

On chassis predictive maintenance and service solutions: An unsupervised machine learning approach

Master's thesis in Complex Adaptive Systems

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Department of Mechanics and Maritime Sciences Division of Vehicle Engineering and Autonomous Systems Vehicle Dynamics group CHALMERS UNIVERSITY OF TECHNOLOGY Gothenburg, Sweden 2019 On chassis predictive maintenance and service solutions: An unsupervised machine learning approach NASTARAN SOLTANIPOUR © NASTARAN SOLTANIPOUR, 2019

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Master's Thesis 2019:41 Department of Mechanics and Maritime Sciences Division of Vehicle Engineering and Autonomous Systems Vehicle Dynamics group Chalmers University of Technology SE-412 96 Gothenburg Sweden Telephone: + 46 (0)31-772 1000 On chassis predictive maintenance and service solutions: An unsupervised machine learning approach NASTARAN SOLTANIPOUR Department of Mechanics and Maritime Sciences Division of Vehicle Engineering and Autonomous Systems Vehicle Dynamics group Chalmers University of Technology

Abstract

Predictive maintenance is a key component in cost reduction in automotive industry and is of great importance. Besides cutting the costs, predictive maintenance can improve feeling of comfort and safety, by early detection, isolation and prediction of prospective failures. That is why automotive industry and fleet managers are turning to predictive analytics to maintain a lead position in industry. In order to predict and mitigate components failure in advance, measurements data from vehicle parts are collected from sensory system mounted on vehicle parts, and employed to evaluate the health condition of the components. An unsupervised learning solution is proposed, in this work, for automatic processing, diagnosis and isolation of faults. This solution is used for advanced analytics of the collected time-series data in the back-end (assuming that faults were reported based on spectral analysis). A literature study on maintenance types with emphasis on predictive maintenance with application to chassis failure is done. Chassis failures and conventional failure detection techniques are also covered in this study.

A data acquisition was done at Hällared test track and labelled data regarding error states of interest were collected. Performance of the proposed method was validated by automatics clustering of two error states, i.e. low tyre pressure and faulty wheel hub.

Key words: predictive maintenance, chassis failure detection, unsupervised machine learning

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1 Introduction

1.1 Background and motivations

Maximizing reliability while keeping the costs down is the key strategy to ensure a leading position in automotive industry. To satisfy these requirements is it essential to develop a framework capable of detecting, isolating and predicting of the prospective failures of the system to ensure reliability of the system as well as cutting the costs of repair or replacement of the components. Predictive maintenance is the key solution to this issue. Predictive maintenance is a condition-based monitoring and maintenance planning solution in which periodic or continuous evaluations of the system is conducted to prevent possible failures of the system. Assessments can be conducted on stored data from data clouds or historic datasets.

Data-driven descriptive and predictive approaches provides us powerful tools to detect, identify and characterize potential system failure modes during operation. These can be directly used for both root-cause analysis (to help system designers to design failure-free designs) and predictive analytics (to help system maintenance/service to diagnose/identify failures based on the unique signatures of each individual) of the potential failure modes in the operation. Machine learning techniques are being spread for service and maintenance solutions, which will be discussed more in the following literature study part. This thesis propose a machine learning based solution for automizing the process of classifiying and clustering of differente errors, after they are detected in the onboard-diagnostic. This, together with detailed discussion about the advantages and limitations of the methods will be discussed in the following.

1.2 Previous work

Considering the above-mentioned issues, the Climate and Motion Control Department (former Vehicle Control and Chassis – Strategy, Innovation and Integration group), within the Research & Development at Volvo Cars, started to investigate & expand the pallet of data analytics and machine learning tools for the purpose of chassis error state detection, monitoring, and maintenance solutions. These advance engineering and method development projects, started from early 2018, resulted in a patent application that was filed under patent Number 16/410,850 [1]. In brief, it consists of three packages:

1) Real-time fault detection solution in an on board diagnosis system, via an spectrum analysis solution.

2) A solution (i.e. a supervised Tree-based machine learning approach) for finding features/signals that are important for classification and root cause analysis of faults/errors of interest. The output of this analysis guides the design/improvement of both detection stage (1) and also isolation/diagnostic for the offline solutions in the back-end in stage (2).

3) Automating the process of fault isolation, clustering and diagnostics via an unsupervised machine learning approach.

This thesis scope is to address the third stage of the-above mentioned patent application. This thesis contributes there, by proposing an Hidden Markov Model-based sequence tagging approach that enables us to automatically annotate/label the collected fault/error data in order to reduce the high amount of cost in labelling of time-series that corresponds to different faults/error states.

1.3 Problem statement

Managers of car sharing and taxi service fleets, are turning to predictive analytics to stay on top of maintenance and predict and mitigate components failure, in advance. Sensors on vehicles collect data for assessing the health of different systems and components. Particularly speaking about chassis, these sensors gather information regarding tyre pressure, wheel hub bearings, hydraulics in the dampers, brakes and so on. However, process of analysis of vehicle sensors in order to diagnose and isolate the faults is an expensive task. The main focus of this thesis is to address this issue by proposing an unsupervised machine learning solution for fault clustering as well as differing different error states as different classes.

1.4 Aims and objectives

This thesis project aims at:

• Literature study on data-driven tools for predictive maintenance solutions in automotive industry.

• Analysis of the available test data, including data cleaning/labelling for creating ground truth labelling of error states of interests. (Collection of more test data, in case needed). Collection and analysis of data concerns low tyre pressure, faulty/noisy wheel hub, and faulty damper error states.

• Develop and apply unsupervised Machine Learning tools for automating the analysis of collected time-series data, to conduct chassis failures detection.

• Using the collected test data (with ground truth labels), validate the performance of the method and results compilation.

1.5 Report structure

The rest of the report is organized as follows: chapter two is dedicated to literature study, mainly discussing different maintenance methods, with the main focus on predictive maintenance, main approaches in predictive maintenance, such as vibration analysis and machine learning approach. Study of different failures occurring in chassis and different machine learning approaches to address clustering problems.

Chapter three discusses the physical phenomenon describing low tyre pressure and faulty wheel hub as well as a brief history of existing detection techniques. Also and the proposed algorithm, data acquisition and data processing is described in this chapter.

