the secondary circuit and $M_{12}$ represents the mutual inductance.

The output impedance $Z_2$ can be expressed as

$$Z_2 = R_L + \frac{1}{s \cdot C_2} \tag{4.3}$$

The current $I_2$ in the secondary circuit can be determined as

$$I_2 = -\frac{V_2}{Z_2} \tag{4.4}$$

By using (4.1) and (4.4), the expression for the current $I_1$ in the primary circuit can be given as

$$I_1 = \frac{V_1 - s \cdot M_{12} \cdot I_2}{s \cdot L_1 + R_1} = \frac{V_1 + \dfrac{s \cdot M_{12} \cdot V_2}{Z_2}}{s \cdot L_1 + R_1} \tag{4.5}$$

To eliminate the currents $I_1$ and $I_2$ from (4.2) by using (4.4) and (4.5), the expression for the output voltage $V_2$ can be written as

$$\begin{aligned} V_2 &= s \cdot M_{12} \cdot \left( \frac{V_1 + \dfrac{s \cdot M_{12} \cdot V_2}{Z_2}}{s \cdot L_1 + R_1} \right) + (s \cdot L_2 + R_2) \cdot I_2 \\ V_2 &= \frac{s \cdot M_{12} \cdot V_1}{s \cdot L_1 + R_1} + \frac{s^2 \cdot M_{12}^2 \cdot V_2}{Z_2 \cdot (s \cdot L_1 + R_1)} - \frac{(s \cdot L_2 + R_2)}{Z_2} \cdot V_2 \end{aligned} \tag{4.6}$$

In order to determine the transfer function $G(s) = \frac{V_2}{V_1}$, (4.6) can be expressed as

$$\frac{s \cdot M_{12}}{s \cdot L_1 + R_1} \cdot V_1 = V_2 \left[ 1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{(s \cdot L_2 + R_2)}{Z_2} \right] \tag{4.7}$$

The final expression for the transfer function $\frac{V_2}{V_1}$ can be given as

$$G(s) = \frac{V_2}{V_1} = \frac{\dfrac{s \cdot M_{12}}{s \cdot L_1 + R_1}}{1 - \dfrac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \dfrac{(s \cdot L_2 + R_2)}{Z_2}} \tag{4.8}$$

The input voltage source $V_S$ in the primary circuit can be written as

$$V_S = V_1 + Z_S \cdot I_1 \tag{4.9}$$

The transfer function $H(s)=\frac{V_2}{V_S}$ of a two winding wireless charging system with two capacitors can be determined as

$$\frac{V_2}{V_S}=\frac{V_2}{V_1}\cdot\frac{V_1}{V_S} \tag{4.10}$$

Equation (4.5) is inserted into (4.9) to make an expression for the input voltage source $V_S$ which can be given as

$$\begin{aligned} V_S &= V_1 + Z_S\cdot\frac{V_1+\frac{s\cdot M_{12}\cdot V_2}{Z_2}}{s\cdot L_1+R_1} \\ V_S &= V_1+\frac{Z_S}{s\cdot L_1+R_1}\cdot V_1+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)}\cdot V_2 \end{aligned} \tag{4.11}$$

In order to get the required transfer function $H(s)=\frac{V_2}{V_S}$, (4.11) can be rearranged as

$$\begin{aligned} V_S &= V_1\left[1+\frac{Z_S}{s\cdot L_1+R_1}\right]+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)}\cdot V_2 \\ \frac{V_S}{V_2} &= \frac{V_1}{V_2}\cdot\left[1+\frac{Z_S}{s\cdot L_1+R_1}\right]+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)} \end{aligned} \tag{4.12}$$

After inserting $\frac{1}{G(s)}=\frac{V_1}{V_2}$ into (4.12), the transfer function $\frac{1}{H(s)}$ can be given as

$$\frac{V_S}{V_2}=\frac{1-\frac{s^2\cdot M_{12}^2}{Z_2\cdot(s\cdot L_1+R_1)}+\frac{(s\cdot L_2+R_2)}{Z_2}}{\frac{s\cdot M_{12}}{s\cdot L_1+R_1}}\cdot\left[1+\frac{Z_S}{s\cdot L_1+R_1}\right]+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)} \tag{4.13}$$

The final expression for the transfer function $H(s)=\frac{V_2}{V_S}$ can be determined as

$$H(s)=\frac{1}{\frac{1-\frac{s^2\cdot M_{12}^2}{Z_2\cdot(s\cdot L_1+R_1)}+\frac{(s\cdot L_2+R_2)}{Z_2}}{\frac{s\cdot M_{12}}{s\cdot L_1+R_1}}\cdot\left[1+\frac{Z_S}{s\cdot L_1+R_1}\right]+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)}} \tag{4.14}$$

The transfer function given in (4.14) can be written as

$$\frac{1}{\left(1-\frac{s^2\cdot M_{12}^2}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\frac{\left(s\cdot L_2+R_2\right)}{Z_2}\right)\cdot\left(1+\frac{Z_S}{s\cdot L_1+R_1}\right)\cdot\left(\frac{s\cdot L_1+R_1}{s\cdot M_{12}}\right)+\left(\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}\right)}$$

$$\frac{1}{\left(1-\frac{s^2\cdot M_{12}^2}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\frac{\left(s\cdot L_2+R_2\right)}{Z_2}\right)\cdot\left(\frac{s\cdot L_1+R_1+Z_S}{\cancel{\left(s\cdot L_1+R_1\right)}}\right)\cdot\left(\frac{\cancel{\left(s\cdot L_1+R_1\right)}}{s\cdot M_{12}}\right)+\left(\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}\right)}$$

$$\frac{1}{\left(\frac{Z_2\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2+\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)}{Z_2\cdot\left(s\cdot L_1+R_1\right)}\right)\cdot\left(\frac{s\cdot L_1+R_1+Z_S}{s\cdot M_{12}}\right)+\left(\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}\right)}$$

$$\frac{1}{\dfrac{\left[Z_2\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2+\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)\right]\cdot\left[\left(s\cdot L_1+R_1\right)+Z_S\right]+s^2\cdot M_{12}^2\cdot Z_S}{s\cdot M_{12}\cdot Z_2\cdot\left(s\cdot L_1+R_1\right)}}$$

$$\frac{s\cdot M_{12}\cdot Z_2\cdot\left(s\cdot L_1+R_1\right)}{\left(\begin{array}{c}Z_2\cdot\left(s\cdot L_1+R_1\right)^2-s^2\cdot M_{12}^2\cdot\left(s\cdot L_1+R_1\right)+\left(s\cdot L_1+R_1\right)^2\cdot\left(s\cdot L_2+R_2\right)+\\ Z_S\cdot Z_2\cdot\left(s\cdot L_1+R_1\right)-\cancel{s^2\cdot M_{12}^2\cdot Z_S}+Z_S\cdot\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)+\cancel{s^2\cdot M_{12}^2\cdot Z_S}\end{array}\right)}$$

$$\frac{s\cdot M_{12}\cdot Z_2\cdot\left(s\cdot L_1+R_1\right)}{\left(\begin{array}{c}s^2\cdot L_1^2\cdot Z_2+2\cdot s\cdot L_1\cdot R_1\cdot Z_2+R_1^2\cdot Z_2-s^3\cdot M_{12}^2\cdot L_1-s^2\cdot M_{12}^2\cdot R_1+s^3\cdot L_1^2\cdot L_2+s^2\cdot L_1^2\cdot R_2+\\ 2\cdot s^2\cdot L_1\cdot R_1\cdot L_2+2\cdot s\cdot L_1\cdot R_1\cdot R_2+s\cdot L_2\cdot R_1^2+R_1^2\cdot R_2+s\cdot Z_S\cdot Z_2\cdot L_1+Z_S\cdot Z_2\cdot R_1+\\ s^2\cdot L_1\cdot L_2\cdot Z_S+s\cdot L_1\cdot R_2\cdot Z_S+s\cdot L_2\cdot R_1\cdot Z_S+R_1\cdot R_2\cdot Z_S\end{array}\right)}$$

$$\frac{s^2\cdot M_{12}\cdot Z_2\cdot L_1+s\cdot M_{12}\cdot Z_2\cdot R_1}{\left(\begin{array}{c}s^3\cdot\left(L_1^2\cdot L_2-M_{12}^2\cdot L_1\right)+s^2\cdot\left(L_1^2\cdot Z_2-M_{12}^2\cdot R_1+L_1^2\cdot R_2+2\cdot L_1\cdot R_1\cdot L_2+L_1\cdot L_2\cdot Z_S\right)+\\ s\cdot\left(2\cdot L_1\cdot R_1\cdot Z_2+2\cdot L_1\cdot R_1\cdot R_2+L_2\cdot R_1^2+Z_S\cdot Z_2\cdot L_1+L_1\cdot R_2\cdot Z_S+L_2\cdot R_1\cdot Z_S\right)+\\ \left(R_1^2\cdot Z_2+R_1^2\cdot R_2+Z_S\cdot Z_2\cdot R_1+R_1\cdot R_2\cdot Z_S\right)\end{array}\right)} \tag{4.15}$$

The transfer function can be defined as

$$H(s) = \frac{b_{\mathrm{m}} \cdot s^{\mathrm{m}} + b_{\mathrm{m-1}} \cdot s^{\mathrm{m-1}} + \ldots + b_1 \cdot s + b_0}{a_{\mathrm{n}} \cdot s^{\mathrm{n}} + a_{\mathrm{n-1}} \cdot s^{\mathrm{n-1}} + \ldots + a_1 \cdot s + a_0}$$

where $m \le n$.

The expressions for $Z_2$ and $Z_{\mathrm{S}}$ are determined as

- $Z_2 = \left( R_{\mathrm{L}} + \frac{1}{s \cdot C_2} \right)$
- $Z_{\mathrm{S}} = \frac{1}{s \cdot C_1}$

and they are inserted into (4.15).

The denominator can be written as

$$a_3 \cdot s^3 = s^3 \cdot \left( L_1^2 \cdot L_2 - M_{12}^2 \cdot L_1 \right)$$

$$\begin{aligned}
a_2 \cdot s^2 &= s^2 \cdot \left( L_1^2 \cdot \left( R_{\mathrm{L}} + \frac{1}{s \cdot C_2} \right) - M_{12}^2 \cdot R_1 + L_1^2 \cdot R_2 + 2 \cdot L_1 \cdot R_1 \cdot L_2 + L_1 \cdot L_2 \cdot \frac{1}{s \cdot C_1} \right) \\
&= s^2 \cdot \left( L_1^2 \cdot R_{\mathrm{L}} + \frac{L_1^2}{s \cdot C_2} - M_{12}^2 \cdot R_1 + L_1^2 \cdot R_2 + 2 \cdot L_1 \cdot R_1 \cdot L_2 + L_1 \cdot L_2 \cdot \frac{1}{s \cdot C_1} \right) \\
&= s^2 \cdot \left( L_1^2 \cdot R_{\mathrm{L}} - M_{12}^2 \cdot R_1 + L_1^2 \cdot R_2 + 2 \cdot L_1 \cdot R_1 \cdot L_2 \right) + s \cdot \left( \frac{L_1^2}{C_2} + \frac{L_1 \cdot L_2}{C_1} \right)
\end{aligned}$$

$$\begin{aligned}
a_1 \cdot s &= s \cdot \left( \begin{array}{c} 2 \cdot L_1 \cdot R_1 \cdot \left( R_{\mathrm{L}} + \frac{1}{s \cdot C_2} \right) + 2 \cdot L_1 \cdot R_1 \cdot R_2 + L_2 \cdot R_1^2 + \\ \frac{1}{s \cdot C_1} \cdot \left( R_{\mathrm{L}} + \frac{1}{s \cdot C_2} \right) \cdot L_1 + L_1 \cdot R_2 \cdot \frac{1}{s \cdot C_1} + L_2 \cdot R_1 \cdot \frac{1}{s \cdot C_1} \end{array} \right) \\
&= s \cdot \left( 2 \cdot L_1 \cdot R_1 \cdot R_{\mathrm{L}} + \frac{2 \cdot L_1 \cdot R_1}{s \cdot C_2} + 2 \cdot L_1 \cdot R_1 \cdot R_2 + L_2 \cdot R_1^2 + \frac{L_1 \cdot R_{\mathrm{L}}}{s \cdot C_1} + \frac{L_1}{s^2 \cdot C_1 \cdot C_2} + \frac{L_1 \cdot R_2}{s \cdot C_1} + \frac{L_2 \cdot R_1}{s \cdot C_1} \right) \\
&= s \cdot \left( 2 \cdot L_1 \cdot R_1 \cdot R_{\mathrm{L}} + 2 \cdot L_1 \cdot R_1 \cdot R_2 + L_2 \cdot R_1^2 \right) + \left( \frac{2 \cdot L_1 \cdot R_1}{C_2} + \frac{L_1 \cdot R_{\mathrm{L}}}{C_1} + \frac{L_1 \cdot R_2}{C_1} + \frac{L_2 \cdot R_1}{C_1} \right) + \frac{1}{s} \cdot \left( \frac{L_1}{C_1 \cdot C_2} \right)
\end{aligned}$$

$$a_0=\left(R_1^2\cdot\left(R_\mathrm{L}+\frac{1}{s\cdot C_2}\right)+R_1^2\cdot R_2+\frac{1}{s\cdot C_1}\cdot\left(R_\mathrm{L}+\frac{1}{s\cdot C_2}\right)\cdot R_1+R_1\cdot R_2\cdot\frac{1}{s\cdot C_1}\right)$$
$$=\left(R_1^2\cdot R_\mathrm{L}+\frac{R_1^2}{s\cdot C_2}+R_1^2\cdot R_2+\frac{R_\mathrm{L}\cdot R_1}{s\cdot C_1}+\frac{R_1}{s^2\cdot C_1\cdot C_2}+\frac{R_1\cdot R_2}{s\cdot C_1}\right)$$
$$=\left(R_1^2\cdot R_\mathrm{L}+R_1^2\cdot R_2\right)+\frac{1}{s}\cdot\left(\frac{R_1^2}{C_2}+\frac{R_\mathrm{L}\cdot R_1}{C_1}+\frac{R_1\cdot R_2}{C_1}\right)+\frac{1}{s^2}\cdot\left(\frac{R_1}{C_1\cdot C_2}\right)$$

The coefficients of the denominator $a_3\cdot s^3$, $a_2\cdot s^2$, $a_1\cdot s$ and $a_0$ can now be added together and written as

$$s^3\cdot\left(L_1^2\cdot L_2-M_{12}^2\cdot L_1\right)+s^2\cdot\left(L_1^2\cdot R_\mathrm{L}-M_{12}^2\cdot R_1+L_1^2\cdot R_2+2\cdot L_1\cdot R_1\cdot L_2\right)+s\cdot\left(\frac{L_1^2}{C_2}+\frac{L_1\cdot L_2}{C_1}\right)+$$
$$s\cdot\left(2\cdot L_1\cdot R_1\cdot R_\mathrm{L}+2\cdot L_1\cdot R_1\cdot R_2+L_2\cdot R_1^2\right)+\left(\frac{2\cdot L_1\cdot R_1}{C_2}+\frac{L_1\cdot R_\mathrm{L}}{C_1}+\frac{L_1\cdot R_2}{C_1}+\frac{L_2\cdot R_1}{C_1}\right)+\frac{1}{s}\cdot\left(\frac{L_1}{C_1\cdot C_2}\right)$$
$$+\left(R_1^2\cdot R_\mathrm{L}+R_1^2\cdot R_2\right)+\frac{1}{s}\cdot\left(\frac{R_1^2}{C_2}+\frac{R_\mathrm{L}\cdot R_1}{C_1}+\frac{R_1\cdot R_2}{C_1}\right)+\frac{1}{s^2}\cdot\left(\frac{R_1}{C_1\cdot C_2}\right)$$

Now, the denominator contains coefficients in descending powers of Laplace domain variable *s*, so it can be expressed as

$$D(s)=s^3\cdot\left(L_1^2\cdot L_2-M_{12}^2\cdot L_1\right)+s^2\cdot\left(L_1^2\cdot R_\mathrm{L}-M_{12}^2\cdot R_1+L_1^2\cdot R_2+2\cdot L_1\cdot R_1\cdot L_2\right)+$$
$$s\cdot\left(\frac{L_1^2}{C_2}+\frac{L_1\cdot L_2}{C_1}+2\cdot L_1\cdot R_1\cdot R_\mathrm{L}+2\cdot L_1\cdot R_1\cdot R_2+L_2\cdot R_1^2\right)+$$
$$\left(\frac{2\cdot L_1\cdot R_1}{C_2}+\frac{L_1\cdot R_\mathrm{L}}{C_1}+\frac{L_1\cdot R_2}{C_1}+\frac{L_2\cdot R_1}{C_1}+R_1^2\cdot R_\mathrm{L}+R_1^2\cdot R_2\right)$$
$$+\frac{1}{s}\cdot\left(\frac{L_1}{C_1\cdot C_2}+\frac{R_1^2}{C_2}+\frac{R_\mathrm{L}\cdot R_1}{C_1}+\frac{R_1\cdot R_2}{C_1}\right)+\frac{1}{s^2}\cdot\left(\frac{R_1}{C_1\cdot C_2}\right)$$

The numerator of the transfer function given in (4.15) can be written as

$$N(s)=s^2\cdot M_{12}\cdot Z_2\cdot L_1+s\cdot M_{12}\cdot Z_2\cdot R_1$$
$$=s^2\cdot M_{12}\cdot\left(R_\mathrm{L}+\frac{1}{s\cdot C_2}\right)\cdot L_1+s\cdot M_{12}\cdot\left(R_\mathrm{L}+\frac{1}{s\cdot C_2}\right)\cdot R_1$$
$$=s^2\cdot M_{12}\cdot L_1\cdot R_\mathrm{L}+s\cdot\left(\frac{M_{12}\cdot L_1}{C_2}+M_{12}\cdot R_1\cdot R_\mathrm{L}\right)+\frac{M_{12}\cdot R_1}{C_2}$$

Now, the transfer function can be determined as

$$H(s)=\frac{s^2\cdot M_{12}\cdot L_1\cdot R_\mathrm{L}+s\cdot\left(\dfrac{M_{12}\cdot L_1}{C_2}+M_{12}\cdot R_1\cdot R_\mathrm{L}\right)+\dfrac{M_{12}\cdot R_1}{C_2}}{\begin{array}{l}s^3\cdot\left(L_1^2\cdot L_2-M_{12}^2\cdot L_1\right)+s^2\cdot\left(L_1^2\cdot R_\mathrm{L}-M_{12}^2\cdot R_1+L_1^2\cdot R_2+2\cdot L_1\cdot R_1\cdot L_2\right)+\\ s\cdot\left(\dfrac{L_1^2}{C_2}+\dfrac{L_1\cdot L_2}{C_1}+2\cdot L_1\cdot R_1\cdot R_\mathrm{L}+2\cdot L_1\cdot R_1\cdot R_2+L_2\cdot R_1^2\right)+\\ \left(\dfrac{2\cdot L_1\cdot R_1}{C_2}+\dfrac{L_1\cdot R_\mathrm{L}}{C_1}+\dfrac{L_1\cdot R_2}{C_1}+\dfrac{L_2\cdot R_1}{C_1}+R_1^2\cdot R_\mathrm{L}+R_1^2\cdot R_2\right)\\ +\dfrac{1}{s}\cdot\left(\dfrac{L_1}{C_1\cdot C_2}+\dfrac{R_1^2}{C_2}+\dfrac{R_\mathrm{L}\cdot R_1}{C_1}+\dfrac{R_1\cdot R_2}{C_1}\right)+\dfrac{1}{s^2}\cdot\left(\dfrac{R_1}{C_1\cdot C_2}\right)\end{array}}$$

After multiplication with $s^2$, the transfer function can be written as

$$H(s)=\frac{s^4\cdot M_{12}\cdot L_1\cdot R_\mathrm{L}+s^3\cdot\left(\dfrac{M_{12}\cdot L_1}{C_2}+M_{12}\cdot R_1\cdot R_\mathrm{L}\right)+s^2\cdot\dfrac{M_{12}\cdot R_1}{C_2}}{\begin{array}{l}s^5\cdot\left(L_1^2\cdot L_2-M_{12}^2\cdot L_1\right)+s^4\cdot\left(L_1^2\cdot R_\mathrm{L}-M_{12}^2\cdot R_1+L_1^2\cdot R_2+2\cdot L_1\cdot R_1\cdot L_2\right)+\\ s^3\cdot\left(\dfrac{L_1^2}{C_2}+\dfrac{L_1\cdot L_2}{C_1}+2\cdot L_1\cdot R_1\cdot R_\mathrm{L}+2\cdot L_1\cdot R_1\cdot R_2+L_2\cdot R_1^2\right)+\\ s^2\cdot\left(\dfrac{2\cdot L_1\cdot R_1}{C_2}+\dfrac{L_1\cdot R_\mathrm{L}}{C_1}+\dfrac{L_1\cdot R_2}{C_1}+\dfrac{L_2\cdot R_1}{C_1}+R_1^2\cdot R_\mathrm{L}+R_1^2\cdot R_2\right)\\ +s\cdot\left(\dfrac{L_1}{C_1\cdot C_2}+\dfrac{R_1^2}{C_2}+\dfrac{R_\mathrm{L}\cdot R_1}{C_1}+\dfrac{R_1\cdot R_2}{C_1}\right)+\left(\dfrac{R_1}{C_1\cdot C_2}\right)\end{array}} \tag{4.16}$$

Finally, the coefficients of the numerator of the transfer function given in (4.16) can be written as

- $b_4\cdot s^4=s^4\cdot\left(M_{12}\cdot L_1\cdot R_\mathrm{L}\right)$
- $b_3\cdot s^3=s^3\left(\dfrac{M_{12}\cdot L_1}{C_2}+M_{12}\cdot R_1\cdot R_\mathrm{L}\right)$
- $b_2\cdot s^2=s^2\cdot\dfrac{M_{12}\cdot R_1}{C_2}$

The coefficients of the denominator of the transfer function given in (4.16) can be written as

- $a_5 \cdot s^5 = s^5 \cdot \left(L_1^2 \cdot L_2 - M_{12}^2 \cdot L_1\right)$
- $a_4 \cdot s^4 = s^4 \cdot \left(L_1^2 \cdot R_\mathrm{L} - M_{12}^2 \cdot R_1 + L_1^2 \cdot R_2 + 2 \cdot L_1 \cdot R_1 \cdot L_2\right)$
- $a_3 \cdot s^3 = s^3 \cdot \left(\dfrac{L_1^2}{C_2} + \dfrac{L_1 \cdot L_2}{C_1} + 2 \cdot L_1 \cdot R_1 \cdot R_\mathrm{L} + 2 \cdot L_1 \cdot R_1 \cdot R_2 + L_2 \cdot R_1^2\right)$
- $a_2 \cdot s^2 = s^2 \cdot \left(\dfrac{2 \cdot L_1 \cdot R_1}{C_2} + \dfrac{L_1 \cdot R_\mathrm{L}}{C_1} + \dfrac{L_1 \cdot R_2}{C_1} + \dfrac{L_2 \cdot R_1}{C_1} + R_1^2 \cdot R_\mathrm{L} + R_1^2 \cdot R_2\right)$
- $a_1 \cdot s = s \cdot \left(\dfrac{L_1}{C_1 \cdot C_2} + \dfrac{R_1^2}{C_2} + \dfrac{R_\mathrm{L} \cdot R_1}{C_1} + \dfrac{R_1 \cdot R_2}{C_1}\right)$
- $a_0 = \left(\dfrac{R_1}{C_1 \cdot C_2}\right)$

However, the transfer function $H(s)$ needs to be modified in order to get the transfer function as a ratio of the output voltage $V_\mathrm{out}$ across the load resistance $R_\mathrm{L}$ and the input voltage source $V_\mathrm{s}$. For that purpose, a new transfer function can be written as

$$\frac{V_\mathrm{out}}{V_\mathrm{s}} = \frac{V_2}{V_\mathrm{s}} \cdot \frac{V_\mathrm{out}}{V_2} = H(s) \cdot \frac{V_\mathrm{out}}{V_2}$$

where $\dfrac{V_\mathrm{out}}{V_2}$ can be determined as

$$\frac{V_\mathrm{out}}{V_2} = \frac{R_\mathrm{L}}{R_\mathrm{L} + \dfrac{1}{s \cdot C_2}} = \frac{1}{1 + \dfrac{1}{s \cdot C_2 \cdot R_\mathrm{L}}} = \frac{s}{s + \dfrac{1}{C_2 \cdot R_\mathrm{L}}}$$

## 4.2 Transfer function of a three winding model with two capacitors

A model of a three winding wireless charging system with two capacitors is presented in Fig. 4.2. The sinusoidal voltage source $v_{s1}(t)$ represents the fundamental component of the inverter's output square wave. A source impedance $Z_S$ consists of a source resistance $R_S$ which will be neglected in this analysis. An output impedance $Z_2$ consists of a capacitor $C_2$ and a load resistance $R_L$. An impedance in the auxiliary circuit is equal to a capacitor $C_3$.

