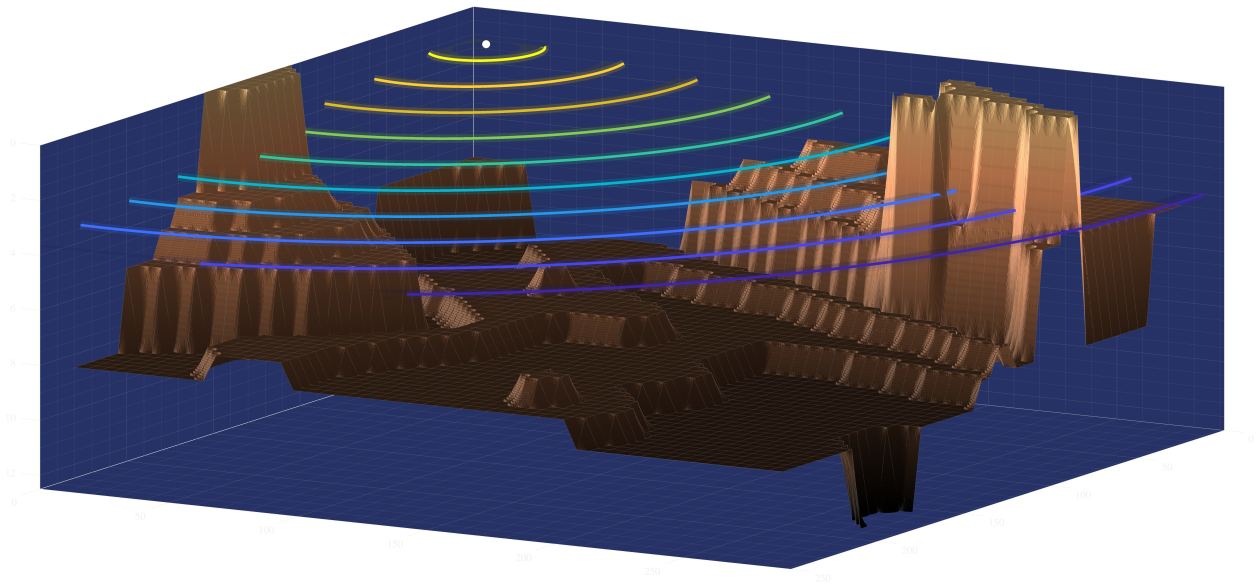




CHALMERS
UNIVERSITY OF TECHNOLOGY



Modelling Underwater Sound Propagation Using the Finite-Difference Time-Domain Method

Master's Thesis in Sound and Vibration

CALLUM CUNNINGHAM

DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
DIVISION OF APPLIED ACOUSTICS

CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2026
www.ta.chalmers.se

MASTER'S THESIS 2026

Modelling Underwater Sound Propagation Using the Finite-Difference Time-Domain Method

Callum Cunningham



CHALMERS
UNIVERSITY OF TECHNOLOGY

Department of Architecture and Civil Engineering
Division of Applied Acoustics
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2026

Modelling Underwater Sound Propagation Using the Finite-Difference Time-Domain Method

Callum Cunningham

© Callum Cunningham, 2026.

Supervisors: Jens Forssén, Division of Applied Acoustics
Carl Andersson, IVL Swedish Environmental Research Institute

Examiner: Jens Forssén, Division of Applied Acoustics

Master's Thesis 2026
Department of Architecture and Civil Engineering
Division of Applied Acoustics

Chalmers University of Technology
SE-412 96 Gothenburg
Telephone: +46 31 772 1000

Cover: Artistic interpretation of sound propagation in a shallow water environment using real bathymetry data from Kosterhavet National Marine Park.

Typeset in L^AT_EX
Printed by Chalmers Digital Printing
Gothenburg, Sweden 2026

Modelling Underwater Sound Propagation Using the Finite-Difference Time-Domain Method

Callum Cunningham

Department of Architecture and Civil Engineering
Chalmers University of Technology

Abstract

Marine life in coastal regions is under environmental pressure from anthropogenic activities such as commercial shipping, offshore wind farms and leisure boats. As part of an ongoing research project to create underwater noise maps for recreational boats, IVL Swedish Environmental Research Institute has measured propagation loss in Kosterhavet marine protected area. These measurements show a variety of acoustic effects that are not captured by simple analytical models. The goal of this project is to produce a numerical model that can provide a more detailed understanding of how noise propagates underwater to the surrounding environment.

A three-dimensional finite-difference time-domain (FDTD) model was developed in MATLAB. The model was validated by comparison to known analytical solutions of increasing complexity. First, it was confirmed that the model reproduces propagation loss due to spherical spreading, agreeing closely with analytical predictions. A nearly perfectly matched layer (NPML) was implemented to avoid unwanted reflections from the boundaries of a truncated domain. The decay factor and layer width were optimised empirically, achieving reduction of reflected wave energy up to 45 dB relative to a rigid boundary condition. Then, several boundary conditions were tested including a rigid wall boundary, a pressure-release condition and a finite-impedance boundary, with the latter two conditions representing the water-air interface and the water-seabed interface respectively. Analytical solutions for each case were calculated and show strong agreement with model output in both frequency and time domains. Finally, an attempt was made to reproduce real world measurements of propagation loss provided by IVL using different configurations of sea floor properties. Initial results show promise however require further investigation and fine-tuning of the model to accurately reproduce measurements.

Keywords: underwater acoustics, finite-difference time-domain (FDTD) method, sound propagation, perfectly matched layer (PML), boundary conditions, numerical modelling, propagation loss, shallow water acoustics, MATLAB simulation.

Acknowledgements

I would like to thank the underwater acoustics team at IVL for suggesting an interesting and challenging thesis and for providing a welcoming space in which to work during the writing of this thesis. Thanks to my supervisors Carl Andersson and Jens Forssén for their guidance. Finally, I would like to thank my family for their continued support over the last two years, I am so grateful.

Callum Cunningham, Gothenburg, August 2026

Table of Contents

List of Figures	vii
List of Tables	ix
List of Acronyms	x
Nomenclature	xi
1 Introduction	1
1.1 Background	1
1.2 Modelling Methods	2
1.3 Objectives	2
2 Theory	4
2.1 Underwater Sound Propagation	4
2.2 Finite-Difference Time-Domain Method	5
2.2.1 Spatial Grids	6
2.2.2 Temporal Grids	9
2.2.3 Considerations and Limitations	12
2.3 Boundary Conditions	13
2.4 Nearly Perfectly Matched Layer	13
3 Validation	17
3.1 Spherical Spreading	17
3.2 Nearly Perfectly Matched Layer	23
3.2.1 Decay Factor	23
3.2.2 Layer Thickness	27
3.3 Boundary Conditions	30
3.3.1 Rigid Boundary	31
3.3.2 Pressure-Release Condition	33
3.3.3 Finite-Impedance Boundary Condition	36
4 Propagation Loss Modelling	41
4.1 Measurement Data	41
4.2 Domain Generation	41

4.3	Comparison with Measurements	45
5	Discussion	48
6	Conclusion	51
	References	52
A	Appendix	A-1

List of Figures

2.1	Spatial finite-difference grids used in FDTD method. The y-z plane is not pictured but follows an equivalent structure.	7
2.2	x-z plane with pressure and velocity variables on grids staggered in time.	10
3.1	Source signal used in the initial stages of validation.	18
3.2	Source signal used in the initial stages of validation.	18
3.3	All possible receivers for spherical spreading validation.	19
3.4	Receivers for which the outgoing wave passes through entirely.	20
3.5	Pressure vs. time at each receiver point. Distance to receiver is displayed in the legend.	20
3.6	(Pressure x radius) vs. time at each receiver point. Distance to receiver is displayed in the legend.	21
3.7	RMS pressure at receivers of varying distances from the source, predicted by FDTD model and analytical solution.	22
3.8	Error in RMS pressure between model output and predicted analytical solution.	22
3.9	Computational domain set-up for the validation of the nearly perfectly matched layer.	23
3.10	Time signal recorded at receiver for varying decay factors η_m	24
3.11	Reflected sound captured at receiver for a broad range of decay factors η_m	25
3.12	Reflected sound captured at receiver for a narrower range of maximum decay factor η_m	26
3.13	Spectra of reflected sound recorded at receiver for a narrower range of η_m values compared to that of a rigid wall BC.	26
3.14	RMS pressure of reflected sound wave for a narrower range of η_m values relative to RMS pressure of a rigid wall BC.	27
3.15	Reflected sound captured at a single receiver for a range of L_{pml}	27
3.16	Spectra of reflected sound recorded at receiver for various L_{pml}	28
3.17	RMS pressure of reflected sound wave for various L_{pml} , relative to RMS pressure of a rigid wall BC.	29
3.18	Physical parameters used to calculate the plane wave reflection factor.	30
3.19	Model set-up for validation of rigid boundary condition.	31
3.20	Comparison of pressure time signals for modelled and analytical solutions of a rigid boundary condition.	32

3.21	Comparison of frequency spectra for modelled and analytical solutions of a rigid boundary condition.	33
3.22	Model set-up for validation of pressure-release condition.	34
3.23	Comparison of pressure time signals for modelled and analytical solutions of a pressure-release boundary condition.	35
3.24	Comparison of frequency spectra for modelled and analytical solutions of a pressure-release boundary condition.	35
3.25	Model set-up for validation of finite-impedance boundary condition.	36
3.26	Comparison of pressure time signals for modelled and analytical solutions of a finite-impedance boundary condition.	37
3.27	Comparison of frequency spectra for modelled and analytical solutions of a finite-impedance boundary condition.	38
3.28	Finite-impedance boundary condition where angle of incidence approaches the critical angle.	39
3.29	Model output compared to analytical solution for receiver in figure 3.28.	39
3.30	Comparison of frequency spectra for time signals in figure 3.29.	40
4.1	Depth map of measurement site with source and receiver locations.	42
4.2	Modelled depth map generated in MATLAB using data provided by IVL.	42
4.3	Modelled depth map generated in MATLAB using data provided by IVL.	43
4.4	Depth map of sub-site 0 generated using linear interpolation.	44
4.5	Depth map of sub-site 0 generate using ‘natural’ interpolation.	44
4.6	A computational domain was created containing the area inside the rectangle.	46
4.7	Frequency spectrum of source signal, receiver signal and propagation loss.	46
4.8	Modelled PL for varying thicknesses of a sea floor composed of very fine silt.	47
A.1	Low pass delta function used as source signal in simulation of real domain.	A-1
A.2	Frequency spectrum of low pass delta function in A.1	A-1

List of Tables

4.1	Material properties of media used in the model.	45
-----	---	----

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

ABC	Absorbing Boundary Condition
BC	Boundary Condition
dB	Decibel
FDTD	Finite-Difference Time-Domain
GPU	Graphics Processing Unit
NPML	Nearly Perfectly Matched Layer
PE	Parabolic Equation
PL	Propagation Loss
PML	Perfectly Matched Layer
RMS	Root Mean Square

Nomenclature

Below is the nomenclature of indices, sets, parameters, and variables used throughout this thesis. Bold symbols denote vector quantities; subscripts x, y, z denote the corresponding Cartesian components (e.g., w_x is the x -component of $\mathbf{w}$).

Indices

i	spatial grid index in the x -direction, $i \in \mathbb{Z}$
j	spatial grid index in the y -direction, $j \in \mathbb{Z}$
k	spatial grid index in the z -direction, $k \in \mathbb{Z}$
l	temporal grid index, $l \in \mathbb{Z}$
α	index denoting the Cartesian direction associated with the PML auxiliary equations, $\alpha \in \{x, y, z\}$

Sets

$\mathbb{Z}$	set of integers
--------------	-----------------

Operators

∂	partial derivative operator
∇	gradient operator

Parameters

b	mass buoyancy	$\text{m}^3 \text{kg}^{-1}$
C	Courant number	
c	spatially-varying sound speed field	m s^{-1}
c_0	reference speed of sound in water	1500 m s^{-1}
c_{max}	highest sound speed of any medium present in the model	m s^{-1}
D	number of grid dimensions	
Δt	temporal grid interval	s
Δx	spatial grid interval in the x -direction	m
Δy	spatial grid interval in the y -direction	m
Δz	spatial grid interval in the z -direction	m
L_{pml}	PML layer width	m
Q	mass source function	$\text{m}^3 \text{s}^{-1}$
$\dot{Q}_s$	time derivative of source function	$\text{m}^3 \text{s}^{-2}$
r	radial coordinate from source to receiver	m
R	reflection factor	
$\mathbf{v}$	velocity of background medium	m s^{-1}
Z	characteristic acoustic impedance of a medium	$\text{kg m}^{-2} \text{s}^{-1}$
η	PML decay factor	s^{-1}
η_m	maximum PML decay factor	s^{-1}
θ_1	angle of incidence	radians
θ_2	angle of refraction	radians
θ_c	critical angle	radians
κ	adiabatic bulk modulus	Pa
ρ	density	kg m^{-3}
ρ_0	reference density of water	1027 kg m^{-3}

Variables

p	acoustic pressure	Pa
PL	propagation loss	dB
P_r	Fourier transform of signal at receiver	dB rel. 1 μ Pa
P_s	Fourier transform of source signal at 1 m in free-field	dB rel. 1 μ Pa
$s(t)$	Source and material terms in the pressure update equation	
t	time	s
$\mathbf{w}$	acoustic particle velocity	m s^{-1}
ξ	generic placeholder representing any of the regular field variables, $\xi \in \{p, w_x, w_y, w_z\}$	
$\bar{\xi}^\alpha$	auxiliary variable corresponding to ξ , associated with direction α , used in the PML formulation	

Introduction

1.1 Background

Life in coastal regions is under large environmental pressure from many human activities including, but not limited to commercial shipping and fishing, military activity, wind farm development and leisure boats. Noise generated by such activities can propagate large distances underwater, especially in the case of low-frequency noise (below 500 Hz) although medium frequency noise (500 Hz to 25 kHz) is still capable of travelling tens of kilometres [1]. Anthropogenic noise sources disrupt acoustic signals used by marine life for navigation and communication, as well as locating prey or avoiding predators. Long term exposure to noise pollution may result in physiological effects such as hearing loss and increased stress. These effects may lead to behavioural changes, for instance, modified migration patterns, changes in reproductive and feeding behaviours, and in the worst case, abandonment of habitat [2].

The underwater environment of Sweden's coastal waters, with its shallow soft-bottom seabed protected from waves by natural formations like archipelagos, provides a breeding ground for many fish species and crustaceans [3], and adequate habitat for marine mammals such as porpoises and seals [4]. This environment is also particularly suited to recreational boating activities, confirmed by the existence of over 700 000 leisure boats kept at almost 110 000 piers and jetties across the country [3]. Whilst much research has been conducted on noise pollution from commercial vessels and there exists an international standard for measurement of underwater noise of ships [5], there is a lack of research addressing noise from leisure boats and its effect on marine organisms. The studies that do exist are difficult to compare due to lack of standard measurement guidelines [3].

As part of an ongoing research project to create underwater noise maps for recreational boats, the underwater acoustics team at IVL Swedish Environmental Research Institute has measured propagation loss (PL) in Kosterhavet marine protected area, a coastal region in western Sweden. These measurements show a variety of acoustic effects that are not captured by data-driven analytical models. It was suggested by the team at IVL that a physics-based numerical model may succeed in better reproducing such effects.

1.2 Modelling Methods

Several methods may be used in the modelling of underwater sound such as ray tracing, wavenumber integration techniques and parabolic equation (PE) methods. Each of these methods have advantages and disadvantages. Ray tracing is less computationally demanding and generally quick to execute however is a high frequency approximation [6]. Wavenumber integration techniques, being a numerical method, are more physically accurate than ray methods however are not suited to determining the wave field for more than one receiver depth. Furthermore, the horizontal stratification inherent to the method excludes low and medium frequency scattering as well as reverberation from ocean boundaries, phenomena which are of significance in modelling of shallow water sound propagation [7]. Parabolic equation methods are frequently used in environmental sound propagation and are very effective at solving range-dependent propagation problems though are limited to one-way propagation [8]. This is generally not a problem for long-range propagation models however in the shallow water environment, scattering will occur from bounding surfaces and other objects, thus PE methods are not suitable. Additionally, PE and wavenumber integration methods are frequency domain methods, meaning that computations are carried out for a frequency of interest. This significantly decreases their usefulness when a model for broadband noise is required [8].

The finite-difference time-domain (FDTD) method has been suggested by IVL due to its ability to determine the wave field across a computational domain throughout time for a broad frequency band. Since it solves the wave equation directly, FDTD involves no high-frequency approximation and, unlike PE methods, inherently models two-way propagation, including backscattering from complex terrain. It is also possible to implement transient source signals, and interactions with bounding surfaces such as the sea surface or sea floor [9]. This versatility does incur a large computational burden, discussed further in section 2.2.3, however for the shallow water environment to be investigated in this project, the burden is deemed appropriate.

1.3 Objectives

The aim of this project is to improve upon the accuracy of the current analytical propagation loss models used by IVL. The model should be able to assess propagation of acoustic waves in the ocean from a specified source in a given environment and accurately model acoustic phenomena. This requires a complex physical model and as such, a finite-difference time-domain model has been developed.

The research questions this project aims to answer are:

1. How well can an FDTD model explain the propagation loss in a complex acoustic environment?
2. How can varying bottom profile (bathymetry) and boundaries be incorporated

in a FDTD model?

3. How sensitive is the FDTD modelling approach to environmental parameters, for example bottom sediment properties?

A satisfactory model could be used to assess propagation of acoustic waves in the ocean from a specified source in a given environment. This could reduce reliance on labour-intensive in-situ measurements and may be employed in a variety of situations such as underwater noise mapping, quantification of vessel source noise in shallow waters or modelling of environmental noise mitigation solutions.

2

Theory

In this chapter the characteristics of underwater sound propagation will be briefly described and assumptions made in this project will be noted. The theory behind the FDTD model and its implementation will be described in detail along with the computational limitations specific to this project. Finally, the boundary conditions prescribed in the FDTD model will be discussed and the implementation of a nearly perfectly matched layer (NPML) will be detailed.

2.1 Underwater Sound Propagation

Sound occurs underwater for a variety of reasons. It is generated by animals and humans alike, both intentionally and unintentionally. Sound is the most efficient means of communication over long distances in the underwater environment since electromagnetic radiation is rapidly attenuated by conductive and suspended materials in seawater. It therefore sees much use in military and commercial contexts for navigation, vessel detection and geological measurements [10][11]. It is also produced as a by-product of marine propulsion systems, offshore drilling and wind farms.

The characteristics of sound propagation underwater are defined by the material properties of water and its interaction with the surfaces that bind it. The speed of sound in water varies according to temperature, depth and salinity and is often calculated empirically using these parameters [10]. In this work a typical ocean sound speed of 1500 m s^{-1} was used in all simulations and calculations. The density of water also affects sound propagation and for simplicity was kept at 1027 kg m^{-3} for all simulations and calculations.

As sound waves travel through water they are subject to several phenomena. Propagation loss describes the decrease in energy of the wave as it travels through water. This loss of energy is due to cylindrical or spherical spreading as well as other attenuation mechanisms such as absorption by water molecules, and scattering from suspended particles, organisms or other inhomogeneities in the water column [12]. Due to the scope of this project, only propagation loss due to spherical spreading was addressed.

Reflections from the sea surface, sea bed and other surfaces interact with the direct sound wave causing interference patterns. The interference pattern due to reflections from the sea surface is a well-known phenomenon called the Lloyd mirror effect and is important to consider in both deep water and shallow water models. A shallow water environment includes both reflections from the sea surface and the sea floor, thereby acting as a waveguide. The wave is continually reflected between the surface and sea bottom. With varying bottom depths and varying sea floor properties, the prediction of acoustic wave propagation becomes very complicated [11]. Reflection phenomena and varying bathymetry are addressed in chapters 3 and 4.

Sound waves may also diffract around objects in the water such as islands causing changes in the sound field beyond said objects. Unfortunately due to the scope of this project, diffraction effects were not investigated.

2.2 Finite-Difference Time-Domain Method

The finite-difference time-domain method is a numerical analysis method originally introduced in the field of electromagnetism by Yee in 1966 as a means to solve Maxwell's equations [13]. Since its inception, its usage has been expanded to other fields of research including acoustics. In the book *Acoustics in Moving Inhomogeneous Media*, Ostashev and Wilson detail the implementation of a two-dimensional FDTD model for sound propagation in the atmosphere [9]. This is the basis for the model described in this paper which has been expanded to three dimensions and adapted for the modelling of underwater sound propagation.

