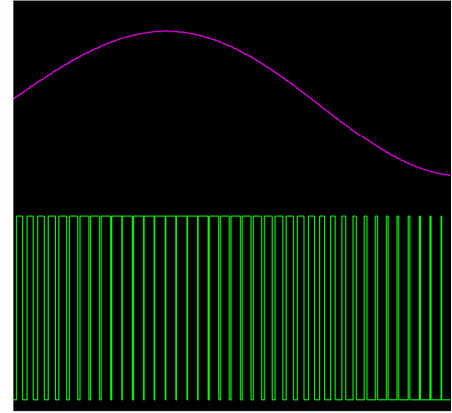
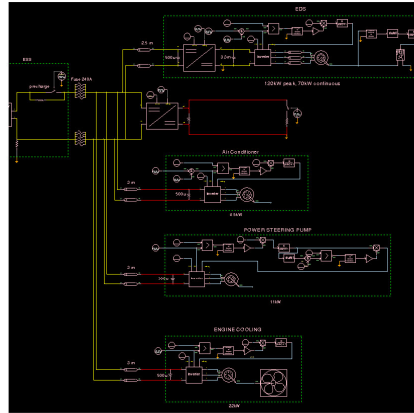


CHALMERS



Modeling and simulation of high voltage distribution in a vehicle

Master Thesis

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Göteborg, Sweden, 2006

Abstract

This report presents several ways of modelling electric system functions in a Typical Heavy Hybrid Vehicle (THHV). The focus is on the dynamics of the system and its components. Presented in this report are dynamic models of inverters, DCDC converters, transmission cables, simple control systems, induction machines, etc. There are several systems that represent different scenarios presented as well including; a complete system run, all loads demand power at the same time, control system fault, breaking of inductive currents, short circuits, cross talk, etc.

Useful comments about what is possible to simulate or not is available throughout the report. The simulation tool used in this report is Synopsis SABER.

The results show that it is fully possible to simulate the electric system in a THHV with an acceptable precision. Predictions of the behaviour of the system are possible make from the results.

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List of abbreviations

| | |
|------|--------------------------------------|
| AC | Air Conditioner |
| ECF | Engine Cooling Fan |
| EDS | Electric Drive System |
| EMI | Electro Magnetic Interference |
| FVCS | Field oriented Vector Control System |
| PE | Power Electronics |
| PSP | Power Steering Pump |
| PWM | Pulse Width Modulation |
| THHV | Typical Heavy Hybrid Vehicle |

List of symbols

| | |
|-------------------|---------------------------------|
| Z_0 | Characteristic impedance |
| ω_0 | Resonance frequency |
| T_{Load} | Loading torque |
| B | Viscous damping |
| R_s | Series resistance |
| X_s | Series reactance based on L_s |
| R_R | Rotor resistance |
| X_R | Rotor reactance based on L_R |
| X_M | Magnetizing reactance |
| J_{motor} | Inertia of electric motor |
| σ_{copper} | Conductance |
| μ_0 | Permeability |
| ε | Permittivity |
| J | Current density |

1 Introduction

The outline of the thesis is presented in the following chapters.

1.1 Problem background

It is likely that electric systems of future THHVs would use considerable higher system voltages compared to the 24 V used today and in this thesis a switch to 600 V is proposed. Currents and voltages levels are analysed in different scenarios in order to predict the behaviour of the system.

1.2 Purpose of work

The aim of this thesis is to describe possible system behaviour of a Typical Heavy Hybrid Vehicle in different situations, called scenarios from now on. An outcome of this is to try to predict system dynamics in these scenarios, for example load responses, electro magnetic emissions and short circuits in different positions in the system. In order to make this possible, models will be designed based on important functions in the vehicle.

Systems representing different scenarios will include these models in order to study their dynamics when working together in a system. One of the goals with the project are the creation of models which can be used further on in other projects or as examples of possible ways to describe system components.

1.3 Outcome of the thesis

There are two results of this master thesis; a comprehensive report (the one you are reading) and a collection of models and systems created in the simulation tool SABER. This report could act as an inspiration for other simulation projects by providing useful approaches by simulation examples of common functions like inverters, DCDC converters, cables etc.

2 System overview

In this master thesis a THHV is used as a source of inspiration but this system is merely an example of an electric architecture. The consequence is that some of the models and parameters are fictive either because they doesn't exist yet or because parameters are not available. The ambition is to make reasonable approximations when needed and to try to use widely spread models of induction machines, inverters and cables.

An overview figure of the system to be simulated is presented in Fig. 2.1 and consists of several boxes representing different functions of the system. These subsystems are connected using transmission cables with a DC-link voltage of 600 V and represent a selection of loads in the example system. The modeled loads are chosen because they represent important functions in the vehicle and their power consumption is generally high. A DCDC converter is used to connect the 600 V system to the traditional 24 V system.

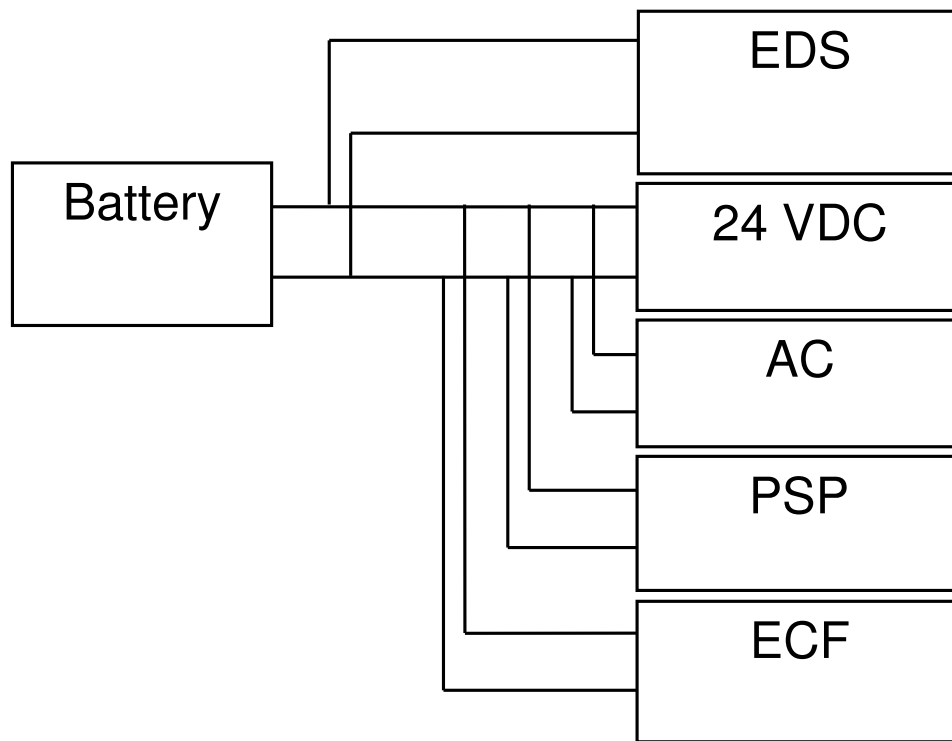


Figure 2.1 System overview

3 Models

Models of electrical loads in the 600 V system of a THHV are presented in this chapter. Some loads are moved from the traditional 24 V system to the 600 V and some mechanical loads are translated to electric 600 V loads. The models are created using Synopsis SABER according to VOLVO Technology's recommendation. Thermal effects are neglected in order to simplify the modeling.

3.1 Power electronics

The concept of Power Electronics (PE) is to control voltages and currents using semiconductor devices. Controllability and efficiency is high using PE in systems. Output voltage levels are created using Pulse Width Modulation (PWM) which is based on switching power transistors in appropriate patterns.

The principle of creating PWM patterns is to compare the reference signal (DC or AC) with a triangular wave and if the reference signal is the highest signal the transistor is turned on. An example of this behaviour can be studied in Fig. 3.1 where the transistor is conducting as long as the reference is larger than the triangular wave. Different PWM patterns are used in different applications, some of them presented below.

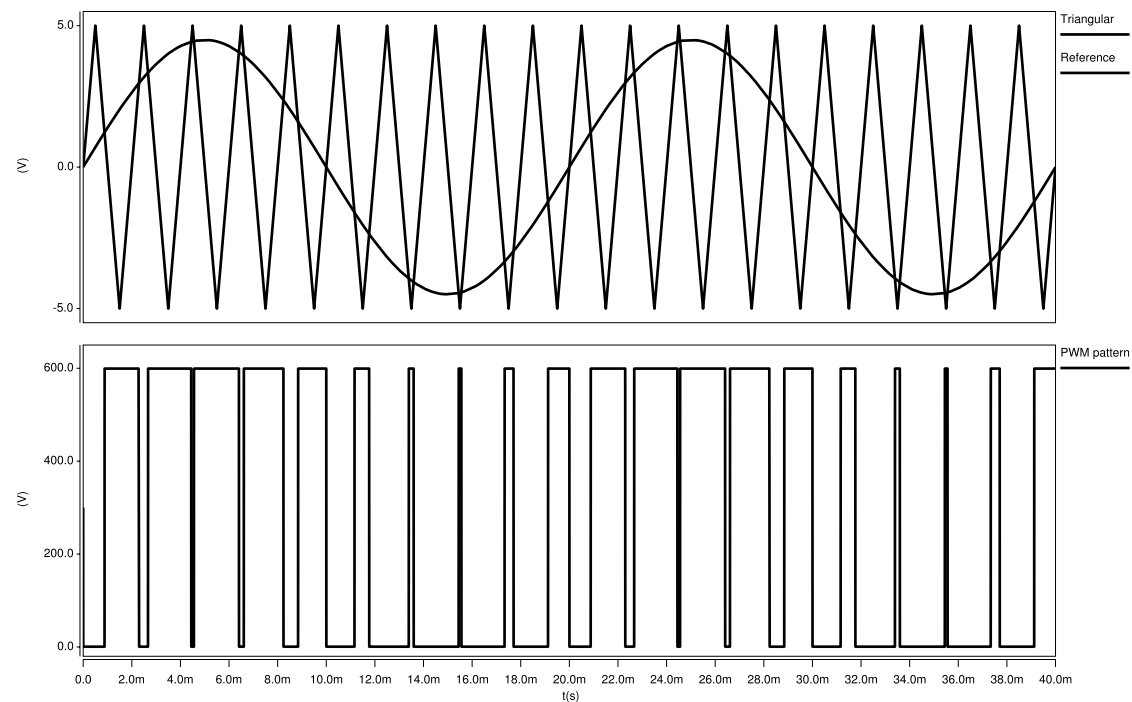


Figure 3.1 PWM pattern of a sine wave

3.1.2 Inverter (AC)

An inverter is used to create controlled AC voltages (frequency and amplitude) from a DC source using PWM in the same manner as described earlier in this chapter. Two models of the inverter are created because of different requirements in different simulation scenarios. The first one includes custom made IGBT transistor models and switching characteristics, the second one neglects switching characteristics. The interested reader is encouraged to study [2] for more details.

Inverter with switched characteristics

In order to keep the transistor model as simple as possible only four parameters were used. The most important parameters of the transistors are the rise and fall times. The equivalent on resistance is considered to be zero and would give an error of approximately a few volts which is acceptable compared to the switched voltage of 450 volt.

Simulation time is reduced by this approximation and a very simple transistor model is available as a diode in series with a switch, shown in Fig. 3.3. Tab. 3.1 presents the transistor parameters used in the simulation.

Table 3.1 IGBT transistor parameters from data sheet

| $R_{\text{equivalent}}$ [m Ω] | R_{off} | T_{rise} [ns] | T_{fall} [ns] |
|---------------------------------------|------------------|------------------------|------------------------|
| 0 | ∞ | 50 | 30 |



Figure 3.3 SABER transistor model

In Fig. 3.4 an inverter model that includes this transistor model, as well as generation of PWM pattern, is shown. A freewheeling diode is added in parallel with each transistor. Three legs, each consisting of two transistors with freewheeling diodes, are required for a 3 phase inverter. The PWM pattern is generated using comparators and the method described in the beginning of this chapter. The chosen switching frequency is 10 kHz.

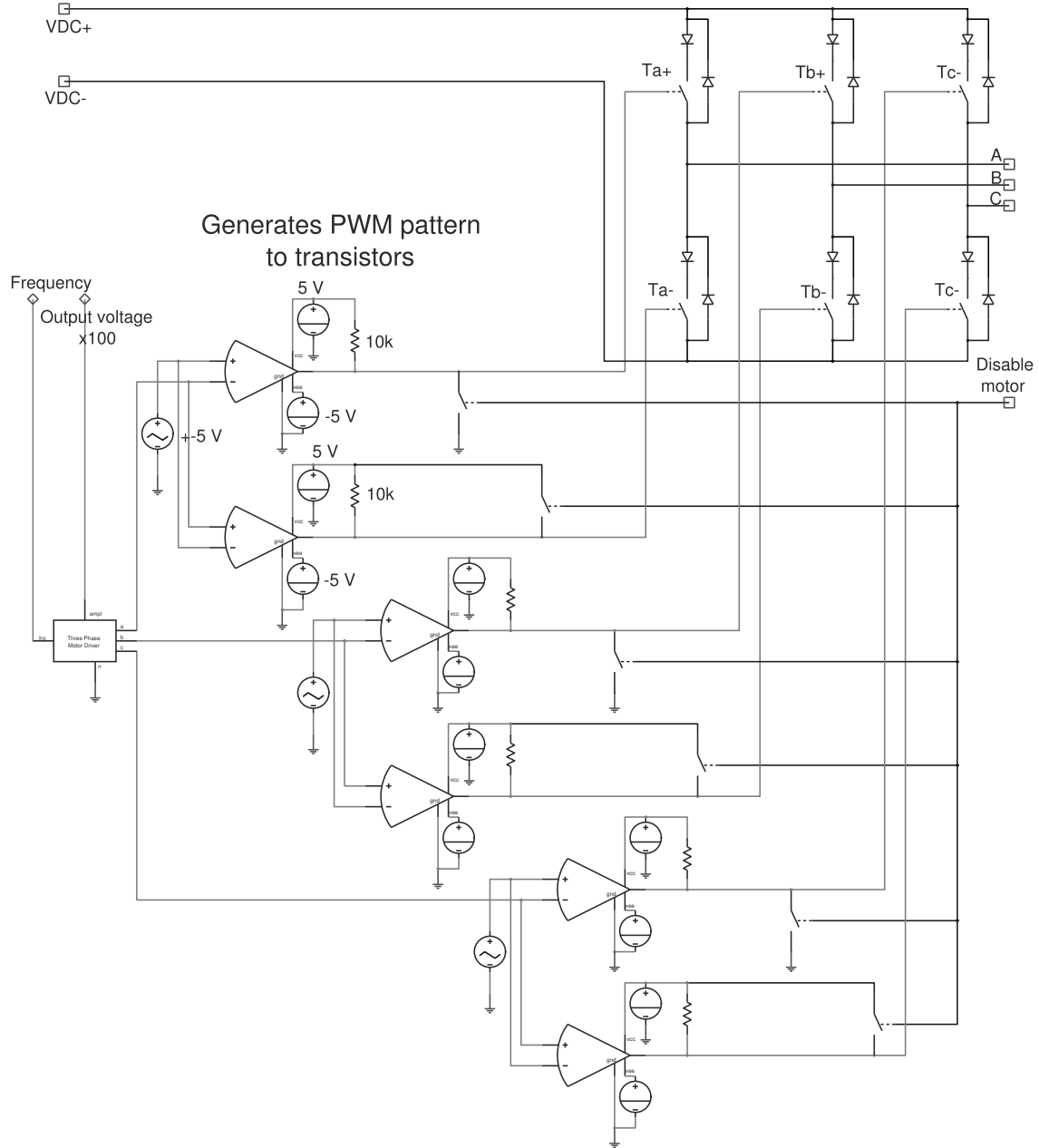


Figure 3.4 Complete inverter with switch transient characteristics

3.2 Induction machine

There are several loads in the system that converts electrical energy to mechanical torque and angular velocity. This is a reason for a reasonable complex motor model; in this case a well known model of an induction machine. Focus will be on the electric parameters but a simple mechanical model is used to model viscous damping and inertia. An overview figure is presented in Fig. 3.6 and an electric equivalent circuit is presented in Fig. 3.7. For more information about how induction machines work, please refer to [3]

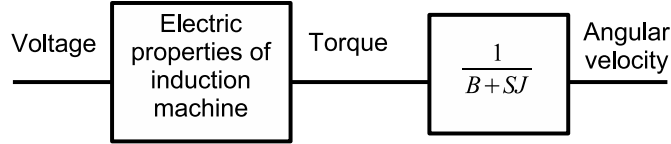


Figure 3.6 Basic idea of the induction machine modeling

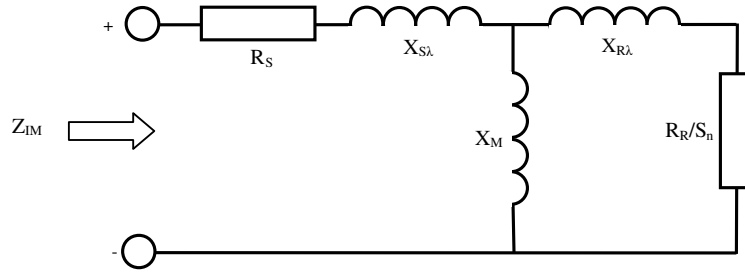


Figure 3.7 Electric phase equivalent model of an induction machine

The schematics in Fig. 3.7 can be described using an equivalent impedance presented as:

$$Z_{IM} = R_s + SX_{s\lambda} + \left((SX_M)^{-1} + \left(\frac{R_R}{s_n} + SX_{r\lambda} \right)^{-1} \right)^{-1} \quad (3.2)$$

3.3 Loads

The models presented below are based on the mechanical characteristics of existing loads that are translated into electrical characteristics. Electrical loads are in contrast to mechanical loads independent of the rotational speed of the diesel engine. When comparing mechanical and electrical loads the most important similarity is the peak power. The electric motors and their parameters are chosen among given examples from [5] when possible. In the cases where no suitable motors were available, an approximation was made.

