

Estimation of Steady State Rollover Threshold for High Capacity Transport Vehicles using RCV Calculation Method

Master's thesis in Automotive Engineering

ABHISHEK SINGH TOMAR

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Göteborg, Sweden 2015

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Schematic roll plane representation of tractor and trailer sprung mass roll motion with fifth-wheel coupling at tractor drive axle

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ABSTRACT

The rollover stability of *high capacity transport (HCT)* vehicles depends on multiple phenomena which result in lateral and vertical shift of the vehicle center of gravity and consequently in the load transfer from inner side tires to outer side tires. Vehicles' roll stability depends on the equilibrium between overturning and restoring moments. When overturning moments become governing, vehicle loses its stability and rolls over. Some of the design parameters which determine the roll stability are the height of center of gravity, track-width, suspension and tire compliance, axle and suspension roll stiffness, fifth-wheel compliance and frame flexibility.

United Nations Economic Commission for Europe (UNECE) Regulation 111 approves vehicles either by tilt table test or a calculation method, which is theoretical way to determine *steady-state rollover threshold (SRT)*. Tilt table test is a good method but limited by the size of the test bed which makes it less preferable for long vehicle combinations with more than one articulation joints and the most preferable way to determine SRT is through a calculation method. However, calculation method involves number of non-validated parameters and simplified equations, which generally predicts higher SRT value than the actual. This overestimation can lead to rollover situation even within safe predicted zone.

This study presents a plausible physical interpretation of UNECE111 calculation method to identify its assumptions and simplifications. An alternative *Roll Compliant Vehicle (RCV)* method is proposed, which includes a more sophisticated approach of compliances influence on roll stability and provides a direct comparison with UNECE111. The SRT value obtained from UNECE111 and RCV methods are then compared with tilt table tests for certain vehicle combinations to determine the degree of UNECE111's SRT overestimation within the study.

Keywords: Heavy commercial vehicles, UNECE111, Steady state rollover threshold, Fifth-wheel compliance

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In this study, rollover of high capacity transport vehicles have been studied to formulate RCV calculation method for steady state rollover threshold estimation. The study has been carried out from February 2015 to June 2015 at the department of Applied Mechanics, Chalmers University of Technology and Volvo Group Trucks Technology (VGTT) in Gothenburg, Sweden.

The work is a part of research project concerning legislation of high capacity transport vehicles, namely, Performance Based Standards for High Capacity Transports in Sweden (PBS-project). The project involves Swedish National Road and Transport Research Institute (VTI), Chalmers University of Technology, VGTT, Scania, Transportstyrelsen, Trafikverket, and Parator Industri AB.

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Gothenburg, June 2015

Abhishek Singh Tomar

NOMENCLATURE

Terminology

The terminology essentially follows ISO-8855 standard. If divergent, then defined by author at respective places.

Dimensions are in SI-units.

All figures are drawn in y-z (i.e. roll) plane. Vehicle is considered to be taking left turn, i.e. rolling towards right side.

Abbreviations

COG	Center of Gravity
EC	European Commission
EU	European Union
HCT	High Capacity Transport
LCV	Long Vehicle Combination
NRTC	National Road Transport Commission
PBS	Performance Based Standards
RCV	Roll Compliant Vehicle
SAR	Standard Axle Repetitions
SPR	Side Pull Ratio
SRT	Steady State Rollover Threshold
SSF	Static Stability Factor
RMD	Roll moment diagram
TTR	Tilt Table Ratio
UNECE	United Nations Economic Commission for Europe
VGTT	Volvo Group Trucks Technology
VTM	Virtual Truck Models

Notations

Notations used in UNECE111 calculation method [2]:

UNECE111 symbol	Adapted symbol	Unit	Description
g	g	[m/s ²]	acceleration due to gravity; = 9.80665
i	i	[-]	axle]bogie index ($i = 1 - n$, front to - axle/bogie; $i = T$, all axles/bogies; $i = M$, stiffest axle/bogie; $i = k$, kingpin)
m_i	$h_{rc,i}$	[m]	nominal suspension roll axis height
q_c	a_{yC}	[g]	corrected lateral acceleration at overturn
q_M	a_{yS}	[g]	lateral acceleration at first wheel lift
q_T	a_{yT}	[g]	lateral acceleration at which all inner wheels lift from ground
A_i	$m_i g$	[kN]	axle/bogie load
C_{DG_i}	$C_{s,i}$	[kNm/rad]	suspension roll stiffness at axle roll axis

C_{DGMi}	$C_{sg,i}$	[kNm/rad]	equivalent suspension roll stiffness at ground level
C_{DRi}	$C_{al,i}$	[kNm/rad]	axle/bogie roll stiffness
C_{DRESi}	$C_{res,i}$	[kNm/rad]	resolved combined suspension roll stiffness at ground level
F_E	s_f	[-]	effective mass factor of stiffest axle/bogie
F_{RVi}	$C_{tv,i}$	[kN/m]	vertical tire rate for each axle/bogie
H_G	h_{cg}	[m]	center of gravity height of complete vehicle
H_N	h_s	[m]	center of gravity height of sprung mass
MA	l_{ttw}	[m]	twin tire width
T_{Ni}	$l_{tn,i}$	[m]	nominal track width
T_i	$l_{t,i}$	[m]	theoretical track width for axle/bogie with twin tires
U_i	$m_{u,i} g$	[kN]	un-sprung weight
θ_i	ϕ_i	[rad]	vehicle pseudo roll angle at wheel lift
β	α	[deg]	equivalent tilt table angle

Notations used in RCV method:

Symbol	Unit	Description
a_y	$[m/s^2]$	lateral acceleration
g	$[m/s^2]$	acceleration due to gravity; = 9.80665
$h_{s,i}$	[m]	center of gravity height of sprung mass at an axle
$h_{cg,i}$	[m]	total center of gravity height at an axle
h_{fw}	[m]	height of fifth-wheel load from ground
$h_{rc,i}$	[m]	height of suspension roll center from ground at an axle
l_{fw}	[m]	width of fifth-wheel
$l_{t,i}$	[m]	nominal track-width at an axle
$l_{teff,i}$	[m]	effective track-width including tire lateral shift at an axle
δl_i	[m]	lateral shift of sprung mass cog at an axle
m_i	[kg]	total mass at an axle
$m_{s,i}$	[kg]	sprung mass at an axle
s_f	[-]	stiffness factor
$C_{al,i}$	[Nm/rad]	axle roll stiffness about axle roll center at an axle
C_{fw1}	[Nm/rad]	fifth-wheel roll stiffness before trailer separation
C_{fw2}	[Nm/rad]	fifth-wheel roll stiffness after trailer separation
$C_{res,i}$	[Nm/rad]	resultant roll stiffness about axle roll center at an axle
$C_{s,i}$	[Nm/rad]	suspension roll stiffness about suspension roll center at an axle
$C_{sg,i}$	[Nm/rad]	equivalent suspension roll stiffness about axle roll center at an axle
$C_{tv,i}$	[N/m]	tire vertical stiffness at an axle
C_{tw}	[Nm/rad]	trailer frame torsional stiffness
$C_{ty,i}$	[N/m]	tire lateral stiffness at an axle
F_l	[N]	vertical load at fifth-wheel left side edge
F_r	[N]	vertical load at fifth-wheel right side edge
$F_{y,i}$	[N]	tire lateral force at an axle
F_{fw}	[N]	fifth-wheel vertical load
$F_{zl,i}$	[N]	vertical reaction force at left side tire at an axle

$F_{zr,i}$	[N]	vertical reaction force at right side tire at an axle
$\phi_{i,max}$	[rad]	total roll angle at an axle
$\phi_{al,i}$	[rad]	roll angle due to tire compliance at an axle
$\phi_{eff,i}$	[rad]	effective roll angle of vehicle COG
ϕ_{fw}	[rad]	fifth-wheel roll angle
ϕ_{is}	[rad]	effective roll angle of vehicle COG due to suspension compliance
$\phi_{s,i}$	[rad]	sprung mass roll angle at an axle
$\phi_{sg,i}$	[rad]	roll angle due to suspension compliance at an axle
ϕ_{tw}	[rad]	trailer frame twist angle
c_i	[-]	location of COG at an axle
$c_{s,i}$	[-]	location of sprung mass COG at an axle
o	[-]	location of axle roll center at an axle
R_i	[-]	location of suspension roll center at an axle
X_{sg}	[-]	suspension compliance factor
Δ_T	[m]	tire lateral shift due to tire lateral compliance
X_i	[-]	slope of effective roll angle versus lateral acceleration
X	[-]	slope of fifth-wheel roll angle versus lateral acceleration curve before trailer separation
X_1	[-]	slope of fifth-wheel roll angle versus lateral acceleration curve after trailer separation
X_{lash}	[-]	instantaneous slope of roll angle versus lateral acceleration due to trailer lash

Subscript, 'i':

1	front axle
2	first unit rear axle group
3	trailer axle group
fw	corresponds to fifth-wheel or its compliance
st	corresponds to trailer sprung mass
T	single axle representation of complete vehicle/vehicle combination
S	single axle representation of vehicle's stiffest axle
$2, tl$	only trailer sprung mass as drive axle sprung mass

' Δ ' when used as prefix to any physical quantity, represents the change in respective physical quantity, if not stated otherwise.

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1 Introduction

Transportation of goods is a backbone of the economy of any society. The effectiveness and safety of road transportation directly affects economy and health. High capacity transport (HCT) vehicles are one of the solution which is being implemented in European Countries to achieve economical effectiveness (viz. traffic flow, cost, and environment) of road transport system. Despite the clear advantages, safety of HCT vehicles remain the resolving factor.

In accidental research report [5], [17], rollover is one of the key safety issue concerning HCT vehicles due to their high complexities and compliant nature. The present work is to study and develop a methodology to estimate the rollover safety for HCT vehicles. The thesis work is a part of Swedish PBS project which aims to provide regulatory description for HCT vehicles in Sweden. A new calculation (roll compliant vehicle) method is proposed to estimate steady state rollover threshold (SRT) for HCT vehicles which shows advantage over United Nations Economic Commission for Europe Regulation No. 111 (UNECE111) method for investigated vehicle combination.

1.1 Problem that Motivates the Thesis

The UNECE111 is regulatory guideline for evaluating SRT for HCT vehicles. It proposes tilt table testing and a calculation method [2]; tilt table testing is a good method but it is limited by availability of testing facilities, length of test-bed and cost and the calculation method can only evaluate single vehicle units (i.e. either tractor or semi-trailer). Therefore, at present there is no method available which can evaluate SRT for HCT vehicles.

The UNECE111 calculation method is suspected to overestimate the SRT [24], [13] compared to tilt table tests even for single vehicle units . In other words, tested vehicle will rollover before the SRT value estimated by calculation method which is dangerous for safety of HCT vehicles and their surroundings. To study and identify the reasons behind the overestimation of SRT, a research is needed. The calculation method undertakes various assumptions and simplifications, which needs to identified and their influence needs to be considered for better SRT estimation.

1.2 Purpose of the Thesis

The main purpose of this thesis is to evaluate UNECE111 calculation method through comparison with tilt table tests and question which physical phenomena that are and should be modelled in such calculation method.

1.2.1 Objectives

The main phenomena to investigate for possible inclusion in SRT estimation for HCT vehicles,

- Influence of tire and axle compliance
- Influence of suspension compliance
- Influence of fifth-wheel lash

- Influence of trailer lash
- How to estimate SRT for roll coupled vehicle combinations

The research questions which can be answered by this thesis work are:

- How much influence and how to include fifth-wheel compliance
- How to consider lateral shift of vehicle cog (inverse pendulum effect)

1.2.2 Deliverables

The deliverables which are achieved within current thesis work are:

- Plausible derivation of UNECE111 calculation method, as physical as possible.
- Sketch of roll compliant vehicle calculation method.
- Evaluation of UNECE111 method.
- Verification of RCV method with roll moment diagram (RMD).
- Validation of RCV method with respect to tilt table tests.

1.3 Overview of Methodology

The roll complaint vehicle (RCV) is a calculation method. The influence of various compliances in the vehicle are derived. Real tilt table tests data are used as comparison to calculations.

1.3.1 Assumptions

Due to static nature and limited number of available inputs for calculation method, following assumptions are considered in RCV method at respective places.

- Suspension elements are considered as a hypothetical torsional spring at respective roll centers.
- Influence of camber angle is not considered.
- Influence of tire pressure is not considered.
- Fifth-wheel lash (i.e. fifth-wheel roll angle about its roll center before bump stops contact) is considered constant as 2 degree.
- Torsional stiffness of frame is not considered.
- Non linearities of suspension and tires are not considered.

1.3.2 Limitations

The thesis work is limited only to investigate steady state rollover threshold for HCT vehicles. Dynamic or transient rollover threshold is not considered in present work.

Vehicle combinations longer than tractor-semitrailer and tractor dolly-semitrailer are not included due to unavailability of tilt table test data.

1.4 Key Terminology

The important terminologies which are used within thesis work are as follows:

Roll Complaint Vehicle: *A vehicle model including effects of roll compliance, such as compliance in frame, axle suspension and tires.*

Fifth-wheel Lash: *The magnitude of roll angle which can be achieved by a fifth-wheel with respect to tractor before fifth-wheel bump stops come in contact.*

Trailer Lash: *The maximum magnitude of angle between trailer and fifth-wheel at the time of trailer separation.*

Steady State Rollover Threshold: *The level of lateral acceleration at which all inner side wheels (i.e. from tractor and semi-trailer both) lift from ground in a vehicle/vehicle combination.*

1.5 Organization of Thesis

In Chapter 2, a brief description of rollover mechanism is presented together with methods to evaluate roll stability of a vehicle.

UNECE111 method recognizes the influence of vehicle compliance on roll stability but SRT estimated with this calculation method differs from tilt table test. One possible reason for this difference can be, calculation method simplifies and ignores some of the effects of vehicle compliance. To identify these simplifications, a physical derivation of calculation method is provided in Chapter 3.

A new calculation method, referred as *Roll Compliant Vehicle (RCV)*, based on physical nature of compliance is presented in Chapter 4 to 6. The aim of RCV method is to provide an alternative approach to UNECE111 calculation method for estimating SRT, especially for HCT vehicles. In order to provide a clear step-by-step comparison between two methods, the structure of RCV method is adapted from UNECE111 calculation method.

Suspension system in HCT vehicles are critical due to heavier sprung mass and higher cog height which can make vehicle more susceptible to roll instability. Roll motion of vehicle's sprung mass is affected by height of suspension roll center, stiffness of suspension system and sprung mass itself. Tires are the only contact between ground and vehicle, and due to their non-linear nature, becomes important to understand the physics of tires during roll motion. It has been observed that not only vertical properties of tire influence roll motion of vehicle but also their lateral properties. One of the consequences of tire lateral properties arrive as reduction in axle track-width, which can reduce roll stability. RCV method includes such effects of suspension and tire compliance in estimating SRT, methodology of these compliance is discussed in Chapter 4.

In HCT vehicles, generally, connection between two units consists of a fifth-wheel, which provides an extra roll degree of freedom between the two units. A step wise description and modeling of fifth-wheel motion is presented in Chapter 5 together with frame flexibility.

The complete RCV method is described in Chapter 6, with prospect of estimating SRT for both single unit vehicles and vehicle combinations.

In Chapter 7, RCV method is first verified using roll moment diagram (RMD) approach, then comparative parameter sensitivity between UNECE111 and RCV method is presented for single unit vehicle. Finally, RCV method is validated against tilt table test and verified using VTM for various vehicle/vehicle combinations.

In Chapter 8, recommendations are presented for future improvements in RCV method.

2 Background

This chapter presents a brief introductory background of high capacity transport vehicles, performance based standards and rollover. Further, rollover mechanism is discussed for rigid and compliant vehicle in roll plane while introducing the importance of vehicle compliance in roll motion.

Road transport vehicles are used to transport various goods, which is one of the basic need for society's survival. With the increasing economy and population, societal demands are increasing. According to European Commission (EC) report [1], road transportation covers about 45.3% of total freight transportation. The improved transportation system will potentially increase the efficiency of demand and supply not only in terms of transportation but also its cost. As it is a value-added cost which varies 20-50% between various sectors of society and freight industry [1].

2.1 High Capacity Transport Vehicles

The transportation of more goods are required with increasing population. Both the size and weight of required vehicles are increasing due to developing need for more transportation of goods. The HCT vehicles can thus be identified with their length and capacity to transport goods. At present only Netherlands, Sweden, Denmark, and Finland allows vehicle combinations of 25.25 m length with maximum gross weight of 60 tonnes [4]. In rest of the Europe, maximum permitted vehicle combination length is 18.75 m but 16.75 m is the most common tractor semi-trailer combination length found in Europe [17]. The common European transport vehicles are presented in Figure 2.1.








Truck combination		Typical length
	Tractor + semitrailer	16.5 m
	Tractor + semitrailer + centre axle trailer	25.25 m
	Tractor + B-double	25.25 m
	Rigid	Varies
	Rigid + centre axle trailer	18.75 m
	Rigid + drawbar trailer	24 m
	Rigid + dolly + semitrailer	25.25 m

Figure 2.1: *Truck combinations in European countries and their permissible lengths*

Due to possibility of trade and transport between European countries, European Union (EU) regulation works to harmonize the transport vehicles in terms of weight and dimensions. According to EU regulation, vehicle combinations of length upto 18.75 m can weigh maximum 40

tonnes and length of 25.25 m can weigh maximum 60 tonnes [11]. Longer vehicle combinations than EU are being used in some countries in North and South America, Africa, Australia and New Zealand. The most common vehicle combination types are A-double and B-double but also C-double and truck-full trailer can be found in these countries. An A-double has a typical tractor semi-trailer (viz. fifth-wheel coupled) connected with either another trailer using draw-bar or another semi-trailer using conventional A-dolly (which has one draw-bar). The C-double has a typical tractor semi-trailer (viz. fifth-wheel coupled) connected with another semi-trailer using converter C-dolly (which has two draw-bars). Figure 2.2 represents a typical A-double or C-double (depending on type of dolly connection) vehicle combination. Although, the influence of both A-dolly (or single draw-bar, in case of no dolly) and C-dolly can be considered similar on roll motion of a vehicle combination in steady state. But, the difference between the two connections can be observed in evasive (or dynamic) maneuvering, where C-dolly is superior than A-dolly in reducing rearward amplification [43].

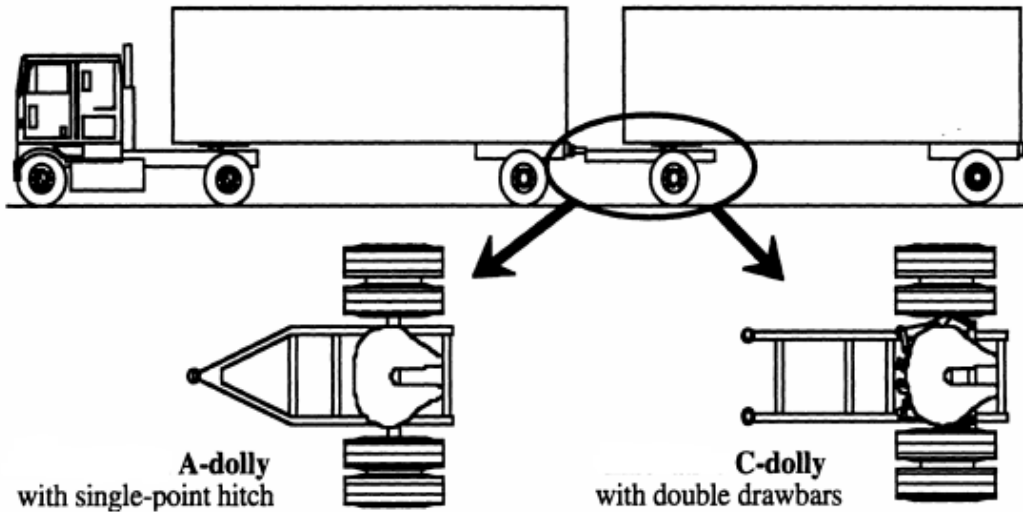


Figure 2.2: *A schematic representation of vehicle combination with A & C dollies [43]*

A B-double represents a semi-trailer to semi-trailer connection using fifth-wheel. The lead semi-trailer (which is connected to tractor via fifth-wheel) (known as B-link trailer or B-semitrailer) has a fifth-wheel at its rear end, through which another semi-trailer is connected as shown in Figure 2.3.

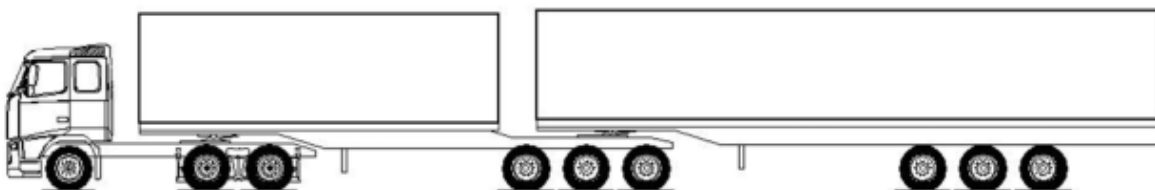


Figure 2.3: *A schematic representation of B-double [11]*

Since, fifth-wheel couples the two connecting units (i.e. tractor and semi-trailer) in roll direction [44], i.e. their roll motion can't be considered separate from each other, therefore, tractor and two semi-trailers in B-double are all roll coupled. A roll coupling system for roll coupling the draw-bar of a trailer has been invented in [45]. However, in current work, it has been assumed that draw-bar doesn't couple the two units (i.e. truck and trailer/dolly), therefore, in A-double and C-double, tractor semi-trailer and dolly semi-trailer/full trailer can be considered separate/independent in roll motion.

The maximum length of these vehicle combinations can reach around 50 m and maximum gross weight can be upto 100 tonnes [11]. Table 2.1 present the definitions of some of the vehicle types used for describing vehicle combinations.

Table 2.1: Vehicle type definitions[11]

Vehicle Types	Definitions
A-double	consists of tractor, semitrailer and full trailer or dolly semitrailer
B-double	consists of tractor, semitrailer with a fifth-wheel at the rear end and a second semitrailer
C-double	consists of tractor, semitrailer, C-dolly and a second semitrailer
C-dolly	is a converter dolly where the coupling of drawbar has only pitch degree of freedom and the axle is steered
Full trailer	has both front and rear running gear, but may also consist of a converter dolly and semitrailer

HCT vehicles doesn't only promote the increased freight capacity but also provides advantages in terms of reduction in fuel consumption and subsequently emission of harmful gases [16]. HCT vehicles occupy comparatively lesser road space for a specific amount of payload with respect to conventional vehicles, thus improved traffic flow. The socio-economic benefits are obvious with HCT vehicles compared to traditional ones [16], however, safety remains the main concern due to increased length and weight for HCT vehicles [37].

The existing accident databases mostly conditional, i.e. there is a lack of accident statistics in a scientifically robust manner [20]. According to the study of Federal Motor Carrier Safety Administration, 69% of the sampled rollover cases involve tractor semitrailer, however, single unit trucks outnumber tractor semitrailer by nearly 3 to 1 [33]. According to accident analysis by Martin et al. [32], the second most frequent type of accident in single unit trucks and tractor semitrailer is rollover (approx. 18%), in which single unit trucks dominate. Rollover accidents might be less frequent but if accident severity involved is compared, rollovers are equally/more severe than rest of the accident types despite their smaller numbers [12]. Therefore, it is necessary to have a robust methodology which can be used to evaluate rollover of HCT vehicles.

2.2 Performance Based Standards

The current vehicle regulations, mainly design based requirements (known as prescriptive vehicle limits), which only provide restriction on vehicle design but does not assess the performance of vehicle combination in terms of their interaction with road network [3]. Performance based standards (PBS) are the alternative way to regulate heavy vehicles according to how they perform, are operated and driven and on the basis of characteristics of road network. PBS key

principle can be understood by a notion “*No one size fits all*” [3]. PBS were first introduced by National Road Transport Commission (NRTC), Australia in 2007 with a purpose of regulating long vehicle combinations (LCV) to make the freight task more efficient without compromising safety or environmental protection. Some of the key parameters for vehicle combinations by which PBS and prescriptive vehicle limit regulate, are presented in Table 2.2.

Table 2.2: Prescriptive limits versus PBS regulations [3]

Prescriptive Limits	PBS Regulation
Overall length limit	Wheel base
Gross mass limit	Center of gravity heights
Axle mass limit	Drawbar length
	Coupling overhang
How they look like?	How they perform?

The objectives and benefits can be achieved by regulating vehicles using PBS are as follows:

- increased productivity through innovation in vehicle design and operation.
- improvements in road safety, traffic operations and asses management.
- a international basis for regulating heavy vehicles.
- better matching of capabilities of vehicles and road systems.
- improvement in environmental damage.

The PBS vehicle regulations include an agreed set of performance measures that can be objectively determined and delivered. Each measure defines a boundary between what is acceptable and unacceptable. PBS assesses a vehicle or vehicle combination mainly in two categories, namely Safety and Infrastructure.

- **Safety** - Starting, stopping, turning, overtaking, ride quality, stability, road space, tracking, tail swing, swept path, rollover
- **Infrastructure** - Pavement and bridge effects - standard axle repetitions (SAR), bridge loading (bending and shear)

Rollover is one of the most frequent accident types in HCT vehicles, The PBS provide regulation for evaluating HCT vehicles for their rollover performance. The PBS measure rollover performance in terms of static rollover threshold, which is 0.35 g for all HCT vehicles with a exception of 0.4 g for vehicles containing dangerous goods.

2.3 Rollover

Rollover is the instability of vehicle which results in at least one 90 degree rotation of vehicle about its roll axis. It can either be the consequence of driver’s manoeuvre or terrain irregularities. In vehicle dynamics terminology, vehicle can rollover in one of the conditions: steady (or

quasi static) state (slowly changing longitudinal speed or steering angle), transient or dynamic (arbitrary manoeuvre and all time derivatives are non-zero). More or less all the vehicle rollovers are generated in one of these conditions.

The most convenient way to categorize rollover is based on the type of rollover phenomenon. The two basic types are tripped and un-tripped rollover [14]. Tripped rollover involves an abrupt impact with another object at vehicle's tires, which induces a rotary motion to the vehicle resulting in rollover. In an un-tripped rollover, vehicle is exposed to a gradual increase of force at tire-ground contact area which, when coupled with vehicle's dynamics and physical properties lifts off the wheel from ground resulting in rollover [23].

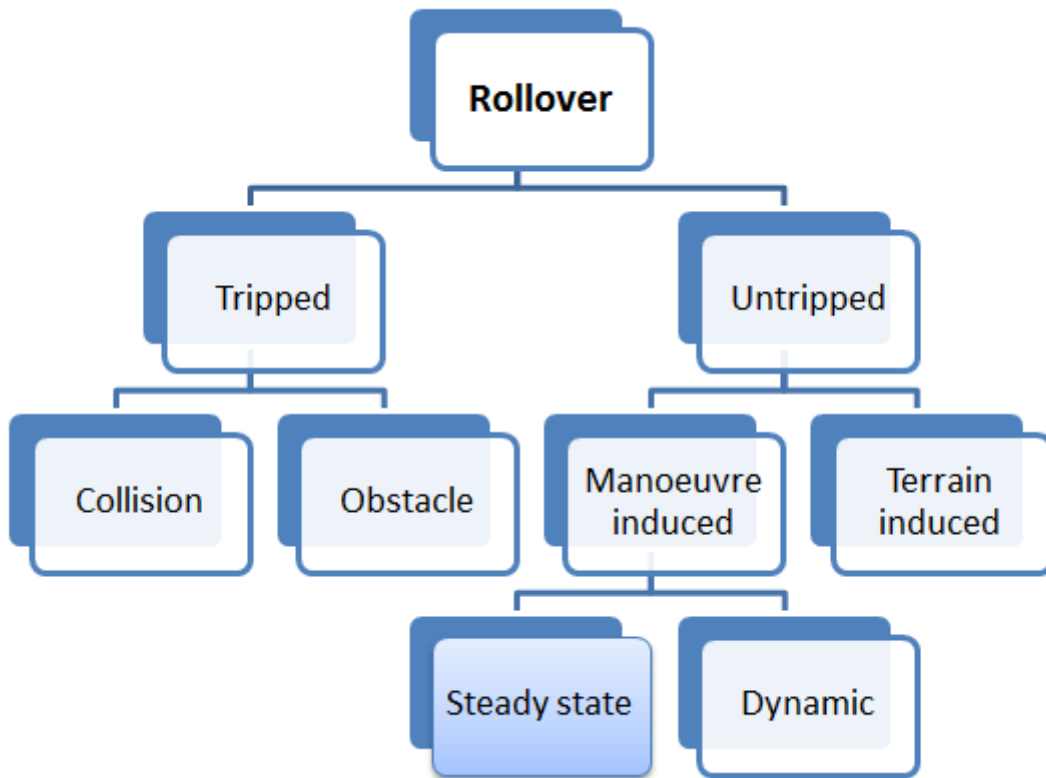


Figure 2.4: *Rollover categorization*

Figure 2.4 shows a simple way to categorize rollover. Generally, un-tripped rollovers are mostly the result of vehicle instability and are most thoroughly documented. Out of which steady state rollover is easiest to analyze due to absence of time derivatives in the vehicle dynamics, therefore, the further study concentrates on steady state rollover.

Vehicle rollover stability is usually assessed only in steady state condition and the methods available such as static stability factor, tilt table ratio and side pull ratio provides a limit after which vehicle are expected to rollover. These methods can be defined as follows [23], [19]:

- **Static Stability Factor** - It is defined as the ratio of half track-width l_t to height of the center of gravity h_{cg} , i.e.

$$SSF = \frac{l_t}{2h_{cg}}$$

- **Tilt Table Ratio** - It is defined as the tangent of the tilt table angle α at which one side of the vehicle wheels lifts-off the table, i.e.

$$TTR = \tan \alpha$$

- **Side Pull Ratio** - It is defined as the ratio of lateral force to vehicle weight at vehicle center of gravity at which one side of the vehicle's wheels lift-off the ground, i.e.

$$SPR = \frac{Lateral\ force}{Vehicle\ weight}$$

The above mentioned methods are all assessing the steady state rollover behaviors but using different techniques. These can all be presented in term of a threshold called *steady state rollover threshold (SRT)*. The SRT is defined as a level of lateral acceleration at which a vehicle's axle lifts-off from one side and it can closely be related to tilt table ratio (TTR). SRT is one of the main criterion to determine the vehicle's roll stability. According to UNECE111, HCT vehicles should not rollover before 0.4 g lateral acceleration irrespective of their size and capacity [2]. However, there is currently no explicit European law, which prescribes a minimum level of rollover threshold as a requirement for vehicle combinations to be operated legally on roads.

When assessing rollover performance of vehicle using PBS, it becomes important to understand what are the parameters contributing to lateral acceleration resulting rollover so that appropriate measures can be considered while designing the vehicle. To identify such factors, numerous parameter sensitivity studies have been done in past [38]. Some of the parameters are enlisted as follows:

- center of gravity (COG) height of tractor and trailer.
- Roll centers height of tractor and trailer.
- Axle roll stiffness of tractor and trailer.
- Effective track-width of tractor and trailer (including dual tire spacing).
- Fifth wheel height.
- Fifth wheel roll stiffness.
- Tire stiffness (vertical, lateral and overturning).
- Frame flexibility especially for trailer

These are only some of the parameters, which certainly influence a vehicle's rollover performance. Moreover whether these parameters have been considered in the assessment procedure or not, needs to be investigated and additional parameter needs to be identified.

