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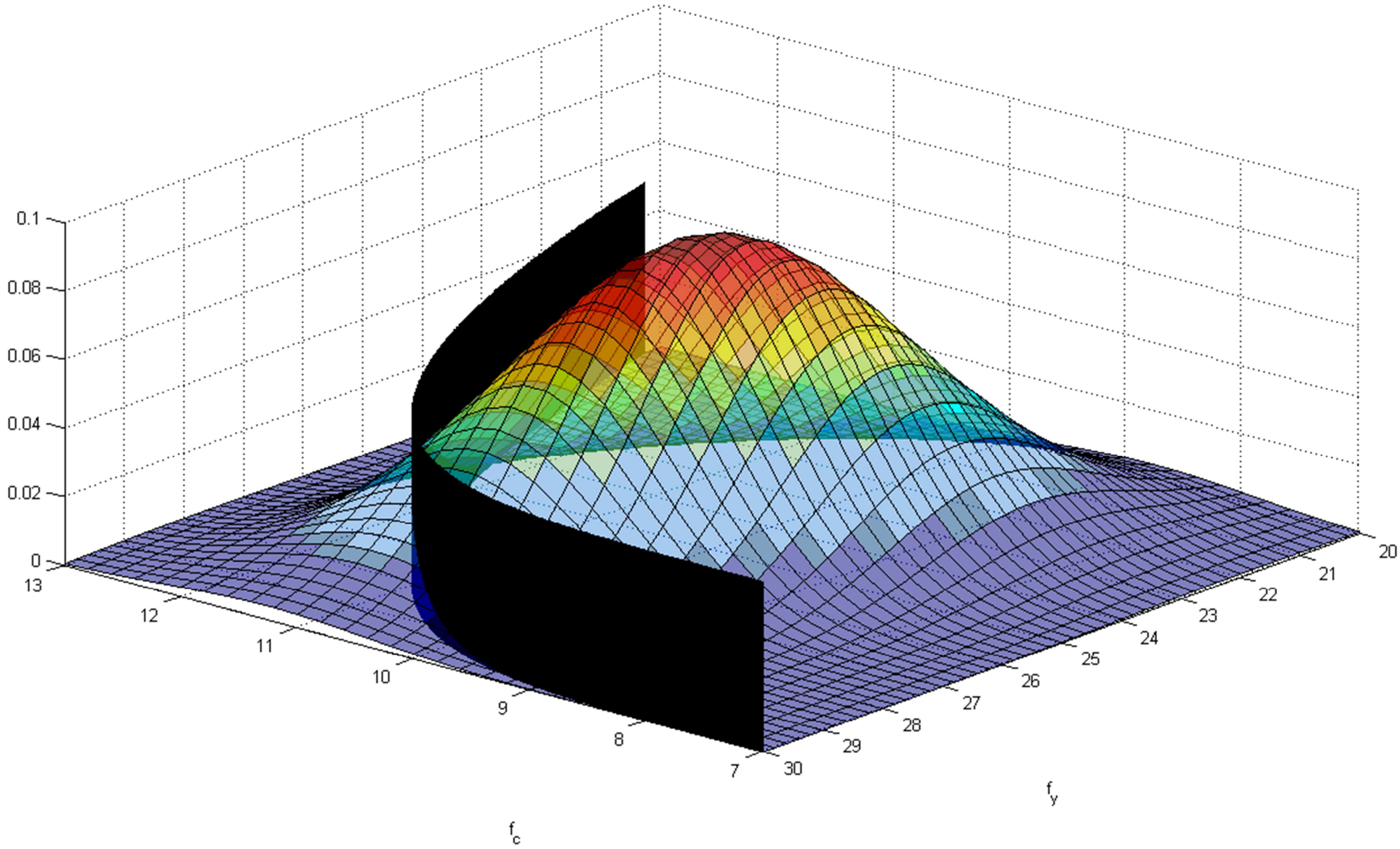

Evaluation of safety formats for non-linear finite element analysis

Master of Science Thesis in the Master's Programme Structural Engineering and Building Performance Design

MIKAEL FURU

RIKARD MÖSE

Department of Civil and Environmental Engineering
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Concrete Structures
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Division of Structural Engineering
Concrete Structures
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

Cover:
Example of capacity distribution.

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ABSTRACT

The use of non-linear finite element (FE) analyses is growing rapidly but still there is limited information concerning safety formats for FE analyses. For design and analysis of concrete structures Eurocode provides two safety formats to obtain the design resistance, CEN(2004a) and CEN(2004b). The Partial Safety Method (PSF) CEN(2004a) is traditionally used for analytical hand calculations and is not appropriate for non-linear FE analyses. In consequence of that, Eurocode suggest the use a safety format with a global resistance safety coefficient, CEN(2004b), which is more suitable for numerical calculations. A weak point of this format is that it does not take into account the varying model uncertainty for different failure modes. Attempts have been made by researchers around the world to develop a safety format that is more suitable for FE analysis.

Chalmers University of Technology has an ongoing research project aiming to develop a new safety format. This master's thesis is closely related to this ongoing research project. The aim of this thesis is to evaluate a new proposed safety format by Schlune *et al* (2010) and compare this with the safety formats in Eurocode and with a previously proposed safety format by Cervenka *et al.* (2007). The results obtained were verified with probabilistic analysis, using a newly developed software module for DIANA called PROBAB which allows for calculating the probability of failure. The aim was also to investigate the potential of PROBAB as a tool to test safety formats.

The comparison of safety formats was conducted on a non-linear reinforced concrete beam in the Ultimate Limit State (ULS) and the Serviceability Limit State (SLS). Furthermore, an attempt was made to do the same on a shear panel. This master's thesis focuses on the resistance i.e. the load uncertainty is not included. Before the comparison, testing of a linear steel beam was performed to verify PROBAB and to evaluate settings for the probabilistic analyses. The failure probability using PROBAB was verified with Monte Carlo (MC) simulation and the results coincided with good accuracy if PROBAB analysis parameter were set adequately.

In the test of the safety formats for the non-linear concrete beams the safety formats according to CEN(2004a,b) gave identical design resistances. The safety format according to H.Schlune *et al* (2010) agreed well with probabilistic results on this structure along with CEN(2004b). Cervenka *et al.* (2007) safety format on the other hand overestimated the design resistance. The same test was also performed only considering material uncertainty and also in this case the proposed safety format and CEN(2004b) agreed well with the probabilistic analyses.

Key words: Safety format, PROBAB, CEN(2004a), CEN(2004b), Cervenka *et al.* (2007), Schlune *et al* (2010)

Contents

Preface

In this study, a new safety format for non-linear FE-analyses is evaluated and verified by probabilistic analyses using PROBAB. The work was carried out as a Master's Thesis project during the period from January 2010 to July 2010. This is a part of a research project "Safety Principles for Structural Design and Assessment with Non-Linear Methods" at Chalmers University of Technology, Gothenburg, Sweden. The research project is supervised by Mario Plos and Kent Gylltoft and carried out by Ph.D.-student Hendrik Schlune; Mario Plos is also the examiner of this Master's Thesis and Hendrik Schlune is the supervisor.

All work was carried out at the Division of Structural Engineering at the Department of Civil and Environmental Engineering, Chalmers University of Technology, Gothenburg, Sweden.

Special thanks to the division and to the IT-support at Chalmers University of Technology for access to the cluster-room. Especially thanks to our supervisor Hendrik Schlune for his support along the project.

Göteborg, June 2010

Mikael Furu

Rikard Möse

Notations

Roman upper case letters

R	resistance [N]
R_d	design resistance [N]
R_k	characteristic resistance [N]
R_n	nominal resistance [N]
R_m	mean resistance [N]
E	action effect [N]
E_d	design action effect [N]
P	applied load [N]
N_{samp}	number of samples
V_m	coefficient of variation of the model uncertainty
V_G	coefficient of variation of the geometric uncertainty
V_f	coefficient of variation of the material uncertainty
V_R	coefficient of variation of the structural resistance uncertainty
X_m	parameter corresponding to the model uncertainty
X_G	parameter corresponding to the geometric uncertainty
X_f	parameter corresponding to the material uncertainty

Roman lower case letters

α_E	weight factor for the action effect
α_R	weight factor for the resistance
β	reliability index
β_R	reliability index for the resistance
u	deflection [m]
γ_s	partial safety factor for steel
γ_c	partial safety factor for concrete
γ_R	resistance safety factor
σ_R	standard deviation of the resistance
σ_E	standard deviation of the action effect
μ_R	mean resistance
μ_E	mean action effect
p_f	Probability of failure
f_c	Concrete compressive strength [Pa]

E_c	modulus of elasticity of concrete [Pa]
ε_{cu}	Concrete ultimate strain
ν_c	poisons ratio for concrete
G_F	concrete fracture energy
f_{sy}	Reinforcement steel yield strength [Pa]
f_{su}	Reinforcement steel ultimate strength [Pa]
E_s	modulus of elasticity of reinforcing steel [Pa]
ν_s	Poisons ratio for reinforcement steel
ε_{su}	Reinforcement steel ultimate strain

1 Introduction

1.1 Background

The finite element (FE) method is an important tool in design of structures for today's engineers and it is expected to become even more common in the future. In design of concrete structures it is often appropriate to perform a non-linear analysis due to the non-linear material behavior. In the context of hand calculation, CEN(2004a) dictates a certain safety format which is based on the concept of partial safety factors. This format is not appropriate for non-linear analysis; as a result a new method was developed, see CEN(2004b), this newer method modifies the input parameters slightly so that they are close to mean values for the analysis and after that the design resistance can be achieved by reducing the obtained resistance. However, the accuracy of this new format is not considered to be sufficient in cases of complex structures where high uncertainty of the structural resistance can be expected. As a result, during the last years several researchers have proposed new safety formats.

Chalmers University of Technology started a research project "Safety Principles for Structural Design and Assessment with Non-Linear Methods" in January 2009 to develop and evaluate a new safety format. The primary aim of the project is to develop a safety format suitable for practicing engineers i.e. safety format that has sufficient accuracy without being too demanding or time-consuming.

1.2 Purpose

The purpose of the Master's Thesis is to evaluate the proposed safety format, compare the proposed safety format with other existing formats and also to verify the proposed safety format with probabilistic analyses using PROBAB. Furthermore, the objective is to evaluate the usefulness of PROBAB to verify safety formats.

1.3 Scope

The evaluation of safety formats was conducted using simple structures. This Master's Thesis focuses on the structural resistance, mainly on material uncertainty, while load has been treated according to Eurocode, see dark grey boxes in Figure 1.1.

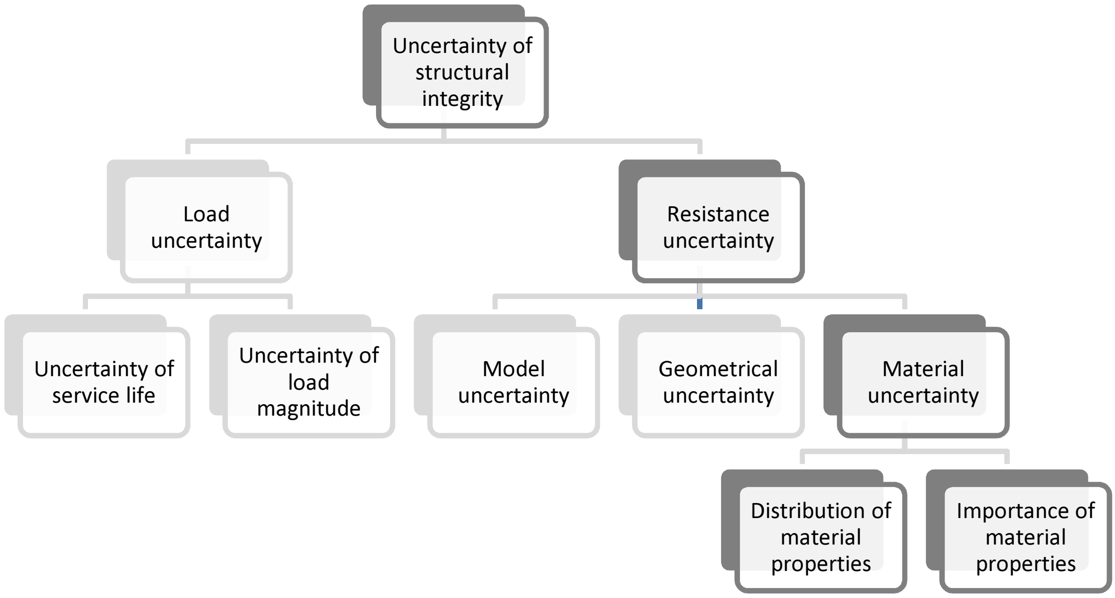


Figure 1.1 Hierarchic illustration of elements influencing the uncertainty of structural integrity. This thesis focuses on the elements marked dark grey.

1.4 Method

The FE analysis has been conducted using software DIANA together with a module called PROBAB, which allows computing the failure probability of reinforced concrete structures. Since PROBAB is a rather untested and not fully developed software, the first step in this thesis was to verify that PROBAB gave results accurate enough. A linear elastic case has been chosen for this study. To ensure that the FE model is correct, it was verified with hand calculations according to CEN(2004a).

The next step was to compare the different safety formats using simple non-linear structures. In this case a concrete beam in bending was chosen. Later, the results from the analysis were evaluated by comparison with a probabilistic analysis using PROBAB which was here considered as a reference.

The safety formats were also tested in a more complex case of a nonlinear concrete shear panel.

2 Uncertainty of structural resistance

When designing structures one has to account for the uncertainty of the calculated resistance capacity. Especially in the case of concrete structures the uncertainty is significant. The uncertainty is a result of several factors that can be categorized into three major groups in accordance with JCSS (2001a):

- Model uncertainty
- Geometric uncertainty
- Material uncertainty

These groups are represented by means of coefficients of variations and together they are causing the uncertainty of the structural resistance V_R, see equation 2.1. These groups will be explained further in this chapter.

$$V_R = \sqrt{V_m^2 + V_G^2 + V_f^2} \qquad (2.1)$$

where

V_R – Coefficient of variation of the structural resistance

V_m – Coefficient of variation of the model uncertainty

V_G – Coefficient of variation of the geometric uncertainty

V_f – Coefficient of variation of the material uncertainty

2.1 Model uncertainty

The model uncertainty is the difference between the response of the model and the actual response. The deviation is caused by all assumptions and idealizations that are made when simulating the behavior of the structure.

The model uncertainty is due to simplifications which are often unavoidable or simplification of mathematical relations e.g. stress-strain relationship of reinforcement steel is often assumed to be bi-linear, which is an idealization of the reality, see Figure 2.1. Limited knowledge on how to model the sectional response in FE analysis can give deviations in results. The result depends on which type of elements that are used, for example if a beam is to be analyzed; beam element may be used (Euler-Bernoulli), by making that choice pure bending is assumed. If the length-depth ratio of the beam is approaching the recommended limitations of a slender beam, failure mode of shear has to be considered, shear failure being unlikely in the case of slender beams. Pure bending can still be assumed but the assumption becomes less accurate. In summary, the choice of element type may contribute to the error of the simulated response and higher model uncertainty. This is just one of many factors causing model uncertainty. Other modeling choices that influence the results can be the assumptions of boundary conditions in the model that should reflect the actual boundary condition, pinned, fixed or intermediate condition. The convergence criterion also influence the result, it is seldom not feasible to perform the analysis accurate enough. The computational effort increases as higher accuracy is demanded.

For example when modeling 3-dimensional structures, one dimension is sometimes omitted if the variation in that dimension is regarded to be negligible. This gives a simplified structure which requires less computational effort. However if the variation of that omitted dimension influences the structural response, that influence will be lost and result in a higher model uncertainty. More complex structures are often simplified in many different ways and alls simplifications add up to considerable model uncertainty.

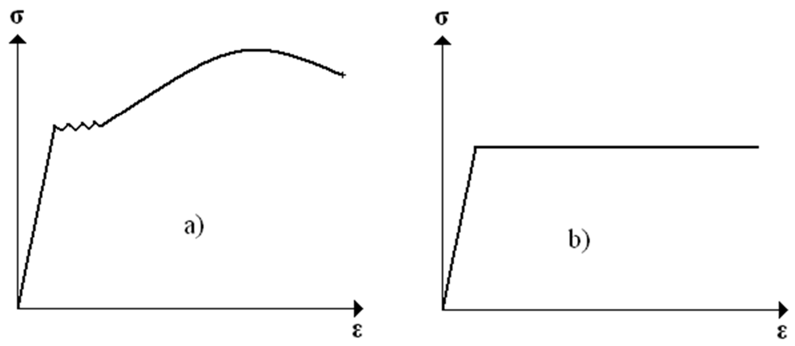


Figure 2.1 *a) Principal stress-strain relationship for hot rolled reinforcement steel. b) Simplified and idealized stress-strain relationship reinforcement for the model.*

2.2 Geometric uncertainty

The geometric uncertainty is the deviation of de facto to the nominal geometry. More specifically, the geometric precision of the execution cannot be one hundred percent accurate at the construction site.

Geometrical uncertainty can for example be the "variation" of the lever arm of the reinforcement in a concrete structure which has a major influence of the structural resistance. This variation can for slender columns become more important for short columns which results in a larger geometrical uncertainty.

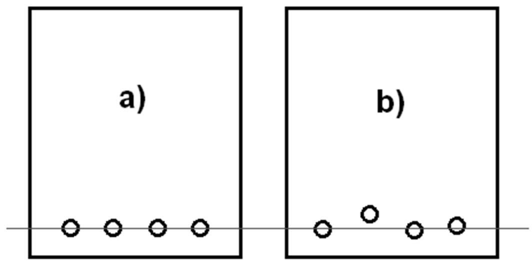


Figure 2.2 *Illustration of the variation of the reinforcement arrangement. a) Nominal geometrics b) De facto geometrics*

2.3 Material uncertainty

All material properties are more or less random. Some are insignificant and can be considered as deterministic without causing a relevant error; others have large influence and must be recognized as stochastic. The properties of building materials such as reinforcement steel and concrete should be considered as stochastic. Steel in general has minor dispersion; it is the concrete which contributes most to the material uncertainty. The coefficient of variation for concrete is commonly estimated to ~15%, this will increase the variation of the structural response. In the case of just one stochastic material parameter it is trivial to derive V_f, but for structures with more than one stochastic parameter the procedure is contentious as different safety formats derive the material uncertainty in different ways.

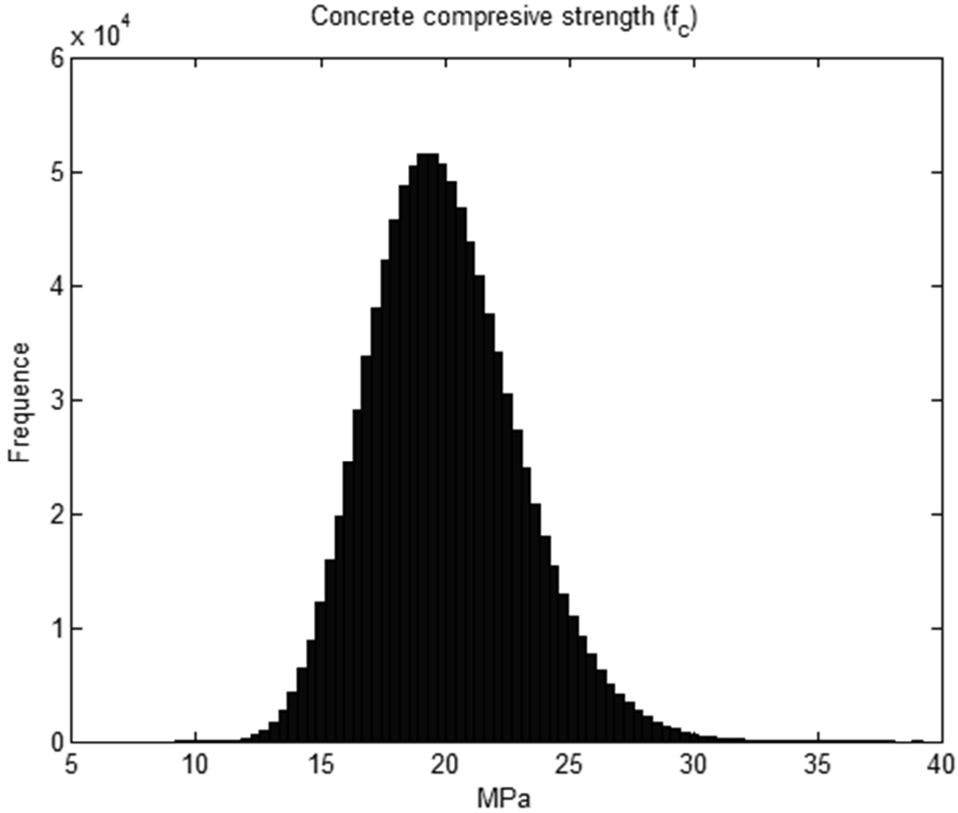


Figure 2.3 Example of the distribution of the concrete compressive strength f_c.

3 Safety formats

3.1 Structural reliability

The purpose of safety formats is to make sure that the structural resistance, *R*, is larger than the action effect, *E*, with enough safety margin to secure a safe structure. To ensure this safety, the resistance must be reduced while the unfavorable action effect must be magnified.

$$E_d \leq R_d \qquad \text{Reliability concept in ULS} \qquad (3.1)$$

Both the action effects and the structural resistance are actually distributed. These distributions should be separated far enough to make sure that the probability for collapse is sufficiently small. Almost all safety formats are based on this concept of probability. Both the action effects and resistances are separated with reliability index β together with sensitivity factors α_E and α_R. These factors are explained later in this chapter.

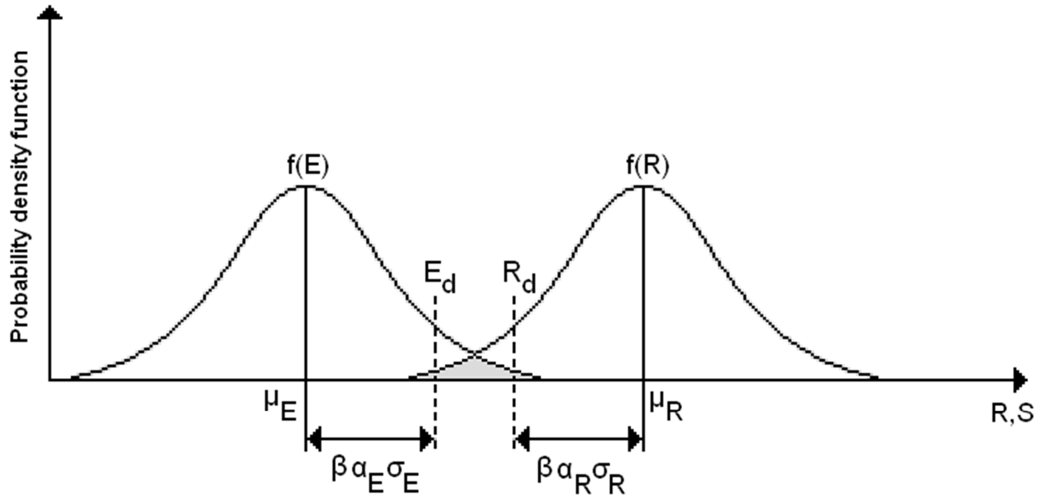


Figure 3.1 The fundamental principles of structural safety. The density function of action effects and resistances should be separated far enough.

In Figure 3.1, it can be seen that the two distributions are overlapping to some extent. This area is a qualitative measure of the probability of failure P_f. In terms of probability of failure this can be expressed as equation 3.2. The fact that there exists such an area indicates that the difference between *R* and *E* will in some cases be negative i.e. collapse of the structure, Haldar and Mahadevan (2000).

$$P_f = P(failure) = P(R < E) \qquad \text{Probability of failure} \qquad (3.2)$$

In order to evaluate the probability of the difference between *R* and *E* being negative the distribution between *R* and *E* should be considered. If *R* and *E* are normally distributed, the difference between *R* and *E* will also be normally distributed according to the following equations CEN (2002):

$$g = R - E \qquad (3.3)$$

g indicates a parameter associated

with *R-E* distribution.

$\mu_g = \mu_R - \mu_E$ (3.4) Mean value of the difference between mean structural resistance and mean action effect.

$\sigma_g = \sqrt{\sigma_R^{\ 2} + \sigma_E^{\ 2}}$ (3.5) Standard deviation of the R-E distribution.

$\beta = \dfrac{\mu_g}{\sigma_g} = \dfrac{\mu_R - \mu_E}{\sqrt{\sigma_R^{\ 2} + \sigma_E^{\ 2}}}$ (3.6) The standardized normal variable, reliability index β.

$p_f = \phi(-\beta)$ (3.7) Probability of failure p_f. Where $\phi(-\beta)$ is the cumulative area under the standardized normal density function from $-\infty$ to $-\beta$.

If a log-normal distribution is assumed instead the reliability index β will be; Madsen *et al.* (2006):

$$\beta = \frac{\mu \log_R - \mu \log_E}{\sqrt{\sigma^2_{\log R} + \sigma^2_{\log E}}} \tag{3.8}$$

The joint probability of *R* and *E* can be separated and visualized as a two dimensional case, see Figure 3.2 for illustration. The limit state equation is the boundary between safe and unsafe regions, hence, equation 3.3. The design point *P* is the closest point to the failure surface from the average resistance point in the normalized space. This is a practical approach if only the resistance or action effect is of interest.

