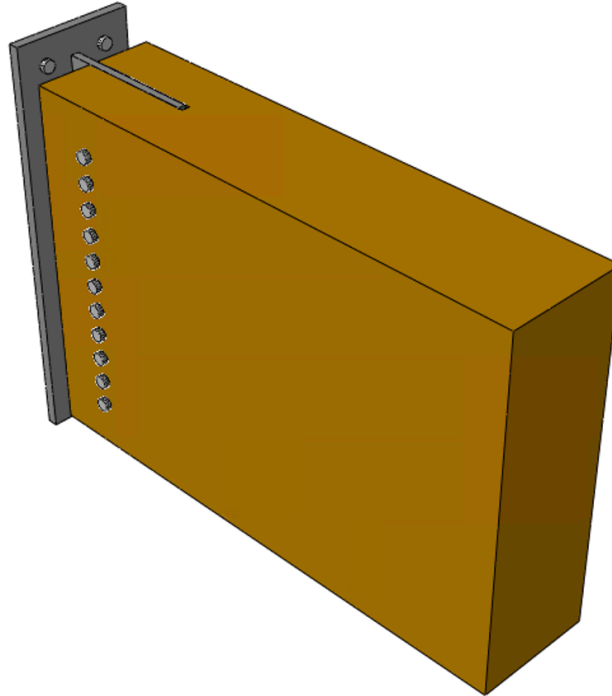




CHALMERS
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Elastic Rotational Stiffness of an Embedded Steel Plate-Timber Connection

A Simplified Solution, Based on an Analytical Study Verified by Numerical Modeling

Master's thesis in Master's Programme Structural Engineering and Building Technology

VICTOR GUSTAFSSON AND KARAM KATERGI

DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
DIVISION OF STRUCTURAL ENGINEERING

CHALMERS UNIVERSITY OF TECHNOLOGY
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ABSTRACT

Connections are among the most complex elements to analyze and design within structural engineering. They consist of multiple interacting components, involve intricate load-transfer mechanisms, and exhibit nonlinear behavior that is difficult to capture using simple assumptions.

Although numerical modeling can be employed to evaluate connection behavior and stiffness, it is often computationally demanding and requires careful verification and interpretation. Consequently, connections are frequently idealized in the design phase as either perfectly pinned or fully fixed, and in some cases, assigned an intermediate stiffness. While such simplifications may be acceptable for simple structural systems, they offer limited insight into the governing parameters that influence connection stiffness. These assumptions restrict the ability to optimize connection design and may be insufficient for structures with demanding performance requirements, such as high-rise hybrid buildings.

The objective of this Master's thesis is to develop a realistic analytical model for the internal parts of a steel-timber connection and the total rotational stiffness in the elastic state. The study considers an embedded steel plate connection, which is widely used due to its aesthetic appeal and ease of on-site installation.

To achieve this objective, load paths and structural models are identified for internal parts in connection. By summing the resulting elastic deformations of each part, the total deformation and rotational response are obtained. The analytical results are subsequently validated and calibrated against numerical simulations, including a 3D solid model and a 2D beam model. Based on this process, a simplified and verified expression for estimating the rotational stiffness of the embedded steel plate connection is derived.

Keywords: embedded steel plate connections, slotted in steel plate-timber connection, elastic rotational stiffness, timber-concrete connection, connection rigidity.

Elastisk rotationsstyvhet hos en insnittsblad stålplåt-träförbindning
En förenklad lösning, baserad på en analytisk studie verifierad med numerisk modellering

Examensarbete inom masterprogrammet Konstruktionsteknik och Byggnadsteknologi

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SAMMANFATTNING

Förbindningar är bland de mest komplexa elementen att analysera och designa inom konstruktionsteknik. De består av flera interagerande komponenter, involverar invecklade lastöverföringsmekanismer och uppvisar ickelinjärt beteende som är svårt att fånga med hjälp av enkla antaganden.

Även om numerisk modellering kan användas för att utvärdera förbindningsbeteende och styvhet, är det ofta beräkningskrävande och kräver noggrann verifiering. Följaktligen idealiseras förbindningar ofta i designfasen som antingen fritt upplagda eller helt inspända, och i vissa fall tilldelas de en mellanliggande styvhet. Även om sådana förenklningar kan vara acceptabla för enkla strukturella system, ger de begränsad insikt i de styrande parametrarna som påverkar förbindningsstyvheten. Dessa antaganden begränsar möjligheten att optimera förbindningens design och kan vara otillräckliga för strukturer med krävande prestandakrav, såsom höga hybridkonstruktioner.

Syftet med denna masteruppsats är att utveckla en realistisk analytisk modell för de interna delarna av en stål-träförbindning och den totala rotationsstyvheten i elastiskt tillstånd. Studien behandlar en insnittsblad stålknivplåtsförbindning, som används flitigt på grund av dess estetiska tilltalande och enkla installation på plats.

För att uppnå detta mål identifieras lastbanor och strukturella modeller för interna delar i sammankoppling. Genom att summera de resulterande elastiska deformationerna för varje del erhålls den totala deformationen och rotationsresponsen. De analytiska resultaten valideras och kalibreras därefter mot numeriska simuleringar, inklusive en 3D-solidmodell och en 2D-balkmodell. Baserat på denna process härleds ett förenklat och verifierat uttryck för att uppskatta rotationsstyvheten hos den insnittsblade stålknivplåtsförbindningen.

