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Optimization of stiffness and damping properties of below-knee prosthesis

GIL SERRANCOLÍ MASFERRER

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Department of Applied Mechanics
Division of Dynamics
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: + 46 (0)31-772 1000

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Department of Applied Mechanics
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ABSTRACT

Nowadays, prosthetic manufacturers have developed new designs of below-knee prosthesis which are anthropomorphic and safe. But more research should be done. This project tries to find out optimized parameters of below-knee prosthesis to walk with as low energy as possible.

A biomechanical optimal problem is formulated to find out suitable stiffness and damping properties of below-knee prosthesis. These properties mean the relation between the torque on the ankle joint of the prosthetic leg with ankle angle and its velocity. The criterion used has been minimizing energy of both on the healthy part of the body as well as external energy with which the prosthesis battery is supplied.

In order to state the optimal problem, firstly an anthropomorphic human gait has been looked for by using computer simulations. It has been chosen a mechanical human model that consist of two legs (with thigh and shank), the trunk and with joints: at ankle, knee and hip. Foot has been modeled as footprint. The system has 7 degrees of freedom. The motion has been parameterized by using polynomial and Fourier series and their parameters have been chosen so that the motion was anthropomorphic. Control torques at joints and ground reaction forces have been determined by using Lagrange equations.

After that, the optimization problem has been considered using energy of the healthy part of the body as cost function, while energy consumed by the prosthesis has been calculated in each variance. Algorithm of solution of optimization problem has been implemented in MATLAB so that optimized kinematic parameters of the gait as well as stiffness and damping parameters of the below-knee prosthesis are determined automatically. *fmincon* function is used to solve the optimal problem and *ode45* function to solve the differential equation to find out kinematics of the trunk.

The analyses of the obtained optimized values of stiffness and damping parameters of a below-knee prosthesis as well as kinematics of the optimized gait are presented. Future work within the project topic is outlined.

Key words: below-knee prosthesis, human gait, Lagrange equations, optimization, cost function, ankle, knee, hip.

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Preface

This Master's Thesis was carried out at the Division of Dynamics at the Department of Applied Mechanics on Chalmers University of Technology during the spring of 2010. It is my final project (in Erasmus program) of my degree, Mechanical Engineering, carried out in Barcelona (Catalonia) in Universitat Politècnica de Catalunya (ETSEIB-UPC).

My supervisor and examiner was Professor Viktor Berbyuk from the Division of Dynamics. I would like to thank him for all guidance and support. Also, I am grateful to Håkan Johansson from the Division of Dynamics who helped me with MATLAB questions and he was very patient in several moments in order to teach me a lot of "secrets" of this program.

Finally, I also would like to thank all my family and friends for their support.

Göteborg, June 2010

Gil Serrancolí Masferrer

Notations

Terms:

α_i	angle from vertical position to thigh
β_i	angle from vertical position to shank
ψ	angle from vertical position to trunk
A_i	ankle joint
a_i	length of the thigh *i*
b_i	length of the shank *i*
C_p	damping of the below-knee prosthesis on the ankle joint
E_h	Energy of the healthy part of the body
E_p	Energy consumed by the below-knee prosthesis
G	centre of mass of the trunk
J_{ai}	moment of inertia of the thigh *i* relative to the Z axis at point O
J_{bi}	moment of inertia of the shank *i* relative to the Z axis at point K_i
K_i	knee joint
K_p	stiffness of the below-knee prosthesis on the ankle joint
L	length of the single step
M	total mass of the body
m_{ai}	mass of the thigh *i*
m_{bi}	mass of the shank *i*
m_{fi}	mass of the foot *i*
O	hip joint
p_i	torque on the ankle joint *i*
q_i	torque on the hip joint *i*
r	distance from the suspension point O of the legs to the centre of mass of the trunk
r_{ai}	distance from O to the centre of mass of the thigh *i*
r_{bi}	distance from K_i to the centre of mass of the shank *i*
R_{1x}	horizontal component of the ground reaction
R_{1y}	vertical component of the ground reaction
T	duration of the single step
u_i	torque on the knee joint *i*
X	horizontal axis
x	horizontal position of the hip
x_1	horizontal position of the prosthetic foot
x_{R1}	horizontal position of the application point of the ground reaction force
x_2	horizontal position of the healthy foot
Y	vertical axis
y	vertical position of the hip
y_1	vertical position of the prosthetic foot
y_{R1}	vertical position of the application point of the ground reaction force
y_2	vertical position of the healthy foot
Z	Z axis, perpendicular to the plane of motion

1 Introduction

A below-knee prosthesis is a prosthetic leg with shank, ankle joint and prosthetic foot. Knee joint is usually healthy, i.e. knee belongs to patient. Somebody who has lost his shank by accident or by illness can wear below-knee prosthesis in order to be able to walk and to do his life more comfortable.

There are some main requirements that lower limb prosthesis should fulfil. It should have a cheap price, a long life, safety and naturally motions. This thesis focuses on more specific and technical backgrounds: energy optimization, such as energy consumption of the healthy part of the body and external energy consumed by battery.

It would be uncomfortable to wear a prosthesis if the patient had to do more effort than normal. It would be non-viable if the patient had to walk in a non-anthropomorphic way. It would be a useless prosthesis if it stops running during its gait, i.e. the battery has to be able to keep enough energy in order to supply the device properly. To sum up, a prosthesis which runs anthropomorphically with as little energy as possible should be achieved.

There are some models which minimize the energy used by a healthy patient in a walking distance within a given time. But it is not common to look for the minimum energy consumption of the prosthesis and the minimum energy consumption of the healthy part of the patient body at the same time. The main target of the master thesis is to find out stiffness and damping properties of a below-knee prosthesis, which avoid wasting energy: external power and healthy power. This means looking for the suitable relation between dynamics (torque) and kinematics (angular position and velocity) on the prosthesis ankle that could be useful to design a new below-knee prosthesis.

Keeping this purpose in mind, several points need to be handled. First of all, a mathematical model of human body motion in a stance phase on a leg with below-knee prosthesis has to be found. Anthropomorphic kinematics and dynamics for initial parameters also need to be found. Every result has to be compared with the literature and to check if these results are anthropomorphic or not. Once an anthropomorphic motion is obtained, energy consumption of human body in a stance phase has to be evaluated. It could also be tested for different stiffness and damping properties and different leg kinematics. Finally, to accomplish our goal, our cost function will be minimized, energy of the body, with suitable restrictions and optimized values of stiffness and damping properties will be obtained.

MATLAB will be used as software supporter. It will help to solve complex relations, differential equations and the optimal problem. It will also help to see different plots to understand the motion.

All this work would be aimed to design a new and more efficient type of lower limb prosthesis. A future work would be the mechanical design of the prosthesis with stiffness and damping properties found in this project.

This report consists of three main parts: an explanation of the mechanical and mathematical models, a description of the initial model found and a development of the optimal solution. It ends up discussing the results obtained.

2 Mechanical and mathematical models

A model as simple as possible but that represents the human gait as real and anthropomorphic as possible should be chosen.

As human gait could be considered periodic, the double step could be studied, stance phase and swing phase of each leg. But since the goal is to look for suitable stiffness and damping properties of the ankle prosthesis, it is enough to take only one step, with a stance phase on the leg with below knee-prosthesis. In our model, it is considered that during swing phase, the ankle torque is null.

It has been chosen a mechanical model with a footprint. The foot is represented as a point, Figure 2.1.

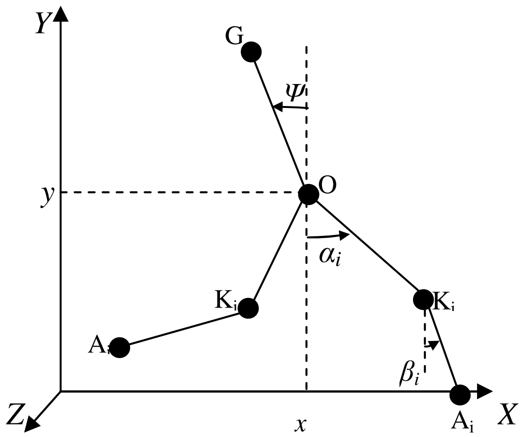


Figure 2.1 Mechanical model

The system contains a trunk (GO) and two legs. Each leg consists of two elements (OK_i and K_iA_i). Our model has several similarities with one used by Berbyuk (2002) [1]. It shows enough points to represent all the data needed to compute in our equations. Prosthesis leg will be represented as leg 1 which is on stance phase. The system would have 7 degrees of freedom, but ankle joint of the prosthesis leg is fixed (A_1), so the system has 5 degrees of freedom.

In addition to the weights of the trunk, thighs and shanks the ground reaction forces and the control moments at the joints of the legs act in the system.

The system moves through the X axis over a horizontal surface (the X-Z plane). To describe the set of generalized coordinates, following notations will be used: x, y, Ψ, α_i, β_i i=1, 2 (are represented in the Figure 2.1); m is the mass of the trunk; r is the distance from the suspension point O of the legs to the centre of mass of the trunk; J is the inertia's moment of the trunk relative to the Z axis at point O; m_{ai}, r_{ai}, a_i, J_{ai} are the mass, the distance from O to the centre of mass, the length and the moment of inertia of the thigh relative to the Z axis at point O, respectively; m_{bi}, r_{bi}, b_i, J_{bi} are the mass, the distance from K_i to the centre of mass, the length and the moment of inertia of the shank relative to the Z axis at point K_i, respectively; m_{fi} is the mass of the foot.

Lagrange equations of the second kind are used to find out the equations of motion of the system:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}+\frac{\partial V}{\partial q}=Q_q \tag{2.1}$$

Where q is a generalized coordinate with corresponding generalized force Q_q. T is kinetic energy and V is potential energy. The equations of motion are written as follows [1]:

By x axis:

$$M\cdot\ddot{x}+\sum_{i=1}^{2}\left[K_{ai}\left(\alpha_i^{'}\cdot\cos\alpha_i\right)^{'}+K_{bi}\left(\beta_i^{'}\cdot\cos\beta_i\right)^{'}\right]-K_r\left(\psi^{'}\cdot\cos\psi\right)^{'}=R_{1x}+R_{2x} \tag{2.2}$$

By y axis:

$$M\left(\ddot{y}+g\right)+\sum_{i=1}^{2}\left[K_{ai}\left(\alpha_i^{'}\cdot\sin\alpha_i\right)^{'}+K_{bi}\left(\beta_i^{'}\cdot\sin\beta_i\right)^{'}\right]-K_r\left(\psi^{'}\cdot\sin\psi\right)^{'}=R_{1y}+R_{2y} \tag{2.3}$$

By Ψ angle:

$$J\ddot{\psi}-K_r\left(\ddot{x}\cdot\cos\psi+\ddot{y}\cdot\sin\psi\right)-g\cdot K_r\cdot\sin\psi=-q_1-q_2 \tag{2.4}$$

By α angle:

$$\begin{aligned}J_i\ddot{\alpha}_i+K_{ai}\left(\ddot{x}\cdot\cos\alpha_i+\ddot{y}\cdot\sin\alpha_i\right)+a_i\cdot K_{bi}\left(\ddot{\beta}_i\cdot\cos(\alpha_i-\beta_i)+\dot{\beta}_i^2\cdot\sin(\alpha_i-\beta_i)\right)+g\cdot K_{ai}\cdot\sin\alpha_i=\\ =q_i-u_i+a_i\cdot\left(R_{ix}\cdot\cos\alpha_i+R_{iy}\cdot\sin\alpha_i\right)\end{aligned} \tag{2.5}$$

By β angle:

$$\begin{aligned}J_{ci}\cdot\beta_i+K_{bi}\left(\ddot{x}\cdot\cos\beta_i+\ddot{y}\cdot\sin\beta_i\right)+a_i\cdot K_{bi}\left(\ddot{\alpha}_i\cdot\cos(\alpha_i-\beta_i)-\dot{\alpha}_i^2\cdot\sin(\alpha_i-\beta_i)\right)+g\cdot K_{bi}\cdot\sin\beta_i=\\ =u_i-p_i+b_i\left(R_{ix}\cdot\cos\beta_i+R_{iy}\cdot\sin\beta_i\right)\end{aligned} \tag{2.6}$$

q_i, u_i and p_i are the torques on hip, knee and ankle respectively.

Where:

$$M=m+m_{a1}+m_{b1}+m_{a2}+m_{b2} \tag{2.7}$$

$$K_r=m\cdot r \tag{2.8}$$

$$K_{ai}=m_{ai}\cdot r_{ai}+a_i\left(m_{bi}+m_{fi}\right) \tag{2.9}$$

$$K_{bi}=m_{bi}\cdot r_{bi}+b_i\cdot m_{fi} \tag{2.10}$$

$$J_i=J_{ai}+a_i^2\left(m_{bi}+m_{fi}\right) \tag{2.11}$$

$$J_{ci}=J_{bi}+b_i^2\cdot m_{fi} \tag{2.12}$$

with i = 1, 2

2.1 Review of biomechanical cost functions

Some authors have written about cost functions used in biomechanical optimizations. The aim here is to find the best suitable function to achieve our objective. To represent the suitable gait pattern, different energy cost functions have been studied.

Then, some cost functions to evaluate optimal energy are presented:

- *Fatigue cost function.*

  M. Ackermann, A. J. van den Bogert, 2009 [2] proposed a family of cost functions that represented different gait patterns, it consist of weighted muscle activations:

$$J = \frac{1}{\sum \omega_i} \frac{1}{T} \sum_{i=1}^{m} \omega_i \int_0^T a_i^{\,p}(t)\, dt \tag{2.13}$$

  where m is the number of muscle groups, a is the muscle activation, and p and ω_i are the exponent of a and weighting factors, respectively. Depending on the muscle action, 8 different cost functions can be differentiated, since it could be p=1, 2, 3 or 10, and there are two sets of weighting factors ω_i.

- *Metabolic energy cost function.*

  Frank C. Anderson and Marcus G. Pandy [3] hypothesized that the suitable motor pattern should be found minimizing the metabolic energy expenditure per unit distance moved. The cost function is as follow:

$$E_d = \frac{\int_0^{t_f} \dot{E}_{total}^{M}}{X_{cm}(t_f) - X_{cm}(0)} = \frac{\int_0^{t_f} \left(\dot{B} + \sum_{m=0}^{54} \left(\dot{A}_m + \dot{M}_m + \dot{S}_m + \dot{W}_m \right) \right) dt}{X_{cm}(t_f) - X_{cm}(0)} \tag{2.14}$$

  where $X_{cm}(0)$ and $X_{cm}(t_f)$ denote the position of the centre of mass of the model at the beginning and at the end of the simulated gait cycle. $\dot{B}$ is the basal metabolic heat rate of the whole body, $\dot{A}_m$, $\dot{M}_m$, $\dot{S}_m$, $\dot{W}_m$ are the activation, maintenance, shortening and mechanical heat rates of each muscle.

  Similar to the previous one, hypothesis of H. Hatze and J. D. Buys [4] consist of minimizing the metabolic energy, expressed as:

$$\dot{E} = \dot{g} + \dot{h} + \dot{s} + \dot{w} + \dot{r} \tag{2.15}$$

  where $\dot{g}$ is the activation heat rate, $\dot{h}$ is the maintenance heat rate, $\dot{s}$ is the shortening heat rate, $\dot{w}$ is the work rate and $\dot{r}$ is the rate of heat dissipated in the parallel structures.

A. E. Minetti and R. McN Alexander [5] considered that the metabolic cost function of muscle activity (metabolic power) was as follow:

$$P = \alpha \cdot T_0 \cdot \omega_{\max} \cdot \Phi\left(\omega / \omega_{\max} \right) \tag{2.16}$$

where α represents a fraction of the muscle's fibres activated, T is the torque on the muscle joint, ω is the angular velocity and they defined a function Φ (ω/ω_{max}) from experimental results:

$$\Phi\left(\omega / \omega_{\max} \right) = \frac{0{,}054 + 0{,}506 \cdot \left(\omega / \omega_{\max} \right) + 2{,}46 \cdot \left(\omega / \omega_{\max} \right)^2}{1 - 1{,}13 \cdot \left(\omega / \omega_{\max} \right) + 12{,}8 \cdot \left(\omega / \omega_{\max} \right)^2 - 1{,}64 \cdot \left(\omega / \omega_{\max} \right)^3} \tag{2.17}$$

- *Muscular force cost function*

A. Pedotti, V. V. Krishman and L. Stark [6] hypothesized that they had to minimize the total muscular force to find out the suitable human locomotion pattern. Their cost functions were as follows:

$$\begin{aligned} J_1 &= \sum_{i=1}^{11} F_i \\ J_2 &= \sum_{i=1}^{11} F_i^2 \\ J_3 &= \sum_{i=1}^{11} \frac{F_i}{F_{\max i}} \\ J_4 &= \sum_{i=1}^{11} \frac{F_i^2}{F_{\max i}^2} \end{aligned} \tag{2.18}$$

where J_1 is a performance criterion related to the initial force required to produce the set of torques. J_2 also minimizes total muscular force but penalizes large individual muscle force severely.

J_3 is similar to J_1 but employs the muscles more efficiently by demanding large force production from the large muscles; indeed, it takes into account the instantaneous state of each muscle, since F_{maxi} depends upon the instantaneous length of muscle as well as its velocity. J_4 is a performance criterion which uses muscles more efficiently while keeping their level of activation as low as possible.

- *Mechanical energy cost function.*

Viktor Berbyuk, Anders Boström, Bogdan Lytwuyn, and Bo Peterson [1] hypothesized that the mechanical energy cost function depended on the torques of the articulations joints: hip, knee, ankle and metatarsal joints and their angular velocities:

$$E = \frac{1}{2L}\int_0^T \left\{ \sum_{i=1}^{2} \left[\left| q_i \left(\dot{\alpha}_i(t) - \dot{\psi}(t) \right) \right| + \left| u_i \left(\dot{\alpha}_i(t) - \dot{\beta}_i(t) \right) \right| + \left| p_i \left(\dot{\beta}_i(t) - \dot{\gamma}_i(t) \right) \right| + \left| w_i \left(\dot{\gamma}_i(t) - \varepsilon_i(t) \right) \right| \right] \right\} dt \tag{2.19}$$

where q_i, u_i, p_i and w_i are the torques that act in the hip, the knee, the ankle and the metatarsal joints respectively. $\dot{\psi}$(t) is the angular velocity between the trunk and the hip, $\dot{\alpha}_i$(t) is the angular velocity between hip joint and each thigh, $\dot{\beta}_i(t)$ is the angular velocity between knee joint and each shank, $\dot{\gamma}_i(t)$ is the angular velocity between ankle joint and each foot and $\varepsilon_i(t)$ is the angular velocity between metatarsal joint and each set of toes. L is the length of the step.

