Investigation of Long Bow Vibrations

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Abstract

The work was about mapping which vibrations occur in bows and how they could affect accuracy. The work was carried out during the Corona pandemic, which meant limited resources for experimental measurement and the experiments were done in an office environment and not in the field. If possible, it was important to find applicable theory behind all experimental results. One goal was to measure with simple means and really understand what was to be measured and how. All experiments and measurements were done on a recurve bow. The measurements therefore began with recording sound using a microphone. Accelerometers were used for further measurements. To increase the understanding of these, a known source of vibration was needed. Thereby, a loudspeaker was modified with the possibility of attaching accelerometers in the middle of the diaphragm. The loudspeaker was powered by a signal generator via a power amplifier. A PC oscilloscope was used to record and interpret signals from the accelerometers. Three different oscillations were recorded, one elemental transverse oscillation of the string, the bending oscillation of the limbs and the third was transverse oscillation laterally with the string and arrow together. All three had a period that was much longer than the shooting. It was then considered important to be able to measure to describe the movement of the handle during the shooting process, which is specific to each combination archer and bow. Experiments on this were done with available equipment that was not sufficient due to too slow serial communication. After creating a theoretical model of the frame, "cushioning pads" could be shown to have a positive but rather small effect.

Key words: Long bow, accelerometer, transverse oscillation, limb vibration, damping eigenfrequensis, loudspeaker as vibrator

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Contents

1 Problem Description

Archery requires the ability to shield oneself from the surroundings and completely focus on the task of repeating all the steps in the shooting cycle with the goal of getting ass little deviation as possible in the target. All disturbing elements in the shooter's environment risk having a negative effect on concentration. By extension, the accuracy.

One of these disturbing moment, which the shooter himself is allowed to influence, is the behavior of the bow in connection with the firing of the arrow. The impact of recoil and vibration is a disturbing event that the shooter has the opportunity to influence at an early stage, when you draw the bow. The archer who has a quiet and vibration-free bow has a great advantage, which can be directly decisive in this sport of very small margins. The difference between a win and a tenth place is very small. Describe the movements of the grip of the bow using exponents and mathematical model to see which vibrations/ signals affect the arc. The grip of the bow is considered as rigid body movement and deformation of the grip is not relevant. bration damping improves the accuracy of an archer. The report is not written by an archer but by an aspiring engineer it can have affect the language selection and the union expression.

2 Metod

Due to the Corona epidemic, no laboratory was available. There were none either opportunities to make measurements under real conditions. The grip of the bow is considered as a rigid body movement and deformation of the grip is not relevant. All experiments were performed in a standard office equipped with a two-channel PC oscilloscope (PicoScope 2000) connected to a computer. All other equipment was developed by the author. If possible, surplus material was used. That meant development of equipment often through interconnection of individual electronic components and modification of previously used electronic equipment. This was often not optimal but given the circumstances it still felt satisfying. All experiments that were done was related to the bow shown in figure 1 with the data that applied to it. The conclusions which were drawn therefore applied only to the bow at hand and could become different for another bow.

Figure 1: The bow used in the experiments.

After interviewing several people who for various reasons have had problems with bows and its problems to do was called vibrations. For that reason, experiments began with examining vibrations such as appeared in the examined bow. It was assumed that the vibrations that could cause problems should have such a frequency and amplitude that they would be easily measurable. For hand and arm, vibrations with a frequency of 16 Hz are experienced most strongly ref [3]. If so, vibrations with this frequency is particularly harmful, so it should of course be avoided during prolonged exposure. For an archer the firing process is very short and is repeated a limited number of times. Since the experience is strong at this frequency, it can cause large involuntary movements and should be avoided for that reason. In the following, the z-direction is used as the direction of the arrow, i.e. the shooting direction. The other two directions x- and y-direction stands for gravitational direction (up and down) and lateral direction, respectively. The board (the target) is thus in the xy-plane and 70 m in z direction in front of the shooter.

3 Experiment

For some theoretical computations an engineer assessment has been made in order to derive meaningful results. Effects such that, the arrow is not positioned In the middle of the string, different stiffnesses of the bow limbs and variations in the string length are assumed to be second order effects and can thus be neglected.

When conducting any experiment it is of grave importance how the constituents of the experiment interact with the environment and how they affect each other. In the context of archery and hence the experiments conducted for this work, it is of importance how the bow is connected to its environment. Since the bow is held the measured responses will vary by the characteristics of the person holding the bow. The following model is presented in order to show and assess the difference with how the bow is operated naturally and the experimental setup. Here the bow is represented by two masses m_1, m_2 and a spring, k, connecting them. The masses represent the bow and the hand holding it. Thusly, the eigenfrequency of the system can be derived as:

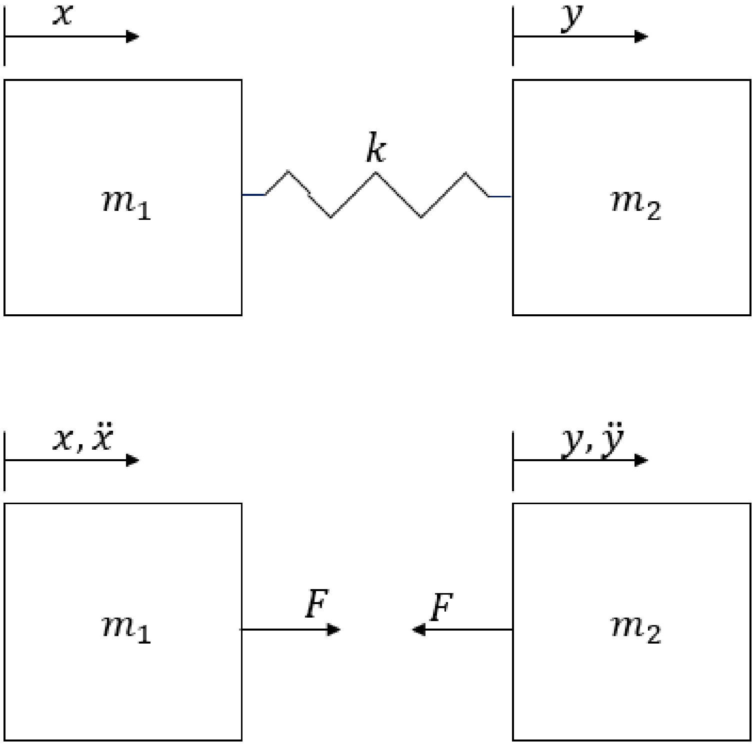

Figure 2: Two bodies spring system

$$F = k(y - x)$$

$$m_2\ddot{y} = -F = -k(y - x)$$

$$m_1\ddot{x} = F = k(y-x)$$

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} -k & k \\ k & -k \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix}$$

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix} = 0$$

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} e^{i\omega t}$$

$$-\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}\omega^2 e^{i\omega t} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} e^{i\omega t} = 0$$

Factoring in

$$\left(-\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}\omega^2 + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\right)\begin{bmatrix} a \\ b \end{bmatrix} e^{i\omega t} = 0$$

$$\begin{vmatrix} k-\omega^2 m_2 & -k \\ -k & k-\omega^2 m_1 \end{vmatrix} = 0$$

$$(k-\omega^2 m_2)(k-\omega^2 m_1) - k^2 = 0$$

$$m_1 m_2 \omega^4 - (m_2 k + m_1 k)\omega^2 = 0$$

$$\omega^2[m_1 m_2 \omega^2 - k(m_1+m_2)] = 0$$

$$\omega = 0 \text{ (Solid body movement)}$$

$$\omega = \sqrt{k\frac{m_1+m_2}{m_1 m_2}} = \sqrt{k(\frac{1}{m_1}+\frac{1}{m_2})} \tag{1}$$

To repeat, the masses m_1 and m_2 represent the masses that are vibrating, i.e. the bow itself and the shooters hand. If the bow is placed in a vice, similar to how the experiments are performed, then one can assume that one of the masses, $m_2 \to \infty$, which gives a new eigenfrequency

$$\omega_0 = \sqrt{\frac{k}{m_1}}$$

If the bow is not held in a vice, then the effect of m_2 should be accounted for according to the expressions above and the relationship between eigenfrequencies for both cases can be written as

$$\frac{\omega}{\omega_0} = \sqrt{(1+\frac{m_1}{m_2})} \tag{2}$$

3.1 Microphone

The first thing you noticed was the sound the bow made when it was "fired off". An audible sound can be recorded using a microphone which has the great advantage that it is non-contact and therefore does not affect the measuring object in any way. For this purpose, a condenser microphone cartridge WM-034, was connected together with a preamplifier which mainly consist of "the well known LM741" and provided with the necessary power supply figure 3.