Nyckelord: Insnittsblade plåtförband, elastisk rotationsstyvhet, förband mellan trä och betong, förbandsstyvhet, hybrida konstruktioner

Preface

This Master's thesis was conducted in collaboration between the Department of Architecture and Civil Engineering, Division of Structural Engineering, at Chalmers University of Technology, and the engineering consultancy company COWI. The work was carried out between January and June 2026.

We would like to express our sincere gratitude to our supervisors at COWI, **Thomas Andersson and Adam Jonsson**, for their continuous support and for sharing their expertise throughout the project. We are also grateful to our examiner, **Mahbube Subhani**, for his dedicated guidance and valuable support. Special thanks are extended to Professor **Muhammad Al-Emrani**, whose expert guidance and constructive feedback were instrumental in this thesis. Finally, we thank the entire team at **COWI** for providing a collaborative and inspiring environment.

Gothenburg, June 2026

Victor Gustafsson and Karam Katergi

Notations

Roman upper case letters

E	Young's modulus of elasticity
EI	Flexural rigidity
F	Force
F_b	Bolt force including prying action
F_t	Applied tensile force
$F_{v,Rk}$	Characteristic shear resistance
$F_{ax,Rk}$	Characteristic axial withdrawal resistance
G	Shear modulus
K	Rotational stiffness/spring stiffness
I	Moment of inertia
M	Bending moment
$M_{y,Rk}$	Characteristic yield moment
Q	Prying force
S_j	Rotational stiffness of the joint
$S_{j,ini}$	Initial rotational stiffness

Roman lower case letters

d	Diameter of bolt/screw/dowel
f	Stress or strength parameter
$f_{h,1,k}$	Characteristic embedment strength
h	Height
k_h	Size factor for timber
k_i	Stiffness coefficient for joint component i
k_{mod}	Modification factor for load duration and service class
k_s	Foundation stiffness modulus
t	Thickness
t_1	Thickness of timber member
z	Lever arm

Greek letters

β	relative stiffness factor
μ	Stiffness ratio

ν	Poisson's ratio
θ	Rotation
σ	Stress
ε	Strain

Acronyms

ABAQUS	Finite Element Analysis software
AI	Artificial Intelligence
BCs	Boundary Conditions
CO ₂	Carbon Dioxide
EC	Eurocode
EWP	Engineered Wood Product
FEM	Finite Element Method
FEM-Design	Finite Element Analysis software
GL30c	Strength class for glulam timber
N.A	Neutral Axis
RH	Relative Humidity
RP	Reference Point
S235	Structural steel grade S235
S355	Structural steel grade S355
ULS	Ultimate Limit State

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1

Introduction

This chapter introduces the background of the research field and provides a detailed description of the problem under investigation. It also presents the purpose and objectives of the study, along with its scope and limitations. Finally, an overview of the proposed methodology is provided.

1.1 Background

Global warming is a global challenge that demands more sustainable solutions across all sectors. The construction industry is a major contributor to greenhouse gas emissions, as well as to the use of materials and land. In Europe, the sector accounts for roughly 30% of total CO₂ emissions, consumes about 40% of the energy, and generates around 40% of the waste (Swedish Wood, 2026b). At the same time, urbanization is increasing, with more people moving to larger cities (SCB, 2015). To accommodate this growing population, cities must construct additional housing and infrastructure. Building taller structures can help meet this demand while limiting the need to reduce land allocated for recreation and other urban functions.

Timber is more environmentally friendly than concrete and steel. However, historical regulations have been a significant obstacle to its use in taller structures across Europe. To encourage the adoption of timber over more carbon-intensive materials, various programs and regulatory changes have been introduced (Swedish Wood, 2026b). Timber offers several advantages due to its high strength-to-weight ratio compared to concrete and its ease of use, making it suitable for a wide range of applications. At the same time, designing tall buildings using only timber can be challenging, particularly when trying to meet requirements for wind resistance and vibration control. As a result, combining timber with concrete in the structural system has become increasingly common.

Hybrid structures combine different materials so each can be used to its strengths. This improves efficiency and can reduce overall material consumption. Such structures exist at both small and large scales. On a smaller scale, different materials are typically combined in details like joints and connections. On a larger scale, hybrid structures refer to the use of multiple materials within a building's main structural system. In large-scale hybrid systems involving timber, it is common to pair timber elements with a concrete core. The concrete core provides stiffness to resist wind loads and control vibrations, benefiting from its higher self-weight. Meanwhile, timber is used for columns and beams to transfer vertical loads to the foundation. This approach makes it possible to use timber in the primary structural system of taller buildings.

1.2 Problem description

Hybrid structural systems offer several advantages. However, the interaction between different materials also introduces significant challenges. In tall timber–concrete hybrid structures, additional complexities arise due to the differing mechanical behaviour of the materials. These complexities are not fully accounted for in current design processes, which may lead to an increased risk of insufficient structural performance.

In current engineering practice, structural connections are commonly idealized as either fully pinned or fully fixed. Previous research has shown that this simplification is not always valid. A master’s thesis by Larsson and Widén (2025) demonstrated through detailed analysis that embedded steel connections exhibit semi-rigid behaviour rather than perfectly pinned behaviour. Their study further indicated that, in tall hybrid timber–concrete structures, this inherent rigidity can lead to high stresses in the connection caused by unintended bending moments. These moments arise from differential vertical deflections between the concrete core and the timber facade columns. Such effects are typically neglected in conventional design approaches and may result in failure of the connection or of adjacent structural elements. Figure 1.1 showcases the differential deformation and the settlement in the horizontal element connecting timber columns with a concrete core.