Viktor Berbyuk, Bogdan Lytwyn and Myroslav Demydyuk [7] took into account very similar mechanical energy cost function, but they hypothesized that mechanical energy only depends on torques on the hip and knee joints (since they considered the control inputs were torques actuators acting only at hip and knee joints), on the velocities of these joints and on the length of the step. This is as follow:

$$E = \frac{1}{2L}\int_0^T \left\{ \sum_{i=1}^{2} \left[\left| q_i \left(\dot{\alpha}_i(t) - \dot{\psi}(t) \right) \right| + \left| u_i \left(\dot{\alpha}_i(t) - \dot{\beta}_i(t) \right) \right| \right] \right\} dt \tag{2.20}$$

2.2 Cost functions selected

The focus will be on the problem of identifying the gait pattern by minimizing a couple of cost functions. These functions will be the energy cost function of the healthy part of the body (E_h), cost function of the energy consumed by the prosthesis (E_p). These functions are defined as follows:

$$E_h = \frac{1}{L}\int_0^T \left\{ \sum_{i=1}^{2} \left[\left| q_i \left(\dot{\alpha}_i(t) - \dot{\psi}(t) \right) \right| + \left| u_i \left(\dot{\alpha}_i(t) - \dot{\beta}_i(t) \right) \right| \right] \right\} dt + \frac{1}{L}\int_0^T \left\{ \left| p_h \left(\dot{\beta}_h(t) \right) \right| \right\} dt \tag{2.21}$$

$$E_p = \frac{1}{L}\int_0^T \left\{ \left| p_p \left(\dot{\beta}_p(t) \right) \right| \right\} dt \tag{2.22}$$

2.3 Statement of the problem

Our system could have two phases on one leg: stance phase [0 , T), when prosthesis leg is on the floor and healthy leg is not; and swing phase [T , 2T), when the prosthesis leg has no contact with the floor. But as mentioned, only the first phase will be represented.

Since human motion is periodic, following boundary conditions could be considered:

$$f(0) = f(T) \quad \dot{f}(0) = \dot{f}(T) \quad f = (y, \psi) \tag{2.23}$$

$$\begin{cases} x(0) = x(T) - L \\ \dot{x}(0) = \dot{x}(T) \end{cases} \tag{2.24}$$

$$\begin{cases} x_2(0) = x_2(T) - 2L \\ \dot{x}_2(0) = \dot{x}_2(T) \end{cases} \tag{2.25}$$

where T is the duration of a single step. α_i and β_i are periodic and their period is $2T$.

As prosthesis leg is on the floor and healthy leg is in swing phase, following boundary conditions can be defined:

$$x_{a1}(t) \equiv x_{a1}^0 \quad y_{a1}(t) \equiv 0 \quad t \in [0, T) \tag{2.26}$$

$$y_{a2}(t) \geq 0 \quad t \in [0, T) \tag{2.27}$$

x_{ai} and y_{ai} are the coordinates of the point A (Figure 2.1).

Without any restriction of the generality the following additional conditions for t=0 and t=T are given by:

$$x_{a1}^0 = L \quad y_{a1}^0 = 0 \tag{2.28}$$

$$x_{a1}(T) = L \quad y_{a1}(T) = 0 \tag{2.29}$$

$$x_{a2}^0 = 0 \quad y_{a2}^0 = 0 \tag{2.30}$$

$$x_{a2}(T) = 2L \quad y_{a2}(T) = 0 \tag{2.31}$$

In order our model to be anthropomorphic some angular displacements constraints could be used. These will be taken from experimental results from literature.

$$\theta_i^0(t) \leq \mu_i^0 \leq \Theta_i^0(t) \tag{2.32}$$

$$\theta_i^k(t) \leq \mu_i^k \leq \Theta_i^k(t) \tag{2.33}$$

$$\theta_i^a(t) \leq \mu_i^a \leq \Theta_i^a(t) \tag{2.34}$$

Where:

$$\mu_i^0(t) = \alpha_i(t) - \psi(t) \tag{2.35}$$

$$\mu_i^k(t) = \alpha_i(t) - \beta_i(t) \tag{2.36}$$

$$\mu_i^a(t) = -\beta_i(t) + \frac{\pi}{2} \tag{2.37}$$

with i = 1, 2

In addition, there are two logical restrictions so that the human gait is anthropomorphic:

$$\alpha_i(t) \geq \beta_i(t), \ \forall t \in [0, T], (i = 1, 2) \qquad (2.38)$$

$$y_2(t) \geq 0, t \in [0, T] \qquad (2.39)$$

There are also dynamic restrictions, reaction force has to be always positive and the ankle prosthesis torque has to be defined as a function of β and $\dot{\beta}$.

$$R_{1y}(t) \geq 0, \forall t \in [0, T] \qquad (2.40)$$

$$p_1(t) = -K_p \cdot \beta_1(t) - C_p \cdot \dot{\beta}_1(t) - K_p \cdot \frac{\pi}{2} \quad \forall t \in [0, T] \qquad (2.41)$$

After defining our mechanical model, a cost function needs to be chosen. Then the optimized motion will be found. In the following section it will be discussed which is the best cost function for us.

Let $Z(t) = \{x, \dot{x}, y, \dot{y}, x_2, \dot{x}_2, y_2, \dot{y}_2, \psi, \dot{\psi}, \alpha, \dot{\alpha}, \beta, \dot{\beta}, i = 1, 2\}$ be a vector of the phase state and $U(t) = \{p_1, q_i, u_i, p_i \ i = 1, 2\}$ be a vector of the controlling stimuli of the system. Once these vectors are defined, the following problem can be stated.

Problem A. Assume that the step length *L* is given, and the duration of the single support phase *T*. It is required to determine the control process $[Z^*(t), U^*(t)]$, for $t \in [0, T]$, which minimize the cost function selected, E_h and E_p (2.21) – (2.22), subject to differential constraints (2.2) – (2.6), boundary conditions (2.23) – (2.34), anthropomorphic constraints (2.38) - (2.39) and dynamic restrictions (2.40) – (2.41).

In the next section, first some functions will be parameterized and then will follow an attempt to find out an initial motion which is anthropomorphic. This motion will be used later as an initial guess in the optimal problem.

3 Initial Model

Before optimizing the model, an initial model that is anthropomorphic must be found. Maths and logic can be used to search for a suitable initial model. Looking for a suitable leg kinematics and dynamics that looks anthropomorphic comes first.

3.1 Leg kinematics

Our mechanical model has to satisfy our equations and constrains. But the motion has to be also anthropomorphic. A shape for our main variables needs to be chosen and it will lead to the other variables as a function of them. Fourier series will be used to define our main variables, these will be x(t), y(t), x_2(t), y_2(t). So, these functions will be set as follows:

$$x(t) = C_{0x} + C_{1x} \cdot t + C_{2x} \cdot t^2 + C_{3x} \cdot t^3 + C_{4x} \cdot t^4 + C_{5x} \cdot t^5 + \\ + \sum_{n=1}^{N_x} \left(a_{nx} \cdot \cos\left(\frac{2\pi n}{T^*} \cdot t \right) + b_{nx} \cdot \sin\left(\frac{2\pi n}{T^*} \cdot t \right) \right) \tag{3.1}$$

$$y(t) = C_{0y} + C_{1y} \cdot t + C_{2y} \cdot t^2 + C_{3y} \cdot t^3 + C_{4y} \cdot t^4 + C_{5y} \cdot t^5 + \\ + \sum_{n=1}^{N_y} \left(a_{ny} \cdot \cos\left(\frac{2\pi n}{T^*} \cdot t \right) + b_{ny} \cdot \sin\left(\frac{2\pi n}{T^*} \cdot t \right) \right) \tag{3.2}$$

$$x_2(t) = C_{0x2} + C_{1x2} \cdot t + C_{2x2} \cdot t^2 + C_{3x2} \cdot t^3 + C_{4x2} \cdot t^4 + C_{5x2} \cdot t^5 + \\ + \sum_{n=1}^{N_{x2}} \left(a_{nx2} \cdot \cos\left(\frac{2\pi n}{T^*} \cdot t \right) + b_{nx2} \cdot \sin\left(\frac{2\pi n}{T^*} \cdot t \right) \right) \tag{3.3}$$

$$y_2(t) = C_{0y2} + C_{1y2} \cdot t + C_{2y2} \cdot t^2 + C_{3y2} \cdot t^3 + C_{4y2} \cdot t^4 + C_{5y2} \cdot t^5 + \\ + \sum_{n=1}^{N_{y2}} \left(a_{ny2} \cdot \cos\left(\frac{2\pi n}{T^*} \cdot t \right) + b_{ny2} \cdot \sin\left(\frac{2\pi n}{T^*} \cdot t \right) \right) \tag{3.4}$$

There are some of these parameters which could be defined a priori because of the boundary conditions.

If taken into account that our motion is periodic, x(t) should satisfy following conditions:

$$\begin{cases} x(0) = x(T) - L \\ \dot{x}(0) = \dot{x}(T) \end{cases} \tag{3.5}$$

Therefore, C_{1x} and C_{2x} will be defined as:

$$C_{1x} = \frac{L}{T} - \left(C_{2x} \cdot T + C_{3x} \cdot T^2 + C_{4x} \cdot T^3 + C_{5x} \cdot T^4 \right) \tag{3.6}$$

$$C_{2x} = -\frac{1}{2} \left(3 \cdot C_{3x} \cdot T + 4 \cdot C_{4x} \cdot T^2 + 5 \cdot C_{5x} \cdot T^3 \right) \tag{3.7}$$

Similar boundary conditions are defined for y(t):

$$\begin{cases} y(0) = y(T) \\ \dot{y}(0) = \dot{y}(T) \end{cases} \tag{3.8}$$

And we could get C_{1y} and C_{2y} as:

$$C_{1y} = -\left(C_{2y} \cdot T + C_{3y} \cdot T^2 + C_{4y} \cdot T^3 + C_{5y} \cdot T^4\right) \tag{3.9}$$

$$C_{2y} = -\frac{1}{2}\left(3 \cdot C_{3y} \cdot T + 4 \cdot C_{4y} \cdot T^2 + 5 \cdot C_{5y} \cdot T^3\right) \tag{3.10}$$

Boundary conditions for $x_2(t)$ are as follows:

$$\begin{cases} x_2(0) = 0 \\ x_2(0) = x_2(T) - 2L \\ \dot{x}_2(0) = \dot{x}_2(T) \end{cases} \tag{3.11}$$

So C_{0x2} , C_{1x2} and C_{2x2} are defined as:

$$C_{0x2} = -\sum_{n=1}^{N_{x2}} a_{nx2} \tag{3.12}$$

$$C_{1x2} = \frac{2L}{T} - \left(C_{2x2} \cdot T + C_{3x2} \cdot T^2 + C_{4x2} \cdot T^3 + C_{5x2} \cdot T^4\right) \tag{3.13}$$

$$C_{2x2} = -\frac{1}{2}\left(3 \cdot C_{3x2} \cdot T + 4 \cdot C_{4x2} \cdot T^2 + 5 \cdot C_{5x2} \cdot T^3\right) \tag{3.14}$$

There are similar boundary conditions for $y_2(t)$:

$$\begin{cases} y_2(0) = 0 \\ y_2(0) = y_2(T) \\ \dot{y}_2(0) = \dot{y}_2(T) \end{cases} \tag{3.15}$$

So C_{0y2}, C_{1y2} and C_{2y2} are defined as:

$$C_{0y2} = -\sum_{n=1}^{N_{y2}} a_{ny2} \tag{3.16}$$

$$C_{1y2} = -\left(C_{2y2} \cdot T + C_{3y2} \cdot T^2 + C_{4y2} \cdot T^3 + C_{5y2} \cdot T^4\right) \tag{3.17}$$

$$C_{2y2} = -\frac{1}{2}\left(3 \cdot C_{3y2} \cdot T + 4 \cdot C_{4y2} \cdot T^2 + 5 \cdot C_{5y2} \cdot T^3\right) \tag{3.18}$$

In order to deal with all other parameters that could change the following vector will be defined:

$$C = \left[C_{x2}^T \; C_{y2}^T \; C_x^T \; C_y^T \right] \tag{3.19}$$

where:

$$C_{x2}^T = \left[C_{3x2}, C_{4x2}, C_{5x2}, a_{nx2}, b_{nx2} \right] \text{ where n = (1...N}_{x2}) \tag{3.20}$$

$$C_{y2}^T = \left[C_{3y2}, C_{4y2}, C_{5y2}, a_{ny2}, b_{ny2} \right] \text{ where n = (1...N}_{y2}) \tag{3.21}$$

$$C_x^T = \left[C_{0x}, C_{3x}, C_{4x}, C_{5x}, a_{nx}, b_{nx} \right] \text{ where n = (1...N}_{x}) \tag{3.22}$$

$$C_y^T = \left[C_{0y}, C_{3y}, C_{4y}, C_{5y}, a_{ny}, b_{ny} \right] \text{ where n = (1...N}_{y}) \tag{3.23}$$

First of all, all fixed parameters that take part in the motion equations (2.2 to 2.12) must be defined. These are taken from [1]. See Table 3.1.

Table 3.1 Fixed data

T	0.57 s	r	0.39 m	J	7.096 Nm^2	L	0.76 m	m	46.7 kg
a_1	0.47 m	r_{a1}	0.258 m	J_{a1}	0.57 Nm^2	m_{a1}	8.49 kg	m_{f1}	1.24 kg
a_2	0.47 m	r_{a2}	0.258 m	J_{a2}	0.57 Nm^2	m_{a2}	8.49 kg	m_{f2}	1.24 kg
b_1	0.53 m	r_{b1}	0.214 m	J_{b1}	0.16 Nm^2	m_{b1}	3.51 kg		
b_2	0.53 m	r_{b2}	0.214 m	J_{b2}	0.16 Nm^2	m_{b2}	3.51 kg		

3.1.1 Effects on the variation of the parameters

How each variable parameter affects in coordinates defined by polynomial parameters and Fourier Series (x_2, y_2, x and y) is studied.

It is important knowing how the motion x_2 changes if Fourier parameters are modified.

$$\begin{cases} x_2(t) \;\; in \; m \\ \dot{x}_2(t) \;\; in \; m/s \end{cases} \tag{3.24}$$

Evaluating above functions, the conclusion is the following:

- Initial velocity is defined by:

$$\dot{x}_2(0) = \frac{2L}{T} + \frac{1}{2} \cdot C_{3x2} \cdot T^2 + C_{4x2} \cdot T^3 + \frac{3}{2} \cdot C_{5x2} \cdot T^4 \tag{3.25}$$

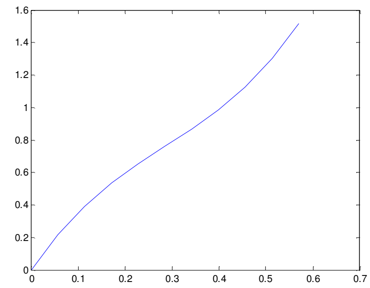
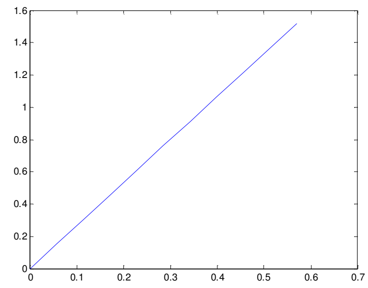
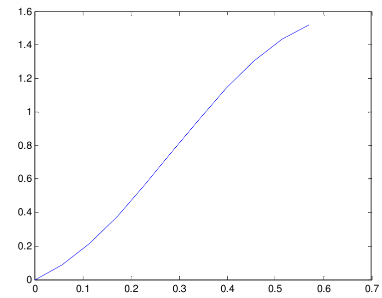
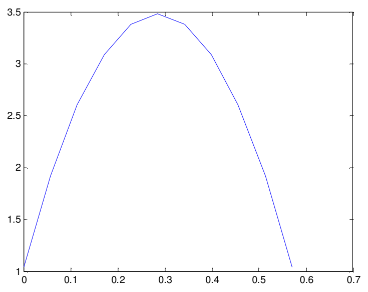
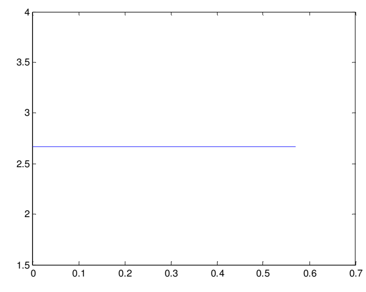
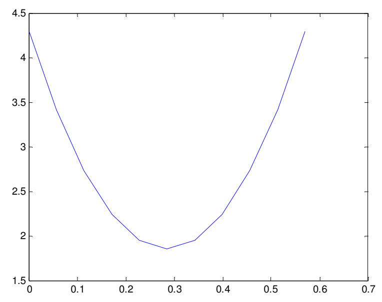
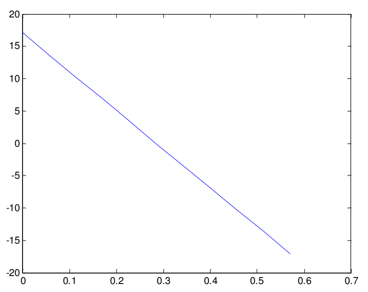
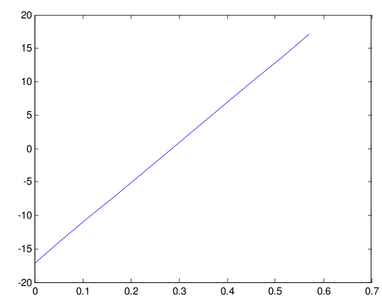
- and the initial acceleration by the expression:

$$\ddot{x}_2(0) = -3 \cdot C_{3x2} \cdot T - 4 \cdot C_{4x2} \cdot T^2 - 5 \cdot C_{5x2} \cdot T^3 \tag{3.26}$$

So varying our parameters, different function shapes are obtained:

For example, varying C_{3x2}. See Table 3.2.

Table 3.2. $x_2(t)$, $\dot{x}_2(t)$ *and* $\ddot{x}_2(t)$ *trials for different* C_{3x2}*, and for* $C_{4x2} = C_{5x2} = 0$.

$C_{3x2} = -10$	$C_{3x2} = 0$	$C_{3x2} = 10$
$x_2(t)$		
		
$\dot{x}_2(t)$		
		
$\ddot{x}_2(t)$		
	Zero acceleration	

Note that only polynomial parameters are varied, not sinus and cosines parameters. For x_2, it will be enough using polynomial parameters.

As sign of parameters C_{ix2} are equal, all parameters will affect in a similar way.

It could be also noted that if T>1, C_{5x2} will have a strong effect since its exponent is higher than others (in absolute value): $\frac{3}{2} \cdot T^4$ on velocity and $5 \cdot T^3$ on acceleration;

while if T<1, C_{3x2} may have a strong effect since its exponent is lower than others (as much as T is close to 0): $\frac{1}{2}\cdot T^2$ on velocity and $3\cdot T$ on acceleration expression.

It could be set that $\dot{x}_2(t)$ has to be always positive: $\dot{x}_2(t) > 0\ m/s$ and it could be defined a maximum possible velocity $\dot{x}_2(t) < 3\ m/s$. See Table 3.3.

