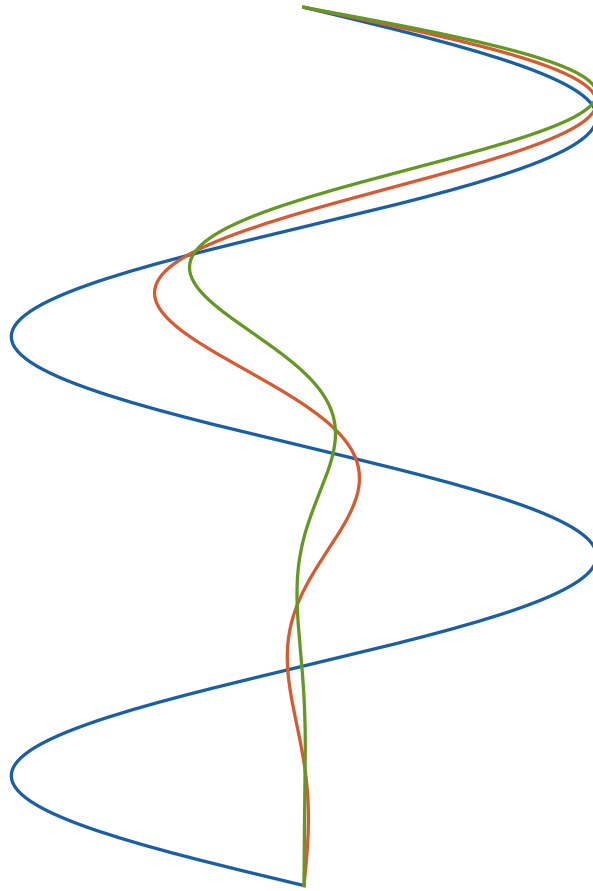




CHALMERS
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Buckling Strength of Steel Piles

Influence of imperfections, soil support, and elasto-plastic soil behaviour

Master's thesis in the Master's Programme Structural Engineering and Building Technology

Mahmoud Ahamad

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
Chalmers University of Technology
Gothenburg, Sweden 2026

MASTER'S THESIS 2026

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Cover:

The cover figure illustrates the main concept of this work: how different soil stiffness conditions influence the instability response of a slender pile embedded in soft soil. The curves show normalised first-mode lateral deflection shapes obtained from linear buckling analysis in ABAQUS. Each curve corresponded to a different undrained shear strength gradient n_h represented through depth-dependent Winkler springs. As n_h increases the lateral deformation becomes more localised near the upper part of the pile, since the stiffer soil at depth provides stronger lateral confinement. This behaviour forms the basis for the parametric study presented in Chapter 3.

Department of Architecture and Civil Engineering

Göteborg, Sweden, 2026

Knäckningsbärförmåga hos stålplålar

Inverkan av imperfektioner, jordens stöd och elasto-plastic jordbeteende

Master's thesis in the Master's Programme Structural Engineering and Building Technology

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ABSTRACT

Long slender displacement piles are widely used in soft clay deposits in Sweden, where the length-to-diameter ratio may exceed 100. Under these conditions, pile stability is strongly influenced by initial geometric imperfections, soil–structure interaction, and boundary conditions. Conventional analytical design approaches in Sweden typically idealise the pile as a beam on a Winkler foundation, assume classical boundary conditions, adopt sinusoidal imperfection shapes, and represent the lateral subgrade stiffness as constant with depth. However, such simplifications may not adequately capture the behaviour of piles embedded in soft clay deposits with depth-dependent stiffness characteristics.

This thesis investigates the geometrically non-linear behaviour of slender piles embedded in soft soil. The study focuses on evaluating the influence of initial geometric imperfections, imperfection mode shapes, soil stiffness distributions, and boundary conditions on the instability response of the pile–soil interaction system. Both constant and depth-dependent soil stiffness profiles are considered in the analyses.

The influence of different boundary-condition configurations on the linear buckling response is also examined. However, the geometrically non-linear analyses are limited to the pinned–pinned (P–P) configuration, since this boundary condition consistently produces the lowest critical buckling resistance and therefore considered the most critical case for instability assessment.

The analyses are performed numerically using the finite element software ABAQUS in combination with a Python-based automation script developed within the study. The pile is modelled using beam element, while the surrounding soil is represented using linear and bilinear Winkler-type springs distributed along the pile. Both lateral and vertical soil springs are considered to represent the interaction between the pile and the surrounding soil, commonly referred to as soil–structure interaction (SSI). This interaction governs how loads and deformations are transferred between the pile and the surrounding ground and has a major role in the stability behaviour of piles embedded in soft clay. Since there are no experimental data existing for direct comparison, the numerical models were first verified against analytical solutions for cases both with and without constant lateral and vertical soil support.

Initial geometric imperfections were introduced using eigenmode shapes obtained from the linear buckling analyses. These imperfections were subsequently incorporated into the geometrically non-linear analyses in order to evaluate their influence on the instability response and ultimate load-carrying capacity of the pile–soil system. The elasto-plastic post-buckling behaviour was captured using the Static RIKS method, enabling investigation of the effect of imperfections and soil–structure interaction on the ultimate load-carrying behaviour of the analysed pile configurations.

The results provide insight into the sensitivity of pile stability to imperfection shape and amplitude, soil stiffness distribution, and boundary conditions. The study highlights limitations associated with simplified design assumptions and contributes to a more realistic assessment of buckling resistance for slender piles

embedded in soft soil.

Keywords: slender piles, pile buckling, soil-structure interaction, geometric imperfections, elasto-plastic post buckling, Winkler springs, finite element analysis, ABAQUS, RIKS method, soft clay.

Knäckningsbärförmåga hos stålplålar

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SAMMANFATTNING

Långa slanka undanträngningspålar används i stor utsträckning i svenska lerområden, där förhållandet mellan pållängd och diameter kan överstiga 100. Under sådana förhållanden påverkas pålarnas stabilitet i hög grad av initiala geometriska imperfektioner, jord-strukturinteraktion och randvillkor. Konventionella analytiska dimensioneringsmetoder i Sverige idealiserar vanligtvis pålen som en balk på elastisk bädd enligt Winklermodellen, antar klassiska randvillkor, använder sinusformade imperfektioner och beskriver den laterala bäddmodulen som konstant med djupet. Sådana förenklingar kan emellertid vara otillräckliga för att beskriva beteendet hos pålar installerade i mjuka leravlagringar med djupberoende styvhetsgenskaper.

Denna studie undersöker det geometriskt olinjära beteendet hos slanka pålar inbäddade i mjuk jord. Fokus ligger på att utvärdera hur initiala geometriska imperfektioner, olika imperfektionsformer, jordstyvhetsfördelningar och randvillkor påverkar instabilitetsresponsen hos pål-jord-systemet. Både konstanta och djupberoende jordstyvhetsprofiler beaktas i analyserna.

Inverkan av olika randvillkorskonfigurationer på det linjära knäckningsbeteendet studeras också. De geometriskt olinjära analyserna begränsas dock till den ledade-ledade (P-P) konfigurationen, eftersom denna randvillkorskombination konsekvent ger den lägsta kritiska knäcklasten och därmed representerar det mest kritiska fallet ur stabilitetssynpunkt.

Analyserna genomförs numeriskt med hjälp av finita elementprogrammet ABAQUS i kombination med ett Python-baserat automatiseringsskript som utvecklats inom ramen för studien. Pålen modelleras med balkelement, medan den omgivande jorden representeras av linjära och bilinjära Winklerfjädrar fördelade längs pålskaftet. Både horisontella och vertikala jordfjädrar inkluderas för att beskriva samspelet mellan pålen och den omgivande jorden, vilket vanligtvis benämns jord-strukturinteraktion (SSI). Denna interaktion styr hur laster och deformationer överförs mellan pålen och omgivande jord samt har stor betydelse för stabilitetsbeteendet hos pålar i mjuk lera. Eftersom experimentella data för direkt jämförelse saknas verifierades de numeriska modellerna först mot analytiska lösningar för fall både med och utan konstant horisontellt och vertikalt jordstöd.

Initiala geometriska imperfektioner introducerades med hjälp av egenmoder erhållna från de linjära knäckningsanalyserna. Dessa imperfektioner infördes därefter i de geometriskt olinjära analyserna för att utvärdera deras inverkan på instabilitetsresponsen och den ultimata bärförmågan hos pål-jord-systemet. Det elastoplastiska efterknäckningsbeteendet modellerades med hjälp av Static RIKS-metoden, vilket möjliggjorde studier av hur imperfektioner och jord-strukturinteraktion påverkar den ultimata lastbärande kapaciteten hos de analyserade pålkonfigurationerna.

Resultaten ger ökad förståelse för hur pålars stabilitet påverkas av imperfektionsform och imperfektionsamplitud, jordstyvhetsfördelning samt randvillkor. Studien belyser begränsningar hos förenklade dimensioneringsantaganden och bidrar till en mer realistisk bedömning av knäckningsmotståndet hos slanka pålar installerade i mjuk jord.

Nyckelord: slanka pålar, pålknäckning, jord–strukturinteraktion, geometriska imperfektioner, elastoplastiskt efterknäckningsbeteende, Winklerfjädrar, finita elementanalys, ABAQUS, RIKS-metoden, mjuk lera.

PREFACE

This master's thesis was carried out at the Department of Architecture and Civil Engineering at Chalmers University of Technology. The work was performed in Gothenburg between January and June 2026 as part of a research project related to the instability behaviour of slender steel piles embedded in soft clay.

We would like to express our sincere gratitude to our supervisor, Doctoral Student Anders Jonefjäll, for his continuous guidance, valuable discussions, and strong engagement throughout the project. His support, availability, and constructive feedback have been highly appreciated and have contributed greatly to the development of this thesis.

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Gothenburg, June 2026

Mahmoud Alhamad
Ayel Osman

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NOTATION

“`latex`”**Roman letters**

Symbol	Description	Unit
A	Cross-sectional area	$[m^2]$
A_b	Pile base area	$[m^2]$
c_u	Undrained clay shear strength	$[kPa]$
$c_u(z)$	Undrained shear strength as a function of depth	$[kPa]$
c_{0u}	Undrained surface clay shear strength	$[kPa]$
$c_{u,b}$	Undrained shear strength at pile base level	$[kPa]$
D	Pile diameter	$[m]$
E	Young’s modulus	$[Pa]$
EI	Flexural rigidity	$[N\,m^2]$
h_k	Characteristic pile cross-sectional dimension	$[m]$
I	Second moment of area	$[m^4]$
K	Effective buckling length factor	$[-]$
KD	Knockdown factor for reduction in failure load ??	$[-]$
k_h	Horizontal spring stiffness	$[kN/m^2]$
k_v	Vertical spring stiffness	$[kN/m^2]$
L	Total or embedded pile length	$[m]$
l_{cr}	Effective/characteristic buckling length	$[m]$
M	Bending moment	$[kNm]$
N	Axial force	$[kN]$
N_c	Bearing capacity factor at the pile base	$[-]$
n_h	Soil stiffness gradient	$[kPa/m]$
N_p	Lateral bearing capacity factor	$[-]$
P	Axial compressive load	$[kN]$
P_{cr}	Critical buckling load	$[kN]$
P_{peak}	Peak load from geometric nonlinear analysis	$[kN]$
$q(x)$	Limited lateral soil reaction	$[kN/m]$
R_{cal}	Calculated geotechnical resistance	$[kN]$
R_d	Design resistance	$[kN]$
R_k	Characteristic resistance	$[kN]$
t	Wall thickness	$[m]$
u	Pile perimeter	$[m]$
u_w	Pore water pressure	$[kPa]$
W	Section modulus	$[m^3]$
x	Coordinate along pile length	$[m]$
$y(x)$	Lateral displacement	$[m]$
z	Depth coordinate	$[m]$

Greek letters

Symbol	Description	Unit	
$\alpha(z)$	Adhesion factor as a function of depth	$[-]$	
α_2	Empirical time-dependent coefficient for k_h	$[-]$	
β	Effective buckling length factor	$[-]$	
γ_{Rd}	Model factor	$[-]$	
γ_t	Resistance partial factor for total pile resistance	$[-]$	
δ	Lateral deformation	$[m]$	
δ_0	Initial imperfection amplitude	$[m]$	““
δ_g	Predefined geometric imperfection	$[m]$	
δ_f	Fictitious imperfection	$[m]$	
Δ	Second-order deformation	$[m]$	
λ	Hetényi foundation parameter	$[m^{-1}]$	
λL	Dimensionless Hetényi parameter	$[-]$	
ξ_3, ξ_4	Correlation factors according to SS-EN 1997-1	$[-]$	
σ	Stress	$[MPa]$	
ϕ	Rotation	$[rad]$	

Abbreviations

Abbreviation	Description
BC	Boundary condition
EC7	Eurocode 7
FEM	Finite Element Method
GNA	Geometrically Non-linear Analysis
LBA	Linear Buckling Analysis
RIKS	Arc-length based nonlinear solution method
SSI	Soil–Structure Interaction

1

Introduction

1.1 BACKGROUND

Pile foundations have been used for centuries to enable construction in areas with weak or compressible ground conditions. While early applications relied primarily on timber piles, advances in construction techniques and materials have enabled the widespread use of modern steel and concrete piles. Today, pile design is governed by standardized codes and guidelines, such as Eurocode 7, as well as Swedish piling approaches represented in Piling commissions[1].

The primary function of piles is to transfer structural loads through weak near-surface soils to deeper layers with higher stiffness and strength, and to mobilize resistance along the pile shaft [2]. However, soil is a complex, stress-dependent material whose stiffness and strength vary with the stress level and loading history. The presence of pore water further influences its behaviour through changes in effective stress, making the prediction of pile–soil interaction considerably more challenging than for conventional structural materials. [3].

A thorough understanding of soil-structure interaction is therefore essential for predicting pile performance. Practically, the lateral support provided by the surrounding soil has a key role in the stability of slender piles[4].

Buckling is a structural instability phenomenon that occurs when a compressive load causes lateral deformation of a slender member. Piles embedded in soils providing substantial lateral confinement generally exhibit increased buckling resistance. In such cases, the ultimate capacity is often governed by either the material resistance of the pile or the geotechnical bearing capacity of the surrounding soil. [5].

In contrast, piles embedded in soft clay may receive only limited lateral confinement from the surrounding soil. Since the stiffness and undrained shear strength of soft clay are generally low, the resistance against lateral pile deformation is reduced. Consequently, the critical buckling load may become comparable to, or even lower than, the geotechnical capacity of the pile–soil system. This is particularly relevant for slender end-bearing piles installed through thick soft clay deposits, where structural instability may become a governing design consideration[4].

1.2 AIM

The aim of this thesis is perform a parametric study to investigate the instability behaviour of slender steel pipe piles embedded in soft clay deposits, with particular focus on the influence of soil–structure interaction on pile stability. The study seeks to improve the understanding of how pile flexural rigidity, soil stiffness distribution, boundary conditions, and geometric imperfections affect the critical buckling load, instability mode shape, effective buckling length, and ultimate load-carrying capacity of the pile–soil system.

Particular attention is given to the transition between short-, intermediate-, and long-pile behaviour and the corresponding shift from Euler-dominated to soil-controlled instability response. Since classical instability theories and Hétényi's λL classification framework are based on the assumption of constant soil stiffness, this thesis investigates how depth-dependent soil stiffness profiles representative of soft clay deposits influence the instability response and the applicability of the classical classification boundaries.

Furthermore, the study examines the sensitivity of the instability response to geometric imperfection amplitude and imperfection mode shape. Linear buckling analyses and geometrically non-linear finite element analyses with elastic and elastic–perfectly plastic soil springs are performed to investigate both the critical buckling behaviour and the post-buckling response. The numerical results are subsequently compared with analytical solutions in order to evaluate the applicability and limitations of simplified design approaches for slender piles embedded in soft clay.

1.3 RESEARCH MOTIVATION

Extensive deposits of soft clay are commonly encountered in Gothenburg, often requiring the use of long and slender piles to transfer structural loads to deeper competent soil layers or bedrock. Under such conditions, the stability of the pile may become an important design consideration, as the limited lateral confinement provided by the surrounding clay can significantly influence the buckling resistance of the pile.

Current design approaches for slender piles are largely based on simplified analytical formulations. These methods commonly assume predefined sinusoidal imperfection shapes, and a constant subgrade reaction coefficient representing the lateral stiffness of the surrounding soil. While such assumptions enable practical design calculations, they may not adequately represent the conditions encountered in soft clay deposits, where both the undrained shear strength and the corresponding soil stiffness typically increase with depth.

In reality, the behaviour of piles is governed by several interacting key factors that influence their stability response, including soil stiffness distribution, boundary conditions, and geometric imperfections. The combined influence of these parameters may significantly affect the critical buckling load, deformation mode shape, and effective buckling length of the pile–soil system.

A more refined analysis is therefore required to adequately capture the buckling response of slender piles in soft soil. This is particularly important for ensuring reliable and safe design of slender pile foundations, as simplified assumptions may lead to inaccurate predictions of stability.

1.4 OBJECTIVES

The main objective of this thesis is to study the buckling behaviour of slender piles embedded in soft clay and to assess the key factors governing their stability, including their influence on both the critical buckling load and the deformation mode shapes. To achieve this, the following objectives are to be defined:

1. To investigate the influence of depth-dependent soil stiffness distributions on the critical buckling load, instability mode shape, and effective buckling length of slender piles.
2. To evaluate the contribution of lateral soil support (k_h) and vertical shaft support (k_v) to the instability response of the pile–soil system.
3. To investigate the transition between short-, intermediate-, and long-pile behaviour and the corresponding shift from Euler-controlled to soil-controlled instability response
4. To assess the applicability of Hétényi’s λL classification framework for piles embedded in depth-dependent soft clay profiles.
5. To evaluate the influence of boundary-condition configurations on the critical buckling load, instability mode shape, and effective buckling length.
6. To investigate the sensitivity of the instability response to different imperfection mode shapes and their influence on post-buckling behaviour. .
7. To evaluate the influence of geometric imperfection amplitude on the ultimate load-carrying capacity and knockdown factor of slender piles.
8. To compare analytical instability solutions based on sinusoidal imperfection shapes with geometrically non-linear finite element analyses in order to evaluate the applicability and limitations of simplified design approaches.

1.5 METHODOLOGY

The instability behaviour of slender steel pipe piles embedded in soft clay is investigated using finite element analyses in ABAQUS. The pile is modelled using beam elements, while the surrounding soil is represented using Winkler-type lateral and vertical springs. Both constant and depth-dependent soil stiffness distributions are considered to evaluate the influence of soil confinement on pile stability. Linear elastic and elastic–perfectly plastic soil spring models are employed to investigate both buckling and post-buckling behaviour.

The adopted methodology consists of the following main steps:

- **Theoretical study and model verification**

Review of the theoretical background governing pile instability, including Euler buckling theory, beam-on-elastic-foundation theory, Hétényi’s λL classification framework, and simplified analytical formulations for pile stability. The finite element model is subsequently verified against analytical solutions for Euler buckling and beams on elastic foundations.

- **Linear buckling analyses and Hétényi classification assessment**

Investigation of the influence of pile stiffness, soil stiffness distribution, soil confinement, and boundary conditions on the critical buckling load, instability mode shape, and effective buckling length. The numerical results are interpreted using Hétényi’s λL classification framework to evaluate the transition between short-, intermediate-, and long-pile behaviour.

- **Geometrically non-linear analyses**

Investigation of post-buckling behaviour using elastic and elastic–perfectly plastic soil spring models to evaluate instability development and load-carrying capacity beyond the critical buckling load.

- **Imperfection sensitivity analyses**

Assessment of the influence of imperfection amplitude and imperfection mode shape on the instability response, post-buckling behaviour, and load-carrying capacity.

- **Analytical–numerical comparison**

Comparison between simplified analytical instability formulations and finite element results to evaluate the applicability and limitations of analytical design approaches for slender piles embedded in soft clay.

The results are subsequently analysed to identify the governing parameters controlling the instability behaviour of slender piles embedded in soft clay and to assess the applicability of classical instability theories to depth-dependent soil profiles.

1.6 LIMITATIONS

This thesis is subject to the following limitations:

- The analyses are based on idealised soft clay conditions representative of those commonly encountered in the Gothenburg region. Therefore, the results may not be directly applicable to sites with substantially different geological or geotechnical conditions.
- The piles are assumed to be primarily subjected axially compressive loading. Effects of cyclic, lateral, and dynamic loading conditions are not considered.
- The soil-structure interaction is modelled using vertical and horizontal springs representing the soil stiffness (Winkler approach). This approach neglects the soil continuity and the three-dimensional effect (solid modelling).
- The effect of pile installation processes, such as driving induced residual stresses and soil deformation, are not considered in the analysis.
- The interaction between multiple piles is not investigated. The analyses are therefore limited to single-pile behaviour and do not account for pile-group effects.
- The boundary condition at the pile's toe is idealized as perfectly fixed or pinned, whereas real soil-pile's toe condition may have an intermediate stiffness.
- The soil behaviour is represented using simplified geotechnical conditions, effects such as soil anisotropy, creep, strain-rate dependency, and other advanced soil behaviours are not explicitly considered.

1.7 ORGANISATION OF THE WORK

The thesis is structured as follows:

- Chapter 1 introduces the thesis by presenting the background and engineering context of pile foundations in soft clay, followed by the aim, research motivation, defined objectives, methodological approach, limitations, and the overall organisation of the work.
- Chapter 2 presents the theoretical framework forming the basis of the study. The chapter includes an overview of pile foundations, geotechnical conditions in Gothenburg, and Swedish pile design practice. Furthermore, theories related to soil–structure interaction, Winkler foundation behaviour, Euler buckling theory, second-order effects, geometric imperfections, and instability behaviour of slender piles are discussed. The chapter also presents the theoretical formulations used to describe axial and lateral soil resistance, geotechnical capacity, and distributed spring stiffness idealisations.

Furthermore, the chapter presents the fundamental components of pile capacity, including shaft resistance and base resistance, as well as the combined effect of axial loading and bending. Theoretical formulations and modelling assumptions forming the basis for the subsequent numerical analysis are also introduced.

- Chapter 3 describes the numerical modelling methodology adopted in the thesis. The finite element modelling approach developed in ABAQUS is presented, including pile material, soil spring implementation, boundary conditions, imperfection modelling, eigenmode extraction, and geometrically non-linear analyses using the Riks method. The verification procedure and modelling assumptions are also discussed.
- Chapter 4 presents the input parameters and material properties used in the numerical analyses, including pile geometry, soil stiffness distributions, spring properties, and modelling configurations.
- Chapter 5 presents and discusses the numerical results. The influence of soil stiffness, boundary conditions, imperfection mode shapes, characteristic buckling length, and soil–structure interaction on the instability response is evaluated. Both linear buckling analyses and geometrically non-linear post-buckling analyses are presented and interpreted in relation to classical stability theory and Swedish pile design assumptions.

2

Theoretical study

This chapter presents the theoretical background used to describe the axial response and geotechnical capacity of piles installed in soft clay. The focus is placed on the interaction between the pile and the surrounding soil, including shaft resistance, load transfer, vertical spring stiffness, and the neutral plane concept. These principles provide the basis for evaluating how soil properties influence the load-bearing behaviour and instability response of slender piles.

2.1 INTRODUCTION TO PILING

Pile foundations are widely used when the near-surface soil possesses insufficient strength or stiffness to support the applied structural loads. In such situations, the loads are transferred to deeper and more competent soil layers or bedrock through a combination of shaft resistance and toe resistance. The interaction between the pile and the surrounding soil governs the load-transfer mechanism and forms an important component of the soil-structure interaction (SSI) governing pile foundation behaviour.[6].

In the design of pile foundations, three fundamental aspects must be considered:

- The installation process, which influences the surrounding soil through displacement, remoulding, and excess pore-pressure generation during pile driving or installation.
- The structural resistance of the pile, which governs the maximum load that can be carried by the pile section without structural failure.
- The geotechnical resistance of the pile-soil system, which governs the load that can be transferred to the surrounding soil through shaft resistance and toe resistance.

These three aspects work together and are interdependent, collectively influencing the overall performance and adequacy of the pile foundation system.

Depending on the relative contribution of shaft resistance, toe resistance, and foundation contact pressure, different foundation concepts can be distinguished [7, 4]:

- **Floating piles:** The applied load is transferred primarily through shaft resistance mobilised along the pile shaft. Such piles are commonly used where no competent bearing layer is available at a reasonable depth and where settlement reduction is an important design objective.
- **Piled raft foundations:** The applied load is shared between the raft foundation and the piles. Both the contact pressure beneath the raft and the shaft and toe resistance of the piles contribute to the overall load-carrying capacity.
- **Compensated raft foundations:** The majority of the load is transferred directly through the raft foundation to the supporting soil. Piles may be introduced locally to support heavily loaded structural elements or to reduce foundation dimensions. This approach is particularly relevant when the weight of the excavated soil is comparable to or exceeds the structural load.

Below the clay, a layer of frictional soil is presented , consisting mainly of gravel and sand, followed by the bedrock[1].

The described soil profile represents challenging geotechnical conditions in which the soft clay deposits largely govern the foundation behaviour. The limited lateral confinement provided by the clay increases the susceptibility of slender piles to instability phenomena, such as buckling, particularly when combined with geometric imperfections and variations in soil stiffness with depth.

2.3 SWEDISH PILE DESIGN PRACTICE

Due to the thick soft clay deposits commonly encountered in Gothenburg, piles are often required to transfer structural loads to deeper competent soil layers or bedrock. Consequently, relatively long piles with slenderness ratios typically ranging from approximately 25 to 100 are frequently employed, causing the piles to behave as slender structural members [9, 1]. Combined with the limited lateral confinement provided by the surrounding clay, these conditions increase the importance of soil-structure interaction and may significantly influence the instability behaviour and buckling resistance of slender piles [10, 11].

Pile foundation may generally be classified into two main categories degree of disturbance induced in the surrounding soil: displacement and non-displacement piles[9]. Displacement piles are installed by forcing the pile into the ground, causing the surrounding soil to be displaced and redistributed. In contrast, non-displacement piles are installed by excavating or removing soil before installing the pile. Both installation methods induce different changes in the surrounding soil and the *SSI* condition, thereby influencing the load-bearing capacity and the ultimate resistance of the pile[9].

In Sweden, pile foundations are usually constructed using driven displacement piles[10]. The most common piling types used in Sweden are driven precast concrete piles and driven steel pipe piles. Statistics presented by the Swedish pile Commissions shows that Driven precast concrete piles represent approximately 60% of installed piles, while driven steel pipe piles account for approximately 23%[10]. Driven steel pipe piles are commonly used in soft clay deposits because they can be installed to significant depths while maintaining high structural capacity and installation efficiency[10].