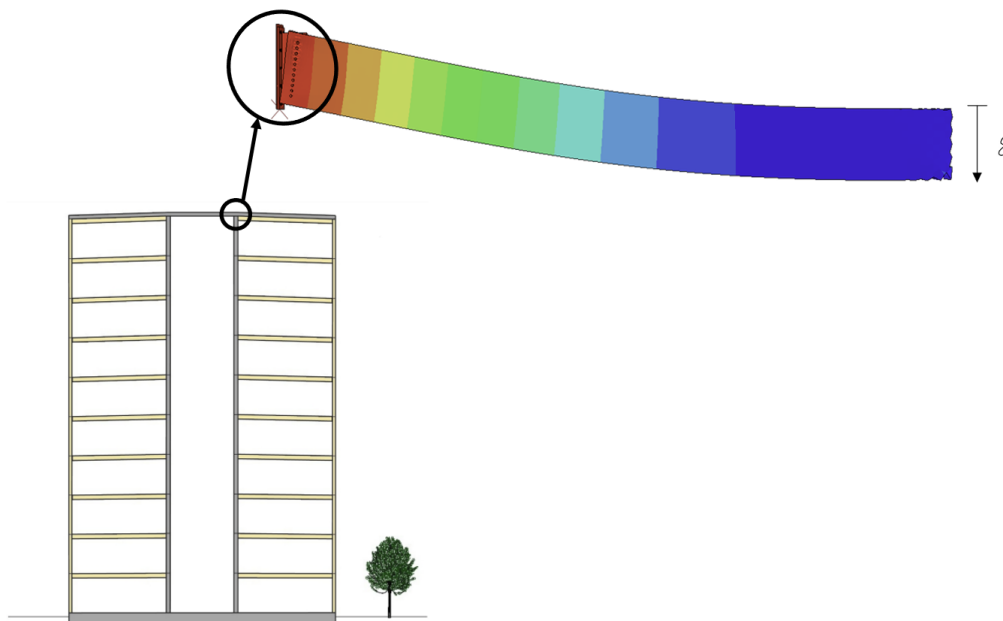


Figure 1.1: Settlement of timber beams connecting timber column with concrete core.

According to Dahlqvist and Kollberg (2023), the differential deformation would reach 20 mm for a 10-storey building and 35 mm for a 20-storey hybrid concrete-timber building as per Figure 1.2.

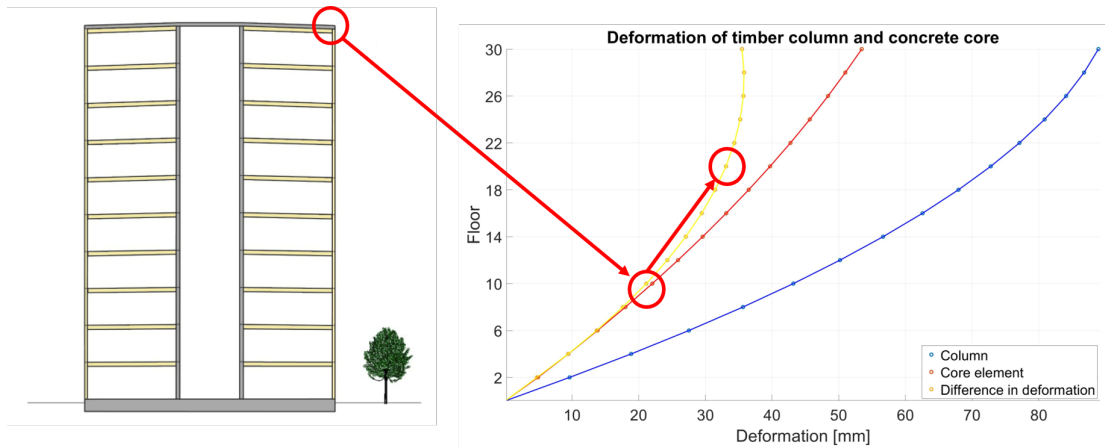


Figure 1.2: The long-term differential deformation for high-rise timber-concrete structures.

Currently, no readily available analytical approach exists for estimating rotational stiffness in connections exhibiting intermediate behavior between pinned and fixed conditions. Although numerical methods can be employed to address this problem, they are generally complex, computationally demanding, and time-consuming to implement. Hence, there is a strong need for a simplified analytical approach that enables practical consideration of actual rotational stiffness in connection design.

1.3 Aim and objectives

The primary aim of this master's thesis is to develop an analytical model for structural connections that accounts for the effects of unintended rotation, verified by numerical modelling, and to contribute to a deeper understanding of the behaviour of such connections. To achieve this aim, a selected connection between a timber beam and a concrete core will be analysed in detail.

To support this aim, the following specific objectives are defined:

- Calculate the connection's rotational stiffness analytically through a model that accounts for the contributions of individual components.
- Verify the analytical model through a parametric study, comparing with numerical results from ABAQUS.
- Identify the parameters that influence the rotational stiffness the most and propose a simplified expression for the analytical model.
- Determine the rigidity of the connection and classify its behaviour as pinned, semi-rigid, or rigid based on the rigidity.

1.4 Scope and limitations

This thesis examines connections in hybrid structural systems composed of a concrete core and timber elements. The objective is to develop an analytical model capable of evaluating the rotational stiffness of these connections. To achieve this, several limitations have been defined for both the overall scope and the numerical modelling.