The FDTD method solves discretised first-order differential equations for acoustic pressure and acoustic particle velocity across a spatial domain at a given point in time. A complete set of linearised fluid dynamics equations is reduced to a set of two equations using a number of assumptions for acoustic waves [14]. The sound wave is assumed to be a small perturbation to the state of the medium. The medium is assumed to behave adiabatically, with no turbulence in its background velocity field. The gradient of pressure in the medium is also neglected which has been deemed a reasonable assumption for application of this method in shallow coastal waters, however it should be noted that pressure gradients are a common and important factor to take into account for many underwater environments.

Equations (2.1) and (2.2) describe acoustic pressure p and acoustic particle velocity $\mathbf{w}$ in a medium and can be used to model the propagation of sound waves. Velocity of the background medium is represented by the vector $\mathbf{v}$, density by ρ and sound speed by c . $\mathbf{F}$ represents a force source acting on the medium and Q represents a mass source.

$$\frac{\partial \mathbf{w}}{\partial t} = -(\mathbf{w} \cdot \nabla) \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{w} - \frac{\nabla p}{\rho} + \frac{\mathbf{F}}{\rho} \quad (2.1)$$

$$\frac{\partial p}{\partial t} = -(\mathbf{v} \cdot \nabla)p - \rho c^2 \nabla \cdot \mathbf{w} + \rho c^2 Q \quad (2.2)$$

These equations are derived for use in a ‘moving medium’, stated by Ostashev and Wilson to be a medium whose motion has non-negligible effects on propagation. Whilst this condition may be appropriate for sound propagation in the atmosphere, where the speed of sound may only be 34-110 times larger than typical wind speed, the presence of the medium velocity $\mathbf{v}$ later complicates the implementation of the FDTD method [15]. Since the speed of sound in water is considerably larger than in air, and ocean currents much slower than wind speeds, it was decided in this project to neglect the motion of the background medium [16]. This simplifies equations (2.1) and (2.2) to equations (2.3) and (2.4). Density and sound speed are also replaced by material properties adiabatic bulk modulus κ ($= \rho c^2$) and mass buoyancy b ($= 1/\rho$).

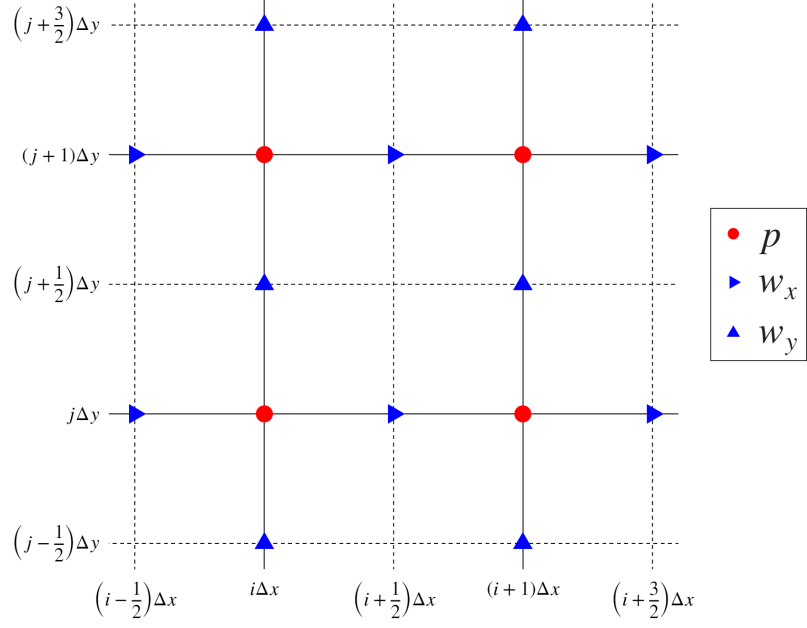
$$\frac{\partial \mathbf{w}}{\partial t} = -b \nabla p \quad (2.3)$$

$$\frac{\partial p}{\partial t} = -\kappa \nabla \cdot \mathbf{w} + \kappa Q \quad (2.4)$$

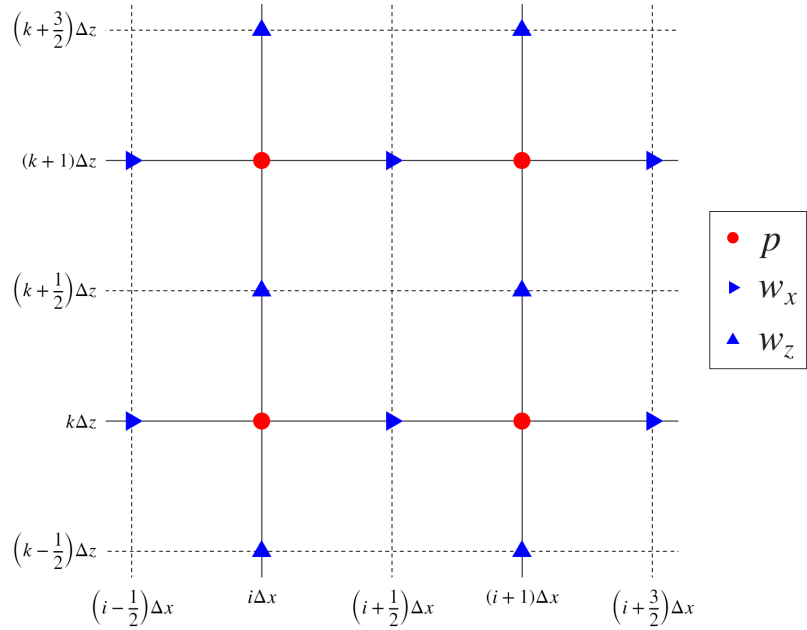
2.2.1 Spatial Grids

The FDTD method discretises the spatial and temporal derivatives in the above equations. The key idea in this method is that acoustic pressure and particle velocity are separated onto grids staggered in space, allowing for (2.3) and (2.4) to be solved using finite differences. Such a staggered grid is shown in figure 2.1. Acoustic pressure is stored at integer nodes $(i\Delta x, j\Delta y, k\Delta z)$ where $i, j, k \in \mathbb{Z}$ and $\Delta x, \Delta y$, and Δz are grid intervals in the given direction. Material properties b, κ and source term Q are also stored at integer nodes, also referred to henceforth as pressure nodes. Particle velocity $\mathbf{w}$ is decomposed into its components w_x, w_y and w_z , each of which are stored on grids staggered by half a grid interval in their respective directions. That is to say, w_x values are stored at $([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z)$, w_y values are stored at $(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z)$ and w_z values are stored at $(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z)$. These grid nodes will be referred to collectively as velocity nodes and if necessary, specified relative to the velocity component with which they correspond.

The separation of $\mathbf{w}$ into its Cartesian components requires the decomposition of (2.3) into equations (2.5) - (2.7). It is then (2.4) - (2.7) that become the equations to be solved in order to describe the propagation of sound through a medium.



(a) x-y plane.



(b) x-z plane.

Figure 2.1: Spatial finite-difference grids used in FDTD method. The y-z plane is not pictured but follows an equivalent structure.

$$\frac{\partial w_x}{\partial t} = -b \frac{\partial p}{\partial x} \quad (2.5)$$

$$\frac{\partial w_y}{\partial t} = -b \frac{\partial p}{\partial y} \quad (2.6)$$

$$\frac{\partial w_z}{\partial t} = -b \frac{\partial p}{\partial z} \quad (2.7)$$

The use of the staggered grid means that the spatial derivatives of pressure required at velocity grid nodes, in equations (2.4) - (2.7) are relatively easily calculated using finite differences. For example, $\partial p / \partial x$ at a location on the w_x grid is determined by calculating the finite difference between the two nearest pressure nodes as in (2.8). Equivalent formulations for $\partial p / \partial y$ and $\partial p / \partial z$ are shown in equations (2.9) and (2.10).

$$\frac{\partial p([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, t)}{\partial x} \approx \frac{p([i + 1]\Delta x, j\Delta y, k\Delta z, t) - p(i\Delta x, j\Delta y, k\Delta z, t)}{\Delta x} \quad (2.8)$$

$$\frac{\partial p(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z, t)}{\partial y} \approx \frac{p(i\Delta x, [j + 1]\Delta y, k\Delta z, t) - p(i\Delta x, j\Delta y, k\Delta z, t)}{\Delta y} \quad (2.9)$$

$$\frac{\partial p(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z, t)}{\partial z} \approx \frac{p(i\Delta x, j\Delta y, [k + 1]\Delta z, t) - p(i\Delta x, j\Delta y, k\Delta z, t)}{\Delta z} \quad (2.10)$$

Similarly, spatial derivatives of acoustic particle velocity that arise in equation (2.4) from the operation $\nabla \cdot \mathbf{w}$ are also required at pressure nodes. These are also derived using the finite difference between the nearest two velocity nodes. These derivations are shown in equations (2.11) - (2.13).

$$\frac{\partial w_x(i\Delta x, j\Delta y, k\Delta z, t)}{\partial x} \approx \frac{w_x([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, t) - w_x([i - \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, t)}{\Delta x} \quad (2.11)$$

$$\frac{\partial w_y(i\Delta x, j\Delta y, k\Delta z, t)}{\partial y} \approx \frac{w_y(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z, t) - w_y(i\Delta x, [j - \frac{1}{2}]\Delta y, k\Delta z, t)}{\Delta y} \quad (2.12)$$

$$\frac{\partial w_z(i\Delta x, j\Delta y, k\Delta z, t)}{\partial z} \approx \frac{w_z(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z, t) - w_z(i\Delta x, j\Delta y, [k - \frac{1}{2}]\Delta z, t)}{\Delta z} \quad (2.13)$$

As mentioned, material properties of the medium are stored at pressure nodes. Nevertheless, the calculations (2.5) - (2.7), which take place on the velocity grid require the material property buoyancy, b . It is thus necessary to determine the buoyancy at velocity nodes. This is done by averaging neighbouring values, for example in equation (2.14) which shows the determination of buoyancy on the w_x grid. In practice, for a uniform medium, buoyancy will not change from one grid point to the next. This operation will be most important at material boundaries such as the water-seabed interface where buoyancy will change.

$$b([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, t) \approx \frac{b([i + 1]\Delta x, j\Delta y, k\Delta z, t) + b(i\Delta x, j\Delta y, k\Delta z, t)}{2} \quad (2.14)$$

This set of finite difference equations (2.8) - (2.14) provides approximations for all spatial derivatives on the right-hand side of equations (2.4) - (2.7). Material properties b, κ and source term Q are assumed to be known quantities and input to the model.

2.2.2 Temporal Grids

With spatial grids alone, one may take an initial pressure condition and use it to calculate the acoustic particle velocity or vice versa for a single time step. In order to simulate sound propagation, it is necessary to advance the model from one time step to the next. Several schemes for temporal advancement are discussed by Ostashev, Wilson and Liu in [9] and [15], each with different computational burden and memory requirements. The scheme used in this model is referred to as the leap-frog method. It was chosen as it is capable of modelling a non-moving media without compromises and since it has relatively low computational burden and memory requirements compared to other methods.

The method is similar to the staggered grid scheme used for the spatial grids. Pressure variables and velocity variables are placed on grids staggered in time. The schematic in figure 2.2 shows the x-z plane staggered in time with pressure nodes at integer interval $l\Delta t$ and velocity nodes at half-integer interval $(l + \frac{1}{2})\Delta t$. This staggering of course applies to the full 3D grid; only a 2D plane of the spatial grid is shown for discernibility.

For pressure stored at whole time steps at velocity and half time steps, the right-hand side of equation (2.4) can be expressed as in equation (2.15), as a function of spatial coordinates, acoustic particle velocities, and source and material terms represented by $s(t)$.

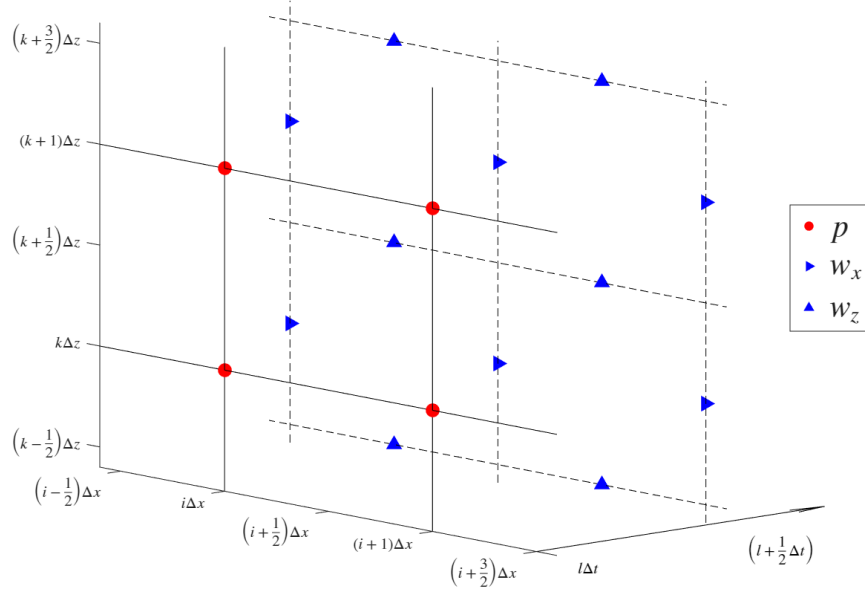


Figure 2.2: x-z plane with pressure and velocity variables on grids staggered in time.

$$\frac{\partial p(i\Delta x, j\Delta y, k\Delta z, [l + \frac{1}{2}]\Delta t)}{\partial t} \approx f_p(i\Delta x, j\Delta y, k\Delta z, \mathbf{w}([l + \frac{1}{2}]\Delta t), s([l + \frac{1}{2}]\Delta t)) \quad (2.15)$$

The time derivative of pressure at this half-integer time step may be calculated by a finite-difference operation using the pressure at integer time steps in equation (2.16). Rearranging this equation allows for a solution to pressure at time step $(l + 1)\Delta t$ using pressure at the previous time step $l\Delta t$ and the intermediate function f_p which exists on the velocity time grid at $(l + \frac{1}{2})\Delta t$.

$$f_p(i\Delta x, j\Delta y, k\Delta z, \mathbf{w}([l + \frac{1}{2}]t), s([l + \frac{1}{2}]t)) \approx \frac{p(i\Delta x, j\Delta y, k\Delta z, [l + 1]\Delta t) - p(i\Delta x, j\Delta y, k\Delta z, l\Delta t)}{\Delta t} \quad (2.16)$$

$$p(i\Delta x, j\Delta y, k\Delta z, [l + 1]\Delta t) \approx p(i\Delta x, j\Delta y, k\Delta z, l\Delta t) + \Delta t f_p(i\Delta x, j\Delta y, k\Delta z, \mathbf{w}([l + \frac{1}{2}]\Delta t), s([l + \frac{1}{2}]\Delta t)) \quad (2.17)$$

A similar procedure yields equivalent solutions for acoustic particle velocity components w_x, w_y and w_z given in equations (2.18) - (2.20).

$$w_x([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, [l + \frac{1}{2}]\Delta t) \approx w_x([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, [l - \frac{1}{2}]\Delta t) + \Delta t f_x([i + \frac{1}{2}]\Delta x, j\Delta y, k\Delta z, p(l\Delta t), s(l\Delta t)) \quad (2.18)$$

$$w_y(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z, [l + \frac{1}{2}]\Delta t) \approx w_y(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z, [l - \frac{1}{2}]\Delta t) + \Delta t f_y(i\Delta x, [j + \frac{1}{2}]\Delta y, k\Delta z, p(l\Delta t), s(l\Delta t)) \quad (2.19)$$

$$w_z(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z, [l + \frac{1}{2}]\Delta t) \approx w_z(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z, [l - \frac{1}{2}]\Delta t) + \Delta t f_z(i\Delta x, j\Delta y, [k + \frac{1}{2}]\Delta z, p(l\Delta t), s(l\Delta t)) \quad (2.20)$$

Let it be stated that f_x, f_y and f_z are equivalent to $\partial w_x/\partial t, \partial w_y/\partial t$ and $\partial w_z/\partial t$. In this scheme, f_p is only required on half-integer time steps and f_x, f_y and f_z are only required on integer time steps. This means that using finite-differences, solutions for each variable are calculated on opposing temporal grids, allowing for the solution to the wave equations to be advanced in time. Pressure at integer time steps is used to calculate velocity at the next half-integer time step, which is then used to update the pressure at the next integer time step and so on, hence the name ‘leap-frog method’. The full procedure for advancing the simulation in time is explained in the steps below:

1. Consider a domain with initial pressure and velocity conditions at time $l\Delta t$. Each pressure node has an associated pressure value, $p(x, y, z, l\Delta t)$.
2. Calculate $\partial p/\partial x, \partial p/\partial y$ and $\partial p/\partial z$ at velocity nodes on the spatial grid using finite-difference equations (2.8) - (2.10).
3. Calculate f_x, f_y and f_z using (2.3).
4. Update w_x, w_y and w_z for all spatial velocity grid nodes for the next half-integer time step $(l + \frac{1}{2})\Delta t$ using (2.18) - (2.20).
5. Calculate $\partial w_x/\partial x, \partial w_y/\partial y$ and $\partial w_z/\partial z$ at $(l + \frac{1}{2})\Delta t$ using (2.11) - (2.13).
6. Calculate $\partial p/\partial t$ using (2.4). Source term only applies at node representing a source.

7. Update p at time step $(l + 1) \Delta t$ using equation (2.17).
8. Repeat to advance the simulation further in time.

2.2.3 Considerations and Limitations

The spatial and temporal grid spacing are defined by the user, however some considerations must be made when deciding upon these values. The spatial grid resolution Δx , equal to Δy and Δz in this model, must be smaller than the shortest wavelength of interest in order to be able to resolve an entire wavelength. It is recommended that the grid spacing must be a small fraction of the wavelength to maintain good numerical accuracy with 20 points per wavelength being cited as high resolution and 10 points per wavelength as acceptable [17] [15]. The temporal grid resolution is also subject to limiting conditions. In order to maintain stability, the time step must be smaller than the time it takes for an acoustic wave to transverse one spatial grid cell. This is a condition known as the Courant-Friedrichs-Lewy condition and maybe be imposed by defining the Courant number,

$$C = c_{max} \Delta t \frac{\sqrt{D}}{\Delta x} \quad (2.21)$$

where c_{max} is the highest sound speed of any medium in the model, Δt is the time step, D is the number of grid dimensions and Δx , the spatial grid spacing [15][18]. This equation is defined for a grid of equal spacing in all directions. The Courant number must remain between 0 and 1 to maintain numerical accuracy. This allows for the determination of the time step using the Courant number, spatial grid spacing and other properties of the model as in equation (2.22).

$$\Delta t = C \cdot \frac{\Delta x}{c_{max} \sqrt{D}} \quad (2.22)$$

With this in mind, the spatial and temporal grid resolution are ultimately determined by desired frequency range of the model. Since the model domain is three-dimensional, doubling the frequency range increases the number of required grid points by a factor of 2^3 . This can very rapidly increase the amount of memory required to store all values required for calculations. The procedure laid in section 2.2.2 details the separate variables necessary for modelling sound propagation. As well as a pressure grid and three velocity grids, computational grids of equal size are required for the variables $\partial p / \partial x$, $\partial p / \partial y$, $\partial p / \partial z$, $\partial p / \partial t$, $\partial w_x / \partial x$, $\partial w_y / \partial y$, $\partial w_z / \partial z$, the material properties b, κ and source term Q . Some memory can be saved by implementing scalar material properties if the whole domain consists of the same material and only a single-column vector source term if the source is located at a singular grid node. Nevertheless, the memory requirements of FDTD simulations are a limiting factor when it comes to possible domain size and frequency range. The volume of calculations required also incurs rather long computational times.

In this project measures were taken to reduce computational burden. For the validation of the model in chapter 3, reasonably sized domains were generated with lower frequency limits to allow for faster computation times. When modelling very large domains or large frequency ranges computation was done on a calculation computer with greater random access memory. This allows the use of MATLAB's 'Parallel Computing Toolbox' which can move very large arrays to the GPU using the function `gpuArray.m`. For extremely large computational domains, the memory requirements can be halved by changing the variable class from 'double' to 'single' using `single.m`. This comes at the cost of floating-point precision which may accumulate rounding errors.

2.3 Boundary Conditions

In order to accurately capture the underwater acoustic environment, a FDTD model needs to consider reflections from boundaries. Boundaries likely to arise in a model of sound propagation in shallow waters are the water-air interface at the sea surface, the water-seabed interface and interactions with hard surfaces such as rock which may appear in the domain as seabed or islands. These possible boundaries and how they are treated in the model are described and validated in section 3.3.

