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Chiral effective theory of spin 1 dark matter direct detection

Master's thesis in Physics

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Abstract

Dark matter (DM) is the collective name for the additional mass needed to explain the data collected from a very wide range of different astronomical observations. Everything from the velocity dispersion of galaxies, gravitational lensing caused by galaxies, the large scale structure of the universe as well as the structure of the microwave background radiation all indicate the existence of DM. The exact nature of DM is however still unknown, but it is largely believed to be new fundamental particle, outside of the current standard model of particle physics. The elusiveness of DM is largely due to the fact that the effects of DM never have been observed at microscopic scales. One promising method for detecting DM particles that permeate the galaxy is in so called direct detection experiments, in which, detectors monitor the recoils of nuclei caused by the scattering of DM which is hitting the Earth [1]. The goal with this work is to provide new theoretical insights into the behavior of scattering between DM and nuclei.

In this work DM is assumed to be a weakly interacting massive particle (WIMP) and that it is non-relativistic. Further, it is also assumed to have spin 1. The cross section for the scattering of DM against nuclei is calculated using chiral effective theory, which has not been done before for spin 1 DM. This methodology has a substantial advantage over non-relativistic theories where the degrees of freedom are limited to nucleons and DM since it also includes mesons and consequently can model the effect of meson exchange. In this work it is shown that the inclusion of the meson exchange is crucial especially when modeling the scattering of DM with heavier elements, e.g. xenon, which is a common choice in direct detection experiments [2]. It is also shown that the non-relativistic operators that span the possible DM-nucleon interactions generally cannot be studied individually in direct detection. This is due to the fact that the interaction operators in the more general relativistic theory match onto several DM-nucleon interaction operators. Several DM-nucleon interaction operators consequently share common coupling constants and must generally be studied together.

Keywords: Dark matter, direct detection, spin 1, chiral effective theory, EFT

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Henric Ernbrink, Gothenburg, June 2021

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Introduction

1.1 Background

One of the biggest mysteries in astronomy and cosmology, but also physics as a whole, is the elusive dark matter (DM). DM is the collective name given to the huge amount of, seemingly invisible, mass that must exist in order to explain a large collection of various observations in astronomy and cosmology. These observations collectively span over an impressive range of scales as evidence for DM have been seen in everything from sub-galactic systems [3][4], such as in the kinematics of stars, to the very structure of the observable universe, such as in the cosmic microwave background (CMB) radiation [1][5][6]. Very little is known about the exact nature of DM but its inability of being detected using normal light based observational techniques indicate that it must interact extremely weakly with the electromagnetic (EM) field which consequently mean that it neither blocks or emits light. There have been several attempts at explaining this DM by considering ordinary matter such as planets, dim stars (brown dwarfs, red dwarfs etc.), and neutron stars. DM candidates such as these are often denoted as MACHOs (massive astrophysical compact halo objects). Through observational results it has however been possible to put an upper bound on how much of the DM mass can attributed to MACHOs. This fraction is only 8% and it thus clear that DM must consist of something more than just MACHOs [7][8]. Fundamental particles already present in The Standard Model, such as the neutrino, which was the most prominent candidate of this type, have been considered, but such objects have also been shown to be inconsistent due to observations and constraints enforced by cosmology [9][1]. Thus by essentially ruling out other alternatives scientists have largely come to the agreement that DM consists of some new type of fundamental particle [1]. The mystery of DM then naturally extend into particle physics, and interestingly there are already questions on that front as well which have led scientists to believe that there should exist additional particles on top the ones already present in our current standard model of particle physics.

The Standard Model of particle physics has been very successful in describing the behavior of the fundamental particles in our universe. There are however some deep questions about the model such as the weak force/gravity hierarchy problem. The problem is the following: The weak force is about 24 orders of magnitude stronger than the gravitational force and the way to explain that with the standard model involves some very contrived methods which has lead people to believe that there is something missing to the standard model [10][11]. In fact, by extending the standard model with the theory of supersymmetry (SUSY) one can give a much more believable explanation to the hierarchy problem [12]. However, a big issue with introducing SUSY is that it predicts the existence of additional fundamental particles we have not yet detected. The fact that there is currently no concrete experimental result that unambiguously points towards the specific necessity for SUSY [1][12] is one of the biggest aspects that challenge the theory of SUSY.

The hope is then naturally that these two open ends of astronomy and particle physics are in fact connected and that DM is made up of one or more of these predicted particles. Establishing this connection does however hinge on detecting DM particles, which has not been done yet [1][12], and then being able to use that data to determine the properties of the DM particles. Even though this is like looking for a needle in a haystack there have at least been great effort put into trying to predict what the needle vaguely should look like using theory and computational methods.

So for example, even though the exact mass of these DM particles is still unknown, combined results from both particle physics and cosmology has made it possible to predict that DM particles, with interactions at the weak scale (so-called Weakly Interacting massive particles, or WIMPs), most likely are relatively massive (>GeV) [1]. Throughout this work DM will be assumed to be a WIMP.

1.2 Direct detection

As mentioned, a natural step to prove the particle nature of DM would be to try to measure the microscopic effects of DM as opposed to the macroscopic effects. This would greatly increase our understanding of DM as this would further strengthen the connection to particle physics. One way of doing this is through so called direct detection experiments. The idea of direct detection experiments is to utilize that the Earth theoretically should be constantly bombarded by DM particles which permeate the galaxy [2]. The goal is then to detect these incoming DM particles with detectors on Earth. The detectors operate by measuring the recoil of nuclei as they are scattered by DM. This scattering can safely be assumed to be non-relativistic since the velocity of the DM particles needs be below approximately 600 km/s ($\approx 10^{-3}$ c) in order to be gravitationally bound to the galaxy [13]. This means that DM particles with higher velocity would fly off into deep space instead of orbiting around the galactic center. It is consequently much more likely that the DM that hit the Earth is non-relativistic rather than relativistic. Further, by applying energy-momentum conservation, one can show that the momentum q transferred in the scattering is below 200 MeV for WIMPs [14]. The recoil momentum is thus a lot smaller than the nucleon masses which means that the nucleons remain non-relativistic in the scattering and that nucleus remains intact [13].

The design and operation of detectors used in direct detection experiments is reviewed in detail in [2]. The type of detector that will be considered in this work is the liquid noble-gas detector, which in first approximation can be modelled as described below. It consists of a large container of some noble element, like xenon, and photon detectors which detect the photons emitted from the nucleus as it is de-excited after the scattering with DM. Molecular gases can also be used, such as in the PICO detector, where Freon is used. The experiments are generally located deep underground, where only particles like DM which interact very weakly will be able to reach, in order reduce the probability of false positives [2].

1.3 Modeling of the DM-nucleus interaction

There are three prominent ways of modeling the DM-nucleus interaction in regards to direct detection experiments. The first method is to use a non-relativistic effective theory which describes the interaction between DM and nucleons. The model is built by constructing the most general Lagrangian obeying Galilean symmetry using a set of Galilean invariant quantum operators. The second method is to instead use a relativistic effective theory and construct the most general Lagrangian which describes the interaction of DM with quarks and gluons while obeying Lorentz symmetry. The third method is to construct a so called simplified model, and extend the standard model with both the DM particle and a mediator particle which mediates the interaction of DM with quarks and gluons. These three methods are related to each other since they each represent different limits of momentum transfer in the DM-nucleus interaction. Specifically, the simplified models reduce to relativistic effective theories in the limit where the momentum transfer is much smaller than the mediator mass. In the limit where the momentum transfer is much smaller than the nucleon mass, and the DM moves at non-relativistic speeds, the relativistic theories reduce to the non-relativistic theories [15].

It can initially seem appropriate to use the non-relativistic effective theory methods since the momentum transfer q will be smaller than the nucleon mass, and since the DM is non-relativistic [14]. However, as shown in [14][16] this can be a fallacy in some scenarios. In these works it is shown that additional, unexpected, constraints are imposed on the non-relativistic theory if one starts from a relativistic effective theory and then evaluate it in the non-relativistic limit. For example, by starting from the non-relativistic theory one can formulate an ordering scheme for the Galilean invariant nucleon-DM operators, that shows which operators are relevant in the low energy limit [17]. However, by starting from the relativistic theory, one can see that several of the low energy operators are in fact not needed at leading orders. Further, starting from relativistic theory reveals that Galilean invariant nucleon-DM operators, which were thought to be independent of each other, are in fact in some cases dependent. These effects have only been studied in the cases of DM with spin 0 and 1/2. One goal of this work is consequently to extend this study to the case of spin 1 DM and see if a similar behavior is observed.

Furthermore, non-relativistic theories which only consider DM and nucleons as the degrees of freedom cannot capture the effects of meson mediated interactions between DM and nucleons, which can occur in DM scattering of nuclei. These effects can consistently be described by applying chiral effective theory to the study of DM scattering by nuclei [16] and this approach has successfully been pursued for spin 0 and spin 1/2 DM [13]. The effect of the meson exchange has also been shown to be very relevant in the cases where spin 0 and spin 1/2 DM scatters against heavy nuclei such as xenon (see [14]). However, this methodology has not been carried out in the case of spin 1 DM. The main purpose of this work is consequently to apply chiral effective theory to model the scattering of spin 1 DM by nuclei, and thus for the first time, account for effects related to meson exchange in nuclei.

2

Theoretical framework

This section will present the theoretical foundation as well as the methodology that will be used to obtain the theoretical results. This includes a quick review of the essential aspects of low energy quantum chromodynamics, that being its symmetry and degrees of freedom. This is then used to give a brief introduction to chiral effective theory. This then culminates in an explanation of how scattering between DM and nucleons can be described using the aforementioned theory.

2.1 Quantum chromodynamics

The Standard Model of particle physics describes the motion of all fundamental particles and all the interactions between them. More specifically this model is a Lagrangian which can be split nicely into different sectors, each one corresponding to the different types of fundamental particles present in our universe. The sector describing the quarks partly consists of terms that represent how the quarks can interact. They can interact via all the four fundamental forces but the strong force is, as the name suggests, much stronger than the other forces. In fact, it is about 100 times stronger than the second strongest force, electromagnetism [18]. It is consequently valid to primarily focus on the interaction mediated by the strong force. The theory of the strong force is called quantum chromodynamics, or QCD, and its Lagrangian $\mathcal{L}_{\text{QCD}}$ is

$$\mathcal{L}_{\text{QCD}} = \sum_{\substack{f=u,d,s\\ c,b,t}} i\bar{q}_f \left(\gamma^\mu \mathcal{D}_\mu - m_f\right) q_f - \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} \tag{2.1}$$

Here q_f are quark spinor fields carrying a flavor index f and each have a mass m_f. QCD has a local $SU(3)$ color symmetry which necessitates the existence of a covariant derivative $\mathcal{D}_\mu$ and a gauge field that form the field strength $\mathcal{G}_{\mu\nu}$ which is called the gluon field strength tensor.

Experimentally we have detected six different quark flavors and these can be split into two categories based on their masses. u, d, s have masses $\ll 1$ GeV and constitute the light quarks while c, b, t have masses > 1 GeV and constitute the heavy quarks. Since low energy QCD is of interest in this analysis it is appropriate to neglect the heavy quarks since the effect from virtual heavy quarks will be very small [19]. By doing this, additional approximate symmetries of $\mathcal{L}_{\text{QCD}}$ can be formulated. These approximate symmetries will be very important since they will give rise to Pseudo Nambu-Goldstone bosons or PNGBs which are critical in the low energy theory.

2.1.1 Pseudo Nambu-Goldstone bosons

On top of the local $SU(3)$ color symmetry there are two global transformations that are of great interest. These two transformations target the flavor index, which is only present on the spinor fields, and as a result the gluon field strength is invariant. Consequently it is enough to just focus on the spinor fields. The first transformation is Λ_V and has the following definition

$$\begin{aligned}\Lambda_V: \quad & q \to e^{-i\Theta_a \frac{\lambda_a}{2}} q \simeq (1 - i\Theta_a \frac{\lambda_a}{2})q \\ & \bar{q} \to \bar{q} e^{+i\Theta_a \frac{\lambda_a}{2}} \simeq \bar{q}(1 + i\Theta_a \frac{\lambda_a}{2})\end{aligned} \tag{2.2}$$

where λ_a are the Gell-Mann matrices which span the Lie algebra of $SU(3)$. The Gell-Mann matrices are used since only the light quark flavors are considered which means that q is a flavor-vector with three elements. Since Λ_V is global it commutes with the covariant derivative in (2.1) and nothing in Λ_V interferes with the gamma matrix. It is then clear that the kinetic term in (2.1) is invariant under the transformation Λ_V. The other global transformation of interest is Λ_A and is defined as follows

$$\Lambda_A: \quad q \to e^{-i\gamma^5 \Theta_a \frac{\lambda_a}{2}} q \simeq (1 - i\gamma^5 \Theta_a \frac{\lambda_a}{2})q \tag{2.3}$$

This time the transformation of $\bar{q}$ needs some care since $\bar{q} = q^\dagger \gamma^0$ and $q^\dagger \to q^\dagger \Lambda_A^\dagger$. Thus $\bar{q} \to q^\dagger \Lambda_A^\dagger \gamma^0$ which combined with the anti-commutation of the gamma matrices yields

$$\Lambda_A: \quad \bar{q} \to \bar{q} e^{-i\gamma^5 \Theta_a \frac{\lambda_a}{2}} \simeq \bar{q}(1 - i\gamma^5 \Theta_a \frac{\lambda_a}{2}) \tag{2.4}$$

The transformation of the kinetic term in (2.1) with respect to Λ_A is then

$$\begin{aligned}\bar{q}\gamma^\mu \mathcal{D}_\mu q \to & \bar{q}(1 - i\gamma^5 \Theta_a \frac{\lambda_a}{2})\gamma^\mu \mathcal{D}_\mu (1 - i\gamma^5 \Theta_a \frac{\lambda_a}{2})q \\ = & \bar{q}\gamma^\mu \mathcal{D}_\mu q - i\Theta_a \frac{\lambda_a}{2}(\bar{q}\gamma^\mu \mathcal{D}_\mu \gamma^5 q + \bar{q}\gamma^5 \gamma^\mu \mathcal{D}_\mu q) + \mathcal{O}(\Theta_a^2) \\ = & \bar{q}\gamma^\mu \mathcal{D}_\mu q\end{aligned} \tag{2.5}$$

where the last equality is once again due to anti-commutation of the gamma matrices. Thus the kinetic term is invariant under a Λ_A transformation. The mass term however is not invariant under transformations of Λ_V and Λ_A. The QCD Lagrangian is consequently only invariant under Λ_V and Λ_A if one approximates that the quark masses are zero, which is the so called chiral limit $m_u, m_d, m_s \to 0$. The masses of the quarks are

$$\begin{aligned} m_u =& 0.005 \text{ GeV} \\ m_d =& 0.009 \text{ GeV} \\ m_s =& 0.175 \text{ GeV} \end{aligned} \tag{2.6}$$

and can be compared to nucleons which have masses on the order of $\approx$ 1 GeV [19]. Since nucleons are the objects of interest in the end, it is appropriate to approximate that quarks are massless, and then treat the mass as a perturbation. Thus $\mathcal{L}_{\text{QCD}}$ has an approximate $SU(3)_V \times SU(3)_A$ symmetry corresponding to Λ_V and Λ_A respectively. The subscripts V and A refers to vector and axial respectively and stems from how the corresponding Noether currents transform under parity. The vector current has positive parity while the existence of the γ^5 results in the axial current having a negative parity and thus transforming as a pseudo-vector.

From experimental observations it is known that the lightest color neutral particles consisting of quarks are the pions, kaons, and eta mesons. They all roughly have the same mass, have negative parity, and together they make up 8 different particles. From the Goldstone theorem it is known that the breaking of a symmetry results in the creation of massless Goldstone bosons. The number of Goldstone bosons and the properties of said bosons depend on the number of generators and the properties of the broken symmetry group. Since $SU(3)_A$ is associated with negative parity and has 8 generators it is reasonable to assume that the 8 observed particles are Goldstone bosons formed from a spontaneous symmetry breaking of the $SU(3)_A$ symmetry. That the observed particles have a non-zero mass can be explained by remembering that the $SU(3)_A$ symmetry only was approximate, which then results in the mass being approximately zero. Goldstone bosons formed due to the breaking of an approximate symmetry are called psuedo Nambu-Goldstone bosons or PNGBs [19]. Since the PNGBs are the lightest color neutral objects in QCD they will be essential in constructing a theory of low energy QCD, which will be seen later.

2.1.2 Chiral symmetry

The above discussion can be recast in a slightly different way using chirality and is nicely explained in [19]. This alternative formulation will become very useful in upcoming sections. As mentioned previously it is appropriate to neglect the quark masses in the low energy limit of QCD which means that the Lagrangian describing low energy QCD, $\mathcal{L}^0_{\text{QCD}}$, is

$$\mathcal{L}^0_{\rm QCD} = i\bar{q}\gamma^\mu \mathcal{D}_\mu q - \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} \tag{2.7}$$

One can then introduce the following set of standard projection operators

$$P_L \equiv \frac{1}{2}(1-\gamma^5) \qquad P_R \equiv \frac{1}{2}(1+\gamma^5) \tag{2.8}$$

which can be used to define the left handed quark $q_L \equiv P_L q$ and the right handed quark $q_R \equiv P_R q$. Using the Weyl (or chiral) basis to represent the gamma matrices one can easily confirm that the only non-zero elements of q_R and q_L are the bottom two and the upper two respectively such that $q = q_L + q_R$. Inserting this decomposition in to $\mathcal{L}^0_{\rm QCD}$ yields

$$\mathcal{L}^0_{\rm QCD} = i\bar{q}_L\gamma^\mu \mathcal{D}_\mu q_L + i\bar{q}_R\gamma^\mu \mathcal{D}_\mu q_R - \frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} \tag{2.9}$$

The cross terms consisting of pairings of q_L and q_R are zero which is once again easily confirmed using the Weyl basis. One can then consider the following two transformations V_L and V_R acting on the flavor indices of the quark fields

$$\begin{aligned} q_L \to \Lambda_L q_L &= \exp\Big(-i\Theta^L_a\lambda_a\Big) q_L \\ q_R \to \Lambda_R q_R &= \exp\Big(-i\Theta^R_a\lambda_a\Big) q_R \end{aligned} \tag{2.10}$$

Λ_L and Λ_R are independent $SU(3)$ matrices and thus $\mathcal{L}^0_{\rm QCD}$ possesses a global $SU(3)_L \times SU(3)_R$ symmetry often referred to as chiral symmetry. This symmetry is in fact equivalent to the previous vector/axial symmetry $SU(3)_V \times SU(3)_A$ [19]. This is most easily seen by calculating the Noether currents associated with the symmetries and observing that they are related by

$$\Lambda_V = \Lambda_R + \Lambda_L \qquad \Lambda_A = \Lambda_R - \Lambda_L \tag{2.11}$$

This shows that the formulation above is just a reformulation of the previous discussion.

$\mathcal{L}^0_{\rm QCD}$ does actually satisfy a larger symmetry than $SU(3)_L \times SU(3)_R$. One can in fact add phases to the transformations in (2.10)

$$\begin{aligned} q_L \to V_L q_L &= \exp\Big(-i\Theta^L_a\lambda_a - i\Theta^L\Big) q_L \\ q_R \to V_R q_R &= \exp\Big(-i\Theta^R_a\lambda_a - i\Theta^R\Big) q_R \end{aligned} \tag{2.12}$$

V_L and V_R are independent $SU(3)\times U(1) = U(3)$ matrices. This additional $U(1)_L \times U(1)_R$ symmetry can in a similar fashion to before be shown to satisfy $U(1)_L \times U(1)_R = U(1)_V \times U(1)_A$. Interestingly, $U(1)_A$ is only a classical symmetry and thus the corresponding Noether current, A^μ, only satisfy $\partial_\mu A^\mu = 0$ before quantization [19]. This $U(1)_A$ anomaly, or *axial anomaly*, must be taken into consideration when constructing the effective Lagrangian describing low energy QCD.

2.2 Heavy vector effective theory

Having described how to model unbound quark/gluon systems, we now focus on the modelling of DM particles. The first step is to address the assumptions regarding the very large mass and the non-relativistic velocity of the DM. These assumptions result in that the typical momentum transfer q that will occur in the scattering is very small compared to the DM mass, $q \ll m_X$. The heavy DM effective theory (HDMET) utilizes this fact to simplify the scattering interaction. HDMET is constructed in very close analogy to Heavy quark effective theory (HQET) and it is instructive to first explain HQET before returning to HDMET.

In HQET the system of interest is a baryon that consist of a heavy quark and light quarks. Here heavy refers to masses much larger than the energy scale of hadronic bound states (Λ_{QCD}=0.2 GeV) which means that the heavy quark will move with close to the same velocity as the baryon. In other words, in the limit of when the heavy quark mass is infinite the heavy quark will be static in the baryon frame. The important observation is then that the heavy quark consists of both massive (or heavy) degrees of freedom (DOF) and massless DOF. Most of the dynamics between the heavy quark and light quarks should consequently be captured by only considering the massless DOF of the heavy quark since the heavy DOF are static [20].

The same principle is applied in HDMET but in this case the baryon is the galaxy, the heavy quark is DM, and the light quarks are the atomic nuclei in the detectors. The analogous thought process is then that in the limit of infinite DM mass the DM is static in the universe and thus only the massless DOF of DM should contribute in the scattering of DM and atomic nuclei. In reality, the mass of DM is not infinite and it is consequently not appropriate to delegate all of the dynamics to the massless DOF; one must also include the heavy DOF. One can however write the heavy DOF as a function of the massless DOF and then expand in an infinite series which is in powers of inverse DM mass. Thus one can include as many terms as one deems necessary based on the assumed DM mass.

In the case of spin 1/2 DM the conversion from HQET to HDMET is relatively straight forward since quarks have spin 1/2, and this has been done in [13]. In the case of spin 1 DM the conversion naturally requires a few more steps which are shown in detail in appendix A, which includes some of the novel contributions to the field of this thesis. In this context the main result from appendix A is that the spin 1 DM field X^μ with mass m_X can be expressed in terms of a massless vector field χ^μ

$$X^\mu = e^{-im_X v\cdot x}\left(\chi^\mu - \frac{iv^\mu}{m_X}\partial_\rho \chi^\rho + \mathcal{O}\left(\frac{1}{m_X^2}\right)\right) \tag{2.13}$$

As described in appendix A, χ^μ does not carry the total momenta p^μ of the massive vector field X^μ. Instead χ^μ carries the small residual momenta $\tilde{p}^\mu$ such that

$$p^\mu = m_X v^\mu + \tilde{p}^\mu \qquad m_X v^\mu >> \tilde{p}^\mu \tag{2.14}$$

where v^μ is a reference vector which can be chosen to be time-like [21], and in later steps where the scattering amplitudes are calculated it will be helpful to impose that. Until then v^μ will be kept general. In this work only terms with one order of inverse DM mass will be considered.

2.3 The QCD-DM Lagrangian

The next step is to formulate the Lagrangian $\mathcal{L}$ describing QCD and its interaction with DM. This will then be converted to an effective low energy Lagrangian. One might expect that the underlying symmetry of the effective Lagrangian should be that of $\mathcal{L}^0_{\text{QCD}}$, i.e. global $U(3)_L \times U(3)_R$. This is however incorrect. Rather, one must consider Green's functions and their Ward identities, or local symmetries, which then extends the theory to include off-shell behavior which is needed to properly describe low energy QCD. This stems from the fact that an effective Lagrangian should not be formulated from postulated symmetries but instead should follow from the Ward identities of the underlying theory [22]. This is outside the scope of this work, hence only a brief outline will be provided.

