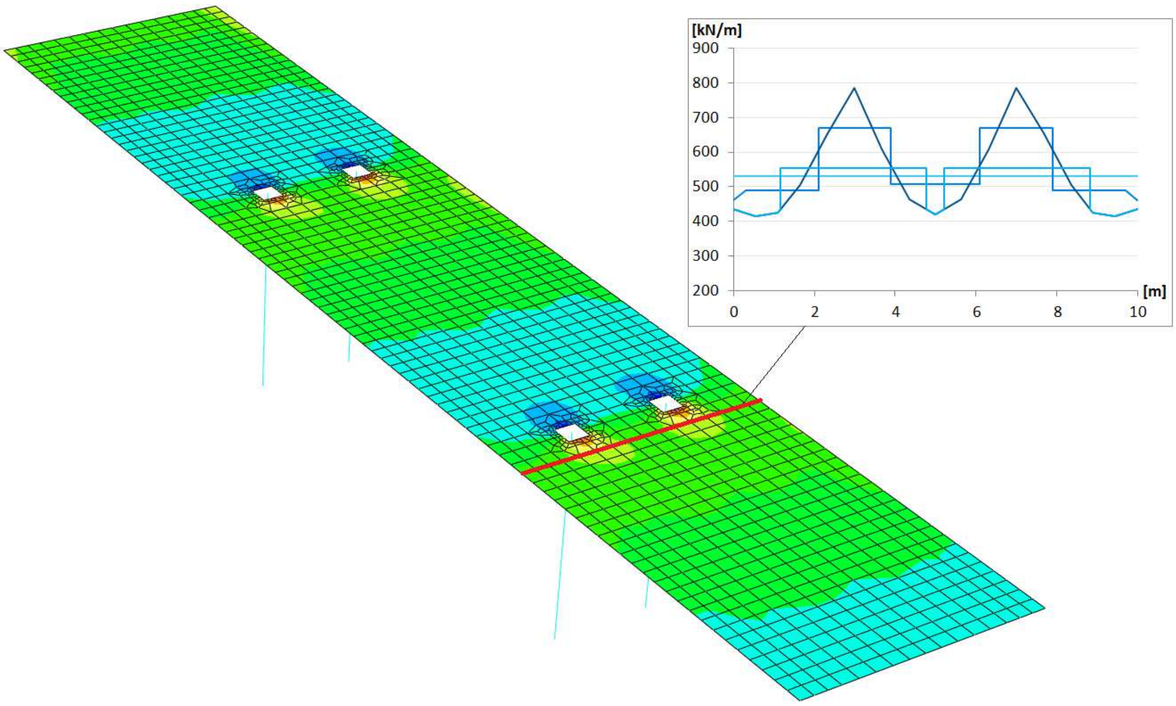
CHALMERS



Shear Force Design of Reinforced Concrete Bridge Decks

Design Choices and their Influence on the Final Design

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

FREDRIK DAVIDSSON

Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2014

Master's Thesis 2014:88

MASTER'S THESIS 2014:88

Shear Force Design of Reinforced Concrete Bridge Decks

Design Choices and their Influence on the Final Design

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

FREDRIK DAVIDSSON

Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
CHALMERS UNIVERSITY OF TECHNOLOGY

Göteborg, Sweden 2014

Shear Force Design of Reinforced Concrete Bridge Decks
Design Choices and their Influence on the Final Design

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

FREDRIK DAVIDSSON

Examensarbete / Institutionen för bygg- och miljöteknik,
Chalmers tekniska högskola 2014:88

Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

Cover:
Finite element model of a continuous slab bridge on column supports, displaying contour plot and post-processing of the maximum shear force envelope in the longitudinal direction.

Chalmers Reproservice, Göteborg, Sweden 2014

Shear Force Design of Reinforced Concrete Bridge Decks
Design Choices and their Influence on the Final Design

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

FREDRIK DAVIDSSON
Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
Chalmers University of Technology

ABSTRACT

The design of reinforced concrete bridge decks is currently mainly based on linear elastic finite element analyses. Since the load effects throughout a certain structure are sought for a great number of load combinations, this approach is suitable. There are, however, some uncertainties related to such an analysis. It is not very well investigated how the shear force design should be executed in order to obtain an optimized and reliable design.

The aim of this thesis has been to investigate how different shear force design choices influence the final design of concrete bridge decks. By designing a specific bridge through different approaches it was possible to highlight differences as well as similarities. Focus has been on comparison between different ways to account for the shear force and how to redistribute obtained peak shear forces, but also related uncertainties and questions have been investigated.

The results show that the differences in terms of the final reinforcement design are small. They also indicate that there is large potential in finding a way to accurately estimate the shear force resistance within the slab as a sum of the resistances provided by the concrete and the reinforcing steel, but current minimum reinforcement requirements prevent this potential to be utilized. These requirements do also eliminate the differences that are obtained when designing the bridge deck slab based on different approaches. This is limiting the potential to optimize the design when shear reinforcement is needed; consequently, the bridge deck should be designed so that shear reinforcement is avoided, if possible.

Key words: Concrete, bridge, design, shear force, distribution, slab, bridge deck, finite element analysis, 3D analysis

Tvärkraftsdimensionering av brobaneplattor i armerad betong
Dimensioneringsval och deras effekt på den slutliga utformningen
Examensarbete inom *Structural Engineering and Building Technology*
FREDRIK DAVIDSSON
Institutionen för bygg- och miljöteknik
Avdelningen för konstruktionsteknik
Betongbyggnad
Chalmers tekniska högskola

SAMMANFATTNING

Dimensioneringen av brobaneplattor i betong sker idag i huvudsak baserat på linjärelastiska finita element analyser. Detta är ett passande angreppssätt eftersom det, i en given konstruktion, är lasteffekterna till följd av många olika lastkombinationer som eftersöks. Det finns dock en del osäkerheter relaterade till en sådan analys. Hur tvärkraftsdimensionering borde gå till för att uppnå en optimerad och pålitlig utformning är inte helt kartlagt.

Målet med detta arbete har varit att undersöka hur olika tvärkraftsdimensioneringsval påverkar den slutliga utformningen av brobaneplattor i betong. Genom att dimensionera en given bro utifrån olika antaganden har det varit möjligt att belysa både skillnader och likheter. Fokus har varit riktat på att jämföra olika sätt att beakta tvärkraften och olika sätt att omfördela toppar av skjuvkrafter, men även relaterade osäkerheter och frågeställningar har undersökts.

Resultaten påvisar att skillnaderna i den slutliga armeringsutformningen är små. De indikerar också att det finns en stor potential i att hitta ett sätt att noggrant kunna uppskatta tvärkraftskapaciteten i plattan som en summa av kapaciteten av betongen och armeringsstålet, men de för närvarande gällande minimiarmeringsreglerna förhindrar att denna potential utnyttjas. Reglerna eliminerar de skillnader som erhålls vid dimensioneringen av brobaneplattan utifrån olika angreppssätt. Detta begränsar potentialen att optimera utformningen när tvärkraftsarmering är nödvändig. Därför borde brobaneplattan, om möjligt, utformas så att tvärkraftsarmering undviks.

Nyckelord: Betong, bro, dimensionering, dimensioneringsval, tvärkraft, fördelning, brobaneplatta, finit element analys, 3D analys

Contents

Preface

This thesis investigates the effect of different shear force design choices for reinforced concrete bridge decks. The work has been carried out at Skanska Sweden AB´s office in Göteborg between January 2014 and June 2014. The thesis has been written in collaboration between the Bridge- and Civil Engineering group of Skanska Teknik and the Division of Structural Engineering at Chalmers University of Technology.

The aim of the thesis was to compare different approaches to use a linear elastic finite element analysis to design for shear force in a reinforced concrete bridge deck. The comparison was made between the final results of the design in order to investigate how much the provided reinforcement amounts would differ. It was also desired to investigate the relation between the level of detailing in the modelling of the load effects compared to the level of approximation used to determine the resistance.

Basically, the result showed that the differences between different approaches to model the load effect are minor, but that the current design methodology to approximate the resistance results in a too conservative design. The requirements of minimum reinforcement amounts are limiting the potential to optimize the design when shear reinforcement is needed; consequently, the bridge deck should be designed so that shear reinforcement is avoided, if possible.

Associate professor Mario Plos, Chalmers, has been examiner for this thesis and PhD Helén Broo, Skanska, has been supervisor. I would like to thank them both for their time, great guidance and valuable advice. I would also like to thank my opponents, Arvid Lindgren and Oscar Pagrotsky, for taking their time to share their helpful thoughts on my work.

I would like to express my deepest gratitude to my fiancée, Sandra Diener, who has given me support and encouragement during the work on this thesis, as well as during my entire academic journey.

Göteborg 2014

Fredrik Davidsson

Notations

Roman Upper Case Letters

A_c	Cross-sectional concrete area
A_s	Cross-sectional bending reinforcement area
A_{sw}	Cross-sectional shear reinforcement area
C_{Rd}	Factor used in shear force resistance determination
D	Slab stiffness
E_c	Young´s modulus for concrete
E_s	Young´s modulus for steel
F_1	Concentrated wheel load
F_2	Concentrated wheel load
F_{td}	Design value of tensile force
L	Span width
L_c	Characteristic span width
M_{Ed}	Design value of the moment
M_{Rd}	Design moment resistance
N_{Ed}	Design value of the normal force
Q_k	Characteristic value of concentrated traffic load
R	Resultant force
SF	Sectional force
SM	Sectional moment
V_{Ed}	Design value of the shear force
V_{Rd}	Design shear force resistance
$V_{Rd.c}$	Design shear force resistance from the concrete
$V_{Rd.s}$	Design shear force resistance from the shear reinforcement
$V_{Rd.max}$	Design value of the maximum possible shear force resistance

Roman Lower Case Letters

a	Support width
a_l	Distance to move the moment curve to take the additional tensile force into account
b_0	Shear resisting control perimeter
b_1	Basic control perimeter for punching shear
b_u	Diameter of a circle with the same area as the basic control perimeter
b_w	Distribution width

b_w	Width of a cross-section
d	Effective height of a cross-section
d_v	Shear resisting effective height of a cross-section
d_g	Maximum aggregate size
e	Eccentricity
e_u	Eccentricity from the centre of the basic control perimeter to the shear force resultant
f_{bd}	Design value of the bond strength
f_{cd}	Design value of the compressive strength of concrete
f_{ck}	Characteristic value of the compressive strength of concrete
f_{yd}	Design value of the yield strength of reinforcing steel
f_{yk}	Characteristic value of the yield strength of reinforcing steel
f_{ywd}	Design value of the yield strength of shear reinforcing steel
g	Acceleration of gravity
h	Slab thickness
k	Factor used in calculation of shear resistance
k_1	Factor used in calculation of shear resistance
k_ε	Factor that takes the state of strain into account
k_ψ	Factor that takes the rotations into account
k_c	Factor used in calculation of shear resistance
k_{dg}	Factor that takes the maximum aggregate size into account
k_e	Coefficient of eccentricity
k_{sys}	Factor that takes the shear cracking and compressive struts into account
k_v	Strength factor
l_{bd}	Anchorage length
m_{Ed}	Design value of the moment (per unit width)
m_{Rd}	Design value of the moment resistance (per unit width)
m_{rx}	Reinforcement moment in x-direction (per unit width)
m_{ry}	Reinforcement moment in y-direction (per unit width)
m_x	Bending moment in x-direction (per unit width)
m_{xy}	Torsional moment (per unit width)
m_y	Bending moment in y-direction (per unit width)
n_l	Number of traffic lanes
p	Vertical load

q_k	Characteristic value of the distributed traffic load
r_s	Position where the radial bending moment is zero
s_w	Spacing of shear reinforcement bars
t	Thickness of paving
v_0	Resultant shear force (per unit width)
v_{min}	Minimum shear resistance (per unit width)
v_x	Shear force in x-direction (per unit width)
v_y	Shear force in y-direction (per unit width)
w	Carriageway width
w	Vertical displacement
w	Load width
w_{ef}	Distribution width for peak values of sectional forces
w_l	Width of traffic lanes
x	Coordinate
x	Distance from the centre of the loading to the design section
x_u	Depth of the neutral axis in ULS
y	Coordinate
z	Coordinate
z	Internal lever arm

Greek Letters

α	Angle between the resultant shear force and the x-axis
α_{cw}	Factor that takes the influence of possible compressive stress into account
α_Q	Adjustment factor for concentrated traffic load
α_q	Adjustment factor for distributed traffic load
β_Q	Adjustment factor for concentrated traffic load
γ_c	Partial factor, concrete
γ_s	Partial factor, steel
ε_x	Longitudinal strain (x-direction)
η_{fc}	Factor that takes the more brittle failure of higher concrete strength into account
θ	Angle of compressive struts in a cracked concrete member
κ_x	Curvature in x-direction
κ_y	Curvature in y-direction
μ_1	Factor used in the determination of reinforcement moment

μ_2	Factor used in the determination of reinforcement moment
ν	Poisson´s ratio
ν_1	Reduction factor of the concrete strength due to shear cracks
ρ_c	Concrete mass density
ρ_l	Reinforcement amount
ρ_{lx}	Reinforcement amount (x-direction)
ρ_{ly}	Reinforcement amount (y-direction)
ρ_{xy}	Torsional curvature
σ_{cp}	Normal stress
σ_{swd}	Stress in the shear reinforcement
ϕ	Bar diameter
ϕ	Column diameter
ϕ_w	Diameter of shear reinforcement bar
ψ	Rotation around supported area

1 Introduction

Previously, the design of bridges was executed with two-dimensional analyses where the behaviour in the different directions was analysed independently from each other. The use of more powerful computer tools together with finite element analyses have now made it possible to take the interaction between the longitudinal and transversal directions into account. However, this has led to other uncertainties in design that need to be investigated.

Today, design methods and recommendations to estimate the design for bending moments in concrete slabs are relatively well established. However, satisfactory investigations of how to model and estimate the shear force behaviour are still lacking. There are several different approaches and design choices to be used in varying design situations, but the effect of these on the final design is not very well investigated.

The finite element analyses, as well as other tools to approximate the load effects, become more and more sophisticated. Since the load effects are to be compared to the local resistance of a given structure, the level of approximation of the resistance needs to be in proportion to these analyses. An advanced analysis of the load effects may be a waste of resources if a too rough estimation of the resistance is made.

1.1 Purpose and Scope

The purpose of this study was to provide a more solid knowledge of different design choices when it comes to shear force distribution and resistance in reinforced concrete bridge decks. The objective was to compare different design methods that are implemented in design today, in order to quantify the effect they have on the final design in terms of reinforcement amounts. The aim was also to investigate the relevance of the relation between the level of detailing in the modelling of the load effects and the level of approximation used to determine the resistance. In order to give a comprehensive picture of the design uncertainties, issues related to the design methodology were also examined.

1.2 Method

With regard to shear force, an investigation of currently used and potential methods to design reinforced concrete bridge decks was made in order to find design methods to be compared. A representative example bridge geometry was created and based on the most relevant design methods, the bridge deck was designed based on linear finite element analysis. The final design, in terms of reinforcement amounts, was compared and the magnitude of the effects of the different design choices was examined. The relevance between the levels of approximation of the load effect and the resistance was investigated by examining the differences between different approaches to determine the shear resistance of the bridge deck. Further investigations and analyses of some related aspects and unanswered questions were also executed.

1.3 Limitations

The response was analysed with linear finite element modelling only, excluding nonlinear analysis. The focus was on the distribution of shear and the design for shear force in concrete bridge decks. The effects of moments were only taken into account in the design to find sufficient bending moment reinforcement. In that way the interaction between concrete, shear force reinforcement and bending reinforcement was investigated. No new shear force design methods were developed. The investigation aimed at comparing existing methods only. The comparison was made for one example bridge geometry. The aim was to create a representative geometry, but it was assumed that all effects could not be examined by one design situation only.

2 Structural Analysis of Concrete Bridge Decks

Today there are several different methods available for design of concrete bridge decks. Currently, the most frequently used methods for structural analysis are based on the assumption of linear elasticity. But there are also methods based on the theory of plasticity and methods where the nonlinearity of the material is taken into account. These methods for structural analysis can be regarded as methods with different levels of accuracy. With each simplification or approximation made, a step away from the reality is taken and therefore the accuracy decreases. On the other hand, with each of these simplifications the analysis becomes simpler and easier to understand. The choice of method should be based on an awareness of these two aspects. The analysis should be as simple as possible but without compromising important effects in the results.

This section aims to describe how the design work is currently executed. The chapter starts with a description of the current design philosophy and with a description of the traffic loads that are used in design. Different design methods and their corresponding assumptions are described as well. Focus is put on the methods based on the assumption of linear elasticity, since these are the most commonly used methods in a design process of a concrete bridge deck. The linear elastic methods presented in this section are a two-dimensional frame method and a three-dimensional finite element method. The latter method is the most relevant method in today´s practice. Plastic methods (the strip method and the yield line method) and the nonlinear method are also treated, but more briefly.

2.1 Design Philosophy

Structural design is performed based on the assumption that the load effect should be lower than, or equal to, the load carrying capacity in each point of the structure. Focus is put on the ultimate limit state where the design criterion is that the structure should have sufficient safety with respect to collapse. The serviceability limit state considerations, where the functionality of the structure is ensured, are secondary and only taken into account by several different checks.

In order to obtain a safe design, conservative assumptions should be made regarding the load effects as well as the resistance. It is essential to accurately determine both the presence and magnitude of the loads that will or could act on the structure during its life span. By the use of load combinations the simultaneous action from the loads is taken into account. The load combinations are made so that the permanent loads and the different variable loads are multiplied with factors that depends on the probability that they will act at the same time and with the assumed magnitudes.

The effects from different loads and combinations of loads are taken into account by using load effect envelopes. The first step is to determine which loads that are relevant for the structure. The second step is to determine which of the loads that could act simultaneously, and create load combinations from these. The envelope is then created as the maximum load effect in each member, connection or cross-section that can occur due to all possible load combinations. This envelope approach is specifically suitable when it comes to bridges, since the moving loads are creating different effects at different locations of the bridge.

Though there are bridge decks made of several different construction materials, the most commonly used material is reinforced concrete. There are a lot of benefits of concrete decks, but there are also some special issues that need certain attention. One of these issues is the nonlinearity of the material, and even though concrete as a material behaves highly nonlinear, the design work is preferably based on linear analyses so that load superposition is applicable. The design of the bridge is therefore based on the assumption that there is sufficient plastic deformation capacity in order for the slab to redistribute stresses so that the chosen distribution of moments and sectional forces is obtained. Normally, the ability of concrete slabs to plastically redistribute stresses is high. This ability is not taken into account when using a linear analysis along with the envelopes, and therefore this method leads to an over-capacity of the structure.

2.2 Traffic Loads on Bridge Decks

The bridge deck is one of the main structural members of a bridge. It is exposed to different loads, such as self weight, wind or earth pressure, and environmental deterioration, such as chlorides or freeze and thaw cycles. All different effects need to be considered in design in order for the bridge to withstand the exposure during its whole life span. However, there is one main load group that a bridge, especially the bridge deck, is exposed to. Often it is the traffic loads that will determine many of the aspects in the design of the bridge. The traffic is moving over the bridge and will induce vertical and horizontal, static and dynamic effects that must be considered in design. The traffic loads will act in many different locations at the bridge and the magnitude of the different effects can vary greatly depending on the amount of traffic that is currently located at the bridge.

A clear methodology of how to apply the loads in design, with respect to the magnitudes and the locations of the loads, must be established in order to achieve an accurate estimation of these. The rules and recommendations given in this section are based on the ones given in *Eurocode 1*, SS-EN 1991-2 (2003). When applying these, all traffic situations that are normally foreseeable should be covered.

2.2.1 Traffic Lanes

On roads and road bridges, the traffic is divided between different lanes. According to SS-EN 1991-2 (2003), the traffic loads should be divided between a specified number of traffic lanes. These traffic lanes are also called notional lanes and the number of these depends on how wide the bridge is. More specifically, the carriageway width, w, should be measured as the width where it is possible for the vehicles to actually drive. The parts of the bridge that do not allow traffic to pass, for example by fences, should be disregarded. The number of traffic lanes, n_l, and the width of these, w_l, should be determined as in Table 2.1.

Table 2.1: Determination of the number of traffic lanes, and the width of these. Adopted from SS-EN 1991-2 (2003).

Carriageway Width (w)	**Number of Traffic Lanes (n_l)**	**Width of a Traffic Lane (w_l)**	**Width of the Remaining Area**
$< 5.4\,m$	1	$3m$	$w - 3m$
$5.4m - 6.0m$	2	$\frac{w}{2}$	0
$6.0m \leq$	$int\left(\frac{w}{3}\right)$	$3m$	$w - 3n_l$

The lanes are numbered so that the worst effects are obtained for each verification with respect to the lanes to be taken into account and the transversal locations. The lanes giving the most, second most and third most unfavourable effects are numbered Lane 1, 2 and 3 respectively, see Figure 2.1. Since the most unfavourable effects are sought, the loads should be applied also in the worst locations longitudinally.

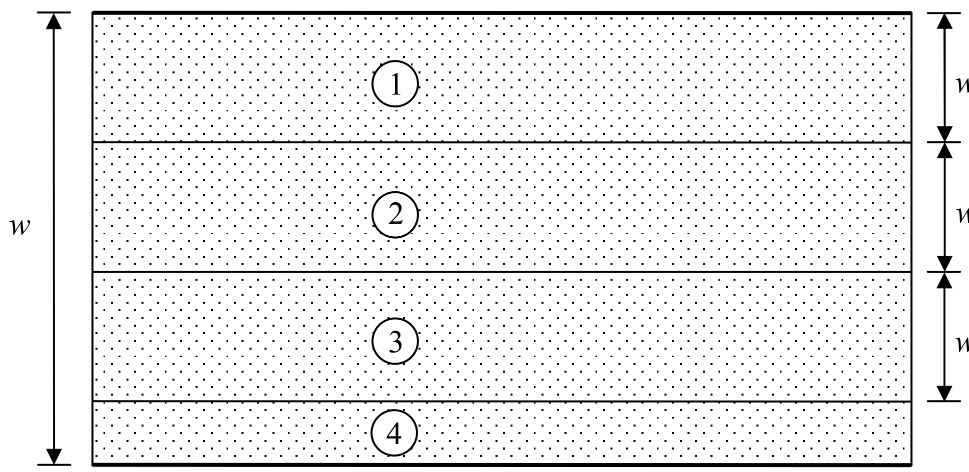


Figure 2.1: Example of the numbering of the traffic lanes. Adopted from SS-EN 1991-2 (2003).

If the carriageway on the bridge deck is permanently divided into two parts, each of the two parts should be divided in accordance with the principle in Figure 2.1. If the division is only made temporarily, the same rules apply as if there were no division at all. Only one numbering should be used regardless of in how many parts the carriageway is divided.

2.2.2 Load Models

Since the traffic can vary both in magnitude and in space, different effects must be considered when designing a bridge deck. The load models defined in SS-EN 1991-2 (2003) aims to cover all the foreseeable traffic situations which could occur at a road bridge. The loads are divided into different models in order to highlight different unfavourable effects that are obtained from different traffic situations. The main load models are number 1 and 2, but there are also other effects that are covered in number 3 and 4. Load Model 1 covers much of the effects coming from lorries and cars, while Load Model 2 covers dynamic effects from the traffic. Load Model 3 and 4 cover the effects from special vehicles and from crowds respectively.

Load Model 1 consists of both concentrated and uniformly distributed loads. The concentrated loads are double-axle loads with the magnitude $\alpha_Q Q_k$ (α_Q is an adjustment factor). The concentrated loads are applied in groups of two axles, with a spacing of 1.2 m, as tandem systems. Only one tandem system should be applied in each traffic lane, and only complete tandem systems should be considered. These should be assumed to travel along a line in the middle of each traffic lane. Each axle consists of two wheels with the wheel load $\alpha_Q Q_k/2$ and a wheel contact area of 0.40·0.40 m^2. The uniformly distributed part of the load model, $\alpha_q q_k$ (α_q is an adjustment factor), should be applied at the whole bridge, both in the traffic lanes and in the remaining area, but only in the areas where unfavourable effects occur. An illustration of how this load model should be applied is presented in Figure 2.2. If a local verification is to be done, the tandem systems should be applied in the most unfavourable way.

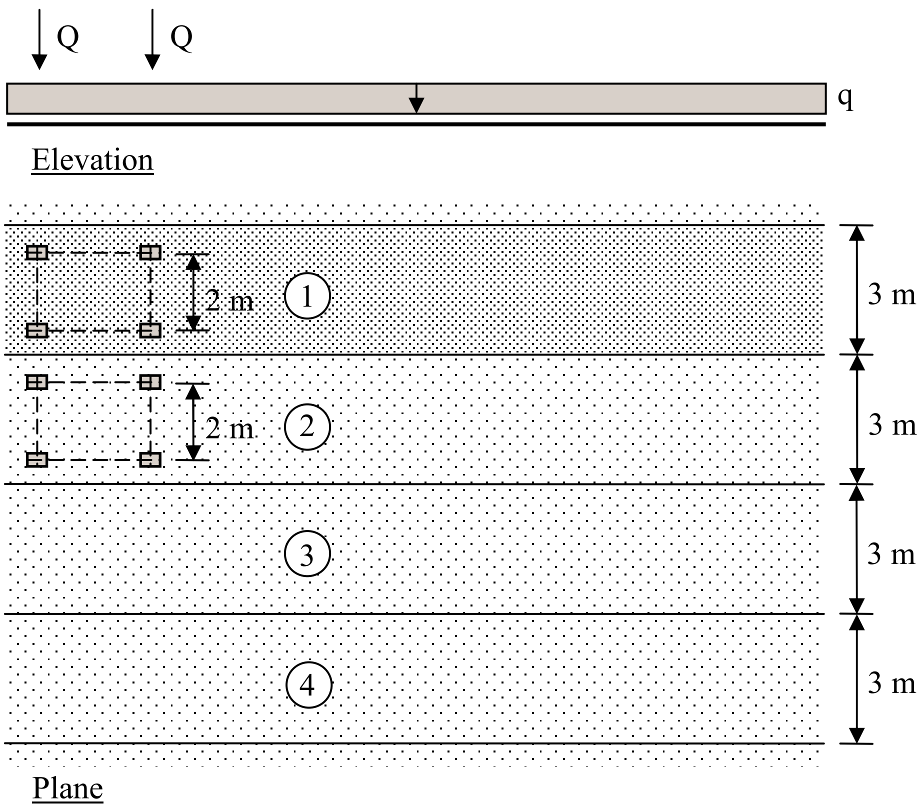


Figure 2.2: Schematic illustration of the application of Load Model 1 on a bridge with traffic lane width $w_l = 3m$. Adopted from SS-EN 1991-2 (2003).

The traffic load in lane i is referred to as Q_{ik} and q_{ik}, while the load on the remaining area is referred to as q_{rk}. The values of the characteristic loads are defined in Table 2.2.

Table 2.2: Summary of the loads in Load Model 1 according to Eurocode and the Swedish annex.

Location	Axle Loads $[kN]$ $(\alpha Q_{ik} = Q)$	Distributed Loads $[kN/m^2]$ $(\alpha q_{ik/rk} = q)$
Lane 1	$0.9 \cdot 300 = 270$	$0.7 \cdot 9 = 6.3$
Lane 2	$0.9 \cdot 200 = 180$	$1.0 \cdot 2.5 = 2.5$
Lane 3	$0 \cdot 100 = 0$	$1.0 \cdot 2.5 = 2.5$
Other Lanes	0	$1.0 \cdot 2.5 = 2.5$
Remaining Area	0	$1.0 \cdot 2.5 = 2.5$

Load Model 2 consists of concentrated loads only, applied as a single axle load, $\beta_Q Q_{ak}$, with a magnitude of 400 kN. The adjustment factor β_Q is recommended to be $\beta_Q = \alpha_{Q1}$. The axle, or the single wheel when applicable, should be applied at any location of the carriageway. The wheel contact surface area differs from Load Model 1. In this case it is 0.35·0.60 m^2 instead.

Load Model 3 and 4 are only applied when they are relevant for specific cases. Model 3 represents special vehicles and can consist of many different combinations of loads. Model 4 represents a crowd load and should be taken into account by applying a uniformly distributed load of 5 kN/m^2 throughout the entire deck.

2.3 Linear Elastic Analysis

In structural engineering, a common approach is to assume a linear relationship between applied load and material response. This assumption could be more or less true depending on which material that is regarded. For example, steel behaves basically linearly elastic up to its yield strength, while concrete starts to behave highly nonlinear due to cracking already for low stress levels. When it comes to plates, linear elastic analyses could either be performed as two- or three-dimensional, where both the complexity and accuracy increases with a higher order analysis.

Linear elastic slab analyses are traditionally based on Kirchhoff´s theory for elastic slabs, Engström (2011b). In addition to the linear elastic stress-strain relationship, this theory is based on four more assumptions. First of all, it is assumed that no stresses or strains take place in the middle plane of the slab. It is also assumed that sections perpendicular to the middle plane remain straight. The stresses perpendicular to the slab, as well as the shear deformations, are assumed to be equal to zero.

2.3.1 Two-Dimensional Frame Analysis

When the bridge geometry is not very complicated a two-dimensional analysis could be suitable, Sustainable Bridges (2007). The principle is to perform two separate linear elastic one-dimensional analyses for the two main directions (longitudinal and transversal) of the bridge. The compatibility between the two directions is not included and therefore the three-dimensional behaviour is not fully related to the linear elastic response. Thus, the moments and shear forces are used for each direction independently of the other, and the torsional moment is excluded entirely.

Since the traffic moves over a bridge, many different load cases and positions must be evaluated and accounted for. Each isolated part and cross-section of the bridge is designed in order to withstand the maximum load effect that could possibly occur, which means that many sections will be designed with an over-capacity for the individual load case, Sustainable Bridges (2007).

2.3.2 Three-Dimensional Finite Element Analysis

When designing a reinforced concrete slab, it is useful to use the finite element method (FEM) as a tool. This method is powerful and simplifies and rationalises the design process, especially when more complicated geometries are to be analysed, Pacoste, Plos & Johansson (2012). Although there are many benefits with such an approach, there are also a lot of inevitable considerations that have to be addressed in order to succeed with the analysis. A three-dimensional finite element analysis becomes more complex and the results from the analysis are hard to interpret and need to be post-processed before they can be used in design, Hillerborg (1996).

For the purpose of analysing a structure through three-dimensional finite element analysis, it is suitable to use a model made from shell and/or beam elements, Sustainable Bridges (2007). In addition to a two-dimensional analysis as described above, a three-dimensional analysis like this will not only fulfil the equilibrium conditions, but the compatibility requirements as well. This makes it possible to reflect the actual behaviour in a more accurate way than in a two-dimensional analysis.

In a finite element analysis it is important to be aware of design choices and how they will influence the results. Some choices may give rise to unwanted effects that may or may not be of importance from a design point of view. One example is the modelling of support conditions, where a certain choice can give rise to a singularity, Pacoste, Plos & Johansson (2012). It is up to the designer to decide which the important and relevant effects are, and which are only present due to the modelling technique and therefore could be ignored.

In Section 3.2 some specific problems related to a three-dimensional finite element analysis of a bridge deck are presented, along with a description of how they can be solved in terms of existing recommendations. The problems treated are the determination of distribution widths and the post-processing of output data from a finite element analysis.

2.4 Plastic Analysis

Reinforced concrete slabs may have different sectional capacities in different directions. The capacities are not only dependent on the material and the slab thickness, but also on the reinforcement amount and layout in the different directions. The behaviour of a reinforced concrete slab is nonlinear, as already described, and in the ultimate limit state the slab undergoes a pronounced plastic phase before the ultimate capacity is reached. With this in mind, the capacity in ULS may be approximated by using plastic analysis, Engström (2011b).

Normally, when solving the sectional forces in strips of a slab the problem is statically indeterminate, Engström (2011b). Thus, the sectional forces could be solved in many different ways if only the equilibrium condition is fulfilled. The equilibrium could be found either by using a lower bound approach (static method) or an upper bound approach (kinematic method). In slab design the most commonly used static and kinematic methods are *the strip method* and *the yield line method* respectively. The two methods are briefly described in Section 2.4.1 and 2.4.2.

2.4.1 The Strip Method

The strip method was originally developed by Hillerborg and presented in 1974. However, a more practical form of the strip method was presented in his handbook from 1996. The method is a lower bound approach, and will therefore give a solution that is either correct or on the safe side. If an inappropriate solution is made, the solution may be too conservative, giving an uneconomical design, Deaton (2005). The aim of this method is to design a slab by finding a solution to the equilibrium equation and reinforce the slab for the sectional moments and forces. The slab is divided into strips where the loads are transferred to the supports through unidirectional action in these strips, excluding torsional effects. This is similar to dividing the slabs into beams and determining how much of the load that should be carried by each beam. Two examples of how this division of the slab may look like are presented in Figure 2.3.

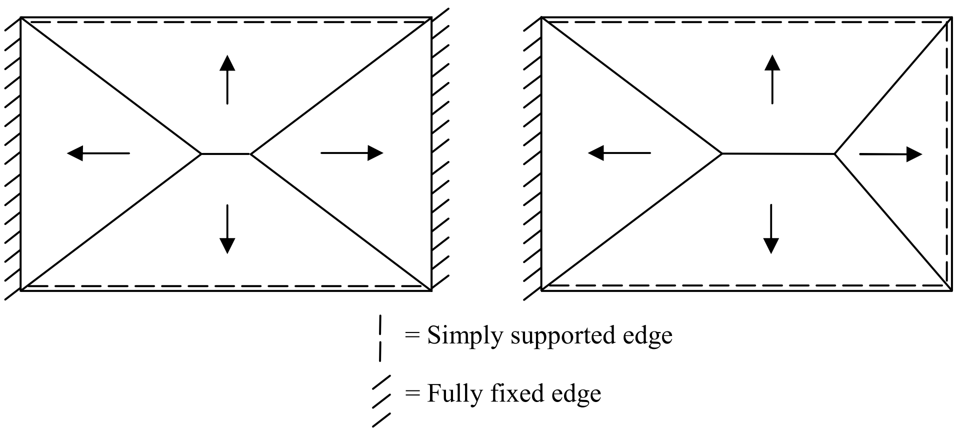


Figure 2.3: Two examples of how the division into strips may look like. Adopted from Engström (2011b).

In order for the design to actually be on the safe side, the slab must be ductile enough, Sustainable Bridges (2007). This means that the reinforcement can yield and sufficient plastic redistributions of the stresses can take place before the slab collapses. In this way the applied load must only be in equilibrium with the capacity of each strip, and the compatibility does not need to be checked. It should be noted, however, that some solutions are more suitable than others. Hillerborg (1996) gives recommendations of how to distribute the loads in order to obtain such a reasonable solution.

2.4.2 The Yield Line Method

Contrary to the strip method, the yield line method is an upper bound approach. This means that a solution obtained from an analysis with this method is either correct or on the unsafe side, Deaton (2005). This method should therefore only be used with care and sufficient experience to assure that the risks of this approach are minimized. The method is mainly used to assess the load-carrying capacity of already existing slabs by assuming a reasonable collapse mechanism for the slab. Many different mechanisms may be possible and therefore all these need to be checked. Two examples of how the assumed collapse mechanisms may look like are presented in Figure 2.4.

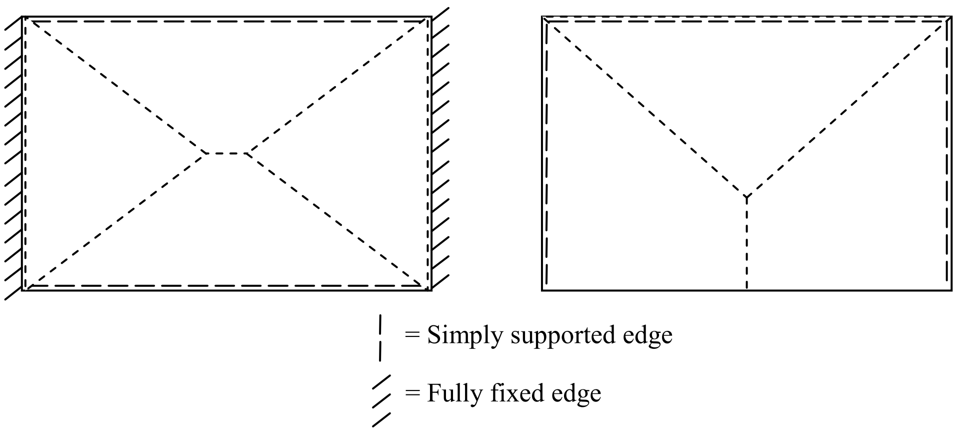


Figure 2.4: Two examples of how collapse mechanisms may look like. Adopted from Engström (2011b).

The assumed collapse mechanism must be possible with respect to the compatibility conditions in ULS. When the mechanism is determined, the maximum load which leads to a failure like this is determined by calculating the work that needs to be done to create the failure mechanism, Sustainable Bridges (2007).

