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Applicability of traditional ice load measurement systems on longitudinally framed ships

Master's thesis in Marine Technology, Mobility Engineering Programme

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DEPARTMENT OF MECHANICS AND MARITIME SCIENCES

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Abstract

This thesis evaluates the applicability of the traditional Influence Coefficient Matrix (ICM) method to ice-load measurement on a longitudinally framed ship structure. ICM is a matrix that creates a relationship between structural responses and external loads. A finite element model was developed in Abaqus and both shear-strain and normal-strain responses were used to establish ICMs. Two rectangular loading patch sizes were investigated, which are 400 mm (transverse) $\times$ 800 mm (longitudinal) and 200 mm (transverse) $\times$ 400 mm (longitudinal). The total equivalent load was 1 kN in each case. The errors varied systematically with whether the loading centre was located on the stringer, a longitudinal stiffener, a plating bay, or the transverse web frame. The normal-strain-based measurement also produced large positive and negative values. These results suggest that the ICM may not be applicable to loading cases different from those used in its construction. Local load transfer and the direct inversion process may also increase the measurement error. The present ICM formulation is therefore not reliable over all studied loading conditions, although accurate measurement remains possible at individual positions. Future work should construct the ICM using a wider range of loading positions and patch sizes; constrained or regularized inverse approaches are recommended.

Key words: ice-induced load, ice-going vessel, longitudinally framed ship, traditional ice-load measurement system, influence coefficient matrix, finite element simulation

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Preface

In this study, the applicability of ICM ice load measurement system to longitudinally framed ships is investigated by FE simulation. The research software is Abaqus. The project is carried out in Helsinki, Finland.

I would like to express my deepest thanks to my supervisors, Professor Jonas Ringsberg and Professor Mikko Suominen, their support, guidance, assistance and patient during the entire thesis period have significant importance for deriving objective results. Meanwhile, I am grateful to the support of Merenkulun säätiö Finland.

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Notations

Symbols

A	Strain-load influence matrix (kN^{-1})
B	Strain-displacement matrix (mm^{-1})
C	Inverse influence coefficient matrix (kN)
E	Young's modulus (GPa)
E_{11}	Normal strain component in the local 1-direction (–)
F	Applied load vector (kN)
F_n	Crushing force (kN)
F_μ	Frictional force (kN)
F_x	Horizontal force (kN)
F_z	Vertical force (kN)
Ψ	Frame angle ($^\circ$)
K	Structural stiffness matrix (kN/mm)
SNEG	Negative shell-surface output position (–)
SPOS	Positive shell-surface output position (–)
X	Structural displacement vector (mm)
v_b	Brine volume fraction (mm)
bf	Flange width (mm)
hw	Web height (mm)
l	Structural-member length (mm)
m	Number of load components (–)
n	Number of strain measurement points (–)
s	Structural-member spacing (mm)
t_f	Flange thickness (mm)
t_w	Web thickness (mm)
x	Transverse in-plane coordinate (mm)
y	Longitudinal in-plane coordinate, parallel to the longitudinal members (mm)
z	Out-of-plane coordinate, perpendicular to the plating (mm)
β	Plate slenderness parameter (–)
γ	Shear strain (–)
ϵ	Structural strain vector (–)
λ	Column slenderness parameter (–)
ν	Poisson's ratio (–)
ρ	Material density (kg/m^3)
σ_c	Uniaxial compressive strength of sea ice (MPa)
σ_f	Flexural strength of sea ice (MPa)

Abbreviations

Abaqus	Finite element analysis software used in this study
FE	Finite element
FEA	Finite element analysis
FSICR	Finnish-Swedish Ice Class Rules
ICM	Influence Coefficient Matrix

1 Introduction

1.1 Background

Recently, sea ice in polar areas is gradually retreating because of global warming, which offers more ice-covered shipping routes to transport resources such as oil, gas and minerals (ISSC, 2012; IPCC, 2021). As a result of more operational opportunities in ice-covered areas, there is an increasing demand for more ice-going vessels to transport resources. Ice-going vessels are subjected to complex ice-induced loads due to the random mechanical interaction between ship hull and ice when operating in ice-covered regions (Daley, 1991; Kujala et al., 2019). These ice-induced loads significantly influence structural safety and operational reliability (Riska et al., 1997). Consequently, accurately measuring ice-induced loads on ship structures is a fundamental requirement for ice-class vessel structure design and the development of classification regulations, such as the Finnish-Swedish Ice Class Rules (FSICR) and the Polar Class regulations developed by International Association of Classification Societies (IACS, 2016).

Ice-induced load and structural response studies have a long history. Generally, ice-induced load study approaches can be divided into three different categories: numerical simulations, laboratory experiments and full-scale measurements, each offering distinct strengths and drawbacks. A numerical model is a simulation of real ship-ice interaction on local structures; it has advantages in studying effects of ice conditions and local breaking processes. However, it requires assumptions about various factors since real ship-ice interaction is a complex and random process. Model-scale testing is an approach that can study local structure-ice interaction, but the number of experimental repetitions is limited. Full-scale measurement is a reliable method because “all variations are embedded, including ice conditions and the ice-breaking process” (Suominen, 2018), however, there are also deviations which are caused by measuring technologies or instruments.

To enhance the accuracy of measuring, a combined application of three approaches is commonly adopted. As shown in Figure 1, all approaches are related to each other, i.e., numerical models are a simulation of full-scale measurement, full-scale measurements are used to validate modelling results. Among them, full-scale measurements provide the most realistic data and have been investigated over a long period, it can be divided into two categories: direct methods and indirect methods.

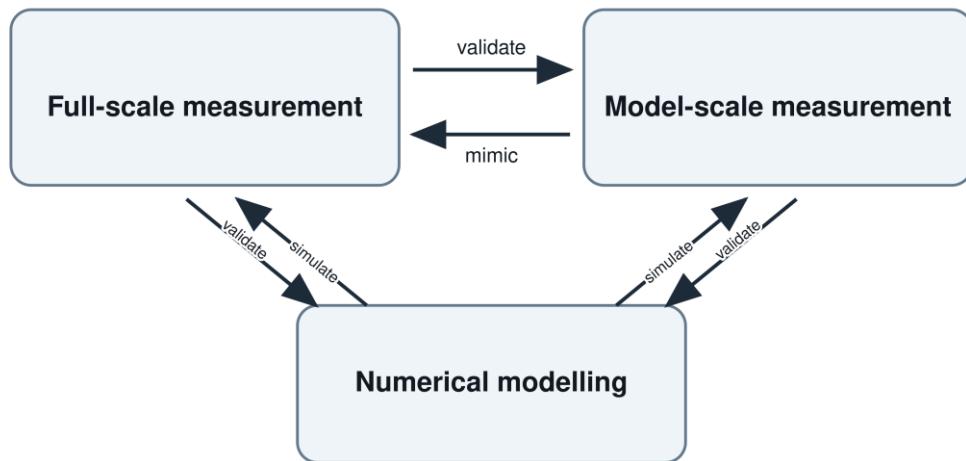


Figure 1 Ice load measurement approaches.

Direct measurement of ice loads acting on ship hull structures remains challenging because ice-ship interaction is highly localized, nonlinear, and stochastic in nature (Daley, 1991). In addition, direct force measurements may be influenced by structural modifications, local stiffness variations, sensor installation uncertainties, and long-term structural degradation (Suominen, 2018). Due to these reasons, indirect measurement techniques based on structural responses have become more widely adopted in full-scale ice load studies (Kujala et al., 2019; Suominen et al., 2017), which are known as inverse methods (Suominen, 2018). One of the currently widely used approaches to derive the relation between structural responses and ice-induced loads is Influence Coefficient Matrix (ICM) (Suominen et al., 2017). Commonly, ICM is defined using a Finite Element (FE) model to infer the applied loads (Suominen et al., 2017).

Most existing ICM measurement studies focus on transversely framed ship structures (see e.g. Riska et al., 1997; Suominen et al., 2017; Suominen, 2018). In these structures, ice loads are primarily transferred through the transverse frames, and the structural responses are well understood. The application of the ICM method to these structures has been demonstrated, although published studies that evaluate its reliability remain limited.

As noted by Rawson and Tupper (2001), modern ships are increasingly large due to the growing demand for cargo transportation. Based on structural mechanics, large ships are mostly designed with longitudinal framing to better withstand loads acting along the ship length (Paik and Thayamballi, 2007). Considering the long period and the high cost of building a new ice-going vessel, the conversion from existing longitudinally framed ships to ice-class vessels represents a growing demand. These conversions require reliable ice-induced load measurement for assessing the structural safety and operational capability in ice-covered regions (Jordaan, 2001).

The applicability of ICM measurement systems is closely related to structural systems, different structures lead to different load transferring mechanisms. However, few studies have focused on longitudinally framed ships (see, e.g., Hayward, 2001). In

particular, the sensitivity of the ICM method to different loading locations, loading patch sizes, and strain measurement approaches in longitudinal structures remains insufficiently understood. This lack of understanding raises a question about the applicability of traditional ice-load measurement methods on longitudinally framed ships.

1.2 Objectives

The thesis contributes to the development and improved understanding of uncertainty sources in ICM ice load measurement systems and provides guidance for selecting suitable strain measurement strategies for full-scale ice load monitoring of ship structures. The main objective is to evaluate the applicability of the Influence Coefficient Matrix (ICM) ice-induced load measurement systems in longitudinally framed ship structures. Structural response components of ice-induced loads consist of both bending and shearing. Different strain measurements capture different structural behaviours. Normal strain-based measurements are sensitive to bending responses and shear strain-based measurements are related to shear force distribution.

Consequently, the applicability and reliability of the ICM measurement system on longitudinally framed ships may depend on different strain responses and the various ice loading scenarios. To achieve the main objective, the ICM is determined using an FE model which simulates ice loads acting on a panel of a longitudinally framed ship. The thesis focuses on how ice-related loading conditions and different strain-response measurements influence the accuracy and reliability of the measured ice loads.

Objective 1: investigate the sensitivity of different ice loading locations on the accuracy of traditional ice load measurement method results. Ice-induced loads commonly act in different positions along the ship hull, which leads to different structural responses. Therefore, the sensitivity of ICM system under loading locations is critical for improving understanding of the applicability of ICM system in longitudinally framed ships. In this study, loading patches are simplified as rectangular uniformly mechanical pressure in FE modelling. Through moving a constant-size loading patch on the objective structure, evaluate the accuracy of measured loading which derived based on different loading locations.

Objective 2: investigate the sensitivity of the traditional ice load measurement method to different loading patch sizes on longitudinally framed ship structures. Due to the high randomness of ice-ship interaction, the contacting regions between ship hull and ice are significantly irregular, which causes the various stress concentration and effect structure responses. Understanding the uncertainties of different loading patch sizes in ICM measurement system provides insight into how contacting regions affect the accuracy of ICM system in longitudinally framed ship. This study simplifies loading patches as rectangular uniformly mechanical pressure loads, two different loading patch sizes are compared

Objective 3: compare the applicability of shear-strain-based and normal-strain-based ICM measurement method on longitudinally framed ship structures. Generally, shear

strain transfers through transverse frames in structures. Thus, shear-strain-based ICM is commonly used for transversely framed ships because it is less influenced by variations or noise. However, the structural responses through longitudinal structures may have different performances. The loading transferring mechanisms are various through different ship structures. Longitudinal structures have both bending and shearing loading transferring, the normal-strain based ICM is relevant to bending while shear-strain based ICM is related to shearing. Comparing the accuracy of both different responses based ICM gives a suitable selection of strain-based ICM and a further understanding of the applicability of ICM measurement system to longitudinally framed ships.

1.3 Limitations

In general, this thesis investigates the applicability of the ICM measuring method rather than structural responses, which means it mainly focuses on the sensitivity or feasibility of ICM ice-induced load measurement method to longitudinally framed ships.

Limitation 1: in this thesis, structures are only considered small deformations and linear elastic material behavior under peak ice loading conditions. Therefore, material nonlinearity, yielding, and failure are not explicitly considered. This simplification complies with the requirements of the ICM method, which relies on linear relationships between load and response.

Limitation 2: ice-induced loads are considered as static in the measurement by ICM method, without accounting for dynamic effects, such as ice impact velocity or fluid-structure interaction. In this analysis, ice-induced loads are simplified as single static uniformly distributed loads.

Limitation 3: the ICM method gives a relationship between structural responses and external loading. Generally, in engineering process the derived ICM relies on empirical adjustment from full-scale or model-scale tests. In this study, the main objectives are focusing on sensitivities of different load positions and different structure responses. Thus, the laboratory experiment is omitted.

Limitation 4: the present study only focuses on single longitudinally framed panel geometry, stiffener and web-frame spacing arrangement and one strain-gauge layout. Therefore, the results are specific to the present structural configuration and not directly generalizable to other ship structures.

Limitation 5: the present load ICM uses direct, unconstrained inversion. Non-negative load constraints, spatial regularization and measurement noise are not included, and the shell surface is selected by using single central reference case. Therefore, the reported measured loads that have a large deviation describe the behaviour of the present inverse formulation rather than the best performance that could be obtained from a constrained or regularized method.

2 Ice Loads on Ship Structures

This chapter provides a theoretical background and a literature review of previous studies to understand ice-induced load measurements on ship structures. It starts with sea ice conditions, sea ice properties and ship-ice interactions, including ice-breaking process and forces on ship hull during the ship operation in ice-covered regions. In addition, this chapter further introduces the general characteristics and influence factors of ice-induced load on ship structures. Finally, the methods for measuring ice loads are reviewed, including both direct and indirect approaches. This chapter explains why indirect measurement systems have been widely applied to transversely framed ships. Furthermore, differences in structural response behaviour between transversely and longitudinally framed ship structures are also discussed, which leads to different sensitivities and uncertainties of traditional measurement systems on longitudinally framed ships. The concepts in this chapter are used in Chapters 3 and 4 to provide numerical simulations of the ICM system applied to longitudinal structures.

2.1 Fundamentals of ship-ice interaction

2.1.1 Sea ice conditions and types

Ice-covered areas are located mainly in the Arctic and Antarctic regions and are also seasonally present in sub-Arctic waters such as the Baltic Sea, the Bohai Sea, and the Sea of Okhotsk (Frederking, 2017). Depending on environmental conditions and ice generation processes, ships operating in ice-covered regions will meet different ice conditions, including level ice, ridged ice, ice channels and broken ice. These ice condition differences lead to significantly various of sea ice geometry, thickness, and mechanical behaviors, which consequently influence ship-ice interaction and the resulting ice-induced loads.

Figure 2 shows the view of level ice. The development of ice-going vessels is commonly studied in the level-ice conditions, since it is the basic assumption of ship operations in ice. Figure 3 shows the ridged ice condition. It can be simply divided into two layers. The top layer consists of consolidated ice, and the keel consists of broken ice pieces.



Figure 2 View of level ice (Railo Technology Inc, 2025).



Figure 3 View of ridged ice (Nygren, 1949).

Figure 4 illustrates the ice channel, which is formed by icebreakers after the ice breaking process. The boundaries of an ice channel consist of loose broken pieces and possibly also a consolidated top layer. Figure 5 is the broken ice condition, the size and concentration of ice floe significantly affect the ship-ice interaction.



Figure 4 View of ice channel (reproduced from ABB, 2019).



Figure 5 View of broken ice (Alandia, 2024).

2.1.2 Ice properties

Sea ice presents critical challenges for ship operations in ice-covered areas due to the interaction between ship hull and ice. It has been historically studied as a natural material with complex and variable properties. The understanding of ice properties and failure mechanisms are important for researching the interaction mechanisms between ship and ice, which is further critical for measuring ice-induced loads on ship.

Sea ice is a complex material which consists of solid ice, brine, and gas and also depends on environmental temperature and solid salt types (Timco and Weeks, 2010). Generally, sea ice is divided into two categories: first-year ice and multi-year ice. First-year ice indicates the ice formed after one winter growing season. In this section, multi-year ice is discussed together with second-year ice, which refers to ice that survives after one summer period. It should be noted there is no clear standard to distinguish between the two categories. First-year ice can have various grain structures depending on different generating processes and the thickness is mainly influenced by environmental conditions (Timco and Weeks, 2010). Different structures lead to different failure mechanisms.

Figure 6 shows a typical first-year ice structure. The structure can be simplified into two layers. The top layer is granular ice, which is usually isotropic, and the bottom layer is columnar. The transition zone on Figure 6 shows the area where the ice structure changes from granular to columnar. The extending direction of columnar ice is related to the c-axis of the grain structure. The fastest growing direction is perpendicular to the c-axis, generally, this means that the columns grow vertically. Compared to granular ice, columnar ice is not isotropic, it can extend through the entire ice sheet thickness.

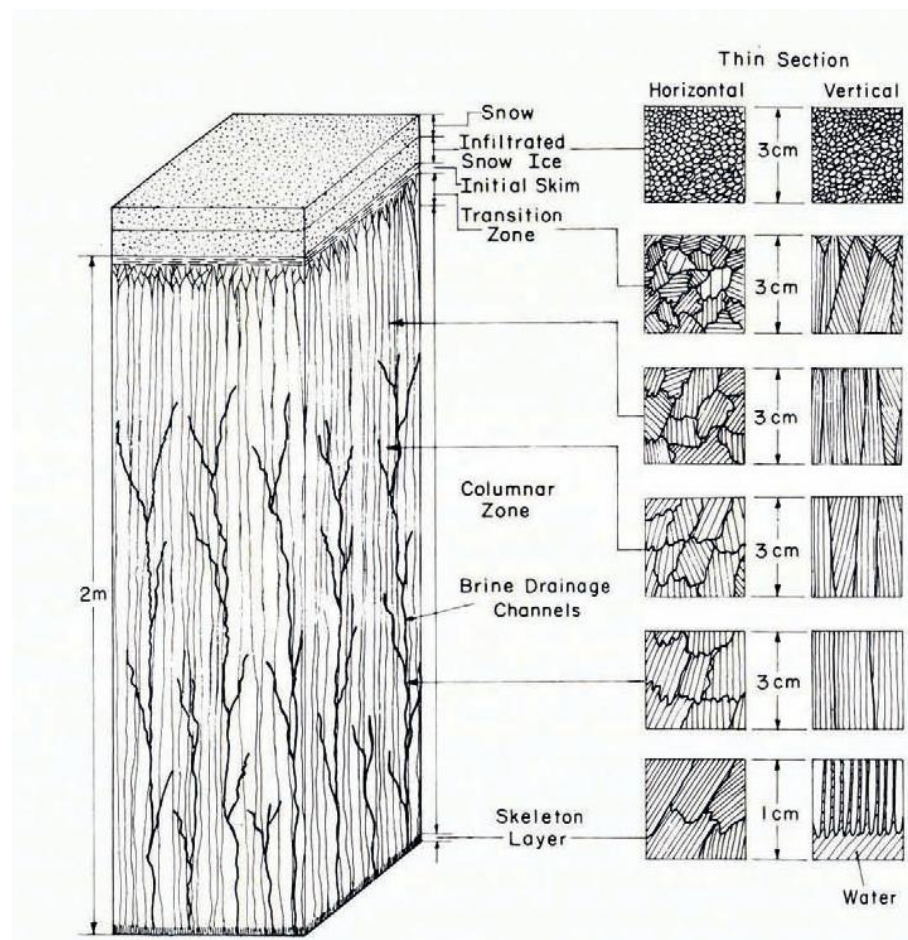


Figure 6 Typical first-year ice structure (Schwarz and Weeks 1977).

Multi-year ice has a composite structure, including a top layer which survives after melting seasons and new first-year ice structure below. Compared to first-year ice, multi-year ice has a lower salinity, which leads to smaller porosity. Therefore, the strength of multi-year ice is higher than first-year ice. The thickness of multi-year ice is influenced by both environmental and mechanical conditions, resulting in different micro grain structures between first-year ice and multi-year ice. Multi-year ice is typically anisotropic due to the preferred orientation of its columnar grain structure (Timco and Weeks, 2010).

The structural difference provides various mechanical behaviours, which is closely associated with ice-induced loads on ship hull. In general, sea ice has four main strengths which are compressive strength, flexural strength, tensile strength and shear strength. Tensile strength and shear strength are rarely measured during ship-ice interaction since they have little influence on the ice-breaking process. Ice-going vessels are commonly designed to break ice through bending as it has shown to lead to better performance in ice. When ice failure mainly occurs through bending, ice breaking process is relevant to ice flexural strength. Thus, flexural strength of sea ice is mainly investigated.

Flexural strength

Flexural strength is regarded as a material response indicator rather than a basic property. However, it is one of the most critical properties of sea ice, since most sea-ice failure occurs in flexure. Sea ice flexural strength test relies on material assumptions such as linear elasticity, plane section and isotropic behaviour. In previous studies, the flexural strength of first-year sea ice can be calculated by equation (1) (Timco and Weeks, 2010).

$$\sigma_f = 1.76 \exp(-5.88\sqrt{v_b}) \quad (1)$$

where σ_f is the flexural strength of the ice (in MPa) and the brine volume (v_b) is expressed as a brine volume fraction, as shown in Figure 7.

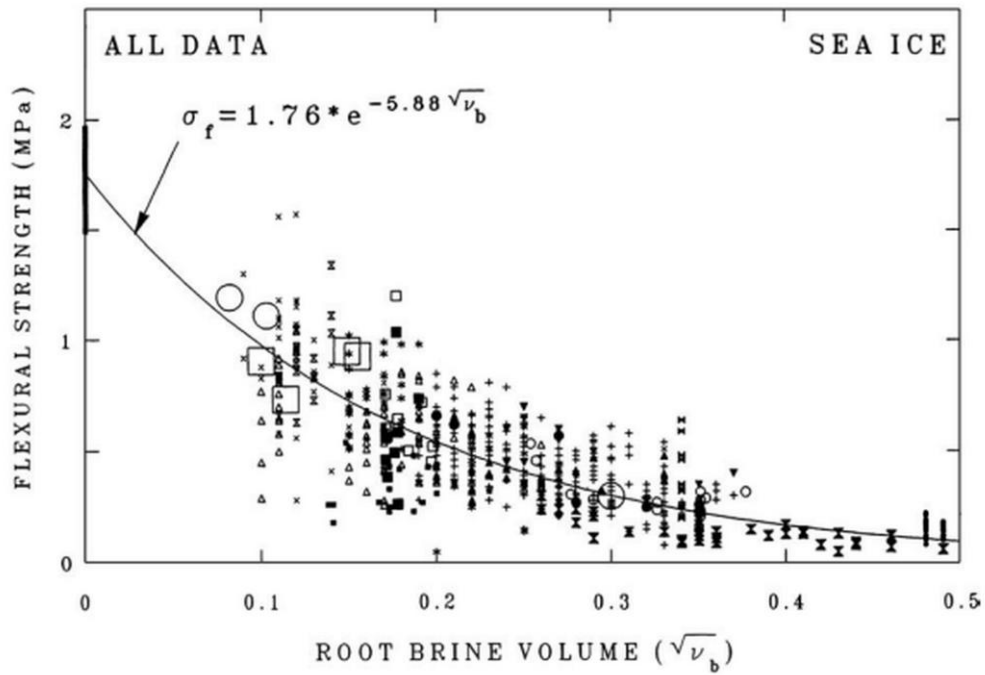


Figure 7 Flexural strength versus the square root of the brine volume for first-year sea ice (Timco and Weeks, 2010).

