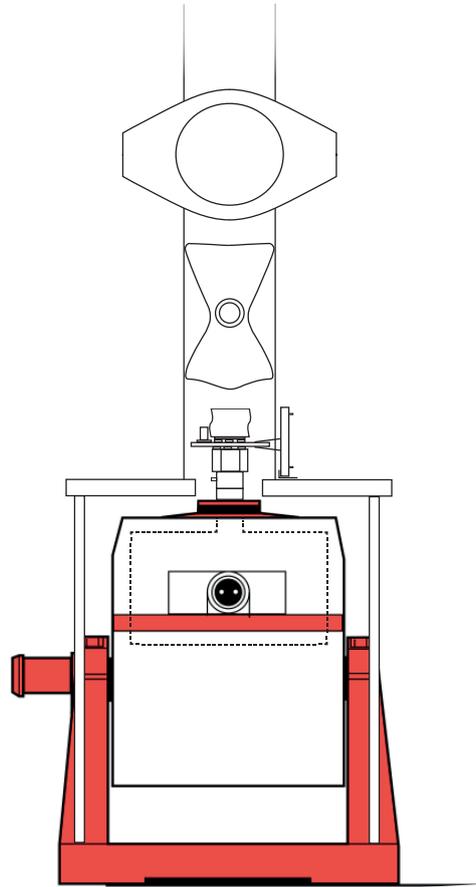


CHALMERS



Controlling a one degree of motion impact

Master's Thesis in the Master Degree programme, Sound and Vibration

ERIK NYSTRÖM

Department of Civil and Environmental Engineering
Division of Applied Acoustics
Vibroacoustics Research Group
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2013
Master's Thesis 2013:130

Controlling a one degree of motion Impact

© ERIK NYSTRÖM, 2013

Master's Thesis 2013:130

Department of Civil and Environmental Engineering
Division of Applied Acoustics
Vibroacoustics Group
Chalmers University of Technology
SE-41296 Göteborg
Sweden

Tel. +46-(0)31 772 2200

Reproservice / Department of Civil and Environmental Engineering
Göteborg, Sweden 2013

MASTER'S THESIS 2013:130

Controlling a one degree of motion impact

Erik Nyström

Department of Civil and Environmental Engineering

Division of Applied Acoustics

Vibroacoustics Research Group

CHALMERS UNIVERSITY OF TECHNOLOGY

Göteborg, Sweden 2013

Controlling a one degree of motion Impact
Master's Thesis in the Master's program in Sound and Vibration
ERIK NYSTRÖM
Department of Civil and Environmental Engineering
Division of Applied Acoustics
Vibroacoustics Research Group
Chalmers University of Technology

Abstract

The impact of two bodies is a source for noise and vibration and could cause material fatigue and breakdown. One way to reduce the impact noise and stress is to use active control. This thesis try to attenuate the energy in a one degree of freedom impact where one body is falling on another, using a basic control law.

First the impact situation has been modelled mathematically, and a control strategy developed using the Hertzian contact theory. Based on the result from simulations a physical experiment has been realised, where one elastic body is dropped on a piece of sylomer attached to a shaker. The shaker is equipped with sensors to register what is happening and then a feed back signal is fed to the shaker to control the impact.

The controller worked good in the sense of attenuating the energy of the fall, and almost all the energy of the falling body was attenuated after a couple of milliseconds. In the situation without any control it took about 0.3 seconds for the energy to die out. The result showed that the control-strategy used here worked, not perfectly but sufficient for this simple set up.

Contents

Abstract	iii
Contents	iv
Acknowledgements	vii
1 Introduction	1
2 Theory	3
2.1 Theory of Impact	3
2.2 Hertzian contact	3
2.3 Equation of motions	4
2.4 Control Strategies	6
3 Simulation	9
3.1 State equation formulation	9
3.2 Discretization	10
3.3 Implementation	10
3.4 Result Simulation	11
4 Experiment	13
4.1 Experiment Setup	13
4.2 Control Implementation	16
4.3 Callibration and Experiment	17
4.4 Result	18
5 Discussion	23
5.1 Further work, and what about the acoustics?	24
5.2 Conclusion	24
References	25

Acknowledgements

The assistance provided by Lars Hansson was greatly appreciated, without his help, the experiment would not be feasible during the limited time of this thesis. I would also like to thank Professor Wolfgang Kropp, my supervisor for his enthusiastic encouragement and useful tips, although he have been a very busy man. My grateful thanks are extended to the Börje Wijk, the technician of the devisions, for his help with equipment and for letting me borrow his tools.

Finally, I wish to thank my family for their support and encouragement throughout my study.

1 Introduction

Inspired by the silent and smooth walk of the family of feline this work is about to attenuate the impact between two bodies. An impact between two bodies can be a foot stamping on the floor, a basketball bouncing on the ground, a robotic arm grabbing a tool or a drumstick hitting the cymbal, and it is a source to sound, noise and vibration and material fatigue and breakdown. The aim for this thesis is to understand the physics of the impact and try to find a way to control it in a simple setup. The impact is modeled with two stiff bodies without any inherent wave propagation, with an elastic layer in between.

The simple setup we could think of was a single degree of freedom impact with one body falling on another, with one control force acting on the latter. The goal is to find a control law to catch the falling body without it bouncing away and to understand what is happening during this moment. The thesis is not about to try to reduce the sound of the impact, it is about to try to control the kinetic and potential energy and to attenuate and kill it as quick as possible. Previous work in this field has been done by for example Tornambè [Tor 96], he models and try to control a one degree of freedom impact, but rejects the control law used here because he claims that is impossible to know if the two bodies are in contact or not. For the setup in the experiment this problem of deciding if the bodies are in contact, or not, has not been a insurmountable problem.

The method to attack the problem is first, in Ch. 2, to describe the contact physics and the equation of motion mathematically, and then find a theoretic control law to control the impact. The controlled impact will then be simulated (Ch. 3) in a computer program were the conditions are ideal. When the control law has been validated in the simulations, a physical experiment will be set up were further validation will be made (in Ch. 4). The result are discussed in Ch. 5

2 Theory

2.1 Theory of Impact

The impact refers to the collision between two bodies. The time interval δ of the impact is often very short and under this short time interval a complex physical process takes place. Even a small change in the contact conditions may have big influence of the behaviour and the outcome after the collision. When the the two bodies collide they deform and the kinetic energy is transformed to potential energy in the deformation, to sound, and to heat. If the bodies are elastic they recover their shapes and the potential energy is transformed back to kinetic energy. If the two bodies are perfectly elastic with a coefficient of restitution of 1 all the energy and momentum prior to the collision is back in the system directly after the collision [Mer 03]. In the simulation in this thesis the approximation that the coefficient of restitution is 1 was done. The equations that describes the behaviour of the system before and after the collision looks like:

$$m_1 \dot{x}_1(t) + m_2 \dot{x}_2(t) = m_1 \dot{x}_1(t + \delta) + m_2 \dot{x}_2(t + \delta) \quad (2.1a)$$

$$m_1 \dot{x}_1(t)^2 + m_2 \dot{x}_2(t)^2 = m_1 \dot{x}_1(t + \delta)^2 + m_2 \dot{x}_2(t + \delta)^2 \quad (2.1b)$$

However if other forces such as a control force is acting during the collision these equations will not hold, and they does not give any information of what happens during the collision and the time of the impact, so they can not be used. Instead the more basic equations of motion that describes what happens before, during and after the impact has to be derived and a model that describes the contact force has to be found.