2.4 Rollover Mechanism

The nature of all rollover events are mostly dynamic and none are of steady state or quasi static. However, accident data analyses suggest a strong correlation between rollover occurrences and steady state roll stability of vehicle [23]. The vehicle's roll stability analysis is discussed by Winkler [41] in simplified and extensive manner.

The roll response of the vehicle is promoted by the deform-ability or irregularity of the road surface which produces vertical inputs (in case of off-road) and paved highways¹, while vehicle is under manoeuvre (cornering, lane change etc.). The lateral tire forces become the significant factor in roll stability, which are the direct consequence of lateral acceleration and lateral load transfer. The roll stability can be simply analyzed by considering a vehicle in only roll plane (i.e. y-z plane), the equilibrium between the moments working in this plane determines vehicle rollover characteristics.

2.4.1 Rigid Vehicle

Rigid vehicle can be described as a vehicle with no suspension (i.e. sprung and un-sprung masses are connected through a solid structure with no deformation) and tires are also rigid (i.e. no vertical and lateral deformation). In other words, the whole vehicle rolls as a single mass lumped at its center of gravity.

Figure 2.5, represents the schematic diagram of a rigid vehicle in roll plane taking a left turn (i.e. load is transferred from left to right side), this vehicle has only one degree of freedom, viz. roll about longitudinal axis.

where,

- m - total mass of vehicle acting at its center of gravity 'c', [kg]
- ϕ - total vehicle roll angle, [rad]
- a_y - lateral acceleration, [g]
- h_{cg} - height of the center of gravity from ground level, [m]
- l_t - vehicle track width, [m]
- F_{zl} - left side tire vertical reaction force, [N]
- F_{zr} - right side tire vertical reaction force, [N]

Lateral force is the one which provides the moment responsible to roll the vehicle and it is contradicted by the moment provided by tire reaction forces, which stabilizes the vehicle. The equilibrium between these two moments especially in case of rigid vehicle are the deciding factor for vehicle rollover. The whole vehicle can be considered to roll about point 'o' at ground level and roll moment equilibrium can be expressed as follows:

$$(F_{zr} - F_{zl}) \frac{l_t}{2} = m a_y \quad (2.1)$$

¹ground plane become inclined with respect to the horizontal

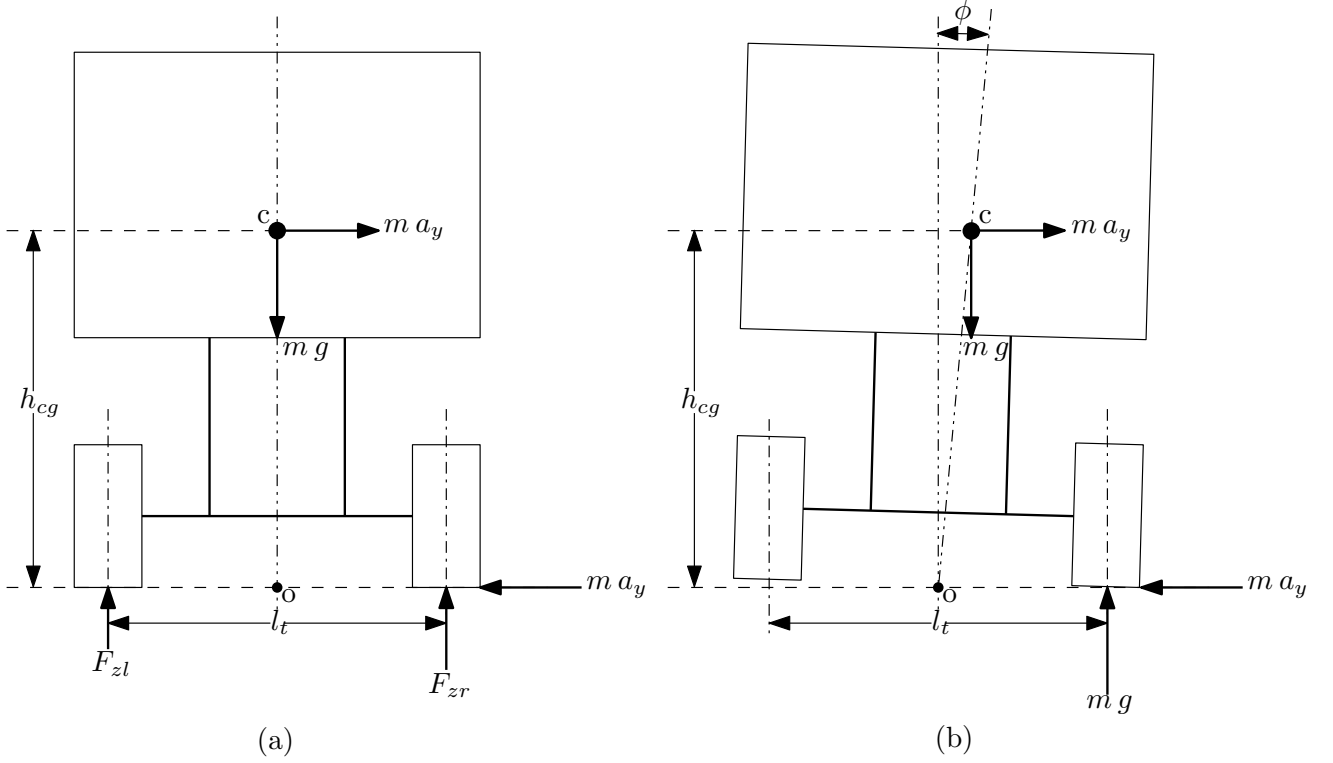


Figure 2.5: *Rigid Vehicle with only one roll degree of freedom: (a) load transfer due to lateral acceleration; (b) wheel lift-off once SRT reached*

In a left turn, lateral acceleration causes load transfer between left to right side of vehicle, i.e. with increasing lateral acceleration vertical reaction force at right side tire continues to increase. At a certain acceleration, complete vehicle load is transferred to right side, i.e. $F_{zl} = 0$ and $F_{zr} = m g$.

$$a_y = \frac{l_t g}{2h_{cg}} \quad (2.2)$$

In case of rigid vehicle, lateral acceleration expressed by equation (2.2) can be defined as steady state rollover threshold, which is the maximum lateral acceleration after which vehicle's left side lifts-off from ground and continues to roll towards right side with increase in lateral acceleration. This SRT value is however maximum value for any vehicle/vehicle combination because rigid vehicles are practically not possible. In reality, a vehicle is combination of various complaint systems such as suspension, tire, chassis, axle etc. which reduces the limit of vehicle's roll stability.

2.4.2 Vehicle with Compliances

Though a vehicle has various compliances but from roll stability perspective, number of compliances can be limited. Figure 2.6 indicates compliances influencing vehicle roll stability. When two units in a vehicle combination are coupled through fifth-wheel, it introduces an additional compliance because fifth-wheel is essentially an angular lash which varies depending

on design. It is highly non-linear in nature but since it is a relation between angle and moment therefore, is still considered as compliance as indicated in Figure 2.6.



Figure 2.6: *Vehicle compliances in roll direction*

Rigid vehicle's SRT is mainly function of track-width and height of center of gravity. The compliances influence roll stability of a vehicle by affecting the track-width or height of center of gravity (COG). Due to suspension and tire compliances, vehicle's COG can shift more laterally outwards or vertically downwards due to lateral acceleration compared to rigid vehicle. This COG shift provides an additional moment about point 'o' at ground level, which decreases the roll stability of vehicle.

Therefore, in a vehicle with compliances, steady state roll equilibrium is maintained by three moments, namely *overturning moment* (lateral force multiplied by COG height, due to lateral acceleration), *restoring moment* (due to lateral load transfer between left and right side tires), and *lateral displacement moment* (due to lateral shift of COG).

In roll plane, due to suspension and tire compliances, vehicle total mass can be seen as sprung and un-sprung mass. Sprung mass which is supported by suspension and which rolls about a point, generally referred as *suspension roll center* above ground. Un-sprung mass which consists of axle and tire, un-sprung mass rotates about a point, generally referred as *axle roll center* at ground level. With this consideration, a vehicle can be seen with two degrees of freedom in roll plane due to tire and suspension compliances. The complete roll motion of the vehicle is the combined effect of both compliances.

A vehicle's roll motion while considering only tire and suspension compliances is presented

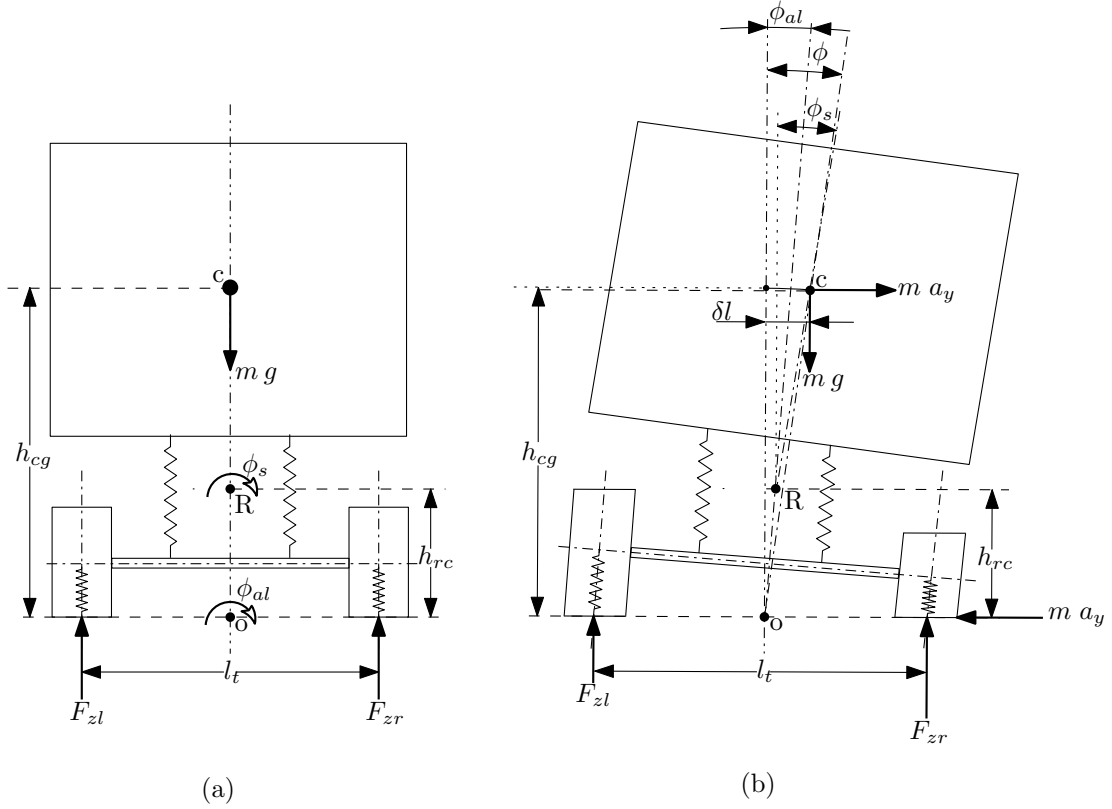


Figure 2.7: *Vehicle with two roll degree of freedom: (a) static condition; (b) rolling condition*

in roll plane as shown in Figure 2.7, where vertical and lateral shift of COG is shown. Since, a strict free body diagram (FBD) terminology has not been adopted in present work to indicate the simple vehicle roll mechanism in single figure, which has led to an assumption of considering rotation around roll centers (i.e. no vertical translation), therefore, this model assumes only rotational deformation around roll centers even after the wheel lift-off. However, RCV method includes calculation steps until wheel lift-off but this assumption is a simplification adapted in RCV method.

As indicated vertical shift of COG is very small compared to lateral shift. Therefore, decrease in COG height is neglected in further study. Also, roll angles can be considered small, therefore, assumption of $\cos \phi \approx 1$ and $\sin \phi \approx \phi$ holds valid for all indicated roll angles.

where,

- O - axle roll center
- R - suspension roll center
- ϕ - total roll angle of vehicle
- ϕ_{al} - roll angle due to tire compliance about axle roll center 'O'
- ϕ_s - roll angle due to suspension compliance about suspension roll center 'R'
- h_{rc} - height of the suspension roll center from ground

- δl - lateral shift of COG

Lateral shift of COG is due to both tire and suspension compliances, which can be determined as follows:

$$\delta l = [h_{rc} \phi_{al} + (h_{cg} - h_{rc}) \phi_s] = h_{cg} \phi \quad (2.3)$$

As expressed in equation (2.3), COG lateral shift is a function of total vehicle roll angle and sprung and un-sprung mass roll angle.

Roll moment equilibrium of a vehicle about point ‘o’ at ground level can be expressed by equation (2.4).

$$m a_y h_{cg} = (F_{zr} - F_{zl}) \frac{l_t}{2} - m g \delta l \quad (2.4)$$

SRT can be determined at a certain roll angle when complete vehicle load is transferred to right side and vehicle lifts off from the left side.

$$SRT = a_y = \frac{\left[\frac{l_t}{2} - h_{rc} \phi_{al} - (h_{cg} - h_{rc}) \phi_s \right] g}{h_{cg}} \quad (2.5)$$

By comparing equations (2.2) and (2.5), it can be observed that with the inclusion of tire and suspension compliance, vehicle's SRT reduces when compared to rigid vehicle. SRT is not only a function of COG height and track-width but also height of roll centers and corresponding roll angles. The roll angles ϕ_{al} and ϕ_s can be determined as a ratio of roll moments due to tires and suspensions and corresponding roll stiffness respectively as shown in below Matlab commands.

```
clear all
syms ay lt hrc hcg phi_al phi_s g m C_al C_s

Eq1 = ay == (lt/2 - hrc * phi_al - (hcg-hrc) * phi_s)*g/hcg;
Eq2 = phi_al == m * ay * hcg / C_al;
Eq3 = phi_s == m * ay * (hcg-hrc) / C_s;
sol = solve(Eq1, Eq2, Eq3, ay, phi_al, phi_s)
sol.ay

ay = (C_al*C_s*g*lt)/(2*(C_al*C_s*hcg + C_al*g*hcg^2*m + C_al*g*hrc^2*m - ...
2*C_al*g*hcg*hrc*m + C_s*g*hcg*hrc*m))
```

Therefore, consideration of various mechanical behaviors affecting roll stability and thereby SRT of vehicle/vehicle combination is necessary.

3 UNECE111 Regulation

This chapter presents procedures in current regulation for evaluating HCT vehicles for rollover. The regulation has mainly two evaluation procedures - tilt table testing and calculation method. An explanatory derivation for UNECE111 calculation method is presented; through which the simplifications and limitations of the calculation method are being identified.

The UNECE111 [2] provides regulatory guidelines for rollover stability of tank and transport vehicles. The tank vehicles can be classified [32] into following categories:

- N2: vehicles to carry on loads with maximum allowed mass between 3.5 and 12 tons
- N3: vehicles to carry on loads with maximum allowed mass greater than 12 tons
- O3: trailers and semitrailers with maximum allowed mass between 3.5 and 10 tons
- O4: trailers and semitrailers with maximum allowed mass greater than 10 tons

A vehicle must fulfill at least one of the criteria set by UNECE111 for approval of a vehicle type with regard to rollover stability, which are defined as follows:

- *Tilt table test*: The tilt table angle at which overturning occurs should be greater than 23 degree. The vehicle has to achieve this angle in three successive tests from both left and right tilt direction.
- *Calculation method*: The rollover stability of the vehicle shall be such that the point at which overturning occurs would not have been passed if a lateral acceleration of 4 m/s² has been reached. The manoeuvre represented by the calculations is a steady state circular test (constant and large radius, constant speed, and consequently constant lateral acceleration).

3.1 Tilt Table Test

This test simulates a non-vibratory steady state turn. It consists of an essentially flat steel deck pivoted longitudinally along one edge, while the other can be lifted by four hydraulic cylinders in simultaneous motion with very gradual increase of tilt table angle i.e. 0.25 deg/s or less up to the required maximum angle or *rollover threshold* (viz. the instant when all the wheels on one side of the vehicle have lost contact with the tilt table surface) as mentioned in Annex-3 [2]. The testing table can accommodate vehicles up to 20 m long and 40 tonne weight. The maximum inclination to which it can be raised is 40 degrees relative to horizontal position.

The vehicle with multiple units is tested while kept in straight line such that their axle longitudinal center line is parallel to tilt table. All axles are locked in longitudinal and lateral direction to prevent forward/backward, lateral movement and turning of wheels in steering direction; as long as locking doesn't influence test results. A schematic representation of tilt table is presented in Figure 3.1.

The tangent of inclination angle (tilt table angle relative to horizontal direction), which is also the ratio of vehicle's lateral force to its normal load, is used to simulate lateral acceleration

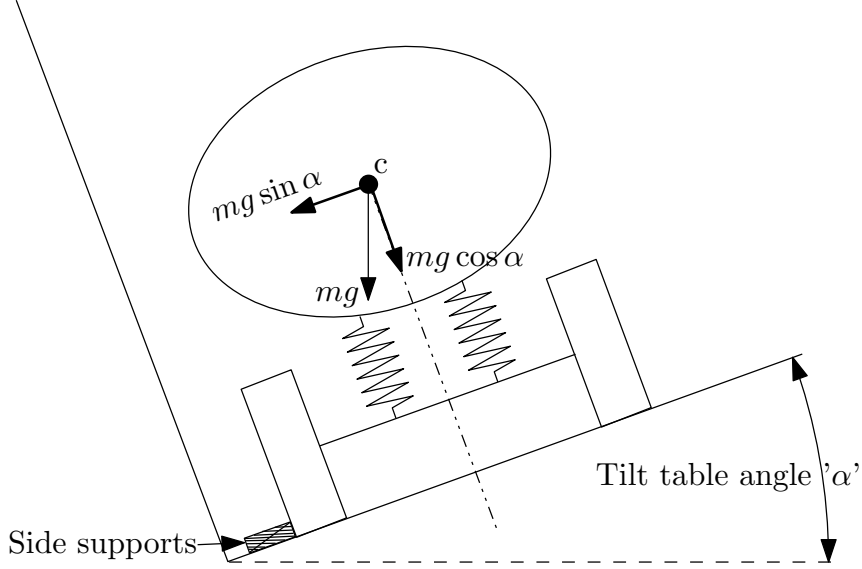


Figure 3.1: Representation of tilt table test set-up

applied to vehicle while making a turn on the road [30]. The tilt table angle ' α ', vehicle's lateral force and normal load can be correlated as presented in equation (3.1).

$$\frac{m g \sin \alpha}{m g \cos \alpha} = \tan \alpha = \frac{m a_y}{m g} \quad (3.1)$$

According to UNECE111, vehicle is considered to be roll stable if the overturning doesn't occur before or at 23 degree of tilt table angle in both directions, i.e. according to equation (3.1) for 23 degree of tilt table angle, vehicle can achieve lateral acceleration of 0.4245 g (approx 4.16 m/s²) before overturning. This level of lateral acceleration is defined as steady state rollover threshold according to tilt table test.

There are also some additional preparatory guidelines which are to be followed during test, whenever applicable, are as follows:

- Tilt table is rigid and flat.
- The total wind velocity must not exceed 5 m/s during test.
- Inflation pressure of tires should be according to laden vehicle condition.
- A filling factor is acceptable between 100% & 70% if fully laden condition can not be achieved.
- In case of vehicle with coupling units, a suitable or referenced tractor must be used as power-driving unit depending on whether it influence the test results or not.
- Accuracy of tilt table angle should be greater than 0.3 degree.
- Restraints systems should be used to prevent final rollover but it must not influence the test result.

To equalize or randomize the influences due to stick-slip of vehicle suspension systems and coupling components, vehicle is removed from the table after each test and driven.

3.2 UNECE111 Calculation Method

The UNECE111 calculation method simulates a steady state circular test in order to provide rollover stability estimation for vehicles according to Annex-4 [2]. The method accounts for the main factors influencing the roll stability of vehicle, such as height of COG, track width and the factors which results in lateral shift of center of gravity (i.e. axle roll stiffness, suspension roll stiffness etc.). In case of semi-trailer, it is simulated with a reference kingpin roll stiffness. A schematic representation of vehicle according to UNECE111 calculation method is presented in Figure 3.2.

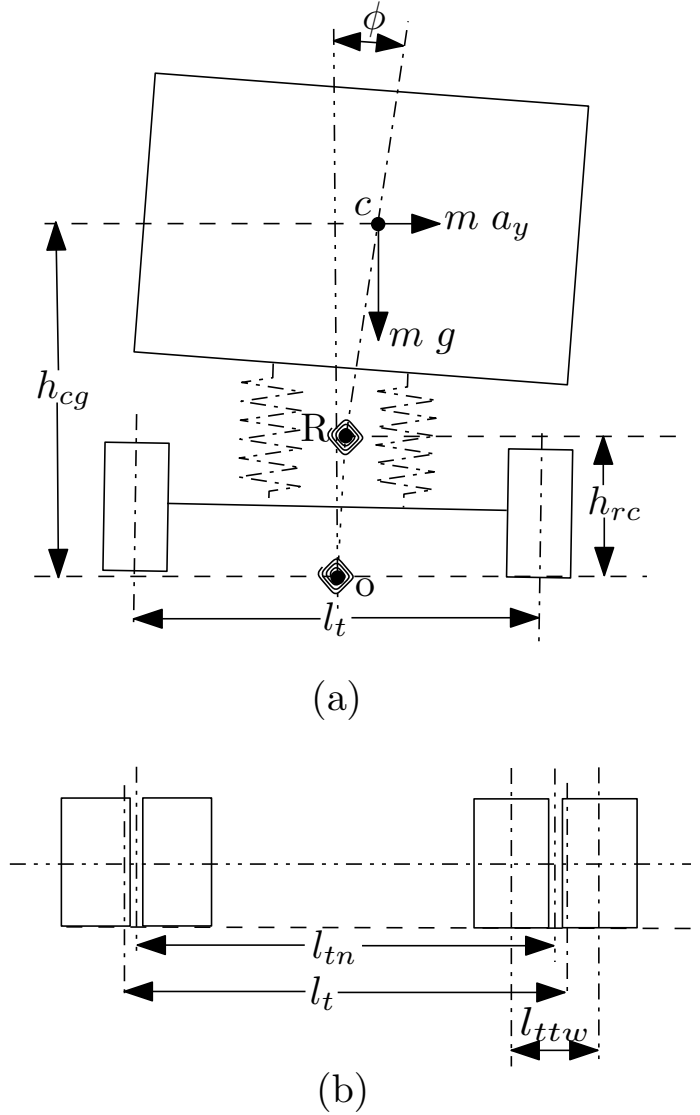


Figure 3.2: Vehicle in roll plane: (a) Representation of torsional springs; (b) Track-width due to dual tire compliance

The roll centers 'R' and 'O' are considered as pivot points, i.e. vertical and lateral compliance at roll centers should not be considered. To indicate the angular compliance in roll plane, two torsional springs due to suspension and tire compliance are considered at suspension

and axle roll center respectively as shown in Figure 3.2(a). The pseudo roll angle ‘ ϕ ’ is due to combined influence of suspension and tire compliance at an axle at wheel lift-off. Figure 3.2(b) represents the dual tire compliance effect on track width. The effective track width l_t can be calculated as a function of nominal track width l_{tn} and twin tire width l_{ttw} according to UNECE111 as presented in equation (3.2), without any explanation. In present work, it has been accepted as it is.

$$l_t = \sqrt{(l_{tn})^2 + (l_{ttw})^2} \quad (3.2)$$

Some of the primary assumptions which has been considered in the calculation method, are as follows:

- Axle roll center is at the ground level.
- Vehicle body is assumed to be rigid.
- Vehicle is symmetrical about its longitudinal center line.
- Tire and suspension deflections are linear.
- Lateral deflection of suspension is zero.

3.2.1 Derivation of Calculation Method for Single Rigid Truck

The calculation method has numerous mathematical equations, which are not validated. UNECE111 also doesn’t provide any explanation for using such equations, therefore, a plausible mathematical derivation of the equations using reverse engineering methodology is presented.

Roll stiffness due to Tire Compliance

An axle of vehicle with rigid suspension (i.e. no suspension compliance) is presented in Figure 3.3. In static condition, both sides of an axle will be loaded equally due to total weight of axle $m g$, causing vertical deflection x of tires as shown in Figure 3.3(a). The force due to lateral acceleration $m a_y$ and axle weight $m g$ acting at axle COG, results in axle roll motion due to only axle and tire compliance. In critical condition, i.e. when left side (or inner) tire lifts-off from ground, these forces cause axle roll angle ϕ_{al} about axle roll center ‘o’ and critical vertical deflection x_{crit} of right side (or outer) tire as shown in Figure 3.3(b).

Since, in UNECE111, roll stiffness due to tire and axle compliance is determined at critical condition, and also, roll compliance is considered entirely due to vertical stiffness of tire, therefore, moment at axle roll center due to critical vertical deflection of right side tire can be determined as expressed in equation (3.3).

$$M_{crit} = C_{tv} x_{crit} \frac{l_t}{2} \quad (3.3)$$

To determine the roll stiffness due to tire and axle compliance, simply a torsional spring can be considered at axle roll center with stiffness C_{al} . If the critical moment M_{crit} resist the

angular deflection¹ of this torsional spring, then critical moment can be considered equivalent to stiffness times angular deflection (viz. equal to ϕ_{al} at critical condition) of torsional spring as shown in equation (3.4).

$$C_{al} \phi_{al} = M_{crit} = C_{tv} x_{crit} \frac{l_t}{2} \quad (3.4)$$

$$C_{al} = C_{tv} \left(\frac{x_{crit}}{\phi_{al}} \right) \frac{l_t}{2}$$

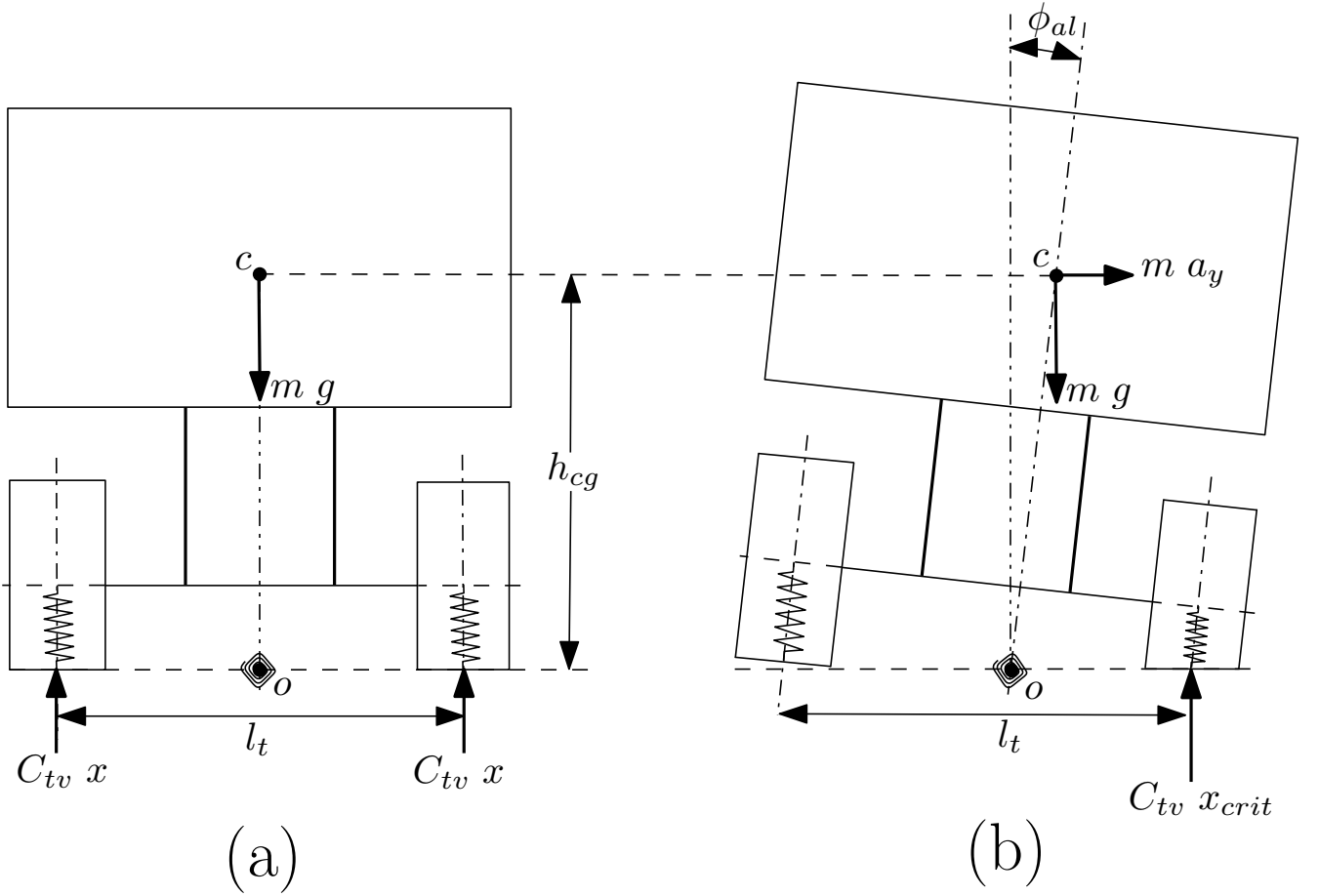


Figure 3.3: *Schematic representation of vehicle with rigid suspension in roll plane: (a) in static condition; (b) in critical condition when left side (or inner) tire lifts-off at an axle*

If axle is considered rigid, then in critical condition, ratio of critical vertical deflection of outer tire to roll angle can be correlated to track-width of axle as

$$\frac{x_{crit}}{\phi_{al}} = l_t$$

Therefore, roll stiffness due to tire and axle compliance can be determined as expressed in equation (3.5).

¹which is being caused by $m a_y$ & $m g$

$$C_{al} = \frac{1}{2} N C_{tv} l_t^2 \quad (3.5)$$

where, N represent the number of tires on each side at an axle, i.e. ($N = 1$) for single tire and ($N = 2$) for double tires.

Roll Stiffness due to Suspension Compliance

Sprung mass is mainly supported by suspension springs and dampers. These suspension elements can be considered to be replaced by a torsional spring with stiffness C_s and the sprung mass m_s can be considered to be rolling about suspension roll center 'R' with the influence of torsional spring. The sprung mass rolls with roll angle ϕ_s about suspension roll center due to torsional spring as shown in Figure 3.4.