Compressive strength

When sea ice failure happens through crushing, such as the milling of ice by ship propellers (Timco and Weeks, 2010), compression is a critical basic property. Empirical formulas below can be used to define the compressive strength of first-year ice. equation (2a) indicates the compressive strength of columnar ice structure with horizontal loading, equation (2b) indicates the compressive strength of columnar ice structure with vertical loading and equation (2c) indicates the compressive strength of granular ice structure (Timco and Weeks, 2010).

$$\sigma_c = 37(\dot{\epsilon})^{0.22} \left[1 - \sqrt{\frac{v_T}{270}}\right] \quad (2a)$$

$$\sigma_c = 160(\dot{\epsilon})^{0.22} \left[1 - \sqrt{\frac{v_T}{220}}\right] \quad (2b)$$

$$\sigma_c = 49(\dot{\epsilon})^{0.22} \left[1 - \sqrt{\frac{v_T}{280}}\right] \quad (2c)$$

where σ_c is uniaxial compressive strength (in MPa), $\dot{\epsilon}$ is strain rate and v_T is salinity which is unit in ppt (parts per thousand).

It has been proved in previous studies that at a low temperature which is lower than -20°C , the strength of multi-year ice is similar to the strength of first-year ice (Timco and Weeks, 2010). When environmental temperature is higher, the strength of multi-year ice can be significantly increased.

2.1.3 Ice breaking process

Sea ice is considered as a natural material for ice-induced load measurement, which means it has statistical properties that depend on different parameters, such as environmental temperature, physical structure and brine volume (Suominen, 2018). Meanwhile, ice conditions have four common varieties. Ship-ice interaction is also progress related to ship hull forms. Therefore, it is difficult to have a standard theoretical method for estimating ice-induced loads for each situation. Historically, ship-ice interaction mostly has been studied theoretically in level-ice conditions. In this section, it is also described in level ice.

Generally, the flexural strength of sea ice is smaller than its compressive strength, therefore, local ice failure happens through bending during the “crushing” process (Suominen, 2018). Generally, ice-going ships are designed to break sea ice through bending failure. In ship-ice interaction, the processes of crushing and ice failure are considered together as “ice breaking” (Riska, 2017). Depending on the studies and research, ice-ship interaction mainly consists of three stages: ice-breaking, submergence and sliding. When a ship operates in level ice, the ice sheet is broken, and then ice pieces are displaced. This ship submerges the broken ice due to the hull shape and then slides ice along the ship bottom before they flocs up to surface.

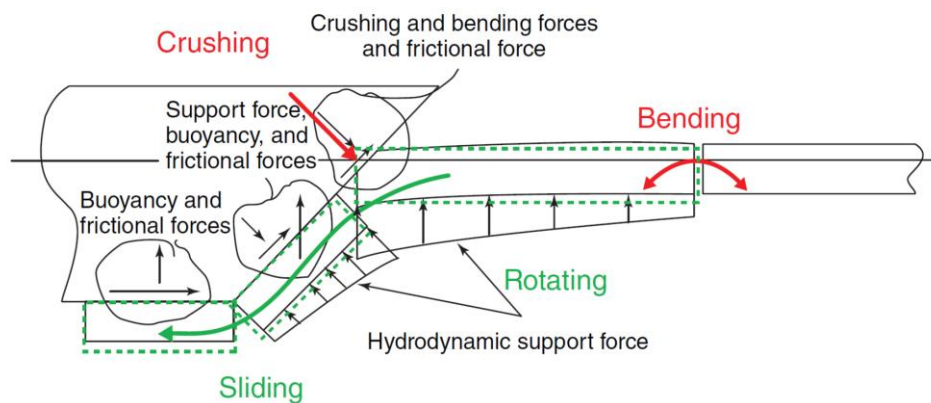


Figure 8 The forces present when a ship is proceeding in ice—illustrated for a two-dimensional case (Riska, 2017).

The ice breaking process is intermittent, since each bending failure generates ice cusps along the ship hull, which is shown in Figure 9. Before the next bending failure happens, these cusps will be crushed first, and the bending force gradually increases until reaching the next bending strength (Riska, 2017). This progressive process leads to a breaking pattern, as shown in Figure 10. From spatial distribution perspective, these crushes commonly occur at vessel stems and shoulders, while bending failure typically occurs at the inclined bow region.

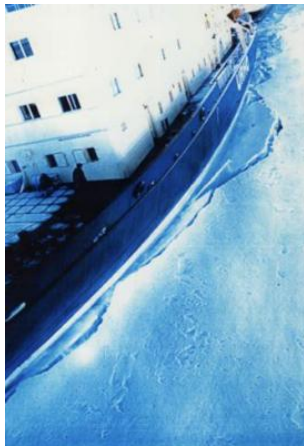


Figure 9 The breaking pattern around the ship hull (reproduced from Suominen (2018); original photograph source not identified.)

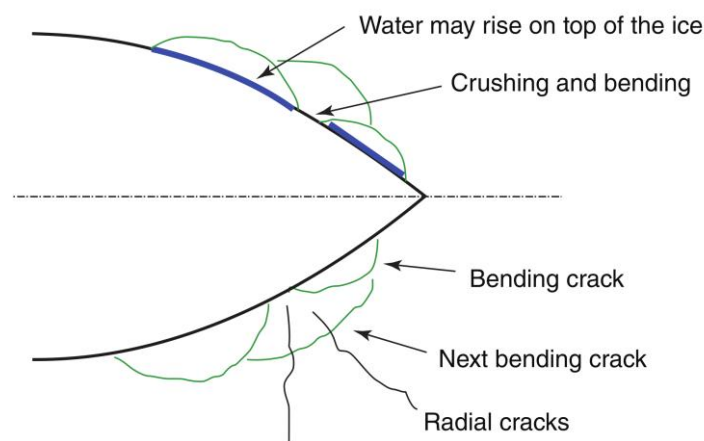


Figure 10 Schematic view on the breaking pattern (Riska, 2017).

It is impossible to accurately predict the breaking pattern through theories, although it has been found to have some regularity during the past few decades. As in recent studies, it illustrates irregularity even with constant ice thickness and ship speed (Riska, 2017). The bold blue line in Figure 11 illustrates the path of sliding and submerging broken ice. It can be seen that these ice floes mainly submerge and slide along the ship hull in the vertical direction. However, due to the role of buoyancy, the floes will move sideways from the original vertical hull line where the breaking happens.

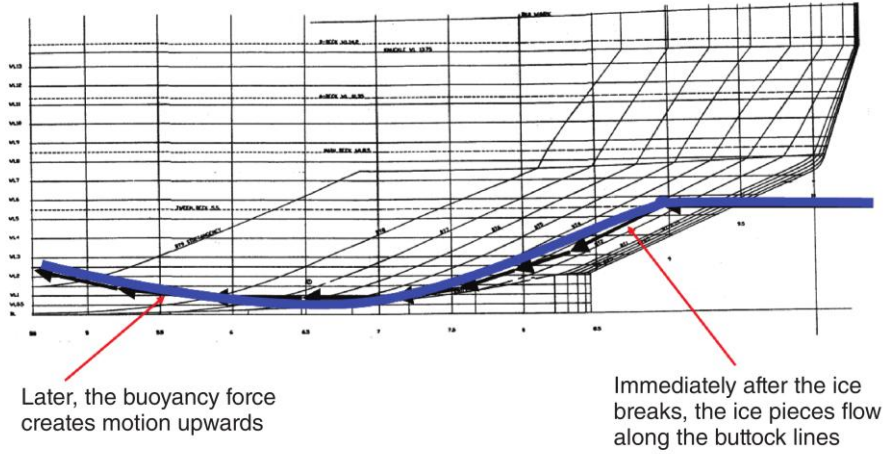


Figure 11 The main path (shown in bold line) of sliding/submerging ice floes (Riska, 2017).

Ice-induced loads are generated during ice breaking process, mainly associated with crushing and bending processes. Figure 12 shows the forces acting on ice during a breaking process, where F_n is the crushing force and F_μ is the frictional force from ship. Based on the bending direction, these two forces are divided into two components: a horizontal force (F_x) and a vertical force (F_z), and can be presented as:

$$F_x = F_n \cos \Psi + F_\mu \sin \Psi \quad (3a)$$

$$F_z = F_n \sin \Psi + F_\mu \cos \Psi \quad (3b)$$

Here, Ψ is defined as the frame angle which is between F_n and the horizontal line, as shown in Figure 13. The vertical force (F_z) will gradually rise along with the increasing contacting area between ship and ice, and reach to the ice flexural strength during the bending process.

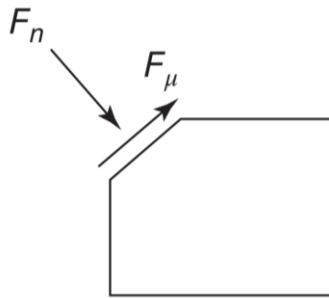


Figure 12 Force components acting on ice during a breaking process (Riska, 2017).

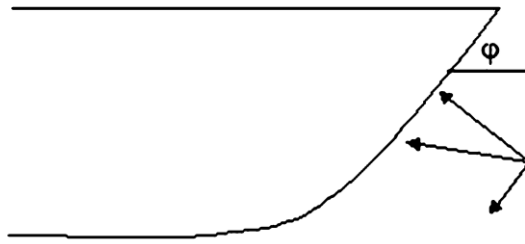


Figure 13 Profile line of a ship and forces on hull in a two-dimensional case (Riska, 2017).

According to Newton's third law, the forces on the ship can also be inferred as the inverse forces of loads on ice. However, when a ship operates in ice-covered regions, ice loads are generated during the cyclic ice-breaking process. Thus, the actual loading magnitude and distribution are further influenced by many factors, such as contact geometry, friction conditions, and operational parameters, which will be discussed in the following section.

2.2 Characteristics of ice-induced loads

Ship-ice interaction is a highly complex process, and ice-induced loads exhibit variabilities in both magnitude and frequency due to various uncertainties, including the discontinuous breaking process during ice failure, the continuous manoeuvring of the vessel, and the variabilities of ice properties, such as thickness, strength, and salinity. These distinguishments lead to a high randomness of contact conditions between ice and hull structures. The temporal and spatial distribution of ice-induced load on ship hull both exhibit irregularities (Zhao et al., 2025).

Generally, ice-induced loads have randomness which are caused by different parameters including ice-related factors and ship operating factors. Meanwhile, ice-induced load generally concentrates on small contact regions between ice and ship hull (Masterson and Frederking, 1993). In addition to the general characteristics, ice-induced loads can also be considered as an event consisting of several individual loading events in the time domain. A typical individual ship-ice interaction process includes both loading and unloading phases, which start with contacting and end up separating. In engineering analysis, extracting loading magnitude in time domain is the basis for ice-induced load measurement.

However, it is inaccurate to extract loading magnitude depending on peak values, since the high-frequency components of the measured signal may contain spurious peaks—so called 'glitches' phenomenon which generated from electrical noise, sensor resonance, or digitization artifacts. (Zhao et al., 2025). The Rayleigh classification is a widely used method to analyse individual ice-induced loads on ships. It identifies local maximum values and local minimum values within the signal and defines loading scenarios by comparing the ratio between a local maximum and minimum (Zhao et al.,

2025). When this ratio exceeds a defined threshold, the corresponding signal is counted as an independent loading cycle. This method significantly reduces the noise influence. Figure 14a and 14b show an example of Rayleigh separation, with a valley value of 10kN. Ice load characteristic parameters including peaks, valleys, time cycles, loading and unloading phases can be observed by using Rayleigh classification method.

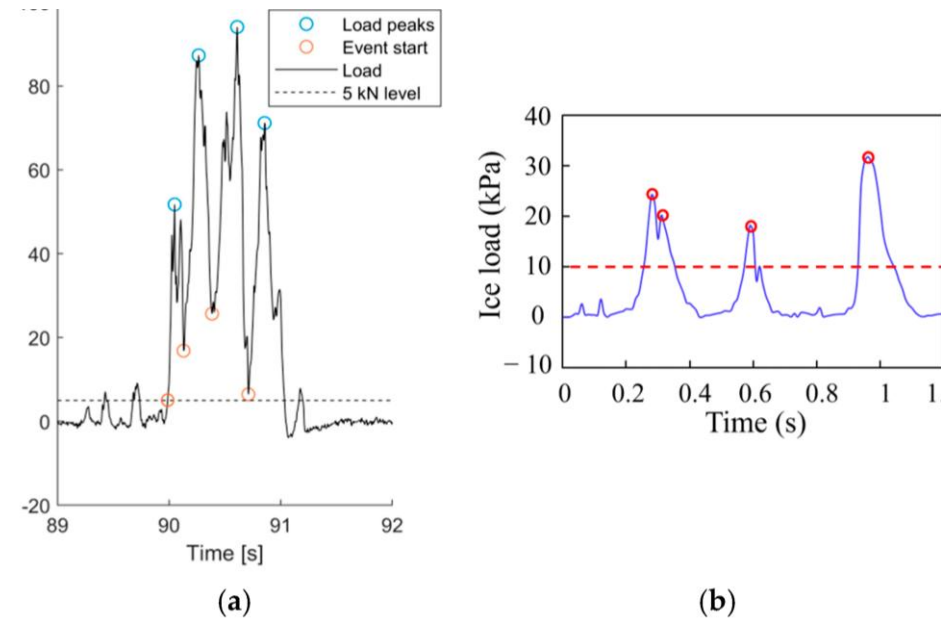


Figure 14 Rayleigh separation (Zhao et al., 2025).

Although ship-ice interaction exhibits certain regular patterns, the general process still shows randomness. However, the overall time-history of ice loads still shows randomness, which is mainly due to the irregular spatial distribution of ice, such as floe sizes, thickness differences, and contacting regions. Figure 15 represents the measured ice load and ship speed during ice-breaking tests of the ‘Sun Yat-sen University Polar’ vessel in the Bohai Sea. It can be observed that ice load has stochastic characteristics.

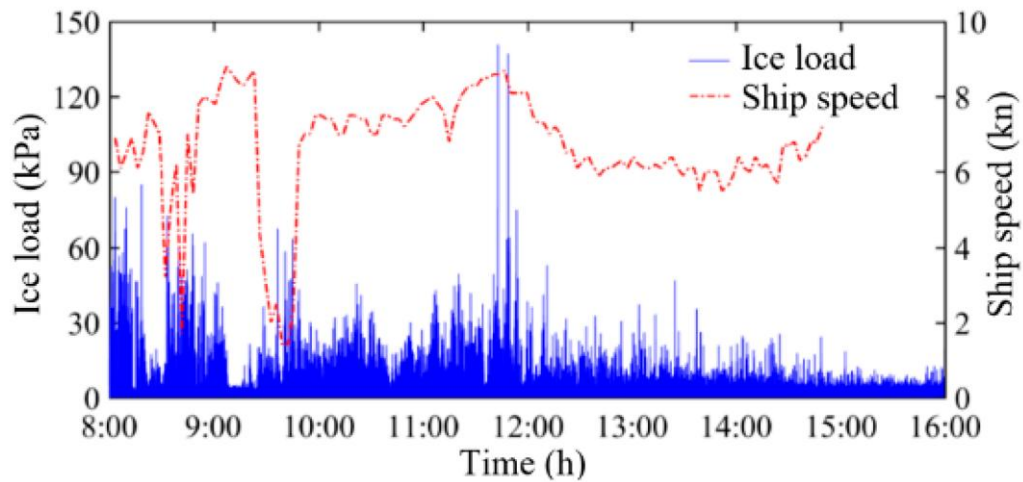


Figure 15 Time-history curve of local ice load and ship speed during icebreaking test. (Zhao et al., 2025).

2.2.1 Ice-induced load temporal distribution

One ice load cycle is defined as the time spacing between single ice load events. During the loading phase, ice-induced load continuously increases until reaches maximum ice flexural strength, when ice failure occurs, the unloading phase begins till the end of this cycle. However, there might be secondary ice breaking between loading and unloading phases, or other interaction between ship and ice. Thus, ice loading variation under time-history can be a single event or a continuous process with intermediate peaks caused by secondary failures during the loading-unloading phases.

Following the classification proposed by Zhao et al. (2025), temporal distribution of ice-induced load can be divided into four categories based on different intermediate peak variations: single-peak ice load, intermediate peak in the loading phase, intermediate peak in the unloading phase, and intermediate peaks in both the loading and unloading phases, which are shown in Figure 16.

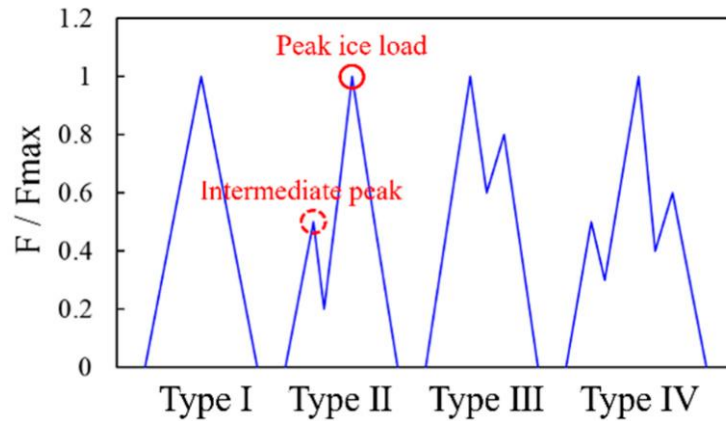


Figure 16 Classification of ice load time-history curves (Zhao et al., 2025).

2.2.2 Ice-induced load spatial distribution

Ice breaking patterns include repeated crushing and ice failure processes. The repeated crushing process applies pressure on ship hull at the area where contact occurs. Figure 17 illustrates a model scale testing result of a pressure distribution along the ship hull during the ice breaking process. It can be observed contact occurs in a narrow line, at areas which are called “hot spots” (Kujala, 2017). These “hot spots” continuously change in both time and space domain because of the randomness of the complicated ship-ice interaction process (Kujala, 2017). Moreover, as the vessel advances in ice-covered area, the contact location gradually moves along the ship hull from bow to stern (Suominen et al., 2017).

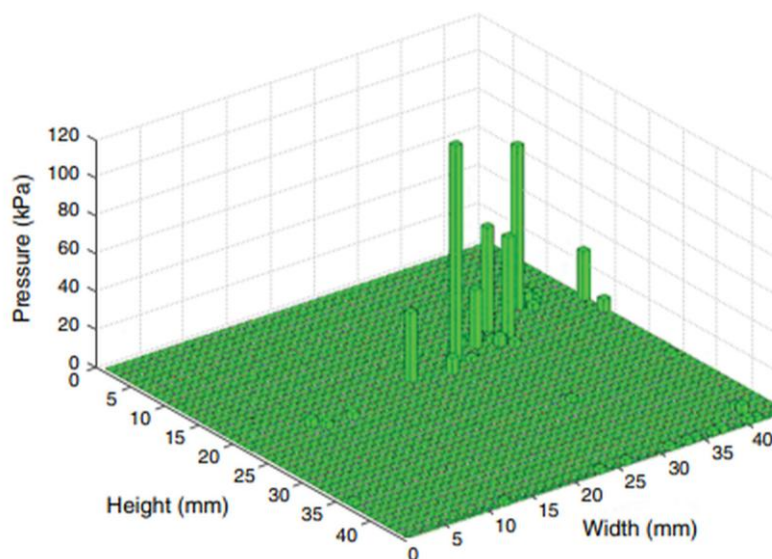


Figure 17 Typical pressure distribution along the ship hull when breaking ice (observed in model scale) (Kujala, 2017).

During a breaking process, contact length is measured in the horizontal direction. It is defined by ice floe size and the breaking processes and is clearly longer than the contact height. The pressure in horizontal direction includes high-pressure zone and low-pressure zone, since it varies significantly in this situation (Suominen, 2018).

The vertical contact location is defined by the ship floating position and the ice thickness which is different from contact length in horizontal direction. Ice edge failure process is shown in Figure 18, β_n is the frame angle of a ship, h_c is contact height and h_i is the ice thickness. Theoretically, contact height can gradually reach ice thickness during the process, however, it has been proved in previous laboratory studies that high-pressure area is clearly narrower than ice thickness, which is normally line-like (Suominen, 2018). However, when the crushing speed is reduced, the contact in the vertical direction becomes higher which is no longer line-like, in this situation, it consists of both high-pressure zone and low-pressure zone, similar to the horizontal contact pattern.

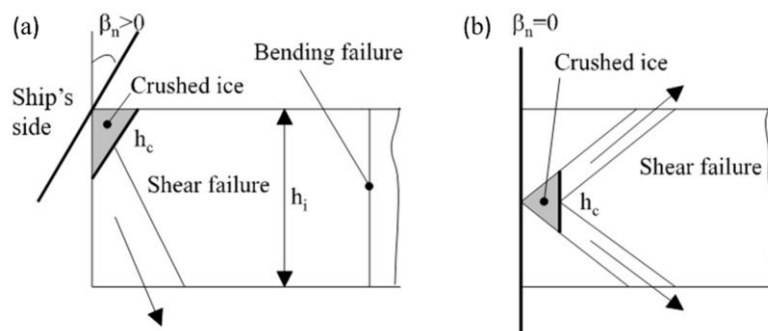


Figure 18 Ice edge failure process (Kujala, 1994).

2.2.3 Parameters influencing ice-induced load

The parameters which affect ice-induced load can be divided into two categories: ship-related factors and ice conditions (Suominen, 2018). Dominant ship-related factors include ship hull geometry and operational factors, while ice conditions mainly include ice thickness, ice concentration and ice failure strength.

Operational factors

From the operational perspective, ship speed and manoeuvring behaviour significantly influence the ice-loading process through dynamic ship-ice interaction, impact effects and strain-rate-dependent ice behaviour (Suominen et al., 2020; Tan et al., 2014). Ship manoeuvring operations affect the contacting area between ship hull and ice, particularly turning motions may increase the frequency and magnitude of ice-induced loads at the stern shoulder and midship regions due to increased side interaction with ice. The increasing ship speed also leads to higher peak ice loads and contact pressures, although the relationship depends on ice thickness and failure mode (Tan et al., 2014).

Hull geometry

The hull geometry is a parameter that closely related to ice-breaking mechanism and local pressure distribution. Ships with a small frame angle at the bow lead to ice bending failure, while vertical hull shapes tend to induce local crushing and high-pressure concentrations (Suominen et al., 2017). The effect of hull geometry on ice loads is therefore independent from ship operation factors.

Ice thickness

Ice thickness directly determines the total resistance during the ship-ice interaction process. It is a dominant factor that affects ice pressure amplitude. Thicker ice sheets require higher forces to break, which means thicker ice presents larger ice-induced loads. (Zhao et al., 2025).

Ice concentration

Higher ice concentrations lead to more frequent and continuous interactions, which affects the total accumulation of ice-induced loads. Moreover, floe size distribution is also relevant to the ice-induced loads. It can be considered a secondary factor compared to concentration and mainly influences the spatial variability of contact conditions during ship-ice interaction process.

Ice failure strength

Ice flexural strength and compressive strength are related to ice failure process. These strengths are highly variable. The most significant influencing factors are environmental temperature and sea ice salinity. Lower temperature generally increases ice strength, while higher salinity reduces it by increasing brine volume within the ice structure (Timco and Weeks, 2010). Therefore, temperature and salinity affect ice loads indirectly through their influence on strength.