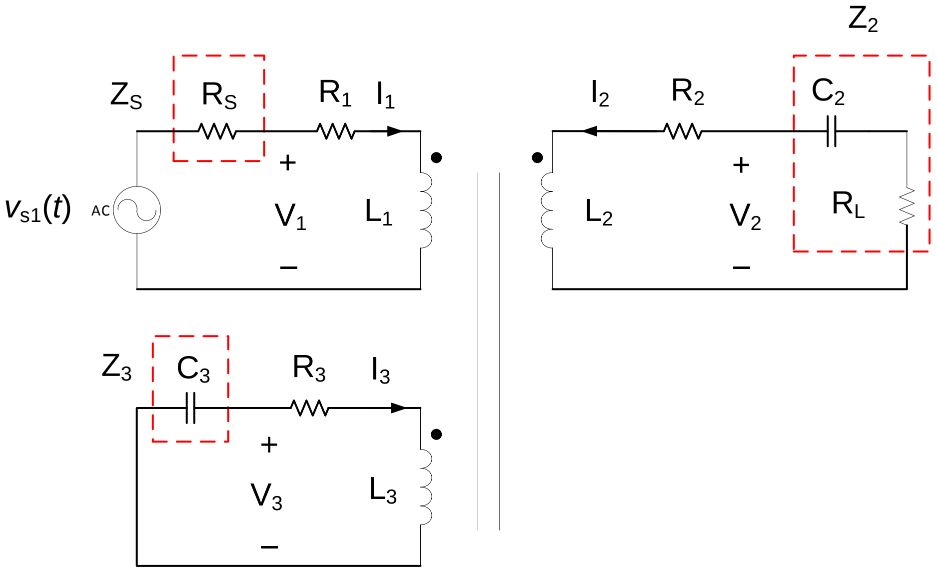


*Figure 4.2: Three winding model with two capacitors*

Kirchhoff's second law applied to three inductors gives

$$V_1 = s \cdot L_1 \cdot I_1 + s \cdot M_{12} \cdot I_2 + s \cdot M_{13} \cdot I_3 + R_1 \cdot I_1 \tag{4.17}$$

$$V_2 = s \cdot L_2 \cdot I_2 + s \cdot M_{12} \cdot I_1 + s \cdot M_{23} \cdot I_3 + R_2 \cdot I_2 \tag{4.18}$$

$$V_3 = s \cdot L_3 \cdot I_3 + s \cdot M_{13} \cdot I_1 + s \cdot M_{23} \cdot I_2 + R_3 \cdot I_3 \tag{4.19}$$

where $V_1$, $V_2$ and $V_3$ are the voltages across the inductors, $R_1$, $R_2$ and $R_3$ are the coil's resistances, $L_1$, $L_2$ and $L_3$ represent the inductance of the coils, $I_1$, $I_2$ and $I_3$ are the currents through the primary, secondary and the tertiary circuit and $M_{12}$, $M_{13}$ and $M_{23}$ represent the mutual inductances.

The output impedance $Z_2$ can be expressed as

$$Z_2 = R_\mathrm{L} + \frac{1}{s \cdot C_2} \tag{4.20}$$

The impedance $Z_3$ in the auxiliary circuit is equal to

$$Z_3 = \frac{1}{s \cdot C_3} \tag{4.21}$$

The current $I_2$ in the secondary circuit can be determined as

$$I_2 = -\frac{V_2}{Z_2} \tag{4.22}$$

The current $I_3$ in the auxiliary winding can be written as

$$I_3 = -\frac{V_3}{Z_3} \tag{4.23}$$

By using (4.17), the expression for the current $I_1$ in the primary circuit can be given as

$$I_1 = \frac{V_1 - s \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{s \cdot L_1 + R_1} \tag{4.24}$$

In order to eliminate the current $I_1$ from the expression given in (4.18), the output voltage $V_2$, after inserting (4.24) into (4.18), can be written as

$$V_2 = \left(s \cdot L_2 + R_2\right) \cdot I_2 + s \cdot M_{12} \cdot \left(\frac{V_1 - s \cdot \left(M_{12} \cdot I_2 + M_{13} \cdot I_3\right)}{s \cdot L_1 + R_1}\right) + s \cdot M_{23} \cdot I_3 \tag{4.25}$$

The currents $I_2$ and $I_3$ in (4.25) can be replaced with expressions given in (4.22) and (4.23). That yields us to an expression for the voltage in the auxiliary circuit $V_3$ and it can be determined as

$$V_2 = -\left(s \cdot L_2 + R_2\right) \cdot \frac{V_2}{Z_2} + \frac{s \cdot M_{12}}{s \cdot L_1 + R_1} \cdot V_1 + \frac{s^2 \cdot M_{12}^2}{s \cdot L_1 + R_1} \cdot \frac{V_2}{Z_2} + \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{s \cdot L_1 + R_1} - s \cdot M_{23}\right) \cdot \frac{V_3}{Z_3}$$

$$V_2 \left[1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot \left(s \cdot L_1 + R_1\right)} + \frac{s \cdot L_2 + R_2}{Z_2}\right] = \frac{s \cdot M_{12}}{s \cdot L_1 + R_1} \cdot V_1 + V_3 \cdot \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot \left(s \cdot L_1 + R_1\right)} - \frac{s \cdot M_{23}}{Z_3}\right)$$

$$V_3 = \frac{V_2 \cdot \left(1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2}\right) - V_1 \cdot \frac{s \cdot M_{12}}{s \cdot L_1 + R_1}}{\left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right)} \tag{4.26}$$

The same procedure can be done with the expression for the voltage across the auxiliary winding which is given in (4.19). The currents $I_1$, $I_2$ and $I_3$ in (4.19) can be replaced with (4.24), (4.22) and (4.23) respectively. The voltage across the tertiary winding can be expressed as

$$V_3 = (s \cdot L_3 + R_3) \cdot I_3 + s \cdot M_{13} \cdot \left(\frac{V_1 - s \cdot (M_{12} \cdot I_2 + M_{13} \cdot I_3)}{s \cdot L_1 + R_1}\right) + s \cdot M_{23} \cdot I_2$$

$$V_3 = -(s \cdot L_3 + R_3) \cdot \frac{V_3}{Z_3} + \frac{s \cdot M_{13}}{s \cdot L_1 + R_1} \cdot V_1 + \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3} \cdot V_3 + \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right) \cdot V_2$$

$$V_3 \cdot \left[1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right] = \frac{s \cdot M_{13}}{s \cdot L_1 + R_1} \cdot V_1 + \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right) \cdot V_2$$

$$V_3 = \frac{\frac{s \cdot M_{13}}{s \cdot L_1 + R_1} \cdot V_1 + \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right) \cdot V_2}{\left[1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right]} \tag{4.27}$$

After inserting (4.26) into (4.27), we can easily find transfer function $\frac{V_2}{V_1}$:

$$\frac{V_2 \cdot \left(1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2}\right) - \frac{s \cdot M_{12} \cdot V_1}{s \cdot L_1 + R_1}}{\left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right)} = \frac{\frac{s \cdot M_{13} \cdot V_1}{s \cdot L_1 + R_1} + \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right) \cdot V_2}{\left[1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right]}$$

$$V_2 \cdot \left(1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2}\right) \cdot \left(1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right) -$$

$$V_1 \cdot \frac{s \cdot M_{12}}{s \cdot L_1 + R_1} \cdot \left(1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right) =$$

$$\left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right) \cdot \left(\frac{s \cdot M_{13}}{s \cdot L_1 + R_1}\right) \cdot V_1 +$$

$$\left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right) \cdot \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right) \cdot V_2$$

The final expression for the transfer function $G(s) = \frac{V_2}{V_1}$ can be given as

$$G(s) = \frac{N_1(s) \cdot N_2(s) + N_3(s) \cdot N_4(s)}{D_1(s) \cdot D_2(s) - D_3(s) \cdot D_4(s)} \tag{4.28}$$

where the $N(s)$ and $D(s)$ represent the parts of numerator and denominator of the transfer function.

$$N_1(s) = \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right)$$

$$N_2(s) = \left(\frac{s \cdot M_{13}}{s \cdot L_1 + R_1}\right)$$

$$N_3(s) = \left(\frac{s \cdot M_{12}}{s \cdot L_1 + R_1}\right)$$

$$N_4(s) = \left(1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right)$$

$$D_1(s) = \left(1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2}\right)$$

$$D_2(s) = \left(1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1) \cdot Z_3}\right)$$

$$D_3(s) = \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right)$$

$$D_4(s) = \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{(s \cdot L_1 + R_1) \cdot Z_2} - \frac{s \cdot M_{23}}{Z_2}\right)$$

In order to get the transfer function of a three winding model with two capacitors in the form of

$$H(s)=\frac{b_m \cdot s^m + b_{m-1} \cdot s^{m-1} + ... + b_1 \cdot s + b_0}{a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + ... + a_1 \cdot s + a_0}$$

mathematical operations will be applied to (4.28).

First, the parts of the numerator $N_1(s)\cdot N_2(s)$ and $N_3(s)\cdot N_4(s)$ will be expressed as

$$\begin{aligned}N_1(s)\cdot N_2(s) &= \left(\frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3}\right)\cdot\left(\frac{s \cdot M_{13}}{s \cdot L_1 + R_1}\right)\\ &= \left(\frac{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)}{Z_3 \cdot (s \cdot L_1 + R_1)}\right)\cdot\left(\frac{s \cdot M_{13}}{s \cdot L_1 + R_1}\right)\\ &= \frac{s^3 \cdot M_{12} \cdot M_{13}^2 - s^2 \cdot M_{23} \cdot M_{13} \cdot (s \cdot L_1 + R_1)}{Z_3 \cdot (s \cdot L_1 + R_1)^2}\end{aligned}$$

$$\begin{aligned}N_3(s)\cdot N_4(s) &= \left(\frac{s \cdot M_{12}}{s \cdot L_1 + R_1}\right)\cdot\left(1 + \frac{s \cdot L_3 + R_3}{Z_3} - \frac{s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1)\cdot Z_3}\right)\\ &= \left(\frac{s \cdot M_{12}}{s \cdot L_1 + R_1}\right)\cdot\left(\frac{(s \cdot L_1 + R_1)\cdot Z_3 + (s \cdot L_1 + R_1)\cdot(s \cdot L_3 + R_3) - s^2 \cdot M_{13}^2}{(s \cdot L_1 + R_1)\cdot Z_3}\right)\\ &= \left(\frac{s \cdot M_{12} \cdot Z_3 \cdot (s \cdot L_1 + R_1) + s \cdot M_{12} \cdot (s \cdot L_1 + R_1)\cdot(s \cdot L_3 + R_3) - s^3 \cdot M_{12} \cdot M_{13}^2}{(s \cdot L_1 + R_1)^2 \cdot Z_3}\right)\end{aligned}$$

Parts of the numerator $N_1(s)\cdot N_2(s) + N_3(s)\cdot N_4(s)$ can be added up and written as

$$N(s) = \frac{-s^2 \cdot M_{23} \cdot M_{13} \cdot (s \cdot L_1 + R_1) + s \cdot M_{12} \cdot Z_3 \cdot (s \cdot L_1 + R_1) + s \cdot M_{12} \cdot (s \cdot L_1 + R_1)\cdot(s \cdot L_3 + R_3)}{(s \cdot L_1 + R_1)^2 \cdot Z_3}$$

After dividing the numerator with $(s \cdot L_1 + R_1)$, it can be determined as

$$N(s) = \frac{s \cdot M_{12} \cdot Z_3 - s^2 \cdot M_{23} \cdot M_{13} + s \cdot M_{12} \cdot (s \cdot L_3 + R_3)}{(s \cdot L_1 + R_1)\cdot Z_3} \tag{4.29}$$

The same procedure will be applied to the denominator. First, the parts of the denominator $D_1(s)\cdot D_2(s)$ and $D_3(s)\cdot D_4(s)$ will be given as

$$D_1(s)\cdot D_2(s)=\left(\frac{Z_2\cdot(s\cdot L_1+R_1)-s^2\cdot M_{12}^2+(s\cdot L_2+R_2)\cdot(s\cdot L_1+R_1)}{Z_2\cdot(s\cdot L_1+R_1)}\right)\cdot$$
$$\left(\frac{Z_3\cdot(s\cdot L_1+R_1)-s^2\cdot M_{13}^2+(s\cdot L_3+R_3)\cdot(s\cdot L_1+R_1)}{(s\cdot L_1+R_1)\cdot Z_3}\right)$$

$$D_1(s)\cdot D_2(s)=\frac{\begin{gathered}Z_2\cdot Z_3\cdot(s\cdot L_1+R_1)^2-s^2\cdot M_{12}^2\cdot Z_3\cdot(s\cdot L_1+R_1)+Z_3\cdot(s\cdot L_1+R_1)^2\cdot(s\cdot L_2+R_2)-\\ s^2\cdot M_{13}^2\cdot Z_2\cdot(s\cdot L_1+R_1)+s^4\cdot M_{13}^2\cdot M_{12}^2-s^2\cdot M_{13}^2\cdot(s\cdot L_2+R_2)\cdot(s\cdot L_1+R_1)+\\ Z_2\cdot(s\cdot L_3+R_3)\cdot(s\cdot L_1+R_1)^2-s^2\cdot M_{12}^2\cdot(s\cdot L_3+R_3)\cdot(s\cdot L_1+R_1)+\\ (s\cdot L_3+R_3)\cdot(s\cdot L_2+R_2)\cdot(s\cdot L_1+R_1)^2\end{gathered}}{Z_2\cdot Z_3\cdot(s\cdot L_1+R_1)^2}$$

$$\begin{aligned}D_3(s)\cdot D_4(s)&=\left(\frac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot(s\cdot L_1+R_1)}-\frac{s\cdot M_{23}}{Z_3}\right)\cdot\left(\frac{s^2\cdot M_{12}\cdot M_{13}}{(s\cdot L_1+R_1)\cdot Z_2}-\frac{s\cdot M_{23}}{Z_2}\right)\\ &=\left(\frac{s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot(s\cdot L_1+R_1)}{Z_3\cdot(s\cdot L_1+R_1)}\right)\cdot\left(\frac{s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot(s\cdot L_1+R_1)}{(s\cdot L_1+R_1)\cdot Z_2}\right)\\ &=\frac{s^4\cdot M_{12}^2\cdot M_{13}^2-2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}\cdot(s\cdot L_1+R_1)+s^2\cdot M_{23}^2\cdot(s\cdot L_1+R_1)^2}{Z_3\cdot Z_2\cdot(s\cdot L_1+R_1)^2}\end{aligned}$$

After subtraction of the denominator's parts $D_1(s)\cdot D_2(s)-D_3(s)\cdot D_4(s)$, it can be written as

$$D(s)=\frac{\begin{gathered}Z_2\cdot Z_3\cdot(s\cdot L_1+R_1)^2-s^2\cdot M_{12}^2\cdot Z_3\cdot(s\cdot L_1+R_1)+Z_3\cdot(s\cdot L_1+R_1)^2\cdot(s\cdot L_2+R_2)-\\ s^2\cdot M_{13}^2\cdot Z_2\cdot(s\cdot L_1+R_1)+\cancel{s^4\cdot M_{13}^2\cdot M_{12}^2}-s^2\cdot M_{13}^2\cdot(s\cdot L_2+R_2)\cdot(s\cdot L_1+R_1)+\\ Z_2\cdot(s\cdot L_3+R_3)\cdot(s\cdot L_1+R_1)^2-s^2\cdot M_{12}^2\cdot(s\cdot L_3+R_3)\cdot(s\cdot L_1+R_1)+\\ (s\cdot L_3+R_3)\cdot(s\cdot L_2+R_2)\cdot(s\cdot L_1+R_1)^2-\cancel{s^4\cdot M_{12}^2\cdot M_{13}^2}+\\ 2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}\cdot(s\cdot L_1+R_1)-s^2\cdot M_{23}^2\cdot(s\cdot L_1+R_1)^2\end{gathered}}{Z_2\cdot Z_3\cdot(s\cdot L_1+R_1)^2}$$

The denominator can be divided with $\left(s\cdot L_1+R_1\right)$, and expressed as

$$\frac{\begin{gathered}Z_2\cdot Z_3\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot Z_3+Z_3\cdot\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)-s^2\cdot M_{13}^2\cdot Z_2-\\ s^2\cdot M_{13}^2\cdot\left(s\cdot L_2+R_2\right)+Z_2\cdot\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot\left(s\cdot L_3+R_3\right)+\\ \left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)+2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^2\cdot M_{23}^2\cdot\left(s\cdot L_1+R_1\right)\end{gathered}}{Z_2\cdot Z_3\cdot\left(s\cdot L_1+R_1\right)}$$

Now, the transfer function given in (4.28) can be determined as

$$\frac{\dfrac{s\cdot M_{12}\cdot Z_3-s^2\cdot M_{23}\cdot M_{13}+s\cdot M_{12}\cdot\left(s\cdot L_3+R_3\right)}{\cancel{\left(s\cdot L_1+R_1\right)}\cdot Z_3}}{\dfrac{\begin{gathered}Z_2\cdot Z_3\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot Z_3+Z_3\cdot\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)-s^2\cdot M_{13}^2\cdot Z_2-\\ s^2\cdot M_{13}^2\cdot\left(s\cdot L_2+R_2\right)+Z_2\cdot\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot\left(s\cdot L_3+R_3\right)+\\ \left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)+2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^2\cdot M_{23}^2\cdot\left(s\cdot L_1+R_1\right)\end{gathered}}{Z_2\cdot\cancel{Z_3\cdot\left(s\cdot L_1+R_1\right)}}}$$

$$\frac{Z_2\cdot\left(s\cdot M_{12}\cdot Z_3-s^2\cdot M_{23}\cdot M_{13}+s\cdot M_{12}\cdot\left(s\cdot L_3+R_3\right)\right)}{\begin{gathered}Z_2\cdot Z_3\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot Z_3+Z_3\cdot\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)-s^2\cdot M_{13}^2\cdot Z_2-\\ s^2\cdot M_{13}^2\cdot\left(s\cdot L_2+R_2\right)+Z_2\cdot\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot\left(s\cdot L_3+R_3\right)+\\ \left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)+2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^2\cdot M_{23}^2\cdot\left(s\cdot L_1+R_1\right)\end{gathered}} \tag{4.30}$$

The next step is to insert the expressions for $Z_2$ and $Z_3$ into (4.30)

- $Z_2=\left(R_L+\dfrac{1}{s\cdot C_2}\right)$
- $Z_3=\dfrac{1}{s\cdot C_3}$

And the transfer function can be written as

$$\frac{\left(R_L+\dfrac{1}{s\cdot C_2}\right)\cdot\left(s\cdot M_{12}\cdot\dfrac{1}{s\cdot C_3}-s^2\cdot M_{23}\cdot M_{13}+s\cdot M_{12}\cdot\left(s\cdot L_3+R_3\right)\right)}{\begin{gathered}\left(R_L+\dfrac{1}{s\cdot C_2}\right)\cdot\dfrac{1}{s\cdot C_3}\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2\cdot\dfrac{1}{s\cdot C_3}+\dfrac{1}{s\cdot C_3}\cdot\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)-\\ s^2\cdot M_{13}^2\cdot\left(R_L+\dfrac{1}{s\cdot C_2}\right)-s^2\cdot M_{13}^2\cdot\left(s\cdot L_2+R_2\right)+\left(R_L+\dfrac{1}{s\cdot C_2}\right)\cdot\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)-\\ s^2\cdot M_{12}^2\cdot\left(s\cdot L_3+R_3\right)+\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)+\\ 2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^2\cdot M_{23}^2\cdot\left(s\cdot L_1+R_1\right)\end{gathered}}$$

A procedure of finding the numerator and the denominator of the transfer function is shown below.

$$\frac{\left(R_{\mathrm{L}}+\frac{1}{s\cdot C_2}\right)\cdot\left(\frac{s\cdot M_{12}}{s\cdot C_3}-s^2\cdot M_{23}\cdot M_{13}+s\cdot M_{12}\cdot\left(s\cdot L_3+R_3\right)\right)}{\begin{gathered}\frac{R_{\mathrm{L}}}{s\cdot C_3}\cdot\left(s\cdot L_1+R_1\right)+\frac{1}{s^2\cdot C_3\cdot C_2}\cdot\left(s\cdot L_1+R_1\right)-\frac{s\cdot M_{12}^2}{C_3}+\frac{\left(s\cdot L_1+R_1\right)\cdot\left(s\cdot L_2+R_2\right)}{s\cdot C_3}-\\ s^2\cdot M_{13}^2\cdot R_{\mathrm{L}}-\frac{s\cdot M_{13}^2}{C_2}-s^3\cdot M_{13}^2\cdot L_2-s^2\cdot M_{13}^2\cdot R_2+R_{\mathrm{L}}\cdot\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)\\ +\frac{\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_1+R_1\right)}{s\cdot C_2}-s^3\cdot M_{12}^2\cdot L_3-s^2\cdot M_{12}^2\cdot R_3+\left(s\cdot L_3+R_3\right)\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)+\\ 2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^2\cdot M_{23}^2\cdot\left(s\cdot L_1+R_1\right)\end{gathered}}$$

$$\frac{\left(R_{\mathrm{L}}+\frac{1}{s\cdot C_2}\right)\cdot\left(\frac{M_{12}}{C_3}-s^2\cdot M_{23}\cdot M_{13}+s\cdot M_{12}\cdot\left(s\cdot L_3+R_3\right)\right)}{\begin{gathered}\frac{s\cdot R_{\mathrm{L}}\cdot L_1}{s\cdot C_3}+\frac{R_{\mathrm{L}}\cdot R_1}{s\cdot C_3}+\frac{s\cdot L_1}{s^2\cdot C_3\cdot C_2}+\frac{R_1}{s^2\cdot C_3\cdot C_2}-\frac{s\cdot M_{12}^2}{C_3}+\frac{s^2\cdot L_1\cdot L_2}{s\cdot C_3}+\frac{s\cdot L_1\cdot R_2}{s\cdot C_3}+\frac{s\cdot L_2\cdot R_1}{s\cdot C_3}\\ +\frac{R_1\cdot R_2}{s\cdot C_3}-s^2\cdot M_{13}^2\cdot R_{\mathrm{L}}-\frac{s\cdot M_{13}^2}{C_2}-s^3\cdot M_{13}^2\cdot L_2-s^2\cdot M_{13}^2\cdot R_2+\\ R_{\mathrm{L}}\cdot\left(s^2\cdot L_1\cdot L_3+s\cdot L_1\cdot R_3+s\cdot L_3\cdot R_1+R_1\cdot R_3\right)+\frac{s^2\cdot L_1\cdot L_3}{s\cdot C_2}+\frac{s\cdot L_1\cdot R_3}{s\cdot C_2}+\frac{s\cdot L_3\cdot R_1}{s\cdot C_2}+\frac{R_1\cdot R_3}{s\cdot C_2}\\ -s^3\cdot M_{12}^2\cdot L_3-s^2\cdot M_{12}^2\cdot R_3+\left(s^2\cdot L_1\cdot L_3+s\cdot L_1\cdot R_3+s\cdot L_3\cdot R_1+R_1\cdot R_3\right)\cdot\left(s\cdot L_2+R_2\right)+\\ 2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^3\cdot M_{23}^2\cdot L_1-s^2\cdot M_{23}^2\cdot R_1\end{gathered}}$$

The numerator of the transfer function can be written as

$$\begin{aligned}N(s)&=\frac{M_{12}}{C_3}\cdot R_{\mathrm{L}}-s^2\cdot M_{23}\cdot M_{13}\cdot R_{\mathrm{L}}+s^2\cdot M_{12}\cdot L_3\cdot R_{\mathrm{L}}+s\cdot M_{12}\cdot R_3\cdot R_{\mathrm{L}}+\\&\quad\frac{M_{12}}{s\cdot C_2\cdot C_3}-\frac{s^2\cdot M_{23}\cdot M_{13}}{s\cdot C_2}+\frac{s^2\cdot M_{12}\cdot L_3}{s\cdot C_2}+\frac{s\cdot M_{12}\cdot R_3}{s\cdot C_2}\\&=\frac{M_{12}}{C_3}\cdot R_{\mathrm{L}}-s^2\cdot M_{23}\cdot M_{13}\cdot R_{\mathrm{L}}+s^2\cdot M_{12}\cdot L_3\cdot R_{\mathrm{L}}+s\cdot M_{12}\cdot R_3\cdot R_{\mathrm{L}}+\\&\quad\frac{M_{12}}{s\cdot C_2\cdot C_3}-\frac{s\cdot M_{23}\cdot M_{13}}{C_2}+\frac{s\cdot M_{12}\cdot L_3}{C_2}+\frac{M_{12}\cdot R_3}{C_2}\\&=s^2\cdot\left(M_{12}\cdot L_3\cdot R_{\mathrm{L}}-M_{23}\cdot M_{13}\cdot R_{\mathrm{L}}\right)+s\cdot\left(M_{12}\cdot R_3\cdot R_{\mathrm{L}}-\frac{M_{23}\cdot M_{13}}{C_2}+\frac{M_{12}\cdot L_3}{C_2}\right)\\&\quad+\left(\frac{M_{12}}{C_3}\cdot R_{\mathrm{L}}+\frac{M_{12}\cdot R_3}{C_2}\right)+\frac{1}{s}\cdot\frac{M_{12}}{C_2\cdot C_3}\end{aligned}$$