The Swedish pile design approach follows the Eurocode framework [12] together with Swedish national annexes [13] and recommendations developed by the Swedish Pile Commission (*Pålkommisionen*). The design process includes a geotechnical verification commonly performed according to Design Approach 2 (*DA2*), while structural stability verification is performed according to Design Approach 3 (*DA3*) [14]; see Appendix A.1.1. For slender piles embedded in soft clay deposits, the structural verification explicitly considers second-order effects, geometric imperfections, and the restraining influence of the surrounding soil [15, 16, 17].

2.4 SOIL RESPONSE AND STRESS REDISTRIBUTION DURING PILE INSTALLATION

Driven piles are installed by forcing piles into the ground using impact driving or vibratory methods. The piles may be installed either by impact driving or vibratory methods depending on the soil conditions and pile type [9]. Both installation methods are commonly used in Sweden for displacement piles [1].

The pile cycle proposed by Randolph and Gourvenec (2011)[18] describes the evolution of soil behaviour in response to pile installation and subsequent loading. The framework illustrates how the installation process alters the stress state and mechanical properties of the surrounding soil, which subsequently influence the load-transfer and deformation behaviour of the pile. The pile cycle consists of four principal stages, illustrated in Figure below:

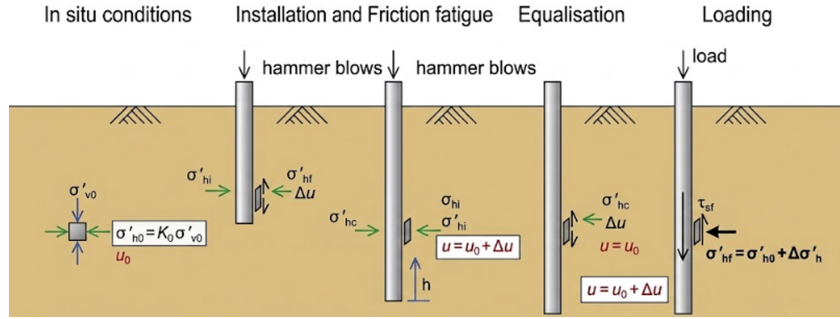


Figure 2.2: Pile installation process in clay

The in-situ stage focuses on analysing the soil state (the initial stress state and mechanical properties).

During the installation stage, the advancing pile displaces the surrounding soil both laterally and vertically, resulting in significant disturbance of the soil in regions close to the pile shaft. This process leads to a redistribution of stresses within the surrounding soil mass and give rise to different phenomena, such as soil arching. Soil arching is a phenomenon where stresses are transferred from more deformable regions to stiffer zones within the soil[7, 19].

In cohesive soils such as clay, piling occurs under predominantly undrained conditions due to the low permeability of the clay[14]. Consequently, the displacement of soil during installation generates excess pore water pressure u , which is according to Terzaghi's effective stress principle [19], temporarily reduces effective stress in the soil:

$$\sigma' = \sigma - u \quad (2.2)$$

Where σ' is the effective stress representing the mechanical behaviour of the soil, σ is the total vertical stress, and u is the pore water pressure. The reduction of the effective stress leads to a corresponding decrease in the undrained shear strength during the installation process.

Since the undrained shear strength depends on both the consolidation history and the effective stress state of the soil, the excess pore pressures generated during installation result in a temporary reduction in effective stress and, consequently, a reduction in shear strength. Furthermore, the high shear strains induced in the soil adjacent to the pile shaft disturb the natural soil structure, leading to remoulding and temporary softening of the clay. As a result, both the stiffness and the undrained shear strength of the surrounding soil are reduced immediately after installation [20].

Following installation, the equalisation stage begins, during which the excess pore pressures gradually dissipate as drainage occurs. This process leads to an increase in effective stress and a progressive recovery of the soil strength and stiffness through reconsolidation [3, 18]. In many cases, the reconsolidation process also increases the shaft resistance of the pile, a phenomenon commonly referred to as pile setup. The pile cycle highlights the importance of considering time-dependent changes in soil properties and pile resistance when evaluating pile behaviour in different soil conditions [3, 18].

2.5 PREDEFINED GEOMETRIC IMPERFECTION AND RESIDUAL STRESSES OF STEEL PILES

Slender steel members are highly sensitive to instability phenomena, where relatively small geometric deviations may significantly influence the buckling response and reduce the critical load capacity [17, 21]. In practice, steel elements are never perfectly straight, and exhibit a combination of geometric imperfections, installation-induced deviations, manufacturing-related irregularities, and residual stresses [17].

For slender steel piles embedded in soft clay, these imperfections become particularly important due to the combined influence of structural instability and soil–structure interaction. Both installation procedures and manufacturing processes — including welding, splicing, transportation, and handling — may further increase the instability sensitivity of the pile [15, 16].

To account for initial imperfections in the design, the Swedish design practice introduces a characteristic initial imperfection value δ_0 that combines both the geometric and material imperfections [16]:

$$\delta_0 = \delta_g \gamma_d + \delta_f \quad (2.3)$$

where δ_g represents a prescribed geometrical initial imperfection, γ_d is a partial safety factor (is usually taken as 1 [16]), and δ_f is a fictitious imperfection that represents the residual stresses within the pile material.

The geometric imperfection is expressed as [16]:

$$\delta_g = \frac{l_{cr}}{600} + n \frac{l_{cr}}{4} v \quad (2.4)$$

where l_{cr} is the buckling length [m], n is the number of joints and splices existing within the buckling length [-], and v is an angular deviation parameter that is usually set to $l_{cr}/300$.

Alternatively, Swedish design recommendations introduce a simplified expression to account for the initial geometric imperfections:

$$\delta_g = \frac{l_{cr}}{h} \quad (2.5)$$

Where h typically varies between 150 and 600, depending on the number of splices, pile type, and whether straightness control is performed or not [15].

The fictitious imperfection associated with residual stresses is defined as:

$$\delta_f = \alpha_h l_{cr} \quad (2.6)$$

Where α_h is a coefficient that depends on the residual stress group, which is determined by the cross-sectional type, manufacturing method, and thickness of the steel section (table 3.6.1a and 3.6.1b in Pålkommission 96:1) [15].

The initial imperfection of slender piles is commonly idealised using sinusoidal deformation shapes over the buckling length, representing the expected global instability mode [16]. In numerical analyses, imperfections are often introduced using

scaled eigenmode shapes[22] in order to represent the sensitivity of slender piles to geometric deviations and second-order effects. The magnitude of the imposed imperfection may significantly influence the non-linear post-buckling response and therefore requires careful consideration in geometrically non-linear analyses. These imperfection idealisations form an important basis for the geometrically non-linear analyses performed in the present study.

2.6 CLASSICAL STABILITY THEORY AND SECOND-ORDER BEHAVIOUR OF SLENDER PILES

This section presents the fundamental structural stability concepts that control the instability behaviour of slender piles under axial compressive loading. Main theories and concepts discussed here are the classical Euler column theory, the second-order effects, geometric imperfections, and the interaction between pile slenderness and surrounding soil confinement. These concepts form the theoretical basis for the geometrically non-linear analyses performed in the present thesis.

2.6.1 Classical Euler buckling and slender piles

Classical Euler buckling theory describes the elastic instability behaviour of slender members subjected to axial loading. As the applied compressive load approaches a critical value, lateral deformations become increasingly amplified due to instability effects. When the critical load P_{cr} is reached, the member loses stability even if the material stresses remain below the yield strength f_y [23, 17].

For an ideal perfectly straight column with linear elastic material behaviour, the Euler critical buckling load may be expressed as:

$$P_{cr} = \frac{\pi^2 EI}{(\beta * L)^2} \quad (2.7)$$

Where E is the Young's modulus, I is the second moment of inertial, L is the pile length and β is the effective length factor depending on the boundary conditions[24].

Appendix A.1.1 shows how the effective buckling length depends strongly on the BC-configurations of the structure. These BC-configurations influence both the deformation mode and the corresponding critical buckling load.

Under classical Euler conditions, instability develops as a global deformation mode extending over the entire member length. In real structural members containing initial geometric imperfections, lateral deformation develops progressively under compressive loading due to eccentricity-induced bending moments. As the applied load approaches the critical buckling load, the deformation becomes increasingly amplified through second-order effects $P - \Delta$, resulting in instability-sensitive behaviour[24].

For piles embedded in soil, However, the instability response differ from the classical Euler theory due to the surrounding soil support. The surrounding soil provides distributed lateral and vertical support along the pile shaft, thereby reducing the effective buckling length l_{cr} and modifying the deformation mode[21]. Consequently, the instability response of piles in soft clay cannot be fully described using classical Euler theory alone and requires explicit consideration of soil-structure interaction effects SSI .

2.6.2 Second-order effects and initial imperfections

The stability assessment of slender piles requires consideration of geometric non-linearity, since the structural response is governed by the interaction between the

applied axial compressive load and the resulting lateral deformation[23]. This interaction is commonly referred to as second-order behaviour, where lateral deflections generate additional bending moments through the $P - \Delta$ effect, thereby amplifying the instability response[21, 17]. Consequently, the load-carrying capacity may become significantly lower than that predicted by linear elastic analysis. [25, 23].

Initial imperfections are commonly idealised using a sinusoidal shape along the pile length [16]:

$$\delta(x) = \delta_0 \sin\left(\frac{\pi x}{l}\right) \quad (2.8)$$

Where δ_0 is the maximum initial lateral deviation, x is the coordinate along the pile length, and l is the pile length. Although a single half-wave sinusoidal imperfection is commonly adopted in classical stability theory[23, 17], piles embedded in soil may exhibit more complex deformation shapes depending on the stiffness distribution and boundary conditions[26]. For this reason, several imperfection mode shapes are evaluated in the numerical analyses presented in Chapter 3.

When axial compressive loading P is applied to an imperfect pile, the initial lateral deviation introduces an eccentricity that generates bending moments even at relatively small load levels. The induced bending moment can be expressed as [16]:

$$M = Pe \quad (2.9)$$

where e is the total eccentricity, including both the initial imperfection and the additional lateral displacement that develops during loading.

As the axial load increases, the induced bending moment causes an additional lateral deflection. This interaction between axial force and lateral deformation is referred to as second-order behaviour or the $(P - \Delta)$ effect and may significantly amplify the instability response of slender piles. The total lateral deflection can be approximated as[16]:

$$y(x) = y_0 \sin\left(\frac{\pi x}{l}\right) \quad (2.10)$$

Where y_0 is the maximum lateral deflection including both the initial imperfection and the additional deflection due to $P - \Delta$ effect.

The initial imperfection shape ($\delta(x)$) and the deformed shape ($y(x)$) illustrates how geometric imperfections influence the response of a compressed pile. As the axial load approaches the critical buckling load (P_{cr}), the amplification of lateral displacements becomes increasingly pronounced, resulting in larger bending moments and increased geometric non-linearity. Consequently, even relatively small initial imperfections may lead to significant lateral deformations and a corresponding reduction in the load-carrying capacity of the pile [23, 26, 25].

The influence of second-order effects is strongly dependent on the slenderness of the pile[17]. For piles with low slenderness ratios, the structural response is primarily governed by material strength and the influence of imperfections remains limited. However, as the slenderness ratio increases, the pile becomes progressively more sensitive to lateral deformation and instability effects. Highly slender piles may therefore experience elastic buckling at stress levels significantly below the material yield strength[23, 21].

2.7 SOIL–STRUCTURE INTERACTION AND WINKLER FOUNDATION BEHAVIOUR

The surrounding soil restraint is commonly idealised using beam-on-elastic-foundation theory, in which the pile is represented as an Euler–Bernoulli beam resting on a continuous elastic medium. In the Winkler idealisation, the soil is discretised into independent springs acting along the pile shaft without reacting with adjacent springs. The soil reaction in this case is assumed to be proportional to the local pile displacement[27]. This approach provides a simplified analytical framework for describing the interaction between pile bending stiffness and the lateral restraint provided by the surrounding soil:

$$q(x) = k_h y(x) \quad (2.11)$$

where $q(x)$ is the lateral soil reaction per unit length [kPa/m], k_h is the lateral subgrade reaction coefficient [kPa/m^2], and $y(x)$ is the lateral displacement of the pile [m]. Despite its simplifications, the model is widely used for pile instability analyses due to its computational efficiency and ability to represent distributed soil restraint behaviour[21, 23, 27].

By combining the Euler–Bernoulli beam theory with the Winkler foundation assumption, the governing differential equation for an axially loaded pile embedded in soil may be expressed as[25]:

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + k_h y = 0 \quad (2.12)$$

where EI is the pile flexural rigidity, P is the applied compressive load, and k_h represents the distributed lateral soil restraint. The equation illustrates that the instability response is governed by the combined interaction between structural stiffness, axial loading, and soil confinement. Although the present study relies primarily on numerical finite-element analyses, this analytical formulation provides the theoretical basis for understanding how the surrounding soil modifies the classical Euler buckling behaviour[23, 25].

The beam-on-elastic-foundation formulation may further be expressed through the classical moment–curvature relationship and equilibrium equations:

$$M = -EI \frac{d^2 y}{dx^2} \quad (2.13)$$

These relationships form the basis of many analytical solutions for laterally supported beams and piles and provide insight into how soil restraint influences the deformation response[25].

Combining the moment-curvature relationship with the beam equilibrium equations yields the classical fourth-order beam equation[25]:

$$q(x) = -EI \frac{d^4 y}{dx^4} \quad (2.14)$$

which relates the distributed loading to the pile deflection and forms the basis of beam-on-elastic-foundation theory[23].

In contrast to classical Euler buckling, where instability develops over the entire member length, the presence of distributed soil restraint modifies the deformation mode. The surrounding soil provides a continuous restoring force that suppresses large global deflections and promotes shorter deformation wavelengths. Consequently, the instability behaviour of piles embedded in soil becomes increasingly localised, with buckling developing within limited regions of the pile shaft rather than over the entire pile length[27, 21]

2.8 CROSS-SECTIONAL PROPERTY

The behaviour of slender piles subjected to axial compression is controlled by an interaction between geometric instability failure (buckling failure) and material nonlinearity (material yielding plasticity). Unlike the classical Euler behaviour, piles exhibit initial geometrical imperfections and interact with the surrounding soil, which provides lateral soil support. As it was mentioned earlier, These initial imperfections induce bending moments as soon as an axial load is applied[16].

As the load increases, the interaction between axial force and lateral deformation gives rise to second-order effects. These effects lead to an amplification of lateral displacements and a corresponding increase in bending moments, resulting in a progressive reduction in the structural stiffness. This interaction between deformation and internal forces is a key factor in determining the governing failure mechanism [21].

The stress distribution within the pile section consists of contributions from both axial compression and bending, where the relative magnitude of these components governs whether the response is controlled by material yielding or geometric instability:

$$\sigma = \frac{N}{A} \pm \frac{M}{W} \quad (2.15)$$

Where A is the cross-sectional area and W is the section modulus.

In soft clay deposits, the limited lateral confinement provided by the surrounding soil reduces the resistance against lateral pile movements. As a result, slender piles may exhibit increased sensitivity to geometric imperfections and second-order effects, causing instability to develop at lower load levels than would be expected in stiffer soil conditions. As lateral deformations increase, the bending stress component becomes increasingly significant, and the structural response becomes governed by instability effects rather than by material yielding alone.

In this regime, the critical buckling load may be reached before the maximum stress in the steel section attains the yield strength. Consequently, failure is governed by geometric instability rather than material yielding [28].

As soil stiffness increases, the lateral confinement becomes more effective, reducing lateral deformation and limiting bending effects. This allows higher axial loads to be carried before instability develops. Consequently, the stresses in the steel section may continue to increase until the material yield strength is reached. When the maximum stress approaches the yield stress $\sigma \simeq f_y$, material yielding becomes the governing failure mechanism [21].

2.8.1 Analytical interpretation of soil-structure interaction

For piles embedded in soil, The critical buckling load is is governed by both the structural stiffness of the pile and the lateral support provided by the surrounding soil. This interaction can be explained using the beam-on-elastic-foundation theory, where the soil is represented by a continuous elastic medium[29].

An approximate analytical expression for the buckling load corresponding to a given deformation mode may be obtained by combining the classical Euler buckling

resistance with the stabilising contribution from the surrounding soil confinement. Assuming a sinusoidal buckling mode with n half-waves along the pile length, the buckling load may be expressed as[30] :

$$P_{cr} = \frac{n^2 \pi^2 EI}{(\beta L)^2} + \frac{k_h (\beta L)^2}{n^2 \pi^2} \quad (2.16)$$

where n is the buckling mode number. This expression illustrates that the buckling resistance depends on both the flexural stiffness of the pile EI and the magnitude of the soil stiffness k_h , the pile length L , and the assumed deformation wavelength. The critical buckling load corresponds to the minimum value obtained among the possible buckling modes.

For piles embedded in Winkler foundation with constant horizontal stiffness k_h , the buckling behaviour differs from classical Euler buckling. As the pile length increases, the deformation tends to localize within a limited region rather than extending over the entire length. This leads to the concept of a characteristic buckling length, which defines the active zone of deformation. The characteristic buckling length is defined as the length of the active buckling zone and is expressed as[16]:

$$l_{cr} = \pi \left(\frac{EI}{k_h} \right)^{(1/4)} \quad (2.17)$$

For sufficiently long piles embedded in linear-elastic Winkler foundation with constant horizontal stiffness k_h , the critical buckling load becomes progressively independent of the total pile length and is instead governed primarily by the interaction between pile stiffness and surrounding soil confinement[16]:

$$P_{cr} = 2(k_h EI)^{(1/2)} \quad (2.18)$$

This expression shows that once the pile length exceeds a certain threshold, the buckling response will be confined to a limited deformation region, and further increase in the length would marginally influence the critical buckling resistance[26, 23].

It should be noted that equations ?? are derived under the assumptions of linear-elastic behaviour, constant soil stiffness. Real piles embedded in soft clay often experience depth-dependent soil stiffness, geometric imperfections, and geometric non-linearity, which may lead to deviations from the idealised analytical solution.

2.8.2 Subgrade reaction coefficient and lateral soil stiffness

The Subgrade reaction coefficient k_h represented in equations ?? is a coefficient that corresponds to the lateral stiffness provided by the surrounding soil, and therefore plays a central role in the instability behaviour of piles. A higher value of k_h corresponds to a stiffer soil, resulting in greater lateral confinement and increased resistance to buckling[21].

The Swedish design approach accounts for the soil stiffness by relating the horizontal subgrade reaction coefficient k_h to the undrained shear strength c_u through

empirical correlations. Consequently, increases in undrained shear strength generally result in higher lateral soil stiffness and greater resistance against lateral pile deformation:

$$k_h = (50 - 200)c_u \quad (2.19)$$

where the selected coefficient depends on the adopted design approach, soil conditions, and level of conservatism.

Although the Winkler model assumes a linear relationship between the soil reaction and the lateral displacement as it is indicated in equation 2.11, the soil behaviour is inherently non-linear. The soil resistance increases with displacement but approaches a limiting value governed by the shear strength[15]:

$$q_{lim} = N_p c_u D \quad (2.20)$$

Where N_p is the bearing capacity coefficient that is typically ranging between long term 6 and short term 9, and D is the pile diameter.

As lateral displacement increases, progressively larger portions of the available soil resistance are mobilised. However, as the soil reaction approaches its limiting value q_{lim} , additional displacement results in progressively smaller increases in resistance. Consequently, the tangent stiffness decreases with increasing deformation. This behaviour is commonly represented using non-linear $p-y$ curves, which provide a more realistic description of soil-structure interaction than the linear Winkler model[31, 18].

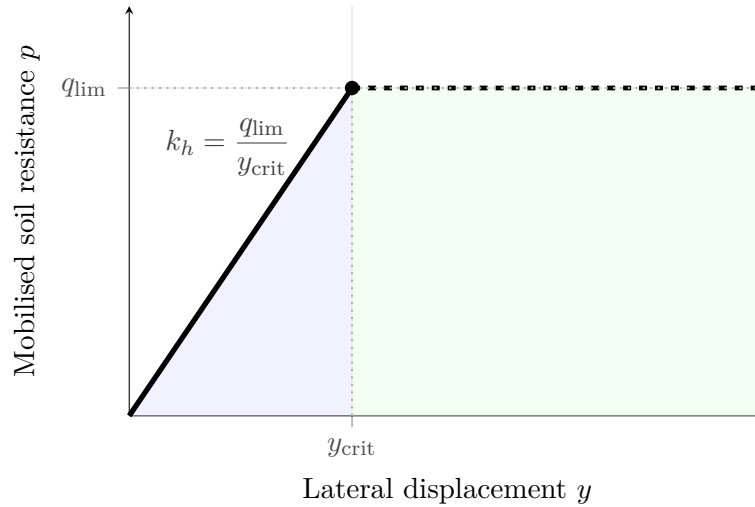


Figure 2.3: Idealised bilinear $p-y$ response for lateral soil resistance. .

In practical design, this non-linear behaviour is often simplified by introducing an equivalent or reduced subgrade reaction coefficient. The Swedish design approach accounts for this by assuming a uniform redistribution of soil resistance along the pile- constant k_h - that represents the overall response [32].

The combination of equation 2.1 and the expression 2.19 shows how the soil stiffness is depth dependent. There the Swedish design approach expresses the horizontal subgrade reaction coefficient k_h as [16]:

$$k_h = (50 - 200)c_u = (50 - 200)(c_{u0} + n * z) \quad (2.21)$$

Where c_{0u} is the undrained shear strength near the ground surface [kPa], and z is the depth coordinate [m].

The influence of depth-dependent soil stiffness forms an important part of the numerical modelling approach adopted in the present thesis, where both uniform and depth-dependent soil stiffness distributions are investigated in order to evaluate their influence on instability behaviour and post-buckling response.

2.9 HÉTÉNYI λL CLASSIFICATION FOR BEAMS ON ELASTIC FOUNDATION

The stability behaviour of beams resting on an elastic Winkler foundation may be characterised using the Hétényi dimensionless parameter λL , which reflects the interaction between the beam flexural rigidity EI and a constant value of the lateral soil stiffness k_h [27]. This classification is used to identify the threshold length at which the pile can be considered a long pile, where the surrounding soil confinement is effectively mobilised and further increase in pile length has limited influence on the critical buckling load. Hétényi defines the foundation parameter λ [m^{-1}] as:

$$\lambda = \left(\frac{k_h}{4EI} \right)^{1/4} \quad (2.22)$$

Where k_h represents the lateral soil stiffness and EI is the beam flexural rigidity.

The dimensionless parameter λL is commonly used to distinguish short-, intermediate-, and long-beam instability responses.

Short-beam regime

$$\lambda L < \frac{\pi}{4}$$

For relatively small values of (λL) , the surrounding soil provides only limited lateral restraint, and the beam response resembles the behaviour of a conventional Euler column. Under these conditions, the critical load is governed primarily by the beam length and the applied boundary conditions. The instability load may therefore be estimated using the classical Euler formulation presented in Equation 2.7.

Intermediate-beam regime

$$\frac{\pi}{4} < \lambda L < \pi$$

Within the intermediate range, both the structural stiffness of the beam and the confining effect of the surrounding soil influence the instability behaviour. The critical load can therefore be interpreted as a combined contribution from classical Euler buckling and the additional restraint provided by the elastic foundation, as described in Equation 2.16. As (λL) increases, the surrounding soil progressively contributes more significantly to the lateral confinement, leading to a gradual transition from structurally controlled behaviour toward foundation-controlled behaviour.

Long-beam regime

$$\lambda L > \pi$$

For large values of (λL) , the instability mode becomes increasingly localized and develops within a finite active buckling zone. In this regime, the overall beam length no longer has a major influence on the instability load. Instead, the critical

load approaches the asymptotic Hétényi solution given in Equation 2.18, where the response is governed mainly by the interaction between beam stiffness and soil confinement. The deformation pattern becomes concentrated over a characteristic buckling length 2.17, and further increases in beam length produce only marginal changes in the critical instability load.

2.10 GEOTECHNICAL CAPACITY

Pile bearing capacity is primarily governed by the geotechnical resistance mobilised in the surrounding soil. It denotes the ability of the pile to transfer the applied load to the soil through shaft friction and, where relevant, base resistance [9, 7]. It should be noted that excessive pile deformation may also lead to structural failure. Large lateral deflections can increase bending moments through second-order effects. If the resulting stresses exceed the material resistance of the pile section, yielding or local failure may occur. However, plasticity of the pile material is not the main focus of this study. Instead, the present work focuses on the surrounding soil properties and how SSI influences the load-bearing capacity of slender piles. Consequently, the linear buckling result should be interpreted as an idealised critical load for studying instability behaviour rather than the actual ultimate failure load [16, 23, 27]

Based on this, the failure load is evaluated with emphasis on the instability response of the pile. The linear buckling analysis provides the critical buckling load and the corresponding instability mode shape. These results are used as a reference for understanding how soil stiffness, shaft resistance, imperfection, and boundary conditions affect the pile response.

In Sweden, piles are often installed in soft clay, where low undrained shear strength, high compressibility, and time-dependent behaviour strongly affect settlement and bearing capacity. Therefore, pile capacity is commonly assessed using the undrained shear strength, $c_u(z)$, and the shaft–soil interaction, often described by the α -method [19, 5, 7].

In some cases, particularly where drained conditions govern the soil response, the geotechnical bearing capacity may instead be evaluated using effective stress parameters. This may be relevant for long-term conditions, overconsolidated soils, or soil layers where drainage can occur during the service life. However, for driven piles in soft clay, the undrained condition is usually governing, and the α -method provides a suitable approach [7]. The design value of the geotechnical bearing capacity is determined by the expression:

$$R_{cal} = \int_0^L \alpha(z) c_u(z) u \, dz + N_c c_{u,b} A_b \quad (2.23)$$

$$(2.24)$$

Where the first term represents the shaft resistance integrated along the embedded pile length, and the second term represents the base resistance. The characteristic resistance is then obtained as:

$$R_k = \min \left(\frac{R_{cal,mean}}{\gamma_{Rd} \xi_3}, \frac{R_{cal,min}}{\gamma_{Rd} \xi_4} \right) \quad (2.25)$$

$$(2.26)$$

Finally, the design value of the geotechnical bearing capacity is determined as:

$$R_d = \frac{R_k}{\gamma_t} \quad (2.27)$$

These expressions are adopted in accordance with *TRVINFRA* [33] and the Swedish national application of Eurocode 7[12]. For cohesion piles, undrained geotechnical resistance is evaluated using the α -method in accordance with *PKR 100*[7]

The geotechnical capacity expressions define the limiting resistance that can be mobilised by the pile. However, they do not describe how this resistance develops with displacement. For the numerical modelling adopted in this study, the shaft resistance must therefore be formulated as an axial load transfer relationship. This is achieved by combining the limiting resistance from α -method with a t - z formulation. Thus, the mobilised shaft resistance is related to the relative displacement between the pile and the surrounding soil. The initial part of this relationship is then represented by an equivalent vertical spring stiffness k_v which is used to model the axial soil structure interaction along the pile shaft.