2.3 Methods for ice load measurement

2.3.1 Direct measurement methods

Direct measurement is a straightforward method that measures contacting pressure or force using instruments such as internal or external pressure gauges. This approach is significantly influenced by measuring instruments themselves. Mechanical wear of instruments normally occurs during repeated high-magnitude loading, which compromises precision (Croasdale et al., 1986). In addition, some instruments will alter the local structure at the measurement area, although the alteration is slight, the structural stiffness has been changed (Croasdale et al., 1986). It is challenging to study the effect of the measurement accuracy of the direct methods.

The key factor of measurement accuracy is pressure sensors, which capture loading in contact areas. Three widely used pressure sensor types are single-point, pressure mapping systems and load panels/ instrumented plates.

Single-point pressure sensors

Single-point pressure sensors are used to measure local ice load at certain locations on the hull, and they can be located at discrete points without affecting each other. Single-point pressure sensors are suitable for long-term observation in harsh environments. However, it has a disadvantage that cannot capture the spatial variability of ice loads, which is critical for analysis (Croasdale et al., 1986).

Pressure mapping systems

Compared to single-point pressure sensors, pressure mapping systems can capture ice load spatial distribution on ship structures since it consists of arrays of small sensing elements. Currently, a system called “I-Scan” manufactured by TekScan is widely used for experimental ice load measurement (Kubiczek et al., 2022). However, these pressure mapping measurement systems have several limitations. The equilibration and calibration can result in measurement uncertainties (Kubiczek et al., 2022). In addition, environmental conditions also influence measurements obtained in contact areas.

Load panels / instrument plates

Load panels or instrument plates are widely used to measure ice-induced load across defined areas. A typical load panel sketch is shown in Figure 19, it consists of one lid and three sensors, two load sensors are placed close to the top, and one is near to the bottom (Hellgren et al., 2020). The solid lid limits elastic deformation based on its high stiffness. The use of three sensors reduces the influence of panel vibration and can define the location of the resultant force more accurately (Hellgren et al., 2020).

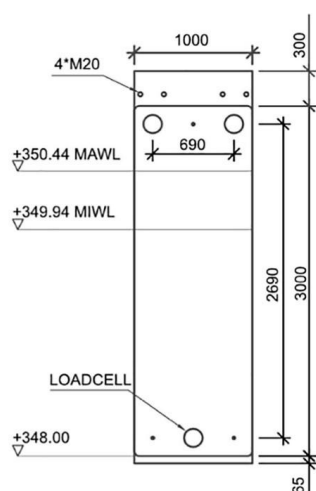


Figure 19 Typical ice load panel sketch (Hellgren et al., 2020).

However, these systems have disadvantages in spatial resolution and have uncertainties in measuring loading peak value due to load averaging over the entire panel area. In addition, these systems are commonly used for measuring compressive pressure instead of bending based on the characteristics.

2.3.2 Indirect measurement methods

From the perspective of indirect measurement methods, the ship structural responses are measured, however, the external loads that caused the deformation are unknown. Consequently, ship-ice interaction is commonly formulated as an inverse problem, where external loads are estimated from measured structural responses. Thus, indirect methods refer to approaches that ice-induced loads are inferred from the measured structural responses rather than measuring loads directly. From indirect method perspective, establishing load-response relationships is an important basis. Commonly, the relationship between external loads and structural responses is defined by FE simulations (Suominen, 2018).

Structural responses used for ice-load measurements include shear strain and compressive strain. In shear-strain measurement, gauges are installed in pairs at the two ends of a frame. According to beam theory, the difference between the shear forces at the two ends is equal to the external transverse load acting on the frame. This shear load is determined from the measured shear strain. Compressive strain measurement aims to measure the local compressive forces at the locations where gauges are installed on a frame. Thus, in order to have a clear understanding of the strain on whole frame, the number of gauges on a frame in compressive strain measurement is normally more than that in shear strain measurement.

2.3.3 Sources of measuring uncertainty

Understanding uncertainties of full-scale measurements is critical to assessing the measurement accuracy. These uncertainties are generated by multiple sources and result in biases of estimated ice-induced loads. This section focuses on how different factors influence the measurement uncertainty.

Ice-related uncertainties

Sea ice is introduced as a highly variable natural material. Variations caused by environmental temperature, grain structures lead to significant uncertainty in the magnitude and distribution of ice-induced loads. These variations influence the size and the shape of the contacting regions between sea ice and ship hull, also further influence the measurement accuracy (Böhm et al., 2021). Since the contacting location leads to internal loading distribution and transition.

If the ice load acts outside the instrumented region of the hull, the measured strain response may not accurately represent the actual load, leading to significant errors in the estimated load magnitude. Similarly, the length of the load patch influences how the load is distributed across adjacent frames, which in turn affects the strain measured at the instrumented sections.

Measurement uncertainties

Similar to direct measurement methods, indirect measurements also rely on sensors, which means the accuracy is affected by factors such as sensor resolution, calibration deviations and installation conditions. This study mainly focuses on the FE simulation process, so the measurement uncertainties are neglected in this paper.

Model-related uncertainties

During FE modelling simulation, assumptions including linear elastic material, static loading and uniformly distributed pressure are used. This generates uncertainties from the differences between simulated and real structural responses.

2.3.4 Applicability to different structural systems

Historically, transverse framed ice-going ships are more common than longitudinally framed ice-going ships. Also, in transverse systems, the load transfer mechanism is more direct compared to longitudinal systems, which leads to a clear relationship between measured strain and external loads. Thus, strain-based methods can be effectively used to measure ice-induced loads. Therefore, indirect full-scale measurements are applied to local transverse frames of ships (see, e.g., Jeon et al., 2018; Suominen et al., 2017).

In contrast with transversely framed ships, the application of indirect measurement methods to longitudinally framed structures have been less studied in previous research. In longitudinal framing systems, external loads are transferred through both longitudinal stiffeners and transverse web frames, resulting in a more complex load transfer mechanism. Consequently, local structural responses are distributed to a wider region compared to transverse framing systems. Therefore, the relationship between measured strains and external ice loads in longitudinal framing systems is more difficult to establish.

Consequently, accuracy assessment and numerical modelling simulation become increasingly significant when studying indirect load measurement methods to longitudinally framed structures. This study aims to investigate the relationship between localized ice-induced loads and structural responses through finite element modelling of a longitudinally framed ship structure.

3 Methodology

This chapter presents the theoretical foundations and methodologies used in this study. The overall relationship between the different methods is illustrated in Figure 20. In FE analysis, unit loads are applied to define load-strain relationships, which depends on the superposition principle. This relationship is required for establishing the Influence Coefficient Matrix (ICM), which is the foundational principle of this study. Two types of strain responses – shear strain difference and local normal strain – are considered as indicators of structural response. The measured strain responses are extracted and converted into equivalent ice loads by using the established ICM.

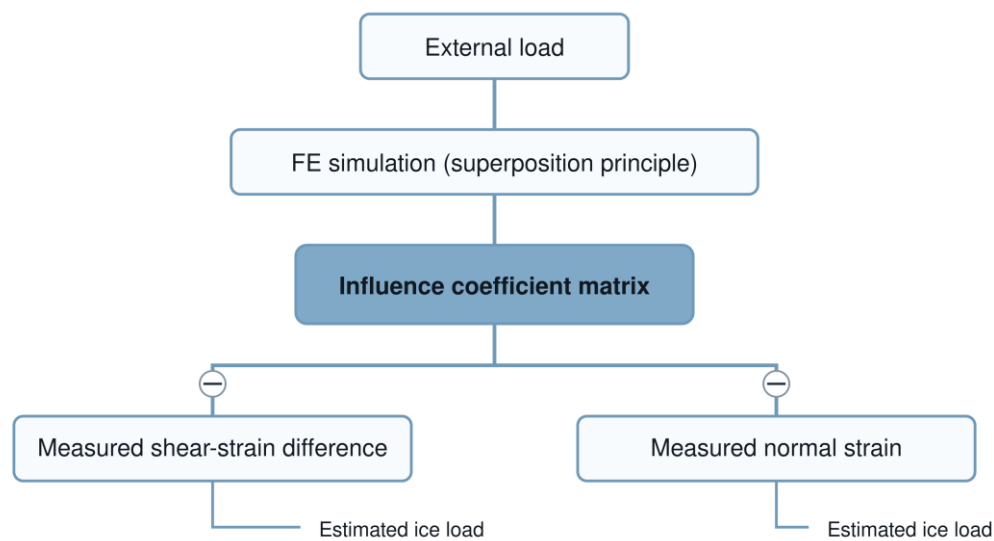


Figure 20 Methods adopted by the study.

3.1 Structural responses under loading

When external loads are applied to a ship structure, structural strain responses are generated. By measuring these strain responses, the external loads can be estimated through the relationship between structural response and loading. In this study, two types of strain responses are considered: shear strain difference and local normal strain. Their physical meanings are introduced in the following sections.

3.1.1 Shear strain difference

Measurement interpretation depends on classical beam theory, where the individual structures such as stiffeners are considered as beam elements. According to Euler-Bernoulli beam theory (Ahmed and Rifai, 2021), the derivative of the bending moment

gives the shear force. For the concentrated load shown in Figure 21a, the shear force has opposite signs on the two sides of the loading point, as shown in Figure 21b. The shear strain varies over the height of the section, but it is proportional to shear force when the strain is extracted at corresponding locations. Therefore, the difference between a pair of measurements taken on opposite sides of the loading region can be used as an indicator of the external transverse load.

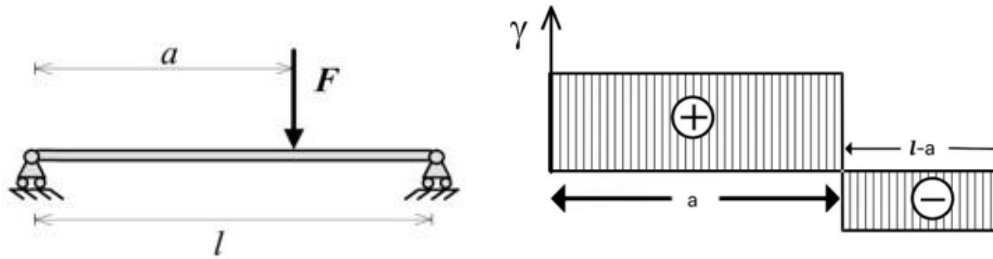


Figure 21 (a) Left: concentrated load on a beam, and (b) right: shear-force distribution for the loading case shown in (a).

3.1.2 Normal strain

External transverse loading also produces bending of the structural members. For a simply supported beam under a concentrated load, the bending moment is zero at the supports and reaches its maximum under the loading point. The normal strain follows the bending-moment distribution: its magnitude increases toward the loading point, and it changes from compression to tension across the neutral axis. Consequently, a normal-strain measurement describes a more local bending response than the paired shear-strain measurement and is more sensitive to the loading position and selected gauge location. In the present FE model, the E11 component is extracted from the stiffener-web shell elements at the predefined gauge positions.

3.2 Influence coefficient matrix (ICM)

According to Kujala (2017), the developed ice-induced loads estimation approaches can be divided into three categories: numerical time-domain simulation methods, energy conservation methods, and semi-empirical methods based on full-scale measurement data. The ICM is an important component of the approach used to account for the impact of surrounding structures in the full-scale measurement process. The main foundation is to establish the ICM through numerical simulations, which makes it possible to determine the ice load. The external ice load is defined as the equivalent resultant force which is inferred from the ICM and full-scale measurements.

Based on mechanical theory, it is easily known that when ice-induced loads apply on the ship hull structure, a portion of the load is directly applied to the frames which are in contact with the ice, while the remaining portion is redistributed to adjacent frames because of the connections between frames. The ICM is an efficient approach to study the relations between ice load and structure responses, which can also improve the accuracy of load identification (Zhou et al., 2025).

With the neglect of high-frequency dynamic effects, when only considering the quasi-static responses, the relationship between the applied load $\mathbf{F}$ and structural displacement vector $\mathbf{X}$ can be mathematically expressed as equation (4). Where $\mathbf{K}$ is the structural stiffness matrix. Structural strain vector $\boldsymbol{\varepsilon}$ is linearly related to displacement vector $\mathbf{X}$ through strain-displacement relationship, as shown in equation (5a), where $\mathbf{B}$ is the strain-displacement matrix. By substituting $\mathbf{X} = \mathbf{K}^{-1} \mathbf{F}$ into equation (5a), strain $\boldsymbol{\varepsilon}$ is linearly related to applied load $\mathbf{F}$, the relationship between strain vector and applied load is shown in equation (5b).

$$\mathbf{F} = \mathbf{K} \mathbf{X} \quad (4)$$

$$\boldsymbol{\varepsilon} = \mathbf{B} \mathbf{X} \quad (5a)$$

$$\boldsymbol{\varepsilon} = \mathbf{B} \mathbf{K}^{-1} \mathbf{F} \quad (5b)$$

By defining the strain-load influence matrix $\mathbf{A}$ as equation (6), the relationship between applied load $\mathbf{F}$ and structural strain $\boldsymbol{\varepsilon}$ can be formed as equation (7a) and (7b). In this study, $\mathbf{F}$ is ice-induced loads that are applied on researching structures, $\mathbf{C}$ is the ICM that is used to estimate external loads from measured strains.

$$\mathbf{A} = \mathbf{B} \mathbf{K}^{-1} \quad (6)$$

$$\boldsymbol{\varepsilon} = \mathbf{A} \mathbf{F} \quad (7a)$$

$$\mathbf{F} = \mathbf{A}^{-1} \boldsymbol{\varepsilon} = \mathbf{C} \boldsymbol{\varepsilon} \quad (7b)$$

3.3 ICM establishment based on the superposition principle

The superposition principle is a fundamental basis of the ICM. The contact area between ice and ship hull is irregular, it is inaccurate to simplify the load as a concentrated force which acts on a small point. Therefore, a large ice load patch can be considered as a combination of small patches, utilizing superposition principle. This implies that structural responses caused by a large ice load patch are equal to the sum of responses caused by unit small patches (Zhu and Suominen, 2025). Meanwhile, the total force applied on the structure is equal to the sum of forces on each small patch. Figure 22 illustrates the distribution of structural strain and the corresponding force, where $\mathbf{C}$ is an ICM.

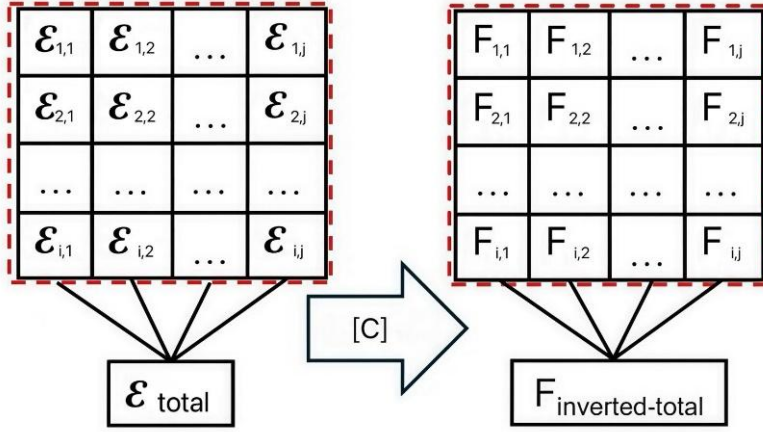


Figure 22 Decomposition of a larger load patch into unit patches (Zhu and Suominen, 2025).

To establish the ICM numerically, the potential ice contact area is divided into a series of sub-patches. A unit load is sequentially applied to each sub-patch while the remaining sub-patches remain unloaded. For each loading case, the strain responses at all measurement locations are extracted from the FE model. Assuming dimensions of the $\mathbf{F}$ matrix is $\mathbf{F} \in \mathbb{R}^{m \times 1}$, $\boldsymbol{\varepsilon}$ matrix is $\boldsymbol{\varepsilon} \in \mathbb{R}^{n \times 1}$, it can be derived that $\mathbf{A} \in \mathbb{R}^{n \times m}$ and $\mathbf{C} \in \mathbb{R}^{m \times n}$. To determine the values of ICM (matrix $\mathbf{C}$) numerically, unit loads are applied to the FE model.

For each simulation, a 1 kN uniformly distributed load is applied to one sub-patch solely, while all other sub-patches remain unloaded. The strains recorded at all measurement locations form one column of matrix $\mathbf{A}$. The simulation is then repeated for every loading sub-patch, so that each column of $\mathbf{A}$ represents one unit-load location and each row represents one strain measurement. After matrix $\mathbf{A}$ is assembled, as illustrated in equation (8a), matrix $\mathbf{C}$ is obtained by inversion and is used to estimate the equivalent loads from the measured strains. During full-scale measurements, the measured strain responses are substituted into equation (8b), the equivalent ice load can be estimated directly.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{Bmatrix} \quad (8a)$$

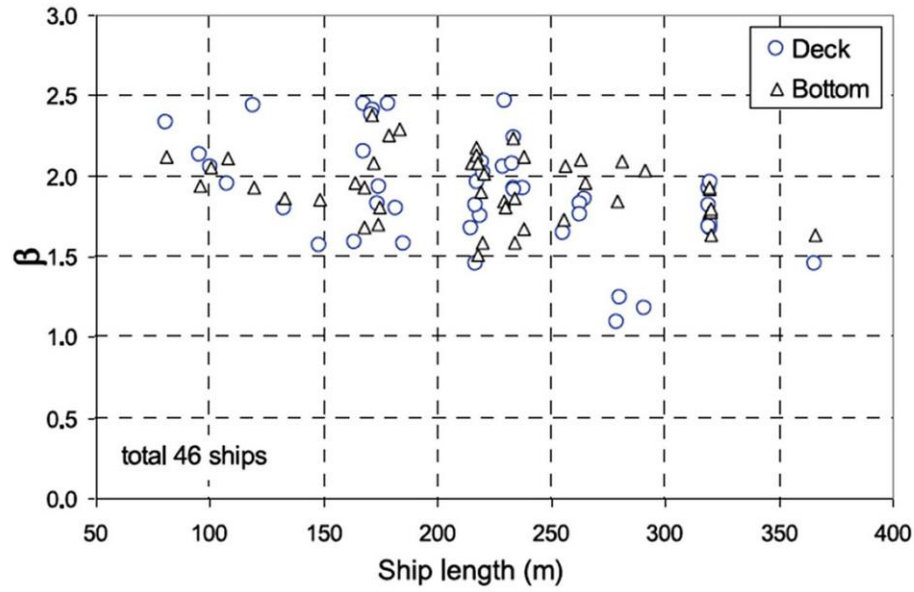
$$\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ C_{m1} & C_{m2} & \dots & C_{mn} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \quad (8b)$$

4 Numerical Model

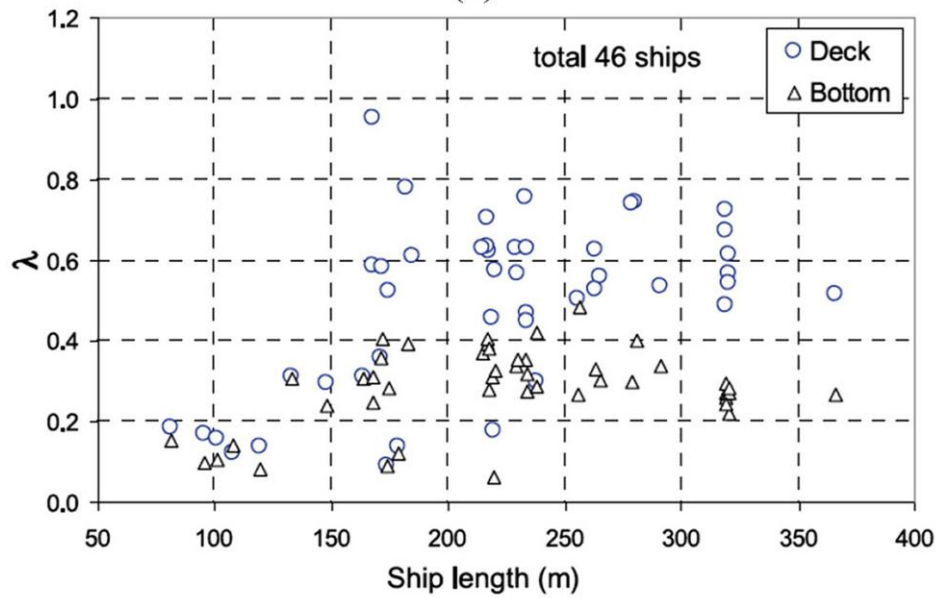
In this study, numerical simulations are performed in Abaqus (Dassault Systèmes Simulia Corp., 2023a). This chapter presents the numerical model of a representative longitudinal ship structure that is used in the study. First, the geometry of the FE model is described. Second, the mesh size adopted in the FE model is presented together with the mesh sensitivity considerations, in order to ensure the accuracy and computational efficiency of the numerical simulations. In addition, the material properties and the boundary conditions are introduced. Finally, the placement of strain gauges for both shear strain and normal strain measurement are defined, which are closely related to the accuracy of established ICM.

4.1 Geometry of FE model

The primary objective of this study is to perform a sensitivity analysis of ICM ice load measurements on longitudinally framed ships, aiming to support the transition from conventional ships to ice-going ships. Therefore, a generally representative panel of longitudinally framed ships is used. Zhang (2016) provided statistical geometric data of 46 longitudinally framed ship structures, including bulk carriers and crude carriers whose length are between 150 m and 400 m. Figure 23 shows the dimensional range of sample ship structures, where β is plate slenderness and λ is column slenderness for the sample ship structures. β and λ are dimensionless parameters commonly used to characterize the slenderness of plating and stiffeners. Based on these statistics together with Polar Class (IACS, 2016), the typical ranges of plate dimensions and for general longitudinally framed vessels are summarized in Table 1, which is adopted by Pan et al. (2019) in their analyses.



(a)



(b)

Figure 23 Dimensional range of the sample ship structures, where β is plate slenderness and λ is column slenderness (Zhang, 2016).

Table 1 Statistical range of structure dimensions.

Length (mm)	Thickness (mm)	Spacing (mm)
2400-6000	12-36	350-450

As shown in Figure 24a, the objective structure which is a panel consists of a hull plating area, two transverse web frames, two longitudinal stringers, and five longitudinal stiffeners. The web frames and stringers are modelled by T-shaped sections, the longitudinal stiffeners are modelled by bulb-flat sections, which are

commonly used in longitudinally framed ship structures. The dimensions of the structural members are shown in Table 2. Depending on the statistics of ship structure dimensions, the objective structure is defined as a representative longitudinally framed ship side panel, which is 2400 mm $\times$ 2400 mm. Figure 24b illustrates the cross-sectional dimensions of web frames and stringers and Figure 24c presents the idealized cross-section of the longitudinal stiffeners used in the shell-element model.

