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Problem Solving in Thinking Classrooms Using Vertical Surfaces

Fostering Critical Thinking and Collaboration through Problem-Solving Activities

Master's thesis in Learning and Leadership

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COMMUNICATION AND LEARNING IN SCIENCE

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Abstract

This phenomenographic study explores how structured problem-solving activities, inspired by the Thinking Classroom model and conducted on vertical surfaces, influence students' learning experiences in upper secondary mathematics education. In recent years, interest in student-centered and collaborative approaches to mathematics teaching has grown, aiming to promote deeper understanding and engagement.

Semi-structured interviews with two teachers and six students, and an analysis of students' written exam responses, were conducted to examine variations in how participants perceive the use of vertical surfaces and group-based tasks. The study focuses on students' mathematical reasoning, collaborative interactions, and attitudes toward problem-solving. It also investigates teachers' experiences regarding the pedagogical benefits and challenges of implementing these methods.

The findings highlight key themes such as increased engagement, improved metacognitive awareness, and enhanced collaboration. The results contribute to understanding how physical learning environments and structured problem-solving approaches can support critical thinking and mathematical development.

Disclaimer: AI tools were used in this thesis for transcribing interview recordings, translating words and short expressions from Swedish to English, and for grammatical editing.

Key words: Phenomenography, Problem-Solving, Vertical Surfaces, Thinking Classroom, Mathematics Education, Collaborative Learning.

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I also extend my heartfelt appreciation to the teachers and students at NTI Gymnasiet who participated in this study. Their openness, enthusiasm, and willingness to share their experiences made this research possible. A special thanks to the dedicated teachers who implemented the Thinking Classroom approach and provided critical reflections on its impact.

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1. Introduction

In today's society, particularly in schools worldwide, including Sweden, problem-solving is a central aspect of education (Skolverket, 2019). Through problem-solving, students have the opportunity to develop their analytical skills, critical thinking, and collaboration abilities. To create a learning environment where these skills can be nurtured and to support students' active participation in their learning, effective tools and methods are required.

In recent years, however, a growing number of researchers have begun to emphasize the collective and context-dependent dimension of problem-solving. Peter Liljedahl (2021) is one of those who clearly highlights that problem-solving in the classroom should not be viewed as an individual or isolated activity, but rather as a social process in which collaboration and access to resources play a crucial role. In real life, problem-solving often takes place in cooperation with others and with the help of various external tools – such as technological aids, collegial support, and access to surrounding materials.

Liljedahl emphasizes the importance of creating classroom environments that stimulate collaboration and the sharing of ideas, as it is through this dynamic interplay that students develop their problem-solving abilities. A central concept in his research is the so-called "Thinking Classroom," where students are encouraged to work together at vertical, non-permanent surfaces (VNPS), such as whiteboards. These surfaces promote movement, interaction, and active engagement in learning, which in turn strengthens both individual and collective thinking (Pruner & Liljedahl, 2021).

However, it is relevant to ask the question: How can we practically implement problem-solving as a collaborative and dynamic process in the classroom, in line with the Thinking Classroom model, and how do students experience this way of learning?

In this report, I focus on how vertical surfaces, combined with a specific lesson structure developed by two teachers and a researcher, drawing inspiration from George Pólya and Akihiko Takahashi (Takahashi, 1996; Szabo et al., 2020), were experienced by students and teachers in upper secondary mathematics education. The aim is to investigate how this method promotes problem-solving and influences students' learning experiences and outcomes.

The implementation of vertical surfaces in problem-solving lessons is part of a practice-based research project (ULF project) led by my supervisor throughout the spring term, with follow-up assessments during the national exam in late May. For my thesis, I have chosen to study this development project. One specific area of mathematics where problem-solving plays a central role is optimization. In this report, optimization has been chosen as the mathematical content for the problem-solving lessons, as it offers rich opportunities for reasoning, discussion, and conceptual exploration, all of which align well with the pedagogical goals of the Thinking Classroom model. Optimization is a field of applied mathematics where its principles and methods are used to solve quantitative problems in disciplines such as biology, engineering, physics, and economics.

However, due to its complexity, optimization can be particularly challenging for students. Common difficulties include a lack of mathematical reasoning skills related to how variables interact, difficulties using appropriate representations (such as graphs or equations), and an overreliance on procedural approaches rather than conceptual understanding. These challenges make optimization a meaningful and relevant topic for investigating how collaborative problem-solving methods can support students' mathematical thinking and learning. Furthermore, the topic requires the integration of several concepts from students' prior mathematical knowledge, making it a suitable test case for exploring how well students engage with and understand mathematics in a Thinking Classroom context.

1.2 Purpose

The purpose of this thesis is to investigate how a systematic approach to problem-solving in groups on vertical surfaces affects high school students' problem-solving skills, mathematical reasoning, and attitudes toward mathematics. Additionally, the study aims to examine mathematics teachers' experiences and perceptions of development work related to teaching with vertical surfaces

1.3 Research Question

1. How do students perceive the use of vertical surfaces as a working method in mathematics education, and in what ways does it affect their problem-solving ability?
2. How do students demonstrate what they have learned from problem-solving lessons in the way they reason and communicate their test solutions?
3. How does teaching through problem-solving affect students' attitudes?
4. What experiences have teachers gained from development work related to teaching with vertical surfaces, and how has it influenced their teaching practice?

1.4 Delimitations

The study is conducted over a short period and includes a limited number of lessons, meaning that only the immediate effects of the method are investigated. Long-term changes cannot be analyzed. The study is also limited in terms of geography and context, as it was conducted at a single school within a specific area. This means that the results are relevant to the specific context, but their generalizability is limited. Due to time constraints, there is no extended follow-up of the results after the study is completed. The analysis therefore focuses on the immediate effects of the observed lessons, without the possibility of assessing the method's long-term impact. The limited time frame, participant selection, and specific mathematical content mean that the results cannot be generalized. Despite these delimitations, the study provides valuable insights into the method's potential and may serve as a foundation for future, more comprehensive research that includes longer time periods and more participants.

Due to time constraints, I am reporting only a part of the project and therefore cannot assess the long-term effects of working with vertical surfaces. It is also challenging to determine the exact impact on students' performance after only three months. I have therefore chosen instead to investigate how teachers and students perceive these lessons and how they believe the method influences their learning and attitudes.

2. Background

This section of the report outlines the documents and theories that have been useful for analysing the interviews. The focus is primarily on problem-solving, the use of vertical surfaces, and a Japanese lesson structure that has served as a source for lesson planning for the upper secondary teacher.

2.1 What Does It Mean to Solve Problems?

Research has shown that problem solving is a central aspect of learning and development, particularly in mathematics, where it is not only about finding the correct answer but also about developing strategic flexibility and conceptual understanding (Kilpatrick, Swafford, & Findell, 2001).

According to Kilpatrick, Swafford, and Findell (2001), problem solving is a fundamental component of mathematics education and should be an integrated part of learning rather than a separate skill. The authors emphasize that effective problem solving requires both strategic competence, the ability to formulate, represent, and solve mathematical problems, and adaptive reasoning, which involves analyzing relationships between different concepts and justifying solutions. These abilities develop when students are exposed to meaningful mathematical challenges and given opportunities to reflect on their solution strategies.

This view of problem-solving as an integrated and dynamic part of learning is also shared by other researchers in the field.

Lester and Cai (2015) present a similar perspective on problem solving but focus particularly on how it can be taught and developed. Problem solving is a cognitive process in which an individual identifies, analyzes, and resolves a task without an immediately apparent solution (Leaster & Cai, 2015). Lester and Cai describe two types of problem solving: routine problems and non-routine problems. Routine problems can be solved using previously learned methods, whereas non-routine problems require productive thinking, in which the individual must devise a new strategy to find a solution. The researchers argue that teaching through problem solving, where students learn mathematics by actively engaging with challenging problems, is more effective than separating problem solving from traditional instruction.

2.1.1 Teaching Through Problem Solving

An effective method for strengthening students' mathematical skills is teaching through problem solving, where the focus is on active exploration rather than solely learning rules and procedures (Lester & Cai, 2015). Research shows that students develop a deeper understanding when they work with authentic and challenging problems, making this approach an important part of modern mathematics education (Lester & Cai, 2015).

Instead of starting from ready-made methods, students in this type of instruction work with real-life tasks that encourage inquiry, reasoning, and shared reflection. The teacher takes on the role of a facilitator rather than a lecturer, creating space for students to discuss various solutions in a social context (Kilpatrick, Swafford & Findell, 2001). By working on tasks rooted in everyday situations, mathematics becomes more relevant and comprehensible to students – they see more clearly its practical application.

Teaching through problem-solving not only develops subject knowledge but also promotes critical and creative thinking. Students are given the opportunity to try different strategies, analyze results, and think beyond given frameworks (Kilpatrick et al., 2001). One example is when students collaborate to solve optimization problems, where multiple solutions may be possible and communication within the group becomes crucial.

This type of teaching aligns well with Carol Dweck's theory of growth mindset, which is based on the belief that intelligence and abilities can be developed through effort and learning (Dweck, 2006; Bates, 2016). In an environment where problem-solving is central, students are encouraged to make mistakes, view challenges as opportunities, and persist even when tasks feel difficult. By taking risks, collaborating, and testing different solutions, the idea that learning occurs through perseverance and reflection is reinforced, which in turn increases motivation and contributes to deeper understanding (Bates, 2016).

A key element of teaching through problem solving is collective learning. When students work together and share their thoughts, both understanding and engagement grow (Niss & Højgaard, 2011). In this way, a learning environment is created where both subject knowledge and the ability to think with numbers can grow.

2.2 Thinking Classrooms

In order to implement teaching through problem solving, a learning environment is required that provides students with real opportunities to engage with problems. Peter Liljedahl (2021), through his research in mathematics education with a particular focus on problem solving, has introduced the concept of the Thinking Classroom – a model aimed at fostering students' creativity, independence, and cognitive development. This model emphasizes the importance of allowing students to work with open-ended and authentic problems, where the focus is on exploring various solution strategies rather than immediately finding the correct answer. To achieve this, tasks should be sufficiently challenging and deliberately designed without detailed instructions or clear hints. By making the tasks somewhat ambiguous, students are compelled to explore the problem on their own, think critically, and collaborate to find possible paths toward a solution (Liljedahl, 2021). To successfully implement the Thinking Classroom model, Liljedahl (2021) identifies several interconnected factors that together create the conditions for effective problem-solving instruction.