2.2 Hertzian contact

The contact behavior is very complex and it is necessary to find a simplified model. The contact model has to take in consideration that the contact area increases with increasing load. The Hertzian contact model predicts how the contact area grows and how the contact force changes with the deformation. The measurement of deformation is modeled as a penetration depth. The body can be looked as a composition of two objects: One stiff with a mass concentrated at the mass centrum, and one flexible without

mass acting as a Hertzian spring. The Hertzian contact force looks like this from [Abr 85]:

$$F_H(t) = cd(t)^{\frac{3}{2}} \quad (2.2)$$

$d(t)$ is the penetration depth and c is contact stiffness that is described by the following equation

$$c = \frac{4}{3}\sqrt{R} \cdot E^* \quad (2.3)$$

where R is the effective radius of the two bodies and E^* is the effective stiffness

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (2.4a)$$

$$\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \quad (2.4b)$$

ν_1 and ν_2 is the Poission's ratio of the spheres with radii R_1 and R_2 .

The Herzian contact model is derived under some assumptions, [Abr 85], with the significant parameters: The contact area a , the effective radius R , the radius of each body R_1 and R_2 , the dimensions of the bodies both lateral and in depth l

1. The surfaces are continuous and non-conforming: $a \ll R$
2. The strains are small: $a \ll R$
3. Each solid can be considered as an elastic half-space: $a \ll R_{1,2}$ and $a \ll l$
4. The surfaces are frictionless

2.3 Equation of motions

Figure 2.1 shows the mechanical system during impact. Body 1 with mass m_1 is the one that falls, and body 2 with mass m_2 is the one attached to the shaker. The force f_1 is the gravitational force and is not controllable. The force f_2 is the force from the shaker and is controllable. $\Phi(\alpha)$ is the function for the reaction force at the contact surfaces, depending on the penetration depth $x_1(t) - x_2(t) + r$, where r is the sum of the radius of body 1 and body 2.

$$r = r_1 + r_2 \quad (2.5)$$

The reaction force function Φ is based on the Hertzian contact model discussed earlier in the text.

There are two different cases for the mechanical system.

1. Body 1 in contact with body 2

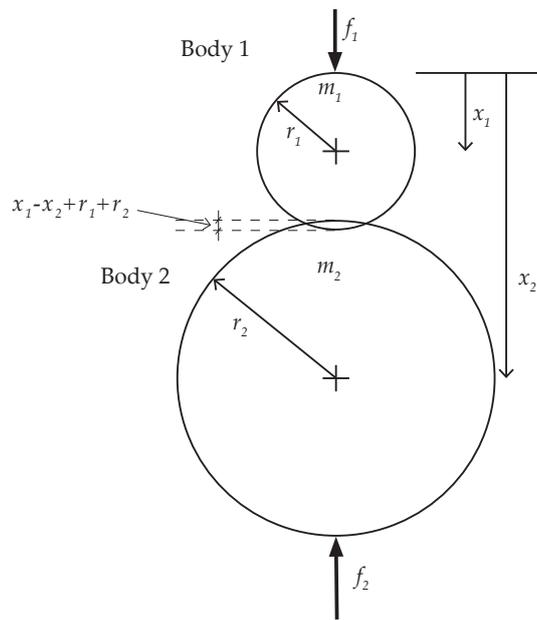


Figure 2.1: Mechanical system

2. Body 1 not in contact with body 2

The condition that decides if the bodies are in contact is:

$$x_1 - x_2 + r \geq 0 \quad (2.6)$$

When the bodies are in contact the equation of motion for the system is:

$$m_1 \ddot{x}_1(t) = -\Phi(x_1(t) - x_2(t) + r) + f_1(t) \quad (2.7a)$$

$$m_2 \ddot{x}_2(t) = \Phi(x_1(t) - x_2(t) + r) - f_2(t) \quad (2.7b)$$

When the bodies are not in contact the equation of motion looks like this:

$$m_1 \ddot{x}_1(t) = f_1(t) \quad (2.8a)$$

$$m_2 \ddot{x}_2(t) = -f_2(t) \quad (2.8b)$$

2.4 Control Strategies

The control force is designed to take control of the falling body with the body attached to the shaker and to attenuate all the energy in the system as quick as possible.

The first step is to extinguish the potential energy in the spring behaviour between the two bodies. To find the control law to achieve this the equation of motion is rewritten with the new variables $y_1 = x_1 - x_2 + r$ to isolate the penetration depth and the potential energy, and $y_2 = x_2$. With this substitution the new equations of motion becomes for the contact case (with $f_1 = m_1 g$):

$$\ddot{y}_1 = -\Phi(y_1) \frac{m_1 + m_2}{m_1 m_2} + g + \frac{f_2}{m_2} \quad (2.9a)$$

$$\ddot{y}_2 = \frac{\Phi(y_1)}{m_2} - \frac{f_2}{m_2} \quad (2.9b)$$

and for the non contact case:

$$\ddot{y}_1 = g + \frac{f_2}{m_2} \quad (2.10a)$$

$$\ddot{y}_2 = -\frac{f_2}{m_2} \quad (2.10b)$$

In the state when the potential contact energy is zero, the contact force $\Phi(y_1)$ is equal to m_1g . y_1 is then the steady state penetration and if the contact behavior is known it can be calculated to a static depth d :

$$d = \left(\frac{m_1g}{c} \right)^{\frac{2}{3}}, \quad (2.11)$$

The first step is to get to this state. We write the reference signal to be tracked as:

$$y_R = d, \quad \dot{y}_R = \ddot{y}_R = 0$$

If the control force is like this when in contact:

$$\frac{f_2}{m_2} = \Phi(y_1) \frac{m_1 + m_2}{m_1 m_2} - g + \ddot{y}_R - h_{11}(\dot{y}_1 - \dot{y}_R) - h_{01}(y_1 - y_R) \quad (2.12)$$

And like this when not in contact:

$$\frac{f_2}{m_2} = -g - \ddot{y}_R - h_{11}(\dot{y}_R - \dot{y}_1) - h_{01}(y_R - y_1) \quad (2.13)$$

The equations 2.9a and 2.10a can be rewritten as:

$$\ddot{y}_1 - \ddot{y}_R + h_{11}(\dot{y}_1 - \dot{y}_R) + h_{01}(y_1 - y_R) = 0 \quad (2.14)$$

We rewrite the tracking error $y_1 - y_R$ as \hat{y} and the equation above becomes:

$$\ddot{\hat{y}} + h_{11}\dot{\hat{y}} + h_{01}\hat{y} = 0 \quad (2.15)$$

If the coefficient h_{11} and h_{01} is chosen correctly the homogeneous equation will converge to zero. How quickly the equation converge is determined by those coefficients.

