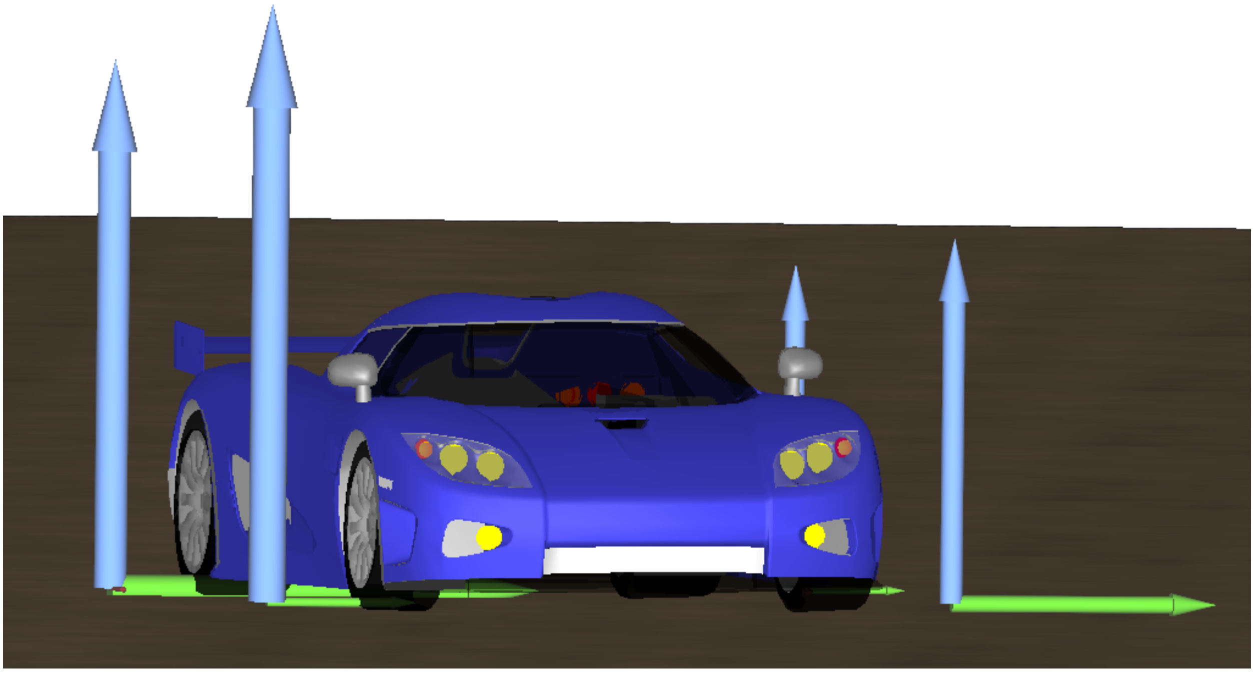

Sensitivity study and optimization of magic formula tire parameters for vehicle handling and steering targets

Master's thesis in Automotive Engineering

VIKRAM PARSHETTI
PRATHAM JAIN

DEPARTMENT OF MECHANICS AND MARITIME SCIENCES

CHALMERS UNIVERSITY OF TECHNOLOGY
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MASTER'S THESIS 2021:20

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VIKRAM PARSHETTI
PRATHAM JAIN

CHALMERS
UNIVERSITY OF TECHNOLOGY

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Division Vehicle Engineering and Autonomous System
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VIKRAM PARSHETTI
PRATHAM JAIN

Industrial Supervisor: Egbert Bakker
Dept 91610 Architects and Product Owners
Volvo Cars Corporation, Göteborg, Sweden

Industrial Supervisor: Kenneth Ekström
Dept 91650 Chassis Integration Attributes
Volvo Cars Corporation, Göteborg, Sweden

Industrial Supervisor: Xin Li
Dept 92540 Wheel, Tyre and Monitoring
Volvo Cars Corporation, Göteborg, Sweden

Academic Supervisor : Ingemar Johansson
Professor of practice,
Division of Vehicle Engineering and Autonomous Systems,
Mechanics and Maritime sciences, Chalmers University of Technology

Examiner: Bengt Jacobson
Professor and group leader Vehicle Dynamics,
Division of Vehicle Engineering and Autonomous Systems,
Mechanics and Maritime sciences, Chalmers University of Technology

Master's Thesis 2021:20
Department of Mechanics and Maritime Sciences
Division of Vehicle Engineering and Autonomous System
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

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Vikram Parshetti
Pratham Jain
Department of Mechanics and Maritime Sciences
Division of Vehicle Engineering and Autonomous System
Chalmers University of Technology

Abstract

Vehicle handling and steering is one of the most important attribute which defines the overall performance and driving dynamics for a car. The factors which primarily define the handling and steering performance are its chassis, suspension and tire characteristics. There are multiple Multi Body Dynamics (MBD) tools currently used in order to better understand and develop the chassis and suspension performance of a car respectively. But the tire development is currently dependent on physical prototyping and testing. This makes the whole process of understanding the tire attributes and its impact on the dynamics of the car difficult and expensive. This calls for further development of tire models and tire simulation tools to aid virtual development of tires. This thesis mainly focuses on developing a tool which optimizes Magic Formula tire parameters to meet the desired vehicle dynamic targets using Magic formula tire 5.2 (PAC2002) which is an empirical tool which defines the forces and moments developed by a tire under the influence of load and slip accurately.Thereby aiding the development and understanding of impact of tire parameters on vehicle handling and steering targets.

This thesis looks into the Magic Formula Tire model 5.2, which is widely used to study vehicle dynamic handling and steering attributes of a car. It is an empirical formula which efficiently captures the physical dynamics of a tire and its impact on a car. This study looks at Magic Formula pure lateral force parameters and their influence on lateral force characteristics of a tire under various load and slip conditions. Simulation software VI-CarRealTime was used in this study in order to integrate Magic Formula tire properties to a vehicle model and to analyse the effect of Magic Formula tire parameters on handling and steering performance of a car. The first part of the study includes the parameter analysis and identification of the most influential Magic Formula tire parameters. Then the study includes identification of suitable maneuvers and vehicle dynamic performance metrics in order to study the influence of Magic Formula tire parameters on handling and steering targets. Once the vehicle dynamic metrics and most influential Magic Formula Tire parameters are identified, a sensitivity study is carried out with Magic Formula Tire parameters varied in equal steps over a fixed range. During the sensitivity run, respective vehicle dynamic metrics and the influence of Magic Formula Tire parameters on these vehicle dynamic metrics are recorded in the form of a objective sensitivity matrix for a reference vehicle used for this study. The objective sensitivity matrix was also developed based on the sensitivity of Magic Formula tire parameters on the axles of a car. Using the objective sensitivity matrix and the weights assigned to the vehicle dynamic metrics by the attribute leaders, an objective function was set up. The minimum value of this objective function will then provide a reference vehicle equipped with the optimized set of tires which will satisfy as many handling and steering targets as possible, including the possibility to optimize a set of tires for individual axles.

Keywords: Magic Formula Tire 5.2 (PAC 2002), Handling and steering targets, optimization, Passenger cars, co-simulation, Matlab API, Parameter and sensitivity study.

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Parshetti,Vikram and Jain,Pratham, Gothenburg, June 2021

Contents

List of Figures

List of Tables

All the other units when not stated are SI Units

Nomenclature

α_y	Lateral slip angle $[rad]$
B_y	Stiffness factor
C_y	Shape factor
C_f	Cornering Stiffness, front axle $[N/rad]$
C_r	Cornering Stiffness, rear axle $[N/rad]$
D_y	Peak Value, Magic Formula tire model
F_y	Lateral tire force $[N]$
F_z	Normal load $[N]$
F_{z0}	Nominal normal load $[N]$
g	Gravitational constant, 9.81 $[m/S^2]$
K_y	Slip stiffness, Magic Formula tire model
μ	Friction coefficient
γ	Camber angle
λ_{Cy}	Scale Factor: Shape factor pure slip F_y
λ_{Ey}	Scale Factor: Curvature factor pure slip F_y
$\lambda_{Ky\gamma}$	Scale Factor: Camber force stiffness pure slip F_y
$\lambda_{Ky\gamma}$	Scale Factor: Cornering stiffness pure slip F_y
λ_{μ_y}	Scale Factor: Peak friction coefficient F_y
λ_{Vy}	Scale Factor: Vertical Shift F_y
λ_{FZ0}	Scale Factor: Nominal load
λ_{Hy}	Scale Factor: Horizontal Shift F_y
L	Wheel Base $[m]$
p_{Cy1}	Shape factor C_{fy} for lateral force
p_{Dy1}	Lateral friction μ_y
p_{Dy2}	Variation of friction μ_y with load
p_{Dy3}	Variation of friction μ_y with squared inclination
p_{Ey1}	Lateral curvature E_{fy} at F_{znom}
p_{Ey2}	Variation of curvature E_{fy} with load
p_{Ey3}	Inclination dependency of curvature E_{fy}
p_{Ey4}	Variation of curvature E_{fy} with inclination
p_{Ky1}	Maximum value of stiffness K_y/F_{znom}
p_{Ky3}	Variation K_{fy}/F_{znom} with inclination
p_{Hy1}	Horizontal shift S_{Hy} at F_{znom}
p_{Hy2}	Variation of shift S_{Hy} with load
p_{Hy3}	Horizontal shift S_{Hy} with inclination
p_{Vy1}	Vertical shift S_{Vy}/F_z at F_{znom}
p_{Vy2}	Variation of shift S_{Vy}/F_z with load

p_{Vy3}	Variation of shift S_{Vy}/F_z with inclination
p_{Vy4}	Variation of shift S_{Vy}/F_z with inclination and load
p_{Ty1}	Peak value of relaxation length
p_{Ty2}	Load at which relaxation reach peak value
m	Mass of the vehicle
a_y	Lateral acceleration $[m/s^2]$
$\dot{v}_y$	Lateral acceleration $[m/s^2]$
v_x	Longitudinal velocity $[m/s]$
w_z	Yaw rate/Rotation about vertical axis
δ_{fl}	Steering angle: front left $[deg]$
δ_{fr}	Steering angle: front right $[deg]$
δ_{rl}	Steering angle: rear left $[deg]$
δ_{rr}	Steering angle: rear right $[deg]$
F_{yflv}	Lateral force front left: vehicle coordinate $[N]$
F_{yfrv}	Lateral force front right: vehicle coordinate $[N]$
F_{yrlv}	Lateral force rear left: vehicle coordinate $[N]$
F_{yrrv}	Lateral force rear right: vehicle coordinate $[N]$
F_{yflw}	Lateral force front left: wheel coordinate $[N]$
F_{yfrw}	Lateral force front right: wheel coordinate $[N]$
F_{yrlw}	Lateral force rear left: wheel coordinate $[N]$
F_{yrrw}	Lateral force rear right: wheel coordinate $[N]$
F_{xflw}	Longitudinal force rear left: wheel coordinate $[N]$
F_{xfrw}	Longitudinal force rear right: wheel coordinate $[N]$

1

Introduction

A car is an amalgamation of various sub systems which inherently define its physical characteristics and performance, e.g. handling and steering performance. These physical characteristics are defined in terms of numerical targets and are generally termed as vehicle DNA. The sub systems e.g. structural systems parts, chassis system(suspension and tires), powertrain not only have their individual influence on dynamics of a car but also on each other. Tire is one important component in the chassis subsystem which induces the preponderance of the DNA attributes, usually caused by the surface it is rolling over. Thus, the understanding of the tires and its effect on other attributes of the car is critical to define the physical characteristics of a car[10].

The characteristics derived from the car can be analyzed in various simulation and testing and for various attributes. Some of the important attributes that an Original Equipment Manufacturer(OEM) looks at are the vehicles energy efficiency, comfort, Noise Vibration and Harshness(NVH), ergonomics, aesthetics, and handling characteristics. Handling of a car can be influenced by various subsystems. Steering, suspension and tires are the main blocks which define most of the handling and steering characteristics and subsequently dynamic safety of a given car. Thus, understanding of tires and its characteristics is critical to understand the handling and steering targets[6].

To develop a successful product for the market, an OEM needs to understand and define its products operating and performance range thus being able to succeed in reaching its targets. This can be realized only if the physics of the car and its response can be modelled and understood at an initial stage of its development by utilizing the tools of Computer Aided Engineering (CAE). Tire is one subsystem which can be modelled empirically rather than defined by physics in-order to capture all its properties and characteristics which could influence the handling and steering of a car[15]. The physics of a tire is difficult to model due to its complex build structure, which includes various layers of different rubber, metal and polymer layers. The formulation of tire digital twin for virtual analysis of vehicle dynamic handling and steering attributes is done by using empirical models[15]. This is due to the efficiency and the effectiveness of such empirical models to capture the most important properties of the tires across linear, nonlinear and saturation region of its operation[15]. The most important property required for handling and steering is the understanding of the mechanism of force and moment build up by the component. Empirical approach rely on the method of fitting a mathematical model into a data set from a flat track test to capture the trend to fit a curve. This also has a lower cost, and is less complicated as compared to that of physical modelling involving complex construction and modelling procedures.

Thus, the use of empirical approach is utilized to capture the effects of tires on handling and steering behavior of a car[15]. But then the method of fitting the curves to a set of measured parameters also inherently includes the set of constraints under which the data set was obtained. There are different models developed under the empirical approach which includes various tire parameters to capture the effect of these tire parameters on the car's handling and steering performance. One of the commonly used models is the Magic Formula tire 5.2 (MF 5.2 / PAC2002) tire model[1]. This tire model captures the relevant parameters required for analyzing the handling and steering performance of the car with respect to tire parameters convincingly in the tires linear to full sliding range[1]. There are newer versions of Magic Formula tire model available to accurately model all the physical characteristics of a given tire under different conditions including temperature effects, Inflation pressure etc which are not included in this study.

1.1 Background

Handling and steering performances are very important within the industry for vehicle dynamic performance and active safety[10]. The handling and steering performance characteristics of a vehicle can be described with metrics like under-steer gradient, frequency response etc. During a vehicle development cycle, the targets for the vehicle dynamics metrics are developed in order to represent a manufacturer's DNA requirement. Determining tire characteristics for a car plays an important role for it to achieve its set vehicle dynamics targets[10]. The modeling of tire forces and tire response characteristics are a crucial in understanding steering and handling dynamics of a car including steering feel and driver confidence [6]. Using good tire models in the initial Vehicle Dynamics simulations especially during the concept design of the chassis and suspension sub systems are very important. Hence there is a need to evaluate and understand the effects of tire parameters on handling and steering targets which in turn influence the performance characteristics of a vehicle and its ability to meet its ambitious DNA targets.

There have been many previous studies on modeling of tire responses and tire models which are used for different applications in the industry. But the Magic Formula tire model is widely used by OEMs and tire manufactures for their respective product development processes [5]. And many previous studies are available on the parameterization and modelling of the Magic Formula tire parameters. Extensive research has also been conducted in understanding the dynamic handling and steering behavior of vehicle for system design and control application[10]s. But very few literature's can be found which investigate the influence of tire parameters such as in Magic Formula tire parameters on the handling and steering metrics. Hence understanding the effects of these Magic Formula tire parameters on the handling and steering performance metrics are important for OEM in order to achieve the intended DNA targets with best compromise. This understanding will enable OEMs be more efficient in their communication with the tire manufacturers such that the manufacturer can develop tires for their specific vehicle needs. Hence OEMs can extract the desired performance characteristics from their vehicles.

Understanding the component requirements for the tire supplier, is in terms of tire parameters and for the OEMs is in terms of vehicle dynamic handling and steering performance targets. This shows a gap existing between the tire suppliers and OEMs. Hence it calls for a tool to analyse the effects of the tire model on handling and steering targets and also OEMs to communicate their desired tire parameters to tire supplier which can achieve the set vehicle dynamic targets.

The optimization tool developed from the study intends to fill the gap in the virtual development of the tires for the attribute of vehicle handling and steering. This can be visualized in the flow chart 1.1, where the highlighted block of 'Tire Evaluation tool' describes the role of the current optimization tool and its intended use in the virtual tire development cycle. This tool can be either used to derive optimized Magic Formula tire parameters or to determine vehicle dynamic targets from a set of Magic Formula tire parameters.

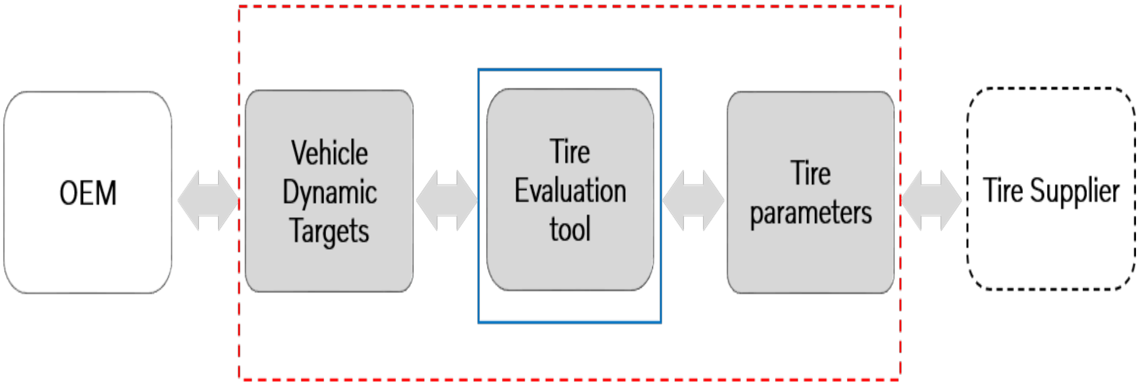

Figure 1.1: Integration of Tire Evaluation tool in the process flow chart

1.2 Deliverables

The overall deliverable of this thesis study can be defined as computing an objective sensitivity matrix to understand the effect of Magic Formula tire parameters on the vehicle handling and steering metrics. And to develop an optimization algorithm which delivers an optimized set of Magic Formula tire parameters in order to achieve a desired vehicle dynamic handling and steering targets. These two main deliverables can be broken down further into following deliverables for the project:

1. Documentation of the information search and record the findings.
2. Run tire parameter pre-study to capture the influence of tire parameters on

the overall Magic Formula lateral force curve.

3. Identify handling and steering performance metrics which captures the required handling and steering characteristics of a vehicle and define the maneuvers which can capture change in the respective handling and steering metrics.
4. Verify the simulated handling and steering results against a reference model targets.
5. Run a sensitivity study in order to capture the influence of Magic Formula tire parameters on the handling and steering metrics and define this with an objective sensitivity matrix.
6. Structure how different Magic Formula tire parameters effect handling and steering performance including its sensitivity to the axle its being changed.
7. To develop an optimization tool which generates an optimized set of Magic Formula tire parameters for a desired set of vehicle dynamic targets and verification of results obtained from the optimization tool.

1.3 Assumptions and limitations

There are many tire models which have been developed for the study of various tire attributes like Magic Formula tire (vX.X), Ftire, CD tire etc. However the study in this thesis has been restricted to only Magic Formula tire 5.2 which is used to study the effect of tires on vehicle handling and steering attributes. Many versions of Magic Formula have been developed in order to incorporate different parameters which effect certain vehicle dynamic attributes. Here the Magic Formula tire 5.2 is used with some restrictions, mainly

1. The Magic Formula tire 5.2 model used in the study does not consider the effects of temperature and inflation pressure on the tire dynamics.
2. This thesis focuses only on the effects of tire parameters on steering and handling attributes of the vehicle. The effects of tires on e.g. Rolling Resistance(RR), Noise Vibration and Harshness (NVH), cost and appearance are not considered. And the balancing of these attributes are also not included in this study.
3. The thesis is limited to the software used in this study which is VI-CarRealTime.

2

Theory

2.1 Tire

2.1.1 Pneumatic Tires

Pneumatic tires are used on a wide range of vehicles, including e.g. cars, trucks, buses etc. Tires provide stability, isolation, steering, and load support to vehicles. Thus allowing them to perform better under application of various stresses and strains. Pressurized air is injected into the tires to provide the load capacity for the tire and to be a structural cushion between the vehicle and the road that absorbs shock and vibrations to ensure a smooth and relaxing journey.

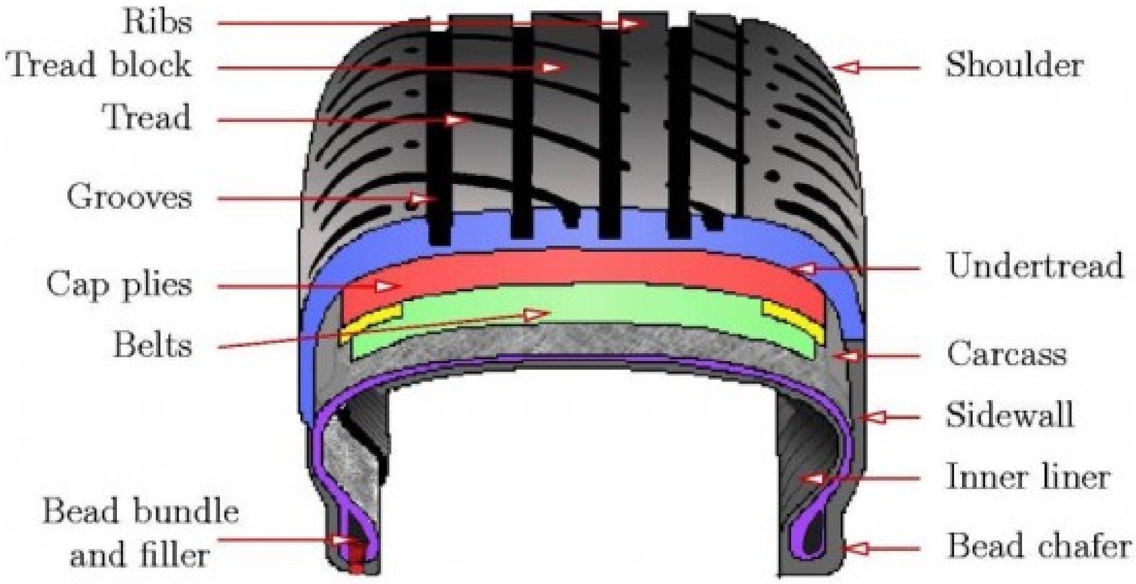

Figure 2.1: Pneumatic tire construction [18]

A pneumatic tire design mainly contains the following construction elements which provide its physical characteristics.