3.3.1 Electric Drive System

In the vehicle used as an example, a THHV, there is a block representing the Electric Drive System (EDS) see Fig. 2-1. It includes a 6-phase, 120 kW induction machine driven by switched power electronics. A DCDC converter, an inverter and a control system are also present to be able to deliver controlled currents and voltages to the induction machine. Regenerative braking is an important function in a THHV and is possible to do with the system. A complete EDS system is presented in Fig. 3.8.

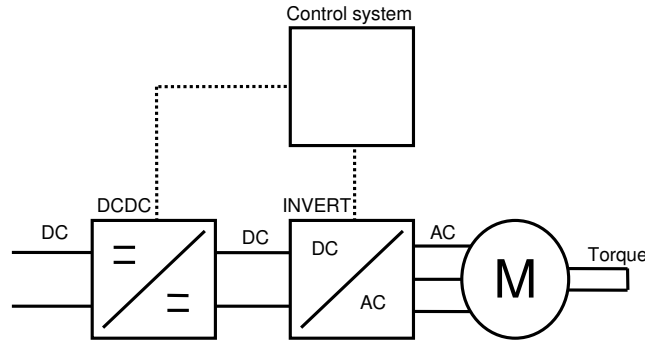


Figure 3.8 Phase equivalent model of an induction machine

Induction machine

SABER has a built in model of a 3-phase motor and by changing its parameters the motor is easy to customise. Since the simulation is an example, a 3-phase motor is used instead of a 6-phase because of simplicity. The peak power of the electric motor (120 kW) is the most important performance requirement to calculate for. Fig. 3.7 shows a schematic of the simulated motor and Tab. 3.2 presents the motor parameters. Values in Tab. 3.2 are based on an average from two other machines, a 55 kW and a 200 kW [5].

Table 3.2 Parameters for a 120 kW motor with impedance values based on $f=50\text{Hz}$ [5]

| Peak power [kW] | R_s [m Ω] | $X_{S\lambda}$ [Ω] | R_R [m Ω] | $X_{R\lambda}$ [Ω] | X_M [Ω] | J_{motor} [kgm ²] | Poles |
|-----------------|---------------------|-----------------------------|---------------------|-----------------------------|--------------------|--|-------|
| 120 | 29.37 | 0.10 | 40.06 | 0.13 | 3.585 | 1.925 | 6 |

Power electronics

The electric motor is to be supplied with a PWM voltage from the inverter. The current is anyway fairly sinusoidal because of the inductive characteristics of an induction machine. The output torque of an induction machine is proportional to the input current and not the voltage. The PWM pattern of the voltage is therefore not a problem for torque ripple.

Implementation in SABER

To be able to control voltages, currents, torque, etc. a sophisticated control system should be used, preferable a Field oriented Vector Control System (FVCS). In order to save simulation time a simple (compared to FVCS) control system was used. A block scheme of the simple control system is available in Fig. 3.9 below.

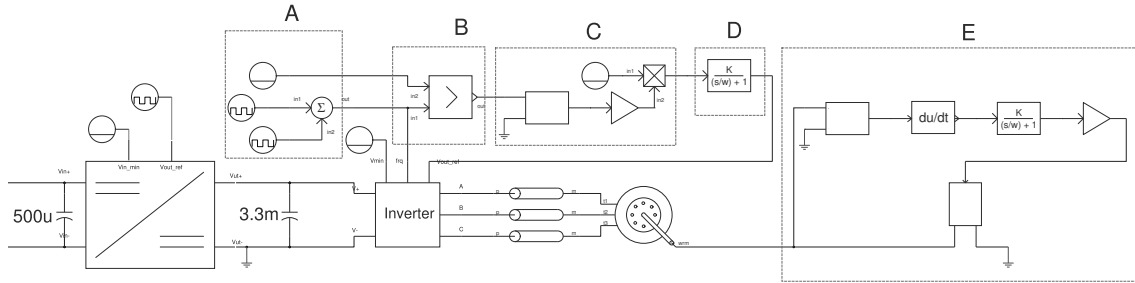


Figure 3.9 SABER implementation of the EDS

The user defines a frequency ramp in one or both of the two lower pulse sources in **A** to be able to create a simple drive cycle. It is connected directly to the inverter to control the output frequency as well as a comparator (**B**). When the frequency exceeds a defined reference in the upper part of **A** ($f_{min}=4$ Hz in this model) the comparator (**B**) gives a 5 V output which is divided down to 1 in **C**. **C** includes a multiplier which acts as a turn on function for the voltage by multiplying the reference voltage (450 V in this case) with “1” if the conditions in **A** and **B** is fulfilled. In **D** a first order lag is added with $\omega_0=1$ to avoid a too high voltage slope which could harm the system. The idea with this control system is to not turn on voltage if the frequency is too low.

In order to simulate the acceleration and regenerative braking torque, a load torque model is added which can be studied in **E**. It measures the derivative of the angular velocity and changes the loading or braking torque accordingly as:

$$T_{load} = K \frac{d\omega}{dt} \quad (3.3)$$

The torque constant K is dimensioned to give a maximum loading torque of 800 Nm which is about the same as in the THHV system.

There is a 500 μ F capacitor at the DCDC converter input and a 3.3 mF capacitor at the converter output. By adding capacitors in an electric drive system, high voltage derivatives can be reduced.

3.3.2 Air condition

To build a model of an electric Air Condition (AC) system several sub models are used. In the model of the air conditioner, a motor, a mechanical load and some power electronics are used to simulate the electrical behavior. The mechanical load is supposed to illustrate a compressor (the very basic function of an air condition system).

Induction machine (4.5 kW)

Peak power of the air conditioner equals 4.5 kW [7]. Motor parameters in the model are presented in Tab. 3.3 and are calculated according to appendix A because there were no reliable parameters available for a machine with the demanded peak power.

Table 3.3 Parameters for a 4.5 kW motor with impedance values based on $f=50\text{Hz}$ [5]

| Peak power [kW] | R_s [m Ω] | $X_{s\lambda}$ [Ω] | R_R [m Ω] | $X_{R\lambda}$ [Ω] | X_m [Ω] | J_{motor} [kgm ²] | Poles |
|-----------------|---------------------|-----------------------------|---------------------|-----------------------------|--------------------|--|-------|
| 4.5 kW | 0.85 | 2.13 | 0.89 | 3.4 | 61.4 | 0.1 | 4 |

Implementation in SABER

The control system is the same as the one used in the EDS but with the output voltage set to 150 V. The air conditioner also lacks a DCDC converter and relies on the inverter alone. The intension is to save costs since the efficiency demand naturally is lower on the AC compared to the EDS.

To model the mechanical part of the load (the compressor) the damping factor in the model of the induction machine was used and set to $B=0.5$, which corresponds to a peak power of 4.5 kW. Fig. 3.10 shows a SABER model of the air condition.

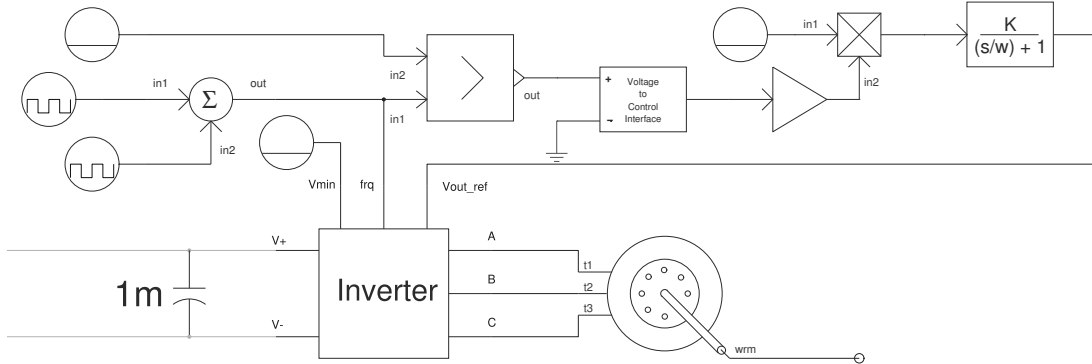


Figure 3.10 SABER model of an 4.5 kW Air Conditioner

3.3.3 Power steering pump

The Power Steering Pump (PSP) is modeled with power electronics, viscous damping and an electric motor. The damping parameter in SABERs motor model represents the mechanical load from the pump. It is important that the startup time of this pump is quick, since it has to be able to deliver torque for the steering with only a short delay.

Induction machine (15 kW)

It is assumed that the peak power for the steering pump equals 11 kW [7] and the motor parameters are presented in Tab. 3.4. Parameters are for a motor with peak power of 15 kW [8].

Table 3.4 Parameters for a 15 kW motor with impedance values based on $f=50\text{Hz}$ [5]

| Peak power | R_s [m Ω] | $X_{s\lambda}$ [Ω] | R_R [m Ω] | $X_{R\lambda}$ [Ω] | X_m [Ω] | J_{motor} [kgm ²] | Poles |
|------------|---------------------|-----------------------------|---------------------|-----------------------------|--------------------|--|-------|
| 15 kW | 0.2518 | 0.567 | 0.2682 | 0.908 | 18.9 | 0.173 | 6 |

Implementation in SABER

The mechanical pump is modeled as a viscous damping in the model of the induction machine in SABER with a damping constant $B=0.7$ that corresponds to a peak power of 11 kW. Output voltage reference is set to 255 V and the rest of the control system is identical to the one used in the EDS chapter, except for the addition in the dashed box in the SABER model shown in Fig. 3.11.

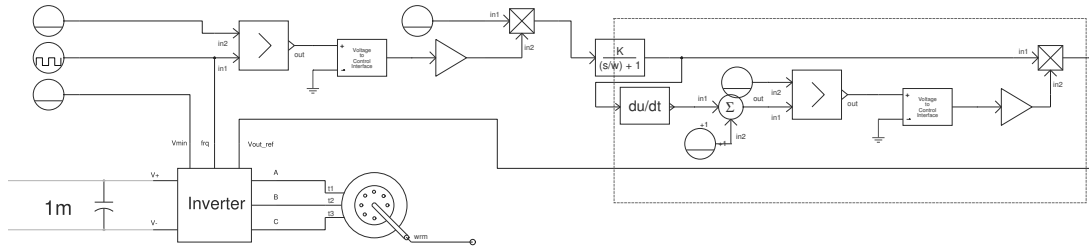


Figure 3.11 SABER model of an 11 kW Power Steering Pump

To be able to start and stop the PSP quickly an addition in the control is made, shown in the dashed rectangle in Fig. 3.11. The added system is designed to have a similar positive slope ($\omega_0=5$) but a much faster negative slope of the voltage reference compared to the EDS.

3.3.4 Engine cooling fan

An Engine Cooling Fan (ECF) does not have to start rapidly but it has to be able to deliver high cooling capability during a long time if needed. The procedure to model this system is similar to the ones used in chapter 3.3.2 and 3.3.3 when modelling an air conditioner and a steering pump.

Induction machine (22 kW)

The peak power of the engine cooling fan equals 22 kW [7] and the parameters for a motor with 22 kW [5] as peak power can be seen in Tab. 3.5.

Table 3.5 Parameters for a 22 kW motor with impedance values based on $f=50\text{Hz}$ [5]

| Peak power | R_s [m Ω] | $X_{s\lambda}$ [Ω] | R_R [m Ω] | $X_{R\lambda}$ [Ω] | X_m [Ω] | J_{motor} [kgm ²] | Poles |
|------------|---------------------|-----------------------------|---------------------|-----------------------------|--------------------|--|-------|
| 22 kW | 0.1245 | 0.358 | 0.1366 | 0.582 | 18.9 | 0.334 | 6 |

Implementation in SABER

Instead of a mechanical load or a mechanical damping, a model of a fan available in SABER is used. Parameters for the fan is constant windage torque $w=0.12$ and constant viscous damping torque $d=0.1$. The rest of the control system is identical to the one used in the EDS except for the voltage which is set to 340 V to give a peak power of 22 kW. The SABER setup is presented in Fig. 3.12.

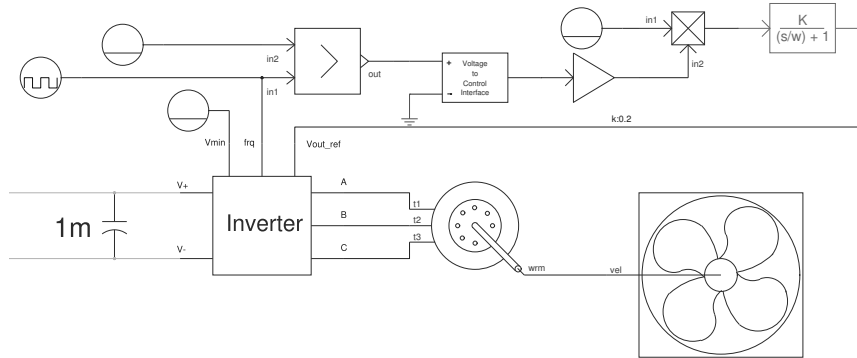


Figure 3.12 SABER model of an 22 kW Engine Cooling Fan

3.4 DC transmission cables

Different types of transmission cables are connecting the electrical sources and loads. Large currents as well as transients will be present at the cables during on- and off switching of electrical loads, short circuit situations and when breaking an inductive current. A dynamic model of the transmission cable is therefore preferred to be able to predict the behaviour in different situations.

3.4.1 Theory

Theoretical models of transmission cables are based on the knowledge on how electromagnetic waves propagate on transmission lines and can be expressed by two differential equations presented as:[1]

$$-\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L \frac{\partial i(z,t)}{\partial t} \quad (3.4)$$

$$-\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C \frac{\partial v(z,t)}{\partial t} \quad (3.5)$$

The 600 V DC cable between the battery and the power electronics for the 120 kW motor has a cross section area of 95 mm². DC cables for the other loads are dimensioned to have similar current density as the cable to the 120 kW motor.

Fig. 3.13 shows a schematic model of a segment of the complete cable that (3.4) and (3.5) explained mathematically.

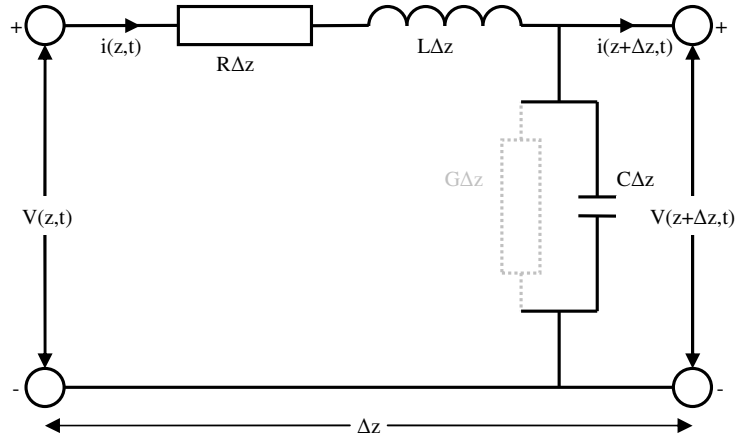


Figure 3.13 Schematic figure of a segment of a transmission cable

The segment shown in Fig. 3.13 is connected in series according to Fig. 3.14.