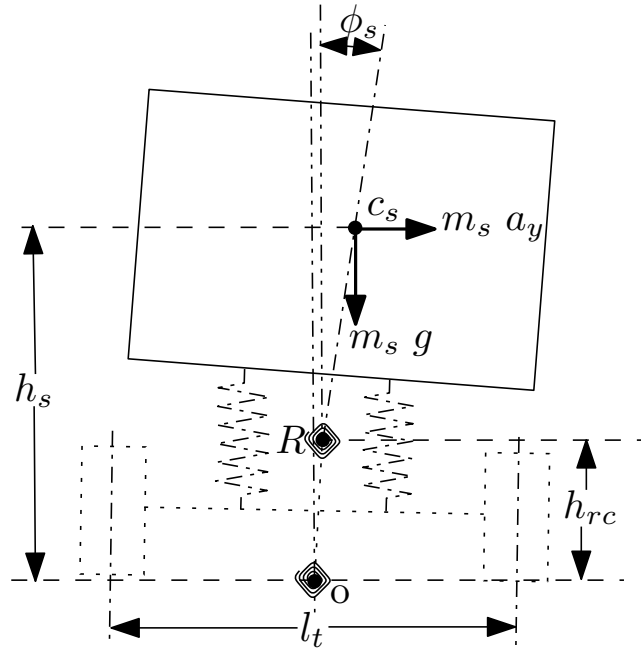


Figure 3.4: *Roll angle due to suspension compliance*

where,

h_s - center of gravity height of sprung mass [m]

h_{rc} - suspension roll center height [m]

ϕ_s - roll angle due to suspension compliance [rad]

C_s - suspension roll stiffness at suspension roll center [kN-m/rad]

C_{sg} - equivalent suspension roll stiffness about axle roll center 'o' at ground level [kN-m/rad]

Since, suspension and tire both influence roll motion of vehicle in similar way but about different roll centers, therefore, it is better to consider the effect of both compliances about same roll center. It can be achieved by transforming suspension roll center to axle roll center, i.e. determining equivalent suspension roll stiffness about axle roll center.

The roll stiffness C_s about suspension roll center ‘R’ can be determined from suspension spring stiffness and its deflection. A roll moment equilibrium about suspension roll center can be established as expressed in equation (3.6).

$$C_s \phi_s = m_s g \phi_s + m_s a_y (h_s - h_{rc}) \quad (3.6)$$

If sprung mass is considered to roll about axle roll center ‘o’ instead of suspension roll center ‘R’, then for same sprung mass roll motion, roll stiffness and roll angle about axle roll center would be different (in other words, same roll motion is only possible if different suspension spring stiffness and deflection is considered). A torsional spring with roll stiffness C_{sg} can also be considered at axle roll center due to which sprung mass rolls with a roll angle ϕ_{sg} . A roll moment equilibrium about axle roll center can be established as expressed in equation (3.7).

$$C_{sg} \phi_{sg} = m_s g \phi_{sg} + m_s a_y h_s \quad (3.7)$$

By solving equations (3.6) and (3.7), roll stiffness about axle roll center due to only sprung mass roll motion can be determined (detailed steps are presented in Section 4.1) as expressed in equation (3.8).

$$C_{sg} = C_s \left(\frac{h_s}{h_s - h_{rc}} \right)^2 \left[1 - \frac{m_s g h_{rc} (h_s - h_{rc})}{C_s h_s} \right] \quad (3.8)$$

In UNECE111, it is being considered as expressed in equation (3.9).

$$C_{sg} = C_s \left(\frac{h_s}{h_s - h_{rc}} \right)^2 \quad (3.9)$$

It can be observed that UNECE111 simplifies the roll stiffness due to suspension which will result in higher roll stiffness.

Resolved Combined Roll Stiffness and Pseudo Roll Angle

Due to the transformation of suspension roll stiffness about axle roll center ‘o’ at ground level, it can be considered to be working in series with axle roll stiffness due to tire compliance. Therefore, resolved combined roll stiffness C_{res} about axle roll center can be determined by considering two torsional springs in series as expressed in equation (3.10).

$$\frac{1}{C_{res}} = \frac{1}{C_{al}} + \frac{1}{C_{sg}}$$

$$C_{res} = \frac{C_{al} C_{sg}}{C_{al} + C_{sg}} \quad (3.10)$$

When moment due to load transfer at axle is equal to torsional spring moment about axle roll center with stiffness equivalent to resolved combined roll stiffness, pseudo roll angle can be determined at wheel lift-off at an axle, i.e.

$$C_{res,i} \phi_i = m_i g \frac{l_{t,i}}{2}$$

Therefore, axle pseudo roll angle ϕ_i can be determined as expressed in equation (3.11).

$$\phi_i = \frac{m_i g l_{t,i}}{2 C_{res,i}} \quad (3.11)$$

Lateral Acceleration at Overturn

After determining the pseudo roll angle and resolved combined roll stiffness at each axle, complete vehicle can be considered as single ‘lumped’ axle with parameters corresponding to summation of individual axle. Maximum lateral acceleration a_{yT} can be determined when this lumped axle lifts-off, i.e. when one side of vehicle lifts-off from ground. The parameters for this axle can be determined as follows:

$$\begin{aligned} \text{Total vehicle mas [ton]} : \quad & m_T = \sum_{i=1}^n m_i \\ \text{Total un-sprung mass [ton]} : \quad & m_{uT} = \sum_{i=1}^n m_{u,i} \\ \text{Total sprung mass [ton]} : \quad & m_{sT} = m_T - m_{uT} \\ \text{Total effective track-width [m]} : \quad & l_{tT} = \frac{\sum_{i=1}^n m_i \cdot l_{t,i}}{m_T} \\ \text{Total roll stiffness [kN-m/rad]} : \quad & C_{resT} = \sum_{i=1}^n C_{res,i} \end{aligned}$$

In a roll plane, all the forces and moments will be in equilibrium for this axle and maximum lateral acceleration can be determined when overturning moments due to lateral acceleration and lateral shift of COG becomes equal to restoring moment due to lateral load transfer.

Overturning moment due to the lateral acceleration can be determined as expressed in equation (3.12).

$$\text{Overturning moment due to lateral acceleration:} \quad OM_{acc} = m_T a_{yT} h_{cg} \quad (3.12)$$

As the vehicle is represented as single ‘lumped’ axle, the lateral shift of vehicle COG can be determined as COG lateral shift of this ‘lumped’ axle, which in reality is the result of both

tire and suspension compliances. In UNECE111 method it has been considered as expressed in (3.13).

$$OM_{cg} = \left[\frac{(m_{sT} g h_s)^2}{(C_{resT} - m_T g h_s)} \right] \frac{a_{yT}}{g} \quad (3.13)$$

It can be observed from equation (3.13), lateral shift of COG is expressed as a function of total sprung mass m_{sT} , height of total sprung mass COG h_s , total resolved combined roll stiffness C_{resT} , and total vehicle mass m_T . If total vehicle mass is considered at sprung mass COG height, then a moment equilibrium can be established at lateral acceleration at overturn about roll center ‘o’ between torsional spring with stiffness C_{resT} and overturning moments as expressed in equation (3.14).

$$C_{resT} \phi_T = m_T g h_s \phi_T + m_T a_{yT} h_s \quad (3.14)$$

$$\phi_T = \left(\frac{a_{yT}}{\left[\frac{C_{resT}}{m_T g h_s} - 1 \right] g} \right) \quad (3.15)$$

Total roll angle ϕ_T can be represented as a function of lateral acceleration as expressed in equation (3.15). Further, if total vehicle mass is considered as sprung mass then moment due to COG lateral shift can be determined as expressed in equation (3.16).

$$OM_{cg} = m_s g h_s \phi_T \quad (3.16)$$

Using equation (3.15), moment due to lateral shift of COG can be simplified as expressed in equation (3.17).

$$OM_{cg} = \left[\frac{(m_{sT} g h_s)^2}{(C_{resT} - m_T g h_s)} \right] \frac{a_{yT}}{g} \quad (3.17)$$

At overturn (i.e. rollover) when the complete load transfer occurs between inner side and outer side. A moment equilibrium equation can be written between overturning moments and restoring moments using equations (3.12) and (3.17) for vehicle represented as single ‘lumped’ axle, as follows:

$$\frac{1}{2} m_T g l_{tT} = \left[(m_T g h_{cg}) + \frac{((m_T - m_{uT}) g h_s)^2}{(C_{resT} - m_T g h_s)} \right] \frac{a_{yT}}{g}$$

Therefore, the maximum optimal theoretical lateral acceleration for vehicle at overturn (rollover) can be expressed as equation (3.18).

$$a_{yT} = \frac{m_T g^2 l_{tT}}{2 \left[(m_T g h_{cg}) + \frac{((m_T - m_{uT}) g h_s)^2}{(C_{resT} - m_T g h_s)} \right]} \quad (3.18)$$

At maximum lateral acceleration vehicle's one side lifts-off from ground and rollover is certain but the SRT have been passed as well. To determine SRT, a minimum lateral acceleration to lift-off just one axle can also be identified by determining the stiffest axle, viz. the axle with smallest pseudo roll angle. To differentiate the axle, all parameters corresponding to stiffest axle are assigned an index 'S'. Since, a vehicle is represented as single 'lumped' axle for determining maximum lateral acceleration at overturn, therefore, vehicle can also be represented as stiffest axle with utilizing an effective mass factor, viz. the ratio of resolved combined roll stiffness of stiffest axle to total vehicle resolved combined roll stiffness. The effective mass factor can be determined as expressed in equation (3.19).

$$s_f = \frac{C_{resS}}{C_{resT}} \quad (3.19)$$

Lateral acceleration at which wheel lift-off occurs at stiffest axle can be determined by moment equilibrium between overturning and restoring moments similar to equation (3.18), however, overturning moment will be different for stiffest axle, which can be represented by utilizing effective mass factor. Therefore, The minimum lateral acceleration required for wheel lift-off at stiffest axle can be determined as expressed in equation (3.20).

$$a_{yS} = \frac{m_S g^2 l_{tS}}{2 \left[(m_T g s_f h_{cg}) + \frac{((m_T - m_{uT}) g s_f h_s)^2}{(C_{resT} - m_T g s_f h_s)} \right]} \quad (3.20)$$

In reality, steady state rollover threshold exists somewhere in between the two lateral acceleration expressed in equation (3.18) and (3.20). SRT can be determined by considering linear relationship between lateral acceleration and ratio of stiffest axle mass to total vehicle mass, i.e. an equation of straight line can be written as expressed in equation (3.21).

$$a_{yC} = k \frac{m_S}{m_T} + c \quad (3.21)$$

k and c represents the slope and intercept of straight line respectively. To determine the slope and intercept of this straight line, two points can be considered as ($\frac{m_S}{m_T} = 1$, $a_{yC} = a_{yS}$), indicating that stiffest axle mass corresponds to total vehicle mass (especially in case of single 'lumped' axle vehicle) and ($\frac{m_S}{m_T} = 0$, $a_{yC} = a_{yT}$), indicating that stiffest axle doesn't carry any load.

Hence, correlated lateral acceleration at overturn (i.e. SRT) can be determined as expressed in equation (3.22).

$$SRT = a_{yC} = a_{yT} - (a_{yT} - a_{yS}) \frac{m_S}{m_T} \quad [g] \quad (3.22)$$

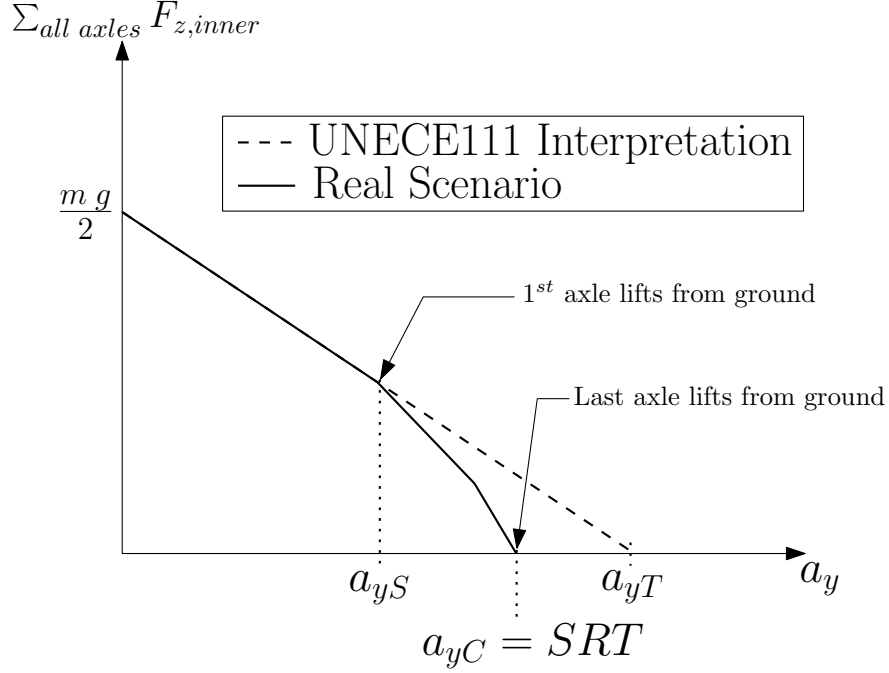


Figure 3.5: *UNECE111 straight line interpretation of steady state rollover*

The corresponding tilt table angle at overturn can be determined as expressed in equation (3.23).

$$\alpha = 57.3 \arctan(a_{yC}) \quad [\text{deg}] \quad (3.23)$$

Equation (3.22) indicates another simplification of UNECE111 approach, i.e. assuming linear behavior of axle lifts off but it doesn't behave linearly in reality. A qualitative pictorial difference is presented in Figure 3.5.

3.2.2 Extension of Calculation Method for Tractor-semitrailer

The UNECE111 calculation method presented in Section 3.2.1 is only valid for single vehicle units without fifth-wheel but it is also possible to include the influence of fifth-wheel (king-pin) into the calculation method. The method considers fifth-wheel as a virtual additional axle without any tires within the unit after the fifth-wheel such that the stiffness of this virtual axle is added to the resolved combined roll stiffness (i.e. C_{res}) of single vehicle unit (eg. total resolved roll stiffness of a 3 axle semitrailer will be the sum of respective resolved roll stiffness of individual 3 axles (which are considered decoupled) and stiffness of virtual axle, i.e. fifth-wheel as shown below) and thereby the two units in a vehicle combination are considered decoupled. The roll stiffness and track-width of the virtual axle corresponding to fifth-wheel can be determined as follows:

$$\begin{aligned} \text{Track-width [m]} : l_{tK} &= \frac{\sum_1^n l_{t,i}}{n} \\ \text{Fifth-wheel roll stiffness [kN-m/rad]} : C_{resK} &= m_K g 4 \end{aligned}$$

where, $m_K g$ represents the fifth-wheel load. The lateral acceleration calculation remains same for all vehicle types as Section 3.1.1. The fifth-wheel only affects the total vehicle parameters, i.e. when vehicle is considered as single ‘lumped’ axle. The effects of fifth-wheel on total resolved combined roll stiffness and track-width can be determined as follows:

$$\begin{aligned} \text{Total vehicle weight [kN]} : m_T &= \sum_{i=1}^n m_i + m_K \\ \text{Effective track-width [m]} : l_{tT} &= \frac{\sum_{i=1}^n m_i l_{t,i}}{m_T} + \frac{m_K l_{tK}}{m_T} \\ \text{Total roll stiffness [kN-m/rad]} : C_{resT} &= \sum_{i=1}^n C_{res,i} + C_{resK} \end{aligned}$$

3.2.3 Limitations and Simplifications of Calculation Method

Tank vehicle rollover stability evaluation using tilt table testing is a good approach, which correlates with real scenario [36] within UNECE111 regulation but calculation method has comparatively more limitations which result in overestimation of SRT (non-conservative nature) [13].

It has been observed that primary assumptions adapted in calculation method are not the only simplifications which affect the SRT estimation for tank vehicles [13], [32]. The calculation method has been simplified at each step, some of them can be concluded based on the plausible mathematical derivation presented in Section 3.2.1.

- Lateral tire properties have not been considered which certainly influence rollover stability.
- Non-linearity of suspension and tire have not been considered.
- Transformation of the suspension roll stiffness doesn’t depend upon its sprung mass.
- Fifth-wheel lash and trailer lash have not been considered.
- Torsional compliance of frame and chassis have not been considered.

Tilt table test is limited by its length of test bed but calculation method should be able to evaluate rollover stability of longer vehicle combinations, which is the most important limitation of calculation method. It is best suited for only single vehicles units. At present there is no regulation method exists by which SRT of existing roll coupled vehicles can be estimated and certainly not for perspective HCT vehicles.

UNECE111 also do not take different roll stiffness at the axles into account in a physical way, but only through the proportioning $\frac{m_S}{m_T}$ between a_{yT} and a_{yS} in Eq (3.21). Exactly how it can be represented for different distribution of axle stiffness has not been possible to express. A physically based calculation could be done by replacing the concept of proportioning between a_{yT} and a_{yS} with actually calculate each knee in Figure 3.5. However, the approach in chapter 4, 5, and finally 6 has instead been to make modifications, so that only a_{yT} and a_{yS} are calculated in a different way, but the proportioning with $\frac{m_S}{m_T}$ is kept.

4 Suspension and Tire Compliance

This chapter presents estimation approach of roll stiffness due to suspension and tire compliance adapted in RCV method. Transformation of suspension torsional spring to axle roll center is estimated using moment equilibrium approach in a roll plane. Lateral tire properties are considered in roll stiffness estimation due to tire compliance.

4.1 Suspension Roll Stiffness about Axle Roll Center

According to [42], height of the vehicle COG h_{cg} or its sprung mass and track-width l_t are the two most important parameters which influence the steady-state rollover the most.

Sprung mass roll motion can be considered entirely due to suspensions. At an axle, it can be analyzed in a roll plane, where a torsional spring can be considered at suspension roll center with roll stiffness equivalent to actual suspension stiffness as presented in Figure 4.1.

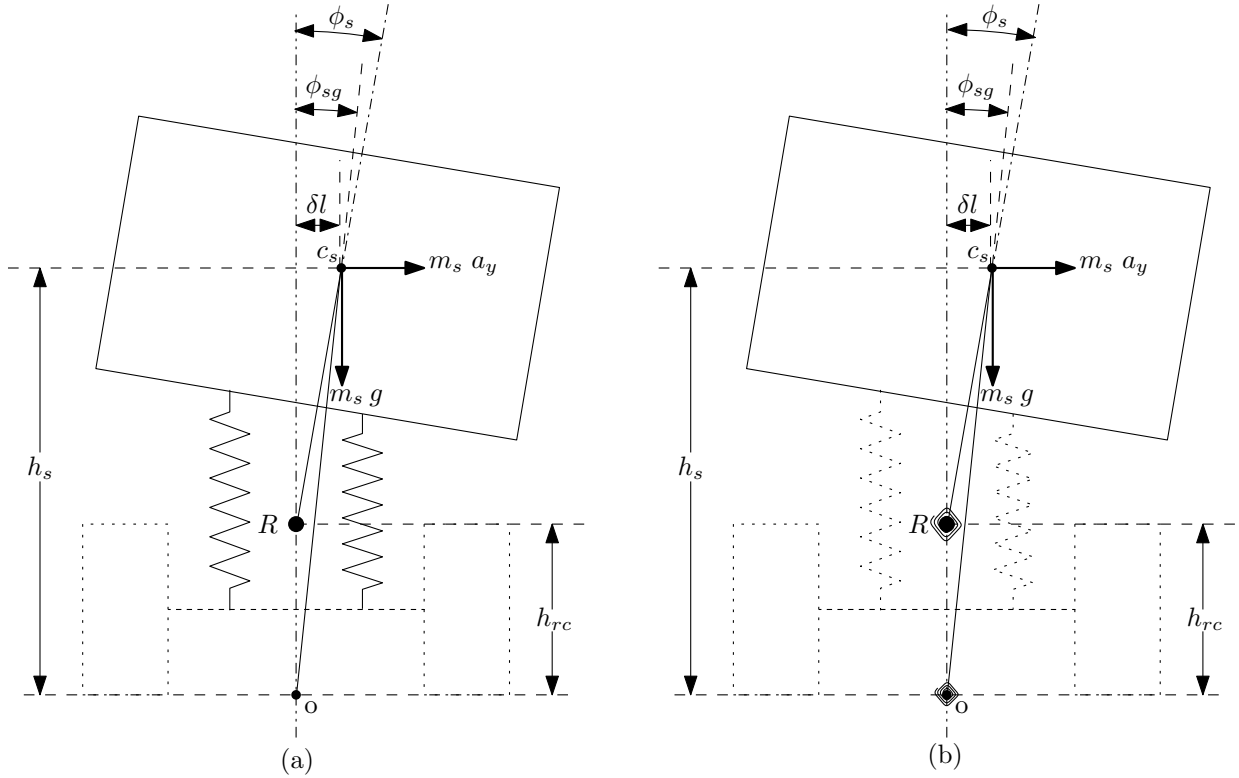


Figure 4.1: *Schematic representation of sprung mass roll motion about suspension roll center: (a) with actual suspensions; (b) with hypothetical torsional spring*

As presented in Figure 4.1(a), sprung mass m_s rolls about suspension roll center ‘R’ with roll angle of ϕ_s due to suspension elements. Sprung mass COG c_s shifts laterally with δl compared to plane symmetry axis due to sprung mass roll motion. The same phenomenon is presented in Figure 4.1(b) but with a torsional spring having stiffness C_s at suspension roll center instead of actual suspension elements. This consideration provides a rather simple approach to determine the roll stiffness due to suspension compliance. The moment of torsional

spring (which is its stiffness ‘ C_s ’ times angular displacement/roll angle ‘ ϕ_s ’) can be considered equivalent to moment due to lateral acceleration force and sprung mass gravitational force. Therefore, a moment equilibrium equation can be established about suspension roll center ‘R’ as expressed in equation (4.1).

$$C_s \phi_s = m_s g \delta l + m_s a_y (h_s - h_{rc}) \quad (4.1)$$

Similarly, if sprung mass is considered to roll about axle roll center ‘o’ instead of suspension roll center, another torsional spring with stiffness C_{sg} can be considered at axle roll center ‘o’ for equal lateral shift of sprung mass COG i.e. δl . In this case, sprung mass roll angle ϕ_{sg} about axle roll center will be different. A moment equilibrium can also be established about axle roll center ‘o’ between respective torsional spring moments (viz. C_s times ϕ_{sg}) and overturning moments (which are moments due to lateral acceleration force and sprung mass gravitational force) as expressed in equation (4.2).

$$C_{sg} \phi_{sg} = m_s g \delta l + m_s a_y h_s \quad (4.2)$$

Since, the roll angles are generally small, therefore, assumption of $\sin \phi_s \approx \phi_s$ and $\cos \phi_s \approx 1$ holds valid. The lateral shift of sprung mass COG can be determined as

$$\delta l \approx (h_s - h_{rc}) \phi_s \approx h_s \phi_{sg}$$

Also by considering same lateral shift of sprung mass, the two sprung mass roll angles with respect to roll centers can be correlated to each other as equation (4.3).

$$\frac{\phi_s}{\phi_{sg}} \approx \left(\frac{h_s}{h_s - h_{rc}} \right) \quad (4.3)$$

By substituting lateral shift of sprung mass COG in terms of roll angle, roll stiffness about roll centers can be represented as a function of respective roll angles and lateral acceleration as expressed in equations (4.4) and (4.5).

$$C_s = m_s g (h_s - h_{rc}) \left[1 + \left(\frac{a_y}{g \phi_s} \right) \right] \quad (4.4)$$

$$C_{sg} = m_s g h_s \left[1 + \left(\frac{a_y}{g \phi_{sg}} \right) \right] \quad (4.5)$$

In UNCEE111 method, roll stiffness about suspension roll center C_s is considered as a known parameters (i.e. used as a input), therefore, for comparison, it can be considered as a known parameter in RCV method as well. Ratio of lateral acceleration to roll angle in case of suspension roll center can be determined from equation (4.4) as expressed in equation (4.6).

$$\frac{a_y}{g \phi_s} = \frac{C_s}{m_s g (h_s - h_{rc})} - 1 \quad (4.6)$$

Using equation (4.3) and (4.6), ratio of lateral acceleration to roll in case of axle roll center can be determined as expressed in equation (4.7).

$$\begin{aligned}\phi_s &= \left(\frac{h_s}{h_s - h_{rc}} \right) \phi_{sg} \\ \frac{a_y}{g \phi_{sg}} &= \left[\frac{C_s}{m_s g (h_s - h_{rc})} - 1 \right] \left(\frac{h_s}{h_s - h_{rc}} \right) \\ \frac{a_y}{g \phi_{sg}} &= \frac{C_s h_s}{m_s g (h_s - h_{rc})^2} - \left(\frac{h_s}{h_s - h_{rc}} \right)\end{aligned}\quad (4.7)$$

Substituting ratio of lateral acceleration to roll angle as expressed in equation (4.7) into equation (4.5), suspension roll stiffness about suspension roll center and axle roll center can be correlated as expressed in equation (4.8).

$$C_{sg} = m_s g h_s \left[1 + \frac{C_s h_s}{m_s g (h_s - h_{rc})^2} - \left(\frac{h_s}{h_s - h_{rc}} \right) \right] \quad (4.8)$$

By further simplification, roll stiffness due to suspension compliance about axle roll center can be expressed as equation (4.9).

$$C_{sg} = C_s \left(\frac{h_s}{h_s - h_{rc}} \right)^2 \left[1 - \frac{m_s g h_{rc} (h_s - h_{rc})}{C_s h_s} \right] \quad (4.9)$$

According to UNECE111 calculation method, roll stiffness due to suspension compliance about axle roll center is determined as expressed in equation (4.10). It is represented here according to RCV method notations to provide comparative difference.

$$C_{sg} = C_s \left(\frac{h_s}{h_s - h_{rc}} \right)^2 \quad (4.10)$$

It can be clearly observed from equation (4.9) and (4.10), that roll stiffness due to suspension compliance considered in UNECE111 method is a simplification, which estimates higher suspension roll stiffness about axle roll center. The estimated suspension roll stiffness by RCV method defers from UNECE111 method by a factor X_{sg} ; defined as *suspension compliance factor* in RCV method. It is an empirical factor depending on parameters at an axle and can be determined as expressed in equation (4.11).

$$X_{sg} = \left(1 - \frac{m_s g h_{rc} (h_s - h_{rc})}{C_s h_s} \right) \quad (4.11)$$

From suspension compliance factor, it can be confirmed that roll stiffness due to suspension compliance also depends on axle sprung mass (indirectly axle load) [29], which is clearly missing in UNECE111 method. With increase in sprung mass, roll stiffness decreases due to suspension compliance factor in RCV method. The suspension roll stiffness at axle roll center mainly depends on:

- sprung mass at an axle
- sprung mass COG height at an axle
- suspension roll center height from ground level at an axle
- suspension roll stiffness about suspension roll center at an axle

It can also be observed from equation (4.9), with increase in suspension roll stiffness about suspension roll center also increases suspension roll stiffness about axle roll center similar to UNECE111 method but not proportionally. The similar influence can be observed from other parameters as well.

Therefore, it is certain that due to suspension compliance factor, roll stiffness estimated by RCV method is lower than UNECE111 method and the dependency of roll stiffness on the other parameters is not directly proportionate.

4.2 Roll Stiffness due to Tire Compliance

Tire is considered as one of the important part of vehicle due to being the only part which is in contact with the road and all the forces and moments in one way or another are related to tire properties. Therefore, it wouldn't be incorrect that rollover is also influenced by tire properties. According to NHTSA report [8], lateral deflection of the tire during rollover results in significant loss of track-width.

To understand the parameters which increase or decrease the track-width which has direct influence on the rollover threshold, a good approach is to start with axles and tires which are generally considered as un-sprung mass and has not been included in Section 4.1.

At an axle, tires are not loaded only with un-sprung mass but also with sprung mass, therefore, better way to analyze tires influence is to consider the complete axle load acting at axle COG height as presented in Figure 4.2.

Tires and axle are considered rigid in lateral direction as shown in Figure 4.2. In static condition, axle load is equally divided at both tires at axle due to only vertical deflection (vertical stiffness) of tires as presented in Figure 4.2(a). During the roll motion, lateral acceleration increases at axle COG which induce the roll motion at an axle and axle COG can be considered to roll about axle roll center 'o' with roll angle ϕ_{al} . Due to this roll motion, axle load is transferred laterally which results in increased vertical deflection (i.e. vertical reaction force) at right side tire as presented in Figure 4.2(b).

Again, a torsional spring can be considered at axle roll center which has stiffness equivalent to tire compliance in a similar approach to suspension compliance. This torsional spring with stiffness C_{al} can be considered to be resisting the overturning moments due to lateral acceleration and lateral shift of COG. The overturning moment M_{al} due to lateral acceleration and lateral shift of COG about axle roll center 'o' can be expressed as equation (4.12).

$$M_{al} = m g h_{cg} \phi_{al} + m a_y h_{cg} \quad (4.12)$$

Load transfer can be realized by increasing vertical deflection of right side tire (outer tire). At the time of wheel lift-off at left side, complete load can be considered to have transferred

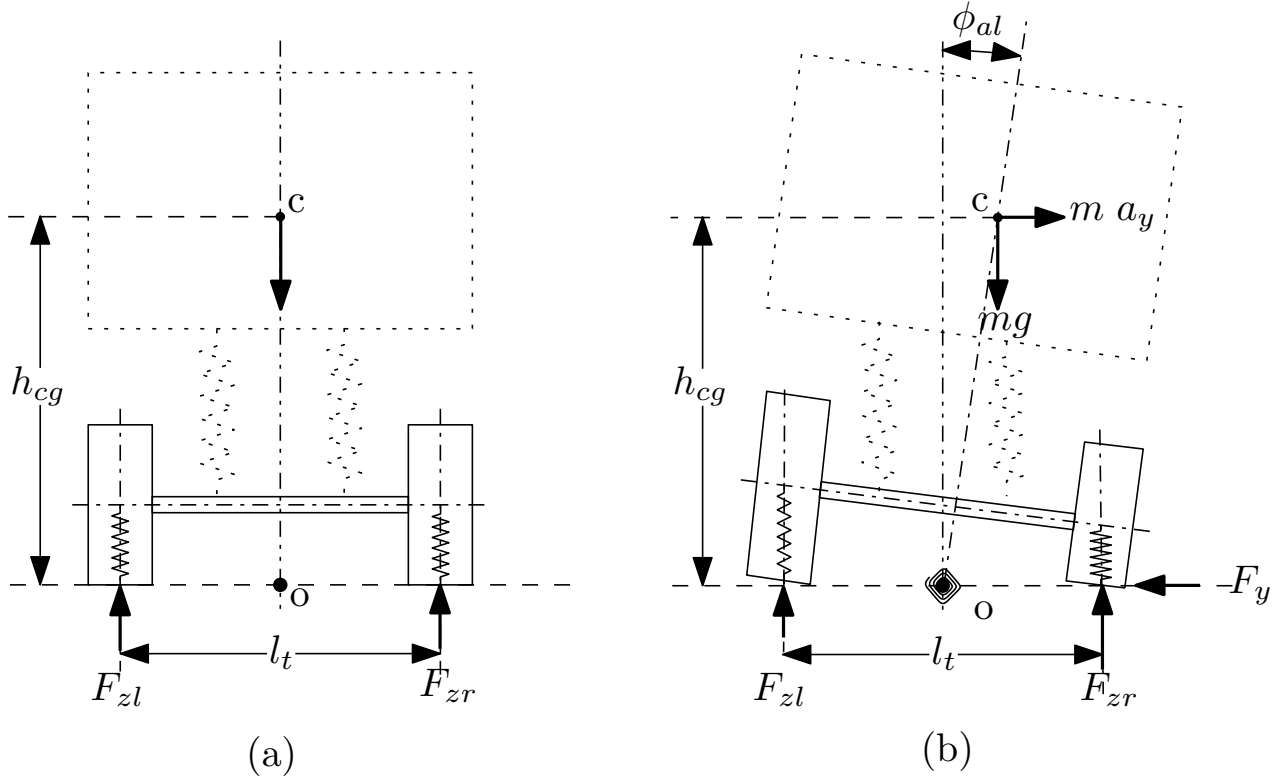


Figure 4.2: Schematic representation of axle load only due to tire compliance: (a) in static condition; (b) in rolling condition

at right side tire, i.e. $F_{zl} = 0$ & $F_{zr} = m g$, resulting in maximum critical vertical deflection x_{crit} of tire. Thus, axle load can be represented in terms of tire vertical stiffness C_{tv} and this critical vertical deflection, i.e.

$$m g = C_{tv} x_{crit}$$

where, ' x_{crit} ' being maximum vertical deflection of outer tire at inner wheel lift-off considering only vertical tire stiffness due to laterally transferred load.