The same procedure will be applied to denominator

$$D(s)=\frac{R_L\cdot L_1}{C_3}+\frac{R_L\cdot R_1}{s\cdot C_3}+\frac{L_1}{s\cdot C_3\cdot C_2}+\frac{R_1}{s^2\cdot C_3\cdot C_2}-\frac{s\cdot M_{12}^2}{C_3}+\frac{s\cdot L_1\cdot L_2}{C_3}+\frac{L_1\cdot R_2}{C_3}+\frac{L_2\cdot R_1}{C_3}+$$
$$\frac{R_1\cdot R_2}{s\cdot C_3}-s^2\cdot M_{13}^2\cdot R_L-\frac{s\cdot M_{13}^2}{C_2}-s^3\cdot M_{13}^2\cdot L_2-s^2\cdot M_{13}^2\cdot R_2+s^2\cdot L_1\cdot L_3\cdot R_L+$$
$$s\cdot L_1\cdot R_3\cdot R_L+s\cdot L_3\cdot R_1\cdot R_L+R_1\cdot R_3\cdot R_L+\frac{s\cdot L_1\cdot L_3}{C_2}+\frac{L_1\cdot R_3}{C_2}+\frac{L_3\cdot R_1}{C_2}+\frac{R_1\cdot R_3}{s\cdot C_2}-$$
$$s^3\cdot M_{12}^2\cdot L_3-s^2\cdot M_{12}^2\cdot R_3+s^3\cdot L_1\cdot L_2\cdot L_3+s^2\cdot L_1\cdot L_2\cdot R_3+s^2\cdot L_2\cdot L_3\cdot R_1+$$
$$s\cdot L_2\cdot R_1\cdot R_3+s^2\cdot L_1\cdot L_3\cdot R_2+s\cdot L_1\cdot R_3\cdot R_2+s\cdot L_3\cdot R_1\cdot R_2+R_1\cdot R_3\cdot R_2+$$
$$2\cdot s^3\cdot M_{12}\cdot M_{13}\cdot M_{23}-s^3\cdot M_{23}^2\cdot L_1-s^2\cdot M_{23}^2\cdot R_1$$

$$D(s)=s^3\cdot\left(L_1\cdot L_2\cdot L_3-M_{13}^2\cdot L_2-M_{12}^2\cdot L_3-M_{23}^2\cdot L_1+2\cdot M_{12}\cdot M_{13}\cdot M_{23}\right)+$$
$$s^2\cdot\left(-M_{13}^2\cdot R_L-M_{13}^2\cdot R_2+L_1\cdot L_3\cdot R_L-M_{12}^2\cdot R_3+L_1\cdot L_2\cdot R_3+L_2\cdot L_3\cdot R_1+L_1\cdot L_3\cdot R_2-M_{23}^2\cdot R_1\right)+$$
$$s\cdot\left(-\frac{M_{12}^2}{C_3}+\frac{L_1\cdot L_2}{C_3}-\frac{M_{13}^2}{C_2}+L_1\cdot R_3\cdot R_L+L_3\cdot R_1\cdot R_L+\frac{L_1\cdot L_3}{C_2}+L_2\cdot R_1\cdot R_3+L_1\cdot R_3\cdot R_2+L_3\cdot R_1\cdot R_2\right)+$$
$$\left(\frac{R_L\cdot L_1}{C_3}+\frac{L_1\cdot R_2}{C_3}+\frac{L_2\cdot R_1}{C_3}+R_1\cdot R_3\cdot R_L+\frac{L_1\cdot R_3}{C_2}+\frac{L_3\cdot R_1}{C_2}+R_1\cdot R_3\cdot R_2\right)+$$
$$\frac{1}{s}\cdot\left(\frac{R_L\cdot R_1}{C_3}+\frac{L_1}{C_3\cdot C_2}+\frac{R_1\cdot R_2}{C_3}+\frac{R_1\cdot R_3}{C_2}\right)+\frac{1}{s^2}\cdot\frac{R_1}{C_3\cdot C_2}$$

After multiplying the numerator and the denominator with $s^2$, the transfer function can be given as

$$\frac{s^4\cdot\left(M_{12}\cdot L_3\cdot R_L-M_{23}\cdot M_{13}\cdot R_L\right)+s^3\cdot\left(M_{12}\cdot R_3\cdot R_L-\frac{M_{23}\cdot M_{13}}{C_2}+\frac{M_{12}\cdot L_3}{C_2}\right)+s^2\cdot\left(\frac{M_{12}}{C_3}\cdot R_L+\frac{M_{12}\cdot R_3}{C_2}\right)+s\cdot\frac{M_{12}}{C_2\cdot C_3}}{\begin{array}{c}s^5\cdot\left(L_1\cdot L_2\cdot L_3-M_{13}^2\cdot L_2-M_{12}^2\cdot L_3-M_{23}^2\cdot L_1+2\cdot M_{12}\cdot M_{13}\cdot M_{23}\right)+\\ s^4\cdot\left(-M_{13}^2\cdot R_L-M_{13}^2\cdot R_2+L_1\cdot L_3\cdot R_L-M_{12}^2\cdot R_3+L_1\cdot L_2\cdot R_3+L_2\cdot L_3\cdot R_1+L_1\cdot L_3\cdot R_2-M_{23}^2\cdot R_1\right)+\\ s^3\cdot\left(-\frac{M_{12}^2}{C_3}+\frac{L_1\cdot L_2}{C_3}-\frac{M_{13}^2}{C_2}+L_1\cdot R_3\cdot R_L+L_3\cdot R_1\cdot R_L+\frac{L_1\cdot L_3}{C_2}+L_2\cdot R_1\cdot R_3+L_1\cdot R_3\cdot R_2+L_3\cdot R_1\cdot R_2\right)+\\ s^2\cdot\left(\frac{R_L\cdot L_1}{C_3}+\frac{L_1\cdot R_2}{C_3}+\frac{L_2\cdot R_1}{C_3}+R_1\cdot R_3\cdot R_L+\frac{L_1\cdot R_3}{C_2}+\frac{L_3\cdot R_1}{C_2}+R_1\cdot R_3\cdot R_2\right)+\\ s\cdot\left(\frac{R_L\cdot R_1}{C_3}+\frac{L_1}{C_3\cdot C_2}+\frac{R_1\cdot R_2}{C_3}+\frac{R_1\cdot R_3}{C_2}\right)+\frac{R_1}{C_3\cdot C_2}\end{array}}$$

Similar to the previous wireless charging system with the two coils and the two capacitors, the transfer function $H(s)$ needs to be modified in order to get the transfer function as a ratio of the output voltage $V_{\mathrm{out}}$ across the load resistance $R_{\mathrm{L}}$ and the input voltage source $V_{\mathrm{s}}$. For that purpose, a new transfer function can be written as

$$\frac{V_{\mathrm{out}}}{V_{\mathrm{s}}} = \frac{V_2}{V_{\mathrm{s}}} \cdot \frac{V_{\mathrm{out}}}{V_2} = H(s) \cdot \frac{V_{\mathrm{out}}}{V_2}$$

where $\frac{V_{\mathrm{out}}}{V_2}$ can be determined as

$$\frac{V_{\mathrm{out}}}{V_2} = \frac{R_{\mathrm{L}}}{R_{\mathrm{L}} + \frac{1}{s \cdot C_2}} = \frac{1}{1 + \frac{1}{s \cdot C_2 \cdot R_{\mathrm{L}}}} = \frac{s}{s + \frac{1}{C_2 \cdot R_{\mathrm{L}}}}$$

## 4.3 Transfer function of a three winding model with three capacitors

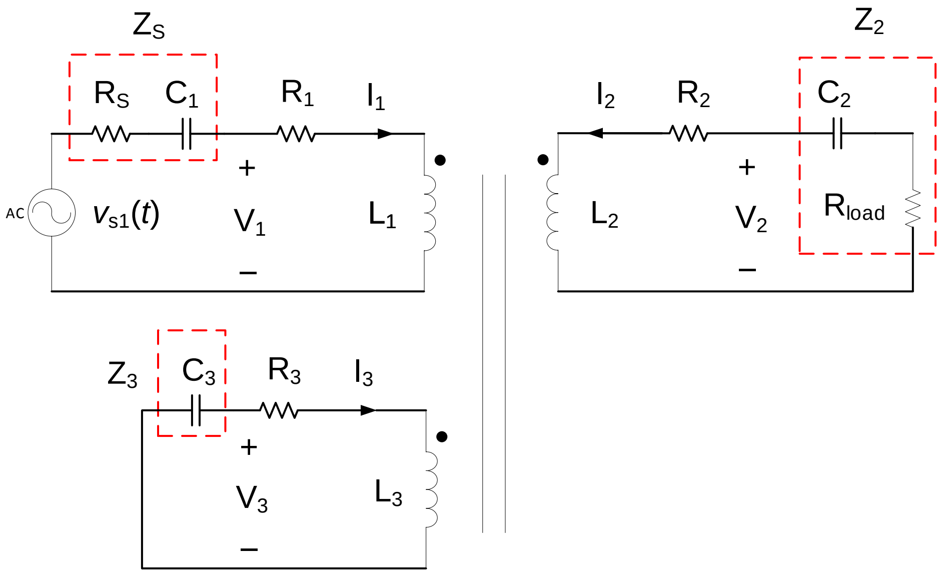


*Figure 4.3: Three winding model with three capacitors*

The same equations that were used in the three winding model with two capacitors can be applied to this model with three capacitors. The only difference between these two models is that the model with three capacitors has a capacitor in the primary circuit so the source impedance consists of a capacitor $C_1$ and a source resistor $R_S$ which will be neglected.

The expression for the source impedance can be given as

$$Z_S = \frac{1}{s \cdot C_1} \tag{4.31}$$

The transfer function $H(s) = \frac{V_2}{V_S}$ of a three winding wireless charging system with two capacitors can be written as

$$\frac{V_2}{V_S} = \frac{V_2}{V_1} \cdot \frac{V_1}{V_S} \tag{4.32}$$

where $V_S$ is the input voltage source in the primary circuit and it can be expressed as

$$V_S = V_1 + Z_S \cdot I_1 \tag{4.33}$$

After inserting the expression for the current $I_1$ given in (4.24) into (4.33), the expression for the input voltage source $V_S$ can be given as

$$V_S = V_1 + Z_S \cdot \frac{V_1 - s \cdot (M_{12} \cdot I_2 + M_{13} \cdot I_3)}{s \cdot L_1 + R_1} \tag{4.34}$$

In order to determine the transfer function $H(s) = \frac{V_2}{V_S}$, expressions for the currents $I_2$ and $I_3$ that were given in (4.22) and (4.23), can be inserted into (4.34), so a new expression for the voltage source $V_S$ can be written as

$$V_S = V_1 + \frac{Z_S \cdot V_1}{s \cdot L_1 + R_1} + \frac{s \cdot M_{12} \cdot Z_S \cdot V_2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot M_{13} \cdot Z_S \cdot V_3}{Z_3 \cdot (s \cdot L_1 + R_1)}$$

$$V_S = V_1 \left( 1 + \frac{Z_S}{s \cdot L_1 + R_1} \right) + V_2 \cdot \frac{s \cdot M_{12} \cdot Z_S}{Z_2 \cdot (s \cdot L_1 + R_1)} + V_3 \cdot \frac{s \cdot M_{13} \cdot Z_S}{Z_3 \cdot (s \cdot L_1 + R_1)} \tag{4.35}$$

Equation (4.35) can be divided with $V_2$ and it can be given as

$$\frac{1}{H(s)} = \frac{V_S}{V_2} = \frac{V_1}{V_2} \left( 1 + \frac{Z_S}{s \cdot L_1 + R_1} \right) + \frac{s \cdot M_{12} \cdot Z_S}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{V_3}{V_2} \cdot \frac{s \cdot M_{13} \cdot Z_S}{Z_3 \cdot (s \cdot L_1 + R_1)} \tag{4.36}$$

Equation (4.36) has in its notation $V_3$, which can be eliminated by using (4.26), and it is written again as

$$V_3 = \frac{V_2 \cdot \left( 1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2} \right) - V_1 \cdot \frac{s \cdot M_{12}}{s \cdot L_1 + R_1}}{\left( \frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3} \right)} \tag{4.37}$$

After inserting (4.37) into (4.36), and replacing $\frac{V_1}{V_2}$ with $\frac{1}{G(s)}$, where $G(s)$ is given in (4.28), the transfer function can be determined as

$$\frac{V_S}{V_2} = \frac{V_1}{V_2} \left( 1 + \frac{Z_S}{s \cdot L_1 + R_1} \right) + \frac{s \cdot M_{12} \cdot Z_S}{Z_2 \cdot (s \cdot L_1 + R_1)} +$$
$$\frac{V_2 \cdot \left( 1 - \frac{s^2 \cdot M_{12}^2}{Z_2 \cdot (s \cdot L_1 + R_1)} + \frac{s \cdot L_2 + R_2}{Z_2} \right) - V_1 \cdot \frac{s \cdot M_{12}}{s \cdot L_1 + R_1}}{V_2 \cdot \left( \frac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \frac{s \cdot M_{23}}{Z_3} \right)} \cdot \frac{s \cdot M_{13} \cdot Z_S}{Z_3 \cdot (s \cdot L_1 + R_1)}$$

$$\frac{V_S}{V_2}=\frac{1}{G(s)}\cdot\left(1+\frac{Z_S}{s\cdot L_1+R_1}-\frac{\dfrac{s^2\cdot M_{12}\cdot M_{13}\cdot Z_S}{Z_3\cdot(s\cdot L_1+R_1)^2}}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot(s\cdot L_1+R_1)}-\dfrac{s\cdot M_{23}}{Z_3}\right)}\right)+\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)}+$$

$$\frac{\left(1-\dfrac{s^2\cdot M_{12}^2}{Z_2\cdot(s\cdot L_1+R_1)}+\dfrac{s\cdot L_2+R_2}{Z_2}\right)}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot(s\cdot L_1+R_1)}-\dfrac{s\cdot M_{23}}{Z_3}\right)}\cdot\frac{s\cdot M_{13}\cdot Z_S}{Z_3\cdot(s\cdot L_1+R_1)}$$

The final expression for the transfer function $H(s)=\dfrac{V_2}{V_S}$ of the three winding model with three capacitors can be determined as

$$\frac{V_2}{V_S}=\frac{1}{\left(\begin{array}{c}\dfrac{1}{G(s)}\cdot\left(1+\dfrac{Z_S}{s\cdot L_1+R_1}-\dfrac{\dfrac{s^2\cdot M_{12}\cdot M_{13}\cdot Z_S}{Z_3\cdot(s\cdot L_1+R_1)^2}}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot(s\cdot L_1+R_1)}-\dfrac{s\cdot M_{23}}{Z_3}\right)}\right)+\dfrac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot(s\cdot L_1+R_1)}+\\ \dfrac{\left(1-\dfrac{s^2\cdot M_{12}^2}{Z_2\cdot(s\cdot L_1+R_1)}+\dfrac{s\cdot L_2+R_2}{Z_2}\right)}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot(s\cdot L_1+R_1)}-\dfrac{s\cdot M_{23}}{Z_3}\right)}\cdot\dfrac{s\cdot M_{13}\cdot Z_S}{Z_3\cdot(s\cdot L_1+R_1)}\end{array}\right)}$$

For further analysis, the transfer function can also be written in the form of

$$\frac{V_2}{V_S}=\frac{1}{\left(\dfrac{1}{G(s)}\cdot D_1(s)+D_2(s)\right)}$$

In order to get the transfer function $\dfrac{V_2}{V_S}$ in the form that can be given as

$$H(s)=\frac{b_m\cdot s^m+b_{m-1}\cdot s^{m-1}+\ldots+b_1\cdot s+b_0}{a_n\cdot s^n+a_{n-1}\cdot s^{n-1}+\ldots+a_1\cdot s+a_0}$$

the denominator of the transfer function needs to be determined.

First, the part of the denominator $D_1(s)$ given below, can be written again as

$$D_1(s) = 1 + \frac{Z_S}{s \cdot L_1 + R_1} - \frac{\dfrac{s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S}{Z_3 \cdot (s \cdot L_1 + R_1)^2}}{\left(\dfrac{s^2 \cdot M_{12} \cdot M_{13}}{Z_3 \cdot (s \cdot L_1 + R_1)} - \dfrac{s \cdot M_{23}}{Z_3}\right)}$$

$$D_1(s) = \frac{(s \cdot L_1 + R_1) + Z_S}{s \cdot L_1 + R_1} - \frac{\dfrac{s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S}{Z_3 \cdot (s \cdot L_1 + R_1)^2}}{\left(\dfrac{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)}{Z_3 \cdot (s \cdot L_1 + R_1)}\right)}$$

$$D_1(s) = \frac{(s \cdot L_1 + R_1) + Z_S}{s \cdot L_1 + R_1} - \frac{s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S}{(s \cdot L_1 + R_1) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)\right]}$$

$$D_1(s) = \frac{\left[(s \cdot L_1 + R_1) + Z_S\right] \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)\right] - s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S}{(s \cdot L_1 + R_1) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)\right]}$$

$$D_1(s) = \frac{\left(\begin{array}{c} s^2 \cdot M_{12} \cdot M_{13} \cdot (s \cdot L_1 + R_1) + \cancel{s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)^2 - \\ s \cdot M_{23} \cdot (s \cdot L_1 + R_1) \cdot Z_S - \cancel{s^2 \cdot M_{12} \cdot M_{13} \cdot Z_S} \end{array}\right)}{(s \cdot L_1 + R_1) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)\right]}$$

$$D_1(s) = \frac{s^2 \cdot M_{12} \cdot M_{13} \cdot (s \cdot L_1 + R_1) - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)^2 - s \cdot M_{23} \cdot (s \cdot L_1 + R_1) \cdot Z_S}{(s \cdot L_1 + R_1) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)\right]}$$

After dividing the above expression with $(s \cdot L_1 + R_1)$, we get

$$D_1(s) = \frac{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1) - s \cdot M_{23} \cdot Z_S}{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)}$$

The expression for $Z_S$ is applied into the above equation, so it can be given as

$$D_1(s) = \frac{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1) - s \cdot M_{23} \cdot \dfrac{1}{s \cdot C_1}}{s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot (s \cdot L_1 + R_1)}$$

$$D_1(s)=\frac{s^2\cdot\left(M_{12}\cdot M_{13}-M_{23}\cdot L_1\right)-s\cdot M_{23}\cdot R_1-\dfrac{M_{23}}{C_1}}{s^2\cdot\left(M_{12}\cdot M_{13}-M_{23}\cdot L_1\right)-s\cdot M_{23}\cdot R_1} \tag{4.38}$$

Equation (4.38) will be later written in the form of

$$\left(\frac{d_2\cdot s^2-d_1\cdot s-d_0}{c_2\cdot s^2-c_1\cdot s}\right)$$

The next part of the denominator of the transfer function is $D_2(s)$ and it can be written as

$$D_2(s)=\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\frac{\left(1-\dfrac{s^2\cdot M_{12}^2}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\dfrac{s\cdot L_2+R_2}{Z_2}\right)}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}}{Z_3\cdot\left(s\cdot L_1+R_1\right)}-\dfrac{s\cdot M_{23}}{Z_3}\right)}\cdot\frac{s\cdot M_{13}\cdot Z_S}{Z_3\cdot\left(s\cdot L_1+R_1\right)} \tag{4.39}$$

Equation (4.39) can be determined again as

$$D_2(s)=\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\frac{\left(\dfrac{Z_2\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2+\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)}{Z_2\cdot\cancel{\left(s\cdot L_1+R_1\right)}}\right)}{\left(\dfrac{s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot\left(s\cdot L_1+R_1\right)}{Z_3\cdot\cancel{\left(s\cdot L_1+R_1\right)}}\right)}\cdot\frac{s\cdot M_{13}\cdot Z_S}{Z_3\cdot\left(s\cdot L_1+R_1\right)}$$

$$D_2(s)=\frac{s\cdot M_{12}\cdot Z_S}{Z_2\cdot\left(s\cdot L_1+R_1\right)}+\frac{\cancel{Z_3}\cdot\left[Z_2\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2+\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)\right]}{Z_2\cdot\left[s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot\left(s\cdot L_1+R_1\right)\right]}\cdot\frac{s\cdot M_{13}\cdot Z_S}{\cancel{Z_3}\cdot\left(s\cdot L_1+R_1\right)}$$

$$D_2(s)=\frac{\left(\begin{array}{c}s\cdot M_{12}\cdot Z_S\cdot\left[s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot\left(s\cdot L_1+R_1\right)\right]+\\ s\cdot M_{13}\cdot Z_S\cdot\left[Z_2\cdot\left(s\cdot L_1+R_1\right)-s^2\cdot M_{12}^2+\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)\right]\end{array}\right)}{Z_2\cdot\left(s\cdot L_1+R_1\right)\cdot\left[s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot\left(s\cdot L_1+R_1\right)\right]}$$

$$D_2(s)=\frac{\begin{array}{c}\cancel{s^3\cdot M_{12}^2\cdot M_{13}\cdot Z_S}-s^2\cdot M_{12}\cdot M_{23}\cdot Z_S\cdot\left(s\cdot L_1+R_1\right)-\cancel{s^3\cdot M_{12}^2\cdot M_{13}\cdot Z_S}+\\ s\cdot M_{13}\cdot Z_S\cdot Z_2\cdot\left(s\cdot L_1+R_1\right)+s\cdot M_{13}\cdot Z_S\cdot\left(s\cdot L_2+R_2\right)\cdot\left(s\cdot L_1+R_1\right)\end{array}}{Z_2\cdot\left(s\cdot L_1+R_1\right)\cdot\left[s^2\cdot M_{12}\cdot M_{13}-s\cdot M_{23}\cdot\left(s\cdot L_1+R_1\right)\right]} \tag{4.40}$$

The expression given in (4.40), can be divided with $\left(s \cdot L_1 + R_1\right)$ and it can be written as

$$D_2(s) = \frac{s \cdot M_{13} \cdot Z_S \cdot Z_2 - s^2 \cdot M_{12} \cdot M_{23} \cdot Z_S + s \cdot M_{13} \cdot Z_S \cdot \left(s \cdot L_2 + R_2\right)}{Z_2 \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot \left(s \cdot L_1 + R_1\right)\right]} \tag{4.41}$$

Now, the expressions for $Z_2$ and $Z_3$ can be applied into (4.41).

$$D_2(s) = \frac{s \cdot M_{13} \cdot \frac{1}{s \cdot C_1} \cdot \left(\frac{1}{s \cdot C_2} + R_L\right) - s^2 \cdot M_{12} \cdot M_{23} \cdot \frac{1}{s \cdot C_1} + s \cdot M_{13} \cdot \frac{1}{s \cdot C_1} \cdot \left(s \cdot L_2 + R_2\right)}{\left(\frac{1}{s \cdot C_2} + R_L\right) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot \left(s \cdot L_1 + R_1\right)\right]}$$

$$D_2(s) = \frac{\frac{M_{13}}{C_1} \cdot \left(\frac{1}{s \cdot C_2} + R_L\right) - \frac{s \cdot M_{12} \cdot M_{23}}{C_1} + \frac{M_{13}}{C_1} \cdot \left(s \cdot L_2 + R_2\right)}{\left(\frac{1}{s \cdot C_2} + R_L\right) \cdot \left[s^2 \cdot M_{12} \cdot M_{13} - s \cdot M_{23} \cdot \left(s \cdot L_1 + R_1\right)\right]}$$

$$D_2(s) = \frac{\left(\frac{M_{13}}{s \cdot C_1 \cdot C_2} + \frac{M_{13} \cdot R_L}{C_1}\right) - \frac{s \cdot M_{12} \cdot M_{23}}{C_1} + \left(\frac{s \cdot L_2 \cdot M_{13}}{C_1} + \frac{M_{13} \cdot R_2}{C_1}\right)}{\left[\frac{s \cdot M_{12} \cdot M_{13}}{C_2} + s^2 \cdot M_{12} \cdot M_{13} \cdot R_L - \frac{M_{23} \cdot \left(s \cdot L_1 + R_1\right)}{C_2} - s \cdot M_{23} \cdot \left(s \cdot L_1 + R_1\right) \cdot R_L\right]} \tag{4.42}$$

Equation (4.42) can be given as

$$D_2(s) = \frac{s \cdot \left(\frac{L_2 \cdot M_{13}}{C_1} - \frac{M_{12} \cdot M_{23}}{C_1}\right) + \left(\frac{M_{13} \cdot R_L}{C_1} + \frac{M_{13} \cdot R_2}{C_1}\right) + \frac{1}{s} \cdot \frac{M_{13}}{C_1 \cdot C_2}}{s^2 \cdot \left(M_{12} \cdot M_{13} \cdot R_L - M_{23} \cdot L_1 \cdot R_L\right) + s \cdot \left(\frac{M_{12} \cdot M_{13}}{C_2} - M_{23} \cdot R_1 \cdot R_L - \frac{M_{23} \cdot L_1}{C_2}\right) - \frac{M_{23} \cdot R_1}{C_2}}$$

After multiplying the numerator and denominator of $D_2(s)$ by *s*, it can be written as

$$D_2(s) = \frac{s^2 \cdot \left(\frac{L_2 \cdot M_{13}}{C_1} - \frac{M_{12} \cdot M_{23}}{C_1}\right) + s \cdot \left(\frac{M_{13} \cdot R_L}{C_1} + \frac{M_{13} \cdot R_2}{C_1}\right) + \frac{M_{13}}{C_1 \cdot C_2}}{s^3 \cdot \left(M_{12} \cdot M_{13} \cdot R_L - M_{23} \cdot L_1 \cdot R_L\right) + s^2 \cdot \left(\frac{M_{12} \cdot M_{13}}{C_2} - M_{23} \cdot R_1 \cdot R_L - \frac{M_{23} \cdot L_1}{C_2}\right) - s \cdot \frac{M_{23} \cdot R_1}{C_2}}$$

The above equation can be expressed as

$$D_2(s) = \frac{f_2 \cdot s^2 + f_1 \cdot s + f_0}{e_3 \cdot s^3 + e_2 \cdot s^2 - e_1 \cdot s}$$

The transfer function of a three winding model with three capacitors can now be determined as

$$H(s)=\frac{1}{\frac{V_1}{V_2}\cdot\left(\frac{d_2\cdot s^2-d_1\cdot s-d_0}{c_2\cdot s^2-c_1\cdot s}\right)+\frac{f_2\cdot s^2+f_1\cdot s+f_0}{e_3\cdot s^3+e_2\cdot s^2-e_1\cdot s}}$$

The transfer function $\frac{V_1}{V_2}$ of a three winding model with two capacitors can be given as

$$\frac{V_1}{V_2}=\frac{b_5\cdot s^5+b_4\cdot s^4+b_3\cdot s^3+b_2\cdot s^2+b_1\cdot s+b_0}{a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0}$$

where the parts of numerator and denominator are

$$\begin{aligned}N(s)=\;&s^5\cdot\left(L_1\cdot L_2\cdot L_3-M_{13}^2\cdot L_2-M_{12}^2\cdot L_3-M_{23}^2\cdot L_1+2\cdot M_{12}\cdot M_{13}\cdot M_{23}\right)+\\&s^4\cdot\left(-M_{13}^2\cdot R_\mathrm{L}-M_{13}^2\cdot R_2+L_1\cdot L_3\cdot R_\mathrm{L}-M_{12}^2\cdot R_3+L_1\cdot L_2\cdot R_3+L_2\cdot L_3\cdot R_1+L_1\cdot L_3\cdot R_2-M_{23}^2\cdot R_1\right)+\\&s^3\cdot\left(-\frac{M_{12}^2}{C_3}+\frac{L_1\cdot L_2}{C_3}-\frac{M_{13}^2}{C_2}+L_1\cdot R_3\cdot R_\mathrm{L}+L_3\cdot R_1\cdot R_\mathrm{L}+\frac{L_1\cdot L_3}{C_2}+L_2\cdot R_1\cdot R_3+L_1\cdot R_3\cdot R_2+L_3\cdot R_1\cdot R_2\right)+\\&s^2\cdot\left(\frac{R_\mathrm{L}\cdot L_1}{C_3}+\frac{L_1\cdot R_2}{C_3}+\frac{L_2\cdot R_1}{C_3}+R_1\cdot R_3\cdot R_\mathrm{L}+\frac{L_1\cdot R_3}{C_2}+\frac{L_3\cdot R_1}{C_2}+R_1\cdot R_3\cdot R_2\right)+\\&s\cdot\left(\frac{R_\mathrm{L}\cdot R_1}{s\cdot C_3}+\frac{L_1}{s\cdot C_3\cdot C_2}+\frac{R_1\cdot R_2}{s\cdot C_3}+\frac{R_1\cdot R_3}{s\cdot C_2}\right)+\frac{R_1}{C_3\cdot C_2}\end{aligned}$$

$$\begin{aligned}D(s)=\;&s^4\cdot\left(M_{12}\cdot L_3\cdot R_\mathrm{L}-M_{23}\cdot M_{13}\cdot R_\mathrm{L}\right)+\\&s^3\cdot\left(M_{12}\cdot R_3\cdot R_\mathrm{L}-\frac{M_{23}\cdot M_{13}}{C_2}+\frac{M_{12}\cdot L_3}{C_2}\right)+\\&s^2\cdot\left(\frac{M_{12}}{C_3}\cdot R_\mathrm{L}+\frac{M_{12}\cdot R_3}{C_2}\right)+s\cdot\frac{M_{12}}{C_2\cdot C_3}\end{aligned}$$

The coefficients $d_2\cdot s^2$, $d_1\cdot s$, $d_0$, $c_2\cdot s^2$ and $c_1\cdot s$ are previously given in (4.38).