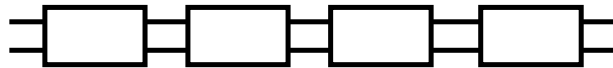


Figure 3.14 Schematic figure of a segment of a transmission cable

3.4.2 Calculating the parameters

The differential equations (3.4) and (3.5) for the cable model are enough to describe the transient behavior in most situations. Parameter values are calculated from:

$$R = \frac{l}{\sigma A} = 181.5 \mu\Omega \quad (3.6)$$

$$L = \frac{\mu}{2\pi} \ln \frac{b}{a} = 124.5 nH \quad (3.7)$$

$$G = \frac{2\pi\sigma}{\ln(b/a)} = (\text{neglected}) \quad (3.8)$$

$$C = \frac{2\pi\epsilon}{\ln(b/a)} = 729 pF \quad (3.9)$$

with

$$f = 10 kHz$$

$$\sigma_{copper} = 5.8 \cdot 10^7$$

$$\mu_0 = 4\pi \cdot 10^{-7}$$

$$\epsilon = 72.28 \cdot 10^{-12}$$

$$J = \text{current density} = 2.63 \text{ MA /m}^2$$

A= cross section area

a = inner radius [8]

b = outer radius (calculated from cross section area) [8]

One cable is to be connected between the battery and the EDS power electronics and is chosen to have a cross section area of 95 mm².

From this the maximum current density in the cable is calculated in order to dimension the other cables in the system with a peak current of 250 A. The current density is 2.63 MA /m² which are reasonable low. The other cables in the system are chosen from cable models listed in [8] based on the same current density. The amount of peak current to each load is based on the known peak power which is presented in corresponding chapters to the loads.

It is important to notice that the parameters listed in (3.6) to (3.9) are “per-meter” values. An example cable could consist of 20 segments with an individual length of 0.1 m, giving a total of 2 m. Tab. 3.6 gives a summary of cable parameters. The lengths of the phase cable and the EDS cable are measured on a real hybrid concept car.

Table 3.6 *DC cable parameters (per meter values)*

| Cable | Length [m] | Area [mm ²] | B [mm] | A [mm] | R [mΩ] | L [nH] | G | C [pF] |
|-----------------|---------------|----------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| EDS | 2.5 | 95 | 10.25 | 5.5 | 0.181 | 124.5 | neglected | 729 |
| Phase cables | 1.5 | 95 | 10.25 | 5.5 | 0.181 | 124.5 | neglected | 729 |
| ECF | 3 | 16 | 5.00 | 2.26 | 1.078 | 158.8 | neglected | 572 |
| PSP | 3 | 10 | 4.50 | 1.78 | 1.724 | 185.5 | neglected | 490 |
| AC | 3 | 10 | 4.50 | 1.78 | 1.724 | 185.5 | neglected | 490 |

3.4.3 Implementation

SABER’s built in transmission cable model is used with the parameters presented in Tab. 3.6. Cable lengths are chosen based on the location of the load compared to the battery. The temperature is assumed to be 20 °C because thermal effects are not considered to be important in this simulation. In SABER a transmission cable is represented by the symbol in shown Fig. 3.15.



Figure 3.15 *SABER model of transmission cable*

4 Scenarios to analyze

The choice of scenarios is based on discussions between the authors of this thesis and the supervisor. A simulation of a well working system is conducted along with simulations of possible fault situations. The intention is that the reader should get an overview of which parameters and parts of the system that have the largest influence in these situations.

4.1 Breaking an inductive current

One of the dangerous situations in these kinds of systems is to break an inductive current since that results in very high voltages. Sources of the breaking of the inductive current might be a broken cable, a blown fuse or a short circuit situation (More information in 5.5.2) The simulations presented in this chapter intend to give a deeper knowledge about system dynamics during the breaking of an inductive current.

4.1.1 Theory

The voltage created is proportional to the current derivative which in this case is very high. It is important to notice that resistive, inductive and capacitive properties are present in all electric circuits and loads, with an overrepresentation of inductive behaviour. These types of systems can be described with the differential equations (4.1) to (4.4).

$$L \frac{di_L(t)}{dt} = V_L(t) \quad (4.1)$$

(4.1) describes the relationship between the voltage over an inductor and the current derivative in a schematic similar to the ones in Fig.4.1. The conclusion that high current derivatives can result in high peak voltages indicates a possible dangerous situation when inductive currents are cut off too quickly.

$$v_C(t) = e^{(-t/\tau)} \cdot Z_0(I_{DC} - I_{L0}) \sin \omega_0(t - t_0) + V_{c0} \cos \omega_0(t - t_0) \quad (4.2)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (4.3)$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad (4.4)$$

I_{DC} is the supplying DC current

I_{L0} and V_{C0} are initial conditions and represent the current through the inductor and the voltage over the capacitor at $t=t_0$.

(4.2) is based from (9-21) in [2] but with a damping factor multiplied to it which corresponds to the resistance in cables and load.

A schematic view of the equations is available in Fig. 4.1. A principle plot of the voltage dynamics can be found in Fig. 4.2 and is as noted only a principle figure with different parameters compared to the system simulated in this chapter.

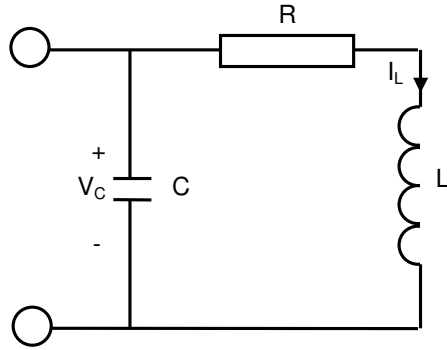


Figure 4.1 Schematic view of (4.1) to (4.4)

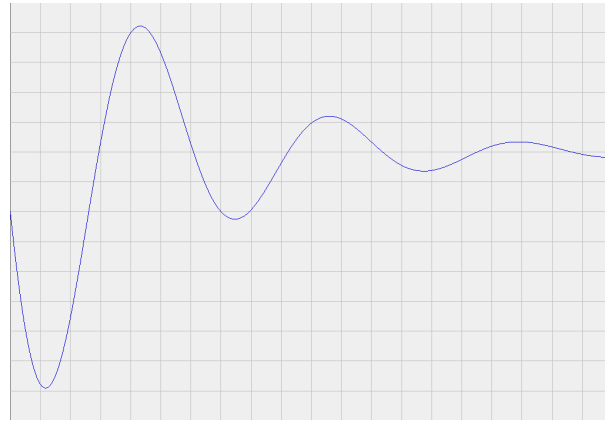


Figure 4.2 A principle plot of (4.2)

4.1.2 Implementation

It is important to have appropriate models of transmission cables and loads. For information about which cables that have been used please refer to chapter 3.4.

One possible scenario is that one of the DC cables to the EDS breaks when it conducts current of approximately 7.5A. This should give a result comparable with the dynamics in (4.2) which is shown in Fig. 4.2 with frequencies and voltage levels depending on inductance, capacitance and resistance of the load and cables.

A hypothetical inductive load is created according to schematic in Fig. 4.1 in SABER with an inductance value chosen to the same level as the magnetizing inductance of the induction machine in the AC (195 mH). The drawn current is assumed to be 7.5 A resulting in a resistance of approximately 85 Ω . Capacitance value is varied between 1 μF and 10 μF to be able to analyze the influence of capacitance value.

$$L=195 \text{ mH}$$

$$R=85 \Omega$$

$$C=1-10 \mu\text{F}$$

4.1.3 Test Setup

Fig. 4.3 shows the implementation in SABER. The built in function *Vary* is used to change the parameter value of the capacitance to be able to present the influence of the component value. **A** in Fig.4.3 is used to start the inductive current and **B** is used to break it.

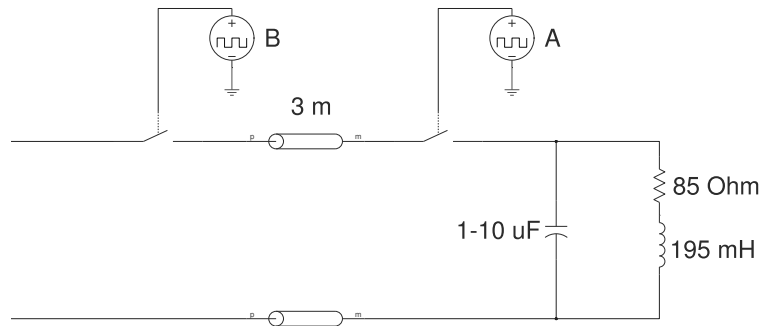


Figure 4.3 SABER model when simulating the break of an inductive current

4.2 Complete System Run

To understand the dynamics of an electric system as the one used in this project a complete system run should be conducted. The aim is to simulate a well working system and present plots of currents, voltages, angular velocities and torques. An appropriate control system have been left out due to time constraints and replaced by a simpler one. This will make the dynamic operation of the system, like start up procedures, less efficient than a system with a proper control system.

4.2.1 Theory

In order to study the behaviour of a well working system, drive cycles should be used. To be able to carry out the simulation, drive cycles are needed to define when to start and stop loads. Drive cycles as well as demanded power levels are inspired by [7] and are presented in Fig. 4.4. The goal is to be able to run the simulation for several minutes to study the system behaviour of torques, motor speeds, voltages and currents. The transients should be investigated with respect to frequency in order to analyze which parameters that could affect the dynamics. (4.1) to (4.4) should be used to analyze the dynamics.

Assumptions have been made about rise times of the different loads, mainly because many of them are fictive equivalents to the current mechanical ones.

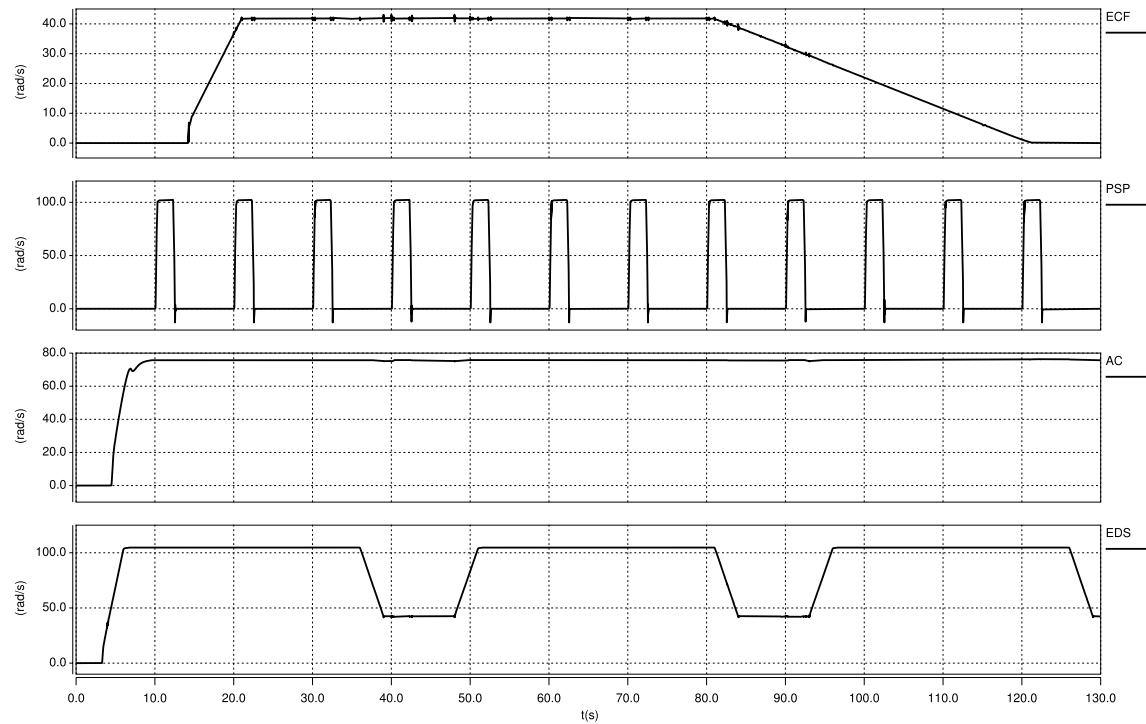


Figure 4.4 Drive cycles for the different loads

4.2.2 Test setup

For information about the different loads, please refer to chapter 3 which presents more details about them. Simulation time is set to 130 s to give an overview of the behaviour of the system. Fig. 4.5 presents the SABER implementation of the system.

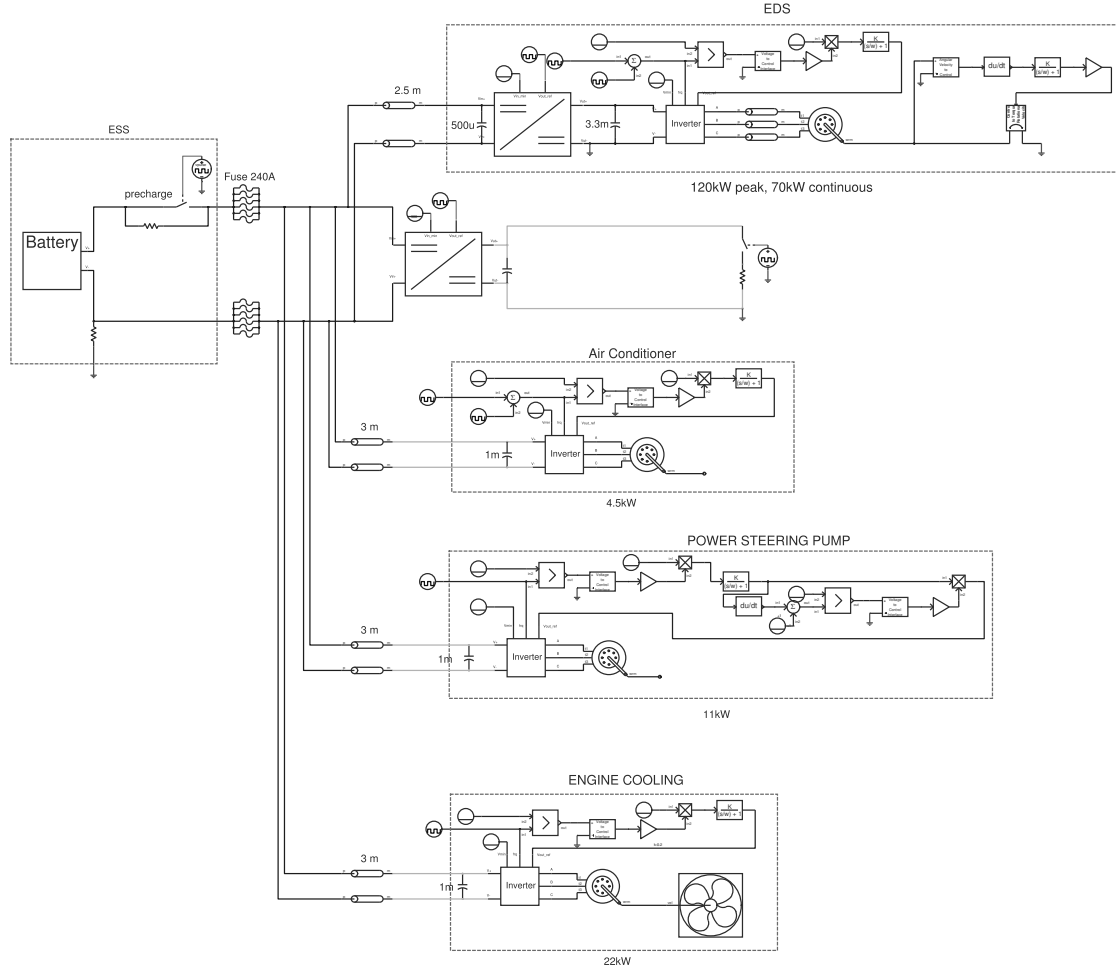


Figure 4.5 Overview picture of the complete test system in SABER

4.3 All loads demand power simultaneously

To simulate this scenario the same system as the one used in chapter 4.2 is used, but with the modification that all loads start at approximately the same time. A study of the DC-link voltage when these loads demand power is the goal of this scenario.

4.4 Failure in control system

The aim of this scenario is to simulate a start up of the 120 kW electric motor, when there is a failure in the control system. To simulate this, the system described in chapter 4.2 is used but when the voltage and frequency starts to ramp, a failure occurs. This could, as an example, correspond to a failure in the control system which in this case is supplying the induction machine with a DC voltage. This is an important scenario since many of the large loads rely on control systems. Fig. 4.6 shows an altered control system where the voltage is raised but with a frequency reference left at zero.

