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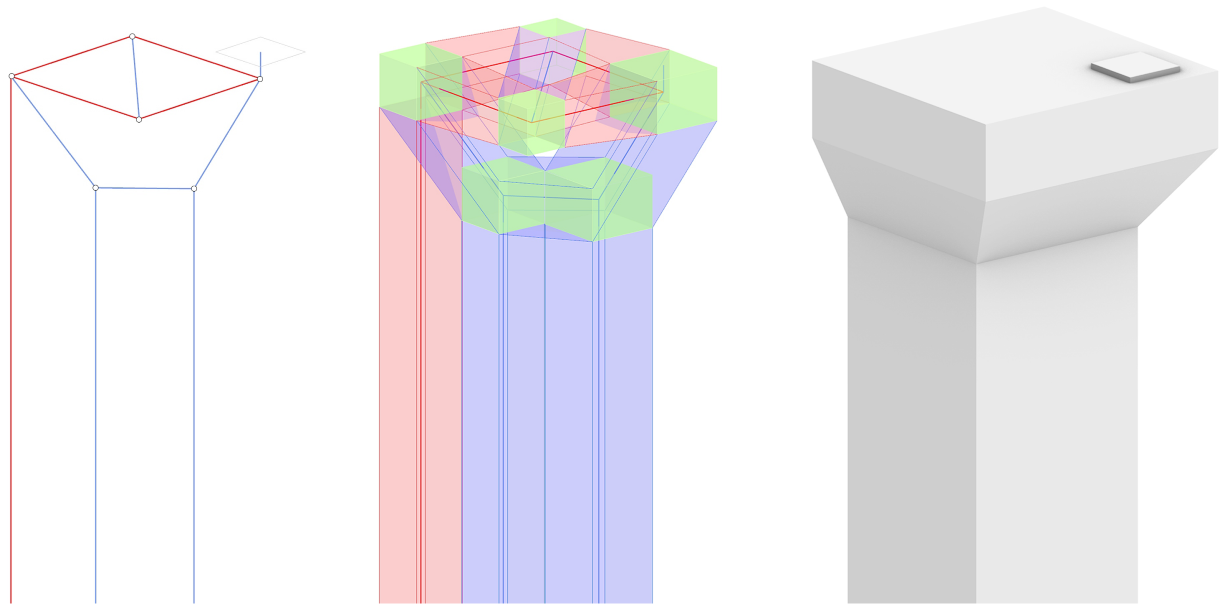

Enhanced 3D strut-and-tie method for design of discontinuity regions

Validation of the method through a case study

Master's Thesis in the Master's Programme Structural Engineering and Building Technology

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Cover: The process of designing a corbel through three dimensional strut-and-tie method.

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Abstract

For many years discontinuity regions in concrete has been calculated by using the strut-and-tie method, where the force flow in a structure is represented as a truss system. Strut-and-tie models can be set up in three dimensions, however in structural engineering practise today the models are simplified to two dimensions in the calculations. This often results in complicated and unnecessarily conservative calculations since the force flow in the structure is not two dimensional. A suggestion of an enhanced method to set up and calculate strut-and-tie models in three dimensions was proposed by Chantelot and Mathern (2010), and presents a way of designing nodal zones that can handle forces in all directions.

A case study was performed in this thesis where the calculations for a bridge detail based on the conventional theory was compared with calculations based upon the enhanced method by Chantelot and Mathern (2010). The three dimensional model was constructed and calculated parametrically with the use of the software Grasshopper, a plugin to the modelling software Rhinoceros 3D.

When comparing the models, it was shown that the model based upon the enhanced method gives a reduction of 8% in the concrete amount and 30% in the reinforcement amount. Furthermore the actual material reduction is larger as the surface reinforcement and formwork material was not included in the comparison. Besides using less materials, the structure is also easier to manufacture at site.

The case study validates the enhanced method for usage in this type of structures. It results in more rational calculations in three dimensional situations compared to the the current standard of calculating. However, it has also been shown in this thesis that the formulation of the rules in the method put unnecessary strict limitations on the model and affects it´s flexibility. A reformulation of the rules through further research would lead to an even better performance.

Keywords: strut-and-tie, strut, tie, D-region, 3D, Rhinoceros 3D, Grasshopper, parametric, optimisation, corbel

Contents

Preface

During the spring of 2021, this master thesis was conducted at Chalmers University of Technology in collaboration with Ramboll Sverige.

We are grateful to our examiner Professor Mario Plos for giving us the opportunity to write on this subject. We also want to give our gratitude to Ramboll and especially our supervisors Victor Andersson and Christoffer Jonsson for sharing their spaces and expertise by always being in our corner whenever we needed assistance. Lastly, we want give a huge gratitude to our family and God for being with us through the whole journey until now.

David Bohman, Nemir Chaudhry
Gothenburg, June 2021

Nomenclature and abbreviations

The list explains abbreviations, symbols and variables that has been used within the body of this master thesis.

Abbreviations

$2D$	Two dimensional
$3D$	Three dimensional
$B - region$	Bernoulli region
$BESO$	Bi-directional evolutionary structural optimisation
$D - region$	Discontinuity region
ESO	Evolutionary structural optimisation
FE	Finite element
FEM	Finite element method
ST	Strut-and-tie
ULS	Ultimate Limit State

Definitions

α	Deviation angle of concentrated forces
α_1	The effect of the form of the bars assuming adequate cover
α_2	The effect of concrete minimum cover
α_3	The effect of confinement by transverse reinforcement
α_4	The influence of one or more welded transverse bars along the design anchorage length
α_5	The effect of the pressure transverse to the plane of splitting along the design anchorage length
α_{cc}	The coefficient taking account of long term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied.
γ_c	The partial safety factor for concrete

ϕ	Bar diameter
ϕ_n	The equivalent diameter of bundled bars
$\sigma_{Rd,max}$	Maximum compression stress
θ_1	Deviation angle of strut meeting single tie
θ_2	Deviation angle of strut meeting two perpendicular ties
A_s	Cross sectional area of reinforcement
d_g	Maximum size of aggregate
f_{cd}	Design value of concrete compressive strength
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days
$l_{0,min}$	The minimum lap length possible for a certain type of reinforcement
l_0	The design lap length
$l_{b,min}$	The minimum anchorage length
$l_{b,rqd}$	The basic required anchorage length
l_{bd}	Design anchorage length
n_b	Number of bars in a bundle

1

Introduction

This introduction intends to provide the reader with an overview of this master thesis. In this chapter the background, purpose, objectives and hypotheses are outlined. Finally, the method and the limitations of this thesis are presented.

1.1 Background

Reinforced concrete, consisting of cement, aggregate, additives and reinforcing steel, is the most used building material in the world. According to Garside M. (2020), approximately 4.1 billion metric tons of cement was produced in the world in 2020. A major byproduct of cement manufacturing process alone is the massive release of carbon dioxide into the atmosphere. There have been estimates that the cement manufacturing in itself stands for around 8 % of the world's total CO_2 emissions (Lehne & Preston, 2018). Similarly, approximately 7 % of the world's total CO_2 emissions are due to iron and steel industry (World Steel Association, 2017). For the future of this earth, these global emissions needs to decrease. As structural engineers, one of the major solutions for this is to minimise the material usage of both concrete and steel by optimising the structural elements.

Structures are often divided into continuity regions, where beam theory applies, and discontinuity regions where the force patterns are more complex (Engström, 2015). One common method for analysis and design of the discontinuity regions is to set up a strut-and-tie (ST) model, where the forces in the concrete are modeled with a truss-like structure. This method is originally based on two-dimensional (2D) situations, so when facing three-dimensional (3D) problems the designer usually sets up a system of 2D ST models in multiple planes. Alternatively a model in 3D is constructed and then simplified to 2D when being calculated. This is not fully efficient because it requires a lot of iterations when several 2D ST models are based on each other. Besides being time consuming, this also results in extensive concrete and reinforcement usage. However, during the previous decade, suggestions for setting up ST models in 3D have been given by researchers such as Chantelot and Mathern (2010). This particular method was tested on pile caps, and resulted in a decrease of the reinforcement usage.

The method of strut-and-tie is heavily focused on geometry, with a fictitious "truss system" that can be chosen in different ways. A possible way of constructing the ST model is by using parametric design, where the struts and ties can be visually generated with help of a software programs such as Rhinoceros 3D and Grasshopper. The ST models could then easily be iterated and compared with each other.

1.2 Aim

The intended outcome of the thesis has been to investigate if the proposed 3D strut-and-tie method by Chantelot and Mathern (2010), is practically implementable for design of three-dimensional structures according to Swedish industry standards, and if it could be further enhanced with parametric modelling. The need for such models has been expressed by structural engineers at Ramboll Sweden in order to find better methods to design three dimensional discontinuity regions.

The following objectives have been defined for this thesis:

- Conduct a literature study of ST modelling, parametrisation and structural optimisation.
- Conduct a case study on a detail of a bridge with the knowledge obtained from the literature study by constructing a parametric ST model in 3D for that specific detail. Further, perform calculations following the regulations set by Eurocode.
- Compare and evaluate the results of the obtained 3D ST model with the results obtained by Ramboll using the conventional ST method.

1.3 Research Questions

The research questions that have been answered in this thesis are the following:

- Can the enhanced 3D strut-and-tie method proposed by Chantelot and Mathern (2010) successfully be used in the design of bridge details?
- Is it possible to generate and optimise a 3D strut-and-tie model of a specific bridge detail in a discontinuity region by the use of parametric design?

1.4 Methodology

The overall methodology for this thesis has been divided into two main phases; a literature study phase and a case study phase. An in-depth description of these two phases is presented below.

Literature study
The main focus of the literature study was on the proposed 3D ST method by Chantelot and Mathern (2010), but also other publications on the subject of 3D ST methods has been studied. In addition, parametric design and structural optimisation was also studied, including different types of optimization methods, for example topology optimisation.

Furthermore, a deeper understanding of the functionality of the software programs Rhinoceros and Grasshopper was needed. This was achieved throughout the thesis, simultaneously with the other steps.

Validation of theory with a case study

The next phase in this master thesis project was to validate if the proposed method by Chantelot and Mathern (2010) worked for a detail other than a pile cap. In collaboration with practicing structural engineers, who had given access to their data for an already designed bridge, a comparison was conducted between the model based upon the proposed method and the model developed with conventional engineering standards. The model based on the proposed method was developed and calculated parametrically, which enabled extensive manual optimisation of the model using parametric software programs such as Rhinoceros 3D, Grasshopper and add-ons.

1.5 Limitations

The general limitations for this thesis are presented below. To note here is that additional limitations are presented in Chapter 3 in connection to development of the ST model for the specific case study in question.

- When the method of 3D ST was evaluated on a case from engineering practice, the work was concentrated on only one type of bridge detail (a corbel).
- The model develop was limited to a design that follows current standards of construction in Sweden.
- The theory presented by Chantelot and Mathern (2010) was validated, but this validation only considered the central parts in the theory that were applicable in the case study of this thesis.
- The software used in this thesis was limited to Rhinoceros 3D and Grasshopper with add-ons.
- Only the Ultimate Limit State (ULS) was investigated in this thesis.

2

Theory

Since the invention of reinforced concrete, the theory of how concrete behaves under loading has been successively developed and refined. During this thesis, a great amount of theory was studied. Some theory is general and can be applied on all types of concrete structures, and some theory is more specific and concerns more rare situations.

In this chapter, a brief background together with the theory of the strut-and-tie (ST) method and structural optimisation is presented.

2.1 B- and D-regions

One way to keep the concrete theory general, although still specific, is to divide the concrete structures into "B-regions" and "D-regions". B-region means "Bernoulli-region" or "beam-region", and is a region where one can apply Bernoulli's hypothesis that plane sections remain plane and therefore the distribution of strain is plane as well (Engström, 2015). Slender beams and plates are two examples of structures that can be designed using this theory. D-region means "discontinuity region", and in these areas Bernoulli's hypothesis can not be used in the design as the distribution of strain is non-linear. This is the case near corners and concentrated loads (Schlaich, Schafer, & Jennewein, 1987). The reason for why a region becomes a D-region depends on the geometry of the region (so-called geometric discontinuities) and how it is loaded or supported (static discontinuities). See Figure 2.1 where highlighted sections are D-regions.

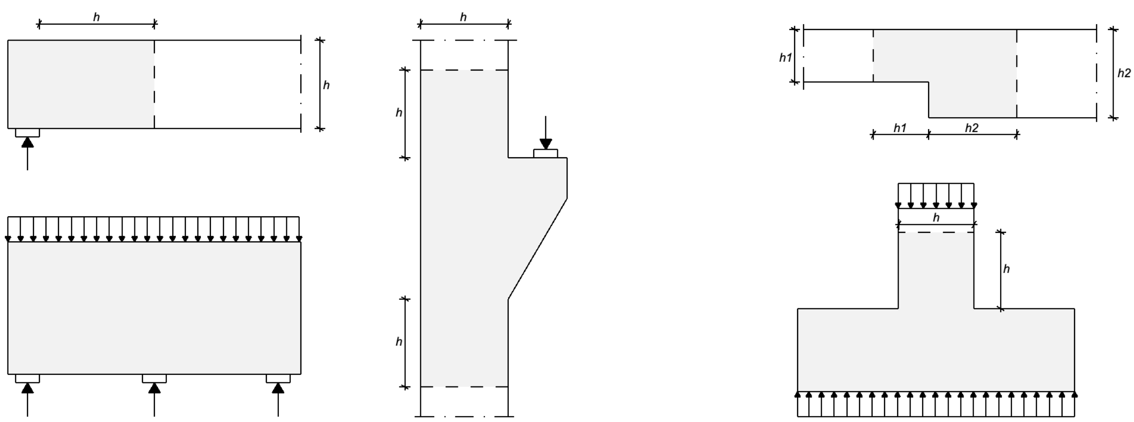


Figure 2.1: *Static discontinuities on the left and and geometrical discontinuities on the right. Based on drawings by Schlaich (1987).*

As an example, if a beam is loaded at the end with a distributed load, the whole beam can be treated as a B-region. But if the beam would be subjected to a concentrated load instead, the outermost part would be a D-region that stretches a specific distance into the beam until the stress has spread out and is linearly distributed over the beam (Engström, 2015). See Figure 2.2.

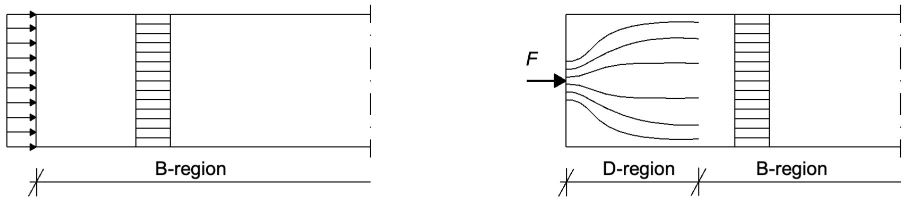


Figure 2.2: *Example of D-regions as described by Schlaich (1987).*

2.2 Strut-and-tie method

The ST method is a method that describes how a cracked reinforced concrete structure carries the internal forces in ultimate limit state (ULS). This method is especially useful when defining and calculating forces in D-regions. A ST model is built up by concrete struts that carry compression forces, and ties of reinforcing steel that carry forces in tension. The struts and ties are then connected by nodes. One can say that the ST model of a structure is a simplification of the stress field in the structure under loading. A simple ST model of a deep beam in loading is shown in Figure 2.3, where the struts are visualized as dashed lines and the ties as continuous lines (which is the convention that will be used further on in this thesis).

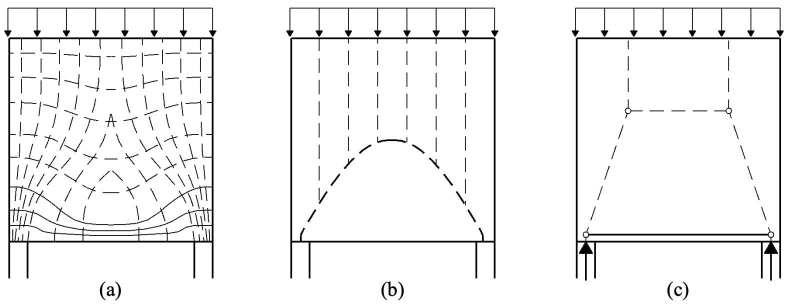


Figure 2.3: *Stress field in deep beam presented as (a) stress trajectories, (b) simplified interpretation and (c) simple ST model based upon interpretation. Adopted from Engström (2015).*

The ST method is a method based on the theorem of plasticity, and thus presents a design method for the ULS. This method underestimates the failure load of the structure compared to reality (on the safe side), and is therefore considered to be a *lower bound approach* (Engström, 2015).

The first step in the search of a ST model of a structure is to do an analysis on how the loads are transferred through the structure, and which stresses that occur. This analysis can be performed either manually by using the load path method, or computer-aided by the use of finite element method.

2.2.1 Load path method

The load path method is a manual approach of simplifying the stress field in a structure subjected to loading. This procedure can be divided in steps as follows (Engström, 2015):

1. The load dividers need to be found. If the load is distributed, use load dividers where the shear stress is zero in order to divide the stress field in appropriate parts. There will be one load path in each part.

2. Make a sketch of the stress field. At a distributed load or support, the stress width of the stress field is of equal size. At a concentrated load or support the stress field is concentrated, but spreads out as it reaches further away from the support.

3. Sketch the resultant of each stress field, use smooth curves.

4. Make sure that the load path sketched is characteristic to the shape of the stress field. If this is not the case, make a refined division of the load. An example of this is shown in Figure 2.4.

5. Make a principal identification of the transversal forces that are located at the curves of the load path.

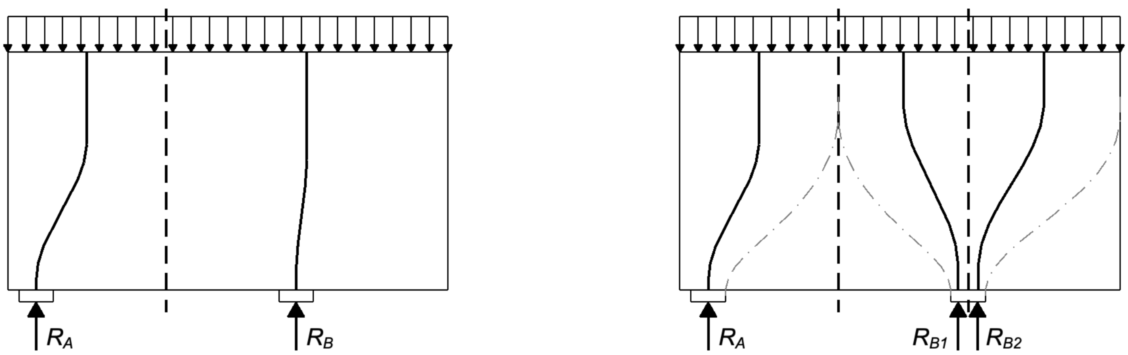


Figure 2.4: *The load path at support R.B at the left figure is over-simplified. The figure to the right show a refined model. Adopted from Engström (2015).*

The rules for obtaining a load path are presented by Engström (2015):

- The load path should represent the resultant of the stress field in each section.
- Load paths cannot cross each other.
- At the boundary of the discontinuity region, the direction of the load path should be the same as the direction of the load or the support reaction.
- Close to a concentrated force, the bend of the load path should be sharp.
- Further out from the concentrated force, the bend of the load path should be soft if it changes direction.

To note here is that the load paths can only change direction when affected by transverse forces (a strut or a tie), due to equilibrium. When the load path takes a sharp bend the transverse forces need to be concentrated, and at a soft bend the transverse forces are distributed on the length of the curve.

When the load paths in the structure have been found and checked, the next step is to translate the model into a ST model. The load paths are transformed to straight struts and ties, that are connected with nodes located at the load paths curves.

2.2.2 Finitie element method

As an alternative, a finite element (FE) analysis of a structure with homogeneous material can be performed to obtain the stress field in the structure. Compared to the manual sketches with the load path method, the stress field obtained by FE analysis offer the possibility of being closer to the real world case, see figure 2.5. This is of course dependent on how detailed the FE model was modelled. FE analysis can also be of great help when the geometry is complicated and the load paths are not intuitive to find. But if the model is not modelled correctly, the FE analysis can give inaccurate results.

One thing to keep in mind is that the FE analysis does not give the ST model automatically; the given stress flow needs to be translated into a ST model manually, just as with the load path method.

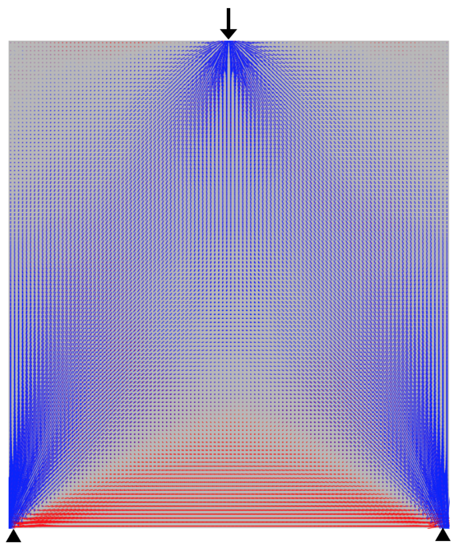


Figure 2.5: *Example of FE analysis showing stress flows in a deep beam subjected to a point load.*

2.3 Eurocode - Rules to follow

The development of a ST model can be done in multiple ways. However in practice, it is important that the design solution is in accordance with the valid design. Today, there are several codes in the world that allow the usage of ST method for design of concrete structures, such as Eurocode 2, Canadian Concrete Code and ACI Building Code.

Eurocode 2 (2004) present rules and checks that must be fulfilled when creating a ST model. Additional presented guidelines for ST models in regards to the angles between different strut and ties has been developed by Shäfer (fib, 1999).

2.3.1 Limitation of angles

A great deal of attention needs to be paid to angles when developing a ST model. According to Chantelot and Mathern (2010), strain compatibility problem can occur if the angles between strut and ties are too small. The same problem can also occur if the angle is too high. Furthermore, the assumption of inappropriate angles may result in high need of plastic redistribution in order to reach equilibrium of the ST model.

To obtain the optimal angles between strut and ties, Shäfer developed recommendations of angles that were published on fib bulletin 3 (1999). These recommendations are presented below and in Figure 2.6.

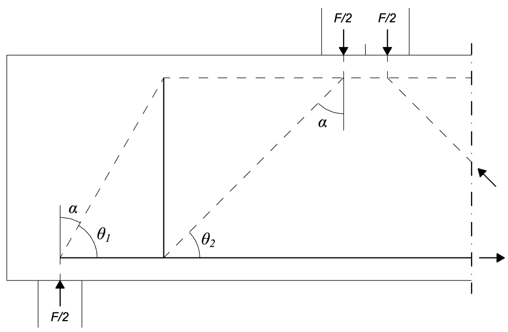


Figure 2.6: *Truss structure with recommended angles defined.*

Deviation angle of concentrated forces

$$\alpha \approx 30^\circ \quad \text{and} \quad \alpha \leq 45^\circ \tag{2.1}$$

Deviation angle of strut meeting single tie

$$\theta_1 \approx 60^\circ \quad \text{and} \quad \theta_1 \geq 45^\circ \tag{2.2}$$

Deviation angle of strut meeting two perpendicular ties

$$\theta_2 \approx 45^\circ \quad \text{and} \quad \theta_2 \geq 30^\circ \tag{2.3}$$

2.3.2 Limitation of strut stresses

In a ST model, the compressive stresses of a concrete structure are carried by the concrete struts with the design strength capacity of these struts being determined by the multi-axial state of stress according to Hendy and Smith (2007). In a multi-axial state of stress, the design strength capacity will be affected positively if the transverse stresses are in compression but if the stresses are in tension, they will affect the capacity negatively. Eurocode 2 (2004) presents two cases on how to obtain the maximum compressive stresses $\sigma_{Rd,max}$, either through transverse tension or transverse compression as seen in Figure 2.7. These cases are simplified and conservative limits for the compressive stress (Hendy & Smith, 2007).