The strip method and the yield line method could be used together in order to find and verify the real load-carrying capacity. If the most critical failure mechanism is found when using the yield line method, this will correspond to the actual capacity. The same applies for the most effective strip division when using the strip method. Thus, when the strip method and the yield line method give the same answers the theoretically correct solution is found.

2.5 Nonlinear Analysis

The most accurate method of structural analysis is a method where the nonlinear behaviour of reinforced concrete is taken into account. This is an advanced method where the behaviour of a given structure can be modelled through all its states; cracking of concrete, yielding of reinforcement and collapse of the whole structure. The nonlinear behaviour starts already for low magnitudes of load, and therefore the linear analysis approach is of limited value for concrete structures, Plos (1996).

The nonlinear method is not one single method, but several different methods based on different assumptions, Engström (2011a). Thus, a nonlinear analysis can be executed on several different levels of accuracy. The modelling level depends on how the nonlinear behaviour is taken into account. As one example, the change of the flexural rigidity can be taken into account by either stepwise changes or by use of nonlinear moment-curvature relationships.

Nonlinear analyses are normally most suitable for assessment of existing structures, Sustainable Bridges (2007). This is a weakness of this structural analysis approach, since the design needs to be defined in detail before the analysis can be performed. However, this method can preferably be used as a validation tool in order to investigate the behaviour of a certain design created from a more simplified design method (linear elastic- or plastic analyses).

3 Finite Element Analysis of Concrete Slabs

Over the past few years the use of the finite element method (FEM) has increased substantially. More powerful computers allow the level of the FE analyses to be much higher than they were before. Due to these tools, many different complex engineering problems can be solved in a simpler way and with a more accurate result. The same development has taken place when it comes to structural analysis of bridge structures. The improvement has taken place relatively fast, and all uncertainties and issues with such analyses are not yet completely investigated. Even though the computer programs available are powerful and simple to use, it is important to understand the theory and assumptions behind the programs, and how different choices may affect the outcome. The freedom to control the analysis and the results varies with different programs, but common for them all are some different aspects that need to be defined. Numerical models are simplifications of the reality, and are only as accurate as the assumptions made when establishing the model, Rombach (2004).

The aim of this chapter is to provide a description of the theory involved in finite element analyses of bridge decks. The first part gives an overview of the plate theory and how sectional forces and moments are handled in a finite element program. The second part handles some specific considerations that the designer has to cope with when designing a bridge deck by means of FEM and a FE program. The considerations presented in this chapter are related to the establishment of the FE model and how to deal with the output results in order to complete the design.

3.1 Plate Theory

Plates can be divided into thick and thin plates. The limit between these two groups is a span-to-thickness ratio of 5, Blaauwendraad (2010). A slab with a ratio larger than this limit is classified as a thin plate, which is the case for normal slabs. The shear deformations are neglected in thin plates, and therefore some simplifications can be made compared to the thick plate case. For a complete derivation of the equations in both thick and thin plate theory the reader is referred to Blaauwendraad (2010). Here follows only a description of what impact the assumption of negligible shear deformations has on the equations.

3.1.1 Sectional Moments and Forces in Plates

First of all, the kinematic relations for thin plates are defined for the three curvatures as in equation (3.1), where w is the vertical displacement.

$$\begin{aligned} \kappa_x &= -\frac{\partial^2 w}{\partial x^2} \\ \kappa_y &= -\frac{\partial^2 w}{\partial y^2} \\ \rho_{xy} &= -2\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{3.1}$$

Since the shear deformation is neglected, the constitutive relations for the slab are only based on the moments as stated in equation (3.2), where m_{xy} is the torsional moment and D is the slab stiffness.

$$
\begin{aligned}
m_x &= D(\kappa_x + \nu\kappa_y) \\
m_y &= D(\nu\kappa_x + \kappa_y) \\
m_{xy} &= \tfrac{1}{2}D(1-\nu)\rho_{xy}
\end{aligned} \tag{3.2}
$$

The equilibrium condition can be stated as in equation (3.3), where p is the vertical load.

$$
-\left(\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2}\right) = p \tag{3.3}
$$

When combining the equations (3.1)-(3.3) a differential equation, called the slab equation, is obtained as in (3.4). In this case the slab stiffness D is only dependent on the flexural stiffness, since the shear is neglected.

$$
D\left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right) w = p \tag{3.4}
$$

The different moment- and shear force components can be expressed explicitly as in equation (3.5).

$$
\begin{aligned}
m_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\
m_y &= -D\left(\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\
m_{xy} &= -(1-\nu)D\frac{\partial^2 w}{\partial x \partial y} \\
v_x &= -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) \\
v_y &= -D\left(\frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^3 w}{\partial y^3}\right)
\end{aligned} \tag{3.5}
$$

In Figure 3.1 it is illustrated how these components act on a plate element. The moment components in the x- and y-directions are leading to stresses parallel to the x- and y-axes respectively, while the torsional moment is rotating the face of the cross section where it is acting. The shear components in the x- and y-directions are both acting in the z-direction.

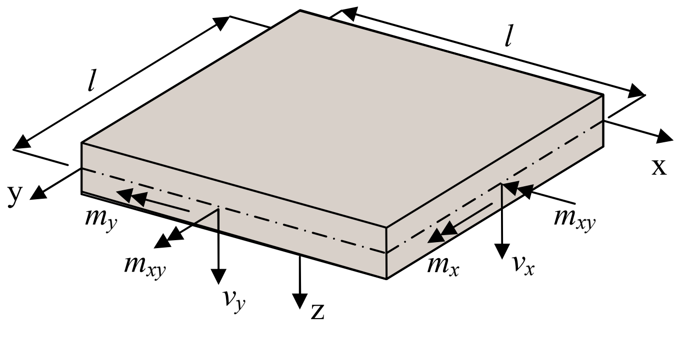


Figure 3.1: Moment- and shear force components acting on a plate element. Adopted from Blaauwendraad (2010).

3.1.2 Sectional Moments and Forces in Reinforced Concrete Slabs

When the bridge deck is made of reinforced concrete much focus needs to be put on the reinforcement design. From a practical point of view, the reinforcement cannot follow the directions of the maximum principal stresses, which would have been an optimal way to reinforce a slab. In order to be able to design the slab with regard to reinforcement, the moments must be expressed in the x- and y-directions only, which correspond to the chosen directions for the reinforcement. According to Blaauwendraad (2010), the bending and torsional moments can be translated into reinforcement moments as in equation (3.6). The shear force components are acting in the x- and y-directions and the resultant shear force can be determined according to equation (3.7). The resultant is acting in the direction corresponding to an angle from the x-axis as defined in equation (3.8).

$$\begin{aligned} m_{rx.pos} &= m_x + \mu_1 |m_{xy}| \\ m_{rx.neg} &= m_x - \mu_2 |m_{xy}| \\ m_{ry.pos} &= m_y + \frac{1}{\mu_1} |m_{xy}| \\ m_{ry.neg} &= m_y - \frac{1}{\mu_2} |m_{xy}| \end{aligned} \tag{3.6}$$

Where: $\mu_1 = \mu_2 = 1$ is normally chosen.

$$v_0 = \sqrt{v_x^2 + v_y^2} \tag{3.7}$$

$$\alpha = arctan\left(\frac{v_y}{v_x}\right) \tag{3.8}$$

In addition to the moment- and shear force components as defined above, in-plane normal force components will also act on a general plate element. The influence from such membrane components is limited for ordinary reinforced concrete slabs. The effects from all these components can be taken into account by a *sandwich model* (Fib Bulletin 45, 2008). The principles of this model are illustrated in Figure 3.2, where the moment-, shear- and membrane components are divided between the different layers of the sandwich.

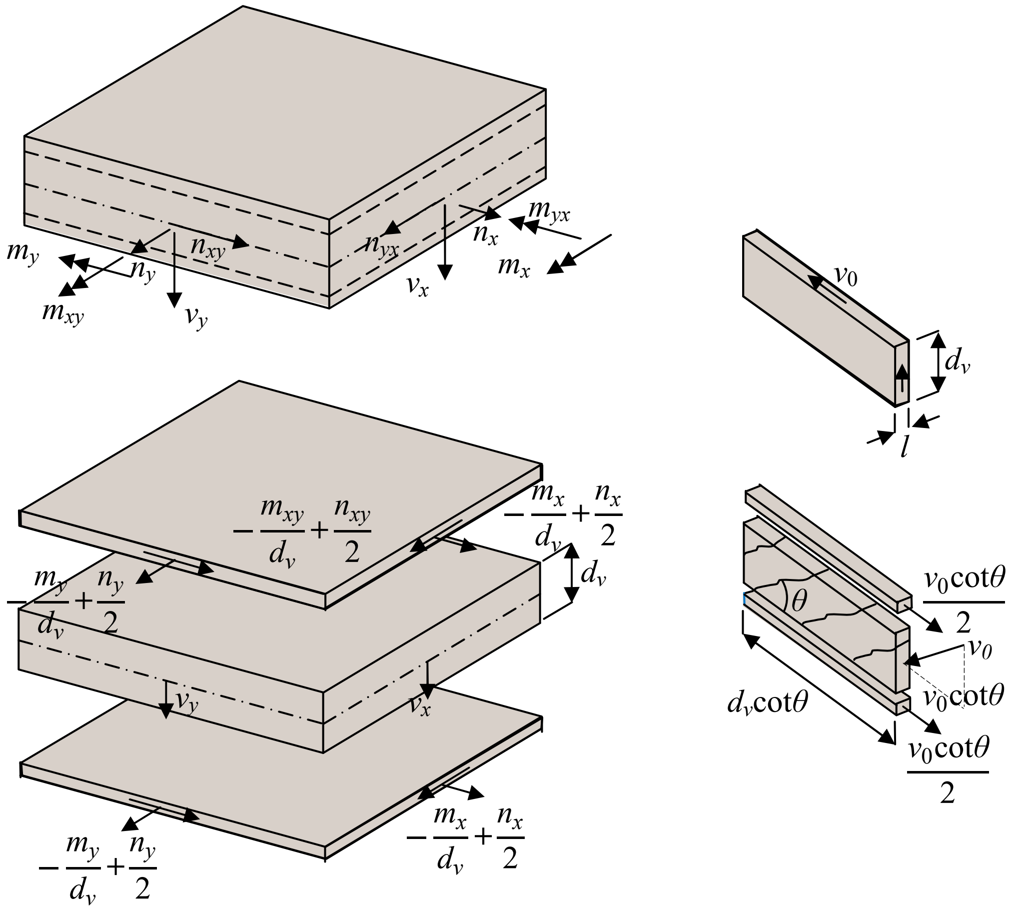


Figure 3.2: Moment-, shear force- and normal force components acting on a plate element. The components are divided between the different layers in the sandwich model. Adopted from Fib Bulletin 45 (2008).

3.2 Finite Element Modelling Considerations Related to Bridge Deck Design

Due to cracking and plastic behaviour of the concrete and yielding of the reinforcement, the behaviour of reinforced concrete is highly nonlinear. Since a bridge deck slab needs to be designed for moving loads and for many different load combinations, it is not possible to use a nonlinear analysis without obtaining a very work-intense design process. Therefore, in order to be able to use load superposition, the design of concrete bridge deck slabs is usually done with linear analysis.

The reader is assumed to be familiar with the fundamental theory behind the finite element method. However, as mentioned earlier, there are some special considerations and issues that need to be treated when a concrete bridge deck is to be designed by means of the linear elastic assumption and the finite element method. One of these considerations has already been covered in the previous section, where the reinforcement moments and the resultant shear force were described. This section aims to highlight some other relevant issues in bridge deck design.

The different steps that are taken when a finite element analysis is to be performed are *idealisation*, *discretization, element analysis, structural analysis, post-processing* and *analysis of results*, Davidson (2003). There are specific considerations to be addressed in all these steps.

3.2.1 Structural Idealisation

When idealising a structure, a structural model of the real geometry is created. Accurate assumptions are needed in order to obtain a model that reflects a realistic behaviour of a specific structure. The designer needs to have knowledge about how different modelling assumptions and simplifications affect the final result. The model should be made as simple as possible without losing the ability to describe important effects, since the aim of finite element modelling is to investigate how a structure behaves when exposed to a certain set of loads. If possible, irregularities such as holes, inclinations and curvatures should be ignored, since they could create effects that are hard to understand in the interpretation of the results.

Special care is needed when supports are idealised, since the boundary conditions influence the results significantly, Pacoste, Plos & Johansson (2012). It is therefore essential to be able to prescribe the restrictions in a way that corresponds to the actual translational and rotational stiffness in the different directions. The modelling of supports is done by fixing one or more of the translational and rotational degrees of freedom, or by adding a certain stiffness to these by springs. It is normally recommended to model supports by prescription in single nodes, as illustrated in Figure 3.3 and Figure 3.4.

Figure 3.3: Recommended ways to model a hinged support for a slab modelled with shell elements. Left: Pinned at the centre of the support. Right: Pinned support with stiffener. From Pacoste, Plos & Johansson (2012).

Figure 3.4: Recommended ways to model a stiff connection between the support and the slab. Left: Stiffener in the support. Right: Stiffener in both the support and the slab. From Pacoste, Plos & Johansson (2012).

It is important to be aware of the singularities than can arise from such an approach, and how to interpret the results in a correct way. This subject is treated in more detail later in this chapter. It is also possible to model the support conditions so that singularities are avoided. However, since this involves prescription of more than one node at each support, such approaches are not treated in this thesis.

3.2.2 Element Types

Applicable elements in a finite element analysis could basically be divided between two groups; structural elements and continuum elements. The element types could also be distinguished between three other groups; one-, two- or three-dimensional elements. In a one-dimensional analysis, *bar* elements are used. In a two-dimensional analysis, however, the structural elements used could be either *beam*-, *plate* or *interface* elements, while continuum elements used could be *plane stress*-, *plane strain*- or *axis-symmetric* elements. In a three-dimensional analysis *beam*-, *shell*-, or *interface* elements can be used as structural elements, or *volume* elements could be used as continuum elements, Davidson (2003).

The choice of element type should be made based on which type of analysis that is to be performed, and which types of results that are of interest. In bridge design it is usually recommended to use shell- and/or beam elements, Sustainable Bridges (2007).

3.2.3 Mesh Density

The element mesh needs to be dense enough to capture the overall response of the structure, as well as important details in the analysis. In order to verify that the mesh is sufficiently dense a convergence study should be performed. In that way the mesh density corresponding to when the results converge represents an upper limit of the allowed size of the elements. Even denser meshes give somewhat more accurate results, but the computational time is also increasing with increasing number of elements. The element mesh density should be chosen with respect to both these aspects. A rule of thumb for shell elements is that the size of the elements should be equal to or less than the thickness of the slab. The choice of the element size should be made so that in each span, there are 12-16 elements in the direction of the traffic lanes, Scanscot Technology (2007).

The preferable mesh density could vary extensively throughout the geometry. Adjacent to supports and critical sections the element mesh should be refined so that there is at least one shell element between the support and the critical section, Pacoste, Plos & Johansson (2012). An example of such a mesh refinement around columns is illustrated in Figure 3.5.

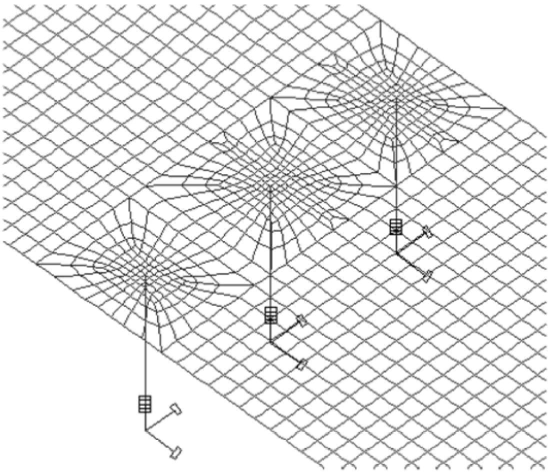


Figure 3.5: Mesh refinement around column supports. From Pacoste, Plos & Johansson (2012).

As mentioned, this refinement of the mesh around supports should be made with respect to the singularities that arise at supports when modelled in single nodes. The refinement creates results that are accurate enough in the nodes adjacent to the support node, which makes it possible to use the results in these nodes to design the bridge deck. The results at the supports are ignored and regarded as inevitable unwanted modelling effects, Sustainable Bridges (2007).

3.2.4 Critical Sections

In order to obtain a sufficiently accurate design, it is important that the choices of result sections are made accurately so that they correspond to the actual behaviour of the structure. Which sections that should be used depend on the support conditions and on which result output that is of interest.

According to Pacoste, Plos & Johansson (2012), the maximum bending moments are overestimated when the supports are modelled in single nodes. The moments acting on adjacent sections are more realistic and do not tend to infinity, as is the case for the moments in the nodes where supports are applied. The exact location of the recommended result section depends on how stiff the connection is between the slab and the support. How the stiffness influences the recommended choice of result section is illustrated in Figure 3.6.

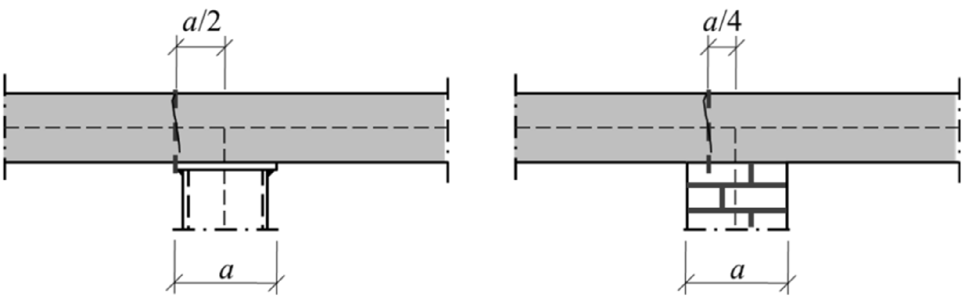


Figure 3.6: Recommended result section for moments. Left: Rigid support. Right: Weak support. From Pacoste, Plos & Johansson (2012).

When the shear force results are of interest, the stiffness of the support is not influencing the choice of result section. Instead it is dependent on where a failure-critical crack could occur. The crack will develop where the highest shear force is acting, and where it is physically possible for the crack to actually develop. With this in mind, the crack cannot be located closer to the support than at the edge of it. The shear force that needs to be transferred to the support is the force that can generate a failure-critical crack. The force that is located closer than a certain distance from the edge of the column is assumed to be transferred directly to the support and will therefore not contribute to the cracking of the member. Therefore the result section should be chosen as illustrated in Figure 3.7.

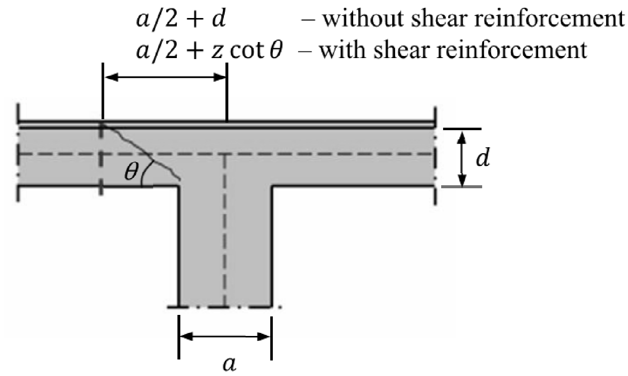


Figure 3.7: Recommended result section for shear force. From Pacoste, Plos & Johansson (2012).

3.2.5 Peak Values of Sectional Forces

Modelling based on linear elasticity assumes a linear relationship between the material response and the applied load. This assumption is true under certain circumstances, but if a slab is loaded until failure the behaviour will vary between the different states. In a linear elastic analysis of a reinforced concrete slab, local peak values of the sectional forces will be obtained at certain locations. These peaks are only fictitious since redistribution of stresses will take place within the slab, mostly due to cracking of concrete and yielding of reinforcement. Therefore, these peaks will be redistributed to other, stiffer, regions.

According to Pacoste, Plos and Johansson (2012), the effects from cracking and yielding can in linear finite element analyses be taken into account by using a certain distribution width. This is a width over which the reinforcement moments and shear forces can be distributed, and the size of the width depends on how ductile the regarded slab is. When the sectional forces are distributed like this the average values over that width can be used to calculate the need for reinforcement. This principle is illustrated in Figure 3.8, where a slab supported on columns is analysed by looking at one result line in each direction, L_1 and L_2.

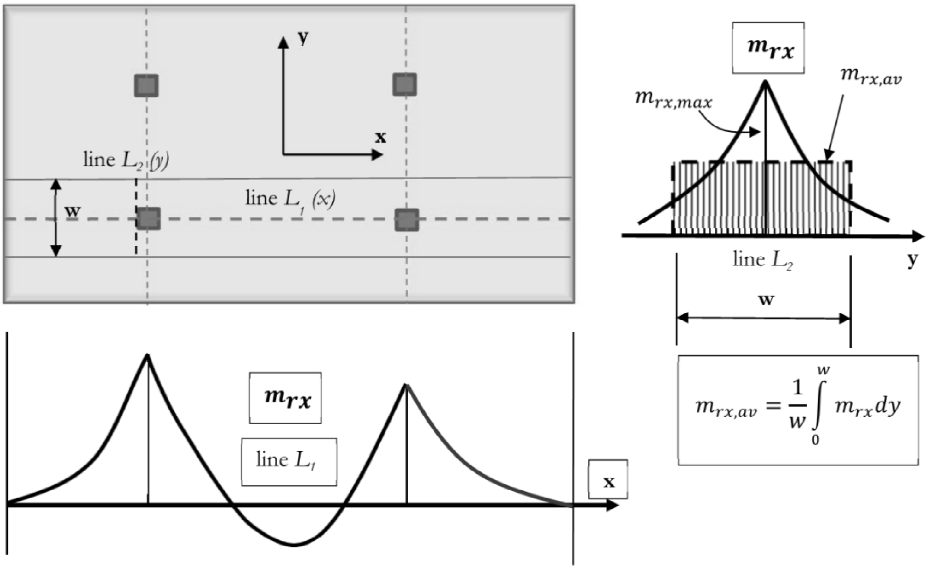


Figure 3.8: Redistribution of the reinforcement moment m_{rx} over a width w. From Pacoste, Plos & Johansson (2012).

The illustration shows the principle of the distribution of moments. However, this principle is applicable on the shear forces as well, but it should be noted that the widths that should be used for the moments and the shear forces do not fully correlate.

It should also be noted that there are other simplifications in the modelling, apart from the linear elastic approach, which could lead to high concentrated forces. The geometry of a structure is always, to some extent, simplified when it should be modelled by means of a finite element analysis. Also here the direct consequence of such simplifications is that it often leads to high peak values of the sectional forces, Pacoste, Plos & Johansson (2012). This could lead to local singularities at the point or lines that should represent supports. Since the singularities occur in these points or lines, and since the results in these points or lines are not of interest, these could simply be disregarded.

The problem related to these peaks is how to modify the results from the finite element analysis in order to obtain realistic moment- and shear force distributions. One reasonable solution to this problem would be to manually distribute the sectional moments and forces over a certain distribution width. The redistribution that would take place in reality due to cracking and plastic behaviour within the materials is then simulated by these distribution widths over which the mean value of the sectional moments and forces are accounted for.

The problem with such an approach is that it is hard to estimate to what extent the redistribution of stresses has taken place in different states, and how much the redistribution can take place ultimately. Since there are many different aspects to take into account when determining the plastic redistribution capacity of a slab, the choice of distribution width could be rather complicated.

Bridge design in Sweden is currently based on the rules and recommendations given by the Swedish Traffic Administration, *Trafikverket*. There is one publication where the technical demands are given, TRVK Bro 11 (2011a) and another where the recommendations are collected, TRVR Bro 11 (2011b). The general recommendation for distribution of moments and forces in slabs is, Trafikverket (2011b):

$$w_{ef} = min\left(3h, \frac{L}{10}\right) \qquad in\ ULS$$
$$w_{ef} = min\left(2h, \frac{L}{10}\right) \qquad in\ SLS \qquad (3.9)$$

Where: w_{ef} is the effective distribution width, h is the slab thickness and L is the span width of the structural member. ULS stands for ultimate limit state and SLS for serviceability limit state.

It is also stated that, in a bridge deck, shear peak values from concentrated wheel loads close to line supports can be distributed on the least of:

- the distance between the points where the shear force according to the calculation with the finite element method is 10% of the peak value and
- a width of:

$$w_{ef} = max(7d + b + t, 10d + 1.3x) \qquad (3.10)$$

Where: d is the effective height of the deck, b is the width of the load, t is the thickness of the cover paving and x is the distance from the centre of the loading to the design section. The principle of this distribution is illustrated in Figure 3.9.

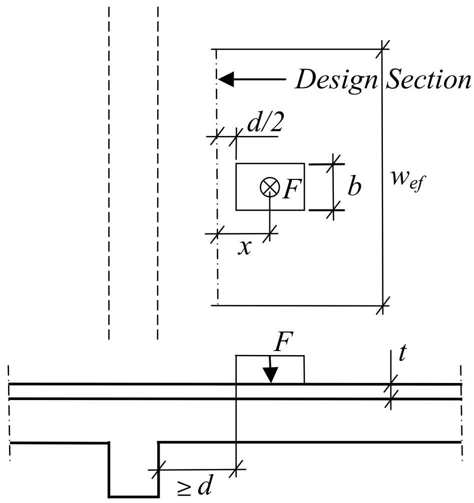


Figure 3.9: Effective width w_{ef} for one concentrated wheel load on a bridge deck close to a line support. Adopted from Trafikverket (2011b).

When two concentrated loads are located so that their respective effective widths are overlapping each other, the shear force per unit length, v, in the design section could be calculated in another way. First the location of the resultant $R(F_1, F_2)$ from F_1 and F_2 is determined. The effective width for this resultant is then determined as:

$$w_{ef} + 2l_{res} \geq max(w_{ef.1}, w_{ef.2}) \qquad (3.11)$$

Where: w_{ef} is the effective width of the larger load and l_{res} is the distance between the resultant $R(F_1, F_2)$ and the larger of the loads.

In addition to the value v from $R(F_1, F_2)$, the shear force per unit width in the result section from other loads should be added. This could for example be the self weight or other concentrated loads. The principle of how to distribute the loads like this is illustrated in Figure 3.10.

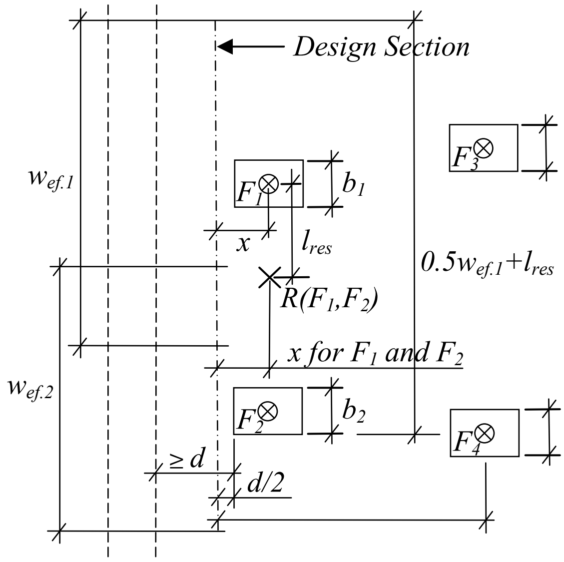


Figure 3.10: Effective width w_{ef} for two concentrated wheel loads on a bridge deck close to a line support. Adopted from Trafikverket (2011b).

The methodology described above is rather simple and is assumed to be conservative. Since the distribution widths given from *Trafikverket* are recommendations and not requirements, other widths may be used if they can be validated. In Pacoste, Plos & Johansson (2012), somewhat more developed recommendations are given. Here the rotational capacity is taken into account by check of the ratio $\frac{x_u}{d}$. The recommendations given for the distribution widths in ULS are given in Equation (3.12).

$$\begin{aligned}
w_{ef} &= min\left(3h, \frac{L_c}{10}\right) && for\ \frac{x_u}{d} = 0.45 \\
w_{ef} &= min\left(5h, \frac{L_c}{5}\right) && for\ \frac{x_u}{d} = 0.30 \\
w_{ef} &= \frac{L_c}{4} && for\ \frac{x_u}{d} = 0.25 \\
w_{ef} &= \frac{L_c}{2} && for\ \frac{x_u}{d} = 0.15 \\
w_{ef} &= min\left(5h, \frac{L_c}{5}\right) && for\ \frac{x_u}{d} = 0
\end{aligned} \tag{3.12}$$

Where: L_c is a characteristic span width and x_u is the depth of the neutral axis in the ultimate limit state. The minimum distribution width should not be less than:

$$w_{ef} \geq w_{min} = 2h + a \tag{3.13}$$

Where: a is the width of the support or the load.

As can be seen in the equations above, the optimal ratio would be $\frac{x_u}{d} = 0.15$. Ratios higher than this value do not allow for equal amounts of plastic redistributions before the reinforcement breaks, and ratios lower than this value corresponds to a section where concrete crushing is the limiting factor. Values between the ratios should be interpolated. The ratio between the averaged and maximum value should not be greater than 0.6. For serviceability limit states the distribution width should be chosen between the two first limits in Equation (3.12). For the shear force distribution the width should not exceed $5h$. For a more complete background to these values the reader is referred to Pacoste, Plos & Johansson (2012). Even though these values are somewhat more refined compared to the recommendations given by Trafikverket (2011b), they are still assumed to be conservative and other widths may still be used if they can be validated.

3.2.6 Post-Processing of Shear Output Data

The results obtained in a finite element analysis needs to be more or less modified before they can be used in the design of the structure. In Section 3.1.2 it was described how the bending- and torsional moments can be combined in order to make it possible to calculate the corresponding reinforcement moments. When this modification has been made, these moments can be used to design the bending reinforcement of the slab. However, the shear force is more complex to handle in order to find a suitable distribution that fulfils the equilibrium conditions and can be obtained in the ultimate limit state through plastic redistributions.

The output obtained from a finite element program consists of shear force components in the two principal directions. Shear failure can take place either by *one-way* or *two-way* shear failure modes. Two-way shear is also referred to as punching shear. In design, both of these failure modes must be taken into account. In principle, the names one- and two-way shear are somewhat confusing since shear as a mechanical quantity is unidirectional, Vaz Rodrigues (2007). In order to better understand the difference between these two types of shear, Vaz Rodrigues presents a schematic illustration of the flow of shear in a slab according to Figure 3.11.

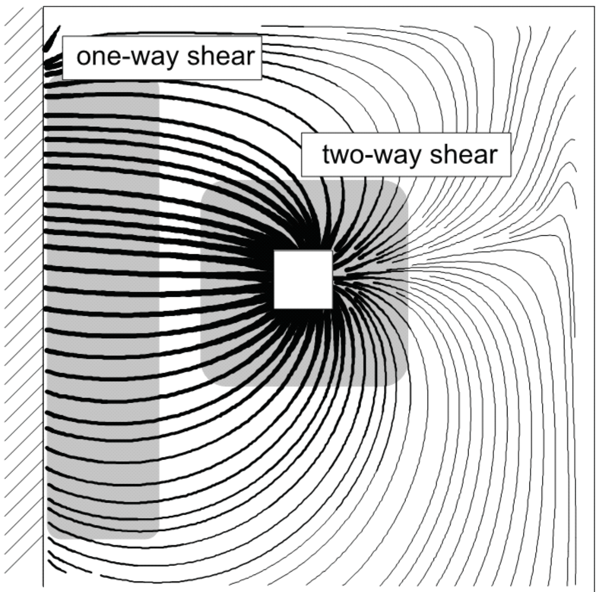


Figure 3.11: Shear flow in a cantilever slab – Illustration of zones of one-way and two-way shear. From Vaz Rodrigues (2007).

Based on this observation, the shear force could either be treated as two components in the two principal directions or as a resultant shear force as described in Section 3.1.2. The two different approaches involve different aspects that have to be taken into account. If the design is based on the components in the two directions the design work is rather similar to what is done for the moments. Different sections are designed with respect to the dominating component, and the secondary component is ignored. In this approach it is important to be able to accurately determine in which regions of the structure that the different components actually are dominating. If this division of the structure is poorly made, there is a risk that the design based on this method will be unconservative.

A more refined methodology is to design for the resultant shear force v_0 instead. Then both components v_x and v_y would be taken into account in each section, and the design would be conservative. However, this approach would give rise to other complications. Since the results from the finite element analysis follow the main x- and y-directions, the resultant shear force should be given in these directions as well. Thus, the use of the resultant shear force involves determining the direction in which it acts. Pacoste, Plos & Johansson (2012) have given a proposal where the angle between the resultant shear force and the shear force component in the longitudinal direction of the structure describes in which direction it acts.

In each point, the resultant shear force and its angle to the main x-axis is determined as in Equation (3.14) and (3.15). An illustration of how they are related is given in Figure 3.12.

$$v_{0.i} = \sqrt{v_{x.i}^2 + v_{y.i}^2} \tag{3.14}$$

$$\alpha_i = arctan\left(\frac{v_{y.i}}{v_{x.i}}\right) \tag{3.15}$$

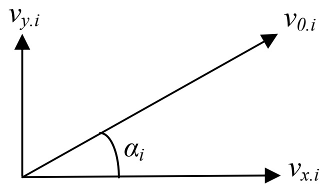


Figure 3.12: Relation between the shear force components, the shear force resultant and the angle used for determination of the direction.

As proposed by Pacoste, Plos & Johansson (2012), the resultant should be approximated to act in one of the main x- and y- directions depending on the angle α. If the angle is less or equal to 45° it is assumed to act in the longitudinal (x) direction, and if it is larger than 45° it is assumed to act in the transversal (y) direction.

When using the resultant shear force instead of the shear force components in design, two different resultant shear forces need to be accounted for. This is because of the fact that the design is made with respect to several different load cases.

$$v_{01.i} = \sqrt{v_{x.max.i}^2 + v_{y.cor.i}^2}$$
$$v_{02.i} = \sqrt{v_{x.cor.i}^2 + v_{y.max.i}^2} \tag{3.16}$$

Where: max stands for the maximum value and

cor stands for the value corresponding to that value (i.e. from the same load case).

It should be noted that when the shear force components are used, $v_{x.max.i}$ and $v_{y.max.i}$ are used directly. The direction of these components does not need to be approximated since they already act in the principal x- and y-directions.

Thus, there are two ways to approximate the shear forces. Either by the shear force components in x- and y-directions independently of each other, or by the resultant shear force where the interaction between the components are taken into account. The shear force component method is straightforward since they already act in the x- and y-directions, while the resultant shear force needs to be approximated to act in one of these directions. A summary of the two methods are given in Table 3.1. The design shear forces are acting in either the x- or y-direction.

Table 3.1: Design shear forces in a section point i, depending on the angle α.

	Case	$\alpha_{1.i}$	$\alpha_{2.i}$	$v_{dx.i}$	$v_{dy.i}$
Shear Force Components	**(1)**	$\leq 45^\circ$	$\leq 45^\circ$	$v_{x.max.i}$	$v_{y.max.i}$
	(2)	$\leq 45^\circ$	$> 45^\circ$	$v_{x.max.i}$	$v_{y.max.i}$
	(3)	$> 45^\circ$	$\leq 45^\circ$	$v_{x.max.i}$	$v_{y.max.i}$
	(4)	$> 45^\circ$	$> 45^\circ$	$v_{x.max.i}$	$v_{y.max.i}$
Resultant Shear Force	**(1)**	$\leq 45^\circ$	$\leq 45^\circ$	$max\ (v_{01.i}, v_{02.i})$	0
	(2)	$\leq 45^\circ$	$> 45^\circ$	$v_{01.i}$	$v_{02.i}$
	(3)	$> 45^\circ$	$\leq 45^\circ$	$v_{02.i}$	$v_{01.i}$
	(4)	$> 45^\circ$	$> 45^\circ$	0	$max\ (v_{01.i}, v_{02.i})$

The method where the shear force components are used is a simple and straightforward method, while the resultant shear force method with the approximation of the directions is a proposed method by Pacoste, Plos & Johansson (2012). According to this method, an upper limit of the distribution widths should be used according to Figure 3.13.

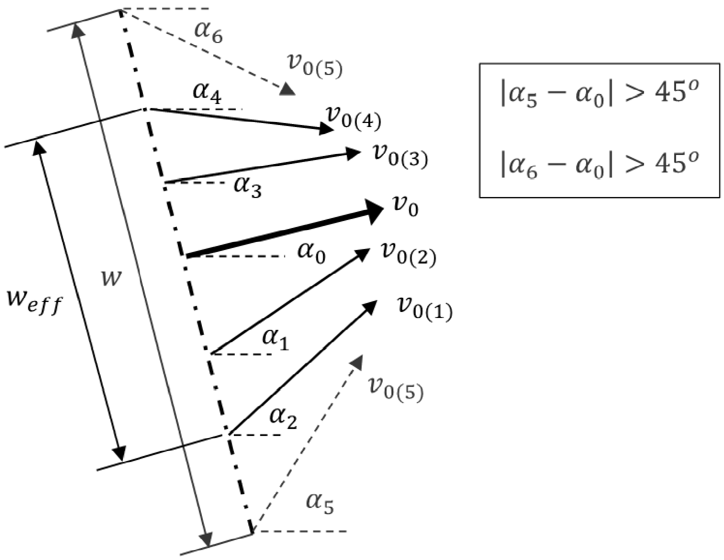


Figure 3.13: Limitation of the distribution width with respect to the direction of the resultant shear force. From Pacoste, Plos & Johansson (2012).

