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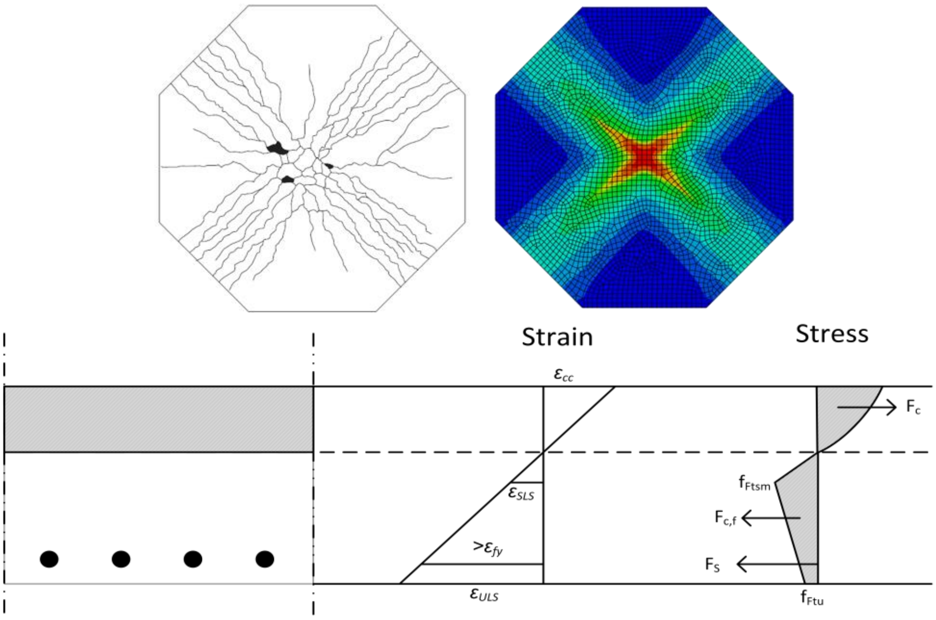

Evaluation of analysis methods for conventional and steel fibre reinforced concrete slabs

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

KARL LEVIN, TIM NILSSON

Department of Civil and Environmental Engineering
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Cover:
Up left: Crack pattern from experimental test of conventional and fibre reinforced concrete slab

Up right: Concrete strain ABAQUS solid elements for steel fibre reinforced concrete with conventional reinforcement.

Down: Cross section analysis for steel fibre reinforced concrete in ultimate state with FIB Model Code linear model.

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ABSTRACT

FEM-design, ABAQUS, strip method and yield-line analysis were used to analyse an octagonal concrete slab simply supported along four edges considering the response and ultimate load. The slab was cast with three different types of reinforcement configurations: conventional reinforcement, fibre reinforcement and a combination of the two. For the hand calculations including fibres FIB Model Code was used. The aim was to compare the methods with regard to work effort and accuracy needed at a design office. The work effort was a combination of consumed time and how complicated the method was perceived.

In FEM design there were limited possibilities to create a fibre reinforced material resulting in an alternative way to implement steel fibres as increased reinforcement area. To model the fibre reinforcement this way did not give sufficiently good results compared to the alternative programs and models, it also required more extensive work.

The most accurate result for the finite elements models was obtained from ABAQUS shell model, while the results obtained from FEM-design shows less accurate results. The best results for the ultimate load were obtained from hand calculations with yield line analysis.

The recommendations considering analysis of fibre reinforced and conventionally reinforced concrete would be to use ABAQUS shell for complicated geometries where bending failure is likely; for cases where time is the most crucial factor FEM-Design would be recommended.

Key words: FIB Model Code, ABAQUS, FEM-Design, steel fibre reinforced concrete, yield line analysis, strip method, two way slab, non-linear analysis

Utvärdering av analysmetoder för konventionellt- och fiberarmerade betongplattor

Examensarbete inom Structural Engineering and Building Technology

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SAMMANFATTNING

FEM-Design, ABAQUS, strimlemetoden och brottlinjemetoden användes för att analysera en åttkantig betongplatta fritt upplagd på fyra stöd med avseende på respons och kritisk last. Tre olika typer av armering studerades: konventionell armering, stålfiberarmering och en kombination av de två. För handberäkning av fiberbetong användes FIB Model Code. Syftet var att jämföra metoderna med avseende på arbetsinsats och den noggrannhet som behövs på en konstruktionsfirma. Arbetsinsatsen utvärderades med avseende på komplexiteten och den använda tiden.

I FEM-Design fanns det begränsade möjligheter att modellera fiberarmerad betong, därför användes en ökad armeringsmängd för att simulera bidraget från fibrerna. Att modellera fiberarmeringen på detta sätt gav inte tillräckligt bra resultat jämfört med de alternativa programen och modellerna.

Resultaten visar att av FEM-programen erhölls de bästa resultaten från ABAQUS med skalelement, medan resultaten från FEM-Design gav mindre exakta resultat. De bästa resultaten med avseende på kritisk last erhölls från handberäkningar med brottlinjemetoden.

Rekommendationer för analys av konventionellt- och fiberarmerad betong är att använda ABAQUS med skalelement för komplicerade geometrier när böjbrott förväntas och FEM-Design när tidsåtgång är en avgörande faktor.

Nyckelord: FIB Model Code, ABAQUS, FEM-Design, stålfiberarmerad betong, brottlinjemetoden, strimlemetoden, två-vägsplatta, olinjär analys

Contents

Preface

The project has been carried out from January 2013 to June 2013, at the department of civil and environmental engineering, Chalmers University of technology, in collaboration with NCC Teknik. The project was divided within the group; Karl's main focus was the hand calculations while Tim was responsible for the finite element modeling.

This master thesis was related to the research project TailorCrete funded from the European Community´s Seventh Framework Programme under agreement NMP2-LA-2009-228663. More information on TailorCrete can be found at www.tailorcrete.com.

We would like to thank our supervisors Linda Cusumano at NCC Teknik, Ph.D. student David Fall and our examiner Karin Lundgren at Chalmers for their help and guidance with our master thesis.

Notations

Roman upper case letters

$A_{c,ef}$	is effective tension concrete area
A_s	is the reinforcement area per meter
$A_{s,new}$	is the new reinforcement area with contribution from fibres
$CMOD_1$	is crack mouth opening displacement, equal to 0.5 mm
$CMOD_3$	is crack mouth opening displacement, equal to 2.5 mm
E_c	is the young's modulus for concrete
E_s	is the young's modulus for steel
F_j	is the load corresponding with $CMOD = CMOD_j$
L	is the span length
M_{CL}	is moment due to concentrated load
M_R	is total moment resistance
M_{SF}	is moment due to self-weight
P	is the point load
W	is the total load on a plate segment

Roman lower case letters

b	is the width RILEM beam
c	is the reinforcement cover thickness
d	is the distance from centre of reinforcement to top the fibre
h_r	is the crack band width
h_{sp}	is the distance between notch tip and the top of RILEM beam
f_{ck}	is the characteristic compressive strength for concrete
f_{ct}	is the tensile strength for concrete
f_{Ftsm}	is the serviceability residual strength
f_{Ftu}	is the ultimate residual strength
f_{Rj}	is the residual flexural tensile strength corresponding with $CMOD = CMOD_j$
f_{R1}	is the residual flexural strength corresponding to $CMOD_1$
f_{R3}	is the residual flexural strength corresponding to $CMOD_3$
f_s	is stress in steel over the yield limit
f_u	is the reinforcement ultimate strength
j	is the distance between service strain and ultimate strain

k_1	is a factor crack distance
k_2	is a factor crack distance
l	is the span length of the RILEM beam
l_{cs}	is the structural characteristic length
l_y	is the length of the yield line
m_b	is the bending moment per unit length of yield line
m_{Rd}	is the ultimate moment resistance for SFRC
$m_{ultimate}$	is the moment capacity in ultimate state in
$m_{ultimate.x}$	is the ultimate moment in x-direction
$m_{ultimate.y}$	is the ultimate moment in y-direction
m_x	is the moment resistance in x-direction
m_y	is the moment resistance in y-direction
q	is the uniformly distributed load on an element
q_x	is the load carried by the strip in x-direction
q_y	is the load carried by the strip in y-direction
s_{rm}	is the main distance value between cracks
w	is the deformation
w_u	is the ultimate crack width
x	is the height of the compressed concrete zone
y	is the distance between neutral axis and tensile side of the cross section
z	is the distance between neutral axis and position of service strain

Greek lower case letters

α	is stress block factor
α_{ri}	is the moment capacity ratio between x- and y-direction
β	is stress block factor for ultimate state
δ	is the virtual displacement of element
Δ_c	is the deflection of the centroid of that segment
ε_1	is strain for effective concrete area
ε_1	is strain for effective concrete area
ε_{Fu}	is the maximum tensile strain in the steel fibre reinforced concrete
ε_s	is the reinforcement strain
ε_{SLS}	is the service strain for SRFC
ε_{su}	is the reinforcement ultimate strain

ε_{sy}	is the reinforcement yield strain
ε_{ULS}	is the ultimate strain for SRFC
η	is stress block factor for ultimate state
θ	is the angle change at the yield line corresponding to the virtual displacement
λ	is stress block factor for ultimate state
$\rho_{s,ef}$	is the effective reinforcement area, $\frac{A_s}{A_{c,ef}}$
ρ_c	is the density of concrete
ρ_s	is the density of reinforcement
φ	is the angle between yield line and reinforcement
Φ_s	is the diameter of one reinforcement bar

Abbreviations

CEB	Euro-International Concrete Committee
CMOD	Crack mouth opening displacement
FEM	Finite element method
FIB	fédération Internationale du béton
FIP	International Federation for Prestressing
FRC	Fibre reinforced concrete
SFRC	Steel fibre reinforced concrete
SP	Technical Research Institute of Sweden

1 Introduction

Time and money are significant factors influencing the decisions made at a designing office concerning what analysis model or program to use. Hence methods and suggestions to improve and streamline the work are of interest. In connection with tests performed at Chalmers University of Technology, a study of how to model and analyse bending in conventionally and steel fibre reinforced concrete slabs were investigated. This study is focused on methods used at a Swedish design office.

The use of fibre reinforced concrete (FRC) in structures has increased during the last decade. Today there is no straight forward approach to include steel fibre reinforcement neither in design nor in analysis. The construction industry is interested in methods of modelling the effect of steel fibres in concrete, especially in finite element software. Furthermore, it is of interest to know if an advanced analysis method is required or if a simplified faster method gives sufficiently accurate result.

Tests of steel fibre reinforced concrete are rarely done at a structural level, but during the spring 2013, Chalmers University of Technology performed such tests. Several concrete slabs with different type of reinforcement were tested. The slabs were reinforced with conventional steel reinforcement, steel fibre reinforcement and a combination of these two.

In collaboration with a Swedish design office and with guidance from Chalmers there was a possibility to test different calculation programs and methods used in practice for analysis of reinforced concrete slabs and compare with the results obtained. There was also a possibility of trying to model fibre reinforcement in commercial finite element software.

1.1 Aim

The aim of the project was to evaluate results obtained from commercial finite element software and analytical methods, considering work effort and accuracy of the model. The project also aimed to find an approach/method to include fibre reinforcement in commercial software used at a design office.

1.2 Method

Literature studies were carried out to search for analytical models to analyse bending of reinforced concrete slabs but also different methods to use finite elements software to carry out the analysis. The investigated analytical methods were strip method and yield line method. The investigation includes the properties and behaviour of conventional and steel fibre reinforced concrete during loading.

The efficiency of the methods was evaluated with regard to work effort and accuracy. The work effort was measured by the time it took to create a finite element model or analyse a slab by hand calculation. The work effort estimation also includes a subjective impression on difficulty.

The different slabs were analysed with the strip method, yield line theory and the finite element programs ABAQUS and FEM-Design, using different assumptions and material specific data. For the slabs with steel fibre reinforcement the contribution from the fibres had to be included. For analytical hand calculation FIB Model Code were used to model the fibre contribution, with ABAQUS it was implemented in the material model and for FEM-Design it was modelled as an increased reinforcement area. Specific material data for the modelled slabs were obtained through material testing performed at Chalmers and Technical Research Institute of Sweden (SP).

The results from finite element models and analytical models were compared with the experimental results from Chalmers. For finite element models the crack pattern, reaction forces, ultimate load, and the response were compared and for the analytical solutions the ultimate load, reaction forces, and the response. To obtain the most accurate results material data from experiments were used.

1.3 Limitations

The evaluation was limited to the commercial finite element software ABAQUS and FEM-Design and the analytical methods yield line analysis and strip method. The analysis was limited to bending of reinforced and steel fibre reinforced concrete slabs. For the steel fibre reinforcement it was decided to use FIB Model Code. It was decided to limit the analytical hand calculations to one strip method model for the plates containing fibre reinforcement.

2 Background theory and tests

This chapter describes the general behaviour of fibre reinforced concrete and the analysed slab experiments.

2.1 Fibre reinforced concrete

Concrete is the most commonly used material in building structures. In itself concrete is a very brittle anisotropic material with high compressive strength and low tensile strength. Reinforcement is used to improve tensile properties; the most commonly used reinforcement is steel bars. Fibre reinforcement has been used in structures for a long time but the use is still low in comparison to conventional reinforcement bars.

Fibres can consist of various material, shapes and size. The most commonly used in structures is steel fibres which were used in this study. In fibre reinforced concrete the fibres are randomly distributed in the matrix.

2.1.1 Influence of fibre reinforcement

Concrete without any kind of reinforcement has low or no capacity after cracking. By adding conventional reinforcement bars a strain hardening behaviour can be obtained, meaning that the reinforced concrete has an increased bearing capacity after cracking. Concrete with only steel fibre reinforcement can show both strain hardening and strain softening behaviour. Strain hardening behaviour for steel fibre reinforcement is only possible if enough fibres are used, approximately about 2% by volume (Jansson, 2011). High fibre content generally requires modified production techniques, normally the amount of fibres in concrete structures is 0.25 – 1.0 % by volume (Löfgren, 2005). In Figure 2.1 the difference between plain concrete and steel fibre reinforced concrete in tension is seen.

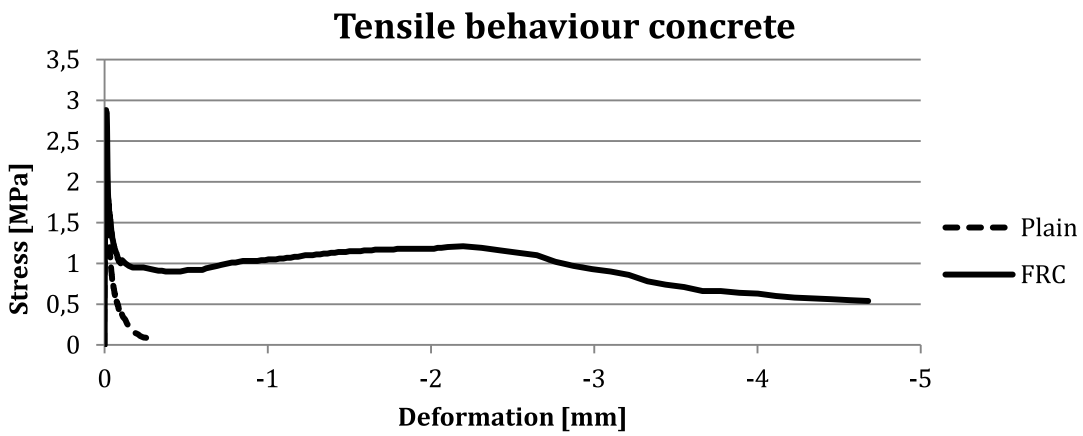

Figure 2.1 Stress–deformation relation for plain and steel fibre reinforced concrete (Rempling et al., 2013).

Steel fibres in concrete will carry some of the load before cracking initiates, the rest is carried by the matrix. It should be possible to increase the strength of the material by adding steel fibres with a higher modulus than the matrix, but experimental studies have shown that that normally used fibre volume in concrete does not lead to an increased strength before cracking (Löfgren, 2005). The major role of the steel fibre reinforcement is to control the cracking of the concrete and give contribution to the capacity after cracking (Bentur & Mindess, 1990). Benefits of steel fibre reinforced concrete are the improved flexural toughness, impact resistance and flexural fatigue endurance. Fibre reinforcement generates improved crack control, decreased crack distance and crack width for concrete (Löfgren, 2005). The tensile deformation capacity is improved, resulting in increased critical crack opening. The critical crack opening is defined as crack opening where no stress can be transferred, illustrated in Figure 2.2.

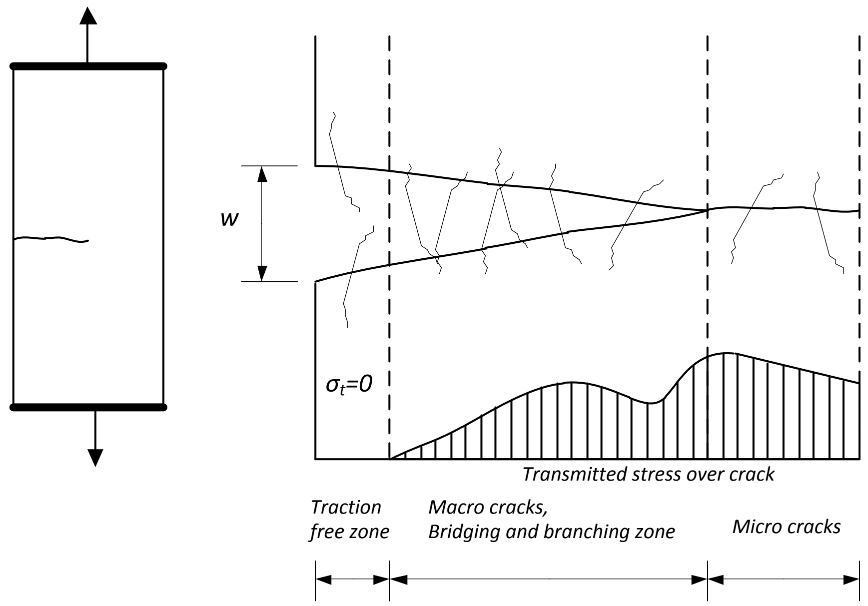

Figure 2.2 Steel fibre reinforced concrete cracking zones (Löfgren, 2005).

The traction free zone Figure 2.2 is where no stress can be transferred over the crack. With fibres, this zone occurs for relatively large crack openings. In the bridging and branching zone stresses are transferred over the crack by fibre pull-out, fibre tension and aggregate bridge. Micro cracking can start before the load is applied and they usually occur in the transition zone between aggregates and cement paste. The micro cracks can be caused by concrete drying shrinkage. Ordinary fibres can only carry load when macro cracks have occurred, which is approximately at 0.05 mm (Löfgren, 2005).

Factors that influence the performance of the steel fibre reinforced concrete is the physical properties of fibres and matrix, the bond strength between fibres and matrix, the amount of fibres, their distribution and orientation (ACI Committee 544, 2002).

2.1.2 Compressive properties

Concrete is strong in compression and until 30% of its ultimate compressive capacity concrete exhibits linear-elastic behaviour, followed by gradual softening up to the concrete compressive ultimate strength. When the compressive ultimate strength is reached the concrete exhibits strain softening behaviour until failure by crushing (Löfgren, 2005).

During compression small tensile cracks can occur in weak zones by frictional sliding between aggregates and cement paste. The tensile cracks propagate with increased compression and become parallel to the direction of the principle compressive stress (Vonk, 1992).With a fibre volume larger than 1% it is possible to reduce this kind of cracks, resulting in increased compressive strength. For normal amount of steel fibre reinforcement the effect on compressive behaviour is generally negligible (ACI Committee 544, 2002).

2.1.3 Application

Due to the properties of steel fibre reinforced concrete the use has increased in flat slabs on grade, where resistance against high load and impact are important. It is also used in shotcrete applications for ground support, rock slope stabilization, tunnelling and repairs (ACI Committee 544, 2002). In structures where the crack control is of importance steel fibre reinforcement can be used as secondary reinforcement, meaning that the fibres are combined with conventional reinforcement (Bentur & Mindess, 1990). Fibre reinforcement reduces the labour cost and production time where it is possible to reduce, or even replace, conventional reinforcement.

Combining steel fibres and self-compacting concrete makes the fibres more evenly distributed. The self-compacting concrete increases the bond strength between fibres and concrete (Löfgren, 2004).

Today there are no standards for design rules of steel fibre reinforced concrete. Work is in progress to develop recommendations for design of steel fibre reinforced concrete, for example in fédération Internationale du béton (FIB) and RILEM committees.

2.2 Analysed slabs

The slabs analysed in this report are octagonal shaped dimensions as shown in Figure 2.3. Three types of reinforcement configurations have been used: conventional reinforcement, fibre reinforcement and a combination of the two.

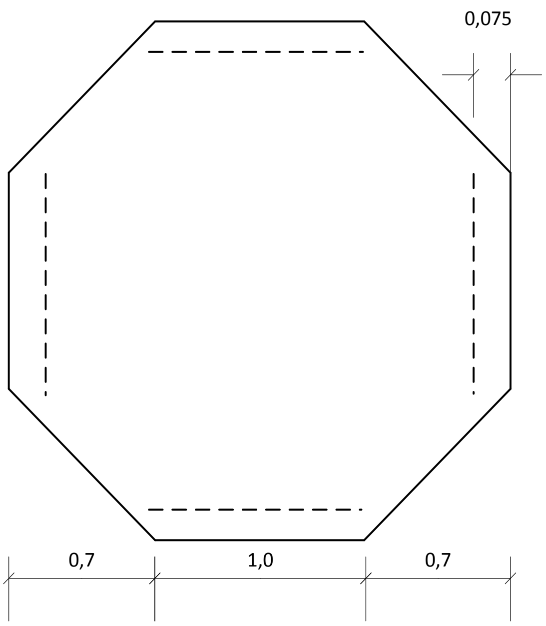

Figure 2.3 Geometry slab, dimensions in [m].

Casting of slabs and experiments has been performed at Chalmers, and is further described in (Fall et al., 2013). The slabs have been tested with a deformation controlled point load in the centre. They were simply supported on four edges and each supports consisted of five steel rollers, placed 75 mm from the slab edges.

The conventional reinforcement bars was placed in two perpendicular layers with spacing 194 mm in y-direction with 26 mm cover thickness and 96 mm spacing in x-direction with 20 mm cover thickness. The bar diameter was 6 mm in both directions. Different amount of reinforcement in x- and y-direction was used to get unequal distribution of the load.

The fibres used in the experiment were Dramix 5D illustrated in Figure 2.4 with a total length of 60 mm and a thickness of 1 mm. Dramix 5D is a new type of steel fibre that is designed to have high bond strength and a tensile failure in the fibre, in opposition to older steel fibre reinforcement which were designed to have a pull out failure (Bekaert, 2012). The fibre content used in the slabs was 0.45 % of the volume.

Figure 2.4 Dramix 5D fibre (Bekaert, 2012).

2.3 Material tests

Material tests have been made on the concrete, the steel fibre reinforced concrete and the reinforcement bars to see the behaviour of the material and obtain necessary data for the analyses. The tests were performed at Chalmers and SP.

The reinforcement was tested in tension. The results seen in Figure 2.5 show that the reinforcement bars have a strain hardening behaviour.

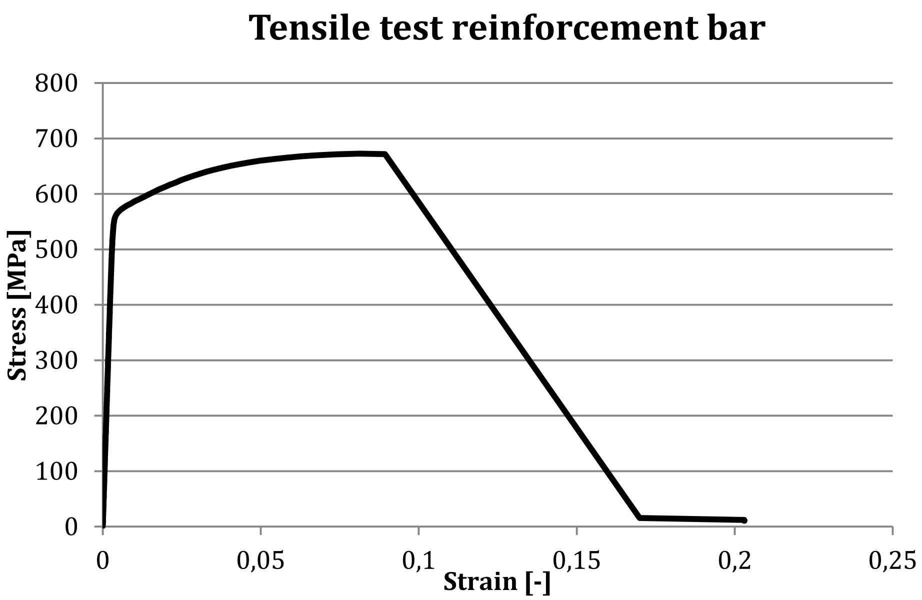

Figure 2.5 Tensile behaviour of reinforcement bar (Rempling et al., 2013)

Compressive tests were performed at cylinder specimens to capture the average compressive stress for plain and steel fibre reinforced concrete see Table 2.1.

Uniaxial tensile tests on cast concrete specimen were made, both on plain concrete and steel fibre reinforced concrete. Measured tensile behaviour from tests for plain concrete is seen in Figure 2.6 and for steel fibre reinforced concrete Figure 2.7. A comparison between plain concrete and steel fibre reinforced concrete is shown in Figure 2.8. A clear difference in the post critical behaviour can be seen. The steel fibre reinforced concrete can take load after cracks occurs, while the plain concrete loses almost the whole capacity after cracking. Uniaxial tensile tests are further described and compared with beam tests in (Rempling et al., 2013).

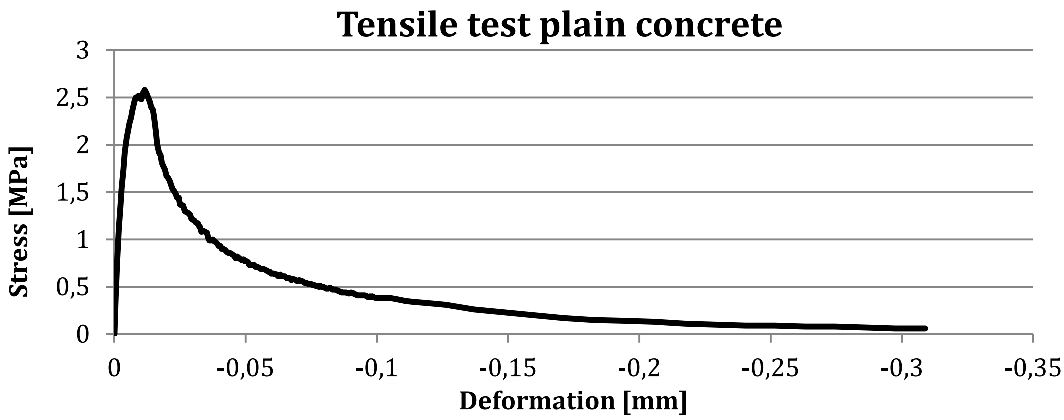

Figure 2.6 Tensile behaviour plain concrete (Rempling et al., 2013).

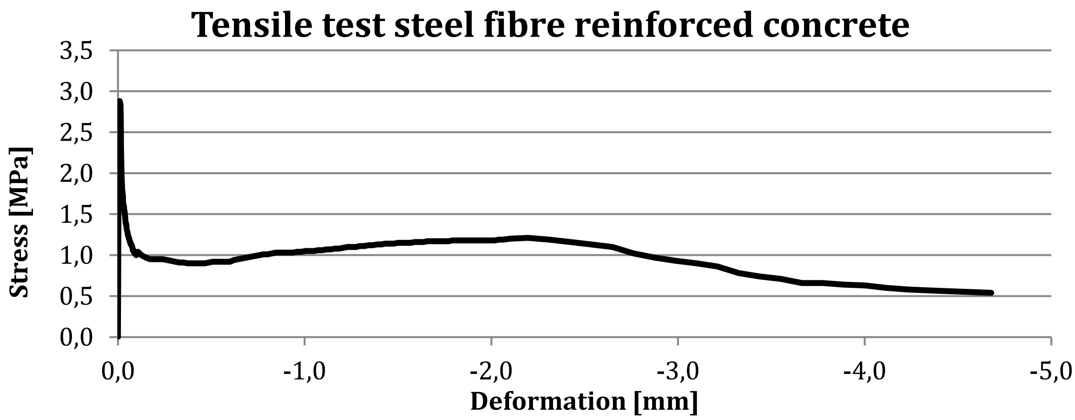

Figure 2.7 Tensile behaviour of steel fibre reinforced concrete (Rempling et al., 2013).

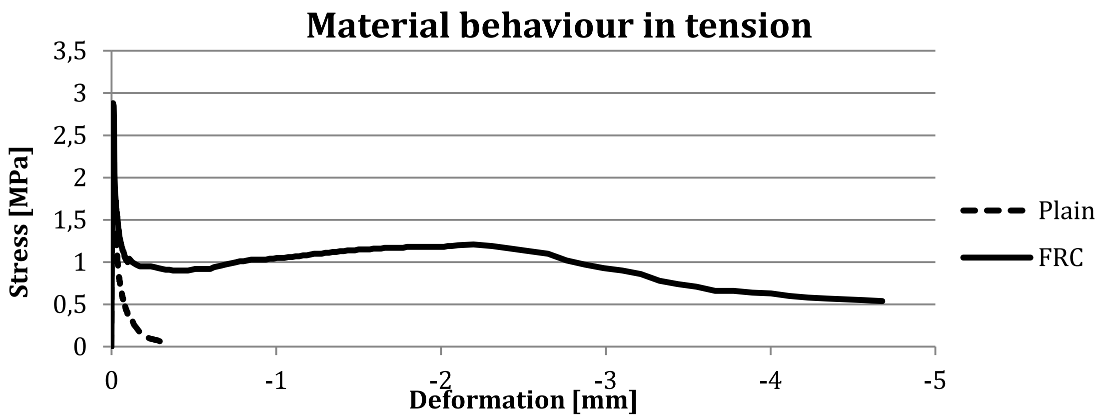

Figure 2.8 Tensile behaviour of plain- and steel fibre reinforced concrete (Rempling et al., 2013).

Calculation for steel fibre reinforced concrete in FIB Model Code was based on material data from bending test on three pointe bending beams in accordance with RIELM. The test method is a standard test to describe the tensile behaviour of fibre reinforced concrete. The test specimen consists of a simply supported beam with a span length of 500 mm and height and depth is 150 mm each (RILEM, 2002). In the middle of the span a notch is placed with a height of 25 mm and a maximum width of 5 mm, seen in Figure 2.9. A concentrated force is placed in the middle of the beam. Under increased load the crack mouth opening displacement (COMD) in the notch were measured. The result from bending test of RILEM beam is shown in Figure 2.10.

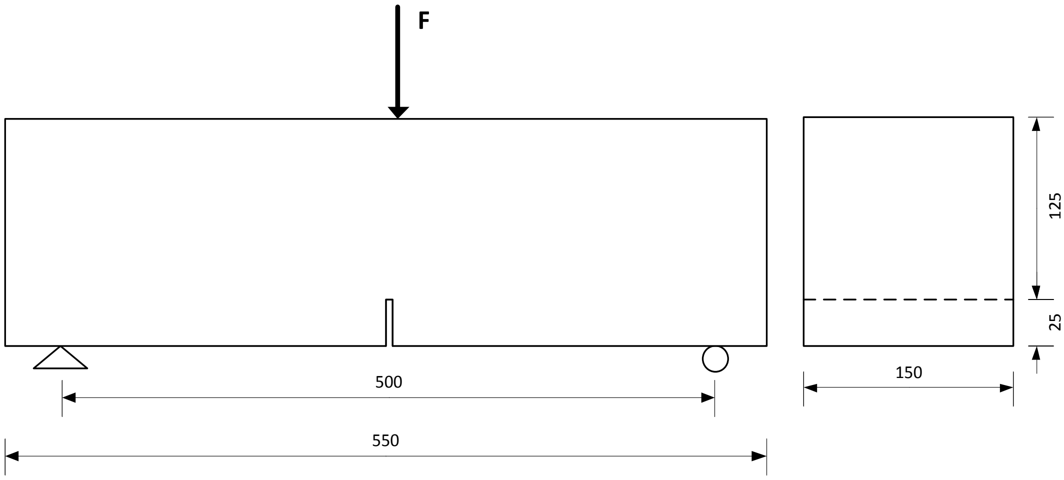

Figure 2.9 Bending test of RILEM beam.

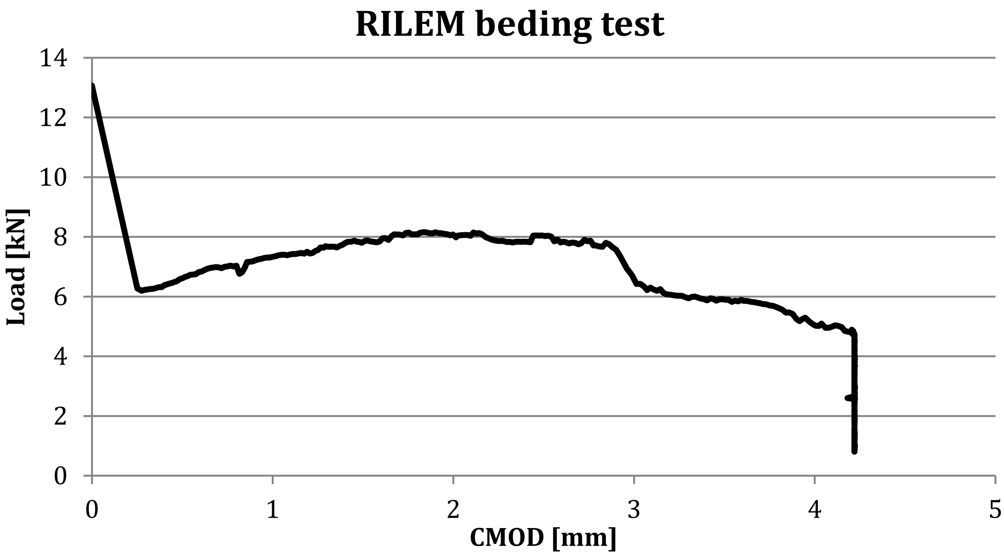

Figure 2.10 Load–CMOD relation from bending test of RILEM beam (Rempling et al., 2013).

2.3.1 Summary of experimental data

Table 2.1 Summary of experimental data.

	Fibre reinforced slab	Conventionally reinforced slab	Fibre+ Conventional reinforcement slab
Cylinder strength, compressive, f_{ck} [MPa]	45.52	51.18	45.52
Tensile strength concrete, f_{ct} [MPa]	3.0	2.7	3.0
Young's modulus concrete, E_c [GPa]	31.0	31.7	31.0
Density Concrete, ρ_c [kg/m^3]	2327	2312	2327
Thickness slab, h [mm]	79.14	83.92	82.37
Yield strength reinforcement, f_y [MPa]	-	560	560
Ultimate strength reinforcement, f_u [MPa]	-	672	672
Young's modulus reinforcement, E_s [GPa]	-	200	200
Density reinforcement, ρ_s [kg/m^3]	-	7850	7850

3 Analytical analysis

Different hand calculation methods were performed to analyse the behaviour of the slab during increased loading. For conventional reinforced concrete a traditional sectional analysis were made. To analyse the slabs in the ultimate state the strip method and the yield line method were used. The bending capacity of cross sections including steel fibre reinforced concrete was analysed using FIB Model Code.

Other considered methods where two-way slab direct design method and two-way slab equivalent frame method (ACI Committee 318, 2008).

3.1 Background and theory

In this section background and theory of the chosen methods is presented.

3.1.1 Plastic analysis

When designing with regard to plastic analysis it is assumed that all the sections of the slab have an ideally plastic moment-curvature relationship (Engström, 2011). Before the yield moment is reached there is no deformation in the slab, and when it is reached there is unlimited deformation and the section starts to rotate without any increased bending moment. When an ideally plastic response is assumed the slab has the ability to have full plastic redistribution until a collapse mechanism is formed (Engström, 2011). With plastic analysis there is no way to analyse the slab in service state, only the behaviour when a collapse mechanism is formed.

When the yield moment is reached the curvature can have any value. This means that the actual collapse load cannot be solved exactly, only by using approximate methods that are on the safe side or the unsafe side of the true solution. Methods that can be used are static methods for lower bound approach and kinematic methods for upper bound approach.

The strip method

The strip method developed by Hilleborg (Hilleborg, 1974) is a static method where a certain load distribution is assumed. Since the slab will find the most efficient way to carry the load the strip method will be on the safe side, as long as equilibrium condition is fulfilled. The accuracy of the model depends on the designer (Hilleborg, 1974).

Results obtained from analysis and design with strip method is under estimated. The load distribution is assumed, and a more accurate load distribution will give a solution closer to the true solution. According to theory of plasticity many different load distribution are possible while designing but equilibrium condition in ultimate limit state must be fulfilled. The strip method is only valid in ultimate state and does not give any information about the deflection or at which load the yielding start (Hilleborg, 1974).

With the strip method the slab is divided in strips in the main directions x and y. Each strip is simplified to act like a beam in the two main directions. The distribution of the load in x- and y-direction is assumed and each strip is analysed for one way action (Engström, 2011). The summation of loads carried in x- and y- direction is equal to the total load acting on the slab seen in equation (3.1). The load distribution is defined with load dividing lines where the shear force is equal to zero, meaning that the moment has maximum value at this line.

$$q = q_x + q_y \tag{3.1}$$

Where

q_x is the load carried by the strip in x-direction

q_y is the load carried by the strip in y-direction

Yield line analysis

The yield line method proposed by Ingerslev and further developed by Johansen is a kinematic method where a collapse mechanism is assumed (Johansen, 1972). Since a possible collapse mechanism is assumed and the slab will find the most efficient way to fail the solution is on the unsafe side. Hence, the accuracy of the model depends on choices done by the designer.

Calculations using yield line analysis are used for analysis of slabs in ultimate state. In contrast to the strip method, where load dividing lines are assumed, yield lines are chosen. The yield line analysis is an ultimate load method which means that the slab will be analysed for failure load (Jones & Wood, 1967).