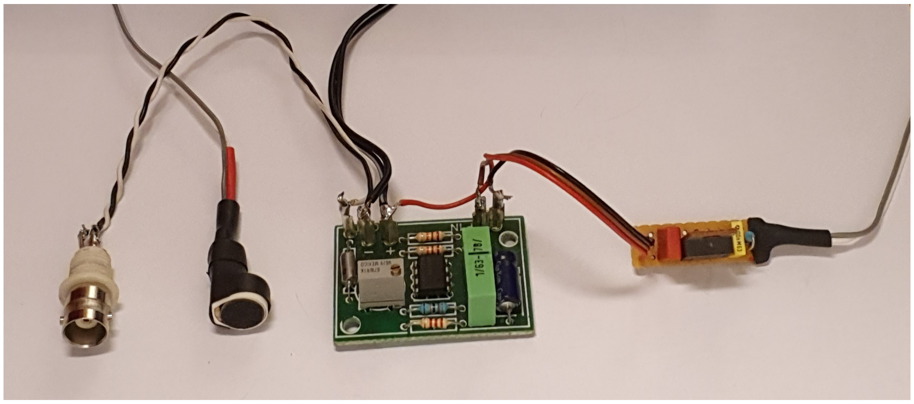

Figure 3: Microphone, amplifier and power source

The frequency response of the microphone and the preamplifier together was 20 Hz to 12 kHz. The only thing that could be distinguished from the noise was a frequency of 120 Hz and a weak signal at 240 Hz, which was assumed to be the transverse oscillation of the string. Any noise produced by the bow limbs was either too weak in amplitude to be distinguished from the noise or was outside the measurable frequency.

To calculate the frequency, f, of a transverse oscillation, the following relationship was used by Hedberg [5]:

$$f = \frac{1}{2L}\sqrt{\frac{F}{m'}}$$

L is the string length, F the string tension in the straight state and m' the mass of the string per unit length.

The length of the string between the interface against the limbs, L, was measured to be 1.39 m. To determine the string tension, F, the bow was hung up under a desk where the string was loaded with a weight with mass of 5 kg whereby a suspension $\delta = 42$ mm was obtained see figure 4

A larger weight would have given a larger deflection and thus easier to measure accurately, but then the string tension deviated more from that when the string is straight and thus the string tension force would change. Which led to the chosen compromise.

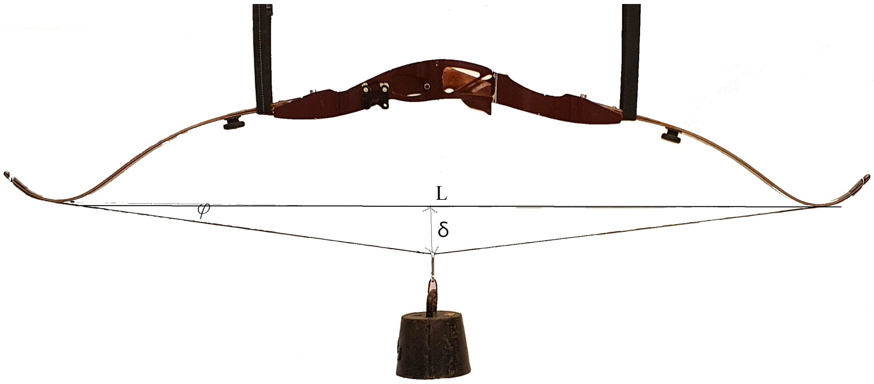

Figure 4: Method used to determine string tension force

The angle φ could then be calculated as

$$\varphi = arctan\frac{\delta}{L/2} = arctan\frac{0.042}{1.39/2} = 3.46°$$

Hence the string tension force could be estimated

$$F = \frac{m \cdot g}{2 \cdot sin\varphi} = \frac{5 \cdot 9.81}{2 \cdot sin3.46} = 407\,\text{N}$$

The string was equipped with loops at the ends which made it difficult to determine its mass per unit length without destroying it. Through measurements, part of the string was measured to determine its weight as well as that the entire string including the lopps at the ends were weighed in order to give cross reference for the weight measurement, $m' = 3.17$ g/m was finally determined.

The frequency of the transverse oscillation could then be calculated according to

$$f = \frac{1}{2L}\sqrt{\frac{F}{m'}} = \frac{1}{2 \cdot 1.39}\sqrt{\frac{407}{0.00317}} = 129\,\text{Hz}$$

Thus, it was assumed that the frequency measured with the microphone was the same as the transverse oscillation and the relatively small deviation could be due to uncertain values of the string tension and especially the mass of the string per unit length.

3.2 Accelerometer

The next step in identifying vibrations was to use accelerometers. An examination showed that they mainly register movement of a mass suspended in some way. The motion is then converted into an electrical quantity using three different physical principles.

- Piezo resistive, are constructed with a material that changes resistance when deforming.
- Piezo electrical, are constructed with a material that produces a voltage when deforming.

- Capacitive, changes capacitance when distance between two surfaces changes.

The output signals from the sensors could be analog or digital and in different directions, in addition the output signals can be linear acceleration and / or rotation (gyro). All three types were considered useful. What became decisive was primarily the possibility of reusing surplus material and then the cost of new procurement. In order to be able to compare and understand different accelerometers, a loudspeaker, signal generator and power amplifier were used to generate known vibrations. In the middle of the loudspeaker diaphragm, a short tube was glued with a circular washer at the end to be able to attach different accelerometers see figure 5.

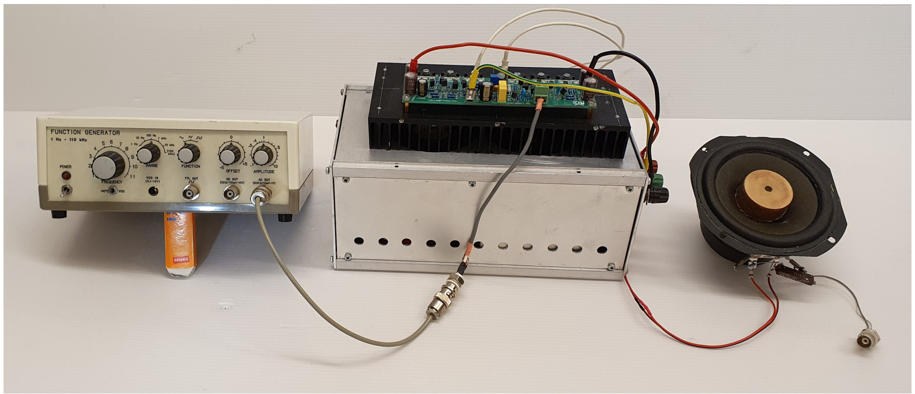

Figure 5: Signal generator, amplifier and loudspeaker

Here it was also possible to measure the input signal to the speaker in order to be able to compare it with the output signal from the current accelerometer. It was primarily intended to examine the speaker's response to a known signal and was used especially with the certified accelerometers described in the next section.