2.10.1 Shaft resistance

From a mechanical perspective, shaft resistance describes how the applied load is transferred from the pile to the surrounding soil, thereby governing the axial load distribution along the pile. As shaft resistance is mobilized, the compressive load decreases with depth. Consequently, the mobilization of shaft resistance is not instantaneous but rather progressive, and it depends on the relative displacement between the pile and the surrounding soil [7]. For very slender piles installed in soft cohesive soils, insufficient shaft resistance may result in a large compressive force acting over extended unsupported lengths. Consequently, this can increase the risk of structural instability depending on the degree of lateral confinement[34].

The α -method describes the mobilised shaft resistance as [9]:

$$\tau = \alpha c_u \quad (2.28)$$

where:

$$\begin{aligned} \tau &= \text{The unite shaft resistance} \\ c_u &= \text{Undrained shear strength of the soil} \end{aligned}$$

2.10.2 Axial soil structure interaction and vertical spring stiffness, k_v

The interaction between the pile and the surrounding soil will be modelled as a one-dimensional axial load-transfer problem. In this framework, the α -method is used to evaluate the available undrained shaft resistance of the cohesion pile. This resistance defines the maximum shaft resistance that can be mobilised along the pile shaft[7].

In order to describe the axial response of the pile, the limiting shaft resistance is combined with a classical t - z formulation. Thus, the local shaft resistance is

related to the relative axial displacement between the pile and the surrounding soil. The α -method therefore defines the ultimate resistance level, while the t - z relationship describes how this resistance is gradually mobilised. In this formulation, t represents the mobilised shaft resistance, while z_{rel} represent the relative displacement between the pile and the surrounding soil [2.29].

This approach makes it possible to derive an equivalent vertical spring stiffness k_v for the pile-soil system under axial loading. The vertical spring stiffness is then used in the numerical model to represent the progressive mobilisation of shaft resistance along the pile shaft [20, 18].

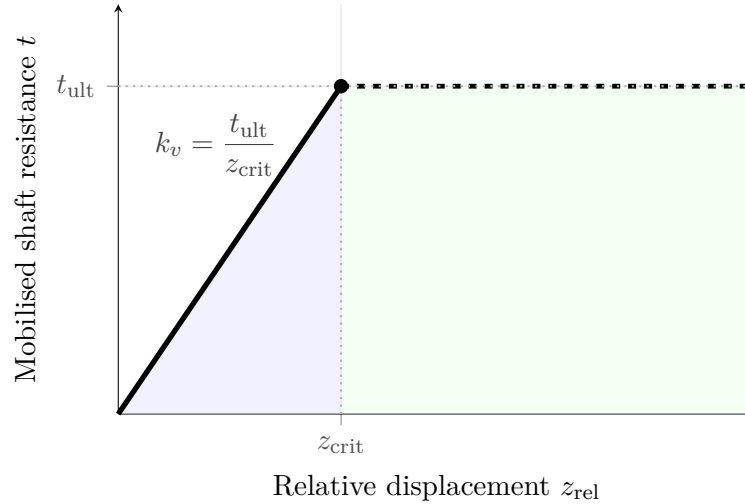


Figure 2.4: Idealised bilinear t - z response for shaft resistance.

2.10.3 Kinematics and neutral plane

The axial pile-soil interaction is governed by load transfer along the pile shaft. Shaft resistance is not mobilised instantly, instead, it is developed progressively with the relative displacement between the pile and the surrounding soil.

This behaviour forms the basis of classical load-transfer theory, represented by t - z and q - z curves. Both relationships describe how the mobilised resistance depends on the local relative axial displacement at a given depth [18]. That can be described as:

$$z_{rel}(z) = w_p(z) - w_s(z). \quad (2.29)$$

where:

$$z = \text{Depth measured downward from the pile head} \quad (2.30)$$

$$z_{rel} = \text{Relative displacement between soil-, and pile displacement} \quad (2.31)$$

$$w_p = \text{The axial settlement of the pile} \quad (2.32)$$

$$w_s = \text{The settlement of the surrounding soil} \quad (2.33)$$

Thus, the neutral plane depth z_{NP} is defined by the depth at which the sign of shaft shear reverses, where the relative displacement between the pile and the soil

is equal to zero [4]. At this depth, the pile and soil move by the same range:

$$z_{\text{rel}}(z_{NP}) = 0 \quad \Leftrightarrow \quad w_p(z_{NP}) = w_s(z_{NP}), \quad (2.34)$$

At the neutral plane, the direction of shaft shear changes. Above this depth, negative skin friction act downward on the pile, while below it, positive shaft resistance may act upward [7]. The maximum axial load in the pile usually occurs at the neutral plane, and can be expressed as:

$$Q_{NP} = Q_d + Q_n, \quad (2.35)$$

where:

Q_d = The sustained/dead load at the pile head

Q_n = The accumulated down-drag load

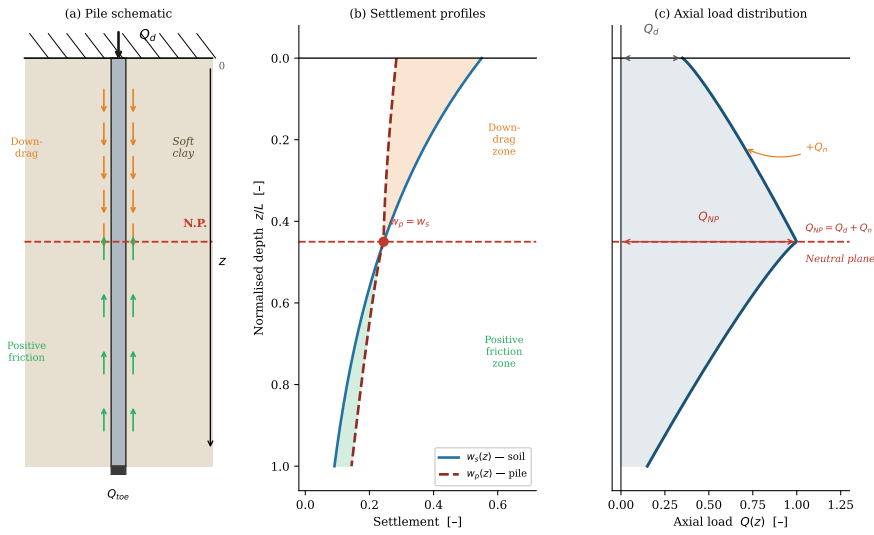


Figure 2.5: Neutral plane concept

2.10.4 Derivation of Vertical Spring Stiffness from Shear Modulus

The limiting unit shaft shear stress is expressed as the adhesion factor multiplied by the shear strength [7]. This means that the maximum shear stress that can be mobilised along the pile shaft depends on the soil strength and the pile soil adhesion factor, which align with the α -method.

$$\tau_{\text{lim}}(z) = \alpha c_u(z), \quad (2.36)$$

To convert this shaft shear stress into a shaft resistance per pile length, it is multiplied by the pile perimeter u . The result is the limiting shaft resistance per unit length

$$q_{\text{lim}}(z) = \tau_{\text{lim}}(z) u \quad [\text{kN/m}], \quad (2.37)$$

The fully mobilised shaft resistance over an embedded pile length L_p is then obtained by integrating the limiting shaft resistance over the total length.

$$R_s(z) = \int_{L_p} q_{lim}. \quad (2.38)$$

A t - z relationship is used to describe how the shaft resistance is mobilised with relative displacement between the pile and the surrounding soil. A simplified bilinear "elasto-plastic" relationship is adopted:

$$q(z) = \begin{cases} k_v(z) z_{rel}(z), & |z_{rel}| \leq z_{s,y}, \\ \pm q_{lim}(z), & |z_{rel}| > z_{s,y}, \end{cases} \quad (2.39)$$

Where z_{rel} is the relative displacement 2.29, and $z_{s,y}$ is the characteristic relative displacement required to mobilize shaft resistance. For piles in clay, this displacement is commonly small and depends on the pile type, installation method, and the soil condition [18]. Thus, the distributed vertical spring is defined as the initial slope of the mobilisation t - z curve 2.4 :

$$k_v(z) = \frac{q_{lim}(z)}{z_{s,y}} \quad [\text{kN/m}^2], \quad (2.40)$$

The k_v represent the axial spring per unite length used to model the progressive mobilisation of shear resistance along the pile shaft.

In this work, the shaft resistance and therefore the vertical spring stiffness is formulated in a way that is consistent with the soil stiffness representation used for the horizontal support. This approach simplifies the numerical modelling procedure and improves the overall workflow. For this reason, the vertical response is represented through the shear modulus G which provides a stiffness-based description of the soil resistance along the pile shaft. After discussion with the supervisor, this was considered a suitable approximation for representing the mobilisation of shaft resistance. The further development of this approach and its alignment with the representation of horizontal soil support is discussed in the following section.

2.10.5 Vertical spring stiffness from the shear modulus

In order to relate $k_v(z)$ to a continuum stiffness parameter, the undrained shear modulus $G(z)$ is adopted. In incremental form, the shear response may be written as

$$d\tau = G d\gamma, \quad (2.41)$$

which is consistent with the modulus-based stiffness description in Verruijt (2006, Chapter 13) [5].

The limiting shaft resistance does not, by itself, describe the initial stiffness governing the elastic mobilised resistance. To define this stiffness, the soil response before slip is idealised as an isotropic linear elastic continuum. From classical elasticity, the shear modulus G , young's modulus E and the Poisson's ration ν are elated by [5]:

$$G(z) = \frac{E(z)}{2(1 + \nu)} \quad (2.42)$$

Poisson's ratio can be taken as $\nu \approx 0.5$, which represents nearly incompressible behaviour. under this condition, the relationship reduces to

$$G(z) = \frac{E(z)}{3} \quad (2.43)$$

The Young's modulus may then be related to the shear strength through an empirical modulus number m according to.

$$E(z) = m c_u(z) \quad (2.44)$$

The Young's modulus is estimated from the undrained shear strength using the empirical relationships recommended in *TRVINFRA-00230*. For cohesive soils, TRVINFRA states that the undrained secant modulus E_{50} for stresses up to approximately half of the failure stress may be estimated from the soil type and the undrained shear strength. The recommended value is $m = 250$ for high plasticity and gyttja-bearing clay [33]

For simplicity, the vertical stiffness is represented using the shear modulus of the surrounding clay. This modelling assumption was adopted following discussion with Anders Jonefjäll, since it provides a practical link between the axial spring stiffness and the deformation properties of the soil.

$$k_v(z) \approx G(z) = \frac{250 c_u(z)}{3}. \quad (2.45)$$

3

Modelling

This chapter presents the modelling strategy performed for the numerical investigation of **slender** piles buckling behaviour. The focus is on evaluating the influence of soil–structure interaction through a parametric study, considering the key factors governing instability in soft soil conditions.

The input parameters are selected to represent typical Swedish design practices that are relevant to the geotechnical conditions in the Gothenburg city and the pile dimensions correspond to standard steel pile sections commonly used in Sweden.

The modelling framework is therefore defined to provide practically relevant insight into the stability behaviour of slender piles under representative field conditions.

3.1 OVERVIEW AND NUMERICAL APPROACH

Finite element analysis was employed to examine the buckling behaviour of **slender** piles embedded in soft soil deposits. The numerical approach was developed to quantify the influence of different key parameters, including boundary conditions, soil stiffness, geometric properties, and initial geometrical imperfections, on the structural response of piles. The modelling framework allows for a systematic assessment of both linear and geometrically non-linear behaviour, allowing the investigation of instability phenomena and the role of soil-structure interaction.

The analyses were performed using ABAQUS (version:CAE 2024.HF2) software, which was selected for its capability to accurately handle geometrically non-linear problems and its flexibility in incorporating user-defined modelling procedures. The software enables the implementation of the second-order effects and the influence of soil-structure interaction within the same numerical model.

The primary objective of the numerical approach is to develop a python-based automation script (version:Python 3.13.5) that efficiently performs the finite element workflow. The script enables automated model generation, execution of simulations, and the feasibility of extracting relevant results. This approach ensures the consistency across all analyses and allows systematic variation of key parameters, such as pile geometry, soil stiffness distribution, imperfection amplitudes, mode shapes, and boundary conditions. Additionally, The use of automation significantly reduces the risk of errors associated with manual modelling.

The numerical approach consists of a structured multi-phase approach. initially, linear eigenvalue buckling analyses are carried out on idealised perfect straight members with predefined geometric properties with and without a constant soil stiffness. These analyses are used to verify the numerical model through a comparison with analytical solutions in terms of critical buckling load and critical buckling length. 3.1.2. Additional analyses are performed to evaluate the influence of the vertical soil stiffness on the critical buckling length.

Following the verification phase, the automated script is extended to incorporate the full set of parameters requires for the parametric study. Geometric Non-linear analyses are then carried out using the arc-length (RIKS) method to capture the post-buckling behaviour and to determine the failure load. These analyses incorporate initial imperfections along the pile length and different mode shapes, allowing evaluation of their influence on load-carrying capacity.

The majority of the analyses are primarily based on geometrically non-linear formulations combined with linear elastic material behaviour. This modelling strategy isolates the effects of geometric instability and soil-structure interaction on the structural response. For selected cases, material non-linearity is performed to assess the influence of yielding on the predicted load-carrying capacity.

Figure 3.1 represents the numerical workflow through out the modelling procedure:

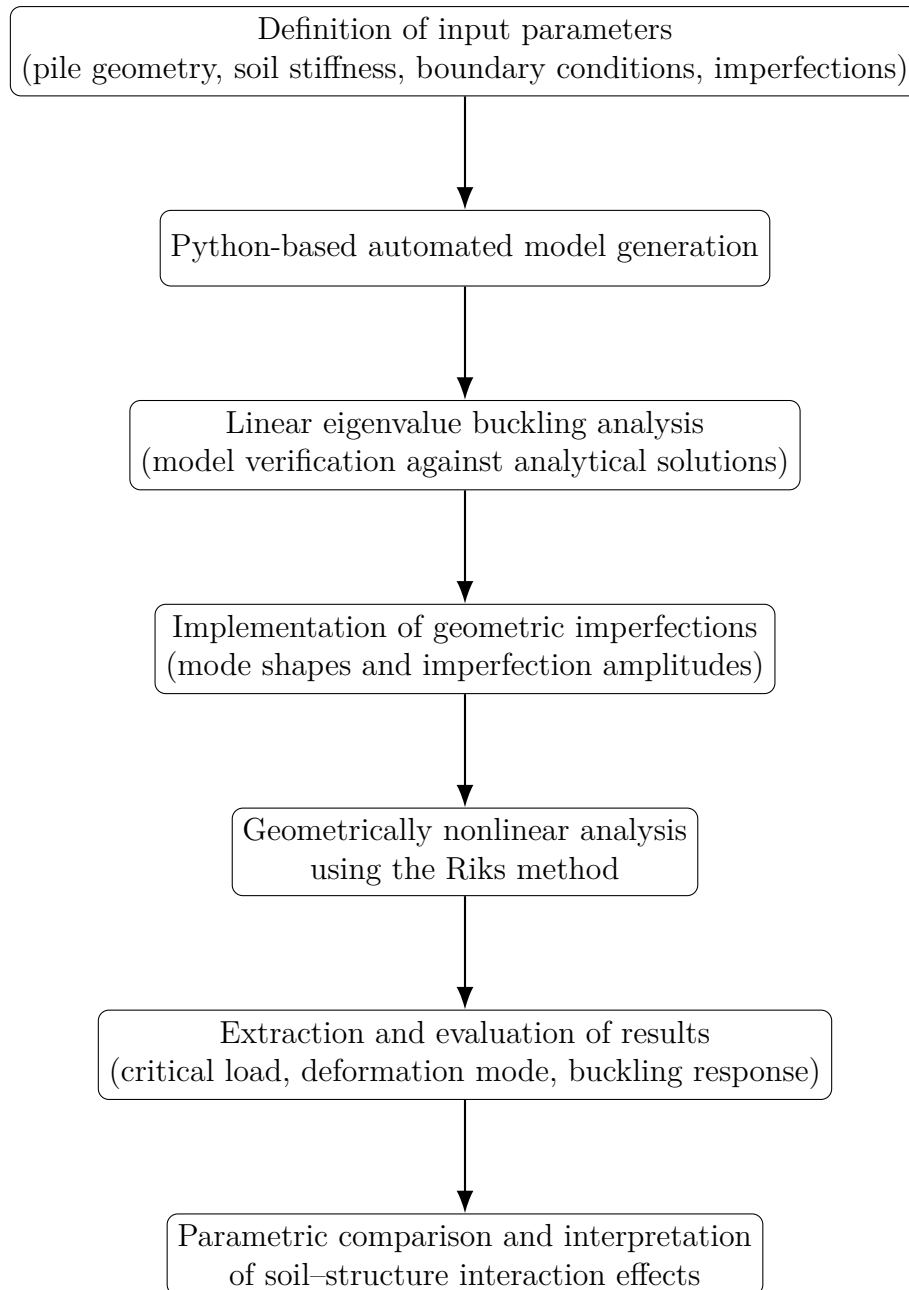


Figure 3.1: Overview of the numerical workflow adopted in the finite element analyses.

3.2 PILE MODELLING

The numerical model adopted in this study is based on a two-dimensional idealisation of the pile–soil system. The pile is represented as a planar wire beam supported by distributed horizontal and vertical Winkler springs along its length. This modelling strategy is suitable for the present study because the main interest is the global instability response of a slender pile, where the behaviour is governed primarily by axial compression, bending deformation, second-order effects, and soil–structure interaction.

The use of a two-dimensional wire beam model provides direct control over the main modelling parameters, including boundary conditions, initial geometric imperfections, pile stiffness, and the variation of soil stiffness with depth. It also allows the spring properties to be assigned along the pile in a transparent way. Compared with three-dimensional shell or solid modelling, this approach significantly reduces the number of degrees of freedom and the computational cost. This is important for the present work, since a large number of parameter combinations are analysed using both linear buckling analysis and geometrically nonlinear Riks analysis.

The pile is discretised using B21 beam elements in ABAQUS. In the Python model, the pile is defined as a two-dimensional planar deformable wire, and the assigned element type is B21. The B21 element is a two-node linear beam element for planar analysis. It is based on a Timoshenko beam formulation and therefore includes axial deformation, bending deformation, and transverse shear deformation within the same element formulation [22, 35]. This makes it appropriate for modelling the pile as a slender beam-column subjected to axial compression and lateral deformation.

This modelling choice is consistent with the objective of the study, which is to evaluate the influence of imperfections, boundary conditions, and soil spring stiffness on the instability load. Local effects such as shell ovalisation, local wall buckling, detailed pile–soil contact, and three-dimensional stress redistribution are not captured by this model. These effects are outside the scope of the present study, where the focus is instead placed on the global buckling and post-buckling behaviour of the pile.

3.3 SOIL SPRING IMPLEMENTATION

The interaction between the pile and the surrounding soil was modelled using distributed spring elements based on a Winkler-type foundation. In this approach, the continuous soil resistance is represented, instead, by discrete spring contributions acting along the pile length. Horizontal springs are used to represent the lateral confinement from the surrounding soil, while vertical springs are used to represent the axial pile–soil interaction along the shaft.

In the initial elastic range, the soil reaction at each spring location is proportional to the local pile displacement. For the non-linear analyses, this elastic response is limited by the adopted bilinear spring formulation, so that the soil resistance reaches a plastic plateau after the limiting displacement is exceeded. This makes it possible to model both the initial stiffness of the soil support and the reduction in effective restraint after soil yielding.

The adopted modelling strategy allows both constant and depth-dependent soil stiffness distributions to be implemented. For the depth-dependent cases, the horizontal stiffness is defined as a function of depth according to Eq. 2.19, while the vertical stiffness is defined according to Eq. 2.45. This provides a simplified but efficient representation of the soil confinement and shaft interaction governing the instability behaviour of slender piles embedded in soft clay.

Several analysis cases were considered in order to evaluate the influence of soil stiffness variation on the pile response. The horizontal springs mainly control the lateral restraint and the active buckling shape, while the vertical springs represent the axial shaft interaction and influence the overall instability response. The investigated cases include different combinations of horizontal and vertical stiffness distributions, as summarised in Table 3.1.

	Diameter 1	Diameter 2
Lengths	L_1, L_2, L_3	L_1, L_2, L_3
Case 1	kh = constant	kv = constant
Case 2	kh > 0	kv = 0
Case 3	kh > 0	kv > 0
Case 4	kh = 0	kv > 0

Table 3.1: Table of cases with different soil stiffness distributions.

3.4 MESHING AND LOADING

To obtain a suitable mesh size, a mesh convergence study was carried out. Several mesh sizes were tested successively, and the convergence of the results was evaluated. The pile model used in this study was relatively simple, which allowed accurate results to be obtained without using an excessively fine mesh. This reduced the computational cost while maintaining sufficient numerical accuracy. Based on the convergence study, a mesh size of 0.1 m was considered adequate for all studied pile lengths and diameters.

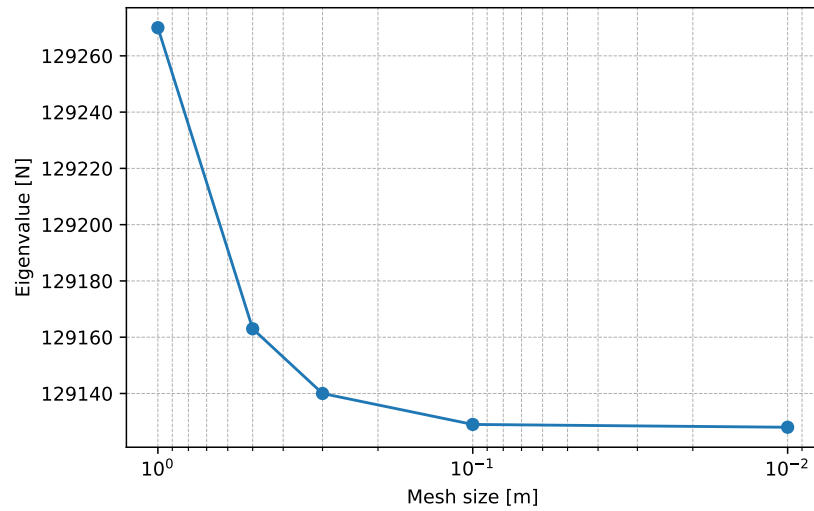


Figure 3.2: Mesh convergence curve for Pipe 220 showing the eigenvalue as a function of mesh size.

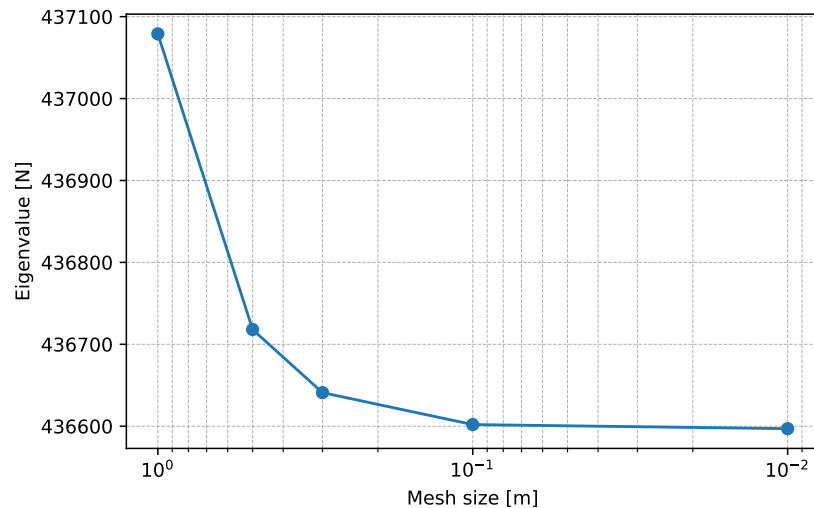


Figure 3.3: Mesh convergence curve for Pipe 321 showing the eigenvalue as a function of mesh size.

3.5 BOUNDARY CONDITIONS

The applied boundary conditions considered in the numerical analyses include fixed-fixed, pinned-pinned, pinned-fixed, and fixed-pinned configurations A.5 for all the cases represented in table 3.1. These boundary conditions were chosen to investigate how different levels of rotational restraint influence the buckling behaviour of piles.

In practical pile foundations, the boundary conditions at the pile toe are more complex than the idealised classical support conditions commonly assumed in structural analysis. The pile toe is not restrained by a discrete structural support, but interacts continuously with the surrounding soil. Soil confinement, shaft resistance, and the mobilisation of soil stiffness along the embedded length all contribute to restraining lateral deformation and rotational movement.

As a result, the behaviour of real piles generally falls between the idealised pinned and fixed support conditions, rather than corresponding to either extreme case. Considering these limiting boundary conditions in the numerical analyses therefore provides a useful framework for evaluating how rotational restraint and soil–structure interaction influence the resulting buckling mode shape and ultimate load-carrying capacity.

3.6 INITIAL IMPERFECTIONS

Initial geometric imperfections may originate from several sources, including manufacturing tolerances, handling, transportation, and installation effects. In practical pile design, the selected imperfection value may therefore depend on the pile type, fabrication quality, installation method, and the relevant design recommendations in Eurocode, TRVINFRA, and Pålkommissionen.

However, the aim of this study is not to define the most probable source or the exact design value of the imperfection. Instead, the focus is to evaluate how the magnitude of the initial imperfection influences the failure load of the pile. For this reason, a range of imperfection amplitudes is considered, from very small near-perfect values to larger conservative values based on the references discussed in Section 1.

The investigated range is defined from $L_{cr}/1000$ to $L_{cr}/150$. Smaller increments are used where the response is expected to change more gradually. The values are varied in steps of 100 from $L_{cr}/1000$ to $L_{cr}/600$, and then in steps of 50 up to $L_{cr}/150$. Minor adjustments to the intermediate values are made where necessary to improve the analysis and provide a clearer interpretation of the results.

3.7 GEOMETRICALLY NON-LINEAR ANALYSIS USING RIKS METHOD

The geometrically non-linear post-buckling behaviour of the pile–soil system was analysed using the Static Riks (arc-length) method available in ABAQUS. This analysis was required to capture the non-linear instability response beyond the elastic critical state and to determine the actual failure load of the imperfect pile.

the linear eigenvalue buckling analysis (LBA) provides an elastic estimation of the critical buckling load P_k , together with the corresponding buckling mode shapes. Since the LBA formulation is based on a perfectly straight geometry and linear structural behaviour, it does not account for the geometric non-linearities, second-order (P^Δ) effects, or the influence of initial imperfections. Moreover, the LBA cannot represent the progressive stiffness degradation that develops after buckling initiation.