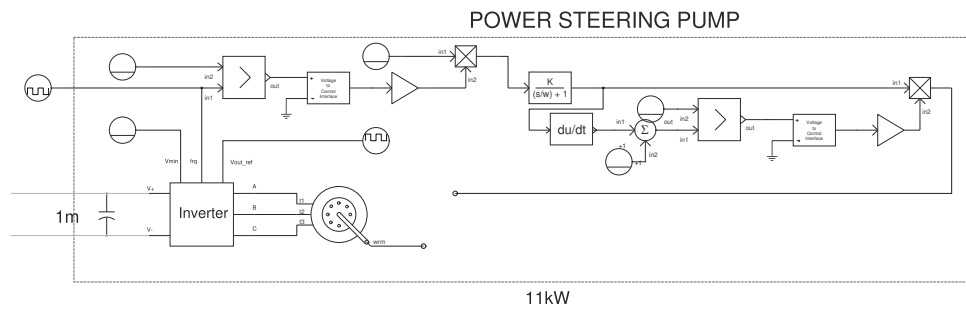


Figure 4.6 SABER model when failure in control system

4.5 Short circuit

There are a number of scenarios which possible could end up as short circuits in different positions in the system. In this chapter several fault scenarios are simulated and analysed to find which scenarios that inflict most damage to systems or persons.

The 600 V DC-link cables have a floating potential compared to ground and that is one of the reasons to the different behaviours in the variety of short circuit scenarios. A floating potential is simulated as a 10 M Ω resistor connected between the negative DC-link cable and ground. For simplicity a “standard” model of a fuse is selected in SABER rated at 240 A (6 · 40 A).

An overview picture of the simulated scenarios is presented in Fig. 4.7. Dashed boxes named **A** and **B** indicates the location of the faults described in the following sub sections.

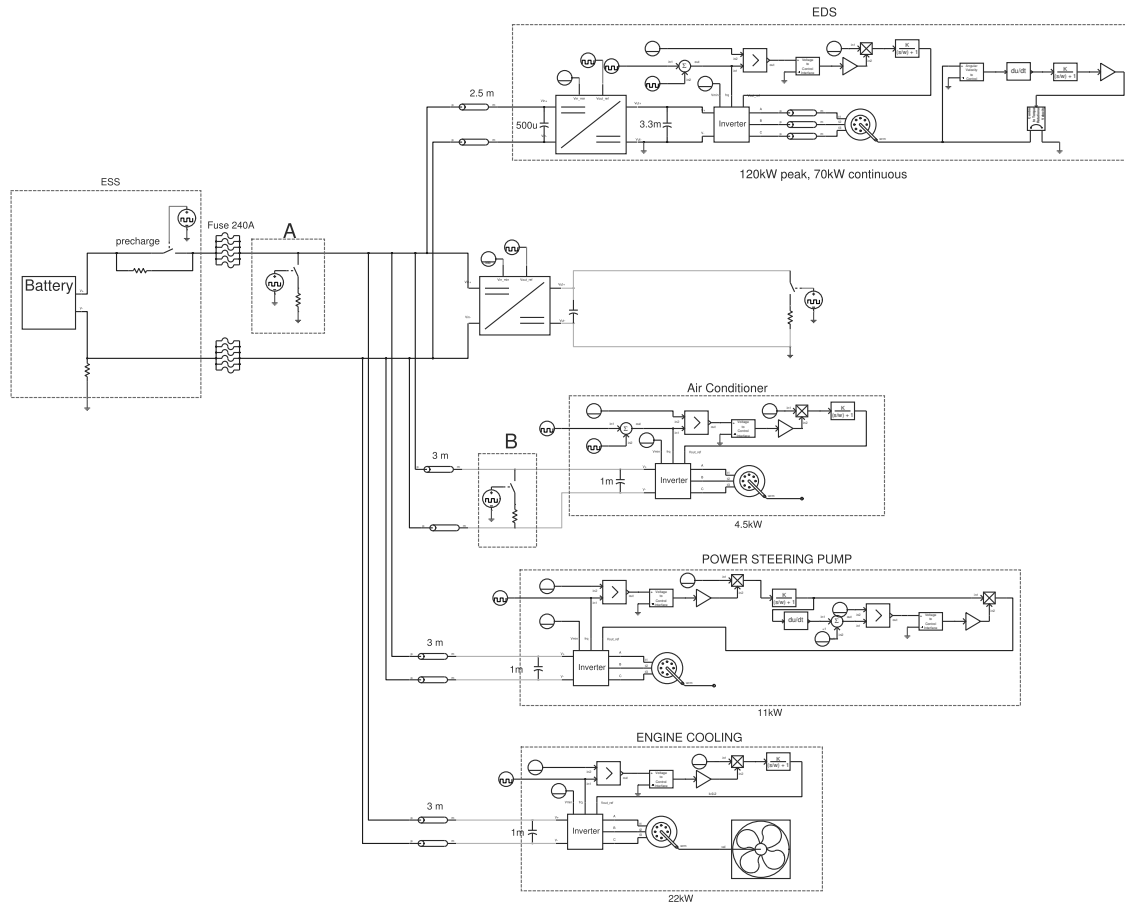


Figure 4.7 Overview figure from SABER of the system

4.5.1 Broken shielding on the EDS DC cables

A simulation of a broken shielding generates a short circuit between the positive DC-link cable and the grounded chassis. In Fig. 4.8, a switch is placed in series with a very small resistor ($1\text{m}\Omega$) and connected to ground to simulate the short circuit. The location of the broken shielding is close to the battery and noted by the dashed box **A** in the overview picture, Fig. 4.7 and the close up picture Fig. 4.8.

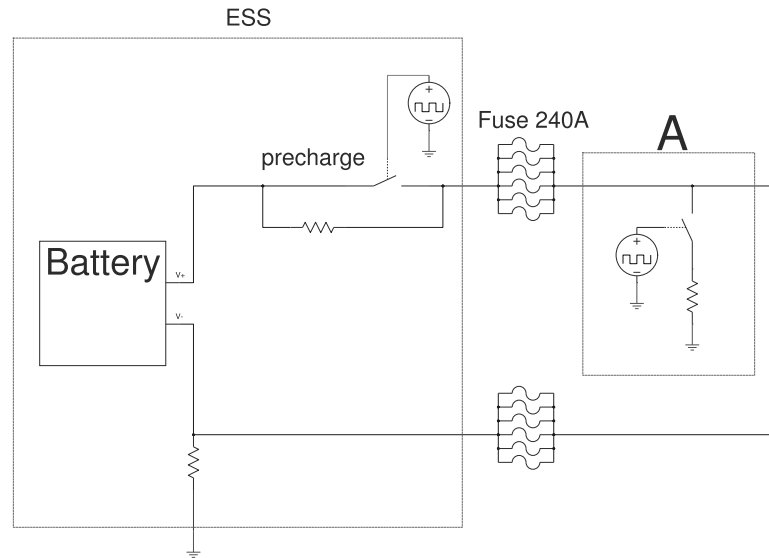


Figure 4.8 A close up picture of box A in Fig. 5.7

4.5.2 Short circuit in a load with a very short duration

It is possible for a short duration short circuit to occur in a load resulting in high peak currents and voltages. One example is a small metal object that is dropped into the electric part of the load. The duration is short (assumed to be 20 ms) because the metal object is just touching the electric part or melts. To simulate the behaviour a model is created according to dashed box **B** in Fig. 4.9.

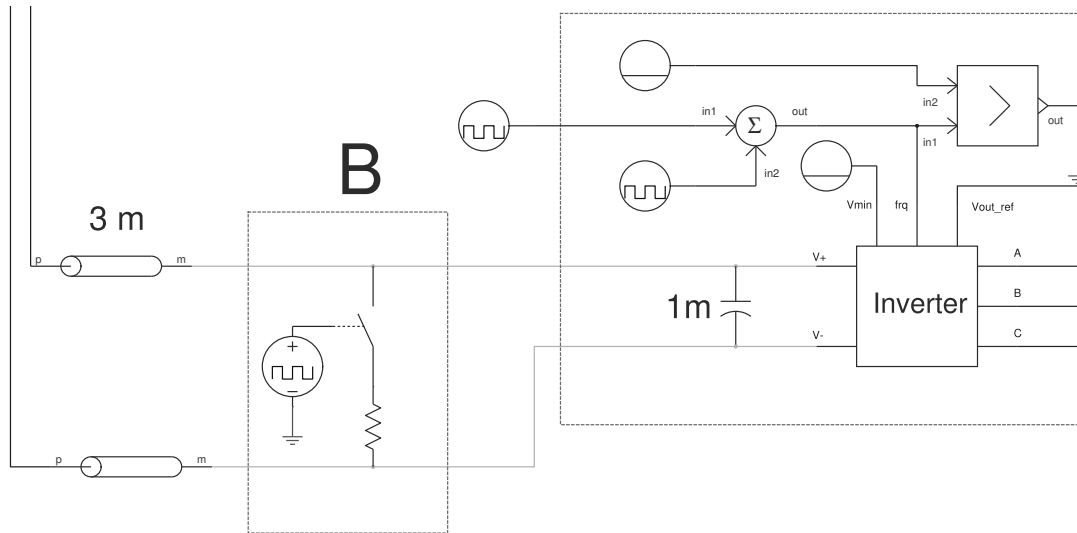


Figure 4.9 A close up picture of box B in Fig. 4.7

4.5.3 Short circuit in a load with a long duration

A short circuit with long duration can occur in a load caused by a failure in the power electronics to mention one example. This scenario is identical to the one in 4.5.2 but with a considerable longer duration.

4.6 Acceleration with switched version of inverter

In this chapter an acceleration simulation of the 120 kW electrical motor will be conducted. The switched version (10 kHz) of the inverter is used to verify the functionality compared to the inverter without switching characteristics. A deeper analysis of the system dynamics, with a simplified version of the system from *chapter 4.2*, is used. The simple system consists of only EDS and battery and is chosen in order to reduce the simulation time and is shown in Fig. 4.10. Fig. 4.11 shows the same system but with the non switched version of the inverter which is used for comparison.

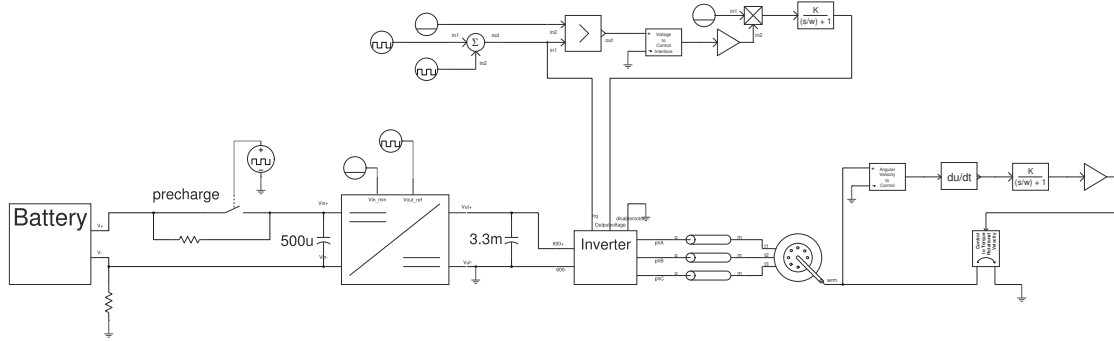


Figure 4.10 SABER model of simplified test system with switched version of inverter

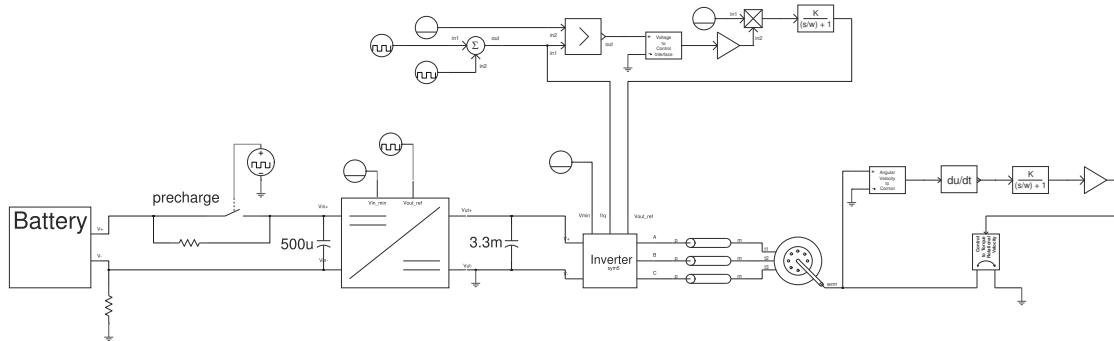


Figure 4.11 SABER model of simplified test system without switched version of inverter

In a “real” vehicle a sophisticated control algorithm for the drive system is preferred. To ensure high performance in a large dynamical range a FVCS could be used. It would control the power electronics using a number of sensors. These sensors could for example measure phase currents, voltages or motor position based on which type of control system it is. For more information on how to control induction machines, please refer to [3].

4.7 Cross-talk

In a dynamic electrical system where currents as well as voltages are changing rapidly, electric and magnetic fields are created around the conductor. An electric field is proportional to a voltage and a magnetic field is proportional to a current. More details about electromagnetic fields can be found in [1]. These fields have a possibility to induce currents and voltages in nearby cables such as sensor systems, etc., and can be a source of disturbances. This type of problems is often called Electro Magnetic Interference (EMI) -problems or cross-talk.

4.7.1 Background

In order to create a model to predict cross-talk behaviour, the time varying electric and magnetic fields are simulated in SABER. An important observation is that the induced voltage in nearby cables is proportional to the time derivative of the voltage and current. The goal with a cross-talk simulation is to find a relationship between distance to the source and the induced voltage.

4.7.2 Implementation

Simulations and parameters are based on a 10 kHz inverter described in chapter 3.1.2, if not noted otherwise. To study the relationship between distance and disturbance, a setup containing 3 phase cables and 5 test cables are used as shown in Fig. 4.12. A theoretical 120 kW load is used, for example uncontrolled start up of an induction machine. The setup was modelled in SABER using the tool *model architect – transmission lines* which generates a component that can be inserted in SABER sketch. This enables the simulation to solve field problems coupled with circuit theory.

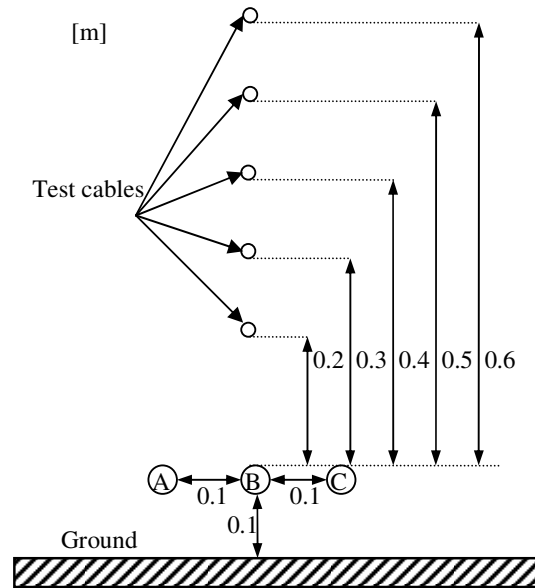


Figure 4.12 Cross section figure of cable placement

In order to study if it is the phase voltage or the phase current that has the largest influence on the disturbance amplitude, two additional simulations were conducted; one with a phase voltage as well the resistance lowered by a factor of 10 and one with the voltage kept at 600 V but with the resistance raised by a factor of 1000. These two simulations are impossible to compare since the rise time of the current is proportional to the input voltage and thereby 10 times larger in the first case. It is not possible to alter the inductance in the system and it is therefore not possible to alter the voltage without changing the rise time of the current, which is required in order to compare the cases.

5 Results

The results of the different scenarios are presented in sub chapters along with comments. Time is measured in seconds throughout the whole chapter if not noted otherwise.

5.1 Breaking of an inductive current

Fig. 5.1 shows a principal plot of the differential equation (4.2) that is compared to Fig. 5.2 which presents the results of the varied capacitance value. It can be seen in Fig. 5.2 that larger capacitance values results in lower voltage transients. Voltage levels in Fig. 5.2 shows that a low capacitance value possible could destroy isolation because of the peak voltage is over 2.5 kV.

