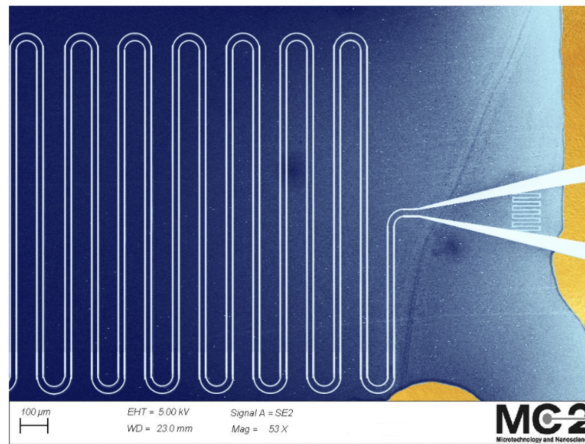
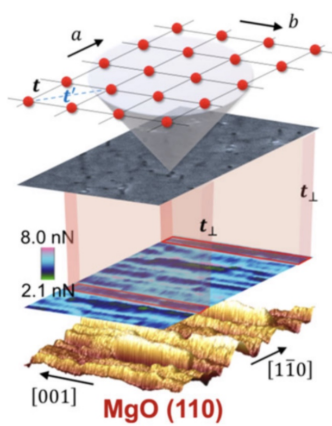
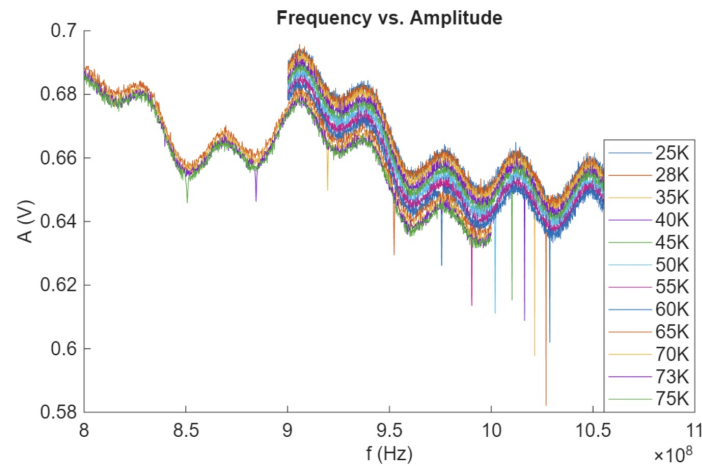




Characterizing Superfluid Stiffness in Cuprate Superconductors

Using Coplanar Waveguide Resonators

Master's thesis in Physics

Carl Oscar Fagrell Alkeskog

DEPARTMENT OF MICROT TECHNOLOGY AND NANOSCIENCE

CHALMERS UNIVERSITY OF TECHNOLOGY

Gothenburg, Sweden 2026

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MASTER'S THESIS 2026

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Carl Oscar Fagrell Alkeskog



Department of Microtechnology and Nanoscience
Quantum Device Physics Laboratory
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Abstract

High Temperature Superconductors (HTS), despite being discovered almost four decades ago, remain an unsolved puzzle that demands innovative approaches and advanced experimental techniques. The fundamental mechanism behind Cooper-pair formation is still unknown, posing a major challenge to efforts aimed at increasing the critical temperature. Recently, the observation of superconductivity in Magic-Angle Twisted Bilayer Graphene (MATBG), driven by moiré-physics and flat-band phenomena, has inspired new perspectives on HTS. This connection arises from striking similarities between cuprates and MATBG, both hosting multiple correlated electronic phases, including superconductivity, although MATBG exhibits a much lower critical temperature.

The increased density of states in flat bands naturally enhance the pairing critical temperature but simultaneously suppress superfluid stiffness due to the diverging effective mass, hindering superconductivity. However, the quantum metric in multi-band systems, such as MATBG, restores finite stiffness, enabling a superconducting state.

Analogous moiré-like effects can be induced in HTS thin films via substrate-driven superpotentials. For instance, quasi-periodic one-dimensional potentials from nanometer-scale facets on (110) MgO substrates significantly influence ultrathin YBCO films, promoting electronic nematicity, a precursor to flat-band physics.

In this thesis, we investigate superfluid stiffness, which is proportional to $1/\lambda^2$ (with λ being the London penetration depth) in YBCO films of varying thickness grown on different substrates. Our measurements reveal a pronounced increase in penetration depth for films on (110) MgO compared to LSAT substrates lacking surface nanostructures, indicating flatter electronic bands. These findings support the possibility of engineering band structures in YBCO thin films, paving the way toward tuning critical temperatures to higher values.

Keywords: superconductivity, YBCO, superpotential, moiré-physics, nano-facets, quantum-metric, resonator, flat-band, nanofabrication.

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This thesis, the hitherto greatest task of my life, would not have been possible without the extraordinary research group QManD, who in addition to tremendous knowledge have given me many laughs and interesting discussions. Nor can I understate the endless support of my beloved family in the roughest of times. I would also like to direct a special thanks to the tireless efforts of Karn Rongrueangkul; not only did I receive the greatest of supervision, but a friend for life.

Carl Oscar Fagrell Alkeskog, Gothenburg, January 2026



To Istis.

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

AFM	Atomic Force Microscopy
BCS	Bardeen-Cooper-Shreiffer (Theory)
CPWR	Coplanar Waveguide Resonator
EBL	Electron Beam Lithography
HTS	High Temperature Superconductor/Superconductivity
LBCO	Lanthanum Barium Copper Oxide
LSAT	Lanthanum Aluminate-Strontium Aluminium Tantalate
LTS	Low Temperature Superconductor/Superconductivity
MATBG	Magic-Angle Twisted Bilayer Graphene
MgO	Magnesium Oxide (110)
PLD	Pulse Laser Deposition
PPMS	Physical Property Measurement System
SEM	Scanning Electron Microscopy
VNA	Vector Network Analyser
YBCO	Yttrium Barium Copper Oxide

Nomenclature

Below is the nomenclature of symbols and parameters used to denote quantities throughout the thesis.

Symbols

θ	Angular offset/phase
θ_{Magic}	Magic Angle
T_c	Critical Temperature
$E(k)$	Energy Band
m_{eff}	Effective Mass
$\hbar$	Reduced Planck's constant
D_s	Superfluid Stiffness
n_s	Superfluid density
e	Elemental charge
λ_L	London Penetration Depth
λ	Wavelength
E_n	Excitation Energy
n	Fermi liquid density OR Natural Number
ω_0	Fundamental Angular Resonance Frequency
f_0	Fundamental Resonance Frequency
A	Amplitude
ω	Angular Frequency
t	Time
Γ	Reflection Coefficient/Amplitude
$\Delta\omega$	Bandwidth
Q	Quality
ΔE	Dissipation Energy

ω_n	n^{th} Angular Resonance Frequency
f_n	n^{th} Resonance Frequency
τ	Relaxation Time
κ	Decay rate
L	Inductance
L_G/L_K	Geometric/Kinetic Inductance
C	Capacitance
C_G	Geometric Capacitance
λ_0	Zero-point penetration depth
γ	Fitting parameter OR Attenuation constant
R	Regular/Internal Resistance
R'	External Resistance
q_s	Supercharge/Cooper-pair
$\vec{E}$	Electric Field
ξ	Coherence length
V	Voltage/Potential
I	Current
J	Conventional Current (density)
J_s	Supercurrent (density)
J_c	Critical Current
e^-	Electron
T	Temperature
F	Force
F_G	Gravitational Force
$\vec{B}$	Magnetic Field
H_c	Critical (Applied) Field
H_{c1}/H_{c2}	Lower/Upper Critical Field
x	Variable
μ_0	Permeability of empty space
Φ_0	Magnetic Flux Quantum
h	Planck's constant
Λ	London Parameter
$\vec{A}$	Magnetic Vector Potential
m_e	Electron mass

ψ	Order parameter/Wave function
f_s	Free Energy density
α/β	Phenomenological coefficients OR real/imaginary attenuation
ϕ	Quantum Mechanical Phase
m	Mass
ψ_s	s -wave order parameter
ψ_d	d -wave order parameter
E_F	Fermi Energy
Δ	Superconducting Band Gap
E	Energy
$\mathcal{V}$	Control Volume
u_k	Occupied State
v_k	Empty State
$\hat{c}^\dagger/\hat{c}$	Creation/Annihilation Operator
$ 0\rangle$	Vacuum State
H	Hamiltonian
n_p	Cooper-pair density
p	Hole doping
a, b, c	Crystal lattice axes
Z_0	Characteristic Impedance
Z_L	Load Impedance
R_l	Resistance per unit length
L_l	Inductance per unit length
C_l	Capacitance per unit length
G_l	Conductance per unit length
Γ_L	(Loaded) Reflection Coefficient
Z_{TL}	Transmission Line Impedance
Z_{RLC}	RLC circuit Impedance
l	Length of resonator strip
a, d, s, g	Resonator dimensions
$K(x)$	Elliptic integral of the first kind
ε_0	Permittivity of empty space
ε_r	Relative permittivity
Q_{int}	Internal Quality

Q_{ext}	External Quality
f	Frequency
v	Velocity
c	Speed of light
ε_{eff}	Effective permittivity
i	Imaginary unit OR index
j	Imaginary unit

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1

Introduction

1.1 Background Motivation

High Temperature Superconductors have been studied for almost 40 years, yet their inner mechanism remains a mystery. The materials can enter a superconducting phase, so much is known, but it is unclear *why* they are able to do so.

Superconductors is a family of materials that can carry a current without developing a voltage, but require extremely low temperatures in order to do so. Low Temperature Superconductivity (LTS) requires as low as liquid Helium range, $\sim 4.2\text{K}$, whilst High Temperature Superconductivity (HTS) can raise this value to the order of liquid Nitrogen, $\sim 77\text{K}$. Though high temperature superconductors significantly reduces operating costs compared to low temperature superconductors, they still require a full cryogenic setup, hindering widespread application into consumer electronics. The true power of superconductivity will not be unleashed until a super-current can be developed under ambient conditions.

In the grand quest toward Room Temperature Superconductivity, the narrow aim of this thesis is to investigate whether it is possible to influence the properties of a certain superconducting material. More specifically, whether a certain substrate surface morphology has an effect on supercharge mobility in a thin film deposition of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, also known as YBCO. Formally, this investigation has been prompted by the Flat Band Hypothesis: substrate MgO surface nano-facets flatten the energy bands of a ultra-thin film deposition of YBCO. The goal of this thesis is to explore the possibility of engineering flat bands in thin film YBCO by growing the films on a substrate with a nano-faceted surface.

1.2 Flat Band Hypothesis

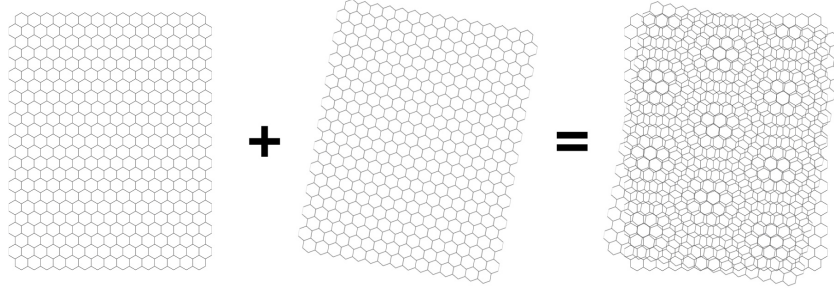


Figure 1.1: Twisted Bilayer Graphene: 2 layers of Graphene superimposed with an angular offset. At this particular offset, the local hexagons produce a self-similar hexagon pattern globally.

Recent discoveries [7] report the observation of superconductivity in Magic-Angle Twisted Bilayer Graphene (MATBG). Superimposing 2 layers of graphene at an angular offset θ (Fig. 1.1), superpositions their individual potentials into a *moiré-pattern* Superpotential¹. At the so called “magic” angles, θ_{Magic} , where superconductivity emerges, the individual potentials of the layers interfere to generate an almost flat electronic band structure. Would it then, hypothetically, be possible to enhance the properties of already known superconductors by flattening out their energy bands?

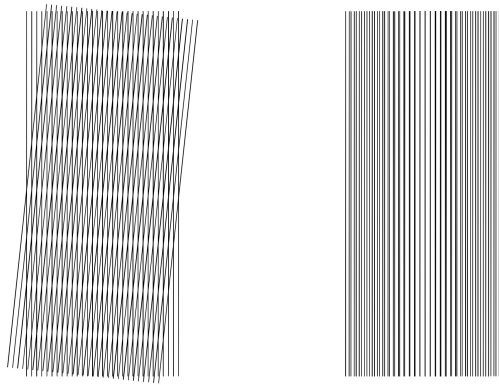
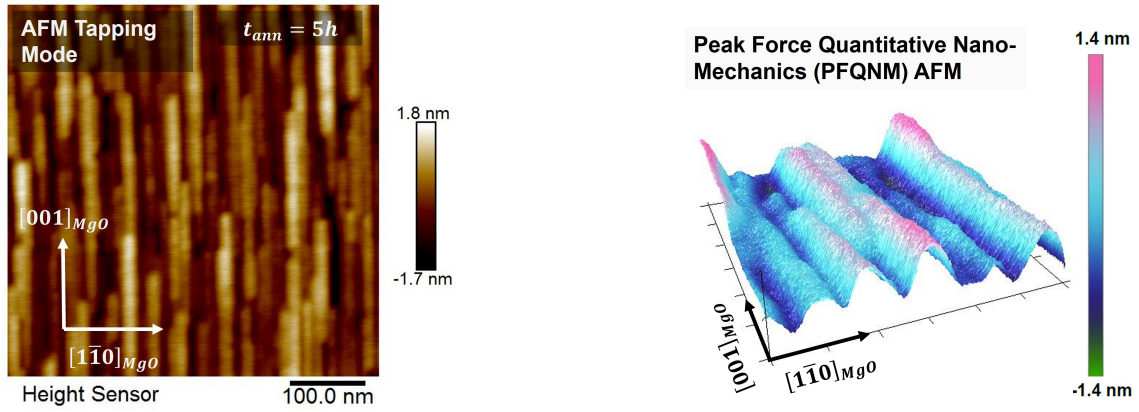


Figure 1.2: Example moiré patterns using parallel lines. An angular offset (left) creates the optical illusion of horizontal wavefronts. Stretching (right) instead creates vertical wavefronts, which can be observed in parallel fences around pedestrian bridges or train stations, if the bars are thin enough!

¹A superpotential is a spatially periodic effective potential with the periodicity of the moiré superlattice (see right panel in Fig. 1.1)



(a) 2D image of MgO nanofacets. Quasi-periodic surface resembling a Moiré pattern. Captured by Riccardo Arpaia.

(b) 3D rendering of MgO nanofacets created using AFM peak force mode. Created by Riccardo Arpaia.

Figure 1.3: Nanofacets observed by QManD on MgO substrate-surface.

The QManD group at Chalmers, spearheaded by professor Floriana Lombardi and specialized in the study of thin films, has observed that heating a MgO (110) substrate above 700°C forms *nano-facets* on its surface. The nano-facets creates a quasi-periodic surface morphology which resembles a moiré pattern, pictured and graphed in Fig. 1.3. It was noticed that the nano-facets induced strain in thin film YBCO deposited on top, which produced an effect on critical temperature T_c opposite to what was expected. Expanding the phase diagram of YBCO around its hole doping p (see section on YBCO) and temperature T , the bottom region contains the empirical “superconducting dome”. In the so called underdoped regime, $p < 0.16$, there exists another phase known as Charge Density Wave (CDW), characterized by a modulation of electron density. CDW competes with superconductivity, deforming the superconducting dome into significantly reducing T_c , graphed below in Fig. 1.4. Introducing strain, one would expect T_c to go down, but the strain induced from nano facets instead partially eliminated the suppression.

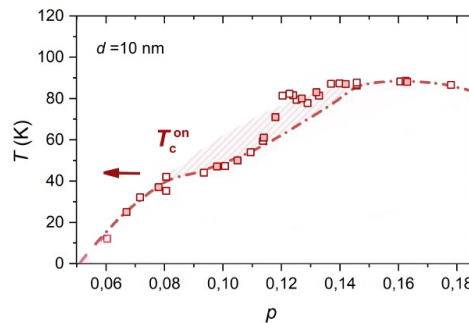


Figure 1.4: Thermodynamic diagram; hole doping p vs. temperature T , of the lower underdoped region of YBCO. Nano-facets boost critical temperature T_c . The dash-dotted line is T_c for a thick film of YBCO, and the squares are T_c -onset values for a thin film on (110) MgO. Image adapted from [1].

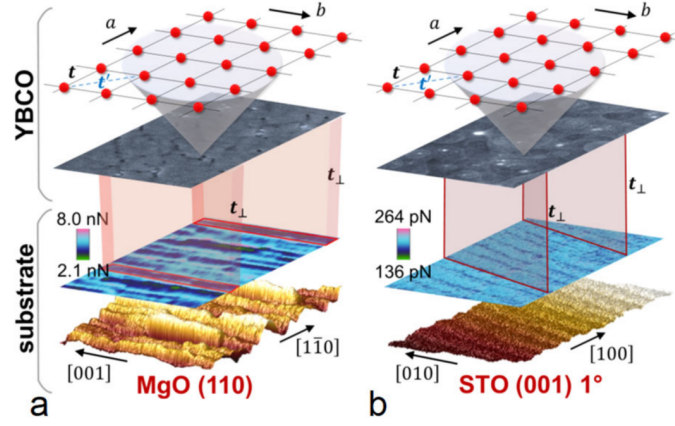


Figure 1.5: Periodic surface structures of MgO (a) and STO (b) which generate strain in YBCO. In both cases, periodicity is only exhibited along 1 direction. Image adapted from [2].

Strain inside YBCO, which is otherwise studied because it produces anisotropic transport (A&B below), can be induced by other substrate morphologies than MgO, e.g. vicinal cut STO illustrated above, but the additional effect on T_c makes the nano-facets unique. According to theoretical predictions made by physicist Götz Seibold at Brandenburg Technical University, the action can be attributed to the nano-facets generating a 1D superpotential [2] which makes the Fermi surface nematic (Fig. 1.6E) and consequently the CDW one-directional, which in turn partially eliminates CDW suppression of superconductivity [3]. Together with the experiments conducted at QManD, it appears that the quasi-periodic nano-facets generate a 1D Superpotential. Recalling the magic angle of twisted bilayer graphene, this poses a question: does the nano-facets also generate a flat band structure?

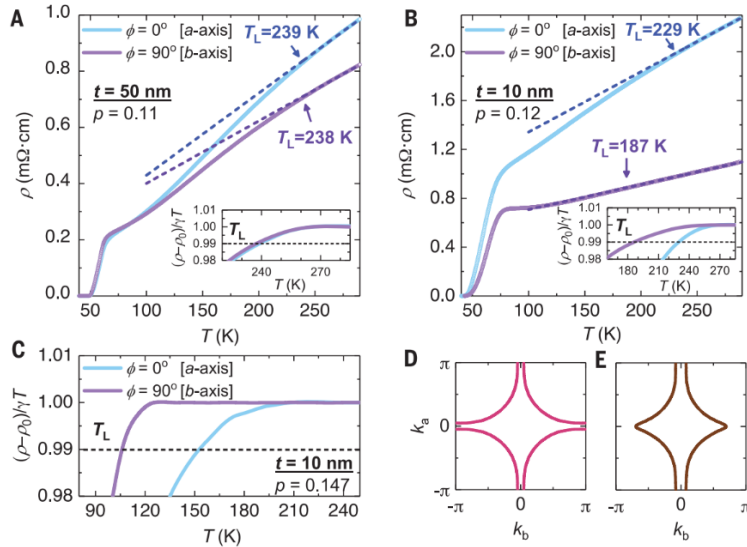


Figure 1.6: (A&B) Anisotropic conductivity induced by strain (D) Simplified model of the typical Fermi surface of cuprates. (E) Theoretical anisotropic Fermi surface model. Image adapted from [3].

In order to answer this question, a measurement needs to be performed. The structure of energy bands are typically measured using spectroscopy, but this will unfortunately not work on YBCO. Angle-Resolved Photoemission Spectroscopy (ARPES) requires a flat crystal structure which cut into smooth surfaces, but YBCO buckles its lattice vectors when deposited on MgO due to lattice mismatch (consequently cutting into rough surfaces). For this reason, the effects on the energy bands $E(k)$ needs to be measured indirectly via its charge transport properties.