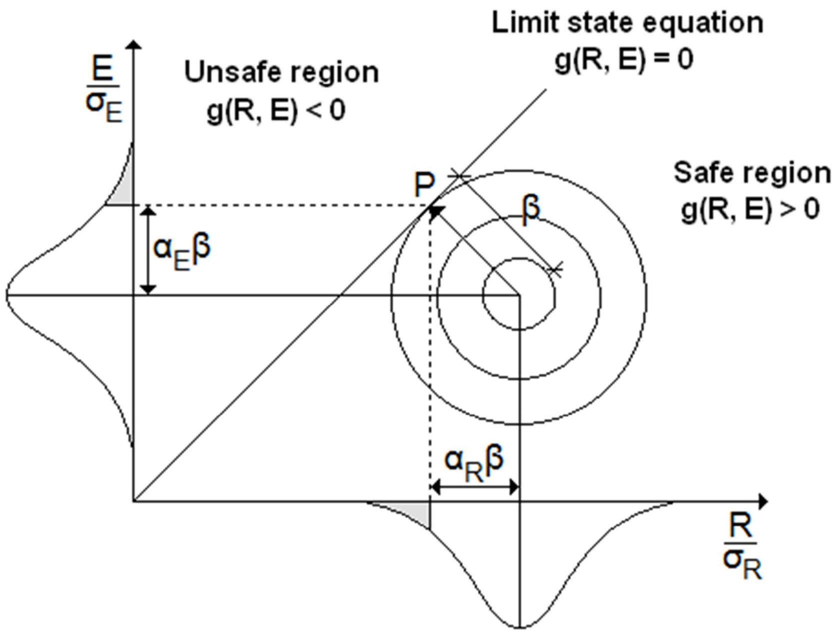


Figure 3.2 Modified from CEN (2002). P is the design point and the limit state equation is the failure boundary g=0 in the normalized space.

$$P(E < E_d) = \phi(\alpha_E \beta) \qquad \textit{Probability that } E_d \textit{ being higher than } E \qquad (3.9)$$

$$P(R > R_d) = \phi(\alpha_R \beta) \qquad \textit{Probability that } R_d \textit{ being lower than } R \qquad (3.10)$$

$$\sqrt{\alpha_E^2 + \alpha_R^2} \leq 1 \qquad (3.11)$$

α_E and α_R are sensitivity factors that separate the resistance and action effects. They are different from case to case and depend on the ratio between the variations of the resistance and action effect. Eurocode assumes a certain ratio which results in sensitivity factors equal to α_E = -0.7 and α_R = 0.8.

The acceptable risk of collapse has been decided to be 1 in a million per year for structures in Reliability Class 2 (RC2), see Table 3.2. Assuming that the life time of the structure is approximately 50 years then the acceptable risk of collapse is 50 in a million assuming design loads with return periods of 50 years, see Table 3.2. Eurocode states a reliability index, β, in the ultimate limit state of 3.8 with a reference period of 50 years for RC2, then the reliability index β_R becomes 3.04, see equation 3.12. CEN (2002)

Consequences Class	Description	Examples of buildings and civil engineering works
CC3	High consequence for loss of human life, or economic, social or environmental consequences very great.	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall).
CC2	Medium consequence for loss of human life, economic, social or environmental consequences considerable.	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building).
CC1	Low consequence for loss of human life, and economic, social or environmental consequences small or negligible.	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses.

Table 3.1 Definition of Consequences Classes according to EN 1990.

The Reliability Classes in table 3.2 may be associated with the Consequences Classes in table 3.1.

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Table 3.2 Recommended minimum values for reliability index β in ULS according to EN 1990.

$$P(R > R_d) = \phi(\alpha_R \beta) = \phi(0.8 \cdot 3.8) = \phi(3.04) \quad (3.12)$$

Hence, when only the design resistance is considered the target reliability index is 3.04 according to CEN (2002). In the context of Serviceability limit state (SLS) Eurocode applies a reliability index of 1.5 which gives a resulting reliability of the resistance of 1.20, see equation 3.13. These two are the targets for the reliability index for the probabilistic analyzes in this thesis.

$$P(R > R_d) = \phi(\alpha_R \beta) = \phi(0.8 \cdot 1.5) = \phi(1.20) \quad (3.13)$$

3.2 Reliability methods

3.2.1 Formulation of reliability problem

The purpose of reliability methods is to compute, or at least check if the probability of failure is small enough. A reliability problem is defined by limit state functions and their pertinent basic variables. A limit state function is a formulation of a desired minimum performance of the structure. Limit state functions may concern ultimate resistance, deflection at service load or any other structural response or features that can be computed and characterized by a number. The basic variables pertinent to a specific limit state function are the material, geometrical or load parameters that influence the structural response regarded. In reliability problems the interesting parameters are the basic variables with both significant influence and with dispersion great enough to affect the distribution of the structural response in question. In the following these basic variables will be referred to simply as parameters and the coordinate system where they are represented, as the parameter domain. In general the number of basic variables and limit state functions may range from one to infinity. If for example the ultimate capacity of the structure is evaluated for values across the parameter domain, the resulting surface is called the response surface with respect to ultimate capacity. In the following examples the ultimate structural capacity is the feature considered. If the desired minimum resistance of the structure is E_{lim}, this constitutes the limit state function. All points in the parameter domain correspond to a set of input parameters, the points rendering a structural resistance less than the desired capacity E_{lim} constitutes a sub domain representing the adverse state that does not meet the desired performance. The limit state equation is the line or surface depending on the dimension of the problem, where the difference between the resulting response and desired capacity is zero. The limit state equation is used to define the subdivision of the parameter domain into the adverse and desired states. The determination of the limit state equation is crucial since the probability of failure is equal to the integrated joint density function in the part of the domain corresponding to the adverse state. The general formulation of the reliability problem is given in equation 3.17 according to Waarts (2000).

Limit state function in the general case:

$$g(x_1, x_2, \ldots, x_n) = R(x_1, x_2, \ldots, x_n) - E_{\text{lim}}(x_{n+1}, x_2, \ldots, x_m) \tag{3.14}$$

Limit state function in the semi probabilistic case where E_{lim} is deterministic:

$$g(x_1, x_2, \ldots, x_n) = R(x_1, x_2, \ldots, x_n) - E_{\text{lim}} \tag{3.15}$$

Limit state equation is constituted by all points where:

$$(x_1, x_2, \ldots, x_n) = (x_1(g(x_1, x_2, \ldots, x_n) = 0), x_2(g(x_1, x_2, \ldots, x_n) = 0), \ldots, x_n(g(x_1, x_2, \ldots, x_n) = 0)) \tag{3.16}$$

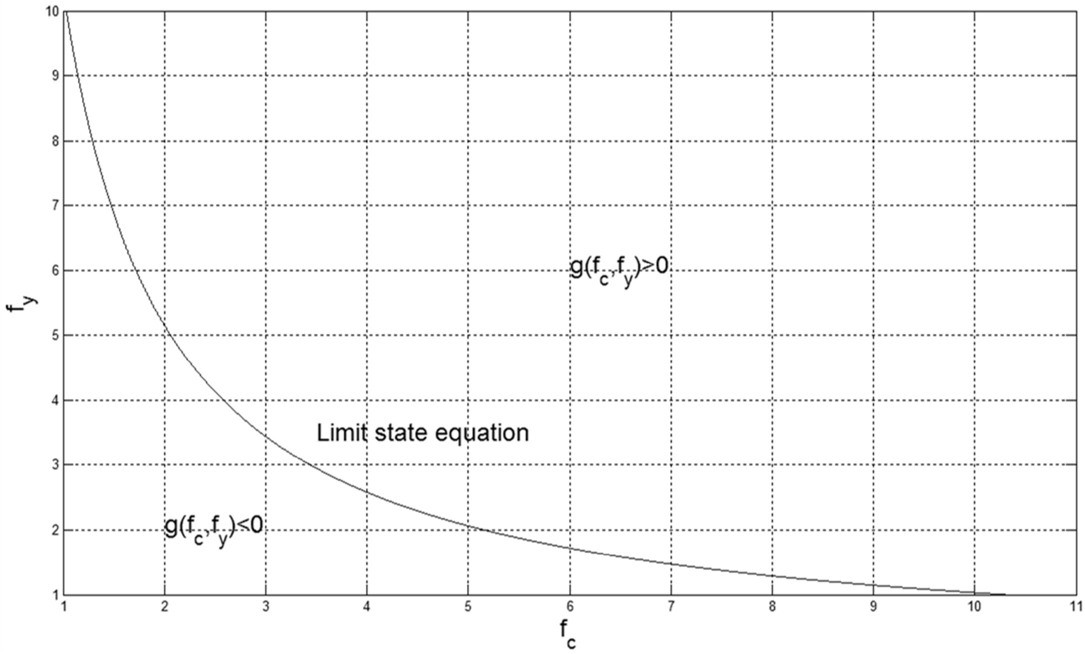


Figure 3.3 Concept of limit state in a case where f_c and f_y are the governing stochastic parameters.

General formulation of the failure probability:

$$p_f = \int_{x_1}\int_{x_2}\dots\int_{x_n} I(g(x_1, x_2, \dots, x_n)) f(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \qquad (3.17)$$

where

$$I(g(x_1, x_2, \dots, x_n)) = 1 \text{ if } g(x_1, x_2, \dots, x_n) \le 0$$

$$I(g(x_1, x_2, \dots, x_n)) = 0 \text{ if } g(x_1, x_2, \dots, x_n) > 0$$

where $f(x_1, x_2, \dots, x_n)$ is the parameter joint density function.

To evaluate the expression in equation 3.17 analytically is cumbersome if not impossible for high dimensional problems and is rarely done in real applications. It is mealy a definition of the failure probability rather than feasible method of computing it.

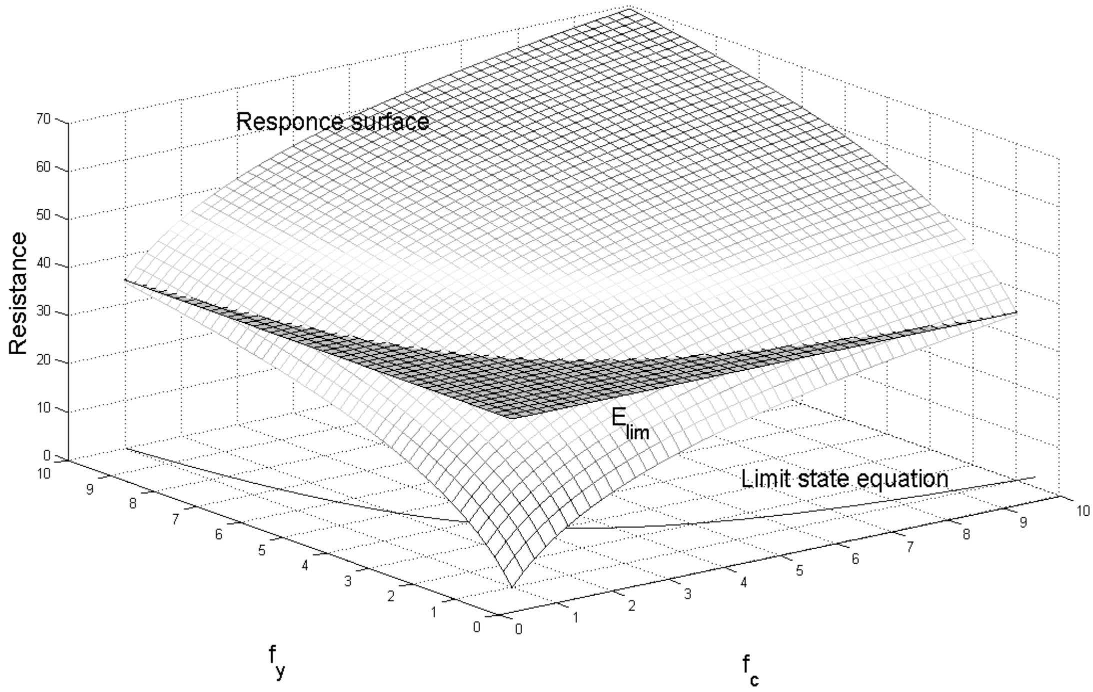


Figure 3.4a Response surface intercepted by limit condition E_{lim}. The corresponding limit state equation is plotted in the f_c-f_y plane.

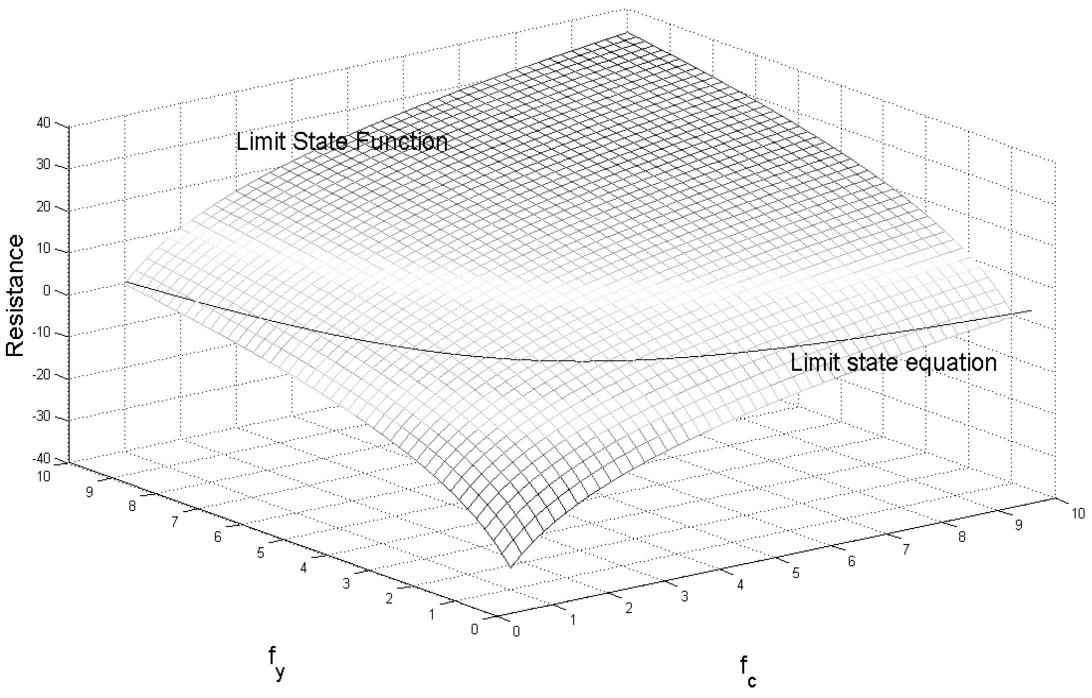


Figure 3.4b Limit State Equation according to equation 3.15 and the corresponding limit state equation is plotted in the f_c-f_y plane as in figure 3.4a.

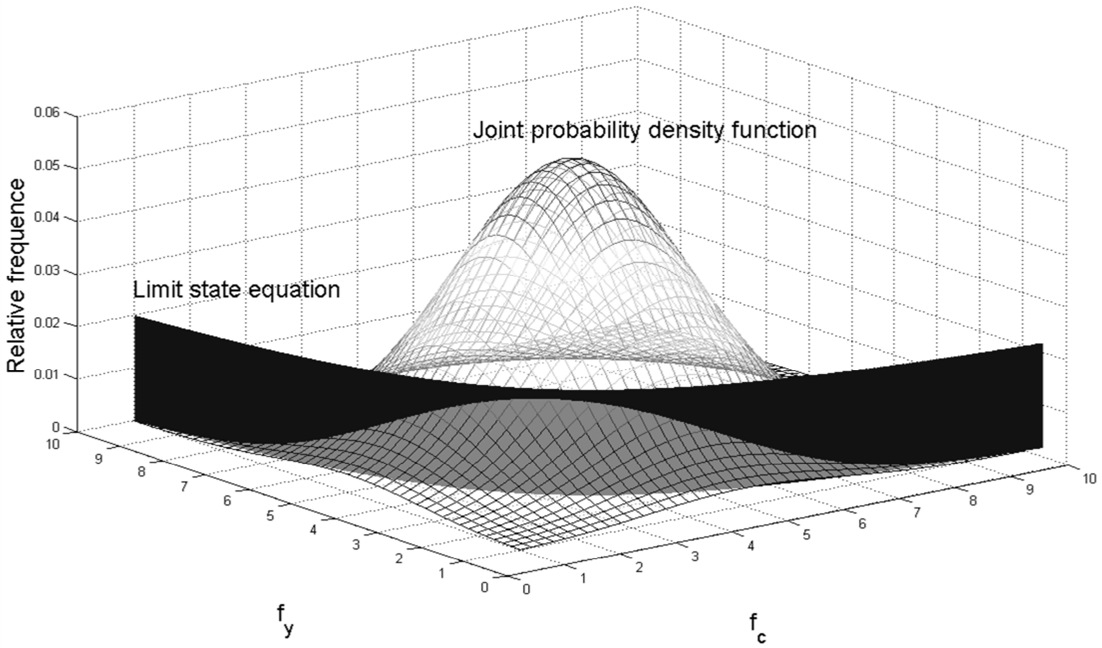


Figure 3.5 Joint probability density function intercepted by the limit state equation displaying the probability-mass of failing specimens.

There are several different ways of computing an approximate value of the failure probability using numerical methods. These methods are in accordance with JCSS categorized with respect to their accuracy into three different levels, level three being the one with highest accuracy. The basic simplifications characterizing each level are stated bellow JCSS (2001b):

- LEVEL I: Without actually computing the failure probability a level I method checks whether the load and resistance are separated far enough.
- LEVEL II: Computes the failure probability approximating the limit state equation with a fitted first or second order polynomial expression.
- LEVEL III: Computes the failure probability using a numerically determined limit state equation of general shape.

3.2.2 LEVEL III

3.2.2.1 Monte Carlo simulation

Monte Carlo simulation is considered to be the most accurate and straightforward reliability method. The derivation of the Monte Carlo method is not based on the general formulation, stated in equation 3.17, and hence the issue of the limit state equation accuracy is omitted. Instead the structure is evaluated for sets of input parameters that are sampled based on their respective distribution, forming the distribution of the structural resistance. As the number of evaluations is increased the relative number of analysis output values that are exceeded by E_{lim} will converge towards the true probability of failure. Advantageous compared to other numerical methods is that the only parameter governing the analysis itself, is the number of evaluated samples, when using other methods the convergence properties of the computed failure probability has to be checked against perhaps several different analysis settings. This scheme is illustrated in Figure 3.6.

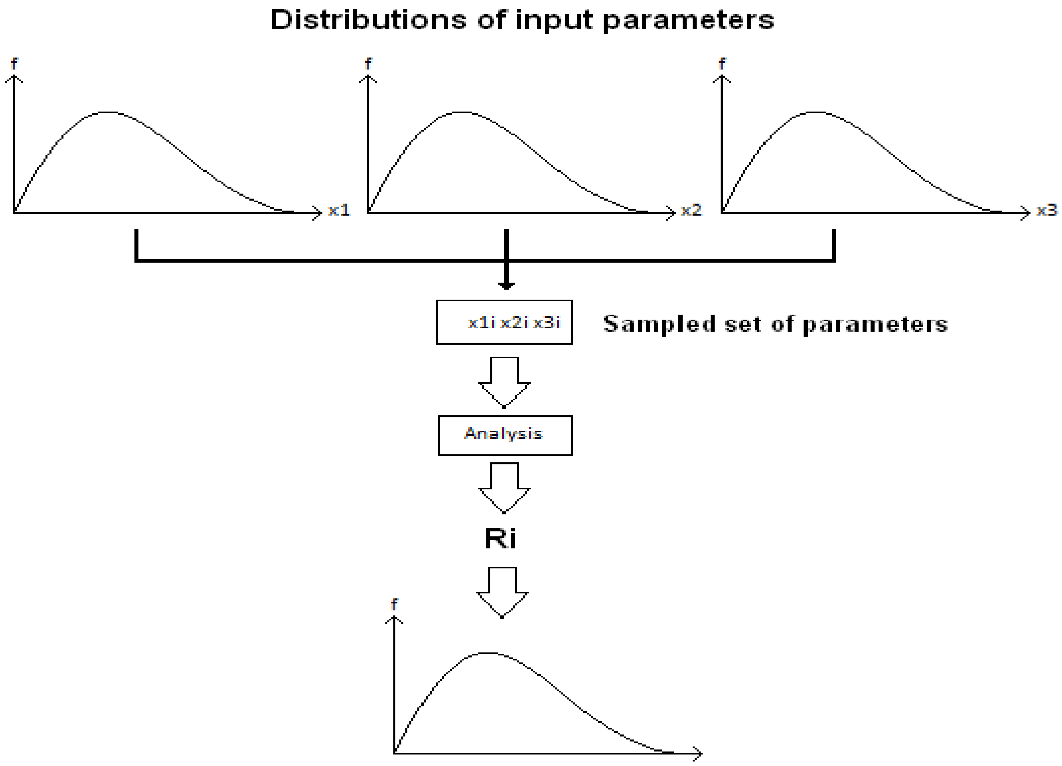


Figure 3.6 *Schematic description of Monte Carlo scheme.*

Once the distribution of the structural capacity is determined the failure probability can be computed as the relative number of samples where the resistance exceeded by the desired minimum performance, see Sorensen (2004)

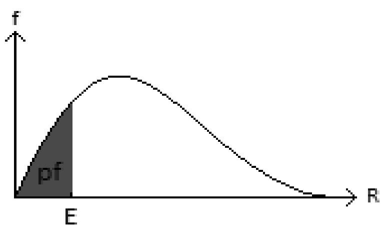


Figure 1.7 *Distribution of the structural resistance with limit and corresponding probability mass of failing specimens.*

3.2.2.2 Numerical Integration

In numerical integration the basis formulation of the reliability problem is approximated according to equation 3.18. Note that the parameters have been transformed from x-space into u-space, x-space and u-space being the non-normalized and the normalized spaces respectively. The range of summation from negative infinity to infinity has to be replaced with whatever range giving the proper convergence.

$$p_f = \int_{x_1}\int_{x_2}\ldots\int_{x_n} I(g(x_1, x_2, \ldots, x_n)) f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \rightarrow$$

$$p_f \approx \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\ldots\sum_{k=-\infty}^{\infty} I(g(u_1, u_2, \ldots, u_n)) f(u_1, u_2, \ldots, u_n) \Delta u_1 \Delta u_2 \ldots \Delta u_n \tag{3.18}$$

where

$$I(g(u_1, u_2, \ldots, u_n)) = 1 \text{ if } g(u_1, u_2, \ldots, u_n) \leq 0$$

$$I(g(u_1, u_2, \ldots, u_n)) = 0 \text{ if } g(u_1, u_2, \ldots, u_n) > 0$$

Since the limit state function has to be evaluated in all elements of size $\Delta u_1 \cdot \Delta u_2 \cdot \ldots \cdot \Delta u_n$ the determination of the adverse and desired sub domains is in no way systematized but instead performed all across the parameter domain and not focused in the area close to the limit state equation. Since the check is of fail pass character it is only interesting to establish the limit state equation separating the two states. Waarts (2000)

3.2.2.3 Directional Sampling

Instead of evaluating the structural resistance for vast number of values across the input domain, the effort is focused on the area where $g(x_1, x_2, \ldots, x_n)$ is close to zero. This approach is more efficient as it is able to determine the limit state equation with higher accuracy when using the same number of evaluations as in Numerical Integration. The general procedure of directional sampling can be summarized in the following four steps.

- Transform the input parameters x-space into the normalized u-space
- Find $g(x_1, x_2, \ldots, x_n) \approx 0$ in different directions with the origin of the normalized parameter domain as starting point.
- Find $g(x_1, x_2, \ldots, x_n) = 0$ thru extensive testing in the area where $g(x_1, x_2, \ldots, x_n) \approx 0$
- Evaluate $p_f = \int_{x_1}\int_{x_2}\ldots\int_{x_n} I(g(x_1, x_2, \ldots, x_n)) f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n$.