Yield lines divide the slab in several elastic plates. At a certain load the yield lines form a plastic mechanism where the slab can deform plastically without increased load. The yield method does not give any information about the deflection or at which load the yielding start (Wight & MacGregor, 2009)

The yield line method is a kinematic method and the assumed collapse mechanism must be possible kinematic, which means that slab fragments must fit together when the slab deflects in the collapse mechanism (Engström, 2011). The collapse mechanism is possible kinematic when the yield line between two slab fragments passes through the intersection of the rotating axes of the two elements, see Figure 3.1.

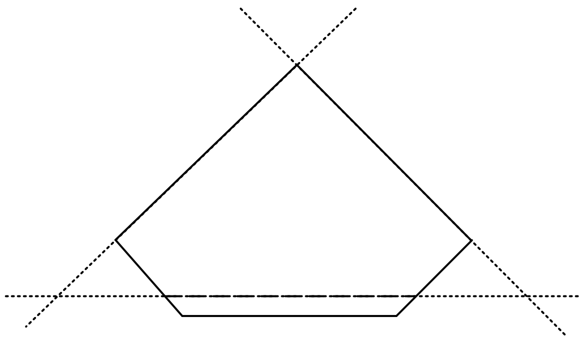

Figure 3.1 Possible kinematic failure mechanism

The virtual work method is used to calculate the ultimate load for yield line analysis. In the virtual work method you assume that at failure there is no loss of energy in the slab, which means that the internal work is equal to the external work (Jones & Wood, 1967) see equation (3.2)-(3.4).

$$external\ work = internal\ work \tag{3.2}$$

$$external\ work = \int\int q\delta\ dx\ dy = \sum(W\Delta_c) \tag{3.3}$$

Where

q is the uniformly distributed load on an element

δ is the deflection of that element

W is the total load on a plate segment

Δ_c is the deflection of the centroid of that segment

$$internal\ work = \sum (m_b l_y \theta) \tag{3.4}$$

Where

m_b is the bending moment per unit length of yield line

l_y is the length of the yield line

θ is the angle change at the yield line corresponding to the virtual displacement δ

The bending moment m_b in the internal work is set to the bending moment resistance in the yield line. The bending moment is assumed uniformly distributed over the yield line. The bending moment resistance can be calculated with equation (3.5).

$$m_b = m_x \sin(\varphi)^2 + m_y \cos(\varphi)^2 \tag{3.5}$$

Where

m_x is the moment resistance in x-direction

m_y is the moment resistance in y-direction

φ is the angle between yield line and reinforcement

3.1.2 Models for bending moment capacity

To analyse the steel fibre reinforced concrete FIB Model Code was used. FIB is a non-profit organization for structural concrete. It is a merger of the Euro-International Concrete Committee (CEB) and the International Federation for Prestressing (FIP). The Model Code for Concrete Structures presented by FIB is a document that is meant to serve as a basis for future design codes (fib Bulletin 55: Model Code 2010).

Properties for steel fibre reinforced concrete were obtained from experimental tests described in section 2.3. Residual flexural tensile strength parameters, f_{Rj}, were evaluated from a load-CMOD relation see Figure 3.2, with equation (3.6).

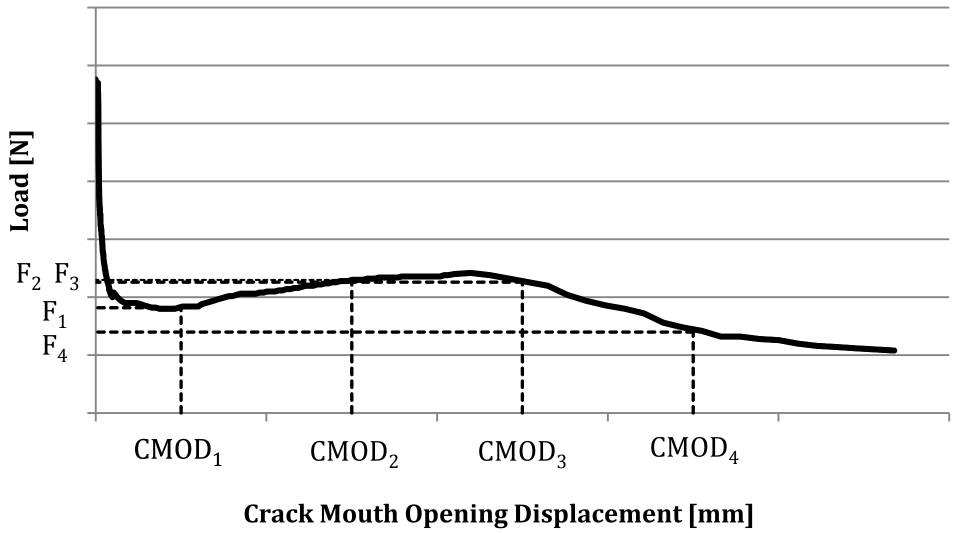

Figure 3.2 *Load – CMOD relation, figure from (fib Bulletin 55: Model Code 2010).*

$$f_{Rj} = \frac{3F_j l}{2bh_{sp}^2} \tag{3.6}$$

Where

f_{Rj} is the residual flexural tensile strength corresponding with $CMOD = CMOD_j$

F_j is the load corresponding with $CMOD = CMOD_j$

l is the span length of the specimen

b is the width of the specimen

h_{sp} is the distance between notch tip and the top of the specimen

In the Model Code there are two simplified stress-crack opening constitutive laws, derived from experimental bending test result; a rigid-plastic behavior and a linear post-cracking behavior seen in Figure 3.3. In Figure 3.4 the cross section in ultimate state for rigid plastic model and linear model can be seen.

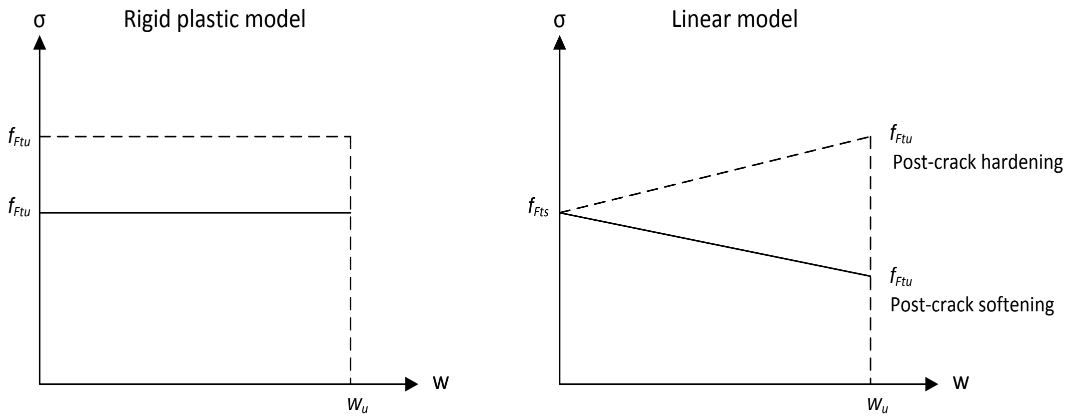

Figure 3.3 Material model for rigid plastic model and linear model (fib Bulletin 55: Model Code 2010).

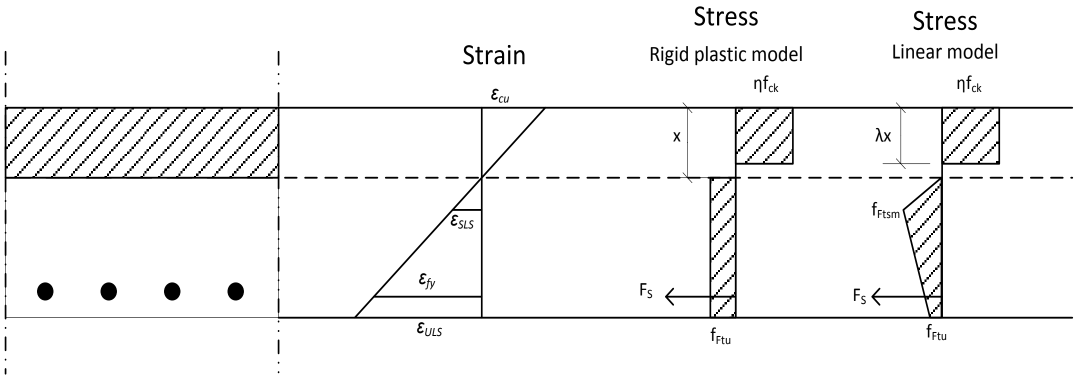

Figure 3.4 Assumptions for cross section analysis with rigid plastic model and linear model (fib bulletin 56: Model Code 2010).

To analyse ultimate moment capacity for conventional reinforced concrete (SS-EN 1992) was used, see Figure 3.5.

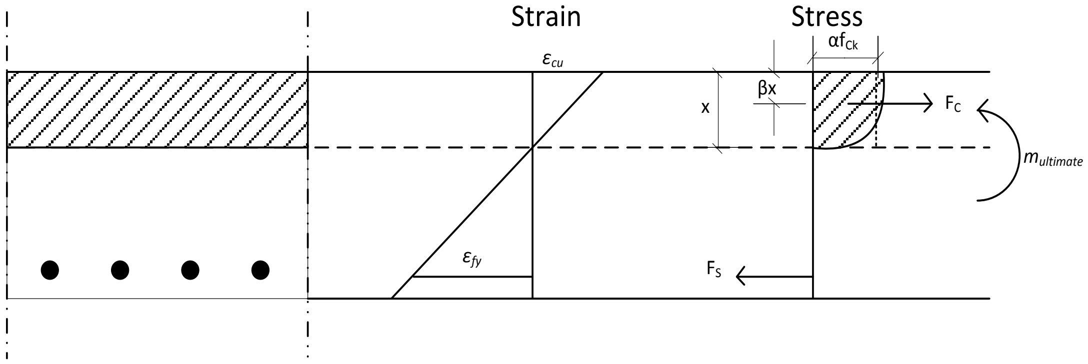

Figure 3.5 Assumptions for cross section analysis for conventional reinforced concrete in ultimate state.

3.2 Material models

Material data used for analysis was obtained from lab tests performed at Chalmers and SP described in section 2.3.

3.2.1 Reinforcement bars

Tensile tests have been made to find the yield limit and the ultimate limit. The material model used in hand calculation with FIB Model Code is a simplified model from the real behaviour. The real behaviour is presented in **Error! Reference source not found.** and the simplified material model in Figure 3.6.

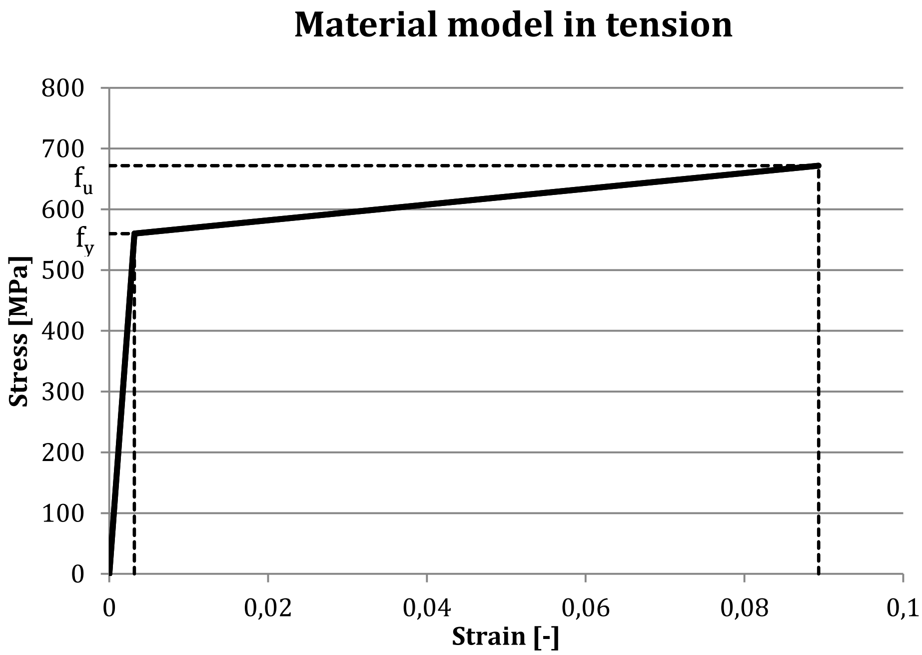

Figure 3.6 Material model for reinforcement bar used in hand calculations.

3.2.2 Plain concrete

Uniaxial tensile tests were done on specimen of cylinders. The real material behaviour is seen in Figure 2.6 and the simplified material model used in the hand calculations is presented in Figure 3.7. The simplified material model assumes a linear elastic behaviour of the concrete until the tensile strength is reached. When the tensile strength achieves no further load can be taken by the concrete in tension.

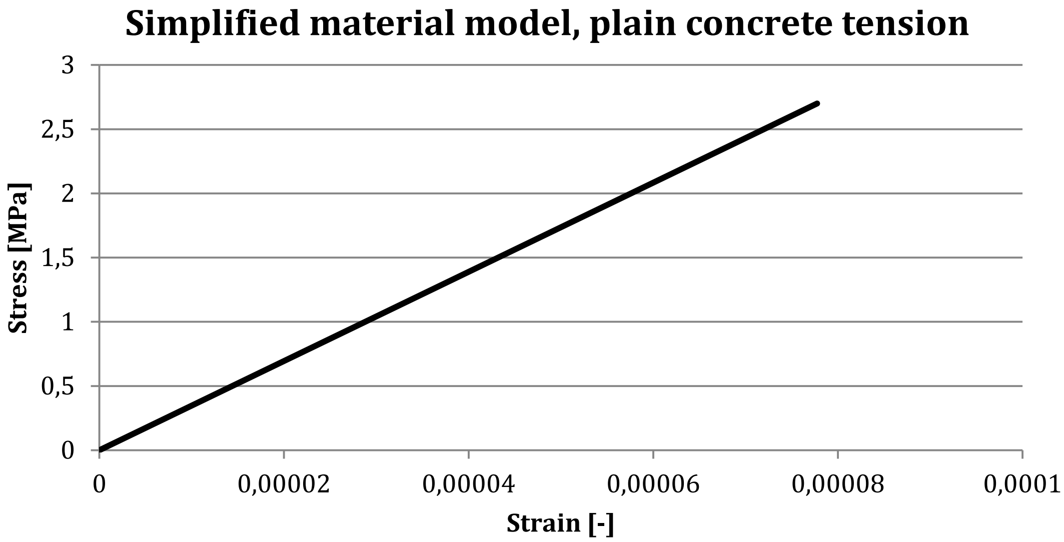

Figure 3.7 Material model for plain concrete used in hand calculations.

The concrete is assumed to have a linear elastic behaviour before cracking. When the cross section is cracked, a nonlinear behaviour of the concrete in compression is assumed, see Figure 3.8. The curve in Figure 3.9 corresponds to the compressed concrete zone where the α-factor is multiplied with the characteristic compressive strength, f_{ck}, found in Table 2.1. The ultimate strain for the concrete is assumed to be 3.5‰. At that point crushing will occur in the concrete.

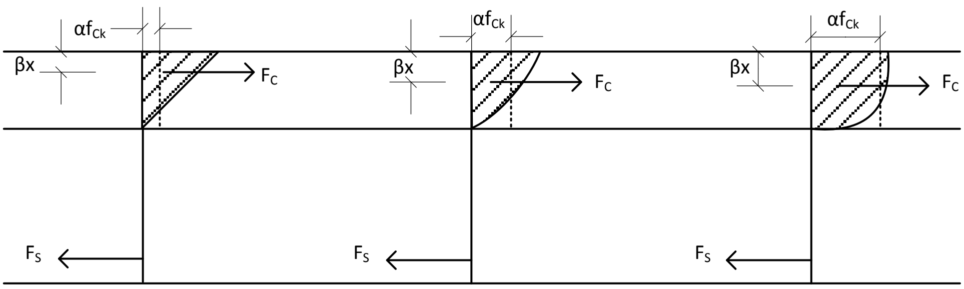

Figure 3.8 Compressed concrete zone under increased load.

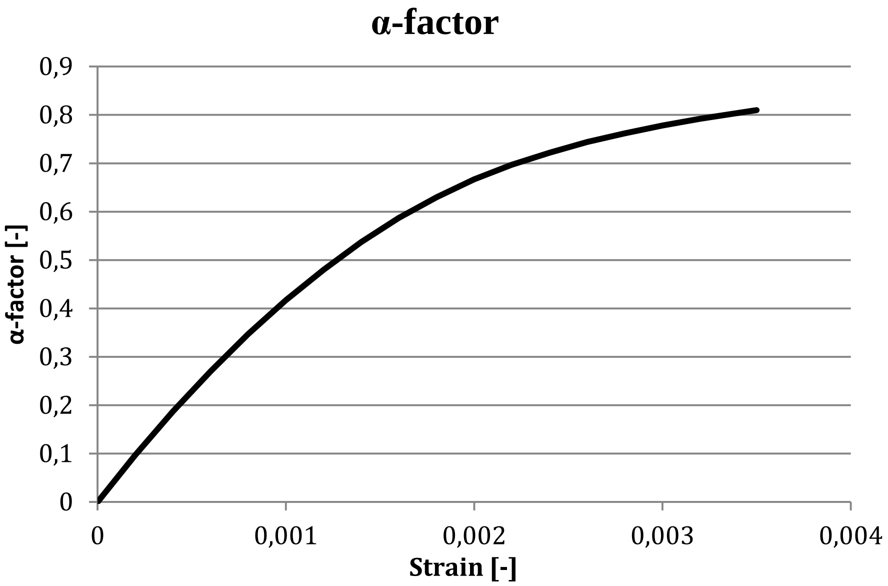

Figure 3.9 α-factor–strain (SS-EN 1992, 2004).

For a certain α-factor there is a corresponding β-factor seen in Figure 3.10. The β-factor is a factor for the position where the resulting compressed concrete force acts.

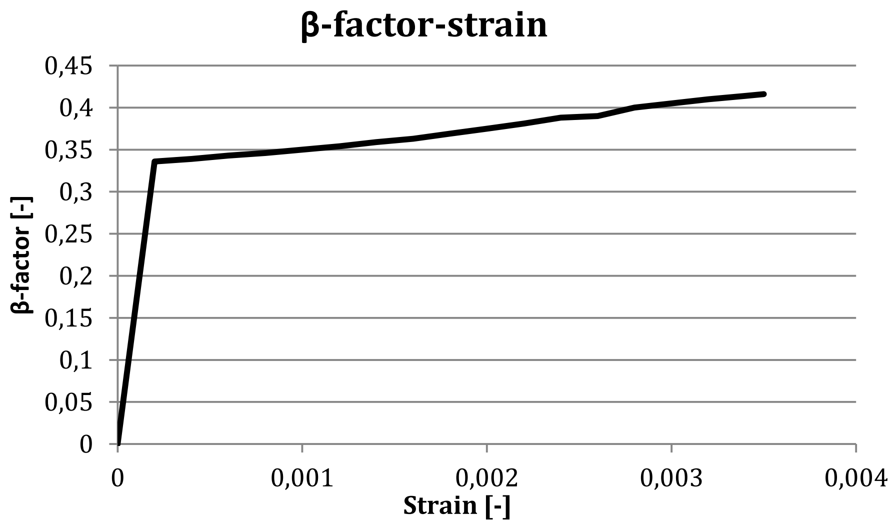

Figure 3.10 β-factor–strain relation (SS-EN 1992, 2004).

3.2.3 Steel fibre reinforced concrete

The material models used for steel fibre reinforced concrete was based on a stress–strain relation for uncracked concrete seen in Figure 3.11 and a load–crack opening relation for cracked concrete seen in Figure 3.12. Same tensile test as for conventional concrete is made on fibre reinforced specimens. The load–crack opening relation was obtained by bending test of RILEM beams, described in section 2.3.

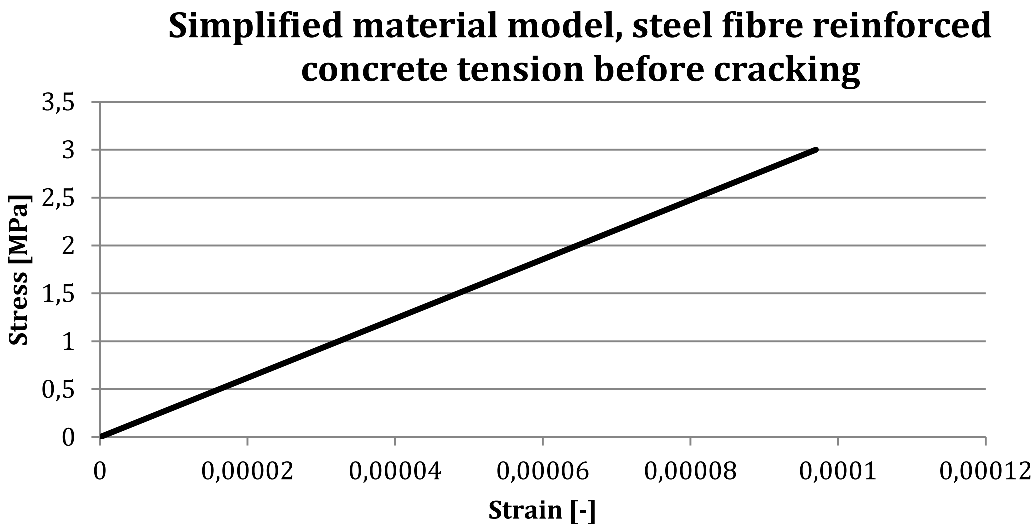

Figure 3.11 Material model for steel fibre reinforced concrete used in hand calculation.

The behaviour of the steel fibre reinforced concrete before cracking was assumed to be linear elastic before cracking. The same material model with α- and β-factors as for plain concrete was used for steel fibre reinforced concrete, with a different characteristic compressive strength, f_{ck}, found in Table 2.1.

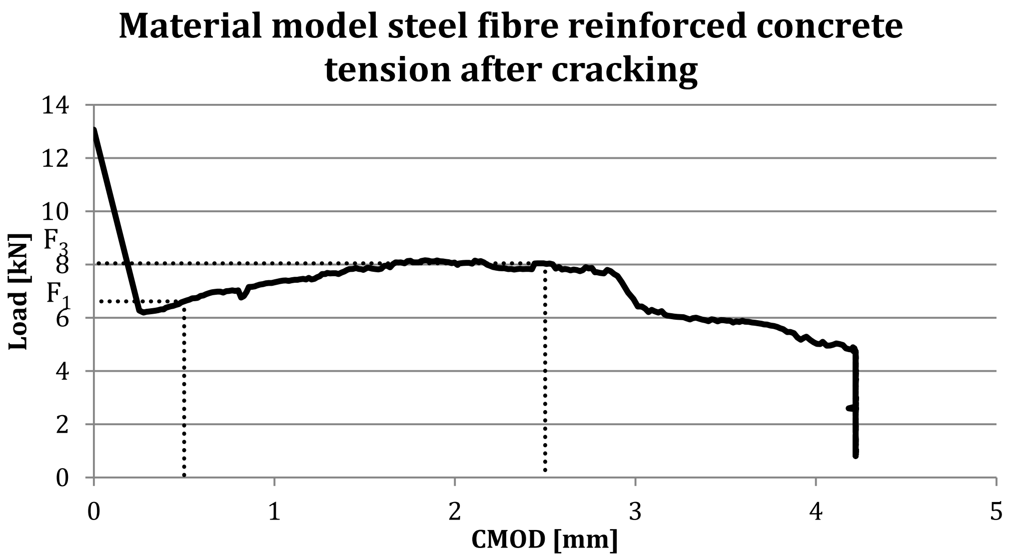

Figure 3.12 Load–CMOD relation used as material model for hand calculation (Rempling et al., 2013).

In Model Code the ultimate residual strength, f_{Ftu}, and the serviceability residual strength, f_{Ftsm}, was used to calculate the capacity of the steel fibre reinforced concrete. f_{Ftu} and f_{Ftsm} is calculated from results in load–crack opening relation of the steel fibre reinforced concrete.

3.3 Analysis of conventional reinforced concrete

The conventional reinforced concrete slab were analysed with traditional cross sectional analysis. The slab was analysed for ultimate load, using both yield line theory and strip method.

3.3.1 Bending moment capacity

The bending moment capacity was calculated assuming the cross-section in state III. State III means that the concrete cross section is cracked in tensioned parts and all tensile stresses in concrete is neglected. In compressed parts there are no effects from eventual cracks. The yield limit is reached for the reinforcement, and/or crushing of the concrete takes place (Al-Emrani, Engström, Johansson, & Johansson, 2010).

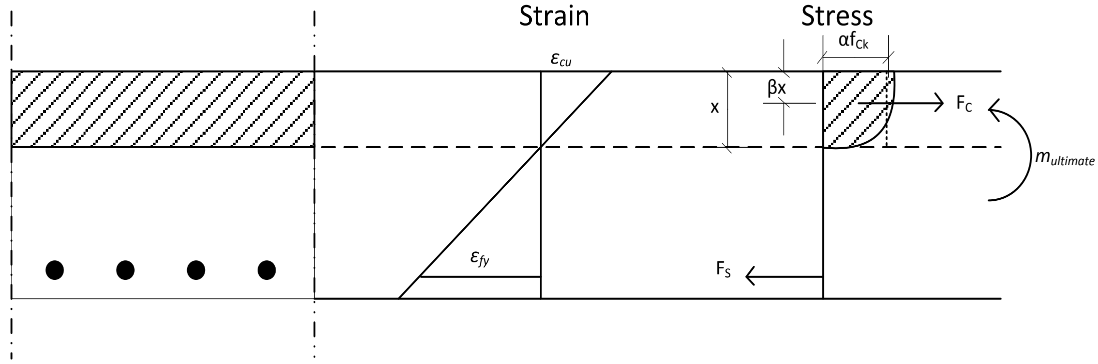

Figure 3.13 Cross section for ultimate load.

The compressive zones were calculated with horizontal force equilibrium in equation (3.7) and the moment capacities were calculated with moment equilibrium with equation (3.9).

$$\alpha \cdot f_{ck} \cdot x = f_s \cdot A_s \tag{3.7}$$

Where

α is stress block factor for ultimate state

f_{ck} is the characteristic compressive strength for concrete

x is the height of the compressed concrete zone

f_s is stress in steel over the yield limit, calculated with expression (3.8)

A_s is the reinforcement area per meter

$$f_s = \frac{(\varepsilon_s - \varepsilon_{sy})}{(\varepsilon_{su} - \varepsilon_{sy})} \cdot (f_u - f_y) + f_y \tag{3.8}$$

Where

ε_s is the reinforcement strain

ε_{sy} is the reinforcement yield strain

ε_{su} is the reinforcement ultimate strain

f_u is the reinforcement ultimate strength

$$m_{ultimate} = \alpha \cdot f_{ck} \cdot x \cdot (d - \beta \cdot x) \tag{3.9}$$

Where

$m_{ultimate}$ is the moment capacity in ultimate state in [Nm/m]

β is stress block factor for ultimate state

d is the distance from centre of reinforcement to top the fibre

The relation for the moment capacities in x- and y-direction was calculated in equation (3.10), to choose reasonable load distributions

$$\alpha_{rx} = \frac{m_{ultimate.x}}{m_{ultimate.x} + m_{ultimate.y}} \tag{3.10}$$

Where

$m_{ultimate.x}$ is the ultimate moment in x-direction [Nm/m]

$m_{ultimate.y}$ is the ultimate moment in y-direction [Nm/m]

The same equation was used to calculate α_{ry}.

3.3.2 Strip method

The strip method was used to analyse the slab in the ultimate state. Since the strip method is a plastic method it is, as previously described, only valid in the ultimate state. A relation between $m_{ultimate,x}$ and $m_{ultimate,y}$was calculated according to equation (3.10) to choose a reasonable load distribution.

The following load distributions for the self-weight were investigated to find the most accurate solution. In all the different alternatives the point load was divided as Alternative I.

Alternative I

In alternative I, it was assumed that the slab acts like one simply supported beam in each direction. Both the self-weight and the point load were assumed to be distributed in relation to the moment capacity in x- and y-direction without any load dividing lines see Figure 3.14.

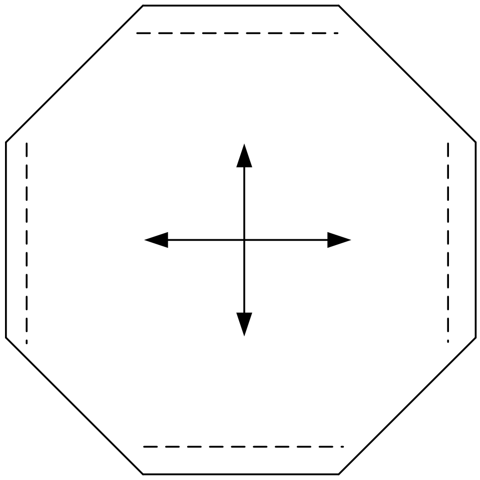

Figure 3.14 Load distribution alternative I for strip method.

Alternative II

In alternative II the point load was distributed in the same way as for alternative I. The distribution of the load from self-weight is seen in Figure 3.15. The strips in the corners between the supports and the strip in the middle were assumed to have a load distribution in relation to the moment capacity in the different direction. The strips adjacent to the supports were assumed to distribute the entire load to the nearest support.

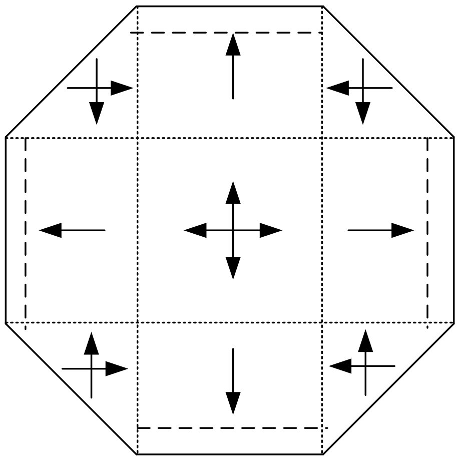

Figure 3.15 Load distribution alternative II for strip method.

Alternative III

Also in alternative III the point load was assumed to be distributed as for the other alternatives. The self-weight was distributed as seen in Figure 3.16. To increase the area of load that goes to the supports in stiffer direction we assumed a distribution based on the different moment capacity in the different directions.

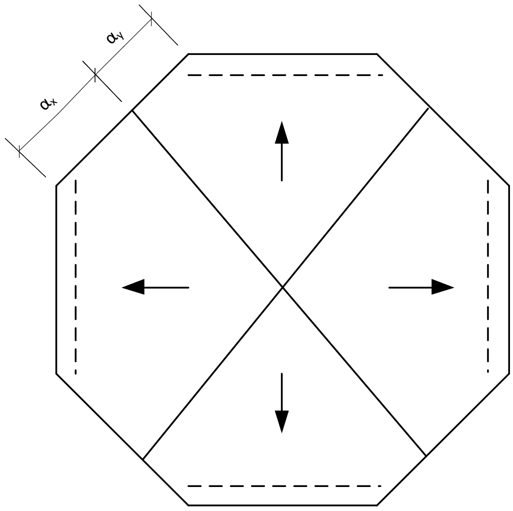

Figure 3.16 Load distribution alternative III for strip method.

The moment from the self-weight, M_{SF}, were calculated from moment equilibrium for the strip method alternatives.

The moment from the self-weight and the point load was combined with superposition, see equation (3.11).

$$M_R = M_{SF} + M_{CL} \tag{3.11}$$

Where

M_R is total moment resistance

M_{SF} is moment due to self-weight

M_{CL} is moment due to concentrated load

The moment due to concentrated load are calculated with elementary case for simply supported beam in equation (3.12).

$$M_{CL} = \frac{\alpha_{ri} \cdot P \cdot L}{4} \tag{3.12}$$

Where

P is the point load

L is the span length

α_{ri} is the moment capacity ratio between x- and y-direction, from equation (3.10)

3.3.3 Yield line analysis

The chosen plastic mechanism for yield line analysis is shown in Figure 3.17. The diagonals represent the assumed yield lines. The capacity in the yield lines is a weighted value between the capacity in x-direction and y-direction calculated with equation (3.5) with φ equal to 45°.

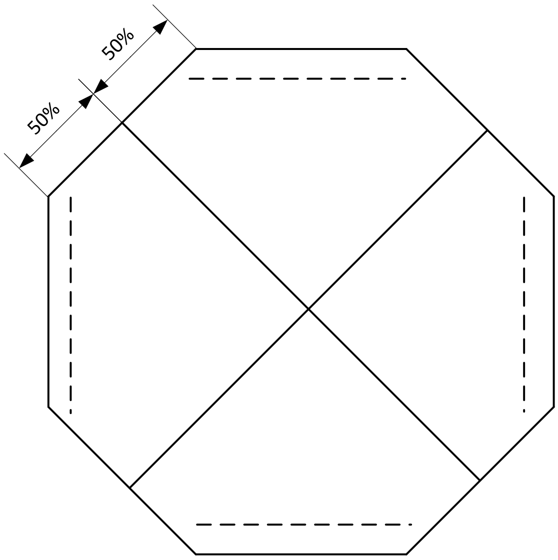

Figure 3.17 Chosen plastic mechanism.

The position of the centroid was calculated for Figure 3.18, assuming the support goes all the way to the edge and the favourable moment from the slab outside the supports was neglected. The centroids position was used to find the moment due to self-weight while the point load was assumed to be divided equally between the supports due to the weighted capacity along the yield lines.

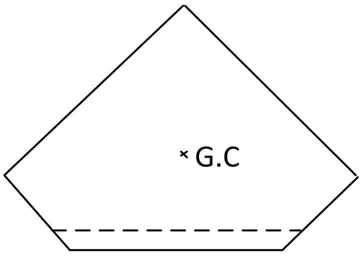

Figure 3.18 Segment between two yield lines.

3.4 Analysis of steel fibre reinforced concrete

FIB Model Code was used to analyse the bending moment capacity of the cross-sections with steel fibre reinforced concrete. Both the rigid-plastic model and linear model described in section 3.1.2 were used. The slabs were analysed in the ultimate state.

3.4.1 Rigid-plastic model

In the rigid-plastic model in (fib Bulletin 55: Model Code 2010) steel fibre reinforced concrete are assumed to have an ideally-plastic behaviour, where the ultimate residual strength, f_{Ftu}, was calculated with equation (3.13).

$$f_{Ftu} = \frac{f_{R3}}{3} \tag{3.13}$$

Where

f_{R3} is the residual flexural strength corresponding to $CMOD_3$ obtained from equation (3.6)

The calculations were done with two different assumptions. The first assumption was that the fibres transfer load over the entire crack. The other assumption was that the fibres only transfer load over the crack where the strain in the cross-section are lower than the ultimate strain for steel fibre reinforced concrete. The calculations are presented parallel in the following sections.

The cross-section was assumed to be in state III for the ultimate load. Two different stages were investigated. The first stage was when the ultimate tensile strain for the steel fibre reinforced concrete, ε_{ULS}, was reached, see Figure 3.19. The second stage was when the ultimate compressive strain in the steel fibre reinforced concrete, ε_{cu}, equal to 3.5 ‰, was reached, see Figure 3.20.

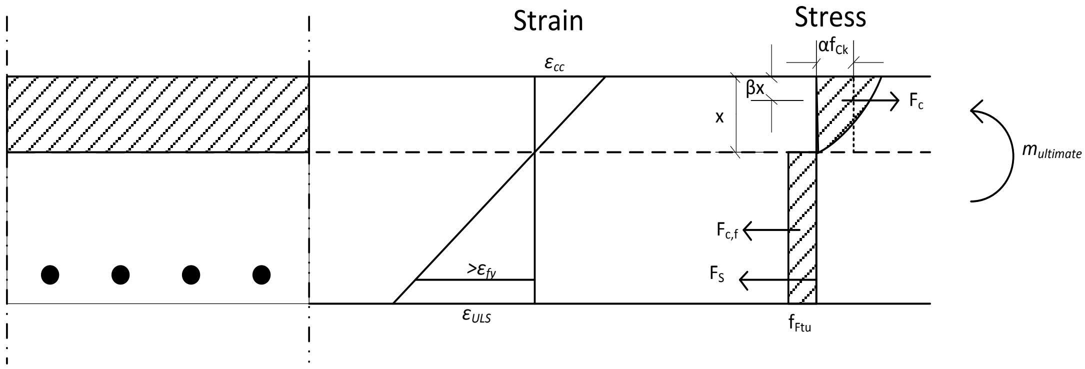

Figure 3.19 Cross section when ultimate strain for steel fibre reinforced concrete was reached in bottom fibre.

The height of the compressed concrete zone was calculated with force equilibrium in equation (3.14) for full load transfer over cracks. The stress block factor α was found by iterations of the force equilibrium in the same way as for the slab without fibre reinforcement, see section 3.3.

$$\alpha \cdot f_{ck} \cdot x = f_{Ftu} \cdot (h - x) + f_s \cdot A_s \tag{3.14}$$

The stress block factor α corresponds to a certain compressive strain. In equation (3.14) both α and x is unknown, and was found by iteration of force equilibrium. The relation of the stress block factor α and strain is seen in Figure 3.9.

The moment resistance was calculated in both directions from moment equilibrium of the cross sections:

$$m_{ultimate} = \alpha \cdot f_{ck} \cdot x \cdot (x - \beta x) + f_s \cdot A_s \cdot (d - x) + f_{Ftu} \cdot \frac{(h-x)^2}{2} \tag{3.15}$$

Two different assumptions for the ultimate state when ε_{cu} was reached in the top fibre can be seen in Figure 3.20 and Figure 3.21. In Figure 3.20 the fibre was assumed to give contribution to the capacity over the whole crack. In Figure 3.21 the zone where the fibres contribute to the capacity was limited to where the tension strain was lower than ε_{ULS}, equal to 2 ‰.