According to Newton's second law of motion, lateral force and lateral acceleration can be correlated as

$$a_y = \frac{F_y}{m}$$

With these considerations, equation (4.12) can be re-written as equation (4.13) at the time of inner wheel lift off, where overturning moment is a function of tire vertical stiffness and lateral force.

$$M_{al,crit} = m g h_{cg} \phi_{al,crit} + C_{tv} x_{crit} \frac{F_y}{m g} h_{cg} \quad (4.13)$$

Since, a torsional spring is considered at axle roll center for identifying roll stiffness due to tire compliance, therefore, stiffness of this torsional spring C_{al} can also be determined at the time of inner wheel lift-off at axle. It can be represented as

$$C_{al} = \frac{M_{al,crit}}{\phi_{al,crit}}$$

Using this, equation (4.13) can be expressed as equation (4.14).

$$C_{al} = \frac{M_{al,crit}}{\phi_{al,crit}} = m g h_{cg} + C_{tv} \left(\frac{x_{crit}}{\phi_{al,crit}} \right) \frac{F_y}{mg} h_{cg} \quad (4.14)$$

The axle can be considered rigid and due to linear deflections, critical axle roll angle and maximum critical vertical deflection at inner wheel lift-off can be correlated with track-width as

$$\frac{x_{crit}}{\phi_{al,crit}} = l_t$$

Therefore, roll stiffness due to tire compliance about axle roll center can be determined as expressed in equation (4.15).

$$C_{al} = h_{cg} \left[m g + C_{tv} l_t \frac{F_y}{m g} \right] \quad (4.15)$$

According to UNECE111 calculation method, roll stiffness due to axle and tire compliance about axle roll center is determined as expressed in equation (4.16). It is represented here in RCV notations to provide comparative difference.

$$C_{al} = \frac{1}{2} C_{tv} (l_t)^2 \quad (4.16)$$

The tire lateral flexibility affects vehicle even in steady state cornering which depends on tire lateral force and vertical load [35]. It can be clearly observed from equation (4.16), UNECE111 method doesn't account for such influence in determining the roll stiffness due to tire compliance. RCV method accounts for above mentioned influence of tire compliance on roll stiffness determination as expressed in equation (4.15). It indicates that roll stiffness due to tire compliance doesn't only depend on vertical tire stiffness and track width but also on axle load, its COG height and lateral force.

4.3 Tire Lateral Shift

During roll motion, an axle experiences load transfer laterally between inner and outer side. Due to this load transfer, tires generate lateral forces at contact patch, which result in the shift of contact patch under the vehicle. It is equivalent to the distance between tire and axle COG, i.e. half track-width. The most important time to look at the track-width change is when the vehicle is about to lift-off at respective axle [39].

This lateral deflection of tire (change in track-width) depends on lateral force, tire inflation pressure and lateral properties of tire. Ellis [28] describes it as the time varying lateral displacement of tire tread in the form of first order differential equation. Heydinger [18], also adopts the Ellis's approach of first order lag but instead of time, he adopts side force lag and further extends this approach to dynamic simulation in terms of second order side force response to slip angles. A multi-body modeling approach of tire lateral flexibility is presented in [31], which represents the static effect of tire lateral flexibility due to modeling of only tire

lateral stiffness and system equations represent the static equilibrium in a roll plane which is close to the approach of RCV method.

The contact patch is the only element to directly contact with the road surface, its deformation significantly affects the cornering characteristics of a tire. The tire elastic deformations are directly related to all of the compliance properties of tread, breaker and sidewall, the lateral deformations are caused due to slip and camber angles during cornering [15],[6]. The actual consideration of tire lateral properties are only possible in dynamic situations because influencing tire parameters (such as slip angles, camber angle, lateral force etc.) are highly dynamic in nature (i.e. time dependent). As it is clear that tire lateral properties also influence the vehicle in steady state condition, therefore, tire lateral shift can be simplified in case of steady state rollover situation as presented in Figure 4.3 [8].

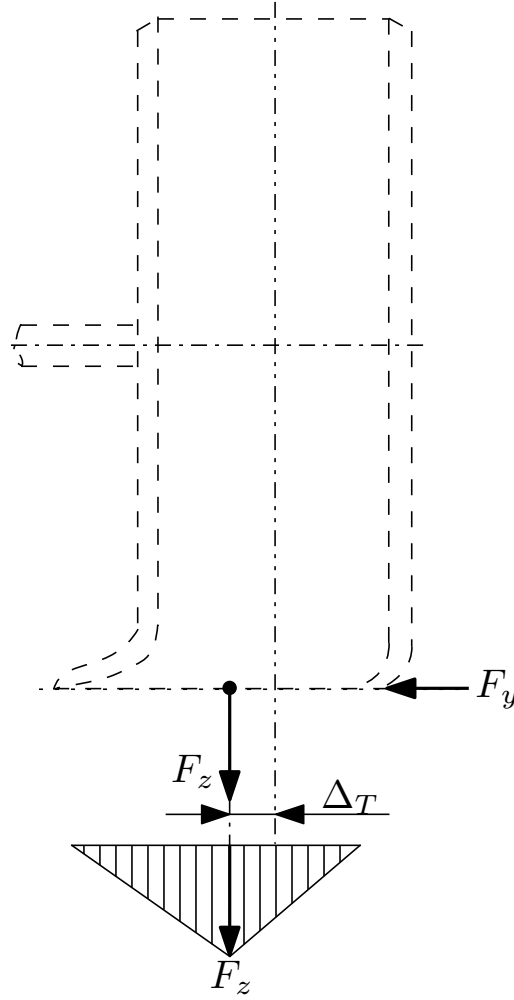


Figure 4.3: *Tire lateral deflection [8]*

Tire lateral shift, as presented in Figure 4.3, is the effect of lateral force induced in tire contact patch due to roll motion of the vehicle. The resultant vertical load at the contact patch can be considered to act at a distance Δ_T inside the tire symmetry axis towards the axle (or vehicle) COG due to lateral flexibility of tire. This tire lateral shift reduce axle track-width which can be determined by considering lateral stiffness of the tire [7]. The change

in track-width, hereafter defined as effective track-width l_{eff} can be determined as expressed in equation (4.17).

$$l_{eff} = (l_t - \Delta_{To} + \Delta_{Ti}) \quad (4.17)$$

where, Δ_{Tj} ($(j = o, i)$ stands for outer and inner side vehicle tire) represents the reduction and promotion of axle track-width due to the lateral shift of the tire contact patch at outer and inner side respectively. Since, this shift is due to the kinematic property of the tire and resulted from the tire side forces, therefore it can be represented in terms of tire properties, which can be measured/obtained from the tire testing machines [8], as expressed in equation (4.18).

$$\Delta_{Tj} = \frac{F_{yj}}{C_{tyj}} \quad (4.18)$$

' C_{ty} ' represents the tire lateral stiffness [kN/m], which can be measured on tire testing machine but lateral force is a dynamic parameter which depends on tire vertical load and during the roll motion vertical load changes due to the lateral load transfer between left and right side tires until the complete load is transferred on right side tire. Therefore, in steady state condition, it is most important to determine the change in track-width at the time wheel lift-off due to lateral flexibility of tire, which would also be convenient from the point of view of determining tire lateral shift at right side tire only.

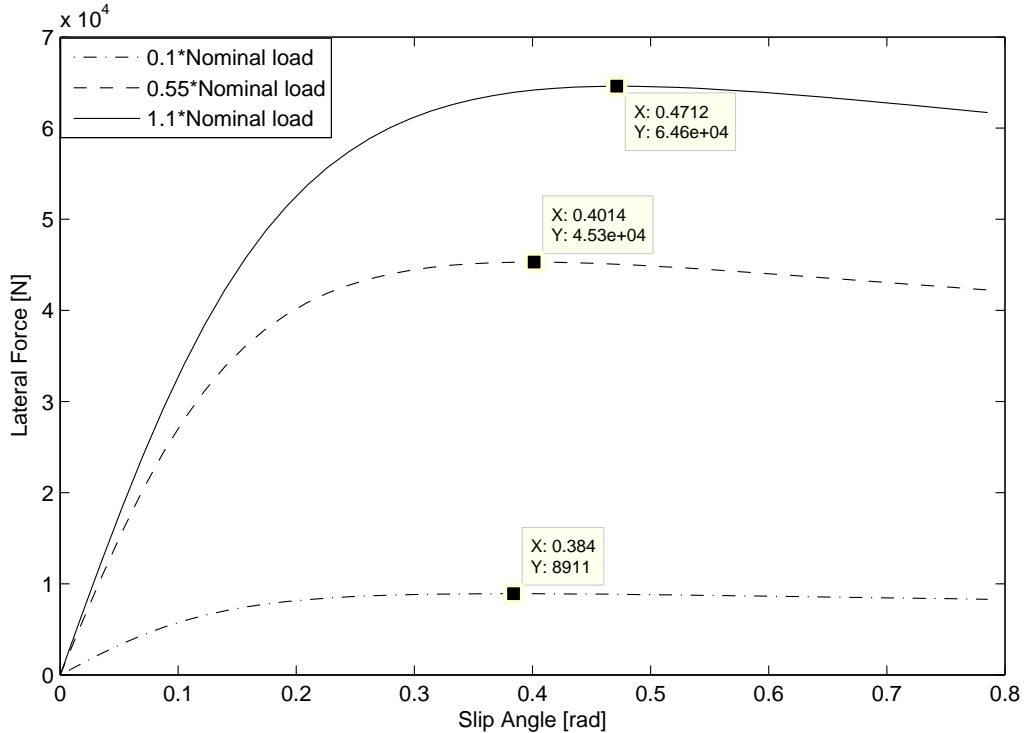


Figure 4.4: *Tire lateral force characteristic with varying tire vertical load*

Tire lateral force decreases with reducing tire vertical load as presented in Figure 4.4, which indicates that at complete load transfer at an axle, left side tire lateral shift can be neglected

due to almost negligible lateral force. Therefore, effective track-width can be determined as expressed in equation (4.19).

$$\begin{aligned} l_{teff} &= (l_t - \Delta_{To}) \\ l_{teff} &= \left(l_t - \frac{F_y}{C_{ty}} \right) \end{aligned} \quad (4.19)$$

Therefore, by considering lateral flexibility of tire, roll stiffness due to tire compliance, which is being considered in RCV method can be determined as expressed in equation (4.20).

$$C_{al} = h_{cg} \left[m g + C_{tv} \left(l_t - \frac{F_y}{C_{ty}} \right) \frac{F_y}{m g} \right] \quad (4.20)$$

5 Fifth-wheel and Frame Torsional Compliance

This chapter presents a modeling and analysis approach for fifth-wheel compliance as adopted in RCV method. Fifth-wheel compliance is presented as a combination of fifth-wheel lash and trailer lash. The roll stiffness due to both lash effects is presented in terms of tractor drive and trailer axles parameters. An approach of considering effects of trailer flexibility (torsional stiffness) is also presented briefly.

5.1 Determination of Fifth-wheel Vertical Load

To estimate the fifth wheel roll stiffness, primary objective is to determine the vertical load on the fifth wheel in a vehicle combination. Lets consider a tractor semi-trailer vehicle with three axles in a static condition¹ as presented in Figure 5.1.

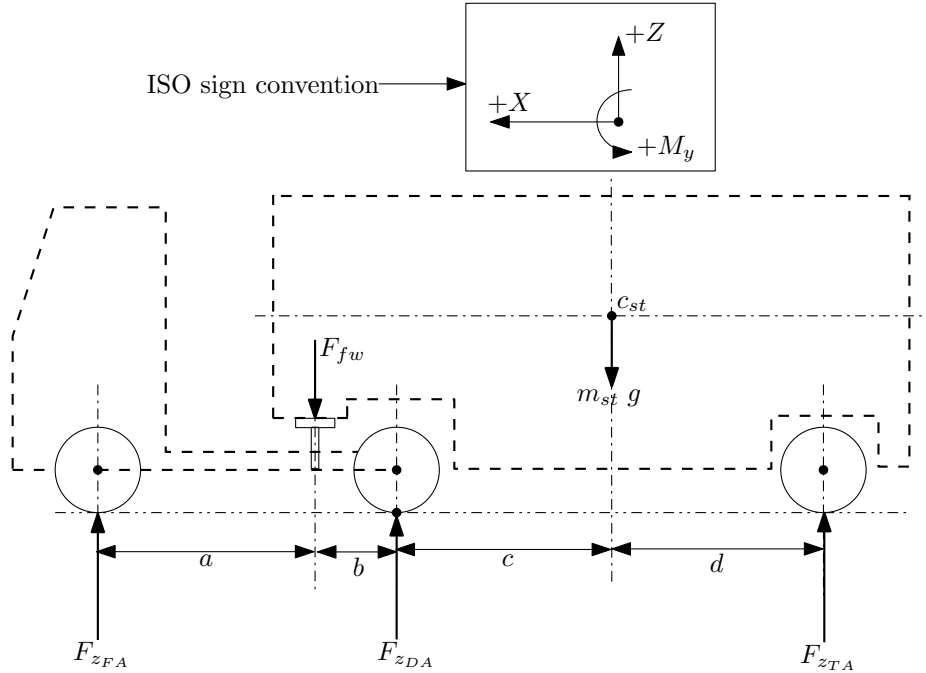


Figure 5.1: *Schematic representation of tractor semi-trailer in static condition*

A tractor semitrailer is represented in x-z plane as presented in Figure 5.1. The axle groups are represented as single axle with respective axle vertical loads. The trailer sprung mass m_{st} is acting at trailer sprung mass cog ' c_{st} ', fifth-wheel is located at distance $(b + c)$ from trailer sprung mass cog. Trailer sprung mass (i.e. superstructure and cargo) is mainly supported by fifth-wheel and trailer axle. Tractor semitrailer can also be simply represented by level diagram as presented in Figure 5.2.

Irrespective of the directions of force vectors², trailer sprung weight $m_{st} g$ can be seen

¹when the vehicle is at rest

²Chosen for the convenience of representing lever diagram

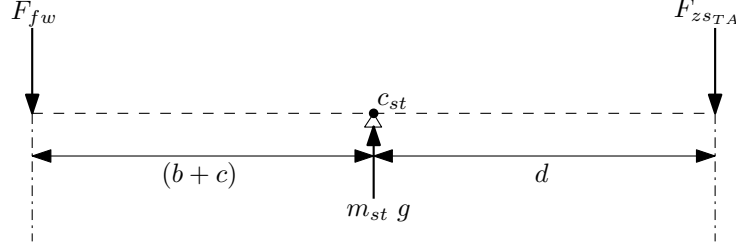


Figure 5.2: *Lever diagram representation of trailer*

as acting at lever point ' c_{st} '³ and the two extreme ends of the lever diagram can be seen as fifth-wheel and trailer axle respectively. Fifth-wheel and trailer axle is located at $(b+c)$ and d distance from trailer sprung mass cog respectively as shown in Figure 5.2. By knowing vehicle longitudinal distances, fifth-wheel vertical load F_{fw} and trailer axle vertical load due to trailer sprung mass can be expressed in terms of trailer sprung mass as represented in equation (5.1) and (5.2) respectively.

$$F_{fw} = \left(\frac{d}{b+c+d} \right) m_{st} g \quad (5.1)$$

$$F_{zsTA} = \left(\frac{b+c}{b+c+d} \right) m_{st} g \quad (5.2)$$

5.2 Estimation of Fifth-wheel Roll Stiffness

Fifth-wheel plays an important role in roll motion of vehicle combinations especially in roll-coupled units as it joins the trailer with tractor. The roll stiffness of fifth-wheel can significantly affect the roll acceleration of tractor and semitrailer [21]. The vehicle combination's roll motion is determined by the fifth-wheel properties, whether the tractor and semitrailer unit behaves as one unit or separate unit in roll direction, thus, can subsequently affect the steady-state rollover threshold.

Law [25] has described a modeling approach for fifth-wheel, where trailer is considered to be supported at two knife-edges, which are located laterally at the extreme ends of fifth-wheel. These knife-edges support the half of the fifth-wheel load at static condition and when the roll motion starts, due to the load transfer between these two edges, roll moment at the fifth-wheel center point is determined. The limitation of this modeling approach is that fifth-wheel roll moment should be considered about fifth-wheel roll center and also trailer lash should have been considered, which is missing.

Therefore, an approach similar to Law [25] has been adapted in this work to model fifth-wheel but with consideration of trailer lash and fifth-wheel roll motion about its roll center. Figure 5.3 represents the static loading condition of tractor drive axle in roll plane, which indicates that drive axle sprung mass is comprised of trailer and tractor sprung masses. The two sprung masses are connected through fifth-wheel, which synchronizes roll motion at drive axle between both sprung masses. Fifth-wheel is considered to have its own roll stiffness through spring elements, which promotes synchronization of tractor and trailer. In static

³trailer sprung mass cog in Figure 5.1

condition, vertical load at fifth-wheel left and right side edges are equal to half the total fifth wheel load in the static condition, i.e.

$$F_l = F_r = \frac{F_{fw}}{2}$$

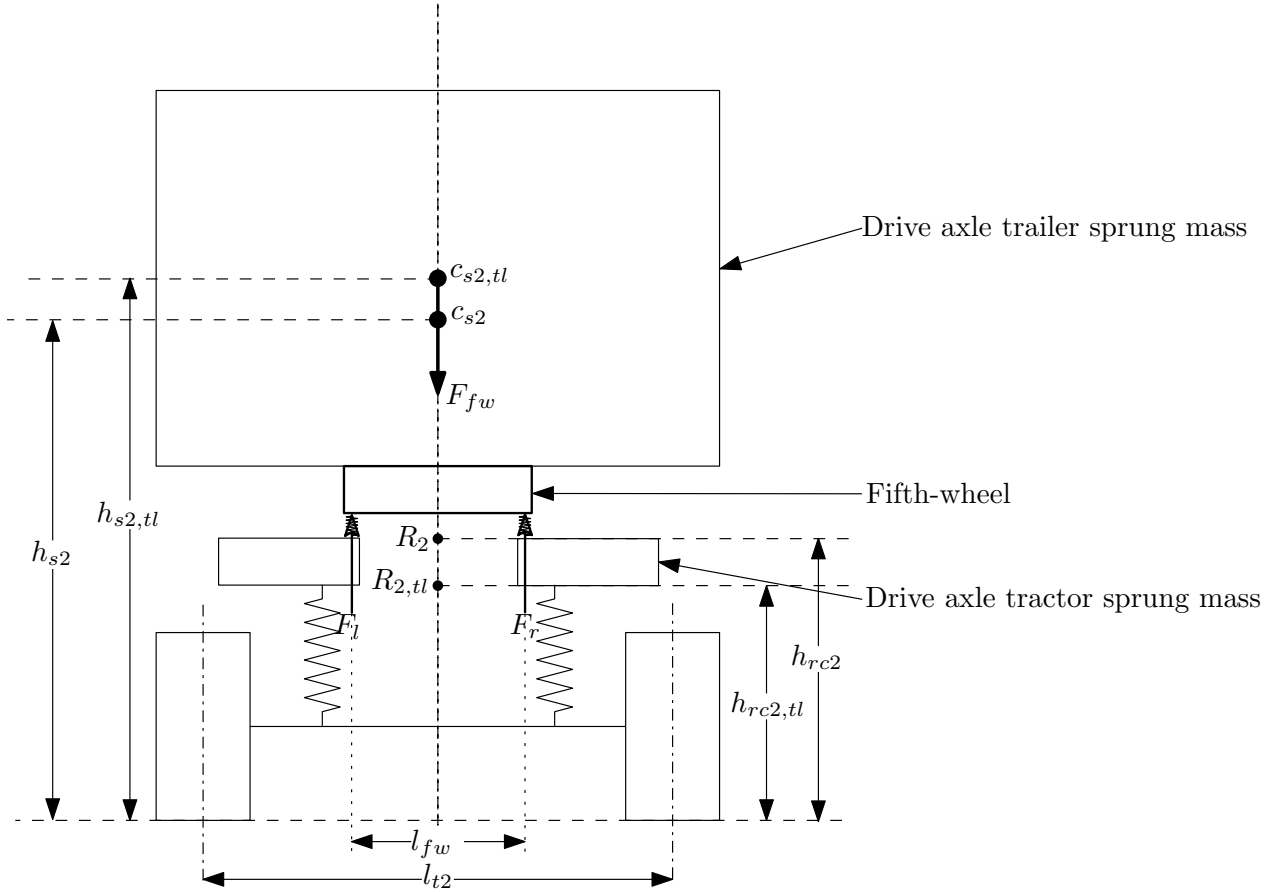


Figure 5.3: *Schematic representation of static loading of fifth-wheel at tractor drive axle*

where,

- $c_{s2,tl}$ - trailer sprung mass center of gravity at drive axle
- c_{s2} - total sprung mass center of gravity at drive axle
- F_{fw} - fifth-wheel load acting at ' $c_{s2,tl}$ ', [kN]
- F_l - vertical load at fifth-wheel left side edge, [kN]
- F_r - vertical load at fifth-wheel right side edge, [kN]
- $R_{2,tl}$ - trailer roll center at drive axle
- R_2 - roll center at drive axle

- $h_{rc2,tl}$ - height of trailer roll center at drive axle ' $R_{2,tl}$ ' from ground, [m]
- h_{rc2} - height of roll center at drive axle ' R_2 ' from ground, [m]
- $h_{s2,tl}$ - height of trailer sprung mass center of gravity at drive axle ' $c_{s2,tl}$ ' from ground, [m]
- h_{s2} - height of sprung mass center of gravity at drive axle ' c_{s2} ' from ground, [m]
- l_{fw} - width of the fifth-wheel [m]
- l_{t2} - track-width at drive axle [m]

Fifth-wheel can be modeled in terms of its roll motion which can be described in a step-wise process as illustrated in Figure 5.4.

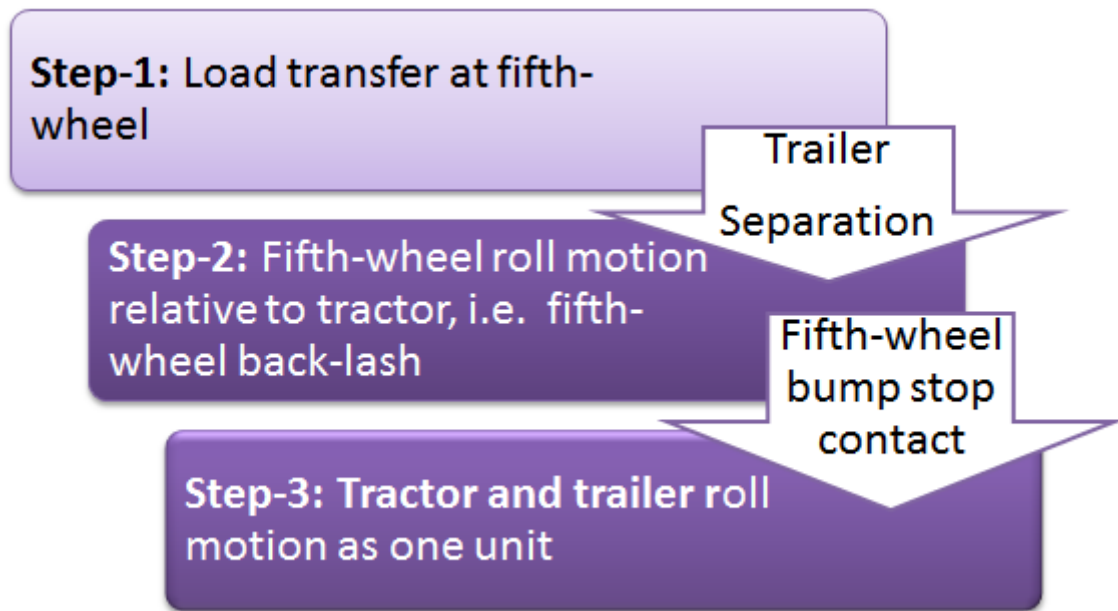


Figure 5.4: *Step-wise process of fifth-wheel roll motion*

- Step-1:** Due to roll motion, load transfer takes place at fifth-wheel from left to right side edge. Since, fifth-wheel load is entirely due to trailer sprung mass at drive axle, therefore, when complete load is transferred between fifth-wheel edges, trailer lifts-off from fifth-wheel left side edge. This phenomenon is known as *trailer separation*.
- Step-2:** After trailer separation, fifth-wheel no longer synchronizes the roll motion between tractor and trailer units. Since, fifth-wheel has its own roll degree of freedom depending up on its bump stop clearances, therefore, it rolls relative to tractor and follows the trailer roll motion until bump stops come in contact with each other. This phenomenon is known as *fifth-wheel lash*.
- Step-3:** Once fifth-wheel has reached its maximum lash, the connection between trailer and tractor becomes rigid, i.e. the trailer and tractor units can no longer roll relative to each

other at drive axle. Both sprung masses at drive axle behaves as one lumped mass and roll together with further increase in lateral acceleration.

To understand the influence of aforementioned fifth-wheel compliance on roll motion of vehicle, above mentioned steps are discussed in detail.

5.2.1 Step-1: Load Transfer at Fifth-wheel

A schematic representation of fifth-wheel roll motion during load transfer between its edges is presented in Figure 5.5.

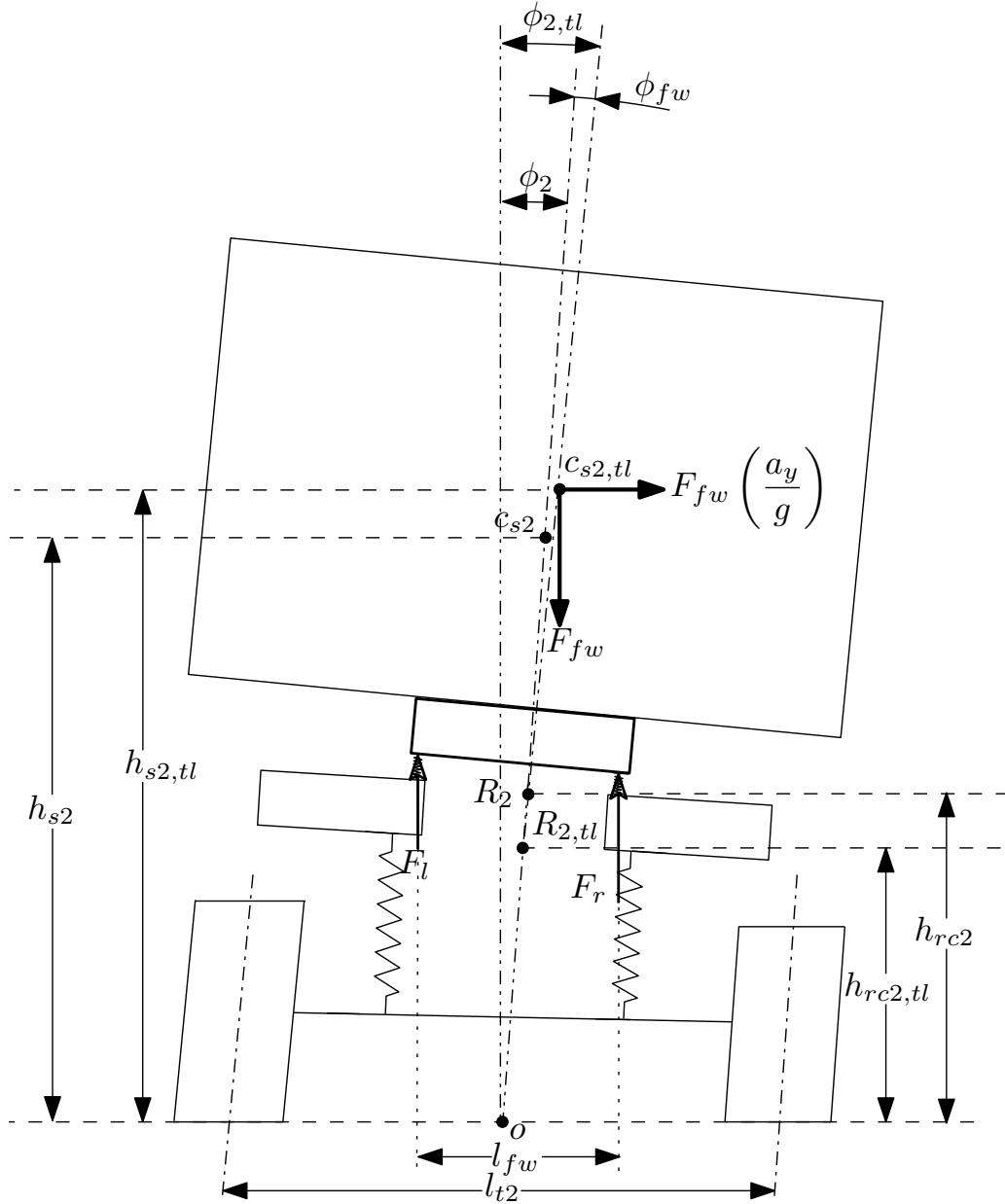


Figure 5.5: *Schematic representation of fifth-wheel roll motion in Step-1 at tractor drive axle*

where,

- $\phi_{2,tl}$ - roll angle of drive axle considering only trailer sprung mass about point ‘o’ at ground level, [rad]
- ϕ_2 - roll angle of drive axle considering total sprung mass about point ‘o’ at ground level, [rad]
- ϕ_{fw} - roll angle of fifth-wheel, [rad]

Load transfer at fifth-wheel depends on trailer sprung mass and total sprung mass motion at drive axle. The difference between their motion is compensated by fifth-wheel in order to maintain the motion synchronicity at drive axle. Fifth-wheel can be considered to be located above drive axle, which provides a simplicity of considering same spring, tire elements and un-sprung mass for trailer and total sprung mass at drive axle. The only difference is that both of the sprung masses roll about different roll centers as shown in Figure 5.5, which results in a relatively small fifth-wheel roll angle. The fifth-wheel roll angle can be represented as a difference between the roll angles of drive axle considering only trailer sprung mass and total sprung mass as drive axle sprung mass respectively, i.e.

$$\phi_{fw} = (\phi_{2,tl} - \phi_2)$$

The roll angles are considered small, i.e. $\cos(\phi_{fw}) \approx 1$ and $\sin(\phi_{fw}) \approx \phi_{fw}$. Roll motion at fifth-wheel can be represented by an equilibrium equation as expressed in equation (5.3).

$$(F_r - F_l) \frac{l_{fw}}{2} = C_{fw1} \phi_{fw} = F_{fw} (h_{fw} - h_{rc,fw}) \phi_{fw} + F_{fw} \frac{a_y}{g} (h_{fw} - h_{rc,fw}) \quad (5.3)$$

$$C_{fw1} = F_{fw} (h_{fw} - h_{rc,fw}) \left[1 + \frac{a_y}{g \phi_{fw}} \right] \quad (5.4)$$

Fifth-wheel roll stiffness C_{fw1} can be determined based on equation (5.4) but fifth-wheel roll angle and lateral acceleration both are variables which needs to be determined first in order to estimated fifth-wheel roll stiffness. One possible way to establish relation between lateral acceleration and fifth-wheel roll angle is to determine the fifth-wheel roll angle at which complete load is transferred between fifth-wheel edges. At that point, $(F_r - F_l)$ can be replaced by fifth-wheel load (F_{fw}) itself, i.e. $F_l = 0$ and $F_r = F_{fw}$ and equation (5.3) can be rewritten as:

$$F_{fw} \frac{l_{fw}}{2} = F_{fw} (h_{fw} - h_{rc,fw}) \phi_{fw} + F_{fw} \frac{a_y}{g} (h_{fw} - h_{rc,fw})$$

$$\frac{l_{fw}}{2(h_{fw} - h_{rc,fw})} = \left[\phi_{fw} + \frac{a_y}{g} \right] \quad (5.5)$$

Equation (5.5) represents the *trailer separation condition*, which indicates that at certain lateral acceleration, when summation of fifth-wheel roll angle and lateral acceleration equals to the ratio of half fifth-wheel width to height of fifth-wheel load above its roll center, then trailer

lifts-off from left edge of fifth-wheel. This boundary condition depends up on the fifth-wheel parameters viz. width and height of load above its roll center, therefore, by changing any of it, trailer separation can be influenced. To determine the fifth-wheel roll stiffness, one more relation needs to be established between fifth-wheel roll angle and lateral acceleration, which can be obtained from determining fifth-wheel roll angle, i.e. by determining drive axle roll angles considering only trailer and total sprung mass respectively. Total roll angle at an axle is due to sprung and un-sprung mass. Un-sprung mass (i.e. axle and tires) remain same for both sprung masses at drive axle.

