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Modeling and Parameter Estimation of Planing and Semi-Planing Marine Vessels under Colored Disturbances

Towards Model-Based Stabilization Control Using Real-Time System Identification

Master's Thesis in Systems, Control and Mechatronics

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Identification

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Cover: Humphree's test vessel, Doris, used for collecting data used in the thesis.

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Abstract

A linear grey-box Multiple Input Multiple Output (MIMO) model describing ship attitude motion is derived from established nonlinear equations for ship motion. The model describes how five control surfaces: two fins, two interceptors and a rudder control the roll, pitch and yaw dynamics of a vessel. The model is developed for ship stabilization control design and consists of 15 parameters, neglecting the translational states surge, sway and heave. These neglected states are concluded to introduce errors in the model through residual correlation analysis, but the model is considered adequate for control design through empirical frequency domain validation using SPAFDR. To ensure the model is applicable to the different operating states of a planing vessel (displacement, hump and planing), it is validated at the operating states respective speed of a test vessel: 10, 15 and 20 kt.

Input-output data is collected in open-loop system identification experiments. To make this possible, an excitation signal for MIMO system identification is proposed as a modified PRBS7 signal. The excitation signal is applied periodically to separate the ship's deterministic output from the noise. The noise is analyzed, modeled as an ARMA process and concluded to be time-variant.

Parameter estimation of the proposed ship model is performed using a batch Prediction Error Method (PEM) in the presence of colored noise. Furthermore, two recursive parameter estimation algorithms are implemented and tested on data with colored noise (specifically a Recursive Output Error Model and a Recursive Instrumental Variable State Varying Filter) to allow for parameter estimation on limited hardware, lacking the ability to use batch estimation methods.

Keywords: Grey-box, MIMO, Open-loop, Parameter Estimation, PEM, RIVSVF, Levenberg-Marquardt Regularization, Colored Noise, Ocean Wave Modeling.

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Oskar Ulstein, Gothenburg, June 2026
Edvin Öhman, Gothenburg, June 2026

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

ACF	Autocorrelation Function
AR	Autoregressive
ARMA	Autoregressive Moving Average
DOF	Degrees of Freedom
EMA	Exponential Moving Average
GN	Gauss-Newton
IV	Instrumental Variables
LM	Levenberg-Marquardt
LQR	Linear Quadratic Regulator
LS	Least Squares
LTI	Linear Time-Invariant
MA	Moving Average
MIMO	Multiple Input Multiple Output
MPC	Model Predictive Control
MSC	Magnitude Squared Coherence
OE	Output Error
PACF	Partial Autocorrelation Function
PEM	Prediction Error Method
PRBS	Pseudo Random Binary Sequence
PSD	Power Spectral Density
RBS	Random Binary Signal
RIV	Recursive Instrumental Variable
RIVSVF	Recursive Instrumental Variable State Variable Filter
RK4	Runge-Kutta 4
RLS	Recursive Least Squares
RMS	Root Mean Square
ROE	Recursive Output Error
SIMO	Single Input Multiple Output
SNR	Signal-to-Noise Ratio
SPAFDR	Spectral Analysis with Frequency-Dependent Resolution
SVF	State Variable Filter

Nomenclature

Below is the nomenclature of indices, sets, parameters, and variables that have been used throughout this thesis.

Indices

$C(\nu)$	Coriolis-centripetal matrix
$D(\nu)$	Damping matrix
$e(t), \varepsilon(t, \theta)$	Prediction error / residual
f, f_e	True wave frequency and encounter frequency (Hz)
$F_{bp}(s)$	Band-pass state variable filter transfer function
$g(\eta)$	Vector of gravitational and restoring forces
H_s	Significant wave height
K_k	Adaptation gain / Kalman gain matrix
M	Inertia matrix (Rigid body + Added mass)
N	Number of data samples
P_k	Parameter covariance matrix
p, q, r	Roll rate, pitch rate, and yaw rate
P_{yy}, P_{uu}, P_{uy}	Power spectral densities and cross-spectral density
$R(N)$	Hessian approximation / Information matrix
S_k	Innovation covariance matrix
T_s	Sampling time
U, U_0	Ship speed / Nominal surge speed
$u(t)$	System input vector
u, v, w	Surge, sway, and heave velocities
$V_N(\theta, Z_N)$	Scalar valued loss function
x	State vector
$y(t)$	True system output vector

$\hat{y}(t \theta)$	Predicted system output
α	Smoothing factor (EMA filter) / Step size (Gauss-Newton)
β	Wave angle of attack
$\gamma_{y:u}^2(f)$	Magnitude Squared Coherence (MSC)
$\zeta(t)$	Instrumental variable vector
η	Position and orientation vector $[x, y, z, \phi, \theta, \psi]^T$
θ	Pitch angle / System parameter vector
$\hat{\theta}$	Estimated parameter vector
λ	Forgetting factor
μ_k	Levenberg-Marquardt damping factor
ν	Translational and angular velocity vector $[u, v, w, p, q, r]^T$
τ	Generalized force vector
ϕ	Roll angle
$\varphi(t)$	Regressor vector
ψ	Yaw angle
$\psi(t, \theta)$	Gradient of the predicted output (sensitivity matrix)
ω, ω_e	Wave frequency and encounter frequency (rad/s)
ω_0	Modal wave frequency (JONSWAP)

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1

Introduction

An important field in marine engineering is stabilization systems for sea vessels [2]. A stabilization system mainly aims to reduce unintended vessel motion at sea, induced by the sea environment consisting of for example waves or wind. A successfully stabilized ship comes with a number of benefits, it can for example improve fuel economy [3], improve onboard safety [4] and it improves the comfort for the passengers by reducing motion sickness. The company Humphree develops stabilization systems that utilize control surfaces such as interceptors and fins, see Figures 1.1-1.2, to achieve stability. These control surfaces have multiple purposes such as stabilizing a vessel in rough seas, but also stabilizing a vessel during a turn. Humphree stabilization systems are commonly mounted on semi-planing and planing boats, which have different and non-linear characteristics depending on their speed [5]. Humphree stabilization systems therefore rely on robust control algorithms. For these algorithms to be optimal, a mathematical model is required. To enable model-based control design, this thesis presents a linear model evaluated at three different speeds: 10, 15 and 20 kt corresponding to the displacement, hump and planing states seen in Figure 1.4. Furthermore, an excitation signal is proposed for parameter identification of this model using both batch and recursive estimation.

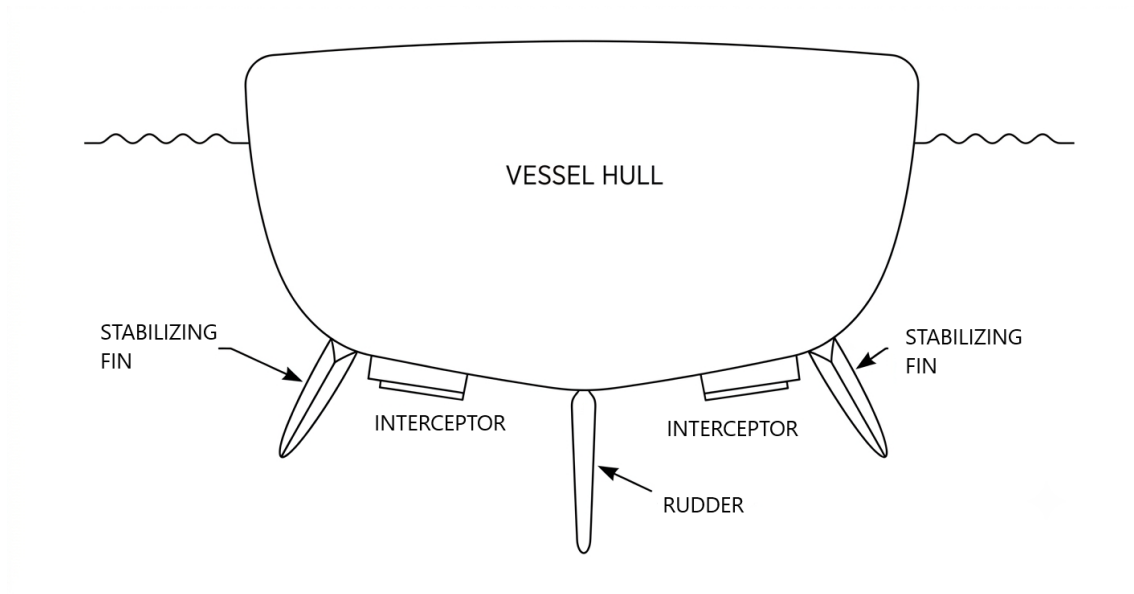


Figure 1.1: A vessel model viewed from the stern with mounted interceptors and fins.

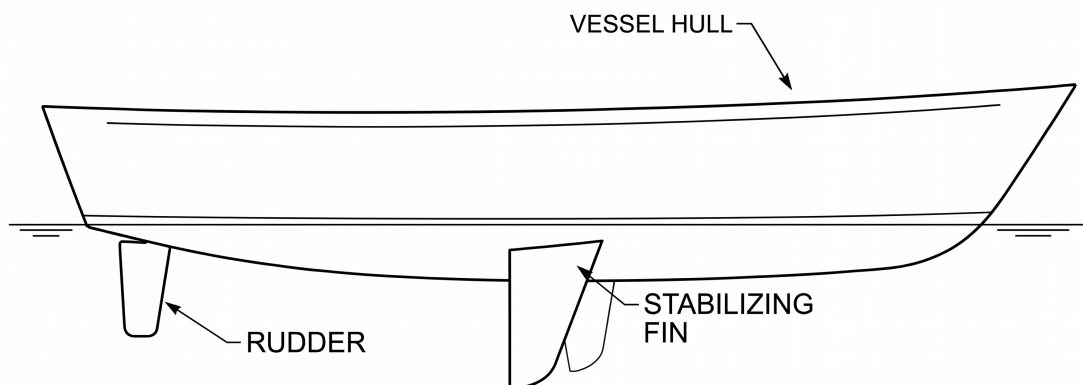


Figure 1.2: A vessel model viewed from the side showing where the fin stabilizers are mounted.

1.1 Background and Motivation

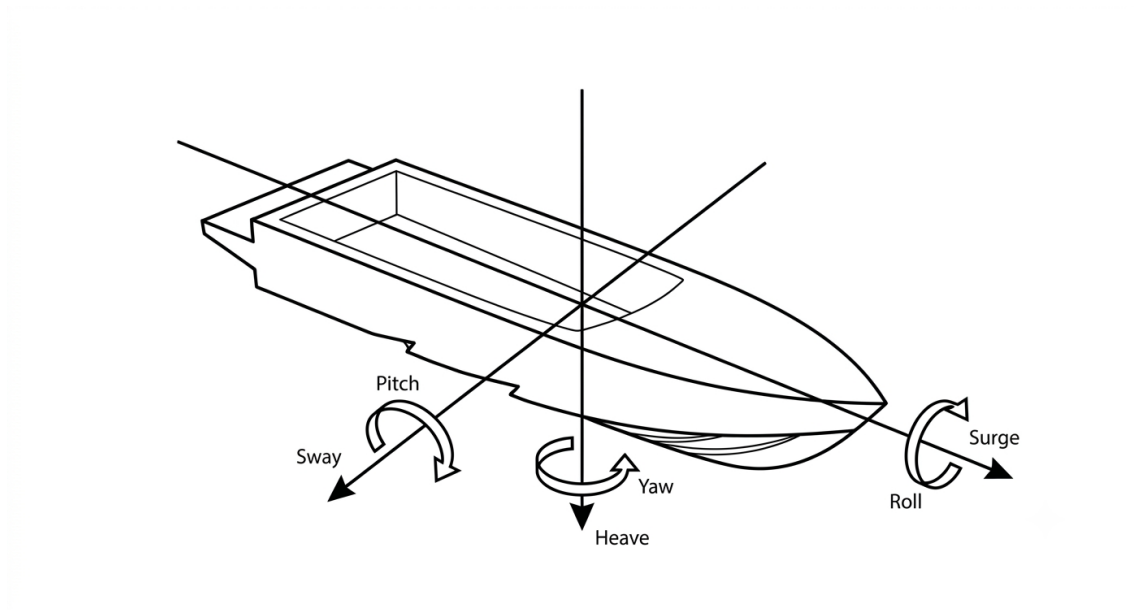


Figure 1.3: Illustration of the vessels degrees of freedom.

Ship motion can be mathematically explained by considering its translational motions: surge, sway and heave, and its attitude motions: roll, pitch and yaw, visualized in Figure 1.3. The dynamics of these degrees of freedom are governed by complex nonlinear equations, see [6] and [5], that depend on vessel speed, and vessel position in the water. These dependencies are especially important when considering the three major phases of a planing vessel: the displacement, hump and planing phase seen in Figure 1.4, which corresponds to below 10, 10-20 and above 20 kt for the test vessel used in this thesis.

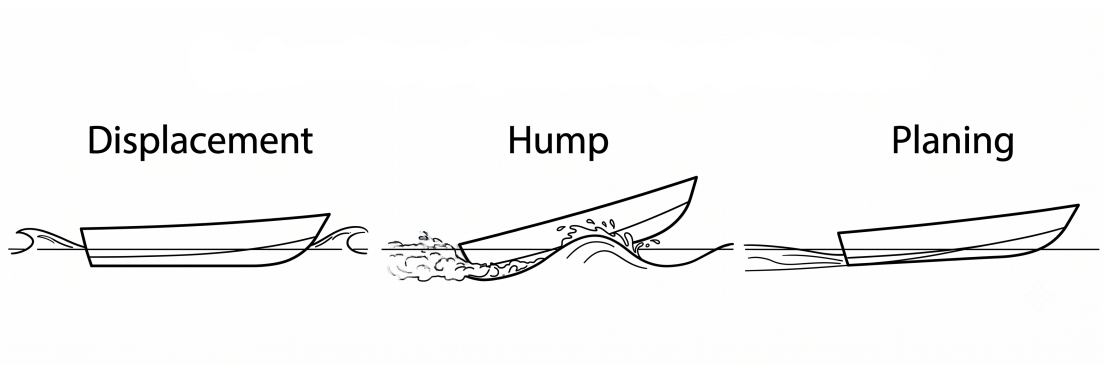


Figure 1.4: The three vessel phases: displacement, hump and planing.

To avoid these complex dynamics in the control design, a suboptimal solution is to neglect model dependent control strategies. However, this omits potential benefits that a model based control design might bring such as increased performance,

scalability across different platforms and easier and faster tuning which in turn decreases the cost of installation [7]. These benefits motivate the development of a mathematical model, which this thesis covers.

To achieve this, the nonlinear vessel equations can be linearized, which is desirable since linear models are easier to work with, see for example the control design in [8, p.226]. However, few examples exist where this is done in a Multiple Input Multiple Output (MIMO) configuration for a planing vessel fitted with a rudder, two fins and two interceptors. Some similar projects have been done but they mainly focus on designing ship course tracking and autopilot controllers, such as in [2], while this thesis focuses on modeling attitude motion specifically for ship stabilization.

This thesis proposes a linear model that utilizes two fins and two interceptors developed by Humphree, together with a rudder to describe ship attitude. The model's parameters are speed dependent and identifiable at the three different phases in Figure 1.4, while remaining generically applicable on a vessel-by-vessel basis. This enables open-loop experiments, acting as a "calibration phase" that calibrates the model at a certain speeds.

Parameter estimation is performed for this model using both batch and recursive estimation methods in the presence of colored noise. The recursive identification is motivated by its ability to estimate the model with limited hardware, that lacks the capability of running batch identification methods. The translational motions (surge, sway and heave) shown in Figure 1.3 are neglected in this model due to the difficulties in measuring them with the available hardware. This is further explained in Section 2.1.

1.2 Contributions

The thesis provides methods to move towards model-based control design by contributing in the following ways:

- **The development of an excitation signal for MIMO system identification:** The PRBS7 [9] (explained in Chapter 3) signal is chosen and modified to fulfill specific requirements listed to account for this specific system.
- **Disturbance modeling:** Colored noise in the form of water waves present during sea trials is estimated and modeled as an ARMA process. It is shown that the frequency content of this noise is time variant, making a fixed noise model difficult to apply.
- **A linear physical grey-box model:** The model is derived from the highly complex and nonlinear equations for ship motion formulated by Fossen [6]. The model is applicable to the three phases shown in Figure 1.4, but its specific parameter values varies for each phase due to the vessel dynamics speed and position dependency discussed in Section 1.1.

- **Parameter identification algorithms:** System identification algorithms are evaluated for identifying the derived grey-box model. Recursive algorithms are tested and compared to standard recursive least squares to see if they can operate in the presence of colored disturbance.

1.3 Structure of This Report

Theoretical Background This chapter introduces key concepts of ship dynamics and some highlighted system identification methods used in this thesis. Wave disturbances are discussed and the project test vessel is specified.

Design of Excitation Signals An excitation signal for MIMO identification is developed from a list of requirements based on data collection, literature and specific system constraints.

Data Collection and Data Analysis The sea trials performed in the project are described along with their purpose. The noise present during data collection is analyzed and modeled. A Single Input Multiple Output(SIMO) rudder model is developed, and used to improve the noise estimation.

Model Identification and Parameter Estimation A linear, lumped grey-box model is derived from Fossens [6] nonlinear ship equations of motion. The parameters of this model are estimated for 10, 15 and 20 kt. The model is validated against traditional system identification metrics such as simulation fit and residual analysis but also empirical frequency domain data.

Recursive Parameter Identification A recursive Output Error Model (OE) identification algorithm and a Recursive Instrumental Variable (RIV) algorithm is evaluated against standard Recursive Least Squares(RLS), for estimating the developed grey-box model in the presence of colored noise.

Discussion and Conclusion A summary of the results in this project is discussed. Conclusions are drawn about the chosen excitation signal, noise estimation, model validation and the parameter identification algorithms used. Potential areas for future work are proposed based on the results in this thesis.

1.4 Ethical and Environmental Aspects

There are potential downsides and ethical complexities of this technology. High-performance planing and semi-planing vessels are frequently utilized in defense and military applications. While the primary scope of this thesis is commercial, the algorithms developed for disturbance rejection and speed-dependent dynamics could theoretically be applied to enhance the performance of vessels used in conflict.

Regarding environmental aspects, boat stabilization systems help reduce resistance for ships traveling in moderate sea conditions [10][3]. Thus improving the control

1. Introduction

performance of such systems help reduce emissions released by ships during sea travel.

2

Theoretical Background

This section explains the test vessel used for data collection, followed by some theory used for noise analysis and noise modeling (ocean waves). Finally, some fundamental system identification methods used in the thesis are explained, along with the problem of colored noise for the standard Least Squares (LS) problem.

2.1 System Definition and Control Surfaces

The boat used for data collection is a Targa 31. It is fitted with two interceptors and two fins connected to Humphrees control system. These control surfaces and some basic ship dynamics are summarized in the following paragraphs to make the coming chapters easier to follow.

Targa 31: The Targa 31 boat has the physical characteristics given in Table 2.1, provided by the manufacturer. This specific boat leans slightly towards the starboard side when stationary introducing a small roll angle bias, this is however not considered to affect the results significantly since the recorded data is detrended before processed. The fins are larger than usual for this boat size (0.6 m^2 for each fin), which gives them unusually high control authority compared to normal system installations. The interceptors are mounted suboptimally, close to the center axis of the stern rather than at the corners of the stern as in Figure 1.1, which yields reduced roll control authority compared to normal system installations.

Specification	Value
Total Mass	6300 kg
Length (Hull)	9.53 m
Beam (Width)	3.15 m
Draught	0.85 m

Table 2.1: Physical specifications of the Targa 31 test vessel.

Roll fins: The fins generate a roll moment towards the port or starboard side depending on how their angle is controlled. To generate maximum power, the fins always work together. For this reason they are seen as a single input to the system in this thesis and thus follows the same reference signal.

Interceptors: The interceptors work by extending their length to induce a pitch and/or roll moment. When being fully retracted they introduce zero moment to the vessel, they can therefore only apply a positive pitch moment to the vessel. To control roll, the port side interceptor is extended to introduce a port roll moment and vice versa for the starboard side. This means that the interceptors are applied together to control the pitch angle, but individually to control the roll angle. They are for this reason considered as two separate inputs with the same characteristics.

State coupling: Vessel movement is governed by highly non-linear dynamics, whose equations are derived by Fossen [6]. To simplify these, the roll and pitch dynamics can be considered to be independent like they do in [2]. The roll, sway and yaw dynamics however are strongly coupled. To control a ship in an optimal way, these couplings are considered important to model, see [8, p.5] and the references therein. The test vessel is equipped with an accelerometer that can be used to estimate translational motion, but the mounting position is unknown and not necessarily on the vessel's center of mass, making its direct measurements inaccurate for this purpose. For this reason translational motion is neglected in the thesis.

2.2 Wave Disturbances

Waves in open water produced by wind can be modeled by a number of different frequency spectra that exist within the frequency band of [0.03-0.7] Hz [11]. These spectra can change rapidly over time due to environmental changes such as wind, changed ship direction or if the ship moves to a new location [8].

A moving ship is however not necessarily disturbed by waves in this frequency band due to the *wave encounter frequency* changing depending on ship direction and speed. The encounter frequency is the frequency that waves hit the boat and can be calculated according to the following equation [8, p.23]:

$$\omega_e = \omega - \frac{\omega^2 U}{g} \cos(\beta) \quad (2.1)$$

where ω_e is the encounter frequency, ω is the wave frequency, U is the ship speed, g the gravitational constant and β the waves angle of attack. Using the [0.03,0.7] Hz frequency band with (2.1) for ship speeds of 10, 15 and 20 kt gives the encounter frequency bands in Table 2.2 where "Following Seas" is when the ship travels along the waves, "Beam Seas" is when the waves hit the ship from the side and "Head Seas" is when the ship is traveling towards the waves.

This means that the frequency bands in Table 2.2 are where waves produced by wind will excite a ship. The upper band of these frequencies will however not be measurable with the roll, pitch and yaw angles due to the disturbance traveling through the system plant (the vessel), which mostly contains dynamics lower than approximately 1.6 Hz due to the large inertia characterizing a ship, effectively acting as a low-pass filter on the disturbances. This process is visually explained in Figure 2.1 and empirically proven in Chapter 3 where it is used to define the frequency band for where the model needs to be accurate.

Table 2.2: Absolute encounter frequency bands (f_e) for a true wave frequency band of $[0.03, 0.7]$ Hz at different ship speeds and angles of attack.

Ship Speed	Following Seas (0°)	Beam Seas (90°)	Head Seas (180°)
10 kt	$[0.03, 0.91]$ Hz	$[0.03, 0.70]$ Hz	$[0.03, 2.31]$ Hz
15 kt	$[0.03, 1.72]$ Hz	$[0.03, 0.70]$ Hz	$[0.03, 3.12]$ Hz
20 kt	$[0.02, 2.53]$ Hz	$[0.03, 0.70]$ Hz	$[0.04, 3.93]$ Hz

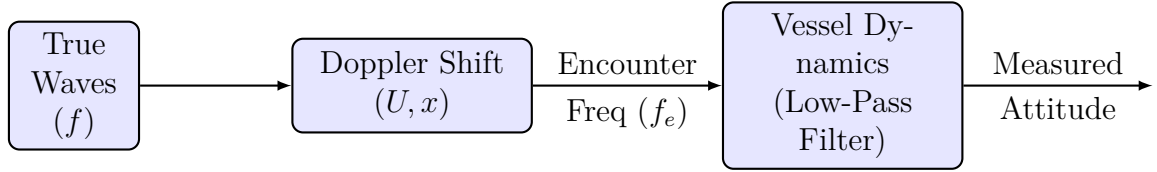


Figure 2.1: Block diagram illustrating the transformation of true ocean waves into measured vessel attitude. High-frequency encounter waves are naturally low-pass filtered by the vessel’s rigid-body inertia.

2.2.1 JONSWAP

While there are many different methods for modeling ocean waves, the JONSWAP spectrum is utilized in this thesis. JONSWAP is an empirical wave energy spectrum used to model fetch-limited (developing) ocean waves. This means that the waves are induced by wind but have not reached their full theoretical steady state, due to the wind blowing over a limited stretch of open water (the fetch). A simplified version of the spectra is given by [12, p. 172]:

$$S(\omega) = 0.2H_s^2\omega_0^4\omega^{-5}\exp(-1.25\omega_0^4\omega^{-4})\gamma^{\exp\left(-\frac{(\omega-\omega_0)^2}{2\sigma^2\omega_0^2}\right)} \quad (2.2)$$

Where:

- H_s is the significant wave height.
- ω_0 is the modal frequency.
- γ is a constant that is usually chosen to be approximately 3.3.
- σ is a parameter where $\sigma = 0.07$ if $\omega < \omega_0$ and $\sigma = 0.09$ otherwise.

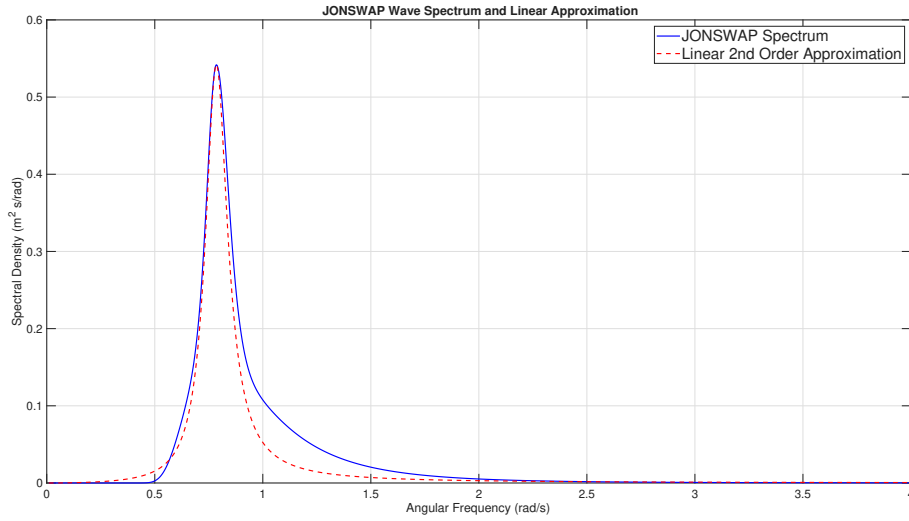


Figure 2.2: Example spectra using $H_s = 1.5$, $\omega_o = 0.785$ and $\gamma = 3.3$

An example spectrum is illustrated in Figure 2.2. The figure also shows a second order linear approximation of the spectrum which is achieved by letting white noise pass through a second order filter [12, p. 172]:

$$H(s) = \frac{Ks}{s^2 + 2\zeta\omega_0s + \omega_0^2} \quad (2.3)$$

Where K is a gain constant used to scale the peak of the approximation to match the JONSWAP spectrum, and ζ is the relative damping ratio. The linear approximation is used to motivate the model structure in Section 4.3.3, while the nonlinear spectrum is used for generating realistic colored noise disturbances in Section 5.1.2, to validate the robustness of the parameter estimation methods.

2.3 System Identification

Some system identification principles that are used in Chapter 5 and 6 are explained.

2.3.1 Prediction Error Method

Prediction error method (PEM) is a family name of system identification methods based on minimizing a prediction error, which is done by solving an optimization problem. The general optimization problem is described by Ljung [13, eq. 7.11, p. 199]:

$$\hat{\theta}_N = \arg \min_{\theta \in D_{\mathcal{M}}} V_N(\theta, Z^N) \quad (2.4)$$

Where:

- θ is the parameter vector.

- $D_{\mathcal{M}}$ is the model set.
- V_N is a scalar valued loss function, typically defined as the sum of squared prediction errors.
- Z^N is the known dataset.

A common PEM method is Least Squares (LS), which has one global minimum that can be solved analytically. However, LS is limited to models where the one-step predictor is linear in the parameters. The following subsections explain why LS estimation fails when exposed to colored noise, and introduce the Gauss-Newton optimizer to cope with this problem.

2.3.1.1 Least Squares and the Colored Noise Problem

This section provides the theoretical explanation to why Recursive Least Squares (RLS) fails in Section 6.3. A linear model of a system can be considered by the following equation:

$$\hat{y}(t) = \varphi^T(t)\theta \quad (2.5)$$

$$\varphi(t) = [-y(t-1), \dots, -y(t-n_a), u(t-1), \dots, u(t-n_b)]^T \quad (2.6)$$

$$\theta = [a_1, \dots, a_{n_a}, b_1, \dots, b_{n_b}]^T \quad (2.7)$$

Where $\hat{y}$ is the predicted output, $\varphi(t)$ is the regressor and θ is the system parameter vector. The prediction error to be minimized is defined by:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t) = y(t) - \varphi(t)^T \theta \quad (2.8)$$

where y is the true system output. ε is used in the cost function (2.4) [13, p.204]:

$$V_N(\theta, Z^N) = \frac{1}{N} \sum_{t=1}^N \frac{1}{2} \varepsilon(t, \theta)^2 = \frac{1}{N} \sum_{t=1}^N \frac{1}{2} [y(t) - \varphi(t)^T \theta]^2 \quad (2.9)$$

(2.9) is minimized analytically by [13, p.204]:

$$\hat{\theta}_N^{LS} = \arg \min V_N(\theta, Z^N) = \underbrace{\left[\frac{1}{N} \sum_{t=1}^N \varphi(t) \varphi^T(t) \right]^{-1}}_{R(N)} \frac{1}{N} \sum_{t=1}^N \varphi(t) y(t) \quad (2.10)$$

Consider a system:

$$y(t) = \varphi(t)^T \theta_0 + v_0(t) \quad (2.11)$$

where θ_0 is the true parameter vector and $v_0(t)$ is considered white noise. The estimation error for LS becomes [13, p.205]:

$$\lim_{N \rightarrow \infty} \hat{\theta}_N^{LS} - \theta_0 = \lim_{N \rightarrow \infty} R^{-1}(N) \frac{1}{N} \sum_{t=1}^N \varphi(t) v_0(t) \quad (2.12)$$

One of the conditions for (2.12) to tend to zero is that the expected value:

$$E[\varphi(t)v_0(t)] = 0 \quad (2.13)$$

(2.13) holds if v_0 is white noise. However, if v_0 is colored noise, the term $v_0(t)$ in (2.11) depends on what happened up to time $t - 1$. Because $\varphi(t)$ contains past outputs $y(t - n)$, φ and v_0 are correlated which means that equation 2.13 $\neq 0$, and the LS estimate is biased.

2.3.1.2 Gauss-Newton

Gauss-Newton (GN) is a numerical search method used for solving nonlinear LS problems. It can be applied to models which do not have a linear in the parameter predictor [14, p. 211-213], which allows for a higher flexibility in modeling the noise. In contrast to the LS problem this cannot be solved analytically. The iterative parameter update step is given by [14, Eq. (7.87), p. 213]:

$$\hat{\theta}^{(k+1)} = \hat{\theta}^{(k)} + \alpha_k \left[\sum_{t=1}^N \psi(t, \hat{\theta}^{(k)}) H \psi^T(t, \hat{\theta}^{(k)}) \right]^{-1} \left[\sum_{t=1}^N \psi(t, \hat{\theta}^{(k)}) H \varepsilon(t, \hat{\theta}^{(k)}) \right] \quad (2.14)$$

where:

- $\hat{\theta}^{(k)}$ is the estimated parameter vector at iteration k .
- α_k is the step size at iteration k .
- N is the total number of data samples.
- t is the discrete time index.
- $\psi(t, \hat{\theta}^{(k)})$ is the gradient of the predicted output with respect to the parameters (the sensitivity matrix).
- H is a symmetric, positive-definite weighting matrix.
- $\varepsilon(t, \hat{\theta}^{(k)})$ is the prediction error.

Since GN is iteratively searching over a non-convex cost function it is critical to provide a good initial parameter estimate $\hat{\theta}^{(0)}$ to prevent the algorithm from converging at a local minima. This is used to identify system parameters in Section 5.2 and 6.1.