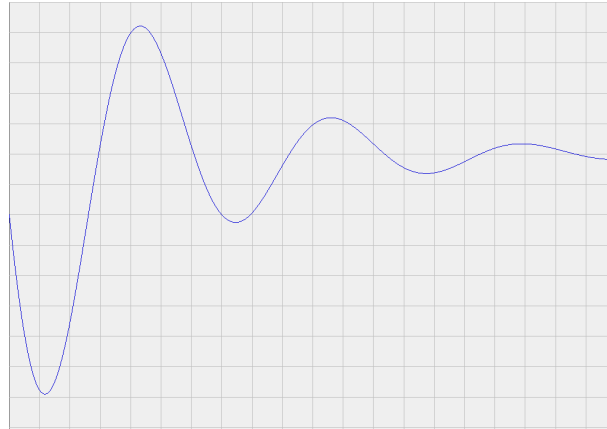


Figure 5.1 A principle plot of equation 4.2

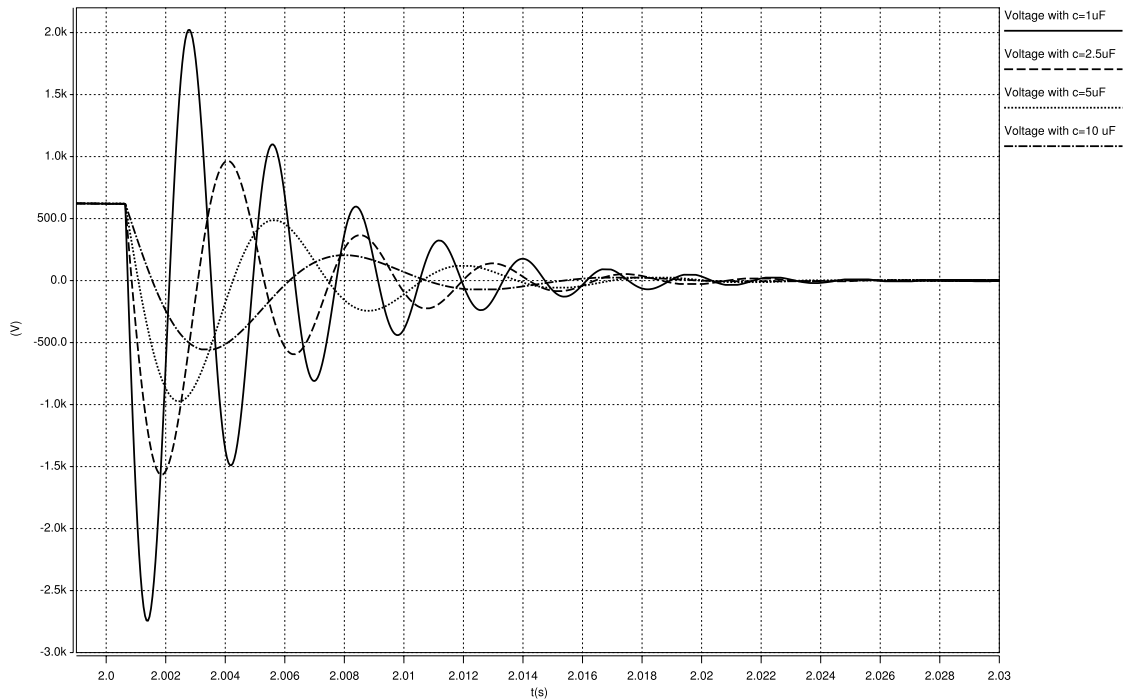


Figure 5.2 Plot of voltage transients when capacitance is varied between $1\mu\text{F}$ and $10\mu\text{F}$

5.2 Complete System Run

The produced torque is bounded to the derivative of the motor speed, which is the equivalence of accelerating the vehicle. The current is proportional to the torque and the motor speed as seen in Fig. 5.3. The voltage variations, on the DC-link, depend on the amount of current drawn by the EDS, which can be both positive and negative. A negative current corresponds to regenerative braking which charges the battery and thereby increases the DC-link voltage.

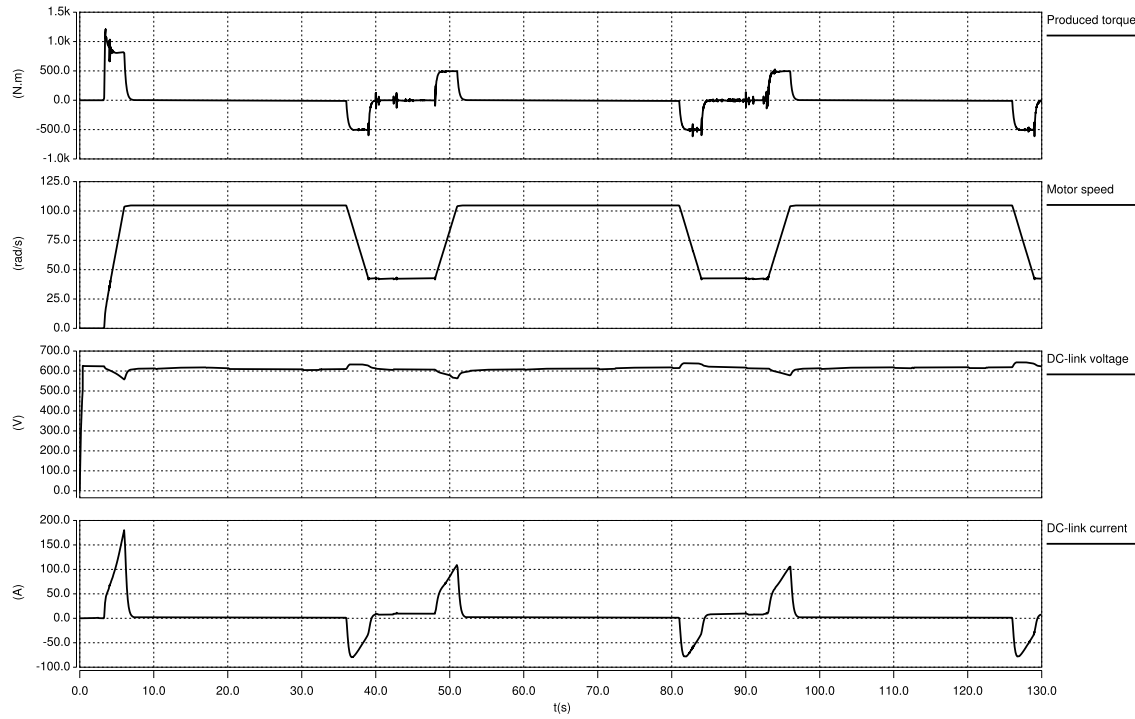


Figure 5.3 Results for EDS in a full working system

Figure 5.4 presents the results from the AC simulation. The amount of current drawn by the AC is directly proportional to the motor speed because the AC compressor is modelled as a viscous damping. The disturbances like the ones seen in DC-link current can be studied in Fig 5.9.

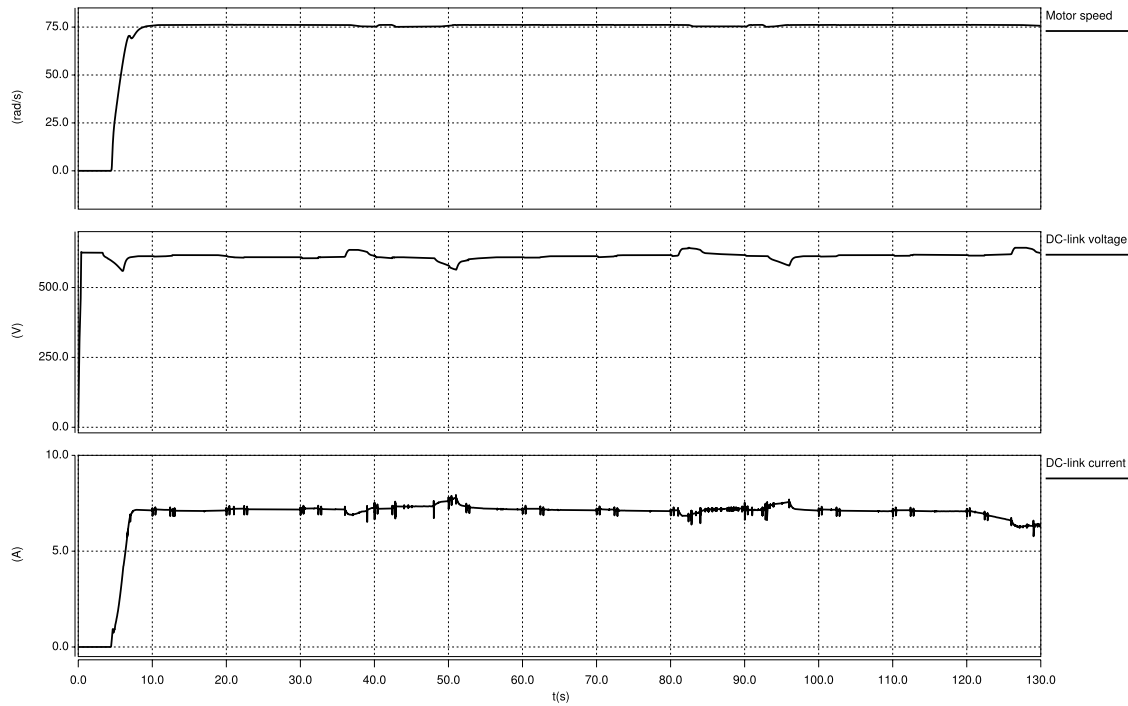


Figure 5.4 Results for AC in a full working system

The upper part of Fig. 5.5 illustrates the rapid acceleration and retardation of the PSP. Because of the rapid changes, current transients occurs and propagates throughout the rest of the system. The voltage is almost not affected. The behaviour of the current can be studied later on in Fig. 5.8.

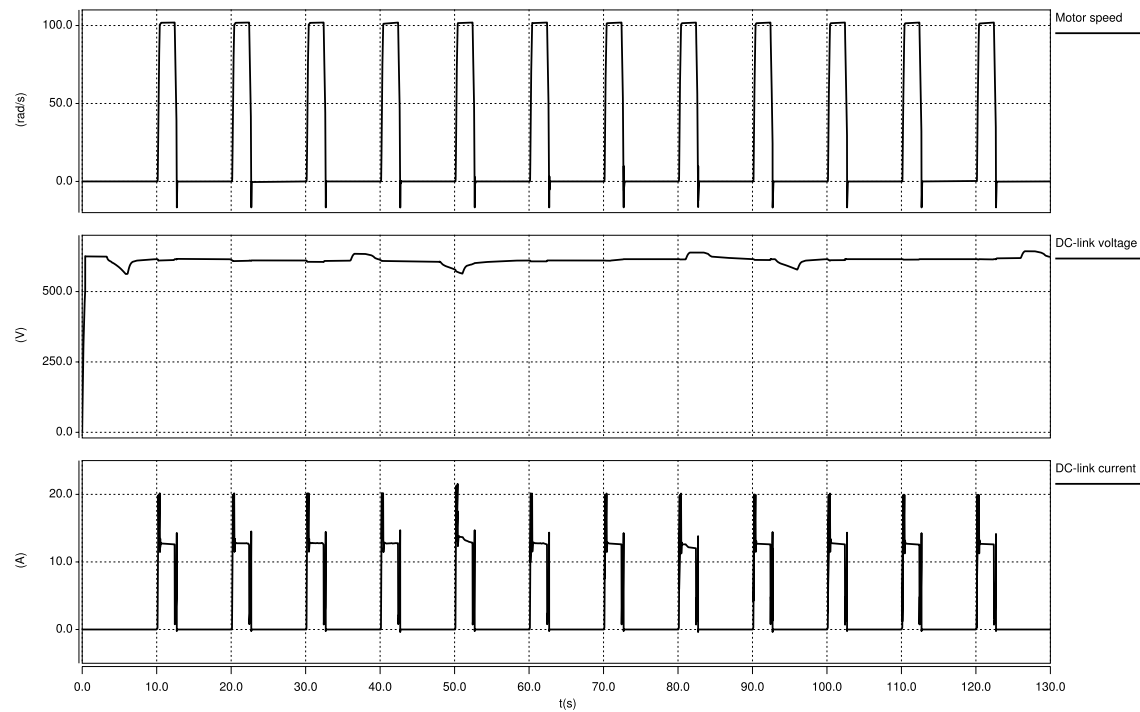


Figure 5.5 Results for PSP in a full working system

Figure 5.6 shows the behaviour of the ECF. The curves representing motor speed and current are bounded to each other. Disturbances in the current depend mainly on the PSP characteristics, among other sources and are explained in Fig. 5.8.

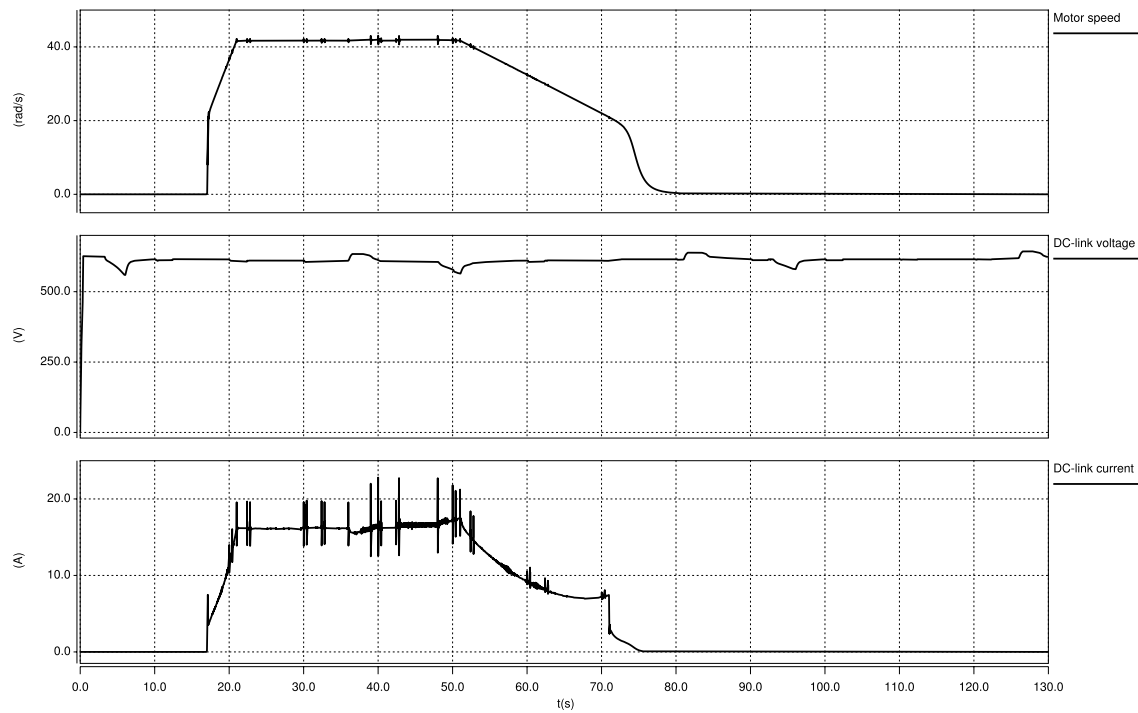


Figure 5.6 Results for ECF in a full working system

In Fig. 5.7, the battery characteristics are presented. An important observation is to compare the voltage differences and compare them with the amount of drawn current. Decreased voltage corresponds to a discharge current and an increase voltage corresponds to a charge current.