(a) Drive Axle Roll Angle While Considering Total Sprung Mass

Roll stiffness due to suspension and tire compliance about axle roll center is determined in Chapter 4. Both of them can be considered to be acting in series and resultant roll stiffness at axle roll center can be determined as expressed in equation (5.6).

$$C_{res,i} = \frac{C_{sg,i} C_{al,i}}{C_{sg,i} + C_{al,i}} \quad (5.6)$$

where,

- i - axle group index (= 1, for steer axle; = 2, for drive axle; = 3, for trailer axle)
- $C_{res,i}$ - resultant roll stiffness about axle roll center 'o' at ground level [kNm/rad]
- $C_{al,i}$ - axle roll stiffness about axle roll center 'o' at ground level [kNm/rad]
- $C_{sg,i}$ - equivalent suspension roll stiffness about axle roll center 'o' at ground level [kNm/rad]

Combined roll motion at drive axle due to sprung and un-sprung mass can be expressed as moment equilibrium equation in roll plane as expressed in equation (5.7).

$$C_{res2} \phi_2 = m_2 g h_{cg2} \phi_2 + m_2 a_y h_{cg2} \quad (5.7)$$

$$[C_{res2} - m_2 g h_{cg2}] \phi_2 = m_2 h_{cg2} a_y$$

$$\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right] \phi_2 = \frac{a_y}{g} \quad (5.8)$$

where,

- m_2 - total mass at drive axle, [ton]
- h_{cg2} - height of cog above ground at drive axle, [m]

Equation (5.8) represents the relation between drive axle roll angle and lateral acceleration. Lets say,

$$Z = \left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right]$$

then

$$\phi_2 = \left(\frac{1}{Z} \right) \frac{a_y}{g}$$

The above expression indicates the relation between drive axle roll angle and lateral acceleration as a straight line with slope $\left(\frac{1}{Z} \right)$.

(b) Drive Axle Roll Angle While Considering Only Trailer Sprung Mass

Trailer sprung mass is less than total sprung mass at drive axle. It can be considered to roll about a different roll center as presented in Figure 5.5. Since, its supported by same spring elements, therefore, suspension roll stiffness can be considered same about suspension roll center. However, suspension roll stiffness about ground level (i.e. axle roll center) will be different due to difference in sprung mass, sprung mass cog height and roll center height. It can be determined according to equation (5.9).

$$C_{sg2,tl} = C_{s2} \left(\frac{h_{s2,tl}}{h_{s2,tl} - h_{rc2,tl}} \right)^2 \left[1 - \frac{m_{s2,tl} g h_{rc2,tl} (h_{s2,tl} - h_{rc2,tl})}{C_{s2} h_{s2,tl}} \right] \quad (5.9)$$

- C_{s2} - suspension roll stiffness about roll center at drive axle, [kNm/rad]
- $C_{sg2,tl}$ - equivalent suspension roll stiffness at ground level considering only trailer sprung mass at drive axle, [kNm/rad]
- $m_{s2,tl}$ - sprung mass at drive axle only considering trailer sprung mass, [ton]

Due to the difference between equivalent suspension roll stiffness about axle roll center, resultant roll stiffness about axle roll center will also be different, which can be determined by equation (5.6), while considering same axle roll stiffness about axle roll center. In this case, moment equilibrium at drive axle can be expressed by equation (5.10).

$$C_{res2,tl} \phi_{2,tl} = m_{2,tl} g h_{cg2,tl} \phi_{2,tl} + m_{2,tl} a_y h_{cg2,tl} \quad (5.10)$$

$$[C_{res2,tl} - m_{2,tl} g h_{cg2,tl}] \phi_{2,tl} = m_{2,tl} a_y h_{cg2,tl}$$

$$\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right] \phi_{2,tl} = \frac{a_y}{g} \quad (5.11)$$

where,

- $m_{2,tl}$ - total mass at drive axle considering only trailer sprung mass, [ton]
- $h_{cg2,tl}$ - height of center of gravity above ground at drive axle considering only trailer sprung mass, [m]

Equation (5.11) represents the relation between drive axle roll angle and lateral acceleration when only trailer sprung mass is considered as drive axle sprung mass. Lets say,

$$Y = \left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]$$

then

$$\phi_{2,tl} = \left(\frac{1}{Y} \right) \frac{a_y}{g}$$

The above expression indicates the relation between drive axle roll angle and lateral acceleration as a straight line with slope $\left(\frac{1}{Y} \right)$.

It can be observed from equation (5.8) and (5.11) that drive axle roll angle has linear relation with lateral acceleration. The slope depends on the ratio of resultant roll stiffness to axle load times the height (i.e. Y or Z). The difference between the two slopes indicate the influence of fifth-wheel at drive axle. Relation between fifth-wheel roll angle and lateral acceleration can also be determined using equations (5.8) and (5.11).

$$\begin{aligned} \phi_{fw} &= \phi_{2,tl} - \phi_2 \\ &= \left(\frac{1}{\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]} \frac{a_y}{g} \right) - \left(\frac{1}{\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right]} \frac{a_y}{g} \right) \\ \phi_{fw} &= \left(\frac{1}{\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]} - \frac{1}{\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right]} \right) \frac{a_y}{g} \\ \phi_{fw} &= \left(\frac{1}{Y} - \frac{1}{Z} \right) \frac{a_y}{g} = X \frac{a_y}{g} \end{aligned} \tag{5.12}$$

where, X represents the slope of fifth-wheel roll angle versus lateral acceleration curve, i.e.

$$X = \left(\frac{1}{Y} - \frac{1}{Z} \right)$$

Fifth-wheel roll angle changes with lateral acceleration according to this slope until trailer separation occurs, which is expressed in equation (5.5). Using equation (5.12), *trailer separation condition* can be expressed either in terms of fifth-wheel roll angle or lateral acceleration. Now, fifth-wheel roll angle and lateral acceleration can be determined when trailer separation occurs.

$$\frac{l_{fw}}{2(h_{fw} - h_{rc, fw})} = \left[X \frac{a_y}{g} + \frac{a_y}{g} \right]$$

$$\frac{l_{fw}}{2(h_{fw} - h_{rc, fw})} = (X + 1) \frac{a_y}{g} \quad (5.13)$$

$$\frac{l_{fw}}{2(h_{fw} - h_{rc, fw})} = \left(1 + \frac{1}{X} \right) \phi_{fw} \quad (5.14)$$

Also, fifth-wheel roll stiffness can be determined by substituting the ratio of lateral acceleration to fifth-wheel roll angle in equation (5.4).

$$C_{fw1} = F_{fw} (h_{fw} - h_{rc, fw}) \left(1 + \frac{1}{X} \right) \quad (5.15)$$

Fifth-wheel behaves as rigid connection before trailer separates, which can be observed from equation (5.15) as well. The fifth-wheel roll angle is very small (order of 0.05 deg), which indicates very small slope of fifth-wheel roll angle versus lateral acceleration curve i.e. $\left(\frac{1}{X} \right)$ and results in high fifth-wheel roll stiffness.

Note: In general, for simplifying the modeling approach, fifth-wheel is considered with infinite roll stiffness before trailer separation, which results in negligible fifth-wheel roll motion, i.e. $(\phi_{fw} \approx 0)$ and lateral acceleration at trailer separation can be determined simply as expressed in equation (5.16).

$$\frac{a_{y1}}{g} = \frac{l_{fw}}{2(h_{fw} - h_{rc, fw})} \quad (5.16)$$

In current study, fifth-wheel is not being considered with infinite roll stiffness and lateral acceleration at trailer separation is determined as expressed in equation (5.17).

$$\frac{a_{y1}}{g} = \frac{l_{fw}}{2(1 + X)(h_{fw} - h_{rc, fw})} \quad (5.17)$$

5.2.2 Step-2: Fifth-wheel Lash

At trailer separation, trailer lifts off from fifth-wheel left edge, which indicates completion of load transfer at fifth-wheel. Roll motion of fifth-wheel can be seen as represented in Figure 5.6. It is only the extension of Figure 5.5 to indicate the fifth-wheel roll motion relative to tractor, therefore, only the change in fifth-wheel roll angle is presented with respect to fifth-wheel roll angle at trailer separation.

where,

- ϕ_{fw1} - fifth-wheel roll angle at trailer separation, [rad]
- $\Delta\phi_{fw}$ - change in fifth-wheel roll angle due to fifth-wheel back-lash, [rad]

while considering only trailer sprung mass.

The relation between drive axle roll angle while considering only trailer sprung mass and lateral acceleration is expressed in equation (5.11). After trailer separation, similar relation can be established between change in roll angle and change in lateral acceleration as expressed in equation (5.18).

$$\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right] \Delta\phi_{2,tl} = \frac{\Delta a_y}{g} \quad (5.18)$$

Moment equilibrium at trailer axle due to sprung and un-sprung mass can be expressed by equation (5.19).

$$\begin{aligned} C_{res3} \phi_3 &= m_3 g h_{cg3} \phi_3 + m_3 a_y h_{cg3} \\ [C_{res3} - m_3 g h_{cg3}] \phi_3 &= m_3 h_{cg3} a_y \end{aligned} \quad (5.19)$$

$$\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right] \phi_3 = \frac{a_y}{g} \quad (5.20)$$

where,

- C_{res3} - resultant roll stiffness at ground level, [kNm/rad]
- ϕ_3 - roll angle at trailer axle, [rad]
- m_3 - total mass at trailer axle, [kg]
- h_{cg3} - height of center of gravity above ground at trailer axle, [m]

Equation (5.20) represents the relation between trailer axle roll angle and lateral acceleration. Change in trailer axle roll angle with lateral acceleration can be expressed by equation (5.21).

$$\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right] \Delta\phi_3 = \frac{\Delta a_y}{g} \quad (5.21)$$

Until trailer separation, trailer axle can be considered to roll independently of drive axle. During fifth-wheel lash, it also contributes in the rolling motion of fifth-wheel and trailer sprung mass above fifth-wheel. Therefore, change in fifth-wheel roll angle can be considered as the summation of change in trailer axle roll angle and change in drive axle roll angle considering only trailer sprung mass due to change in lateral acceleration, i.e.

$$\Delta\phi_{fw} = \Delta\phi_{2,tl} + \Delta\phi_3$$

By using equation (5.18) and (5.21), a relation between change in fifth-wheel roll angle and change in lateral acceleration can be established during fifth-wheel lash.

$$\begin{aligned}
\Delta\phi_{fw} &= \Delta\phi_{2,tl} + \Delta\phi_3 \\
&= \left(\frac{1}{\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]} \frac{\Delta a_y}{g} \right) + \left(\frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} \frac{\Delta a_y}{g} \right) \\
\Delta\phi_{fw} &= \left(\frac{1}{\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]} + \frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} \right) \frac{\Delta a_y}{g} \tag{5.22}
\end{aligned}$$

Lets say,

$$X_1 = \left(\frac{1}{\left[\frac{C_{res2,tl}}{m_{2,tl} g h_{cg2,tl}} - 1 \right]} + \frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} \right)$$

then

$$\Delta\phi_{fw} = X_1 \frac{\Delta a_y}{g} \tag{5.23}$$

where, ‘ X_1 ’ represents the slope of change in fifth-wheel roll angle versus change in lateral acceleration curve, which is also a straight line. If compared the slope of fifth-wheel roll angle versus lateral acceleration curve before and after trailer separation, it can be clearly concluded that after trailer separation fifth-wheel roll angle increases with very high slope compared to before trailer separation due to lateral acceleration. It also indicates that after trailer separation, roll stiffness of fifth-wheel decreases quite significantly. This fifth-wheel roll stiffness can be determined by applying equation (5.4) during fifth-wheel lash.

$$\begin{aligned}
C_{fw2} &= F_{fw} (h_{fw} - h_{rc, fw}) \left[1 + \frac{\Delta a_y}{g \Delta\phi_{fw}} \right] \\
C_{fw2} &= F_{fw} (h_{fw} - h_{rc, fw}) \left(1 + \frac{1}{X_1} \right) \tag{5.24}
\end{aligned}$$

Equation (5.24) represents the fifth-wheel roll stiffness after trailer separation during fifth-wheel lash. It can be concluded based on the slope of $\left(\phi_{fw} \text{ v/s } \frac{a_y}{g} \right)$ curve, i.e. ($X_1 \gg X$) that fifth-wheel roll stiffness after trailer separation is quite low compared to the fifth-wheel roll stiffness before trailer separation, i.e. ($C_{fw2} \ll C_{fw1}$).

Note: In general, change in lateral acceleration during fifth-wheel lash (depending on fifth-wheel design, approx. 2 degree of fifth-wheel lash) is of the order of 0.05 g [26] and the roll stiffness of fifth-wheel can be simplified during fifth-wheel lash as

$$C_{fw2} \approx F_{fw} (h_{fw} - h_{rc, fw}) \left(1 + \frac{0.05}{0.035} \right) \approx 2.43 (h_{fw} - h_{rc, fw}) F_{fw}$$

5.2.3 Trailer Lash

During the load transfer between fifth-wheel edges, fifth-wheel synchronizes the roll motion between tractor and trailer units. At trailer separation (i.e. complete load transfer at fifth-wheel), trailer becomes free to roll at fifth-wheel which results in additional roll angle, defined as *trailer lash*. Due to lack of trailer frame flexibility, trailer sprung mass at fifth-wheel and trailer axle is connected rigidly, which means at trailer separation, trailer lash is the roll angle difference between trailer axle and trailer sprung mass roll angle at drive axle at trailer separation. The roll angle of trailer axle can be represented as a function of lateral acceleration as expressed in equation (5.25).

$$\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right] \phi_3 = \frac{a_{y1}}{g} \quad (5.25)$$

Trailer sprung mass at fifth-wheel (or drive axle) can be considered to roll due to influence of drive axle suspensions only at trailer separation. Its roll angle can be represented as a function of lateral acceleration as expressed in equation (5.26).

$$\left[\frac{C_{sg2}}{F_{fw} h_{fw}} - 1 \right] \phi_{sg,tl} = \frac{a_{y1}}{g} \quad (5.26)$$

Trailer lash can be determined as difference of two roll angles expressed in equations (5.25) and (5.26).

$$\begin{aligned} \phi_{tl,lash} &= \phi_3 - \phi_{sg,tl} \\ &= \left(\frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} \frac{a_{y1}}{g} \right) - \left(\frac{1}{\left[\frac{C_{sg2}}{F_{fw} h_{fw}} - 1 \right]} \frac{a_{y1}}{g} \right) \\ \phi_{tl,lash} &= \left(\frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} - \frac{1}{\left[\frac{C_{sg2}}{F_{fw} h_{fw}} - 1 \right]} \right) \frac{a_{y1}}{g} \end{aligned} \quad (5.27)$$

Therefore, roll angle due to trailer lash can be determined as presented in equation (5.27) as a function of lateral acceleration at trailer separation.

Theoretically trailer lash should occur instantaneously but in reality, it is also a function of time which means there will a range lateral acceleration during which trailer lash will occur. However, in RCV method, it has been considered to occur instantaneously with slope ' X_{lash} ' for simplicity. The slope can be determined as expressed in equation (5.28).

$$X_{lash} = \left(\frac{1}{\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right]} - \frac{1}{\left[\frac{C_{sg2}}{F_{fw} h_{fw}} - 1 \right]} \right) \quad (5.28)$$

vehicle. The roll moment equilibrium for complete vehicle as one unit can be expressed as equation (5.30).

$$\begin{aligned} C_{resT} \Delta\phi_T &= m_T g h_{cgT} \Delta\phi_T + m_T \Delta a_y h_{cgT} \\ [C_{resT} - m_T g h_{cgT}] \Delta\phi_T &= m_T g h_{cgT} \Delta a_y \end{aligned} \quad (5.30)$$

$$\left[\frac{C_{resT}}{m_T g h_{cgT}} - 1 \right] \Delta\phi_T = \frac{\Delta a_y}{g}$$

$$\Delta\phi_T = \left(\frac{1}{\left[\frac{C_{resT}}{m_T g h_{cgT}} - 1 \right]} \right) \frac{\Delta a_y}{g} \quad (5.31)$$

where,

- $\Delta\phi_T$ - change in total vehicle roll angle after fifth-wheel back-lash, [rad]
- C_{resT} - total resultant roll stiffness of the vehicle combination including the fifth-wheel, [kNm/rad]
- m_T - total vehicle combination mass, [kg]
- h_{cgT} - total vehicle center of gravity height above ground level, [m]
- $\frac{\Delta a_y}{g}$ - change in lateral acceleration after fifth-wheel back-lash, [g]

Equation (5.31) represents the change in roll angle of complete vehicle combination due to change in lateral acceleration after fifth-wheel lash, i.e. after fifth-wheel lash, roll angle of all the axle groups within vehicle combination will change according to equation (5.31) due to change in lateral acceleration.

Fifth-wheel characteristics, which also represents the concept of fifth-wheel formulation and modeling are presented in Figure 5.8.

5.3 Frame Torsional Compliance

During rollover, substantial torsional flexibility of tractor and trailer frames have been observed. Such torsional compliance serves to reduce the rollover stability of vehicle combination and therefore, it is important to account for frame flexibility. There have been various modeling approaches adapted by researchers to account for frame torsional compliance. Mcnaull [34] has modeled frame compliance as a spring and damper system connected longitudinally between two supporting edges of body. Eric [22] has also adapted the same modeling approach with a difference of connecting multiple torsional springs rather than connecting one spring and damper system between two supporting edges of body. The tractor and trailer sprung masses

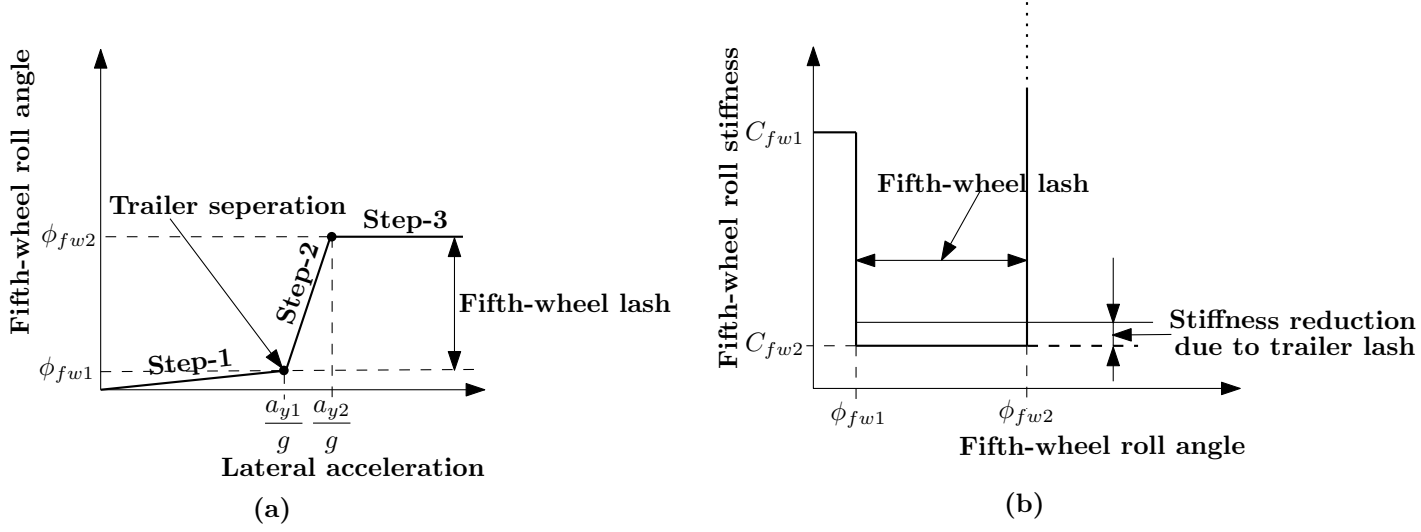


Figure 5.8: *Fifth-wheel characteristics: (a) Fifth-wheel roll angle versus lateral acceleration; (b) Fifth-wheel roll stiffness versus fifth-wheel roll angle*

are discretized into multiple sprung masses, each containing corresponding sprung mass. The two adjacent discretized sprung masses are then connected with torsional springs.

It has been suggested by Lawson [27] that torsional stiffness and weight distribution of tractor and trailer frames are unknown functions and vary along the length of the frame, therefore, discretized method is better for approximating the actual tractor and trailer system. However, the accuracy of this method highly depends on number and location of the discretion points, which needs to be determined based on the experimental data.

In present study, a simple methodology similar to Mcnaull [34] approach is adapted and *frame flexibility* is defined as the maximum roll motion (i.e. roll angle) of tractor or trailer unit at one axle/axle group relative to another axle/axle group when the unit frame is supported by these two axle/axle groups. These two axle groups can be considered to be connected by a torsional spring. A schematic representation of trailer frame is presented in Figure 5.9.

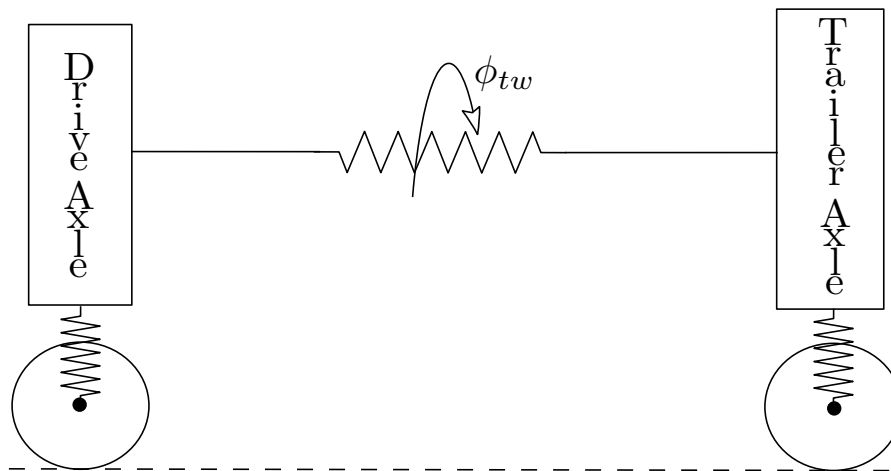


Figure 5.9: *Schematic representation of trailer frame supported by drive and trailer axle*

Figure 5.9 represents trailer frame as a mass less torsional spring, having stiffness in roll direction and trailer sprung mass is divided correspondingly into drive and trailer sprung masses.

Drive and trailer axles generally have different roll properties which depends on the various parameters such as vertical load on axle, height of the center of gravity, suspension and tire compliance, which results in different behavior of two axles with respect to lateral acceleration. If trailer axle is considered more stiffer than drive axle, which means trailer axle will lift-off first from ground; trailer frame twist angle ϕ_{tw} can be determined as difference between maximum trailer axle roll angle $\phi_{3,max}$ and drive axle roll angle ϕ_2 at corresponding lateral acceleration, i.e.

$$\phi_{tw} = (\phi_{3,max} - \phi_2)$$

Trailer axle maximum roll angle⁴ can be determined by the equilibrium of roll moments, i.e. restoring moment due to load transfer at axle and overturning moment due to lateral acceleration and cog lateral shift. The overturning moment is also equal to resultant roll stiffness times maximum roll angle at an axle. The roll moment equilibrium is expressed in equation (5.32).

$$m_3 g \frac{l_{t3}}{2} = C_{res3} \phi_{3,max} \quad (5.32)$$

Therefore, maximum trailer axle roll angle is,

$$\phi_{3,max} = \frac{m_3 g l_{t3}}{2C_{res3}} \quad (5.33)$$

Lateral acceleration at trailer axle wheel lift-off can be determined by the relation between trailer axle roll angle and lateral acceleration as described in equation (5.20), viz.

$$\left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right] \phi_3 = \frac{a_y}{g}$$

By substituting the value for maximum trailer axle roll angle, corresponding lateral acceleration can be determined.

$$\begin{aligned} \left[\frac{C_{res3}}{m_3 g h_{cg3}} - 1 \right] \frac{m_3 g l_{t3}}{2C_{res3}} &= \frac{a_y}{g} \\ \frac{a_y}{g} &= \frac{l_{t3}}{2h_{cg3}} - \frac{m_3 g l_{t3}}{2C_{res3}} \end{aligned} \quad (5.34)$$

Lateral acceleration expressed in equation (5.34) represents trailer axle wheel lift-off. Drive axle roll angle at this lateral acceleration can be determined from the relation between drive axle roll angle and lateral acceleration as described in equation (5.8), viz.

$$\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right] \phi_2 = \frac{a_y}{g}$$

⁴when trailer axle wheel lifts-off from ground

$$\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right] \phi_2 = \frac{l_{t3}}{2h_{cg3}} - \frac{m_3 g l_{t3}}{2C_{res3}}$$

The drive axle roll angle at the time of trailer axle maximum roll angle is,

$$\phi_2 = \frac{\left[\frac{l_{t3}}{2h_{cg3}} - \frac{m_3 g l_{t3}}{2C_{res3}} \right]}{\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right]} \quad (5.35)$$

The trailer frame twist angle can be determined by simply negation of equation (5.33) and (5.35).

$$\begin{aligned} \phi_{tw} &= (\phi_{3,max} - \phi_2) \\ \phi_{tw} &= \left[\frac{m_3 g l_{t3}}{2C_{res3}} - \frac{\left[\frac{l_{t3}}{2h_{cg3}} - \frac{m_3 g l_{t3}}{2C_{res3}} \right]}{\left[\frac{C_{res2}}{m_2 g h_{cg2}} - 1 \right]} \right] \end{aligned} \quad (5.36)$$

It is uncertain whether frame twist angle represented by equation (5.36) is maximum or not because once trailer axle has lifted off from ground, it continues to roll and frame twist angle will depend on whether trailer axle rolls with same rate as drive axle after lift-off or not. But in both cases either this frame twist angle is maximum or not, roll stiffness of the frame can be determined. The trailer frame roll moment can be considered equal to the difference of trailer and drive axle overturning moments.

$$C_{tw} \phi_{tw} = C_{res3} \phi_{3,max} - C_{res2} \phi_2$$

$$C_{tw} = \left(\frac{C_{res3} \phi_{3,max} - C_{res2} \phi_2}{\phi_{tw}} \right) \quad (5.37)$$

It is of utmost importance to indicate that frame torsional stiffness (or trailer frame roll stiffness) expressed in equation (5.37) is only valid when there is no fifth-wheel lash and trailer lash, which in reality not possible for roll coupled units. Therefore, the best approach to include fifth-wheel and trailer frame compliance is to know at least roll stiffness due to one compliance in advance and then determine the other roll stiffness by considering the influence of former compliance.

6 RCV Calculation Method

This chapter presents the complete approach of estimation of SRT for single vehicle units and vehicle combination. Lateral force is presented as a function of known input static parameters by solving a quadratic equation. Lateral shift of cog is presented as a function of roll angle due to suspension, tire and fifth-wheel compliance for vehicle/vehicle combination.

Steady State Rollover Threshold (SRT) can be defined as the lateral acceleration at or beyond which vehicle/vehicle combination losses its roll stability and rollover occurs as the ultimate consequence during steady state condition.

The exact location of arrival of roll instability is not clearly defined, as some researchers determine SRT when all axles have lifted off from one side and some determine at the first axle lift off. Both approaches lead to an significant error in SRT determination compared to actual one, former approach is of non-conservative nature and latter is of conservative nature. However UNECE111 regulation determine SRT by linearly interpolating between these two extreme approaches, which obviously better than adapting the former approaches. One thing remains unclear is, at what point roll instability occurs as in physical sense.

To identify this roll instability point, current study defines, SRT as the level of lateral acceleration required to lift more than 50% axle groups/bogies from one side in vehicle/vehicle combinations. In case of three axle group vehicle, SRT can be identified when one side of at least two axle groups lift-off from ground.

According to RCV method, a vehicle/vehicle combination can be perceived as the number of axles with their respective sprung and un-sprung mass and combined vehicle motion is represented by the ability of relative motion between these axles depending on either unit frame torsional flexibility or fifth-wheel coupling between two units. To identify SRT, first axle roll stability needs to be determined, i.e. the roll angle at which respective axle lifts-off from ground.

6.1 Roll Angle of Axle at Lift-off

As described in Chapter 3, an axle is the combination of sprung and un-sprung masses. The roll motion of sprung mass is often interpreted as the suspension compliance in roll direction and un-sprung mass roll motion is interpreted as tire compliance in roll direction. The effects of both of these compliance on roll motion have been discussed in Chapter 3. Roll stiffness due to suspension compliance about axle roll center as expressed in equation (6.1).

$$C_{sg,i} = C_{s,i} \left(\frac{h_{s,i}}{h_{s,i} - h_{rc,i}} \right)^2 \left[1 - \frac{m_{s,i} g h_{rc,i} (h_{s,i} - h_{rc,i})}{C_{s,i} h_{s,i}} \right] \quad (6.1)$$

and roll stiffness due to tire compliance about axle roll center as expressed in equation (6.2).

$$C_{al,i} = h_{cg,i} \left[m_i g + C_{tv,i} l_{teff,i} \frac{F_{y,i}}{m_i g} \right] \quad (6.2)$$

The two compliance effect can be considered to be acting in series at an axle, the resultant effect of these compliance can be determined at ground level by considering two torsional

spring in series with stiffness equal to roll stiffness due to suspension and tire compliance. The resultant roll stiffness can then be determined as expressed in equation (6.3):

$$C_{res,i} = \frac{C_{sg,i} C_{al,i}}{C_{sg,i} + C_{al,i}} \quad (6.3)$$

In other words, if an imaginary torsional spring with roll stiffness equivalent to stiffness expressed by equation (6.3) can be considered at ground level at each axle, then, each axle (which originally has two roll degrees of freedom) can be transformed into single roll degree of freedom model.