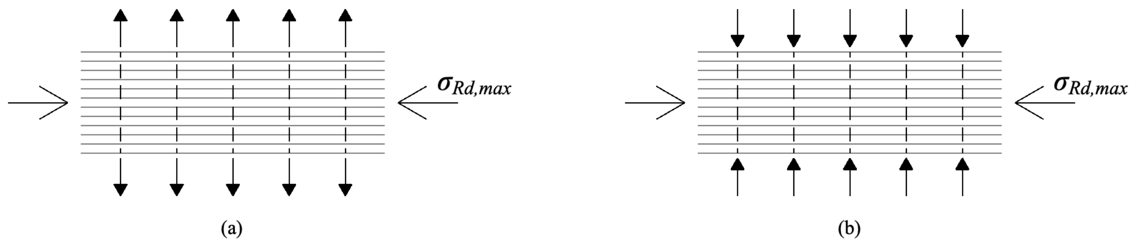


Figure 2.7: *Struts subjected to (a) transverse tension (b) transverse compression. Adopted from Eurocode 2.*

(a) Transverse compression or no transverse stress

$$\sigma_{Rd,max} = f_{cd} \quad \text{where} \quad f_{cd} = \alpha_{cc}\frac{f_{ck}}{\gamma_c} \tag{2.4}$$

According to Hendy and Smith (2007), this type of stresses are usually not possible since transverse tension can occur easily by the bulging of a compression strut between nodes.

(b) Transverse tension or Cracked compressive zones

$$\sigma_{Rd,max} = 0,6v^{'}f_{cd} \quad \text{where} \quad v^{'} = 1 - \frac{f_{ck}}{\gamma_c} \quad \text{and} \quad f_{cd} = \frac{f_{ck}}{\gamma_c} \tag{2.5}$$

When transverse tension occurs in a concrete strut or the strut is in a cracked compression zone, Eurocode 2 (2004) suggests a reduction of the design strength of the compressive stress as seen in Equation 2.5. This equation gives the most conservative amount of reduction and can therefore be used as an approximation. However, this value can be changed depending on the application. Hendy and Smith (2007) specifies several situation and defines reduction values for each of these situation.

2.3.3 Limitation of tie stresses

In a concrete structure, the tensile stresses are usually carried through steel reinforcement and can be symbolically represented as ties in a ST model. The design strength of the tensile stresses in a ST model is determined by the amount of required area of reinforcement. This is defined by the following equation:

$$A_s \leq \frac{T}{f_{yd}} \tag{2.6}$$

Here, T is the tensile stresses in the tie and f_{yd} is the design yield strength of the steel reinforcement.

In addition, when defining the limitation of tie stresses, it is important to investigate anchorage of the steel reinforcement. In Section 2.3.5, steel reinforcement and the requirements in a structure according to Eurocode, is presented.

2.3.4 Nodes

In a ST model, the struts and ties intersect each other at nodes. According to Hendy and Smith (2007), a node can be described as a volume of concrete where strut and ties intersect with dimensions determined by the geometry rules defined in above chapters. Furthermore, for a ST model to be valid, it shall be in equilibrium and this requires that all nodes also should be in equilibrium with regards to the forces that meet in the node (Engström, 2015). In this thesis, a node is defined as the point where struts and ties meet, in other words, a coordinate. The nodal zone is defined as the zone around the node where the geometry of the struts and ties are intersecting (as their centroidal axes meet). The nodal zone is an area in 2D ST models, and a volume in 3D ST models. See Figure 2.8 for a comparison of a node and nodal zone.

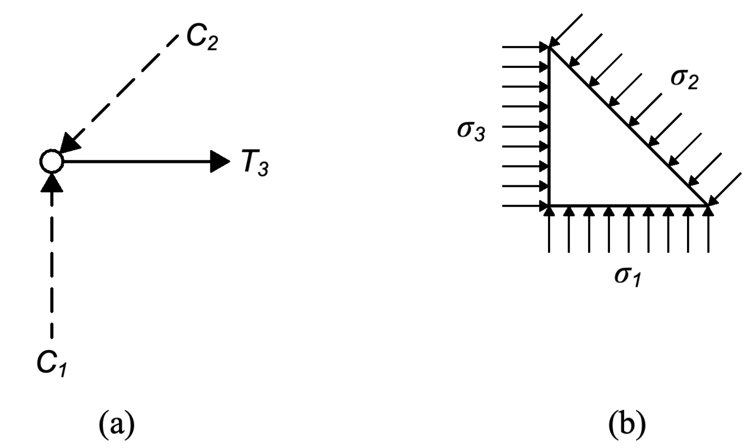


Figure 2.8: *Example of: (a) Forces acting on node (b) Stresses acting on nodal zone.*

Generally, a node can be either a concentrated or a distributed node. A concentrated node occurs where concentrated forces act such as external loads, supports or reaction forces. A distributed node appears when the stresses are distributed over a larger area, e.g. at distributed loads or reactions, or where distributed stress fields meet. The concentrated nodes need to be checked with Equation 2.7 due to the risk of concrete overstressing (Engström, 2015).

Similar to limitations of angles between struts and ties, Eurocode 2 (2004) has defined the maximum allowed compressive stresses for different types of concentrated nodes that are summarised in Figure 2.9. Abbreviations for describing the different types of nodes are usual, where C stand for compression, and T for tension. In nodes with more than 3 struts or ties, usual in 3D cases, the nodes are defined with a number instead that indicates the number of elements of each type. For example 2C2T is a node with two struts and two ties.

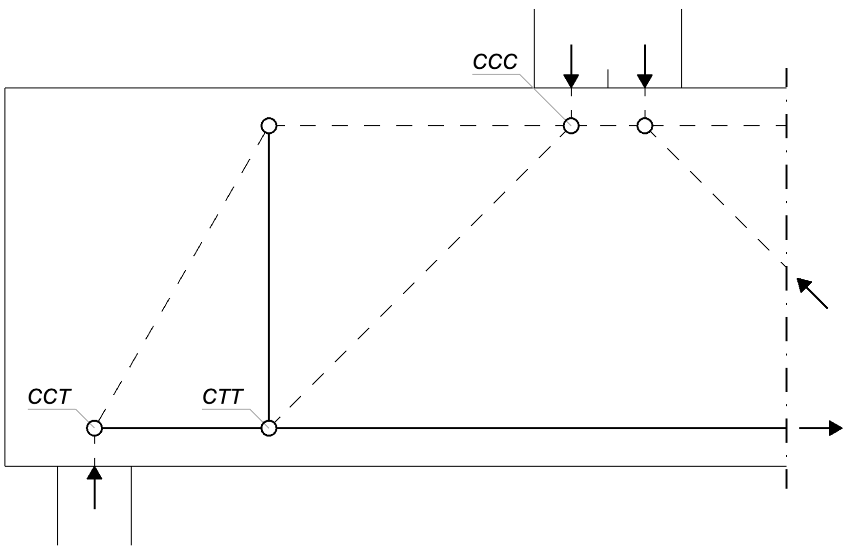


Figure 2.9: *Examples of node types, in a deep beam subjected to concentrated loads.*

Compression - Compression - Compression (CCC) node
A CCC node is characterized by only having compressive struts intersecting.

$$\sigma_{Rd,max} = k_1 v' f_{cd} \quad \text{where} \quad k_1 = 1 \tag{2.7}$$

Compression - Compression - Tension (CCT) node
A CCT node is characterized by having one of the intersecting elements in tension while the rest are in compression.

$$\sigma_{Rd,max} = k_2 v' f_{cd} \quad \text{where} \quad k_2 = 0,85 \tag{2.8}$$

Compression - Tension - Tension (CTT) node
A CTT node is characterized by having more than one of the intersecting elements in tension.

$$\sigma_{Rd,max} = k_3 v' f_{cd} \quad \text{where} \quad k_3 = 0,75 \tag{2.9}$$

In regards to a CTT node, Eurocode 2 (2004) has defined that if at least one of the following criterion applies, the design compressive stress can be increased with 10%:

- The node has triaxial compression.
- All angles between struts and ties in a node are 55° or more.
- The stresses are uniform at supports or point loads, and the node is confined by stirrups.
- Arrangement of steel reinforcement is in more than one layer.
- The node is reliably confined by means of bearing arrangement or friction

Triaxially compressed node
Eurocode also states that for triaxial compression, the capacity can be increased with the use of the equations for confined concrete (Equation 3.24 and 3.25 in Eurocode 2 (2004)) and the following equation.

$$\sigma_{Rd,max} \leq k_4 v' f_{cd} \quad \text{where} \quad k_4 = 3 \tag{2.10}$$

2.3.5 Reinforcement

In the design of ST models, it is important to do reinforcement detailing checks. For example, how much reinforcement that will be needed in a particular tie, and check if the amount of reinforcement bars does fit within the given tie region. Another aspect to check with the detailing is the anchorage and the shape of reinforcement bars for a particular section of the detail. In Figure 2.10, a set of different shapes of reinforcement bars is shown. The naming of these shapes has been defined according to Swedish building industry in this thesis (BE Group, 2017).

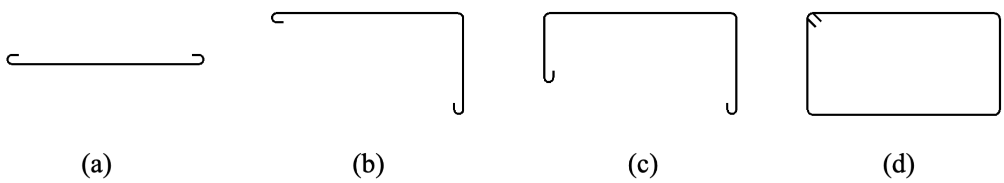


Figure 2.10: *Examples of different shapes of reinforcement bars: (a) Type A (b) Type B (c) Type C (d) Type N.*

2.3.5.1 Minimum bar spacing

According to Eurocode 2 (2004), the minimum spacing horizontally and vertically between bars shall have a minimum clear distance according to Equation 2.11.

$$\mathrm{s_{min}} = \max(1,0 \cdot \phi \ ; \ d_g + 5\mathrm{mm} \ ; \ 20\mathrm{mm}) \tag{2.11}$$

The above equation gives the theoretical minimal bar spacing. However in most cases, the spacing becomes a challenge in terms of the practicality of using it in real world construction. To adjust for practical reasons, a minimum bar spacing value of approximately 100 millimeters is aimed at in the Swedish building sector, according to structural engineers at Ramboll Sweden (2021).

2.3.5.2 Anchorage

Reinforcing bars in a concrete structure need to be anchored correctly so that the forces can safely be transmitted to the concrete without the risk of cracking or spalling. Independently of the shape of the anchorage, a design anchorage length needs to be determined according to Equation 2.12.

$$l_{bd} = \alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ l_{b,rqd} \geq l_{b,min} \tag{2.12}$$

Here, α_1 , α_2 , α_3 , α_4 , α_5 are coefficients given in Eurocode 2 (2004). $l_{b,rqd}$ is the basic required anchorage length for anchoring the force T from Section 2.3.3. $l_{b,min}$ is the minimum anchorage length. The definition for how to calculate $l_{b,rqd}$ and $l_{b,min}$ can be found in Eurocode 2 (2004).

The anchorage of a reinforcement bar is possible not just with straight bars but also with other methods such as bending or looping. This can be useful when e.g. there is limited space in and behind the node where the force should be anchored. If the anchorage is bent, it is vital to check that the minimum mandrel diameter of the bend is fulfilled to prevent damage to reinforcement. Further, anchorage length in bent bars should be measured along the center line of the reinforcement bar.

2.3.5.3 Laps

In the real world, reinforcement bars have a limit to their lengths and dimensions due to transportation and workability. Thus, a way is needed to transmit the forces from one bar to another. A common way in the building industry is to transmit forces through lapping of bars. According to Eurocode 2 (2004), the design of these laps needs to detailed properly with the transmission of forces being safely assured. This is done by calculating the lap length (Equation 2.13) which determines the distance of overlap needed between two bars to have a safe transmission of forces.

$$l_0 = \alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_5 \ \alpha_6 \ l_{b,rqd} \geq l_{0,min} \tag{2.13}$$

α_1 , α_2 , α_3 , α_5 , α_6 are coefficient given in Eurocode 2 (2004). $l_{b,rqd}$ is the basic required anchorage length for anchoring the force A_s from Section 2.3.3. $l_{0,min}$ is the minimum lap length possible for a certain type of reinforcement. The definition for how to calculate $l_{b,rqd}$ and $l_{0,min}$ can be found in Eurocode 2 (2004).

2.3.5.4 Bundled bars

Sometimes it is not possible to fit the required bars within a given tie geometry with a minimum bar spacing. Fortunately, this can be resolved by bundling up to four bars into one bundled bar with an equivalent diameter (Eurocode 2, 2004). This equivalent diameter can be calculated according to Equation 2.14.

$$\phi_n = \phi\sqrt{n_b} \leq 55\text{mm} \tag{2.14}$$

Here, n_b is the number of bars in a bundle. According to Eurocode 2 (2004), this value can be up to 4 for vertical bars in compression and for bars in a lapped joint. If this requirement is not fulfilled, the value can not be greater than 3.

Regarding anchorage for bundled bars, the individual bars with in the bundle should be staggered from each other with a distance greater than 1,3 times $l_{b,rqd}$. Further, in regards to lapping of bundled bars, it should be known that bundles of more than three bars should not be lapped (Eurocode 2, 2004).

2.4 Force actions

To develop a ST model, the designer needs to define several parameters with one of them being the mechanism for transfer of forces. One of the forces that will govern how a ST model should look like is the shear transfer forces where Chantelot and Mathern (2010) have presented six types of shear transfer mechanisms based upon the models of Muttoni *et al.* (2008). However the conclusion was that only two of the mechanisms are relevant for transfer of shear forces in a practical ST model: direct arch action and truss action.

2.4.1 Direct arch action

The mechanism of direct arch action is based on that the forces from a load to a support is only transferred through compression in the concrete struts and tension in the main steel reinforcement, see Figure 2.11.

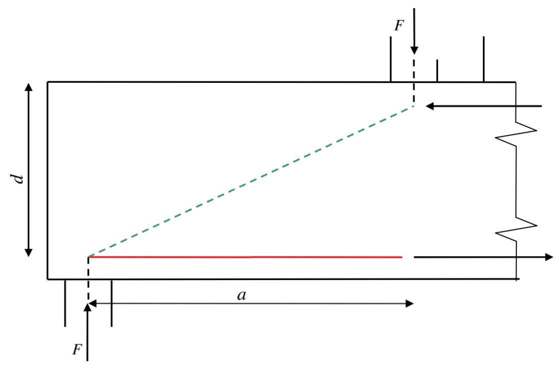


Figure 2.11: *Direct arch action. Reproduced from Chantelot and Mathern with permission.*

The simplicity of this mechanism makes it preferable in design. However Chantelot and Mathern (2010) emphasises that the use of direct arch action in design is limited due to the stress distribution capacity of concrete and the angle of the strut.

2.4.2 Truss action

Compared to direct arch action, a truss action mechanism transfers shear forces by combining compressive concrete struts and tensile reinforcement ties with stirrups acting as ties as illustrated in Figure 2.12.

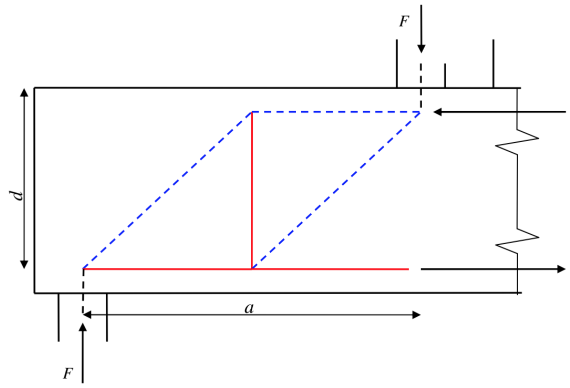


Figure 2.12: *Truss action. Reproduced from Chantelot and Mathern with permission.*

A conventional ST method is usually based upon this type of truss action models developed from the requirements on geometrical angles, struts, ties and nodes according to building codes as explained in Chapter 2.3.

Even if it is preferable to use a direct arch action approach when developing a ST model, this is in most cases not possible while fulfilling all the requirements in building codes. For many cases, the only solution left to use is the truss action approach usually due to the angle requirements set by Eurocode 2 (2004) (Engström, 2015).

In Figure 2.12, the stirrups are placed as vertical ties, but stirrups can also be placed with an inclination. Using inclined shear reinforcement is often more beneficial from a structural point of view according to Chantelot and Mathern (2010) who have concluded that inclined stirrups can increase the shear strength capacity of the struts by 60 percent compared to vertical stirrups. To use inclined stirrups is in practice not common due to the addition of complexity in the construction phase, but it is possible through using other shapes of reinforcement bars that could fulfil the same purpose.

2.4.3 Strut actions

When developing a ST model, it is essential to know the path that the forces will take from one point to another. Regardless of force action mechanism, there are three types of strut actions that can occur in a concrete structure, see Figure 2.13.

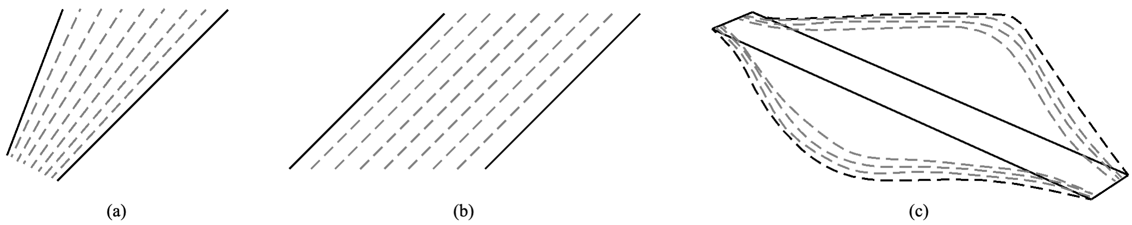


Figure 2.13: *Different types of strut actions. (a) Fan-shape strut (b) Prismatic strut (c) Bottle-shape strut*

2.5 Strut-and-tie method in 3D

The theory about the ST method that has been presented until now has been two-dimensional. The calculation of struts, ties and nodes has either been in the same plane or projected onto one plane and calculated that way. But, in reality, most structures carries forces in all three dimensions. When designing them by the use of the ST method, the usual way is to make a simplification of the force flow in 2D in order to be able to apply the ST method and the rules that are presented in Eurocode 2 (2004). These simplifications are often sufficient in design, although it underestimates the real capacity of the structure. There have been methods proposed for how to construct and calculate ST models in three dimensions, for example a simplified method presented by Klein (2002) and a more refined version presented by Chantelot and Mathern (2010). The method presented by Chantelot and Mathern provides a consistent way in defining the nodal zones in 3D and will be presented further on in this thesis.

2.5.1 Method today

ST models in 3D can be divided into two subcategories, ST models in multiple planes and ST models with 3D actions, where the first category consists of several 2D ST models acting in combination rather than in real 3D action.

Structures that are being built up by plate-like structures are three dimensional in themselves, but the structure can be simplified into 2D planes that are assembled together in a 3D space. A ST model in 2D can then be set up for every such plane, acting in its own plane. This results in a structure that is distributed in 3D, but consists of elements that are modelled by ST models in 2D, see Figure 2.14a.

Traditionally, 3D ST models have been calculated by the use of 2D ST models in orthogonal planes. In this way the struts and ties that are in 3D are divided into components instead, that take the loads in their directions. As an example, the 3D ST model in Figure 2.14b can be calculated by using a diagonal section in order to make the model two-dimensional. By this division of the model, the general rules for nodes apply.

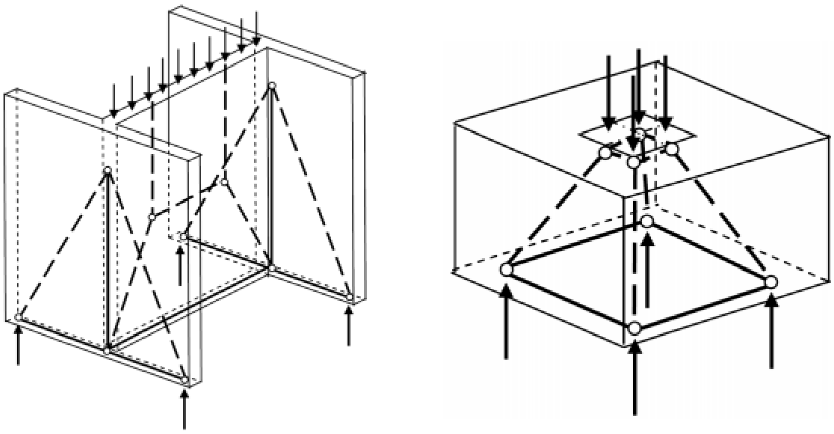


Figure 2.14: *a) 2D ST model in multiple planes b) ST model in 3D (Engström, 2015).*

2.5.2 The enhanced strut-and-tie method

One of the main concerns when considering ST models in 3D is how to design and check the nodal zones. The geometry gets more complex when one node is the meeting point of several struts and ties from different directions. One proposed method, that also was part of the American building code (ACI Building Code, 2008), was to simplify the node geometry and not demand an exact compatibility between the geometry of the struts and ties that meet in a node (Chantelot & Mathern, 2010). However this particular method is insufficient since the method does not correlate to the real world case accurately.

In the Master thesis *Strut-and-tie modelling of reinforced concrete pile caps* (Chantelot & Mathern, 2010), a way to consistently define the 3D nodal zones is proposed and is further presented in this chapter. This method takes into consideration that the struts and ties are three dimensional and presents how these elements can meet consistently in the node. Chantelot and Mathern (2010) formulates methods for calculations and proposes methods for solving different situations that occurs in 3D. Note that the method presented further in this chapter treats the central part of the theory developed by Chantelot and Mathern, and other parts that are of interest in the case study that was performed in this thesis. Other parts of the theory that are not presented in this chapter is neither used nor validated. From now on, this part of the method proposed by Chantelot and Mathern will be mentioned as *the enhanced method*

Furthermore, the thesis written by Chantelot and Mathern (2010) focused on pile caps and thus the geometry of the examples that are presented in this chapter are based on pile cap situations.

In order to distinguish between the different types of ST models, they will now be referred to as the *skeleton* and *expanded* model. The skeleton model is the original ST model where the forces are visualized as one dimensional lines (the figure to the right in Figure 2.15). In the expanded model, the three dimensional volume of the strut and nodal zone is visualised (the figure to the left in Figure 2.15).

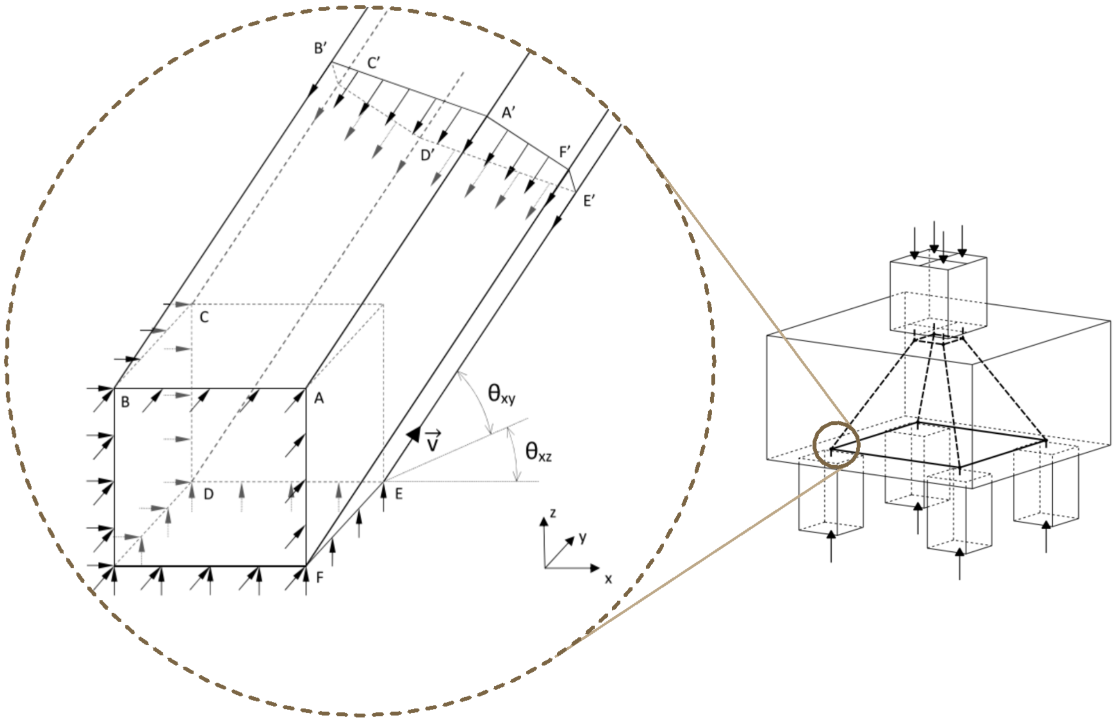


Figure 2.15: *Nodal zone, 2C2T. Reproduced from Mathern et al. (2017) with permission.*

According to the enhanced method, a consistent nodal zone with forces in three dimensions needs to fulfill three criterion:

- All the faces of the nodal zone need to be under stress, either by compression or tension. This is needed in order to keep every node in equilibrium.
- The centroid axes of the three dimensional struts and ties used in the model should correspond to the axis that the ST model consists of.
- When multiple struts meet in a nodal zone, they should not overlap before reaching the boundary of the nodal zone.