2.4 Nearly Perfectly Matched Layer

As well as reflecting boundaries there are cases where a propagating waves are expected to continue beyond the modelled domain. In such a case there should not be any reflections at the domain boundaries. Due to the nature of the FDTD calculations, if a boundary condition is not implemented, either pressure or acoustic particle velocity at the domain boundary will be zero since finite differences cannot be calculated for the most extreme grid points. This results in either a pressure-release condition or a rigid wall condition, both of which send a reflected sound wave back into the model domain. It is thus required to implement an absorbing boundary condition (ABC) that imitates radiation into free space.

Several methods for implementing ABCs exist. For this model, it was chosen to use the nearly perfectly matched layer (NPML), a variant of the perfectly matched layer originally devised for application in electromagnetism by Berenger in 1994 [19]. Compared to other ABCs, the perfectly matched layer can be applied with a smaller absorbing region, which decreases computational burden for an already computation-intensive method (FDTD) [20]. It also achieves more significant reduction than other ABCs for wave-boundary interactions at angles other than normal incidence [21].

The NPML adds a layer beyond the real domain of the the simulation, that is to say the area of the computational domain that represents a physical space. In this layer, and only in this layer, are auxiliary variables introduced to the equations of state along with a decay factor, η . The introduction of the decay factor causes the oscillating wave in the real domain to become an exponentially decaying wave

inside the NPML. In theory, the layer can be made very thin by introducing a large decay factor. However in practice, discretisation introduces discontinuities at the boundary between the domain and NPML. Therefore, it is required to find an optimal decay factor η , that allows for sufficient reduction in energy whilst maintaining computational efficiency. In this project, the ideal layer width and decay factor were found empirically using a two-part iterative manual search. This approach is detailed and analysed in section 3.2. In this section the method used to implement the NPML is described. This method is based on a paper published in *Geophysics* and is formulated specifically for acoustic wave modelling [21].

In the nearly perfectly matched layer regular state variables p and $\mathbf{w}$ in the governing equations (2.3) are replaced by auxiliary variables which are related to the regular variables using the relation,

$$\frac{\partial \bar{\xi}^\alpha}{\partial t} + \eta_\alpha \bar{\xi}^\alpha = \frac{\partial \xi}{\partial t} \quad (2.23)$$

where $\alpha \in \{x, y, z\}$, $\xi \in \{p, w_x, w_y, w_z\}$ and η is the decay factor. The new governing equations are thus,

$$\frac{\partial \mathbf{w}}{\partial t} = -b \nabla \bar{p}^\alpha \quad (2.24)$$

$$\frac{\partial p}{\partial t} = -\kappa \nabla \cdot \bar{\mathbf{w}}^\alpha + \kappa Q \quad (2.25)$$

which result in six auxiliary variables, $\bar{p}^x, \bar{p}^y, \bar{p}^z, \bar{w}_x^x, \bar{w}_y^y, \bar{w}_z^z$, and six corresponding differential equations from 2.23 to be solved. The decay factor η^α is non-zero only in the NPML and ramps from a maximum value η_m at the end of the layer to zero at its border with the real domain. The ramping function $\eta_\alpha(\alpha)$ is defined

$$\eta_\alpha(\alpha) = \begin{cases} \eta_m [(L_{pml} - \alpha)/L_{pml}]^3 & \text{for } \alpha \leq L_{pml} \\ 0 & \text{for } L_{pml} < \alpha \leq L_{pml} + \alpha_m \\ \eta_m [(\alpha - L_{pml} - \alpha_m)/L_{pml}]^3 & \text{for } \alpha \geq L_{pml} + \alpha_m \end{cases} \quad (2.26)$$

where $\alpha \in \{x, y, z\}$, L_{pml} is the thickness of the layer and α_m is the width of the medium in a given direction.

Due to how the leapfrog scheme is carried out, the update of the regular variables p and $\mathbf{w}$ use variables on the previous and opposing half-step time grid. However, equation (2.23) introduces a damping term $\eta_\alpha \bar{\xi}^\alpha$ which means the auxiliary variable

depends on itself. This requires choosing which time level to evaluate $\eta_\alpha \bar{\xi}^\alpha$ at. Using $\bar{\xi}_l^\alpha$ evaluates the damping one step behind the time derivative, and using $\bar{\xi}_{l+1}^\alpha$ evaluates it one step ahead. Both introduce an error proportional to Δt because the damping term is offset from the midpoint of the time derivative. A solution to this problem is to evaluate the damping at the midpoint $l + \frac{1}{2}$, which can be approximated by averaging its two neighbours:

$$\bar{\xi}_{l+\frac{1}{2}}^\alpha \approx \frac{\bar{\xi}_{l+1}^\alpha + \bar{\xi}_l^\alpha}{2} \quad (2.27)$$

This results in a discretised update equation derived from (2.23) as follows:

$$\begin{aligned} \frac{\bar{\xi}_{l+1}^\alpha - \bar{\xi}_l^\alpha}{\Delta t} + \eta_\alpha \bar{\xi}_{l+\frac{1}{2}}^\alpha &= \frac{\partial \xi}{\partial t} \\ \frac{\bar{\xi}_{l+1}^\alpha - \bar{\xi}_l^\alpha}{\Delta t} + \eta_\alpha \frac{\bar{\xi}_{l+1}^\alpha + \bar{\xi}_l^\alpha}{2} &= f_\xi \\ \bar{\xi}_{l+1}^\alpha \left(1 + \frac{\Delta t \eta_\alpha}{2}\right) - \bar{\xi}_l^\alpha \left(1 - \frac{\Delta t \eta_\alpha}{2}\right) &= \Delta t f_\xi \\ \bar{\xi}_{l+1}^\alpha &= \frac{\Delta t f_\xi + \bar{\xi}_l^\alpha \left(1 - \frac{\Delta t \eta_\alpha}{2}\right)}{\left(1 + \frac{\Delta t \eta_\alpha}{2}\right)} \end{aligned} \quad (2.28)$$

This process is implemented during the process listed in section 2.2.2. Regular variables are replaced with the auxiliary variables which are used to update variables on the opposing spatial and temporal grids. A revised list is written below with the additional treatment required to implement the NPML.

1. Consider a domain with initial pressure and velocity conditions at time $l\Delta t$. Each pressure node has an associated pressure value, $p(x, y, z, l\Delta t)$.
2. Pressure is converted to the auxiliary variables $\bar{p}^x, \bar{p}^y, \bar{p}^z$ with (2.28).
3. Calculate $\partial \bar{p}^x / \partial x, \partial \bar{p}^y / \partial y$ and $\partial \bar{p}^z / \partial z$ at velocity nodes on the spatial grid using finite-difference equations (2.8) - (2.10).
4. Calculate f_x, f_y and f_z using (2.28).
5. Update w_x, w_y and w_z for all spatial velocity grid nodes for the next half-integer time step $(l + \frac{1}{2})\Delta t$ using (2.18) - (2.20).

6. Convert regular velocity variables to auxiliary variables $\bar{w}_x^x, \bar{w}_y^y$ and $\bar{w}_z^z$ using (2.23).
7. Calculate $\partial \bar{w}_x^x / \partial x, \partial \bar{w}_y^y / \partial y$ and $\partial \bar{w}_z^z / \partial z$ at $(l + \frac{1}{2}) \Delta t$ using (2.11) - (2.13).
8. Calculate f_p using (2.25). Source term only applies at node representing a source.
9. Update p at time step $(l + 1) \Delta t$ using equation (2.17).
10. Repeat to advance the simulation further in time.

The width of the NPML in the model is a parameter that is decided by the user. The NPML may be implemented at all boundaries on the real physical domain or only at selected boundaries. It may have different thicknesses at different boundaries. The maximum decay factor η_m is also a user parameter. Section 3.2 covers trials conducted to empirically optimise NPML thickness and η_m .

3

Validation

In the following section, the accuracy of the FDTD model is validated by way of comparison to analytical solutions for problems of increasing complexity. Initially, it is confirmed that the amplitude of an acoustic pressure wave in ‘unbounded space’ (i.e. without reflections) decreases linearly with respect to radius. Then, in order to avoid reflections at the edge of the computational domain, a nearly perfectly matched layer (NPML) is implemented according to the method outlined in [21] and its efficacy is discussed. Subsequently, reflections generated at boundaries due to a pressure-release condition, a rigid boundary condition and a finite-impedance boundary condition are validated using known analytical solutions. These three conditions represent the sea surface, an infinitely hard sea floor and a realistic sea floor, respectively.

3.1 Spherical Spreading

A general solution to the wave equation for an spherically symmetric outgoing wave can be written

$$p(r, t) = \frac{f(r - ct)}{r} \quad (3.1)$$

where c is the speed of sound in the medium, t is time and r the radial coordinate from the source to the point of measurement [22]. A numerical model must, then, be able to predict the pressure at a given point at a specific time if the source function is known. The accuracy of the model can be readily compared with an analytical solution calculated using this simple relationship.

To test the validity of the FDTD model a rudimentary model was produced. A cubic domain was generated with sides of length 49.8750 m. A radiating source was placed in the centre of the domain. To avoid reflections from the edge of the domain, which would affect the computed pressure, the simulation was run for a limited time ensuring that a sound wave would not reach the edge of the domain. In order to avoid complex behaviour at this early stage of implementation, the source function is a simple Gaussian pulse introduced in MATLAB using a squared sine function. The

signal was further windowed to smooth discontinuities. This time signal is displayed in figure 3.1. The frequency range was limited to allow for reasonable calculation time which is seen in the frequency spectrum in figure 3.2.

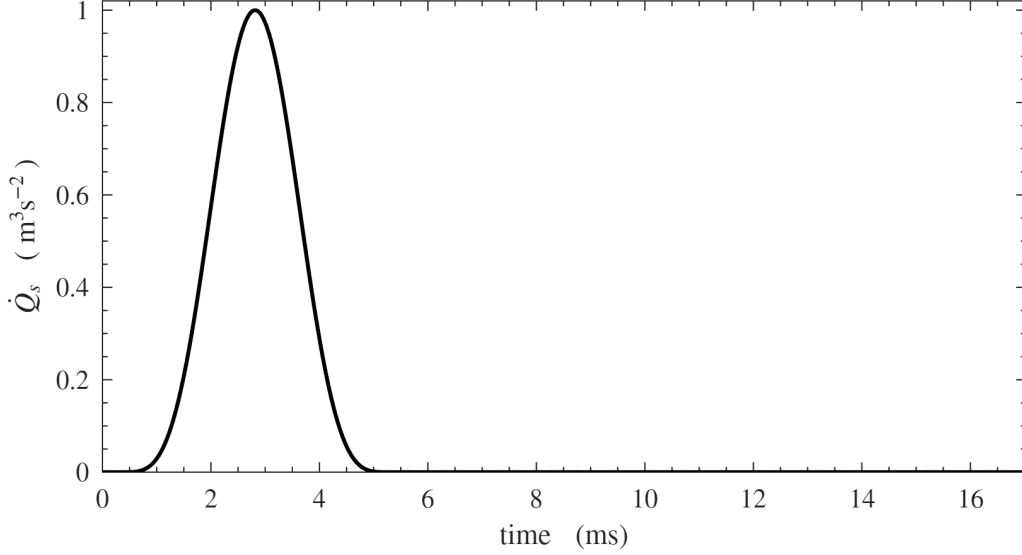


Figure 3.1: Source signal used in the initial stages of validation.

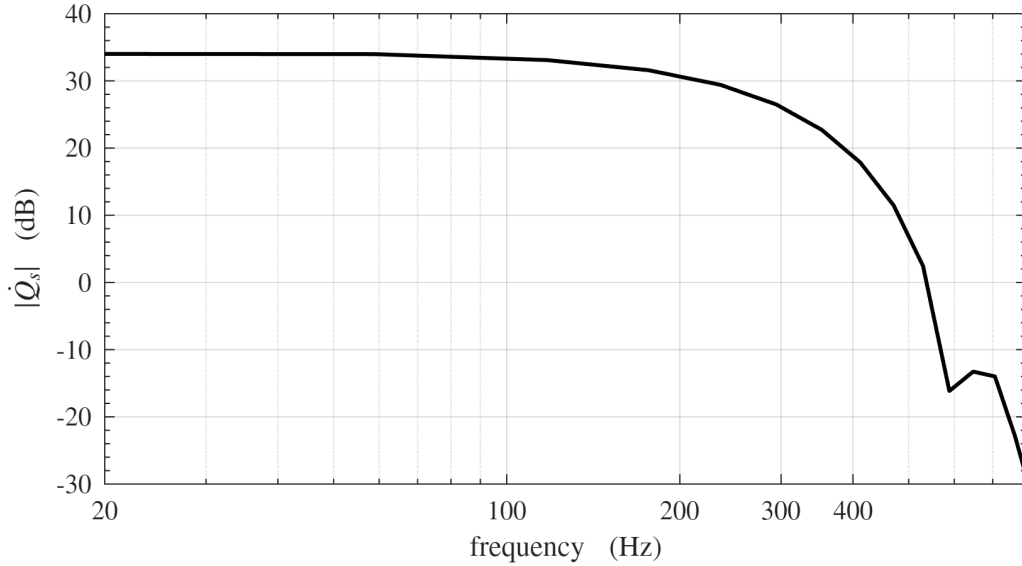


Figure 3.2: Source signal used in the initial stages of validation.

For each time step of the simulation, the calculated pressure at all grid points within several two-dimensional slices was stored. For simplicity in calculations, slices were taken at the source node coordinate for the x-y plane, x-z plane and y-z plane. This allows for the pressure at each grid point to be investigated for any time step during the simulation.

Coordinates of a number of ‘receivers’ across the grid were generated. The positions of the receivers in the aforementioned slice of the x-y plane are shown in figure 3.3. The pressure at these positions was used to validate the accuracy of the model. Due to the width of the propagating wavefront, not all of these receivers are able to be used in the validation. Receivers used in this stage of validation were those for which the wavefront had completely passed through by the end of the simulation time. Figure 3.4 shows a snapshot of the x-y plane at the end of the simulation. The pressure wave is seen projected onto the plane and the valid receivers lie within the sphere. Due to the spacing of the receivers, there will of course be duplicate receivers which share the same distance from the source and differ only in angle. Duplicates are later omitted to show only receivers with differing radii.

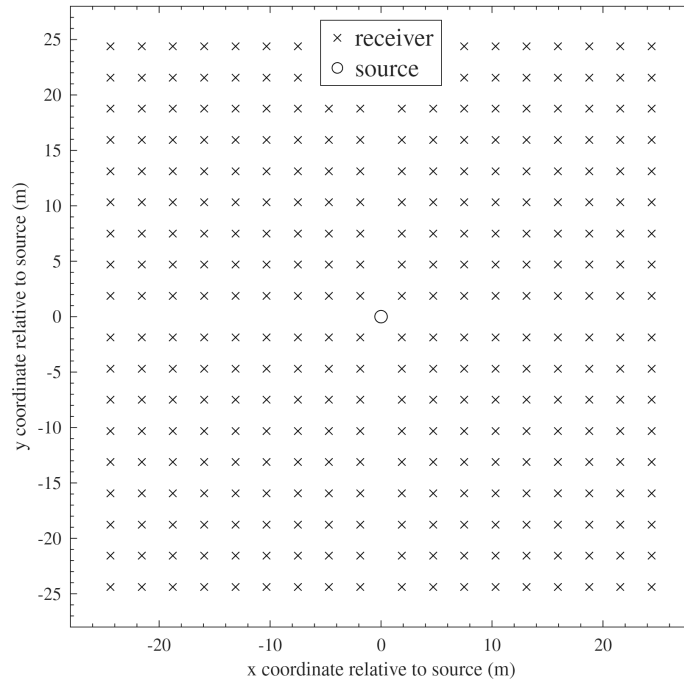


Figure 3.3: All possible receivers for spherical spreading validation.

For all valid receiver points, pressure as a function of time, as computed by the model, is plotted in figure 3.5. The plots in this graph are limited to points with unique radii as to avoid overlapping time signals which occur at points of equal distance from the source. This graph shows a clear decrease in amplitude according to increasing radius and even appears to show an inverse relationship to radius. If this relationship is accurate, then multiplying these time signals by the distance from the source of the given point should yield a set of curves with equal amplitude. Figure 3.6 confirms this relationship and in doing so confirms the ability of the model to predict propagation losses due to spherical spreading.

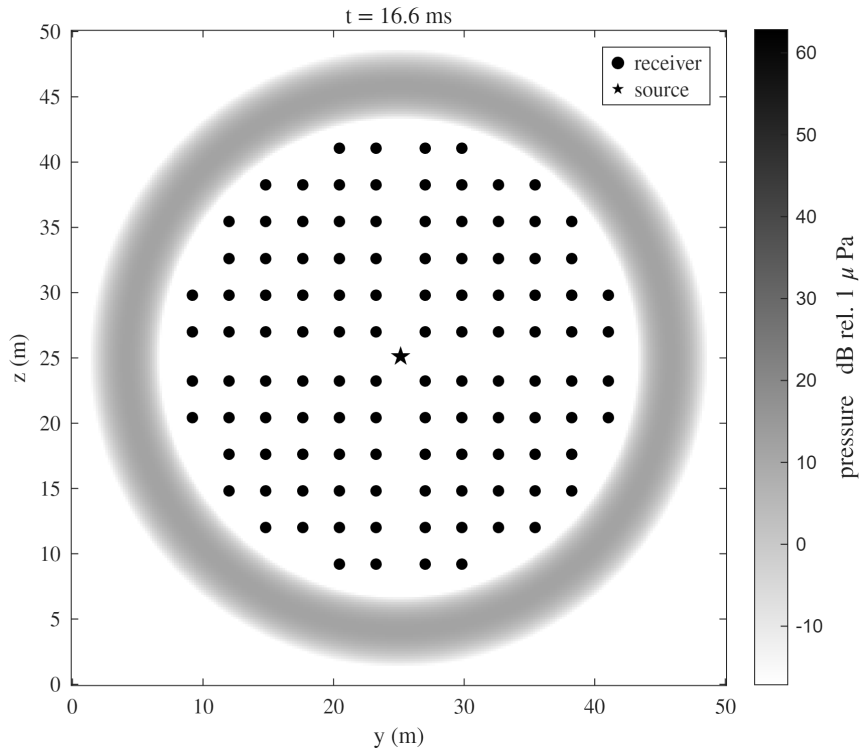


Figure 3.4: Receivers for which the outgoing wave passes through entirely.

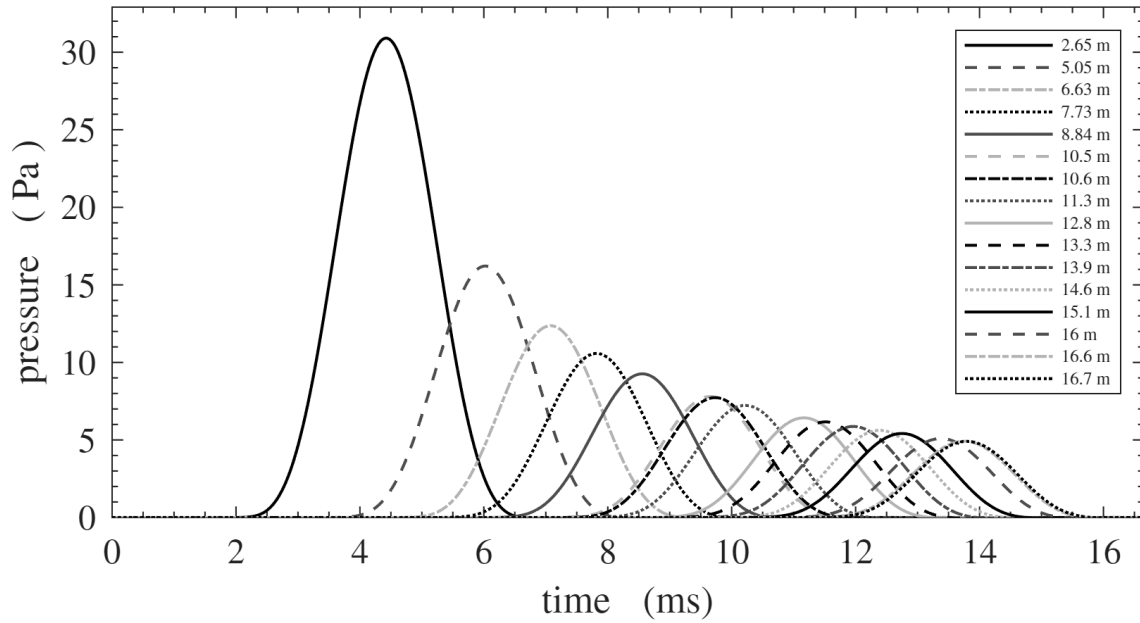


Figure 3.5: Pressure vs. time at each receiver point. Distance to receiver is displayed in the legend.