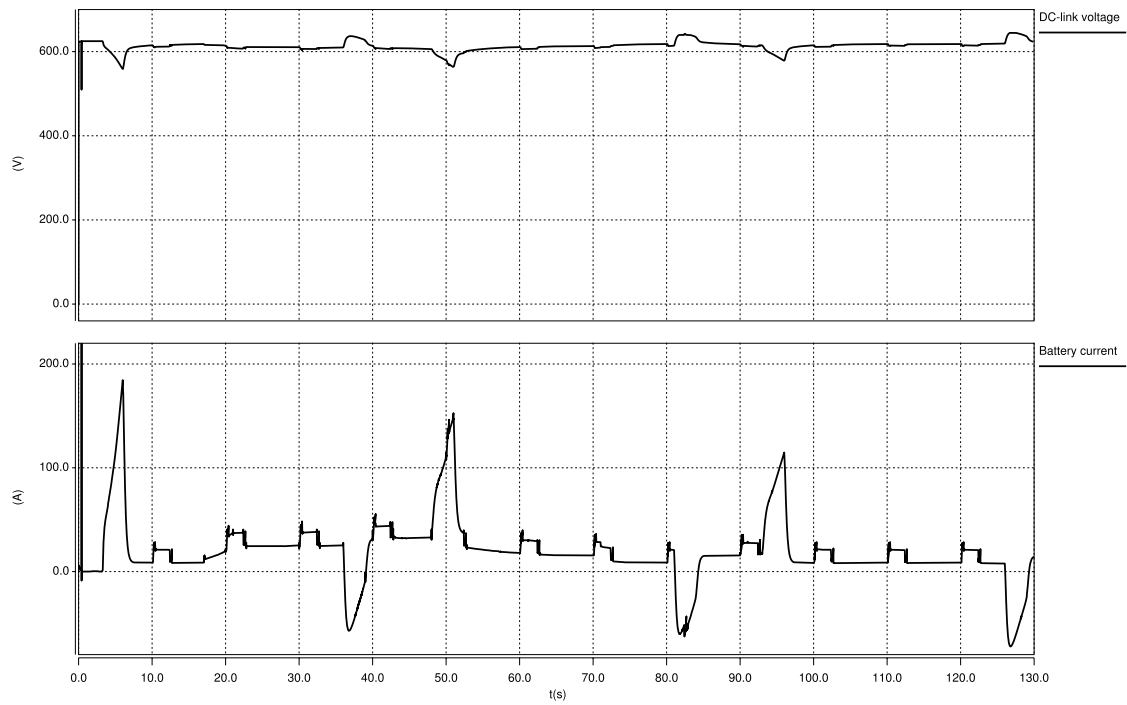


Figure 5.7 Results for Battery in a full working system

Fig. 5.8 shows battery current as well as current drawn by the four loads. The largest amount of current is naturally drawn by the EDS but the largest disturbances on the system are caused by the rapid start and stop of the PSP. Current derivative of the PSP is very high compared to the EDS causing disturbances, which propagate throughout the system. Examples of this can be studied in curves representing the AC and the ECF.



Figure 5.8 All currents in the system

Fig. 5.9 presents a detailed view of the currents to the ECF to investigate the frequency of the disturbances. The frequency should be based on the inductance and capacitance seen from the ECF. An approximation of the frequency is to use the capacitance of the ECF ($0.5\mu\text{F}$) and the magnetizing inductance of the induction machine (39.8mH). This gives, using (4.3), a frequency of approximately 35 Hz and Fig. 5.9 gives a frequency of 31Hz. Using (4.3) is thereby proven to be a relatively good approximation with a simulated result only 13% from the calculated one.

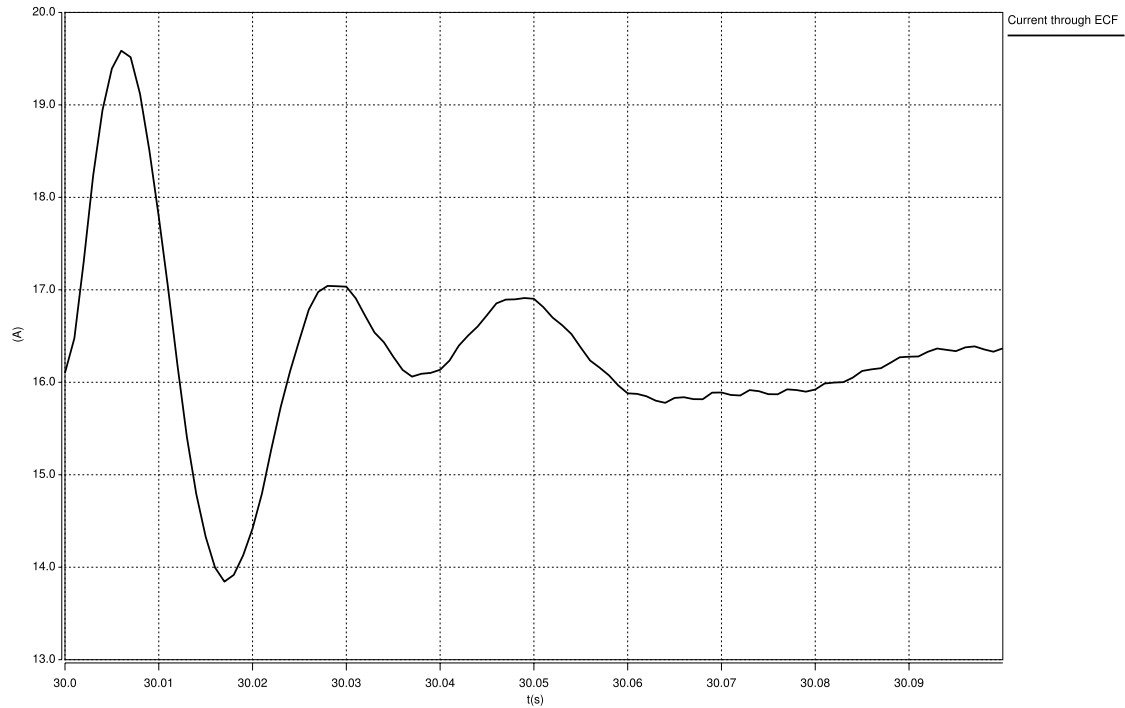


Figure 5.9 Zoomed picture of ECF current in figure 5.8

5.3 All loads demand power simultaneously

In Fig. 5.10 motor speeds of the individual loads are presented along with DC-link voltage. By studying DC-link voltage and motor speeds a relationship between the EDS motor speed and voltage drop can be drawn. The largest influence on the system voltage is the usage of the EDS because of the large peak power of the device.

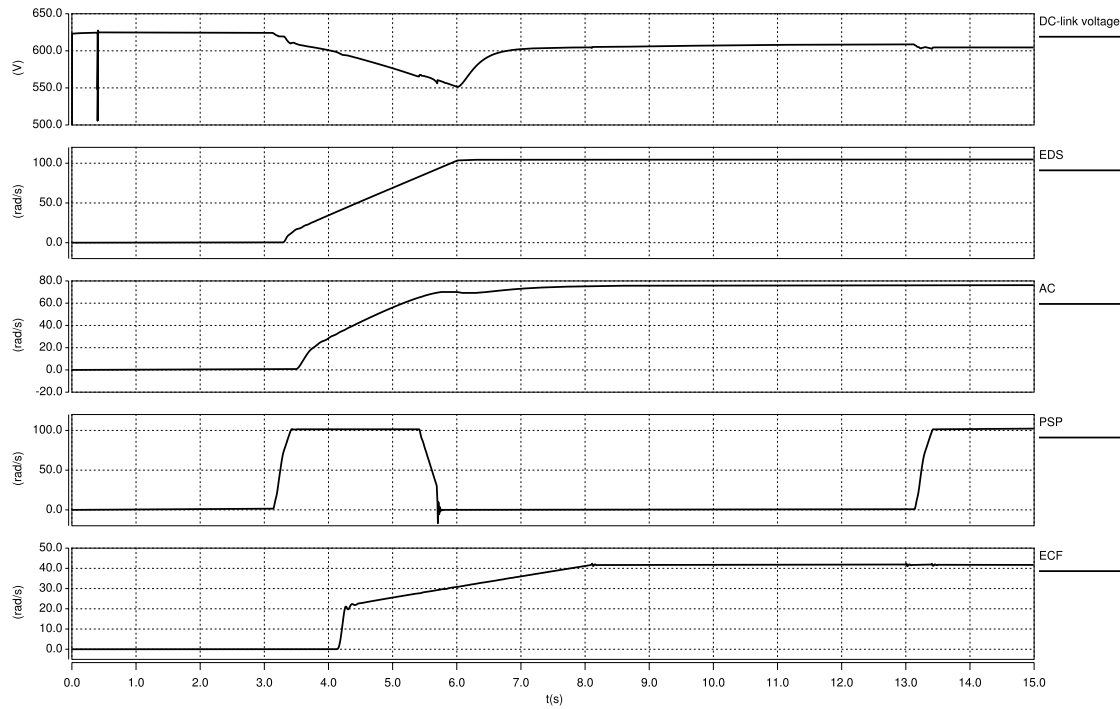


Figure 5.10 Results of simulation when all loads demands power at the same time.

5.4 Failure in the control system

Fig. 5.11 shows motor speed, voltages and currents during control system failure. The motor speed is left at zero throughout the whole course of events since the supply frequency is zero. At $t=9$ s phase currents starts to rise as noted in Fig. 5.11 and DC-link values starts to change accordingly. The DC-link current exceeds the rated value of the fuses which finally melts resulting in a large current transient. When the fuses are blown, voltage and current are lowered to zero after the large transients. The final values of the current and the voltage should be zero but it is not available in this figure due to limitations in the simulation tool. SABERs solving algorithm is sensitive to high derivatives and is thereby not able to calculate values after the fuses have been blown.

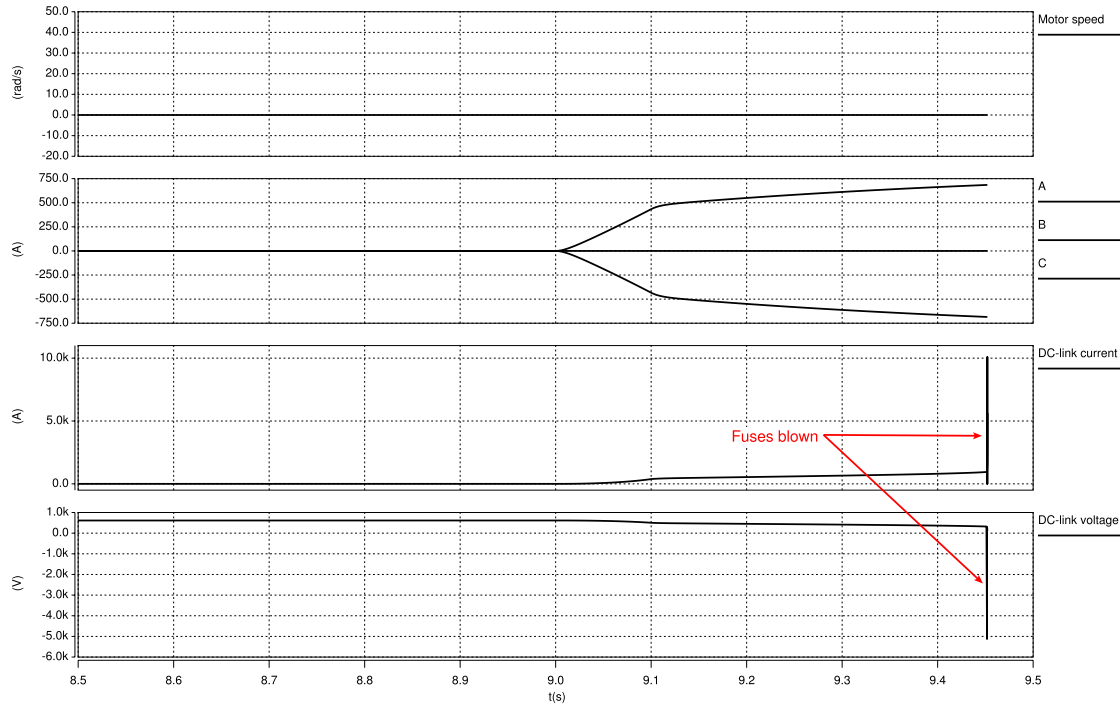


Figure 5.11 Currents, voltage and motor speed when a control system failure occurs

5.5 Short circuit

This chapter presents results from the different short circuit scenarios along with comments.

5.5.1 Broken shielding on the 600 VDC cables

The potential on the positive DC link cable is affected by the fault and the voltage compared to ground is zero as can be seen in the upper part of Fig. 5.12. If the differential voltage on the DC link bus is studied in the mid section of Fig. 5.12 it becomes clear that it is not affected at all by the broken shielding. The lower part of Fig. 5.12 shows the current through the cable during the short circuit. The amplitude along with the duration is negligible.

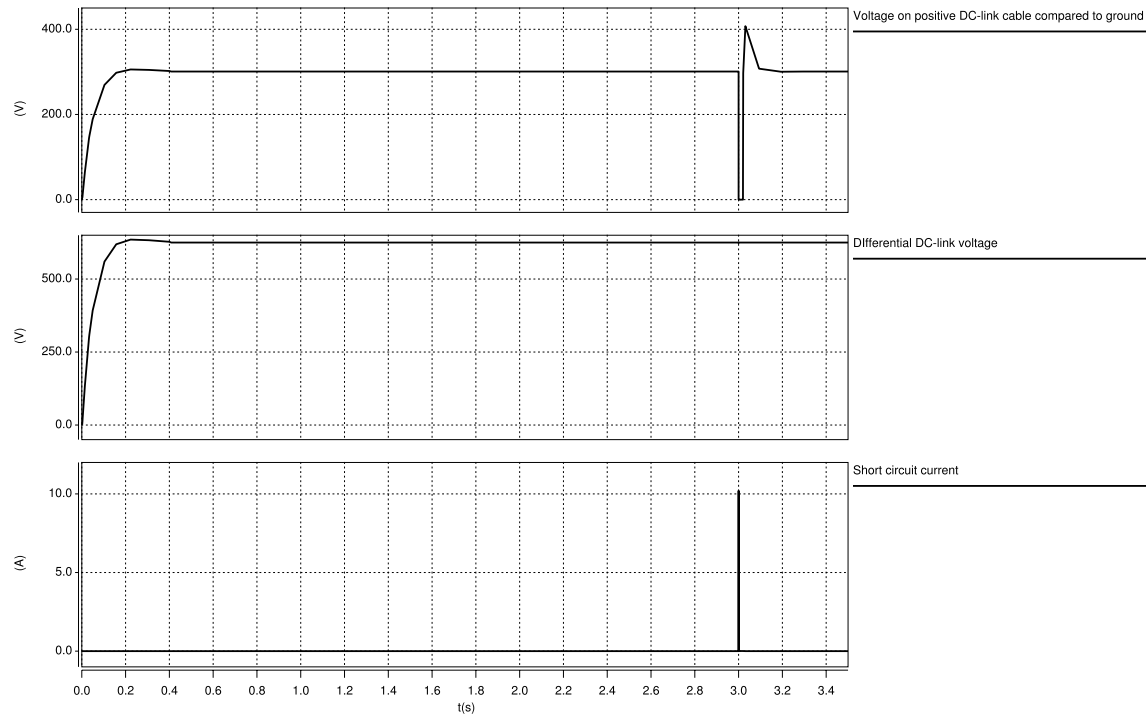


Figure 5.12 Transients during short circuit between ground and positive DC-link cable

5.5.2 Short circuit in a load with a very short duration

The short circuit is considered to last after approximately 20 ms and the results are presented in Fig. 5.13. At the time instant for the short circuit the voltages and currents change dramatically with considerable overshoots. The differential voltage at the battery is lowered to zero with a slight overshoot at the beginning, and a larger one at the end of the short circuit. In the mid section of the figure it is shown that the amplitude of the battery current increases to a very high level with a high slope. When the differential voltage at the load is studied the nature of inductive currents becomes apparent. The highest transient occurs when the short circuit ends which can be studied in the lower part of Fig. 5.13. Some theory about inductive behaviour is available in chapter 4.1.

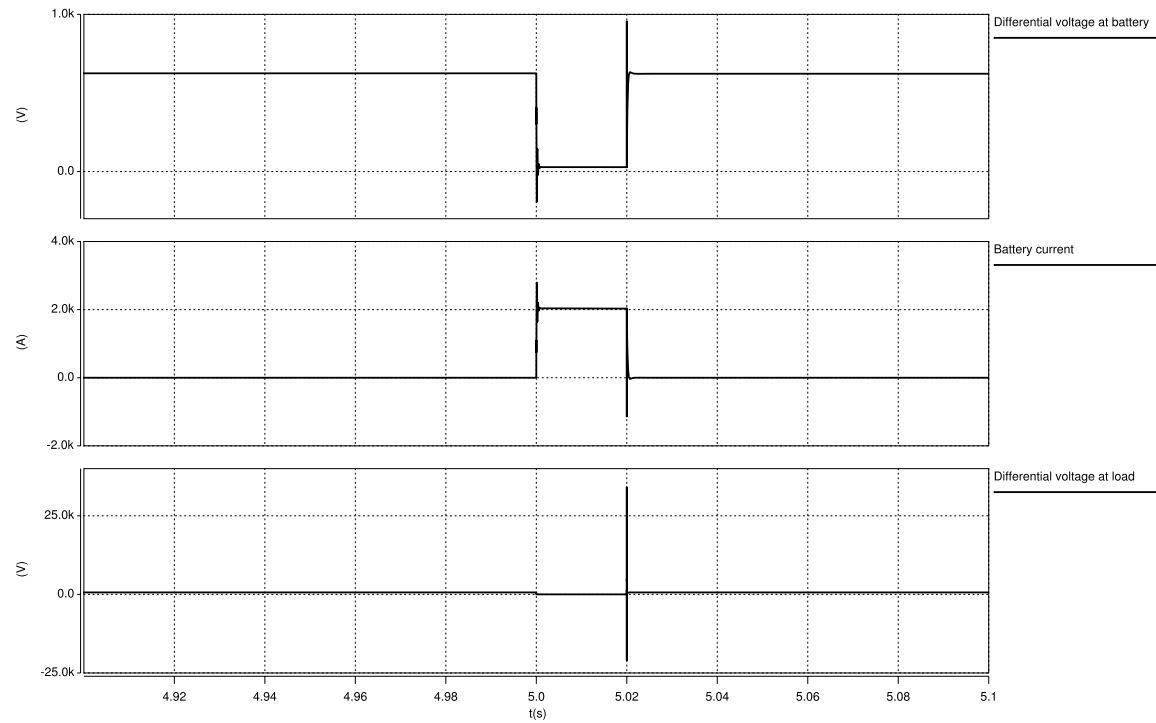


Figure 5.13 Transients during short duration short circuit

Fig. 5.14 presents a zoom of the voltage and current transients around $t=5.0$ s. Voltage transients of magnitudes and duration similar to the one seen in Fig. 5.13 cannot be avoided using normal melting fuses. A melting fuse needs more time to melt than 20 ms, the duration in this case. Transients in magnitudes of several kV or above are very likely to destroy semiconductor devices connected to the DC-link.