3.2.1 Piezoelectrisc sensor

It was possible to use surplus material in the form of three single-axis piezoelectric sensors with the designation IMI sensor 608A11 [10]. They were of the "industrial quality" type and certified see appendix A. In order to be used, the sensors were connected in series with a resistor of 6.3 kohm and fed with 24 V. The output signal was taken out via a capacitor of 1 μF which was large enough to transmit low frequencies. The output signal was connected in parallel with a resistor of 10 Mohm, seen in figure 6.

When examining a sensor on the speaker, the same shape (sine) was obtained on the output signal from the sensor as the input signal to the speaker down to a frequency of about 20 Hz. The deviation at the low frequency was assumed to be due to the proximity of the natural frequency of the speaker diaphragm (spring) with the mass of the accelerometer. This was not further investigated due to the difficulty of measuring the stiffness of the speaker diaphragm which, moreover, was probably not constant but due to the amplitude. It was enough to be aware of it.

A special bracket for the sensors was constructed with space for the three accelerometers in the three directions. The bracket was supplemented with one special screw to be able to be attached to the handle of the bow see figure 7.

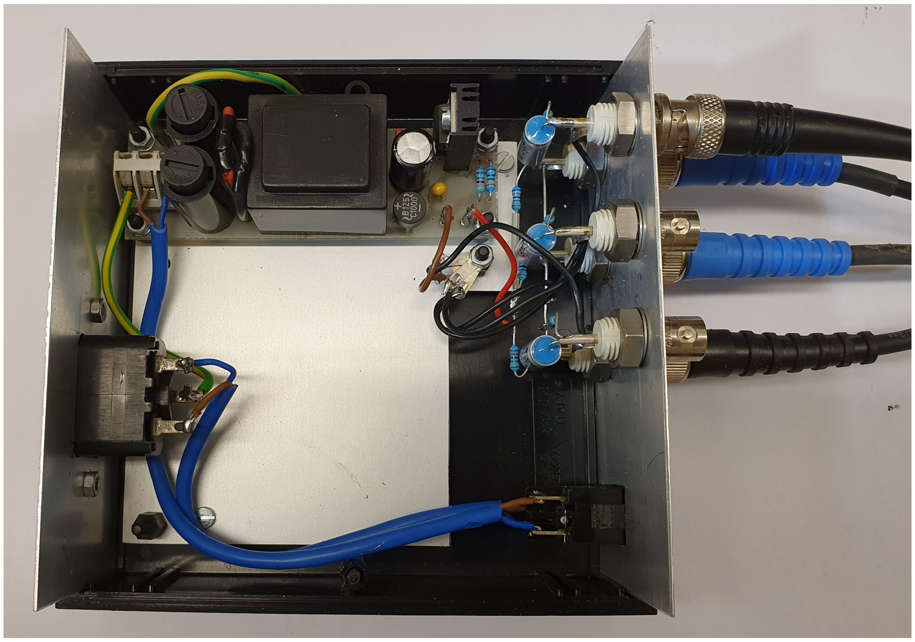

Figure 6: Electrical box for the piezoelectric sensors

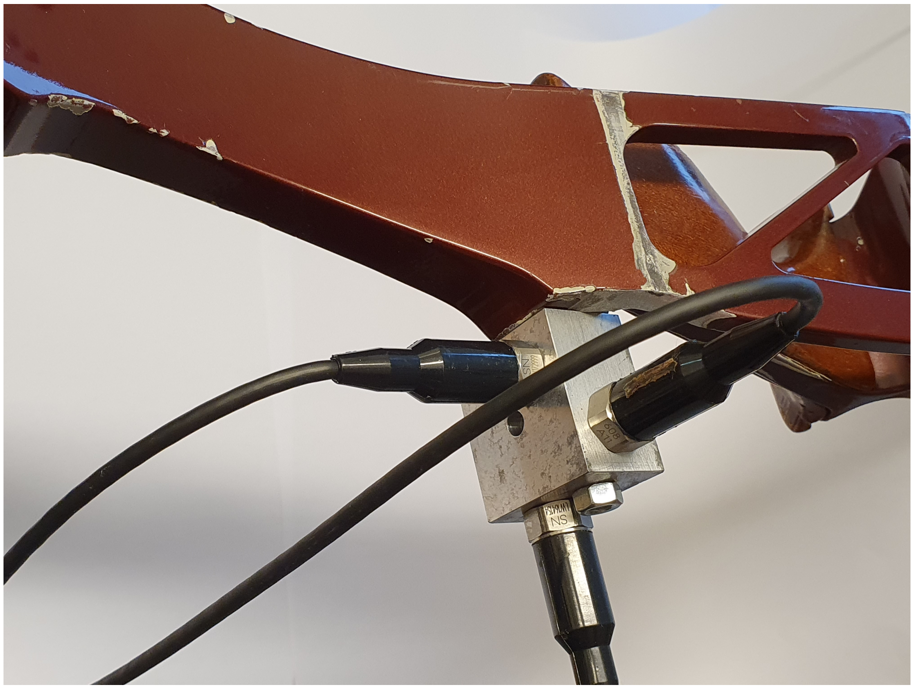

Figure 7: Grip of the bow with piezoelectric sensors conected

The bracket with the three sensors weighed a total of 175 grams and was considered too heavy to be placed anywhere on the bow and possibly had too large a measuring range ±50g to detect weak signals.

The only measurement that could be distinguished from the noise, which was very low, was the transverse oscillation of the string with some harmonics.

3.2.2 Capacitive sensor

To find frequencies with lower amplitude, sensors with higher resolution were needed. Lower weight would also mean possibility of freer placement with less influence on the frequency. A cheap capacitive sensor was procured with three analog outputs with measuring range of ±3g. The designation of the sensor was ADXL335 Analog Devices [1]. Each output signal was internally connected in series with its own resistance of 32 kohm. Thereby it was possible to equip the outputs with a RC-filter to minimize noise. It was also done through putting a capacitor of 10 nF over each output which meant a bandwidth of 500 Hz according to the sensor specifications.

Together with the power supply, everything was built into a box see figure 8.

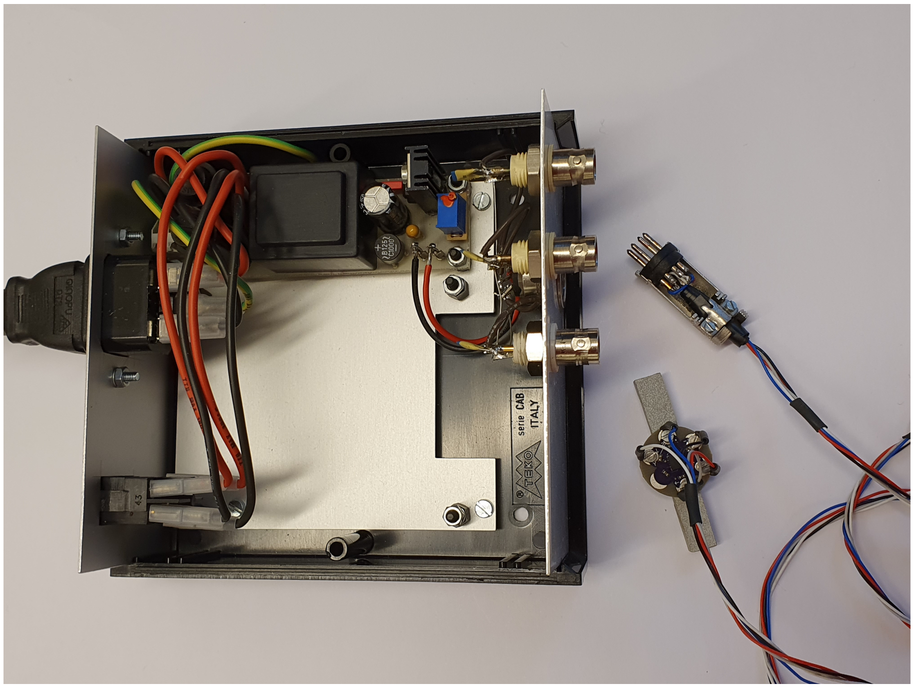

Figure 8: Electrical box for the capacitive sensor

For the sensor a number of different attachment brackets were designed and constructed, figure 9, in order to freely attach the sensor to the bow.