Tire construction elements	Description
Tread	Tread & The paper by [18] explains that the tread is the tire's outermost layer. Treads, as a result must be built to protect the tire's inner construction. They also define the tire characteristics in different friction conditions and aids in clearing the water when the road surface is wet.
Carcass	The carcass is a boundary for compressed air inside, but still is flexible enough to withstand external excitation's. The tire's carcass is made up of a variety of rubber-coated webbed layers known as plies. The tire's ability to stretch is decided by the properties of its carcass.[18]
Side Wall	The tire carcass is protected by the side wall, which are made of flexible rubber. Side wall must be capable of being pliant without failure.[18]
Plies	Layers of plies are made up of highly tensile nylon cords that have been loosely woven together and coated with a rubber compound on both sides. Plies define the strength of a tire.[18]
Beads	The beads secure the tire to the rim[18]
Belts	Stabilizer bias ply belts under the base rubber in radial tires provide additional support for the radial plies underneath and decide the footprint shape of the tires.[18]
Liner	This is made up of two or more layers of rubber in tubeless tires and is intended to hold air under pressure. Tubeless tires have lined inner walls. The liner is made of thick rubber material. It's made of an air-impermeable rubber paste which is like tubes.[18]

2.1.2 Tire Designs

The pneumatic tires are designed differently for various car applications.The tire construction mainly depends on the environment it is designed for. Some of the common types of constructions currently being used widely in market are listed below.

Summer Tires

Tires developed for high grip and provide good handling and steering behaviour are known as summer tires. For better grip and traction, they also have a softer rubber compound, particularly useful for cornering and vehicle dynamics performance attributes[19].

Winter Tires

In cold (under 7 degrees) conditions, winter tires are engineered to stay pliable. In summer tires the rubber glazes at around 7 degrees and the winter tire have their glazing temperature lower than the summer tires. Fine grooves are commonly seen in the tread patterns, which are built to grip into ice and snow on the road. Winter tires should be used in winter conditions and low temperatures. In warmer and dry conditions winter tires have less grip and handling performance due to their soft compound. Many winter tires are studded for extra traction on ice[19].

All Season Tires

All-season tires are used on some cars. These tires are designed to meet the needs of most road conditions; they have the deep water channels as found in winter tires, but they also have a tougher rubber compound for longer tire life in hot weather. All-season tires are designed to strike a balance between grip, stiffness, durability, wet weather capability, and comfort[19]. Currently OEMs intend to concentrate more into all season tires to be used as standard tires in their products.

Figure 2.2: Types of pneumatic tires [20]

2.1.3 Magic Formula Tire Model (Empirical Model)

The paper by [21] states that Magic Formula Tire or PAC2002 is the Delft-Tire standard implementation of the well-known Pacejka's Magic Formula. It can simulate validated steady-state and transient actions up to 8Hz with Magic Formula Tire, making it a good tire model for vehicle handling and control development [21]. The effects of cornering, braking and combined conditions are captured well through this model. Magic Formula Tire's extrapolation robustness makes it ideal for simulating outside the measurement conditions. These characteristics make the Magic Formula Tire a good tire model for development analysis of handling and steering performance[21].

Advantages of Magic Formula tire 5.2 modeling for vehicle handling and steering simulations:

- Characteristics of Magic Formula Slip
- Effects of advanced non-linear relaxation
- Communication between the tire and the road in a single point
- Possibilities for combined slip calculation

The general form of the model for given values of vertical load and camber angles is given in equation 2.1

$$y = D\ \sin[C\ \arctan Bx - E(Bx - \arctan Bx)] \tag{2.1}$$

$$Y(X) = y(x) + S_V \tag{2.2}$$

$$x = X + S_H \tag{2.3}$$

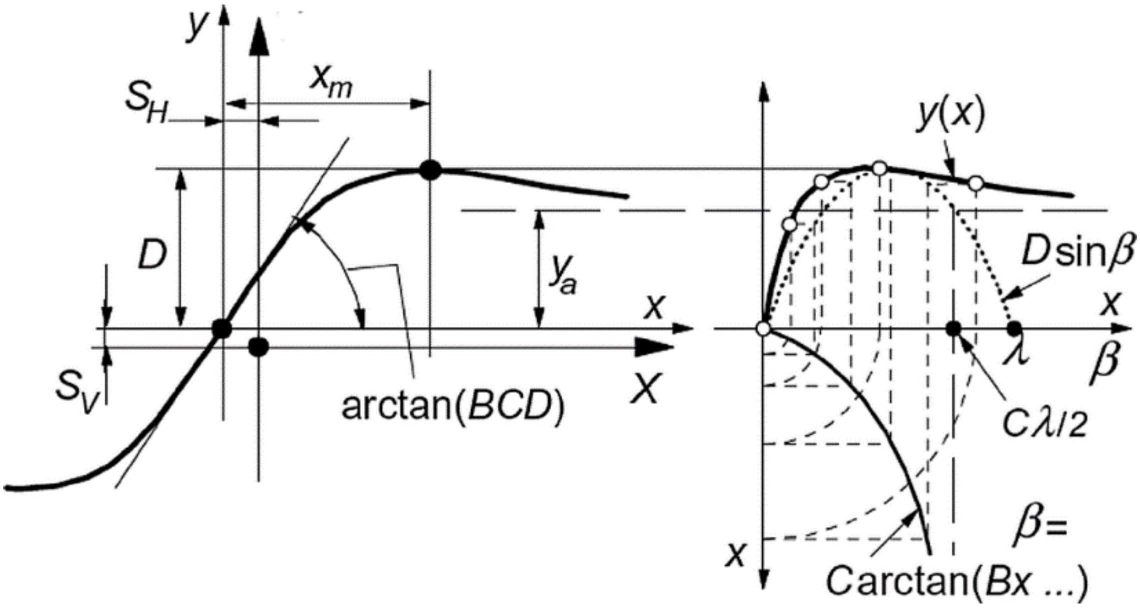

Figure 2.3: Tire force curve from [21]

Where,
Y is the output variable; F_x or F_y
X is the input variable; tanα or k
B is the stiffness factor
C is the shape factor
D is the peak value
E is the curvature factor
S_H is the horizontal shift
S_V is the vertical shift

The constants in the formulae have been revised over time to incorporate new factors, such as inflation strain to boost the model's validity range. MSC Software created one of the first versions, named as "Tires and Vehicle Characteristics" based on the PAC2002 model in the year 2002. The formula is used to measure both pure tire forces and combined tire forces. The pure lateral force equation can be expressed by the equation 2.4. The generic Magic Formula tire parameters B,C,D and E are further defined by sub parameters like p_{KY1}, p_{DY1}, p_{CY1}, p_{EY1} etc. The equation for sub parameters have been referred from [1] and for further details of each parameters refer list of symbols.

$$F_{y0} = D_y\ \sin[C_y\ \arctan\{B_y\alpha_y - E_y\ (B_y\alpha_y - \arctan(B_y\alpha_y))\}] + S_{Vy} \tag{2.4}$$

$$\alpha_y = \alpha^* + S_{Hy} \tag{2.5}$$

$$C_y = p_{Cy1} \cdot \lambda_{cy} \tag{2.6}$$

$$D_y = \mu_y \cdot F_z \cdot \zeta_2 \tag{2.7}$$

where,
F_{y0} is the Lateral tire force
B_y is the Stiffness factor
C_y is the Shape factor
D_y is the Peak value factor
E_y is the curvature factor
p_{Cy1} is the shape factor
α_y is the lateral slip angle
λ_{Cy} is the scale factor for shape factor C_y
F_z is the normal load
μ_y is the coefficient of friction

The parameters B_y,C_y,D_y and E_y can be further described using magic formula coefficients as below,

$$\mu_y = (p_{Dy1} + p_{Dy2}\ df_z)(1 - p_{dy3}\ \gamma^{*2}) \cdot \lambda^*_{\mu_y} \tag{2.8}$$

$$E_y = (p_{EY1} + p_{Ey2}\ df_z)\{1 + -(p_{Ey3} + p_{Ey4}\gamma^*)sgn(\alpha_y)\} \cdot \lambda_{Ey} \tag{2.9}$$

$$K_{y0} = p_{Ky1}F'_{z0} \sin\left[p_{Ky4}\ \arctan\{\frac{F_z}{p_{Ky2}F'_{z0}}\}\right] \cdot \lambda_{Ky\alpha} \tag{2.10}$$

$$K_y = K_{y0} \cdot (1 - p_{Ky3}\gamma^{*2}) \cdot \zeta_3 \tag{2.11}$$

$$B_y = \frac{K_y}{C_yD_y + \epsilon_y} \tag{2.12}$$

where,
p_{Dy1} is the lateral friction μ_y
p_{Dy2} is the variation of friction μ_y with load
p_{Dy3} is the variation of friction μ_y with squared inclination
$\lambda_{\mu y}$ is the Scale Factor: Peak friction coefficient F_y
p_{Ey1} is the lateral curvature E_{fy} at F_{znom}
p_{Ey2} is the variation of curvature E_{fy} with load

p_{Ey3} is the inclination dependency of curvature E_{fy}
p_{Ey4} is the variation of curvature E_{fy} with inclination
λ_{Ey} is the scale factor: Curvature factor pure slip F_y
K_y is the slip stiffness, Magic Formula tire model
p_{Ky1} is the maximum value of stiffness K_y/F_{znom}
p_{Ky3} is the variation K_y/F_{znom} with inclination
γ is the camber angle
$\lambda_{\mu y}$ is the scale factor: Peak friction coefficient F_y
$\lambda_{Ky\alpha}$ is the scale factor: cornering stiffness pure slip F_y
λ_{FZ0} is the scale factor: Nominal load
F_{z0} is the Nominal normal load

The horizontal and vertical shift parameters can be described as below

$$S_{Vy} = Fz.\ \{(p_{Vy1} + p_{Vy2}df_z).\ \lambda_{Vy} +\ (p_{Vy3} + p_{Vy4}df_z)\gamma^*.\ \lambda_{Ky\gamma}\}\lambda^{'}_{\mu y}\zeta_2 \tag{2.13}$$

$$S_{Hy} = (p_{Hy1} + p_{Hy2}df_z)\ \lambda_{Hy} + p_{Hy3}\gamma^* \cdot \lambda_{Ky\gamma}\zeta_0 + \zeta_4 - 1 \tag{2.14}$$

$\lambda_{Ky\gamma}$ is the Scale Factor: Camber force stiffness pure slip F_y
λ_{Vy} is the Scale Factor: Vertical Shift F_y
λ_{Hy} is the Scale Factor: Horizontal Shift F_y
p_{Hy1} is the Horizontal shift S_{Hy} at F_{znom}
p_{Hy2} is the Variation of shift S_{Hy} with load
p_{Hy3} is the Horizontal shift S_{Hy} with inclination
p_{Vy1} is the Vertical shift S_{Vy}/F_z at F_{znom}
p_{Vy2} is the Variation of shift S_{Vy}/F_z with load
p_{Vy3} is the Variation of shift S_{Vy}/F_z with inclination
p_{Vy4} is the Variation of shift S_{Vy}/F_z with inclination and load

Here p_{KY1}, p_{KY2}, p_{DY1}, p_{DY2}, p_{CY1}, p_{EY1}, p_{EY2}, p_{TY1}, p_{TY2} are the dimensionless input variables of the Magic Formula tire 5.2 model used for further study and optimization.

2.2 Vehicle Dynamics

Vehicle Dynamics has always been an essential area of study for OEMs in order to define the handling and steering characteristics of its products[10]. Companies need to understand the dynamic behaviour of a vehicle under different driving conditions like extreme limit driving to city ride conditions[10]. These vehicle dynamic studies are done regularly to extract data which help OEMs to improve performance of the vehicle in terms of ride, vehicle handling and steering performance. With focus on specific type of behavior of their vehicles, OEM define their philosophy of vehicle performance and behavior that attracts the customers. An OEMs in the future, cannot afford to develop a model specifically concentrated on a single vehicle attribute like NVH, Vehicle Energy Efficiency etc. but rather develop a product which incorporates a significant bit of all attributes. Successful OEMs would be the manufacturers who could balance majority of the attributes into their product. Better understanding of tires will play an important part in the future of developing cars to incorporate better attribute balancing.

Vehicle Handling is one of the important aspect that affect vehicle directional control, safety, vehicle stability and driver confidence [8]. The lateral control of vehicle is the major focus study in this thesis. When the vehicle makes a turn from straight line motion into a corner and the other way around where the vehicle enters a transient phase of its motion. The behavior of the vehicle in such steady state and transient situations describes its handling performance and stability characteristics. Automotive industries develop the vehicle with certain desired handling and steering performance characteristics. These characteristics can be described with vehicle dynamic objective metrics like under-steer gradient, frequency response, steering response time etc. These metrics are computed using different transient and steady state maneuvers. These standard maneuvers are defined by companies and standard organisations e.g. ISO, NHTSA[14].
For further explanation on how this method was used in this study, please refer the methodology section 3.1.1.1.

2.3 Simulation Environment

Physical testing of vehicles with test drivers or steering robots used to be the way to assess and develop the dynamic behavior of vehicle under different conditions. However with the evolution of modern computers and many mathematical modelling and simulation software's, it is now possible to develop a realistic digital twin models of vehicles. These software's not only aid in the studying the dynamics behavior of vehicles but also provide a platform to develop and evaluate many vehicle systems. Competition among various automotive companies in the global market also necessitates continuous reduction of the total development cycle time and also development cost [5]. Thus OEMs can test a car for its attribute characteristics before building a physical prototype. Thus saving enormous costs from prototyping and it significantly reduce the time to market.

2.3.1 VI- CarRealTime

Vehicle simulation environments is one of the main tools which is essential in the development of a car. One of the examples of integrating the vehicle model into the simulation environment is through Adams (Automated Dynamic Analysis of Mechanical Systems) Car model. But these are usually computationally costly models and need a lot of computational power and time to run simulations. Hence it calls for a simulation tool that requires less computational power and can effectively run the vehicle simulation, setting along with efficiently managing all the intended simulation setups. One of the tools which satisfies these requirements is VI-CarRealTime from VI-Grade. VI-CarRealTime is a vehicle simulation software that is based on look-up tables [13]. The VI-CRT has on average 96 output variables from the model. This thesis focuses on developing repeatable objective performance matrix with offline open loop and closed loop simulations which are repeatable and accurate with every other simulation run.

VI-CarRealTime vehicles models are created by loading vehicle data from an existing ADAMS model but this is not included in this thesis. Different subsystems are individually accessible to modify and run different combination for optimization or testing. It is also possible to visualize the simulation and the output signal through Graphical User Interface (GUI).

Different open loop and closed loop maneuvers are preset in the platform and which can be modified as per requirements of the user. It also includes possibility building user defined maneuvers using feature called VI-EventBuilder to meet real world testing requirement.

VI-CarRealTime generates simulation files with signals that can be extracted again in matlab and simulink using Application Program Interface (API) and co-simulation for post processing.

2.3.2 Matlab API Toolbox

This toolbox enables the user to create and automate design of experiments in to order to find best vehicle configuration by modifying the vehicle model. In this toolbox, it is possible to manage the entire simulation process by interacting directly with the model input files (*.xml*) and modify the data directly in the Matlab environment[16]. These function packages can be used to interact with VI-CarRealTime models from Matlab and can be divided into three groups,

- Functions to modify systems and subsystems input files
- Functions to update the VI-CarRealTime Databases
- Functions to run the simulation and analyse

For further explanation on how this method was used in this study, please refer the methodology section 3.10.

3

Methodology

3.1 Maneuvers and standards:

The maneuvers included in this thesis mainly reflect the maneuvers used to calculate the vehicle dynamic performance under the standards defined by ISO and NHTSA[14]. The maneuvers are also tailored from the standard set to match the requirements set by the OEM. The maneuvers include Constant Radius maneuver (CR), Frequency Response (Sine Sweep), Sine with Dwell (SWD) and J Turn. Vehicle data that is recorded to derive the vehicle dynamics metrics are for example, steering wheel angle(SWA), velocities, accelerations, displacements and timestamps. Detailed explanations regarding the maneuvers can be found in section 3.3.

3.2 Magic Formula tire model and Parameter study

The Magic Formula tire model will be used for modeling the tire in these simulations. Magic Formula tire is an empirical model derived by fitting a model to measured data. These are non dimensional parameters which determine the shape of the tire force curve and captures the dynamics of the whole tire. Hence, this calls for a different approach to analyze the effects of these Magic Formula tire parameters on the vehicle performance parameters. Magic Formula tire has varying range of complexity in its parameters in order to capture most of the characteristic of tire. Additional parameters are included in order to capture the effect of load transfer, camber and combined slip phenomenon.

To understand the basic Magic formula tire with the generic parameters B, C, D and E, the study finds the influence of these parameters on the force vs slip curve. This shows the influence of theses parameters on the specific areas of the tire force graph.

The maneuver considered here for analyzing the effect of Magic Formula 5.2 tire parameters on the tire force curve is ramp maneuver at a speed of 80 kilometers per hour for 30 seconds in a simulink environment with a low fidelity vehicle model.

3.2.1 Simulation Environment

The simulation environment used for the Magic Formula tire generic parameter pre-study mainly involves a two track low fidelity model built in Simulink. Tires in this two track model are modelled with Magic Formula tire B,C,D and E parameters and the driver model is designed so as to replicate a ramp manoeuvre.

3.2.2 Vehicle model

The vehicle model is a two track model with lateral compliance built into the model. The roll effects on the overall dynamics of the car are also built into the model. The vehicle uses a nonlinear tires model with Magic Formula tire parameters. The slip model uses a switch for a transfer function block to simulate the relaxation behaviour of the tire.

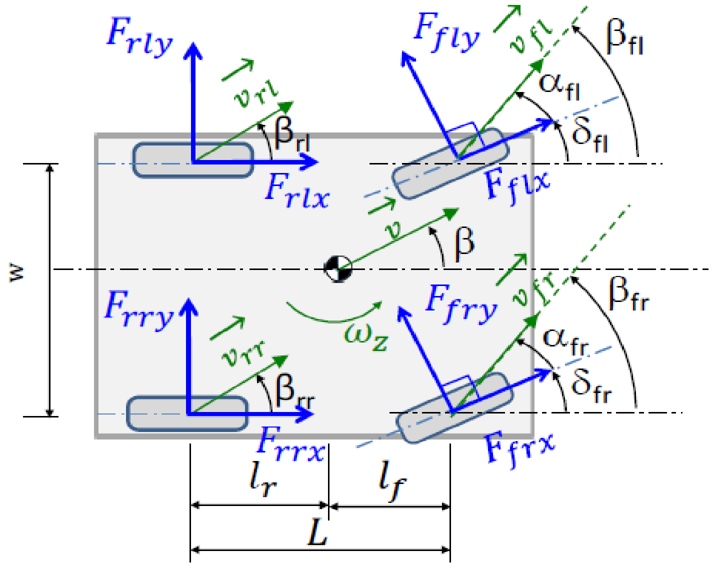

Figure 3.1: Free body diagram of a two track model[3], where $\vec{v}$ indicates velocity vector, β indicates body slip, α indicates tire slip angle, δ indicates steering angle and F indicates the tire forces

the lateral equilibrium for model in the figure 3.1 is expressed as

$$\sum F_{yv} = \text{m}.\, a_y = \text{m}.(\dot{v}_y - w_z.\, v_x) = F_{yflv} + F_{yfrv} + F_{yrlv} + F_{yrrv} \tag{3.1}$$

Transformation equation between vehicle and wheel:

$$\begin{aligned} F_{xv} &= \cos(\delta) F_{xw} - \sin(\delta) F_{yw} \\ F_{yv} &= \sin(\delta) F_{xw} + \cos(\delta) F_{yw} \end{aligned} \tag{3.2}$$

where,
m is the mass of the vehicle
a_y is the lateral acceleration
$\dot{v}_y$ is the lateral acceleration
w_z is the yaw rate
w is the width of the vehicle
l_f is the distance from CG to front axle

l_r is the distance from CG to rear axle
L is the wheel base of the vehicle
v_x is the longitudinal velocity
F_{yflv} is the lateral force front left: vehicle coordinate
F_{yfrv} is the lateral force front right: vehicle coordinate
F_{yrlv} is the lateral force rear left: vehicle coordinate
F_{yrrv} is the lateral force rear right: vehicle coordinate
F_{yv} is the lateral force: vehicle coordinate
F_{xv} is the longitudinal force: vehicle coordinate
F_{xw} is the longitudinal force: wheel coordinate
F_{yw} is the lateral force: wheel coordinate

Since no steering available at rear wheel, and assuming equal steering on front wheels, we have

$$\delta_{rl} = \delta_{rr} = 0 \tag{3.3}$$

$$\delta_{fl} = \delta_{fr} = \delta_f \tag{3.4}$$

where,
δ_{rl}, δ_{rr} are the rear wheel steering angle (left and right respectively)
δ_{fl}, δ_{fr} are the front wheel steering angle (left and right respectively)

Hence, the lateral equilibrium will be represented as

$$\begin{gathered}\sum F_{yv} = m.\, a_y = m.(\dot{v}_y - w_z.\, v_x) = (F_{yflw} + F_{yfrw})\ \cos(\delta) + \\ (F_{xflw} + F_{xfrw})\ \sin(\delta) + F_{yrlw} + F_{yrrw}\end{gathered} \tag{3.5}$$

3.2.3 Driver model

The driver model used here generates the ramp manoeuvre and is initiated at a constant speed of 80 kph for 30 seconds.