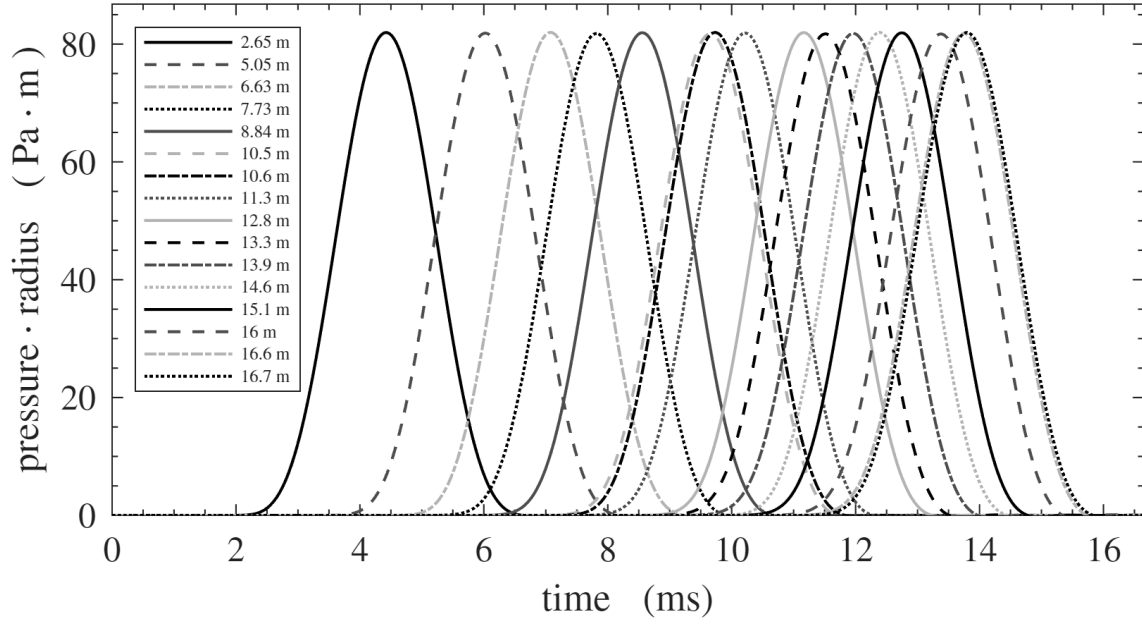


Figure 3.6: (Pressure x radius) vs. time at each receiver point. Distance to receiver is displayed in the legend.

Until this point, the amplitude of the pressure wave has not been of great importance, it need only demonstrate the expected decay with increasing radius. However, the model should of course agree with predictions made by an analytical solution. In this case, for a volume source at the origin in an unbounded region, the analytical solution for the pressure at a receiver is calculated as follows,

$$p = \frac{\rho}{4\pi r} \dot{Q}_s \left[t - \frac{r}{c} \right] \quad (3.2)$$

where ρ is the density of the medium, $\dot{Q}_s$ is the source function shown in figure 3.1, and $t - \frac{r}{c}$ accounts for the time taken for the wave to reach the receiver, also known as the retarded time [22]. For the time signal recorded at receiver point, the root mean square (RMS) pressure was calculated and compared with the analytically derived RMS pressure. This comparison is shown in figure 3.7. At all receiver distances the modelled RMS pressure calculated agrees with the analytical solution, confirming that the model not only reproduces spherical spreading of sound waves but also does so whilst maintaining accuracy. The error between the model output and analytical solution in decibels was calculated using,

$$e_{dB} = 20 \log_{10} \left| \frac{p_{model} - p_{analytic}}{p_{analytic}} \right| \quad (3.3)$$

where pressures are the RMS pressures shown in figure 3.7. The error for receivers at different distances from the source is shown in figure 3.8. The variation in error across receiver points can be attributed to numerical dispersion inherent to the FDTD

grid scheme. For a rectangular grid, the numerical speed of sound varies slightly depending on the angle of propagation relative to the grid axes. Receivers lying at oblique angles accumulate more phase error than those lying on or close to the source node grid axes, resulting in varying RMS pressure errors across the receiver array. With that being said, the largest error is more than satisfactory with the discrepancy between model and analytical solution being only a fraction of a decibel. This further justifies the success of the model of reproducing results predicted by an analytical solution.

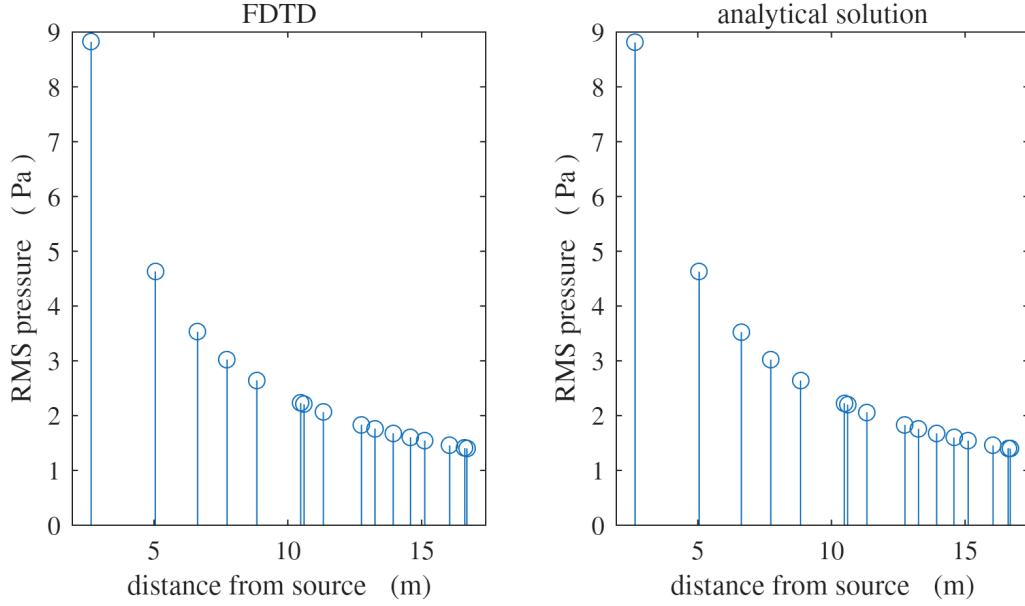


Figure 3.7: RMS pressure at receivers of varying distances from the source, predicted by FDTD model and analytical solution.

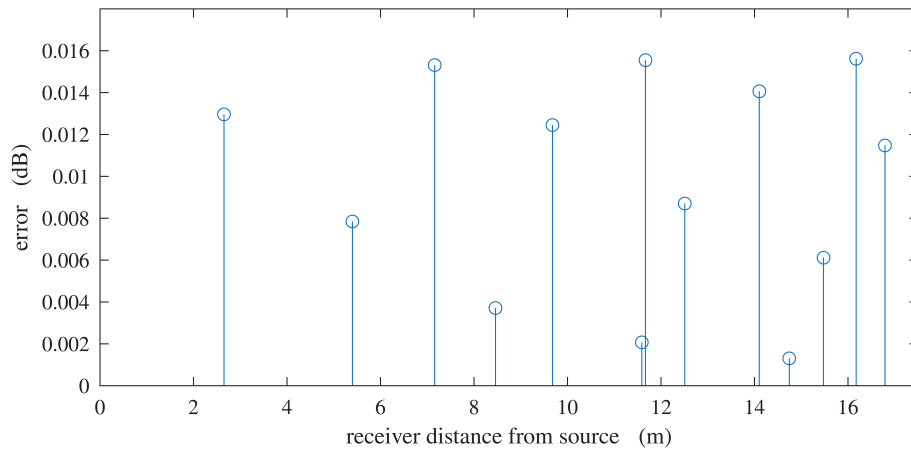


Figure 3.8: Error in RMS pressure between model output and predicted analytical solution.

3.2 Nearly Perfectly Matched Layer

In order to accurately capture the underwater acoustic environment, a FDTD model needs to consider reflections from boundaries such as the sea surface, sea floor and other potential reflecting surfaces such as rocky outcrops. Whilst the sea surface and sea floor are likely to be present in most simulations, reflecting surfaces at the sides of the domain may not be. There will be cases where a propagating wave is expected to continue beyond the truncated domain. In such a case there should be not be any reflections at the domain boundaries.

One solution to this case is the nearly perfectly matched layer. The theory behind this absorbing boundary condition is discussed in section 2.4. To successfully implement the NPML, it is required to find an optimal decay factor η_m and layer thickness L_{pml} , that allow for sufficient reduction whilst maintaining computational efficiency. In this work, the ideal layer width and decay factor were found empirically using a two-part iterative process. This approach is detailed below.

3.2.1 Decay Factor

As in the previous section, a cubic domain was generated with sides of length 49.8750 m. The source was placed at coordinates (24.9375, 24.9375, 9.9375) m. A receiver was placed between the source and the nearest boundary at a distance of 4 m from the source. The source function was the same used in the previous validation, shown in figure 3.1. The simulation run time was limited to ensure that only reflections from the nearest domain boundary reach the receiver. Figure 3.9 shows the configuration of the model used in the trial and error process.

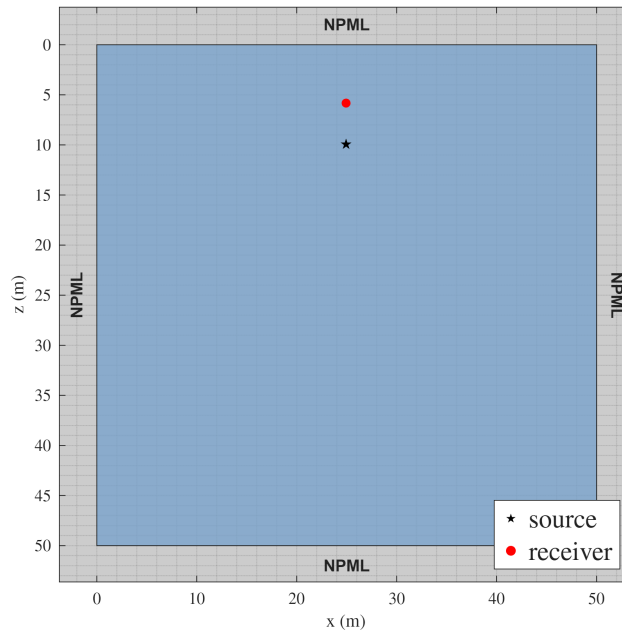


Figure 3.9: Computational domain set-up for the validation of the nearly perfectly matched layer.

Initially, an arbitrary layer width of 3.75 m (8% of the domain width) was chosen. For this layer width the simulation was run several times using different values of η_m ranging from zero to 10^5 . For each run of the simulation, the resulting time signal at the receiver was recorded. The time signal was then split into direct sound and reflected sound and the frequency spectrum of each was calculated. For the reflected sound, the RMS pressure was also calculated. This allowed for the reduction due to the NPML to be compared for different values of η_m using the time signals, spectra and RMS pressure.

Figure 3.10 shows the time signal recorded at the receiver for a broad range of η_m . The amplitude of the direct sound peaks at around 9 ms, consistent with the time taken for the outgoing wave to reach the receiver. This will, of course, be the same for all trials. In order to better see the effects of the NPML, figure 3.11 shows only the reflected sound. There is a clear difference in efficacy of the NPML dependent on η_m . There is also a noticeable difference in when the reflections reach the receiver.

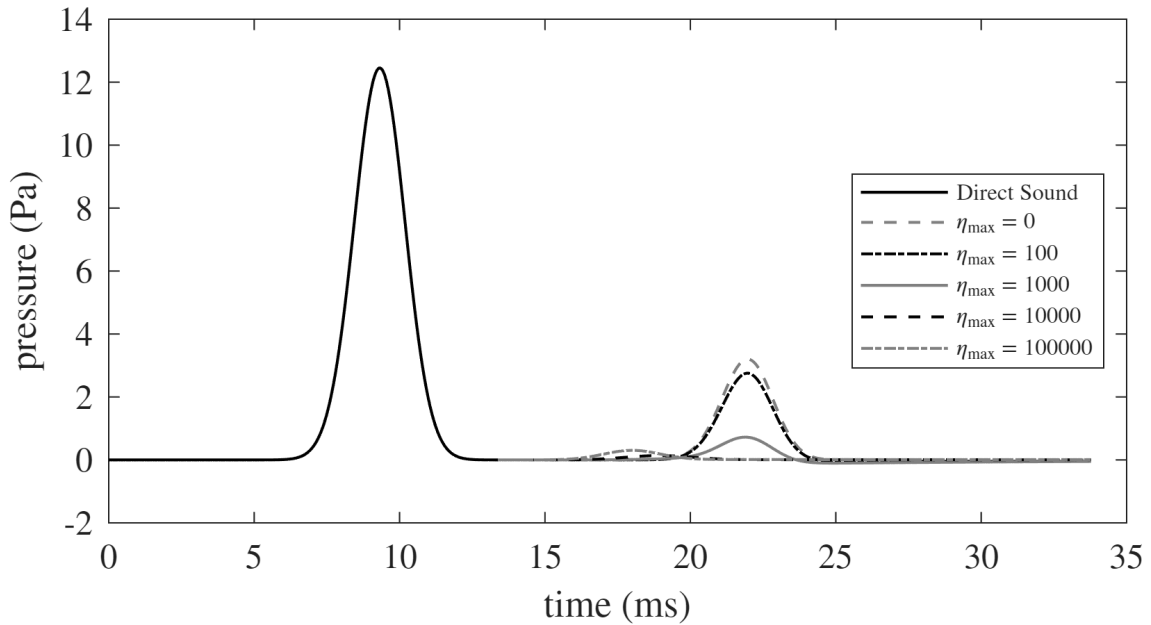


Figure 3.10: Time signal recorded at receiver for varying decay factors η_m .

Consider the case where the decay factor η_m is equal to zero. This essentially means that there is no decay happening in the NPML whatsoever; the propagating wave passes straight through and reflects off the boundary of the domain. For the opposite case, where the decay factor is very large, there is a discontinuity between the real part of domain (where $\eta_m = 0$) and the NPML which results in the wave reflecting off this boundary instead. Hence, the larger the decay factor, the earlier the arrival of the reflected sound.

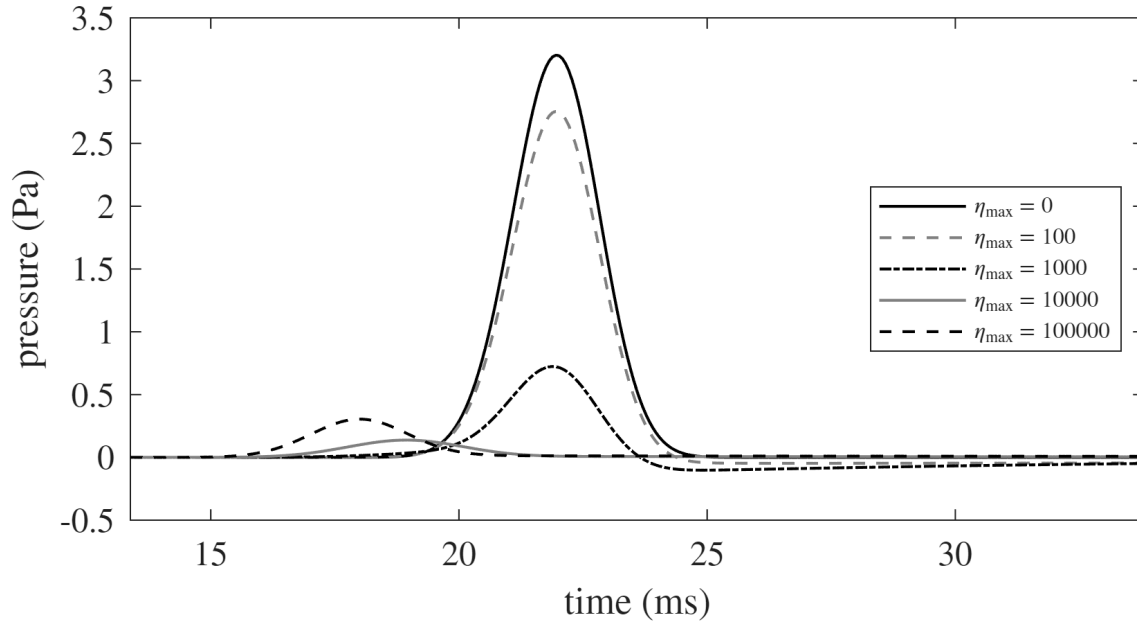


Figure 3.11: Reflected sound captured at receiver for a broad range of decay factors η_m .

Figure 3.11 indicates that the ideal η_m lies between 1000 and 10000. More trials were conducted using values at regular intervals in this narrower range. The results of this trial are shown in figure 3.12. Trials were conducted for a large number of η_m values, yet in this figure only a selected number of values are shown in order to improve discernibility. At $\eta_m = 2000$, reflections at the receiver are still arriving later than for other trials, suggesting that the wave is still mostly passing through the layer and reflecting off the truncated domain. For the larger values η_m , reflections arrive earlier indicating reflection from the NPML boundary. For η_m values of 3600 and 4200, the pressure peaks are broader suggesting some reflection from the NPML boundary and some reflection from the domain boundary.

The spectra of these time signals and the RMS pressure are shown in figure 3.13 and 3.14 respectively. The frequency spectra are shown alongside the frequency spectrum of a reflected wave due to a rigid wall boundary condition (i.e. no NPML) and the RMS pressures are plotted in dB relative to the RMS pressure of the rigid wall BC. From these three graphs it can be deduced that a NPML with a decay factor maximum of $\eta_m = 3600$ is the the most effective value (of those trialled) at reducing reflections from the domain boundary. The RMS pressure may be reduced by more than 30 dB compared to a rigid BC and for certain frequencies this reduction may extend to upwards of 50 dB.

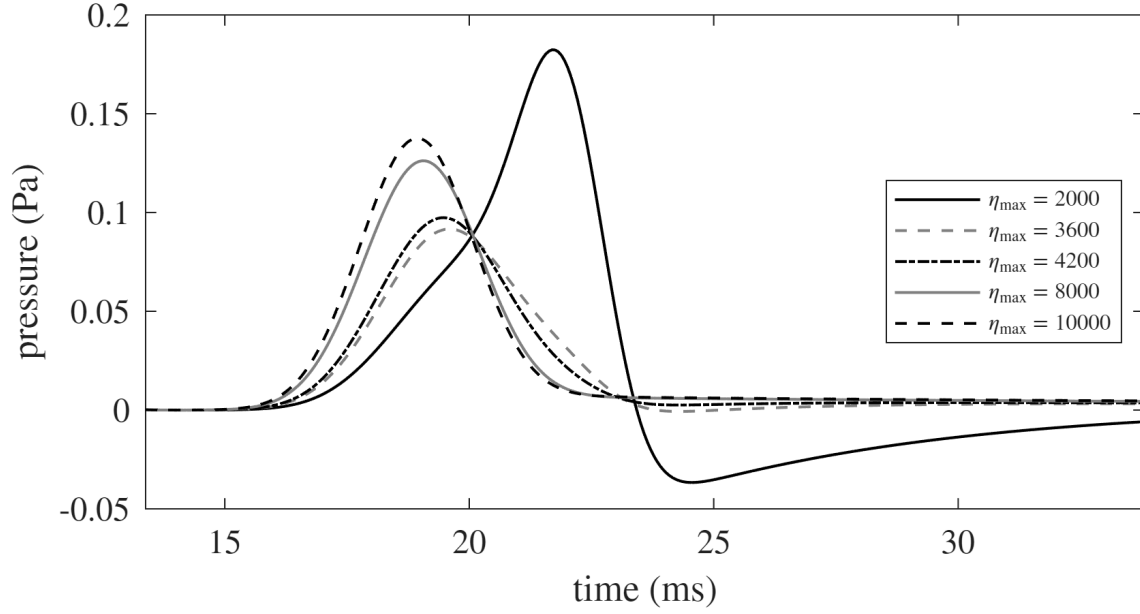


Figure 3.12: Reflected sound captured at receiver for a narrower range of maximum decay factor η_m .