One key factor in enabling problem-solving-focused instruction and creating a Thinking Classroom is how students are grouped. Group formation plays a central role in fostering student engagement and creative thinking. According to Liljedahl (2021), students should not work in static groups but rather in temporary, randomly assigned groupings. The aim is to disrupt habitual social patterns and encourage collaboration among different classmates, thereby increasing overall participation in the classroom. Effective groups should consist of three students, a size that strikes a balance between idea exchange and responsibility distribution. This group size is small enough to prevent passivity but large enough to support a diversity of perspectives (Liljedahl, 2021). By using random methods for group formation, such as playing cards, color codes, or digital tools, the risk of social hierarchies negatively impacting collaboration is reduced. This group structure supports a learning environment where students' shared exploration of problems is prioritized over teacher-led instruction and individual performance.

Another essential aspect of designing a Thinking Classroom is the use of vertical working surfaces. Liljedahl (2021) points out that it is not only the content of the instruction that matters but also how and where students work. When students work while standing at vertical and non-permanent surfaces, such as whiteboards or whiteboard film applied to walls and windows, their thinking becomes visible to themselves, their peers, and the teacher. This visibility fosters metacognition and allows the teacher to provide formative feedback in real-time. Creating workstations with standing areas, ideally near vertical whiteboards, facilitates student participation in the problem-solving process. The physical environment should clearly signal that active participation, exploration, and collaboration are the norms in the classroom.

A third key factor highlighted by other researchers is the crucial role of the teacher in successful problem-solving instruction, as emphasized by Peter Liljedahl (2021). A vital part of the teacher's responsibility is to carefully select discussable tasks, form groups, and create an environment that supports both thinking and collaboration. In a Thinking Classroom, the teacher shifts from being a lecturer to serving as a guide who poses questions, offers hints, and responds to students' thought processes. The focus is not on how quickly groups reach solutions but on how they reason and collaborate. The teacher organizes instruction to promote reflection and active thinking. To build a successful Thinking Classroom, the teacher must understand the importance of fostering an atmosphere where students feel free to express their ideas and discuss their solutions. This requires giving students space to think independently, take risks, and sometimes fail, only to reflect and improve next time. Liljedahl emphasizes that such a learning environment not only enhances subject knowledge but also promotes a deeper understanding of the problem-solving process.

Collaborative learning has strong theoretical foundations, with Vygotsky's sociocultural theory playing a central role (Vygotsky,1962 ; Bates, 2016). Vygotsky emphasized that cognitive development occurs through interaction with others in social contexts and that learning is a deeply social process. He highlighted the importance of the zone of proximal development, in which a student can expand their understanding and skills with support from others, such as teachers, peers, or more knowledgeable individuals. According to Vygotsky, language is a vital tool for thinking and knowledge construction, making dialogue between individuals a key component of the learning process (Bates, 2016).

2.2.3 Understand Mathematical Relationships Through Well-Planned Problems and Discussions

In order to enable teaching that emphasizes problem solving, previous sections have addressed how the learning environment should be structured and which didactic factors are essential in the classroom. This section presents the theoretical and practical foundation underlying the lesson design used in the development project conducted in collaboration between the supervisor and the participating teachers.

The lesson design presented is based on the work of Japanese researcher Akihiko Takahashi, whose model provides a clear structure for planning and implementing instruction focused on students' problem-solving abilities. Takahashi's model aims to support students' understanding of the problem-solving process they are expected to develop during lessons, while also offering a systematic framework for lesson planning and the teacher’s didactic considerations (Takahashi, 2013).

A key aspect of Takahashi’s method is the concept of *Kyozaikenkyu*, which involves a thorough analysis of the teaching materials. This includes a deep understanding of the mathematical content, an expectation of student responses, and a strategy to optimally support student learning. This method allows for the refinement of teaching practices and the sharing of effective strategies for implementing pedagogical approaches, such as Teaching Through Problem-Solving (TTP). Through collaboration (Lesson study), teachers can develop new teaching materials and mathematical tasks while ensuring that the problems presented to students are sufficiently challenging. Additionally, teachers are prepared for potential misunderstandings and incorrect solutions that may arise during the lesson (Takahashi, 1996). Thus, Lesson Study supports the continuous development of teaching practices and is considered a crucial mechanism through which Japanese teachers have improved their mathematics teaching and pedagogy in general (Takahashi, 1996).

Through his research, Takahashi has shown that effective mathematics teaching is not just about presenting formulas and procedures, but about allowing students to discover and understand mathematical relationships through well-planned problems and discussions (Takahashi, 2013). Takahashi’s lesson design is structured in several phases aimed at promoting a deeper understanding of mathematical concepts (see Figure 1). The lesson begins with the teacher creating a context and activating students' prior knowledge to spark their interest and engagement. Then, a mathematical problem is introduced, encouraging students to think critically and draw on their previous experiences. Students are then given the opportunity to work individually or in smaller groups to solve the problem, followed by a collective discussion where different solutions and strategies are compared.

This discussion allows students to verbalize their thought processes and reasoning, leading to a deeper understanding of the mathematical concepts. The lesson concludes with a summary where the central insights from the discussion are highlighted and connected to broader mathematical contexts, further reinforcing students’ understanding (Takahashi, 2013).

The effectiveness of a TTP lesson largely depends on the richness of the problem.

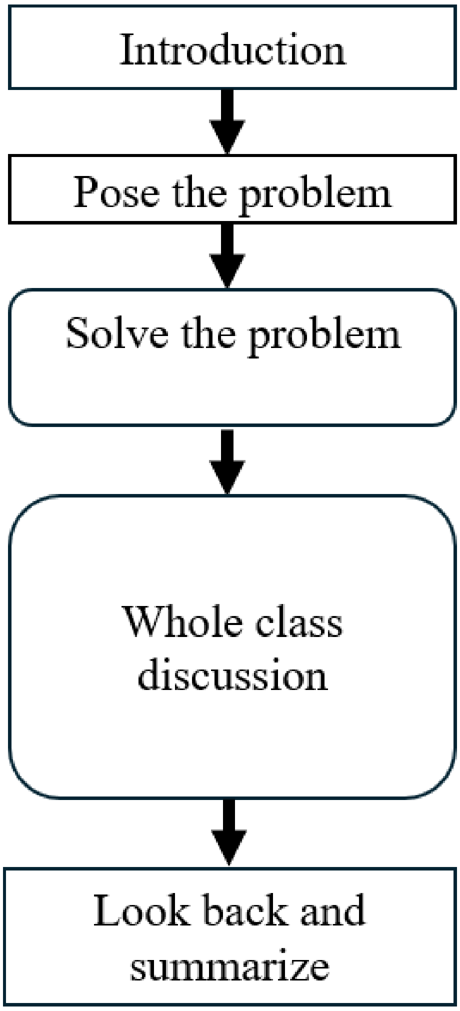


Figure 1 illustrates the Japanese Teaching Through Problem-Solving (TTP) lesson structure (Takahashi 1996).

Teachers must consider students' prior knowledge and design lessons that promote understanding and engagement, which is grounded in the principles of Lesson Study a method that emphasizes collaborative planning, teaching, and reflection among teachers.

2.3 Implementation

This section presents the lesson plan, which is the instructional design developed by the teachers in collaboration with researcher. The plan has been created with particular consideration for the student group, who experience difficulties in mathematics and problem-solving. The aim of the lesson plan is to support students' understanding of how to approach mathematical problems, as well as to provide them with strategies for identifying where and how to begin solving them. The instruction also aims to develop the students' ability to present their solutions in a correct and mathematically structured manner.

2.3.1 Theoretical Framework of the Lesson Design

The design of the learning environment during problem-solving instruction is inspired by the research of Peter Liljedahl. His ideas regarding how student groups should be organized, how classrooms and writing surfaces should be arranged, and how appropriate problems should be selected form the foundation of this approach. The structure and content of the problem-solving lessons are in turn inspired by Akihiko Takahashi’s instructional models.

The elements included in the lesson plan, as well as the aspects students need to consider during problem-solving, are inspired by George Pólya’s four-step problem-solving model as described in *How to Solve It* (Polya, 1990). This model has been further developed and adapted to the teaching context of Mathematics 3c, specifically within the topic of optimization, by the teachers in collaboration with a researcher in mathematics education.

In a report described by Szabo et al. (2020), concrete examples are presented to demonstrate how Pólya's four-step problem-solving model can be applied in a broader context. By incorporating and practicing specific methods for solving mathematical problems in the learning process, students are allowed to develop a way of thinking that allows them to approach and solve problems successfully, not only in mathematics but also in other life situations. This method can be seen as part of the instructional structure described in the report, which has been adapted to promote students' ability to solve problems systematically and creatively.

The students involved in the project, specifically those in classes working on problem-solving using vertical surfaces within the mathematics 3c course, have received instruction on identifying maximum and minimum points, a central topic in the optimization unit. The adapted version of the model has resulted in a clear structure that supports students in their problem-solving process (see Figure 2).

The following section presents how the lesson structure is linked to George Pólya’s four-step model for problem-solving.