Table 3.3. Expressions that satisfies the restriction

1	$\dot{x}_2(t) > 0$	$\frac{2L}{T}+\frac{1}{2}\cdot C_{3x2}\cdot T^2 + C_{4x2}\cdot T^3 + \frac{3}{2}\cdot C_{5x2}\cdot T^4 - \left(3\cdot C_{3x2}\cdot T + 4\cdot C_{4x2}\cdot T^2 + 5\cdot C_{5x2}\cdot T^3\right)\cdot t + {}$ $+3\cdot C_{3x2}\cdot t^2 + 4\cdot C_{4x2}\cdot t^3 + 5\cdot C_{5x2}\cdot t^4 > 0\ \forall t$
2	$\dot{x}_2(t) < 3$	$\frac{2L}{T}+\frac{1}{2}\cdot C_{3x2}\cdot T^2 + C_{4x2}\cdot T^3 + \frac{3}{2}\cdot C_{5x2}\cdot T^4 - \left(3\cdot C_{3x2}\cdot T + 4\cdot C_{4x2}\cdot T^2 + 5\cdot C_{5x2}\cdot T^3\right)\cdot t + {}$ $+3\cdot C_{3x2}\cdot t^2 + 4\cdot C_{4x2}\cdot t^3 + 5\cdot C_{5x2}\cdot t^4 < 3\ \forall t$

For an initial optimization $C_{3x2} = 1$, $C_{4x2} = 0$ and $C_{5x2} = -1$ are chosen.

$$\begin{cases} y_2(t)\ \ in\ m \\ \dot{y}_2(t)\ \ in\ m/s \end{cases} \tag{3.27}$$

This coordinate has a special constraint, because it has to be positive, it will be anthropomorphically wrong if the foot would introduce under the ground.

As a first approximation, the function y_2 (t) has been set as a parabola. Therefore maximum height is in the middle of the step. In addition, the initial velocity with positive value and final velocity with negative have also been set. A maximum height $\left(y_2(t) < 0.05m\right)$ and a maximum velocity $\left(\dot{y}_2(t) < 3m/s\right)$ can be defined. See Table 3.4.

Table 3.4. Expressions that satisfies the restrictions

1	$\dot{y}_2(\frac{T}{2}) = 0$ (singular point at the middle)	$-\frac{1}{4}\cdot C_{3y2} - \frac{1}{2}\cdot C_{4y2}\cdot T - \frac{11}{16}\cdot C_{5y2}\cdot T^2 = 0$
2	$\ddot{y}_2(\frac{T}{2}) < 0$ (singular point has to be a maximum)	$C_{4y2} > -\frac{5}{2}\cdot C_{5y2}\cdot T$
3	$\dot{y}_2(0) \ge 0$ and $\dot{y}_2(T) \le 0$	$\frac{C_{3y2}}{2} + C_{4y2}\cdot T + \frac{3}{2}\cdot C_{5y2}\cdot T^2 = 0$
4	$y_2(t) < 0.05\,m$ $\forall t$	$\left(\frac{1}{2}\cdot C_{3y2}\cdot T^2 + C_{4y2}\cdot T^3 + \frac{3}{2}\cdot C_{5y2}\cdot T^4\right)\cdot t - \frac{1}{2}\cdot\left(3\cdot C_{3y2}\cdot T + 4\cdot C_{4y2}\cdot T^2 + 5\cdot C_{5y2}\cdot T^3\right)\cdot t^2 + {}$ $+C_{3y2}\cdot t^3 + C_{4y2}\cdot t^4 + C_{5y2}\cdot t^5 < 0{,}05\ \forall t$
5	$\lvert\dot{y}_2\rvert < 3\ m/s$	$\lvert\frac{1}{2}\cdot C_{3y2}\cdot T^2 + C_{4y2}\cdot T^3 + \frac{3}{2}\cdot C_{5y2}\cdot T^4 - \left(3\cdot C_{3y2}\cdot T + 4\cdot C_{4y2}\cdot T^2 + 5\cdot C_{5y2}\cdot T^3\right)\cdot t + {}$ $3\cdot C_{3y2}\cdot t^2 + 4\cdot C_{4y2}\cdot t^3 + 5\cdot C_{5y2}\cdot t^5\rvert < 3\ \forall t$

For an initial optimization $C_{3y2} = -9.12$, $C_{4y2} = 8$ and $C_{5y2} = 0$ is chosen.

$$\begin{cases} x(t) \;\; in \; m \\ \dot{x}(t) \;\; in \; m/s \end{cases} \tag{3.29}$$

Regarding $x(t)$, it is similar than $x_2(t)$, but an initial value has to be chosen. This initial value is given by $C_{0x} = 0$, it represents the situation of the hip at t=0.

So,

$$0 \; m < C_{0x} < 0.76 m \tag{3.30}$$

Arbitrary but logically, $C_{0x} = 0.4$ is chosen. Similar restrictions like $\dot{x}_2(t)$ are set now. See Table 3.5.

Table 3.5. Expressions that satisfies the restrictions

1	$\dot{x} > 0$	$\frac{L}{T} + \frac{1}{2} \cdot C_{3x} \cdot T^2 + C_{4x} \cdot T^3 + \frac{3}{2} \cdot C_{5x} \cdot T^4 - \left(3 \cdot C_{3x} \cdot T + 4 \cdot C_{4x} \cdot T^2 + 5 \cdot C_{5x} \cdot T^3\right) \cdot t +$ $+ 3 \cdot C_{3x} \cdot t^2 + 4 \cdot C_{4x} \cdot t^3 + 5 \cdot C_{5x} \cdot t^4 > 0 \;\; \forall t$
2	$\dot{x} < 3$	$\frac{L}{T} + \frac{1}{2} \cdot C_{3x} \cdot T^2 + C_{4x} \cdot T^3 + \frac{3}{2} \cdot C_{5x} \cdot T^4 - \left(3 \cdot C_{3x} \cdot T + 4 \cdot C_{4x} \cdot T^2 + 5 \cdot C_{5x} \cdot T^3\right) \cdot t +$ $+ 3 \cdot C_{3x} \cdot t^2 + 4 \cdot C_{4x} \cdot t^3 + 5 \cdot C_{5x} \cdot t^4 < 3 \;\; \forall t$

For an initial optimization $C_{3x} = C_{4x} = C_{5x} = 0$ will be chosen. So, velocity will be constant $\left(\frac{L}{T} = 1.33 m/s\right)$ and acceleration will be zero.

$$\begin{cases} y(t) \;\; in \; m \\ \dot{y}(t) \;\; in \; m/s \end{cases} \tag{3.31}$$

Hip should be between a height of $0.75 m < y(t) < 1 m$. So,

$$0.75 m < C_{0y} < 1 \; m \tag{3.32}$$

$C_{0y} = 0.91$ is chosen.

Sinus and cosines parameters are used, instead of polynomial parameters, because otherwise, the acceleration of hip would not be anthropomorphic. To start just a couple of parameters are taken, so $N_y = 1$. T^* will be $T^* = 2T$, because it is a parabola between t=0 and t=T.

$\dot{y}(t)$ should be slower than $\dot{y}_2(t)$, therefore it is set $|\dot{y}(t)| < 0.5 m$. See Table 3.6.

Table 3.6. Expressions that satisfies the restrictions

1	$0.75 m < y(t) < 1 m \;\; \forall t$	$0.75 m < C_{0y} + a_{1y} \cdot \cos\left(\frac{\pi}{T}\right) + b_{1y} \cdot \sin\left(\frac{\pi}{T}\right) < 1 m$
2	$\lvert\dot{y}(t)\rvert < 0.5 m/s$	$\left\lvert - a_{1y} \cdot \frac{\pi}{T} \cdot \sin\left(\frac{\pi}{T} \cdot t\right) + b_{1y} \cdot \frac{\pi}{T} \cdot \cos\left(\frac{\pi}{T} \cdot t\right)\right\rvert < 0.5 m/s$

For an initial optimization $C_{0y} = 0.91$, $a_{1y} = 0$, $b_{1y} = 0.08$ will be chosen.

In order to define angular motions of the links, our model needs to be taken into account (Figure 2.1) and following features of the motion (see Figure 3.1). Only one step is studied. Leg 1 is supposed to wear the below knee prosthesis. Foot 2 moves one step while foot 1 is supported on the floor. Next step would be almost the same. Now, angles from healthy leg would be the same like prosthesis leg in the last step and vice versa.

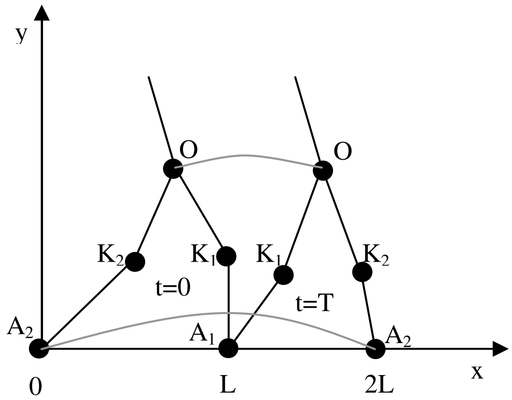


Figure 3.1. Definition of the studied motion.

So, to find out angular motions, the following system of equations has to be solved:

$$\begin{cases} x(t) = x_i(t) - b_i \cdot \sin\beta_i - a_i \cdot \sin\alpha_i \\ y(t) = y_i(t) + b_i \cdot \cos\beta_i + a_i \cdot \cos\alpha_i \end{cases} \tag{3.33}$$

Angles will be as follows:

$$\alpha_i(t) = \arcsin\left(\frac{1}{a_i}\left(x_i(t) - x(t) - b_i \cdot \sin\beta_i\right)\right) \tag{3.34}$$

$$\beta_i = \arcsin\left(\frac{\frac{1}{2b_i}\left((x_i - x)^2 + (y - y_i)^2 - a_i^2 + b_i^2\right)}{\sqrt{(x_i - x)^2 + (y - y_i)^2}}\right) - \arctan\left(\frac{y - y_i}{x_i - x}\right) \tag{3.35}$$

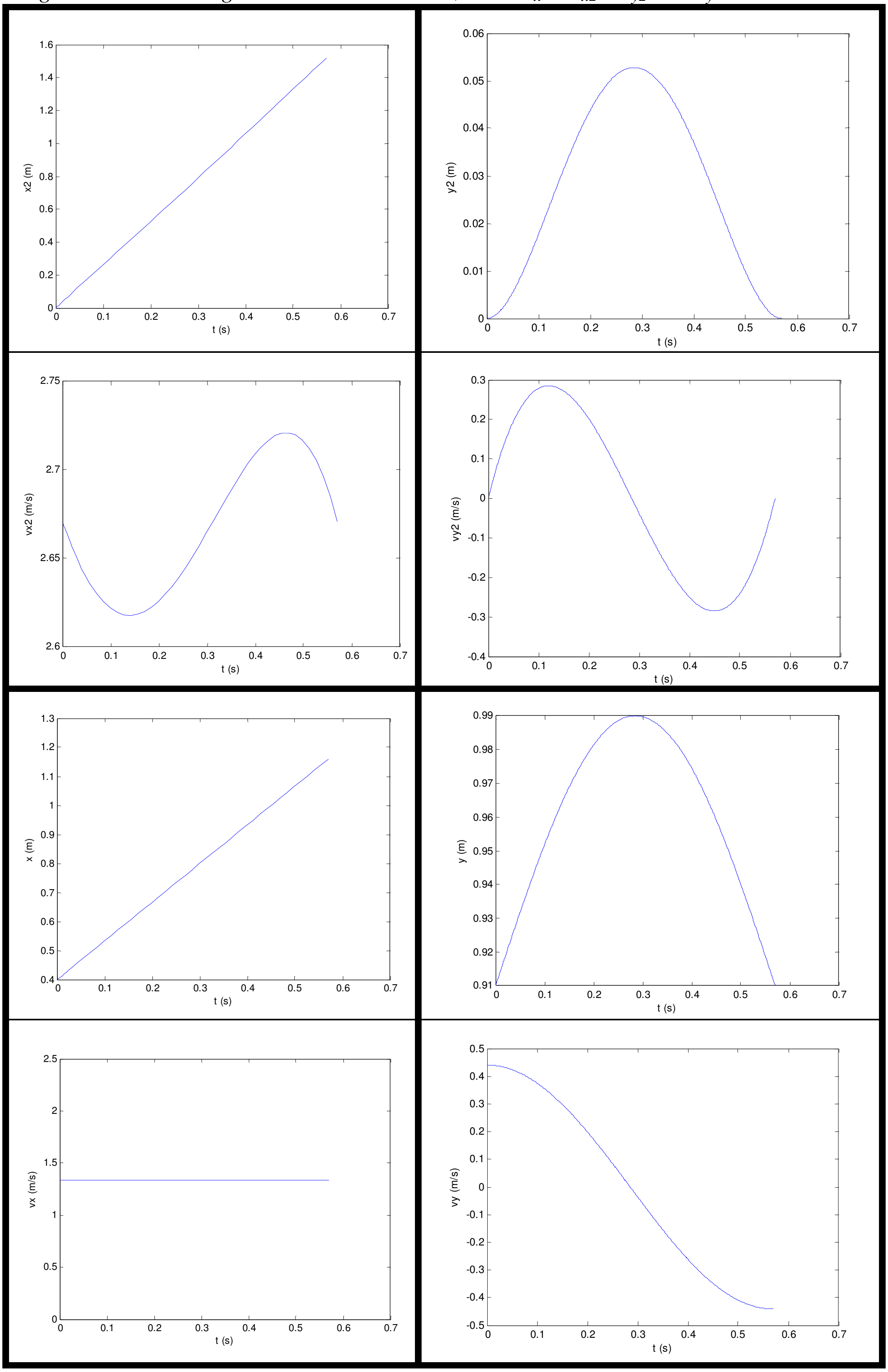
Velocity and acceleration of these angles will be their time derivation.

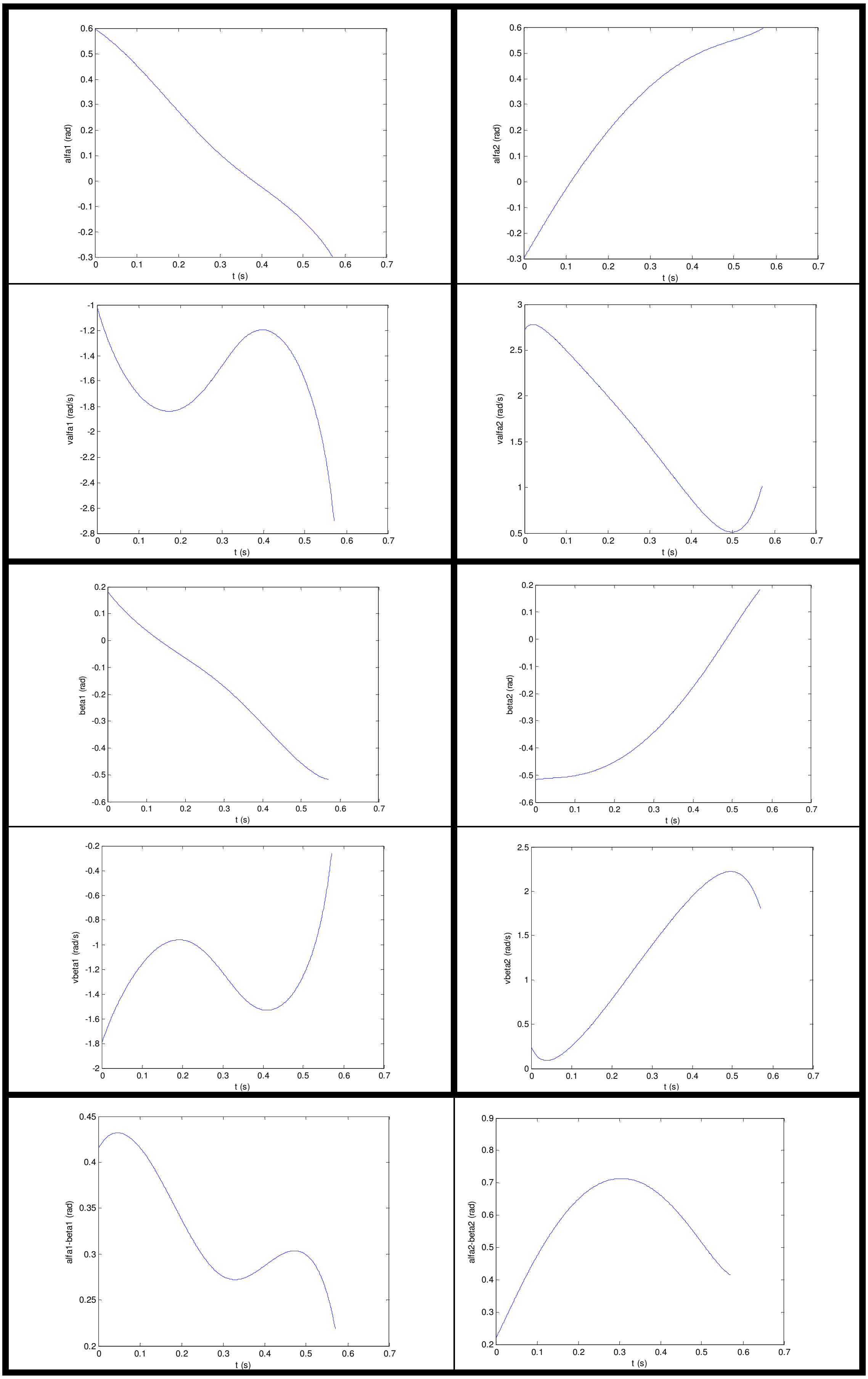
As mentioned on (2.28) in order to be an anthropomorphic model, angle $\beta_i < \alpha_i$.

In section 3.1.2, all leg kinematics parameters are shown. See Figure 3.2.