Below two alternative ways of performing general procedure given in the four points are described. First standard directional sampling (DS) and also Directional Adaptive Response Surface Sampling (DARS), which is developed by Waarts (2000)

3.2.2.3.1 Standard Directional Sampling

In the Directional Sampling method all stochastic analysis parameters are transformed into the u-space. The origin of the u-space, which by definition corresponds to the point where all parameters are at their mean values, is used as starting point when the directions are sampled. In each randomly chosen direction a point is chosen. The structure is evaluated using the parameter values corresponding to the point chosen. If the resistance is greater than the considered limit (E_{lim}) this point is in the safe part of the domain and a new point is chosen further away from the origin. If the new set of input parameters renders a resistance exceeded by the limiting load, that point is in the part of the domain representing the adverse state. It also means that the limit state function is somewhere between the first and the second point. A third point is now chosen, this one is closer to the origin than the second one but further away than the first. This iteration is continued until the limit state equation is bracketed with sufficient accuracy in this direction. Each direction contributes to a more accurate estimation of the failure probability. The procedure is described in the following eleven steps, by Waarts (2000):

1. Sample a random direction in the normalized parameter domain.
2. Choose a point in this direction at some distance from the origin of the normalized domain.
3. Transform this set of normalized parameter values from u-space to x-space.
4. Evaluate the structural resistance for this set of parameters.
5. Evaluate $g(x_1, x_2, \ldots, x_n)$.
6. If $g(x_1, x_2, \ldots, x_n)$ is less than zero, pick a new point closer to the origin and vice versa.
7. Start over from step (2) until the limit state function is bracketed with sufficient accuracy.
8. When the distance from origin to the limit state function (λ_i) is determined in the sampled direction, evaluate the sample value $P_i = 1 - \chi_i^2(\lambda_i^2)$ and start over from step (1) until a sufficient number of samples are performed. χ_i^2 is the squared density function in the direction of the sample.
9. Estimate the probability of failure $P_f = \frac{1}{N}\sum_{i=1}^{N} P_i$
10. Estimate the standard deviation of the failure probability $\sigma^2(P_f) = \frac{1}{N(N-1)}\sum_{i=1}^{N}\left(P_i - E(P_f)^2\right)$
11. Form a confidence interval assuming normal distribution: $E(P_f) - z\sigma(P_f) < P_f < E(P_f) + z\sigma(P_f)$

3.2.2.3.2 Directional Adaptive Response Surface Sampling

The Directional Adaptive Response Surface Sampling, DARS method, is a further development of DS method, made to reduce the calculation time. First the structure is evaluated for parameter values in the axis directions. This procedure is called Axis directional integration (ADI). Once the limit state function is found in these directions a quadratic response surface is approximated. Just as in the regular directional sampling different directions are sampled, but this time directions with higher influence on the computed probability of failure are given more importance. That is, the determination of the limit state function is focused in areas where it is close to the origin. This is sensible since the computed probability of failure is more sensitive to changes of the limit state equation in areas where the parameter joint density function assumes higher values. The information about the response surface gained during the procedure is added to the one from the initial evaluation. The information from the initial evaluation of the response surface in the axis direction is used when the importance sampling is made. Also information gained when previously directions where considered. Once a direction is chosen and an initial guess is made, the procedures are identical to standard directional sampling. Waarts (2000)

1. Find $g(x_1, x_2, \ldots, x_n) = 0$ in the directions of the parameters axis.
2. Sample directions where directions where the limit state equation is chose to the u-space origin using information from (1) and possibly from evaluation of other directions.
3. Choose a point in this direction at distance from the origin based on the information gained in (1) and possibly from evaluation of other directions.
4. Transform this set of normalized parameter values from u-space to x-space.
5. Evaluate the structural resistance for this set of parameters.
6. Evaluate $g(x_1, x_2, \ldots, x_n)$.
7. If $g(x_1, x_2, \ldots, x_n)$ is less than zero, pick a new point closer to the origin and vice versa.
8. Start over from step (2) until the limit state function is bracketed with sufficient accuracy.
9. When the distance from origin to the limit state function (λ_i) is determined in the sampled direction ϕ, evaluate the sample value $P_i = \frac{1 - \chi_i^2(\lambda_i^2)}{h(\phi)}$ where $h(\phi)$ is the importance given the angel ϕ, then start over from step (1) until a sufficient number of samples are performed.
10. Estimate the probability of failure $P_f = \frac{1}{N}\sum_{i=1}^{N} P_i$
11. Estimate the standard deviation of the failure probability $\sigma^2(P_f) = \frac{1}{N(N-1)}\sum_{i=1}^{N}(P_i - E(P_f))^2$
12. Form a confidence interval: $E(P_f) - z\sigma(P_f) < P_f < E(P_f) + z\sigma(P_f)$

One disadvantage of both DS and DARS compared to a Monte Carlo simulation is that the full distribution of the structural resistance is not obtained. The Monte Carlo simulation demands more computational effort but it renders the distribution of the structural resistance which can be used to quickly compute the failure probability for any given limit state. Figures 3.8, 3.9 and 3.10 illustrate different approaches in the determination of the limit state equation. Figure 3.8 shows how the limit sate evaluations are spread across the domain in the case of Numerical Integration and Monte Carlo simulation. It is quite clear that many of the limit state evaluations do not add information about the limit state equation. In Figure 3.9 it is shown how the DS method evaluates the limit state function in randomly chosen directions and through iterations in manages to focus the effort in the area close to the limit state equation. Figure 3.10 shows the DARS sampling scheme which focuses the evaluations close to the limit state equation the same way DS does but also gives higher importance to directions in which the limit state equation is closer to the u-space origin.

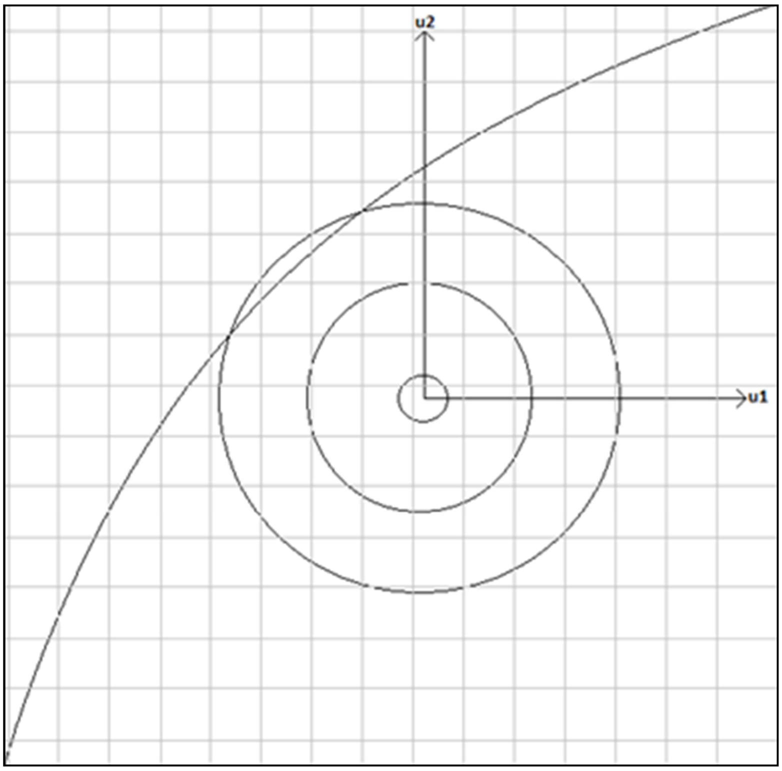


Figure 3.8 *The grid of grey lines illustrates how the structural response is evaluated across the parameter domain.*

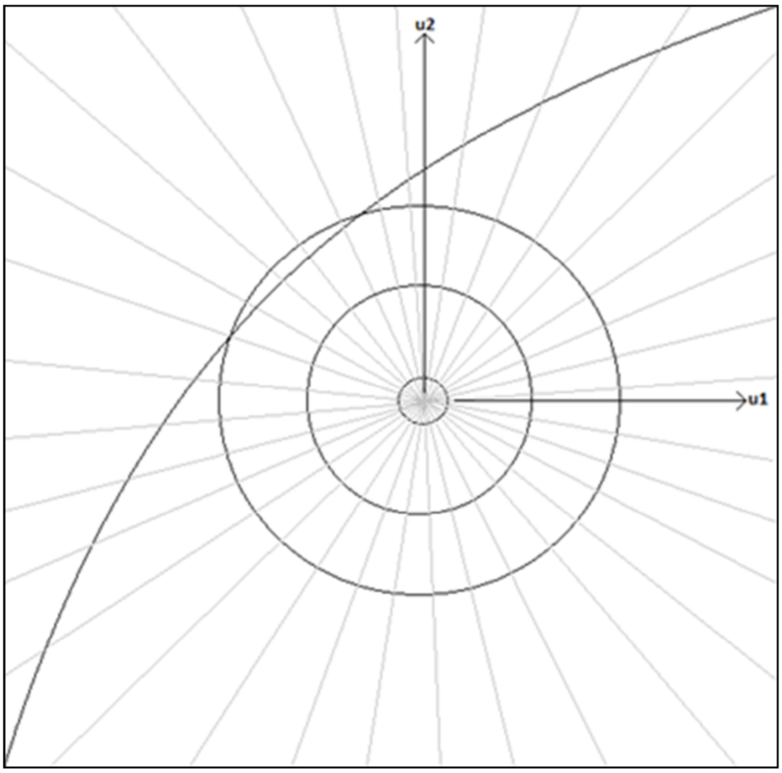


Figure 3.9 Illustration of how the DS method chooses random direction in which the distance from the origin to the limit state equation is determined.

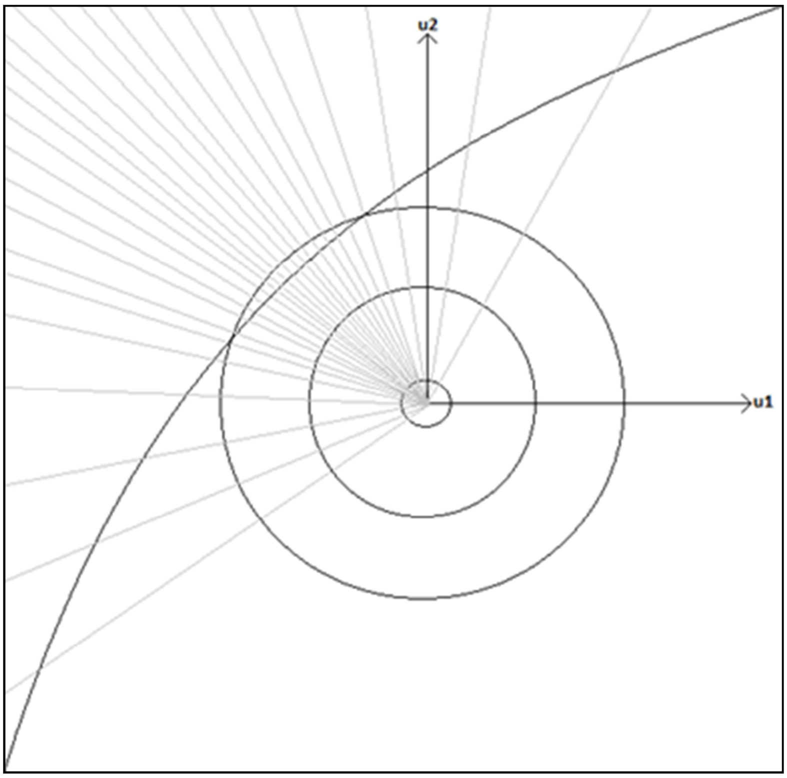


Figure 3.10 Illustration of how the DARS method chooses directions of higher importance in which the distance from the origin to the limit state equation is determined.

3.2.3 LEVEL II

3.2.3.1 FORM

First order reliability methods use the first order Taylor expansion to approximate the limit state function. Here a FORM method called 'The mean value first order' is described.

Limit state function evaluated at mean values of the parameters.

$$\mu_g = g\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right) \tag{3.19}$$

In the case of uncorrelated variables the variance can be computed according to equation 3.20.

$$\sigma_g^2 = \sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i}\right)^2 \sigma_{x_i}^2 \tag{3.20}$$

Once the mean value and standard deviation is determined the probability of failure can be computed as the mass of the distribution less than zero. The point in the parameter domain closest to the point $\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right)$ is called the design point. The distance perpendicular to the limit state function between the design point and $\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right)$ is equal to the reliability index β. If the performance of the structure is a linear function of the parameters the limit state function and limit state equation will be linear and FORM is in that case correct and not an approximation. The FORM method will also perform well if joint probability density function decays rapidly in the direction parallel to the limit state equation, Haldar and Mahadevan (2000).

3.2.3.2 SORM

In second order reliability methods a second order Taylor series is used to approximate the limit state function. The limit state function is approximated at the design point $\left(x_1^*, x_2^*, \ldots, x_n^*\right)$. The design point is found by minimizing the distance between the approximated limit state equation and the point $\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right)$ through iterative procedures.

$$g(x_1, x_2, \ldots, x_n) = g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{n}(x_i - x_i^*)\frac{\partial g}{\partial x_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_i^*)(x_j - x_j^*)\frac{\partial^2 g}{\partial x_i \partial x_j} \quad (3.21)$$

Given the beta value, which follows from the determination of the design point, and the principle curvatures of the limit state function at the design point an approximate value of the failure probability can be computed according to Breitung (1984) to

$$p_f \approx \Phi(-\beta)\prod_{i=1}^{n-1}(1+\beta\kappa_i)^{-\frac{1}{2}} \quad . \quad (3.22)$$

3.2.4 LEVEL I

Level I methods do not compute the probability of failure; instead they check that at least a target reliability is achieved. The target reliability is represented by its corresponding reliability index β. The circle with midpoint at the u-space origin and radius equal to β define a domain of parameter sets, if it can be verified that all of those points renders at least the desired performance, the reliability is at least equal to the target reliability. This is because the cumulative value of the joint probability function in the circle is equal to $\Phi(\beta)$. In the general case the limit state function has to be evaluated in all points on the domain boundary. If the limit state function assumes positive values in all boundary points the limit state equation cannot possibly enter the circle which means all points inside the circle renders sufficient performance. If the ratio between the two parameters is known the point on the boundary most likely to fail can be determined. This makes the method a lot more practical when only one point has to be checked. CEN (2004a) is based on the assumption that the ratio of load and resistance standard distribution is known. This makes it possible to determine weight factors (α-values) that describing the point in normalized load-resistance domain as seen in Figure 3.2, which has to be checked in order to verify that the reliability is at least the desired one.

3.3 Existing and proposed safety formats

For a long time the Eurocode dictated the use of Partial Safety Factors, CEN(2004a). It has been used for classic analytical design approach but also for non-linear approach. This format has later been developed further to improve the response when it comes to FE-analyses, CEN(2004b). Together with minor partial safety factors on the material parameters this format uses a resistance safety factor to obtain the design resistance after the structure has been analyzed. There have been other formats suggested for FE-analyses and one well known is the safety format by Cervenka *et al.* (2007) which is an Estimate Coefficient of Variation (ECOV) method. The proposed safety format by H.Schlune *et al* (2010) is also using the ECOV concept. These four safety formats will be explained in this chapter.

3.3.1 CEN(2004a) PSF method – partial safety factors

In the partial safety factor method the resistance can be estimated by using design values of the materials based on cylinder values reduced with safety factors. This method is commonly used for the classic analytical design approach. CEN(2004a)

Design resistance according to CEN(2004a):

$$R_d = R(f_{cd}, f_{yd}, ...) = R\left(\frac{f_{ck}}{\gamma_c}, \frac{f_{yk}}{\gamma_s}, ...\right) \quad (3.23)$$

where $\gamma_c = 1.50$ Partial safety factor for concrete

$\gamma_s = 1.15$ Partial safety factor for steel

R Resistance from FE software

If the distribution is assumed to be log-normal the partial safety factors are obtained from

$$\gamma_s = \exp\left(\alpha_R \beta V_R - 1.64 \cdot V_f\right) \quad \text{For steel} \quad (3.24)$$

Concrete has shown to give lower in-situ strength than the strength from specimen test according to CEN(2004a) and therefore an additional factor of 1.15 has been introduced for the concrete. The number 1.64 in equation 3.24 is a reliability index which represents the fifth percentile.

$$\gamma_c = 1.15 \cdot \exp\left(\alpha_R \beta V_R - 1.64 \cdot V_f\right) \quad \text{For concrete} \quad (3.25)$$

The partial safety factors are based on the uncertainties in the model V_m, geometric V_g and material uncertainties V_f. V_R is the resistance coefficient of variation, see equation 3.27 below. Action effects and resistance are assumed to be separated by different fixed sensitivity factors, α_R is the sensitivity factor for resistance reliability and β is the reliability index. These parameters are set to β=3.8 with a reference period of 50 year and α_R=0.8 according to CEN(2002).

This method is built on assumption that the resistance R and nominal resistance R_n have the relation:

$$R = \chi_m \chi_G \chi_f R_n \quad (3.26)$$

where χ_m Parameter corresponding to the model uncertainty

χ_G Parameter corresponding to the geometry uncertainty

χ_f Parameter corresponding to the material uncertainty

With this assumption the coefficient of variation of the resistance can be calculated as:

$$V_R = \sqrt{V_m^{\ 2} + V_G^{\ 2} + V_f^{\ 2}} \tag{3.27}$$

where V_m Coefficient of variation of the model uncertainty

V_G Coefficient of variation of the geometric uncertainty

V_f Coefficient of variation of the material uncertainty

These coefficients can be found in JCSS and depends on assumed variations of uncertainties that are calibrated against statistics. Inserting the values of these coefficients from Table 3.3 into equation 3.24 and 3.25 leads to the partial factors 1.15 and 1.5 for the steel and concrete parameters which are used in CEN(2004a).

Uncertainty	Steel	Concrete
Model V_m	2.5%	5%
Geometric V_G	5%	5%
Material V_f	4%	15%

Table 3.3 Coefficients of variation which lead to the partial factors according to CEN(2004a).

Using design values of materials in non-linear analyses give material parameters that are reduced and much lower than in reality. This can give unrealistic load distribution and deviations in the structural response, e.g. wrong failure mode. Therefore, it is inappropriate to use this safety format for non-linear analyses, Carlsson *et al.* (2008a).

3.3.2 CEN(2004b) safety format

Instead of using design material parameters as in CEN(2004a) to calculate the resistance, CEN(2004b) uses a resistance safety factor γ_R and material parameters which are modified to be close to mean values. These material parameters will give a structural response closer to the response using mean values i.e. more realistic response. After analyses, the design resistance can be given by reducing the obtained resistance with a resistance safety factor γ_R.

Design resistance CEN(2004b):

$$R_d = R(\tilde{f}_{ym}, \tilde{f}_{pm}, \tilde{f}_{cm}, ..., S) / \gamma_R \tag{3.28}$$

where

$\tilde{f}_{ym} = 1.1 f_{yk}$ Steel yield strength

$\tilde{f}_{pm} = 1.1 f_{pk}$ Prestressing steel strength

$\tilde{f}_{cm} = \gamma_{cf} f_{ck}$ Concrete compressive strength

$$\gamma_{cf} = 1.1 \cdot \alpha_{cc} \frac{\gamma_s}{\gamma_c} = 1.1 \cdot 1.0 \cdot \frac{1.15}{1.5} = 0.843$$

γ_s=1.15 and γ_c=1.5 which are partial safety factors for steel and concrete. The coefficient α_{cc} is taking into account for how the load is applied and long term effects.

When the limit deformation in steel is reached the resistance safety factor γ_R becomes; $1.15 \cdot 1.1 = 1.27$, when the limit deformation is reached in concrete; $1.5 \cdot 0.843 = 1.27$. This shows that it does not matter if the limit deformation is reached in steel or concrete, global safety coefficient is the same, γ_R=1.27.

By using a resistance safety factor to obtain the design resistance after the beam has been analyzed is a great improvement for non-linear analysis. The response of the analysis is better than CEN(2004a) and closer to the realistic structural response. The format does still not consider the importance of each material.

3.3.3 Cervenka *et al.* (2007) safety format

The Cervenka *et al.* (2007) safety format is a new proposed format developed by V. Cervenka, J. Cervenka and R. Pukl. This method referred to as an ECOV method, i.e. estimated coefficient of variation. Cervenka *et al.* (2007)

This method is based on a random distribution of resistance with the assumption that it is lognormal distributed. This distribution is described by the coefficient of variation, V_R. The coefficient is calculated from mean resistance R_m and characteristic resistance R_k, which are estimated by two separate non-linear analysis using mean and characteristic material parameters.

Assuming a log-normal distribution of the resistance, the coefficient of variation can be computed.

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right)$$ β 1.65 corresponds to 5^{th} and 50^{th} kvantile. (3.29)

$$R_m = R(f_m, ...)$$ R_m - The structural resistance evaluated for material parameters at their mean values. (3.30)

$$R_k = R(f_k, ...)$$ R_k – The structural resistance evaluated for material parameters at their characteristic values. (3.31)

The method uses a resistance safety factor, γ_R, which can be calculated according to equation 3.32. Here α_R is the sensitivity factor for resistance reliability and β is the reliability index. Cervenka *et al.* (2007) recommends to use β=4.7 and α_R=0.8 from EN.

$$\gamma_R = \exp(\alpha_R \beta V_R)$$ Global safety factor (3.32)

The design resistance can then be obtained by the mean resistance, R_m, and the resistance safety factor, γ_R:

$$R_d = \frac{R_m}{\gamma_R}$$ The design resistance (3.3)

This safety format can easily be used, only two analyses are required to estimate the coefficient of variation of the resistance and obtain the global safety factor. This format assumes that all uncertainties are captured by these two analyses and the major uncertainty groups are not introduced in an explicit manner. Cervenka et al. (2007) suggests that the model uncertainty can be covered elsewhere.

3.3.4 Proposed safety format by Schlune *et al* (2010)

As in the ECOV method by Cervenka *et al.* (2007) the idée is to estimate the coefficient of variation of the resistance by considering the resulting resistance when the structure is evaluated for input values which corresponding to different fractiles. Schlune *et al* (2010) format is adapted to fit Eurocode, only the material uncertainty is reconsidered otherwise it is identical.

Schlune *et al* (2010) format estimates the variation of structural resistance based on the distribution and importance of each material parameter. In other words the partial derivatives of the resistance with respect to all parameters with significance influence are used to weight the importance of each parameter.

If the material parameters are uncorrelated, the variance of the resistance can be calculated to

$$\sigma_{Rf} \approx \sqrt{\left(\frac{\partial R(x_1, x_2, \ldots, x_n)}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial R(x_1, x_2, \ldots, x_n)}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \ldots + \left(\frac{\partial R(x_1, x_2, \ldots, x_n)}{\partial x_n}\right)^2 \sigma_{x_n}^2} \tag{3.34}$$

where *n* is the number of material parameters, and the partial derivatives are evaluated numerically as:

$$\frac{\partial R(x_1, x_2, \ldots, x_n)}{\partial x_1} = \frac{R(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}) - R(\mu_{x_1} - \Delta x_1, \mu_{x_2}, \ldots, \mu_{x_n})}{\Delta x_1} \tag{3.35}$$

For log-normal material distributions Δx_i:

$$\Delta x_i = \mu_i \exp(\beta V_{x_i}) \tag{3.36}$$

In case of normal distributed parameters:

$$\Delta x_i = \mu_i - \beta \sigma_{x_i} \tag{3.37}$$

Number of analysis that has to be performed is equal to:

$$n_{analysis} = n_{distributed\ parameters} + 1 \tag{3.38}$$

The coefficient of variation of the material uncertainty of the resistance can then be calculated with:

$$V_{Rf} = \frac{\sigma_{Rf}}{R(\mu_{x_1}, \mu_{x_2}, \ldots \mu_{x_n})} \tag{3.39}$$

Schlune *et al* (2010) method for concrete

In the case of non-linear concrete structures the resistance concerning the material parameters depends on:

- f_c - Concrete compressive strength
- f_{ct} – Concrete tensile strength

- E_c – Concrete Young's modulus
- ε_{cu} – Concrete ultimate strain
- v_c – Poisons ratio for concrete
- G_F - Fracture energy
- f_{sy} – Reinforcement yield strength
- f_{su} – Reinforcement ultimate strength
- E_s – Reinforcement Young's modulus
- v_s – Poisons ratio for reinforcement
- ε_{su} – Reinforcement ultimate strain

Every material parameter that is added requires an additional analysis where the parameter in question is the only one being altered, see equation 3.34. To minimize the computational effort only the material parameters with importance should be selected. Some of the material parameters have very low dispersion and can therefore be set as deterministic with no effect on the resulting resistance. The influence of the material parameters are in some cases of less influence and importance, they can therefore also be set as deterministic.

In the case of bending failure the design resistance is governed by two material parameters, concrete compressive strength, f_c, and the reinforcement yield strength f_y. The distribution of other material parameters has a negligible influence and can be set deterministically.

The coefficient of the variation of the resistance then becomes:

$$\sigma_{\mathrm{Rf}} \approx \sqrt{\left(\frac{R(\mu_{fc},\mu_{fy}) - R(\mu_{fc}-\Delta fc,\mu_{fy})}{\Delta fc}\right)^2 \sigma_{fc}^2 + \left(\frac{R(\mu_{fc},\mu_{fy}) - R(\mu_{fc}y,\mu_{fy}-\Delta fy)}{\Delta fy}\right)^2 \sigma_{fy}^2} \tag{3.40}$$

When calculating Δf_c and Δf_y, Schlune *et al* (2010) suggests to use β 2.15 and log-normal distribution of the material parameters.

The coefficient of variation of the structural resistance can be computed in the same way as in Eurocode, see equation:

$$V_R = \sqrt{V_G^2 + V_m^2 + V_{Rf}^2} \tag{3.41}$$

where

$$V_{Rf} = \frac{\sigma_{Rf}}{R(\mu_{fy},\mu_{fc})}$$

4 Comparison of safety formats

The first step in the comparison of safety formats and verification with probabilistic analyzes is to make sure that the probabilistic analyses are sufficiently accurate. The software which was intended to be used for the probabilistic analyzes is called PROBAB. PROBAB is a module for the FE software DIANA, which is a not fully developed. In the first part of the chapter a linear elastic beam is used to test PROBAB; and later the safety formats are compared and verified with PROBAB in the case of a nonlinear concrete beam both in ULS and SLS and in the case of a nonlinear shear panel in ULS.