Therefore, the RIKS method was employed to obtain a more realistic representation of the instability behaviour and to evaluate the influence of **imperfections and soil-structure interaction** on the load-carrying capacity.

The non-linear analyses were performed with geometric non-linearity activated (NLGEOM=on) to capture large displacement effects and second-order behaviour. The same boundary conditions adopted in the linear analyses (LBA) were maintained in the non-linear analyses, allowing a consistent comparison between the elastic eigenvalue predictions and the geometrically non-linear responses.

Geometric imperfections were introduced prior the RIKS step using the buckling mode shapes obtained in LBA analyses. For the selected eigenmode, several imperfection amplitudes were considered in order to assess imperfection sensitivity. The imperfection amplitudes were implemented directly within the automation script and applied along the entire pile length before initiating the non-linear analysis step.

In the non-linear analyses, the distributed springs were assigned the same initial stiffness used during the LBA analyses. This maintains consistency between the modelling stages.

To obtain more realistic representation of the soil response at large lateral displacements, non-linear spring behaviour with an elastic-perfectly plastic formulation was introduced. The spring response therefore consisted of an initial linear elastic region followed by a yielding plateau corresponding to the limiting soil resistance. This approach allows progressive mobilisation of soil resistance while preventing unrealistically large confining forces during the post-buckling response.

In this step, Steel properties were assumed linear elastic except for one representative verification case where an elastic-plastic model was used to assess the influence of yielding on the post-buckling response.

4

Input data

The input data used in this thesis is primarily obtained from technical documentation provided by SSAB[36] and represents standard Swedish steel pipe pile dimensions commonly used in foundation engineering practice. The selected pile geometries include the cross-sectional properties and dimensions required for the analytical and numerical investigations presented in this study.

The SSAB data serves as the basis for defining the structural properties of the piles, including the outer diameter, wall thickness, cross-sectional area, moment of inertia, and flexural rigidity. These parameters are subsequently used in the stability analyses, finite-element models, and soil–structure interaction assessments performed throughout the thesis.

The parameters representing the undrained shear strength at the clay surface, c_{u0} , and the strength gradient with depth, n_h , were calibrated based on geotechnical investigations conducted for the Västlänken and Haga Station projects, reported in 2016 and 2022, respectively[8, 1].

5

Results And Discussion

5.1 LINEAR ELASTIC ANALYSES

This section presents the linear elastic FEM analyses performed in this thesis, where the effects of both horizontal k_h and vertical k_v Winkler-type springs that are assigned along the pile length were examined separately and then in combination in order to distinguish their separate influence on the instability response of the pile. Particular focus was placed on the influence of soil–structure interaction, pile flexural rigidity, BC-configurations, and soil stiffness distribution on the critical buckling load, instability deformation mode, and effective buckling length.

A comparison between the Hétényi dimensionless parameter λL and the FEM analyses is further presented and interpreted.

Lastly, the relationship between the effective buckling length and the pile slenderness ratio is evaluated and interpreted for both uniform and depth-dependent soil profiles.

5.1.1 Influence of depth-dependent soil stiffness variation on the critical buckling load P_{cr}

The following figure presents the influence of BC-configurations and depth-dependent soil gradient ($n_h = 1.1$ and $n_h = 3$ kPa/m) on the critical buckling load P_{cr} that is a function of pile length range $2 < L < 40$ m, with length step $\Delta L = 2$ m. The analyses were performed using geometrically linear eigenvalue buckling analyses with linear elastic lateral Winkler springs activated ($k_h > 0$).

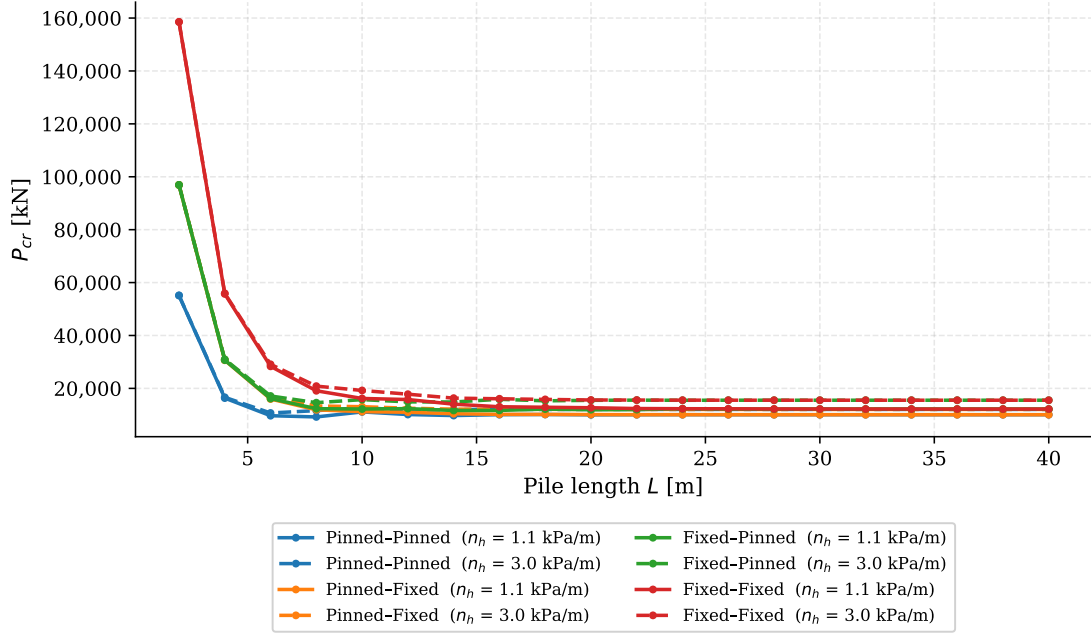


Figure 5.1: Effect of Boundary condition and soil gradient

Figure 5.1 shows that the critical buckling load decreases significantly with increasing pile length for all BC-configurations. For relatively short piles, the instability response is strongly governed by the applied boundary conditions, resulting in noticeable differences in the critical buckling load P_{cr} values. This behaviour is characteristic of classical Euler buckling, where the instability response is highly dependent on the effective buckling length and end restraints (KL). The fixed-fixed configuration develops the highest buckling resistance due to the increased rotational restraint at both pile ends, while the pinned-pinned configuration exhibits the lowest critical loads because of its reduced rotational restraint. The fixed-pinned and pinned-fixed configurations exhibit overlapping curves, indicating that the rotational restraint at either pile end provides an equivalent influence on the instability response within this pile length range.

As the pile length increases, the surrounding lateral soil confinement becomes progressively mobilised over a larger portion of the pile shaft, causing the soil contribution to increasingly govern the instability response. Consequently, the critical buckling load decreases for all BC-configurations until the curves gradually converge towards approximately constant buckling loads. This behaviour indicates a transition from classical Euler buckling, where the instability response is primarily governed by the effective buckling length and end restraints, towards soil-controlled instability behaviour dominated by soil-structure interaction. Under these conditions, the instability deformation progressively starts to localise within a limited active buckling region, reducing the influence of both the total pile length and the BC-configurations on the critical buckling resistance. This transition may be interpreted using the analytical expression 2.16, where both the classical Euler buckling contribution and the stabilising influence of the surrounding lateral soil support are represented.

For sufficiently long piles, all BC-configurations converge towards nearly constant

critical buckling loads, indicating that the instability response becomes almost independent of the BC-configurations and total pile length. Under these conditions, the surrounding soil confinement governs the instability behaviour by restraining large portions of the pile shaft and limiting the instability deformation to a localised active buckling region (l_{cr})^{2.17}. Consequently, the instability response no longer behaves as a global Euler buckling mode extending over the entire pile length, but instead develops as a soil-supported localised instability mode primarily controlled by the interaction between the pile flexural rigidity EI and the surrounding lateral soil support k_h .

Figure 5.1 further shows that increasing the soil stiffness gradient n_h increases the critical buckling load mainly within the transition and long-pile regions, where soil-structure interaction progressively governs the instability behaviour. This demonstrates that stronger depth-dependent soil confinement provides additional lateral restraint against instability deformation and therefore enhances the overall buckling resistance. The figure also indicates that the soil stiffness gradient n_h has a negligible influence on the critical buckling load within the short-pile region, as the curves corresponding to both soil gradients largely overlap. This behaviour confirms that the instability response within this region is primarily governed by classical Euler buckling and boundary-condition effects rather than soil-structure interaction.

Within the long-pile region, the curves corresponding to the fixed-head configurations overlap and develop higher critical buckling loads compared with the pinned-head configurations. This behaviour illustrates that, for sufficiently long piles, the influence of the toe boundary condition gradually diminishes, while the rotational restraint at the pile head remains the governing boundary-condition parameter affecting the instability response.

5.1.2 Characteristic buckling length and deformation localization

Figure 5.2 illustrates how different distributions of soil stiffness ($n_h = 0 \text{ kPa/m}$, $n_h = 1.1 \text{ kPa/m}$, and $n_h = 3 \text{ kPa/m}$) influence the mode shape and the critical buckling length for a pile length $L = 24 \text{ m}$ and a surface undrained shear strength $c_{0u} = 12 \text{ kPa}$. The pile length chosen belongs to the long-pile regime as it is presented in figure 5.1.

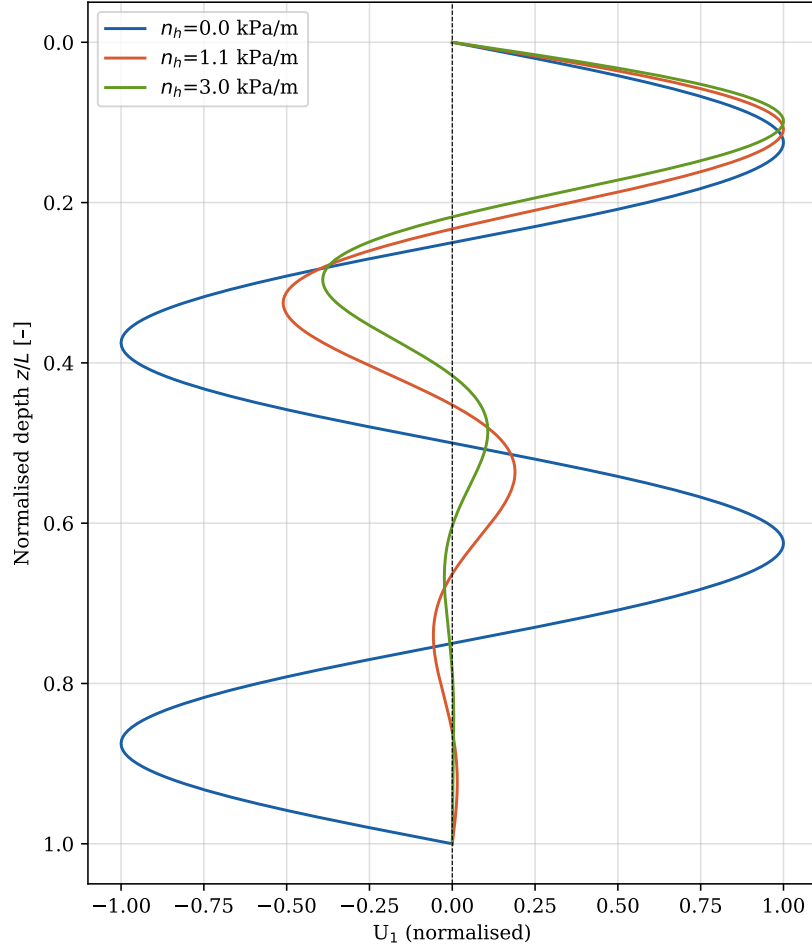


Figure 5.2: Effect of the soil support on buckling behaviour

For a constant soil stiffness along the pile length ($n_h = 0 \text{ kPa/m}$), the deformation shape extends over a relatively large portion of the pile length, leading to a broader and symmetric mode shape along the entire length. Since the surrounding soil stiffness is constant with depth, the pile shaft is uniformly confined by the surrounding soil, allowing the instability deformation to develop over a larger active region.

As the soil stiffness gradients increase, the mode shape becomes progressively more localised towards the upper part of the pile, where the maximum lateral displacement becomes increasingly suppressed. As expression 2.1 stated, this behaviour occurs because the surrounding soil stiffness increases with depth, leading to a stronger lateral confinement in the deeper soil layers and limiting the lateral movement in the lower embedded region.

The trend observed in figure 5.2 shows that increasing the soil stiffness with depth not only increases the critical buckling load, but also shortens the critical buckling wavelength and concentrates the instability deformation within a shorter active zone. This result is consistent with equation 5.1.2, where the critical buckling length decreases with increasing surrounding soil stiffness.

5.1.3 Influence of soil confinement on instability mode and effective buckling length

Following the previous discussion regarding the influence of BC-configurations and soil stiffness on critical buckling load, figure 5.3 further illustrates the influence of the lateral (k_h) and vertical (k_v) soil springs on the effective buckling length L_{cr} for Pinned-Pinned BC-configuration.

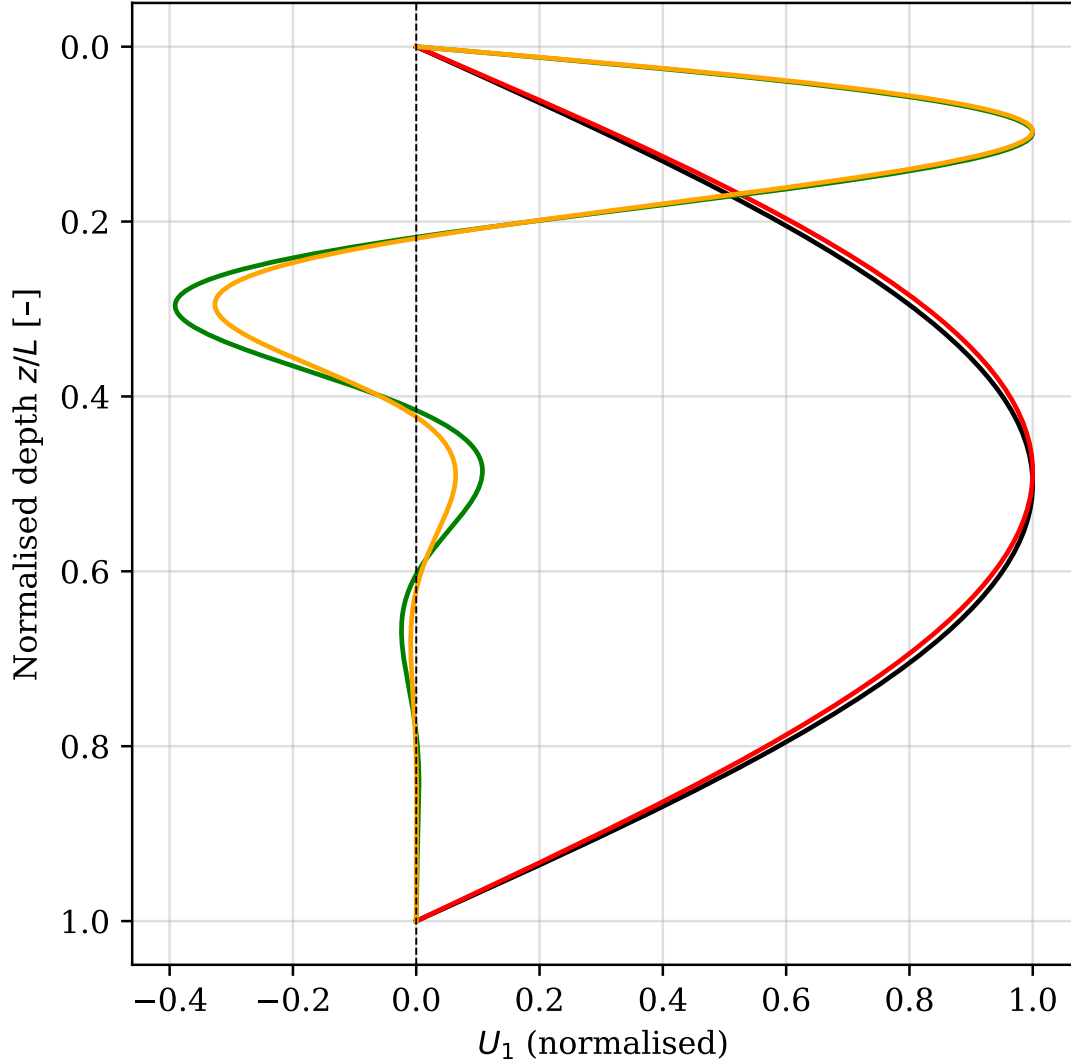


Figure 5.3: The influence of lateral and vertical soil stiffness on the critical buckling length. BLACK: k_h -off & k_v -off, RED: k_h -off & k_v -on, GREEN: k_h -on & k_v -off, YELLOW: k_h -on & k_v -on

The figure shows that when the lateral soil springs are not activated (red and black curves), a global instability mode extending over the entire pile length is developed, resembling the classical Euler buckling behaviour. Activating only the vertical soil springs (red curve) produces only a minor influence on the instability deformation shape, while the overall mode shape remains global.

In contrast, activating the lateral soil springs significantly modifies the instability

response. Both the green and yellow curves illustrates that the lateral deformation progressively localises within a limited active buckling region rather than extending over the entire pile length. This behaviour demonstrates the development of a characteristic buckling length caused by the surrounding lateral soil confinement.

The figure 5.3 further illustrates that the mode shapes corresponding to the green and yellow curves are nearly identical, indicating that the lateral soil springs dominate the instability behaviour, while the contribution of the vertical soil springs remains limited to a minor reduction in the instability deformation response. Consequently, the instability response and effective buckling length are governed primarily by the interaction between pile flexural rigidity and the surrounding lateral soil stiffness.

The active buckling length for the cases where the lateral soil springs (k_h) are activated is localised closer to the upper portion of the pile, where the lateral confinement is weakest according to the equation 2.1. Figure 5.3 The figure shows that the instability deformation no longer extends over the entire pile length, but instead concentrates within a limited active deformation region near the pile head. This behaviour occurs because the undrained shear strength, and consequently the lateral soil stiffness, increases progressively with depth. As a result, the deeper portions of the pile receive stronger lateral restraint from the surrounding soil, suppressing large lateral deformations at greater depths and forcing the instability mode to localise within the weaker upper soil layers. Consequently, this further confirms the influence of the pile head configuration on the instability response of long piles observed in figure 5.1, since the active buckling region develops close to the upper boundary where the surrounding soil confinement is lowest.

5.1.4 Influence of vertical shaft resistance k_v on buckling behaviour

The influence of the vertical shaft resistance k_v on the critical buckling load was assessed by comparing models with and without vertical Winkler springs. The k_v sensitivity was expressed as following:

$$\Delta k_v = \frac{P_{cr,kv,on} - P_{cr,kv,off}}{P_{cr,kv,off}} * 100\% \quad (5.1)$$

Expression 5.1 is computed for each combination of pile length, shear strength $c_{0u} = 24kPa$, soil strength gradient $n_h = 1.1kPa$, pile cross-sections $D_1 219/t_1 10$ m and $D_2 324/t_2 10$ m, and boundary condition configurations. The results are represented in figure 5.4 for pile length range $5m \leq L \leq 100m$ with $\Delta L = 5m$ as a length step:

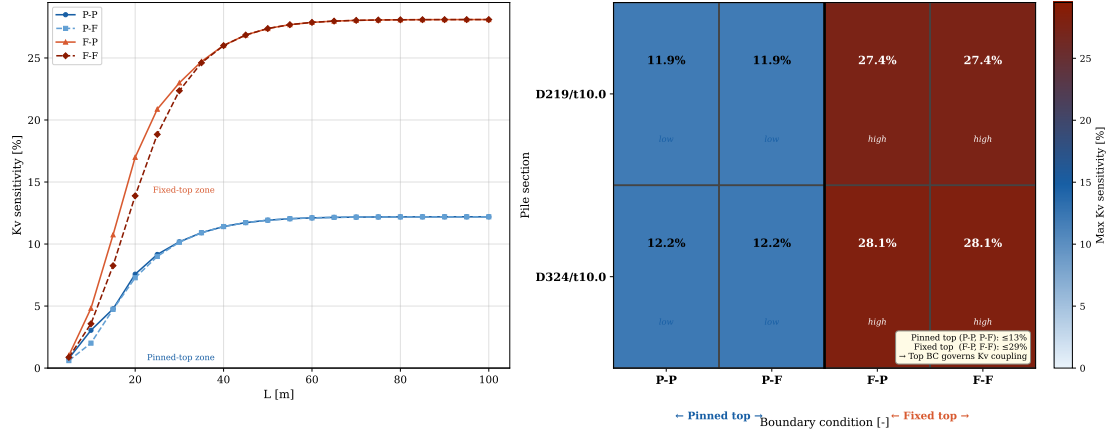


Figure 5.4: Enter Caption

Figure 5.4 shows that the influence of the vertical springs depends strongly on the pile head boundary condition. Regarding the pinned top configuration ($P-P$ and $P-F$), the influence of k_v is relatively limited over the entire pile-length range. $k_v \approx 12\%$ as the maximum influence on the critical buckling load P_{cr} for both diameters, and the current soil conditions indicating that the instability response is primarily governed by the lateral deformation and rotational flexibility at the pile head. Since the pile head is free to rotate, the vertical shaft resistance would contribute marginally to the global buckling resistance.

While for the fixed head configurations ($F-P$ and $F-F$), the maximum influence on the critical buckling load P_{cr} increases to $\approx 27\% - 28\%$ for both diameters, indicating a larger sensitivity to the vertical shaft resistance. This shows that the fixed pile head restricts rotational deformation and therefore increases the mobilisation of the surrounding soil resistance, resulting in a larger contribution from the vertical shaft springs.

The figure 5.4 also shows that k_v sensitivity increases progressively with increasing pile length, until approaching approximately constant plateau values (the value increase as both the shear strength and soil strength gradient increase). This indicates that the vertical shaft resistance becomes increasingly mobilised as a larger part of the pile length interacts with the surrounding soil. At approximately $L \approx 40m - 50m$, the curves stabilise and no further influence of k_v on the critical buckling load takes place, suggesting that the vertical shaft resistance is fully developed and any further increase in the pile length provides only limited additional contribution to the buckling load.

Regarding slender piles, both pile sections show similar plateau values within the same boundary-condition configurations, indicating that the influence of the vertical shaft resistance is mainly controlled by the rotational support conditions rather than the pile flexural stiffness. Appendix A.4.2 shows further validation for the influence of k_v on the critical buckling load.

5.1.5 Hétényi λL Classification: Complete Soil-Structure Interaction

Figure 5.5 presents the instability classification of piles using the Hétényi dimensionless parameter λL framework, for different pile cross-sections and BC-configurations and with both the lateral and vertical soil springs included. The vertical lines at $\lambda L = \pi/4$ and $\lambda L = \pi$ represents the theoretical boundaries between short-, intermediate-, and long-pile regions according to classical Hétényi beam-on-elastic-foundation theory 2.9. The lateral soil springs k_h used in λ -equation 2.22 is assumed constant with depth ($c_u = 12 \text{ kPa}$), while the k_h used in the FEM-analyses is depth-dependent with soil gradient $n_h = 1.1 \text{ kPa/m}$.

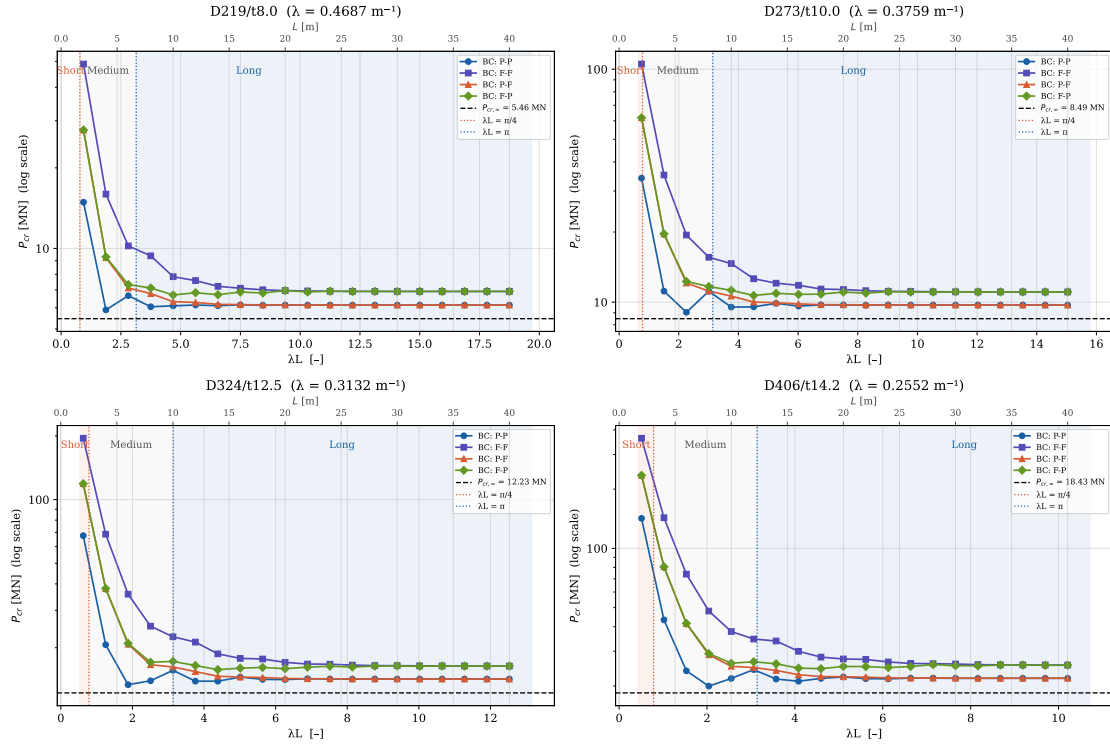


Figure 5.5: The theoretical classification using λL framework vs. FEM results using a depth-dependent soil profile

Figure 5.5 summarises the instability interpretations discussed previously regarding the influence of boundary conditions, and the transition between the short-, intermediate-, and long-pile regions. Comparison with the classical Hétényi classification limits shows that the numerical FEM results exhibit a more gradual transition between the short-, intermediate-, and long-pile regions. The FEM curves do not converge exactly at the theoretical limits proposed by Hétényi, but instead continue to evolve beyond these limits before approaching approximately constant buckling load plateau. This indicates that the transition between Euler-controlled and soil-controlled instability behaviour occurs progressively rather than abruptly, particularly for different boundary-condition configurations and varying pile flexural rigidities.