4 Resistance in Reinforced Concrete Slabs

In the previous chapters, different methods to determine the load effects that will act on a certain structure were described. In this chapter focus is put on how to determine the resistance of the structure. Basically, the resistance of the structure needs to be equal to or greater than the applied load effects. Similar as with the different methods of structural analysis, the resistance can be calculated in several different ways. Different assumptions and methods can lead to different levels of accuracy, but also different levels of work amount.

When designing a reinforced concrete slab, ductile failure behaviour is desired. This is because the ductility allows for large deformations and redistributions of stresses to take place before the structural member fails. The redistribution of local peak stresses makes it possible to reach a higher load before a global collapse mechanism is formed. When the design is not properly done, the failure mode could be brittle. This is a dangerous failure mode since it propagates fast and without warning.

In conventionally reinforced concrete slabs, no pronounced membrane effects will be present. However, the applied loads create large bending moments and shear forces within the slab that need to be controlled and designed for. This chapter gives a brief description of how the moment resistance is currently determined. Since the aim of this thesis is to investigate shear force distribution and modelling principles in concrete slabs, a more thorough description of the shear- and punching shear resistance determination is given. The methodology given is mainly based on the one given in *Eurocode 2*, SS-EN 1992 (2005). For the shear force calculations, however, methods to determine the resistance with a higher level of accuracy are given as well. The different design levels are adopted from *Model Code for Concrete Structures 2010*, CEB-FIP (2013).

4.1 Moment Resistance

The moment resistance in concrete structural members depends highly on the cross-sectional width (w) and height (h), as well as on the reinforcement area (A_s), Al-Emrani *et al* (2011). These parameters must be determined so that the moment resistance is sufficient in order to carry the loads that are applied on the member. Different combinations of these parameters could give sufficient resistance, and therefore the design of the member will be an optimization problem. However, in slabs the moment resistance is preferably determined per unit width. If also the slab height is fixed the only parameter that could be modified to increase or decrease the resistance is the reinforcement amount.

The moment resistance of a concrete structural member is obtained by an inner force couple, coming from compression in the concrete and tension in the reinforcement, Al-Emrani *et al* (2011). The moment resistance can therefore be determined as the force in the reinforcement multiplied by the inner lever arm, as in Equation (4.1).

$$\begin{aligned} M_{Rd} &= f_{yd} \cdot A_{s.tot} \cdot z \\ m_{Rd} &= f_{yd} \cdot A_s \cdot z \end{aligned} \tag{4.1}$$

Where: M_{Rd} is the design value of the moment resistance of the cross-section, m_{Rd} is the resistance per unit width, f_{yd} is the design value of the yield strength in the reinforcement and z is the internal lever arm.

The required reinforcement area can then be approximated in a simplified way. For cross sections with rectangular compression zones the inner lever arm can be approximated as 90% of the effective height of the cross section. The approximation of the required reinforcement area can then be calculated as in Equation (4.2).

$$A_{s.tot} = \frac{M_{Ed}}{f_{yd} \cdot 0.9d}$$
$$A_s = \frac{m_{Ed}}{f_{yd} \cdot 0.9d} \tag{4.2}$$

The reinforcement area should be limited both downwards as well as upwards in order for the reinforcement to be able to yield before it breaks or before concrete crushing occurs. The reader is referred to *Eurocode 2* for more information about these limits. The reinforcement should also be able to carry some additional tensile force due to the presence of inclined cracks. This is treated more in detail in the next section. One way to take this into account is to move the moment curve a certain distance, a_l, when performing the curtailment of the bending reinforcement. In that way the need for bending reinforcement becomes greater in adjacent sections within this distance.

4.2 Shear Resistance

As mentioned in Section 3.2.6, shear can be regarded as either one-way or two-way shear (see Figure 3.11). In this section the one-way shear resistance is treated and referred to as ordinary shear resistance. In the next section two-way shear resistance is treated and referred to as punching shear resistance.

One-way shear action is relevant when the slab is supported by line supports and when the slab is loaded with a line load or a distributed load. An illustration of how one-way shear failure may look like is presented in Figure 4.1.

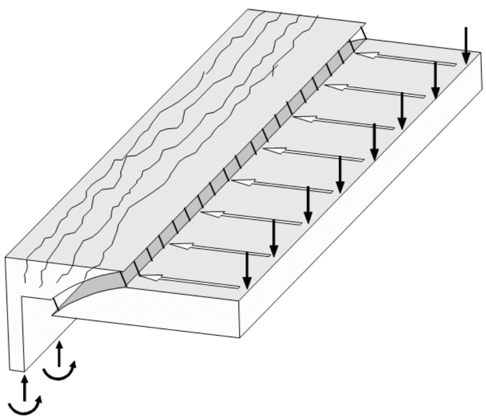


Figure 4.1: One-way shear flow and failure. From Vaz Rodrigues (2007).

The design shear force resistance can in principal be calculated as a sum of the resistance provided by the concrete and the reinforcing steel as stated in Equation (4.3), CEB-FIB (2013).

$$V_{Rd} = V_{Rd.c} + V_{Rd.s} \geq V_{Ed} \tag{4.3}$$

In this equation the term V_{Rd} represents the design value of the shear resistance and V_{Ed} the design value of the applied shear force, while the terms $V_{Rd.c}$ and $V_{Rd.s}$

represents the resistance provided by the concrete and the shear reinforcement respectively.

Depending on the situation, different design approaches can be applicable. The design can be based on a strut-and-tie model, a stress field analysis or a cross-sectional approach where the design section is located at a certain distance from the end of a concentrated load or support.

One-way slabs, similar to beams, cannot redistribute internal stresses transversally and therefore the shear resistance in these members is not as high as in two-way slabs, where such redistribution is possible, CEB-FIP (2013). When two-way slabs are exposed to concentrated loads close to a support line, the effect of redistribution can be accounted for by assuming a uniform distribution of the local peak values of the sectional forces. The extent of this distribution depends on the support condition and is controlled by a load distribution angle. This angle should be taken as 45° or 60° for clamped or simply supported edges respectively, CEB-FIP (2013). The distribution angle is illustrated in Figure 4.2 together with the distribution width b_w.

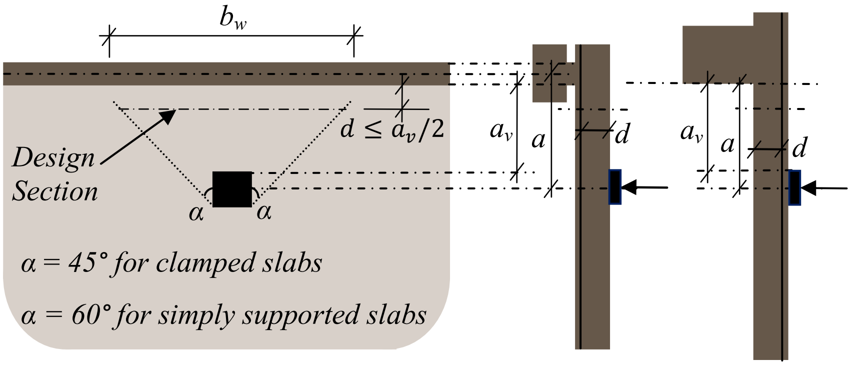


Figure 4.2: (a) Distribution angle α, control section d and distribution width b_w. (b) Simply supported slab (c) Clamped slab. Adopted from CEB-FIP (2013).

When a concentrated load is applied close to a support line as in Figure 4.2, both the punching shear resistance at the load location and the shear resistance of the support location have to be checked.

4.2.1 Members without Shear Reinforcement

If no shear reinforcement is used, the shear resistance is fully dependent on the resistance of the concrete and the bending reinforcement. The following expression could be used to determine the shear resistance of a member without shear reinforcement, CEB-FIP (2013):

$$V_{Rd.c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} z b_w \qquad (4.4)$$

Where: k_v is a strength factor, f_{ck} is the characteristic value of the concrete compressive strength and γ_c is a partial factor for concrete.

As stated in Section 4.1, the bending reinforcement may have to carry an additional tensile force due to the inclined cracks coming from the shear force. If no shear reinforcement is used the longitudinal reinforcement has to be designed to be able to carry the additional tensile force as in Equation (4.5)

$$\Delta F_{td} = V_{Ed} \tag{4.5}$$

The factor k_v is determined differently depending on the level of approximation. In CEB-FIP (2013) there are two levels of approximation described. The factor is determined as stated in Equation (4.6) and (4.7).

Level 1 Approximation

$$k_v = \frac{180}{1000+1.25z} \qquad z \text{ in [mm]} \tag{4.6}$$

Level 2 Approximation

$$k_v = \frac{0.4}{1+1500\varepsilon_x} \cdot \frac{1300}{1000+k_{dg}z} \qquad z \text{ in [mm]} \tag{4.7}$$

The factor k_{dg} is a factor that takes the aggregate size into account. The size of the aggregates is relevant since larger aggregates yield a higher resistance due to interlocking in a crack. The factor is taken as 1.0 if the maximum aggregate size, d_g, is not less than 16 mm. Otherwise it can be calculated as: $k_{dg} = \frac{32}{16+d_g} \geq 0.75$.

The longitudinal strain, ε_x, are taken into account in the level 2 approximation since it also affects the interlocking in the crack. A higher strain gives less interlocking between the two sides of a certain crack. If a cross-sectional design is performed this strain can be calculated as: $\varepsilon_x = \frac{1}{2E_sA_s}\left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed}\left(\frac{1}{2} \pm \frac{\Delta e}{z}\right)\right)$. There are several conditions that need to be fulfilled in order to be able to use this equation and the reader is referred to CEB-FIP (2013) for a description of these.

The level 1 approximation is based on the same equation as level 2, but with some pre-defined assumptions made with respect to the longitudinal strain and the maximum aggregate size. These assumptions are made in order for the design process to be as simple as possible.

In Eurocode 2, SS-EN 1992-2 (2005) the shear resistance in a member without shear reinforcement is calculated based on another equation compared to the one described above (Equation (4.4)). The main difference between the approaches made in CEP-FIP (2013) and SS-EN 1992-2 (2005) is that the latter takes the bending reinforcement into account. The resistance is then calculated as in Equation (4.8).

$$V_{Rd.c} = \left(C_{Rd.c}k(100\rho_l f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right)b_w d \tag{4.8a}$$

The resistance should not be less than:

$$V_{Rd.c} = \left(v_{min} + k_1\sigma_{cp}\right)b_w d \tag{4.8b}$$

Where:

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \qquad d \text{ in [mm]}$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0.02$$

A_{sl} is the area of the tensile reinforcement according to Figure 4.3.

b_w is the smallest width of the cross section in the tensile zone [mm]

$\sigma_{cp} = N_{Ed}/A_c$ A_c is the concrete cross-sectional area

The values of $C_{Rd.c}$, v_{min} and k_1 can be found in the National Annex. Recommended values are: $C_{Rd.c} = 0.18/\gamma_c$, $v_{min} = 0.035k^{3/2} \cdot f_{ck}^{1/2}$ and $k_1 = 0.15$.

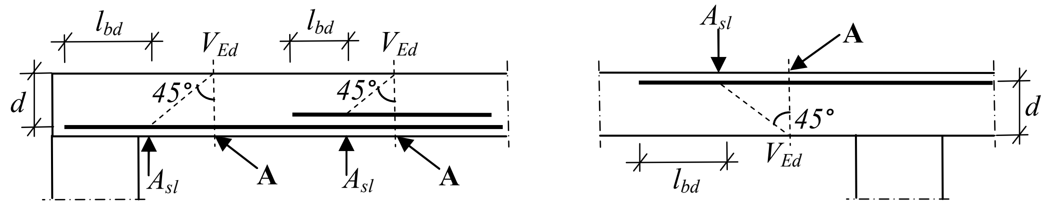


Figure 4.3: Determination of A_{sl} in expression (4.8). Adopted from SS-EN 1992-2 (2005).

4.2.2 Members with Shear Reinforcement

When shear reinforcement is provided within a beam or a slab, the design shear resistance can be determined as the sum of the resistance from the concrete itself and the resistance from the shear reinforcement. According to CEB-FIP (2013) the resistance can be determined as stated in Equation (4.3). There is, however, an upper limit of the design shear resistance due to crushing of the compressive concrete struts, and can be determined as in Equation (4.9):

$$V_{Rd.max} = k_c \frac{f_{ck}}{\gamma_c} b_w z \sin\theta \cos\theta \tag{4.9}$$

The angle θ represents the inclination of the compressive struts and the factor k_c is a strength reduction factor determined as: $k_c = k_\varepsilon \eta_{fc}$. The factor k_ε takes the state of strain into account and is determined differently depending on which level of approximation that is adopted. The second factor, η_{fc}, is a factor that takes the more brittle failure of concrete with higher strength into account and is determined as: $\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1.0$.

The resistance of the concrete itself is calculated as stated before in Equation (4.4) and the resistance provided by the shear reinforcement is determined as in Equation (4.10):

$$V_{Rd.s} = \frac{A_{sw}}{s_w} z f_{ywd} \cot\theta \tag{4.10}$$

Where: A_{sw} is the area of the shear reinforcement, s_w is the spacing of the shear reinforcement and f_{ywd} is the design value of the shear reinforcement strength.

When shear reinforcement is used, the bending reinforcement has to be designed to be able to carry the additional tensile force created by the shear force, similar as to what is stated in Equation (4.5). In this case the additional tensile force is calculated as in Equation (4.11).

$$\Delta F_{td} = \frac{V_{Ed}}{2} cot\theta \tag{4.11}$$

In design, the inclination of the compressive struts may be chosen between certain limits depending on which level of approximation that are adopted. The angle can be chosen as:

$$\theta_{min} \leq \theta \leq 45^{\circ} \tag{4.12}$$

According to CEB-FIP (2013) four different approximation levels can be distinguished when calculating the shear resistance in a member with shear reinforcement.

Level 1 Approximation

The level 1 approximation is based on a variable angle truss model approach. In this approach the larger of the resistances obtained from the shear reinforcement or the concrete is regarded separately:

$$V_{Rd} = max\ (V_{Rd.c}, V_{Rd.s}) \tag{4.13}$$

The value of the factor k_{ε} is in this approximation level put as: $k_{\varepsilon} = 0.55$ and the minimum allowed inclination of the compression field, θ_{min}, is for reinforced concrete members limited to: $\theta_{min} = 30^{\circ}$.

Level 2 Approximation

The level 2 approximation is based on a generalised stress field approach. The resistance is also in this approach determined as the larger of the resistances provided by the shear reinforcement or the concrete, see Equation (4.13).

The value of the factor k_{ε} is in this approximation level put as: $k_{\varepsilon} = \frac{1}{1.2+55\varepsilon_l} \leq 0.65$. The value of ε_l is put as: $\varepsilon_l = \varepsilon_x + (\varepsilon_x + 0.002)\, cot^2\, \theta$. The inclination of the compressive stress field is chosen within the limits as given in Equation (4.12), with a minimum value of: $\theta_{min} = 20^{\circ} + 10000\varepsilon_x$.

Level 3 Approximation

The level 3 approximation is based on the simplified modified compression field theory. In this approach the shear resistances obtained from the shear reinforcement and the concrete are added together in order to better describe the actual resistance, see Equation (4.3). The resistance provided by the shear reinforcement is determined as in the level 2 approximation. The design shear resistance of the concrete itself is determined as in Equation (4.4), where the factor k_v is taken as: $k_v = \frac{0.4}{1+1500\varepsilon_x}\Big(1 -$ *VEdVRd.maxθmin≥0.*

Level 4 Approximation

The approximation on this level is very advanced and often requires extensive knowledge and experience in order to verify that the modelling is correctly done. This approximation is a nonlinear response approach where the shear resistance is determined based on equilibrium-, constitutive- and compatibility conditions. Due to the complexity with a nonlinear approach and because of the fact that nonlinear analyses are not commonly used in design, such an approach is not described further in this thesis.

The approach made in Eurocode 2, SS-EN 1992-1-1 (2005), is based on a truss model. As in both level 1 and 2 described earlier, the shear resistance obtained from the reinforcement is approximated as the resistance of the whole member, excluding the resistance of the concrete itself. However, the value of $V_{Rd.max}$ is determined by a different expression compared to the approaches described earlier. This limiting resistance is instead calculated as (to be compared with Equation (4.9)):

$$V_{Rd.max} = \alpha_{cw} b_w z v_1 f_{cd} / (cot\theta + tan\theta) \quad (4.14a)$$

Where:

α_{cw} is a coefficient that takes the influence of compressive stress into account.

v_1 is a reduction factor of the concrete strength due to shear cracks.

Equation (4.14a) can be rewritten as:

$$V_{Rd.max} = \alpha_{cw} v_1 \frac{f_{ck}}{\gamma_c} b_w z \ sin\theta \ cos\theta \quad (4.14b)$$

This makes it clear that the only difference between Equation (4.9) and Equation (4.14) is the use of the strength factor k_c instead of the two factors α_{cw} and v_1 in the expression above. A more detailed description of the assumptions, recommendations and guidelines according to Eurocode 2 can be found in SS-EN 1992-1-1 (2005).

For members where shear reinforcement is needed, there is a minimum amount of shear reinforcement that must be added to the structural member. This amount needs to be provided in order to prevent brittle failure, SS-EN 1992-1-1 (2005). The limit is as given in Equation (4.15) and the actual shear reinforcement amount is calculated as in Equation (4.16).

$$\rho_{min} = 0.08 \frac{f_{ck}^{0.5}}{f_{yk}} \quad (4.15)$$

$$\rho_w = \frac{A_{sw}}{s \cdot b_w \cdot sin\alpha} \quad (4.16)$$

There are also rules regarding the maximum spacing between the reinforcement bars, but the reader is referred to Eurocode for a complete description of these.

4.3 Punching Shear Resistance

When a concentrated load is applied at a slab or when a slab is supported on concentrated supports, a shear punching failure could occur. This is a brittle failure mode and is therefore highly unwanted since it can take place without any considerable deformations. Therefore, it is important to design the slab with sufficient safety against punching shear. An illustration of such two-way failure is presented in Figure 4.4.

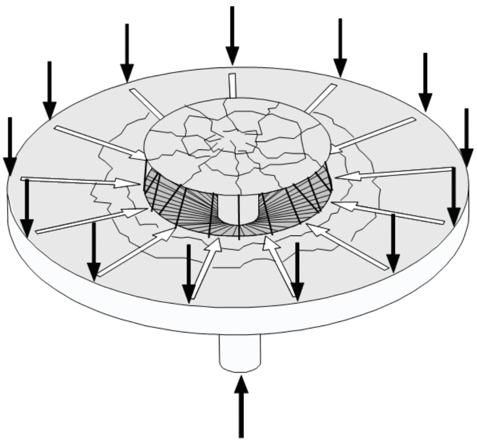


Figure 4.4: Two-way shear flow and failure (punching shear). From Vaz Rodrigues (2007).

It can be made certain that the safety against punching is sufficient by either creating a large enough loading area, or by providing punching shear reinforcement within the member, CEB-FIP (2013). It should also be made certain that progressive collapses when adjacent supports may cause punching shear failure is avoided. In order to avoid this, the deformation capacity should be high so that internal stresses can be redistributed to other regions. Otherwise, if the deformation capacity is limited, integrity reinforcement should be provided.

The design punching shear force that a slab must be able to resist should be calculated at the *basic control perimeter,* b_1, which is a perimeter located at a certain distance from the edge of the supported or loaded area. This distance can in general be taken as $0.5d_v$ from the edge, CEB-FIP (2013), see Figure 4.5. It can be seen that the perimeter is limited by the slab edge in (d).

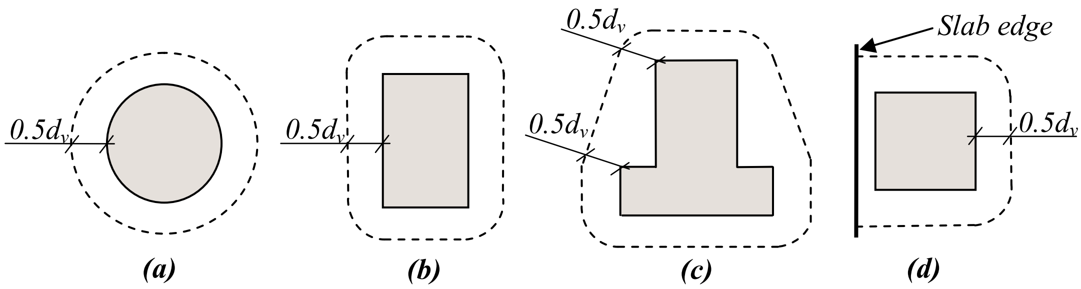


Figure 4.5: Examples of control perimeters in different situations. Adopted from CEB-FIP (2013).

The resistance, however, should be calculated at a perimeter named the *shear-resisting control perimeter,* b_0, which takes into account the non-uniform shear force distribution along b_1. It can be obtained as:

$$b_0 = \frac{V_{Ed}}{v_{perp.d.max}} \tag{4.17}$$

Where: $v_{perp.d.max}$ is the maximum shear force per unit length perpendicular to the basic control perimeter.

Another dimension that is needed in the determination of the punching shear resistance is the *shear-resisting effective depth,* d_v, which depends on the geometry of the slab. The effective depth with respect to shear can be seen in Figure 4.6, where also the effective depth with respect to bending is illustrated.

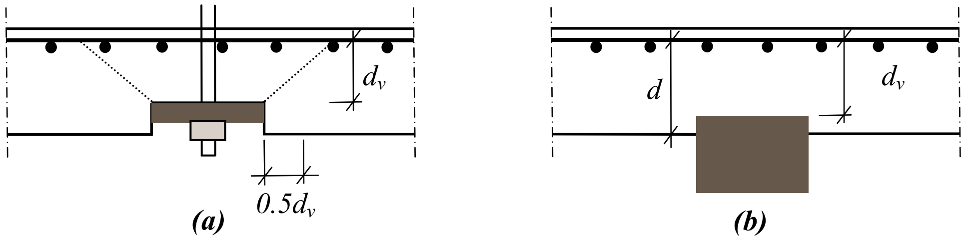


Figure 4.6: Effective depth in different situations in a slab. Adopted from CEB-FIP (2013).

According to CEB-FIP (2013) the punching shear resistance can, similar to the shear resistance as described above, be determined as a sum of the resistance provided by the concrete and by the reinforcement, see Equation (4.3). The resistance provided by the concrete depends, in addition to the geometry and the compressive strength, on the rotations of the slab. More specifically this resistance can be calculated as:

$$V_{Rd.c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{4.18}$$

Where: k_ψ is a parameter that takes the rotations into account and can be determined as: $k_\psi = \frac{1}{1.5+0.9k_{dg}\psi d} \leq 0.6$.

The rotation of the slab, ψ, can be calculated on a basis of different levels of approximation, making the approximation of the resistance more or less accurate depending on which level that is adopted. The levels of approximation and the corresponding assumptions are presented in the following parts. Similar to the ordinary shear resistance, the punching resistance is also dependent on the maximum aggregate size, d_g. This is taken into account by the factor k_{dg}, which is calculated equally as for the shear resistance, see Section 4.2.

The resistance provided by the shear reinforcement is dependent on the force that the reinforcement carries and the eccentricity of this force, and can be calculated as:

$$V_{Rd.s} = \sum A_{sw}\, k_e \sigma_{swd} \tag{4.19}$$

Where: k_e is a coefficient that takes the eccentricity into account and can be calculated as: $k_e = \frac{1}{1+e_u/b_u}$. e_u is the eccentricity from the centre of the basic control perimeter to the shear force resultant and b_u is the diameter of a circle of the same area as the basic control perimeter. σ_{swd} is the stress in the shear reinforcement and is determined as: $\sigma_{swd} = \frac{E_s\psi}{6}\left(1 + \frac{f_{bd}}{f_{ywd}} \cdot \frac{d}{\phi_w}\right) \leq f_{ywd}$. ϕ_w is the diameter of the shear reinforcement bars and f_{bd} is the bond strength.

If punching shear reinforcement is needed, a minimum limit of the amount of punching shear reinforcement is required because of the risk of insufficient ability to deform. This limit can be determined as:

$$\sum A_{sw}\, k_e f_{ywd} \geq 0.5 V_{Ed} \tag{4.20}$$

The upper limit for the total design punching resistance is determined as the capacity of the concrete struts to resist crushing and can be calculated as:

$$V_{Rd.max} = k_{sys} k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \leq \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{4.21}$$

The coefficient k_{sys} takes the shear cracking and compression struts into account and can be put to 2.0 if the reinforcement is properly detailed.

Similar as with ordinary shear, CEB-FIP (2013) presents four different approximation levels that could be adopted to reach a more or less accurate estimation of the resistance against punching shear. The different approximations aim at describing the rotation of the slab more accurately with increasing level.

Level 1 Approximation

In the level 1 approximation, the rotation failure is estimated on the safe side for a slab designed based on a linear elastic analysis with considerable stress redistributions. The rotation failure depends on the location of the zero moment section (with respect to the support axis), and can in this case be calculated as:

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{yd}}{E_s} \tag{4.22}$$

Where: r_s is the position where the radial bending moment is zero. The value needs to be the highest possible when applying a level 1 approximation and it can be estimated as $0.22L$ for each main direction separately (if the ratio between the spans in the two directions is between 0.5-2.0).

Level 2 Approximation

In the level 2 approximation the bending moment redistribution is taken into account and the slab rotation can then be determined in each direction in a reinforced slab as:

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{yd}}{E_s} \left(\frac{m_{Ed}}{m_{Rd}}\right)^{1.5} \tag{4.23}$$

The width of the support strip can be determined as (if no restriction due to an adjacent edge is relevant): $b_s = 1.5\sqrt{r_{s.x} r_{s.y}} \leq L_{min}$.

Where: L_{min} is the least of the span widths in x- and y-directions.

Level 3 Approximation

For slabs where the ratio between the span lengths in the different directions is not between $0.5 - 2.0$ or for irregular slabs, an approximation on level 3 should be made. In the two first approximations, a coefficient of 1.5 was used. This value can be replaced with 1.2 if r_s and m_{Ed} are calculated according to linear elastic modelling. A minimum value of $r_s \geq 0.67 b_{sr}$ should be used at edge- or corner columns.

Level 4 Approximation

If an even more detailed approximation of the rotation ψ is to be made, this could be based on a nonlinear analysis with consideration to cracking and plastic behaviour within the slab. Such approaches are not within the scope of this thesis and therefore it is not treated more here.

Eurocode Approach

The approach made in SS-EN 1992-1-1 (2005) is quite similar as to what was described earlier However, the basic control perimeter b_1 is defined somewhat differently. Typical basic control perimeters are defined in Figure 4.7. It can be seen that the distance from the face of the support to the perimeter is $2d$ instead of $0.5d_v$ as in Figure 4.5. Observe that the basic control perimeter is referred to as u_1 instead of b_1 as it was earlier.

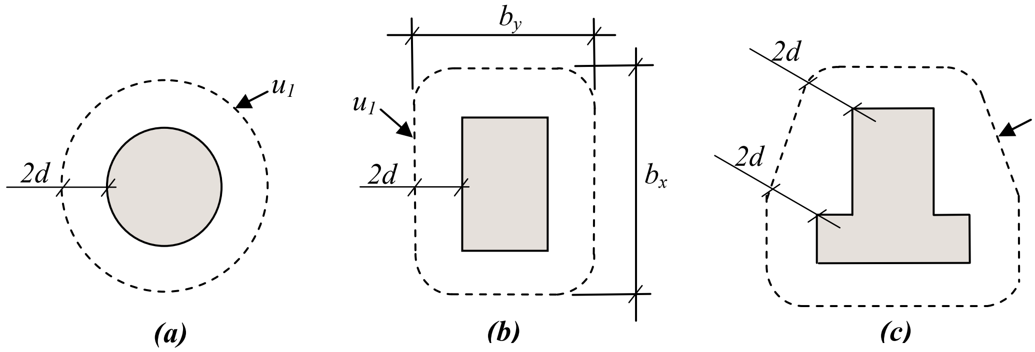


Figure 4.7: Examples of control perimeters in different situations. Adopted from SS-EN 1992-1-1 (2005).

The effective depth can in this case be determined as the mean value between the effective depths in the longitudinal (x) and transversal (y) directions.

$$d_{eff} = \frac{(d_x + d_y)}{2} \tag{4.24}$$

In SS-EN 1992-1-1 (2005), the procedure for punching shear calculation involves controls at the basic control perimeter as defined above, but also at the face of the column. If shear reinforcement is needed, an additional perimeter should be found further out from the support where reinforcement is no longer needed.

The punching shear resistance of the concrete itself is determined as in Equation (4.25), and it can be seen that it is basically the same equation that is used for ordinary shear resistance, see Equation (4.8a). The difference is that for punching shear the resistance is determined per square meter. This is due to the fact that several checks, with different lengths of the perimeter, are needed.

$$v_{Rd.c} = C_{Rdc} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \tag{4.25}$$

In this case the longitudinal reinforcement as well as the transversal reinforcement needs to be taken into account:

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lx}} \leq 0.02 \tag{4.26}$$

If shear reinforcement is required, the resistance should instead be calculated according to Equation (4.27):

$$v_{Rd.cs} = 0.75 v_{Rd.c} + 1.5 \frac{d}{s_r} A_{sw} f_{ywd.ef} \frac{1}{b_1 d} \, sin\alpha \tag{4.27}$$

Where: s_r is the radial spacing of perimeters of the shear reinforcement, $f_{ywd.ef}$ is the effective design strength of the punching shear reinforcement ($f_{ywd.ef} = 250 + 0.25d \leq f_{ywd}$) and α is the angle between the shear reinforcement and the plane of the slab.

5 Example Bridge

This thesis aims to investigate how different design choices are influencing the final design of concrete bridge decks in terms of shear reinforcement amounts. In order to be able to make calculations and design choices that reflect the reality, they should be made for a realistic design situation so that the magnitudes of the effects are relevant for a real design situation. Thus, the bridge that should be investigated must be able to represent real bridges.

One way to find a realistic design situation is to perform a case study where the real geometry and site conditions could be implemented. However, in this thesis a fictitious bridge has been created based on realistic parameters and choices. In that way it has been possible to keep the bridge as simple as possible and therefore also to avoid the presence of complicated results and effects that are hard to interpret. Only one bridge type and geometry has been investigated, and therefore all possible influences have not been covered. However, the choices have been made in order for the bridge to cover as many effects as possible.

In this chapter a description of how the example bridge was created is given. Section 5.1 contains a description of the choices made regarding the geometry. In Section 5.2 the establishment of the finite element model is described, and in Section 5.3 it is described how the bending reinforcement was designed for the bridge.

5.1 Bridge Geometry

The geometry of the bridge is highly influencing the behaviour of the whole structure. First of all, there are many different bridge types that carry the loads in different ways. Moreover, there are also many different types of supports that yield different support conditions and different critical aspects. The dimensions of the bridges in general, and the span widths specifically, are determining how the bridge may be designed in terms of reinforcement and structural solution.

5.1.1 Bridge Concept

There are two types of bridges that are frequently used in both smaller and larger bridge structures; slab bridges and beam bridges. Since one of the main objectives for this thesis is to investigate different ways to model the shear force distribution in concrete bridge deck slabs, the slab bridge was chosen. It should be noted, however, that the shear force distribution issue is also relevant for beam bridges where the slab is acting as a cantilever slab. The distribution of the shear force from concentrated wheel loads close to the line support (the beam) is one of the major considerations in shear force design.

The type of support is also influencing the type of design considerations that have to be treated. The line support, mentioned above, is one of the supports that would create a certain set of considerations. Another important support type is the point supports, where both ordinary shear force and punching shear force failure modes are relevant. In this thesis the slab bridge is chosen to be supported by columns, which is translated into point supports.

Basically, the chosen bridge concept is a continuous slab bridge on column supports. An illustration of this bridge concept is presented in Figure 5.1.

Figure 5.1: Example of a continuous plate bridge on column supports.

5.1.2 Dimensions

Apart from the bridge- and support type, choices have been made regarding the dimensions of the bridge. The dimensions have been chosen so that they follow recommendations and reasonable values.

The outer spans in a continuous bridge should be between 50-90% of the inner span, Vägverket (1996). A reasonable value of the outer span width is 80% of the inner span. Moreover, the inner span width should not exceed 25 m, Jernström (2012).With these considerations in mind, the bridge was chosen to consist of three spans, one inner span of 20 m and two outer spans of 16 m each. In that way the outer spans are 80% of the inner spans, while the inner span width is slightly lower than the recommended maximum value. The width of the bridge is not an equally important parameter, and could vary between different bridges depending on what kind of traffic that is expected on the bridge. Widths between 5-25 m are common and in this case a width of 10 m was chosen, which is somewhere between.

The thickness of the deck in slab bridges is normally constant and chosen to be about 5% of the inner span length, Vägverket (1996). In this case it would correspond to a thickness of 1.0 m, which is also the value that was chosen. The columns are often made as circular columns, and by investigating existing bridges the column diameter was chosen to be 1 m, which corresponds quite well with the studied bridges. For simplicity, the thickness of the columns was kept constant. For bridge widths up to about 10 m it is possible to use only one column in each support. However, in order to make the design more effective, two columns were chosen instead. An illustration of the chosen bridge geometry is presented in Figure 5.2.

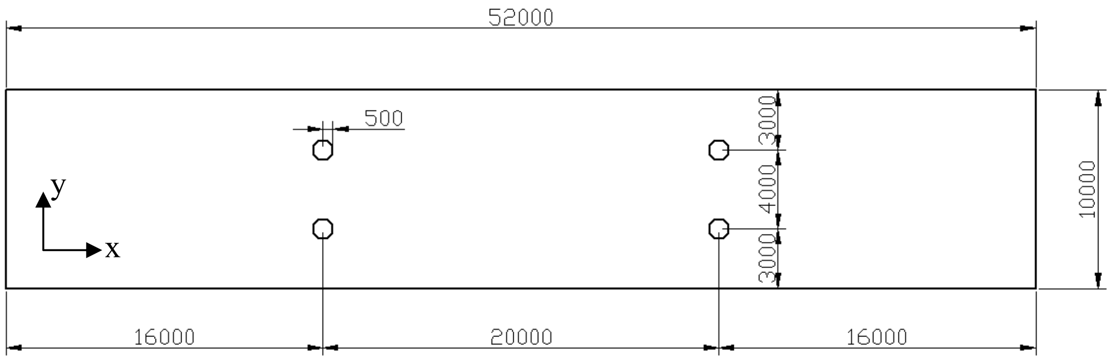


Figure 5.2: Sketch of the chosen geometry for the example bridge, seen from above.

5.2 Finite Element Model of the Bridge

Based on the geometry obtained in the previous section, a three-dimensional finite element analysis was performed in the finite element program Brigade/Plus. The model was created based on the considerations described in Section 3.2, and the loads have been applied based on the load models described in Section 2.2. In this section follows a description of how these assumptions were implemented into a finite element model of the bridge.

5.2.1 Modelling

Since the geometry was kept relatively simple compared to real bridges, the structural idealisation of the geometry and support conditions was relatively straight forward. As described in Section 3.2.2, in bridge design it is recommended to use shell- and beam elements. In this case the deck was modelled with three-dimensional shell elements and the columns with two-dimensional beam elements.

The columns were rigidly attached to the deck, which corresponds to a stiff connection between the support and the deck, see Figure 3.4. This is reasonable since it corresponds to a situation where the columns and the deck are cast together. The end supports were modelled as hinged line supports by preventing the nodes to move in the x-, y-, and z-directions in one support and in the y- and z-directions in the other, see Figure 5.2. In the bottom of all four columns a fully fixed situation was modelled by preventing the movement in all three principal directions and rotation about the three principal axes. It should be noted that it was assumed that the support conditions do not have a pronounced effect on the results regarding the shear force distribution, which is one of the main results of interest in this study. The end supports were located far from the area of interest, which was the area around the columns, and the fixation in the bottom of the columns should primarily influence the bending moment and not the shear force distribution within the slab.

The columns, which were circular with a diameter of 1 m, were simplified in the analysis. Because of practical reasons, the cross-section of the columns were translated into equivalent square sections with a side length, a, as stated in Equation (5.1):

$$a = \frac{\sqrt{\pi}\phi}{2} = \frac{\sqrt{\pi}\cdot 1}{2} = 0.886\ m \tag{5.1}$$

This simplification made it easier to determine result sections for the shear force and the bending moments. The sections chosen for the design were taken as in Figure 3.6 and Figure 3.7. In order to find the exact values of the effective height d in the longitudinal- and transversal directions, the bending reinforcement had to be designed. The process where the bending moment reinforcement was designed is presented in the next section. The results obtained from the bending reinforcement design were used afterwards to find the critical sections for the shear force. An illustration of the finite element model with the different result sections are presented in Figure 5.3. As can be seen, the slab has been partitioned in the x- and y directions at both sides of each column. Since the slab is double symmetric, only the results from the sections connected to one column have been used in design.