4.1 Testing of PROBAB on a linear elastic beam

4.1.1 Calibration of PROBAB parameters

To make sure that PROBAB delivers accurate result of the probability of failure P_f, a number of analysis parameters have to be calibrated. A Monte Carlo (MC) simulation is used as reference when comparing failure probability with PROBAB. Given that the number of samples used in the Monte Carlo simulation is high enough it will generate values of high accuracy. The structural feature considered in the limit state function is the deflection at mid span. MC results are based on analytical computation while PROBAB results are based on numerical methods. To make a fair comparison between the two probabilistic methods, the analytical and the numerical computation of the deflection must be equal.

To make sure that the PROBAB analysis is performed with sufficient accuracy, parameters concerning the analysis such as coefficient of variation of the probability of failure, *COV*, and the iteration convergence criteria, *conv*, are studied. Analysis methods DS and DARS are also studied. The evaluation of analysis settings are illustrated in Figure 4.1.

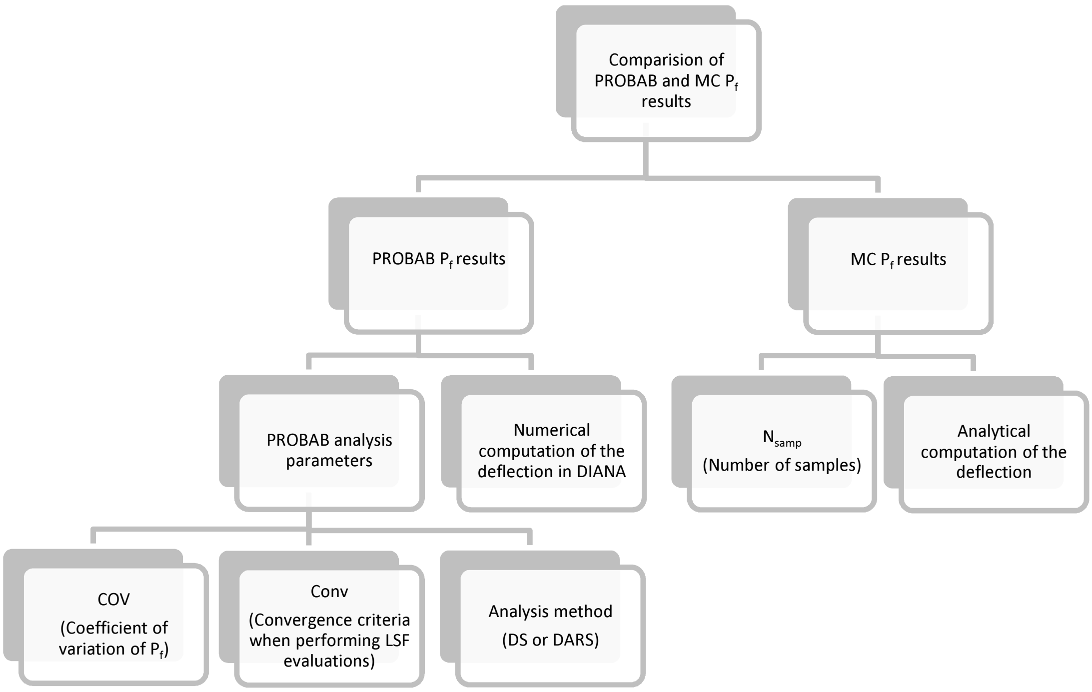


Figure 4.1 Dependency scheme over the different parameters the comparison is sensitive to.

4.1.2 Model description

The model used in the analysis is a linear-elastic simply supported steel beam, with a span of 4 meters. It is subjected to a point load at the middle of the span. Both Young's modulus, E_s and the load, P, are lognormal distributed, other parameters are set constant. This is a static structural 2-dimensional problem which does not depend on the number of elements. It is therefore enough to use only two beam elements.

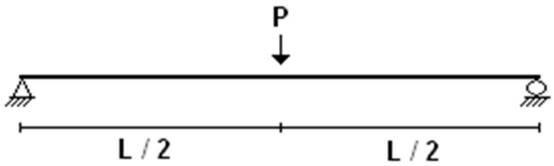


Figure 4.2 Beam model used in test of PROBAB.

Cross-section [mm^2]	100x100
Length [mm]	4000
Load P [kN]	Mean: 100; std: 15
Youngs modulus E_s [GPa]	Mean: 200; std: 10
Poison ratio	0.3
Element type	L7BEN (2D - Beam elements)
Number of elements	2

Table 4.1 Model and material parameters

4.1.3 Comparison between numerical and analytical deflections

The maximum deflection, u, can be calculated analytically as:

$$u_{max} = \frac{PL^3}{48EI} \quad (4.1)$$

Using mean values of E_s and the mean load P gives:

$$u_{analytical} = 0.080m \quad (4.2)$$

The deflection from numerical computation gives:

$$u_{DIANA} = 0.080 \mp 5 \times 10^{-4} m \quad (4.3)$$

The results show that the deflection is equal i.e. proving the two methods to be equal.

4.1.4 Settings for Monte Carlo simulation

A Monte Carlo simulation was done in MATLAB to calculate the probability of failure with the same limit state and material parameters as in PROBAB. Cases with higher values of deflection limit states and hence the lowest P_f requires 10 billion samples according to equation 4.4 where the *COV* is set to 0.01. The Monte Carlo simulations gives very reliable values for the probability of failure and therefore the MC is used as a reference in this analysis.

The number of samples for MC simulation can be calculated according to Waarts (2000) :

$$N_{samp} = \frac{1}{COV(P_f)^2}\left(\frac{1}{P_f} - 1\right) \qquad (4.4)$$

4.1.5 Sensitivity of P_f to COV and conv

The probability of failure, P_f, has been calculated for a number of analyses while changing the *conv* and *COV* for both DS and DARS method.

The limit state function is defined by the deflection of the beam. The limit state is deterministic and set to 100 mm while running all analyses, which is a bit over the mean deflection of the beam 80 mm. This is to only get results concerning the altered PROBAB parameters without taking the limit state into account. A limit state corresponding to lower reliability will increase the number of calculation before PROBAB reach results i.e. longer calculation time. Therefore the limit state is set to 100 mm and not higher.

As seen in a Figure 4.3 and 4.4 both DS and DARS method gives higher P_f than MC. The result is sensitive to both *conv* and *COV*. If *COV* and *conv* are decreased both DS and DARS results approaches MC values. As seen in Figure 4.3, the accuracy of DS is decreasing as *COV* is altered from 0.3 to 0.2 this indicates that the analysis is unreliable for higher values of *COV*, on the other hand large values of *COV* are not interesting in most cases.

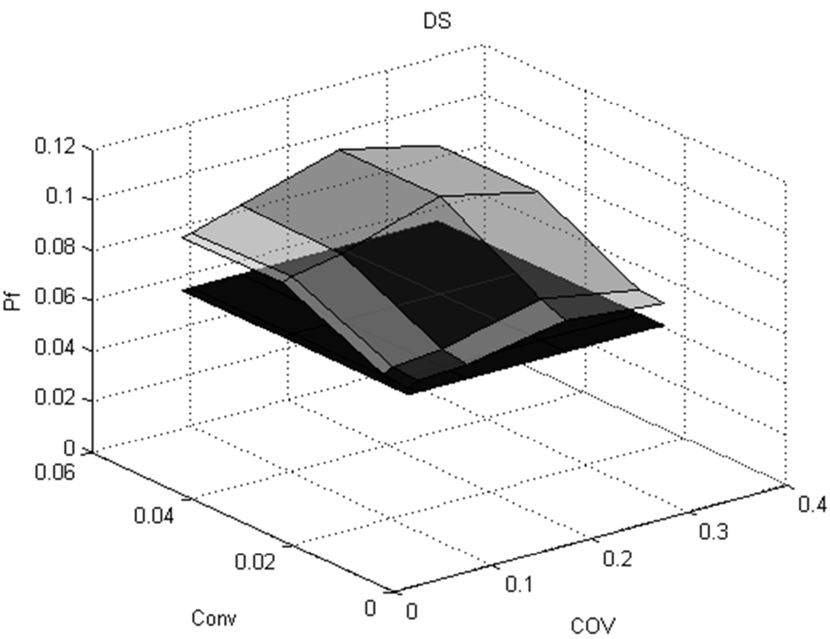


Figure 4.3 *P_f using DS Method. The blue surface is the result of the MC simulation.*

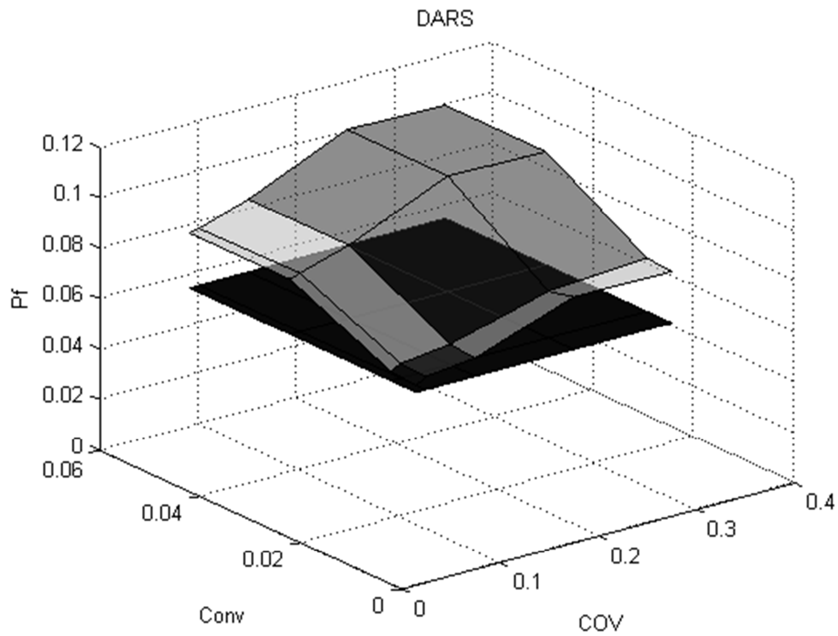


Figure 4.4 *P_f using DARS Method. The blue surface is the result of the MC simulation.*

For both DS and DARS method the *conv* seems to have converged at 0.01 while *COV* still tributes to a P_f difference. Figure 4.5 below takes a closer look at the development in *COV* direction at *conv* equal to 0.01. It can be seen that DARS method follows DS method very closely as *COV* decreases.

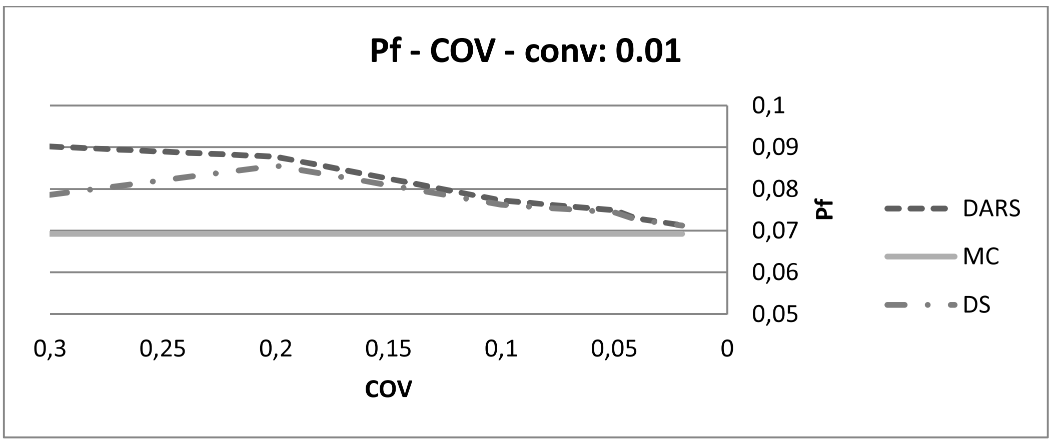


Figure 4.5 *P_f plotted against COV at conv: 0.01.*

The difference in P_f between DS and DARS method is illustrated in Figure 4.6. As can be seen, DS has better accuracy for all values of *conv* and *COV*. However, this difference is negligible for higher convergence demands. In Figure 4.7 and 4.8 the computation time for DS and DARS is plotted while changing *COV* values. DARS method is almost twice as efficient as DS method with almost the same accuracy at low *COV* and *conv* values and therefore the DARS method is preferred.

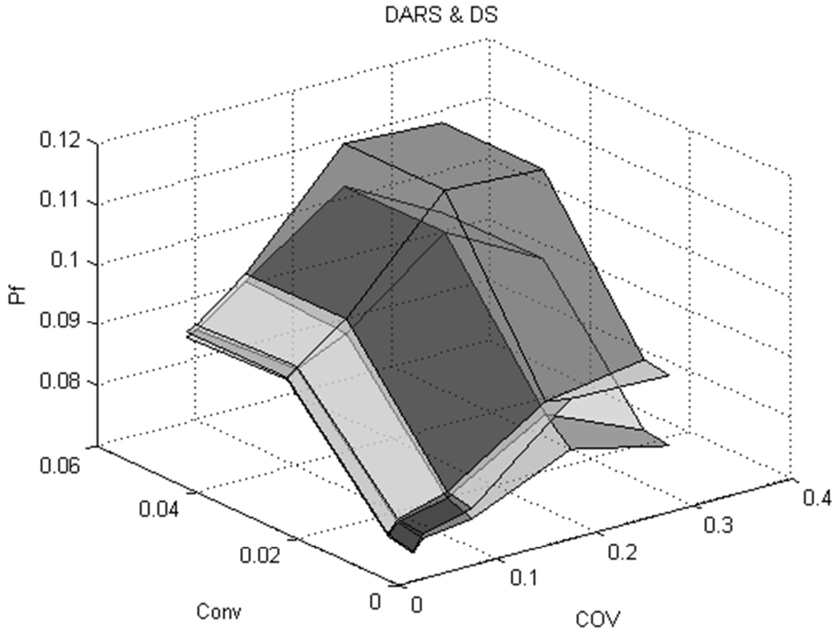


Figure 4.6 DARS & DS method. DARS is the upper surface and DS is the lower surface.

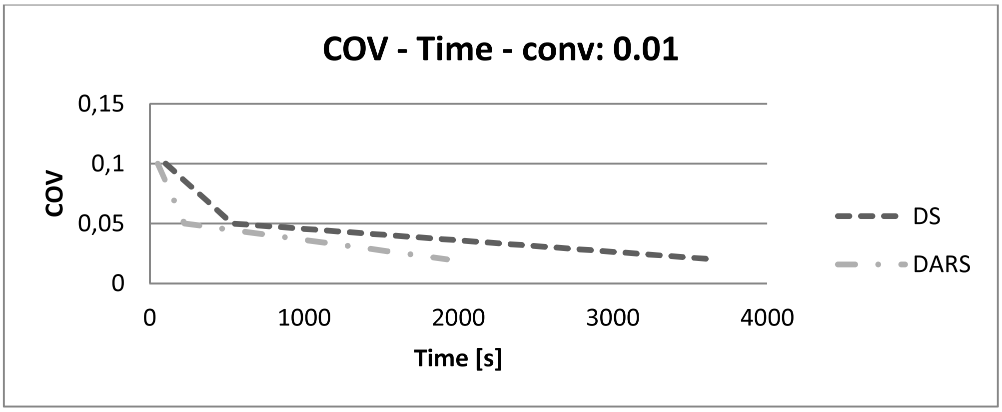


Figure 4.7 COV plotted against computation Time at conv 0.01.

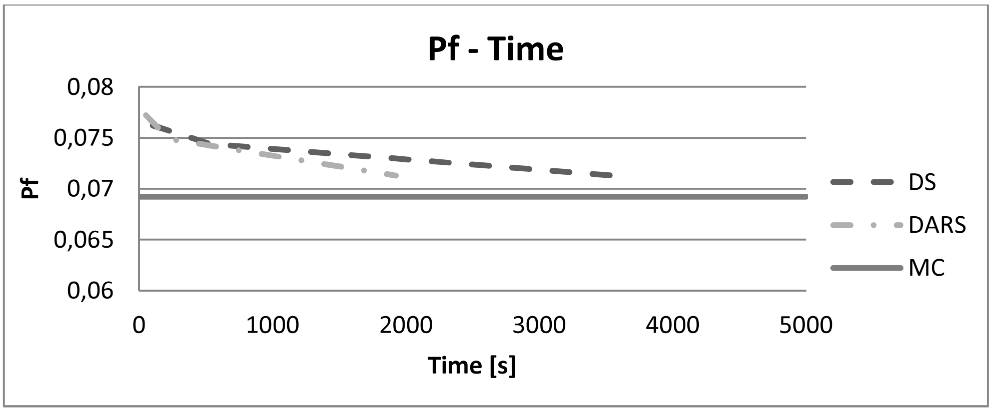


Figure 4.8 P_f *plotted against computation times corresponding to different COV values at conv 0.01.*

The relative error of DARS is shown in Figure 4.9. It can be seen that the P_f is converging with respect to *COV* values lower than 0.05. Whether the values of P_f are good enough or not are difficult to tell in absolute manner. However, the overall trend seems to be that the P_f value is converging in both *conv* and *COV* direction, definitely in *conv* direction. The behavior in *COV* direction is perhaps less convincing. The relative error is around 2.9 % for *COV* 0.02 with *conv* 0.01 which should not be neglected, see Figure 4.10. With even lower *COV* the P_f error is expected to be smaller but considering how much computation time this requires it is not feasible.

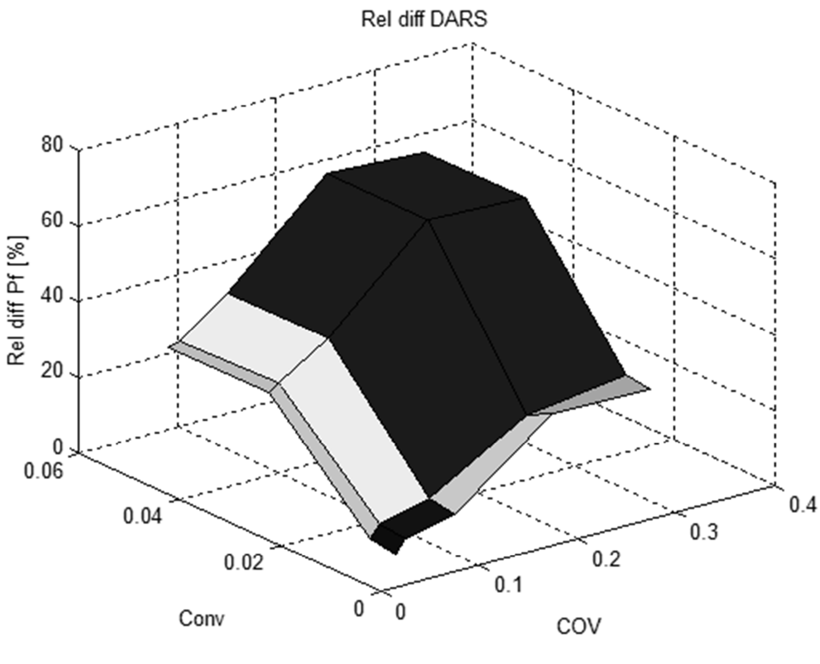


Figure 4.9 *Relative differences between DARS and MC.*

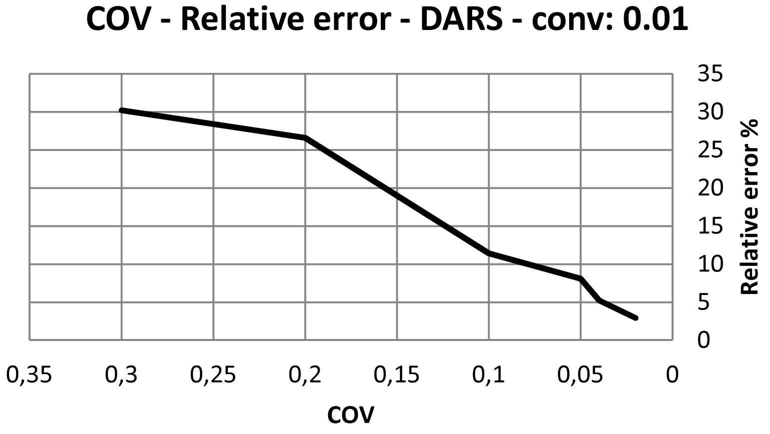


Figure 4.10 Relative error of DARS at conv 0.01

4.1.6 Sensitivity of P_f to limit state

Figure 4.11 shows that the P_f with PROBAB follows MC for different values limit states. The results form PROBAB are performed with *COV* 0.05 and *conv* 0.01. As can be seen in figure 4.12 the relative error of PROBAB results are in ranges from 22% to –8%. Concerning the deviation of the two results at different limit states, PROBAB should deliver less accurate results for limit states close to the mean deflection and for limit states extremely far away. MC is very accurate for limit states close to mean values and even less accurate for limit states further away where the P_f is significantly low. But having in mind that a certain relative error for a limit state further away corresponds to a smaller error in absolute values than the same relative error in a limit state closer to mean values. i.e. the relative error of 22% at limit state 170 mm corresponds to a absolute error of just 1.24E-07. For better results the values of *COV* and *conv* should be set lower.

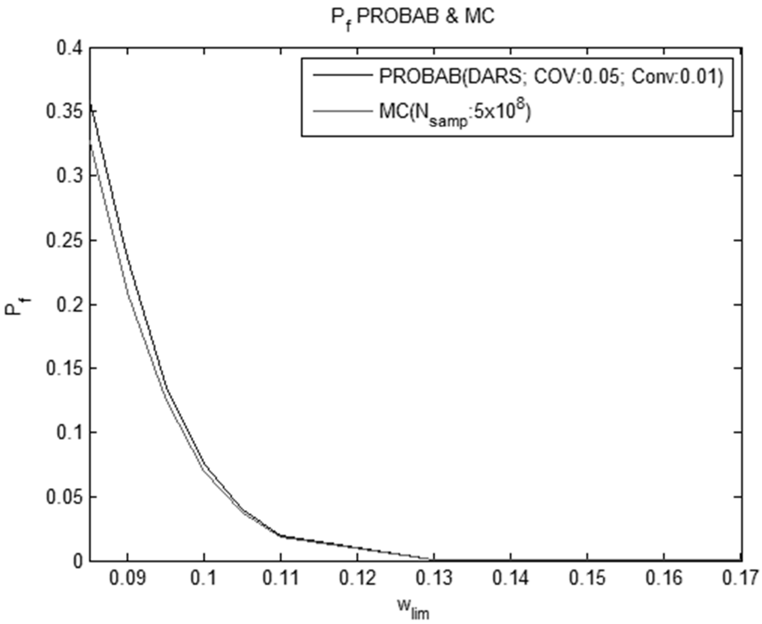


Figure 4.11 *P_f from PROBAB using DARS method with COV 0.05 and conv 0.01 and MC simulation with Nsamples: $10x10^9$, plotted against deflection limit.*

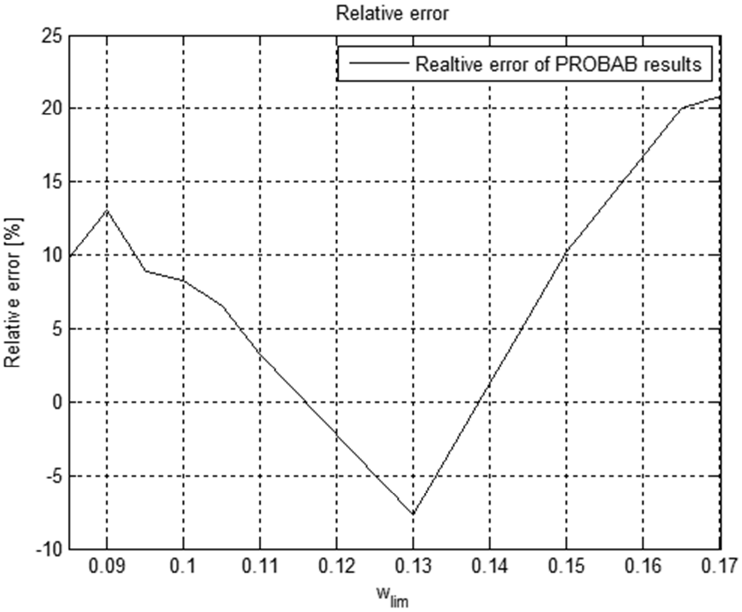


Figure 4.12 Relative error of P_f computed by PROBAB plotted against deflection limits.

4.1.7 Conclusion

This study of PROBAB parameters was made in order to gain understanding about how the results will be affected and how PROBAB should be used with reasonable settings for sufficient accuracy. This is very important for analyses that are more complex, for example non-linear concrete beams or even larger structures like whole bridges because the computation time will be tremendous. This analysis of a simple linear steel beam only takes approximately one second to perform one FE-analysis. This is of course not even close to how long a more complex FE-analysis will take. By choosing *COV* and *conv* with a better judgment the computation time can be reduced a lot, also reduced by choosing DARS instead of DS. It is important to have knowledge about how the PROBAB output is sensitive to parameters governing the analysis.

This report proposes a value of *conv* to 0.01. The value of *COV* is harder to propose but at least a value below 0.05 is needed for the relative error to be below 10%, *COV* 0.02 gives 2.9% relative error but this would increase the computation time a lot with reservations of this particular case not being of general validity.

4.2 Test of safety formats on a nonlinear concrete beam

To be able to compare the performance of the different safety formats they were tested on a nonlinear concrete beam. The test was done using both numerical and analytical methods. In the test a simple statically determined structure was used, i.e. simply supported reinforced concrete beam with a point load acting in the middle of the span, the beam is governed by failure mode of bending. The numerical analyses were made with FE software DIANA. The safety formats were also compared with a probabilistic analysis using PROBAB as a reference. The evaluation of different safety formats are carried out against both ULS and SLS criteria.