One possibility to obtain information about the energy bands is to measure the *superfluid stiffness* D_s , or equivalently the London penetration depth λ_L (see Section 2.1 and 2.4). Superfluid stiffness quantifies supercurrent response, i.e. how “stiff” the associated superfluid is, and is given by:

$$D_s = \frac{n_s e^2}{m_{\text{eff}}} \propto \frac{1}{\lambda_L^2}$$

where m_{eff} is the *effective mass* of the charge carriers. From conventional band theory, effective mass is a quantitative measure of how an electron behaves under the influences of forces inside a solid and is represented as the curvature of associated energy bands:

$$m_{\text{eff}} = \hbar^2 \left(\frac{\partial^2 E}{\partial k^2} \right)^{-1}.$$

If the energy bands are flat, whereby above derivative approaches 0, the effective mass diverges and charge carriers will remain static. However, new theoretical developments point towards that a non-trivial *Quantum Geometry* can allow bound *electron-pairs* in multi-orbit systems, even in the case of flat bands! Supercharges are electron-pairs (see section 2.1), so this would allow supercharges to attain a finite effective mass, consequently a finite superfluid stiffness and therefore dynamic charge carriers [8] [9] [10]. Furthermore, because superfluid stiffness is inversely proportional to the square of penetration depth λ_L , which is a measurable quantity, estimating λ_L experimentally allows determining how the nano-facets affects the energy bands compared to a flat substrate. An experiment capable of measuring penetration depth λ_L can be conducted with the help of a Coplanar Waveguide Resonator (CPWR), illustrated in Fig. 1.7.

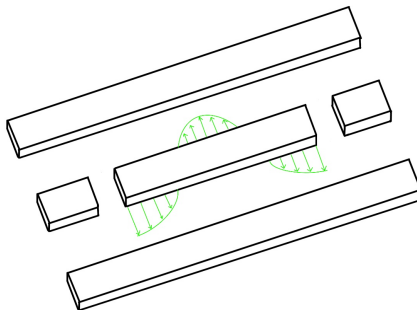


Figure 1.7: Illustration of a Coplanar Waveguide Resonator (CPWR) with an oscillating electric field in green. The image depicts the first mode of a half-wave $\lambda/2$ resonator; two open ends, no shorts.

1.3 Resonator Experiment

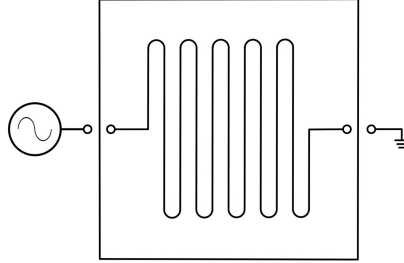


Figure 1.8: Example sketch of a resonator cavity meandering over a chip with superconducting thin film. Sending in an electrical signal excites the device. The strip is typically made very long to lower the resonance frequency of the cavity.

A coplanar waveguide is a type of transmission line (with imposed boundary conditions) that can be printed on a circuit board. Transmission lines are used to convey electrical signals in the microwave range (around 300 MHz to 300 GHz) across a wide range of electronic devices. A resonator is a geometry capable of manifesting a standing wave phenomena, for example a flute would be an acoustic resonator. A coplanar waveguide with the appropriate boundary conditions therefore yields a Coplanar Waveguide Resonator (CPWR). Since a coplanar waveguide is a type of transmission line, a CPWR is a Transmission Line Resonator, a type of electric resonator.

If a Transmission Line Resonator is fabricated on a chip with superconducting thin film, see Fig. 1.8, and cooled below T_c , the device can be excited by sending in an electric signal $Ae^{i\omega t}$. Figuratively, the resonator can be represented as a photon bouncing back and forth elastically inside the resonator cavity. Performing a frequency sweep over reflected power Γ (see section 2.3), a portion of the power is absorbed only around the *resonance frequencies*, producing a frequency dip. In the ideal case, where the device is critically coupled, 100% of the signal is absorbed under a full 360° phase shift, but in reality it is some fractional value (see Fig. 1.9).

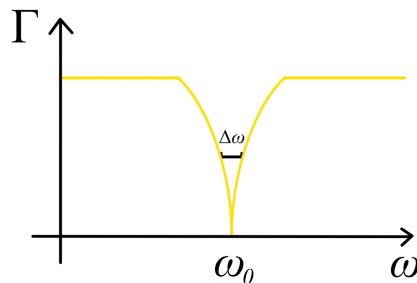


Figure 1.9: Reflected power Γ of signal around the fundamental resonance frequency ω_0 . The bandwidth $\Delta\omega$ is the Full Width Half Maximum of the dip. Their quotient $\frac{\omega_0}{\Delta\omega_0}$ approximates the quality factor Q of the resonator.

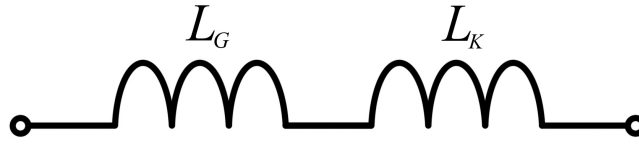


Figure 1.10: Geometric Inductance L_G and Kinetic Inductance L_K are different properties, represented as a series circuit: $L = L_G + L_K$

Because a coplanar waveguide is a type of transmission line, it can be analysed as one using a *distributed element* model. This means, explained in Section 2.3, that a CPWR will be similar to a lumped parallel RLC circuit around resonance, with the resonance frequency ω_0 given by the electric properties Inductance L and Capacitance C . However, represented in Fig. 1.10, transitioning into a superconducting phase introduces *Kinetic Inductance* $L_K(\lambda_L)$ separate from the conventional Geometric Inductance L_G , which will affect the fundamental resonance frequency:

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{[L_G + L_K(\lambda_L)]C_G} \quad (1.1)$$

where $L = L_G + L_K$ and $C = C_G$ correspond to the *total*² Inductance and Capacitance of the resonator, respectively. Again, as further explained in Section 2.3, both Geometric Inductance L_G and Capacitance C_G can be approximated analytically using elliptic integrals, thus measuring the resonance frequency yields the contribution from Kinetic Inductance L_K .

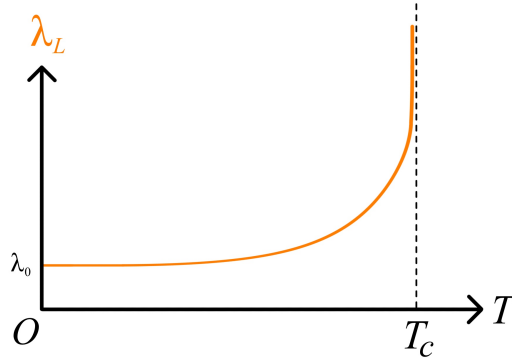


Figure 1.11: London Penetration Depth λ_L with respect to Temperature T . The parameters λ_0 and T_c represent the penetration depth at zero temperature and the critical temperature of the superconducting phase transition, respectively.

Kinetic Inductance is the Kinetic Energy of superconducting charge carriers, which is proportional to the square of the London penetration depth λ_L . Penetration depth can be expressed as a function of Temperature with parameters zero-point value λ_0 and critical temperature T_c :

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1 - (T/T_c)^\gamma}}$$

²Capacitance is assumed to remain unaffected when transitioning to a superconducting phase

with γ being an additional fitting parameter. The archetypical shape of the function is sketched in Fig. 1.11. Decreasing temperature to T_c introduces both London penetration depth and kinetic inductance at their maximum values. Decreasing past T_c the penetration depth, and therefore kinetic inductance, decreases, which increases the resonance frequency. The sketch below illustrates how measuring a resonance at different temperatures displaces its position in the frequency spectrum. Plotting resonance vs. temperature, the London parameters λ_0 , T_c and γ can be extracted via data processing.

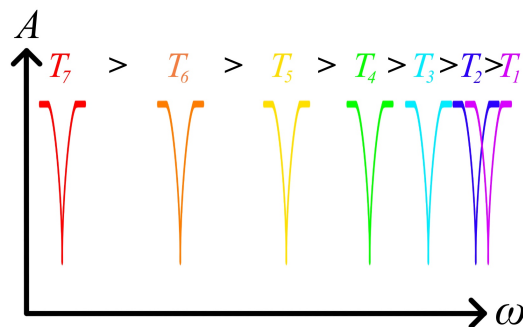


Figure 1.12: The same Resonance frequency over different Temperatures. Increasing Temperature increases penetration depth, which in turn increases the kinetic inductance. Increase in inductance implies decrease in Resonance frequency.

Resonances, like those drawn above, can be measured using a Vector Network Analyser (VNA). Further analysis of the shape of the individual dips allows extracting the *Quality Factor* Q of the resonator: energy stored E_n (excitation Energy), divided by energy loss per cycle ΔE_n (dissipation Energy), which correlates with the bandwidth $\Delta\omega$ of a resonance dip. If the average number of photons inside the cavity is large, it can be approximated as the quotient between resonance frequency and bandwidth [11]:

$$Q = 2\pi \frac{E_n}{\Delta E_n} \simeq \frac{\omega_n}{\Delta\omega_n} = \frac{f_n}{\Delta f_n}$$

This means that a resonator of high quality only resonates at very specific frequencies, which corresponds to the figurative photon oscillating for longer before dissipating all its Energy. Thus Q is associated with some relaxation time $\tau = 1/\kappa$, where κ is the decay rate: $Q = \omega_n/\kappa$. Performing dimensional analysis, Q is a non-dimensional number that equates to the number of oscillations completed inside a cavity before dissipating all its energy, see Fig. 1.13, which is a reflection of the losses associated with the dissipation process. Losses can be represented as an Effective Resistance $R_{\text{eff}} = R + R'$ of the resonator circuit, where R and R' corresponds to the internal and external losses, respectively:

$$\frac{1}{Q} \simeq \omega_n \frac{C}{1/R + 1/R'}$$

The Q -value of a resonator can be extracted via curve-fitting reflection data measured by the VNA.

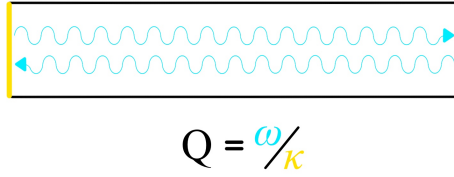


Figure 1.13: The Quality Q of a resonator cavity is the number of oscillations completed before decaying. Given a frequency ω , the decay rate κ sets the number of oscillations per excitation, which is a reflection of the losses associated with the process.

Because λ_L and Q can be extracted from measurement data, these values can be compared if other aspects of the resonator are varied³. Comparing values provides insight into how the superconducting material comprising the resonator is affected.

1.4 Thesis Structure

This thesis has been structured into 3 parts: physical theory, experimental method and observed results. Hence, the conclusion is presented.

The theory section dives into a few of the essential ideas surrounding superconductivity to introduce the subject, both physically and mathematically. The experimental device is modelled to derive the quantities of experimental interest. The Flat Band Paradigm is given a rigorous treatment.

The method contains an overview of all empirical work conducted during the scope of this thesis. Thin film of superconducting material has been deposited, characterized, and optimized. Films have undergone nanofabrication to create structures in the material. Fabricated devices have been connected and measured electrically. Finally, the data has been processed by writing a computer program.

In results and discussion, the outcomes of the project are highlighted with relevant observations. When the results have been presented, they are tied up into a final conclusion.

³In this thesis film thickness and substrate material

2

Theory

For the uninitiated follows a brief overview on the idea of superconductivity. Whence delving into the electromagnetism of the resonator device, the chapter is tied up with the Flat Band Hypothesis.

2.1 Superconductivity

This section is based upon past lectures, with inspiration drawn from Richard Behiel's video *Superconductivity and the Higgs Field*. [12]

2.1.1 Physics

Any real theory within the universe must, without exception, be rooted in empirical observation.

2.1.1.1 Fundamental Phenomenon

Superconductivity is characterized by 3 fundamental phenomena: quantization of supercharge, magnetic field expulsion and quantization of magnetic flux.

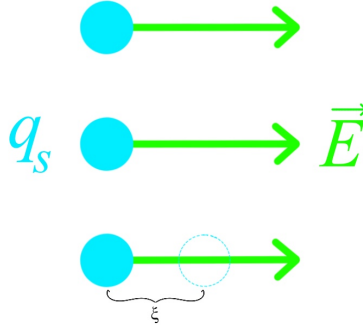


Figure 2.1: Supercharges q_s accelerate individually without resistance when applying a DC electric field $\vec{E}$: the Onnes effect. Supercharge quanta only phase into existence over a Coherence Length ξ , which is conceptually similar to a mean free path.

Conductivity $1/R$ is a quantitative measure of a material's ability to transport electric charge q , famously given by Ohm's law:

$$\frac{I}{V} = \frac{1}{R}$$

where V is Voltage, I is current. If a material undergoes a *Superconducting Phase Transition*, it can carry a DC current without developing a Voltage, breaking Ohm's law:

$$\frac{I}{V} = \lim_{R \rightarrow 0} \frac{1}{R} \rightarrow \infty$$

A superconducting phase allows charge to be *accelerated* individually, rather than collectively¹, via the formation of a new type of quantum: *supercharge* q_s (Fig. 2.1). These are separate from conventional charges, making the difference between

¹In a conventional current, the collective flow of charge carriers is very fast, but individually they move very slowly atom to atom. A supercurrent allows individual charge carriers to move very fast relative the atomic lattice

a supercurrent (density) J_s and a conventional current (density) J the type of charge they carry. Crucially, the supercurrent is maximally bounded by a value J_c called the critical current, where conduction returns to normal. Conventional charges q are electrons; e^- , and supercharges are entangled electron-pairs:

$$q_s \simeq 2e^-$$

which stems from BCS Theory discussed further down.

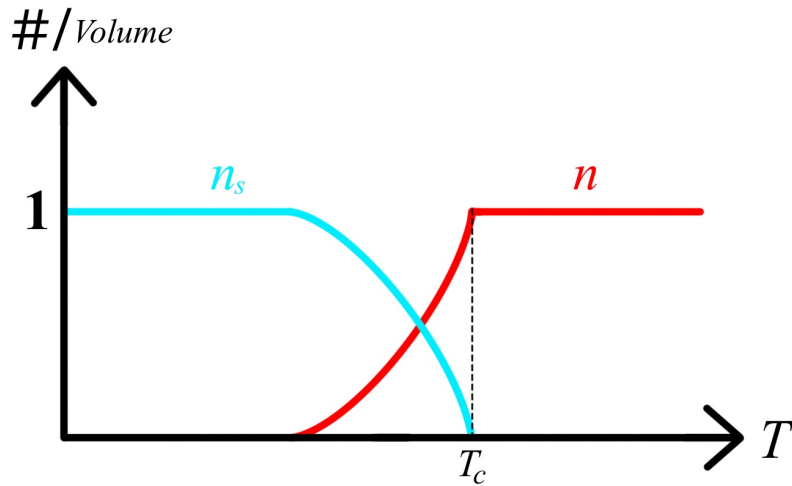


Figure 2.2: Two Fluid Model. A total amount of charges separated into two categories; conventional- and super-charge, which are distributed into two fractions: n and n_s respectively.

Splitting charge carriers into two separate objects is commonly referred to as the Two Fluid Model and sketched in Fig. 2.2. Mathematically, it is a fractional distribution, $n_{tot} = n + n_s = 1$, where the conventional, n , and superconducting, n_s , charges occupying a volume $\mathcal{V}$ are split into a Fermi liquid and a Superfluid Bose-Einstein condensate. If $\mathcal{V}$ contains N total charges, the fractional distributions can be represented as functions of temperature:

$$\begin{aligned} n &= (T/T_c)^\gamma \\ n_s &= (1 - (T/T_c)^\gamma) \end{aligned}$$

Here γ is a fitting parameter² and T_c is the *Critical Temperature*; the temperature where the transition occurs. The model is originally a macroscopic theory on Superfluidity, which is what conditions supercharges to collectively behave as a Bose-Einstein condensate.

²Typically between 2 and 4

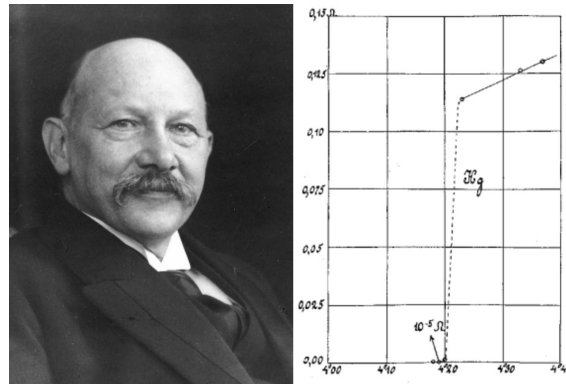


Figure 2.3: Kamerling Onnes and his original plot of the sharp drop in Resistance of Mercury with respect to Temperature.

The Two Fluid Model quantifies the first empirical observations ever made on superconductivity, conducted by Kamerlingh Onnes in 1911. Having previously constructed an apparatus capable of liquefying Helium, Onnes tried submerging a sealed tube of Mercury into the liquid whilst conducting a current. As the Temperature of Mercury approached the surrounding liquid Helium at 4.17K, he observed the Resistance R decrease with Temperature as expected. However, just as the Mercury was about to thermalize with the surrounding liquid, at around 4.2K, the resistance suddenly vanished completely, pictured in Fig. 2.3. Noting down his findings, Onnes went on to find that this remarkable phenomenon was reproduced in various metals. These observations laid the experimental groundwork for superconductivity, or at least the first half.

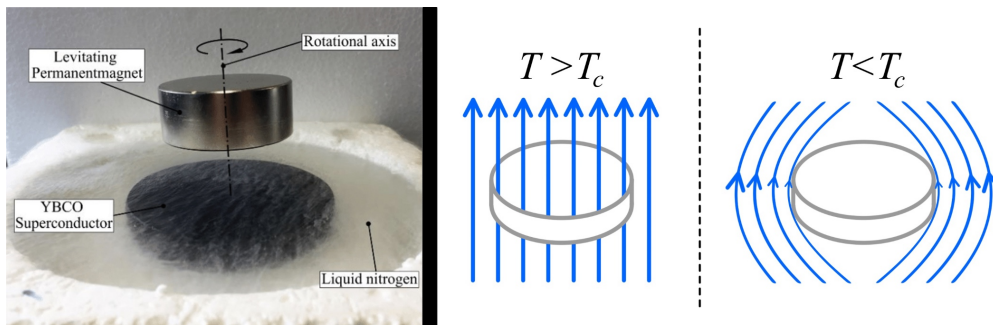


Figure 2.4: Illustration of the Meissner Effect (Right). Magnetic field is expelled from a superconductor below the critical temperature T_c , represented as the bending of field lines. This Action can generate the phenomenon of magnetic levitation (Left). The superconductor expels the field lines produced by the magnet, creating an opposing force to gravity.

The second half deals with magnetic fields instead of currents. Superconductors are not only perfect conductors, they are also perfect diamagnets. Dubbed the Meissner effect, but technically discovered by both Walther Meissner and Robert Ochsenfeld back in 1933, a superconductor cooled below its critical temperature T_c will expel a magnetic field $\vec{B}$ up to a maximum applied field H_c ; the *Critical Field*. This can be thought of as the field lines bending around the superconductor like in

Fig. 2.4, but more realistically the field is opposed by circulating eddy supercurrents. These opposing eddies penetrate into the superconductor, successively reducing the field strength before eliminating it completely. The rapid reduction of field strength can be modelled as an exponential decay and the effect quantified by the London Penetration Depth λ_L :

$$\vec{B} = \vec{B}_0 e^{-x/\lambda_L} : |\vec{B}| < \mu_0 H_c$$

i.e. the $1/e \sim 37\%$ decay length of the field strength, sketched in Fig. 2.5. The typical length scale is typically in the of hundreds of nanometers, $\lambda_L \sim 100\text{nm}$.

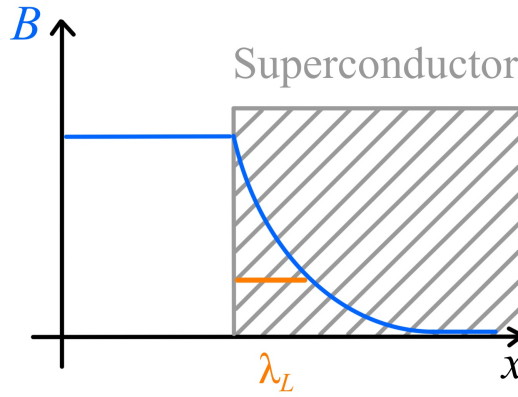


Figure 2.5: Illustration of the definition of London Penetration Depth λ_L . Magnetic field $\vec{B}$ decays exponentially over length x .