3.2.4 Results

The results from the simulation model are then used to understand the effects of the Magic Formula tire B,C,D & E parameters on the component level (tire level) metric. The metric used is the lateral force curve where, the lateral force developed by the tire is plotted against the side slip. This defines the tire force curve capturing the effects from changing B,C,D and E parameters +/- 50 percent from its reference value.

3.2.5 Procedure

The maneuver is run for the two track vehicle model with the Magic Formula tire model which includes reference parameter for B,C,D and E. The individual parameter values are varied for +/- 50 percent across both axles to investigate sensitivity of the parameters. Then results from the simulation shows the affect of Magic Formula tire parameters on the tire force curve.

3.3 Maneuvers and identification of handling and steering performance metrics

The change in the tire parameters will affect the overall handling and steering characteristics of a vehicle [3]. There is a need to define metrics which could be studied in order to relate the handling and steering characteristics of the vehicle to its sensitivity on Magic Formula tire parameters. Once the influential handling and steering metrics are identified, their targets values are obtained from OEM. The maneuvers used for vehicle dynamic tests are standard for OEMs. Right measures of input parameters defining the maneuver are used to simulate through its digital twin in VI-CarRealTime. A post processing script is created in accordance to the standard vehicle dynamic metrics derived by OEM. A reference model is used to check for its feasibility to run on simulation environment and to rightly capture the desired parameters.

3.3.1 Constant Radius Cornering

This is a quasi steady state maneuver with constant radius and increasing velocity. The test technique entails driving the vehicle at various speeds along a circular course with a defined radius. The circle's radius should be between 30 and 100 meters, with a tolerance of 0.5 meters. A radius of 40 meters was employed in the test for this investigation. The speed is gradually raised while maintaining vehicle in the intended direction, such that the lateral acceleration does not change at a rate faster than the 0.1 m/s^2. The vehicle's speed is increased until the vehicle can no longer maintain the planned course. For the sake of repeatability, many runs are undertaken in each direction. Steering wheel angle, steering wheel torque, yaw velocity, lateral acceleration, roll angle, side slip angle, and vehicle velocity/side slip angle should all be included in the measurement data[14].

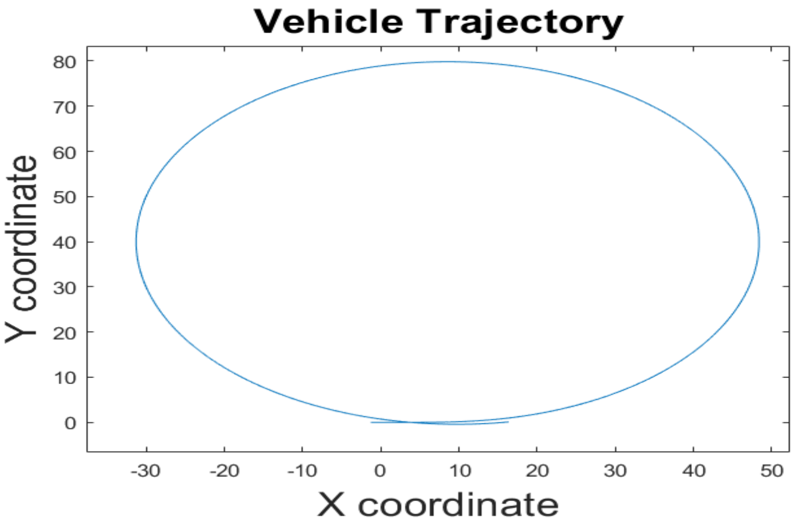

Figure 3.2: Vehicle Trajectory for constant radius maneuver

3.3.2 J Turn

J-Turn is a transient state maneuver developed by NHTSA to evaluate the lateral stability of the vehicle during the ESC intervention[14]. The maneuver is conducted at a speed of 120 kmph and the maximum lateral acceleration achieved is measured.

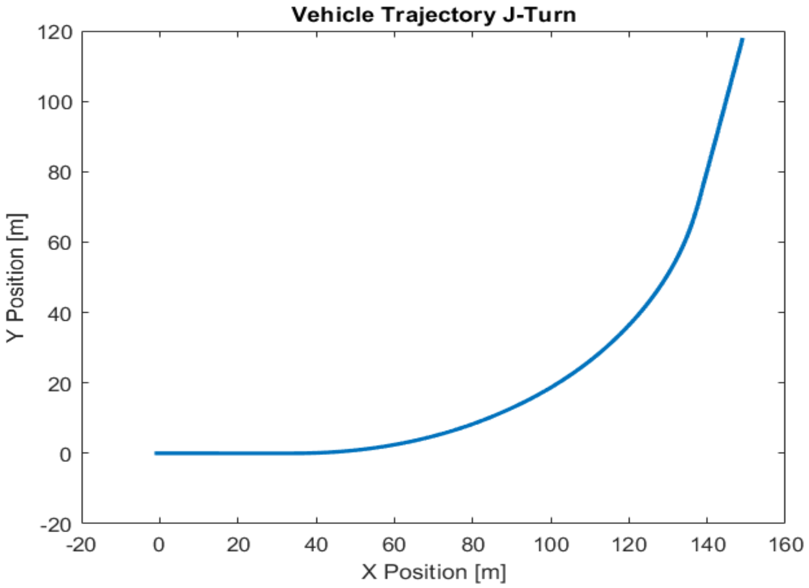

Figure 3.3: Vehicle Trajectory for J-Turn maneuver

3.3.3 Frequency Response

The frequency domain is used to investigate system reactions to sinusoidal steering excitation for different steering frequencies. The outputs, amplification factor and phase delay are functions of frequency and are visualized through bode plots. Modern passenger vehicles only exhibit appreciable sensitivity to parameter shifts in the range of 0.2 Hz – 4 Hz, according to this frequency analysis. The lower limit is defined as the difference between quasi static and dynamic responses, while the upper limit is justified by the point where the steering input covers the vehicle reaction bandwidth.

3.3.4 Sine with Dwell

Sine with Dwell is a transient state maneuver developed by NHTSA to evaluate the lateral stability of the vehicle during the Electronic Stability Control(ESC) intervention like the J-Turn maneuver[14]. It involves a sinusoidal steering input with a dwell in the second half of the cycle. The metrics for the sine with dwell maneuver is described in the table 3.1.

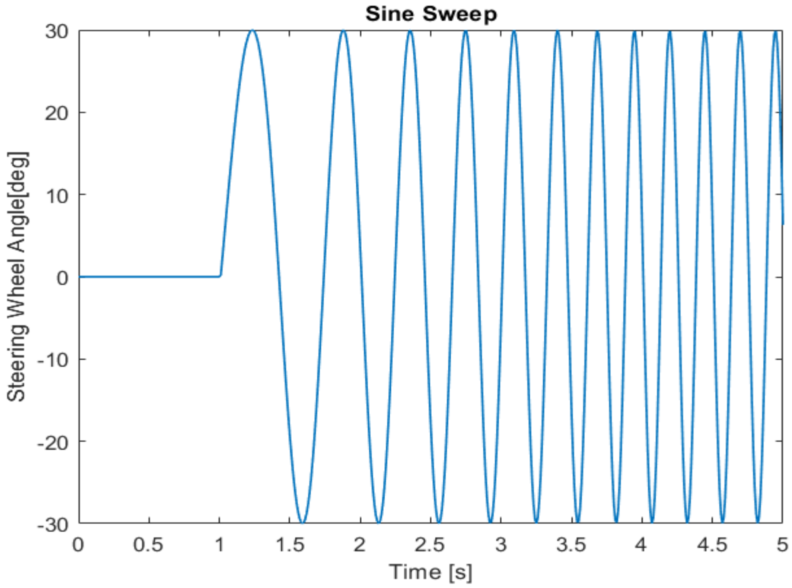

Figure 3.4: Steering input for Frequency Response maneuver

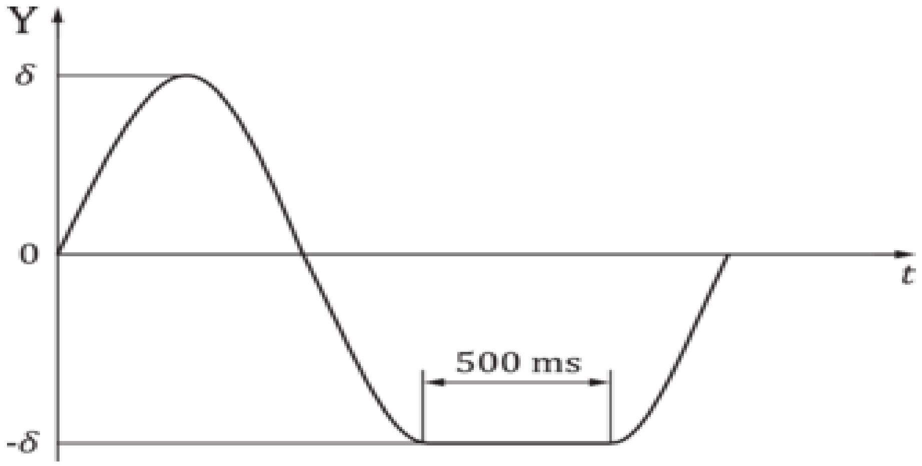

Figure 3.5: Steering input$(\delta)forsinewithdwellmaneuver$[11]

Parameter	Constant Radius	J-Turn	Frequency Response	Sine with Dwell
Initial Velocity(kmph)	0.0	120	120	80
Turn Radius(m)	40	-	-	-
Final Velocity(kmph)	80	120	120	80
Sweep Amplitude(deg)	-	-	30	-
Initial Frequency (Hz)	-	-	0.2	0.7
Final Frequency (Hz)	-	-	4	0.7
Nominal Lateral Acceleration(g)	-	-	-	0.4
Dwell Duration(ms)	-	-	-	500

Table 3.1: Parameters for the standard maneuvers

Maneuver	**Vehicle Dynamic metrics**
Constant Radius	Linear Understeer Coefficient
	Non-Linear Understeer Coefficient
	Progressivity
	Road Holding Capacity
J-Turn	Maximum Lateral Acceleration
Frequency Response	Yaw time lag @ 45deg phase delay
Sine with Dwell	Time lag between SWA and Yaw rate
	Time lag between SWA and Lateral Acceleration
	Time lag between Yaw rate and Lateral Acceleration

Table 3.2: Final vehicle dynamic metrics used and the maneuvers used to extract them

3.4 Verification of handling and steering targets with reference model

To verify the method used to compute the vehicle dynamic metrics using the standard maneuvers, the simulation results are verified with the results from a benchmarked model. Here the simulation environment (VI-CRT) and the results from the post processing (Matlab) script which quantifies the handling and steering metrics are compared against already measured vehicle dynamic handling and steering target. If the measured parameter is in range of the reference model, the script can then be used to conduct further sensitivity analysis of Magic Formula tire parameters to vehicle dynamic handling and steering targets.

3.4.1 Vehicle dynamic handling and steering metrics and targets

Linear Understeer Gradient:

The understeer gradient can be defined as the additional steering angle needed per increase of lateral force in a steady state cornering maneuver. For a linear two track model it can be computed with following expression.

$$Ku = \frac{C_f \, \mathrm{l}_f - C_r \, \mathrm{l}_r}{C_f \, C_r \, \mathrm{L}} \tag{3.6}$$

where,
Ku is the understeer coefficient
C_f is the front wheel cornering stiffness
C_r is the front wheel cornering stiffness
l_f is the distance from CG to front axle
l_r is the distance from CG to rear axle
L is the wheel base

The understeer gradient can also be computed from the handling diagram as the slope of the Steering angle vs Lateral Acceleration and can be computed as below.

$$Ku = \frac{Slope\, of\, SWA\, vs\, a_y}{Steering\, ratio} \tag{3.7}$$

for linear understeer this is computed in the range of 0.1 - 0.35g of the SWA vs a_y.

Non-Linear Understeer Gradient:

Like the linear understeer gradient, non-linear understeer gradient is computed with the expression 3.6 in the range of 80% of maximum lateral acceleration +/- 0.1g.

Progressivity:

Progressivity is defined as the ratio of non-linear understeer gradient to linear understeer gradient.

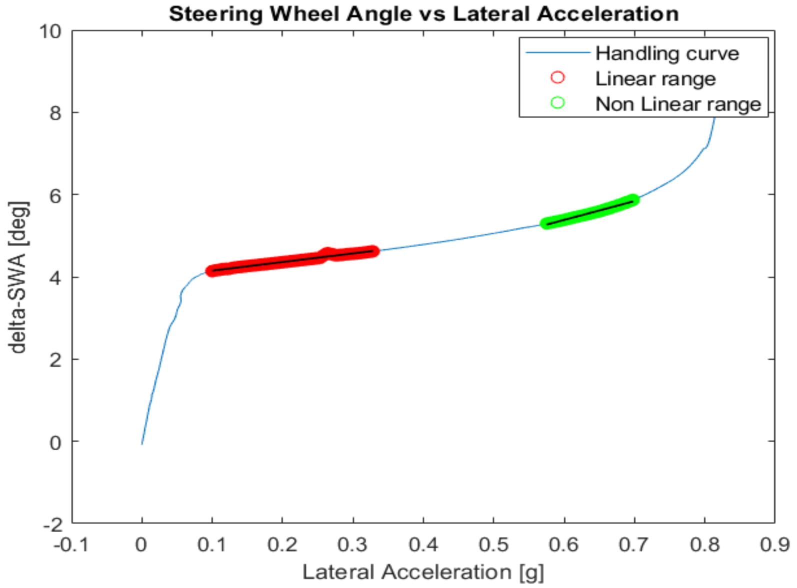

Figure 3.6: SWA vs Lateral Acceleration for constant radius maneuver that is used to compute the linear(green) and non-linear(red) understeer gradient

Road Holding Capacity:

The road holding capacity is the steady state maximum lateral acceleration achieved during the constant radius maneuver.

Bode Plot:

The result from the frequency response maneuver is analyzed through a bode plot. Bode plots show the characteristics of a system under different steering frequency excitations. The resulting output to these excitations determine the vehicle reaction bandwidth. The amplitude and phase characteristics of yaw rate and lateral acceleration are analyzed to derive vehicle dynamic metrics such as yaw time lag @45 degrees etc.

The gains are mainly Yaw rate gain dB(deg/sec),Lateral Acceleration gain dB(m/sec2). And the phase plot contains yaw rate delay (deg) and lateral acceleration delay (deg). The vehicle dynamic metric currently derived from the bode plot using frequency response maneuver is:

Yaw rate time lag @ 45deg:

The yaw rate time lag from the vehicle is derived from the phase plot, where it defines the lag time when the yaw rate phase is -45 *deg*. The time lag is obtained by dividing the inverse of that frequency by 8, as the frequency cycle is denoted by 360 degrees and -45 degrees contributes 1/8th the whole cycle time.

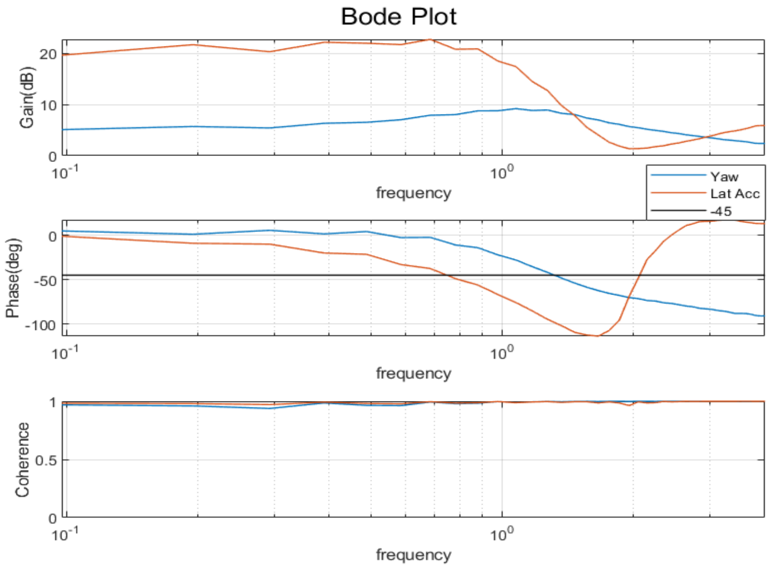

Figure 3.7: Bode Plot for frequency response maneuver

Maximum Lateral Acceleration (Max LatAcc):

The maximum lateral acceleration of the vehicle is considered as one of the vehicle dynamic metric for this study. The difference with the metric which measures the lateral acceleration under constant radius maneuver as road holding, is that the current metric defines the lateral acceleration limit for a vehicle when the tires across both axles are saturated. The maximum lateral acceleration recorded during this event is taken as the value for this metric. This metric thus defines the maximum lateral acceleration a vehicle can pull under any avoidance maneuver.

Time lag between Steering Wheel Angle (SWA) and yaw rate:

The time lag between Steering Wheel Angle (SWA) and yaw rate can be measured in several ways using Sine With Dwell (SWD) maneuver. In this study the measurement is taken as the time difference when the SWA crosses the zero line (when the steering wheel records zero degrees) after the first peak to that of the yaw rate (when the yaw rate crosses the zero line) of the system. The calculation for this metric follows the procedure, where the first peak of SWD event is considered. The time lag between the SWA and yaw rate at the time it crosses the zero line is considered to calculate this metric in terms of milliseconds.

Time lag between Steering Wheel Angle (SWA) and Lateral Acceleration:

The time lag between Steering Wheel Angle (SWA) and lateral acceleration can be measured in several ways using Sine With Dwell (SWD) maneuver. In this study the measurement is taken as the time difference when the SWA crosses the zero line (when the steering wheel records zero degrees) after the first peak to that of the lateral acceleration (when the lateral acceleration crosses the zero line) of the

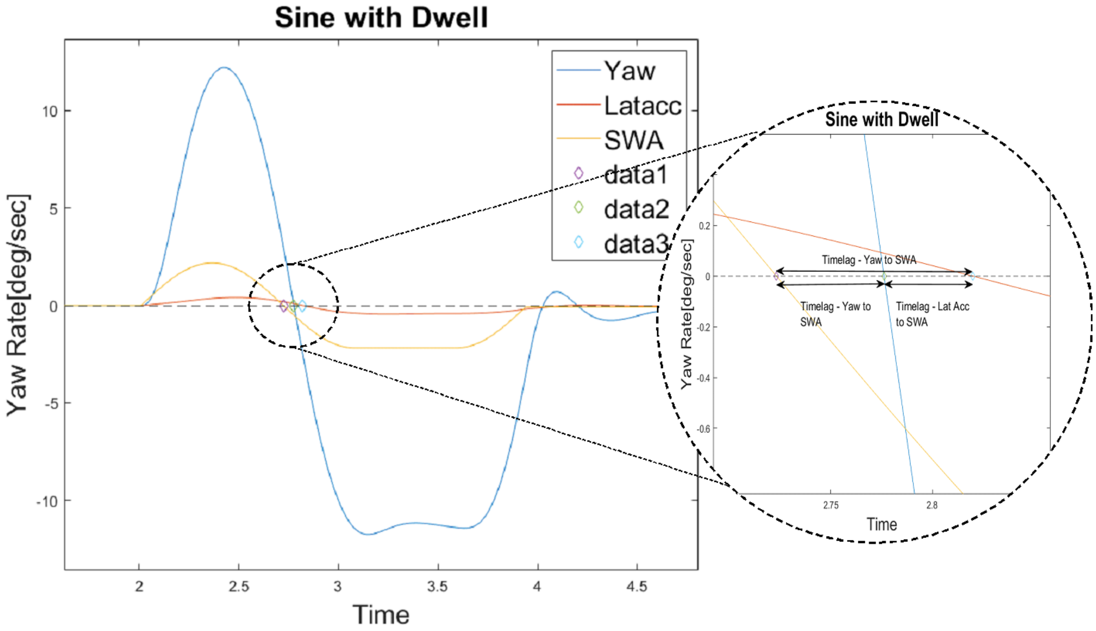

Figure 3.8: Plot of SWA, yaw rate and Lateral acceleration for sine with dwell maneuver

system. The calculation for this metric follows the procedure where the first peak of SWD event is considered. The time lag between the SWA and lateral acceleration at the time it crosses the zero line is considered to calculate this metric in terms of milliseconds.

Time lag between Yaw rate and Lateral Acceleration:

The consequence of time lag between SWA and lateral acceleration and, SWA and yaw rate being considered as metric explains the time lag between yaw rate and lateral acceleration. The result is computed from the SWD event where the difference in the timestamps are measured between lateral acceleration and yaw rate as it crosses the zero line after its first peak.This metric is measured in terms of milliseconds.

3.5 Automation of experiment process

This study of sensitivity of tire parameters requires a Design of Experiments (DoE) which involves different combinations of vehicle dynamic metrics and tire parameters. This involves running VI-CarRealTime simulation for different values of tire parameters individually across the front, rear and both axle configurations. Simulating these configurations manually consumes time. The study also includes running the percentage sweep over different tire parameters individually to study the sensitivity of parameter on the vehicle dynamic handling and steering performance. This necessitates developing a DoE that facilitates the simulation to run in a loop which includes the post processing script to extract the vehicle dynamic results using the output signal of VI-CarRealTime simulation software and the rewriting of the input files in order to conduct the sweep. In order to expedite this process an automation script is developed using the Matlab API interface from the VI-CarRealTime library. This script allows the user to modify the tire parameters, run predefined standard maneuvers and extract the required vehicle dynamic results. The flow of the automation process is described in the figure 3.10

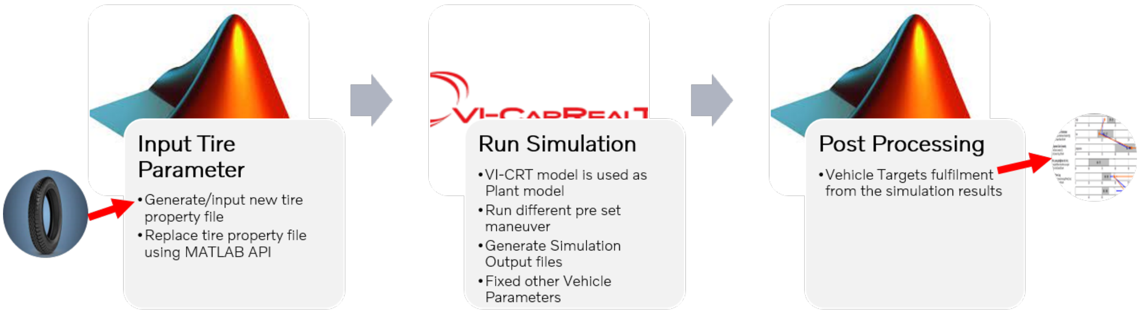

Figure 3.9: Automation process flow

The input to this automation script is the tire property file in *.tir* format and output are the results of the identified vehicle dynamic metric.