2.3.1 Locally chiral symmetric QCD

Suppose that the system of interest with Lagrangian $\mathcal{L}$ has Noether currents J^μ_i numerated by i stemming from the symmetry group G. Using these currents one can formulate the Green's functions through the use of external fields $f^i_\mu(x)$. The inclusion of these external fields can be viewed as modifications to the initial Lagrangian such that $\mathcal{L} \to \mathcal{L} + f^i_\mu J^\mu_i$. The Ward identities of the system are in fact equivalent to the Lagrangian being locally invariant under transformations of G which of course puts conditions on how the external fields must transform. This is very helpful as it allows one to contextualize the construction of the effective Lagrangian in terms of including external fields to the modified Lagrangian instead of studying Ward identities and Green's functions. One caveat is when anomalies are present in the symmetries of the system. In this case some of the external fields will transform non-trivially under G [22].

As previously mentioned $\mathcal{L}^0_{\text{QCD}}$ has a global $SU(3)_V \times U(1)_V \times SU(3)_A \times U(1)_A$ symmetry which corresponds to the following Noether currents [19]

$$V^{\mu,a} = \bar{q}\gamma^\mu \frac{\lambda^a}{2} q \qquad V^\mu = \bar{q}\gamma^\mu q \tag{2.15}$$

$$A^{\mu,a} = \bar{q}\gamma^\mu\gamma^5 \frac{\lambda^a}{2} q \qquad A^\mu = \bar{q}\gamma^\mu\gamma^5 q \tag{2.16}$$

Using the prescription above one should then modify $\mathcal{L}^0_{\text{QCD}}$ by including external fields contracted to the currents such that

$$\mathcal{L} = \mathcal{L}^0_{\text{QCD}} + \bar{q}(x)\gamma^\mu\Big[\nu_\mu(x) + \gamma_5 a_\mu(x)\Big]q(x) \tag{2.17}$$

where

$$\nu_\mu(x) = \frac{\lambda^a}{2}\bar{\nu}^a_\mu(x) + \tilde{\nu}_\mu(x) \qquad a_\mu(x) = \frac{\lambda^a}{2}\bar{a}^a_\mu(x) + \tilde{a}_\mu(x) \tag{2.18}$$

where the two terms in each equality corresponds to the external fields of the $SU(3)$ current and $U(1)$ current respectively. Equation (2.18) is included for transparency and in the future $\nu_\mu(x)$ and $a_\mu(x)$ will be referred to as the external fields. Since $\mathcal{L}$ must be locally invariant under $U(3)_L \times U(3)_R$ it forces $\nu_\mu(x)$ and $a_\mu(x)$ to transform. By writing the external fields in terms of left/right external fields such that

$$\nu_\mu(x) = \frac{1}{2}(r_\mu + l_\mu) \qquad a_\mu(x) = \frac{1}{2}(r_\mu - l_\mu) \tag{2.19}$$

and writing the quarks in terms of q_L and q_R it is clear that l_μ and r_μ must transform as [19][23]

$$r_\mu \rightarrow V_R r_\mu V_R^\dagger - i\partial_\mu V_R V_R^\dagger \tag{2.20}$$

$$l_\mu \rightarrow V_L l_\mu V_L^\dagger - i\partial_\mu V_L V_L^\dagger \tag{2.21}$$

The derivatives in equations (2.20) and (2.21) cancel the terms generated in the transformation of $\mathcal{L}^0_{\text{QCD}}$. In later steps it will be useful to consider ν_μ, a_μ rather than l_μ, r_μ, and is consequently useful to instead write the above transformation as

$$\nu_\mu + a_\mu \rightarrow V_R(\nu_\mu + a_\mu)V_R^\dagger - i\partial_\mu V_R V_R^\dagger \tag{2.22}$$

$$\nu_\mu - a_\mu \rightarrow V_L(\nu_\mu - a_\mu)V_L^\dagger - i\partial_\mu V_L V_L^\dagger \tag{2.23}$$

2.3.2 Spurions

At this stage it is appropriate to acknowledge that $\mathcal{L}^0_{\text{QCD}}$ is approximate since it is only valid for vanishing quark masses. One should therefore include a mass term in the Lagrangian and treat it as a perturbation. Inclusion of such a term would be an example of explicit symmetry breaking as it clearly breaks the symmetries in $\mathcal{L}^0_{\text{QCD}}$. The above framework supports the addition of terms like this by simply writing $\mathcal{L} \to \mathcal{L} + f^i_\mu J^\mu_i + m_\alpha(x)O^\alpha$ where O^α is the symmetry breaking term weighted by $m_\alpha(x)$. $m_\alpha(x)$ will be referred to as spurions. For example in the case of a mass term: $O = \bar{q}q$ and $m(x) = m_f$. $\mathcal{L}$ must still be locally invariant under transformations of G which means that spurions $m_\alpha(x)$ must transform contragradiently to their corresponding operators O^α [22]. The interaction with DM will also be seen as a perturbation and it is thus necessary to also include spurions which represent all the ways DM can interact with QCD. Extending $\mathcal{L}$ with the addition of vector ($\hat{\nu}_\mu$), axial ($\hat{a}_\mu$), scalar (s), pseudoscalar (p), and gluon (s_G) spurions gives

$$\begin{aligned}\mathcal{L} = \mathcal{L}^0_{\text{QCD}} &+ \bar{q}(x)\gamma^\mu\Big[\nu_\mu(x) + \gamma_5 a_\mu(x)\Big]q(x)\\ &+\bar{q}(x)\gamma^\mu\Big[\hat{\nu}_\mu(x) + \gamma_5\hat{a}_\mu(x)\Big]q(x) - \bar{q}(x)\Big[s(x) - i\gamma_5 p(x)\Big]q(x)\\ &+s_G(x)\frac{\alpha_s}{12\pi}\mathcal{G}^{\mu\nu}\mathcal{G}_{\mu\nu}\end{aligned} \tag{2.24}$$

where α_s is the strong coupling constant. s_G does not transform since $\mathcal{G}_{\mu\nu}$ do not transform. Similarly, no enforcement of transformation are imposed on $\hat{\nu}_\mu$ and $\hat{a}_\mu$ since the chiral quark transformations cancel each other which is made clear by noting that

$$\bar{q} \to \bar{q}(P_L V_R^\dagger + P_R V_L^\dagger) \qquad P_L\gamma^\mu = \gamma^\mu P_R \tag{2.25}$$

$$q \to (P_L V_L + P_R V_R)q \qquad P_R\gamma^\mu = \gamma^\mu P_L \tag{2.26}$$

Because there are no constraints on $\hat{\nu}_\mu$ and $\hat{a}_\mu$ the need for writing them out explicitly is diminished. Instead, they are here reabsorbed into a redefinition of ν_μ and a_μ, as it is typically done in the literature. This streamlines the upcoming steps but one needs keep in mind that transformations of ν_μ and a_μ corresponds to the external fields and that no constraints are needed on the spurions. On the other hand, in the terms involving s and p the chiral quark transformations do not cancel and it consequently forces s and p to transform contragradiently. Specifically, $s \to V_R s V_L^\dagger$ (or by considering the Hermetian conjugate: $s \to V_L s V_R^\dagger$) and analogously for p.

As previously mentioned the initially neglected quark mass will be added as a spurion. This can be considered a fundamental spurion since it is the missing piece in the QCD theory. DM interactions will be added as spurions later but these are optional modifications and the QCD theory should of course function without DM.

The quark mass is in s and must therefore transform as $\mathcal{M}_q \to V_R \mathcal{M}_q V_L^\dagger$, where $\mathcal{M}_q = \text{diag}(m_u, m_d, m_s)$. Then, in order to avoid imposing transformation constraints on the DM fields, all DM interaction spurions which belong in s and p will be multiplied by $\mathcal{M}_q$.

The axial anomaly must also be handled. One can write V_L and V_R in terms of vector generators $\alpha(x)$ and axial generators $\beta(x)$

$$V_L = e^{i\alpha(x) - i\beta(x)} \qquad V_R = e^{i\alpha(x) + i\beta(x)} \tag{2.27}$$

As explained in [24] the axial generator $\beta(x)$ is problematic since it induces non trivial transformations which involve the winding number density $\mathcal{G}_{\mu\nu}\tilde{\mathcal{G}}^{\mu\nu}$, where $\tilde{\mathcal{G}}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\mathcal{G}_{\rho\lambda}$. This is dealt with by extending the Lagrangian with an additional term

$$\mathcal{L} \to \mathcal{L} - \frac{1}{(4\pi)^2}\theta(x)\mathcal{G}_{\mu\nu}\tilde{\mathcal{G}}^{\mu\nu} \tag{2.28}$$

where $\theta(x)$ is an external field which transforms as

$$\theta(x) \to \theta(x) - 2\,\text{Tr}\,\beta(x) \tag{2.29}$$

As explained in [25] β must transform as

$$\beta(x) = \frac{\theta(x)}{2}\frac{\mathcal{M}_q^{-1}}{\text{Tr}\left(\mathcal{M}_q^{-1}\right)}, \tag{2.30}$$

The final expression for $\mathcal{L}$ is then [13]

$$\begin{aligned} \mathcal{L} =& \mathcal{L}^0_{\text{QCD}} + s_G(x)\frac{\alpha_s}{12\pi}G^a_{\mu\nu}G^{a\mu\nu} + \bar{q}(x)\gamma^\mu\Big[\nu_\mu(x) + \gamma_5 a'_\mu(x)\Big]q(x) \\ &- \bar{q}(x)\Big[s(x) - i\gamma_5 p'(x)\Big]q(x) \end{aligned} \tag{2.31}$$

where

$$a'_\mu = a_\mu + \frac{\partial_\mu\theta}{2}\frac{\mathcal{M}_q^{-1}}{\text{Tr}\left(\mathcal{M}_q^{-1}\right)} \qquad p' = p + \frac{\theta}{\text{Tr}\left(\mathcal{M}_q^{-1}\right)} \tag{2.32}$$

2.4 Effective field theory

The main difficulty when working with QCD is that the associated coupling constant is large when the energy is low which means that "usual" perturbation theory no longer is a valid tool when doing low energy QCD calculations. In fact, it does not yet exist an *ab initio* (or from first principles) analytical method for describing low energy QCD. One can however construct a perturbative field theory describing low energy QCD by using an effective field theory (EFT). The virtue of using an EFT is that scattering amplitudes can be expanded in the ratio between a small-energy and a high-energy scale, and only involve degrees of freedom that are relevant at the low-energy scale where measurements are performed. This approach circumvents problems arising from an expansion of the S-matrix in powers of the QCD coupling constant [19].

This EFT must of course still encapsulate the theory of QCD. This is achieved by considering the most general Lagrangian possible which obeys the (local) symmetries of QCD. Since it is fully general, it will consist of an infinite number of terms. Not all of these terms are equally important however, and a big aspect of EFT is consequently to find a way of ordering the terms based on their importance. One must also decide which DOF to use in the EFT, or in other words which fields the Lagrangian should be a function of. One could in principle use quarks and gluons but imposing color confinement makes such a theory complicated. Instead it is more appropriate to consider color neutral objects such as mesons and nucleons. Since a low energy theory is needed the best choice for the mesons is to use the PNGBs since they are the objects closest to being massless as discussed previously. The effective theory describing the PNGBs is called Chiral perturbation theory or ChPT [26]. Nucleons can be added to this theory and will be addressed later.

2.4.1 Chiral perturbation theory

As mentioned, a crucial aspect of establishing an EFT is to have a way of ordering the infinite number of terms based on their importance. This relies on that there exists a separation of energy scales in the system of interest. In the case of ChPT, where the DOF are the PNGBs, a natural transition is the energy at which additional heavier vector mesons become prominent. At this chiral symmetry breaking scale, $\Lambda_{\text{ChEFT}} \approx 1$ GeV, the assumed DOF and the assumption of chiral symmetry are no longer appropriate and the EFT breaks down. The second task is then to find a small ($\ll \Lambda_{\text{ChEFT}}$) quantity present in the system of interest that will aid in the ordering the infinite terms. The natural choice in ChPT is the small PNGB momenta and mass, both of which are referred to as Q. Thus the terms in the Lagrangian can be ordered in powers of $Q/\Lambda_{\text{ChEFT}} \approx 0.3$ [13], or in other words, based on the number of derivatives and powers of mass. Further, interactions between the PNGBs must always involve derivatives of the fields or in other words their momenta must be non-zero. This is because the PNGBs are degenerate with the vacuum and their interactions must consequently vanish in the limit of zero momenta [26]. (This is only true if the chiral symmetry is considered to be exact, which it is in this case since the quark mass is introduced as a perturbation). Instead of writing out each power it is more elegant and useful to consider the field $U(x)$ which collects all the PNGBs and all their powers by utilizing the Taylor expansion of the exponential.

$$U(x) \equiv \exp\left(\frac{i\sqrt{2}}{f}\Pi\right) \qquad \Pi \equiv \sum_a \lambda_a \pi_a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} \tag{2.33}$$

where f is a constant related to the pion decay and π_a are the PNGB fields [13]. The Lagrangian is then built using U, derivatives, and mass matrices and the terms are ordered using the same scheme.

However, since the broken chiral symmetry must be turned into a local symmetry, as explained above, one needs to introduce a covariant derivative and associated gauge fields which will be the external fields ν_μ and a_μ. Since U transforms as $U \to V_R U V_L^\dagger$ one can construct the following covariant derivative [19]

$$\nabla_\mu U = \partial_\mu - i(\nu_\mu + a_\mu)U + iU(\nu_\mu - a_\mu) \tag{2.34}$$

By also including the spurions s and p the leading order Lagrangian satisfying local broken chiral symmetry is (see [19][24][27])

$$\mathcal{L}^{(2)}_{\text{ChPT}} = \frac{f^2}{4}\,\text{Tr}\left(\nabla_\mu U^\dagger \nabla^\mu U\right) + \frac{B_0 f^2}{2}\,\text{Tr}\left[(s - ip')U + (s + ip')U^\dagger\right] \tag{2.35}$$

where B_0 is a constant which can be determined from the quark condensate expectation value $\langle \bar{q}q \rangle \approx -f^2 B_0$ [13]. The superscript "(2)" refers to that the leading order is of order 2 in momentum. The s_G spurion is not yet present but can be added to $\mathcal{L}^{(2)}_{\text{ChPT}}$ in a consistent manner. The specific form of the term is chosen such that the QCD energy-momentum tensor is reproduced at this order of the chiral effective theory [13]. Including this term and expanding $\mathcal{L}^{(2)}_{\text{ChPT}}$ while only keeping terms linear in the spurions gives [13]

$$\begin{aligned}
\mathcal{L}_{\chi,\text{ChPT}} = &- \frac{if^2}{2}\,\text{Tr}\left[\left(U\partial_\mu U^\dagger + U^\dagger \partial_\mu U\right)\nu^\mu_\chi + \left(U\partial_\mu U^\dagger - U^\dagger \partial_\mu U\right)a^\mu\right] \\
&+ \frac{B_0 f^2}{2}\,\text{Tr}\left[s(U + U^\dagger) - ip(U - U^\dagger) - \frac{i\theta}{\text{Tr}\left(\mathcal{M}_q^{-1}\right)}(U - U^\dagger)\right] \\
&- \frac{if^2}{4}\frac{\partial^\mu \theta}{\text{Tr}\left(\mathcal{M}_q^{-1}\right)}\,\text{Tr}\left[\left(U\partial_\mu U^\dagger - U^\dagger \partial_\mu U\right)\mathcal{M}_q^{-1}\right] \\
&+ s_G\left(\frac{f^2}{6}\,\text{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{2B_0 f^2}{27}\,\text{Tr}\left[\mathcal{M}_q(U + U^\dagger)\right]\right)
\end{aligned} \tag{2.36}$$

Equation (2.36) is a low-energy expression for $\mathcal{L}$ given in equation (2.24). The expressions contracted to the spurions will be referred to as hadronized quark/gluon currents. “Hadronized” refers to that they are in terms of mesons in $\mathcal{L}_{\chi,\mathrm{ChPT}}$, while the corresponding quark/gluon currents in $\mathcal{L}$ are in terms of quarks and gluons. This reformulation will be referred to as hadronization of the quark/gluon current. Next let’s introduce the theory for how the quark/gluon currents are hadronized to nucleons.

2.4.2 Heavy baryon ChPT

After having established the theory for PNGB the next step is to add nucleons (or more generally baryons) to the theory. The large mass of the nucleons do however pose a problem since the fraction $Q/\Lambda_{\mathrm{ChEFT}}$, where Q now refers to baryon mass and momentum, no longer is small. This complication is handled by acknowledging that the baryons are much more massive than the PNGBs and consequently can be treated as static, taking inspiration from HQET [28]. The chiral theory for baryons is thus aptly named heavy baryon ChPT or HBChPT. The methodology is similar to ChPT and is explained in [13][28]. The result is in essence similar to equation (2.36) with the main difference being that the spurions now contracts to quark/gluon currents which instead have been hadronized to nucleons.

2.5 Scattering

After having described the more abstract parts of the theoretical framework it is now time to introduce how to link this to DM scattering.

2.5.1 Interaction Lagrangian

The next step is to construct the interaction Lagrangian $\mathcal{L}_\chi$ which describes how QCD and DM interact in the most general way. Since there are infinitely many terms in $\mathcal{L}_\chi$ one must find a way to order the terms based on their importance analogously to the situation described before in the context of QCD. In the case of $\mathcal{L}_\chi$ the ordering is achieved by considering a mediator particle with mass m_G which interacts with both DM and QCD [13]. In the low energy limit the propagator associated with this mediator particle will simply be described by the inverse mass of the mediator particle. Thus any interaction between DM and QCD can in the limit small momentum transfer q (small referring to $q^2 \ll m_G^2$) be described by some operator consisting only of DM and QCD weighted by some power of inverse mediator mass. Specifically

$$\mathcal{L}_\chi = \sum_{n,d,q} \hat{C}^{(d)}_{n,q} \mathcal{Q}^{(d)}_{n,q} \qquad \hat{C}^{(d)}_{n,q} = \frac{C^{(d)}_{n,q}}{m_G^{d-4}} \tag{2.37}$$

where $\mathcal{Q}^{(d)}_{n,q}$ are interaction operators of mass dimension d associated to the quark flavor q enumerated by n with coupling constants $\hat{C}^{(d)}_{n,q}$ [13]. By then assuming that m_G is much larger than the momentum of the DM as well as the energy scale of QCD the terms in $\mathcal{L}_\chi$ can be ordered in importance based on their mass dimension.

This then leads to the formulation of the relativistic EFT for DM-quark/gluon interactions, which was referenced in the introduction. All the leading order interaction operators are calculated and are given in the results section 4.1.1.

2.5.2 Interaction Lagrangian and spurions

Having constructed the interaction Lagrangian $\mathcal{L}_\chi$ it is then straight forward to determine the spurions by simply cross referencing to the general QCD-DM Lagrangian $\mathcal{L}$ in equation (2.24). Based on the structure of the interaction Lagrangian in equation (2.24) it is natural to define the following quark/gluon currents

$$J_q^S = \bar{q}q \qquad J_q^P = \bar{q}i\gamma^5 q \tag{2.38}$$

$$J_q^{V,\mu} = \bar{q}\gamma^\mu q \qquad J_q^{A,\mu} = \bar{q}\gamma^\mu\gamma^5 q \tag{2.39}$$

$$J^G = \frac{\alpha_s}{12\pi}\mathcal{G}^{a\mu\nu}\mathcal{G}^a_{\mu\nu} \qquad J^\theta = \frac{\alpha_s}{8\pi}\mathcal{G}^{a\mu\nu}\widetilde{\mathcal{G}}^a_{\mu\nu} \tag{2.40}$$

In each of the interaction operators in $\mathcal{L}_\chi$ it is possible to find one of the quark/gluon currents. The remaining part of the operator is a part of the corresponding spurion associated to that quark/gluon current. By using the upcoming results in section 4.1.1 one can see for example that the psuedo-scalar spurion p is simply given, to this order of mass dimension and order of inverse DM mass, by $p = \mathcal{M}_q\bar{C}_2^{(6)}\chi_\mu^*\chi^\mu$, where $\bar{C}_n^{(d)} = \mathrm{diag}(\hat{C}_{n,u}^{(d)}, \hat{C}_{n,d}^{(d)}, \hat{C}_{n,s}^{(d)})$ is used.

In addition to working with spurions, which are flavor matrices, it is beneficial to simultaneously think of $\mathcal{L}_\chi$ in terms of quark/gluon currents and DM currents J_χ (i.e. vectors) as done in equation (2.41). The usage of DM currents and spurions will be somewhat concurrent.

$$\mathcal{L}_\chi = \sum_{q=u,d,s} J_q^S J_\chi^S + J_q^P J_\chi^P + J_q^V \cdot J_\chi^V + \dots \tag{2.41}$$

where for example (once again using the upcoming results from section 4.1.1)

$$\sum_q J_q^P J_\chi^P = \sum_q m_q \bar{q}i\gamma^5 q\left(\hat{C}_{2,q}^{(6)}\chi_\mu^*\chi^\mu\right) + \dots \tag{2.42}$$

The ellipsis refers to higher order terms. It is useful to factor out the quark/gluon currents since they will be hadronized to either nucleons or mesons depending on the specific type of scattering that occurs. This will be described further in the next section. Quark/gluon currents hadronized to mesons will be denoted by "hat", like for example $\hat{J}_q^S$, whereas hadronization to nucleons will be denoted by a tilde, like for example $\tilde{J}_q^S$. The expression for the mesons hadronized currents are obtained by studying $\mathcal{L}_{\chi,\text{ChPT}}$ in equation (2.36) and comparing the terms to the ones in $\mathcal{L}_\chi$. So for example, the expression multiplied by the scalar spurion s in $\mathcal{L}_{\chi,\text{ChPT}}$ is $\hat{J}_q^S$ and the expression contracted to ν^μ is $\hat{J}_{q\mu}^V$ etc. As a concrete example consider

$$J^V_{q,\mu} = \bar{q}\gamma_\mu q \xrightarrow[\text{hadronization}]{\text{Meson}} \frac{-if^2}{2}\,\text{Tr}\left(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U\right) = \hat{J}^V_{q,\mu} \tag{2.43}$$

Obtaining expressions for the nucleon hadronized currents is in essence achieved by using the same methodology i.e. comparing $\mathcal{L}_\chi$ to the heavy baryon ChPT Lagrangian $\mathcal{L}_{\chi,\text{HBChPT}}$. Compared to the meson hadronization this is however more complicated and the reader is referred to [13] for a detailed description.

2.5.3 Feynman diagrams

At leading order there are two Feynman diagrams describing the scattering of DM with a nucleus [13]. These are given in figure 2.1.

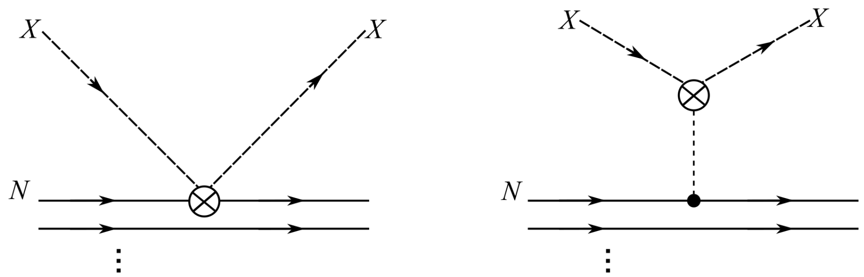

Figure 2.1: The leading order diagrams describing the scattering of DM with a nucleus. The dashed lines with arrows denote spin 1 DM, the solid lines with arrows denote nucleons, and the dashed line denote mesons. The ellipsis represent additional nucleons potentially present in the nucleus. The effective DM-nucleon and DM-meson interaction is denoted by a crossed circle whereas the meson-nucleon interaction is denoted by a dot.