2.3.2 Instrumental Variables

An alternative method for handling colored disturbances is the use of correlation based methods such as Instrumental Variables (IV). To avoid the biased estimate result from section 2.3.1.1, LS can be extended to use IV [13, p.224]. This is done by introducing a correlation vector ζ called the Instrument, which should be correlated

with the regression vector φ and uncorrelated with the noise v_0 [13, p.225]. Using the instrument ζ , equation 2.10 can be reformulated as:

$$\hat{\theta}_N^{IV} = \underbrace{\left[\frac{1}{N} \sum_{t=1}^N \zeta(t) \varphi^T(t) \right]^{-1}}_{R_{IV}(N)} \frac{1}{N} \sum_{t=1}^N \zeta(t) y(t) \quad (2.15)$$

which yields:

$$\lim_{N \rightarrow \infty} \hat{\theta}_N^{IV} - \theta_0 = \lim_{N \rightarrow \infty} R_{IV}^{-1}(N) \frac{1}{N} \sum_{t=1}^N \zeta(t) v_0(t) \quad (2.16)$$

that removes the bias in the estimation error if v_0 is colored, given that ζ is chosen correctly [13, p.225]. This is used in Section 6.2.

3

Design of Excitation Signal

In order to identify the dynamics of the vessel, an appropriate excitation signal needs to be applied to the vessel's control surfaces. In this chapter, historical data collected by Humphree prior to this thesis is evaluated. Limitations and flaws with this data are identified and used together with literature and system constraints to formulate a list of requirements for a new excitation signal. An excitation signal is chosen and validated against the formulated requirements. Finally, the frequency bandwidth where the vessel dynamics exist is determined.

3.1 Available Data from Previous Sea Trial

Data collected from a sea trial before this thesis utilized a Random Binary Signal (RBS) as input when the vessel was traveling at 10 kt. This signal is plotted in Figure 3.1. The RBS is only applied to the interceptors and fins and not to the rudder. The reason is that the rudder is controlled manually by the captain and can thus not have these inputs.

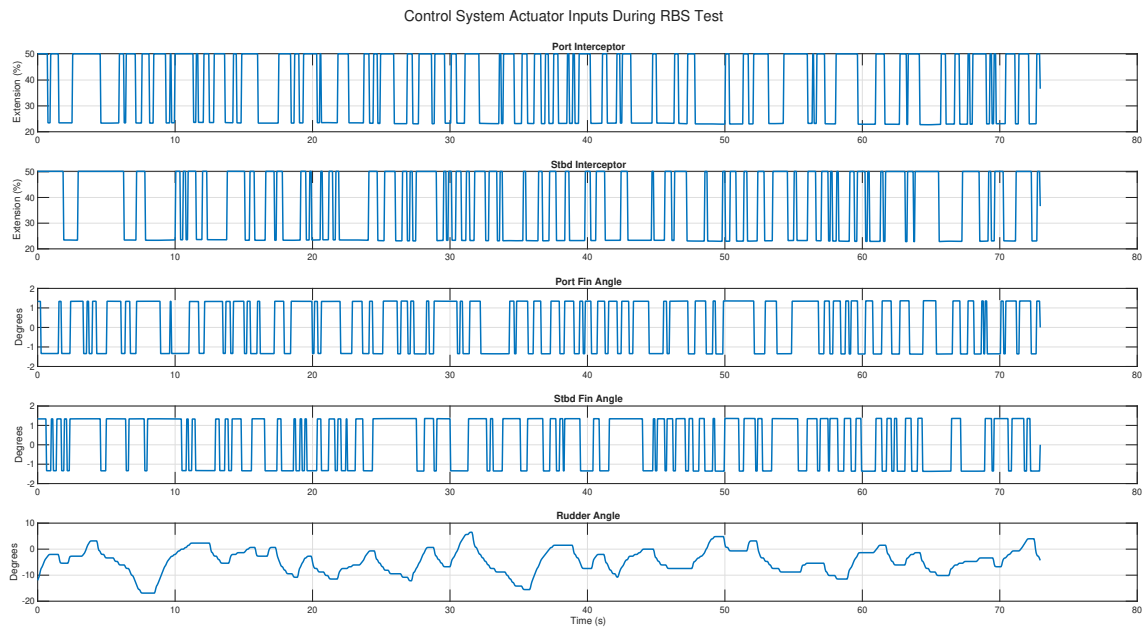


Figure 3.1: Input reference signals during RBS test.

The RBS Power Spectral Densities (PSDs) are plotted in Figure 3.2 together with the PSD of the roll and pitch output angles. Figure 3.2 shows that the inputs with the RBS reference have frequency properties with higher energy for lower frequencies that tails off gradually along higher frequencies. The outputs PSD tails off faster with almost all energy existing at frequencies of 1 Hz and lower. This is due to the vessel acting as a low-pass filter effectively damping higher frequency content (shown in Section 3.4). A low Signal to Noise Ratio (SNR) might contribute to this but cannot be concluded due to the noise being unknown.

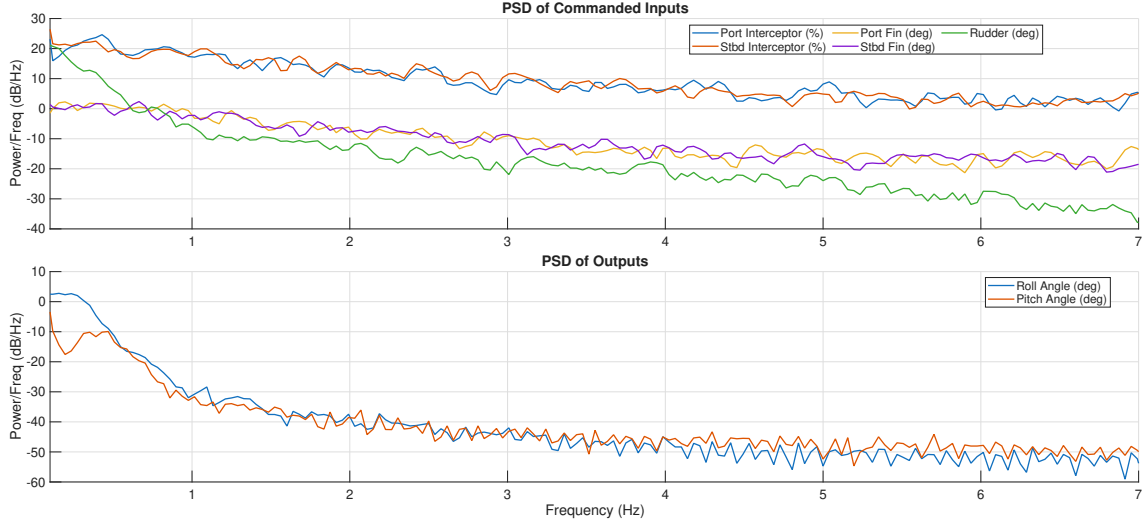


Figure 3.2: PSD for fin reference during RBS test. The commanded inputs(reference for the inputs) contains higher frequencies than they can physically execute. The Roll and Pitch outputs are only excited significantly at less than approximately 1 Hz.

Since the data is recorded at a constant speed, the dynamics are expected to be mostly linear (confirmed in Section 5.3.4). The SNR is therefore evaluated by considering the Magnitude Squared Coherence (MSC) estimate. MSC takes values in the interval of $[0, 1]$ where a value approaching 1 indicates a linear relationship in the data and a value approaching 0 instead indicates a non-linear relationship in the data [15]. This non-linear relationship can be explained by non-linear dynamics or noise corrupting the data. The MSC estimate is calculated for the input-output data according to:

$$\gamma_{y:u}^2(f) = \frac{\mathbf{P}_{yu}(f)\mathbf{P}_{uu}^{-1}(f)\mathbf{P}_{uy}(f)}{P_{yy}(f)} \quad (3.1)$$

where y is the output, u is the input, f is the frequency, $P_{yy}(f)$ is the auto-spectral density of the output, $\mathbf{P}_{uu}(f)$ is the $q \times q$ spectral density matrix of the q inputs, $\mathbf{P}_{uy}(f)$ is the $q \times 1$ cross-spectral density vector between the inputs and the output, and $\mathbf{P}_{yu}(f)$ is its conjugate transpose.

Figure 3.3 shows that the MSC drops significantly at approximately 0.3 Hz for the

roll angle output and approximately at 0.16 Hz for the pitch angle output. The MSC value is also consistently smaller for pitch compared to roll. Because the input-output dynamics are expected to be linear, the drop in MSC is explained by low SNR.

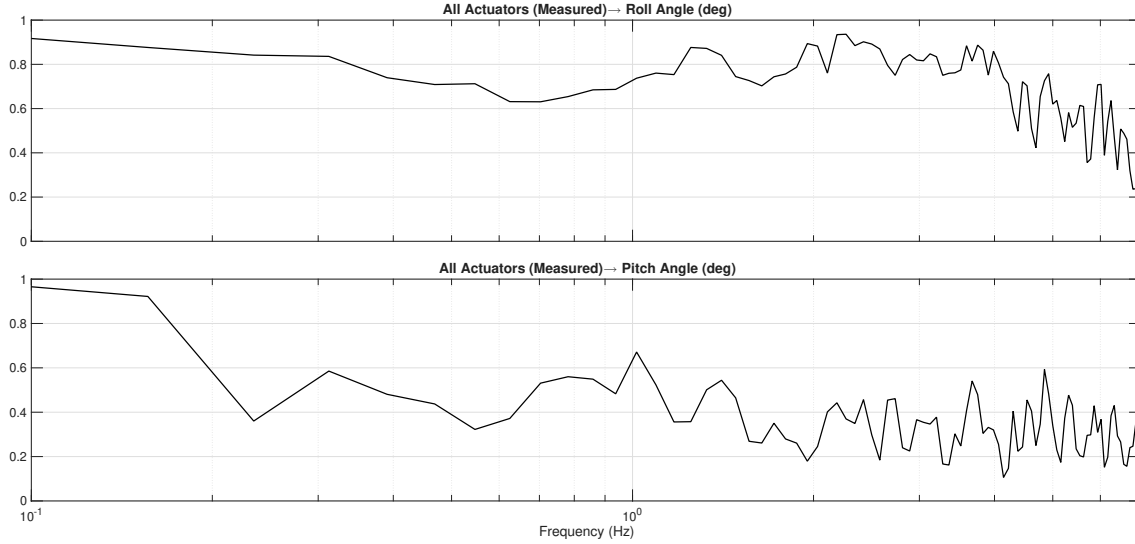


Figure 3.3: The MSC plot for all inputs to roll and pitch angles. The MSC drops significantly at approximately 0.3 Hz for roll and at 0.16 Hz for pitch.

A summary of what is learned from this data:

- The vessel's frequency properties are concentrated to approximately $[0,1]$ Hz, based on the PSD in Figure 3.2.
- The SNR seems to be too low for the inputs to have considerable effects on the output, especially for pitch based on Figure 3.3.

These conclusions are considered along literature when choosing the new excitation signal in the following section.

3.2 Specification of Excitation Signal Requirements

To design the new excitation signal that allows for system identification, a list of requirements is formulated. The list was developed by considering the literature ([13, p.418-423] [16, 9]), previous sea trials (Section 3.1) and physical system constraints. The selected signal (Pseudo Random Binary Sequence (PRBS)) is then adapted to fulfill these requirements. A summary of the listed requirements is provided in Table 3.1.

Table 3.1: Summary of Excitation Signal Requirements and Fulfillment

Requirement Specification	Filtered PRBS
1. High SNR	
1.1 Maintains high actuator amplitude	✓
1.2 Excite the full frequency band for the control actuators	✓
2. Deterministic and periodic signal for data averaging	✓
2.1 Used to increase SNR	✓
2.2 Used to estimate noise	✓
3. System Dynamics Excitation	
3.1 Dominant energy focused in the 0–2 Hz band	✓
3.2 Statistically uncorrelated input signals for MIMO system identification:	✓
4. Actuator Constraints	
Smooth transitions to minimize mechanical wear	✓

Table 3.1 is discussed and explained further in the following text.

1. High SNR

1.1 Maintains high actuator amplitude: It is possible to use a high amplitude for the input signal. This increases the actuator effects on the output which helps overpower output effects caused by noise.

1.2 Excites the full frequency band where the control actuators: In Section 2.2 it is established that the wave disturbances can appear anywhere in the frequency band where the vessel dynamics exist. If the wave disturbance overpowers the control actuator induced output, the system may still be identified at other frequencies [17]. For this reason it is desired that the excitation signal excites the full frequency band where the vessel dynamics exist, or at least at the frequencies where the control surfaces actually can operate.

2. Deterministic and periodic signal for data averaging:

2.1 Used to increase SNR: With a periodic input signal, the SNR can be increased. This is done by running the signal periodically for n periods. Due to the system output being deterministic, the data can then be averaged over the n periods which increases the SNR by a factor of n [13].

2.2 Used to estimate noise: Using the period average, the noise can be estimated as well. This is done by removing the average from each period which leaves an estimate of the noise useful for noise analysis.

3. System dynamics excitation

3.1 Dominant energy focused in the 0–2 Hz band: Section 3.1 showed that the system dynamics exist at frequencies within approximately 0-1 Hz. To excite this frequency band two actions are taken:

- The signal is low-pass filtered at a cutoff frequency of 2 Hz. This concentrates the signal energy to the $[0, 2]$ Hz frequency band, intentionally exceeding the system dynamics frequency band to have some margin for error. The actuators are not expected to be efficient at frequencies over 2 Hz due to system constraints, which also motivates this choice of cutoff frequency.
- The clock frequency is lowered (holding each input state over several samples) - This acts as a low-pass filter [13, p.422], making the reference change less often, giving the actuators more time to reach the target reference signal. It is advisable to choose the clock frequency to be 2.5 times the bandwidth covered by the signal [13, P.422], which yields a clock frequency of $2 \cdot 2.5 = 5$ Hz for this case.

3.2 Statistically uncorrelated input signals for MIMO system identification: To be able to model specific input-output dynamics the input signals applied to a MIMO system needs to be uncorrelated with each other [13]. A PRBS signal is a deterministic signal and only maintains white noise PSD properties when the full sequence is executed [13, p.421-422]. This means that the same PRBS cannot be applied to all inputs simultaneously for MIMO system identification, because the inputs would be fully correlated with each other. To solve this, the PRBS can be time shifted for every input to the system [1]. This results in input signals that are uncorrelated, making the PRBS possible to use for MIMO identification.

4. Actuator constraints, wear and tear

The initial data collection from Section 3.1 utilized standard random binary input signals. These signals contain aggressive, instantaneous step changes that induce mechanical stress on the control surfaces and are physically impossible for the actuators to execute perfectly. To solve this the excitation signal must be smoother than a raw binary signal. This is accounted for by:

- The low-pass filter previously introduced to solve the "System dynamics excitation" requirement makes the signal smoother, avoiding sudden and aggressive step changes.
- The chosen clock frequency of 5 Hz means that the signal holds the value for longer resulting in less aggressive changes.

The modified PRBS signal is examined and compared to the previous excitation signal in the following section to validate that it fulfills the listed requirements in Table 3.1.

3.3 Validation of Chosen Excitation Signals

A PRBS can have varying lengths depending on the number of bits used to generate the sequence (derived in [9]). An overview is provided in Table 3.2.

# bits	Length	Sequence time (s) with 5 Hz clock freq.	Sequence time (s) with 3 Hz clock freq.
L	$2^L - 1$	$0.2 \times (2^L - 1)$	$1/3 \times (2^L - 1)$
4	15	3.0	5.0
5	31	6.2	10.3
6	63	12.6	21.0
7	127	25.4	42.3
8	255	51.0	85.0
9	511	102.2	170.3
10	1023	204.6	341.0

Table 3.2: Resulting length of PRBS and period time (at 5 Hz and 3 Hz clock frequencies) depending on the number of bits used for generation.

As stated in Requirement 3.2, the PRBS needs to be time shifted in order to be used for MIMO system identification. A prerequisite for this is that the longest settling time of all channels needs to be shorter than the time shift of one PRBS sequence [16]. This ensures that the transient dies off before a new one is introduced, resulting in uncorrelated input signals, fulfilling Requirement 3.2 in Table 3.1. The system's transient lengths are unknown and therefore estimated to be around 15 seconds based on empirical observations during the sea trial discussed in Section 3.1. Considering this, the PRBS9 is chosen which has a length of 102.2 seconds according to Table 3.2. This yields a time shift of $\frac{102.2}{3} \approx 34$ seconds when applied to the interceptors and fin control actuators, resulting in a $34 - 15 = 19$ seconds margin for error in the time shift.

The amplitude should be chosen as high as possible to increase SNR according to Requirement 1.1, while also considering system limitations in terms of limited actuator bandwidth. The actuators bandwidths needs to be considered since if the requested amplitude is too large, the actuators will be required to move beyond their frequency limits. The actuators have their own closed loop control system described by the block diagram in Figure 3.4.

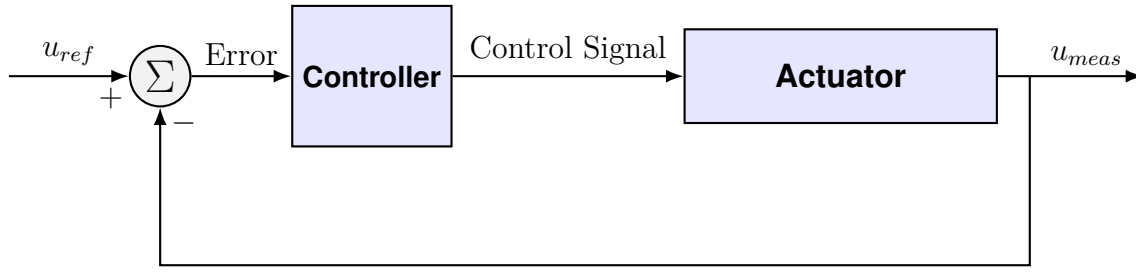


Figure 3.4: Closed-loop block diagram of the integrated actuator unit. The actuator moves according to the control signal and its movement is limited to the actuator bandwidth.

The exact duration of this time delay in Figure 3.4 is unknown. Therefore the amplitude is initially chosen to be 25% for the interceptors and 3° for the fins (the amplitude choice is elaborated upon in Section 3.4 and discussed in Section 7.1.1) based on manual testing prior to the data collection. The generated, time shifted sequences are plotted in Figure 3.5.

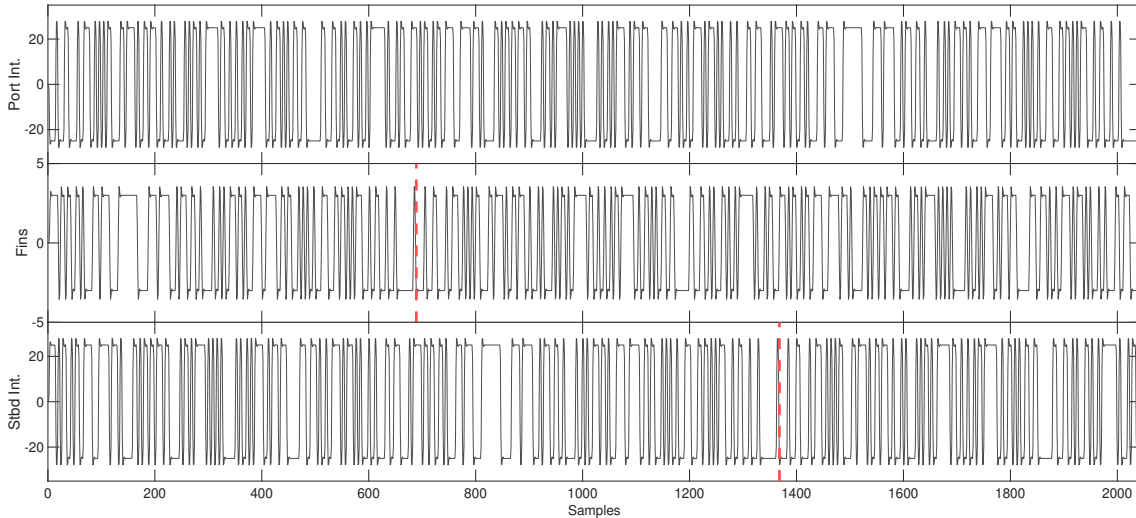


Figure 3.5: The same PRBS9 signal is applied to all inputs, with the starting point shifted to make the inputs uncorrelated. The time shifts are marked with a red dashed line.

To confirm the signals in Figure 3.5 are uncorrelated, the cross correlation between them is calculated and plotted in Figure 3.6. The figure shows that the signals are uncorrelated except at the time lags of 680 and 1360, which aligns with the time shift shown in Figure 3.5. These correlation peaks do not cause correlated excitation signals if the time shift is greater than the systems longest transient [1]. For this reason the input signals are considered to be uncorrelated.

3. Design of Excitation Signal

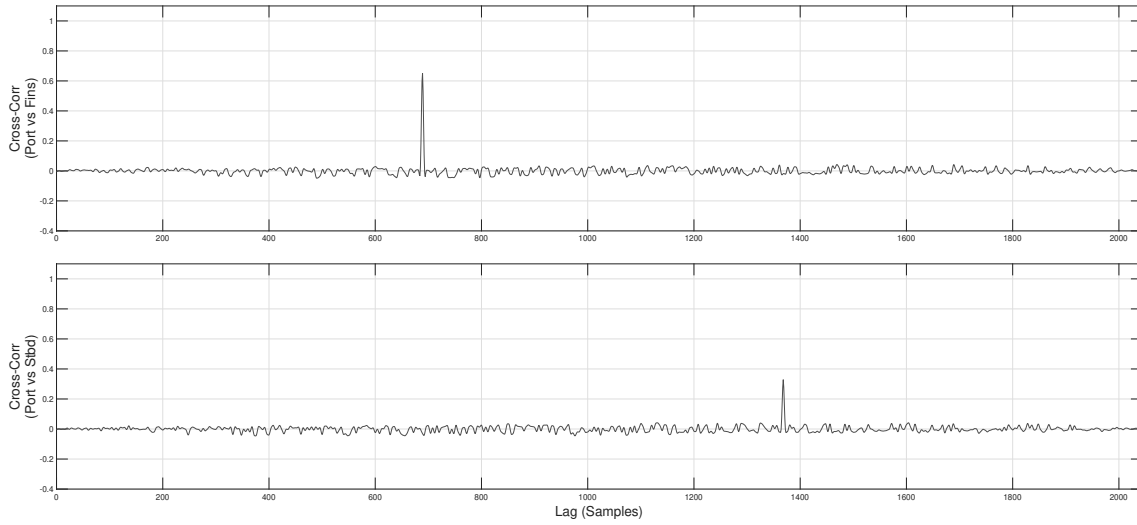


Figure 3.6: Cross correlation between the shifted PRBS sequences. The time shift has successfully yielded uncorrelated input signals, except at lag 680 and 1360. This is expected since the signals are the same sequences, only differing in amplitude and time shift. They do however not cause correlated excitation signals if the time shifts are greater than the systems longest transient [1]. For this reason the excitation is considered to be uncorrelated and fulfills Requirement 3.2.

The PSD of the generated PRBS9 is plotted in Figure 3.7 together with the actuators measured position to confirm that the PRBS9 excites the actuators full frequency bands (Requirement 1.2). This is confirmed because the actuators PSDs does not contain the same power as the reference for frequencies above ≈ 1.2 Hz for the interceptors and above ≈ 2 Hz for the fins. Hence, the actuators are saturated and therefore excited at their full bandwidth.

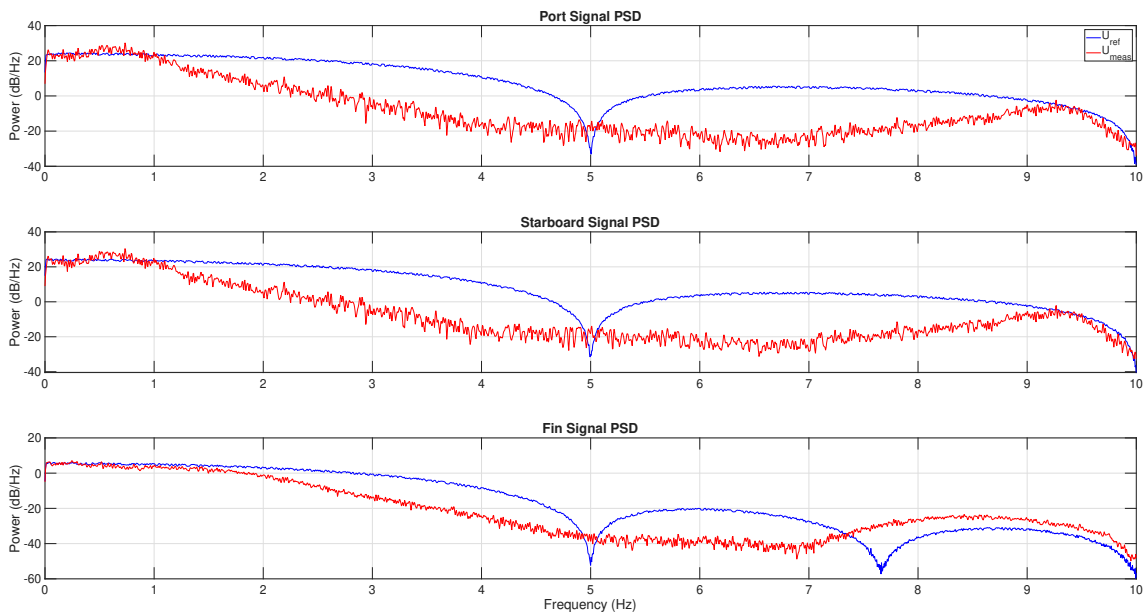


Figure 3.7: PSD of PRBS9 reference signal, and the actuators measured position.

3.4 Improving the Selected Excitation Signal

A sea trial was performed with the validated excitation signal from Section 3.3. The MSC estimate is calculated on the input-output data from this sea trial, and plotted in Figure 3.8.

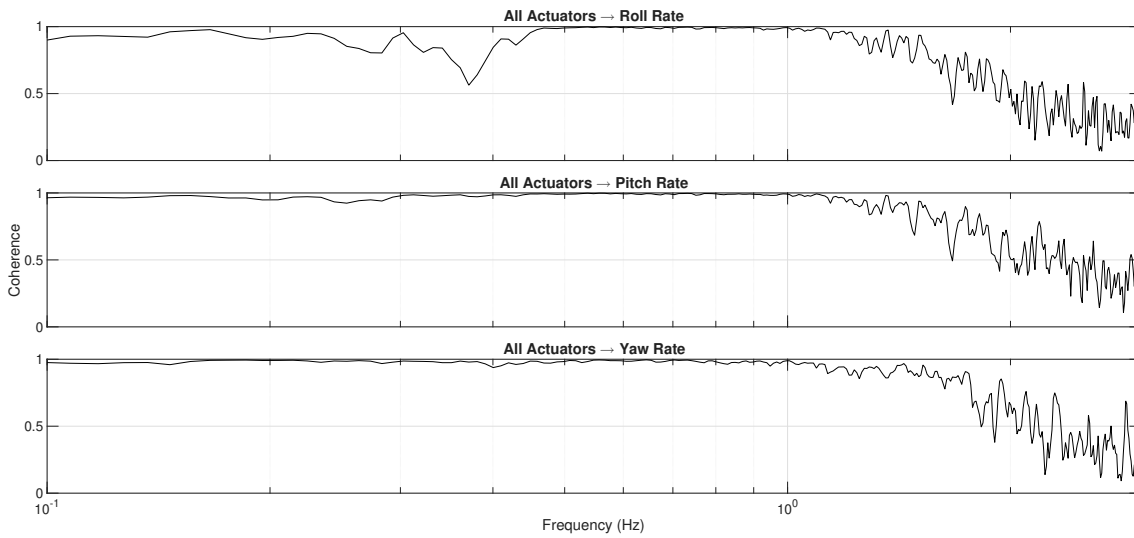


Figure 3.8: MSC of the input-output data collected with the PRBS9 excitation signal. As explained in Section 3.1, a value close to one indicates a strong linear relationship between the signals. Based on this the input-output relationship is linear except at approximately 0.37 Hz for the roll rate.

Figure 3.8 suggests that the maximum frequency for when the system is linearly excited by all of the control surfaces is approximately 1 Hz. This means that the clock frequency in Requirement 3.1 can be reduced to $1 \cdot 2.5 = 2.5$ Hz, but is chosen to 3 Hz to keep some margin for error. The MSC for roll rate in Figure 3.8 shows a drop at approximately 0.37 Hz. This non-linearity lacks physical explanation since the vessel is traveling at a constant speed, and is therefore expected to be colored noise corrupting the data (confirmed by Figure 3.12).

The PRBS9 reference signal is plotted against the measured actuator position in Figure 3.9 to see if the actuators can keep up with the frequencies at the chosen amplitude.

3. Design of Excitation Signal

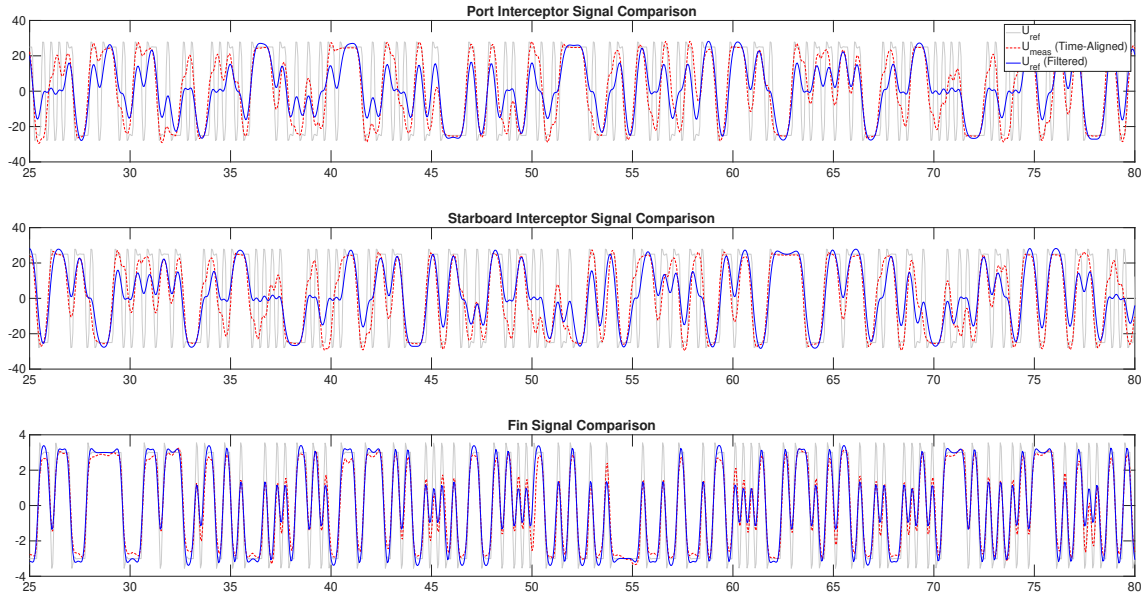


Figure 3.9: The figure shows the actuator reference signal (U_{ref}), the Measured actuator position (U_{meas} Time-Aligned) and the low-pass filtered reference signal (U_{ref} Filtered) for the PRBS9. Since there exists a time delay between the requested actuator position and the actuator response, U_{meas} is time shifted to align with U_{ref} to make it easier to see the effect of the low-pass filter (U_{ref} Filtered). The figure shows that the low-pass filter makes the reference signal align with the measured actuator position.

In Figure 3.9 it is clear that the requested frequencies are too high for the actuators to keep up with the set amplitude. To confirm this, the signal is low-pass filtered to see if this makes the reference align with the measured actuator position, with the result plotted in Figure 3.9. The low-pass filter cutoff frequencies are set to 2 Hz for the fins and 1.2 Hz for the interceptors. These values are based on the highest frequency at which the control surfaces follows the PSD of the reference signal, see Figure 3.7. This further motivates lowering the clock frequency to 3 Hz, but also to lower the requested amplitude.

The amplitude is chosen through empirical testing at different vessel speeds. Since hydrodynamic forces increase at higher speeds (due to the increased water flow introducing more pressure on the actuators) control authority increases at higher vessel speed. The amplitude should therefore be decreased as the speed increases. Based on this the amplitude for the control inputs are chosen according to Table 3.3. Note that the interceptors are biased to 50% extension. This is strictly necessary at 10 kt since the amplitude is $\pm 50\%$ and the PRBS would otherwise not be executable within the actuator limit of 0% to 100%. This bias is kept at 15 and 20 kt so the interceptors remain at the same operating point.