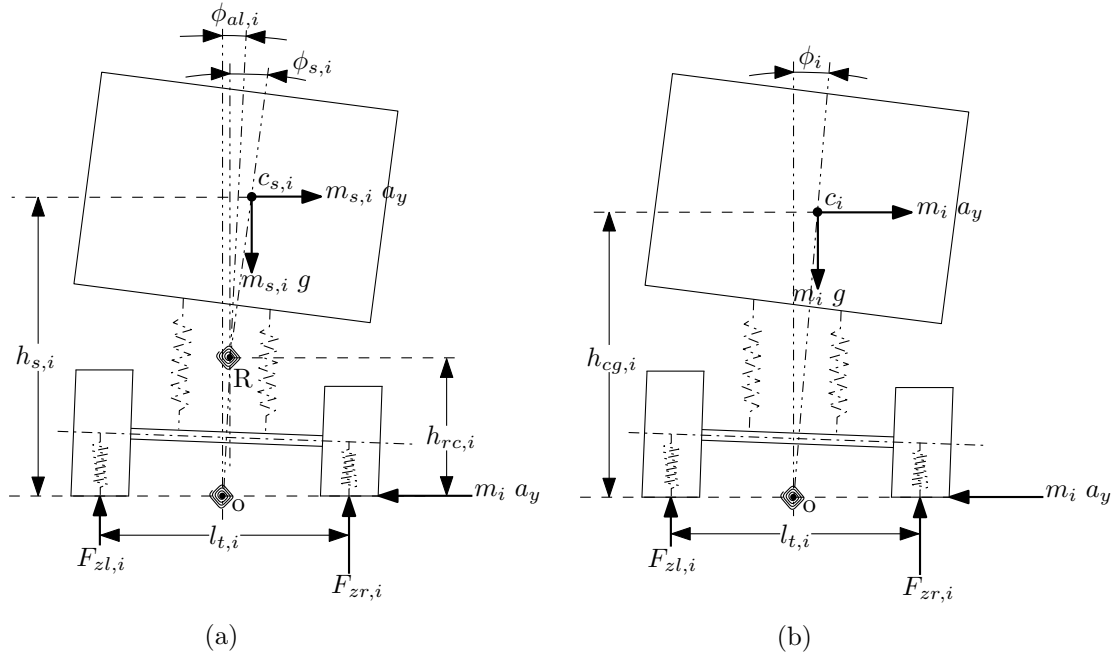


Figure 6.1: Schematic representation of an axle: (a) two degree of roll freedom model; (b) equivalent one degree of roll freedom model

Figure 6.1(a) represents one degree of roll freedom due to sprung mass motion about suspension roll center 'R' and second degree of roll freedom due to tire compliance about axle roll center 'o' at ground level. Compliance due to suspension and tire elements are indicated by two torsional springs with stiffness $C_{s,i}$ and $C_{al,i}$ at respective roll centers. Figure 6.1(b) represents only one degree of roll freedom about point 'o' at ground level due to the combined effect of suspension and tire compliance. The resultant compliance is indicated by a torsional spring with stiffness $C_{res,i}$ at ground level.

By considering a torsional spring with stiffness equivalent to $C_{res,i}$ at ground level provides an additional equation to relate the restoring and overturning moments separately through moment generated by this torsional spring, i.e. the torsional moment of the spring can be considered equal to overturning moments due to lateral acceleration and lateral shift of cog at an axle and also to the restoring moment due to lateral load transfer at an axle. The maximum roll angle of axle at which axle wheels lift-off can then be determined by the moment equilibrium between torsional spring moment and maximum possible restoring moment due to maximum possible load transfer, viz. $\max.(F_{zr} - F_{zl}) = m_i g$

$$C_{res,i} \phi_i = \max.(F_{zr} - F_{zl}) \frac{l_{teff,i}}{2}$$

$$\phi_i = \frac{m_i g l_{teff,i}}{2 C_{res,i}} \quad (6.4)$$

Equation (6.4) represents the maximum roll angle, that an axle can sustain due to its suspension and tire compliance before it lifts off from ground. The lateral acceleration required to lift off the axle from ground can be determined by equating overturning moments to torsional spring moment.

$$\begin{aligned} C_{res,i} \phi_i &= m_i g h_{cg,i} \phi_i + m_i a_y h_{cg,i} \\ [C_{res,i} - m_i g h_{cg,i}] \phi_i &= m_i a_y h_{cg,i} \end{aligned}$$

$$\left[\frac{C_{res,i}}{m_i g h_{cg,i}} - 1 \right] \phi_i = \frac{a_y}{g} \quad (6.5)$$

Equation (6.5) represents the relation between roll angle and lateral acceleration at an axle. As expressed, the relation between axle roll angle and lateral acceleration is a straight line and slope of the line depends on resultant roll stiffness, total mass and height of cog at an axle. Also, lateral acceleration at wheel lift-off at an axle can be determined by substituting maximum possible roll angle from equation (6.4) into equation (6.5).

6.2 Determination of Lateral Force at Each Axle

As expressed in equation (6.2), roll stiffness due to tire compliance is a function of lateral force and consequently resultant roll stiffness at an axle is also a function of lateral force. Since, in RCV method, lateral force can not be used as input parameter because of its dynamic nature and due to lack of other dynamics input parameters, such as vehicle speed, time, tire slips etc., which are generally used to estimate lateral force. It is essential to determine lateral force in terms of other static parameters, one possible way is to use Newton's second law, i.e. lateral force is equal to product of mass and its lateral acceleration at an axle.

$$F_{y,i} = m_i a_y$$

Equation (6.5) can be rewritten by replacing lateral acceleration to lateral force and roll angle can be replaced from equation (6.4),

$$\begin{aligned}
\left[\frac{C_{res,i}}{m_i g h_{cg,i}} - 1 \right] \frac{m_i g l_{teff,i}}{2 C_{res,i}} &= \frac{F_{y,i}}{m_i g} \\
\frac{l_{teff,i}}{2 h_{cg,i}} - \frac{m_i g l_{teff,i}}{2 C_{res,i}} &= \frac{F_{y,i}}{m_i g} \\
\frac{l_{teff,i}}{2 h_{cg,i}} - \frac{F_{y,i}}{m_i g} &= \frac{m_i g l_{teff,i}}{2 C_{res,i}} \\
C_{res,i} &= \left[\frac{m_i g l_{teff,i}}{2 \left(\frac{l_{teff,i}}{2 h_{cg,i}} - \frac{F_{y,i}}{m_i g} \right)} \right]
\end{aligned} \tag{6.6}$$

Equation (6.6), represents one relation between lateral force and resultant roll stiffness at an axle. Another relation can be obtained from equation (6.3), as resultant roll stiffness depends on roll stiffness due to tire compliance which in itself is a function of lateral force. Therefore,

$$C_{res,i} = \left[\frac{C_{sg,i} h_{cg,i} \left(m_i g + C_{tv,i} l_{teff,i} \frac{F_{y,i}}{m_i g} \right)}{C_{sg,i} + h_{cg,i} \left(m_i g + C_{tv,i} l_{teff,i} \frac{F_{y,i}}{m_i g} \right)} \right] \tag{6.7}$$

By equating equations (6.6) and (6.7), lateral force can be expressed only in static parameters, which can be used as inputs for the RCV method.

$$\left[\frac{m_i g l_{teff,i}}{2 \left(\frac{l_{teff,i}}{2 h_{cg,i}} - \frac{F_{y,i}}{m_i g} \right)} \right] = \left[\frac{C_{sg,i} h_{cg,i} \left(m_i g + C_{tv,i} l_{teff,i} \frac{F_{y,i}}{m_i g} \right)}{C_{sg,i} + h_{cg,i} \left(m_i g + C_{tv,i} l_{teff,i} \frac{F_{y,i}}{m_i g} \right)} \right]$$

The effective track width also depends on lateral force, by substituting it as a function of lateral force will result in fifth order equation. Therefore, for simplicity, instead of effective track width, nominal track width is used, which result a quadratic equation of lateral force as expressed in equation (6.8).

$$\left[\frac{2 C_{sg,i} C_{tv,i} h_{cg,i} l_{t,i}}{(m_i g)^2} \right] (F_{y,i})^2 + \left[C_{tv,i} h_{cg,i} (l_{t,i})^2 + 2 C_{sg,i} h_{cg,i} - \frac{C_{sg,i} C_{tv,i} (l_{t,i})^2}{m_i g} \right] F_{y,i} + (m_i g)^2 h_{cg,i} l_{t,i} = 0 \tag{6.8}$$

In RCV method, first lateral force is determined at each axle by solving equation (6.8) with Shri Dharacharya method. To account for change in axle track width due to tire compliance, effective track width can be calculated using this lateral force and tire lateral stiffness as

$$l_{teff,i} = (l_{t,i} - \frac{F_{y,i}}{C_{ty}})$$

and then the resultant roll stiffness and maximum roll angle at each axle can be calculated by incorporating corresponding lateral force and effective track width value.

6.3 Estimation of Steady State Rollover Threshold

SRT can be identified, when overturning moments (due to lateral acceleration and lateral shift of cog) become governing with respect to restoring moment (due to lateral load transfer). In RCV method, due to respective discretion of total mass at axles, SRT would rather become simple to identify if it is considered equal to the lateral acceleration required to lift-off more than 50% axles from ground. The equilibrium of moments at an axle at wheel lift-off is expressed in equation (6.9).

$$m_i g \frac{l_{teff,i}}{2} = m_i g h_{cg,i} \phi_i + m_i a_{y,i} h_{cg,i} \quad (6.9)$$

By replacing axle roll angle ϕ_i in equation (6.9) with maximum roll angle at an axle using equation (6.4), lateral acceleration at an axle can be determined. However, this approach will result in overestimation due to the fact that without considering frame or chassis torsional compliance, the sprung mass will have same roll angle at each axle. The simplification of mass discretion at each axle will not hold valid. The concept of maximum axle roll angle is only good for determining the order of axle lift-off but not for estimating the lateral acceleration at which axles actually lift-off. Therefore, it is necessary to determine either the actual roll angle at each axle or effect of lateral shift of vehicle cog.

6.3.1 Lateral Shift of Center of Gravity

Lateral shift of COG can be seen as the phenomenon which differentiates compliant vehicle from rigid vehicle. In other words, COG's lateral shift is the effect of vehicle compliances and the moment due to lateral shift of COG provides the reduction in restoring moment which reduces SRT. Therefore, it is important for better estimation of SRT, this phenomenon is accounted properly.

In a compliant vehicle, total roll angle is the contribution from axle and tire compliance and suspension compliance. Roll angle due to tire compliance is estimated at ground level being total axle mass located at actual COG height so roll angle contribution of tire compliance can be considered as it is. However, suspension roll angle contribution needs to be corrected due to the fact that roll angle of sprung mass will be different from vehicle mass because of its height and magnitude.

As illustrated in Figure 6.2, axle sprung mass which is located at sprung mass COG ' $c_{s,i}$ ' rolls about point 'o' at ground level with roll angle $\phi_{sg,i}$ with equivalent suspension roll stiffness of $C_{sg,i}$ at an axle. The total axle mass located at COG ' c_i ' will roll about point 'o' at ground level with different roll angle ϕ_{is} due to equivalent suspension roll stiffness of $C'_{sg,i}$ at an axle. Since, both masses are rolling about same point due to same roll stiffness in a single roll plane, hence, can be considered in a roll equilibrium. The moments due to lateral shift of COG and lateral acceleration can then be considered equivalent to torsional spring moment for respective masses, which can be represented in terms of moment equilibrium equation for both masses.

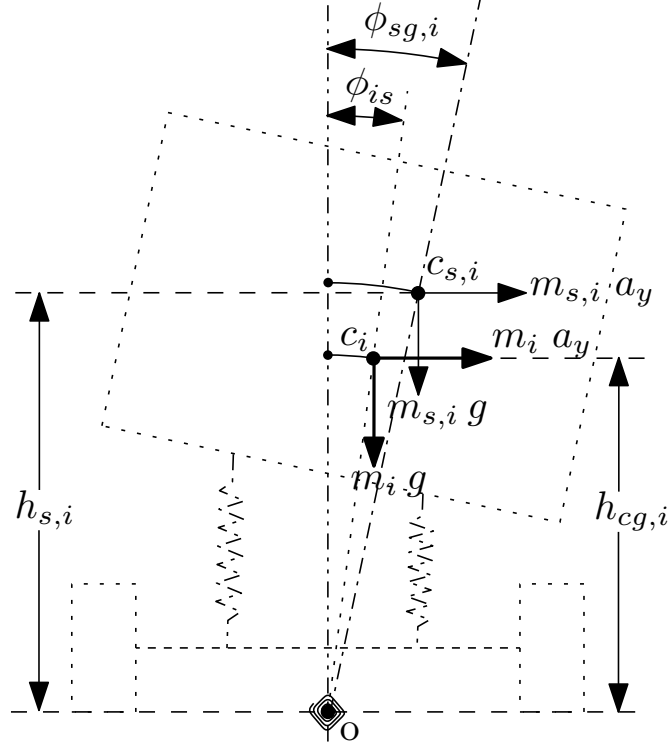


Figure 6.2: Schematic representation of roll angle of axle COG due to suspension compliance

Sprung mass:

$$C_{sg,i} \phi_{sg,i} = m_{s,i} g h_{s,i} \phi_{sg,i} + m_{s,i} a_y h_{s,i}$$

$$\phi_{sg,i} = \frac{\frac{a_y}{g}}{\left[\frac{C_{sg,i}}{m_{s,i} g h_{s,i}} - 1 \right]} \quad (6.10)$$

Total mass:

$$\begin{aligned} C_{sg,i} \phi_{sg,i} &= m_i g h_{cg,i} \phi_{is} + m_i a_y h_{cg,i} \\ \phi_{is} &= \left(\frac{C_{sg,i} \phi_{sg,i}}{m_i g h_{cg,i}} - \frac{a_y}{g} \right) \end{aligned} \quad (6.11)$$

The actual total COG roll angle due to contribution of suspension compliance can be determined by substituting sprung mass roll angle from equation (6.10) to equation (6.11).

$$\phi_{is} = \left[\frac{C_{sg,i}}{m_i g h_{cg,i}} \frac{1}{\left[\frac{C_{sg,i}}{m_{s,i} g h_{s,i}} - 1 \right]} - 1 \right] \frac{a_y}{g}$$

The roll angle contribution of sprung mass (suspension compliance) in roll angle of total COG at an axle can be expressed as equation (6.12).

$$\phi_{is} = \left[\frac{C_{sg,i} m_{s,i} h_{s,i}}{m_i h_{cg,i} (C_{sg,i} - m_{s,i} g h_{s,i})} - 1 \right] \frac{a_y}{g} \quad (6.12)$$

Therefore, effective roll angle due to tire and suspension compliance at an axle can be obtained by simply adding the roll angle contribution of tire and suspension compliance.

$$\phi_{eff,i} = \phi_{al,i} + \phi_{is}$$

$$\phi_{eff,i} = \left(\frac{1}{\left[\frac{C_{al,i}}{m_i g h_{cg,i}} - 1 \right]} + \left[\frac{C_{sg,i} m_{s,i} h_{s,i}}{m_i h_{cg,i} (C_{sg,i} - m_{s,i} g h_{s,i})} - 1 \right] \right) \frac{a_y}{g}$$

By further simplification, effective roll angle can be expressed as equation (6.13).

$$\phi_{eff,i} = \left(\frac{m_i g h_{cg,i}}{(C_{al,i} - m_i g h_{cg,i})} + \frac{C_{sg,i} m_{s,i} h_{s,i}}{m_i h_{cg,i} (C_{sg,i} - m_{s,i} g h_{s,i})} - 1 \right) \frac{a_y}{g} \quad (6.13)$$

The moment due to lateral shift of COG can then be expressed as

$$m_i g h_{cg,i} \phi_{eff,i}$$

6.3.2 SRT for Single Vehicle Units

Although a vehicle can be seen as combination of axles with their corresponding discrete masses and independent roll motions, but in reality roll motion of all axles are interconnected through sprung mass. The discretion of corresponding sprung mass at each axle is just an approach to analyze roll motion of vehicle which otherwise be rather complex.

The better approach for estimating vehicle's SRT would be consider a vehicle as single axle with parameters equivalent to all axles lumped together.

Maximum Lateral Acceleration

Maximum lateral acceleration for a vehicle can be defined when all the wheels at one side lifts-off from ground. To estimate this, equivalent parameters for vehicle as single axle can be determined as follows:

Total vehicle mass;	$m_T = \sum_{i=1}^n m_i$
Total vehicle sprung mass;	$m_{sT} = \sum_{i=1}^n m_{s,i}$
Total track-width ;	$l_{tT} = \frac{\sum_{i=1}^n m_i l_{teff,i}}{m_T}$
Total COG height ;	$h_{cgT} = \frac{\sum_{i=1}^n m_i h_{cg,i}}{m_T}$
Total sprung mass COG height ;	$h_{sT} = \frac{\sum_{i=1}^n m_{s,i} h_{s,i}}{m_{sT}}$
Total axle roll stiffness;	$C_{alT} = \sum_{i=1}^n C_{al,i}$
Total suspension roll stiffness;	$C_{sgT} = \sum_{i=1}^n C_{sg,i}$
Total resultant roll stiffness;	$C_{resT} = \sum_{i=1}^n C_{res,i}$

The effective roll angle can be determined using equation (6.13) with corresponding equivalent parameters as follows:

$$\phi_{effT} = \left(\frac{m_T g h_{cgT}}{(C_{alT} - m_T g h_{cgT})} + \frac{C_{sgT} m_{sT} h_{sT}}{m_T h_{cgT} (C_{sgT} - m_{sT} g h_{sT})} - 1 \right) \frac{a_{yT}}{g}$$

$$\phi_{effT} = X_T \frac{a_{yT}}{g}$$

‘ X_T ’ represents the linear relation between effective roll angle and lateral acceleration for complete vehicle as single lumped axle, which depends on various vehicle parameters. Roll moment equilibrium for the vehicle can be expressed as equation (6.14).

$$m_T g \frac{l_{tT}}{2} = m_T g h_{cgT} \left(\phi_{effT} + \frac{a_{yT}}{g} \right) \quad (6.14)$$

Since, effective roll angle is a function of lateral acceleration, therefore, maximum lateral acceleration at which one side of vehicle lifts-off can be expressed as equation (6.15).

$$\frac{a_{yT}}{g} = \frac{l_{tT}}{2 h_{cgT} (1 + X_T)} \quad (6.15)$$

Although at this maximum lateral acceleration, rollover of a vehicle is certain. But it can't be considered as SRT because vehicle rolls over even before this lateral acceleration is achieved.

Lateral Acceleration at Stiffest Axle

The stiffest axle means the first axle of vehicle which lifts-off from ground at a certain lateral acceleration. This lateral acceleration is the minimum acceleration at which vehicle starts to rollover. It can be determined using the same approach as maximum lateral acceleration but with parameters corresponding to stiffest axle and by utilizing a stiffness factor. The stiffness factor is used to represent the vehicle as single axle but corresponding to stiffest one.

The *stiffness factor* can be defined as ratio of resultant roll stiffness of stiffest axle (C_{resS}) to resultant roll stiffness of complete vehicle (C_{resT}).

$$\text{Stiffness factor; } s_f = \frac{C_{resS}}{C_{resT}}$$

Using this stiffness factor, the one axle which represents the complete vehicle as in case of maximum lateral acceleration can now be replaced as stiffest axle. The effective roll angle for stiffest axle can then be determined as expressed in equation (6.16).

$$\begin{aligned} \phi_{effS} &= \left(\frac{s_f m_T g h_{cgT}}{(C_{atS} - s_f m_T g h_{cgT})} + \frac{C_{sgS} m_{sT} h_{sT}}{m_T h_{cgT} (C_{sgS} - s_f m_{sT} g h_{sT})} - 1 \right) \frac{a_{yS}}{g} \\ \phi_{effS} &= X_S \frac{a_{yS}}{g} \end{aligned} \quad (6.16)$$

‘ X_S ’ represents the relation between effective roll angle and lateral acceleration for vehicle as single stiffest axle. Roll moment equilibrium for the vehicle can be expressed as equation (6.17).

$$m_S g \frac{l_{tS}}{2} = s_f m_T g h_{cgT} \left(\phi_{effS} + \frac{a_{yS}}{g} \right) \quad (6.17)$$

The effective roll angle for stiffest axle is also a function of lateral acceleration as expressed in equation (6.16). Therefore, lateral acceleration for which respective stiffest axle in a vehicle combination will lift-off from ground can be determined as expressed in equation (6.18).

$$\frac{a_{yS}}{g} = \frac{m_S l_{tS}}{s_f m_T h_{cgT} (1 + X_S)} \quad (6.18)$$

Steady State Rollover Threshold

The lateral acceleration expressed by equations (6.15) and (6.18) are the two cases which defines the boundary of rollover. In other words, a vehicle crosses over from stable roll to unstable roll motion between these two lateral acceleration values, which indicates actual SRT lies between these two lateral acceleration values.

Since, it is difficult to pin point the exact moment at which vehicle becomes unstable in roll motion. Therefore, for simplicity, in RCV method, the point of unstable roll motion is considered to arrive when more than 50% axles lift-off from ground in a vehicle. Using this simplification, SRT can be determined by linear interpolation between maximum lateral

acceleration and lateral acceleration at stiffest axle using their respective masses. It is expressed as follows:

$$\begin{pmatrix} \frac{a_{yT}}{g} - \frac{a_{yC}}{g} \\ \frac{a_{yT}}{g} - \frac{a_{yS}}{g} \end{pmatrix} = \begin{pmatrix} m_T - m_C \\ m_T - m_S \end{pmatrix}$$

Therefore, steady state rollover threshold for a vehicle can be determined as expressed in equation (6.19).

$$SRT = \frac{a_{yC}}{g} = \frac{a_{yT}}{g} - \left(\frac{a_{yT}}{g} - \frac{a_{yS}}{g} \right) \left(\frac{m_T - m_C}{m_T - m_S} \right) \quad (6.19)$$

‘ m_C ’ corresponds to the axle mass which lifts-off when 50% axles in a vehicle has already lifted-off from ground. During the verification of RCV method, it was observed that for vehicles in which m_C is greater than m_S , equation (6.19) holds true but in case of m_C being less than m_S , SRT obtained indicates significant error. Therefore, initial attempt to determine the physical location of SRT can’t be generalized and hence equation (6.19) is discarded further in study.

However, a general form of SRT can be obtained by simple linear interpolation between the minimum and maximum lateral acceleration using similar approach but with an assumption of

$$\left(\frac{m_T - m_C}{m_T - m_S} \right) \approx \frac{m_S}{m_T}$$

Though in reality, this assumption is only valid when m_S being very small compared to m_C and m_T . By implementing this assumption in equation (6.19), SRT can be expressed as equation (6.20), which is used in RCV method for SRT estimation.

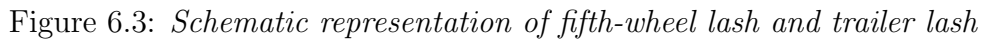
$$SRT = \frac{a_{yT}}{g} - \left(\frac{a_{yT}}{g} - \frac{a_{yS}}{g} \right) \frac{m_S}{m_T} \quad (6.20)$$

6.3.3 SRT for Vehicle Combinations

The SRT expressed by equation (6.20) is only valid for single vehicle units. In general, HCT vehicles are combination of more than one units and the two units are connected either via fifth-wheel or draw-bar. Draw-bar coupling between two units doesn’t influence the units in roll direction, i.e. it doesn’t provide any roll compliance and the units can be considered to be rolling independently with respect to each other. SRT for such vehicle combinations (eg. A-double etc.) can be determined by selecting the minimum SRT value among individual single vehicle units when estimated independently.

However, fifth-wheel couple the two units even in roll direction and they can not be treated separately, i.e. all units in a vehicle combination will influence each other in roll direction. Therefore, such vehicle combinations (eg. B-double etc.) have to be considered as one unit. The fifth-wheel coupling provides an additional roll compliance which results in further reduction of SRT. The compliance due to fifth-wheel lash and trailer lash has been discussed in detail in Chapter 4.

In RCV method, the compliance due to fifth-wheel lash and trailer lash are interpreted as change in effective roll angle, which influence the moment due to lateral shift of COG. By this consideration vehicle combination can be treated as single axle similar to single vehicle unit approach but the relation between effective roll angle and lateral acceleration will change, resulting in respective change in SRT.



As described in Chapter 4, the equivalent roll stiffness due to these effects can be determined as expressed in equation (6.21).

where, X_{lash} represents the instantaneous slope of trailer lash versus lateral acceleration

curve. The total effect of fifth-wheel coupling is the combination of these two effects, therefore, a moment equilibrium can be established between torsional spring moment and overturning moments about fifth-wheel roll center ' R_{fw} ', viz.

$$C_{res,fw} \phi_{fw} = F_{fw} (h_{fw} - h_{rc,fw}) \phi_{fw} + F_{fw} (h_{fw} - h_{rc,fw}) \frac{a_y}{g}$$

$$\phi_{fw} = \frac{\frac{a_y}{g}}{\left[\frac{C_{res,fw}}{F_{fw} (h_{fw} - h_{rc,fw})} - 1 \right]} \quad (6.22)$$

Therefore, the effective roll angle in case of vehicle combination with fifth-wheel coupling can be determined using equation (6.22) as

$$\phi_{eff,i} = \phi_{al,i} + \phi_{is} + \phi_{fw}$$

$$\phi_{eff,i} = \left(\frac{m_i g h_{cg,i}}{(C_{al,i} - m_i g h_{cg,i})} + \frac{C_{sg,i} m_{s,i} h_{s,i}}{m_i h_{cg,i} (C_{sg,i} - m_{s,i} g h_{s,i})} - 1 \right) \frac{a_y}{g} + \frac{1}{\left[\frac{C_{res,fw}}{F_{fw} (h_{fw} - h_{rc,fw})} - 1 \right]} \frac{a_y}{g}$$

By further simplification, effective roll angle can be represented as function of lateral acceleration and SRT for vehicle combination can be determined by similar approach as single vehicle units. The effective roll angle for complete vehicle combination considered as lumped single axle can be expressed as equation (6.23). The equivalent parameters can be determined similar to single vehicle units.

$$\begin{aligned} \phi_{effTfw} &= \left(X_T + \frac{F_{fw} (h_{fw} - h_{rc,fw})}{[C_{res,fw} - F_{fw} (h_{fw} - h_{rc,fw})]} \right) \frac{a_{yTfw}}{g} \\ \phi_{effTfw} &= X_{Tfw} \frac{a_{yTfw}}{g} \end{aligned} \quad (6.23)$$

Similarly, the effective roll angle for complete vehicle combination considered as stiffest single axle can be determined by equation (6.24).

$$\begin{aligned} \phi_{effSfw} &= \left(X_S + \frac{F_{fw} (h_{fw} - h_{rc,fw})}{[C_{res,fw} - F_{fw} (h_{fw} - h_{rc,fw})]} \right) \frac{a_{ySfw}}{g} \\ \phi_{effSfw} &= X_{Sfw} \frac{a_{ySfw}}{g} \end{aligned} \quad (6.24)$$

X_{Tfw} and X_{Sfw} represents the relation between effective roll angle and lateral acceleration when vehicle combination with fifth-wheel coupling is interpreted as single axle corresponding to complete vehicle and stiffest axle respectively. The maximum lateral acceleration required for lifting the one side of vehicle can be determined using equation (6.25).

$$\frac{a_{yTfw}}{g} = \frac{l_{tT}}{2 h_{cgT} (1 + X_{Tfw})} \quad (6.25)$$

The minimum lateral acceleration which is required to lift-off only stiffest axle in vehicle combination can be determined using equation (6.26).

$$\frac{a_{ySfw}}{g} = \frac{m_S l_{tS}}{s_f m_T h_{cgT} (1 + X_{Sfw})} \quad (6.26)$$

Therefore, SRT for any vehicle combination with fifth-wheel coupling can be determined using equation (6.27).

$$SRT = \frac{a_{yTfw}}{g} - \left(\frac{a_{yTfw}}{g} - \frac{a_{ySfw}}{g} \right) \frac{m_S}{m_T} \quad (6.27)$$

7 Results and Discussions

This chapter presents the verification of RCV method using roll moment diagram and VTM models in comparison with UNECE111 method. The RCV method is also validated against tilt table tests for single vehicle units. The RCV method is also compared with VTM models for various vehicle combinations.

The proposed RCV method is a numerical calculation method which considers the various vehicle compliance due to suspension, tire and fifth-wheel. The SRT estimated by RCV method is less than the one estimated by UNECE111 method, which is a significant difference. The RCV method also indicates that more vehicle parameters influence the rollover of vehicle/vehicle combination than proposed by UNECE111. Before determining the difference of estimated SRT using RCV method with respect to UNECE111 method, RCV method is primarily verified using roll moment diagram (RMD) analysis.

7.1 Roll Moment Diagram

RMD is a analytical approach first introduced by Wilkins [40] and the graphical form used in current study is originated by Chalasani [42]. It can be used to study the roll stability of vehicle combinations. In other words, it is a graphical representation of roll moment equilibrium equation in a single roll plane, viz.

$$m_T h_{cgT} a_y = (F_{zrT} - F_{zlT}) \frac{l_{tT}}{2} - m_T g \phi_{effT}$$

This method is also based on the principle of considering complete vehicle as single lumped axle in roll plane. SRT can be identified as the lateral acceleration at which destabilizing moment due to lateral acceleration equals to the deference of maximum restoring moment due to lateral load transfer and moment due to lateral shift of cog.

This approach can also be applied for multiple axle vehicles. In such a case, the total moment due to lateral load transfer is the summation of load transfers at each axle but the moment due to lateral shift of cog remains same as in for a complete vehicle. In RMD, roll angle is indicated at positive x-axis in right hand side figure, and lateral acceleration is indicated at positive x-axis in right hand side figure and restoring and overturning moments are indicated at positive y-axis as shown in Figure 7.1.

Single Vehicle Units

This category of vehicles can be identified as single vehicle units with two or more number of axles but without any coupling (fifth-wheel or draw bar), i.e. these vehicles can be considered to have at least suspension and tire compliance and the parameters respective to only these compliance influence the roll behavior of vehicle. The reference vehicle (viz. Volvo FH12 6x2 with rigid superstructure) which is being used for verification of RCV method in this study.

The other vehicle parameters required for SRT calculation are lateral force, maximum roll angle at each axle and resultant roll stiffness at each axle, which can be estimated by RCV method as follows:

Individual Axle Calculations

Equivalent suspension roll stiffness at ground level can be determined using equation (7.1) for each axle as discussed in Chapter 3.

$$C_{sg,i} = C_{s,i} \left(\frac{h_{s,i}}{h_{s,i} - h_{rc,i}} \right)^2 \left[1 - \frac{m_{s,i} g h_{rc,i} (h_{s,i} - h_{rc,i})}{C_{s,i} h_{s,i}} \right] \quad (7.1)$$

Lateral force at each axle can be determined by solving equation (7.2).

$$\left[\frac{2 C_{sg,i} C_{tv,i} h_{cg,i} l_{t,i}}{(m_i g)^2} \right] (F_{y,i})^2 + \left[C_{tv,i} h_{cg,i} (l_{t,i})^2 + 2 C_{sg,i} h_{cg,i} - \frac{C_{sg,i} C_{tv,i} (l_{t,i})^2}{m_i g} \right] F_{y,i} + (m_i g)^2 h_{cg,i} l_{t,i} = 0 \quad (7.2)$$

Effective axle track-width and axle roll stiffness due to tire compliance can be determined using equations (7.3) and (7.4) as discussed in Chapter 3.

$$l_{teff,i} = \left(l_{t,i} - \frac{F_{y,i}}{C_{ty,i}} \right) \quad (7.3)$$

$$C_{al,i} = h_{cg,i} \left[m_i g + C_{tv,i} \frac{F_{y,i}}{m_i g} l_{teff,i} \right] \quad (7.4)$$

Resultant roll stiffness at each axle can be determined using equation (7.5).

$$C_{res,i} = \frac{C_{al,i} C_{sg,i}}{C_{al,i} + C_{sg,i}} \quad (7.5)$$

Maximum axle roll angle can be determined using equation (7.6) at which one side of wheels lift-off at each axle.

$$\phi_{max,i} = \frac{m_i g l_{teff,i}}{2 C_{res,i}} \quad (7.6)$$

The RCV method estimates individual axle parameters using equations (7.1) to (7.6), estimated parameters for referenced vehicle are presented in Table 7.1.