Note: The publications CEN(2004a) and CEN(2004b) referred to describes safety formats EN1992-1-1 and EN1992-2 respectively. In the figure and plot legends the name of the safety format is used instead of the publication.

4.2.1 Point-loaded concrete beam

The concrete beam is modeled using element type CL9BE which is a curved 3 node 2D beam element with embedded reinforcement. The geometry and specification of the beam can be seen in Table 4.2. For the numerical calculations a step size of 100 N has been chosen to ensure accurate results. It is important to reduce the computational time for the FE analysis when performing probabilistic analysis with PROBAB, therefore a convergence study on the element sizes was carried out to reduce the number of elements while ensuring sufficient accuracy and stability of the analysis. The built up with a fine mesh in the mid-span where the deformation hinge can be expected and coarse mesh in the rest of the beam. Boundary conditions and loads are applied in accordance with Figure 4.13.

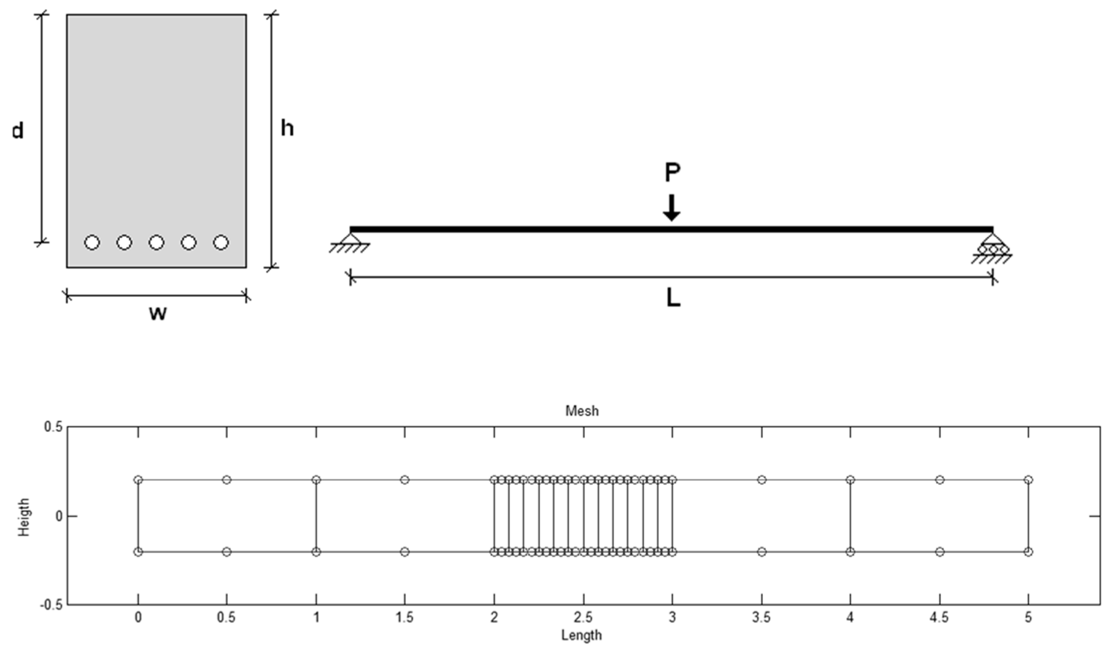


Figure 4.13 Point-loaded concrete beam model.

a) Cross-section b) Boundary conditions c) Mesh sizes

	E_{cm} [GPa]	f_{ctk} [MPa]	f_{ctm} [MPa]	f_{ck} [MPa]	f_{cm} [MPa]	GF [Nm/m^2]	CrackB [m]	Poison
Concrete 30/37 - cyl	33	2.0	2,9	30	38	75	0.200	0.2
Standard deviation σ	1.47	-	0.55	-	4.86	6.9	-	-
	E_s [GPa]	f_{syk} [MPa]	σ_{fy} [MPa]	Poison				
Reinforcement B500B	200	500	30	0,3				
	h [mm]	w [mm]	d [mm]	L [mm]	A_s [mm^2]			
Geometry	400	250	360	5000	1005			

Table 4.2 Properties of the analyzed concrete beam

4.2.2 Material parameters

Both CEN(2004a,b) safety formats consider the difference between concrete material parameters in-situ and the cylinder values. The difference is caused due to different curing conditions and results in a greater dispersion for in-situ test. CEN(2004a,b) considers this uncertainty through their partial safety factors and therefore cylinder values should be used as input. Hence also Schlune *et al* (2010) safety format should consider this uncertainty. This is done by converting the concrete material input parameters to in-situ values according to JCSS (2001a) and Thelandersson, Carlsson *et al.* (2008b). Cervenka *et al.* (2007) safety format uses cylinder values as input without any in-situ considerations.

In-situ material parameters are obtained using the following formulas:

The in-situ concrete strength $\mu_{fc,is}$ is related to the cylinder concrete strength $\mu_{fc,cyl}$; see to equation 4.5, where μ_κ is based on tests and set to 0.85 according to EN 13791.

$$\mu_{fc,is} = \mu_K \mu_{fc,cyl} \quad (4.5)$$

According to (2001a), κ is lognormal distributed with mean value μ_κ and with coefficient of variation V_κ = 0.06. The in-situ relation can then be described with equation 4.6.

$$V_{fc,is} = \sqrt{V_{fc,cyl}^2 + V_\kappa^2} \quad (4.6)$$

The mean value of the concrete tensile strength $\mu_{fct,is}$ and the mean characteristic modulus of elasticity in-situ $\mu_{Ec,is}$ can according to CEN(2004a) be calculated according to equation 4.7 and 4.8. Where $f_{ck,is}$ is the characteristic concrete compressive strength in-situ and $\mu_{fc,is}$ is the mean value of the concrete strength in-situ.

$$\mu_{fct,is} = 0.3 f_{ck,is}^{\frac{2}{3}} \tag{4.7}$$

$$\mu_{Ec,is} = 22000\left(\frac{\mu_{fc,is}}{10}\right)^{0.3} \tag{4.8}$$

Reinforcement material parameters, f_y and f_u, can according to (2001a) be approximated with lognormal distribution and a standard deviation of 30 MPa. E_s is considered to be constant. Reinforcement parameters are not affected by in-situ conditions.

		Analytical methods		Numerical Methods		Fractiles				
		EN 1992-1	EN 1992-2	EN 1992-1	EN 1992-2	Mean	Beta 1.65	Beta 2.15	Beta 3.04	Beta 3.76
	Beta	-	-	-	-	0	-1,65	-2,15	-3,04	-3,76
Safety factor	γ_c	1,5	0,843	1,5	0,843	-	-	-	-	-
	γ_s	1,15	1,1	1,15	1,1	-	-	-	-	-
	γ_R	-	1,27	-	1,27	-	-	-	-	-
Concrete 30/37 cylinder	f_c [MPa]	20	25,3	20	25,3	38	30	27,9	24,6	22,1
	Ec [GPa]	33	33	33	33	32,8	30,6	29,9	28,8	27,9
	fct [MPa]	-	-	1,33	1,69	2,9	2	1,78	1,46	1,24
	GF [Nm/m2	-	-	43 ,2	54,6	76,4	64,7	61,5	56,3	52,3
	Poison	-	-	0,2	0,2	0,2	0,2	0,2	0,2	0,2
Concrete 30/37 in-situ	fc,is [Mpa]	-	-	-	-	32,3	25	23,1	20,1	18
	Ec,is [MPa]	-	-	-	-	31,3	29	28,3	27,1	26,2
	fct,is [MPa	-	-	-	-	2,57	1,57	1,35	1,03	0,83
	GF,is [Nm/	-	-	-	-	68,2	57	53,9	48,9	45,2
Reinforcement B500B	Es [GPa]	200	200	200	200	200	200	200	200	200
	fsy normal	435	550	435	550	549	500	485	458	437
	fsy lognorn	-	-	-	-	552	500	485	460	440
	Poison	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3

Table 4.3 *Material parameters for various fractiles and methods.*

The resistance of the structure depends on the stochastic material parameters. In order to evaluate the sensitivity of the resistance of the structure with respect to material parameters with a significant stochastic distribution, a parameter study is performed. This is done through changing of the material parameters with stochastic distribution to values corresponding to certain fractiles while keeping other constant according to Table 4.4.

Material	Concrete				Steel
Parameter	$E_{c,is}$	$f_{c,is}$	$f_{ct,is}$	$GF_{,is}$	$f_{y,is}$
Concrete: mean ; Steel: Beta 1.65	mean	mean	mean	mean	β 1.65
Concrete: mean ; Steel; Beta 2.15	mean	mean	mean	mean	β 2.15
Concrete: mean ; Steel: Beta 3.04	mean	mean	mean	mean	β 3.04
Concrete: mean ; Steel: Beta 3.76	mean	mean	mean	mean	β 3.76
Steel: mean ; Concrete: Beta 1.65	β 1.65	β 1.65	β 1.65	β 1.65	Mean
Steel: mean ; Concrete: Beta 2.15	β 2.15	β 2.15	β 2.15	β 2.15	Mean
Steel: mean ; Concrete: Beta 3.04	β 3.04	β 3.04	β 3.04	β 3.04	Mean
Steel: mean ; Concrete: Beta 3.76	β 3.76	β 3.76	β 3.76	β 3.76	Mean
All: mean ; f_c: Beta 1.65	mean	β 1.65	mean	mean	Mean
All: mean ; f_c: Beta 2.15	mean	β 2.15	mean	mean	Mean
All: mean ; f_c: Beta 3.04	mean	β 3.04	mean	mean	Mean
All: mean ; f_c: Beta 3.76	mean	β 3.76	mean	mean	Mean

Figure 4.4 *Table of performed analyses and their input values. Note that in-situ values are used.*

The parameter study is conducted using values that are corresponding to certain β index. The β index chosen for the fractiles here are:

0.00 (mean)

1.65 (characteristic)

2.15 (Proposed value for Schlune *et al* (2010) safety format)

3.04

3.76

The mean value gives the most realistic response of the structure.

4.2.3 Comparison with only the material uncertainty

The difference between CEN(2004a,b) safety format and Schlune *et al* (2010) safety format is the way the material uncertainty is derived, geometric and model uncertainty is accounted for identically in the same way. To make a clear comparison between CEN(2004a,b) and Schlune *et al* (2010) one part of the comparison is focused on the material uncertainty X_f. The probabilistic analysis with PROBAB can therefore be simplified and much faster with only material parameters set as stochastic.

In the following part the material uncertainty was isolated from the resistance uncertainty.

4.2.3.1 CEN(2004a)

By only taking the material uncertainty into account when deriving the partial safety factors it is possible to calculate the design resistance with considerations of the material uncertainty only, see equation 4.9. The coefficient of variation of the material uncertainty for steel and concrete is 4% and 15%, respectively according to JCSS (2001b).

$$V_R = V_f \tag{4.9}$$

According to CEN(2004a) the partial safety factors can be calculated with equation 4.10, and 4.11. The partial safety factors when only considering the material uncertainty become:

For steel:

$$\gamma_s = \exp(\alpha_R \beta V_R - 1.64 \cdot V_f) = \exp(0.8 \cdot 3.8 \cdot 0.04 - 1.64 \cdot 0.04) = 1.058 \tag{4.10}$$

For concrete:

$$\gamma_c = 1.15 \cdot \exp(\alpha_R \beta V_R - 1.64 \cdot V_f) = 1.15 \exp(0.8 \cdot 3.8 \cdot 0.15 - 1.64 \cdot 0.15) = 1.419 \tag{4.11}$$

4.2.3.2 CEN(2004b)

CEN(2004b) is based on the partial safety factors in CEN(2004a). By using the safety factors from CEN(2004a) when only considering the material uncertainty, new material parameters and a new resistance safety factor can be calculated as:

$$\gamma_{cf} = 1.1 \cdot \alpha_{cc} \frac{\gamma_s}{\gamma_c} = 1.1 \cdot 1.0 \cdot \frac{1.058}{1.419} = 0.820 \tag{4.12}$$

When the limit deformation is reached in steel:

$$\gamma_R = \gamma_s \cdot 1.1 = 1.058 \cdot 1.1 = 1.163 \tag{4.13}$$

When the limit deformation is reached in concrete:

$$\gamma_R = \gamma_c \cdot \gamma_{cf} = 1.419 \cdot 0.820 = 1.163 \tag{4.14}$$

The resistance safety factor becomes identical for whichever limit deformation is reached.

4.2.3.3 Schlune *et al* (2010)

The safety format derives the material uncertainty mainly and adds both the geometrical and model uncertainty as parameters to the coefficient of variation of the resistance; this makes it easy to separate the material uncertainty and neglect the other uncertainties.

$$V_R = V_f \tag{4.15}$$

4.2.3.4 Cervenka *et al.* (2007)

Cervenka *et al.* (2007) safety format is calculating the coefficient of variation of the resistance according to equation 4.16. It is not possible to separate the material uncertainty or any uncertainty from Cervenka *et al.* (2007) method. The V_R is directly based on the analyses using mean and characteristic material parameters. Therefore this safety format cannot be compared with the other safety formats when only the material uncertainty is considered.

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \tag{4.16}$$

4.2.4 Evaluation of structural sensitivity to material parameters

The difference of structural resistance when the concrete in-situ strength and cylinder strength is used can be seen in Figure 4.14. The resistance when using in-situ values is lower than the one obtained when using cylinder values as expected. The difference is too great to be neglected and influence the structural resistance to some extent.

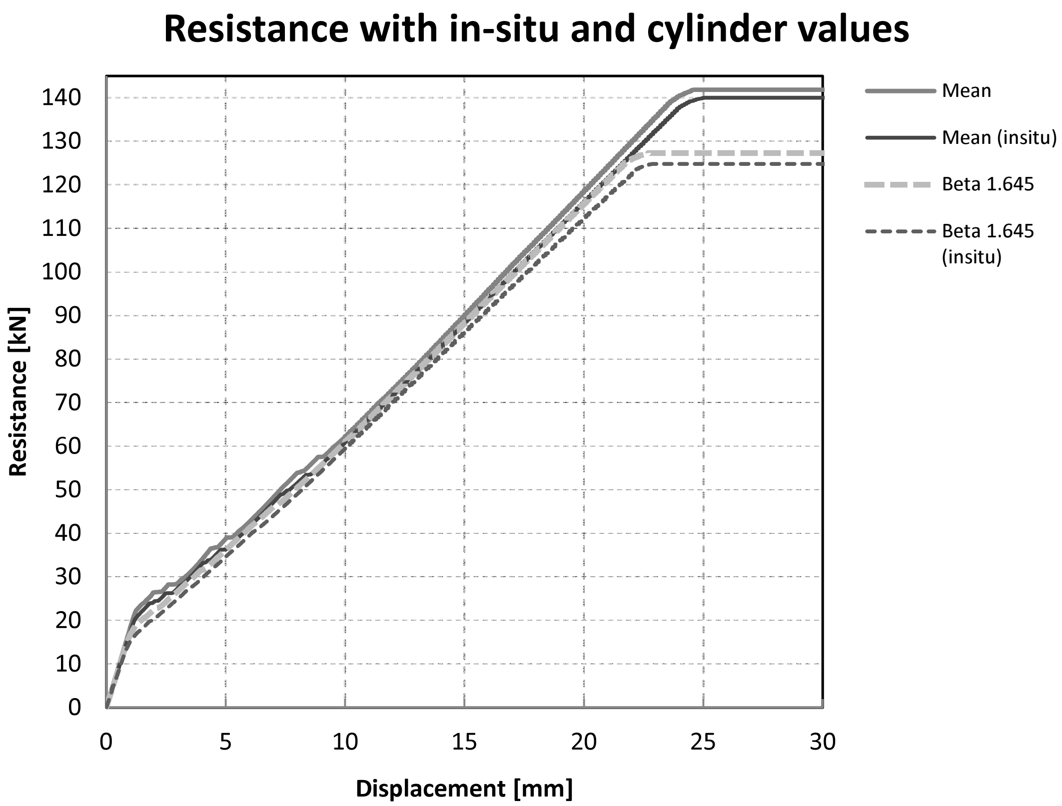


Figure 4.14 Comparison of the structural resistance with in-situ and cylinder values on the material parameters.

Figure 4.15 shows the response of the structural resistance when all material parameters are altered. The resistance is decreasing as the reliability index is increasing as expected.

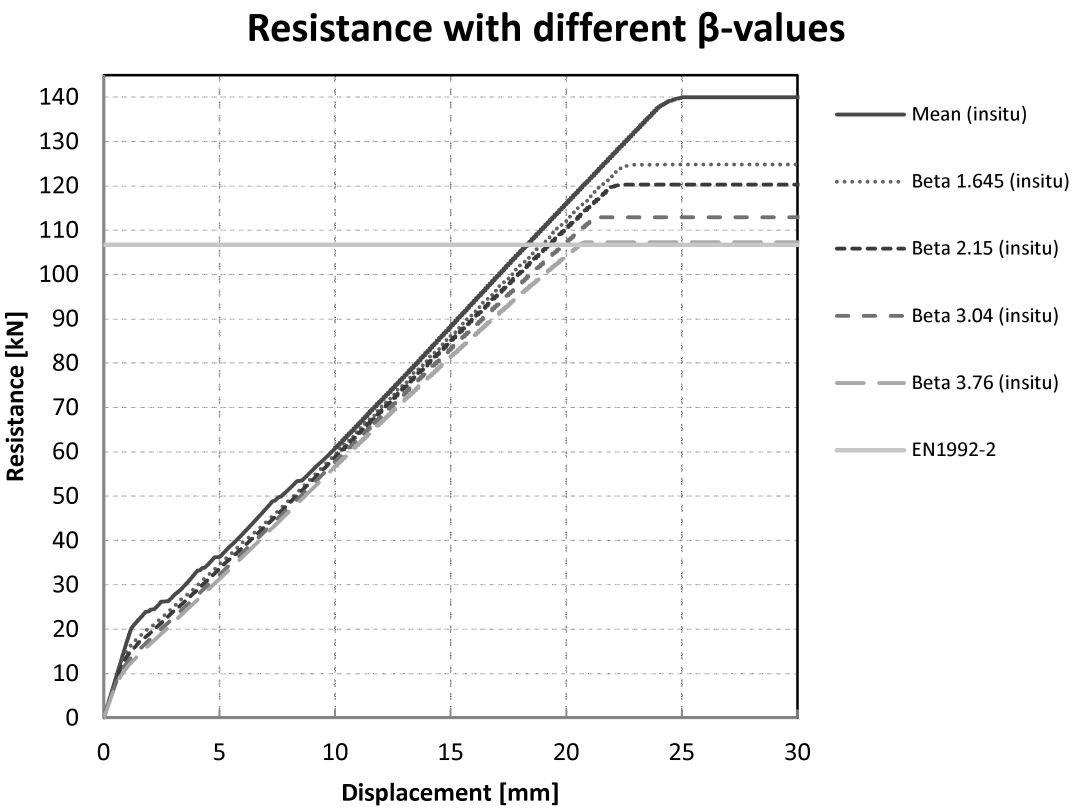


Figure 4.15 The structural resistance is given for material parameters corresponding to different Beta values.

Figure 4.16 shows the sensitivity of structural resistance against altering of f_y. The resistance of the structure decreases when changing the reinforcement strength corresponding to fractiles further away. The only stochastic steel material parameter is f_y, E_s is set constant, distribution of E_s is neglected. The stiffness of the beam is not affected by the steel strength and this can be seen as the response with all different fractiles is the same until yielding. The diagram clearly shows that the ultimate resistance is very much dependent on the reinforcement steel, i.e. the yield strength f_y.

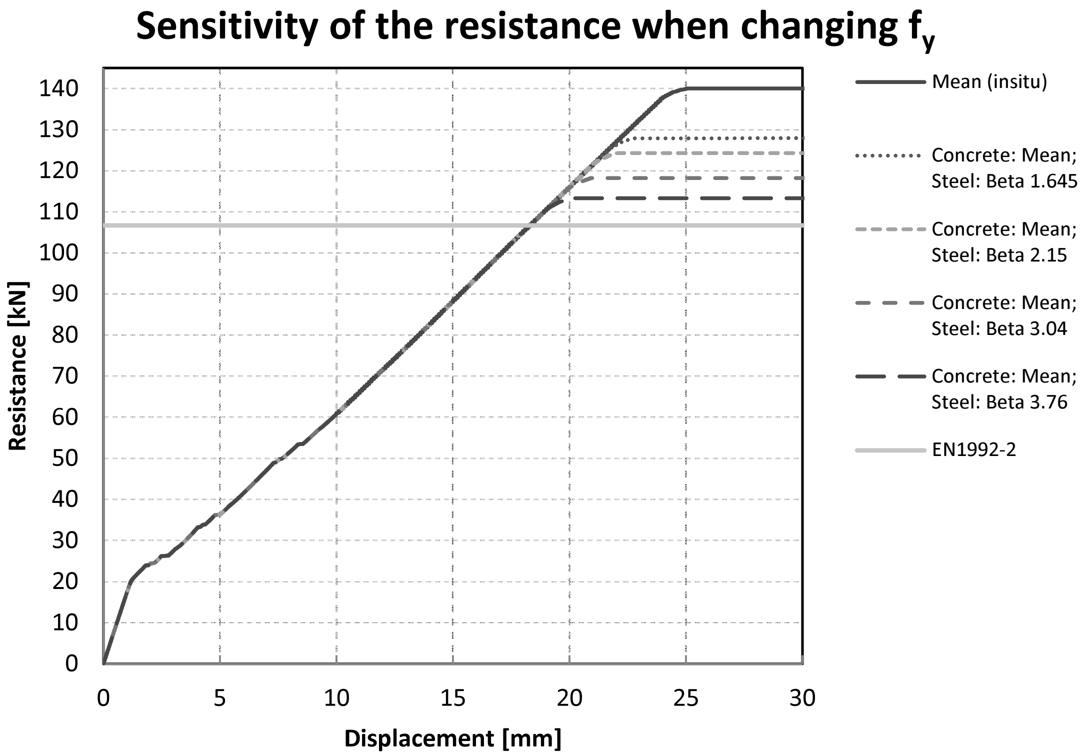


Figure 4.16 The structural resistance when the concrete parameters are held at mean with reinforcement parameters corresponding to different fractiles.

The result when looking at the sensitivity of the structural resistance when f_c is altered can be seen in Figure 4.17 and with a closer look in Figure 4.18, the figures show the dependence of the concrete when the reinforcement has been set to mean values. The result shows that the stiffness is deviating when using different concrete compressive strengths. As can be seen in Figure 4.17, according to the FE simulation the structure has the same ultimate resistance even if all concrete parameters are altered or if only the concrete compressive strength f_c is altered.

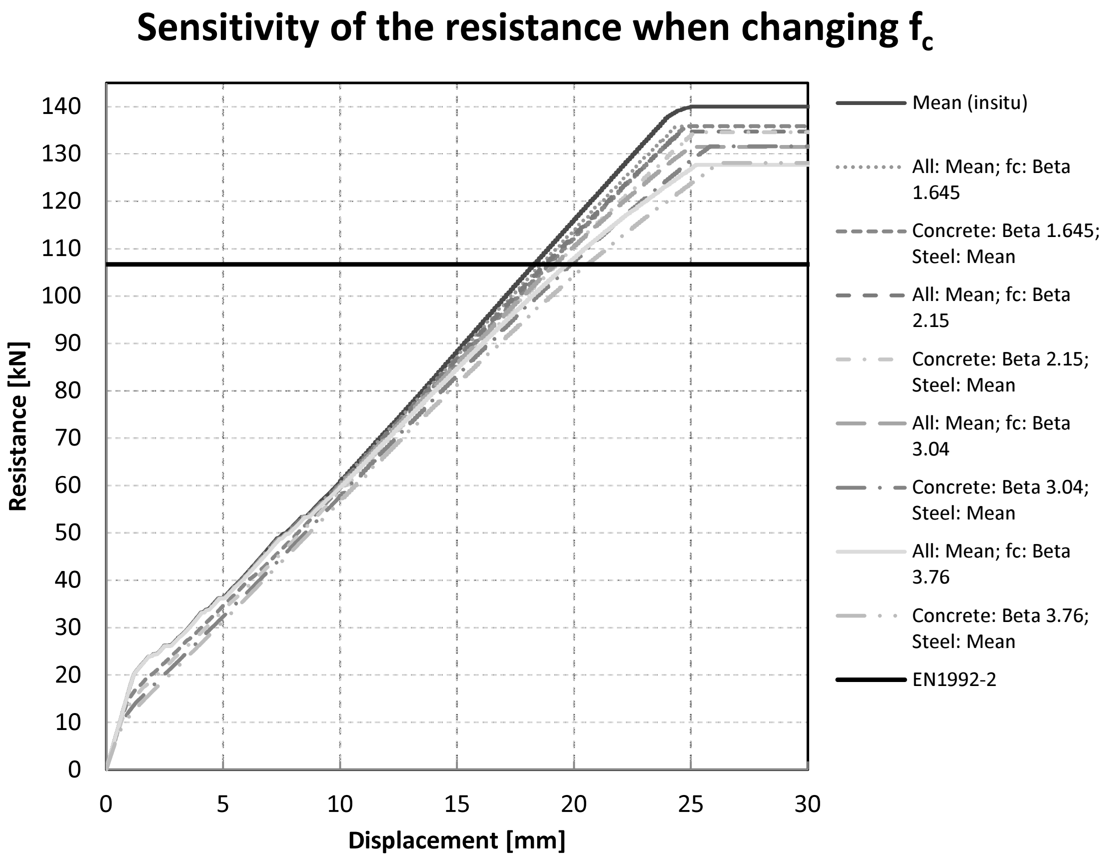


Figure 4.17 The structural response is compared when changing all the concrete compressive strength.