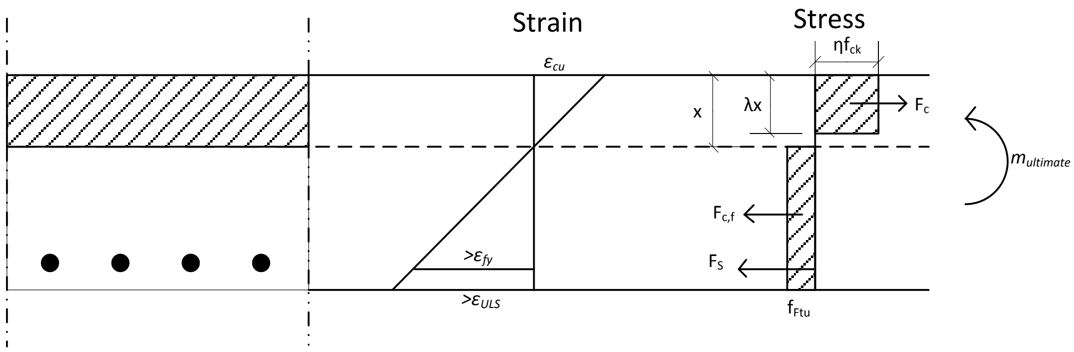

Figure 3.20 Cross section when ultimate compressive strain for the concrete is reached with the assumption of full load transfer over cracks.

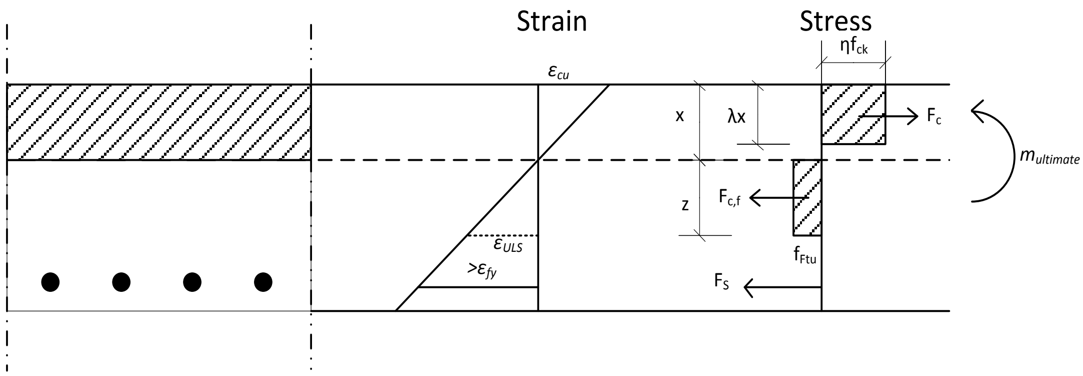

Figure 3.21 Cross section when ultimate compressive strain for the concrete is reached with the assumption of limited load transfer over cracks.

The height of the compressed concrete zone with assumption of full load transfer over cracks was calculated with equation (3.21). The height of the compressed concrete zone with the assumption of limited load transfer over cracks was calculated with force equilibrium for the cross section by combining equation (3.22) and (3.23).

$$\eta \cdot f_{ck} \cdot \lambda x = f_{Ftu} \cdot (h - x) + f_s \cdot A_s \qquad (3.21)$$

$$\eta \cdot f_{ck} \cdot \lambda x = f_{Ftu} \cdot z + f_s \cdot A_s \qquad (3.22)$$

Where

η is stress block factor for ultimate state

λ is stress block factor for ultimate state

The height of the zone where the fibres contribute to the capacity was calculated with the strain relation in expression (3.23).

$$\frac{\varepsilon_{cu}}{x} = \frac{\varepsilon_{ULS}}{z} \qquad (3.23)$$

The moment capacities in the ultimate state were calculated with moment equilibrium in equation (3.24) for full load transfer and equation (3.25) for limited load transfer.

$$m_{ultimate} = \eta \cdot f_{ck} \cdot \lambda x \cdot \left(x - \frac{\lambda x}{2}\right) + f_{Ftu} \cdot \frac{(h-x)^2}{2} + f_s \cdot A_s \cdot (d-x) \quad (3.24)$$

$$m_{ultimate} = \eta \cdot f_{ck} \cdot \lambda x \cdot (x - \frac{\lambda x}{2}) + f_{Ftu} \cdot \frac{z^2}{2} + f_s \cdot A_s \cdot (d-x) \quad (3.25)$$

For slab without conventional reinforcement the same equations were used but removing the contribution from the conventional reinforcement.

From (fib bulletin 56: Model Code 2010) the recommendation to calculate moment capacity in ultimate state for a slab without conventional reinforcement is to consider a rigid plastic relationship with equation (3.26).

$$m_{Rd} = \frac{f_{Ftu} \cdot h^2}{2} \quad (3.26)$$

When the ultimate point load were calculated, strip method alternative I was used for the self-weight, assuming load distribution with regard to moment capacity in x- and y-direction. The point load was assumed to have distribution to the supports based on the moment capacity in the different directions. See section 3.3 for method and assumptions.

3.4.2 Linear model

In the linear model both the ultimate residual strength, f_{Ftu}, and the serviceability residual strength, f_{Fts}, were calculated. The serviceability residual strength was defined as the post-cracking strength for serviceability crack opening (fib Bulletin 55: Model Code 2010). The residual strengths were calculated with equation (3.27) and (3.28).

$$f_{Fts} = 0.45 f_{R1} \quad (3.27)$$

Where

f_{R1} the residual flexural strength corresponding to $CMOD_1$, obtained from material test seen in section 3.2.3

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3}(f_{Fts} - 0.5 f_{R3} + 0.2 f_{R1}) \geq 0 \quad (3.28)$$

Where

w_u is the ultimate crack width, obtained from equation (3.29)

$CMOD_3$ is crack mouth opening displacement, equal to 2.5 mm

$$w_u = l_{cs} \cdot \varepsilon_{Fu} \tag{3.29}$$

Where

l_{cs} is the structural characteristic length, obtained in equation (3.30)

ε_{Fu} is the maximum tensile strain in the steel fibre reinforced concrete

ε_{Fu} can be assumed to be 2 % for variable strain distribution along the cross section.

$$l_{cs} = \min(s_{rm}, y) \tag{3.30}$$

Where

s_{rm} is the main distance value between cracks, from equation (3.31)

y is the distance between neutral axis and tensile side of the cross section. For fibre reinforced concrete without conventional reinforcement, $y = h$ can be assumed

The mean spacing between cracks, s_{rm}, was calculated with expression (3.31) from (Boverket, 2004).

$$s_{rm} = 50 + k_1 \cdot k_2 \cdot \frac{6}{\rho_{s,ef}} \tag{3.31}$$

Where

k_1 is a factor calculated with equation (3.32)

k_2 is a factor calculated with equation (3.33)

$\rho_{s,ef}$ is the effective reinforcement area, $\frac{A_s}{A_{c,ef}}$

$$k_1 = 0.125 \cdot \frac{(\varepsilon_1 + \varepsilon_2)}{\varepsilon_1} \tag{3.32}$$

$$k_2 = 0.25 - \frac{A_{c,ef}}{8 \cdot (h - x)} \tag{3.33}$$

ε_1 and ε_2 is strain seen in Figure 3.22

$A_{c,ef}$ is effective tension concrete area calculated with equation (3.34)

$$A_{c,ef} = \min(2.5 \cdot \left(c + \frac{\Phi_s}{2}\right), \frac{(h - x)}{3}) \tag{3.34}$$

Where

c is the cover thickness

Φ_s is the diameter of one reinforcement bar

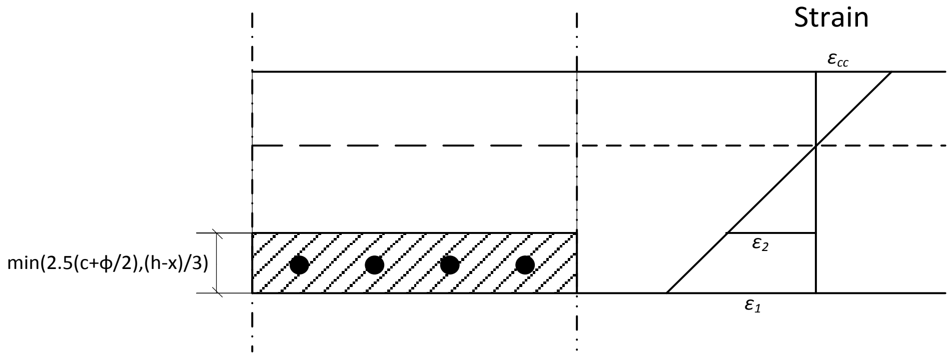

Figure 3.22 The effective concrete area.

The strain corresponding to the serviceability residual strength, f_{Fts}, are calculated with equation (3.35) and the ultimate strain, ε_{ULS}, corresponding to the ultimate residual strength and equal to ε_{FU}.

$$\varepsilon_{SLS} = \frac{CMOD_1}{l_{cs}} \tag{3.35}$$

Since the service strain depends on the compressed concrete zone, a new service strain was calculated for all investigated states.

The cross section in ultimate state was assumed to be in state III. As for rigid-plastic model two different cases were investigated in the ultimate state. The first when the ultimate tensile strain for the steel fibre reinforced concrete, ε_{ULS}, was reached seen in Figure 3.23 and the second stage when the ultimate compressive strain in the concrete, ε_{cu}, was reached.

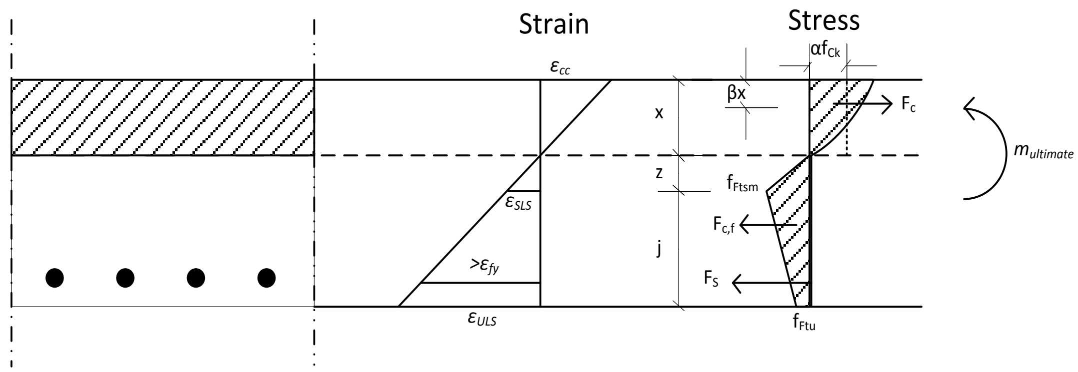

Figure 3.23 Cross section when the ultimate strain of steel fibre reinforced concrete was reached in bottom fibre.

The method with stress block factor α was used described in 3.2.2. The position of the serviceability strain was found by geometrical relation in equation (3.39).

$$\frac{\varepsilon_{SLS}}{z} = \frac{\varepsilon_{ULS}}{h - x} \tag{3.39}$$

$$\alpha \cdot f_{ck} \cdot x = f_{Ftu} \cdot j + \frac{1}{2} \cdot f_{Ftsm} \cdot z + \frac{1}{2} \cdot (f_{Ftsm} - f_{Ftu}) \cdot z + f_s \cdot A_s \tag{3.40}$$

By combining equation (3.39) and (3.40) the compressive stress zone was found.

The moment capacity was found by moment equilibrium in the cross section.

$$m_{ultimate} = \alpha \cdot f_{ck} \cdot x \cdot (x - \beta x) + f_{Ftsm} \cdot \frac{z^2}{2} + (f_{Ftsm} - f_{Ftu}) \cdot j \cdot \left(z + \frac{j}{2}\right) + f_{Ftu} \cdot j \cdot \left(z + \frac{j}{2}\right) + f_s \cdot A_s \cdot (d - x) \tag{3.41}$$

The second stage in the ultimate state was calculated with two different assumptions of the contribution from the fibre reinforcement. The assumptions were the same as for the rigid-plastic models, where the fibres contribute to the capacity over the whole crack in the first assumption and where the strain is lower than the ε_{ULS} in the second.

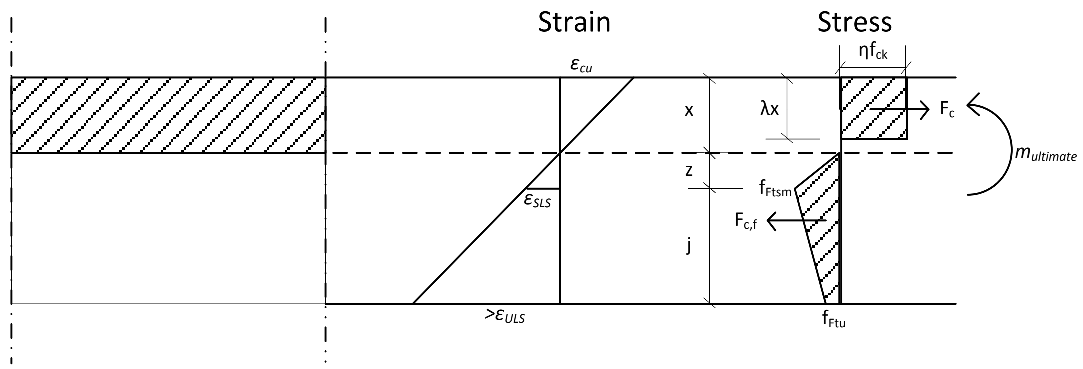

Figure 3.24 Cross section when ultimate compressive strain for the concrete was reached with the assumption of full load transfer over cracks.

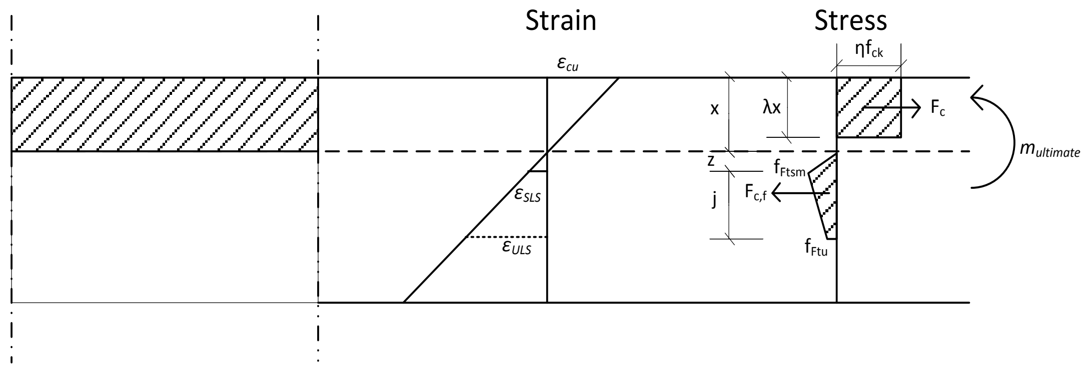

Figure 3.25 Cross section when ultimate compressive strain for the concrete was reached with the assumption of limited load transfer over cracks.

To calculate the point load, strip method alternative I was used, assuming load distribution with regard to moment capacity.

4 FEM analysis

Two different finite elements programs were used to analyse the concrete slabs, FEM-Design and ABAQUS. The programs and modelling specific information is presented in this chapter.

In total nine different analyses were made, each slab type was analysed with three different finite element models.

4.1 Constitutive models

In general there are three types of constitutive models used to describe nonlinear behaviour in concrete (Plos, 1995). Two of them are normally available in conventional designer software: smeared- and discrete crack model. In both FEM-Design and ABAQUS the smeared crack approach was used.

The discontinuity caused by cracking is smeared out over the element by a constitutive model for the material including cracks (Plos, 1996). FEM-Design considers the crack directions in each element perpendicular to the tensile principle stress direction by reducing the stiffness (Strusoft, 2010). The material response in smeared crack model is described by continuum constitutive relations in terms of stress and strain including elastic-plastic response (Plos, 1995).

4.2 Incrementation

To follow the nonlinear behaviour, different incrementation methods can be used. In this project load controlled incrementation was used in the finite element analysis.

For each user defined load step it is calculated whether the slab is cracked in a certain point. If the slab is cracked the direction of the crack is set perpendicular to the principle stress and the stiffness of the cracked element is reduced and iterations are done to find equilibrium. In the next step the procedure is repeated (Strusoft, 2010)

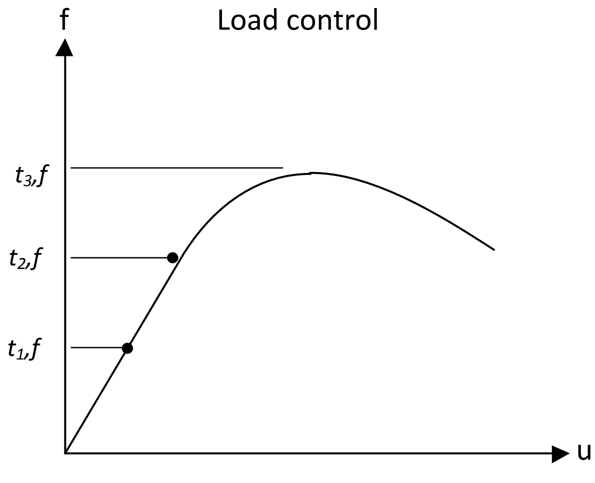

Figure 4.1 Load control incrementation (Plos, 1996).

4.3 ABAQUS modelling

Two different models were used in ABAQUS, the first with solid elements for the slab and truss element for the reinforcement and the other with shell elements. For each slab setup the average thickness and measured material parameters were used to facilitate the comparison between analyses and test results.

The following simplifications were made. The supports were modelled as one long roller instead of five small, and the reinforcement in the solid model was modelled with full interaction. In the shell model, the concentrated load was modelled as a distributed load on the slab with the same distribution as in reality. For the solid model a steel plate was used to distribute the concentrated load; the steel plate was modelled with full interaction with the concrete slab.

The following boundary conditions were used in ABAQUS: along the supports the vertical displacements were prevented and in the midpoint rotational displacements around z-axis and horizontal displacements in x and y direction were prevented, see Figure 4.2. In all ABAQUS models an adaptable increment size was used with a maximum of 100 iterations.

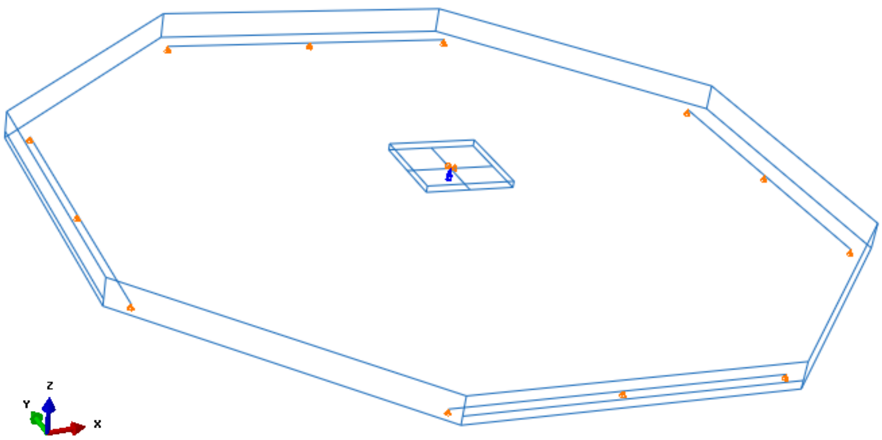

Figure 4.2 Boundary conditions slab in ABAQUS.

4.3.1 Material models

The non-linear material responses were obtained from tests, see section 2.3. The tests resulted in a non-linear stress-deformation relation. The stress-strain relation used in the material models were calculated with equation (4.1) from stress-deformation relation in uniaxial tensile tests, see Figure 2.6 and Figure 2.7.

$$\varepsilon = \frac{\sigma}{E} + \frac{w}{h_r} \tag{4.1}$$

Where

w is the deformation

h_r is the crack band width

The crack band width was set to the element height. The models had to be simplified to fit the material input in finite element software. The behaviour for steel fibre reinforced concrete and ordinary concrete were modelled in the same way but with different input describing their behaviour. Thus, they were both considered to be homogeneous materials meaning that cement paste, aggregate and fibre reinforcement were not considered separately. The simplified material models for concrete damage plasticity in tension can be seen in Figure 4.3 and Figure 4.4. The material model used for tensile response is linear elastic before plastic stress is reached.

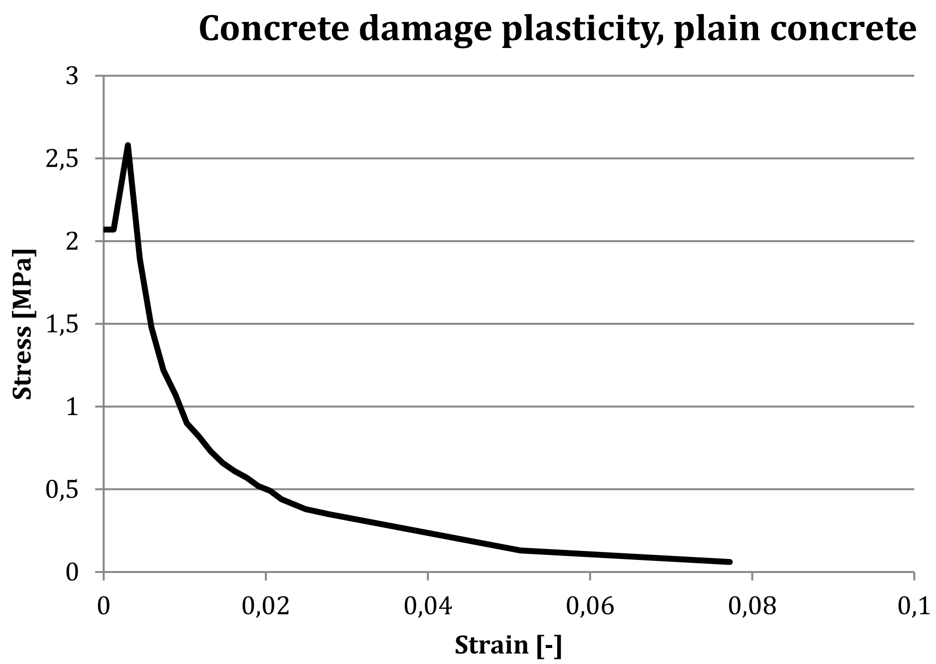

Figure 4.3 *Stress-strain relation plain concrete.*

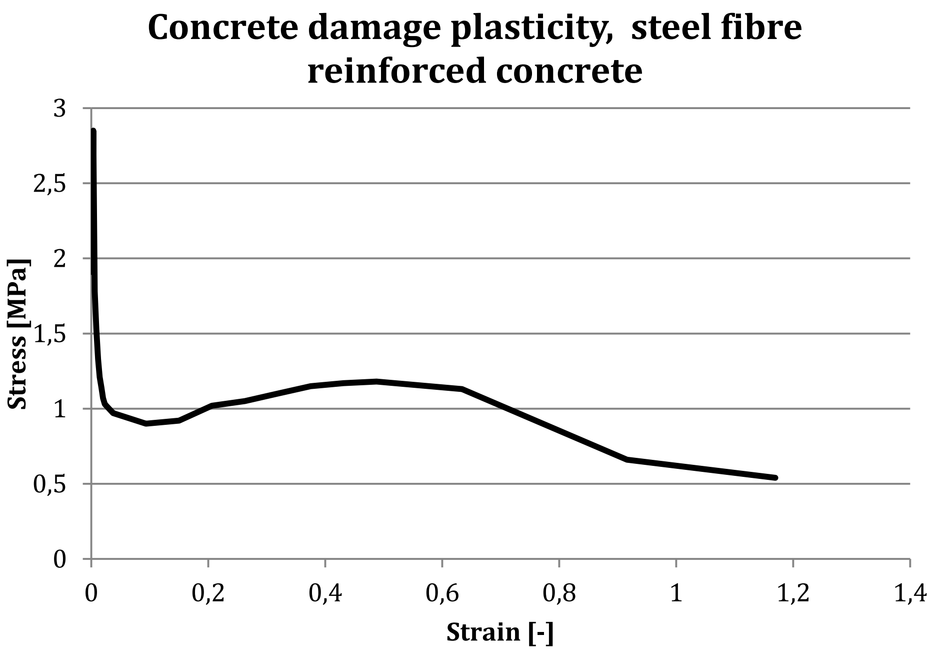

Figure 4.4 *Stress-strain relation steel fibre reinforced concrete.*

The material model for concrete in compression was assumed to be linear elastic until the compressive strength was reached. After that the concrete was assumed to have a plastic response, see Figure 4.5.

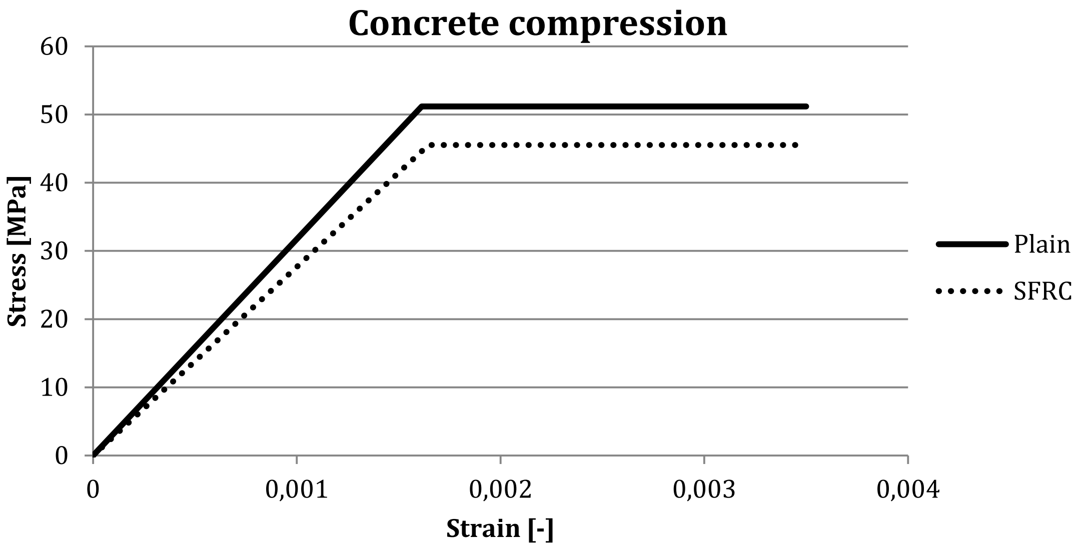

Figure 4.5 Material model for plain concrete in compression.

The reinforcement was modelled using a linear elastic-plastic model seen in Figure 3.6.

4.3.2 Modelling with solid elements and 3D truss elements

Solid elements were used for the concrete and steel fibre reinforced concrete together with embedded truss elements for the reinforcement. Solid elements are good at describing shear response and able to describe bending as well if a sufficient amount of higher order elements are used over the height of the structure. Solid elements are computationally expensive due to the number of elements needed to describe bending (Simulia, 2010).

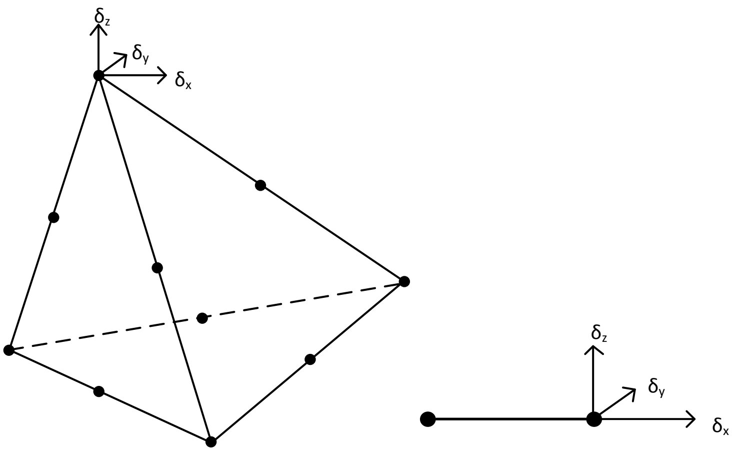

Figure 4.6 C3D10M solid element (left), 3D truss element (right).

The 3D truss element describes the behaviour of reinforcement well and is computationally cheap compared to the alternative frame element. Truss elements can only take stresses in axial direction. The element type makes it possible to model embedded reinforcement together with solid elements in ABAQUS (Simulia, 2010).

The solid element used was solid 10 node tetrahedron second order element with three degrees of freedom in each node Figure 4.6. The reason this element was used is that it describes bending better than the other available alternatives due to that they have more nodes over the height (Simulia, 2010). The element size was approximately 20 mm on the height and 40 mm on the base.

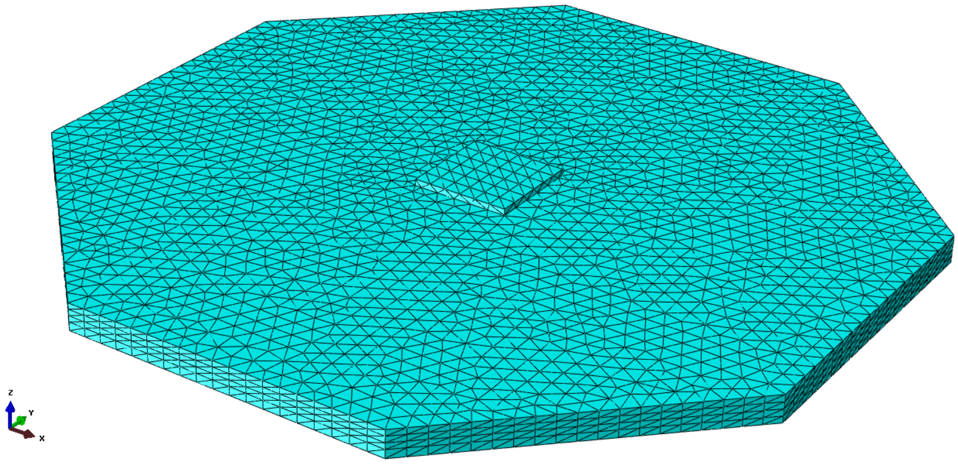

Figure 4.7 Mesh solid elements.

4.3.3 Modelling with shell elements

The shell element used was a S4R: A4- node with smeared reinforcement. The shell element has 6 degrees of freedom in each node, 3 displacements and 3 rotational seen in Figure 4.8. Shell elements are good to describe bending where the thickness is significantly smaller than the other dimensions (Simulia, 2010).

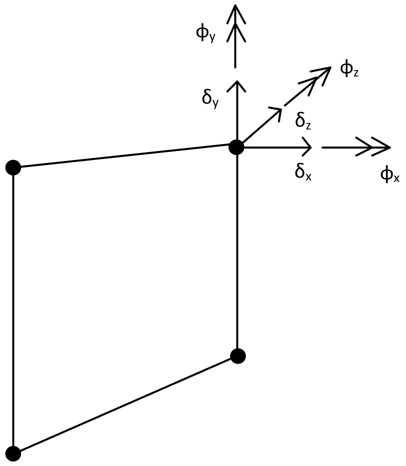

Figure 4.8 Shell element.

The reason for using shell elements is that they are generally better than solid elements to describe bending (Simulia, 2010). The reinforcement was smeared out over the elements with the commando rebar. The element base was approximately 40 mm.

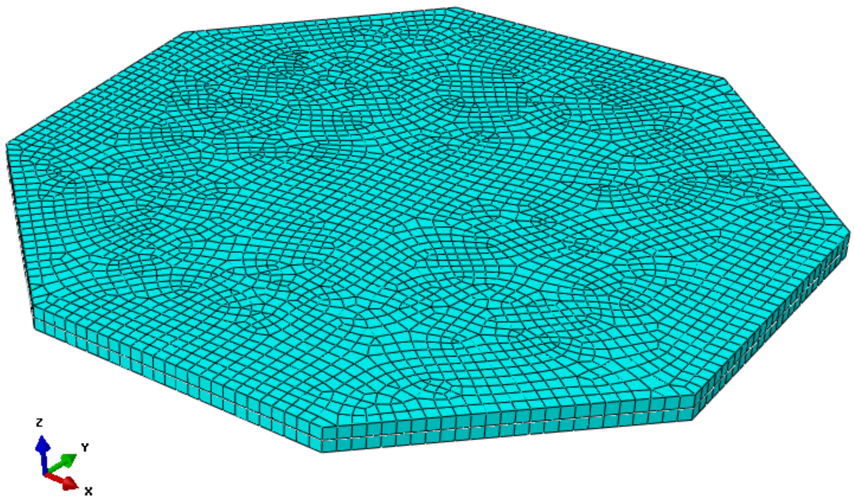

Figure 4.9 Mesh shell elements.

4.4 FEM-Design

The membrane element used in FEM-Design has a various number of nodes depending on mesh shape. It is computationally cheap with only 3 degrees of freedom in each node, consisting of nodal displacement δ_z and nodal rotations Φ_x and Φ_y (Strusoft, 2010) seen in Figure 4.10. The possibility to use different models considering material models and elements were limited in FEM design. The material models were only possible to modify by the means of a few parameters.

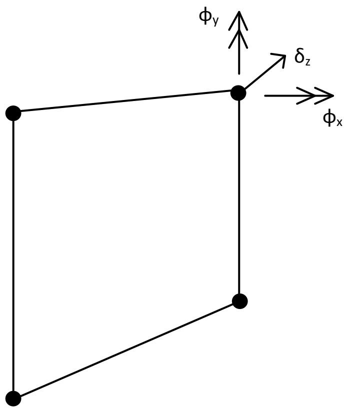

Figure 4.10 Membrane element.

The mesh used in FEM-Design is seen in Figure 4.11. The mesh has an average element size of 0.01 m^2.

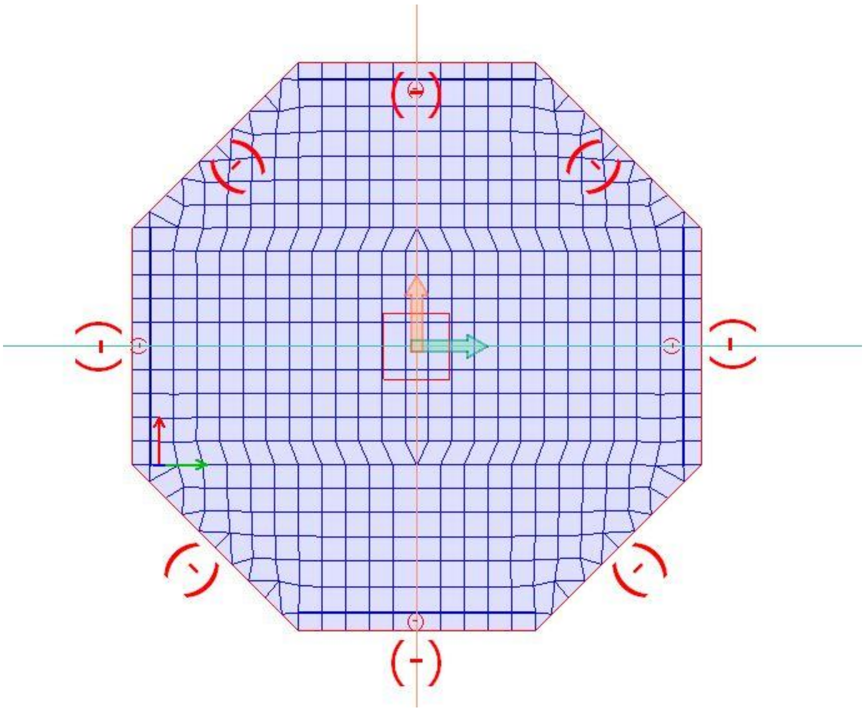

Figure 4.11 Mesh in FEM-Design.

The load increment used in FEM-Design is 10% of the totally applied load, and the number of iterations to find equilibrium was set to 20 (Strusoft, 2010). As boundary conditions displacement in z-direction was locked in all supports. The load was applied as a distributed load where the steel plate was in reality. The boundary condition used in FEM-Design is seen in Figure 4.12.

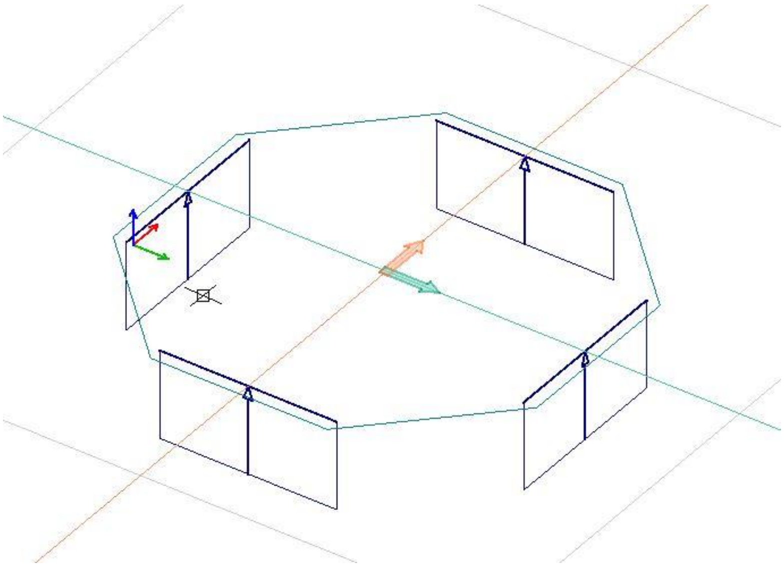

Figure 4.12 Boundary conditions FEM-Design.

The membrane elements were used in combination with smeared reinforcement. It works in a similar way as shell elements. The material models used in FEM-Design is a simplification of the test results from section 2.3. The behaviour of the reinforcement was assumed to have a linear elastic-plastic behaviour, similar as in hand calculation, shown in Figure 3.6. The constitutive model of the concrete in tension was bilinear, reducing from uncracked to fully cracked stiffness when the tensile strength was reached for the concrete. Compared to the material models for concrete tension in ABAQUS there is no post critical capacity.