3.5.1 VI-CarRealTime

The maneuvers required to study the sensitivity of tires and to extract the vehicle dynamic results are from the VI-CarRealTime software. The standard maneuvers are available in Test Mode tab. The maneuvers are extracted from the software as *.xml* system files using the *files only* mode in order to run the maneuver externally from matlab environment.

3.5.2 MATLAB

Matlab is the programming language that has been used to process the on track and the virtual test and simulations data[14]. In this study, a matlab script has been developed in such a way that, with any given tire parameters as input it was possible to simulate the process and extract the results. A model defined in the database of the VI-CarRealTime is loaded into matlab environment using the

$.xml$. And a system structure is generated using the function $generateSystemStruct$ available in the Matlab API toolbox. The script enables the user to define desired tire parameters or range of values of tire parameters as input to the system. These parameters are used to create a custom tire property file ($.tir$) using user developed function $CreateInput$. The tire property file in the model system file is replaced by the custom tire property file generated by the user using the $frontTireSetPropertyFile/rearTireSetPropertyFile$ function and is saved in the main system struct file. With new tire property file with user defined tire parameters being uploaded in the system files of the model, this model is then simulated using $runViCrt$ function. By defining the $baseFringeprint$ using standard maneuver xml files saved from the VI-CarRealTime, $runViCrt$ function allows the user to run the selected maneuvers to generate output signals in the matlab environment. These output signals are then used by the post-processing script to compute the vehicle dynamic results as described in section 3.4.

3.6 Sensitivity analysis of tire parameters on defined handling and steering metrics

Once the parameter pre study of generic Magic Formula tire model and description of the handling and steering metrics are defined and studied, the effect of change in the tire parameters on the overall handling and steering metrics of the vehicle needs to be analysed. From the basic understanding of the individual tire parameters effect on the tire force curve and verified pre processing script which captures the right vehicle dynamic metric, an objective sensitivity matrix was created from the DoE results using automated script.

In order to observe the changes to the vehicle dynamic metrics by changing the Magic Formula Tire parameters, a maneuver catalog was devised. This was defined so that a standard procedure could be performed across all the significant Magic Formula Tire parameters and record the respective changes in the vehicle dynamic metrics. In order to identify the significant Magic Formula Tire parameters from the set of all the Magic Formula Tire variables available from the PAC2002 set, an initial parameter sensitivity run is conducted to record the effect of these parameters on the force curve. And logging the data from the study and from discussions with some of the attribute leaders and thesis supervisors in the organization, a conclusion was derived where the initial DoE will only include lateral force empirical equation parameters while not including on aligning moment and longitudinal force parameters. The parameters in the lateral force equations were then decided to be limited to most influential magnitude parameters (p_{KY1}, p_{DY1}, p_{CY1}, p_{EY1} and p_{TY1}) and load sensitive parameters (p_{KY2}, p_{DY2}, p_{EY2} and p_{TY2}) respectively from the initial tests made.

The standard operating procedure for experimentation in order to record the vehicle dynamic results is then devised. The methodology expands as follows:

3.6.1 Set up of Experimentation Model

Firstly, a reference vehicle model from ADAMS was uploaded into the simulation software which is not included in this study. Then the set of maneuvers were loaded into the tool chain which is used by the simulation software to analyze the reference model for the defined events. The reference model includes a reference tire which is used for the study and this tire is input to the simulation as a .tir file which has all the properties modelled in terms of its Magic Formula tire 5.2 parameters. The simulation is then run where the solver solves the vehicle model across the maneuvers and publish the respective results in-terms of its standard 96 output variables. The results are then carried to a custom post-processing script where the outputs from the simulation is processed in order to compute the defined vehicle dynamic metrics. The post processing script is written in MATLAB and the simulation results are exported from VI CRT to MATLAB environment which is seamlessly possible through the prescribed API method.

3.6.2 Varying the Tire properties

Individual tire property parameters(Magic Formula Tire Parameters) like p_{KY1} ,p_{KY2} ,p_{DY1} ,p_{DY2}, p_{CY1} ,p_{EY1} ,p_{EY2} ,p_{TY1} and p_{TY2} (Section 2.1.3.1) are varied across +/-20 percent from their reference value in order to capture their influence on the vehicle dynamic metrics. Once these individual tire parameters are varied across +/-20 percent from their reference value across both the axles to study their influence on the vehicle dynamic metrics. The same DoE is followed in order to capture the influence of Magic Formula tire parameters on vehicle dynamic metrics by changing tire parameters only on front axle and then only on rear axle. The simulation software sets the same tire properties mirrored on the same axle(left and right tires), but can insert a different set of tires i.e. different tire property file (.tir) across different axles for the sensitivity study.

3.6.3 Sensitivity study

The sensitivity study is done across 3 axle combinations by changing tire parameters on front axle tires, on rear axle tires and on both axles tires to look for the influence of Magic Formula tire parameters on vehicle dynamic metrics. The study included firstly studying the sensitivity of Magic Formula Tire parameters for the front axle alone by keeping the rear axle standard to that of the reference model and varying the rear axle properties alone by keeping the front axle properties the same as that of the reference model. Finally all the tires are changed across both the axles to study the sensitivity of these Magic Formula Tire parameters on the selected vehicle dynamic targets of the car. The sensitivity is defined as delta change or a percentage change. Thus the tire parameter in every test case is varied for 20 percent and the vehicle dynamic results are recorded subsequently for the changes in the tire parameters and the sensitivity is calculated by finding the difference of the newly obtained value to the reference value divided with the original value. We obtain the delta percentage sensitivity for every test case.

3.6.4 Sensitivity analysis across axle combinations

The results from the sensitivity study across all the axle combinations are recorded and are compared across with the results from the other combination by normalizing the sensitivity values by the percentage change in the input variation, which in this case is 20 percent. The final normalized sensitivities are recorded and then compared against each other to order them according to their sensitivities based on their axle characteristics. The sensitivity matrix obtained after running the DoE for the handling and steering results is then plotted against the bench marked criterion and will be analyzed accordingly. The analysis aims to come up with an output chart which captures the effect of Magic Formula tire parameters on individual tire curves and then its effect on the vehicle handling and steering. The resulting analysis of the obtained tire curves to the bench marked curves will aid in its communication with the tire suppliers and then the further implication of this on the vehicle handling and steering performance will help the design engineers in balancing the attribute targets early in the product development cycle.

3.7 Optimization of Magic Formula tire parameters for vehicle handling and steering targets

The aim of this optimization algorithm revolves around the fact that one can derive a set of Magic Formula Tire (PAC2002) parameters by setting the vehicle dynamic targets specified in the vehicle level by using optimization algorithm. As the current study includes only lateral force equations of Magic Formula tire equations, the optimization is done only for the lateral force parameters of the tire parameter file for the pre defined vehicle dynamic targets.

3.7.1 Objective function

An objective function is the most important element in setting up an optimization algorithm.A function that transfers an event or the values of one or more input variables onto a real number that intuitively represents some "cost" connected with the occurrence or an output requirement is known as an objective function. A loss function is minimized in an optimization scheme.The objective function here represents the relationship between the input Magic Formula tire parameters and the output vehicle dynamic metric targets variables.

The major part of the optimization algorithm is constructing the right equations for the objective function which defines a relationship between the Magic Formula tire parameters to the vehicle dynamic targets. Considering that currently there is no physical relationship between the input and output variables namely vehicle dynamic metrics and Magic Formula Tire lateral parameters, the objective function then need to be set up with a generic mathematical relations which has a convergence or can show a minimal value in its formulation. A linear regression method with variables for each factor being the difference of the computed value to the target value is used in the objective function and also to avoid the error in the overall function due to the influence of sign between the computed and the target value. Thus the final factor of the objective function includes a linear regression function with target and computed value which in whole has a convex trend. Considering there are multiple vehicle dynamic targets and the optimization algorithm needs to optimize the Magic Formula Tire parameters for all the vehicle dynamic targets, the final objective function is the sum of the squares of the individual difference in the vehicle dynamic metrics as shown in equation 3.9. Since the units associated with the vehicle dynamic targets are different, a weight factor is needed to normalize the units. This factor called as normalization factor is explained in the next section.Here X is the basic form of the objective function which needs to be further improved to capture the right convergence profile like a convex or concave function with a local minimum or maximum.

$$X = \sum_{i=1}^{9} (y_i - h_i)^2 \tag{3.8}$$

where, i is the no of vehicle dynamic metrics, h_i is the predicted value and the y_i is

the actual value.
The equation 3.8, can be adapted to this project with computed vehicle dynamic metrics and targets as,

$$X = \sum_{i=1}^{9} (Target_i - CAE\ Result_i)^2 \tag{3.9}$$

where, $CAE\ Result_i$ are the vehicle dynamic results from the simulations and $Target_i$ are the defined targets.

3.7.2 Normalization factor

Considering there are multiple targets included in X equation 3.9, the final sum of the objective function for it to output a reasonable value as its cost needs to be normalized. Thus a normalizing factor needs to be included in the equation. There are multiple individual vehicle dynamic metric values independent of each other in the objective function. A single normalizing factor is thus not sufficient. The normalization needs to be done for every vehicle dynamic target in the objective function. To derive a reasonable normalizing factor in the objective function, each metric component of the function is divided by the absolute value of the difference between the maximum value of that vehicle dynamic metric to its minimum. Normalization makes sure that the output from the ratio remains non dimensional value. This way the maximum and minimum of these normalizing factor can be derived by running the simulation for the maximum and minimum values of the tire parameters possible (upper and lower bounds) and then derive the vehicle dynamic results for the same. The absolute difference of these individual factors resulted in the final normalization factor for the objective function as described in equation 3.10.

$$Normalising\ Factor_{Tgti} = |CAE\ Result_i(X_{max}) - CAE\ Result_i(X_{min})| \tag{3.10}$$

where,

$X_{max} = 1.1 * (p_{KY1}, p_{KY2}, p_{DY1}, p_{DY2}, p_{CY1}\ , p_{EY1}, p_{EY2}, p_{TY1}, p_{TY2})$

$X_{max} = 0.9 * (p_{KY1}, p_{KY2}, p_{DY1}, p_{DY2}, p_{CY1}, p_{EY1}\ ,p_{EY2}\ , p_{TY1}, p_{TY2})$

$CAE\ Result_i(X_{max})$ is the vehicle dynamics metric value measured with X_{max} tire parameters as input

$CAE\ Result_i(X_{min})$ is the vehicle dynamics metric value measured with X_{min} tire parameters as input

Using the equation 3.10 in equation 3.9, the objective function is defined as below

$$Objective\ function = \sum_{i=1}^{9} \frac{(Target_i - CAE\ Result_i)^2}{Normalising\ Factor\ _{Tgti}} \tag{3.11}$$

3.7.3 Weighing factors

The final part of the objective function is to introduce the weights for the factors in the objective function. The derivation of the weights were split into two different categories.The units for individual weights will be (1 / unit of vehicle dynamic target).

1. Computing weights from the sensitivity matrix

To improve the structure in the optimization algorithm, clear dependencies between the vehicle dynamic metrics needs to be specified and should be weighed accordingly giving a hierarchy to the influence of tire parameter to the vehicle dynamic target. The sensitivity matrix derived from the sensitivity study conducted is used in the optimization algorithm to weigh the tire parameters with the vehicle dynamic metrics. The sensitivity matrix will be different for different axle configurations. There will be three sensitivity matrices included in the algorithm, were for any optimization with respect to axle configuration a specific sensitivity matrix can be chosen.

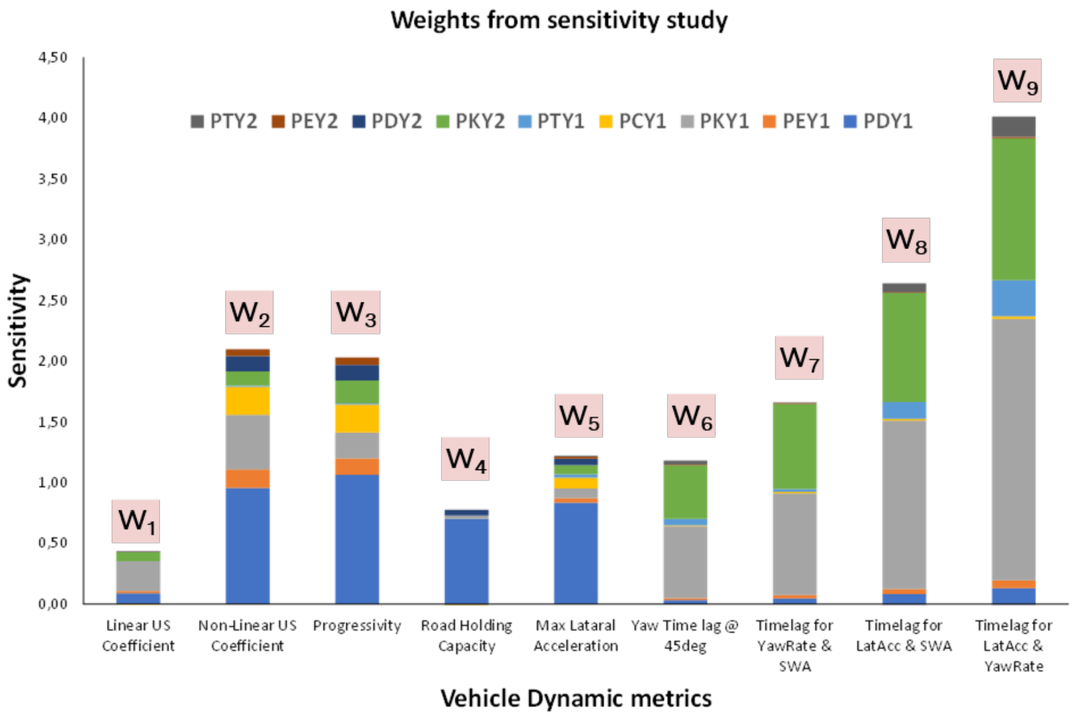

Figure 3.10: Weights derived from sensitivity study results

2. Introducing the weights from the attribute leaders

In order to categorize the vehicle dynamic metrics as a whole based on their importance with respect to each other in determining the DNA of a car, weights are introduced based on how important is the target to the overall performance of the car. These weights can only be defined by the attribute leaders of that Original Equipment Manufacturer (OEM). The weights of these will be of the highest magnitude and thus has the highest weight factor.

$$Weights = [W_{VD1}\, W_{VD2}\, W_{VD3}\, W_{VD4}\, W_{VD5}\, W_{VD6}\, W_{VD7}\, W_{VD8}\, W_{VD9}] \tag{3.12}$$

With the weights describing the dependencies with the tire parameters and the vehicle dynamic metrics and the ones with overall performance requirement of the car defined, a final objective function is formed as below equation. The

$$Objective\ Function = \sum_{i=1}^{9} [W_i + W_{VDi}]' * \frac{(Target_i - CAE\ Result_i)^2}{Normalising\ Factor\ _{Tgti}} \tag{3.13}$$

where,

W$_1$, W$_2$.... are the weights for tire parameters derived from sensitivity study

W$_{VD}$ weights from the attribute leaders for vehicle dynamic target optimization

3.7.4 Optimization scheme:

The optimization schemes considered for this thesis were the schemes ranging from FMINCON, LSQNONLIN and Genetic Algorithm schemes(GA).

Genetic Algorithm (GA) is an optimization scheme where the solution for constrained and unconstrained objective functions are made based on the natural selection rather than the gradient as in the case of FMINCON and LSQNONLIN.A population of individual solutions is repeatedly modified by the genetic algorithm. At each phase, the genetic algorithm chooses parents at random from the current population and utilizes them to generate the following generation's children. The population "evolves" toward an ideal solution over generations. This usually takes a large amount of time in order to converge to a minimum in the case of multiple output and input parameters.Thus this schemes takes the highest time among FMINCON, LSNONLIN and GA.

FMINCON and LSQNONLIN are similar kind of optimization schemes if the optimization relies of providing the gradient at a particular point.But the algorithms were then pilot tested on the current current objective function along with the bound and the selection of initial guess point helps in getting to the closest local minimum possible, particularly if the case contains multiple local minimums. Between the two schemes FMINCON runs the optimization in considerably faster as compared to LSQNONLIN, where by giving similar results in the end.

Once the objective function is decided then the optimization function is set accordingly. The optimization scheme considered for this problem is the ´FMINCON´ scheme.

FMINCON is an optimization scheme for multi variable constrained nonlinear objective function. The optimization follows the scheme whereby the slope at an initial guess is taken and based on the slope of the curve the steps are taken until the values converge to a minimum point. The input variables to this optimization scheme includes an initial guess point, bounds between which the optimization scheme can operate and the step size the scheme can take between each iteration:

The explanations of the input variables used in this scheme are as follows:

1. Lower and Upper Bounds:

The optimization to converge with a suitable output after optimization and to reduce the overall time taken for the optimization, bounds for individual input tire parameters are pre-loaded into the algorithm. All the Magic Formula tire input parameters are given a lower and an upper bound between which the optimization algorithm can work. Currently the bound are input as 10 percent from their reference state. This can be further improved with the data from the respective tire manufactures.

2. Initial Guess point:

The input variables in this case the Magic Formula Tire parameters are given an initial guess around which the algorithm tries to look for the minimum cost. The initial guess could either be random or the values close to a reference tire around which the optimization will try to find the optimum result. If the initial guess is random then the optimization algorithm will try to find optimum point around the guess but can venture out based on the topology of the objective function. The initial guess is rather important where there are chances of having multiple local optimums and there are high chances that the optimization algorithm find the closest local optimum to the initial guess point. It is feasible to feed in the reference tire parameters as the initial guess as the scheme tries to find a optimum point close to the initial guess and the initial guess is sure to be realizable tire.

3. Step size between each optimization iteration:

The step size between each optimization iteration can also be set in this process as one of the tune-able input parameters. The correction in the optimization and the accuracy can be increased based on the step size of the scheme. Currently the step size is maintained at 0.01.

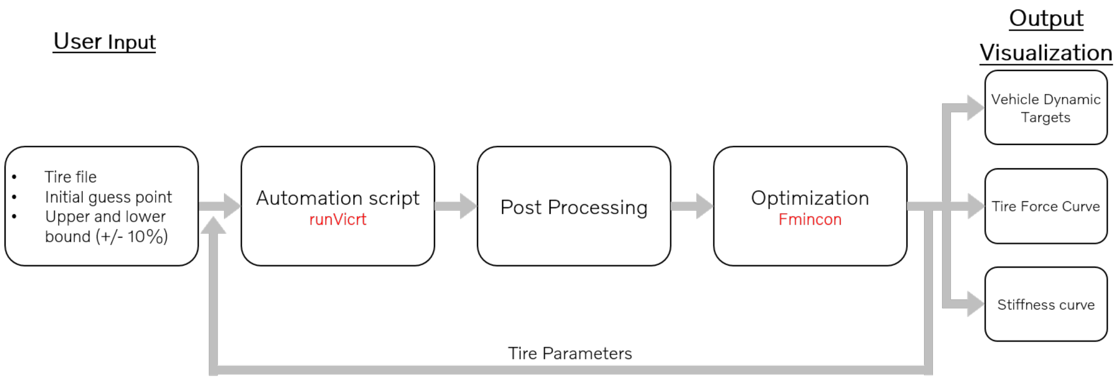

Figure 3.11: Flow of final optimization process

3.8 Output Verification

3.8.1 Vehicle Dynamic Target Verification

The optimization scheme is run based on the input parameters mentioned previously and the scheme outputs the optimized Magic Formula Tire pure Lateral force parameters for the specified vehicle dynamic targets. The tire properties obtained from the objective function is run through the simulation model and verified if the vehicle dynamic results are met or are closer as compared to that of the reference tire.

3.8.2 Stiffness Curve

Another important property which characterizes a tire is its stiffness curve.Stiffness curve defines the cornering stiffness of a given tire which includes the maximum cornering stiffness limit and the load at which the peak cornering stiffness is achieved.The stiffness curve can be plotted for the original tire and compared against the optimized tire to look for the feasibility for the tire manufacturer to manufacture the compound.

3.8.3 Lateral Force curve and Load Sensitivity

Another most important property that defines the property of a given tire are their lateral force curves for a range of slip. Thus, the optimized tire from the algorithm is then simulated in the virtual flat track environment in order to obtain the lateral force curve for varying normal load cases, camber and inclination conditions. And the optimized tire is compared against the reference tire looking for its characteristics.

Another test that is conducted to validate the optimized tire is the derivation of its ‘g-g’ diagram (Lateral Acceleration vs Longitudinal Acceleration). This simulates its maximum grip properties with reference to it reference tire. This ‘g-g’ diagram defines the friction ellipse for a given tire and the experimentation conditions are maintained identical and the optimized tire is then compared against the reference tire. Thus, defining its friction ellipse and its comparison with the reference tire.

4

Results and Discussion

4.1 Magic Formula Tire model parameter study

The influence of each Magic Formula tire parameter is studied using the changes to the tire force curve with respect to the variation in the generic Magic Formula parameters such as B,C,D and E. These tire force responses will provide an understanding and insight into the expected behavior of the vehicle dynamics performance.