Chapter four represent the results from low tyre pressure detection, and isolation of faulty wheel hub and low tire pressure states in form of figures, tables and confusion matrices

Chapter five is dedicated to the summary of the thesis, conclusions and proposals for the future works in chassis failure detection techniques.

1.6 Limitations and assumptions

The dataset employed in this study contains the normal driving state as well as data collected from the car while driving on faulty components. We assume that the error states are isolated and known and the data is collected from proper road surface conditions with no pothole or roughness. The data is not reflecting neither the error states occurring simultaneously in the car nor the transient phases between the normal and faulty states. The datasets collected from test tracks are stored in predefined circumstances and since the procedure of repeating the test is time consuming and needs access to the proving ground the data is limited to 4 different speed levels, three test tracks, and three different states, one normal and two faulty. On the other hand the test has been conducted in only one vehicle and one set of tire.

2 **Predictive maintenance**

2.1 Maintenance and service solutions

Over the course of years numerous maintenance methods have been proposed and employed in industries. All these methods share a common goal: increasing productivity and quality while preventing failure as well as limiting the downtime. All different maintenance methods fall into two main categories, namely: scheduled and unscheduled maintenance categories[2].

The first class of maintenance is corrective maintenance also known as "Run-to-Failure" and "Breakdown Maintenance". This type of maintenance is conducted when the machine suffers a breakdown or malfunction. This also can lead to multiple damages to the equipment or the system. This type of maintenance can be significantly expensive due to repair and replacement costs alongside with non-productive duration of downtime of the equipment[3]. It is estimated that only in US the cost of corrective maintenance every year is more than 200 million dollars which can be prevented with the help of more intelligent approaches[2].

Preventive maintenance is the second approach in maintenance classes which is a type of scheduled maintenance. Preventive maintenance tends to prevent failures, safety violation situations, production losses, unnecessary repairs, and aims to preserve original materials and equipment. This can be achieved by replacing the consumables at predefined time intervals and conducting regular time-based inspection of the system as well as identifying the rate of failures in the system[4].

The last and the most advanced maintenance and the most cost effective method is predictive maintenance. In this method the data from sensory system is evaluated continuously to find the trends in the data to assess the health of the system as well as to predict the prospective failures of the system. This is discussed more in the next following section[5].

2.1.1 Predictive maintenance

Predictive maintenance is generally described simply as estimation of the optimal time for performing maintenance on the system. In predictive maintenance the ultimate goal is to detect failure before they occur.

Predictive maintenance is capable and efficient in detection of the possible failures during early stages, which leads to decrease in unscheduled downtime. Shorter downtime means increase in productivity as well as quality and safety improvement. Predictive maintenance is capable of significant reduction of cost in industry. According to McKinsey Global Institute, predictive maintenance can save up to \$200 billion to \$600 billion for manufacturers by year 2025[6]. Figure (1) depicts an example of a comparison between corrective and predictive maintenance methods.

Predictive maintenance has become feasible, thanks to lower sensor prices and wireless data acquisition and data delivery mechanisms such as telematics and data clouds.

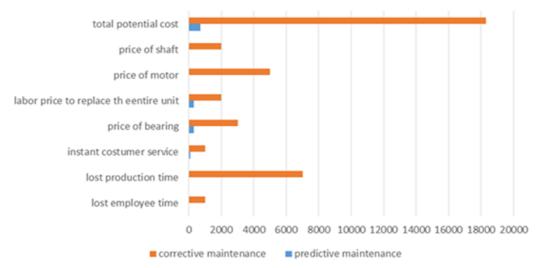


Figure 1: An example of cost (in US dollar) reduction in a factory. Figure borrowed from [6]

2.1.1.1 Predictive maintenance Technologies

Based on the nondestructive testing and detection methods employed to conduct condition monitoring in predictive maintenance, this maintenance class falls into different categories as follows[8][9]:

- Infrared-based analysis: Infrared (IR) Thermography is the process of generating visual images that represent variations in IR radiance of the surfaces of objects which is used in a wide range of applications and is the most cost efficient detection method. Infrared has the potential of detecting both electrical and mechanical failures.
- Acoustic-based analysis: it can be conducted both in sonic and ultrasonic levels and can be employed to detect friction and stress in machineries. Ultrasoundsbased analysis: Ultrasounds are sound waves that have a frequency of greater than 20 kHz. Non-contact Ultrasonic Detectors are utilized in PdM to detect airborne ultrasound. Contact probes are used for diagnostics with bearings, steam traps, etc. One of the main advantages of this method is the ability to detect high frequency sound which are not audible to human ear.

Vibration-based analysis: These solutions are used to identify faults in machinary/structures, plan repairs and service, which is the most commonly method used in industry and can provide information to identify prospective failures. This is done by utilizing different vibration sensors (accelerometers, velocity transducers and displacement probes). Common faults treated are rotary components imbalanace, resonance, bent shafts, rotor/stator faults, mechanical loosness, bearing failures and etc.

2.1.1.2 Predictive maintenance approaches

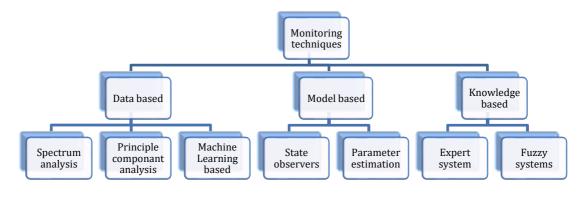


Figure 2: Predictive maintenance methods based on monitoring techniques. Figure borrowed from review paper [9]

This section is dedicated to the main approaches in predictive maintenance, there are three main categories in condition monitoring techniques, namely: Data methods, Model Based Methods, and Knowledge Based Methods. Data based method employ available historical data to conduct failure detection. Principal component analysis (PCA) is an instance of such methods in which orthogonal transformations are employed to diminish the dimension of possible correlated observations to a reduced number of uncorrelated variables. In fact PCA is a dimension reduction method which makes it easier to process and monitor large dimensional datasets. Fault detection can be conducted by change detection in the converted data[10].

The main idea in model based methods is to compare the actual measured output of the system with the output from the mathematical model. State observer method is a model based method functioning upon this fact that changes in input-output behavior directly affect the state variables of the state space model. State observer can be employed for fault detection if it is possible to model the error as changes in state variables[10].