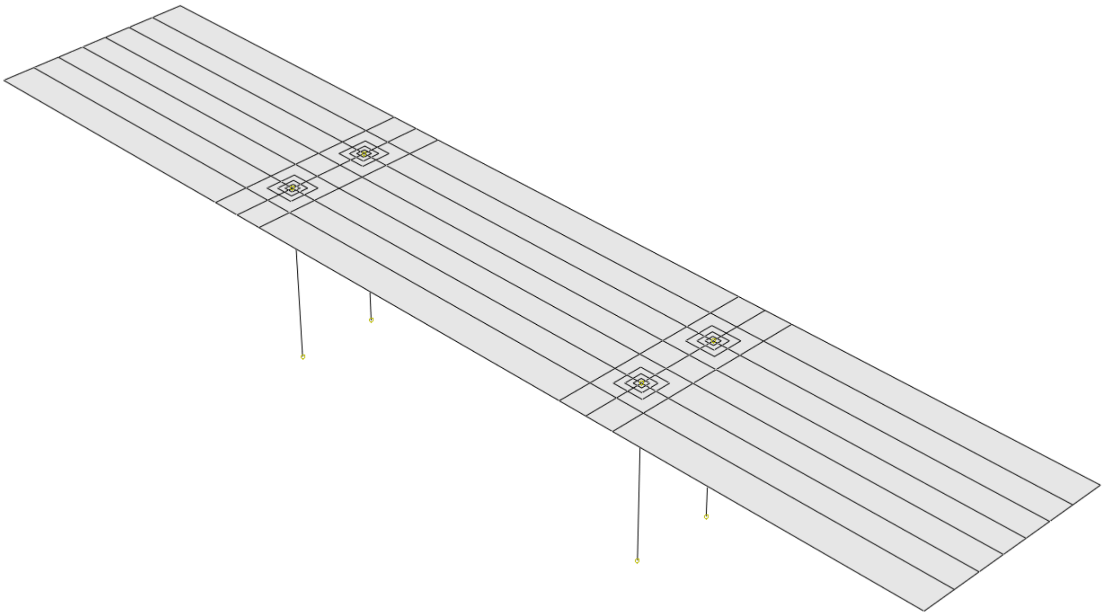


Figure 5.3: The finite element model with the result lines of interest.

As was described in Section 3.2.3, the chosen mesh density should be good enough to capture the overall behaviour and the important effects of the structure, but without creating too long computational times. It was also described how the modelling of supports in one node creates singularities in the FE results. The mesh density must be dense enough for the adjacent regions not to be affected by the singularity in the support node. Therefore, a mesh refinement was created around the column supports. The final element size of the overall structure was approximately 0.60·0.60 m, but in the support regions the element size was much smaller. The refinement of the element mesh is illustrated in Figure 5.4.

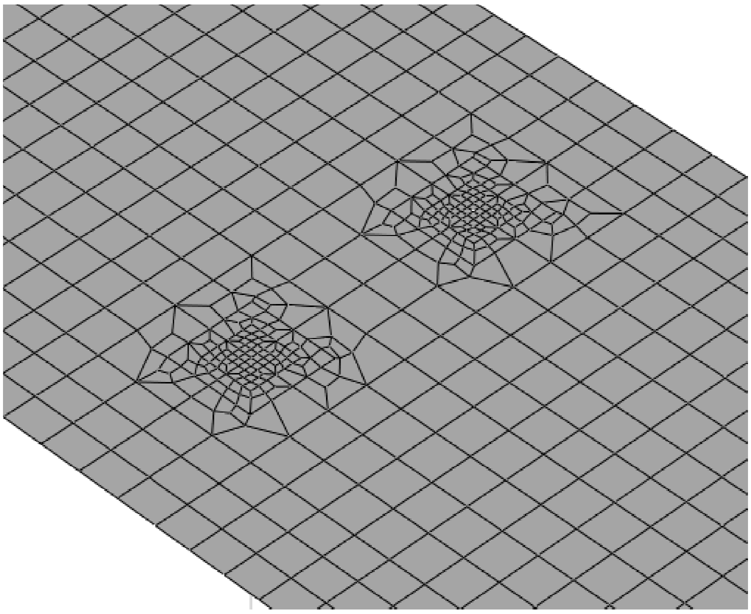


Figure 5.4: Mesh refinement around column supports.

The material properties for the concrete were chosen to correspond to concrete class C35/45. The modulus of elasticity, E_c, was set to 34 GPa, while the Poisson´s ratio, ν, was set to 0.2. The concrete class chosen, as well as the material properties, is a realistic choice since it is frequently used in real bridges. Since the analysis performed is a linear elastic analysis the cracking of the concrete is not included in the material properties given.

5.2.2 Applied Loads

The loads that were applied on the bridge were the self weight and the traffic loads as described in Section 2.2. The bridge in this thesis was 10 m wide, and therefore it was divided into three traffic lanes of 3 m each, and the rest was defined as the remaining area. The traffic should be applied in the centre of each lane, and therefore a traffic load application line was created in the centre of each lane. In order to find the most critical load combinations, the traffic should be applied in every point along these lines. This was simulated by application of the traffic loads along the lines at every point with a distance 0.25 m in between. In the design work an envelope of the highest forces and moments obtained was used. The traffic lines, as well as the mesh density of the overall structure, can be seen in Figure 5.5.

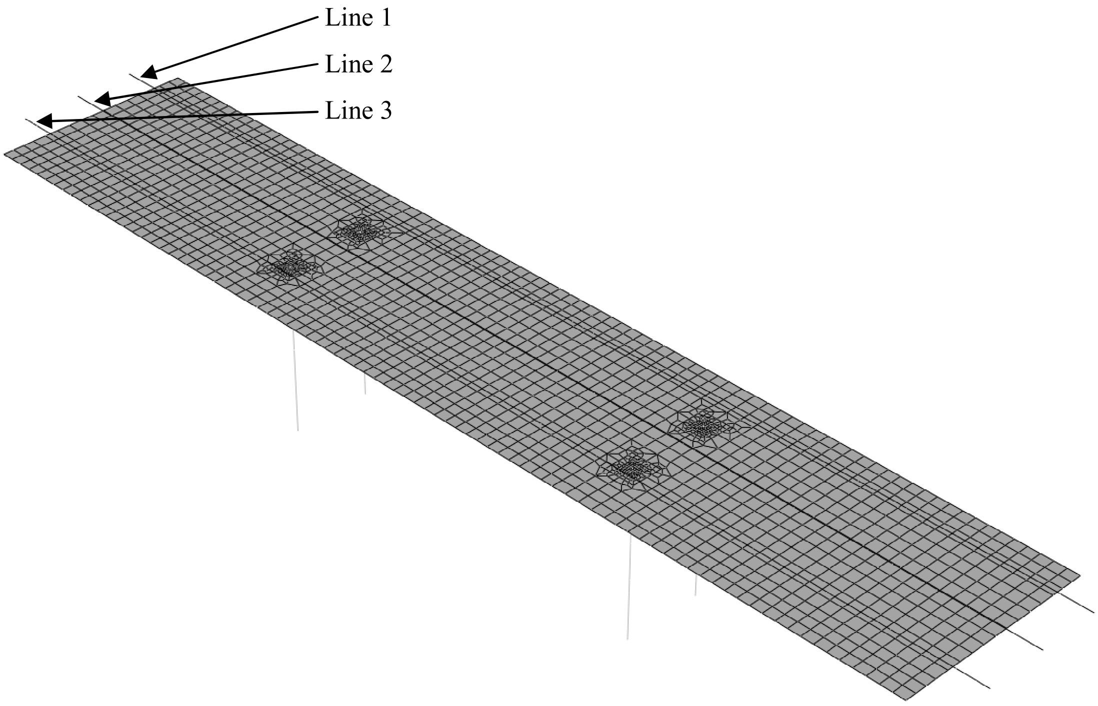


Figure 5.5: Overview of the element mesh for the bridge and traffic lines for application of the traffic loads.

The load combination used in design consisted of the self weight of the bridge itself, as well as the traffic loads defined by Load Model 1 and 2. The self weight was applied by defining the density of the concrete, ρ_c, to 2500 kg/m^3, and the acceleration of gravity, g, to 9.81 m/s^2. In the load combination for ULS, the self weight was multiplied by a factor of 1.35. The traffic loads consisted of both axle loads and distributed loads. According to Load Model 1, there are three magnitudes of the axle loads. They were set to 270 kN, 180 kN and 0 kN, where the highest was applied in the most critical traffic lane (number 1), the second highest in the second most critical lane (number 2), and the lowest in the least critical lane (number 3). The distributed loads applied were 6.3 kN/m^2 for lane number 1 and 2.5 kN/m^2 for lane number 2 and 3 and the remaining area. Load Model 2, however, consisted only of one axle load with the magnitude 360 kN. The traffic loads are more thoroughly described in Section 2.2.2. The traffic loads were in the ULS load combination multiplied by a factor of 1.5. The envelope was created by adding the effects from the self weight with the most adverse of the effects created by the two load models. In that way the most adverse effects in all locations throughout the bridge were obtained.

5.3 Bending Reinforcement in the Bridge Deck

The shear force resistance provided by the concrete is dependent on the amount of bending reinforcement. Therefore, the bending reinforcement had to be designed before the shear force design could take place. In this section follows a description of how the design of bending reinforcement was executed, and the resulting bending reinforcement layout. It should be noted that the bending reinforcement was designed based on the ultimate limit state, disregarding the serviceability limit state considerations. The amount and layout of the bending reinforcement might have been different if the serviceability limit state would also have been included.

5.3.1 Contour Plots and Result Lines for Design

According to Section 3.1.2, the moments should be calculated for the two principal directions only, since it is reasonable to reinforce a slab in these two directions. The reinforcement moments were in the analysis calculated by combining the moments in the two principal directions with the torsional moments, according to Equation (5.2).

$$\begin{aligned} M_{max.1.dir} &= max(SM1) + |SM3| \\ M_{min.1.dir} &= min(SM1) - |SM3| \\ M_{max.2.dir} &= max(SM2) + |SM3| \\ M_{min.2.dir} &= min(SM2) - |SM3| \end{aligned} \tag{5.2}$$

Where: SMi is the sectional moment. SM1 corresponds to the longitudinal moments, SM2 to the transversal moments, and SM3 to the torsional moments. This is corresponding to what was stated in Equation (3.6).

With the reinforcement moments determined, contour plots could be created for each of the reinforcement layers in the two directions. A contour plot of $M_{max.1.dir}$ was used to design the bottom reinforcement in the longitudinal direction, while a plot of $M_{min.1.dir}$ was used to design the top reinforcement in the same direction. The same methodology was applied for the transversal direction. Critical sections were then chosen based on these plots for the longitudinal and transversal directions. When the reinforcement had been designed for each of these critical sections, they were curtailed based on result lines that were also chosen from the contour plots. The design was made so that all the most critical effects were covered in order to create a conservative design. The contour plots used for the design of the bending reinforcement, together with the different result lines used, are presented in Figure 5.6 - Figure 5.9.

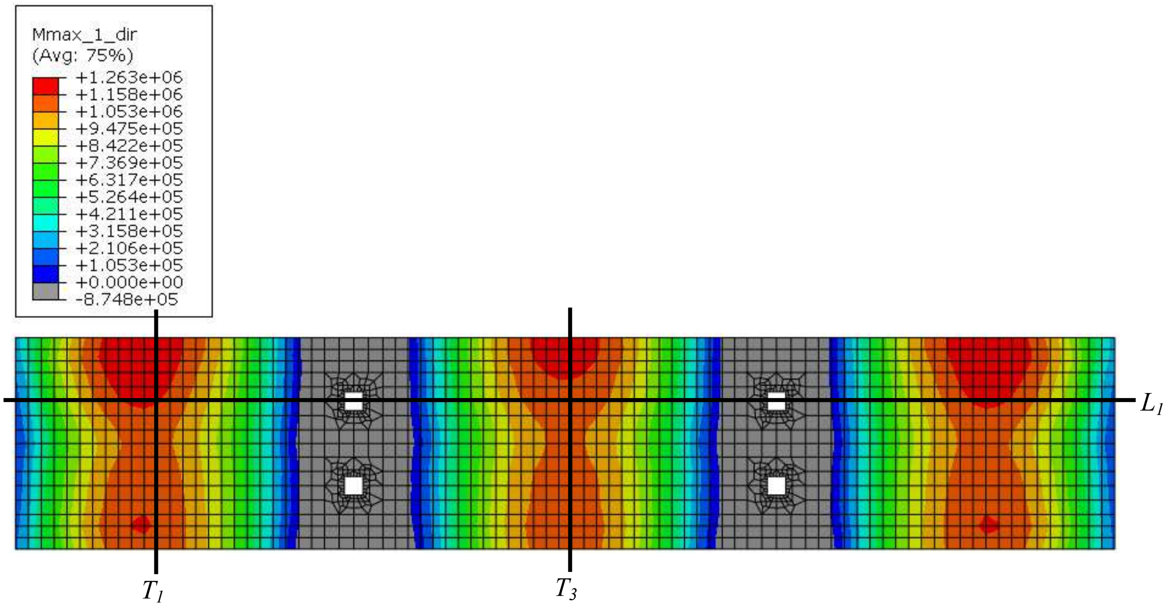


Figure 5.6: Contour plot of the reinforcement moments [Nm/m] for the design of the bottom reinforcement in the longitudinal direction, seen from above. The results for the critical cross-sections are taken from line T_1 and T_3. The curtailment is made along line L_1.

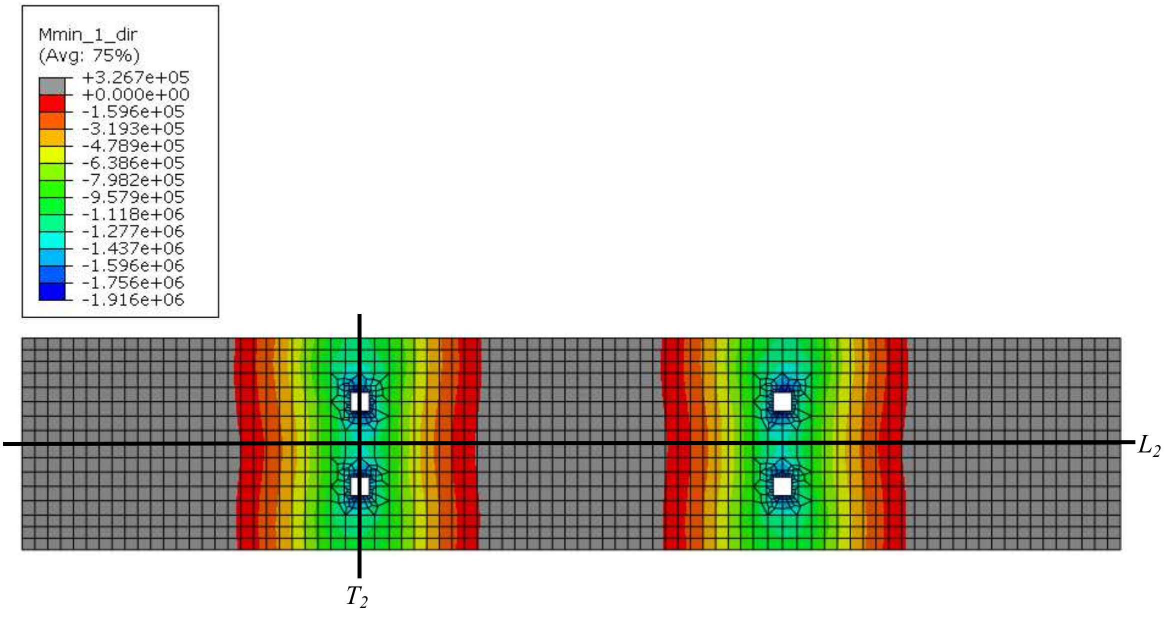


Figure 5.7: Contour plot of the reinforcement moments [Nm/m] for the design of the top reinforcement in the longitudinal direction. The results for the critical cross-section are taken from line T_2. The curtailment is made along line L_2.

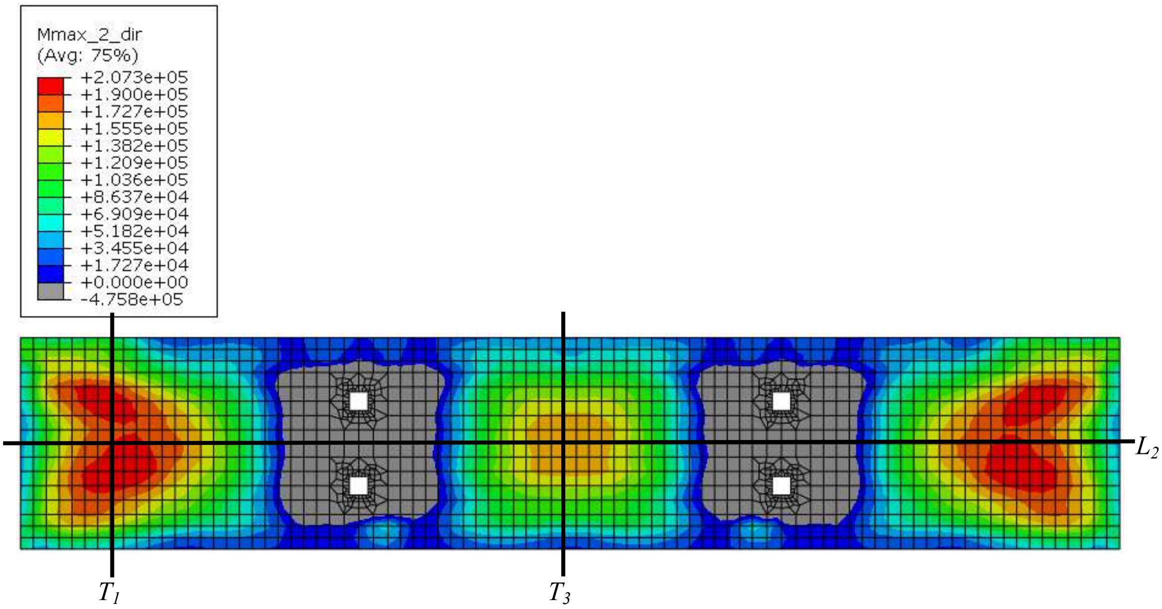


Figure 5.8: Contour plot of the reinforcement moments [Nm/m] for the design of the bottom reinforcement in the transversal direction. The results for the critical cross-section are taken from line L_2. The curtailment is made along line T_1 and T_3.

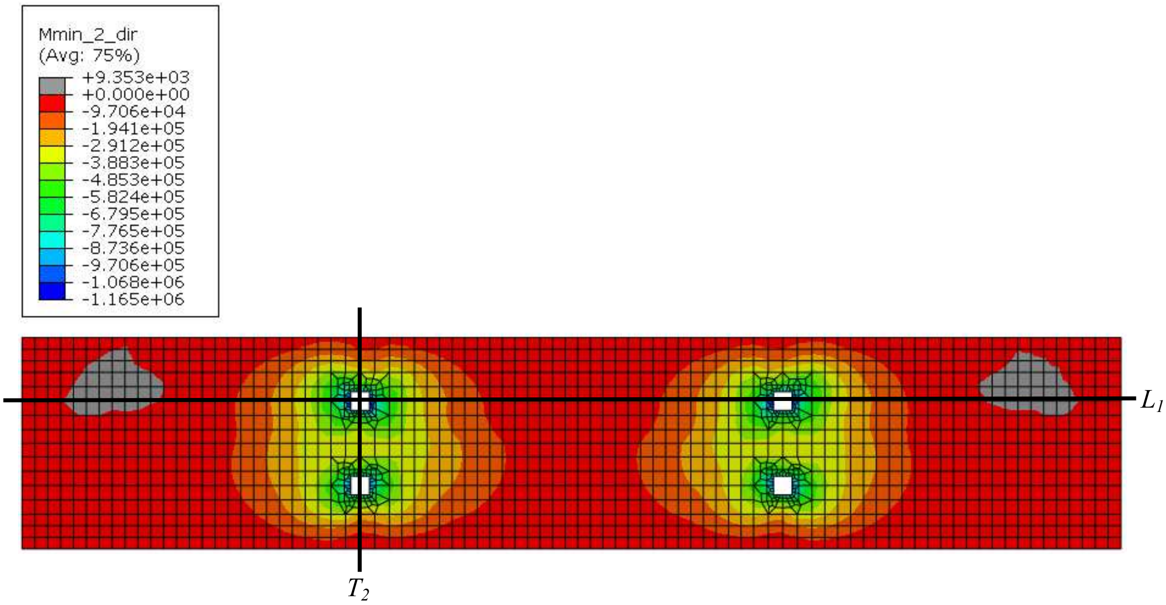


Figure 5.9: Contour plot of the reinforcement moments [Nm/m] for the design of the top reinforcement in the transversal direction. The results for the critical cross-section are taken from line L_1. The curtailment is made along line T_2.

5.3.2 Design of Critical Cross-Sections

From all four contour plots, bending moment diagrams were created for each critical cross-section. For the longitudinal reinforcement the line T_1, T_2 and T_3 corresponds to the critical cross-sections, while line L_1 and L_2 represents the critical cross-sections for the transversal bending reinforcement. From the diagrams the required reinforcement amounts after redistribution of sectional forces, determined according to the recommendations from Pacoste, Plos & Johansson (2012) (presented in Section 3.2.5), were determined. Thus, the distribution widths used were determined according to Equation (3.12), with a minimum width according to Equation (3.13).

In the longitudinal direction, the smallest distribution width obtained was more than 5 m, which is the width of half of the bridge. Since there is one peak value at each of the two bridge halves in the longitudinal direction, the redistribution creates a situation where the mean values can be accounted for in design. In the transversal direction the distribution widths were controlled by the minimum width according to Equation (3.13), which in this case gave a width of 2.89 m.

The redistributed curves represent the required capacity, and the provided capacity needs to be higher than these values in each point. The reinforcement capacities were calculated according to Equation (4.1) and (4.2). First of all, the minimum allowed reinforcement amounts according to the rules in *Eurocode 2* and *TRVK Bro* were determined for the bridge in the two different principal directions. It was found that for the longitudinal direction the minimum reinforcement area was 1560 mm^2/m (Eurocode 2, (9.3-4)), and for the transversal direction 832 mm^2/m (Eurocode 2, 7.3). These values correspond approximately to ϕ25 bars with spacing 300 mm (1636 mm^2/m) and ϕ16 bars with spacing 240 mm (838 mm^2/m) respectively. In the regions where the minimum reinforcement was not enough, additional suitable capacities were provided.

The diagrams for each of these cases are presented in Figure 5.10 - Figure 5.13. It should be noted that for the diagrams coming from T_1-T_3, the curve was modified in order to create symmetric curves. The result output is presented together with the required capacity and the provided capacity within the cross-sections.

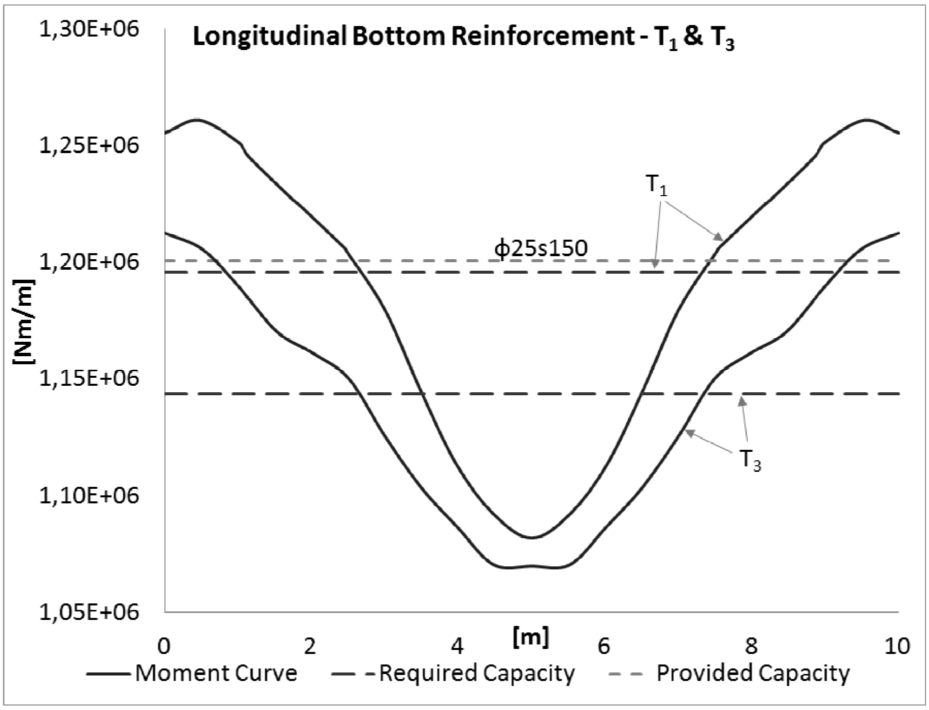


Figure 5.10: Bottom reinforcement moment in the longitudinal direction. Symmetric moment curve, required moment capacity after redistribution and provided moment capacity within the cross-section.

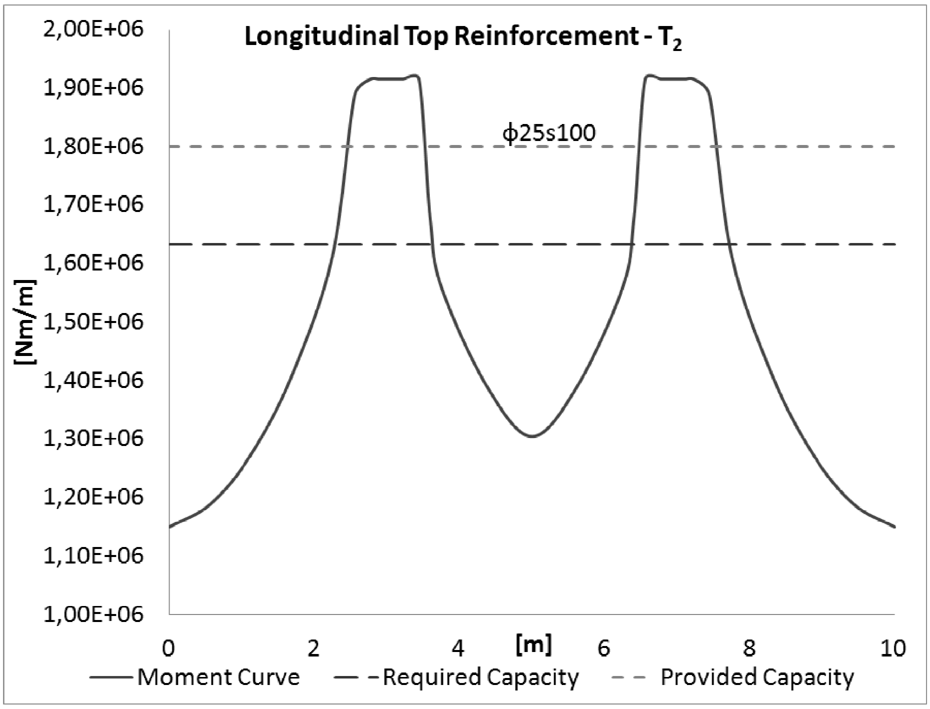


Figure 5.11: Top reinforcement moment in the longitudinal direction. Symmetric moment curve, required moment capacity after redistribution and provided moment capacity within the cross-section.

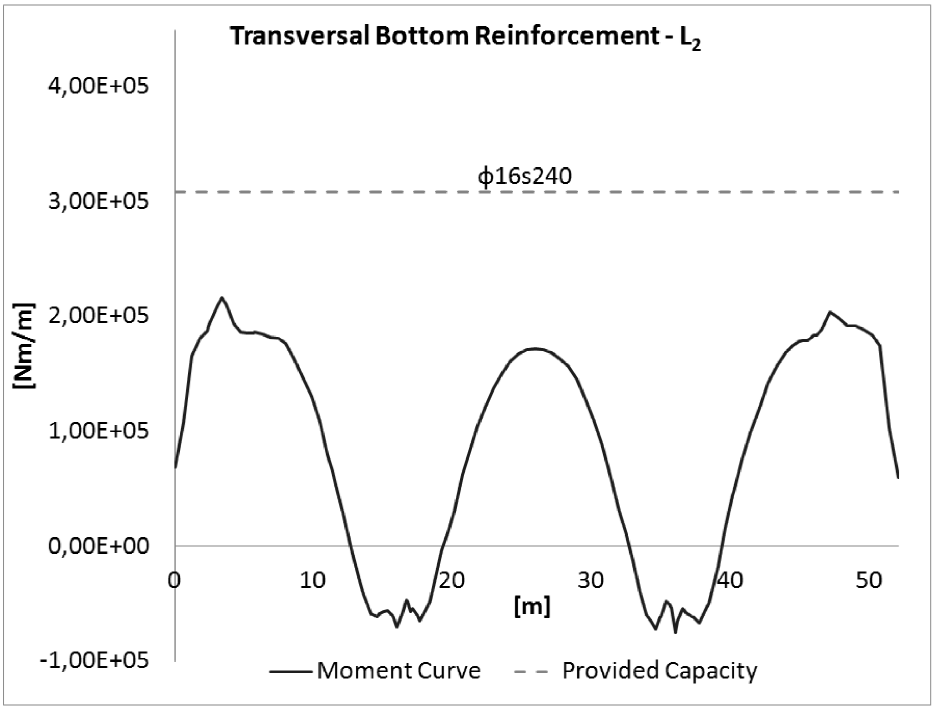


Figure 5.12: Bottom reinforcement moment in the transversal direction. Moment curve and provided moment capacity within the cross-section. The redistribution of the moments has not been considered since the minimum reinforcement capacity is enough in all regions.

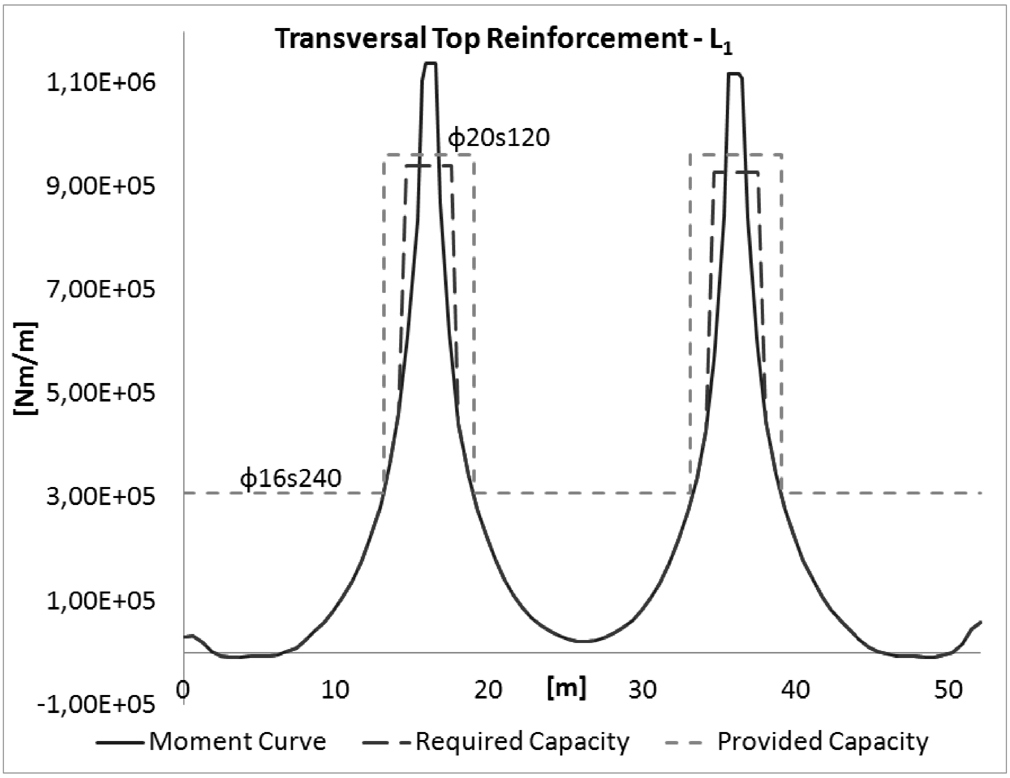


Figure 5.13: Top reinforcement moment in the transversal direction. Moment curve, required moment capacity after redistribution and provided moment capacity within the cross-section.

5.3.3 Curtailment

When the reinforcement amounts in each of the critical cross-sections had been determined, curtailment of each reinforcement layer in the different directions was performed. This was to avoid the over-capacity that would be obtained if all sections where to be designed based on the most critical sections. The curtailment was made along chosen result lines according to Section 5.3.1. The moment diagrams parallel to each reinforcement direction are the basis for the curtailment design. The curve in the diagram was modified and moved horizontally a distance, a_l, in order to take into account the additional tensile force coming from inclined cracks. In this way the bending reinforcement was designed to be able to carry this additional force. The distance was determined according to, SS-EN 1992-1-1 (2005):

$$a_l = \frac{z(cot\theta - cot\alpha)}{2} = 422\ mm \tag{5.3}$$

In addition to the distance a_l, the anchorage lengths for the different cases needed to be determined before the curtailment could be made. The anchorage lengths were determined as, SS-EN 1992-1-1 (2005):

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_{b.rqd} = 700\ mm \qquad \text{(Longitudinal bottom)}$$

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_{b.rqd} = 1000\ mm \qquad \text{(Longitudinal top)} \tag{5.4}$$

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_{b.rqd} = 659\ mm \qquad \text{(Transversal top)}$$

The curtailment is presented in Figure 5.14 - Figure 5.17, where the moment diagram along the longitudinal section is presented. In these curves the required capacity is also plotted, as well as the provided capacities.

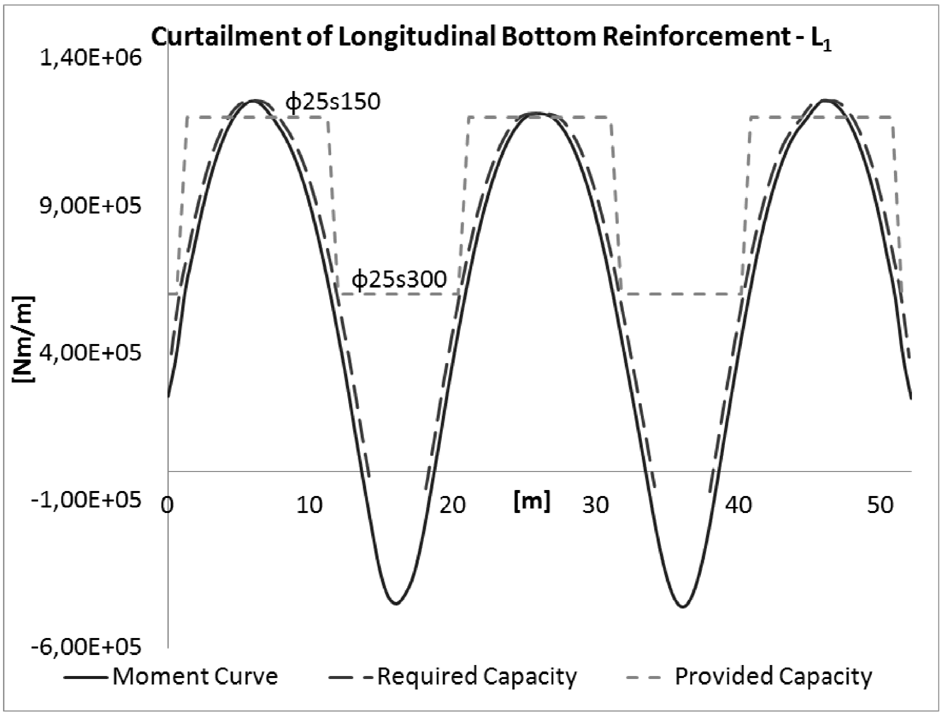


Figure 5.14: Curtailment of the longitudinal bottom reinforcement. Moment curve, moment curve with consideration of the additional tensile force and provided moment capacity along the regarded result line. Note that in the peaks where the required capacity exceeds the provided capacity, transversal redistribution of the moments takes place. Therefore there is enough capacity in these sections as well.