Vessel Speed (kt)	Interceptor Amplitude (%)	Fin Amplitude (deg)
10 (Displacement)	± 50	± 10
15 (Hump)	± 25	± 6
20 (Planing)	± 25	± 2

Table 3.3: Chosen amplitudes for the PRBS excitation signal across different operating speeds. The amplitude is reduced at higher speeds to compensate for increased hydrodynamic control authority.

During the sea trial, step tests were performed in calm water to investigate the settling time of the vessel. These step tests (provided in Appendix B) suggest that the settling time is shorter than 7 seconds. Further, the identified models in section 5.3 give an estimated longest settling time of approximately 4.37 (10 kt), 2.91 (15 kt) and 3.20 s (20 kt) by step test simulation, confirming that the settling time is less than 7 seconds. This means that the PRBS sequence needs to be at least $7 \cdot 3 = 21$ seconds. This motivates choosing PRBS6 with the 3 Hz clock frequency according to Table 3.2, but since it leaves no margin for error the PRBS7 is chosen yielding $\frac{42.3}{3} = 14.1$ seconds time shifts. This shorter signal comes with the benefit of collecting more periods of data during the same time sequence, yielding more accurate data averaging for Requirement 2 in Table 3.1.

To validate the new PRBS7 signal, it is plotted in the time and frequency domain in Figures 3.10 - 3.11.

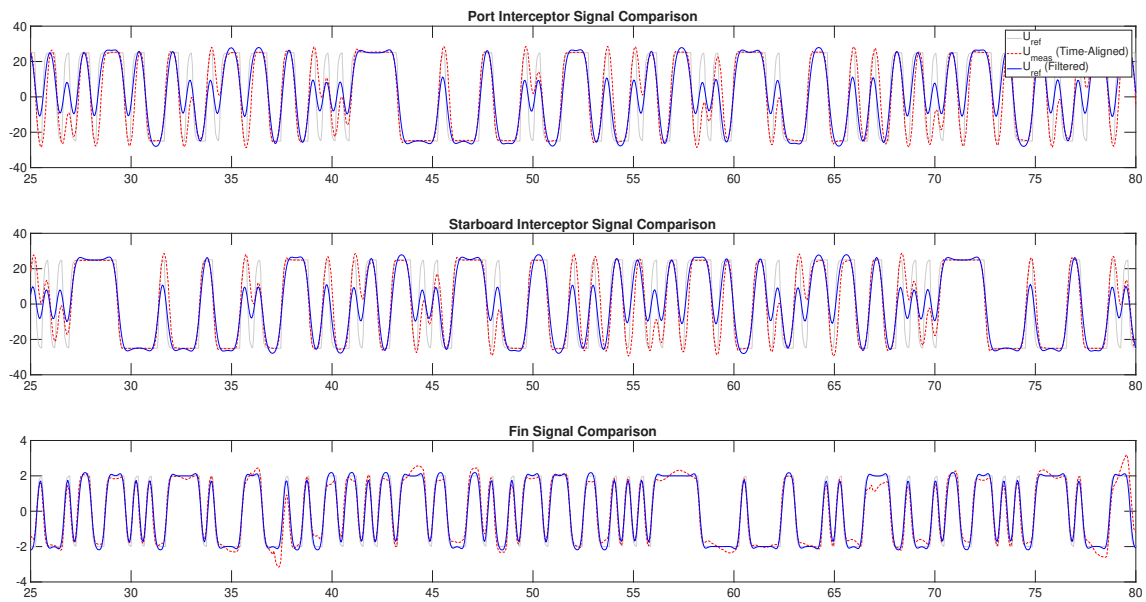


Figure 3.10: Same as in Figure 3.9 but for the PRBS7 signal.

3. Design of Excitation Signal

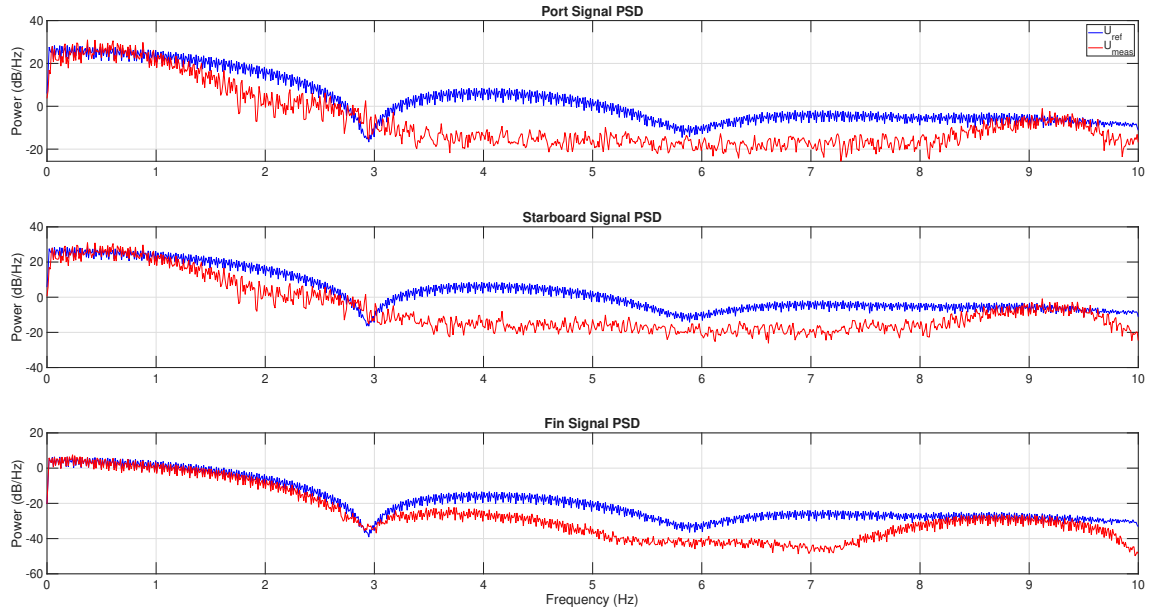


Figure 3.11: PSD of PRBS7 reference signal, and the actuators measured position.

Figure 3.10 shows that the fins manages to follow the reference much better compared to the PRBS9 signal. Figure 3.11 shows that the measured fin signal overlaps with the requested signal PSD, further implying that the reference is tracked correctly. The interceptors are however not following the reference completely, but this frequency and amplitude is considered necessary to excite the system enough to allow for identifying the interceptor parameters in following chapters. Figure 3.12 shows that the input-output data is linear, suggesting that the MSC drop in Figure 3.8 is caused by some disturbance.

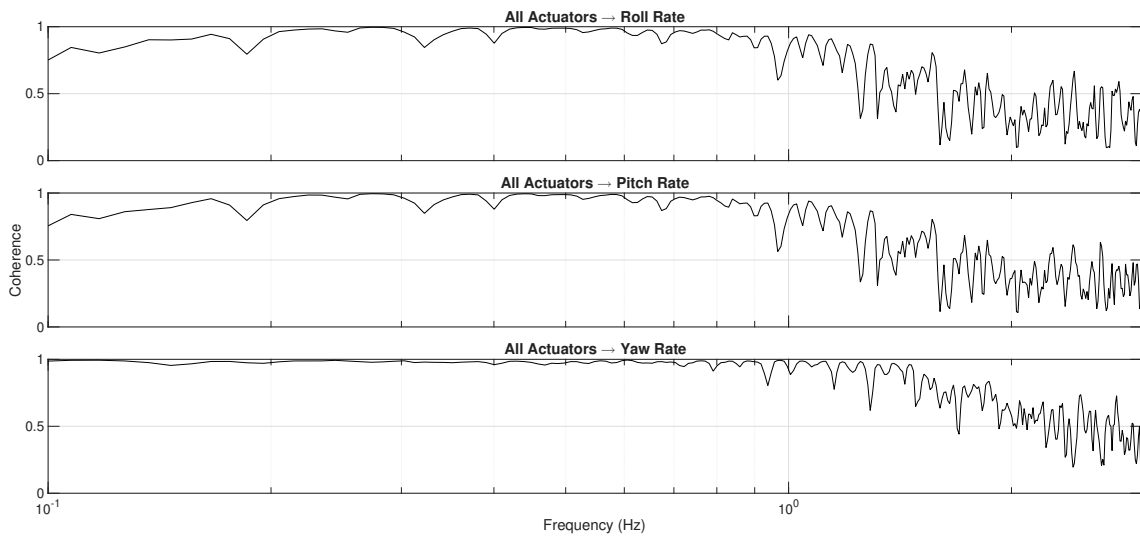


Figure 3.12: MSC of the input-output data collected with the PRBS7 excitation signal. As explained in Section 3.1, a value close to one indicates a strong linear relationship between the signals.

3.5 System Occupied Bandwidth

As explained in Section 2.2, the system dynamics do not occupy the full spectrum due to the vessel’s low-pass filter effect. It is relevant to determine the occupied spectrum of the system dynamics, so the model can be validated for its control purpose (done in Section 5.3.2). To determine this spectrum the system occupied bandwidth is used, which is defined by the bandwidth that contains 99% of the dataset’s spectral energy. The system occupied bandwidth is calculated for the outputs of the collected data at the speeds of 10, 15 and 20 kt, provided in Table 3.4 and highlighted with plots in Figures 3.13 – 3.15. Table 3.4 provides bandwidths approximately between 0 and 1.7 Hz, with the exception of pitch rate at 10 and 15 kt. These intervals stretch up to roughly 6 Hz. This is because the interceptors are simply too weak to control the pitch rate at these speeds. As established in Section 3.4, control authority is highly speed-dependent. Consequently, this results in a low SNR at 10 and 15 kt, which means that the 99% occupied bandwidth consists mostly of noise.

Table 3.4: 99% Occupied Bandwidth of Output Signals across Vessel Speed Regimes

Output Signal	99% Occupied Bandwidth (Hz) [Lower, Upper Limits]		
	10 kt (Displacement)	15 kt (Hump)	20 kt (Planing)
List Angle	0.9448 [0.00, 0.95]	0.9378 [0.01, 0.95]	0.6700 [0.00, 0.67]
Trim Angle	0.8535 [0.00, 0.86]	1.0862 [0.00, 1.09]	1.0857 [0.00, 1.09]
Roll Rate	1.4575 [0.09, 1.55]	1.6061 [0.04, 1.65]	1.6129 [0.03, 1.64]
Pitch Rate	5.9168 [0.08, 6.00]	5.2832 [0.07, 5.36]	1.5787 [0.14, 1.72]
Yaw Rate	0.5545 [0.00, 0.56]	0.7602 [0.00, 0.76]	0.7397 [0.01, 0.75]

3. Design of Excitation Signal

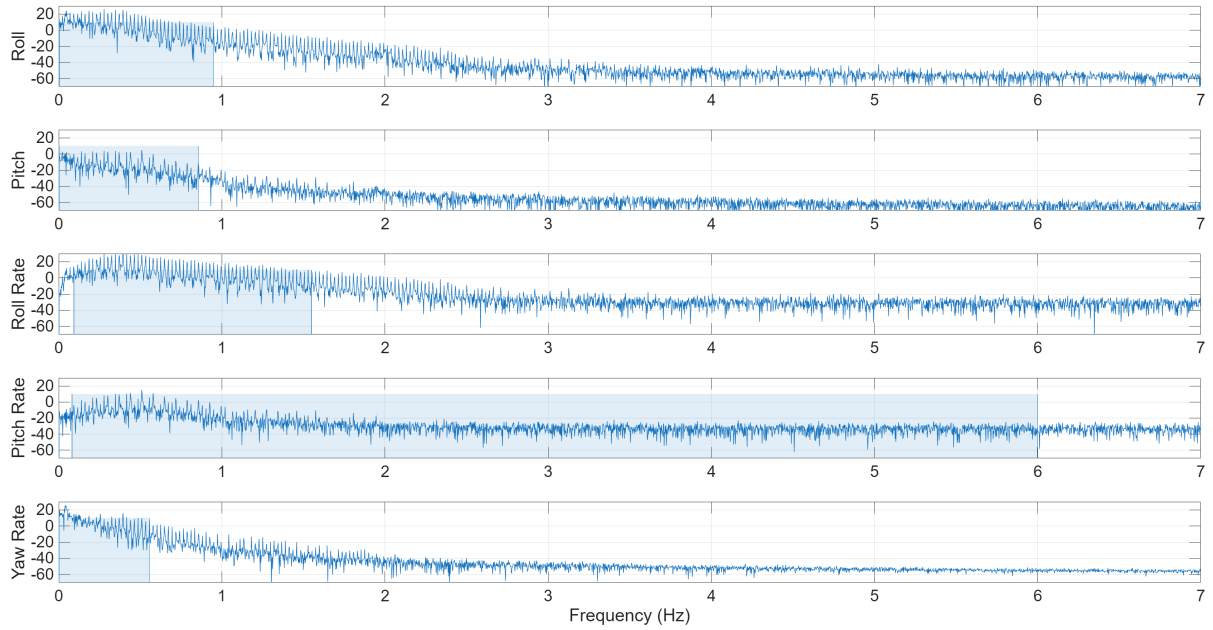


Figure 3.13: Output PSD for data collected at 10 kt, with 99% of the occupied bandwidth highlighted.

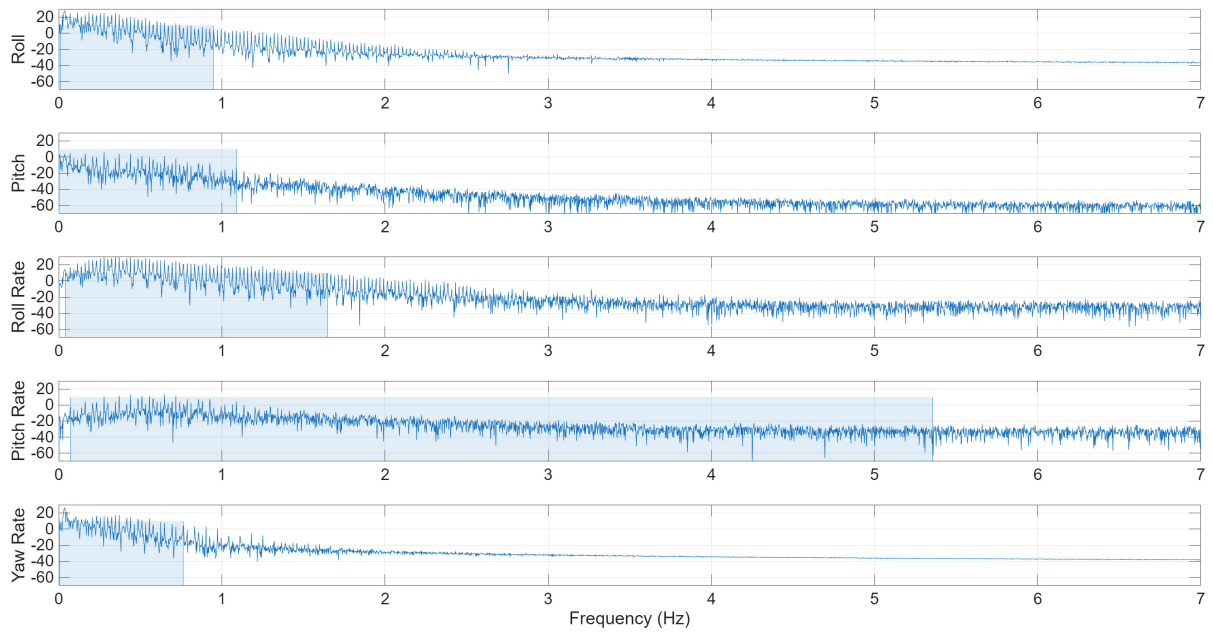


Figure 3.14: Output PSD for data collected at 15 kt, with 99% of the occupied bandwidth highlighted.

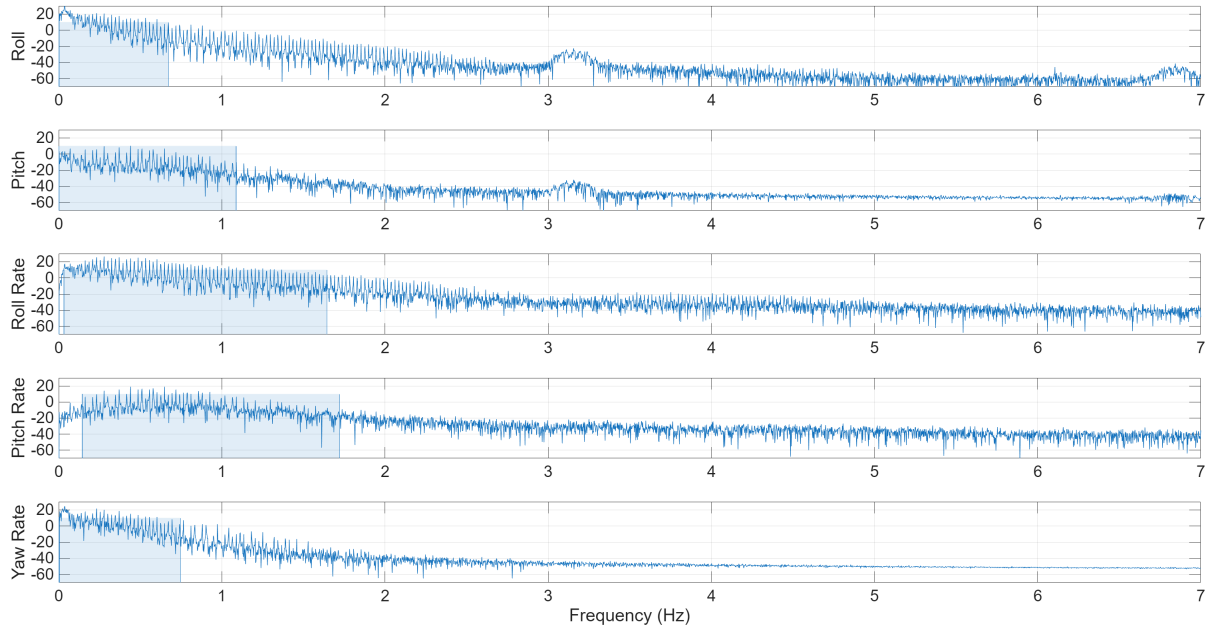


Figure 3.15: Output PSD for data collected at 20 kt, with 99% of the occupied bandwidth highlighted.

3.6 Summary

To summarize this chapter an excitation signal for MIMO identification (shifted and low-pass filtered PRBS7) using 3 inputs has been designed to fulfill the requirements in Table 3.1. The bandwidth for where the model needs to be accurate has also been defined based on collected input-output data and is provided in Table 3.4.

4

Data Collection and Noise Modeling

The sea trials conducted to collect data are explained followed by an analysis of the present colored noise. Different noise models are evaluated by comparing the autocorrelation of their residual and the noise is shown to be time variant.

4.1 Sea Trials

Several tests are performed during sea trials with different excitation signals. The tests are performed for 3 different speeds 10, 15, and 20 kt to collect data for the displacement, hump and planing states of the vessel (explained in Chapter 1). The tests are explained in the following list along with their purpose.

1. **ZIG-ZAG:** This test is performed to identify a SIMO model for the rudder dynamics. A ZIG-ZAG maneuver is performed by applying a symmetric step input to the rudder angle over several periods effectively steering the vessel in a ZIG-ZAG manner, while the interceptor and fins are deactivated. This is needed because the rudder is manually controlled by the vessel captain and can not have a precise computer controlled reference signal (such as a PRBS), like the other control surfaces. This is an established method for identifying the isolated rudder-yaw dynamics [18]. Because of the known coupling between vessel roll and yaw dynamics [8] it is used for rudder-roll dynamic estimation as well.
2. **MIMO:** This test uses the MIMO excitation signal developed in Chapter 3. It is performed by applying the PRBS7 signal for 10 consecutive periods. During the first two PRBS7 periods, 6 ZIG-ZAG steps are applied to the rudder. This excites the rudder, fin and interceptors to allow simultaneous identification of their dynamics (see Chapter 5). For the remaining 8 periods the rudder is excited as little as possible (sometimes it has to be used to avoid collision) to not interfere with the interceptor and fin excitation.
3. **RAMP:** This test uses the same PRBS7 excitation signal executed in the MIMO test. It differs from the MIMO test by only using the PRBS7 for 5

periods and keeping the rudder angle constantly 0 for all periods. The actuator amplitude is increased by 20% of the MIMO amplitude for each period. The purpose of this test is to validate if the systems control actuators are linear across different amplitudes. The RAMP test excitation signal is plotted in Figure 4.1.

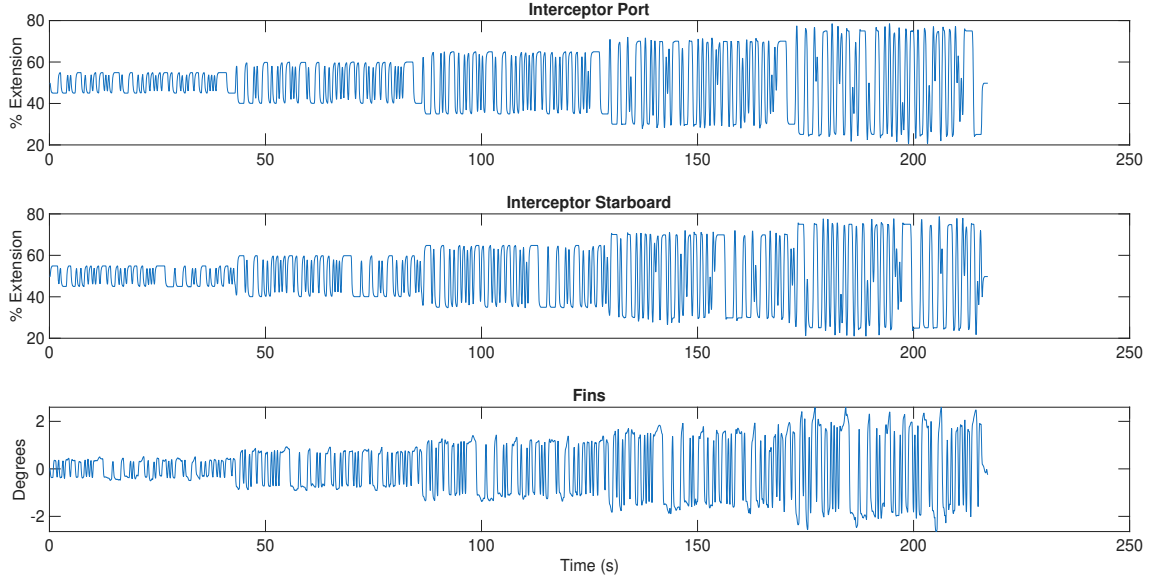


Figure 4.1: The measured actuator signal for each control actuator during the RAMP test.

4.2 Data Pre-processing

To model the noise present in the data set collected from the MIMO test (performed at 20 kt) explained in the previous section, it needs to be pre-processed. This is explained in the following subsections which consists of detrending, rudder compensation and periodic averaging.

4.2.1 Detrending

Before using the data for any model estimation, the mean of each input and output is removed. This removes potential bias in the data, making it easier to identify pure input-output relations [13, p. 459].

4.2.2 Rudder Compensation

As mentioned in Section 4.1, the rudder sometimes has to be used during tests to avoid collision, even though it is meant to stay constant at 0 degrees. The ZIG-ZAG test is used to train a simple rudder model that can be used to reduce undesired rudder effects during sea trials explained in Figure 4.2.

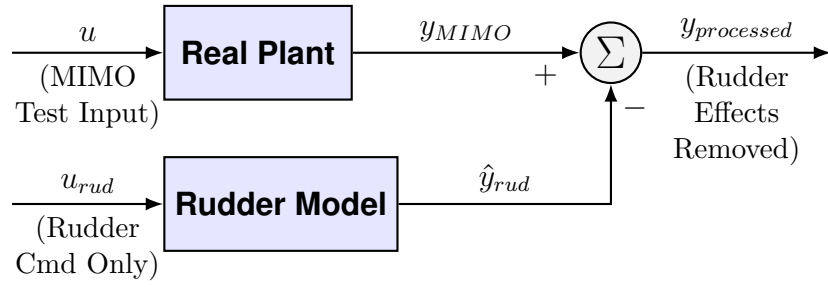


Figure 4.2: Block diagram illustrating the procedure for decoupling and removing rudder effects from the measured MIMO test data.

The rudder can be assumed to only control roll and yaw resulting in a Single Input Multiple Output (SIMO) structure:

$$\begin{bmatrix} Y_{roll}(s) \\ Y_{rollrate}(s) \\ Y_{yawrate}(s) \end{bmatrix} = \begin{bmatrix} G_{roll}(s) \\ G_{rollrate}(s) \\ G_{yawrate}(s) \end{bmatrix} U_{rudder}(s) \quad (4.1)$$

The transfer functions in (4.1) are trained with the ZIG-ZAG test data plotted in Figure 4.3.

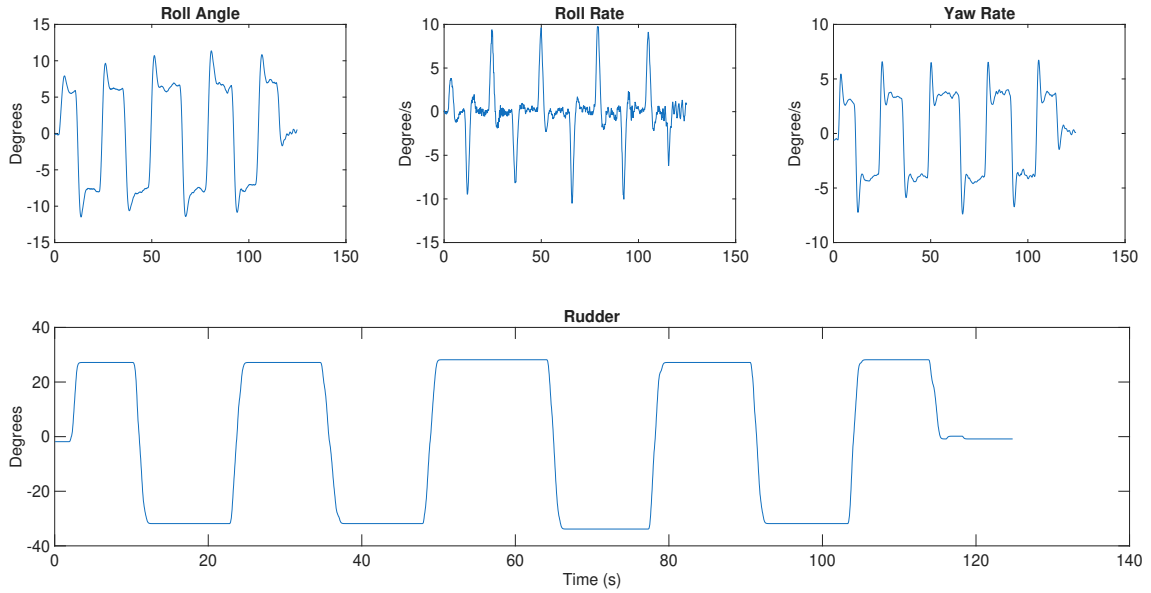


Figure 4.3: Data recorded during ZIG-ZAG test at 20knt. Used for training rudder transfer function model.

The rudder transfer function model is validated on a MIMO test plotted in Figure 4.4. The figure shows that the signal with applied rudder reduction is unaffected when the system is purely excited by the PRBS signal and the rudder is inactive, and where the rudder is active, the compensated signal has become more similar to when the pure PRBS excitation is active. This suggests that the rudder model is effective for reducing any unwanted rudder excitations during a sea trial, which is

useful for estimating pure noise models in the data. This is explained further in the following section.

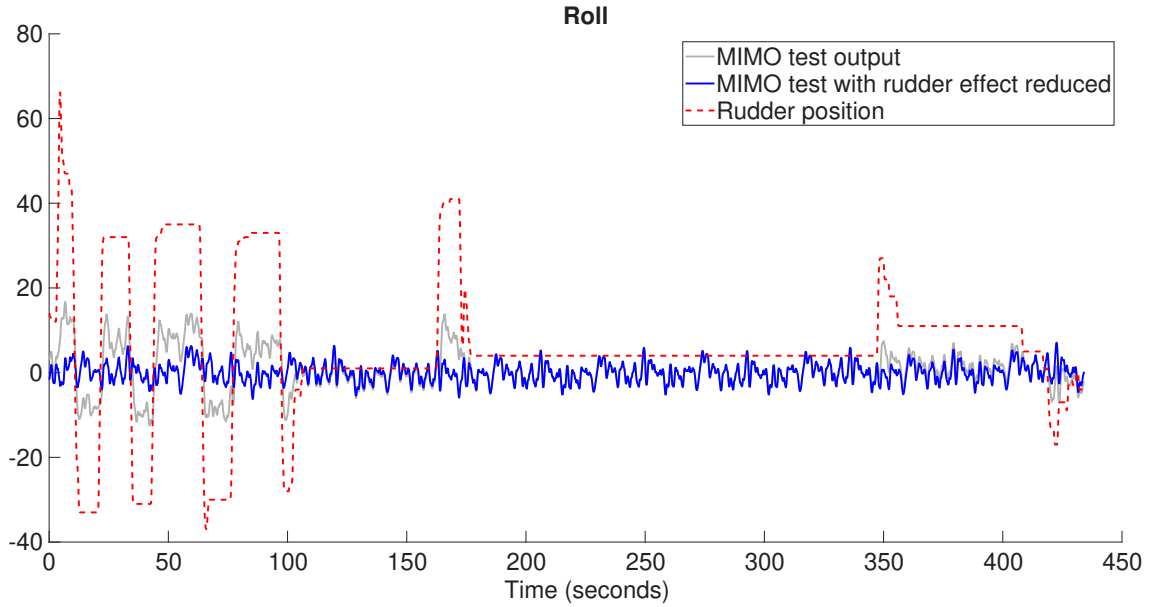


Figure 4.4: A comparison for the MIMO test with reduced rudder excitation for the roll output. The blue line is the roll angle with the rudder effect reduced according to Figure 4.2. The rudder compensation seems to work as intended since the signals align when the rudder is inactive, and where there is rudder excitation a more "normal" signal has been achieved.

4.2.3 Periodic Averaging

Requirement 2 in Section 3.2 states a method that can be used for increased SNR and noise estimation. As explained in Section 4.1, the rudder is not used periodically in the MIMO test like the other inputs that follow a PRBS. This non-periodic excitation is problematic when averaging the data across the recorded periods, since some periods become "outliers". Because the rudder is intentionally excited in period 1 and 2 of the MIMO test, only the last 8 periods of the data are used for averaging. This is motivated by the fact that the rudder model is linear and is thus not fully accurate for the true non-linear vessel dynamics, meaning that all data in period 1 and 2 would be inaccurate if rudder compensation was applied since the rudder is active during those entire periods. However, neither of the last 8 periods have rudder excitation for an entire period so they are not completely "corrupted" by rudder excitation and can be used for noise estimation. A comparison with and without rudder compensation is plotted in Figure 4.5 for the roll angle output to analyze the rudder compensation effectiveness for each period.

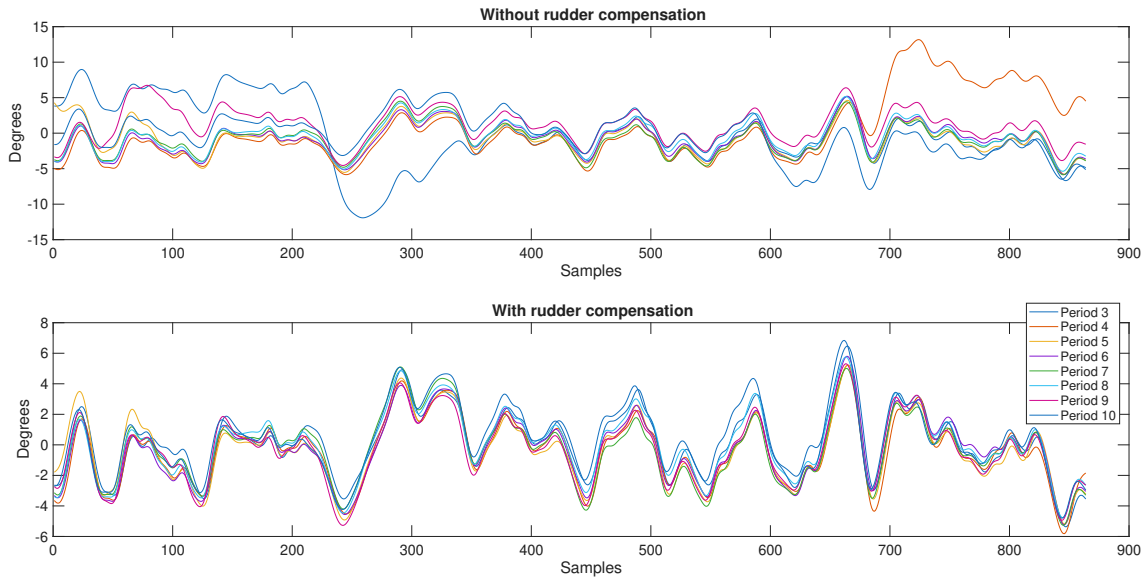


Figure 4.5: A comparison of the data with and without rudder effect reduced. The figure shows that the signals in each period align more when the rudder effect is reduced. Some signals still differ substantially at isolated time windows, but this is expected to effect the noise modeling in the following section insignificantly, since its only for short time windows and the differences are so small that the average signal is also effected insignificantly.