Finally, in 1935, Lev Shubnikov discovered what nowadays is referred to as Type-II superconductivity. The hitherto known superconductors were Type-I, meaning that the entire applied field is expelled up to the critical value, H_c . Shubnikov demonstrated that a single crystal Lead-Thallium (PbTl) possessed 2 critical fields, H_{c1} and H_{c2} . The first, H_{c1} , dubbed the Meissner state, is the same full expulsion as Type-I. The second $H_{c2} > H_{c1}$, dubbed the Abrikosov state, allows the applied field to penetrate in discrete quanta, known as Abrikosov vortices³. The vortices formed in the superconducting condensate is a circular flow of supercurrent, which corresponds to a discrete magnetic flux quantum:

$$\Phi_0 = \frac{h}{2e}$$

where h is Planck's constant and e is the fundamental charge. Fig. 2.6 illustrates the basic model of a flux quantum and includes an actual photograph taken of fluxons, as they are also known as, penetrating a sample of YBCO.

³after Alexei Abrikosov who developed their mathematical theory

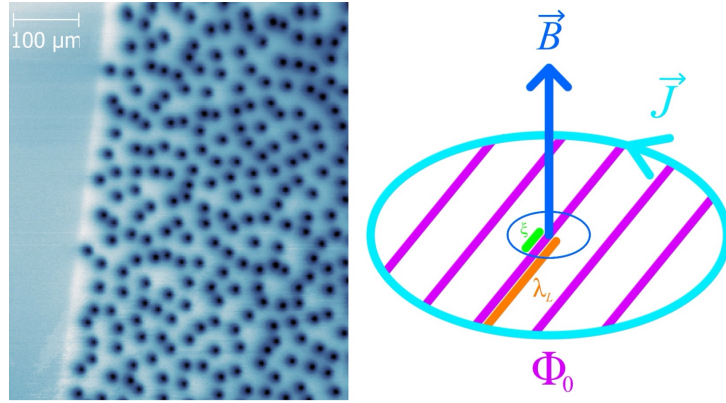


Figure 2.6: (Right) Basic illustration of a magnetic flux quantum Φ_0 , also known as a fluxon. When the magnetic field $\vec{B}$ manages to penetrate the superconductor, it corresponds to an Eddy supercurrent $\vec{J}_s$ with a discrete quantum of flux. The size of the “normal” magnetic core $\vec{B}$ is bounded by the Coherence Length ξ and the flux area encircled by supercurrent $\vec{J}_s$ by London Penetration Depth λ_L . (Left) Actual image taken of fluxons penetrating through YBCO [4].

2.1.1.2 Important Ideas

The two core phenomena of superconductivity; Onnes⁴ and Meissner effect, are characterized by the 2 London equations [13]:

$$\begin{aligned}\vec{E} &= \Lambda \frac{d}{dt} \vec{J}_s \\ \vec{B} &= -\Lambda \nabla \times \vec{J}_s\end{aligned}$$

which can be summarized as a single equation using vector calculus⁵:

$$\vec{J}_s = -\frac{1}{\Lambda} \vec{A} = -\frac{n_s e^2}{m_e} \vec{A}.$$

Here J_s is the supercurrent density, $\vec{A}$ is the magnetic vector potential and $\Lambda = \frac{m_e}{n_s e^2}$ is a constant known as the London parameter, whose reciprocal $\frac{1}{\Lambda}$ will later be identified as the superfluid stiffness D_s of free electron-pairs. Both London equation and penetration depth are named after brothers Fritz and Heinz London who derived them in the same year Shubnikov discovered Type-II, 1935. It is therefore crucial to point out that the London model does not account for Type II superconductivity, but serves as a good first approximation. Incorporating Type II into theory requires a thermodynamic perspective.

⁴As in no resistance, not to be confused with the concept of Rollin film

⁵See section 2.1.2.2 on Mathematics

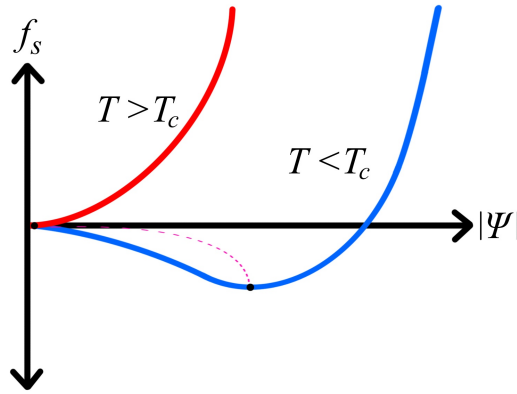


Figure 2.7: Sketch of the Ginzburg-Landau free Energy f_s with respect to the order parameter ψ . When the temperature T is decreased below the critical value T_c , the free Energy curve transforms to yield a negative free Energy minimum. The family of associated curves is represented by their minima, following the pink dashed line.

In 1950, Lev Landau and Vitaly Ginzburg presented a thermodynamic approach to superconductivity and the best description so far: Ginzburg-Landau theory. But it is not a mechanical theory of superconductivity, rather a phenomenological framework based upon Lev Landau's previously established theory on second-order phase transitions. They argued that the free Energy density f_s of a superfluid n_s can be expressed in terms of a complex order-parameter field, $\psi = \sqrt{n_s}e^{i\phi}$:

$$f_s = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m_{\text{eff}}} \left| (-i\hbar\nabla - q_s\vec{A})\psi \right|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$$

where m_{eff} is the effective mass of a supercharge $q_s \simeq 2e^-$. As illustrated in Fig. 2.7, the shape of this function varies depending on if the second- or fourth-order term dominate the expression. As discussed in more detail during Section 2.1.2.2 on mathematics, applying various physical principles can derive much of the mathematics surrounding superconductivity. For example, the London equation is recovered if the electromagnetic terms of f_s dominate, but Ginzburg-Landau is still **not** a mechanical theory on superconductivity. Ginzburg-Landau gives you all of the *how* regarding superconductivity, but none of the *why*.

Why superconductivity manifests is, at time of writing, still a convoluted question, but it has something to do with the crystal lattice. High Temperature Superconductivity is mostly a mystery, but Low Temperature Superconductivity is well described by BCS theory and can be attributed to electron-phonon coupling. This can be observed via the so-called Isotope Effect. The same year Ginzburg and Landau published their theory, 1950, Emanuel Maxwell demonstrated how different isotopes of Lead Pb exhibit different critical temperatures, decreasing as an inverse square of molecular mass:

$$T_c \sim \frac{1}{\sqrt{m}}$$

The idea at the time was that the molecular mass correlates with its Debye modes ω_D , suggesting electron-phonon coupling as critical for superconducting phenomena.

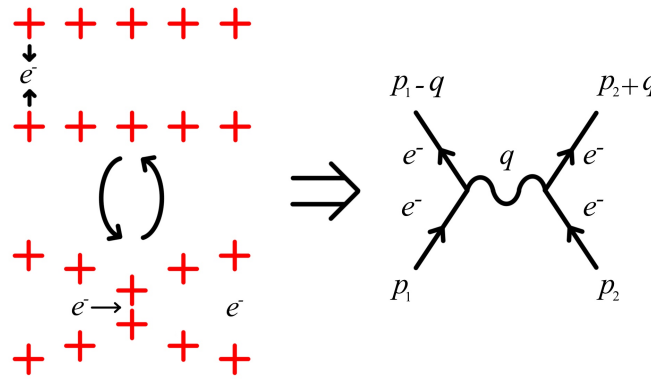


Figure 2.8: BCS pairing mechanism in state space. An electron in motion through a superconducting crystal lattice attracts neighbouring lattice nodes. The subsequent concentration in charge distribution exerts an increased attraction on a following electron, causing a relative attraction between the sequential electrons. The process can be represented as a Feynman diagram, where q is a force mitigating phonon that exchanges momentum between the particles. Time moves up in the diagram.

The idea of electron-phonon coupling would be further expanded upon in 1957 by John Bardeen, Leon Cooper, and Robert Schrieffer (BCS), in their theory on Phonon-Mediated Electron Pairing. In a simplified nutshell illustrated above in Fig. 2.8, an electron moving through a lattice will attract the positively charged atomic nodes of the lattice. The attraction causes a concentration of positive charge which attracts a next-coming electron. The interaction between the electrons and the atomic lattice enables pairs of electrons to exchange momentum which correlates their spins over a s -wave order parameter⁶ ψ_s . When many identical pairs are formed, they overlap very strongly to form a collective condensate (Bose-Einstein condensate). Because of this, the energy required to break one pair is identical to breaking all of the pairs, which means that lattice excitations (at sufficiently low temperatures) are not strong enough to affect the condensate nor the individual pairs. Therefore, the electrons remain paired and avoid scattering, which causes the total electron flow to not experience any resistance.

Zooming out a little bit, the compound phenomena of charge concentration inside the lattice mimics lattice vibrations. In a picture (Fig. 2.9), the concentrated regions overcome the Coulomb repulsion and induce a small attractive potential between two electrons. This causes them to pair up into a single bosonic particle, a *Cooper-pair*⁷. In fact, even an arbitrarily small potential attraction between electrons can cause a paired state as the Energy state E will be lower than their individual Fermi Energies E_F :

$$E = 2E_F - \Delta.$$

Here $\Delta > 0$ represents the size of the band gap between the normal and superconducting state of 2 electrons.

⁶The spins of the two electrons become entangled, such that the spins point in opposite directions via Pauli's exclusion principle (and add to 0).

⁷The name derives from Cooper, who gave the mathematical proof on pairing

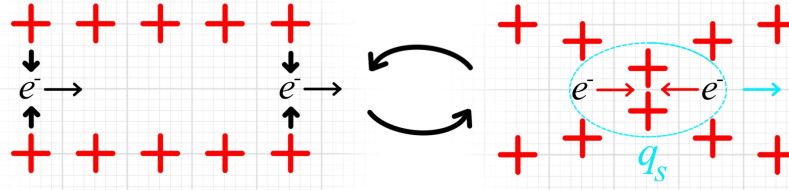


Figure 2.9: BCS pairing mechanism in physical space. Negatively charged electrons traversing the superconducting crystal lattice pull on the positively charged atoms. The lattice vibrations induced from the perturbation concentrates positive charge which overcomes the Coulomb repulsion of the electrons, causing them to pair up into a single particle; a Cooper pair q_s .

BCS provides a microscopic theory on Low Temperature Superconductivity, but not all superconductors can be described via the mechanics of BCS. In particular, none of the known High Temperature Superconductors. Perhaps this is why the theory is commonly associated with predicting a maximum critical temperature: $T_c \lesssim 30$, but this is not true. Exempli gratia, the superconductor MgB_2 exhibits a $T_c = 39\text{K}$ and is known to be BCS. There is nothing preventing a critical temperature T_c above 30K within BCS itself, though it is an OK approximation when classifying known superconducting materials according to critical temperature.

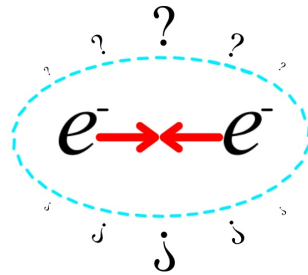


Figure 2.10: The general mechanism responsible for the pairing of Cooper pairs is unknown.

As a final mention, the crucial aspect of superconductivity is the pairing mechanism. Whenever BCS is mentioned, it strictly refers to the pairing mechanism responsible for pairing up electrons into a supercharge being electron-phonon coupling. In BCS, the action is attributed to phonon mediated attractive interaction between quasiparticles (electrons), but experiments demonstrate clear deviation from this paradigm, namely High Temperature Superconductors. Besides being known to produce Cooper-pairs, no one knows why these materials can become superconducting.

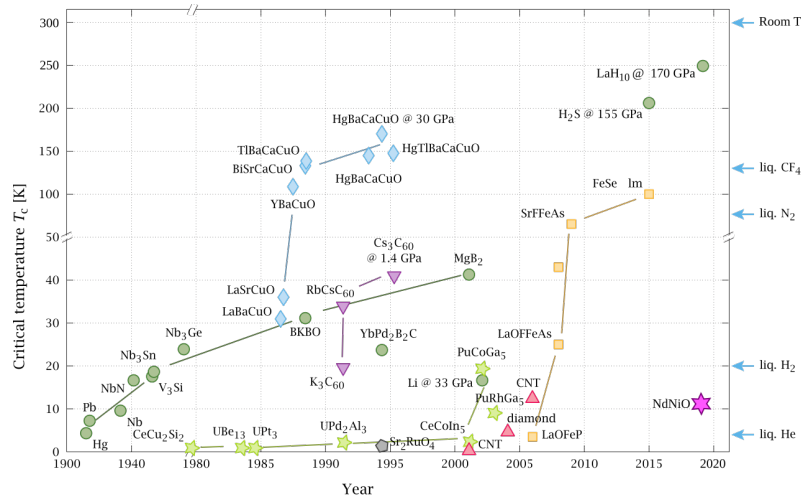


Figure 2.11: A map of the timeline of superconductivity, created by Pia Jensen Ray and cited as [5] per request, but found and retrieved from Wikipedia. The image depicts the critical Temperature T_c of various compounds discovered versus when they were discovered.

2.1.1.3 High Temperature Superconductivity

The name “High Temperature” might seem a bit misleading at first, as the relevant temperature range lies just above the boiling point of liquid Nitrogen, $T \sim 77\text{K}$. However, from an empirical perspective, dealing with liquid Nitrogen is a lot easier than liquid Helium. First and foremost, it is abundant, literally drawn from the air we breathe, making it significantly cheaper, but it is also simpler to produce and handle. Liquid Nitrogen being cheaper than liquid Helium enables more research and practical applications, granting the name high-temperature Superconductivity (HTS).

In 1986, researchers Georg Bednorz and Alex Müller made the first discovery of HTS in Lanthanum Barium Copper Oxide, $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$; LBCO. Technically, they “only” achieved a critical temperature $T_c \simeq 35\text{K}$, but their proof of concept was quickly modified by Paul Chu to realize $T_c \simeq 92\text{K}$ in Yttrium Barium Copper Oxide, $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$; YBCO. This made YBCO the first material found to be superconducting above liquid Nitrogen, making it the first High Temperature Superconductor ever discovered.

Conductors are traditionally metals, but Bednorz, Müller, and Chu had been studying a new class of ceramics known as Copper Oxides, or *Cuprates*. These materials consist of stacked planes of Copper Oxide CuO_2 planes with some sort of doping in between, making the materials effectively 2-dimensional. High- T_c superconductors also possess the ability to retain superconductivity in higher magnetic fields compared to Low- T_c . The Cooper pairs correlate over a d -wave order parameter ψ_d , reflected in the materials being highly anisotropic. A collection of known superconducting materials are depicted in Fig. 2.11.

2.1.2 Mathematics

Starting from Maxwell's equations, superconductivity can be mathematically modelled via the Ginzburg-Landau Free Energy Density.

2.1.2.1 Prerequisites

Recall the Maxwell equations for a medium with permittivity ε and permeability μ :

$$\begin{aligned}\nabla \cdot \vec{E} &= \rho/\varepsilon \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{d\vec{B}}{dt} \\ \nabla \times \vec{B} &= \mu\vec{J} + \mu\varepsilon\frac{d\vec{E}}{dt}\end{aligned}\tag{2.1}$$

and the definition of the *Magnetic Vector Potential*:

$$\vec{B} = \nabla \times \vec{A}\tag{2.2}$$

Utilizing the relation on Faraday's law and that derivatives commute, we write the general field solutions as:

$$\begin{aligned}\vec{E} &= -\Delta V - \frac{1}{c}\frac{d\vec{A}}{dt} \\ \vec{B} &= \nabla \times \vec{A}\end{aligned}\tag{2.3}$$

such that $c = \frac{1}{\sqrt{\mu\varepsilon}}$ is the speed of light in the medium. These solutions are *Gauge Invariant*; they take the same shape independent of scaling:

$$\begin{aligned}V' &= V - \frac{dX}{dt} \\ \vec{A}' &= \vec{A} + \nabla X\end{aligned}\tag{2.4}$$

Formally, the associated Lagrangian is symmetric under transformation X .

2.1.2.2 Physical Modelling

Consider the Two-Fluid model, a temperature dependent fractional distribution $n_{tot} = n + n_s = 1$ with a conventional Fermi liquid n and a superconducting Bose-Einstein condensate n_s following the densities:

$$\begin{aligned} n &= (T/T_c)^\gamma \\ n_s &= 1 - (T/T_c)^\gamma \end{aligned}$$

such that T_c is the Critical Temperature where superconducting phenomena emerge. The structure associated with the Superfluid Bose-Einstein condensate is modelled as a wave function with phase ϕ and density n_s as its square modulus, commonly known the Ginzburg-Landau order parameter:

$$\psi = \sqrt{n_s} e^{i\phi},$$

which associates with the free energy density of the superconducting system. The Ginzburg-Landau free energy (density) f_s is a quartic function in order parameter ψ :

$$f_s = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m_{\text{eff}}} \left| (-i\hbar\nabla - q_s\vec{A}) \psi \right|^2 + \frac{1}{2\mu_0} |\vec{B}|^2.$$

where the parameters m_{eff} and q_s are the effective mass and charge of the supercharges comprising n_s . The first 2 terms of the expression are associated with being a condensate (since $|\psi|^2 = n_s$), where $\alpha(T)$ and $\beta(T)$ are phenomenological functions of Temperature T . The remaining 2 terms are associated with the electromagnetic interaction. Identifying the inside of the bracket as the momentum of a charged particle inside an electromagnetic field⁸, the third term represents the Kinetic Energy of the charges. Finally, recall the fourth as the Energy stored within a magnetic field.

If the electromagnetic terms of f_s dominate, such that condensate terms can be neglected, the London model can be retrieved from f_s . Variations in the magnetic vector potential $\delta\vec{A}$ over a control volume $\mathcal{V}$ yields the aforementioned vector potential equation⁹:

$$\Lambda \vec{J}_s = \frac{\hbar}{q_s} \nabla \phi - \vec{A} = \frac{\Phi_0}{2\pi} \nabla \phi - \vec{A}$$

albeit in a slightly modified form. The new term, $\nabla \phi$; *Phase Gradient*, is a reflection of Gauge Invariance and emerges because the superconducting system of Ginzburg-Landau f_s breaks $U(1)$ -symmetry [12]. Adopting the London Gauge; $\nabla \cdot \vec{A} = 0$ sets the phase gauge $\phi = 0$, yielding the regular:

$$\Lambda \vec{J}_s = -\vec{A} \tag{2.5}$$

In the language of complex analysis, the phase is multi-valued, which has profound consequence. Since a supercurrent is continuous, any closed loop integral inside bulk superconductor is zero; $\oint \vec{J}_s = 0$, which quantizes magnetic flux:

$$\oint \vec{A} \cdot d\vec{l} = \frac{\hbar}{q_s} \oint \nabla \theta \cdot d\vec{l} = \frac{\hbar/2e}{2\pi} \cdot k2\pi = k\Phi_0 : k \in \mathbb{N}$$

⁸essentially the associated Lorentz force

⁹Can also be derived from the quantum mechanical definition of a current density

Now, if Maxwell's equations consist of four¹⁰ differential relations describing physical phenomena, the London model of superconductivity consists of two¹¹, the London equations:

$$\begin{aligned}\vec{E} &= \Lambda \frac{d}{dt} \vec{J}_s \\ \vec{B} &= -\Lambda \nabla \times \vec{J}_s\end{aligned}$$

with Λ being the London parameter. The first differential relation can be obtained by applying the time derivative to (2.5). Combining Faraday's law with the definition of the magnetic vector potential and utilizing that derivatives commute yields:

$$\Lambda \frac{d}{dt} \vec{J}_s = -\frac{d\vec{A}}{dt} : \nabla \times \vec{E} = \nabla \times \left(-\frac{d}{dt} \vec{A} \right)$$

Hence applying appropriate boundary conditions¹² over a control volume $\mathcal{V}$, the differential relation describes the accelerated response of a supercurrent $\vec{J}_s$ when applying an electric field $\vec{E}$. Since Λ is a constant, acceleration is linear with respect to time until reaching a maximum value; the Onnes effect. The second relation is obtained by instead applying the curl operator:

$$\vec{B} = \nabla \times \vec{A} = \nabla \times -\Lambda \vec{J}_s$$

Again applying appropriate boundary conditions over a control volume $\mathcal{V}$, it describes the Meissner effect. Applying another curl and utilizing Ampère's law with double curl-identity yields:

$$\begin{aligned}\nabla \times [\vec{B} &= -\Lambda \nabla \times \vec{J}_s] \\ \implies \nabla^2 \vec{B} &= \frac{\mu_0}{\Lambda} \vec{B} = \frac{1}{\lambda_L^2} \vec{B} \\ \iff \nabla^2 \vec{J}_s &= \frac{\mu_0}{\Lambda} \vec{J}_s = \frac{1}{\lambda_L^2} \vec{J}_s.\end{aligned}$$

where λ_L is the London penetration depth. The solution is an exponential/hyperbolic depending on the relative width of the control volume compared to λ_L :

$$\vec{B} \sim \vec{B}_0 e^{-x/\lambda_L}$$

This means that supercurrent variations are confined within a length scale of λ_L , implying that both magnetic field and supercurrent are concentrated close to the surface of the superconducting material, known as the *Superconducting Skin Effect*.