3.1.2 Leg kinematic plots

Figure 3.2. Initial legs kinematical solution, with $N_x = N_{x2} = N_{y2} = 0$. $N_y = 1$.




alfa1 (rad)
t (s)
alfa2 (rad)
t (s)
valfa1 (rad/s)
t (s)
valfa2 (rad/s)
t (s)
beta1 (rad)
t (s)
beta2 (rad)
t (s)
vbeta1 (rad/s)
t (s)
vbeta2 (rad/s)
t (s)
alfa1-beta1 (rad)
t (s)
alfa2-beta2 (rad)
t (s)

3.2 Trunk kinematics

Before looking for trunk motion defined by ψ angle, a p_1 torque value must be assigned, since it will be needed to find out ψ.

p_1 is the ankle joint torque of the prosthesis leg and it is wanted that depends on β angle and its angular velocity, $\dot{\beta}$, related with stiffness and damping properties. So, following assumption is done:

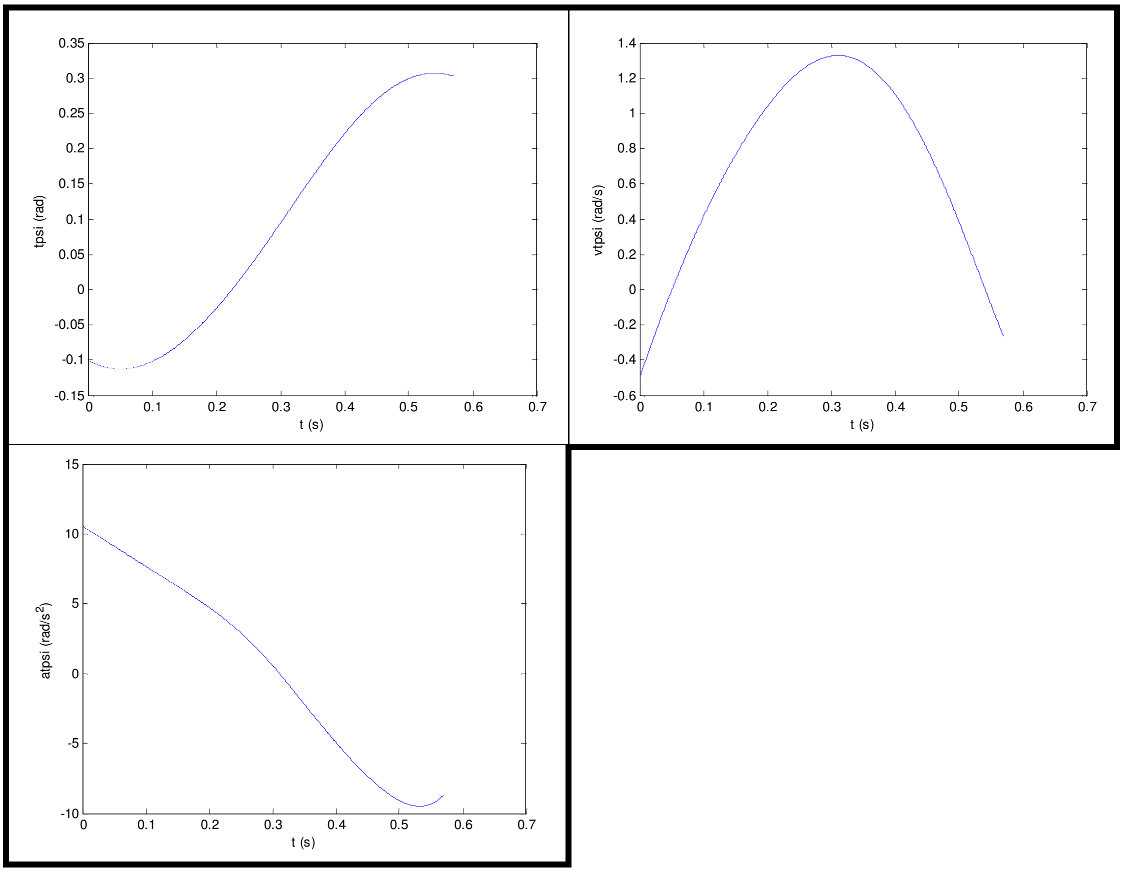
$$p_1(t) = -K_p \cdot \beta_1(t) - C_p \cdot \dot{\beta}_1(t) - K_p \cdot \frac{\pi}{2} \tag{3.36}$$

Once defined $\beta_1(t)$ and $\dot{\beta}_1(t)$ suitable values for K_p and C_p have to be chosen. As in [1], p_1 reaches up to -2·M [Nm], and starts at p_1 = 0 Nm, where M is the mass of the whole body in kg. In our model, M = 73,18 kg, so if chosen C_p = 158,13 Nms/rad and K_p= 162,41 Nm/rad, a suitable solution similar than the one found out in [1] will be obtained. Our p_1 starts at 0 Nm and reaches -130 Nm, see Figure 3.5.

Once defined p_1, ψ could be looked for. Combining equations 2.1 to 2.5 a differential equation whose unknowns are only ψ, $\dot{\psi}$ and $\ddot{\psi}$ could be achieved. In the following section the trunk kinematics plots can be seen. See Figure 3.3. As initial guess $\psi_0 = -0.1\,rad$ and $\dot{\psi}_0 = -0.5\,rad/s$ have been chosen.

3.2.1 Trunk kinematics plots

Figure 3.3. ψ, $\dot{\psi}$ and $\ddot{\psi}$ plots.



Our kinematics results can be summarized in a plot where it can be seen whole single step. It seems anthropomorphic but one also assumes that it is improvable in order to be more natural, since the trunk motion is strange. It looks like uncomfortable and unstable. Maybe, this problem will be able to be fixed when the optimal problem is solved.

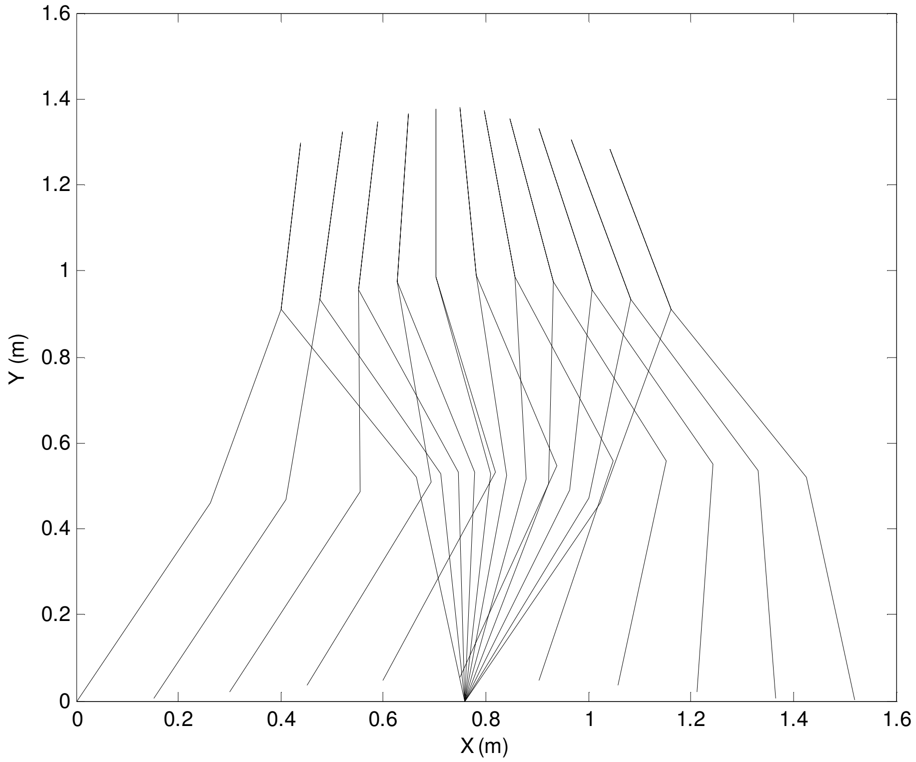


Figure 3.4. Draft of single step of this initial model.

3.3 Leg dynamics

In section 3.2 it has already been seen how to find p_1. Now, all other dynamics variables (p_2, u_1, u_2, q_1, q_2, R_{1x}, R_{1y}, R_{2x} and R_{2y})can be obtained.

First of all, some of these variables are 0 between t=0 and t=T, since one foot is in swing phase and it does not touch the floor. These variables are $p_2=0$ since the ankle has been represented as a footprint and it does not touch the floor, so its torque is null. For the same reason, reaction force on the floor of healthy leg (leg 2) is null too, so $R_{2x}=R_{2y}=0$.

Reaction force has two coordinates: x and y, because it does not work in a fixed point but it moves along the implicit foot. x_{R1} and y_{R1} are coordinates that define the position of the application point. It has been supposed $y_{R1}=0$. See Figure 3.5.

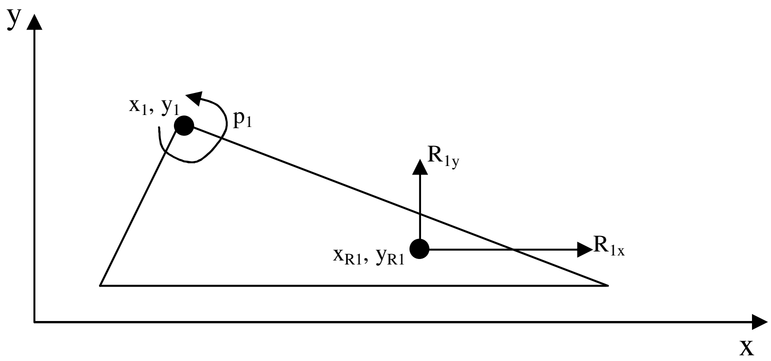
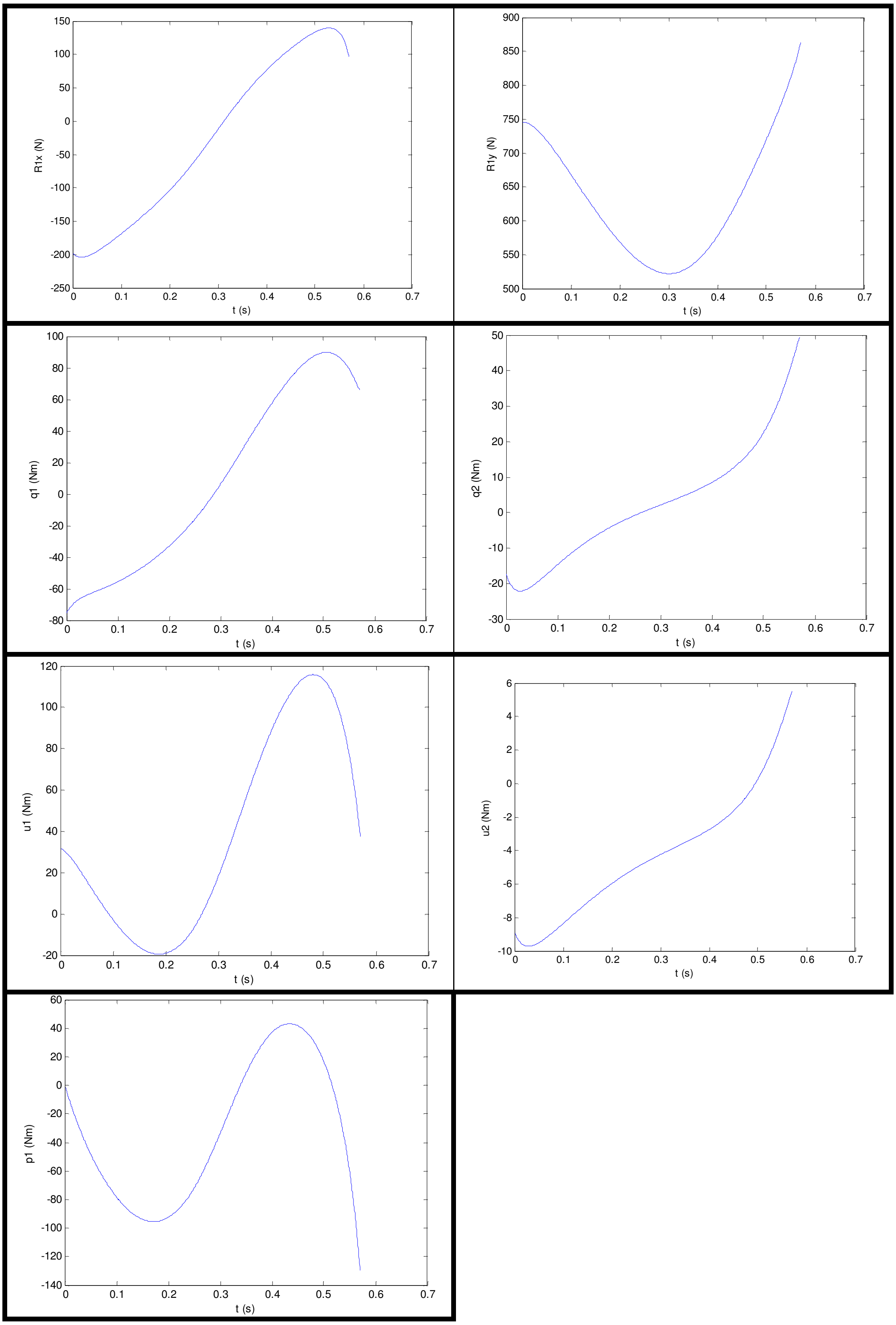


Figure 3.5. Implicit foot (in our model x_1=L, y_1=y_{R1}=0).

Substituting known variables in equation (2.2), R_{1x} is obtained. And the same with R_{1y} in equation (2.3). Combining equations (2.5) and (2.6) hip torques can be found out: q_1 and q_2; and knee torques u_1 and u_2. In the following section dynamic plots can be seen. See Figure 3.6.

3.3.1 Dynamic plots

Figure 3.6. Dynamic plots



4 Optimization problem

In order to achieve our main goal, which is finding out stiffness and damping properties of the below-knee prosthesis, our optimal problem has to be designed accurately.

As mentioned in 2.1 and 2.3, two cost functions need to be minimized at the same time (Problem A). This would lead to discuss which the best solution is.

There are 18 or more varying parameters to optimize our cost functions. 14 or more of these parameters belongs to leg kinematic parameters, 2 of them belongs to trunk kinematic parameters and 2 of them to control parameters. These are:

- Vector $C = \left[C_{x2}^T \; C_{y2}^T \; C_x^T \; C_y^T\right]$, as it has been defined in (3.20)-(3.23). These are 14 or more leg kinematic parameters, it depends on whether sinus and cosines parameters are taken into account or not.

- ψ_0 and $\dot{\psi}_0$, which are initial conditions of the differential equation to find out ψ motion (trunk kinematic parameters).

- K_p and C_p, stiffness and damping properties of the below-knee prosthesis. Finding them out is the main goal of our work.

It is required to proceed step by step. Firstly just few parameters will be varied optimizing just one cost function: energy of the healthy part of the body. Once optimized, these will be used as initial values. Then the cost function will be again optimized with these values and few more parameters. All parameters need to be optimized. Step by step and if it is necessary new restrictions are introduced.

Next diagram represents our followed steps.

Min Eh → Variables ≡ {Kp, Cp}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

1

ψ_0, $\dot{\psi}_0$, Kp, Cp as initial parameters

Min Eh → Variables ≡ {Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

$0m < y_2 < 0{,}1\text{m}$

$\dot{y}_2(T/2) = 0$

2

ψ_0, $\dot{\psi}_0$, Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2} as initial parameters

Min Eh → Variables ≡{Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

$0\,m < y_2 < 0{,}1\,\mathrm{m}$

$\dot{y}_2(T/2) = 0$

$|\psi(0) - \psi(T)| < 0{,}01\,rad$

3

ψ_0, $\dot{\psi}_0$, Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2} as initial parameters

Min Eh → Variables ≡{Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}, C_{0y}, b_{1y}}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

$0\,m < y_2 < 0{,}1\,\mathrm{m}$

$\dot{y}_2(T/2) = 0$

$|\psi(0) - \psi(T)| < 0{,}01\,rad$

$0{,}9\,m < y < 1\,\mathrm{m}$

$0{,}9 < C_{0y} < 0{,}95$

4

ψ_0, $\dot{\psi}_0$, Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}, C_{0y}, b_{1y} as initial parameters

Min Eh → Variables ≡{Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}, C_{0y}, b_{1y}, $\psi_0, \dot{\psi}_0$}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

$0\,m < y_2 < 0{,}1\,\mathrm{m}$

$\dot{y}_2(T/2) = 0$

$|\psi(0) - \psi(T)| < 0{,}01\,rad$

$0{,}9\,m < y < 1\,\mathrm{m}$

$0{,}9 < C_{0y} < 0{,}95$

5

ψ_0, $\dot{\psi}_0$, Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}, C_{0y}, b_{1y}, as initial parameters

Min Eh → Variables ≡{Kp, Cp, C_{3y2}, C_{4y2}, C_{5y2}, C_{0y}, b_{1y}, $\psi_0, \dot{\psi}_0$, C_{3x2}. C_{4x2}. C_{5x2}}

Restrictions:

$|\psi| < 0{,}35\,rad$

$Kp, Cp > 0$

$0\,m < y_2 < 0{,}1\,\mathrm{m}$

$\dot{y}_2(T/2) = 0$

$|\psi(0) - \psi(T)| < 0{,}01\,rad$

$0{,}9\,m < y < 1\,\mathrm{m}$

$0{,}9 < C_{0y} < 0{,}95$

6

Figure 4.1. Organization chart of our optimal problem.

Variances 2, 3 and 4 could vary slightly if sinus and cosines parameters are used instead of polynomial parameters.

In each variance several aspects need to be taken into account. First of all the solutions given must be anthropomorphic; if these are not, it is not worth continuing with that. In each variance the energy consumed by the prosthesis needs to be checked. Both cost functions have to be minimized at the same time. The solutions should have low value of energy of the healthy part of the body and at the same time a low value of the energy consumed by the prosthesis. Also which is our kinematics and dynamics in each variance should be taken into consideration compared with previous variances.

Finally, the final motions found, should be anthropomorphic. Therefore, these should be compared with experimental motions from literature.

4.1 Matlab Program

Matlab code has been written to solve the optimal problem (Problem A) which has been set in previous section.

It had to be feasible but also easy to use as well as manageable.

Our Matlab program can be splitted in several parts. On the first one, our fixed (length of body parts, time step...) and variable parameters are set. Here changing the value of every variable parameter is easy. On the next one, initial leg and trunk kinematics are found. Then, dynamics of the healthy part of the body as well as torque on the ankle of the prosthesis leg are obtained.

Next part is the main section; it is where the optimal problem is going to be solved. We call *fmincon* function and Matlab should optimize our parameters to minimize the cost function. So, our cost function as well as restrictions, constraints and boundary conditions need to be introduced. A single cost function (instead of two cost functions –Pareto front) is optimized in order to know how the value of the consumed energy changes while some parameters are varied.

Once optimized parameters are obtained, these results must be evaluated. New kinematics and dynamics of the system will be found. Then the plots that are needed should be seen.

In figure 4.2 all parts of the program are present.