The crossed circle in the left diagram of figure 2.1 denote the effective interaction between DM and nucelons. Thus when calculating the amplitudes generated from the left diagram the quark/gluon currents must be hadronized to nucleons, and when using the right diagram the currents must be hadronized to mesons. Only specific terms in the interaction Lagrangian $\mathcal{L}_\chi$ give contributions to the two diagrams in figure 2.1 at leading order. Specifically, the left diagram get leading order contributions from terms involving J_q^S, J_q^A, J^G, and J^θ. The right diagram get leading order contributions from terms involving J^P, J^A, and J^θ [13]. The right diagram is especially interesting as it has not been studied before in the case of spin 1 DM. One can also consider diagrams which involve multiple nucleons and/or multiple meson exchanges, but these are next to leading order (NLO) corrections [13] and will be neglected in this work. The scattering amplitudes are calculated in appendix C.

2.5.4 Momentum and constraints

The momenta involved in the scattering is, for both diagrams in figure 2.1, described by figure 2.2

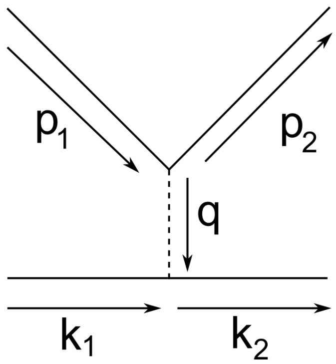

Figure 2.2: Figure showcasing the momenta involved in the DM-nucleon scattering. p_1 and p_2 denote the momentum of the incoming and outgoing DM. k_1 and k_2 denote the momentum of the incoming and outgoing nucleon. q is the momentum transferred to the nucleon in the scattering.

In figure 2.1 there are seemingly 4 independent momenta: p_1^μ, p_2^μ, k_1^μ, and k_2^μ. The system is however constrained by both frame independence and momentum conservation which means that there are only two momenta which are linearly independent. It this stage it is important to note that the HDMET operators in section 4.1.2, and consequently also the amplitudes in appendix C, are non-relativistic even though they are written in Lorentz covariant notation. The appropriate symmetry is therefore actually Galilean symmetry which means that the scattering amplitudes will be in terms of objects which obey Galilean symmetry. The Galilean invariant vectors q^μ and $v_\perp^\mu$ are often chosen as the two independent basis vectors [29]. $v_\perp^\mu$ is defined as

$$v_\perp^\mu = \frac{1}{2}\left(\frac{\tilde{p}_1^\mu + \tilde{p}_2^\mu}{m_X} - \frac{\tilde{k}_1^\mu + \tilde{k}_2^\mu}{m_N}\right) \tag{2.44}$$

where m_N is the nucleon mass. However, due to the fact that the underlying theory is Lorentz invariant there will be terms in the scattering amplitudes which break Galilean invariance. These symmetry breaking terms only start to show up at NLO in momentum [14]. Since non-relativstic dynamics is of main interest, only terms with leading order in momentum will be considered in each amplitude which means that the final result will obey Galilean symmetry.

2.5.5 Coherent enhancement

Since scattering against nuclei is considered the effect of coherent enhancement can be important. Coherent enhancement is a phenomenon that occurs when the (de

Broglie) wavelength of an incoming particle is of the same order of magnitude as the target nuclei it scatters against. Since the position of the incoming particle is "smeared" over a large region it interacts with several nucleons simultaneously [30]. If the interaction is the same with each of the nucleons the contributions add up and lead to the cross section scaling as A^2 in the long wavelength limit, where A is the number of nucleons in the nucleus. If the interaction for example is spin-dependent then the interaction is not the same with each nucleon and contributions no longer add up, thus negating the enhancement [29].

2.5.6 Non-relativistic effective Lagrangian

After having calculated the scattering amplitudes it is useful to write the result in terms of an effective scattering Lagrangian $\mathcal{L}_{\text{eff}}$

$$\mathcal{L}_{\text{eff}} = \sum_i \left(c_{i,p}^{\text{NR}}(q^2)\, \mathcal{O}_{i,p} + c_{i,n}^{\text{NR}}(q^2)\, \mathcal{O}_{i,n} \right) \tag{2.45}$$

Here $\mathcal{O}_i$ are non-relativistic (NR) operators enumerated by i, which together form a basis that spans all non-relativistic DM-nculeon interactions. The basis operators are weighted by the coefficients c_i. The two terms in equation (2.45) corresponds to DM-proton scattering and DM-neutron scattering respectively. The general $\mathcal{O}_i$ up to order q^2 have been formulated in [17][15] and are given below. There is a relative sign difference in the definition of $\boldsymbol{q}$ in this work as compared to the definition used in [17][15]. Letters in bold refer to spatial 3-vectors. The operators below have been written using the definition of $\boldsymbol{q}$ used in this work.

$$\mathcal{O}_{1,N} = 1_X 1_N \qquad \mathcal{O}_{2,N} = \left(v_\perp\right)^2 1_X 1_N \tag{2.46}$$

$$\mathcal{O}_{3,N} = 1_X\, \boldsymbol{S}_N \cdot \left(\boldsymbol{v}_\perp \times \frac{i\boldsymbol{q}}{m_N}\right) \qquad \mathcal{O}_{4,N} = \boldsymbol{S}_X \cdot \boldsymbol{S}_N \tag{2.47}$$

$$\mathcal{O}_{5,N} = \boldsymbol{S}_X \cdot \left(\boldsymbol{v}_\perp \times \frac{i\boldsymbol{q}}{m_N}\right) 1_N \qquad \mathcal{O}_{6,N} = \left(\boldsymbol{S}_X \cdot \frac{\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N \cdot \frac{\boldsymbol{q}}{m_N}\right) \tag{2.48}$$

$$\mathcal{O}_{7,N} = 1_X \left(\boldsymbol{S}_N \cdot \boldsymbol{v}_\perp\right) \qquad \mathcal{O}_{8,N} = \left(\boldsymbol{S}_X \cdot \boldsymbol{v}_\perp\right) 1_N \tag{2.49}$$

$$\mathcal{O}_{9,N} = \boldsymbol{S}_X \cdot \left(\frac{i\boldsymbol{q}}{m_N} \times \boldsymbol{S}_N\right) \qquad \mathcal{O}_{10,N} = -1_X \left(\boldsymbol{S}_N \cdot \frac{i\boldsymbol{q}}{m_N}\right) \tag{2.50}$$

$$\mathcal{O}_{11,N} = -\left(\boldsymbol{S}_X \cdot \frac{i\boldsymbol{q}}{m_N}\right) 1_N \qquad \mathcal{O}_{12,N} = \boldsymbol{S}_X \cdot \left(\boldsymbol{S}_N \times \boldsymbol{v}_\perp\right) \tag{2.51}$$

$$\mathcal{O}_{13,N} = -\left(\boldsymbol{S}_X \cdot \boldsymbol{v}_\perp\right)\left(\boldsymbol{S}_N \cdot \frac{i\boldsymbol{q}}{m_N}\right) \qquad \mathcal{O}_{14,N} = -\left(\boldsymbol{S}_X \cdot \frac{i\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N \cdot \boldsymbol{v}_\perp\right) \tag{2.52}$$

$$\mathcal{O}_{15,N} = -\left(\boldsymbol{S}_X \cdot \frac{\boldsymbol{q}}{m_N}\right)\left[\left(\boldsymbol{S}_N \times \boldsymbol{v}^\perp\right) \cdot \frac{\boldsymbol{q}}{m_N}\right] \tag{2.53}$$

The basis operators which are unique to spin 1 DM are

$$\mathcal{O}_{17,N} = i\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{v}_\perp 1_N \qquad \mathcal{O}_{18,N} = i\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{S}_N \tag{2.54}$$

$$\mathcal{O}_{19,N} = \frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N} \qquad \mathcal{O}_{20,N} = \left(\boldsymbol{S}_N \times \frac{\boldsymbol{q}}{m_N}\right)\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N} \tag{2.55}$$

with $N = p, n$. Further, $\boldsymbol{\mathcal{S}}_{ij} = \frac{1}{2}(e'_i e_j + e_i e'_j)$ where e' and e is the polarization vector of the outgoing and incoming DM respectively.

The benefit of writing the result in terms of this non-relativistic effective Lagrangian is two-fold. Firstly, as described in the introduction a primary goal of this work is to study the constraints that are imposed on the non-relativistic theory from the relativistic theory. Writing the result in terms of non-relativistic objects will naturally make this easier. Secondly, it will allow for a more streamlined computation of observables since there already exists a Mathematica package [17] that utilizes the above basis operators to compute observables related to DM direct detection.

3

Computational framework

After having formulated the effective scattering Lagrangian $\mathcal{L}_{\text{eff}}$ the last step is to calculate measurable quantities, which in this work will be how often a nucleus is scattered, with some recoil energy, by DM. More specifically the quantity of interest is the total event rate $\mathcal{R}$ which describes the expected number of DM-nucleus scattering events per unit recoil energy per unit detector mass per unit time and is obtained by

$$\frac{d\mathcal{R}}{dE_R} = \frac{\rho_X}{m_A m_X} \int_{v_{\min}} \frac{d\sigma}{dE_R} v f_E(\vec{v}) d^3 v \tag{3.1}$$

where m_A is the mass of the nucleus, ρ_X is the local density of DM (i.e. density of DM in the vicinity of the Earth), $d\sigma/dE_R$ is the differential cross section, and f_E is the DM velocity distribution relative the Earth's frame [14][17]. $v_{\min}$ refers to the minimum velocity required to induce a recoil energy E_R. The recoil energy E_R refers to the energy transferred to the nucleus in the scattering with DM and is related to the momentum transfer q through $E_R = q^2/2m_A$ [17]. The local DM density is approximately $\rho_X \approx 0.4$ GeV/cm^{-3} [31]. The velocity distribution is obtained by utilizing the standard halo model which states that the velocity distribution should be a Maxwell-Boltzmann distribution where the average value is set to the local orbital velocity relative to the galactic center ($v_0 = 220$ km/s) [2]. In order to account for the escape velocity of the galaxy the distribution is truncated at $v_{\text{esc}} = 544$ km/s [2].

It is important to point out that all scattering amplitudes that have been calculated describe the scattering between DM and nucleons. Additional steps are thus needed to describe the scattering between DM and nuclei. As previously mentioned this step will be automated through the use of the Mathematica package written by Anand et al. [17], which is built around $\mathcal{L}_{\text{eff}}$ and the associated non-relativistic basis operators in equations (2.46)-(2.55). One simply needs to input the coefficients associated to the basis operators, the DM density, and the assumed velocity distribution of the DM. One also need to specify which type of nucleus that the DM scatters against. This is important since the mass of the nucleus will greatly influence the predicted total event rate. This will be shown explicitly in the results section. The choice of nucleus also alter the DM-nucleus interaction in more subtle ways since intricacies of the nuclear structure can influence how the nucleus responds to the scattering. This is modeled through so called nuclear response functions that partly depends on density matrices which encapsulates the nuclear physics of each nucleus. This is explained in more detail here [17].

Another reason for why the choice of nucleus is important is due to its strong connection to experiments. As previously mentioned, the detectors that are used in the direct detection experiments considered in this work can, if one greatly simplifies, be represented by containers filled with homogeneous nuclei. One can thus rather easily make predictions about what a specific detector theoretically should detect by simply choosing the nucleus used in that detector. Another aspect to keep in my mind however is that each detector have a limited range of recoil energies that it can detect. Where relevant, the two detectors that will be used as examples are LUX and PICO. LUX uses xenon and has the following sensitivity region $E_R \in [3, 50]$ keV. PICO uses chlorofluorocarbon molecules (Freon) which contain carbon, chlorine, and fluorine atoms. As done in [14], these molecules will be approximated by only considering the contribution of fluorine. The sensitivity region of PICO is $E_R > 3.3$ keV [14]. This choice of detectors makes it possible to clearly showcase how the mass of the nucleus influences the behavior. It also allows for a more direct comparison to the results obtained in work similar to this, where spin 0 and spin 1/2 DM were investigated, since they considered the same detectors [14]. XENON1T is a more modern experiment as compared to LUX. However, since XENON1T also uses xenon, all the xenon related results in this work are equally as applicable to XENON1T as they are to LUX. The sensitivity region of XENON1T is also very similar to that of LUX $E_R \in [4, 50]$ [32]. LUX is just used in order to establish a more direct comparison to the work of Bishara et al. [14].

It is important to note that ^{19}F is the only stable isotope of fluorine and is consequently the species that will be used in the analysis. Xenon on the other hand has eight stable isotopes. Due to this, scattering involving xenon will be calculated for each isotope and then be combined using their natural abundance as weights.

In order to properly analyze spin 1 DM the Mathematica package needs to be slightly modified to include support for the additional non-relativistic operators that are unique to spin 1 DM (these are given in equations (2.54) and (2.55)). This is done by adding the additional terms listed for the DM response functions in [15] to the DM response functions in the source code for the Mathematica package. This is described more closely in appendix E.

The analysis in the results chapter in general revolves around setting all but one of the coupling constants $\hat{C}_n$ to zero in order to isolate specific behaviors and/or to showcase specific comparisons. Two types of plots will be used to visualize the results. In the first type is the DM mass fixed to some value and $d\mathcal{R}/dE_R$ is plotted against E_R. Different graphs of $d\mathcal{R}/dE_R$ will be superimposed where the scattering cross section have been calculated using different sets of scattering amplitudes that carry the same $\hat{C}_n$. Further, in order to make it more clear if potential differences are experimentally interesting, the E_R-sensitivity bands for LUX and PICO will be visualized together with the graphs. The magnitude of the graphs are normalized. This is not an issue since it is only the behavior with respect to E_R and the relative difference compared to other graphs that are of interest in this work.

The second type plot visualizes how a change in DM mass influences $d\mathcal{R}/dE_R$. In this case an example is in order. Imagine that the relative importance of two scattering amplitudes A_1 and A_2 is sought after. Let's also assume that these two amplitudes stem from the same interaction operator and thus carry the same coupling constant $\hat{C}$. $d\mathcal{R}/dE_R$ can be calculated including just the contribution from A_1 or by including the contribution from both amplitudes. Generally one can summarize the result as

$$\frac{d\mathcal{R}}{dE_R}(A_1) = \hat{C}^2 f(q, m_X) \qquad \frac{d\mathcal{R}}{dE_R}(A_1, A_2) = \hat{C}^2 g(q, m_X) \tag{3.2}$$

where f, g are general functions. One can then impose a bound b, which is the same for both rates, that represents how large the rates can be while still avoiding experimental detection. Both sides of the equations are then integrated with respect to q using the sensitivity span set by the relevant experiment (so in the case of LUX $q \in [3, 50]$ keV). This then gives two functions of the DM mass for the coupling constant that describes how strong the coupling can be and still avoid detection given some bound b.

$$\hat{C}^2_{\mathcal{O}_4} = \frac{b \int dq}{\int dq\ f(q, m_X)} \qquad \hat{C}^2_{\mathcal{O}_4, \mathcal{O}_6} = \frac{b \int dq}{\int dq\ g(q, m_X)} \tag{3.3}$$

Dividing the two functions and studying the fraction is helpful as it removes the need of specifying b.

$$\frac{\mathcal{O}_4}{\mathcal{O}_4 \text{ and } \mathcal{O}_6} \equiv \frac{\hat{C}^2_{\mathcal{O}_4}}{\hat{C}^2_{\mathcal{O}_4, \mathcal{O}_6}} = \frac{\int dq\ g(q, m_X)}{\int dq\ f(q, m_X)} \tag{3.4}$$

Studying this fraction as a function of the DM mass is very useful as it allows one to evaluate the importance of including the additional amplitude A_2. If the ratio is 1 then the required value for the coupling constant obtained from just using A_1 is the same as the value obtained from using both A_1 and A_2. Thus if the ratio is 1 then the contribution of A_2 is zero and is not important. As the ratio moves further away from 1 the importance of A_2 increases. As will be seen later in the results section, this type of analysis can be used to compare a non-relativistic EFT approach (where one amplitude A_i at the time is considered) to a relativistic EFT approach which predicts that A_i can be related to some other amplitude A_j such that the two amplitudes must be analyzed together.

4

Results and discussion

This section will be split in to two parts. The first part will consist of theoretical results on the cross section for DM-nucleus scattering in a relativistic EFT for spin-1 DM, quarks and gluons. The second part will focus on the numerical evaluation and physical interpretation of such theoretical predictions.

4.1 Theoretical results

A first important theoretical result of this thesis is the systematic classification of DM-quark/gluon interactions for spin-1 DM (section 4.1.1). They define the relativistic EFT for DM-quark/gluon interactions used in this work for the calculation of DM-nucleus scattering rates. A second important result, is the heavy field expansion (HDMET) of such interaction operators (section 4.1.2). By using this expansion, the amplitude for DM-nucleon scattering is computed for each relativistic operator considered in this work, and the result is then matched onto a non-relativistic effective scattering Lagrangian (section 4.1.3). The latter is the starting point for the numerical results presented in the next section.

4.1.1 Interaction operators

In this subsection, all the leading order (i.e. of lowest mass dimension) spin-1 DM-quark and -gluon interaction operators, that are identified in this thesis, are listed. All interaction operators must be Hermetian, Lorentz invariant and CPT invariant. Leveraging on the work of [13][15][33], the leading order (mass dimension 6) operators are found to be

$$\mathcal{Q}_{1,q}^{(6)} = m_q \bar{q} q X_\mu^* X^\mu \qquad \mathcal{Q}_{2,q}^{(6)} = i m_q \bar{q}\gamma^5 q X_\mu^* X^\mu \tag{4.1}$$

Type a

$$\mathcal{Q}_{3,q}^{(6)} = i\bar{q}\gamma^\mu q X_\nu^* \overset{\leftrightarrow}{\partial}_\mu X^\nu \qquad \mathcal{Q}_{4,q}^{(6)} = i\bar{q}\gamma^\mu\gamma^5 q X_\nu^* \overset{\leftrightarrow}{\partial}_\mu X^\nu \tag{4.2}$$

Type b

$$\mathcal{Q}_{5,q}^{(6)} = \bar{q}\gamma_\mu q X_\nu^* \overset{\leftrightarrow}{\partial}_\rho X_\lambda \epsilon^{\mu\nu\rho\lambda} \qquad \mathcal{Q}_{6,q}^{(6)} = \bar{q}\gamma_\mu\gamma^5 q X_\nu^* \overset{\leftrightarrow}{\partial}_\rho X_\lambda \epsilon^{\mu\nu\rho\lambda} \tag{4.3}$$

Type c

$$\mathcal{Q}^{(6)}_{7,q} = i\bar{q}\gamma_\mu q \partial_\rho (X^*_\nu X_\lambda)\epsilon^{\mu\nu\rho\lambda} \qquad \mathcal{Q}^{(6)}_{8,q} = i\bar{q}\gamma_\mu \gamma^5 q \partial_\rho (X^*_\nu X_\lambda)\epsilon^{\mu\nu\rho\lambda} \tag{4.4}$$

Type d

$$\mathcal{Q}^{(6)}_{9,q} = \bar{q}\gamma^\mu q \left[X^*_\nu \partial^\nu X_\mu + \text{c.c}\right] \qquad \mathcal{Q}^{(6)}_{10,q} = \bar{q}\gamma_\mu \gamma^5 q \left[X^*_\nu \partial^\nu X_\mu + \text{c.c}\right] \tag{4.5}$$

Type e

$$\mathcal{Q}^{(6)}_{11,q} = \bar{q}\gamma^\mu q \left[iX^*_\nu \partial^\nu X_\mu + \text{c.c}\right] \qquad \mathcal{Q}^{(6)}_{12,q} = \bar{q}\gamma_\mu \gamma^5 q \left[iX^*_\nu \partial^\nu X_\mu + \text{c.c}\right] \tag{4.6}$$

At this order there are two operators involving gluons. One consists of ordinary gluon field strength tensors whereas the other involves the dual gluon field strength tensor $\widetilde{\mathcal{G}}_{\mu\nu} = \frac{1}{2}\mathcal{G}^{\rho\lambda}\epsilon_{\mu\nu\rho\lambda}$.

$$\mathcal{Q}^{(6)}_{1} = \frac{\alpha_s}{12\pi}\mathcal{G}^{a\mu\nu}\mathcal{G}^a_{\mu\nu}X^*_\rho X^\rho \qquad \mathcal{Q}^{(6)}_{2} = \frac{\alpha_s}{8\pi}\mathcal{G}^{a\mu\nu}\widetilde{\mathcal{G}}^a_{\mu\nu}X^*_\rho X^\rho \tag{4.7}$$

The following notation were and will be used a lot in the remaining part of this work: $X^*_\mu \overleftrightarrow{\partial}_\nu X_\rho = X^*_\mu \partial_\nu X_\rho - \partial_\nu X^*_\mu X_\rho$. Operators containing $\partial_\mu X^\mu$ are zero due to the Proca equation. It is important to note that $\partial_\mu \chi^\mu \neq 0$ since χ^μ is not a massive vector field and consequently does not obey the Proca equation. The operators involving quarks now also carry the index $q = u, d, s$ to reflect the fact that the interaction can occur with either a up, down, or strange quark.