Similar to the previous two wireless charging systems, the transfer function $H(s)$ needs to be modified in order to get the transfer function as a ratio of the output voltage $V_{\mathrm{out}}$ across the load resistance $R_{\mathrm{L}}$ and the input voltage source $V_{\mathrm{s}}$. For that purpose, a new transfer function can be written as

$$\frac{V_{\mathrm{out}}}{V_{\mathrm{s}}} = \frac{V_2}{V_{\mathrm{s}}} \cdot \frac{V_{\mathrm{out}}}{V_2} = H(s) \cdot \frac{V_{\mathrm{out}}}{V_2}$$

where $\frac{V_{\mathrm{out}}}{V_2}$ can be determined as

$$\frac{V_{\mathrm{out}}}{V_2} = \frac{R_{\mathrm{L}}}{R_{\mathrm{L}} + \frac{1}{s \cdot C_2}} = \frac{1}{1 + \frac{1}{s \cdot C_2 \cdot R_{\mathrm{L}}}} = \frac{s}{s + \frac{1}{C_2 \cdot R_{\mathrm{L}}}}$$

# 5 Measurements

## 5.1 Set-up

The resonant frequency $f_0$ that was chosen for the set-up is 85 kHz. According to [14], this frequency is used in wireless charging systems for electric and plug-in hybrid vehicles. The frequency band for this nominal frequency, according to SAE J2954 standard, is from 81,38 to 90 kHz and the standard also defines the three maximum input power classes that are shown below:

- 3,7 kW
- 7,7 kW
- 22 kW

However, this set-up is not designed for such high power, it will operate within medium power level, which is according to the new Qi standard, from 5 W to 120 W. Fig. 5.1 shows three transmitter coils. In this thesis, only the coils $L_{T1}$ and $L_{T2}$ will be used. The coil that is marked in Fig. 5.1 as $L_{T1}$ will be used as a transmitter coil and the coil marked as $L_{T2}$ will be used as a third auxiliary winding. The receiver coil $L_R$ is presented in Fig. 5.2 and the electrical properties of all four coils are provided in Table 5.1.

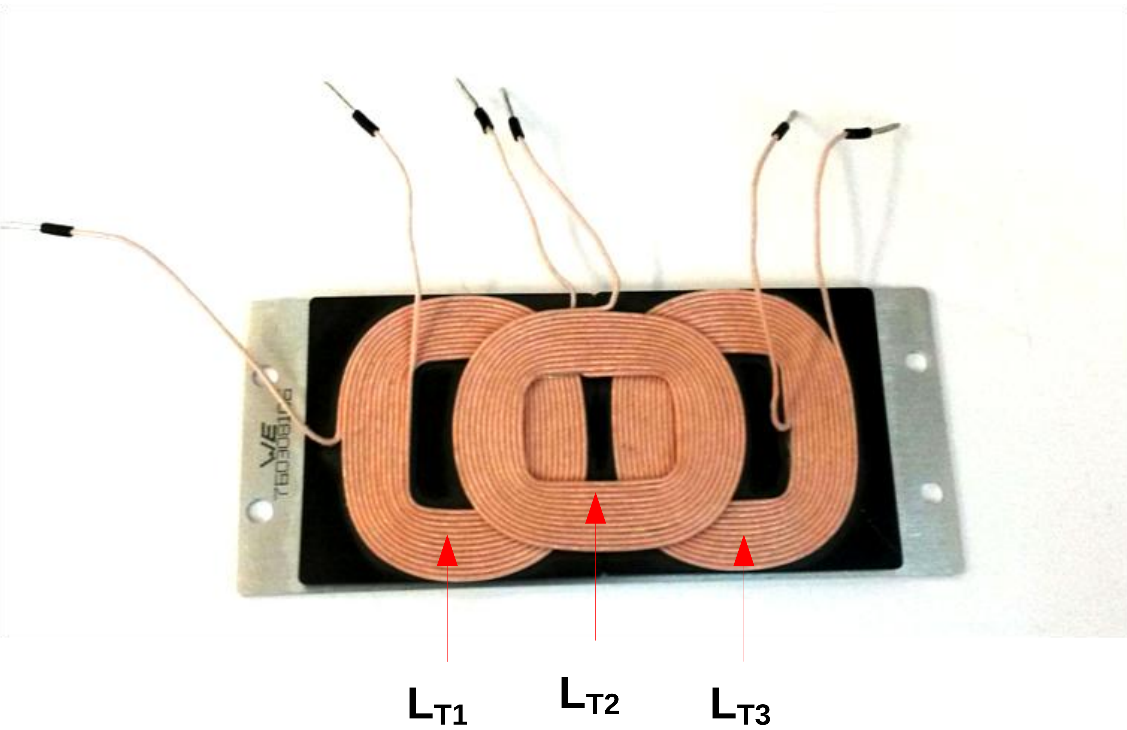


*Figure 5.1: Transmitter coils*


*Figure 5.2: Receiver coil*

The electrical properties of all four coils are given in Table 5.1. The maximum current that can go through the transmitter and the auxiliary coil is limited to 9 A, so the input current through the transmitter coil will be limited to 6 A, as it is the maximum current that can be provided by the dc power source.

*Table 5.1: Electrical properties of coils provided by the manufacturer*

| Parameter | Symbol | Value | | | | Unit |
|---|---|---|---|---|---|---|
| | | $L_{T1}$ | $L_{T2}$ | $L_{T3}$ | $L_R$ | |
| Inductance (@ 125 kHz) | L | 12.5 | 11.5 | 12.5 | 6.3 | µH |
| Rated current | $I_R$ | 9 | 9 | 9 | 13 | A |
| DC resistance | $R_{DC}$ | 55 | 55 | 55 | 17 | mΩ |
| Max DC resistance | $R_{DC}$ | 65 | 65 | 65 | 25 | mΩ |
| Self resonant frequency | $f_{res}$ | 14 | 14 | 14 | 20 | MHz |

The measured values of the self-inductance and the resistance of the coils at the resonant frequency $f_0 = 85$ kHz are shown in Table 5.2. The equipment that was used in this measurement is listed below

- Würth Elektronik three transmitter coils (manufacturer's serial number 760308106)
- Würth Elektronik receiver coil (manufacturer's serial number 760308111)
- Bode 100 Vector Network Analyzer
- PC with Bode Analyzer Suite
- Bode 100 User Manual
- USB cable
- 3 x BNC cable 50 Ω
- B-WIC Impedance Adapter

*Table 5.2: Measured values of the inductance and the resistance of the coils at 85 kHz*

| Coil | Self-inductance [μH] | AC resistance [mΩ] | DC resistance [mΩ] |
|---|---|---|---|
| $L_{T1}$ | 12,377 | 188,132 | 54,783 mΩ |
| $L_{T2}$ | 11,180 | 172,231 | 53,697 mΩ |
| $L_{T3}$ | 12,335 | 184,680 | 54,335 mΩ |
| $L_R$ | 5,967 | 106,136 | 16,512 mΩ |

As it can be seen from Table 5.2, the measured values of the dc resistance of each coil correspond to the values given in Table 5.1. However, the measured values of the ac resistances are significantly higher than the values of the dc resistance, which will later result in high copper losses. The coils that were used are shown in Fig. 5.3 where the air gap was 2,5 cm and the third coil is placed on top of the transmitter coil.

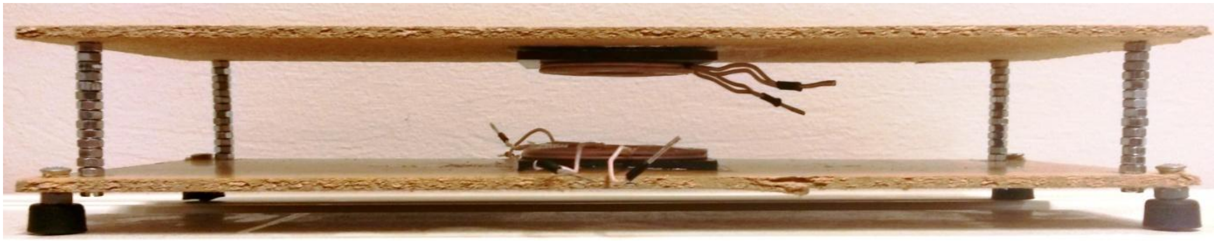

*Figure 5.3: Coils with 2,5 cm air gap*

## 5.2 Measurements of the mutual inductance and the self-inductance

Table 5.3 shows the measured values of the inductances for the different air gap ($h$) and misalignment ($x$) . In order to find the mutual inductance between the coils and to determine the coupling coefficients, it was necessary to measure the self-inductances $L_1$, $L_2$ and $L_3$ . After finding those values, we can measure the inductance across one winding while the second winding is shorted. Therefore, the inductance $L_{s12}$ represents the measured inductance across $L_1$ with $L_2$ shorted and it can be defined as

$$L_{s12} = L_1 - \frac{M_{12}^2}{L_2} \tag{5.1}$$

where $M_{12}$ is the mutual inductance between the transmitter and the receiver coil.

*Table 5.3: Measured values of the inductances*

| $h$ [cm] | $x$ [cm] | $L_1$ [µH] | $L_2$ [µH] | $L_3$ [µH] | $L_{s12}$ [µH] | $L_{s21}$ [µH] | $L_{s13}$ [µH] | $L_{s31}$ [µH] | $L_{s23}$ [µH] | $L_{s32}$ [µH] |
|---|---|---|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 15,9 | 8,6 | 14,7 | 10,8 | 6,1 | 6,6 | 5,6 | 5,9 | 8,8 |
| 0,5 | 1 | 15,5 | 8,4 | 14,4 | 13,2 | 7,4 | 6,5 | 5,5 | 7,3 | 11,7 |
| 0,5 | 2 | 14,8 | 8,0 | 13,7 | 14,3 | 7,9 | 6,5 | 5,3 | 7,9 | 13,1 |
| 0,5 | 3 | 14,5 | 7,2 | 13,2 | 14,5 | 7,2 | 6,7 | 5,7 | 7,2 | 13,2 |
| 1 | 0 | 13,9 | 7,1 | 12,1 | 11,8 | 6,3 | 6,9 | 5,5 | 6,2 | 10,1 |
| 1 | 1 | 14,0 | 7,2 | 12,1 | 12,9 | 6,7 | 7,0 | 5,6 | 6,6 | 10,8 |
| 1 | 2 | 13,8 | 7,1 | 12,2 | 13,7 | 6,9 | 7,0 | 5,6 | 7,0 | 12,0 |
| 1 | 3 | 13,4 | 6,9 | 11,9 | 13,4 | 6,9 | 6,9 | 5,5 | 6,9 | 11,9 |
| 1,5 | 0 | 13,0 | 6,8 | 11,4 | 12,1 | 6,5 | 6,8 | 5,4 | 6,4 | 10,4 |
| 1,5 | 1 | 13,2 | 6,8 | 11,6 | 12,7 | 6,6 | 7,0 | 5,6 | 6,4 | 11,1 |
| 1,5 | 2 | 13,2 | 6,8 | 11,4 | 13,1 | 6,7 | 7,0 | 5,6 | 6,7 | 11,3 |
| 1,5 | 3 | 13,2 | 6,7 | 11,4 | 13,2 | 6,7 | 7,1 | 5,6 | 6,7 | 11,4 |
| 2 | 0 | 12,9 | 6,7 | 11,1 | 12,5 | 6,5 | 7,1 | 5,4 | 6,4 | 10,6 |
| 2 | 1 | 12,8 | 6,6 | 11,0 | 12,6 | 6,5 | 7,0 | 5,3 | 6,5 | 10,7 |
| 2 | 2 | 12,8 | 6,6 | 11,2 | 12,8 | 6,6 | 7,0 | 5,5 | 6,6 | 11,2 |
| 2 | 3 | 12,8 | 6,6 | 11,1 | 12,8 | 6,6 | 7,0 | 5,5 | 6,6 | 11,1 |
| 2,5 | 0 | 12,5 | 6,5 | 10,9 | 12,3 | 6,4 | 6,7 | 5,3 | 6,4 | 10,6 |
| 2,5 | 1 | 12,5 | 6,5 | 10,8 | 12,4 | 6,4 | 6,7 | 5,2 | 6,4 | 10,6 |
| 2,5 | 2 | 12,7 | 6,5 | 11,0 | 12,7 | 6,5 | 7,0 | 5,4 | 6,5 | 11,0 |
| 2,5 | 3 | 12,7 | 6,6 | 10,9 | 12,7 | 6,6 | 7,0 | 5,4 | 6,6 | 10,9 |

The mutual inductance $M_{12}$ can be determined as

$$M_{12} = \sqrt{L_2 \cdot \left(L_1 - L_{s12}\right)} \tag{5.2}$$

and the similar expressions can be applied to the other mutual inductances ($M_{21}$, $M_{13}$, $M_{31}$, $M_{23}$ and $M_{32}$). Calculated values of all mutual inductances for the different air gap ($h$) and misalignment ($x$) are presented in Table 5.4.

*Table 5.4: Calculated values of the mutual inductances*

| $h$ [cm] | $x$ [cm] | $M_{12}$ [μH] | $M_{21}$ [μH] | $M_{13}$ [μH] | $M_{31}$ [μH] | $M_{23}$ [μH] | $M_{32}$ [μH] |
|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 6,62 | 6,30 | 11,69 | 12,02 | 6,30 | 7,12 |
| 0,5 | 1 | 4,39 | 3,94 | 11,38 | 11,75 | 3,98 | 4,76 |
| 0,5 | 2 | 2,0 | 1,22 | 10,66 | 11,15 | 1,17 | 2,19 |
| 0,5 | 3 | 0 | 0 | 10,15 | 10,43 | 0 | 0 |
| 1 | 0 | 3,86 | 3,33 | 9,20 | 9,58 | 3,30 | 3,77 |
| 1 | 1 | 2,81 | 2,65 | 9,20 | 9,54 | 2,69 | 3,06 |
| 1 | 2 | 0,84 | 1,17 | 9,11 | 9,54 | 1,10 | 1,19 |
| 1 | 3 | 0 | 0 | 8,79 | 9,26 | 0 | 0 |
| 1,5 | 0 | 2,47 | 1,97 | 8,41 | 8,83 | 2,14 | 2,61 |
| 1,5 | 1 | 1,84 | 1,62 | 8,48 | 8,90 | 2,15 | 1,84 |
| 1,5 | 2 | 0,82 | 1,15 | 8,41 | 8,75 | 1,07 | 0,82 |
| 1,5 | 3 | 0 | 0 | 8,34 | 8,75 | 0 | 0 |
| 2 | 0 | 1,64 | 1,61 | 8,02 | 8,57 | 1,82 | 1,83 |
| 2 | 1 | 1,15 | 1,13 | 7,99 | 8,54 | 1,05 | 1,41 |
| 2 | 2 | 0 | 0 | 8,06 | 8,54 | 0 | 0 |
| 2 | 3 | 0 | 0 | 8,02 | 8,47 | 0 | 0 |
| 2,5 | 0 | 1,14 | 1,12 | 7,95 | 8,37 | 1,04 | 1,39 |
| 2,5 | 1 | 0,81 | 1,12 | 7,91 | 8,37 | 1,04 | 1,14 |
| 2,5 | 2 | 0 | 0 | 7,92 | 8,43 | 0 | 0 |
| 2,5 | 3 | 0 | 0 | 7,88 | 8,36 | 0 | 0 |

The arithmetic mean of the mutual inductances $M_{12}$ and $M_{21}$, $M_{13}$ and $M_{31}$ and between $M_{23}$ and $M_{32}$ for the diferent air gap and misalignment are shown in Table 5.5.

*Table 5.5: Arithmetic mean of the mutual inductances*

| $h$ [cm] | $x$ [cm] | $M_{12}$ [μH] | $M_{13}$ [μH] | $M_{23}$ [μH] |
|---|---|---|---|---|
| 0,5 | 0 | 6,46 | 11,86 | 6,71 |
| 0,5 | 1 | 4,17 | 11,57 | 4,37 |
| 0,5 | 2 | 1.61 | 10,91 | 1,68 |
| 0,5 | 3 | 0 | 10,29 | 0 |
| 1 | 0 | 3,60 | 9,39 | 3,54 |
| 1 | 1 | 2,73 | 9,37 | 2,88 |
| 1 | 2 | 1,01 | 9,33 | 1,15 |
| 1 | 3 | 0 | 9,03 | 0 |
| 1,5 | 0 | 2,22 | 8,62 | 2,38 |
| 1,5 | 1 | 1,73 | 8,69 | 1,99 |
| 1,5 | 2 | 0,99 | 8,58 | 0,95 |
| 1,5 | 3 | 0 | 8,55 | 0 |
| 2 | 0 | 1,63 | 8,30 | 1,83 |
| 2 | 1 | 1,14 | 8,27 | 1,23 |
| 2 | 2 | 0 | 8,30 | 0 |
| 2 | 3 | 0 | 8,25 | 0 |
| 2,5 | 0 | 1,13 | 8,16 | 1,22 |
| 2,5 | 1 | 0,97 | 8,14 | 1,09 |
| 2,5 | 2 | 0 | 8,18 | 0 |
| 2,5 | 3 | 0 | 8,12 | 0 |

The arithmetic mean of the mutual inductances $M_{12}$ and $M_{13}$ for different heights and misalignments are shown in Fig. 5.4 and in Fig. 5.5. The reason why the mutual inductance $M_{12}$ for $h=1$ cm has lower values than for $h=1{,}5$ cm between $x$=2 and $x$=3 cm is because of the cubic spline data interpolation in Matlab.

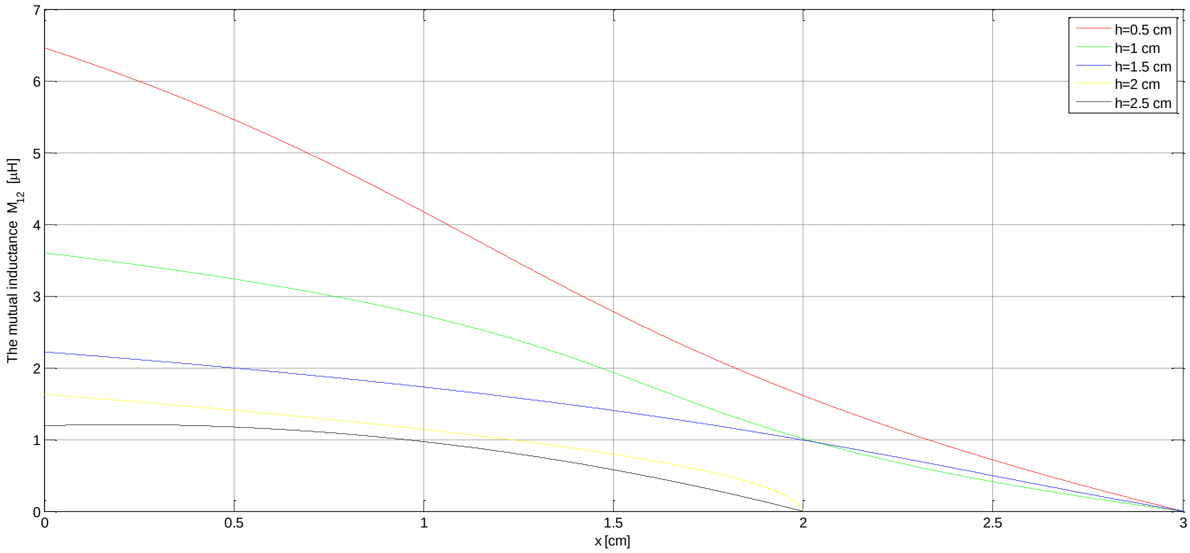


*Figure 5.4: The mutual inductance $M_{12}$ for different heights and misalignments*

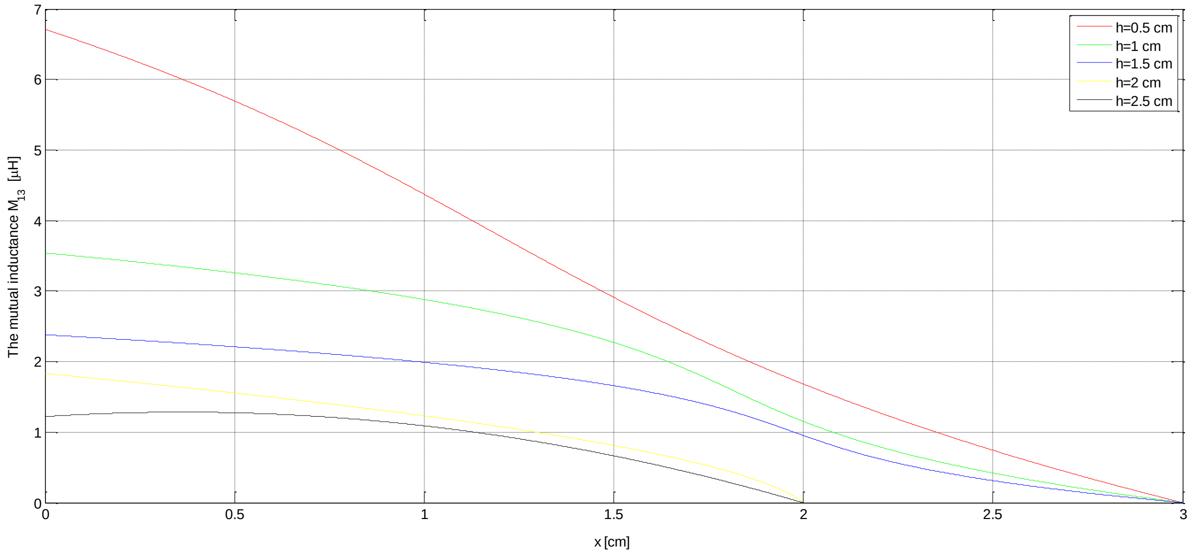


*Figure 5.5: The mutual inductance $M_{13}$ for different heights and misalignments*

The coupling coefficient $k_{12}$ can be determined as

$$k_{12} = \frac{M_{12}}{\sqrt{L_1 \cdot L_2}} \tag{5.3}$$

and the similar expressions can be applied to the other coupling coefficients $k_{21}$, $k_{13}$, $k_{31}$, $k_{23}$ and $k_{32}$ whose values are presented in Table 5.6.