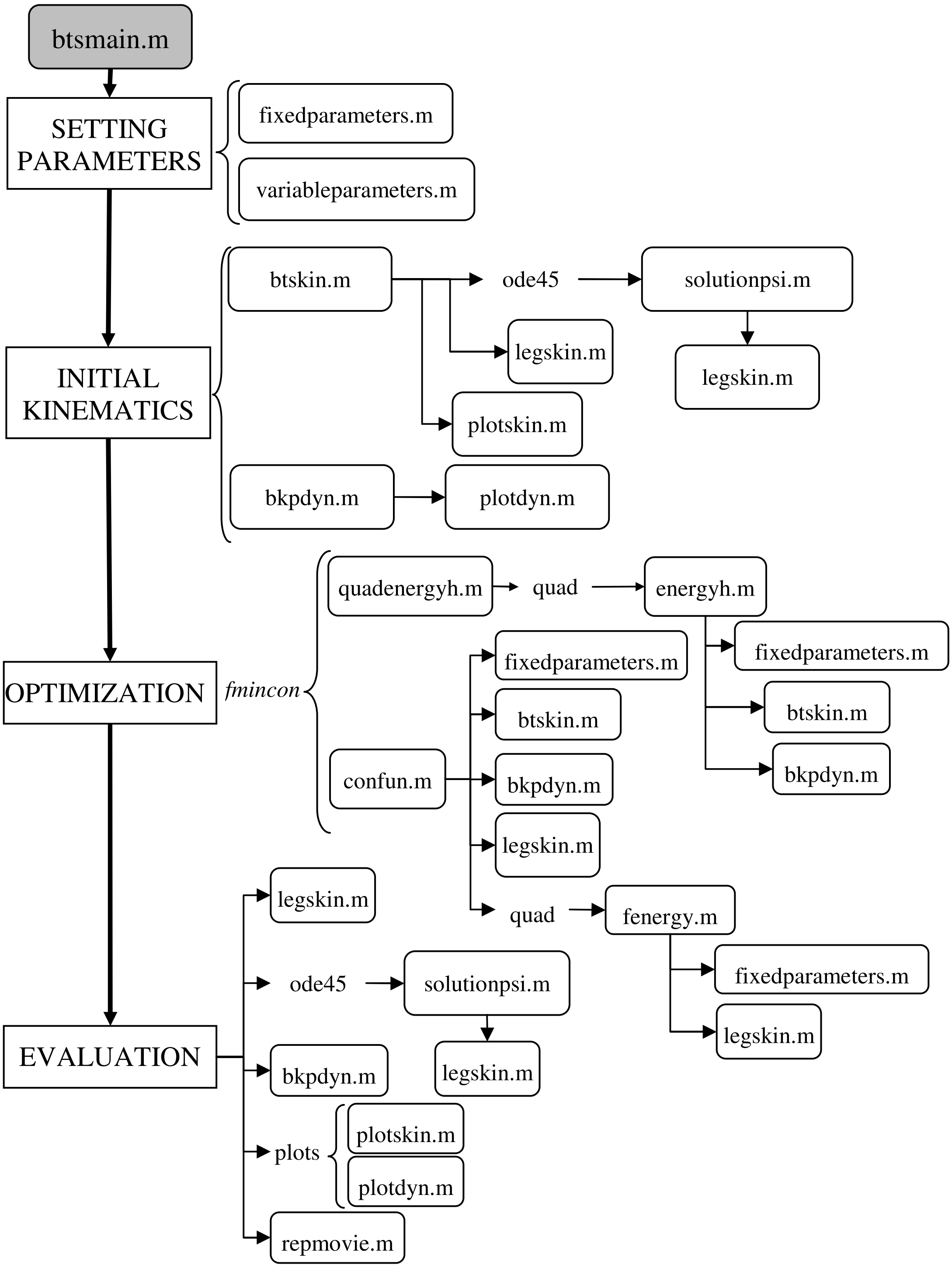


Figure 4.2. Organization chart of the Matlab program.

bkpdyn.m: it finds out all dynamics variables.

btskin.m: this program finds out all kinematics variables.

btsmain.m: it is the main program which calls each subprogram.

confun.m: there are restrictions, equalities and inequalities for parameters which have to be optimized. With *quadenergyh.m*, both are input data for *fmincon* function.

data.m: ask if user wants to see plots of kinematics, dynamics or others.

energyh.m: it is the definition of the power consumed by the healthy part of the body.

expdata.m: it is where the experimental data has to be introduced

fenergyp.m: it is the power consumed by the below-knee prosthesis. If the integral is applied, the energy consumed by the prosthesis during one single step will be obtained.

fixedparameters.m: all fixed parameters are defined: T, a_1, a_2, b_1, b_2, L, m, m_{a1}, m_{b1}, m_{f1}, m_{a2}, m_{b2}, m_{f2}, r, r_{a1}, r_{a2}, r_{b1}, r_{b2}, J_{a1}, J_{a2}, J_{b1}, J_{b2} and J.

legskin.m: this program finds out all leg kinematics, in *btskin.m* this subprogram is used to obtain these variables as vectors, with the same time steps like the solution of the differential equation.

plotdyn.m: it makes dynamics-time plots.

plotskin.m: it makes leg kinematics-time plots.

plotothers.m: it makes different kinds of plots. K_p versus C_p, N (number of variance) versus E_p, ψ versus $\dot{\psi}$, N (number of variance) versus E_h.

quadenergyh.m: it is the cost function of the healthy part of the body. It is an implementation of the integral of the power consumed by the healthy part of the body during the time of a single step.

repmovie.m: it makes a little movie with the gait motion with the aid of *getframe* command and then, a movie file with *movie2avi* command can be obtained.

solutionpsi.m: this program has ψ differential equation. Every variable has to be defined as a scalar in order to solve the differential equation, so *legskin.m* finds out x, y, x_2, y_2, α, β and their velocities and accelerations as scalars.

variableparameters.m: parameters which could be changed in optimization phase are defined in this file. These are stiffness and damping values (K_p and C_p), vector $C = \left[C_{x2}^T \; C_{y2}^T \; C_x^T \; C_y^T\right]$ and initial values of ψ: ψ_0 and $\dot{\psi}_0$.

variableparameters1.m ... variableparameters7.m: variable parameters that have been optimized during optimal problem in each variance.

5 Results

Once one has worked with MATLAB, results can be looked for. The main goal can't be forgotten: looking for the optimal stiffness and damping properties of a below-knee prosthesis, with which patient use as less energy as possible, as well as the prosthesis use the minimum energy as possible.

However, our model has to be anthropomorphic and consistent. So, kinematic and dynamic results of our model need to be compared with literature models. All external data that it has been used in this section comes from [8] (David A. Winter, 1991). This experimental data was tested from healthy people. It consists of 53 trials carried out in a laboratory. They did trials walking in three different velocities, between 80 to 130 steps/min: natural walkers with an average cadence of 105,3 steps/min with a stride length of 1,51 m; fast walkers with an average of cadence of 123,1 steps/min and a stride length of 1,64 m; slow walkers with an average cadence of 86,8 steps/min and a reduced stride length of 1,38 m.

These data consist of ankle, knee and hip angles, so α, β and ψ angles from our model have been able to obtain; ankle, knee and hip torques as well as ground reaction (horizontal and vertical).

To solve the optimal problem variances mentioned at Section 4 have been followed. Solutions already had a low value of our cost function (energy of the healthy part of the body).

In next table, these variances can be seen with some of main results, optimized values of stiffness and damping properties (K_p and C_p), minimum values of the healthy part of the body (E_h) and energy consumed by the below-knee prosthesis with that configuration.

	Variance 1	Variance 2	Variance 3	Variance 4	Variance 5	Variance 6
K_p (Nm/rad)	65,58	89,19	77,60	24,22	46,64	40,01
C_p (Nms/rad)	34,48	66,52	65,28	0,0031	16,22	23,21
E_h (J/m)	66,44	16,74	47,20	13,43	14,07	2,62
E_p (J/m)	43,96	36,62	24,40	31,35	40,19	22,46

Table 5.1. Optimal stiffness and damping values (K_p and C_p), and energy values (E_h and E_p) for each optimal configuration.

Then, looking into our results one can check if they are anthropomorphic enough. Kinematics results are shown in next section.

5.1 Kinematic results

Coordinates *x* and *y* (motion of the hip) seems to be anthropomorphic enough, as one can see in Figure 5.1.

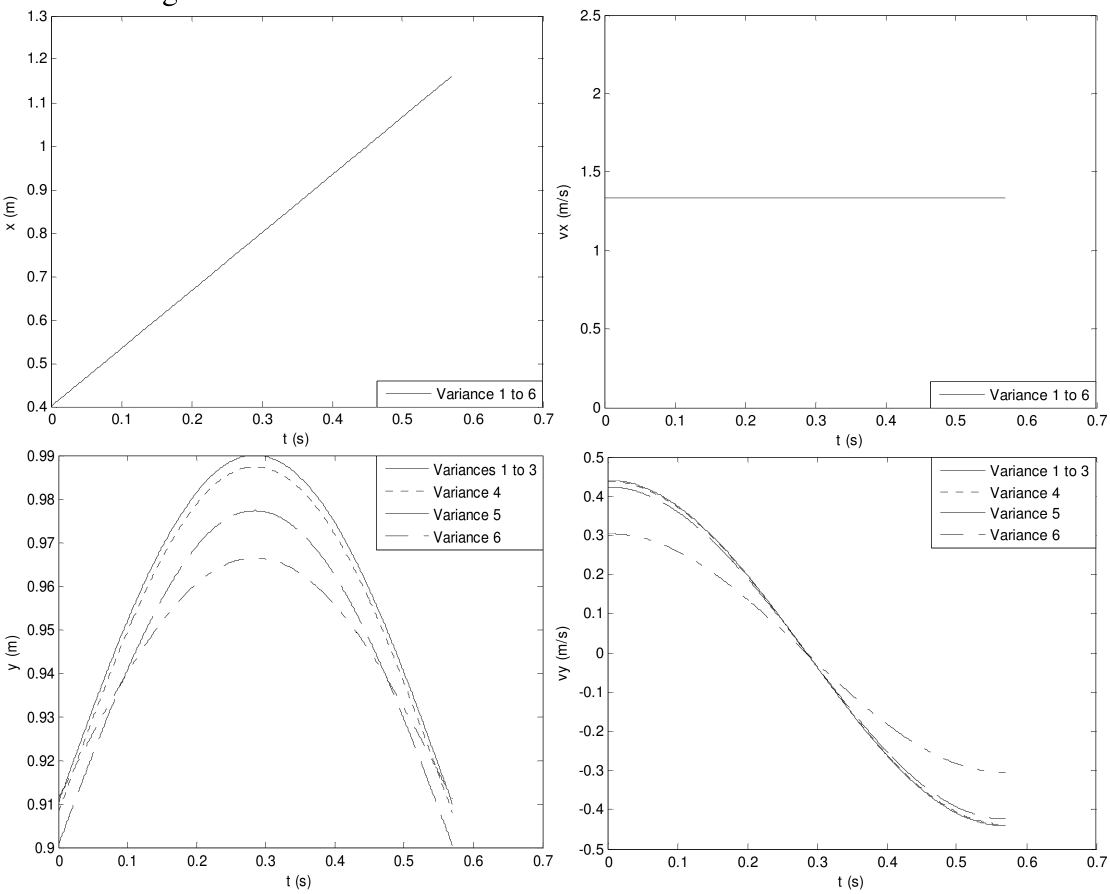


Figure 5.1. Motion of the hip. $x, \dot{x}, y$ *and* $\dot{y}$.

Horizontal motion of the hip has a constant velocity, it starts at x=0,4 m and finishes when it has gone through a length equal to one step (L). Vertical position, coordinate *y*, decreases along optimal steps. It could be guessed that if *y* path is shorter, it spends less global energy.

Following, plots of the healthy ankle foot path can be seen, during its swing phase. See figure 5.2 (a) and (b).

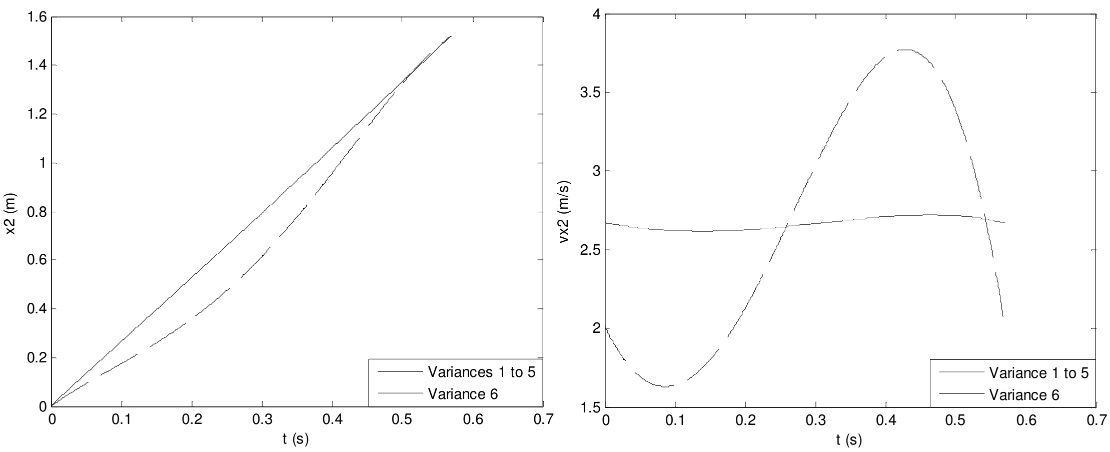


Figure 5.2.(a)

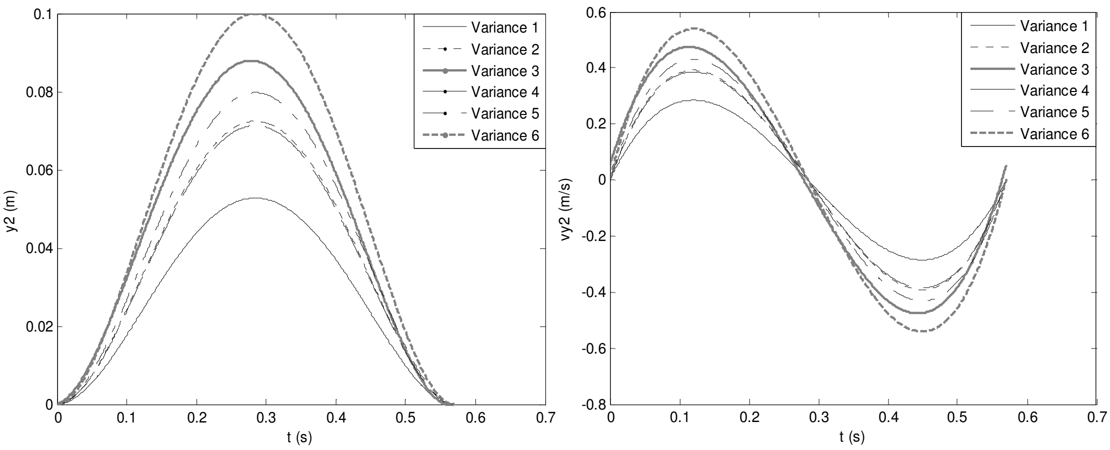


Figure 5.2 (b). Motion of the healthy ankle in its swing phase. $x_2, \dot{x}_2, y_2$ *and* $\dot{y}_2$.

Its horizontal coordinate haven't been optimized until variance 6, where can be seen that there is a change in its velocity. Its height increase during optimal process, therefore vertical velocity increase too.

Experimental data for angles from literature [8] is available, so these will be compared with our results. Note that experimental results that are available belongs to double step, but our results are from only one step; when the prosthesis leg is on its stance phase and the healthy leg is on its swing phase. With the aid of [1], one can conclude that our single step took place between 10% and 50% of the double step (support phase on both heel and metatarsal joint) for prosthesis leg and from 60% to 100% (swing phase of the foot over the surface) for healthy leg.

Before proceeding to compare our results with literature results, one assumption has to be explained which is supposed in order to get data. In [8], the mechanical model used is like in Figure 5.3. As it can be seen, it is different than our model (Figure 2.1), since angle of the foot θ_{ft} angle between horizontal and a line along the bottom of the foot measured from the distal end (5th metatarsal phalangeal joint) have not been considered. In order to get β angle, foot angle has to be approximate.

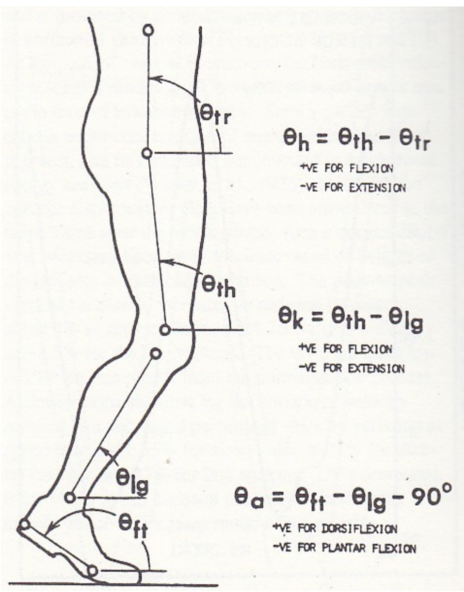


Figure 5.3. Mechanical model of [8]. θ_a *is the ankle angle.*

To approximate this angle, the time plot of the double step (Figure 5.4) has been taken into account. Looking the position of the heel and metatarsal during the step (see Figure 5.5), some angles during the double step have been decided, see Table 5.2. Then, interval times between these values have been approximated lineally.

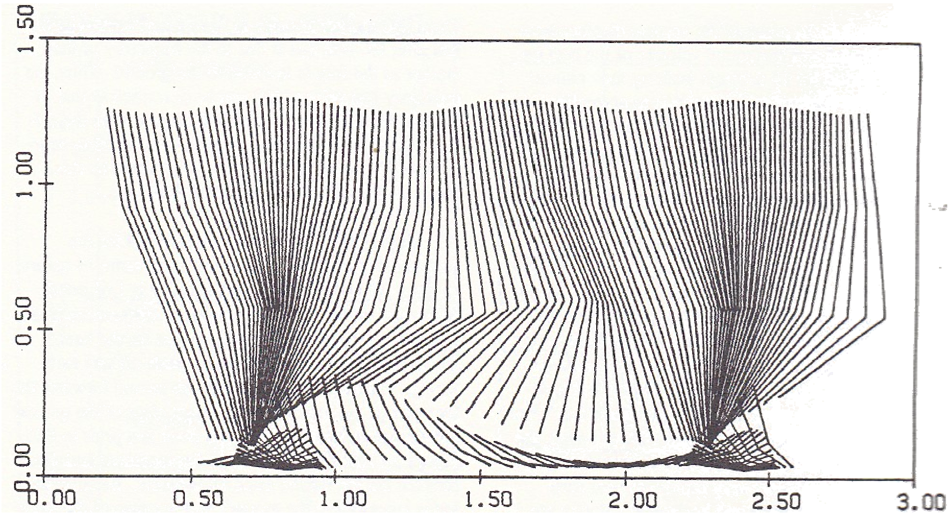


Figure 5.4. Time plot of the double step on one leg, Data from [8].