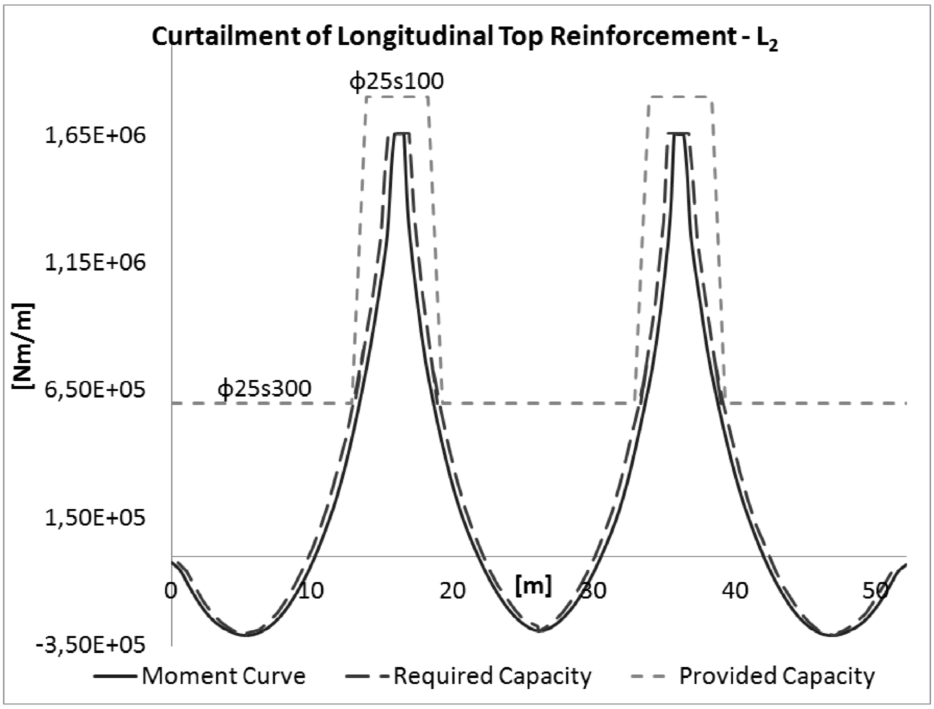


Figure 5.15: Curtailment of the longitudinal top reinforcement. Moment curve, moment curve with consideration of the additional tensile force and provided moment capacity along the regarded result line.

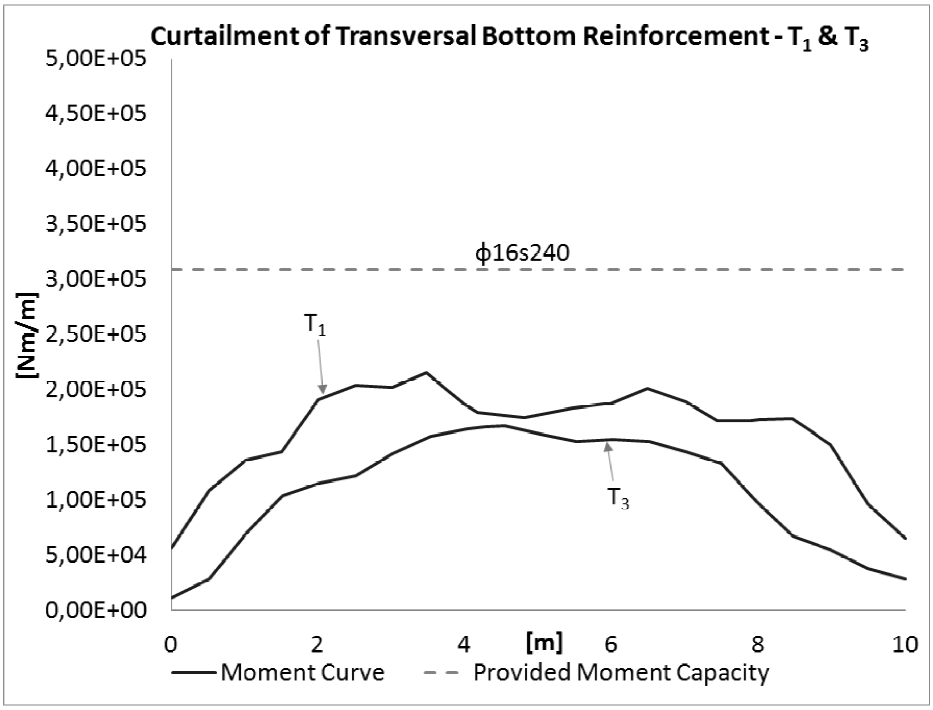


Figure 5.16: Curtailment of the transversal bottom reinforcement. Moment curve and provided moment capacity along the regarded result line. No consideration of the additional tensile force has been taken since the minimum reinforcement capacity is enough in all regions.

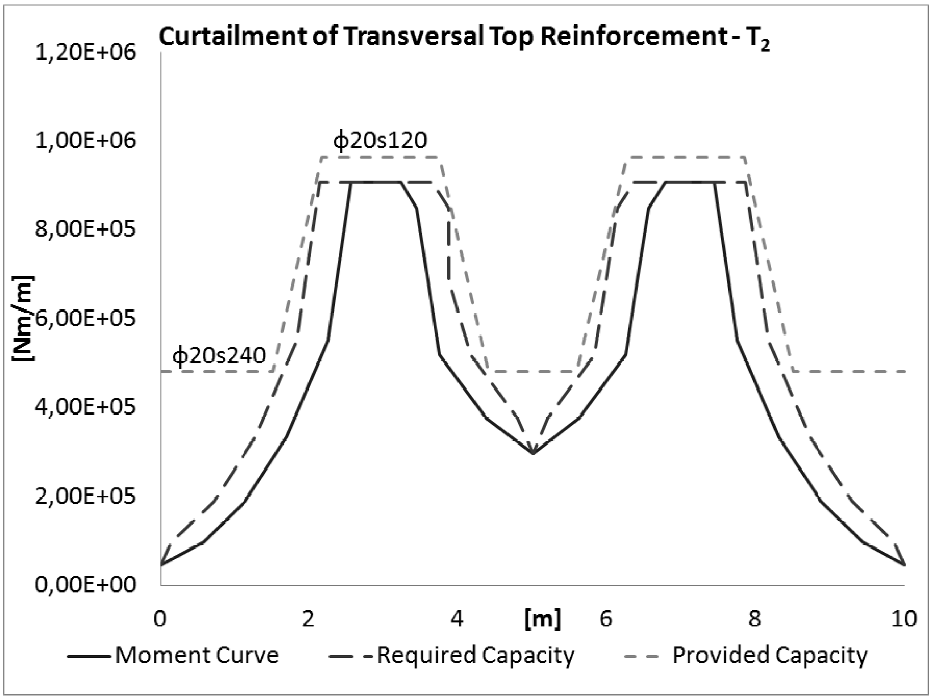


Figure 5.17: Curtailment of the transversal top reinforcement. Moment curve, moment curve with consideration of the additional tensile force and provided moment capacity along the regarded result line.

6 Comparison between Shear Force Design Approaches

The example bridge described in Chapter 5 was studied and designed with respect to shear. The design of the shear force reinforcement was made based on the different design approaches that have been described earlier in this report. A summary of the investigated approaches is given in Section 6.2. Based on the different resulting designs the benefits and drawbacks of these have been analysed. The design work also gave rise to further questions and uncertainties, and the most important of these were investigated and are presented in the following chapters.

Section 6.1 gives a brief description of the punching shear check of the bridge deck. Section 6.2 gives a description of how the design work has been carried out. All assumptions and approximations are described, as well as the equations and theory used. In Section 6.3 the results from the different approaches are presented, and in Section 6.4 a discussion of the results and some questions regarding the design is given.

6.1 Punching Shear Check

The first step in the shear force design was to check the punching shear resistance of the bridge deck. In order to do that, the vertical reaction forces along with the sectional moments where used. The reaction forces were taken in the bottom of the column, and therefore the self weight of the column itself was disregarded, and the sectional moments in the two principal directions were taken in the top of the column. The presence of moments in the top of the column indicates an eccentric load case, and the shear load was modified accordingly. The output data were taken and treated as presented in Equation (6.1).

$$\begin{aligned} V_{Ed} &= RF3_{bottom} - G_{column} = 5060\ kN \\ M_{Ed} &= max\left(SM1_{top}, SM2_{top}\right) = 278\ kNm \\ V_{Ed.mod} &= \beta V_{Ed} = 5172\ kN \end{aligned} \tag{6.1}$$

The bending reinforcement, both the longitudinal and the transversal, contribute to the punching shear resistance of the slab. Therefore, the combined effect from the longitudinal and transversal bending reinforcement was taken into account, see Equation (4.24). The punching resistance should, according to Eurocode, be checked at the at the basic control perimeter, u_1. However, the resistance is limited to the crushing resistance of the concrete, which is checked at the face of the column. The capacities were calculated according to Equation (6.2).

$$\begin{aligned} V_{Rd.c} &= v_{Rd.c} \cdot d_{eff} \cdot u_1 = 5736\ kN \\ V_{Rd.max} &= v_{Rd.max} \cdot d_{eff} \cdot u_0 = 9178\ kN \end{aligned} \tag{6.2}$$

When comparing the result in Equation (6.1) with the one in Equation (6.2) it is obvious that there was no need for punching shear reinforcement within the slab: $V_{Rd.max} < V_{Rd.c} < V_{Ed.mod}$.

6.2 Shear Force Reinforcement Design Methodology

As has been described in previous parts of this report, the shear force design can be executed in several different ways. The shear force components could either be used one by one, or by combining these to use the resultant shear force. Both approaches contain strengths as well as uncertainties. The use of the components is straightforward and the directions of the different sectional forces are easy to follow. However, the interaction between the two components is disregarded and the design may be unconservative in some sections, as one of the components is always ignored. The resultant shear force, on the other hand, takes the interaction between the components into account, but in this case it is hard to fully design for each direction of the resultant, since it varies in all points of the slab. One solution to this problem has been presented in Pacoste, Plos & Johansson (2012), as has been described in Section 3.2.6, where an approximation of the direction for each resultant shear force is given.

The redistribution of the peak shear forces that are obtained in a linear elastic analysis is also an uncertain aspect that is a part of the investigation. This redistribution of the peaks should be approximated in a way that corresponds to the reality in order to avoid too conservative or too unconservative designs. Different approaches to account for this redistribution have also been discussed, and the effects of these have also been investigated.

In order to investigate the effects from both the interaction between the shear force components and the choice of distribution widths, eight different design methods have been used to design the bridge. They are all summarized in Table 6.1, where the resultant shear force is used in the four first approaches, and the shear force components in the other four. For each of these methods, the redistribution of the peak values is made in four different ways.

Table 6.1: Summary of the design approaches used in the comparison.

Approach	Shear Force Method	Redistribution Method
(1)	Resultant	Pacoste *et al* (2012)
(2)	Resultant	TRVR Bro 11 (2011)
(3)	Resultant	No redistribution
(4)	Resultant	Redistribution in chosen regions
(5)	Components	Pacoste *et al* (2012)
(6)	Components	TRVR Bro 11 (2011)
(7)	Components	No redistribution
(8)	Components	Redistribution in chosen regions

Resultant Shear Forces

The method where the resultant shear forces are used (Approach 1-4) has been used as recommended by Pacoste, Plos & Johansson (2012). A more detailed description of the resultant shear force method is given in Section 3.2.6.

Shear Force Components

The shear force component method (Approach 5-8) was used similarly to the resultant shear force method, but the associated shear force components in the other direction were ignored in design. A more detailed description of the shear force components methods are given in Section 3.2.6.

Redistribution according to Pacoste et al (2012)

This method (Approach 1 and 5) is performed based on the recommendations given by Pacoste, Plos & Johansson (2012). These recommendations are the most generous ones adopted in this thesis.

Redistribution according to TRVR Bro 11 (2011)

This method (Approach 2 and 6) follows the recommendations given in Trafikverket (2011b) and should be regarded as the most commonly used guidelines.

No Redistribution

In Method 3 and 7, no distribution widths have been used. However, the shear resistance for the concrete section without shear reinforcement has been used where possible.

Chosen Redistribution

In Method 4 and 8, active choices were made with respect to in which regions the peaks should be redistributed. This is a suggested method based on the results from the other approaches, where an attempt to create the most favourable distributions possible is made. Basically, the regions chosen are only those where the shear force exceeds the shear resistance for the concrete section without shear reinforcement. The redistribution within these regions is made without any restrictions in order to highlight the effects from such methodology.

The slab was designed according to these eight approaches by approximating that the slab transfers the shear forces similar to a beam in the two principal directions. The design of the slab was therefore divided in two different directions, the longitudinal (x) direction where the loads are transferred to each support and in the transversal (y) direction where the loads are divided between the columns within each support. An illustration of how the load was assumed to be transferred within the slab is presented in Figure 6.1.

Figure 6.1: Assumed transfer of shear forces within the bridge deck slab.

The dashed lines in the figure correspond to the critical sections with respect to shear in each main direction, see Section 3.2.4, and all output data of interest were therefore taken from these sections. In this way the design was made based on the same output data, but with different methods for post-processing of the data. In order to find the data of interest, two transversal result lines (T_1 and T_2) and two longitudinal result lines (L_1 and L_2) were used. An illustration of these result lines is presented in Figure 6.2. Since the bridge is double symmetric, these lines are enough to cover the critical sections for shear for the structure. It should be noted that even though the longitudinal result lines are reaching across the whole bridge, the only parts used are those in between the two transversal result lines, as indicated in Figure 6.1.

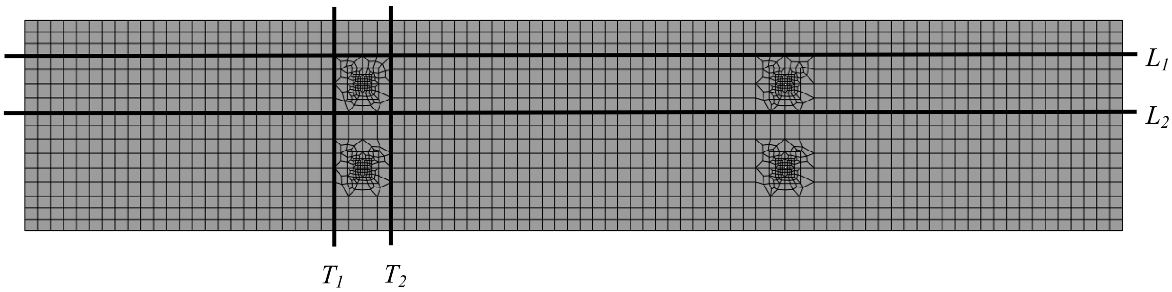


Figure 6.2: Result lines used in the shear reinforcement design.

The output data taken from the finite element program and used for design was basically the maximum (positive) and minimum (negative) shear force components in the x- and y-directions. However, it is also of interest to use the associated shear force components to these outputs. Thus, for the maximum or minimum shear force envelope in x-direction, the associated shear force envelope in y-direction was also taken as output from the program and vice versa. The different approaches were then based on how the output was handled in the post-processing of the data. How the data was post-processed is described in more detail in Section 3.2.6.

Since the aim of the design work was to compare the different design approaches with each other, the inclination of the compressive struts was for simplicity chosen to be 45°, even though it is possible for the designer to choose a considerably lower angle. Regardless of which angle that is chosen, the results would be the same since the same angle would have been used for all approaches. However, if a comparison was to be made with an approach where the inclination cannot be chosen, the results would be misleading since the required reinforcement amounts are lower for a smaller angle.

In addition to the design of the critical sections, the extension of the regions where shear reinforcement need to be provided must be determined as well. This was made so that the total shear force reinforcement amounts could be compared within the bridge. For both the resultant shear force and the shear force component approach the distance from the critical sections to the point where there is no need for shear force reinforcement was determined. This was done by creating four more result lines in the model, see Figure 6.3.

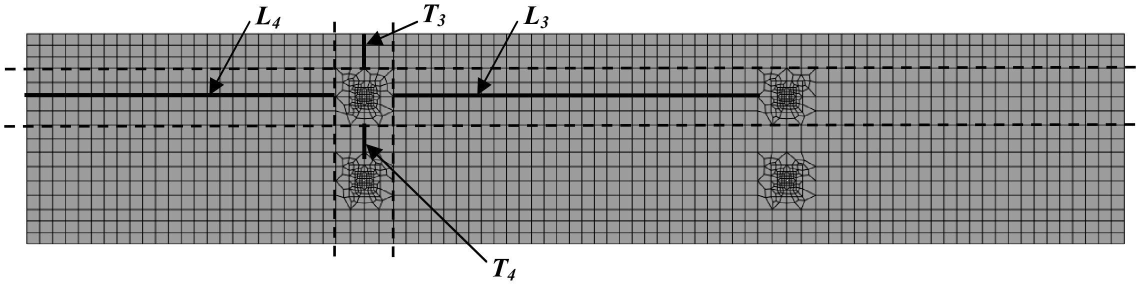


Figure 6.3: Result lines used in the curtailment of the shear force reinforcement.

The transversal result lines, T_3 and T_4, were used to determine the distance from the critical sections in y-direction that needed to be provided with shear force reinforcement. Similarly, the result lines L_3 and L_4 were used to determine this distance for the shear force in x-direction. This is a simplified approach, since the distance would not be equal for the whole sections, but it is a conservative approach that gives results that does not deviate much from the shear reinforcement amounts really needed. Another conservative simplification that was made was that the same shear resistance was provided all the way to the point where shear reinforcement was no longer needed.

The methodology described above for the extension of the shear reinforcement regions was used in the design based on all eight different approaches. The results, in terms of the reinforcement amounts, were then compared in order to find similarities and differences as well as benefits and drawbacks with the different approaches. In this way the accuracy of the different assumptions could be determined. Furthermore, conclusions could be drawn on whether it is beneficial or not to adapt more detailed analysis methods to be used in design.

6.3 Shear Reinforcement Design Results

In this section the results from the different steps of the design are presented. Choices that have been made in the design work are highlighted and motivated where needed. Since the creation of the finite element model of the bridge was already made, see Section 5.2, the first step was to create the output data of interest. This data was then post-processed in order to create design values of the load effects, and the required shear resistance was thereafter determined based on this modified data.

6.3.1 Output Data

The maximum and minimum shear force components in the different directions are assigned according to Equation (6.3), and the associated shear force components according to Equation (6.4), with notations according to Brigade/Plus.

$$
\begin{aligned}
v_{x.max} &= max(SF4) = V_{max.1.dir} \\
v_{y.max} &= max(SF5) = V_{max.2.dir} \\
v_{x.min} &= min(SF4) = V_{min.1.dir} \\
v_{y.min} &= min(SF5) = V_{min.2.dir}
\end{aligned}
\tag{6.3}
$$

$$
\begin{aligned}
v_{y.assoc.to.vxmax} &= SF5\ assoc.to\ max(SF4) = V_{assocTo.Vmax.1.dir} \\
v_{x.assoc.to.vymax} &= SF4\ assoc.to\ max(SF5) = V_{assocTo.Vmax.2.dir} \\
v_{y.assoc.to.vxmin} &= SF5\ assoc.to\ min(SF4) = V_{assocTo.Vmin.1.dir} \\
v_{x.assoc.to.vymin} &= SF4\ assoc.to\ min(SF5) = V_{assocTo.Vmin.2.dir}
\end{aligned}
\tag{6.4}
$$

In the design work the notations used were those to the right in the equations above. These are the notations that are created by the macro used in Brigade/Plus. The contour plots for these result outputs are presented in Figure 6.4 - Figure 6.11. In the finite element model the bridge was not symmetrically loaded as the traffic lanes were applied for traffic in one direction only. The design was, however, made with consideration of possible traffic in both directions. Therefore, the shear force reinforcement around the worst loaded column corresponded to the actual design situation for all columns. In this case it meant that the design load effect that was acting on the upper half of Figure 6.4 - Figure 6.11 was also the basis for design on the lower half.

In the plots illustrated in Figure 6.4 - Figure 6.11 the results in the regions inside the equivalent square cross-section, see Equation (5.1), were omitted. This was made since these regions were not relevant for the shear reinforcement design and since it gives a more clear contour plot. If these regions would have been included the regions of interest would not be illustrated with the same accuracy due to the high shear values in the column regions.

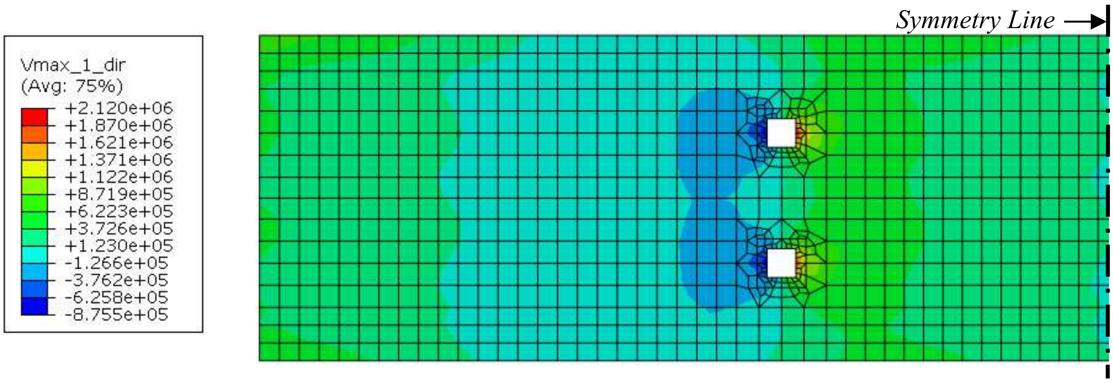


Figure 6.4: Maximum envelope of the shear component in the longitudinal direction, [N/m]

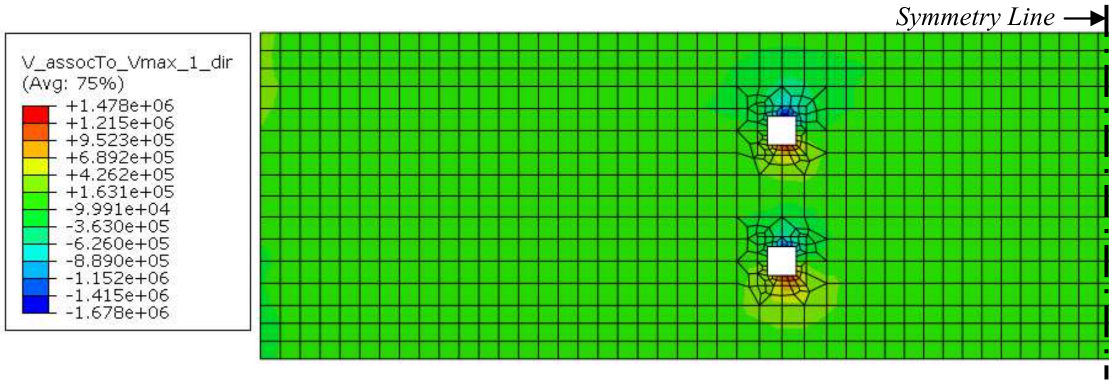


Figure 6.5: Shear component in the transversal direction associated to the maximum shear component in the longitudinal direction, [N/m]

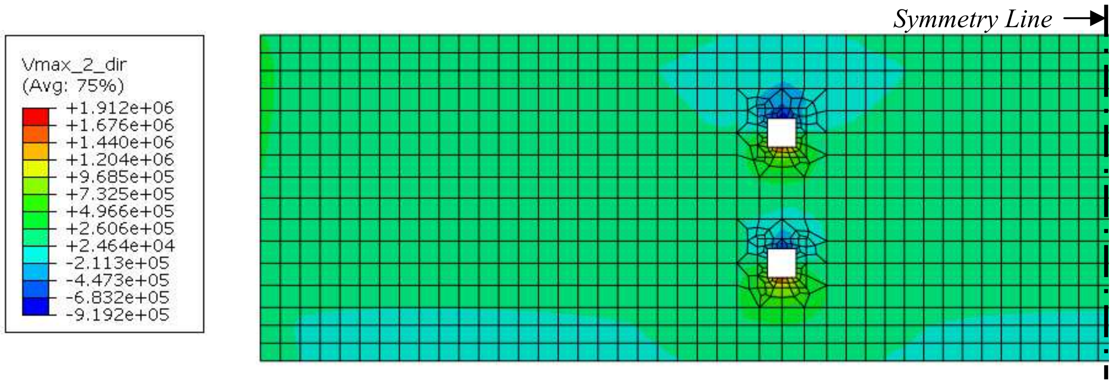


Figure 6.6: Maximum envelope of the shear component in the transversal direction, [N/m]

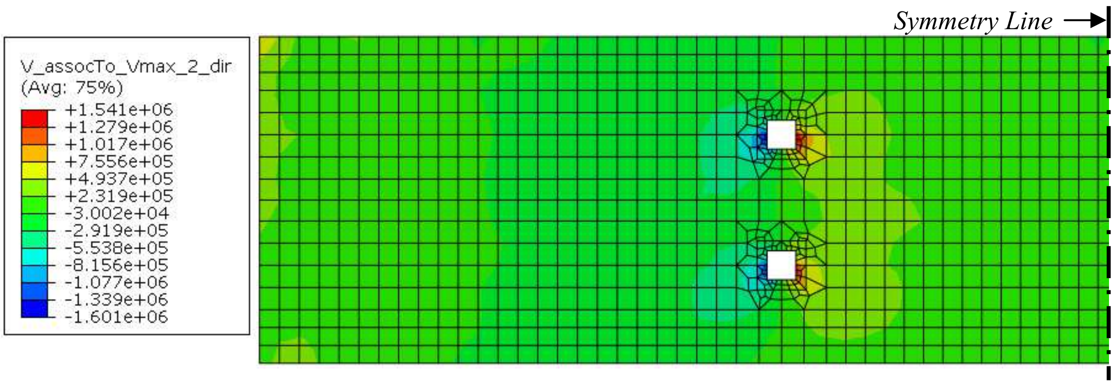


Figure 6.7: Shear component in the longitudinal direction associated to the maximum shear component in the transversal direction, [N/m]

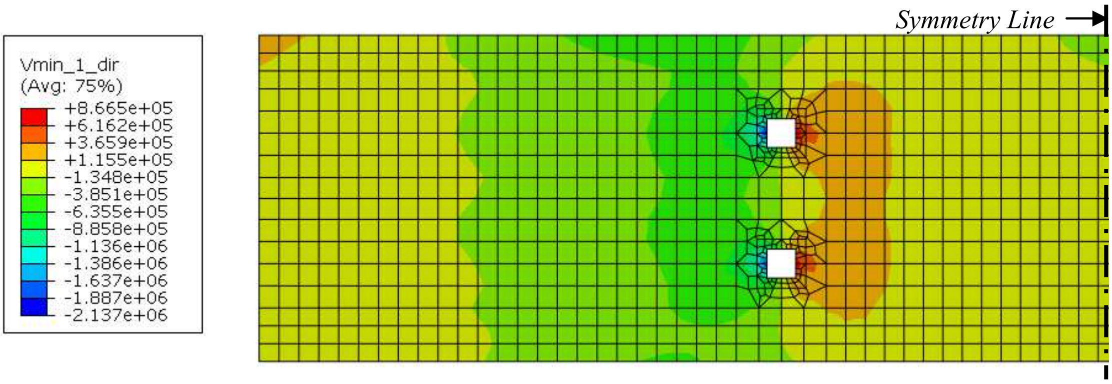


Figure 6.8: Minimum envelope of the shear component in the longitudinal direction, [N/m]

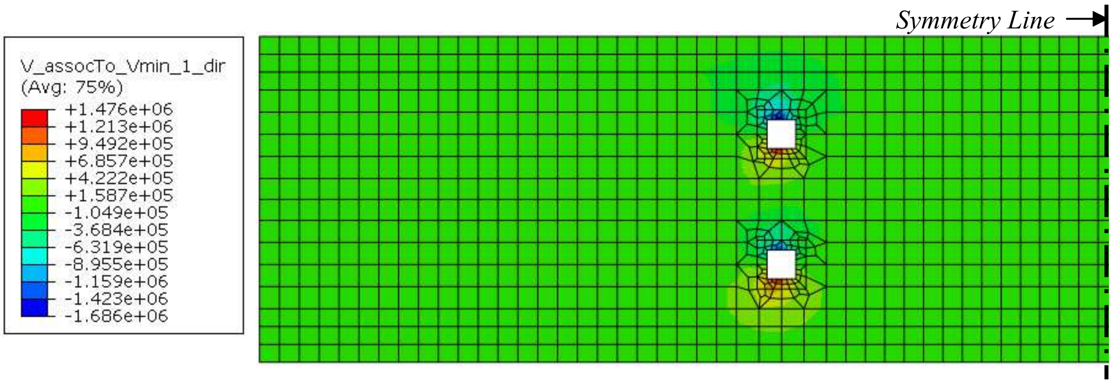


Figure 6.9: Shear component in the transversal direction associated to the minimum shear component in the longitudinal direction, [N/m]

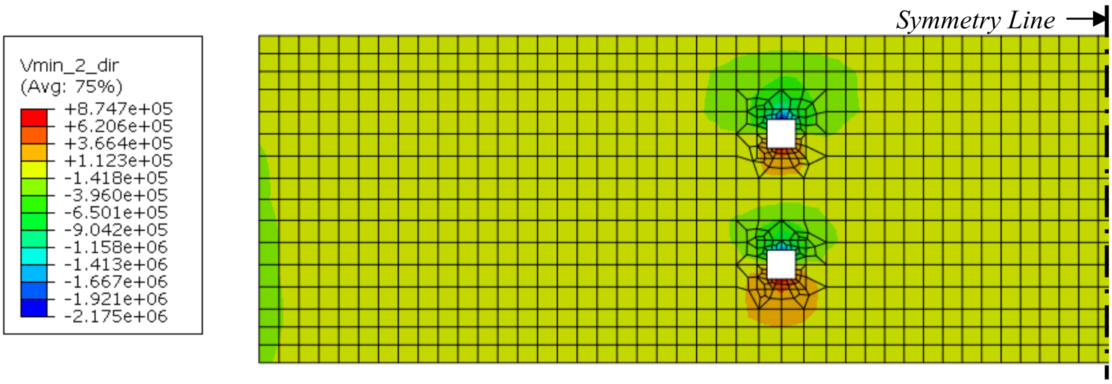


Figure 6.10: Minimum envelope of the shear component in the transversal direction, [N/m]

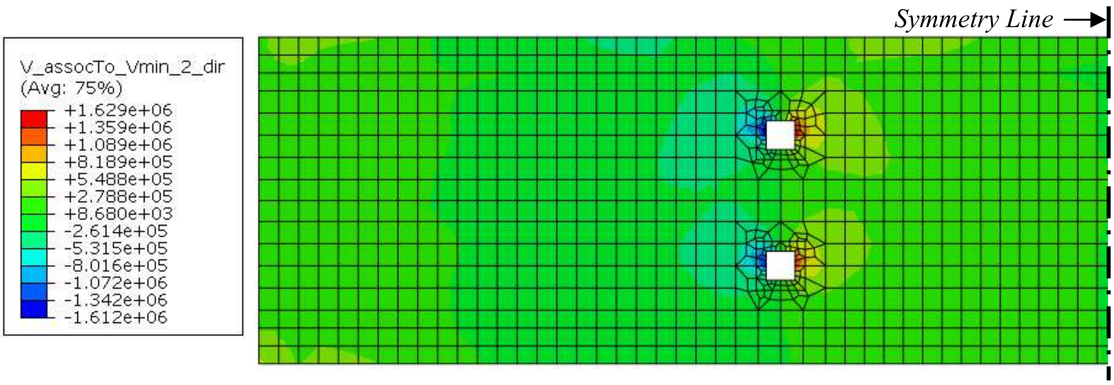


Figure 6.11: Shear component in the longitudinal direction associated to the minimum shear component in the transversal direction, [N/m]

6.3.2 Post-Processing of Output Data

For the four result lines illustrated in Figure 6.2 the output data according to Equation (6.3) and (6.4) were exported from the finite element program. The data was then treated in two different ways to correspond to the two different shear force determination methods as given in Table 6.1.

The first method, used in Approach 1-4, was based on the resultant shear forces in each node of the result line. Exactly how these resultant shear forces were handled, and how the design shear force in each node was determined, is described in Section 3.2.6. The second method, used in Approach 5-8, is a considerably simpler method. The design curves in the two different directions were obtained by simply picking the maximum absolute values in each point along the result lines according to Equation (6.5).

$$\begin{aligned} v_{dx.i} &= max(abs(v_{x.max.i}), abs(v_{x.min.i})) \\ v_{dy.i} &= max(abs(v_{y.max.i}), abs(v_{y.min.i})) \end{aligned} \tag{6.5}$$

The result from these steps is presented in the diagrams in Figure 6.12 and Figure 6.13 for the adopted example bridge and the result lines used in this thesis. A comparison between the methods is given in Figure 6.14. Note that even though the loading was not symmetrically applied in the model, the curve was created as a symmetrically loaded case by mirroring the design curves, since this should be the basis for the design.

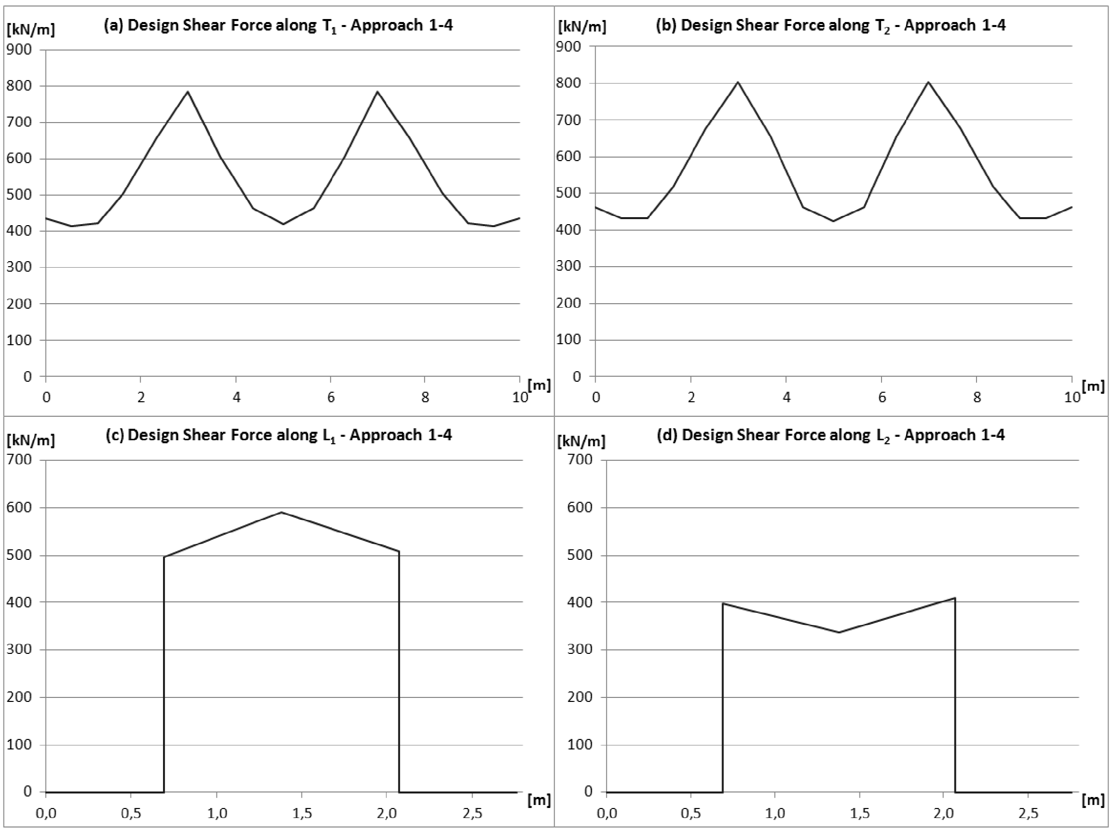


Figure 6.12: Design shear force curves according to Approach 1-4 along the used result lines. (a) and (b) represents the shear force across the whole bridge width and are relevant for the design in the longitudinal (x) direction. (c) and (d) represents the shear force in the isolated area around the columns and are relevant for the design in the transversal (y) direction.