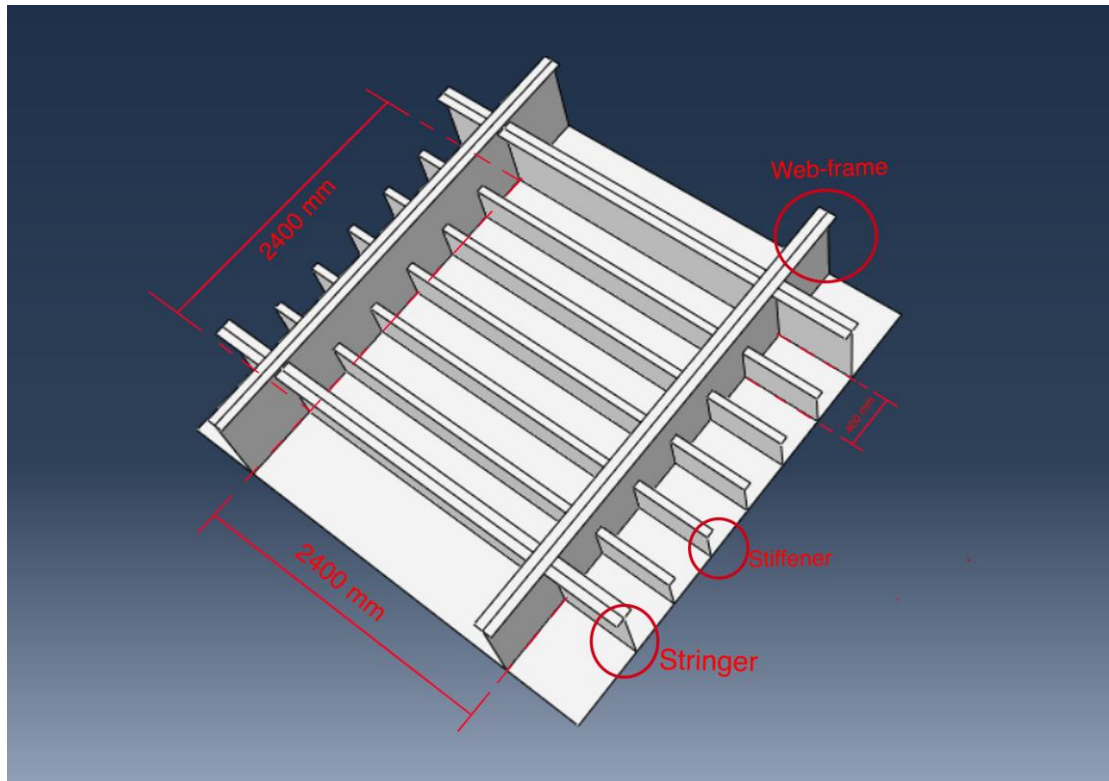


Figure 24a The view of objective structure.

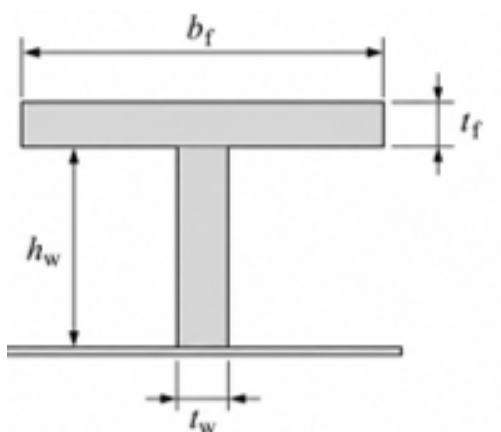


Figure 24b Cross-sectional dimensions of web frames and stringers.

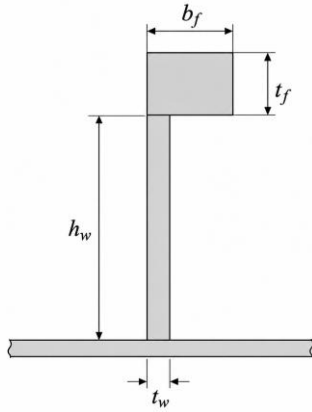


Figure 24c Idealized cross-section of the longitudinal stiffeners used in the shell-element model.

Table 2 Dimensions of structure components: l -length, s -spacing, h_w -height, t_w -web thickness, b_f -flange width, t_f -plate thickness.

	l (mm)	s (mm)	h_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)
Web-frames	2400	2400	800	22	140	24
Stringers	2400	2400	660	18	140	20
Stiffeners	2400	400	340	14	63	35

Regarding efficient calculations and maintaining acceptable engineering accuracy, shell elements are more common to be used in Finite Element (FE) model investigation compared to solid elements. As presented in the Abaqus documentation (Dassault Systèmes Simulia Corp., 2023b), the four-node general-purpose shell (S4) employs a fully integrated scheme, which eliminates the need for hourglass control and avoid non-physical strain modes. Meanwhile, it ensures accurate solutions for in-plane bending behavior. In addition, the ice-induced load may produce localized peak strains on the structure or cause a large strain gradient, shell element types that reduced the integration points such as S4R may influence the strain prediction. Therefore, considering that longitudinally framed ship structures involve significant inner-plane bending and stretching of the shell plating, S4 shell element is used in this study for all structural components.

According to Song et al. (2025), when establishing the FE models, the detailed dimensions of structure components are selected based on “3Co principle”. It ensures correspondence, comparability and competence between the numerical model and the actual structure. In order to reduce the influence of boundary conditions, an extended region has been considered. Single panel is defined as the hull plating area bounded by two adjacent transverse web frames and two adjacent longitudinal stringers, as illustrated in Figure 24. The entire FE model consists of nine panels arranged in a 3×3 configuration, as shown in Figure 25. The central trimmed region in the red circle is the objective structure, which is also the central panel. The geometry size of the FE modelling is shown in Table 3.

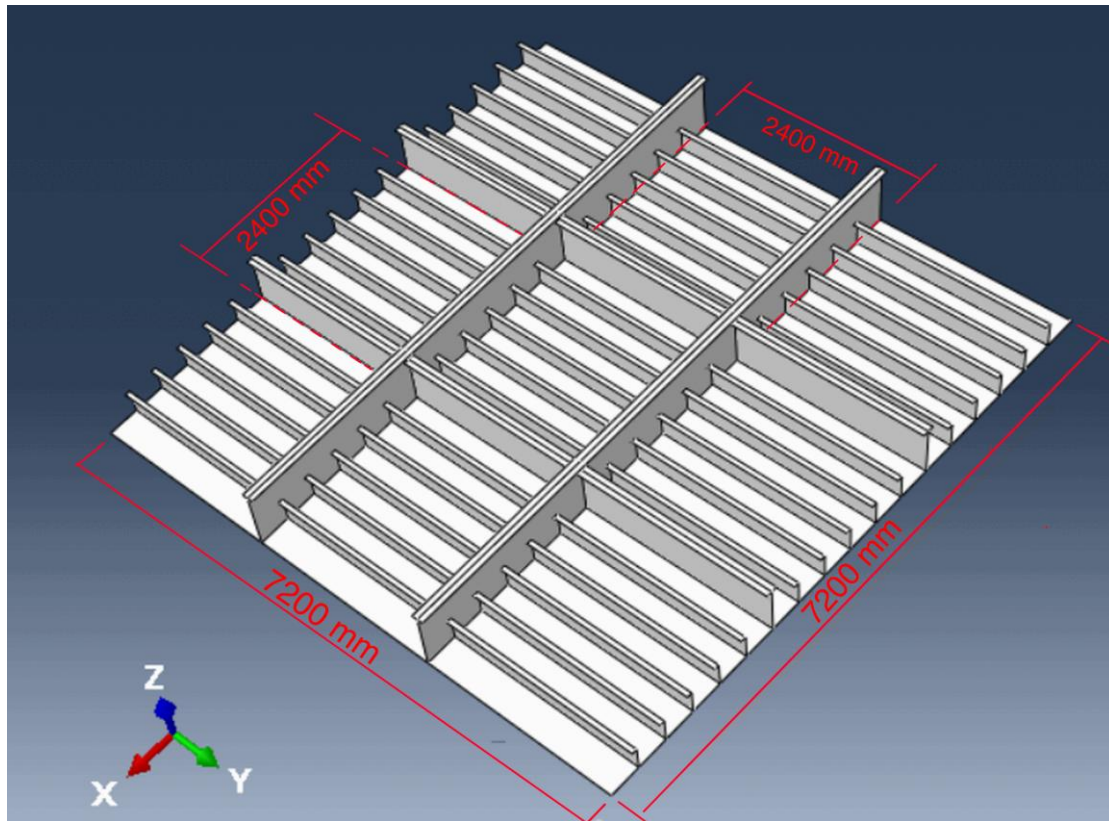


Figure 25 Nine-panel FE model with the central 2400 mm × 2400 mm objective panel and principal model dimensions.

Table 3 Dimensions of entire FE model including central objective structure and the extended region.

Panel length (mm)	2400	Model length (mm)	7200
Panel width (mm)	2400	Model width (mm)	7200
Stiffener spacing (mm)	400	Web-frame spacing (mm)	2400

4.2 Mesh size

The FE model is created by S4 shell elements. Each S4 element has four in-plane integration points, and five section points were used through the shell thickness based on Simpson's integration rule. Strain responses are extracted at the two outer section points, SNEG (section point 1) and SPOS (section point 5). For each surface, the strain assigned to a gauge location is calculated as the average of the four in-plane integration-point values of the corresponding element. The mesh size of FE model should ensure the accuracy of the simulated strain responses, meanwhile capturing local peak values without excessive averaging.

The mesh size of the FE model affects the extracted strain responses. Even with identical strain gauge positions, the average strain over an element can vary

significantly when the mesh sizes are different. To evaluate mesh convergence, a single representative loading case was analysed using different mesh sizes, while the loading and boundary conditions, material properties, and strain-gauge positions remain identical. As illustrated in Figure 26a, a uniformly distributed load of 1 kN is applied over a 400 mm area in the transverse x-direction and an 800 mm area in the longitudinal y-direction, which is along the longitudinal members. The loaded region is shown in blue and is located at the centre of the plating. The shear strain difference of the central stiffener at the predefined gauge locations is extracted and used as the reference quantity, as shown in Figure 26b.

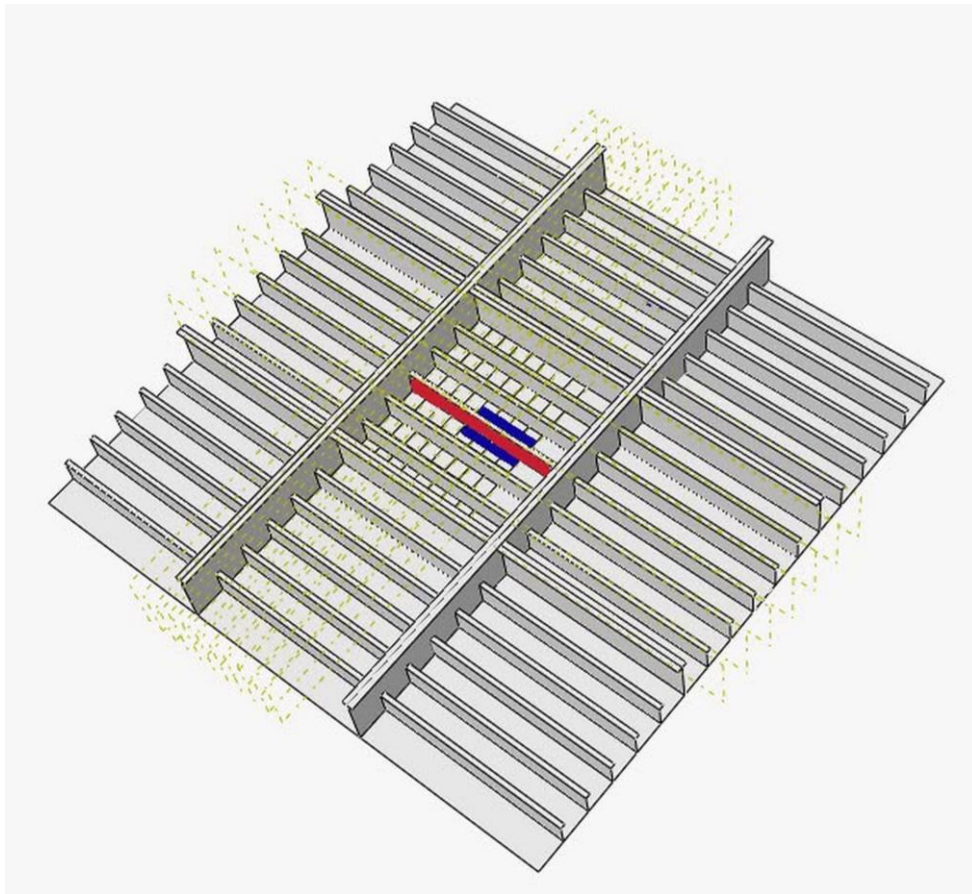


Figure 26a The FE-model region used for the mesh-convergence analysis.

Figure 26a shows the FE-model region used for the mesh-convergence analysis. The blue area indicates the central 800 mm × 400 mm loading patch subjected to a uniformly distributed total load of 1 kN, while the red-highlighted central stiffener indicates the region from which the shear-strain response was evaluated. Figure 26b represents the central stiffener which is red-highlighted in Figure 26a, the locations of the shear-strain evaluation points on the central longitudinal stiffener in the global FE coordinate system are indicated as two points P1 and P2. P1 = (3600, 2700, 150) mm in global coordinate system and P2 = (3600, 4500, 150) mm, where the x-, y-, and z-axis indicate the transverse, longitudinal, and plating-normal directions.

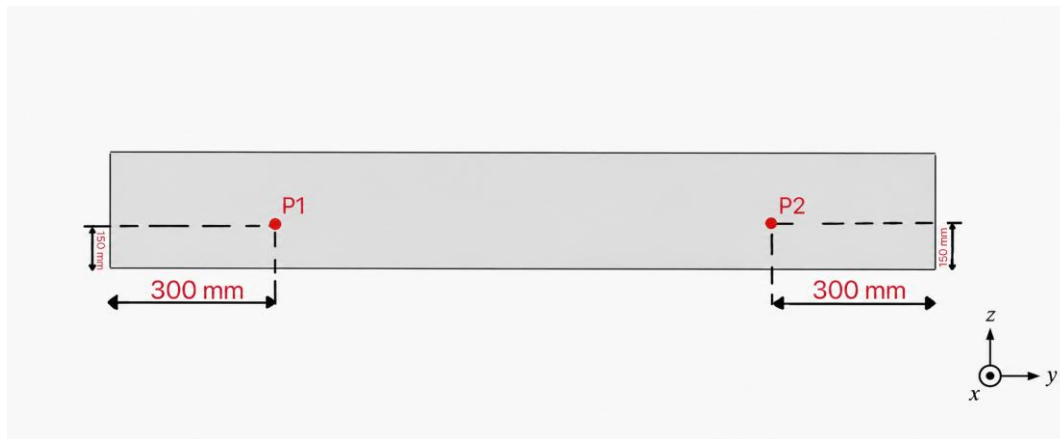


Figure 26b The locations of the shear-strain evaluation points on the central longitudinal stiffener in the global FE coordinate system.

Figure 27 presents the convergence analysis of the central stiffener shear strain difference with different mesh sizes. When the mesh size is larger than 45 mm, the average simulation value of a single element may deviate significantly from the true local strain. It can be observed that the percentage difference between adjacent element-wise strain values gradually decreases as the mesh size is reduced. When the mesh is refined to 35 mm, the computed shear strain difference converges to a stable value.

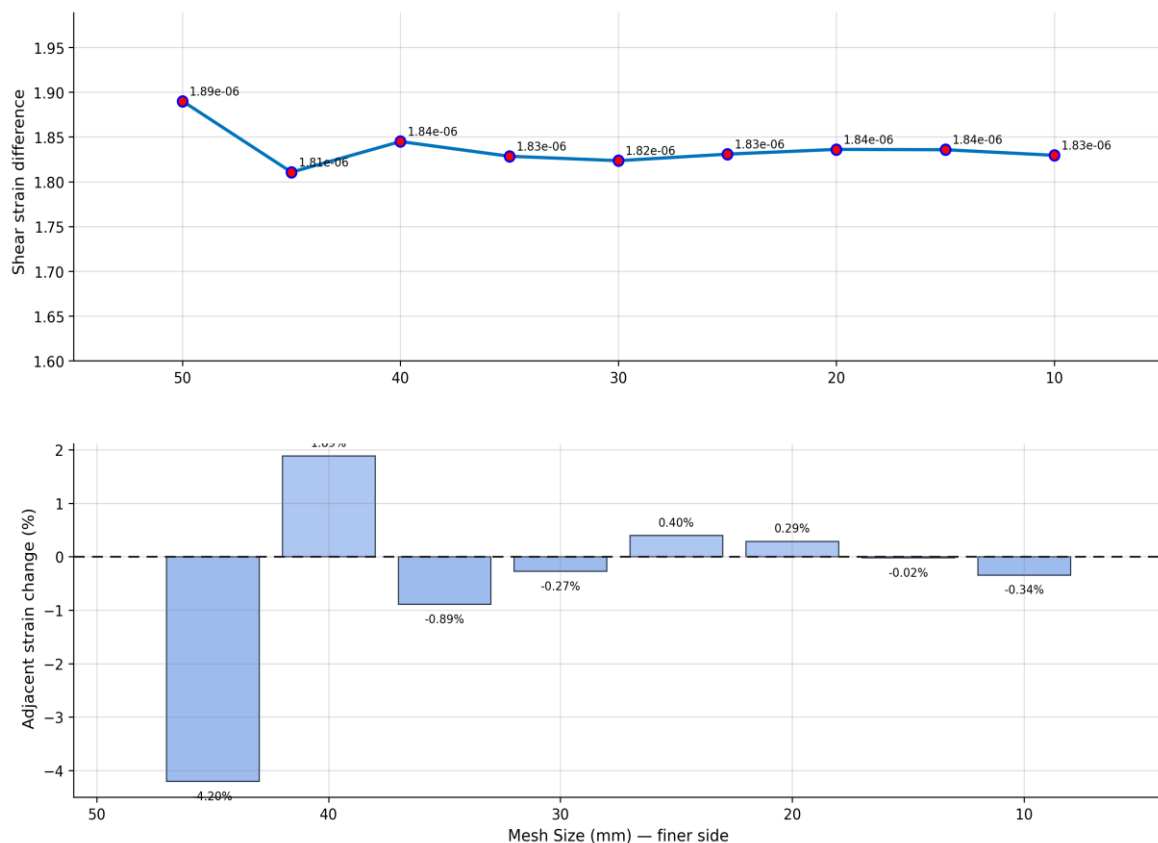


Figure 27 Convergence analysis of the central stiffener shear-strain difference.

To further investigate suitable mesh size, the relationships between measuring locations and strain measurements under each mesh size are established. Using central stiffener as an example, the selected analysis patch is 300 mm outwards from web frame boundary and along the vertical direction, which is the vertical strain-extraction line shown in Figure 28, the coordinate origin ($z = 0$) is defined at the connection between the central stiffener and the plating, which lies within the modelled frame length.

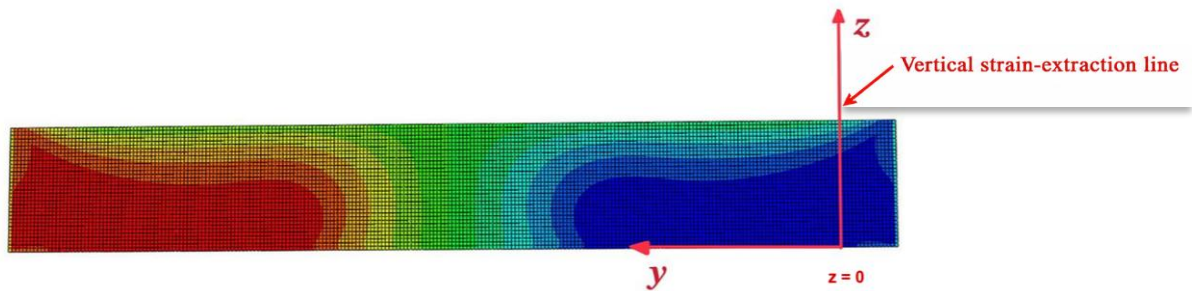


Figure 28 Location of the vertical strain-extraction line in the local FE coordinate system.

Figure 28 shows the location of the vertical strain-extraction line used in the mesh-convergence analysis. The line is located on the web of the central longitudinal stiffener, 300 mm from the adjacent transverse web frame. The z -coordinate is measured from the stiffener–plating connection, where $z = 0$, the vertical strain-extraction line is also along the local z -axis in Figure 28.

The shear strains of the elements along the selected vertical sampling line are extracted for different mesh sizes and fitting equations relating the sampling locations to the corresponding strain values are established. At each sampling location, the strain is obtained from the single S4 element containing the location and is calculated as the average of its four in-plane integration-point values at the selected shell section point 1 (SNEG), corresponding to the bottom shell surface. The resulting strain curves are presented in Figure 29. The vertical axis represents the z -coordinate, defined as the height above the plating, while the horizontal axis represents the corresponding shear strain, E12. Mesh convergence is evaluated by comparing the shear-strain curves obtained with different mesh sizes. The curves show that the simulated response becomes sufficiently convergent when the mesh size is below 25 mm. For each mesh size, a curve-fitting equation describing the variation of shear strain along the z -coordinate from 20 to 200 mm above the plating is established. The shear strain at $z = 150$ mm is then calculated from the corresponding fitting equation, and the fitting equations and calculated values are presented in Table 4.

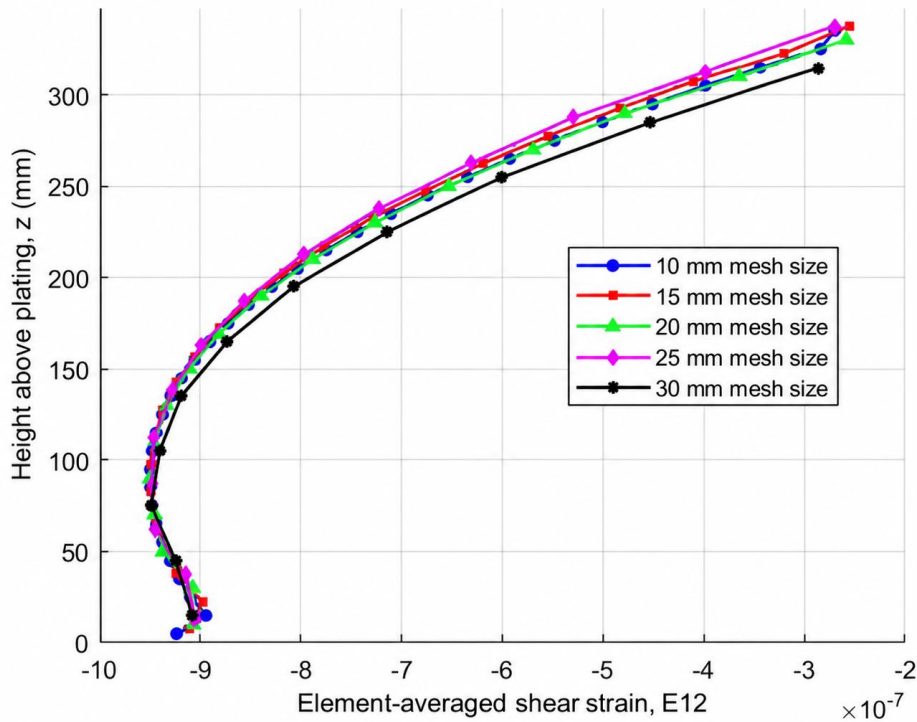


Figure 29 Element-averaged E12 shear-strain profiles along the vertical sampling line on the central longitudinal stiffener for mesh sizes of 10–30 mm.