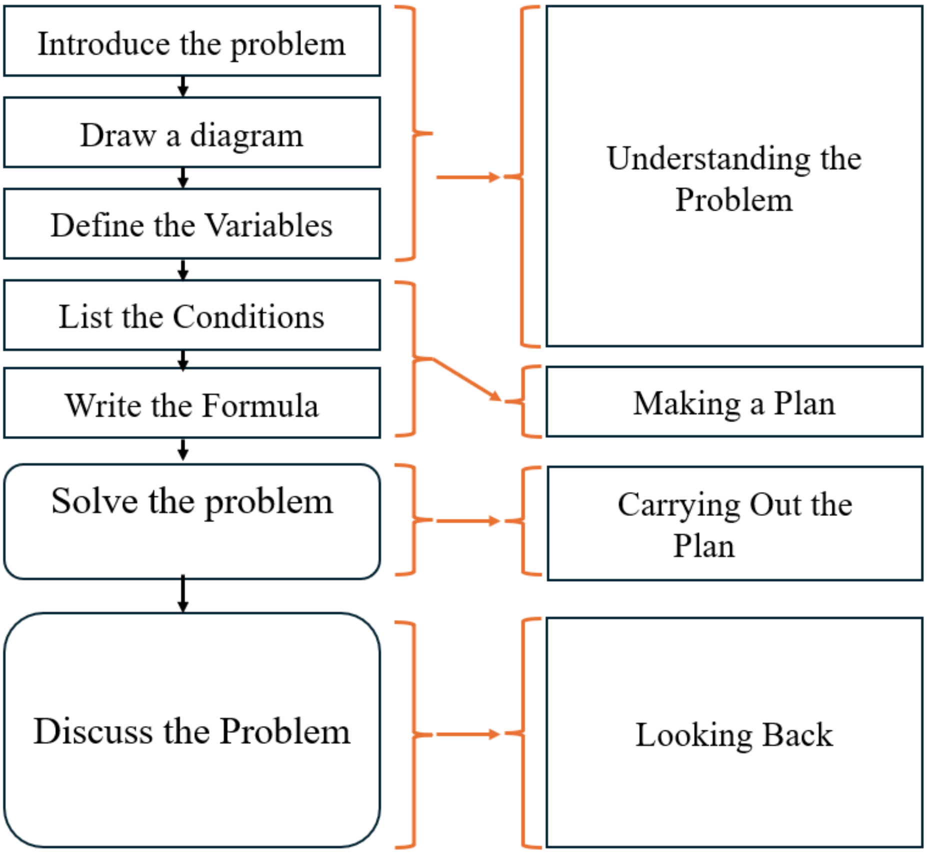


Figure 2 Lesson structure based on Pólya's four-step problem-solving model.

The problem is initially introduced by the teacher, corresponding to the first step in George Pólya's four-step model for problem-solving: understanding the problem. At this stage, emphasis is placed on the importance of students developing a deep understanding of the problem's meaning, purpose, and given conditions before beginning their work. To support this, students are encouraged to visualize the situation by drawing a figure, which helps clarify the task and deepen their understanding. Identifying relevant variables and systematically listing all given conditions creates a clear structure and serves as the necessary groundwork before planning the solution.

The second step in the model, devising a plan, is reflected in the instruction as students formulate a solution strategy, often in the form of a formula, and justify their choice of mathematical methods. During this stage, students are encouraged to connect previously learned knowledge with new strategies, which strengthens their ability to reason mathematically and identify appropriate approaches.

This is followed by the third step, carrying out the plan, where students carry out their calculations step by step. They are expected to work systematically and clearly present each part of the solution to ensure traceability and logical consistency.

The final step, looking back, is implemented in the instruction through the component "discussing the solution." Here, students reflect on the reasonableness of their answer, check their calculations, and consider alternative solution methods. This phase is essential for developing students' metacognitive abilities and their capacity to evaluate and improve their solutions.

The following figure presents the connections between the lesson structure and Akihiko Takahashi's instructional model. The structure of the problem-solving lessons has been designed with inspiration from Takahashi's methodology, which emphasizes a well-thought-out lesson design that supports students' mathematical reasoning and problem-solving skills.

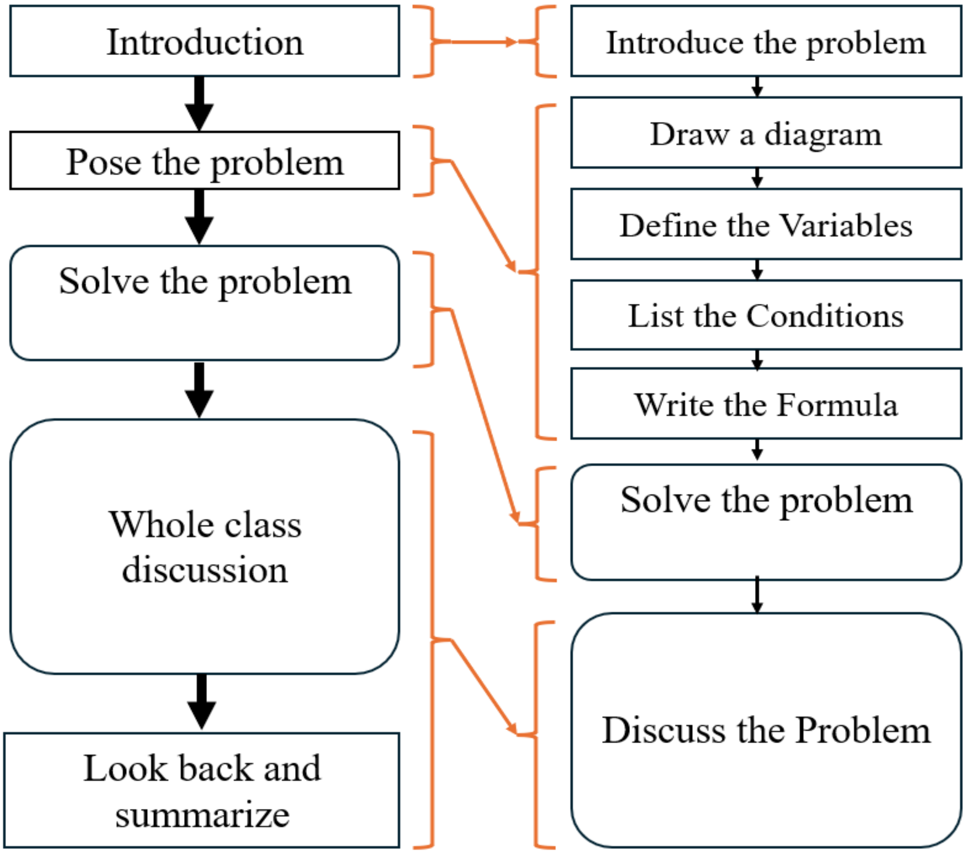


Figure 3 Lesson design inspired by Takahashi's Teaching Through Problem-Solving model.

To support students' development of mathematical reasoning during problem-solving lessons and to promote a clear and structured presentation of their solutions, explicit rules are applied regarding how problem-solving should be carried out. For example, students are not allowed to write down their thoughts until the entire group has agreed on a common strategy, and they are not allowed to sit down – the work must take place standing at the boards.

3. Method

This chapter presents an overview of the conducted work, including a description of the lessons and how the design is applied in mathematics teaching. Additionally, details about the interviews and their implementation are provided.

3.1 Data Collection

To gain a deeper understanding of how a systematic approach to problem-solving in groups on vertical surfaces impact teaching, interviews will be conducted with both teachers and students who have participated in the development project. Through these interviews, their experiences, challenges, and perceptions of the method will be examined. Additionally, students' written solutions to a test question will be analyzed to identify patterns in their reasoning and problem-solving skills, with a particular focus on the aspects they have practiced during problem-solving lessons. The collected data will serve as a basis for analyzing how the use of vertical surfaces can influence learning and teaching over time.

Furthermore, semi-structured interviews will be conducted with students from both classes to gain insights into their experiences of working on mathematical problems using vertical surfaces. The focus of these interviews will be on understanding how students perceive this method and how they believe it impacts their understanding and learning of mathematics. These conversations are expected to provide a deeper understanding of students' experiences with the teaching approach and the effects it has had on their problem-solving abilities (Bryman, 2017).

Semi-structured interviews will also be conducted with the teaching staff. The purpose of these interviews, together with the interviews with students, is to generate empirical data that enables a multifaceted exploration of the research questions from both the teacher's and the student's perspectives. Teacher interviews will investigate how problem-solving lessons have been planned, implemented, and experienced from a didactic and pedagogical standpoint. The student interviews will complement this by shedding light on students' experiences of the teaching approach and the perceived effects on their learning and problem-solving skills. The combination of these perspectives will contribute to a more comprehensive understanding of how teaching with vertical surfaces and problem-solving-oriented methods impacts learning processes in mathematics instruction.

Semi-structured interviews have been chosen because they allow for a flexible and open dialogue between the researcher and participants, creating space to explore specific topics while also allowing for new insights and perspectives to emerge during the conversation (Bryman, 2017). This method also enables follow-up questions, which can deepen understanding and generate more detailed responses.

This approach is particularly suitable for me as it allows for active listening and helps identify patterns in students' reasoning. By using a semi-structured interview method, I can systematically collect and analyze students' responses, providing a clearer and more detailed picture of their experiences. The open structure of the interviews also allows me to adapt questions based on students' answers, enabling me to capture important aspects of their learning and understanding in a more dynamic way (Bryman, 2017).

3.2 Selection

The selection of participants in this study is carried out using a purposive sampling method, meaning that students and teachers from classes that have worked with vertical surfaces are chosen based on their experiences and knowledge of this specific teaching method (Bryman, 2017). According to Bryman (2017), purposive sampling involves selecting participants based on their experience and knowledge of the specific phenomenon being studied. This method allows for a deeper understanding of the experiences and perceptions related to the use of vertical surfaces in teaching.

To ensure breadth and variation in the collected data, both students who have experienced success with the method and those who have encountered difficulties using it will be included in the selection. This distinction was made based on classroom observations as well as consultations with the involved teachers. Since the teachers have worked closely with these students for about two years, they have a good understanding of which students have found the method challenging and which have succeeded. This approach provides a more nuanced picture of how the method works for different students and their learning processes.

Three different groups of students will be selected from each class: students who demonstrate high knowledge and performance, corresponding to grade levels A and B; students with performance at an intermediate level, corresponding to grade levels C and D; and students with lower performance, corresponding to grade levels E and F. The exact grades will not be disclosed, but only the performance levels of the students will be indicated.