General scope limitations

- The analysis is restricted to the linear-elastic range; plastic deformation and force redistribution are not considered.
- Environmental and external influences, such as moisture variation, dynamic loading, and temperature effects, are excluded.
- The study focuses specifically on an embedded steel plate connection between a timber beam and a concrete core.
- The results are not intended for direct design application but rather to provide conceptual insight into how rotational stiffness in hybrid connections can be assessed and which parameters most significantly influence it.

Numerical modelling limitations

- Only the steel connection components and the timber beam are explicitly modelled. The concrete core is represented as a rigid surface.
- Construction tolerances for screw and bolt holes are neglected to avoid introducing unintended displacements in the analysis.
- Contact interactions between components are modelled to behave linearly, preventing any overlap between parts. Consequently, friction is not considered in the model.
- No capacity verification is carried out, as linear-elastic behaviour is assumed. Instead, results are extracted and evaluated manually.

1.5 Methodology

The methodology of this thesis is divided into several stages. The first stage consists of a comprehensive literature review, covering relevant standards and design guidelines, as well as previous research on the design and structural behaviour of connections in hybrid concrete–timber structures. This stage also includes a review of material properties.

The second stage involves numerical modelling and analysis of the selected embedded steel-plate connection. These analyses are used to evaluate the stiffness of the connection and to quantify the influence of individual connection components on the overall stiffness response. Also in the second stage, an analytical model to evaluate the rotational stiffness is developed using the component method.

In the final stage, a parametric study is conducted to verify the analytical model, so it accurately represents the structural behaviour of the connection. If the analytical model does not match the numerical results, it is to be refined until it does.

1.6 Use of AI

Artificial intelligence (AI) has been used as a supportive tool throughout this master's thesis. It has primarily assisted with correcting grammar and spelling errors, as well as generating code for specific LaTeX commands to improve the overall quality of the report. AI tools were also consulted when encountering errors in ABAQUS, helping to clarify issues and suggest possible solutions.

2

Literature review

This chapter begins with a brief explanation of the material properties relevant to timber-to-concrete connections and a brief introduction to engineered wood products. It then presents common types of connections between a timber beam and a concrete core. The chapter continues with a review of previous research and design codes related to the stiffness of an embedded plate connection. In the final part, a basic overview of prying action and beams resting on an elastic foundation is provided.

2.1 Material properties

Timber, concrete, and steel each have distinct material properties that make them unique. When combining these materials in a hybrid structure, it is essential to understand their differences in order to prevent challenges and to select the appropriate material for each structural purpose. The material properties presented in this text are those considered most relevant for the design and performance of timber-to-concrete connections.

2.1.1 Timber

Historically, wood has been one of the most important construction materials (Swedish wood, 2026a). In the past, its use was largely limited to smaller buildings due to regulatory constraints and competition from alternative materials. However, during the 21st century, regulatory changes have been introduced to promote the increased use of wood in the construction sector, as it is a more sustainable and the only renewable structural material (Swedish wood, 2026b).

Timber is an anisotropic material, exhibiting mechanical properties that vary with direction (Swedish wood, 2022a). Conventionally, three principal directions are distinguished: longitudinal, radial, and tangential (Figure 2.1). The variation in mechanical properties for the radial and tangential directions is typically small and often considered collectively. Consequently, for most structural analyses, timber behavior is described in terms of two primary categories: parallel to the grain (longitudinal) and perpendicular to the grain (radial–tangential plane).

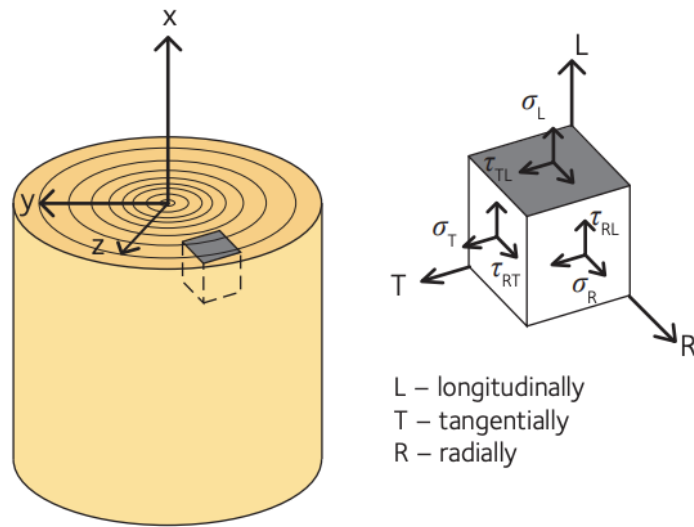


Figure 2.1: Structural directions of timber (Swedish wood, 2022a)

Timber exhibits markedly higher strength and stiffness when loaded parallel to the grain than when loaded perpendicular to the grain (Swedish wood, 2022a). This anisotropic behavior is illustrated in Figure 2.2, which presents the stress–strain relationships for clear wood subjected to tension load both parallel and perpendicular to the grain. The figure further demonstrates that timber fails in a brittle manner under tensile loading, as the stress–strain curve does not display a plastic plateau at peak stress but instead terminates shortly after the linear elastic region.