In Figure 2.15 a sketch of a 3D nodal zone is presented where two struts meets two ties (2C2T). In this node two ties and a vertical strut have rectangular cross-sections, and the centre of the nodal zone is found where the centroid of the struts and ties meet. The shape of the inclined strut and its cross-sectional area can then be calculated based on the nodal zone geometry. If for example, the vertical strut is given by a support, the loading- and support areas need to be defined and serve as a first input parameter. Secondly, the height of the node is defined, and is based on the horizontal member in the node. This is the height of the influence area (in case of a tie) or the height of the strut. When these initial conditions has been defined, the corners of the nodal zone can be identified. In some examples as Figure 2.15

shows, the corners are given by these conditions (A-F in the figure). In other cases further assumptions have to be done before all the corners can be identified.

In Figure 2.16 two versions of 3D ST models are presented. The example in this figure shows how vertical forces from e.g. a column is transferred to the piles through a pile cap. Note that only the two struts in the front are visualized in this figure.

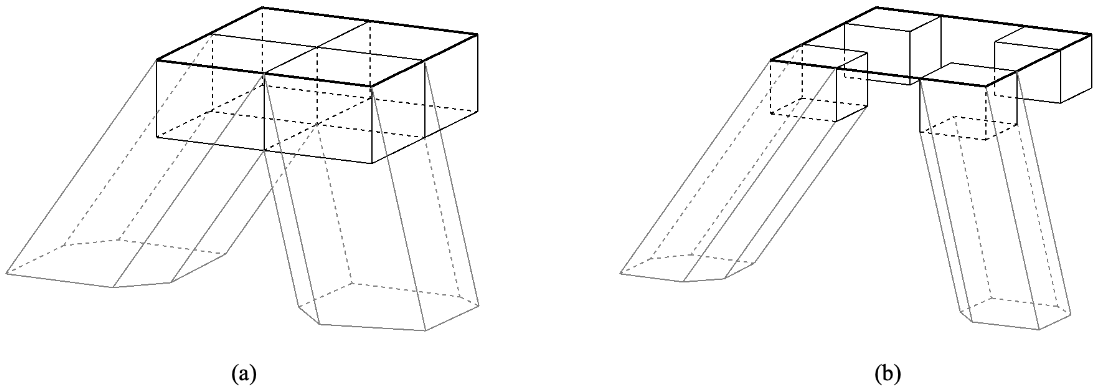


Figure 2.16: *Similar models with different sizes of the nodal zones.*

When nodal zones are placed tightly together they seem to form a joint nodal zone, but it is important to understand that this is not the case. Figure 2.17 shows a clarification of how the nodal zones in Figure 2.16a are placed.

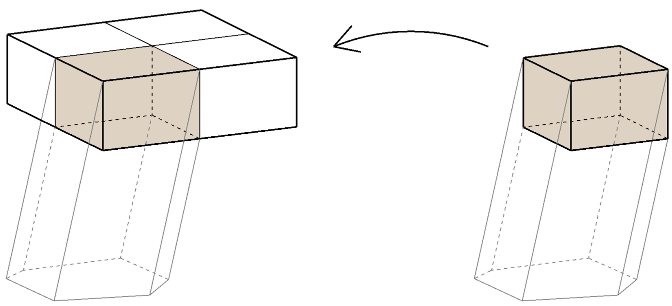


Figure 2.17: *Four nodal zones placed together*

If nodal zones of more complex geometries are used, the design process will also be more complex. As the intersection of the central axis for the struts and ties in a node will then need to be identified, the position of the node might be difficult to establish. In order to keep the model simplified it is suggested by Chantelot and Mathern (2010) that the nodal zone geometry in the 2C2T-node in Figure 2.15 is used in 3D ST models, and therefore this thesis is based on this type of nodal zones. For further information about complex nodal zone geometries, see the work by Chantelot and Mathern (2010). The nodal geometry in Figure 2.15 is called the *cuboid nodal zone*, and it is stated that every three-dimensional concentrated nodal zone can be built up by using cuboid nodal zones, the two dimensional elementary 3C-node, and the method for combining struts. The method for combining struts is not used in this thesis, and is therefore not mentioned in this theory part either. As a tie can also be seen as a strut acting from the other direction, the cuboid nodal zone can be seen as a 4C-node.

2.5.2.1 Cross-sectional area of hexagonal struts

It was presented in Chapter 2.3.2 that the nodes in the ST model need to be checked to make sure that the stress does not exceed the given capacity. In 2D ST, this is rather simple as the width of the strut is the length of a line. In 3D ST, an extra dimension is added so the stress is found by dividing the force with the cross-sectional area of the strut. When using cuboid nodal zones the strut has hexagonal cross-section shape that is not as intuitive to calculate as in 2D ST, see Figure 2.18.

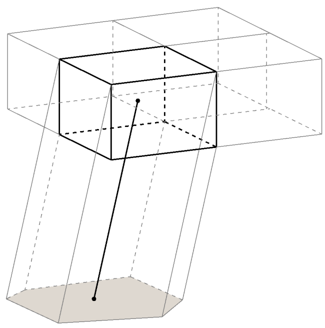


Figure 2.18: *Hexagonal cross section area of struts, as in the highlighted part.*

One method for calculating the hexagonal cross-section manually is based on the direction vector of the strut, and the length of the original sides of the parallelepiped that the hexagonal strut is meeting (see Figure 2.15). The area is then calculated by dividing it into triangles and using Heron´s formula (see Equation 2.15).

$$Area(triangle) = \frac{1}{4}\sqrt{(a^2+b^2+c^2)^2 - 2(a^4+b^4+c^4)} \tag{2.15}$$

Chantelot and Mathern (2010) uses this method to make a comparison of the strut areas in 2D and 3D for a 2C2T-node over a pile. In the comparison struts with varying angles were compared, and they found that the area with the enhanced model in 3D always resulted in larger areas (up to 29% larger). A larger area results in most cases in a higher lever arm, that lowers the horizontal force. The enhanced method therefore has potential to directly reduces material usage, as a lower horizontal stress reduce the amount of flexural reinforcement needed.

2.5.2.2 Design of nodal zones

As mentioned in Chapter 2.5.2, one of the rules in design of consistent nodal zones is that *the centroid axis of the three dimensional struts and ties used in the model should correspond to the axis that the ST model consists of.* The meaning of this sentence is that the center line of the expanded strut geometry should coincide with the line between the center of gravity of each nodal zone. In many cases, when using coboidal nodal zones, this line is the same as the original skeleton strut line. Another way of describing this is that if a cross-section is taken anywhere on the line between the center of the nodal zones, the center of the cross-section will coincide with the mentioned line. The cutting plane for the cross-section must be orthogonal to the line.

During the writing of this thesis, the authors found out that in order to follow this rule, the cuboidal nodal zones in each end of the strut need to be designed to fulfil one of the following criteria:

- The cuboidal nodal zones should have the same dimensions in all three directions
- The cuboidal nodal zones should have the same proportions, but not the same dimensions (the strut is *fan-shaped*). In this case the strut inclination in all the three directions also needs to be 45 or 0 degrees.

As an example, the rules are followed for the two nodal zones with the following dimensions and angles:

- **Nodal zones (x*y*z)** - (50*100*200) and (25*50*100)
- **Angle of the strut (x,y,z)** - (0, 0, 45) degrees

Figure 2.19 shows an example where the nodal zones have the same shape but not the same dimensions, and the angle of the strut is not 45 degrees. The solid line is the line between the center points of the nodal zones. The dashed line is the center line of the expanded strut, obtained by using a cutting plane orthogonal to the brown line.

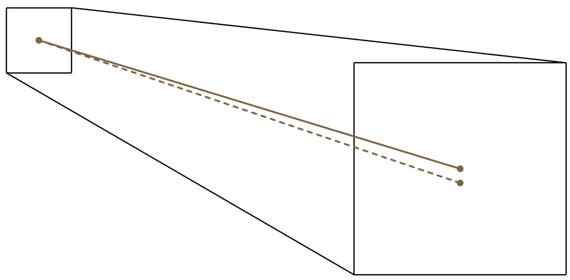


Figure 2.19: *Two nodal zones of different sizes in two dimensions. The center line of the expanded strut (dashed) and the skeleton strut (solid) must coincide. Illustrative figure.*

Biaxial and triaxial nodes

When designing 2D ST models, the model is usually placed in one plane and can be defined and explained in a two dimensional coordinate system. The elements that meet in a node can have different directions, but all the elements exist in the same plane. Therefore the node is defined as *biaxial*, see Figure 2.20a. In the enhanced method the model is designed and calculated in three dimensions. The nodal zones are three dimensional, as well as the placement of the elements. Nodal zones that connects elements in three dimensions are therefore defined as *triaxial*, see Figure 2.20b. Note that nodes in the enhanced method can be both bi- and triaxial, but nodes in 2D ST are limited to biaxial design.

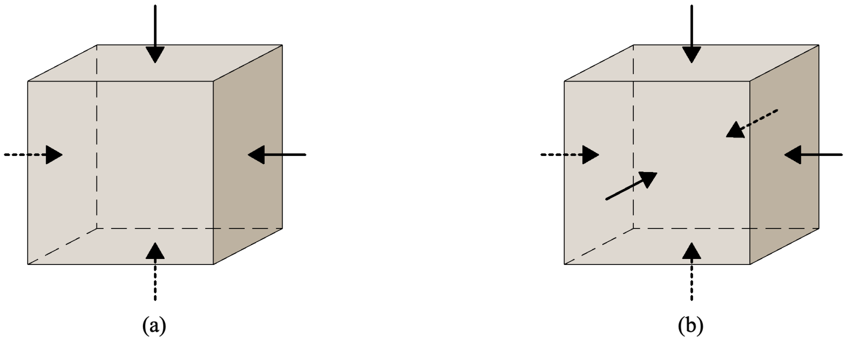


Figure 2.20: *(a) 3D biaxial node. (b) 3D triaxial node.*

The nodal zones in each end of a strut can under specific circumstances have different dimensions, when designing according to the enhanced method. As the strut has a larger cross section area in one end, the stress spreads out in a direction orthogonal to the strut-axis and induces tension in the nodal zone. When using triaxial nodes this stress can be handled in the node, as every face of the node is covered in stress from the elements meeting there. Of course these forces need to be accounted for in the calculations as well.

A special situation occurs when fan-shaped struts are used at biaxial nodal zones in a model based on the enhanced method, see Figure 2.21. If the strut only spreads out in the same plane as the elements that meet in the node, the stress can be accounted for in the equilibrium calculations of the node. But if the strut spreads out in the direction perpendicular to the nodal plane as well, there will be a need for adding reinforcement in the nodal zone to handle the tension stress. If the stress is below the tension capacity of the concrete, this can theoretically be neglected.

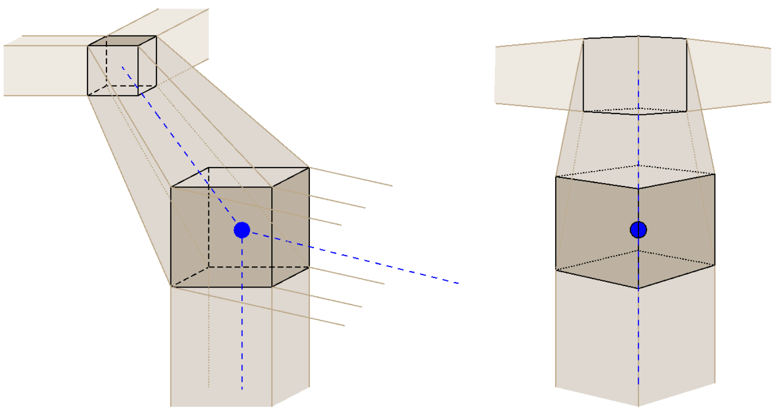


Figure 2.21: *A 2D biaxial nodal zone with a fan-shaped strut in a 3D environment*

2.5.2.3 Adaption of Eurocode 2 in 3D cases

According to Chantelot and Mathern (2010), the enhanced method can adopt the rules of 2D cases as determined by Eurocode 2. For example in a concentrated node, the angle between a strut and a tie should follow the same as Equation 2.2. Compared to 2D cases, the angle limitations for 3D cases applies to the real angle between strut and ties instead of the angle in a specific plane. Chantelot and Mathern has presented a formula to calculate the real angle between struts and ties using their vectors.

$$\theta = \text{acos}\left(\frac{\vec{v_{strut}} \cdot \vec{v_{tie}}}{|\vec{v_{strut}}| \cdot |\vec{v_{tie}}|}\right) \quad \text{where} \quad \theta \approx 60° \quad \text{and} \quad \theta \geq 45° \tag{2.16}$$

In the 3D case of a ST model, the nodes can sometimes be subjected to triaxial compression and thus, the maximum design compressive strength needs to be adjusted. Eurocode 2 (2004) has defined the limit in the case of triaxial compression if the distribution of the load is known for all directions for the struts, see Equation 2.10. In this equation, it can be seen that in the case of a 4C node in a 3D ST model, the node will have a higher concrete design capacity compared to 2D cases presented earlier in Chapter 2.3.4. The theory behind this is that in the case of a triaxial compression, the concrete in the nodal zone will be compressed in all directions and it is known through experiments that concrete as building material has much more strength in compression compared to tension. This phenomena is summarised in Eurocode with the coefficient k which changes depending what type of transverse stress is applied on the node.

When the nodes in a 3D case is not subjected to a triaxial compression, the rules of the 2D cases according to Eurocode 2 will be used.

2.5.3 Other proposed methods in strut-and-tie modelling

As shown in previous sections, Chantelot and Mathern (2010) have developed a method on how to set up and calculate ST models in 3D through primarily a refinement of the nodal zone theory. However, throughout the years, several other papers have been published with improvements, or suggestion of improvements, on ST modelling. Demeyere (2018) has listed and compared 11 methods that have been published throughout the years regarding how to develop ST models, such as Yun, Kim, and Ramirez (2018) and Meléndez (2017). While Meléndez (2017) focuses on the upper non-fixed nodes in 3D, similar to Chantelot and Mathern (2010), Yun et al. (2018) uses finite element and considers triaxial stress state of concrete to develop 3D ST models. The common theme for these papers is the usage of finite element software programs to develop ST models. Although there are other methods that can be used, the method of enhanced nodal zone geometry proposed by Chantelot and Mathern is the most comprehensive method in 3D ST theory published to this date according to the knowledge of the authors of this thesis.

2.6 Structural optimisation

A natural goal within the field of structural engineering, is to design structures that are safe and robust. However these structures are not always the most optimal ones, as they need to be on the "safe side". The world is in an era where all aspect of life should aim to be sustainable, whether it is environmental, economical or social sustainability. In the field of structural engineering the need of concrete in structures may decrease but will probably not vanish, thus the need of sustainable usage of reinforced concrete needs to be implemented. One of the solution is to use structural optimisation in order to minimize the material usage.

Structural optimisation can be described as finding the most optimal design of a structure based on given criterion. For example in the case of a concrete bridge, optimisation may be defined as minimising cost and/or environmental impact while still fulfilling all aspects regarding safety and functionality in regards to codes. According to Christensen and Klarbring (2019), structural optimisation can be summarised in three categories: size optimisation, shape optimisation and topology optimisation.

In this thesis, topology optimisation was used as the main optimisation procedure to find an initial stress topology. This decision was taken by the authors of this thesis based upon several reports for the last couple of years that have concluded that topology optimisation is a powerful tool to use for developing an optimised ST model such as X. Huang, Y. M. Xie, and M. C. Burry (2007) and Yang, Moen, and Guest (2015). See Section 2.6.1 for more on topology optimisation.

While topology optimisation is powerful in finding the optimal topology of a ST model, size or shape optimisation may also be useful in finding the most optimal sub-nodal zones. However due to limited time for writing this thesis, size and shape optimisation has not been investigated and could be grounds for further optimisation of ST models in 3D.

2.6.1 Topology optimisation

The use of topology optimisation has become one of the most used methods in finding optimal ST models. The reason behind this is that topology optimisation as a method have less requirement of input data to find the optimal design in comparison to the other optimisation types and thus being defined as the most general type of structural optimisation (Christensen & Klarbring, 2019). See Figure 2.22 for an example of structural topology optimisation. Within topology optimisation, there are several different types of method that can be used such as Homogenization method, Solid Isotropic Material Penalization (SIMP) and Evolutionary Structural Optimization (ESO) method. In general, what all of these methods have in common is that topology optimisation is an iterative design process which is automated through mathematical formulations. In other words, the method can be described as a computer aided approach.

In this thesis, the method of *Bi-directional evolutionary structural optimization*

(BESO) was explored for finding the most optimal ST model in preliminary stages. Below, the general theory behind the BESO method is explained.

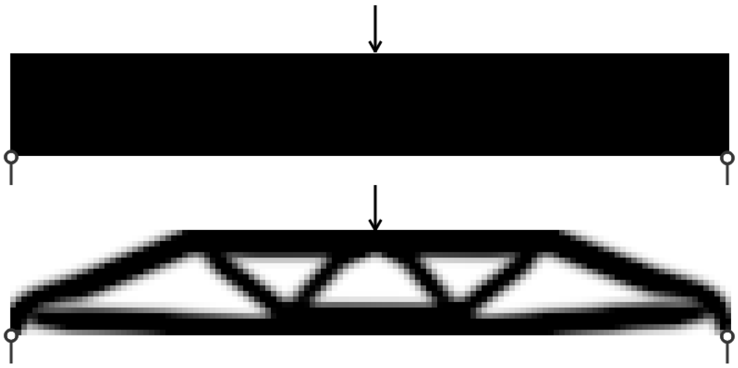


Figure 2.22: *Example of a simple topology optimisation of a beam.*

Bi-directional Evolutionary Optimisation

Bi-directional Evolutionary Optimisation (BESO) is a method developed directly from the evolutionary structural optimization method (ESO). ESO is a method that is based upon gradually removing elements in a structure which are ineffective in carrying loads within the given structure in order to find the most optimal design (Shobeiri & Ahmadi Nedushan, 2017). In a master thesis written by Nilsson and Öhman (2019) this method was explained in FE design terminology as removing ineffective meshed elements from the overall meshed structure so that only the effective elements remain in the end. But removal of elements through iterations can have negative impact on the overall optimisation in later stages. The elements which are removed in the beginning, based upon the initial criteria set by the designer, may affect the final optimisation negatively (Shobeiri & Ahmadi Nedushan, 2017). To fix this issue in the ESO method, BESO method was evolved. The two methods are quite similar but the BESO method both removes ineffective elements and adds elements next to areas that demonstrate high stresses. This is an advantage in terms of the robustness of the final model found through the BESO method. Another difference is the efficiency of the method with regard to computer performance.

For further in-depth understanding, the readers are referred to Shobeiri and Ahmadi Nedushan (2017).

2.7 Parametrisation

In structural engineering, a structure is defined through several parameters that control the design e.g. geometry, ST model, design loads and material properties. Often these parameters needs to be changed during the design process, and if traditional manual calculation are used this process can become time consuming. The purpose of the parametric approach is to make adjustments of the parameters easy. This approach is most suitable when the need to investigate different options is important. In the case of designing with a ST model, the parametric approach is helpful to find an optimal model by trying different combinations of angles, struts and ties.

In the second decade of 21st century, the parametric approach in the architecture and engineering sector has taken a more prominent role in the way a designer works. For architects, it can be to generate complex geometries while for an engineer it can be to structurally design the generated complex geometry.

In this thesis, the parametric approach was used to generate and analyse a bridge detail.

2.7.1 Software

In the following sections, a brief introduction to key parametric software programs and plugins used in this thesis is presented.

2.7.1.1 Rhinoceros 3D

Rhinoceros 3D (Rhino) is a 3D CAD software that is based upon the mathematical method of NURBS which stands for "Non-Uniform Rational Basis Spline". With NURBS, simple 2D geometry as well as complex 3D geometry can be produced. Rhino is a software that has a wide range of usability and different disciplines are using it today, from engineers and architects to graphic designers and jewelers (Carlota V, 2020).

In this thesis, Rhino was used as a visualisation aid for the parametric calculations performed in Grasshopper.

2.7.1.2 Grasshopper

Grasshopper is a graphical algorithm editor within Rhino, and is a powerful tool for parametric modelling. It can be described as a plugin to Rhino. What makes Grasshopper unique is that it offers new ways to develop and analyse different geometries. For example by using mathematical functions to generate geometry, the design can easily be changed by adjusting a few parameters (Reilly, 2014).

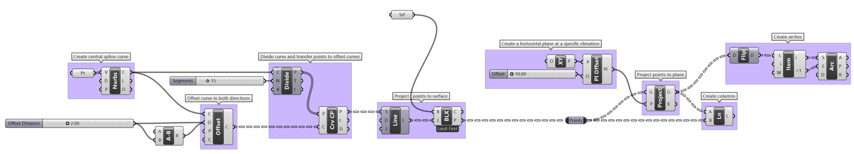


Figure 2.23: *Example of how a Grasshopper algorithm can look like.*

The advantages of Grasshopper are several, and therefore been largely implemented by different disciplines. Firstly, Grasshopper has a graphical interface so that the user need no prior knowledge of programming or scripting (Reilly, 2014). Secondly, since Grasshopper has become widely used, several hundred add-ons have been developed for different purposes (Food4Rhino, 2021). In the field of structural engineering, it is possible to use add-ons that, for example, can calculate the best optimised shape according to the laws of physics or can use FE-analysis to find the forces within a complex geometry.

In this thesis, Grasshopper was used to generate the geometry and develop the ST model of the given geometry. Grasshopper has further been used to calculate and validate angles, forces and stresses.

2.7.1.3 Millipede

Millipede is an add-on to Grasshopper and enables structural optimisation analysis for a given geometry. In this thesis, only a fraction of this add-on's features was used and it is the topology optimisation feature. The type of method used for topology optimisation in Millipede is the BESO method where the software does a computational analysis of several iterations to find the optimal force flow for a given geometry set in Rhinoceros 3D or in Grasshopper.

In this thesis, Millipede was used to find the most optimal stress fields within a geometry based upon a boundary given by the authors of this thesis. The final results of the stress field are used as a reference for the development of a ST-model for the 3D case explained in Chapter 3.

2.8 Remarks

As mentioned earlier, there are different methods to define a ST model in 3D. The traditional (and conservative) method is to transform the model to 2D models before the calculations. The method developed by Chantelot and Mathern (2010) is the most promising according to the authors of this thesis. This method may be time consuming and require extensive modelling and calculations, but the theory behind the method seems to be more developed and applicable than other methods studied. In this thesis, the enhanced method proposed by Chantelot and Mathern was the only method used for development of ST models in the case study phase.

3

Case study and applied methodology of 3D strut and tie

Based on the theory of strut-and-tie (ST) methods obtained in previous chapters, this chapter contains a case study where a three-dimensional ST model was developed for a corbel originally designed by the engineering company Ramboll. As a first step, an introduction to the case study is presented such as project description, dimensions, loads, assumptions and limitations. Further, two different methods to obtain the ST model for the corbel are presented with the first method being the conventional method of using 2D models in multiple planes. The second approach was based upon the enhanced method proposed by Mathern et al. (2017) with adjustments to current standards of manufacturing in Sweden.

For the ease of the reader, the two different ST models presented in this thesis have been assigned with different names:

- **Model A** - ST model based upon the conventional method of 2D ST models in multiple planes, as applied by practicing engineers.
- **Model B** - ST model based upon the enhanced ST method proposed by Mathern et al. (2017) with reinforcement placed in a reasonable manner for manufacturing.

Figure 3.1: *Early 3D rendition of the bridge studied in this thesis. Reproduced from Ramboll (2019) with permission.*

3.1 Case study description

The bridge is located in mid-west Sweden in the city of Trollhättan, and is a double-leaf bascule bridge consisting of steel box girders resting on concrete columns and a concrete foundation. At one of the supports, the bearings are placed upon a crossbeam that also works as a corbel, and the forces are transferred vertically to the ground through concrete columns (see Figure 3.3). This is the detail that was chosen for this case study, and is from now on referred to as *the corbel*.