Figure 4.5 suggests that the rudder effects are reduced by the rudder model from Section 4.2. The figure shows that especially periods 3, 4 and 9 align more with the remaining periods after the rudder compensation is applied. They still differ notably at some isolated time instants due to the rudder model neglecting the known non-linear properties of the real system. However, using the rudder model is considered an improvement compared to ignoring the rudder effects, since the most obvious rudder induced deviations are significantly reduced in terms of amplitude. The periods that still differ notably in amplitude are not judged to substantially effect the periodic averaging, since the mathematical effect of the period averaging practically attenuates these small isolated differences.

4.3 Noise Estimation and Noise Modeling

This section explains a methodology to estimate the noise and model it with a suitable autonomous model structure. The methodology is explained for the roll angle noise, from data recorded at a 20 kt vessel speed, but can be applied to any output at any vessel speed.

4.3.1 Noise Estimation

By removing the mean of all periods in Figure 4.5 (using the data with rudder compensation), from each period an estimate of the noise is achieved, plotted in Figure 4.6.

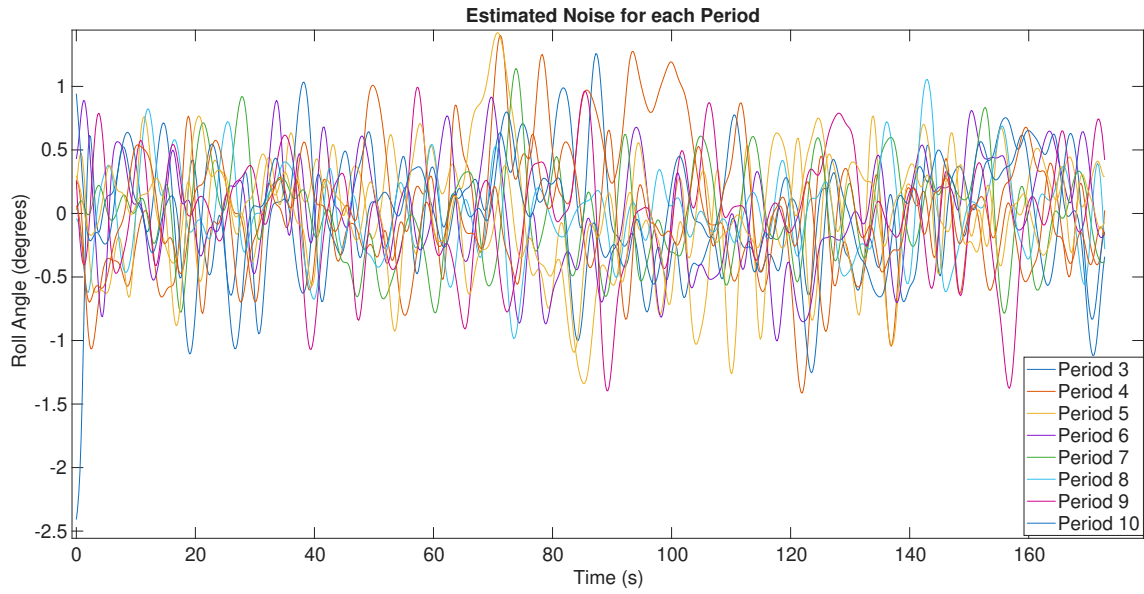


Figure 4.6: The estimated noise for each period. The estimations are achieved by removing the mean of all periods, from each period.

The estimated noise in Figure 4.6 can be used for estimating a noise model. To understand what type of noise model is suitable, the autocorrelation and partial autocorrelation of the data are plotted in Figure 4.7. They can be used together to decide suitable model structures and orders according to Table 4.1 [19].

Model	Autocorrelation Function (ACF)	Partial Autocorrelation Function (PACF)
$AR(p)$	Tails off (decays exponentially or sine wave)	Cuts off to zero after lag p
$MA(q)$	Cuts off to zero after lag q	Tails off (decays exponentially or sine wave)
$ARMA(p, q)$	Tails off	Tails off

Table 4.1: Theoretical behavior of the ACF and PACF for stationary time series models.

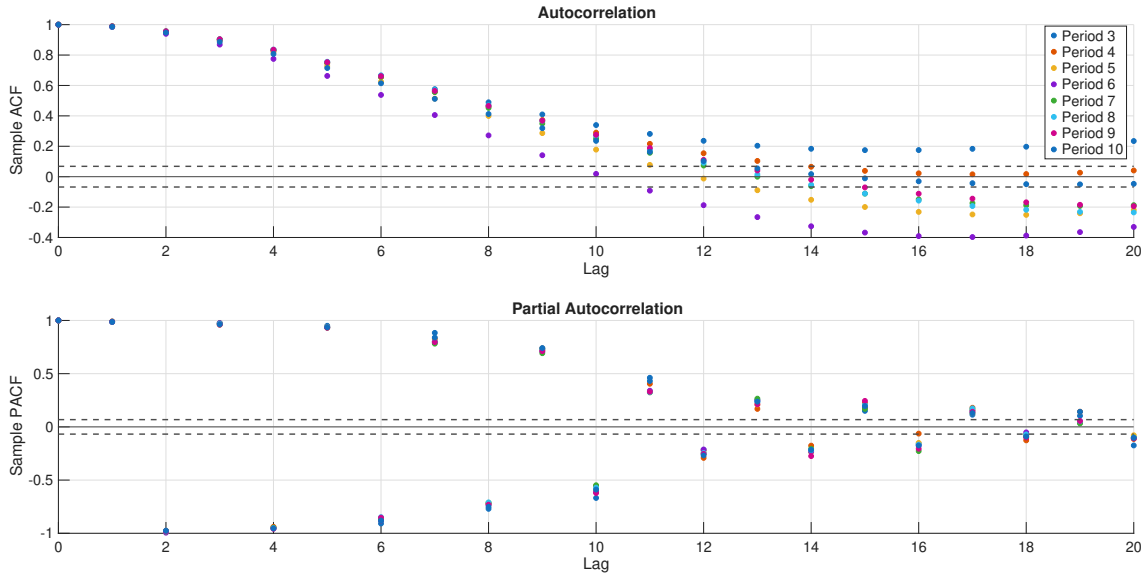


Figure 4.7: Autocorrelation and Partial autocorrelation function for the estimated noise.

Figure 4.7 and Table 4.1 suggests an Autoregressive (AR) model structure of different orders depending on the period, proving that the noise is colored.

4.3.2 Signal Decimation for Noise Modeling

Figure 4.8 shows the 99% occupied bandwidth for the PSD of the estimated noise. This estimated noise represents the system's output signal when all inputs are set to zero, since it is produced by removing the periodic means explained in Section 4.2.3. This occupied bandwidth spans approximately from 0 to 1.3 Hz, suggesting that the 20 Hz sampling frequency is unnecessarily high, since the relevant dynamics appear far below the sampling Nyquist frequency of 10 Hz. With an unnecessarily high sampling frequency the discrete time frequency axis (from 0 to π) is utilized poorly, since it spans from 0 to the Nyquist frequency (10 Hz). Consequently, estimated model poles are clustered close to $z = 1$ on the unit circle in the z -plane, which obscures the result in Section 4.3.4. For this reason, the data is decimated by a factor of 4. This is done by first applying a low-pass filter at 2.5 Hz, and then downsampling which yields a 5 Hz sampling frequency. The cutoff frequency of 2.5 Hz is chosen since it is the Nyquist frequency of 5 Hz, avoiding potential aliasing from corrupting the data and makes sure the occupied bandwidth is covered (since $2.5 \geq 1.3$). With the lowered sampling frequency, oversampling is avoided and the frequency axis spans from 0 to 2.5 Hz which is better suited for the occupied bandwidth of 0 to 1.3 Hz.

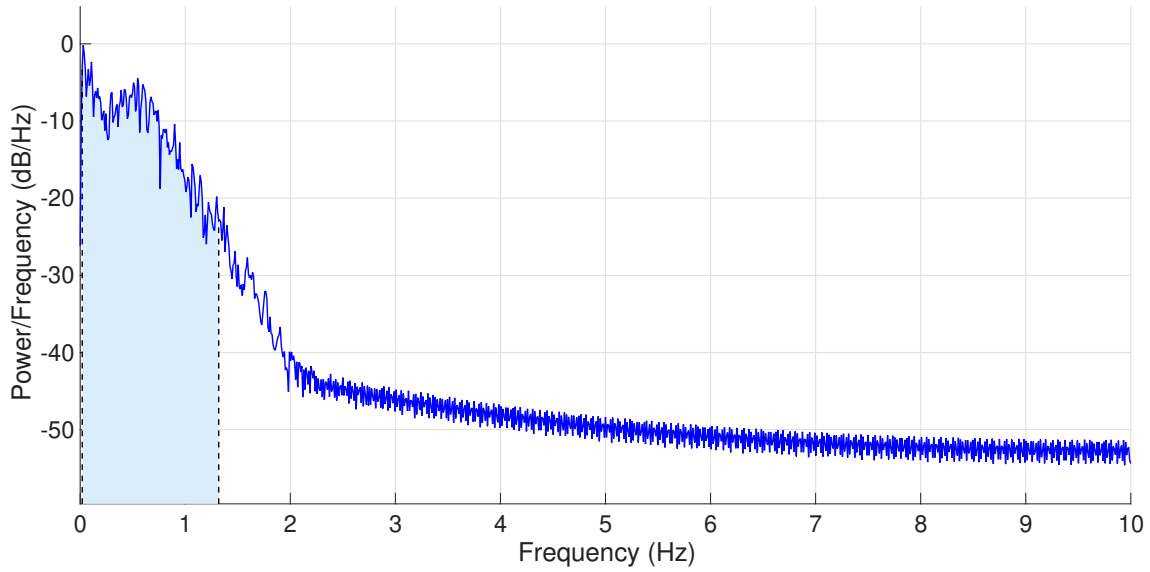


Figure 4.8: PSD of the estimated noise with 99% of the occupied bandwidth highlighted.

4.3.3 Model Structure and Model Order selection

To choose model structure and model order, the autocorrelation and partial autocorrelation of the decimated data are plotted in Figure 4.9. Based on Figure 4.9 and table 4.1 an AR model of order 2 or arguably an Autoregressive Moving Average (ARMA) model is suitable. To decide model structure, AR and ARMA models are estimated on period 9 and validated on period 10 with residual plots in Figure 4.10.

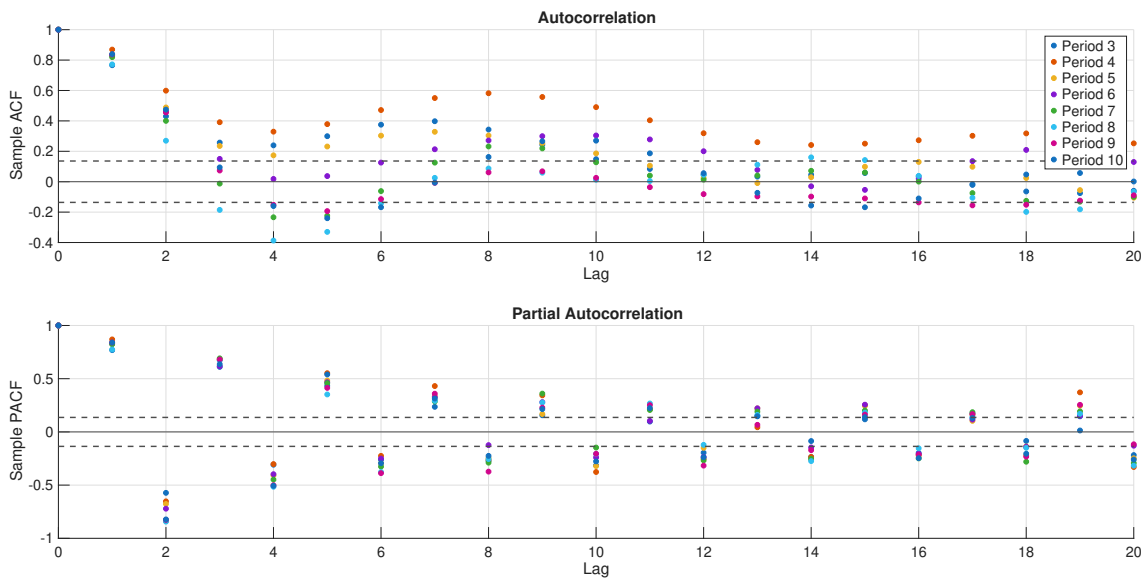


Figure 4.9: Autocorrelation and partial autocorrelation plotted for the decimated data. The plots suggests an AR or ARMA structure according to Table 4.1.

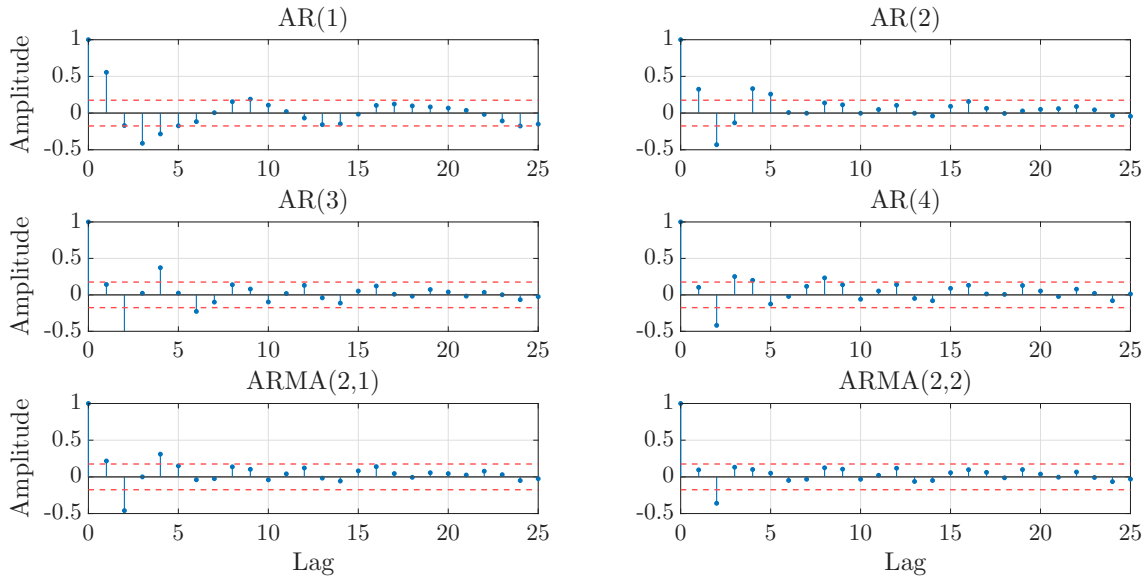


Figure 4.10: Estimated AR and ARMA models of different orders validated with autocorrelation residual plots, with the 99 % confidence bound. The models are estimated on the data from period 9, and validated on the data from period 10.

Based on Figure 4.10, the ARMA(2,2) model captures most of the colored noise in the validation data. The two poles of an ARMA(2,2) structure also aligns with the number of poles of a linear JONSWAP approximation, explained in Section 2.2.1. To validate it further, the residual plot from validating the model on every estimated noise period is provided in Figure 4.11 which reveals that it does not perform equally across all periods. Noticeably, period 5-8 has larger residual correlations compared to period 3,4 and 9. These increased residual correlations are believed to occur due to the waves hitting the stern during these periods (see Figure 4.12), instead of the side of the boat which is the case in period 9 (which is the period the ARMA(2,2) model is trained on). This result suggests that the colored noise is time-variant which is shown in the following section.

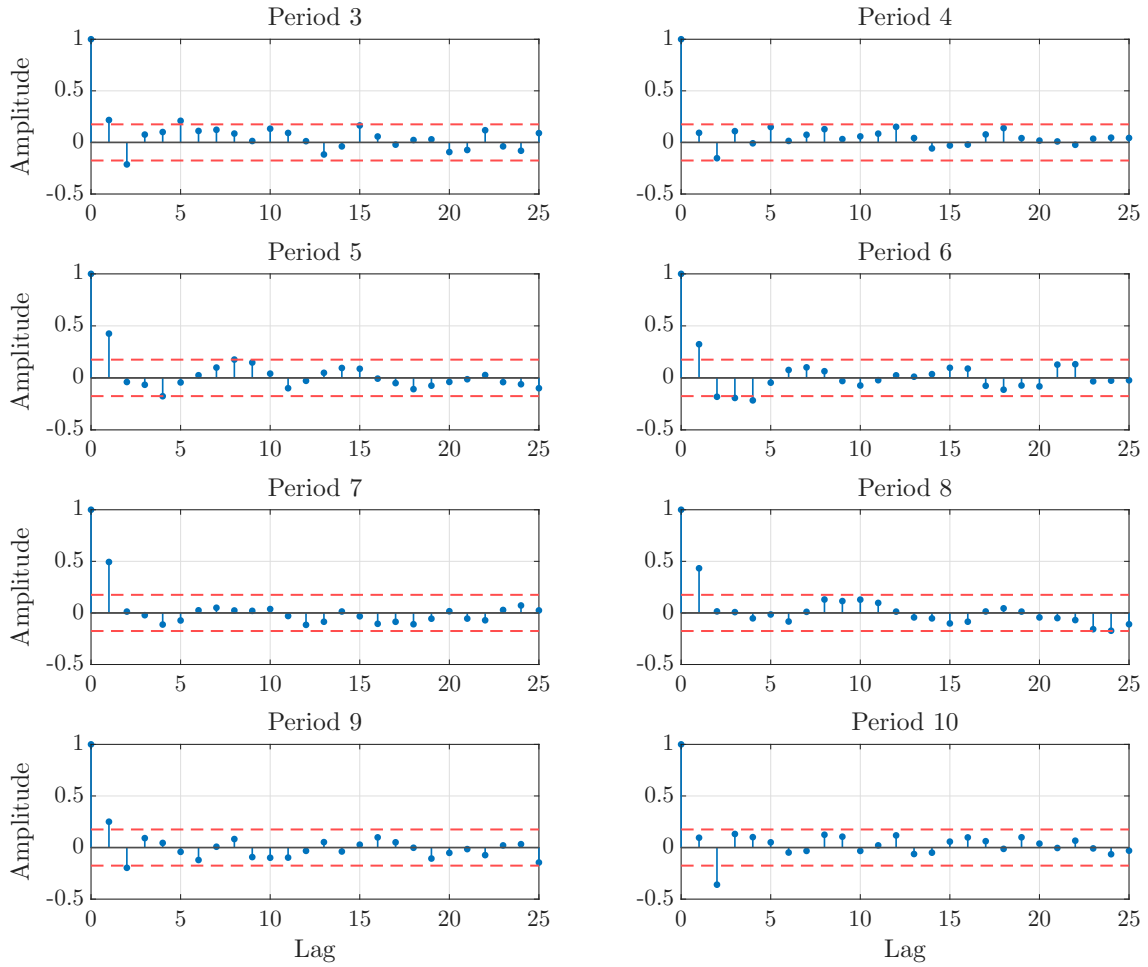


Figure 4.11: ARMA(2,2) model validated on all 8 noise periods.

4.3.4 Changing Disturbance Analysis

As explained in Section 2.2, wave disturbances can change based on encounter frequency and environmental changes. To see if this is the case, 500 ARMA(2,2) models are iteratively trained on a small set of the data using a sliding window approach. The 500 ARMA(2,2) models estimated with the sliding windows are plotted with the vessel movement in Figure 4.12. Figure 4.12 shows that the poles of the estimated ARMA(2,2) models move according to the theoretical wave disturbance covered in Section 2.2. In the beginning, the waves are hitting the starboard side, when the boat is turning the ARMA poles move away from the real axis meaning less damping which aligns with the waves now hitting the stern. Finally as the boat turns and the waves starts hitting the starboard side, the poles move towards the real axis again, increasing the damping. The magnitude of the poles is instead large in the beginning of the data set and become smaller towards the end of the data set since the poles moves closer to origin. This is expected to be due to the wave height changing during the test but this can not be confirmed since the wave height is not accurately measured during the test.

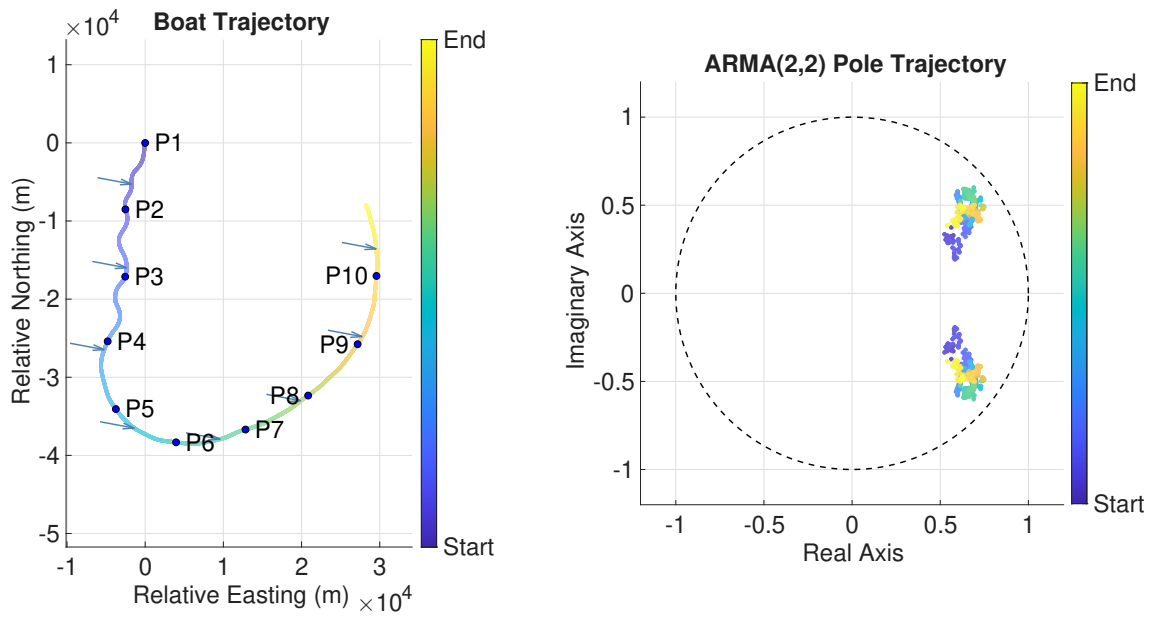


Figure 4.12: Vessel movement during the test and ARMA(2,2) models pole plot for different sliding windows. The blue arrows indicate the direction of the waves. The black dots mark where each period of the PRBS begins.

5

Model Identification and Parameter Estimation

5.1 Grey-box Modeling

This section explains how the linear physical grey-box model is achieved from Fossens [6] nonlinear equations of motion for ship dynamics. The test vessel characteristics is also used to estimate an initial guess for the parameters in the linearized model.

5.1.1 Fossen's Marine Craft Model

Fossen's marine craft model is a widely used mathematical framework for describing the maneuvering dynamics of different marine crafts [6]. The general 6-DOF equation of motion is expressed as:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \quad (5.1)$$

where:

- $\eta = [x, y, z, \phi, \theta, \psi]^T$ is the position and orientation vector.
- $\nu = [u, v, w, p, q, r]^T$ is the translational and angular velocity vector in the body frame. u , v and w correspond to surge, sway and heave.
- M is the inertia matrix (Rigid Body + Added Mass).
- $C(\nu)$ is the Coriolis-centripetal matrix.
- $D(\nu)$ is the damping matrix.
- $g(\eta)$ is the vector of gravitational and hydrostatic restoring forces, which naturally push the vessel back toward an upright equilibrium.
- τ is the generalized force acting on the vector, for instance control inputs, waves, and propulsion.

The time derivative of the position and orientation vector η is related to the body-

fixed velocity ν through a transformation matrix $J(\eta)$, such that $\dot{\eta} = J(\eta)\nu$. In the following linearizations, the vessel attitude is assumed to be small, allowing this transformation to be approximated as an identity matrix $J(\eta) \approx I$.

The translational degrees of freedom (surge u , sway v and heave w) in (5.1) are removed, and the model parameters are lumped as much as possible to reduce the number of parameters to estimate. By linearizing the remaining rotational dynamics (as detailed in Appendix A.1), the resulting simplified state space model ($\dot{x} = Ax + Bu$) is derived as:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_1 & 0 & 0 & a_2 & 0 & a_3 \\ 0 & a_4 & 0 & 0 & a_5 & 0 \\ a_6 & 0 & 0 & a_7 & 0 & a_8 \end{bmatrix}}_A \underbrace{\begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix}}_x + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_1 & -b_1 & -b_2 & b_3 \\ b_4 & b_4 & 0 & 0 \\ -b_5 & b_5 & b_6 & b_7 \end{bmatrix}}_B \underbrace{\begin{bmatrix} Int_{port} \\ Int_{stbd} \\ Fins \\ Rudder \end{bmatrix}}_u \quad (5.2)$$

Because this state-space model is linear in the parameters $a_1 \dots a_8$ and $b_1 \dots b_7$, it can be rewritten on regressor form. This allows the model to be used with standard methods like LS or IV. While the states x (angles and angular rates) are measured directly by the vessel's sensors, the angular accelerations $\dot{x}$ must be estimated or handled by the estimation algorithm.

Note that while the full state vector x has dimension 6, the kinematic relationships ($\dot{\phi} = p, \dot{\theta} = q, \dot{\psi} = r$) are already known and contain no unknown parameters. Therefore, the regressor equation only needs to formulate the dynamics for the three angular accelerations:

$$\begin{bmatrix} \ddot{p} \\ \ddot{q} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} \Phi_x & \Phi_u \end{bmatrix} \theta \quad (5.3)$$

Where the state and input sub-matrices are:

$$\Phi_x = \begin{bmatrix} \phi & p & r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta & q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi & p & r \end{bmatrix} \quad (5.4)$$

$$\Phi_u = \begin{bmatrix} (u_1 - u_2) & -u_3 & u_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (u_1 + u_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (-u_1 + u_2) & u_3 & u_4 \end{bmatrix} \quad (5.5)$$

where $u_1 = Int_{port}$, $u_2 = Int_{stbd}$, $u_3 = Fins$ and $u_4 = Rudder$. The parameter vector to be estimated is:

$$\theta = [a_1 \ a_2 \ \dots \ a_8 \ b_1 \ b_2 \ \dots \ b_7]^T \quad (5.6)$$

5.1.2 Disturbance Simulation

To evaluate the robustness of the parameter estimation method a disturbance simulation is introduced. With this simulation it is possible to test and evaluate the performance of the parameter estimation algorithm in colored disturbances, without completing multiple costly sea trials. The complete architecture of this simulation process is illustrated in Figure 5.1.

Based on the theory in Sections 2.2-2.2.1, wave surface elevation is modeled using the JONSWAP spectrum. The generated wave sequence is dynamically adjusted based on vessel speed and heading to calculate the encounter frequency (2.1) and then differentiated to acquire the wave slope. This slope is used together with the vessel's restoring forces, if the vessel is tilted relative to the water surface, the restoring forces push the vessel to be orthogonal to the water surface. The restoring forces are extracted from the simplified Fossen model trained on calm data with high SNR covered in Section 5.3.1.

To integrate this physical wave interaction into the state-space model, the model is extended with the wave angles as new inputs. The first two columns in the A matrix represent the mapping between the roll and pitch angle to their respective derivatives via the restoring forces. Because the wave slope acts as an angular offset to the flat water surface, the wave disturbance matrix B_{wave} is constructed simply by negating these columns of the A matrix. c_{roll} and c_{pitch} are tuning parameters to control how much wave disturbance is introduced.

$$\mathbf{B}_{wave} = \begin{bmatrix} -\mathbf{A}_{:,1}c_{roll} & -\mathbf{A}_{:,2}c_{pitch} \end{bmatrix} \quad (5.7)$$

The slope angle is separated based on the encounter angle β :

$$\begin{aligned} \alpha_{roll}(t) &= \alpha_{wave}(t) \sin(\beta(t)) \\ \alpha_{pitch}(t) &= \alpha_{wave}(t) \cos(\beta(t)) \end{aligned} \quad (5.8)$$

$$\mathbf{B}_{total} = \begin{bmatrix} \mathbf{B}_{act} & \mathbf{B}_{wave} \end{bmatrix}, \quad \mathbf{u}_{total}(t) = \begin{bmatrix} \mathbf{u}_{act}(t) \\ \alpha_{roll}(t) \\ \alpha_{pitch}(t) \end{bmatrix} \quad (5.9)$$

Finally, to create a realistic noise injection the simulated wave response is combined with the true measured data. This is done by simulating the extended disturbance model and subtracting the simulated model without the disturbance extension. This isolates the disturbance data which is then added to the measured output data:

$$\mathbf{y}_{noisy}(t) = \mathbf{y}_{calm}(t) + \left(\mathbf{y}_{sim,total}(t) - \mathbf{y}_{sim,calm}(t) \right) \quad (5.10)$$

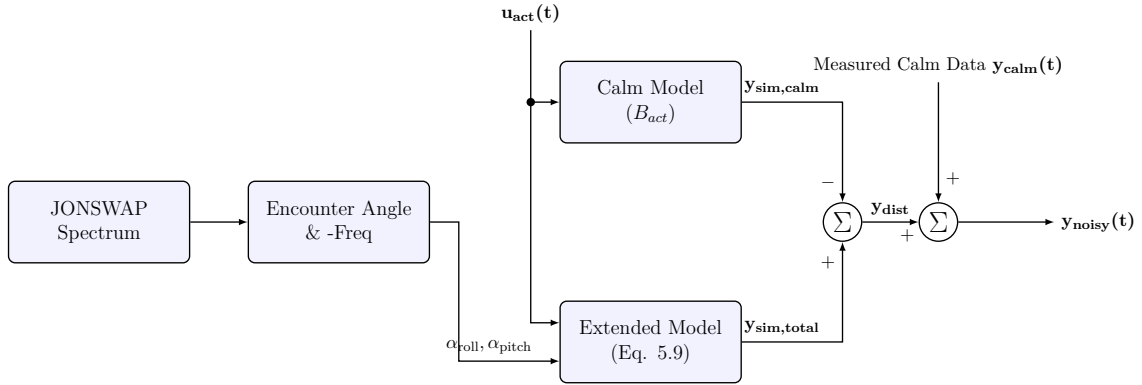


Figure 5.1: Block diagram illustrating the disturbance simulation process described in Section 5.1.2. The simulated wave disturbance is isolated by comparing the extended model against the calm model, and then superimposed onto the measured calm water data to generate a realistic noisy dataset.

5.2 Parameter Estimation

To identify the physical parameters of the system, the Prediction Error Method (PEM) was selected. While the true physical system exhibits nonlinear hydrodynamic properties, the PEM algorithm ensures that the resulting grey-box parameters converge to the statistically optimal linear approximation of the true system that is contained within the model structure [13, p. 264]. As Ljung showed in [13, eq 8.103, p.273] the PEM parameter estimate $\hat{\theta}_N$ converges with probability 1 to the minimum of the expected cost function:

$$\hat{\theta}_N \rightarrow \arg \min_{\theta \in D_M} \bar{E} [\ell(\varepsilon(t, \theta), \theta)] \quad \text{w.p. 1 as } N \rightarrow \infty \quad (5.11)$$

Where:

- $\hat{\theta}_N$ is the parameter vector estimated by the algorithm using a dataset of length N .
- D_M represents the allowed parameter search space defined by the chosen model structure
- $\ell(\varepsilon(t, \theta), \theta)$ is the scalar loss function applied to the prediction errors.