Table 4 Shear strain calculation under different mesh size.

Mesh size (mm)	Fitting equation	Calculated shear strain at 150 mm height
30	$E12 = (1.1994e - 11) * z^2 + (-2.0901e - 9) * z + (-8.5594e - 7)$	-8.9960e-07
25	$E12 = (1.0713e - 11) * z^2 + (-2.0017e - 9) * z + (-8.5683e - 7)$	-9.1605e-07
20	$E12 = (1.1206e - 11) * z^2 + (-2.0261e - 9) * z + (-8.5927e - 7)$	-9.1142e-07
15	$E12 = (1.0626e - 11) * z^2 + (-1.9393e - 9) * z + (-8.6267e - 7)$	-9.1449e-07
10	$E12 = (1.0599e - 11) * z^2 + (-1.8813e - 9) * z + (-8.6734e - 7)$	-9.1207e-07

In Table 4, the changes between shear strain at a vertical position of 150 mm (i.e., $z = 150$ mm measured from the plating) under different mesh sizes are minor, thus, suitable simulation mesh size in this study is lower than 30 mm. The length and width of physical strain gauges are generally smaller than 10 mm, therefore, to better match the reality of ice-induced load measurement, modelling mesh size is defined as 10 mm. In order to ensure that 10 mm mesh size is adopted to the study, which does not ignore the

peak strain under the applied load, examine a single element mesh where the shear strain gauge is installed, the values of four in-plane integration points which are extracted at bottom surface (SNEG), the average value and the percentage of difference between them are shown in Table 5, it can be seen that the percentage of difference between four integration points and the average value are all lower than 0.1%, which illustrates that under 10 mm mesh size, the strain value of a single element can be derived as the average value of four integration points in the FE simulation.

Table 5 Shear-strain values at the four in-plane integration points of selected S4 element.

Strain value of in-plane integration point	Average value	Percentage of difference
1.52082e-06	1.51951e-06	0.09%
1.51936e-06		-0.01%
1.51965e-06		0.01%
1.51819e-06		-0.09%

4.3 Material properties and boundary conditions

It has been discussed that structures are only considered small deformations and linear elastic material behaviours. To simulate the general situation that ice-induced loads on ship hull, common steel material is defined. In addition, material properties are defined as average values in this section, where steel density is 7850 kg/m³, and Poisson's ratio is 0.3. When using shell elements to simulate the hull structure, the material distribution mode is selected to be homogeneous.

This study mainly focuses on the uncertainty and sensitivity of ICM methods under different loading patch sizes and loading locations. Therefore, the same boundary condition is used for all research cases so that the results are less influenced by other factors. In addition, to reduce the effect on local structural responses, a fixed boundary condition is placed at the area that is sufficiently far away from objective structure. As shown in Figure 30, a general fixed boundary condition is applied to the extended FE model, all translational degrees of freedom are constrained along the outer edges of the model, while the central objective structure remains free from direct boundary constraints. The red region is the objective structure where the strain responses are extracted.

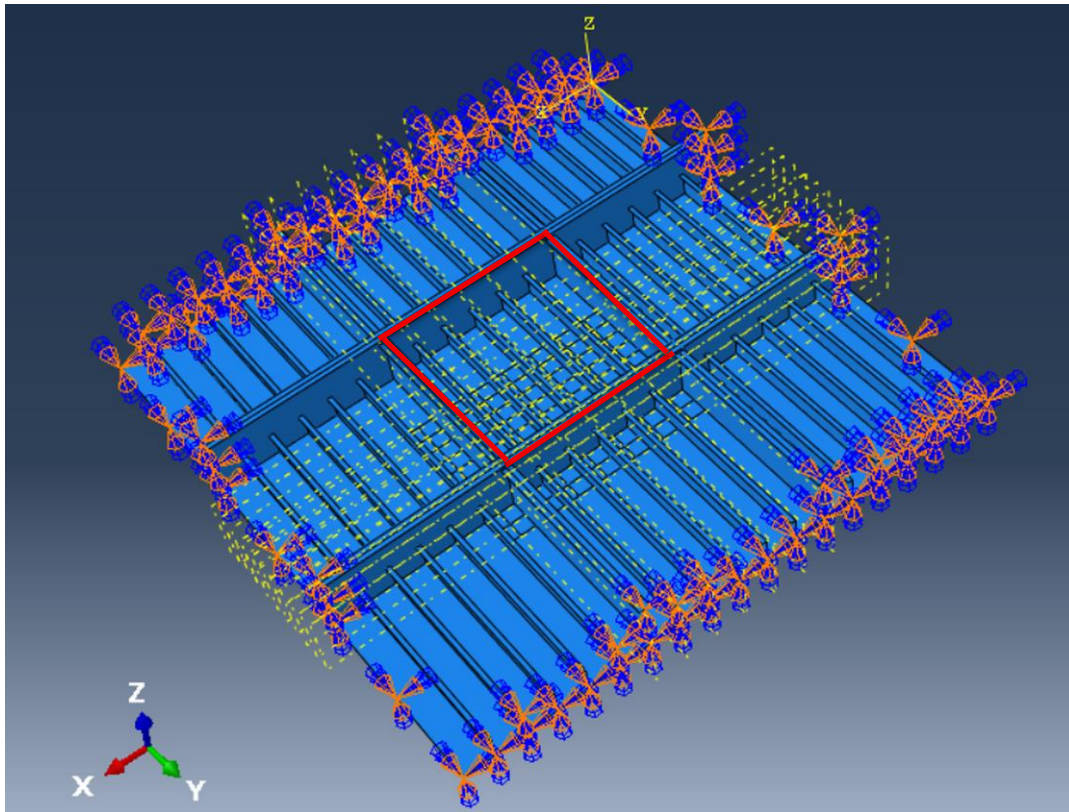


Figure 30 Boundary condition that all translational degrees of freedom are constrained along the outer edges of the model.

Considering decreasing the boundary condition influence, the entire FE model includes an extended region in addition to the objective structure. For simulating the loading conditions applying on the structure of a ship, the extended region is also composed of plates with the same structural as the objective structure. As an example, a unit uniformly distributed pressure load of 1 kN is applied at the centre of the structure. Figure 31 presents the distribution of shear strain, with a maximum strain value of $1.52\text{e-}6$. It can be observed that the strain gradients near the fixed boundaries are localized and decline rapidly within the extended region. Consequently, the strain field at the objective structure is unaffected by the boundary conditions.

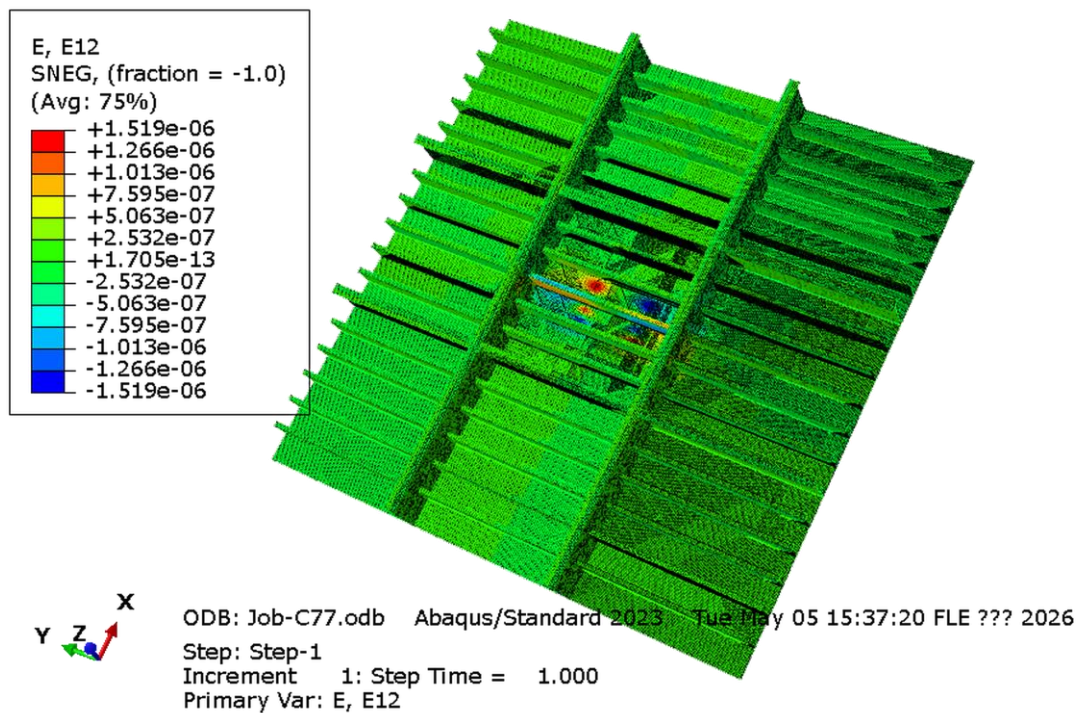


Figure 31a Shear-strain distribution under the central 1 kN reference load.

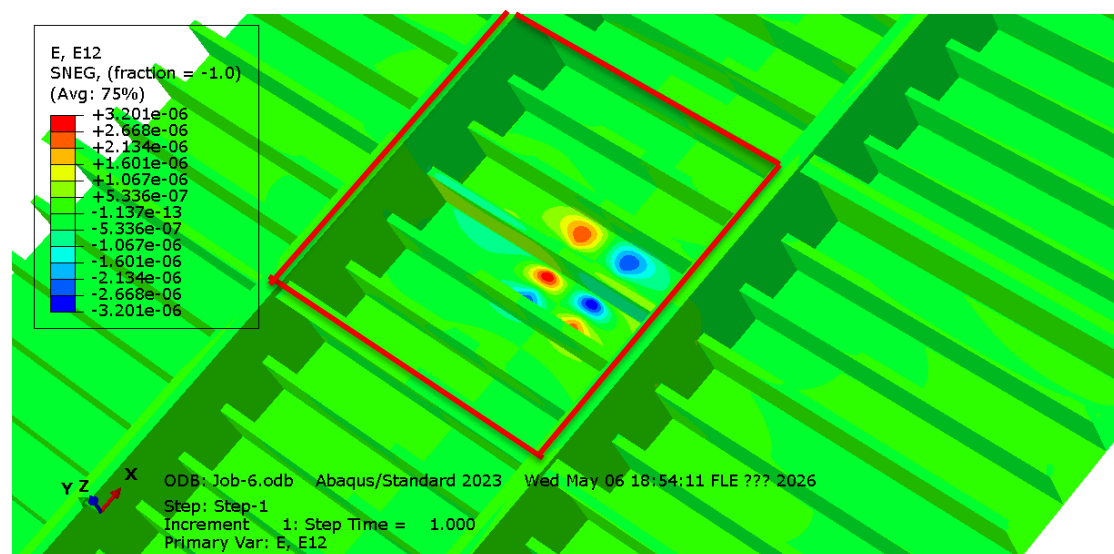


Figure 31b Enlarged view of the central objective region showing the E12 shear-strain distribution.

4.4 Placement of strain gauges

4.4.1 Shear strain gauges

From shear strain perspective, strain gauges are installed in pairs on each longitudinal stiffener of objective structure, symmetrical along the central line of the structure.

Figure 32 illustrates the corresponding gauge positions. The red circles indicate the gauge locations, while the x - and y -axis show the directions of the global system. The length of each longitudinal structure member is 2400 mm, the defined lengths of ice-induced load patches along transverse x -direction in this study are 200 mm and 400 mm. Along the horizontal y -direction direction, the selection of gauge positions is governed by the strain gradients which are induced by two factors: the boundaries of the ice load patch and the constraints imposed by the adjacent web frames.

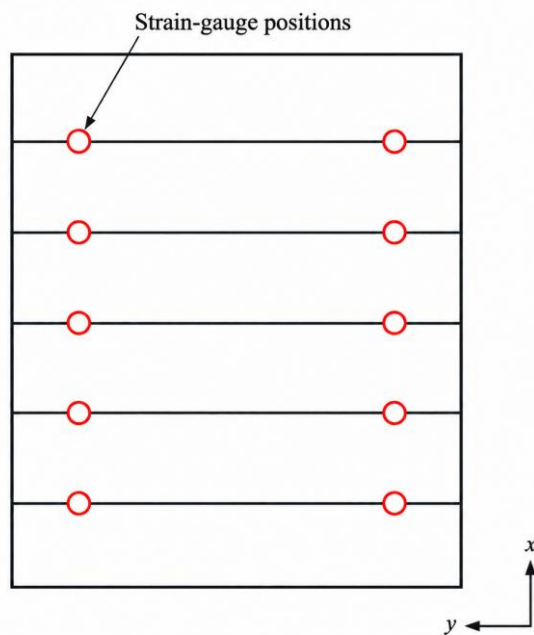


Figure 32 Positions of the paired shear-strain gauges on the longitudinal stiffeners.

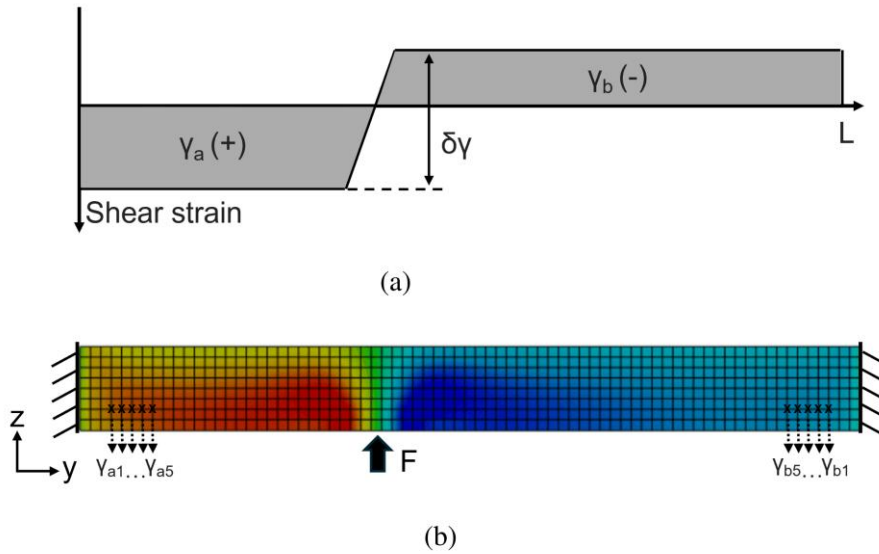


Figure 33 Influence of gauge pair placements (Zhu and Suominen, 2025).

The gauge vertical placement adopted in this study follows the configuration investigated by Zhu and Suominen (2025). As illustrated in Figure 33, their work examined concentrated force and investigated five pairs of strain gauges. Figure 33a presents the theoretical shear strain distribution and the loading condition, while Figure 33b shows the specific positions of the five gauge-pairs. It has been proved that moving the gauge pairs outward to the boundary can increase the effective measurement area without compromising the results (Zhu and Suominen, 2025). Thus, the shear strain gauges in this study are installed at 300mm outward to web-frames in horizontal direction. Along the vertical direction, strain gauges are installed at the place without compressive or tensile stress, to avoid interference of normal stress on shear strain measurement. In this study, it is 150 mm outward to plane in vertical direction which is z-axis.

4.4.2 Normal strain gauges

Compared to shear stains, the normal strain is more sensitive to boundaries. The normal strain considered in this study is the E11 component, which represents the normal strain in the shell local 1-direction. According to the Abaqus sign convention, tensile strain is positive, whereas compressive strain is negative. Furthermore, the normal strain matrix relies on the direct strain measurement at selected local points. In this study, inverse matrix is derived by normal strain along z-axis direction. Thus, the determination of gauge locations is important for accurately measuring local force. The objective structure has five longitudinal stiffeners, for reducing the effects of boundaries, the normal strain gauges are installed on the three central longitudinal stringers. Each stringer has 11 gauges on it and horizontal spacing which is y-axis direction between gauges is 200 mm, spacing between side gauges and web-frames are also 200 mm. In total there are 33 gauges on the structure for normal strain ICM.

Moreover, the compressive strain declines significantly away from the contact region. If the gauges positions are half width from the boundary along z-axis, there will be local stiffener bending or torsion influence. However, the normal strain at positions that are tightly close to plate is affected by local stress concentrations and torsion. Therefore, the gauge positions along z-axis direction should be between 0.3-0.5 stiffener web height. To ensure the gauge locations for creating ICM, using the central stiffener as an example, a uniformly distributed load is applied over a rectangular load patch measuring 200 mm in the transverse x-direction and 400 mm in the longitudinal y-direction, which is along the stiffener length.

Figure 34 illustrates the central stiffener under 10mm mesh, Figure 34(a) shows the normal strain E11 which is the local normal strain component, perpendicular to the beam length direction. Figure 34(b) is the zooming local central part of the stiffener, which helps to investigate the local strain gradient near the plating. The strain is highly localized within 20 mm of the plating; beyond approximately 40 mm, the gradient diminishes and the strain becomes stable. Thus, it can be observed that compressive normal strain at locations which are 40 mm offset from plating is suitable for ICM creation.

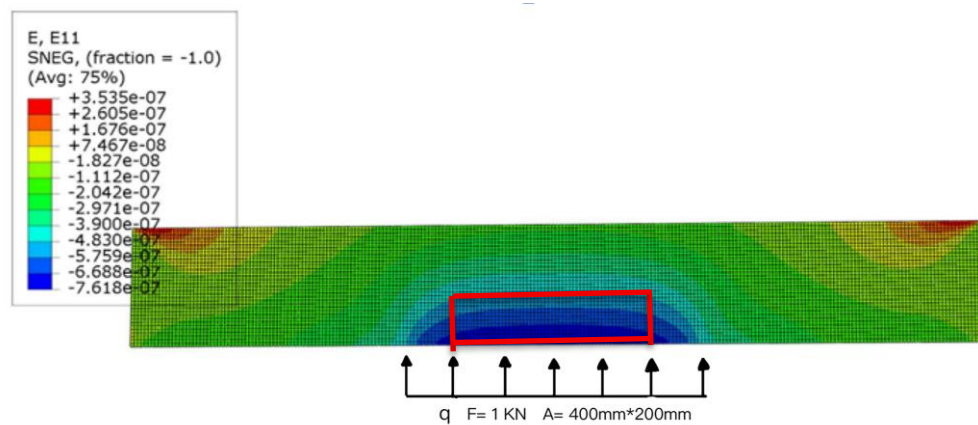


Figure 34a Normal strain distribution in z-axis of the central stiffener.

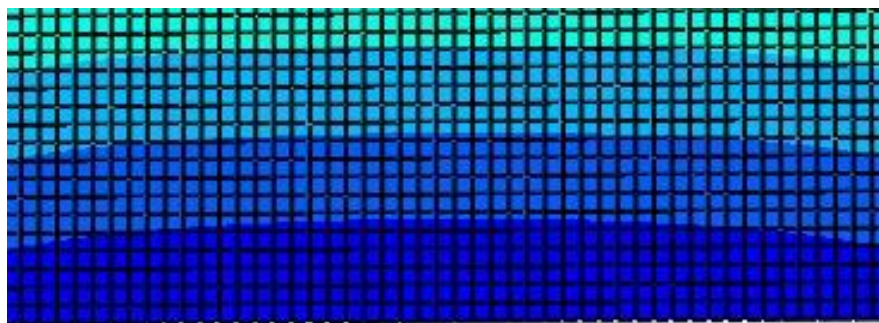


Figure 34b Local normal strain (E11) of the central stringer, the red region in Figure 34a.

5 Results

5.1 Shear-strain-based ICM

In the ICM approach, the strain values extracted from the FE model should correspond to the locations of the strain gauges in full-scale measurements. On a real ship structure, strain gauges are typically installed at the internal surfaces. However, in this study, the strain gauges are placed on the stiffeners, both SPOS and SNEG shell surfaces represent internal structural surfaces. Therefore, in FE analysis, ICMs that established by using both shell surface values including top (SPOS) and bottom (SNEG) are compared, and the one that shows more accurate load measurement is adopted for sensitivity analysis.

Two shear-strain ICMs are established by applying uniformly distributed loads with a total force of 1 kN over loading patches measuring 400 mm in the transverse x -direction and 800 mm in the longitudinal y -direction. The shear-strain differences at each frame are extracted from both shell surfaces. The following equation (9a) is the matrix defined by top element shear strain values and equation (9b) is the matrix of bottom element shear strain values. For comparing the accuracy of these two ICMs, the unit uniformly-pressure load which is 1 kN in 400 mm $\times$ 400 mm is applied to the centre of the structure. Table 6 illustrates the comparison of both measuring results.

$$\begin{bmatrix} 335460 & -7750 & 430 & 90 & 310 \\ -76290 & 332550 & -7610 & 80 & -90 \\ 25680 & -75260 & 332470 & -7530 & 130 \\ -9360 & 25290 & -75260 & 332390 & -7530 \\ 3290 & -9140 & 25110 & -74680 & 330780 \end{bmatrix} \quad (9a)$$

$$\begin{bmatrix} 541090 & -159670 & 69360 & -29040 & 13950 \\ 10520 & 538220 & -158950 & 68730 & -30490 \\ -700 & 10580 & 538170 & -159070 & 72020 \\ -30 & -880 & 10270 & 538360 & -166960 \\ -510 & 100 & -420 & 9690 & 555690 \end{bmatrix} \quad (9b)$$

Table 6 Comparison of both ICMs.

Applied load (kN)	Load measurement (SPOS) (kN)	Load measurement (SNEG) (kN)
1.00	1.0009	0.9921

The load measurements from both matrixes are 1.0009 from top and 0.9921 from bottom. SPOS value represents the surface in the direction of the positive shell normal, which faces the positive transverse x -direction for the stiffener-web elements that are used for strain extraction; SNEG value denotes the opposite surface. The ICM of element bottom values has approximately 1 percentage of distribution deviation. Inverse matrix that is derived from element top side values is more precise compared to which is derived by bottom values. Thus, in this study, shear-strain based ICM method considers top side of element values.

5.2 Normal -strain-based ICM

As discussed in Section 4.4, there are 33 normal-strain gauges on the three central longitudinal stiffeners, with 11 gauges on each stiffener. The distance between adjacent gauges is 200 mm. The E11 strain at each gauge position is calculated from the average value of four integration points in the shell element. To establish the ICM, a 1 kN uniformly distributed load is applied at 33 calibration positions over a patch measuring 200 mm in the transverse x -direction and 400 mm in the longitudinal y -direction. Under each loading case, the 33 normal-strain values are extracted. Thus, a 33×33 normal-strain ICM is established. The complete SPOS and SNEG matrices are provided in Appendix A.

To compare the accuracy of two matrixes, a reference uniformly-pressure load is applied to the structure, which is 1 kN over $400 \text{ mm} \times 400 \text{ mm}$ area at the centre of objective panel. The measured load from element top value matrix and bottom value matrix are 0.999 kN and 0.986 kN. It can be observed that the measurement depending on top value ICM is more accurate. Thus, the normal-strain based matrix that established by element top values is adopted in this study.