For the steel fibre reinforced concrete an increased amount of ordinary reinforcement was applied to describe the increase in bending strength due to fibres. For the transformation of fibres into conventional reinforcement the Model Code rigid plastic model in the state where the ultimate strain for the steel fibre reinforced concrete is reached in the bottom fibre was used. This was done by calculating the amount of conventional reinforcement needed to obtain the same moment capacity as for the steel fibre reinforced concrete, by combining equation (4.2) and (4.3) where x and $A_{s,new}$ were unknown.

$$\alpha \cdot f_{ck} \cdot x = f_y \cdot A_{s,new} \tag{4.2}$$

$$m_{ultimate} = \alpha \cdot f_{ck} \cdot x \cdot (x - \beta \cdot x) + f_y \cdot A_{s,new} \cdot (d - x) \tag{4.3}$$

Where

$A_{s,new}$ is the new reinforcement area with contribution from fibres

$m_{ultimate}$ is the moment capacity in ultimate state for rigid plastic model

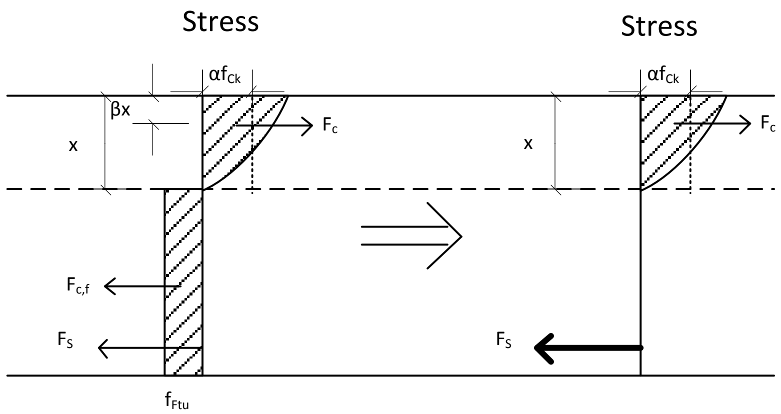

Figure 4.13 Recalculation of fibre contribution to increased reinforcement area.

Due to the limits in the program it was not possible to follow the load history. The problem was solved by increasing the load manually and plotting the load-displacement relation.

5 Results

The experimental results used for a comparison were taken from representative test (Fall et al., 2013). There were three slabs tested for each slab type. The point measured in the experiment is positioned approximately 150 mm from the middle. For all calculations a point in the middle of the slabs was used to compare with the test results.

5.1 Conventional reinforcement

The experimental and calculated load-deflection behaviour is presented in Figure 5.1. The graph clearly shows where the concrete cracks, at approximately 26 kN. After cracking the stiffness of the plate decreases with increased deflection, seen by the decreasing slope of the curve. The slab was loaded until failure in the reinforcement.

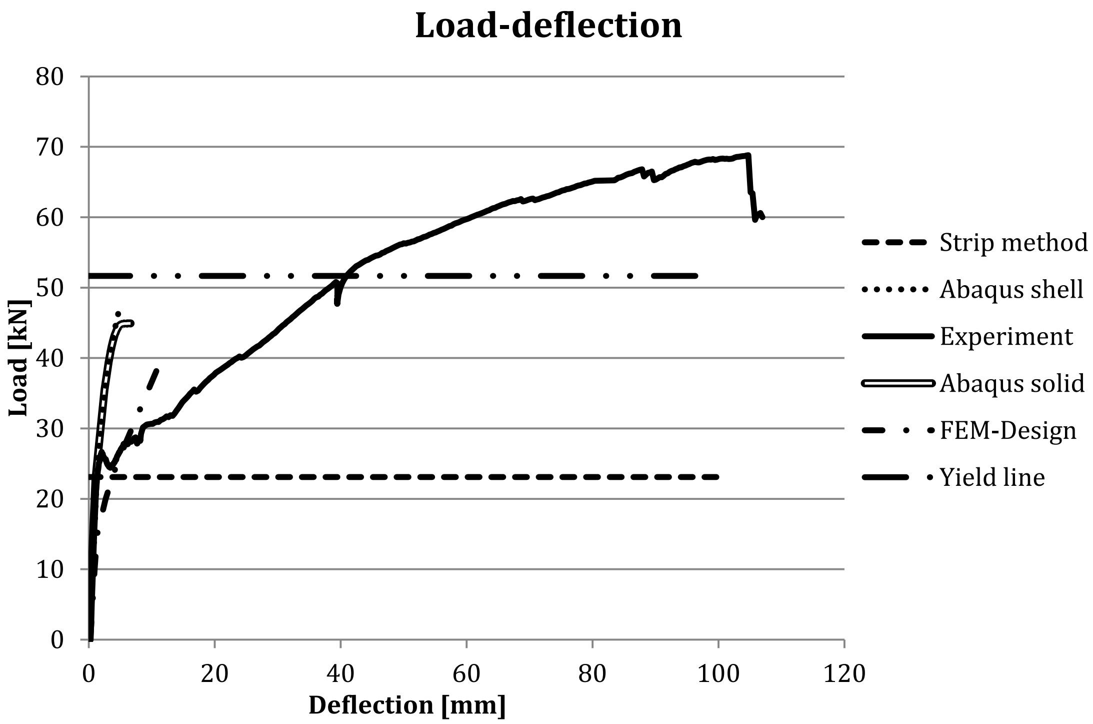

Figure 5.1 Summary of results for all analysis models for concrete with conventional reinforcement.

Results obtained from both solid and shell elements in ABAQUS show a lower capacity than in reality. The slab stiffness was close to reality before cracking but shows a higher stiffness in the post critical behaviour.

Results obtained from FEM-Design show a capacity that is significantly lower than in reality. The stiffness was lower than the ABAQUS models and reality and the concrete cracked for a lower concentrated load.

The result from hand calculation with strip method presented in Figure 5.1 is for the load distribution alternative I. This because alternative I probably are used more often due to its simplicity and the variation of the results are rather low. The results with the other load distributions are presented in Table 5.1. As expected, calculations with yield line analysis gave a higher ultimate load than the strip method.

In Figure 5.2 the load-deflection behaviour for the experimental result and FEM results for small deflection are presented.

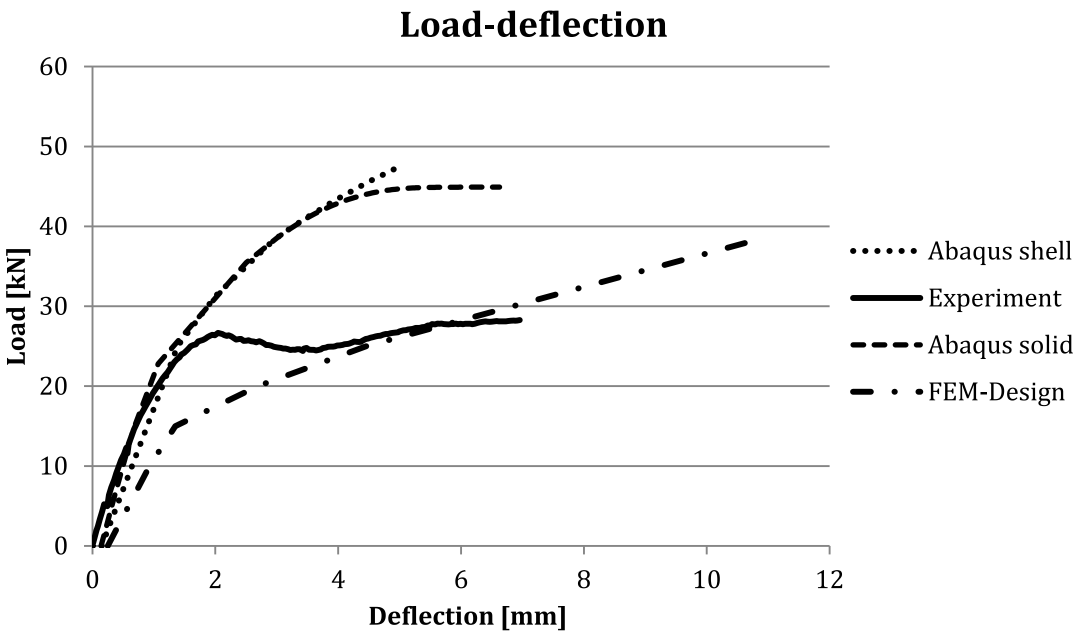

Figure 5.2 Summary of FEM results and experimental result for small deflections.

Measured reaction forces on supports in x- and y-directions are shown in Figure 5.3. After cracking no more load is taken by the y-direction, which indicates that reinforcement in y-direction yields immediately after cracking. The reaction forces in the figure are measured at one support in each direction.

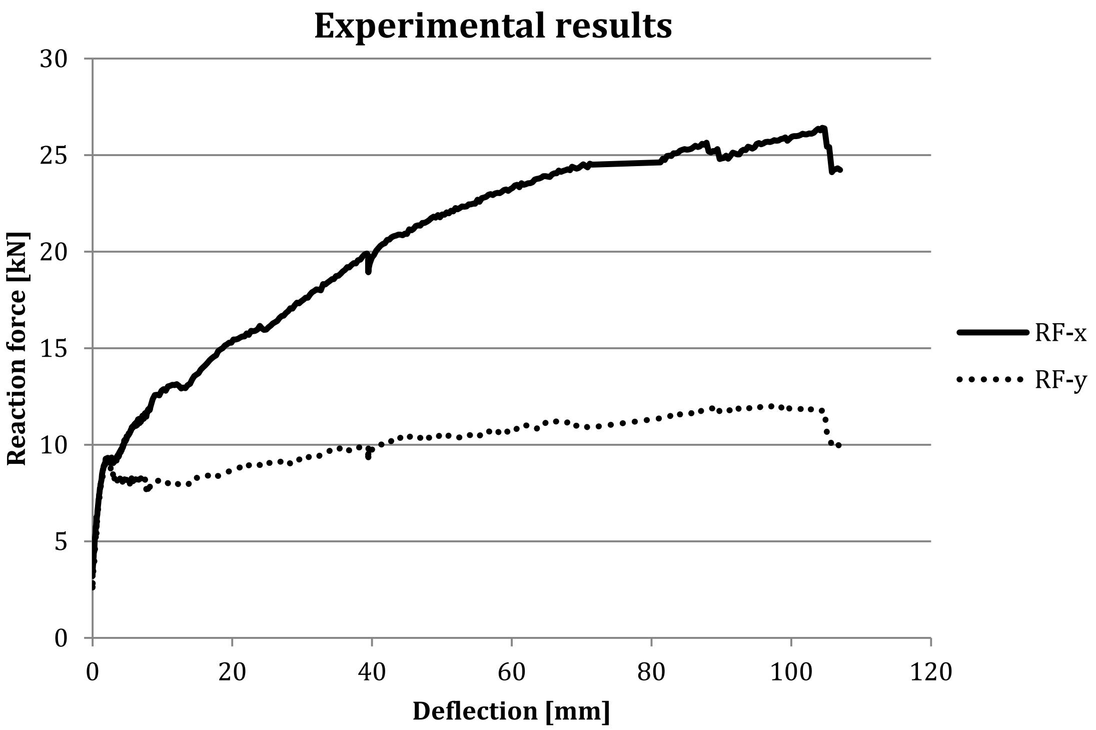

Figure 5.3 Experimental result reaction forces on supports in x- and y-direction (Fall et al., 2013).

Reaction forces from FEM-Design in Figure 5.4 and ABAQUS in Figure 5.5 and Figure 5.6 are presented to see the load redistribution. The results from FEM-Design indicate that there was almost no redistribution of the load. In ABAQUS the results shows a redistribution of the load. Before failure the reaction forces on the supports in y-direction increase. It should be noted that only a few load-steps were used for the graphs because the data for every load-step were exported manually for the plots.

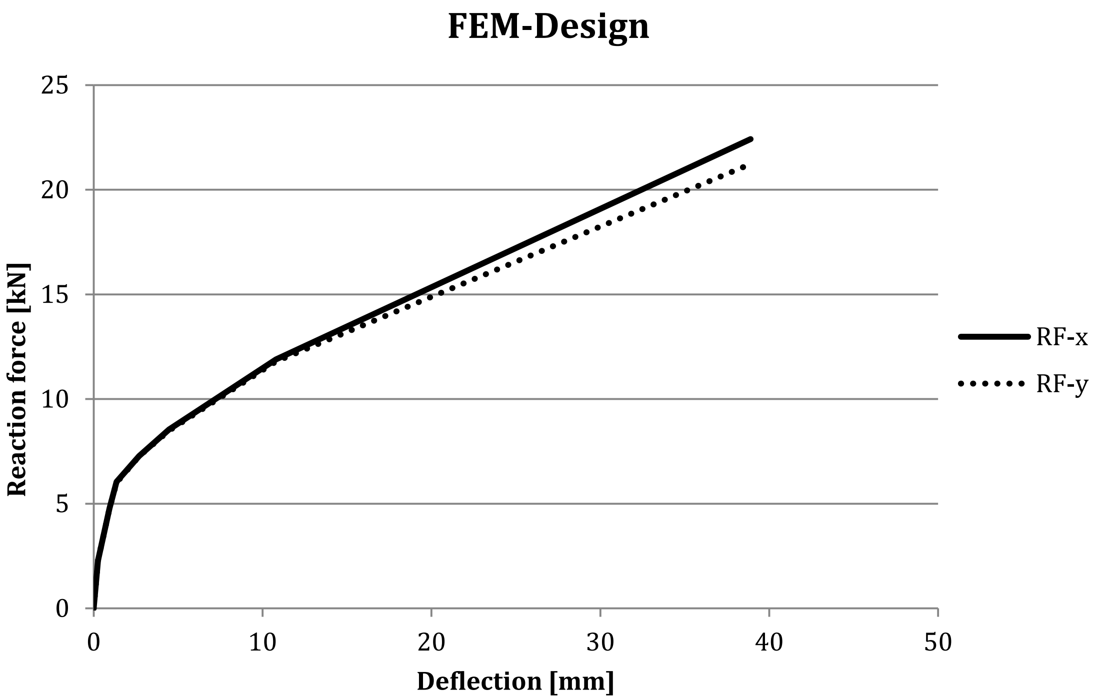

Figure 5.4 Reaction forces on supports in x- and y-direction from FEM-Design.

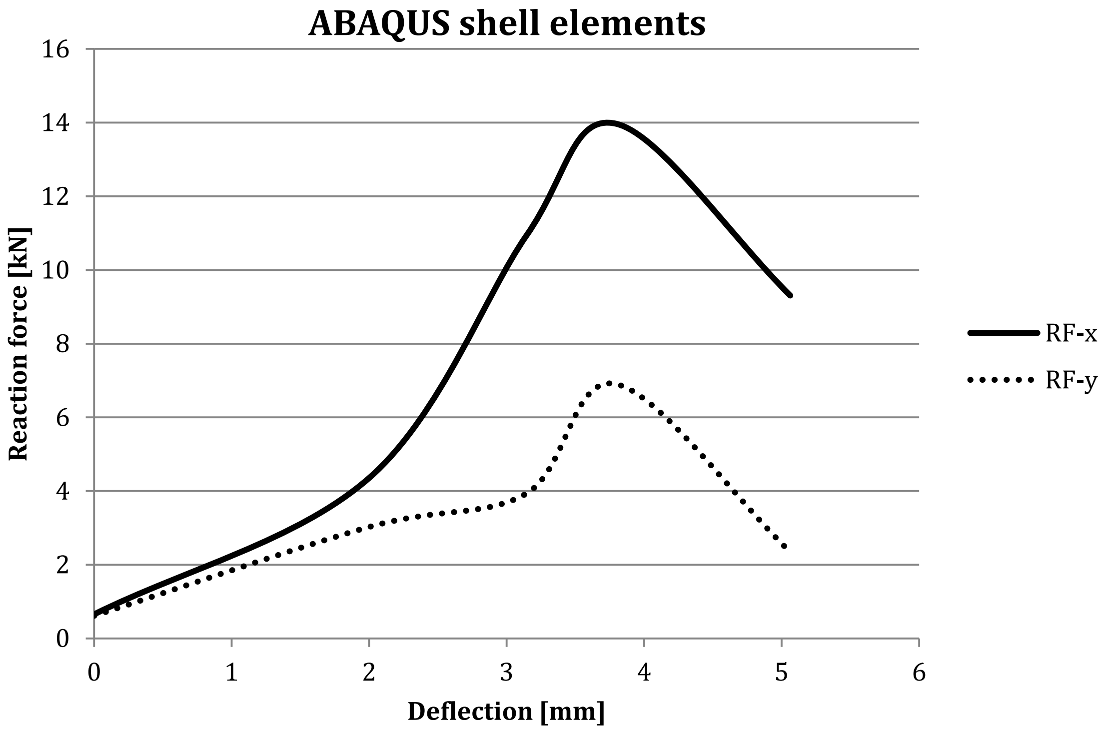

Figure 5.5 *Reaction forces on supports in x- and y-direction from ABAQUS with shell elements.*

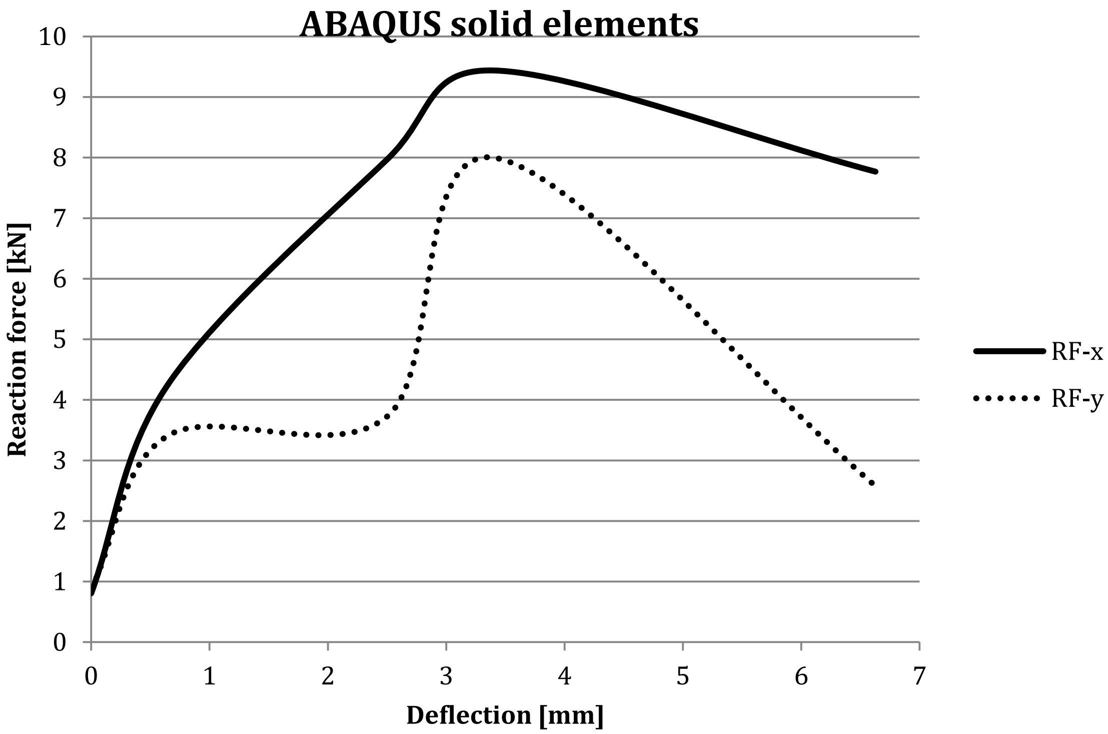

Figure 5.6 *Reaction forces on supports in x- and y-direction from ABAQUS with solid elements.*

5.2 Steel fibre reinforcement

The experimental and calculated load-deflection behaviour is presented in Figure 5.7. The graph clearly shows where the concrete cracks, at approximately 30 kN. For the steel fibre reinforced concrete slab the crack load and the ultimate load were the same. After cracking the capacity of the plate decreased and the contribution from the fibres gave the slab a ductile behaviour.

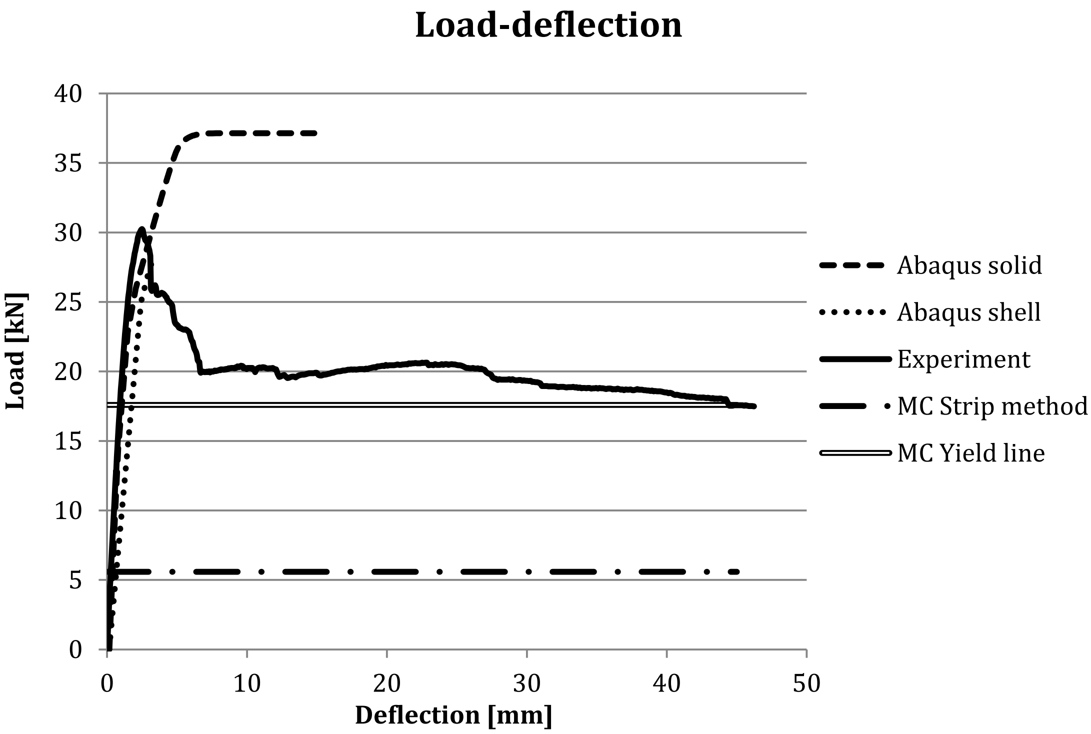

Figure 5.7 Summary of results for all analysis models for FRC without conventional reinforcement.

Results obtained from solid elements in ABAQUS show a higher ultimate capacity and the shell elements a slightly lower ultimate capacity than reality.

The possibilities to model steel fibre reinforced concrete in FEM-Design were limited with the alternative we tried. For the steel fibre reinforced slab without conventional reinforcement, there was no possibility to create a working model in FEM-Design, therefore no investigation including only fibre reinforcement are presented in the results. The model created where unable to carry the self-weight without any concentrated load. The reinforcement amount that was used to replace the fibre contribution in FEM-Design was calculated to 88 mm^2/m with a cover thickness of 20 mm in both directions.

The results from hand calculation in Figure 5.7 are both from the strip method and the yield line analysis. With FIB Model Code the highest ultimate load were obtained with the linear model. Results for the rigid plastic model are presented in Table 5.2. The results from FIB Model Code with the strip method were calculated with load distribution alternative I.

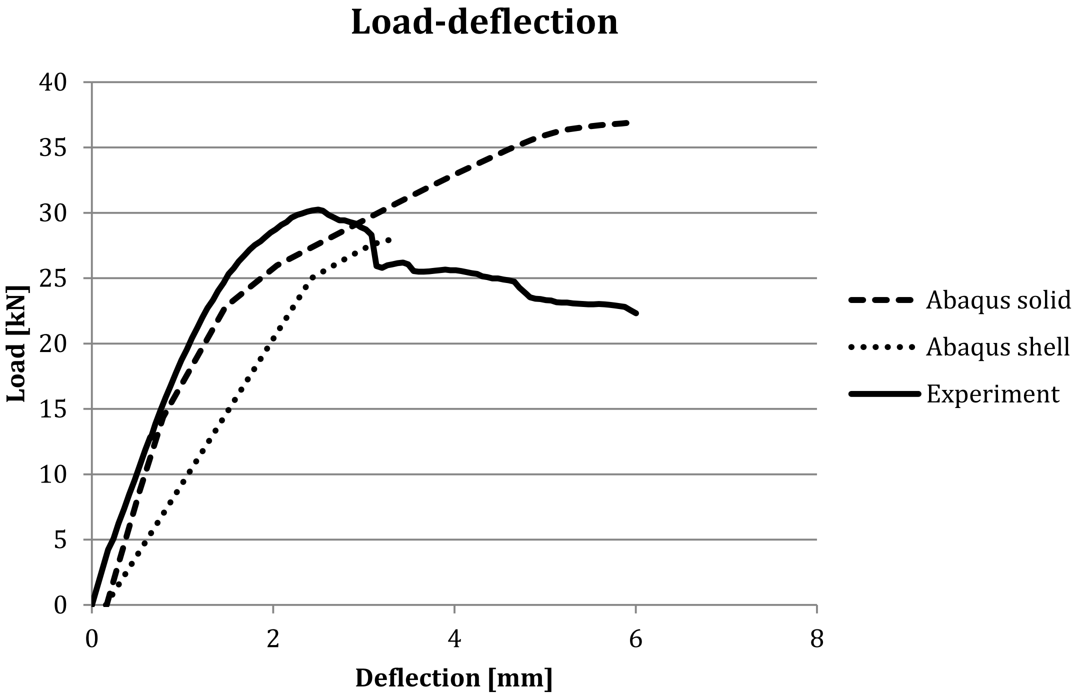

Figure 5.8 Summary of FEM results and experimental result for small deflections.

The slab stiffness for ABAQUS model with solid elements was almost the same as in reality until the ultimate load was reached for the experiment see Figure 5.8. In the shell model the slab stiffness was slightly lower than in reality.

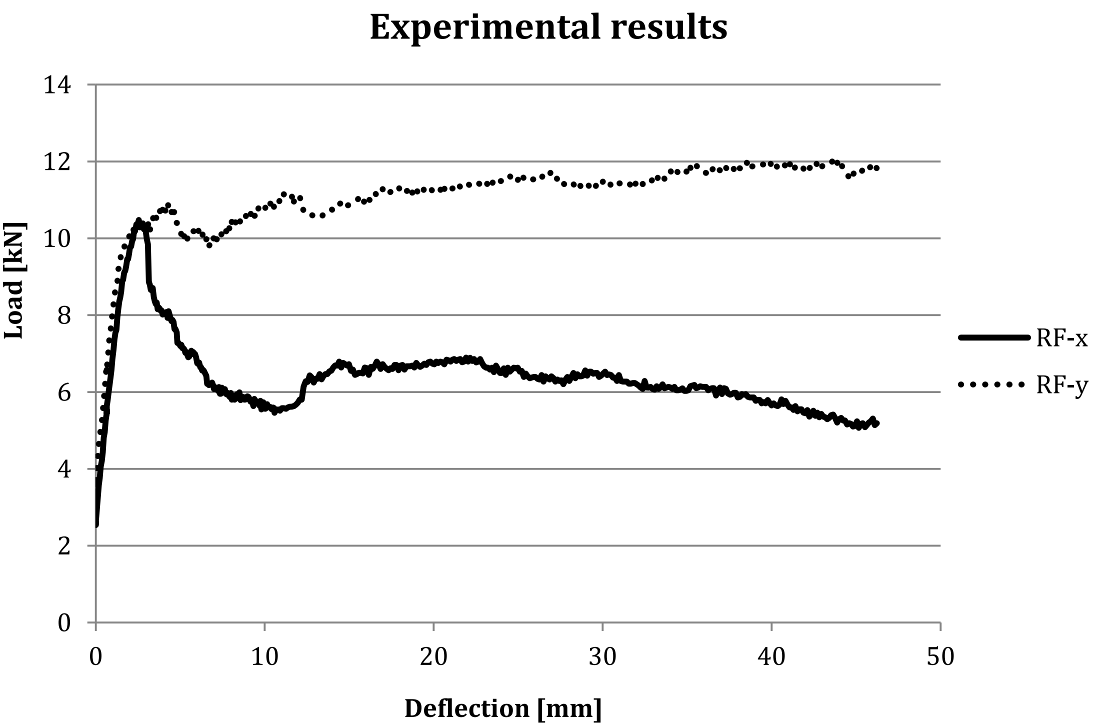

Figure 5.9 Experimental result reaction forces on supports in x- and y-direction (Fall et al., 2013).

Reaction forces from experiment are presented in Figure 5.9. In ABAQUS the reaction forces were equal on all supports, which was expected. The same distribution of reaction forces was seen in the experiment up to ultimate load. After the ultimate load was reached, there was a load redistribution which indicates an uneven fibre distribution or some other imperfections. This behaviour was not included in the FE models.

5.3 Conventional and steel fibre reinforcement

The response for the slab with conventional and fibre reinforcement from experiments and calculations are presented in Figure 5.10. The ultimate load in experiments was reached just before the first reinforcement bar failed.

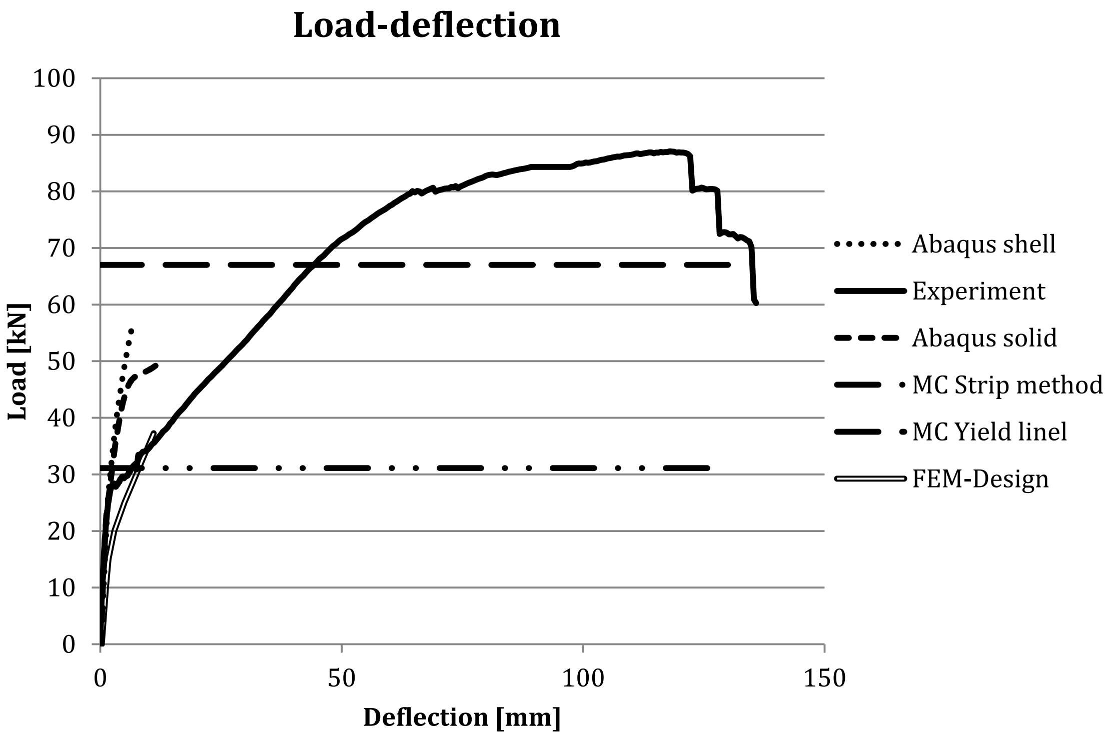

Figure 5.10 Summary of results for all analysis models for FRC with conventional reinforcement.

The results from hand calculations with FIB Model Code in Figure 5.10 shows the highest results obtained from the strip method and the yield line analysis. For both the strip method and the yield line analysis the highest values were obtained for the linear model assuming full load transfer over cracks. Both of the ultimate loads were obtained when the ultimate compressive strain for concrete was reached. For the assumption of limited load transfer over cracks the highest ultimate load were obtained when the ultimate strain for steel fibre reinforced concrete was reached in the bottom fibre for both plastic and linear model. In Table 5.3 all the results are presented. All results from FIB Model Code with the strip method were calculated with load distribution alternative I.

In Figure 5.11 the load-deflection behaviour for the experimental and FEM results for small deflection are presented. The slabs in ABAQUS and experiments show a similar behaviour before cracking. In the post cracking stage the ABAQUS model shows a higher stiffness than in experiments. FEM-Design model shows a lower stiffness and cracks for a lower load than in experiments. After cracking the behaviour is linear until the ultimate capacity of the reinforcement was reached in the FEM-Design analysis.

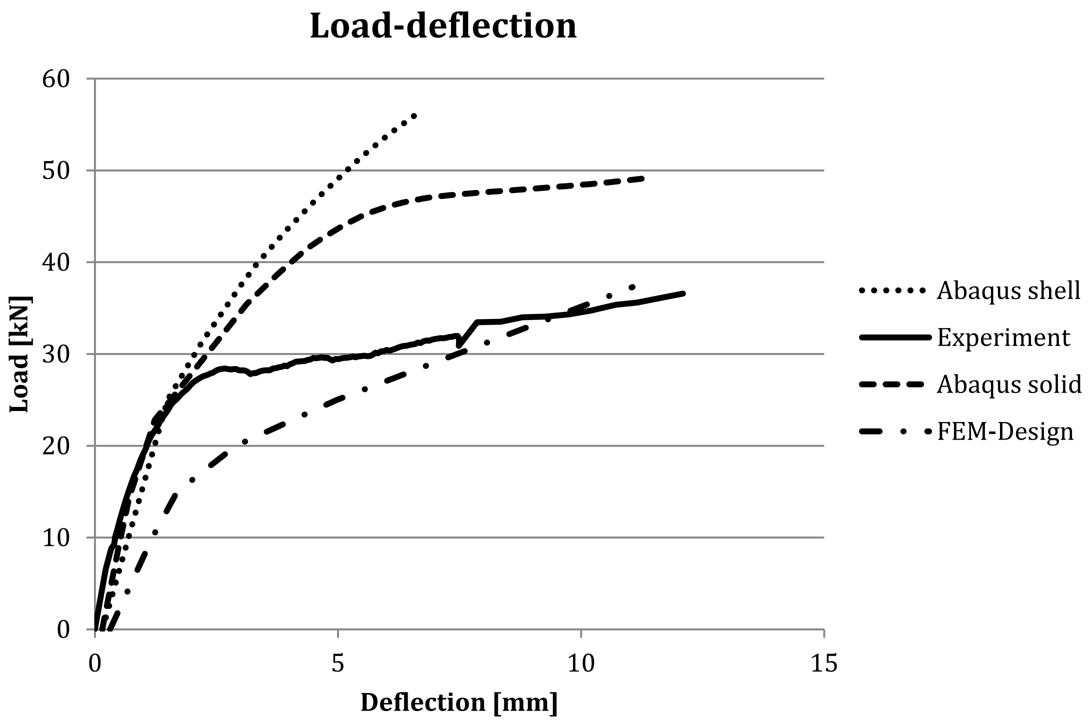

Figure 5.11 Summary of FEM results and experimental result for small deflections.

The points where the slab cracks can be seen at the first notch, at approximately 28 kN for the experimental result.

Figure 5.12 shows the measured response for the reaction forces from a representative test done by (Fall et al., 2013). Considering the reaction forces it could be seen that the stiffness ratio between x- and y-direction increases with increased load. Unlike the conventional reinforced slab, the capacity in y-direction could be increased after cracking.

Reaction forces obtained from FE-models in ABAQUS and FEM-Design were similar to those obtained from conventional reinforcement.

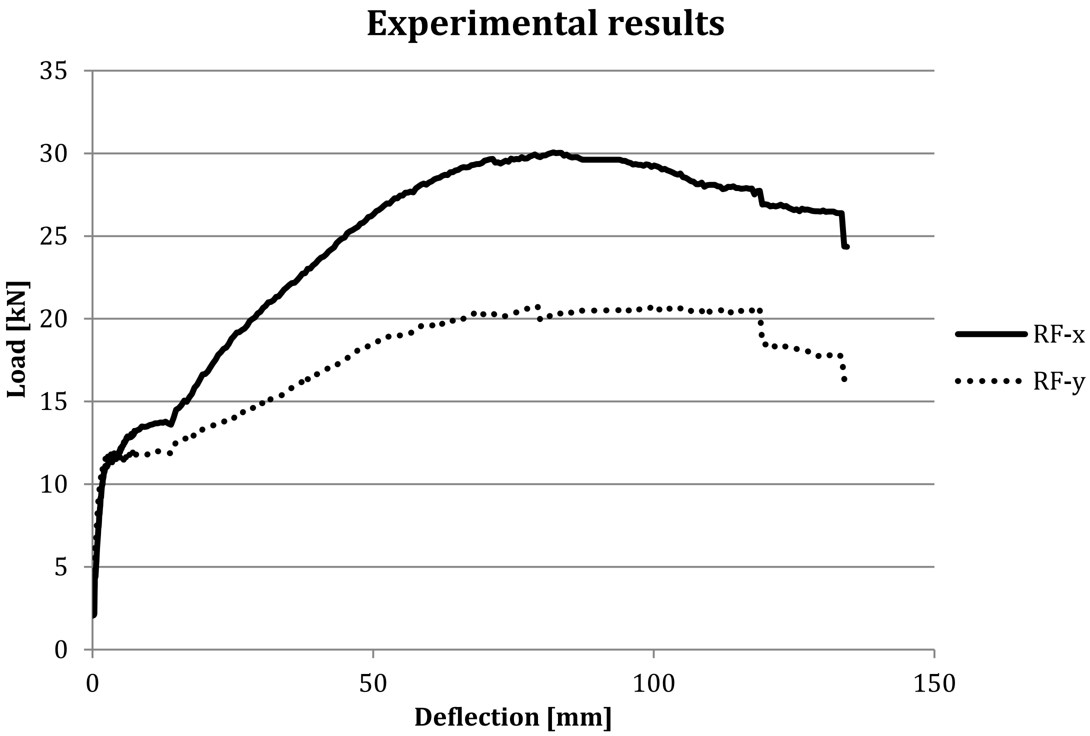

Figure 5.12 Experimental result reaction forces on supports in x- and y-direction (Fall et al., 2013).

5.4 Crack pattern

Crack pattern from experiments are shown in Figure 5.13. The slabs with only steel fibre reinforcement have one distinct crack in each direction. This indicates that the steel fibre itself is not capable to distribute the load. The slabs with conventional reinforcement shows cracks distributed over a larger area, which indicate distribution of the load in the slabs.