For this parameter estimation task, the primary objective is to accurately model the physical dynamics of the system. PEM assumes that the process and disturbance models G and H are linear and time invariant. While the process model has been simplified to fulfill this requirement, this is not possible for the disturbance model. The wave disturbance will significantly change over time, see Section 4.3.4, for example the zig-zag maneuver changes the wave encounter frequency and angle. This effectively changes the energy spectra of the environmental disturbance, a

change that a time-invariant model cannot capture. To account for this an output-error (OE) structure is proposed. In an OE structure the noise dynamics are not explicitly parameterized and thus the linear model can be described as:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \quad (5.12a)$$

$$y(t) = Cx(t) + e(t) \quad (5.12b)$$

Where the observation matrix $C \in \mathbb{R}^{5 \times 6}$ is defined to map the state vector $x = [\phi, \theta, \psi, p, q, r]^T$ to the 5 measured sensor outputs (Roll Angle, Pitch Angle, Roll Rate, Pitch Rate, and Yaw Rate), omitting the unmeasured absolute yaw angle:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.13)$$

This implies that all disturbances, unmodeled dynamics and colored environmental noise are lumped together and treated as a direct additive error on the output measurement, $e(t)$. If the colored noise is uncorrelated with the inputs, PEM estimation using this model will provide an asymptotically unbiased linear estimate of the true system dynamics $G_0(q)$ (see Appendix A.2 for proof). Consequently, this method only works when collecting data in open loop and is therefore only suitable for the calibration phase of the model. Another downside is that the statistical efficiency property of PEM is broken due to the unmodeled disturbance dynamics [13]. This will make the parameter variance higher and more data will be necessary for finding a good estimation of the parameters. The outcome of this parameter estimation and the subsequent evaluation of the model's performance are presented in Section 5.3.

Instrumental variables (IV) methods were excluded from this batch calibration due to their vulnerability to unmodeled dynamics. Because the model is a heavy simplification of the true nonlinear system, unmodeled dynamics will show in the residuals. IV methods do not minimize simulation error, instead they solve for an orthogonality condition [13]. Forcing this orthogonality when the model can not represent the true system can severely distort the parameters and provides unreliable model estimates [20].

5.2.1 Initial Parameter Estimation

Although the derived state space model is linear in its parameters, identifying the system parameters from input-output data using an OE approach is a non-convex optimization problem [21]. Because the model output is simulated iteratively, the cost function becomes non-linear with respect to the parameters. The solvers are therefore susceptible to converging on local minima if the parameters are initialized

randomly or set to zero. To prevent this, a good initial guess of the true system parameters is necessary.

The specification data for the test vessel provided in (2.1) are used for calculating the initial parameter estimation. The variables L , B and T correspond to length, width and draught respectively. The estimation is based on the assumption that the vessel is a rectangular cuboid. The inertias are calculated as:

$$J_x = \frac{m}{12}(B^2 + 4T^2) \quad (5.14a)$$

$$J_y = \frac{m}{12}(L^2 + 4T^2) \quad (5.14b)$$

$$J_z = \frac{m}{12}(L^2 + B^2) \quad (5.14c)$$

The uncoupled roll and pitch dynamics of the vessel can be approximated as two linear second-order spring mass damper systems expressed as:

$$\ddot{x} = -2\zeta\omega_n\dot{x} - \omega_n^2x + u \quad (5.15)$$

Where ζ is the damping ratio and ω_n is the natural frequency. This equation can be mapped to the parameters in the A matrix in the simplified Fossen model derived in A.1. Looking at the model, a_1 and a_4 represents restoring accelerations and corresponds to the stiffness term ($-\omega_n^2$). These can be approximated using the transverse and longitudinal metacentric heights ($\overline{GM_T}$ and $\overline{GM_L}$) further specified in [6, Eq. 4.23]

$$a_1 = -\frac{mg\overline{GM_T}}{J_x} \quad (5.16a)$$

$$a_4 = -\frac{mg\overline{GM_L}}{J_y} \quad (5.16b)$$

The metacentric heights are calculated based on Example 4.2 in [6, p. 67]

$$\overline{GM_T} = \frac{B^2}{12T} - \frac{T}{2} \quad (5.17a)$$

$$\overline{GM_L} = \frac{L^2}{12T} - \frac{T}{2} \quad (5.17b)$$

The linear damping parameters (a_2 and a_5) correspond to the $-2\zeta\omega_n$ term of the spring mass damper system. Substituting $\omega_n = \sqrt{-a_1}$, the initial damping parameters are formulated by choosing damping ratios for roll (ζ_ϕ) and pitch (ζ_θ):

$$a_2 = -2\zeta_\phi\sqrt{-a_1}, \quad \text{assumption : } \zeta_\phi = 0.1 \quad (5.18a)$$

$$a_5 = -2\zeta_\theta\sqrt{-a_4}, \quad \text{assumption : } \zeta_\theta = 0.3 \quad (5.18b)$$

Because yaw dynamics lack hydrostatic restoring forces, it can not be modeled with a spring mass damper. Instead, Nomoto's first order model is used, explained in [6, p. 152], and expressed as:

$$\dot{r} = -\left(\frac{U}{L}\right) \frac{1}{T'} r + \left(\frac{U}{L}\right)^2 \frac{K'}{T'} \delta \quad (5.19)$$

where δ represents the rudder input. The constants K' and T' are typically in between $0.5 < K' < 2$ and $0.5 < T' < 2$. For simplicity both are chosen to be 1. Parameter a_8 is then calculated as:

$$a_8 = -\frac{U}{L} \quad (5.20)$$

All cross couplings are assumed to be zero,

$$a_3 = 0, \quad a_6 = 0, \quad a_7 = 0 \quad (5.21)$$

The control parameters are heavily influenced by hydrodynamics and mounting position. Instead of explicitly trying to calculate these, they will just be set to a small constant. The optimizer has to find these. The only exception is the rudder-yaw parameter which is extracted from the Nomoto model (5.19)

$$b_{1-6} = 0.1, \quad b_7 = \left(\frac{U}{L}\right)^2 \quad (5.22)$$

5.3 Model validation and Identified Model

This section validates the identified models (5.23 - 5.25). The model is confirmed to be stable for all three vessel speeds, and it is confirmed that the model is speed dependent by examining the poles of the identified models for the three vessel speeds. Because the purpose of the model is to be used for control design, traditional metrics such as correlation residual analysis is not suitable to validate the model on its own, its more interesting to study the model in the frequency domain [22] [23]. For this reason the model is also validated with empirical data in the frequency domain using Spectral Analysis with Frequency-Dependent Resolution (SPAFDR).

5.3.1 Parameter Convergence and System Stability

The parameters of the state-space model were identified using the PEM approach detailed in Section 5.2 for the speeds of 10, 15 and 20 kt, resulting in the following models:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -13.3 & 0 & 0 & -3.1 & 0 & -11.1 \\ 0 & -9.5 & 0 & 0 & -3.2 & 0 \\ -0.8 & 0 & 0 & -0.5 & 0 & -2.2 \end{bmatrix}}_{10 \text{ kt model}} \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0 & 0.0 & -4.5 & 2.9 \\ -0.1 & -0.1 & 0 & 0 \\ 0.0 & 0.0 & -0.3 & 0.4 \end{bmatrix} \begin{bmatrix} Int_{port} \\ Int_{stbd} \\ Fins \\ Rudder \end{bmatrix} \quad (5.23)$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -9.6 & 0 & 0 & -3.4 & 0 & -3.0 \\ 0 & -9.8 & 0 & 0 & -4.1 & 0 \\ -1.1 & 0 & 0 & -0.6 & 0 & -2.2 \end{bmatrix}}_{15 \text{ kt model}} \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.1 & -0.1 & -7.2 & 2.3 \\ -0.1 & -0.1 & 0 & 0 \\ 0.0 & 0.0 & -0.5 & 0.6 \end{bmatrix} \begin{bmatrix} Int_{port} \\ Int_{stbd} \\ Fins \\ Rudder \end{bmatrix} \quad (5.24)$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4.9 & 0 & 0 & -2.8 & 0 & -0.1 \\ 0 & -10.8 & 0 & 0 & -2.8 & 0 \\ -1.6 & 0 & 0 & -0.5 & 0 & -1.8 \end{bmatrix}}_{20 \text{ kt model}} \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.2 & -0.2 & -8.2 & 1.3 \\ -0.2 & -0.2 & 0 & 0 \\ 0.0 & 0.0 & -1.0 & 0.7 \end{bmatrix} \begin{bmatrix} Int_{port} \\ Int_{stbd} \\ Fins \\ Rudder \end{bmatrix} \quad (5.25)$$

The initial validation step of the resulting models is to examine their mathematical stability and the physical plausibility of the estimated parameters. Since the hydrodynamic effects on marine vessels differ significantly between different surge speeds, three different speeds (10, 15 and 20 knots) are used for validation. Figure 5.2 presents the pole-zero map of the final identified models which represent the displacement, hump and planing states respectively. As seen in the figure, all identified poles are in the left half plane, confirming overall mathematical stability. Furthermore the yaw dynamics correctly exhibit a pole at the origin, reflecting that the yaw angle is a pure integration of yaw rate. The models also capture expected physical shifts across the different operating states. For instance, pitch damping increases significantly as the vessel pushes through the hump phase (15 knots), while the roll natural frequency notably decreases during planing (20 knots) due to the expected loss of hydrostatic restoring stiffness as the hull lifts out of the water. This is consistent with established high-speed hydrodynamic theory [5].

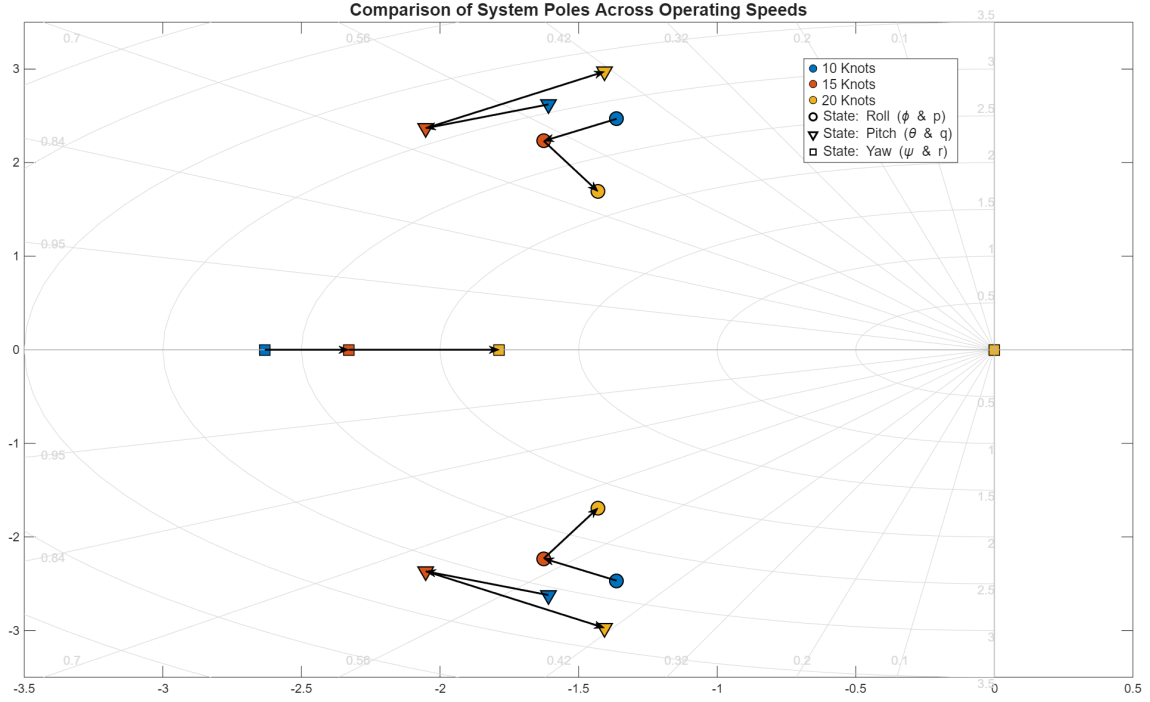


Figure 5.2: Pole-zero map comparing the pole locations between different operating points, illustrating that the system dynamics are highly speed dependent.

To ensure a concise presentation, the following evaluation focuses exclusively on 20 kt results. The results from 10 and 15 knots can be found in Appendix C. Three different datasets are used: the first is collected at a calm sea state with low colored disturbances, the second is collected using the same input but with more disturbances and the final dataset uses the calm dataset with added simulated disturbances. The simulated disturbance is described in Section 5.1.2.

To evaluate the relative strength of the disturbances, a Signal-to-Noise Ratio (SNR) was calculated for both pitch and roll. A baseline signal strength (RMS_{signal}) was established by calculating the standard deviation of the period averaged response from the data recorded at a calm sea state. Let y_i represent the i -th sample of this period-averaged baseline signal which consists of N_{period} samples. Let $\hat{y}_{k,i}$ represent the corresponding i -th sample within the k -th period of the test data. For a test dataset containing P total periods, the number of samples is $N = P \times N_{\text{period}}$.

$$RMS_{\text{true}} = \sqrt{\frac{1}{N_{\text{period}}} \sum_{i=1}^{N_{\text{period}}} y_i^2}, \quad RMSE_{\text{noise}} = \sqrt{\frac{1}{N} \sum_{k=1}^P \sum_{i=1}^{N_{\text{period}}} (y_i - \hat{y}_{k,i})^2} \quad (5.26)$$

Because these values represent the signal amplitudes, the SNR is computed in decibels using the following ratio:

$$SNR_{dB} = 20 \log_{10} \frac{RMS_{\text{signal}}}{RMSE_{\text{noise}}} \quad (5.27)$$

The SNR for each dataset is shown in Table 5.1.

State	Calm	Real Dist.	Simulated
Roll	9.21 dB	3.37 dB	-2.42 dB
Pitch	5.6 dB	0.55 dB	-10.03 dB

Table 5.1: Comparison of SNR between the three different data sets used for validation.

While Table 5.1 presents the broadband RMS estimates of the SNR, the signal quality is highly frequency-dependent, as illustrated in Figure 5.3. Although the vessel was operating in disturbed sea states with a distinct dominant wave frequency (As detailed in 2.2), the continuous turning maneuvers of the vessel resulted in constantly shifting encounter frequency. This behavior was demonstrated by the time-varying noise dynamics in Section 4.3.4. When averaged over the time window used for the MSC calculation, this non-stationary wave energy was spectrally smeared, transforming what would typically be narrow band disturbance notch into a broadband noise. Consequently, while the signal quality for the calm dataset remains high throughout the operational bandwidth (established in Section 3.5), the disturbed datasets exhibits reduced MSC across the entire bandwidth due to the spectrally smeared wave disturbance.

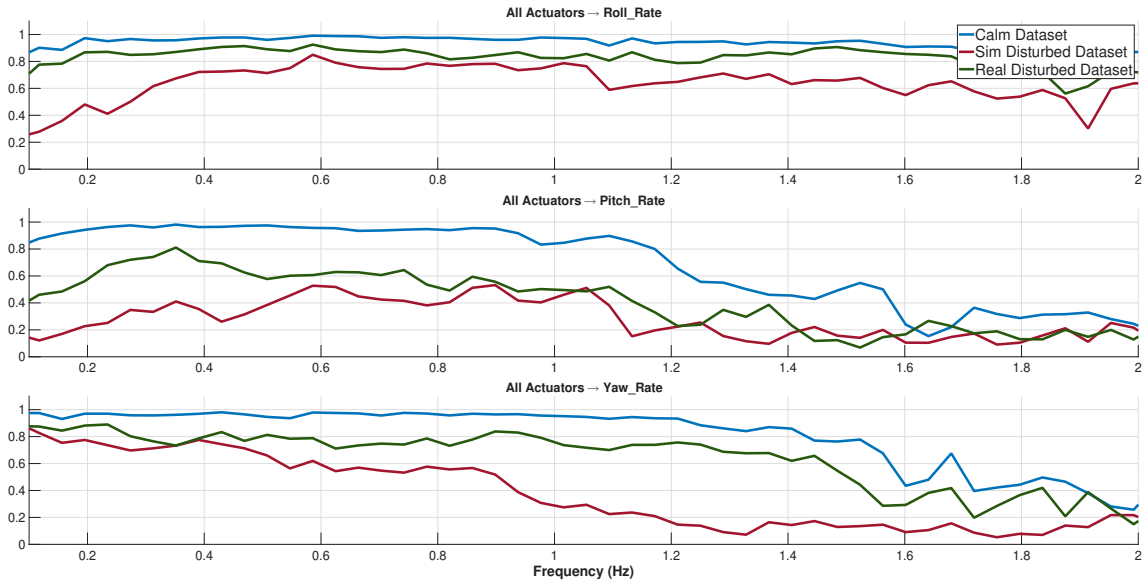


Figure 5.3: MSC for roll, pitch and yaw rates, illustrating the frequency dependent SNR. The high-frequency drop in coherence occurs as the 3 Hz PRBS input naturally decays. Furthermore, the broadband appearance of the disturbed dataset is a result of spectral smearing. Continuous vessel maneuvers cause a shifting encounter frequency, flattening the expected narrow band wave notch into a broadband noise profile.

Figure 5.4 illustrates the final identified pole locations for the models trained on 20 knot data with varying levels of disturbance. As seen in the figure, the unmodeled

colored wave noise introduces a systematic bias, causing the system poles for the simulated (red) and real (green) disturbed datasets to converge to slightly offset locations compared to the calm water baseline (blue). The impact of this environmental noise is further reflected in the parameter convergence shown in Figure 5.5, where models trained under stronger disturbances exhibits higher variance and require more data to stabilize. For clarity, the figure shows the convergence of two representative parameters (a_4 and b_1). The full convergence plots for all system parameters can be found in Appendix D.1. Because the shifted pole locations dictate the fundamental system dynamics, it raises the critical question of whether models identified in disturbed condition remain valid for control design. This is evaluated in the frequency domain in Section 5.3.2.

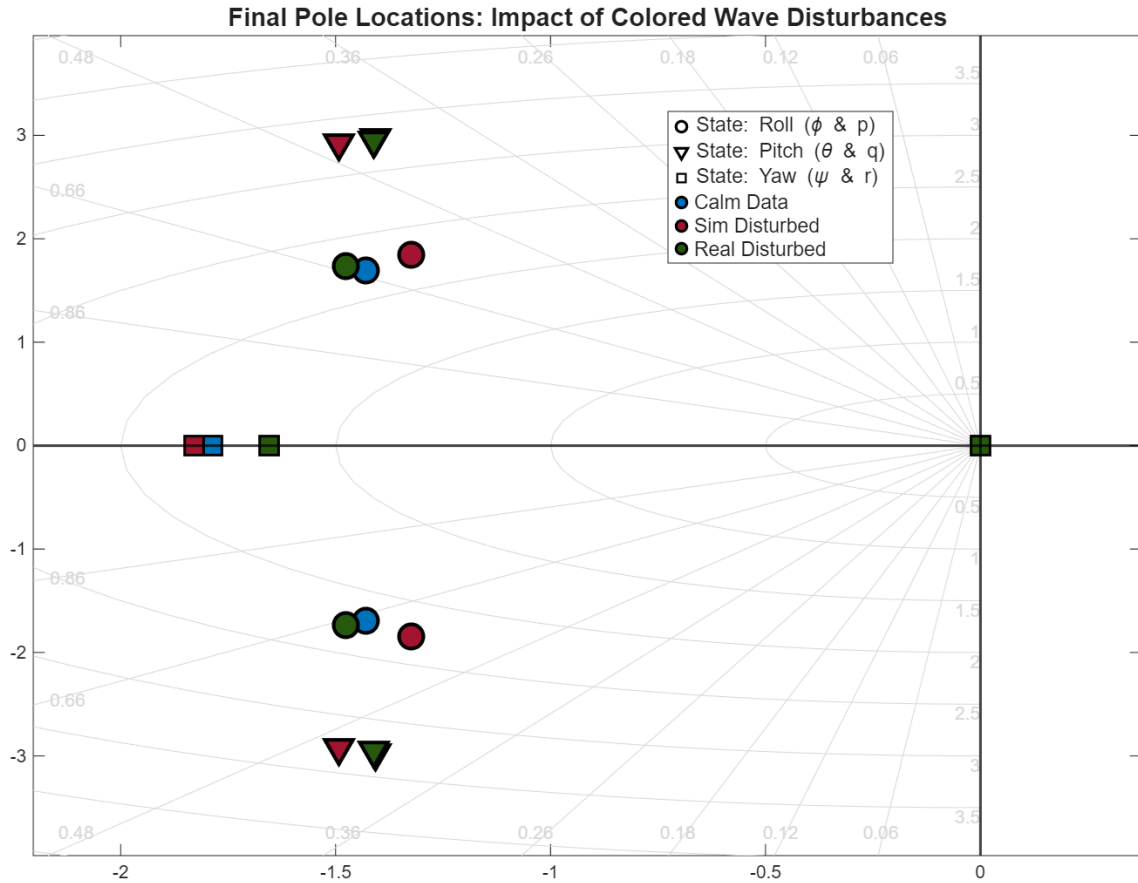


Figure 5.4: Final identified pole locations at 20 knots evaluated across varying disturbance levels. The observed shift in pole locations indicates that the output-error identification structure is biased by the unmodeled colored wave disturbances, compensating for the noise by altering the system poles.

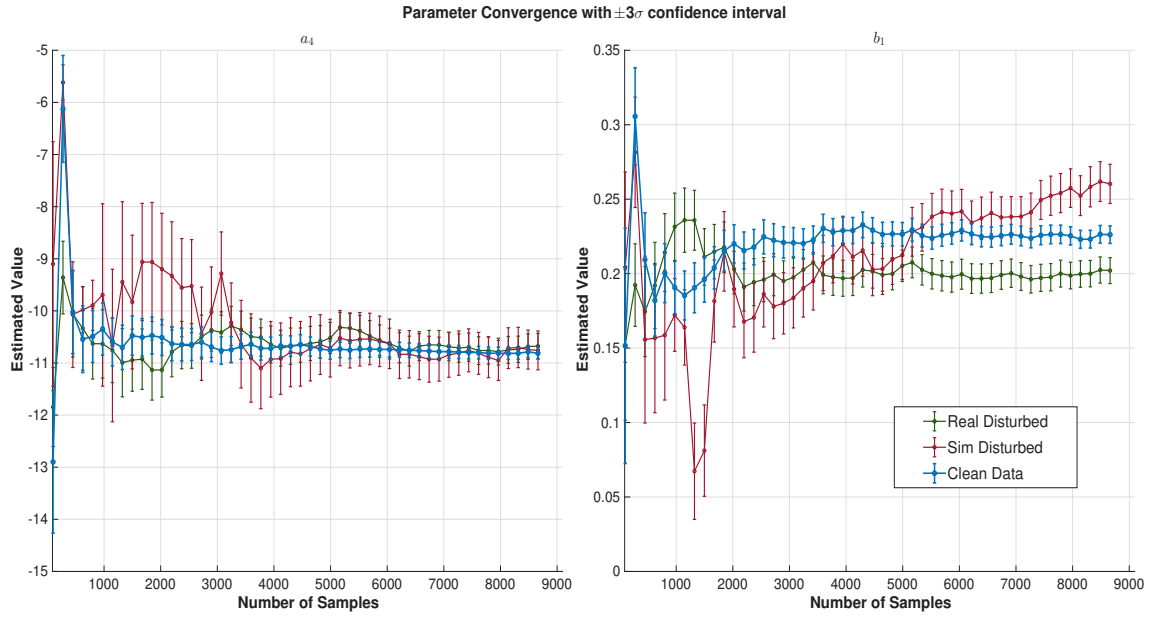


Figure 5.5: Convergence of representative parameters a_4 and b_1 over increasing amounts of training data. The vertical error bars represent the $\pm 3\sigma$ confidence intervals of the parameter estimates. Comparing the calm baseline (blue) to the disturbed datasets (red and green), the disturbed data exhibits wider error bars (higher variance) and requires more samples to flatten and stabilize. Additionally, the plot for b_1 illustrates the systematic bias introduced by the unmodeled wave dynamics, as the disturbed models converges to a slightly offset value.

5.3.2 Frequency Domain Validation

To evaluate the models performance within the limited bandwidth of the actuators, the frequency responses of the trained models were compared against an empirical, data-driven spectral estimate.

The empirical frequency response was calculated using Spectral Analysis with Frequency-Dependent Resolution (SPAFDR). Unlike standard empirical transfer function estimates (ETF) or standard spectral analysis which apply a fixed-width smoothing window across the entire frequency range, SPAFDR adapts its frequency resolution. It applies higher frequency resolution at lower frequencies to capture the exact rigid-body dynamics of the vessel, while applying heavier smoothing at higher frequencies to suppress the variance introduced by the wave disturbances [24].

Figure D.2 presents the Bode plots comparing this empirical SPAFDR against the three identified parametric models (trained on calm, simulated disturbance, and real disturbance data). For visual clarity, only a representative input-output dynamic is shown (Fins to Roll Rate). The complete frequency response matrix is available in Appendix D.2.

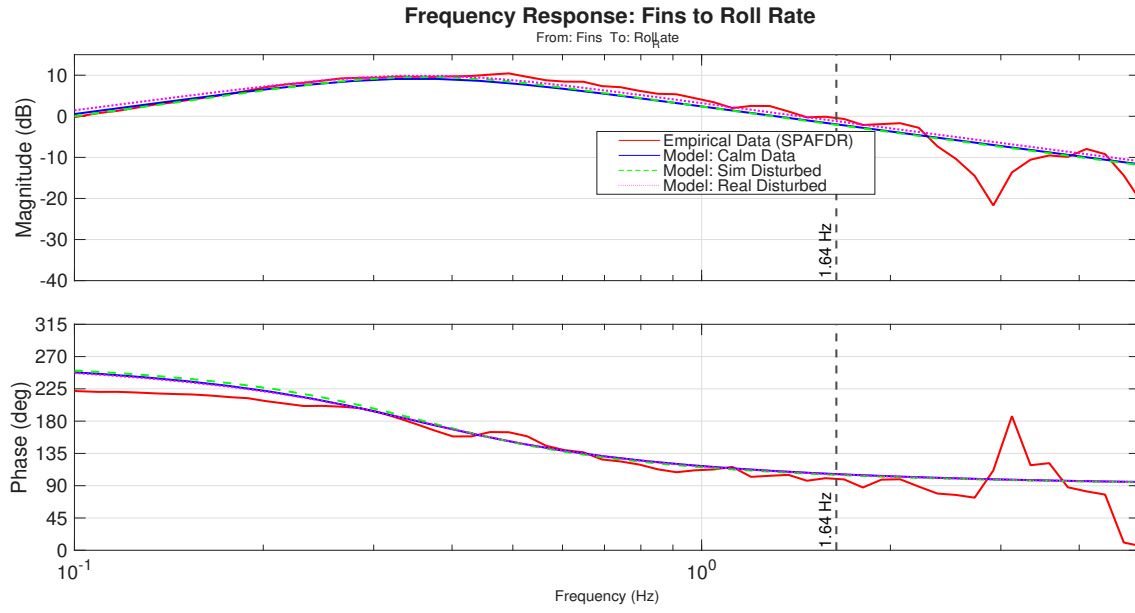


Figure 5.6: Frequency response comparing the identified parametric grey-box models to the empirical SPAFDR estimates (solid red) for fins to roll rate. To distinguish the parametric models, distinct line styles are used: calm data (solid blue), simulated disturbance (dashed green), and real disturbance (dotted magenta). The upper limit of the relevant frequency band determined in Section 3.4 is marked with a dashed black line (1.64 Hz). This figure illustrates that the models accurately capture the true vessel dynamics within the operational bandwidth despite the different disturbances.

As shown in the figure, the frequency response across the three parametric models overlap almost perfectly. This near perfect overlap visually demonstrates that despite the shift in pole locations identified in Section 5.3.1, the parameter identification extracts the similar system dynamics within the relevant operational bandwidth (below 1.64 Hz), regardless the environmental noise. Furthermore, the parametric models track the SPAFDR very closely, indicating that the true underlying physics in this bandwidth are well captured by the linear model.

5.3.3 Time-Domain Validation

The models trained with disturbed data are validated against the calm data set, to see if the model can be successfully identified in the presence of noise. This is done using a time domain validation metric called simulation fit, defined as:

$$\text{FIT} = 100 \left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|} \right) \quad (5.28)$$

where y is the validation data, $\hat{y}$ is the model output and $\bar{y}$ is the mean of y . The model trained on the calm data is used as a baseline for simulation fit comparison. The results are presented in Figure 5.7.

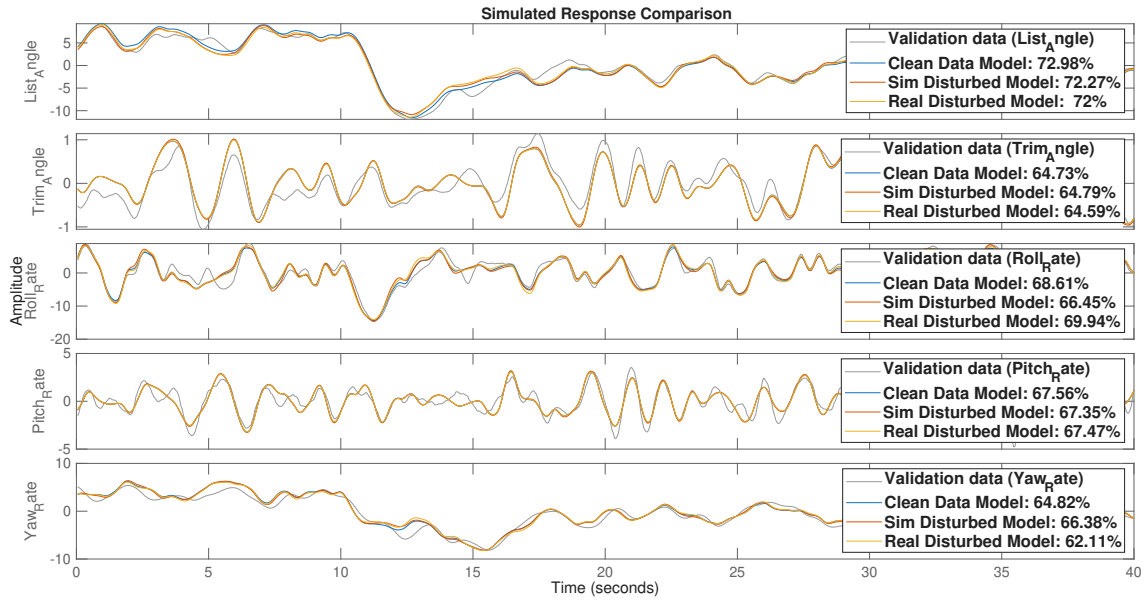


Figure 5.7: Time-domain cross-validation comparing simulated model outputs against measured sensor data on an unseen dataset. As shown by the simulation fit percentages in the legend, all three models perform equally well. This indicates that the shift in pole placements caused by the disturbances (previously shown in Figure 5.4) did not significantly affect the model’s predictive performance within the operational bandwidth.

The figure shows that all models perform equally well in terms of simulation fit, which aligns with the result from the frequency domain validation. This further showcases that the shift in pole placement seen in Figure 5.4 did not significantly alter the dynamics within the operating bandwidth.

In Figure 5.8 the simulation fit (5.28) is plotted for different amounts of training data. This is presented to show how the severity of the disturbance affect the amount of data necessary for the model to converge. The figure shows that the model identified in the clean data set converges in terms of simulation fit for all angles with ≈ 200 samples, while the real disturbance model requires more than ≈ 400 and the simulated disturbance model more than ≈ 1500 . This result and the SNR presented in Table 5.1 shows that the simulation fit convergence requires more data when the disturbance is greater, summarized in Table 5.2.

State	Calm	Real Dist.	Simulated
Roll	9.21 dB	3.37 dB	-2.42 dB
Pitch	5.6 dB	0.55 dB	-10.03 dB
Nr. of samples required for simulation fit convergence	≈ 200	≈ 400	≈ 1500

Table 5.2: Comparison of SNR between the three different data sets used for validation and the number of samples required to achieve simulation fit convergence. The table shows that the amount of data required for simulation fit convergence decreases when the SNR increases.