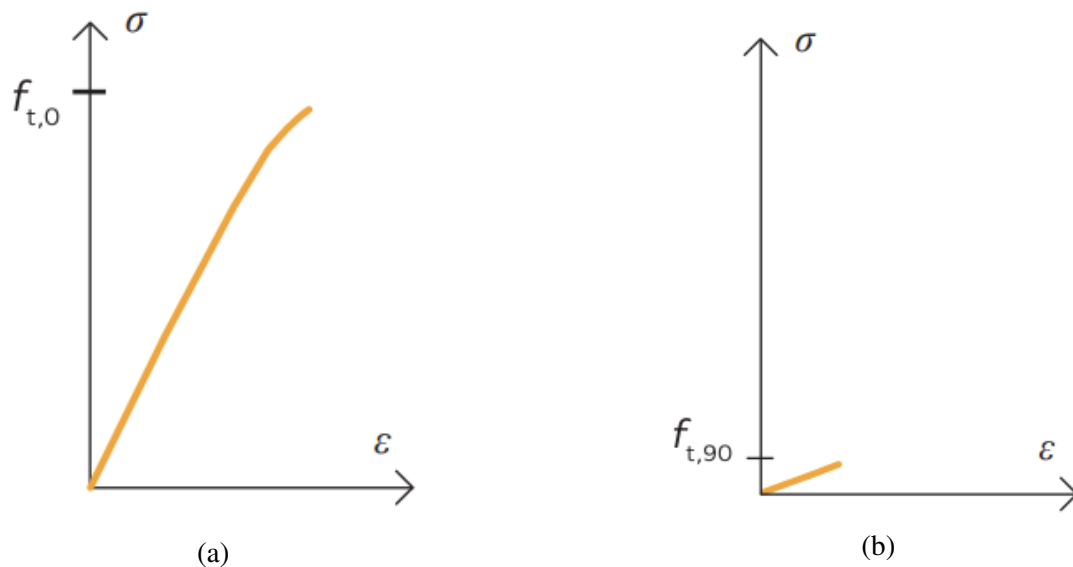


Figure 2.2: Stress-strain relationship for clear wood loaded in tension, a) parallel to the grain, b) perpendicular to the grain (Swedish wood, 2022a).

Similar to tensile loading, timber subjected to compression exhibits higher strength and stiffness when loaded parallel to the grain than when loaded perpendicular to the grain (Swedish wood, 2022a), although the strength ratio is less pronounced than in tension.

Figure 2.3 presents the stress–strain relationships for clear wood loaded in compression, both parallel and perpendicular to the grain. In contrast to tensile loading, timber displays more ductile behavior in compression. As shown in the figure, the stress–strain curve reaches a maximum stress followed by a plateau, indicating significant plastic deformation prior to ultimate failure.

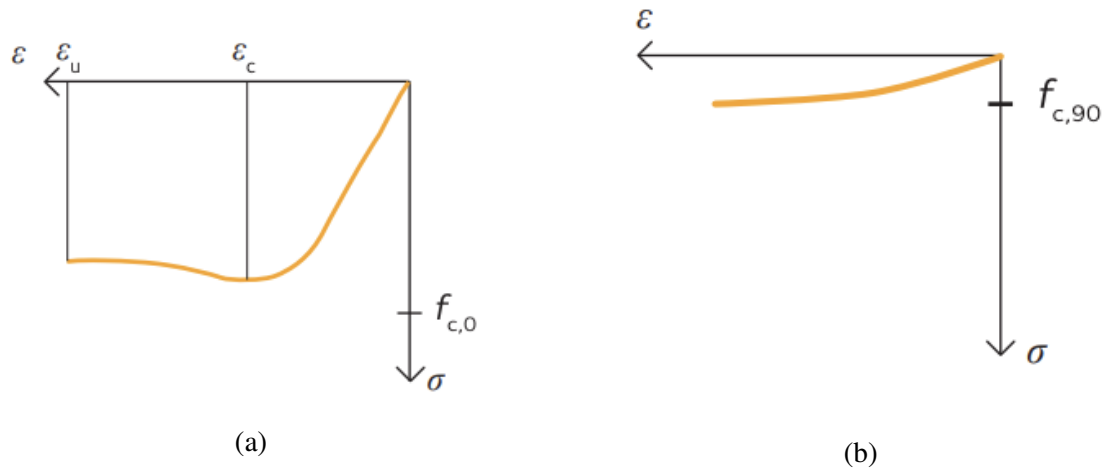


Figure 2.3: Stress-strain relationship for clear wood in loaded in compression, a) parallel to the grain b) perpendicular to the grain.(Swedish wood, 2022a)

The strength of timber is influenced not only by the orientation of the load relative to the grain but also by several other factors, including moisture content, temperature, duration of loading, inherent imperfections, and member size (Swedish wood, 2022a). Increased moisture content generally reduces both the stiffness and strength of timber. In addition, higher moisture content causes swelling, whereas lower moisture content leads to shrinkage. The surrounding environmental conditions significantly affect the timber’s moisture content. This effect is addressed by assigning structural elements to a service class, which reflects the highest moisture content the element is expected to experience during its service life. Then, reducing the design strength with a factor depending on the service class (Eurocode 5, 2004a).

Temperature and time also affect timber strength. Both stiffness and strength decrease as temperature rises (Swedish wood, 2022a), but in the range -30°C to 95°C , this effect is relatively minor and usually not included in the design process. In terms of time, longer load durations reduce the timber’s strength. In design, the effect of load duration is combined with the service class into a single factor k_{mod} (Eurocode 5, 2004a).

Large timber members do not contain perfectly straight fibres and typically include natural imperfections, such as knots, which act as local weaknesses and induce stress concentrations (Swedish wood, 2022a). Consequently, the mechanical behaviour of clear wood cannot be directly applied to structural-sized sawn timber elements. Larger members are more likely to contain such defects, and according to the weakest-link theory, their characteristic strength is therefore reduced compared to smaller specimens. The influence of member size is accounted for by the size factor k_h , which increases the strength for smaller cross-sections (Eurocode 5, 2004a).