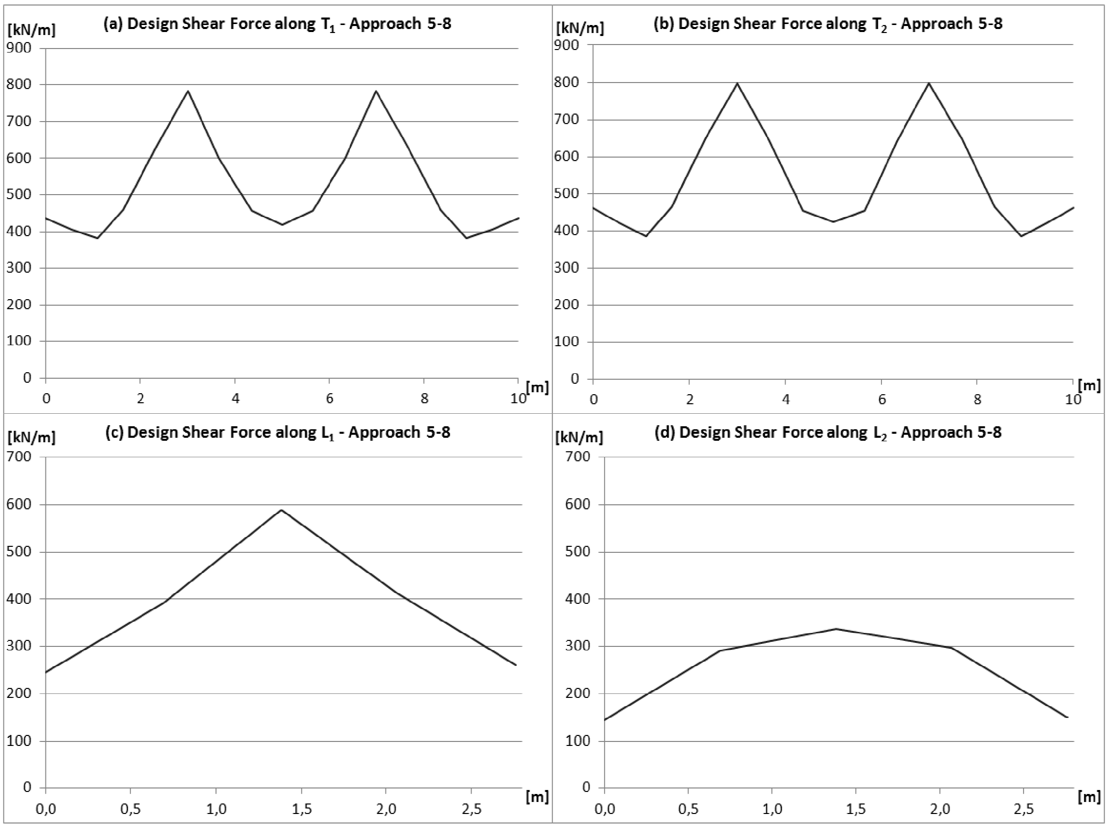


Figure 6.13: Design shear force curves according to Approach 5-8 along the used result lines. (a) and (b) represents the shear force across the whole bridge width and are relevant for the design in the longitudinal (x) direction. (c) and (d) represents the shear force in the isolated area around the columns and are relevant for the design in the transversal (y) direction.

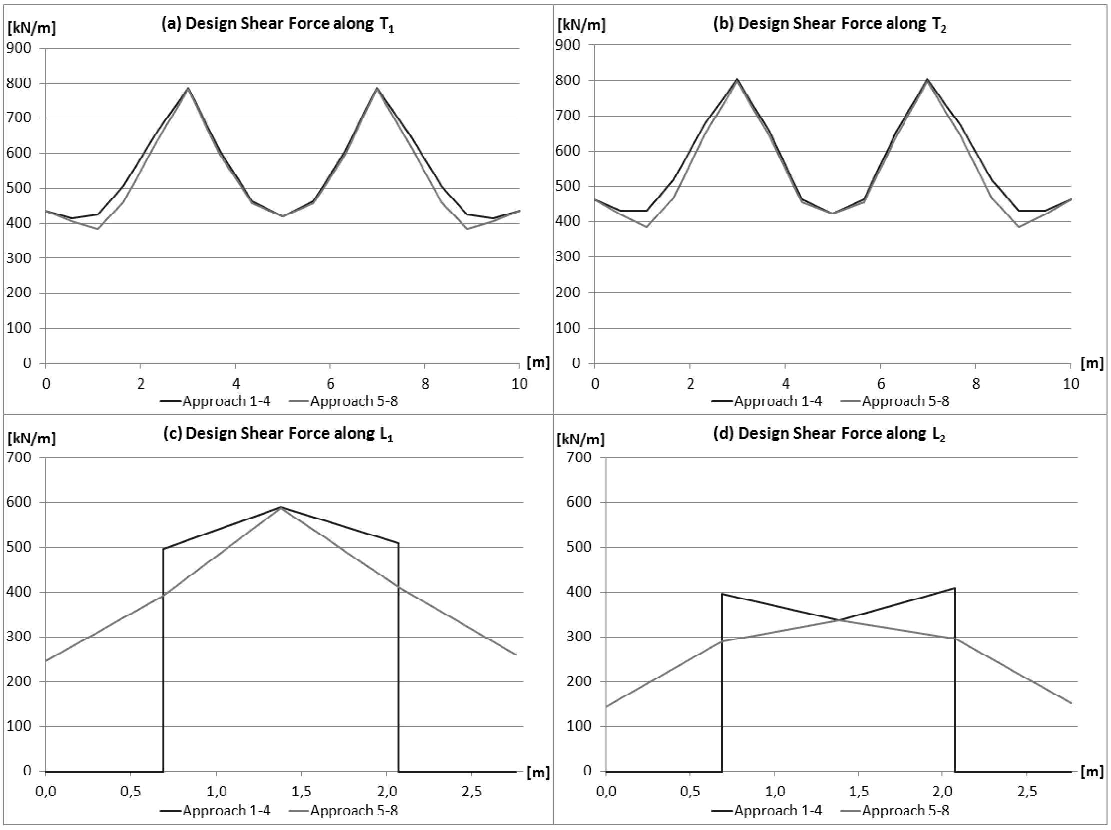


Figure 6.14: Comparison between the resultant shear force method (Approach 1-4) and the shear force component method (Approach 5-8)

As mentioned, for each of the two shear force determination methods illustrated above, four different distribution width methods were adopted. The shear force curves presented above were then modified so that the peaks in the curves were smeared out over the applied distribution width. In that way the design curves illustrating the actually required shear force resistance became different from approach to approach.

The widths were calculated according to the equations and restrictions described in Section 3.2.5. For this specific study, the used widths for the eight different approaches were according to Table 6.2. It should be noted that the widths used in Approach 1-3 and 5-7 were calculated directly from the equations. For Approach 4 and 8, however, active choices were also influencing the width. The choices were made so that the most favourable design would be obtained. This was made by comparing the curves with the shear resistance of the concrete section without shear reinforcement, and only the shear force in regions where shear reinforcement actually was needed.

The reason for including Approach 4 and 8 was that it was found that it could be unfavourable to use too wide distribution widths. According to Eurocode, the entire shear force must be resisted by shear reinforcement where resistance provided by the concrete section (without any shear reinforcement) is insufficient, SS-EN 1992-1-1 (2005). With too wide distribution widths, shear reinforcement may be needed in order to meet the averaged shear force action, also in parts where the shear force is smaller than the shear resistance of the concrete section.

Table 6.2: Distribution widths for the shear force according to the different design approaches

Approach	Result Line T_1 $[m]$	Result Line T_2 $[m]$	Result Line L_1 $[m]$	Result Line L_2 $[m]$
(1)	5.0	5.0	1.38	1.38
(2)	1.8	1.8	0.35	0.35
(3)	0	0	0	0
(4)	3.62	3.62	1.38	1.38
(5)	5.0	5.0	2.76	2.76
(6)	1.8	1.8	0.35	0.35
(7)	0	0	0	0
(8)	3.27	3.27	1.9	0

The required shear reinforcement was determined by implementation of the different distribution widths given in Table 6.2 into the sets of design diagrams illustrated in Figure 6.12 and Figure 6.13. In that way eight different design diagram sets were obtained. The first step to find an appropriate reinforcement layout was to determine the shear resistance of the concrete section without shear reinforcement, and to compare this with the required shear resistance diagrams in the different approaches. In that way the regions that needed to be provided with shear reinforcement could easily be distinguished. The concrete shear force resistance in the two different principal directions were determined in accordance with Equation (4.8) as presented in Equation (6.6).

$$\begin{aligned} v_{Rd.c.x} &= (C_{Rd.c.x}k_x(100\rho_{l.x}f_{ck})^{\frac{1}{3}}d_x = 434.4\ kN/m \\ v_{Rd.c.y} &= (C_{Rd.c.y}k_y(100\rho_{l.y}f_{ck})^{\frac{1}{3}}d_y = 346.6\ kN/m \end{aligned} \qquad (6.6)$$

In the regions that needed to be provided with shear reinforcement the concrete section resistance was not accounted for in accordance with the currently valid requirements. The amount of shear reinforcement needed to prevent a shear failure was determined for each specific case according to Equation (4.9). The minimum required amount of shear reinforcement in the critical crack was calculated according to Equation (4.15) as presented in Equation (6.7), and corresponding reinforcement area and spacing was determined from Equation (4.16) as presented in Equation (6.8).

$$\rho_{min} = 0.08\frac{f_{ck}^{0.5}}{f_{yk}} = 0.095\% \qquad (6.7)$$

$$\begin{aligned} \rho_{w.x} &= \frac{A_{sw.x}}{s_x \cdot b_{w.x} \cdot sin\alpha} = 0.101\% \qquad (40\ \emptyset 16\ bars) \\ \rho_{w.x} &= \frac{A_{sw.y}}{s_y \cdot b_{w.y} \cdot sin\alpha} = 0.100\% \qquad (12\ \emptyset 12\ bars) \end{aligned} \qquad (6.8)$$

There are also requirements of the maximum spacing in the longitudinal and transversal directions of a critical crack. For this specific slab, the maximum spacing for the design in the x- and y-directions was obtained as presented in Equation (6.9).

$$\begin{aligned} s_{l.max.x} &= d_x = 937.5\ mm \\ s_{l.max.y} &= d_y = 915\ mm \\ s_{t.max.x} &= 1.5d_x = 1406.3\ mm \\ s_{t.max.y} &= 1.5d_y = 1372.5\ mm \end{aligned} \tag{6.9}$$

The actual provided amount of bars was chosen so that an adequate design layout was obtained and so that the requirements regarding minimum reinforcement area and maximum spacing were fulfilled. The aim of the design work was in each approach to use as little reinforcement as possible, but without creating an unsuitable or inappropriate design from a practical point of view. For instance, only one bar diameter was used in each approach, since it would have been inappropriate to use a range of different diameters just to somewhat decrease the total amount of reinforcement. Based on this the final designs for each of the eight different approaches were obtained according to Figure 6.15 - Figure 6.22.

The *Shear Curve* corresponds to the linear distribution of the shear force, while the *Required Capacity* curve is the averaged shear force over the distribution width. The *Provide Capacity* curve is the shear resistance provided by the shear reinforcement, and the *Concrete* curve is the capacity provided by the concrete section without shear reinforcement.

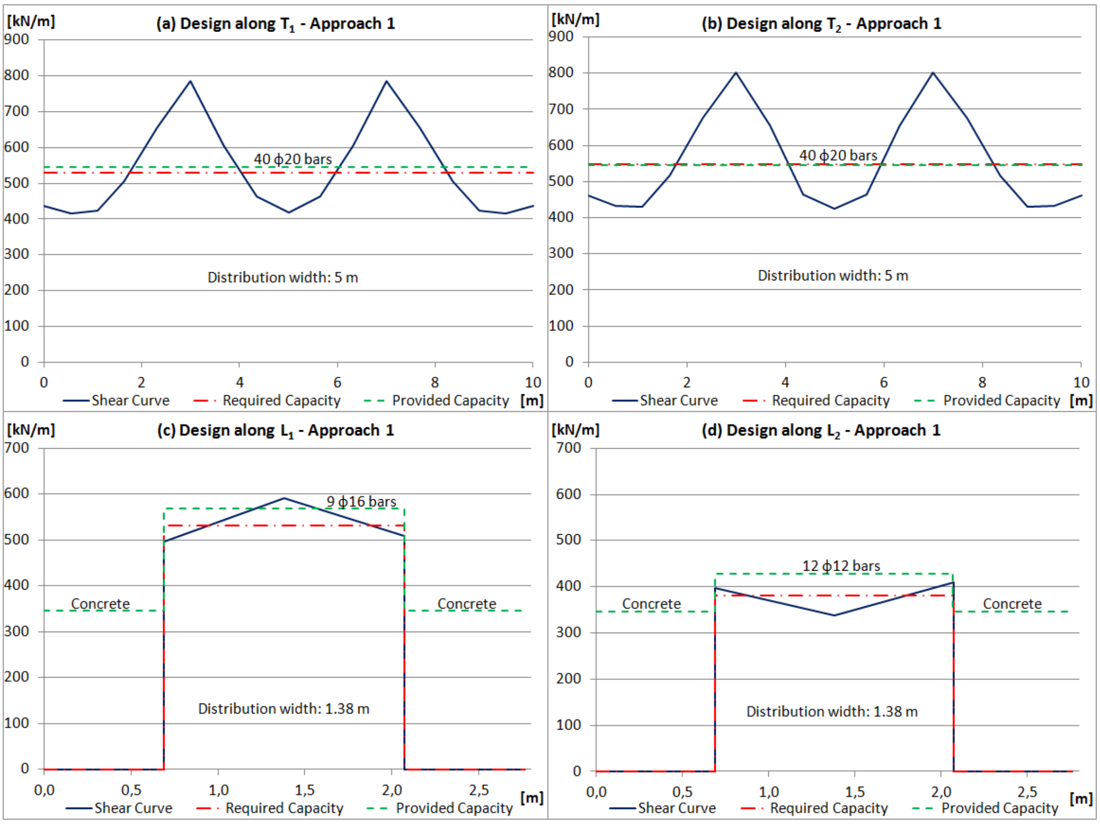


Figure 6.15: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 1

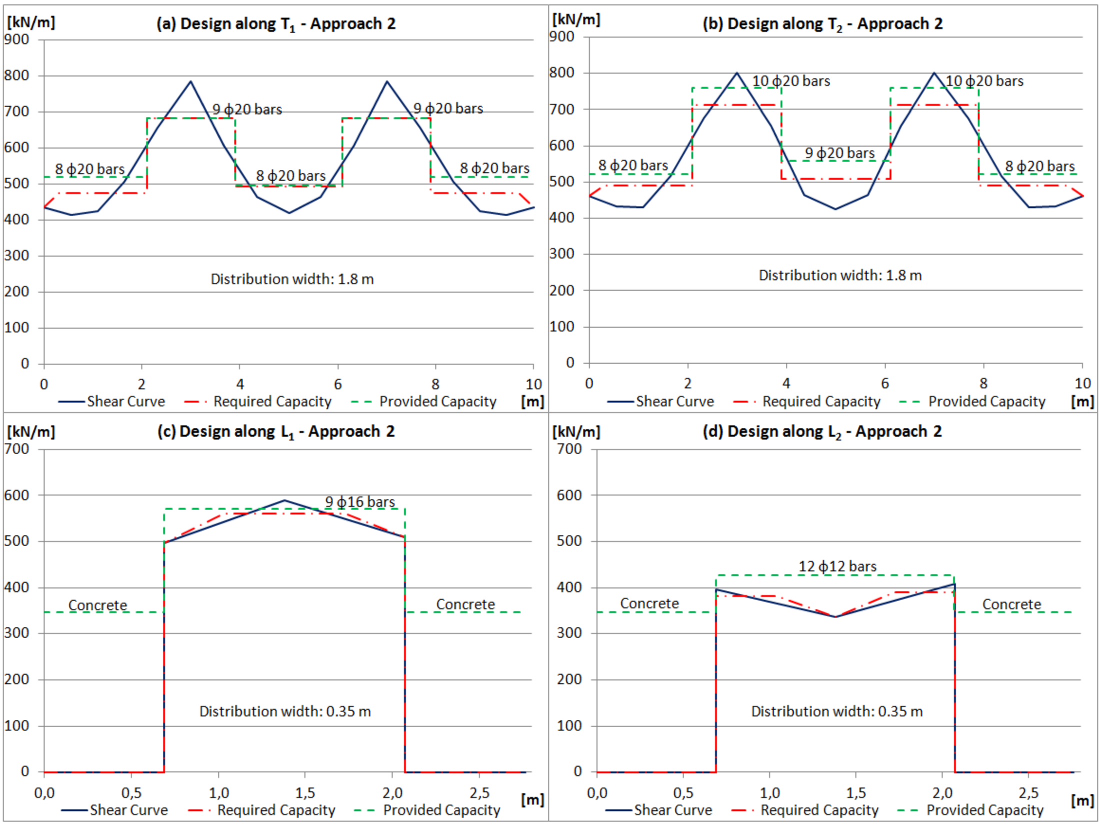


Figure 6.16: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 2

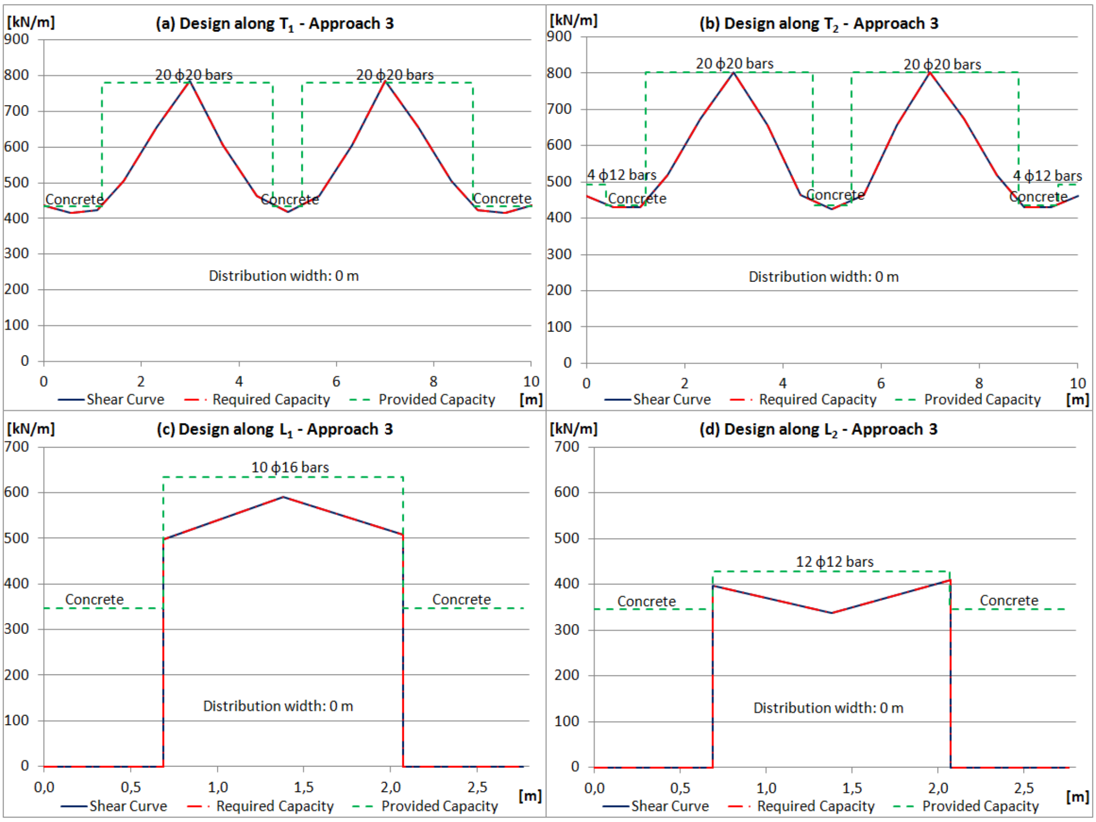


Figure 6.17: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 3

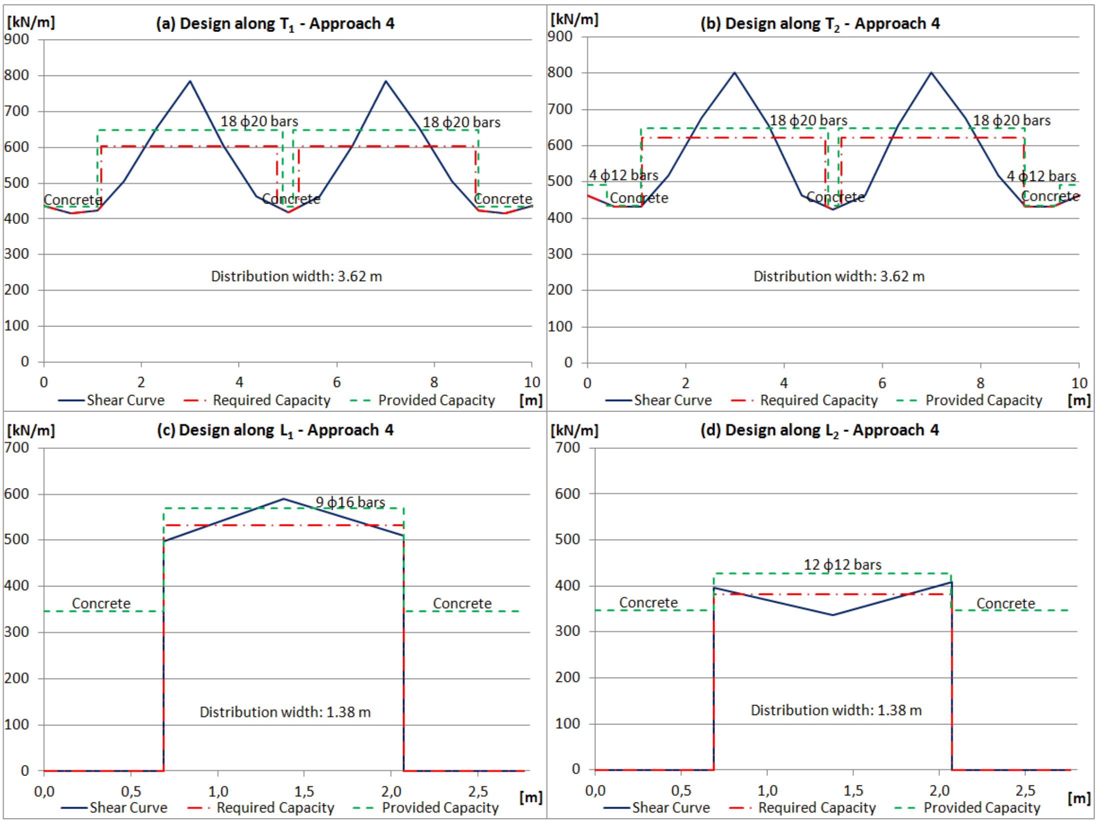


Figure 6.18: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 4

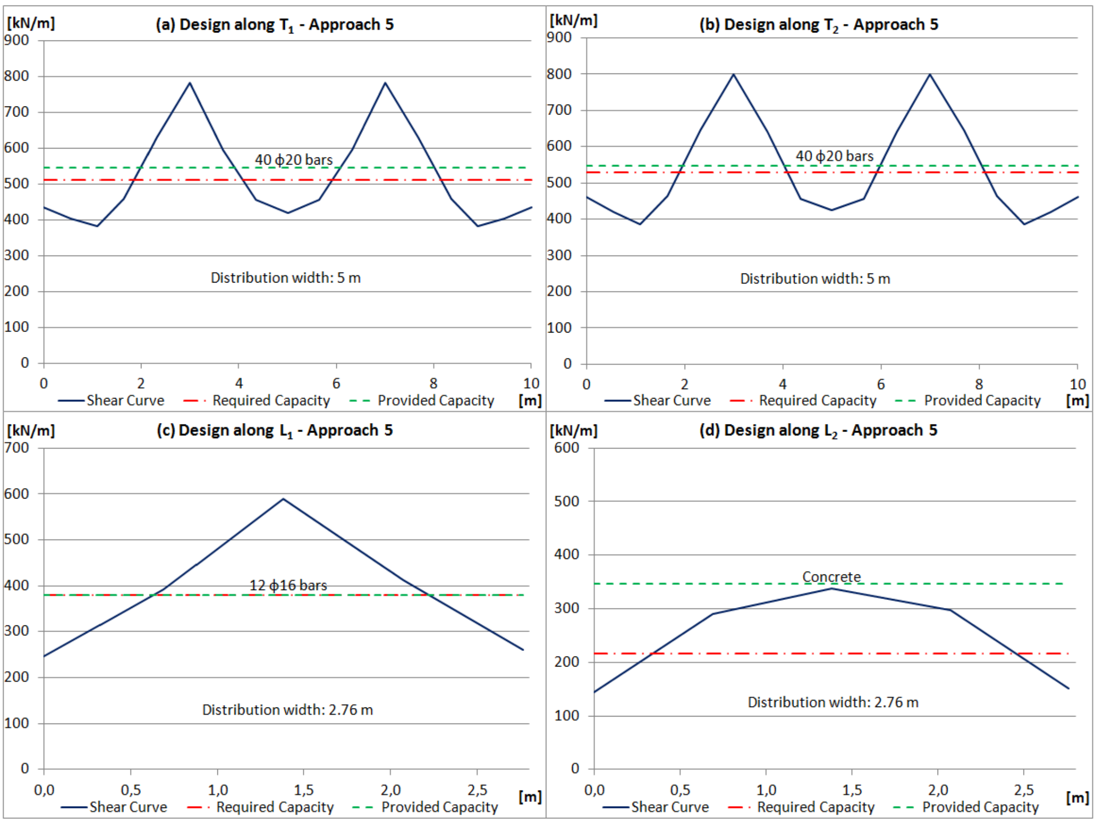


Figure 6.19: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 5

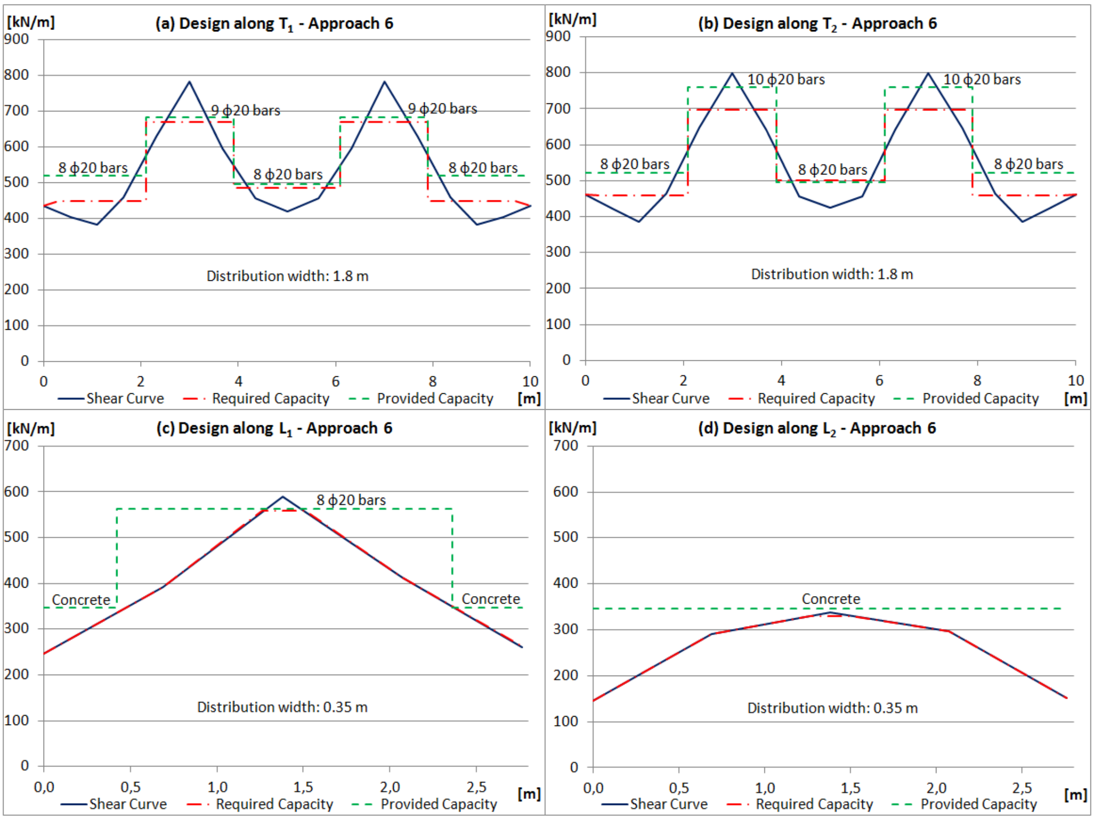


Figure 6.20: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 6

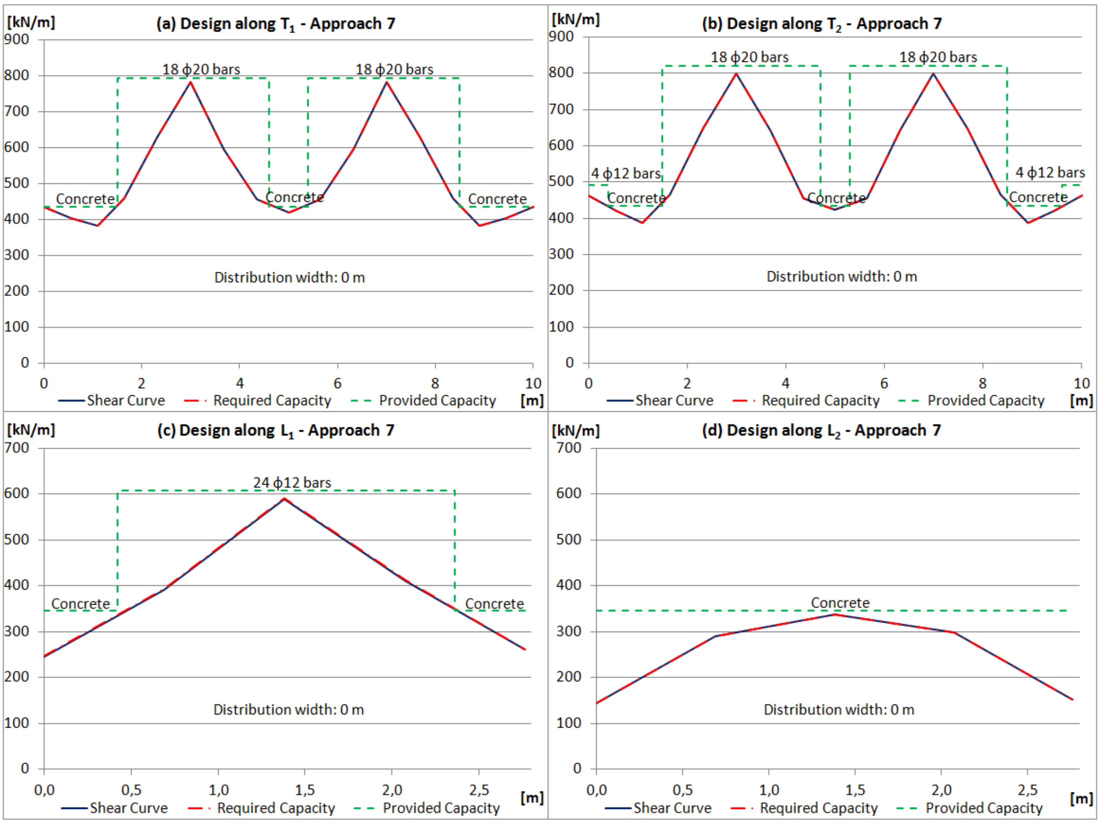


Figure 6.21: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 7

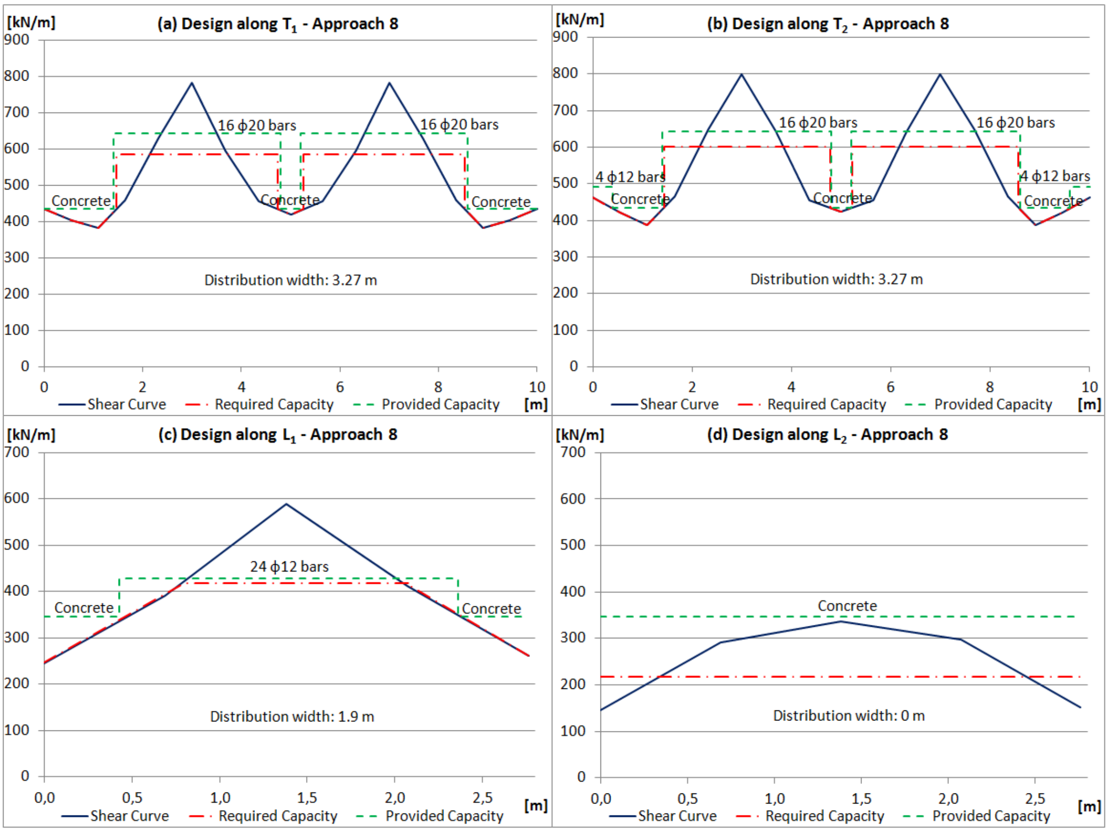


Figure 6.22: Shear reinforcement design along the transversal and longitudinal result lines according to Approach 8

From the diagrams presented in Figure 6.4 - Figure 6.11, the number of shear reinforcement bars used in the critical shear cracks is given. However, the total reinforcement amount is also dependent on the extension of the area where shear reinforcement is needed. The area with need for shear reinforcement is reaching a certain distance from the critical section before the required capacity becomes small enough to be resisted by the concrete section without shear reinforcement. In order to find this distance, the result lines presented in Figure 6.3 were used. The same output data as used for the design in the critical sections were used for determining the extension of the shear reinforced area as well. The post-processing of the output data was also executed in the same manner. In this way it could be found how the need for shear capacity decreased the further from the critical section that the result was taken. The principle is illustrated for one of the result lines in Figure 6.23.

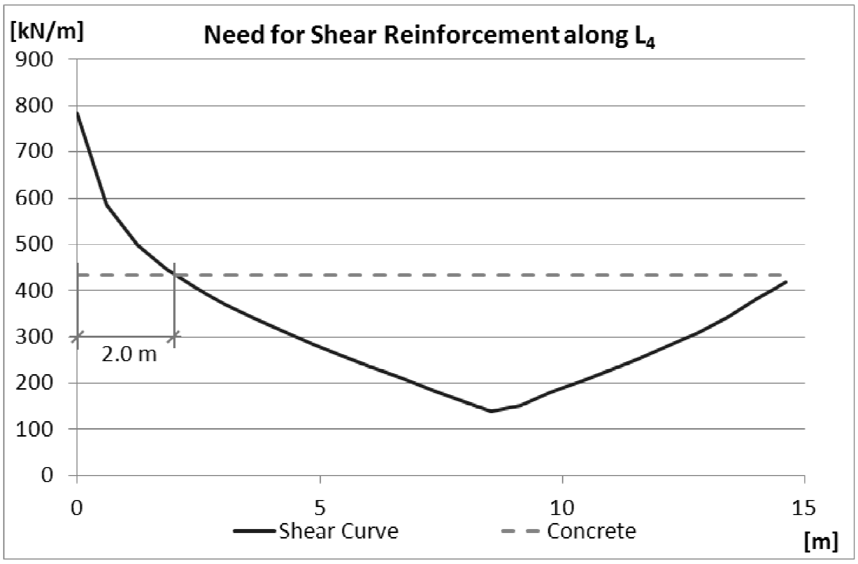


Figure 6.23: Need for shear force reinforcement along result line L_4. The critical section is located to the left (x=0). The distance from the critical section that needs to be reinforced with respect to shear is 2.0 m.

It should be noted that the implemented method for the extension of the area where shear reinforcement is needed is conservative as no redistributions of stresses was accounted for in the shear force variation curve in Figure 6.23. Moreover, the same shear resistance was provided all the way to the section where reinforcement was no longer needed, even though the required shear capacity decreased continuously.

For the comparison the total amounts of shear reinforcement was calculated for all approaches. Both the reinforcement used in the critical shear sections only, and the total amount of reinforcement in the whole areas where shear reinforcement was needed was calculated and compared. The results in terms of shear reinforcement cross-section area are summarized in Table 6.3 - Table 6.5.

Table 6.3: Shear reinforcement design in the longitudinal direction. Summary of the results in terms of reinforcement area in the critical shear crack and with curtailment considered.