3.2.1 Respondent Description

The interviews will be conducted anonymously, and participants' identities will be protected by assigning fictitious names, such as "Student 1," "Teacher 1," "Student 2," etc., in the final report. This anonymization ensures that no personal data can be linked to participants' responses, helping to create a safe and trusting environment where participants can express their opinions and reflections openly without fear of negative consequences.

The respondents in this study are divided into two groups: teachers and students. To ensure anonymity, each participant has been assigned an acronym, providing a shortened identification. A summary of the participating respondents is presented below to facilitate the presentation of results.

TE1: The respondent has worked as a teacher for approximately four years and holds a teaching degree. The respondent is qualified to teach both physics and mathematics and has taught all mathematics courses at the upper secondary level.

TE2: The respondent has worked as a teacher for a total of seven years. During the first three years, the Respondent did not have a teaching degree, but after obtaining their teaching qualification, they worked for an additional four years. The respondents are qualified to teach both programming and AI, which is a new course at the school. The respondent has also taught all mathematics courses at the upper secondary level.

ST1, ST2, ST3, ST4, ST5, ST6: All these respondents are the same age and take the same courses. ST1, ST2, and ST3 are in a parallel class to the other respondents. For ST1, ST2, and ST3, TE1 is their mathematics teacher. These respondents are currently studying Mathematics 3C.

ST1: A student with an average performance level (corresponding to grade levels C and D).
ST2: A student with a high performance level (corresponding to grade levels A and B).
ST3: A student with a low performance level (corresponding to grade levels E and F).
ST4: A student with an average performance level (corresponding to grade levels A and B).
ST5: A student with a low performance level (corresponding to grade levels E and F).
ST6: A student with a high performance level (corresponding to grade levels C and D).

3.3 Interview Implementation

In order to answer the research questions, two appendices with interview questions were developed. First, questions for teacher interviews were designed, see Appendix 2.1. The initial questions for the teachers aim to understand their career background, their interest in the current development project, as well as their experience with the lesson's methodology and their perspective on potential areas for improvement. The first interviews were conducted with the teachers, as their perspectives could help formulate relevant questions for the students.

After the teacher interviews, an interview appendix for students was developed, see Appendix 2.2. The interview questions began by exploring students' opinions on mathematics, vertical surfaces, as well as the solution process and lesson design. The interviews ended with the students being asked to suggest areas for improvement and express their interest in this type of teaching.

After the interviews were conducted, all interviews were transcribed from the audio recordings. The transcriptions have been useful for identifying patterns, opinions, and experiences regarding the current lesson planning and their revision methods. It has also helped highlight both positive and negative feedback about the lessons, as well as the strengths and weaknesses of the lesson planning. Another interesting area was to investigate whether the students' interest in mathematics had changed after the lessons and if their opinions aligned with those of the teachers.

3.4 Phenomenographic Analysis

One way to identify and understand variations in how individuals experience and comprehend a specific phenomenon is phenomenography, a qualitative research methodology. Phenomenography helps capture diverse experiences (Martha, o.a., 2023). For my study, which focuses on problem-solving skills and students' experiences from a broad range of perspectives, this approach may be appropriate.

Phenomenography interviews can assist a researcher in developing "descriptive categories", which qualitatively present how a phenomenon, in this case, problem-solving using vertical surfaces, is perceived (Foster, 2013). These categories help structure the data while also revealing the complexity and diversity of experiences.

In order to analyse the interviews, it is important to understand the methodology and application within qualitative research. Phenomenography analysis focuses on understanding the different ways in which people perceive or understand specific phenomena (Lunn & Ross, 2021). In this case, it is about understanding the experiences and opinions of both students and teachers. To analyze a phenomenographic interview study, the following methodological steps should be followed.

One of the first steps is transcription, ensuring that all interviews are transcribed verbatim (Foster, 2013). It is then important to become familiar with the data by reading through the transcriptions several times to understand the content and identify significant statements and phrases that reflect different ways of experiencing the phenomenon. The next step is to categorize and group similar perceptions to understand how people make sense of the phenomenon. This step is about identifying variations in experiences and thoughts about the phenomenon (Foster, 2013). Finally, one needs to look for patterns in the data to understand what overlaps or appears in different contexts. This analysis will make it easier to interpret what the results mean and how they can be understood.

3.5 Societal, Ethical and Environmental Considerations

The teachers at the high school where I am conducting my research are responsible for teaching mathematics using vertical surfaces, as mentioned earlier in the text. My work primarily focuses on investigating the effects of this teaching method, as well as how students experience the lessons and the results they have achieved.

The use of vertical surfaces can promote inclusion and create an environment where all students have the opportunity to express their ideas and work on problem-solving, regardless of their current level. This can lead to a more equitable education, where students with varying backgrounds have the same opportunity to participate and develop. Furthermore, this method can help develop essential social skills in students, such as collaboration, communication, and critical thinking, which will assist them in facing future challenges both inside and outside of school.

This teaching method can foster a more open and transparent learning environment, where all students are given the chance to participate and receive feedback. It is ethically important to ensure that the teaching is fair, and that no student is overlooked or made invisible. By encouraging students to express their thinking and openly discuss their solutions, the method can contribute to increased self-esteem and a sense of belonging. It is also crucial to provide students with constructive feedback that promotes learning and reflection, rather than simply offering right or wrong answers.

An important ethical consideration in this work relates to the handling of data related to students and teachers. Since lessons in the school are mandatory for students, and the teaching method (e.g., using vertical surfaces) is part of the regular curriculum, it is essential to clearly explain how data will be collected and how privacy will be ensured. The data collected will only be accessible to me as a student and my supervisor, and all information will be anonymized in the report to ensure that no individual students or teachers can be identified. Participation in the lessons was mandatory, but participating in the study was voluntary. Since the lessons were recorded as part of the research project, all students were asked to complete a consent form to approve the use of their participation in the study. It is also important, as a researcher, to be aware of the power dynamics in the classroom so that no student feels personally scrutinized or judged as an individual in connection with data collection. The data collected focuses on understanding the teaching method as a whole and the students' general experiences, rather than individual performances. In this way, the study aims to handle the content ethically and responsibly, following research ethics principles.

The use of vertical surfaces may also have benefits from a resource utilization perspective. If whiteboards or other writable surfaces are used instead of paper, this can reduce waste and contribute to more sustainable working processes. It can also be a step toward reducing the school’s environmental impact by using less disposable material. Problem-solving-based teaching, where students actively work on and discuss solutions, can further contribute to developing their critical thinking regarding sustainability and environmental issues.

4. Results and Analysis

In this section, the results of the interview study are presented, followed by an analysis based on the literature review. The complete questionnaire for teachers and students can be found in Appendix 1 and Appendix 2. After the presentation of the results, an analysis section follows.

The respondents are divided into two groups: teachers and students. To ensure anonymity, they have been assigned acronyms. To simplify the presentation of the results, a summary of the participating respondents is shown in Table 1.

4.1 The Teacher's Perspective

In this section, the interview results from teachers regarding their role in development projects, their experiences, and their thoughts on problem-solving will be presented and analysed.

4.1.1 The Teacher's Perspective on Vertical Surfaces

TE1 has implemented vertical surfaces to promote collaboration among students. TE1 believes that these surfaces make students more active and engage with each other in ways they otherwise wouldn’t. The respondent describes a benefit of vertical surfaces as a tool to encourage students to think out loud and collaborate.

> *"What I like about the vertical surfaces is that they force the students to discuss and work together. It becomes a more natural way for them to help each other and think aloud about the solutions."*

For TE1, another important aspect is how students begin to explain their solutions to each other.The respondent sees this as not only beneficial for weaker students, but also for more advanced ones, as it encourages them to structure their thoughts more clearly. This is viewed as a way to enhance understanding by allowing students to 'see their own thinking' through joint discussion.

TE2, views vertical surfaces as a way to increase student engagement and improve their problem-solving skills. TE2 expresses that students struggle to get started with problem-solving, particularly those who are less confident. According to TE2, vertical surfaces create an opportunity for students to articulate their thoughts more openly, which in turn facilitates their understanding of the material.

> *"I notice that when we use vertical surfaces, it becomes easier for the students to get started, and they get a chance to share their thoughts in a way that prevents them from getting stuck as quickly."*

For TE2, vertical surfaces are not just about students expressing their thoughts verbally, but also about creating a more interactive learning environment where students learn from each other. TE2 also sees the benefit of the teacher being able to observe students' thought processes more directly, which makes it easier to provide quick feedback.

Both TE1 and TE2 see vertical surfaces as a valuable tool to engage students and help them improve their problem-solving techniques. For them, it is not just a physical surface, but a way to create a learning environment that encourages collaboration and discussion. Both emphasize the importance of providing students the opportunity to express their thoughts in a safe environment.

> *TE1: "When they work together on the vertical surface, it feels less stressful for them to show what they actually know."*

> *TE2: "It's also a sense of security for students to be able to write and think together, as they feel that they are not alone in their thinking."*

4.1.2 Engagement and Challenges According to Teachers

Both TE1 and TE2 emphasize the importance of using vertical surfaces in mathematics teaching to increase student engagement and promote collaboration. TE1 describes how vertical surfaces enable a more dynamic and interactive classroom.

> *"I think it's incredibly important. I don't think it's the only way, but it likely engages more students. It's really hard to collaborate around a piece of paper on the table. With a vertical surface, you can point at something and say, 'What do you mean by that? Have we thought this through correctly?'" (TE1)*

TE1 believes that vertical surfaces make it easier for students to visualize and discuss mathematical problems together, leading to increased engagement. TE2 shares this view and adds that students become more active and tired after lessons where vertical surfaces are used, which is interpreted as a sign that they are thinking and working harder.

> *"I see that they work more, they are tired afterward, so they've been thinking. I think they think more and have to work harder and push themselves more." (TE2)*

Both teachers highlight that vertical surfaces foster collaboration and create a safe environment where students feel comfortable participating, even if they are unsure of their answers. TE2 describes how group work at the board can reduce students' fear of making mistakes.

> *"It's really great, because many students are afraid of being wrong. When working in a group, that fear fades. Suddenly, you're not as afraid to make a mistake." (TE2)*

TE1 points out how vertical surfaces allow students with different levels of knowledge to contribute to problem-solving.