The above interaction operators must be converted using the framework of HDMET in order to reflect the assumed non-relativistic nature of DM. This is done in appendix B and the results of this expansion are presented in section 4.1.2. The expansion is done to next to leading order which means that

$$X^\mu \approx e^{-im_X v\cdot x}\left(\chi^\mu - \frac{iv^\mu}{m_X}\partial_\rho \chi^\rho\right) \tag{4.8}$$

4.1.2 HDMET interaction operators

The interaction operators expanded to NLO using HDMET are

$$\mathcal{Q}^{(6)}_{1,q} = m_q \bar{q} q \chi^*_\mu \chi^\mu \qquad \mathcal{Q}^{(6)}_{2,q} = i m_q \bar{q} \gamma^5 q \chi^*_\mu \chi^\mu \tag{4.9}$$

4.1.2.1 Type a

$$\begin{aligned}\mathcal{Q}^{(6)}_{3,q} =& 2m_X \bar{q}\gamma^\mu q v_\mu \chi^*_\nu \chi^\nu \\ &+ i\bar{q}\gamma^\mu q \chi^*_\nu \overleftrightarrow{\partial}_\mu \chi^\nu \\ &+ \frac{2}{m_X}\bar{q}\gamma^\mu q v_\mu \partial^\rho \chi^*_\rho \partial_\lambda \chi^\lambda\end{aligned} \qquad \begin{aligned}\mathcal{Q}^{(6)}_{4,q} =& 2m_X \bar{q}\gamma^\mu \gamma^5 q v_\mu \chi^*_\nu \chi^\nu \\ &+ i\bar{q}\gamma^\mu \gamma^5 q \chi^*_\nu \overleftrightarrow{\partial}_\mu \chi^\nu \\ &+ \frac{2}{m_X}\bar{q}\gamma^\mu \gamma^5 q v_\mu \partial^\rho \chi^*_\rho \partial_\lambda \chi^\lambda\end{aligned} \tag{4.10}$$

4.1.2.2 Type b

$$\begin{aligned}\mathcal{Q}^{(6)}_{5,q} =& -2m_X i\bar{q}\gamma_\mu q v_\rho \chi^*_\nu \chi_\lambda \epsilon^{\mu\nu\rho\lambda} \\ &+ \bar{q}\gamma_\mu q \chi^*_\nu \overleftrightarrow{\partial}_\rho \chi_\lambda \epsilon^{\mu\nu\rho\lambda} \\ &+ \frac{1}{m_X}\bar{q}\gamma_\mu q v_\nu \left(i\chi^*_\lambda \overleftrightarrow{\partial}_\rho \partial_\sigma \chi^\sigma + \text{c.c}\right)\epsilon^{\mu\nu\rho\lambda}\end{aligned} \qquad \begin{aligned}\mathcal{Q}^{(6)}_{6,q} =& -2im_X \bar{q}\gamma_\mu \gamma^5 q v_\rho \chi^*_\nu \chi_\lambda \epsilon^{\mu\nu\rho\lambda} \\ &+ \bar{q}\gamma_\mu \gamma^5 q \chi^*_\nu \overleftrightarrow{\partial}_\rho \chi_\lambda \epsilon^{\mu\nu\rho\lambda} \\ &+ \frac{1}{m_X}\bar{q}\gamma_\mu \gamma^5 q v_\nu \left(i\chi^*_\lambda \overleftrightarrow{\partial}_\rho \partial_\sigma \chi^\sigma + \text{c.c}\right)\epsilon^{\mu\nu\rho\lambda}\end{aligned} \tag{4.11}$$

4.1.2.3 Type c

$$\begin{aligned}\mathcal{Q}^{(6)}_{7,q} =& i\bar{q}\gamma_\mu q \partial_\rho(\chi^*_\nu \chi_\lambda)\epsilon^{\mu\nu\rho\lambda} \\ &- \frac{1}{m_X}\bar{q}\gamma_\mu q v_\nu [\partial_\rho(\chi^*_\lambda \partial_\sigma \chi^\sigma) + \text{c.c}]\epsilon^{\mu\nu\rho\lambda}\end{aligned} \qquad \begin{aligned}\mathcal{Q}^{(6)}_{8,q} =& i\bar{q}\gamma_\mu \gamma^5 q \partial_\rho(\chi^*_\nu \chi_\lambda)\epsilon^{\mu\nu\rho\lambda} \\ &- \frac{1}{m_X}\bar{q}\gamma_\mu \gamma^5 q v_\nu [\partial_\rho(\chi^*_\lambda \partial_\sigma \chi^\sigma) + \text{c.c}]\epsilon^{\mu\nu\rho\lambda}\end{aligned} \tag{4.12}$$

4.1.2.4 Type d

$$\begin{aligned}\mathcal{Q}^{(6)}_{9,q} =& \bar{q}\gamma^\mu q[\partial^\nu(\chi^*_\nu \chi_\mu) + \text{c.c}] \\ &- \frac{1}{m_X}\bar{q}\gamma^\mu q v_\mu [i\partial^\nu(\chi^*_\nu \partial_\lambda \chi^\lambda) + \text{c.c}] \\ &+ \frac{1}{m_X}\bar{q}\gamma^\mu q [i\partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \text{c.c}]\end{aligned} \qquad \begin{aligned}\mathcal{Q}^{(6)}_{10,q} =& \bar{q}\gamma^\mu \gamma^5 q[\partial^\nu(\chi^*_\nu \chi_\mu) + \text{c.c}] \\ &- \frac{1}{m_X}\bar{q}\gamma^\mu \gamma^5 q v_\mu [i\partial^\nu(\chi^*_\nu \partial_\lambda \chi^\lambda) + \text{c.c}] \\ &+ \frac{1}{m_X}\bar{q}\gamma^\mu \gamma^5 q [i\partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \text{c.c}]\end{aligned} \tag{4.13}$$

4.1.2.5 Type e

$$\begin{aligned}\mathcal{Q}^{(6)}_{11,q} =& i\bar{q}\gamma^\mu q[\partial^\nu(\chi^*_\nu \chi_\mu) - \text{c.c}] \\ &+ \frac{1}{m_X}\bar{q}\gamma^\mu q v_\mu [\partial^\nu(\chi^*_\nu \partial_\lambda \chi^\lambda) + \text{c.c}] \\ &- \frac{1}{m_X}\bar{q}\gamma^\mu q [\partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \text{c.c}]\end{aligned} \qquad \begin{aligned}\mathcal{Q}^{(6)}_{12,q} =& i\bar{q}\gamma^\mu \gamma^5 q[\partial^\nu(\chi^*_\nu \chi_\mu) - \text{c.c}] \\ &+ \frac{1}{m_X}\bar{q}\gamma^\mu \gamma^5 q v_\mu [\partial^\nu(\chi^*_\nu \partial_\lambda \chi^\lambda) + \text{c.c}] \\ &- \frac{1}{m_X}\bar{q}\gamma^\mu \gamma^5 q [\partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \text{c.c}]\end{aligned} \tag{4.14}$$

The operators containing gluons become

$$\mathcal{Q}_1^{(6)} = \frac{\alpha_s}{12\pi} \mathcal{G}^{a\mu\nu} \mathcal{G}^a_{\mu\nu} \chi^*_\rho \chi^\rho \qquad\qquad \mathcal{Q}_2^{(6)} = \frac{\alpha_s}{8\pi} \mathcal{G}^{a\mu\nu} \widetilde{\mathcal{G}}^a_{\mu\nu} \chi^*_\rho \chi^\rho \tag{4.15}$$

4.1.3 Matching to the effective scattering Lagrangian

The next step is to calculate the scattering amplitudes by using the two Feynman diagrams in figure 2.1 and utilizing the Feynman rules. The vertices that need to be considered are all the HDMET interaction operators listed in section 4.1.2. Not all vertices are however needed for both diagrams at first order as explained in section 2.5.3. The quark/gluon current of the operators also needs to be hadronized to either nucelons or mesons depending on whether the left or right diagram in figure 2.1 is considered, as explained in section 2.5.3. A detailed calculation of all diagrams using all leading order vertices is provided in appendix C. As mentioned, it is advantageous to summarize the results from the various scattering amplitudes using an effective scattering Lagrangian $\mathcal{L}_{\text{eff}}$. As opposed to appendix C however, the following result is written in a manifestly non-relativistic notation using the conversions in appendix D.

$$\mathcal{L}_{\text{eff}} = \sum_i \left(c^{\text{NR}}_{i,p}(q^2)\, \mathcal{O}_{i,p} + c^{\text{NR}}_{i,n}(q^2)\, \mathcal{O}_{i,n} \right) \tag{4.16}$$

Here $\mathcal{O}_i$ are non-relativistic (NR) operators enumerated by i, which together form a basis which is weighted by the coefficients c_i. The two terms in equation (4.16) corresponds to DM-proton scattering and DM-neutron scattering respectively. The relevant basis operators are (using the same definitions as in [17] and [15])

$$\mathcal{O}_{1,N} = 1_X 1_N \qquad\qquad \mathcal{O}_{4,N} = \boldsymbol{S}_X \cdot \boldsymbol{S}_N \tag{4.17}$$

$$\mathcal{O}_{5,N} = \boldsymbol{S}_X \cdot \left(\boldsymbol{v}_\perp \times \frac{i\boldsymbol{q}}{m_N} \right) 1_N \qquad\qquad \mathcal{O}_{6,N} = \left(\boldsymbol{S}_X \cdot \frac{\boldsymbol{q}}{m_N} \right) \left(\boldsymbol{S}_N \cdot \frac{\boldsymbol{q}}{m_N} \right) \tag{4.18}$$

$$\mathcal{O}_{7,N} = 1_X \left(\boldsymbol{S}_N \cdot \boldsymbol{v}_\perp \right) \qquad\qquad \mathcal{O}_{8,N} = \left(\boldsymbol{S}_X \cdot \boldsymbol{v}_\perp \right) 1_N \tag{4.19}$$

$$\mathcal{O}_{9,N} = \boldsymbol{S}_X \cdot \left(\frac{i\boldsymbol{q}}{m_N} \times \boldsymbol{S}_N \right) \qquad\qquad \mathcal{O}_{10,N} = -1_X \left(\boldsymbol{S}_N \cdot \frac{i\boldsymbol{q}}{m_N} \right) \tag{4.20}$$

$$\mathcal{O}_{11,N} = -\left(\boldsymbol{S}_X \cdot \frac{i\boldsymbol{q}}{m_N} \right) 1_N \qquad\qquad \mathcal{O}_{14,N} = -\left(\boldsymbol{S}_X \cdot \frac{i\boldsymbol{q}}{m_N} \right) \left(\boldsymbol{S}_N \cdot \boldsymbol{v}_\perp \right) \tag{4.21}$$

$$\mathcal{O}_{17,N} = i \frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{\mathcal{S}} \cdot \boldsymbol{v}_\perp 1_N \qquad\qquad \mathcal{O}_{18,N} = i \frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{\mathcal{S}} \cdot \boldsymbol{S}_N \tag{4.22}$$

$$\mathcal{O}_{19,N} = \frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{\mathcal{S}} \cdot \frac{\boldsymbol{q}}{m_N} \qquad\qquad \mathcal{O}_{20,N} = \left(\boldsymbol{S}_N \times \frac{\boldsymbol{q}}{m_N} \right) \cdot \boldsymbol{\mathcal{S}} \cdot \frac{\boldsymbol{q}}{m_N} \tag{4.23}$$

$$\mathcal{O}_N^{(3)} = -\left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{\mathcal{S}} \cdot \frac{\boldsymbol{q}}{m_N} \right) \frac{i\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_N \tag{4.24}$$

Here $N = p, n$, $\boldsymbol{S}_N$ is the nucleon spin operator, and $\boldsymbol{S}_X$ is the DM spin operator. Further, $\boldsymbol{\mathcal{S}}_{ij} = \frac{1}{2}(e'_i e_j + e_i e'_j)$ where e' and e is the polarization vector of the outgoing and incoming DM respectively. All of the scattering amplitudes can be written in terms of these basis operators. Basis operators of orders higher than 2 in q are not defined in [17][15]. In this work there is however necessary to include one order 3 operator $\mathcal{O}_N^{(3)}$. The coefficients corresponding to the basis operators are

$$
\begin{aligned}
c_{1,p}^{\mathrm{NR}} =& -\left(\hat{C}_{1,u}^{(6)}\sigma_u^p + \hat{C}_{1,d}^{(6)}\sigma_d^p + \hat{C}_{1,s}^{(6)}\sigma_s^p\right) - 2m_X(2\hat{C}_{3,u}^6 + \hat{C}_{3,d}^6) + \frac{2m_G}{27}\hat{C}_1^{\prime(6)} \\
c_{4,p}^{\mathrm{NR}} =& -2m_X\hat{\mathcal{A}}_p(\hat{C}_{6,q}^6) + q^2\hat{\mathcal{W}}_p(\hat{C}_{11,q}^6) \\
c_{5,p}^{\mathrm{NR}} =& -m_N(2\hat{C}_{11,u}^6 + \hat{C}_{11,d}^6) \\
c_{6,p}^{\mathrm{NR}} =& -2m_N^2 m_X\hat{\mathcal{B}}_p(\hat{C}_{6,q}^6) - m_N^2\hat{\mathcal{W}}_p(\hat{C}_{11,q}^6) \\
c_{7,p}^{\mathrm{NR}} =& 2m_X\hat{\mathcal{A}}_p(\hat{C}_{4,q}^6) \\
c_{8,p}^{\mathrm{NR}} =& 2m_X(2\hat{C}_{5,u}^6 + \hat{C}_{5,d}^6) \\
c_{9,p}^{\mathrm{NR}} =& -2m_N m_X\hat{\mathcal{W}}_p(\hat{C}_{5,q}^6) - m_N\hat{\mathcal{A}}_p(\hat{C}_{12,q}^6)
\end{aligned}
\tag{4.25}
$$

$$
\begin{aligned}
c_{10,p}^{\mathrm{NR}} =& -m_N B_0\Big[g_a\frac{1}{m_\pi^2 - q^2}\left(\hat{C}_{2,u}^{(6)}m_u - \hat{C}_{2,d}^{(6)}m_d\right) \\
&+ \frac{(\Delta u + \Delta d - 2\Delta s)}{3}\frac{1}{m_\eta^2 - q^2}\left(\hat{C}_{2,u}^{(6)}m_u + \hat{C}_{2,d}^{(6)}m_d - 2\hat{C}_{2,s}^{(6)}m_s\right)\Big] \\
&- m_N\hat{C}_2^{(6,0)}\Big\{-\Big[D\Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_s}\Big) + F\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_s}\Big) + G\Big] \\
&+ \frac{q^2}{2}\Big[g_a\frac{1}{m_\pi^2 - q^2}\left(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\right) + \frac{(\Delta u + \Delta d - 2\Delta s)}{3}\frac{1}{m_\eta^2 - q^2}\left(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\right)\Big]\Big\}
\end{aligned}
\tag{4.26}
$$

$$
c_{11,p}^{\mathrm{NR}} = m_N(2\hat{C}_{7,u}^6 + \hat{C}_{7,d}^6) \qquad c_{14,p}^{\mathrm{NR}} = m_N\hat{\mathcal{A}}_p(\hat{C}_{8,q}^6) \tag{4.27}
$$

$$
c_{17,p}^{\mathrm{NR}} = 2m_N(2\hat{C}_{9,u}^6 + \hat{C}_{9,d}^6) \qquad c_{18,p}^{\mathrm{NR}} = -2m_N\hat{\mathcal{A}}_p(\hat{C}_{10,q}^6) \tag{4.28}
$$

$$
c_{19,p}^{\mathrm{NR}} = -\frac{m_N^2}{m_X}(2\hat{C}_{11,u}^6 + \hat{C}_{11,d}^6) \qquad c_{20,p}^{\mathrm{NR}} = -2m_N^2\hat{\mathcal{W}}_p(\hat{C}_{9,q}^6) - \frac{m_N^2}{m_X}\hat{\mathcal{A}}_p(\hat{C}_{8,q}^6) \tag{4.29}
$$

$$
c_p^{\mathrm{NR}(3)} = 2m_N^3\hat{\mathcal{B}}_p(\hat{C}_{10,q}^6) \tag{4.30}
$$

Coefficients corresponding to DM-neutron interactions are obtained by letting $p \to n, u \to d$. $\hat{\mathcal{A}}_p$, $\hat{\mathcal{W}}_p$, and $\hat{\mathcal{B}}_p$ are defined as

$$
\hat{\mathcal{A}}_p(\hat{C}_{n,q}^d) \equiv 2\Big((D + F + G)\hat{C}_{n,u}^d + G\hat{C}_{n,d}^d + (D - F + G)\hat{C}_{n,s}^d\Big) \tag{4.31}
$$

$$
\hat{\mathcal{W}}_p(\hat{C}_{n,q}^d) = \Big((g_4 - g'_4)\hat{C}_{n,u}^d - g'_4\hat{C}_{n,d}^d - 3\frac{\mu_s}{m_N}\hat{C}_{n,s}^d\Big)\Big) \tag{4.32}
$$

$$\begin{aligned}\hat{\mathcal{B}}_p(\hat{C}^d_{n,q}) \equiv & g_a \frac{1}{q^2 - m_\pi^2}\left(\hat{C}^{(d)}_{n,u} - \hat{C}^{(d)}_{n,d}\right) \\ & \frac{(\Delta u + \Delta d - 2\Delta s)}{3} \frac{1}{q^2 - m_\eta^2}\left(\hat{C}^{(d)}_{n,u} + \hat{C}^{(d)}_{n,d} - 2\hat{C}^{(d)}_{n,s}\right)\end{aligned} \tag{4.33}$$

The numerical values and physical connection for the various constants can be found in appendix C of [13].

4.1.4 Non-relativistic operators

The non-relativistic operators $\mathcal{O}_i$ that span the basis are formulated in [17][15]. These are constructed by considering all the ways that spin 1 DM can interact with nucleons up to order q^2 under the constraint of Galilean symmetry. However, from studying the above result one can see that not all the non-relativistic operators are needed in order to span the scattering amplitudes. (The unused operators are $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_{12}$, $\mathcal{O}_{13}$, and $\mathcal{O}_{15}$) This behavior can also not simply be attributed to the operators being at higher order in q since some operators of order q^2 are retained while some of order q^0 are missing (e.g. $\mathcal{O}_{12}$). This is interesting as it shows that some of the constraints from the relativistic theory carries over when considering the non-relativistic limit. Moreover, this result is very helpful from a strictly practical viewpoint as well since it reduces the number of interaction operators one has to consider when comparing against experimental data, at least to leading orders.

4.2 Computational results

In this section DM-nucleus interactions are analyzed. This is done by utilizing the Mathematica package developed by Anand et al. [17].

4.2.1 Meson exchange

The first result that will be introduced is that the inclusion of mesons, and consequently the meson exchange, can be a critical component in properly describing how DM scatters against nuclei. This is exemplified by studying the interaction operator $\mathcal{Q}^{(6)}_{6,q}$ first introduced in section 4.1.1. The coupling constant associated to this interaction operator is $\hat{C}^{(6)}_{6,q}$. After enforcing the non-relativistic nature of DM through the use of HDMET, calculating the scattering amplitude, and writing in terms of a non-relativistic effective Lagrangian, one can see that $\hat{C}^{(6)}_{6,q}$ shows up in the coefficient of two different non-relativistic basis operators. These two coefficients are $c^{\mathrm{NR}}_{4,N}$ and $c^{\mathrm{NR}}_{6,N}$ (see equation (4.25)). This means that the relativistic operator $\mathcal{Q}^{(6)}_{6,q}$ matches onto the two non-relativistic operators $\mathcal{O}_{4,N}$ and $\mathcal{O}_{6,N}$. This is important since it shows that it is generally dangerous to analyse the behavior of $\mathcal{O}_{4,N}$ on its own as the inclusion of $\mathcal{O}_{4,N}$ necessitates that also $\mathcal{O}_{6,N}$ should be considered since they share the same coupling constant $\hat{C}^{(6)}_{6,q}$. It is also important to note that the term in

$c^{\rm NR}_{6,N}$ that involves $\hat{C}^{(6)}_{6,q}$ stems from an amplitude generated from the diagram that involves a meson exchange (see figure 2.1). In other words, that amplitude would not be present if one were to use a non-relativistic theory which does not include mesons as a DOF. Thus starting from the non-relativistic theory means that one would be completely oblivious to the fact that $\mathcal{O}_{4,N}$ and $\mathcal{O}_{6,N}$ are connected. The question is however how big the correction is when one includes both operators as opposed to just one.

It is at this stage instructive to write out the operators $\mathcal{O}_{4,N}$ and $\mathcal{O}_{6,N}$.

$$\mathcal{O}_{4,N} = \boldsymbol{S}_X \cdot \boldsymbol{S}_N \qquad \mathcal{O}_{6,N} = \left(\boldsymbol{S}_X \cdot \frac{\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N \cdot \frac{\boldsymbol{q}}{m_N}\right) \tag{4.34}$$

Naively one might expect that operator $\mathcal{O}_{6,N}$ could be neglected since it suppressed by two powers in the small momentum $q \ll m_N$. However, when combined with the associated coefficient, which contain a meson propagator, the term is of order $\mathcal{O}(q^2/m^2_{\pi/\eta})$, where $m^2_{\pi/\eta}$ is the mass of the π and η meson which both are of the same order of magnitude as q. This means that the contribution from $\mathcal{O}_{N,6}$ generally cannot be neglected. To be more explicit $d\mathcal{R}/E_R$ is calculated for both instances, first using only the amplitude with basis operator $\mathcal{O}_{4,N}$, and then using both the amplitudes. The result is presented in figure 4.1.

From studying the left panel of figure 4.1 it is clear that the inclusion of mesons can cause a rather large correction when DM scatters against xenon. The drastic part of the alteration to the graph is also well within the sensitivity region of currently operating xenon based experiments such as XENON1T. This correction is however dependent on the DM mass which can be seen in the right panel of figure of 4.1. This is due to fact that the meson exchange correction scales as $\mathcal{O}(q^2/m^2_{\pi/\eta})$ and thus depends on the amount of momentum that is exchanged. So as the DM mass goes to zero, q goes to zero, which means that correction term becomes negligible and ratio approaches 1. Conversely, as the DM mass increases, q increases, and the effect of the meson exchange becomes important. One can however see that the increase in importance of the meson exchange tapers off as the masses of two bodies coincide at $m_X \approx m_{\rm Xe} \approx 100$ GeV. This makes sense since the momentum transfer does not change much when only one of the masses increases. This also explains why the meson exchange correction is so small for scattering on fluorine; fluorine is less massive which means that momentum transfer is smaller. This can also be explained more specifically by using the equation $q_{\rm max} = 2\mu v$, where $q_{\rm max}$ is the maximum momentum transfer in the scattering, μ is the reduced DM-nucleus mass and v is a given DM velocity in the laboratory frame. If the DM mass is much larger than the target nucleus mass, $q_{\rm max} \approx 2M_T v$, where M_T is the target nucleus mass. Consequently, for $m_X \gg M_T$ $q_{\rm max}$ becomes independent of m_X and grows linearly with M_T. Both effects are apparent in the right panel of figure 4.1. The fact that the ratio is less than 1 may seem contrary to the result in the left panel of figure 4.1 as the rate is smaller when both operators are included. The reason that it is less than 1 is however due to that it is the coupling constants that are compared (see section 3), where the rates end up in the denominator hence inverting the expression.

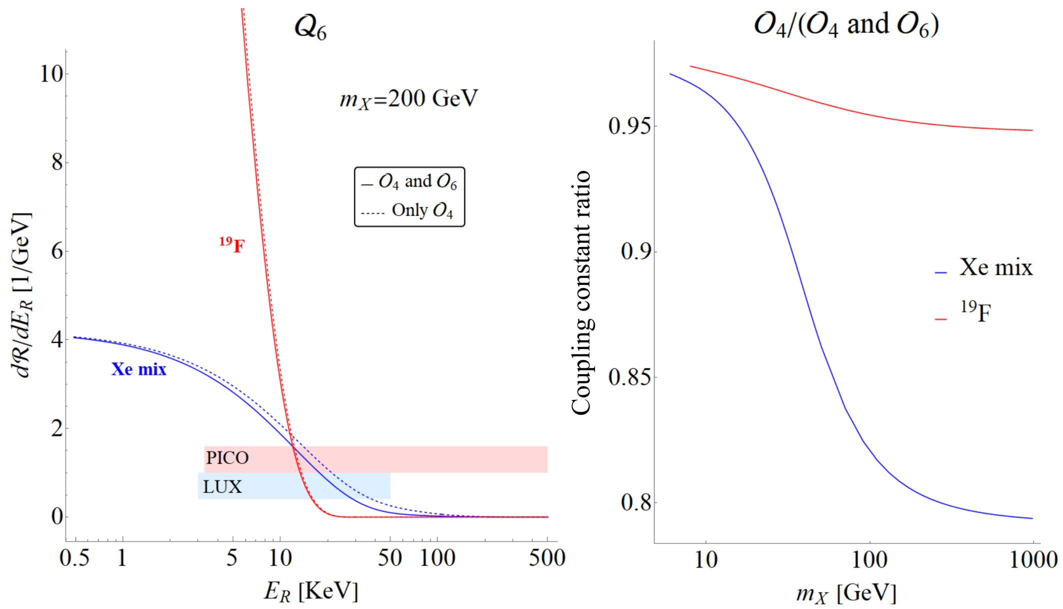

Figure 4.1: Left Plot showcasing the rate $d\mathcal{R}/E_R$ associated to the interaction operator $\mathcal{Q}^{(6)}_{6,q}$ plotted against E_R in 4 different scenarios. The graphs with solid lines are calculated using the contribution from both $\mathcal{O}_{N,4}$ and $\mathcal{O}_{N,6}$. The graphs with dashed lines are calculated using only the contribution from $\mathcal{O}_{N,4}$. The red and blue graphs represent scattering against fluorine and xenon respectively. The shaded rectangles indicate the E_R-sensitivity regions for PICO (fluorine) and LUX (xenon). The coupling constants are chosen to be $\hat{C}^{(6)}_{6,u} = -\hat{C}^{(6)}_{6,d} = 1$ and $\hat{C}^{(6)}_{6,s} = 0$ as to match the analysis done in [14]. **Right** Plot showcasing the relative importance of using just $\mathcal{O}_{N,4}$ compared to using both $\mathcal{O}_{N,4}$ and $\mathcal{O}_{N,6}$, for different values of DM mass (see section 3). The y-axis is the ratio between the two coupling constants.