The figure further indicates that increasing the pile cross-sectional stiffness delays

the transition toward fully soil-controlled instability behaviour. Piles with larger flexural rigidity EI therefore require greater pile lengths before the FEM curves stabilise and converge toward approximately constant critical buckling loads. Consequently, the instability response of stiffer piles remains more sensitive to the applied boundary conditions over a wider pile-length range before the surrounding soil confinement becomes dominant.

The following figure 5.6 follows the same analysis procedure as 5.5, but considers a single pile cross-section ($D = 324mm$ and $t = 10mm$), two soil stiffness gradients ($n_h = 1.1kPa/m$ and $n_h = 3kPa/m$), and an undrained shear strength near the ground surface $c_{0u} = 12kPa$:

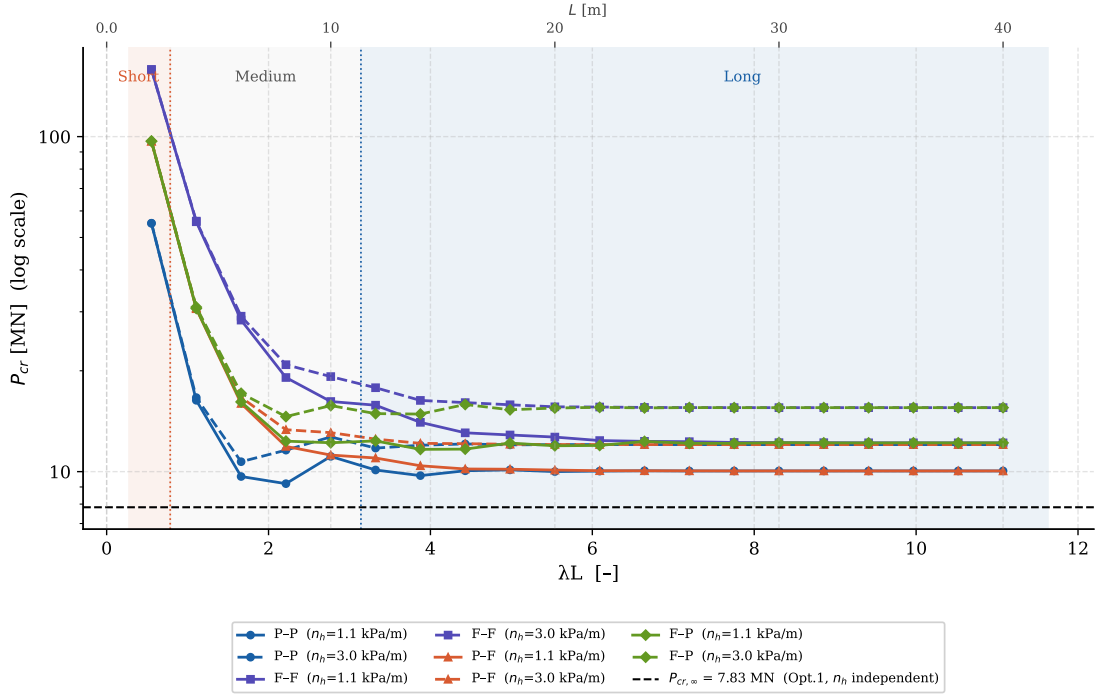


Figure 5.6: Influence of soil stiffness gradient n_h vs. Hétényi λL instability classification for different boundary-condition configurations

Comparison between the different analyses 5.6 and 5.5 further indicates that variations in pile flexural rigidity EI produce a significantly larger influence on the transition between Euler-controlled and soil-controlled instability behaviour compared with variations in the soil strength parameters c_{0u} and n_h . Although increasing soil strength enhances the surrounding lateral confinement, the instability response remains fundamentally governed by the bending stiffness of the pile. Increasing EI directly increases the resistance against curvature and delays localisation of the instability deformation, thus requiring larger pile lengths before fully soil-controlled behaviour develops. In contrast, increasing c_{0u} and n_h primarily modifies the surrounding confinement stiffness and therefore produces only moderate shifts in the transition regions.

5.1.6 Relation between effective buckling length L_{cr} and the slenderness ratio L/D for different soil gradient values

The following figure illustrates the relation between the effective buckling length factor β with the slenderness ratio L/D for two different pile diameters ($D_1 = 324\text{mm}$ and $D_2 = 406\text{mm}$). The soil profiles used are with $c_{0u} = 24\text{kPa}$ and two soil gradients $n_h = 1.1\text{kPa/m}$ and $n_h = 3\text{kPa/m}$.

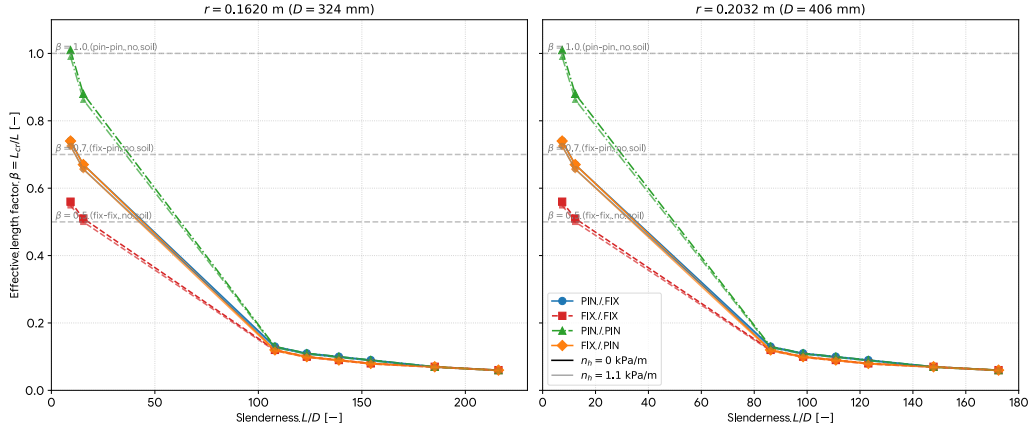


Figure 5.7: Effective length factor β vs. slenderness ratio L/D

Figure 5.7 illustrates how the effective buckling factor β decreases with increasing slenderness ratio L/D . For low slenderness ratios, β factor is relatively large, indicating that short piles behave similarly to classical Euler column associated with the applied boundary conditions shown in Appendix A.2.1. As the slenderness ratio increases, β decreases rapidly before gradually approaching approximately constant low values ranging $\beta \approx 0.06 - 0.1$ for both pile diameters within the long-pile regime.

This behaviour indicates that the equivalent Euler buckling length becomes progressively smaller relative to the total pile length as the surrounding soil confinement increasingly mobilises. As a result, the instability deformation gradually changes from a global buckling mode involving most of the pile length to a more localized deformation pattern confined to a limited active buckling zone.

The figure 5.7 further shows that both soil profiles follow nearly identical trends over the entire slenderness range. However, higher soil gradient $n_h = 1.1\text{kPa}$

consistently produces slightly smaller effective length factor β than the uniform-stiffness case $n_h = 0kPa$. This indicates that increasing soil stiffness with depth provides a moderate increase in the lateral confinement at greater depths, resulting in shorter equivalent buckling lengths for the same slenderness ratio.

Comparison between the two pile cross-sections further shows that larger diameter ($D = 406mm$) consistently produces slightly larger β -factors than the smaller diameter ($D = 324mm$) over the same slenderness ratio.

This further confirms the behaviour observed in figure 5.5, where increasing pile flexural rigidity causes the instability deformation to remain distributed over a larger portion of the pile before the surrounding soil confinement becomes dominant.

5.2 GEOMETRIC NON-LINEAR ANALYSIS

5.2.1 Mode shape sensitivity

The instability behaviour was further investigated using geometrically non-linear analyses (STATIC/RIKS) with bilinear lateral springs and linear vertical springs. The analyses were performed to evaluate the post-buckling response and identify the dominant instability mode and corresponding eigenvalue beyond the linear buckling analysis.

Figure 5.8 presents the load–displacement ($p - y$) response for the first three instability modes using bilinear lateral springs with $N_p = 6[-]$. The analyses were performed for a slender pile with $L = 35m$, $D = 324mm$, uniform-soil profile with undrained shear strength $c_{0u} = 24kPa$, and a $P - P$ configuration. Initial geometric imperfection corresponding $L_{cr}/300$ and $L_{cr}/1000$ were applied.

The reference critical buckling load $P_{cr,1}$ presented in the figures corresponds to the first eigenvalue obtained from linear buckling analysis assuming linear elastic behaviour for both the pile material and the soil springs. Additional detailed numerical modelling are presented in AppendixA.3.1.

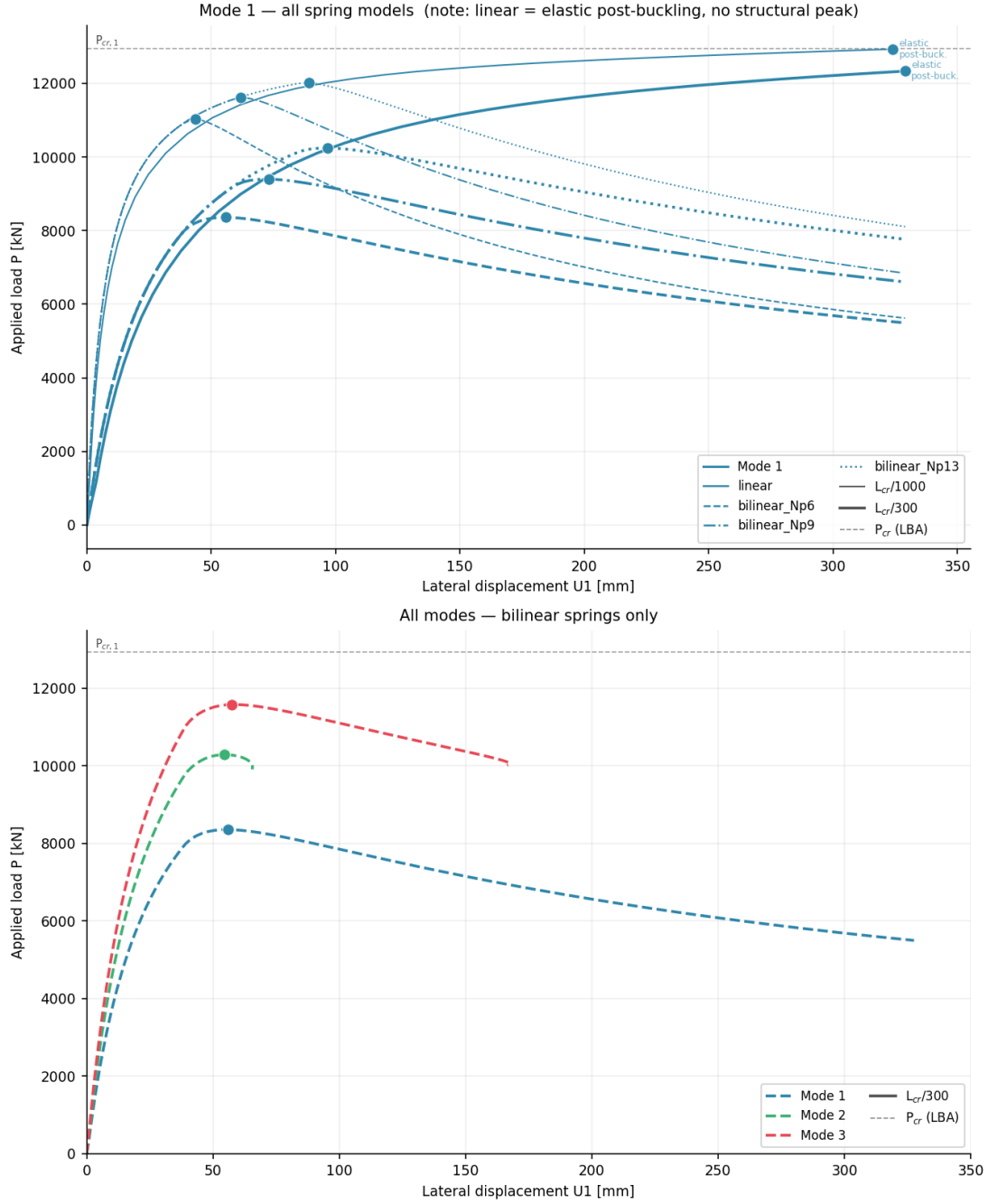


Figure 5.8

The figure shows that Mode 1 produces smoothest and most stable GNA response, where the applied load gradually increases towards a peak value followed by moderate post-buckling softening. This indicates the progressive mobilization and yielding of the surrounding bilinear soil springs during the development of the instability response.

In contrast, modes 2 and 3 exhibit increasingly rapid post-buckling stiffness degradation compared to mode 1 once $P_{cr,1}$ is exceeded. This suggests that higher-order modes create more localised deformation, which causes the surrounding soil springs to lose stiffness more rapidly after instability starts. While mode 1 is most sta-

ble and physically dominant instability mode, where surrounding soil confinement is mobilised progressively over larger portion of the pile length, Appendix A.5.1 further validates this section.

The results further indicate that the governing instability mechanism is controlled primarily by the first buckling mode. Consequently, mode 1 was adopted for the subsequent geometrically non-linear analyses, while the higher-order modes were used mainly to assess localisation sensitivity and post-buckling behaviour.

The upper figure 5.8 governs the post-buckling response for mode 1 and different N_p - values together with corresponding analyses using linear elastic lateral springs. The solid blue curves correspond to geometrically non-linear analyses performed with linear elastic lateral and vertical springs for two different initial imperfections, $L_{cr}/1000$ and $L_{cr}/300$.

The results show that the smaller imperfection ($L_{cr} = 1000$) produces a stiffer response and allows higher load levels before significant lateral displacements occur. The larger imperfection ($L_{cr} = 300$), on the other hand, leads to earlier lateral deformation and a more gradual stiffness reduction due to increased geometric sensitivity of the pile system.

The elastic spring models further exhibit stable elastic post-buckling behaviour, where the applied load continues increasing with increasing displacement. This behaviour occurs because the linear elastic soil springs continuously provide increasing lateral resistance without reaching a limiting soil capacity.

In contrast, the bilinear spring models exhibit a distinct peak load followed by progressive post-buckling softening. This behaviour indicates yielding and gradual degradation of the surrounding lateral soil resistance during instability development. The results further show that increasing the N_p -value produces a smoother post-buckling response, delays stiffness degradation at larger lateral displacements, and results in slightly higher peak loads.

When introducing a depth-dependent soil gradient ($n_h = 1.1 \text{ kPa/m}$) A.14, the post-buckling response becomes noticeably stiffer and more stable for all N_p -values (Appendix A.5.1). This behaviour indicates that the increasing soil stiffness with depth provides stronger lateral confinement during instability development, thereby reducing the rate of post-buckling degradation. This observation is consistent with the previous result sections discussing the influence of depth-dependent soil confinement on the instability response. Despite these differences, both soil profiles follow the same overall instability mechanism and post-buckling trend.

5.2.2 Imperfection effect and knockdown of the critical load

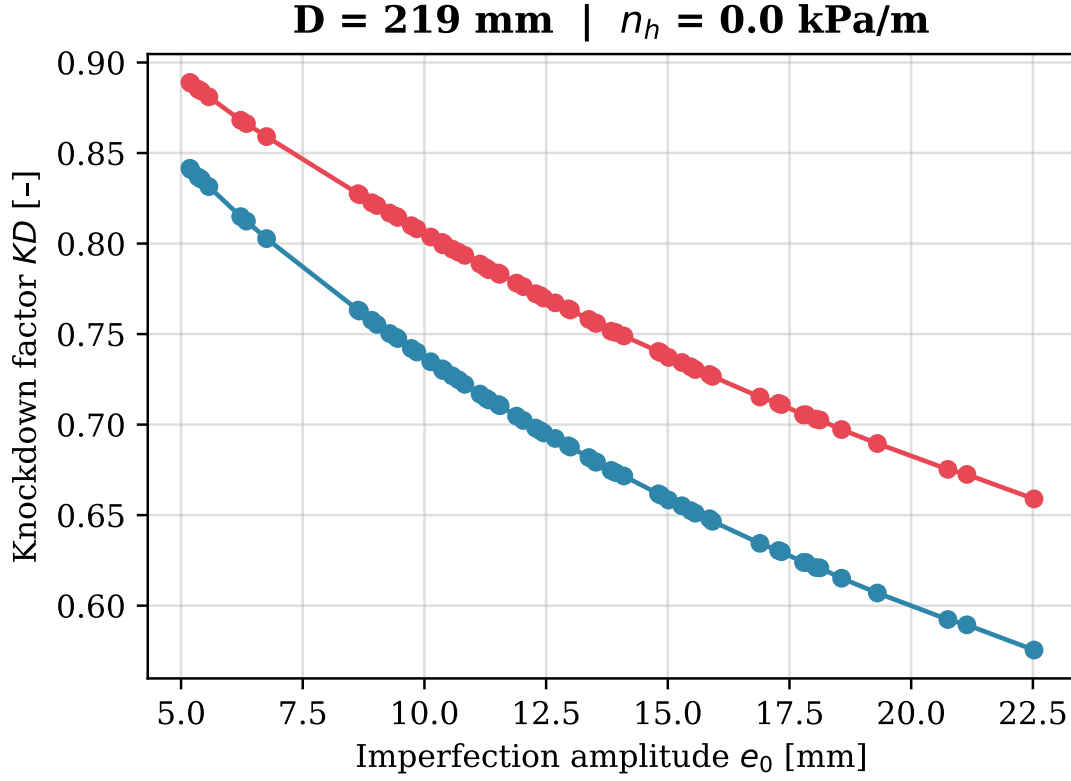


Figure 5.9: Knockdown factor KD as a function of geometric imperfection amplitude e_0 .

Figure 5.9 presents the influence of the initial geometric imperfection amplitude e_0 on the knockdown factor KD A.5.2. The analyses were performed to examine how increasing imperfection reduces the failure load of slender piles in the long-pile regime. Two pile lengths, $L = 25$ and $L = 35$, were considered together with the two pile diameters, $D = 219\text{mm}$ and $D = 324\text{mm}$. The soil conditions were varied using surface undrained shear strength $c_{u,0} = 10, 17, 20, 24$ [kPa], soil strength gradients $n_h = 0.0$ and $n_h = 1.1$ [kPa/m], and the lateral bearing capacity factors $N_p = 6$ and $N_p = 9$. All cases were analysed using linear elastic pile material behaviour, a bilinear Winkler spring model, Mode 1 as the imperfection shape, and pinned-pinned boundary conditions see A.15.

The imperfection amplitudes applied in the analyses were selected to cover a broad range of possible initial geometric imperfections. This was done to obtain a comprehensive understanding of how imperfection amplitude influences the knockdown factor and the post buckling failure load. Design guidance from the Swedish Pile Commission accounts for different imperfection levels depending on pile type, number of splices, straightness control, and residual stresses, while EC-based steel design also recognises the importance of geometric imperfections in stability verification [16, 15, 17]. Therefore, the selected imperfection range was not limited to a single recommended design value, but was instead varied systematically to

evaluate the sensitivity of the pile response to increasing initial curvature.

The results show a consistent decrease in the knockdown factor KD with increasing imperfection amplitude for all analysed parameter combinations. This confirms that the initial geometric imperfection has a strong influence on the post buckling load carrying capacity. No clear plateau or saturation is observed within the investigated amplitude range. For example, for $D = 219\text{mm}$ and $n_h = 0.0[\text{kPa/m}]$ KD decreases from approximately 0.84 at $e_0 = 5\text{mm}$ to approximately 0.52 at $e_0 = 22.5\text{mm}$ for $N_p=6$. This corresponds to a reduction of about 32% over the studied imperfection range.

The lateral spring capacity factor N_p has a clear and consistent influence on the knockdown factor. For the same imperfection amplitude, cases with $N_p = 9$ give KD values that are approximately 5% – 13% 5.1 higher than those obtained with $N_p = 6$. The curves remain almost parallel, indicating that increasing N_p shifts the response upward without significantly changing the rate at which KD decreases with imperfection amplitude. This behaviour is consistent with the confining effect of the Winkler springs. A higher lateral resistance limits the development of lateral displacement and reduces the second order bending effects caused by the initial imperfection.

Table 5.1: Selected case pairs used to compare the effect of the lateral bearing capacity factor on the knockdown factor.

Riks ID	LBA ID	L [m]	n_h [kPa/m]	c_{u0} [kPa]	N_p [-]	D [mm]	L_{cr} [m]	KD [-]
9	1	25	0.0	10	6	219	6.3434	0.8124
17	1	25	0.0	10	9	219	6.3434	0.8662
Deviation								6.22%
592	25	35	0.0	17	6	219	6.7570	0.5755
600	25	35	0.0	17	9	219	6.7570	0.6589
Deviation								12.67%

The influence of the soil stiffness gradient n_h is comparatively limited 5.2. By comparing the curves for each pile diameter, see A.15, it can be observed that introducing a depth dependent soil strength gradient $n_h = 1.1[\text{kPa/m}]$ produces only a very small shift in the knockdown factor KD 5.2. The curves remain nearly similar in both slope and level. This indicates that, for the pile lengths and soil conditions considered, the imperfection sensitivity is governed mainly by the lateral spring capacity factor rather than by the detailed depth distribution of the soil stiffness.

Table 5.2: Selected case pairs used to compare the influence of the soil strength gradient on the knockdown factor.

Row	Riks ID	LBA ID	n_h [kPa/m]	c_{u0} [kPa]	N_p [-]	D [mm]	L_{cr} [m]	KD [-]
34	393	17	0.0	17	6	219	5.3517	0.8367
35	417	18	1.1	17	6	219	4.8952	0.8463
Deviation								1.14%
2	9	1	0.0	10	6	219	6.3434	0.8124
3	33	2	1.1	10	6	219	5.2000	0.8379
Deviation								3.05%
18	17	1	0.0	10	9	219	6.3434	0.8662
19	41	2	1.1	10	9	219	5.2000	0.8851
Deviation								2.13%

The larger pile diameter $D = 324mm$ shows a wider imperfection amplitude range. This is mainly because the imperfection amplitude is scaled from the critical buckling length, $e_0 = Lcr/1000$ and larger sections generally produce longer critical lengths. However, the resulting KD values remain comparable to those obtained for $D = 219mm$ 5.3. This indicates that, when the imperfection amplitude is related to the buckling length, the sensitivity to imperfections is similar for both cross sections. The reduction in capacity is therefore governed more by pile slenderness and imperfection level than by the absolute pile diameter.

Table 5.3: Selected case pairs used to compare the effect of pile diameter on the knockdown factor.

Riks ID	LBA ID	L [m]	n_h [kPa/m]	c_{u0} [kPa]	N_p [-]	r [m]	KD [-]
9	1	25	0.0	10	6	0.10955	0.8124
105	5	25	0.0	10	6	0.16195	0.8296
Deviation							2.08%
17	1	25	0.0	10	9	0.10955	0.8662
113	5	25	0.0	10	9	0.16195	0.8801
Deviation							1.57%
393	17	25	0.0	17	6	0.10955	0.8367
489	21	25	0.0	17	6	0.16195	0.8461
Deviation							1.12%
201	9	35	0.0	10	6	0.10955	0.8149
297	13	35	0.0	10	6	0.16195	0.8232
Deviation							1.01%

The parametric study shows that the surface undrained shear strength c_{u0} has a limited influence on the knockdown factor KD ???. Increasing c_{u0} from 10 to 24[kPa], which corresponds to a 140% increase in strength results in only a small increase in KD , typically around 3 – 4% 5.4 across the investigated imperfection amplitudes and pile configurations. This effect is considerably smaller than the influence of the imperfection amplitude and the lateral bearing capacity factor, N_p , which produce much larger changes in the knockdown response. The weak

statistical correlation, $r(c_{u0}, KD)=0.132$, also indicates that c_{u0} explains less than 2% of the total variation in KD . Therefore, c_{u0} may be regarded as a secondary parameter for the cases studied. For preliminary analyses, a representative or conservative value of c_{u0} is generally sufficient, provided that the soil profile remains within the range of soft clay conditions considered in this study.

Table 5.4: Selected case pairs used to compare the effect of surface undrained shear strength on the knockdown factor.

Riks ID	LBA ID	L [m]	n_h [kPa/m]	c_{u0} [kPa]	N_p [-]	D [mm]	KD [-]
9	1	25	0.0	10	6	219	0.8124
393	17	25	0.0	17	6	219	0.8367
Deviation							2.91%
9	1	25	0.0	10	6	219	0.8124
57	3	25	0.0	20	6	219	0.8409
Deviation							3.39%
9	1	25	0.0	10	6	219	0.8124
441	19	25	0.0	24	6	219	0.8416
Deviation							3.47%

The parametric study confirms that the geometric imperfection amplitude is the dominant parameter controlling the knockdown factor KD . Within the investigated range, increasing the imperfection amplitude leads to reductions of up to approximately 32%. The lateral bearing capacity factor N_p has a clear secondary influence, producing changes in KD of about 5 – 13%. In contrast, the soil strength gradient, n_h pile diameter D and surface undrained shear strength c_{u0} produce only minor variations, generally below 4%.

5.2.3 Comparison Between Analytical and FEM Peak Load Predictions

Figure 5.10 compares the analytical peak load with the corresponding peak load obtained from the geometrically nonlinear FEM analyses. The purpose of this comparison is to evaluate how well the simplified analytical method can predict the nonlinear instability response of piles embedded in soft clay.

The analytical method is based on a simplified Swedish design-type formulation A.5.3. In this approach, the soil support is represented by a representative lateral stiffness, together with an initial imperfection amplitude and a limiting soil yield displacement. The imperfection shape is assumed to be sinusoidal over the critical buckling length. The steel yield limit is not included in this comparison. This is done in order to make the analytical model consistent with the FEM analyses, where the pile material is treated as linear elastic while the soil response is allowed to yield through bilinear Winkler springs.

The FEM model provides a more detailed representation of the pile–soil interaction. It includes the spring response along the full pile length and introduces the initial imperfection by scaling the buckling mode shape obtained from the linear buckling analysis. Therefore, the comparison reflects both the influence of the

simplified soil representation in the analytical method and the difference between a sinusoidal imperfection shape and a mode-shape-based imperfection.

The comparison includes two pile diameters, $D_1 = 219$ mm and $D_2 = 324$ mm, both with a wall thickness of $t = 10$ mm. Two imperfection amplitudes are considered, $e_{01} = L_{cr}/1000$ and $e_{02} = L_{cr}/300$, together with two lateral bearing capacity factors, $N_p = 6$ and $N_p = 9$. The analytical peak loads are obtained using the formulation presented in Section 2.6.2, while the FEM peak loads are taken from the GNIA/Riks analyses.