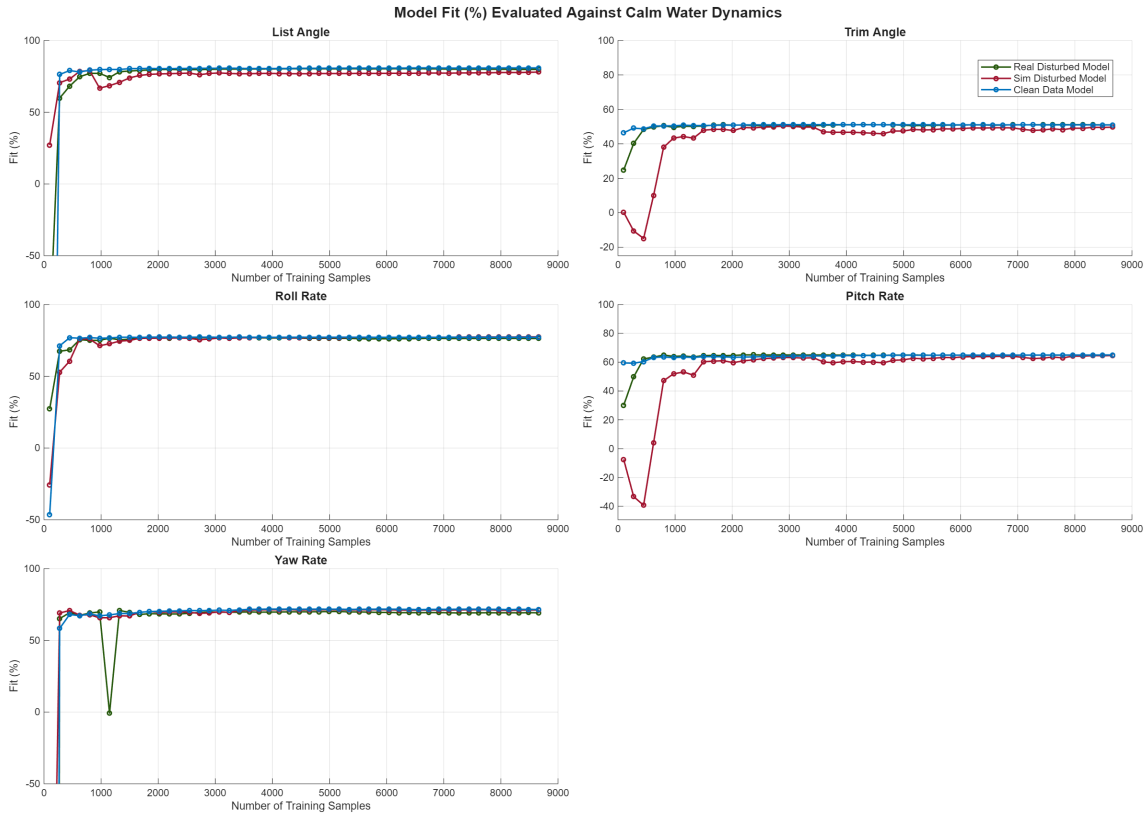


Figure 5.8: Simulation fit for different amounts of training data. Larger disturbances needs more data to converge.

5.3.4 Residual Analysis

To determine whether the grey-box model successfully captures all relevant linear dynamics, a residual analysis was conducted. To isolate and evaluate the impact of neglected translational motion, the grey-box residuals were benchmarked against those of a black-box transfer function matrix. By fitting an independent transfer function for every input-output combination according to:

$$\mathbf{Y}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) & G_{34}(s) \\ G_{41}(s) & G_{42}(s) & G_{43}(s) & G_{44}(s) \\ G_{51}(s) & G_{52}(s) & G_{53}(s) & G_{54}(s) \end{bmatrix} \mathbf{U}(s) \quad (5.29)$$

the black-box model implicitly captures all system cross couplings. This is achieved because the transfer functions are not structurally limited to exclusively rotational motion unlike the physics based grey-box models (5.23-5.25)). Instead each transfer function inherently accounts for translational motion and any other unmodeled dynamics by directly mapping the true input-output relationship.

Each transfer function in (5.29) consists of 3 poles and 2 zeros which yields a total of 120 learnable parameters across all transfer functions. As defined by [13, p. 412], the number of parameters to be estimated simultaneously requires an input signal that is persistently exciting of an order greater than the number of parameters. In physical terms persistence of excitation corresponds to the signal's spectral richness, if it injects enough frequencies to uniquely stimulate all dynamics of the system. To ensure that the excitation signal remains persistently exciting for the transfer function, a PRBS9 MIMO dataset collected at 20kt in calm waters was used. Since this signal consists of 511 bits it is persistently exciting of order 511 which is well above the 120 parameters. The residual plots for the grey-box and transfer-function models are shown in Figure 5.9 and Figure 5.10 respectively.

Both figures show a high correlation over multiple time lags in the auto correlation. This is expected since the models do not account for any colored disturbances. The cross correlations shows that the grey-box model have correlation between residuals and inputs, which is not the case for the transfer function model (with the exception of fin-pitch rate and port interceptor to pitch angle). This shows that the missing dynamics in the grey-box model are not caused by nonlinear dynamics and indicates that the model structure misses some cross couplings. This is further discussed in Section 7.1.3

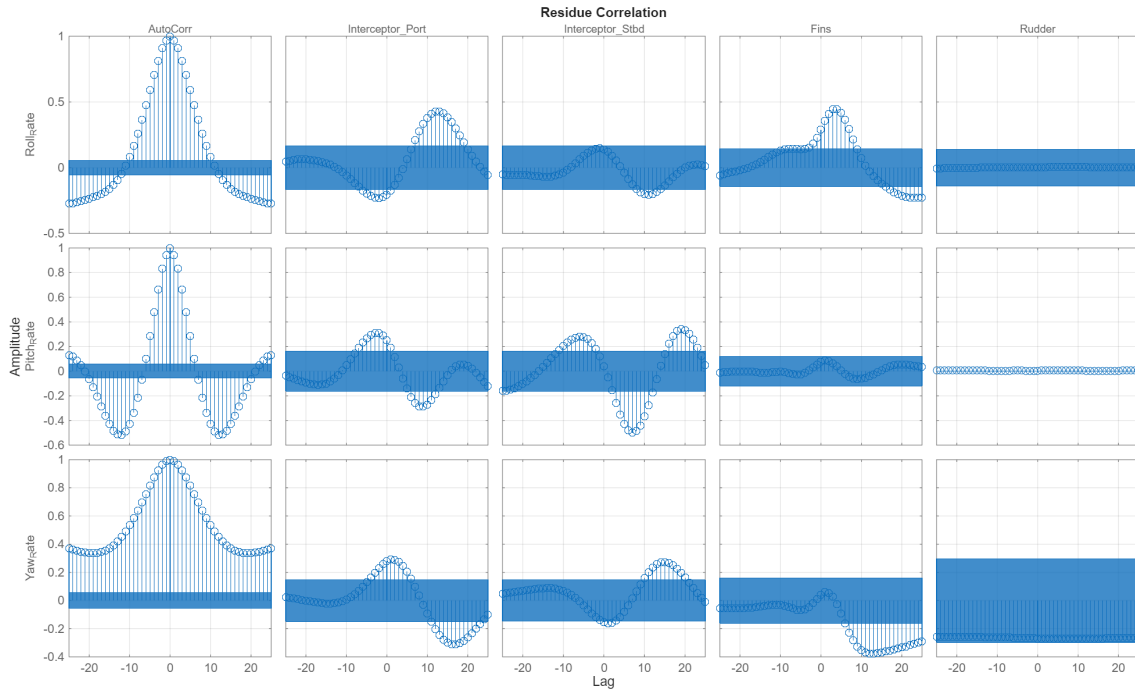


Figure 5.9: Residual auto and cross correlation for the grey-box model trained on the first 4 periods of PRBS9 mimo dataset. The cross correlation indicates that the model misses some dynamics that is excited by the control surfaces.

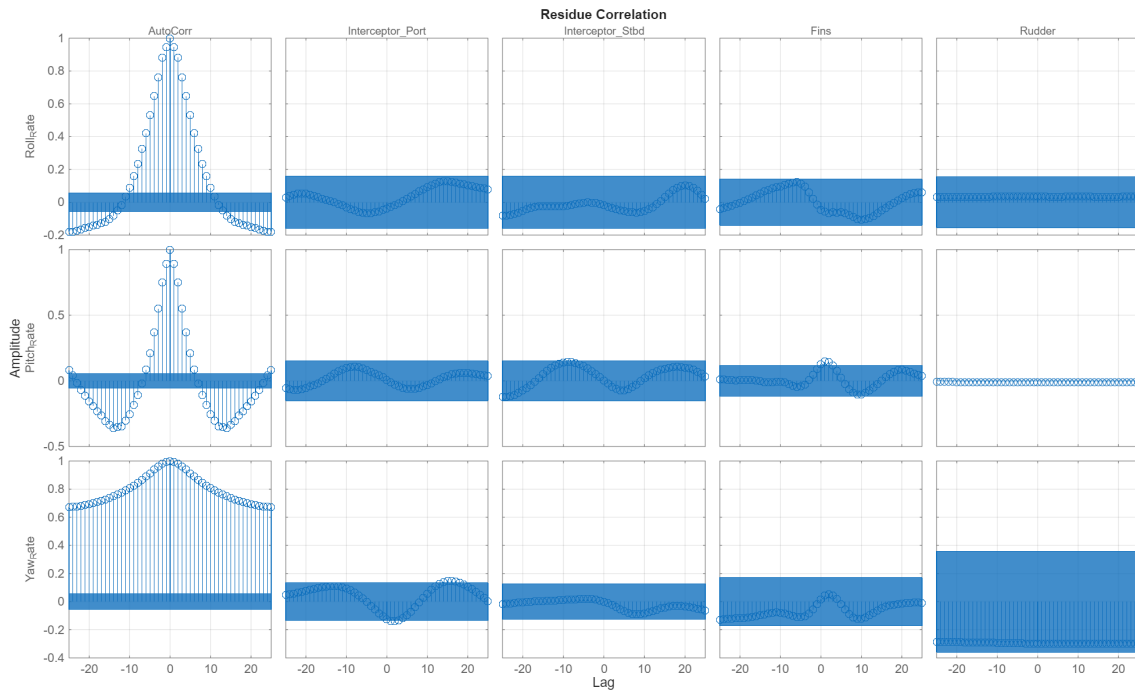


Figure 5.10: Residual auto and cross correlation for the transfer function model trained on the first 4 periods of PRBS9 mimo dataset. The black-box transfer function model captures most of the input-output dynamics.

5.3.5 Evaluation of Nonlinear Amplitude Dependence

To evaluate system linearity across different control actuator amplitudes, the model is cross-validated against the ramp dataset. The model, which was initially identified using the calm water dataset, is evaluated against each period with different amplitudes independently.

The time-domain simulation fit for each amplitude are presented in Table 5.3 and visually summarized in Figure 5.11. To establish a baseline, the fit percentages from the original training data are included as a reference. Each period of the ramp dataset corresponds to a discrete scaling of the training data's actuator amplitude, ranging from 20% to 100%.

State	Train	20%	40%	60%	80%	100%
ϕ	80.92	29.25	60.44	63.07	84.82	76.79
θ	50.97	12.26	26.32	39.84	43.77	44.02
p	77.16	52.23	66.54	74.15	76.74	76.57
q	65.01	27.82	50.60	63.05	67.29	69.93
r	71.51	57.69	61.57	65.91	77.24	52.53

Table 5.3: Time-domain model fit (%) evaluated across varying actuator amplitudes (Ramp test) compared against the training data baseline.

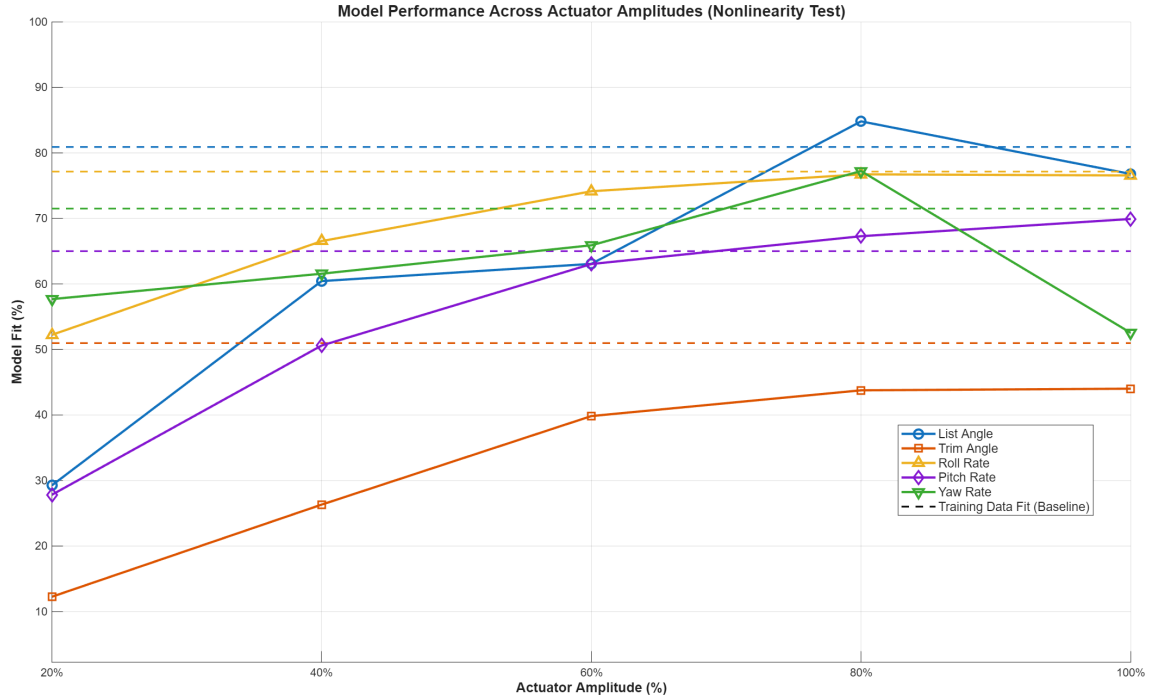


Figure 5.11: Simulation fit of model trained on the calm dataset and evaluated across varying actuator amplitudes. The dashed lines represents the model validated on the training data. The simulations fit increases with amplitude which could be caused by nonlinear amplitude dependence or increasing SNR. This figure is not sufficient to draw any conclusions about the amplitude dependence.

In Figure 5.11, the model performance initially appears degraded at the lower actuator amplitudes. However, this degradation is not necessarily indicative of poor model performance. The lower amplitudes reduces SNR, with low control forces the vessel's measured response is dominated by unmodeled environmental disturbances.

Furthermore, the sharp drop in model fit observed at the 100% amplitude period for the yaw rate (r) and roll angle (ϕ) is primarily caused by a severe rudder input required to avoid an obstacle during the collection of this data set. This forced the vessel into a sudden roll angle 15° away from the linearized operating point, pushing the system into significant nonlinear dynamics. The rudders effect on roll and speed are presented in Figure 5.12.

5. Model Identification and Parameter Estimation



Figure 5.12: The rudders effect on roll angle and speed in the RAMP dataset, the vertical red dashed lines indicates the time where the periods end and transition to a higher amplitude. A large rudder disturbance is recorded at the end of period 4.

To decouple the environmental noise and unintended rudder input from the true underlying physics, the empirical SPAFDR was calculated for each amplitude period independently. Figure D.3 compares the resulting frequency responses with the parametric baseline model. For visual clarity, only a representative input-output pair (fins to roll rate) is presented here. The complete frequency response matrix detailing all control actuators is available in Appendix D.3

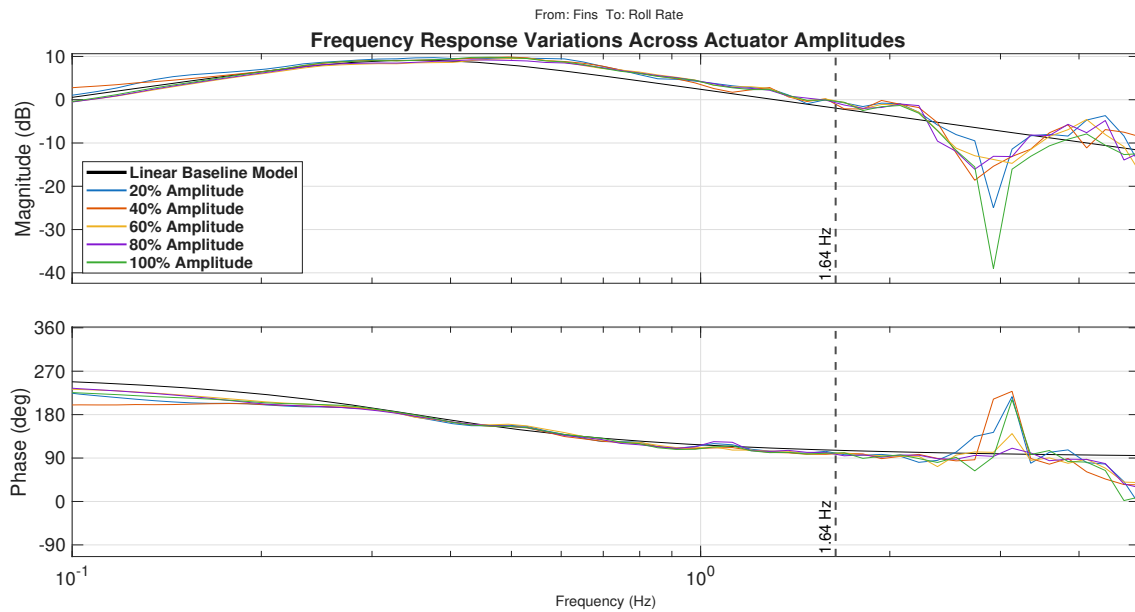


Figure 5.13: Frequency response variations across different amplitudes compared against the linear baseline model for fins to roll rate dynamics. This spectral estimates cluster tightly around the parametric model which indicates that amplitude does not significantly shift the frequency dynamics across the operational bandwidth.

The spectral estimates for the different amplitudes cluster tightly around the parametric baseline model near the vessels resonance frequencies. The absence of significant frequency shifting or change in magnitude around these resonance peaks

demonstrates that the fundamental dynamics remains largely linear across tested actuator amplitudes.

6

Recursive Parameter Identification

While the previous chapter utilized batch estimation to identify the model, practical real-time applications require a different approach. Transitioning to a recursive implementation provides significant advantages. Unlike batch estimation, which processes an entire historical dataset simultaneously to find an optimum, recursive identification updates the parameter estimates sequentially at each discrete time step as new data points become available. The primary benefit is that it allows the estimation to run directly on embedded hardware without the need to store datasets. It also opens up the possibility to model time-varying dynamics. This chapter evaluates two recursive identification algorithms: the Recursive Output Error (ROE) method and the Recursive Instrumental Variable State Variable Filter (RIVSVF).

6.1 ROE

The recursive output error algorithm builds upon the Gauss-Newton optimizer described in Section 2.3.1.2. At each iteration, a standard Runge-Kutta 4 (RK4) numerical integration scheme is utilized to propagate the estimated system states forward in time by one discrete time step.

Furthermore, to prevent low-frequency drift and steady state DC offsets from biasing the parameter estimates, the input and output measurements are recursively detrended. This is achieved by first calculating the slow-moving components using an exponential moving average (EMA) filter, and then subtracting these means from the raw measurements to effectively high-pass filter the data:

$$\bar{u}_k = \alpha \bar{u}_{k-1} + (1 - \alpha) u_k \quad (6.1a)$$

$$\bar{y}_k = \alpha \bar{y}_{k-1} + (1 - \alpha) y_k \quad (6.1b)$$

$$\tilde{u}_k = u_k - \bar{u}_k \quad (6.1c)$$

$$\tilde{y}_k = y_k - \bar{y}_k \quad (6.1d)$$

where $\bar{u}_k$ and $\bar{y}_k$ are the estimated means at time step k and α is the smoothing factor (set to 0.999 in this implementation). The variables $\tilde{u}_k$ and $\tilde{y}_k$ represent the resulting detrended signals. This recursive high-pass filtering removes low-frequency wave drift while preserving the operational system dynamics identified in Section 3.5.

To prevent the initial filter transients from corrupting the parameter calibration, the parameter updates are initially paused for 300 samples to let the filter reach steady state.

The optimizer utilizes a Levenberg-Marquardt (LM) regularization, which dynamically interpolates between Gauss-Newton updates and gradient descent steps using a damping factor, μ_k . To prevent erratic parameter updates when severe physical disturbances are introduced (e.g surge waves or aggressive rudder inputs) μ_k is defined as a function of the squared norm of the prediction error:

$$\mu_k = 0.01 + 0.05||e_k||^2 \quad (6.2)$$

When an unmodeled disturbance causes an instantaneous spike in the prediction error the damping gain forces the algorithm into an conservative gradient-descent step, effectively slowing down the learning and protecting the previously learned parameters. The mathematical validity of using the squared residual to bound the system and guarantee stable convergence is formally described in [25]. To execute this continuously, the algorithm relies on the recursive Levenberg-Marquardt formulation established in [26].

$$S_k = \psi(k, \hat{\theta}_{k-1})P_{k-1}\psi^T(k, \hat{\theta}_{k-1}) + \mu_k I \quad (6.3a)$$

$$K_k = P_{k-1}\psi^T(k, \hat{\theta}_{k-1})S_k^{-1} \quad (6.3b)$$

$$P_k = \frac{1}{\lambda} [P_{k-1} - K_k\psi(k, \hat{\theta}_{k-1})P_{k-1}] \quad (6.3c)$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + P_k\psi^T(k, \hat{\theta}_{k-1})e_k \quad (6.3d)$$

Here, $\hat{\theta}_k$ is the estimated parameter vector, $\psi(k, \hat{\theta}_{k-1})$ represents the prediction gradient (Jacobian matrix), P_k is the parameter covariance matrix approximation, S_k is the innovation covariance, K_k is the update gain, I is the identity matrix, and λ is the chosen forgetting factor (choosing λ is further discussed in Section 6.3). Given the choice of μ_k , calculating the parameter update using (6.3d) is problematic since:

$$\begin{aligned} P_k\psi^T(k) &= \frac{1}{\lambda} [P_{k-1} - K_k\psi(k)P_{k-1}] \psi^T(k) \\ &= \frac{1}{\lambda} (P_{k-1}\psi^T(k) - K_k [\psi(k)P_{k-1}\psi^T(k)]) \\ &= \frac{1}{\lambda} (K_k S_k - K_k [S_k - \mu_k I]) \\ &= \frac{1}{\lambda} \mu_k K_k \end{aligned} \quad (6.4)$$

Substituting this back into (6.3d) and expanding μ_k shows that the parameter up-

date has a cubed dependency on the residual:

$$\begin{aligned}\hat{\theta}_k &= \hat{\theta}_{k-1} + \frac{1}{\lambda} \mu_k K_k e_k \\ &= \hat{\theta}_{k-1} + \frac{1}{\lambda} \left(0.01 + 0.05 \|e_k\|^2\right) K_k e_k\end{aligned}\quad (6.5)$$

This scaling is not desired since the parameters will update very erratic when exposed to a large residual, which is the opposite of the intended physical shock-absorbing behavior. Instead K_k is used as in [13, p. 382-383].

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k e_k \quad (6.6)$$

The full psudo code is shown in Algorithm 1.

Algorithm 1 Recursive Output Error (ROE) Parameter Estimation

Require: Data $\mathbf{u}, \mathbf{y}$ (length N), Initials $\mathbf{p}_{init} \in \mathbb{R}^{15}$, $\mathbf{P}_{init}$, Forgetting factor λ

Ensure: Final estimated parameters $\mathbf{p}$, Final covariance $\mathbf{P}$

Initialize: $\mathbf{p} \leftarrow \mathbf{p}_{init}$, $\mathbf{P} \leftarrow \mathbf{P}_{init}$, $\mathbf{x} \leftarrow \mathbf{0}_{6 \times 1}$, $\mathbf{S} \leftarrow \mathbf{0}_{6 \times 15}$

Define Jacobians $\frac{\partial \mathbf{A}}{\partial p_i}$, $\frac{\partial \mathbf{B}}{\partial p_i}$, EMA vars $\bar{\mathbf{u}} \leftarrow \mathbf{u}_1$, $\bar{\mathbf{y}} \leftarrow \mathbf{y}_1$

for $k = 1$ to N **do**

Detrend: $\bar{\mathbf{u}} \leftarrow 0.999\bar{\mathbf{u}} + 0.001\mathbf{u}_k$, $\mathbf{u}_k \leftarrow \mathbf{u}_k - \bar{\mathbf{u}}$ (*Repeat for* $\mathbf{y}_k$)

Predict: Extract $\mathbf{A}_c, \mathbf{B}_c$ using $\mathbf{p}$. $\hat{\mathbf{y}}_k \leftarrow \mathbf{C}\mathbf{x}$

Residual & Sensitivity: $\mathbf{e}_k \leftarrow \mathbf{y}_k - \hat{\mathbf{y}}_k$, $\Psi \leftarrow \mathbf{C}\mathbf{S}$

Adaptation Gain (LM): $\mu \leftarrow 0.01 + 0.05 \|\mathbf{e}_k\|^2$

$\mathbf{S}_{mat} \leftarrow \Psi \mathbf{P} \Psi^T + \mu \mathbf{I}$, $\mathbf{K} \leftarrow \mathbf{P} \Psi^T \mathbf{S}_{mat}^{-1}$

$\mathbf{p}_{prop} \leftarrow \mathbf{p} + \mathbf{K} \mathbf{e}_k$

Stability Gate: Extract test matrix $\mathbf{A}_{test}$ using $\mathbf{p}_{prop}$

if $k \geq 400$ **and** $\max(\text{real}(\text{eig}(\mathbf{A}_{test}))) \leq 0$ **then**

$\mathbf{p} \leftarrow \mathbf{p}_{prop}$, $\mathbf{P} \leftarrow \frac{1}{\lambda} (\mathbf{P} - \mathbf{K} \Psi \mathbf{P})$

$\triangleright$ Accept update

end if

State Integration (RK4): $\mathbf{f}(\mathbf{x}) = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u}_k$

$\mathbf{k}_1 \leftarrow \mathbf{f}(\mathbf{x})$, $\mathbf{k}_2 \leftarrow \mathbf{f}(\mathbf{x} + 0.5 T_s \mathbf{k}_1)$

$\mathbf{k}_3 \leftarrow \mathbf{f}(\mathbf{x} + 0.5 T_s \mathbf{k}_2)$, $\mathbf{k}_4 \leftarrow \mathbf{f}(\mathbf{x} + T_s \mathbf{k}_3)$

$\mathbf{x}_{next} \leftarrow \mathbf{x} + \frac{T_s}{6} (\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)$

Sensitivity Integration (Euler): $\mathbf{A}_d \leftarrow \mathbf{I} + \mathbf{A}_c T_s + 0.5 \mathbf{A}_c^2 T_s^2$

for $i = 1$ to 15 **do**

$\mathbf{S}_{next}^{(:,i)} \leftarrow \mathbf{A}_d \mathbf{S}^{(:,i)} + \left(\frac{\partial \mathbf{A}}{\partial p_i} \mathbf{x} + \frac{\partial \mathbf{B}}{\partial p_i} \mathbf{u}_k \right) T_s$

end for

Advance Time Step: $\mathbf{x} \leftarrow \mathbf{x}_{next}$, $\mathbf{S} \leftarrow \mathbf{S}_{next}$

end for

return $\mathbf{p}, \mathbf{P}$

6.2 RIVSVF

The RIVSVF is an algorithm specifically designed for recursive identification of continuous-time systems operating in colored noise [27]. It combines an instrumental variable approach with a State Variable Filter (SVF).

Continuous-time identification relies on differential equations which require the time derivative of the measured signal. These derivatives can be calculated using numerical differentiation (such as finite differences), but they can severely amplify high frequency noise [28]. The SVF solves this problem by applying a continuous-time band-pass filter, which provides smooth noise filtered derivatives while also detrending the data. The band-pass filter is defined by the following transfer function:

$$F_{bp}(s) = \left(\frac{s}{s + \omega_{hp}} \right) \left(\frac{\omega_{lp}}{s + \omega_{lp}} \right) = \frac{\omega_{lp}s}{s^2 + (\omega_{hp} + \omega_{lp})s + \omega_{hp}\omega_{lp}} \quad (6.7)$$

To simplify the notation and prepare for time-domain implementation, the filter coefficients are defined as follows:

$$a_0 = \omega_{hp}\omega_{lp} \quad (6.8)$$

$$a_1 = \omega_{hp} + \omega_{lp} \quad (6.9)$$

$$b_1 = \omega_{lp} \quad (6.10)$$

Substituting these coefficients into the transfer function yields:

$$F_{bp}(s) = \frac{b_1s}{s^2 + a_1s + a_0} \quad (6.11)$$

When a signal $X(s)$ is passed through this filter, the filtered output is $X_f(s) = F_{bp}(s)X(s)$. To avoid explicit differentiation of the input (which causes noisy derivatives), the transfer function is transformed into a state-space representation using the controllable canonical form. This is achieved by introducing an artificial intermediate variable $V(s)$ that splits the transfer function into a denominator part (the system dynamics) and a numerator part (the output combination):

$$X(s) = (s^2 + a_1s + a_0)V(s) \quad (6.12)$$

$$X_f(s) = (b_1s)V(s) \quad (6.13)$$

Transforming Equation 6.12 into the time domain yields:

$$x(t) = \ddot{v}(t) + a_1\dot{v}(t) + a_0v(t) \quad (6.14)$$

To convert this second-order differential equation into a system of first-order equations, a state vector is defined:

$$\mathbf{v}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} v(t) \\ \dot{v}(t) \end{bmatrix} \quad (6.15)$$

Since $v_2(t) = \dot{v}_1(t)$, it follows that $\dot{v}_2(t) = \ddot{v}(t)$. The time-domain dynamics is rearranged to solve for the highest derivative:

$$\dot{v}_1(t) = v_2(t) \tag{6.16}$$

$$\dot{v}_2(t) = -a_0v_1(t) - a_1v_2(t) + x(t) \tag{6.17}$$

Finally, the output Equation 6.13 is transformed into the time domain. Since $X_f(s) = b_1sV(s)$, the time-domain filtered signal is $x_f(t) = b_1\dot{v}(t)$. By substituting the defined states v_1 and v_2 , the filtered signal and its smoothed derivative can be extracted algebraically:

$$x_f(t) = b_1v_2(t) \tag{6.18}$$

$$\dot{x}_f(t) = b_1\dot{v}_2(t) = b_1(-a_0v_1(t) - a_1v_2(t) + x(t)) \tag{6.19}$$

This formulation bypasses numerical differentiation to calculate the derivative, avoiding potentially noisy derivatives produced by numerical differentiation [28].

The algorithm works by applying this SVF to the measured input-output data and the data generated by an auxiliary model. The instrument regressor is then built using the auxiliary model output, using the currently estimated parameters. The IV regressor is then utilized in the recursive instrumental variable update equations [13, p.369] to update the parameters and covariance matrix. The RK4 numerical method is used to integrate the continuous-time auxiliary model states forward in time, generating the predicted output for the next iteration. Finally the SVF states are updated. The complete pseudo-code is provided in Algorithm 2.