The fact that the inclusion of the meson exchange actually decreases the rate, rather than increase it, is interesting and warrants some additional comments. This behavior stems from the interference between $\mathcal{O}_{N,4}$ and $\mathcal{O}_{N,6}$. One can easily calculate this interference by subtracting the rates obtained from just using either $\mathcal{O}_{N,4}$ or $\mathcal{O}_{N,6}$ from the rate obtained from using both.

$$\frac{d\mathcal{R}}{dE_R}(\mathcal{O}_{N,4}, \mathcal{O}_{N,6}) - \frac{d\mathcal{R}}{dE_R}(\mathcal{O}_{N,4}) - \frac{d\mathcal{R}}{dE_R}(\mathcal{O}_{N,6}) = \text{Interference} \tag{4.35}$$

The interference is plotted in figure 4.2 and one can clearly see that the interference has a negative sign, which then explains why the combined rate, that includes the meson exchange, is lower than when the mesons exchange is excluded. It is also interesting to note that it is actually the interference caused by $\mathcal{O}_{N,6}$ that leads to the big discrepancy seen in figure 4.1 rather than the contribution of $\mathcal{O}_{N,6}$ itself. This is clear from comparing the rate obtained from using $\mathcal{O}_{N,6}$ to the interference, in figure 4.2.

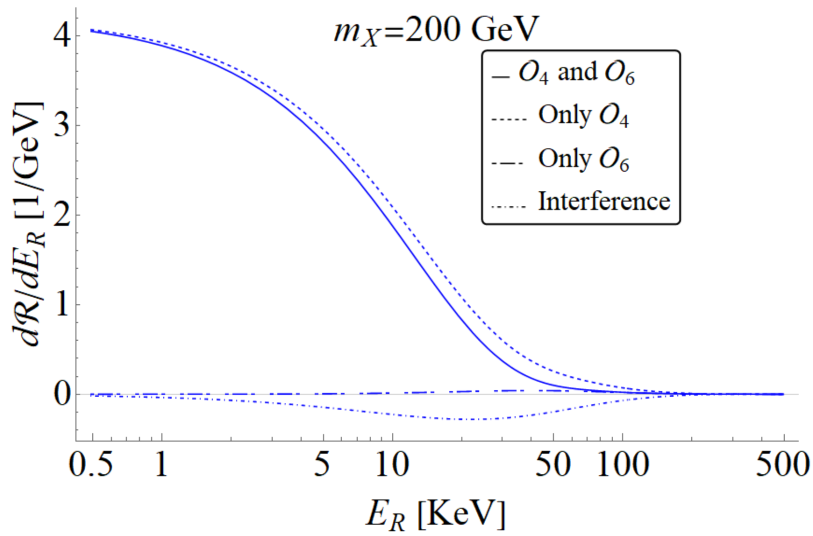

Figure 4.2: Plot showcasing the rate $d\mathcal{R}/E_R$ associated to the interaction operator $\mathcal{Q}^{(6)}_{6,q}$ plotted against E_R using the contribution from different combinations of amplitudes. The interference (see equation (4.35)) between $\mathcal{O}_{N,4}$ and $\mathcal{O}_{N,6}$ is also visualized. In this figure only scattering against xenon is considered.

4.2.2 Matching onto non-relativistic operators without mesons

There are also examples, which have nothing to do with the meson exchange, where the relativistic operators match onto several non-relativistic operators. Thus, as opposed to the last example, the non-relativistic theory knows about the amplitudes, but the relativistic theory shows that some carry the same coupling constant. One example is $\mathcal{Q}^{(6)}_{9,q}$ which matches onto both $\mathcal{O}_{N,17}$ and $\mathcal{O}_{N,20}$. $\mathcal{O}_{N,17}$ and $\mathcal{O}_{N,20}$ are unique to spin 1 DM and can thus only arise from a relativistic theory that specifically is in terms of spin 1 DM.

$$\mathcal{O}_{17,N} = i\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{v}_\perp 1_N \qquad \mathcal{O}_{20,N} = \left(\boldsymbol{S}_N \times \frac{\boldsymbol{q}}{m_N}\right)\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N} \tag{4.36}$$

Determining the relative importance of these two terms is bit more complicated compared to the last example. First of all, $\mathcal{O}_{N,20}$ is q-suppressed relative to $\mathcal{O}_{N,17}$, but $\mathcal{O}_{N,17}$ consists of a $\boldsymbol{v}_\perp$. The magnitude of $\boldsymbol{v}_\perp$ is $\mathcal{O}(q/m_N)$ [29] which results in that there is no net suppression between the two operators. $\mathcal{O}_{N,17}$ is however coherently enhanced as opposed to $\mathcal{O}_{N,20}$ which is dependent on the nucleon spin. $\mathcal{O}_{N,17}$ is consequently expected to dominate over $\mathcal{O}_{N,20}$ when DM scatters against heavier nuclei, but equalize in importance when DM scatters against lighter nuclei. This exact behavior is seen in figure 4.3.

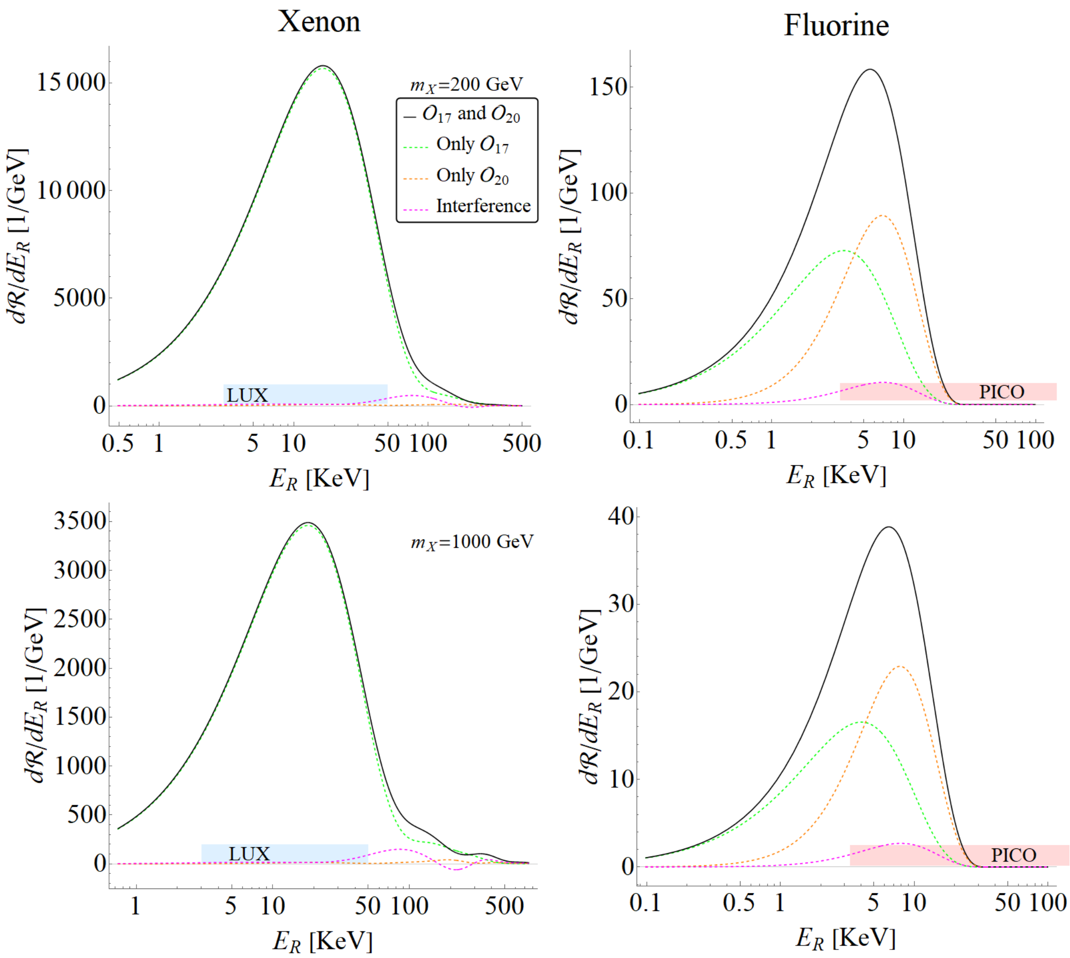

Figure 4.3: Plots showing different $d\mathcal{R}/E_R$ associated to the interaction operator $\mathcal{Q}^{(6)}_{9,q}$. The different rates are calculated using different combinations of operators. The shaded boxes indicate the E_R-sensitivity region for LUX and PICO respectively. **Left** Scattering against xenon. **Right** Scattering against fluorine. **Top** $m_X = 200$ GeV. **Bottom** $m_X = 1000$ GeV.

From studying figure 4.3 it is clear that $\mathcal{O}_{N,17}$ almost captures all of the dynamics when DM scatters against xenon. There is however some amount of interference between $\mathcal{O}_{N,17}$ and $\mathcal{O}_{N,20}$ at around $\approx$ 100 KeV. Around this E_R region it is not correct to just use $\mathcal{O}_{N,17}$ and this is a good example of why using the relativistic theory is so important. Dismissing the importance of $\mathcal{O}_{N,20}$, in the case of DM-xenon scattering, is at the surface very sound. Using the non-relativistic theory does however not show that $\mathcal{O}_{N,17}$ and $\mathcal{O}_{N,20}$ in fact occur together and consequently give rise to interference that is not negligible. The range of recoil energy where the deviation occurs is however very small which means that it is fairly unlikely that this will be of much aid when one analyzes experimental data. This is due to the fact that many interactions must occur specifically at around $\approx$ 100 keV which is unlikely since the total number of interactions, over the whole E_R range ($q <$ 200 MeV), is expected to be rather low. The previous example with the mesons is for example more useful in this regard since the deviation occurs over a larger range of recoil energies. This could potentially change in the future if detectors that have much larger detector media masses, compared to current generation of detectors, are built in order to increase the number of observed interactions. This deviation at around 100 keV is also outside the sensitivity region of XENON1T, but the result could be important to other xenon detectors with a broader region of sensitivity.

In the case of DM-fluorine scattering the two operators are, as expected, almost equally important. $\mathcal{O}_{N,17}$ does however dominates in the low E_R limit while $\mathcal{O}_{N,20}$ dominates in the large E_R limit. It is important to remember that both operators are approximately equally suppressed in this case of fluorine which means that it is meaningful to study their dependence in small and large q. The observed behavior is thus simply due to the fact that $\mathcal{O}_{N,20}$ depends on q^2 whereas $\mathcal{O}_{N,17}$ only depends on q. For most recoil energies however, both operators are needed in order to describe the actual behavior.

5

Conclusion

In this work I formulated all the leading order relativistic interaction operators between spin 1 dark matter (DM) and quarks/gluons. These operators are as general as possible and are only constrained by the enforcement of being Hermetian, Lorentz invaraint, and CPT invariant. The assumed non-relativistic nature of DM was then incorporated through the use of heavy DM effective theory. I then calculated all the scattering amplitudes using the two leading order Feynman diagrams for DM-nucleon scattering in chiral perturbation theory. One of the diagrams describes a contact interaction between DM and a nucleon whereas the other diagram involves a virtual meson which mediates the interaction between the DM and nucleon. I then matched the calculated scattering amplitudes onto an effective Lagrangian consisting of non-relativistic basis operators which made it straight forward to compare the results obtained from the relativistic theory to results that would be obtained from a non-relativistic theory that does not include mesons a degrees of freedom.

I showed that some of the DM-quark/gluon interaction operators match onto several non-relativistic operators. In some circumstances, studying individual operators, which actually occur in conjunction with another operator, can lead to scattering rates that are very incorrect. This is both due to simply not taking into account the contribution coming from the other operator, and, more subtlety, not taking into account the interference effects between the operators. I also showed that the inclusion of mesons, and consequently the meson exchange interaction, is important when DM scatters against heavy nuclei, such as xenon. Interestingly, this meson exchange contribution, which is not always negligible, is associated with a non-relativistic operator that is q-suppressed.

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A

Heavy vector fields

Consider a massive complex vector field X^μ. This field can be decomposed into to two constituent fields by defining the following projection operators proposed by [34].

$$\mathcal{P}_\perp^{\mu\nu} \equiv g^{\mu\nu} - v^\mu v^\nu \qquad \mathcal{P}_\parallel^{\mu\nu} \equiv v^\mu v^\nu \tag{A.1}$$

Where v is a 4-velocity and thus satisfy $v^2 = 1$. These are projection operators since they obey the normal projection operator identities ($P^2 = P$ and $P_\perp P_\parallel = 0$)

$$\begin{aligned}\left(\mathcal{P}_\perp^2\right)^{\mu\lambda} =& g_{\nu\rho}\mathcal{P}_\perp^{\mu\nu}\mathcal{P}_\perp^{\rho\lambda} = g_{\nu\rho}(g^{\mu\nu} - v^\mu v^\nu)(g^{\rho\lambda} - v^\rho v^\lambda) \\ =&(\delta^\mu_\rho - v^\mu v_\rho)(g^{\rho\lambda} - v^\rho v^\lambda) = g^{\mu\lambda} - v^\mu v^\lambda - v^\mu v^\lambda + v^2 v^\mu v^\lambda \\ =& g^{\mu\lambda} - v^\mu v^\lambda = \mathcal{P}_\perp^{\mu\lambda}\end{aligned} \tag{A.2}$$

$$\left(\mathcal{P}_\parallel^2\right)^{\mu\lambda} = g_{\nu\rho}\mathcal{P}_\parallel^{\mu\nu}\mathcal{P}_\parallel^{\rho\lambda} = g_{\nu\rho}v^\mu v^\nu v^\rho v^\lambda = v^\mu v^\lambda = \mathcal{P}_\parallel^{\mu\lambda} \tag{A.3}$$

$$\begin{aligned}g_{\nu\rho}\mathcal{P}_\perp^{\mu\nu}\mathcal{P}_\parallel^{\rho\lambda} =& g_{\nu\rho}(g^{\mu\nu} - v^\mu v^\nu)v^\rho v^\lambda = (\delta^\mu_\rho - v^\mu v_\rho)v^\rho v^\lambda = v^\mu v^\lambda - v^\mu v^2 v^\lambda \\ =&0\end{aligned} \tag{A.4}$$

At this point it is necessary to introduce the concept of residual, or *soft*, momenta $\tilde{p}^\mu$ such that total momenta p^μ of the heavy vector field at low energies is

$$p^\mu = m_X v^\mu + \tilde{p}^\mu \qquad m_X v^\mu >> \tilde{p}^\mu \tag{A.5}$$

where m_X is the mass of the field and v^μ is a reference vector which can be chosen to be time-like. The idea is that only the soft momenta change during the scattering whereas the large momentum component $m_X v^\mu$, associated to the large mass, does not change and is therefore irrelevant to the dynamics [21]. Due to this, the large momentum component will be factored out from the field such that the field only carries soft momenta. Consider two such fields defined as

$$\chi^\mu \equiv e^{im_X v\cdot x}\mathcal{P}_\perp^{\mu\nu}X_\nu \qquad X_\parallel^\mu \equiv e^{im_X v\cdot x}\mathcal{P}_\parallel^{\mu\nu}X_\nu \tag{A.6}$$

It is clear that one can use χ^μ and $X_\parallel^\mu$ and formulate X^μ as the following

$$X^\mu = e^{-im_X v\cdot x}(\chi^\mu + X_\parallel^\mu) \tag{A.7}$$

In the upcoming steps it will however be preferable to write

$$X_\parallel^\mu = v^\mu \mathbb{X} \quad \Rightarrow \quad \mathbb{X} \equiv e^{im_X v\cdot x}v^\nu X_\nu \tag{A.8}$$

such that

$$X^\mu = e^{-im_X v\cdot x}(\chi^\mu + v^\mu \mathbb{X}) \tag{A.9}$$

The fact that $v_\mu \chi^\mu = 0$ will also be important.

$$\begin{aligned} v_\mu \chi^\mu =& e^{im_X v\cdot x} v_\mu P_\perp^{\mu\nu} X_\nu = e^{im_X v\cdot x} v_\mu (g^{\mu\nu} - v^\mu v^\nu) X_\nu \\ =& e^{im_X v\cdot x}(v^\nu - v^2 v^\nu) X_\nu = 0 \end{aligned} \tag{A.10}$$

To see why the decomposition of X^μ in (A.9) is useful one must consider the Proca Lagrangian $\mathcal{L}_p$ which describes a free massive complex vector field X^μ

$$\mathcal{L}_p = -\frac{1}{2}\Big(\partial_\mu X_\nu^* - \partial_\nu X_\mu^*\Big)\Big(\partial^\mu X^\nu - \partial^\nu X^\mu\Big) + m_X^2 X_\mu^* X^\mu \tag{A.11}$$

Inserting the expression for X^μ given in equation (A.9)

$$\partial_\mu X_\nu^* = e^{im_X v\cdot x}\left[\partial_\mu \chi_\nu^* + v_\nu \partial_\mu \mathbb{X}^* + im_X v_\mu(\chi_\nu^* + v_\nu \mathbb{X}^*)\right] \tag{A.12}$$

Using this the positive terms look like

$$\begin{aligned}\partial_\mu X^*_\nu \partial^\mu X^\nu =& e^{im_X v\cdot x} e^{-im_X v\cdot x}[\partial_\mu \chi^*_\nu + v_\nu \partial_\mu \mathbb{X}^* + im_X v_\mu(\chi^*_\nu + v_\nu \mathbb{X}^*)]\cdot \\ & \cdot [\partial^\mu \chi^\nu + v^\nu \partial^\mu \mathbb{X} - im_X v^\mu (\chi^\nu + v^\nu \mathbb{X})] \\ =& \partial_\mu \chi^*_\nu \partial^\mu \chi^\nu + \partial_\mu \mathbb{X}^* \partial^\mu \mathbb{X} + [-imm_X(v\cdot\partial \chi^*_\nu \chi^\nu + v\cdot\partial \mathbb{X}^* \mathbb{X}) + \text{c.c}] \\ & + m_X^2(\chi^*_\nu \chi^\nu + \mathbb{X}^*\mathbb{X})\end{aligned} \tag{A.13}$$

The negative cross terms look like

$$\begin{aligned}\partial_\mu X^*_\nu \partial^\nu X^\mu =& e^{im_X v\cdot x} e^{-im_X v\cdot x}[\partial_\mu \chi^*_\nu + v_\nu \partial_\mu \mathbb{X}^* + im_X v_\mu(\chi^*_\nu + v_\nu \mathbb{X}^*)]\cdot \\ & [\partial^\nu \chi^\mu + v^\mu \partial^\nu \mathbb{X} - im_X v^\nu (\chi^\mu + v^\mu \mathbb{X})] \\ =& \partial_\mu \chi^*_\nu \partial^\nu \chi^\mu + v\cdot\partial \mathbb{X}^* v\cdot\partial \mathbb{X} + [-im_X(\partial_\mu \mathbb{X}^* \chi^\mu + v\cdot\partial \mathbb{X}^* \mathbb{X}) + \text{c.c}] \\ & + m_X^2 \mathbb{X}^*\mathbb{X}\end{aligned} \tag{A.14}$$

Note that there is no mass term generated for χ^μ in the negative cross terms. This means that only a (negative) mass term for χ^μ is generated from the kinetic term of $\mathcal{L}_p$. The mass term in $\mathcal{L}_p$ is simply

$$m_X^2 X^*_\mu X^\mu = m_X^2(\chi^*_\mu \chi^\mu + \mathbb{X}^*\mathbb{X}) \tag{A.15}$$

Once again using the fact that $v_\mu \chi^\mu = 0$. Using all of the above $\mathcal{L}_p$ can be reformulated as

$$\begin{aligned}\mathcal{L}_p =& -\frac{1}{2}\Big(\partial_\mu \chi^*_\nu - \partial_\nu \chi^*_\mu\Big)\Big(\partial^\mu \chi^\nu - \partial^\nu \chi^\mu\Big) - \partial_\mu \mathbb{X}^* \partial^\mu \mathbb{X} + m_X^2 \mathbb{X}^*\mathbb{X} \\ & + v\cdot\partial \mathbb{X}^* v\cdot\partial \mathbb{X} + [im_X v\cdot\partial \chi^*_\nu \chi^\nu + \text{c.c}] + \text{interaction}\end{aligned} \tag{A.16}$$

This Lagrangian describes a massless vector field χ^μ, a massive scalar field $\mathbb{X}$, and their interaction. Thus it is clear that the DOF for X^μ can be decomposed into massless DOF which correspond to χ^μ, and massive (or *heavy*) DOF which correspond to $\mathbb{X}$. This is similar to the result obtained for spin 1/2 fields within heavy quark effective theory [20]. These heavy DOF can be eliminated by using the equations of motion for X^μ (the Proca equation). The Proca equation is obtained by solving the Euler-Lagrange equation for the Proca Lagrangian $\mathcal{L}_p$ and is as follows

$$\partial_\mu(\partial^\mu X^\nu - \partial^\nu X^\mu) + m_X^2 X^\nu = 0 \tag{A.17}$$

Evaluating the derivative of the reformulation of X^μ from equation (A.9) gives

$$\partial^\mu X^\nu = e^{-im_X v\cdot x}[\partial^\mu \chi^\nu + v^\nu \partial^\mu \mathbb{X} - im_X v^\mu(\chi^\nu + v^\nu \mathbb{X})] \tag{A.18}$$

Which means that $\partial_\mu\partial^\mu X^\nu$ is

$$\begin{aligned}\partial_\mu\partial^\mu X^\nu = e^{-im_X v\cdot x}\Big[&-im_X v_\mu\Big(\partial^\mu\chi^\nu + v^\nu\partial^\mu\mathbb{X} - im_X v^\mu(\chi^\nu + v^\nu\mathbb{X})\Big)\\ &+\partial_\mu\partial^\mu\chi^\nu + v^\nu\partial_\mu\partial^\mu\mathbb{X} - im_X v^\mu(\partial_\mu\chi^\nu + v^\nu\partial_\mu\mathbb{X})\Big]\end{aligned} \tag{A.19}$$

The other double derivative term is

$$\begin{aligned}\partial_\mu\partial^\nu X^\mu = e^{-im_X v\cdot x}\Big[&-im_X v_\mu\Big(\partial^\nu\chi^\mu + v^\mu\partial^\nu\mathbb{X} - im_X v^\nu(\chi^\mu + v^\mu\mathbb{X})\Big)\\ &+\partial_\mu\partial^\nu\chi^\mu + v^\mu\partial_\mu\partial^\nu\mathbb{X} - im_X v^\nu(\partial_\mu\chi^\mu + v^\mu\partial_\mu\mathbb{X})\Big]\end{aligned} \tag{A.20}$$

Using $v_\mu\chi^\mu = 0$ simplifies the first row of equation (A.20). The Proca equation is then

$$\begin{aligned}m_X^2 v^\nu\mathbb{X} + &im_X(-2v_\rho\partial^\rho\chi^\nu - v^\nu v^\rho\partial_\rho\mathbb{X} + \partial^\nu\mathbb{X} + v^\nu\partial_\rho\chi^\rho)\\ &+\partial_\rho\partial^\rho\chi^\nu + v^\nu\partial_\rho\partial^\rho\mathbb{X} - \partial_\rho\partial^\nu\chi^\rho - v^\rho\partial_\rho\partial^\nu\mathbb{X} = 0\end{aligned} \tag{A.21}$$