The framework of Ginzburg-Landau not only recovers the London model and quantizes magnetic flux, it also introduces a new fundamental property: *Coherence Length* ξ . By the 2nd law of thermodynamics, the Free Energy of a system approaches a minimum value at equilibrium. This corresponds to minimizing the volume integral

¹⁰Gauss's law for electricity and magnetism, Faraday's law and Ampère's law

¹¹Onnes and Meissner effect

¹²Speed v of charge inside a control volume must be finite, $v_{\max} < c$

$\int f_s d\mathcal{V}$ over spatial variations $\delta\psi^*$. Whence also varying ψ with $\delta\psi$ to probe the coherence of the order parameter obtains the following differential relation:

$$\nabla^2 \delta\psi \simeq 2 \frac{|\alpha| 2m_{\text{eff}}}{\hbar^2} \delta\psi = \frac{2}{\xi^2} \delta\psi$$

The existence of a coherence length implies that the supercharge quanta q_s exists over a length ξ .

Having derived both coherence length ξ and penetration depth λ_L , superconductivity can be classified into 2 types:

$$\begin{aligned} \text{Type-I} : \frac{\lambda_L}{\xi} &< \frac{1}{\sqrt{2}} \\ \text{Type-II} : \frac{\lambda_L}{\xi} &> \frac{1}{\sqrt{2}} \end{aligned}$$

which quantifies critical field splitting: if the quotient is greater than the inverse root of 2, magnetic flux penetration is allowed. The quotient is sometimes denoted $\kappa = \frac{\lambda_L}{\xi}$.

Finally, the general Energy state of a superconductor is unknown, but Low Temperature Superconductors can be represented by the BCS Energy state. It is a second quantization such that particles may or may not occupy a point in a field. Creation (and annihilation) of electron-pairs is represented as coupled creation operators $\hat{c}_k^\dagger$ and $\hat{c}_{-k}^\dagger$ (and their Hermitian conjugates), with probabilities of empty/occupied states; u_k/v_k . The total state is the product of all possible states acting on the vacuum state $|0\rangle$:

$$|\psi_{\text{BCS}}\rangle = \prod_k \left(u_k + v_k e^{i\phi} \hat{c}_{k\uparrow}^\dagger \hat{c}_{-k\downarrow}^\dagger \right) |0\rangle.$$

This wave function is technically not an exact eigenfunction of the associated BCS Hamiltonian H_{BCS} , but it is close enough to be used as an approximation for computations. The total of all occupied states corresponds to a spatial density of Cooper pairs n_p , which coming full circle is proportional to the Superfluid Bose-Einstein condensate of the Two-Fluid model:

$$n_s = 2n_p.$$

2.2 YBCO

Yttrium Barium Copper Oxide, $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, illustrated in Fig. 2.12 and commonly abbreviated YBCO, is a High Temperature Type-II Superconductor. It has an unknown pairing mechanism and a maximum critical temperature $T_c \sim 93\text{K}$.

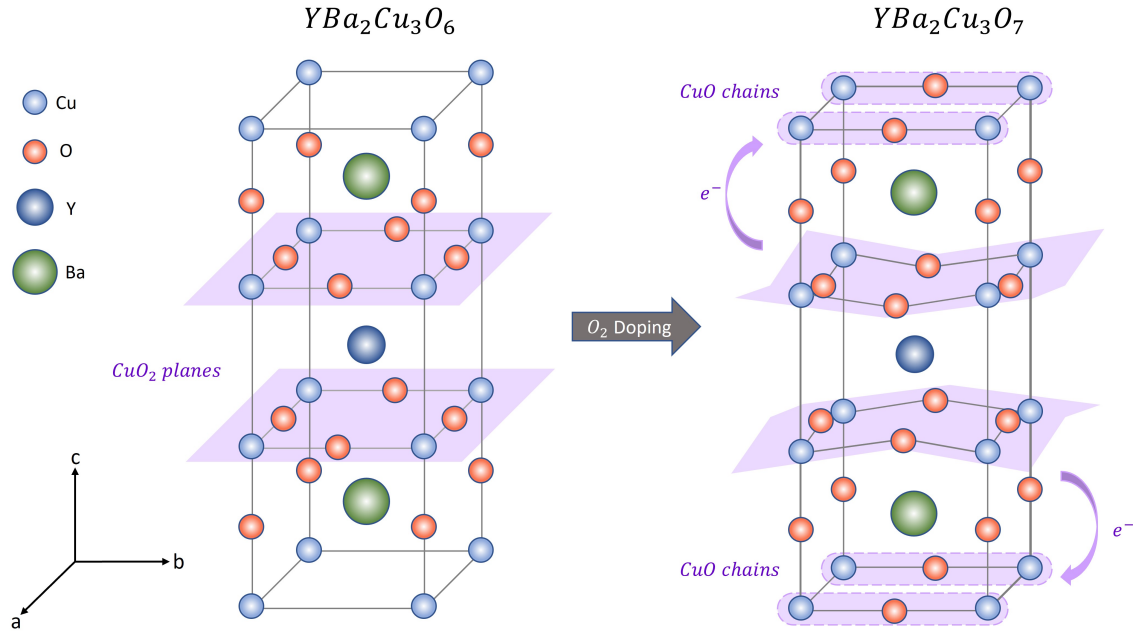


Figure 2.12: YBCO Crystal Structure. Image created by Karn Rongrueangkul [6].

Structurally, the material is a defect perovskite cell, consisting of stacked layers of 2D molecular planes: CuO_x , BaO , CuO_2 and Y . Here the Oxygen content of the Copper Oxide chains is a variable indicated by x , but in the unit cell it is instead represented as the total Oxygen content $7 - \delta$, which maps to a *hole doping* p ; hole per Copper atom in the lattice. The unit cell of YBCO can be represented as 3 perovskite cells stacked on top of each other, with crystal lattice vectors a , b and c varying with hole doping. For example, $a = b = 3.86\text{\AA}$ if $\delta = 1$, but $a \neq b$ if $\delta < 1$. Superconductivity occurs in the CuO_2 -planes, with the CuO -chains acting as charge reservoirs.

The phase diagram of YBCO is extremely complicated, being an entire field of study on its own. It is expanded around Temperature T and hole doping p and depicted in Fig. 2.13. For example, if $p \approx 0.16$ the compound is optimally doped, yielding a $T_c \approx 93\text{K}$, but if $\delta = 1$ the material stops being superconducting altogether, becoming an Anti-Ferromagnetic Mott-Hubbard Insulator. Mapping the phase diagram reveals highly unconventional phases, such as Strange Metal or the manifestation of Charge Density Waves.

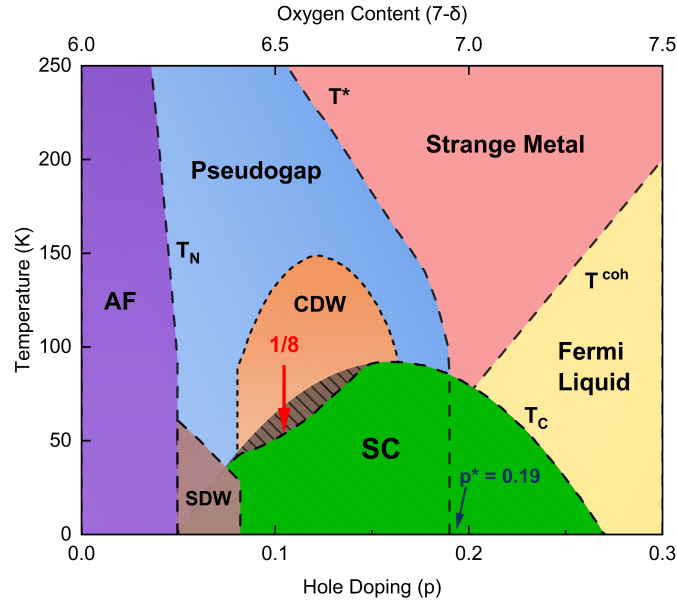


Figure 2.13: YBCO Phase Diagram: Temperature vs. hole doping. Image created by Karn Rongrueangkul [6].

The YBCO within the scope of this thesis is superconducting thin films. They exist within the green dome of the phase diagram in Fig. 2.13 and are categorized as either underdoped, optimally doped, or overdoped. Optimal doping refers to $p = 0.16$ and under-/overdoped to less/more. Hole doping does not only affect the conductive properties, but also indicates how stable the chemical composition is. For example, an underdoped film is a bit more delicate and decays quickly over time compared to the higher dopings¹³. The underdoped part of the phase diagram contains a plethora of intertwined local order, including charge density wave which is well-known to compete with superconductivity. The competition suppresses superconductivity, reaching maximum suppression around $p \sim 1/8$ hole doping.

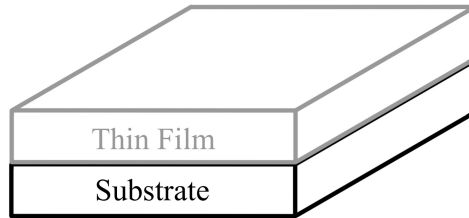


Figure 2.14: Illustration of a thin film of YBCO on a substrate.

Another important aspect to consider when depositing thin film YBCO is the quality of deposition. Because YBCO is so sensitive, it requires a very clean environment under optimal conditions to not induce any impurities or defects into the

¹³Oxygen can leak out from the film into the surrounding atmosphere, ruining the doping

material. The deposition parameters require optimization to generate qualitative crystal growth. The cleanliness of the film can be observed by employing an Optical Microscope, but visualizing impurities and defects requires a Scanning Electron Microscope in order to resolve relevant features on the surface. The quality of crystal growth is further evaluated using X-Ray Diffraction (XRD) to scan the lattice.

2.3 Coplanar Waveguide Resonators

This section is largely based on the PhD-thesis of Martin Göppl [14].

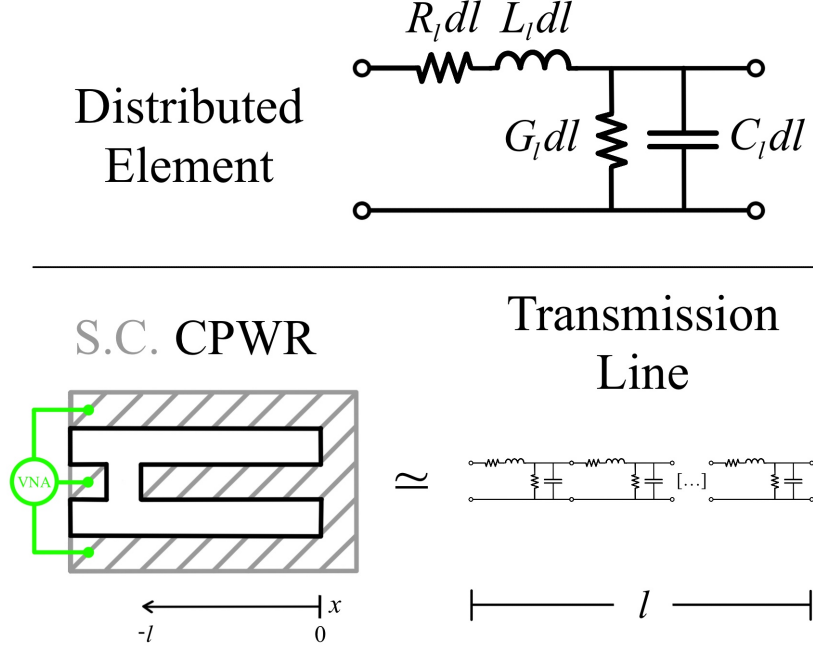


Figure 2.15: A superconducting coplanar waveguide resonator may be analysed using a distributed element model, which makes it isomorphic to a transmission line. The image depicts a quarter-wave $\lambda/4$ resonator; one open end and one short. The open end is shunted by a 50Ω VNA and couples capacitively to the resonator strip (cavity).

Superconductors are typically very homogeneous materials, which means that a superconducting coplanar waveguide resonator (CPWR) may be analysed via a *distributed element model*. The resonator, sketched above in the bottom left figure panel, can be divided into small equal circuit elements connected in series with electrical properties given per unit length. An element is shown in the top panel above. Distributed element models are typically employed when dealing with transmission lines, which means that a CPWR is isomorphic to a transmission line resonator and may therefore be analysed as one.

The length of the resonator strip is much greater than the length of the component and may therefore be considered similar to an infinitely long transmission line. Assuming small losses and leakage, the characteristic impedance Z_0 of an infinitely long transmission line can be approximated as:

$$Z_0 = \sqrt{\frac{R_l + i\omega L_l}{G_l + i\omega C_l}} \approx \sqrt{\frac{L_l}{C_l}}$$

where $R_l/G_l/L_l/C_l$ are the electrical properties per unit length of the distributed element model [14]. The characteristic impedance of the component should ideally match the impedance of the input 50Ω power line shunt:

$$\sqrt{\frac{L_l}{C_l}} = 50\Omega$$

to minimize a phenomena known as *impedance mismatch*. Basically, when a wave travels from one medium to another, i.e. one impedance to another, a portion of the wave is reflected if the transition is not homogeneous.

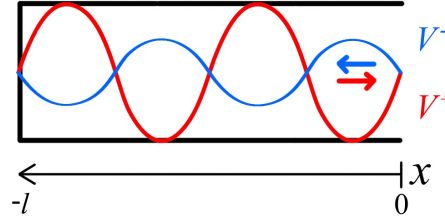


Figure 2.16: Forward (red) and backward (blue) moving wave components inside a resonator of length l . The load impedance of the resonator is defined at its output, i.e. $x = 0$.

The strip can hence be modelled as a transmission line with forward/backward moving wave components (Fig. 2.16) attenuated along a distance x :

$$\begin{aligned} V(t, x) &= V^+ e^{i\omega t} e^{-\gamma x} + V^- e^{i\omega t} e^{\gamma x} \\ I(t, x) &= I^+ e^{i\omega t} e^{-\gamma x} + I^- e^{i\omega t} e^{\gamma x} \\ &= \frac{V^+}{Z_0} e^{i\omega t} e^{-\gamma x} - \frac{V^-}{Z_0} e^{i\omega t} e^{\gamma x} \end{aligned}$$

where γ is an attenuation constant. V^+/I^+ and V^-/I^- denote the Voltage/Current amplitude for ingoing and outgoing wave of frequency ω , respectively. The effective load impedance Z_L of the entire component is defined at its output, which corresponds to the end of the resonator strip:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V^+ + V^-}{V^+ - V^-} Z_0.$$

This expression can be rewritten to define the Reflection Coefficient Γ :

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

which represents the reflected fraction of an incoming electromagnetic wave. For an ideal resonator, all waves are completely reflected; $\Gamma = 1$, or completely absorbed if the frequency matches the resonance frequency; $\Gamma = 0$, because the load impedance is frequency dependent. The impedance of a transmission line resonator around resonance can be approximated as [14]:

$$Z_{TL} \approx \frac{Z_0}{\alpha l + i\frac{\pi}{2} \Delta\omega/\omega_0}$$

where the attenuation has been split up into real and imaginary part, $\gamma = \alpha + i\beta$, such that $\beta \approx \frac{\pi}{2} \Delta\omega/\omega_0$. This expression is similar to a lumped RLC circuit around resonance, $\omega_0 + \Delta\omega$:

$$\frac{1}{Z_{RLC}} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$\Rightarrow Z_{RLC} \approx \frac{R}{1 + 2j\Delta\omega RC}$$

Promoting the resonance frequency to include overtones, $\omega_0 \rightarrow \omega_n = n\omega_0$, the properties per unit length (of the resonator cavity sketched in Fig. 2.17 below) can be mapped to the lumped properties:

$$R = Z_0/\alpha l$$

$$L = \frac{8L_l l}{n^2 \pi^2} \equiv L_n$$

$$C = \frac{C_l l}{2}$$

The Resistance R reflects the dissipation losses of the resonator, Inductance L and Capacitance C yields the resonance frequency. Inductance depends on which harmonic of the resonance frequency is considered, indicated by a subscript n . Capacitance is divided by 2 because the geometry of the resonator distributes the electric field symmetrically between both sides along the cavity.

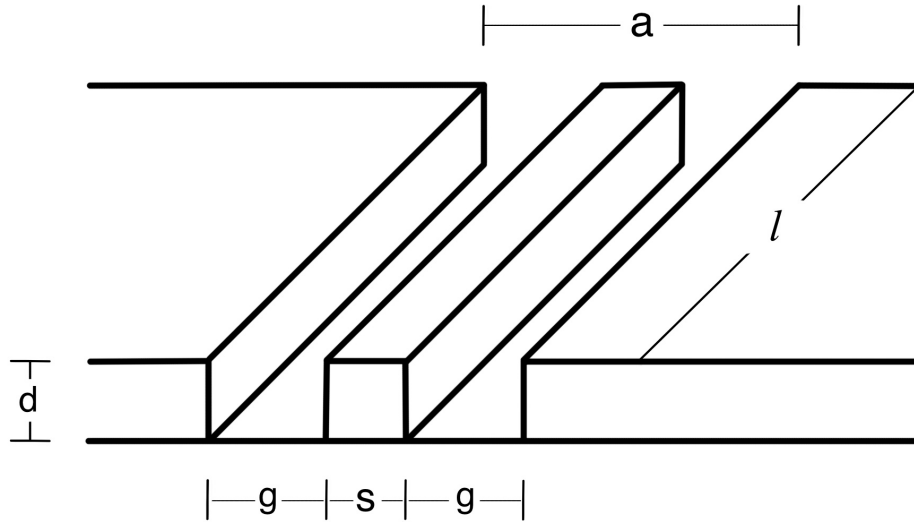


Figure 2.17: Resonator Dimensions. The parameters are: strip width s , gap width g , film thickness d , strip length l and cavity width a .

The superconducting phase introduces Kinetic Inductance L_K , separate from the conventional geometric property. Concretely, it is the Kinetic Energy of supercharges, which can be assumed to not affect the capacitance:

$$L_l = L_G + L_K$$

$$C_l = C_G$$

Both geometric inductance and capacitance can be computed using elliptic integrals¹⁴ [14] of the first kind, $K(k)$:

$$L_G = \mu_0 \frac{K(\sqrt{1 - (s/a)^2})}{4K(s/a)}$$

$$C_G = \varepsilon_0 \varepsilon_{\text{eff}} \frac{4K(s/a)}{K(\sqrt{1 - (s/a)^2})}.$$

Here s is the width (see Fig. 2.17) of the resonator strip, $a = s + 2g$ is the total width of the resonator cavity, ε_0/μ_0 is the permittivity/permeability of free space and ε_{eff} is a special permittivity that emerges from the interplay between the thin film and the substrate. It is calculated with the following formula [14]:

$$\varepsilon_{\text{eff}} = 1 + \frac{(\varepsilon_{\text{sub}} - 1)}{2} \frac{K(k_1)}{K(\sqrt{1 - k_1^2})} \frac{K(\sqrt{1 - (s/a)^2})}{K(s/a)}$$

$$k_1 = \frac{\sinh(\frac{\pi s}{4h})}{\sinh(\frac{\pi a}{4h})}$$

where ε_{sub} and h is the permittivity and thickness of the substrate, respectively.