We can only control the relative movement of body 1 and 2 with this force, not how body 2 moves, but when \hat{y} has converged to zero equation 2.9b becomes:

$$\ddot{y}_2 = \frac{\Phi(d)}{m_2} - \Phi(d) \frac{m_1 + m_2}{m_1 m_2} + g = \frac{\Phi(d)}{m_2} - \frac{\Phi(d)}{m_2} - \frac{\Phi(d)}{m_1} + g = 0 \quad (2.16)$$

The acceleration of body 2 are zero but, we do not know anything about the velocity or the position. We can now add a second condition that moves body 2 back to its initial position with a slower convergence rate than eq. 2.15. The control force would then look like this when the two bodies are in contact:

$$f_2 = m_2(\Phi(y_1) \frac{m_1 + m_2}{m_1 m_2} - g + \ddot{y}_R - h_{11}(\dot{y}_1 - \dot{y}_R) - h_{01}(y_1 - y_R)) - h_{12}\dot{y}_2 - h_{02}y_2 \quad (2.17)$$

and like this when they are not in contact:

$$f_2 = m_2(-g - \ddot{y}_R - h_{11}(\dot{y}_R - \dot{y}_1) - h_{01}(y_R - y_1)) - h_{12}\dot{y}_2 - h_{02}y_2 \quad (2.18)$$

and the equation of motion would become:

$$\ddot{\hat{y}} + h_{11}\dot{\hat{y}} + h_{01}\hat{y} + h_{12}\dot{y}_2 + h_{02}y_2 = 0 \quad (2.19)$$

This control will make the potential energy in the contact go to zero, and move body 2 followed by body 1 to its initial position if the coefficients h_{01} , h_{11} , h_{02} , h_{12} are chosen correctly.

3 Simulation

3.1 State equation formulation

(NOTE: y here is not the same as the y in the control law derivation)

$$y_1 = x_1 \quad (3.1a)$$

$$y_2 = \dot{x}_1 \quad (3.1b)$$

$$y_3 = x_2 \quad (3.1c)$$

$$y_4 = \dot{x}_2 \quad (3.1d)$$

The equations of motion, 2.7, is formulated as state equations. For simplicity the reference points x_1 and x_2 are shifted from the center of the bodies to the surfaces close to each other, this makes the constant distance r disappear. To write the equations on matrix form the contact force $c(y_1 - y_2)^{3/2}$ has to be written as $(c\sqrt{y_1 - y_3})(y_1 - y_3)$ with $c\sqrt{y_1 - y_3} = \sigma(t)$ inside the matrix and y_1 and y_3 on the outside. f_2 is the control force described by equation 2.17 and 2.18, and f_1 is m_1g

The equation of motion is written for the contact case as:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{\sigma(t)}{m_1} & 0 & \frac{\sigma(t)}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\sigma(t)}{m_2} & 0 & -\frac{\sigma(t)}{m_2} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2(t) \end{bmatrix} \quad (3.2)$$

and for the non contact case the state equations look like

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2(t) \end{bmatrix} \quad (3.3)$$

In compact form equation 3.2 and 3.3 is written as:

$$\dot{\mathbf{y}}(t) = \mathbf{A}(t)\mathbf{y}(t) + \mathbf{B}\mathbf{f}(t) \quad (3.4)$$

3.2 Discretization

To solve this with Matlab the simple approximation of the derivative, is used:

$$\dot{\mathbf{y}}(t) = \lim_{T \rightarrow 0} \frac{\mathbf{y}(t+T) - \mathbf{y}(t)}{T} \quad (3.5)$$

which give the solution in the next step as:

$$\mathbf{y}(t+T) = \mathbf{y}(t) + \dot{\mathbf{y}}T \quad (3.6)$$

The combination of equation 3.4 and 3.6 gives us the solution of the equation of motion in the next step as

$$\mathbf{y}(t+T) = \mathbf{y}(t) + T[\mathbf{A}(t)\mathbf{y}(t) + \mathbf{B}\mathbf{f}(t)] \quad (3.7)$$

With $t = nT, n = 0, 1, 2, \dots$ and T the sample time, the equation becomes:

$$\mathbf{y}[(n+1)T] = [\mathbf{I} + T\mathbf{A}(nT)]\mathbf{y}(nT) + T\mathbf{B}\mathbf{f}(nT) \quad (3.8)$$

This equation is easy to solve in an iterative way with a computer.

3.3 Implementation

The tricky part was to find good time parameters $h_{01}, h_{11}, h_{02}, h_{12}$. It is hard to do an optimisation calculation to find the best parameter so instead different values around a first educated guess was tried out in a nested for loop. The parameter that controls the potential energy is h_{01} and for taking control over the mechanical system this parameter is the most important one and should have the highest value and in the order of contact stiffness coefficient c . Many sets of time parameters gave good result but one set with low control force and short convergence time had the following values:

- $h_{01} = 3 \cdot 10^6$
- $h_{11} = 400$
- $h_{02} = -3 \cdot 10^4$
- $h_{12} = -1000$

3.4 Result Simulation

In the figure below (figure 3.2) the behaviour of the system, when controlled by the force derived in equation 2.17 and 2.18 with the time parameters in table 3.3, can be seen. The time starts at $T = 0$ when the distance between the two bodies is zero. The controller beautifully stops the momentum of the falling body 1. It makes it still after around 50 ms (figure 3.2 (b)) and the bodies are in contact all the time. Figure 3.2 (a) shows how the energy changes during the first 7 ms. In this figure it can be seen that the controller cancels out the potential contact energy already after 2 ms and after this the bodies follow each other at the constant tracking distance d calculated in equation 2.11. Figure 3.2 (c) shows how the velocity of the two bodies changes over time. Body 2 controlled by the shaker accelerate as quick as possible up to the speed of the falling body 1. When they have the same speed the shaker stops the movement. Figure 3.2 (d) shows how the displacement of the bodies changes over time. Body 2 moves down to intercept the falling body 1 and then it moves back to its initial position. Figure 3.2 (e) shows the time response of the force. It can be seen that the force response has to be very quick and this is going to be the big challenge in the experiment later. Figure 3.2 (f) shows the frequency response of the force. The most important information is in the range from 20 Hz to 1000 Hz. In figure 3.1 it can be seen how the acceleration of body 1 change when the control is switched off compared to when it is on.