*Table 5.6: Calculated values of the coupling coefficients*

| $h$ [cm] | $x$ [cm] | $k_{12}$ | $k_{21}$ | $k_{13}$ | $k_{31}$ | $k_{23}$ | $k_{32}$ |
|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 0,574 | 0,539 | 0,768 | 0,783 | 0,561 | 0,627 |
| 0,5 | 1 | 0,385 | 0,345 | 0,762 | 0,786 | 0,361 | 0,431 |
| 0,5 | 2 | 0,184 | 0,112 | 0,748 | 0,783 | 0,112 | 0,209 |
| 0,5 | 3 | 0 | 0 | 0,743 | 0,763 | 0 | 0 |
| 1 | 0 | 0,389 | 0,335 | 0,709 | 0,738 | 0,356 | 0,406 |
| 1 | 1 | 0,279 | 0,264 | 0,707 | 0,733 | 0,288 | 0,328 |
| 1 | 2 | 0,085 | 0,118 | 0,702 | 0,735 | 0,118 | 0,128 |
| 1 | 3 | 0 | 0 | 0,696 | 0,733 | 0 | 0 |
| 1,5 | 0 | 0,263 | 0,210 | 0,691 | 0,725 | 0,243 | 0,296 |
| 1,5 | 1 | 0,194 | 0,171 | 0,691 | 0,726 | 0,242 | 0,207 |
| 1,5 | 2 | 0,087 | 0,121 | 0,686 | 0,713 | 0,122 | 0,093 |
| 1,5 | 3 | 0 | 0 | 0,679 | 0,713 | 0 | 0 |
| 2 | 0 | 0,176 | 0,173 | 0,670 | 0,716 | 0,211 | 0,212 |
| 2 | 1 | 0,125 | 0,123 | 0,673 | 0,719 | 0,123 | 0,165 |
| 2 | 2 | 0 | 0 | 0,673 | 0,713 | 0 | 0 |
| 2 | 3 | 0 | 0 | 0,673 | 0,711 | 0 | 0 |
| 2,5 | 0 | 0,126 | 0124 | 0,681 | 0,717 | 0,123 | 0,165 |
| 2,5 | 1 | 0,090 | 0,124 | 0,681 | 0,720 | 0,124 | 0,136 |
| 2,5 | 2 | 0 | 0 | 0,670 | 0,713 | 0 | 0 |
| 2,5 | 3 | 0 | 0 | 0,669 | 0,710 | 0 | 0 |

The arithmetic mean of the coupling coefficients $k_{12}$, $k_{13}$ and $k_{23}$ for the diferent air gap and misalignment are shown in Table 5.7. The coupling coefficients $k_{12}$ and $k_{23}$ are also shown in Fig. 5.6 and in Fig. 5.7.

*Table 5.7: Arithmetic mean of the coupling coefficients*

| $h$ [cm] | $x$ [cm] | $k_{12}$ | $k_{13}$ | $k_{23}$ |
|---|---|---|---|---|
| 0,5 | 0 | 0,557 | 0,776 | 0,594 |
| 0,5 | 1 | 0,365 | 0,774 | 0,396 |
| 0,5 | 2 | 0,148 | 0,766 | 0,161 |
| 0,5 | 3 | 0 | 0,753 | 0 |
| 1 | 0 | 0,362 | 0,724 | 0,381 |
| 1 | 1 | 0,272 | 0,720 | 0,308 |
| 1 | 2 | 0,102 | 0,719 | 0,123 |
| 1 | 3 | 0 | 0,715 | 0 |
| 1,5 | 0 | 0,237 | 0,708 | 0,270 |
| 1,5 | 1 | 0,183 | 0,709 | 0,225 |
| 1,5 | 2 | 0,104 | 0,700 | 0,108 |
| 1,5 | 3 | 0 | 0,696 | 0 |
| 2 | 0 | 0,175 | 0,693 | 0,212 |
| 2 | 1 | 0,124 | 0,696 | 0,144 |
| 2 | 2 | 0 | 0,693 | 0 |
| 2 | 3 | 0 | 0,692 | 0 |
| 2,5 | 0 | 0,125 | 0,699 | 0,143 |
| 2,5 | 1 | 0,107 | 0,701 | 0,130 |
| 2,5 | 2 | 0 | 0,692 | 0 |
| 2,5 | 3 | 0 | 0,690 | 0 |

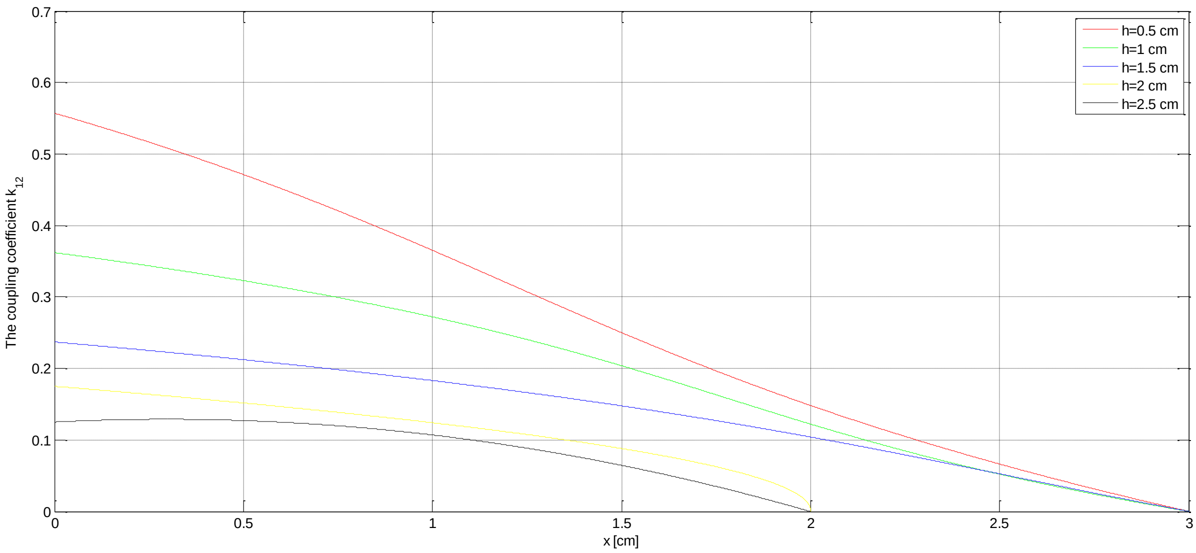


*Figure 5.6: The coupling coefficient $k_{12}$ for different heights and misalignments*

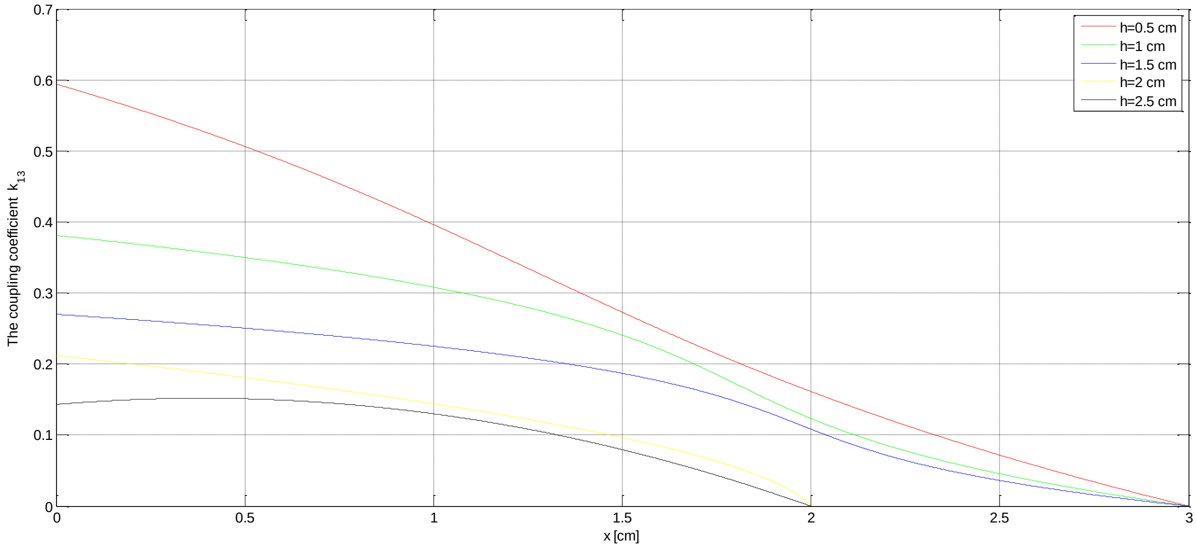


*Figure 5.7: The coupling coefficient $k_{13}$ for different heights and misalingments*

As it can be seen from Table 5.7, the coupling coefficient $k_{23}$ has higher values than the coupling coefficient $k_{12}$, because the auxiliary coil $(L_3)$ was placed on top of the transmitter coil $(L_1)$, so the auxiliary coil has lower air gap than the transmitter coil.

## 5.3 Wireless charging system with two coils and two capacitors

In order to compensate for the reactive power, capacitors in the primary and in the secondary circuit are needed. The value of each of the capacitors $C_1$ and $C_2$ is set to cancel the self-inductance in each circuit and it can be written as

$$C_1 = \frac{1}{\omega_0^2 \cdot L_1} = \frac{1}{(2\pi \cdot f_0)^2 \cdot L_1} \tag{5.4}$$

$$C_2 = \frac{1}{\omega_0^2 \cdot L_2} = \frac{1}{(2\pi \cdot f_0)^2 \cdot L_2} \tag{5.5}$$

where $L_1$ and $L_2$ are the self-inductances of the transmitter and the receiver coil for the best case when the air gap is $h = 0{,}5$ cm and the misalignment is $x = 0$ cm and their values are given in Table 5.3. By using (5.4) and (5.5), the calculated values of the capacitors $C_1$ and $C_2$ are equal to

$$C_1 = \frac{1}{(2\pi \cdot 85000)^2 \cdot 15{,}9 \cdot 10^{-6}} = 220{,}49 \text{ nF}$$

$$C_2 = \frac{1}{(2\pi \cdot 85000)^2 \cdot 8{,}6 \cdot 10^{-6}} = 407{,}66 \text{ nF}$$

In order to achieve the value of the capacitor $C_1$, four capacitors were connected in parallel and their capacitance values are 47, 47, 56 and 68 nF. The measured value of the equivalent capacitance is 220,9 nF and the capacitors in the primary circuit are shown in Fig. 5.8.


*Figure 5.8: Capacitors in the transmitter circuit*

To get the value of the capacitor $C_2$ in the receiver circuit, six equal capacitors were connected in parallel and the value of each capacitor is 68 nF. The measured value of the equivalent capacitance is 411,1 nF and the capacitors in the receiver circuit are shown in Fig. 5.9.


*Figure 5.9: Capacitors in the receiver circuit*

The measured values of the equivalent capacitors $C_1$ and $C_2$ will be used in the further analysis as well as for the Bode plot of this wireless charging system which is shown in Fig 5.10 and the .m script that was used to create this plot is available in the appendix.

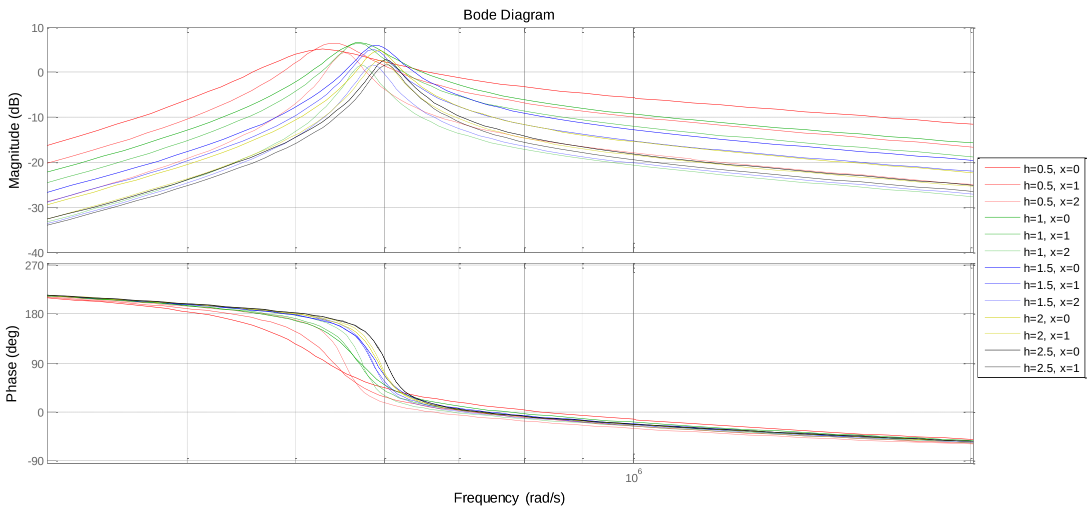


*Figure 5.10: Bode plot of the transfer function of a two winding model with two capacitors*

The peak response that were given by Bode plot from Matlab are shown in Table 5.8 as well as the measured output and input voltage values. The ratio of the output to input voltage is shown in Fig. 5.11 along with the Bode plot of the transfer function for the best case.

*Table 5.8: Measured values of the magnitude*

| $V_{S,rms}$ [V] | $V_{out,rms}$ [V] | $20\log\left(\frac{V_{out,rms}}{V_{S,rms}}\right)$ [dB] | $f_{meas}$ [kHz] |
|---|---|---|---|
| 3,361 | 3,424 | 0,16 | 73,212 |
| 5,594 | 8,724 | 3,86 | 79,699 |
| 3,303 | 5,847 | 4,96 | 84,216 |
| 4,377 | 7,009 | 4,09 | 89,643 |
| 5,910 | 7,945 | 2,57 | 94,027 |

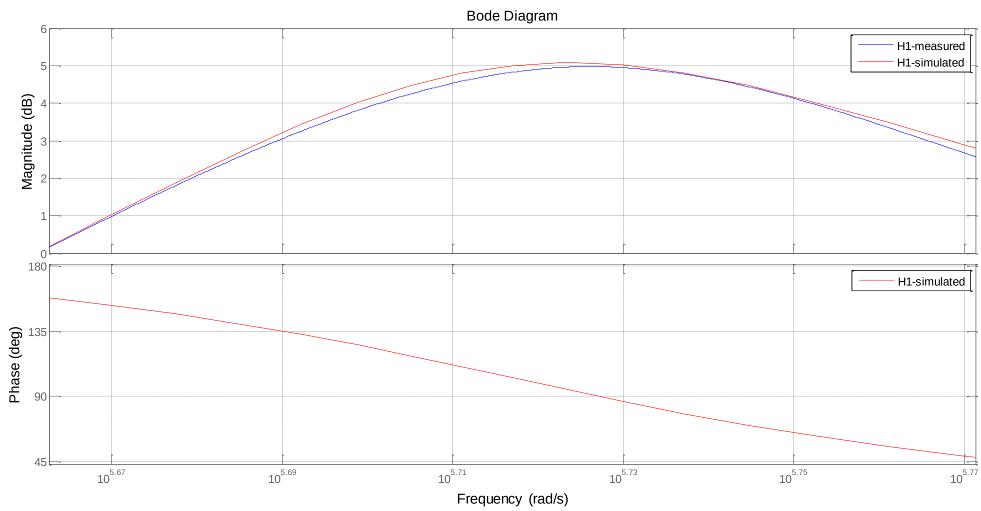


*Figure 5.11: Simulated and measured values of output/input voltage*

*Table 5.9: Efficiency for fixed input voltage*

| f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|
| 81,737 | 12,0 | 4,01 | 48,12 | 15,781 | 1,972 | 31,130 | 0,647 |
| 82,515 | 12,0 | 3,92 | 47,04 | 15,362 | 1,920 | 29,495 | 0,627 |
| 83,696 | 12,0 | 3,77 | 45,24 | 15,201 | 1,900 | 28,882 | 0,638 |
| 84,491 | 12,0 | 3,65 | 43,80 | 15,027 | 1,878 | 28,221 | 0,644 |
| 85,220 | 12,0 | 3,52 | 42,24 | 14,802 | 1,850 | 27,384 | 0,648 |
| 86,837 | 12,0 | 3,29 | 39,48 | 14,275 | 1,784 | 25,467 | 0,645 |
| 87,540 | 12,0 | 3,18 | 38,16 | 14,048 | 1,756 | 24,668 | 0,646 |
| 88,657 | 12,0 | 3,04 | 36,48 | 13,680 | 1,710 | 23,393 | 0,641 |
| 89,928 | 12,0 | 2,85 | 34,20 | 13,261 | 1,657 | 21,973 | 0,642 |

*Table 5.10: Efficiency for different air gap and misalignment*

| $h$ [cm] | $x$ [cm] | f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 85,220 | 12,0 | 3,52 | 42,24 | 14,802 | 1,850 | 27,384 | 0,648 |
| 0,5 | 1 | 87,928 | 12,0 | 2,95 | 35,40 | 12,470 | 1,559 | 19,441 | 0,549 |
| 0,5 | 2 | 81,692 | 12,0 | 2,79 | 33,48 | 6,009 | 0,751 | 4,513 | 0,138 |
| 1 | 0 | 84,654 | 12,0 | 3,51 | 42,12 | 12,561 | 1,570 | 19,721 | 0,468 |
| 1 | 1 | 91,839 | 12,0 | 3,31 | 39,72 | 11,398 | 1,425 | 16,242 | 0,409 |
| 1 | 2 | 90,604 | 12,0 | 2,92 | 35,04 | 4,992 | 0,624 | 3,115 | 0,089 |
| 1,5 | 0 | 88,863 | 12,0 | 3,02 | 36,24 | 9,557 | 1,194 | 11,411 | 0,315 |
| 1,5 | 1 | 88,509 | 12,0 | 3,15 | 37,80 | 8,491 | 1,061 | 9,009 | 0,238 |
| 1,5 | 2 | 92,192 | 12,0 | 3,13 | 37,56 | 4,020 | 0,503 | 2,022 | 0,054 |
| 2 | 0 | 92,771 | 12,0 | 3,28 | 39,36 | 7,745 | 0,968 | 7,497 | 0,190 |
| 2 | 1 | 91,581 | 12,0 | 2,71 | 32,52 | 5,622 | 0,703 | 3,952 | 0,122 |
| 2,5 | 0 | 93,628 | 12,0 | 3,27 | 39,24 | 5,519 | 0,689 | 3,803 | 0,097 |
| 2,5 | 1 | 92,795 | 12,0 | 2,70 | 32,40 | 4,064 | 0,508 | 2,065 | 0,064 |

As it can be seen from Table 5.9, the overall efficiency for different frequency is between 62% and 64% for the best case when the air gap is $h = 0{,}5$ cm and the misalignment is $x$=0 cm. However, from Table 5.10 it can be seen that the overall efficiency gets significantly lowered if the air gap is increased or the misalignment is increased.

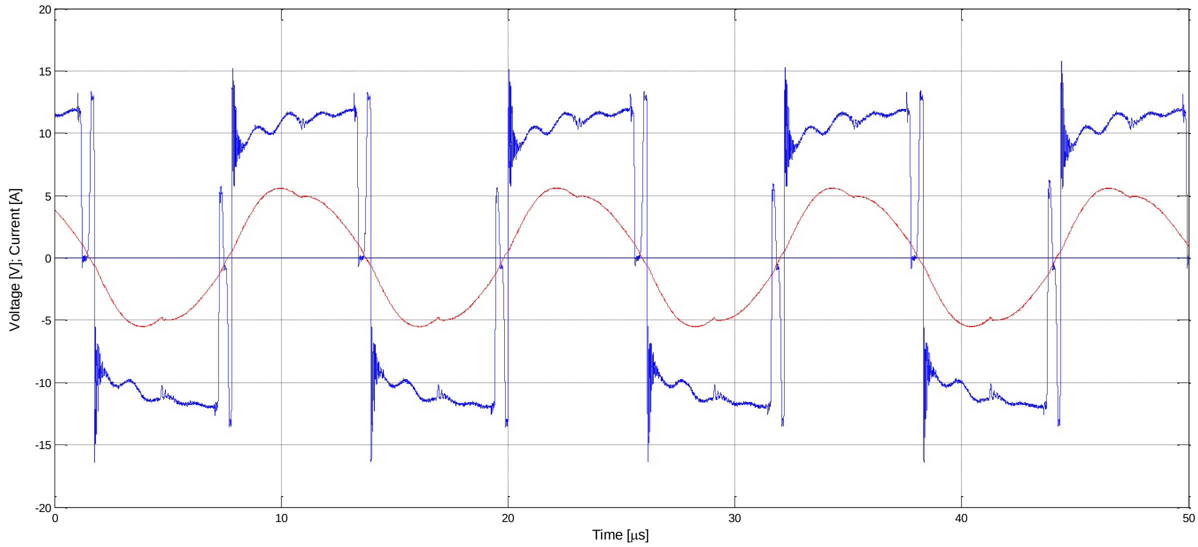


*Figure 5.12: Voltage and current waveform from the output of the inverter - measured*

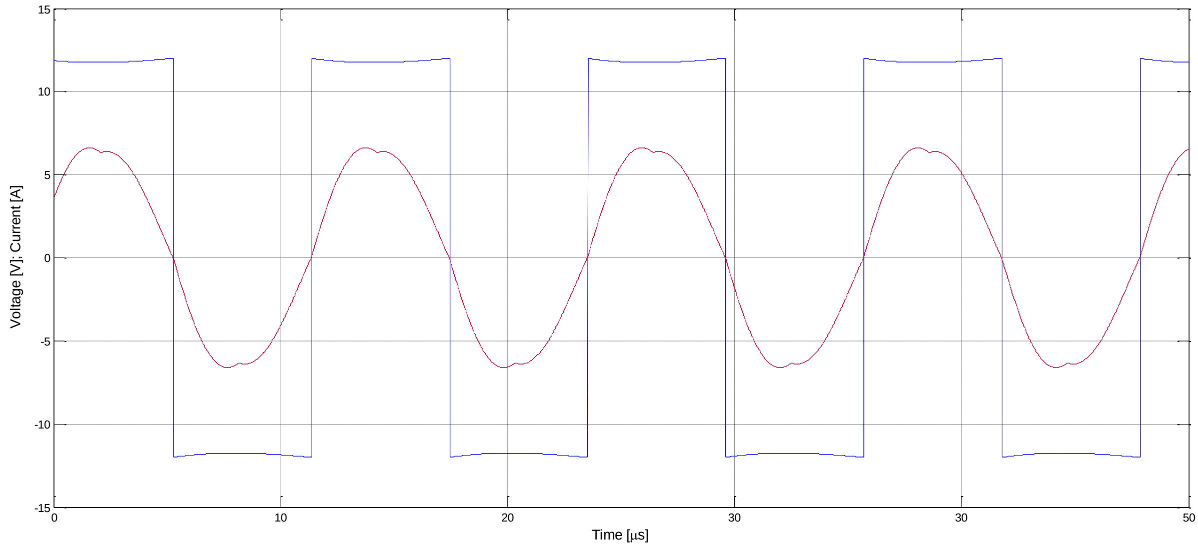


*Figure 5.13: Voltage and current waveform from the output of the inverter - simulation*

Fig. 5.12 shows the voltage and the current waveform that are measured at the output of the inverter. The same waveforms are obtained by Simulink model are shown in Fig. 5.13. As it can be seen, there is a resonance in the primary circuit, because there is no phase shift between the voltage and the current. Matlab/Simulink model that was used in this thesis is shown in Fig. 5.14.

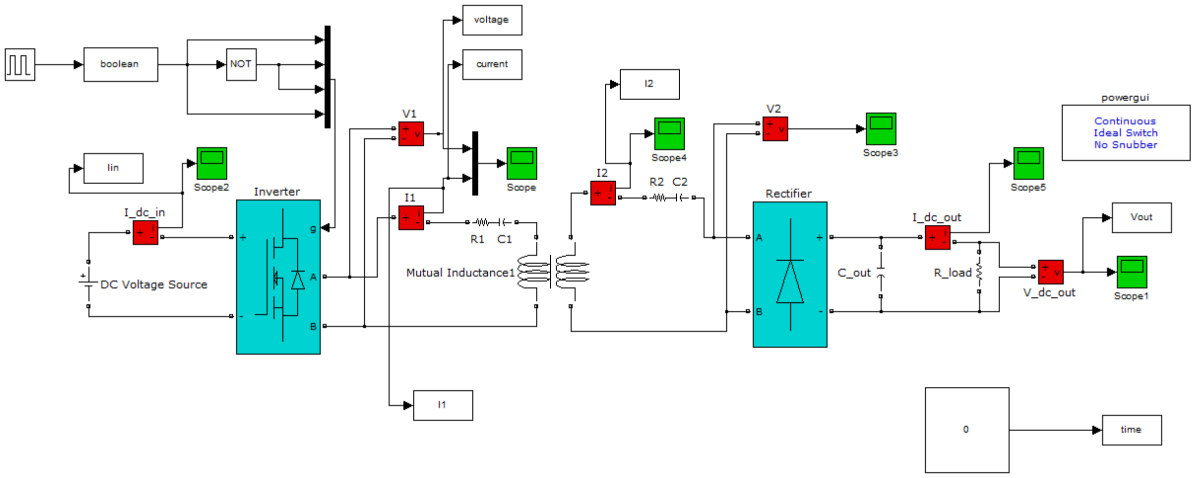


*Figure 5.14: Matlab/Simulink model*

## 5.4 Wireless charging system with three coils and two capacitors

In the wireless charging system with three coils and two capacitors, there is a capacitor $C_2$ in the receiver circuit and a capacitor $C_3$ in the auxiliary circuit, which are used to compensate the reactive power. The value of each of the capacitors $C_2$ and $C_3$ is set to cancel the self-inductance in each circuit and it can be written as

$$C_2 = \frac{1}{\omega_0^2 \cdot L_2} = \frac{1}{\left(2\pi \cdot f_0\right)^2 \cdot L_2} \tag{5.6}$$

$$C_3 = \frac{1}{\omega_0^2 \cdot L_3} = \frac{1}{\left(2\pi \cdot f_0\right)^2 \cdot L_3} \tag{5.7}$$

where $L_2$ and $L_3$ are the self-inductances of the receiver and the third auxiliary coil for the best case when the air gap is $h = 0{,}5$ cm and the misalignment is $x = 0$ cm and their values are given in Table 5.3. The value of the capacitor $C_2$ is equal as in the previous wireless charging system with only two coils and its measured value is 411,1 nF. By using (5.7), the calculated value of the capacitor $C_3$ is equal to

$$C_3 = \frac{1}{\left(2\pi \cdot 85000\right)^2 \cdot 14{,}5 \cdot 10^{-6}} = 241{,}79 \text{ nF}$$

In order to achieve the calculated value of the capacitor $C_3$, five capacitors with capacitance of 47 nF and one capacitor with capacitance of 5,2 nF were connected in parallel. The measured value of the equivalent capacitance is 239,3 nF and these capacitors are shown in Fig. 5.15.