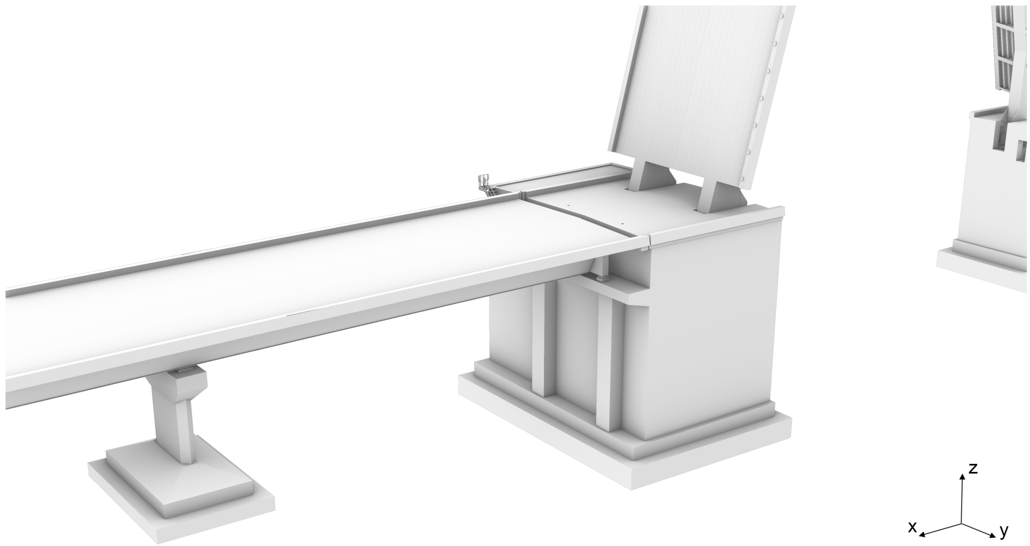


Figure 3.2: *Illustration on the general design of the bridge.*

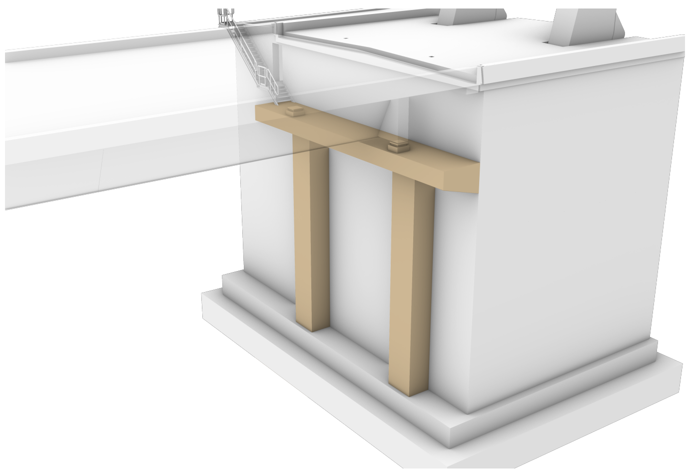


Figure 3.3: *Illustration of the original corbel detail that was studied in this thesis.*

The corbel is located in connection to one of the chambers that houses the counter weight and the mechanics for one of the bascules. This corbel acts as a standalone substructure that takes all the forces from the superstructure of the steel box girders to the foundation through a horizontal beam resting on two columns.

3.2 Input data and description of the case study

Compared to an ordinary column or crossbeam, the corbel in this case study is a mixture of these two. As it can be seen in the elevation in Figure 3.4, there is an eccentricity between the vertical load to the center line of the columns. This results in the need of transferring the load horizontally in x-direction before it can be transferred vertically through the column.

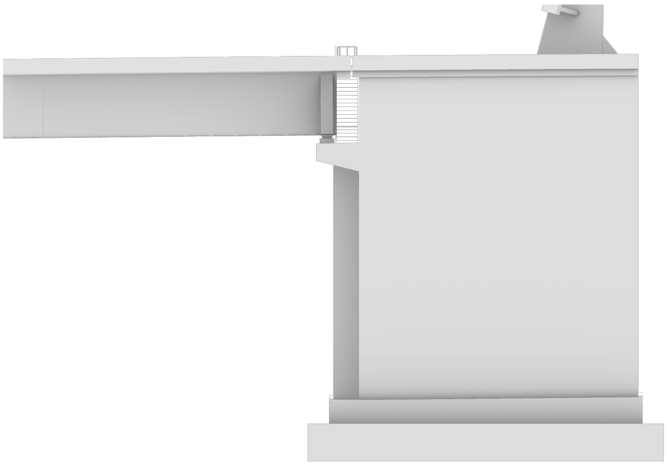


Figure 3.4: *Illustrative elevation of the bridge from the side.*

When viewed in section (Figure 3.5a), the two positions of the vertical forces from the bridge deck are indicated. The position of the vertical load vary depending on situation, thus the bridge needs to be dimensioned for these cases:

- **The permanent loading situation** - The load from the bridge deck is transferred vertically to the corbel through bearings. The bearings consists of a steel cylinder that rest upon a thick steel plate. This detail serve as a "roller support" in order to allow the bridge to move horizontally in its longitudinal direction due to temperature movements. The bearing locations are aligned with the columns, which enables the load to be transferred in the XZ-plane in the longitudinal direction of the bridge (same as the elevation plane in Figure 3.5b).
- **The temporary loading situation** - The permanent bearings have a more limited lifespan than the rest of the bridge, and need to be changed regularly. This is done by placing temporary hydraulic jacks next to the bearings, and lift the bridge. The vertical force will then be located at the position of the jacks, which are not aligned with the columns (see Figure 3.5b).

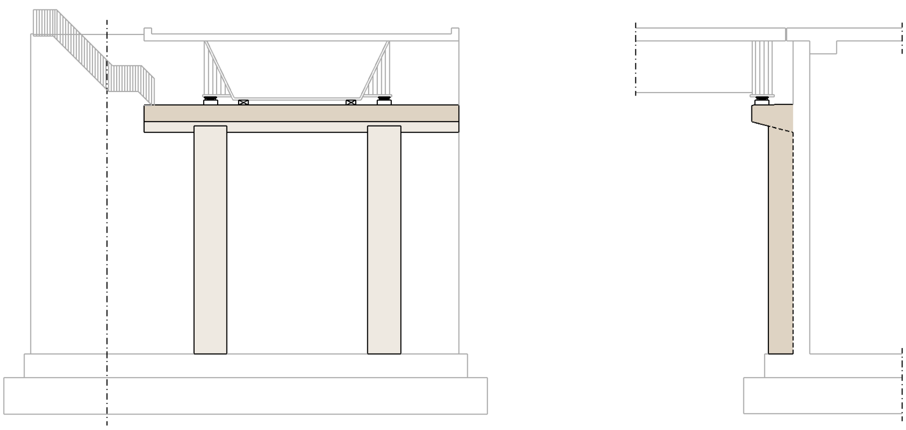


Figure 3.5: *Left: Section from X-axis. Right: Section from Y-axis*

The most important input data are presented in Table 3.1. In Appendix A, other input data can be found that was relevant for the developing of ST models and the calculations of them.

Table 3.1: *Input data for Model A and Model B*

Max design load from bearing and jack:	4.38 MN
Distance between bearings and jacks:	1200 mm
Distance between bearing/jacks and chamber wall:	1500 mm

Several advantages can be found with using this particular case study, with the main advantage being the access to original drawings and calculations for this project. This allows comprehensive comparison between the traditional and enhanced method of developing ST models. Further it allows for dialogue with the structural engineers of this project with experience from the current engineering practice in Sweden, and thus assistance in verifying the calculations and assumption made in this thesis.

3.3 Assumptions and limitations in the case study

3.3.1 Comparison limitations

A primary aim of this thesis was to study if the enhanced 3D ST method proposed by Chantelot and Mathern could be used in a bridge detail, and compare this method to the original method used in the already planned bridge. Therefore, in this thesis new calculation iterations was performed based on the theory of the enhanced method. These iterations focused on the part of the calculations that concerned the 3D ST model, and other calculation parameters such as anchorage length, concrete cover, concrete class, etc, was kept unchanged in order to make the comparison possible. Another example of how the assumptions in the original calculations was kept unchanged is that the chamber back wall that the corbel and the columns is connected to was assumed not to carry any forces from the bearings or jacks. In reality, the main function of the back wall is to support the weight of the movable part of the bridge but a secondary function is that it can possibly take care of some of the forces from the bearings or jacks. However to simplify the ST models, it was assumed in the design that all the forces coming from the bearings or jacks will be transferred down to the bottom plate through the columns. This assumption gives a conservative approach to the problem but it is on the safe side as the back wall is not designed to handle the forces from the bridge deck.

3.3.2 Geometry and material usage

Model A and B was also compared with regard to material usage, by calculating the concrete and steel volume in both models. In order to make the comparison correct, only the actual material used for carrying the load was taken into account. That means that the amount of reinforcement calculated do not include the surface reinforcement inside the volume parts that are removed, even though it can be counted as additional decrease in steel. The concrete volume in Model B was based upon the ST-model, and a cover thickness of 50 mm was added on the nodal zones.

The geometrical changes that was done to Model B, compared to the original design (Model A) was:

- **Column centre** - The centre of the columns below the corbel was shifted with 578 mm. They were also extended with 200 mm in width in x-direction, but reduced with 100 mm in width in y-direction.
- **The steel plate under the bearings and jacks** - The area of the steel plate in Model A was 470*470 for the bearings, and 450*450 for the jacks. The plates in Model B was chosen to 550*550 mm both for the bearings and the jacks.
- **Removal of the cross beam** - The cross beam was replaced with steel deck, as it had no load-bearing function in Model B.

3.3.3 Reinforcement

The layout of the reinforcement was limited so that the reinforcement was placed in the main orthogonal directions of the bridge only, from now on mentioned as the main directions. This means that ties was only placed along the bridge, in 90 degrees across the bridge or vertically. The reason for this limitation was primarily based upon the objective of this thesis that the 3D ST model created needs to result in a reinforcement layout suitable for the construction phase, i.e. have orthogonal placed reinforcement.

For a fair comparison, concrete and steel classes was assumed to be the same for the two models, as well as anchorage lengths, lap lengths and reinforcement bar diameters. In Table 3.2, the design choices for the reinforcement is presented.

Table 3.2: *Assumptions of reinforcement design for Model A and Model B.*

Reinforcement bar diameter [mm]	25
Steel Reinforcement class	K500
Anchorage Length [mm]	824
Lap Length [mm]	1236

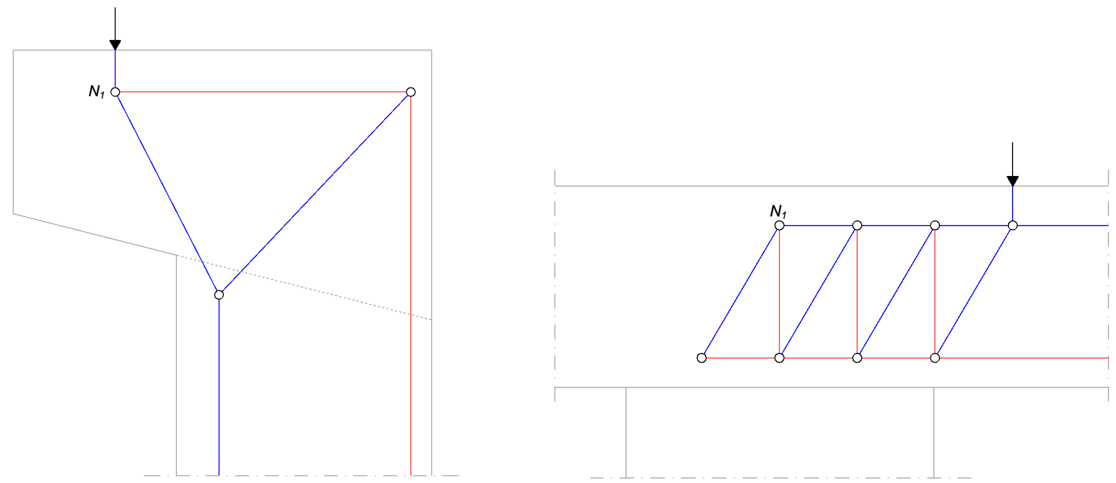
Furthermore, with the high magnitude of the forces it was vital to check spalling of the concrete in the calculations. In Section 3.4.2, spalling is further presented.

3.4 Model A - Conventional strut and tie method

In this section, the ST model obtained through the conventional method is presented. This method is the standard method used in engineering practice today in Sweden. Model A was based upon the ST model constructed by engineers at Ramboll Sweden for the bridge studied, and adjusted to the assumptions and limitations for this thesis.

3.4.1 Methodology

Model A was designed as two ST models in orthogonal planes. The first model was placed in the elevation plane (see Figure 3.6a) and handle the vertical forces from the permanent bearings. From now on, this model is referred to as Model A-1 and was based on the assumption that the structure works as a corbel.



(a) *Model A-1.* **(b)** *A section of Model A-2*

Figure 3.6: *ST model of Model A in different planes.*

When the bearings are being exchanged, the jacks are placed on the inside of the bearings and the crossbeam is being activated (see Figure 3.3a). The structure is viewed as a freely supported beam that rests on top of the corbels at each end. The ST model that describe this behaviour was designed as a truss-system that is connected along the cross beam for equilibrium reasons (see Figure 3.6b). This ST model transfer the load to a point just below the bearing, where the two ST models connects (See the overall illustration of Model A-1 and Model A-2 in Figure 3.7). From now, this model is referred to as Model A-2.

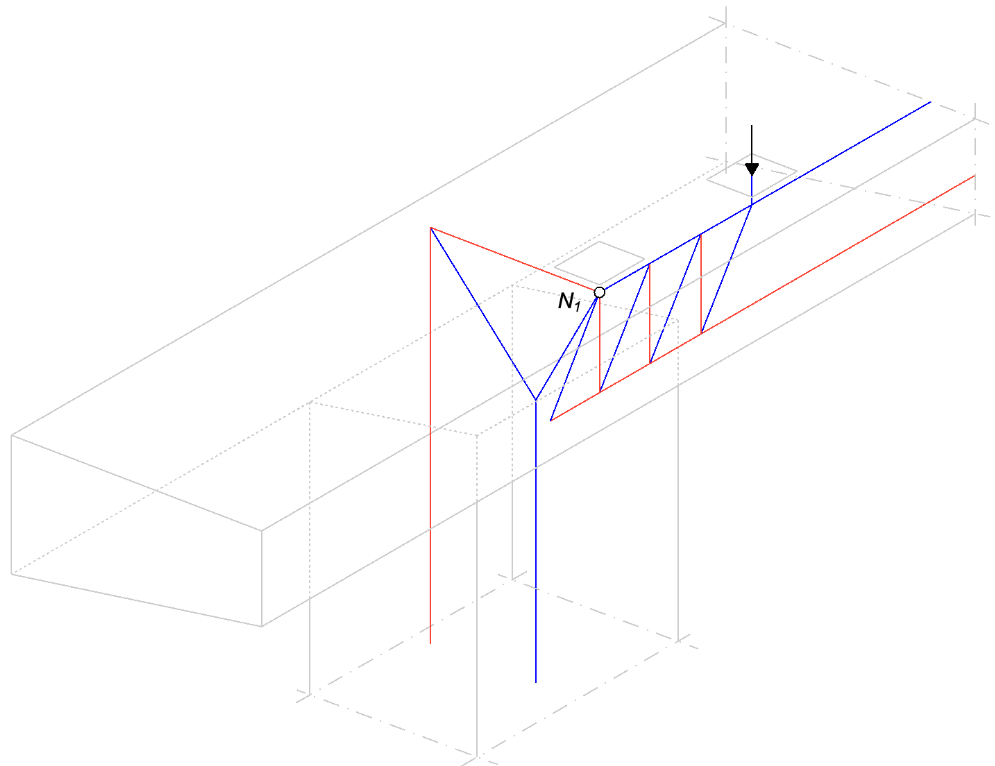


Figure 3.7: *ST model of complete Model A-2 in 3D environment.*

3.4.2 Calculations regarding spalling

When calculating a ST model in 2D, the effect of spalling in both directions need to be taken into account if the force spreads out. In Model A-1, the force from the bearings spreads out perpendicular to Model A-1 until it reaches the height of the first node (see Figure 3.8). This was visualised and calculated as a local ST model. The reason for this is that the 2D model is located in one plane in itself, but the force flow is three dimensional in reality.

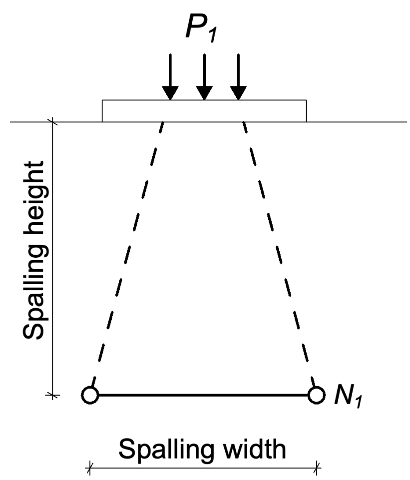


Figure 3.8: *Illustrative figure of spalling.*

3.5 Model B - The enhanced strut and tie method

In this section, the method for obtaining the ST model through the enhanced method is presented. This model is not bound to strictly follow the orthogonal planes as the previous ST model. The models have similarities though, as Model B in the temporary situation can be seen theoretically as a rotated and further developed version of Model A-1.

3.5.1 Three-dimensional strut and tie model

As presented in Section 3.2, the corbel needs to be designed to resist two different load cases, the permanent loading situation (Model B-1) and the temporary loading situation (Model B-2). In the permanent situation the load is transferred in one plane, thus Model B-1 was handled in a same manner as Model A-1. There is basically no need for applying 3D ST theory when the model is in one plane. When studying the temporary situation however, the load need to be transferred in three orthogonal directions and therefore Model B-2 requires to be solved with 3D ST theory.

The basic ST geometry of Model B-2 is similar to Model B-1, but rotated around the vertical tie, see Figure 3.9. The vertical strut is divided in two compared to Model B-1, in order to enable the design of the nodal zones. The horizontal tie is split up into a system of four separate ties to ensure the use of orthogonal reinforcement. This can seem to be unnecessary and even a disadvantage but that has not been the case. Firstly, one of the ties in Model B-2 corresponds to the horizontal tie in Model B-1 (see Figure 3.6a). Secondly it is preferable to work in orthogonal directions due to easier manufacturing on location during construction (Ramboll, 2021), see Figure 3.10.

The position of the concrete column was adjusted, and centered closer to the center line of the bridge deck. Also the concrete dimensions are slightly increased in both directions, see Figure 3.9.

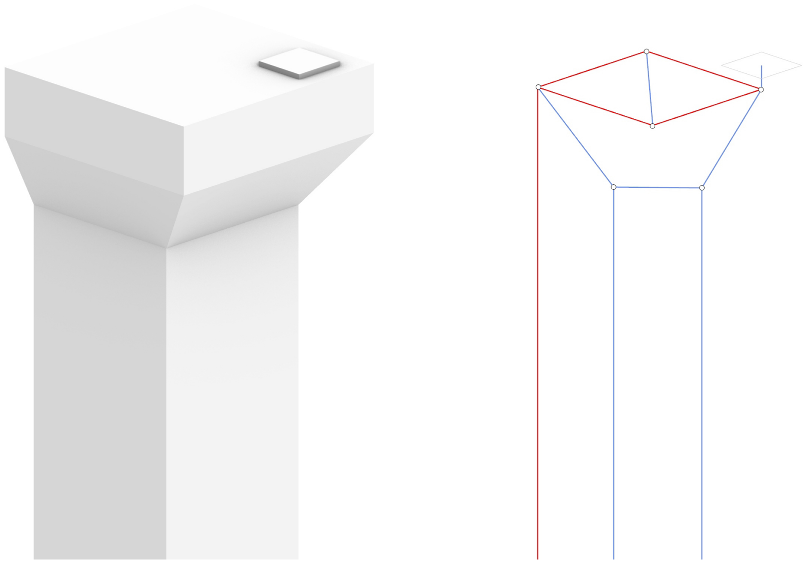


Figure 3.9: *3D view of Model B-2 in skeleton form.*

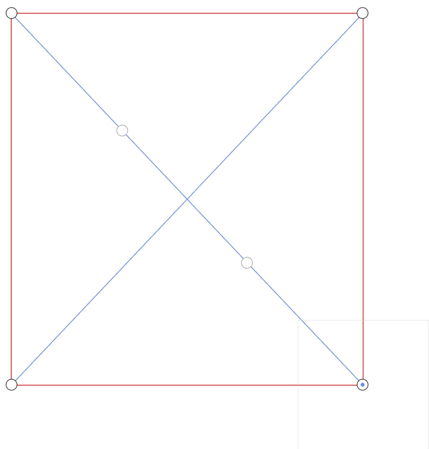


Figure 3.10: *Top view of Model B-2 in skeleton form.*

The nodes in Model B-2 are of two types, biaxial and triaxial. Node 1 (CTT) and 4 (1C3T) in Figure 3.11 are triaxial and act in three dimensions. Node 2 and 3 are both biaxial CTT-nodes, and node 5 and 6 are biaxial CCC-nodes. The nodal zones, which can be seen in Figure 3.12 in the ends of each strut need to have the same size, in order to fulfill the check presented in Chapter 2.5.2.2 (the center line of the skeleton strut must align with the center line of the expanded strut). Node 5 and 6 (connected with the horizontal fan-shaped strut C5) is not of the exact same size, but this was neglected because the difference between the axes is very small. The tension stress that this strut gives rise to in the node was checked as well to make sure that it is well below the tension capacity of concrete.

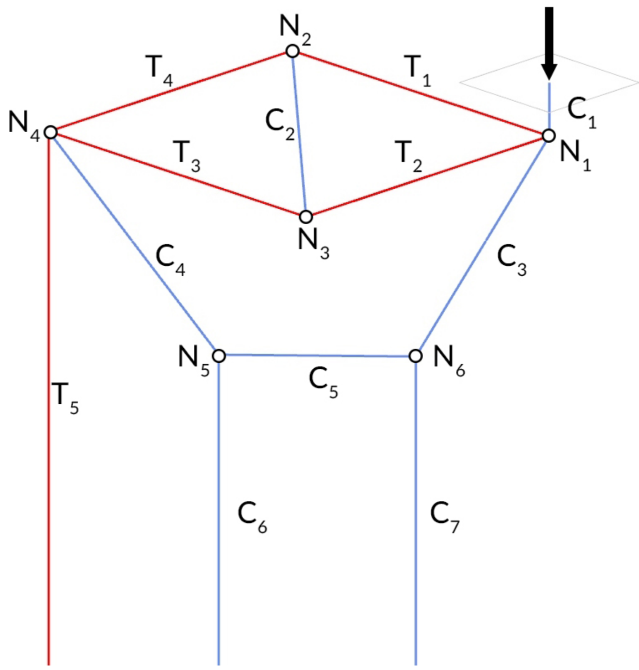


Figure 3.11: *Labelling of each element in Model B-2. N indicates Nodes, C indicates struts and T indicated ties.*

In Figure 3.12, Model B-2 is presented with visualized struts and ties. As one can see, the dimensions of the struts need to be rather large to withstand the forces. Neither the struts nor the nodal zones should intersect with each other, which affects the position of the nodes and their sizes and pushes the 3C-nodes (node 5 and node 6) further away from each other. In both Figure 3.12 and Figure 3.13, this placement of the nodal zones is shown. Strut C1 is vertically leading the load from the jack to the first node in the ST model, N1. Although it is not clearly visible in Figure 3.12, as the height of N1 is almost reaching up to the steel plate under the jack.

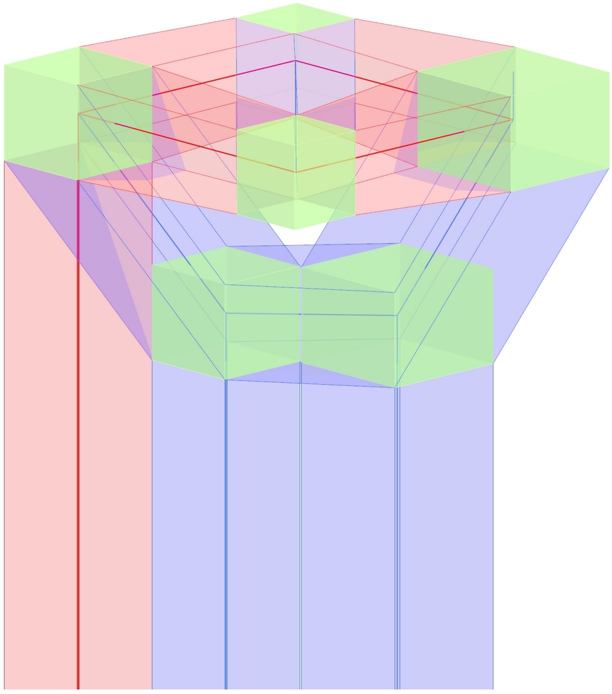


Figure 3.12: *3D view of Model B-2 with visualized struts and ties in expanded form.*

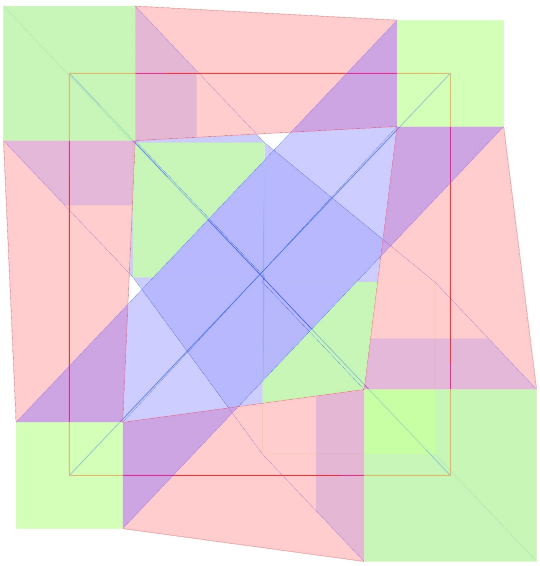


Figure 3.13: *Top view of Model B-2 with visualized struts and ties in expanded form.*

3.5.2 Reinforcement design

In the expanded ST-model the width and volume of the struts and ties are visualized. This visualisation was helpful for the design of the ST-model where one can graphically show the elements and for example find out if struts intersect with each other. This was possible as the struts show the geometry of solid concrete in compression.