The tire parameter B in equation 2.1 defines the stiffness of the tire. This can be interpreted from the slope or gradient at zero slip of the tire force curve as described in the figure 2.3. Hence variation in the magnitude of parameter B changes the slope or gradient at zero slip of the lateral tire force. This variation is directly proportional to parameter B i.e. with increasing magnitude of B, the stiffness of the tire increases as evident from the figure 4.1.

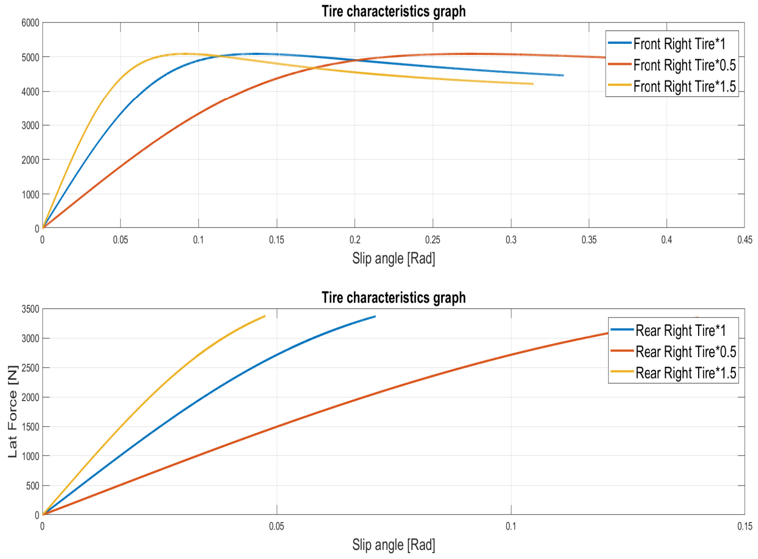

Figure 4.1: Illustration of variation in front and rear lateral tire force curve for 50% (red), 100% (blue) and 150% (yellow) of reference magic tire parameter B value

The behavior can be more evident when with plot the cornering stiffness of the tire as in figure 4.2. The peak of the cornering stiffness increases with parameter B exhibiting its relationship with the cornering stiffness for a tire which is defined by the relation $arctan(BCD)$ in figure 2.3.

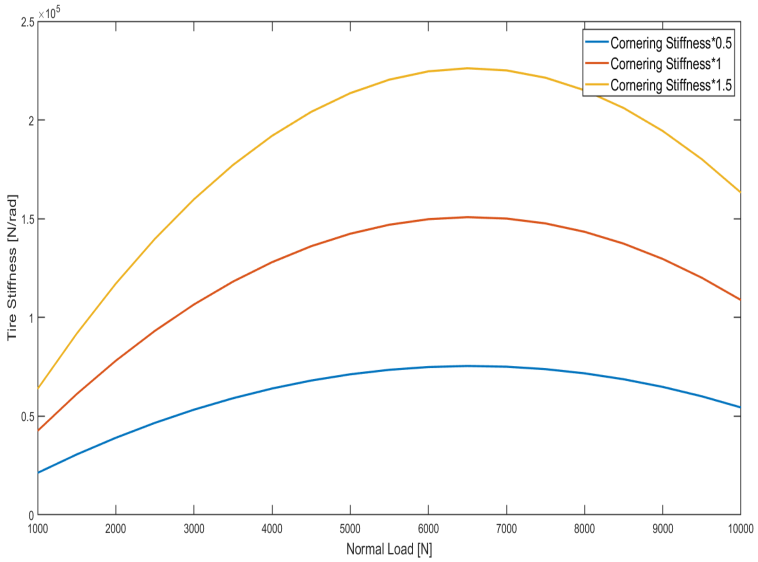

Figure 4.2: Illustration of variation in cornering stiffness of tire for 50% (red), 100% (blue) and 150% (yellow) of reference magic tire parameter B value

The peak value of the tire force curve is defined by the tire parameter D as clearly understood from the equation 2.1. The parameter D is the amplitude for sine function. Hence the maximum lateral force is directly proportional magnitude of the D. This behavior is described in the figure 4.3, with increasing peak value of the lateral force with increasing magnitude of D.

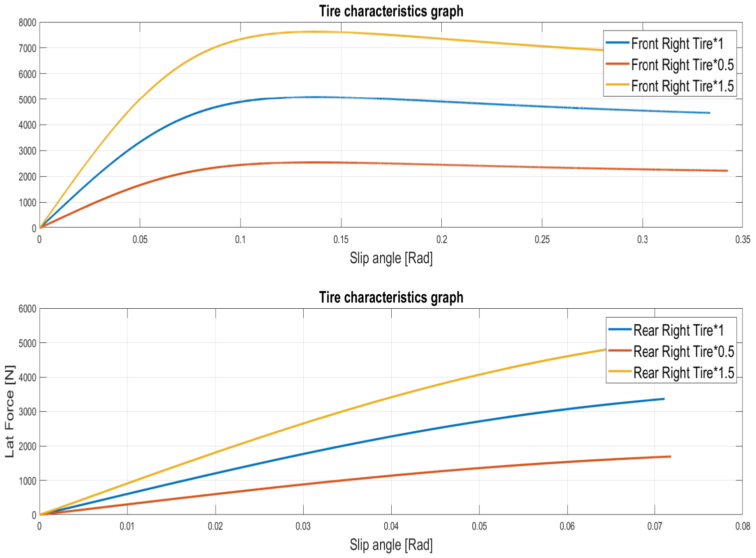

Figure 4.3: Illustration of variation in front and rear lateral tire force curve for 50% (red), 100% (blue) and 150% (yellow) of reference magic tire parameter D value

The parameter C is the shape factor, and controls the limits of the range of the sine function[1] of the equation 2.1. Hence the magnitude controls shape of the the asymptote of the tire force curve. The increase in the parameter C pushes the curve to drop after the peak value and decreases the saturation force influencing the tire limit behavior. The value usually is maintained between 1 to 2.

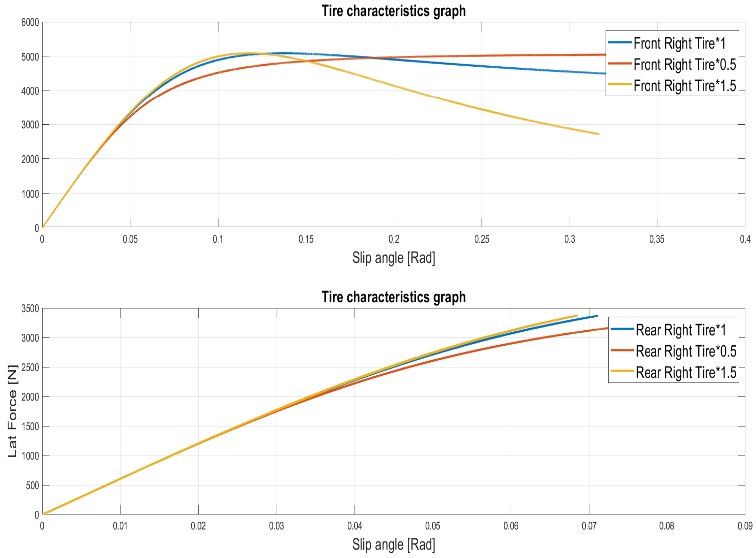

Figure 4.4: Illustration of variation in front and rear lateral tire force curve for 50% (red), 100% (blue) and 150% (yellow) of reference magic tire parameter C value

The parameter E is introduced to control the curvature at the peak and the horizontal position of the peak [1]. It controls the shape of the transition from non linear to saturation region.

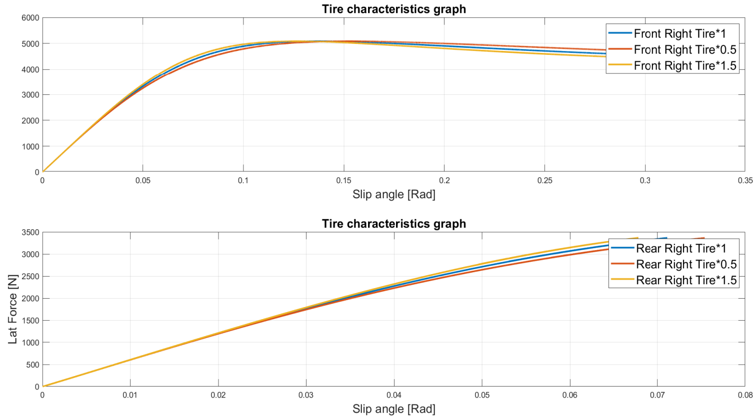

Figure 4.5: Illustration of variation in front and rear lateral tire force curve for 50% (red), 100% (blue) and 150% (yellow) of reference magic tire parameter E value

4.2 Verification of handling and steering targets with reference model

Different vehicle dynamic performance metrics were explored to identify the pre-eminent metric which captures the handling and steering characteristics influenced by change in tire properties, these targets of these metrics are verified against the bench-marked vehicle.

4.2.1 Constant Radius Cornering

Constant radius maneuver is a steady state maneuver and this maneuver is used to identify the handling and steering target Understeer Gradient. This metric exhibits the tendency of the vehicle in a steady state maneuver to be under-steering or over-steering and is measured as deg/g. This computed value of the metric for the bench-marked vehicle is compared against the target mentioned in table 4.1 in-order to validate the tool chain starting from maneuver input to post processing the individual vehicle dynamic handling and steering metric. Both linear and non-linear understeer gradient metric are in good correlation with the target requirement standard.Progressivity is the ratio between non linear understeer coefficient to the linear understeer coefficient. Progressivity value is in good co-relation to the target requirement.

Another metric that is extracted from this maneuver is the road holding capacity. This is the maximum lateral acceleration achieved in the quasi steady state maneuver. Road holding capacity measure in terms g force is observed to be within requirement range.

Metric	Simulation Value	Target requirement
Linear Understeer Gradient [deg/g]	2.099	2.10
Non-Linear Understeer Gradient [deg/g]	4.631	4.2
Progressivity	2.207	2.0
Road Holding Capacity [g]	0.780	0.8

Table 4.1: Verification of vehicle dynamic results from constant radius maneuver with targets

4.2.2 Frequency Response Maneuver

Frequency Response maneuver is used to evaluate the system response to sinusoidal steering excitation's of constant amplitude and varying frequency. The handling and steering target computed with this maneuver is yaw time lag at -45 degree phase delay. The target requirement and the value extracted from the current model in this project has good correlation as described in table 4.2.

Metric	Simulation Value	Target requirement
Yaw time lag at 45 deg phase lag [ms]	94.807	94

Table 4.2: Verification of vehicle dynamic results from frequency response maneuver with targets

4.2.3 J Turn - NHTSA

The J Turn maneuver developed by NHTSA, saturates the tires across the axles and hence signifies the limit performance of the tire. The maximum lateral acceleration is achieved through this maneuver is used as a vehicle dynamic metric in this study. Though this metric is not part of the standard performance requirement for OEM, in this thesis this serves as good indication to study the influence of tire parameters under peak and full slip conditions.

Metric	Simulation Value	Target requirement
Max Lateral Acceleration vs SWA [g]	0.910	Not a defined target

Table 4.3: Verification of vehicle dynamic results from J-turn maneuver with targets

4.2.4 Sine with Dwell

This is transient maneuver is used to extract the time lags for Yaw rate and Lateral Acceleration. This is an important handling and steering target when developing a vehicle.The computed values of the metric are compared against the standard are mentioned in table 4.4. It is seen to be in close agreement with the requirements.

Metric	Simulation Value	Target requirement
Time lag Yaw vs SWA [ms]	57.266	60
Time lag Lateral Acceleration vs SWA [ms]	98.124	100
Time lag Yaw vs Lateral Acceleration [ms]	40.858	40

Table 4.4: Verification of vehicle dynamic results from sine with dwell maneuver with targets

4.3 Sensitivity Analysis of tire parameters on defined targets

This section describes the sensitivity of the tire parameters on the vehicle dynamics targets described in the section 4.2. The sensitivity is explained for three axle configurations used in the study,

- Changing tire parameters on both axles
- Changing tire parameters on front axle only
- Changing tire parameters on rear axle only

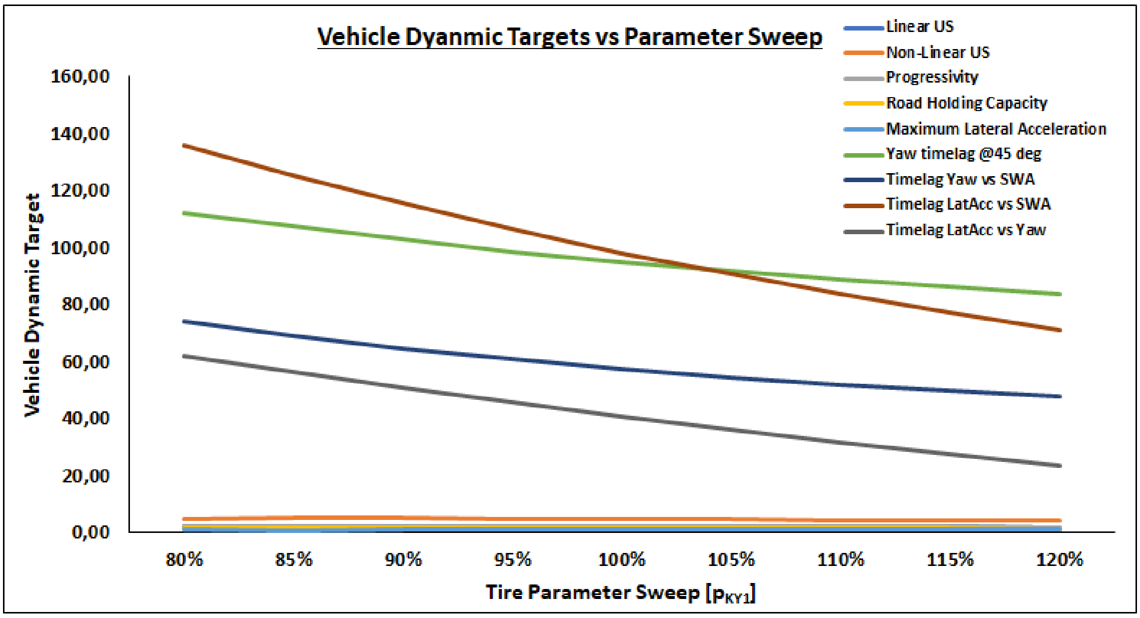

Figure 4.6: Trend of all vehicle dynamic results over sweep of tire parameter values

The sensitivity was conducted by sweeping over the range of individual tire parameters and observing the trend of variation of each result. From figure 4.6, the variation of all the vehicle dynamic results are linear with respect to the sweep for parameter p_{KY1}. Similarly it was found to be linear with all the tire parameters considered in the study. This result is used to define the normalizing factor to derive the sensitivity matrix.

The sensitivity results are described graphically as stacked bar graphs indicating both the magnitude and direction of influence. The tire parameters for the particular vehicle dynamic target are also arranged in the descending order of their sensitivity in the table. And a table is used to build up to the final sensitivity matrix for further study in the optimization.The study conducted is specific to a single vehicle model and one standard tire model.

The sensitivities are computed as, ratio of the final output of the vehicle dynamic metric in terms of it percentage to its reference value to the percentage change in the input tire parameter.

$$Sensitivity = \frac{\%\, change\, in\, vehicle\, dynamic\, metric}{\%\, change\, in\, tire\, parameter\; (20)} \tag{4.1}$$

where the percentage change in the vehicle dynamic metric for every change in tire parameter is computed with below method,

$$\%\, change\, in\, metric = \frac{Vehicle\, metric\, computed\; - Reference\, Vehicle\, metric}{Reference\, Vehicle\, metric} \tag{4.2}$$

4.3.1 Linear Understeer Coefficient

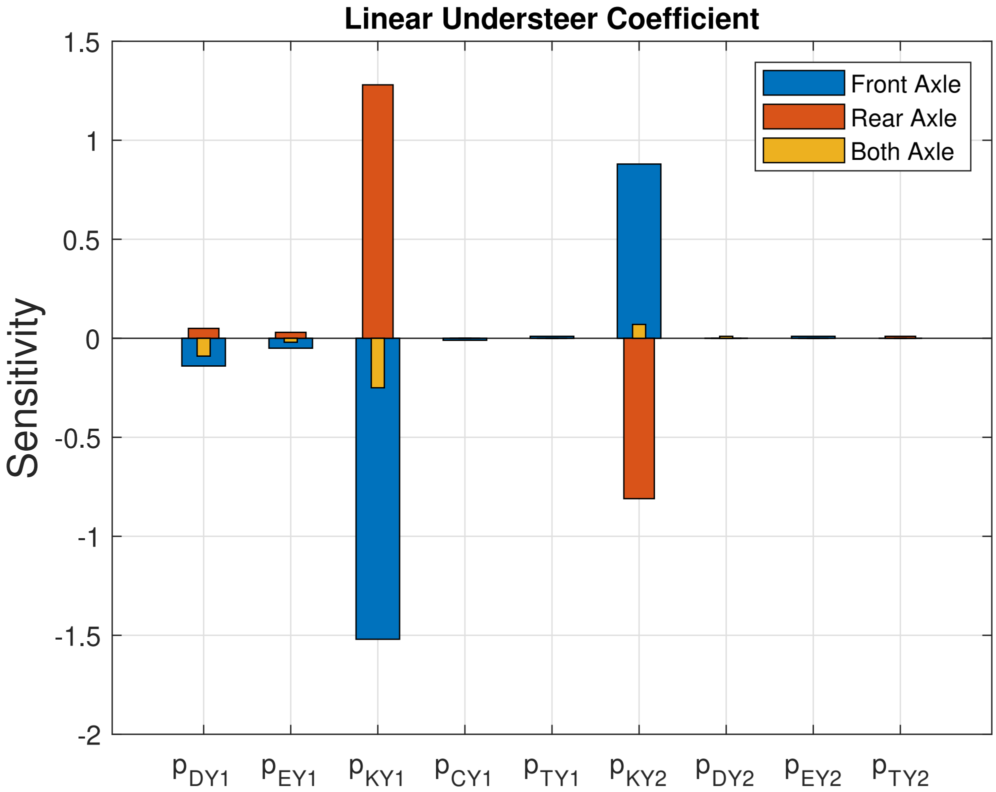

Figure 4.7: Illustration of sensitivity of magic formula tire parameters on Linear understeer metric for front axle(blue), rear(red) and both axle(yellow) configuration

Both axle configuration

From the figure 4.7, it can be seen that p_{KY1} is the most sensitive tire parameter influencing the linear understeer coefficient. It can be observed from the force vs slip figure 2.3, the stiffness of the tire is represented by the slope of the curve at

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Linear US Coefficient	-0.25	-0.09	0.07	-0.02	0.01	0.00	0.00	0.00	0.00

Table 4.5: Sensitivity scale of tire parameters on Linear Understeer - Both axles

zero slip,hence variation in the magnitude of p_{KY1} influences the stiffness of the tire. The negative sign indicating that increase in the magnitude of p_{KY1} reduces linear understeer gradient and the vehicle tends move towards neutral steer. The parameter p_{KY2} which defines the normal load at which the peak of the stiffness curve occurs does not show any major influence on the target as the maneuver is quasi static and does not induce any major load transfer. The second most sensitive parameter is observed to be p_{DY1}. Though the vehicle operates in the linear range of the Lateral vs Slip curve, the peak value of the curve has small influence on the slope of the curve at the 0.5 g peak value of the curve.

Front axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Linear US Coefficient	-1.52	0.88	-0.14	-0.05	-0.01	0.01	0.01	0.00	0.00

Table 4.6: Sensitivity scale of tire parameters on Linear Understeer - Front axle only

Similar to both axle simulation result, it is seen that p_{KY1} is the most sensitive among all the studied Magic Formula tire parameters. It is also evident that sensitivity p_{KY1} has the highest magnitude across all the axle simulation results, and this can be understood by that fact that understeer coefficient is defined as the difference between the front and rear axle stiffness. Hence any increase in the magnitude of p_{KY1} increases the stiffness of the front axle, making vehicle more neutral steer. The sensitivity trend across other tire parameters remain the same expect for that of the magnitude of the sensitivity. This can be attributed to fact that this vehicle dynamics target is sensitive to the differential of cornering stiffness due to the difference in the tire properties across both the axles.

Rear axle configuration

The influence of tire parameters variation follows the front axle but only to have an opposite effect on the vehicle dynamic metric. Similar to previous two axle combinations, p_{KY1} has the highest influence on the linear under-steer coefficient but the effect is reversed as the increase in the stiffness at the rear axle makes the vehicle more under-steer. Unlike front axle,p_{KY2} which define the normal load at

Vehicle Dynamic Metric	Tire Parameters Sensitivity								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Linear US Coefficient	1.28	-0.81	0.05	0.03	0.01	0.00	0.00	0.00	0.00

Table 4.7: Sensitivity scale of tire parameters on Linear Understeer - Rear axle only

which the peak of the stiffness for a given tire occurs, is moved to a larger normal loads. Creating an offset with front axle where the front axle generates higher lateral force for a given slip angle. Hence by increasing p_{KY2} only on the rear axle the vehicle tends to be more neutral steered.