2.1.2 Steel

Steel is one of the most widely used structural materials due to its high strength, ductility, and reliability. It is applied in a wide range of structures, from buildings and bridges to industrial facilities. Steel enables slender structural elements and long spans. Its predictable mechanical behavior makes it well-suited for engineered design (Al-Emrani et al., 2013). Steel has notable environmental impacts, contributing to pollution and accounting for up to 11% of global CO_2 emissions. The carbon footprint of steel largely depends on the production method used, making the manufacturing process the most significant factor influencing its overall environmental impact (Arbor, 2025).

From a material perspective, steel is characterized by high strength in both tension and compression. For structural steel, the yield strength typically governs design and commonly ranges from about 235 MPa to 460 MPa depending on the steel grade (Al-Emrani et al., 2013). The compressive strength of steel is essentially equal to its tensile strength, as it is an isotropic and homogeneous material. However, for compression members, instability phenomena such as buckling can govern the design rather than material failure.

The stress–strain relationship for structural steel is typically idealized as elastic–plastic. Up until the yield strength, steel behaves linearly elastic according to Hooke’s law, with a modulus of elasticity E of approximately 210 GPa. Beyond the yield point, plastic deformation occurs, allowing significant strains at nearly constant stress before strain hardening and eventual fracture (Al-Emrani et al., 2013). The stress-strain relationship for structural steel of S235 is shown in Figure 2.4.

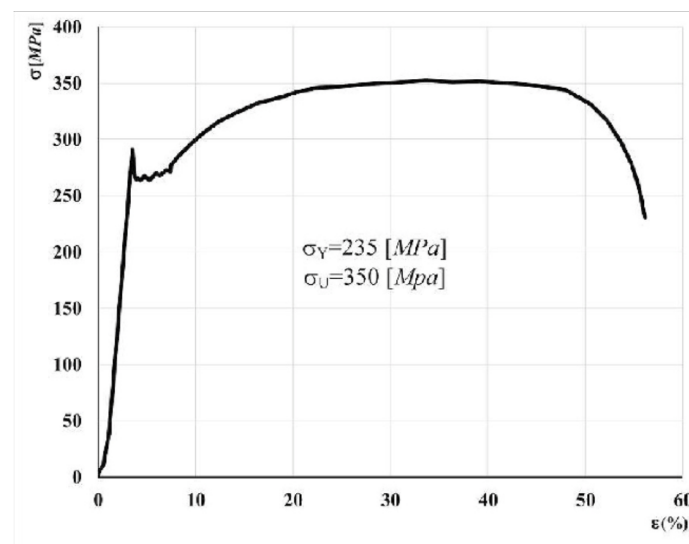


Figure 2.4: Stress-strain relationship for structural steel S235, (Moaieri & Khedmati, 2021)

For fasteners such as bolts and screws, higher-strength steel is typically used (Al-Emrani et al., 2011). Structural bolts are often produced in high-strength classes with an ultimate tensile strength of around 800 MPa or higher. This allows them to transfer significant tensile and shear forces in connections, making them essential components in steel and timber–steel joints.

2.1.3 Concrete

Another commonly used structural material is concrete. It is a composite material made primarily of cement, water, and aggregates. Its main strength lies in its high compressive capacity, durability, and ability to be cast into almost any shape (Al-Emrani et al., 2013). Concrete performs very well under compression, which makes it ideal for columns, foundations, and slabs. However, its tensile strength is relatively low, so it is commonly reinforced with steel to form reinforced concrete.

Despite its technical advantages, concrete has significant negative environmental impacts. The production of cement is energy-intensive and generates large amounts of carbon dioxide CO₂ (Cheng et al., 2023). The calcination of limestone and the high-temperature kiln processes contribute substantially to global greenhouse gas emissions. In addition, concrete production consumes large quantities of natural resources such as sand, gravel, and water. This can also lead to habitat destruction and landscape degradation. The high impact on the environment results in a strong demand to use concrete efficiently in design.

Concrete undergoes time-dependent deformation due to creep and shrinkage (Engström, 2014). Both phenomena are influenced by environmental conditions and evolve over time. However, a key distinction is that creep depends on loading conditions, whereas shrinkage occurs independently of applied loads.

Shrinkage is composed of both drying shrinkage and autogenous shrinkage (Engström, 2014). Drying shrinkage results from the loss of moisture from hardened concrete to the surrounding environment. As water evaporates from the pores, volume reduction occurs. This type of shrinkage depends mainly on the relative humidity, the notional size of the member, curing conditions, and time. It develops gradually over a long period and is strongly influenced by environmental exposure. Drying shrinkage does not occur homogeneously through the specimen since most of the drying happens at the surface.

Autogenous shrinkage is caused by the chemical process of cement hydration (Engström, 2014). As hydration progresses, internal self-desiccation occurs, leading to volume reduction even without moisture exchange with the environment. This effect is particularly significant in high-strength concretes with low water–cement ratios and develops mainly at early ages. The total shrinkage, according to Eurocode 2 (2004a) the sum of the autogenous and drying shrinkage.

Creep is load-dependent and does not occur in the absence of sustained loading (Engström, 2014). Unlike elastic deformation, which occurs immediately upon load application, creep develops gradually over time and continues as long as the load is maintained. The effect is more pronounced at early ages, particularly when the load is applied soon after casting. Relative humidity (RH) also plays a significant role. Lower RH accelerates creep because increased moisture loss from the concrete is required to reach equilibrium with the surrounding environment.