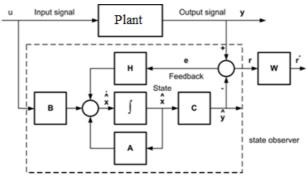


Figure 3: State observer detection structure [9]

Knowledge Based Methods are feasible when experts and experience is available. However acquiring deep understanding of a system is costly and hard if not impossible. Fuzzy logic is a knowledge based system that is capable of providing different level of severity of the fault in the alarm system. In figure (2) different detection methods are introduced[10]. All the methods above fall into two broader categories as: rule based methods and machine learning methods. Rule based maintenance also known as condition monitoring is the continuous analysis of the sensory data for triggering an alarm signal upon a predefined rule or threshold. In this method once the common reasons for system failure are stablished having the model of the system network, engineers can build up a set of "if- then" rule to control the behavior and interdependencies of the system. Like fuzzy logic and spectrum analysis methods like fast Fourier transform[11].

Though these systems provide some level of autonomy to the system, still these sort of predictive maintenance algorithms need expert knowledge to derive the rules based on the system requirements [11].

Unlike rule based methods, in machine learning methods the historical data or data from online clouds are fed as inputs to a number of different algorithms namely: classification and clustering algorithms, pattern recognition systems using neural networks, and PCA data analysis method. The general architecture of different intelligent methods includes slightly same steps: data acquisition, data clean up, feature extraction or feature reduction and in the final step failure detection or prediction. The structure is presented in figure (4) bellow[10][11].



Figure 4: General structure of machine learning approaches

2.2 Vehicle hardware failures

Automotive failures are prevalent enough to be experienced by almost everyone during lifetime. These failures can be minor or catastrophic, and stem from different component in the vehicle. Analysis of distribution of failures in the vehicle shows that chassi, as the main framework of the vehicle, and the mounted components on chassi namely, suspension, brake and steering are responsible for 30% of the failures occurring in the car. Also aging and abuse is the case of almost 40% of the failures which is a gradual effect and can be detected before it reaches a catastrophic stage. The distributions of failures based on causes and component are depicted in figure (5) and figure (6) bellow[12]:

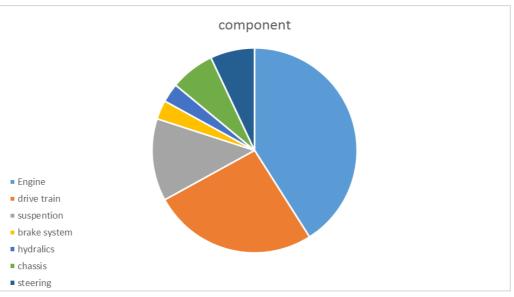


Figure 5: Distributions of failures regarding components. Figure borrowed from [11].

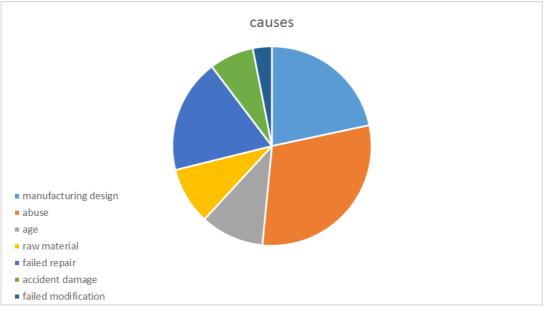


Figure 6: Distributions of failures root cause. Figure borrowed from [11].

2.2.1 Chassis failures

Chassis is one of the most crucial components in vehicle, not only one of the biggest parts but also responsible to hold up all main components of the vehicle together such as power train, suspension system and brake system. Thus safety in the chassis components should be guaranteed by early stage detection of possible failures of the components. Chassis can suffer from a number of broken or faulty components such as: damper, wheel hub, spring and tie rod. Also the error may stem from the wheels like flat spots, wheel imbalance or under inflation[13]. From safety and cost points of view these errors are in different stages of importance. In the next two consecutive sections low tyre pressure and faulty wheel hub phenomena are discussed further.

2.2.2 Investigated chassis failure

The main focus in this thesis is to develop a detection method capable of detecting and isolating low tire pressure and faulty wheel hub error states since both can lead to catastrophic circumstances in vehicles if not detected and also both low tyre pressure and faulty wheel hub can have similar effects on vibration analysis that makes it difficult to isolate the errors using conventional detection techniques.

2.2.2.1 Low tire pressure

A tire is a rubber structure reinforced with nylon fabrics and metal steel belts[14]. Tires can be modelled as springs with stiffness depending on the direction the load is applied in this sense each tire will have its specific stiffness that is directly related to the level of inflation of the tire. Higher pressures inducing higher stiffness and vice versa[15]. The spring model is depicted bellow:

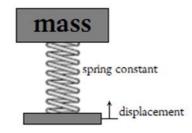


Figure 7: The spring model of the tire

In this model mass is the un-sprung mass which is mainly weight of the parts not supported by the suspension system, namely: wheels, wheels axle and all components connected to them. In some studies in the literature tire is modeled considering the damping effect too but the simplified model we are using here considers only the spring which is sensitive to tire pressure level[15].

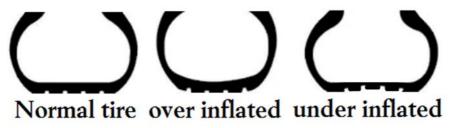


Figure 8: Effect of different Inflation levels on tire figure borrowed from [16].

Distribution of vehicle weight on the tire, thus stability of the vehicle is directly related to the level of inflation of the tires. Over or under inflation can affect handling of the car as well as cornering and stopping. An over or under inflated tire wear out unevenly and faster which increases the repair expenses. Under inflated tires are slower in response hence affects the normal performance of the vehicle as well as safety. Over inflated tires become stiffer and influence the road-tire contact which in turn causes noise and make the vehicle more prone to tar roads and potholes[16]. In figure (8) the effect of over and under inflation on the tire is depicted.