4.3.2 Non-Linear Understeer Coefficient

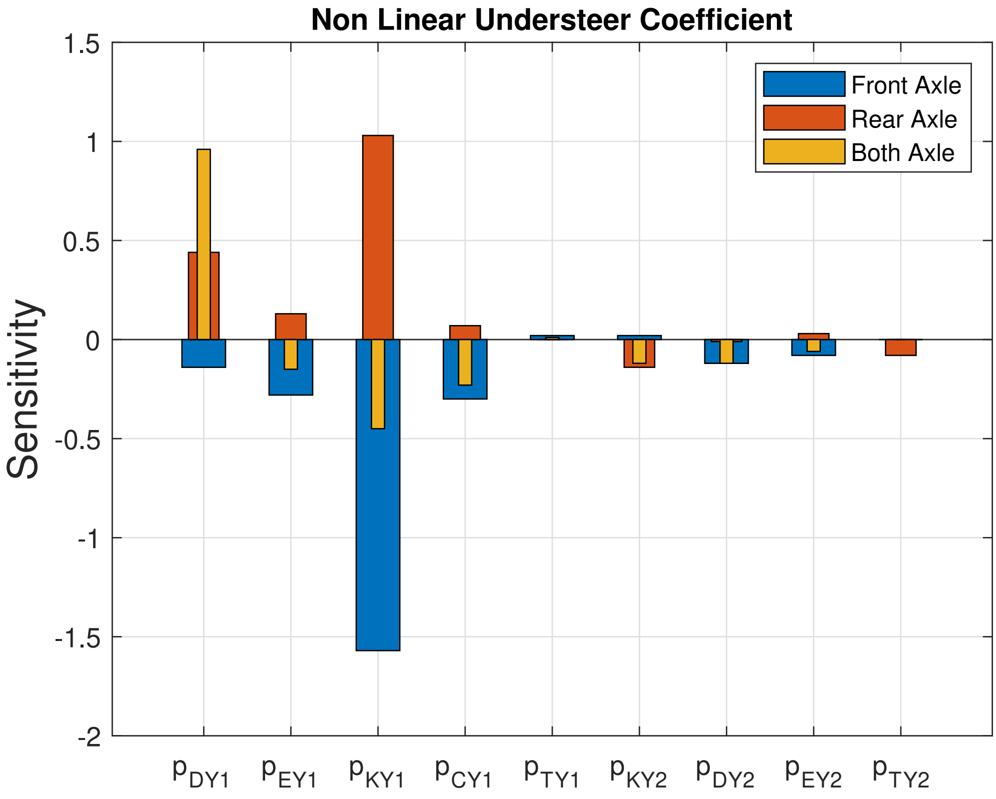

Figure 4.8: Sensitivity of magic formula tire parameters on non-linear understeer metric

Both axle configuration

The most sensitive tire parameter for non linear understeer coefficient from changing tire parameters for both axles is p_{DY1} from figure 4.8. This is because the slope of

Vehicle Dynamic metric	**Tire Parameters Sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Non Linear US Coefficient	0.96	-0.45	-0.23	-0.15	-0.12	-0.12	-0.06	0.01	0.00

Table 4.8: Sensitivity scale of tire parameters on Non-Linear Understeer - Both axles

the lateral force vs slip curve in the non linear region (0.5 g - peak value g) is highly sensitive to the tire parameter p_{DY1} when the differential tire stiffness across the axle remains the same. Hence the increase in the magnitude of p_{DY1} increases the non linear understeer coefficient. The second most sensitive tire parameter is p_{KY1}, as this parameter influences the slope of the lateral force vs slip curve. Hence increase in p_{KY1} makes the vehicle more neutral steer. The next sensitive tire parameters are p_{CY1} and p_{EY1}, which defines the saturation region and the region of transition from peak value to saturation value of the force curve.

Front axle configuration

Vehicle Dynamic metric	**Tire Parameters Sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Non Linear US Coefficient	-1.57	-0.30	-0.28	-0.14	-0.12	-0.08	0.02	0.02	0.00

Table 4.9: Sensitivity scale of tire parameters on Non-Linear Understeer - Front axle only

Changing tire parameters only on front axle, p_{KY1} shows highest influence on non linear understeer coefficient. This is due to the difference in the tire stiffness between front and rear axle hence making the vehicle more neutral steer with increase in p_{KY1}. The next most sensitive parameters are p_{CY1} and p_{EY1}, because the characteristic of the front axle is determined by the weight distribution of the vehicle, which in this case is biased towards front. Front axle being steered axle, the non linear characteristic mainly depends on the saturation and the transition region. Thus it can be seen that, with the increase in parameters p_{CY1} and p_{EY1}, this vehicle dynamic metric decreases and making vehicle more towards neutral steer.

Rear axle configuration

Similar to front axle, the most sensitive tire parameter here is p_{KY1}, but the effect of its variation is reversed. This is because the increase in p_{KY1} at rear axle makes the vehicle more under steered. Another factor that influences this effect is that the rear axle not being steered, does not reach higher slip angle as achieved by the front axle. Unlike the front axle, the second most sensitive parameter is p_{DY1}, and this can be attributed to the rear tire not being saturated. Unlike the front axle which

Vehicle Dynamic metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Non Linear US Coefficient	1.03	0.44	-0.14	0.13	-0.08	0.07	0.03	-0.01	0.00

Table 4.10: Sensitivity scale of tire parameters on Non-Linear Understeer - Rear axle only

has a higher slip angle due to the axle being steered, the non linear characteristics of the rear axle is mainly determined by p_{DY1} parameter and has less of an influence with p_{CY1} and p_{EY1} parameters.As p_{DY1} increases the vehicle tends to be more understeered.

4.3.3 Progressivity

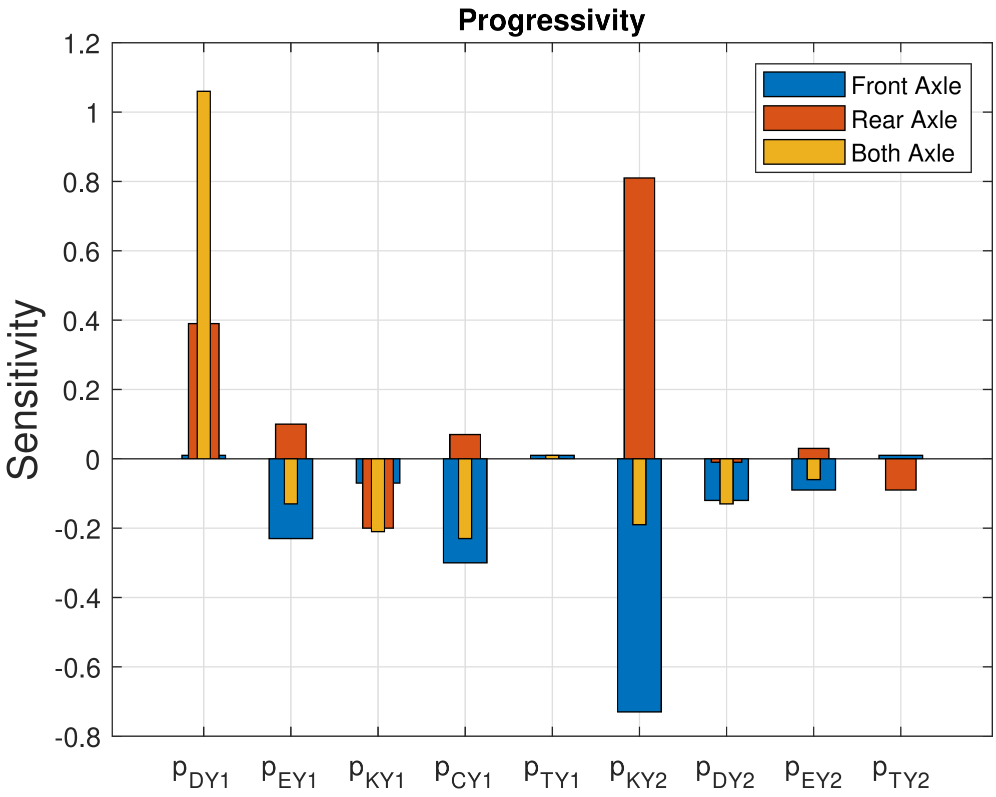

Figure 4.9: Sensitivity of magic formula tire parameters on progressivity metric

Both axle configuration

Progressivity is the ratio between the non linear understeer coefficient to the linear understeer coefficient. The most sensitive tire parameter for this vehicle dynamic

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Progressivity	1.06	-0.23	-0.21	-0.19	-0.13	-0.13	-0.06	0.01	0.00

Table 4.11: Sensitivity scale of tire parameters on Progressivity - Both axles

target for both axles is p_{DY1}. The understeer coefficients primarily dependent on the differential stiffness across the axles, when the tire properties are varied across both the axles the most sensitive parameter for progressivity is p_{DY1} parameter. As p_{DY1} increases the progressivity ratio also increases. The other influential parameters influencing the target are p_{CY1} and p_{KY1} parameters. The non linear understeer coefficient dominates the ratio, the trend is then a reflection of the influence of the tire parameters on the non linear understeer coefficient as an vehicle dynamic target from the previous section 4.3.2.

Front axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Progressivity	-0.73	-0.30	-0.23	-0.12	-0.09	-0.07	0.01	0.01	0.00

Table 4.12: Sensitivity scale of tire parameters on Progressivity - Front axle only

The most sensitive parameter for the front axle alone is the p_{KY2} parameter. The p_{KY2} parameter defines the load at which the maximum cornering stiffness occurs. The front axle being the steered axle the influence of the saturation parameters and the transition parameters are consider to be the second and third most sensitive parameters for this vehicle dynamic target under this axle configuration.Increase in the value of these parameters there is a reduction in the measured vehicle dynamic target.

Rear axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Progressivity	0.81	0.39	-0.20	0.10	-0.09	0.07	0.03	-0.01	0.00

Table 4.13: Sensitivity scale of tire parameters on Progressivity - Rear axle only

The most sensitive parameter for the vehicle dynamic target is the p_{KY2} parameter and the explanation reflects the previously made statement for the other axle combinations that the tread follows the trend of the non linear understeer coefficient and thus it hold true in this case as well.As the p_{KY2} parameter is increased then there is an increase in the progressivity ratio which also implies that the non linear understeer coefficient also increases. The progressivity increases as it the ratio of the non linear understeer coefficient to the linear understeer coefficient.The second and the third most sensitive parameters for the rear axle for this particular vehicle dynamic target are the p_{DY1} and p_{KY} parameters. As the rear axle does not saturate as it is not a steered axle, the non linear region is mainly influenced by the peak and the stiffness parameters,which is reflected in the analysis.

4.3.4 Road Holding Capacity

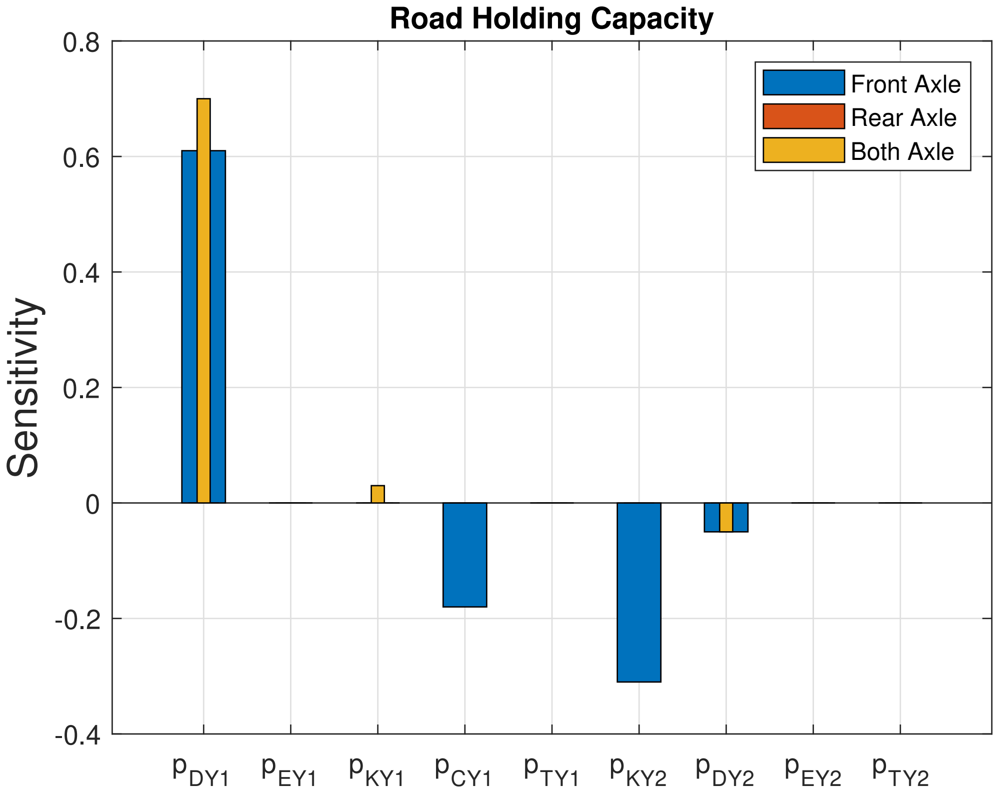

Figure 4.10: Sensitivity of tire parameters on Road Holding Capacity metric

Both axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Road Holding Capacity	0.71	-0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00

Table 4.14: Sensitivity scale of tire parameters on Road Hold capacity - Both axles

The most sensitive parameter for the Road holding metric is p_{DY1}.Road holding as an vehicle dynamic target measures the maximum lateral force that the vehicle can sustain without offtracking. Thus the p_{DY1} and p_{DY2} parameters which define the peak of the lateral force curve are the most sensitive parameters for this vehicle dynamic target and for this particular axle combination.

Front axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Road Holding Capacity	0.61	-0.31	-0.18	-0.05	0.00	0.00	0.00	0.00	0.00

Table 4.15: Sensitivity scale of tire parameters on Road Hold capacity - Front axle only

The most sensitive parameter for this vehicle dynamic target at front axle alone is still the lateral force limit parameter which is the p_{DY1} parameters and the magnitude remains similar to that of the sensitivity with both the axles. So as the value in the parameter p_{DY1} is increased then the road holding capacity of the vehicle increases. By changing only the p_{KY2} parameter in the front axle the sensitivity of this parameter for this particular axle combination is higher as compared to that of the previous both axle combination.

Rear axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Road Holding Capacity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4.16: Sensitivity scale of tire parameters on Road Hold capacity - Rear axle only

The rear axle does not have any influence for this vehicle dynamic target.

4.3.5 Maximum Lateral Acceleration

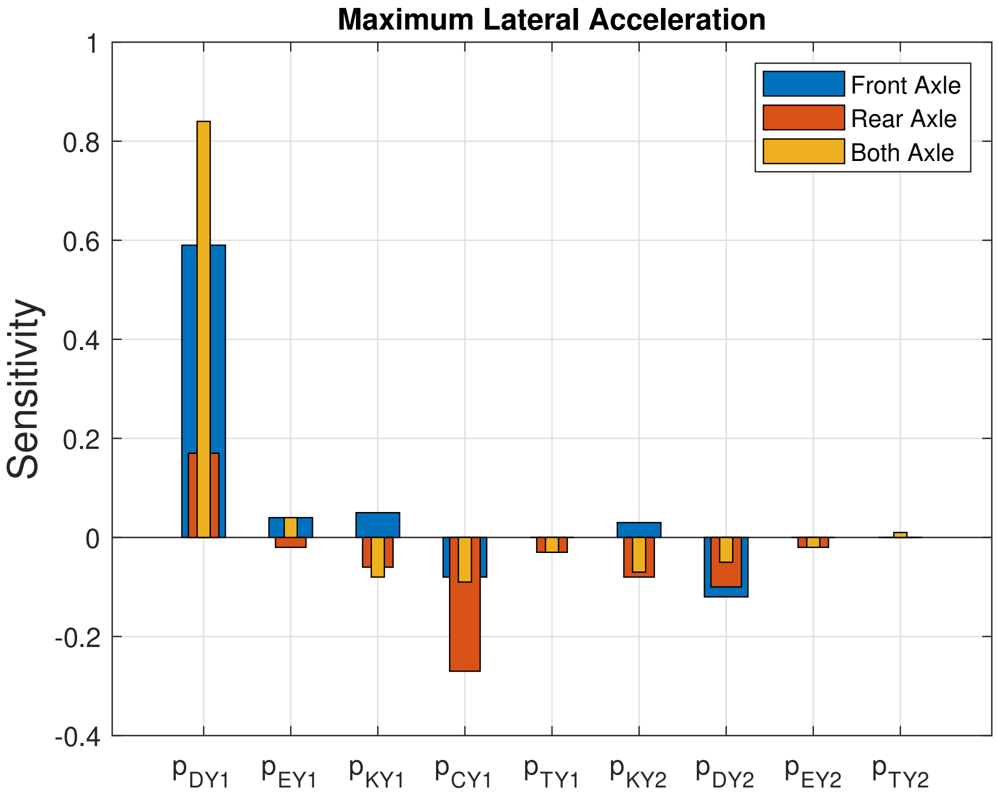

Figure 4.11: Sensitivity of tire parameters on Maximum Lateral Acceleration metric

Both axle configuration

Vehicle Dynamic metric	Tire Parameters sensitivity								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Maximum Lateral Acceleration	0.84	-0.09	-0.08	-0.07	-0.05	0.04	-0.03	-0.02	0.01

Table 4.17: Sensitivity scale of tire parameters on Max lateral acceleration - Both axles

The maximum lateral acceleration is measured as the maximum g's a vehicle can pull during cornering. But unlike the previous target the current target is measured with J turn maneuver where all the tires are saturated and maximum lateral forces are generated. In this case the most sensitive parameter remains the p_{DY1} parameter for across both axles and the other sensitive parameters are significantly less in magnitude as compared to the others.

Front axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Maximum Lateral Acceleration	0.59	-0.12	-0.08	0.05	0.04	0.03	0.00	0.00	0.00

Table 4.18: Sensitivity scale of tire parameters on Max lateral acceleration - Front axle only

The trend continuing from the previous set of results the front axle resembles the characteristics of the both axles as the influence of the rear axle in determining the lateral force is limited. The most sensitive parameter for the front axle for this particular vehicle dynamic target is p_{DY1}. As p_{DY1} increases it is also observed that there is also an increase in the maximum lateral acceleration capacity of the vehicle.

Rear axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Maximum Lateral Acceleration	-0.27	0.17	-0.10	-0.08	-0.06	-0.03	-0.02	-0.02	0.00

Table 4.19: Sensitivity scale of tire parameters on Max lateral acceleration - Rear axle only

The sensitivity of the rear axle as a non steered axle has the highest sensitivity for maximum lateral acceleration capacity for the parameter p_{CY1}. The rear tires are saturated at its maximum side slip values and shows the maximum sensitivity for that tire parameter.Increasing p_{CY1} parameter reduces the vehicle dynamic target. And the second most sensitive parameter still remains p_{DY1} parameter which defines the maximum lateral force limit for that set of tires.

4.3.6 Yaw rate time lag @45 deg phase lag

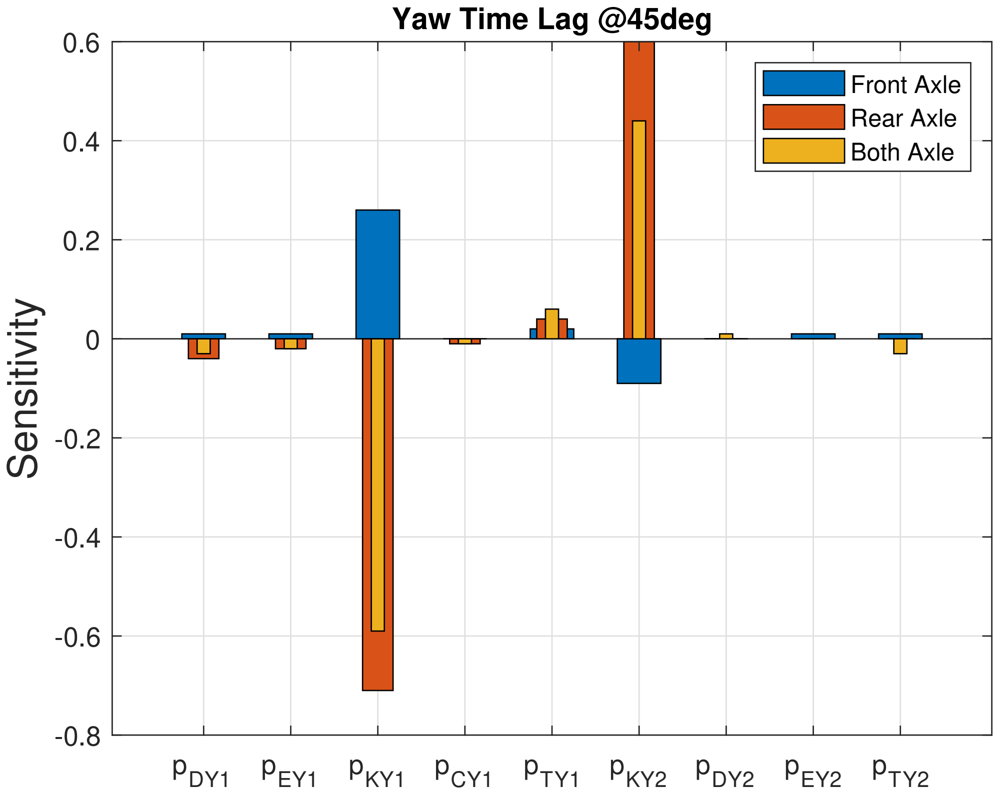

Figure 4.12: Sensitivity of tire parameters on Yaw Rate time lag @45 deg metric

Both axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Yaw time lag @45 deg Acceleration	-0.59	0.44	0.06	-0.03	-0.03	-0.02	-0.01	0.00	0.00

Table 4.20: Sensitivity scale of tire parameters on Yaw time lag @45 - Both axles

The most sensitive tire parameter for yaw rate time lag target are p_{KY1} and p_{KY2}. This is mainly due to the maneuver which is an on center maneuver where the steering input does not exceed the slip angles going to the nonlinear region of the lateral force curve. The major influential parameters are the stiffness parameters. Then the other influential parameter is the relaxation parameter where larger the relaxation length the time lag in increased. This vehicle dynamic target captures the effect of relaxation parameter.