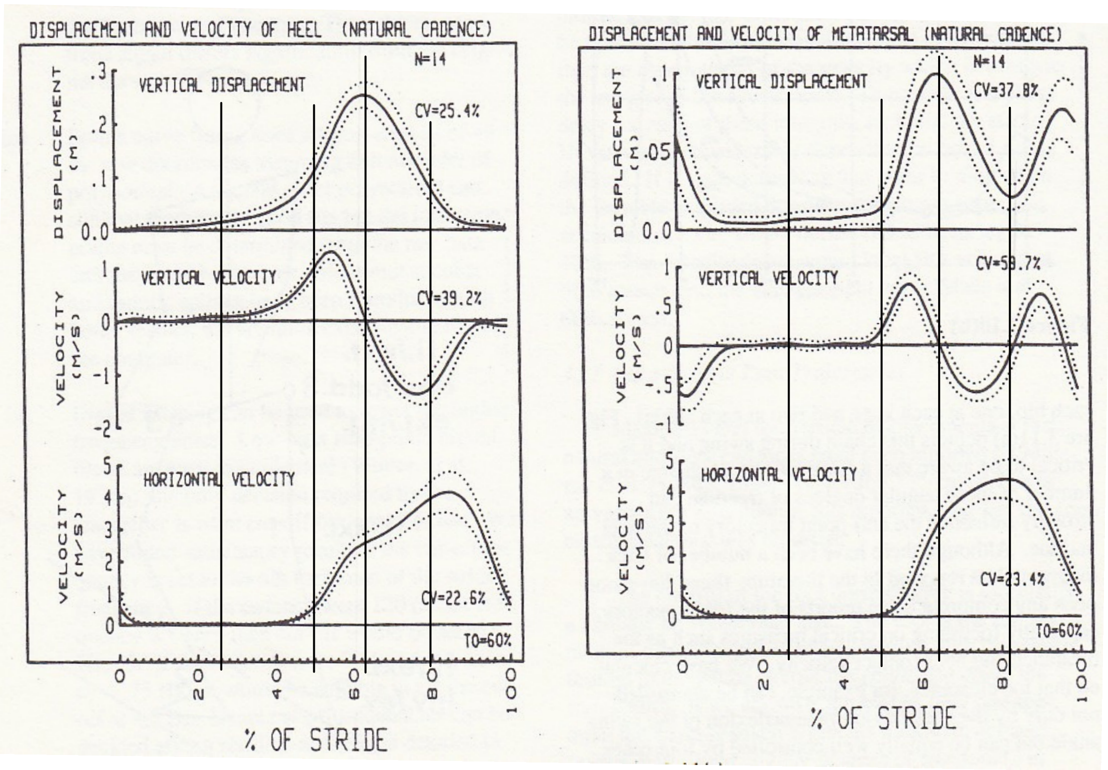


Figure 5.5. Time plots of displacements of heel and metatarsal.

Time (%)	0	26	50	64	80	100
Angle (°)	185	180	120	145	180	185

Table 5.2. Approximated values of foot angle. Standard Deviation of these data null has been assumed.

According Figure 5.3, the following relations have been assumed:

$$\beta = \theta_{ft} - \theta_a - 180° \tag{5.1}$$

$$\alpha = \theta_{ft} - \theta_a + \theta_k - 180° \tag{5.2}$$

$$\psi = \theta_{ft} - \theta_a + \theta_k - \theta_h - 180° \tag{5.3}$$

So, once α, β and ψ experimental data are found, one can compare our results with them. β angle on the leg which has the prosthesis is compared, see Figure 5.5.

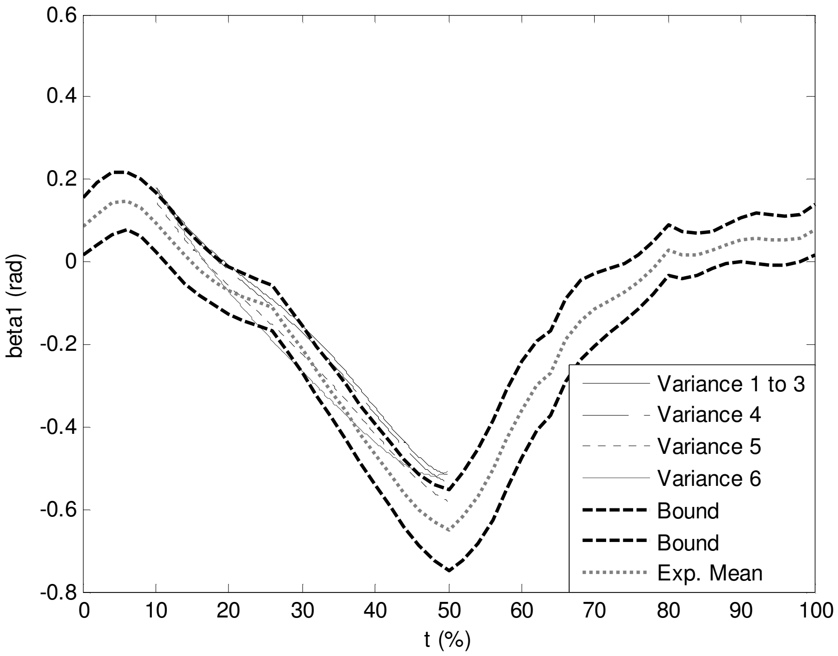


Figure 5.5. β angle on the prosthetic leg with pipe of the experimental results.

In the plot, "bound" means the value of magnitude calculated plus or minus one standard deviation. Almost all of our results are inside the pipe of the experimental results.

In Figure 5.6, results of $\dot{\beta}$ can be seen. There are no special comments in this results, just note that step 6 changes a little the shape of the plot; it starts with a lower value than the other variances and it finishes with higher value. Experimental results from this data are not available, so they can be compared.

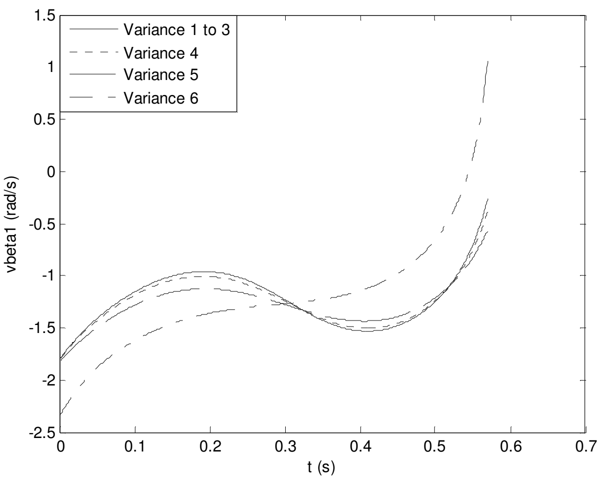


Figure 5.6. Plot of $\dot{\beta}$ on the prosthetic leg.

Now, α angle on the leg which has the prosthesis is going to be compared, see Figure 5.7.

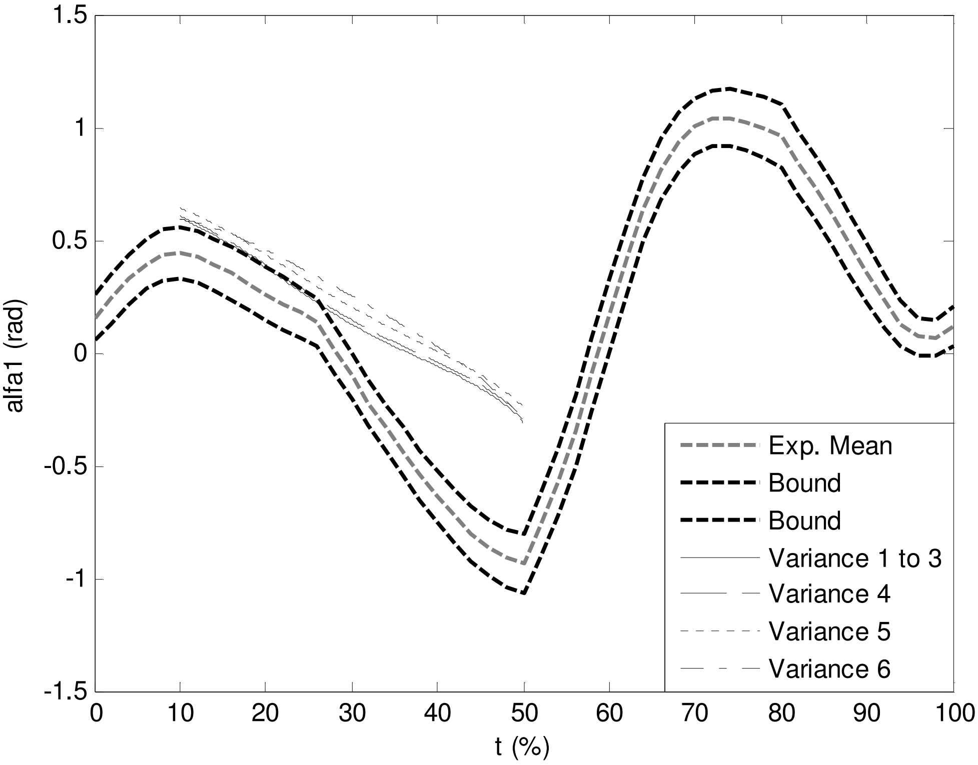


Figure 5.7. α angle on the prosthetic leg with pipe of the experimental results.

Our results are close to the pipe that determines the mean of experimental results plus and minus one standard deviation ($\overline{\alpha} \pm st.dev$). One can conclude that at the end of the single step thigh of the prosthetic leg is not as flexed as in experimental results. It can be also seen that in first variance the flexion of the thigh was higher than latest variances.

In Figure 5.8. One can see the plot of velocity of alpha angle, $\dot{\alpha}$.

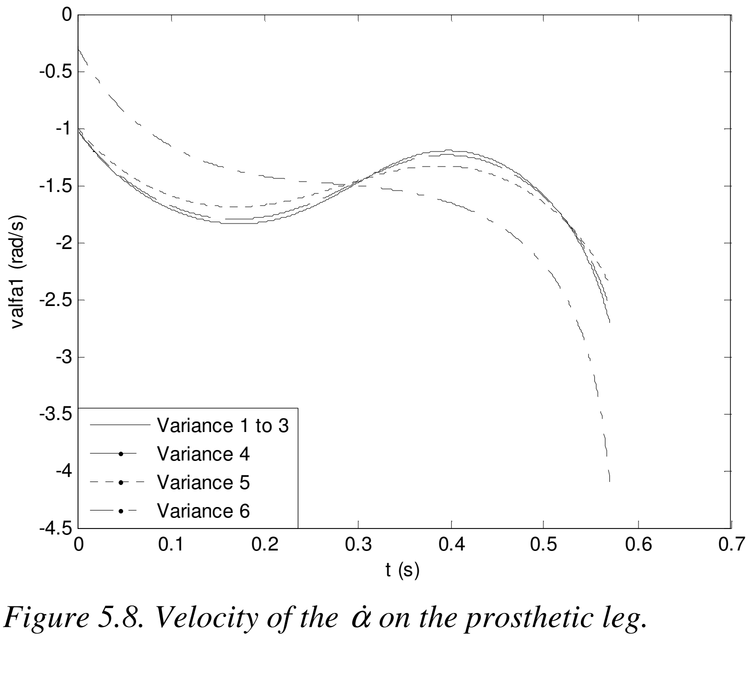


Figure 5.8. Velocity of the $\dot{\alpha}$ on the prosthetic leg.

Like in $\dot{\beta}$ (Figure 5.6) on the same leg, there are no special comments in this results, just note that step 6 changes a little the shape of the plot.

Regarding swing phase on the healthy leg, most of our results are under the pipe of the experimental results. See β angle on this leg, Figure 5.9. On variance 6, flexion of the healthy shank is larger.

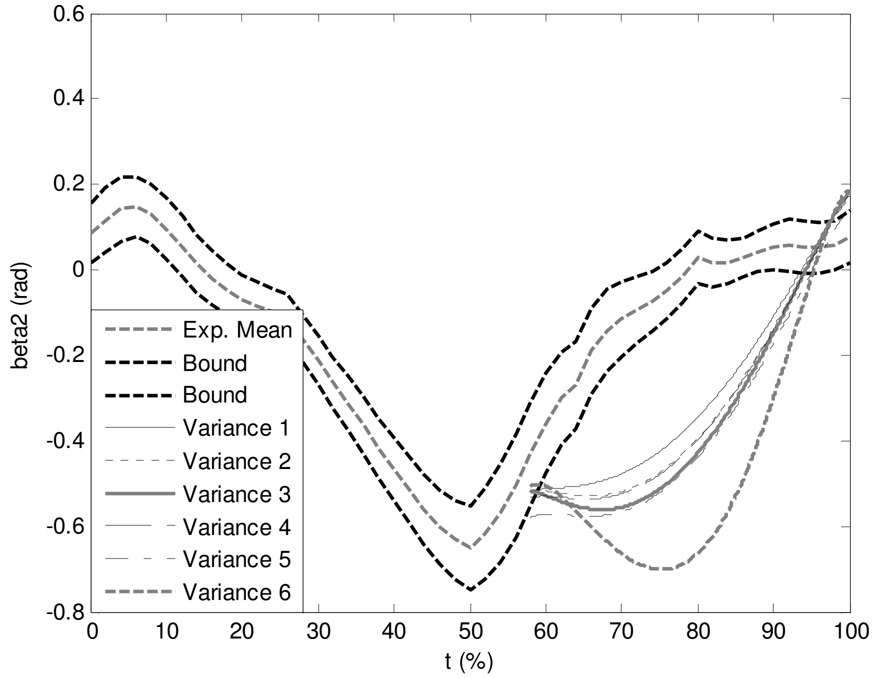


Figure 5.9. β angle on the healthy leg.

Experimental results from this data are not available, so our results can't be compared with them. However one can see in order that shank of the healthy leg gets more flexion, its velocity is also higher. See Figure 5.10.

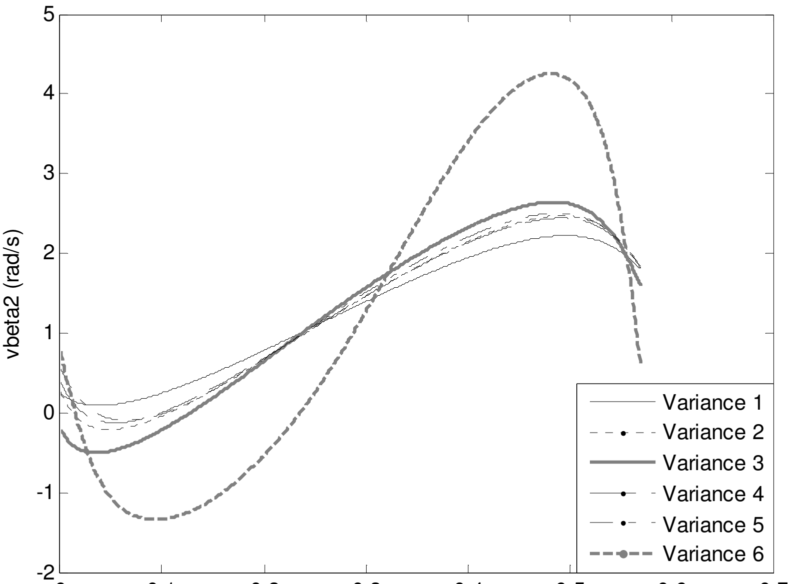


Figure 5.10. Plot of $\dot{\beta}$ on the healthy leg.

If α angle results are compared with literature, one also can see that at the beginning of the swing phase it starts at almost the same angle, but then, experimental results increase more than our results. See Figure 5.11.

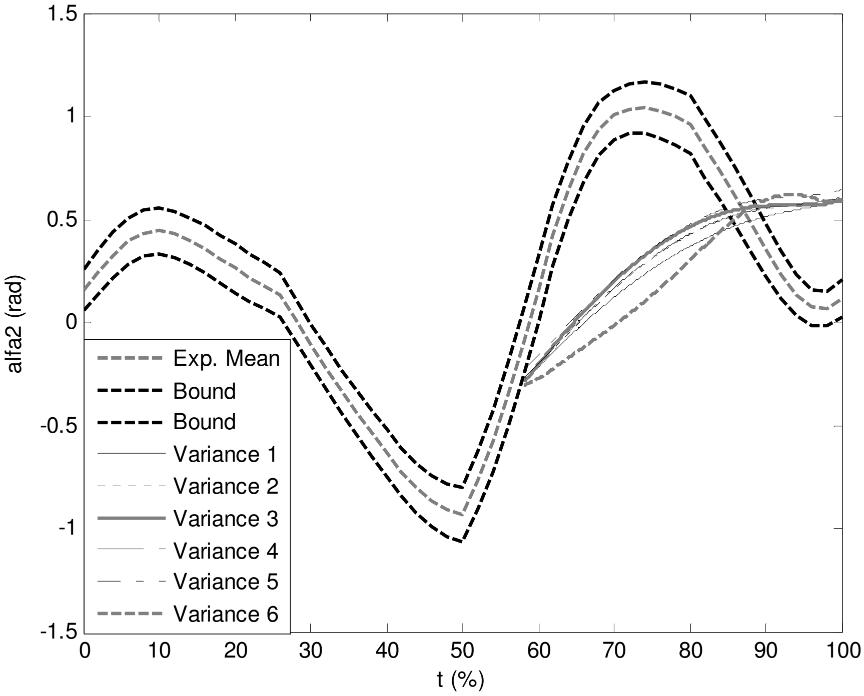


Figure 5.11. Plot of α angle on the healthy leg.

Regarding $\dot{\alpha}$ angular velocity on the healthy leg, like angular velocities on the prosthetic leg, no special comments can be stated, because no experimental data is available as a reference. However one can see that variance 6 differs from other results.

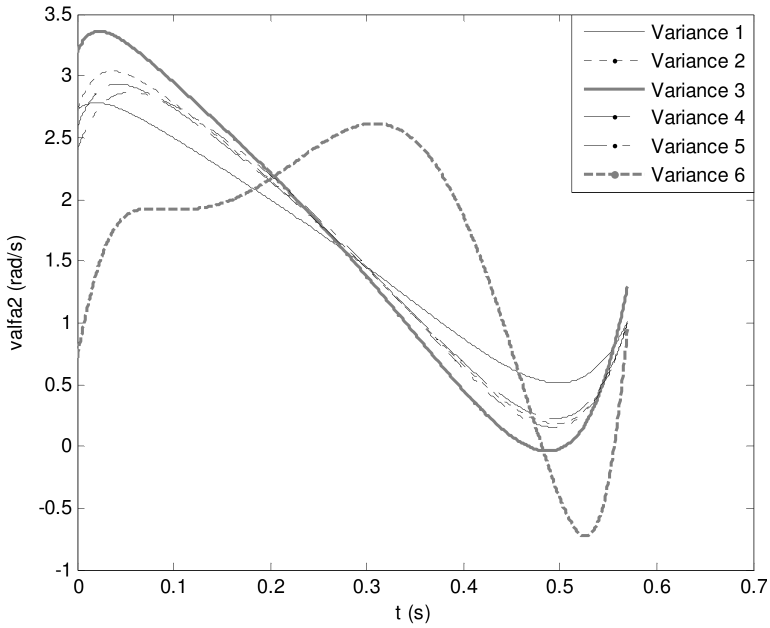


Figure 5.12. Plot of $\dot{\alpha}$ on the healthy leg.

Last angle that is left to be analyzed is ψ. In Figure 5.13., our results can be seen with experimental data. One can state that our results at the beginning of the single step are close to be anthropomorphic. Experimental results flex the trunk more than ours.

As it is known, if the trunk is in a vertical position, the gait will be more safe and stable. In [8], one also can learn that in human gait as a first approximation the trunk could be considered almost vertical (actually it is biased slightly forward of vertical). In our results, trunk is close to be vertical, especially variances 2, 3 and 4. But experimental results show that in anthropomorphic gait there is a movement more forward and backward. It should be also taken into account that foot angle has been approximated; probably real anthropomorphic pipe has a smoother shape.