In Figure 5.10, the solid diagonal line represents perfect agreement between the analytical and FEM results. The dashed and dotted lines indicate the selected deviation intervals. The results show that the analytical method gives lower peak load predictions than the FEM model for all studied cases. This indicates that the simplified analytical formulation is conservative. The mean deviation is approximately -12.6% , while the maximum difference is about 16.3% .

The results are also approximately parallel to the one-to-one agreement line. This suggests that the deviation is mainly systematic rather than random. The main reasons are likely the use of a representative soil stiffness and the assumed sinusoidal imperfection shape in the analytical model. Larger imperfection amplitudes tend to increase the difference between the two methods, since the response becomes more influenced by second-order effects and soil yielding. Increasing the lateral bearing capacity factor from $N_p = 6$ to $N_p = 9$ slightly reduces the difference, because the higher lateral soil resistance delays the onset of soil yielding.

Overall, the analytical method gives conservative peak load predictions but captures the same general trend as the FEM analyses. It can therefore be useful for preliminary assessment, while the FEM model provides a more detailed description of the nonlinear pile–soil interaction.

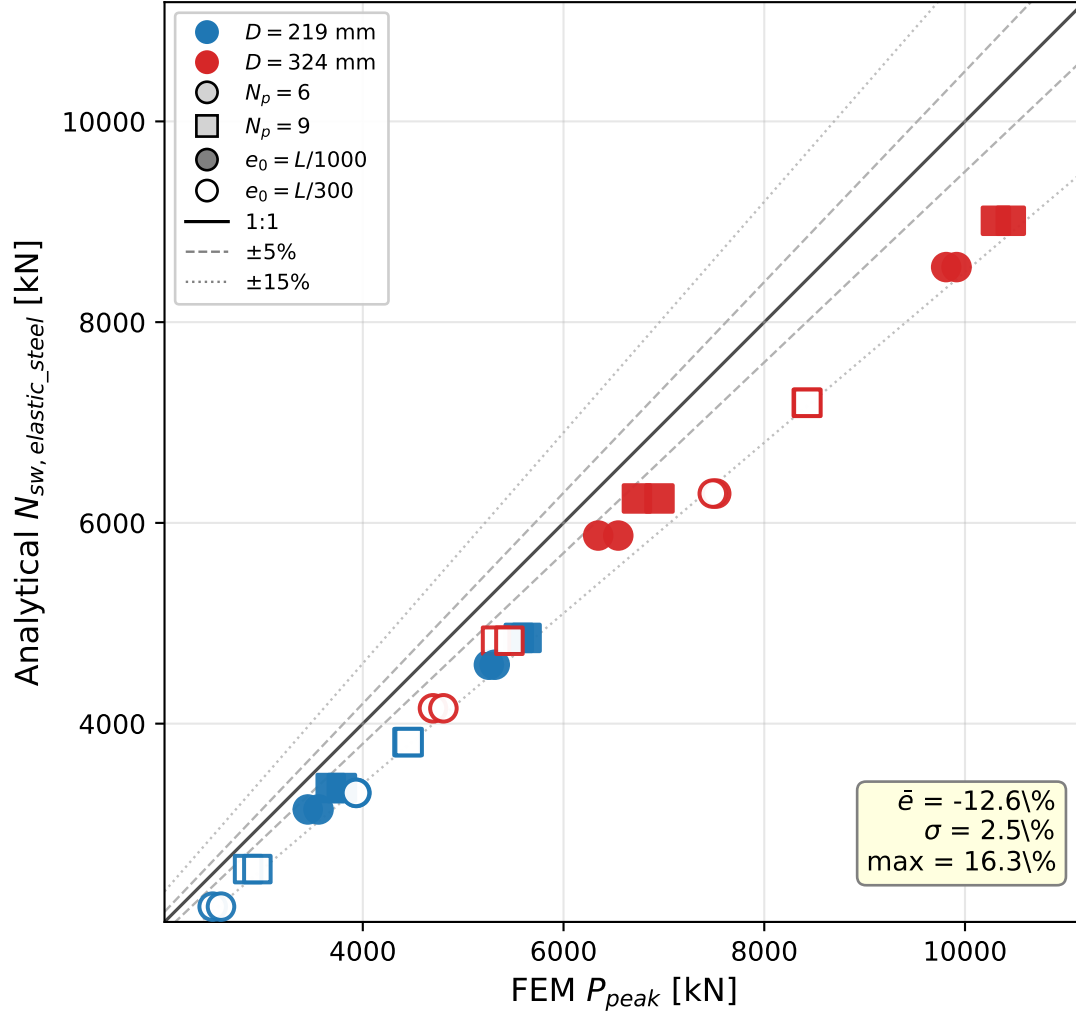


Figure 5.10: Comparison of analytical and numerical peak load predictions

The analytical method gives lower peak load predictions than the FEM model for all studied cases. The mean error is -12.6% , while the maximum absolute error is 16.3% . This indicates that the analytical method is conservative for the investigated parameter range.

The points in Figure 5.10 form a narrow band that is approximately parallel to the 1:1 agreement line. This suggests that the deviation is mainly systematic. The difference is attributed to the simplifications introduced in the analytical model, including the use of a representative constant soil stiffness, a sinusoidal imperfection shape, and a simplified treatment of the elasto-plastic soil response.

Table 5.5 summarises the comparison by parameter group. The values are averaged over the two pile lengths for each group.

Table 5.5: Comparison between FEM peak load and analytical elastic-steel peak load. Values are averaged over $L = 25$ and 35 m for each parameter group.

D [mm]	N_p [-]	e_0	$\bar{P}_{\text{FEM}}$ [kN]	$\bar{N}_{\text{sw,el}}$ [kN]	Error [%]	$\bar{K} \bar{D}_{\text{FEM}}$ [-]
219	6	$L_{\text{cr}}/1000$	4395	3866	-11.7	0.825
219	6	$L_{\text{cr}}/300$	3240	2744	-15.2	0.607
219	9	$L_{\text{cr}}/1000$	4665	4106	-11.7	0.876
219	9	$L_{\text{cr}}/300$	3677	3183	-13.2	0.689
324	6	$L_{\text{cr}}/1000$	8155	7211	-11.1	0.840
324	6	$L_{\text{cr}}/300$	6128	5222	-14.4	0.629
324	9	$L_{\text{cr}}/1000$	8613	7627	-11.0	0.888
324	9	$L_{\text{cr}}/300$	6911	6011	-12.6	0.711
<i>Overall mean</i>					-12.6	—

Two trends can be observed from Table 5.5. First, the difference is generally larger for $e_0 = L_{\text{cr}}/300$ than for $e_0 = L_{\text{cr}}/1000$. This shows that the analytical idealisation becomes less accurate as the imperfection amplitude increases. At larger imperfections, the pile develops larger lateral deflections, and the response becomes more influenced by second-order effects and soil yielding.

Second, increasing the lateral bearing capacity factor from $N_p = 6$ to $N_p = 9$ slightly reduces the error in most cases. A higher value of N_p increases the limiting lateral soil resistance and delays the onset of soil yielding. The response therefore remains closer to the elastic range, where the analytical approximation is more representative.

Overall, the analytical method provides conservative peak load predictions, with an average underestimation of about 12.6 %. It should therefore not be interpreted as a direct replacement for the FEM. However, it captures the same general trend and provides a useful estimate of the imperfection sensitivity of an elastic steel pile supported by elasto-plastic soil.

6

Conclusion

6.1 KEY CONCLUSIONS FROM THE PARAMETRIC STUDY

This thesis investigated the instability behaviour of piles embedded in clay deposits, with particular focus on the transition between short-, intermediate-, and long-pile behaviour, corresponding to the transition from Euler-controlled to soil-controlled instability response. The study further investigated the governing parameters influencing the instability behaviour, including pile flexural rigidity, uniform and depth-dependent soil stiffness distribution, BC-configurations, and soil–structure interaction. Numerical analyses were performed using finite element modelling in ABAQUS together with automated Python scripting, where the surrounding soil was represented using lateral and vertical Winkler-type springs (k_v and k_h) with both constant and depth-dependent stiffness profiles.

The results obtained have shown that the instability behaviour is strongly controlled by the different parameters studied. For short piles, the response was governed primarily by the applied BC-configurations and resembled classical Euler instability behaviour. In this regime, the effective buckling length extended over a large portion of the pile, and the critical buckling load was highly sensitive to the rotational restraints at the pile both ends.

As the pile length increases, the surrounding soil stiffness became progressively mobilised, causing the instability deformation to localise within a limited active buckling region. This produced a gradual transition from BC-controlled Euler buckling towards soil-controlled instability behaviour. The results further proved that the transition between short-, intermediate-, and long-pile behaviour occurred progressively rather than abruptly, this indicates that the theoretical Hétényi λL classification ($\lambda L = \pi/4$ and $\lambda L = \pi$) should be interpreted as approximate and conservative transition limits rather than strict boundaries, since the transition behaviour is strongly influenced by parameters such as pile flexural rigidity, soil stiffness distribution, and boundary conditions. These parameters may therefore shift the transition regions and delay or accelerate the development of fully soil-controlled instability behaviour.

The results further showed that the flexural rigidity of the pile has a larger influence on the instability response and transition behaviour than the investigated variation in the soil stiffness profiles. Increasing EI , through larger pile cross-sectional dimensions, maintained higher critical buckling loads over a wider pile-length range and delayed the convergence toward fully soil-controlled instability behaviour. Consequently, piles with larger flexural rigidity remained more sensi-

tive to BC-configurations before surrounding soil confinement became fully mobilised and dominant.

By comparing with the influence of pile flexural rigidity, variation of soil stiffness profiles mainly resulted in moderate increase in the critical buckling load together with minor shifts in the transition regions. Although increasing soil stiffness with depth enhanced the lateral confinement at greater depths, causing the instability deformation to become concentrated within a smaller region closer to the upper portion of the pile, its influence on the overall instability response remained considerably smaller than the influence of pile flexural rigidity.

Increasing the soil gradient (n_h) further caused the active buckling region to localise closer to the ground surface, where the surrounding soil confinement is considered the weakest, resulting in a shorter effective buckling length l_{cr} . Consequently, the pile head configuration continued to influence the instability response even within the long-pile regime, whereas the effect of the pile toe-configuration gradually diminished as the pile length increased.

The analyses additionally showed that the lateral soil springs k_h governed both the instability behaviour and the critical buckling length, while the influence of the vertical soil springs k_v mainly contributed to a moderate reduction in the deformation magnitude without substantially altering the governing instability response.

The geometrically non-linear (GNA) with bilinear lateral springs analyses showed that the post buckling response is strongly influenced by the selected mode shape and the surrounding soil behaviour. Mode 1 governed the overall instability response and resulted in the smoothest and most stable post-buckling behaviour, while higher order modes resulted in faster degradation of the surrounding soil resistance once the peak load is reached. The results also showed clear differences between the linear elastic and bilinear lateral soil spring idealisations. The linear elastic springs continued providing increasing lateral resistance, leading to stable elastic post-buckling behaviour, whereas the bilinear springs produced a more realistic response with distinct peak loads followed by gradual post-buckling softening due to yielding of the surrounding soil springs.

Furthermore, introducing depth-dependent soil stiffness improved the overall post-buckling stability by providing stronger lateral confinement at greater depths and reducing the rate of stiffness degradation during instability development.

6.2 KNOCKDOWN FACTOR SENSITIVITY AND BASIS FOR A SIMPLIFIED MODEL

The parametric study presented in this section provides a systematic evaluation of how the initial geometric imperfection amplitude and SSI-parameters influence the reduction in the failure load of slender steel piles embedded in soft clay. The purpose is not to establish a final predictive design equation, but to identify the main trends that could form the basis for a simplified knockdown-factor model.

The results show that the imperfection amplitude, e_0 , is the dominant parameter controlling the knockdown factor KD . Across the investigated pile diameters, soil profiles, and spring capacity levels KD decreases almost linearly as e_0 increases. The maximum reduction in KD , relative to the near-perfect imperfection case, reaches approximately 32% within the studied imperfection range. No clear plateau is observed. This indicates that the sensitivity to imperfection amplitude remains significant even at larger values of e_0 .

The observed trend suggests that, for a given pile configuration, the knockdown factor may be approximated by a simple linear relationship. In its most general form, the dependency may be written conceptually as

$$KD = f(e_0, N_p, D, c_{u,0}, n_h, L), \quad (6.1)$$

where e_0 is the initial imperfection amplitude, N_p is the lateral bearing capacity factor, D is the pile diameter, $c_{u,0}$ is the surface undrained shear strength, n_h is the soil strength gradient, and L is the pile length. This expression is not intended as a calibrated design equation. It is used only to describe the parameters that may influence KD .

The numerical results allow this general dependency to be simplified by considering the relative influence of each parameter. Within the investigated range, the pile length, L , the surface undrained shear strength, $c_{u,0}$, and the soil strength gradient, n_h , show only a limited influence on KD . Their effect generally remains below approximately 5%. The global statistical correlations are also weak, with $r(c_{u,0}, KD) = 0.132$ and $r(n_h, KD) = 0.121$. These values indicate that each of these parameters explains only a small part of the total variation in KD in the present dataset. Their influence is therefore treated as secondary at this stage and may be absorbed into a calibration constant, giving the reduced conceptual dependency

$$KD \approx f(e_0, N_p, D, a), \quad (6.2)$$

where a represents the combined contribution of the marginal parameters.

The pile diameter, D , produces a small but consistent shift in KD between the two investigated cross-sections. The variation is approximately 2–4%. When the imperfection amplitude is related to the critical buckling length, the responses for $D = 219\text{ mm}$ and $D = 324\text{ mm}$ follow nearly parallel trends. This suggests that the diameter effect does not strongly interact with the imperfection amplitude. It may therefore be included as a secondary correction to the intercept:

$$KD \approx f(e_0, N_p, a_D), \quad (6.3)$$

where a_D is a calibrated constant that includes the influence of the pile diameter and the other secondary parameters.

The lateral bearing capacity factor, N_p , has a clearer influence. Increasing N_p from 6 to 9 shifts the knockdown factor upward by approximately 5–13%. The shift is nearly parallel over the investigated imperfection range. This means that N_p mainly affects the intercept of the KD – e_0 relationship, while the slope remains approximately unchanged. The influence of N_p can therefore be represented conceptually by a linear intercept term:

$$KD \approx f(e_0, \kappa + \chi N_p), \quad (6.4)$$

where κ includes the contribution of the secondary parameters and χ describes the increase in KD associated with the lateral bearing capacity factor.

Based on these observations, a simplified representative form of the knockdown-factor relationship can be written as

$$\boxed{KD(e_0, N_p) \approx (\kappa + \chi N_p) - \zeta e_0} \quad (6.5)$$

where κ is the base intercept, χ is the correction associated with N_p , and ζ is the slope coefficient describing the reduction in KD per unit increase in imperfection amplitude. The expression should be interpreted as a representative trend model only. It is not a final calibrated equation, and the coefficients are not intended for direct design use without further calibration.

This simplified expression provides a practical way to describe the observed influence of imperfection amplitude and lateral soil capacity on the knockdown factor. With further development, this representative expression could form the basis for an alternative prediction method to the analytical approach used in this study. This would require a wider calibration range and validation against additional pile geometries, soil profiles, spring properties, boundary conditions, and imperfection amplitudes. A broader study would make it possible to include the influence of the secondary parameters more explicitly, instead of absorbing them into calibration constants. In that case, the model could become more sensitive to the full set of governing parameters and may provide more accurate predictions of the knockdown factor than the simplified analytical method.

6.3 CONCLUSION ON THE ANALYTICAL–FEM PEAK LOAD COMPARISON

The comparison shows that the simplified analytical approach, based on the Swedish pile buckling method, gives conservative peak-load predictions for the investigated cases. The analytical peak load, $N_{\text{sw,el}}$, is consistently lower than the FEM peak load, P_{peak} , with an average difference of approximately 12.6 %. This result applies to the elastic-steel instability response only, since material yielding in the pile is not included in either the analytical comparison value or the FEM Riks analyses.

This comparison should therefore not be interpreted as a validation or assessment of the complete Swedish design resistance. In the original Swedish design procedure, the final pile capacity may also be limited by the steel axial force–bending moment interaction. This limit is intentionally omitted here in order to compare the instability response under the same basic material assumption, namely linear elastic steel with elasto-plastic lateral soil support.

The remaining difference between the analytical and numerical results is mainly related to the level of idealisation. The analytical method uses a sinusoidal imperfection over the critical buckling length and a representative constant soil stiffness. In contrast, the FEM model uses the first LBA buckling mode as the imperfection shape and resolves the nonlinear spring response along the pile length. These modelling differences explain the systematic offset between the two approaches.

Overall, the analytical method provides a conservative estimate of the elastic-steel peak load for the studied parameter range. It is therefore useful as a simplified comparison method, but it should not be used as a direct replacement for the numerical analysis when the detailed nonlinear pile–soil response is of interest.

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A

Appendix

A.1 APPENDIX A

A.1.1 Swedish design approach

The following figure illustrates the Swedish design methodology employed for the design of pile foundations.[14]: The Swedish design is based on:

1. Analytical method
2. Infinite long pile
3. Sinusoidal shape
4. Initial curvature
5. Elastic lateral soil stiffness
6. Elastic pile material.
7. Sources used: PKR-81, -81:a, -96:1+ECs

A.1.2 Verifications

Case	Diameter	Length
V1 – No soil support	Diameter 1	L_1
		L_2
		L_3
	Diameter 2	L_1
		L_2
		L_3
V2 – Constant soil stiffness	Diameter 1	L_1
		L_2
		L_3
	Diameter 2	L_1
		L_2
		L_3

Table A.1: Hierarchy of the verification analyses.

The following figure illustrates the verification of the linear elastic FEM analyses performed by comparing numerically obtained critical buckling load P_{crFEM}

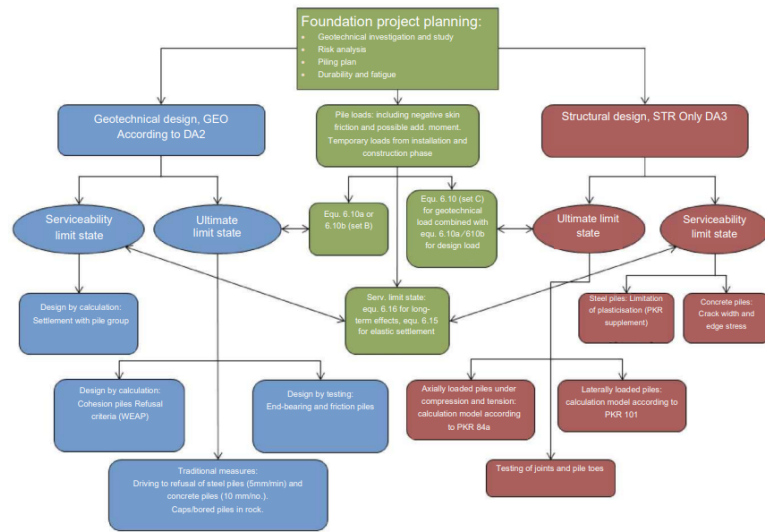


Figure A.1: The Swedish design approach of piling, picture is taken from Pålkommission 106[14]

with the corresponding analytical solutions for all BC-configurations and beam-on-elastic foundation:

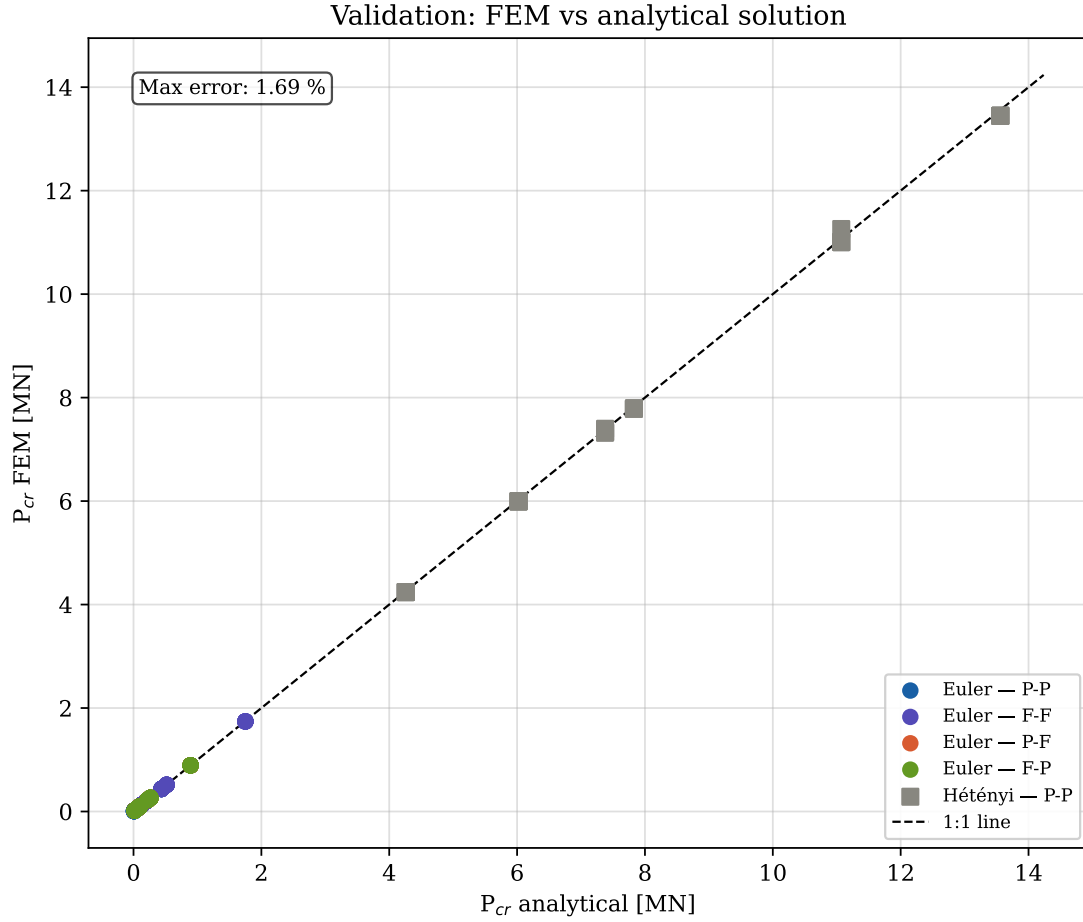


Figure A.2: Linear elastic finite element method (FEM) analysis of the critical buckling load in comparison with analytical solutions derived from the classical critical buckling load formulations proposed by Euler and by Hétényi.

The dashed 1 : 1 line represents perfect agreement between the analytical and numerical solutions. The figure illustrates that the maximum deviation between FEM results and the analytical solutions is approximately 1.69%, showing that the numerical modelling approach accurately captures both classical Euler instability behaviour and lateral soil-supported buckling response. The formulas used for the analytical solutions are:

$$P_{Euler,cr} = \frac{\pi^2 EI}{(KL)^2} \quad (\text{A.1})$$

$$P_{Hetenyi,cr} = 2\sqrt{k_h EI} \quad (\text{A.2})$$

The first eigenvalue obtained in the FEM analyses corresponds $P_{FEM,cr}$.

The following figure illustrates the influence of pile length L [m], flexural rigidity EI [kN m²], and the constant lateral soil support k_h [kPa] on the critical buckling load for both FEM model and the corresponding analytical solutions for pinned-pinned BC-configuration:

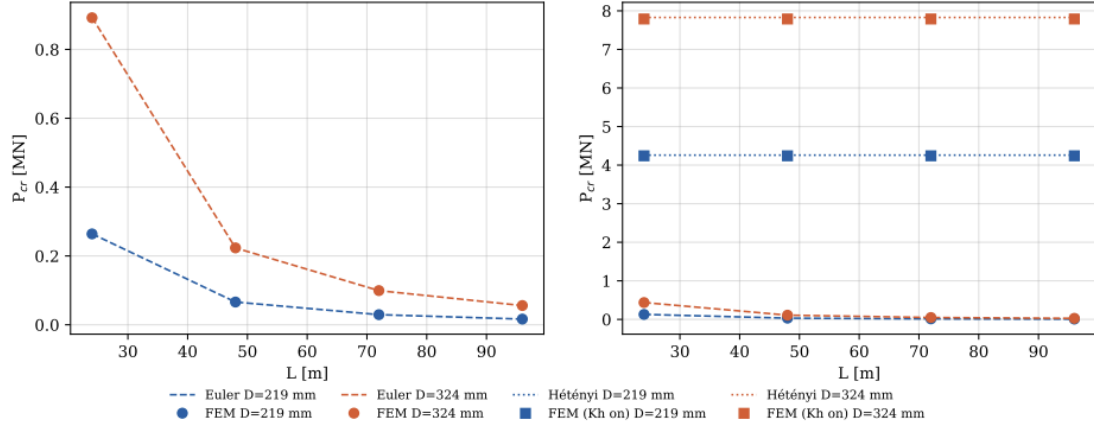


Figure A.3: Verification of FEM and analytical critical buckling loads for pinned–pinned piles with and without lateral soil support

The figure on the left presents the classical Euler buckling response with $k_h = 0$. Both the analytical and the FEM solutions show that the critical buckling load decreases with increasing length. The figure further shows that larger EI develops higher P_{cr} confirming the accuracy of the numerical implementation for classical instability behaviour.

The right figure presents the corresponding results when lateral soil support $k_h > 0$ is included using Winkler foundation springs. Here the critical buckling load is almost independent of the pile length.

The right figure further shows that the lateral soil support provides the pile with greater support leading to higher P_{cr} compared to the corresponding Euler solutions.

A.2 APPENDIX B

A.2.1 Slenderness ratio

Slenderness is often expressed as a non-dimensional expression[17]:

$$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{P_k}} = \frac{l_k}{i} \cdot \frac{1}{\lambda_1} \quad (\text{A.3})$$

Where $\bar{\lambda}$ is the slenderness ratio, P_k is the critical buckling load, l_k is the buckling length in the buckling plane considered, λ_1 is the reference slenderness, and i is the radius of gyration about the relevant axis, determined using the properties of the gross cross-section. The reference slenderness is referred as[17]:

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 93.9 \varepsilon \quad (\text{A.4})$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \quad (f_y \text{ in N/mm}^2) \quad (\text{A.5})$$

Where f_y is the yield strength of steel (in MPa), and ε is a material factor that accounts for the influence of steel strength on buckling behaviour.

A.2.2 Second order-effects

Real structural members have initial geometrical imperfections that introduce eccentricity δ_0 . When an axial compressive load is applied to an imperfect member, the initial eccentricity generates bending moments and corresponding lateral deformations. As the axial load increases, the lateral deformation becomes progressively amplified due to geometric nonlinearity, resulting in second-order behaviour [24].