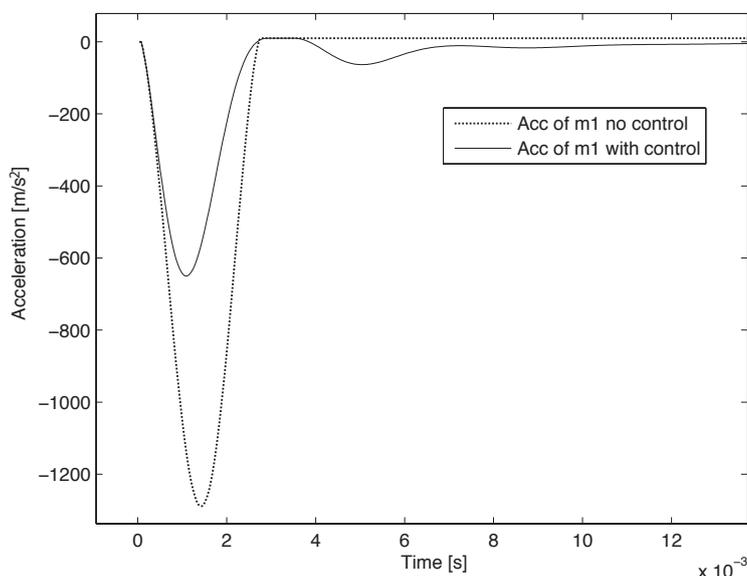


Figure 3.1: The figure shows a comparison between the acceleration of body 1 with control and without control

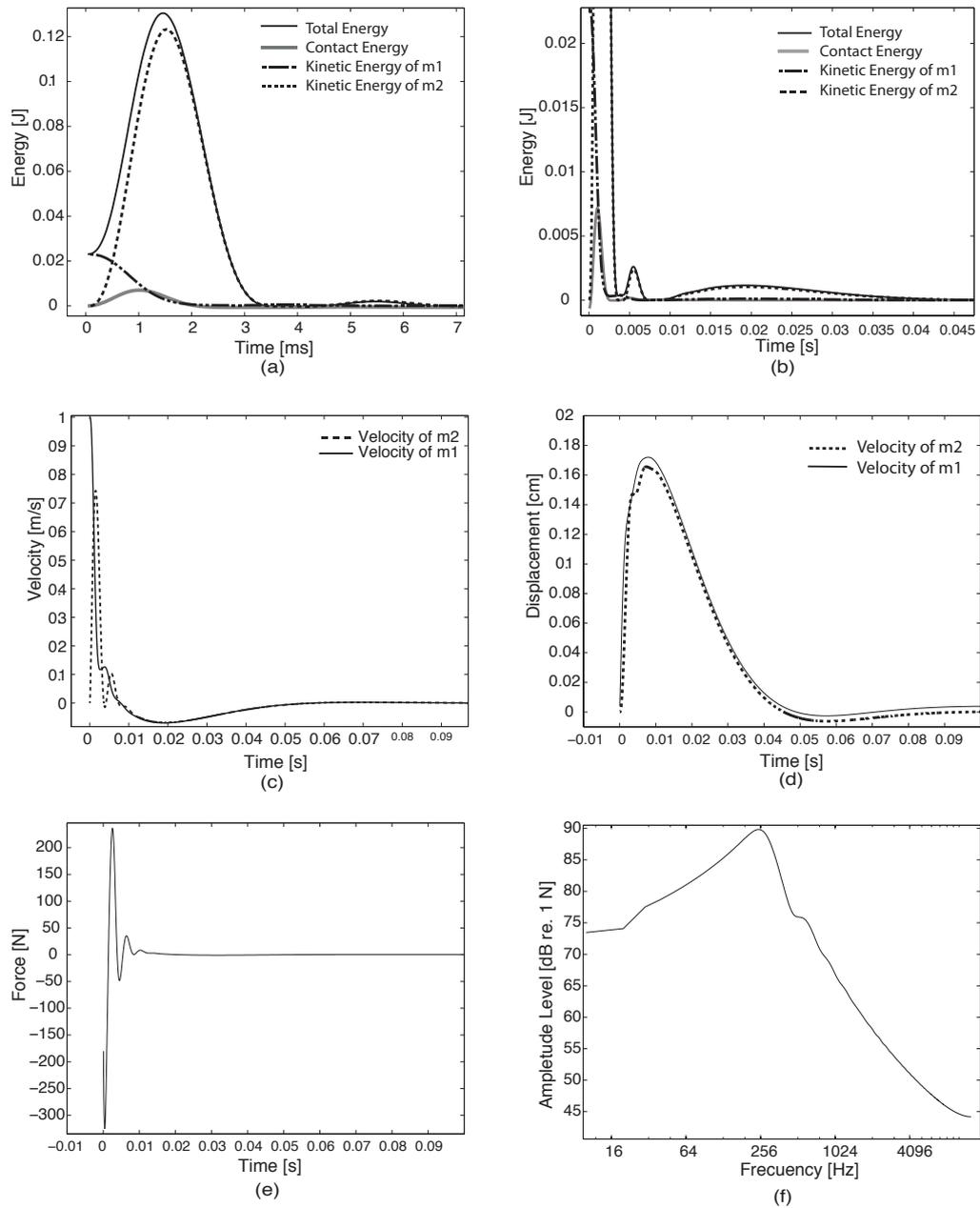


Figure 3.2: Figure a, b, c, d and e shows the behavior of the system when controlled by the force derived in equation 2.17 and 2.18 with the time parameters in table 3.3

4 Experiment

4.1 Experiment Setup

To see if the control strategy would work in reality an experiment was set up. To get the most basic one degree of freedom movement of the bodies the falling body 1 was dropped in a tube of plexiglass. The shaker that was used was a LDS V406/8 PA 100E permanent magnet shaker and amplifier with a peak sine force of 98 Newton and a frequency range from 5 to 9000 Hz. To drop body 1 from the same height every time an electro magnet drop relay was constructed. Body 1 had magnets on the sides.

Figure 4.1 is showing the setup of the experiment. A is a switchable electromagnet that drops B, body 1 in figure 2.1. C is a piece of sylomer that is the face of body 2 in figure 2.1. Different materials were tried with requirement of being soft and have low damping. The best compromise was the sylomer even though the damping was quite high. The coefficient of restitution was not measured but was lower than the simulated value of 1. D is an accelerometer, E is a force transducer and F is a potentiometer and these are used to observe the system. With knowledge of the mass m_2 of body 2, the contact stiffness c and the information from the sensors, all the parameters that describes the system can be calculated when the two bodies are in contact. When the bodies are not in contact the mass of body 1 must also be known. The signals from the sensors is fed via amplifiers to a D-space control unit which is loaded with a program written in Simulink. Calculations are made according to the control laws of equations 2.17 and 2.18. A force is fed back to body 2 via a power amplifier and the shaker G. The schematic signal flow can be seen in figure 4.2