2.2.2.2 Faulty wheel bearing

Faulty wheel hub is a state of the hub which can occur due to faults stemming from hub units. Wheel hub components are vulnerable to aging and wear out over the course of time. Hub bearing is one of the essential components in wheel hub which can cause speed dependant noise upon being worn. Wheel bearing can fail due to damages on the rollers surface or the bearing race. The roller and the race surfaces are both polished so precisely to let roller pass easily over the race but over time the bearing wears slightly causing microscopic metal particles into the bearing grease. Metal particles ae well as any other kind of contamination causes deformation of the surfaces. Since the bearing are subjected to so much weight even on very small deformations of the surface it can cause a lot of noise[17]. In figure (9) a wheel hub is depicted.

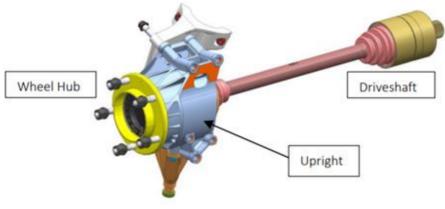


Figure 9: Wheel hub schematic

In wheel hub bearing, each ball bearing can be modelled as a set of parallel springs and dampers[18]. Any deformation in the ball bearing can change the stiffness and damping of the system which can lead to changes in frequency response of the system. Spring-damper model of a bearing is depicted bellow:



Figure 10: Wheel bearing spring-damper model

Wheel hub bearing is crucial in safety and handling characteristics of the vehicle. It enables the car to turn freely and plays an important role in smoothness of the ride and fuel consumption as well as performance of ABS brake system. If the wheel bearing wears out leads to increase in the friction and cause the wheel to wobble[17].

10

2.3 Machine learning in predictive maintenance

The rapid progress in field of predictive maintenance is due to recent advances in internet technologies, sensory systems and capability of systems in handling big datasets which enables employment of machine learning techniques. In machine learning approaches not only the process data is collected but also health condition parameters are stored such as vibration, pressure, temperature, acoustics, viscosity, and flow rate to evaluate the future condition of the system as well as conducting fault prediction or prediction[6].

Machine learning is a branch of artificial intelligence which can be describe as an algorithm capable to learn with some sort of autonomy. The main attribute of machine learning methods which make ML so marvellous is their capability to model nonlinearities and complex interdependencies of the systems which is crucial in systems suffering from large amount of fluctuations and variabilities or systems with complex networks of machines and daisy chained sequences.

2.3.1 Machine learning techniques

Machine learning is applied to a wide range of application namely, big data, vision, speech recognition, and robotics. Machine learning methods fall into three different categories. Supervised learning is a popular approach in machine learning in which predictors and responses are known hence labelling the data is feasible. And based on these labels is that system is trained. In unsupervised learning only the responses are known and not the labels. And in the reinforcement learning actions and consequences of an action is learned interacting with the environment[20].

2.3.1.1 Supervised learning

Supervised learning is employed when the input - output pairs are available in the data. Main approaches in supervised learning are classification and regression which is slightly the same as classification but is done in a continuous manner. Main task in supervised learning is to derive a mapping function between the input-output pairs and based on this function the algorithm can predict the prospective outputs based on the inputs fed to the algorithm. In other words supervised learning is usually used where historical data predicts prospective future events. In figure (11) the general structure of a supervised algorithm is illustrated[21].

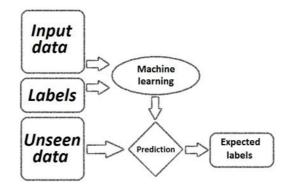


Figure 11: General structure of a supervised algorithm [19]

The most popular methods in supervised learning are: K-nearest Neighbour, Naive Bayes classifiers, artificial neural networks, and Decision tree. Decision tree is a treelike model in which the decisions and their prospective outcomes as well as possibility of each outcome are employed to build up a decision making system.

While working with datasets it is always a must to evaluate the usefulness of the variables in the data. Decision tree is an effective algorithm for feature selection this algorithm conduct feature selection while classification and this can be done in several ways. One way is feature ranking calculation based on the calculation of sum of improvements in all the nodes containing that particular attribute[21].

2.3.1.2 Unsupervised approaches

Unsupervised learning is employed to learn features from the unlabelled data. There is no target values or supervision in this method due to high cost of labelling. Upon feeding new data the algorithm employs the previous features extracted from data to assign the newly introduced data to a class. The main applications of unsupervised learning is in clustering and feature reduction as well as anomaly detection. In clustering tasks instead of using feedbacks commonalities observed in the data are used to classify the data. Most popular clustering methods are presented in the next section[22].

2.3.2 Clustering techniques

Clustering is the task of assigning a set of data points or data features into categories so that in each category or cluster, members share same attributes. Clustering is an unsupervised task and is widely applied in statistical data analysis. Data points are assigned to different clusters using similarity measures or distance measures like: Euclidean distance, Manhattan distance, Minkowski distance and Cosine similarity. Clustering is conducted so that it minimizes the distance between data points inside a cluster and maximizes the distance between data points across different clusters. Clustering algorithms fall into three different categories. Partitional clustering which is the most popular approach in which the data is divided into different partitions or clusters using objective functions. K-means clustering is an example of partitioned clustering which starts with random initializing of cluster centres. In the next step using distance equations each data point is assigned to the closest centre and then the consecutive new centres are calculated using averaging over all dimensions. Until the improvement in each step falls down a predefined value. Though this method is fast and requires low computational cost, the number of clusters is chosen by user and it does not necessarily represent the true number of clusters in the data. Other partitional clustering method is hidden Markov model which will be discussed thoroughly in next section[23].

The other approach is Density based Clustering. In this method clusters are defined as regions where the density of the data point exceeds a predefined value. DBSCAN is an example of density based clustering. Hierarchical Clustering is the last method which tends to divide or merge partitions in the data to form optimal number of clusters.

Mixture of Hidden Markov Models, is also of great importance, which is employed in the proposed algorithm to conduct clustering task. This method is a probabilistic method, assigning the points to the clusters with a probability. Closer the point to the centre of a cluster, higher the probability that the point belong to the cluster. This method employs variances as well as means of the clusters so ellipse shapes are also possible for the clusters. The number of the clusters should be predefined like K mean algorithm but in this algorithm the points are assigned softly to the clusters so it is possible to have overlapping clusters which is not possible in K means clustering[23].