Studied carefully, it can be seen that the curves are in pairs of two for the same value of β. The change of resistance is governing by the concrete compressive strength f_c, other concrete parameters have minor influence. When looking at the response it is clear that the stiffness when moving all concrete parameters is decreasing more than when only changing f_c but this is not influencing the ultimate resistance.

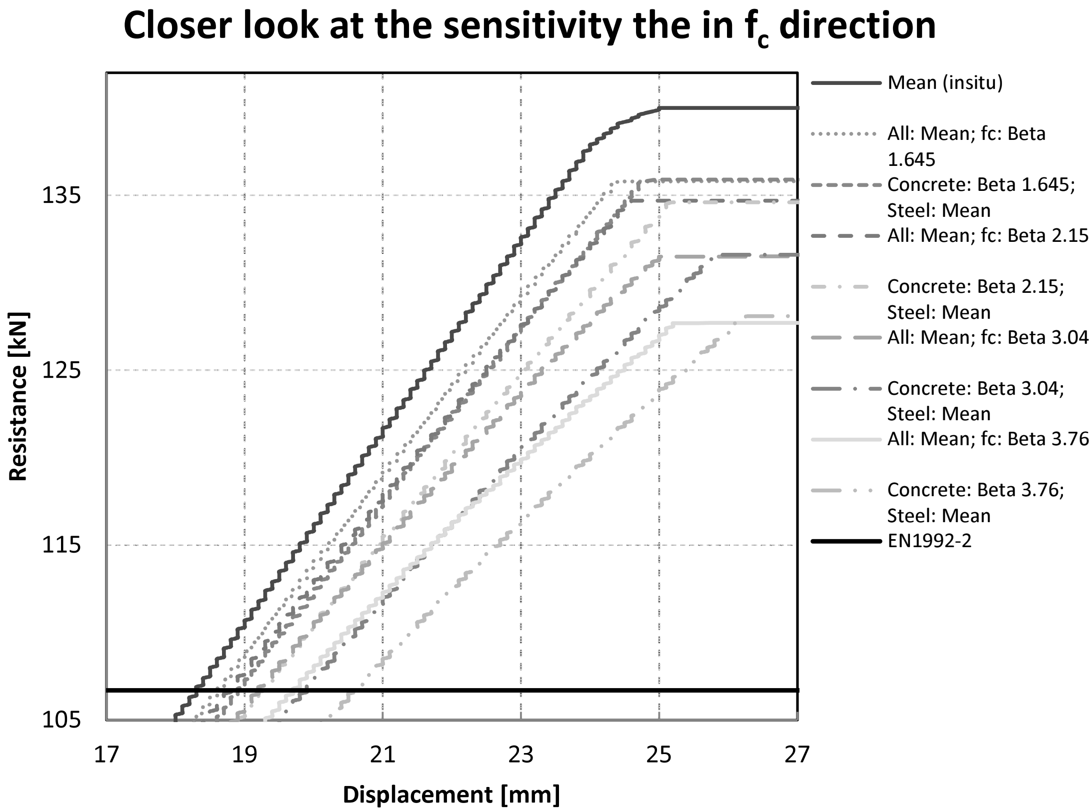


Figure 4.18 A closer look on the design resistance. The concrete compressive strength is compared with all concrete parameters.

4.2.5 Results of the safety format comparison

4.2.5.1 EN safety formats

It can be seen in Figure 4.19 that both CEN(2004a,b) safety formats give approximately the same design resistance. CEN(2004a) has been analyzed both numerically and analytically in order to be compared. Their design resistances are almost equal and this confirms that the accuracy of the FE analysis is sufficient. The basic idea of CEN(2004b) is to use input values of the material parameters close to mean values to get a response close to the realistic response and to be able to use the ultimate resistance with a resistance safety factor to get the design resistance. But still there is a difference in response compared with the mean response, even if it is relatively close. The difference in response appears more in the beginning and when reaching closer the ultimate resistance. Compared with CEN(2004a) the response is a great improvement in CEN(2004b). However, the design resistances of both CEN(2004a,b) safety formats coincide very well with each other.

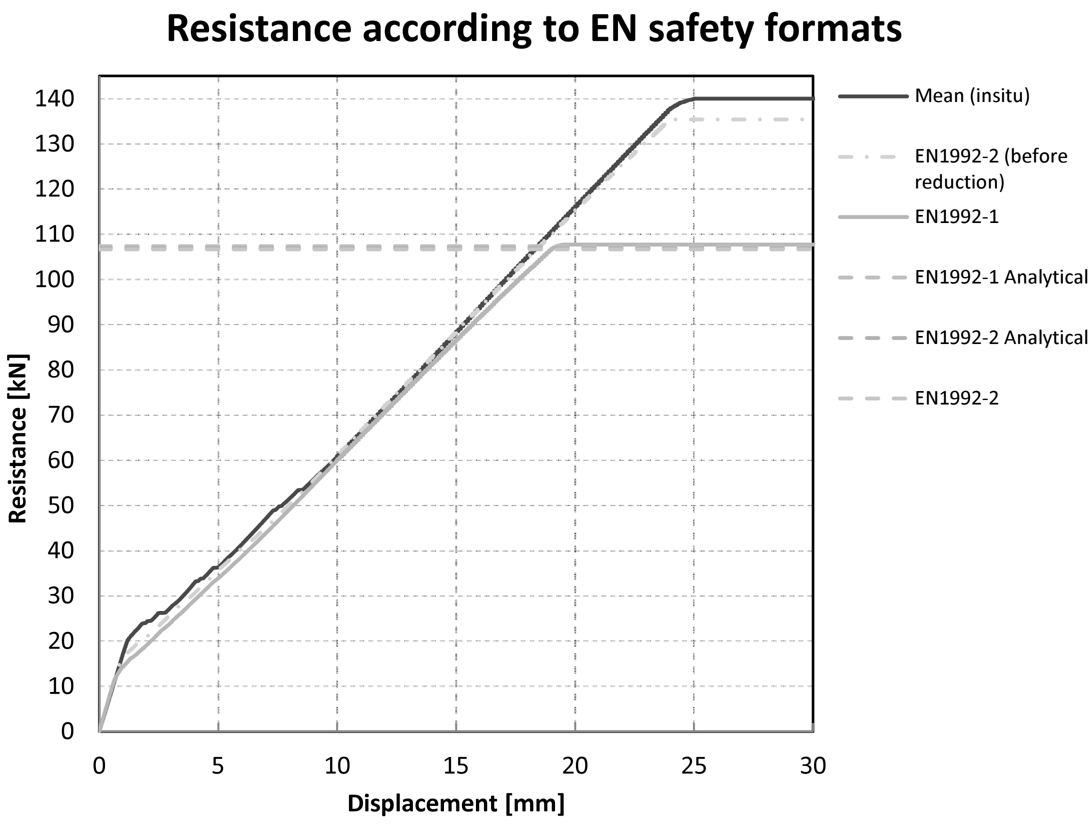


Figure 4.19 Comparison of the structural resistance according to CEN(2004a,b). CEN(2004a,b) safety formats have been performed both analytically and numerically.

4.2.5.2 Cervenka *et al.* (2007) safety format

Cervenka *et al.* (2007) safety format clearly overestimates the design resistance compared with CEN(2004b).

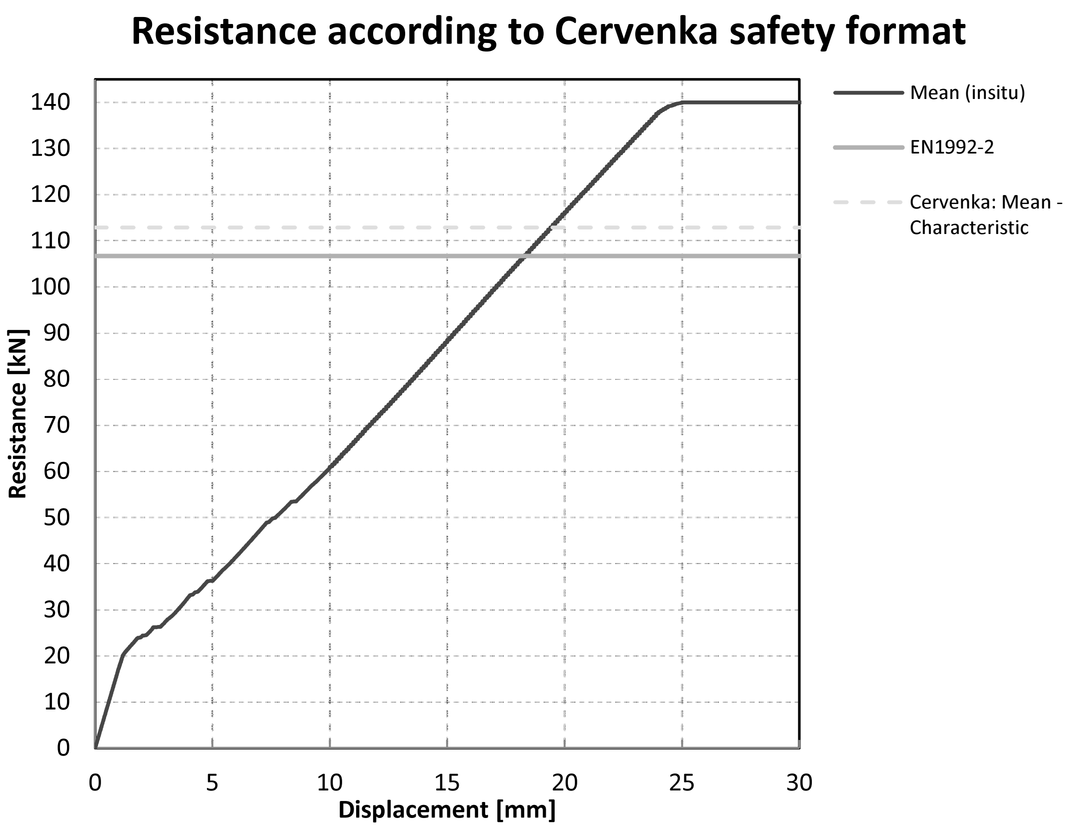


Figure 4.20 Comparison of the structural resistance between Cervenka et al. (2007) and CEN(2004b) safety format.

4.2.5.3 Schlune *et al* (2010) safety format

The resistance according to the safety format by Schlune *et al* (2010) can be seen in Figure 4.21. CEN(2004b) is compared with Schlune *et al* (2010) safety format using numerically partial derivatives based on simulations where inputs of different β-values have been used. Schlune *et al* (2010) suggests a β value of 2.15 but other β-values does also show to give similar results. The difference of the structural resistance when compared with both CEN(2004b) and all different partial derivatives are negligible.

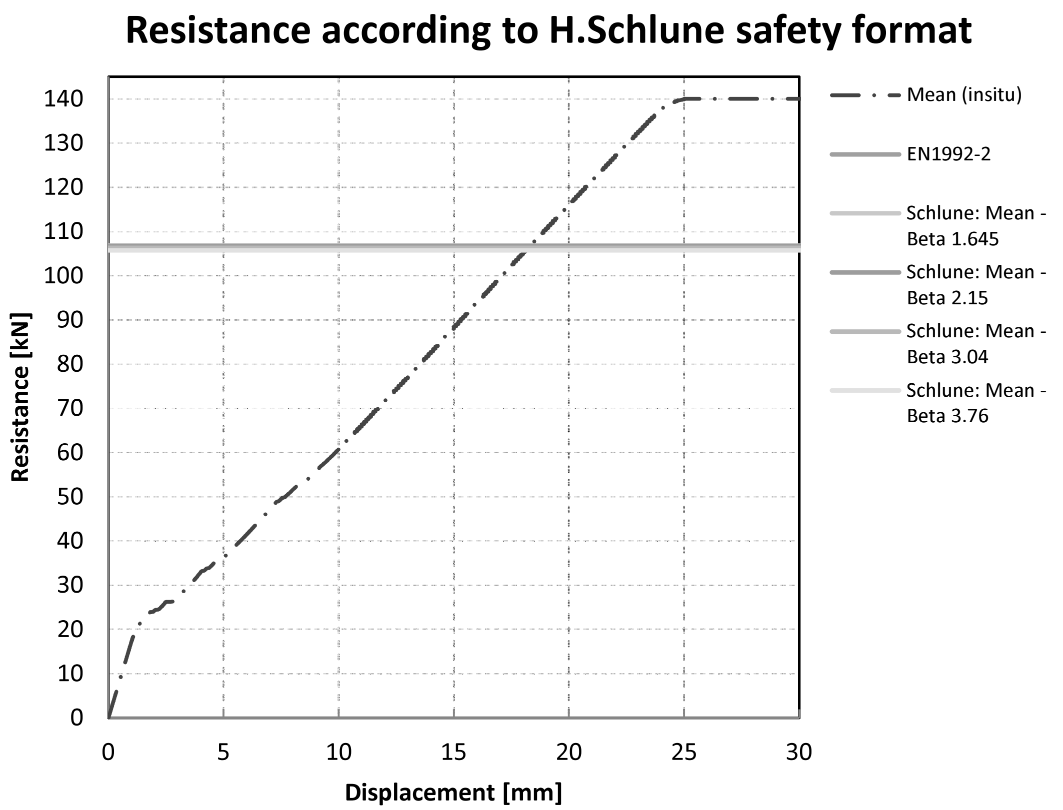


Figure 4.21 The structural resistance according to the proposed safety format by Schlune et al (2010).

Figure 4.22 reveals that the design resistances with different β-values are almost equal. Figure 4.23 shows that the relative difference is small. Schlune *et al* (2010) suggests the use of β 2.15 which give a design resistance closest to CEN(2004b).

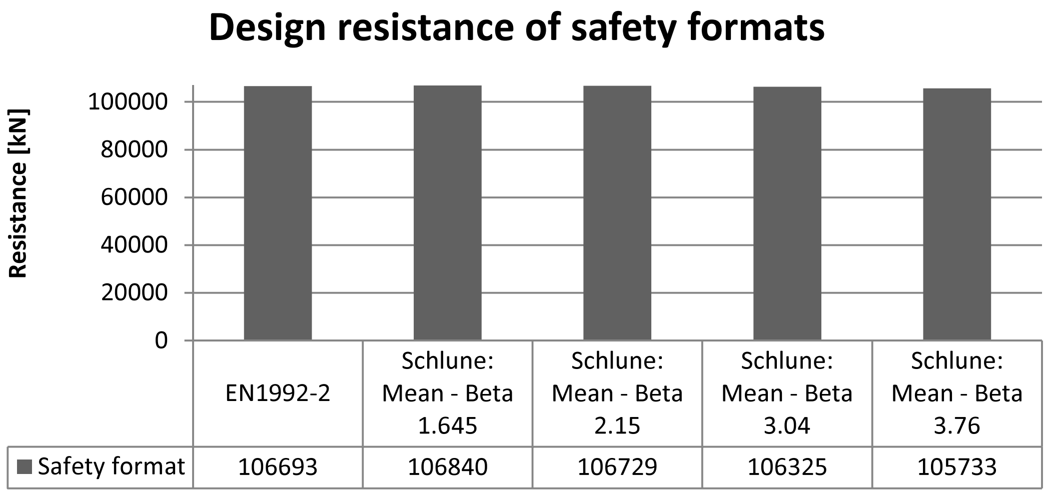


Figure 4.22 The design resistance of Schlune et al (2010) safety format for different values of β and CEN(2004b).

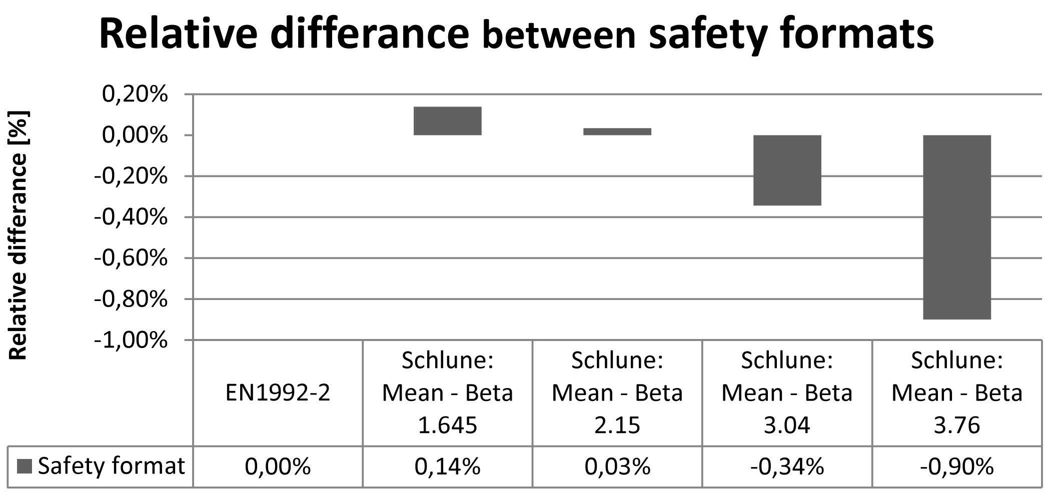


Figure 4.23 The relative difference between the design resistance of Schlune et al (2010) safety format for different values of β and CEN(2004b).

If making the same comparison as above when only taking the material uncertainty X_f into account the results are still in the same range. The relative difference is only 1 % or 2 %, see Figure 4.24 and 4.25. The derivatives using β 2.15 do not give the most accurate result in this case. All β values in this test give resistances that are too high. The Schlune *et al* (2010) design resistance is approaching CEN(2004b) resistance as the β-value is increased.

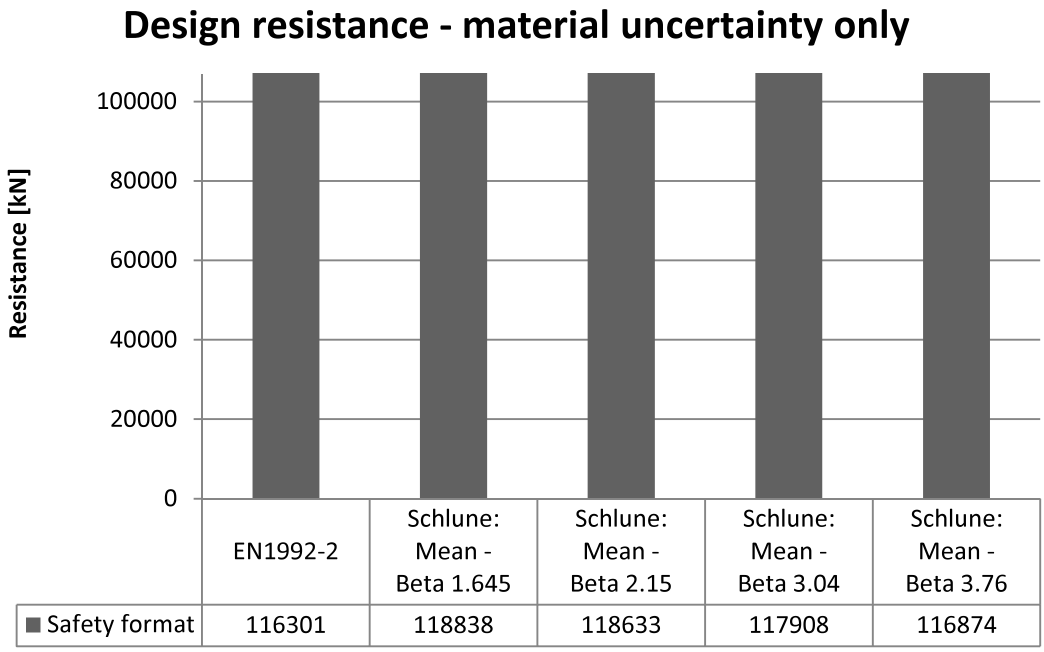


Figure 4.24 The structural resistance when only considering the material uncertainty. CEN(2004b) is compared with Schlune et al (2010) safety format using different β-values.

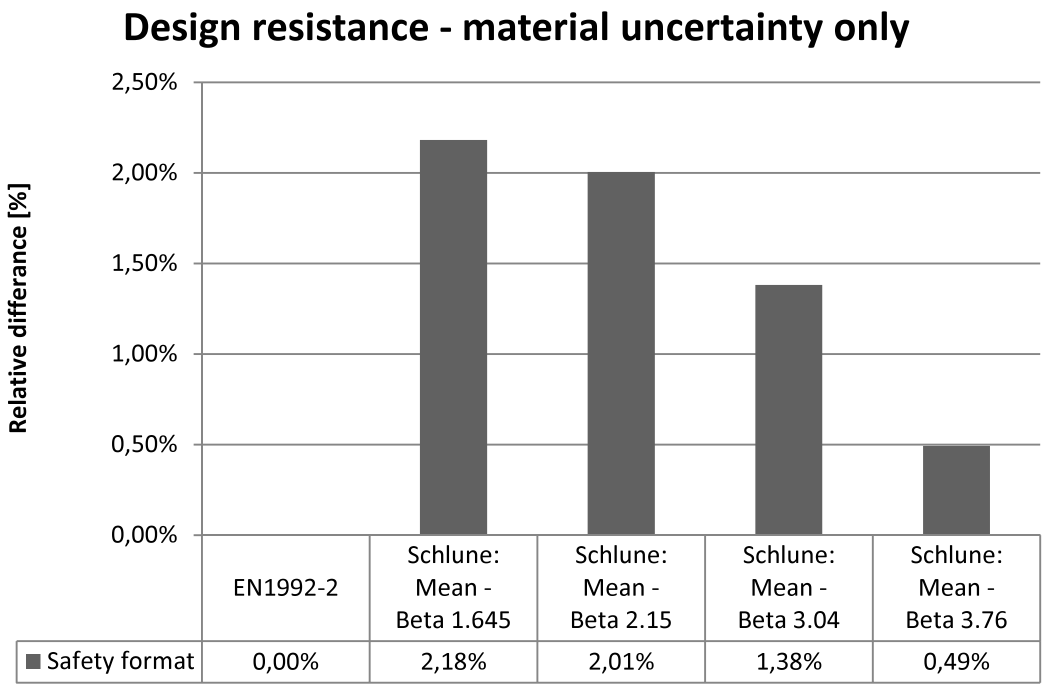
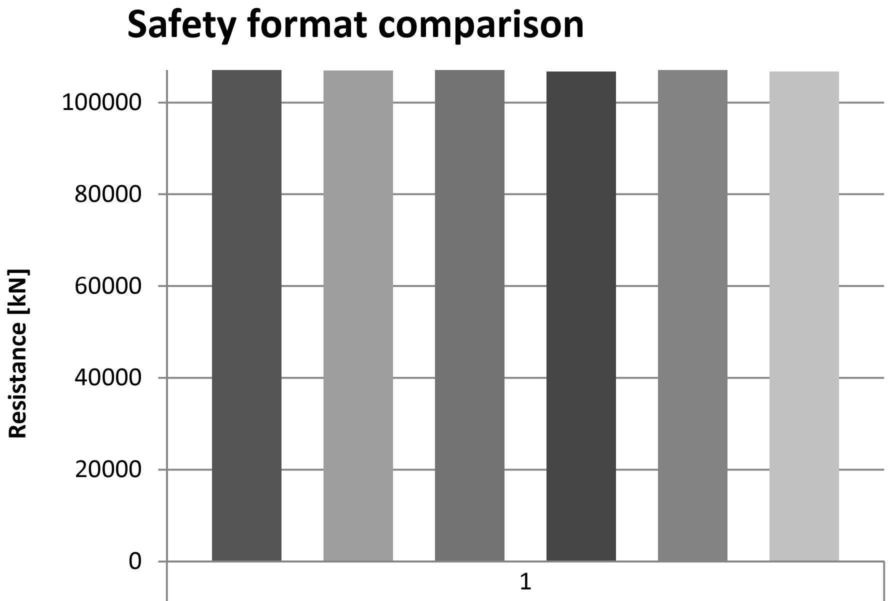


Figure 4.25 The relative difference in the structural resistance when only considering the material uncertainty. CEN(2004b) is compared with Schlune et al (2010) safety format using different β-values.

4.2.5.4 Comparison of safety formats

When studying all design resistances obtained in accordance with the different safety formats, see Figure 4.26, it can be seen that they all give a design resistance within the range of a few percent. According to this comparison the relative difference of all safety formats perform within 1 % with exception of Cervenka *et al.* (2007) format which gives higher deviation of 5.5 %, see Figure 4.27. In this test CEN(2004b) is regarded as a reference.



	1
EN1992-1 ANALYTICAL	107386
EN1992-2 ANALYTICAL	106957
EN1992-1	107700
EN1992-2 (ref)	106693
CERVENKA	112852
SCHLUNE	106729

Figure 4.26 The design resistance of all tested safety formats, both analytically and nummerically.