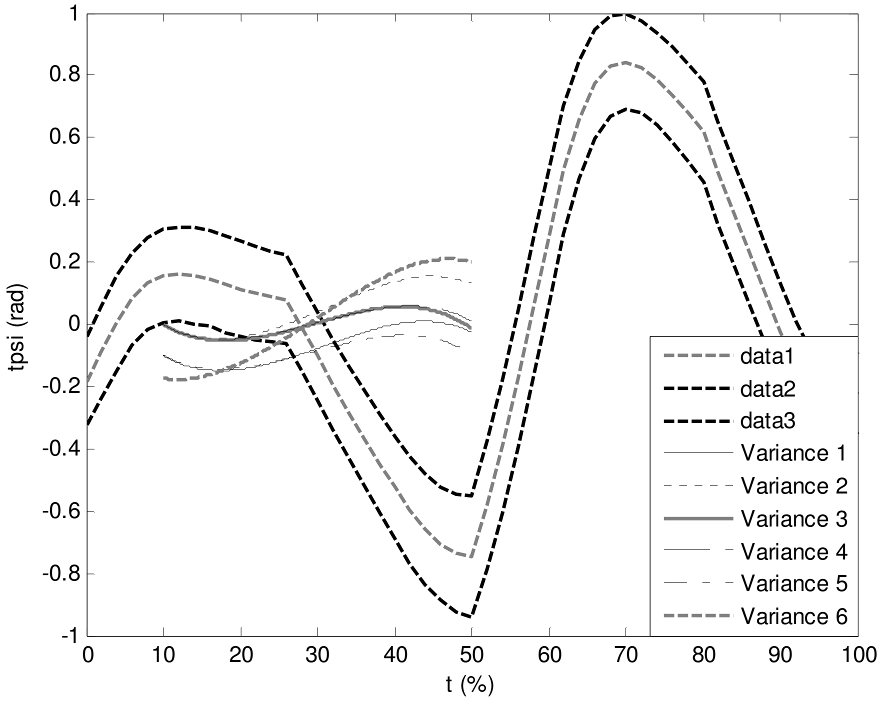


Figure 5.13. Trunk angle (ψ) plot, with pipe of the experimental results.

Regarding $\dot{\psi}$, from the first until fifth variance, results start nearly at the same velocity. Variance 6 starts with a lower velocity (in absolute value), but then, during the step, it increases its velocity more than other variances. See figure 5.14.

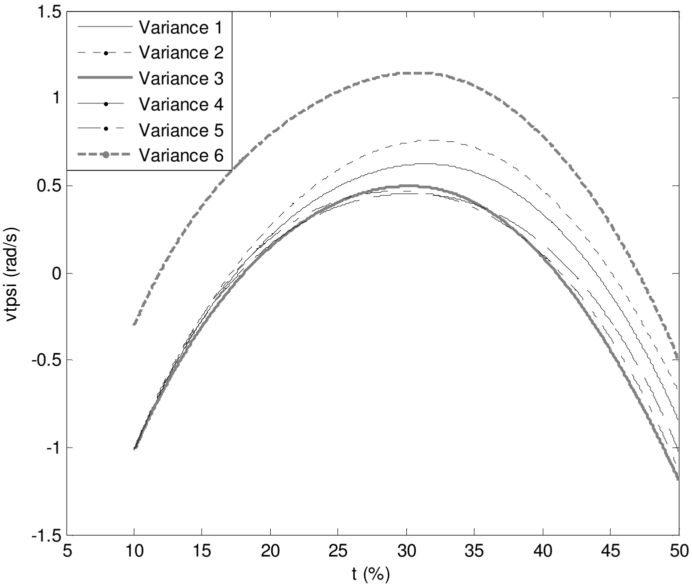


Figure 5.14. Velocity of the trunk angle, $\dot{\psi}$.

In figure 5.15 an interesting plot can be seen, it consist of the relation between ψ and $\dot{\psi}$. From the 3rd variance a new restriction is introduced: $|\psi(0)-\psi(T)| < 0{,}01\ rad$ so that, trunk angle at the beginning and at the end of our step were almost the same. One can bear out this fact in this plot. Also, as mentioned few lines above, trunk angle would be better as much vertical is (or slightly forward of vertical). So, one can consider variances 3, 4 or 5 more anthropomorphic, in addition, they have almost the same ψ and $\dot{\psi}$ on the bounds.

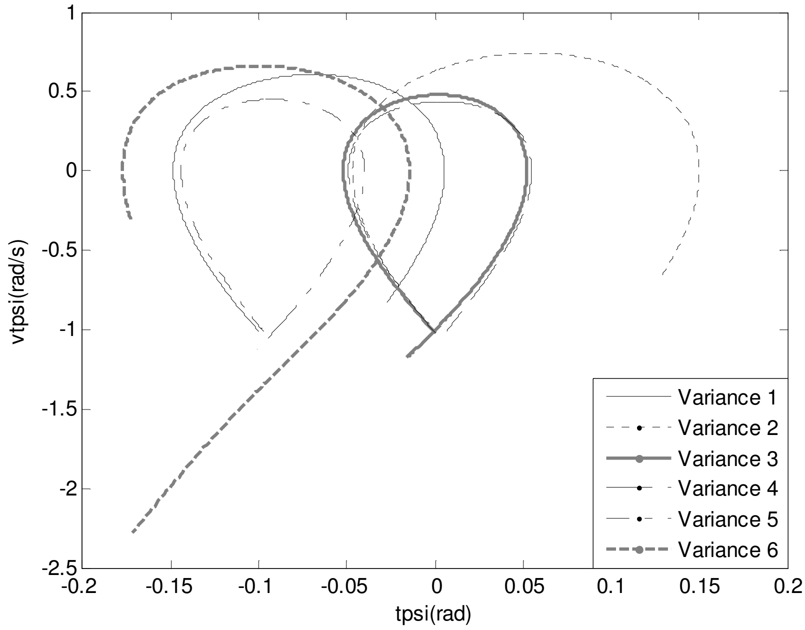
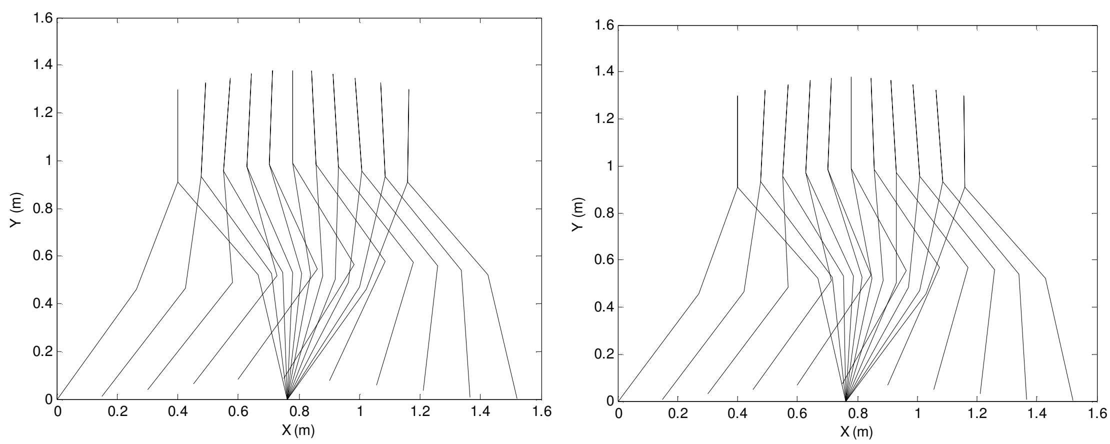


Figure 5.15. Relation ψ versus $\dot{\psi}$.

Kinematics results can be seen in one plot where the single step is represented. Plots from variances 3, 4 and 5 are available, which are the most anthropomorphic. Figure 5.16.



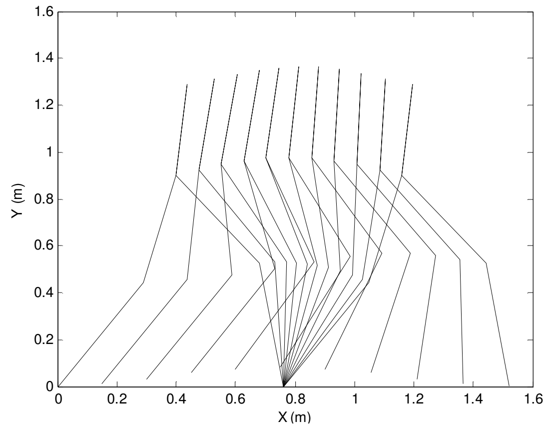


Figure 5.16. Variance 3 (above left), Variance 4 (above right) and Variance 5 (down at the middle).

One can see that variances 3 and 4 are almost the same, like it has been seen in Figure 5.15. In variance 5 the trunk is slightly forward to vertical. As a first approximation it seems more anthropomorphic and natural than variances 3 and 4.

To finish comparing and discussing kinematics plots, x_{R1} coordinate is studied, horizontal position of the point where ground reaction acts. In Figure 5.17 its temporal progress can be seen. In Figure 3.5 "the implicit foot" have been defined and how to calculate this coordinate x_{R1} is known, y_{R1} = 0 m have also been defined, therefore it is easy to find out x_{R1}. Looking this plot and Figure 3.5 one can guess that p_1, the torque of the prosthesis leg, will be negative, since ground reaction force is applied in front of the ankle (x_{R1}>0.76 m). These results are anthropomorphic, as this point of application would be on a real foot, since the maximum value of x_{R1} is in variance 3 (x_{R1}=0.8814) and it would be at a reasonable distance from ankle joint: 0,8814-0,76=0,1214 m. Distance between ankle joint and toes is higher than 0,1214m, so our results could be considered anthropomorphic.

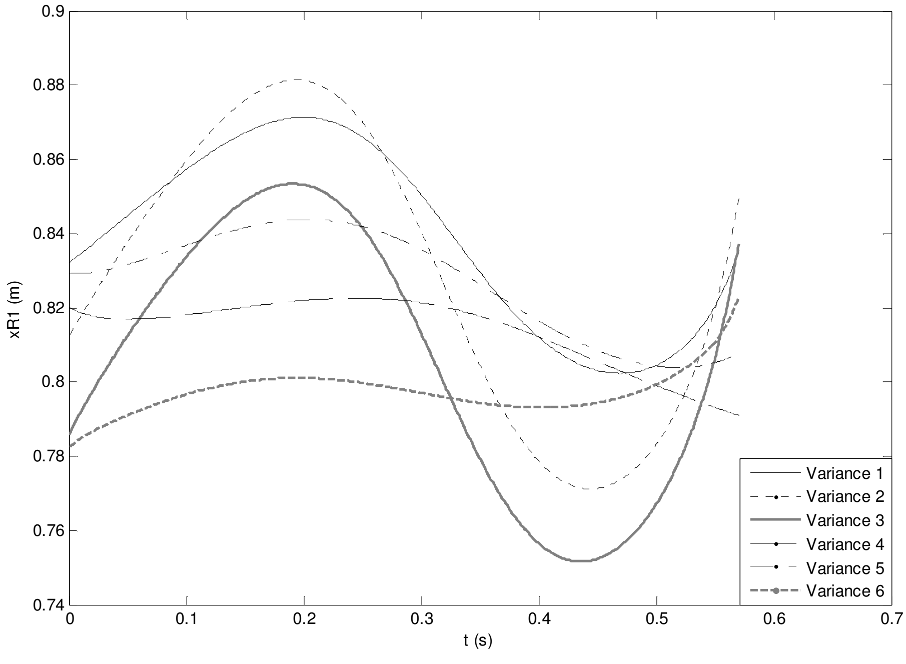


Figure 5.17. Horizontal position of the point application of the ground reaction.

5.2 Dynamic results

To start with dynamics, results of the torque on the ankle of the prosthetic leg are compared with the ankle torque of a healthy body from experimental results. It is assumed that it won't be the same, but they should be similar in order that the patient can walk in an anthropomorphic way. In Figure 5.18 one can see that the shape of p_1 is completely different from experimental results, however, values from the beginning of the step are inside of "the experimental pipe". One also can state that depending on variances, the shape of our p_1 differs a little.

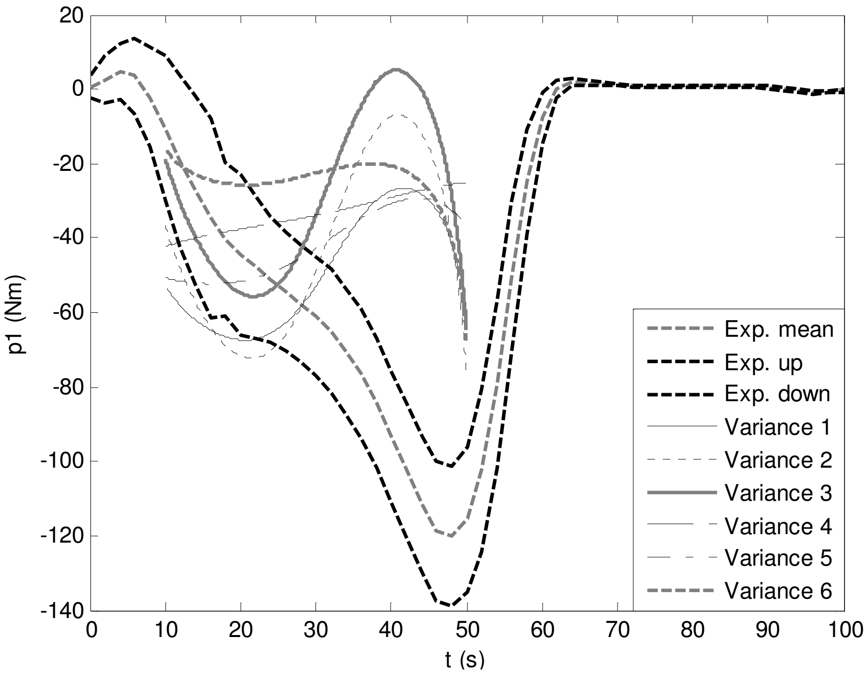


Figure 5.18. Torque on the ankle joint of the prosthetic leg, p_1.

As foot has been considered as a footprint, torque ankle of the healthy leg during its swing phase will be null.

As regards hip torque on the prosthetic leg (q_1), in Figure 5.19 one can see that the most results are inside or close to the pipe of the experimental results. Variance 6 has a significant difference, because at the end of the step, its hip torque goes up fast. This fact can't be anthropomorphic. It is a result of the optimal problem; maybe if new restrictions are added in this variance, the problem will be able to be solved.

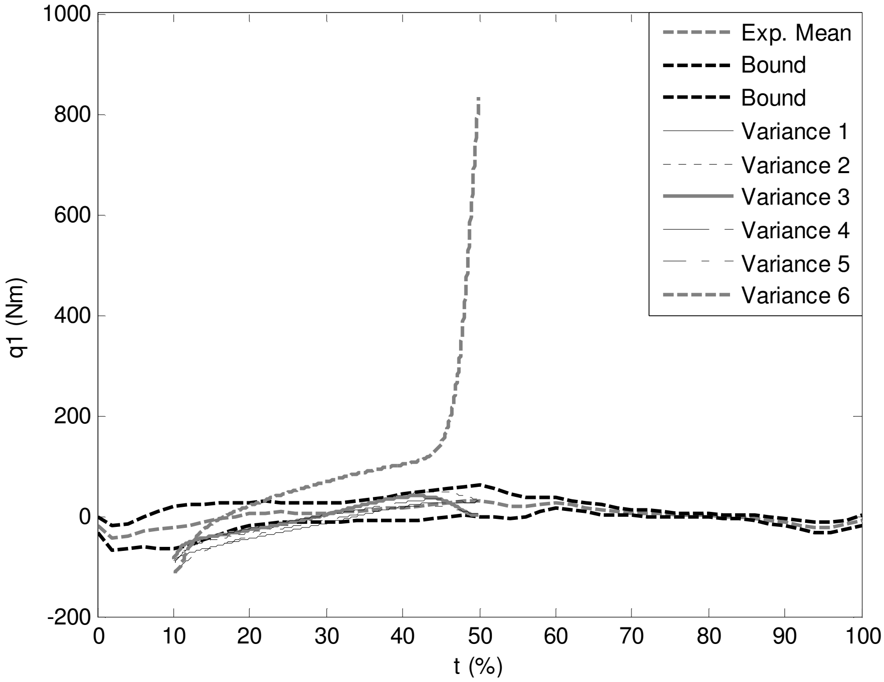


Figure 5.19. Hip torque on the prosthetic leg, q_1.

Knee torque on the leg that has the prosthesis (u_1) has some parts that can be considered anthropomorphic, but at the end of the step, they get out of the anthropomorphic pipe. Like in hip torque on the same leg, variance 6 goes up at the end of the step unreasonably. See Figure 5.20.

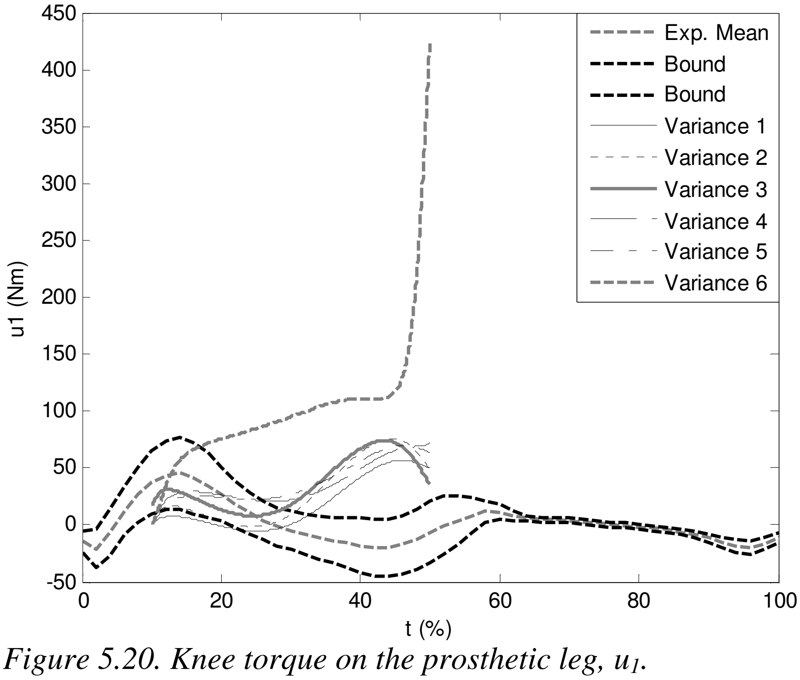


Figure 5.20. Knee torque on the prosthetic leg, u_1.

Results about hip torque on the healthy leg (q_2) don't have strong comments to be stated. Just like q_1 and u_1, variance 6 differs from other variances. See Figure 5.21.