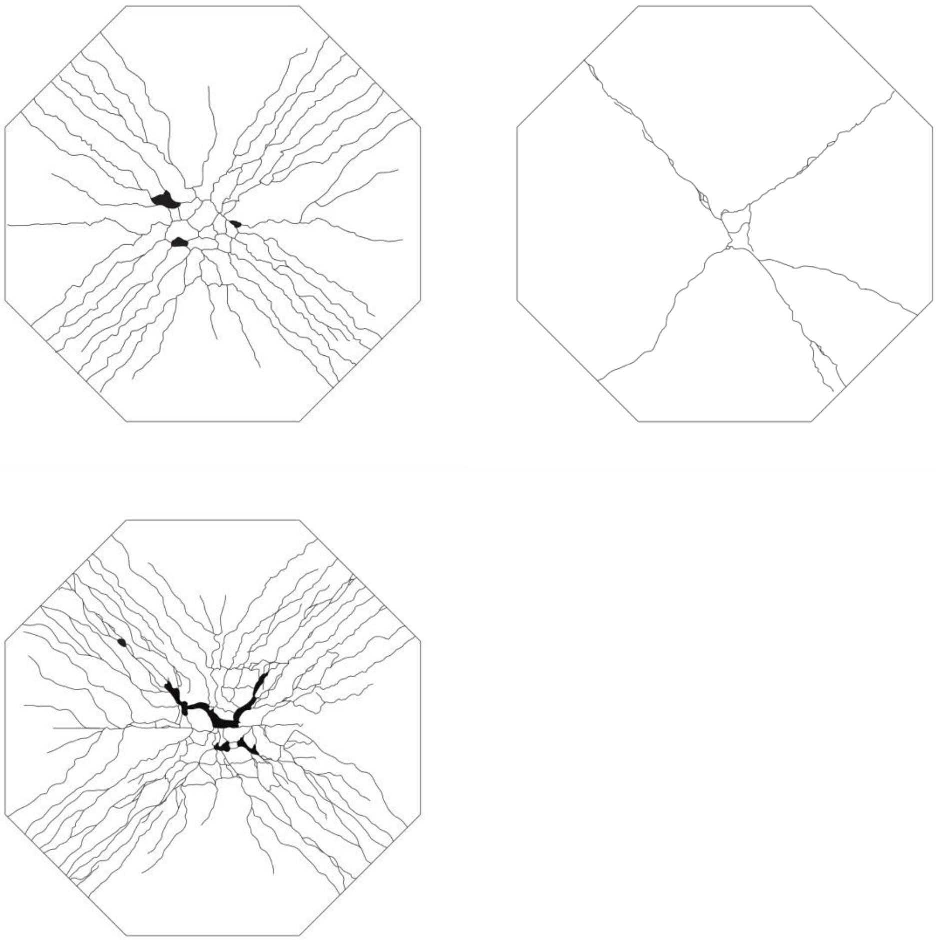

Figure 5.13 Crack pattern from experiments, conventional reinforcement (up left), steel fibre reinforcement (up right) and combination of steel fibre and conventional reinforcement (down right) (Fall et al., 2013).

The crack pattern from ABAQUS with solid elements indicate in Figure 5.14 corresponds well to the experimental failure mechanism with a square of cracks underneath the steel plate and a concentrated crack pattern.

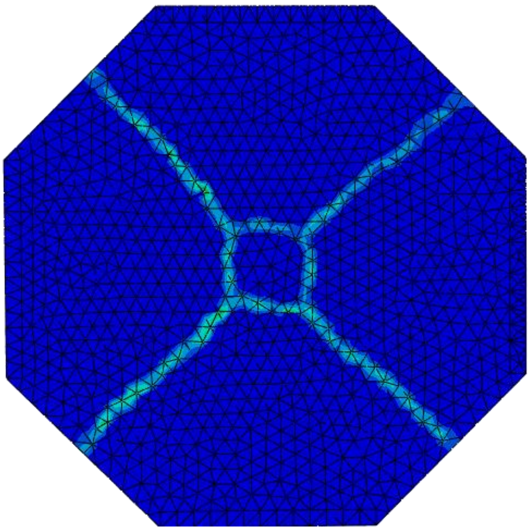

Figure 5.14 Concrete strain ABAQUS solid elements for steel fibre reinforced concrete with conventional reinforcement.

Figure 5.15 shows contour plots of the strain in the slabs which indicates where cracks occurred. The strains were obtained for ultimate state. Looking at the figures it is possible to see the differences in stiffness ratio. In the slabs with conventional and fibre reinforcement the cracks are slightly turned due differences in stiffness. In the steel fibre reinforced slab the stiffness was equal in both directions resulting in evenly distributed cracks. The difference between the crack pattern from solid and shell elements depends on various scale, since the reinforcement was modelled in the solid model.

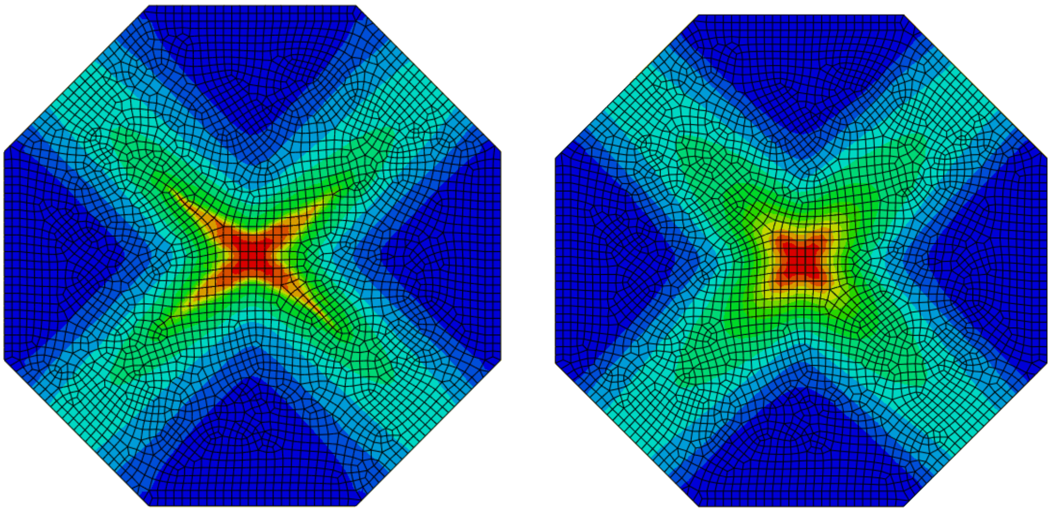

Figure 5.15 Strain shell elements, steel fibre reinforced concrete with conventional reinforcement (left) and steel fibre reinforced concrete (right).

The crack pattern obtained from the FEM-Design analysis shows precise and crack openings see Figure 5.16 but the crack openings fully depend on the element size. The crack pattern corresponds to reality considering the differences in stiffness.

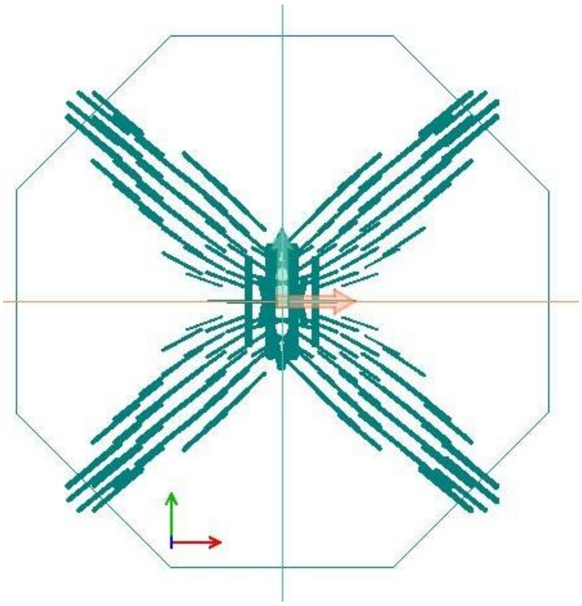

Figure 5.16 Crack pattern FEM-Design with conventional reinforcement concrete.

5.5 Work effort

Evaluating the required work effort for each model was complicated, due to differing knowledge about the different programs and analytical methods. The modelling times in FEM-Design were tested by our supervisor Linda Cusumano, the ABAQUS models were tested by Tim Nilsson and the analytical models by Karl Levin. However, considering the work effort by a single person does not generate statistically significant results. Hence it is not possible to reach strong conclusions concerning the work time spent. The work effort also includes a subjective assessment on the difficulty of each model.

The three different alternatives of load distribution with strip method gave similar results. A reason for that is that only the uniformly distributed load was divided with the strip method. The concentrated load for all alternatives was distributed on each support with regard to moment capacities in each direction. Since the concentrated load was higher than the self-weight of the slab the difference in load distribution resulted in small differences in ultimate load. The work effort for the strip method alternatives was very different. Alternative I was the simplest and gave similar results to the more complicated load distributions.

Considering the material models used for hand calculations they became more extensive when fibres were included. The linear model in Model Code were more time consuming than the plastic model but gave ultimate loads closer to reality.

The yield line analysis was perceived as more complex due to the kinematic theory and the time used for this analysis was longer than for strip-method. The gravity centre calculation was the time consuming part for the calculations. With a simplification for the gravity centre the time could be reduced.

The non-linear analysis in FEM-Design was the fastest to model with 15 min and also the simplest model including less information than the others. Except for the cases where fibre reinforcement was included, where understanding the behaviour of fibre reinforcement and how to transfer of it in to reinforcement was required. The transformation of fibre reinforcement into conventional reinforcement also increases the time consumed.

For the ABAQUS analysis a higher understanding about material behaviour and finite element method was required. The material models are more accurately describing a post critical behaviour resulting in more extensive input data to create an analysis. In the case where solid elements were used 3D truss elements had to be modelled and meshed separately increasing the time consumption and complexity compared to the model where shell elements were used. The time used to create the models were 25 minutes for ABAQUS shell and 30 minutes for ABAQUS solid.

The computational time for the different nonlinear analysis varies, but this was not considered to be a crucial factor since it is possible to run the analysis while working with other things. This is in contrast to the analytical methods, where the designer has to be active when calculating the response and ultimate capacity.

5.6 Summary of results

As could be seen in Table 5.1, Table 5.2 and Table 5.3 where the work effort is presented, the time consumption to create a finite element model or an analytical model does not differ much. The difference in work effort is remarkably small when Model Code was excluded. The work effort for hand calculations could possibly be reduced with a worksheet, since many calculations are the same.

Table 5.1 Summary of results for plain concrete with conventional reinforcement.

	Ultimate load [kN]	Work effort [min]	Comment
Experimental	68.8	-	-
Strip method I	23.1	15	Easy calculations
Strip method II	24.1	20	Rather easy
Strip method III	24.6	30	Complicated
Yield line analysis	51.7	20	Underestimated reality
ABAQUS solid	44.9	25	Complex
ABAQUS shell	47.7	20	
FEM-Design	38.2	15	Easy

Table 5.2 Summary of results for FRC without conventional reinforcement.

	Ultimate load [kN]	Work effort [min]	Comment
Experimental	30.3	-	-
Strip-method alt. I, Model Code Linear	5.6	35	Linear model more complicated
Strip-method alt. I, Model Code Plastic	4.3	25	Plastic model simpler
Yield line analysis, Model Code Linear	17.6	40	
Yield line analysis, Model Code Plastic	15.1	30	
ABAQUS Solid	37.7	20	Overestimated reality, shorter modelling time than with reinforcement bars
ABAQUS Shell	27.9	20	

Table 5.3 Summary of results for FRC with conventional reinforcement.

	Ultimate load [kN]	Work effort [min]	Comment
Experimental	87.1		
Strip-method alt. I, Model Code Linear, Full load transfer	31.1	35	
Strip-method alt. I, Model Code Linear, Limited load transfer	28.9	35	
Strip-method alt. I, Model Code Plastic, Full load transfer	28.9	25	
Strip-method alt. I, Model Code Plastic, Limited load transfer	28.6	25	
Yield line analysis, Model Code Linear, Full load transfer	67.0	40	
Yield line analysis, Model Code Linear, Limited load transfer	62.8	40	
Yield line analysis, Model Code Plastic, Full load transfer	62.7	30	
Yield line analysis, Model Code Plastic, Limited load transfer	62.2	30	
ABAQUS Solid	49.2	25	
ABAQUS Shell	56.0	20	
FEM-Design	37.3	20	Fibre calculations

In Figure 5.17, Figure 5.18 and Figure 5.19, the work effort and the ultimate load are presented in bar graphs for all analysis methods. For the slabs with steel fibre reinforcement, the ultimate load calculated with FIB Model Code is presented for the assumption of limited load transfer over cracks.

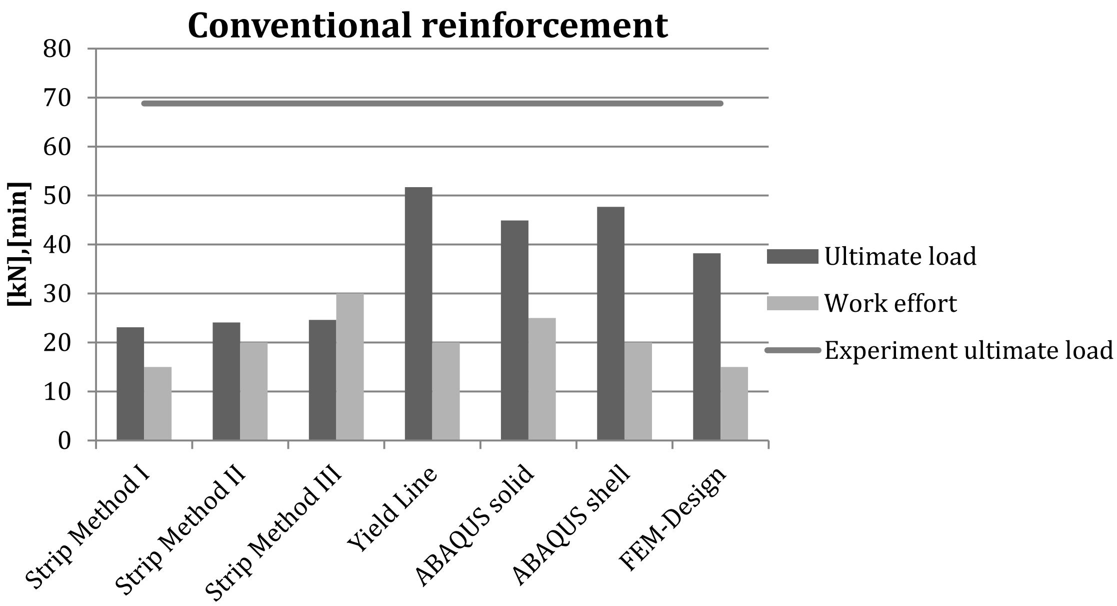

Figure 5.17 Work effort and ultimate load for conventional reinforced concrete slab

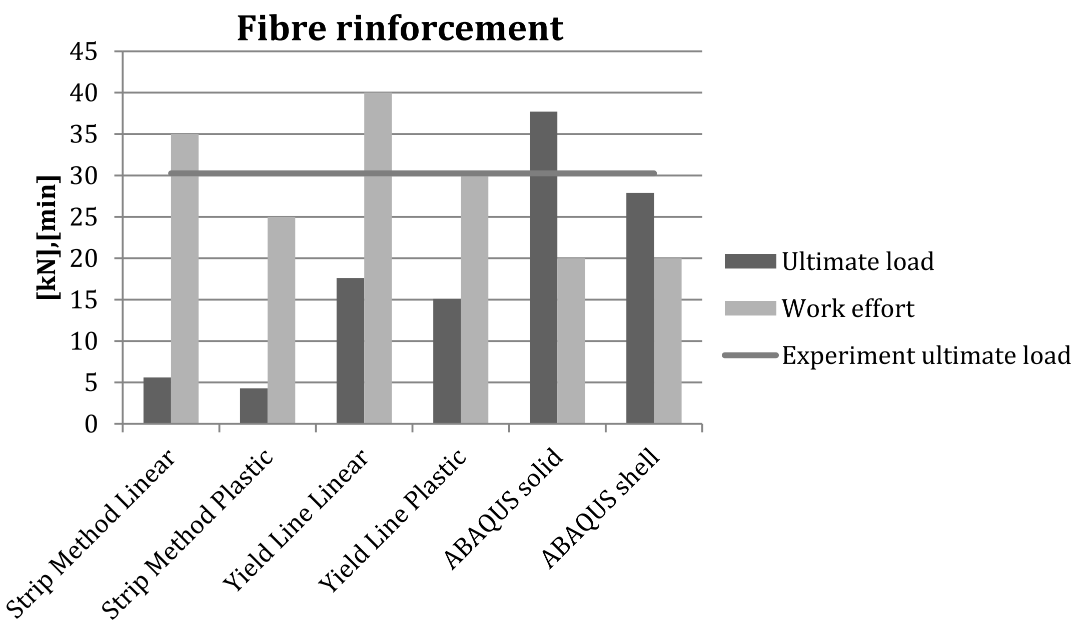

Figure 5.18 Work effort and ultimate load for fibre reinforced concrete slab

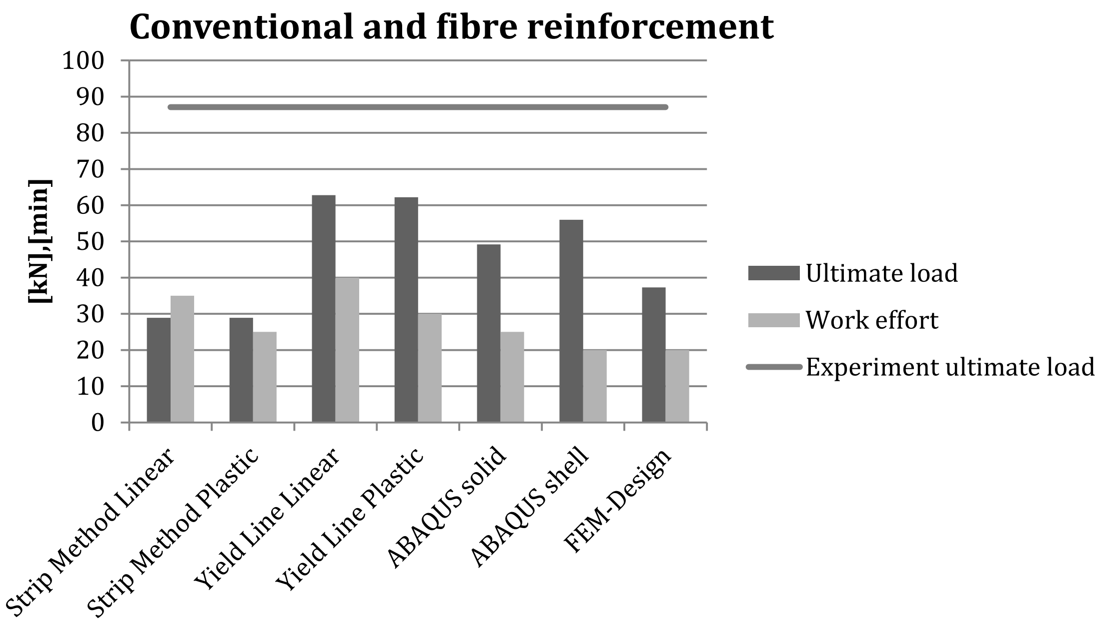

Figure 5.19 *Work effort and ultimate load for conventional and fibre reinforced concrete slab*

6 Discussion

The results obtained in model deviates from the experiments, how much depends on what model or type of analysis that was used. The different methods required various work effort and knowledge by the designer. While evaluating the accuracy of the models the ultimate loading capacity has been considered the most important aspect since none of the finite element models gave reasonable deflections and for the analytical models there was no simple method to calculate the deflections.

Comparing the different models with the experimental results, it can be noted that all except one underestimated the ultimate capacity. The exception was the ABAQUS model with solid elements for the steel fibre reinforced slab.

The stiffness was overestimated for all models after the cracking load was reached, resulting in a significantly smaller deflection than in experiments. The stiffness in both ABAQUS models follows the real behaviour well until the experimental slab cracks. Since the behaviour was very similar for shell and solid elements there is possibly a need for further investigations on the material models used for concrete in combination with mesh shape, element type and reinforcement strength. For example one sensitive factor while calculating the material response was crack band width. There are different recommendations on how to choose the crack band width. The element height was used in this study as the crack band width. Since the critical cracking was obtained in the plain, the crack band width might have been set to the length of an element to describe the response of the slabs better. Looking at the results obtained in ABAQUS compared to the experimental it can be seen that a softer response after cracking would be necessary to better describe the response. With smaller crack band width, higher deflection and ultimate capacity were obtained in ABAQUS.

The possible choices in ABAQUS can increase the accuracy of the models if used correctly, but there is also a risk if bad choices are made. For example the designer has to be aware of the strength and weaknesses for different element types. In FEM-Design there are limited options for the designer; this facilitates the modelling.

Combining analytical hand calculations with the FIB Model Code, the ultimate load calculated from strip method was lower than for the experiment. The ultimate loads calculated with yield line analysis were relatively close to reality and on safe side in opposition to the theory. This is probably due to the simplifications made, e.g. the concrete was assumed not to carry tension forces after cracking, which is not true since load can be transferred in the uncracked concrete between the cracks. Furthermore, there might have been membrane forces in the experiments, creating a circle with beneficial stresses around the concentrated load.

The calculations with the linear model in FIB Model Code were more time consuming but gave results closer to the real behaviour than the plastic model. Assumptions with full load transfer over cracks gave results closer to reality, but the assumption of limited load transfer is recommended because it describes reality better.

Calculating the ultimate capacity for fibre reinforced concrete slabs with plastic analyses might not be secure. Considering the measured deflections, the slab with only fibre reinforcement shows smaller deflections, resulting in a smaller rotational capacity which is needed for plastic analysis. In the analysed slabs the plastic analyses results in a capacity on the safe side but there is no guarantee it will work for other geometries.

The ultimate capacity for the fibre reinforced slabs without conventional reinforcement was in experiments obtained for the cracking load. It may therefore be better to make an elastic analysis for fibre reinforced concrete slabs where elementary cases are available, which was not the case here.

The indicated crack patterns obtained from FEM-Design and both ABAQUS models looks reasonable. Considering the number of cracks and crack spacing in FEM-Design it can be concluded that the crack width and crack distance depends on the element size. Membrane element, with no deformation in x- and y-direction might in some cases have problem to describe cracking.

ABAQUS indicates a crack pattern by the maximum principle strain in concrete. For the solid elements models the crack pattern was more concentrated while the indicated crack patterns for the shell elements was more smeared out. In the solid model it is also possible to follow the stresses and strain in the embedded reinforcement.

It can be seen that the results do not deviate significantly more from the experimental when fibres are used except for FEM-Design. Reasons for this could be the possibility to model post critical behaviour for concrete in tension. This indicates that our model is not suitable for this purpose in FEM-Design.

The investigated slab had a very simple geometry and obvious boundary conditions along supports, in reality the situations are often more complicated. Houses are rarely built with geometry as simple as this slab or with rollers as supports. Instead there are irregular shapes, point supports and partially fixed supports that have to be considered. Changes like that will simplify the use and work effort for finite element programs compared to analytical methods. It is important to remember that the work effort investigation was based on our own experiences of difficulty and the time is tested by one person for each model.

Possible deviations from the experiments can be fibre distributions in the slab and the material tests. This could affect the material models used for analysis and the results from the experiments.

7 Conclusions

Evaluating the models considering both work effort and accuracy it was noticed that FEM Design were a faster instrument than ABAQUS and the hand calculations. Complicated geometries and reinforcement placement increase the benefits of finite elements programs. For a simple geometry, however, there were small differences in work effort using a simple load distribution and possibly a work sheet for strip and yield line method.

For the ultimate concentrated load obtained in the different finite element models, the best result was from the nonlinear ABAQUS analysis with shell elements. What we did not expect were the low ultimate point load obtained from the response in FEM-Design which was far from reality. This might be because FEM-Design does not take account of any plastic redistribution which could be seen on the ratio between reaction forces.

Following the load deflection behaviour in the finite element models, it could be noted that the deflections were remarkably lower for the ultimate load compared to the real slab for the slabs including conventional reinforcement. This is a crucial error when deflection has to be considered in constructions due requirement on maximum deflection.

The weak point in doing nonlinear analysis in ABAQUS is all data required. It is harder to obtain material data since it is not tabulated and has to be investigated from material testing. If there were deformation curves suggested for concrete and steel fibre reinforced concrete, it would simplify the analysis and thereby increase the possibility to use nonlinear analysis at construction offices.

The results obtained from strip method are lower than the results obtained with yield line analysis in accordance with the theory. Yield line analysis is considered more suitable for analysis of concrete slabs and strip method for design. Yield line analysis is also suitable for more complicated geometries.

Considering the aim to investigate the possibility of modelling fibre reinforcement in commercial software it was simple in ABAQUS with possibility to describe the post critical behaviour in concrete. FEM Design did not have the same possibility in modelling the post critical behaviour of concrete. This resulted in a trial and error process to combine analytical methods for fibre reinforcement and translate it to increased capacity and stiffness in the section. Modelling fibre reinforcement in FEM-Design with increased capacity and stiffness did not work out well. It shows a lower capacity than the slab with conventional reinforcement. Therefore, further investigations are needed on this point.

All models showed capacities that were significantly lower than in experiments. One reason might be membrane forces which is a circle with compressive forces around the point-load creating beneficial multi-axial stresses.

The recommendations considering analysis of fibre reinforced and conventionally reinforced concrete would be to use ABAQUS shell for complicated geometries where bending failure is likely; for cases where time is the most crucial factor FEM-Design would be recommended.

In reality a combination of finite element software for load calculations and analytical methods for designing of concrete and reinforcement are commonly used.

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Appendix

APPENDIX A: ANALYSIS OF CONVENTIONAL REINFORCED CONCRETE SLAB

APPENDIX B: ANALYSIS OF STEEL FIBRE REINFORCED CONCRETE WITH FIB MODEL CODE

APPENDIX C: CALCULATION OF CONTRIBUTION FROM STEEL FIBRE REINFORCEMENT IN FEM-DESIGN

Appendix A
Analysis of conventional reinforced concrete slab

Indata

Geometry

$n_x := 25$ number of reinforcement bars x-direction
$n_y := 13$ number of reinforcement bars y-direction

$\phi := 6\ mm$ Diameter reinforcement

$h := 83.92\ mm$ Height slab
$l_s := 1.0\ m$ Length support
$l_e := 0.7\ m$ Length
$l_a := 75\ mm$ Length support to edge
$l_{e2} := l_e - l_a = 0.625\ m$
$l := l_s + 2 \cdot l_e = 2.4\ m$ Total length slab
$l_2 := l_s + 2 \cdot l_{e2} = 2.25\ m$ Length between supports
$c_x := 20\ mm$ Cover thickness reinforcement x-direction
$c_y := 26\ mm$ Cover thickness reinforcement y-direction

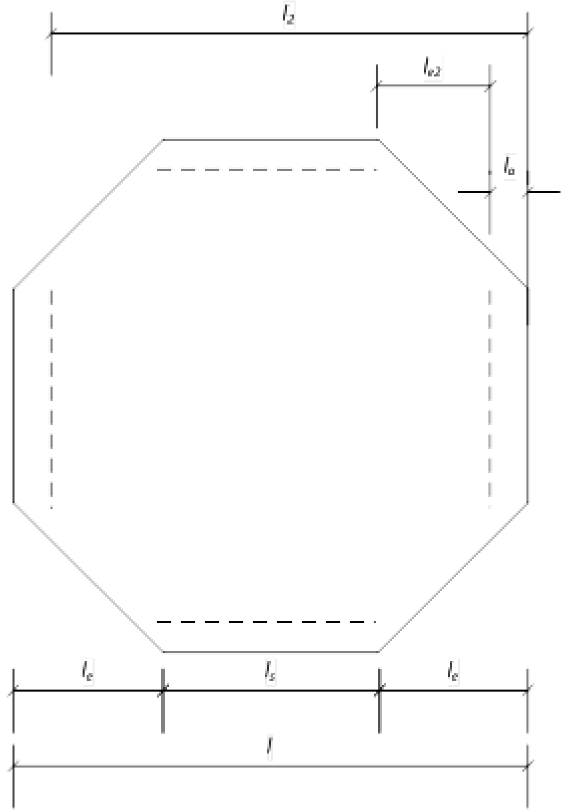
$$A_{sx} := \frac{\left(\frac{\phi}{2}\right)^2 \cdot \pi \cdot n_x}{l} = 294.524\ \frac{mm^2}{m}$$ Area reinforcement per meter x-direction

$$A_{sy} := \frac{\left(\frac{\phi}{2}\right)^2 \cdot \pi \cdot n_y}{l} = 153.153\ \frac{mm^2}{m}$$ Area reinforcement per meter y-direction

$$A_{slab} := l^2 - 4 \cdot \frac{l_e^{\,2}}{2} = 4.78\ m^2$$ Total area slab

$$d_x := h - \frac{\phi}{2} - c_x = 60.92\ mm$$ Distance top slab to ceneter of reinforcement x-direction

$$d_y := h - \frac{\phi}{2} - c_y = 54.92\ mm$$ Distance top slab to ceneter of reinforcement y-direction



Concrete

$f_{ck} := 51.18\ MPa$	Characteristic compression strength, value from lab result
$E_c := 31.7267\ GPa$	Youngs modulus concrete, value interpolated from table
$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$	Ultimate strain in concrete
$f_{ctm} := 2.70\ MPa$	Mean tensile strength

$$k := 1.6 - \frac{h}{1000\ mm}$$

$f_{ctm.b} := k \cdot f_{ctm} = 4.093\ MPa$ Bending strength

$\rho_c := 2311.804\ \frac{kg}{m^3}$ Density concrete

Steel

$f_y := 560\ MPa$	Yield strength reinforcement bars
$f_u := 672\ MPa$	Ultimate strength reinforcement bars
$E_s := 200\ GPa$	Youngs modulus steel
$\varepsilon_{syk} := \frac{f_y}{E_s} = 2.8 \cdot 10^{-3}$	Yield strain steel
$\varepsilon_{su} := 0.089339$	Ultimate strain

$\rho_s := 7850 \ \frac{kg}{m^3}$ Density steel

$\alpha_c := \frac{E_s}{E_c} = 6.304$ Relation youngs modulud

$\alpha_\rho := \frac{\rho_s}{\rho_c} = 3.244$ Relation density

$\alpha := 0.810$ Stress-block factor

$\beta := 0.416$ Stress-block factor

Loads

$\rho_{cs} := \frac{(A_{sx} + A_{sy}) \cdot (\alpha_\rho - 1) \cdot \rho_c}{h} + \rho_c = (2.339 \cdot 10^3) \ \frac{kg}{m^3}$ Density including reinforcement

$Q_g := \rho_{cs} \cdot A_{slab} \cdot h \cdot g = (9.203 \cdot 10^3) \ N$ Total load due to self weight

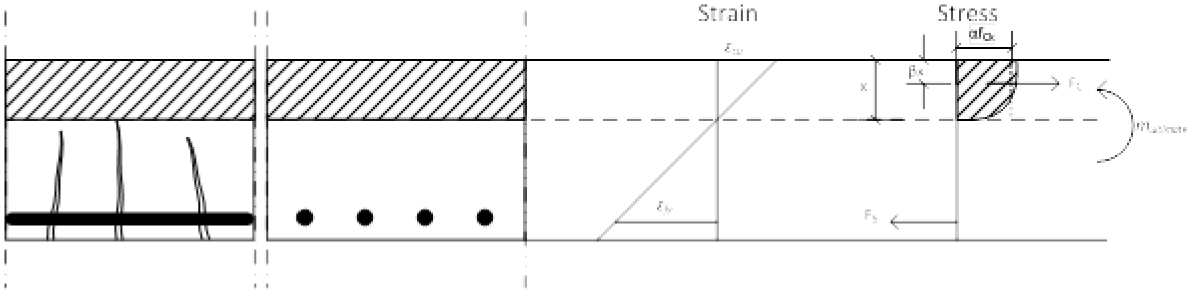
$q_g := \frac{Q_g}{A_{slab}} = (1.925 \cdot 10^3) \ \frac{N}{m^2}$ Distributed load self weight

Ultimate load

Failure occurs when ultimate compressive strain in the concrete is reached.

Assuming concrete in state III

- Cracked section
- Linear elastic limit is reached in either concrete or reinforcement
- Influence of concrete below neutral axis is neglected



Calculating compressed concrete area, force equilibrium

Guess Values

$$x_{x.u} := 0.1\ m \qquad \varepsilon_s := 3 \cdot 10^{-3}$$

Constraints

$$\alpha \cdot f_{ck} \cdot x_{x.u} = \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sx} \qquad \frac{\varepsilon_{cu}}{x_{x.u}} = \frac{\varepsilon_s}{d_x - x_{x.u}}$$

Solver

$$X := \text{Find}\left(x_{x.u}, \varepsilon_s\right) = \begin{bmatrix} 4.369 \\ 45.299\ \frac{1}{m} \end{bmatrix} mm$$

$$x_{x.u} := X(0) = 4.369\ mm$$

$$\varepsilon_s := X(1) = 0.045$$

Controll if the ultimate strain is reached

$$\varepsilon_s < \varepsilon_{su} = 1$$

Guess Values

$x_{y.u} := 0.1\ m \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$\alpha \cdot f_{ck} \cdot x_{y.u} = \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sy} \qquad \frac{\varepsilon_{cu}}{x_{y.u}} = \frac{\varepsilon_s}{d_y - x_{y.u}}$$

Solver

$$X := \mathbf{Find}(x_{y.u}, \varepsilon_s) = \begin{bmatrix} 2.419 \\ 75.972\ \frac{1}{m} \end{bmatrix} mm$$

$x_{y.u} := X(0) = 2.419\ mm$

$\varepsilon_s := X(1) = 0.076$

Controll if the ultimate strain is reached

$\varepsilon_s < \varepsilon_{su} = 1$

Moment capacity

$$m_{Rx} := \alpha \cdot f_{ck} \cdot x_{x.u} \cdot (d_x - \beta \cdot x_{x.u}) = 10.705\ \frac{kN \cdot m}{m}$$

$$m_{Ry} := \alpha \cdot f_{ck} \cdot x_{y.u} \cdot (d_y - \beta \cdot x_{y.u}) = 5.406\ \frac{kN \cdot m}{m}$$

Check if reinforcement yields

$$\varepsilon_{sx} := \frac{d_x - x_{x.u}}{x_{x.u}} \cdot \varepsilon_{cu} = 0.045 \qquad \varepsilon_{sy} := \frac{d_y - x_{y.u}}{x_{y.u}} \cdot \varepsilon_{cu} = 0.076$$

$\varepsilon_{sx} \geq \varepsilon_{syk} = 1 \qquad \varepsilon_{sy} \geq \varepsilon_{syk} = 1$

Center of gravity

$$z_{gc.x} := \frac{\frac{{x_{x.u}}^2}{2} + \alpha_c \cdot A_{sx} \cdot d_x}{x_{x.u} + \alpha_c \cdot A_{sx}} = 19.7\ mm \qquad z_{gc.y} := \frac{\frac{{x_{y.u}}^2}{2} + \alpha_c \cdot A_{sy} \cdot d_y}{x_{y.u} + \alpha_c \cdot A_{sy}} = 16.532\ mm$$

Moment of inertia cracked section

$$I_x := \frac{x_{x.u}^{3}}{12} + \left(z_{gc.x} - \frac{x_{x.u}}{2}\right)^2 \cdot x_{x.u} + \alpha_c \cdot A_{sx} \cdot \left(d_x - z_{gc.x}\right)^2 = \left(4.502 \cdot 10^6\right) \frac{mm^4}{m}$$

$$I_y := \frac{x_{y.u}^{3}}{12} + \left(z_{gc.y} - \frac{x_{y.u}}{2}\right)^2 \cdot x_{y.u} + \alpha_c \cdot A_{sy} \cdot \left(d_y - z_{gc.y}\right)^2 = \left(1.992 \cdot 10^6\right) \frac{mm^4}{m}$$

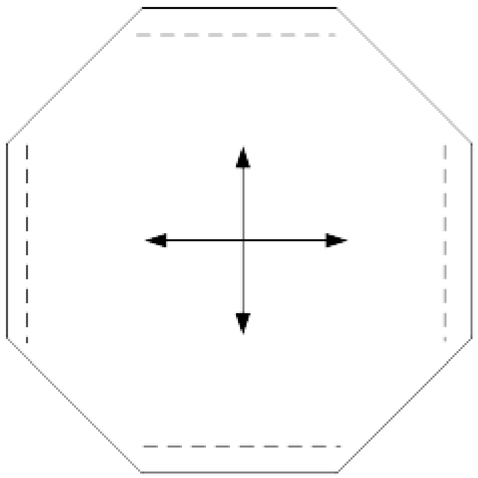
Relation of stiffness in x- and y-direction for cracked section

$$\alpha_x := \frac{I_x}{I_x + I_y} = 0.693 \qquad \alpha_y := \frac{I_y}{I_x + I_y} = 0.307$$

$$\alpha_x := \frac{m_{Rx}}{m_{Rx} + m_{Ry}} = 0.664 \qquad \alpha_y := \frac{m_{Ry}}{m_{Rx} + m_{Ry}} = 0.336$$

Strip method

Load distribution alternative I



Moment and reaktion force due to self weight

$$R_{AI.sw} := \alpha_x \cdot \frac{Q_g}{2} = \left(3.058 \cdot 10^3\right)\ N$$

$$R_{BI.sw} := \alpha_y \cdot \frac{Q_g}{2} = \left(1.544 \cdot 10^3\right)\ N$$

$$M_{xI.sw} := R_{AI.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_x \cdot q_g \cdot \left(\frac{l_s}{2} \cdot l_2 \cdot \frac{l_s}{4} + l_s \cdot l_{e2} \cdot \left(\frac{l_s}{2} + \frac{l_{e2}}{2}\right) + {l_{e2}}^2 \cdot \left(\frac{l_{e2}}{3} + \frac{l_s}{2}\right)\right) = 2.076\ kN \cdot m$$

$$M_{yI.sw} := R_{BI.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_y \cdot q_g \cdot \left(\frac{l_s}{2} \cdot l_2 \cdot \frac{l_s}{4} + l_s \cdot l_{e2} \cdot \left(\frac{l_s}{2} + \frac{l_{e2}}{2}\right) + {l_{e2}}^2 \cdot \left(\frac{l_{e2}}{3} + \frac{l_s}{2}\right)\right) = 1.048\ kN \cdot m$$