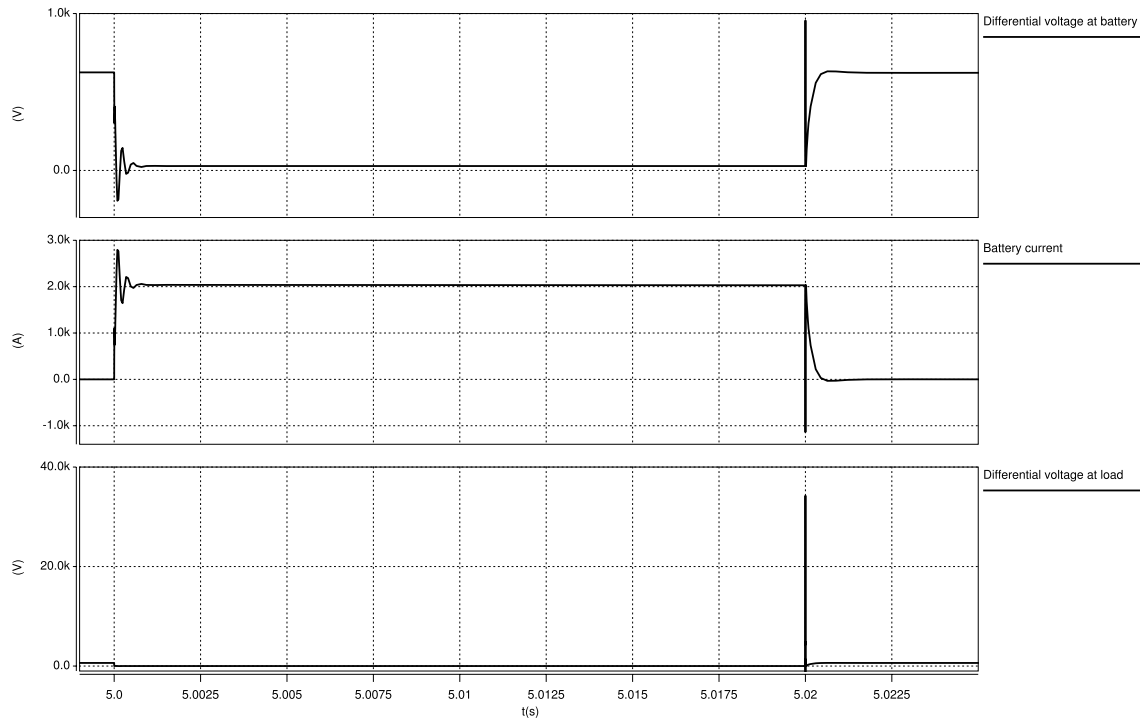


Figure 5.14 Close up picture of transients during short duration short circuit

5.5.3 Short circuit in a load with a long duration

In the scenario with a long duration short circuit the fuses are blown after 30 ms. The two upper plots in Fig. 5.15 presents current as well as voltage at the battery. The differential voltage at the load is presented in the lowest part of the figure and shows that there are no harmful voltage transients. Because the fuses are blown, no harmful voltage transients occurred on the DC-link (please compare to 5.5.2).

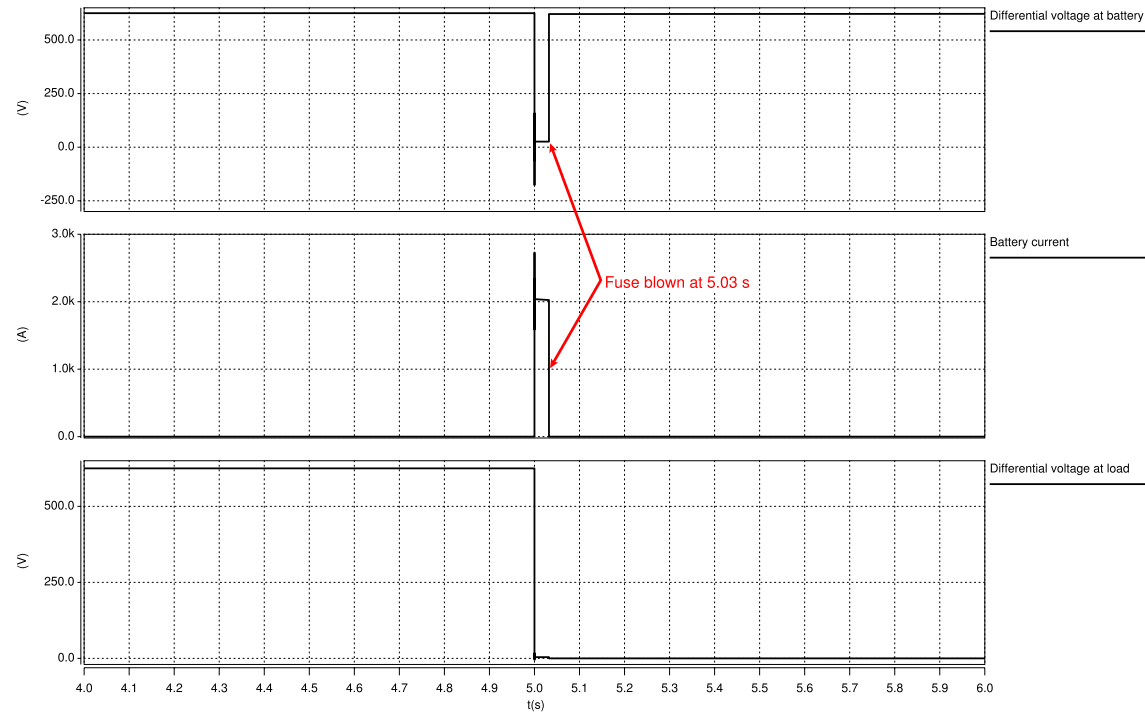


Figure 5.15 Transients during long duration short circuit

Fig. 5.16 is a close up view of the oscillations in Fig. 5.15.

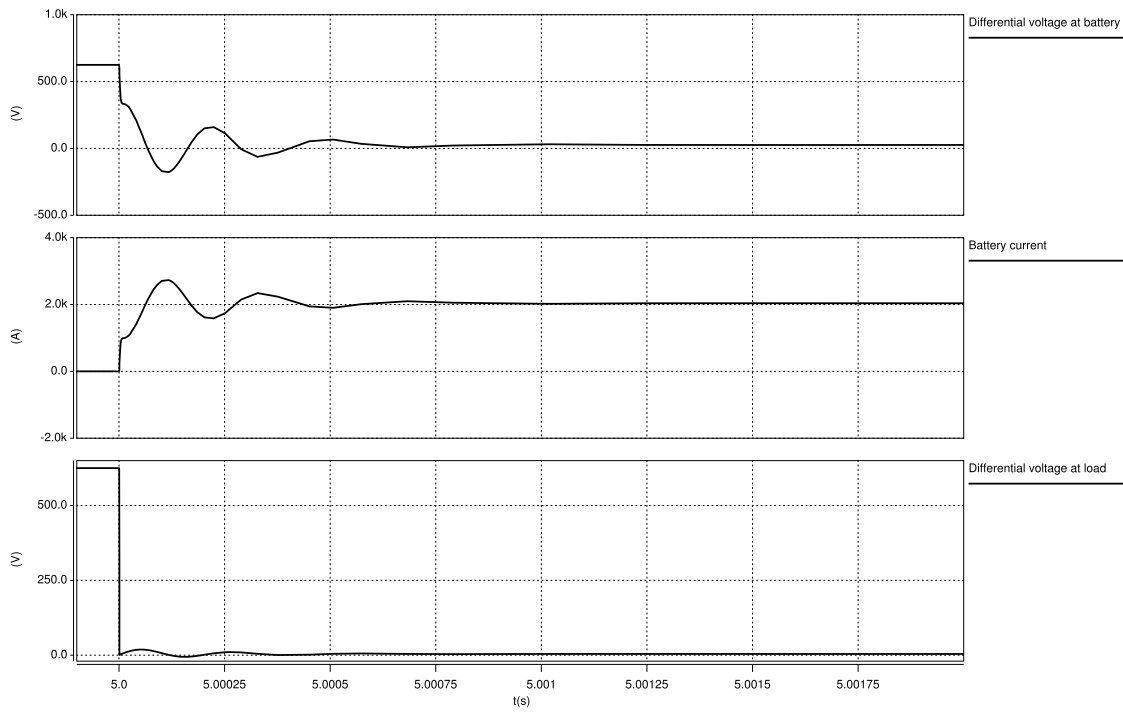


Figure 5.16 Close up picture of transients during long duration short circuit

5.6 Acceleration with switched version of inverter

Fig. 5.17 shows a comparison between the results with two versions of the inverter, one version with switching characteristics and one without. If the two versions are compared with respect to the DC-link current some differences becomes apparent. The non switched version has a much larger starting current and a faster accelerating motor shaft. A higher starting current should correspond to a higher acceleration due to the fact that motor torque is proportional to the phase current. This is in the end proportional to the DC-link current as well.

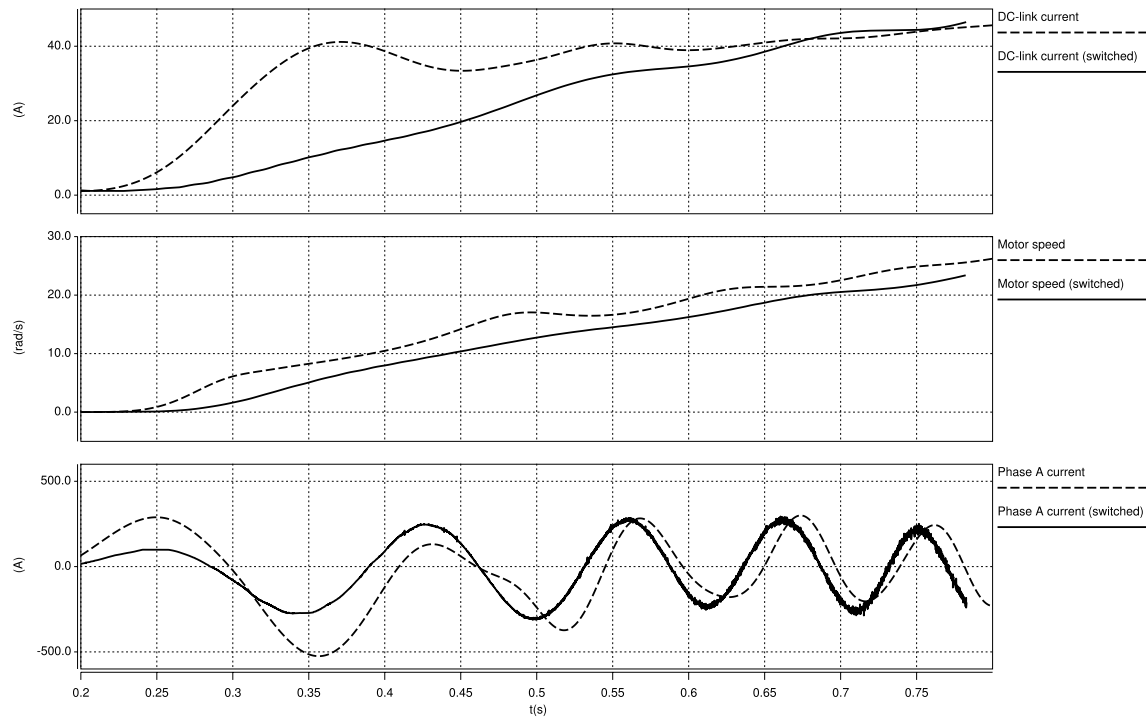


Figure 5.17 Currents and motor speed during acceleration

Fig. 5.18 shows phase voltages and currents with switched inverter and phase voltages in the non switching inverter model.

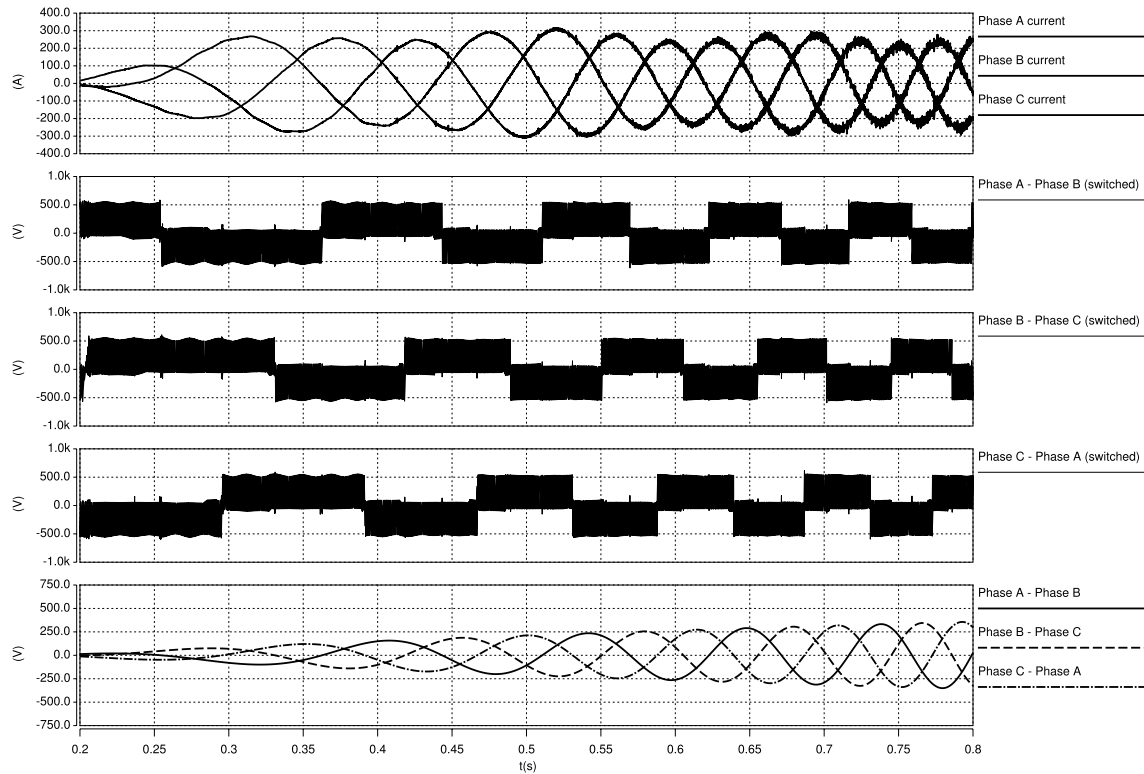


Figure 5.18 Phase currents and voltages with switched inverter and phase voltage without switched inverter

5.7 Cross-talk

Figure 5.19 shows transients during one switching period and Fig. 5.20 shows a more detailed view of the switching dynamics. The phase current and the voltage are drawn in the lower part of Fig. 5.20. Transients in the test cables can be seen in the upper part of the figure where it is clearly shown that the distance between phase cables and test cables have a direct impact of the amplitude of the disturbances. By studying Fig. 5.20 one conclusion is that the disturbances are proportional to the currents time derivative, in this particular test setup. That is an important observation because shielded cables are unable to stop the magnetic fields created by high currents.