Algorithm 2 Recursive Instrumental Variable with State Variable Filter (RIVSVF)

Require: Data $\mathbf{u}, \mathbf{y}$ (length N), Initials $\boldsymbol{\theta}_{init} \in \mathbb{R}^{15}$, $\mathbf{P}_{init}$, Forgetting factor λ

Ensure: Final estimated parameters $\boldsymbol{\theta}$, Final covariance $\mathbf{P}$

Initialize: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta}_{init}$, $\mathbf{P} \leftarrow \mathbf{P}_{init}$, $\mathbf{x}_{sim} \leftarrow \mathbf{0}_{6 \times 1}$, $\tilde{\mathbf{y}}_f \leftarrow \mathbf{0}_{5 \times 1}$

Define SVF parameters a_0, a_1, b_1 . Init all filter states $\mathbf{v}_1, \mathbf{v}_2 \leftarrow \mathbf{0}$ for $\mathbf{y}, \mathbf{u}, \tilde{\mathbf{y}}$

First-sample shift: $\mathbf{u}_{start} \leftarrow \mathbf{u}_1$, $\mathbf{y}_{start} \leftarrow \mathbf{y}_1$

for $k = 1$ to N **do**

Shift Data: $\mathbf{u}_k \leftarrow \mathbf{u}_k - \mathbf{u}_{start}$, $\mathbf{y}_k \leftarrow \mathbf{y}_k - \mathbf{y}_{start}$

Filter Real Data (SVF): (*Applied to both $\mathbf{y}_k$ and $\mathbf{u}_k$*)

$\dot{\mathbf{v}}_{1,y} \leftarrow \mathbf{v}_{2,y}$, $\dot{\mathbf{v}}_{2,y} \leftarrow -a_0 \mathbf{v}_{1,y} - a_1 \mathbf{v}_{2,y} + \mathbf{y}_k$

$\mathbf{y}_f \leftarrow b_1 \mathbf{v}_{2,y}$, $\dot{\mathbf{y}}_f \leftarrow b_1 \dot{\mathbf{v}}_{2,y}$, $\mathbf{u}_f \leftarrow b_1 \mathbf{v}_{2,u}$

Extract noisy measurement: $\mathbf{y}_{noisy} \leftarrow [\dot{y}_{f,3}, \dot{y}_{f,4}, \dot{y}_{f,5}]^T$

Build Regressors:

$\Phi_k \leftarrow \text{regressor}(\mathbf{y}_f, \mathbf{u}_f)$

▷ Noisy regressor

$\mathbf{Z}_k \leftarrow \text{regressor}(\tilde{\mathbf{y}}_f, \mathbf{u}_f)$

▷ Clean instrument

Adaptation Gain & Parameter Update (RIV):

$\mathbf{K} \leftarrow \mathbf{P} \mathbf{Z}_k^T (\lambda \mathbf{I} + \Phi_k \mathbf{P} \mathbf{Z}_k^T)^{-1}$

$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \mathbf{K}(\mathbf{y}_{noisy} - \Phi_k \boldsymbol{\theta})$

$\mathbf{P} \leftarrow \frac{1}{\lambda} (\mathbf{P} - \mathbf{K} \Phi_k \mathbf{P})$

Auxiliary Model Simulation (RK4): Extract $\mathbf{A}_c, \mathbf{B}_c$ using $\boldsymbol{\theta}$

$\mathbf{f}(\mathbf{x}) = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u}_k$

$\mathbf{k}_1 \leftarrow \mathbf{f}(\mathbf{x}_{sim})$, $\mathbf{k}_2 \leftarrow \mathbf{f}(\mathbf{x}_{sim} + 0.5 T_s \mathbf{k}_1)$

$\mathbf{k}_3 \leftarrow \mathbf{f}(\mathbf{x}_{sim} + 0.5 T_s \mathbf{k}_2)$, $\mathbf{k}_4 \leftarrow \mathbf{f}(\mathbf{x}_{sim} + T_s \mathbf{k}_3)$

$\mathbf{x}_{sim} \leftarrow \mathbf{x}_{sim} + \frac{T_s}{6} (\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)$

Filter Simulated Data (SVF): $\tilde{\mathbf{y}}_{raw} \leftarrow \mathbf{C} \mathbf{x}_{sim}$

$\dot{\mathbf{v}}_{1,\tilde{y}} \leftarrow \mathbf{v}_{2,\tilde{y}}$, $\dot{\mathbf{v}}_{2,\tilde{y}} \leftarrow -a_0 \mathbf{v}_{1,\tilde{y}} - a_1 \mathbf{v}_{2,\tilde{y}} + \tilde{\mathbf{y}}_{raw}$

$\tilde{\mathbf{y}}_f \leftarrow b_1 \mathbf{v}_{2,\tilde{y}}$

▷ Phase-matched for next instrument

Forward Euler Integration: (*Applied to $\mathbf{y}, \mathbf{u}$, and $\tilde{\mathbf{y}}$*)

$\mathbf{v}_1 \leftarrow \mathbf{v}_1 + T_s \dot{\mathbf{v}}_1$, $\mathbf{v}_2 \leftarrow \mathbf{v}_2 + T_s \dot{\mathbf{v}}_2$

end for

return $\boldsymbol{\theta}, \mathbf{P}$

6.3 ROE and RIVSVF Results and Tuning

The RIVSVF and ROE algorithms are using the disturbance data set as training data and the calm dataset as validation data. To run the algorithms, a suitable forgetting factor λ needs to be determined. When choosing the forgetting factor, it is a matter of choosing between how adaptable the parameters should be to sys-

tem changes, and how robust to noise they should be [29]. A larger λ means less adaptability and more noise robustness and vice versa. If the algorithm is meant to run during a calibration phase as stated in Chapter 1, the model to be identified can be assumed to be time invariant, adaptability can therefore be neglected which motivates choosing $\lambda = 1$. When choosing $\lambda = 1$ all samples are weighted equally which is similar to batch estimation. Besides $\lambda = 1$ the initial parameters are chosen according to the derivation in Section 5.2.1 and the initial covariance matrix P is the identity matrix. To illustrate the learning behavior, the parameters are plotted against the number of samples used for estimation. For visual clarity, a representative subset of the identified parameters is displayed in Figure 6.1, while the complete 15-parameter convergence matrix is provided in Appendix E. Furthermore, the model simulation fit using the parameters estimated at different sample intervals is plotted in Figure 6.2. These figures demonstrate that the model simulation fit converges before all of the parameters do. Furthermore, many of the parameters of the RIVSVF and ROE converges to different values compared to the batch OE estimation, but the resulting models are still performing similarly (a behavior analogous to the disturbance-induced pole shifts analyzed in Section 5.3.1).

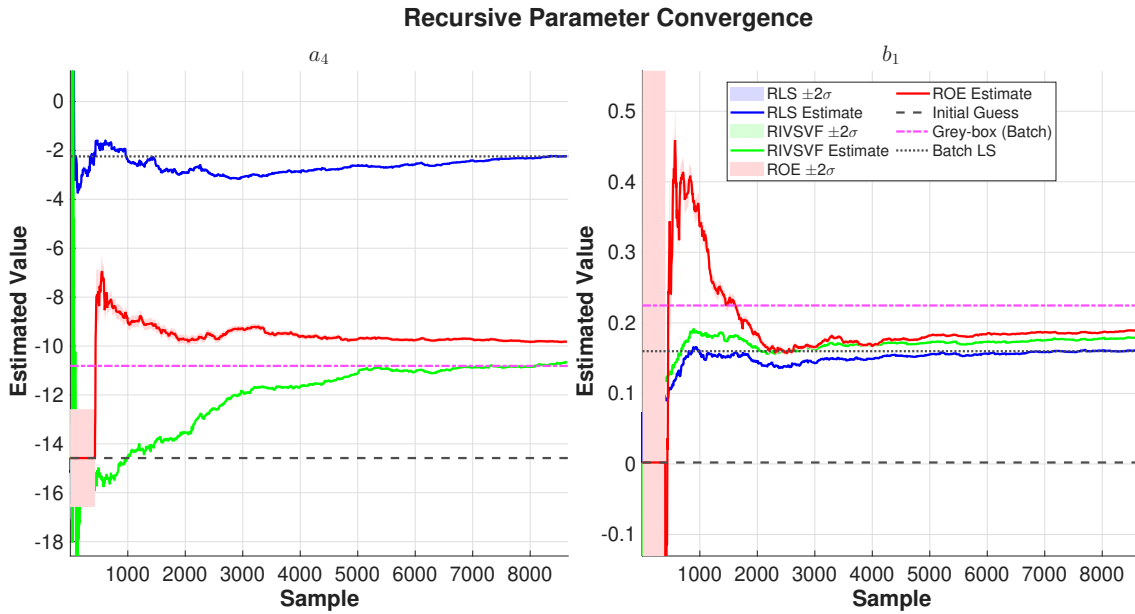


Figure 6.1: Parameter convergence for the recursive algorithms illustrates that the parameters of ROE and RIVSVF converge to different solutions compared to the batch OE estimation. The complete 15-parameter grid is provided in Appendix E.

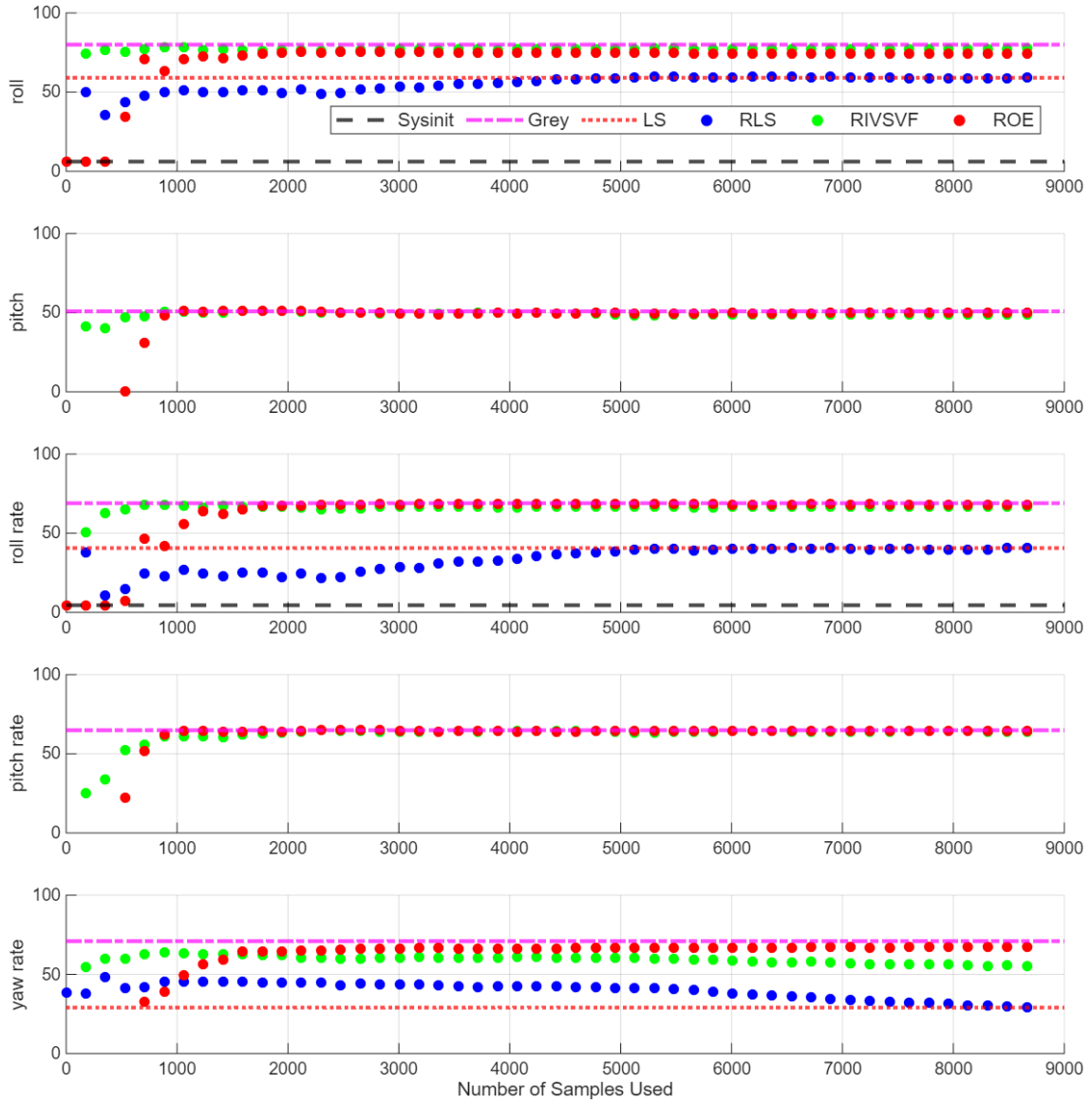


Figure 6.2: Model simulation fit with the estimated parameters at different number of samples. The simulation fit is similar between ROE and RIVSVF even though the parameters differ.

6.4 Algorithm Robustness to Simulated Noise

The proposed recursive algorithms are meant to be applicable even at moderate sea conditions with significant colored noise. Because there is only one recorded data set with disturbances, the algorithms are tested on generated data using the disturbance simulation from Section 5.1.2. This allows for quick testing with multiple different datasets, which is significantly more cost effective than conducting hundreds of sea trials. The algorithms robustness are validated by calculating the model simulation fit for each generated dataset. The result is plotted in Figure 6.3 where 100 separate datasets are calculated. The blue box highlights where 50% of the simulation fit

values are, with the outliers marked with a red cross in the rest of the scale. Figure 6.3 further emphasizes that the RIVSVF algorithm is less robust than the ROE, but both algorithms can still provide similar simulation fit performance. All of the generated datasets are at the speed of 20 kt. The parameters for the model are thus assumed to be constant which motivates the choice of $\lambda = 1$ for these simulation tests.

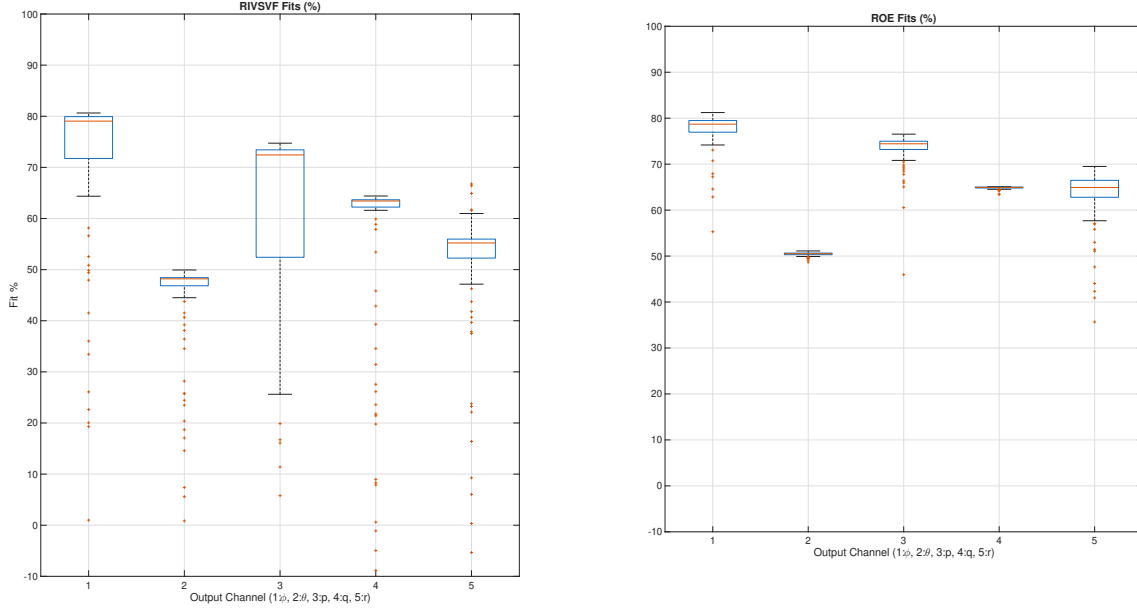


Figure 6.3: Model simulation fit across 100 simulated datasets with colored noise, evaluating the robustness of the ROE and RIVSVF algorithms. The ROE is significantly more robust to disturbances compared to the RIVSVF as demonstrated by the spread of the simulation fit, since the blue boxes containing 50% of the simulation fits are smaller and concentrated at higher simulation fit percentages compared to the RIVSVF.

7

Discussion and Conclusion

This chapter discusses the results provided in previous chapters of this thesis. It is structured by a discussion of the results from each chapter, followed by some conclusions and a future work section.

7.1 Discussion of Methodology and Results

The methodology used in each chapter is discussed and connected to the results. This discussion is used to draw the conclusions in Section 7.2.

7.1.1 Design of Excitation Signal and Data Collection

The excitation signal consisted of a PRBS7 modified with a clock frequency and low-pass filter at 3 Hz. It provided open-loop data suitable for parameter identification of the MIMO system used in this thesis. In Section 2.1, it is noted that the test vessel was fitted with unusually large fins for its size, and with interceptors mounted sub optimally. This introduces some important aspects of the data collection:

1. The high SNR gained with the large fins provides data that contains mostly system dynamics which is desired from a system identification perspective. However, this means they can excite the system at large roll angles, which introduces nonlinearities in the data, explained further in [30] and [8].
2. With the interceptors mounted near the rudder and propeller of the boat, rather than far out in the corners of the stern, they receive less roll control authority.

These aspects make conclusions about choosing excitation signal amplitude by considering ship size difficult to make. If the test vessel was fitted with an installation tailored to its size, stricter conclusions could be drawn about the necessary amplitudes required for parameter estimation. The fact that the control authority increases with faster vessel speeds further complicates the choice of amplitude, since this means that less actuator excitation induces larger output angles at faster vessel speeds.

7.1.2 Noise Modeling

Chapter 4 showed that the disturbance present at sea is indeed colored. The results align with the theoretical expectation explained in Section 2.2, specifically that the sea state can change, partly because if the vessel turns the wave encounter frequency changes, and partly because the waves can change in both frequency and power. The disturbances present in the data recorded for this project had a natural frequency of approximately 0.5 Hz, but theoretically it can appear at any frequency where the vessel dynamics exist as shown in Section 2.2.

The method used to estimate noise in this thesis relies on batch estimation and periodic inputs, and is not applicable recursively. Chapter 4 showed that the disturbance may change during a sea trial which motivates to estimate the disturbance online. To estimate noise recursively, a noise model may be introduced as a second order oscillator like they do in [8, p.228-233], or a recursive ARMA model may be introduced, since that is the most suitable model structure to the wave disturbance seen in Section 4.3.3. Box Jenkins models are common to estimate noise, but since the noise shares poles with the plant as explained in Section 2.2 (also empirically proven in Section 3.5), an AR or ARMA is more suitable. However, if the noise and the process model are to be identified simultaneously, some difficulties may arise further discussed in Section 7.3.

7.1.3 Batch Identification and Model Validation

The linear model proposed in this thesis is a simplified version of Fossens equations [6] for marine vessel dynamics. The model simulation fit analyzed in Section 5.3.3 provides promising results while the correlation residual analysis in Section 5.3.4 confirms that the model lacks some of the true system dynamics.

This discrepancy is likely due to the significant simplifications performed to derive the physical model. The most probable cause is the exclusion of the heave and sway degree of freedom, which ignores the strong dynamic cross-coupling with the system's surge-heave-pitch and sway-roll-yaw outputs [6]. In contrast, the transfer function approach detailed in Section 5.3.4 models all cross coupling implicitly by fitting an independent transfer function for each input-output combination, allowing it to learn the linear effect of heave and sway on the modeled outputs. Surge is mainly constant during the sea trials in this thesis, therefore its absence in the grey-box model is not considered to introduce any major correlation with the residuals in Figure 5.9.

While the transfer function performs better than the grey-box model in terms of simulation fit and residual analysis, it introduces significant downsides. First, using this for control design is highly impractical due to the massive number of parameters spread across 20 independent transfer functions. The state space structure used for the physics informed grey-box model is the standard, preferred model structure for control methods like LQR and MPC [7]. Furthermore, the number of parameters in the transfer function model complicates the estimation problem. This is a problem

for the recursive estimation algorithms as their computational complexity scales quadratically with the number of parameters, $\mathcal{O}(p^2)$ [26]. Because the black-box transfer function model requires significantly more parameters than the grey-box model (120 versus 15), its computational cost per time step is 64 times higher ($\frac{120^2}{15^2} = 64$). Finally, this increase in tunable parameters make the transfer function model sensitive to overfitting the noise. Given these practical limitations regarding both system identification and control design, the gray-box model is the far better choice even if the simulation performance is notably worse.

Regarding the model structure and estimator, utilizing an OE structure has been shown to converge to an accurate model within the systems bandwidth, even in presence of heavy colored noise. As demonstrated in Section 5.3.1, lower SNR led to slower convergence rates and increased parameter uncertainty due to the disturbance model mismatch explained in Section 5.2. However, an explicit noise model was omitted because the environmental disturbances were shown to be time variant in Sections 4.3.3 and 4.3.4. Since the parameter identification is designed to be done during a calibration phase these downsides can be accepted, since, as long as the model converges to an accurate physical representation across varying sea conditions, the time of convergence is of secondary importance.

7.1.4 Recursive Identification

The recursive identification in this thesis evaluated two algorithms, a Recursive Output Error (ROE) algorithm and the Recursive Instrumental Variable State Variable Filter (RIVSVF). It is shown that the ROE relies on more data to converge than the RIVSVF, but the RIVSVF yields more inconsistent parameter estimates. The RIVSVF likely fails due to the auxiliary model not capturing all of the system dynamics, as shown in Section 5.3.4. This means that the instruments do not correlate fully with the true system which is required for an instrumental variable method to converge to the correct parameter solution [20].

The calibration phase motivates choosing the forgetting factor $\lambda = 1$, which gives the recursive algorithms the ability to "remember" all of the data. Considering this, the ROE is a more suitable algorithm since the RIVSVF loses its advantage in requiring less data to converge, provided that the calibration time is significant enough for the ROE to converge.

7.2 Conclusions

The developed excitation signal is adequate for open-loop parameter estimation of the model in this thesis, provided it has sufficient SNR. Strict conclusions about the required excitation signal amplitude to achieve this are difficult to make for the following reasons:

- The suboptimal control actuator installation for the test vessel explained in Section 2.1.

- Control authority depending on vessel speed, seen in Section 3.3.
- The time variant noise seen in Section 4.3.4.

A practical solution to determine a sufficient amplitude is to iteratively increase it until the required SNR is achieved.

The ocean wave disturbances are colored and time variant, making a fixed noise model inappropriate for a calibration test. Recursive ARMA model estimation may solve this problem, but this method is not tested in this thesis.

Considering the model's intended control application, the linear grey-box model structure is superior to a pure black-box model even though the black-box model may capture more of the system dynamics. This is due to the significantly fewer parameters required with the grey-box approach, making control tuning easier and the recursive parameter identification less computationally expensive. The developed grey-box model lacks some of the true system dynamics (surge, sway and heave) which introduces correlation between the input and prediction error, but frequency domain validation suggests that the model is adequate for control design over the system occupied bandwidth.

To decide the most appropriate recursive identification algorithm, the application has to be considered. The ROE is superior to the RIVSVF, provided that there is an open-loop calibration phase with enough data for it to converge. If there are limited data, the RIVSVF may converge faster than the ROE but provides inconsistent estimates.

7.3 Future Work

This section provides some potential areas for future work, using this thesis as foundation.

7.3.1 Closed Loop Identification

To avoid a calibration phase, the identification might be extended to happen during closed loop control. This is however more difficult to achieve due to the problem becoming significantly more complex. If closed loop identification is considered necessary, it should be noted that the ROE algorithm is not suitable due to the requirement of the noise to be uncorrelated with the input. Therefore the RIVSVF could be applied instead, but it would likely require an improved auxiliary model to make the instruments fully correlated with the true system. This includes modeling surge, sway and heave which relies on accelerometer measurements. The accelerometer in the current hardware is placed differently in each boat/installation introducing varying lever arms from the vessels center of mass, which makes the states difficult to measure. To account for this, more accelerometer sensors should be considered, as it may improve estimation for these states [31]. It could be investigated if these sensors can be integrated to the existing servos for the interceptors and fins, removing the

need to mount individual sensors for every installation.

7.3.2 Online Parameter Estimation

The current Output Error (OE) architecture with LM regularization successfully absorbs the wave disturbances under open-loop condition with enough data. However, treating colored noise as an unmodeled disturbance reduces statistical efficiency and increases parameter variance and estimation time. To accelerate calibration identification, future research should transition to include a time varying noise model.

To introduce online parameter estimation, specific parameters identified during the calibration phase could be "locked" if they are expected to stay the same over time. Parameters that rely on physical system properties that change slowly such as mass (due to fuel consumption and cargo etc.) might then be estimated online to continuously update and improve the model over time.

If an accurate noise model is considered necessary, it is worth considering introducing new sensors. Sensors such as radars could be mounted on the vessel to predict incoming waves. These sensors are expected to yield a more accurate wave prediction compared to using ARMA models estimated based on ship motion, since they rely on measuring an incoming wave directly. This would also allow for feedforward control strategies which could improve the stabilization system.

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A

Model Derivations

A.1 Fossen Marine Craft Model Simplification

Fossen's equation of motion is expressed as

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \quad (\text{A.1})$$

where:

- $\eta = [x, y, z, \phi, \theta, \psi]^T$ is the position and orientation vector.
- $\nu = [u, v, w, p, q, r]^T$ is the linear and angular velocity vector in the body frame. u, v and w corresponds to surge, sway and heave.
- M is the inertia matrix (Rigid Body + Added Mass).
- $C(\nu)$ is the Coriolis-centripetal matrix.
- $D(\nu)$ is the damping matrix.
- $g(\eta)$ is the vector of gravitational and restoring forces.
- τ is the generalized force acting on the vector, for instance control inputs, waves, and propulsion.

For the purpose of designing controllers for stabilization systems, the rotational subsystem is isolated.

$$\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \quad \nu_1 = [u, v, w]^T \text{ (Linear)}, \quad \nu_2 = [p, q, r]^T \text{ (Rotational)} \quad (\text{A.2})$$

The vessel is assumed to have constant speed and travels primarily in surge ($u \approx U_0, v \approx 0, w \approx 0$). By doing this simplification it reduces the number of parameters that had to be estimated but it misses to model the coupling between lateral and rotational dynamics. This simplification were necessary since it wasn't possible to get measurements of heave and sway.

The $\mathbf{M}$ matrix is the sum of the rigid body mass $\mathbf{M}_{RB}$ and the added mass $\mathbf{M}_A$ from hydrodynamic drag

$$\mathbf{M} = \begin{bmatrix} m\mathbf{I}_3 & -m\mathbf{S}(\mathbf{r}_g^b) \\ m\mathbf{S}(\mathbf{r}_g^b) & \mathbf{I}_b^b \end{bmatrix} + \mathbf{M}_A \quad (\text{A.3})$$

where $\mathbf{S}$ is a skew-symmetry matrix and $\mathbf{r}_g^b$ is the distance from the sensor to the center of mass:

$$\mathbf{S}(\mathbf{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad \mathbf{r}_g^b = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} \quad (\text{A.4})$$

For a planing vessel, symmetry is assumed about the xz -plane ($y_g = 0$) and $I_{xy} = I_{yz} = 0$.

$$\mathbf{M}_{rot} = \begin{bmatrix} I_{xx} - K_{\dot{p}} & 0 & -I_{xz} - K_{\dot{r}} \\ 0 & I_{yy} - M_{\dot{q}} & 0 \\ -I_{xz} - N_{\dot{p}} & 0 & I_{zz} - N_{\dot{r}} \end{bmatrix} \quad (\text{A.5})$$

For system identification, it is difficult to separate rigid body inertia from added mass. Lumped parameters can be defined to describe the total effect:

$$\begin{aligned} J_x &= I_{xx} - K_{\dot{p}} & J_{xz} &= I_{xz} + K_{\dot{r}} \\ J_y &= I_{yy} - M_{\dot{q}} & J_z &= I_{zz} - N_{\dot{r}} \end{aligned}$$

Resulting in the identified mass matrix:

$$\mathbf{M}_{sys} = \begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{bmatrix} \quad (\text{A.6})$$

The linearized Coriolis and centripetal force matrix is calculated using syms in Matlab.

$$\mathbf{C}_{lin} = \begin{bmatrix} 0 & 0 & -U_0 m z_g \\ 0 & U_0(M_{\dot{w}} + m x_g) & 0 \\ -U_0 K_{\dot{v}} & 0 & -U_0(N_{\dot{v}} - m x_g) \end{bmatrix} \quad (\text{A.7})$$

For the hydrodynamic damping matrix we remove the non linear effects and let the different linearization of the speed handle this. Since we assume that the vessel is symmetric in the xz plane we can remove the parameters that models that.

$$\mathbf{D} = - \begin{bmatrix} K_p & 0 & K_r \\ 0 & M_q & 0 \\ K_r & 0 & N_r \end{bmatrix} \quad (\text{A.8})$$

And the matrix for restoring forces $\mathbf{G}$ will be simplified to only have a restoring force in roll and pitch since we are not modelling heave.

$$G = \begin{bmatrix} K_\phi & 0 & 0 \\ 0 & M_\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad (\text{A.9})$$

To properly account for the forward-speed effects captured by the linearized Coriolis matrix $\mathbf{C}_{lin}(U_0)$, we lump these speed-dependent terms together with the linear hydrodynamic damping matrix $\mathbf{D}$.

$$K_p^* = K_p \quad (\text{A.10})$$

$$K_r^* = K_r - mz_g U_0 \quad (\text{A.11})$$

$$M_q^* = M_q + U_0(M_{\dot{w}} + mx_g) \quad (\text{A.12})$$

$$N_p^* = N_p - U_0 K_{\dot{v}} \quad (\text{A.13})$$

$$N_r^* = N_r - U_0(N_{\dot{v}} - mx_g) \quad (\text{A.14})$$

By substituting these lumped parameters into the rotational subsystem, the final state-space model structure is defined. Note that while the system is physically non-linear and highly dependent on forward speed, for the purpose of identification, it is treated as a Linear Time-Invariant (LTI) system operating at a fixed nominal speed U_0 .

The simplified, lumped equation of motion becomes:

$$\underbrace{\begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{bmatrix}}_{\mathbf{M}_{sys}} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \underbrace{\begin{bmatrix} K_p^* & 0 & K_r^* \\ 0 & M_q^* & 0 \\ N_p^* & 0 & N_r^* \end{bmatrix}}_{\mathbf{C}_{lin}(U_0)+\mathbf{D}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \underbrace{\begin{bmatrix} K_\phi & 0 & 0 \\ 0 & M_\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \boldsymbol{\tau} \quad (\text{A.15})$$

The number of parameters can be reduced by dividing with the inertia terms: J_x , J_y and J_z :

$$\underbrace{\begin{bmatrix} 1 & 0 & \frac{-J_{xz}}{J_x} \\ 0 & 1 & 0 \\ \frac{-J_{xz}}{J_z} & 0 & 1 \end{bmatrix}}_{\mathbf{M}_{sys}} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{K_p^*}{J_x} & 0 & \frac{K_r^*}{J_x} \\ 0 & \frac{M_q^*}{J_y} & 0 \\ \frac{N_p^*}{J_z} & 0 & \frac{N_r^*}{J_z} \end{bmatrix}}_{\mathbf{C}_{lin}(U_0)+\mathbf{D}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{K_\phi}{J_x} & 0 & 0 \\ 0 & \frac{M_\theta}{J_y} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \boldsymbol{\tau} \quad (\text{A.16})$$

This way two parameters has been removed. To be able to use this model for system identification and control systems it is rewritten onto the state space form:

$$A \cdot x = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{J_z K_\phi}{J_{xz}^2 - J_x J_z} & 0 & 0 & \frac{J_z K_p + J_{xz} N_p}{J_{xz}^2 - J_x J_z} & 0 & \frac{J_z K_r + J_{xz} N_r}{J_{xz}^2 - J_x J_z} \\ 0 & -\frac{M_\theta}{J_y} & 0 & 0 & -\frac{M_q}{J_y} & 0 \\ \frac{J_{xz} K_\phi}{J_{xz}^2 - J_x J_z} & 0 & 0 & \frac{J_{xz} K_p + J_x N_p}{J_{xz}^2 - J_x J_z} & 0 & \frac{J_{xz} K_r + J_x N_r}{J_{xz}^2 - J_x J_z} \end{bmatrix} \begin{bmatrix} \phi \text{ (Roll)} \\ \theta \text{ (Pitch)} \\ \psi \text{ (Yaw)} \\ p \text{ (Roll Rate)} \\ q \text{ (Pitch Rate)} \\ r \text{ (Yaw Rate)} \end{bmatrix} \quad (\text{A.17})$$

By lumping together the complex parameter combinations the final linear in the parameter model with 15 parameters is acquired:

$$A \cdot x = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_1 & 0 & 0 & a_2 & 0 & a_3 \\ 0 & a_4 & 0 & 0 & a_5 & 0 \\ a_6 & 0 & 0 & a_7 & 0 & a_8 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} \quad (\text{A.18})$$

The B matrix is constructed to map the actuator positions to the angular accelerations produced by the forces induced by the control surfaces.

$$B \cdot u = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_1 & -b_1 & -b_2 & b_3 \\ b_4 & b_4 & 0 & 0 \\ -b_5 & b_5 & b_6 & b_7 \end{bmatrix} \begin{bmatrix} Int_{port} \\ Int_{stbd} \\ Fins \\ Rudder \end{bmatrix} \quad (\text{A.19})$$

Since this state space equation is linear in the parameters it is possible to rewrite it on regressor form. This makes it possible to use with regressor based system identification algorithms such as least squares and instrumental variables.