*Figure 5.15: Capacitors in the auxiliary circuit ($C_3$=239,3 nF)*

The measured values of the equivalent capacitors $C_2$ and $C_3$ are also used to make the Bode plot of the transfer function of a three winding model with two capacitors which is shown in Fig. 5.16.

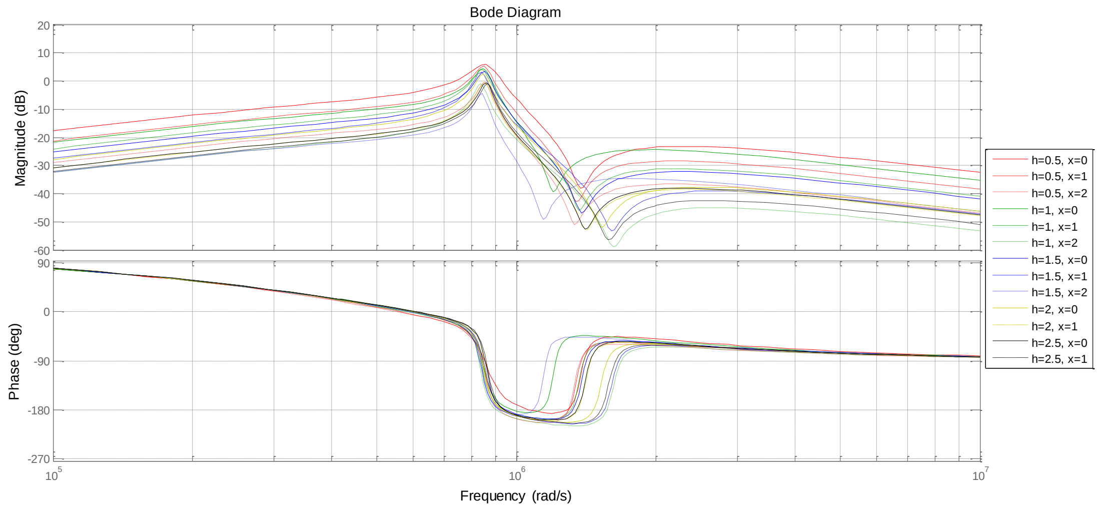


*Figure 5.16: Bode plot of the transfer function of a three winding model with two capacitors ($C_3$=239,3 nF)*

*Table 5.11: Efficiency for the best case ($C_3$=239,3 nF)*

| f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|
| 81,322 | 12,0 | 0,59 | 7,08 | 4,628 | 0,579 | 2,679 | 0,378 |
| 82,241 | 12,0 | 0,58 | 6,96 | 4,520 | 0,565 | 2,554 | 0,367 |
| 83,960 | 12,0 | 0,51 | 6,12 | 4,449 | 0,556 | 2,474 | 0,404 |
| 85,098 | 12,0 | 0,49 | 5,88 | 4,456 | 0,557 | 2,482 | 0,422 |
| 86,616 | 12,0 | 0,47 | 5,64 | 4,409 | 0,551 | 2,429 | 0,431 |
| 87,298 | 12,0 | 0,46 | 5,52 | 4,397 | 0,550 | 2,418 | 0,438 |
| 88,572 | 12,0 | 0,45 | 5,40 | 4,380 | 0.548 | 2,400 | 0,444 |
| 89,442 | 12,0 | 0,46 | 5,52 | 4,463 | 0,558 | 2,490 | 0,451 |

As it can be seen from Fig. 5.16, the peak gain of each of the transfer function is between 800 000 and 900 000 rad/s, which is equal to the frequency interval of 127-143 kHz. However, because of the frequency range that was previously defined from 81,38 to 90 kHz, we would like to see how would this wireless charging system behave if all the peak gains are inside required frequency interval. In order to achieve that, it was necessary to change the value of the capacitor $C_3$ in the auxiliary circuit. After running several simulations, it was found that the value of the capacitor $C_3$

should be around 600 nF. In order to accomplish that value four capacitors with capacitance of 150 nF and one capacitor with capacitance of 6,8 nF were connected in parallel. The measured value of the equivalent capacitance in the auxiliary circuit was 595,2 nF and these capacitors are shown in Fig 5.17. That value of equivalent capacitance $C_3$ was used to make another Bode plot of the transfer function of a three winding model with two capacitors which is shown in Fig. 5.18.


*Figure 5.17: Capacitors in the auxiliary circuit ($C_3$=595,2 nF)*

*Table 5.12: Efficiency for fixed input voltage ($C_3$=595,2 nF)*

| f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|
| 81,170 | 12,0 | 2,03 | 24,36 | 7,908 | 0,989 | 7,821 | 0,321 |
| 82,844 | 12,0 | 2,19 | 26,28 | 8,326 | 1,041 | 8,667 | 0,329 |
| 83,304 | 12,0 | 2,28 | 27,36 | 8,454 | 1,057 | 8,936 | 0,327 |
| 84,436 | 12,0 | 2,51 | 30,12 | 8,748 | 1,094 | 9,570 | 0,318 |
| 85,305 | 12,0 | 2,83 | 33,96 | 9,047 | 1,131 | 10,232 | 0,301 |
| 86,864 | 12,0 | 3,05 | 36,60 | 9,261 | 1,158 | 10,724 | 0,293 |
| 87,335 | 12,0 | 2,77 | 33,24 | 8,611 | 1,076 | 9,265 | 0,278 |
| 88,157 | 12,0 | 2,77 | 33,24 | 8,528 | 1,066 | 9,091 | 0,273 |
| 90,842 | 12,0 | 2,83 | 33,96 | 8,534 | 1,067 | 9,106 | 0,268 |

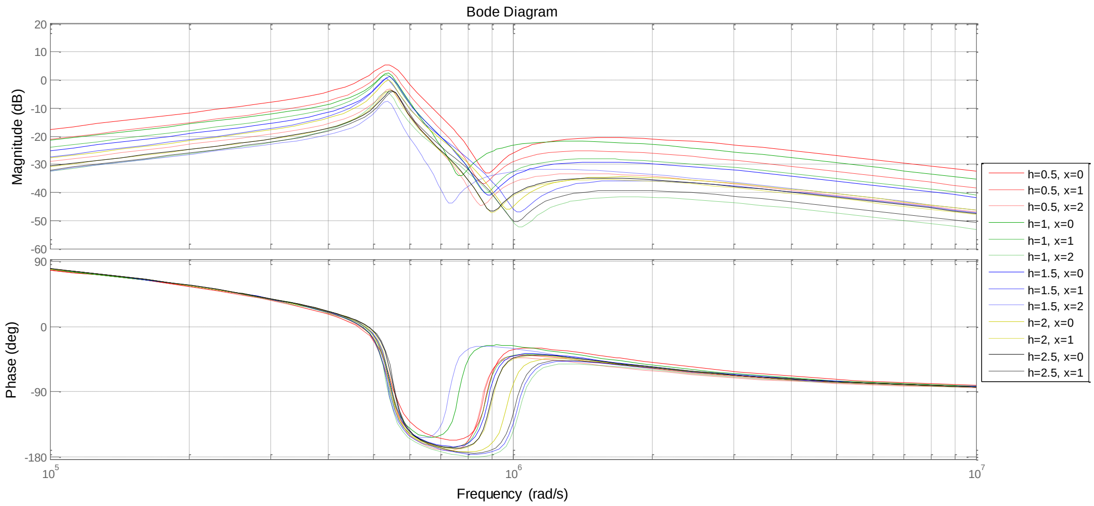


*Figure 5.18: Bode plot of the transfer function of a three winding model with two capacitors ($C_3$=595,2 nF)*

As it can be seen from Fig. 5.18, the peak gain of each of the transfer function is inside the interval of 500 000 to 600 000 rad/s, which is equal to frequency range of 79,5 to 95,5 kHz. However, it is important to say that the third auxiliary circuit is no longer in resonance if we operate inside the required frequency range, because the inductive reactance is higher than the capacitive reactance and it can be written as

$$
\begin{aligned}
Z_3 &= j\omega \cdot L_3 + \frac{1}{j\omega \cdot C_3} \\
Z_3 &= j\left(85000 \cdot 2\pi\right) \cdot 14{,}5 \cdot 10^{-6} + \frac{1}{j\left(85000 \cdot 2\pi\right) \cdot 595{,}2 \cdot 10^{-9}} = j \cdot 4{,}6\ \Omega
\end{aligned}
\tag{5.8}
$$

If there is no resonance in the third auxiliary circuit, it consequently brings higher impedance and the current $I_3$ is therefore lower. The aim of the third auxiliary circuit is to provide the reactive power, so the current in this circuit can be much higher than in the other two circuits. In case the current in the auxiliary circuit is lower than the currents in the transmitter and in the receiver circuit, a direct consequence of that would be that the current $I_3$ is not producing the magnetic flux as much as it could be. If there is less magnetic flux that can be captured by the receiver coil, then the purpose of the third winding is not fulfilled. However, model with three coils and two capacitors does not show good efficiency for the best case, as it can be seen from Table 5.11 and Table 5.12. Therefore, this model will not be tested for different air gap and different misalignment.

## 5.5 Wireless charging system with three coils and three capacitors

In the wireless charging system with three coils and three capacitors, there is a capacitor $C_1$ in the transmitter circuit, a capacitor $C_2$ in the receiver circuit and a capacitor $C_3$ in the auxiliary circuit, which are used to compensate the reactive power. The value of each of the capacitors $C_1$, $C_2$ and $C_3$ is set to cancel the self-inductance in each circuit and it can be written as

$$C_1 = \frac{1}{\omega_0^2 \cdot L_1} = \frac{1}{(2\pi \cdot f_0)^2 \cdot L_1} \tag{5.9}$$

$$C_2 = \frac{1}{\omega_0^2 \cdot L_2} = \frac{1}{(2\pi \cdot f_0)^2 \cdot L_2} \tag{5.10}$$

$$C_3 = \frac{1}{\omega_0^2 \cdot L_3} = \frac{1}{(2\pi \cdot f_0)^2 \cdot L_3} \tag{5.11}$$

where $L_1$, $L_2$ and $L_3$ are the self-inductances of the transmitter, receiver and the third auxiliary coil for the best case when the air gap is $h = 0{,}5$ cm and the misalignment is $x = 0$ cm and their values are given in Table 5.3. The value of each of the capacitors is equal as in the previous two wireless charging systems, so the measured values are shown again below

$$C_1 = 220{,}9 \text{ nF}$$

$$C_2 = 411{,}1 \text{ nF}$$

$$C_3 = 239{,}3 \text{ nF}$$

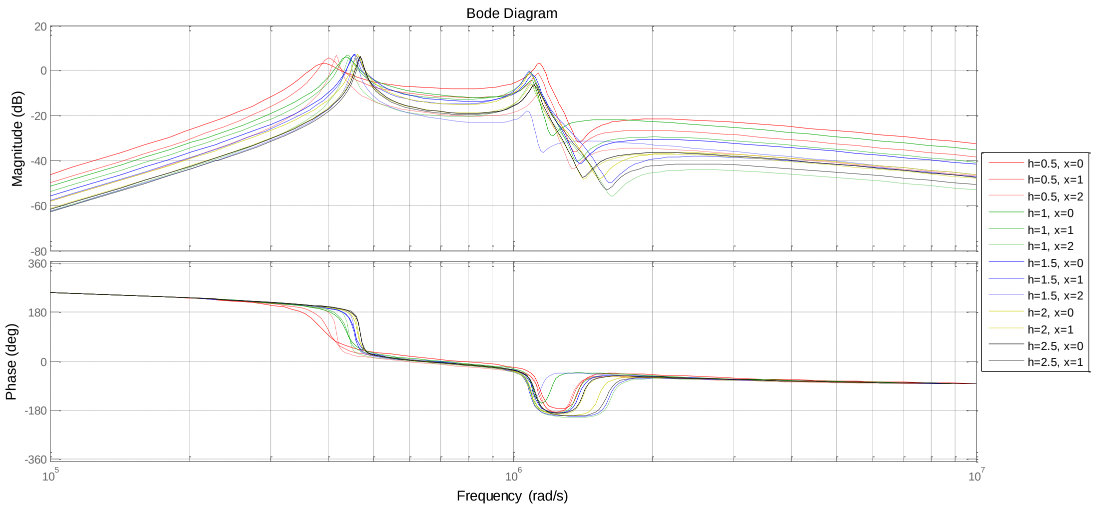


*Figure 5.19: Bode plot of the transfer function of a three winding model with three capacitors*

A bode diagram of the transfer function of a three winding model with three capacitors is shown in Fig. 5.19. As it can be seen from Fig 5.19, the transfer functions have two peak gains, one at the interval between 400 000 and 500 000 rad/s and the other is at approximately $1,1 \cdot 10^6$ rad/s. At the required interval from 81,38 kHz to 90 kHz, the gain of all the transfer functions is below 0 dB which means that the ratio of the output voltage to the input voltage is less than one. In case we want to have the same output voltage for all three wireless charging systems, the input voltage for the wireless charging system with three coils and three capacitors should be increased. If the input voltage is increased, then it is possible to have lower values of the input currents, which will results in almost the same input power for all the wireless charging systems, and therefore it needs to be verified with the measurements.

*Table 5.13: Efficiency for the best case*

| f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|
| 57,443 | 12,0 | 1,82 | 21,84 | 9,690 | 1,211 | 11,734 | 0,537 |
| 59,326 | 12,0 | 2,05 | 24,60 | 10,385 | 1,298 | 13,479 | 0,548 |
| 61,294 | 12,0 | 2,09 | 25,08 | 10,616 | 1,327 | 14,087 | 0,562 |
| 63,749 | 12,0 | 1,85 | 22,20 | 10,012 | 1,252 | 12,535 | 0,566 |
| 81,892 | 12,0 | 0,69 | 8,28 | 5,085 | 0,636 | 3,234 | 0,391 |
| 83,301 | 12,0 | 0,60 | 7,20 | 4,785 | 0,598 | 2,861 | 0,397 |
| 85,696 | 12,0 | 0,53 | 6,36 | 4,573 | 0,572 | 2,616 | 0,411 |
| 87,442 | 12,0 | 0,48 | 5,76 | 4,396 | 0,549 | 2,413 | 0,419 |
| 89,686 | 12,0 | 0,41 | 4,92 | 4,201 | 0,525 | 2,206 | 0,448 |

*Table 5.14: Output and input power for different air gap and misalignment*

| $h$ [cm] | $x$ [cm] | f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $I_{3,rms}$ [A] |
|---|---|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 90,326 | 36 | 1,32 | 47,52 | 13,994 | 1,749 | 24,476 | 4,795 |
| 0,5 | 1 | 81,551 | 36 | 1,56 | 56,16 | 12,363 | 1,545 | 19,101 | 6,036 |
| 0,5 | 2 | 82,865 | 36 | 0,97 | 34,92 | 3,748 | 0,469 | 1,758 | 6,213 |
| 1 | 0 | 87,309 | 36 | 1,23 | 44,28 | 12,440 | 1,555 | 19,344 | 5,318 |
| 1 | 1 | 81,344 | 36 | 1,67 | 60,12 | 11,766 | 1,471 | 17,308 | 6,910 |
| 1 | 2 | 81,413 | 36 | 1,08 | 38,88 | 3,973 | 0,497 | 1,975 | 6,965 |
| 1,5 | 0 | 87,019 | 36 | 1,08 | 38,88 | 9,589 | 1,199 | 11,497 | 5,847 |
| 1,5 | 1 | 83,714 | 36 | 1,22 | 43,92 | 8,452 | 1,057 | 8,934 | 6,619 |
| 1,5 | 2 | 85,630 | 36 | 1,12 | 40,32 | 4,009 | 0,501 | 2,009 | 7,585 |
| 2 | 0 | 83,371 | 36 | 1,22 | 43,92 | 7,733 | 0,967 | 7,478 | 6,998 |
| 2 | 1 | 89,954 | 36 | 0,88 | 31,68 | 5,576 | 0,697 | 3,886 | 6,709 |
| 2,5 | 0 | 82,196 | 36 | 1,32 | 47,52 | 5,542 | 0,693 | 3,841 | 7,518 |
| 2,5 | 1 | 83,728 | 36 | 1,04 | 37,44 | 4,061 | 0,508 | 2,063 | 6,957 |

In order to get the peak gain of the transfer functions in the required interval, it is necessary to change the value of the capacitor $C_3$ and it was found through several simulations that the best value of $C_3$ is 25 nF. To achieve that value, six capacitors with capacitance of 150 nF were connected in parallel. The measured value of the equivalent capacitance was 24,48 nF and the new capacitors in the third auxiliary coil are shown in Fig. 5.20.

*Figure 5.20: Capacitors in the auxiliary circuit ($C_3$=24,48 nF)*

The measured values of the output and the input voltages as well as their magnitude are shown in Table 5.9 for the case when the equivalent capacitance is $C_3$=24,48 nF. The measured values are used to find the transfer function for the best case when the misalignment is $x$=0 cm and the air gap is $h$=0,5 cm. The Bode plot of the measured and the simulated transfer function is shown in Fig. 5.21.

*Table 5.15: Measured values of the magnitudes*

| $V_{S,rms}$ [V] | $V_{out,rms}$ [V] | $20\log\left(\frac{V_{out,rms}}{V_{S,rms}}\right)$ [dB] | $f_{meas}$ [kHz] |
|---|---|---|---|
| 6,883 | 6,440 | -0,58 | 70,232 |
| 6,857 | 9,122 | 2,48 | 73,930 |
| 6,499 | 10,588 | 4,24 | 77,303 |
| 4,778 | 9,041 | 5,54 | 81,685 |
| 4,574 | 8,171 | 5,04 | 85,277 |
| 4,685 | 6,772 | 3,20 | 90,561 |
| 4,701 | 5,973 | 2,08 | 93,900 |

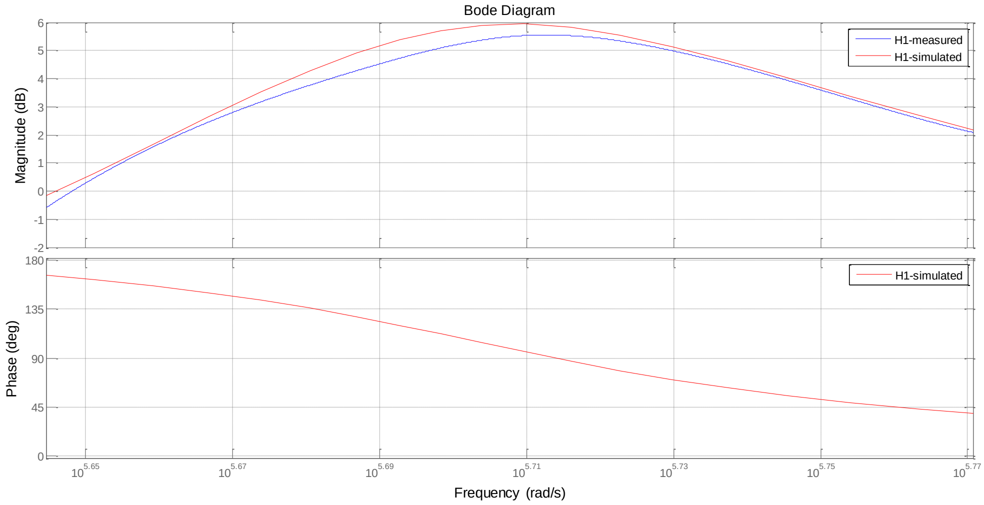


*Figure 5.21: Bode plot along with the measured values of output/input voltage*

*Table 5.16: Efficiency for different  air gap and misalignment*

| f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $\eta = \frac{P_{out}}{P_{in}}$ |
|---|---|---|---|---|---|---|---|
| 79,722 | 12,0 | 3,57 | 42,84 | 14,872 | 1,859 | 27,647 | 0,645 |
| 81,441 | 12,0 | 3,29 | 39,48 | 14,390 | 1,799 | 25,888 | 0,656 |
| 83,356 | 12,0 | 3,07 | 36,84 | 13,817 | 1,727 | 23,862 | 0,648 |
| 84,308 | 12,0 | 2,82 | 33,84 | 13,210 | 1,651 | 21,809 | 0,644 |
| 85,814 | 12,0 | 2,64 | 31,68 | 12,608 | 1,576 | 19,870 | 0,627 |
| 87,673 | 12,0 | 2,34 | 28,08 | 12,003 | 1,500 | 18,005 | 0,641 |
| 89,714 | 12,0 | 2,09 | 25,08 | 11,294 | 1,411 | 15,936 | 0,635 |

As it can be seen from Table 5.16, the overall efficiency is close to the efficiency for the system with two coils. It is because the current in the auxiliary winding is very low, because there is no longer resonance in the auxiliary circuit if the value of the capacitor $C_3$ is changed from 239,3 nF to 24,48 nF.

Table 5.17 shows the output and the input power for different air gap and different misalignment. By using the capacitor $C_3$ whose value is 24,48 nF, it can be seen that the current in the primary circuit is slightly reduced, therefore causing this system with three coils to behave like system with two coils.

*Table 5.17: Output and input power for different air gap and misalignment*

| $h$ [cm] | $x$ [cm] | f [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $P_{in}$ [W] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $P_{out}$ [W] | $I_{3,rms}$ [A] |
|---|---|---|---|---|---|---|---|---|---|
| 0,5 | 0 | 85,142 | 12,0 | 2,66 | 31,92 | 12,507 | 1,563 | 19,553 | 0,935 |
| 0,5 | 1 | 84,126 | 12,0 | 2,90 | 34,80 | 12,446 | 1,556 | 19,366 | 1,004 |
| 0,5 | 2 | 81,692 | 12,0 | 2,49 | 29,88 | 6,058 | 0,757 | 4,586 | 1,154 |
| 1 | 0 | 82,234 | 12,0 | 3,44 | 41,28 | 12,459 | 1,557 | 19,399 | 1,049 |
| 1 | 1 | 85,907 | 12,0 | 3,19 | 38,28 | 11,519 | 1,439 | 16,576 | 1,133 |
| 1 | 2 | 85,668 | 12,0 | 2,72 | 32,64 | 4,972 | 0,622 | 3,093 | 1,229 |
| 1,5 | 0 | 88,768 | 12,0 | 2,79 | 33,48 | 9,631 | 1,204 | 11,596 | 1,128 |
| 1,5 | 1 | 86,075 | 12,0 | 3,04 | 36,48 | 8,470 | 1,059 | 8,969 | 1,361 |
| 1,5 | 2 | 89,434 | 12,0 | 2,86 | 34,32 | 4,088 | 0,511 | 2,089 | 1,256 |
| 2 | 0 | 89,369 | 12,0 | 3,17 | 38,04 | 7,828 | 0,979 | 7,664 | 1,293 |
| 2 | 1 | 86,464 | 12,0 | 2,35 | 28,20 | 5,524 | 0,691 | 3,817 | 1,470 |
| 2,5 | 0 | 91,849 | 12,0 | 2,83 | 33,96 | 5,502 | 0,688 | 3,785 | 1,194 |
| 2,5 | 1 | 92,731 | 12,0 | 2,31 | 27,72 | 4,053 | 0,506 | 2,051 | 1,138 |

# 6 Analysis

## 6.1 Copper losses

The amount of copper that was used in this set-up can be calculated as

$$m = V \cdot \rho \tag{6.1}$$

where *m* is the mass, *V* is volume and $\rho$ is the density of the copper and it is equal to 8,96 g/cm$^3$. In order to find the volume of each wire or coil, a mean radius is needed. For the receiver coil, which is drawn with its dimensions in Fig. 6.1, the mean radius can be determined as

$$R = \frac{r_2 - r_1}{2} + r_1 \tag{6.2}$$

where $r_2$ is the outer radius, and $r_1$ is the inner radius of the coil.

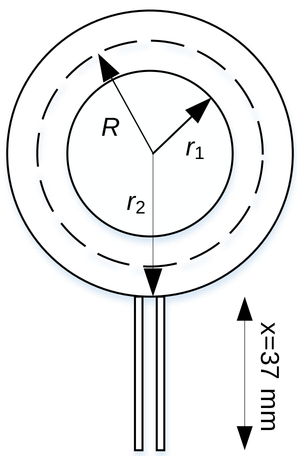


*Figure 6.1: The mean radius of the receiver coil*

From the datasheet that was previously given in [8], the inner radius $r_1$ of the receiver coil is 20,5 mm, and the outer radius $r_2$ is 43,0. Therefore, by using (6.2), the mean radius *R* is 37,5 mm. The total length of the copper wire that was used for the receiver coil can be expressed as

$$L = 2\pi \cdot R \cdot N + 2 \cdot x \tag{6.3}$$

where *N* is the number of turns of the receiver coil and it is equal to $N_2 = 10$, and x can be determined from the Fig. 6.1 and it is equal to 37 mm. By using (6.3), the total length of the copper wire in the receiver coil is 2430,19 mm.