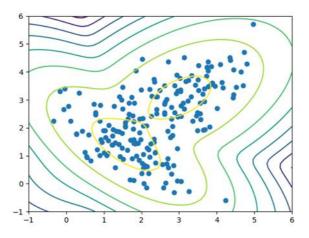


Figure 12: General mixture model example

2.4 Hidden Markov model

Conducting a clustering task on time series stored from sensory systems are quite complex and different from clustering individual data points. Time series collected from sensors can be faulty and have different temporal length and sample rates. These differences in length and sample rate make it hard or impossible in some circumstances to apply conventional similarity or distance measures. On the other hand these time series might be chaotic in nature showing different characteristics in different segments and sequences of the data. Thus it is desirable to develop a method capable of addressing the issues mentioned above. The proposed solution to address these complexity is a Hidden Markov Model. A Hidden Markov Model is a statistical model developed for systems that evolve through a finite number of states. It is assumed that the observations of these systems are generated by a hidden stochastic process which can be model using a Markov chain. This hidden states can be decoded by analysing the observations of the system to determine probabilistically which state the system is in. Hidden Markov models have been used successfully in a wide range of applications such as: body gesture recognition, sign language interpretation as well as time series analysis[24].

In a hidden Markov model a sequence of observations $O = \{O_1, O_2, ..., O_n\}$ are generated by an unknown hidden process. Each observation is the result of a specific hidden state represented as: $X = \{X_1, X_2, ..., X_m\}$. Considering the hidden Markov model as $\lambda = \{\Pi, \alpha, \beta\}$ where Π is the initial probability of each state showing how probable it is for a new input sequence to start from a particular state, α is the state transition matrix defining the probability of moving from one hidden state to another and β is the observation matrix and is the probability of each observation to be the result of a particular state. The sum of all start probabilities is equal to one as well as the sum of elements in each row in the transition probability matrix. An example of hidden Markov model is presented below, assume that an individual decisions on what to do during day is highly dependent on the weather, for instance on a rainy day the individual does the cleaning with a higher probability and on a sunny day goes for a walk. The emission and transition probabilities of this example is depicted in figure (13):

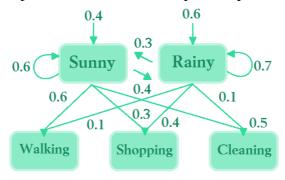


Figure 13: Example of a hidden markov model

As can be seen the states in this example are sunny and rainy weathers, and the observations are if the decision made by the individual is going walking, shopping or doing the cleaning. Hidden Markov model is capable to estimate the current state of the system as well as the sequence of states based on the observations.

There are three different problem formulation for a given hidden Markov model:

First problem: to compute the probability of the observation sequence as $P(O|\lambda)$, given the model and the observation sequence. Solution to this problem is using forwardbackward procedure to calculate the probability of the partial observation sequence before time t and state S_i given the model λ .

Second: to compute the most probable state sequence $\{X_1, X_2, \dots, X_m\}$ given the model and the observation sequence. Solution to this problem is Viterbi algorithm. This algorithm finds the best sequence of the states based on the observations by finding the most probable chain arriving at state S_i Third: model parameter selection to maximize probability of the observation sequence $P(0|\lambda)$. Though there is no optimal solution for this problem it is possible to choose $\lambda = \{\Pi, \alpha, \beta\}$ such that it is locally minimized using Baum-Welch algorithm. This algorithm starts by assigning random probabilities and adjust them later on such that the probability that the model assigns to the training set is maximized[24]. 2.4.5. Model selection

As mentioned in section (2.3.1.1) dedicated to unsupervised clustering methods, in majority of the methods, the number of clusters is assumed to be known. This assumption is usually made upon observing the data. Nevertheless, for high dimensional datasets where a visual realization of the data is impossible or in complex datasets with overlapping clusters, assuming the number of clusters if not impossible is hard. Thus some model selection algorithms are proposed in the literature to find the optimal number of clusters in the data. The Bayesian information criterion (BIC) is a criteria for selecting the optimal model among a finite number of possible models. The model is selected which produces the lowest BIC. BIC criteria is defined as follows:

$BIC = \ln(n) k - 2\ln(L) \qquad (1)$

In this equation L is the maximum likelihood function of the model which is described as the plausibility of a value for the parameter, given some data. k is the number of free parameters in the model, such as: number of states in each hidden Markov model, number of hidden Markov models in the mixture model, and the mixture models gains, and n is the number of inputs fed to the algorithm. In this thesis BIC is employed for model selection[25].

3 Proposed framework

3.1 Fault detection stage

This part is not the main focus of this thesis however, this stage is briefly explained for presentation of the whole picture to the reader. The main idea is that any change in hardware of the chassis components, that have significant effect on stiffness of the car corner, will directly affect the natural frequencies and local vibration behaviours of the chassis at corresponding car quarter.

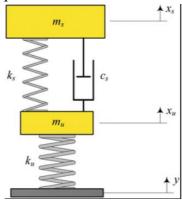


Figure 14: Car quarter model

Any significant change in chassis hardware (due to system degradation, failure and etc.) can change the natural frequency and vibration fingerprint of the car quarter model. As a result the level and frequency of fluctuations in wheel speed will change due to changing of the spring constant, as mentioned in equation bellow:

$$\omega_r = \sqrt{\frac{K - \Delta_K}{M}} \qquad (2)$$

Here ω_r is the resonance angular velocity, *M* is the mass and Δ_K is the change in spring constant that can be the result of change in stiffness of different components at chassis corner. This stiffness is representing both K_u as tyre stiffness and K_s as chassis spring stiffness. It is worth to mention that mass of the system can be assumed as constant before and after the occurrence of the error states, and the stiffness is the main parameter that will be affected by hardware failure/degradation. So as can be inferred, applying fast Fourier transform (FFT) to wheel speed signals can be considered as a good measure for detecting any change in local vibration fingerprint of the car quarter and used as a measure for detection of faults like low tyre pressure and changes in suspension system.