2.2 Glulam

To mitigate the inherent variability and defects of solid timber and to enable construction with larger dimensions, a range of Engineered Wood Products (EWPs) has been developed (Swedish wood, 2022a). EWPs are manufactured from sawn timber, veneers, particles, or fibres bonded together using adhesives to form products with enhanced and more predictable mechanical properties. A commonly used EWP is glued laminated timber (glulam), which is the material used for the beam in this study.

Glued laminated timber is the oldest form of engineered wood product and remains one of the most widely used (Swedish wood, 2022a). It is manufactured by bonding multiple layers of sawn timber with their grain directions aligned parallel to one another, as illustrated in Figure 2.5. The thickness of each layer is typically 45 mm, and the beams generally have a maximum width of 215 mm. Wider beams can be achieved by gluing multiple glulam beams together. While glulam exhibits mechanical properties comparable to those of solid sawn timber in smaller cross-sections, its primary advantage lies in the ability to produce large structural members with more uniform and predictable properties. This is achieved by distributing natural timber imperfections throughout the member during manufacturing. Glulam behaves predominantly as a linear structural element and is therefore primarily used for beams and columns in load-bearing systems (Quesnel, 2020).

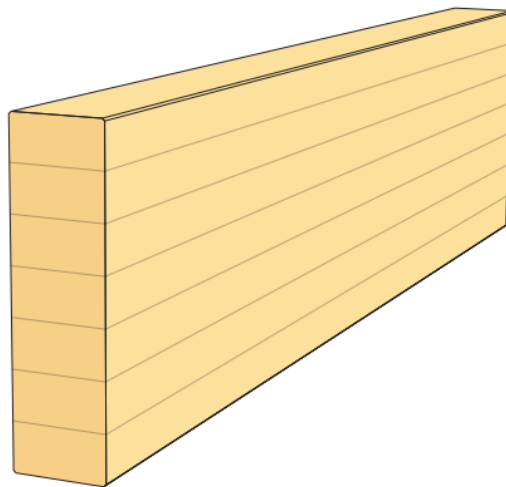


Figure 2.5: Glulam beam with seven layers of sawn timber, (Swedish wood, 2024)

2.3 Challenges in hybrid structures

Building a hybrid structure poses several technical challenges due to the different behaviors of its materials. A major concern is moisture interaction between timber and concrete (Swedish wood, 2022b). Timber is highly sensitive to changes in humidity and can swell when exposed to moisture, whereas concrete experiences drying shrinkage and creep as moisture leaves the material and moves into the surroundings. If not carefully detailed, freshly cast concrete can introduce additional moisture into adjacent timber elements, potentially causing swelling, deformation, or long-term durability issues. Therefore, controlling moisture transfer is essential in hybrid design. This can be achieved by incorporating barriers between timber and concrete and by allowing for material movement. Steel components are often used as an intermediate layer between concrete and timber to help prevent moisture transfer.

Another challenge arises from the structural interaction between materials with different stiffness and long-term behavior (Al-Emrani et al., 2013). Concrete and timber respond differently to loading and moisture. Concrete elements may undergo creep and shrinkage over time, whereas timber exhibits swelling and shrinkage that is influenced by moisture conditions. The difference in stiffness also affects the deflection of the different members. Timber is a more flexible material and will deflect more under the same load. In taller buildings, these differences can lead to uneven settlements or internal stresses due to restraint if not properly considered in the design. High stresses resulting from differential vertical deflections between timber and concrete were observed in Larsson and Widén's (2025) master's thesis, highlighting the significance of this issue in taller structures.

Materials behave very differently when exposed to fire. Concrete is non-combustible and heats up slowly due to its low thermal conductivity (Eurocode 2, 2004b). This allows the inner core to retain load-bearing capacity for a relatively long time. Although its strength reduces at high temperatures and spalling may occur, it generally provides good inherent fire resistance without additional protection. Timber is a combustible material, but it chars at a predictable rate (Eurocode 5, 2004b). The char is forming a protective layer that insulates the core and allows it to retain its structural capacity for a defined period. Failure, therefore, occurs gradually rather than suddenly. Steel is also non-combustible but heats up very quickly, and its strength and stiffness decrease significantly in high temperatures (Eurocode 3, 2004). Without fire protection, it can lose load-bearing capacity rapidly, so it usually requires insulation or coatings to achieve the required fire resistance.

This difference in fire response must be considered when designing a timber structure that incorporates steel components. By concealing the steel elements, they are shielded from direct exposure to a potential fire. This can be achieved by effectively embedding the steel components within timber elements or by applying additional fire protection measures.

2.4 Timber-concrete connections

Connections are essential in a structural system to ensure that loads are effectively transferred between members. A complete structure relies on various types of connections linking its different components. This study focuses on a beam-to-wall connection between a timber beam and a concrete core. The following section presents some of the most common connection types, together with their challenges and advantages.

2.4.1 Beam hanger

A common detail for connecting a timber beam to a concrete wall is a beam hanger. This connection transfers vertical loads from the beam into the wall (WoodWorks, 2024). Typically, the beam is seated in a steel hanger and secured with dowel-type fasteners. The hanger is welded to a steel plate, which is anchored to the concrete wall using dowels, see Figure 2.6a. As an alternative to a steel hanger, an embedded plate combined with an angle bracket may be used, see Figure 2.6b. The load-carrying capacity of these connections is often governed by the timber's strength perpendicular to the grain. However, the shear capacity of the dowels in the steel hanger or embedded plate must also be verified, along with the tensile capacity of the anchors embedded in the concrete. In addition, the effects of timber swelling and shrinkage should be considered in the design, as moisture-related movement can influence the connection's performance. Finally, if the beam is not placed centrally in the support, an eccentricity will occur and introducing additional moment in the connection.