Because of the geometry of the sylomer, the assumptions made using the Hertzian contact model was not followed strictly so the calculated stiffness would probably be incorrect, therefore the stiffness was also measured using the set up in figure 4.3. The acceleration of the falling body was measured from the time t_0 when the body 1 hit the sylomer to the time t_1 when maximum deflection occurred and the contact stiffness c was calculated from equation 2.2 according to:

$$c = \frac{a_1 m_1}{\Delta x^{\frac{3}{2}}} \quad (4.1)$$

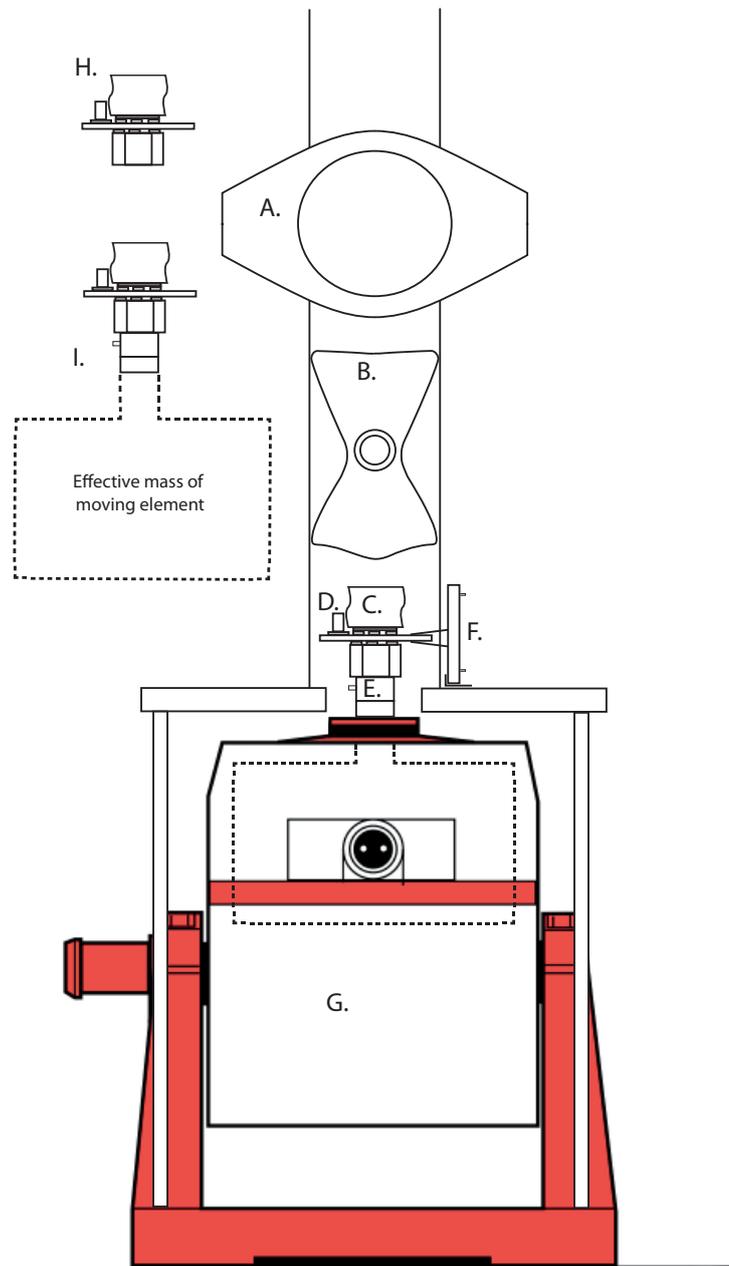


Figure 4.1: Setup of the experiment.

with

$$\Delta x = \int_{t_0}^{t_1} \int_{t_0}^{t_1} a_1(t) dt dt \quad (4.2)$$

The stiffness was measured and then calculated to be $2 \cdot 10^6 \left[\frac{N}{m^{3/2}} \right]$

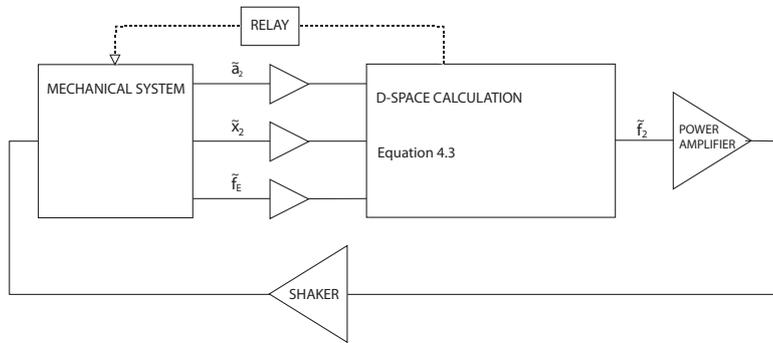


Figure 4.2: Schematic pictures of the setup.

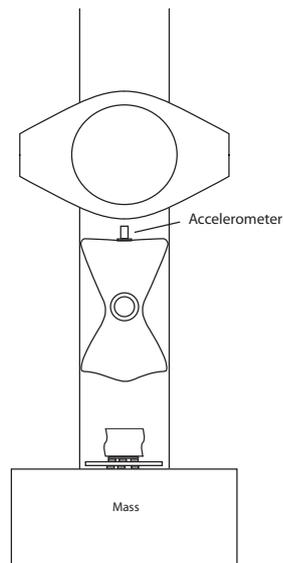


Figure 4.3: The setup for measuring the contact stiffness.

4.2 Control Implementation

The feedback control was implemented with a Simulink program that was loaded into the D-space unit. The Simulink program has been developed with starting point from the Matlab program. The equation for the control force was described by:

$$f_2 = m_2(c(\tilde{x}_1 - \tilde{x}_2)^{3/2} \frac{m_1 + m_2}{m_1 m_2} - g - h_{11}(\tilde{x}_1 - \tilde{x}_2) - h_{01}(\tilde{x}_1 - \tilde{x}_2 - d)) \quad (4.3)$$

with \tilde{x} being the approximation of x . (h_{12} and h_{02} were set to zero because the controller worked only in the contact state and the shaker find back to its rest position during the non contact period because of the suspension).

The first challenges was to decide whether the bodies was in contact or not. This was solved by using the information of the force from transducer E. and the acceleration from accelerometer D. using equation 2.7b to write the a condition:

$$m'_2 \ddot{x}_2 + f_2 > 0 \quad (4.4)$$

If equation 4.4 is true then the bodies are in contact. m'_2 is the part of mass m_2 on top of the force transducer (H in figure 4.1). To get this condition stronger and faster the derivative of $m'_2 \ddot{x}_2 + f_2$ was taken. This made the edge in the moments of transition from non contact to contact and from contact to non contact very steep and easy to distinguish. The contact conditions were set so when the bodies was in the rest mode, when bodie 1 rested on body 2 it registered non contact, and then the rest penetration d was set to zero.

A big difficulty was that the contact event happened during a very short amount of time and only once the same way and the actuators had to response immediatly so no averaging was possible, and this made the program very sensitive to the noise in the signals from the sensors. To clean up the signals from the noise, filters were used, the filters though added time delays to the signals. So there has to be a compromise between noise and time delays. The biggest problem with this delay and the noise was that it was impossible to get the information of the speed of body 1 in the transition from the contact to the non contact state which was necessary for the determination of the position and velocity of body 1, so the description of the mechanical system in the non contact state was not correct. The controller could therefore only operate during the time when the two bodies were in contact with each other.

The response of the shaker and power amplifier was not linear at all, and that colored the signal that were fed back to the system. This made the controller unusable and had to be taken care of, so the response of the shaker and amplifier was measured and an

approximate inverse filter was designed. The response of the shaker and amplifier with the inverse filter applied was measured again, and this new conditions was implemented in the Matlab program to find good time constants $h_{i,j}$.