To formulate the linear regressor, we define our input vector as $u = [u_1, u_2, u_3, u_4]^T$, where $u_1 = Int_{port}$, $u_2 = Int_{stbd}$, $u_3 = Fins$, and $u_4 = Rudder$. The regressor equation is then given by:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \Phi_x & \Phi_u \end{bmatrix} \theta \quad (\text{A.20})$$

Where the state and input sub-matrices are:

$$\Phi_x = \begin{bmatrix} \phi & p & r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta & q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi & p & r \end{bmatrix} \quad (\text{A.21})$$

$$\Phi_u = \begin{bmatrix} (u_1 - u_2) & -u_3 & u_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (u_1 + u_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (-u_1 + u_2) & u_3 & u_4 \end{bmatrix} \quad (\text{A.22})$$

And the parameter vector is:

$$\theta = [a_1 \ a_2 \ \dots \ a_8 \ b_1 \ b_2 \ \dots \ b_7]^T \quad (\text{A.23})$$

A.2 OE Convergence in Colored Disturbance

Assume that the true system is given by:

$$y(t) = G_0(q)u(t) + v(t) \quad (\text{A.24})$$

the optimal one step predictor is given by:

$$\hat{y}(t|\theta) = G(q, \theta)u(t) \quad (\text{A.25})$$

The prediction error is:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t|\theta) = [G_0(q) - G(q, \theta)]u(t) + v(t) \quad (\text{A.26})$$

The Prediction Error Method (PEM) asymptotically minimizes the variance of the error, $\bar{V}(\theta) = \mathbb{E}[\varepsilon^2(t, \theta)]$.

$$\varepsilon^2(t, \theta) = ([G_0(q) - G(q, \theta)]u(t))^2 + 2[G_0(q) - G(q, \theta)]u(t)v(t) + v^2(t) \quad (\text{A.27})$$

$$\bar{V}(\theta) = \mathbb{E} \left[([G_0(q) - G(q, \theta)]u(t))^2 \right] + 2\mathbb{E} [[G_0(q) - G(q, \theta)]u(t)v(t)] + \mathbb{E} [v^2(t)] \quad (\text{A.28})$$

If $u(t)$ and $v(t)$ is uncorrelated and zero mean:

$$2\mathbb{E} [[G_0(q) - G(q, \theta)]u(t)v(t)] = 0 \quad (\text{A.29})$$

the cost function simplifies to:

$$\bar{V}(\theta) = \mathbb{E} \left[([G_0(q) - G(q, \theta)]u(t))^2 \right] + \mathbb{E} [v^2(t)] \quad (\text{A.30})$$

The colored noise does only induce an offset which does not change the minimum value of the cost function. The parameter vector is therefore not biased by the colored noise when $N \rightarrow \infty$.

B

Estimating System Settling Time

Step tests are performed to estimate the settling time of the plant which is used to choose the optimal PRBS order. Two step tests are performed - one applying a step to the fin actuators to estimate the roll settling time - the other is applied to the two interceptors simultaneously to estimate the pitch settling time. This is done for 10 periods with a period time of 10 and 7 seconds. The step tests are plotted in Figures B.1-B.2

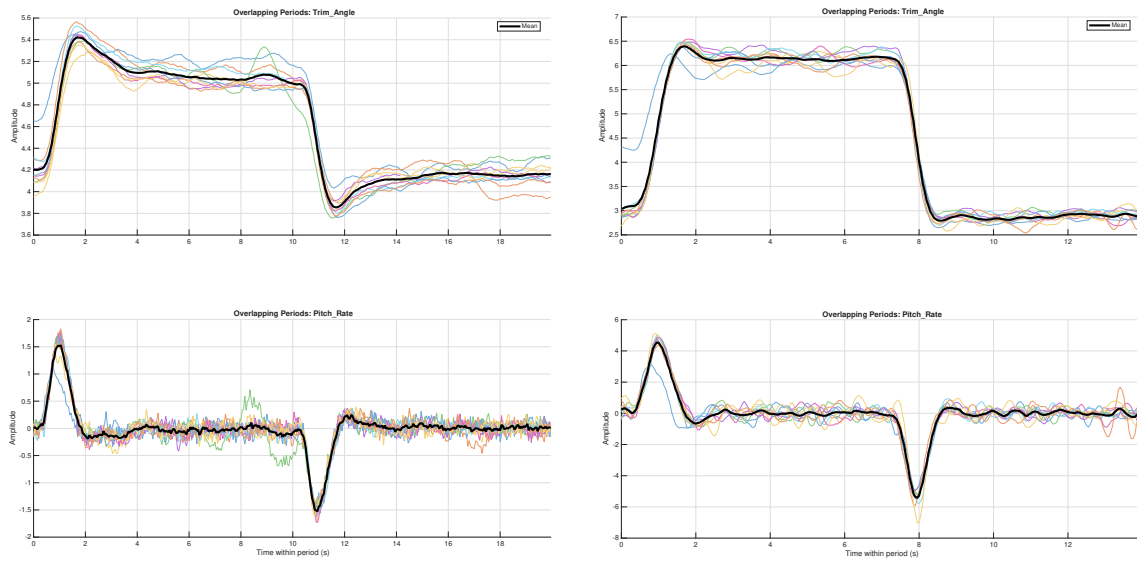


Figure B.1: Step responses for Pitch (10 kt left, 20 kt right)

B. Estimating System Settling Time

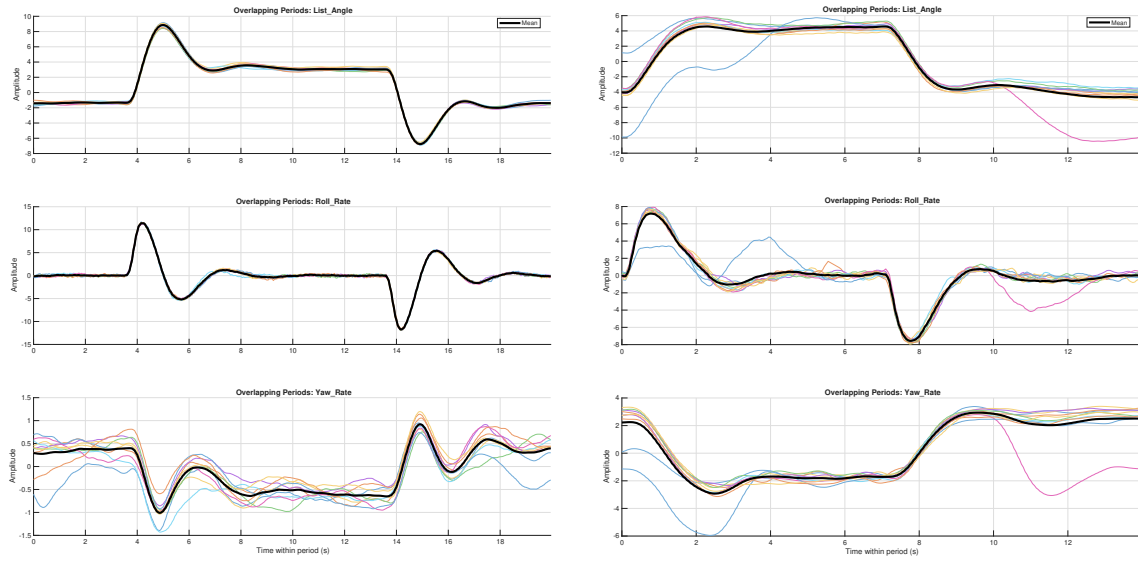


Figure B.2: Step responses for Roll and Yaw (10kt left, 20kt right)

C

Results from 10 and 15 knots

In this chapter the result from fitting the linear model at 10 and 15 kt are presented.

The result for these speeds are similar to the 20 knot model validated in Chapter 5. Figures C.1-C.2 illustrate the simulation fit for two models, which were trained on calm data with and without simulated disturbance and validated against the same calm dataset. While similar performance between the speeds it reveals a clear pitch yaw-rate coupling. During the initial rudder steps, time 0-100 seconds, there are clear pitch dynamics induced that the model is not able to capture. This is the largest difference in performance compared to the model fitted at 20 knots where this was significantly less prominent. As illustrated in Figures C.3-C.4 the speed changes slightly during these steps. Since there is a strong coupling between surge, heave and pitch[6] this would explain the missing pitch dynamics.

To evaluate the models performance at the operating bandwidth the models are compared against the SPAFDR of the calm dataset. The results are shown in Figure C.5-C.6. Both models struggle to learn roll dynamics from the interceptors. It can be seen by the spread of the frequency response of the models trained on different disturbances. The SPAFDR response are also very noisy which explains why the model struggles to find the correct physics. This is expected since the interceptors are mounted suboptimally (as detailed in 2.1). This suboptimal placement, combined with reduced water flow and subsequently decreased performance leads to a very low SNR in the roll axis. The remaining input-output combination exhibit performance similar to the 20 knot model.

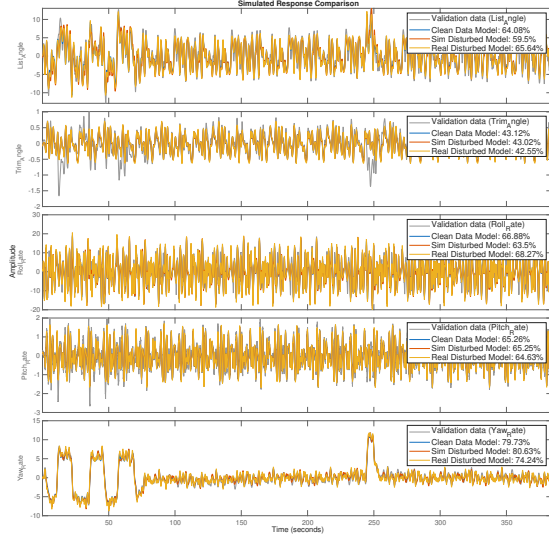


Figure C.1: Simulation performance of models train on 10 kt data with different disturbances

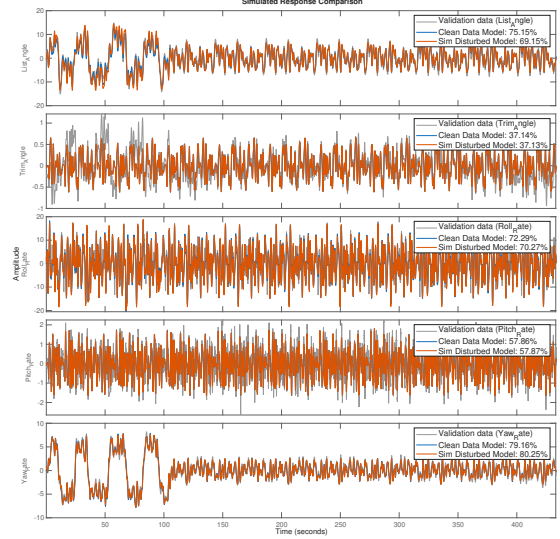


Figure C.2: Simulation performance of models train on 15 kt data with different disturbances

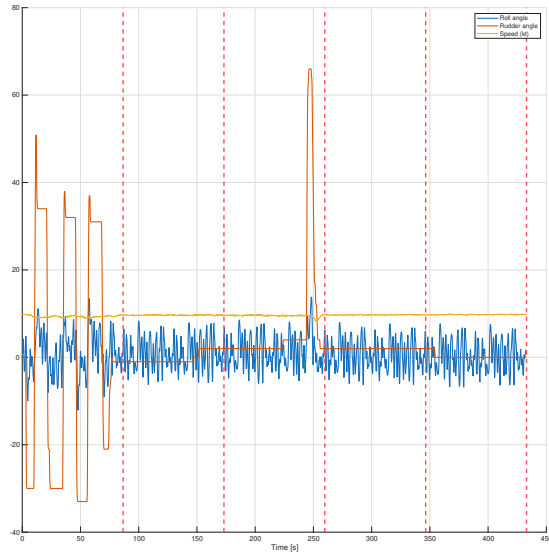


Figure C.3: Roll angle, rudder angle and speed (kt), for the 10 knot dataset. The vertical red dashed lines represent the end of each period.

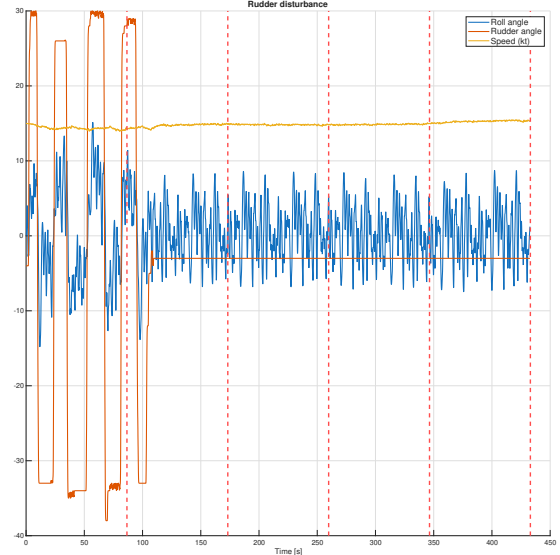


Figure C.4: Roll angle, rudder angle and speed (kt), for the 15 knot dataset. The vertical red dashed lines represent the end of each period.

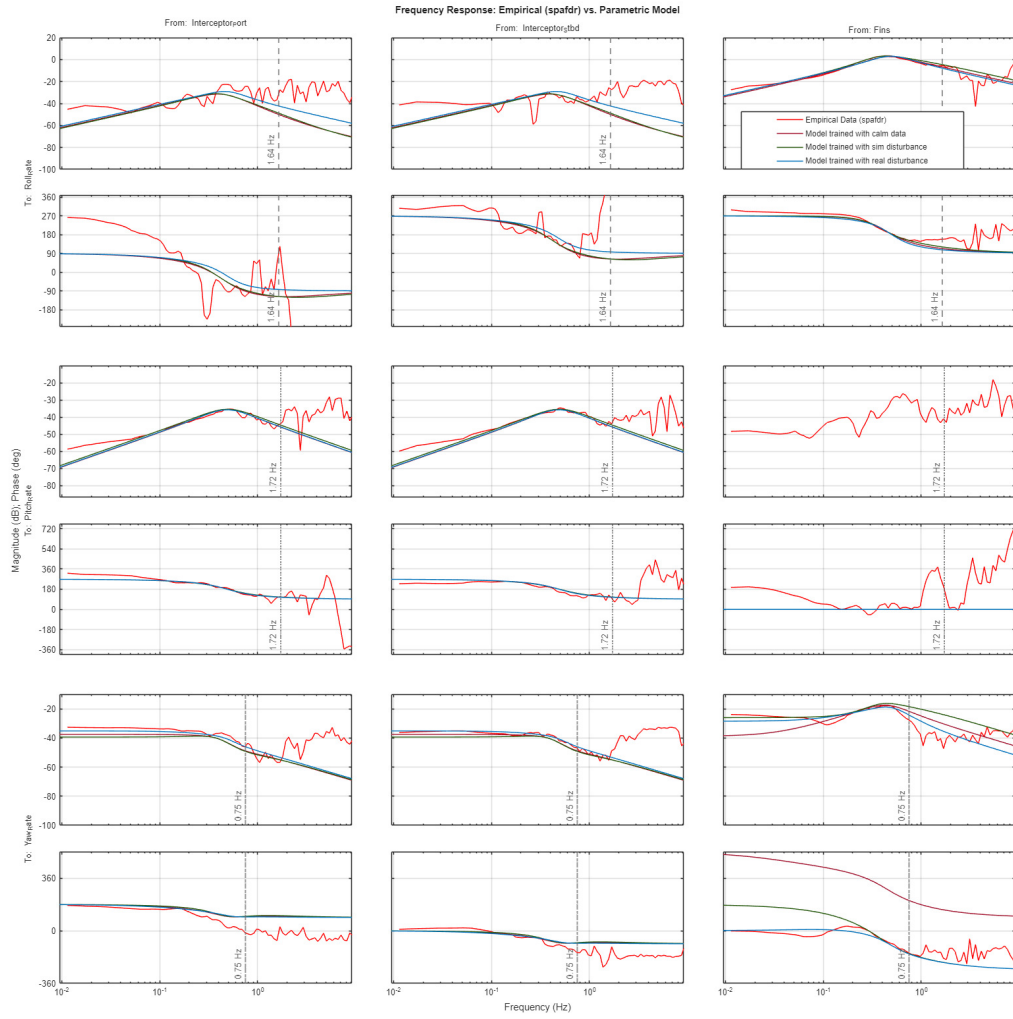


Figure C.5: SPAFDR compared with models trained on 10 kt datasets with different disturbances.

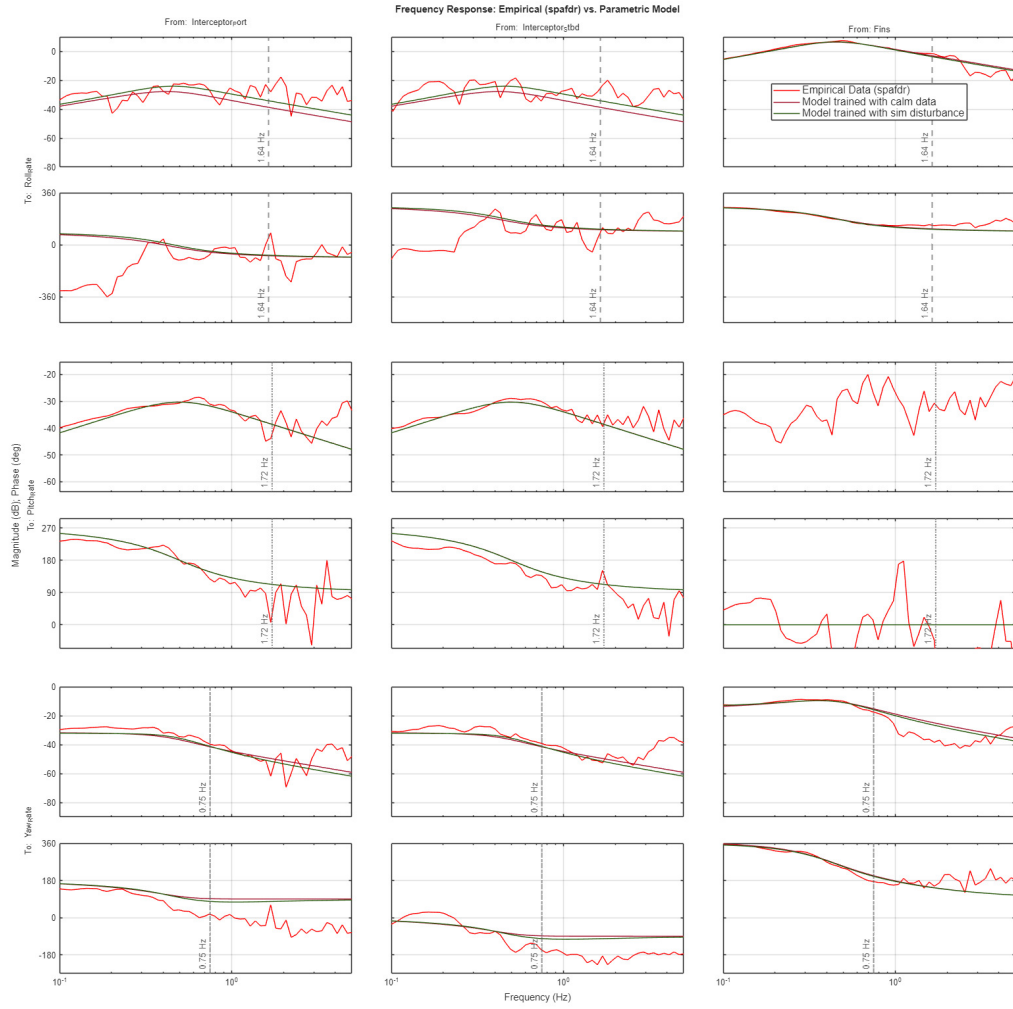


Figure C.6: SPAFDR compared with models trained on 15 kt datasets with different disturbances.

D

Extended Model Validation Plots

This appendix provides the complete, expanded visual data for the model validation discussed in Chapter 5.

D.1 Full Parameter Convergence

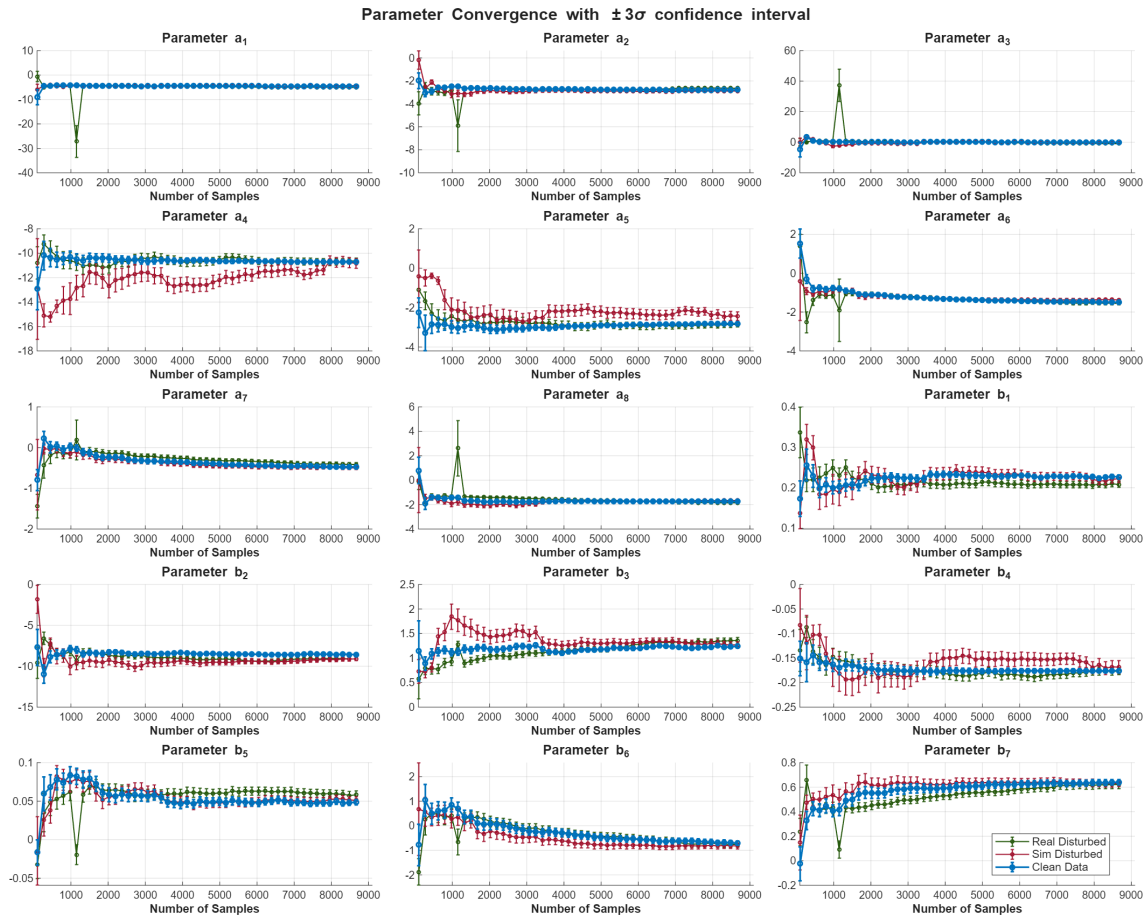


Figure D.1: Convergence plots for all identified system parameters over increasing amounts of training data, complete with $\pm 3\sigma$ confidence intervals. Refer to Section 5.3.1.

D.2 Complete Frequency Response Matrix

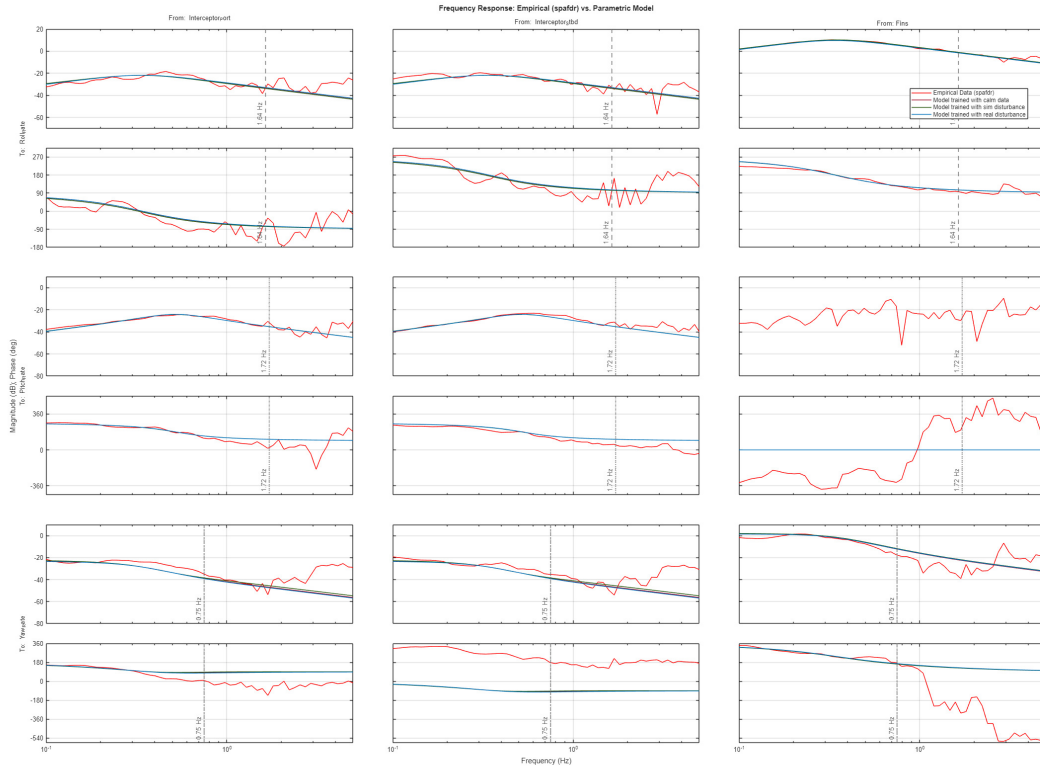


Figure D.2: Frequency response matrix comparing the grey-box models to the empirical SPAFDR data (red) for the vessel's angular rates. The upper limit of the relevant frequency bands determined in Section 3.5 are marked with a dashed black line (1.64 Hz for roll rate, 1.72 Hz for pitch rate and 0.75 Hz for yaw rate). This figure illustrates that the model across different disturbances accurately captures the true vessel dynamics. Refer to Section 5.3.2

D.3 Full Amplitude Dependence Matrix

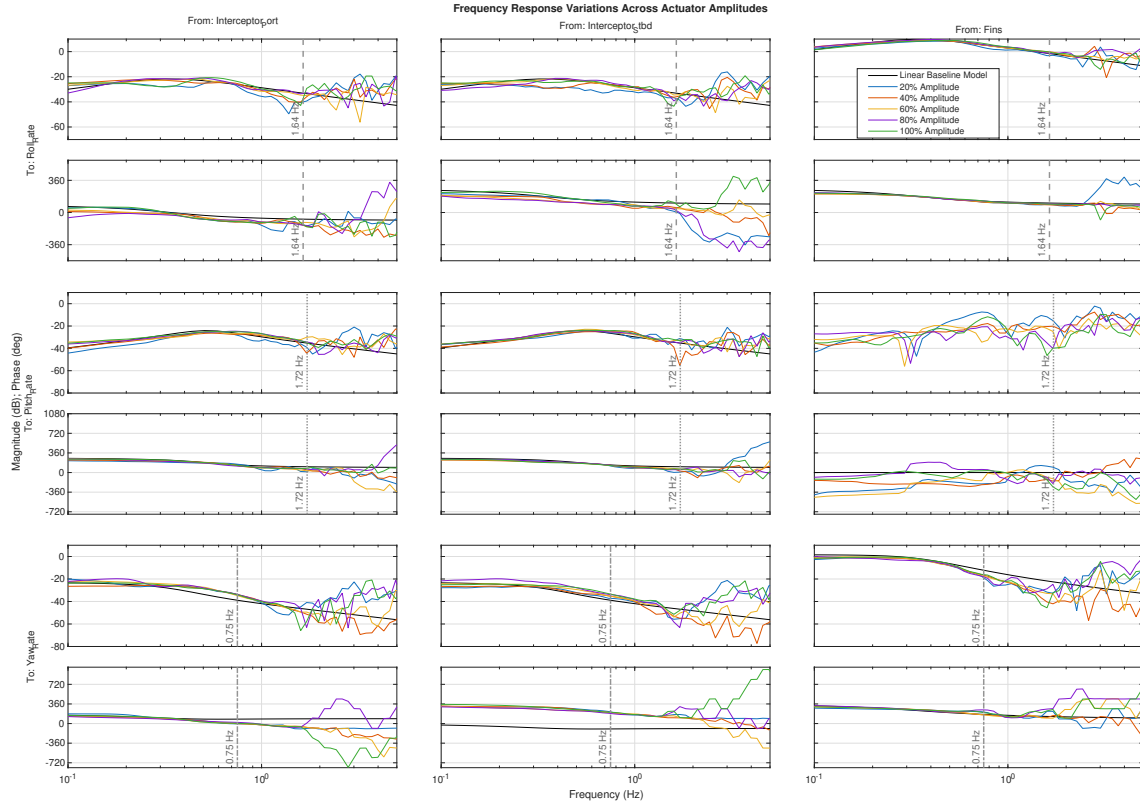


Figure D.3: Frequency response variations across different amplitudes compared against the linear baseline model. This figure indicates that amplitude does not significantly shift the frequency dynamics across the operational bandwidth indicating a linear amplitude dependence. Refer to Section 5.3.5

E

Extended Recursive Validation Plots

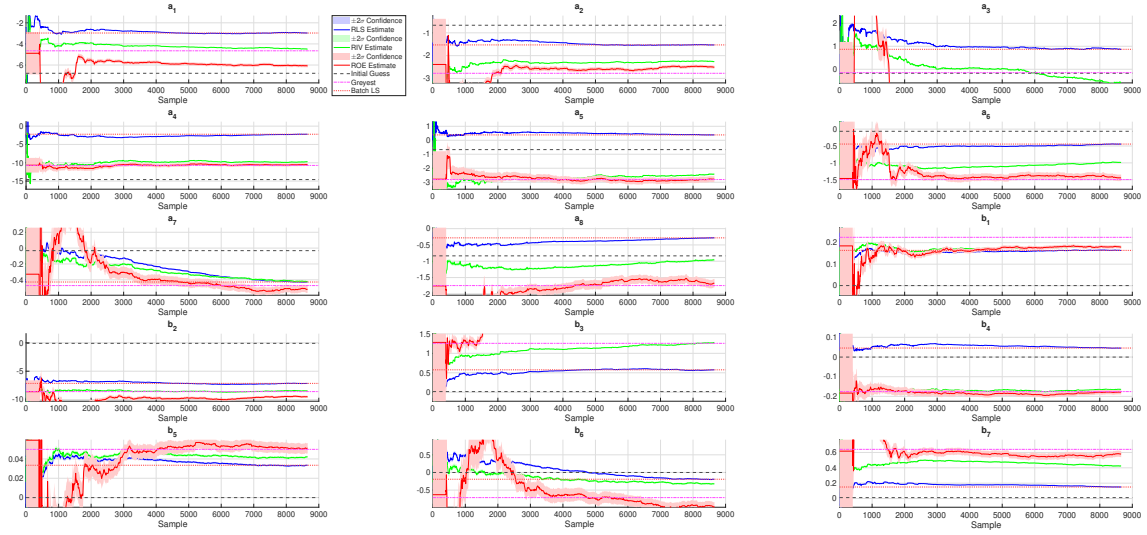


Figure E.1: Parameter convergence for the recursive algorithms illustrates that the parameters of ROE and RIVSVF converge to different solutions. Refer to Section 6.3

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