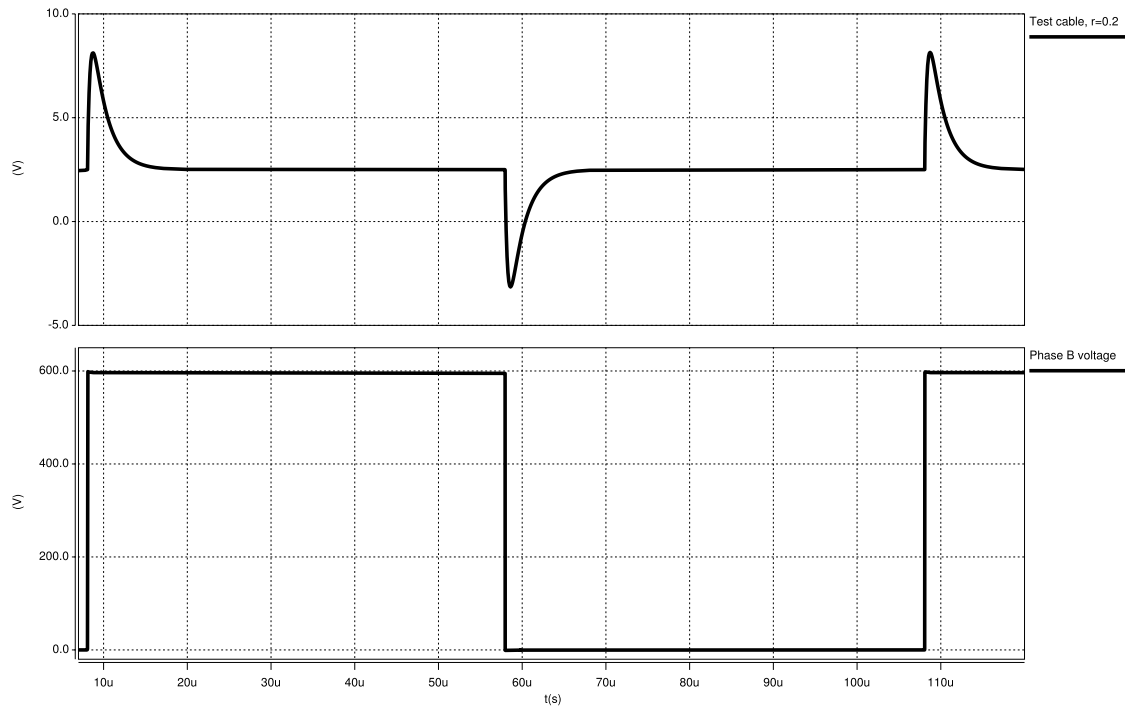


Figure 5.19 Voltages during one switching period

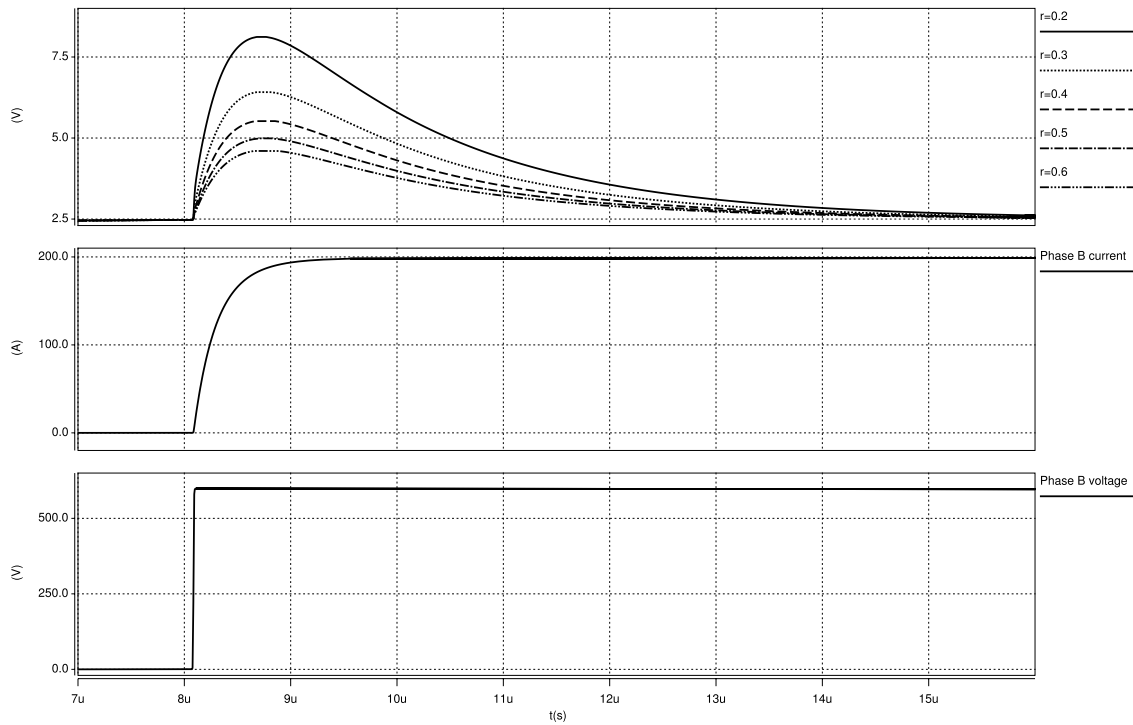


Figure 5.20 Detailed figure of switch transients

Two simulations were conducted; one with lowered current and one with lowered voltage, and the results are presented in Fig. 5.21 and Fig. 5.22. By studying the magnitude of the disturbance in Fig. 5.21 and Fig. 5.22 a conclusion can be drawn that the rise time of the current is proportional to the applied voltage. The voltage in Fig. 5.21 is 10 times lower than the voltage in Fig. 5.22 and the current rise time is therefore considerable longer in Fig. 5.21, as expected when studying (4.1) with a unaltered inductance. It is impossible to change the inductance in this simulation since all inductive behaviour is caused by the cable and not the load.

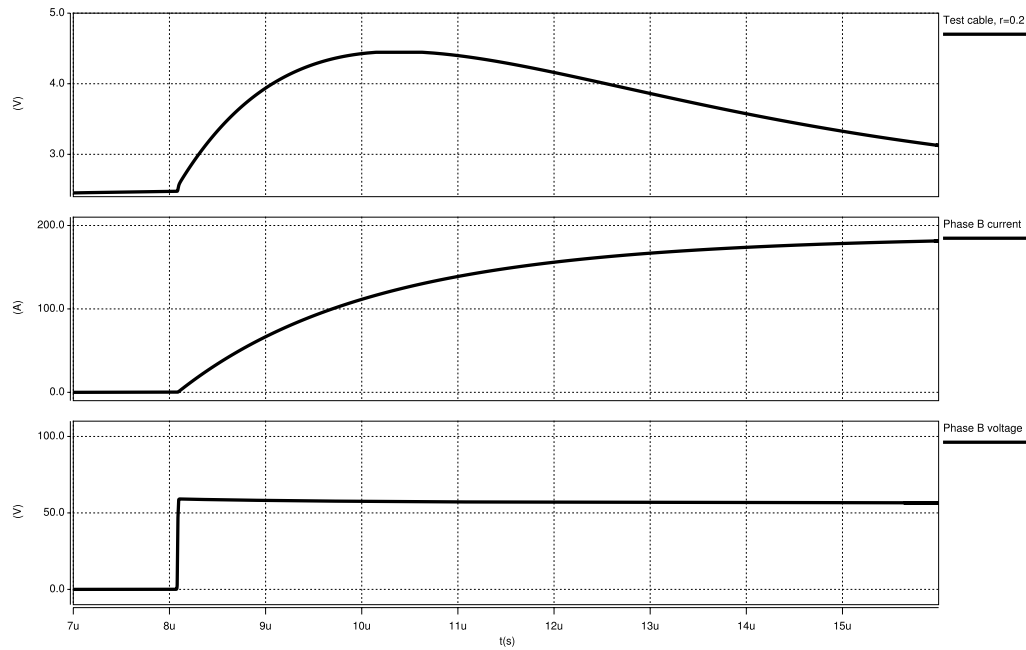


Figure 5.21 Detailed figure of switch transient with lowered voltage

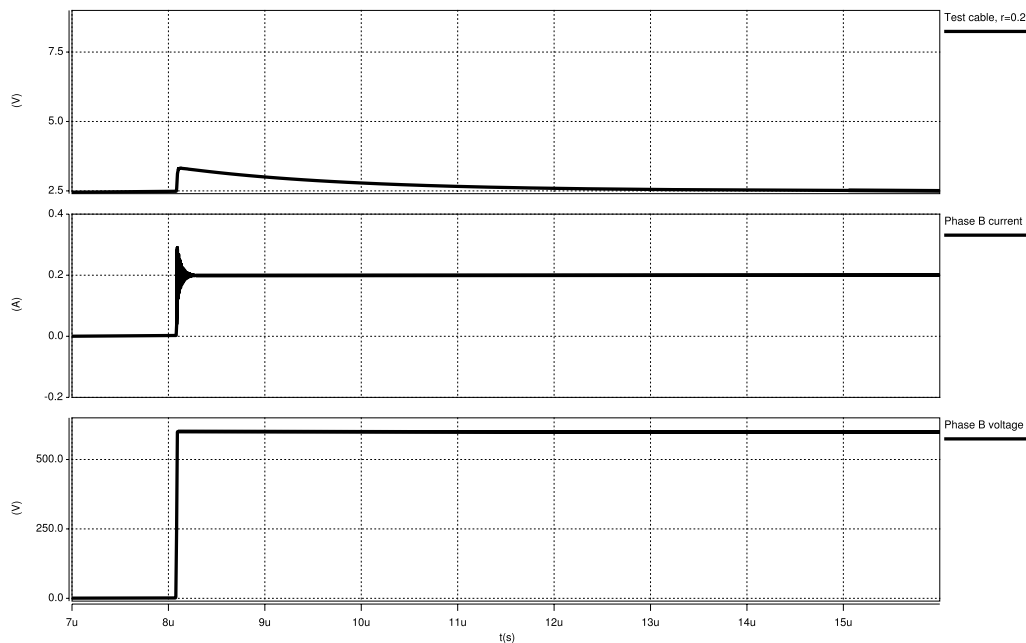


Figure 5.22 Detailed figure of switch transient with lowered current

6 Conclusions

The following sub chapters will present the conclusions of the simulations performed in this thesis.

6.1 Breaking of an inductive current

Massive transients will be present in the system if an inductive current is broken instantly. The theory described in chapter 4.1 is corresponding well with the simulations with the possibility to protect the system from high voltages using large capacitances. A suitable value can be calculated using (4.1) to (4.4) if a higher protection of overvoltages is needed.

6.2 Complete System Run

It is interesting to study the dynamics in the system when loads are switching on and off in a fully working environment. This is a good point of view to be able to compare with in the other scenarios where different faults have occurred. The system also features regenerative braking which makes it realistic.

Voltage drops is mainly caused by the EDS because it has a peak power several times larger than the other loads and thereby also the largest peak current. This also proves that the models for cables and the battery are good as they provide a realistic voltage drop when accelerating as an example. The PSP is, despite its moderate peak current, the largest influence to transients that travels throughout the system mostly because of the rapid start and stop of the PSP.

Studies of the transients (frequencies, etc.) are showing behavior as expected with oscillating frequencies that is possible to predict using the differential equation described in (4.2).

Results from simulations seem reasonable and provides an overview of the system dynamics.

6.3 All loads demand power simultaneously

All loads have control systems for start and stop that prohibit massive current transients in a fully functional system. The control system ramps the voltages and frequencies up to their final values with relatively low derivatives. When all loads were switched on at the same time no harmful levels were detected in the simulation and DC-link current wasn't able to blow the fuse. A voltage drop of approximately 80 V was present but that is not harmful for the equipment. The EDS has the largest influence of the voltage drop because its large peak current.

6.4 Failure in control system

A failure in the control system that occurs according to the scenario in chapter 4.3 (frequency =0 Hz) will melt the fuses that hopefully protects the system components. Notice that high voltage and current transients occurs only in SABER and that there is a lag before the fuses are blown.

6.5 Short circuit

The consequence of the short circuit is highly depending on its location and duration.

Broken shielding on 600 V DC

This fault alone is not harmful for the system or persons. In order to create a dangerous fault scenario a combination with an additional fault is required. This possible dangerous situation is created because the shielding is grounded which means that the floating DC-link potentials are not floating any more.

Short circuit in a load with a short duration

The dominating transient in this case is the one occurring when the inductive current is broken which means that the short circuit has ended. No fuses were blown because of the short duration (20ms) of the short circuit.

Short circuit in a load with a long duration

The fuses are blown after a relatively short period of time (30ms) which protects the system components and prevents it from large voltage transients such as the one seen in the short duration short circuit. The current transient is similar in both cases.

6.6 Acceleration with switched version of inverter

This scenario provides a useful example of comparing the switched inverter with the one without switching characteristics. A main difference is that the non switching variant is using more current at start and is thereby accelerating faster. This difference could possibly be compensated for with a good control system or improvements in switching patterns.

It is possible that the switched inverter is more realistic and produces results closer to a real system. Another important observation is that the phase current is sinusoidal and thereby the torque as well. The conclusion is that it is possible to simulate a PWM system with realistic and believable results.

6.7 Cross-talking

The amplitude of the transient is decreasing with a larger distance from the source (phase cables). The influence of the current derivative is larger than the influence of the voltage derivative as seen in the results. A consequence of this is that the shielding is not very effective, but a better alternative could be to have the phase cables very close to each other. The magnetic fields in this case should sum up to zero.

7 Discussion

In order to make successful simulations of a real world system, one has to be able to identify the most important aspects of different scenarios and to understand the underlying physics. By having a good knowledge of the physics behind different phenomenon, better models can be achieved by skipping effects that are not important for the specified purpose. Thermal effects are, as an example, neglected in our simulation because the purpose is to only study voltages, currents and angular velocities during various simulations.

One has to remember that it is impossible to count for every aspect of the real world physics when creating a theoretical model, which means that a choice has to be made about which effects that are the most important. Our belief is that the models in this thesis are a good compromise between precision and simplicity.

Is SABER an adequate simulation tool for this purpose?

The used simulation tool SABER is rather competent with a possible usage in many different technology areas and the possibility to create coupled simulations between mechanics and electronics is very useful in our thesis. There are flaws, as with every simulation tool, which in some cases had to be taken into account when deciding how to build a model. The approach of modeling IGBT transistors is an example of this, where capacitive effects were cut out in order to make a model that converged. We have the opinion, despite flaws in models and the simulation tool, that the outcome of the simulations conducted is believable and could prove useful when designing an electric system in a THHV.

What kind of experience has been collected while working on this thesis?

After several months of work on this thesis with a heavy focus on simulation and modelling, a great deal of knowledge has been collected by us. Important aspects is, as an example, the ability to prioritize among parts and thereby deciding which parts that will make it to the thesis and which parts that will not. To be able to do this one has to be able to plan the work quite well and predict the amount of time needed for specific tasks. Other things learned during the work process are a deeper knowledge of the dynamics between power electronics, control systems, induction machines and to model electric circuits.

What is left to do?

It is important to be aware of possible ways to continue a certain project or how to open up for new ones following the one that has ended. One possible way to improve this model is to design a FVCS in order to improve the performance of the EDS model. A FVCS would require a lot of time to design and would be much more time consuming to simulate. Other ways to improve is to build additional loads and add them to any of the systems for different scenarios (short circuit, all loads demand power at the same time, etc). Finally, the 24 V system could be added and simulated together with the 600 V system.

8 References

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Appendix A

To approximate parameters for a 4.5 kW induction machine a 15 kW machine from [5] is used. The results are presented below.

$$R_{s4.5} = \frac{15 \text{ kW}}{4.5 \text{ kW}} \cdot R_{s15} = 3.33 \cdot 0.25 = 0.83$$

$$R_{r4.5} = \frac{15 \text{ kW}}{4.5 \text{ kW}} \cdot R_{r15} = 3.33 \cdot 0.27 = 0.9$$

$$X_{s4.5} = \frac{4.5 \text{ kW}}{15 \text{ kW}} \cdot X_{s15} = 3.33 \cdot 0.57 = 1.9$$

$$X_{r4.5} = \frac{15 \text{ kW}}{4.5 \text{ kW}} \cdot X_{r15} = 3.33 \cdot 0.91 = 3$$

$$X_{M4.5} = \frac{15 \text{ kW}}{4.5 \text{ kW}} \cdot X_{M15} = 3.33 \cdot 18.9 = 63$$

$$J_{4.5} = \frac{15 \text{ kW}}{4.5 \text{ kW}} \cdot J_{15} = 0.33 \cdot 0.17 = 0.1$$