4.3 Calibration and Experiment

The shaker had a limited force output (98 N in the data sheet for sine peak, but when tested for impulses it could put out approximately 200 N depending on the duration of the pulses) so the time parameters had to be optimized with this in consideration. To get the Matlab simulation as close as possible to real mechanical system, the force signals inside the Matlab program were convolved with the response of the inverse filtered amplifier/shaker. The Matlab controller was also changed so it only operated during the contact time. This made that the time parameters used in the ideal simulation in figure 3.2 did not work.

Different time parameters were tried out in a loop. The result was plotted in a three dimensional plot with the force on the y-axis, the speed of body 1 in the moment when the bodies left each other the first time on the x-axis, and a index for the time parameters on the z-axis. Figure 4.4 shows the result. The span of the time parameters that were tested was h_{01} from 4 to 1000, and h_{11} from $1 \cdot 10^6$ to $3 \cdot 10^6$ From the figure it can be seen that it is possible to stop the falling body totally with many sets of parameter, but then the control force would be higher than allowed by the shaker. With a control force around 200 N it is possible to theoretically lower the speed to 30% of the speed before the collision moment. In practice the material also have quiet high damping so the speed would than be lower than 30 %.

The controller was then updated with new values of h_{01} and h_{11} and body 1 was dropped. To get a better overview how the control law worked in reality and how well it matched the simulation the Simulink program was modified with a part that automatically dropped body 1, bounced it back to the electromagnet, changed the values of the time parameters and dropped it again. In this way different parameters could be tested automatically. The events was recorded both with a acquisition station and with a video camera for later evaluation. When a good set of values for h_{01} and h_{11} was found, the sequence with these values was recorded with a high speed camera.

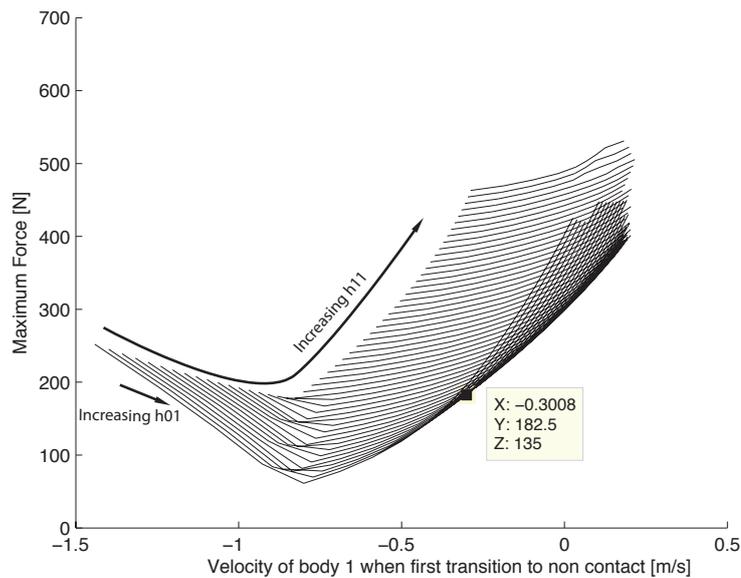


Figure 4.4: Time parameters

4.4 Result

In figure 4.5 (a, b, c, d, e and f) the control force from the simulations is compared with the measured control force from the experiment. The pictures are showing the force during the first impact for three different sets of time parameter, the parameters are:

- (a) and (d): $h_{01} = 1.2 \cdot 10^6$ and $h_{11} = 18$
- (b) and (e): $h_{01} = 1.8 \cdot 10^6$ and $h_{11} = 18$
- (c) and (f): $h_{01} = 2.3 \cdot 10^6$ and $h_{11} = 18$

The control force has the same shape when comparing the experiment with the simulation for the different pairs of values for the time parameters, and the duration is also approximately the same, but the measured force amplitude is about 30 % lower in the experiment. The parameters in figure 4.5 (a) and (b) did not work any good as it did not converge. In the experiment the controller made body 1 bounce uncontrollable, and the simulation showed that the velocity of body 1 became higher and higher after every bounce which can be seen in figure 4.7. This divergence was not spotted in the experiment because the damping of the sylomer was high and because of the threshold for the contact condition and the force output limit of the shaker. The parameters used in figure 4.5 (b) and (e) worked better and after a couple of bounces the bodies ended in a rested state. With

the parameters in figure 4.5 (c) and (f) the controller captured and stopped the falling body very quickly, for the eyes it looked like it did it during the first contact event. In the simulation, see figure 4.6, it can be seen that the speed of body 1 slowed down to about 5 % of the falling speed after 2 bounces and then the conditions stabilises.

Figure 4.8 shows how the energy changes over time. In Figure 4.8 (a) and (b) a comparison is made for the experiment when the control law is applied and when it is not applied. In 4.8 (a) the comparison is made during a longer time, and in 4.8 (b) the comparison is made during the time when the program registered contact. The speed of body 1 could only be measured during contact so how the energy would look when it is not in contact is not showed. The plots are showing that the energy of body 1 dies out quickly when the controller is switched on. In the figure it looks like it stops completely during the contact time, but in the video from the high speed camera is possible to see that it gets a little speed in the upward direction. The shaker adds relatively much energy to body 2 and the force from the shaker drives this body downwards. It look almost like body 1 is having the same mass as body 2 and that it is the kinetic energy of the falling body that hits the second body down, but in reality the mass of body 1 is $\frac{1}{10}$ of the mass of body 2 so the control force is fooling the eyes. The suspension brings body 2 back and the two bodies meet again a settle. The total energy is totally attenuated after 0.1 seconds when the controller is on compared to about 0.35 seconds without control. The time 0.1 seconds is a little bit unfair because the most part of the energy is in the resonance of the body 2/shaker setup. If only the energy of the falling body would be measured the extinct time would have been shorter and most part of the energy of body 1 is attenuated after a couple of milliseconds. In figure 4.8 (b) it can be seen that when the controller is on, the contact time is much shorter and that the contact tension holds less energy then if the controller is switched of. The energy of body 1 is calculated with help of the time T between the bounces according to this formula:

$$E_1 = \frac{g^2 T^2 m_1}{8} \quad (4.5)$$

And during the bounces it is linearly interpolated. g is the gravitational constant.

Figure 4.8 (c) shows how the energy is divided during the contact time when the controller is on. Figure 4.8 (d) is the same plot as figure 3.2 (a) and shows how the energy changes over time when the controller works with ideal conditions in the simulation. The energy in the contact looks similar, but for the kinetic energy of body 1 it look like the energy in the experiment dies out faster than in the simulation.