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Yaw time lag @45 deg Acceleration	0.26	-0.09	0.02	0.01	0.01	-0.01	0.00	0.00	0.00

Table 4.21: Sensitivity scale of tire parameters on Max lateral acceleration - Front axle only

Front axle configuration

The sensitivity for the front axle for this particular vehicle dynamic target also follows the tread of the previous axle where the front axle is mostly influential in defining the time lag characteristics of a vehicle. Due to the on center maneuver as p_{KY1} and p_{KY2} has the most influence on the vehicle dynamic target and then relaxation length accounts for the next sensitive parameter. While the trend remains the same in terms of the parameters influence on the target the effect of the stiffness parameters are inverse to that of the previous case. Here increasing p_{KY1} increases the time lag and increasing p_{KY2} decreases the time lag respectively.

Rear axle configuration

Vehicle Dynamic metric	**Tire Parameters sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Yaw time lag @45 deg Acceleration	0.71	0.60	0.04	0.04	0.02	0.01	0.00	0.00	0.00

Table 4.22: Sensitivity scale of tire parameters on Max lateral acceleration - Rear axle only

The rear axle also follows the similar trend as by the other axle combination in terms of the most influential parameters in the previous axle combinations. The most sensitive parameters still remain the stiffness parameters where unlike the front axle effects increasing p_{KY1} increases the time lag and increasing p_{KY2} also increases the time lag in the vehicle for Yaw at -45 deg.The next most sensitive parameters again follow the trend of Relaxation parameter and the maximum lateral acceleration parameter respectively.While the trend for relaxation parameter remains the same where if there is an increase in the value for the value of relaxation parameter then there is a direct correlation to an increase in the time lag irrespective of the axle combination in question.The result of the effect of p_{DY1} remains the same for individual axle cases where the increase in p_{DY1} will increase the time lag but in case of all the tires being varied for p_{DY1} then there is an reduction in the time lag or yaw rate at -45 degrees.

4.3.7 Time Lag Yaw vs SWA

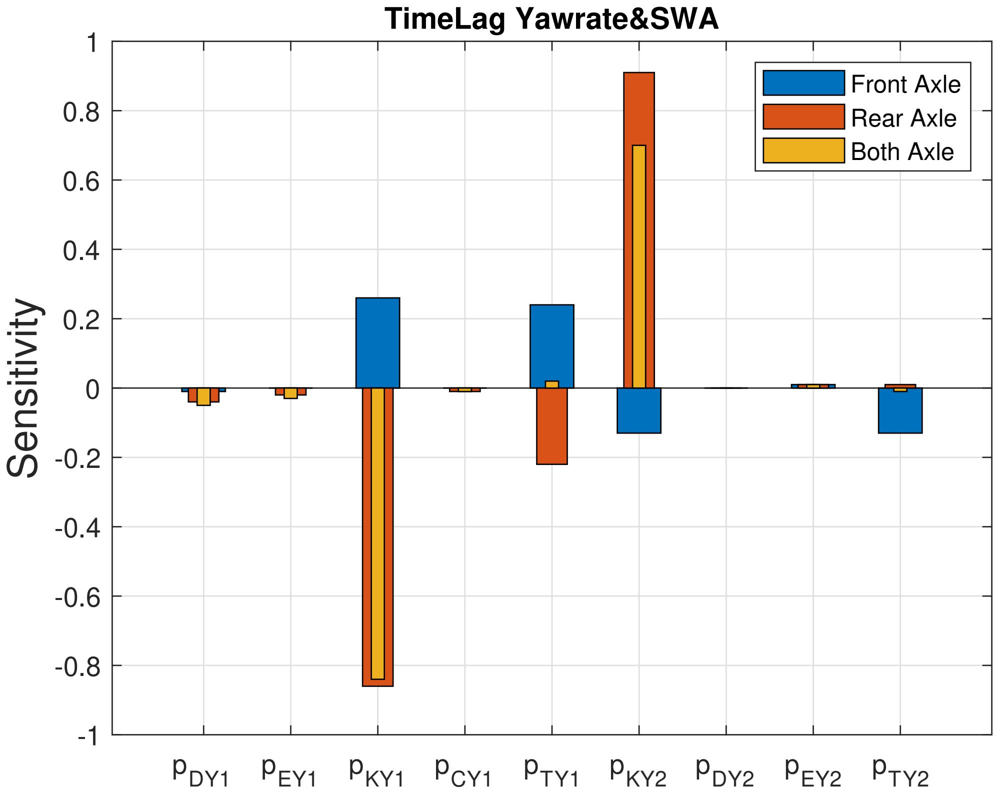

Figure 4.13: Sensitivity of tire parameters on Timelag Yaw Rate Vs SWA metric

Both axle configuration

Vehicle Dynamic metric	Tire Parameters sensitivity								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & SWA	-2.15	1.17	-0.30	-0.16	-0.13	-0.06	0.02	0.01	0.00

Table 4.23: Sensitivity scale of tire parameters on Timelag Yaw Rate vs SWA - Both axles

The most sensitive tire parameter for the vehicle dynamic target Time lag between Steering Wheel Angle (SWA) and yaw rate when all the tires are of same specification are the stiffness parameters. The maneuver used to derive this particular vehicle dynamic target is the Sine With Dwell (SWD) maneuver.It is a transient maneuver which characterizes the dynamic behavior if the vehicle rather than the quasi steady state maneuvers seen in constant radius characterizations.The most sensitive parameter other than the stiffness parameters are the relaxation parameters.The most important vehicle dynamic target which defines the driving dynamics of the car has a very high influence from the relaxation parameters from the tires when the same set of tires with same specifications are used in all corners of the car.

Front axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & SWA	0.26	0.24	-0.13	-0.13	-0.01	-0.01	0.00	0.00	0.00

Table 4.24: Sensitivity scale of tire parameters on Timelag Yaw Rate vs SWA - Front axle only

The same vehicle dynamic target time lag between yaw rate and SWA is considered for the sensitivity across only front axle alone. The the most sensitive tire parameter still remains the stiffness parameter but then as the front axle being the steered axle shows a significantly higher sensitivity to relaxation parameters and increase in relaxation length has a direct impact on the increase in the time lag experienced from SWA to yaw rate on the car.While the load sensitive parameters also has a significant influence on this particular vehicle dynamic target and it also shows the significance of the sensitivity of the front axle to the determination of this vehicle dynamic target at the vehicle level.After the stiffness parameters and the relaxation parameters due to the maneuver under consideration being a transient maneuver the D and E parameters also show a small influence in the sensitivity of the front axle towards this particular vehicle dynamic target.

Rear axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & SWA	0.91	-0.86	-0.22	-0.04	-0.02	0.01	0.01	0.01	0.00

Table 4.25: Sensitivity scale of tire parameters on Timelag Yawrate vs SWA - Rear axle only

The most sensitive parameter of time lag between yaw rate and SWA with respect to rear axle alone are the stiffness parameters. The rear axle being the non steered axle and the fact that the normal load on the rear axle is lesser than the front axle the load sensitive parameter of stiffness and the magnitude parameter of the stiffness parameters are equally sensitive to this particular vehicle dynamic target for this particular axle combination.The relaxation parameter also is sensitive to this vehicle dynamic target for this particular axle combination, the increase in the value for the relaxation parameter in the rear axle results in in reduction in time lag between yaw rate and SWA at the vehicle level.The rear axle also shows the similar trend when compared against the front axle with the other tire parameters influencing the timelag between Yaw rate vs SWA.The tire parameters p_{DY1} and p_{EY1} show a small influence on the sensitivity of timelag between yaw rate and SWA, where increase in the values of these particular tire parameters results in reduction of the time lag between yaw rate and SWA.

4.3.8 Time Lag Lateral Acceleration vs SWA

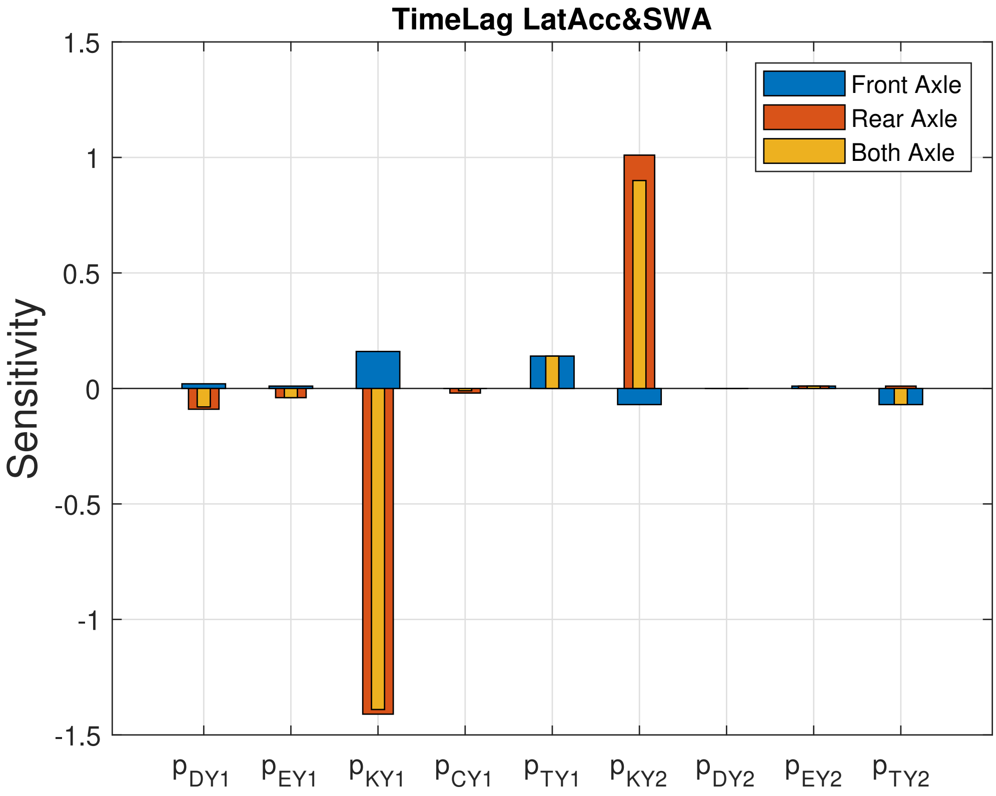

Figure 4.14: Sensitivity of tire parameters on Timelag LatAcc Vs SWA metric

Both axle configuration

Vehicle Dynamic Metric	Tire Parameters Sensitivity								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between LatAcc & SWA	-1.39	0.90	0.14	-0.08	-0.07	-0.04	0.01	0.01	0.00

Table 4.26: Sensitivity scale of tire parameters on Timelag Lateral Acceleration vs SWA - Both axles

The trend for the vehicle dynamic metric timelag between lateral acceleration and SWA follows the similar trajectory to that of 4.3.7 where the maximum sensitivity for this particular vehicle dynamic target are the stiffness parameters and relaxation parameters for this axle combination where all the tires in the vehicle model are identical.The most significant parameter is the stiffness parameter where increase in stiffness will result in reduction in time lag between the SWA and the Lateral Acceleration response of the vehicle.

Front axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Time lag between LatAcc & SWA	0.16	0.14	-0.07	-0.07	0.02	0.01	0.01	0.00	0.00

Table 4.27: Sensitivity scale of tire parameters on timelag Lateral Acceleration vs SWA - Front axle only

The sensitivity analysis done for time lag between lateral acceleration and SWA follows the same trend as that of the previous axle combination where the tire properties were changes on both axles. The maximum sensitivity for timelag between lateral acceleration and SWA is shown by the stiffness parameter. The second influential Magic Formula tire parameter for this particular vehicle dynamic target and for front axle configuration are the relaxation parameters. This is mainly due to the front axle being the steered axle.Thus the major influential parameters affecting the vehicle dynamic metrics in terms of tire parameters are the stiffness and the relaxation parameters like the previous mentioned analysis for this target. The sensitivity follows the previous results of time lag between lateral acceleration and SWA, where the maneuver events being the transient events the influence of p_{DY1} and p_{EY1} parameters are also visible in the study.

Rear axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Time lag between LatAcc & SWA	-1.41	1.01	-0.09	-0.04	-0.02	0.01	0.01	0.00	0.00

Table 4.28: Sensitivity scale of tire parameters on Timelag Lateral Acceleration vs SWA - Rear axle only

The most sensitive parameter for this particular vehicle dynamic target for rear axle configuration is the stiffness parameter p_{KY1} where if there is an increase in the stiffness parameter then there is a significant reduction in the time lag between lateral acceleration and the SWA. And the load sensitive parameter for stiffness p_{KY2}

also has the second most influence in the time lag between lateral acceleration and SWA for rear axle configuration.Unlike the front axle sensitivity where the second most influential parameters were the relaxation parameters due to the front axle being steered axle.The rear axle, as the result of it not being the steered axle has its sensitivity after the stiffness parameters p_{KY1} and p_{KY2}, the lateral force limit parameter p_{DY1}.Therefore increase in the lateral force limit parameter p_{DY1} for the rear axle has an higher influence in the over all reduction of the time lag of lateral acceleration with respect to SWA at the vehicle level after the stiffness parameters.

4.3.9 Time Lag Yaw Rate vs Lateral Acceleration

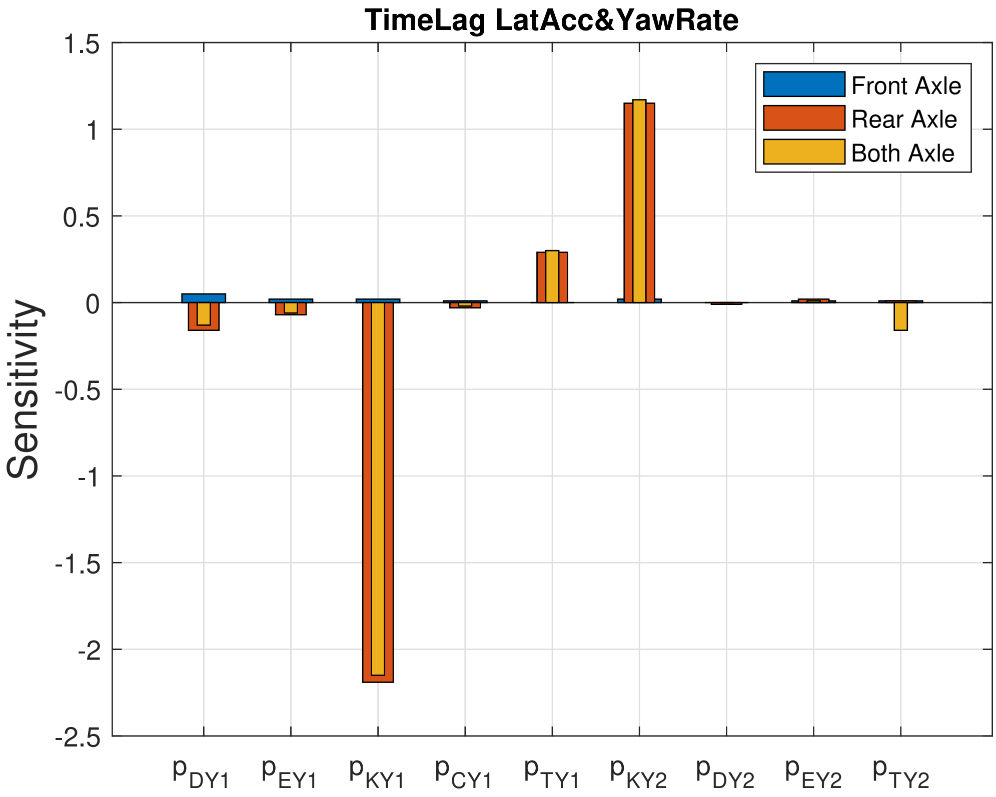

Figure 4.15: Sensitivity of tire parameters on Timelag Lat Acc Vs Yaw Rate metric

Both axle configuration

Vehicle Dynamic Metric	Tire Parameters Sensitivity								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & LatAcc	-2.15	1.17	0.30	-0.16	-0.13	-0.06	0.02	0.01	0.00

Table 4.29: Sensitivity scale of tire parameters on timelag lateral acceleration vs yaw rate- both axles

The most sensitive parameter for this particular vehicle dynamic target form the maneuver Sine With Dwell(SWD) is the stiffness parameter and the relaxation parameters. The derivation of this particular vehicle dynamic target is an reflection of the previous vehicle dynamic metrics of time lag between SWA and lateral acceleration, and SWA and yaw rate.The most sensitive parameters are the stiffness parameters p_{KY1} and p_{KY2}. As the stiffness increases across all the tires the time lag between the lateral acceleration and the yaw rate decreases and opposite to that of the second most influential parameter where the influence is opposite due to the fact that the stiffness curve is then moved to a higher normal load sensitivity thus resulting in the increase in the time lag between the lateral acceleration and yaw rate.The second most influential parameter after the stiffness parameters are the relaxation parameters.But the sensitivity is mainly due to the sensitivity of rear axle to this vehicle dynamic metric by changing the tire properties across both axles.

Front axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & LatAcc	0.05	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00

Table 4.30: Sensitivity scale of tire parameters on Time lag Lateral Acceleration vs Yaw rate- Front axle only

The front axle shows very little to no sensitivity to time lag between lateral acceleration and yaw rate.

Rear axle configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Time lag between Yaw rate & LatAcc	-2.19	1.15	0.29	-0.16	-0.07	-0.03	0.02	0.01	-0.01

Table 4.31: Sensitivity scale of tire parameters on time lag Lateral Acceleration vs Yaw - Rear axle only

The sensitivity of time lag between lateral acceleration and yaw rate recorded under the both axle configuration is reflected here in the pure rear axle case. This shows that the sensitivity of the vehicle dynamic metrics are majorly influenced by the rear axle alone.The sensitivities from the table 4.29 , 4.30 and 4.31, the trend shows the magnitude and direction of the individual tire parameters sensitivities to the time lad between lateral acceleration and yaw rate remains same for both 4.29 and 4.31 while the front axle has little to no effect. The most sensitive parameters are the stiffness parameters and then the relaxation parameters.While increasing stiffness reduces the time lag between lateral acceleration and yaw rate, increasing relaxation length increases time lag between lateral acceleration and yaw rate.

The sensitivity values are extracted from the sensitivity analysis which objectively defines the influence of the most influential pure lateral Magic Formula tire parameters to the vehicle dynamic metrics. Sensitivity data is then used for optimization where the sensitivities of these individual Magic Formula tire parameters to the vehicle dynamic metrics are used to develop a suitable objective sensitivity matrix to be used as weights for the optimization algorithm which derives an optimized tire from the vehicle targets defined. Considering sensitivities are grouped in a way where it can be used as weights ,the direction to optimize based on the influence of the particular tire parameter on a given vehicle dynamic target can be used in the objective function.

Three matrices which contain the magnitude of the sensitivities for three different axle configurations can be used to develop optimization process. This optimization process can be run for staggered setup or to define an optimized set of tires for both the axles. The matrix values are absolute values as the simulation only requires the weights for the objective function and the direction of change will be inherently considered by running the simulation software in loop where it gives out real time data based on the direction of change. The three matrices which define the weights to the objective function parameters are mentioned in tables 4.32 (front axle configuration), 4.33 (rear axle configuration) and 4.34 (both axle configuration).

4.3.10 Front axle Configuration

Vehicle Dynamic Metric	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Linear US Coefficient	-0,14	-0,05	-1,52	-0,01	0,01	0,88	0,00	0,01	0,00
Non-Linear US Coefficient	-0,14	-0,28	-1,57	-0,30	0,02	0,02	-0,12	-0,08	0,00
Progressivity	0,01	-0,23	-0,07	-0,30	0,01	-0,73	-0,12	-0,09	0,00
Road Holding Capacity	0,61	0,00	0,00	-0,18	0,00	-0,31	-0,05	0,00	0,00
Maximum Lateral Acceleration	0,59	0,04	0,05	-0,08	0,00	0,03	-0,12	0,00	0,00
Yaw Time Lag @ 45 deg	0,01	0,01	0,26	0,00	0,02	-0,09	0,00	0,00	-0,01
Time lag for Yaw rate and SWA	-0,01	0,00	0,26	0,00	0,24	-0,13	0,00	0,01	-0,13
Time lag for Lateral Acceleration and SWA	0,02	0,01	0,16	0,00	0,14	-0,07	0,00	0,01	-0,07
Time lag for Lateral Acceleration and Yaw rate	0,05	0,02	0,02	0,01	0,00	0,02	0,00	0,01	0,01

Table 4.32: Sensitivity matrix for front axle configuration

4.3.11 Rear axle Configuration

Vehicle Dynamic Metrics	**Tire Parameters Sensitivity**								
	p_{KY1}	p_{KY2}	p_{DY1}	p_{EY1}	p_{CY1}	p_{TY1}	p_{EY2}	p_{DY2}	p_{TY2}
Linear US Coefficient	0,05	0,03	1,28	0,00	0,00	-0,81	0,00	0,00	0,01
Non-Linear US Coefficient	0,44	0,13	1,03	0,07	0,00	-0,14	-0,01	0,03	-0,08
Progressivity	0,39	0,10	-0,20	0,07	0,00	0,81	-0,01	0,03	-0,09
Road Holding Capacity	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Maximum Lateral Acceleration	0,17	-0,02	-0,06	-0,27	-0,03	-0,08	-0,10	-0,02	0,00
Yaw Time Lag @ 45 deg	-0,04	-0,02	-0,71	-0,01	0,04	0,60	0,00	0,00	0,00
Time lag for Yaw rate and SWA	-0,04	-0,02	-0,86	-0,01	-0,22	0,91	0,00	0,01	0,01
Time lag for Lateral Acceleration and SWA	-0,09	-0,04	-1,41	-0,02	0,00	1,01	0,00	0,01	0,01
Time lag for Lateral Acceleration and Yaw rate	-0,16	-0,07	-2,19	-0,03	0,29	1,15	-0,01	0,02	0,01

Table 4.33: Sensitivity matrix for rear axle configuration

4.3.12 Both axle configuration

Vehicle Dynamic Metrics	**Tire Parameters Sensitivity**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Linear US Coefficient	-0,09	-0,02	-0,25	0,00	0,00	0,07	0,01	0,00	0,00
Non-Linear US Coefficient	0,96	-0,15	-0,45	-0,23	0,01	-0,12	-0,12	-0,06	0,00
Progressivity	1,06	-0,13	-0,21	-0,23	0,01	-0,19	-0,13	-0,06	0,00
Road Holding Capacity	0,70	0,00	0,03	0,00	0,00	0,00	-0,05	0,00	0,00
Maximum Lateral Acceleration	0,84	0,04	-0,08	-0,09	-0,03	-0,07	-0,05	-0,02	0,01
Yaw Time Lag @ 45 deg	-0,03	-0,02	-0,59	-0,01	0,06	0,44	0,00	0,00	-0,03
Time lag for Yaw rate and SWA	-0,05	-0,03	-0,84	-0,01	0,02	0,70	0,00	0,01	-0,01
Time lag for Lateral Acceleration and SWA	-0,08	-0,04	-1,39	-0,01	0,14	0,90	0,00	0,01	-0,07
Time lag for Lateral Acceleration and Yaw rate	-0,13	-0,06	-2,15	-0,02	0,30	1,17	0,00	0,01	-0,16

Table 4.34: Sensitivity matrix for both axle configuration

4.4 Optimization of Magic Formula Tire Parameters for vehicle handling and steering targets

The optimization script is formed to determine optimized set of Magic Formula tire parameters for a set vehicle dynamic targets. The optimization result shown below are using one realistic case.