Complete Vehicle Calculation

The complete vehicle is considered as single axle in one roll plane in RMD analysis. The restoring moment due to lateral load transfer for complete vehicle can be obtained by summation of all the individual axle restoring moments. The moment due to lateral shift of cog can simply be estimated as $(m_T g h_{cgT} \phi_{effT})$, which will increase linearly with roll angle.

The net restoring moment is the difference of restoring moments due to lateral load transfer and moment due to lateral shift of cog. When the overturning moment due to lateral

Table 7.1: Rigid truck axle calculations

Estimated axle parameters	Axle 1	Axle 2	Axle 3
Axle roll stiffness [kNm/rad]	1596.6	2972.3	1734.4
Effective track width [m]	1.9033	1.7512	1.8861
Equivalent suspension roll stiffness [kNm/rad]	822.587	2833.4	1740
Lateral force [kN]	29.012	48.37	34.185
Maximum roll angle [rad]	0.1201	0.0671	0.0804
Resultant roll stiffness [kNm/rad]	542.887	1450.6	868.597

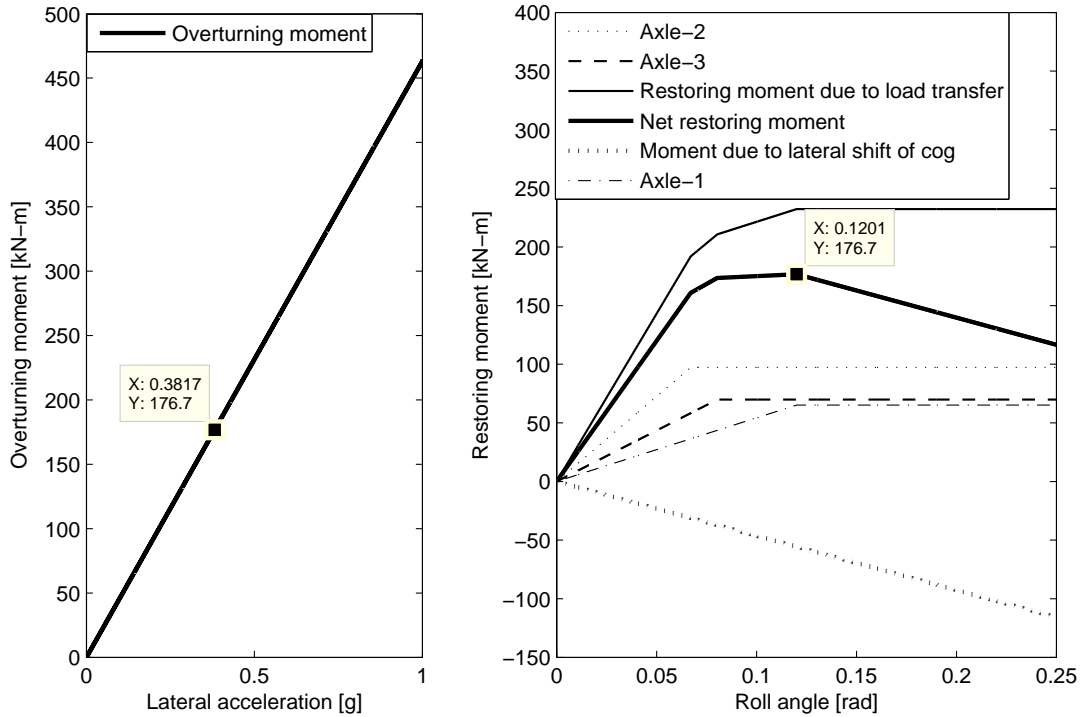


Figure 7.1: Roll moment diagram for rigid truck with input parameters using RCV method

acceleration (viz. $m_T a_y h_{cgT}$) equals the maximum net restoring moment, the corresponding lateral acceleration can be considered as SRT according to RMD analysis.

The RMD requires input parameters such as maximum roll angle of lift-off at each axle and their corresponding masses to calculate the restoring moment due to lateral load transfer, these inputs can be provided using either RCV or UNECE111 method. Matlab script for generating RMD is presented in Appendix C.

RMD predicts the SRT for rigid truck¹ as 0.3817 g as indicated in Figure 7.1 and RCV method estimates the SRT as 0.4063 g with a percentage error of 6.08% with respect to RMD analysis.

Similar analysis is performed with input data obtained from UNECE111 calculation method for RMD as presented in Figure 7.2. In this case, RMD predicts the SRT for referenced

¹vehicle data is confidential

vehicle as 0.4143 g while UNECE111 method estimates SRT as 0.419 g with a percentage error of 1.15% with respect to RMD analysis.

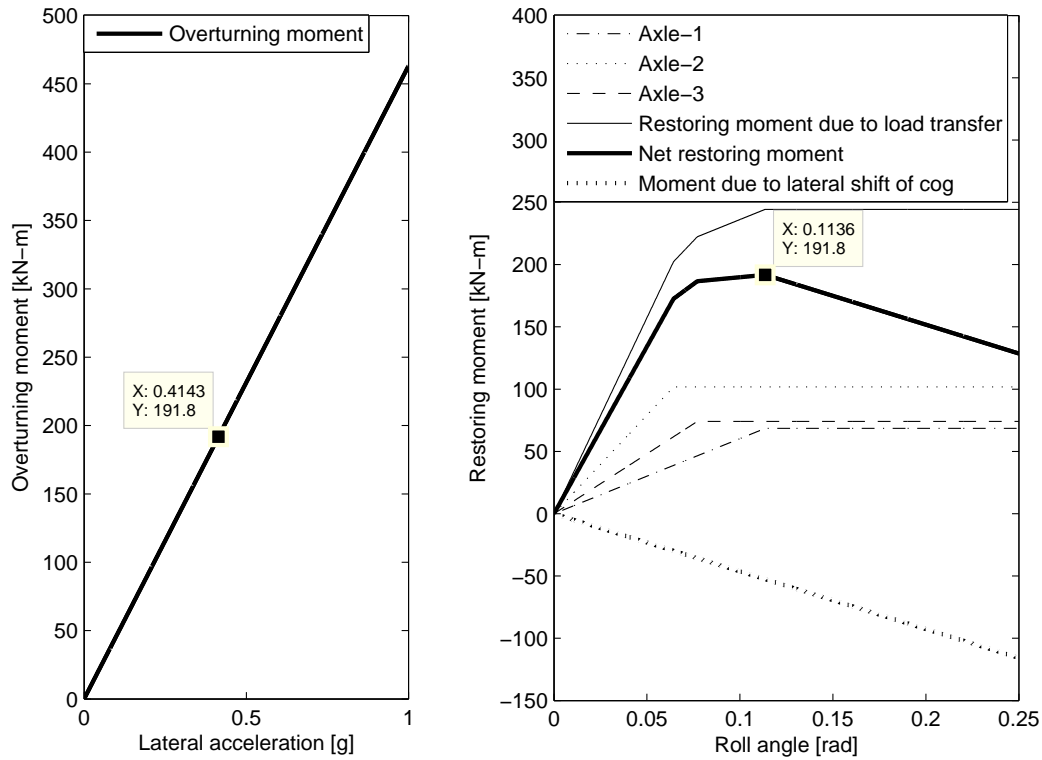


Figure 7.2: Roll moment diagram for rigid truck with input parameters using UNECE111 method

In the RMD analysis, vehicle is simply interpreted as a single mass rolling about a point at ground level with certain stiffness, i.e. a single roll degree of freedom. However, actual vehicle has at least two roll degrees freedom due to sprung and un-sprung mass roll motion. The effect of all type of vehicle compliance can be interpreted as moment due to lateral shift of cog which is represented as linear function of roll angle in RMD analysis. One of the important thing is the individual maximum axle roll angle at which respective axles lift-off. These maximum roll angles are calculated either from RCV or UNECE111 method and used as a input for generating respective RMDs. The lateral load transfer at each axle represent restoring moment, which being a function of individual axle roll angles also represent vehicle compliance due to suspension and tire at each axle.

It is certain that due to the various vehicle compliance, vehicle's SRT decreases, or in other words, with better consideration of vehicle compliances, net restoring moment due to lateral load transfer decreases which result in reduction of SRT, i.e. a vehicle rolls over at relatively lower lateral acceleration. By comparing Figure 7.1 and 7.2, it can be observed that RCV method estimates lower restoring moments for higher roll angles.

Since, RMD requires input from either RCV or UNECE111 method as aforementioned, therefore, it is better to compare two methods with their respective RMD. The difference between a method (i.e. RCV or UNECE111) and its RMD entirely depend on the consideration of vehicle compliances in respective method. RMD approach defers from rigid vehicle only

in terms of its consideration of vehicle COG lateral shift due to various vehicle compliances. Higher the SRT difference between a method and its respective RMD, better will be the estimation method. Since, for reference vehicle, difference in SRT predicted by RCV and its respective RMD is higher, therefore, it can be concluded that RCV method provides a better estimation of SRT for at least long single vehicle units than UNECE111 method due to its better consideration of various vehicle compliances.

7.2 Influence of Vehicle Parameter on SRT

It has been observed that various vehicle parameters influence the roll motion of a vehicle either directly or indirectly. In previous chapters, the effect of these parameters have been represented in mathematical equations. The RCV method proposes different set of mathematical relations for roll stiffness due to suspension and tire compliance than UNECE111 method and also include additional parameters which consequently indicate diverse effect of these vehicle parameters on SRT.

Since, UNECE111 method is best suited for single long unit vehicles, therefore, it is in best interest to compare the influence of various parameters on SRT of single long unit vehicles (such as referenced vehicle) using RCV and UNECE111 method. For parameter sensitivity analysis, the respective parameters are varied within 20% range of referenced vehicles parameter value and the difference between RCV and UNECE111 method is presented in terms of SRT estimation error. The error is calculated with respect to UNECE111 method and represented as percentage error at respective places. The parameters chosen for sensitivity analysis are as follows:

- height of the center of gravity
- height of the sprung mass center of gravity
- height of suspension roll center
- suspension roll stiffness
- vehicle total mass
- sprung mass
- tire vertical and lateral stiffness

Winkler [41] classified vehicle suspensions (i.e. axles) into three categories based on RMD analysis: *First*, axles whose tires lift-off before (i.e. the smaller roll angle) the peak of the net restoring moment, *Second*, axles whose tires lift-off at the peak of net restoring moment, and *Third*, axles whose tires lift-off after the peak of net restoring moment. The roll stability (i.e. SRT) can only be improved by stiffening the suspensions (i.e. by increasing resultant axle roll stiffness) of the latter two types but first type of axles doesn't contribute if stiffened.

Therefore, axles of type Second and Third have been selected for parameter sensitivity analysis based on RMD of reference vehicle, which is presented in Figure 7.1 and 7.2 with input parameters using RCV and UNECE111 method respectively. In both cases, axle 3 lifts-off first and before the peak of net restoring moment and peak of the net restoring moment arrives at

lift-off of axle 1. Therefore, axle 1 is the only one which can be classified under Second type of axles, i.e. all the vehicle parameters which increase or decrease the resultant roll stiffness of axle 1, will also influence SRT in respective manner. In other words, parameters influencing equivalent suspension roll stiffness (i.e. equation (7.1)) and axle roll stiffness (i.e. equation (7.4)) of axle 1 influence the resultant roll stiffness (i.e. equation (7.5)) and subsequently influence the SRT of vehicle.

7.2.1 Vehicle COG Height

In a rigid vehicle (i.e. vehicle without any roll compliance), COG height has contradictory effect on SRT, i.e. higher the cog height lower the SRT. It also plays an important role in vehicle having roll compliances which has throughout been discussed in RCV method formulation. It is expected that cog height will have contradictory effect on vehicle's SRT, i.e. with increase in cog height, vehicle's SRT will reduce. To indicate the influence of COG height predicted by both RCV and UNECE111 methods, it has been varied within 20% range about its real value for reference vehicle.

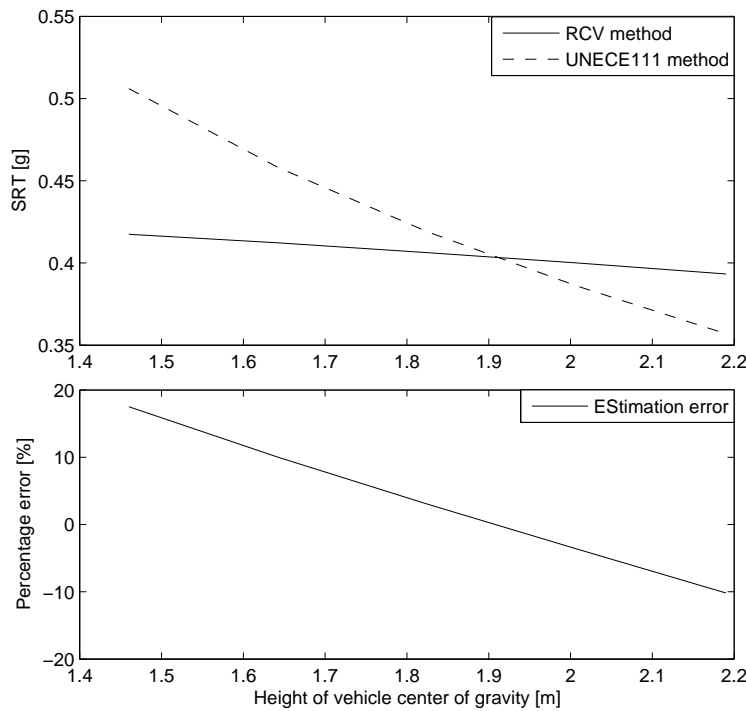


Figure 7.3: *Influence of vehicle cog height on rigid truck SRT*

As presented in Figure 7.3, SRT of reference vehicle unquestionably decreases with increase in COG height. It is of interest to observe that UNECE111 method indicates the effects of COG height on SRT very similar to rigid vehicle, which is evident from approx 35% change in SRT between minimum and maximum value of COG height considered. RCV method however indicates the same contrasting behavior but with less sensitivity (only approx 20% compared to UNECE111) towards COG height. The difference between two methods increases with both

increase and decrease in COG height due to highly sensitive nature of UNECE111 method.

7.2.2 Sprung Mass COG Height

Sprung mass COG height certainly influence the sprung mass roll motion, in both calculation methods, primarily the sprung mass COG height is used in calculating equivalent suspension roll stiffness at ground level. It is expected that sprung mass COG height has inverse effect on vehicle roll stability. To indicate the influence of sprung mass COG height on vehicle roll behavior, it is varied within 20% range about its real value for reference vehicle.

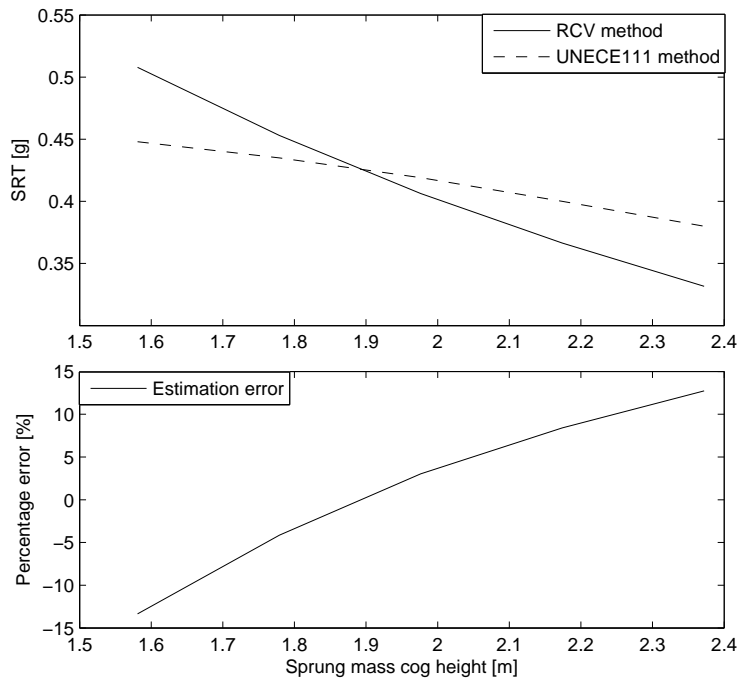


Figure 7.4: *Influence of sprung mass cog height on rigid truck SRT*

As presented in Figure 7.4, with increase in sprung mass COG height, SRT decreases which indicates the reduced roll stability of vehicle. Both methods indicate that vehicle's roll stability is highly sensitive to sprung mass COG height especially RCV method is more sensitive than UNECE111 method. It can also be observed that, the difference between the two methods increases with both increase or decrease in sprung mass COG height. At lower values of sprung mass COG height, RCV method predicts the higher roll stability and at higher values UNECE111 predicts the higher roll stability.

7.2.3 Suspension Roll Center Height

Sprung mass is generally considered to roll about a point above ground, which is known as suspension roll center. A vehicle can also be visualize as a pendulum in a circular motion. Due to inverse pendulum effect of sprung mass about suspension roll center, it is expected that with increase in suspension roll center height above ground, roll stability of vehicle should

increase similar to the length of a pendulum. To indicate the influence of suspension roll center height on vehicle roll behavior, suspension roll center height of axle 1 is varied within 20% range about its real value for reference vehicle.

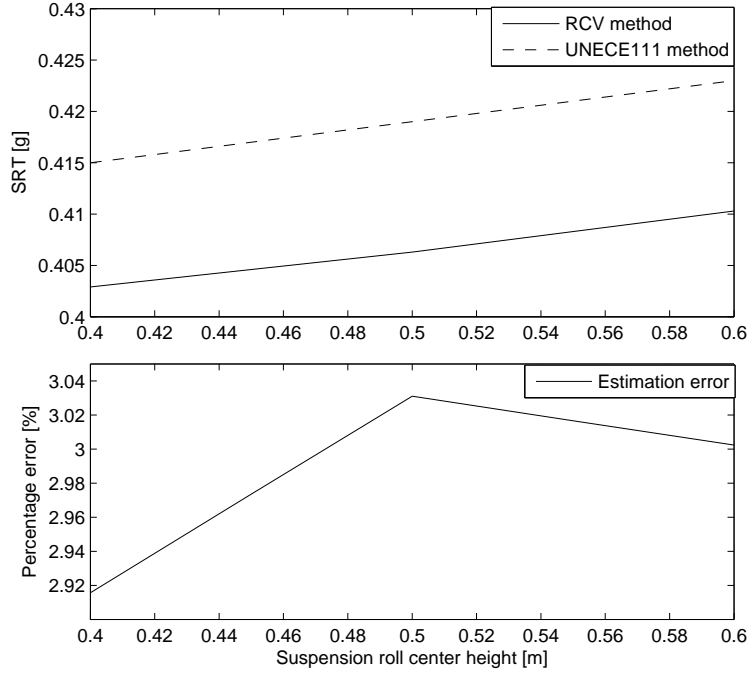


Figure 7.5: *Influence of suspension roll center height on rigid truck SRT*

As presented in Figure 7.5, with increase in suspension roll center height at axle 1, SRT of vehicle increases which indicates the improvement in vehicle roll stability. The effect of suspension roll center height is small (approx 1.5%), which can be observed in both methods. UNECE111 method indicates a constant slope response with change in suspension roll center height but RCV method indicates higher increase in roll stability than decrease in roll stability with increase and decrease in suspension roll center height respectively. Also, the difference between the two methods increases with both increase and decrease in suspension roll center height.

7.2.4 Suspension Roll Stiffness

Sprung mass of vehicle primarily rolls due to suspension roll stiffness which in turns decides the degree of suspension compliance for a certain vehicle. To indicate the influence of suspension roll stiffness on vehicle roll behavior, suspension roll stiffness of axle 1 is varied within 20% range about its real value for reference vehicle. It is expected that by increasing suspension roll stiffness of axle 1, roll stability of vehicle should increase.

As presented in Figure 7.6, with increase in suspension roll stiffness of axle 1, SRT of vehicle increases which indicates the improved roll stability of vehicle. However, the increase in SRT is small (approximately 3% between maximum and minimum value of suspension roll stiffness) which is indicated by both methods respectively. It can also be observed that

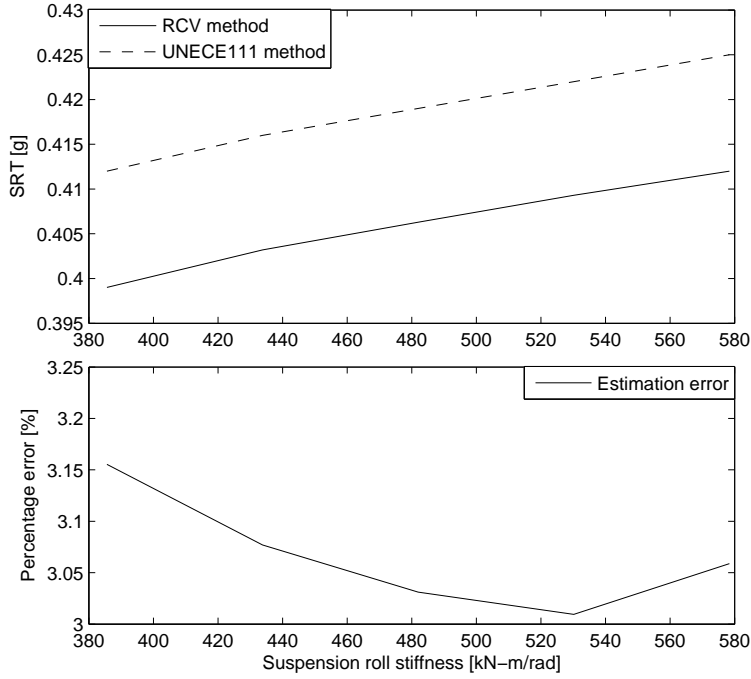


Figure 7.6: *Influence of suspension roll stiffness on rigid truck SRT*

difference between the two methods is higher at lower suspension roll stiffness and again at higher suspension roll stiffness, difference starts to increase, which indicates that UNECE111 method is rather more sensitive to suspension roll stiffness than RCV method.

7.2.5 Axle/Total mass

Axle total mass is combination of sprung and un-sprung mass at an axle. Both suspension and tire compliances depend on axle mass which makes it important from roll stability point of view. It is expected that with increase in axle mass, roll stability of vehicle should decrease because it will increase the magnitude of overturning moments. To indicate the influence of axle mass on vehicle roll behavior, total mass of axle 1 is varied within 20% range about its real value for reference vehicle.

As presented in Figure 7.7, with increase in axle mass, SRT of vehicle decreases which indicates the reduced roll stability of vehicle. The effect of axle mass on roll stability of vehicle is comparatively higher than other parameters (approx. 7% between maximum and minimum values), which indicates its importance. Although both methods, indicates the same response with respect to change in axle mass but UNECE111 is more sensitive. At lower axle mass, the difference between two methods is high which reduces with increase in axle mass.

7.2.6 Tire Vertical Stiffness

Tire is the only part of a vehicle which is in contact with road and therefore, forces and reactions from surrounding are transferred through tires only. Axle roll stiffness is a function of

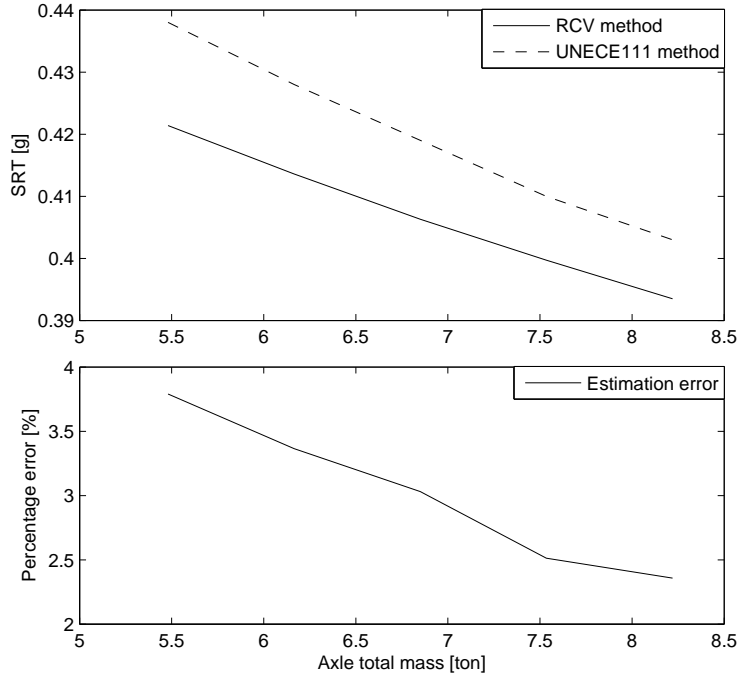


Figure 7.7: *Influence of axle total mass on rigid truck SRT*

vertical tire stiffness, which mainly contribute to the tire compliance of vehicle. It is expected that with increase in tire vertical stiffness, vehicle's roll stability should be improved because increased tire vertical stiffness means less vertical deflection of tires and consequently less roll angle due to tire compliance and less overturning moments. To indicate the influence of tire vertical stiffness on vehicle roll behavior, tire vertical stiffness of axle 1 is varied within 20% range about its real value for reference vehicle.

As presented in Figure 7.8, with increase in tire vertical stiffness, SRT of vehicle increases which indicates the improved roll stability of vehicle. However, the improvement is rather small (approx. 1.6% between maximum and minimum value) but it is significant due to overall small contribution of tire compliance in roll stability. It can be observed that, RCV method indicates a higher increase in SRT between minimum and maximum tire vertical stiffness value as if SRT is directly proportional to it. Although, in UNECE111 method axle roll stiffness is directly proportional to tire vertical stiffness but it indicates higher decrease than increase in roll stability with decrease and increase in tire vertical stiffness respectively. It can also be observed from decreasing difference indicated by percentage error between two methods.

7.2.7 Tire Lateral Stiffness

In steady state cornering/cornering condition, tire doesn't only experience vertical and longitudinal forces but also lateral forces. Since, tire is not infinitely rigid in lateral direction, therefore, it deflects laterally due to action of lateral forces. The lateral deflection of tire can result in reduced axle track width which will reduce the restoring moment due to lateral load transfer, resulting in reduced roll stability. Even in tilt table testing, it can be expected that

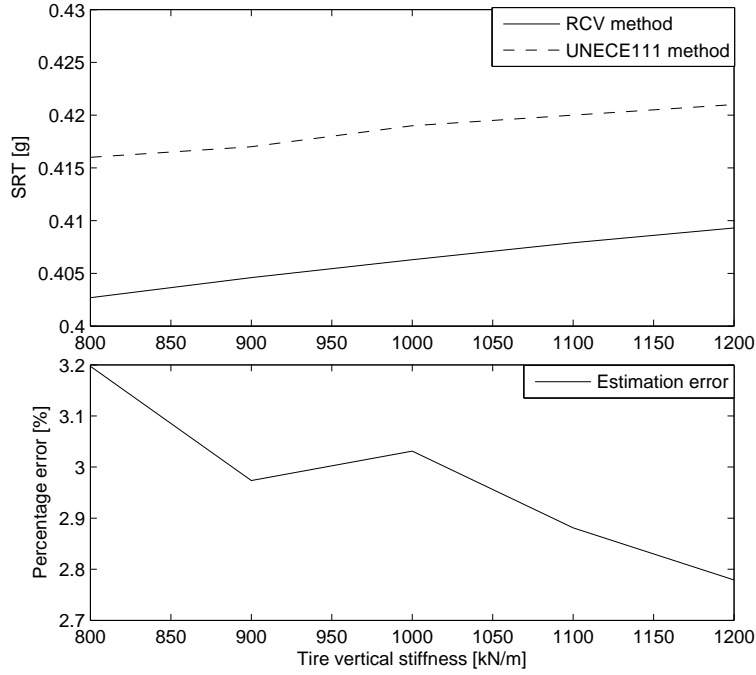


Figure 7.8: *Influence of tire vertical stiffness on rigid truck SRT*

tires will deflect because of lateral forces may be rather small but it is a significant effect of tire compliance which needs to be included. It is expected that with increase in tire lateral stiffness, vehicle's roll stability should increase. To indicate the influence of tire lateral stiffness on vehicle roll behavior, tire lateral stiffness of axle 1 is varied within 20% range about its real value for reference vehicle.

The UNECE111 method doesn't include the influence of tire lateral stiffness, which is evident from no change in SRT. But, RCV method includes the effect of tire lateral stiffness and as presented in Figure 7.9, with increase in tire lateral stiffness, SRT certainly increases which indicates that vehicle's roll stability improve with increase in tire lateral stiffness. The effect may be rather small (approx. 1%) but it is due to lack of dynamics in the calculation method. It is expected that this effect influence the SRT rather more than what is being observed. It can also be observed that with increase in lateral stiffness, difference between two methods decreases and at relatively very high value (approx. 10 times the actual value) the difference would be zero but it would not represent the real tire property.

7.3 Validation of RCV Method

The RCV method estimates comparatively lower SRT value than UNECE111 method for reference vehicle used in parameter sensitivity study. Both methods are calculation methods based on mathematical interpretation of a vehicle/vehicle combination using physical equations. The RCV method is developed to reduce the assumptions and simplifications considered in UNECE111 method, which means RCV method should be able to estimate SRT for a vehicle with more precision and close to real value. To evaluate whether proposed RCV method provides

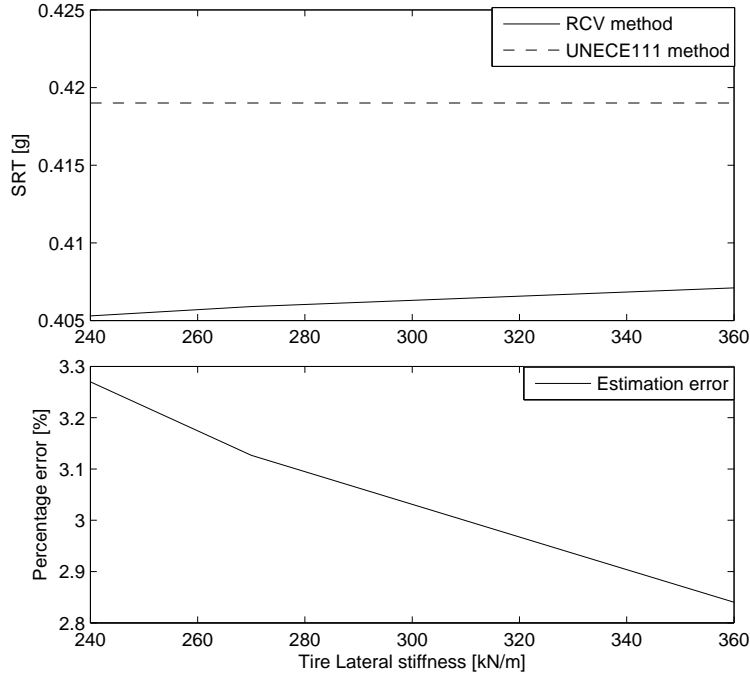


Figure 7.9: *Influence of tire lateral stiffness on rigid truck SRT*

a better estimation than UNECE111 method; the SRT estimated by two methods are further compared with SRT measured by tilt table tests for various vehicles/vehicle combinations. The vehicle types considered within study are single vehicle units (either truck, tractor or trailer), double unit (tractor and semitrailer) vehicles.