The visualisation of the ties are on the other hand not equally "true", as the ties symbolise reinforcement bars. The reinforcement handle the tension, and can in the enhanced ST theory be seen as a strut acting on the opposite side. What the visualisation shows when it comes to ties, is the three dimensional space where the reinforcement need to act in order to set the node in equilibrium. The cross-section of the ties at the nodes define the area where the reinforcement need to be placed inside, see Figure 3.6b. If this cross-sectional area is too small for the reinforcement to be placed within, the nodal zone dimensions need to increase and thus increase the cross-sectional area. In Eurocode 2 (2004), a definition for the minimum bar spacing has been defined as explained in Section 2.3.5.1, however no strict requirements has been found for minimum cross-sectional area. Therefore, the minimum cross-sectional area was defined through the amount of bars needed and the practical minimum bar spacing of approximately 100 millimeters.

3.5.3 Development of the model using parametric design

Model B have been developed and calculated parametrically in Grasshopper, where the checks are more or less being made simultaneously in order to make the development process as smooth as possible. This was possible since the chosen model was statically determinate, so when the node positions changed the new forces in the elements followed automatically.

All the important parameters in the model could be adjusted with instant feedback on the calculations. The resulting stresses in the nodes were presented on panels that also indicated with colours if the stress was below the limit or not. The angles between the elements were checked in the same way, as well as that the struts were not intersecting. This way of designing the geometry made it possible to find a manually optimized ST model.

3.5.4 Methodology

To be able to obtain a 3D ST model for this specific case study, a methodology was created. There were certain checks that needed to be fulfilled in the final ST model, so when changes were made during the development these checks needed to be verified again. The process of developing and refining a ST model has therefore been an iterative process which is summarised in a flowchart presented in Figure 3.16 at the end of this section. Below, the methodology is presented in detail for obtaining the 3D ST model for this specific case study.

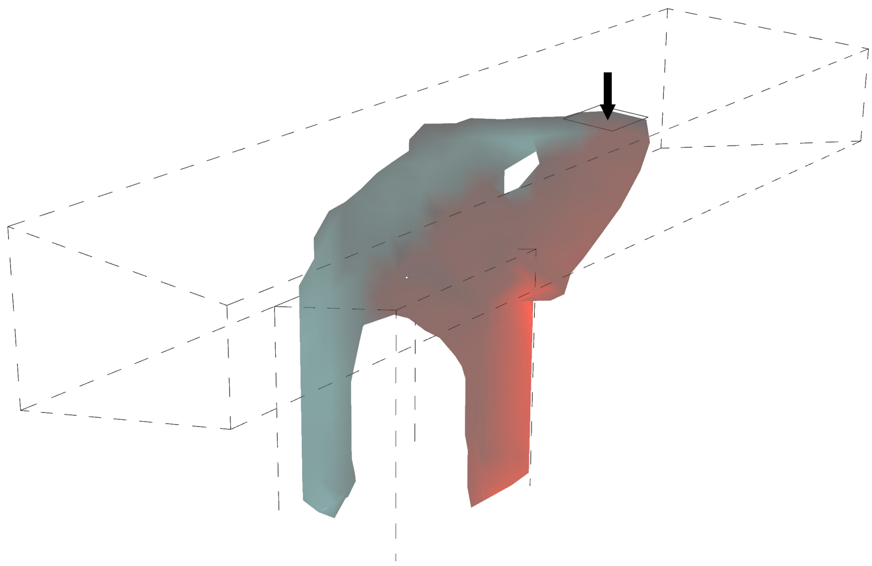


Figure 3.14: *Preliminary Topology optimisation for localising the force flow in the structure. Red is compression and teal is tension forces.*

1. The first step in the process was to localise the force flow in the structure and construct a ST-model based on this. In this thesis, topology optimization was used as a tool for getting a hint of how the structure would look like (See Figure 3.14), and a ST-model was then chosen manually based upon this optimization. As mentioned in Chapter 3.3, the reinforcement had to be placed in the three main axis of the bridge, and therefore the ties were designed accordingly.

2. When the ST-model was chosen, the angles between the elements were checked according to Eurocode. The angles were continuously checked during the following steps as well.

3. As the model was statically determinate, the force in each strut and tie could be calculated with equilibrium equations. There were no need of further assumptions of the force division.

4. In the next step the nodal zones were designed. In this thesis cuboid nodal zones were chosen at every node, with varying sizes and proportions. However, of practical reasons the height of the nodal zones in each layer were chosen to be the same. The geometry of the struts and ties were then created by connecting the nodal zones to each other

5. The cross-sectional area of the struts at the nodes were calculated, and the concrete stresses were checked. Also the position of the struts were checked so the struts did not intersect with each other.

6. In the ST-model that was chosen, there was one fan-shaped strut between two biaxial nodes that resulted in tension stress perpendicular to the strut´s main axis. The tension stress in the node was calculated and checked against the tension capacity of concrete.

7. The next step was to design the reinforcement and ensure that the chosen design was able to build. The reinforcement dimensions were chosen and the amount of reinforcement needed was calculated. The distance between the bars and the anchorage method was chosen based upon the the calculated minimum distance and minimum anchorage length according to Eurocode 2 (2004). It was checked that:

- The reinforcement would fit in the smallest cross-section of the tie (see Figure 3.15).
- The reinforcement from the ties did not collide in the nodes.
- There was enough space for anchorage.
- There was enough space for the reinforcement splices.

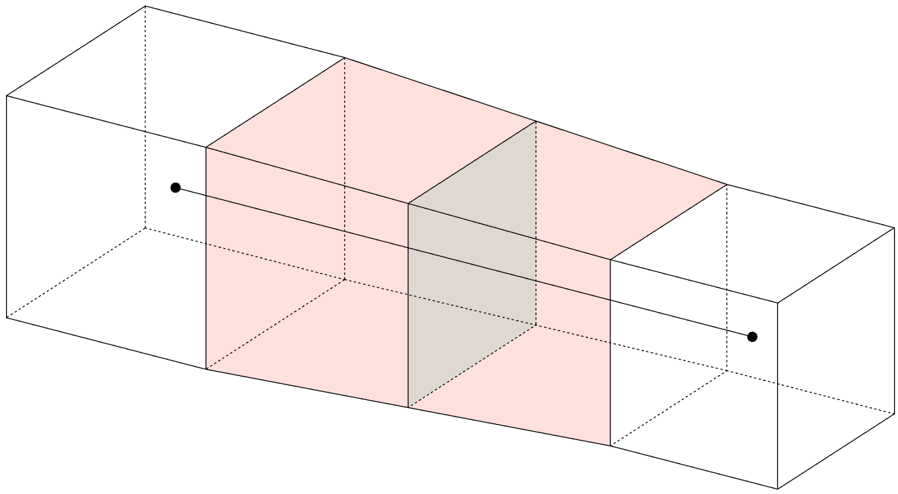


Figure 3.15: *The reinforcement needed to be placed inside the expanded tie (within the highlighted area).*

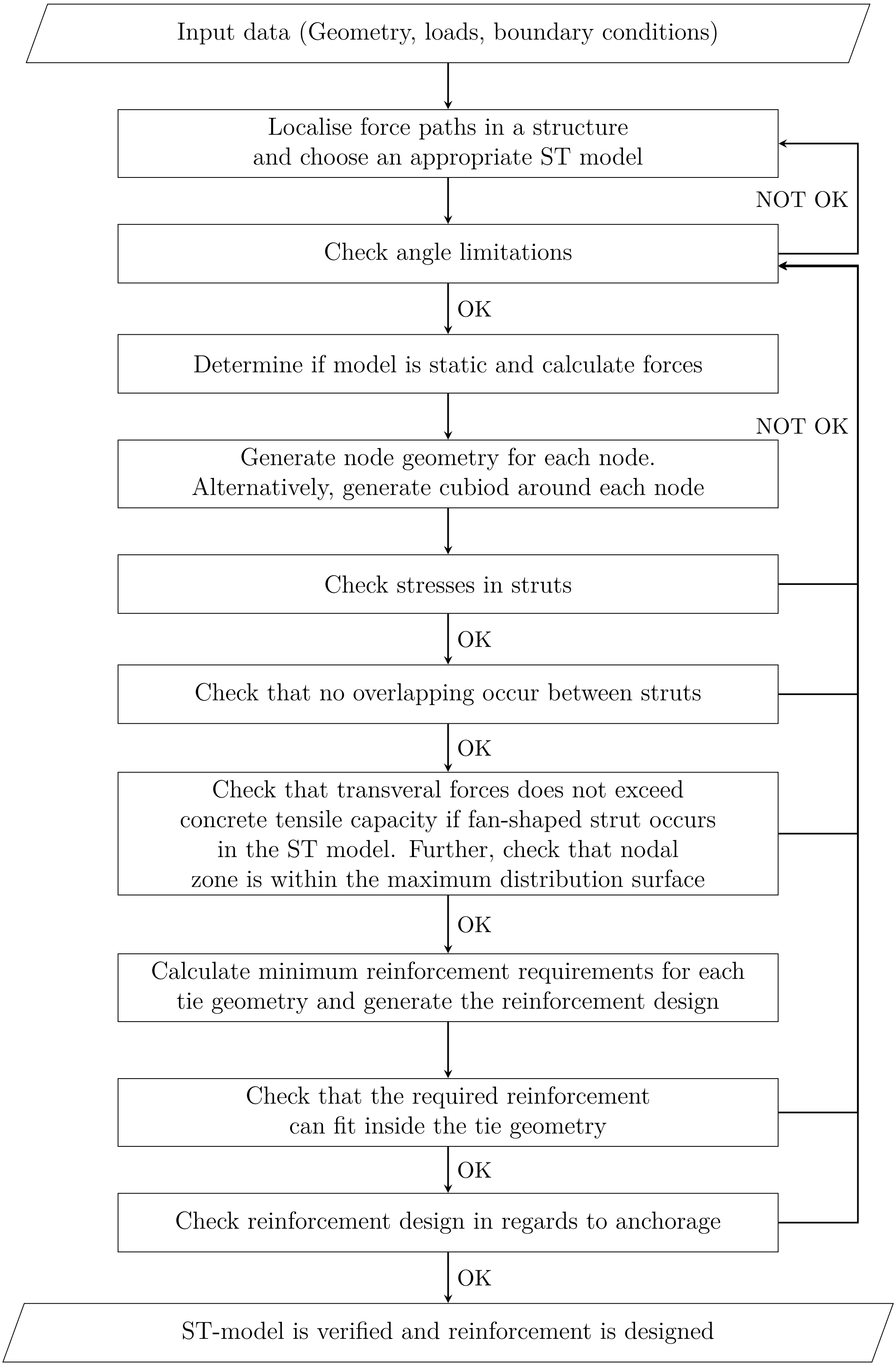


Figure 3.16: *Flowchart for developing a 3D Strut and Tie model.*

4
Results

In this chapter the results are presented for the different case study models described in the previous chapter. The results that are presented are the strut and tie (ST) models, analysis of forces, checks of nodes and a suggestion to the reinforcement design. At the end of this chapter, the material usage for both models are presented.

For the ease of the reader, the illustration of the final design of each model within the context are presented in Figure 4.1 and Figure 4.2.

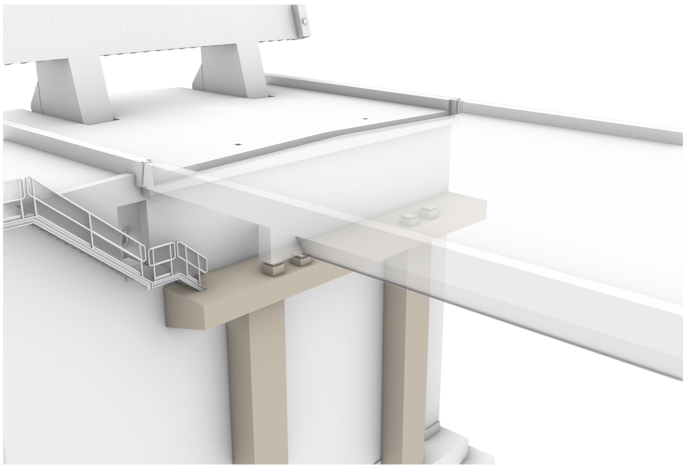


Figure 4.1: *Final design of Model A*

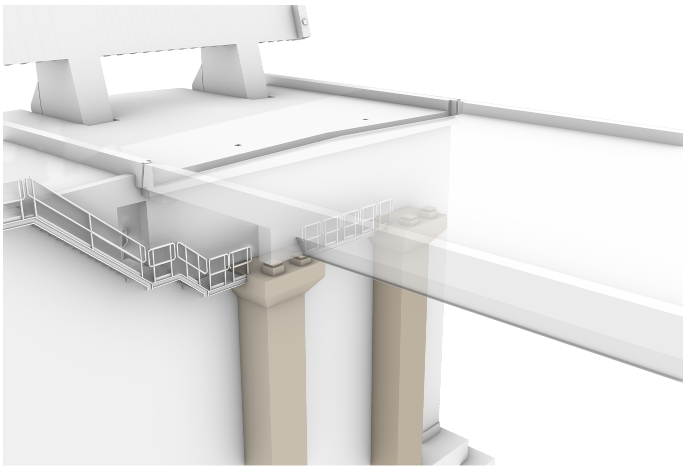


Figure 4.2: *Final design of Model B*

4.1 Model A

In this section, the results are presented of the ST model based upon the conventional method of 2D ST models in multiple planes.

4.1.1 Final Strut and Tie Model - Model A

The final ST model for Model A consists of two separate 2D ST models in two different planes (Model A-1 and A-2). See Figure 4.3 and Figure 4.4 for detailed sections of the two ST models with location and designation of main struts, ties, nodes and external forces. Figure 4.5 presents the full ST-model in 3D for both models.

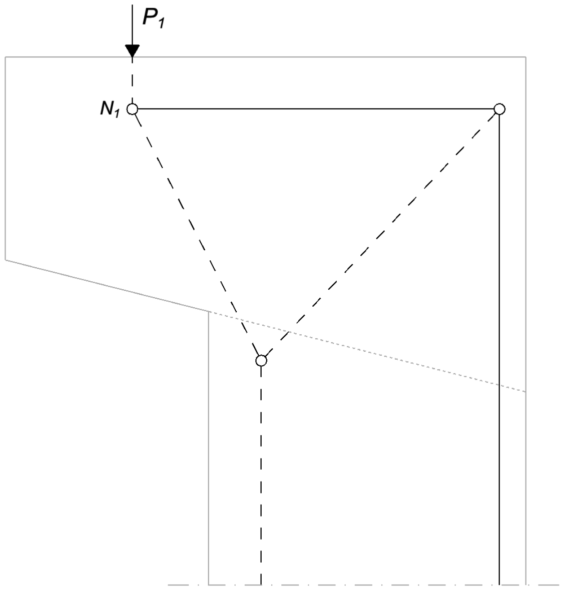


Figure 4.3: *ST model for Model A-1, transferring the forces from the permanent bearing to the column.*

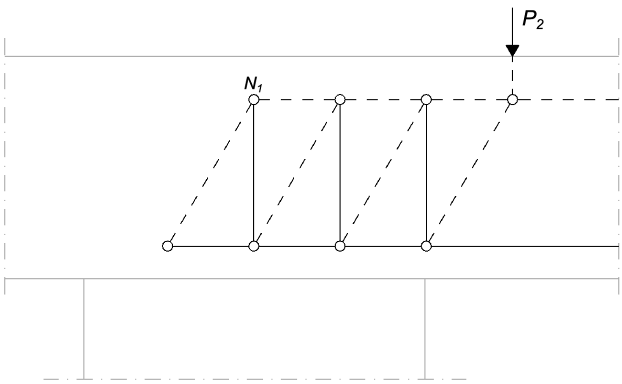


Figure 4.4: *ST model for Model A-2, transferring the forces from the temporary bearing onto Model A-1.*

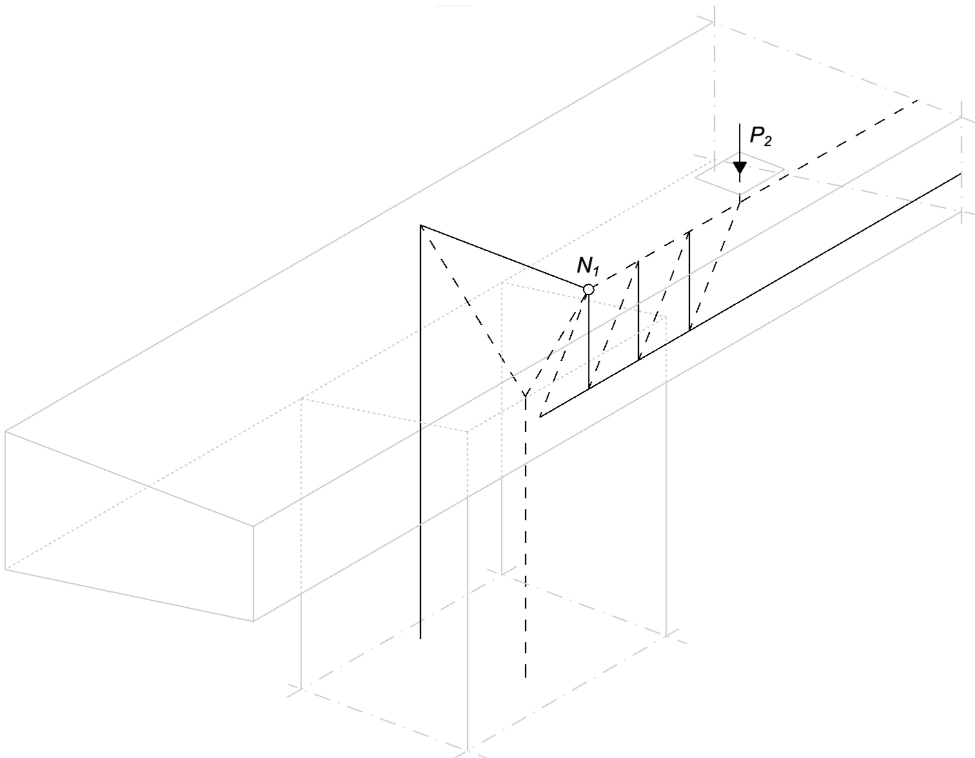


Figure 4.5: *Overview of full ST model for Model A-2 in 3D where the forces goes from the temporary bearings to the column through Model A1.*

4.1.2 Reinforcement design - Model A

A suggestion of structural reinforcement design for Model A is presented below in Figure 4.6. The total volume of steel reinforcement and concrete for the suggested design is presented in Table 4.4.

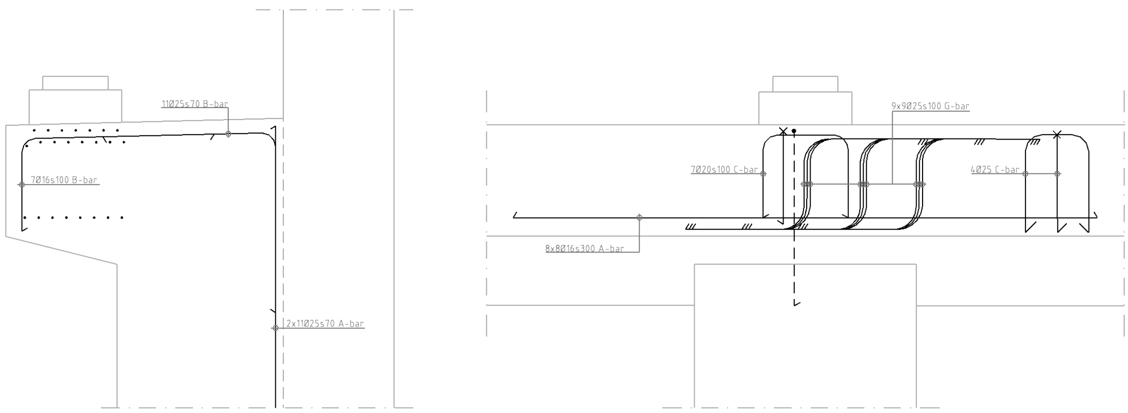


Figure 4.6: *Suggestion of reinforcement design for Model A. To note here is that on the left section, G-bars are not shown due to readability of the drawing in this scale.*

4.2 Model B

In this section the results from the 3D ST model, based upon the enhanced method proposed by Chantelot and Mathern (2010), are presented.

4.2.1 Final Strut and Tie Model - Model B

The final ST model for Model B can only be seen in total in a 3D environment. See Figure 4.7 and 4.8 for an illustrative overview of the ST models with a skeleton model for better understanding with labeling of each strut, tie, node and external forces. In figure 4.9, the expanded ST model including the thickness of each strut and tie is presented.