3.2 Fault isolation and diagnosis stage

Spectral analysis (e.g. FFT-based analysis) is a fast way of detecting faults, but not sufficient for isolation, diagnosis and classification of different faults since many of the faults can have similar effects on wheel speed signals in time and frequency domain. Thus in this stage, time-series data is stored just before/after the fault is detected and reported in on-board diagnostics and sent to cloud in the back-end. An unsupervised

learning solution is adopted, in this thesis, for automatic fault isolation/diagnosis, for the purpose of more efficient and lower cost predictive/descriptive solutions for service and maintenance in the back-end. This method is proposed in [28] for clustering Vehicle Maneuver Trajectories The performance of the method is validated by diagnosis of two different error states, i.e. low tyre pressure and faulty wheel hub, using the collected data at test track. A mixture of hidden Markov models is employed in order to conduct clustering task, where the configuration of the model is specified by a model selection procedure, based on BIC model selection criteria [29]. To achieve this, the experiment is repeated for different number of hidden states and different number of Markov models in the mixture model. The model yielding smallest value of BIC is chosen. The initial emission and transient probabilities are chosen randomly which is updated and optimized during the training process. In this experiment forward algorithm is employed to calculate the log probability of the model. The whole dataset is fed to the algorithm for training since no verification was needed due to unsupervised nature of training algorithm.

In this thesis unsupervised learning is employed to give the algorithm the freedom to detect as many clusters as there are in the data since the actual number of fault clusters is unknown in real world applications.

3.3 Data collection process

The time series are collected from flex-ray of a V90 Volvo passenger car. First the data is examined for possible corrupted data and the data is cleaned up from empty runs and corrupted data then labelled based on different states of driving this labelling system is employed for the verification of the method and is not used in a supervised manner as targets. Then data is sorted based on different driving profiles so the functionality of the method can be evaluated in different driving modes such as: turns and straight forwards.

3.4 Datasets

The datasets are collected by Volvo Car Corporation in Hällered proving ground in Sweden. The tests are conducted on different test tracks for three different circumstances. One state represents normal driving situation, the other state is 25% under inflation in one wheel then test is repeated for a faulty wheel hub. The tire pressure initially is set to 275 kPa for nominal inflated pressure for normal driving test and then reduced to 220kpa for low tyre pressure test. Data acquisition frequency is set to 100 Hertz.

First dataset is collected from the vehicle conducting a test on the country road test track which is straight forward driving track. This track contains no curve and no bump and there was an intention to keep the speed constant throughout the test to minimize the effect of other interfering factors. Country road data-set contains normal, low tyre pressure and faulty wheel hub states.

The second dataset is collected from handling road test track, which contains moderate turning and uphill and downhill driving. The dataset contains time series collected from normal, low tire pressure and faulty wheel hub tests. Each test is conducted for two different scenarios: one scenario is to log the data at fixed spped of 60 Kmph and the other scenario is to log data when the vehicle starts with 30 Kmph but then elevate the speed to 60 Kmph and then in the next step elevate the spped to 80 Kmph in order to include the effect of acceleration in the dataset. Then both scenarios are repeated for two trials.

City traffic test track is the last track we have used for collecting data. Since there is a speed limit for this test track due to sharp turns, the data set only contains one speed level and time series on normal and low tire pressure. For each state the experiment is repeated twice.

The manouvers where selected in different test tracks to log the data needed to investigate different situation excited in the chassis and tyres the list of signals investigated is as bellow:

| Front left wheel speed 30kmph | Front right wheel speed 30kmph | Rear right wheel speed | Rear left wheel speed 30kmph |
|--------------------------------|---------------------------------|------------------------|-------------------------------|
| | | 30kmph | |
| Front left wheel speed 60kmph | Front right wheel speed60kmph | Rear right wheel speed | Rear left wheel speed 60kmph |
| | | 60kmph | |
| Front left wheel speed 80kmph | Front right wheel speed 80kmph | Rear right wheel speed | Rear left wheel speed 80kmph |
| | | 80kmph | |
| Front left wheel speed 100kmph | Front right wheel speed 100kmph | Rear right wheel speed | Rear left wheel speed 100kmph |
| | | 100kmph | |

Table 1:Speed levels of speed signals fed tp the algorithm

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4 **Results**

4.1 Fault diagnosis result for low tyre pressure

We evaluated the functionality of our proposed algorithm using the low tyre pressure and normal driving from the three datasets described in the previous section. As mentioned in the algorithm description, BIC method is employed for model selection. BIC results depicted in figure (15) shows the mixture model containing two hidden Markov model with each Markov model having one state yields the best value of BIC.

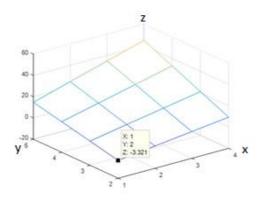


Figure 15:BIC values for different number of states(x axis) and HMMs(y axis)

The results show that the dataset contains two cluster in nature which corresponds to two state of the system. On the other hand the Markov model has one state showing the pattern in the data is monotone and is not changing over time. The experiment is done in some steps: first data of the normal and low tyre pressure from each test track is fed to the mixture model and the results are depicted for each test track. In the next step data of handling road and country road are mixed together and fed to the mixture model to evaluate the functionality of the algorithm while different data from different driving profiles are fed to it.

The result of training of the algorithm upon data collected from country road is depicted in figure (16). The green signals illustrate the normal driving while the red ones are representing low tyre pressure and as can be seen the algorithm is capable to detect the low tyre pressure time series with 100% accuracy. The signal that is employed here is a mixture of wheel speed signals. The table depicted in table (2) show the clustering results versus the labels assigned manually based on the test information. In this experiment 8 time series correspond to normal driving on four different speed levels and 8 signals represent low tyre pressure. Here the column 0 is representing the manual labelling while column 1 contains the predicted labels.