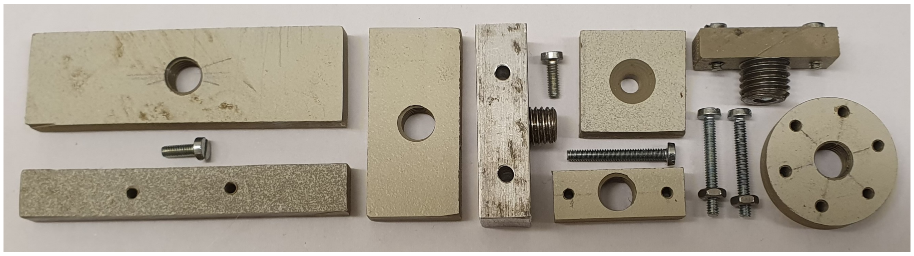

Figure 9: Brackets

The brackets each weighed about the same amount and together with the sensor a total of 6 grams, and can be seen down to the right in figure 8. When the sensor was placed on the handle, in addition to the transverse oscillation of the string, a frequency of about 20 Hz could be registered. After placing the sensor on one of the limbs it could be stated that it was the natural frequency of the limbs. It was different depending on how the bow was held, when one holds the bow with the hand the measured eigenfrequency is 18.3 Hz and when the bow is fixed the eigenfrequency is 15.6 Hz. The measured values seems reasonable especially taking into account the reasoning that iead to equation 2 and its consequences. Also as mentioned i ref [3] the human arm and hand are most sensitive to frequencies from 10 to 20 Hz which is in the same range as the measured frequency.

So far it can be stated that there are only two vibrations (oscillations) with such amplitude and frequency that they could have any significance. A third which is a transverse oscillation of the string with a part of the arrow in the xy plane is explained later.

3.3 Motion

The time period for the measurable oscillation (vibration) with the highest frequency (transverse oscillation) was 1/f = 1/120 = 0.0083 s. Assuming that it is excited at the moment the arrow leaves the contact with the string (the string is straight) the arrow has a distance of 0.2 m before it leaves the contact with the handle.

With an output speed of 70 m/s, the time becomes 0.2/70 = 0.0029 s, which is approximately a third of the period time. If you do the same for the swing of the limbs, the period time will be 1/20 = 0.05 s. It is excited when the string is released, i.e. a little earlier. The extra distance is 0.2 m and if you count the average speed 70/2 = 35 m/s the time will be 0.2/35 = 0.0057 s. Together with the time before it will be a total of 0.0029 + 0.0057 = 0.008 s which is less than one sixth of the period.

Thus, one can then suspect that it is not vibrations that primarily affect the accuracy. The sound perceived by the bow emanates after the arrow has left the bow, and the reduction of this sound is of the "a good feeling" type rather than a performance increase. It then seems to be more important to be able to describe the movement of the handle during the firing process. The firing process means the time from which the string is released until the arrow has left contact with the handle. This is the time that the shooter through the bow can affect the arrow. The movement is then assumed to be caused by the "force change" when the string is released and the recoil (opposite to the amount of motion the arrow receives) in combination with involuntary movements. It should then be pointed out that the movement to be described is unique to each shooter with a particular bow.

Before releasing the string, one hand holds the string backwards and the other holds the handle forward with forces. Both are equal in magnitude 265 N and opposite on the z-axis. When the string is released, the natural oscillation of the limbs is excited, which only causes movement in the z-direction (harmless) if they are in phase. In reverse phase, they would cause distortion around y-axis and degrade the hit image in height. Opposition phasing seems impossible even theoretically.

Depending on how the string is released, it is conceivable that the string is given an excitation laterally (y-direction) and would then impair the accuracy. By a theoretical model of this, the string was considered as before and with a part of the mass of the arrow as point mass in the middle of the string. Here one can imagine that the rigidity of the shooting moment is next 0. Which also gives the natural frequency almost 0. The moment the arrow leaves the string, the stiffness of the string in the y-direction becomes the same as in the x-direction.

The following calculations can be made.

$$\omega = \sqrt{\frac{c}{m}}$$

Previous measured quantities can be used to estimate stiffness

$$c = \frac{F}{\delta} = \frac{407}{0.042} = 9690\,\mathrm{N/m}$$

An engineering approximation gives that one third of the mass of the arrow (0.019/3 kg) can be assumed as point mass on the string. Thus, the frequency can be calculated as:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{9690 \cdot 3}{0.019}} = 197\ \mathrm{Hz}$$

Together with the previously measured and calculated transverse oscillation frequency of the string is obtained according to Dunkerly's Method [4]:

$$f = \sqrt{\frac{1}{\frac{1}{120^2} + \frac{1}{197^2}}} = 102\ \mathrm{Hz}$$

The arrow is thus excited with a frequency (in the y-direction) that starts with zero when the string is released to 102 Hz when the arrow releases the contact with the string. This can then be a reason why the arrow behaves like a swimming fish up to the goal. In some archery communities, it is called the paradox of the arrow, which is the same as a free-swinging beam that can be described by the Euler-Bernoulli beam equations ref [9].

$$\frac{\partial^4 y(z,t)}{\partial z^4} = -\frac{m'}{k}\frac{\partial^2 y(z,t)}{\partial t^2}$$

This has been done for a special case by Meyer [8]. An arrow was considered to be a thin, uniform, free, vibrating beam with the result for the frequency of the fundamental mode as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m'}}\left(\frac{4.730}{L}\right)^2$$

Where k is the stifness, L length an m' the mass per unit length.

This gives a free vibrating eigenfrequency of 97 Hz, as mentioned previously this work does not consider the motion of the arrow. To further investigate the characteristics of the arrow is not part of this work. But the estimated frequency is reasonable since it is between the case of point mass, which gives zero frequency and the frequency that was obtained through Dunkerlys method, 102 Hz.

The recoil is the same as the momentum $\mathbf{p}$ given to the arrow and equal to the product of the mass of the arrow and its initial velocity which gives:

$$\mathbf{p} = 0.019 \cdot 70 = 1.33\,\text{kgm/s}$$

The bow gets an equal amount of movement in the other direction that should be constrained and damped by the users hand or arm. In order to increase the understanding of the momentum one can write it as $\int F dt$, i.e. a product of force and time. This is highlighted by the fact that kgm/s = Ns. It is often spoken about that one wants to achieve a recoil free apparatus but this is not achievable if not an equal momentum is released in the opposite direction (back blow principle). One way to achieve a recoil free sensation is to deliver the force under a long duration (long path), the momentum does not change but the recoil is not felt as intrusive. Since the recoil is in the z-direction and for the most part travels through the center of mass it should not directly affect the users accuracy.

3.3.1 Accelerometers

By this time, valuable knowledge and experience had been gained by accelerometers. It then felt natural to continue to use such to register movement. In order to be able to describe the movement in total (three-dimensionally), measurement of all six degrees of freedom must be made. There was no equipment to be able to receive six signals simultaneously. For that reason, the possibility of serial communication was investigated. An accelerometer with the designation GY-521 MPU-6050 [11] was procured. It gave both acceleration and rotation around x, y and z with I2C protocol. The measurement range could be set to ±2g, ±4g, ±8g or ±16g.

To decode the serial signal, an Arduino was used with the source code (in Appendix B) from the Arduino library [2].