Acting with $\mathcal{P}_{\|\mu\nu}$ on equation (A.21) yields

$$\Big(\partial_\rho\partial^\rho + m_X^2 - (v\cdot\partial)^2\Big)\,\mathbb{X} = (v\cdot\partial - im_X)\partial_\lambda\chi^\lambda \tag{A.22}$$

Focusing on the left hand side and considering $\mathbb{X}$ as a plane wave one obtains the following eigenvalue equation

$$\Big(\partial_\rho\partial^\rho + m_X^2 - (v\cdot\partial)^2\Big)\,\mathbb{X} = \Big(-k^2 + m_X^2 + (v\cdot k)^2\Big)\,\mathbb{X} \tag{A.23}$$

The eigenvalues of the operator acting on $\mathbb{X}$ are real and one can thus divide with the operator granted one includes a small complex offset $i\epsilon$. Using this in equation (A.22) one obtains the relation

$$\mathbb{X} = \frac{1}{\partial_\rho\partial^\rho + m_X^2 - (v\cdot\partial)^2 + i\epsilon}(v\cdot\partial - im_X)\partial_\lambda\chi^\lambda \tag{A.24}$$

The fraction can be expanded in a series

$$
\begin{aligned}
\frac{1}{\partial_\rho \partial^\rho + m_X^2 - (v\cdot\partial)^2 + i\epsilon} =& \frac{1}{m_X^2} \frac{1}{\frac{\partial_\rho \partial^\rho - (v\cdot\partial)^2 + i\epsilon}{m_X^2} + 1} \\
=& \frac{1}{m_X^2} \left(1 - \frac{\partial_\rho \partial^\rho - (v\cdot\partial)^2 + i\epsilon}{m_X^2} + \mathcal{O}\left(\frac{1}{m_X^4}\right)\right)
\end{aligned}
\tag{A.25}
$$

Where the Taylor expansion of $1/(x+1)$ was utilized. It converges for $|x| < 1$ which is true in this case since m_X is assumed large. By only considering one inverse power of m_X one can write $\mathbb{X}$ as

$$
\mathbb{X} = -\frac{i}{m} \partial_\lambda \chi^\lambda + \mathcal{O}\left(\frac{1}{m_X^2}\right) \tag{A.26}
$$

This result coincides with the result obtained here [34]. Inserting this back into the reformulation of X^μ in equation (A.9) one obtains an expression for X^μ as a function of only χ^μ.

$$
X^\mu = e^{-im_X v\cdot x} \left(\chi^\mu - \frac{iv^\mu}{m_X} \partial_\lambda \chi^\lambda + \mathcal{O}\left(\frac{1}{m_X^2}\right)\right) \tag{A.27}
$$

B

HDMET conversions

Heavy DM effective theory (HDMET), in the case of spin 1 DM, was described in detail in appendix A. One result from appendix A is that a massive vector X^μ with mass m_X can be written in terms of a massless vector field χ^μ.

$$X^\mu \approx e^{-im_X v\cdot x}\left(\chi^\mu - \frac{iv^\mu}{m_X}\partial_\rho\chi^\rho\right) \tag{B.1}$$

In this appendix some commonly used expressions involving the DM vector field X^μ are converted using heavy DM effective theory (HDMET) to be in terms of the massless vector field χ^μ.

$$\begin{aligned} X^*_\mu X^\mu \approx& \left(\chi^*_\mu + \frac{iv_\mu}{m_X}\partial^\rho\chi^*_\rho\right)\left(\chi^\mu - \frac{iv^\mu}{m_X}\partial_\lambda\chi^\lambda\right) \\ =&\chi^*_\mu\chi^\mu + \frac{1}{m_X^2}\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda \end{aligned} \tag{B.2}$$

$$X^*_\mu X_\nu \approx \chi^*_\mu\chi_\nu - \frac{i}{m_X}v_\nu\chi^*_\mu\partial_\lambda\chi^\lambda + \frac{i}{m_X}v_\mu\chi_\nu\partial^\rho\chi^*_\rho + \frac{v_\mu v_\nu}{m_X^2}\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda \tag{B.3}$$

$$\begin{aligned} X^*_\nu\overleftrightarrow{\partial}_\mu X^\nu =& X^*_\nu\partial_\mu X^\nu - \partial_\mu X^*_\nu X^\nu \\ \approx& \left(\chi^*_\nu + \frac{i}{m_X}v_\nu\partial^\rho\chi^*_\rho\right)\left(\partial_\mu\chi^\nu - \frac{i}{m}v^\nu\partial_\mu\partial_\lambda\chi^\lambda - im_Xv_\mu(\chi^\nu - \frac{i}{m_X}v^\nu\partial_\lambda\chi^\lambda)\right) \\ &- \left(\partial_\mu\chi^*_\nu + \frac{i}{m}v_\nu\partial_\mu\partial^\rho\chi^*_\rho + im_Xv_\mu(\chi^*_\nu + \frac{i}{m_X}v_\nu\partial^\rho\chi^*_\rho)\right)\left(\chi^\nu - \frac{i}{m_X}v^\nu\partial_\lambda\chi^\lambda\right) \\ =&\chi^*_\nu\partial_\mu\chi^\nu - im_Xv_\mu\chi^*_\nu\chi^\nu + \frac{1}{m^2}\partial^\rho\chi^*_\rho\partial_\mu\partial_\lambda\chi^\lambda - \frac{i}{m_X}v_\mu\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda \\ &-\partial_\mu\chi^*_\nu\chi^\nu - \frac{1}{m_X^2}\partial_\mu\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda - im_Xv_\mu\chi^*_\nu\chi^\nu - \frac{i}{m_X}v_\mu\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda \\ =&\chi^*_\nu\overleftrightarrow{\partial}_\mu\chi^\nu - 2im_Xv_\mu\chi^*_\nu\chi^\nu - \frac{2i}{m_X}v_\mu\partial^\rho\chi^*_\rho\partial_\lambda\chi^\lambda + \frac{1}{m_X^2}\partial^\rho\chi^*_\rho\overleftrightarrow{\partial}_\mu\partial_\lambda\chi^\lambda \end{aligned} \tag{B.4}$$

$$
\begin{aligned}
X^*_\nu \overleftrightarrow{\partial}_\rho X_\lambda \epsilon^{\mu\nu\rho\lambda} =&(X^*_\nu \partial_\rho X_\lambda - \partial_\rho X^*_\nu X_\lambda)\epsilon^{\mu\nu\rho\lambda} \\
\approx&\Big(\chi^*_\nu \partial_\rho \chi_\lambda + \frac{i}{m_X} v_\nu \chi^*_\lambda \partial_\rho \partial_\sigma \chi^\sigma - i m_X v_\rho \chi^*_\nu \chi_\lambda + \frac{i}{m_X} v_\nu \partial^\gamma \chi^*_\gamma \partial_\rho \chi_\lambda \\
&-(\partial_\rho \chi^*_\nu \chi_\lambda + \frac{i}{m_X} v_\nu \partial_\rho \chi^*_\lambda \partial_\sigma \chi^\sigma + i m_X v_\rho \chi^*_\nu \chi_\lambda + \frac{i}{m_X} v_\nu \partial_\rho \partial^\gamma \chi^*_\gamma \chi_\lambda)\Big)\epsilon^{\mu\nu\rho\lambda} \\
=&(-2 i m_X v_\rho \chi^*_\nu \chi_\lambda + \chi^*_\nu \overleftrightarrow{\partial}_\rho \chi_\lambda + \frac{i}{m_X} v_\nu \chi^*_\lambda \overleftrightarrow{\partial}_\rho \partial_\sigma \chi^\sigma - \frac{i}{m_X} v_\nu \chi_\lambda \overleftrightarrow{\partial}_\rho \partial^\gamma \chi^*_\gamma)\epsilon^{\mu\nu\rho\lambda}
\end{aligned}
\tag{B.5}
$$

Where $v_\nu v_\rho \epsilon^{\mu\nu\rho\lambda} = 0$ was used.

$$
\begin{aligned}
\partial_\rho (X^*_\nu X_\lambda)\epsilon^{\mu\nu\rho\lambda} =&(X^*_\nu \partial_\rho X_\lambda + \partial_\rho X^*_\nu X_\lambda)\epsilon^{\mu\nu\rho\lambda} \\
\approx&\Big(\partial_\rho(\chi^*_\nu \chi_\lambda) + \frac{i}{m_X} v_\nu \partial_\rho(\chi^*_\lambda \partial_\sigma \chi^\sigma) + \frac{i}{m_X} v_\nu \partial_\rho(\chi_\lambda \partial^\gamma \chi^*_\gamma)\Big)\epsilon^{\mu\nu\rho\lambda}
\end{aligned}
\tag{B.6}
$$

$$
\begin{aligned}
X^*_\nu \partial^\nu X_\mu \approx&\chi^*_\nu \partial^\nu \chi_\mu - \frac{i}{m_X} v_\mu \chi^*_\nu \partial^\nu \partial_\lambda \chi^\lambda + \frac{i}{m_X} \partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \frac{1}{m_X^2} v_\mu \partial^\rho \chi^*_\rho v\cdot\partial \partial_\lambda \chi^\lambda \\
&+\partial^\rho \chi^*_\rho \chi_\mu - \frac{i}{m_X} v_\mu \partial^\rho \chi^*_\rho \partial_\lambda \chi^\lambda \\
=&\partial^\nu(\chi^*_\nu \chi_\mu) - \frac{i}{m_X} v_\mu \partial^\nu(\chi^*_\nu \partial_\lambda \chi^\lambda) + \frac{i}{m_X} \partial^\rho \chi^*_\rho v\cdot\partial \chi_\mu + \frac{1}{m_X^2} v_\mu \partial^\rho \chi^*_\rho v\cdot\partial \partial_\lambda \chi^\lambda
\end{aligned}
\tag{B.7}
$$

C

Scattering amplitudes

C.1 Feynman rules and definitions

To begin with a few general comments are needed. The derivatives acting on χ^μ produce soft momentum $\tilde{p}$ since the hard momentum have been factored out from the field as explained previously. It is also worth mentioning that, in contrast to QED for example,

$$p\cdot\epsilon \neq 0 \tag{C.1}$$

where the ϵ is the polarization vector of the DM field. This is because no gauge invariance is assumed for the DM field which consequently means that the corresponding Ward identity does not exist.

The vertices that will be used are all very similar. The derivation of one will be given in more detail, the others follow analogously. Consider the interaction $\mathcal{L}_{\text{int}} = \chi^*_\nu \overleftrightarrow{\partial}_\mu \chi^\nu = \chi^*_\nu \partial_\mu \chi^\nu - \partial_\mu \chi^*_\nu \chi^\nu$. The sign of the momentum produced by the derivative depends on if the field it acts on is incoming or outgoing which is clear from studying the quantized expansion of χ_ν

$$\chi_\nu = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s \left(a^s_{\mathbf{p}} \epsilon^s_\nu e^{-ip\cdot x} + a^{s\dagger}_{\mathbf{p}} \epsilon^{s*}_\nu e^{+ip\cdot x} \right) \tag{C.2}$$

Thus by using the Feynman rules for external vector fields

$$\begin{aligned} \mathcal{L}_{\text{int}} =& \chi^*_\nu \partial_\mu \chi^\nu - \partial_\mu \chi^*_\nu \chi^\nu \to i \left(\epsilon^*_\nu (-i\tilde{p}_{1\mu}) \epsilon^\nu - (+i\tilde{p}_{2\mu}) \epsilon^*_\nu \epsilon^\nu \right) \\ =& - i^2 (\tilde{p}_{1\mu} + \tilde{p}_{2\mu}) \epsilon^*_\nu \epsilon^\nu \end{aligned} \tag{C.3}$$

The additional i comes from the usual vertex coefficient. In the case of a Hamiltonian it would be $-i$ but since $\mathcal{L}_{\text{int}}$ is a Lagrangian a Legendre transformation must be done which generates an additional minus sign. One may also define

$$\tilde{P}_\mu \equiv \tilde{p}_{1\mu} + \tilde{p}_{2\mu} \tag{C.4}$$

in contrast to

$$q_\mu = \tilde{p}_{1\mu} - \tilde{p}_{2\mu} \tag{C.5}$$

The rules of interest are

$$\begin{aligned} \chi^*_\nu \overleftrightarrow{\partial}_\mu \chi^\nu &\to i(-i)\tilde{P}_\mu \epsilon^*_\nu \epsilon^\nu \\ \partial_\mu(\chi^*_\nu \chi^\nu) &\to i(-i) q_\mu \epsilon^*_\nu \epsilon^\nu \\ \partial^\mu \chi^*_\mu \partial_\nu \chi^\nu &\to i \tilde{p}^\mu_2 \tilde{p}^\nu_1 \epsilon^*_\mu \epsilon_\nu \end{aligned} \tag{C.6}$$

$$\chi^*_\lambda \overleftrightarrow{\partial}_\rho \partial_\sigma \chi^\sigma \to i(-1)\tilde{P}_\rho \epsilon^*_\lambda \epsilon\cdot\tilde{p}_\epsilon \qquad \chi_\lambda \overleftrightarrow{\partial}_\rho \partial_\sigma \chi^{*\sigma} \to i(-1)\tilde{P}_\rho \epsilon_\lambda (\epsilon\cdot\tilde{p}_\epsilon)^* \tag{C.7}$$

$$\partial_\rho(\chi^*_\lambda \partial_\sigma \chi^\sigma) \to i(-1) q_\rho \epsilon^*_\lambda \epsilon\cdot\tilde{p}_\epsilon \qquad \partial_\rho(\chi_\lambda \partial_\sigma \chi^{*\sigma}) \to i q_\rho \epsilon_\lambda (\epsilon\cdot\tilde{p}_\epsilon)^* \tag{C.8}$$

Where $\epsilon\cdot\tilde{p}_\epsilon$ is defined such that

$$\epsilon\cdot\tilde{p}_\epsilon \equiv \epsilon\cdot\tilde{p}_1 \qquad (\epsilon\cdot\tilde{p}_\epsilon)^* \equiv \epsilon^*\cdot\tilde{p}_2 \tag{C.9}$$

It will be beneficial to introduce the following operators

$$\mathcal{S}_{\mu\nu} \equiv \frac{1}{2}\left(\epsilon^*_\mu \epsilon_\nu + \epsilon_\mu \epsilon^*_\nu\right) \qquad \hat{\mathcal{S}}^{\mu\nu} \equiv \epsilon^*_\rho \epsilon_\lambda \epsilon^{\mu\nu\rho\lambda} \qquad \bar{\mathcal{S}}_{\mu\nu} \equiv \frac{1}{2}\left(\epsilon^*_\mu \epsilon_\nu - \epsilon_\mu \epsilon^*_\nu\right) \tag{C.10}$$

Using this, one can write some commonly occurring expressions in a nicer way

$$\begin{aligned} [\epsilon^*_\nu \epsilon_\mu + \text{c.c}] &= 2\mathcal{S}_{\mu\nu} \\ q^\mu[\epsilon^*_\mu \epsilon\cdot\tilde{p}_\epsilon + \text{c.c}] &= q\cdot\mathcal{S}\cdot\tilde{P} \\ q^\mu[\epsilon^*_\mu \epsilon\cdot\tilde{p}_\epsilon - \text{c.c}] &= q\cdot\mathcal{S}\cdot q + q_\mu \tilde{P}_\nu \bar{\mathcal{S}}^{\mu\nu} \end{aligned} \tag{C.11}$$

The second row is calculated using $\tilde{p}^\mu_1 = \frac{1}{2}(\tilde{P}^\mu + q^\mu)$ and $\tilde{p}^\mu_2 = \frac{1}{2}(\tilde{P}^\mu - q^\mu)$ such that

$$\begin{aligned} q^\mu[\epsilon^*_\mu \epsilon\cdot\tilde{p}_\epsilon + \text{c.c}] &= q^\mu(\epsilon^*_\mu \epsilon\cdot\tilde{p}_1 + \epsilon_\mu \epsilon^*\cdot\tilde{p}_2) \\ &= \frac{1}{2} q^\mu \left(\epsilon^*_\mu \epsilon\cdot(\tilde{P}+q) + \epsilon_\mu \epsilon^*\cdot(\tilde{P}-q)\right) \\ &= \frac{1}{2}\left(q^\mu \tilde{P}^\nu(\epsilon^*_\mu \epsilon_\nu + \epsilon_\mu \epsilon^*_\nu)\right) = q^\mu \mathcal{S}_{\mu\nu} \tilde{P}^\nu \end{aligned} \tag{C.12}$$

The third row is obtained through an analogous calculation.

C.2 Amplitudes

Throughout this section only the amplitudes for the DM-proton scattering will be written out. The DM-neutron amplitude is obtained by letting $p \to n, u \to d$ in the DM-proton amplitudes.

As stated previously in section 2.5.4, only the terms with leading order in momenta are guaranteed to obey Galilean symmetry [14]. Since DM is assumed to be non-relativistic it is therefore appropriate to neglect higher order terms in the final expression. Further, it is important to note that q_0 (and consequently also $v \cdot q$, if v chosen to be timelike) is of order 2 in momentum. This is due to that the energy of the incoming and outgoing DM can be written as

$$E_{\text{in}} \approx m_X + \frac{\boldsymbol{p}_1^2}{2m_X} \qquad\qquad E_{\text{out}} \approx m_X + \frac{\boldsymbol{p}_2^2}{2m_X} \tag{C.13}$$

which means that

$$\begin{aligned} q_0 =& E_{\text{in}} - E_{\text{out}} = m_X - m_X + \frac{1}{2m_X}(\boldsymbol{p}_1^2 - \boldsymbol{p}_2^2) \\ =& \frac{1}{2m_X}\boldsymbol{q}\cdot\boldsymbol{P} \end{aligned} \tag{C.14}$$

C.2.1 Contact interaction diagram (left diagram in Fig. 2.1)

Terms consisting of J_q^S, J_q^V, J_q^A, J^G and J^θ contribute to the left diagram [13]. In the case of the left diagram the quark/gluon currents have to be hadronized in terms of nucleons. The expressions for these currents will be taken from [13]. The numerical values for the constants that are needed in the currents can also be found in appendix C of [13].

C.2.1.1 Scalar current

Using equation (B45) and (B54) in [13] (where now $q = u, d$)

$$\begin{aligned} \tilde{J}_q^S =& -2b_0 m_q \bar{N}N - 2(b_D + b_F) m_q \bar{N}_q N_q + \cdots \\ \tilde{J}_s^S =& -2\Big(b_0 + b_D - b_F\Big) m_s \bar{N}N + \cdots \end{aligned} \tag{C.15}$$

where $\bar{N}N = \bar{p}_v p_v + \bar{n}_v n_v$, $N_u = p_v$ and $N_d = n_v$.

The DM-proton interaction is then

$$\mathcal{L}_{\chi,p}^{S(6)} = \chi_\mu^* \chi^\mu \bar{p}_v p_v \Big[\hat{C}_{1,u}^{(6)} \sigma_u^p + \hat{C}_{1,d}^{(6)} \sigma_d^p + \hat{C}_{1,s}^{(6)} \sigma_s^p\Big] \tag{C.16}$$

The DM-neutron interaction is obtained by simply letting $p \to n, u \to d$.

The constants are defined as

$$\sigma_u^p = -2m_u(b_0 + b_D + b_F) \qquad \sigma_d^n = -2m_d(b_0 + b_D + b_F) \tag{C.17}$$
$$\sigma_u^n = -2m_u b_0 \qquad \sigma_d^p = -2m_d b_0 \tag{C.18}$$
$$\sigma_s = -2m_s(b_0 + b_D - b_F) \tag{C.19}$$

and the numerical values can be found in appendix C of [13]. The scattering amplitude is then

$$iM_p^S = i\epsilon_\mu^* \epsilon^\mu \bar{p}_v p_v \Big[\hat{C}_{1,u}^{(6)}\sigma_u^p + \hat{C}_{1,d}^{(6)}\sigma_d^p + \hat{C}_{1,s}^{(6)}\sigma_s^p\Big] \tag{C.20}$$

C.2.1.2 Vector current

Using equation (B46) and (B52) in [13]

$$\begin{aligned}\tilde{J}_q^{V,\mu} = &\Big(v^\mu + \frac{\tilde{K}^\mu}{2m_N}\Big)\Big(\bar{N}_q N_q + \bar{N}N\Big)\\ &+ i\epsilon^{\alpha\beta\lambda\mu} v_\alpha q_\lambda \Big(g_4 \bar{N}_q S_{N\beta} N_q - g_4' \bar{N} S_{N\beta} N\Big) + \cdots\end{aligned} \tag{C.21}$$

$$\tilde{J}_s^{V,\mu} = -i(g_4 - g_5 + g_4')\epsilon^{\alpha\beta\lambda\mu} v_\alpha q_\lambda \bar{N} S_{N\beta} N + \cdots \tag{C.22}$$

It is beneficial to define

$$\mathcal{W}_\mu(v,q) \equiv v^\alpha q^\lambda S_N^\beta \epsilon_{\mu\alpha\beta\lambda} \tag{C.23}$$

such that

$$\mathcal{W}_\mu q^\mu = 0 \qquad \mathcal{W}_\mu v^\mu = 0 \tag{C.24}$$

Another useful definition is

$$\hat{\mathcal{W}}_p(\hat{C}_{n,q}^d) = \Big((g_4 - g_4')\hat{C}_{n,u}^d - g_4'\hat{C}_{n,d}^d - 3\frac{\mu_s}{m_N}\hat{C}_{n,s}^d\Big)\Big) \tag{C.25}$$

C.2.1.2.1 Type a

$$\begin{aligned} iM_p^{Va} =& i\Big[2m_X\epsilon^*_\mu\epsilon^\mu\bar{p}_v p_v v^\mu\Big(v_\mu + \frac{\tilde{K}_\mu}{2m_N}\Big)(2\hat{C}^6_{3,u}+\hat{C}^6_{3,d}) \\ &+i(-i)\tilde{P}_\mu\epsilon^*_\nu\epsilon^\nu\Big(\bar{p}_v p_v\Big(v^\mu+\frac{\tilde{K}^\mu}{2m_N}\Big)(2\hat{C}^6_{3,u}+\hat{C}^6_{3,d})+i\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{3,q})\Big) \\ &+\frac{2}{m_X}\tilde{p}_2^\mu\tilde{p}_1^\nu\epsilon^*_\mu\epsilon_\nu\bar{p}_v p_v(2\hat{C}^6_{3,u}+\hat{C}^6_{3,d})\Big] \end{aligned} \tag{C.26}$$

The leading order in momentum is zero, hence only the first row contributes to leading order. Writing in terms of basis operators

$$iM_p^{Va} \approx 2im_X\epsilon^*_\mu\epsilon^\mu\bar{p}_v p_v(2\hat{C}^6_{3,u}+\hat{C}^6_{3,d}) \tag{C.27}$$

C.2.1.2.2 Type b

$$\begin{aligned} iM_p^{Vb} =& i\epsilon^{\mu\nu\rho\lambda}\Big(\bar{p}_v p_v\Big(v_\mu+\frac{\tilde{K}_\mu}{2m_N}\Big)(2\hat{C}^6_{5,u}+\hat{C}^6_{5,d})+i\bar{p}_v\mathcal{W}_\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{5,q})\Big)\Big[\\ &-2im_X v_\rho\epsilon^*_\nu\epsilon_\lambda \\ &+(-i)\tilde{P}_\rho\epsilon^*_\nu\epsilon_\lambda \\ &+\frac{i}{m_X}(-1)v_\nu\tilde{P}_\rho[\epsilon^*_\lambda\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}] \end{aligned} \tag{C.28}$$