> *"Even if you have some knowledge gaps, you can still contribute an idea. There's usually something you can add." (TE1)*

Despite the positive effects, both TE1 and TE2 mention some challenges with using vertical surfaces in teaching. TE2 notes that it requires more effort from the teacher to manage a classroom where students are working at the boards. TE1 highlights that students are not always accustomed to this method, which requires a clear introduction and a shift in the classroom environment.

4.1.3 Structured Lessons for Student Growth

Both TE1 and TE2 emphasize the importance of having a clear problem-solving process during lessons. TE1 describes the use of a structured process in which students first draw a picture, introduce variables, write formulas and conditions, and finally solve the problem. According to TE1, this structure provides students with a clear framework to work within and helps them approach problem-solving systematically

> *"It also becomes a good guide in this conversation. You have the students with you for that. Yes, but if I go up to a group and see that they haven't drawn a figure, for example, I can just ask, do you think you have followed the structure? Is there a step in the structure missing?" (TE1)*

TE2 shares this view and describes having previously used a similar structure, but notes that students often do not follow it unless it is integrated into the lesson. This method is seen as providing students with an opportunity to truly understand and apply the structure.

Both teachers highlight the importance of a clear introduction to the lesson, where they explain the structure and expectations to the students. TE1 describes introducing the lesson by explaining what the students will do, followed by group work on the problems. This approach is seen as providing students with a clear direction, making it easier for them to get started.

> *"I personally liked it, but I think it's actually quite smooth. It's making sure the rules are on the board. Seeing that the structure is on the board, and then it's really just an introduction where you try to get them to figure out what they are and then get them into the lesson." (TE1)*

An important aspect of the lesson structure is that students are responsible for their own learning. TE1 describes allowing students to follow the structure independently and to identify on their own if they have missed a step. This is seen to promote student independence and to equip them with tools to solve problems even outside the classroom.

> *"And then instead of me pointing out, 'you've forgotten the figure,' I take away from them the opportunity to solve their own problem somehow. So, this becomes a very good way because you can just turn around and say, 'Yes, no, we missed the figure.' (TE1)*

TE2 also emphasizes the importance of students trying on their own first and having time to reason about the problems before they get help. This is considered to strengthen students' ability to think critically and solve problems independently.

> *"It is incredibly important that they get to try for a while. Maybe they are the ones leading the reasoning a bit." (TE2)*

Both TE1 and TE2 discuss the importance of adjusting the lesson structure according to students' needs and levels of knowledge. TE1 describes using the strategy of handing the pen to the student who needs to be more active in the group, thereby fostering an inclusive environment where all students are allowed to participate. TE2 explains how group compositions are adjusted to support students who find mathematics challenging. Emphasis is placed on creating a safe and inclusive environment where all students feel involved, regardless of their prior knowledge.

"I haven't completely randomized the groups, but I've pulled a little to make them comfortable. The ones who struggle a bit should be with someone they like being with, so it feels safe." (TE2)

4.2 The Student's Interview Results

This section presents the interview results regarding experiences and effects, focusing on students' interviews and opinions on vertical surfaces, the lesson process, and the problem-solving process.

4.2.1 What do students think about problem-based lessons and the use of vertical surfaces during lessons?

The use of vertical surfaces (whiteboards) was perceived by the majority of students as a positive and engaging method for mathematics instruction. Several students emphasized that the whiteboards promote collaboration and understanding by allowing them to see and discuss each other's solutions. The students believe that working with vertical surfaces helps them understand and solve problems by expressing their ideas and explaining them to others. They also think that vertical surfaces and problem-based lessons have their advantages, especially in allowing them to see how others approach tasks. This made it easier for them to understand the problems and provided a broader perspective on how tasks can be solved.

ST1, ST4 och ST6 described how working with whiteboards made it easier to express their ideas and find inspiration from others' thinking. Seeing different solutions helped them better understand the task and how to approach it. As ST4 puts it:

"If you have no idea how to solve the task, you could see and get inspiration." (ST4)

ST3 appreciated the opportunity to learn from other groups:
"You get to see different ways of thinking. Everyone can arrive at the right answer in different ways, and that helps you learn different ways of thinking." (ST3)

ST5 also found problem-based teaching with vertical surfaces to be both engaging and educational. The student learned more by observing how others solved the tasks, which confirmed whether they were on the right track or helped them discover new ways to approach a problem. This was especially valuable when the student got stuck or was unsure how to proceed.

ST2 has mixed feelings about working with vertical surfaces and the problem-based approach. The student felt that the method was easier to follow at a basic level, but as an A-level student, they found it less challenging.

> *"It was easier to keep up and understand the concepts at a basic level. But at the same time... I'm in the A-level, and it felt like the teaching was more tailored for the whole class to follow." (ST2)*

ST2 feels that the tasks done on the whiteboard were often at an E to C level and that the pace was too slow for him. The student would have preferred more individual instruction and a quicker introduction to new concepts. ST2 believes that the use of vertical surfaces did not always contribute to understanding and effectively solving the problems. Sometimes, it felt more time-consuming to try to solve the tasks without a clear explanation at the start. Despite this, ST2 acknowledged that the group work was enjoyable and that other students appreciated the method. However, for ST2, it was not the most effective way to learn.

4.2.2 Students' Experience of Learning and Challenges

Students' engagement varied depending on their preferences and group dynamics. Many felt that problem-based learning increased their activity and motivation, while others felt limited by the composition of the group. Students appreciated both sharing their own ideas and listening to others, recognizing the value of being active participants who both contribute to and learn from their group members. All students, except for ST2 and ST4, found the group work overall positive, though with some challenges. Working with individuals who had different working styles created small obstacles, but students were able to work through these and focus on the common goals.

> ST1, ST3, and ST5 described how collaboration made them more involved:
> *"You automatically become more engaged because you have to participate."* (ST3)

> ST6 appreciated how time flew when fully engaged:
> *"When we worked like this, we were engaged the whole time. I didn't even think about the clock.*

Overall, ST2 expresses frustration with the collaboration in his group and how it impacted the work process. The student often feels that they alone took on the responsibility of thinking and explaining, while the others in the group did not contribute actively. ST2 perceives that the group members were passive and instead copied others' solutions without reflecting themselves, which is seen as a barrier to learning. Since the student does not believe the others developed through their own thinking, this negatively affects their own learning experience.

ST2 believes it would be better if everyone in the group had clearer roles and responsibilities to create a more fair and effective collaboration. The student appears to value challenges and working with different people to develop, which is why choosing their own group is not preferred. This approach helps avoid falling into a comfort zone where collaboration is limited to familiar people. For ST2, it is also important that everyone is active and involved in the group work to achieve a good result.

> ST2 criticized uneven group work:
> *"It often ends up that only one or two people in the group do something."*
> *"People looked more at what others wrote than thinking for themselves."*

ST4 provides a genuine and nuanced perspective on engagement when working with vertical surfaces. A central part of the reflection is the dual effect of the communication requirement. On one hand, it is challenging to always explain thoughts before writing them down, but on the other hand, it is recognized that this forces a deeper understanding of the subject. A recurring challenge highlighted by ST4 is the difficulty of taking initiative in passive groups, which affects both engagement and the work process.

> ST4 mentioned that communication didn't always work:
> *"If no one else starts, I won't either."*
> *"Sometimes no one wanted to talk, and then nothing happened."*

The students feel that working with vertical surfaces and problem-based teaching generally increases their engagement in mathematics. This approach seems to activate all students, but there are also challenges that need to be addressed to make the learning process even more effective and inclusive.

4.2.3 Lesson Design and Problem-Solving Process According to the Students

The students had different opinions about the lesson structure, but several highlighted that structured problem-solving and clear rules provided them with support. The students give a collective yet nuanced view of the lesson structure with vertical surfaces and problem-based learning. In general, they appreciate the active and collaborative-oriented structure, where the opportunity to work in groups at the boards and see each other's solutions creates a more engaging learning environment.

ST1 appears to be a strong advocate for this teaching method, according to the student's own words. The student's positive attitude is based on how collaboration in groups creates a dynamic learning process. For ST1, mathematics becomes not just individual work but a joint exploration where ideas emerge through dialogue. The student particularly emphasizes how this way of working forces one to clarify their thoughts to others, which in turn strengthens their own understanding.

ST3 takes a balanced position based on the student's own reflections. The student's view of the new method shows value, but ST3 is not blind to its limitations. ST3 particularly appreciates the visual dimension of the work and the opportunity to learn from others' solutions, suggesting that ST3 is a visual and social learner.

> ST1 and ST3 appreciated the problem-solving steps and the lesson structure.
>
> *"Discussing with friends helped a lot, it became a dialogue around the task." (ST1)*

ST4 seems to think that the lesson structure and problem-solving process are helpful, but with some reflections on improvements. They particularly appreciate working with boards and having the solutions presented step by step. This makes it easier to understand and follow the solution to the problems. The student also seems to appreciate having a clear structure for problem-solving, although they sometimes do not follow it exactly in order.

> *"It is a good method where I can practice communicating... Explaining with words instead of just showing on the board." (ST4)*

ST6 believes that the lesson structure and problem-solving process work well and are effective. ST6 appreciates working step by step with problem-solving, where they draw figures, introduce variables, and write expressions. ST6 feels that these steps help them solve tasks and make the process more structured. This way of working makes it easier to understand and solve problems, which ST6 sees as a strength. Regarding the lesson structure, ST6 likes the longer lessons (80 minutes), as it provides more time to work on tasks. ST6 feels that the lessons go by faster and that they are more engaged when working in groups, which helps them forget about the time. This means they have more time to learn and do not feel stressed about rushing through the tasks.

ST6 thought the lessons were the right length:

"The lesson went by much faster when we worked this way." (ST6)

ST2 sees both advantages and disadvantages in following a structured method for solving math problems. ST2 thinks it's good to use structure, especially for more difficult problems, as it helps to solve them in an organized way. At the same time, ST2 feels that it can sometimes be time-consuming, and one might want to go directly to the solution. ST2 believes that the structure is especially helpful for those who find math difficult, as they can follow the steps and arrive at a solution. However, ST2 thought that some steps took unnecessary time.