Kinetic inductance, on the other hand, does not follow a general expression. The mathematical description is very sensitive with respect to geometry, which is why different sources might cite very different expressions. All dimensions of length matter, but the dominant variable is the thickness of the film. For the dimensions implemented in this thesis, Kinetic Inductance L_K is calculated using the following (complicated) expression [15]:

$$L_K(\lambda_L) = \frac{\mu_0 \lambda_L}{4K(s/a)} \left[\frac{C}{AD} \left(\frac{1.7}{\sinh(d/2\lambda_L)} + \frac{0.4}{\sqrt{((B/A)^2 - 1)(1 - (B/D)^2)}} \right) \right].$$

where¹⁵

$$\begin{bmatrix} A = -\frac{d}{\pi} + \sqrt{(\frac{d}{\pi})^2 + \frac{s^2}{4}} \\ B = \frac{s^2}{4A} \\ C = B - \frac{d}{\pi} + \sqrt{(\frac{d}{\pi})^2 + \frac{(2g)^2}{4}} \\ D = 2\frac{d}{\pi} + C \end{bmatrix}$$

Finally, London Penetration Depth λ_L is proportional to Temperature T :

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1 - (T/T_c)^\gamma}}$$

which means that a resonance frequency of the resonator is Temperature dependent:

$$\omega_n^2(T) = \frac{1}{L_n(\lambda_L(T))C}$$

¹⁴These are estimates rather than exact, and might deviate from the actual value, but at any rate good approximations

¹⁵Do not confuse this C with capacitance, it is just a notation coincidence

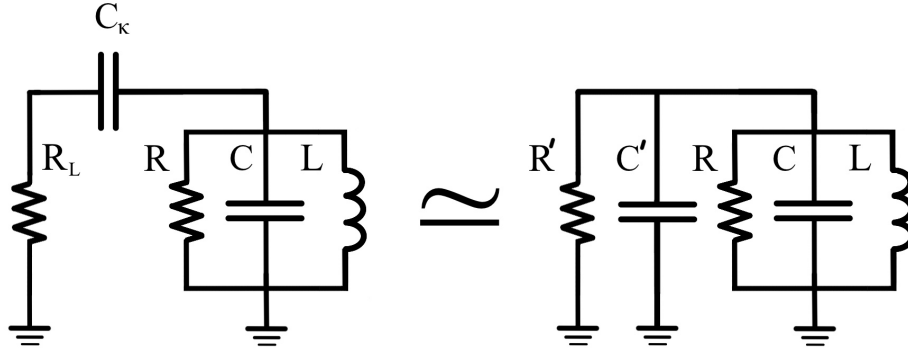


Figure 2.18: Norton equivalent of the applied Thevenin circuit. The Load Resistance $R_L = 50\Omega$ and Coupling Capacitance C_k used to connect to the device in series are transformed to instead parallel the circuit with R' and C' . This represents the environmental interaction of the device, with R' being the effective resistance of environmental leakage. The large Resonator Capacitance C dominates over the small Coupling Capacitance C_k , which implies $C \gg C'$, so it can be neglected.

The device can be coupled to capacitively. This means that the connection from instrument to device not only experiences a Load Resistance R_L , but also a Coupling Capacitance C_k , which can be represented as a Thevenin circuit. If the Thevenin circuit is transformed into its Norton equivalent (Fig. 2.18 above), all components become parallel with respect to ground, which allows defining environmental leakage as the external Resistance R' :

$$R' = \frac{1 + \omega_n^2 C_k^2 R_L^2}{\omega_n^2 C_k^2 R_L^2} R_L$$

$$C' = \frac{C_k}{1 + \omega_n^2 C_k^2 R_L^2}.$$

Assuming $\omega_n^2 C_k^2 R_L^2 \ll 1$, which is typically is, both expressions can be approximated:

$$R' \approx \frac{R_L}{\omega_n^2 C_k^2 R_L^2}$$

$$C' \approx C_k$$

The capacitive effects of C' are negligible, because the size of the coupling capacitance C_k is very tiny compared to the capacitance of the resonator C . The Quality factor of the resonator can now be expressed using standard definitions of the quality factor for an RLC circuit:

$$Q_{\text{int}} = \omega_n RC$$

$$Q_{\text{ext}} = \omega_n R' C$$

However, here the Quality factor has been split up into Internal, Q_{int} , and External, Q_{ext} , which represent Internal and External losses. The meaning of Q_{int} is the conventional definition mentioned in the introduction: internal power loss (per cycle),

with Q_{ext} instead representing external power leakage (per cycle). Because these are orthogonal channels of dissipation, they need to be connected in parallel to yield the total power loss (per cycle):

$$\frac{1}{Q} = \frac{1}{Q_{\text{int}}} + \frac{1}{Q_{\text{ext}}} \simeq \frac{1/R + 1/R'}{\omega_n C}$$

The relative magnitude of the factors indicate the relative magnitude of resistive losses. There are 3 scenarios:

$$\frac{Q_{\text{int}}}{Q_{\text{ext}}} = \begin{cases} < 1, & \text{Undercoupled} \\ = 1, & \text{Critically Coupled} \\ > 1, & \text{Overcoupled} \end{cases}$$

If $Q_{\text{int}} = Q_{\text{ext}}$, the resonator is critically coupled, indicating maximum Energy transfer to the resonator. If the resonator instead is undercoupled, $Q_{\text{ext}} > Q_{\text{int}}$, external losses are weak compared to internal losses and little Energy enters or exits the resonator. Finally, if the resonator is overcoupled, $Q_{\text{int}} > Q_{\text{ext}}$, external losses dominate. As further touched upon during Section 3.4.3 on data processing, the 3 scenarios of coupling given by the Quality factors can be visualized via the following function [11]:

$$\Gamma(x) = \frac{1 - x^2 + jx(1/Q_{\text{int}} - 1/Q_{\text{ext}})}{1 - x^2 + jx(1/Q_{\text{int}} + 1/Q_{\text{ext}})}$$

It describes a resonance dip around resonance, reflection Γ vs. $x = f/f_0$, as a complex plot. The graph is a circle inside the complex unit circle, tangent to $z = 1$. The 3 scenarios correspond to 3 different characteristic sizes of circles, sketched in figure 2.19.

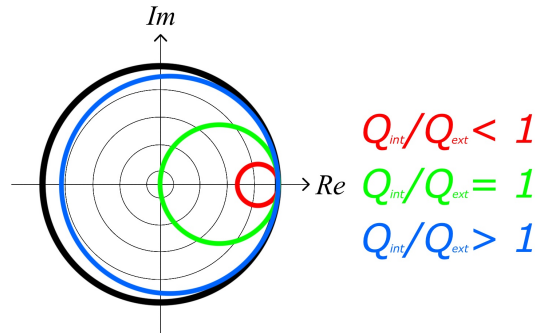


Figure 2.19: Sketch of under-/critically/over-coupled as red/green/blue plots in the complex plane, bounded by a unit circle. The quotient of Q -factors is not the radius of the circle, it just happens on overlap in the critical scenario. This graph emerges when mapping reflection measurements and will be further discussed during data processing.

For the experiment conducted in this thesis, the resonator is ideally overcoupled, so that it is easy to communicate with the device.

2.4 Flat Band Hypothesis

The *effective mass* m_{eff} of charge carriers inside a material represents the inertial response to an applied electric force. For dissipative transport, where energy is exchanged with the environment, inertial response is quantified by the curvature of Energy bands $E(k)$ in k -space¹⁶:

$$m_{\text{eff}} = \hbar^2 \left(\frac{\partial^2 E(k)}{\partial k^2} \right)^{-1}.$$

Reactive transport is however not an exchange of energy with the environment, but an energy cost for inducing quantum mechanical phase fluctuations in the superconducting condensate. This energy corresponds to the kinetic energy E_{kin} of supercharges, which is represented by kinetic inductance L_K :

$$\begin{aligned} L_K &= \mu_0 l \frac{\lambda_L^2}{wd} \\ \implies E_{\text{kin}} &= \frac{1}{2} L_K I^2 = \frac{1}{2} \mu_0 l \frac{\lambda_L^2}{wd} I^2 \end{aligned}$$

where $l/w/d$ is the length/width/depth of a (superconducting) rectangular block. Computing kinetic energy E_{kin} per unit volume $\mathcal{V}$ and simplifying:

$$\begin{aligned} \frac{E_{\text{kin}}}{\mathcal{V}} &= \frac{\frac{1}{2} \mu_0 l \frac{\lambda_L^2}{wd} I_s^2}{lwd} = \frac{\mu_0 \lambda_L^2}{2} \left(\frac{I_s}{wd} \right)^2 \\ &= \frac{1}{2} \mu_0 \lambda_L^2 J_s^2 \end{aligned}$$

yields an expression proportional to supercurrent density squared. Utilizing the equation for current density with zero vector potential and diverting extra attention to the London parameter Λ :

$$J_s = \frac{-1}{\mu_0 \lambda_L^2} \frac{\hbar}{2e} \nabla \phi : \mu_0 \lambda_L^2 = \frac{m_s}{n_s e^2} \quad (= \Lambda)$$

substituting J_s yields:

$$\begin{aligned} \frac{E_{\text{kin}}}{\mathcal{V}} &= \frac{1}{2} \mu_0 \lambda_L^2 \left(\frac{-1}{\mu_0 \lambda_L^2} \frac{\hbar}{2e} \nabla \phi \right)^2 = \frac{1}{2} \frac{n_s e^2}{m_{\text{eff}}} \Phi_0^2 \left(\frac{\nabla \phi}{2\pi} \right)^2 \\ &= \frac{1}{2} D_s \Phi_0^2 \left(\frac{\nabla \phi}{2\pi} \right)^2. \end{aligned}$$

where Φ_0 is the magnetic flux quantum. The London parameter has been redefined as Superfluid Stiffness D_s :

$$D_s = \frac{n_s e^2}{m_{\text{eff}}}.$$

to account for a variable effective mass of supercharge carriers. The nominal London parameter corresponds to the superfluid stiffness of free electrons pairs, $m_{\text{eff}} = 2m_e$.

¹⁶The radius of curvature at a point on a curve is inversely proportional to its second derivative

The expression still contains the effective mass, which means that if the energy band is flat, whereby its curvature is 0, the effective mass diverges:

$$\frac{\partial^2 E}{\partial k^2} \rightarrow 0 \implies m_{\text{eff}} \rightarrow \infty \implies D_s \rightarrow 0$$

and yields a trivial infinitesimal superfluid stiffness. However, if the associated eigenstates in state space possess a non-trivial geometric structure, i.e. not orthogonal to each other, hybridized orbitals enter the band structure. If an electron-pair occupies a mixed state, an infinitesimal change $d\vec{k}$ in lattice momentum $\vec{k}$ can yield a finite change ΔS in state space S [9]. The non-zero “quantum length” ΔS between two states arbitrarily close to each other manifests from a transformation of basis vectors, because the hybridized orbital can be expressed by multiple combinations of non-orthogonal basis orbitals. These multi-orbital systems possess a non-zero *Quantum Metric* which have been found to allow for bound electron pairs [10], yielding a non-zero Superfluid Weight and a finite effective mass, even for flat energy bands:

$$D_s > 0 : m_{\text{eff}} < \infty$$

Now, recall deriving the 2nd London equation during the theory section:

$$\begin{aligned} \nabla \times \left[\vec{B} = -\frac{1}{D_s} \nabla \times \vec{J}_s \right] \\ \implies \nabla^2 \vec{J}_s = \frac{D_s}{1/\mu_0} \vec{J}_s = \frac{1}{\lambda_L^2} \vec{J}_s \end{aligned}$$

and notice the use of D_s instead of Λ . Rearranging, superfluid weight is inversely proportional to the square of the penetration depth:

$$D_s = \frac{1/\mu_0}{\lambda_L^2}.$$

or equivalently the effective mass of a supercharge can be expressed as:

$$m_{\text{eff}} = n_s e^2 \mu_0 \lambda_L^2$$

which in context corresponds to an effective curvature of the Energy Bands $E(k)$.

If the penetration depth λ_L of a material changes, one can compute the relative change in superfluid stiffness D_s to determine the action on the effective curvature of the energy bands. If a separate parameter, in this thesis surface morphology, causes penetration depth to change, the curvature will change.

3

Methods

All fabrication discussed in this section has been carried out in Myfab - Nanofabrication Laboratory, MC2 Chalmers.

3.1 Thin Film Deposition and Characterization

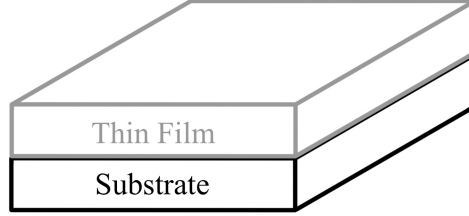


Figure 3.1: A thin film of YBCO deposited on a substrate.

Before a device can be manufactured out of a thin film of YBCO, sketched above in Fig. 3.1, the film itself requires optimization. First and foremost, because YBCO is a very complicated material, its properties highly depend on the choice of substrate when deposited as a thin film. Optimal conditions emerge from qualitative *c*-axis growth, which requires a substrate surfaces which approximately matches the *a* and *b* lattice vectors of the YBCO. Historically at QManD, the substrate Strontium Titanate (STO) has been used as a shared reference because of its favourable crystal growth, but it is not very suitable for a resonator. STO has a large relative permittivity ϵ_r that varies a lot with Temperature, reaching from around 250-300 at room temperature to an extreme high of almost 20000 at 4K. This has detrimental impact on a resonator, because the resonance frequencies depend on the permittivity of the substrate. Instead, this thesis opted to use the substrates Magnesium Oxide (MgO) and Lanthanum aluminate-Strontium Aluminium Tantalate, $(\text{LaAlO}_3)_{0.3}(\text{Sr}_2\text{TaAlO}_6)_{0.7}$, (LSAT), which are also well-studied substrates with favourable crystal growth. These exhibit small relative permittivities ϵ_r , roughly 9.8 for MgO and 25 for LSAT.

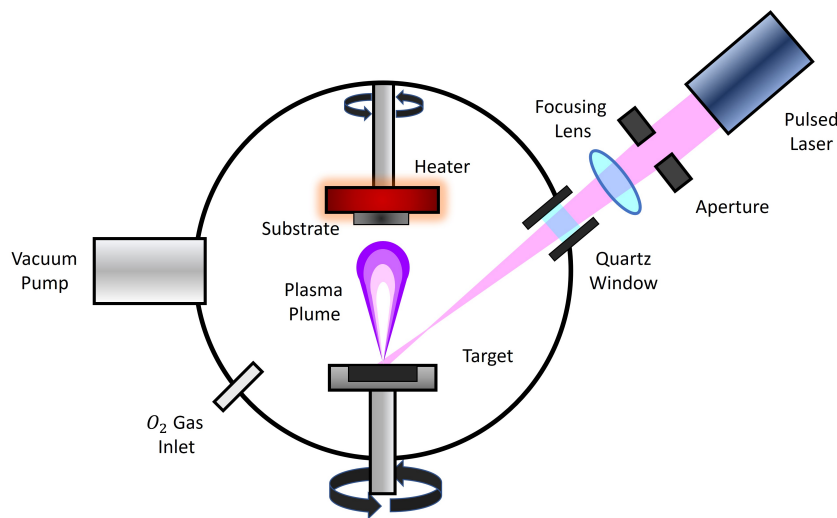


Figure 3.2: Illustration of a PLD setup, adapted from [6].

Thin film YBCO was deposited onto a substrate using a process called Pulse

Laser Deposition (PLD), illustrated in Fig. 3.2. The substrate is placed inside a vacuum chamber via a rail transport-system placed inside vacuum tubes. Once in place, the chamber was sealed off from the rest of system and oxygenated to a set pressure. After setting some parameters and waiting, a target of YBCO is exposed to energetic photons that scatter molecules in the form of a plasma plume onto the substrate. There are multiple delicate parameters to optimize for this process to work, including temperature, pressure and position. From previous experiments, QManD has deduced that a certain number of pulses at a Laser Energy around 55mJ can yield a desired film-thickness, stoichiometry and doping. This requires the substrate to be mounted at a particular height above the target, with subsequent planar freedom acting as a tweaking parameter. Furthermore, the substrate requires intense heating, reaching temperatures between 700-800°C, to generate qualitative crystal growth. Using MgO (110) as a substrate, heating somewhere around 5 hours before deposition optimizes creation of nano-facets. As discussed in the Theory section, the Oxygen content of YBCO is crucial, so the chamber requires a specific inlet rate and output pump of Oxygen gas to achieve the correct doping conditions. If these parameters are tuned correctly, the process yields a thin film of YBCO with a desired thickness and doping.

After deposition, the quality of crystal growth is measured using X-Ray Diffraction (XRD). The sample film is mounted and tilted relative a Röntgen source in a sweeping manner to reflect the photons to a sensor, effectively mapping the crystal structure via its Bragg-angles. Qualitative crystal is characterized by alignment of lattice layers in both horizontal and vertical direction, but a real sample will never be perfectly aligned throughout the crystal. When performing an XRD-measurement of YBCO, layers throughout the crystal will have a slight angular offset as illustrated below to the left, which need to be accounted for during the measurement. The rate of c -axis alignment of the lattice layers is quantified by the so called Rocking Curve graphed below to the right.

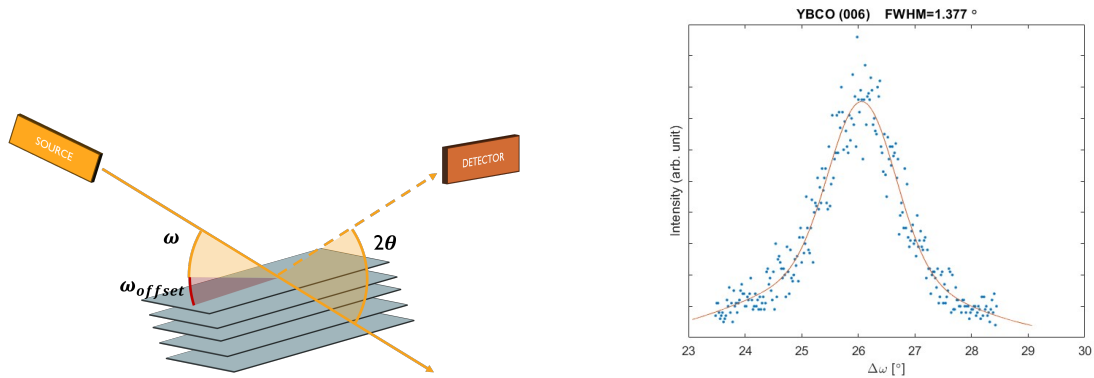


Figure 3.3: (Left) Principle XRD measurement. Light from a source is scattered off of lattice layers at its Bragg angles and measured by a detector. A crystal in reality will not be perfect, so the lattice layers will have a slight angular offset. (Right) Rocking Curve: light intensity vs lattice layer offset. The spread of the measured distribution corresponds to rate of alignment of the lattice layers. The blue dots are recorded measurements and the orange curve is a known reference for high quality YBCO in the (006) direction.

3. Methods

The main idea behind tuning the parameters of the PLD process is to obtain qualitative crystal growth, but equally important is the quality of deposition. For this particular scenario, besides random dirt, quality of deposition specifically refers to insulating cavities; impurities, that may form when the material particles are deposited. Cavities can be observed using a Scanning Electron Microscope (SEM), where they appear as bright blobs scattered on a dark background. The insulating cavities reflect incoming electrons instead of transmitting them through the film, which is what yields the electrical data used to produce an image. By observing the films in an SEM, deposition parameters can be further optimized.

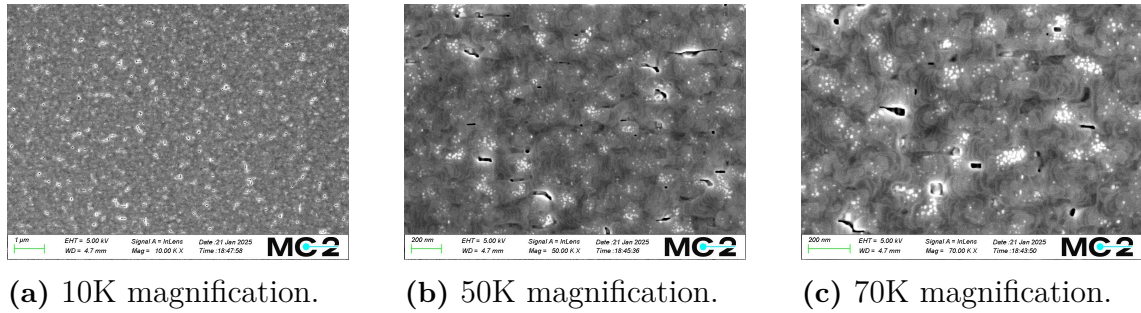


Figure 3.4: 40nm film YBCO on MgO.

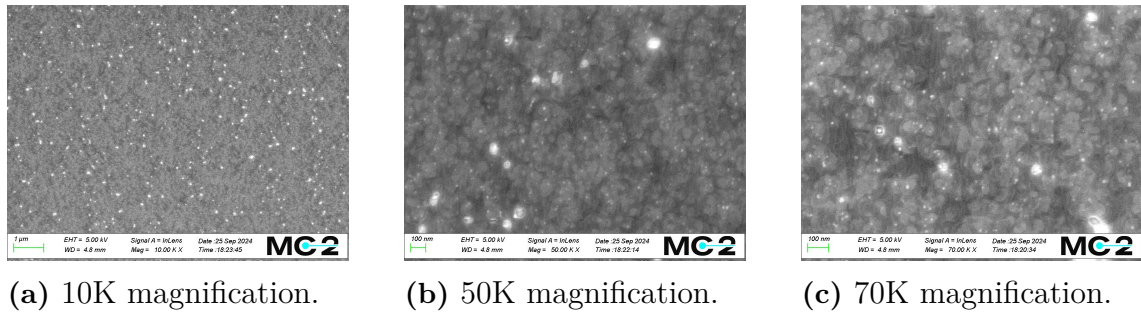


Figure 3.5: 10nm film YBCO on MgO.