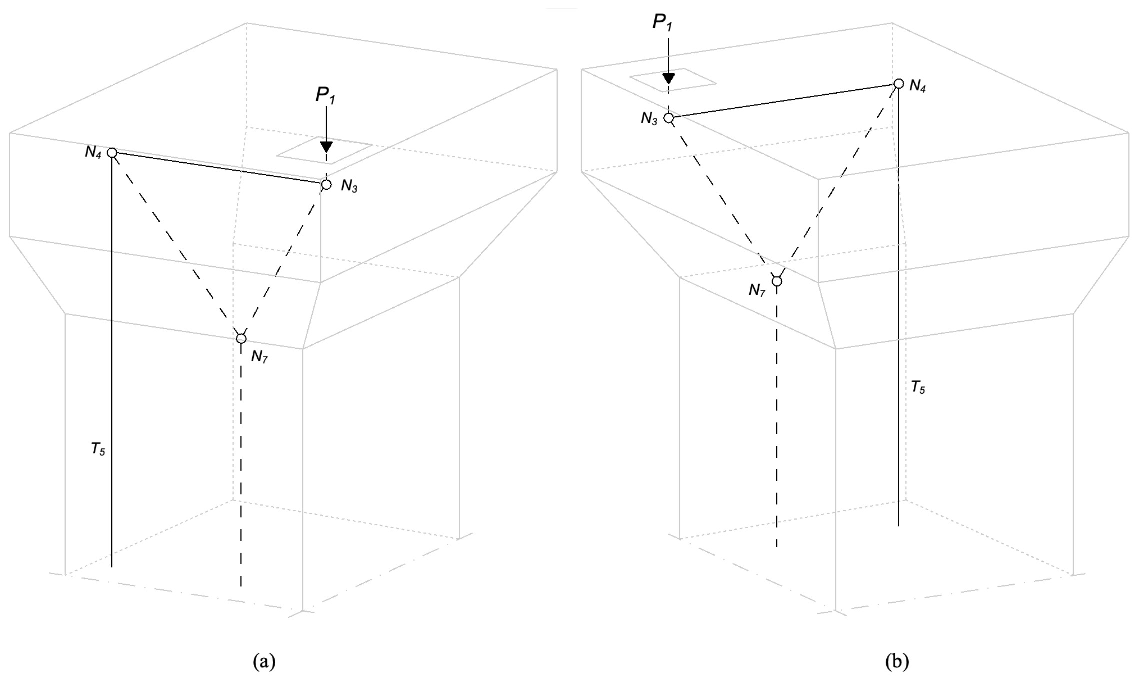


Figure 4.7: *Skeleton form of the final ST model for Model B-1 from two perspectives.*

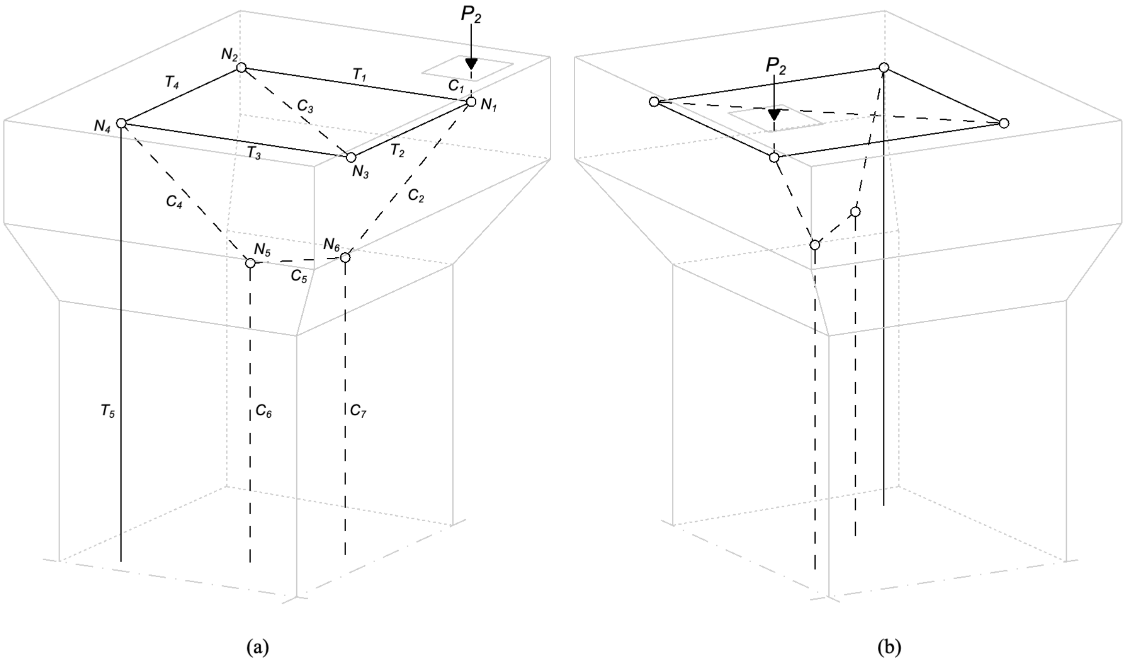


Figure 4.8: *Skeleton form of the final ST model for Model B-2 from two perspectives.*

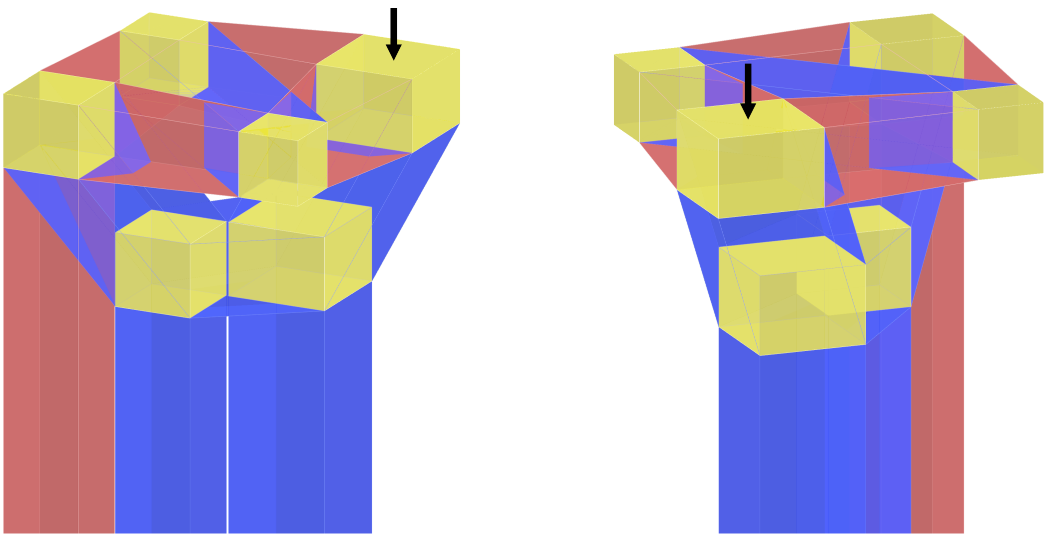


Figure 4.9: *Expanded form of the final ST model for Model B-2 from two perspectives.*

In Table 4.1 the magnitudes of forces in struts and ties are presented as well as the magnitude of loads.

Table 4.1: *Magnitude of forces and loads for Model B-2.*

Strut	[kN]	Tie	[kN]	Loads	[kN]
C_1	4380	T_1	1917	P_1	4380
C_2	3130	T_2	2474	P_2	4380
C_3	5384	T_3	1917		
C_4	4937	T_4	2474		
C_5	3130	T_5	3818		
C_6	3818				
C_7	4380				

For more in depth calculations on how the forces within the ST model was obtained, please see Appendix 2.

4.2.2 Check of nodes - Model B

The compressive stresses in the nodes needs to be checked to determine if the node can handle the stresses without reaching concrete crushing. These checks are based upon the requirements set by Eurocode 2 (2004) as described in Chapter 2. In Table 4.3, the notations of the critical stresses are defined with the first index being the label of the strut, and the second index being the node where the cross section of the strut was taken (this is only relevant for fan-shaped struts). For more in depth calculations and walk through on how the checks of stresses was performed, see Appendix 2.

Table 4.2: *Maximum stresses according to Eurocode 2 (2004).*

CCC-node	[MPa]	CCT-node	[MPa]	CTT-node	[MPa]
$\sigma_{Rd,max}$	20,07	$\sigma_{Rd,max}$	17,06	$\sigma_{Rd,max}$	15,05

Table 4.3: *Checks of critical stresses in struts at the nodes according to requirements from Eurocode 2 (2008) for Model B-2.*

	[MPa]	Utilization rate
σ_{C_1,N_1}	14.48	96%
σ_{C_2,N_2}	14.68	98%
σ_{C_3,N_1}	12.65	84%
σ_{C_4,N_4}	12.71	84%
σ_{C_5,N_5}	10.48	52%
σ_{C_6,N_5}	14.40	72%
σ_{C_7,N_6}	14.48	72%

4.2.3 Reinforcement design - Model B

A suggestion of structural reinforcement design and layout for Model B is presented below in Figure 4.10 where it is possible to interpret the amount and size of bar in cross section. Further, in Figure 4.11, the type of reinforcement bar used in the detail are presented. The total volume of steel reinforcement and concrete based upon the suggested reinforcement design is presented in Table 4.4.

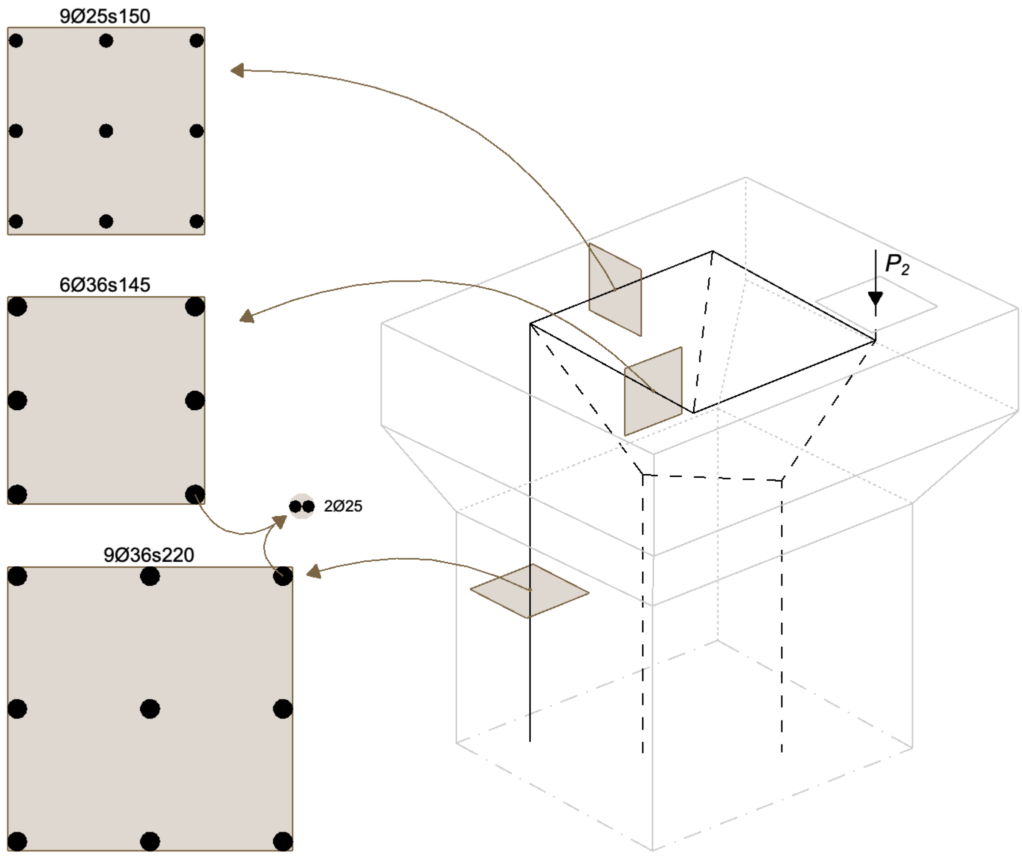


Figure 4.10: *Illustrative suggestion of reinforcement design inside the minimum cross section for tie elements in Model B.*

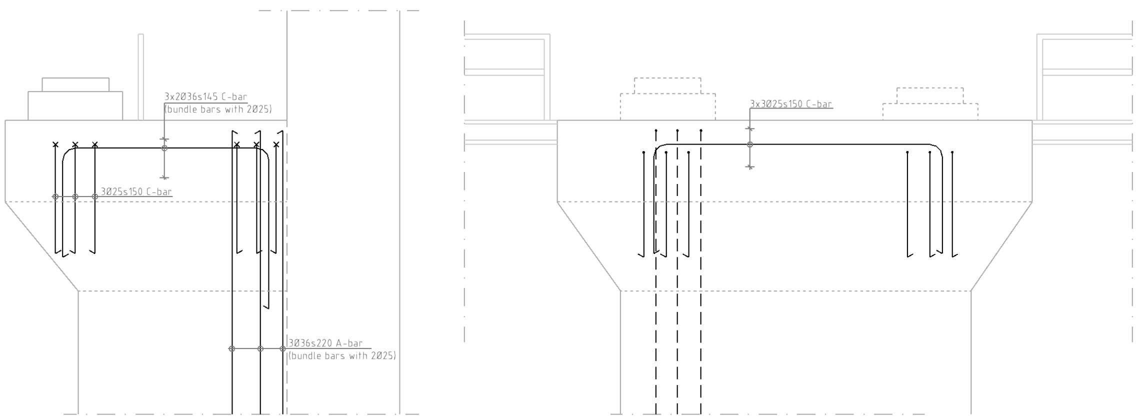
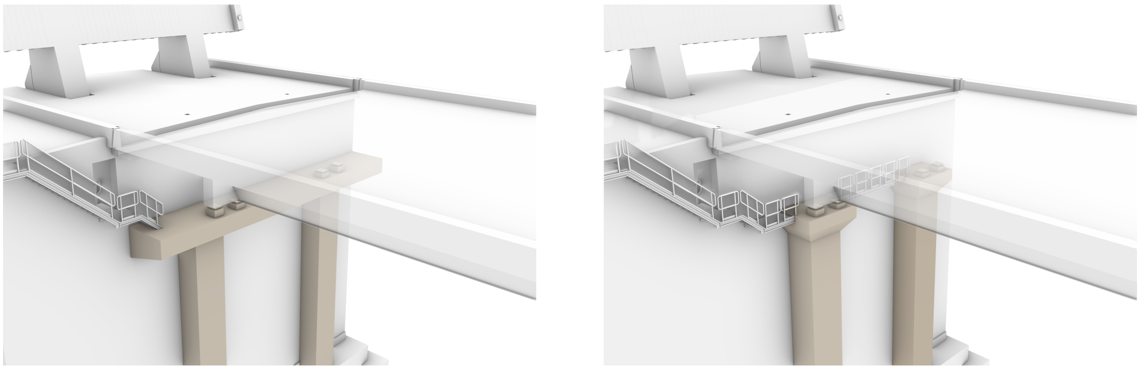


Figure 4.11: *Detailed suggested reinforcement design in 2D sections for Model B.*

4.3 Material usage of Model A and Model B

Based upon the ST models and the suggested reinforcement design for each model presented in earlier sections, the material usage of each model are presented in this section. In Figure 4.12, the final designs of the detail (within the overall context) are presented. Furthermore, in Table 4.4, the total reinforcement and concrete volumes are presented. In addition, the reduction of material volume is shown with a percentage comparison between the models.



(a) *Model A* **(b)** *Model B*

Figure 4.12: *Side by side comparison of the concrete volumes of the two models.*

Table 4.4: *Total material usage of Model A and Model B.*

Model A	Total Reinforcement Volume [m^3]	0.224
	Total Concrete Volume [m^3]	36.450
Model B	Total Reinforcement Volume [m^3]	0.158
	Total Concrete Volume [m^3]	33.650
Savings in Model B compared to Model A	Reinforcement Volume [%]	29.5
	Concrete Volume [%]	7.7

5
Discussion

In Chapter 3 and 4, a comparative case study was performed with two different strut-and-tie (ST) models. In this chapter, an analysis of the results is conducted.

5.1 Theory of the enhanced method

The case study in this thesis was based on the enhanced method (Chantelot & Mathern, 2010), and the relevant theory for the case study in this thesis was presented in Chapter 2. As described in Section 2.5.2.2, the design of the nodal zones in each end of a strut was limited by the criteria that *the centroid axis of the three dimensional struts and ties used in the model should correspond to the axis that the ST model consists of.* In practice, this means that nodal zones that are connected to each other with struts need to have the same dimensions.

What is worth to mention is that in the report where the enhanced method was presented originally, the above mentioned criterion was not fulfilled as nodes of different dimensions were used. In that report it was assumed that the centroids of the struts would always coincide with the axis between the nodal zone center points, which is not true. Although in reality, the distance between the two axes will be rather small if the inclination is close to 45 degrees and the proportions of the nodal zones are similar. But the fact remains that these axes will not coincide exactly (See Figure 2.19).

The limitations in design that this problem induces could possibly be solved by developing an acceptable ratio for the distance between the axes. In order to formulate this ratio, further studies in the behaviour of concrete is needed.

5.1.1 The use of triaxial compression

One of the main advantages with the enhanced method has been that it presents an easy way to design nodes with triaxial compression, see the top nodes in Figure 2.14(b) for an example . As $k_4 = 3$, the capacity of the node is significantly higher. However, the case study in this thesis (Model B) does not contain any nodes with triaxial compression and therefore this high capacity can not be utilised. But in other structures with triaxial compression (e.g. pile caps) the construction has potential to be further optimized by implementing this factor.

5.1.2 Theory not treated in this thesis

The theory presented by Chantelot and Mathern (2010) treats many different situations. All the parts of that theory/method was not applied in the case study performed in this thesis. The central parts of the theory, like the design of nodal zones and the checks that need to be controlled, was applied in this work. But some parts of the theory that considers specific situations, like joining multiple struts in the same quadrant or the combination of truss and arch action, was not used in this report. Therefore the authors of this thesis can not claim that the theory presented by Chantelot and Mathern (2010) has been validated in its fullest, but at least the central and most important parts of it has been treated and validated.

5.2 Case study

In this section, the discussion relating to the case study is presented with focus on the model based upon the enhanced method (Model B).

5.2.1 Assumed geometry of the structure

When setting up 2D ST models in two perpendicular planes, it is of great help if the geometry is assumed on beforehand to be orthogonal, with sides parallel to the orthogonal coordinate system. In real world cases in the design procedure, a preliminary geometry is often set on beforehand, and then the reinforcement need is calculated and the geometry is further refined if suitable.

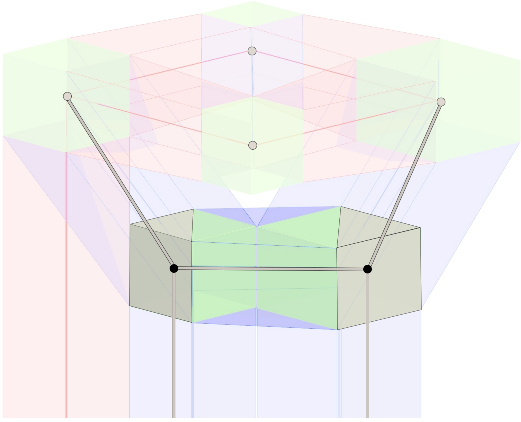
However this procedure was not optimal for the 3D ST model (Model B). Relative early in the iterations of Model B, it was obvious that the ST model could not fit into the original corbel design due to the theory of the enhanced 3D method. It was clear that for this model to be implemented, the ST model needed to govern the corbel design.

5.2.2 Geometry of the nodal zones

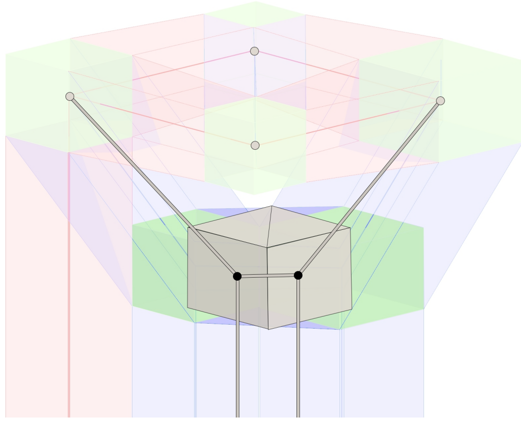
The horizontal strut between node N5 and N6 is fan-shaped (see Figure 4.9), and the nodal zones are of same shape but the angle is 51 degrees. This has actually been a marginally deviation from the rules, as it need to be either 0 or 45 degrees. But it was neglected as it was not central for the model, the tension stress was below the capacity of the concrete and it did not affect the results in any considerable way. It could also in practice have been solved by moving the suggested position of the jack 300 mm in y-direction to obtain exactly 45 degrees.

The limiting part of the 3D ST model was the dimensions of the nodal zones. The CCC-nodes N5 and N6 demanded tremendous space because the nodal zones must not overlap and needed to be cuboidal as well. Seen from above, the areas of the nodal zones needed to be large enough to handle the vertical stresses. A more compatible design of the nodal zones would result in a more flexible design, and give the possibility to a more compact design as well. Figure 5.1 show an illustrative example of two more compatible nodal zone shapes, in this case in the shape of two prismatic triangles. Here nodal zones N5 and N6 can be placed closer to each other, or even next to each other which results in a more flexible model and a smarter use of the space. The nodal zones shown in this picture could not be used in the case study though, as they was not compatible with the enhanced method.

Generally one can conclude that if the dimensions of the structure would have been larger, then the shape and design of the nodal zones are of less importance.



(a) *Nodes at the original positions*



(b) *Nodes placed together*

Figure 5.1: *Illustrative picture of how cuboidal nodal zones can be replaced by more compatible nodal zone shapes*

5.2.3 3D strut-and-tie with non-orthogonal reinforcement

As mentioned in chapter 3.3, the reinforcement layout was limited to the orthogonal directions of the bridge due to constructability. If this limitation would not exist, Model B-2 could follow the topology optimized geometry (see Figure 3.14) even closer and apply a diagonal tie directly between node N1 and N4. This would result in more efficient reinforcement usage, and also removing the horizontal strut that limits the width of the diagonal struts. However, if this limitation was removed, the reinforcement in Model B-2 would not coincide with the reinforcement needed for Model B-1, and therefore the difference would probably be marginal.

5.2.4 Comparison between Model A and Model B

In this section the two different models is compared with regard to different aspects.

5.2.4.1 Material usage - Model A versus Model B

The comparison of material usage was a central part in this thesis, as it offers a way to compare the outcome when calculating a structure by using the ST method in 2D and in 3D. But questions may arise how these models can actually be compared when new choices was made in Model B.

The reason for why the comparison is still interesting is based on the following aspects:

- **The loads** - Model B was designed to carry the same loads as Model A, with the same position of the loads.

- **The design of the corbel** - Model B was designed to follow the same way of action as Model A, and has a similar appearance. If Model B would had been designed as a simple column that led the forces from the bridge deck vertically to the ground, this would not have been the case.
- **The new choices did not affect the rest of the bridge considerably** - The width of the columns were slightly widened in both directions, the position of the columns was marginally moved and the angle of the front of the corbel was slightly changed as well. Also the horizontal beam-like elements of the corbel was removed and replaced with steel decks. These changes would not affect the structural design of the rest of the bridge and at most only need slight changes in the original design.

One have to keep in mind that this case study do only treat one specific detail in one specific bridge. The results presented in chapter 4 show that a large amount of concrete and reinforcement can be saved, but this has to do with other factors as well.

One of the factors that affects the results was that Model A was based upon the calculation of practising professional engineers and might not have been a fully optimized design from the beginning due to e.g. lack of enough time. The process for Model A was that a preliminary geometry of the corbel was given and then the reinforcement need was calculated based on the loads. Therefore the concrete amount might have been able to optimised further in the 2D case. The same goes for the reinforcement design, where large amounts of reinforcement in Model A was used to carry the load during the exchange of the bearings. There might exist methods of solving this in a better way with the 2D ST method. With this said, the authors of this thesis have only made speculations of what *possibly could have been done.* In the end, this thesis treats a detail that was actually projected and intended for construction by professional engineers, and therefore this design was deemed as valid for comparison.

Another factor was that some material parts are not included in the summation. This affects the material difference between the models in both directions. Some examples:

- The minimum and surface reinforcement was not included in the summation of reinforcement steel, but the usage of this reinforcement was much lower in Model B as large parts of the corbel was removed. This *decreases* the material usage in Model B. This also applies for the formwork.
- The concrete parts between the columns was replaced with a steel deck in Model B. This *increases* the material usage of steel in Model B.

5.2.4.2 Construction - Model A versus Model B

One important factor for the comparison have been how complicated the design will be to constructed on site. This regards both the concrete formwork and the reinforcement placement. Model B has less reinforcement and concrete in volume and mass than Model A, but this does not necessarily have to mean that this model will be easier to build. Some other factors are the shape of the reinforcement, the dimensions (the lighter the better), the placement in the formwork, the distance between the bars and the shape of the formwork. These factors are of course dependent from case to case, but Model B has some general advantages compared to Model A within this area:

- Fewer amount of bars results in less physical work.
- Fewer types of bars means saves time when manufacturing the bars into the right shape, and will also result in less time for mounting.
- Easier placement. The huge amount of tightly placed reinforcement in Model A, designed for replacement of bearings, does not exist in Model B.
- Less concrete volume and a more simple shape means less formwork construction
- Less surface reinforcement

Another difference in Model B, compared to Model A, is that the reinforcement was bundled. This can be both an advantage and a disadvantage. On one hand, heavier bars are harder to handle, but on the other hands fewer bars need to be handled. With all this in mind, the conclusion regarding manufacturing process is that Model B can be judged to be an easier and less time consuming design.

5.3 Implementation of parametric approach in structural engineering

The usage of parametric modelling and calculations was central in this thesis as it vastly reduced the time spent on the iterative process. Strut-and-tie calculations in 2D can more or less be performed with the help of pen and paper. The models are often placed in one plane only and can be easily sketched for the sake of understanding and communicating. When constructing and calculating 3D ST models, this is not as easy and straight forward. Geometries in 3D are more complex to handle and sketch, and it also demands more complicated calculations. As an example, we can study how to calculate the cross-sectional area of a 3D strut. With conventional programming, using Python or MATLAB, this would probably be calculated with geometric formulas (see Equation 2.15). With the use of Grasshopper on the other hand, this was done by using a pre-programmed component that calculates the cross-sectional area of an arbitrary 3D geometry in a given plane. Further, the results could also be visualized in 3D instantly for instant visual review.

One reason for why the 3D ST theory has not been formulated and implemented in industry might be due to both the lack of the intuitive tools that are needed, and the lack of knowledge on existing tools. Parametric modelling software programs such as Grasshopper has existed for over a decade though and is rapidly developing, especially in the field of Architecture and Structural Engineering. But in order to make the 3D theory as intuitive and easy to use as the existing theory, some sort of modelling environment would need to be developed. The Grasshopper script that was developed during the work of this thesis is an example of a modelling and calculation environment for 3D ST. It can not be used for any type of structure, but it serves as an example and a template of how the calculation and modelling part can be performed for any given type of concrete structure.

5.4 Practicality of 3D strut-and-tie - Eurocode and the Swedish Transport Administration

When building bridges today, the calculations are required to follow the current standards and rules. Further, in most cases, the calculations are required to be reviewed and accepted by the Swedish Transport Administration which is the government agency responsible for the long-term planning of the national transport infrastructure in Sweden. If a calculation method has not been used before, a more rigorous validation of the calculations are required. In a specific project, it is often hard to find time and resources for such a validation. This limits the use of new design methods, even if they would facilitate more efficient and sustainable structures.