5.3 Loading location sensitivity analysis

Due to uncertainties in ice-structure interaction, the actual contact region between ice and the ship hull cannot be accurately determined. Therefore, in the present study, the ice contact region is simplified as a series of rectangular loading patches applied to the objective structure. In Chapters 5.3 and 5.4, the dimensions are reported consistently as transverse x -direction $\times$ longitudinal y -direction, where the y -direction is along the longitudinal members. This section focuses on the measurement sensitivity of loading locations on longitudinal framed structures. Considering the large number of possible loading locations, representative loading cases are selected, researching loading moves in 1/4 area on the objective panel. Figure 35 is the top view of modelling in Abaqus, the grid area is studying region on the plating where the external loads are applied at. The external loads are also applied at plating where they connect with a longitudinal stiffener and a transverse web-frame.

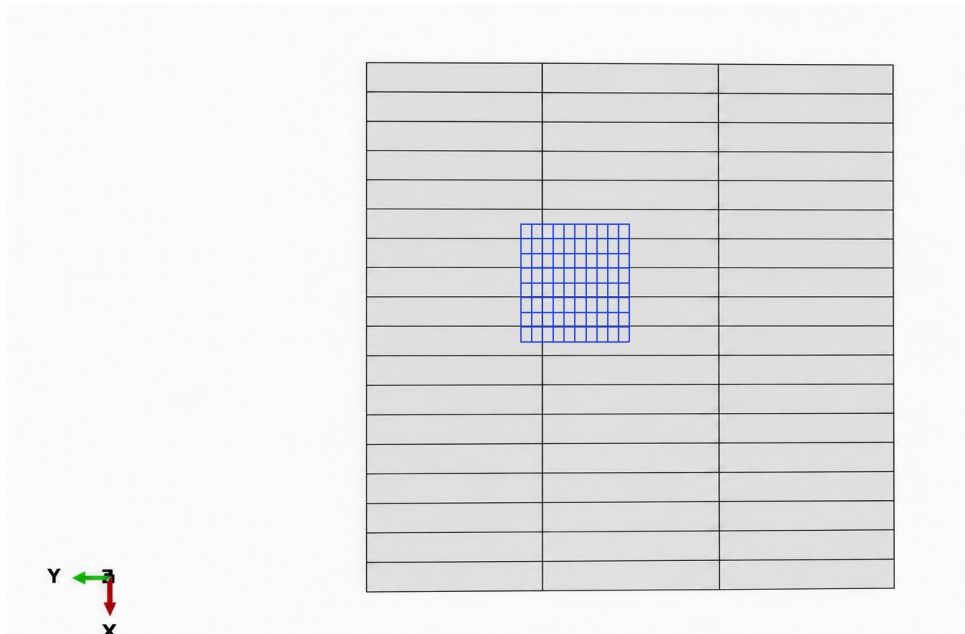


Figure 35 Loading patch locations.

Figure 36 shows the detailed C-series loading locations. The distance between adjacent loading centres is 200 mm in both in-plane directions. Each C-series patch measures 400 mm in the transverse x -direction and 800 mm in the longitudinal y -direction. Since the moving distance is smaller than the patch dimensions, adjacent patches overlap. The row coordinate describes the transverse structural position: row C1m is cantered on the longitudinal stringer, rows C3m, C5m and C7m are cantered on longitudinal stiffeners, and rows C2m, C4m and C6m are cantered in the plating bays. The column coordinate describes the longitudinal position: column Cn1 is cantered on the transverse web frame and column Cn7 is located at the mid-span centre line between web frames. In total, 49 loading cases are studied and named C11-C77, where the first digit represents the row and the second digit indicates the column.

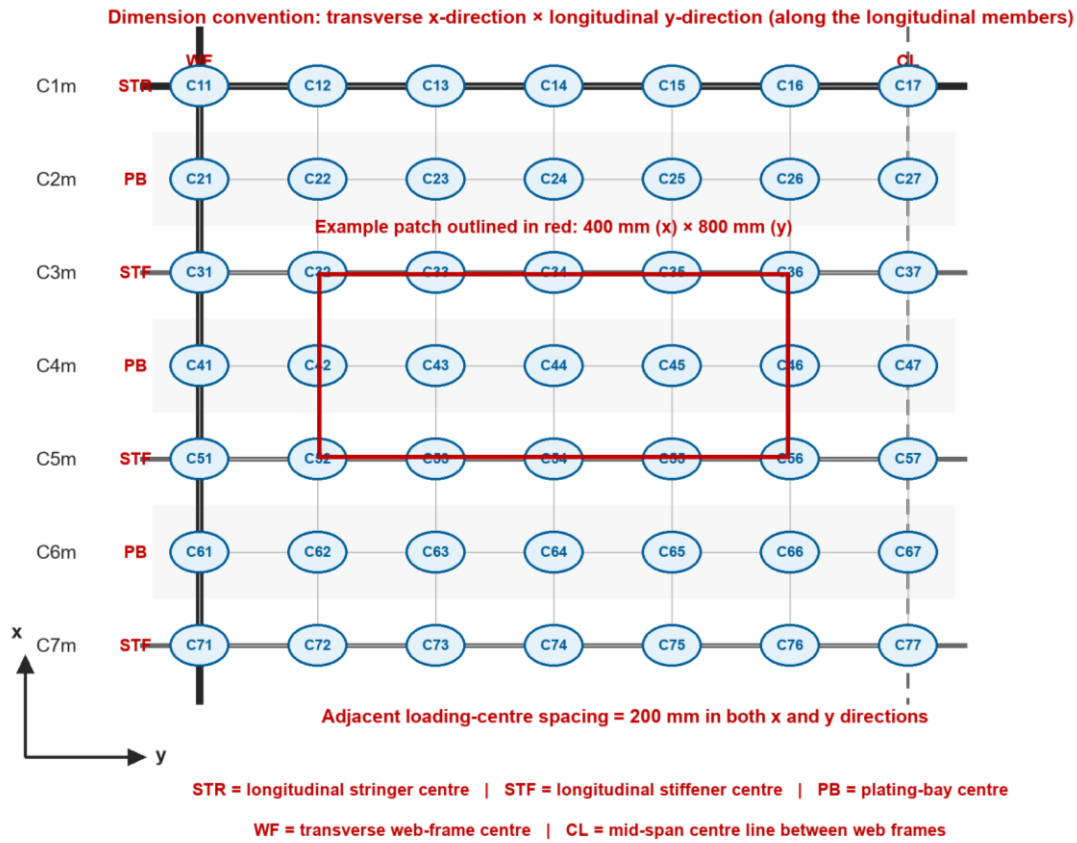


Figure 36 C-series loading centres for the 400 mm (transverse) × 800 mm (longitudinal) patches.

5.3.1 Shear strain measurement

The measured shear-strain loads are presented directly in Figure 37, and their percentage deviations from the applied load are presented in Figure 38. The applied load remains 1 kN in each case. Therefore, the difference between each measured value and 1 kN represents the measurement error. The numerical value shown in each heat-map cell corresponds to the loading centre identified by its row and column coordinates.

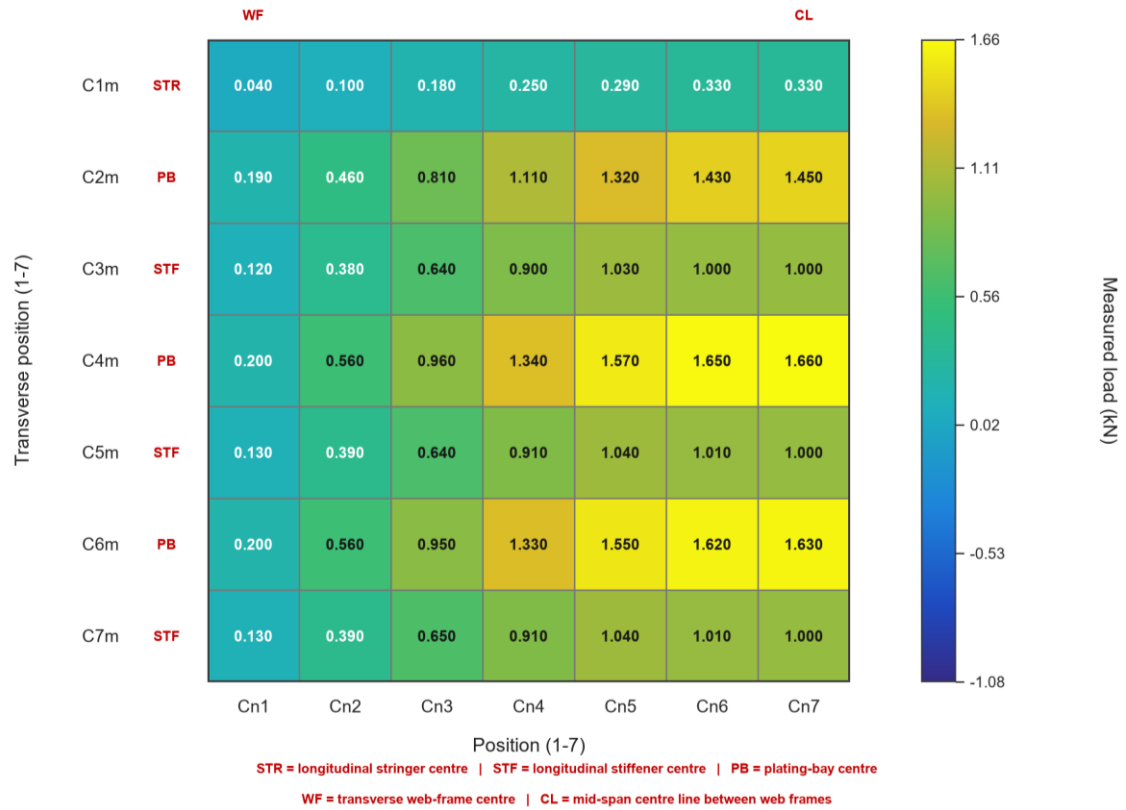


Figure 37 Shear-strain measurement for the 400 mm (transverse) $\times$ 800 mm (longitudinal) C-series patches.

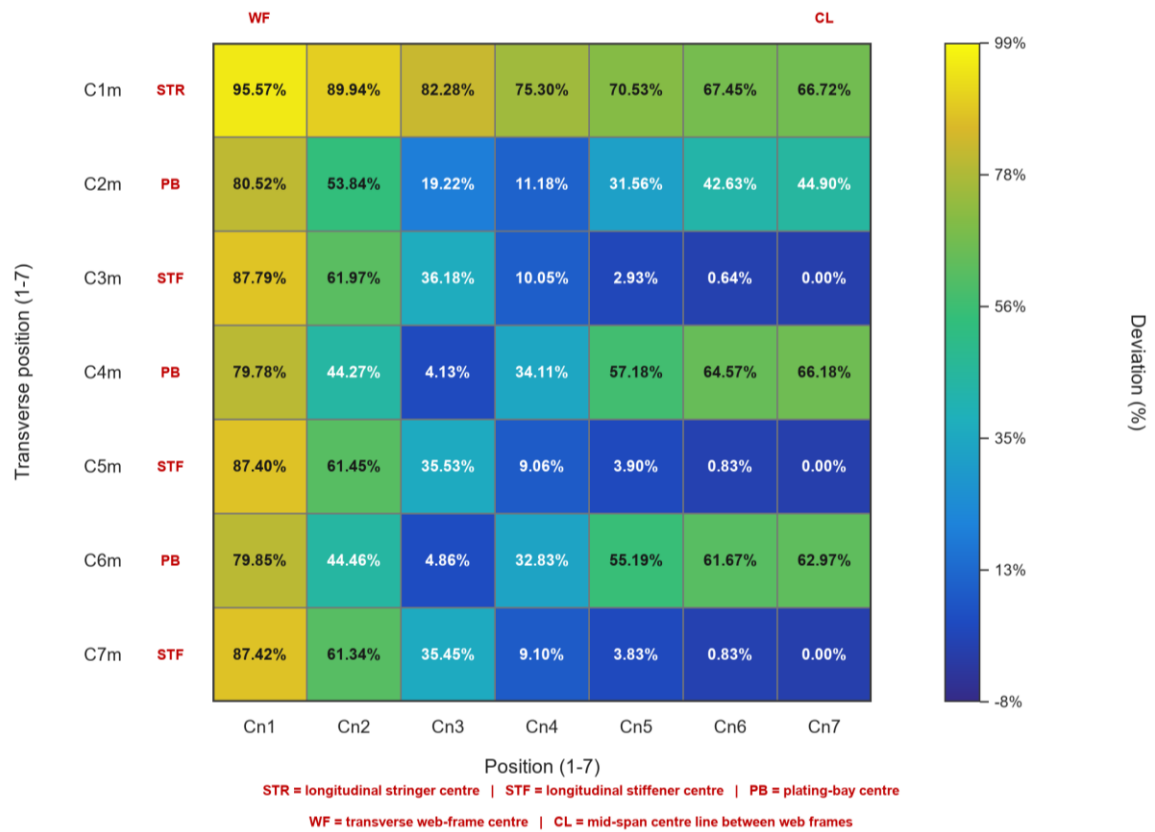


Figure 38 Percentage-deviation heat map for the C-series shear-strain measurement.

It can be observed that the shear-strain measured loads range from 0.04 kN to 1.66 kN. Among the 49 cases, 14 cases have a deviation lower than or equal to 10%, and the median absolute error is 0.44 kN. Some cases, such as C36, C37, C57 and C77, measure approximately 1.00 kN. However, C11 only measures 0.04 kN and C47 measures 1.66 kN. The errors show a clear relationship with the loading locations. Thus, the shear-strain ICM can accurately measure some locations, but it cannot keep the same accuracy over the entire studied region.

5.3.2 Normal strain measurement

The measured ice load results based on normal strain ICM of all 49 cases are shown directly in Figure 39, and the corresponding percentage deviations are presented in Figure 40. The structural row and column labels are the same as those defined in Figure 36. For the normal-strain method, only the three central longitudinal stiffeners are instrumented, with 11 gauges on each stiffener.



Figure 39 Normal-strain measurement for the 400 mm (transverse) $\times$ 800 mm (longitudinal) C-series patches.



Figure 40 Percentage-deviation heat map for the C-series normal-strain measurement.

Compared with the shear-strain results, the normal-strain results still show significantly larger changes. The measured loads range from -5.88 kN in C47 to 7.48 kN in C67. Among the 49 results, 14 values are negative and 8 cases have a deviation lower than or equal to 10%. The median absolute error is 0.84 kN. Negative values occur mainly at the right side of the C3 and C4 rows, while the C6 row produces large positive values. Several cases in the C5 and C7 rows are close to 1 kN. Therefore, the large deviations are not randomly distributed but change with the loading location. The negative values are numerical measurement results and do not represent physical tensile ice loads.

It should also be noted that C54 to C57 and C74 to C77 measure between 0.99 kN and 1.02 kN and have deviations lower than or equal to 10%. This indicates that the normal-strain ICM can work under some loading conditions. However, the accuracy of these cases does not continue at the nearby loading locations. Possible reasons for this difference are discussed in Chapter 6.

5.4 Ice-loading patch size sensitive analysis

Results from Section 5.3 show that when ice-induced loading applying on structures where the plating connects with web-frames, longitudinal stringers or the spacing

between two longitudinal stiffeners, the measurement is highly sensitive. However, ice-related uncertainties also include other factors such as contacting regions. To further investigate the applicability of traditional measurement system, a smaller V-series patch measuring 200 mm in the transverse x-direction and 400 mm in the longitudinal y-direction is investigated. The total load remains 1 kN. Its area is one-fourth of the 400 mm (transverse) $\times$ 800 mm (longitudinal) C-series patch, so the applied pressure is four times larger. The V-series loading centres use the same 200 mm coordinate spacing as the C-series grid, but only 24 selected perimeter positions are analysed. The studied region is shown in Figure 41.

In addition, in this section, compared to previous studying cases, loading locations focus on sensitive regions where the measured results show a significant error, including plating bay between two stiffeners, connection area between plating and web-frame or longitudinal stringer. Researching loading moves in one-fourth area on the objective panel. Locations of applying loads are shown in Figure 41, grid area is the loading regions on the top view of the structure.

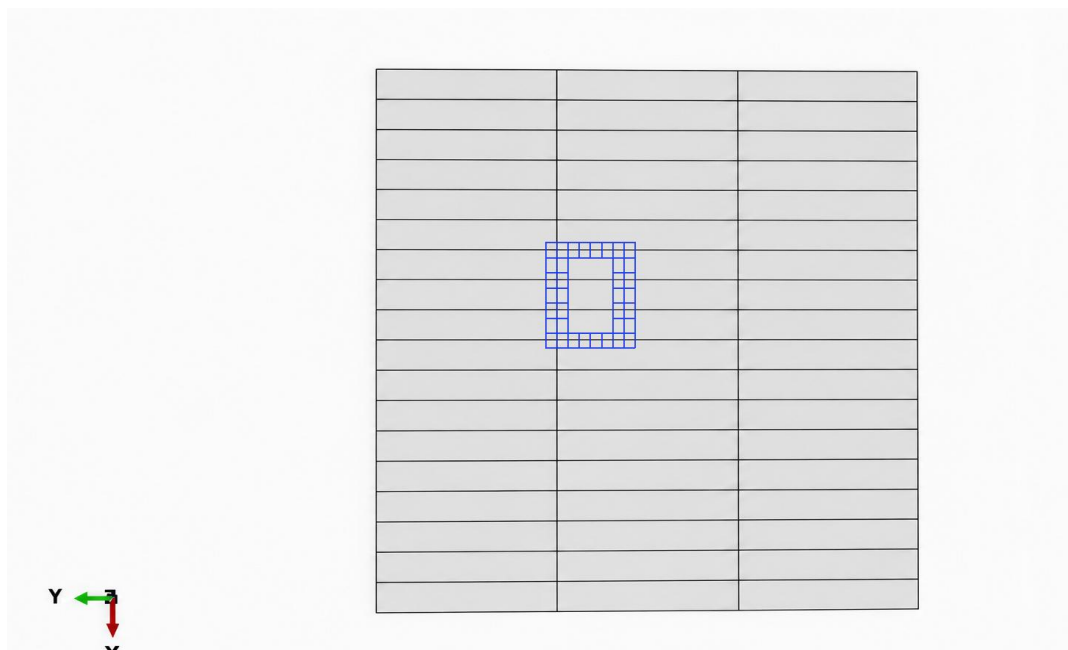


Figure 41 Studying panel region for the V-series loading-patch analysis.

In total, 24 V-series cases are studied. V11-V17 form the complete row cantered on the longitudinal stringer, and V71-V77 form the complete row cantered on the outer longitudinal stiffener. The remaining cases V21-V61 and V27-V67 form the two side columns: column Vn1 is cantered on the transverse web frame and column Vn7 is located at the mid-span centre line between web frames. Rows V3m and V5m are cantered on longitudinal stiffeners, whereas rows V2m, V4m and V6m are cantered in plating bays. Thus, the grey interior positions in Figure 42 are coordinate references and were not analysed in the V series.

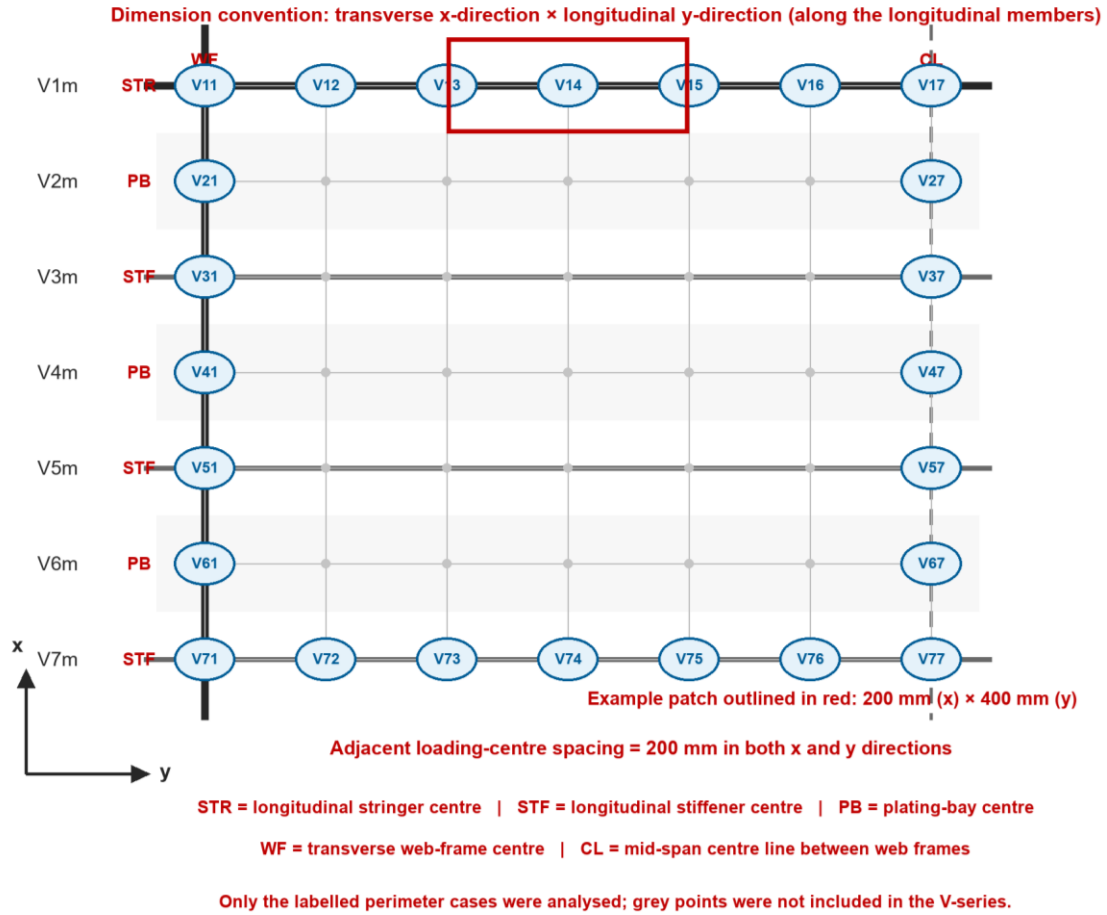


Figure 42 Selected V-series loading centres for the 200 mm (transverse) × 400 mm (longitudinal) patches.

5.4.1 Shear-strain measurement

The same shear-strain ICM and the same total load of 1 kN are used when the loading-patch size is changed. The measured loads of the 24 selected V-series cases are presented directly in Figure 43, and their percentage deviations are presented in Figure 44. The numerical values are shown only for the analysed perimeter positions identified in Figure 42.