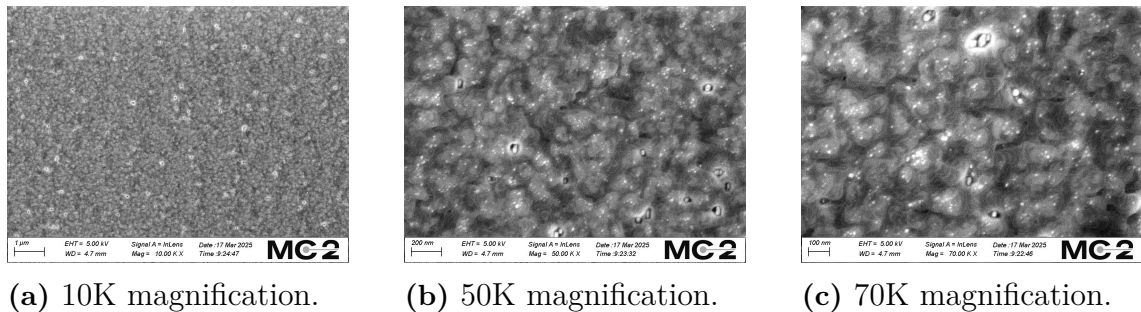
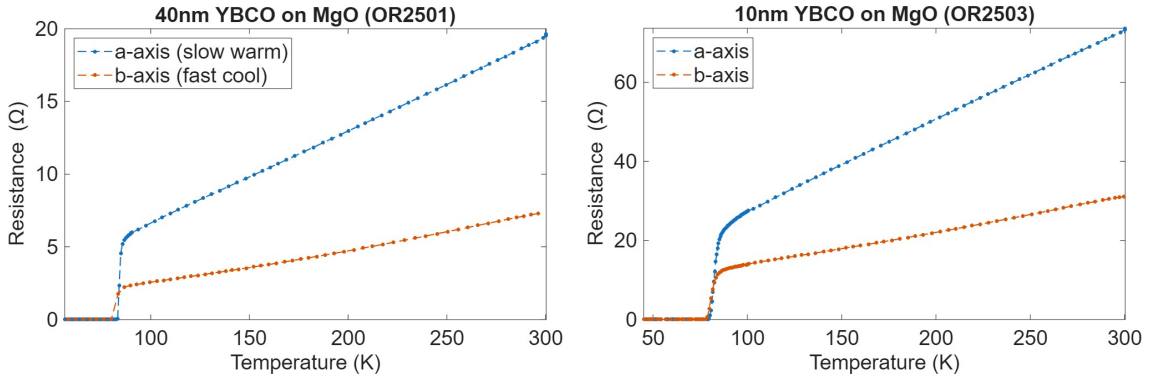
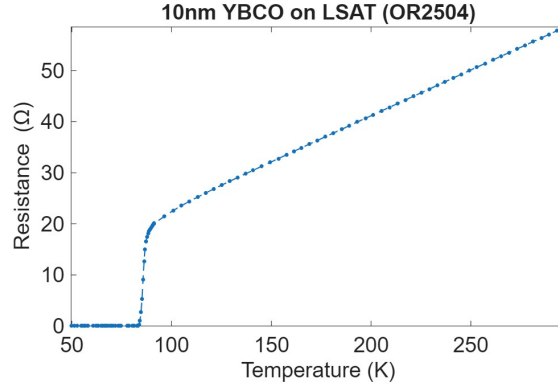


Figure 3.6: 10nm film YBCO on LSAT.

The electric properties of the films were characterized using a Physical Property Measurement System (PPMS), by measuring the Resistance $R(T)$ along the films from room temperature down to $T = 4\text{K}$. This results in a long tail; the normal state resistance, and a sharp drop at T_c ; the superconducting transition. The interesting features of the graph is the magnitude of the normal state resistance, the value of T_c and how broad the transition is. The magnitude of the normal state is an indicator of film-quality, as impurities and defects affect the resistance. Critical temperature indicates where the sharp superconducting transition occurs, but in thin films the transition is not instant like in bulk material. Ultra thin films, $\sim 10\text{nm}$, typically have a broader transition than regular thin film, $\sim 50\text{nm}$, around 5K instead of $1\text{-}2\text{K}$. The cause of this effect is unknown and if deposition parameters are off, the singular transition might even split into 2 separate transitions.



(a) 40nm YBCO on MgO. Corresponding film was used to fabricate resonator OR2501. (b) 10nm YBCO on MgO. Corresponding film was used to fabricate resonator OR2503.



(c) 10nm YBCO on LSAT. Corresponding film was used to fabricate resonator OR2504.

Figure 3.7: Resistance versus Temperature, $R(T)$, measurements for relevant films obtained using a PPMS. All 3 films exhibited a critical temperature $T_c \approx 84\text{K}$.

3.2 Nanofabrication

From an empirical perspective, this thesis is centered around *Nanofabrication*. It is what powers all modern computational devices. Fundamentally, the process deals with manipulating material on the nano-meter scale: $1\text{nm} = 10^{-9}\text{m}$.

3.2.1 Fundamental Idea

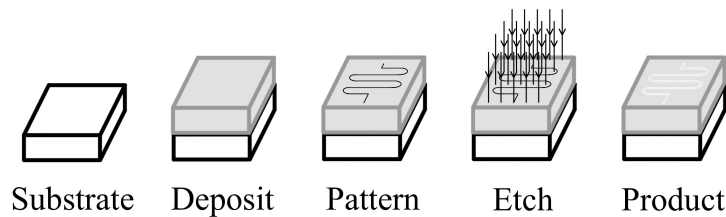


Figure 3.8: The fundamental idea behind nanofabrication. Starting with a base substrate, material is deposited, patterned, and etched to yield a finished product.

Nanofabrication utilizes THE most complex machinery so far constructed by humans, but the basic idea illustrated above is fairly simple. Any process can be split up into 3 categories:

- **Deposition** - Add material. Mainly conducted via evaporation; molecules of a target material are converted into gas form which deposit onto a substrate placed above the target. However, YBCO is deposited using Pulse Laser Deposition (PLD): a laser is aimed towards a target of YBCO, creating localized plasma formations consisting of atoms, ions, clusters and particles which eject from the target, depositing onto a substrate placed above.
- **Patterning** - Draw a design on material. Patterning *is* Lithography. A beam of photons/electrons is directed towards a resist coating and traces out a geometry specified by an input CAD-file. Exposing the resist to energetic particles changes the material properties of the resist, which allows removing all material not part of the specified design.
- **Etching** - Remove material. Can be done in a variety of ways. The simplest being wet etching; chemical reactions take material as reactant, and dry etching; particles with momentum knock away localized material-particles via impulse, e.g. Argon Ion-Milling and Oxygen Plasma Exposure.

In the simplest scenario, these 3 steps are carried out in order to yield a finished structure. The steps can also be repeated and interchanged sequentially to create even more complex structures. Exactly how depends on what machinery is available and what structure is desired.

3.2.2 Fabrication Process

The goal is to fabricate a YBCO Coplanar Waveguide Resonator on the micro-meter scale, $1\mu\text{m} = 10^{-6}\text{m}$ on a $5 \times 5\text{mm}$ substrate. The fundamental recipe to execute such a task is as follows:

1. Deposit YBCO and Gold contacts. The latter is done so that the device can be connected to electrically later on, during measurement.
2. Deposit Carbon hard mask to protect the YBCO, but the hard mask also help create sharp features in the resulting structure. Spin coat a double layer positive resist and get ready for EBL exposure.
3. Patterning with Electron Beam Lithography (EBL) using a 100kV electron beam.
4. Deposit Chromium for resist lift-off to define the final pattern.¹
5. Etch the exposed parts of the resist using chemicals to define the pattern in Chromium; resist lift-off. Removing resist also removes the corresponding Chromium on top, exposing the Carbon hard mask.
6. Etch the pattern into the exposed Carbon using Oxygen Plasma. This exposes the YBCO instead.
7. Etch the pattern into the exposed YBCO using Argon Ion Milling. This process requires liquid Nitrogen cooling and very gentle (barely etching) parameters, because a lot of heat can be produced and cause Oxygen to leak out of the YBCO. During this step, the remaining Chromium on top is also removed, exposing leftover Carbon.
8. Etch leftover Carbon using Oxygen Plasma. The etching parameters must be gentle, especially if the film is ultra thin.

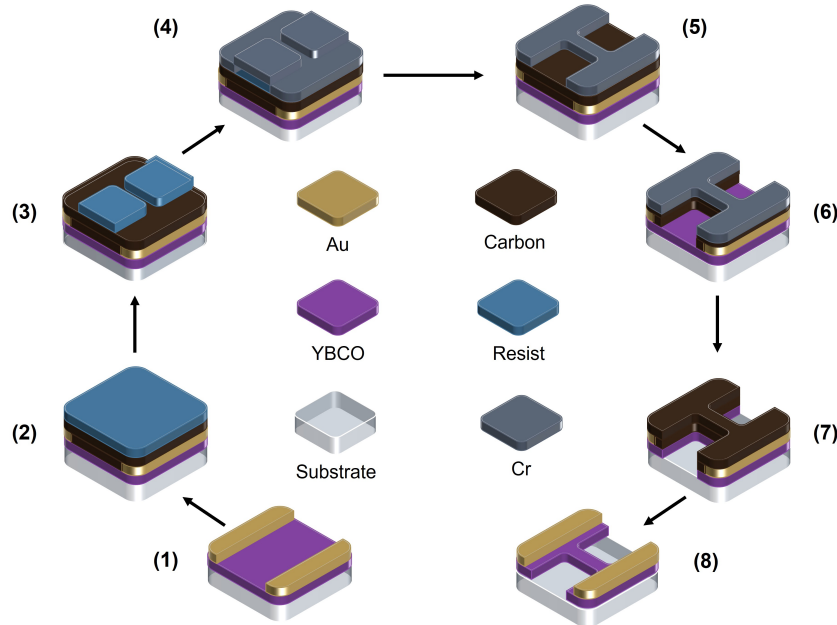


Figure 3.9: Illustration of fabrication cycle. Image created by Karn Rongrueangkul.

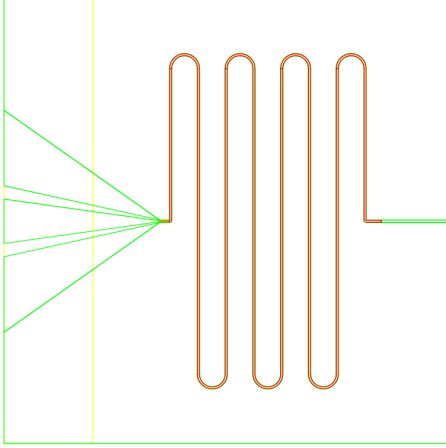
¹Positively charged Chromium repels positively charged Oxygen plasma after lift-off.

3.2.3 Resonator Design

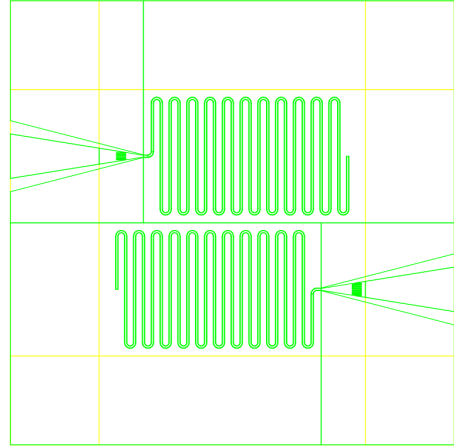
Resonators were designed using the CAD software AutoCAD. The CAD file can be fed into the EBL machine, where the computer recognizes closed surfaces for lithography exposure. The shape of the resonator strip does, in general, not matter, as long as it does not meander too closely too itself, since this could induce capacitive effects. What is important is the length l of the strip, because it sets the fundamental frequency. Neglecting any Kinetic Inductance, the fundamental frequency can be estimated using the following formula [14]:

$$f = \frac{v}{\lambda} = \frac{c/\sqrt{\varepsilon_{\text{eff}}}}{4l}.$$

This expression demands the strip to be fairly long compared to the chip-size $5 \times 5\text{mm}$; see Fig. 3.10, since millimeters yields an order of magnitude above the GHz range: $f \sim \frac{10^8}{10^{-3}} = 10^{11}$. A long length over a small area is easily achieved by *meandering*² the strip across the chip.



(a) Resonator Design Version 1.



(b) Resonator Design Version 2.

Figure 3.10: CAD-designs of resonators. Green signifies High Current exposure, and red Low Current exposure. Yellow represents the Gold contact used to connect the chip to electrical instruments.

A total of 2 designs were created during this thesis, illustrated above. Both were quarter-wave $\lambda/4$. Version 1 contained 1 resonator, and Version 2 incorporated 2. Both versions implemented the same strip design of width $s = 20\mu\text{m}$ with $g = 5\mu\text{m}$ wide gaps on each side. These parameters are important because they set the (geometric) electric properties, C_G and L_G , which together with (an estimate of) the Kinetic Inductance L_K are ideally matched to the characteristic impedance of the power-line to minimize mismatch reflections (see section 2.3):

$$Z_0 = \sqrt{\frac{L_G + L_K}{C_G}} \simeq 50\Omega$$

²A meander is a self-avoiding curve which crosses a line a certain number of times, i.e. “snakes” across an area.

The lengths of the strips were slightly different, $l = 27.675\text{mm}$ for Version 1 and $l = 30.105\text{mm}$ for Version 2, due to how they were drawn. The exact length is, however, not very important, as long as the order of magnitude yields a resonance frequency within the measuring range of the equipment.

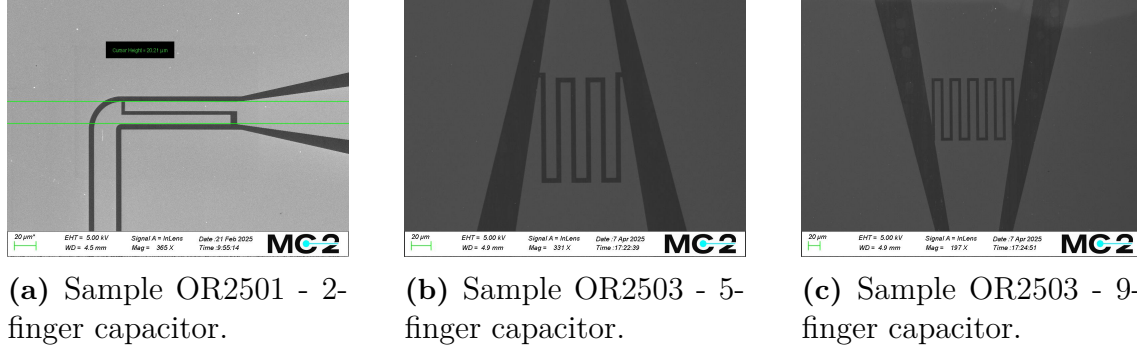
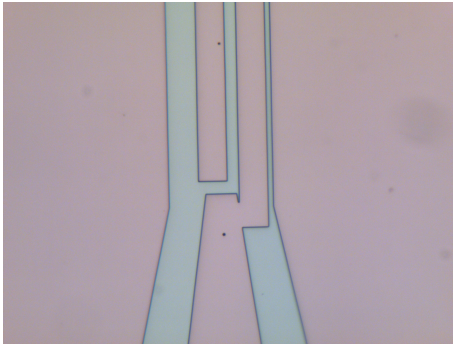


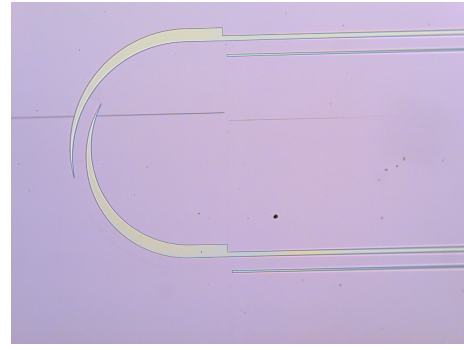
Figure 3.11: Coupling capacitances of finger design. Images captured using SEM.

Version 1 in 3.10a was successful, but it was realized that the meander could be squeezed a lot more tightly. This led to Version 2 in Fig. 3.10b where 2 resonators were fitted onto the same chip. The main advantage of having 2 resonators is that they can be fitted with different coupling capacitances C_k , pictured in Fig. 3.11. It was a priori unknown how large these should be, so they needed to be guessed. Since the coupling capacitances implemented a finger design, it was a bit unclear how thick and how many fingers there should be. Version 1 utilized 2 fingers and Version 2 utilized 5, respectively 9, but these would all prove too small. Gold was only deposited as a 1mm thick strip at the base of the resonator in Version 1, whilst in version 2 around all edges. Using only 1 Gold strip on 1 edge worked fine, but since the more compact design in Version 2 increased the free surface area, it was done to improve electric grounding.

Another improvement from Version 1 to 2 was the exclusion of Low Current exposure. It was originally thought to help achieve sharp features in the strip, but turned out to be unnecessary since there was little difference in observing the edges of Low and High Current. The reason was that the EBL machine had an extremely high resolution, such that Low Current is only needed for sharp features on the nanometre scale, making High Current sufficient for a micrometre scale $\mu\text{m} = 10^{-6}\text{m}$ resonator. Excluding Low Current exposure significantly reduces exposure time, greatly enhancing the efficiency of the process whilst retaining the quality of exposure.



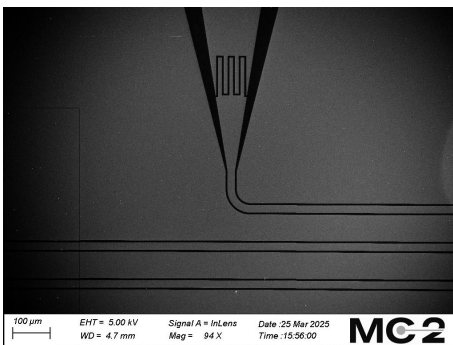
(a) Sample OR2501 - Distorted capacitor.



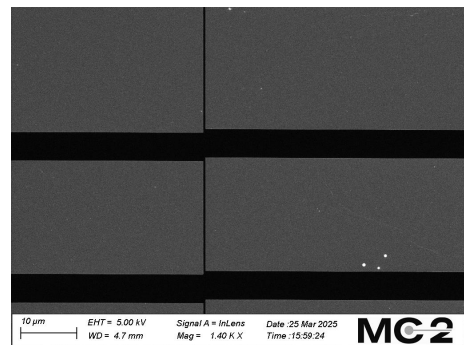
(b) Sample OR2501 - Distorted meander.

Figure 3.12: Unsuccessful exposure yielding a distorted structure.

The dose, or the total amount of charge delivered per area, during lithography exposure is very important to complete a successful lift-off afterwards. This became evident during the first fabrication attempt of Version 1; Fig. 3.12, as the resist structure became distorted. Removing all resist and repeating the same exposure with a higher dose turned out to be a lucky fix, but only foreshadowed the actual problem. When fabricating Version 2, the initial attempt seemed successful looking through an optical microscope, but further examination using Scanning Electron Microscopy (SEM), see Fig. 3.13, revealed a *stitching problem*: misalignment of the exposure pattern where the beam shifts from one area to another. The EBL does not expose the entire chip in one go, but rather the exposure is separated into finite elements. This demands the elements to overlap correctly in order to yield a homogeneously exposed area. If the elements overlap incorrectly, there is a problem stitching them together. The root cause of this problem was identified as the proximity scattering effect. The substrate is insulating, which causes the electrons from the electron beam to backscatter, affecting the resist. In addition, since (almost) the entire chip was exposed, the effect is enhanced. The issue was ultimately resolved by utilizing a double exposure with a slight offset, smearing out the exposure to close up misalignments.



(a) Sample OR2502 - Stitching problem.



(b) Sample OR2502 - Zoom in on stitching problem.

Figure 3.13: Fabrication error caused by Proximity scattering effect; Stitching error.

3.3 Electrical Measurement

Having completed the nanofabrication of the *sample*, the local aim shifts to measuring the *chip*.

3.3.1 Connecting to the chip

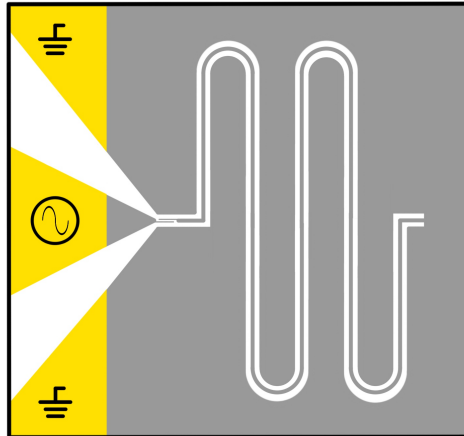


Figure 3.14: Sketch of a Superconducting Resonator from an electrical perspective. Standing waves emerge in the middle strip with the surrounding plane acting as ground. The sketch depicts a quarter-wave $\lambda/4$ geometry, used in this thesis; a short and not a shunt. Yellow is Gold and grey is YBCO.