It might not be clear whether 3D calculations by using the enhanced method is supported by Eurocode or not. Nonetheless, all rules and guidelines presented in Eurocode regarding ST was followed in this thesis, such as angle limitations, partial coefficients for specific node types and reinforcement design. The enhanced method do actually present a more conservative nodal design as all the faces in the nodes need to be covered with stresses. On the other hand, the rules in Eurocode are developed for 2D cases and might not therefore be directly applicable for 3D situations. The enhanced method has not been extensively tested in construction practice and can not be backed up by many already built structures or practical tests.

If future studies shows that calculations in 3D are more conservative than the 2D calculations, it would indicate that the enhanced method applied in this thesis can safely be used in practice.

6

Conclusion

As it was stated in the introduction, the purpose of this master thesis was to investigate if the 3D ST method proposed by Chantelot and Mathern could be applied when designing a bridge corbel, and study if this could be further enhanced with parametric modelling.

A case study was performed where a comparison was made between the conventional 2D strut-and-tie (ST) method and the enhanced 3D ST method. The three dimensional strut-and-tie model in the case study was generated and manually optimised parametrically by the use of software programs Rhino and Grasshopper.

The design based on the enhanced method resulted in reduced material usage compared to the conventional method for the studied structural detail. Further, the shape of the load caring structure was slimmed and resulted in a decrease of concrete volume with approximately 8%. The reduction in reinforcement volume was even larger with approximately 30%. Furthermore, the final shape of the detail resulted in less complicated formwork, and thus judged to reduce the time consumption of constructing the detail. Not only was the amount of material and cost reduced, the labour at the construction site was also estimated to be reduced due to less complicated reinforcement design and less framework.

In this particular case study, the ST model based on the enhanced method, did not utilise the possibility to use triaxial compression that a 3D environment offers, due to the model not containing any nodes of this type. This indicates that there is a potential for the material usage to decrease even further in other structures where triaxial compression occurs.

When it comes to the aesthetics, the shape of the detail based on the enhanced method, has a similar appearance as the middle supports of the bridge. This affects the aesthetics of the bridge positively with the overall bridge design being more coherent.

In terms of flexibility, the enhanced method has some limitations and puts high demands on the dimensions and geometry of the nodal zones. With a new formulation of the basic criterion about strut axes, this would probably make the theory even more useful. The method demanded changes to the original shape of the detail, but do generally present a more rational way when designing discontinuity regions in three dimensional details. In the end, this thesis has validated the majority of the proposed theory of enhanced three dimensional strut-and-tie models and concludes that the method can successfully be used in the design of bridge details.

7
Further Research

During the making of this master thesis, several aspects was found that can be further investigated.

One primary area is a more in-depth investigation of the enhanced 3D ST method proposed by Chantelot and Mathern (2010). As explained in Section 5.1, the model created in this thesis was based upon the fact that the nodal zones in each end of the struts needed to have the same dimensions. If the enhanced method would be able to have the potential to be widely spread and used in construction industry, this is a central part to further develop. One suggestion is to find a ratio that limits how far these axes can be placed from each other without imposing stress in the concrete.

The geometry of the nodal zones should be studied further as well (see Section 5.2.2. For the geometry studied in the case study, cuboidal nodal zones was used. This limits the possibility to make use of the entire concrete volume and limits the internal level arm for the reinforcement. For geometries where the ST model is not aligned with the geometric boundaries of the structure, and where the position of the compression struts is limited by the boundaries, other types of nodal geometries might be more optional.

A parametric approach was chosen in the development of the ST models with manual iterations for obtaining a optimised ST model. This approach was implemented through the software Grasshopper. However, it could be beneficial to investigate the possibility of structural optimisation with automated processes, for example by using the Grasshopper add-on Galapacos.

In this thesis, the comparison between the traditional 2D ST method and the enhanced 3D ST method was done by comparing the volume of concrete and steel reinforcement. Further aspect that can be researched and compared is the environmental and economical impact of the two methods. This can be done through comparative Life Cycle Analysis (LCA) calculations and Life Cycle Cost (LCC) calculations of concrete and steel.

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A

Appendix A
INPUT DATA

General

Table A.1: *General input data*

Max design load from bearing and jack:	4.38 MN
Reinforcement bar diameter:	25 mm
Steel Reinforcement class:	K500
Concrete class:	C35/45
Anchorage Length:	824 mm
Lap Length:	1236 mm

Table A.2: *Material input data - Concrete*

Concrete class	C35/45
E_{cm}	35.2 GPa
f_{ck}	35 MPa
f_{ctk005}	2.2 MPa
f_{cd}	23.3 MPa
f_{ctd005}	1.5 MPa

Table A.3: *Material input data - Steel reinforcement*

Steel class	K500
f_{yk}	500 MPa
f_{yd}	434.8 MPa

Model A

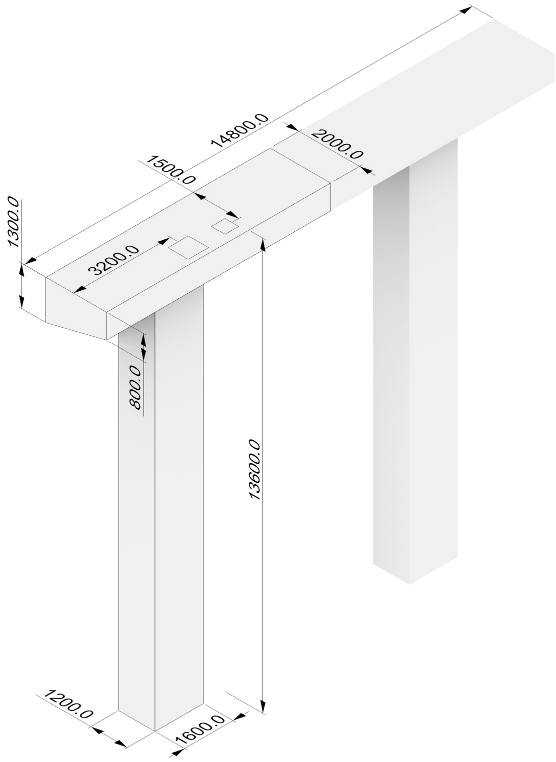

Figure A.1: *Geometry of Model A*

Table A.4: *Input data for Model A*

Dimensions of bearing:	470 x 470 mm
Dimensions of jack:	450 x 450 mm

Model B

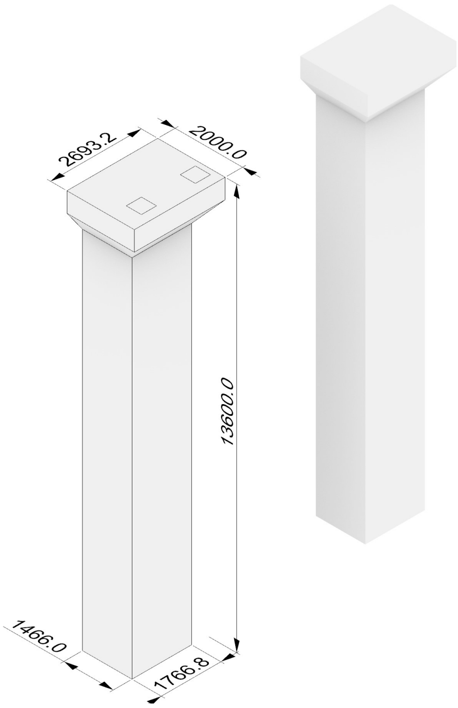

Figure A.2: *Geometry of Model B*

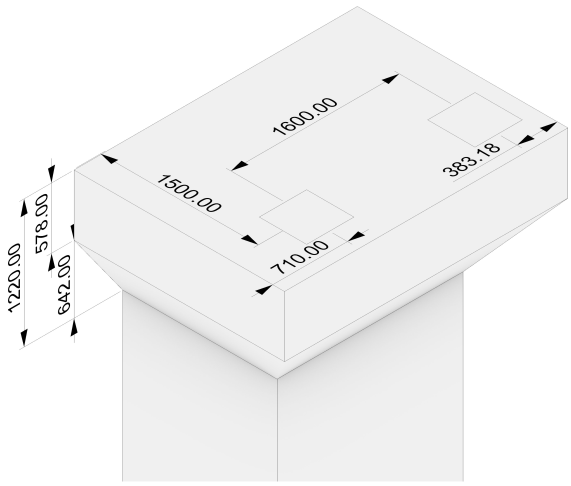

Figure A.3: *Geometry of Model B - Detailed around the corbel*

Table A.5: *Input data for Model B*

Dimensions of bearing:	550 x 550 mm
Dimensions of jack:	550 x 550 mm

B

Appendix B

CALCULATIONS MODEL B

The grasshopper definition is presented in Figure B.1 and B.2 and divided into different blocks. The blocks are defined in the numbering below.

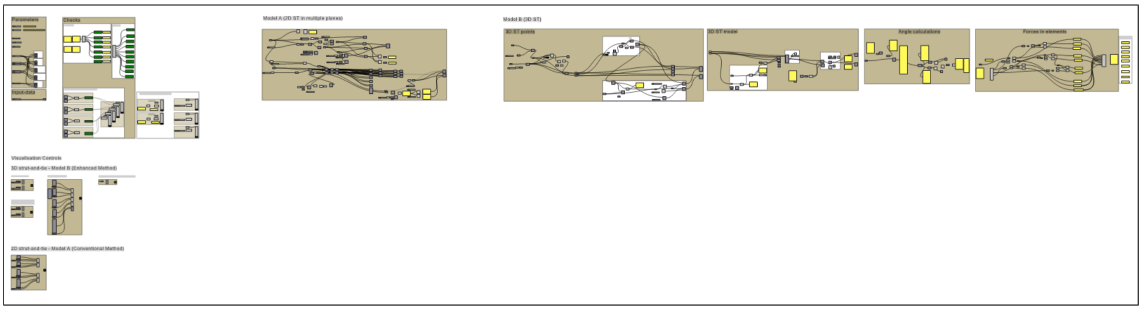

Figure B.1: *First part of the grasshopper definition*

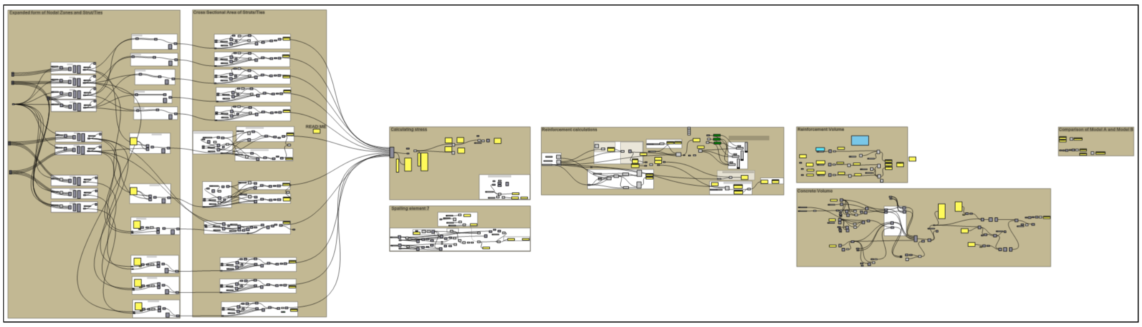

Figure B.2: *Second part of the grasshopper definition*

1. Parameters, controls and results
2. Controls for displaying all geometry
3. Model A - nodes and elements
4. Model B - nodes
5. Model B - node and element indexing
6. Calculation of angles
7. Calculations of forces in elements
8. Constructing and calculating nodal zones and elements in between
9. Calculation of stresses in struts

10. Reinforcement design and calculations
11. Reinforcement volume
12. Concrete geometry- and volume calculation
13. Comparison between models

1. Parameter, controls and results

The parameters that was subjected to manual adjustments was placed together with the results and the controls. In this way instant feedback was provided when the geometries and node positions was changed. The panels (in green) that presents the values was color-coded and switched color to red instantly if any value exceeded the limits.

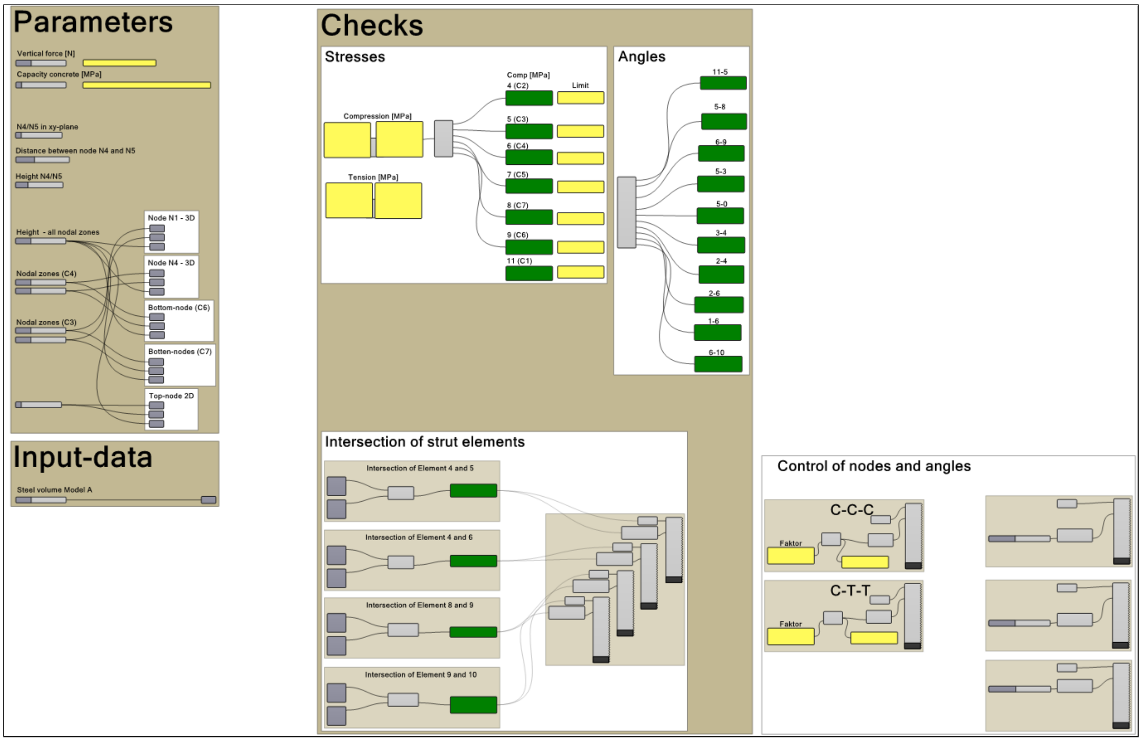

Figure B.3: *The code for the parameters and the controls*

2. Controls for displaying all geometry

This part contain the controls for the graphic visualization of all geometry. Includes:

- Model A
- Model B
- 3D-ST model for Model B-2 without orthogonal reinforcement
- The concrete volume for Model B

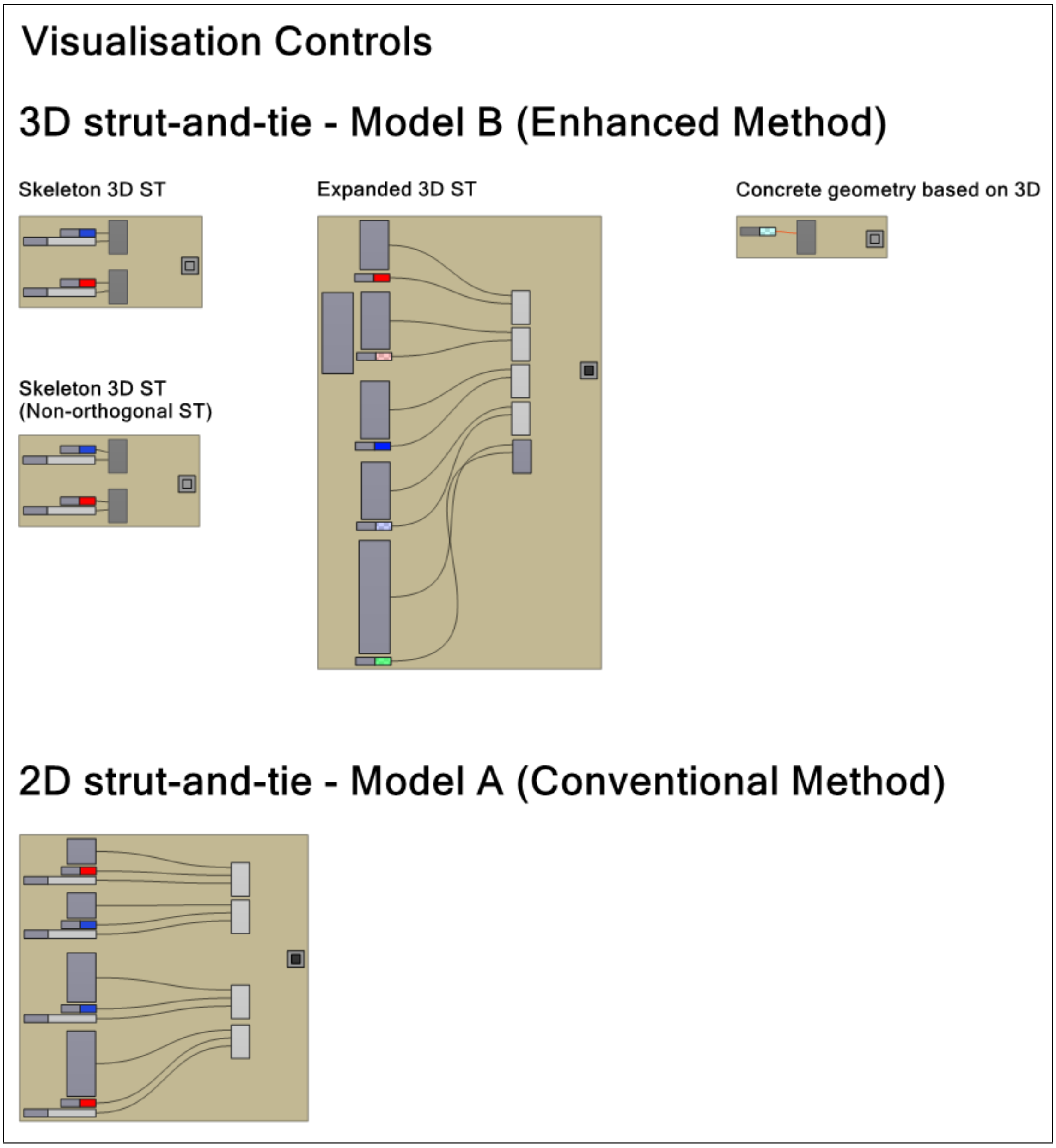

Figure B.4: *The code for displaying the model geometries*

3. Model A - Geometry

The geometry of the ST-model in two perpendicular planes was defined. This model was based on the calculations made by Ramboll, and therefore the geometry is set from the start. Includes:

- The position of the loads
- The position of all nodes in Model A

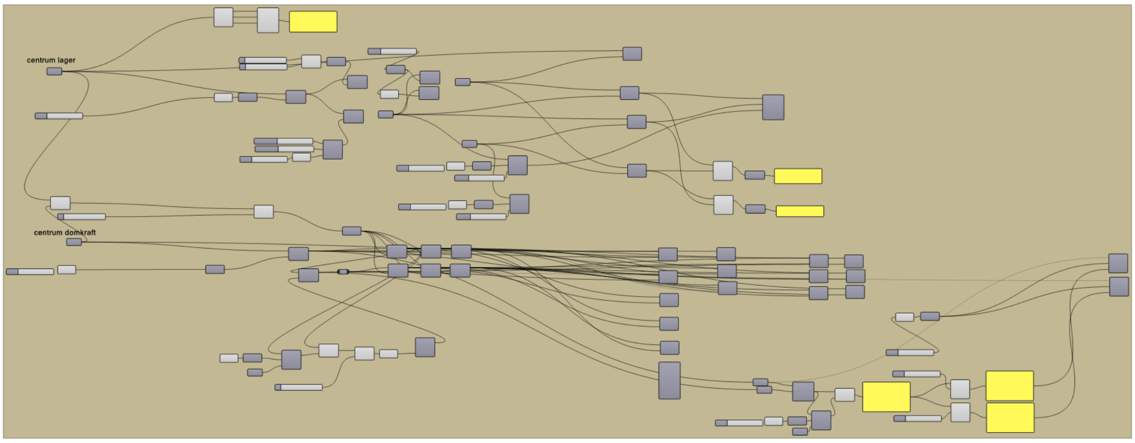

Figure B.5: *The code for producing the geometry of Model A*

4. Model B - nodes

The node positions in Model B are defined. Includes:

- The position of all nodes in Model B

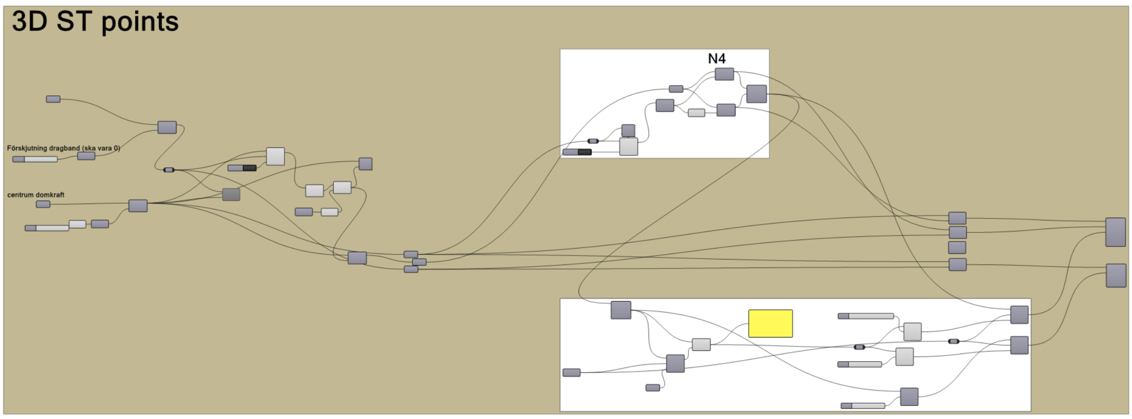

Figure B.6: *The code for producing the geometry of Model B*

5. Model B - node and element indexing

The elements in model B are defined, and the indexing for the nodes and elements are defined. Includes:

- Construction of the lines between the nodes (the elements)
- Indexing of the nodes and elements

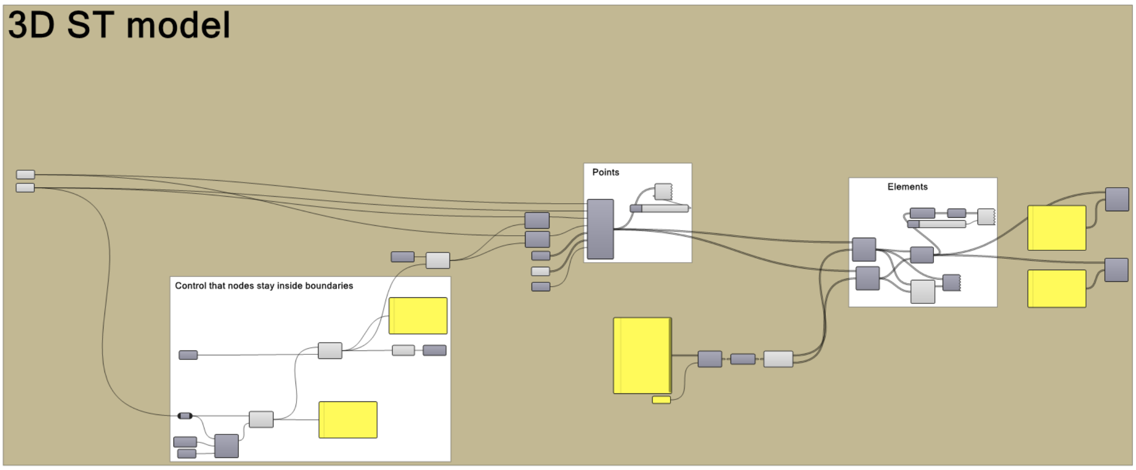

Figure B.7: *The code for defining and indexing all points and elements*

6. Calculation of angles

The relevant elements are defined in a list and the angle between them is calculated by using the component "Angle". The angles presented are the angle between the direction vectors in the elements

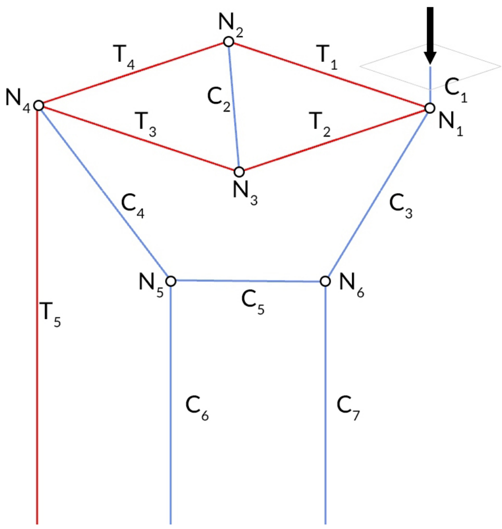

Figure B.8: *Labelling of each element in Model B-2. N indicates Nodes, C indicates struts and T indicated ties.*

Table B.1: *Angles between elements*

Elements	Angle
C3-C1	30
C3-C7	30
C3-C5	60
C4-C6	36
C4-C5	54
C3-T1	69
C3-T2	70
T1-C2	43
T4-C2	47
T4-C4	66
T3-C4	65
C4-T5	36

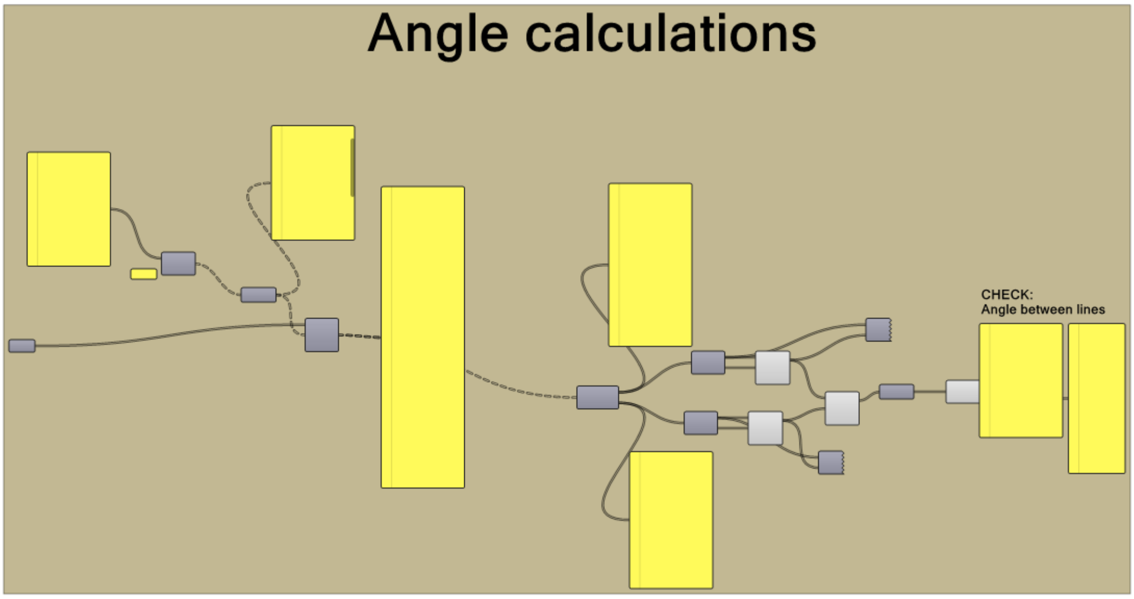

Figure B.9: *The code for calculating the angles between the elements*

7. Calculations of forces in elements

The force in every element was calculated by trigonometric calculations, as the system is statically determinate.