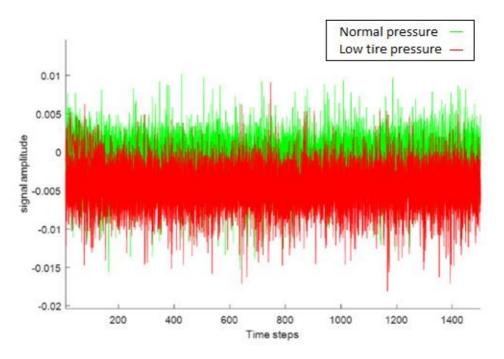


Figure 16: Clustering result of the algorithm trained using country road dataset

As can be seen in Figure (16) the figure contains the time series from normal pressure and low tire pressure data, and Hidden Markov Model has successfully clustered the time series as two instinct clusters. The difference between the two cluster can be seen as a minor deviation between the green and red time series which is actually due to the different diameter of the normally inflated and under inflated tires.

| | 0 | 1 |
|---------------------------------|---------------------------------------------------------------|--------------------------------------------------------------------|
| 0 | 1 1 1 1 | 1 1 1 1 1 |
| 1 2 3 4 5 6 7 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 1 |
| 6 | 1 | 1 |
| 7 | 1 | 1 |
| 8 9 | 0 | 0 |
| | 0 | 0 |
| 10 | 0 | 0 |
| 11 | 0 | 0 |
| 10 11 12 | 0 | 0 |
| 13 | 0 | 0 |
| 14 15 | 1 1 1 0 0 0 0 0 0 0 0 0 0 | 1 1 1 0 0 0 0 0 0 0 0 0 0 0 |
| 15 | 0 | 0 |

Table 2:Comparison of the resultant clusters and the manually labelled data for country road data

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The results of feeding handling road dataset is depicted in figure (18). The proposed algorithm is able to conduct the clustering task with no error. This dataset contains only 8 time series 4 representing normal and the other four corresponds to low tyre pressure but each signal is a mixture of different speed levels and transient phases in acceleration too.

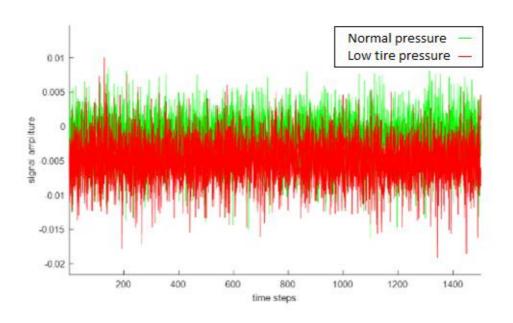


Figure 17: Clustering result of the algorithm trained using handling road dataset

| | 0 | 1 |
|--------|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 0 | 0 |
| 2 3 | 0 | 0 |
| 4 | 1 | 1 |
| 4 5 | 1 | 1 |
| 6 | 0 | 0 |
| 7 | 0 | 0 |

Table 3:Comparison of the resultant clusters and the manually labelled data for handling road data

The result of the third test track named city traffic is depicted in (20). This dataset contains only four time series due to speed limit in the track. Algorithm yield 100% accuracy.

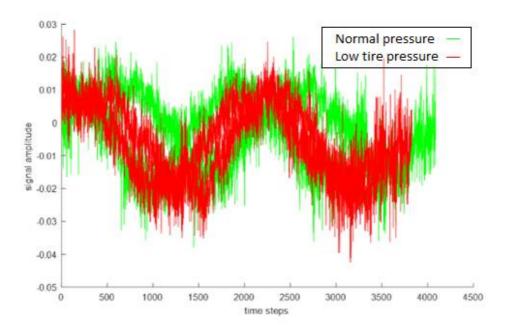


Figure 18: Clustering result of the algorithm trained using city traffic road dataset

| | 0 | 1 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |

Table 4:Comparison of the resultant clusters and the manually labelled data for city traffic road data

In the last experiment in low tyre pressure detection the mixed data of country road and handling road is employed. Due to mixed characteristics of the data fed to the algorithm there is a misclassification error of 12.5%. The results are depicted below:

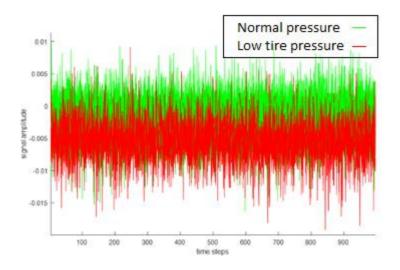


Figure 19: Clustering result of the algorithm trained using Handling road and country road data

| | 0 | 1 |
|------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 1 |
| 6 | 1 | 1 |
| 7 | 1 | 1 |
| 8 | 1 | 0 |
| 9 | 1 | 0 |
| 10 | 0 | 0 |
| 11 | 1 | 0 |
| 12 | 0 | 0 |
| 13 | 0 | 0 |
| 14 | 0 | 0 |
| 15 | 0 | 0 |
| 16 | 1 | 1 |
| 17 | 1 | 1 |
| 18 | 0 | 0 |
| 19 | 0 | 0 |
| 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 | 0 1 1 1 1 1 1 1 1 1 1 1 1 1 | 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 |
| 21 | 1 | 1 |
| 22 | 0 | 0 |
| 23 | 0 | 0 |

Table 5:Comparison of the resultant clusters and the manually labelled data for handling road and country road data

As can be seen from the table (24) three time series out of 24 time series are misclassified. The algorithm could not detect the low tyre pressure for two time series out of 12 time series corresponding to low tyre pressure and also sends out low tire pressure warning when no error has occurred. The results are summarised in the contingency matrix bellow:

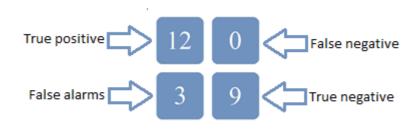


Figure 20: Contingency matrix of the mixed data clustering result

Regarding the contingency some evaluation metrics are calculated as bellow: the table shows the number of true negative, true positive and false alarms.

Sensitivity of the system also known as detection rate and true positive rate is 100%. Sensitivity is the portion of errors detected among the number of errors actually occurred in the system. Specificity of the system is 75% which is the proportion of actual negatives that are correctly identified also known as true negative rate. Accuracy of the system 87.5% which is the capability of the algorithm to assign the time series to the right cluster.

4.2 Fault diagnosis result for faulty wheel hub

The algorithm employed to detect faulty wheel hub is exactly similar to the algorithm utilized to detect low tyre pressure. The model selection procedure for faulty wheel hub yields same configuration with two hidden Markov model and one hidden state in each model. The result of BIC method for model selection is illustrated in figure (25).