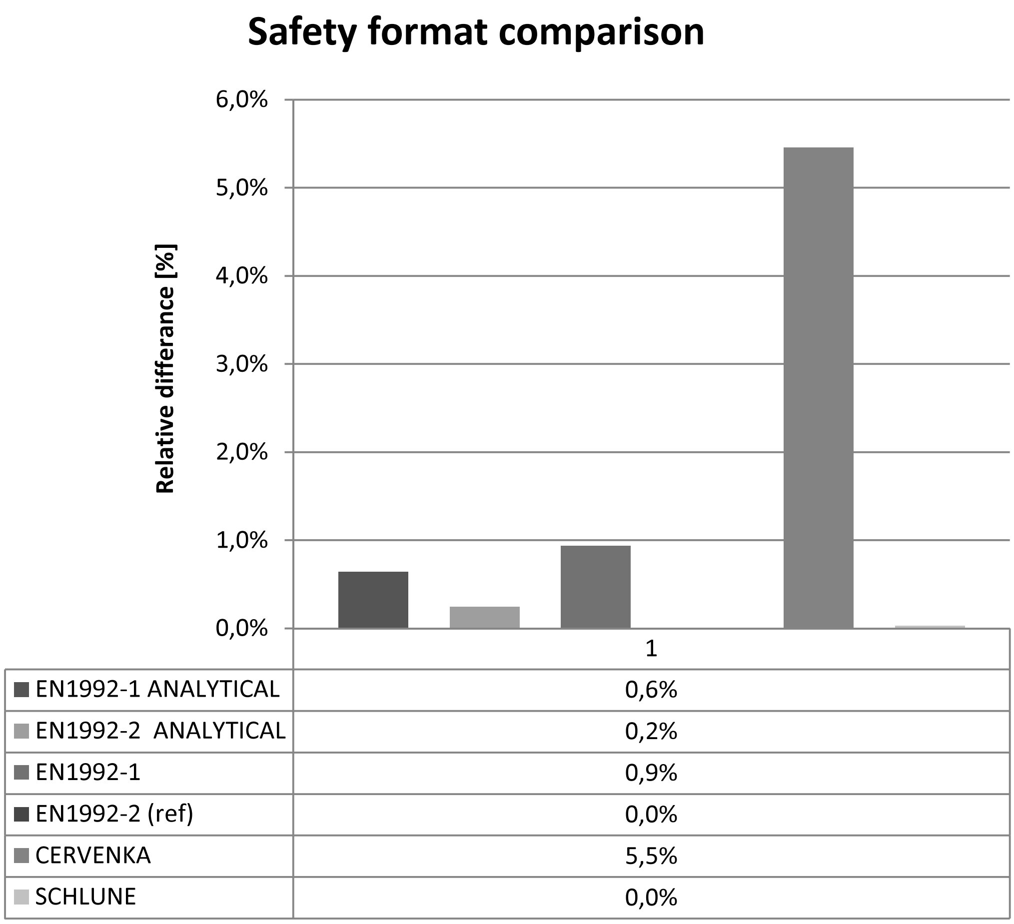


Figure 4.27 The relative difference of structural resistance of the safety formats, CEN(2004b) is used as a reference.

4.2.6 Probabilistic comparison of safety formats with only consideration of the material uncertainty

4.2.6.1 Probabilistic analysis with PROBAB

A probabilistic analysis has to be performed to get the actual design resistance of the concrete beam and be able to evaluate response from the different safety formats. This is achieved by using PROBAB. The settings for PROBAB parameters should be used according to Chapter 4.1, where *conv* and *COV* should be set to 0.02 and 0.10 or better. The beam in this analysis is relatively fast to compute numerically and therefore it can be feasible to use even tougher *conv* and *COV* for the probabilistic analyze, i.e. more accurate results. Also the DARS method should be used instead of DS to save computation time.

Concrete parameters are correlated which should be considered in a probabilistic analysis. The correlation is not explicitly stated in the JCSS model code. Instead formulas to compute all input parameters given a concrete compressive strength. The correlation between the parameters can be obtained by the following equations from JCSS (2001a)

The concrete compressive strength f_c is given by the following expression:

$$f_{c,ij} = \alpha(t,\tau)(f_{co,ij})^{\lambda} Y_{1,j} \qquad (4.18)$$

where

$Y_{i,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in-situ concrete at job j, see table 4.6.

λ = lognormal variable with mean 0.96 and coefficient of variation 0.005

$\alpha(t,\tau)$ = is a deterministic function which takes into account the concrete age at the loading time t [days] and the duration of loading τ [days];

$$\alpha(t,\tau) = \alpha_1(\tau)\alpha_2(t)$$

$\alpha_1(\tau) = 0.8$ can be used in most applications

$$\alpha_2(t) = a + b\ln(t)$$

a = 0.6 and b = 0.12 can be used for normal conditions

$f_{co,ij}$ = lognormal variable, independent of $Y_{1,j}$, see equation 4.19

The function $f_{co,ij}$ depends on parameters which are given in Table 4.5, the parameters are student t-distributed. The concrete used in this test is C30/37 and by interpolation it can easily be seen that the strength parameters in the table becomes; m'=3.75, n'=3.0, s'=0.105 and v'= 10.

$$f_{co,ij} = \exp(m'' + t_v s''(1+\frac{1}{n''})^{0.5}) \text{ [MPa]} \qquad (4.19)$$

Concrete type	Concrete grade	Parameters			
		m'	n'	s'	v'
Ready mixed	C25	3.65	3.0	0.12	10
	C35	3.85	3.0	0.09	10

Table 4.5 Prior parameters for concrete strength distribution (Part of table from JCSS)

The concrete tensile strength f_{ct} and E-modulus E_c can be calculated by the following equations:

$$f_{ct,ij} = 0.3 f_{c,ij}^{2/3} Y_{2,j} \tag{4.20}$$

$$E_{c,ij} = 10.5 f_{c,ij}^{1/3} Y_{3,j} (1 + \beta_d \varphi(t,\tau))^{-1} \tag{4.21}$$

Variable	Distribution type	Mean	Coefficient of variation	Related to
$Y_{1,j}$	LN	1.0	0.06	compression
$Y_{2,j}$	LN	1.0	0.30	tension
$Y_{3,j}$	LN	1.0	0.15	E-modulus
$Y_{4,j}$	LN	1.0	0.15	ultimate strain

Table 4.6 Data for parameters Y_i (Table from JCSS)

With help of Matlab it is possible to calculate the concrete parameters and their distribution and also the correlation between the stochastic concrete parameters. The resulting correlation can be seen in table 4.7.

Parameter	f_c	f_{ct}	E_c
f_c	1	0.3121	0.3157
f_{ct}	0.3121	1	-
E_c	0.3157	-	1

Table 4.7 Resulting correlation between the concrete parameters f_c, f_{ct} and E_c.

According to the JCSS formulas above the correlation of the material parameters is nonlinear; in PROBAB it is only possible to introduce linear correlations.

The material parameters of preeminent importance on the structural resistance are f_c and f_y when it comes to beams subjected to bending. Other stochastic material parameters have shown to be negligible and can be set deterministically (Which has also been confirmed by results in the previous chapter). Therefore, only these two were sampled and should give results with enough accuracy. When only f_c is a stochastic parameter in the concrete there is no need for correlation and there is no correlation between f_c and f_y. In-situ values of the material parameters were used in order to make a fair comparison between the safety formats which also take in-situ conditions into consideration. Settings for the input parameters can be seen in Table 4.8.

Material parameter	Value	Distribution	Standard deviation
E_{sm}	200 GPa	Deterministic	-
f_{ym}	552 MPa	Lognormal	30 MPa
$E_{cm,is}$	31 GPa	Deterministic	-
$f_{cm,is}$	32 MPa	Lognormal	5.03 MPa
$f_{ctm,is}$	2.6 MPa	Deterministic	-
$GF_{m,is}$	68 Nm/m^2	Deterministic	-

Table 4.8 Input parameters for PROBAB.

Limit state:

The limit state should be defined by a parameter that corresponds to the collapse in the pertinent failure mode. The most straightforward way of achieving this is to assign distributions to the material parameters of importance and a limit state based on sudden increase of deflection or stain, whatever represents the collapse, this approach is illustrated in Figure 4.2.6.1. In this simulation where the failure mode is bending, there are two occupancies are possible, crushing of the concrete or yielding of the reinforcement. Both these possibilities should be accounted for by limit states.

Instead of distributing the yield strength as a material parameter, the condition of collapse at yielding is equivalently captured through distribution of the pertinent limit state and a deterministic material parameter. The concrete compressive strength is still distributed as a material parameter as illustrated in Figure 4.2.6.2.

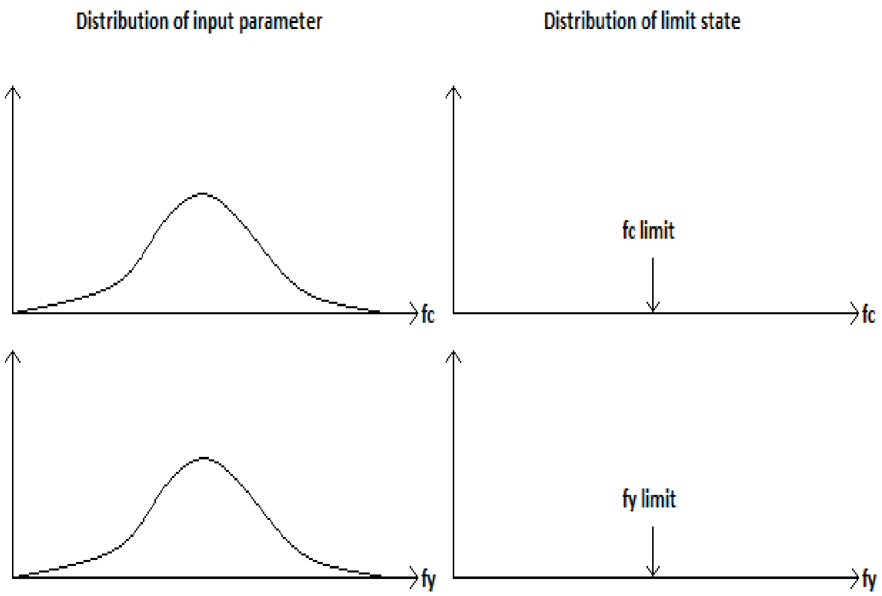


Figure 4.2.6.1 *Traditional approach of limit state formulation with distributed input parameters and deterministic limit states.*

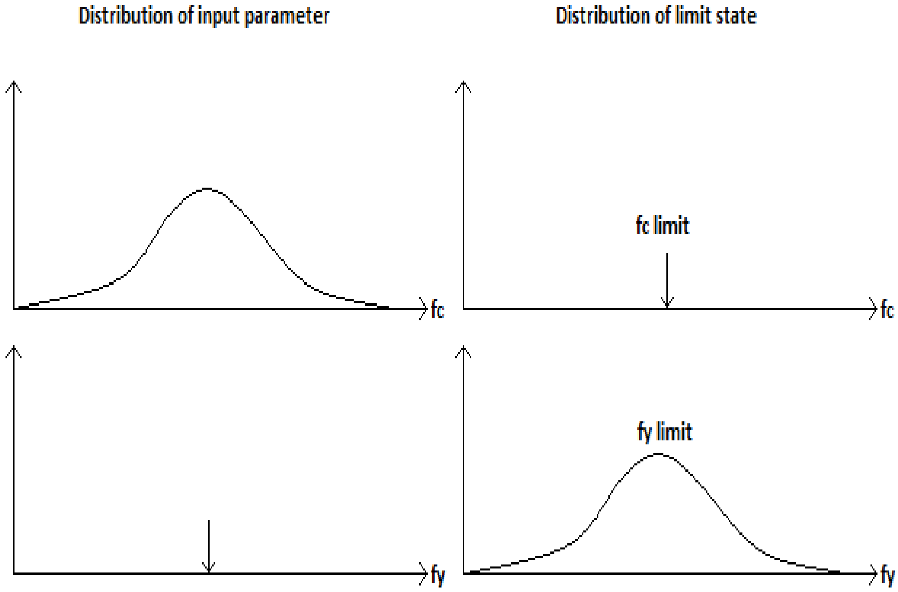


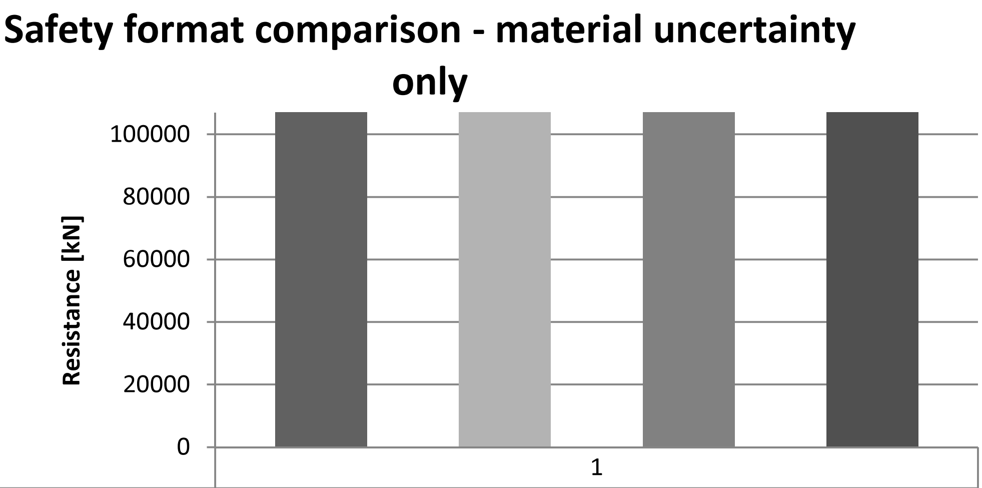
Figure 4.2.6.2 *Formulation of limit states used in this simulation.*

Reliability target:

The aim of the probabilistic analyze in this test is to obtain the resistance corresponding to a reliability index β of 3.04.

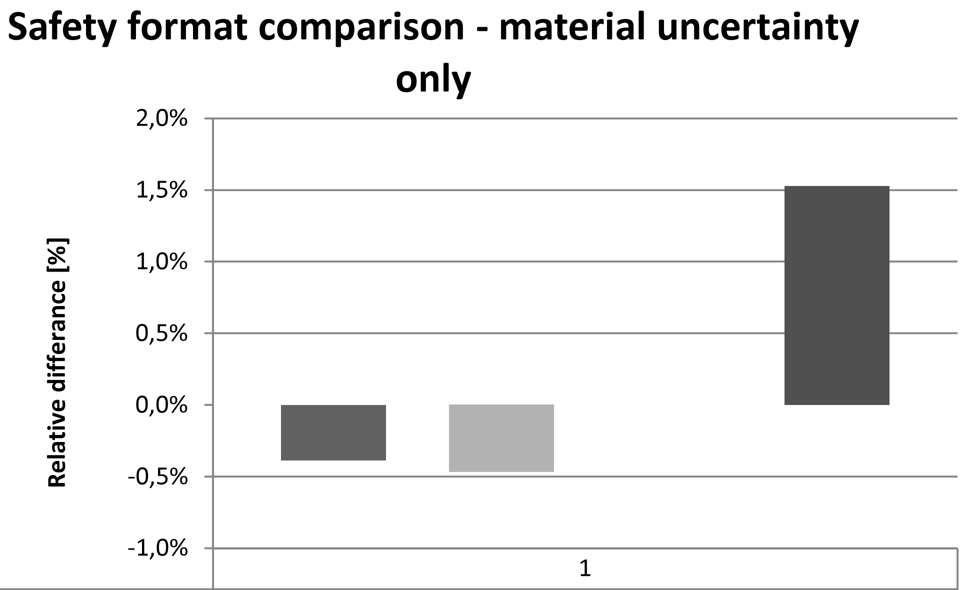
4.2.6.2 Results comparison against probabilistic analysis

The result shows the resistance regarding only the material uncertainty X_f.. As mentioned earlier this was done in order to isolate the material uncertainty which is the focus of this investigation. Also Schlune *et al* (2010) safety format exclusively concerns X_f. The probabilistic analyze using PROBAB was taken as reference. The design resistances according to CEN(2004a,b) and Schlune *et al* (2010) safety format performs well and render results close to the probabilistic results see Figure 4.28. When taking a closer look it can be seen that Schlune *et al* (2010) method is overestimating the design resistance with 1.5%.



	1
EN1992-1	116400
EN1992-2	116301
PROBAB Beta 3.04 (ref)	116850
SCHLUNE	118633

Figure 4.28 The structural resistance with regard to material uncertainty X_f only. PROBAB is used as a reference.



	1
EN1992-1	-0,4%
EN1992-2	-0,5%
PROBAB Beta 3.04 (ref)	0,0%
SCHLUNE	1,5%

Figure 4.29 The relative difference between CEN(2004a,b) safety formats, Schlune et al (2010) safety format and a probabilistic analyze using PROBAB.

4.2.7 Schlune *et al* (2010) format with altered reinforcement amount

To investigate the general validity of the comparison between Schlune *et al* (2010) and CEN(2004b) safety format, over and under reinforced beams where tested with both safety formats. PROBAB analyses where used to calculate the probability of failure P_f and the β-value for both cases. To be able to compare the results of the safety formats with the probabilistic analyses only the material uncertainty is considered.

The concrete beam studied so far in the test has an according to Eurocode adequate amount of reinforcement which is ~1005 mm^2 for a beam of this geometrics. Another under reinforced beam with 500 mm^2 and an over reinforced beam with 2000 mm^2 reinforcement are also tested.

The limit state is formulated through the reinforcement yield strength and the vertical deflection of the beam at mid-span.

The resistance of the beam is calculated for both safety formats with different amounts of reinforcement and these resistances are used to calculate the reliability index with PROBAB.

The results show that both CEN(2004b) and Schlune *et al* (2010) overestimate the capacity when the beam is strongly under reinforced and underestimates when the beam is over reinforced. Schlune *et al* (2010) safety format is then less conservative than CEN(2004b).

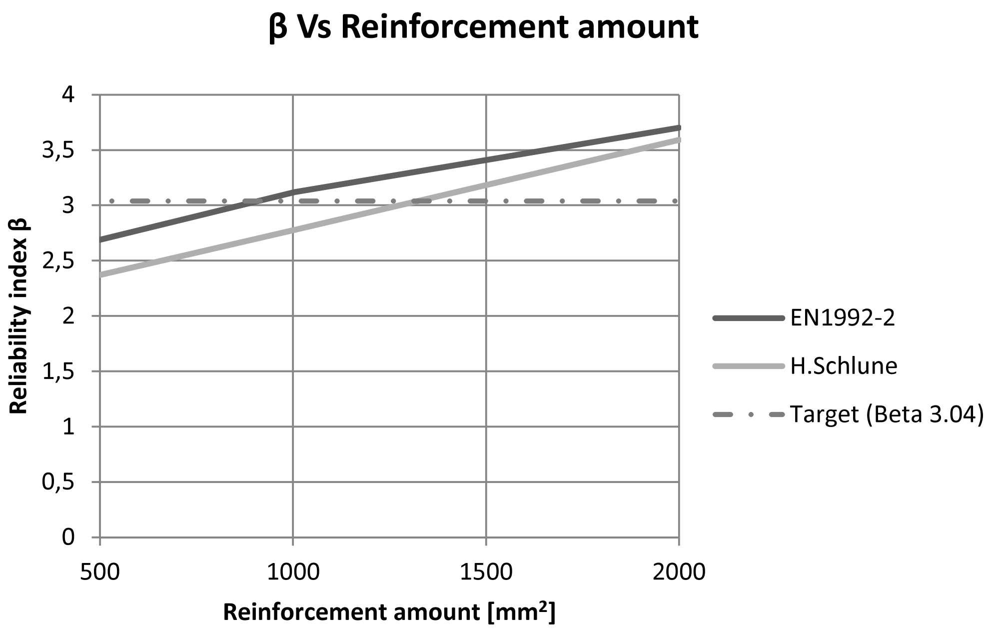


Figure 4.30 Results using PROBAB to calculate the β-value when changing reinforcement amount.

4.2.8 Test of safety formats in SLS

Schlune *et al* (2010) safety formats is compared with CEN(2004b) in Serviceability Limit State (SLS) using the same concrete beam as in ULS testing. CEN(2004b) requirements in SLS concerns limiting crack width and deflection. Here the deflection criterion is considered. Eurocode states that the maximum deflection limit is $L/400$ with reliability index β of 1.5. In SLS there are no safety factors corresponding to the ones in ULS that are fitted to achieve the target reliability. Instead CEN(2004b) states that the limiting deflection is not to be exceeded when using characteristic material parameters. With a beam length of 5 meters the deflection limit becomes 12.5 mm.

A sensitivity study of the material parameters where carried out in order to determine which stochastic parameters are of importance for the structural deflection. The study showed that E_c, f_c, f_{ct} and G_F influenced the deflection and should be used for the safety format.

Schlune *et al* (2010) suggests that the partial derivatives are evaluated at reliability index of 0.85 however the test is also performed using reliability index 2.15 as in ULS tests. Again the in-situ material parameters where used in the test. For the same reasons as in the ULS testing the comparison is also conducted where the model and geometric uncertainties are excluded. This in order to enable verification by PROBAB, it is not possible to run PROBAB with respect to all uncertainties. As explained above the resistance according to CEN(2004b) is not governed by safety factors based on the target reliability, this makes it impossible to separate the material uncertainty.

When conducting the probabilistic analysis the target reliability is β 1.2. The probabilistic analysis is again using PROBAB´s DARS method with settings *COV* 0.1 and *Conv* 0.01. As in the previous test the stochastic material parameters are in accordance with JCSS guidelines. Unfortunately the correlation feature of PROBAB is out of order. The results from PROBAB where not changing as the feature where used. Therefore the analysis is carried out using uncorrelated material parameters. The parameters used are stated bellow in Table 4.9.

Table 4.9 Stochastic material parameters according to JCSS

Material parameter	Value	Distribution	Standard deviation
E_{sm}	200 GPa	Deterministic	-
f_{ym}	552 MPa	Deterministic	-
$E_{cm,is}$	31 GPa	Lognormal	4.69 GPa
$f_{cm,is}$	32 MPa	Lognormal	5.03 MPa
$f_{ctm,is}$	2.6 MPa	Lognormal	0.77 MPa
$GF_{m,is}$	68 Nm/m^2	Lognormal	7.98 Nm/m^2

4.2.8.1 Results in serviceable limit state

Results of CEN(2004b) and Schlune *et al* (2010) safety format:

Table 4.10 Resulting resistance with all uncertainties and with the material uncertainty X_f only.

	Safety format	Resistance [N]
All uncertainties	CEN(2004b)	74323
	Schlune β 0.85	70258
	Schlune β 2.15	70229
X_f only	Schlune β 0.85	73167
	Schlune β 2.15	73294

Schlune *et al* (2010) safety format give a lower resistance in SLS compared to CEN(2004b) for both β 0.85 and β 2.15 with all uncertainties in consideration.

Results from the probabilistic analyze:

Table 4.11 Results from PROBAB compared with Schlune et al (2010) safety format

	Resistance [kN]	Difference [%]
PROBAB β 1.20 (ref)	70.500	0.00%
Schlune β 0.85	73.167	3.78%
Schlune β 2.15	73.294	3.96%

Compared with the results from the probabilistic analysis the resistances according to Schlune *et al* (2010) format are roughly 4 % higher. Both β 0.85 and β 2.15 renders almost no difference i.e. this value does not depend on where the partial derivatives are evaluated. Note that the obtained probabilistic results can only be compared with safety formats where only the material uncertainty is considered, CEN(2004b) cannot be compared with the probabilistic results.

4.2.9 Conclusion

The intention with the test of the non-linear concrete beam was to evaluate Schlune *et al* (2010) safety format.

When comparing all safety formats with consideration of all uncertainties, the design resistances agree well with each other except for Cervenka *et al.* (2007) safety format which overestimates the design resistance by 5.5%, see Figure 4.27.

When only considering material uncertainty, the design resistance with Schlune *et al* (2010) safety format is 1.5 % higher than the probabilistic analyses and CEN(2004b) 0.5 % lower than the probabilistic analyses, see Figure 4.29.

When the reinforcement amount is increased both CEN(2004b) and Schlune *et al* (2010) tends to give higher reliability index. At reinforcement amount adequate to Eurocode recommendations, the reliability of CEN(2004b) coincides with the target value better than Schlune *et al* (2010) safety format. Overall, when looking at the reinforcement amount CEN(2004b) is more conservative than Schlune *et al* (2010) safety format.

In SLS Schlune *et al* (2010) format is conservative compared to CEN(2004b) according to table 4.10. In turn Schlune *et al* (2010) overestimates the resistance according to the probabilistic analysis. Hence it is quite clear that CEN(2004b) is non-conservative. All safety formats could not be compared with probabilistic results, but the resistance according to Schlune *et al* (2010) safety format was roughly 4 % higher than the probabilistic results.