"It felt like we spent the whole lesson on something I could have learned in five minutes." (ST2)

ST5 seems to have mixed feelings about the lesson structure and problem-solving process. ST5 appreciates working in a group, although there is a preference to choose group members independently. It is seen as effective when everyone in the group is active, but challenges arise when some members do not fully understand the task and require assistance from others. Regarding the problem-solving process, ST5 thinks this approach is beneficial because it provides the opportunity to think more thoroughly before sharing ideas, which increases confidence. Although it can feel somewhat frustrating not to write down thoughts immediately, it is understood that this process encourages deeper reflection before communicating ideas to others. Additionally, explaining something to someone else is seen to gain a better understanding of the material, which in turn leads to a more thorough comprehension of the task.

"It works well when everyone in the group is active, but it is challenging when some don't fully understand the task." (ST5)

4.3 Students Learning Outcomes

The students took an exam after having had four lessons focused on problem-solving and the use of vertical surfaces. I have chosen to present one of the exam questions and analyze how the students answered it. I selected this particular task because it resembled what the students had worked on during the lessons, and also because it was the first question in the exam – the first thing the students encountered. This may have contributed to them being energized, focused, and having clear thoughts when answering the question.

The test question is as follows:

A municipality is planning to build a rectangular dog park next to a high wall. To do this, the contractor has purchased 220 meters of fencing, which will be used for the three sides of the park that are not bordered by the wall. The municipality wants the park to be as large as possible. How large can the park be?

4.3.1 Examples of Student Solutions

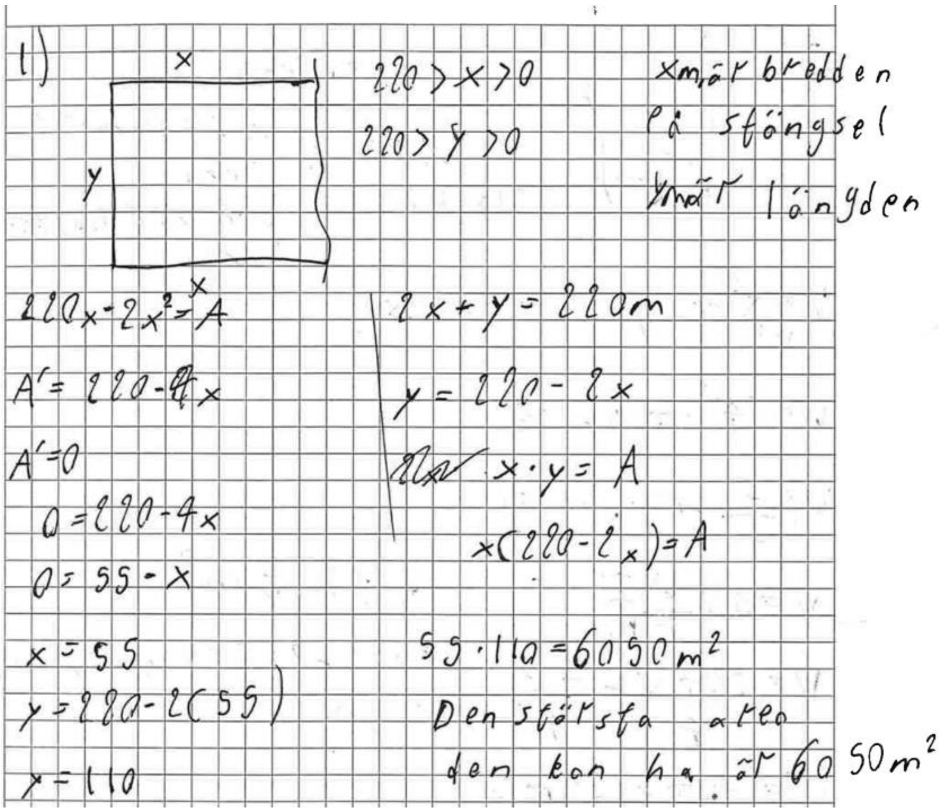


Figure 4 above shows the response from Student 5 (ST5) to the test question.

The image above shows the response from Student 5 (ST5) to the test question. The student drew a figure as a visual representation of the problem and defined variables as well as specified the conditions relevant to the task. ST5 followed a systematic problem-solving strategy, where the first step was to define variables for the sides of the dog park. The student then formulated an equation for the total length of the fencing and expressed one variable in terms of the other. They constructed a function to represent the area of the park.

To find the maximum possible area, the student differentiated the area function and set the derivative equal to zero to identify the maximum point. Finally, the student calculated the maximum area and provided a correct answer.

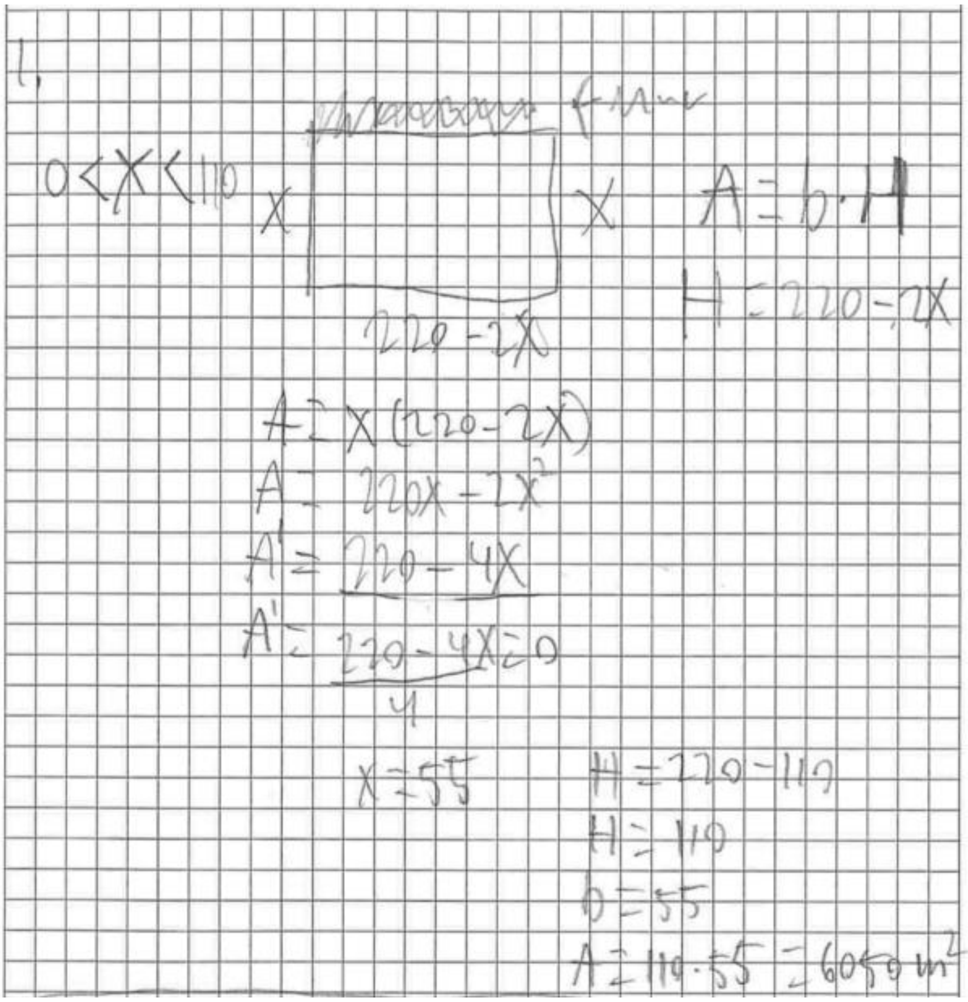
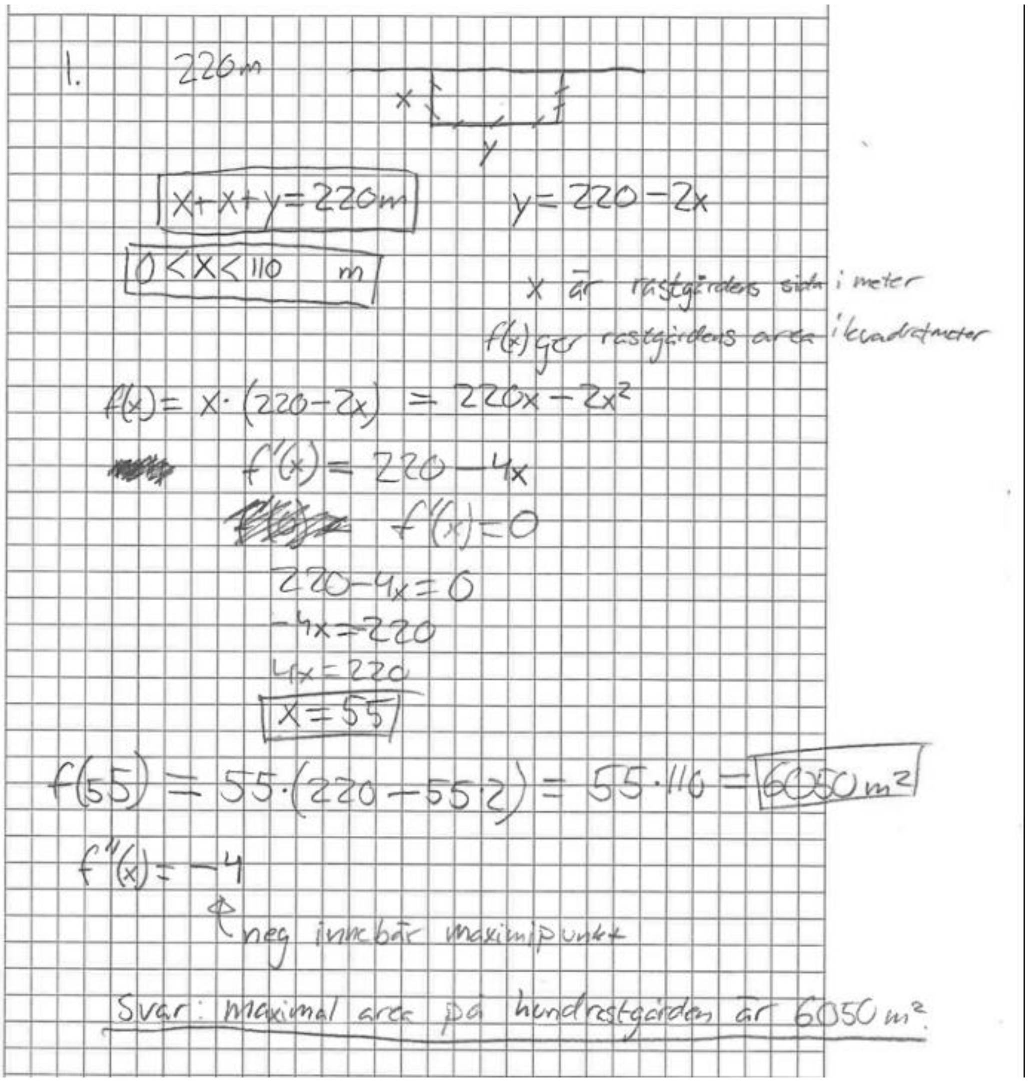


Figure 5 above shows the response from Student 6 (ST6) to the test question.