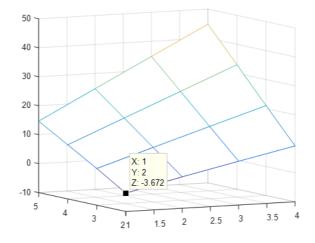


Figure 21: BIC values for different number of states(x axis) and HMMs(y axis)

The experiment is conducted for country road and handling test tracks. Figure (26) depicts the results for faulty wheel hub detection conducted on handling test tracks. The dataset contains 4 time series representing faulty wheel hub and the other four correspond to low tyre pressure.

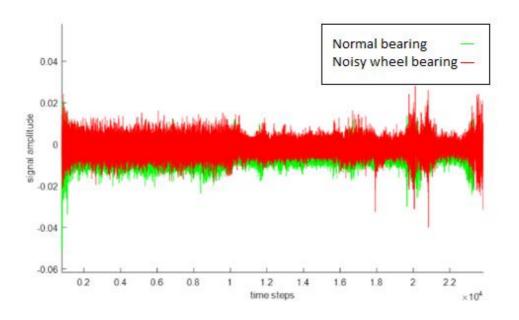


Figure 22: Clustering result of the algorithm trained using Handling road data

| | 0 | 1 |
|--------|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 3 | 1 | 1 |
| 3 | 1 | 1 |
| 4 5 | 0 | 0 |
| | 0 | 0 |
| 6 | 0 | 0 |
| 7 | 0 | 0 |

Table 6:Comparison of the resultant clusters and the manually labelled data for handling road data

The experiment is repeated for country road test track and the results are depicted in figure (28) as can be seen the algorithm is capable of isolating faulty wheel hub from low tyre pressure with 100% accuracy.

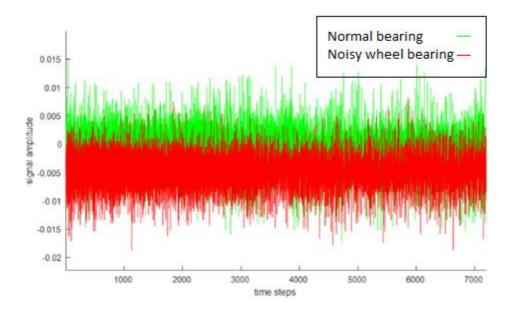


Figure 23: Clustering result of the algorithm trained using country road data

| | 0 | 1 |
|----------------------------|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 1 | 1 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 2 3 4 5 6 7 | 0 | 0 |
| 7 | 0 | 0 |

Table 7: Comparison of the resultant clusters and the manually labelled data for countryg road data

In the last attempt the datasets of handling and country road are mixed and fed to the algorithm to evaluate the functionality of the algorithm in different driving profiles. As can be seen in figure (30) the algorithm is able to detect faulty wheel hub from low tyre pressure with 100% accuracy.

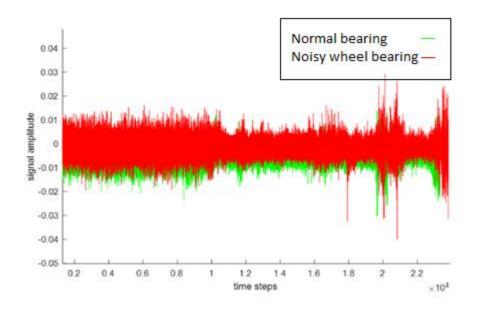


Figure 24: Clustering result of the algorithm trained using country road and handling road data

| | 0 | 1 |
|-------------------------------------------|------------------------------------------------|-------------------------------------------|
| 0 | 1 | 1 1 1 1 1 1 1 1 |
| 1 | 1 | 1 |
| 1 2 3 4 5 6 7 8 9 | 1 1 1 | 1 |
| 3 | 1 1 1 | 1 |
| 4 | 1 | 1 |
| 5 | 1 | 1 |
| 6 | 1 | 1 |
| 7 | 1 | 1 |
| 8 | 0 | 0 |
| | 0 | 0 |
| 10 | 0 | 0 |
| 11 | 0 | 0 |
| 12 | 0 | 0 |
| 12 13 14 15 | 1 0 0 0 0 0 0 0 0 0 | 1 0 0 0 0 0 0 0 0 |
| 14 | 0 | 0 |
| 15 | 0 | 0 |

 Table 8:Comparison of the resultant clusters and the manually labelled data for countryg road and handling road data

The summary of the results on faulty wheel hub is depicted in the confusion matrix bellow.



Figure 25: Contingency matrix of the mixed data clustering result

Based on the contingency matrix specificity, sensitivity and accuracy of the system is 100%.

5 Conclusions

5.1 Summary of the work done

Chassis failure detection is crucial regarding safety and maintenance costs. It is also vital for automotive companies to address the issue to be able to maintain a leading position in the industry, so detection and prevention of such failures is of great importance. In this thesis a novel machine learning method for chassis failure detection was proposed.

First data collected from the test tracks was investigated and cleaned up from corrupted data and empty runs, and labelled using Mat-lab software. Then the data was fed to the unsupervised machine learning algorithm developed in pomegranate library in python which was a mixture model of hidden Markov models.

The experiment was conducted for low tyre pressure detection as well as isolating of low tyre pressure from faulty wheel hub for different datasets. The algorithm was capable to detect low tyre pressure with 100% accuracy for handling, country road, and city traffic road, and yield 87.5% accuracy when employed for a mixture of different driving profiles. The results of isolating faulty wheel hub from low tyre pressure were more promising where the algorithm was capable to detect faulty wheel hub with 100% accuracy for all datasets as well as mixed dataset.

5.2 Recommendations for future work

Next step to enhance this method can be collecting a new dataset containing transient states of the signals from different driving scenarios as well as different speeds and different road conditions so the algorithm can be trained upon a broad range of data that can lead to better understanding of effect of different conditions on the algorithm such as effect of turns or rough roads. On the other hand the data is logged from a particular vehicle and this can effect the robustness of the algorithm in the future we aim to train the algorithm based on the data logged form different vehicles.

In this thesis the proposed method has been applied to detect individual error states, such as low tyre pressure and faulty wheel hub. The other track for future work can be applying this method to detect different error states as well as mixed errors as a separate state of the system. In this situation algorithm will be trained to detect clusters that are representing combination of different errors.

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