To describe the procedure, a direction is selected, for example, x with the velocity $\dot{x}$, the acceleration $\ddot{x}$ and the time between each measured value Δt. If position, speed and acceleration are known at the time i and the acceleration is measured after the time Δt, the speed can be approximated to that at the time i plus the mean of the accelerations multiplied with Δt i.e.

$$\dot{x}_{i+1} = \dot{x}_i + \Delta t(\ddot{x}_i + \ddot{x}_{i+1})/2$$

In the same way, the position can be obtained as:

$$x_{i+1} = x_i + \Delta t(\dot{x}_i + \dot{x}_{i+1})/2$$

Since it cannot be taken for granted that the movement is harmonic, many measured values are required during the firing process. This means the highest possible sampling frequency that thus gives as small Δt as possible.
Then the question arose how often did each measured value return? Was it possible to get enough measurement signals during the firing process to be able to describe the movement? This was determined by looking at the signal from the accelerometer to the Arduino without any "delay" in the program see figure 10.

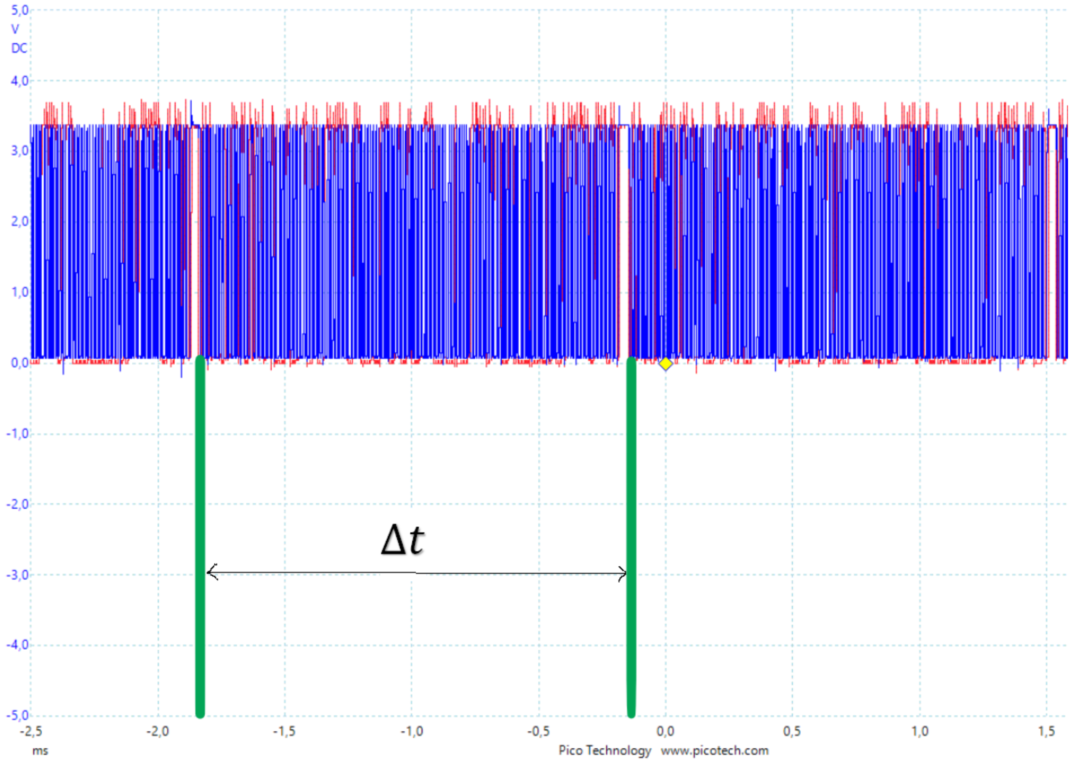

Figure 10: Picture from oscilloscope

In the figure you can see that all measured values returned at 1.7 ms intervals. This meant that during the current time, 4/1.7, i.e. less than 3 signals could be obtained, which made it impossible to describe the movement. At least 10 preferably 20 of each measured value during the firing process would have made it possible to describe the movement satisfactorily.

3.3.2 Further work

In order to continue with accelerometers possibility with measurement of all six degrees of freedom must be made and ten to twenty values of each during the firing process.

There are different ways to measure and record movement. When the corona restrictions began to subside, there was an opportunity to meet and talk to people who were going be able to have knowledge of measuring and registering movement. The following was suggested:

- **GPS** Using GPS will not be useful since the accuracy of the current best GPS technologies is approximately 1 meter.
- **Xsens suit** A body suit made by Xsens can be used to measure the body movements, unfortunately this method is both expensive and not sufficiently accurate. The accuracy is of 0.1 meters magnitude.
- **Force sensor** Large, heavy affects measurement with its own presence. Force sensors could be used to measure the forces generated by the bow when shooting an arrow. Especially the forces experienced in the grip as these forces affect the shooter and hence the shot accuracy. The main issue with the invrstigated force sensor is their weight and large size. It would be cumbersome to use them without affecting the user of the bow significantly.
- **”Laser arm”** A measurement arm can also be used to measure the positions/displacement of the bow. The measurement arm can non-intrusively be attached to any part of the bow and will measure the motion through a shot cycle. These measurement arms can measure at a micrometer level of accuracy.
- **High speed recording** In order to improve the positional measurements of the bow action one could employ techniques of high speed recording. A reference system would then need to be developed so that the bow positions could be read from each frame of the recording as well as a read out of th time stamp of each frame, that is needed in order to create a motion path.
- **LIDAR** Lidar could be used to measure the bow and arrow motion but as some of the previous mentioned methods it is very expensive and only has an accuracy up to 1 mm.

4 Theoretical model

4.1 Introduction

Several researchers and academics have wondered how to model the bow and arrow, during the late 20th century many of these researchers have developed ideas and methods of how to mechanically describe the bow and arrow system.

The first mathematical model of a bow and arrow system was constructed by Hickman [ref 207503341.pdf (core.ac.uk)]. Hickman investigated a traditional longbow; he reduced the bow to a mechanical system using the following assumptions. Firstly, it was assumed that the bow was completely symmetric along a horizontal line splitting the bow in half. The arrow is assumed to be a point mass with half of each mass belonging to each half of the bow. Moreover, Hickman prescribes the motion of the bow as a circular motion, i.e., the limbs of the bow trace a circle. Here it is assumed that the bow limb can be divided into two parts where both of them are considered to be slender and rigid. One part attached rigidly to the grip of the bow is one third of the length of the bow. The other part, being two thirds of the limb length, is attached to the first part by a revolute joint and a spring. The revolute joint forces the upper part of the bow limb to trace a circular motion and the spring that connects the two parts of the limb represents the elasticity of the bow. The mass of the limb is placed at the tip of the limb. Furthermore, the bow string that attaches the two limbs is assumed to be inextensible and massless. Hickman's analysis of the bow and arrow system deduces that the system is of one degree of freedom and therefore only has one mode of motion and one eigenfrequency. This a large discrepancy from the actual bow which of course has elastic limbs and might not be horizontally symmetric.

Another simplified bow and arrow model was developed by Marlow ref [7]. Marlow produces a two degree of freedom model of the bow and arrow system, where one degree of freedom is the position of the arrow and the other degree of freedom is the bow string half-length. As Hickman Marlow assumes that the bow only operated in a plane and that the bow is horizontally symmetric, as well as that the arrow is considered to be a point mass that can evenly be split between the two parts of the bow. But, Marlow considers that the bow string has an uniform mass distribution and that it is extensible.

Furthermore, the limbs of the bow have a known mass distribution and potential energy, i.e. elasticity relation. The extensible string is forced to be straight during the entire motion of the bow. This assumption entails that longitudinal and transverse signal velocities in the string have infinite speed ref [7]. Using Marlow's model one can rederive the Hickman model with similar mass distributions for the limbs and string and similar potential energy. One can can also expand on the Hickman model by assuming that the string has mass. A more complex model is also derived by Marlow assuming that the string has both mass and elasticity.