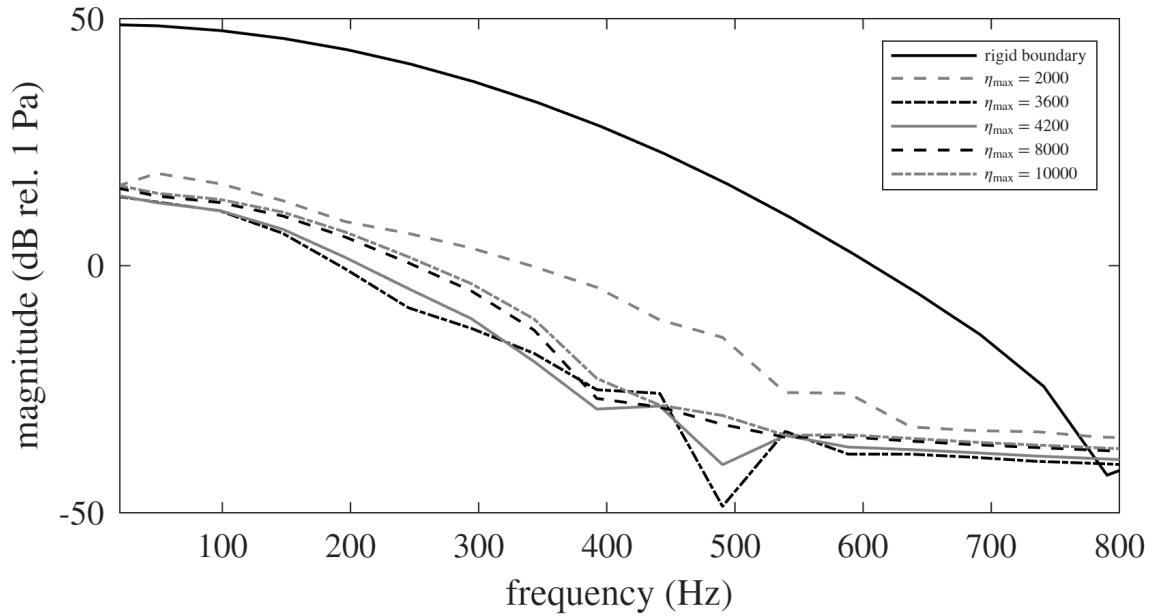


Figure 3.13: Spectra of reflected sound recorded at receiver for a narrower range of η_m values compared to that of a rigid wall BC.

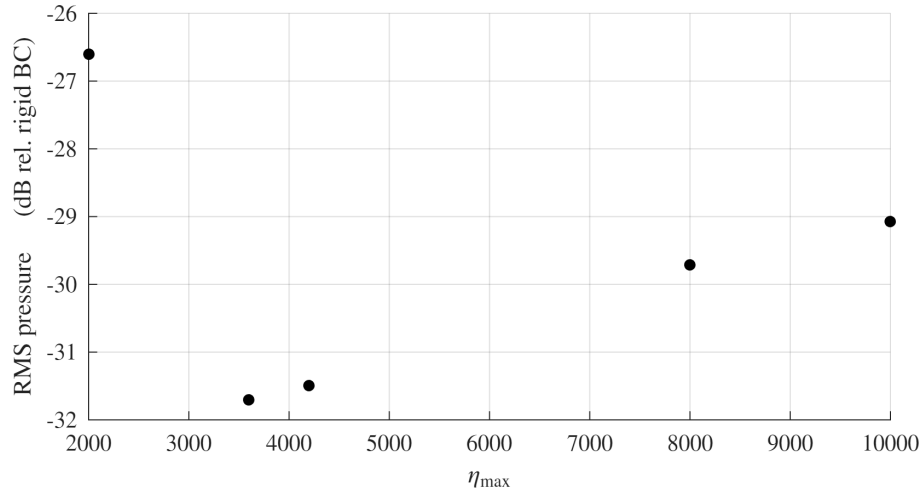


Figure 3.14: RMS pressure of reflected sound wave for a narrower range of η_m values relative to RMS pressure of a rigid wall BC.

3.2.2 Layer Thickness

With an optimal η_m of 3600, a second trial was conducted to document the effect of increasing the thickness of the nearly perfectly matched layer, L_{pml} . The obvious expectation of increasing the thickness of the damping layer is that the amplitude of the reflected wave will decrease proportionally. The set-up from the previous trial was unchanged although η_m was held constant at 3600 and instead the width of the NPML was gradually increased. In this trial, a NPML width of 0 m is the control case, effectively identical to the rigid boundary condition in the previous trial.

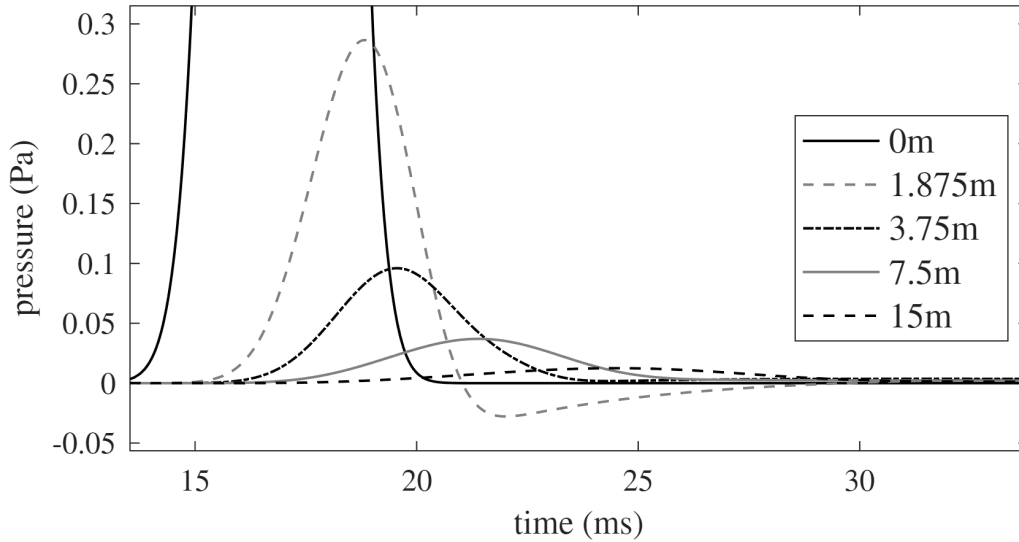


Figure 3.15: Reflected sound captured at a single receiver for a range of L_{pml} .

Figures 3.15, 3.16 and 3.17 show the pressure recorded at the receiver, frequency spectra and RMS pressure in dB relative the rigid BC. As expected the amplitude of the reflected wave decreases for thicker NPMLs. As is seen in figure 3.17, the RMS pressure of the reflected wave decreases exponentially as L_{pml} increases. This is due to the nature of the NPML which introduces a decay factor that increases exponentially from zero at the NPML interface to η_m at the edge of the computational domain.

Propagation loss due to spherical spreading still takes effect even in the NPML. Shorter wavelengths relative to L_{pml} experience much greater reduction than larger wavelengths. The total distance a wave travels in the NPML is equivalent to double the NPML width, from interface to domain edge and back again. It would be expected that wavelengths less than this distance will experience greater reduction as a combination of decay due to the NPML and spherical spreading. This is evident, for example, in figure 3.16. For a sound speed of 1500 m s^{-1} , the frequency corresponding to a wavelength of 15 m is 100 Hz. This is the frequency at which the magnitude of reflected energy begins a sharp decrease for a 7.5 m layer width. At its most effective, consider the point at around 250 Hz on this same curve, the reduction is upwards of 60 dB. This behaviour can also be seen for other NPML widths.

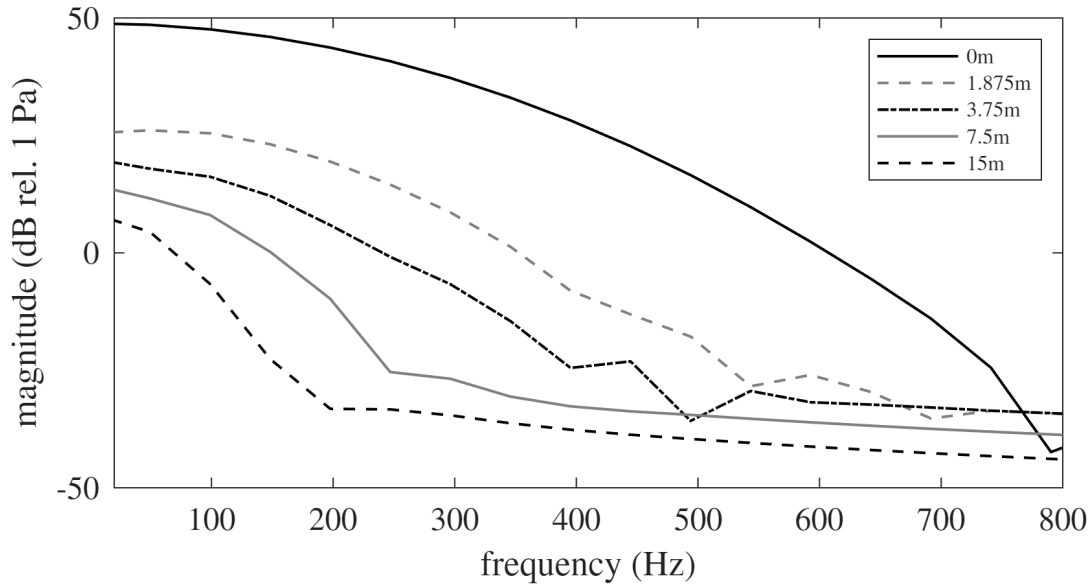


Figure 3.16: Spectra of reflected sound recorded at receiver for various L_{pml} .

The maximum reduction in RMS pressure achieved by the largest NPML width is around 45 dB. Since this curve of the data is exponential, the reduction would not be much greater for larger PML widths. As noted above, the reduction for certain frequencies maybe be much larger depending on the ratio between wavelength and NPML width. This is an important phenomenon that can be advantageous to consider when deciding NPML thickness. The size of the NPML may add considerable width to the total computational domain. In this trial the largest NPML width was 15 m, 30% of the width of the domain. With the awareness that the model

may require perfectly match layers at each edge of the ‘real domain’, the NPML may become a large proportion of the total computational domain. For relatively small domains this may pose a problem as the addition of NPMLs requires extension of the calculation grids, thereby increasing memory requirements and computation time. For larger domains, the total space dedicated to NPMLs might only be a small proportion of the total domain, in which case additional memory requirements may not be such a concern.

In further simulations in this project, it was decided to use a NPML width of 3.75 m, a compromise between efficacy and computational burden. If this width is adjusted, it will be stated in the text.

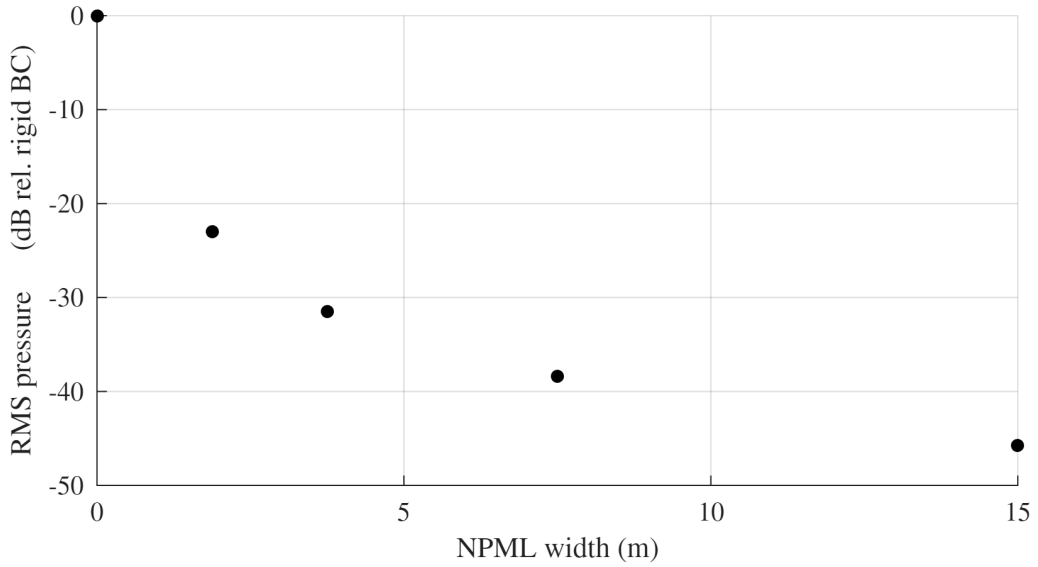


Figure 3.17: RMS pressure of reflected sound wave for various L_{pml} , relative to RMS pressure of a rigid wall BC.

3.3 Boundary Conditions

At any boundary between materials with an impedance mismatch, reflections occur. The difference in impedances dictates the amplitude and phase of the reflected wave. The characteristic acoustic impedance of a lossless non-dispersive material may be determined by its density and speed of sound in the material as in equation (3.4). It is clear from this equation that acoustic impedance will be larger for materials with higher densities or higher sound speeds.

$$Z_c = \rho \cdot c_0 \quad (3.4)$$

In the context of underwater sound propagation, examples of materials with differing characteristic acoustic impedances that may be encountered include water, air, sandy sea floor, clay sea floor and bedrock. All of these materials have different characteristic acoustic impedances and as such sound waves travelling from one medium to another will experience reflections. Depending on the direction of flow of sound, an acoustic wave may travel from a medium of higher impedance to a medium of lower impedance or vice versa. It is important that the FDTD model is able to accurately model both cases.

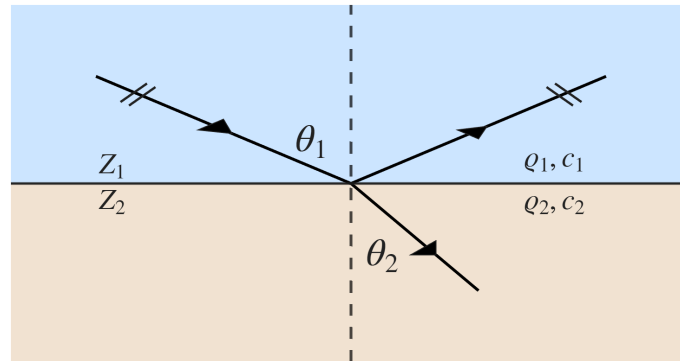


Figure 3.18: Physical parameters used to calculate the plane wave reflection factor.

In the following sub-sections, the cases of an infinitely hard wall (rigid BC), a pressure-release condition and a hard boundary are implemented in the model. The computed results are compared with analytical solutions in order to determine the accuracy of the model. The analytical solution assumes that at the interface between media the sound wave is at great enough distance from the source to be considered a plane wave. It also assumes that in the second medium, free wave propagation occurs resulting in Rayleigh reflection. This allows for the plane wave reflection factor, which determines the amplitude of the reflected wave, to be calculated using the characteristic impedances of the intersecting media:

$$R = \frac{Z_2 \cos(\theta_1) - Z_1 \cos(\theta_2)}{Z_2 \cos(\theta_1) + Z_1 \cos(\theta_2)} \quad (3.5)$$

where Z_1 and Z_2 are the impedances of the first and second media, θ_1 is the angle of incidence and θ_2 the angle of refraction [23]. A schematic of the interface with relative parameters is shown in figure 3.18.

3.3.1 Rigid Boundary

A rigid boundary occurs when the normal component of the velocity at the interface between two media is zero [24]. This is a theoretical case where the characteristic impedance of the second medium is infinite, however for cases where the impedance of the second medium is much greater than the first it can be a reasonable assumption. Such an assumption leads to a reflection factor of +1, i.e the wave is reflected at the boundary with equal amplitude.

To validate the model's ability to reproduce a rigid boundary, a domain was set up similar in dimension to that in section 3.2. In this case, NPMLs were set up to enclose the domain on all sides except for the $z = 0$ m plane where instead a rigid boundary was implemented. A source and a receiver were placed in the domain at (14.8125, 24.9375, 5.8125) m and (26.8125, 24.9375, 8.8125) m respectively. A 2D slice of this set-up is displayed in figure 3.19. Again, the source produced a single pulse as shown in figure 3.1. For a given point in the model domain, the pressure measured at that point will receive contributions both directly from the source, but also from the wave reflected in the rigid boundary. These contributions should create an interference pattern visible in the frequency spectrum that should also be predicted by an analytical solution.

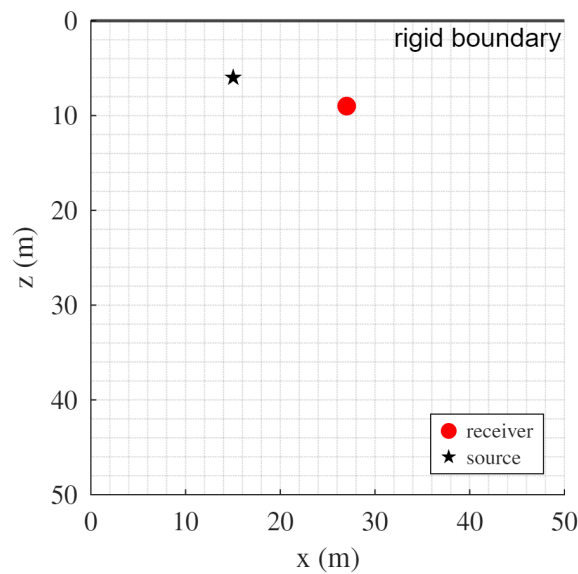


Figure 3.19: Model set-up for validation of rigid boundary condition.

The analytical solution was calculated using equation (3.6), a superposition of the pressure due to the direct sound from the source and pressure due to an image source which corresponds to a reflection at the boundary.

$$p_{tot} = p_{dir} + p_{img} \quad (3.6)$$

Each component may be calculated using equation (3.2) using the appropriate radial distance between source (or image source) and receiver. The wave from the image source is multiplied with a reflection factor R which, as previously stated, in the simple case of a perfectly reflecting hard boundary, is 1. This results in equation (3.7), which provides the analytical solution to be compared with the model output.

$$p_{tot}(\mathbf{r}, t) = \frac{\rho}{4\pi} \left(\frac{\dot{Q}_s[t - \frac{r_{dir}}{c}]}{r_{dir}} + R \frac{\dot{Q}_s[t - \frac{r_{img}}{c}]}{r_{img}} \right) \quad (3.7)$$

Figure 3.20 shows the pressure at the receiver point as computed by the FDTD model and as predicted by the analytical solution. The initial pulse from the source is seen, followed closely by the reflection from the rigid boundary. The agreement between the two methods is very good. There may be a very slight mismatch, hardly visible in the plots, due to how quantities are calculated in the FDTD scheme.

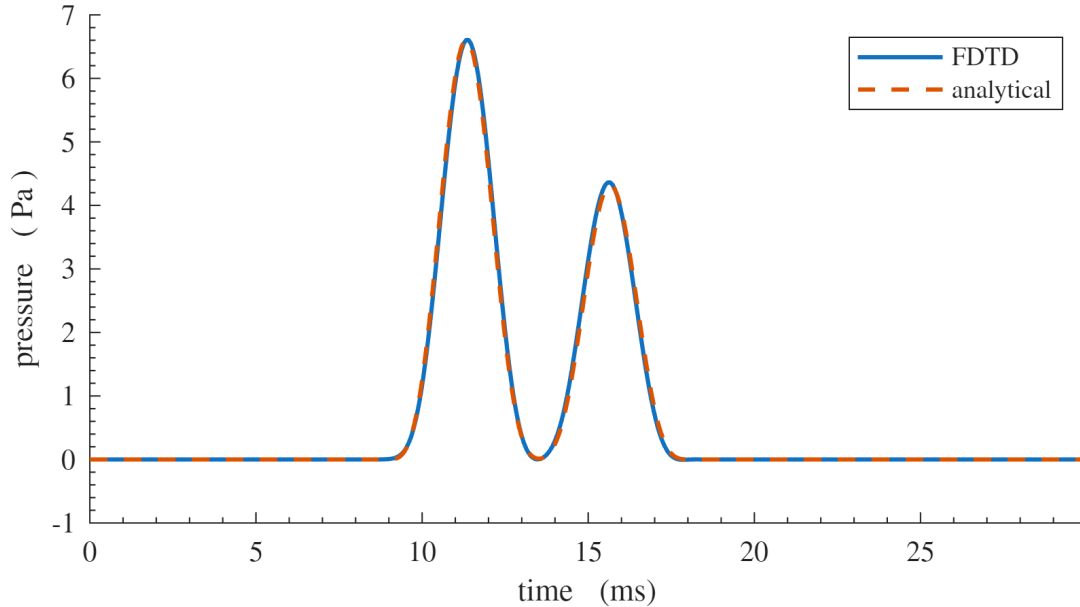


Figure 3.20: Comparison of pressure time signals for modelled and analytical solutions of a rigid boundary condition.