For an initially imperfect member, the bending moment generated under axial compression may be expressed as:[24]

$$M = P(\delta + y) \quad (\text{A.6})$$

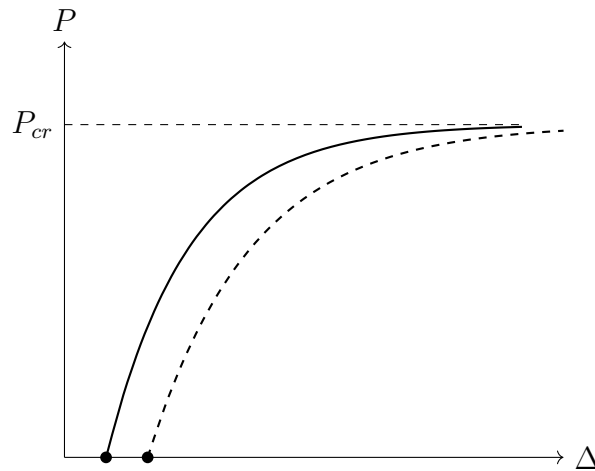


Figure A.4: Load-displacement response of a compressed member including second-order effects for different levels of initial geometric imperfections. Both curves approach the critical load P_{cr} asymptotically.

Figure A.4 illustrates the linear elastic response of axially compressed steel members in the presence of geometric imperfections, commonly referred to as second-order effects. Members with larger imperfections exhibit greater lateral deformations at lower load levels, resulting in earlier instability amplification and reduced buckling resistance[24].

A.2.3 Critical buckling length factor for steel columns

The following figure illustrates the five classical Euler buckling cases for columns subjected to axial compressive loading under different boundary conditions. The figure shows how the rotational and translational restraints at the column ends influence both the deformation mode and the corresponding critical buckling load[37].

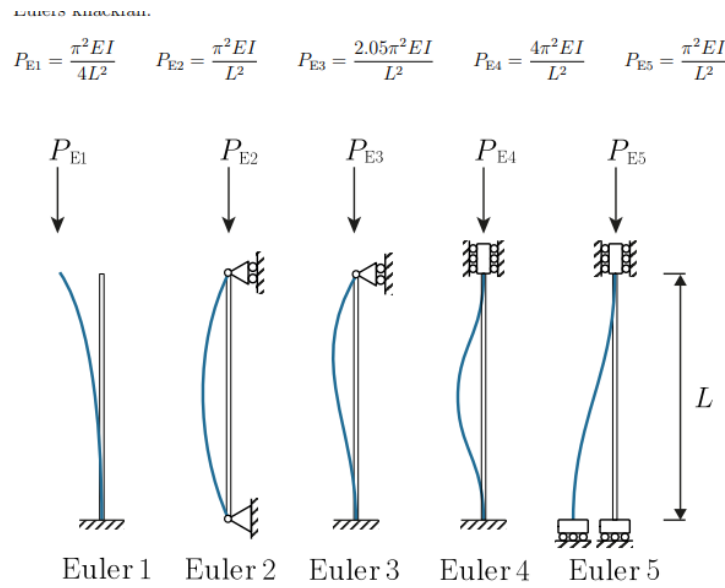


Figure A.5: : Critical buckling length factors for different BC-configurations

A.3 APPENDIX C

A.3.1 Geometric non-linear analysis (GNA with bilinear horizontal springs): Dominant eigenvalue and mode shape with bilinear horizontal springs

The numerical methodology used in this result section is based on Geometrically non-linear analysis (STATIC/RIKS) performed on imperfect geometry, with linear elastic pile material. The surrounding soil is modelled using bilinear lateral springs ($p - y$) per node along the pile shaft, while the vertical shaft resistance is represented by linear vertical springs.

The initial lateral stiffness is:

$$k_h = 50c_u(z)\Delta l \quad (\text{A.7})$$

And the ultimate load assumed is:

$$q_{lim}(z) = N_p c_u(z) \Delta l D \quad (\text{A.8})$$

There:

- k_h is the depth-dependent bilinear lateral spring stiffness per node [N/m].
- $c_u(z)$ is the undrained shear strength varying with depth [kPa]
- Δl is the distance between nodes along the pile shaft [m].
- $q_{lim}(z)$ defines plastic plateau for the ultimate lateral spring force at different depths [N].
- N_p is the lateral bearing capacity factor for undrained clay [-]. Value used is 50, which is the most conservative.
- D is the Outer diameter of the pile [m].

The bilinear spring model consists of an initial elastic stiffness followed by yielding and stiffness softening after the ultimate lateral spring resistance q_{lim} is reached. This allows progressive degradation of the surrounding soil confinement during the post-buckling response.

A.4 APPENDIX D

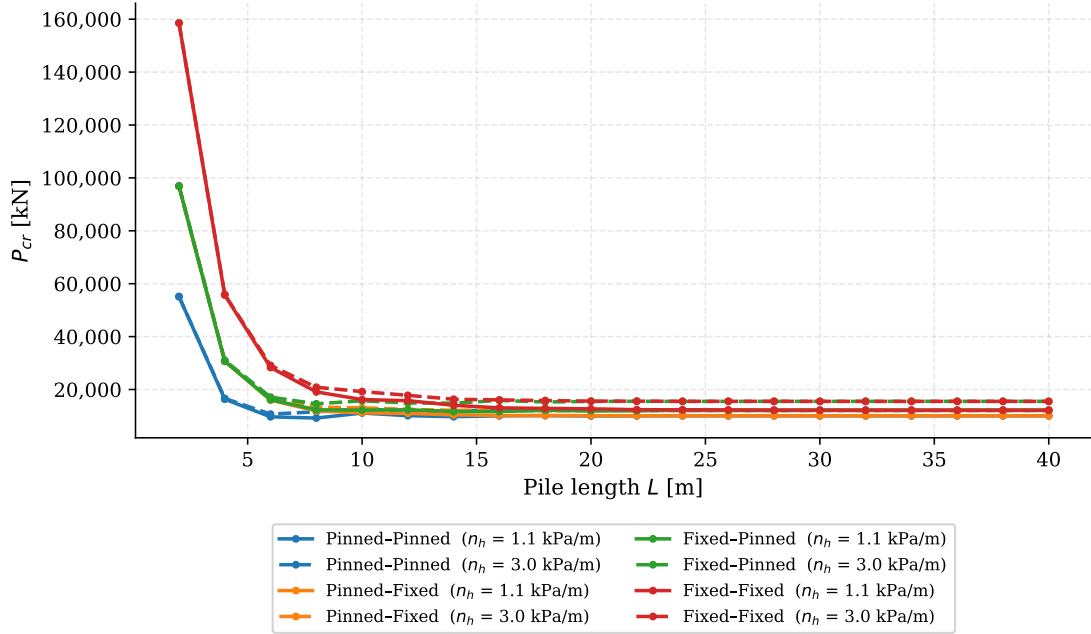
A.4.1 Lateral soil resistance k_h 

Figure A.6: Different BC-configuration response on the critical buckling load for a $c_{0u} = 12kPa$

Figure A.6 shows The variation of critical buckling load with the pile length for all BC-configurations and soil stiffness gradients $n_h = 3kPa/m$ and $n_h = 1.1kPa/m$. The figure provides further insight into the mechanism governing the reduction in BC-sensitivity discussed in section3. The following trends are observed:

1. All the BC-configurations follow the same three-regime pattern:
 - For short pile lengths, the instability response is governed primarily by structural behaviour and corresponds closely to classical Euler buckling

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad (A.9)$$

. In this regime, the surrounding soil stiffness is not yet fully mobilized, and the critical buckling load remains highly sensitive to the applied boundary conditions

- Intermediate-pile: As the pile length increases, the surrounding soil progressively contributes to the lateral restraint. The instability response can therefore be interpreted as a combined effect of the classical Euler contribution and the stabilizing influence of the surrounding soil stiffness:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} + \frac{(KL)^2 k_h}{\pi^2} \quad (A.10)$$

. Within this region, the curves progressively converge due to increasing mobilisation of the lateral soil confinement k_h .

- Long-pile regime is soil-controlled and additional increases in pile length produce only minor changes in the critical buckling load, The theoretical critical buckling load is expressed as

$$P_{cr} = 2\sqrt{EI k_h} \quad (\text{A.11})$$

2. The difference between F-F and P-P configurations is largest for short piles and progressively decreases until it stabilises, This demonstrates that the influence of rotational end restraint becomes less significant as the surrounding soil stiffness becomes increasingly mobilised.
3. The influence of soil stiffness gradients increases as the pile length increases for all BC-configurations. Thus, the influence of the soil stiffness gradient becomes more significant for longer pile lengths, where the higher soil stiffness gradient consistently leads to larger critical buckling loads for all BC-configurations.
4. Lastly, the figure confirms that the influence of restraint at the pile toe decreases more rapidly compared to the pile head, suggesting that the upper soil layers control the remaining BC-sensitivity in the long-pile regime.

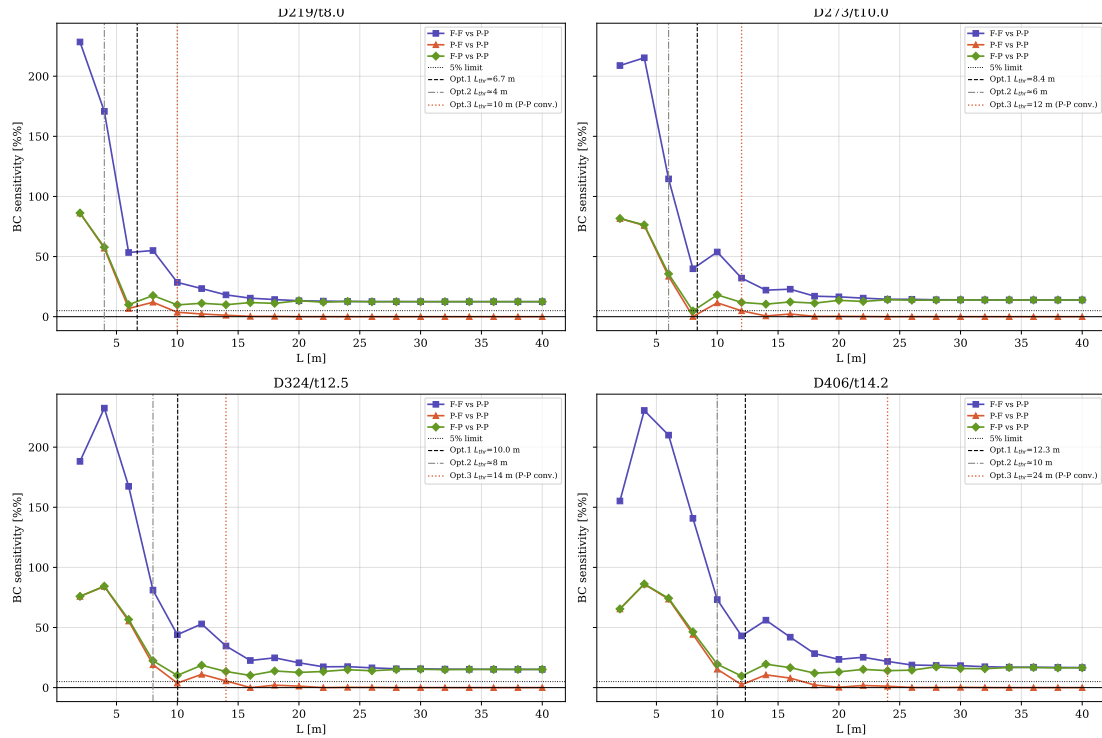


Figure A.7: BC-sensitivity on different cross sectional dimensions, $c_{0u} = 24kPa$ and $n_h = 1.1kPa$

FigureA.7 presents the variation of BC-sensitivity with pile length for four different pile cross-sections subjected only to lateral soil support k_h . To thoroughly inves-

tigate the influence of depth dependent lateral support and the transition from BC-controlled behaviour towards soil-controlled behaviour, three different threshold definitions were introduced as part of the present interpretation framework for the FEM results:

- Option 1: Constant surface undrained shear strength $c_{0u} = 24kPa$, based on the Hétényi formulation.
- Option 2: Average value of the depth-dependent lateral support $k_{h,avg}$.
- Option 3: Threshold defined directly from the FEM results as the pile length where the BC-sensitivity curves begin to converge and approach stabilized behavior..

The results obtained from figure A.7 are presented in the following table:

Table A.2: Summary of BC-sensitivity behaviour and threshold lengths for all analysed pile sections (k_h on, k_v off, $c_{u0} = 24kPa$, $n_h = 1.1kPa/m$).

Pile section	Long-pile plateau	Opt.1 sensitivity	Opt.1	Opt.2	Opt.3
	F-F vs P-P [%]	F-F vs P-P [%]	L_{thr} [m]	L_{thr} [m]	L_{thr} [m]
D273/t10.0	≈ 215	≈ 14	8.4	6.0	12
D324/t12.5	≈ 230	≈ 15	10.0	8.0	14
D406/t14.2	≈ 230	≈ 16	12.3	10.0	24

TableA.2 shows that the transitions region becomes larger as the pile flexural rigidity increases. For the smaller pile sections ($D219/t8.0$ and $D273/t10.0$), the BC-sensitivity stabilizes relatively early, whereas the larger sections ($D324/t12.5$ and $D406/t14.2$) require longer pile lengths before approaching approximately constant plateau values. This behaviour indicates that stiffer piles require larger embedded lengths before the surrounding soil stiffness becomes fully mobilized.

FigureA.7 show that the asymmetric BC-configurations $P - F$ and $F - P$ behave similarly within the short-pile regime and for all pile cross-sections, there both configurations exhibit comparable BC-sensitivity levels, indicating that the instability response is governed primarily by the global Euler-type behaviour, where both pile ends contribute similarly to the rotational restraint.

As the pile diameter increases, the separation between the $P - F$ and $F - P$ curves occurs at larger pile lengths. For the smaller pile sections, the separation starts relatively early within the transition region, while the larger sections exhibit similar behaviour over a longer length interval before separating. This behaviour indicates that larger pile diameters delays the transition from BC-controlled to soil-controlled response.

As the pile length increases, The $F - P$ configuration gradually approaches the $F - F$ curve, whereas the $P - F$ configuration approaches the $P - P$ response. This behaviour becomes particularly clearer within the long-pile regime, where the residual BC-sensitivity remains significantly larger for configurations with fixed-head restraint, and the influence of the end boundary-condition disappears.

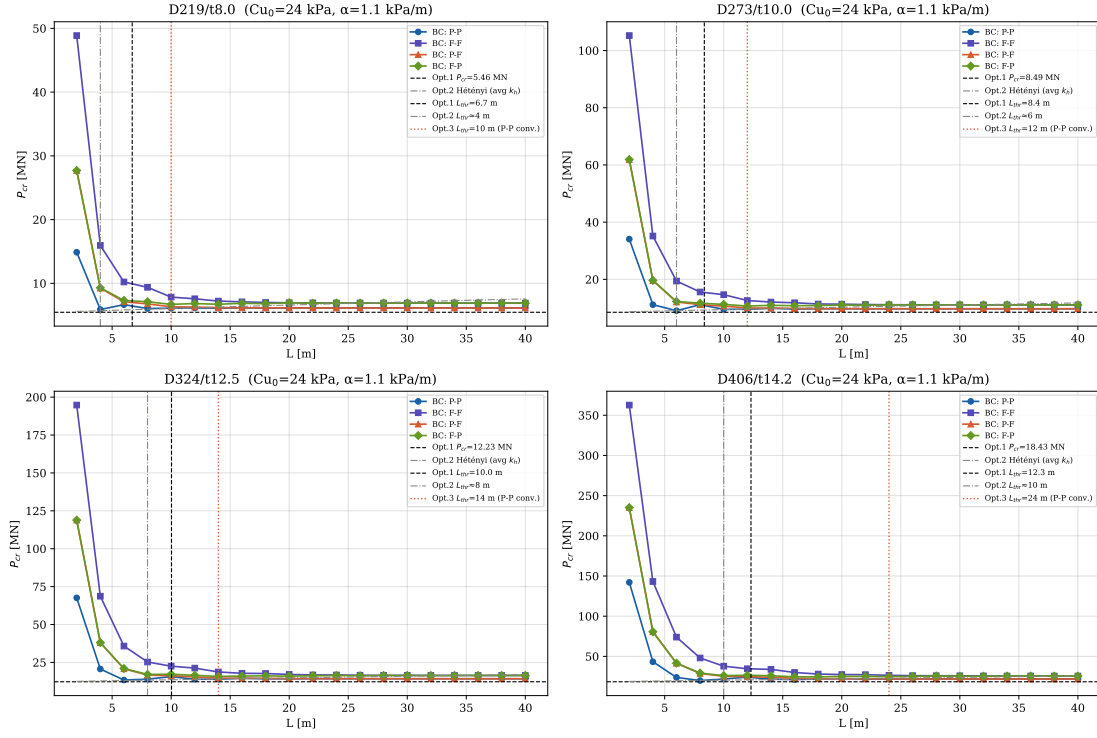


Figure A.8: Critical buckling load vs. pile length for different cross sectional dimensions

Figure A.8 shows that as the pile dimension increases, the Euler controlled response continues to be the controlling response over a longer pile length.

A.4.2 Vertical soil resistance k_v

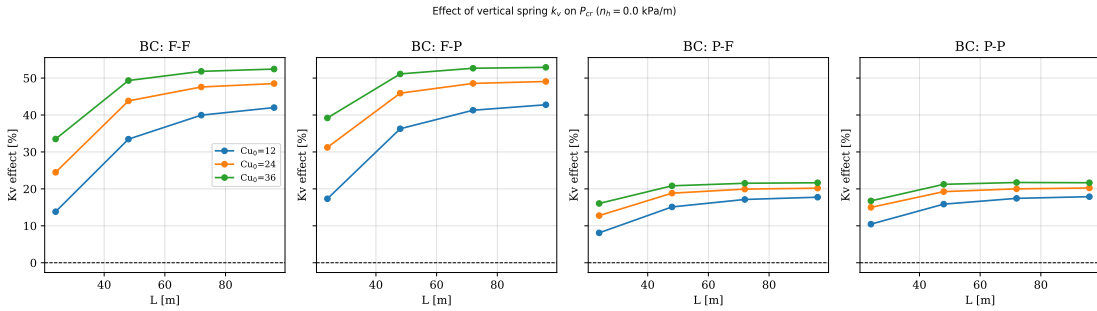


Figure A.9: Influence of vertical shaft resistance k_v on the critical buckling load

Figure A.9 presents the detailed influence of the vertical shaft resistance k_v for each BC-configurations and different surface undrained shear strength c_{0u} . This figure confirms that the k_v -springs contribution is significantly larger for configurations with fixed-head configurations than for pinned-head configurations. Increasing undrained shear strength further increases the contribution from k_v , while the curves gradually converges to plateau values for larger pile lengths, indicating progressive mobilization of the vertical shaft resistance.

A.5 EFFECTIVE LENGTH FACTOR β FOR DIFFERENT BOUNDARY CONDITIONS AND PILE LENGTHS

The following map-figures summarize the variation of the effective length factor β for different BC-configurations and pile lengths for the $D = 324\text{mm}$ and $D = 406\text{mm}$ section with $n_h = 0.0kPa$:

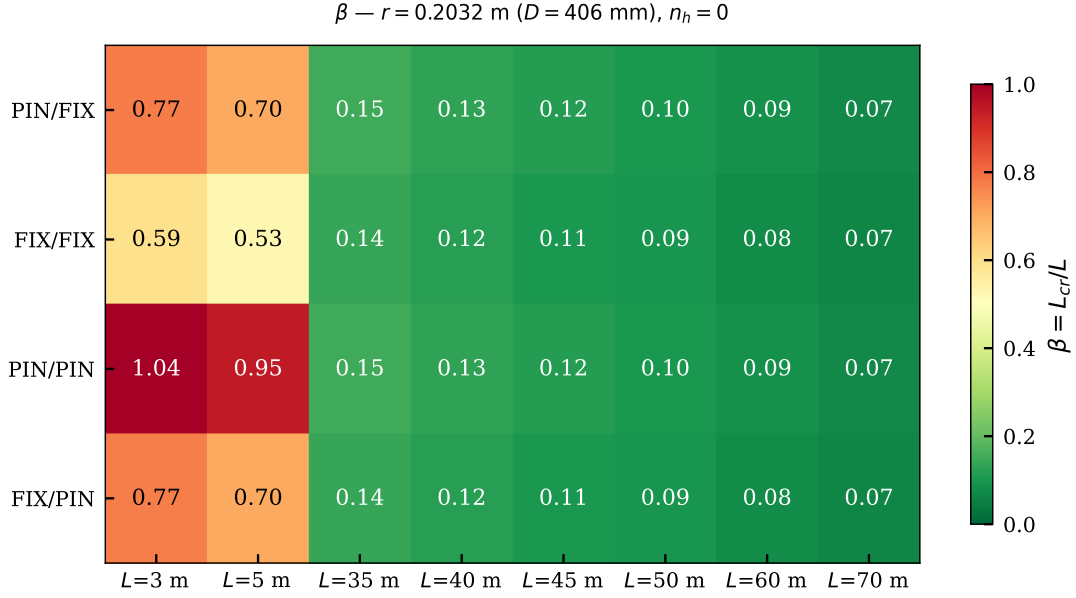


Figure A.10

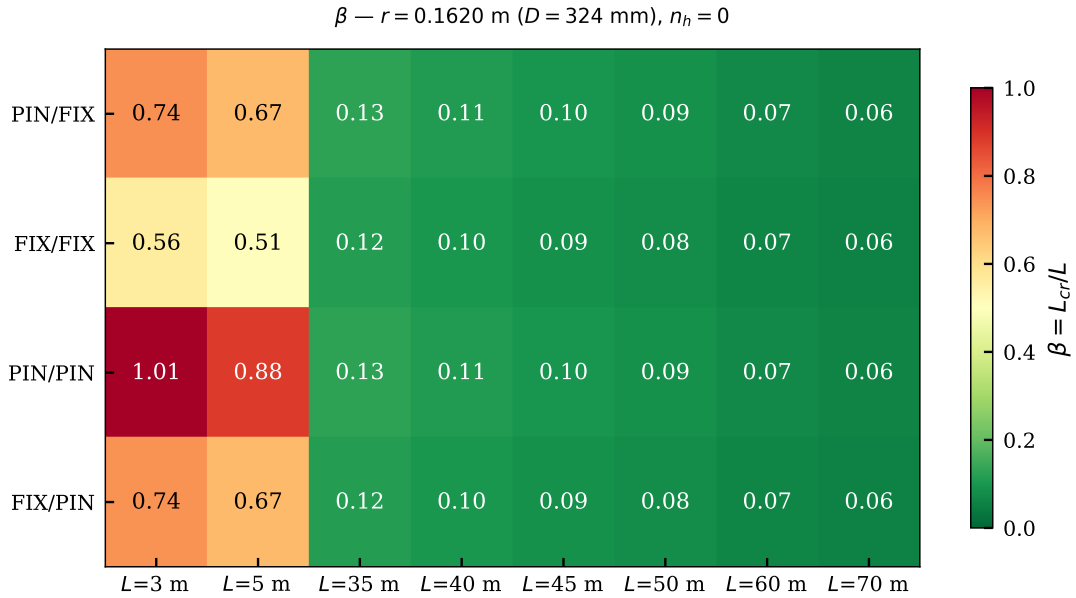


Figure A.11

The results from both map-figures show that short piles exhibit relatively large effective length factors β , particularly for the $P - P$ configuration where $\beta \approx 1$ at $L = 3$. This indicates a global buckling response that resembles the classical

Euler buckling behaviour, where the instability deformation extends over most of the pile length.

As the pile length increases, β decreases rapidly for all BC-configurations and gradually approaches low stabilised values within the long-pile regime. For both pile cross-sections, β reduces to approximately 0.10~0.13 at $L = 35\sim 40m$, after which only minor reductions occur. This behaviour indicates a gradual transition from global instability behaviour toward a localised deformation mode where the total pile length becomes less influential.

The figures further show that the influence of the boundary conditions decreases significantly with increasing pile length, as the β -values progressively converge for slender piles.

Comparison between the two pile cross-sections shows that the larger diameter section ($D = 406mm$) consistently produces slightly β -values than the $D = 324mm$ cross-section over the same pile lengths. This indicates that the larger flexural rigidity (EI) delays the localisation of the instability deformation and maintains a slightly larger active buckling region before the surrounding soil confinement becomes dominant

A.5.1 Mode shape sensitivity

Before performing the geometrically nonlinear analyses, the relevant initial imperfection shape had to be selected. Since the imperfection distribution is commonly based on the buckling mode shapes obtained from the linear buckling analysis (LBA), the first five eigenmodes were evaluated for the studied boundary-condition configurations, pile diameters, and soil profiles.

Figure A.12 and A.13 shows the normalised buckling mode shapes for $D = 219\text{ mm}$ and $D = 324\text{ mm}$, considering both a uniform soil profile and a depth-dependent profile with $n_h = 1.1\text{ kPa/m}$. For short and intermediate piles, the first eigenvalue was clearly lower than the following eigenvalues. In these cases, the first mode could therefore be identified as the governing instability mode.

For long piles, however, the behaviour was more sensitive. As the soil–structure interaction became fully mobilised, the buckling deformation became more localised and the eigenvalues of the first modes became closer to each other. Therefore, the governing imperfection shape could not be selected from the eigenvalues alone. For this reason, additional GNIA analyses were performed using Modes 1–3 as initial imperfection shapes.