Lastly, a model was developed by Kooi [ref]. Similar to the other models, this model also only permits motion in a plane and that the bow is horizontally symmetric. The arrow is considered to be a point mass which is split between the upper and lower parts of the bow. Kooi assumes that the string is massless, and its elasticity is governed by a Hooke type constitutive equation. The limbs of the bow are modeled as Euler-Bernoulli beams. Further the mass of the limbs is concentrated as point masses at the limb tips, similar to Hickman. The Kooi model results in a system of differential equations that must be solved numerically. However, this model can describe a more complex motion that the other two mentioned models.

4.2 The used model

In the previous chapters the experimental procedures and conclusions were established as well as the human perception of vibrations. To recap, it was concluded that the only one frequency in the field of perception could be measured, namely the eigenfrequency of the bow at 15.6 - 18.3 Hz. No other frequencies were measured and other frequencies had amplitudes of orders of magnitude lower than the eigenfrequency at 15.6 - 18.3 Hz. This suggests that one can model the limb of the bow as a rigid body connected to the grip with a rotational spring, this gives one eigenfrequency for the limb and thus one degree of freedom.

The simplified model used in this work is based on the Hickman model ref [6], where the bow is split into two parts using a revolute joint see figure 11. As the Hickman model, the mass of the limb is concentrated at the tip, m_L, and the elasticity of the limb is modeled by a rotational spring, k_L. Furthermore, the bow is assumed to symmetric and therefore only half the bow is analyzed. The grip is assumed to be a rigid body with mass m_h that is constrained to move only in the direction of the arrow, due to symmetry. Moreover, the arm of the archer is modeled as a mass-spring-damper system, parametrized by the parameters with subscript a. The effects of the string is in order to simplify the model. The two main reasons for being able to neglect the string, are that the bow without string is a more conservative system since the string prohibits the motion of the limbs. Further, the simulation times used are significantly short so that the string has a limited impact, this is mainly due to the fact that the arrow leaves the bow relatively fast. Finally, the vibration damping device is attached to the grip and it is also modeled as a mass-spring-damper system, with subscripts d as is seen in figure 11. In the model the effects of gravity are neglected.

This simple pendulum type model is used to evaluate whether attaching a vibration damping device to the grip will have a meaningful effect in minimizing the motion of the grip and hence the disturbance felt by the archer. The experimental measurements of the bow discussed in the previous chapters can be used to parametrize the simplified bow model. The masses of the grip, assuming symmetry i.e., half the mass, moving part of limb and its length are measured directly to be 0.8 kg, 60 g and 400 mm, respectively. Moreover, the rotational stiffness of the bow is estimated from the measured eigenfrequencies as 127 Nm/rad. The characteristics of the vibrational damping device is dependent on its material choice, for this particular

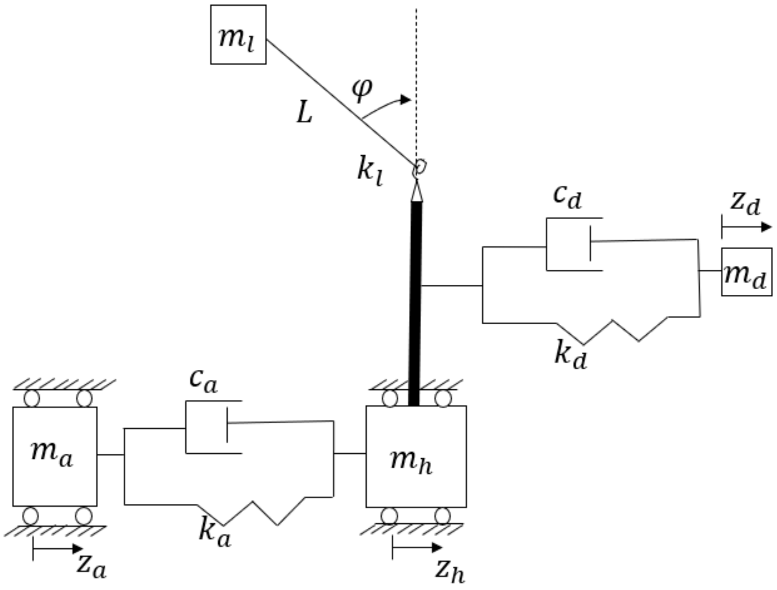

Figure 11: The used model

work the stiffness of the device is varied and the damping coefficient is obtained by the rule of thumb $c_d = k_d/100$. Meaning that the damping coefficient is 1% of the stiffness. The mass of the vibration damping device is limited by the following rule of thumb, it can maximally be 10% of half of the bow weight. This maximal limit is enforced so that the total weight of the bow doesn t increase too much since this will negatively impact archer performance. Hence, the maximal vibration damping device weight m_d is 100 g.

The characterization of the archer arm is a non-trivial matter that is not covered by the scope of this thesis. Studies has been made of different procedure to model the arm of a human [ref arm]. Unfortunately, these models are quite complicated and would not provide any clear insight in our simplified model. Hence, for simulating the bow model the extremum case is considered where the grip of the bow is allowed to move freely, i.e. the arm holding the grip has no stiffness. Since a comparative study is made between the case with no vibration damping device and with a device it is argued that the relative difference between both cases is not significantly affected by the modelling of the arm, thus it can be neglected.

The equations of motion for the simplified bow model are derived using a Lagrangian formalism. Here, four generalized coordinates are employed to describe the motion of the four inertial masses, φ for m_L, z_d for m_d, z_h for m_h and z_a for m_a, respectively. Using these four generalized coordinates the kinetic and potential energy as well as the Rayleigh potential of the system can be written as follows

$$\vec{v}_L = (L\dot{\varphi}\cos\varphi + \dot{z}_h)\mathbf{e}_z + L\dot{\varphi}\sin\varphi\,\mathbf{e}_y$$

$$T = \frac{1}{2}m_a\dot{z}_a^2 + \frac{1}{2}m_h\dot{z}_h^2 + \frac{1}{2}m_d\dot{z}_d^2 + \frac{1}{2}m_L([\dot{z}_h + L\dot{\varphi}\cos\varphi]^2 + L^2\varphi^2\sin^2\varphi) =$$

$$= \frac{1}{2}m_a\dot{z}_a^2 + \frac{1}{2}m_h\dot{z}_h^2 + \frac{1}{2}m_d\dot{z}_d^2 + \frac{1}{2}m_L(\dot{z}_h^2 + L^2\dot{\varphi}^2 + 2\dot{z}_h L\dot{\varphi}\cos\varphi)$$

$$V = \frac{1}{2}k_a(z_h - z_a)^2 + \frac{1}{2}k_d(z_d - z_h)^2 + \frac{1}{2}k_L\varphi^2$$

Rayleigh dissipation function

$$R = \frac{1}{2}c_a(\dot{z}_h - \dot{z}_a)^2 + \frac{1}{2}c_d(\dot{z}_d - \dot{z}_h)^2$$

Furthermore the Lagrangian of the system can be written as

$$\mathcal{L} = T - V$$

And employing the Euler-Lagrange equation for each generalized coordinate

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial \dot{q}_i} + \frac{\partial R}{\partial \dot{q}_i} = 0, \quad \dot{q}_i = \{z_a, z_h, z_d, \varphi\}$$

Gives the four equations of motion as

$\underline{z_a:}$

$$m_a\ddot{z}_a + k_a(z_a - z_h) + c_a(\dot{z}_a - \dot{z}_h) = 0$$

$\underline{z_h:}$

$$(m_h+m_L)\ddot{z}_h+m_L L\ddot{\varphi}\cos\varphi-m_L L\dot{\varphi}^2\sin\varphi+k_a(z_h-z_a)+k_d(z_h-z_d)+c_a(\dot{z}_h-\dot{z}_a)+c_d(\dot{z}_h-\dot{z}_d) = 0$$