Pressure and acoustic particle velocity are staggered by half a grid step. The rigid boundary is implemented by setting the velocity at the boundary to zero, causing the reflective plane to occur on the velocity grid and not the pressure grid. This will cause a slight time difference (albeit a very small one) between the analytical solution and the modelled solution for the pressure of the reflected wave which is just visible in figure 3.20. Frequency spectra shown in figure 3.21 show good coherence between the output of the model with the analytical prediction. Small differences occur but most significantly at the upper end of the frequency range which is to be expected as the wavelength approaches the resolution of the model's spatial grids. Error may also be compounded at higher frequencies by the slight time mismatch of the reflected wave.

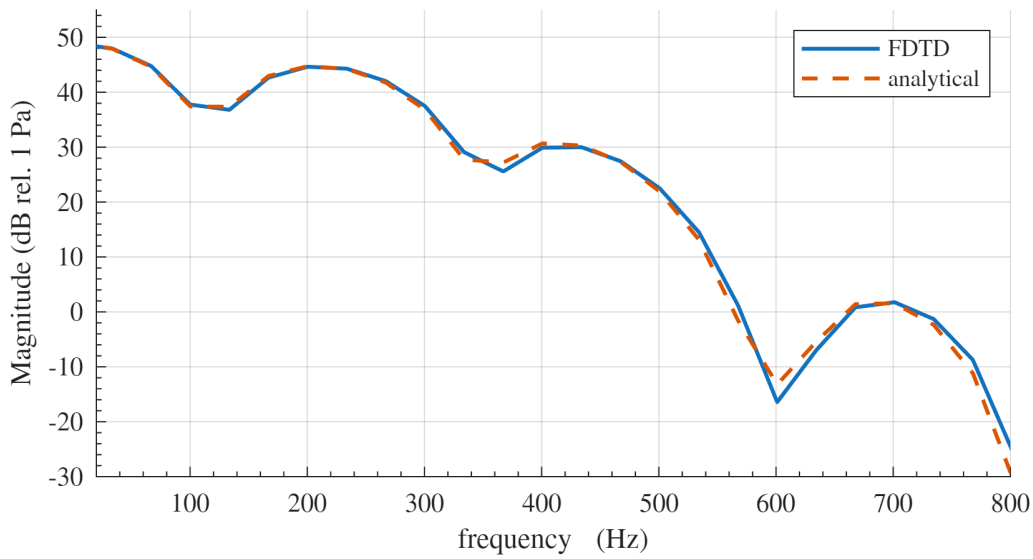


Figure 3.21: Comparison of frequency spectra for modelled and analytical solutions of a rigid boundary condition.

3.3.2 Pressure-Release Condition

A pressure-release condition occurs when a sound wave travels from a medium with a higher impedance to one with a much lower impedance. Should the difference be several orders of magnitude, one can note from equation (3.5), the reflection factor will tend towards -1 meaning that the wave is reflected from the boundary with equal but opposite amplitude. For sound travelling from water to air, the impedance change is such that the pressure-release condition is a reasonable boundary condition [25]. This boundary condition is therefore very likely to be employed in an FDTD model of underwater sound propagation as it simplifies computations whilst remaining physically accurate, unlike the rigid boundary condition which is less likely to occur in the underwater environment due to the relatively high density and sound speed of water.

Such a boundary condition is implemented in the FDTD model by setting the pressure at the desired boundary to zero. The procedure to validate the model's efficacy at replicating a pressure-release BC is much the same as the validation of the rigid BC. A 2D slice of the set-up is shown in figure 3.22. The only difference is the change of the rigid boundary to the 'sea surface' where pressure is zero. Source and receiver location remain unchanged. The source function is as in figure 3.1. The analytical solution was calculated using equations (3.6) and (3.7) with a reflection factor $R = -1$.

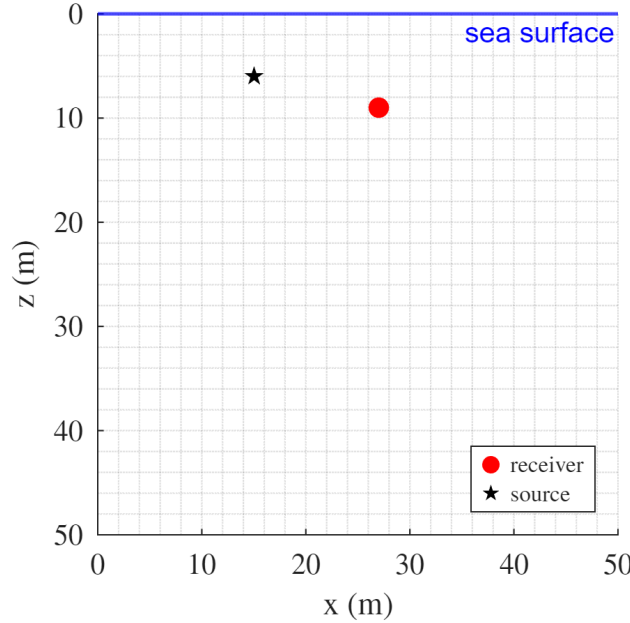


Figure 3.22: Model set-up for validation of pressure-release condition.

The pressure signals computed by the model and predicted by the analytical solution are plotted in figure 3.23. The initial pulse from the source is followed by a reflected pulse with negative amplitude, as expected for a pressure-release condition. The model agrees very well with the analytical solution for both the direct and reflected waves. Since the boundary condition is implemented on the pressure grid, there is no time mismatch between solutions for the reflected wave as for the rigid BC. This is reflected in figure 3.24 where the spectra match extremely well with only very slight deviation in the upper frequencies where the model approaches its frequency limit. These graphs show the the model is quite capable of reproducing a pressure-release boundary condition.

For this validation, both simulation and analytical solution assume air is essentially a vacuum. A more accurate solution would actually simulate the impedance difference by changing the material properties of a section of the model domain to that of air. This would however, require a larger computational domain therefore increasing computational burden. As stated earlier, the pressure-release is a common approach to the sea surface condition due to a large impedance mismatch between air and water and, due to its computational simplicity, was used in further simulations.

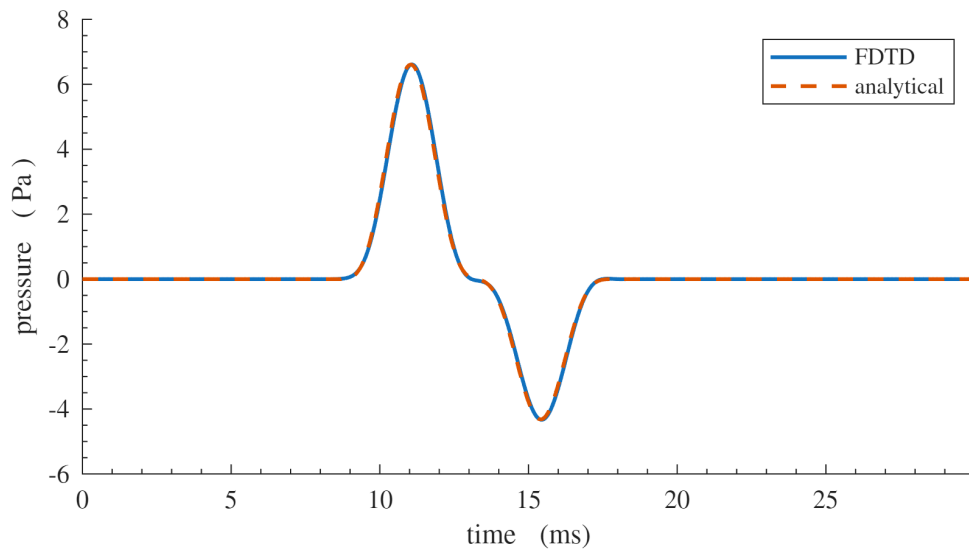


Figure 3.23: Comparison of pressure time signals for modelled and analytical solutions of a pressure-release boundary condition.

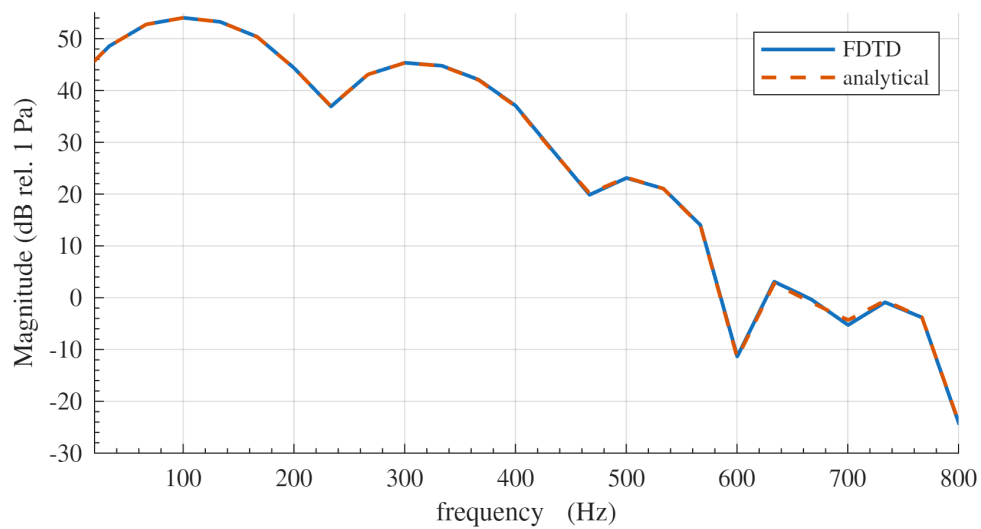


Figure 3.24: Comparison of frequency spectra for modelled and analytical solutions of a pressure-release boundary condition.

3.3.3 Finite-Impedance Boundary Condition

A boundary with finite impedance is a boundary whose properties lie in between the two previous cases of a rigid boundary and a pressure-release condition. The second medium may have a lower or higher impedance than the first medium. In the context of underwater sound propagation, since the water-air boundary may be represented by the pressure-release condition, the case in which water meets a harder surface such as sand or rock is more likely to be modelled as a finite-impedance boundary. In this case, Z_2 in equation (3.5) will be larger than Z_1 and the reflection factor R , will lie between zero and one. An impinging sound wave will be reflected with a positive but reduced amplitude. This boundary condition will be used to model the sea floor, a medium with a higher density and sound speed than water.

In the FDTD model, the finite-impedance BC is readily implemented simply by adjusting the density and sound speed of the domain area that comprises the second medium. This adjustment will in turn change the buoyancy and bulk modulus of the material which are used in the equations of state to calculate pressure and velocity changes over time. As a sound wave propagates through the model, this change of material properties at the interface of the two media will cause reflections whose amplitude ought to be consistent with the predictions of an analytical model.

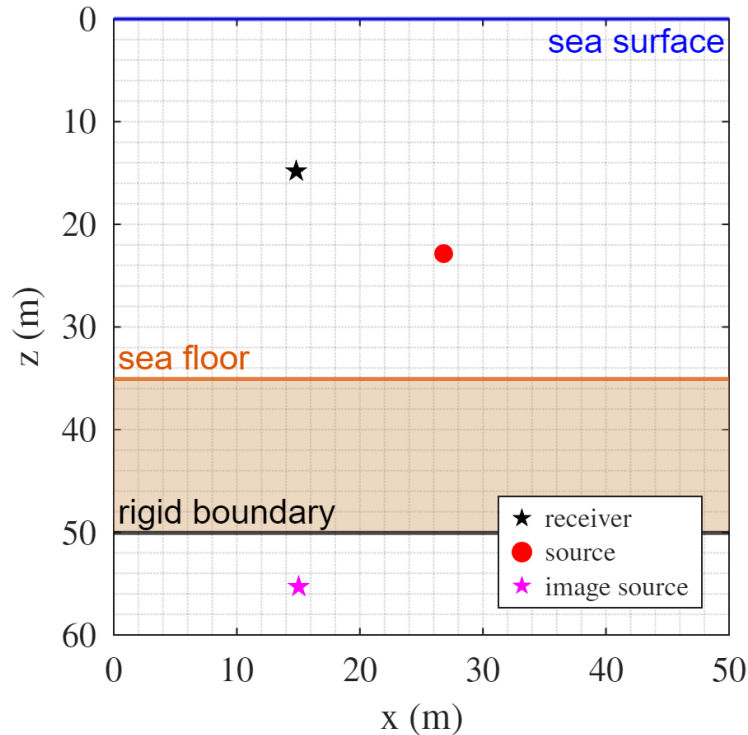


Figure 3.25: Model set-up for validation of finite-impedance boundary condition.

As in previous validation processes, a cubic domain of side length 49.8750 m was generated with a source and receiver located in the same x-y plane. A finite-impedance boundary was implemented at a depth of 34.875 m by changing the material properties (density and sound speed) to those of coarse silt as specified in Annex D of ISO 17208-3 [26]. This area is displayed in figure 3.25 labelled as ‘sea floor’. The rigid boundary at the end of the computational domain is also noted as well as the location of the image source used in the analytical solution.

The material properties of coarse silt, $\rho = 1.702 \rho_0$ and $c = 1.0772 c_0$ where ρ_0 and c_0 are the density and sound speed in water, and the relative geometry of the source, receiver and reflective boundary are used in equation (3.5) resulting in a reflection factor $R = 0.2293$. This allows for the analytical solution for pressure at the receiver to be calculated using equations (3.6) and (3.7).

The simulation was run for a long enough time to ensure that both the direct wave and reflected wave pass through the receiver point, but short enough that reflections from other boundaries did not reach the receiver. The time signal at the receiver is shown in figure 3.26. The prediction provided by the analytical solution is overlaid on the model output and shows very good agreement. This agreement is also seen in the frequency spectrum where the model follows the analytical prediction extremely well.

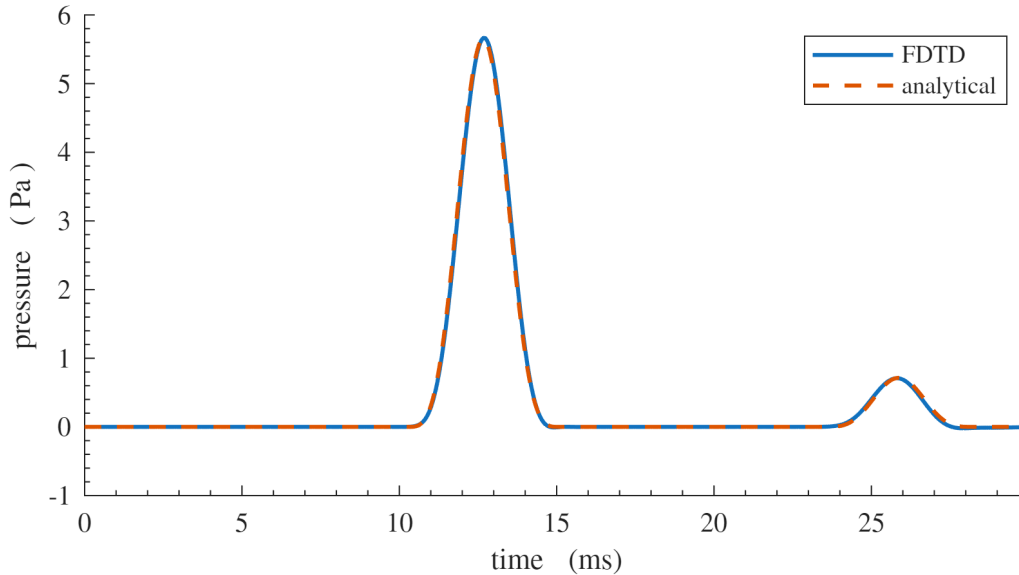


Figure 3.26: Comparison of pressure time signals for modelled and analytical solutions of a finite-impedance boundary condition.

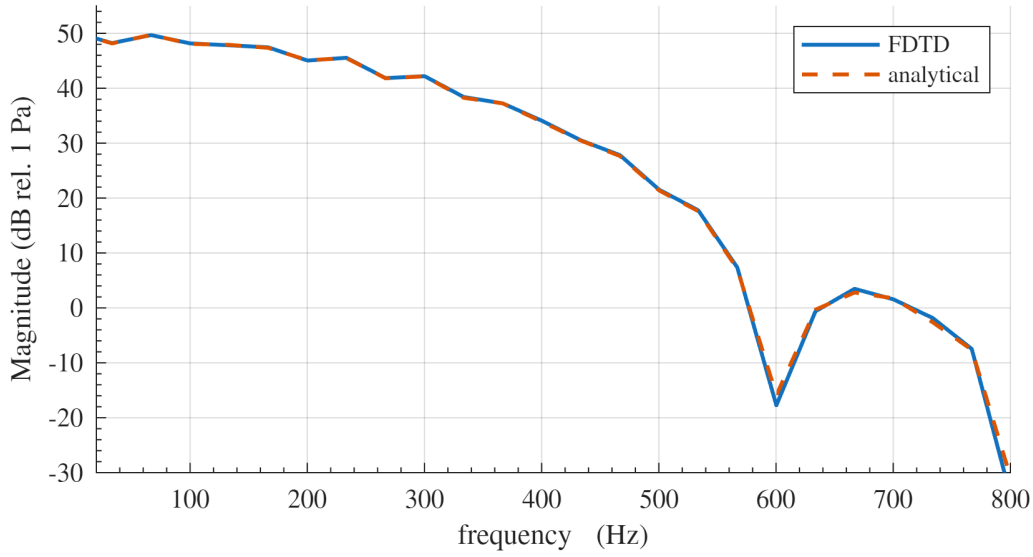


Figure 3.27: Comparison of frequency spectra for modelled and analytical solutions of a finite-impedance boundary condition.

As mentioned at the beginning of this section, the analytical solution relies on a plane wave reflection factor. This trial employed a geometry of source and receiver that ensured an angle of incidence well below the so-called critical angle, the angle of incidence beyond which the plane-wave reflection coefficient transitions to total reflection. If the geometry is changed, and the angle of incidence approaches or exceeds the critical angle, the analytical solution will no longer be valid. For a sea floor made of coarse silt, used in this validation, the critical angle is calculated using equation (3.8), where c_1 is the first medium and c_2 the second medium, to be $\theta_c = 68.2^\circ$.

$$\theta_c = \sin^{-1} \left(\frac{c_1}{c_2} \right) \quad (3.8)$$

A subsequent trial was carried out using the geometry shown in figure 3.28. This results in an angle of incidence $\theta_1 = 63.6^\circ$. Figures 3.29 and 3.30 show the resulting time signal and frequency spectrum recorded at the receiver. It is clear that the analytical solution deviates from the model output. For such cases, where the angle of incidence approaches the critical angle, a comparison is needed with a more complete analytical solution, such as the spherical reflection factor derived by Sommerfeld. Due to the time constraints and scope of this thesis, it was decided not to investigate more complex cases and instead remain aware of this possible limitation in further simulations.

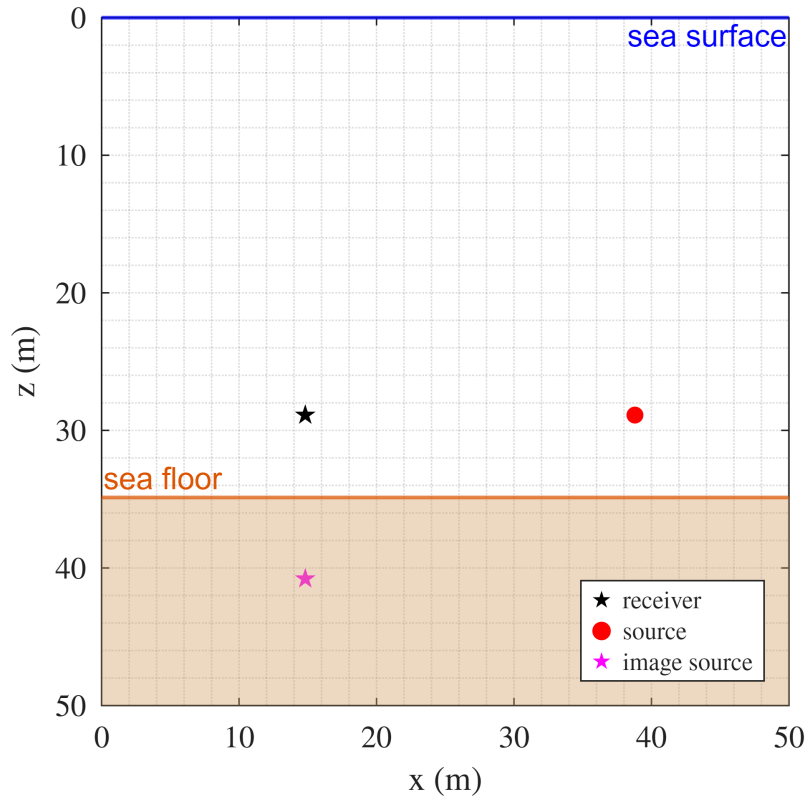


Figure 3.28: Finite-impedance boundary condition where angle of incidence approaches the critical angle.