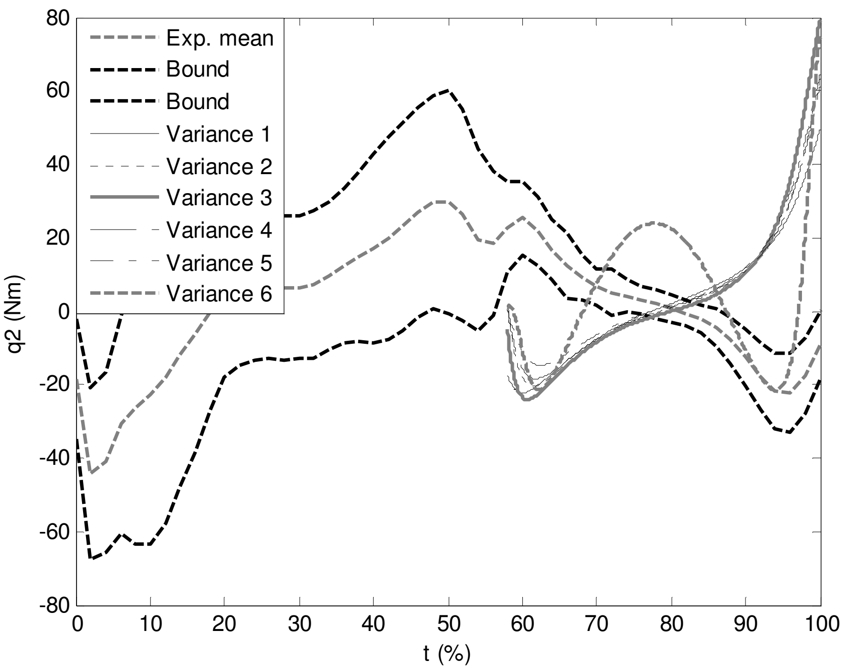


Figure 5.21. Hip torque on the healthy leg, q_2.

The same for the knee torque on the healthy leg. See Figure 5.22.

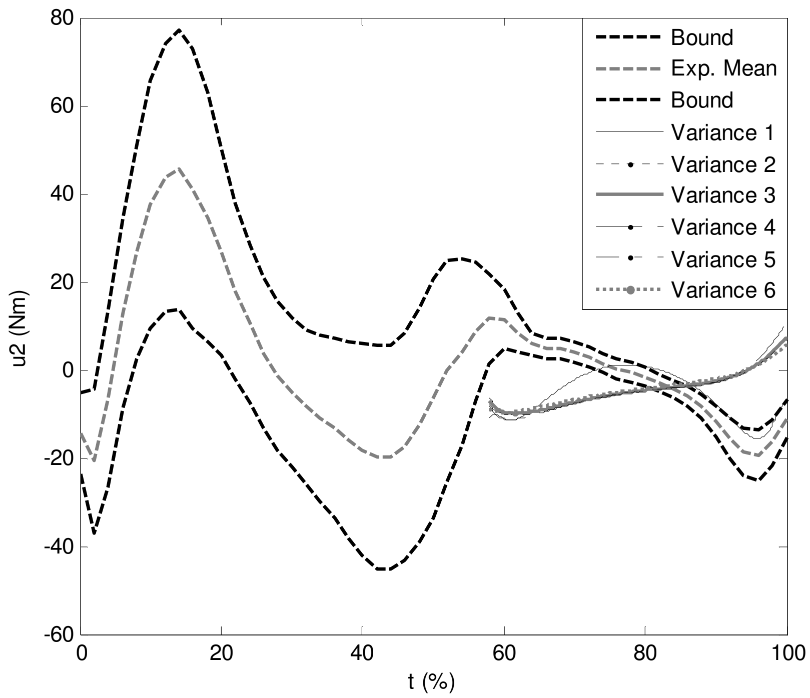


Figure 5.22. Knee torque on the healthy leg, u_2.

Ground reaction force is close to pipe of the experimental results. As mentioned, variance 6 is not anthropomorphic, more constraints should be put. Nevertheless, other results seem to be anthropomorphic. If one look on horizontal component (R_{1x}) on Figure 5.23, one can see that firstly decelerate the bipedal system (when $R_{1x}<0$) and then accelerate it (when $R_{1x}>0$). Regarding vertical component (R_{1y}), at the beginning and at the end of our studied period it has the highest values. See Figure 5.24.

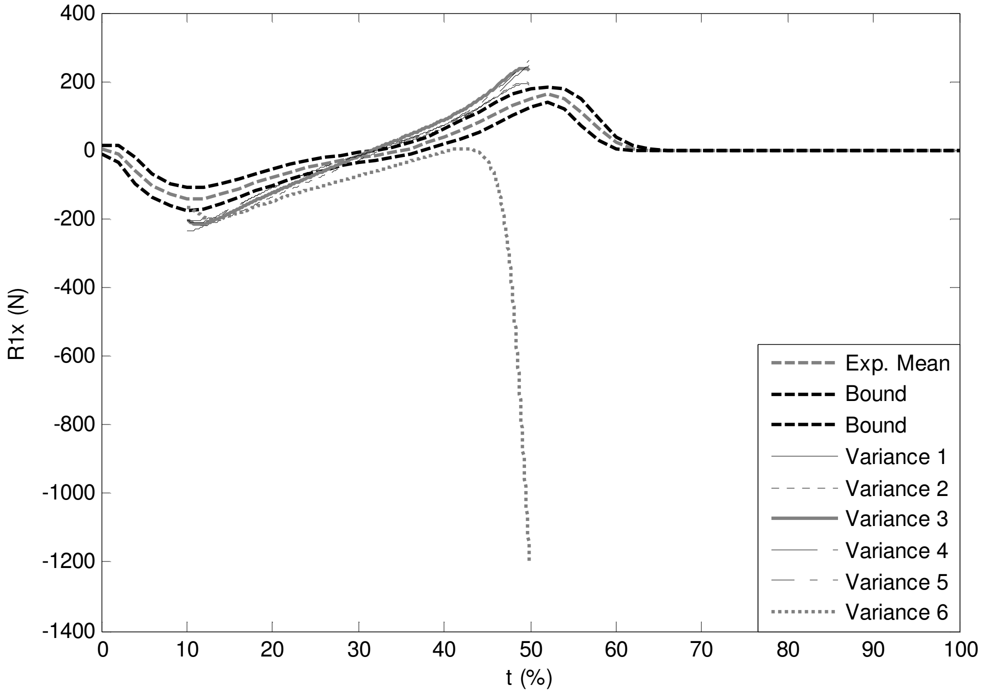


Figure 5.23. Horizontal component of the ground reaction force. R_{1x}.

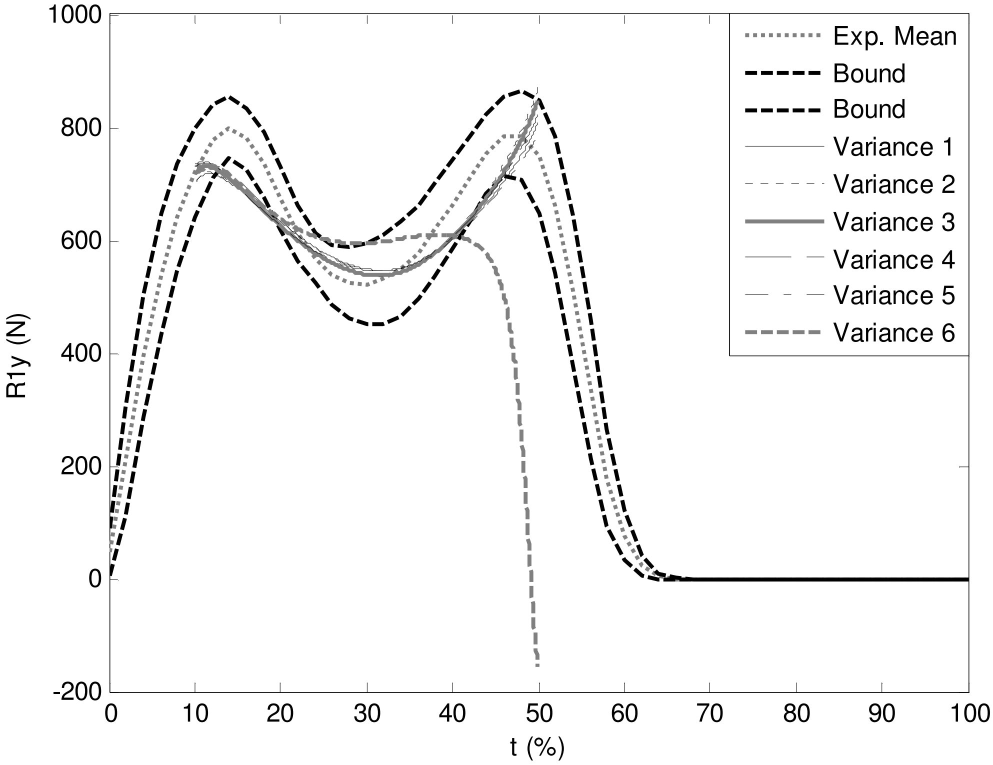


Figure 5.24. Vertical component of the ground reaction force. R_{1y}.

Once kinematics and dynamics of all variances have been discussed, it could be stated that, taking into account all comments that have been said said, as a first approximation all of them are anthropomorphic enough except variance 6.

5.3 Other results

Now, focus on results that will help to try to solve Problem A. Hence, if one takes a look on Figure 5.25, one can see that energy of the healthy part of the body (E_h) decreases when more parameters to optimize are introduced. From variances 2 to 3 no, because no parameters were introduced, just new restrictions.

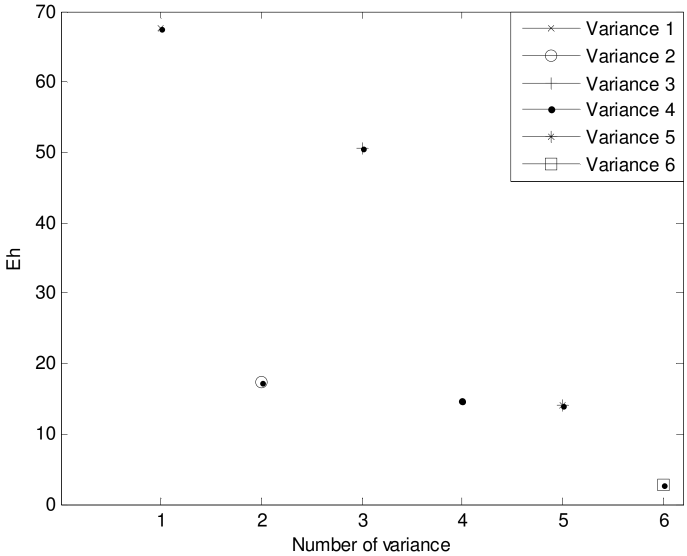


Figure 5.25. Evolution of energy of the healthy part of the body (E_h) during variances in optimal problem.

The evolution of E_h could have been guessed, but one doesn't know anything about the evolution of E_p, the energy consumed by the below-knee prosthesis. It could be supposed that if one spends less healthy energy, energy consumed by the prosthesis would be higher. If Figures 5.25 and 5.26 are compared, from 2 to 5 variance one could state that it is true, but also depend on which is the configuration of all parameters to optimize, since variances 1-2 and 5-6 our statement it is not true.

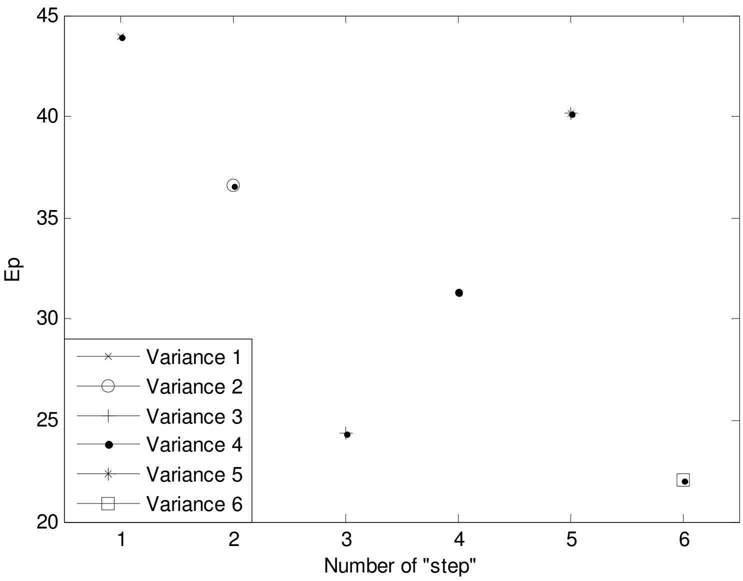


Figure 5.26. Evolution of the energy consumed by the below-knee prosthesis (E_p) during variances in optimal problem.

In Figure 5.27 the relation E_p versus E_h can be seen and one can conclude that range of E_h is from 2,62 to 66,44 J/m and range of E_p is from 22,46 to 43,96 J/m.

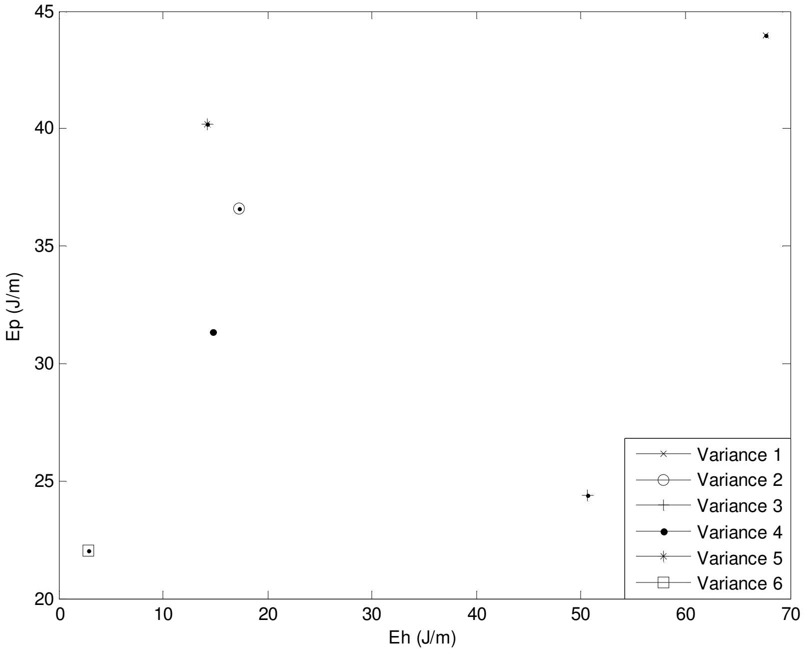


Figure 5.27. Relation between E_h and E_p.

Finally, the most important relation can be plotted, damping (C_p) versus stiffness (K_p) properties of results that have been obtained. Our results shows that for obtained optimized kinematics of the motion stiffness values are between 24,22 Nm/rad and 89,19 Nm/rad and damping results are between 0,0031 Nms/rad and 43,96 Nms/rad. See Figure 5.28.

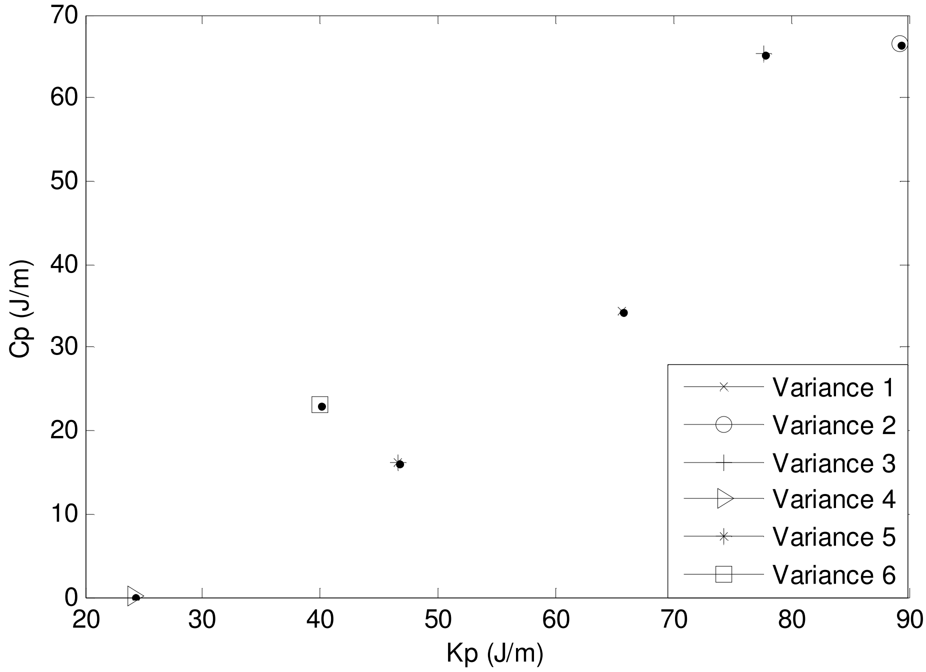


Figure 5.28. Relation between damping and stiffness properties (Cp versus Kp).

6 Conclusions

This master thesis has objectives to contribute in designing a new type of actively controlled below-knee prosthesis which spends as low energy as possible. The knowledge about suitable stiffness and damping properties of the below-knee prosthesis is needed.

In the report an optimization problem has been formulated which includes estimation of optimized stiffness and damping parameters of the below-knee prosthesis as well as determination of kinematics of human body motion minimizing energy consumption of the healthy part of the body (E_h). Numerical algorithm has been developed and MATLAB (*fmincon* function) has been used to solve the complex optimization problem.

Apart from one variance in our optimization problem, satisfactory results have been achieved, since obtained results are almost anthropomorphic. At the same time low values of E_h have been obtained, while E_p value has been taken into account in order that it doesn't increase too much. If one looks on Table 5.1, one could state that a suitable configuration would be among variances 3 to 5, because they have a low value of E_h while they keep a low value of E_p; especially 3 and 4 would be the motions more stable, with a vertical trunk position.

Finally, it has been managed to find out a reasonable range of values for stiffness (K_p) and damping (C_p) properties. This fact means that possible relations between torque applied on the ankle joint of the below-knee prosthesis leg with its angle and angular velocity have been obtained.

However, our model differ from experimental data [8] that have been used, since the angle of foot has been approximated in order to get experimental data that was needed. So, comparisons that it has been done aren't extremely accurate, because real motion of the angle foot is not available. This is a typical problem when a mechanical system is simulated.

6.1 Future work

Our model is two-dimensional, that is the mechanical system motion is considered only in a sagittal plane. A possible next step of the work would be simulation of human motion and estimation of stiffness and damping properties of the below-knee prosthesis within the frame of 3D model.

One also could spend more time refining MATLAB code, looking for different restrictions or changing initial conditions in order to find new anthropomorphic motion which consumes a lower energy value and thus finding out new combinations of K_p and C_p possible values.

To end our work, it has to be said that this is a part of a contribution to design a new type of prosthesis. Our research could be continued designing the prosthesis with actively controlled ankle joint. To build this new types of prosthesis a suitable configuration of mechanical elements: spring, damper, shaft, bearings…. should be designed.

7 References

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