The leading order in momentum is linear, hence only the terms on the second and third row contributes to leading order. Writing in terms of basis operators

$$\begin{aligned} iM_p^{Vb} \approx& 2m_X v_{\perp\mu}v_\nu\hat{\mathcal{S}}^{\mu\nu}\bar{p}_v p_v(2\hat{C}^6_{5,u}+\hat{C}^6_{5,d}) \\ &-2im_X\hat{\mathcal{S}}^{\mu\nu}v_\nu\bar{p}_v\mathcal{W}_\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{5,q}) \end{aligned} \tag{C.29}$$

C.2.1.2.3 Type c

$$\begin{aligned} iM_p^{Vc} =& i\epsilon^{\mu\nu\rho\lambda}\Big(\bar{p}_v p_v\Big(v_\mu+\frac{\tilde{K}_\mu}{2m_N}\Big)(2\hat{C}^6_{7,u}+\hat{C}^6_{7,d})+i\bar{p}_v\mathcal{W}_\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{7,q})\Big)\Big[\\ &+i(-i)q_\rho\epsilon^*_\nu\epsilon_\lambda \\ &-\frac{i}{m_X}iv_\nu q_\rho[\epsilon^*_\lambda\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}]\Big] \end{aligned} \tag{C.30}$$

The leading order in momentum is linear, hence only the term on the second row contributes to leading order. Writing in terms of basis operators

$$iM_p^{Vc} \approx -iq_\nu v_\mu\hat{\mathcal{S}}^{\mu\nu}\bar{p}_v p_v(2\hat{C}^6_{7,u}+\hat{C}^6_{7,d}) \tag{C.31}$$

C.2.1.2.4 Type d

$$\begin{aligned}iM_p^{Vd} =& i\Big[(-i)q^\nu[\epsilon_\nu^*\epsilon_\mu + \text{c.c}]\Big(\bar{p}_v p_v\frac{\tilde{K}^\mu}{2m_N}(2\hat{C}^6_{9,u} + \hat{C}^6_{9,d}) + i\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{9,q})\Big)\\ &+\frac{i}{m_X}q^\nu[\epsilon_\nu(\epsilon\cdot\tilde{p}_\epsilon)^* + \text{c.c}]v^\mu\Big(\bar{p}_v p_v\Big(v_\mu + \frac{\tilde{K}_\mu}{2m_N}\Big)(2\hat{C}^6_{9,u} + \hat{C}^6_{9,d})\Big)\\ &+\frac{i}{m_X}[(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu - \text{c.c}]\Big(\bar{p}_v p_v\frac{\tilde{K}^\mu}{2m_N}(2\hat{C}^6_{9,u} + \hat{C}^6_{9,d}) + i\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{9,q})\Big)\Big]\end{aligned} \tag{C.32}$$

The leading order in momentum is quadratic, hence only the terms on the first and second row contributes to leading order. Writing in terms of basis operators

$$\begin{aligned}iM_p^{Vd} \approx& -2v_\perp\cdot\mathcal{S}\cdot q\bar{p}_v p_v(2\hat{C}^6_{9,u} + \hat{C}^6_{9,d})\\ &+i2q^\nu\mathcal{S}_{\mu\nu}\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{9,q})\end{aligned} \tag{C.33}$$

C.2.1.2.5 Type e

$$\begin{aligned}iM_p^{Ve} =& i\Big[i(-i)q^\nu[\epsilon_\nu^*\epsilon_\mu - \text{c.c}]\Big(\bar{p}_v p_v\frac{\tilde{K}^\mu}{2m_N}(2\hat{C}^6_{11,u} + \hat{C}^6_{11,d}) + i\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{11,q})\Big)\\ &-\frac{1}{m_X}q^\nu[\epsilon_\nu^*\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}]v^\mu\Big(\bar{p}_v p_v\Big(v_\mu + \frac{\tilde{K}_\mu}{2m_N}\Big)(2\hat{C}^6_{11,u} + \hat{C}^6_{11,d})\Big)\\ &-\frac{1}{m_X}[(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu + \text{c.c}]\Big(\bar{p}_v p_v\frac{\tilde{K}^\mu}{2m_N}(2\hat{C}^6_{11,u} + \hat{C}^6_{11,d}) + i\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{11,q})\Big)\Big]\end{aligned} \tag{C.34}$$

The leading order in momentum is quadratic, hence only the terms on the first and second row contributes to leading order. Writing in terms of basis operators

$$\begin{aligned}iM_p^{Ve} \approx& \Big(-2iq^\nu v_\perp^\mu\bar{\mathcal{S}}_{\mu\nu} - i\frac{q}{m_X}\cdot\mathcal{S}\cdot q\Big)\bar{p}_v p_v(2\hat{C}^6_{11,u} + \hat{C}^6_{11,d})\\ &-2q^\nu\bar{\mathcal{S}}_{\mu\nu}\bar{p}_v\mathcal{W}^\mu p_v\hat{\mathcal{W}}_p(\hat{C}^6_{11,q})\end{aligned} \tag{C.35}$$

C.2.1.3 Axial current

Using equation (B47) and (B53) in [13].

$$\tilde{J}_q^{A,\mu} = 2(D+F)\bar{N}_q\Big(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{K}\cdot S_N\Big)N_q + 2G\bar{N}\Big(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{K}\cdot S_N\Big)N + \cdots \tag{C.36}$$

$$\tilde{J}_s^{A,\mu} = 2(D - F + G)\bar{N}\Big(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{K}\cdot S_N\Big)N + \cdots \tag{C.37}$$

Note that $S_N \cdot v = 0$.

Let

$$\hat{\mathcal{A}}_p(\hat{C}^d_{n,q}) \equiv 2\Big((D + F + G)\hat{C}^d_{n,u} + G\hat{C}^d_{n,d} + (D - F + G)\hat{C}^d_{n,s}\Big) \tag{C.38}$$

C.2.1.3.1 Type a

$$\begin{aligned} i\tilde{M}_p^{Aa} = i\hat{\mathcal{A}}_p(\hat{C}^6_{4,q})\Big[&-2\epsilon^*_\mu\epsilon^\mu\frac{m_X}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v \\ &+i(-i)\epsilon^*_\mu\epsilon^\mu\bar{p}_v\tilde{P}\cdot S_N p_v \\ &-2\epsilon^*_\mu p_2^\mu\epsilon_\nu p_1^\nu\frac{1}{2m_N m_X}\bar{p}_v\tilde{K}\cdot S_N p_v\Big] \end{aligned} \tag{C.39}$$

The leading order in momentum is linear, hence only the terms on the first and second row contributes to leading order. Writing in terms of basis operators

$$i\tilde{M}_p^{Aa} \approx 2im_X\epsilon^*_\mu\epsilon^\mu\bar{p}_v v_\perp\cdot S_N p_v\hat{\mathcal{A}}_p(\hat{C}^6_{4,q}) \tag{C.40}$$

C.2.1.3.2 Type b

$$\begin{aligned} i\tilde{M}_p^{Ab} = i\epsilon^{\mu\nu\rho\lambda}\hat{\mathcal{A}}_p(\hat{C}^6_{6,q})\Big[&-2im_X v_\rho\epsilon^*_\nu\epsilon_\lambda\bar{p}_v S_{N\mu}p_v \\ &+(-i)\tilde{P}_\rho\epsilon^*_\nu\epsilon_\lambda(\bar{p}S_{N\mu}p_v - \frac{v_\mu}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v) \\ &+(-1)\frac{i}{m_X}\tilde{P}_\rho v_\nu[\epsilon^*_\lambda\epsilon\cdot p_\epsilon - \text{c.c}]\bar{p}_v S_{N\mu}p_v \end{aligned} \tag{C.41}$$

The leading order in momentum is zero, hence only the first row contributes to leading order. Writing in terms of basis operators

$$i\tilde{M}_p^{Ab} \approx -2m_X\hat{\mathcal{S}}^{\mu\nu}v_\nu\bar{p}_v S_{N\mu}p_v\hat{\mathcal{A}}_p(\hat{C}^6_{6,q}) \tag{C.42}$$

C.2.1.3.3 Type c

$$\begin{aligned} i\tilde{M}_p^{Ac} = i\epsilon^{\mu\nu\rho\lambda}\hat{\mathcal{A}}_p(\hat{C}^6_{8,q})\Big[&i(-i)q_\rho\epsilon^*_\nu\epsilon_\lambda(\bar{p}S_{N\mu}p_v - \frac{v_\mu}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v) \\ &+\frac{1}{m_X}v_\nu q_\rho[\epsilon^*_\lambda\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}]\bar{p}_v S_{N\mu}p_v\Big] \end{aligned} \tag{C.43}$$

The leading order in momentum seems to be linear but is actually quadratic due to $\hat{\mathcal{S}}^{\mu\nu} q_\nu S_{N,\mu} = \mathcal{O}(q^2)$ (see appendix D). All the terms in the amplitude are therefore necessary. Simplifying

$$\begin{aligned} i\tilde{M}_p^{Ac} &= i\hat{\mathcal{A}}_p(\hat{C}^6_{8,q})\Big[- q_\nu \hat{\mathcal{S}}^{\mu\nu}(\bar{p}S_{N\mu}p_v - \frac{v_\mu}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v) \\ &\quad + \epsilon^{\mu\nu\rho\lambda}\frac{1}{m_X}v_\nu q_\rho(\bar{\mathcal{S}}_{\lambda\kappa}\tilde{P}^\kappa + \mathcal{S}_{\lambda\kappa}q^\kappa)\bar{p}_v S_{N\mu}p_v\Big] \\ &= i\hat{\mathcal{A}}_p(\hat{C}^6_{8,q})\Big[- q_\nu \hat{\mathcal{S}}^{\mu\nu}\bar{p}S_{N\mu}p_v + \frac{1}{2m_N}(-i)(-1)S_X^\nu q_\nu \bar{p}_v\tilde{K}\cdot S_N p_v \\ &\quad + \epsilon^{\mu\nu\rho\lambda}\frac{1}{m_X}v_\nu q_\rho(\bar{\mathcal{S}}_{\lambda\kappa}\tilde{P}^\kappa + \mathcal{S}_{\lambda\kappa}q^\kappa)\bar{p}_v S_{N\mu}p_v\Big] \end{aligned} \tag{C.44}$$

C.2.1.3.4 Type d

$$\begin{aligned} i\tilde{M}_p^{Ad} = i\hat{\mathcal{A}}_p(\hat{C}^6_{10,q})\Big[&(-i)q^\nu[\epsilon^*_\nu\epsilon_\mu + \text{c.c}]\bar{p}_v S_N^\mu p_v \\ &+ i\frac{1}{m_X}q^\nu[\epsilon_\nu(\epsilon\cdot\tilde{p}_\epsilon)^* + \text{c.c}]\frac{1}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v \\ &+ \frac{i}{m_X}[(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu - \text{c.c}]\bar{p}_v S_N^\mu p_v\Big] \end{aligned} \tag{C.45}$$

The leading order in momentum is linear, hence only the first row contributes to leading order. Writing in terms of basis operators

$$i\tilde{M}_p^{Ad} \approx 2\mathcal{S}_{\mu\nu}q^\nu\bar{p}_v S_N^\mu p_v \hat{\mathcal{A}}_p(\hat{C}^6_{10,q}) \tag{C.46}$$

C.2.1.3.5 Type e

$$\begin{aligned} i\tilde{M}_p^{Ae} = i\hat{\mathcal{A}}_p(\hat{C}^6_{12,q})\Big[&(-i)iq^\nu[\epsilon^*_\mu\epsilon_\nu - \text{c.c}]\bar{p}_v S_N^\mu p_v \\ &- \frac{1}{m_X}q^\nu[\epsilon^*_\nu\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}]\frac{1}{2m_N}\bar{p}_v\tilde{K}\cdot S_N p_v \\ &- \frac{1}{m_X}[(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu + \text{c.c}]\bar{p}_v S_N^\mu p_v\Big] \end{aligned} \tag{C.47}$$

The leading order in momentum is linear, hence only the first row contributes to leading order. Writing in terms of basis operators

$$i\tilde{M}_p^{Ae} \approx 2i\bar{\mathcal{S}}_{\mu\nu}q^\nu\bar{p}_v S_N^\mu p_v \hat{\mathcal{A}}_p(\hat{C}^6_{12,q}) \tag{C.48}$$

C.2.1.4 Gluon current

Using equation (B56) in [13].

$$iM_p^{G(6)} = -i\frac{2m_G}{27}\epsilon_\mu^*\epsilon^\mu\bar{p}_v p_v\hat{C}_1^{(6)} \tag{C.49}$$

C.2.1.5 Dual gluon current

Using equation (B57) in [13].

$$iM_p^{\theta(6)} = -i\bar{p}_v iq\cdot S_N p_v\epsilon_\mu^*\epsilon^\mu\hat{C}_2^{(6)}\left[D\left(\frac{\tilde{m}}{m_u}+\frac{\tilde{m}}{m_s}\right)+F\left(\frac{\tilde{m}}{m_u}-\frac{\tilde{m}}{m_s}\right)+G\right] \tag{C.50}$$

C.2.2 Meson exchange diagram (right diagram in Fig. 2.1)

J_q^P, J_q^A and J^θ contribute to the right diagram [13].

The hadronized QCD Lagrangian is given in equation (B23) in [13].

$$\mathcal{L}_{\text{HBChPT}}^{(1),\text{QCD}} \supset \frac{g_a}{f}\Big(\bar{p}_v\, iq\cdot S_N p_v - \bar{n}_v\, iq\cdot S_N n_v\Big)\pi^0 + \frac{\Delta u+\Delta d-2\Delta s}{\sqrt{3}f}\Big(\bar{n}_v\, iq\cdot S_N n_v + \bar{p}_v\, iq\cdot S_N p_v\Big)\eta + \cdots \tag{C.51}$$

The meson propagator is given in equation (A.47) in [26]

$$\frac{i}{q^2 - m_{(\pi/\eta)}^2} \tag{C.52}$$

C.2.2.1 Pseudo-scalar current

Using equation (B44) in [13]

$$\hat{J}_{u,d}^P = B_0 f m_{u,d}\Big(\pm\pi^0 + \frac{1}{\sqrt{3}}\eta\Big) + \cdots \qquad \hat{J}_s^P = -\frac{2}{\sqrt{3}}B_0 f m_s\eta + \cdots \tag{C.53}$$

$$\begin{aligned}\mathcal{L}_\chi^{P(6)} =& fB_0\chi_\mu^*\chi^\mu\Big[\pi^0\left(\hat{C}_{2,u}^{(6,0)}m_u - \hat{C}_{2,d}^{(6,0)}m_d\right)\\ &+\frac{\eta}{\sqrt{3}}\left(\hat{C}_{2,u}^{(6,0)}m_u + \hat{C}_{2,d}^{(6,0)}m_d - 2\hat{C}_{2,s}^{(6,0)}m_s\right)\Big]\end{aligned} \tag{C.54}$$

The scattering amplitude is

$$
\begin{aligned}
iM_p^P =& \bar{p}_v iq\cdot S_N p_v \epsilon_\mu^* \epsilon^\mu B_0 i^2 \Big[g_a \frac{i}{q^2 - m_\pi^2} \left(\hat{C}_{2,u}^{(6)} m_u - \hat{C}_{2,d}^{(6)} m_d \right) \\
& + \frac{(\Delta u + \Delta d - 2\Delta s)}{\sqrt{3}\sqrt{3}} \frac{i}{q^2 - m_\eta^2} \left(\hat{C}_{2,u}^{(6)} m_u + \hat{C}_{2,d}^{(6)} m_d - 2\hat{C}_{2,s}^{(6)} m_s \right) \Big]
\end{aligned}
\tag{C.55}
$$

The additional i^2 comes from the usual vertex coefficient (two vertices in this case). It is also important to note that quark current is $J^P = \bar{q} i\gamma^5 q$ and thus includes an i. The DM-neutron interaction is obtained by letting $p \to n, u \to d$.

C.2.2.2 Axial current

Using equation (B43) in [13]

$$
(\hat{J}_{u,d}^A)_\mu = f\Big(\mp \partial_\mu \pi^0 - \frac{\partial_\mu \eta}{\sqrt{3}} \Big) + \cdots \quad (\hat{J}_s^A)_\mu = \frac{2f}{\sqrt{3}} \partial_\mu \eta + \cdots \tag{C.56}
$$

Let

$$
\begin{aligned}
\hat{\mathcal{B}}_p(\hat{C}_{n,q}^d) \equiv & g_a \frac{1}{q^2 - m_\pi^2} \Big(\hat{C}_{n,u}^{(d)} - \hat{C}_{n,d}^{(d)} \Big) \\
& \frac{(\Delta u + \Delta d - 2\Delta s)}{\sqrt{3}\sqrt{3}} \frac{1}{q^2 - m_\eta^2} \left(\hat{C}_{n,u}^{(d)} + \hat{C}_{n,d}^{(d)} - 2\hat{C}_{n,s}^{(d)} \right)
\end{aligned}
\tag{C.57}
$$

Note the sign difference occurring in $\hat{\mathcal{A}}_p(\hat{C}_{n,q}^d)$ on the first row of equation (C.57) when letting $p \to n, u \to d$, which must be done to get the expression for the DM-neutron interaction. It is also important to note that $\partial_\mu \pi = \partial_\mu \eta = iq_\mu$.

C.2.2.2.1 Type a

$$
\begin{aligned}
iM_p^{Aa} = i^2 \bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}_{4,q}^6) \Big[& 2m_X v\cdot q \epsilon_\mu^* \epsilon^\mu \\
& + i(-i) \epsilon_\mu^* \epsilon^\mu \tilde{P}\cdot q \\
& + 2\frac{1}{m_X} v\cdot q \epsilon_\mu^* \tilde{p}_2^\mu \epsilon_\nu \tilde{p}_1^\nu \Big]
\end{aligned}
\tag{C.58}
$$

The leading order in momentum would be quadratic but due to $v\cdot q = \mathcal{O}(q^2)$ it is actually order 3. One can simplify the expression

$$
\begin{aligned}
iM_p^{Aa} = i^2\bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}^6_{4,q})\Big[&2\epsilon^*_\mu \epsilon^\mu \tilde{P}\cdot q \\
&+2\frac{1}{m_X} v\cdot q\epsilon^*_\mu \tilde{p}_2^\mu \epsilon_\nu \tilde{p}_1^\nu\Big]
\end{aligned}
\tag{C.59}
$$

and see that is not Galilean invariant. This is however not an issue since this amplitude is associated to the interaction operator $\mathcal{Q}^{(6)}_{4,q}$ which has its leading order contribution from the left diagram (see equation (C.43)).

C.2.2.2.2 Type b

$$
\begin{aligned}
iM_p^{Ab} = i^2\bar{p}_v iq\cdot S_N p_v \epsilon^{\mu\nu\rho\lambda}\hat{\mathcal{B}}_p(\hat{C}^6_{6,q})\Big[&-2m_X iq_\mu v_\rho \epsilon^*_\nu \epsilon_\lambda \\
&+(-i)\tilde{P}_\rho q_\mu \epsilon^*_\nu \epsilon_\lambda \\
&+(-1)\frac{i}{m_X}\tilde{P}_\rho q_\mu v_\nu [\epsilon^*_\lambda \epsilon\cdot p_\epsilon - \text{c.c}]\Big]
\end{aligned}
\tag{C.60}
$$

The leading order in momentum is quadratic, hence only the first row contributes to leading order. Writing in terms of basis operators

$$
iM_p^{Ab} \approx -2im_X \hat{\mathcal{S}}^{\mu\nu} q_\mu v_\nu \bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}^6_{6,q}) \tag{C.61}
$$

C.2.2.2.3 Type c

$$
\begin{aligned}
iM_p^{Ac} = i^2\bar{p}_v iq\cdot S_N p_v \epsilon^{\mu\nu\rho\lambda}\hat{\mathcal{B}}_p(\hat{C}^6_{8,q})\Big[&i(-i)q_\rho q_\mu \epsilon^*_\nu \epsilon_\lambda \\
&-\frac{i}{m_X} iv_\nu q_\rho q_\mu [\epsilon^*_\lambda \epsilon\cdot\tilde{p}_\epsilon - \text{c.c}]\Big]
\end{aligned}
\tag{C.62}
$$

Both terms are zero due to $q_\rho q_\mu \epsilon^{\mu\nu\rho\lambda} = 0$.

C.2.2.2.4 Type d

$$
\begin{aligned}
iM_p^{Ad} = i^2\bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}^6_{10,q})\Big[&(-i)q^\mu q^\nu [\epsilon^*_\nu \epsilon_\mu + \text{c.c}] \\
&+i\frac{1}{m_X} v\cdot q q^\nu [\epsilon_\nu (\epsilon\cdot\tilde{p}_\epsilon)^* + \text{c.c}] \\
&+\frac{i}{m_X} q^\mu [(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu - \text{c.c}]
\end{aligned}
\tag{C.63}
$$

The leading order in momentum is cubic, hence only the first row contributes to leading order. Writing in terms of basis operators

$$iM_p^{Ad} \approx 2iq\cdot\mathcal{S}\cdot q\bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}^6_{10,q}) \tag{C.64}$$

DM-neutron interaction is obtained by letting $p \to n, u \to d$.

C.2.2.2.5 Type e

$$\begin{aligned} iM_p^{Ae} =& i^2\bar{p}_v iq\cdot S_N p_v \hat{\mathcal{B}}_p(\hat{C}^6_{12,q})\Big[\\ & +i(-i)q^\mu q^\nu[\epsilon^*_\nu\epsilon_\mu - \text{c.c}] \\ & -\frac{1}{m_X}v\cdot qq^\nu[\epsilon^*_\nu\epsilon\cdot\tilde{p}_\epsilon - \text{c.c}] \\ & -\frac{1}{m_X}q^\mu[(\epsilon\cdot\tilde{p}_\epsilon)^* v\cdot\tilde{p}_{\epsilon 1}\epsilon_\mu + \text{c.c}]\Big] \end{aligned} \tag{C.65}$$

The leading order in momentum would be cubic but the term in second row is zero due to antisymmetrization. The term on the third row is actually of order 4 due to $v\cdot q = \mathcal{O}(q^2)$. The remaining term on the last row is of order 3 but is not Galilean. This is however not an issue since this amplitude is associated to the interaction operator $\mathcal{Q}^{(6)}_{12,q}$ which has its leading order contribution from the left diagram (see equation (C.47)).