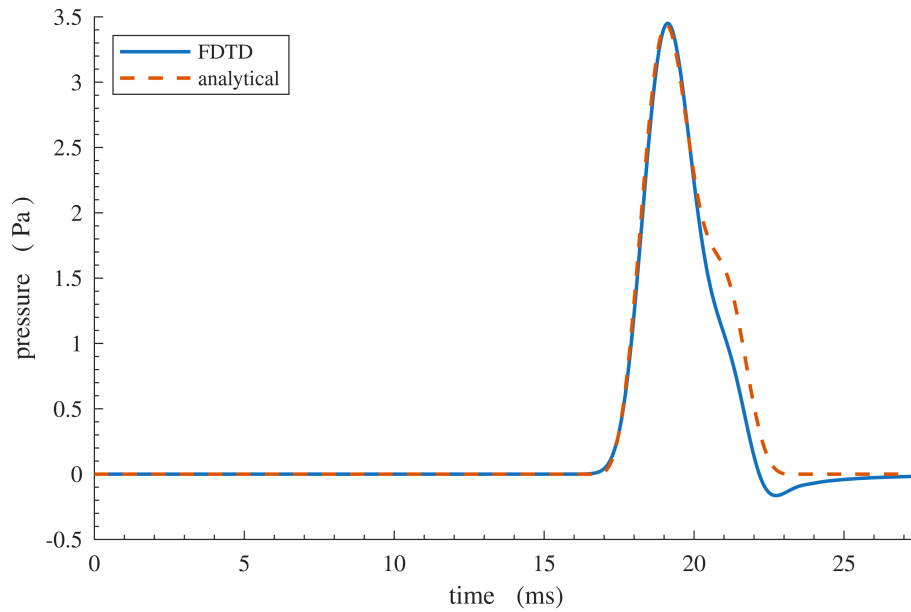


Figure 3.29: Model output compared to analytical solution for receiver in figure 3.28.

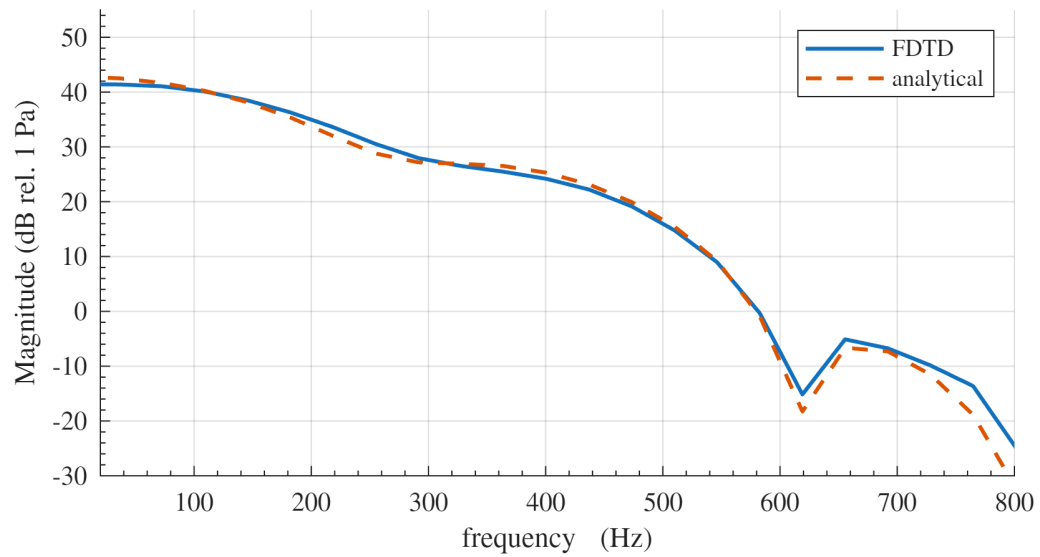


Figure 3.30: Comparison of frequency spectra for time signals in figure 3.29.

4

Propagation Loss Modelling

In this section, the output of the FDTD model will be compared with measurements of propagation loss obtained by the IVL underwater acoustics team in the Kosterhavet marine national park. A FDTD grid was generated in MATLAB that replicates the real physical domain in which measurements were conducted. Simulations were run with varying sea bottom parameters in an attempt to reproduce the measured results and compare the effect of different sea floor properties.

4.1 Measurement Data

Sea depth data for a region of the Koster Islands was provided by IVL. This site is shown in figure 4.1. Axes are given in northing and easting coordinates with depth indicated by the colour mapping. The resolution of the easting and northing coordinates is 5 m. Sources and receivers were placed at the locations indicated on the map. Sources were placed at a depth of 2 m with receivers placed at varying depths, this data was also provided by IVL. For each source the propagation loss at the receivers was measured in third octave bands from 125 Hz - 4 kHz. The composition of the seabed at the measurement sites is not known and is therefore investigated in section 4.3.

4.2 Domain Generation

In order to model the area shown in figure 4.1, a 3D domain had to be generated in MATLAB using the depth data. First, the northing and easting coordinates were converted into distances in metres with the origin placed at the minimum easting and northing coordinates. The model grid was then generated with grid spacing Δx . The grid spacing is dependent on the desired upper frequency limit of the model, as discussed in section 2.2.3. Even for relatively low frequencies the resolution of the grid required to maintain numerical stability will be considerably higher than the resolution of the provided depth map. It is therefore necessary to use interpolation to determine the water depth at each point on the x-y plane.

This was done in MATLAB using the function `scatteredInterpolant.m` which creates an interpolating function from the available depth data. This function was

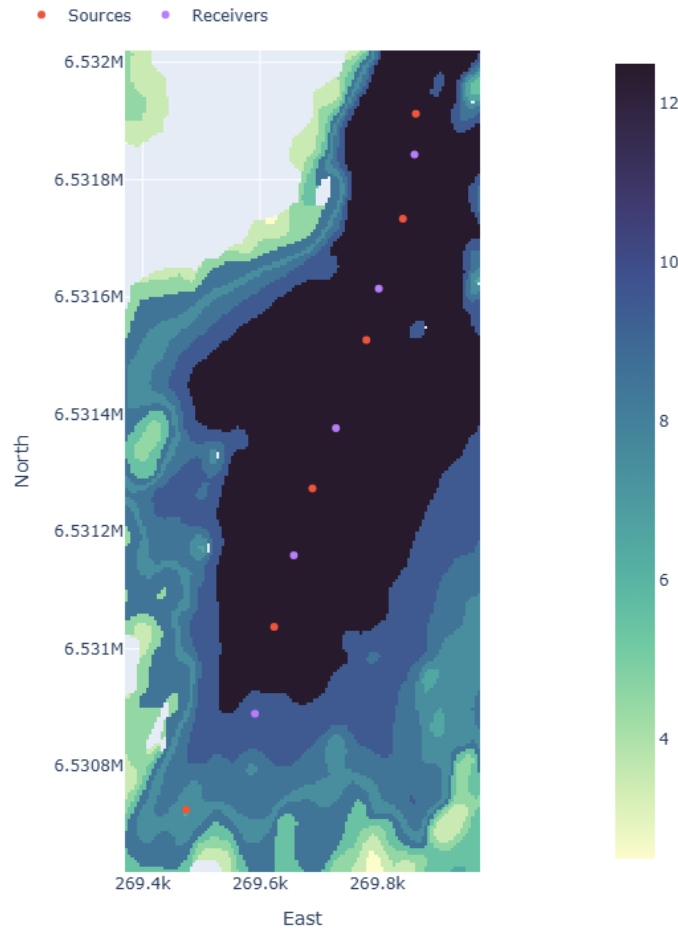


Figure 4.1: Depth map of measurement site with source and receiver locations.

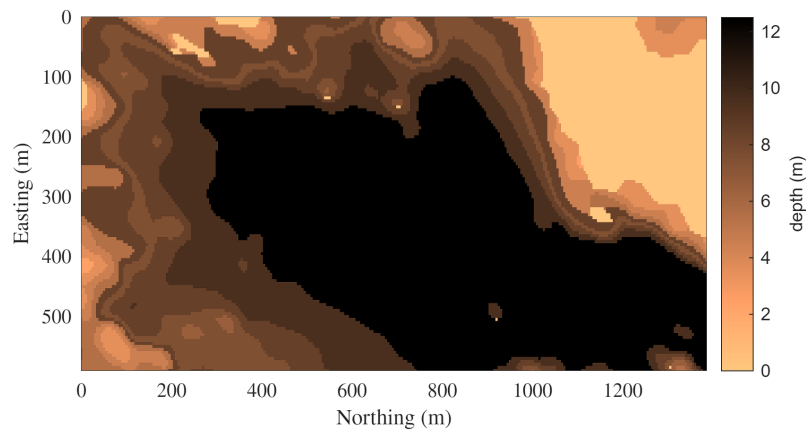


Figure 4.2: Modelled depth map generated in MATLAB using data provided by IVL.

applied to the generated x-y plane to produce a depth map with the resolution required by the model. The interpolating function allows the user to specify the interpolation method as ‘linear’, ‘nearest’ or ‘natural’. The ‘linear’ method only allows for depth values present in the original data to be used in the interpolation. This retains the precision of the available data though results in sharp changes in depth, leading to staircase formations in the model grid. ‘Nearest’ and ‘natural’ provide increasingly smoothed boundaries between varying sea floor depths. These methods result in some grid points having depths which are not present in the original depth data, but provide a more continuous boundary that one expects to be more consistent with a real physical environment. Since propagating waves will interact with boundaries differently depending on their geometries, this is an important factor to consider when attempting to model the real physical domain.

Figure 4.2 shows the 2D depth map generated in MATLAB using data provided by IVL. Easting and northing coordinates have been converted to distance in metres, with labels retain for comparison to figure 4.1. A linear interpolating method was used hence the similarity to the latter figure. A 3D depth map is shown in figure 4.3, the staircase structure as a result of the interpolating method is clearly visible.

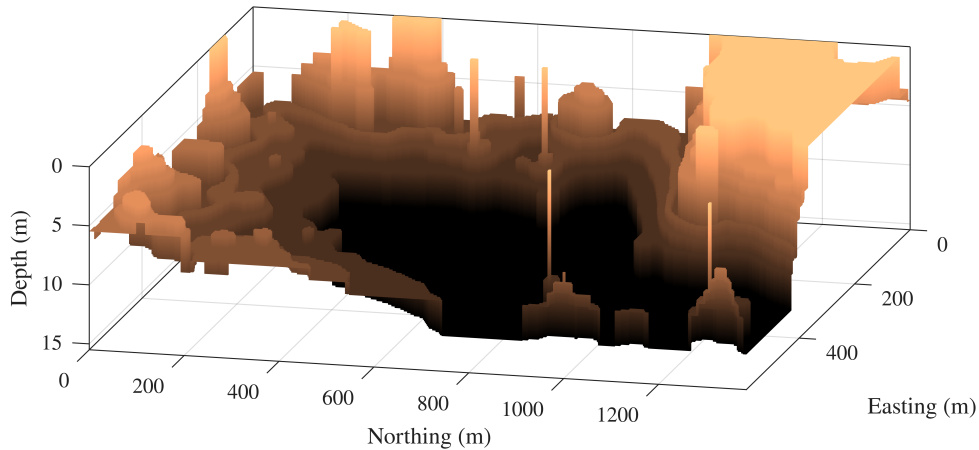


Figure 4.3: Modelled depth map generated in MATLAB using data provided by IVL.

The entire measurement site is unfortunately too large to be modelled in a single simulation. A single 3D finite-difference grid of the entire site volume requires 252.4 GB of memory. To efficiently run simulations, the site was split into sub-sites. Figures 4.4 and 4.5 show 3D depth maps of ‘sub-site 0’ which is the south-west corner of the domain shown in figure 4.1 encapsulating the first source and receiver. Figure 4.4 shows depth computed using the ‘linear’ interpolation method whereas in figure 4.5 depth was computed according to the ‘natural’ interpolation method. Variations in depth are significantly smoothed in the latter plot although still subject to the grid resolution as determined by the model parameters, leading to some staircasing still visible.

The generated depth map was used to implement the sea floor in the FDTD model. A sea floor thickness L_{floor} was defined which extends the domain in the z-direction from the sea floor's lowest point. A finite-impedance boundary was implemented in the model by changing the material properties of points that lie below the sea floor to those of the desired sea floor material such as silt or bedrock. In the model code, this means altering the buoyancy and bulk modulus grids. A PML may also be added below the sea floor to avoid unwanted reflections from the domain end.

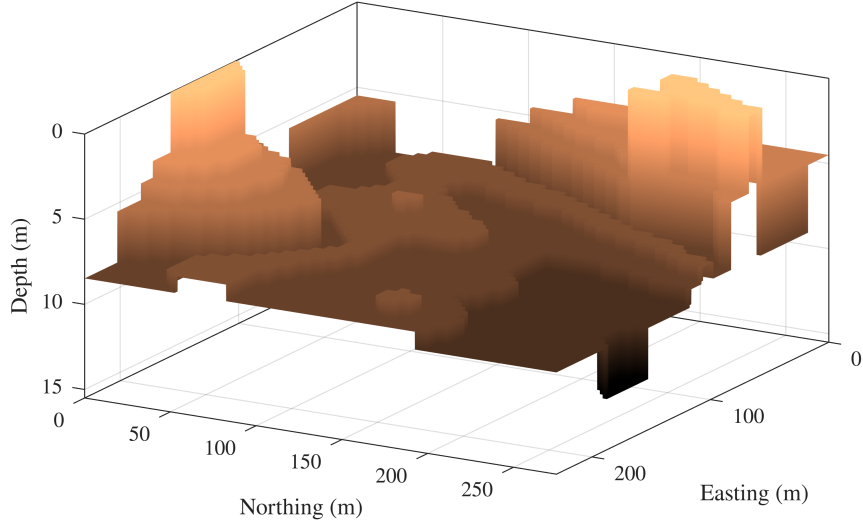


Figure 4.4: Depth map of sub-site 0 generated using linear interpolation.

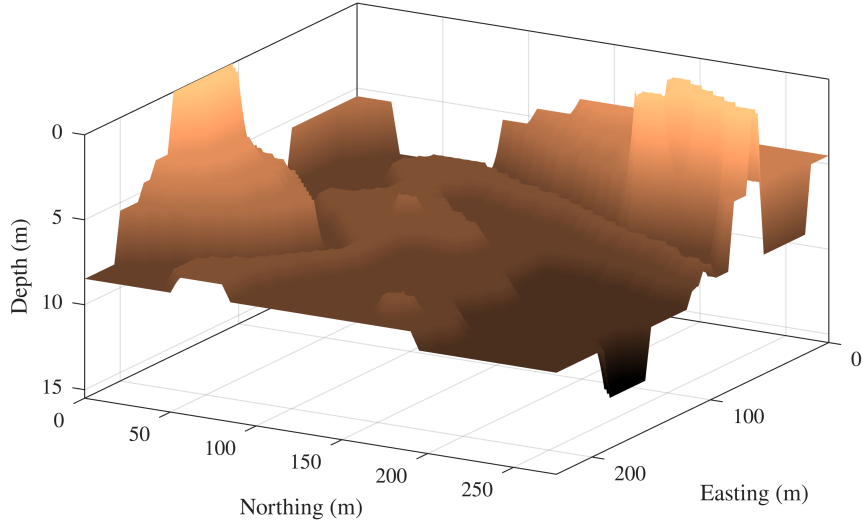


Figure 4.5: Depth map of sub-site 0 generate using 'natural' interpolation.

4.3 Comparison with Measurements

For the initial comparison with measurements, the sub-site with the least complicated bathymetry was chosen. A rectangular area containing a source located at (269.689k, 6.531273M) and receiver at (269.657k, 6.531159M) was chosen. Water depth in the area is a uniform 12.5 m. The sub-site is shown inside the yellow rectangle in figure 4.6.

Several simulations were run for this sub-site with varying sea floor material properties taken from Annex D of ISO 17208-3 [26], shown in table 4.1 alongside the properties of water used in this model. The sea floor extends a further 15 m from the lowest point stated in the available depth data. NPMLs were placed at all boundaries except for the sea surface.

Table 4.1: Material properties of media used in the model.

Medium	Density, ρ (kg/m ³)	Sound Speed, c (m/s)	Layer Thickness (m)
Water	1027	1500	12.5
Fine Silt	$1.513 \rho_{\text{water}}$	$1.0239 c_{\text{water}}$	15
Very Fine Silt	$1.0054 \rho_{\text{water}}$	$1.439 c_{\text{water}}$	15
Coarse Sand	$1.2530 \rho_{\text{water}}$	$2.231 c_{\text{water}}$	15
NPML	—	—	3.975

The source node was driven with a low-passed delta function which provides a flat frequency response within the desired frequency limit. The time signal and frequency spectrum of this function are found in appendix A. The simulation was run for 100 ms allowing enough time for the direct wave and subsequent reflections to reach the receiver. Propagation loss between the receiver and source is calculated using the equation,

$$PL = 20 \log_{10} \left(\frac{|P_s|}{|P_r|} \right) \quad (4.1)$$

where P_s is the Fourier transform of the source signal measured at 1 m in a free-field and P_r is the Fourier transform of the signal measured at the receiver.

Figure 4.7 shows the propagation loss calculated at the receiver for the different sea floor properties specified in table 4.1 as well as propagation loss at the receiver measured by IVL. There are significant differences in the PL depending on the sea floor properties with the closest to real measurements being a sea floor composed of very fine silt. The general trend of the modelled data for very fine silt is similar

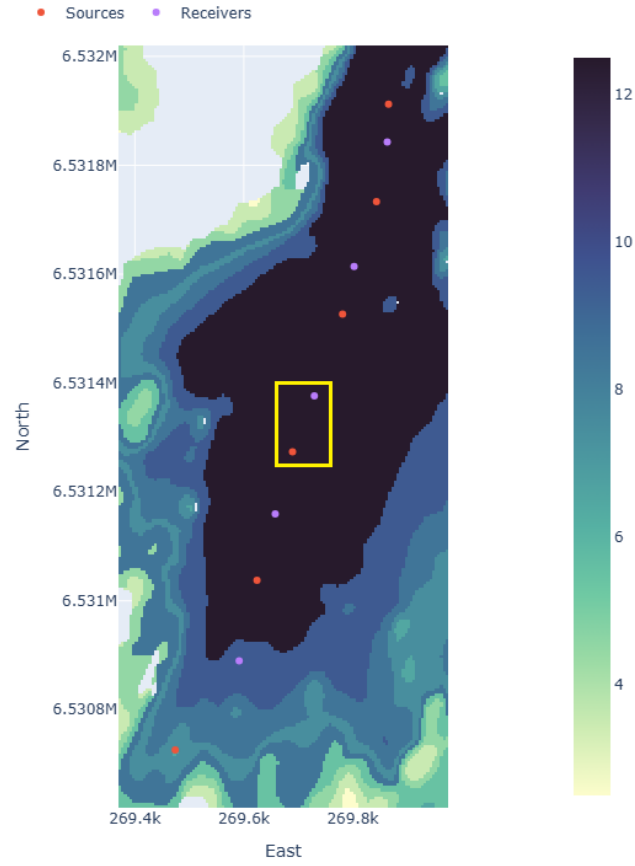


Figure 4.6: A computational domain was created containing the area inside the rectangle.