Each turn of the receiver coil consists of two wires and each wire has a cross section of 0,75 $\text{mm}^2$. Volume $V$ of the copper wire used for the receiver coil can be given as

$$V = L \cdot A \tag{6.4}$$

where $L$ is the length and $A$ is the total cross section and is equal to 1,5 $\text{mm}^2$. By using (6.4), the total volume of the copper used in the receiver coil is equal to 3645,285 $\text{mm}^3$ or 3,645285 $\text{cm}^3$. Finally, the mass of the copper used in the receiver coil can be determined from (6.1), and is equal to 32,662 g.

The same analysis will be applied to the transmitter and the third auxiliary coil. Both of the coils have equal shape which is shown in Fig. 6.2.

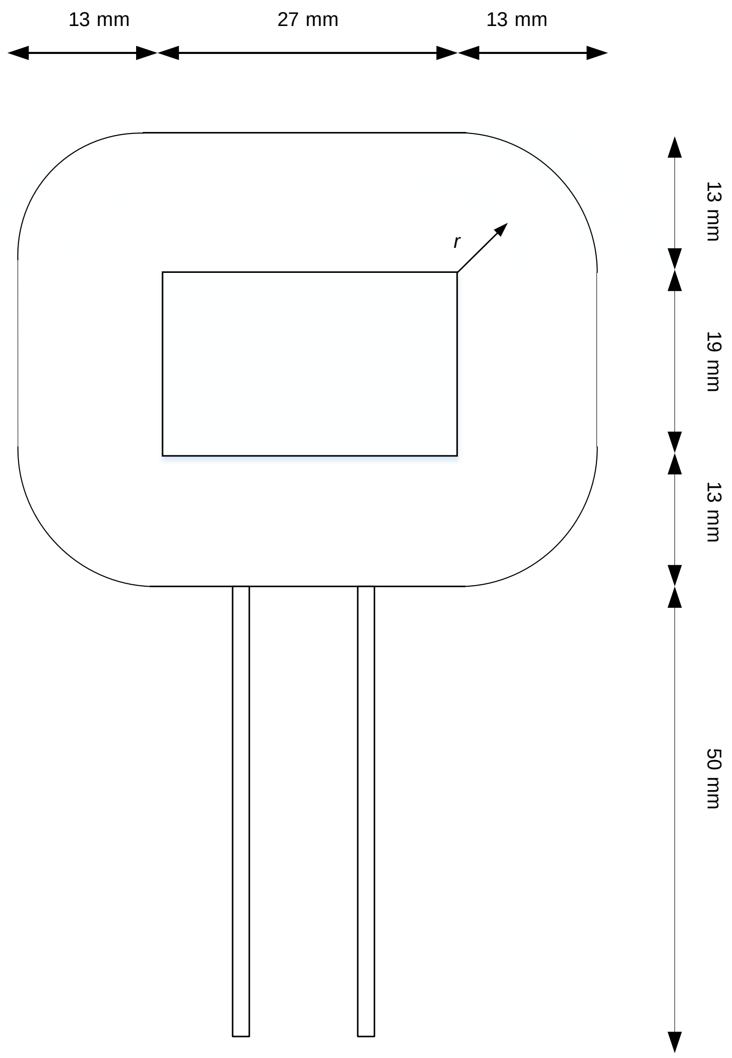


*Figure 6.2: Shape of the transmitter and the auxiliary coil*

The total length of one turn of the coil given in Fig. 6.2 can be determined as

$$L = N \cdot \left[ (a+b) \cdot 2 + 2\pi \cdot r \right] + 2 \cdot x \tag{6.5}$$

where $N$ is the number of turns of the coil, and $a$, $b$ and $x$ are equal to 27, 19 and 50 mm, respectively. The radius $r$ can be expressed as

$$r = \frac{13}{2} = 6{,}5 \text{ mm} \tag{6.6}$$

The number of turns of both coils is $N_1 = N_3 = 12$, so the total length of the copper wire used in the transmitter and the auxiliary coil can be given as

$$L = 12 \cdot \left[ (27+19) \cdot 2 + 2\pi \cdot 6{,}5 \right] + 2 \cdot 50 = 1694{,}09 \text{ mm}$$

The cross section of the coil given in Fig. 6.2 is $0{,}75 \text{ mm}^2$ and by using (6.4), the total volume of the copper used for the transmitter and the auxiliary coil can be determined as

$$V = L \cdot A$$
$$V = 1694{,}09 \cdot 0{,}75 = 1270{,}57 \text{ mm}^2$$

Finally, the mass of the copper used in the transmitter and the auxiliary coil can be determined from (6.1), and is equal to 11,384 g.

The set-up with three coils and three capacitors is shown in Fig. 6.3. As it can be seen from Fig. 6.3, there are three wires in the primary circuit that go from the inverter to the transmitter coil. In the auxiliary circuit, there are two wires connecting the capacitors with the third coil. High frequency current will go through three wires in the receiver circuit, and after the rectifier, dc current will go through four wires. This number of wires is important as it will be used to find the copper losses. The cross section of each wire is $2{,}5 \text{ mm}^2$ and the length is 250 mm. Therefore, the volume of copper inside one wire is $625 \text{ mm}^3$, and for all twelve wires, the total volume is 7500 $\text{mm}^3$ or $7{,}5 \text{ cm}^3$. The mass of copper that was used in the set-up with three coils can be given as

$$m = V \cdot \rho$$
$$m = 7{,}5 \cdot 8{,}96 = 67{,}2 \text{ g}$$

The total amount of copper in the set-up with three coils and three capacitors can be determined as the sum of the copper in the coils plus the copper in the wires which can be written as

$$m_{33} = m(L_1) + m(L_2) + m(L_3) + m(\text{wires})$$
$$m_{33} = 11{,}384 + 32{,}662 + 11{,}384 + 67{,}2 = 122{,}63 \text{ g}$$

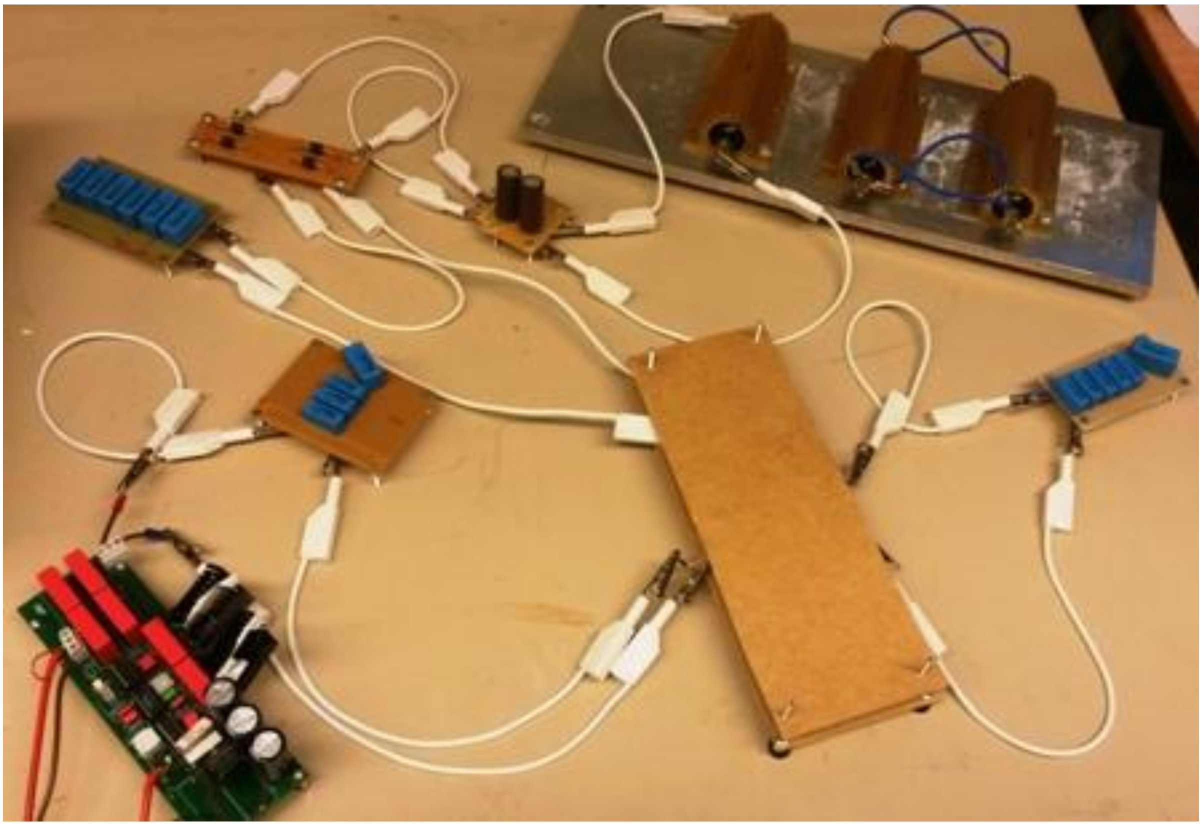

*Figure 6.3: Set-up with three coils and three capacitors*

In the set-up with two coils and two capacitors, the auxiliary coil with two wires was not used, therefore, the total amount of the copper can be determined as

$$m_2 = m(L_1) + m(L_2) + m(\text{wires}) \cdot \frac{10}{12}$$

$$m_2 = 11{,}384 + 32{,}662 + 67{,}2 \cdot \frac{10}{12} = 100{,}046 \text{ g}$$

By comparing the total mass of the copper in both systems, it can be seen that the set-up with three coils and three capacitors has 22,6% more copper.

In the set-up with three coils and two capacitors, there is only one wire less than in the setup with three coils and three capacitors, so the total amount of copper that was used can be given as

$$m_{32} = m(L_1) + m(L_2) + m(L_3) + m(\text{wires}) \cdot \frac{11}{12}$$

$$m_{32} = 11{,}384 + 32{,}662 + 11{,}384 + 67{,}2 \cdot \frac{11}{12} = 117{,}03 \text{ g}$$

It can be seen that the set-up with three coils and two capacitors has 17% more copper comparing to a system with two coils.

The ac and the dc resistances of the wires and the coils that were used are presented in Table 6.1.

*Table 6.1: AC and DC resistance of the coils and the wire*

| | AC resistance | DC resistance |
|---|---|---|
| Transmitter coil | $R_{\mathrm{AC,1}} = 188{,}132\ \mathrm{m\Omega}$ | $R_{\mathrm{DC,1}} = 54{,}783\ \mathrm{m\Omega}$ |
| Receiver coil | $R_{\mathrm{AC,2}} = 106{,}136\ \mathrm{m\Omega}$ | $R_{\mathrm{DC,2}} = 16{,}512\ \mathrm{m\Omega}$ |
| Auxiliary coil | $R_{\mathrm{AC,3}} = 172{,}231\ \mathrm{m\Omega}$ | $R_{\mathrm{DC,3}} = 54{,}335\ \mathrm{m\Omega}$ |
| White wire | $R_{\mathrm{AC,wire}} = 79{,}729\ \mathrm{m\Omega}$ | $R_{\mathrm{DC,wire}} = 653{,}213\ \mathrm{\mu\Omega}$ |

For the wireless charging system with two coils and two capacitors, the copper losses can be divided into losses in the transmitter and the receiver coil and the losses in the wires. In this system, the high frequency current in the primary circuit $i_1$ goes through the three wires, as well as the high frequency current $i_2$ in the receiver circuit which also goes through three wires. As the dc resistance of the wire can be neglected due to its very small value, the copper loss in the wires after the rectifier will not be calculated. So, the total copper losses can be given as

$$P_{\mathrm{Cu}} = i_{\mathrm{1,rms}}^2 \cdot \left(R_{\mathrm{AC,1}} + 3 \cdot R_{\mathrm{AC,wire}}\right) + i_{\mathrm{2,rms}}^2 \cdot \left(R_{\mathrm{AC,2}} + 3 \cdot R_{\mathrm{AC,wire}}\right) \tag{6.7}$$

For the system with two coils, the measured rms values of the high frequency currents in the primary and in the receiver circuit, as well as the dc input and the output currents and voltages are given in Table 6.2.

*Table 6.2: Measured values of currents and voltages*

| | $f_{\mathrm{s}}$ [kHz] | $V_{\mathrm{DC,in}}$ [V] | $I_{\mathrm{DC,in}}$ [A] | $I_{\mathrm{1,rms}}$ [A] | $V_{\mathrm{DC,out}}$ [V] | $I_{\mathrm{DC,out}}$ [A] | $I_{\mathrm{2,rms}}$ [A] |
|---|---|---|---|---|---|---|---|
| measured | 83,209 | 12,0 | 3,82 | 4,205 | 15,291 | 1,911 | 2,149 |

By using the rms values for the current given in Table 6.2, the total copper losses can be given as

$$P_{\mathrm{Cu}} = 4{,}205^2 \cdot \left(0{,}188132 + 3 \cdot 0{,}079729\right) + 2{,}149^2 \cdot \left(0{,}106136 + 3 \cdot 0{,}079729\right)$$
$$P_{\mathrm{Cu,meas}} = 7{,}556 + 1{,}595 = 9{,}151\ \mathrm{W}$$

The winding loss for the system with two coils can be given as

$$P_{\mathrm{Cu}}' = i_{\mathrm{1,rms}}^2 \cdot R_{\mathrm{AC,1}} + i_{\mathrm{2,rms}}^2 \cdot R_{\mathrm{AC,2}}$$
$$P_{\mathrm{Cu}}' = 4{,}205^2 \cdot 0{,}188132 + 2{,}149^2 \cdot 0{,}106136$$
$$P_{\mathrm{Cu}}' = 3{,}327 + 0{,}490 = 3{,}817\ \mathrm{W}$$

For the wireless charging system with three coils and two capacitors, the total copper losses are calculated as the sum of the losses inside three coils plus the wire losses. The total number of wires used in this system is eleven, of which two are in the primary circuit, three are in the secondary circuit and two are in the auxiliary circuit. The other four wires are in the dc circuit and they are neglected because the dc resistance can be neglected compared to the ac resistance. The total copper losses can be given as

$$P_{Cu} = i_{1,rms}^2 \cdot \left(R_{AC,1} + 2 \cdot R_{AC,wire}\right) + i_{2,rms}^2 \cdot \left(R_{AC,2} + 3 \cdot R_{AC,wire}\right) + i_{3,rms}^2 \cdot \left(R_{AC,3} + 2 \cdot R_{AC,wire}\right) \quad (6.8)$$

For this system, the measured rms values of all high frequency currents, as well as the dc input and the output currents and voltages are given in Table 6.3.

*Table 6.3: Measured values of currents and voltages*

| | $f_s$ [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $I_{1,rms}$ [A] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $I_{2,rms}$ [A] | $I_{3,rms}$ [A] |
|---|---|---|---|---|---|---|---|---|
| measured | 84,204 | 32 | 1,60 | 2,784 | 15,301 | 1,913 | 2,398 | 4,553 |

By using (6.8) and the values from Table 6.3, the total copper losses are equal to

$$P_{Cu} = 2{,}784^2 \cdot (0{,}188132 + 2 \cdot 0{,}079729) + 2{,}398^2 \cdot (0{,}106136 + 3 \cdot 0{,}079729) + 4{,}553^2 \cdot (0{,}172231 + 2 \cdot 0{,}079729)$$

$$P_{Cu} = 2{,}694 + 1{,}986 + 6{,}876 = 11{,}556 \text{ W}$$

The winding loss for the system with three coils and two capacitors can be given as

$$P_{Cu}' = i_{1,rms}^2 \cdot R_{AC,1} + i_{2,rms}^2 \cdot R_{AC,2} + i_{3,rms}^2 \cdot R_{AC,3}$$

$$P_{Cu}' = 2{,}784^2 \cdot 0{,}188132 + 2{,}398^2 \cdot 0{,}106136 + 4{,}553^2 \cdot 0{,}172231$$

$$P_{Cu}' = 1{,}458 + 0{,}610 + 3{,}570 = 5{,}638 \text{ W}$$

For the wireless charging system with three coils and three capacitors, the copper losses are calculated as the sum of the losses inside all three winding plus the wire losses. As there are three wires in the primary circuit, three wires in the secondary circuit and two wires in the auxiliary circuit, the copper losses in this system will be the highest comparing to other two systems. The total copper losses can be given as

$$P_{Cu} = i_{1,rms}^2 \cdot \left(R_{AC,1} + 3 \cdot R_{AC,wire}\right) + i_{2,rms}^2 \cdot \left(R_{AC,2} + 3 \cdot R_{AC,wire}\right) + i_{3,rms}^2 \cdot \left(R_{AC,3} + 2 \cdot R_{AC,wire}\right) \quad (6.9)$$

For this system, the measured rms values of all high frequency currents, as well as the dc input and the output currents and voltages are given in Table 6.4.

*Table 6.4: Measured values of currents and voltages*

| | $f_s$ [kHz] | $V_{DC,in}$ [V] | $I_{DC,in}$ [A] | $I_{1,rms}$ [A] | $V_{DC,out}$ [V] | $I_{DC,out}$ [A] | $I_{2,rms}$ [A] | $I_{3,rms}$ [A] |
|---|---|---|---|---|---|---|---|---|
| measured | 64,061 | 17,0 | 2,80 | 3,316 | 15,289 | 1,911 | 2,128 | 4,049 |

By using (6.9), the total copper losses are equal to

$$P_{Cu} = 3{,}316^2 \cdot \left(0{,}188132 + 3 \cdot 0{,}079729\right) + 2{,}128^2 \cdot \left(0{,}106136 + 3 \cdot 0{,}079729\right) + 4{,}049^2 \cdot \left(0{,}172231 + 2 \cdot 0{,}079729\right)$$

$$P_{Cu} = 4{,}699 + 1{,}564 + 5{,}4381 = 11{,}701 \text{ W}$$

The winding loss for the system with three coils and three capacitors can be given as

$$P_{Cu}' = i_{1,rms}^2 \cdot R_{AC,1} + i_{2,rms}^2 \cdot R_{AC,2} + i_{3,rms}^2 \cdot R_{AC,3}$$
$$P_{Cu}' = 3{,}316^2 \cdot 0{,}188132 + 2{,}128^2 \cdot 0{,}106136 + 4{,}049^2 \cdot 0{,}172231$$
$$P_{Cu}' = 2{,}069 + 0{,}481 + 2{,}824 = 5{,}374 \text{ W}$$

## 6.2 Power semiconductors losses

For the system with two coils, the conduction losses in the converter can be determined as

$$\mathrm{P_{cond,total}} = 4 \cdot \mathrm{P_{cond}} = 4 \cdot \frac{1}{4} \cdot \mathrm{R_{DS,on}} \cdot \mathrm{i_{D,pk}^2} = \mathrm{R_{DS,on}} \cdot \mathrm{i_{D,pk}^2}$$
$$\mathrm{P_{cond,total}} = 0.018 \cdot 5{,}947^2 = 0{,}637 \text{ W}$$

The switching losses in the converter can be found as

$$P_{switch} = 4 \cdot \left( \frac{v_{DS} \cdot i_{D,pk} \cdot T_{on}}{2} + \frac{v_{DS} \cdot i_{D,pk} \cdot T_{off}}{2} \right) \cdot f_s = 2 \cdot v_{DS} \cdot i_{D,pk} \cdot f_s \cdot \left(T_{on} + T_{off}\right)$$

where the turn on and the turn off time are measured and are equal to

$$T_{on} = 149{,}443 \text{ ns}$$

$$T_{off} = 145{,}579 \text{ ns}$$

Therefore, the switching losses can be given as

$$P_{switch} = 2 \cdot 12 \cdot 5{,}947 \cdot 83209 \cdot \left(149{,}443 + 145{,}579\right) \cdot 10^{-9} = 3{,}504 \text{ W}$$

The total losses in the converter are equal to

$$P_{\text{total,meas}} = 0{,}637 + 3{,}504 = 4{,}141 \text{ W}$$

The conduction loss in the bridge rectifier can be determined as

$$P_{\text{cond,total}} = 4 \cdot v_{D0} \frac{i_{D,pk}}{\pi} + R_D \cdot i_{D,pk}^2$$

$$P_{\text{cond,total}} = 4 \cdot 0{,}82 \cdot \frac{2{,}149\sqrt{2}}{\pi} + 0{,}01 \cdot \left(2{,}149\sqrt{2}\right)^2 = 3{,}265 \text{ W}$$

For the system with three coils and two capacitors, the conduction losses in the converter can be determined as

$$\mathrm{P}_{\text{cond,total}} = 4 \cdot \mathrm{P}_{\text{cond}} = 4 \cdot \frac{1}{4} \cdot \mathrm{R}_{\text{DS,on}} \cdot \mathrm{i}_{\text{D,pk}}^2 = \mathrm{R}_{\text{DS,on}} \cdot \mathrm{i}_{\text{D,pk}}^2$$

$$\mathrm{P}_{\text{cond,total}} = 0.018 \cdot \left(2{,}784\sqrt{2}\right)^2 = 0{,}279 \text{ W}$$

The switching losses in the converter can be determined as

$$P_{\text{switch}} = 4 \cdot \left( \frac{v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot T_{\text{on}}}{2} + \frac{v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot T_{\text{off}}}{2} \right) \cdot f_{\text{s}} = 2 \cdot v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot f_{\text{s}} \cdot \left(T_{\text{on}} + T_{\text{off}}\right)$$

$$P_{\text{switch}} = 2 \cdot 32 \cdot 2{,}784\sqrt{2} \cdot 84204 \cdot \left(149{,}443 + 145{,}579\right) \cdot 10^{-9} = 6{,}260 \text{ W}$$

The total losses in the converter are equal to

$$P_{\text{total,meas}} = 0{,}279 + 6{,}260 = 6{,}539 \text{ W}$$

The conduction loss in the bridge rectifier can be determined as

$$P_{\text{cond,total}} = 4 \cdot v_{D0} \frac{i_{D,pk}}{\pi} + R_D \cdot i_{D,pk}^2$$

$$P_{\text{cond,total}} = 4 \cdot 0{,}82 \cdot \frac{2{,}382\sqrt{2}}{\pi} + 0{,}01 \cdot \left(2{,}382\sqrt{2}\right)^2 = 3{,}631 \text{ W}$$

For the system with three coils and three capacitors, the conduction losses in the converter can be determined as

$$\mathrm{P}_{\text{cond,total}} = 4 \cdot \mathrm{P}_{\text{cond}} = 4 \cdot \frac{1}{4} \cdot \mathrm{R}_{\text{DS,on}} \cdot \mathrm{i}_{\text{D,pk}}^2 = \mathrm{R}_{\text{DS,on}} \cdot \mathrm{i}_{\text{D,pk}}^2$$

$$\mathrm{P}_{\text{cond,total}} = 0.018 \cdot \left(3{,}316\sqrt{2}\right)^2 = 0{,}396 \text{ W}$$

The switching losses in the converter can be determined as

$$P_{\text{switch}} = 4 \cdot \left( \frac{v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot T_{\text{on}}}{2} + \frac{v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot T_{\text{off}}}{2} \right) \cdot f_{\text{s}} = 2 \cdot v_{\text{DS}} \cdot i_{\text{D,pk}} \cdot f_{\text{s}} \cdot \left( T_{\text{on}} + T_{\text{off}} \right)$$

$$P_{\text{switch}} = 2 \cdot 17 \cdot 3{,}316\sqrt{2} \cdot 64061 \cdot \left(149{,}443 + 145{,}579\right) \cdot 10^{-9} = 3{,}013 \text{ W}$$

The total losses in the converter are equal to

$$P_{\text{total,meas}} = 0{,}396 + 3{,}013 = 3{,}409 \text{ W}$$

The conduction loss in the bridge rectifier can be determined as

$$P_{\text{cond,total}} = 4 \cdot v_{D0} \frac{i_{D,pk}}{\pi} + R_D \cdot i_{D,pk}^2$$

$$P_{\text{cond,total}} = 4 \cdot 0{,}82 \cdot \frac{2{,}128\sqrt{2}}{\pi} + 0{,}01 \cdot \left(2{,}128\sqrt{2}\right)^2 = 3{,}233 \text{ W}$$

## 6.3 Comparison of losses

The results that were obtained by doing analysis are provided in Table 6.5, where $m_{\mathrm{Cu}}$ is the copper weight, $P_{\mathrm{rectifier}}$ are the rectifier losses, $P_{\mathrm{converter}}$ are the converter losses and $P_{\mathrm{loss,total}}$ are the total losses. In order to make comparison, the output power was approximately the same in all three cases and as it can be seen, the best efficiency has the system with two coils. The highest amount of copper, as it was expected was used in the system with three coils and three capacitors and that system also has the highest copper loss. Due to high input voltage in the system with three coils and two capacitors, there are, comparing to other two systems, high switching losses, which increased the total losses inside the converter. This comparison could not fulfill the requirement of having the same input voltage in all three cases, because the output power would not be the same.