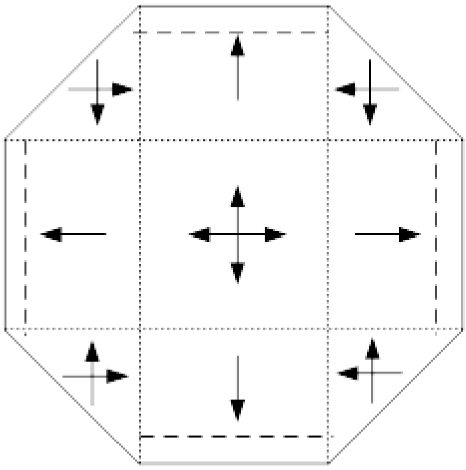
Calculation of point load in ultimate state

$$M_{I.P} = \frac{P \cdot l_2}{4} \qquad\qquad m_R \cdot l_2 = M_{I.sf} + M_{I.P}$$

$$P_{x.u.I} := \left(m_{Rx} \cdot l_s - M_{xI.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.309 \cdot 10^4\right)\ N$$

$$P_{y.u.I} := \left(m_{Ry} \cdot l_s - M_{yI.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.309 \cdot 10^4\right)\ N$$

Load distribution alternative II



Moment and reaktion force due to self weight

$$R_{AII.sw} := \frac{\alpha_x \cdot q_g \cdot \left(l_s^{\,2} + 2 \cdot l_e^{\,2}\right) + q_g \cdot 2 \cdot l_s \cdot l_e}{2} = \left(2.614 \cdot 10^3\right)\ \boldsymbol{N}$$

$$R_{BII.sw} := \frac{\alpha_y \cdot q_g \cdot \left(l_s^{\,2} + 2 \cdot l_e^{\,2}\right) + q_g \cdot 2 \cdot l_s \cdot l_e}{2} = \left(1.987 \cdot 10^3\right)\ \boldsymbol{N}$$

$$M_{xII.sw} := R_{AII.sw} \cdot \frac{l_2}{2} - q_g \cdot \left(l_s \cdot l_{e2} \cdot \left(\frac{l_s + l_{e2}}{2}\right) + \alpha_x \cdot l_s \cdot \frac{l_s}{2} \cdot \frac{l_s}{4} + \alpha_x \cdot l_{e2}^{\,2} \cdot \left(\frac{l_{e2}}{3} + \frac{l_s}{2}\right)\right) = 1.449\ \boldsymbol{kN \cdot m}$$

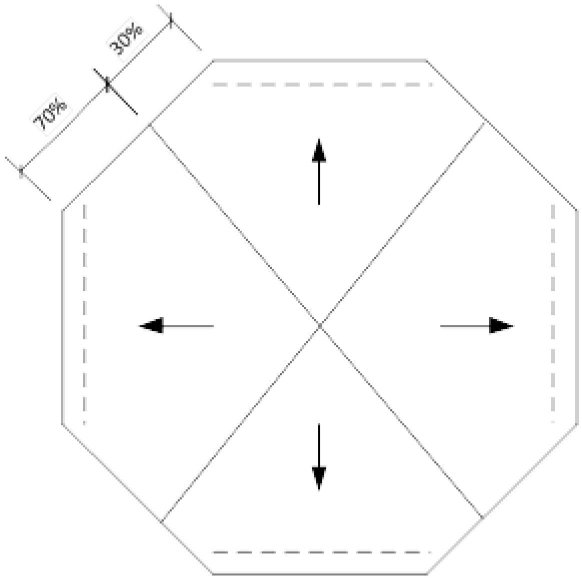
$$M_{yII.sw} := R_{BII.sw} \cdot \frac{l_2}{2} - q_g \cdot \left(l_s \cdot l_{e2} \cdot \left(\frac{l_s + l_{e2}}{2}\right) + \alpha_y \cdot l_s \cdot \frac{l_s}{2} \cdot \frac{l_s}{4} + \alpha_y \cdot l_{e2}^{\,2} \cdot \left(\frac{l_{e2}}{3} + \frac{l_s}{2}\right)\right) = 0.998\ \boldsymbol{kN \cdot m}$$

Calculation of point load

$$P_{x.u.II} := \left(m_{Rx} \cdot l_s - M_{xII.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.476 \cdot 10^4\right)\ \boldsymbol{N}$$

$$P_{y.u.II} := \left(m_{Ry} \cdot l_s - M_{yII.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.335 \cdot 10^4\right)\ \boldsymbol{N}$$

Load distribution alternative III



$$a := \sqrt{2} \cdot l_e = 0.99\ m$$

$$b := \sqrt{2} \cdot \frac{l}{2} = 1.697\ m$$

$$c := \frac{\sqrt{2} \cdot l_e}{2} = 0.495\ m$$

$$d := b - c = 1.202\ m$$

$$i_x := \frac{2}{3} \cdot \alpha_x \cdot a \cdot \frac{1}{\sqrt{2}} = 0.31\ m$$

$$i_y := \frac{2}{3} \cdot \alpha_y \cdot a \cdot \frac{1}{\sqrt{2}} = 0.157\ m$$

$$j := \frac{1}{\sqrt{2}} \cdot \frac{1}{3} \cdot d = 0.283\ m$$

$$k_x := j + i_x = 0.593\ m$$

$$k_y := j + i_y = 0.44\ m$$

Moment and reaktion force due to self weight

$$R_{AIII.sw} := \frac{q_g \cdot \left(l_s \cdot \left(\frac{l_s}{2} + l_e\right) + 2 \cdot a \cdot \alpha_x \cdot d\right)}{2} = \left(2.678 \cdot 10^3\right)\ N$$

$$R_{BIII.sw} := \frac{q_g \cdot \left(l_s \cdot \left(\frac{l_s}{2} + l_e\right) + 2 \cdot a \cdot \alpha_y \cdot d\right)}{2} = \left(1.924 \cdot 10^3\right)\ N$$

$$M_{xIII.sw} := R_{AIII.sw} \cdot \frac{l_2}{2} - q_g \cdot \left(\frac{l_s}{2} \cdot \frac{l_2}{2} \cdot \frac{l_2}{2} \cdot \frac{2}{3} + \alpha_x \cdot a \cdot d \cdot \left(\frac{l_2}{2} - k_x\right)\right) = 1.391\ kN \cdot m$$

$$M_{yIII.sw} := R_{BIII.sw} \cdot \frac{l_2}{2} - q_g \cdot \left(\frac{l_s}{2} \cdot \frac{l_2}{2} \cdot \frac{l_2}{2} \cdot \frac{2}{3} + \alpha_y \cdot a \cdot d \cdot \left(\frac{l_2}{2} - k_y\right)\right) = 0.826\ kN \cdot m$$

$$P_{x.u.III} := \left(m_{Rx} \cdot l_s - M_{xIII.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.492 \cdot 10^4\right)\ N$$

$$P_{y.u.III} := \left(m_{Ry} \cdot l_s - M_{yIII.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.427 \cdot 10^4\right)\ N$$

Yield line method

$\omega := 45\ deg$ Angle between reinforcementbars

$m_b := m_{Rx} \cdot \sin(\omega)^2 + m_{Ry} \cdot \cos(\omega)^2 = 8.056\ \frac{kN \cdot m}{m}$ Moment capacity along yieldline

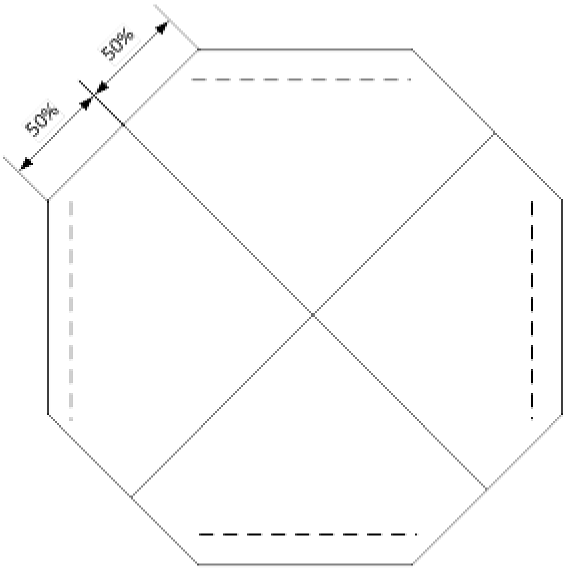
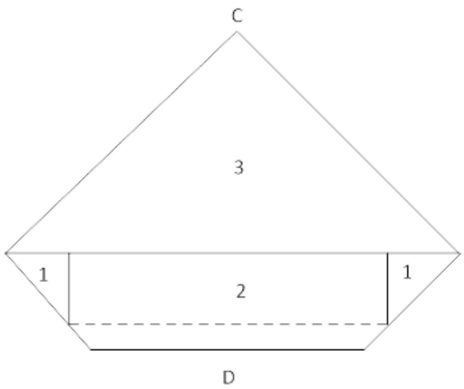


Figure showing choosen failuer mechanism

Calculating the possition of the centroid of the segment on each side of the yieldline
Areas and gravity center for each part of a slab segment



$$A_1 := \frac{\left(\frac{l_e}{2} - l_a\right)^2}{2} = 0.038\ \boldsymbol{m}^2$$

$$A_2 := \left(\frac{l_e}{2} - l_a\right) \cdot \left(l_s + 2 \cdot l_a\right) = 0.316\ \boldsymbol{m}^2$$

$$A_3 := \frac{\left(\frac{l_s}{2} + \frac{l_e}{2}\right) \cdot \left(l_s + l_e\right)}{2} = 0.723\ \boldsymbol{m}^2$$

$$x_{gc.1} := \frac{2}{3} \cdot \left(\frac{l_e}{2} - l_a\right) = 0.183\ \boldsymbol{m}$$

$$x_{gc.2} := \left(\frac{\frac{l_e}{2} - l_a}{2}\right) = 0.138\ \boldsymbol{m}$$

$$x_{gc.3} := \left(\frac{l_e}{2} - l_a + \frac{1}{3} \cdot \left(\frac{l_s}{2} + \frac{l_e}{2}\right)\right) = 0.558\ \boldsymbol{m}$$

$$x_{gc} := \frac{2 \cdot A_1 \cdot x_{gc.1} + A_2 \cdot x_{gc.2} + A_3 \cdot x_{gc.3}}{2 \cdot A_1 + A_2 + A_3} = 0.413\ \boldsymbol{m}$$

$$l_d := \sqrt{l^2 + l^2} = 3.394\ \boldsymbol{m}$$ Diagonal of sqare slab

$$d := l_d - \sqrt{\left(\frac{l_e}{2}\right)^2 + \left(\frac{l_e}{2}\right)^2} = 2.899\ \boldsymbol{m}$$ Diagonal octagonal

$$l_P := \frac{l_2}{2} = 1.125\ \boldsymbol{m}$$ Lever arm to point load

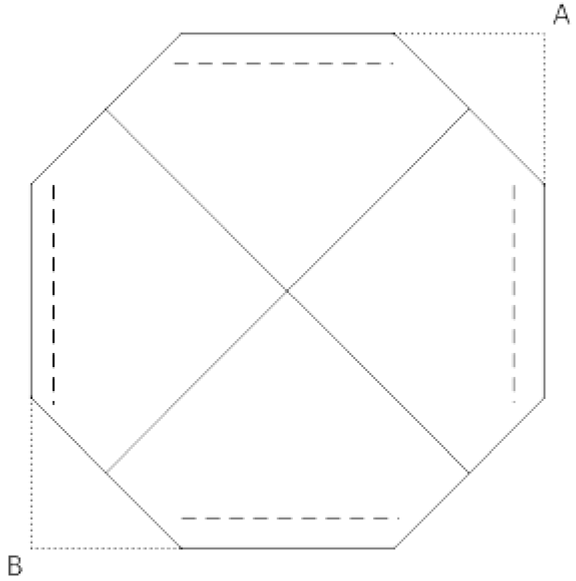


Figure showing the geometry and the square diagonal length between A and B

Solving P from EW=IW

Guess Values

$$P := 40\ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_g}{l_P} + P$$

Solver

$$P := \mathbf{find}(P) = 51.665\ kN$$

Appendix B
Analysis of steel fibre reinforced concrete with FIB Model Codel

Indata

Geometry

$n_x := 25$	Number of reinforcement bars x-direction
$n_y := 13$	Number of reinforcement bars in y-direction
$\phi := 6\ mm$	Diameter reinforcement bar
$h_f := 79.14\ mm$	Height slab fiber reinforcement
$h_c := 82.37\ mm$	Height slab fiber + conventional reinforcement
$l_s := 1.0\ m$	Length support
$l_e := 0.7\ m$	Length
$l_a := 75\ mm$	
$l_{e2} := l_e - l_a = 0.625\ m$	
$l := l_s + 2 \cdot l_e = 2.4\ m$	Total length slab
$l_2 := l_s + 2 \cdot l_{e2} = 2.25\ m$	Length between supports
$c_x := 20\ mm$	Cover thickness reinforcement x-direction
$c_y := 26\ mm$	Cover thickness reinforcement y-direction

$$A_{sx} := \frac{\left(\frac{\phi}{2}\right)^2 \cdot \pi \cdot n_x}{l} = 294.524\ \frac{mm^2}{m}$$ Area reinforcement per meter x-direction

$$A_{sy} := \frac{\left(\frac{\phi}{2}\right)^2 \cdot \pi \cdot n_y}{l} = 153.153\ \frac{mm^2}{m}$$ Area reinforcement per meter y-direction

$$A_{slab} := l^2 - 4 \cdot \frac{{l_e}^2}{2} = 4.78\ m^2$$ Total area slab

$$d_x := h_c - \frac{\phi}{2} - c_x = 59.37\ mm$$ Distance top slab to ceneter of reinforcement x-direction

$$d_y := h_c - \frac{\phi}{2} - c_y = 53.37\ mm$$ Distance top slab to ceneter of reinforcement y-direction

Concrete

$f_{ck} := 45.52\ MPa$ Characteristic compressive strength, value from lab resault
$E_c := 30.93\ GPa$ Youngs modulus FRC
$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$ Ultimate strain in concrete
$f_{ctm} := 3.00\ MPa$ Mean tensile strength

$$k_c := 1.6 - \frac{h_c}{1000\ mm} = 1.518$$

$$k_f := 1.6 - \frac{h_f}{1000\ mm} = 1.521$$

$f_{ctm.bc} := k_c \cdot f_{ctm} = \left(4.553 \cdot 10^6\right)\ Pa$ Bending strength conventional + fibre

$f_{ctm.bf} := k_f \cdot f_{ctm} = \left(4.563 \cdot 10^6\right)\ Pa$ Bending strength fibre

$\rho_c := 2326.599\ \frac{kg}{m^3}$ Density FRC

Steel

$f_y := 560\ MPa$ Yield strength reinforcement bar
$E_s := 200\ GPa$ Youngs modulus steel

$\varepsilon_{syk} := \frac{f_y}{E_s} = 2.8 \cdot 10^{-3}$ Yield strain steel

$f_u := 672\ MPa$ Ultimate strength reinforcement bar

$\varepsilon_{su} := 0.089339$ Ultimate strain

$\rho_s := 7850\ \frac{kg}{m^3}$ Density steel

$\phi_s := 6\ mm$ Diameter reinforcement

$\alpha_c := \frac{E_s}{E_c} = 6.466$ Relation youngs modulus

$\alpha_\rho := \frac{\rho_s}{\rho_c} = 3.224$ Relation density

$\alpha := 0.810$ Stress-block factor ultimate load
$\beta := 0.416$ Stress-block factor ultimate load

Loads

Fibre reinforcement

$Q_{gf} := \rho_c \cdot A_{slab} \cdot h_f \cdot g = \left(8.631 \cdot 10^3\right)\ N$ Total load due to self weight

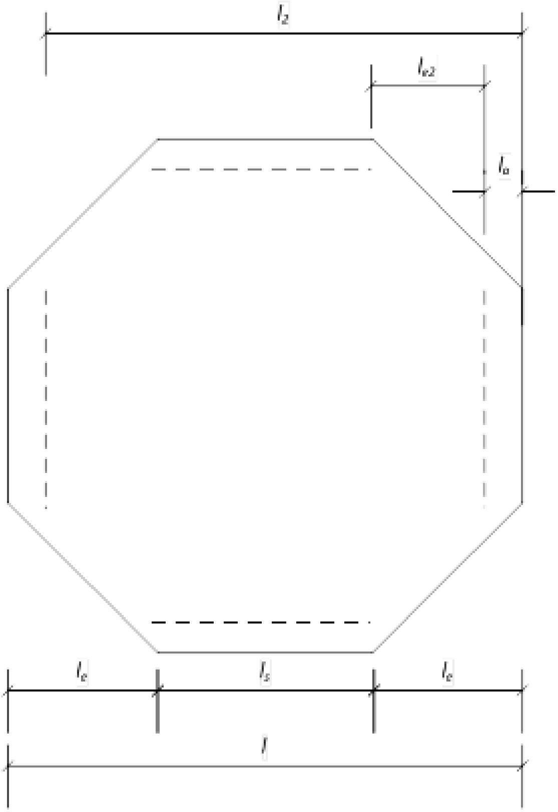
$q_{gf} := \dfrac{Q_{gf}}{A_{slab}} = \left(1.806 \cdot 10^3\right)\ \dfrac{N}{m^2}$ Distributed load self weight

Fibre + conventional reinforcement

$\rho_{cs} := \dfrac{\left(A_{sx} + A_{sy}\right) \cdot \left(\alpha_\rho - 1\right) \cdot \rho_c}{h_c} + \rho_c = \left(2.355 \cdot 10^3\right)\ \dfrac{kg}{m^3}$ Density including reinforcement

$Q_{gc} := \rho_{cs} \cdot A_{slab} \cdot h_c \cdot g = \left(9.092 \cdot 10^3\right)\ N$ Total load due to self weight

$q_{gc} := \dfrac{Q_{gc}}{A_{slab}} = \left(1.902 \cdot 10^3\right)\ \dfrac{N}{m^2}$ Distributed load self weight



Data from experimental test of RILEM beam

$l_z := 500\ mm$ Span length beam
$b := 150\ mm$ Width of the beam
$h_{sp} := 125\ mm$ Distance between tip of the notch and the top of cross section

$CMOD_1 := 0.5\ mm$ Crack mouth opening displacement
$CMOD_3 := 2.5\ mm$

$F_1 := 6.62\ kN$ Load corresponding to CMOD1
$F_3 := 8.02\ kN$ Load corresponding to CMOD3

$$f_{R1} := \frac{3 \cdot F_1 \cdot l_z}{2 \cdot b \cdot h_{sp}^{\ 2}} = \left(2.118 \cdot 10^6\right)\ Pa$$

$$f_{R3} := \frac{3 \cdot F_3 \cdot l_z}{2 \cdot b \cdot h_{sp}^{\ 2}} = \left(2.566 \cdot 10^6\right)\ Pa$$

$w_u := 2.5\ mm$ Ultimate crack opening

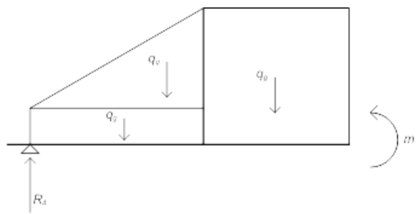
$\eta := 1$ Stress block factor
$\lambda := 0.8$ Stress block factor

$\varepsilon_{Fu} := 0.02$ Ultimate strain

Rigid plastic model

Only fibre reinforcement

$$R_{swf}:=\frac{Q_{gf}}{4}=\left(2.158\cdot 10^{3}\right)\ \boldsymbol{N}$$



$$M_{gf}:=q_{gf}\cdot\left(\frac{l_s}{2}\cdot l_2\cdot\frac{l_s}{4}+l_s\cdot l_{e2}\cdot\left(\frac{l_s}{2}+\frac{l_{e2}}{2}\right)+l_{e2}{}^{2}\cdot\left(\frac{l_{e2}}{3}+\frac{l_s}{2}\right)\right)$$

$$M_{swf}:=R_{swf}\cdot\left(l_{e2}+\frac{l_s}{2}\right)-0.5\cdot M_{gf}=1.465\ \boldsymbol{kN\cdot m}$$

Ultimate load

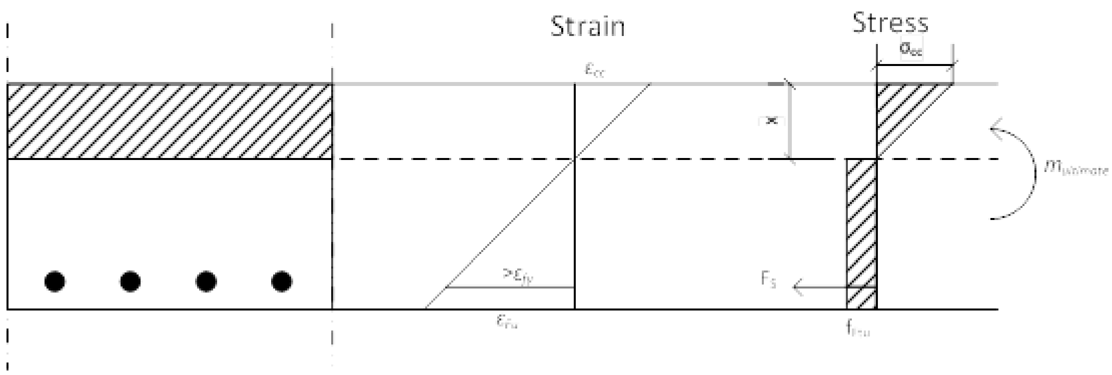
Assuming concrete in state III

- Cracked section
- Linear elastic limit is reached in either concrete or reinforcement
- Influence of concrete below neutral axis is neglected

Assume that fibres transfear load over the whole crack

Assume that failure occurs when the ultimate strain for the FRC is reached in the bottom fibre.

Assume that the plastic compressive strain for the concrete is not reached.



$$f_{Ftu.pm}:=\frac{f_{R3}}{3}=\left(8.55467\cdot 10^{5}\right)\ \boldsymbol{Pa}$$ Ultimate residual strength

Iteration process until force equilibrium is found

Assume a strain in top fibre when yield strain in reinforcement is reached

$\varepsilon_{cc} := 0.944987 \cdot 10^{-3}$

$\alpha_u := 0.3977455$ Stress block factor coresponding to chosen strain

$\beta_u := 0.3488997$ Stress block factor coresponding to chosen strain

Compressed concrete zone

$$x_u := \frac{\varepsilon_{cc}}{\varepsilon_{cc} + \varepsilon_{Fu}} \cdot h_f = 3.571\ \boldsymbol{mm}$$

Controll equilibrium

$$f_{Ftu.pm} \cdot \left(h_f - x_u\right) - \alpha_u \cdot x_u \cdot f_{ck} = -0.041\ \boldsymbol{m \cdot Pa}$$

Moment capacity

$$m_{Rd.pm} := f_{Ftu.pm} \cdot \frac{\left(h_f - x_u\right)^2}{2} + \alpha_u \cdot f_{ck} \cdot x_u \cdot \left(x_u - \beta_u \cdot x_u\right) = 2.593\ \frac{\boldsymbol{kN \cdot m}}{\boldsymbol{m}}$$

Moment due to self weight

Assuming equal load distribution in both x- and y-direction

$R_{swf} = \left(2.158 \cdot 10^3\right)\ \boldsymbol{N}$ $M_{swf} = 1.465\ \boldsymbol{kN \cdot m}$

Moment due to point load

Assuming equal distribution in both x- and y-direction

$M_P = \dfrac{P \cdot l}{4}$ $m_R \cdot l = M_{sf} + M_P$

Maximum point load

$$P_{u.I} := \left(m_{Rd.pm} \cdot l_s - M_{swf}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{0.5} = \left(4.009 \cdot 10^3\right)\ \boldsymbol{N}$$

Analysis of point load with yield line analysis

$\omega := 45\ deg$ Angle between reinforcementbars

$m_b := m_{Rd.pm} \cdot \sin(\omega)^2 + m_{Rd.pm} \cdot \cos(\omega)^2 = 2.593\ \frac{kN \cdot m}{m}$ Moment capacity along yieldline

$$A_1 := \frac{\left(\frac{l_e}{2} - l_a\right)^2}{2} = 0.038\ m^2 \qquad A_2 := \left(\frac{l_e}{2} - l_a\right) \cdot \left(l_s + 2 \cdot l_a\right) = 0.316\ m^2$$

$$A_3 := \frac{\left(\frac{l_s}{2} + \frac{l_e}{2}\right) \cdot \left(l_s + l_e\right)}{2} = 0.723\ m^2$$

$$x_{gc.1} := \frac{2}{3} \cdot \left(\frac{l_e}{2} - l_a\right) = 0.183\ m \qquad x_{gc.2} := \left(\frac{\frac{l_e}{2} - l_a}{2}\right) = 0.138\ m$$

$$x_{gc.3} := \left(\frac{l_e}{2} - l_a + \frac{1}{3} \cdot \left(\frac{l_s}{2} + \frac{l_e}{2}\right)\right) = 0.558\ m$$

$$x_{gc} := \frac{2 \cdot A_1 \cdot x_{gc.1} + A_2 \cdot x_{gc.2} + A_3 \cdot x_{gc.3}}{2 \cdot A_1 + A_2 + A_3} = 0.413\ m$$

$l_d := \sqrt{l^2 + l^2} = 3.394\ m$ Diagonal of sqare slab

$d := l_d - \sqrt{\left(\frac{l_e}{2}\right)^2 + \left(\frac{l_e}{2}\right)^2} = 2.899\ m$ Diagonal octagonal

$l_P := \frac{l_2}{2} = 1.125\ m$ Lever arm to point load

Guess Values

$P := 40\ kN$

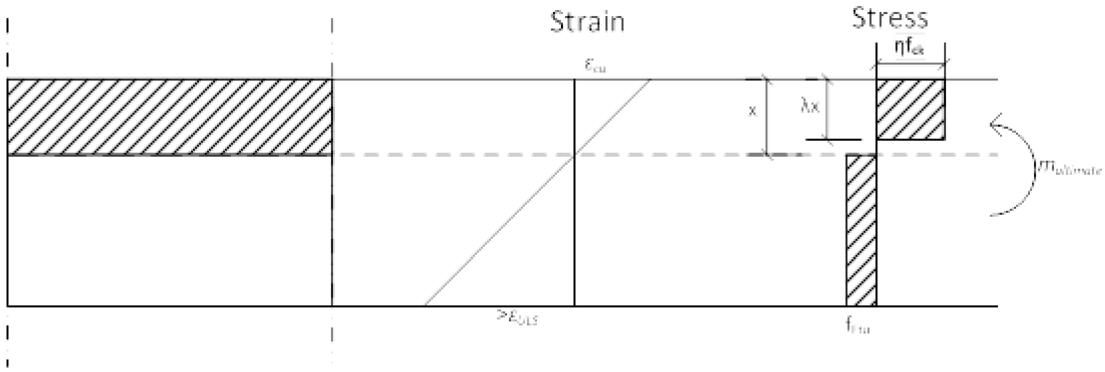
Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gf}}{l_P} + P$$

Solver

$P := \text{find}(P) = 14.547\ kN$

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Compressed concrete area, force equilibrium

Guess Values

$x_u := 10\ mm$

Constraints

$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.pm} \cdot (h_f - x_u)$

Solver

$x_u := \mathbf{find}(x_u) = 1.816\ mm$

$x_1 := x_u$

Moment capacity

$$m_{Rd.pm} := f_{Ftu.pm} \cdot \frac{(h_f - x_u)^2}{2} + f_{ck} \cdot \lambda \cdot x_u \cdot \left(x_u - \frac{\lambda \cdot x_u}{2}\right) = 2.629\ \frac{kN \cdot m}{m}$$

Moment due to self weight

Assuming equal load distribution in both x- and y-direction

$R_{swf} = (2.158 \cdot 10^3)\ N$ $M_{swf} = 1.465\ kN \cdot m$

Moment due to point load

Assuming equal distribution in both x- and y-direction

$$M_P = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{u.II} := \left(m_{Rd.pm} \cdot l_s - M_{swf}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{0.5} = \left(4.139 \cdot 10^3\right)\ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rd.pm} \cdot \sin(\omega)^2 + m_{Rd.pm} \cdot \cos(\omega)^2 = 2.629\ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40\ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gf}}{l_P} + P$$

Solver

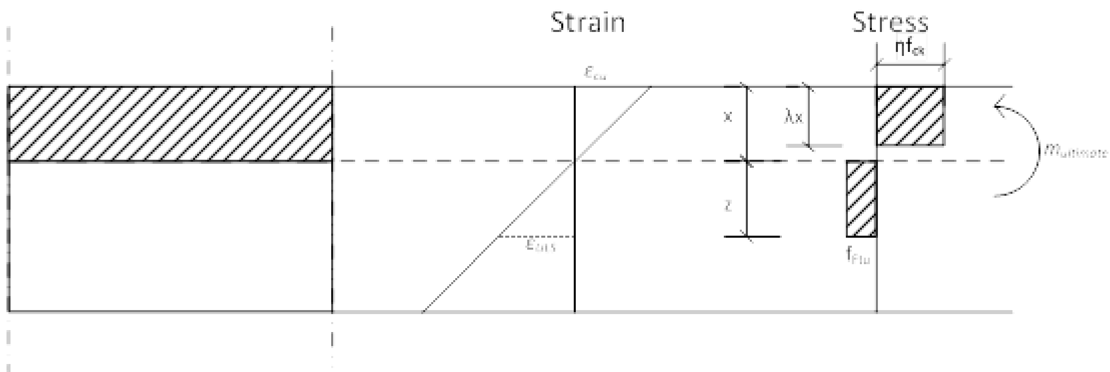
$$P := \mathbf{find}(P) = 14.796\ kN$$

Assuming that fibres only transfear load over cracks where the strain in the cross-section are lower than the ultimate strain for FRC

Ultimate load

Same ultimate load for failure when the ultimate strain for the FRC is reached in the bottom fibre for assumption of full load transfer.

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Iteration is used to find the compressed concrete area and possition of service and ultimate strain since no force equilibrium can be found

Iteration 1

Controlling strain in bottom fiber

$$\varepsilon := \frac{h_f - x_u}{x_u} \cdot \varepsilon_{cu} = 0.149 \qquad z := \frac{\varepsilon_{Fu} \cdot x_u}{\varepsilon_{cu}} = 10.38\ mm$$

Guess Values

$x_u := 10\ mm$

Constraints

$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.pm} \cdot z$

Solver

$x_u := \mathbf{find}\left(x_u\right) = 0.244\ mm$

$x_2 := x_u$

Iteration 2

Controlling strain in bottom fiber

$\varepsilon := \dfrac{h_f - x_u}{x_u} \cdot \varepsilon_{cu} = 1.132$

$z := \dfrac{\varepsilon_{Fu} \cdot x_u}{\varepsilon_{cu}} = 1.393\ mm$

Guess Values

$x_u := 10\ mm$

Constraints

$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.pm} \cdot z$

Solver

$x_u := \mathbf{find}\left(x_u\right) = 0.033\ mm$

$x_3 := x_u$

Iteration 3

Controlling strain in bottom fiber

$$\varepsilon := \frac{h_f - x_u}{x_u} \cdot \varepsilon_{cu} = 8.459 \qquad z := \frac{\varepsilon_{Fu} \cdot x_u}{\varepsilon_{cu}} = 0.187\ mm$$

Guess Values

$$x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.pm} \cdot z$$

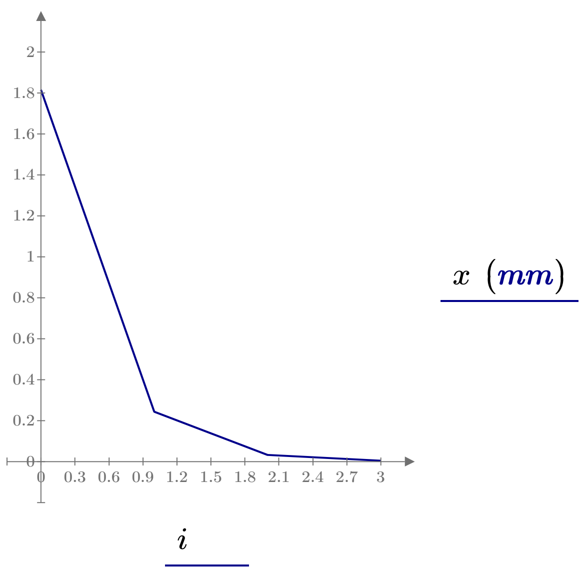
Solver

$$x_u := \mathbf{find}\left(x_u\right) = 0.004\ mm$$

$$x_4 := x_u$$

Convergens studie

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.816 \\ 0.244 \\ 0.033 \\ 0.004 \end{bmatrix} mm \qquad i := \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$



The compressed concrete area is going to the limit zero

Moment capacity

From moment equilibrium

$$m_{Rd.pm} := f_{Ftu.pm} \cdot \frac{(z - x_u)^2}{2} + f_{ck} \cdot \lambda \cdot x_u \cdot \left(x_u - \frac{\lambda \cdot x_u}{2}\right) = \left(1.469 \cdot 10^{-5}\right) \frac{kN \cdot m}{m}$$

No force equilibrium was found, the compressed concrete zone is converge to zero.

Moment capacity from expression in Model Code

$$m_{Rd.pm} := \frac{f_{Ftu.pm} \cdot {h_f}^2}{2} = 2.679 \frac{kN \cdot m}{m}$$

Moment due to self weight

Assuming equal load distribution in both directions

$$R_{swf} = \left(2.158 \cdot 10^3\right) N \qquad M_{swf} = 1.465 \; kN \cdot m$$

Moment due to point load

Assuming equal distribution in both directions

$$M_{I.P} = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{I.sf} + M_{I.P}$$

Maximum point load

$$P := \left(m_{Rd.pm} \cdot l_s - M_{swf}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{0.5} = \left(4.315 \cdot 10^3\right) N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rd.pm} \cdot \sin(\omega)^2 + m_{Rd.pm} \cdot \cos(\omega)^2 = 2.679 \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$P := 40\ kN$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gf}}{l_P} + P$$

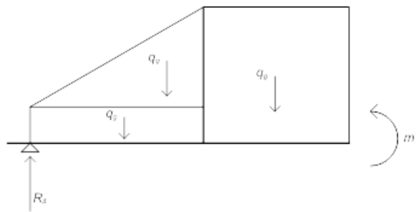
Solver

$P := \mathbf{find}(P) = 15.134\ kN$

Fibre reinforcement combined with conventional reinforcement

Assuming that fibres transfear load over the whole crack

Moment and reaktion force due to self weight



$$R_{swc} := \frac{Q_{gc}}{4} = \left(2.273 \cdot 10^{3}\right)\ N$$

$$M_{gc} := q_{gc} \cdot \left(\frac{l_s}{2} \cdot l_2 \cdot \frac{l_s}{4} + l_s \cdot l_{e2} \cdot \left(\frac{l_s}{2} + \frac{l_{e2}}{2}\right) + {l_{e2}}^{2} \cdot \left(\frac{l_{e2}}{3} + \frac{l_s}{2}\right)\right)$$

$$M_{swc} := R_{swc} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - 0.5 \cdot M_{gc} = 1.544\ kN \cdot m$$

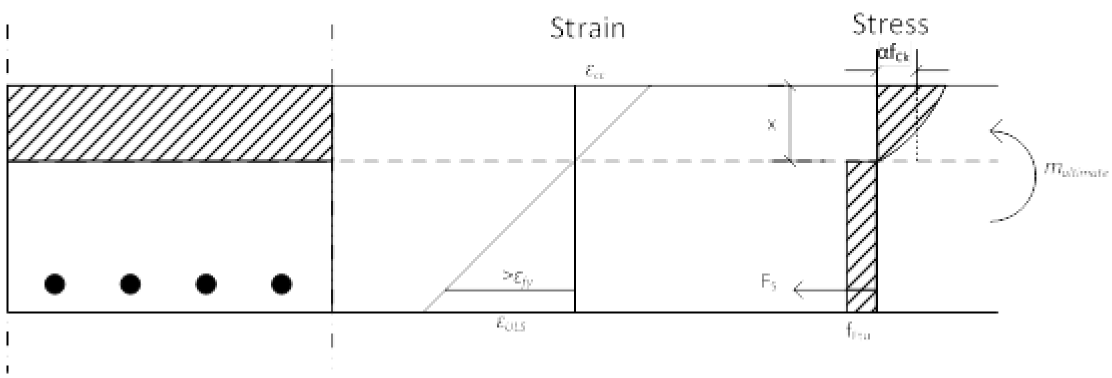
Ultimate load

Assuming concrete in state III

- Cracked section
- Linear elastic limit is reached in either concrete or reinforcement
- Influence of concrete below neutral axis is neglected

Assume that failure occurs when the ultimate strain for the FRC is reached in the bottom fibre.

Assume that the plastic compressive strain for the concrete is not reached.