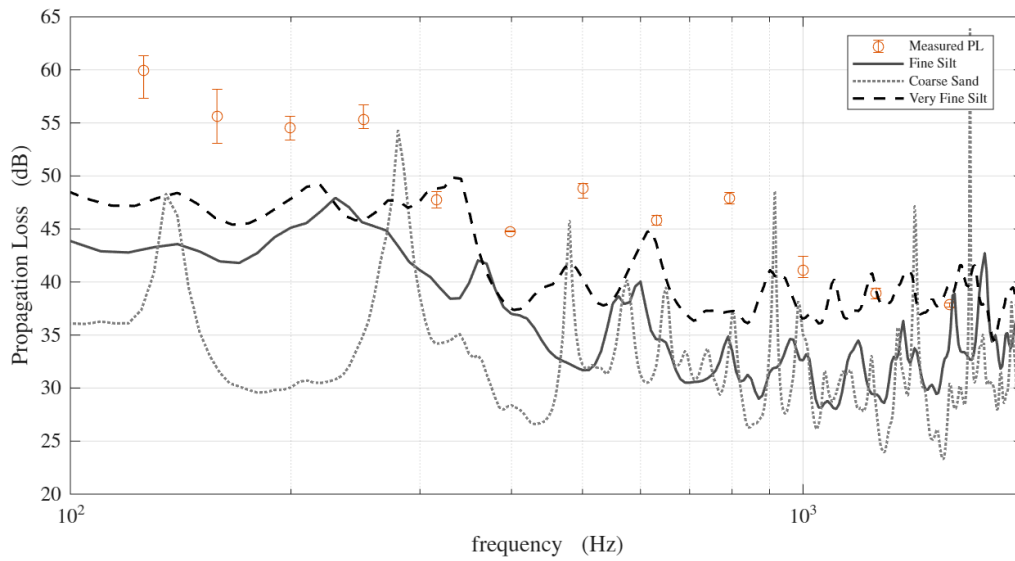


Figure 4.7: Frequency spectrum of source signal, receiver signal and propagation loss.

to the measured data but differs by over 10 dB at some frequencies. This may be due to several factors. The sea floor may not be as uniform as the supplied depth data suggests; the sea floor may have a different thickness or be composed of several different layers; there may be remaining energy in the domain due to ineffective NPMLs.

In order to investigate the possibility of stratified sediment in the sea floor, development and validation of the model's capabilities beyond the scope of this project is required. Instead it was decided to take the best performing material properties, those of very fine silt, and investigate the effect of different sea floor thicknesses.

Figure 4.8 shows the modelled propagation loss for a sea floor of varying thicknesses composed of very fine silt. For thicknesses upwards of 15 m the propagation loss results converge, especially towards the upper end of the frequency range. This suggests that beyond a certain layer thickness, sound waves no longer return to the water column due to the combined effects of geometric spreading and attenuation as the wave travels through the sediment and the PML. Since the previous simulations, comparing different sea floor properties, used a sea floor layer of 15 m, the results of this run of simulations do not bring the model any closer to reproducing measurements. This means that the discrepancy between model output and measured results must arise from other processes.

Due to the scope of this thesis and the time available, further work to reproduce measured propagation loss in Kosterhavet was not carried out. Suggestions of further work are discussed in the following chapter.

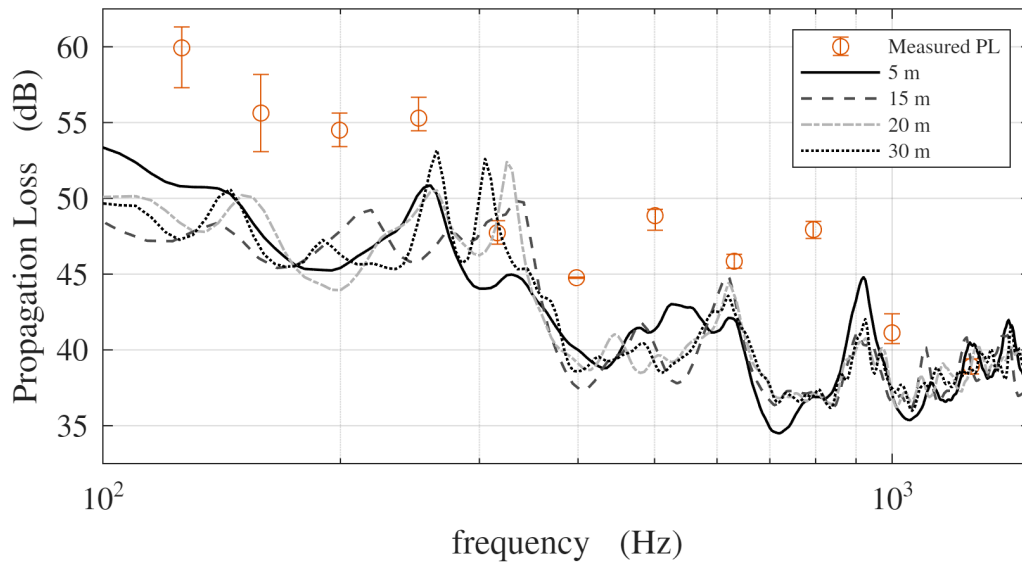


Figure 4.8: Modelled PL for varying thicknesses of a sea floor composed of very fine silt.

5

Discussion

In this project a finite-difference time-domain model was developed for the purpose of modelling propagation of acoustic waves in shallow water environments. Such environments are complex domains for sound propagation due to reflections produced by interactions with the sea surface and the sea floor.

It was demonstrated in chapter 3 that the model is capable of reproducing losses due to spherical spreading with high accuracy in both the time and frequency domains. The model also shows the capability to accurately reproduce reflections from rigid boundaries, pressure-release boundaries and finite-impedance boundaries. With regards to the finite-impedance boundary, the validation confirmed that the model agrees with analytical solutions for reflections and angles of incidence below the critical angle. For cases where the angle of incidence approaches the critical angle, the coherence between the analytical solution and the model breaks down. This is likely a fault of the analytical solution which uses a plane wave reflection factor. Although, since no attempt was made using a more complex analytical solution, it cannot be presumed that the model is entirely accurate for these cases either. A continuation of this work would certainly investigate finite-impedance boundaries further, attempting to validate the model output for all possible interactions with such a boundary.

A further validation that was not implemented in this project but may be of particular interest in the shallow water environment would be to analyse diffraction phenomena and compare with analytical solutions. Diffraction occurs underwater when sound waves encounter objects such as rock formations and islands. When this thesis was proposed by IVL, it was suggested that this could be one of the acoustic effects that would be accurately modelled by the FDTD model. Regretfully, this was not investigated but should be considered in further work.

In order to avoid spurious reflections from domain edges, a perfectly matched layer was implemented which mimics the radiation of the acoustic wave into free space by adding layers around the real domain in which the oscillating wave decays. A reasonable reduction in energy reflected from the limits of the domain was achieved, although concessions in layer width had to be made to avoid high demands on computational memory. The performance of the NPML is dependent on the decay

factor η , layer thickness and the ramping function that increases η across the layer. For the boundary where water meets the end of the physical domain, η and thickness were optimised empirically using a trial and error process. This process was performed, but not documented here, for a seabed composed of coarse silt, but not for any other media due to the time consuming process. This is of course not ideal, and may have reduced the performance of the NPML beneath the seabed for cases where the seabed was composed of a different material. Unfortunately, due to the scope of this project, further optimisation was overlooked. Further work could include empirical optimisation of the ramping function and optimisation of all parameters for other materials. An investigation into analytical solutions for optimising NPML parameters could also be carried out.

Chapter 4 documents an attempt to reproduce measurements of propagation loss conducted by the IVL underwater acoustics team in Kosterhavet national park. The real physical domain was converted into a finite-difference grid to be used by the model. Given the size of the entire measurement volume, roughly (1.4 km x 600 m x 12.5 m), it was necessary to split the domain into sub-domains to avoid overwhelming memory requirements. The size of these sub-domains (e.g. 150 m x 55 m x 12.5 m) is still large enough to impose an upper frequency limit of around 2 kHz due to memory constraints. The upper frequency limit of measurements was 4 kHz meaning that just over half of the frequency range was unable to be modelled. The constraints of domain size and frequency range are inherent in FDTD models as a result of the many computational grids required for calculations. Attempts were made where possible to condense the number of grids; a calculating computer with large RAM was employed; and precision was occasionally sacrificed by using single-precision floating-point values. However, large memory demands are the reality of dealing with highly accurate numerical models. Ostashev & Wilson mention that FDTD computations are parallelisable and future development of the model could look into how to optimise memory usage in order to be able to increase domain size and broaden frequency range [9].

For the chosen sub-site, simulations were run with varying material properties of the sea floor. Propagation loss was calculated at the receiver for a range of frequencies for each different sea floor. This allowed for the composition of the sea floor to be estimated by comparing with measured PL values, as well as demonstrating the stark differences in propagation loss depending on the sea floor material properties. Whilst the best fit to measurements came from a sea floor composed of very fine silt, there were still discrepancies of over 10 dB at some frequencies. This suggests that additional processes not captured by the model contribute to further propagation loss. Nevertheless, these best-fit properties were taken and additional simulations were run with varying sea floor layer thicknesses ranging from 5 m - 30 m. Results of modelled propagation loss converged for layer thicknesses above 15 m, demonstrating that beyond this, little energy is returned into the water column from the sea bed. This has implications for further testing, providing a lower limit of required seabed depth which may reduce memory requirements.

Aside from memory requirements, modelling a real physical domain is complicated. In reality, there are many unknown factors. High-resolution bathymetry data is sensitive information meaning that the modelled sea floor may not accurately represent the real sea floor. There may be variations in the material properties of the seabed as well as its bathymetry. To know the thickness of the sea floor is also difficult without geological surveys, and there may be several layers of different sediment. In this project, many of these parameters were estimated or in the case of the bathymetry, interpolated from lower-resolution data. The process of interpolation may lead to more or less ‘staircasing’ depending on the interpolation method used which may have consequences for reflections from the seabed. This is yet another factor that could be further investigated.

Despite all this complexity, the comparison of modelled propagation loss against measured propagation loss is promising. The model is close to measured values for certain frequencies, although up there is up to 15 dB difference at other frequencies. With more investigation and refinement, it is possible that the model could come even closer to reproducing real-world measurements.

6

Conclusion

This thesis work has presented the development and validation of a three-dimensional finite-difference time-domain model for the simulation of underwater sound propagation in shallow-water environments. The objective of the work was to investigate whether a physics-based numerical model could better represent the complex acoustic behaviour observed in measured data, from the Kosterhavet marine national park, than data-driven analytical propagation loss models.

The model was validated by comparison to a series of analytical solutions of increasing complexity. It was able to accurately reproduce losses due to spherical spreading and demonstrated excellent agreement with analytical predictions for rigid, pressure-release and finite-impedance boundary conditions. These validation exercises confirmed that the FDTD approach can reliably model wave reflections from the sea surface and seabed, important acoustic phenomena for shallow-water conditions.

To enable simulations of unbounded domains, a nearly perfectly matched layer was implemented and optimised. The NPML significantly reduced unwanted reflections from computational boundaries whilst maintaining an acceptable computational cost. The study showed that both the decay factor and layer thickness strongly influence performance and must be carefully selected to balance accuracy and efficiency.

The model was further extended to incorporate real bathymetry data provided by IVL Swedish Environmental Research Institute. Comparison with measured propagation loss demonstrated that the model captures the overall trends observed in the field data, although noticeable discrepancies remain at certain frequencies. These differences are likely due to uncertainties in seabed properties, bathymetric resolution, boundary treatment, and computational limitations on frequency range and simulation domain size.

Overall, the results show that the FDTD approach can be used to model underwater sound propagation in environments with varying bathymetry and seabed properties. Further work is needed to improve the environmental representation, reduce computational limitations, and evaluate the model against a wider range of measurement data.

References

- [1] J. A. Hildebrand, “Anthropogenic and natural sources of ambient noise in the ocean,” *Marine Ecology Progress Series*, vol. 395, pp. 5–20, 2009. Accessed: 2026-01-22. [Online]. Available: <http://www.jstor.org/stable/24874238>.
- [2] S. M. K. Wijesinghe et al., “Marine pollution: The global challenge of ocean contaminants and mitigation efforts,” *Anthropocene Coasts*, vol. 9, p. 5, 2026. DOI: 10.1007/s44218-025-00107-8. [Online]. Available: <https://link.springer.com/article/10.1007/s44218-025-00107-8>.
- [3] P.-O. Moksnes et al., “Fritidsbåtars påverkan på grunda kustekosystem i sverige,” Havsmiljöinstitutet, Technical Report 2019:3, 2019.
- [4] Kosterhavets nationalpark, *Naturvärden i havet*, <https://www.kosterhavet.se/om-nationalparken/naturvarden-i-havet/>, Accessed: 2026-01-29.
- [5] *Underwater acoustics — quantities and procedures for description and measurement of underwater sound from ships — part 1: General requirements*, en, International Organization for Standardization, 2016.
- [6] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, “Ray methods,” in *Computational Ocean Acoustics*, ser. Modern Acoustics and Signal Processing, 2nd ed., New York, NY: Springer, 2011, ch. 3, pp. 155–232. DOI: 10.1007/978-1-4419-8678-8.
- [7] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, “Wavenumber integration techniques,” in *Computational Ocean Acoustics*, ser. Modern Acoustics and Signal Processing, 2nd ed., New York, NY: Springer, 2011, ch. 4, pp. 233–336. DOI: 10.1007/978-1-4419-8678-8.
- [8] V. E. Ostashev and D. K. Wilson, “Acoustics in moving inhomogeneous media,” in 2nd. CRC Press / Taylor & Francis, 2015, ch. Chapter 11, pp. 387–426.
- [9] V. E. Ostashev and D. K. Wilson, “Acoustics in moving inhomogeneous media,” in 2nd. CRC Press / Taylor & Francis, 2015, ch. Chapter 12, pp. 427–447.
- [10] R. F. W. Coates, “Sound transmission fundamentals,” in *Underwater Acoustic Systems* (New Electronics), 1st ed., New Electronics. Red Globe Press London, 1990, ch. 1, pp. 1–15. DOI: 10.1007/978-1-349-20508-0.

- [11] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, “Fundamentals of ocean acoustics,” in *Computational Ocean Acoustics*, ser. Modern Acoustics and Signal Processing, 2nd ed., New York, NY: Springer, 2011, ch. 1, pp. 1–64. DOI: 10.1007/978-1-4419-8678-8.
- [12] T. Neighbors, “Chapter 4 - absorption of sound in seawater,” in *Applied Underwater Acoustics*, T. H. Neighbors and D. Bradley, Eds., Elsevier, 2017, pp. 273–295. DOI: <https://doi.org/10.1016/B978-0-12-811240-3.00004-7>. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/B9780128112403000047>.
- [13] S. C. Hagness, A. Taflove, and S. D. Gedney, “Finite-difference time-domain methods,” in *Numerical Methods in Electromagnetics*, ser. Handbook of Numerical Analysis, vol. 13, Elsevier, 2005, pp. 199–315. DOI: [https://doi.org/10.1016/S1570-8659\(04\)13003-2](https://doi.org/10.1016/S1570-8659(04)13003-2). [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1570865904130032>.
- [14] V. E. Ostashev and D. K. Wilson, “Acoustics in moving inhomogeneous media,” in 2nd. CRC Press / Taylor & Francis, 2015, ch. Chapter 2, pp. 25–62.
- [15] D. Wilson and L. Liu, “Finite-difference, time-domain simulation of sound propagation in a dynamic atmosphere,” U.S. Army Engineer Research and Development Center, Tech. Rep., 2004.
- [16] *Esa - eduspace - weather and climate - sea surface currents — esa.int*, https://www.esa.int/SPECIALS/Eduspace_Weather_EN/SEMSM2L1YHH_0.html, [Accessed 01-06-2026].
- [17] V. E. Ostashev, D. K. Wilson, L. Liu, D. F. Aldridge, N. P. Symons, and D. Marlin, “Equations for finite-difference, time-domain simulation of sound propagation in moving inhomogeneous media and numerical implementation,” *The Journal of the Acoustical Society of America*, vol. 117, no. 2, pp. 503–517, Jan. 2005. DOI: 10.1121/1.1841531. eprint: https://pubs.aip.org/asa/jasa/article-pdf/117/2/503/8096010/503_1_online.pdf. [Online]. Available: <https://doi.org/10.1121/1.1841531>.
- [18] J. O. A. Robertsson and J. O. Blanch, “Numerical methods, finite difference,” in *Encyclopedia of Solid Earth Geophysics*, H. Gupta, Ed., Springer, 2011, p. 889.
- [19] J.-P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *Journal of Computational Physics*, vol. 114, no. 2, pp. 185–200, 1994. DOI: <https://doi.org/10.1006/jcph.1994.1159>. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0021999184711594>.
- [20] V. E. Ostashev and D. K. Wilson, “Acoustics in moving inhomogeneous media,” in 2nd. CRC Press / Taylor & Francis, 2015, ch. Chapter 12, p. 443.
- [21] W. Hu, A. Abubakar, and T. Habashy, “Application of the nearly perfectly matched layer in acoustic wave modeling,” *Geophysics*, vol. 72, Sep. 2007.
- [22] A. D. Pierce, “Basic linear acoustics,” in *Springer Handbook of Acoustics*, ser. Springer Handbooks, T. D. Rossing, Ed., New York, NY: Springer, 2015, pp. 25–108.

- [23] M. Kleiner, *1.8.3 normal reaction*, 2012. [Online]. Available: <https://app.knovel.com/hotlink/pdf/id:kt00U8S552/acoustics-audio-technology/solutions--stationary>.
- [24] V. E. Ostashev and D. K. Wilson, “Acoustics in moving inhomogeneous media,” in 2nd. CRC Press / Taylor & Francis, 2015, ch. Chapter 12, p. 439.
- [25] A. D. Pierce, “Acoustics: An introduction to its physical principles and applications,” in 3rd. Cham: Springer Nature Switzerland, 2019, ch. 3.6, p. 153.
- [26] International Organization for Standardization, “Underwater acoustics — Quantities and procedures for description and measurement of underwater sound from ships — Part 3: Requirements for measurements in shallow water,” International Organization for Standardization, Geneva, Switzerland, Standard ISO 17208-3:2025, 2025, Annex D, Section D.4, Table D.1.

A

Appendix

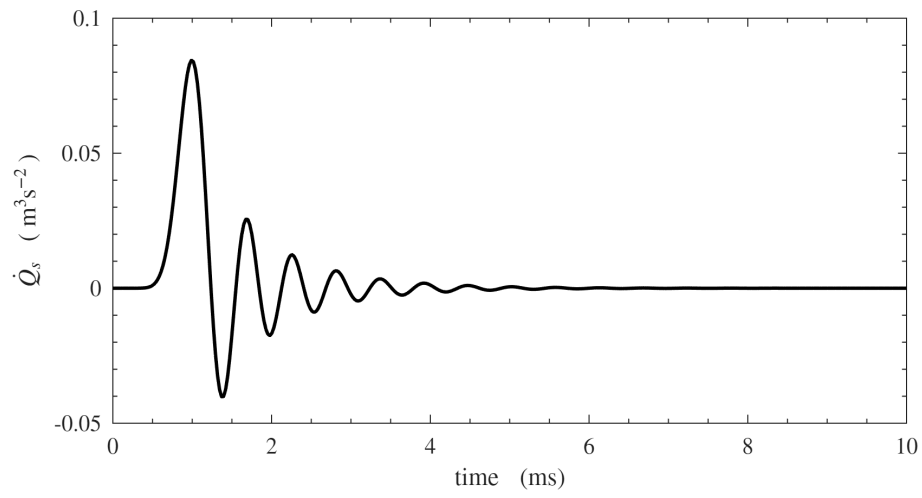


Figure A.1: Low pass delta function used as source signal in simulation of real domain.

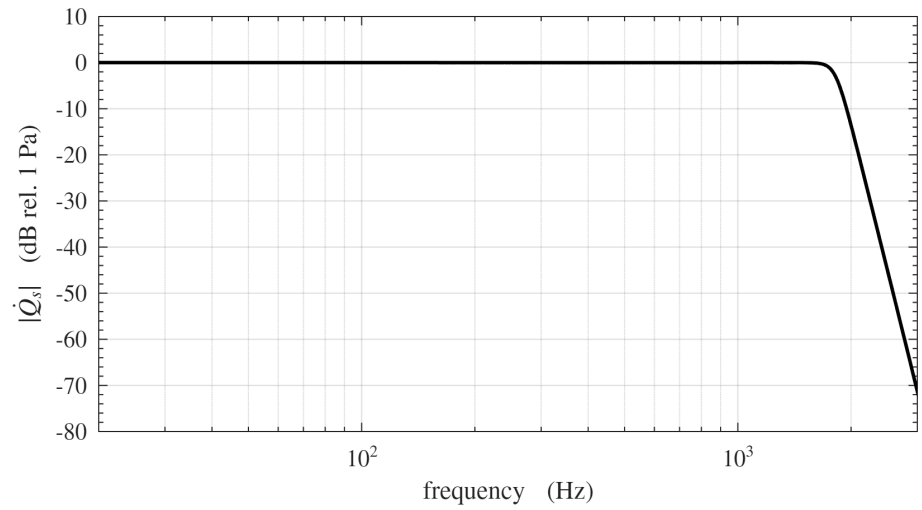


Figure A.2: Frequency spectrum of low pass delta function in A.1

DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
CHALMERS UNIVERSITY OF TECHNOLOGY

Gothenburg, Sweden
www.ta.chalmers.se



CHALMERS
UNIVERSITY OF TECHNOLOGY