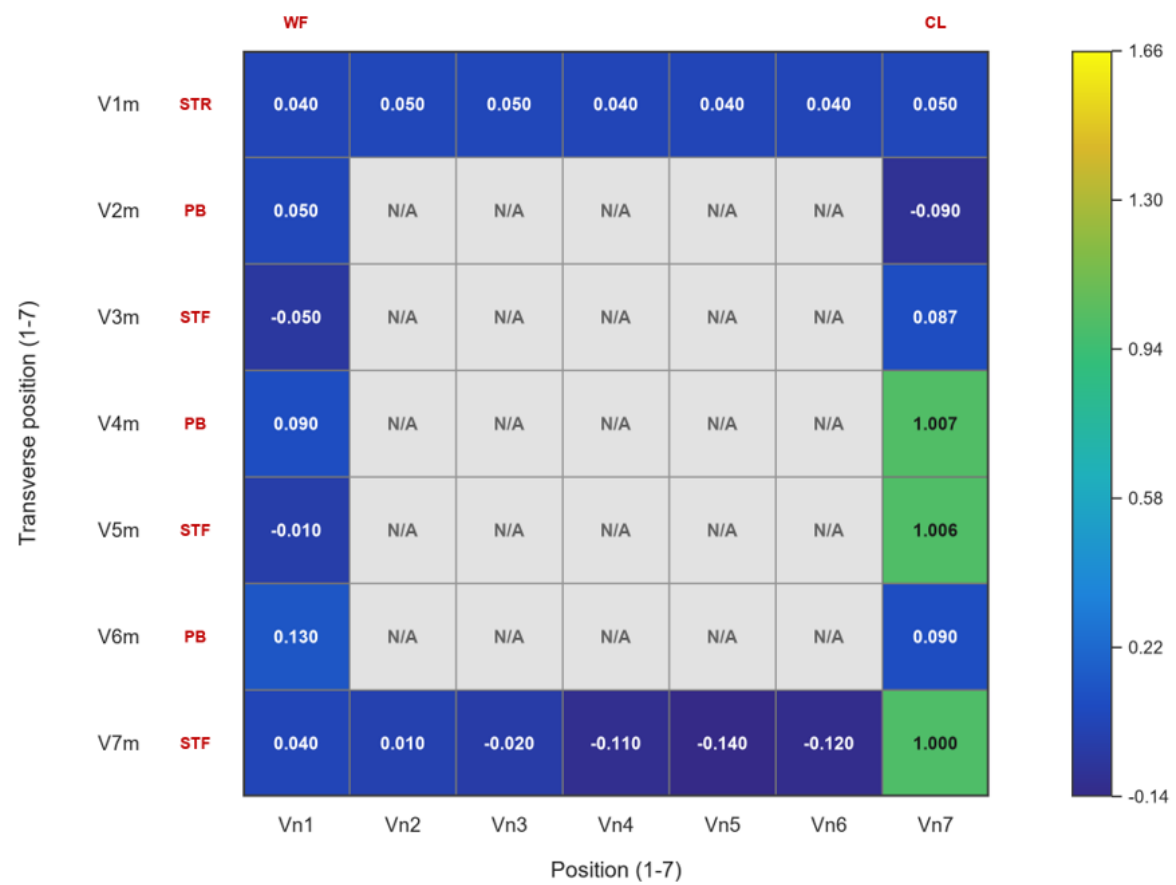


Figure 43 Shear-strain measurement for the 200 mm (transverse) $\times$ 400 mm (longitudinal) V-series patches.

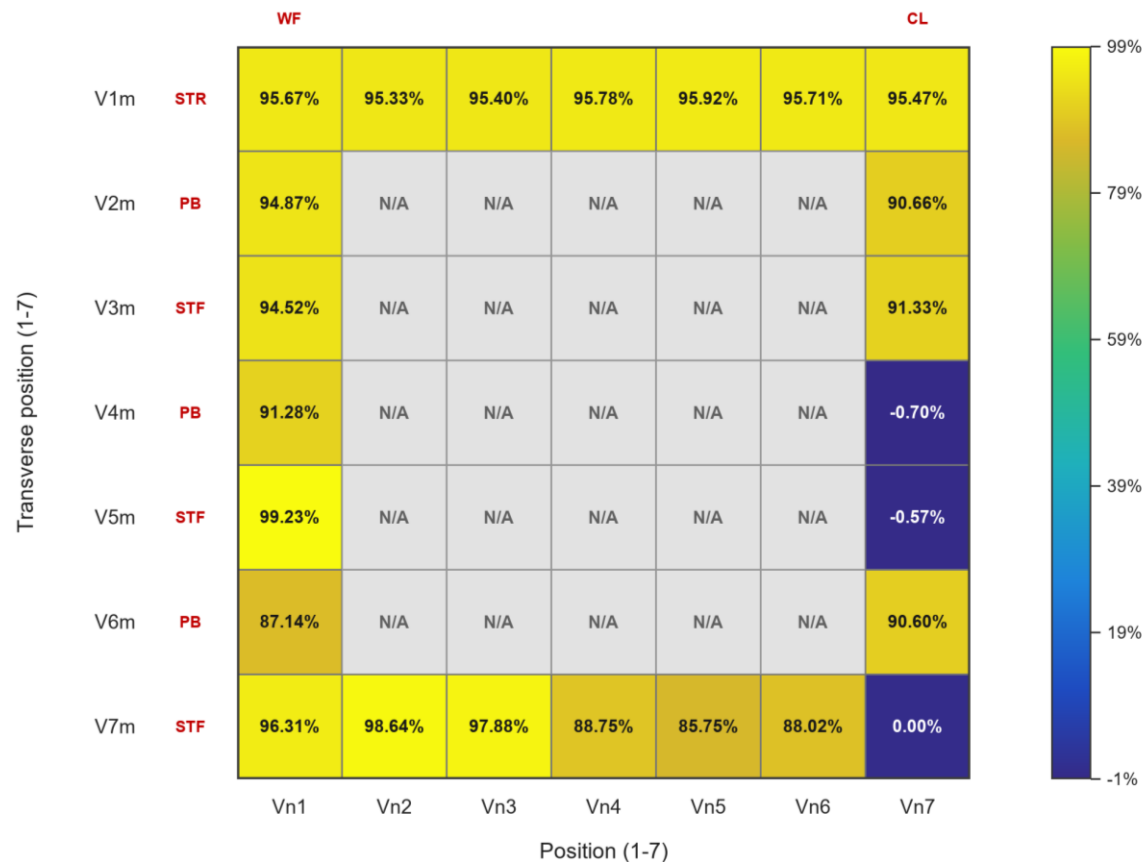


Figure 44 Percentage-deviation heat map for the V-series shear-strain measurement.

For the 200 mm (transverse) $\times$ 400 mm (longitudinal) loading patches, the shear-strain measured loads range from -0.14 kN to 1.01 kN. Three cases, V47, V57 and V77, have deviations lower than or equal to 10%, and the median absolute error remains 0.96 kN. Seven of the 24 measured values are negative. Most results are still close to zero instead of the applied load of 1 kN. V47 and V57 measure 1.007 kN and 1.006 kN, respectively, while V37 measures only 0.087 kN. Therefore, the smaller patch is not inaccurate at every location, but its measurement accuracy is still highly dependent on the loading position.

5.4.2 Normal strain measurement

The total applied load remains 1 kN in the normal-strain measurement. The same 24 selected V-series positions are used. The measured loads are presented directly in Figure 45 and the percentage deviations are presented in Figure 46. The normal-strain measurement of the smaller patch also shows large deviations. The measured loads range from -9.86 kN in V47 to 11.48 kN in V67. Four of the 24 values are negative, and only V76 and V77 have deviations lower than or equal to 10%. Along the V7 path, the measured load increases from 0.18 kN in V71 to 0.98 kN in V77. The largest

negative and positive values appear in the V4 and V6 paths. Therefore, the smaller loading patch is not uniformly less accurate for the normal-strain method. The measurement accuracy mainly depends on the loading location and whether its strain response can be represented by the calibration matrix.

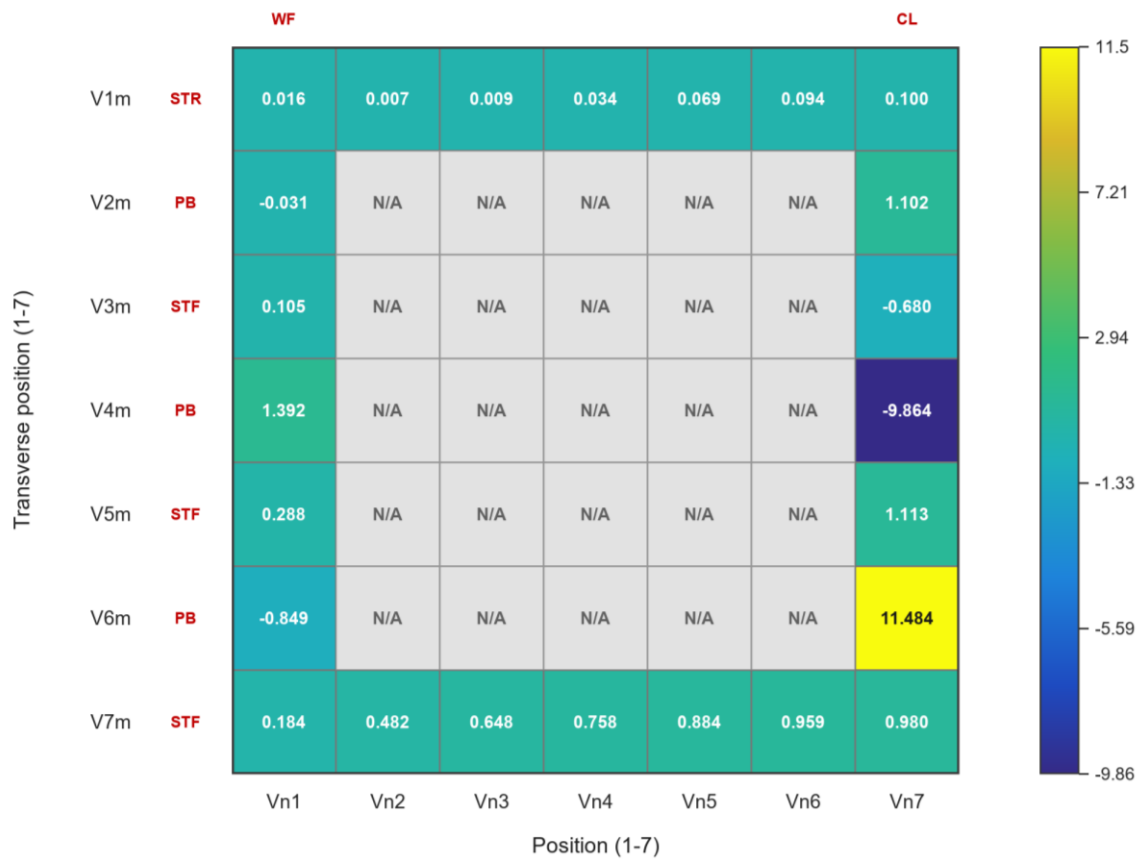


Figure 45 Normal-strain measurement for the 200 mm (transverse) $\times$ 400 mm (longitudinal) V-series patches.

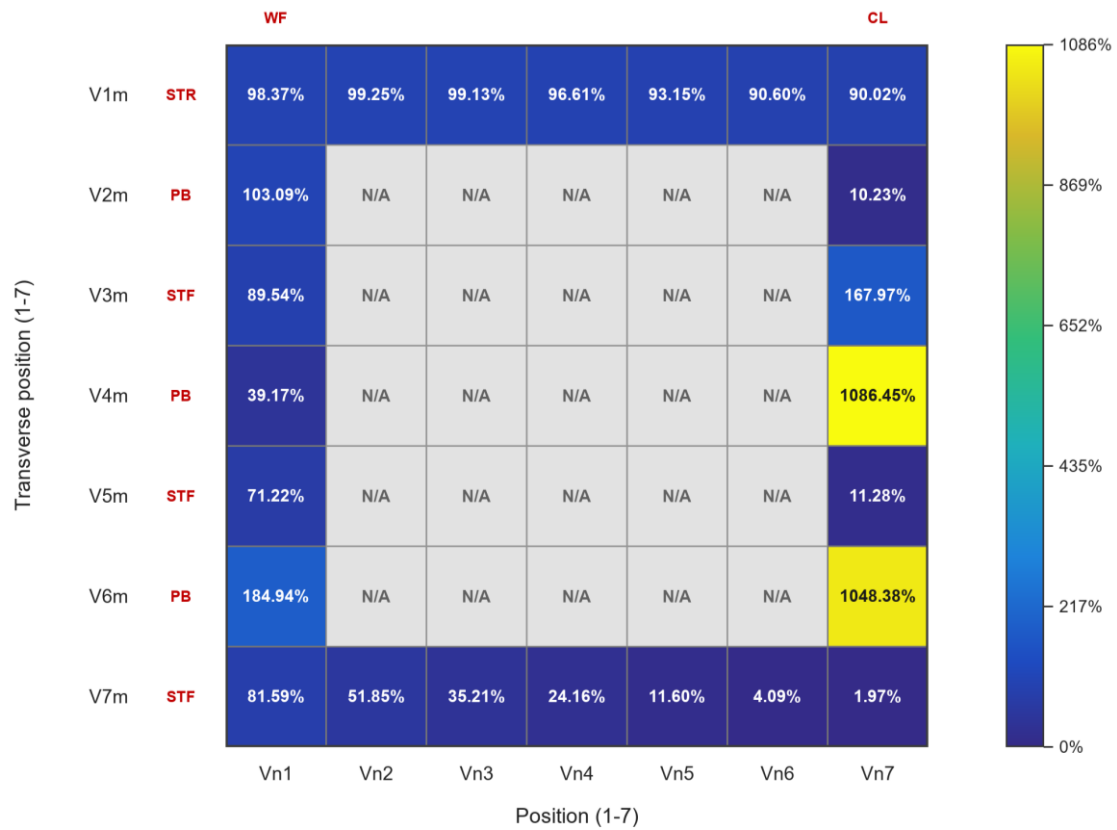


Figure 46 Percentage-deviation heat map for the V-series normal-strain measurement.

6 Discussion

The results show that both ICM methods are sensitive to the loading location. For the 400 mm (transverse) $\times$ 800 mm (longitudinal) C-series patches, the median absolute errors are 0.44 kN for the shear-strain ICM and 0.84 kN for the normal-strain ICM. Thirteen shear-strain cases and eight normal-strain cases have deviations lower than or equal to 10%. For the 200 mm (transverse) $\times$ 400 mm (longitudinal) V-series patches, the median absolute errors are 0.96 kN and 0.90 kN, and the corresponding numbers of cases within 10% are three and two.

Figures 35 to 45 show that the deviations follow the structural positions. For the C series, the shear-strain method is within 10% at C35-C37, C43, C54-C57, C63 and C74-C77. The normal-strain method is within 10% only at C54-C57 and C74-C77. For the V series, the corresponding accurate cases are V47, V57 and V77 for shear strain and V76 and V77 for normal strain. These accurate positions are mainly located on instrumented stiffeners and away from the transverse web-frame boundary, although V47 also shows that an individual plating-bay case can be accurately measured. Loads on the stringer, close to the web frame and at several plating-bay positions generally give larger errors. This occurs because the plating, stringer, web frames and longitudinal stiffeners carry different parts of the load at different positions. The shear-strain difference represents a less local response, while the normal strain is more strongly affected by local bending, torsion and the selected gauge position.

The measurement is applicable at locations where the applied load produces a strain distribution close to the distributions included in the ICM. For the normal-strain ICM, the reliable C54-C57 and C74-C77 cases are centered on instrumented longitudinal stiffeners and are located away from the transverse web-frame boundaries. At these positions, bending of the instrumented stiffeners defines the main measured response. When the load is applied near the stringer, web frame or plating bays, part of the load is transferred through other members, also, local torsion or plate bending changes the strain distribution. The ICM is established from the strain responses generated by the predefined unit-load cases. If a new loading case produces a substantially different strain distribution, direct inversion may produce unrealistically large positive or negative load estimates. The shear-strain-based ICM measures fewer local responses of shear and is accurate at more C-series positions, but it also fails when the load is carried by other structural members or distributed differently between the paired gauge locations. V47 is an exception: its shear result is accurate although it is located in a plating bay, because its strain pattern happens to be close to a combination represented by the ICM. Consequently, the results should be considered reliable only when the loading position and patch dimensions are similar to those used to establish the ICM and when small changes in the input strains do not cause large changes in the measured load.

The normal-strain matrices have full rank, but this only means that a mathematical inverse exists. The condition number is approximately 93 for the SPOS matrix and 6.8 for the SNEG matrix. The SPOS matrix gives the more accurate result for the selected

central reference case, but it is also more sensitive to small differences in the input strains. Since direct inverse does not require non-negative loads or a smooth distribution between neighbouring positions, small strain differences can be enlarged into alternating positive and negative measured loads. The negative values are therefore numerical results and do not represent physical tensile ice loads.

The two rectangular patches are numerical representations of uniform contact areas rather than direct models of all ice loads described in Chapter 2. They may represent a single idealized local contact of similar size, but not line-like high-pressure zones, several separated hot spots, irregular contact boundaries or a contact area moving with time. The two patch sizes were selected for the numerical sensitivity analysis and should not be considered as the complete range of ice-contact sizes occurring under full-scale conditions. Therefore, the 200 mm (transverse) $\times$ 400 mm (longitudinal) V-series patch and the 400 mm (transverse) $\times$ 800 mm (longitudinal) C-series patch are defined as two sensitivity cases rather than lower or upper limits for real ice contact. For the same total load, reducing the patch dimensions increases the applied pressure. Based on the expected structural response, a smaller patch may produce a more concentrated strain distribution, whereas a larger patch may distribute the load over more plating and supporting members. However, patches smaller than the V-series patch and larger than the C-series patch were not analysed. Therefore, the present results cannot determine whether the measurement accuracy would improve or decrease outside the two studying patch sizes. This should be investigated by applying several patch sizes at the same loading positions and using the same ICM. The C and V series also use different sets of loading locations. Thus, their error statistics cannot be used to consider the patch-size effect alone.

The limitations of current study also affect the magnitude and distribution of the reported errors. The linear-elastic model makes superposition possible, but yielding or structural damage might change the local stiffness and make the ICM dependent on load history. The present inverse relationship might overestimate or underestimate the load instead of scaling linearly. The static analysis does not include inertia, vibration or strain-rate effects. Under an impact load, the measured strain peak may not occur at the same time as the external load peak, and a static ICM could give an incorrect peak magnitude or position. The numerical model also excludes gauge noise, installation differences and calibration drift. Including these effects would probably increase the error, especially for the SPOS normal-strain matrix, because direct inversion amplifies small strain changes. Since no laboratory or full-scale validation was carried out, modelling errors in stiffness, boundary conditions or gauge positions may shift the locations that appear accurate in the numerical results. Different panel geometry, stiffener spacing or gauge layout also change the load paths, so the listed accurate cases cannot be transferred directly to another structure. Finally, a constrained or regularized inverse can reduce the negative and alternating loads produced by direct inversion, although strong regularization may also smooth a highly local ice load and underestimate its peak.

Overall, the results do not show that the ICM approach is inaccurate under each loading condition, because several individual cases are measured accurately. They show that

the present ICM, which is directly inverted, cannot maintain the same accuracy when the loading location or patch dimensions vary. The method is therefore not sufficiently reliable for general application to the studied longitudinally framed panel in its present form.

7 Conclusions

This study investigated the applicability of traditional ICM ice-induced load measurement methods on longitudinally framed ships. ICM of both shear strain and normal strain are derived by FE model in Abaqus, the objective structure is a panel with longitudinal stringers. Sensitivity analysis of loading locations and different loading patch sizes are performed to evaluate the accuracy of measurement results. The main conclusions of this study are summarized as follows:

- For the complete C-series grid, the shear-strain ICM has a median absolute error of 0.44 kN, and 13 of the 49 cases have deviations lower than or equal to 10%. These cases are C35-C37, C43, C54-C57, C63 and C74-C77. For the selected V-series positions, the median absolute error increases to 0.96 kN and only V47, V57 and V77 are within 10%. The shear-strain-based ICM is most accurate when the loading response is represented by the calibration matrix and is generally less accurate near the stringer and transverse web-frame boundary.
- The normal-strain ICM is more unstable. Its median absolute errors are 0.84 kN for the C series and 0.90 kN for the V series. The accurate cases are limited to C54-C57, C74-C77, V76 and V77. Other positions can produce large positive and negative measured loads. The higher condition number of the selected SPOS matrix and the local character of the E11 responses help to explain this behaviour.
- Because the C and V series do not contain the same complete cases of loading centres, the effect of patch size cannot be separated completely from the effect of loading location. Within the studied cases, neither ICM method provides reliable accuracy for all loading conditions. Accurate results at several individual positions show that the ICM principle can still work when the applied strain response is sufficiently represented by the calibration matrix, but the present direct-inversion formulation is not reliable outside that range.

8 Future Work

The current objective structures only consider bulb shape stiffeners, which are commonly used in ice-going ship structures. Different stiffener types such as T-stiffeners or flat bars have different stiffness and load transferring characteristics, which may influence the structural strain response and the establishment of the ICM. The applicability of the ICM to different stiffener types should therefore be investigated in future work.

Only one quarter of the loading region was analysed because the idealized panel geometry, boundary conditions, gauge arrangement and loading grid are symmetric about the centre line. Under the present linear model, corresponding cases in the remaining quarters would be mirror cases and would mainly repeat the simulations. A full-scale ship structure may contain different details, boundary conditions and loading. Therefore, the complete panel should be examined in future work when these asymmetries are included.

In the present study, the ICM is established by applying predefined rectangular unit loading patches. The size and location of these unit loading patches may affect the load transferring behaviour between adjacent structural members. Further work is required to investigate the effect of different unit loading patch dimensions on the accuracy of the ICM.

Only two loading patch sizes were considered in this study. Since the contact area between ice and the ship hull varies continuously during ice-ship interaction process, additional loading patch sizes and shapes should be investigated to evaluate their influence on load measurement accuracy.

Finally, the present study is based on a single structural configuration. Future studies should investigate different panel dimensions, stiffener spacing and structural arrangements to evaluate whether the conclusions obtained in this study are generally applicable to other longitudinally framed ship structures.

The direct inversion should be compared with constrained and regularized approaches. Tikhonov regularization, truncated singular-value decomposition and a non-negative load constraint could be investigated to reduce unrealistic alternating loads. The regularization parameter should be selected using independent FE cases, an L-curve or generalized cross-validation rather than the final test cases. Both the measured-load error and the remaining strain residual should be reported because strong regularization may smooth a highly local load.

In addition, in Section 5.4.1, it should be investigated why some loading patches centred on longitudinal stiffeners still produce large errors, as observed along the V7 path. The individual gauge strains and paired shear-strain differences can be compared at different loading positions, in order to examine possible response cancellation and the influence of nearby web frames.

Declaration of artificial intelligence use

OpenAI's ChatGPT and Codex were used during the preparation of this thesis to support the drafting and revision of parts of the text, language correction, improvement of clarity and structure, development and debugging of MATLAB code, checking of data processing and calculations, and the identification and formatting of references.

All AI-assisted outputs were critically reviewed, edited and, where applicable, checked against the original data and publications. AI-assisted calculations and code suggestions were verified against the source data by the author. The finite element models, research methodology, selection of input data, interpretation of the results, conclusions and final content of the thesis remain the responsibility of the author.

9 References

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Appendix A

A.1 Normal-strain based ICM from SPOS values

1.0e+03

Table A.1 Normal-strain based ICM from SPOS values.