$\underline{z_d:}$

$$m_d\ddot{z}_d + k_d(z_d - z_h) + c_d(\dot{z}_d - \dot{z}_h) = 0$$

$\underline{\varphi:}$

$$m_L L^2\ddot{\varphi} + m_L\ddot{z}_h L\cos\varphi + k_L\varphi = 0$$

Results

Using the derived equations of motion the bow model was simulated for two cases, one where the vibration damping devices stiffness is varied and on where its mass is varied. For both cases a simulation time of 3 ms and 10 ms is chosen see figure 12 and figure 13. The time of 3 ms is chosen since that is the time it takes the arrow to leave the bow and a longer time 10 ms is also investigated to see if the vibration damping device will impart a good feeling to the archer. For the case when stiffness, k_d, is varied the vibration damping mass, m_d, is held constant at 100 g. Similarly, when the mass, m_d, is varied the stiffness, k_d, is held constant at 10000 N/m. For all cases the bow is drawn so that the limb makes an 20 degree angle with the vertical.

As can clearly be seen from the four studied cases that employing a vibration damping device reduces the motion of the grip. However, it can also be noted that for

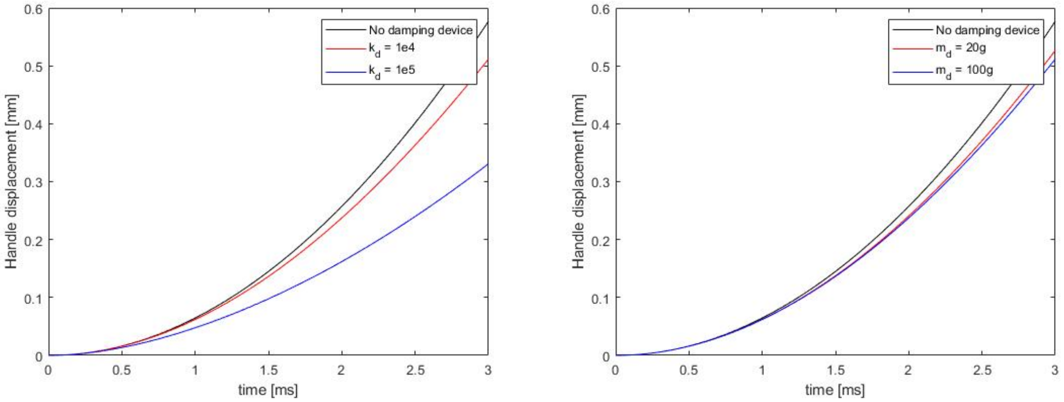

Figure 12: Vibration damping during the firing process with varied damping parameters.

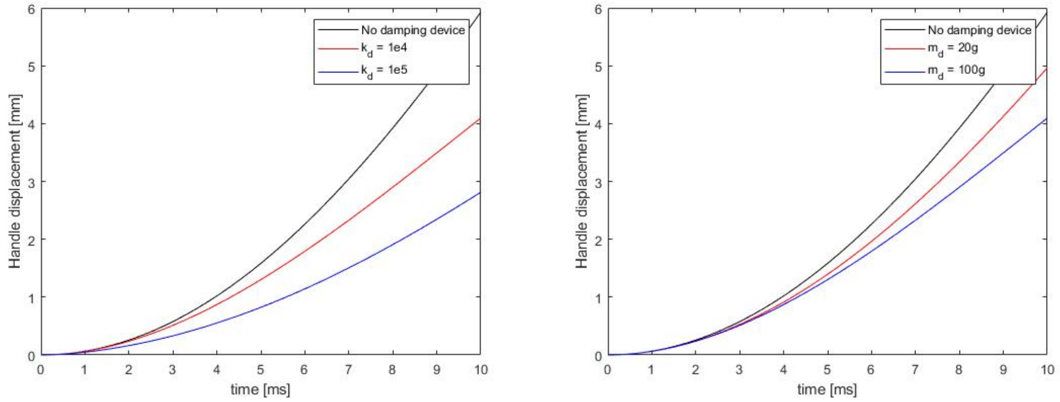

Figure 13: Vibration damping during longer than the firing process with varied damping parameters.

both simulation times that the result is more sensitive to stiffness than to mass. It should be noted that this effect is also dependent on the rule of thumb that the damping coefficient is 1% of the corresponding stiffness. In the case of the arrow just leaving the bow, 3 ms, the reduced displacement of the grip is about 40%, which is an substantial amount. For the case of 10 ms simulation time the archer feels a significant reduction of grip displacement. This reduction does not affect the arrow since it has already left the bow but the reduction will give the archer a good feeling and a sense of stability. Moreover, the presented results consider only the extremum case where the archers arm does not impact the motion of the grip. If the arm is considered in the simulation then both cases of with and without vibration damping device will show smaller grip displacements. Furthermore, this part of the analysis is heavily dependent on each individual archer and their own arm composition. If a vibration damping device is to be employed by an archer it should be tuned both for the bow as well as the archer.

5 Conclusion

The work was about mapping what vibrations occur in bows and how the impact of it could affect accuracy as little as possible. Experiments and measurements were done on a specific bow. There we found three different oscillations, an elementary transversal oscillation of the string, the limb bending and the third was the transverse oscillation in the sideways of the string and arrow together. The period time with the oscillation with the highest frequency was much longer than the actual shooting time and to be able to describe how the shooter can be affected by the handle during the time that the arrow has contact with the bow during the shooting procedure.

The elimination of the vibrations are of good feeling type and do not affect precision. The question was whether the inclusion of a vibration damping device would increase performance. After having made calculation of the theoretical model it was concluded to be a small effect.

The result was that it is not the vibrations that are decisive, but movement of the bow during the shooting process and that it is individual for combination of archer and bow. Further work would be to use more precise measuring equipment to measure the movement of the grip.