Iteration process until force equilibrium is found

Assume a strain in top fibre when ultimate strain in FRC is reached

$\varepsilon_{cc.x} := 2.036998 \cdot 10^{-3}$ $\varepsilon_{cc.y} := 1.538544 \cdot 10^{-3}$

Stress block factors coresponding to chosen strain

$\alpha_{x.u} := 0.6725497$ $\qquad$ $\alpha_{y.u} := 0.5716360$

$\beta_{x.u} := 0.3761099$ $\qquad$ $\beta_{y.u} := 0.3617709$

Compressed concrete zone

$$x_{x.u} := \frac{\varepsilon_{cc.x}}{\varepsilon_{cc.x} + \varepsilon_{Fu}} \cdot h_c = 7.614\ mm \qquad x_{y.u} := \frac{\varepsilon_{cc.y}}{\varepsilon_{cc.y} + \varepsilon_{Fu}} \cdot h_c = 5.884\ mm$$

Strain in reinforcement

$$\varepsilon_{sx} := \varepsilon_{cc.x} \cdot \frac{\left(d_x - x_{x.u}\right)}{x_{x.u}} = 0.014 \qquad \varepsilon_{sy} := \varepsilon_{cc.y} \cdot \frac{\left(d_y - x_{y.u}\right)}{x_{y.u}} = 0.012$$

Controll equilibrium

$$\left(\frac{\left(\varepsilon_{sx} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sx} + f_{Ftu.pm} \cdot \left(h_c - x_{x.u}\right) - f_{ck} \cdot \alpha_{x.u} \cdot x_{x.u} = 0.211\ m \cdot Pa$$

$$\left(\frac{\left(\varepsilon_{sy} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sy} + f_{Ftu.pm} \cdot \left(h_c - x_{y.u}\right) - f_{ck} \cdot \alpha_{y.u} \cdot x_{y.u} = -0.259\ m \cdot Pa$$

Moment capacity

$$m_{cx} := \alpha_{x.u} \cdot f_{ck} \cdot x_{x.u} \cdot \left(x_{x.u} - \beta_{x.u} \cdot x_{x.u}\right) \qquad m_{Ftux} := f_{Ftu.pm} \cdot \frac{\left(h_c - x_{x.u}\right)^2}{2}$$

$$m_{sx} := f_y \cdot A_{sx} \cdot \left(d_x - x_{x.u}\right)$$

$$m_{cy} := \alpha_{y.u} \cdot f_{ck} \cdot x_{y.u} \cdot \left(x_{y.u} - \beta_{y.u} \cdot x_{y.u}\right) \qquad m_{Ftuy} := f_{Ftu.pm} \cdot \frac{\left(h_c - x_{y.u}\right)^2}{2}$$

$$m_{sy} := f_y \cdot A_{sy} \cdot \left(d_y - x_{y.u}\right)$$

$$m_x := m_{cx} + m_{Ftux} + m_{sx} = \left(1.203 \cdot 10^4\right) \frac{N \cdot m}{m}$$

$$m_y := m_{cy} + m_{Ftuy} + m_{sy} = \left(7.15 \cdot 10^3\right) \frac{N \cdot m}{m}$$

Relation of stiffness in x- and y-direction for ultimate section

$$\alpha_x := \frac{m_x}{m_x + m_y} = 0.627 \qquad \alpha_y := \frac{m_y}{m_x + m_y} = 0.373$$

Moment and reaktion force due to self weight

$$R_{AI.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = \left(2.852 \cdot 10^3\right)\ \boldsymbol{N}$$

$$R_{BI.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = \left(1.694 \cdot 10^3\right)\ \boldsymbol{N}$$

$$M_{xI.sw} := R_{AI.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_x \cdot M_{gc} = 1.937\ \boldsymbol{kN \cdot m}$$

$$M_{yI.sw} := R_{BI.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_y \cdot M_{gc} = 1.151\ \boldsymbol{kN \cdot m}$$

Calculation of point load in when reinforcement yields

$$M_P = \frac{P \cdot l_2}{4} \qquad m_R \cdot l_2 = M_{I.sf} + M_{I.P}$$

$$P_{x.u.I} := \left(m_x \cdot l_s - M_{xI.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.862 \cdot 10^4\right)\ \boldsymbol{N}$$

$$P_{y.u.I} := \left(m_y \cdot l_s - M_{yI.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.862 \cdot 10^4\right)\ \boldsymbol{N}$$

Analysis of point load with yield line analysis

$$m_b := m_x \cdot \sin(\omega)^2 + m_y \cdot \cos(\omega)^2 = 9.592 \ \frac{kN \cdot m}{m}$$

Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

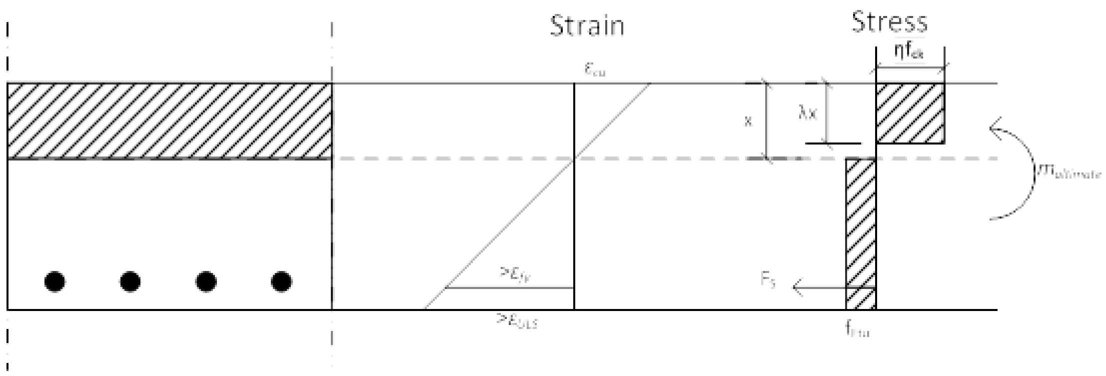
Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 62.203 \ kN$$

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Compressed concrete area, force equilibrium

Guess Values

$x_{x.u} := 10\ mm \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f_{ck} \cdot \lambda \cdot x_{x.u} = f_{Ftu.pm} \cdot (h_c - x_{x.u}) + \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sx} \qquad \frac{\varepsilon_{cu}}{x_{x.u}} = \frac{\varepsilon_s}{d_x - x_{x.u}}$$

Solver

$$X := \mathbf{find}(x_{x.u}, \varepsilon_s) = \begin{bmatrix} 6.575 \\ 28.106\ \frac{1}{m} \end{bmatrix} mm$$

$x_{x.u} := X(0) = 6.575\ mm \qquad \varepsilon_s := X(1) = 0.028$

Guess Values

$x_{y.u} := 10\ mm \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f_{ck} \cdot \lambda \cdot x_{y.u} = f_{Ftu.pm} \cdot (h_c - x_{y.u}) + \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sy} \qquad \frac{\varepsilon_{cu}}{x_{y.u}} = \frac{\varepsilon_s}{d_y - x_{y.u}}$$

Solver

$$X := \mathbf{find}(x_{y.u}, \varepsilon_s) = \begin{bmatrix} 4.385 \\ 39.101\ \frac{1}{m} \end{bmatrix} mm$$

$x_{y.u} := X(0) = 4.385\ mm \qquad \varepsilon_s := X(1) = 0.039$

Moment capacity

$$m_{cx} := f_{ck} \cdot \lambda \cdot x_{x.u} \cdot \left(x_{x.u} - \frac{\lambda \cdot x_{x.u}}{2} \right) \qquad m_{Ftux} := f_{Ftu.pm} \cdot \frac{\left(h_c - x_{x.u} \right)^2}{2}$$

$$m_{sx} := f_y \cdot A_{sx} \cdot \left(d_x - x_{x.u} \right)$$

$$m_{cy} := f_{ck} \cdot \lambda \cdot x_{y.u} \cdot \left(x_{y.u} - \frac{\lambda \cdot x_{y.u}}{2} \right) \qquad m_{Ftuy} := f_{Ftu.pm} \cdot \frac{\left(h_c - x_{y.u} \right)^2}{2}$$

$$m_{sy} := f_y \cdot A_{sy} \cdot \left(d_y - x_{y.u} \right)$$

$$m_{Rdx.pm} := m_{cx} + m_{Ftux} + m_{sx} = \left(1.211 \cdot 10^4 \right) \frac{\boldsymbol{N} \cdot \boldsymbol{m}}{\boldsymbol{m}}$$

$$m_{Rdy.pm} := m_{cy} + m_{Ftuy} + m_{sy} = \left(7.223 \cdot 10^3 \right) \frac{\boldsymbol{N} \cdot \boldsymbol{m}}{\boldsymbol{m}}$$

Load distribution due to moment capacity

$$\alpha_x := \frac{m_{Rdx.pm}}{m_{Rdx.pm} + m_{Rdy.pm}} = 0.626$$

$$\alpha_y := \frac{m_{Rdy.pm}}{m_{Rdx.pm} + m_{Rdy.pm}} = 0.374$$

Moment due to self weight

$$R_{Ax.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = \left(2.848 \cdot 10^3 \right) \boldsymbol{N}$$

$$R_{By.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = \left(1.698 \cdot 10^3 \right) \boldsymbol{N}$$

$$M_{x.sw} := R_{Ax.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_x \cdot M_{gc} = 1.934 \ \boldsymbol{kN} \cdot \boldsymbol{m}$$

$$M_{y.sw} := R_{By.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_y \cdot M_{gc} = 1.153 \ kN \cdot m$$

Moment due to point load

$$M_{I.P} = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{I.sf} + M_{I.P}$$

Maximum point load

$$P_{x.u.II} := \left(m_{Rdx.pm} \cdot l_s - M_{x.sw} \right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.888 \cdot 10^4 \right) \ N$$

$$P_{y.u.II} := \left(m_{Rdy.pm} \cdot l_s - M_{y.sw} \right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.888 \cdot 10^4 \right) \ N$$

Reaction force

$$R_{x.u.II} := R_{Ax.sw} + \alpha_x \cdot \frac{P_{x.u.II}}{2} = \left(1.189 \cdot 10^4 \right) \ N$$

$$R_{y.u.II} := R_{By.sw} + \alpha_y \cdot \frac{P_{x.u.II}}{2} = \left(7.093 \cdot 10^3 \right) \ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rdx.pm} \cdot \sin(\omega)^2 + m_{Rdy.pm} \cdot \cos(\omega)^2 = 9.666 \ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 62.71 \ kN$$

Assuming that fibres only transfear load over cracks where the strain in the cross-section are lower than the ultimate strain for FRC

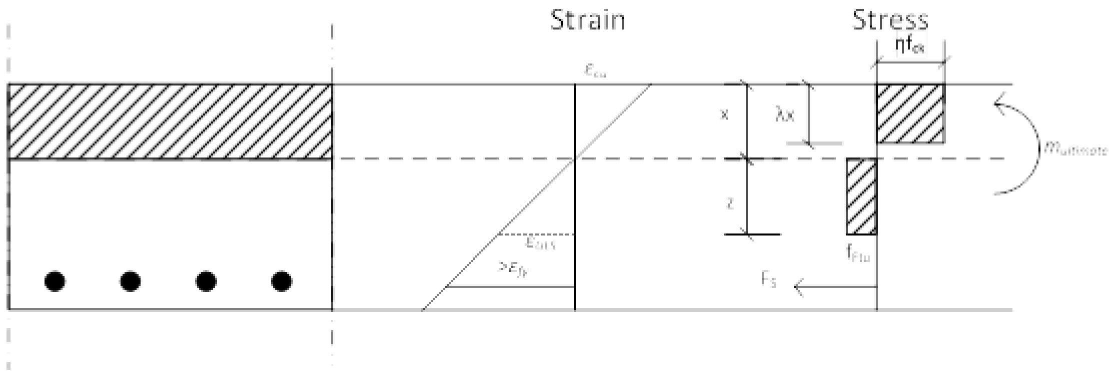
Controll strain

$$\varepsilon := \frac{h_c - x_{x.u}}{x_{x.u}} \cdot \varepsilon_{cu} = 0.04 \qquad \varepsilon := \frac{h_c - x_{y.u}}{x_{y.u}} \cdot \varepsilon_{cu} = 0.062$$

Ultimate load

Same ultimate load for failure when the ultimate strain for the FRC is reached in the bottom fibre for assumption of full load transfer.

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Compressed concrete area, force equilibrium

Guess Values

$$x_{x.u} := 10\ mm \qquad \varepsilon_s := 3 \cdot 10^{-3}$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_{x.u} = f_{Ftu.pm} \cdot \frac{\varepsilon_{Fu} \cdot x_{x.u}}{\varepsilon_{cu}} + \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sx} \qquad \frac{\varepsilon_{cu}}{x_{x.u}} = \frac{\varepsilon_s}{d_x - x_{x.u}}$$

Solver

$$X := \mathbf{find}\left(x_{x.u}, \varepsilon_s\right) = \begin{bmatrix} 5.604 \\ 33.583\ \frac{1}{m} \end{bmatrix} mm$$

$x_{x.u} := X(0) = 5.604\ mm$ $\varepsilon_s := X(1) = 0.034$

Guess Values

$x_{y.u} := 10\ mm$ $\varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f_{ck} \cdot \lambda \cdot x_{y.u} = f_{Ftu.pm} \cdot \frac{\varepsilon_{Fu} \cdot x_{y.u}}{\varepsilon_{cu}} + \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y \right) \cdot A_{sy} \qquad \frac{\varepsilon_{cu}}{x_{y.u}} = \frac{\varepsilon_s}{d_y - x_{y.u}}$$

Solver

$$X := \mathbf{find}(x_{y.u}, \varepsilon_s) = \begin{bmatrix} 3.064 \\ 57.464\ \frac{1}{m} \end{bmatrix} mm$$

$x_{y.u} := X(0) = 3.064\ mm$ $\varepsilon_s := X(1) = 0.057$

$$z_x := \frac{\varepsilon_{Fu} \cdot x_{x.u}}{\varepsilon_{cu}} = 32.02\ mm \qquad z_y := \frac{\varepsilon_{Fu} \cdot x_{y.u}}{\varepsilon_{cu}} = 17.509\ mm$$

$$m_{Rdx.pm} := f_{Ftu.pm} \cdot \frac{z_x^{\ 2}}{2} + f_{ck} \cdot \lambda \cdot x_{x.u} \cdot \left(x_{x.u} - \frac{\lambda \cdot x_{x.u}}{2} \right) + f_y \cdot A_{sx} \cdot (d_x - x_{x.u}) = (9.993 \cdot 10^3)\ \frac{N \cdot m}{m}$$

$$m_{Rdy.pm} := f_{Ftu.pm} \cdot \frac{z_x^{\ 2}}{2} + f_{ck} \cdot \lambda \cdot x_{y.u} \cdot \left(x_{y.u} - \frac{\lambda \cdot x_{y.u}}{2} \right) + f_y \cdot A_{sy} \cdot (d_y - x_{y.u}) = (4.958 \cdot 10^3)\ \frac{N \cdot m}{m}$$

Load distribution due to moment capacity

$$\alpha_x := \frac{m_{Rdx.pm}}{m_{Rdx.pm} + m_{Rdy.pm}} = 0.668$$

$$\alpha_y := \frac{m_{Rdy.pm}}{m_{Rdx.pm} + m_{Rdy.pm}} = 0.332$$

Moment due to self weight

$$R_{Ax.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = (3.038 \cdot 10^3)\ N$$

$$R_{By.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = (1.508 \cdot 10^3)\ N$$

$$M_{x.sw} := R_{Ax.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_x \cdot M_{gc} = 2.063 \ kN \cdot m$$

$$M_{y.sw} := R_{By.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_y \cdot M_{gc} = 1.024 \ kN \cdot m$$

Moment due to point load

$$M_P = \frac{P \cdot l}{4} \qquad\qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{x.u.II} := \left(m_{Rdx.pm} \cdot l_s - M_{x.sw} \right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.109 \cdot 10^4 \right) \ N$$

$$P_{y.u.II} := \left(m_{Rdy.pm} \cdot l_s - M_{y.sw} \right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.109 \cdot 10^4 \right) \ N$$

Reaction force

$$R_{x.u.II} := R_{Ax.sw} + \alpha_x \cdot \frac{P_{x.u.II}}{2} = \left(1.009 \cdot 10^4 \right) \ N$$

$$R_{y.u.II} := R_{By.sw} + \alpha_y \cdot \frac{P_{x.u.II}}{2} = \left(5.005 \cdot 10^3 \right) \ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rdx.pm} \cdot \sin(\omega)^2 + m_{Rdy.pm} \cdot \cos(\omega)^2 = 7.475 \ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 47.74 \ kN$$

Linear model
Indata

Values from experimental tests

$f_{Ftsm} := 0.45 \cdot f_{R1} = (9.533 \cdot 10^5)\ Pa$ Serviceability residual strength

$$A_{c.efx} := min\left(2.5 \cdot \left(c_x + \frac{\phi_s}{2}\right), \frac{(h_c - x_{x.u})}{3}\right) = (2.559 \cdot 10^4)\ \frac{mm^2}{m}$$

$$A_{c.efy} := min\left(2.5 \cdot \left(c_y + \frac{\phi_s}{2}\right), \frac{(h_c - x_{y.u})}{3}\right) = (2.644 \cdot 10^4)\ \frac{mm^2}{m}$$

Effective area of concrete in tension, x-value from ultimate section in plastic model

$$\rho_{s.efx} := \frac{A_{sx}}{A_{c.efx}} = 0.012$$

Effective reinforcement ratio

$$\rho_{s.efy} := \frac{A_{sy}}{A_{c.efy}} = 0.006$$

$\tau_{bm} := 1.8 \cdot f_{ctm} = 5.4\ MPa$ Mean bond stress between concrete and reinforcement

$$l_{s.max.x} := \frac{1}{4} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}} \cdot \frac{\phi_s}{\rho_{s.efx}} = 49.395\ mm$$

$$l_{s.max.y} := \frac{1}{4} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}} \cdot \frac{\phi_s}{\rho_{s.efy}} = 98.133\ mm$$

Length over width the slip between steel and concrete occurs

$$\varepsilon_{1x} := \varepsilon_{cu} \cdot \frac{(h_c - x_{x.u})}{x_{x.u}} \qquad \varepsilon_{2x} := \varepsilon_{cu} \cdot \frac{(h_c - x_{x.u} - A_{c.efx})}{x_{x.u}}$$

$$\varepsilon_{1y} := \varepsilon_{cu} \cdot \frac{(h_c - x_{y.u})}{x_{y.u}} \qquad \varepsilon_{2y} := \varepsilon_{cu} \cdot \frac{(h_c - x_{y.u} - A_{c.efy})}{x_{y.u}}$$

$$k_{1x} := 0.125 \cdot \frac{(\varepsilon_{1x} + \varepsilon_{2x})}{\varepsilon_{1x}} \qquad k_{2x} := 0.25 - \frac{A_{c.efx}}{8 \cdot (h_c - x_{x.u})}$$

$$k_{1y} := 0.125 \cdot \frac{(\varepsilon_{1y} + \varepsilon_{2y})}{\varepsilon_{1y}} \qquad k_{2y} := 0.25 - \frac{A_{c.efy}}{8 \cdot (h_c - x_{y.u})}$$

$$s_{rm.x} := \left(50 + k_{1x} \cdot k_{2x} \cdot \frac{6}{\rho_{s.efx}}\right) \cdot mm = 72.625\ mm$$

Mean crack distance

$$s_{rm.y} := \left(50 + k_{1y} \cdot k_{2y} \cdot \frac{6}{\rho_{s.efy}}\right) \cdot mm = 94.95\ mm$$

$y := h_c$ Distance between neutral axis and tensil side of cross section. Can be assumed to be te height of the cross section for slabs

$l_{cs.x} := min\left(s_{rm.x}, y\right) = 72.625\ mm$ Characteristic length of the structural element

$l_{cs.y} := min\left(s_{rm.y}, y\right) = 82.37\ mm$

$w_{u.x} := \varepsilon_{Fu} \cdot l_{cs.x} = 1.453\ mm$ Ultimate crack width

$w_{u.y} := \varepsilon_{Fu} \cdot l_{cs.y} = 1.647\ mm$

$\varepsilon_{ULS.x} := \frac{w_{u.x}}{l_{cs.x}} = 0.02$ Ultimate strain

$\varepsilon_{ULS.y} := \frac{w_{u.y}}{l_{cs.y}} = 0.02$

The service strain will be calculated in each step since it will varies with the compressed concrete zone

Ultimate residual strength

$$f_{Ftu.lmx} := f_{Ftsm} - \frac{w_{u.x}}{CMOD_3} \cdot \left(f_{Ftsm} - 0.5 \cdot f_{R3} + 0.2 \cdot f_{R1}\right) = \left(8.988 \cdot 10^5\right)\ Pa$$

$$f_{Ftu.lmy} := f_{Ftsm} - \frac{w_{u.y}}{CMOD_3} \cdot \left(f_{Ftsm} - 0.5 \cdot f_{R3} + 0.2 \cdot f_{R1}\right) = \left(8.915 \cdot 10^5\right)\ Pa$$

Fibre reinforcement combined with conventional reinforcement

Assuming that fibres transfear load over the whole crack.

Assuming to have ultimate concrete strain in top fiber and ultimare FRC strain in bottom fiber. The position of the SLS strain and the neutral axis is calculated with force equilibrium.

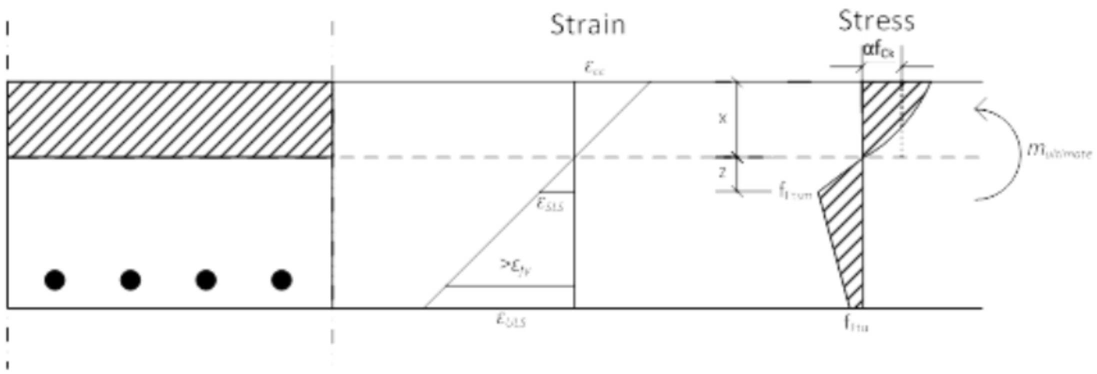
Ultimate load

Assume concrete in state III

- Cracked section
- Linear elastic limit is reached in either concrete or reinforcement
- Influence of concrete below neutral axis is neglected

Assume that failure occurs when the ultimate strain for the FRC is reached in the bottom fibre.

Assume that the plastic compressive strain for the concrete is not reached.



The service strain for the FRC must be found for the state when the ultimate strain for the FRC is reached in the bottom fibre. The x-value which are used to find effective concrete area in tension is the value calculated for the state when reinforcement yields found with an iteration process.

$x_x := 7.477\ mm$ $\qquad$ $x_y := 5.768\ mm$

$$A_{c.efx} := min\left(2.5 \cdot \left(c_x + \frac{\phi_s}{2}\right), \frac{(h_c - x_x)}{3}\right) = \left(2.496 \cdot 10^4\right) \frac{mm^2}{m}$$

$$A_{c.efy} := min\left(2.5 \cdot \left(c_y + \frac{\phi_s}{2}\right), \frac{(h_c - x_y)}{3}\right) = \left(2.553 \cdot 10^4\right) \frac{mm^2}{m}$$

Effective area of concrete in tension, x-value from ultimate section in plastic model

$$\rho_{s.efx} := \frac{A_{sx}}{A_{c.efx}} = 0.012$$ Effective reinforcement ratio

$$\rho_{s.efy} := \frac{A_{sy}}{A_{c.efy}} = 0.006$$

$$\varepsilon_{1x} := \varepsilon_{cu} \cdot \frac{(h_c - x_x)}{x_x} \qquad \varepsilon_{2x} := \varepsilon_{cu} \cdot \frac{(h_c - x_x - A_{c.efx})}{x_x}$$

$$\varepsilon_{1y} := \varepsilon_{cu} \cdot \frac{(h_c - x_y)}{x_y} \qquad \varepsilon_{2y} := \varepsilon_{cu} \cdot \frac{(h_c - x_y - A_{c.efy})}{x_y}$$

$$k_{1x} := 0.125 \cdot \frac{(\varepsilon_{1x} + \varepsilon_{2x})}{\varepsilon_{1x}} \qquad k_{2x} := 0.25 - \frac{A_{c.efx}}{8 \cdot (h_c - x_x)}$$

$$k_{1y} := 0.125 \cdot \frac{(\varepsilon_{1y} + \varepsilon_{2y})}{\varepsilon_{1y}} \qquad k_{2y} := 0.25 - \frac{A_{c.efy}}{8 \cdot (h_c - x_y)}$$

$$s_{rm.x} := \left(50 + k_{1x} \cdot k_{2x} \cdot \frac{6}{\rho_{s.efx}}\right) \cdot mm = 72.073\ mm$$

$$s_{rm.y} := \left(50 + k_{1y} \cdot k_{2y} \cdot \frac{6}{\rho_{s.efy}}\right) \cdot mm = 93.417\ mm$$

$y := h_c$ Distance between neutral axis and tensil side of cross section. Can be assumed to be te height of the cross section for slabs

$$l_{cs.x} := min(s_{rm.x}, y) = 72.073\ mm$$ Characteristic length of the structural element

$$l_{cs.y} := min(s_{rm.y}, y) = 82.37\ mm$$

$$\varepsilon_{SLS.x} := \frac{CMOD_1}{l_{cs.x}} = 0.00693738$$ Service strain

$$\varepsilon_{SLS.y} := \frac{CMOD_1}{l_{cs.y}} = 0.00607$$

Iteration process until force equilibrium is found

Assume a strain in top fibre when yield strain in reinforcement is reached

$\varepsilon_{cc.x} := 1.996833 \cdot 10^{-3}$ $\varepsilon_{cc.y} := 1.506081 \cdot 10^{-3}$

Stress block factors coresponding to chosen strain

$\alpha_{x.u} := 0.6664141$ $\alpha_{y.u} := 0.5635203$

$\beta_{x.u} := 0.3749050$ $\beta_{y.u} := 0.3611216$

Compressed concrete zone

$x_{x.u} := \frac{\varepsilon_{cc.x}}{\varepsilon_{cc.x} + \varepsilon_{Fu}} \cdot h_c = 7.477\ mm$ $x_{y.u} := \frac{\varepsilon_{cc.y}}{\varepsilon_{cc.y} + \varepsilon_{Fu}} \cdot h_c = 5.768\ mm$

$z_x := \frac{\varepsilon_{SLS.x}}{\varepsilon_{Fu}} \cdot \left(h_c - x_{x.u}\right) = 25.978\ mm$ $z_y := \frac{\varepsilon_{SLS.y}}{\varepsilon_{Fu}} \cdot \left(h_c - x_{y.u}\right) = 23.249\ mm$

$j_x := h_c - x_{x.u} - z_x = 48.915\ mm$ $j_y := h_c - x_{y.u} - z_y = 53.352\ mm$

Strain in reinforcement

$\varepsilon_{sx} := \varepsilon_{cc.x} \cdot \frac{\left(d_x - x_{x.u}\right)}{x_{x.u}} = 0.014$ $\varepsilon_{sy} := \varepsilon_{cc.y} \cdot \frac{\left(d_y - x_{y.u}\right)}{x_{y.u}} = 0.012$

Controll equilibrium

$f_{sx} := \left(\frac{\left(\varepsilon_{sx} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right)$ $f_{sy} := \left(\frac{\left(\varepsilon_{sy} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right)$

$$\left(f_{Ftsm} - f_{Ftu.lmx}\right) \cdot j_x \cdot \frac{1}{2} + f_{Ftsm} \cdot z_x \cdot \frac{1}{2} + f_{Ftu.lmx} \cdot j_x + f_{sx} \cdot A_{sx} - \alpha_{x.u} \cdot f_{ck} \cdot x_{x.u} = -0.404\ m \cdot Pa$$

$$\left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot j_y \cdot \frac{1}{2} + f_{Ftsm} \cdot z_y \cdot \frac{1}{2} + f_{Ftu.lmy} \cdot j_y + f_{sy} \cdot A_{sy} - \alpha_{y.u} \cdot f_{ck} \cdot x_{y.u} = -1.024\ m \cdot Pa$$

Moment capacity

$$m_{cx} := \alpha_{x.u} \cdot f_{ck} \cdot x_{x.u} \cdot \left(x_{x.u} - \beta_{x.u} \cdot x_{x.u}\right) \qquad m_{sx} := f_y \cdot A_{sx} \cdot \left(d_x - x_{x.u}\right)$$

$$m_{Ftux} := f_{Ftu.lmx} \cdot j_x \cdot \left(\frac{j_x}{2} + z_x\right) \qquad m_{Ftsmx1} := \frac{1}{2} \cdot f_{Ftsm} \cdot {z_x}^2 \cdot \frac{2}{3}$$

$$m_{Ftsmx2} := \frac{1}{2} \left(f_{Ftsm} - f_{Ftu.lmx}\right) \cdot j_x \cdot \left(\frac{j_x}{3} + z_x\right)$$

$$m_{cy} := \alpha_{y.u} \cdot f_{ck} \cdot x_{y.u} \cdot \left(x_{y.u} - \beta_{y.u} \cdot x_{y.u}\right) \qquad m_{sy} := f_y \cdot A_{sy} \cdot \left(d_y - x_{y.u}\right)$$

$$m_{Ftuy} := f_{Ftu.lmy} \cdot j_y \cdot \left(\frac{j_y}{2} + z_y\right) \qquad m_{Ftsmy1} := \frac{1}{2} \cdot f_{Ftsm} \cdot {z_y}^2 \cdot \frac{2}{3}$$

$$m_{Ftsmy2} := \frac{1}{2} \left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot j_y \cdot \left(\frac{j_y}{3} + z_y\right)$$

$$m_{Rdx.lmI} := m_{cx} + m_{sx} + m_{Ftux} + m_{Ftsmx1} + m_{Ftsmx2} = \left(1.211 \cdot 10^4\right) \ \boldsymbol{N}$$

$$m_{Rdy.lmI} := m_{cy} + m_{sy} + m_{Ftuy} + m_{Ftsmy1} + m_{Ftsmy2} = \left(7.242 \cdot 10^3\right) \ \boldsymbol{N}$$

Load distribution due to moment capacity

$$\alpha_x := \frac{m_{Rdx.lmI}}{m_{Rdx.lmI} + m_{Rdy.lmI}} = 0.626 \qquad \alpha_y := \frac{m_{Rdy.lmI}}{m_{Rdx.lmI} + m_{Rdy.lmI}} = 0.374$$

Moment due to self weight

$$R_{Ax.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = \left(2.845 \cdot 10^3\right) \ \boldsymbol{N}$$

$$R_{By.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = \left(1.701 \cdot 10^3\right) \ \boldsymbol{N}$$

$$M_{x.sw} := R_{Ax.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_x \cdot M_{gc} = 1.932 \ \boldsymbol{kN \cdot m}$$

$$M_{y.sw} := R_{By.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_y \cdot M_{gc} = 1.155\ kN \cdot m$$

Moment due to point load

$$M_P = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{x.u.I} := \left(m_{Rdx.lmI} \cdot l_s - M_{x.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.891 \cdot 10^4\right)\ N$$

$$P_{y.u.I} := \left(m_{Rdy.lmI} \cdot l_s - M_{y.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.891 \cdot 10^4\right)\ N$$

Reaction force

$$R_{x.u.I} := R_{Ax.sw} + \alpha_x \cdot \frac{P_{x.u.I}}{2} = \left(1.189 \cdot 10^4\right)\ N$$

$$R_{y.u.I} := R_{By.sw} + \alpha_y \cdot \frac{P_{x.u.I}}{2} = \left(7.112 \cdot 10^3\right)\ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rdx.lmI} \cdot \sin(\omega)^2 + m_{Rdy.lmI} \cdot \cos(\omega)^2 = 9.675\ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40\ kN$$

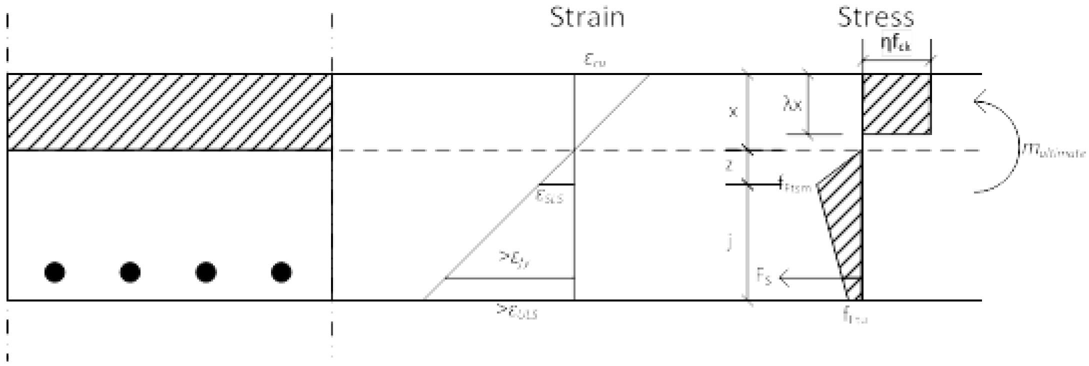
Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 62.768\ kN$$

Assume that failure occurs when the ultimate comressive strain for the concrete is reached



The service strain for the FRC must be found for the state when the ultimate compressive strain for the concrete is reached. The x-value which are used to find effective concrete area in tension is the value calculated for the state when reinforcement yields found with an iteration process.