Shear Force	Distribution	Approach	Result Line	Shear Reinforcement Area in Critical Section [$\cdot 10^3\ mm^2$]	Total Shear Reinforcement Area [$\cdot 10^3\ mm^2$]
Resultant	**Pacoste *et al* (2012)**	**(1)**	T_1	12.6	47.1
			T_2	12.6	47.1
	TRVR Bro 11 (2011)	**(2)**	T_1	13.2	50.9
			T_2	14.1	55.3
	No redist.	**(3)**	T_1	12.6	47.1
			T_2	13.5	53.9
	Chosen regions	**(4)**	T_1	11.3	42.4
			T_2	12.2	45.8
Components	**Pacoste *et al* (2012)**	**(5)**	T_1	12.6	47.1
			T_2	12.6	47.1
	TRVR Bro 11 (2011)	**(6)**	T_1	13.2	50.9
			T_2	13.8	53.4
	No redist.	**(7)**	T_1	11.3	42.4
			T_2	12.2	49.2
	Chosen regions	**(8)**	T_1	10.1	40.2
			T_2	11.0	43.6

Table 6.4: Shear reinforcement design in the transversal direction. Summary of the results in terms of reinforcement area in the critical shear crack and with curtailment considered.

Shear Force	Distribution	Approach	Result Line	Shear Reinforcement Area in Critical Section [$\cdot 10^3\ mm^2$]	Total Shear Reinforcement Area [$\cdot 10^3\ mm^2$]
Resultant	Pacoste *et al* (2012)	(1)	L_1	1.8	3.6
			L_2	1.4	1.8
	TRVR Bro 11 (2011)	(2)	L_1	1.8	3.6
			L_2	1.4	1.8
	No redist.	(3)	L_1	2.0	4.0
			L_2	1.4	1.8
	Chosen regions	(4)	L_1	1.8	3.6
			L_2	1.4	1.8
Components	Pacoste *et al* (2012)	(5)	L_1	2.4	4.0
			L_2	0	0
	TRVR Bro 11 (2011)	(6)	L_1	2.5	5.0
			L_2	0	0
	No redist.	(7)	L_1	2.7	3.4
			L_2	0	0
	Chosen regions	(8)	L_1	1.4	2.3
			L_2	0	0

Table 6.5: Shear reinforcement design in the bridge. Summary of the results in terms of total reinforcement area in the critical shear cracks and with curtailment considered.

Shear Force	Distribution	Approach	Total Shear Reinforcement Area in Critical Sections $[\cdot 10^3\ mm^2]$	Total Shear Reinforcement Area in the whole Bridge $[\cdot 10^3\ mm^2]$
Resultant	Pacoste *et al* (2012)	(1)	62.9	210.2
	TRVR Bro 11 (2011)	(2)	67.3	240.4
	No redist.	(3)	65.5	225.4
	Chosen regions	(4)	59.7	198.1
Components	Pacoste *et al* (2012)	(5)	59.9	204.6
	TRVR Bro 11 (2011)	(6)	64.1	228.7
	No redist.	(7)	57.9	196.8
	Chosen regions	(8)	47.5	176.7

6.4 Discussion of the Shear Reinforcement Design Results

The main aims of the comparison were to investigate differences in the final design when using different methods to determine the design shear forces and to approximate the distribution widths. In addition to these two parameters, the scope has also been to find and investigate special issues and uncertainties in the design based on today´s design philosophy. The discussion in this section is therefore focused on these three topics; the different methods to *determine the design shear forces*, the different methods to *approximate the distribution widths for peak shear forces* and *uncertainties found that need to be further investigated*. The uncertainties and questions that are discussed here are further investigated in coming chapters.

6.4.1 The Influence of Different Methods to Determine the Design Shear Forces

The different methods to determine the shear force distribution between the two principal directions show many similarities as well as deviations. When comparing Figure 6.12 with Figure 6.13, it becomes quite obvious that the two methods give very similar results for the global behaviour in the longitudinal direction, while pronounced differences are obtained in the local behaviour in the transversal direction. This is even more obvious when looking at the provided reinforcement results in Table 6.3 and Table 6.4. It seems that the behaviour in the longitudinal direction is similar to what is expected when regarding the bridge as a beam, while the local behaviour in the transversal direction is more complicated. This is not a surprising result, since the areas around the supports could be regarded as discontinuity regions. The behaviour in such regions is not as simple to predict since the forces are transferred irregularly to the support. The magnitudes of the shear components in both principal directions become high within these areas, leading to an underestimation of the shear force if one of them is ignored. In other words, in the regions where two-way shear is relevant, see Figure 3.11, the behaviour differs to a higher extent than in the regions where one-way shear is dominating.

The irregularity is present in all regions of the bridge deck, but not as pronounced as around the columns. For the design in the longitudinal direction, the ignored components are relatively small, and therefore the shear force curve when only the main component is accounted for becomes similar to the approach where the resultant shear force is used. Thus, for the longitudinal design it seems that either of these two methods could be used to obtain a realistic shear force approximation. It should, however, be emphasized that in this thesis the geometry was kept simple, and therefore no geometric irregularities exist. So-called discontinuity regions could be created by such irregularities and therefore the fact that the two approaches correlate may not be valid for more complicated geometries. However, for many bridges the bridge decks are close to completely straight and regular between each support. For such bridges the choice of methodology for determination of the shear force is of minor importance.

With the observations discussed above kept in mind, there could be reason to question the design methodology used in this thesis for the shear force reinforcement design. It was assumed that the bridge deck acts similar to a beam in the longitudinal direction regarding the transfer of shear forces to each support, and as a beam in the transversal direction regarding the division of the shear forces between the columns in each support. The results indicate that the global behaviour is in fact corresponding to the beam approximation, which is not the case for the local behaviour at the support regions. Therefore, the design methodology may be questionable for the transversal design. It seems that the design at these regions should be made with more care, and therefore the resultant shear force method should be considered. This method should be conservative since it takes the two-way shear into account in a more accurate way than when looking at one shear force component at a time. Since the support regions are quite small in comparison to the whole bridge, it may be reasonable to ignore the angle criterion according to Figure 3.13. This makes the design more straightforward and more conservative, and since the regions are that small the extra reinforcement amount that is obtained will not be much compared to the total amount provided in the bridge deck.

6.4.2 The Influence of Different Methods to Approximate the Distribution Widths for Peak Shear Forces

At first glance, the different methods to approximate the distribution widths for the peak shear forces show considerable differences. This can be seen when comparing the *Required Capacity* curves in Figure 6.15 - Figure 6.22. For instance, the redistribution of the peak shear forces for the longitudinal design in Approach 1 and 5 creates a completely horizontal curve representing the mean value across the width of the bridge. On the other end, no redistribution of these peak forces is permitted in Approach 3 and 7. Since the design curves look very different compared to each other it was assumed that the final design in terms of reinforcement amount and layout would also differ to a quite high extent. However, when comparing the results obtained in Table 6.3 the differences between all approaches are small. No matter which of the shear force determination methods that are used, the obtained differences between maximum and no redistribution of peak shear forces are minor in terms of the amounts.

The differences between the approaches are more visible when comparing the design diagrams presented in Figure 6.22. To compare the maximum redistribution case (Pacoste *et al* (2012)) with the case where no redistribution is permitted Figure 6.15 and Figure 6.17 can be compared. The amounts are similar in both these approaches but the layouts differ. In Approach 3 the reinforcement is more concentrated to the column regions than in Approach 1. Since Approach 1 should be regarded as the more favourable way to design the slab, it might be surprising that the amounts do not differ more. It should also be emphasized that the reinforcement layout in Approach 1 could preferably be concentrated to the column regions. The shear forces should be led to the column and if the reinforcement is concentrated to these regions the redistribution of the forces does not need to be fully utilized. Therefore, the layout of the reinforcement is not of equal priority as the amount of reinforcement provided within the slab.

The redistribution of the peak shear forces is even of less importance for the transversal shear reinforcement design. In Approach 1-4, which are representing the resultant shear force method, almost no differences at all are obtained for the different redistribution methods. This should be an effect of the angle criterion where the direction of the resultant in each point is determined. According to this criterion there are only a few points next to the column that are approximated to transfer shear forces in the transversal direction. The magnitude of these does not differ much since they are located at a small distance from each other, and therefore redistribution of the forces does not influence the results enough to create differences in the final design.

Despite the differences in the results between Approach 1-4 and 5-8, they all indicate that the shear component in the longitudinal direction is dominating in almost all points across result line L_1 and L_2. The magnitude of the peaks for the transversal shear forces is of the same order, showing that in the peaks it is the transversal shear component that is dominating. But this effect is very local since it is in fact the only regions where this component is pronounced. Therefore, even though it seems that the shear component method underestimates the shear, the total design of the bridge may still be sufficient. This is because the shear reinforcement in the longitudinal direction continues from the critical section in to the support edge, covering much of the regions where the shear forces are underestimated. Thus, even though some parts are underestimated while looking at the transversal direction only, the design might be safe enough anyway, since the regions for the longitudinal and transversal shear reinforcement are overlapping.

According to Table 6.5, the total amount of reinforcement put into the bridge is not differing much. The largest effect was obtained when active choices for the redistribution widths are taken. In that way the concrete could be used to carry the shear forces in some regions, while shear reinforcement is only needed in the remaining regions. Even though redistribution of peak shear forces is permitted in Approach 2 and 6, the results from Approach 3 and 7 are more favourable. This is because of the fact that it was possible to carry the shear forces in some parts of the cross-section with the concrete resistance only. This fact seems to be very favourable since these parts are quite limited. For instance, when comparing Figure 6.16 (a) and Figure 6.17 (a), it is clear that it is unfavourable to reinforce areas of the cross-section that would have been able to carry the shear forces without reinforcement. Based on this, the design should be made based on a case where the redistribution is used only for shear forces exceeding the concrete section resistance, as was done in Approach 4 and 8. Regardless of which of the methods that is used, it seems that the potential for optimization lies within the ability to utilize the resistance of the concrete section, even when shear reinforcement is needed in other areas.

6.4.3 Uncertainties and Questions related to the Design Methodology

Today´s design practice is based on the requirements given in Eurocode. According to these requirements, the concrete capacity can be fully utilized up to the point where shear reinforcement is required, but it is not permitted to account for any concrete resistance when this limit is exceeded. Based on the discussion above, where it was stated that there is a lot of potential in using the concrete resistance in isolated areas, it would be of interest to actually be able to use a part of the concrete resistance in all areas. As is described in the theory part of this thesis, there are methods to do so, and therefore it would be of interest to actually see how a more extensive use of the concrete resistance would influence the results.

When determining if shear reinforcement is required in different sections, the concrete capacity in one of the main directions is used. However, the shear forces are not acting in one direction only, as the resultant shear force is always acting in an angle related to these directions. In addition to this, the concrete resistance is used to control the need for reinforcement in one of the principal directions, and the same concrete is used to control the need in the other direction as well. In that way the concrete resistance could be used twice without any restrictions of the interaction between the shear force components. With these considerations kept in mind, it would be of interest to investigate how different interaction models could be used for these checks instead.

As is described in Section 2.1, bridge design is made based on load envelopes where the maximum or minimum forces and moments are summed up to create a complete loading of the bridge. In the shear reinforcement design, mean values are used when redistributing the shear forces. Since the forces in a load envelope is the highest possible in each node of the finite element model, the mean values over a certain width will be overestimated. This means that the bridge is actually designed for a load case that will never be relevant for the bridge, since the traffic will not act at all locations at the same time. In fact, the traffic load models do only permit one simultaneous vehicle in each lane. Therefore, it would be of interest to investigate how much the load envelope design methodology influences the final design.

7 Comparison of Techniques to Approximate the Resistance in Concrete Slabs

As given in the discussion in Section 6.4.3, the methods to determine the resistance of the concrete section without shear reinforcement is of interest in determining the total shear force resistance of a reinforced concrete slab. If the resistance of the concrete section itself may be added to the resistance provided by the reinforcement, a lower amount of shear reinforcement would be required in design. Since it is the shear force reinforcement amounts that were the main aim of this thesis, it is most relevant to investigate how an additive design methodology would influence the results.

7.1 Methodology of Comparison

In order to investigate the influence of an additive shear resistance approximation, three different methods were compared. Method 1 corresponds to the method that was used in design according to the previous chapter, and is the same method as is given in Eurocode (see Section 4.2). In this method the concrete and shear reinforcement resistances are separated and used independently of each other. However, in this comparison a modification is made compared to the previous chapter. Since it is allowed to choose the inclination of the compressive struts between certain limits in this method, the most favourable inclination was chosen. In Method 2 the concrete and shear reinforcement resistances were simply added together. This method corresponds to the method used in the previous Swedish concrete code, BBK 04 (2004). It should be noted that this method is currently not permitted in bridge design in Sweden, but it was still used in the comparison to quantify the effects. The third method in the comparison is the method according to CEB-FIP (2013), and is described in detail in Section 4.2. This method contains different approximation levels, but it is approximation 3 that is used in the comparison, as approximation 1 and 2 do not permit addition of the two resistances, and approximation 4 is too complex to handle within this thesis.

Based on these three methods, a comparison was executed in order to find differences and similarities between them. The minimum reinforcement amounts and maximum reinforcement spacing criteria from Eurocode were applied to all three methods. Since only the differences between these methods are sought, only one design approach was used from the precious chapter. Thus, Approach 1, see Figure 6.15, was adopted and modified according to the three methods described. Contrary to the previous comparison, this comparison was executed based on different design sections. The result lines were in this case chosen according to Figure 7.1. As can be seen in the illustration, only transversal result lines have been used in this comparison. The result lines with index *a* correspond to the same result lines as was used in the previous comparison, while the result lines with index *b* correspond to the new ones. The distance from the face of the column to these two sections was *d* and *2.5d* respectively.

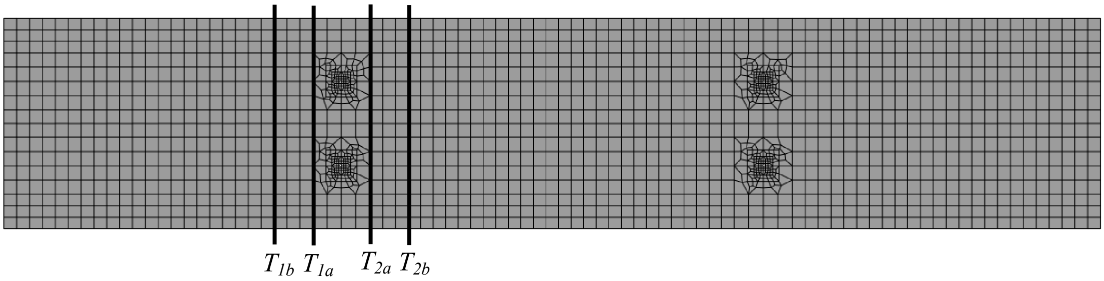


Figure 7.1: Result lines used in the comparison.

Based on the results obtained in the result lines illustrated in Figure 7.1, the design was created in the same manner as has already been described, see Section 6.2. The outcome of the design work is presented in the Section 7.2.

7.2 Results from Comparison

In this section the results from the different steps are presented. Choices that have been made in the design work are highlighted and motivated where needed. Firstly, the design diagrams used in this comparison are presented in Figure 7.2. The output data and the post-processing of the output data in order to obtain these curves have already been treated in Section 6.3.

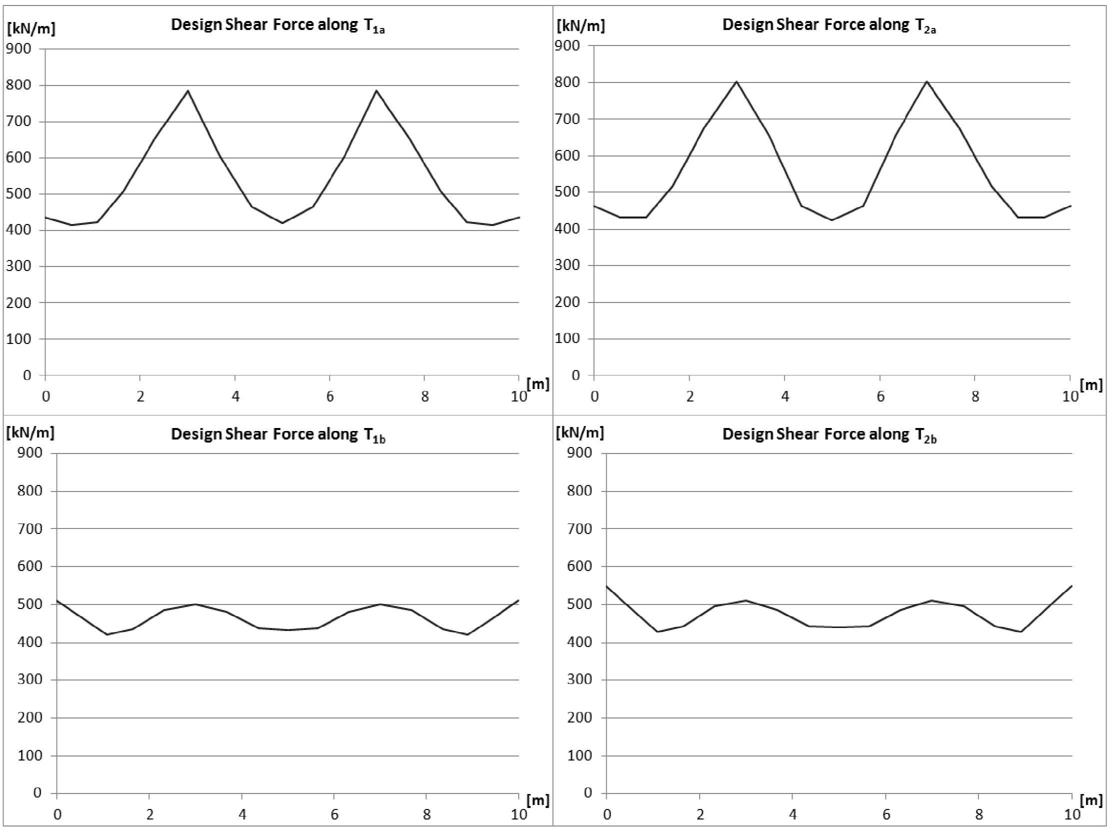


Figure 7.2: Design shear force curves for the different result lines used in the comparison.

The designs according to Method 1-3 are presented in Figure 7.3 - Figure 7.5.

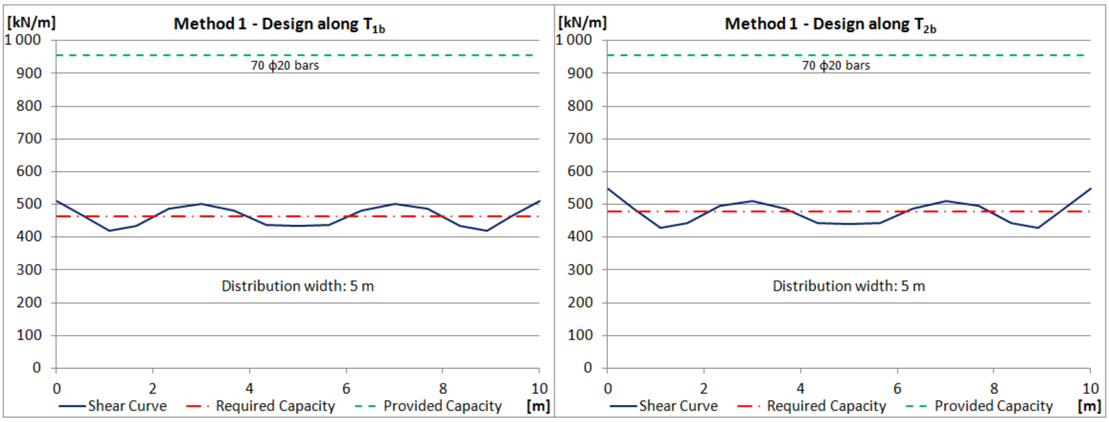


Figure 7.3: Shear force reinforcement design along the transversal result lines T_{1b} and T_{2b} according to Method 1.

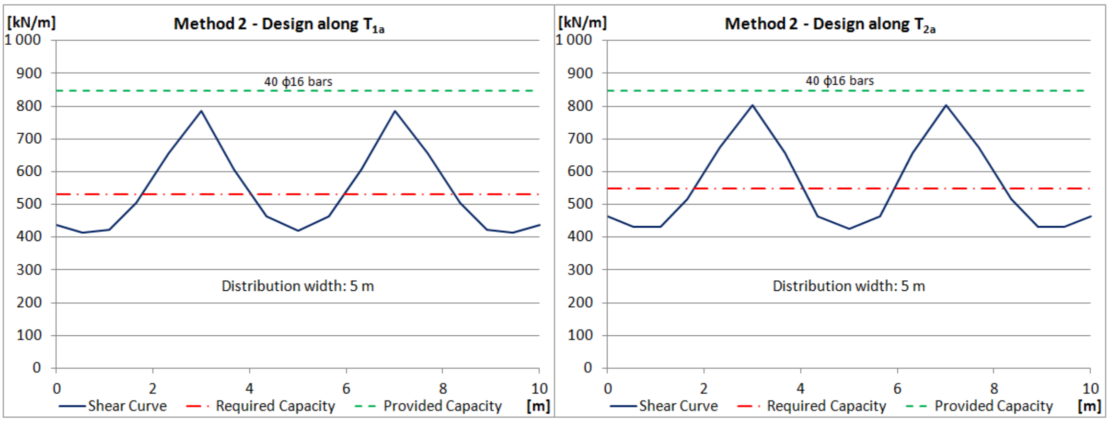


Figure 7.4: Shear force reinforcement design along the transversal result lines T_{1a} and T_{2a} according to Method 2.

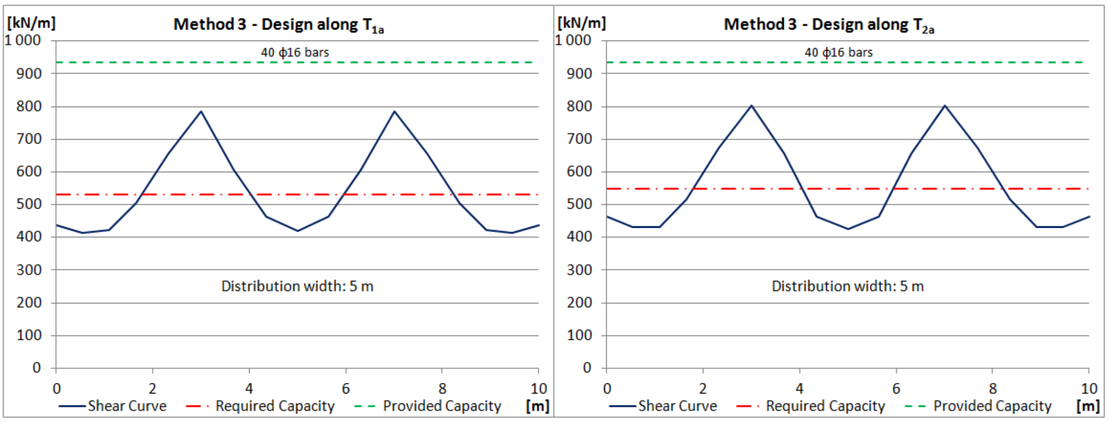


Figure 7.5: Shear force reinforcement design along the transversal result lines T_{1a} and T_{2a} according to Method 3.

It should be noted that there are other methods that could have been implemented and compared as well. The method according to CEB-FIP (2013) was now used in a result line where the inclination of the struts is 45°. According to this code, more favourable inclinations could be chosen as well, similar to what was done in Method 1. However, the effect from a more favourable inclination choice was assumed to be covered with Method 1 only.

The shear reinforcement design, as presented in the diagrams in Figure 7.3 - Figure 7.5, could not be directly compared since different result sections have been adopted. In order to compare the methods the total amount of reinforcement had to be regarded. The total shear reinforcement amounts obtained from Method 1-3 are presented in Table 7.1. Observe that in all three cases a high over-capacity was obtained, which is a consequence from the minimum reinforcement amounts stated in Eurocode.

Table 7.1: Shear reinforcement design in the longitudinal direction. Summary of the results in terms of reinforcement area in the critical shear crack and with curtailment considered.

Method	Result Line	Shear Reinforcement Area in Critical Section $[\cdot 10^3\ mm^2]$	Total Shear Reinforcement Area $[\cdot 10^3\ mm^2]$
(1)	T_{1b}	22.0	28.3
	T_{2b}	22.0	28.3
(2)	T_{1a}	8.0	30.2
	T_{2a}	8.0	30.2
(3)	T_{1a}	8.0	30.2
	T_{2a}	8.0	30.2

7.3 Discussion of the Comparison

It is clear that the methods adopted in the comparison above all resulted in lower reinforcement amounts than in Chapter 6. This conclusion could be drawn already before the design was made. However, when implementing the methods in design, it also became clear that the minimum reinforcement criteria quickly become the governing factor. This resulted in an extensive over-capacity, as can be seen in Figure 7.3 - Figure 7.5. By that reason it seems that it does not matter which of the methods that are used in design. The result will be similar independent of the design method, which can be seen in Table 7.1. Method 1, which is the currently used design method in bridge engineering in Sweden, gave a lower reinforcement amount compared to the other two. Since this is an already used and verified method, there is not much relevance in developing and verifying the other two. It should also be noted that Method 3, which should be regarded as the most advanced method, is also a more work-intense method.

The minimum reinforcement, since it is quickly becoming the governing factor, prevents an optimization of the design. The reason to choose a smaller inclination of the compressive struts is to optimize the design in terms of the reinforcement amounts. The results according to Table 7.1 in comparison to the results in Table 6.3

clearly show that lower reinforcement amounts can be used. However, when looking at Figure 7.3 - Figure 7.5 it is also clear that the design may be optimized even further. The results show that it is favourable to adopt one of the methods in this chapter, but the reinforcement amounts cannot become much smaller due to the minimum reinforcement rules. In order to optimize the design even further, the minimum shear reinforcement rules need to be modified. The fact that minimum reinforcement criteria are used is important, since the reinforcement needs to be able to withstand the forces when the crack is created. However, a more modified or more advanced approximation of the minimum reinforcement would give rise to the possibility of more optimized designs. How the minimum reinforcement rules for slabs should look like should be further investigated, since the rules used today are for beams specifically. For cases where the required shear force resistance is larger compared to the minimum reinforcement capacity it could be more of interest to use refined methods to determine the shear force resistance.

In all three methods the design is determined by the minimum reinforcement rules. Therefore, none of the design methods actually use much of the concrete resistance. Only a minor part of the calculated concrete resistance has to be used in order to reach the required shear resistance. This is an indication of the reasonability of the design approaches, since the concrete should be able to provide some resistance even after the crack is created. It is not straightforward to approximate to what extent the concrete resistance can be accounted for, but since the concrete does not need to carry much of the shear force by itself, it should be reasonable to design the slab according to all three of these methods. However, this needs to be further investigated and verified before it is implemented in design.

It seems that the most important conclusion to be drawn from this study concerns the current design methodology. When looking at Figure 7.3 it is obvious that no matter of which design approach that is adopted, the result in terms of reinforcement amounts would be the same. The choice of design approach is, as was pointed out in Chapter 6, of minor importance with respect to the shear reinforcement amounts. The result in this chapter indicates that there is no importance at all to choose one method specifically. The result would have been the same anyway.

8 Investigation of Techniques to Approximate the Shear Force Distribution between the Main Directions

The discussion in Section 6.4.3 concerns the fact that the shear force is acting in one direction only, but is approximated to act in the two principal directions by the shear force components. This fact gave rise to some questions concerning the shear force resistance of the concrete. The resistance is also divided between these two directions, but the same concrete can be used to carry shear forces in both of these directions. Therefore it was of interest to investigate to what extent this assumption corresponds to or differs from interaction models.

8.1 Methodology of the Investigation

The method used in design in the comparison according to Chapter 6 was based on a division of the slab into two principal directions. The concrete capacity was then calculated for these two directions, accounting for the bending reinforcement area in the respective direction, and used independently of each other. The criterion for this approximation was that the ratio between the shear force and the concrete capacity in the regarded direction should be less than or equal to 1 in all points, see Equation (8.1).

$$\eta_{x.comp} = \frac{abs(V_{Ed.x})}{V_{Rd.c.x}} \leq 1 \qquad \eta_{y.comp} = \frac{abs(V_{Ed.y})}{V_{Rd.c.y}} \leq 1 \qquad \textit{Component Criterion} \qquad (8.1)$$

In this approach the two components are treated without any consideration of the interaction between them. Since the shear force is actually only one resultant shear force, the criterion should be based on the resultant shear force instead. The problem with such an approach is that the concrete resistance in the same direction as the resultant shear force is hard to determine. Therefore, in lack of better alternatives, the minimum value of the two concrete resistance values was used, since this is a concervative assumption. The criterion used was that the ratio between the resultant shear force and the minimum shear force resistance in the two directions should be less than or equal to 1 in all points, see Equation (8.2).

$$\eta_{res} = \frac{\sqrt{V_{Ed.x}^2 + V_{Ed.y}^2}}{min\ (V_{Rd.c.x}; V_{Rd.c.y})} \leq 1 \qquad \textit{Resultant Criterion} \qquad (8.2)$$

A more refined approximation would be to use the resultant shear force approach, but with both the capacities and the components in the different directions. In that way the components are not treated completely isolated from each other and the resistance is better approximated. The criterion for this approach would be according to Equation (8.3).

$$\eta_{int} = \sqrt{\left(\frac{V_{Ed.x}}{V_{Rd.c.x}}\right)^2 + \left(\frac{V_{Ed.y}}{V_{Rd.c.y}}\right)^2} \leq 1 \qquad \textit{Interaction Criterion} \qquad (8.3)$$

In order to compare these three approaches, contour plots were created for the region around one of the columns. For each of the different approaches, four different contour plots were used. The output data that was used in these contour plots are stated in Table 8.1.

Table 8.1: Summary of the different criteria for the concrete resistance control. Four different plots for each criterion are needed because of limitations in the software.

	Direction	Criterion	Location
Component Criterion	Longitudinal (x)	$\eta_{comp.1} = \frac{abs(V_{max1})}{V_{Rd.c.x}}$	To the right in Figure 8.1
	Longitudinal (x)	$\eta_{comp.2} = \frac{abs(V_{min1})}{V_{Rd.c.x}}$	To the left in Figure 8.4
	Transversal (y)	$\eta_{comp.3} = \frac{abs(V_{max2})}{V_{Rd.c.y}}$	Lower part in Figure 8.7
	Transversal (y)	$\eta_{comp.4} = \frac{abs(V_{min2})}{V_{Rd.c.y}}$	Upper part in Figure 8.10
Resultant Criterion	Longitudinal (x)	$\eta_{res.1} = \frac{\sqrt{V_{max1}^2+V_{assocToVmax1}^2}}{min\,(V_{Rd.c.x};V_{Rd.c.y})}$	To the right in Figure 8.2
	Longitudinal (x)	$\eta_{res.2} = \frac{\sqrt{V_{min1}^2+V_{assocToVmin1}^2}}{min\,(V_{Rd.c.x};V_{Rd.c.y})}$	To the left in Figure 8.5
	Transversal (y)	$\eta_{res.3} = \frac{\sqrt{V_{max2}^2+V_{assocToVmax2}^2}}{min\,(V_{Rd.c.x};V_{Rd.c.y})}$	Lower part in Figure 8.8
	Transversal (y)	$\eta_{res.4} = \frac{\sqrt{V_{min2}^2+V_{assocToVmin2}^2}}{min\,(V_{Rd.c.x};V_{Rd.c.y})}$	Upper part in Figure 8.11
Interaction Criterion	Longitudinal (x)	$\eta_{int.1} = \sqrt{\left(\frac{V_{max1}}{V_{Rd.c.x}}\right)^2 + \left(\frac{V_{assocToVmax1}}{V_{Rd.c.y}}\right)^2}$	To the right in Figure 8.3
	Longitudinal (x)	$\eta_{int.2} = \sqrt{\left(\frac{V_{min1}}{V_{Rd.c.x}}\right)^2 + \left(\frac{V_{assocToVmin1}}{V_{Rd.c.y}}\right)^2}$	To the left in Figure 8.6
	Transversal (y)	$\eta_{int.3} = \sqrt{\left(\frac{V_{max2}}{V_{Rd.c.y}}\right)^2 + \left(\frac{V_{assocToVmax2}}{V_{Rd.c.x}}\right)^2}$	Lower part in Figure 8.9
	Transversal (y)	$\eta_{int.4} = \sqrt{\left(\frac{V_{min2}}{V_{Rd.c.y}}\right)^2 + \left(\frac{V_{assocToVmin2}}{V_{Rd.c.x}}\right)^2}$	Upper part in Figure 8.12

8.2 Results from the Investigation

The created contour plots with the criteria summarized in Table 8.1 are given in Figure 8.1 - Figure 8.12. The black regions correspond to regions where shear reinforcement is not needed.