The response from Student 6 (ST6) is similar to that of Student 5 (ST5). The student began the solution by drawing a figure and writing down the variables but did not specify what the variables represent. ST6 correctly formulated an expression for the area and differentiated it correctly. The student also correctly identified the maximum point and calculated the maximum area.



1. 220m

x y

$x+x+y=220m$ $y=220-2x$

$0<x<110$ m

x är rastgårdens sida i meter

f(x) ger rastgårdens area i kvadratmeter

$f(x)=x\cdot(220-2x) = 220x-2x^2$

$f'(x)=220-4x$

$f'(x)=0$

$220-4x=0$

$-4x=-220$

$4x=220$

$x=55$

$f(55)=55\cdot(220-55\cdot2)=55\cdot110=6050\ m^2$

$f''(x)=-4$

← neg innebär maximipunkt

Svar: maximal area på hundrastgården är 6050 m².

Figure 6 above shows the response from Student 2 (ST2) to the test question.

Here we have ST2's solution, which is very detailed. It is clear that the student started by drawing a figure, defining variables, and writing both a function and a domain. The student has also clearly stated what x represents and what f(x) describes. The solution is completely correct, and it is evident that the problem-solving strategies practiced during the lessons have clearly influenced the student's approach.

5. Discussion

This section discusses the results and analysis in relation to previous research. Based on the interview results and the analysis, a summary of the answers to the research questions is presented, along with my final conclusion.

5.1 General Discussion

5.1.1 Experiences and Insights from Teachers on Using Vertical Surfaces in Teaching

The results show that both TE1 and TE2 have gained valuable insights from working with vertical surfaces, insights that have directly influenced and developed their teaching practices. The reflections shared by the two teachers highlight how development work with vertical surfaces has significantly shaped their instructional approaches and perspectives on student learning. One key finding is that vertical surfaces facilitate students' engagement in problem-solving processes. Both teachers observe that students are quicker to get started, more willing to collaborate, and more reflective when working in this way. This aligns with Kilpatrick, Swafford, and Findell’s (2001) view that effective problem-solving requires strategic competence and adaptive reasoning, both of which are fostered when students externalize and discuss their thinking.

An important impact on teaching practice is the shift towards more student-centered instruction. TE2 notes that allowing students to work independently on vertical surfaces gives them greater ownership of the problem-solving process. This aligns with Liljedahl’s (2021) concept of a "thinking classroom," where students are encouraged to explore ideas actively and make sense of problems collaboratively. TE1 similarly emphasizes the use of structured problem-solving steps when working with vertical surfaces, helping students systematically reason through complex tasks. This suggests that teachers have adapted their instruction to better support student autonomy while still providing necessary scaffolds.

Another important experience concerns the social dynamics in the classroom. Both TE1 and TE2 report that vertical surfaces lower the threshold for participation and reduce students' fear of making mistakes. Working in groups on visible, non-permanent surfaces promotes a safe and inclusive environment, supporting Vygotsky’s (1978) theory of the social origins of learning. In this way, vertical surfaces have influenced teaching practice by encouraging more frequent and meaningful peer interactions.

Increased physical and cognitive engagement is another recurring theme. TE2's observation that students are mentally fatigued after lessons with vertical surfaces supports findings by Sesen and Smith (2017), suggesting that the method demands and fosters greater cognitive effort. For the teachers, this experience has underscored the value of active, physically engaged learning environments, prompting them to rethink traditional seated classroom arrangements.

However, the teachers also highlight challenges. TE1 points out that teaching with vertical surfaces requires more active classroom management and greater flexibility from the teacher. Similarly, TE2 notes that students initially need guidance to adapt to this more open-ended and collaborative way of working. These experiences suggest that while vertical surfaces offer clear pedagogical benefits, they also require teachers to invest more in planning, classroom organization, and student support, consistent with Liljedahl's (2016) observations about implementing thinking classrooms.

Finally, both TE1 and TE2 emphasize that vertical surfaces have helped integrate problem-solving more deeply into everyday mathematics teaching, rather than treating it as an isolated activity. Students are encouraged not only to find solutions but also to justify and reflect on their reasoning. This aligns closely with the theoretical framework suggesting that teaching through problem solving, supported by tools like vertical surfaces, strengthens both conceptual understanding and problem-solving abilities (Kilpatrick et al., 2001; Lester & Cai, 2015).

Teachers' experiences with vertical surfaces are also predominantly positive. They describe how this teaching model leads to increased engagement and more active learning. By making students' thoughts and solutions visible on the boards, teachers have better opportunities to follow their thought processes and provide immediate feedback. This facilitates formative assessment and creates a more dynamic teaching environment. At the same time, teachers emphasize that the model requires flexibility and support, especially for students who need more adjustments in order to participate on equal terms. Therefore, a clear structure and teacher support are crucial for the model to be inclusive.

5.1.2 Students' Reasoning and Attitudes through Problem Solving

Students' experiences with problem-solving lessons on vertical surfaces reflect how this approach impacts both their reasoning and attitudes towards learning. Several students, like ST1 and ST3, felt they could express their ideas more effectively and learned a lot by seeing others' solutions. They described how they could reflect on and discuss their own and others' solutions in real-time, which helped them understand the problems more deeply and strengthen their creative thinking. This supports the idea that learning occurs through collaboration, which is a central aspect of Liljedahl’s (2021) "Thinking Classroom" – creating an environment where students actively share and reflect on their solutions.

ST5 described how it became more engaging to see others solve problems, especially when they encountered difficulties themselves. Feedback from peers and the quick adaptation of their ideas within the group strengthened their solutions and made learning more dynamic and engaging. This highlights the importance of interactive work and the opportunity for students to receive quick feedback to continue developing. According to Liljedahl’s theory, failure is an important part of learning, and the ability to process these failures through group discussions enhances students' problem-solving skills.

However, ST2 felt that the tasks were too easy, and the pace was too slow, which made them feel insufficiently challenged. This suggests that vertical surfaces and problem-solving work don’t always work for all students if the tasks are not appropriately tailored to their level. ST2's experience reminds us of the importance of creating the right balance in challenges to keep students engaged, as Liljedahl points out in his "Thinking Classroom" model.

Regarding group collaboration, some students, like ST2, expressed frustration with the lack of equal participation within their group. ST2 felt they were the only one thinking and explaining solutions, which made the collaboration less effective. This points to a barrier for the creative and open learning environment that Liljedahl advocates – all students need to be active for the social and interactive learning process to function at its best.

According to Dweck’s (2006) theory of a growth mindset, students like ST3 and ST6 reflect a positive change in their attitudes as they actively collaborated and reflected on their solutions. By engaging in group discussions, their view of learning shifted from being a passive process to an active and collaborative experience. ST4 also demonstrated how explaining their thoughts to others before writing them down helped them reflect on their ideas and strengthened their understanding, which is a clear example of a growth mindset in practice.

An interesting aspect is that students' attitudes towards mathematics have generally changed in a positive direction. Many describe math as more engaging and less stressful with the new approach. The social interaction and physical activity seem to help reduce performance anxiety. However, it also emerges that some students perceive the pace as slow and the tasks as insufficiently challenging. This suggests that the method does not suit all students equally well, highlighting the importance of differentiation and individualized instruction in teaching.

The results of the study show that the majority of students perceive working with vertical surfaces in combination with group-based problem-solving as having a positive impact on their learning, which can be seen in the Results section 4.2 The Student's Interview Results. The students particularly highlight benefits such as increased collaboration and improved communication. The interviews clearly show that working with whiteboards makes mathematics more concrete and accessible, which in turn creates a safe environment where students feel comfortable testing their ideas without fear of making mistakes. This sense of security helps strengthen students' confidence in problem-solving, while also promoting mathematical reasoning and interaction in the classroom.

5.1.3 Students' Responses to the Exam Question

In the analysis of the students' responses to the exam question about constructing a dog park next to a tall wall, clear connections emerge between their performance, the set learning objectives, and the teaching design implemented during the lessons. The students ST2, ST5, and ST6 demonstrate how well they have applied the problem-solving strategies and mathematical methods that were addressed during the lessons.

All students started by drawing a figure to visualize the problem, an approach emphasized through George Pólya's four-step model for problem-solving. By creating a visual representation, they were able to better understand the given conditions and the mathematical relationships within the task. This step was highlighted in the lesson plan as a key tool for promoting conceptual understanding. ST5 went a step further by clearly defining variables, which made the solution easier to follow. ST2 also did this in a similar way and showed clear structure. ST6, on the other hand, lacked this clarity, which made their solution somewhat harder to interpret. This difference points to the fact that students still need to practice expressing their mathematical reasoning in a structured and systematic way, which is an important part of the specified learning objectives.