$x_x := 6.562\ mm \qquad x_y := 4.43\ mm$

$$A_{c.efx} := min\left(2.5 \cdot \left(c_x + \frac{\phi_s}{2}\right), \frac{(h_c - x_x)}{3}\right) = \left(2.527 \cdot 10^4\right)\frac{mm^2}{m}$$

$$A_{c.efy} := min\left(2.5 \cdot \left(c_y + \frac{\phi_s}{2}\right), \frac{(h_c - x_y)}{3}\right) = \left(2.598 \cdot 10^4\right)\frac{mm^2}{m}$$

Effective area of concrete in tension, x-value from ultimate section in plastic model

$$\rho_{s.efx} := \frac{A_{sx}}{A_{c.efx}} = 0.012$$

Effective reinforcement ratio

$$\rho_{s.efy} := \frac{A_{sy}}{A_{c.efy}} = 0.006$$

$$\varepsilon_{1x} := \varepsilon_{cu} \cdot \frac{(h_c - x_x)}{x_x} \qquad \varepsilon_{2x} := \varepsilon_{cu} \cdot \frac{(h_c - x_x - A_{c.efx})}{x_x}$$

$$\varepsilon_{1y} := \varepsilon_{cu} \cdot \frac{(h_c - x_y)}{x_y} \qquad \varepsilon_{2y} := \varepsilon_{cu} \cdot \frac{(h_c - x_y - A_{c.efy})}{x_y}$$

$$k_{1x} := 0.125 \cdot \frac{(\varepsilon_{1x} + \varepsilon_{2x})}{\varepsilon_{1x}} \qquad k_{2x} := 0.25 - \frac{A_{c.efx}}{8 \cdot (h_c - x_x)}$$

$$k_{1y} := 0.125 \cdot \frac{(\varepsilon_{1y} + \varepsilon_{2y})}{\varepsilon_{1y}} \qquad k_{2y} := 0.25 - \frac{A_{c.efy}}{8 \cdot (h_c - x_y)}$$

$$s_{rm.x} := \left(50 + k_{1x} \cdot k_{2x} \cdot \frac{6}{\rho_{s.efx}}\right) \cdot mm = 72.343\ mm$$

$$s_{rm.y} := \left(50 + k_{1y} \cdot k_{2y} \cdot \frac{6}{\rho_{s.efy}}\right) \cdot mm = 94.176\ mm$$

$y := h_c$ Distance between neutral axis and tensil side of cross section. Can be assumed to be te height of the cross section for slabs

$l_{cs.x} := min(s_{rm.x}, y) = 72.343\ mm$ Characteristic length of the structural element

$l_{cs.y} := min(s_{rm.y}, y) = 82.37\ mm$

$\varepsilon_{SLS.x} := \frac{CMOD_1}{l_{cs.x}} = 0.00691$ Service strain

$$\varepsilon_{SLS.y} := \frac{CMOD_1}{l_{cs.y}} = 0.00607$$

Compressed concrete area, force equilibrium

Guess Values

$x_{x.u} := 10\ mm \qquad f := 10\ \frac{kg}{s^2} \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f = f_{Ftsm} \cdot \frac{\frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u}}{2} + (f_{Ftsm} - f_{Ftu.lmx}) \cdot \frac{\left(h_c - \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u} - x_{x.u}\right)}{2} \qquad \frac{\varepsilon_{cu}}{x_{x.u}} = \frac{\varepsilon_s}{d_x - x_{x.u}}$$

$$f_{ck} \cdot \lambda \cdot x_{x.u} = f_{Ftu.lmx} \cdot \left(h_c - \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u} - x_{x.u}\right) + f + \left(\frac{(\varepsilon_s - \varepsilon_{syk})}{(\varepsilon_{su} - \varepsilon_{syk})} \cdot (f_u - f_y) + f_y\right) \cdot A_{sx}$$

Solver

$$X := \text{find}(x_{x.u}, f, \varepsilon_s) = \begin{bmatrix} 6.562 \\ (7.889 \cdot 10^6)\ Pa \\ 28.164\ \frac{1}{m} \end{bmatrix} mm$$

$x_{x.u} := X(0) = 6.562\ mm$

$z_x := \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u} = 12.959\ mm$ $\qquad j_x := h_c - z_x - x_{x.u} = 62.848\ mm$

$\varepsilon_{sx} := X(2) = 0.028$

Guess Values

$x_{y.u} := 10\ mm \qquad f := 10\ \frac{kg}{s^2} \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f = f_{Ftsm} \cdot \frac{\frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u}}{2} + \left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot \frac{\left(h_c - \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u} - x_{y.u}\right)}{2} \qquad \frac{\varepsilon_{cu}}{x_{y.u}} = \frac{\varepsilon_s}{d_y - x_{y.u}}$$

$$f_{ck} \cdot \lambda \cdot x_{y.u} = f_{Ftu.lmy} \cdot \left(h_c - \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u} - x_{y.u}\right) + f + \left(\frac{\left(\varepsilon_s - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sy}$$

Solver

$$X := \mathbf{find}\left(x_{y.u}, f, \varepsilon_s\right) = \begin{bmatrix} 4.43 \\ \left(5.833 \cdot 10^6\right)\ Pa \\ 38.662\ \frac{1}{m} \end{bmatrix} mm$$

$x_{y.u} := X(0) = 4.43\ mm$

$z_y := \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u} = 7.684\ mm$ $\qquad j_y := h_c - z_y - x_{y.u} = 70.256\ mm$

$\varepsilon_{sy} := X(2) = 0.039$

Moment capacity

$m_{cx} := f_{ck} \cdot \lambda \cdot x_{x.u} \cdot \left(x_{x.u} - \frac{\lambda \cdot x_{x.u}}{2}\right) \qquad m_{sx} := \left(\frac{\left(\varepsilon_{sx} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sx} \cdot \left(d_x - x_{x.u}\right)$

$m_{Ftsm1x} := f_{Ftsm} \cdot {z_x}^2 \cdot \frac{2}{3}$

$m_{Ftsm2x} := \left(f_{Ftsm} - f_{Ftu.lmx}\right) \cdot \frac{j_x}{2} \left(\frac{j_x}{3} + z_x\right)$

$m_{Ftux} := f_{Ftu.lmx} \cdot j_x \cdot \left(\frac{j_x}{2} + z_x\right)$

$$m_{cy} := f_{ck} \cdot \lambda \cdot x_{y.u} \cdot \left(x_{y.u} - \frac{\lambda \cdot x_{y.u}}{2} \right) \qquad m_{sy} := \left(\frac{\left(\varepsilon_{sy} - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y \right) \cdot A_{sy} \cdot \left(d_y - x_{y.u}\right)$$

$$m_{Ftsm1y} := f_{Ftsm} \cdot {z_y}^2 \cdot \frac{2}{3}$$

$$m_{Ftsm2y} := \left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot \frac{j_y}{2} \left(\frac{j_y}{3} + z_y \right)$$

$$m_{Ftuy} := f_{Ftu.lmy} \cdot j_y \cdot \left(\frac{j_y}{2} + z_y \right)$$

$$m_{Rdx.lmII} := m_{cx} + m_{sx} + m_{Ftsm1x} + m_{Ftsm2x} + m_{Ftux} = \left(1.283 \cdot 10^4\right) \frac{N \cdot m}{m}$$

$$m_{Rdy.lmII} := m_{cy} + m_{sy} + m_{Ftsm1y} + m_{Ftsm2y} + m_{Ftuy} = \left(7.761 \cdot 10^3\right) \frac{N \cdot m}{m}$$

Load distribution due to moment capacity

$$\alpha_x := \frac{m_{Rdx.lmII}}{m_{Rdx.lmII} + m_{Rdy.lmII}} = 0.623$$

$$\alpha_y := \frac{m_{Rdy.lmII}}{m_{Rdx.lmII} + m_{Rdy.lmII}} = 0.377$$

Moment du to self weight

$$R_{Ax.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = \left(2.833 \cdot 10^3\right) N$$

$$R_{By.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = \left(1.713 \cdot 10^3\right) N$$

$$M_{x.sw} := R_{Ax.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_x \cdot M_{gc} = 1.924 \ kN \cdot m$$

$$M_{y.sw} := R_{By.sw} \cdot \left(l_{e2} + \frac{l_s}{2} \right) - \alpha_y \cdot M_{gc} = 1.163 \ kN \cdot m$$

Moment due to point load

$$M_P = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{x.u.II} := \left(m_{Rdx.lmII} \cdot l_s - M_{x.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(3.112 \cdot 10^4\right)\ N$$

$$P_{y.u.II} := \left(m_{Rdy.lmII} \cdot l_s - M_{y.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(3.112 \cdot 10^4\right)\ N$$

Reaction force

$$R_{x.u.II} := R_{Ax.sw} + \alpha_x \cdot \frac{P_{x.u.II}}{2} = \left(1.253 \cdot 10^4\right)\ N$$

$$R_{y.u.II} := R_{By.sw} + \alpha_y \cdot \frac{P_{x.u.II}}{2} = \left(7.577 \cdot 10^3\right)\ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rdx.lmII} \cdot \sin(\omega)^2 + m_{Rdy.lmII} \cdot \cos(\omega)^2 = 10.297\ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40\ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 67.02\ kN$$

Assuming that fibres only transfear load over cracks where the strain in the cross-section are lower than the ultimate strain for FRC

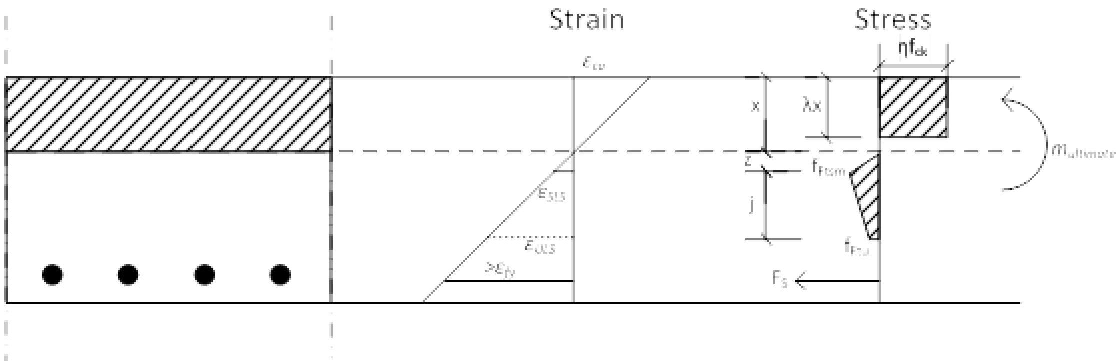
Controll strain bottom fiber

$$\varepsilon_x := \frac{\varepsilon_{SLS.x} \cdot (h_c - x_{x.u})}{z_x} = 0.04 \qquad \varepsilon_y := \frac{\varepsilon_{SLS.y} \cdot (h_c - x_{y.u})}{z_y} = 0.062$$

Ultimate load

Same ultimate load for failure when the ultimate strain for the FRC is reached in the bottom fibre for assumption of full load transfer.

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



The service strain for the FRC must be found for the state when the ultimate compressive strain for the concrete is reached. The x-value which are used to find effective concrete area in tension is the value calculated for the state when reinforcement yields found with an iteration process.

$$x_x := 5.526\ mm \qquad x_y := 3.031\ mm$$

$$A_{c.efx} := min\left(2.5 \cdot \left(c_x + \frac{\phi_s}{2}\right), \frac{(h_c - x_x)}{3}\right) = (2.561 \cdot 10^4)\ \frac{mm^2}{m}$$

$$A_{c.efy} := min\left(2.5 \cdot \left(c_y + \frac{\phi_s}{2}\right), \frac{(h_c - x_y)}{3}\right) = (2.645 \cdot 10^4)\ \frac{mm^2}{m}$$

Effective area of concrete in tension, x-value from ultimate section in plastic model

$$\rho_{s.efx} := \frac{A_{sx}}{A_{c.efx}} = 0.011$$ Effective reinforcement ratio

$$\rho_{s.efy} := \frac{A_{sy}}{A_{c.efy}} = 0.006$$

$$\varepsilon_{1x} := \varepsilon_{cu} \cdot \frac{\left(h_c - x_x\right)}{x_x} \qquad \varepsilon_{2x} := \varepsilon_{cu} \cdot \frac{\left(h_c - x_x - A_{c.efx}\right)}{x_x}$$

$$\varepsilon_{1y} := \varepsilon_{cu} \cdot \frac{\left(h_c - x_y\right)}{x_y} \qquad \varepsilon_{2y} := \varepsilon_{cu} \cdot \frac{\left(h_c - x_y - A_{c.efy}\right)}{x_y}$$

$$k_{1x} := 0.125 \cdot \frac{\left(\varepsilon_{1x} + \varepsilon_{2x}\right)}{\varepsilon_{1x}} \qquad k_{2x} := 0.25 - \frac{A_{c.efx}}{8 \cdot \left(h_c - x_x\right)}$$

$$k_{1y} := 0.125 \cdot \frac{\left(\varepsilon_{1y} + \varepsilon_{2y}\right)}{\varepsilon_{1y}} \qquad k_{2y} := 0.25 - \frac{A_{c.efy}}{8 \cdot \left(h_c - x_y\right)}$$

$$s_{rm.x} := \left(50 + k_{1x} \cdot k_{2x} \cdot \frac{6}{\rho_{s.efx}}\right) \cdot mm = 72.648\ mm$$

$$s_{rm.y} := \left(50 + k_{1y} \cdot k_{2y} \cdot \frac{6}{\rho_{s.efy}}\right) \cdot mm = 94.969\ mm$$

$y := h_c$ Distance between neutral axis and tensil side of cross section. Can be assumed to be te height of the cross section for slabs

$$l_{cs.x} := min\left(s_{rm.x}, y\right) = 72.648\ mm$$ Characteristic length of the structural element

$$l_{cs.y} := min\left(s_{rm.y}, y\right) = 82.37\ mm$$

$$\varepsilon_{SLS.x} := \frac{CMOD_1}{l_{cs.x}} = 0.00688$$ Service strain

$$\varepsilon_{SLS.y} := \frac{CMOD_1}{l_{cs.y}} = 0.00607$$

Compressed concrete area, force equilibrium

Guess Values

$x_{x.u} := 10\ mm \qquad f := 10\ \frac{kg}{s^2} \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f = f_{Ftsm} \cdot \frac{\frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u}}{2} + \left(f_{Ftsm} - f_{Ftu.lmx}\right) \cdot \frac{\left(\frac{\varepsilon_{ULS.x}}{\varepsilon_{cu}} \cdot x_{x.u} - \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u}\right)}{2} \qquad \frac{\varepsilon_{cu}}{x_{x.u}} = \frac{\varepsilon_s}{d_x - x_{x.u}}$$

$$f_{ck} \cdot \lambda \cdot x_{x.u} = f_{Ftu.lmx} \cdot \left(\frac{\varepsilon_{ULS.x}}{\varepsilon_{cu}} \cdot x_{x.u} - \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u}\right) + f + \left(\frac{\left(\varepsilon_s - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sx}$$

Solver

$$X := \mathbf{find}\left(x_{x.u}, f, \varepsilon_s\right) = \begin{bmatrix} 5.526 \\ \left(5.743 \cdot 10^{6}\right)\ Pa \\ 34.105\ \frac{1}{m} \end{bmatrix} mm$$

$x_{x.u} := X(0) = 5.526\ mm$

$z_x := \frac{\varepsilon_{SLS.x}}{\varepsilon_{cu}} \cdot x_{x.u} = 10.866\ mm \qquad j_x := \frac{\varepsilon_{ULS.x}}{\varepsilon_{cu}} \cdot x_{x.u} - z_x = 20.71\ mm$

$\varepsilon_{sx} := X(2) = 0.034$

Guess Values

$x_{y.u} := 10\ mm \qquad f := 10\ \frac{kg}{s^2} \qquad \varepsilon_s := 3 \cdot 10^{-3}$

Constraints

$$f = f_{Ftsm} \cdot \frac{\frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u}}{2} + \left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot \frac{\left(\frac{\varepsilon_{ULS.y}}{\varepsilon_{cu}} \cdot x_{y.u} - \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u}\right)}{2} \qquad \frac{\varepsilon_{cu}}{x_{y.u}} = \frac{\varepsilon_s}{d_y - x_{y.u}}$$

$$f_{ck} \cdot \lambda \cdot x_{y.u} = f_{Ftu.lmy} \cdot \left(\frac{\varepsilon_{ULS.y}}{\varepsilon_{cu}} \cdot x_{y.u} - \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u}\right) + f + \left(\frac{\left(\varepsilon_s - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y\right) \cdot A_{sy}$$

Solver

$$X := \mathbf{find}\left(x_{y.u}, f, \varepsilon_s\right) = \begin{bmatrix} 3.031 \\ \left(2.878 \cdot 10^{6}\right)\ Pa \\ 58.135\ \frac{1}{m} \end{bmatrix} mm$$

$x_{y.u} := X(0) = 3.031\ mm$

$z_y := \frac{\varepsilon_{SLS.y}}{\varepsilon_{cu}} \cdot x_{y.u} = 5.256\ mm \qquad j_y := \frac{\varepsilon_{ULS.y}}{\varepsilon_{cu}} \cdot x_{y.u} - z_y = 12.062\ mm$

$$\varepsilon_s := X(2) = 0.058$$

Moment capacity

$$m_{cx} := f_{ck} \cdot \lambda \cdot x_{x.u} \cdot \left(x_{x.u} - \frac{\lambda \cdot x_{x.u}}{2} \right) \qquad m_{sx} := \left(\frac{\left(\varepsilon_s - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y \right) \cdot A_{sx} \cdot \left(d_x - x_{x.u}\right)$$

$$m_{Ftsm1x} := f_{Ftsm} \cdot {z_x}^2 \cdot \frac{2}{3}$$

$$m_{Ftsm2x} := \left(f_{Ftsm} - f_{Ftu.lmx}\right) \cdot \frac{j_x}{2} \left(\frac{j_x}{3} + z_x \right)$$

$$m_{Ftux} := f_{Ftu.lmx} \cdot j_x \cdot \left(\frac{j_x}{2} + z_x \right)$$

$$m_{cy} := f_{ck} \cdot \lambda \cdot x_{y.u} \cdot \left(x_{y.u} - \frac{\lambda \cdot x_{y.u}}{2} \right) \qquad m_{sy} := \left(\frac{\left(\varepsilon_s - \varepsilon_{syk}\right)}{\left(\varepsilon_{su} - \varepsilon_{syk}\right)} \cdot \left(f_u - f_y\right) + f_y \right) \cdot A_{sy} \cdot \left(d_y - x_{y.u}\right)$$

$$m_{Ftsm1y} := f_{Ftsm} \cdot {z_y}^2 \cdot \frac{2}{3}$$

$$m_{Ftsm2y} := \left(f_{Ftsm} - f_{Ftu.lmy}\right) \cdot \frac{j_y}{2} \left(\frac{j_y}{3} + z_y \right)$$

$$m_{Ftuy} := f_{Ftu.lmy} \cdot j_y \cdot \left(\frac{j_y}{2} + z_y \right)$$

$$m_{Rdx.lmII} := m_{cx} + m_{sx} + m_{Ftsm1x} + m_{Ftsm2x} + m_{Ftux} = \left(1.116 \cdot 10^4\right) \frac{N \cdot m}{m}$$

$$m_{Rdy.lmII} := m_{cy} + m_{sy} + m_{Ftsm1y} + m_{Ftsm2y} + m_{Ftuy} = \left(5.213 \cdot 10^3\right) \frac{N \cdot m}{m}$$

Load distribution due to moment capacity

$$\alpha_x := \frac{m_{Rdx.lmII}}{m_{Rdx.lmII} + m_{Rdy.lmII}} = 0.682 \qquad \alpha_y := \frac{m_{Rdy.lmII}}{m_{Rdx.lmII} + m_{Rdy.lmII}} = 0.318$$

Moment due to self weight

$$R_{Ax.sw} := \alpha_x \cdot \frac{Q_{gc}}{2} = \left(3.099 \cdot 10^3\right) N$$

$$R_{By.sw} := \alpha_y \cdot \frac{Q_{gc}}{2} = \left(1.447 \cdot 10^3\right) N$$

$$M_{x.sw} := R_{Ax.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_x \cdot M_{gc} = 2.104 \ kN \cdot m$$

$$M_{y.sw} := R_{By.sw} \cdot \left(l_{e2} + \frac{l_s}{2}\right) - \alpha_y \cdot M_{gc} = 0.983 \ kN \cdot m$$

Moment due to point load

$$M_P = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{x.u.II} := \left(m_{Rdx.lmII} \cdot l_s - M_{x.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_x} = \left(2.363 \cdot 10^4\right) \ N$$

$$P_{y.u.II} := \left(m_{Rdy.lmII} \cdot l_s - M_{y.sw}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{\alpha_y} = \left(2.363 \cdot 10^4\right) \ N$$

Reaction force

$$R_{x.u.II} := R_{Ax.sw} + \alpha_x \cdot \frac{P_{x.u.II}}{2} = \left(1.115 \cdot 10^4\right) \ N$$

$$R_{y.u.II} := R_{By.sw} + \alpha_y \cdot \frac{P_{x.u.II}}{2} = \left(5.207 \cdot 10^3\right) \ N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rdx.lmII} \cdot \sin(\omega)^2 + m_{Rdy.lmII} \cdot \cos(\omega)^2 = 8.188 \ \frac{kN \cdot m}{m}$$ Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gc}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 52.611 \ kN$$

Only fibre reinforcement

$f_{Ftsm} = (9.533 \cdot 10^5)\ Pa$ Serviceability residual strength

$\varepsilon_{Fu} = 0.02$ Ultimate strain

$s_{rm} := 82.37\ mm$ Mean distance between cracks

$l_{cs} := min(s_{rm}, h_c) = 82.37\ mm$ Characteristc length

$l_{cs} := h_c = 82.37\ mm$

$w_u := \varepsilon_{Fu} \cdot l_{cs} = 1.647\ mm$ Ultimate crack width

$\varepsilon_{ULS} := \frac{w_u}{l_{cs}} = 0.02$ Ultimate strain

$\varepsilon_{SLS} := \frac{CMOD_1}{l_{cs}} = 0.00607$ Service strain

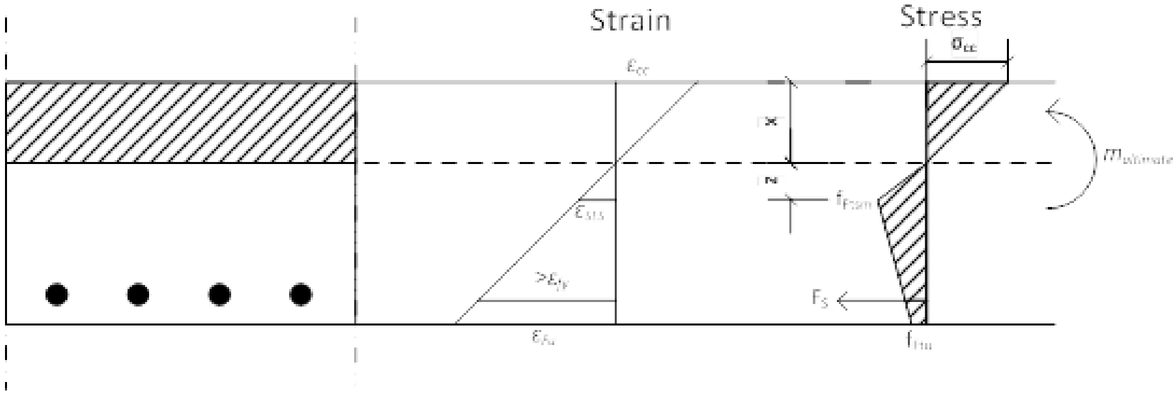
$$f_{Ftu.lm} := f_{Ftsm} - \frac{w_u}{CMOD_3} \cdot (f_{Ftsm} - 0.5 \cdot f_{R3} + 0.2 \cdot f_{R1}) = (8.915 \cdot 10^5)\ Pa$$

Ultimate residual strength

Ultimate load

Assume that failure occurs when the ultimate strain for the FRC is reached in the bottom fibre.

Assume that the plastic compressive strain for the concrete is not reached.



Iteration process until force equilibrium is found

Assume a strain in top fibre when yield strain in reinforcement is reached

$\varepsilon_{cc} := 0.902909 \cdot 10^{-3}$

Stress block factors coresponding to chosen strain

$\alpha_u := 0.3830182$

$\beta_u := 0.3480582$

Compressed concrete zone

$$x_u := \frac{\varepsilon_{cc}}{\varepsilon_{cc} + \varepsilon_{Fu}} \cdot h_c = 3.558\ mm$$

$$z := \frac{\varepsilon_{SLS}}{\varepsilon_{Fu}} \cdot \left(h_c - x_u\right) = 23.92\ mm \qquad j := h_c - x_u - z = 54.892\ mm$$

Controll equilibrium

$$\left(f_{Ftsm}-f_{Ftu.lm}\right)\cdot j\cdot\frac{1}{2}+f_{Ftsm}\cdot z\cdot\frac{1}{2}+f_{Ftu.lm}\cdot j-\alpha_u\cdot f_{ck}\cdot x_u=-0.848\ \boldsymbol{m\cdot Pa}$$

Moment capacity

$$m_c:=\alpha_u\cdot f_{ck}\cdot x_u\cdot\left(x_u-\beta_u\cdot x_u\right) \qquad m_{Ftu}:=f_{Ftu.lm}\cdot j\cdot\left(\frac{j}{2}+z\right)$$

$$m_{Ftsm1}:=\frac{1}{2}\cdot f_{Ftsm}\cdot z^2\cdot\frac{2}{3} \qquad m_{Ftsm2}:=\frac{1}{2}\left(f_{Ftsm}-f_{Ftu.lm}\right)\cdot j\cdot\left(\frac{j}{3}+z\right)$$

$$m_{Rd.lm}:=m_c+m_{Ftu}+m_{Ftsm1}+m_{Ftsm2}=\left(2.911\cdot10^3\right)\ \boldsymbol{N}$$

$$R_{swf}=\left(2.158\cdot10^3\right)\ \boldsymbol{N} \qquad M_{swf}=1.465\ \boldsymbol{kN\cdot m}$$

Moment due to point load

$$M_P=\frac{P\cdot l}{4} \qquad m_R\cdot l=M_{sf}+M_P$$

Maximum point load

$$P_{u.I}:=\left(m_{Rd.lm}\cdot l_s-M_{swf}\right)\cdot\frac{4}{l_2}\cdot\frac{1}{0.5}=\left(5.14\cdot10^3\right)\ \boldsymbol{N}$$

Reaction force

$$R_{u.I}:=R_{swf}+\frac{P_{u.I}}{4}=\left(3.443\cdot10^3\right)\ \boldsymbol{N}$$

Analysis of point load with yield line analysis

$$m_b := m_{Rd.lm} \cdot \sin(\omega)^2 + m_{Rd.lm} \cdot \cos(\omega)^2 = 2.911 \ \frac{kN \cdot m}{m}$$

Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

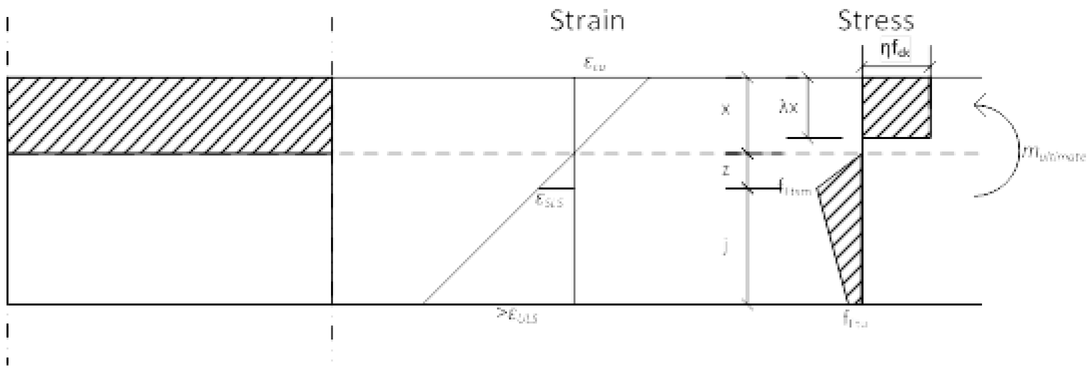
Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gf}}{l_P} + P$$

Solver

$$P := \text{find}(P) = 16.719 \ kN$$

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Compressed concrete area, force equilibrium

Guess Values

$$x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.lm} \cdot \left(h_c - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u - x_u\right) + f_{Ftsm} \cdot \frac{\frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u}{2} + \left(f_{Ftsm} - f_{Ftu.lm}\right) \cdot \frac{\left(h_c - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u - x_u\right)}{2}$$

Solver

$$x_u := \mathbf{find}\left(x_u\right) = 1.994\ mm$$

$$z := \frac{\varepsilon_{SLS} \cdot x_u}{\varepsilon_{cu}} = 3.457\ mm \qquad j := h_c - x_u - z = 76.919\ mm$$

Moment capacity

$$m_c := f_{ck} \cdot \lambda \cdot x_u \cdot \left(x_u - \frac{\lambda \cdot x_u}{2}\right)$$

$$m_{Ftsm1} := f_{Ftsm} \cdot z^2 \cdot \frac{2}{3}$$

$$m_{Ftsm2} := \left(f_{Ftsm} - f_{Ftu.lm}\right) \cdot \frac{j}{2} \left(\frac{j}{3} + z\right)$$

$$m_{Ftu} := f_{Ftu.lm} \cdot j \cdot \left(\frac{j}{2} + z\right)$$

$$m_{Rd.lm} := m_c + m_{Ftsm1} + m_{Ftsm2} + m_{Ftu} = \left(3.038 \cdot 10^3\right) \frac{N \cdot m}{m}$$

Moment due to self weight

$$R_{swf} = \left(2.158 \cdot 10^3\right) N \qquad M_{swf} = 1.465 \ kN \cdot m$$

Moment due to point load

$$M_P = \frac{P \cdot l}{4} \qquad m_R \cdot l = M_{sf} + M_P$$

Maximum point load

$$P_{u.II} := \left(m_{Rd.lm} \cdot l_s - M_{swf}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{0.5} = \left(5.5916 \cdot 10^3\right) N$$

Reaction force

$$R_{u.II} := R_{swf} + \frac{P_{u.II}}{4} = \left(3.556 \cdot 10^3\right) N$$

Analysis of point load with yield line analysis

$$m_b := m_{Rd.lm} \cdot \sin(\omega)^2 + m_{Rd.lm} \cdot \cos(\omega)^2 = 3.038 \frac{kN \cdot m}{m}$$

Moment capacity along yieldline

Guess Values

$$P := 40 \ kN$$

Constraints

$$\frac{8 \cdot d \cdot m_b}{l_d} = \frac{x_{gc} \cdot Q_{gf}}{l_P} + P$$

Solver

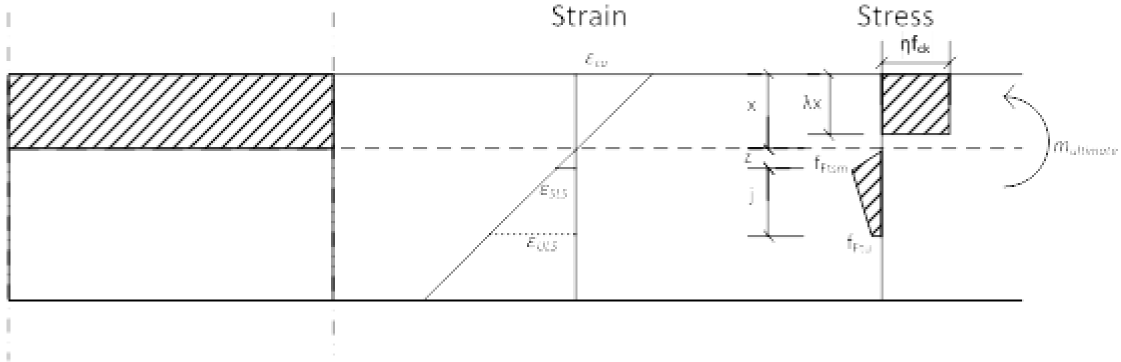
$$P := \text{find}(P) = 17.587 \ kN$$

Assuming that fibres only transfear load over cracks where the strain in the cross-section are lower than the ultimate strain for FRC

Ultimate load

Same ultimate load for failure when the ultimate strain for the FRC is reached in the bottom fibre for assumption of full load transfer.

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Iteration is used to find the compressed concrete area and possition of service and ultimate strain since no force equilibrium can be found

Controll strain bottom fiber

$$\varepsilon := \frac{\varepsilon_{SLS} \cdot \left(h_c - x_u\right)}{z} = 0.141$$

Distance neutral axis to position with ultimate strain for FRC

$$k := \frac{\varepsilon_{ULS}}{\varepsilon_{cu}} \cdot x_u = 11.392\ mm$$

$$x_1 := x_u$$

Force equilibrium

Guess Values

$$x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.lm} \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right) + \frac{1}{2}\ f_{Ftsm} \cdot \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u + \frac{1}{2}\ \left(f_{Ftsm} - f_{Ftu.lm}\right) \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right)$$

Solver

$$x_u := \mathbf{find}\left(x_u\right) = 0.283\ mm$$

$$x_2 := x_u$$

Calculating new z-value

$$k := \frac{\varepsilon_{ULS}}{\varepsilon_{cu}} \cdot x_u = 1.615\ mm$$

Calculating new x-value

Guess Values

$$x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.lm} \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right) + \frac{1}{2}\ f_{Ftsm} \cdot \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u + \frac{1}{2}\ \left(f_{Ftsm} - f_{Ftu.lm}\right) \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right)$$

Solver

$$x_u := \mathbf{find}\left(x_u\right) = 0.04\ mm$$

$$x_3 := x_u$$

Calculating new z-value

$$k := \frac{\varepsilon_{ULS}}{\varepsilon_{cu}} \cdot x_u = 0.229\ mm$$

Calculating new x-value

Guess Values

$$x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_{Ftu.lm} \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right) + \frac{1}{2}\ f_{Ftsm} \cdot \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u + \frac{1}{2}\ \left(f_{Ftsm} - f_{Ftu.lm}\right) \cdot \left(k - \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u\right)$$

Solver

$$x_u := \mathbf{find}\left(x_u\right) = 0.006\ mm$$

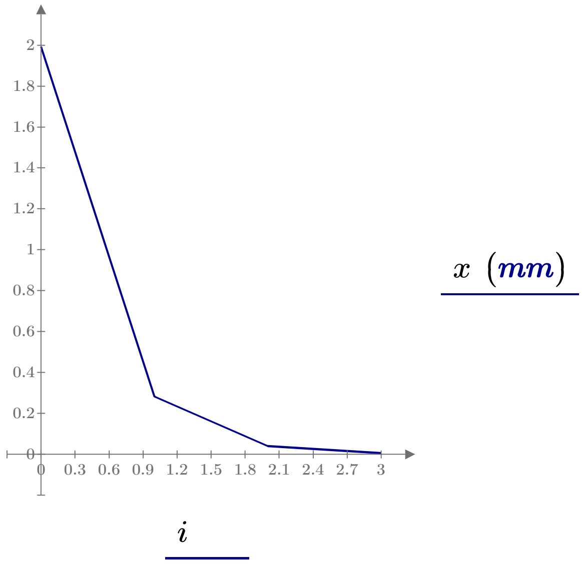
$$x_4 := x_u$$

$$k := \frac{\varepsilon_{ULS}}{\varepsilon_{cu}} \cdot x_u = 0.032\ mm \qquad z := \frac{\varepsilon_{SLS}}{\varepsilon_{cu}} \cdot x_u = 0.01\ mm$$

$$j := k - z = 0.023\ mm$$

Convergens studie

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.994 \\ 0.283 \\ 0.04 \\ 0.006 \end{bmatrix} mm \qquad i := \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$



The compressed concrete area is going to the limit zero

No force equilibrium was found. The compressed concrete zone converge to zero.

Moment capacity

$$m_c := f_{ck} \cdot \lambda \cdot x_u \cdot \left(x_u - \frac{\lambda \cdot x_u}{2} \right)$$

$$m_{Ftsm1} := f_{Ftsm} \cdot z \cdot \frac{1}{2} \cdot z \cdot \frac{2}{3}$$

$$m_{Ftsm2} := \left(f_{Ftsm} - f_{Ftu.lm} \right) \cdot \frac{j}{2} \left(\frac{j}{3} + z \right)$$

$$m_{Ftu} := f_{Ftu.lm} \cdot j \cdot \left(\frac{j}{2} + z \right)$$

$$m_{Rd.lm} := m_c + m_{Ftsm1} + m_{Ftsm2} + m_{Ftu} = 0.001\ \frac{N \cdot m}{m}$$

Moment due to self weight

Assuming equal load distribution in both direction

$R_{swf} = \left(2.158 \cdot 10^{3}\right)\ \boldsymbol{N}$ $\qquad$ $M_{swf} = 1.465\ \boldsymbol{kN \cdot m}$

Moment due to point load

$M_P = \dfrac{P \cdot l}{4}$ $\qquad$ $m_R \cdot l = M_{sf} + M_P$

Maximum point load

$$P := \left(m_{Rd.lm} \cdot l_s - M_{swf}\right) \cdot \frac{4}{l_2} \cdot \frac{1}{0.5} = -5.21 \cdot 10^{3}\ \boldsymbol{N}$$

Appendix C
Calculation of contribution fromm steel fibre reinforcement in FEM-Design

$f_{ck} := 45.52\ MPa$ Characteristic compression strength, value from lab resault

$\lambda := 0.8$

$f_{Ftu.pm} := 8.55467 \cdot 10^5\ Pa$ Ultimate residual strength

$f_y := 560\ MPa$ Yield strength steel reinforcement

$E_s := 200\ GPa$ Youngs modulus steel

$\varepsilon_{syk} := \frac{f_y}{E_s} = 2.8 \cdot 10^{-3}$ Yield strain steel

$h := 82.37\ mm$

$A_{sx} := 295\ \frac{mm^2}{m}$ Area reinforcement per meter , values from FEM-design

$A_{sy} := 146\ \frac{mm^2}{m}$

$c_x := 20\ mm$ Cover thickness reinforcement x-direction

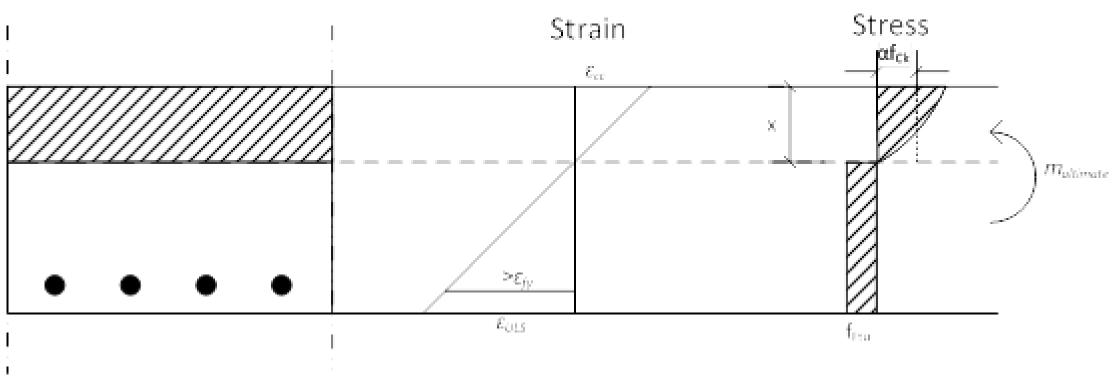
$c_y := 26\ mm$ Cover thickness reinforcement y-direction

$\phi := 6\ mm$ Diameter reinforcement

$d_x := h - \frac{\phi}{2} - c_x = 59.37\ mm$ Distance top slab to ceneter of reinforcement x-direction

$d_y := h - \frac{\phi}{2} - c_y = 53.37\ mm$ Distance top slab to ceneter of reinforcement y-direction

Assuming that failure occurs when the ultimate comressive strain for the concrete is reached



Calculating new reinforcement area

$$m_{Rdx.pm} := 1.203 \cdot 10^4 \ \frac{N \cdot m}{m}$$

$$m_{Rdy.pm} := 0.715 \cdot 10^4 \ \frac{N \cdot m}{m}$$

$\alpha_{x.u} := 0.6725497$ $\alpha_{y.u} := 0.5716360$

$\beta_{x.u} := 0.3761099$ $\beta_{y.u} := 0.3617709$

Guess Values

$$A_{sx} := 500 \ \frac{mm^2}{m} \qquad x_{x.u} := 10 \ mm$$

Constraints

$$\alpha_{x.u} \cdot f_{ck} \cdot x_{x.u} = f_y \cdot A_{sx}$$

$$m_{Rdx.pm} = \alpha_{x.u} \cdot f_{ck} \cdot x_{x.u} \cdot \left(x_{x.u} - \beta_{x.u} \cdot x_{x.u}\right) + f_y \cdot A_{sx} \cdot \left(d_x - x_{x.u}\right)$$

Solver

$$X := \text{find}\left(A_{sx}, x_{x.u}\right) = \begin{bmatrix} 3.784 \cdot 10^{-4} \\ 0.007 \end{bmatrix} m$$

$$A_{sx} := X(0) = 378.43 \ \frac{mm^2}{m}$$

$$x_{x.u} := X(1) = 6.922 \ mm$$

Guess Values

$$A_{sy} := 500 \ \frac{mm^2}{m} \qquad x_{y.u} := 10 \ mm$$

Constraints

$$\alpha_{y.u} \cdot f_{ck} \cdot x_{y.u} = f_y \cdot A_{sy}$$

$$m_{Rdy.pm} = \alpha_{y.u} \cdot f_{ck} \cdot x_{y.u} \cdot \left(x_{y.u} - \beta_{y.u} \cdot x_{y.u}\right) + f_y \cdot A_{sy} \cdot \left(d_y - x_{y.u}\right)$$

Solver

$$X := \text{find}\left(A_{sy}, x_{y.u}\right) = \begin{bmatrix} 2.482 \cdot 10^{-4} \\ 0.005 \end{bmatrix} m$$

$$A_{sy} := X(0) = 248.221\ \frac{mm^2}{m}$$

$$x_{y.u} := X(1) = 5.342\ mm$$

Calculating reinforcement for fibre reinforced concrete without conventional reinforcement

$$m_{Rd.pm} := \frac{f_{Ftu.pm} \cdot h^2}{2} = 2.902\ \frac{kN \cdot m}{m}$$

Guess Values

$$A_s := 500\ \frac{mm^2}{m} \qquad x_u := 10\ mm$$

Constraints

$$f_{ck} \cdot \lambda \cdot x_u = f_y \cdot A_s$$

$$m_{Rd.pm} = f_{ck} \cdot \lambda \cdot x_u \cdot \left(x_u - \frac{\lambda \cdot x_u}{2}\right) + f_y \cdot A_s \cdot (d_x - x_u)$$

Solver

$$X := \text{find}(A_s, x_u) = \begin{bmatrix} 6.558 \cdot 10^{-5} \\ 0.001 \end{bmatrix} m$$

$$A_s := X(0) = 88.092\ \frac{mm^2}{m}$$

$$x_u := X(1) = 1.355\ mm$$