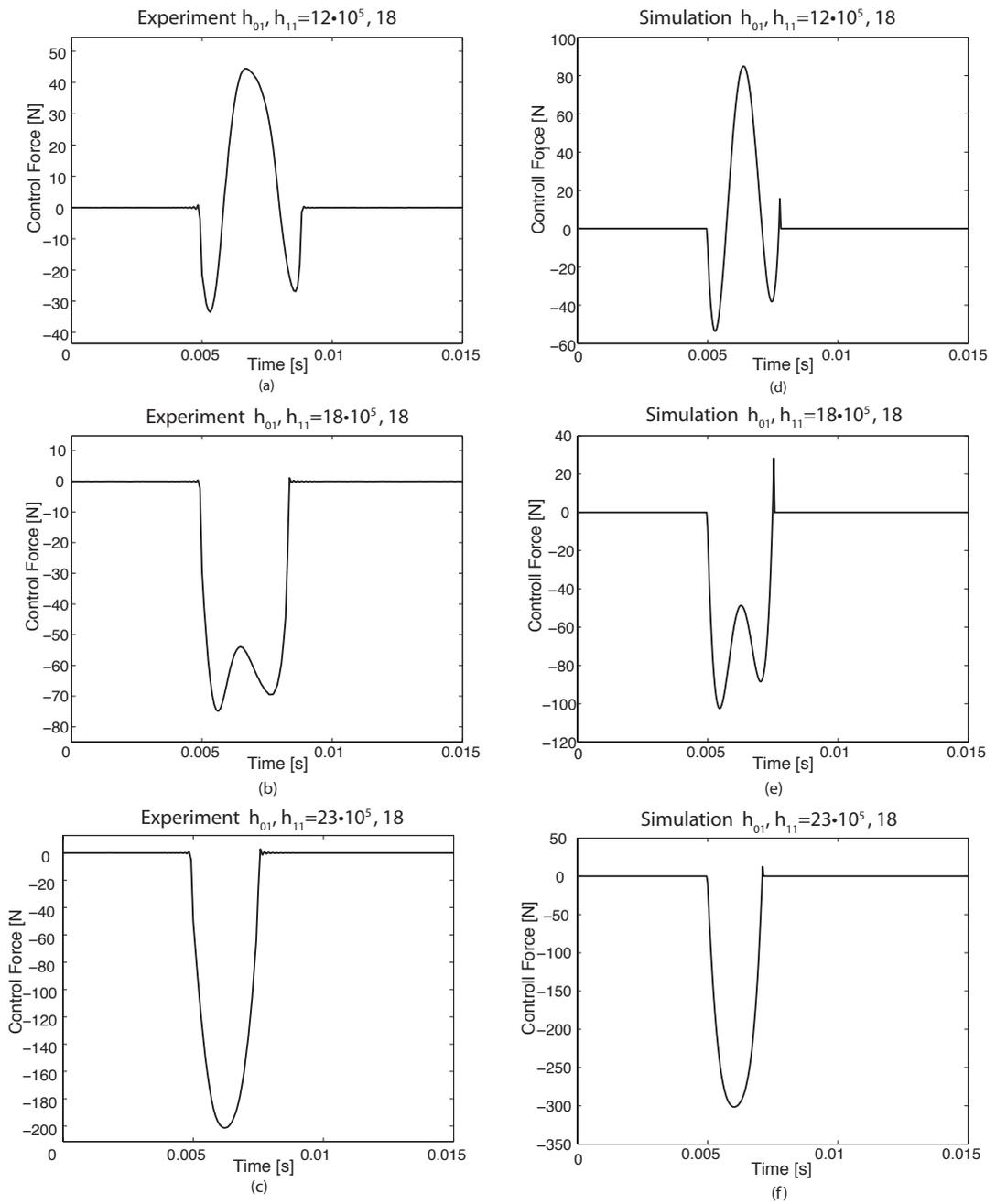


Figure 4.5: Experiment and Simulation (simulation filtered with response of mechanical system). The three different sets of time parameters are from the table in chapter 4.4.

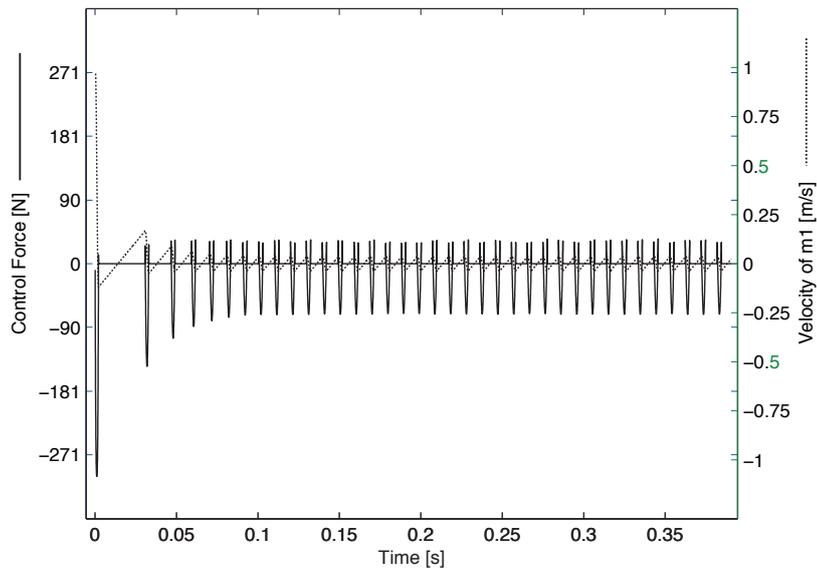


Figure 4.6: Simulation with one set of parameters in which gave good result: $h_{11}, h_{01} = 18, 23e5$)

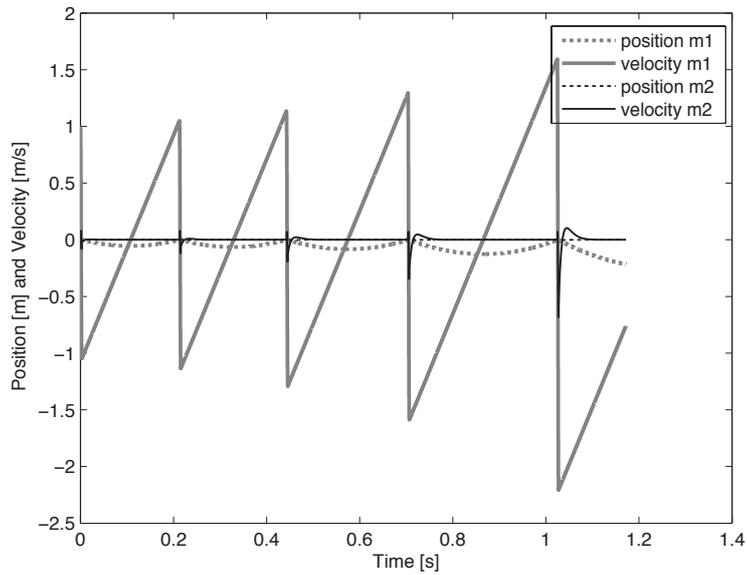


Figure 4.7: Simulation with one set of parameters which diverge, $h_{11} = 18$ and $h_{01} = 1.2 \cdot 10^6$. Body 1 bounce higher and higher for every jump.