C.2.2.2.6 Dual gluon current Using equation (B45) in [13]

$$\hat{J}^\theta = \frac{f}{2}\Big[\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\Big)\partial^2\pi^0 + \Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\Big)\frac{\partial^2\eta}{\sqrt{3}}\Big] \tag{C.66}$$

$$\mathcal{L}_\chi^{\theta(6)} = \Big(\chi^*_\mu\chi^\mu\hat{C}_2^{(6)}\Big)\frac{f}{2}\Big[\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\Big)\partial^2\pi^0 + \Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\Big)\frac{\partial^2\eta}{\sqrt{3}}\Big] \tag{C.67}$$

The scattering amplitude is

$$\begin{aligned} iM_p^\theta =& \Big(\bar{p}_v iq\cdot S_N p_v \epsilon^*_\mu\epsilon^\mu\hat{C}_2^{(6)}\Big)\cdot \\ & \frac{i^2q^2}{2}\Big[g_a\frac{i}{q^2-m_\pi^2}\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\Big) + \frac{(\Delta u + \Delta d - 2\Delta s)}{\sqrt{3}\sqrt{3}}\frac{i}{q^2-m_\eta^2}\Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\Big)\Big] \end{aligned} \tag{C.68}$$

C.3 Effective Lagrangian and basis operators

The effective scattering Lagrangian $\mathcal{L}_{\text{eff}}$ is

$$\mathcal{L}_{\text{eff}} = \sum_{i,d} \left(c_{i,p}^{(d)}(q^2)\, Q_{i,p}^{(d)} + c_{i,n}^{(d)}(q^2)\, Q_{i,n}^{(d)} \right) \tag{C.69}$$

where $Q_i^{(d)}$ are operators of momentum order d enumerated by i, which together form a basis which is weighted by the coefficients $c_i^{(d)}$. The two terms in equation (C.69) corresponds to DM-proton scattering and DM-neutron scattering respectively. The basis operators of order zero are

$$Q_{1,p}^{(0)} = \epsilon_\mu^* \epsilon^\mu \bar{p}_v p_v \qquad Q_{2,p}^{(0)} = S_X^\mu \bar{p}_v S_{N\mu} p_v \tag{C.70}$$

The basis operators of order one are

$$Q_{1,p}^{(1)} = \epsilon_\mu^* \epsilon^\mu \bar{p}_v iq \cdot S_N p_v \qquad Q_{2,p}^{(1)} = iq \cdot S_X \bar{p}_v p_v \tag{C.71}$$

$$Q_{3,p}^{(1)} = \epsilon_\mu^* \epsilon^\mu \bar{p}_v v_\perp \cdot S_N p_v \qquad Q_{4,p}^{(1)} = v_\perp \cdot S_X \bar{p}_v p_v \tag{C.72}$$

$$Q_{5,p}^{(1)} = i\epsilon^{\alpha\beta\mu\nu} v_\alpha q_\beta S_{X\mu} \bar{p}_v S_{N\nu} p_v \qquad Q_{6,p}^{(1)} = \bar{\mathcal{S}}_{\mu\nu} q^\nu v_\perp^\mu \bar{p}_v p_v \tag{C.73}$$

$$Q_{7,p}^{(1)} = q^\nu \bar{\mathcal{S}}_{\mu\nu} \bar{p}_v S_N^\mu p_v \qquad Q_{8,p}^{(1)} = v_\perp \cdot \mathcal{S} \cdot q \bar{p}_v p_v \tag{C.74}$$

$$Q_{9,p}^{(1)} = q^\nu \mathcal{S}_{\mu\nu} \bar{p}_v S_N^\mu p_v \tag{C.75}$$

where $i\hat{\mathcal{S}}^{\mu\nu} v_\nu \equiv S_X^\mu$ was used. The basis operators of order two are

$$Q_{1,p}^{(2)} = iq \cdot S_X \bar{p}_v iq \cdot S_N p_v \qquad Q_{2,p}^{(2)} = q^\nu \bar{\mathcal{S}}_{\mu\nu} \bar{p}_v \mathcal{W}^\mu p_v \tag{C.76}$$

$$Q_{3,p}^{(2)} = q \cdot \mathcal{S} \cdot q \bar{p}_v p_v \qquad Q_{4,p}^{(2)} = q^\nu \mathcal{S}_{\mu\nu} \bar{p}_v \mathcal{W}^\mu p_v \tag{C.77}$$

$$Q_{5,p}^{(2)} = \epsilon^{\mu\nu\rho\lambda} v_\nu q_\rho \mathcal{S}_{\lambda\kappa} q^\kappa \bar{p}_v S_{N\mu} p_v \tag{C.78}$$

$$Q_{6,p}^{(2)} = -q_\nu \hat{\mathcal{S}}^{\mu\nu} \bar{p} S_{N\mu} p_v + \frac{i}{2m_N} S_X^\nu q_\nu \bar{p}_v \tilde{K} \cdot S_N p_v + \epsilon^{\mu\nu\rho\lambda} \frac{1}{m_X} v_\nu q_\rho \bar{\mathcal{S}}_{\lambda\kappa} \tilde{P}^\kappa \tag{C.79}$$

The basis operators of order three are

$$Q_{1,p}^{(3)} = q \cdot \mathcal{S} \cdot q \bar{p}_v iq \cdot S_N p_v \tag{C.80}$$

An analogous set of operators corresponding to DM-neutron interaction is obtained by letting $p \to n$. All the scattering amplitudes can be written in terms of these basis operators. The coefficients of order zero are

$$
\begin{aligned}
c_{1,p}^{(0)} =& \Big[\hat{C}_{1,u}^{(6)}\sigma_u^p + \hat{C}_{1,d}^{(6)}\sigma_d^p + \hat{C}_{1,s}^{(6)}\sigma_s^p\Big] + 2m_X(2\hat{C}_{3,u}^6 + \hat{C}_{3,d}^6) - \frac{2m_G}{27}\hat{C}_1^{(6)} \\
c_{2,p}^{(0)} =& 2m_X\hat{\mathcal{A}}(\hat{C}_{6,q}^6)
\end{aligned}
\tag{C.81}
$$

The coefficients of order one are

$$
\begin{aligned}
c_{1,p}^{(1)} =& B_0\Big[g_a\frac{1}{m_\pi^2 - q^2}\left(\hat{C}_{2,u}^{(6)}m_u - \hat{C}_{2,d}^{(6)}m_d\right) \\
&+ \frac{(\Delta u + \Delta d - 2\Delta s)}{3}\frac{1}{m_\eta^2 - q^2}\left(\hat{C}_{2,u}^{(6)}m_u + \hat{C}_{2,d}^{(6)}m_d - 2\hat{C}_{2,s}^{(6)}m_s\right)\Big] \\
&+\hat{C}_2^{(6,0)}\Big\{-\Big[D\Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_s}\Big) + F\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_s}\Big) + G\Big] \\
&+ \frac{q^2}{2}\Big[g_a\frac{1}{m_\pi^2 - q^2}\left(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\right) + \frac{(\Delta u + \Delta d - 2\Delta s)}{3}\frac{1}{m_\eta^2 - q^2}\left(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\right)\Big]\Big\}
\end{aligned}
\tag{C.82}
$$

$$
c_{2,p}^{(1)} = (2\hat{C}_{7,u}^6 + \hat{C}_{7,d}^6) \qquad c_{3,p}^{(1)} = 2m_X\hat{\mathcal{A}}(\hat{C}_{4,q}^6) \tag{C.83}
$$

$$
c_{4,p}^{(1)} = -2m_X(2\hat{C}_{5,u}^6 + \hat{C}_{5,d}^6) \qquad c_{5,p}^{(1)} = 2m_X\hat{\mathcal{W}}_p(\hat{C}_{5,q}^6) \tag{C.84}
$$

$$
c_{6,p}^{(1)} = -2(2\hat{C}_{11,u}^6 + \hat{C}_{11,d}^6) \qquad c_{7,p}^{(1)} = 2\hat{\mathcal{A}}(\hat{C}_{12,q}^6) \tag{C.85}
$$

$$
c_{8,p}^{(1)} = 2i(2\hat{C}_{9,u}^6 + \hat{C}_{9,d}^6) \qquad c_{9,p}^{(1)} = -2i\hat{\mathcal{A}}(\hat{C}_{10,q}^6 \tag{C.86}
$$

The coefficients of order two are

$$
c_{1,p}^{(2)} = 2m_X\hat{\mathcal{B}}_p(\hat{C}_{6,q}^6) \qquad c_{2,p}^{(2)} = 2i\hat{\mathcal{W}}_p(\hat{C}_{11,q}^6) \tag{C.87}
$$

$$
c_{3,p}^{(2)} = -\frac{1}{m_X}(2\hat{C}_{11,u}^6 + \hat{C}_{11,d}^6) \qquad c_{4,p}^{(2)} = 2\hat{\mathcal{W}}_p(\hat{C}_{9,q}^6) \tag{C.88}
$$

$$
c_{5,p}^{(2)} = \frac{1}{m_X}\hat{\mathcal{A}}_p(\hat{C}_{8,q}^6) \qquad c_{6,p}^{(2)} = \hat{\mathcal{A}}_p(\hat{C}_{8,q}^6) \tag{C.89}
$$

The coefficients of order three are

$$
c_{1,p}^{(3)} = 2\hat{\mathcal{B}}_p(\hat{C}_{10,q}^6) \tag{C.90}
$$

Coefficients corresponding to DM-neutron interactions are obtained by letting $p \to n, u \to d$.

D

Manifestly non-relativistic basis operators

As previously mentioned the basis operators in section C.3 are non-relativistic and obey Galilean symmetry but are written in a Lorentz covariant notation. In order to compare these basis operators with the results obtained from calculations using non-relativistic theory as the underlying theory the basis operators will be rewritten here in a manifestly non-relativistic notation. This will also be useful since it enables streamlined usage of the Mathematica package in [17]. In the non-relativistic limit the polarization vector can be written as [15]

$$\epsilon_s^\mu \simeq \begin{pmatrix} \frac{1}{2m_X}(\boldsymbol{P}+\boldsymbol{q})\cdot\boldsymbol{e}_s \\ \boldsymbol{e}_s \end{pmatrix} \qquad \epsilon_{s'}^{\mu*} \simeq \begin{pmatrix} \frac{1}{2m_X}(\boldsymbol{P}-\boldsymbol{q})\cdot\boldsymbol{e}'_{s'} \\ \boldsymbol{e}'_{s'} \end{pmatrix} \tag{D.1}$$

where $\boldsymbol{e}_s$ is the non-relativistic polarization vector with polarization s. Explicit mention of the polarization states will be omitted below. The "mostly negative" metric convention will be used such that $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. Using this, and only keeping terms with leading order in momentum, $\epsilon_\mu^*\epsilon^\mu$ is

$$\begin{aligned} \epsilon_\mu^*\epsilon^\mu \simeq& \frac{1}{4m_X^2}(\boldsymbol{P}+\boldsymbol{q})\cdot\boldsymbol{e}(\boldsymbol{P}-\boldsymbol{q})\cdot\boldsymbol{e}' - \boldsymbol{e}'\cdot\boldsymbol{e} \\ \approx& -\boldsymbol{e}'\cdot\boldsymbol{e} \end{aligned} \tag{D.2}$$

The operator $\mathcal{W}_\mu = v^\alpha q^\lambda S_N^\beta \epsilon_{\mu\alpha\beta\lambda}$ can be simplified when the reference vector is chosen to be timelike

$$\begin{aligned} \mathcal{W}_\mu =& v^\alpha q^\lambda S_N^\beta \epsilon_{\mu\alpha\beta\lambda} = q^\lambda S_N^\beta \epsilon_{\mu 0\beta\lambda} = -q^\lambda S_N^\beta \epsilon_{0\mu\beta\lambda} \\ =& \begin{cases} (\boldsymbol{q}\times\boldsymbol{S}_N)_\mu & \mu = 1,2,3 \\ 0 & \mu = 0 \end{cases} \end{aligned} \tag{D.3}$$

Hereafter the indices $i = 1, 2, 3$. Similarly, $i\hat{\mathcal{S}}^{\mu\nu} v_\nu$ can be simplified if v is chosen to be timelike

$$i\hat{\mathcal{S}}^{\mu\nu} v_\nu = -i(\boldsymbol{\epsilon}^* \times \boldsymbol{\epsilon})^i = -i(\boldsymbol{e}' \times \boldsymbol{e})^i \equiv S_X^i \tag{D.4}$$

Which then makes it clear that $i\hat{\mathcal{S}}^{\mu\nu} v_\nu = S_X^\mu$, where S_X^μ is the DM spin operator [15]. The other operator which involves $\hat{\mathcal{S}}$ is $\hat{\mathcal{S}}^{\mu\nu} q_\nu \bar{p}_v S_{N,\mu} p_v$.

$$\begin{aligned}\hat{\mathcal{S}}^{\mu\nu} q_\nu S_{N,\mu} \approx& e_i' e_j q_\nu S_{N,\mu} \epsilon^{ij\mu\nu} = -e_i' e_j q_0 S_{N,k} \epsilon^{ijk0} - e_i' e_j q_k S_{N,0} \epsilon^{ij0k} + e_i' e_j S_{N,k} q_l \epsilon^{ijkl} \\ =& (\boldsymbol{e}' \times \boldsymbol{e}) \cdot \boldsymbol{S}_N \, q^0 - (\boldsymbol{e}' \times \boldsymbol{e}) \cdot \boldsymbol{q} \, S_N^0 \\ =& i \frac{\boldsymbol{q} \cdot \boldsymbol{P}}{2m_X} \boldsymbol{S}_X \cdot \boldsymbol{S}_N\end{aligned} \tag{D.5}$$

where the last equality was obtained using $S_N^0 = 0$ which is due the reference vector being set to timelike.

Lastly let's address the operators involving $\bar{\mathcal{S}}_{\mu\nu} = \frac{1}{2}(\epsilon_\mu^* \epsilon_\nu - \epsilon_\mu \epsilon_\nu^*)$. First note that $\epsilon_\mu^* \epsilon_\nu = e_i' e_j + \mathcal{O}(q)$ which means that

$$\bar{\mathcal{S}}_{\mu\nu} \xrightarrow{\text{Leading order}} \frac{1}{2}(e_i' e_j - e_i e_j') = \frac{1}{2}(\boldsymbol{e}' \times \boldsymbol{e})^k \epsilon_{ijk} = \frac{i}{2} \boldsymbol{S}_X^k \epsilon_{ijk} \tag{D.6}$$

So for example

$$\bar{\mathcal{S}}_{\mu\nu} q^\nu v_\perp^\mu \approx \frac{i}{2} \boldsymbol{S}_X^k \epsilon_{ijk} v_\perp^i q^j = \frac{i}{2} \boldsymbol{S}_X \cdot (\boldsymbol{v}_\perp \times \boldsymbol{q}) \tag{D.7}$$

Another example is

$$\bar{\mathcal{S}}_{\mu\nu} \mathcal{W}^\mu q^\nu \approx \frac{i}{2} \boldsymbol{S}_X \cdot (\boldsymbol{q} \times (\boldsymbol{q} \times \boldsymbol{S}_N)) = \frac{i}{2} S_X^k \left(q_k (\boldsymbol{q} \cdot \boldsymbol{S}_N) - S_{N,k} q^2 \right) \tag{D.8}$$

Lastly, consider the term

$$
\begin{aligned}
&\epsilon^{\mu\nu\rho\lambda}\frac{1}{m_X}v_\nu q_\rho(\bar{\mathcal{S}}_{\lambda\kappa}\tilde{P}^\kappa+\mathcal{S}_{\lambda\kappa}q^\kappa)S_{N\mu} \\
=&\frac{1}{m_X}\epsilon^{\mu 0\rho\lambda}S_{N,\mu}q_\rho\tilde{P}^\kappa\bar{\mathcal{S}}_{\lambda\kappa}+\frac{1}{m_X}\epsilon^{\mu 0\rho\lambda}S_{N,\mu}q_\rho q^\kappa\mathcal{S}_{\lambda\kappa} \\
\approx&\frac{-i}{m_X}\epsilon^{0\lambda\mu\rho}\epsilon_{\lambda ij}S_X^j\tilde{P}^iS_{N,\mu}q_\rho-\frac{1}{m_X}\epsilon^{0\lambda\mu\rho}S_{N,\mu}q_\rho q^\kappa\mathcal{S}_{\lambda\kappa} \\
=&\frac{-i}{m_X}(\delta_i^\mu\delta_j^\rho-\delta_i^\rho\delta_j^\mu)S_x^j\tilde{P}^iS_{N,\mu}q_\rho-\frac{1}{m_X}(\boldsymbol{S}_N\times\boldsymbol{q})\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{q} \\
=&-\frac{i}{2m_X}\boldsymbol{S}_X\cdot\boldsymbol{q}\boldsymbol{P}\cdot\boldsymbol{S}_N+\frac{i}{2m_X}\boldsymbol{S}_X\cdot\boldsymbol{S}_N\boldsymbol{q}\cdot\boldsymbol{P}-\frac{1}{m_X}(\boldsymbol{S}_N\times\boldsymbol{q})\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{q}
\end{aligned}
\tag{D.9}
$$

D.1 Relation to the basis of Anand et al.

In the non-relativistic limit, almost all of the basis operators in section C.3 coincide with the basis operators in [17], up to some small modification such as sign (The ones which does not coincide involves $\mathcal{S}$ and will be addressed in the next section). $\boldsymbol{q}$ is defined differently in [17] which leads to difference in sign when compared to the definition used in this work. The operators in [17] will be rewritten using the definition of $\boldsymbol{q}$ used here. The relevant operators from [17] are

$$\mathcal{O}_{1,N}=1_X1_N \qquad \mathcal{O}_{4,N}=\boldsymbol{S}_X\cdot\boldsymbol{S}_N \tag{D.10}$$

$$\mathcal{O}_{5,N}=\boldsymbol{S}_X\cdot\left(\boldsymbol{v}_\perp\times\frac{i\boldsymbol{q}}{m_N}\right)1_N \qquad \mathcal{O}_{6,N}=\left(\boldsymbol{S}_X\cdot\frac{\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N\cdot\frac{\boldsymbol{q}}{m_N}\right) \tag{D.11}$$

$$\mathcal{O}_{7,N}=1_X\left(\boldsymbol{S}_N\cdot\boldsymbol{v}_\perp\right) \qquad \mathcal{O}_{8,N}=\left(\boldsymbol{S}_X\cdot\boldsymbol{v}_\perp\right)1_N \tag{D.12}$$

$$\mathcal{O}_{9,N}=\boldsymbol{S}_X\cdot\left(\frac{i\boldsymbol{q}}{m_N}\times\boldsymbol{S}_N\right) \qquad \mathcal{O}_{10,N}=-1_X\left(\boldsymbol{S}_N\cdot\frac{i\boldsymbol{q}}{m_N}\right) \tag{D.13}$$

$$\mathcal{O}_{11,N}=-\left(\boldsymbol{S}_X\cdot\frac{i\boldsymbol{q}}{m_N}\right)1_N \qquad \mathcal{O}_{14,N}=-\left(\boldsymbol{S}_X\cdot\frac{i\boldsymbol{q}}{m_N}\right)(\boldsymbol{S}_N\cdot\boldsymbol{v}_\perp) \tag{D.14}$$

with $N=p,n$. The relation between the coefficients associated to the basis operators in section C.3 and the coefficients associated to the basis introduced in [17] is presented in the following equations.

$$-c_{1,N}^{(0)} = c_{1,N}^{\mathrm{NR}} \qquad -c_{2,N}^{(0)} = c_{4,N}^{\mathrm{NR}} \tag{D.15}$$
$$-m_N c_{1,N}^{(1)} = c_{10,N}^{\mathrm{NR}} \qquad m_N c_{2,N}^{(1)} = c_{11,N}^{\mathrm{NR}} \tag{D.16}$$
$$c_{3,N}^{(1)} = c_{7,N}^{\mathrm{NR}} \qquad -c_{4,N}^{(1)} = c_{8,N}^{\mathrm{NR}} \tag{D.17}$$
$$-m_N c_{5,N}^{(1)} = c_{9,N}^{\mathrm{NR}} \qquad \frac{m_N}{2} c_{6,N}^{(1)} = c_{5,N}^{\mathrm{NR}} \tag{D.18}$$
$$-\frac{m_N}{2} c_{7,N}^{(1)} = c_{9,N}^{\mathrm{NR}} \qquad \frac{iq^2}{2\mu_N} c_{8,N}^{(1)} = c_{4,N}^{\mathrm{NR}} \tag{D.19}$$
$$-m_N^2 c_{1,N}^{(2)} = c_{6,N}^{\mathrm{NR}} \qquad \frac{im_N^2}{2} c_{2,N}^{(2)} = c_{6,N}^{\mathrm{NR}} \tag{D.20}$$
$$m_N c_{6,N}^{(2)} = c_{14,N}^{\mathrm{NR}} \qquad -\frac{iq^2}{2} c_{2,N}^{(2)} = c_{4,N}^{\mathrm{NR}} \tag{D.21}$$

where $\mu_N = m_X m_N/(m_X + m_N)$.

D.1.1 Additional operators

The operators which consist of $\mathcal{S}$ are yet to be addressed. In [15] the list of basis operators of Anand et al. has been extended to include the non-relativistic operators which consist of $\boldsymbol{\mathcal{S}}$. The relevant ones are

$$\mathcal{O}_{17,N} = -i\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{v}_\perp 1_N \qquad \mathcal{O}_{18,N} = -i\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\boldsymbol{S}_N \tag{D.22}$$
$$\mathcal{O}_{19,N} = \frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N} \qquad \mathcal{O}_{20,N} = \left(\boldsymbol{S}_N \times \frac{\boldsymbol{q}}{m_N}\right)\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N} \tag{D.23}$$

Basis operators of orders higher than 2 in q are not defined in [15]. In this work there is however necessary (see section C.2.2.2.4) to include one order 3 operator

$$\mathcal{O}_N^{(3)} = -\left(\frac{\boldsymbol{q}}{m_N}\cdot\boldsymbol{\mathcal{S}}\cdot\frac{\boldsymbol{q}}{m_N}\right)\frac{i\boldsymbol{q}}{m_N}\cdot\boldsymbol{S}_N \tag{D.24}$$

The relations between the coefficients are

$$-im_N c_{9,N}^{(1)} = c_{17,N}^{\mathrm{NR}} \qquad -im_N c_{10,N}^{(1)} = c_{18,N}^{\mathrm{NR}} \tag{D.25}$$
$$m_N^2 c_{3,N}^{(2)} = c_{19,N}^{\mathrm{NR}} \qquad -m_N^2 c_{4,N}^{(2)} = c_{20,N}^{\mathrm{NR}} \tag{D.26}$$
$$-m_N^2 c_{5,N}^{(2)} = c_{20,N}^{\mathrm{NR}} \qquad m_N^3 c_{1,N}^{(3)} = c_N^{\mathrm{NR}(3)} \tag{D.27}$$

E

Modification of the Mathematica package

In this appendix it is explained how to modify the source code of the Mathematica package by Anand et al. [17] so that the non-relativistic interaction operators unique to spin 1 DM (equation (2.54) and (2.55)) are incorporated into the script. The inclusion of the additional operators will alter the expression for the DM response functions $R^{\tau\tau'}$ as described in [15].

The modification of the script starts with extending the dimensions of the vectors `cnvector` and `cpvector` from 15 to 20 (or to whatever the new total number of non-relativistic operators is). They are both defined in the beginning of the script.
Next, search for
`For[iii=1,iii<=15,iii++,`
and change it to
`For[iii=1,iii<=20,iii++,`
Next, search for
`If[Op==1, coeff=(4mN*mchiFORMAL/mV^ 2)coeffdimless;];`
and add both
`If[Op==17, coeff=(4mN*mchiFORMAL/mV^ 2)coeffdimless/mN;];`
and
`If[Op==20, coeff=(4mN*mchiFORMAL/mV^ 2)coeffdimless/mN^ 2;];`
Next, search for
`If[!(Integer[Op]&&1<=Op&&Op<=15),`
and change it to
`If[!(Integer[Op]&&1<=Op&&Op<=20),`
An optional step is to also expand the list where all the symbolic expressions for the operators are defined, as to include the new operators. Doing this will make it so that the Lagrangian that is printed out when running the script is correct. The list is found by searching for `Op[1]="1";`. I would recommend extending the list by something simple like `Op[17]="Op17";` for each additional operator.

After having fixed the script to allow for a greater number of operators, the final step is to extend the DM response functions in the script. The DM response functions are found by searching for `DMResponseCoeff[MJ]`. Then simply extend the response functions with the new terms associated to the new operators.

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