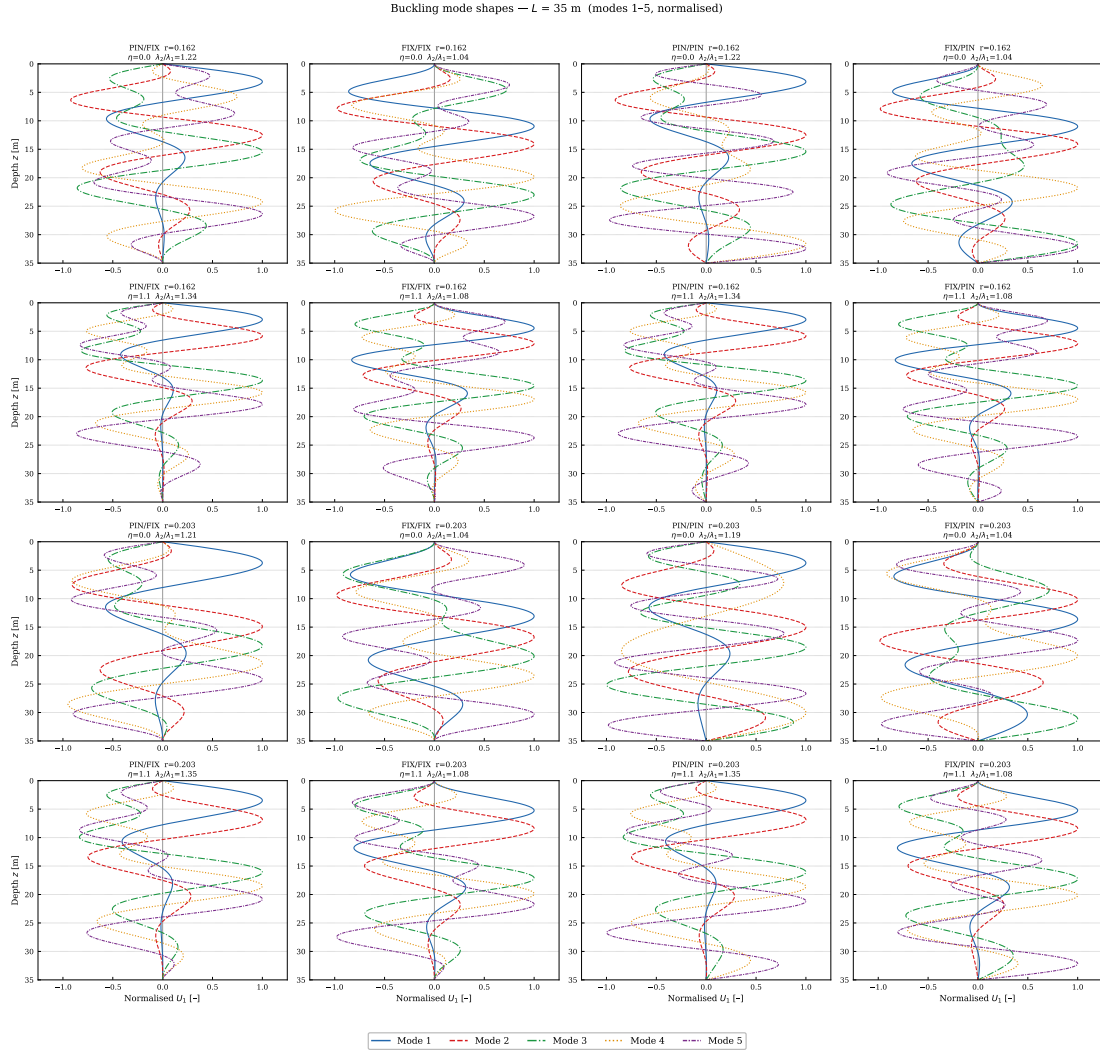


Figure A.12: Buckling mode shapes for the first five eigenmodes

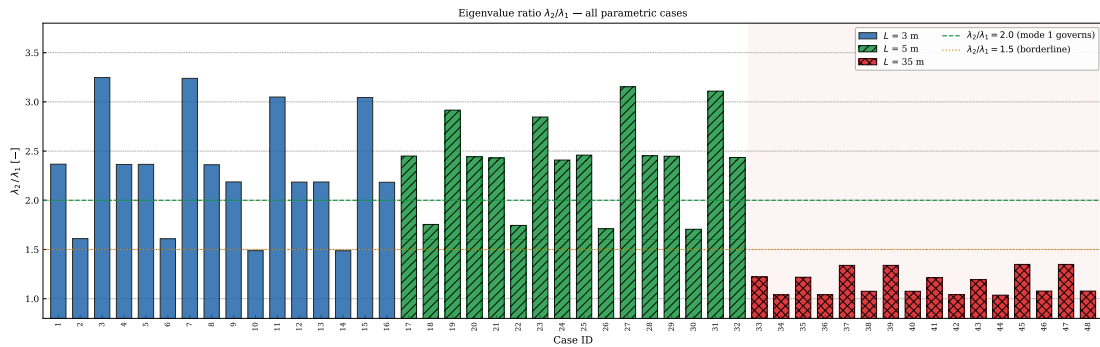


Figure A.13: Eigenvalue ratio

The GNIA analyses A.14 were performed with bilinear Winkler springs to confirm which mode shape gives the governing nonlinear response. From the nonlinear analyses, Mode 1 produced the smoothest and most stable response. The load increased gradually towards a peak value, followed by moderate post-buckling

softening. In contrast, the higher modes showed faster stiffness degradation after the onset of buckling.

Different soil properties were also examined to check whether the numerical response was physically reasonable. The results showed that stronger soil confinement gives a stiffer and more stable post-buckling response. This is expected, since stronger soil provides higher lateral resistance to the pile. The effect of the imperfection level was also clear. A smaller imperfection, such as $L_{cr}/1000$, gave a stiffer response and allowed a higher load level. A larger imperfection, such as $L_{cr}/300$, caused earlier lateral deformation and reduced the peak load. Based on these observations, Mode 1 was selected as the governing imperfection shape for the following GNIA analyses.

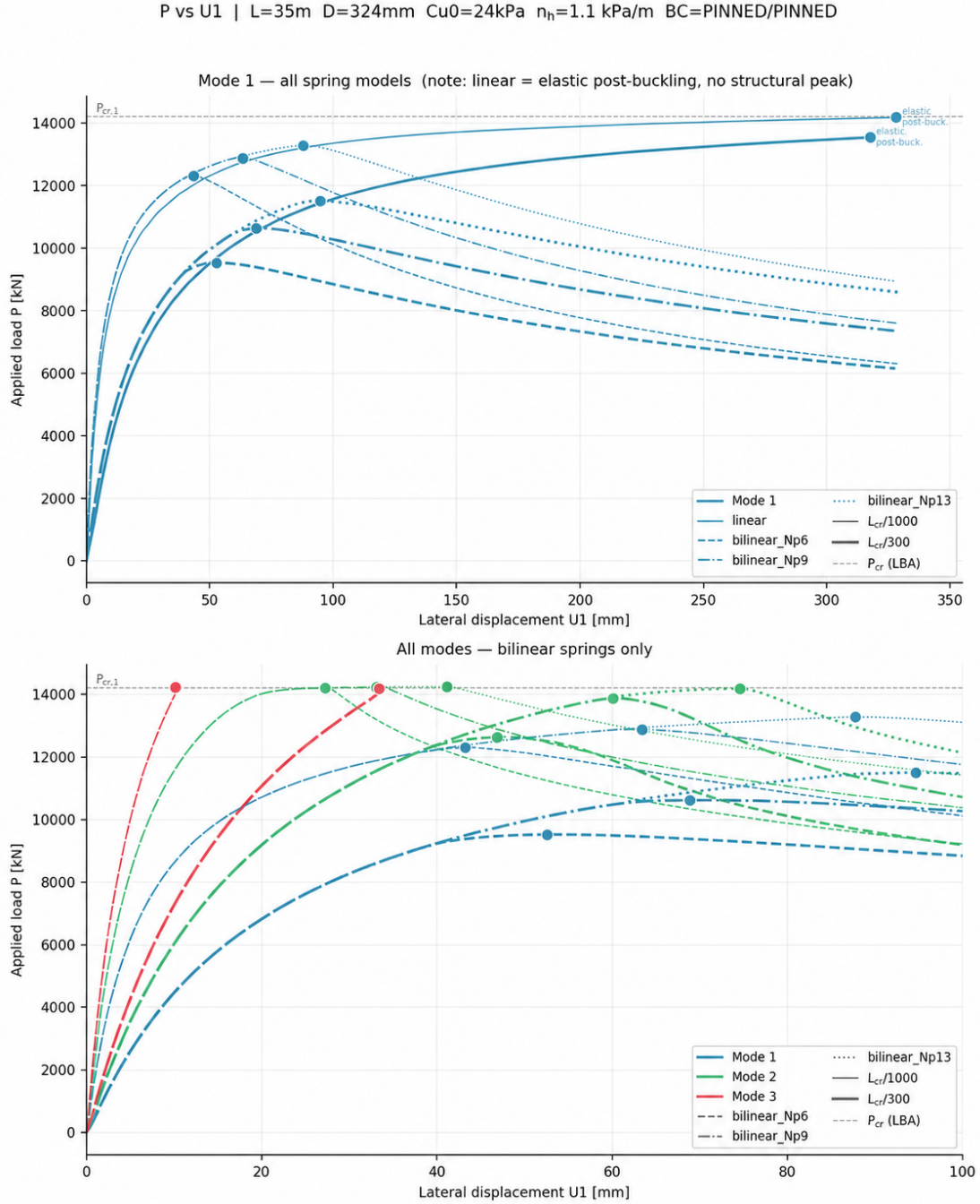


Figure A.14: GNIA load–displacement response

A.5.2 Imperfection effect and knockdown of the critical load

After selecting the relevant imperfection shape, the influence of imperfection amplitude on the nonlinear failure load was evaluated. The analyses were performed for long piles with $L = 25$ m and $L = 35$ m, two pile diameters, and different soil conditions. The soil parameters included variations in undrained shear strength, stiffness gradient, and lateral bearing capacity factor. All cases in this part were analysed using pinned–pinned boundary conditions and bilinear Winkler springs.

In this study, the knockdown factor, KD , is used to quantify the reduction in failure load caused by initial geometric imperfections and bilinear soil behaviour. It is defined as the ratio between the peak load obtained from the nonlinear Riks analysis and the corresponding critical load obtained from the linear buckling analysis. The Riks analysis includes the prescribed initial imperfection and the elasto-plastic soil spring response, while the LBA result represents the ideal elastic critical load of the perfect system. The knockdown factor is therefore expressed as

$$KD = \frac{P_{\text{Riks}}}{P_{\text{LBA}}}, \quad (\text{A.12})$$

where P_{Riks} is the peak load from the nonlinear analysis and P_{LBA} is the critical load from the LBA. A value of $KD = 1.0$ indicates that the nonlinear imperfect model reaches the same load as the ideal LBA model. Values lower than 1.0 indicate a reduction in the load-carrying capacity.

Figure A.15 shows the knockdown factor as a function of the imperfection amplitude. The main observation is that KD decreases as the imperfection amplitude increases. This confirms that the initial geometric imperfection has a strong influence on the failure load. No clear plateau is observed within the investigated range, which means that the pile remains sensitive to increasing imperfection amplitude.

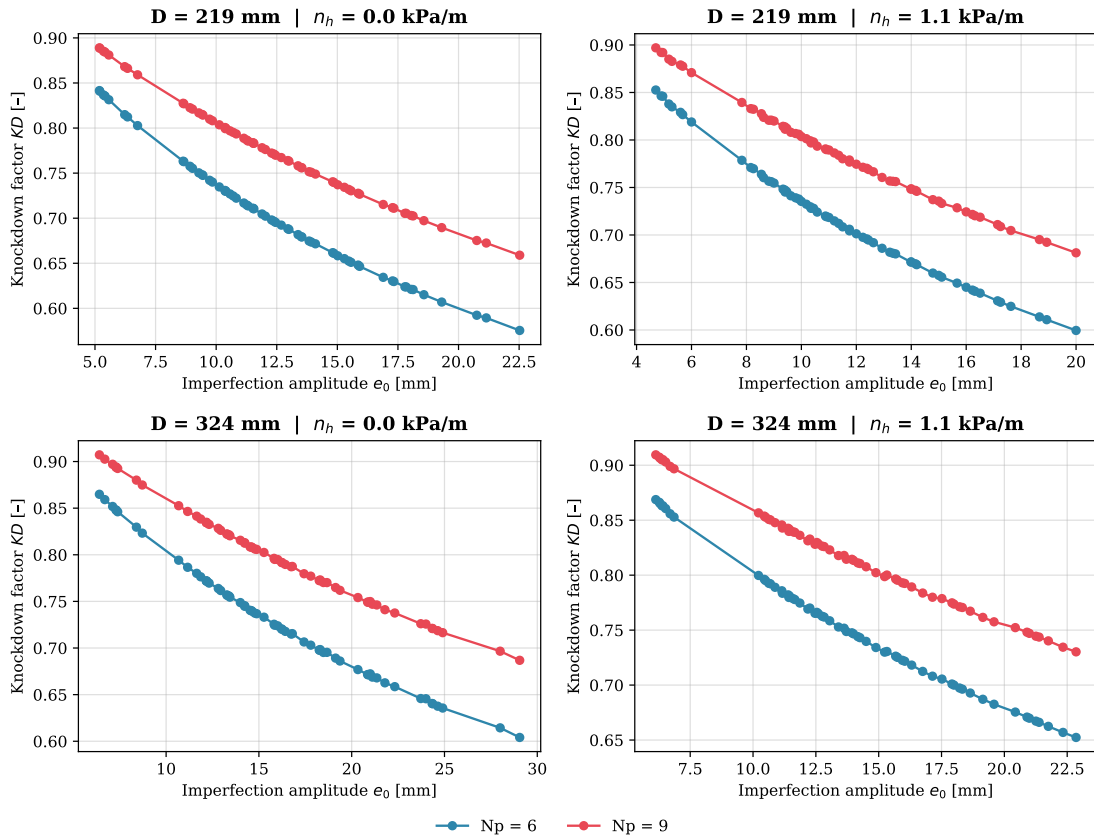


Figure A.15: Knockdown factor KD as a function of geometric imperfection amplitude e_0 .

Figure A.16 compares the influence of the main studied parameters. Increasing the lateral bearing capacity factor from $N_p = 6$ to $N_p = 9$ increases the knockdown factor. This is expected, since a higher N_p gives a larger limiting lateral soil resistance. However, the curves remain almost parallel, indicating that N_p mainly shifts the response upward without strongly changing the rate of reduction with imperfection amplitude.

The influence of the soil stiffness gradient, n_h , is more limited. The curves keep nearly the same shape and slope when the gradient changes. This indicates that the imperfection amplitude is the dominant parameter controlling the reduction in failure load, while N_p has a clear secondary effect. The influence of soil gradient, pile length, and cross-section is smaller within the investigated range.

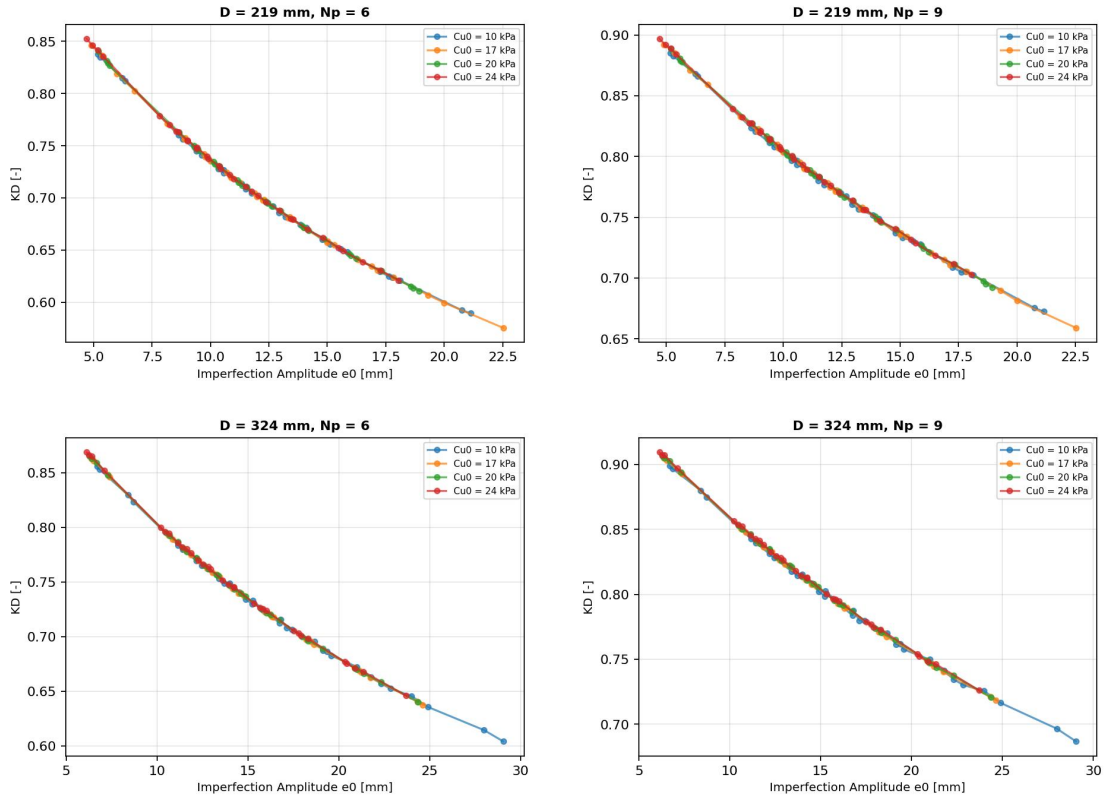


Figure A.16: Influence of D , N_p , n_h , and c_{u0} on the knockdown factor KD .

““

A.5.3 Analytical method and basis for comparison

The FEM peak loads obtained from the Riks analyses are compared with an analytical method based on the Swedish design approach for steel piles in soft clay. The method follows the principles used in the Swedish pile commission and the classical beam-on-elastic-foundation formulation, where the pile is idealised as a beam supported by lateral soil resistance [26, 15]. The purpose of the comparison is not to reproduce the full structural design resistance of the pile, but rather to

compare the instability response obtained from the simplified analytical method with the response obtained from the numerical model.

In the original Swedish design procedure, the pile response is evaluated by combining the buckling behaviour with the structural resistance of the steel section. This means that the final capacity may be limited either by instability or by the steel yield limit expressed through the axial force–bending moment interaction [17, 15]. In the present study, however, this steel yield limit is intentionally removed from the analytical evaluation. The aim is to isolate the peak load associated with instability while the steel remains linear elastic and the surrounding soil is allowed to behave elasto-plastically. This makes the analytical result directly comparable to the numerical analyses, in which the pile material is modelled as linear elastic and the soil response is represented by bilinear Winkler springs.

A key difference between the two approaches is the way in which the initial geometric imperfection is introduced. In the Swedish analytical method, the initial imperfection is represented by a sinusoidal half-wave distributed over the critical buckling length, L_{cr} [15]:

$$y_{\text{imp}}(z) = e_0 \sin\left(\frac{\pi z}{L_{cr}}\right), \quad (\text{A.13})$$

where e_0 is the imperfection amplitude and z is the coordinate measured along the buckling zone. In the FEM model, the imperfection is instead introduced by scaling the first buckling mode shape obtained from the linear buckling analysis (LBA). The comparison therefore includes not only the load level, but also the effect of using a sinusoidal imperfection in the analytical method and a mode-shape-based imperfection in the FEM model.

The analytical prediction is obtained using the following sequence, while maintaining the steel pile in the elastic range throughout the calculation.

Step 1 – Cross-section properties. The pile bending stiffness EI is determined from the steel pipe geometry and the elastic modulus of steel. In the original Swedish design framework, the steel resistance may subsequently be checked against a structural yield limit [17, 15]. In the present study, however, this yield limit is not applied, since the objective is to isolate the elastic-steel instability response. Therefore, the analytical failure load is denoted as $N_{\text{sw,el}}$, where “sw” refers to the Swedish analytical approach and “el” indicates that the steel material is kept linear elastic.

Step 2 – Representative lateral soil stiffness. In the Swedish analytical approach, the lateral soil support is represented by a constant bedding modulus. Therefore, a representative value is adopted instead of a depth-dependent stiffness. For long-term loading in clay, the bedding modulus is expressed as [15]

$$k_{k,\text{rep}} = \frac{\alpha_2 c_{u0}}{D}, \quad (\text{A.14})$$

where $k_{k,\text{rep}}$ is the representative bedding modulus, c_{u0} is the undrained shear strength at the ground surface, D is the pile diameter, and $\alpha_2 = 50$ is the long-term

stiffness coefficient 2.8.2. The corresponding distributed lateral spring stiffness per unit pile length is then obtained by multiplying the bedding modulus by the pile diameter:

$$k_{h,\text{rep}} = k_{k,\text{rep}} D = \alpha_2 c_u 0. \quad (\text{A.15})$$

This representative stiffness is used in the analytical buckling calculation in order to remain consistent with the Swedish design formulation. It should be noted that this stiffness only defines the initial elastic part of the lateral soil response. The mobilised soil reaction must therefore be limited by the ultimate lateral soil resistance, q_{lim} , in accordance with the bilinear soil model 2.8.2. Thus, the lateral soil reaction is not allowed to increase indefinitely with displacement, but is capped once the limiting resistance is reached.

Step 3 – critical load. The elastic critical load of a geometrically perfect pile on an elastic Winkler foundation is obtained from the classical solution for a beam on an elastic foundation [26]:

$$N_{\text{cr,H}} = 2\sqrt{EI k_{h,\text{rep}}} = 2\sqrt{EI k_{k,\text{rep}} D}. \quad (\text{A.16})$$

This load represents the analytical critical load of the perfect pile and serves as the reference level for the subsequent imperfection analysis.

Step 4 – Soil yield displacement and limiting soil resistance. The bilinear soil model assumes that the lateral soil response remains elastic up to a limiting displacement, after which the soil resistance reaches a plastic plateau. In the Swedish formulation, this limiting displacement is defined as the ratio between the limiting lateral soil pressure and the bedding modulus [15]:

$$y_{\text{lim}} = \frac{q_k}{k_{k,\text{rep}}}. \quad (\text{A.17})$$

For long-term loading in clay, the Swedish approach gives ($q_k = 6c_u$) and ($k_k = 50c_u/D$). This gives

$$y_{\text{lim}} = \frac{6c_u}{50c_u/D} = 0.12D. \quad (\text{A.18})$$

In the present study, the lateral bearing capacity factor is also varied in order to investigate the sensitivity to the limiting lateral soil resistance. The limiting pressure is therefore written in the general form

$$q_k = N_p c_u, \quad (\text{A.19})$$

which gives

$$y_{\text{lim}} = \frac{N_p c_u}{\alpha_2 c_u / D} = \frac{N_p D}{\alpha_2}. \quad (\text{A.20})$$

For the reference long-term Swedish case, $N_p = 6$ and $\alpha_2 = 50$, which again gives y_{lim} . In this study, both $N_p = 6$ and $N_p = 9$ are considered in order to investigate the sensitivity to the assumed lateral soil resistance.

It should be noted that the representative spring stiffness $k_{h,\text{rep}}$ only defines the initial elastic part of the soil response. The mobilised lateral reaction must be limited by the ultimate lateral soil resistance, q_{lim} , in accordance with the bilinear soil model adopted in 2.8.2. Consequently, the soil reaction increases linearly with displacement only up to the limiting resistance, after which a plastic plateau is reached:

$$q(y) = \begin{cases} k_{h,\text{rep}} y, & |k_{h,\text{rep}} y| \leq q_{\text{lim}}, \\ \pm q_{\text{lim}}, & |k_{h,\text{rep}} y| > q_{\text{lim}}. \end{cases} \quad (\text{A.21})$$

Step 5 – Load–deflection trajectory. The axial load carried by the imperfect pile is described as a function of the lateral deflection (y) by a Swedish design-type buckling trajectory [15]:

$$N_{\text{Ek}}(y) = N_{\text{cr,H}} \Phi(y) \left(1 - \frac{e_0}{e_0 + y} \right). \quad (\text{A.22})$$

The term $\left(1 - \frac{e_0}{e_0 + y} \right)$ describes the influence of the initial geometric imperfection. The function $\Phi(y)$ accounts for the reduction in effective lateral restraint after the soil yield displacement is exceeded. In the simplified implementation used in this study, this reduction is represented by the ratio between the yield displacement and the current lateral displacement:

$$\Phi(y) = \begin{cases} 1, & y \leq y_{\text{lim}}, \\ \frac{y_{\text{lim}}}{y}, & y > y_{\text{lim}}. \end{cases} \quad (\text{A.23})$$

This approximation gives full elastic support before soil yielding and a reduced effective support after the limiting displacement is exceeded. As a result, the load–deflection curve increases initially, reaches a peak, and then decreases as the effective lateral soil support is reduced.

Step 6 – Analytical peak load with elastic steel. In the original Swedish design method, the load trajectory may subsequently be combined with a steel yield limit based on the axial force–bending moment interaction of the pile section [17, 15]. In the present study, this step is intentionally omitted. Instead, the analytical quantity used for comparison is defined as the maximum value of the trajectory while the steel remains elastic:

$$N_{\text{sw,el}} = \max_{y \geq 0} N_{\text{Ek}}(y). \quad (\text{A.24})$$

This value represents the highest axial load that the pile can carry in the analytical

model before instability develops, under the assumption of linear elastic steel and elasto-plastic soil support.

The analytical peak load $N_{sw,el}$ is compared directly with the FEM peak load P_{peak} , obtained from the Riks analysis. In both cases, the quantity of interest is the maximum axial load associated with instability while the pile material remains elastic. The soil is allowed to yield through a bilinear spring formulation in both approaches. The main differences are therefore the level of idealisation and the representation of the initial imperfection.

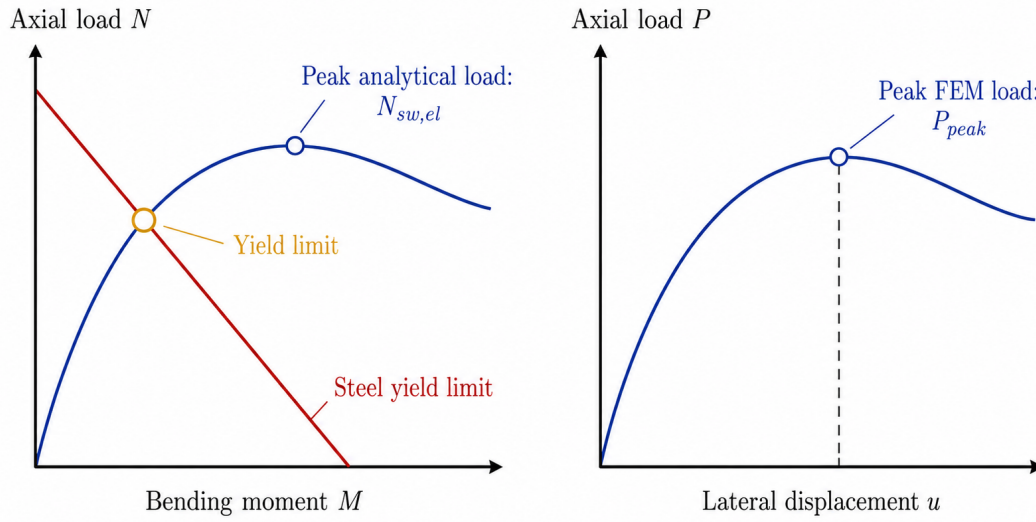


Figure A.17: Conceptual comparison of the analytical and numerical approaches.