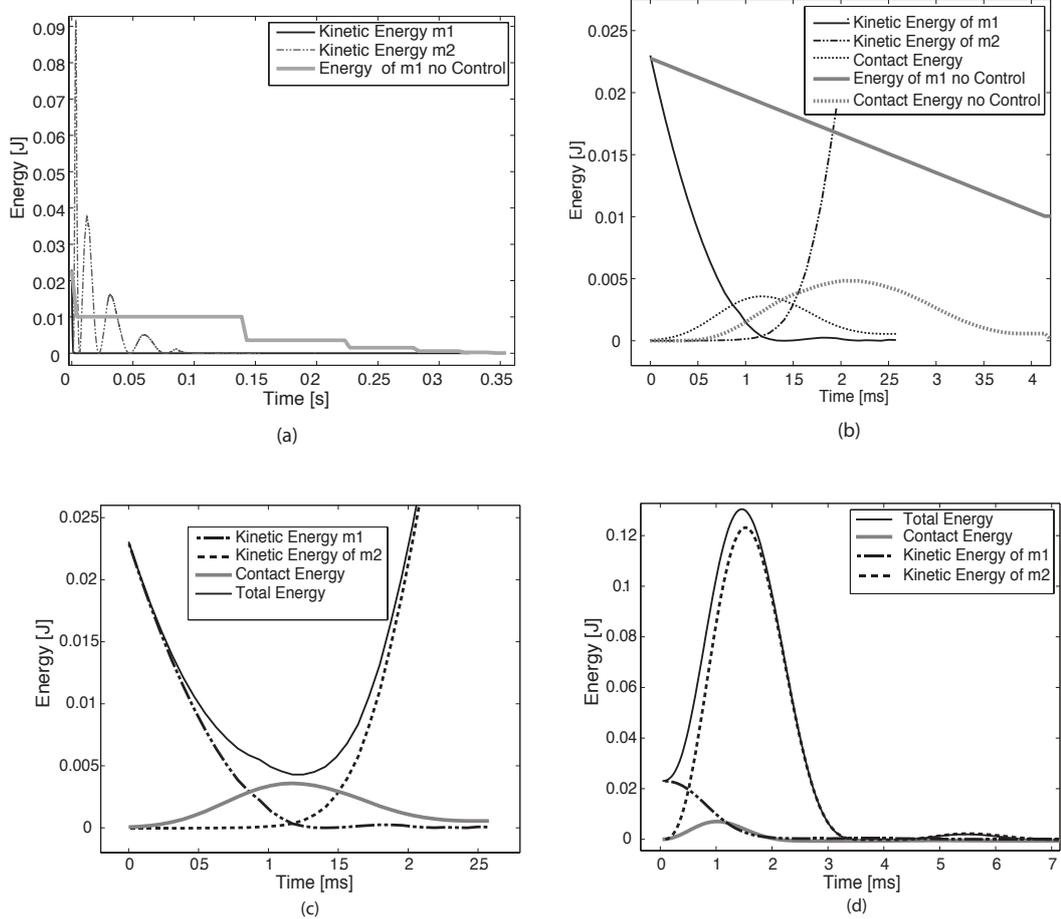


Figure 4.8: The figures a and b shows how the energy is divided in the experiment and comparisons are made between when the control is switched on respective when it is switched off. Figure c shows the energy behaviour during the time when the controller registered contact and figure d shows how a ideal simulated system could work .

5 Discussion

The controller was designed to take control over the impact and stop a falling body as quickly as possible and prevent it to bounce away, almost like a catcher in baseball but without any grabbing. The result from the experiment showed that the controller did not work as perfect as it did in the simulation. The controller made it stop much quicker than if no control was acting, but the two bodies was not in contact all the time, which was intended. To control the impact the system has to be fast. The catching body (body 2) has to accelerate to have a speed so it compensates for the contact tension release between the two bodies so it is not throwing away the falling body. The shaker was loaded with a quite heavy mass which demanded a high force from it to quickly be able to follow the movement for control. Maybe with the right time parameters this force could be within the limits of the shaker but the heavy mass made also so that the system had a long impulse response.

It was also problematic with the sensor because the sensor that was used was designed for vibrations and not for short impacts. Any averaging was impossible because of the single events. The signals was polluted with noise and filtering them was necessary. The control signals had its most important information in the frequency range of 20 to 1000 Hz so it was possible to bandpass the other parts away, but the filtering made the time response slower. It was therefore a delicate balance act to find the best filter parameters for good clean signals versus fast response. This fine tuning was time consuming but at last good filters was found that made the controller work sufficiently good.

If the exit speed of body 1 when it left body 2 could be decided the control could have worked better and body 2 could have "glued" better on body 1. This speed could maybe have been found if it had been approximated from a typical curve of how body 1 behaved from certain parameters. If the problem had been solved in this way the controller maybe had been less general. The controller should be able to dampen the fall in many situation without any tuning and with as little information as possible. The falling body could not be too heavy and not fall from a too high height because then the force should be higher than the shaker could handle, but because the sylomer was soft the information of the stiffness of body 1 was not relevant because all compression occurred in the sylomer. Therefore many bodies could have dropped upon the controller and still be controlled. Tornamb talks in [Tor 96] that the conditions whether the bodies were in contact or not could be hard to distinguish but in this case

it worked rather good. Maybe it would be much harder if a force not as simple as the gravitational force would have act upon the incoming body like the case with a tyre road contact would be.

5.1 Further work, and what about the acoustics?

The next step would be to try to set up an experiment with something that has a much faster and flatter response than the shaker. Maybe a stack of piezo electric disks that could change shape almost immediately would do the trick. Then the controller would have much easier to follow the desired curve and the stack would also probably have much weaker resonances. With a setup with piezo disks the controller could probably prevent the incoming body to bounce away and stop it immediately. To solve the problem with how to minimise the impact noise it is still much work to do. Most of the sound is coming from the sudden acceleration during the impact [Hoe 08]. The first step on the way to implement this in the tyre/road situation would probably be to imitate how something that rolls interact with a surface and then find the equation of motion for this situation and modify the control law.

5.2 Conclusion

The controller worked sufficiently good to attenuate the impact between the two bodies in this one degree of freedom setup. After one small bounce the falling body rested on the body attached to the shaker.

References

- [Tor 96] Tornambè, A. : *Modelling and controlling one-degree-of-freedom impacts*, IEE Proc -Control Theory Appl, Vol 143, No I, January 1996
- [Ind 01] Indri, M.: *Impact Modelling and Control of Robotic Links*, S. Nicosia et al. (Eds.): RAMSETE, LNCIS 270, pp. 155-179, 2001
- [Mer 03] Meriam, J. L. and Kraige, L.G.: *Engineering Mechanics Dynamics Fifth Edition*, John Wiley & Sons, Inc. 2003
- [Abr 85] Johnson, K.L.: *Contact Mechanics*, Cambridge University Press 1985
- [Hoe 08] Hoever, C: *Active Structural Control of Impacts With a View Towards Noise Control*, Diplomarbeit Technischen Universität Berlin 2008