The input parameters for the optimization tool are the tire parameters for an already existing tire and the upper and lower bounds for the selected Magic Formula tire parameters in the table 4.35

Inputs	**Tire Parameters**								
	$\boldsymbol{p_{KY1}}$	$\boldsymbol{p_{KY2}}$	$\boldsymbol{p_{DY1}}$	$\boldsymbol{p_{EY1}}$	$\boldsymbol{p_{CY1}}$	$\boldsymbol{p_{TY1}}$	$\boldsymbol{p_{EY2}}$	$\boldsymbol{p_{DY2}}$	$\boldsymbol{p_{TY2}}$
Initial point	-24.5551	1.6447	1.0882	1.00	1.9923	2.3346	-0.3314	-0.20	1.6704
Upper bound	0.9*p_{KY1}	1.1*p_{KY2}	1.1*p_{DY1}	1.1*p_{EY1}	1.1*p_{CY1}	1.1*p_{TY1}	0.9*p_{EY2}	0.9*p_{DY2}	1.1*p_{TY2}
Lower bound	1.1*p_{KY1}	0.9*p_{KY2}	0.9*p_{DY1}	0.9*p_{EY1}	0.9*p_{CY1}	0.9*p_{TY1}	1.1*p_{EY2}	1.1*p_{DY2}	0.9*p_{TY2}

Table 4.35: Initial start point, upper bound and lower bounds for all tire parameters defined in one of the example case for optimization

The normalizing factor and the weights for the selected vehicle dynamic handling and steering targets used for the optimization for one realistic case is mentioned below in the table 4.36.

Vehicle Dynamic Metrics	**Normalization factor**	**Weights**
Linear Understeer Coefficient	0.1525 deg/g	6.45 1/(deg/g)
Non-Linear Understeer Coefficient	0.0328 deg/g	8.10 1/(deg/g)
Progressivity	0.1964	10.02
Road Holding Capacity	0.1092 g	7.78 1/g
Maximum Lateral Acceleration	0.1802 g	8.23 1/g
Yaw Timelag @ 45 deg	17.5814 ms	10.18 1/ms
Time lag for Yaw rate and SWA	14.4324 ms	8.65 1/ms
Time lag for Lateral Acceleration and SWA	48.7222 ms	9.63 1/ms
Time lag for Lateral Acceleration and SWA	34.2898 ms	13.99 1/ms

Table 4.36: Normalization factor (with units) and weights defined for vehicle dynamics metrics in one of the example case

4.4.1 Vehicle Dynamic Target verification

The optimization is run by taking a standard tire and optimizing it to a set of desired vehicle dynamic targets. The optimization algorithm then cycles through multiple combinations of tire parameters in order to satisfy the intended full vehicle targets which is expressed in terms of the objective function. Then optimization function delivers a optimized set of tire parameters with the minimum value for the respective objective function mentioned in the algorithm.

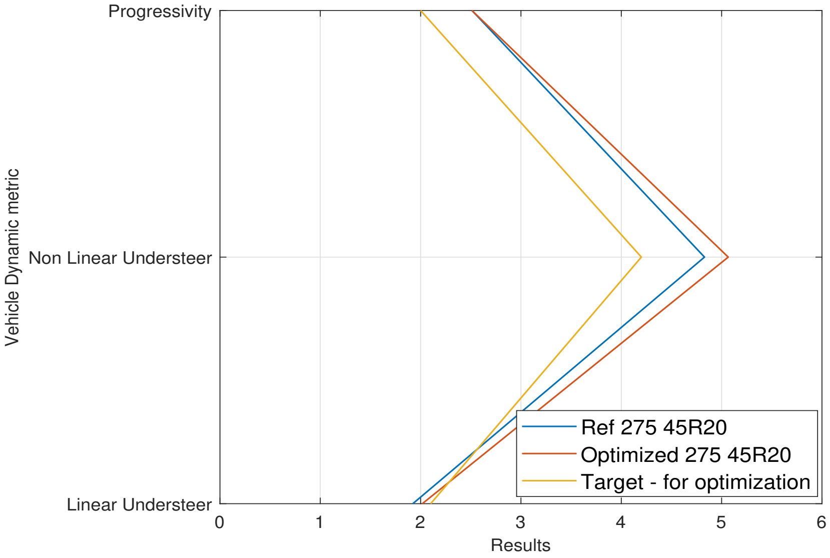

Figure 4.16: Comparison of Linear Understeer, Non-linear Understeer and Progressivity metric from reference tire(blue) and optimized tire(red) with targets values(yellow)

In order to validate the optimized tire output from the optimization tool chain, the newly optimized tire with the set of optimized tire properties are then introduced in the same vehicle model through VI-CRT.The vehicle model which is similar to the reference model but for the tires are simulated with the maneuvers defined in this study in order to derive the results for the vehicle dynamic metrics. This set of vehicle dynamic metric results are compared against the targets. Verifying the outputs if it satisfies the intended optimization target where the vehicle dynamic metric results with the new tire is closer to the target as from the original tire.

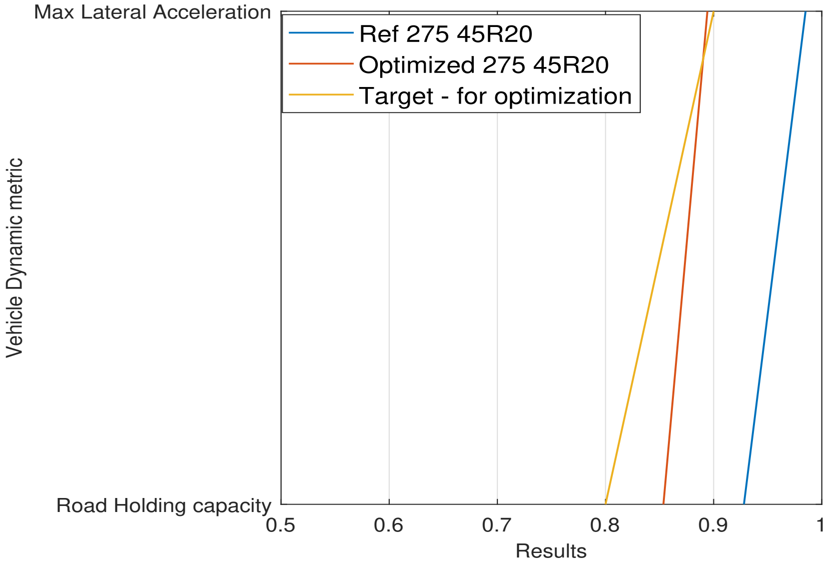

Figure 4.17: Comparison of Road Holding capacity and Maximum lateral acceleration metrics from reference tire(blue) and optimized tire(red) with targets values(yellow)

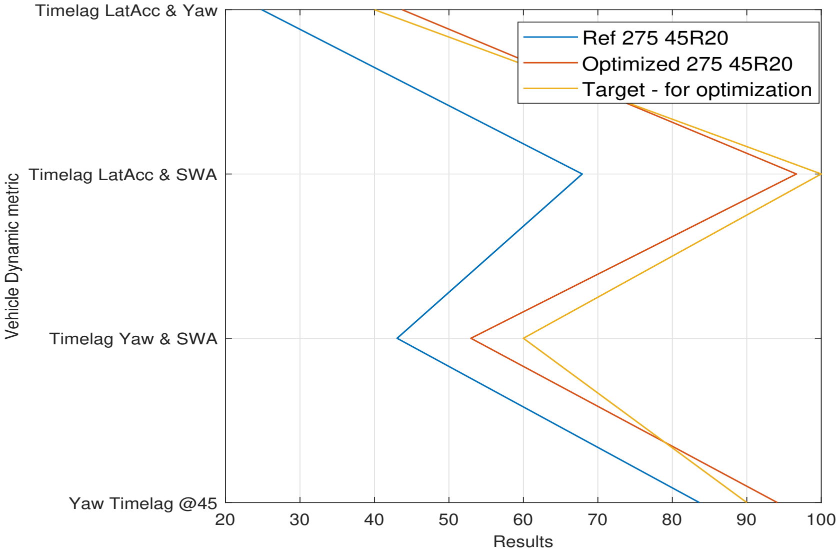

Figure 4.18: Comparison of Yaw time lag, time lag between Yaw & SWA, time lag between Lateral Acceleration & SWA and time lag between Lateral acceleration & Yaw from reference tire(blue) and optimized tire(red) with targets values(yellow)

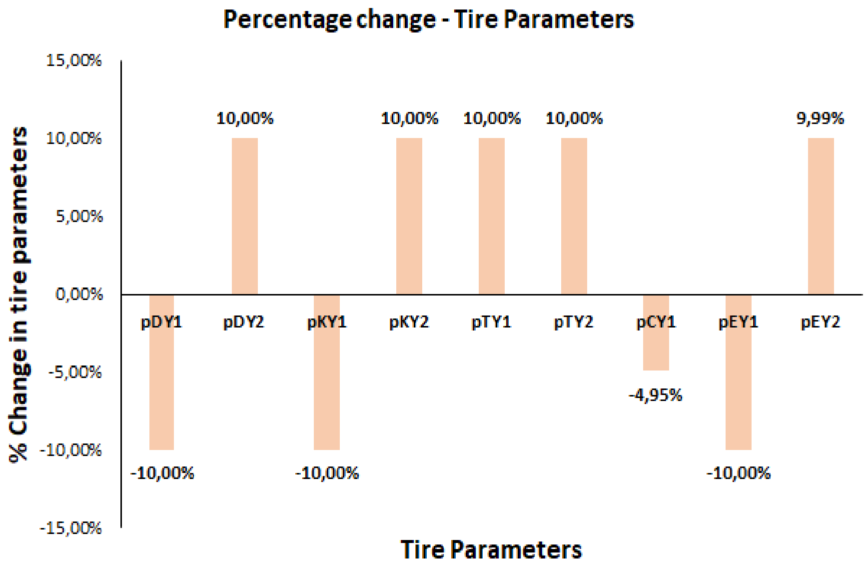

Figure 4.19: Percentage change in the tire parameters after optimization from the reference tire value

4.4.2 Stiffness Curve

Then optimized tire can also be validated by plotting its stiffness curve where the optimized tire is again compared against the reference tire before optimization. The maximum stiffness and the load at which the maximum stiffness is reached can also be compared. This can be used to validate the optimised tire if it is a reasonable tire to be realized physically with the constraints on the manufactures to produce such a tire.And this can also be a good medium for communication with the tire manufacturers.

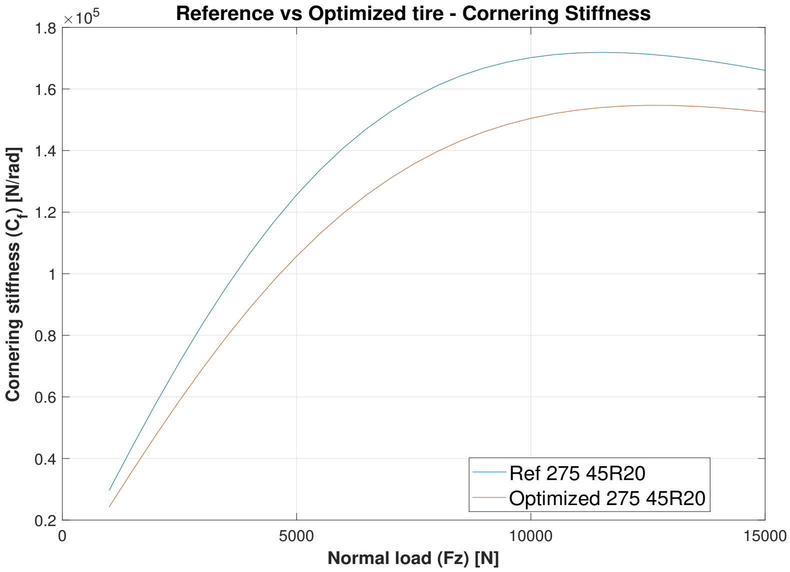

Figure 4.20: Plot of cornering Stiffness curve for reference tire(blue) and optimized tire(red)

4.4.3 Lateral Force Curve

The final output of the Magic formula tire model are the force and moment curves which then can be shared with individual tire manufactures to better categorize the tires required in order to meet the vehicle dynamics handling and steering targets set from the OEM.The tire force curve is the plot of lateral force generated by the tire with respect to the side slip from linear to full sliding conditions. The optimized tire model is run for various normal load cases and respective lateral force cure is obtained for the optimized tire.

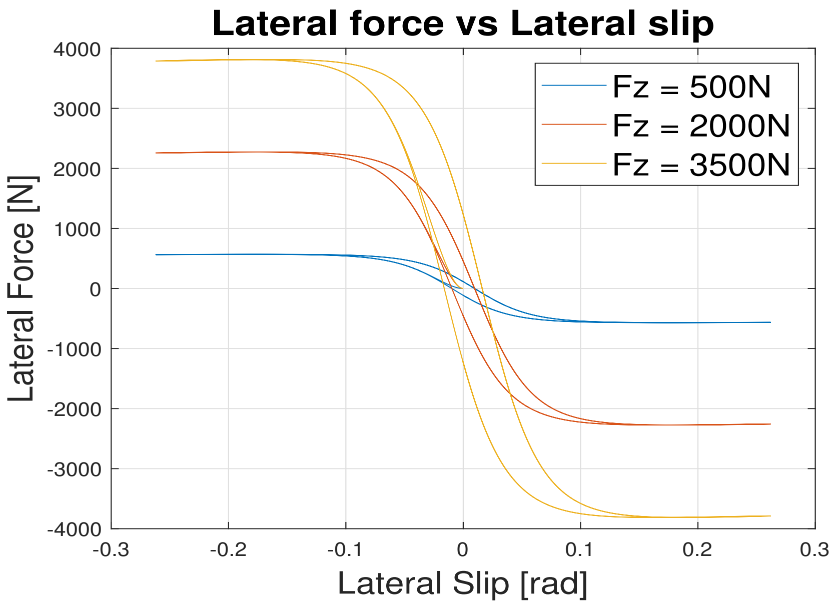

Figure 4.21: Tire Force Curve for the optimized tire with 3 cases of normal loads

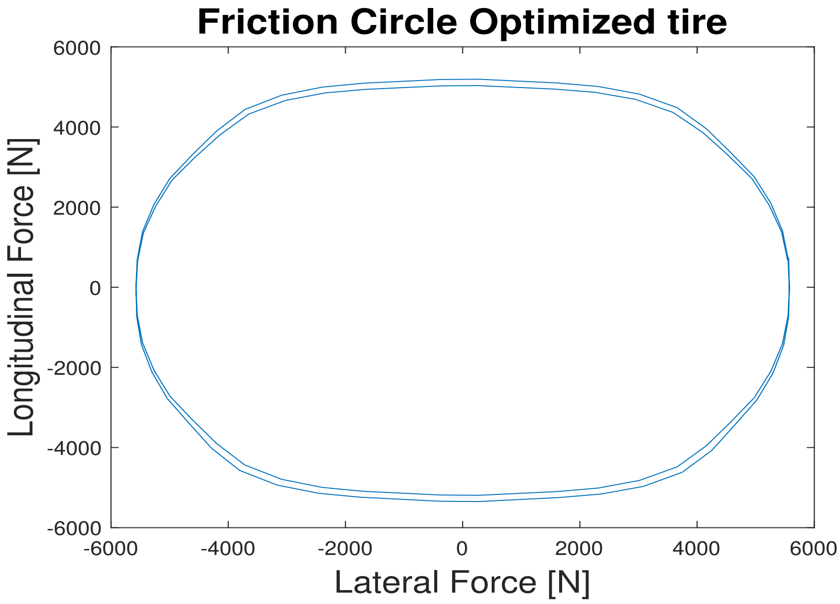

Figure 4.22: Friction circle for the optimized tire

5

Deliverables and Conclusions

The main deliverables of this thesis were parameter study to conduct sensitivity study of Magic Formula tire parameters 5.2 to the vehicle handling and steering targets and to develop an optimization tool.

5.1 Deliverables

- An information search was conducted where the scope of this thesis was defined, aiming to fill the gap in the understanding of the effect of individual Magic Formula tire 5.2 tire parameters on vehicle handling and steering metrics.

- The generic Magic Formula tire parameters B,C,D and E were then studied with a low fidelity SIMULINK two track model in order to understand the influence of these parameters on the output lateral force versus side slip curve.
- After the initial parameter study the sub parameters of Magic Formula Tire 5.2 were then studied individually to quantify their influence to the lateral force curve.
- Once the tire model was decided along with the parameters to be studied,a set of vehicle dynamic metrics were identified based on the requirement setting by the attribute leaders and its importance in defining the DNA of a car.

- Using a reference vehicle model, a post processing script was developed which derives the identified vehicle dynamic performances indicators that were compared with the ones derived from physical testing. .

- The identified Magic Formula tire parameters were varied +/- 20 percent from their reference value in order to study the sensitivity of these tire parameters compared to the Vehicle dynamic metrics. The output of this study is the sensitivity matrix which describes the relative sensitivities between the selected Magic Formula Tire lateral force parameters to that of the vehicle dynamic handling and steering performance indicators.

- The computed sensitivity matrices were further used to develop an optimization tool. The optimization tool developed computes a set of tire parameters to create an optimized tire to meet the defined vehicle dynamic targets. The optimization was demonstrated and verified to work in one realistic case

5.2 Conclusion

- The initial literature study was helpful in understanding the structure of the empirical equation used to model a tire and the parameters used in the formulation of the equation and their significance on the output of the empirical equation. The main conclusions from this literature study was the realization of the gap had previously existed and thus scope for this thesis to be able to fill the gap currently found form this thesis.

- The conclusion from this parameter study is that the influence of these parameters are quantified as to show the order of the most influential to the lesser influential generic Magic Formula tire parameters.

- The output from this study is the identification of the most important sub parameters of the Magic Formula tire parameters and the physical significance of these influential on the lateral force curve.

- Identifying and defining certain key vehicle dynamic performance indicator which could be helpful in understanding the effect of tires through its force scale on the vehicle by these vehicle level metrics.

- After verifying the computed values from the post processing script with the measured metrics, this model was considered as the reference case for the further sensitivity study

- Sensitivity matrices were computed for three axle configurations namely front axle, rear axle and both axles. The conclusions from the sensitivity study was that the identification of p_{KY1} among the Magic tire parameters as one of the most important parameters influencing the handling and steering metrics. Another main observation from the sensitivity matrix is the stark difference in the vehicle handling and steering characteristics based on the axle configurations. The metrics sensitive to the differential properties between axles and the metrics showing sensitivity to the additive properties across axles are visualized from this study and will be helpful in deciding future tire selection process and compromises to achieve certain vehicle dynamic targets.

- Constraints established through the upper and lower bounds on tire parameters are randomly defined at +/-10 percent from the reference value. These can be more accurately defined with the inputs from tire manufacturer. Another uncertainty in the optimization due to its multi parameter input and output structure is the possibility of multiple local minimums. Thus the result will be greatly biased to the initial guess point and in this study the initial guess is defined with the reference tire available with model. The optimization tool currently optimizes tires for individual axles. Thus an optimized set of staggered tires either for the front axle or for the rear axle can be developed from this tool as all the sensitivity matrices are derived in the sensitivity study.

6

Future Work

The future work proposed as a result of the parameter study, sensitivity analysis and optimization of Magic Formula tire pure lateral force parameters for vehicle handling and steering targets are as follows:

1. The study mainly concentrates on Magic Formula tire parameters in version 5.2 or PAC2002. The study can also be done for other versions of Magic Formula (ex: 6.1) which has some extra tire parameters and some parameters modelled differently (ex. Relaxation parameter).

2. The study also mainly concentrates on Lateral force parameters of Magic Formula 5.2 (PAC 2002).The Magic Formula 5.2 also includes Longitudinal force parameters and Aligning moment parameters which also plays a very important role in defining the handling and steering characteristics of a vehicle. Especially Aligning moment factors.

3. The study mainly focuses on the pure lateral force parameters in the Magic Formula tire and currently does not include combined force parameters. These could also be included in the further optimization algorithms.

4. The study as mentioned in the introduction only considers the most influential parameters of pure lateral fore parameters of Magic Formula tire 5.2 (PAC2002).The other parameters which influence the tire behaviour with camber and shift parameters could also be included in the optimization scheme.

5. The work currently involves only 9 vehicle dynamics targets with four manoeuvres. This work can be expanded by incorporating more complete and additional vehicle dynamic metrics using different manoeuvres.

6. The optimization method incorporated in this study is unit sensitive. The future optimization methods can include incorporating equations for objective function which are unit independent.

7. Adding requirements from other attributes such as energy consumption, NVH etc. to better select a optimum tire for car could be considered in the future study.

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