The safety format according to CEN(2004b) and Schlune *et al* (2010) lead to quite similar resistances when all uncertainties are included. In the ULS CEN(2004b) agrees better with the full probabilistic analysis when only the material uncertainty is considered. However, the strength of the safety format according to Schlune *et al* (2010) is to account more properly for the model uncertainty which has not been included in this study. There is reason to believe that the nature of Schlune *et al* (2010) format suggests that it will perform better in more complex situations

4.2.10 Discussion of non-linear concrete beam test

Parameter study in ULS:

According to Chapter 4.2.4 the concrete compressive strength and reinforcement steel yield strength are the two parameters with influence on the structural resistance superior to others. That is, they are the two most important parameters in relation to their coefficient of variation. The parameter study performed in Chapter 4.2.4 does not guarantee that the structural resistance is insensitive to all other parameters. For example, if the steel modulus of elasticity is set to zero it will have a tremendous impact on the results. On the other hand, the event of such a value of the modulus of elasticity is well beyond highly unlikely. In a probabilistic analysis it is only meaningful to describe a parameter as distributed if it has importance in an absolute way and in addition has a coefficient of variation high enough to cause the resulting structural capacity to be distributed. What is considered as output in the sensitivity analysis in Chapter 4.2.4 is the ultimate structural resistance. If instead the ultimate deflection was considered the conclusion may have been different. In terms of ultimate deflection the difference is in fact several percent when comparing the result from the analysis using a concrete compressive strength at a certain fractile to the result received when all concrete parameters corresponds to that same fractile, this is because of the altered modulus of elasticity. When comparing the ultimate resistance in the same way the relative difference is per thousandths. So have in mind that this is a result valid in the case of ultimate bending capacity. If for example ultimate shearing-load or crack-load is considered the importance of the different material parameters may be different.

ULS results:

The results from the test of the nonlinear concrete beam show that both CEN(2004b) and Schlune *et al* (2010) safety format works well according to results from the probabilistic analysis. CEN(2004b) safety format originate in CEN(2004a) which has been developed under a long period of time and has been composed on years of testing and experience from many different researchers i.e. empiric based safety format. This format has in that way more or less been adapted for common cases as beams in bending with adequate amount of reinforcement and therefore it is no surprise that CEN(2004b) gives good agreement with the probabilistic analysis. Schlune *et al* (2010) safety format also gives good agreement but dissimilar to CEN(2004a,b) is based on theory, this is very interesting and shows that theory coincides well with the empiric based safety format for the studied structure. How they perform in a more complex structure is hard to say, whether a theory or empiric based safety format will result in a more accurate design resistance.

The theory behind Schlune *et al* (2010) safety format seems reasonable, it takes consideration to the importance of the stochastic parameters with significant influence of the resistance, and therefore this format should not only work with adequate amount of reinforcement but also for all amount of reinforcement. This theory yields also for other parameters with different importance for the resistance.

Schlune *et al* (2010) suggests estimating the importance of each stochastic parameter by changing the parameter with the β of 2.15 to get a good estimation. In this case with a concrete beam in bending with adequate amount of reinforcement this choice works well, other β values do not give much difference and could also be used.

The Cervenka *et al.* (2007) safety format is built up on the idea to estimate the coefficient of variation of the resistance. Only the ratio between characteristic and mean resistance has been taken into consideration when estimating the coefficient of variation V_R which then is used to calculate the resistance safety factor for the resistance with reliability index β and α_R of 4.7 and 0.8, respectively, see Table 3.2. For information about reference periods see Chapter 3.1. Cervenka *et al.* (2007) dictates 4.7 as a value of β but why this is taken is not clear, Eurocode suggest β 3.8 in the ultimate limit state which corresponds to failure probability of 50 in a million. Instead if β of 3.8 is used with Cervenka *et al.* (2007) safety format the design resistance would be grossly overestimated. In Figure 4.31, the design resistance is plotted against β which clearly shows the difference between CEN(2004b), Cervenka β 3.8 and Cervenka β 4.7.

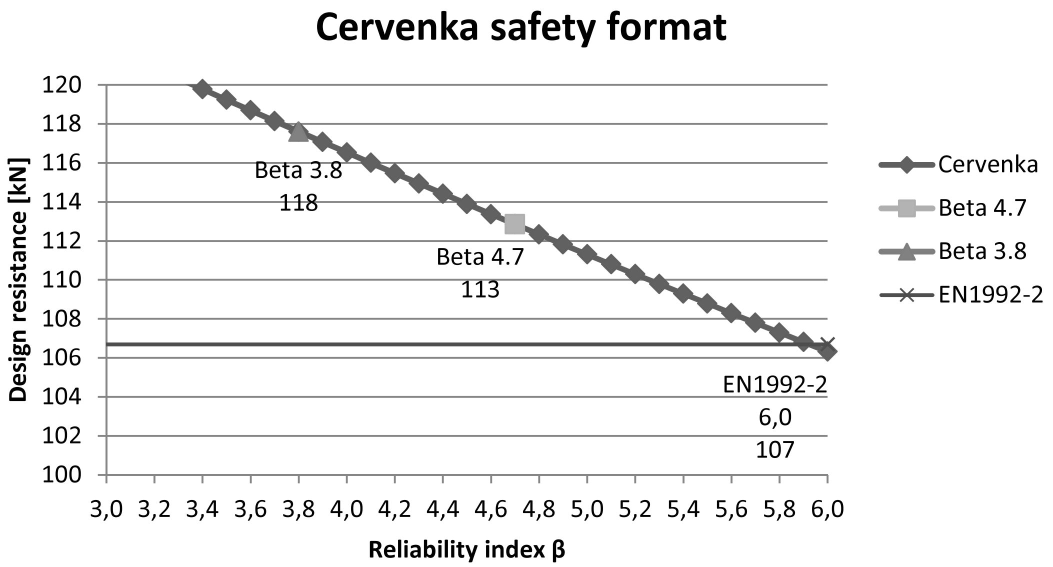


Figure 4.31 The resistance for Cervenka et al. (2007) safety format Vs reliability index β. The horizontal line is the design resistance according to CEN(2004b).

When it comes to geometric and model uncertainty the guidance today in this subject is diffuse. The geometric uncertainty is often small but the model uncertainty can be of great significance. It can in many cases be hard to estimate how big the uncertainties are, especially for the model uncertainty. For more complex structures this becomes even harder, for these structures the importance of the model uncertainty can be essential and much larger than the material uncertainty. If the model uncertainty is much larger than the material uncertainty, the uncertainty of the structure becomes almost solely dependent on the model uncertainty while the material uncertainty becomes negligible. Research about model uncertainty and better guidance is important and needed in the future to be able to improve the evaluation of the structure design resistance. This is the reason why in this comparison of safety formats it was decided to separate the model and geometric uncertainty from the results and focused only on the material uncertainty.

PROBAB is still under development and far from being finished probabilistic FE software. There is almost no guidance or help able about the software and even the error messages do not give much help. In the end, with knowledge based on trial and error, which is extremely time consuming because of probabilistic analysis, the software works fine and the results were very accurate when they are compared with Monte Carlo simulations. By only considering the material uncertainty many stochastic parameters can be neglected and this has speeded up the probabilistic analyses a lot. This has not only made it possible in the matter of time to perform probabilistic analyzes but also leads to less time demanding analyses and therefore the opportunity to run convergence criterions of higher accuracy of the analyses that was performed. Neglecting the geometric and model uncertainty also reduces the risk of choosing wrong input or setting for each probabilistic analysis i.e. reduces the error in results.

It is of importance that safety formats are feasible, i.e. easy to accomplish and understand, and not too time consuming. This might be the reason why the safety format by Cervenka *et al.* (2007) has been well know and suggested in a short period of time. The Cervenka *et al.* (2007) format only requires two analyses and some simple calculation to obtain the design resistance, however this design resistance does not seem to be accurate but the safety format is still very feasible. Schlune *et al* (2010) format is still under development which shall be taken into consideration but as we speak the safety format uses three analyzes for the same case where Cervenka *et al.* (2007) uses two and CEN(2004b) uses only one. And it should be noted that also the stochastic parameters with significant influence must be recognized and analyzed with parameters corresponding to fractiles of β 2.15. This might not be a problem to find the parameters with significant influence with some experience for simple structures like bending beams for example, but for more complex structures they are harder to recognize. It is of course possible to use all stochastic parameters but for each parameter added another analysis is required. The in-situ consideration can also be tricky. Instead of converting the cylinder values to in-situ values according to Carlsson *et al.* (2008b) and JCSS (2001a) an easier way of taking the in-situ into consideration would be great, maybe by some constants or simple formulas. However Schlune *et al* (2010) safety format works very well in this test, further test of other structures is needed to verify that this is the case in other tests.

SLS results:

According to the results, Schlune *et al* (2010) overestimates the capacity and CEN(2004b) gives a higher resistance than Schlune *et al* (2010) when considering all uncertainties. Meaning that Schlune *et al* (2010) is closer to the obtained probabilistic results, hence, the comparison may not be completely fair since CEN(2004b) cannot be compared directly with the probabilistic analyze but anyway shows that it is very likely that Schlune *et al* (2010) performs more accurate than CEN(2004b).

Unfortunately it was not possible to correlate the stochastic material parameters, if this had worked the dispersion of the result would have been greater as the concrete parameters are positively correlated and all have a positive influence of the structural resistance. Again, PROBAB is a software under development and it is not clear why the analysis did not work when using the correlation feature.

Simplifications of the probabilistic model:

In reality the concrete compressive capacity and the concrete modulus of elasticity are correlated quite strongly, the same goes for the reinforcement yield strength and it´s modulus of elasticity. These correlations are not represented in the probabilistic model due to the fact that only f_c and f_y are considered to be distributed in ULS. Furthermore, in this probabilistic simulation the quality of both the reinforcement and concrete is assumed to be the same in the entire structure. A more realistic model would be to divide the geometry into a certain number of elements and given the material assign each element values of its material parameters. Naturally this sub-division of the geometry should coincide with the FE-mesh. If this is done the FE-analysis can be run with a more realistic variation of the material throughout the geometry. Although the parameter values should not be sampled randomly according to the distributions specified in Table 4.2 as if the different elements where completely independent of each other. The quality of the material in adjacent elements are correlated and the variation along a reinforcement bar is considerably less than what stated in Eurocode where distributions are based on tests on reinforcement bars out of different batches from different manufactures. If the quality of reinforcement bars would vary within a batch to the same extent as it does globally the element of chance would be eliminated as the number of bars is increased in a section and the average capacity would approach the global mean value. Now this is not the case, the variation within a batch is smaller than, and biased in relation to the global one. In this probabilistic model the variation steel and concrete quality is assumed to be zero and based to the global average as an individual sample.

4.3 Test of safety formats on a non-linear shear panel

To gain further insight in the functionality of established and newly proposed formats other structures than simple beams should be considered. A commonly used constructing element is the shear panel and it is therefore a suitable object of testing when verifying the general validity of the safety formats.

The different stages of this test are performed in the same order as in the previous one with a non-linear concrete beam. First a sensitivity study is performed to recognize the sensitivity of each material parameter with respect to the structural resistance. After that, the safety formats are compared and finally the safety formats are evaluated with probabilistic analyses. Only CEN(2004b) and Schlune *et al* (2010) safety format are compared in this test.

4.3.1 Model description

The shear panel is an orthogonally reinforced shear panel where the reinforcement mesh is rotated 45 degrees in relation to the boundaries. The panel is subjected to loads perpendicular to the edges, this in combination with the reinforcement arrangement simulates a shearing action, see Figure 4.32. This shear panel is known as a so called 'Houston shear panel' described in Broo *et al.* (2008), a panel that has been built and tested in reality. The panel is modeled with 4 node 2D curved shell elements with embedded reinforcement which are able to describe shear response. The same material properties are applied as in the test of the non-linear concrete beam, see Chapter 4.2.2 for the material properties, and again, in-situ parameters are used. The panel is subjected to 3 uniformly distributed loads with a total step-size of 100N per load. Load and boundary conditions can be seen in Figure 4.32.

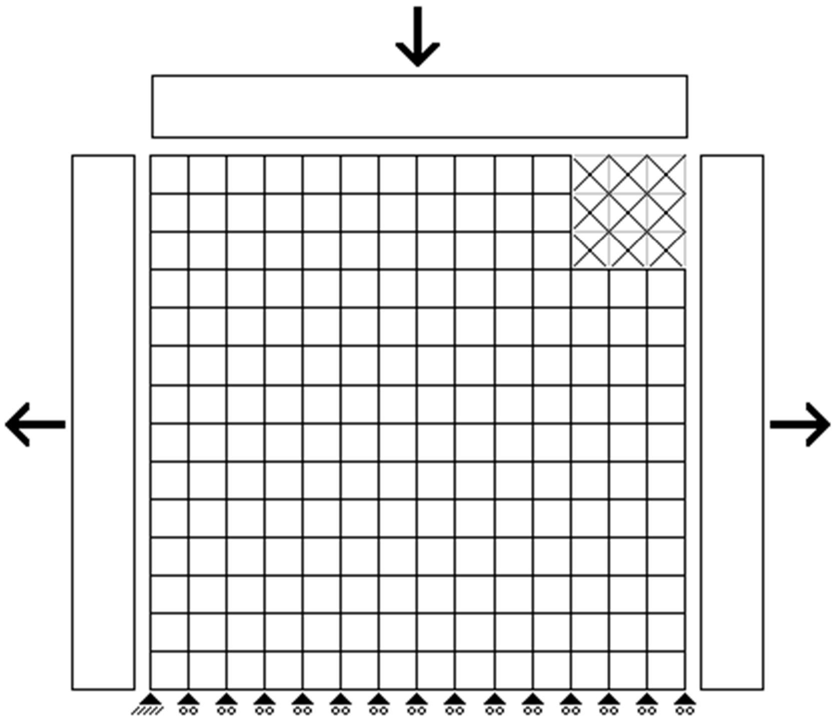


Figure 4.32 Houston shear panel. The reinforcement is rotated 45 degrees, see right upper corner, and the panel is loaded with distributed loads in the directions indicated by the arrows.

4.3.2 Results from test of shear panel

By studying the deflection in the y-direction when reducing each stochastic material parameters one by one with the distance of β 2.15 it can be seen that the response when decreasing f_c 'f_c β 2.15' is almost the same as when decreasing all material parameters at the same time 'All β 2.15', see Figure 4.33. The ultimate resistance is almost only influenced by f_c, however, f_{ct} shows to give some influence, all the other material parameters have minor influence.

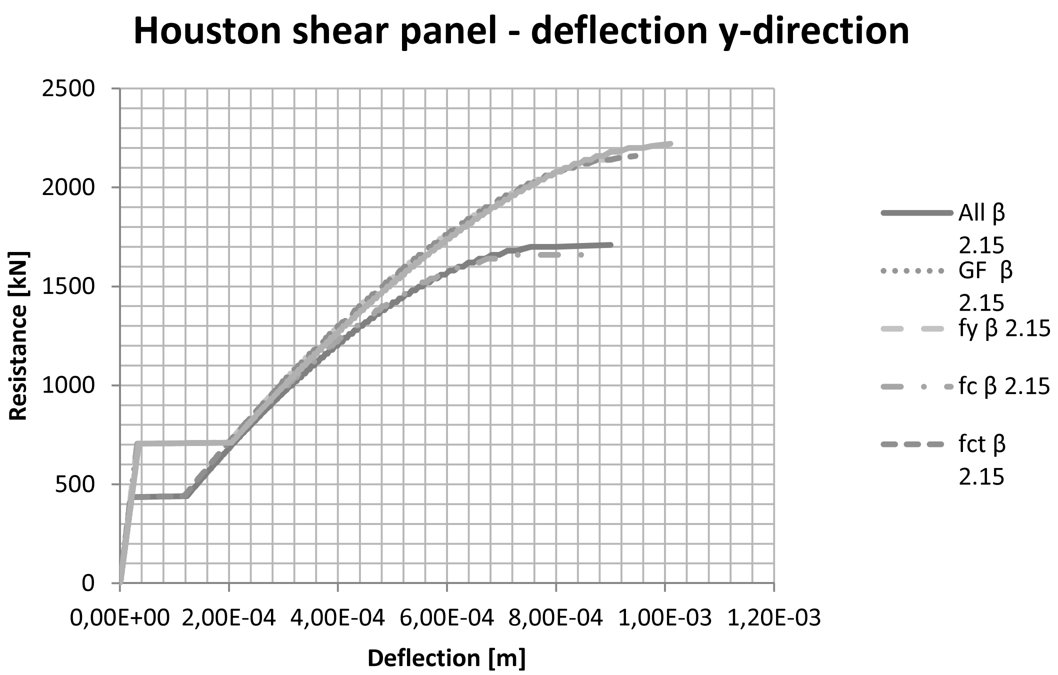


Figure 4.33 Sensitivity of the structural response of Houston shear panel when altering. 'f_c β 2.15' and 'All β 2.15' are the two bottom graphs, the others are above.

As shown in previous tests it can be seen that the structural response of CEN(2004b) deviates from the response with mean values, see Figure 4.34. The deviation is about 20 % in this test and about 7 % in the previous test.

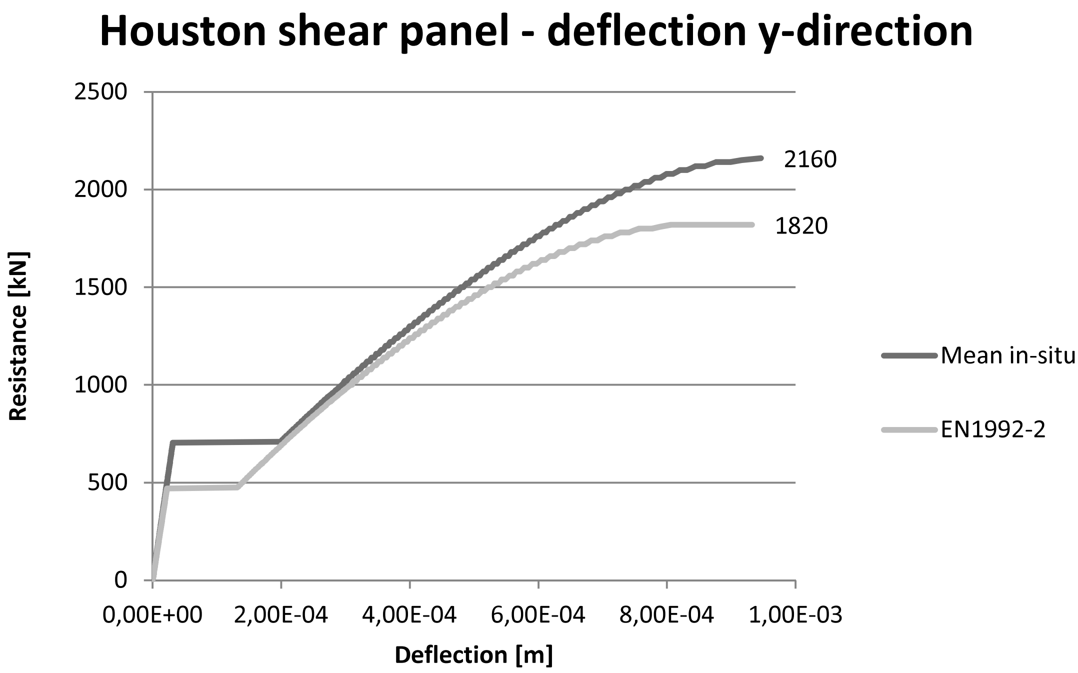


Figure 4.34 The structural response of CEN(2004b) safety format. (Note that the ultimate resistance should be divided by the resistance safety factor to obtain the design resistance.)

Schlune *et al* (2010) design resistance has been calculated with respect to the sensitivity in f_c-and f_{ct}-direction. Again as in precious test, both safety formats have been considered with only respect to the material uncertainty X_f. This was done to be able to verify the results later with probabilistic analyses using PROBAB.

Safety format	Design resistance [kN]	Resistance, material uncertainty only [kN]
Schlune *et al*	1249	1305
CEN(2004b)	1437	1599

Table 4.9 Results of the Houston shear panel. The design resistance and the resistance when only considering material uncertainty for Schlune et al (2010) and CEN(2004b) safety format.

These resulting resistances have to be compared with results from probabilistic analyses to determine which one who is closest to the 'true' resistance. Unfortunately the PROBAB analysis was not successful, see discussion 4.3.3 for further information.

4.3.3 Discussion and conclusion

Several attempts of probabilistic analyses were made. The reason why the probabilistic analyses were not successful is the well known problem of numerically defining a collapse (limit state) with FE analysis together with limitations in the software PROBAB. This is very unfortunate since the shear panel study is quite interesting because the resulting design resistance obtained using Schlune *et al* (2010) deviates from CEN(2004b). But without a reference value of the design resistance which should have been computed with PROBAB it is not known which safety format is the more accurate one. As mentioned before, PROBAB is not a fully developed software and the reasons why certain analyses fails are not always explained in the error messages. Therefore it can be hard to understand why the analyses did not run successfully. The method of 'trial and error' can be very time consuming. However, the problem with defining the limit state using PROBAB is explained in more detail bellow:

Deformation limit

It is not possible to formulate collapse criteria through a deformation limit state. This is because the deformation associated with the collapse varies when material parameters are sampled and it is not possible to set a general limit defining a collapse. Another idea is to choose a deformation limit well beyond the collapse. This will not work either since the PROBAB iteration technique updates the guess based on the previous error. Using this approach there will be difficulties converging in a case of this nature where the errors suddenly becomes tremendous, see Figure 4.35.

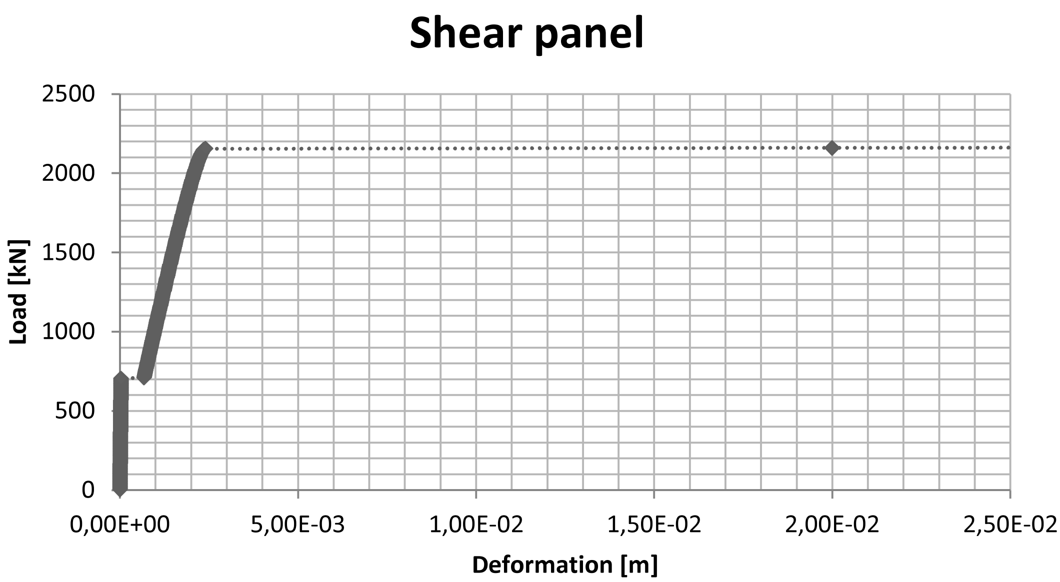


Figure 4.35 Load Vs deformation for a reinforced shear panel. Note the deformation of the last load step is much larger than in the previous step, next load step results in even larger deflection.

Strain limit

PROBAB allows strain formulation of limit state. The strain corresponds to the deformation in such a way that the problem encountered when trying to set the deformation as limit state is the same for strain.

Reinforcement yield limit

A limit state defined by yielding is not possible either. PROBAB does not allow formulating a criterion when all the reinforcement has yielded in every reinforcement element i.e. collapse of the shear panel. The only way to set a limit in PROBAB is choosing one or several elements/nodes and defines the collapse as the event of one of these elements/nodes is exceeding the limit. If it was possible to demand that all of these reinforcement elements are yielding this would be a way to define a collapse of the shear panel.

Concrete stress limit

It is not possible to define a stress limit because of the multi-axial stress state. For example, the concrete compressive strength is reached faster when the concrete is subjected to tension force at the same time. The relationship between equivalent stress at collapse and the concrete compressive strength is diffusely, this cannot be predicted using Von Mises formula because of cracking.

5 Conclusion

One objective of this master's thesis was to evaluate PROBAB as a tool to verify safety formats. However, the usefulness of the program for this was found to be very limited. The reason for this is that there are limited possibilities to formulate collapse criterions in the version used. The software is very powerful in its structure but PROBAB will be a lot more useful if more options were available in the user interface. It seems to be the first version of the software and there is almost no information or guidance on how to use PROBAB properly. Error messages were few and difficult to interpret. A big concern was that no information about how the DIANA analysis has to be performed was given. This was figured out by 'trial and error' which was very time consuming. The overall opinion is that PROBAB is not ready for use in general cases.

The Schlune *et al* (2010) concept seems accurate enough according to the non-linear beam test and was not proven wrong. The safety format agrees well with that of CEN(2004b) and with the probabilistic results. By the nature of the Schlune *et al* (2010) safety format, there are reasons to believe that this format will perform better than that of CEN(2004b) for other failure modes and structures than beams in bending. The test performed on the shear panel shows that the CEN(2004b) and Schlune *et al* (2010) does not agree. This is interesting and also expected as the safety format of Schlune *et al* (2010) is able to adapt to general situations in contrast to the format of CEN(2004b). It could not be proven that Schlune *et al* (2010) format gives a higher accuracy due to the fact that the probabilistic analysis failed. Other ways to perform probabilistic analyzes should be investigated so that CEN(2004b) and Schlune *et al* (2010) safety format can be tested in cases of shear panels and other structures.

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