*Table 6.5: Comparison of three wireless charging systems*

| | Two coils and two capacitors | Three coils and two capacitors | Three coils and three capacitors |
|---|---|---|---|
| $V_{\mathrm{DC,in}}$ [V] | 12,0 | 32,0 | 17,0 |
| $I_{\mathrm{DC,in}}$ [A] | 3,82 | 1,61 | 2,80 |
| $P_{\mathrm{DC,in}}$ [W] | 45,84 | 51,52 | 47,60 |
| $V_{\mathrm{DC,out}}$ [V] | 15,291 | 15,301 | 15,289 |
| $I_{\mathrm{DC,out}}$ [A] | 1,911 | 1,913 | 1,911 |
| $P_{\mathrm{DC,out}}$ [W] | 29,221 | 29,271 | 29,217 |
| $\eta = \frac{P_{\mathrm{DC,out}}}{P_{\mathrm{DC,in}}}$ | 0,637 | 0,568 | 0,614 |
| $m_{\mathrm{Cu}}$ [g] | 100,046 | 117,030 | 122,630 |
| $P_{\mathrm{Cu}}$ [W] | 9,151 | 11,556 | 11,701 |
| $P_{\mathrm{rectifier}}$ [W] | 3,265 | 3,631 | 3,233 |
| $P_{\mathrm{converter}}$ [W] | 4,141 | 6,539 | 3,409 |
| $P_{\mathrm{loss,total}}$ [W] | 16,557 | 21,726 | 18,343 |
| $P_{\mathrm{DC,out}} + P_{\mathrm{loss,total}}$ [W] | 45,778 | 50,997 | 47,560 |

The efficiency of three different wireless charging systems is presented in Table 6.6. It was calculated as the input power minus the losses in the power semiconductor devices and the winding loss $P_{\mathrm{Cu}}'$ and that can be written as

$$\eta = \frac{P_{\mathrm{in}} - P_{\mathrm{Cu}}' - P_{\mathrm{rectifier}} - P_{\mathrm{converter}}}{P_{\mathrm{in}}} \tag{6.10}$$

*Table 6.6: Calculated efficiency when the AC resistance of the wires is neglected*

| | Two coils and two capacitors | Three coils and two capacitors | Three coils and three capacitors |
|---|---|---|---|
| $V_{DC,in}$ [V] | 12,0 | 32,0 | 17,0 |
| $I_{DC,in}$ [A] | 3,82 | 1,61 | 2,80 |
| $P_{DC,in}$ [W] | 45,84 | 51,52 | 47,60 |
| $P_{Cu}'$ [W] | 3,817 | 5,638 | 5,374 |
| $P_{rectifier}$ [W] | 3,265 | 3,631 | 3,233 |
| $P_{converter}$ [W] | 4,141 | 6,539 | 3,409 |
| $P_{loss,total}$ [W] | 11,223 | 15,808 | 12,016 |
| $\eta$ | 0,755 | 0,693 | 0,748 |

As it can be seen from Table 6.6, the overall efficiency of each wireless charging system is significantly increased when the ac resistance is neglected. For the system with two coils the efficiency is increased from 63,7% to 75,5% and for the system with three coils and two capacitors the efficiency is increased from 56,8% to 69,3%. The highest increase in the efficiency is in the system with three coils and three capacitors, which is from 61,4% to 74,8%. Therefore, it can be concluded that the selection of the wires was not adequate, because the AC resistance of the wires caused additional copper losses, which resulted in lower efficiency.

# 7 Conclusions

In this thesis, three different wireless charging systems were investigated. The first model that was taken into consideration was a simple two winding wireless transfer system with capacitors connected in series with the coils. By adding a third winding close to the transmitter coil, there were several possibilities regarding the number of the capacitors and their connection in the circuit, therefore only two different designs of the three winding wireless transfer systems were analyzed in this thesis. The first model of a three winding wireless transfer system had a capacitor connected in series in the receiver loop and in the auxiliary loop, while the second model had a capacitor connected in series with each coil.

The transfer function of each wireless transfer system was determined in oder to describe the relation between the output and the input voltage. By increasing the air gap between the transmitter and the receiver coil from 0,5 cm to 2,5 cm and changing the misalignment from 0 cm to 3 cm, there were 20 different cases, of which only 13 of them had the mutual inductances $M_{12}$ and $M_{23}$ different from zero. The coils were quite sensitive when the air gap and the misalignment were changed, which resulted in lower overall efficiency. Another disadvantage of the given coils is the relatively high ac resistance which resulted in high copper loss. Therefore, a recommendation for some future work would be to use the Finite Element Method (FEM) in order to make the most suitable design of the coils.

It was decided that the output voltage should be equal in each case in order to make a valid comparison of three different wireless charging models, so the output voltage was chosen to be 15 V. With this requirement, it was found that the wireless charging system with three coils and two capacitors needs to have higher input voltage (32 V) in order to fulfill this requirement, compared to a basic two winding system, which had the input voltage of 12 V. However, by having higher input voltage in the system with three coils and two capacitors it was possible to reduce the input current from 3,80 A in the system with two coils to 1,61 A. By adding a third winding it is possible to downrate the copper losses in the transmitter loop, but on the other hand, there are high copper losses in the auxiliary circuit. It is important to say that if the input voltage is kept fixed in each system, then the currents in the primary and in the secondary circuit are signifacntly lowered in models with three winding compared to a basic two winding model. Therefore, the input voltage was increased in both of the three winding models in order to have the same output voltage.

It was found that the best overall efficiency had a two winding model, which was 63,7% and by neglecting the copper loss of the wires, the efficiency increased to 75,5%. The three winding model with two capacitors had 56,8% overall efficiency and it could increase to 69,3% if the copper loss of the wire is neglected. The last model with three coils and three capacitors had 61,4% efficiency, and it can also be increased to 74,8% if the copper loss of the wires is neglected. In order to increase the efficiency of each system, it would be usefull to use the multilevel inverter to supress the harmonics and to change the used connecting wires with Litz wires to have lower copper loss. Afterward it could be possible to have a high efficiency wireless charging system that can be scaled for the higher power in order to charge the batteries of the electric vehicles.

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# Appendices

## Matlab code used for calculating the currents

```
f = input('Enter frequency in Hz: ');
N = input('Enter number of coils: ');
if(N==3)
    N_C=input('Enter the number of capacitors (2 or 3): ');
end
V_DC = input('Enter DC voltage in V: ');
L1 = input('Enter self inductance L1 in H: ');
L3 = input('Enter self inductance L3 in H: ');
M13 = input('Enter mutual inductance M13 in H: ');
R1 = input('Enter self resistance R1 in Ohm: ');
R3 = input('Enter self resistance R3 in Ohm: ');
R_load=input('Enter load resistance R_load in Ohm: ');

w=2*pi*f;
V_in=(4*(V_DC)/pi);
fprintf('Input sinusoidal voltage V_in is %0.2f V.\n',V_in);

if(N~=2 && N~=3)
    disp('Number of coils N is not correct.')
end

while(N==2 || N==3)
if(N==2)
    if((M13/sqrt(L1*L3))>1 || (M13/sqrt(L1*L3))<0)
    disp('The value of L1 or L3 or M13 is not correct.')
    break
    end
    C1=1/((w^2)*L1);
    fprintf('Capacitor C1 in transmitter coil is %0.5f uF
.\n',(C1)*1e6);
    C3=1/((w^2)*L3);
    fprintf('Capacitor C3 in receiver coil is %0.5f uF
.\n',(C3)*1e6);
    I3=(1i*2*pi*f*M13*V_in)/((1i*2*pi*f*M13)^2-
R1*(R3+R_load));
    disp(sprintf('Current I3 is: %s.',num2str(I3)));
    fprintf('Current I3 in receiver coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I3), (angle(I3)*180/pi));
    I1=-I3*(R3+R_load)/(1i*2*pi*f*M13);
    disp(sprintf('Current I1 is: %s.',num2str(I1)));
    fprintf('Current I1 in transmitter coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I1), (angle(I1)*180/pi));
    H=(1i*2*pi*f*M13*R_load)/((1i*2*pi*f*M13)^2-
R1*(R3+R_load));
    disp(sprintf('Transfer function H is: %s.',num2str(H)))
```

```
    V_out=H*V_in;
    fprintf('Output voltage is %0.2f V and the phase angle is
%0.2f degrees.\n',abs(V_out),(angle(V_out)*180/pi));
    break
end

if(N==3 && N_C==2)
    L2 = input('Enter self inductance L2 in H: ');
    M12 = input('Enter mutual inductance M12 in H: ');
    M23 = input('Enter mutual inductance M23 in H: ');
    R2 = input('Enter self resistance R2 in Ohm: ');
    if((M12/sqrt(L1*L2))>1 || (M12/sqrt(L1*L2))<0)
    disp('The value of L1 or L2 or M12 is not correct.')
    break
    end
    if((M23/sqrt(L2*L3))>1 || (M23/sqrt(L2*L3))<0)
    disp('The value of L2 or L3 or M23 is not correct.')
    break
    end
    C2=1/((w^2)*L2);
    fprintf('Capacitor C2 in auxiliary coil is %0.5f uF
.\n',(C2)*1e6);
    C3=1/((w^2)*L3);
    fprintf('Capacitor C3 in receiver coil is %0.5f uF
.\n',(C3)*1e6);
    I3=(((-
1i*w*M13*V_in)/(R1+1i*w*L1))+((1i*w*M12*V_in)/(R1+1i*w*L1)/(R2
+((w*M12)^2)/(R1+1i*w*L1)))*((1i*w*M23)+(((w)^2)*M13*M12/(R1+1
i*w*L1))))/((R3+R_load+(((w*M13)^2)/(R1+1i*w*L1)))-
(((1i*w*M23)+(((w)^2)*M13*M12/(R1+1i*w*L1)))^2)/(R2+((w*M12)^2
)/(R1+1i*w*L1)));
    disp(sprintf('Current I3 is: %s.',num2str(I3)));
    fprintf('Current I3 in receiver coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I3), (angle(I3)*180/pi));
    I2=(((-1i*w*M12*V_in)/(R1+1i*w*L1))-
I3*((1i*w*M23)+((w^2)*M13*M12/(R1+1i*w*L1))))/(R2+((w*M12)^2)/
(R1+1i*w*L1));
    disp(sprintf('Current I2 is: %s.',num2str(I2)));
    fprintf('Current I2 in auxiliary coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I2), (angle(I2)*180/pi));
    I1=(V_in-1i*w*(M12*I2+M13*I3))/(R1+1i*w*L1);
    disp(sprintf('Current I1 is: %s.',num2str(I1)));
    fprintf('Current i1 in transmitter coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I1), (angle(I1)*180/pi));
    H=(I3*R_load)/V_in;
    disp(sprintf('Transfer function H is: %s.',num2str(H)))
    V_out=H*V_in;
    fprintf('Output voltage is %0.2f V and the phase angle is
%0.2f degrees.\n',abs(V_out),(angle(V_out)*180/pi));
    break
end
```

```
if(N==3 && N_C==3)
    L2 = input('Enter self inductance L2 in H: ');
    M12 = input('Enter mutual inductance M12 in H: ');
    M23 = input('Enter mutual inductance M23 in H: ');
    R2 = input('Enter self resistance R2 in Ohm: ');
    if((M12/sqrt(L1*L2))>1 || (M12/sqrt(L1*L2))<0)
    disp('The value of L1 or L2 or M12 is not correct.')
    break
    end
    if((M23/sqrt(L2*L3))>1 || (M23/sqrt(L2*L3))<0)
    disp('The value of L2 or L3 or M23 is not correct.')
    break
    end
    C1=1/((w^2)*L1);
    fprintf('Capacitor C1 in transmitter coil is %0.5f
uF.\n',(C1)*1e6);
    C2=1/((w^2)*L2);
    fprintf('Capacitor C2 in auxiliary coil is %0.5f
uF.\n',(C2)*1e6);
    C3=1/((w^2)*L3);
    fprintf('Capacitor C3 in receiver coil is %0.5f
uF.\n',(C3)*1e6);
    I3=(((-1i*w*M13*V_in)/(R1))
*(R2+((w*M12)^2)/(R1))+((1i*w*M12*V_in)/(R1)*((1i*w*M23)+((w^2
)*M13*M12/(R1)))))/((R3+R_load+(((w*M13)^2)/(R1)))*(R2+((w*M12
)^2)/(R1))- (((1i*w*M23)+((w^2)*M13*M12/(R1)))^2));
    disp(sprintf('Current I3 is: %s.',num2str(I3)));
    fprintf('Current I3 in receiver coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I3), (angle(I3)*180/pi));
    I2=(((-1i*w*M12*V_in)/(R1))-
I3*((1i*w*M23)+((w^2)*M13*M12/(R1))))/(R2+((w*M12)^2)/(R1));
    disp(sprintf('Current I2 is: %s.',num2str(I2)));
    fprintf('Current I2 in auxiliary coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I2), (angle(I2)*180/pi));
    I1=(V_in-1i*w*(M12*I2+M13*I3))/(R1);
    disp(sprintf('Current I1 is: %s.',num2str(I1)));
    fprintf('Current i1 in transmitter coil is %0.5f A and the
phase angle is %0.2f degrees.\n',abs(I1), (angle(I1)*180/pi));
    H=(I3*R_load)/V_in;
    disp(sprintf('Transfer function H is: %s.',num2str(H)))
    V_out=H*V_in;
    fprintf('Output voltage is %0.2f V and the phase angle is
%0.2f degrees.\n',abs(V_out),(angle(V_out)*180/pi));
    break
end

end
```

### Transfer function of a two winding model with two capacitors

```
    for n=1:13
    R1=0.188132;
    L1=[15.9 15.5 14.8  13.9 14.0 13.8  13.0 13.2 13.2  12.9
12.8  12.5 12.5 ]*1e-6;
    C1=220.9e-9;
    R2=0.106136;
    L2=[8.5 8.4 8.0  7.1 7.2 7.1  6.8 6.8 6.8  6.7 6.6  6.5
6.5 ]*1e-6;
    C2=411.1e-9;
    Rload=8;
    M12=[6.46 4.17 1.61  3.6 2.73 1.01  2.22 1.73 0.99  1.63
1.14  1.13 0.97 ]*1e-6;

    b4=M12(n)*L1(n)*Rload;
    b3=M12(n)*L1(n)/C2+M12(n)*R1*Rload;
    b2=M12(n)*R1/C2;
    a5=((L1(n))^2)*L2(n)-((M12(n))^2)*L1(n);
    a4=((L1(n))^2)*Rload-
((M12(n))^2)*R1+((L1(n))^2)*R2+2*L1(n)*R1*L2(n);

a3=((L1(n))^2)/C2+L1(n)*L2(n)/C1+2*L1(n)*R1*Rload+2*L1(n)*R1*R
2+L2(n)*((R1)^2);

a2=2*L1(n)*R1/C2+L1(n)*Rload/C1+L1(n)*R2/C1+L2(n)*R1/C1+((R1)^
2)*Rload+((R1)^2)*R2;
    a1=L1(n)/(C1*C2)+((R1)^2)/C2+Rload*R1/C1+R1*R2/C1;
    a0=R1/(C1*C2);

    s = tf('s');

H(n)=(b4*s^4+b3*s^3+b2*s^2)/(a5*s^5+a4*s^4+a3*s^3+a2*s^2+a1*s+
a0)*(s/(s+1/(Rload*C2)));
end

    bode(H(1),{3e5,2.01e6},'-r');
    hold on;
    bode(H(2),{3e5,2.01e6},'--r');
    hold on;
    bode(H(3),{3e5,2.01e6},':r');
    hold on;
    bode(H(4),{3e5,2.01e6},'-g');
    hold on;
    bode(H(5),{3e5,2.01e6},'--g');
    hold on;
    bode(H(6),{3e5,2.01e6},':g');
    hold on;
    bode(H(7),{3e5,2.01e6},'-b');
    hold on;
```

```
    bode(H(8),{3e5,2.01e6},'--b');
    hold on;
    bode(H(9),{3e5,2.01e6},':b');
    hold on;
    bode(H(10),{3e5,2.01e6},'-y');
    hold on;
    bode(H(11),{3e5,2.01e6},'--y');
    hold on;
    bode(H(12),{3e5,2.01e6},'-k');
    hold on;
    bode(H(13),{3e5,2.01e6},'--k');
    hold on;
    grid on;

    legend('h=0.5, x=0','h=0.5, x=1','h=0.5, x=2','h=1,
x=0','h=1, x=1','h=1, x=2','h=1.5, x=0','h=1.5, x=1','h=1.5,
x=2','h=2, x=0','h=2, x=1','h=2.5, x=0','h=2.5,
x=1','location','BestOutside');
```

**Transfer function of three winding model with two capacitors**

```
for n=1:13

    R1=0.188132;
    L1=[15.9 15.5 14.8  13.9 14.0 13.8  13.0 13.2 13.2  12.9
12.8  12.5 12.5 ]*1e-6;
    R2=0.106136;
    L2=[8.5 8.4 8.0  7.1 7.2 7.1  6.8 6.8 6.8  6.7 6.6  6.5
6.5 ]*1e-6;
    C2=411.1e-9;
    Rload=8;
    M12=[6.46 4.17 1.61  3.6 2.73 1.01  2.22 1.73 0.99  1.63
1.14  1.13 0.97 ]*1e-6;
    R3=0.172231;
    L3=[14.5 14.4 13.7  12.1 12.1 12.2  11.4 11.6 11.4  11.1
11.0  10.9 10.8 ]*1e-6;
    M13=[11.86 11.57 10.91  9.39 9.37 9.33  8.62 8.69 8.58
8.30 8.27  8.16 8.14 ]*1e-6;
    M23=[6.71 4.37 1.68  3.54 2.88 1.15  2.38 1.99 0.95  1.83
1.23  1.22 1.09 ]*1e-6;
    C3=595.2e-9;

    b4=M12(n)*L3(n)*Rload-M23(n)*M13(n)*Rload;
    b3=M12(n)*R3*Rload-M23(n)*M13(n)/C2+M12(n)*L3(n)/C2;
    b2=M12(n)*Rload/C3+M12(n)*R3/C2;
    b1=M12(n)/(C2*C3);
    a5=L1(n)*L2(n)*L3(n)-((M13(n))^2)*L2(n)-
((M12(n))^2)*L3(n)-((M23(n))^2)*L1(n)+2*M12(n)*M13(n)*M23(n);
    a4=-((M13(n))^2)*Rload-((M13(n))^2)*R2+L1(n)*L3(n)*Rload-
((M12(n))^2)*R3+L1(n)*L2(n)*R3+L2(n)*L3(n)*R1+L1(n)*L3(n)*R2-
((M23(n))^2)*R1;
    a3=-((M12(n))^2)/C3+L1(n)*L2(n)/C3-
((M13(n))^2)/C2+L1(n)*R3*Rload+L3(n)*R1*Rload+L1(n)*L3(n)/C2+L
2(n)*R1*R3+L1(n)*R3*R2+L3(n)*R1*R2;

a2=L1(n)*Rload/C3+L1(n)*R2/C3+L2(n)*R1/C3+R1*R3*Rload+L1(n)*R3
/C2+L3(n)*R1/C2+R1*R2*R3;
    a1=Rload*R1/C3+L1(n)/(C2*C3)+R1*R2/C3+R1*R3/C2;
    a0=R1/(C3*C2);

    s = tf('s');

G(n)=(b4*s^4+b3*s^3+b2*s^2+b1*s)/(a5*s^5+a4*s^4+a3*s^3+a2*s^2+
a1*s+a0);
    H(n)=G(n)*(s/(s+1/(Rload*C2)));
end

    bode(H(1),{1e5,1e7},'-r');
    hold on;
    bode(H(2),{1e5,1e7},'--r');
```

```
    hold on;
    bode(H(3),{1e5,1e7},':r');
    hold on;
    bode(H(4),{1e5,1e7},'-g');
    hold on;
    bode(H(5),{1e5,1e7},'--g');
    hold on;
    bode(H(6),{1e5,1e7},':g');
    hold on;
    bode(H(7),{1e5,1e7},'-b');
    hold on;
    bode(H(8),{1e5,1e7},'--b');
    hold on;
    bode(H(9),{1e5,1e7},':b');
    hold on;
    bode(H(10),{1e5,1e7},'-y');
    hold on;
    bode(H(11),{1e5,1e7},'--y');
    hold on;
    bode(H(12),{1e5,1e7},'-k');
    hold on;
    bode(H(13),{1e5,1e7},'--k');
    hold on;
    grid on;

    legend('h=0.5, x=0','h=0.5, x=1','h=0.5, x=2','h=1,
x=0','h=1, x=1','h=1, x=2','h=1.5, x=0','h=1.5, x=1','h=1.5,
x=2','h=2, x=0','h=2, x=1','h=2.5, x=0','h=2.5,
x=1','location','BestOutside');
```

### Transfer function of three winding model with three capacitors

```
    for n=1:13
    R1=0.188132;
    L1=[15.9 15.5 14.8  13.9 14.0 13.8  13.0 13.2 13.2  12.9
12.8  12.5 12.5 ]*1e-6;
    C1=220.9e-9;
    R2=0.106136;
    L2=[8.5 8.4 8.0  7.1 7.2 7.1  6.8 6.8 6.8  6.7 6.6  6.5
6.5 ]*1e-6;
    C2=411.1e-9;
    Rload=8;
    M12=[6.46 4.17 1.61  3.6 2.73 1.01  2.22 1.73 0.99  1.63
1.14  1.13 0.97 ]*1e-6;
    R3=0.172231;
    L3=[14.5 14.4 13.7  12.1 12.1 12.2  11.4 11.6 11.4  11.1
11.0  10.9 10.8 ]*1e-6;
    M13=[11.86 11.57 10.91  9.39 9.37 9.33  8.62 8.69 8.58
8.30 8.27  8.16 8.14 ]*1e-6;
    M23=[6.71 4.37 1.68  3.54 2.88 1.15  2.38 1.99 0.95  1.83
1.23  1.22 1.09 ]*1e-6;
    C3=24.48e-9;

    a4=M12(n)*L3(n)*Rload-M23(n)*M13(n)*Rload;
    a3=M12(n)*R3*Rload-M23(n)*M13(n)/C2+M12(n)*L3(n)/C2;
    a2=M12(n)*Rload/C3+M12(n)*R3/C2;
    a1=M12(n)/(C2*C3);
    b5=L1(n)*L2(n)*L3(n)-((M13(n))^2)*L2(n)-
((M12(n))^2)*L3(n)-((M23(n))^2)*L1(n)+2*M12(n)*M13(n)*M23(n);
    b4=-((M13(n))^2)*Rload-((M13(n))^2)*R2+L1(n)*L3(n)*Rload-
((M12(n))^2)*R3+L1(n)*L2(n)*R3+L2(n)*L3(n)*R1+L1(n)*L3(n)*R2-
((M23(n))^2)*R1;
    b3=-((M12(n))^2)/C3+L1(n)*L2(n)/C3-
((M13(n))^2)/C2+L1(n)*R3*Rload+L3(n)*R1*Rload+L1(n)*L3(n)/C2+L
2(n)*R1*R3+L1(n)*R3*R2+L3(n)*R1*R2;

b2=L1(n)*Rload/C3+L1(n)*R2/C3+L2(n)*R1/C3+R1*R3*Rload+L1(n)*R3
/C2+L3(n)*R1/C2+R1*R2*R3;
    b1=Rload*R1/C3+L1(n)/(C2*C3)+R1*R2/C3+R1*R3/C2;
    b0=R1/(C3*C2);

    d2=M12(n)*M13(n)-M23(n)*L1(n);
    d1=M23(n)*R1;
    d0=M23(n)/C1;
    c2=M12(n)*M13(n)-M23(n)*L1(n);
    c1=M23(n)*R1;

    f2=L2(n)*M13(n)/C1-M12(n)*M23(n)/C1;
    f1=M13(n)*Rload/C1+M13(n)*R2/C1;
    f0=M13(n)/(C1*C2);
    e3=M12(n)*M13(n)*Rload-M23(n)*L1(n)*Rload;
```

```
    e2=M12(n)*M13(n)/C2-M23(n)*R1*Rload-M23(n)*L1(n)/C2;
    e1=M23(n)*R1/C2;

    s = tf('s');

G(n)=1/((b5*s^5+b4*s^4+b3*s^3+b2*s^2+b1*s+b0)/(a4*s^4+a3*s^3+a
2*s^2+a1*s)*(d2*s^2-d1*s-d0)/(c2*s^2-
c1*s)+(f2*s^2+f1*s+f0)/(e3*s^3+e2*s^2-e1*s));
    H(n)=G(n)*(s/(s+1/(Rload*C2)));
end

    bode(H(1),{1e5,1e7},'-r');
    hold on;
    bode(H(2),{1e5,1e7},'--r');
    hold on;
    bode(H(3),{1e5,1e7},':r');
    hold on;
    bode(H(4),{1e5,1e7},'-g');
    hold on;
    bode(H(5),{1e5,1e7},'--g');
    hold on;
    bode(H(6),{1e5,1e7},':g');
    hold on;
    bode(H(7),{1e5,1e7},'-b');
    hold on;
    bode(H(8),{1e5,1e7},'--b');
    hold on;
    bode(H(9),{1e5,1e7},':b');
    hold on;
    bode(H(10),{1e5,1e7},'-y');
    hold on;
    bode(H(11),{1e5,1e7},'--y');
    hold on;
    bode(H(12),{1e5,1e7},'-k');
    hold on;
    bode(H(13),{1e5,1e7},'--k');
    hold on;
    grid on;

    legend('h=0.5, x=0','h=0.5, x=1','h=0.5, x=2','h=1,
x=0','h=1, x=1','h=1, x=2','h=1.5, x=0','h=1.5, x=1','h=1.5,
x=2','h=2, x=0','h=2, x=1','h=2.5, x=0','h=2.5,
x=1','location','BestOutside');
```