As stated in step number 4 of the recipe, Gold was deposited on top of YBCO. This is done so that the chip can be connected to a circuit board by bonding them with Gold wires³. The circuit board needs to be mounted on a holder that both fixate the chip and connect to measuring equipment, see below picture.

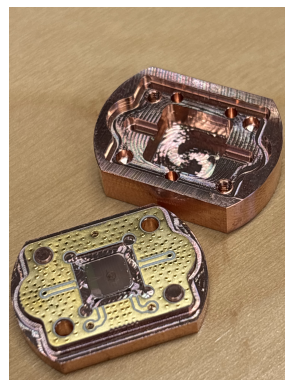


Figure 3.15: Holder that was manufactured to hold the chip. The circuit board is bonded to the chip via Gold wires.

³using a special Gold Bonding Machine

3. Methods

Fortunately for this thesis, similar measurements have been carried out previously at the MC2 department, but unfortunately they had been conducted using 7×7 mm chips, so the available holder was too big. However, the design could easily be transformed to create a new 5×5 mm holder locally at the department. There were also spares of the associated circuit board available, so only new connector ports had to be ordered. Hence assembling the holder, it could be connected to a Vector Network Analyser (VNA).

The instrument employed for measurement was a Rohde Schwarz VNA model ZNB20, photographed below. A VNA is a type of wave-generator that can send in and measure the reflection an electric signal at different ports. However, in this thesis reflection was measured on the same port as the input, because the additional feature of measuring at different ports is typically required when dealing with mK temperatures. It is still very important to properly attenuate the signals though (see next page).

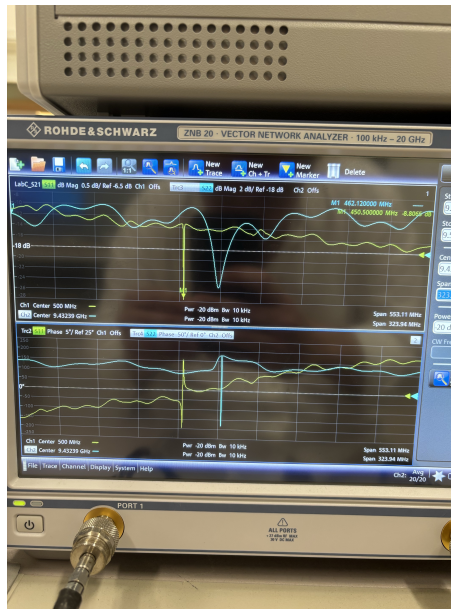


Figure 3.16: An example resonance measured using a Rohde Schwarz VNA model ZNB20. Upper half is Amplitude and lower half is phase. The two different colours represents different channels for the two different resonators on Resonator Design Version 2.

3.3.2 Cooling the chip

Two different cryogenic setups were utilized in this thesis: a Dilution Refrigerator and a Physical Property Measurement System (PPMS). Both are viable options for measuring the resonance frequency of a superconducting resonator at different temperatures, each with its own advantages and disadvantages. The main advantage of a Dilution Refrigerator is that it can reach temperatures as low as 20mK, but it is tricky to measure above 30K and it cools very slowly. A PPMS on the other hand cools very fast and measures above 30K with ease, but it can “only” reach a minimum of around 4K.

The original plan was to conduct all measurements using the PPMS, but there were some unfortunate setbacks, including the limited availability of the tool and unforeseen issues during the fabrications. Another available alternative was to use the Dilution Refrigerator pictured below to measure the first sample (OR2501), because there was already a setup implemented from the previous resonator measurements, including automated measurement software. The chip was measured from $T = 73\text{K}$ to $T = 20\text{K}$ during a descent down to 20mK.

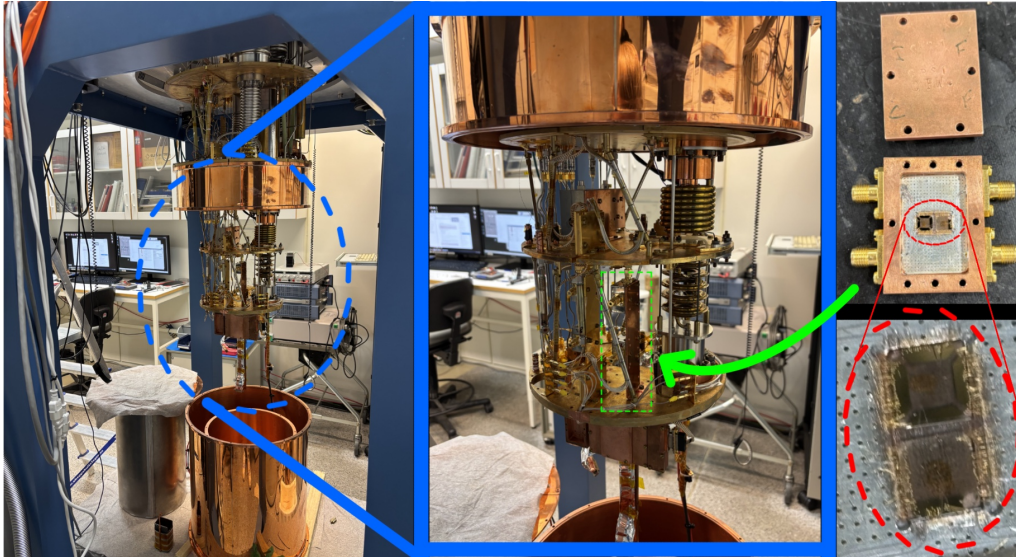


Figure 3.17: Experimental setup of the Dilution Refrigerator. The red encirclement contains the mounted chips in a holder exclusive to the fridge. The green arrow indicates where the holder was mounted.

The quality of the data from the Dilution Refrigerator would prove a lot better than the PPMS, but this was only due to an unfortunate hardware oversight. Because the Dilution Refrigerator was equipped to measure at 20mK, it was fitted with attenuators and amplifiers to deal with the extreme temperature, but a resonator measurement had yet to be conducted using the PPMS. As such, though discussing it over, attenuation was falsely thought to be unnecessary. It turned out very necessary, because the resonator became saturated, i.e. distorting the resonance dips in the frequency spectrum. This could be a non-linear response due to probing at high powers. It unfortunately also renders it impossible to deduce the quality factor

Q of these resonances. Luckily, only the actual location of the resonance is needed to extract London penetration depth λ_L .

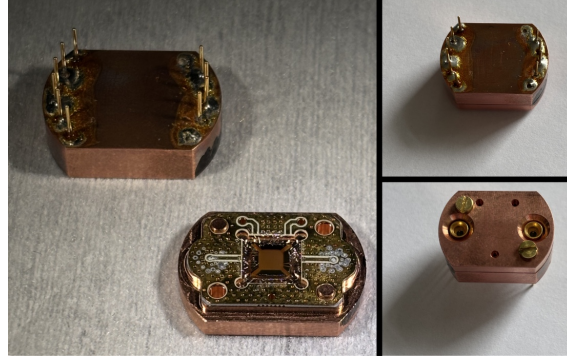


Figure 3.18: Close up of the holder. The soldered pins fit into the grounding of the measurement instrument. The device is electrically connected via the 2 ports on top. Each port is responsible for 1 resonator.



Figure 3.19: A picture of the PPMS setup. The holder is mounted on the RF-stick and inserted vertically into the cylindrical cryogenic chamber.

Samples OR2503 and OR2504 were measured using the PPMS pictured above. The samples were mounted inside the holder, the holder onto an RF-stick and the RF-stick into the PPMS vertically like in the picture above. The VNA was then connected to connection ports on top of the stick, the system sealed and hence cooled down to $T = 4\text{K}$. Starting at the lowest temperature, instead of the highest, was done to minimize time. If the system is cooled down to $T = 4\text{K}$ and given time to settle into equilibrium, increasing in increments up to $T = 65\text{K}$ significantly reduces the thermalization time compared to decreasing decrement, because it is easier for the system to settle into equilibrium if Entropy is increased rather than decreased. Each measurement had to be done manually, because a program could not be created in time for the measurements.

3.4 Data Processing

With *measurements* completed, focus was directed towards processing the *data*.

3.4.1 Locating Resonances

The data from the Dilution Refrigerator and the PPMS came in different file formats: `.mat` respectively `.csv`, but both store a complex-valued signal $Ae^{j\theta}$ over frequency f . Here j is used to represent the imaginary unit instead of i , which will instead be reserved for *index*.

The data was processed using MATLAB. Both file formats could be loaded as a `struct` containing arrays of Amplitude A , phase θ , and frequency f . The `.mat`-files had A in units of Volt and contained all measurements of a given Temperature T , whilst the `.csv`-files contained individual measurements with A in units of Decibel. Having loaded all data into memory, the aim is to extract London penetration depth λ_L and quality factor Q from the arrays.

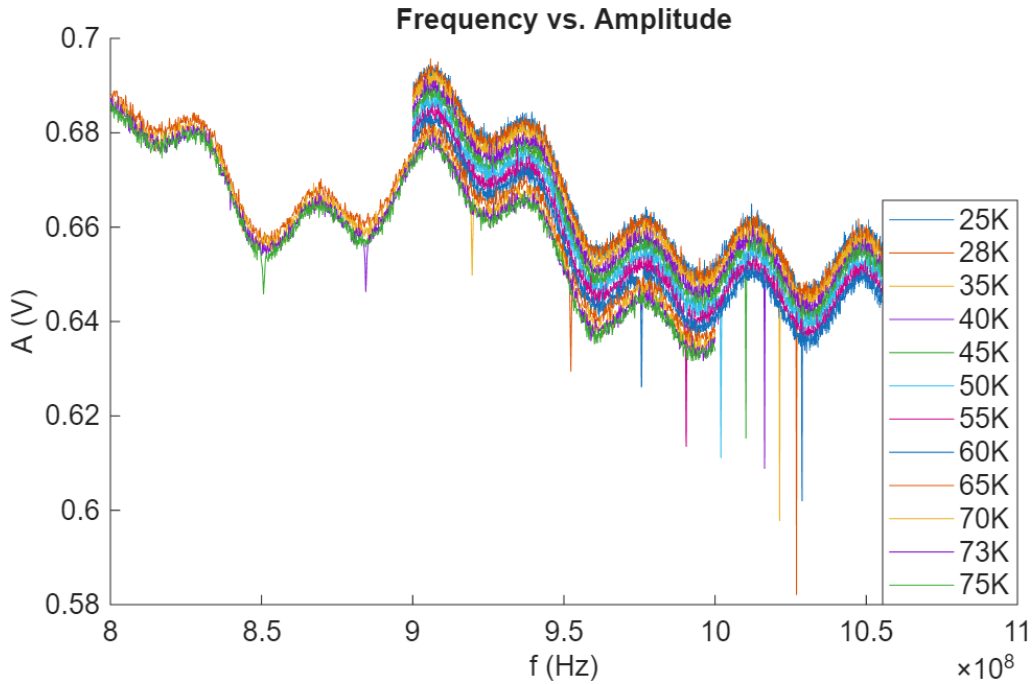


Figure 3.20: Example raw data plot for the fundamental frequency of sample OR2501 at different temperatures; reflected Voltage Amplitude A vs. frequency f . The different colours are different measurement series, i.e. frequency sweeps with a dip at different temperatures. The legend top to bottom corresponds to the dips right to left.

Extraction requires locating and analysing the dips. Upon plotting the data graphed above in Fig. 3.20; a set of frequency sweeps at different temperatures, a non-constant background was observed. A non-constant background makes extracting dips non-trivial, because the local minima is not necessarily the same as the global minima. At any rate, if an arbitrarily broad dip could be located at f_0 , it

could be analysed by simply guessing an appropriate span Δf from the plot. This does hinder full automation of the process, because a variable input is required, but the number of data sets to be analysed was not that large. Optimizing an automated software would have taken much more time compared to requiring 1 input.

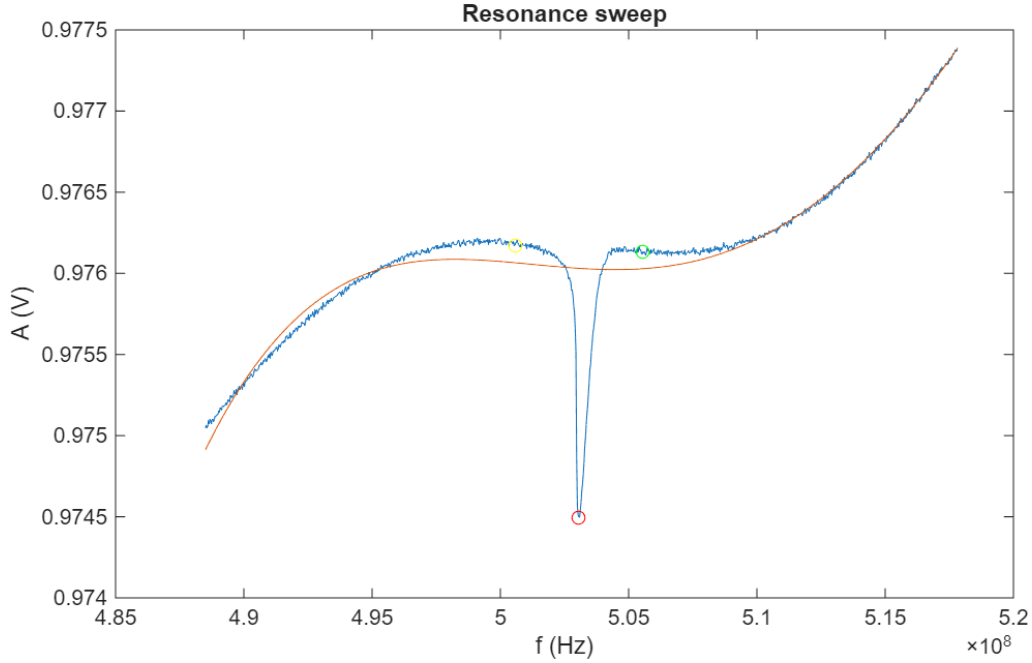


Figure 3.21: Example image of algorithm locating and extracting a dip, here depicting the fundamental frequency for sample OR2503 at $T = 4\text{K}$. The background fit is not perfect and does not have to be. The vertical scale in this is technically in decibel, but it is referring to Volts.

If there was no background B , locating the dip would be trivial as it would correspond to the smallest element of the Amplitude array A . When there is a background, the local minima we want to extract could be overtaken by some global minima stemming from the background. If B can be estimated or guessed, the issue can be circumvented by normalizing the data. Dividing the background from the amplitude per index i :

$$A_{\text{Norm}}(i) = A(i)/B(i)$$

yields a function that is 1 everywhere except the dip⁴. The background was estimated using the toolbox function `fit`. If the user can observe the plot of the data, it can guess an approximate background fit, if it is a `fittype`. The guess does not have to be exact so long it follows the general trend of the background, see Fig. 3.21. After normalizing, the desired minima is guaranteed to be located at the minimum of the array:

$$f_0 \equiv f(i_{\min}) : i_{\min} \equiv \min_i A_{\text{Norm}}(i)$$

⁴If the background fit B is perfect

3.4.2 London Penetration Depth

Having located a set of resonance frequencies f_0 , penetration depth can be extracted by solving the electric resonance equation (Eq. (1.1) in the introduction) as a root problem:

$$\frac{1}{(L_G + L_K(\lambda_L))C_G} - (2\pi f_0)^2 = 0$$

using toolbox function `fzero`. This yields measured values of λ_L with respect to temperature T , plotted in Fig. 3.22 below. As detailed in the theory section, Geometric Inductance L_G and Capacitance C_G are known values and Kinetic Inductance L_K is a known function with respect to penetration depth.

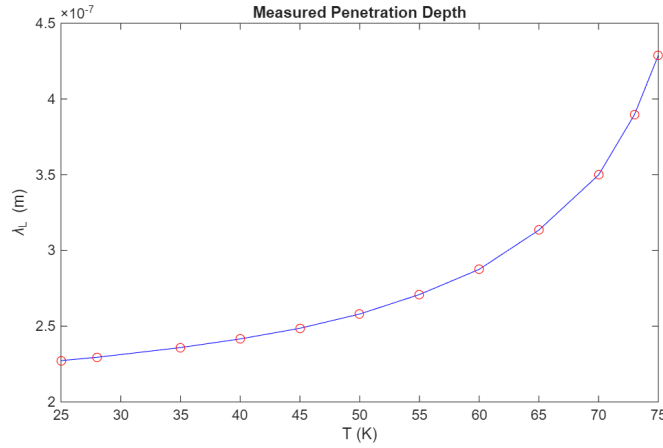


Figure 3.22: Measured Penetration Depth of OR2501 obtained by solving electric resonance equation.

The plot of points can be curve-fitted to the following expression:

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1 - (T/T_c)^\gamma}}$$

where the parameters λ_0 , T_c , γ were extracted via least squares using toolbox function `lsqcurvefit`.

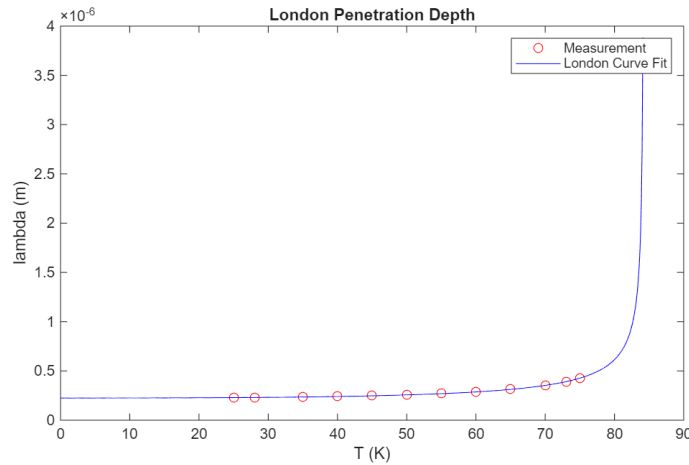


Figure 3.23: Curve fit of OR2501 measurements.

3.4.3 Quality Factor Q

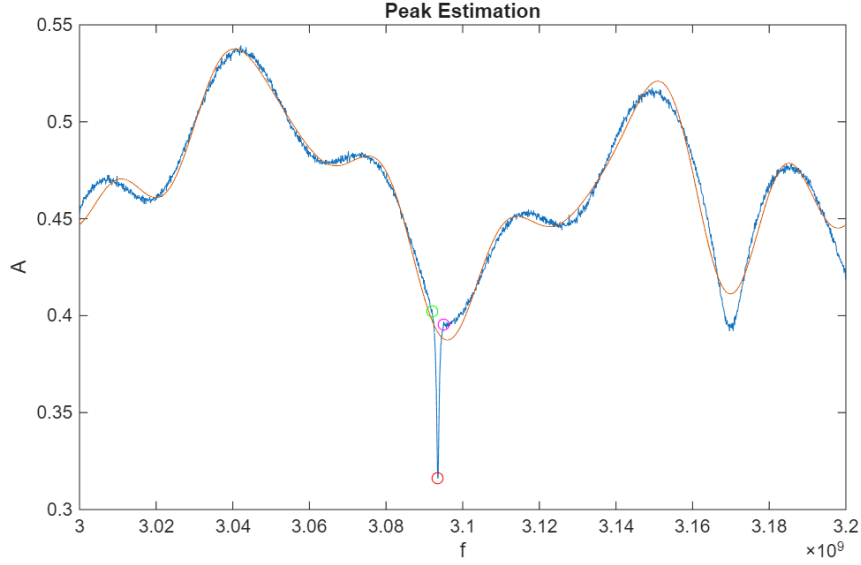


Figure 3.24: Dip extraction for the fundamental frequency of sample OR2501.

In order to extract quality factor Q , dips needed to be analysed individually. Having guessed a symmetric span Δf around f_0 , it can be converted into an index shift k :

$$k = \frac{\frac{1}{2}\Delta f}{\delta f} : \delta f \equiv f(i+1) - f(i)$$

which can be used to extract the dip by picking out the elements of A and f surrounding the minimum index $i_{\min}$:

$$\text{Dip} \equiv [f(i), A(i)] : i_{\min} - k \leq i \leq i_{\min} + k$$

The dip now needed to be normalized. Theoretically, it would be sufficient to use the aforementioned A_{Norm} , but this would require a perfect background fit⁵. Because the background was unknown and came in many different forms throughout the data, it was better to extract and normalize separately. The Amplitude extracted from the dip was normalized via linearly interpolating the left/right ends of the dip, i.e. $A(i_{\min} \pm k)$, and dividing subsequent straight line out of the Amplitude. This is a good approximation as long as the background does not vary significantly relative to the width of the dip. The Phase was normalized by instead subtracting its average value, because dividing signals subtracts their phases. This sets a completely reflected wave as the reference point, i.e. $\Gamma = 1$, and scales the dip accordingly, directly yielding the fractional absorption of the resonator.