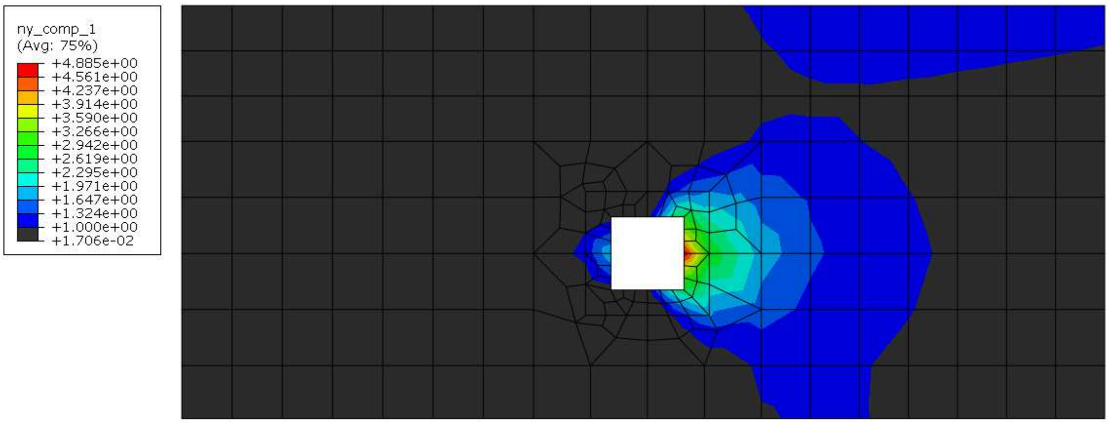


Figure 8.1: Contour plot of the utilization ratio of the concrete resistance to the right of the support (mid span) according to the component criterion, [-]

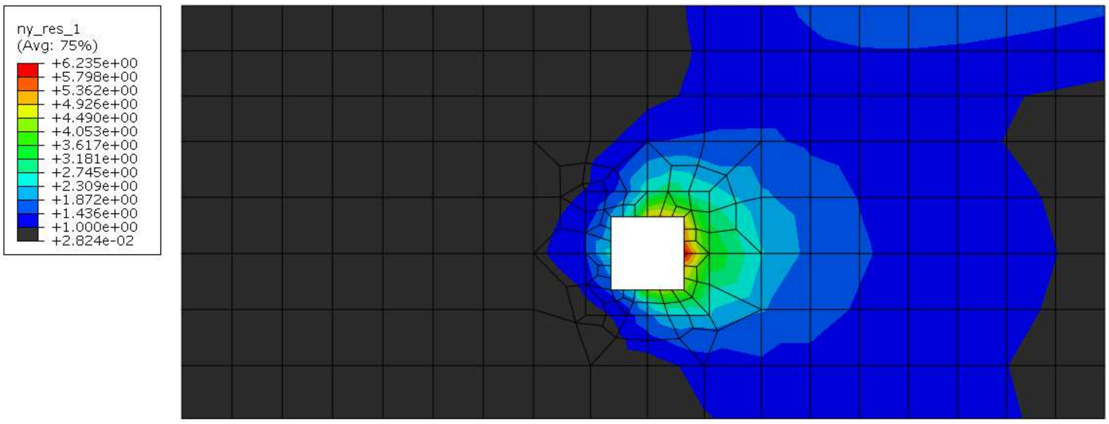


Figure 8.2: Contour plot of the utilization ratio of the concrete resistance to the right of the support (mid span) according to the resultant criterion, [-]

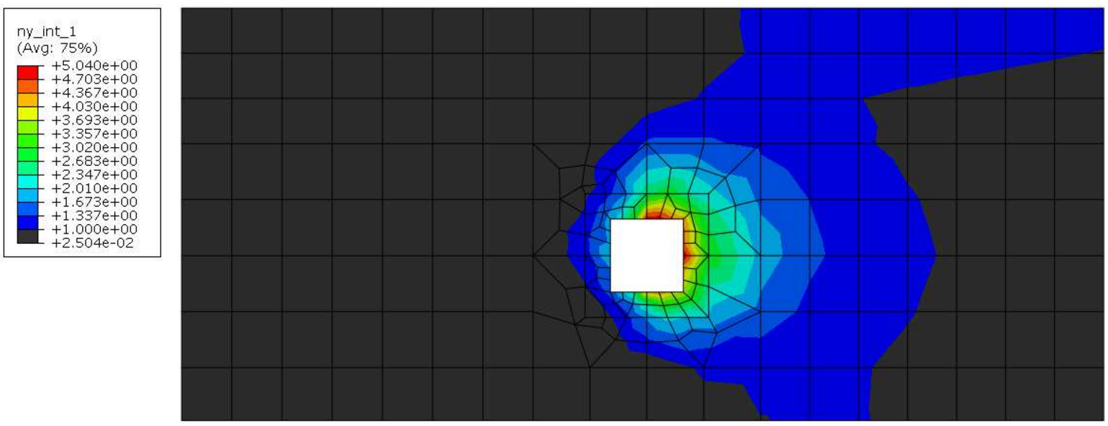


Figure 8.3: Contour plot of the utilization ratio of the concrete resistance to the right of the support (mid span) according to the interaction criterion, [-]

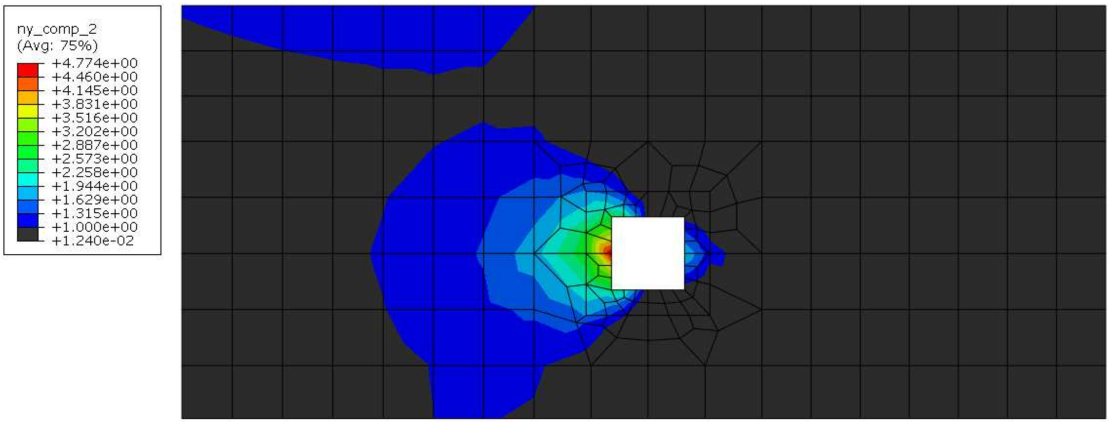


Figure 8.4: Contour plot of the utilization ratio of the concrete resistance to the left of the support (outer span) according to the component criterion, [-]

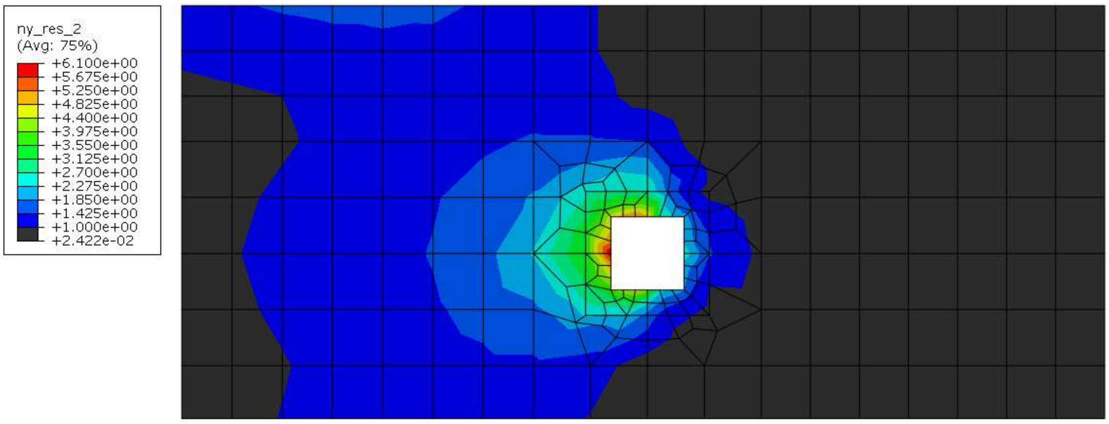


Figure 8.5: Contour plot of the utilization ratio of the concrete resistance to the left of the support (outer span) according to the resultant criterion, [-]

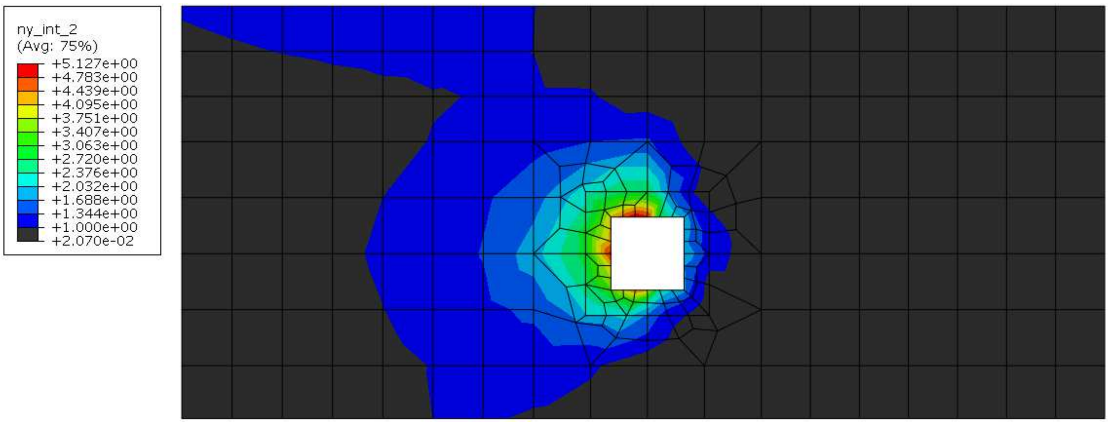


Figure 8.6: Contour plot of the utilization ratio of the concrete resistance to the left of the support (outer span) according to the interaction criterion, [-]

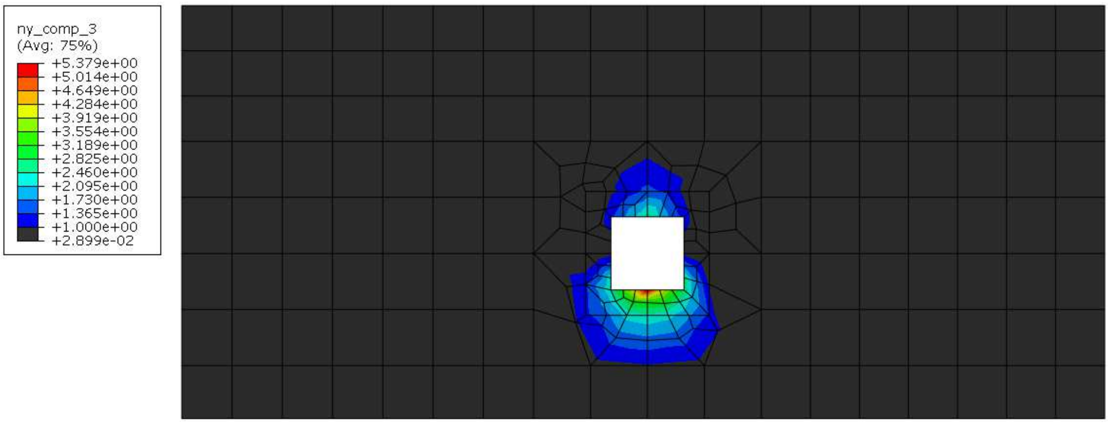


Figure 8.7: Contour plot of the utilization ratio of the concrete resistance below the support (mid span) according to the component criterion, [-]

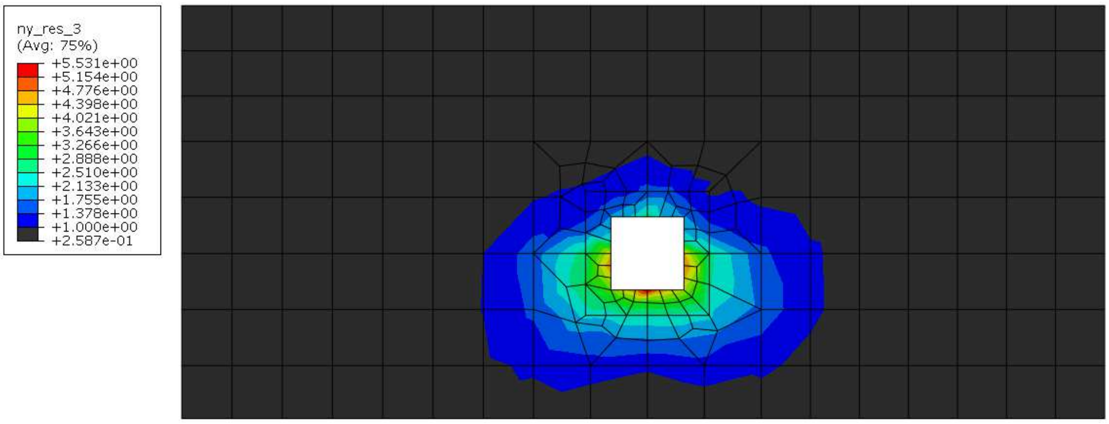


Figure 8.8: Contour plot of the utilization ratio of the concrete resistance below the support (mid span) according to the resultant criterion, [-]

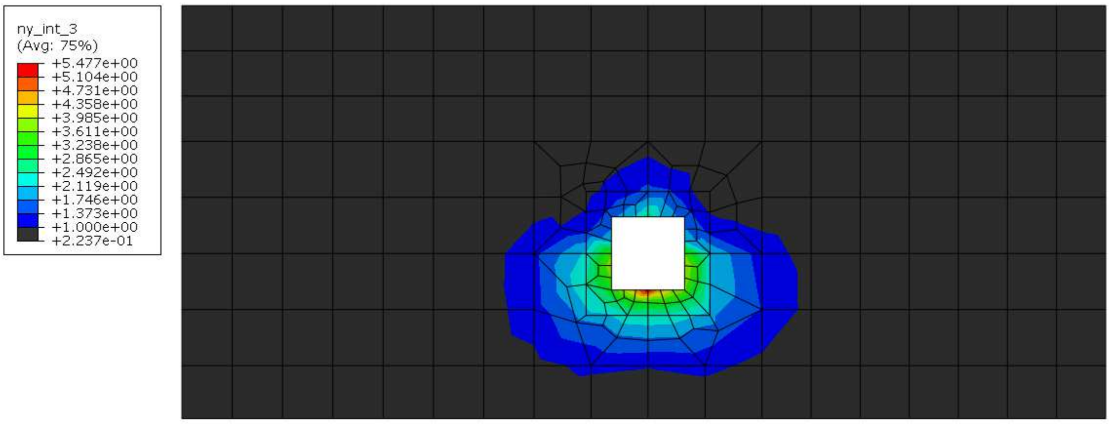


Figure 8.9: Contour plot of the utilization of the concrete resistance below the support (mid span) according to the interaction criterion, [-]

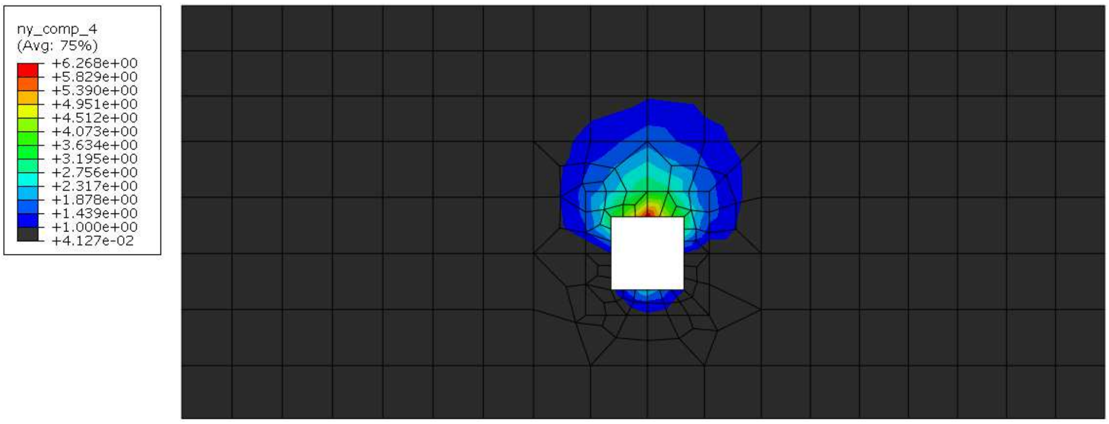


Figure 8.10: Contour plot of the utilization of the concrete resistance above the support (outer span) according to the component criterion, [-]

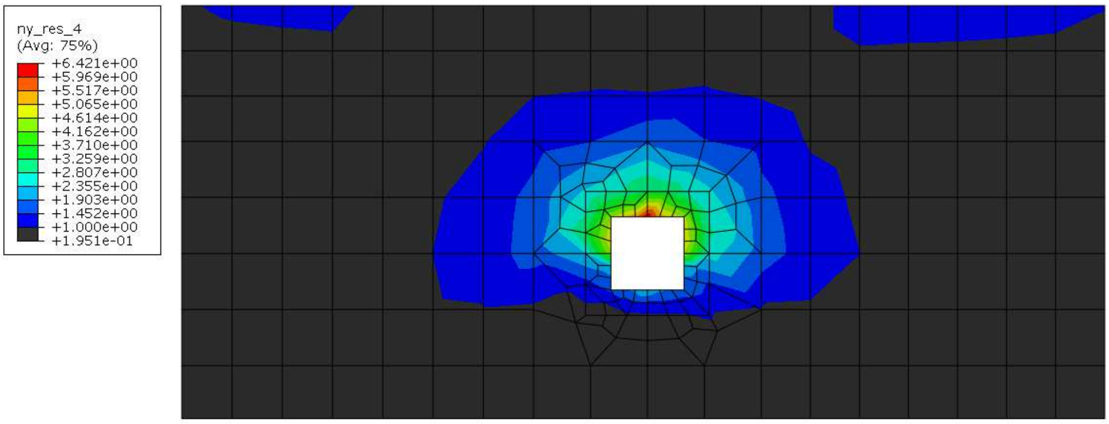


Figure 8.11: Contour plot of the utilization of the concrete resistance above the support (outer span) according to the resultant criterion, [-]

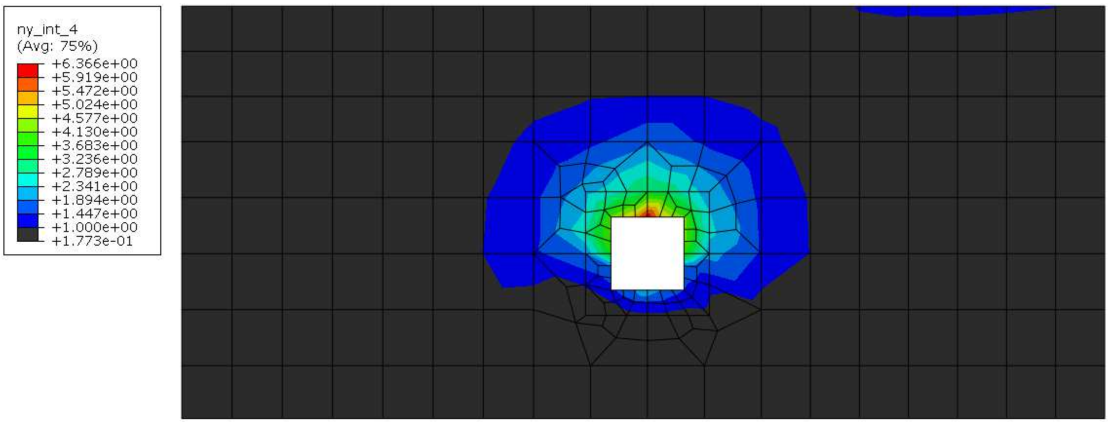


Figure 8.12: Contour plot of the utilization of the concrete resistance above the support (outer span) according to the interaction criterion, [-]

8.3 Discussion of the Investigation

The investigation shows both similarities and differences between the different examined methods that were used to determine the areas that need to be reinforced. As can be seen when comparing the different contour plots, the overall behaviour of the slab is similar independently of the chosen method, but the size of the critical areas varies relatively much.

For the determination of the need of reinforcement in the longitudinal direction, the utilization ratio based on the shear force components, Method 1, give approximately the same result as the one based on the interaction model, Method 3. This can be seen when comparing Figure 8.1 with Figure 8.3 and Figure 8.4 with Figure 8.6. However, both the resultant- and interaction methods give more conservative results than the component method, which is the method that was used in design in this thesis. Therefore, it could be questioned if the design was made based on reasonable assumptions, or if one of the other methods should be preferred. The most conservative method is the resultant method, Method 2. When comparing for instance Figure 8.2 with Figure 8.1 and Figure 8.3, this becomes obvious. The areas that need to be provided with shear reinforcement are significantly larger according to this method compared to the other two. This is a misleading result, as the minimum capacity is used in lack of better ways to approximate the capacity in the directions that the resultant forces act in. Therefore the preferred method should be the interaction method, since this seems to be the most accurate one.

The results for the design in the transversal direction are more distinguished than for the longitudinal. As can be seen when comparing for instance Figure 8.7 with Figure 8.8 and Figure 8.9, Method 1 gives by far the smallest area that need to be provided with shear reinforcement. This is the same tendency that could be seen in the longitudinal direction, but the differences are more distinguished. In this case it is even more relevant to question the design assumption, as the area given by Method 2 and 3 are more than twice as large as the one given by the method used. It should, however, be noted that in the design approach the design in the transversal direction only took place in an isolated part between the critical sections for the design in the longitudinal direction. With this in mind, all regions to the left and to the right of these two critical sections are irrelevant for the design work, as these are taken into account in the design in the longitudinal direction. In that way the results obtained from the three different methods correlates to a higher extent. However, also in this case it seems that the most reasonable and accurate way to approximate the need of shear reinforcement is by the interaction method, even though the resultant method gives more reasonable results for this case than for the previously discussed one.

The difficulties in determining the critical areas are related to the directions. The shear force is to be compared to the concrete resistance and where the shear force exceeds the resistance shear reinforcement is needed. However, the shear force acts in different directions in each point of the slab, and the resistance can only be determined in the two principal directions. In order to relate these to each other, either the shear force needs to be divided into its components, or the capacity needs to be determined for different directions in all different points. The latter alternative is not reasonable and therefore it is preferred to use the shear force components. In the interaction method the components in the different directions are directly connected to the corresponding capacity. When handling more complex geometries it should be even more advantageous to use this method.

9 Investigation of the Load Envelope Design Approach

As discussed in Section 6.4.3, the load envelope design approach could be misleading in bridge design. The summation of the worst loads in all positions creates a higher mean value of the loads than what is actually possible. The vehicles cannot act in all these points at the same time, and therefore it was of interest to investigate to what extent the envelope approach agrees with an approach where only one critical load position is regarded.

9.1 Methodology of Investigation

In order to investigate the similarities and differences between the load envelope and critical load position methodologies, the finite element model was modified. The worst critical position with respect to the design in the longitudinal direction corresponds exactly to the envelope design method, and was therefore not relevant for the investigation. However, the design in the transversal direction is influenced by the envelope design since different load positions are used in different points along the result lines used. An illustration of how this looks like for result line L_1 is presented in Figure 9.1.

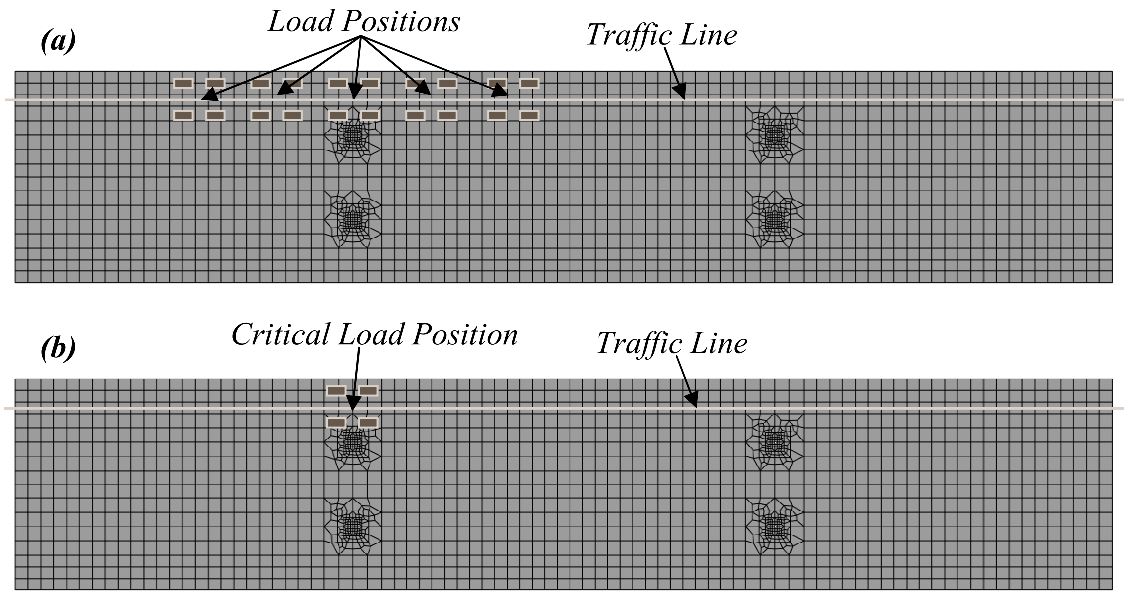


Figure 9.1: Illustration of the envelope design method compared to the critical design method. (a) illustrates the load envelope design method where summation of all possible load position is made. (b) illustrates the critical load position method where only the most critical load position is used for a specific design situation.

The finite element model was modified in a way where a new load case was created. Instead of summation of the moving traffic loads, a single load position was created by application of point loads. The investigation was performed for the outer part of the slab according to Figure 9.1, which corresponds to the design along result line L_1. In order to be able to place the point loads in the desired locations, new partitions of the geometry was made, and therefore a new, denser mesh needed to be created. The results were then taken from the finite element model and post-processed according to the same methodology as described earlier in this thesis, but for both the envelope and for the critical load position method.

9.2 Results from the Investigation

The results from the comparison between the two methods are presented in Figure 9.2 and Figure 9.3.

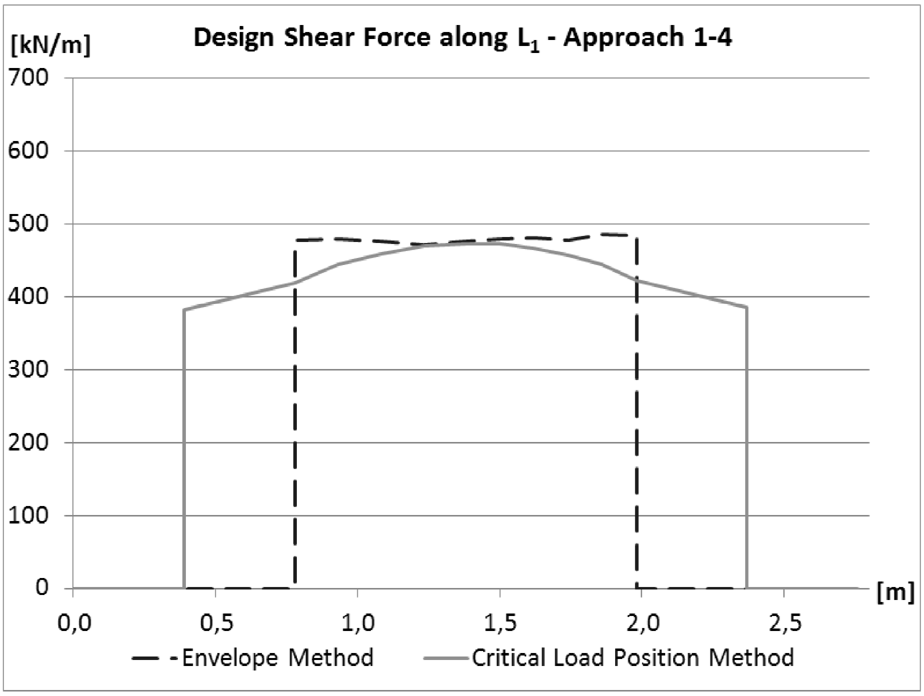


Figure 9.2: Design shear force along result line L_1 according to Approach 1-4 for both the load envelope method and the critical load position method.

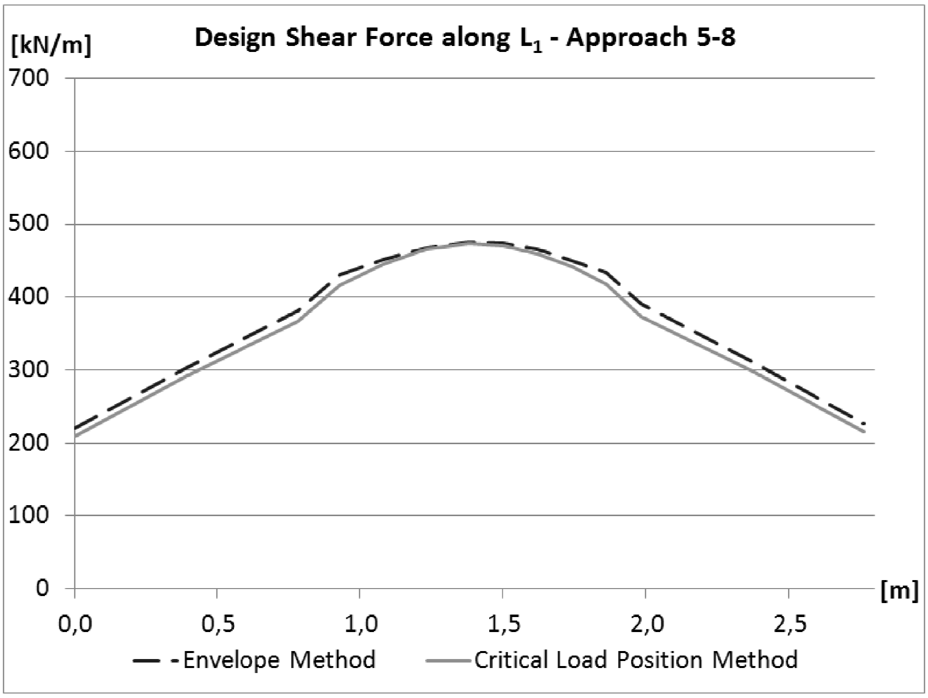


Figure 9.3: Design shear force along result line L_1 according to Approach 5-8 for both the load envelope method and the critical load position method.

9.3 Discussion of Investigation

The investigation shows that there are differences between designing based on the load envelope method or on the critical load position method. However, it also shows that the differences are not significant, and should therefore not be a major influence when it comes to the final design. However, it can be seen that the load envelope design method was the most conservative in one approach, but the most critical one in the other. This can be seen when comparing the curves in Figure 9.2 and Figure 9.3. The behaviour according to Figure 9.3 is what was assumed, and the reason for this investigation to take place.

The reason for the differences in the results between Approach 1-4 and 5-8 is the angle criterion that is used to determine the direction of the resultant shear force in each point, see Figure 3.13. The difference is that the load envelope design method creates large components in the longitudinal direction, while the critical load position method only creates minor components in this direction. This is because the load position exactly at the support does not contribute to any significant shear stresses in the longitudinal direction. This is the same for both the critical load position and the envelope method. However, when looking at points further out from the support, the envelope approach considers other load positions that create stresses in this direction. When the shear components in the longitudinal direction become dominant, the angle criterion assigns these resultant shear forces to act in the longitudinal direction instead. Therefore, the envelope design curve for the transversal direction becomes zero earlier than the critical load position design curve.

For this specific bridge geometry, the load envelope design method seems to give accurate and optimized results compared to the critical load position method. According to Figure 9.3 the shear force components in the transversal direction are approximately the same independently of which of the methods that are used. The load envelope method gave slightly higher values, as could be expected, but the difference is so small that it could be disregarded. However, in the design according to Approach 1-4, see Figure 9.2, larger differences are visible. At first glance, the load envelope design method seems to give unconservative values as the critical load position give high shear values further out from the support. However, this should be a more optimized way to design the reinforcement within the slab, since the resultant shear force was approximated to be acting in the longitudinal direction instead. In that way it can be avoided to reinforce the same concrete area for shear force in both directions. Thus, regardless of which of the approaches that are used to determine the shear force within the slab, the load envelope design method could preferably be used in design. It seems that isolated critical load positions are only relevant for supports that are larger in the longitudinal direction, for example line supports in a console plate deck.

10 Conclusion

The aim of this thesis was to investigate how different shear force design choices influence the final design of reinforced concrete bridge decks in terms of shear reinforcement. In general, the results show that there is not much difference between the different methods to approximate the shear force and the distribution widths for peak shear forces obtained in a linear analysis.

The choice of whether to approximate the shear force as either two shear force components or as a resultant shear force does not lead to any substantial differences in the final design. The total amount of shear reinforcement is similar independently of which of the methods that are implemented. However, locally there is a risk to miss some important regions where the shear force components in the two principal directions are both contributing to a large resultant shear force. One of the shear force components alone may not be critical for design, but together they could exceed the concrete resistance.

From an optimization point of view the choice between these two methods are not important, but from the safety aspect the resultant shear force approach should be chosen in design, even if the work amount is somewhat higher with this method. The resultant shear force approach seems to capture the need for shear reinforcement, both globally and locally, to a higher extent than the shear component approach. Moreover, with the angle criterion according to Pacoste *et al* (2012) that is used to determine the direction of the shear force, the design is also more optimized as some areas are avoided to be double-reinforced. It should also be noted that this tendency will be more distinguished in a bridge with more irregular and complex geometry. If the slab is curved or when the width of the slab varies higher fluctuations are expected.

Currently there are many uncertainties when it comes to the redistribution of peak shear forces within the slab. Due to the linear elastic assumption within the finite element analyses these peaks cannot be avoided and therefore a well founded methodology to determine the distribution widths is preferred. The results show that only small differences in the final design can be distinguished between implementing large or small distribution widths. In fact, almost the same shear reinforcement amounts are implemented regardless of which distribution width that is chosen. It is more a matter of how to distribute the reinforcement within the slab. The reinforcement layout obtained when using small distribution widths may even be preferable since the reinforcement is localised into those regions where the shear forces are highest.

Instead of focusing on the actual distribution width that should be used in the design, the aim should be to find an optimized design. The results show that the shear reinforcement amounts are lower when active choices have been made of regions for redistribution that really benefit from it. Regions in which the shear forces do not exceed the concrete section resistance, without any shear reinforcement, should not be included in the redistribution of the shear forces. If such regions are included together with regions with higher shear forces, the risk is that the average shear force will exceed the concrete resistance. Consequently, the whole region has to be provided with shear reinforcement that resists the average shear force, and the concrete section resistance may no longer be accounted for in the design.

Since it was noticed that there was a large potential for optimization in the use of the concrete resistance, it was investigated how much the reinforcement amounts could be decreased with other methods to approximate the shear capacity of the slab. The results show that there is a potential to increase the accuracy in the capacity determination and in that way optimize the design. Compared to the problem of load effect determination more possibilities are distinguished when investigating this aspect. The current design procedure seems not to be well balanced as the level of accuracy is high on the load effect determination side, but relatively rough on the capacity determination side. More focus should be put on developing a more advanced, but still simple, methodology where the concrete resistance and the shear force reinforcement resistance could be added together. Instead, it should be sufficient to implement simple and conservative assumptions when approximating the shear force and its distribution.

An aspect that had to be handled continuously throughout the thesis is the direction of the shear force related to the main directions of the bridge. One of the questions that came up related to the directions was if other more sophisticated approaches to determine the utilization of the concrete resistance could be used. The results show that for this rather simple and plain geometry the difference between looking at one direction at a time and using an interaction approach was not very significant. However, the interaction method seems to be both realistic and simple, as the main directions are used for both the shear force and the capacity, and the criterion takes into account the interaction between these two directions.

Another question that came up related to the optimization of the design was how much the average shear forces were overestimated in the load envelope design procedure. The results show that the load envelope design approach gave similar or even better results from an optimization point of view compared to an approach where only a critical load position was considered. It seems that the load envelope design method is a reliable approach for this kind of bridge. However, for cantilever slabs the load envelope design should be complemented with a more detailed design work based on critical load positions.

One of the most important conclusions to be drawn from this study is that the results indicate that the current design methodology does not allow for any further significant optimizations. With current minimum reinforcement criterion, there is no need for improving the approaches to approximate the load effects in terms of magnitudes, directions or distributions of the shear force. The study shows that if shear reinforcement is needed and if the inclination of the struts is chosen to be as favourable as possible, an over-capacity will be obtained due to the minimum reinforcement requirements. This means that the only way to optimize the design would be to avoid the need for shear reinforcement. Based on all this, the check of the concrete resistance should be made as accurate as possible, but if shear reinforcement is needed, the magnitude and direction of the shear force, as well as the redistribution of the peak shear forces, are of minor importance. It seems that the need for shear resistance will be covered anyway.

11 Further Work within the Subject

Although several investigations have been made within this thesis, many areas related to this topic need further investigations. Some of the more important areas connected to the work in this thesis are listed below.

- Similar comparisons on more complex bridge geometries in order to investigate the magnitude of differences in such cases. The geometry in this thesis was kept as simple as possible and the results in this study may not always be relevant for other bridges.
- Development of simple but more accurate methods to determine the combined resistance provided by the concrete and the reinforcement. The results in this thesis showed that there is much potential from an optimization point of view in utilizing the concrete resistance and adding it together to the shear force reinforcement resistance.
- Investigation of the relation between shear force failure and punching shear failure. A more complete picture regarding the design for the shear force and the punching shear is desired. In principal, the shear force and the punching shear is the same force, but still the design work is made isolated for each of them.
- Investigation of how reasonable the current minimum reinforcement rules are for a reinforced concrete bridge deck. The current rules are rather strict for a slab and if models where the concrete and reinforcement capacities can be added together are used, the minimum rules quickly become governing.

12 References

Al-Emrani, M., Engström, B., Johansson, M., & Johansson, P. (2011): *Bärande konstruktioner – Del 1* (Load bearing structures – Part 1. In Swedish). Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

BBK 04 (2004): *Boverkets handbok om betongkonstruktioner, BBK 04.* (Hand Book for Concrete Structures. In Swedish) Vällingby

Blaauwendraad, J. (2010): *Plates and FEM – Surprises and Pitfalls.* Springer, Dordrecht

CEB-FIP (2013): *Model Code for Concrete Structures 2010,* Ernst and Sohn

Davidson, M. (2003): *Strukturanalys av betongkonstruktioner med finita elementmetoden.* (Structural Analysis of Concrete Structures with the Finite Element Method. In Swedish) Brosamverkan Väst, Göteborg, Sverige.

Deaton, J.B. (2005): *A Finite Element Approach to Reinforced Concrete Slab Design.* School of Civil and Environmental Engineering, Georgia Institute of Technology.

Engström, B. (2011a): *Design and Analysis of Continuous Beams and Columns.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Engström, B. (2011b): *Design and Analysis of Slabs and Flat Slabs.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Fib Bulletin 45 (2008): *Practitioner´s Guide to Finite Element Modelling of Reinforced Concrete Structures.* CEB-FIB Lausanne.

Hillerborg, A. (1996): *Strip Design Method Handbook.* London, Great Britain: E & FN Spon

Jernström, G. (2012): *Broprojektering, riktlinjer.* (Bridge Design, Guidelines. In Swedish.) Lecture in the bachelor´s degree course "Bridge Design". Held at Chalmers University of Technology, 2012.

Pacoste, C., Plos, M., & Johansson M. (2012): *Recommendations for Finite Element Analysis for the Design of Reinforced Concrete Slabs.* KTH, Stockholm, Sweden.

Plos, M. (1996): *Finite Element Analyses of Reinforced Concrete Structures.* Department of Civil and Environmental Engineering, Chalmers University of Technology, Göteborg.

Vaz Rodrigues, R. (2007): *Shear Strength of Reinforced Concrete Bridge Deck Slabs.* Ph.D. Thesis, EPFL, N^{o} 3739, Lausanne, 2007.

Rombach G. A. (2004): *Finite Element Design of Concrete Structures.* Thomas Telford, Cornwall.

Scanscot Technology AB (2007): *Introduction to Brigade/Plus.* Scanscot Technolofy AB, Sweden.

SS-EN 1991-2 (2003): *Eurocode 1: Actions on Structures – Part 2: Traffic Loads on Bridges.* European Standard, Brussels.

SS-EN 1992-1-1 (2005): *Eurocode 2: Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings.* European Standard, Brussels.

SS-EN 1992-2 (2005): *Eurocode 2: Design of Concrete Structures – Part 2: Concrete Bridges – Design and Detailing Rules.* European Standard, Brussels.

Sustainable Bridges (2007): *Nonlinear Analysis and Remaining Fatigue Life of Reinforced Concrete Bridges.* Sustainable Bridges – Assessment for Future Traffic Demands and Longer Lives.

Trafikverket (2011a): *TRVK Bro 11 – Trafikverkets tekniska krav Bro* (Technical Requirements for Bridges. In Swedish). Trafikverket, Borlänge.

Trafikverket (2011b): *TRVR Bro 11 – Trafikverkets tekniska råd Bro* (Technical recommendations for Bridges. In Swedish). Trafikverket, Borlänge.

Vägverket (1996): *Broprojektering – En handbok.* (Bridge Design – A Handbook. In Swedish.) Vägverket, Borlänge.