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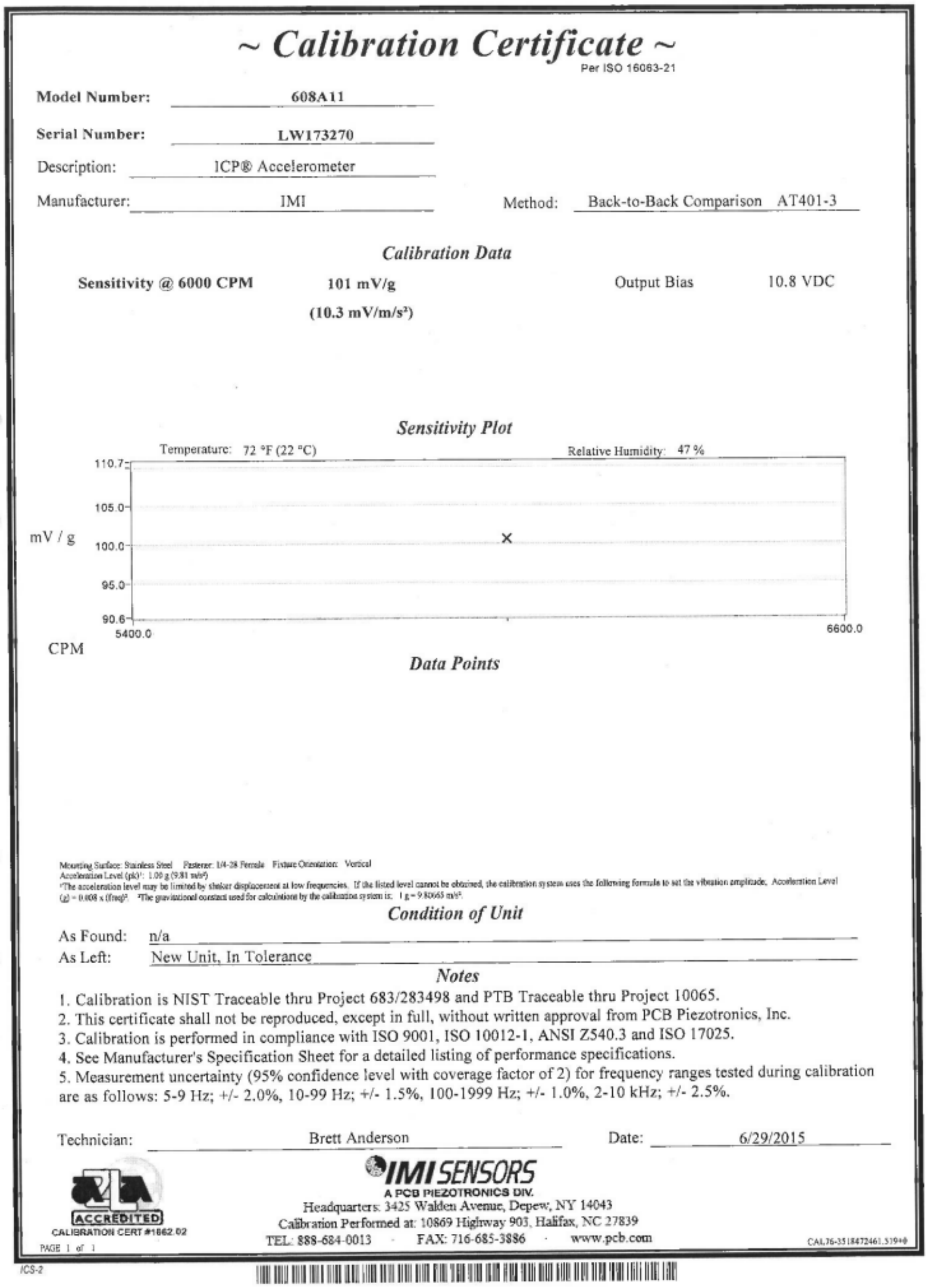
A Appendix A



~ Calibration Certificate ~

Per ISO 16063-21

Model Number: 608A11

Serial Number: LW173270

Description: ICP® Accelerometer

Manufacturer: IMI

Method: Back-to-Back Comparison AT401-3

Calibration Data

Sensitivity @ 6000 CPM 101 mV/g (10.3 mV/m/s²)

Output Bias 10.8 VDC

Sensitivity Plot

Temperature: 72 °F (22 °C)

Relative Humidity: 47 %

Data Points

Mounting Surface: Stainless Steel Fastener: 1/4-28 Female Fixture Orientation: Vertical
Acceleration Level (pk)[1]: 1.00 g (9.81 m/s²)
[1]The acceleration level may be limited by shaker displacement at low frequencies. If the listed level cannot be obtained, the calibration system uses the following formula to set the vibration amplitude: Acceleration Level (g) = 0.008 x (freq)². [2]The gravitational constant used for calculations by the calibration system is: 1 g = 9.80665 m/s².

Condition of Unit

As Found: n/a

As Left: New Unit, In Tolerance

Notes

1. Calibration is NIST Traceable thru Project 683/283498 and PTB Traceable thru Project 10065.
2. This certificate shall not be reproduced, except in full, without written approval from PCB Piezotronics, Inc.
3. Calibration is performed in compliance with ISO 9001, ISO 10012-1, ANSI Z540.3 and ISO 17025.
4. See Manufacturer's Specification Sheet for a detailed listing of performance specifications.
5. Measurement uncertainty (95% confidence level with coverage factor of 2) for frequency ranges tested during calibration are as follows: 5-9 Hz; +/- 2.0%, 10-99 Hz; +/- 1.5%, 100-1999 Hz; +/- 1.0%, 2-10 kHz; +/- 2.5%.

Technician: Brett Anderson

Date: 6/29/2015

ACCREDITED
CALIBRATION CERT #1862.02

IMI SENSORS
A PCB PIEZOTRONICS DIV.
Headquarters: 3425 Walden Avenue, Depew, NY 14043
Calibration Performed at: 10869 Highway 903, Halifax, NC 27839
TEL: 888-684-0013 · FAX: 716-685-3886 · www.pcb.com

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CAL76-3518472461.539+0

ICS-2

B Appendix B Arduino Code

```
// (c) Michael Schoeffler 2017, http://www.mschoeffler.de

#include ”Wire.h” // This library allows you to communicate with I2C devices.

const int MPU_ADDR = 0x68; // I2C address of the MPU-6050. If AD0 pin is set
to HIGH, the I2C address will be 0x69.

int16_t accelerometer_x, accelerometer_y, accelerometer_z; // variables for accelerom-
eter raw data
int16_t gyro_x, gyro_y, gyro_z; // variables for gyro raw data
int16_t temperature; // variables for temperature data

char tmp_str[7]; // temporary variable used in convert function

char* convert_int16_to_str(int16_t i)  // converts int16 to string. Moreover, resulting
strings will have the same length in the debug monitor. sprintf(tmp_str, ”%6d”, i);
return tmp_str;



void setup()
Serial.begin(9600);
Wire.begin();
Wire.beginTransmission(MPU_ADDR); // Begins a transmission to the I2C slave
(GY-521 board)
Wire.write(0x6B); // PWR_MGMT_1 register
Wire.write(0); // set to zero (wakes up the MPU-6050)
Wire.endTransmission(true);

void loop()
Wire.beginTransmission(MPU_ADDR);
Wire.write(0x3B); // starting with register 0x3B (ACCEL_XOUT_H) [MPU-6000
and MPU-6050 Register Map and Descriptions Revision 4.2, p.40]
Wire.endTransmission(false); // the parameter indicates that the Arduino will send
a restart. As a result, the connection is kept active.
Wire.requestFrom(MPU_ADDR, 7*2, true); // request a total of 7*2=14 registers

// ”Wire.read()¡¡8 — Wire.read();” means two registers are read and stored in the
same variable
accelerometer_x = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x3B (AC-
CEL_XOUT_H) and 0x3C (ACCEL_XOUT_L)
accelerometer_y = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x3D (AC-
CEL_YOUT_H) and 0x3E (ACCEL_YOUT_L)
accelerometer_z = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x3F (AC-
CEL_ZOUT_H) and 0x40 (ACCEL_ZOUT_L)
temperature = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x41 (TEMP_OUT_H)
and 0x42 (TEMP_OUT_L)
gyro_x = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x43 (GYRO_XOUT_H)
```

```
and 0x44 (GYRO_XOUT_L)
gyro_y = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x45 (GYRO_YOUT_H)
and 0x46 (GYRO_YOUT_L)
gyro_z = Wire.read()¡¡8 — Wire.read(); // reading registers: 0x47 (GYRO_ZOUT_H)
and 0x48 (GYRO_ZOUT_L)


// print out data
Serial.print(”aX = ”); Serial.print(convert_int16_to_str(accelerometer_x));
Serial.print(” — aY = ”); Serial.print(convert_int16_to_str(accelerometer_y));
Serial.print(” — aZ = ”); Serial.print(convert_int16_to_str(accelerometer_z));
// the following equation was taken from the documentation [MPU-6000/MPU-6050
Register Map and Description, p.30]
Serial.print(” — tmp = ”); Serial.print(temperature/340.00+36.53);
Serial.print(” — gX = ”); Serial.print(convert_int16_to_str(gyro_x));
Serial.print(” — gY = ”); Serial.print(convert_int16_to_str(gyro_y));
Serial.print(” — gZ = ”); Serial.print(convert_int16_to_str(gyro_z));
Serial.println();


// delay
delay(1000);
```