-926.83	-292.71	-534.93	-418.05	-237.58	11.19	256.69	420.91	463.36	380.00	209.23	-307.85	3.61	-29.55	-50.90	-14.08	6.00	30.18	45.28	50.95	33.12	25.00	-561.46	-555.01	-515.96	-468.77	-261.26	17.47	284.42	468.21	512.03	417.92	252.95
-333.23	-1543.70	-919.04	-939.34	-487.51	63.07	593.42	949.74	1031.49	837.12	455.47	-11.12	-323.16	-81.26	-77.57	-63.91	9.74	59.92	93.13	108.04	70.10	49.97	-676.48	-1240.93	-1231.46	-978.68	-551.59	57.98	647.06	1042.27	1131.31	915.32	546.88
-643.23	-952.93	-1977.66	-1028.07	-669.28	98.35	788.68	1247.28	1352.46	1096.45	594.37	-47.03	-60.11	-387.72	-89.03	-52.82	-5.05	82.70	121.75	141.11	91.69	64.72	-688.32	-1284.51	-1732.19	-1363.08	-684.29	83.03	850.50	1368.13	1479.94	1196.16	713.19
-507.40	-1004.81	-1033.09	-1696.89	-444.81	64.44	762.88	1192.75	1285.31	1040.62	562.92	-73.24	-67.76	-90.02	-360.01	-34.94	18.25	59.72	118.14	131.99	86.39	60.82	-630.20	-1033.51	-1379.23	-1436.41	-724.17	113.09	809.53	1297.48	1404.80	1132.23	673.74
-279.20	-511.84	-683.71	-427.46	-959.70	223.68	465.36	782.38	834.28	670.23	361.74	-24.45	-61.59	-55.37	-29.04	-289.69	31.09	57.31	58.46	87.70	54.40	38.37	-343.45	-583.66	-689.27	-725.16	-628.46	3.94	551.51	834.14	900.74	727.33	430.10
41.21	85.00	119.39	69.33	245.13	-557.90	236.01	87.18	128.92	95.60	47.65	2.90	11.21	-8.76	19.36	36.32	-250.34	31.29	19.32	-4.87	10.12	4.20	47.87	81.95	102.23	132.10	11.68	-189.84	18.34	139.20	116.80	92.97	54.47
353.48	661.54	830.65	786.31	467.55	254.22	-937.81	-420.99	-657.18	-503.56	-276.42	37.60	53.36	86.69	56.43	58.71	37.16	-288.35	-32.27	-56.71	-58.15	-26.19	419.98	720.21	895.60	834.28	566.92	23.86	-604.49	-701.55	-674.38	-569.03	-340.52
557.58	1043.14	1298.64	1212.49	792.33	83.21	-403.87	-1678.16	-1035.78	-995.88	-515.08	60.39	86.44	129.02	122.53	58.28	20.70	-28.17	-357.72	-98.08	-68.69	-70.75	667.85	1137.02	1417.43	1319.60	838.90	150.24	-694.28	-1416.50	-1368.20	-1033.93	-635.04
594.70	1108.71	1380.49	1280.74	824.78	129.99	-654.52	-1003.82	-1976.82	-977.51	-646.45	64.52	93.06	139.52	125.01	86.38	-6.47	-51.41	-81.65	-392.61	-65.97	-50.39	713.00	1213.18	1507.32	1405.20	891.60	118.44	-654.89	-1352.76	-1734.19	-1292.03	-706.67
461.52	858.17	1065.75	988.27	629.05	86.22	-474.42	-951.27	-917.35	-1562.77	-368.19	50.77	71.74	108.27	97.75	61.76	12.72	-66.42	-76.18	-80.12	-325.63	-17.57	554.67	940.02	1167.58	1083.91	687.89	87.47	-539.70	-979.83	-1248.24	-1259.86	-694.09
216.29	397.25	489.80	448.24	279.41	23.62	-237.24	-424.65	-557.79	-295.11	-930.38	25.37	34.76	52.06	47.41	31.85	5.63	-13.14	-54.51	-33.11	9.53	-307.89	261.05	437.85	539.77	497.78	309.38	32.61	-259.65	-481.40	-528.67	-571.52	-578.69
293.11	-118.88	-61.84	-25.17	-29.75	-0.74	21.06	36.37	41.00	34.12	19.14	-370.69	221.59	-78.97	4.28	-3.58	-6.31	-1.27	0.29	1.44	1.31	1.46	-308.94	-5.35	-30.29	-53.99	-17.65	1.91	26.26	42.99	47.52	38.73	24.84
-123.05	193.34	-152.14	-84.88	-25.91	-3.37	42.96	69.70	76.46	62.67	34.01	246.02	-343.52	208.74	-63.51	14.66	9.12	7.48	10.42	10.02	5.60	3.98	-0.45	-322.23	-62.66	-64.86	-56.70	7.40	48.34	79.18	86.32	69.88	42.11
-71.59	-181.56	135.02	-185.59	-79.68	14.39	59.01	108.64	119.36	98.12	53.32	-86.79	223.46	-334.62	211.85	-57.01	23.88	18.70	15.90	17.24	10.22	6.26	-47.79	-78.82	-387.30	-92.65	-55.09	-7.98	80.43	124.34	135.90	110.87	66.76
-26.79	-100.06	-186.50	163.21	-132.12	-14.86	70.61	90.88	107.58	87.88	47.67	16.80	-75.85	226.83	-325.84	222.10	-46.82	32.07	22.82	16.76	11.02	6.54	-69.43	-77.43	-81.79	-357.24	-32.75	16.66	54.44	115.96	122.41	99.33	60.93
-35.14	-34.32	-79.12	-141.37	221.92	-76.18	21.35	76.28	64.83	59.26	31.69	3.56	18.95	-68.71	237.07	-315.84	230.91	-41.36	32.81	20.20	8.31	6.19	-22.91	-69.39	-54.41	-30.35	-290.38	29.90	57.33	61.36	86.23	67.42	41.75
4.08	-2.85	20.50	-12.94	-81.85	257.69	-75.05	-12.27	18.78	-0.14	3.84	0.47	9.21	28.93	-58.43	245.58	-310.85	230.83	-45.95	24.84	9.81	1.58	5.77	13.98	-7.43	20.26	36.49	-250.31	31.34	19.86	-3.72	13.15	6.90
29.65	56.64	59.60	76.39	20.27	-81.16	225.58	-130.16	-74.31	-33.44	-31.45	6.57	7.06	19.11	36.52	-53.86	245.58	-315.32	222.82	-55.70	14.98	4.38	39.29	63.25	84.21	55.74	57.32	37.33	-286.32	-30.61	-50.15	-62.86	-22.82
47.82	89.42	110.99	92.21	76.90	-12.59	-137.47	163.28	-181.13	-101.60	-31.63	6.05	11.51	15.85	22.80	36.54	-58.87	236.87	-325.58	211.86	-63.24	11.74	61.07	101.61	124.66	121.26	55.84	20.64	-25.65	-357.04	-88.62	-80.05	-67.85
52.48	97.09	119.63	110.21	59.11	19.68	-76.22	-189.07	138.03	-173.78	-70.80	6.35	9.65	17.25	15.26	18.23	28.31	-69.04	227.09	-334.61	208.47	-72.21	65.61	109.98	135.77	124.62	84.38	-7.21	-50.36	-83.86	-384.84	-82.24	-49.60
34.78	64.49	79.16	72.92	46.44	-3.85	-22.57	-86.27	-160.46	191.77	-118.24	3.72	6.07	9.58	11.08	6.77	8.58	18.89	-75.84	223.38	-342.96	225.37	43.12	72.12	89.64	82.97	51.15	11.42	-58.49	-64.55	-58.46	-325.10	-7.70
18.68	34.37	41.60	36.71	21.91	1.05	-31.14	-20.85	-60.28	-126.82	290.05	1.91	1.03	1.88	0.43	-0.44	-6.24	-3.75	9.74	-91.44	234.88	-358.30	24.13	38.96	47.65	43.68	27.02	2.40	-14.99	-56.05	-29.52	2.47	-306.45
-567.25	-430.68	-353.17	-341.57	-185.78	16.66	209.15	340.02	372.95	304.72	167.17	294.74	-107.26	-56.73	-18.55	-25.04	1.43	21.14	32.53	38.75	24.85	18.64	-917.07	-238.31	-472.89	-366.08	-206.15	11.42	226.44	372.40	407.65	332.76	200.89
-543.85	-1185.98	-1046.06	-796.68	-463.24	52.04	541.20	864.69	940.38	764.13	415.75	-135.57	192.34	-168.67	-96.67	-32.10	-0.68	53.75	83.58	96.31	62.87	44.63	-382.78	-1544.37	-920.68	-944.17	-487.30	62.49	595.19	956.13	1035.11	836.46	499.93
-500.21	-1087.21	-1592.54	-1146.24	-551.86	69.23	711.50	1135.45	1230.44	998.58	542.09	-75.70	-164.12	134.48	-182.64	-77.64	16.33	61.15	106.30	124.26	80.54	56.65	-698.53	-950.21	-1975.72	-1031.80	-668.41	97.60	787.82	1253.44	1354.84	1092.86	651.04
-484.04	-841.69	-1164.23	-1340.68	-618.49	114.05	673.54	1079.58	1169.62	947.08	512.56	-34.20	-91.38	-193.54	160.07	-134.10	-13.24	75.13	93.38	116.82	75.38	52.58	-555.44	-1003.15	-1031.92	-1701.13	-444.82	61.93	762.13	1197.20	1286.95	1037.00	615.93
-258.61	-489.76	-555.41	-617.27	-672.43	-10.64	478.86	692.01	754.58	611.95	329.60	-38.91	-26.93	-80.14	-140.57	222.72	-75.63	21.71	74.01	66.36	47.46	32.07	-306.43	-513.05	-685.09	-432.10	-961.60	221.88	464.18	785.92	836.67	669.00	396.65
38.89	71.25	81.17	128.80	-5.88	-306.60	1.78	136.55	98.96	83.63	45.45	1.80	-5.35	19.07	-13.93	-82.34	258.09	-75.17	-12.58	18.14	-2.83	2.11	44.31	82.90	116.22	66.37	242.14	-559.89	236.16	88.63	131.01	96.91	53.20
320.82	604.13	751.13	690.03	493.52	4.17	-652.20	-597.49	-543.09	-476.80	-256.53	31.49	47.30	62.06	77.53	21.51	-81.35	223.92	-131.93	-80.31	-29.01	-36.39	387.08	660.92	830.43	788.80	467.08	255.09	-939.02	-424.62	-659.41	-502.43	-303.28
508.31	948.87	1182.44	1098.25	695.23	143.49	-593.96	-1323.48	-1153.04	-845.88	-485.47	51.80	75.79	114.98	93.75	79.15	-12.48	-139.57	162.21	-189.72	-90.85	-38.46	610.42	1041.47	1297.72	1216.92	791.89	85.16	-402.82	-1682.24	-1036.17	-991.71	-563.20
542.47	1010.60	1256.10	1166.97	745.60	93.42	-526.47	-1137.51	-1592.39	-1096.44	-518.27	56.86	81.36	123.04	110.81	60.96	20.59	-77.31	-187.29	130.49	-158.51	-75.90	651.39	1108.48	1380.52	1286.98	825.72	132.89	-652.17	-1008.00	-1977.82	-972.22	-703.10
421.95	784.67	974.48	901.36	575.65	73.72	-456.28	-796.46	-1059.24	-1205.40	-556.72	45.27	64.65	96.39	87.42	56.51	-2.25	-29.56	-97.18	-180.04	191.32	-130.61	507.44	860.82	1070.25	996.35	632.63	89.64	-474.02	-957.44	-922.13	-1562.77	-418.69
175.97	324.43	400.76	368.98	231.24	27.52	-189.09	-359.22	-367.22	-449.40	-585.28	19.53	26.54	40.54	35.29	22.72	2.86	-27.88	-16.76	-59.32	-117.72	289.37	211.69	354.87	438.17	403.91	252.19	23.93	-208.88	-379.17	-502.75	-244.47	-925.12

A.2 Normal-strain based ICM from SNEG values

1.0e+03

Table A.2 Normal-strain based ICM from SNEG values.

-672.19	154.50	-19.21	27.06	17.72	10.48	8.07	5.71	5.34	5.96	7.00	309.26	-80.88	-18.32	12.45	-9.10	-2.99	-2.90	-3.06	-1.34	-1.12	0.72	-350.38	-59.46	69.97	21.20	21.95	16.64	9.37	6.75	5.09	5.19	4.91
155.50	-638.80	154.13	-11.75	28.33	17.08	8.18	5.20	2.72	2.67	3.70	-88.07	259.89	-69.77	-10.78	16.71	-3.38	1.74	0.43	0.17	-0.02	0.96	-81.68	-340.74	-44.67	68.27	22.61	20.59	14.53	7.22	4.50	2.78	2.66
-45.25	166.13	-614.77	168.02	-2.71	32.80	18.78	8.84	4.95	2.50	1.99	-15.94	-78.04	258.94	-73.03	-12.04	14.48	-5.56	0.02	-0.64	-0.94	0.33	53.49	-48.48	-322.93	-27.09	78.21	27.43	22.84	14.50	6.76	3.62	1.38
15.90	-22.58	181.05	-604.67	173.82	-0.04	33.01	17.94	7.60	3.41	1.20	18.54	-12.54	-79.13	260.13	-72.94	-12.74	14.25	-5.90	-0.25	-0.48	-0.51	-1.03	69.39	-27.94	-308.58	-17.58	83.73	29.76	23.10	13.89	5.33	1.03
7.18	26.80	-12.51	188.84	-600.35	175.93	0.58	32.64	17.11	6.43	3.13	-8.42	18.22	-12.36	-79.16	259.61	-73.06	-13.04	14.17	-5.70	0.28	-0.30	7.76	14.40	82.03	-18.08	-302.34	-14.52	85.33	29.07	21.56	11.89	3.25
2.61	14.12	33.31	-8.31	191.30	-599.49	175.85	-0.75	31.41	14.38	3.11	1.43	-7.12	17.33	-12.98	-79.63	259.11	-73.35	-12.94	14.56	-5.34	0.65	6.76	17.52	22.92	88.74	-14.24	-300.34	-14.85	83.38	26.16	17.27	6.72
2.90	6.13	16.73	35.30	-7.32	191.44	-600.09	173.61	-4.80	24.91	7.47	-0.45	0.95	-7.37	17.02	-13.04	-79.62	259.63	-72.96	-12.36	15.30	-6.28	2.26	12.01	21.95	26.18	90.55	-14.25	-302.00	-18.28	77.11	17.54	7.80
1.52	3.62	7.20	17.67	36.04	-7.53	189.24	-604.34	166.20	-15.33	13.47	-0.43	-0.58	0.64	-7.86	16.99	-12.72	-79.24	259.95	-72.66	-12.54	15.24	1.47	4.83	14.17	23.58	26.87	89.81	-17.67	-308.09	-28.31	64.32	2.08
2.02	2.17	5.20	8.01	18.39	35.21	-9.81	183.37	-614.81	151.74	-36.64	0.26	-1.07	-0.60	0.80	-7.38	17.58	-12.49	-78.79	258.94	-71.80	-14.98	1.53	3.40	6.17	14.81	22.89	24.76	84.33	-26.03	-322.43	-48.38	47.52
3.83	2.68	2.70	5.54	7.83	16.96	30.74	-18.12	170.21	-637.80	138.00	0.82	0.27	-0.36	0.55	1.98	-5.54	19.64	-11.28	-76.26	260.36	-81.90	2.64	2.98	4.41	6.91	14.91	20.98	19.81	74.60	-43.24	-340.20	-78.80
6.67	5.79	4.74	5.49	7.91	9.60	17.58	29.32	-27.46	168.53	-664.04	0.83	-1.21	-1.20	-2.39	-2.86	-1.58	-10.85	15.71	-17.00	-89.31	305.45	4.88	4.75	4.96	6.36	8.82	16.24	22.33	17.36	75.29	-58.39	-355.08
-276.15	42.64	24.90	-4.19	13.49	7.42	6.58	5.81	4.32	3.95	3.48	-374.87	216.09	-79.13	6.49	-0.81	-3.68	-0.36	-0.01	0.15	-0.17	0.37	308.55	-82.69	-17.51	13.11	-8.35	-2.08	-2.24	-1.79	-0.74	0.52	0.92
47.40	-238.31	36.13	20.56	-6.82	9.11	3.54	2.77	1.92	0.74	0.56	240.42	-353.85	197.27	-73.85	4.44	-2.49	-5.04	-1.69	-1.06	-0.51	-0.08	-85.49	260.81	-69.36	-11.16	16.28	-4.22	0.87	-0.36	-0.82	-0.50	-0.17
18.58	41.56	-235.76	42.81	24.69	-2.94	11.78	5.08	3.93	2.73	1.29	-87.92	213.36	-349.47	193.43	-75.35	4.24	-2.34	-4.19	-0.79	-0.38	0.51	-16.11	-78.88	259.15	-72.84	-12.16	14.64	-5.57	-0.13	-0.61	-0.63	0.38
-14.74	21.57	47.37	-232.78	45.69	26.41	-1.87	12.33	4.70	3.01	1.30	15.72	-85.31	207.51	-353.99	189.92	-78.16	2.24	-3.51	-4.80	-0.87	0.31	19.19	-12.84	-78.91	260.33	-72.91	-12.64	14.43	-5.94	-0.04	-0.58	-0.54
8.25	-9.65	24.41	50.62	-230.63	47.03	26.82	-2.35	11.27	3.45	0.83	1.94	10.04	-87.90	205.13	-355.75	188.76	-78.58	2.40	-2.77	-4.29	0.92	-7.94	17.63	-11.92	-79.05	259.94	-72.93	-12.90	14.49	-5.62	0.20	-0.31
0.38	10.58	-6.80	26.92	52.34	-229.67	47.11	26.16	-4.12	9.03	1.10	-2.76	-1.58	8.13	-89.85	203.75	-356.42	188.67	-78.11	3.59	-1.47	-2.13	1.57	-7.36	17.40	-12.54	-79.31	259.69	-72.93	-12.74	14.55	-5.49	0.71
0.71	2.64	12.84	-5.12	27.65	52.20	-230.43	45.12	24.07	-6.90	6.59	1.41	-4.69	-3.10	7.00	-90.63	203.80	-355.85	190.31	-76.27	5.51	1.94	-0.53	0.86	-7.41	17.19	-12.89	-79.20	259.88	-72.87	-12.17	15.07	-5.91
1.36	3.03	3.83	13.81	-4.72	27.46	50.91	-232.28	41.68	20.95	-11.44	0.10	-0.21	-5.42	-3.62	6.89	-90.12	204.99	-354.11	192.70	-73.57	10.95	-0.38	-0.49	0.44	-7.75	17.31	-12.64	-78.96	260.35	-72.67	-13.14	15.87
1.24	2.33	3.99	4.69	13.45	-5.68	25.46	47.86	-235.52	36.68	16.99	0.31	-0.64	-0.10	-5.21	-2.46	8.59	-87.64	208.28	-349.85	198.02	-74.11	0.06	-0.91	-0.54	0.64	-7.41	17.51	-12.33	-79.13	259.42	-72.48	-15.26
0.45	1.26	2.01	2.58	3.13	10.85	-9.15	21.83	42.10	-239.05	42.40	0.22	-0.47	-1.33	-0.79	-5.59	-2.55	9.18	-85.99	212.36	-353.36	219.60	-0.01	-0.24	-0.61	-0.66	1.35	-6.36	19.03	-11.33	-76.00	261.04	-79.61
3.42	3.36	4.28	5.72	6.00	6.60	14.73	-7.48	24.62	49.68	-274.06	0.18	-0.12	0.16	-0.43	0.23	-4.44	-1.22	11.65	-91.72	229.11	-362.73	0.91	0.25	-0.61	-1.69	-2.17	-0.82	-10.19	16.28	-16.74	-90.78	305.41
-259.61	-67.83	47.24	18.46	15.72	12.50	7.12	6.19	6.04	7.63	9.91	-276.02	44.71	24.41	-4.75	13.29	6.87	6.47	4.95	4.13	2.30	3.07	-673.69	149.67	-20.43	27.71	19.13	11.53	8.86	6.72	6.10	6.60	6.97
-81.89	-239.50	-41.39	56.01	24.68	18.44	12.35	5.93	3.71	3.04	3.63	44.51	-239.42	35.29	20.61	-6.75	9.73	3.84	3.39	2.58	1.19	1.61	160.40	-637.72	154.93	-10.62	29.12	17.96	9.05	5.42	3.21	2.50	2.54
32.38	-47.04	-220.12	-25.18	65.78	28.79	18.73	11.35	3.62	1.43	0.81	19.04	42.31	-235.87	42.51	24.85	-3.15	12.09	5.28	3.57	2.49	1.06	-44.02	165.30	-615.11	167.87	-2.30	33.18	19.05	8.48	4.75	1.29	1.25
-0.31	53.53	-26.68	-205.46	-15.70	70.39	30.50	18.61	10.40	2.78	-0.25	-15.53	22.20	46.70	-232.58	45.37	26.55	-2.33	12.23	4.81	2.72	1.36	15.96	-23.88	180.31	-604.31	174.05	0.12	33.52	17.94	7.16	3.60	0.25
2.26	16.86	66.93	-16.70	-199.47	-12.90	71.20	29.60	17.08	8.10	0.05	7.79	-9.35	24.22	50.12	-230.73	46.53	26.94	-2.55	10.85	3.59	0.78	6.73	26.10	-12.71	188.37	-599.97	176.12	0.53	32.79	16.71	5.30	2.15
2.57	12.66	25.00	72.97	-13.09	-197.93	-13.10	69.07	27.02	12.82	2.51	0.20	10.62	-6.77	26.72	51.71	-229.86	46.38	26.14	-4.19	9.31	0.78	1.99	13.12	32.55	-8.04	190.95	-598.74	175.82	-1.05	30.74	13.08	2.58
-0.36	8.31	16.48	28.21	74.70	-12.81	-199.38	-16.25	63.22	19.05	2.65	0.57	3.07	12.44	-5.29	27.52	51.83	-230.58	44.84	23.74	-6.65	6.13	1.99	5.10	16.46	35.00	-6.86	191.12	-600.07	173.79	-5.39	23.99	7.14
-0.05	2.24	10.53	18.38	29.04	74.39	-15.62	-205.37	-26.06	50.13	1.25	1.40	2.51	4.30	13.43	-4.78	27.15	50.64	-232.63	41.57	21.32	-12.03	0.53	3.28	6.91	17.88	35.78	-6.68	189.37	-604.47	165.86	-16.37	13.32
1.13	1.54	3.75	11.15	18.69	26.97	70.03	-24.29	-219.93	-46.50	28.57	1.08	2.35	3.59	4.74	13.46	-5.63	25.33	48.04	-235.99	37.55	17.45	1.15	1.63	4.73	8.15	18.62	35.52	-9.84	183.84	-615.15	150.52	-35.19
3.32	2.85	3.63	5.80	12.24	18.51	23.07	60.53	-40.95	-238.60	-79.78	1.36	1.40	2.63	3.60	3.38	11.42	-8.94	21.87	41.09	-239.75	39.57	2.59	2.29	2.80	5.85	8.62	17.84	31.92	-17.54	171.00	-637.17	142.48
9.81	7.38	5.67	5.89	6.85	12.28	15.73	16.53	51.53	-66.92	-262.39	3.16	1.99	3.75	5.13	5.98	6.15	14.60	-7.99	24.38	51.27	-274.16	6.79	6.41	5.81	6.19	8.87	10.55	18.89	30.00	-28.13	164.11	-665.71



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