$F_{C3} = 5.04MN$	Force in strut C3
$\alpha_{C3,C7} = 29.66°$	Angle between strut C3 and strut C7
$F_{C7} = F_{C3} * cos(\alpha_{C3,C7}) = 4.38MN$	Force in strut C7

Table B.2: *Forces in elements*

Element	Force [MN]
T2	1.71
T3	1.81
T4	1.71
T1	1.81
C2	2.49
C3	5.04
C4	4.25
C5	2.49
C7	4.38
C6	3.44
T5	3.44
C1	4.38

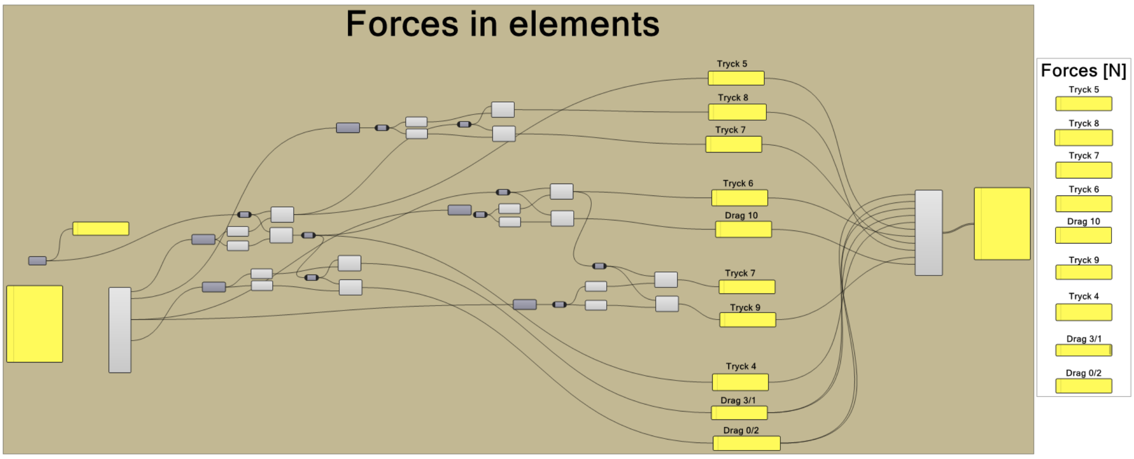

Figure B.10: *The code for calculating the forces in the elements*

8. Constructing and calculating nodal zones and elements in between

The geometry of the nodal zones were defined with the node as center point. The corners of the nodal zones were connected with lines that defined the expanded struts and ties. The cross-sectional area of the struts were calculated by using the component "plane normal" to define a cutting plane orthogonal to the struts direction vector. The area of the cross-section was calculated with the component "area".

The cross-sectional area for every element. This is only interesting for the struts (element C3-C7).

Table B.3: *Elements cross-sectional area*

Elements	Area [mm^2]
T2	183727
T3	152800
T4	154862
T1	183250
C2	192258
C3	417717
C4	287305
C5	271110
C7	302500
C6	180600
T5	180600
C1	302500

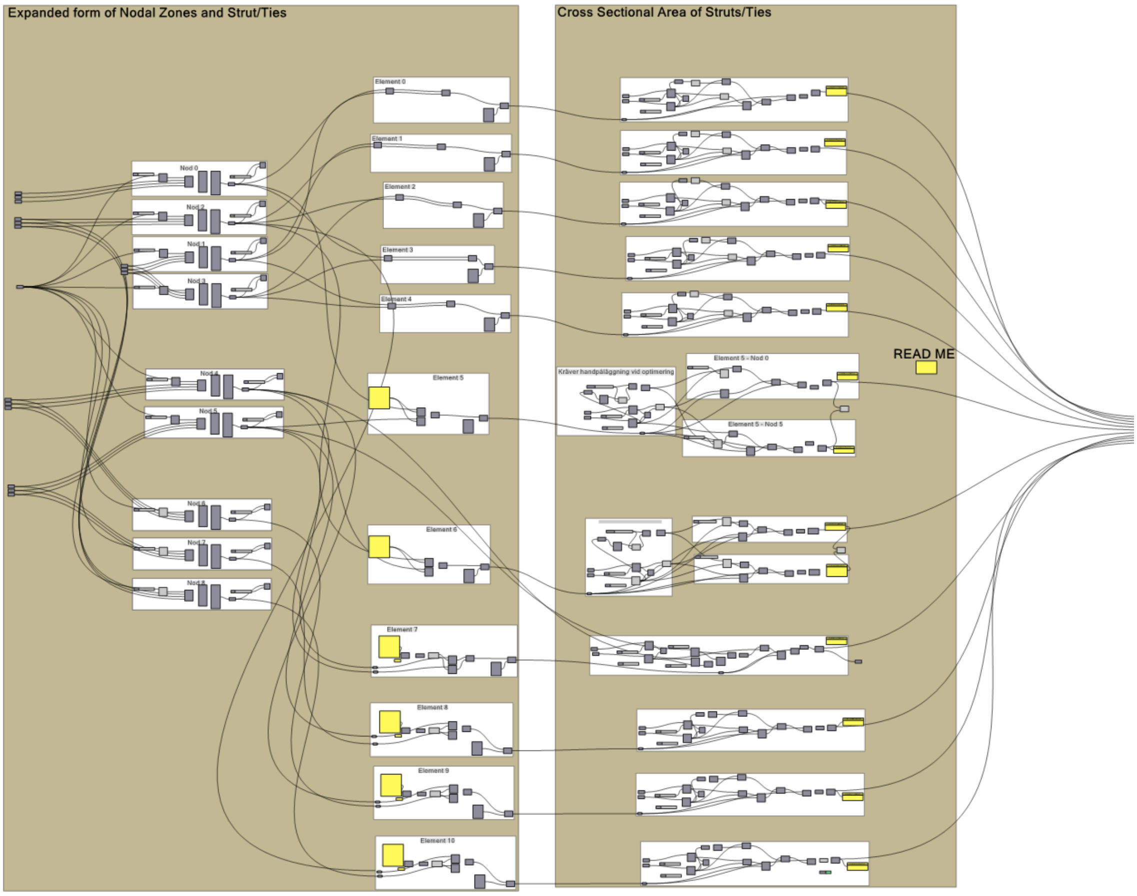

Figure B.11: *The code for constructing nodal zones, struts, ties, and calculate the cross-sectional area*

9. Calculation of stresses in struts

The force in every strut was divided with the cross-sectional area to obtain the stress. Element number in the panel to the left, and the force in the panel to the right

Table B.4: *Stress in struts*

Elements	Stress [MPa]
C2	14.7
C3	12.1
C4	14.8
C5	9.2
C7	14.5
C6	19.1

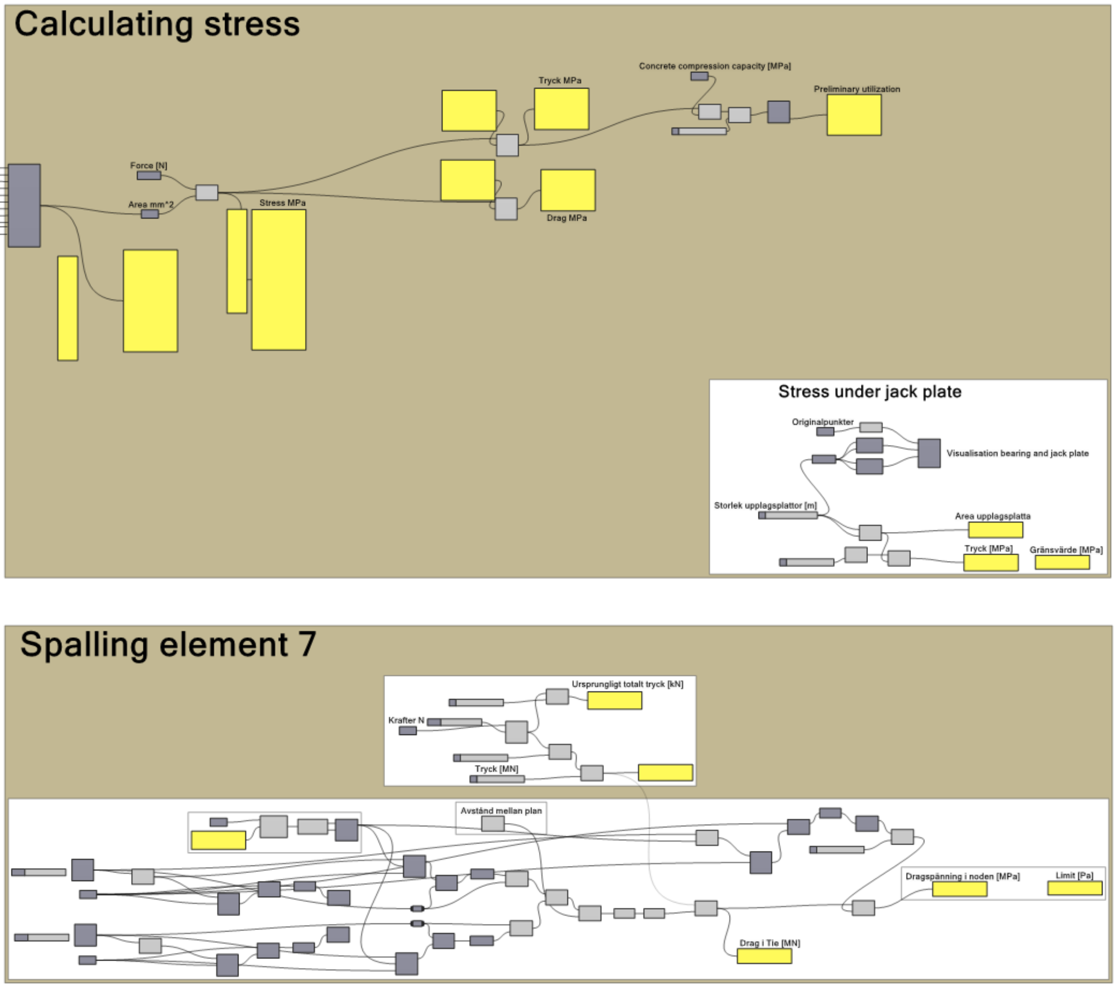

Figure B.12: *The code for calculating the stresses in the struts*

10. Reinforcement design and calculations

The reinforcement calculations was based on the following parameters: Material:

- Concrete C35/45
- Steel reinforcement K500
- Reinforcement bar diameter 25 mm
- Maximum size of aggregate 0.027m

Geometry and forces:

- The force in each tie
- The crossection of the nodal zones at the ends of the tie

The force was divided with the steel capacity, and the required steel was obtained:

Table B.5: *Required steel area*

Elements	**Area [mm^2]**
T5	14.7
T1 & T3	12.1
T2 & T4	14.8

Then the steel area was divided with the area of one bar to obtain the number of bars needed

Table B.6: *Minimum number of bars needed*

Elements	**Bars**
T5	17
T1 & T3	9
T2 & T4	9

The minimum bar spacing was calculated, and the used bar spacing was chosen based on this distance and industry standard in Sweden.

Table B.7: *Chosen bar spacing*

Minimum bar spacing	**Chosen bar spacing**
32 mm	$\approx 100mm$

The minimum cross-sectional area of the reinforcement in the ties was calculated, and compared with the provided cross-sectional area in the nodal zones. If the area needed was larger than the provided area, bundled reinforcement was needed.

Table B.8: *Minimum cross sectional area of reinforcement in ties*

Elements	Area needed [mm^2]	Provided area [mm^2]
T5	80063	180600
T1 & T3	40033	30090
T2 & T4	40033	30090

The equivalent diameter of bundled bars and the amount of bundled bars was calculated. The calculations was based on the following parameters:

- Amount of bars in each bundle
- Bar diameter
- Number of bars needed

The amount of bars needed in Model B is compared with the amount of bars needed in Model A. The ties in Model A coincide with two of the ties in model B, and therefore the largest need of reinforcement is chosen for the design. Custom scripts is used to picking out the largest reinforcement need out of these two models.

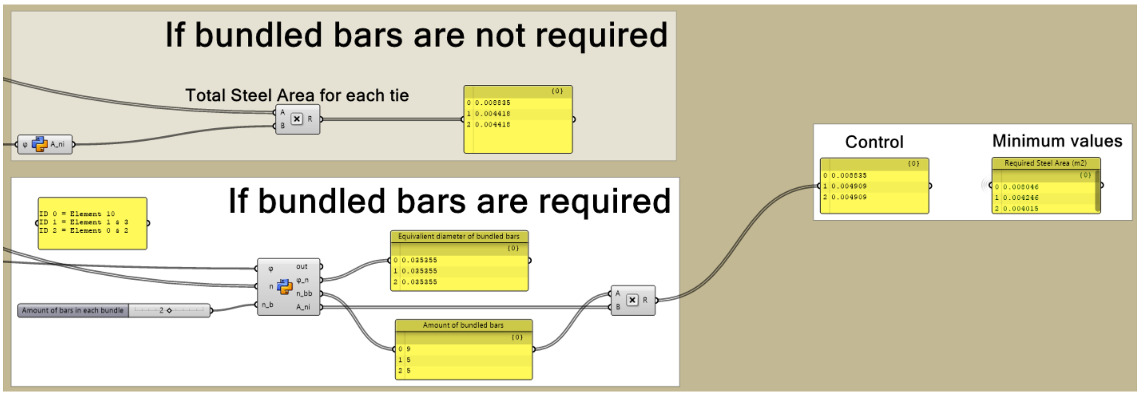

Figure B.13: *The code for calculating required reinforcement for Model B.*

The final dimensions for the reinforcement in each tie is presented in Table B.9. For more information of the reinforcement layout, see Figure 4.11

Table B.9: *Reinforcement dimensions*

Elements	Number of total bars	Bar **Area** [mm^2]
T1 & T3	9	25 mm
T2 & T4	9	25 mm
T5	18	25 mm

The anchorage length and lap length was calculated based on the following parameters:

- Type of anchorage (tension)
- Bar diameter
- Concrete type (f_{ctd})
- Steel type (K500)

Table B.10: *Anchorage and lap length*

Design anchorage length	824 mm
Lap length	1236 mm

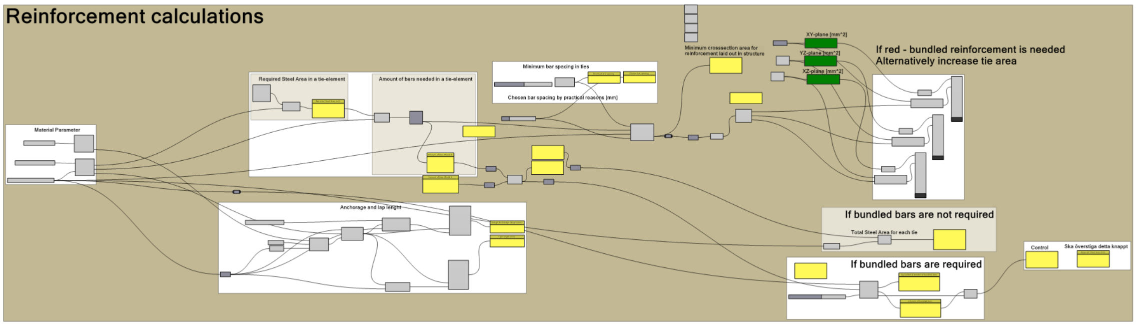

Figure B.14: *The code for the overall reinforcement design*

11. Reinforcement volume

The total amount of reinforcement volume was calculated. This was based on:

- The length of the ties
- The anchorage length on both sides
- The cross-sectional area of the bar chosen

Table B.11: *Reinforcement volume*

Elements	Volume [m^3]
T5	0.105
T1& T3	0.026
T2& T4	0.025
Total	**0.156**

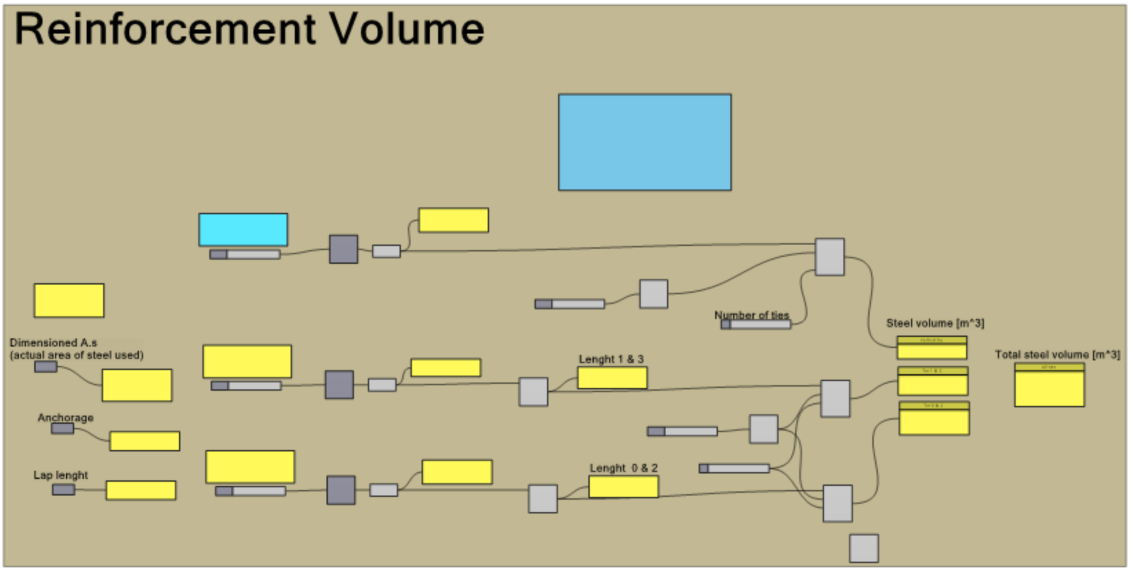

Figure B.15: *The code for summing up the total amount of reinforcement used*

12. Concrete geometry- and volume calculation

The concrete volume in Model B was chosen to follow the strut-and-tie model, but the original shape of the corbel and column as well as the manufacturing part was also taken into consideration. The parameters:

- Concrete cover over the nodal zones, chosen to 50 mm
- Height of the edge in the front, chosen to 178mm

Table B.12: *Concrete volume*

Model	Volume [m^3]
Model A	36.450
Model B	28.020

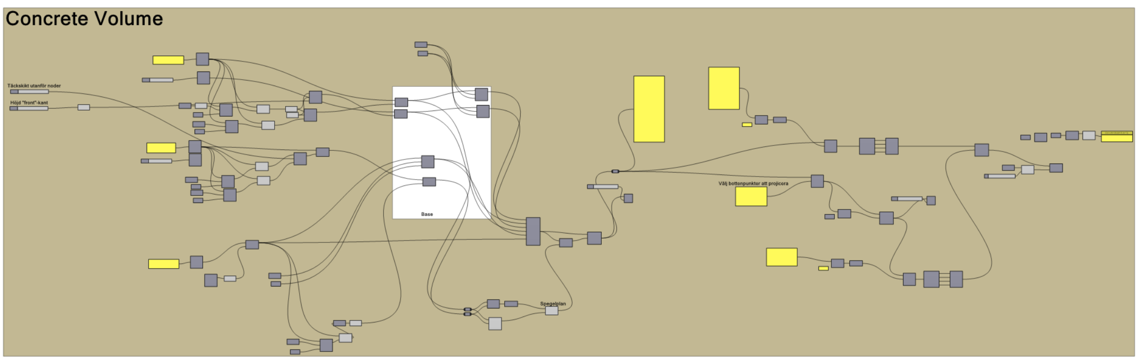

Figure B.16: *The code for summing up the total amount of concrete used*

12. Comparison between models

In the end, a comparison was made between Model A and Model B in regards to steel reinforcement volume and concrete volume. The code is presented below.

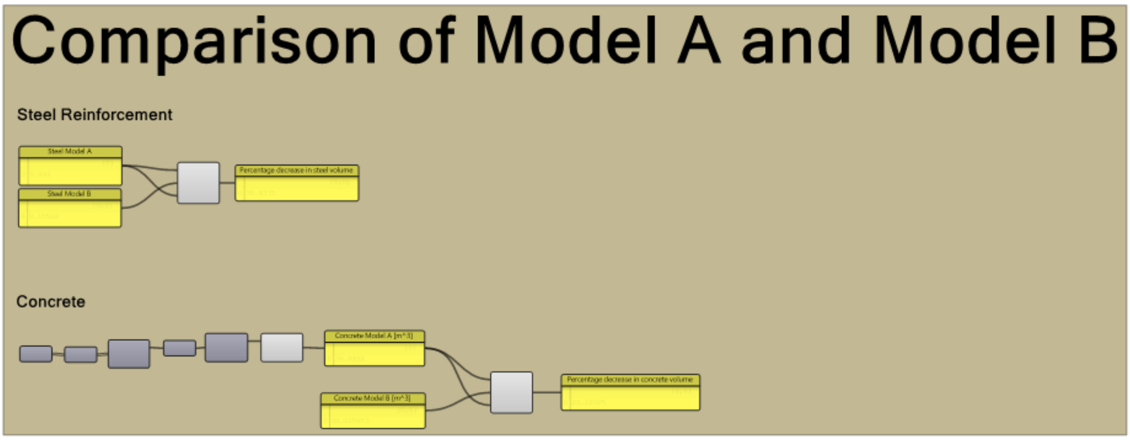

Figure B.17: *The code for comparison between Model A and Model B*