⁵essentially a direct measurement of the background

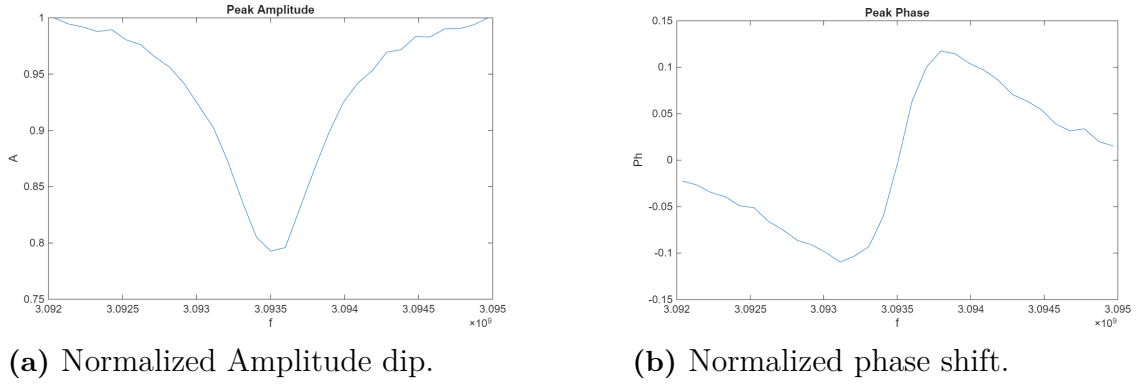


Figure 3.25: Normalized data plot of the 1st overtone of sample OR2501.

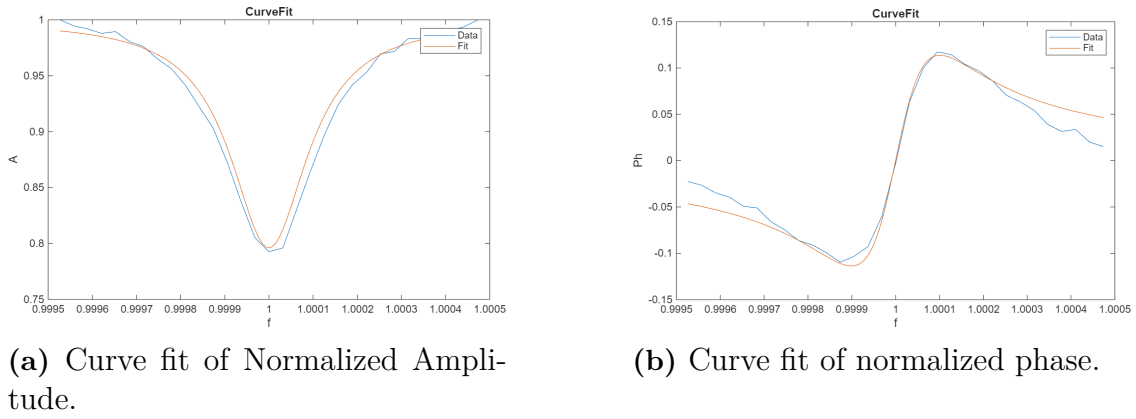


Figure 3.26: Curve fit of the 1st overtone of sample OR2501.

Extracting Q was done by fitting the complex-valued dip $Ae^{j\theta}$ to the expression [11]:

$$\Gamma(x) = \frac{1 - x^2 + jx \left(1/Q_{int} - 1/Q_{ext} \frac{1-kx^2}{1-k} \right)}{1 - x^2 + jx \left(1/Q_{int} + 1/Q_{ext} \frac{1-kx^2}{1-k} \right)}$$

where Γ is the measured reflection such that:

$$\begin{aligned} \frac{1}{Q} &= \frac{1}{Q_{int}} + \frac{1}{Q_{ext}} \\ x &= \frac{\omega}{\omega_0} = \frac{f}{f_0} \\ k &= \frac{C}{C + C_\kappa} \end{aligned}$$

Here f_0 is the resonance frequency, C is the total capacitance of the resonator and C_κ is the coupling capacitance. Assuming $C \gg C_\kappa \Leftrightarrow k = 1$ and only analysing the dip around resonance, whereby $x \simeq 1$, the expression simplifies to the following:

$$\Gamma(x) = \frac{1 - x^2 + jx (1/Q_{int} - 1/Q_{ext})}{1 - x^2 + jx (1/Q_{int} + 1/Q_{ext})}$$

Though it is hard to spot the plot of this function at first glance, it is simply a complex circle of radius $r \leq 1$ tangential to $z = 1$ in the complex plane. If $r < \frac{1}{2}$, the resonator is undercoupled, if $r < \frac{1}{2}$, it is overcoupled and if $r = \frac{1}{2}$, touching the origin, it is critically coupled. As mentioned during the section on design, the devices in this thesis turned out to be severely undercoupled. This caused the circle to be very small and more difficult to fit, but still worked pretty well after tweaking the tolerances of the solver.

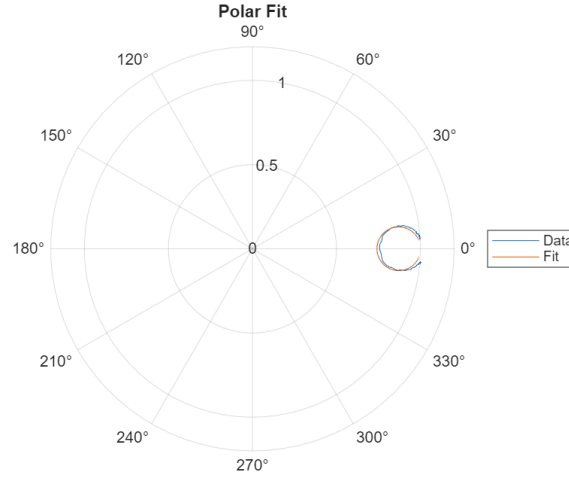


Figure 3.27: Polar plot of Amplitude and phase. The Amplitude dip and phase shift corresponds to a circle in the complex plane, again the 1st overtone of OR2501 is depicted.

The fitting was done in multiple different ways using toolbox functions `lsqcurvefit` and `lsqnonlin`. The reason for trying multiple ways was to attempt fitting the distorted amplitude dips, which looked more like an ellipse. The idea was to allow Q to be complex-valued, such that Q_{int} and Q_{ext} could be complex, but this turned out to have little effect. Allowing complex quality factors effectively only displaces the real-valued parameter version, not skewering it.

4

Results and Discussion

4.1 Resonator Fabrication

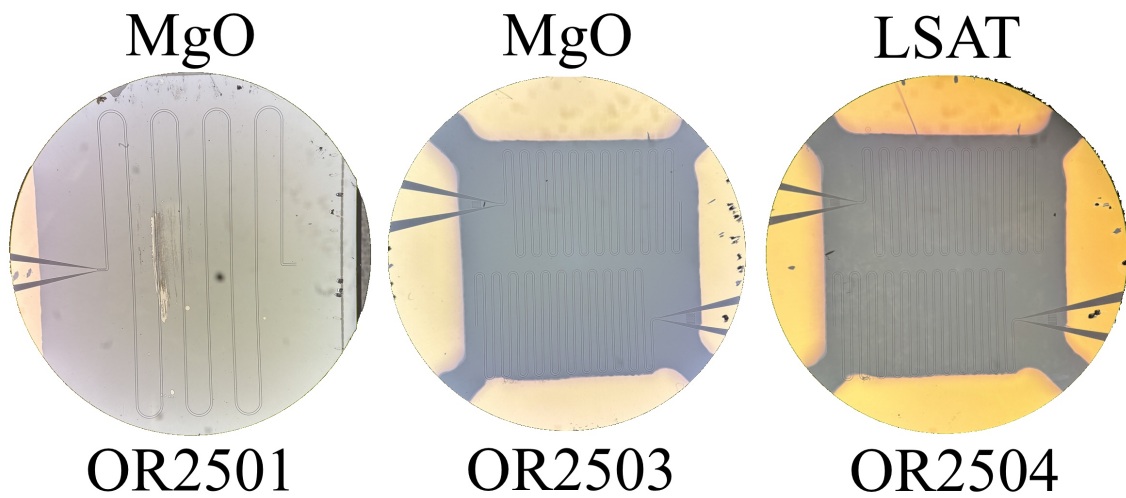


Figure 4.1: Optical images of the resonators fabricated during this thesis. They are a bit dirty because the photos were taken a while after measurements. The first can be seen contaminated by vacuum grease, which fortunately did not ruin the device.

A total of 3 resonators were fabricated and measured during this thesis:

- OR2501: 40nm YBCO on MgO substrate
- OR2503: 10nm YBCO on MgO substrate
- OR2504: 10nm YBCO on LSAT substrate

There was a sample OR2502, but that fabrication was unsuccessful, as described in the Method chapter. The first sample, OR2501, featured a Version 1 Resonator Design, the others a Version 2. Version 2 was an improvement from Version 1. The data from OR2501 was a lot better due to better experimental setup.

Fabrication was overall successful. A reliable process for fabricating resonators was identified. QManD had already developed a general recipe for the fabrication on YBCO, so the main objective was to design a functioning resonator structure.

4.2 Resonator Properties

The two properties extracted from data processing, London Penetration Depth λ_L and quality factor Q , correlate with the Superfluid Stiffness D_s and the Resistive losses $R + R'$, respectively.

4.2.1 London Penetration Depth

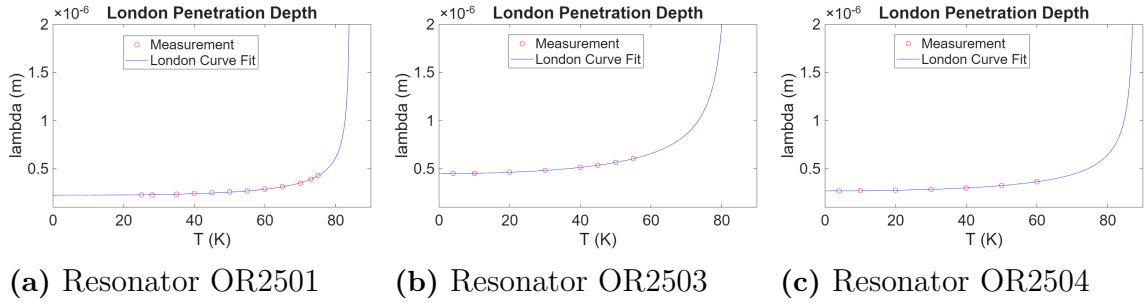


Figure 4.2: London Penetration Depth curves extracted from data, plotted together with measurement points.

Extracting London Penetration Depth explicitly refers to the values λ_0 , T_c and γ , obtained from the curve fit. The following London parameters were extracted for the fundamental frequency¹ of the resonators:

Resonator	Substrate	d (nm)	λ_0 (nm)	T_c (K)	γ
OR2501	MgO	40	225	84.19	2.83
OR2503	MgO	10	450	81.15	2.06
OR2504	LSAT	10	270	87.87	2.07

Table 4.1: London parameters for YBCO resonators. Parameter d is the thickness of the thin film YBCO on top of the substrate.

¹0th harmonic

4.2.2 Quality Factor

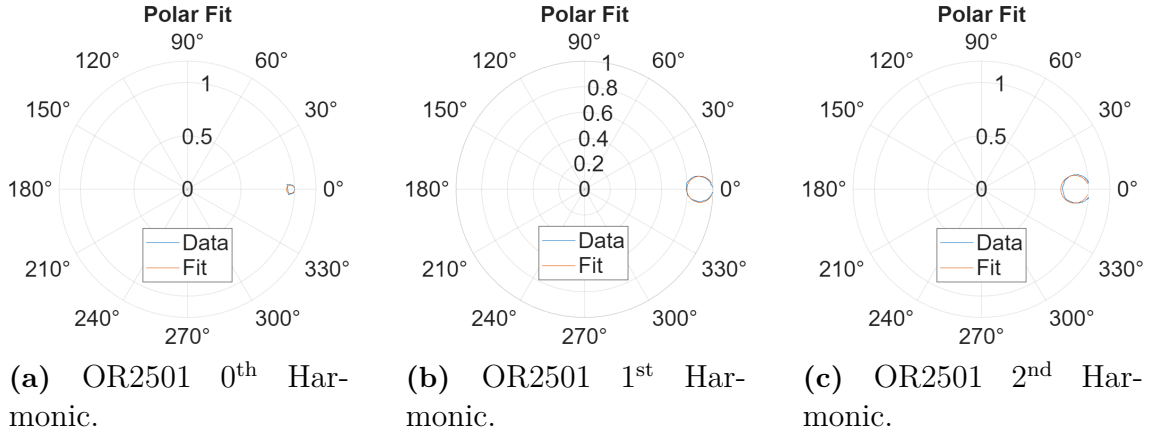


Figure 4.3: Polar plots of the first 3 Harmonics of OR2501.

Extracting the Quality factors explicitly refers to the values Q_{int} and Q_{ext} . They are obtained by curve fitting individual dips in both Amplitude and phase. At each temperature, a number of Harmonics were measured, starting from the fundamental. The following quality factors were obtained at the lowest measured Temperature $T = 25\text{K}$ for sample OR2501:

Resonator	Substrate	Harmonic	ω_0	T (K)	Q_{int}	Q_{ext}
OR2501	MgO	Fundamental	$1.03 \cdot 10^9$	25	$6.27 \cdot 10^3$	$1.57 \cdot 10^5$
OR2501	MgO	1 st	$3.09 \cdot 10^9$	25	$4.92 \cdot 10^3$	$4.33 \cdot 10^4$
OR2501	MgO	2 nd	$5.15 \cdot 10^9$	25	$3.27 \cdot 10^3$	$2.19 \cdot 10^4$

Table 4.2: Internal and external quality factors for sample OR2501 at $T = 25\text{K}$. The values did not vary significantly with temperature.

4.3 Discussion

How does the results relate to the hypothesis?

4.3.1 London Penetration Depth

Observing the zero-temperature penetration depth λ_0 of YBCO on MgO, 225nm for 40nm film and 450nm for 10nm film, reducing film thickness by 4 doubles λ_0 , or rather its square λ_0^2 is quadrupled. Recalling the relation $D_s \propto 1/\lambda_0^2$, reducing thickness by 4 reduces superfluid stiffness by 4:

$$d \rightarrow \frac{d}{4} \implies D_s \rightarrow \frac{1}{4}D_s.$$

In other words, a quarter reduction of YBCO film thickness yields a quarter reduction in superfluid stiffness, supporting the flat band hypothesis: MgO surface nano-facets does flatten the energy bands of a ultra-thin film deposition of YBCO. Under the assumption of a varying quantum metric, the effect on superfluid stiffness is an indicator that the quantum metric is affected.

Comparing the 10nm ultra-thin film of YBCO on MgO with the one on LSAT, 450nm and 270nm, the increase in penetration depth is significantly less on LSAT, only increasing 20% compared to the bulk value 225nm. The smaller increase in penetration depth yields a smaller decrease in superfluid stiffness:

$$d \rightarrow \frac{d}{4} \implies D_s \rightarrow \frac{7}{10}D_s.$$

To be clear, though it has not been explicitly measured, a 40nm thick film of YBCO on LSAT should exhibit $\lambda_0 = 225\text{nm}$ just like 40nm on MgO². The 10nm extracted value $\lambda_0 = 270\text{nm}$ only deviates 20%, which only decreases superfluid stiffness about 30%, compared to 75% on MgO. The slight reduction could be explained by other factors, such as crystal growth, and is negligible compared to the much stronger decrease on MgO with nano-facets. This further strengthens the flat band hypothesis, because LSAT does not have any nano-facets on its surface and does not experience a significant change in penetration depth.

Observing the remaining parameter values, T_c compare to the independent characterization of $T_c \approx 84\text{K}$, but there is a stark contrast³ in the fitting parameter γ . Both OR2503 and OR2504 correspond to a d -wave symmetry, but γ being closer to 3 in the case of OR2501 could suggest an orthogonal component of s -wave gap symmetry in the order parameter:

$$\psi \simeq \psi_d + i\psi_s.$$

Important to point out is that OR2501 lacked some of the lower Temperatures $T < 10\text{K}$, which could affect the accuracy of the fit.

²The bulk value of YBCO is cited around 200 to 300nm in literature

³For clarity, $\gamma = 4$ correspond to a pure s -wave BCS superconductor (LTS) and $\gamma = 2$ correspond to a pure d -wave symmetry (HTS)

4.3.2 Quality Factors

Both internal and external Quality decrease when increasing harmonic, but the external varies significantly more than the internal. With these values, the internal R and external R' components of the total (effective) Resistance $R_{\text{eff}} = R + R'$ can be calculated by rewriting the expressions given for the quality factors during the Theory Section:

$$R = \frac{Q_{\text{int}}}{\omega_0 C}$$

$$R' = \frac{Q_{\text{ext}}}{\omega_0 C}$$

However, the absolute resistance per se is not very interesting. The idea was to yet again compare values when varying substrate, but the distorted measurement data made it impossible. Below is an example fit of remaining samples where Q_{int} is given a lower bound, $Q_{\text{int}} > 10^3$, estimated from the processing of OR2501:

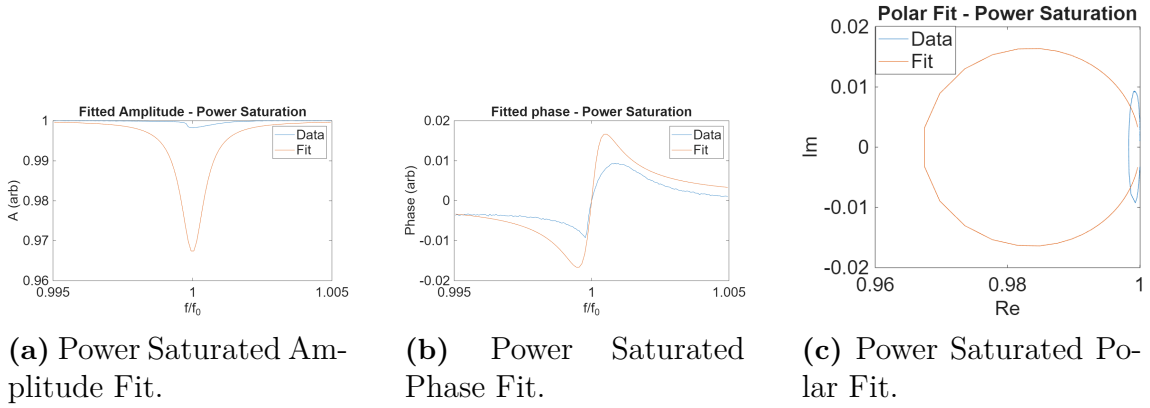


Figure 4.4: Bounded fit of 0th harmonic of OR2503 at $T = 4\text{K}$, suffering from power saturation.

Bounding by $Q_{\text{int}} > 10^3$ is a bit arbitrary, but it highlights the problem with the measurements. If we assume that the phase fit is somewhat accurate, as it is on the same order of magnitude as OR2501, the amplitude appears very tiny in comparison. This type of distortion in amplitude is typical during power saturation, which could be an explanation. A few creative ideas was tried in order to make sense of the measurement data, but such approaches were deemed speculative. The scientific method relies on objective observations, not subjective transformation of data. Even comparing the 2 distorted 10nm films is inaccurate, because the distortion not only saturates, but also skews the resonance dip into distributing into a mixed state of Amplitude and phase.

5

Conclusion

5.1 Thesis Project

The surface nano-facets on an MgO substrate reduced the Superfluid Stiffness D_s in ultra-thin film YBCO compared to a non-faceted LSAT surface. This supports the Flat Band Hypothesis: substrate MgO surface nano-facets flatten the energy bands of a ultra-thin film deposition of YBCO. Flatter energy bands could be an indicator for variations in the underlying quantum metric.

It was possible to estimate the effective losses methodically, but not the relative effect between substrates. This was due to distorted measurement data caused by an incorrect experimental setup.

5.2 Idea for Future Work

Though the hypothesis of flat bands was confirmed, the idea of boosting critical temperature lies within the underdoped regime of the phase diagram. How are the energy bands affected at the illustrious $1/8$ doping?

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