Comparison with Tilt Table Tests

The more globally accepted criterion to evaluate the SRT for vehicle/vehicle combinations are Tilt-table testing, which is also the basis of both calculation methods, i.e. in absence of tilt-table testing, the calculation method can be utilized to estimate the SRT for a vehicle.

The primary limitation for UNECE111 method is that it is not suitable to estimate the SRT for vehicle combinations (i.e. vehicles with more than one unit). It can only estimate SRT for single vehicle unit i.e. either for truck, tractor or trailer. In other words, UNECE111 method is best suited for only long single vehicle units. On the other hand, RCV method claims to evaluate all type of HCT vehicles even long roll coupled ones.

To verify this claim, RCV method is compared with tilt table tests. However, not being a part of present study, tilt table tests data are obtained from previous tests performed by Volvo Group Trucks Technology (VGTT) on various types of vehicles [9],[10],[24]. The vehicle types and their respective SRT values estimated or measured from UNECE111, RCV and tilt table tests are presented in Table 7.2. The SRT estimated by both methods deviate from tilt table tests, this deviation is also presented in former tabular as percentage error of each method with respect to tilt table test value for respective vehicle type.

A qualitative behavior of RCV and UNECE111 calculation method with reference to

Table 7.2: Comparison of RCV and UNECE111 method with tilt table test

Vehicle No.	Vehicle types	Steady state rollover threshold [m/s ²]			SRT estimation error relative to tilt table test [%]	
		UNECE111 method	RCV method	Tilt table test	UNECE111 error	RCV error
1	3-axle truck	4.109	3.984	3.979	-3.278	-0.148
2	4-axle turn-table trailer	3.756	3.599	3.609	-4.076	0.272
3	5-axle turn-table trailer	3.687	3.529	3.275	-12.574	-7.754
4	2-axle dolly and 3-axle semitrailer	3.491	3.246	3.275	-6.587	0.898
5	3-axle tractor semitrailer	3.707	3.501	3.530	-5.0	0.833
6	5-axle tractor semitrailer	4.129	3.679	3.697	-11.671	0.477

tilt table test can be observed from Figure 7.10 for SRT estimation. UNECE111 method clearly overestimates SRT for all tested vehicles but RCV method estimates SRT within close proximity of tilt table tests except for vehicle 3. The vehicle types and their respective SRT values either estimated or measured are presented in Table 7.2. The first three vehicles are single vehicle units and next three are combination of at least two units coupled together with fifth-wheel. The deviation of RCV and UNECE111 calculation methods are also presented in term of percentage error compared to tilt table test, which clearly indicates that RCV method provide better SRT estimation than UNECE111 method for all tested vehicles.

The RCV method estimates SRT within 1% deviation with respect to tilt table test except for vehicle 3. The reason for larger deviation of vehicle 3 can be either the vehicle data which has been used for estimation is not accurate or the SRT measured during the testing was not accurately measured. There could be another explanation for such deviation but it is difficult to understand that which effect can possibly result in approx. 8% error in a single vehicle unit.

The HCT vehicles such as A-double², B-double³, C-double⁴ etc. [11] are generally combination of two or more vehicle types presented in Table 1. In case of draw-bar coupling, the two units can be considered decoupled in roll motion and minimum SRT among units would be the SRT for respective vehicle combination. With this argument, RCV method is also suitable for HCT vehicles except B-double (because all units are roll-coupled).

²consists of tractor semitrailer and full trailer connected with draw-bar

³consists of tractor, semitrailer with fifth-wheel and semitrailer

⁴tractor semitrailer, C-dolly and semitrailer

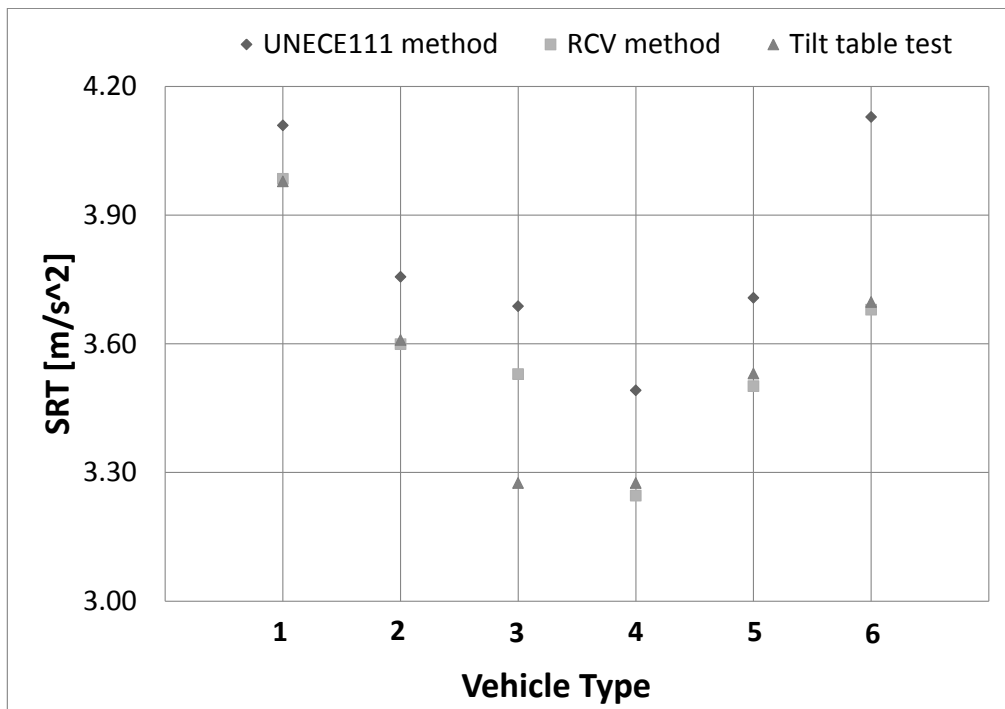


Figure 7.10: Comparison of steady state rollover threshold for various vehicles using UNECE111, RCV and Tilt table tests

8 Summary and Recommendations

All vehicle compliance included in RCV method are based on purely physical relations, which makes RCV method more realistic and increases its applicability for HCT vehicles. Due to its similar structure, RCV method can be compared and evaluated at each calculation step with UNECE111 method. RCV method uses sophisticated equations in calculating roll stiffness due to suspension and tire compliance which result in better SRT estimation. It shows potential for estimating SRT not only for single vehicle units but also for vehicle combinations which can be observed from the variety of vehicle types presented in Chapter 7.

In RCV method, roll stiffness due to suspension compliance at ground level is calculated utilizing a suspension factor which is simplified in UNECE111 method. The suspension factor depends on axle sprung mass, sprung mass COG height, suspension roll center height and suspension stiffness as determined. Due to this suspension factor, reduction of about 5-6% can be observed in roll stiffness due to suspension at each axle with respect to UNECE111 method for referenced single vehicle unit.

In RCV method, the influence of tire compliance on SRT is improved by utilizing the lateral properties of tires which otherwise is neglected in UNECE111 method. A tire is not stiff in lateral direction and due to the implication of lateral force, tires deflect laterally which in turn changes the track-width at an axle. The roll stiffness due to tire compliance which depends only on vertical tire stiffness and axle track-width in UNECE111 method, also depends on axle total load, axle COG height, lateral tire stiffness and lateral force in RCV method. Due to this relation, for referenced single vehicle unit, a reduction of about 12-20% can be observed in axle roll stiffness due to tire compliance with respect to UNECE111 method.

The resultant roll stiffness at an axle is due to the combined effect of suspension and tire compliance. Due to which for referenced single vehicle unit, a reduction of about 8-10% can be observed in resultant roll stiffness at an axle compared to UNECE111 method.

In UNECE111 method, fifth-wheel is treated as an additional axle and roll stiffness of fifth-wheel is considered as an empirical relation which only depends on fifth-wheel load. In RCV method, fifth-wheel roll motion is described in three steps. Fifth-wheel compliance is considered due to the combination of fifth-wheel lash and trailer lash. Fifth-wheel lash mainly depends on vertical distance available between fifth-wheel bump stops which is not studied in detail and simplified as 2 deg. The trailer lash is described as a function of the axles parameters which supports the trailer. Fifth-wheel roll stiffness considered in RCV method is not only the function of fifth-wheel load but also depends on height of fifth-wheel load, fifth-wheel roll center height and angles due to fifth-wheel lash and trailer lash. In RCV method, fifth-wheel is not considered as additional axle and fifth-wheel stiffness doesn't contribute to vehicle total resultant roll stiffness, which is otherwise included in UNECE111 method.

The fifth-wheel compliance is considered to influence the lateral shift of COG (inverse pendulum effect). The lateral shift of COG is considered due to tire and suspension compliance in single rigid unit. The fifth-wheel compliance is additionally added in case of roll coupled vehicle combinations for determining lateral shift of COG.

Due to consideration of these effects in RCV method, it is suitable to estimate SRT for any type of HCT vehicle, which can be confirmed from the results presented in Chapter 7. In this work, RCV method is verified and validated only to indicate the applicability of the calculation method. Despite the advantages, RCV method require additional input compared to UNECE111 method, which are as follows:

- Tire lateral stiffness [kNm/rad]
- Fifth-wheel half width [m]
- Roll center height at fifth-wheel coupling [m]

Tire lateral stiffness is required to determine the reduction in track-width, roll center height at fifth-wheel is required to determine trailer lash, fifth-wheel lash and lateral shift of COG in case of vehicle combinations. Fifth-wheel half width is required to determine the lateral acceleration at which trailer separation occurs.

Recommendations for Future Work

It can be clearly reflected that RCV method is better than UNECE111 method for SRT evaluation of single unit vehicles and vehicle combinations and it has potential to be implemented for all type of HCT vehicles. However, it is far from being perfect and have some simplifications which needs to be researched and implemented appropriately in RCV method.

- Suspension variations have not been considered and investigated, which will certainly influence the suspension compliance. There will be suspension lash in case of leaf suspension which is expected to further reduce SRT.
- Influence of suspension geometries will produce additional compliance and will also affect the tire properties.
- Pressure distribution at tire contact patch changes due to lateral load transfer which may be of contradictory nature to tire lateral shift considered using tire lateral stiffness, i.e. this phenomenon is expected to shift the point of action of tire vertical load outwards, which is expected to improve SRT may be rather small.
- Fifth-wheel geometries is not investigated, in order to include better compliance due to fifth-wheel lash, it is important to develop a relation including bump-stops and other fifth-wheel parameters.
- Torsional compliance of frame and chassis will influence trailer lash resulting in changed lateral shift of COG which is expected to reduce SRT further.

The RCV method presented in this thesis work is developed in such a way that RCV can be directly compared with UNECE111 even in the intermediate calculations steps. However, a more sophisticated physical model can be developed if concept of a_{yS} and a_{yT} is removed from the RCV method. It can be done by introducing chassis and frame torsional stiffness between two consecutive axles. This introduction of chassis and frame torsional stiffness will remove the simplification of assuming a vehicle/vehicle combination as single ‘lumped’ axle to calculate a_{yS} and a_{yT} and it will be possible to actually calculate the knees shown in Figure 3.5 and thus, SRT could be identified when inner side of all axles lift from ground. This would be the extension of RCV method but then the only possibility of comparing RCV method with UNECE111 would be in terms of SRT, not the intermediate calculation steps.

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A UNECE111 Calculation Method

```
% UNECE111 script for 3-axle Volvo FH12 with rigid structure.

% SRT measured through Tilt Table Test is 0.4057 g (22.1 deg).

% Vehicle combination input parameter required (each axle)

A = Confidential ; % axle/bogie load [kN]
T_n = Confidential ; % nominal track width [m]
F_rv = Confidential ; % vertical tire rate [kN/m]
MA = Confidential ; % twin tire width [m]
U = Confidential ; % unsprung axle load [kN]
m = Confidential ; % nominal suspension roll axis height [m]
Hgch = Confidential ; % cog height of complete chassis [m]
Huch = Confidential ; % cog height of unsprung axle load [m]
Hnss = Confidential ; % cog height of superstructure incl. cargo [m]
Wch = Confidential ; % complete chassis weight [kN]
Cdg = Confidential ; % suspension roll stiffness at axle
roll axis [kNm/rad]
Ak = Confidential ; %king-pin load [kN]

At=sum(A)+Ak;
Ut=sum(U);

for i = 1:length(m)
T(i)=(T_n(i).^2+MA(i).^2).^0.5;
end

%centre of gravity height of the sprung mass of the chassis

if Wch==Ut
    Hnch=sum(U.*Huch)/Ut;
else
    Hnch=(Hgch*Wch-sum(U.*Huch))/(Wch-Ut);
end

%centre of gravity heights of the complete vehicle

Wss=At-Wch;
Hg=(Hgch*Wch+Hnss*Wss)/At;
Hn=(Hnch*(Wch-Ut)+Hnss*Wss)/(At-Ut);
```

```

Cdr=0.5*F_rv.*T.^2;
Cdgm=Cdg.*(Hn./(Hn-m)).^2;
Cdres=Cdgm.*Cdr./(Cdgm+Cdr);
teta=0.5*A.*T./Cdres;

% King-pin or fifth wheel effects on rollover is calculated as follows.

if Ak > 0
    At = sum(A)+Ak;
    Tk = mean(T);
    Tt = sum((A.*T))/At+(Ak*Tk)/At;
    Cdresk = Ak *4;
    Cdrest = sum(Cdres)+Cdresk;
else
    At = sum(A);
    Tt = sum((A.*T))/At;
    Cdrest = sum(Cdres);
end

%Axle, which has the smallest roll angle at wheel lift, lifts first

j=find(teta==min(teta));

if length(j)~=1
    j=j(1);
end

Am=A(j);
Um=U(j);
Tm=T(j);
Cdresm=Cdres(j);

%The effective mass factor is

Fe=Cdresm/Cdrest;

%The lateral acceleration at first wheel lift is

qm=0.5*Am*Tm/(Fe*At*Hg+((At-Ut)*Fe*Hn)^2/(Cdresm-At*Fe*Hn));

%The maximum optimal theoretical lateral acceleration at overturn is

qt=0.5*At*Tt/(At*Hg+((At-Ut)*Hn)^2/(Cdrest-At*Hn));

%The corrected lateral acceleration is

```

```

qc=qt-(qt-qm)*Am/At;

%The corresponding equivalent tilt table angle at overturn is

betha=57.3*atan(qc);

['The rollover limit = ',num2str(qc,'%3.3f'),' g']
['The tilt table angle at rollover = ',num2str(betha,'%3.1f'),' deg']

```

B RCV Calculation Method

```
% RCV script for tractor B-double with 3-axle B-link vehicle.

% SRT obtained from VTM is approx 0.525 g (with ramp slope of 0.005 and
% velocity vx0 = 40 m/s).

% Vehicle input parameters

m = Confidential ; % axle mass [ton]
l_tn = Confidential ; % nominal track width [m]
C_tv = Confidential ; % vertical tire stiffness [kN/m]
MA = Confidential ; % twin tire width [m]
U = Confidential ; % unsprung axle mass [ton]
h_rc = Confidential ; % nominal suspension roll axis height [m]
h_s = Confidential ; % center of gravity height of sprung mass [m]
h_cgT = Confidential ; % center of gravity height of complete vehicle [m]
huch = Confidential ; % cog height of unsprung axle load [m]
C_s = Confidential ; % suspension roll stiffness at axle roll axis [kNm/rad]
C_ty = Confidential ; % lateral tire stiffness [kN/m]
g = 9.81; % gravitational acceleration [m/s^2]
F_fw1 = Confidential ; % first fifth-wheel load [kN]
F_fw2 = Confidential ; % second fifth-wheel load [kN]
h_rcfw1 = Confidential ; % roll center height at fifth-wheel [m]
h_rcfw2 = Confidential ; % roll center height at second fifth-wheel [m]
h_cg = Confidential ; % cog height at each axle [m]
h_fw1 = Confidential ; % height of first fifth-wheel [m]
h_fw2 = Confidential ; % height of second fifth-wheel [m]
```

Individual axle calculations

```
% Roll stiffness due to suspension complaine at ground level

for i = 1:length(m)

    m_s(i) = m(i) - U(i); % axle sprung mass [ton]

    X_s(i) = [1 - (m_s(i).*g.*h_rc(i).*(h_s - h_rc(i))./ (C_s(i).*h_s))];
    % Additional suspension compliance factor compared to UNECE111 method

    C_sg(i) = C_s(i).*(h_s./(h_s - h_rc(i)))^2.*X_s(i);

end

m_sT =sum(m_s);
```

```

% lateral force and effective track width at each axle
% quadratic equation in lateral force can be seen as
%  $a(i)*F_y(i)^2 + b(i)*F_y(i) + C(i) = 0$ , which is solved using Shri
% Dharacharya formula

for i = 1:length(m)

    l_t(i) = (l_tn(i)^2+(MA(i)^2))^0.5;

    %h_cg(i) = (U(i).*huch(i) + m_s(i).*h_s)/m(i);

    a(i) = (2*C_sg(1)*C_tv(1)*h_cg(1)*l_t(1)/(m(1)*g)^2);
    b(i) = (C_tv(1)*h_cg(1)*l_t(1)^2+2*C_sg(1)*h_cg(1)-(C_sg(1)*C_tv(1)*...
        l_t(1)^2/m(1)/g));
    c(i) = (m(1)*g)^2*h_cg(1)*l_t(1);

    F_y(i) = (-b(i) + (b(i)^2-4.*a(i).*c(i))^0.5)/(2.*a(i)); % axle lateral
    % force

    l_teff(i) = l_t(i)- (F_y(i)./C_ty(i)); % effective track width at axle
end

% axle roll stiffness

for i = 1:length(m)

    C_al(i) = h_cg(i).*(m(i)*g + C_tv(i).*l_teff(i).*F_y(i)./(m(i)*g));

end

% resultant roll stiffness and maximum roll angle

for i = 1:length(m)

    C_res(i) = (C_al(i).*C_sg(i)./(C_al(i)+C_sg(i)));

    phi_max(i) = m(i)*g.*l_teff(i)./(2*C_res(i));

end

```

Complete vehicle calculation as single lumped axle

```

C_alT = sum(C_al); % total axle roll stiffness [kNm/rad]
C_sgT = sum(C_sg); % total suspension roll stiffness [kNm/rad]
C_resT = sum(C_res); % total resultant roll stiffness [kNm/rad]
m_T = sum(m); % total vehicle mass [ton]

```

```

l_teffT = sum(m.*l_teff)/m_T; % equivalent effective track width of lumped
% axle [m]

% To determine the lateral shift of cog, effective roll angle of cog can be
% expressed as function of lateral acceleration with fifth-wheel compliance
%  $\phi_{effT} = X_T * a_{yT}/g$ 

if F_fw1 == 0

    X_T = (m_T*g*h_cgT)/(C_alT - m_T*g*h_cgT) + (m_sT*h_s*C_sgT)/...
        (m_T*h_cgT*(C_sgT - m_sT*g*h_s)) - 1; % ( $\phi_{effT} = \phi_{alT} + \phi_{is}$ )

else if F_fw1 > 0 && F_fw2 == 0

    X_lash1 = 1/( 1/((C_res(4))/(m(4)*g*h_cg(4))-1)+...
        1/((C_res(5))/(m(5)*g*h_cg(5))-1)+...
        1/((C_res(6))/(m(6)*g*h_cg(6))-1)-...
        (1/(((C_sg(2)+C_sg(3))/(F_fw1*h_fw1))-1))); % instantaneous slope of
% lateral acceleration versus roll angle due to trailer lash,
% i.e.  $X_{lash1} * \phi_{tl,lash} = a_y/g$  or  $X_{lash1} = (a_y/g)/(\phi_{tl,lash})$ 

    C_resk1 = F_fw1*(h_fw1-h_rcfw1)*(2.43 + (1 + X_lash1)); % fifth-wheel
% roll stiffness due to fifth-wheel and trailer lash

    X_T = (m_T*g*h_cgT)/(C_alT - m_T*g*h_cgT) + (m_sT*h_s*C_sgT)/...
        (m_T*h_cgT*(C_sgT - m_sT*g*h_s)) - 1 + F_fw1*(h_fw1-h_rcfw1)/...
        (C_resk1 - F_fw1*(h_fw1-h_rcfw1));

else if F_fw1 > 0 && F_fw2 > 0

    X_lash1 = 1/( 1/((C_res(4))/(m(4)*g*h_cg(4))-1)+...
        1/((C_res(5))/(m(5)*g*h_cg(5))-1)+...
        1/((C_res(6))/(m(6)*g*h_cg(6))-1)-...
        (1/(((C_sg(2)+C_sg(3))/(F_fw1*h_fw1))-1))); % trailer lash at first
% fifth-wheel

    X_lash2 = 1/( 1/((C_res(8))/(m(8)*g*h_cg(8))-1)+...
        1/((C_res(9))/(m(9)*g*h_cg(9))-1)+...
        1/((C_res(7))/(m(7)*g*h_cg(7))-1)-...
        (1/(((C_sg(4)+C_sg(5)+C_sg(6))/(F_fw2*h_fw2))-1))); % trailer lash at
% second fifth-wheel

    C_resk1 = F_fw1*(h_fw1-h_rcfw1)*(2.43 + (1 + X_lash1)); % roll
%stiffness of first fifth-wheel

    C_resk2 = F_fw2*(h_fw2-h_rcfw2)*(2.43 + (1 + X_lash2)); % roll

```

```

%stiffness of second fifth-wheel

X_T = (m_T*g*h_cgT)/(C_alT -m_T*g*h_cgT) + (m_sT*h_s*C_sgT)/...
(m_T*h_cgT*(C_sgT-m_sT*g*h_s)) - 1 + F_fw1*(h_fw1-h_rcfw1)/...
(C_resk1-F_fw1*(h_fw1-h_rcfw1)) + F_fw2*(h_fw2-h_rcfw2)/...
(C_resk2-F_fw2*(h_fw2-h_rcfw2));
    end
end
end

% maximum lateral acceleration required to lift-off one side of vehicle,
% i.e. lateral acceleration at which this lumped axle lift -off
% Moment equilibrium equation:
%m_T*g*l_teffT/2 = m_T*g*h_cgT*(phi_effT + a_yT/g)

a_yT = l_teffT/(2*h_cgT*(1+X_T)); % in [g]

```

To determine minimum lateral acceleration at which first axle lift-off

in a vehicle/vehicle combination, vehicle considered as lumped axle but corresponding to stiffest axle.

%To identify the stiffest axle, i.e. axle with smallest maximum roll angle

```

j = find(phi_max==min(phi_max));

if length(j)~=1
    j=j(1);
end

m_S = m(j);
U_S = U(j);
l_teffS = l_teff(j);
C_resS = C_res(j);
C_alS = C_al(j);
C_sgS = C_sg(j);

% stiffness factor

sf = C_resS/C_resT;

% phi_effS = X_S * a_yS/g (for corresponding stiffest axle)

if F_fw1 == 0

    X_S = (sf*m_T*g*h_cgT)/(C_alS -sf*m_T*g*h_cgT) + (m_sT*h_s*C_sgS)/...

```

```

(m_T*h_cgT*(C_sgS-sf*m_sT*g*h_s)) - 1;

else if F_fw1 > 0 && F_fw2 == 0

    X_lash1 = 1/( 1/((C_res(4))/(m(4)*g*h_cg(4))-1)+...
    1/((C_res(5))/(m(5)*g*h_cg(5))-1)+...
    1/((C_res(6))/(m(6)*g*h_cg(6))-1)-...
    (1/(((C_sg(2)+C_sg(3))/(F_fw1*h_fw1))-1))); % instantaneous slope of
% lateral acceleration versus roll angle due to trailer lash,
% i.e. X_lash1 * \phi_tl,lash = a_y/g or X_lash1 = (a_y/g)/(\phi_tl,lash)

    C_resk1 = F_fw1*(h_fw1-h_rcfw1)*(2.43 + (1 + X_lash1)); % fifth-wheel
% roll stiffness due to fifth-wheel and trailer lash

    X_S = (sf*m_T*g*h_cgT)/(C_alS -sf*m_T*g*h_cgT) + (m_sT*h_s*C_sgS)/...
    (m_T*h_cgT*(C_sgS-sf*m_sT*g*h_s)) - 1 + F_fw1*(h_fw1-h_rcfw1)/...
    (C_resk1-F_fw1*(h_fw1-h_rcfw1));

else if F_fw1 > 0 && F_fw2 > 0

    X_lash1 = 1/( 1/((C_res(4))/(m(4)*g*h_cg(4))-1)+...
    1/((C_res(5))/(m(5)*g*h_cg(5))-1)+...
    1/((C_res(6))/(m(6)*g*h_cg(6))-1)-...
    (1/(((C_sg(2)+C_sg(3))/(F_fw1*h_fw1))-1))); % trailer lash at first
% fifth-wheel

    X_lash2 = 1/( 1/((C_res(8))/(m(8)*g*h_cg(8))-1)+...
    1/((C_res(9))/(m(9)*g*h_cg(9))-1)+...
    1/((C_res(7))/(m(7)*g*h_cg(7))-1)-...
    (1/(((C_sg(4)+C_sg(5)+C_sg(6))/(F_fw2*h_fw2))-1))); % trailer lash at
% second fifth-wheel

    C_resk1 = F_fw1*(h_fw1-h_rcfw1)*(2.43 + (1 + X_lash1)); % roll
%stiffness of first fifth-wheel

    C_resk2 = F_fw2*(h_fw2-h_rcfw2)*(2.43 + (1 + X_lash2)); % roll
%stiffness of second fifth-wheel

    X_S = (sf*m_T*g*h_cgT)/(C_alS -sf*m_T*g*h_cgT) + (m_sT*h_s*C_sgS)/...
    (m_T*h_cgT*(C_sgS-sf*m_sT*g*h_s)) - 1 + F_fw1*(h_fw1-h_rcfw1)/...
    (C_resk1-F_fw1*(h_fw1-h_rcfw1)) + F_fw2*(h_fw2-h_rcfw2)/...
    (C_resk2-F_fw2*(h_fw2-h_rcfw2));
    end
end
end

```



```

% minimum lateral acceleration required to lift-off one axle of vehicle,
% i.e. lateral acceleration at which this lumped axle lift -off
% Moment equilibrium equation:
%  $m_S * g * l_{teffS} / 2 = sf * m_T * g * h_{cgT} * (\phi_{effS} + a_{yS} / g)$ 

a_yS = m_S * l_teffS / (2 * sf * m_T * h_cgT * (1 + X_S)); % in [g]

% Steady state rollover threshold by linear interpolation between maximum
% and minimum lateral acceleration

SRT = a_yT - (a_yT - a_yS) * (m_S / m_T);

% The corresponding tilt table angle is.

Beta = 57.3 * atan(SRT);

% Display rollover threshold and tilt table angle.
['The Rollover Threshold = ', num2str(SRT), 'g']
['The tilt table angle = ', num2str(Beta), 'deg']

```

C Roll Moment Diagram Script

```
% To generate the roll moment diagram, input parameters are evaluated
% using RCV method and script generate RMD for 5-axle tractor semitrailer.
```

```
% input vehicle parameters
```

```
m = Confidential ; % mass of axle [ton]
m_T = sum(m); % total mass of vehicle [ton]
g = 10; % gravitational acceleration [m/s^2]
h_cgT = Confidential ; % vehicle center of gravity height [m]
l_teff = Confidential ; % effective track width at each axle [m]
phi_max1 = Confidential ; % maximum roll angle of axle 1 at lift-off [rad]
phi_max2 = Confidential ; % maximum roll angle of axle 2 at lift-off [rad]
phi_max3 = Confidential ; % maximum roll angle of axle 3 at lift-off [rad]
phi_max4 = Confidential ; % maximum roll angle at axle 4 at lift-off [rad]
phi_max5 = Confidential ; % maximum roll angle at axle 5 at lift-off [rad]
F_fw = Confidential ; % fifth wheel load [kN]
```

```
% roll angle and lateral acceleration range for roll moment diagram
```

```
phi = 0:0.0001:0.25; % roll angle [rad]
a_y = 0:0.0001:1; % lateral acceleration [g]
```

Restoring moment due to lateral load transfer at each axle

```
% Axle 1
```

```
for i = 1:length(phi)

    if phi(i) < phi_max1
        RM1(i) = m(1)*g*l_teff(1).* phi(i)/(2*phi_max1);

    else if phi(i) >= phi_max1
        RM1(i) = m(1)*g*l_teff(1)/2;

    end
end
end
```

```
% Axle 2
```

```
for i = 1:length(phi)

    if phi(i) < phi_max2
```

```

        RM2(i) = m(2)*g*l_teff(2).* phi(i)/(2*phi_max2);

    else if phi(i) >= phi_max2
        RM2(i) = m(2)*g*l_teff(2)/2;

    end
end
end

% Axle 3

for i = 1:length(phi)

    if phi(i) < phi_max3
        RM3(i) = m(3)*g*l_teff(3).* phi(i)/(2*phi_max3);

    else if phi(i) >= phi_max3
        RM3(i) = m(3)*g*l_teff(3)/2;

    end
end
end

%Axle 4

for i = 1:length(phi)

    if phi(i) < phi_max4
        RM4(i) = m(4)*g*l_teff(4).* phi(i)/(2*phi_max4);

    else if phi(i) >= phi_max4
        RM4(i) = m(4)*g*l_teff(4)/2;

    end
end
end

% Axle 5

for i = 1:length(phi)

    if phi(i) < phi_max5
        RM5(i) = m(5)*g*l_teff(5).* phi(i)/(2*phi_max5);

    else if phi(i) >= phi_max5
        RM5(i) = m(5)*g*l_teff(5)/2;

```

```

        end
    end
end

% Moment due to fifth-wheel

for i = 1:length(phi)

    if phi(i) < 0.0416
        RMfw(i) = 0;
    else if phi(i) < 0.0766
        RMfw(i) = -F_fw*12.33.* (phi(i)-0.0416);

    else if phi(i) >= 0.0766
        RMfw(i) = -F_fw*12.33*0.035;

        end
    end
end
end

```

Evaluation of net restoring moment for complete vehicle

```

for i = 1:length(phi)

    RM(i) = RM1(i)+RM2(i)+RM3(i)+RM4(i)+RM5(i); % maximum restoring moment

    % Destabilizing momnet due to lateral shift of vehicle cog
    if F_fw > 0

        OM_cog(i) = - m_T*g*h_cgT*phi(i) + RMfw(i);
    else
        OM_cog(i) = - m_T*g*h_cgT*phi(i);
    end

    RM_net(i) = RM(i)+ OM_cog(i); % net available restoring moment
end

```

Overturning moment due to lateral acceleration

```

for j = 1:length(a_y)

    OM(j) = m_T*g*h_cgT*a_y(j);

end

```

Determination of SRT

```
k = find(RM_net==max(RM_net));
```

```
RM_max = RM_net(k);
```

```
z = find(OM >= RM_max);
```

```
SRT = a_y(z(1));
```

```
['The Rollover Threshold = ', num2str(SRT), 'g']
```