Both ST5 and ST6 followed a systematic strategy for approaching the problem. ST2 did the same. They broke the task into smaller components and applied previous knowledge in algebra and differentiation. ST5 showed a higher degree of independence by deriving a function and using calculus to optimize the area, a skill that directly aligns with the teaching focus on optimization techniques and identifying critical points. This progression shows that the teaching successfully promoted an understanding of both theoretical and practical applications of derivatives.

The teaching was designed to support students' ability to identify mathematical problems, formulate solution strategies, and present their reasoning clearly. By integrating Pólya's model and Takahashi's problem-solving-based teaching methodology, the lesson structure offered students opportunities to develop both mathematical and metacognitive skills. The students worked in groups where they discussed their solutions before documenting them. ST2, ST5, and ST6 demonstrated that they understood the structure of the problem and were able to develop solution strategies using figures and variables, which is evidence that this component of the lesson was effective.

However, there were differences in how students reflected on their solutions. ST5 demonstrated a more developed metacognitive ability by conducting a thorough analysis of the maximum area and reasoning about the plausibility of the results. ST6 lacked this deeper reflection, which suggests that there is still a need to strengthen students' ability to critically evaluate their solutions. However, the elements that were meant to be practiced during the lessons and that influenced their proposed solutions during the exam were met, and traces of what students practiced during the lessons can be seen. Illustrating a picture, defining variables, gathering information, and then solving the problem suggests that the students at least have a grasp of how to approach a problem-solving task.

The results from the test analysis show that the students have largely adopted the strategies that the problem-solving lessons aimed to develop. In the students' test responses – particularly evident in Figures 4 and 5 – it is clear that many students have learned to structure their solutions by drawing figures, defining variables, and gradually building up their reasoning. These elements directly reflect the topics emphasized during the problem-solving lessons. It is also noticeable how students break down the problem into smaller parts and clearly explain their thought processes using both text and mathematical expressions. Through their approach to solving the task, it can be seen that they have understood – or at least gained an understanding of – the methods and approaches introduced during the lessons. This is especially notable considering that the lesson series was relatively short and involved a limited number of classes.

5.2 Methodological Reflection

The chosen qualitative research method, which consisted of semi-structured interviews and phenomenographic analysis, proved to be suitable for investigating both students' and teachers' experiences of working with vertical surfaces and group-based problem-solving. The interviews provided a deeper understanding of the participants' perspectives, particularly because they allowed for follow-up questions and adaptability during the conversation. This made it possible to capture nuances in their experiences that might not have emerged through a more structured method. The phenomenographic analysis was especially useful for identifying patterns and variations in how students and teachers perceived the teaching method and its impact on learning and engagement.

The pilot analysis of students' written solutions complemented the interviews by providing concrete examples of how their problem-solving strategies had developed and changed. This gave a more tangible picture of how students applied what they had learned in practical situations and how vertical surfaces influenced their working processes.

Semi-structured interviews enabled in-depth and spontaneous responses, allowing a more thorough exploration of participants' thoughts and feelings. At the same time, there is a risk that the researcher's own expectations and preconceptions may influence the interpretation of the interview responses. To minimize this influence, it is important to remain aware of the researcher's own biases and to carefully consider alternative interpretations of the data. Another limitation is the small sample size, only eight participants (two teachers and six students), which may affect the generalizability of the results. A larger and more diverse group could provide a broader picture of how vertical surfaces and group-based problem-solving affect students with different conditions and needs. Additionally, a more in-depth analysis of gifted students could offer further insights into how the method can be adapted to meet their specific needs.

5.3 Significance and Implications

What do the results mean in practice?

The results of this study show that working with vertical surfaces and group-based problem-solving can be an effective method for engaging students and promoting their mathematical thinking. By making mathematics more concrete and interactive, students can develop both their problem-solving skills and their confidence. This teaching model also contributes to a more dynamic and inclusive learning environment where students actively participate and support one another.

How can the results be used?
The results can be used to further develop mathematics instruction by integrating more vertical surfaces and group-based activities in classrooms. Teachers can use this method to create a more interactive and visible learning environment that gives students opportunities to reflect on and discuss their ideas. For the model to be effective, it is important that it is adapted to students' individual needs, which may include providing support for those who need more guidance or for those who feel that the pace is too slow.

Who benefits from my conclusions?
The conclusions of this study can be valuable for both teachers and school leaders who aim to develop and adapt their teaching methods. The findings can provide educators with insight into how vertical surfaces and group-based work can improve learning and engagement. Furthermore, the research can be useful for teacher education programs that want to prepare future teachers to work with inclusive and interactive teaching models. Students, especially those who struggle with traditional teaching methods, can also benefit from instruction that is more tailored to their needs and learning styles.

5.4 Suggestions for Future Research

There are several interesting areas that could be further explored to deepen the understanding of how vertical surfaces and group-based problem-solving impact learning, as well as to develop methods for addressing students' diverse needs. One important aspect that requires further investigation is how this teaching model affects students with different prerequisites. In particular, students with various learning styles, especially those who are gifted or have special educational needs, could be examined in greater detail. It would be valuable to explore whether the method can be adapted to meet these students' specific learning needs and how it influences their engagement and performance.

Another interesting research question would be to examine how the use of vertical surfaces and group work affects students' mathematical abilities and attitudes toward the subject over time. Since this teaching model was only tested over a short period (two months) in this study, it would be interesting to investigate its effects over a longer duration, perhaps an entire academic year. A longitudinal study could assess whether the positive effects on collaboration and problem-solving persist or change when students work on different types of tasks or in other learning environments. Additionally, it would be worthwhile to explore how the method could be applied in earlier mathematics courses.

Another key question is how the use of vertical surfaces changes the teacher's role in the classroom. What new skills and strategies do teachers need to develop to successfully implement this teaching model? Such a study could provide insights into the need for teacher training and professional development when working with more dynamic and interactive learning environments.

Currently, there is a lack of deeper understanding regarding how vertical surfaces affect students with different learning needs in the long term. Although the method appears to have a positive impact on students' attitudes and engagement, it remains unclear how these effects develop and whether they lead to lasting changes in students' learning. More research is also needed on how vertical surfaces can be integrated with technological tools and how these can support both individual and collaborative learning.

Since this method was only tested over a relatively short period (two months) in this study, it would be interesting to examine its long-term effects. A year-long study would provide a more suitable timeframe to determine whether the results hold and how students progress. It would also be valuable to investigate how the method could be applied in earlier mathematics courses, which could offer a more comprehensive picture of its impact on student learning.

These suggestions and identified gaps highlight important areas where future research could contribute to improving and developing teaching methods that promote active, inclusive, and long-term learning for all students.

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Appendix 1: The interview

Appendix 2.1: The untranslated interview questions

Intervju frågor till lärare

Inledning

Hur länge har du jobbat som lärare?

Vilka andra ämnen har du?

Vilka kurser har du undervisat i matematik?

Varför vill du delta i utvecklingsprojektet om att undervisa om problemlösning?

Vertikala ytor:

1. Hur tror du att vertikala ytor användning påverkar elevernas problemlösningsförmåga?

2. Har du observerat några skillnader i elevernas sätt att resonera och lösa problem när de arbetar med vertikala ytor jämfört med innan?

3. På vilket sätt kan man uppmuntra eleverna att använda vertikala ytor i problemlösning men även allmänt? Tycker du att det är bra metod?

4. Hur tror du att vertikala ytor påverkar elevernas engagemang och samarbete under lektionerna?

5. Finns det några särskilda utmaningar med att använda vertikala ytor i undervisningen? Om så, hur hanterar du dem?

Lektionsprocess

6. Vad tycker du om den lektionsdesignen som används för att stödja elevernas utveckling av problemlösningsförmåga? Finns det något i designen som du anser är särskilt effektivt?

7. Har du märkt några skillnader i elevernas beteende i prov/lektion situationer beroende på deras kunskaper i matematik? Hur påverkar det sättet du planerar och genomför lektionerna?

8. Hur säkerställer du att alla elever, oavsett deras tidigare kunskaper i matematik, känner sig inkluderade och delaktiga under lektionen?

9. Vad anser du vara de viktigaste komponenterna i en lektion för att eleverna ska kunna utveckla sina matematiska resonemang och problemlösningsförmåga?

10. Vilka förväntningar har du på hur eleverna ska utvecklas under dessa lektioner?

Appendix 2.2: The untranslated interview questions

Intervju frågor till Elever

Inledning

Vad tycker du är roligast med matematik?

Vad tycker du är svårast med matematik?

Vertikala ytor:

1. Hur upplevde du att arbetet med problembaserad undervisning jämförs med traditionell undervisning?

2. Hur hjälpte de vertikala ytorna dig att förstå och lösa problemet?

3. Vilka strategier använde din grupp för att lösa problemet?

4. Var det någon del av uppgiften som var särskilt utmanande? Hur löste ni det?

5. Tycker du att du lär dig mer av att se hur andra löser problem på tavlan? Hur påverkar synligheten av allas arbete ditt eget lärande?

Grupparbete

6. Vad tycker du om att grupperna görs random, att du inte får välja vem du vill arbeta med?

7. Vad tycker du om att du inte får skriva upp dina idéer utan måste berätta till någon annan först?

8. Vad tycker du om att förklara någon annans grupps lösning inför klassen istället för att förklara sitt eget?

9. Brukar du fråga din klasskompis på en vanlig lektion eller bara läraren?

10. Hur fungerade samarbete i gruppen?

11. Hur delade ni upp arbetet?

12. Kände du att du kunde bidra aktivt? Varför eller varför inte?

Lektionsprocess

13. Tycker du att strukturen på problemlösning lektionerna hjälper dig att lösa matematikproblem? Är det något som gör det lättare eller svårare?

14. Har du märkt några skillnader i dina resultat på problemlösningsuppgifter?

15. Känner du att lektionen är lagom lång för att hinna förstå problemen?

Framtida förbättringar

16. Vad skulle du vilja förändra i undervisningen för att göra den ännu mer effektiv?

17. Skulle du vilja arbeta på detta sätt igen? Varför eller varför inte?

18. Finns det något du skulle göra annorlunda nästa gång du arbetar i en sådan här miljö?

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