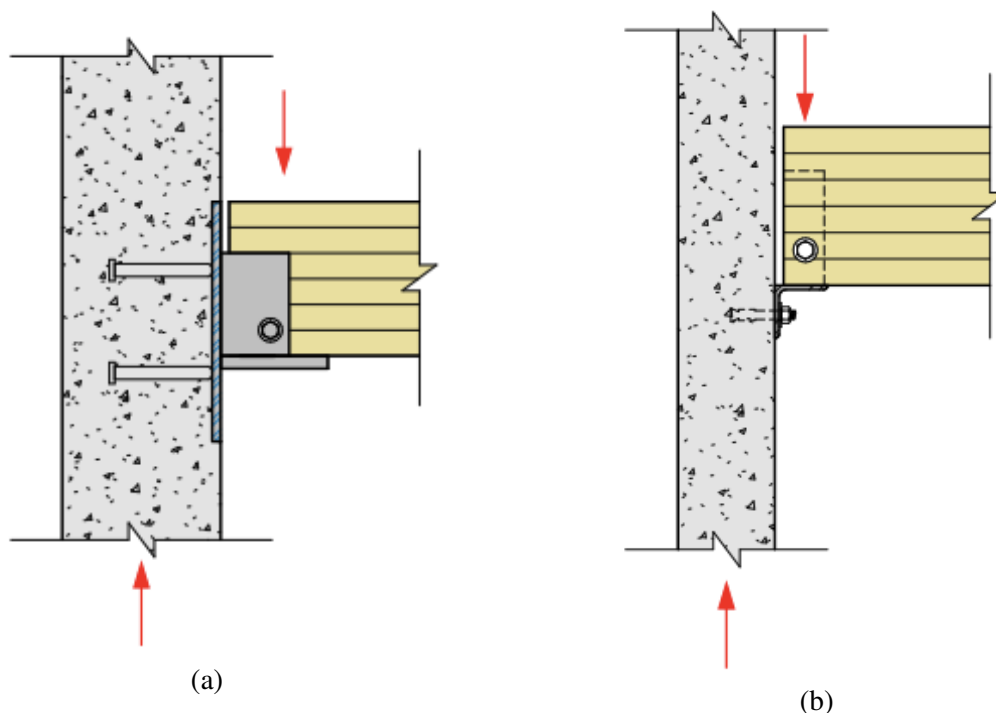


Figure 2.6: Connection of a beam perpendicular to the wall where the beam is connected with, a) Beam hanger b) embedded plate on angle bracket (WoodWorks, 2024).

To increase the capacity and redundancy of a simple beam hanger connection and to improve resistance to eccentric loading, the hanger can be provided with a top plate (WoodWorks, 2024). This plate is placed in a pocket in the concrete wall and secured with grout, see Figure 2.7. When designing the anchorage of the top plate, it is important to consider the risk of edge failure in the concrete wall, since limited edge distances can significantly reduce the anchorage capacity.

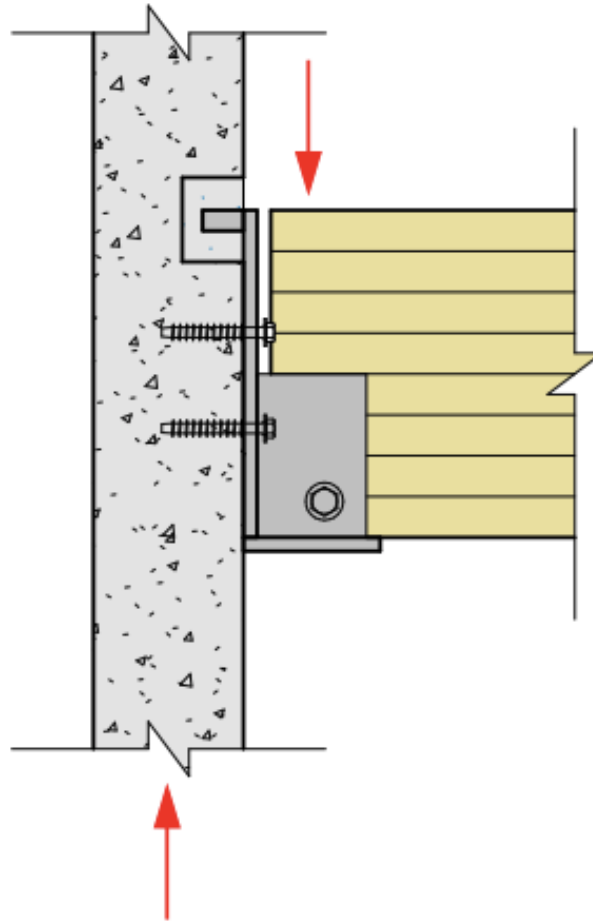


Figure 2.7: Beam hanger with top bearing, (WoodWorks, 2024)

2.4.2 Wall notch

The wall notch connection is designed to transfer vertical loads from a timber beam to a concrete wall. The concrete wall is cast with a rectangular recess, forming a pocket to receive the connection (WoodWorks, 2024). The timber beam is bolted either to an embedded plate or to side plates, which are welded to a bearing plate equipped with threaded studs. During installation, the beam–plate assembly is placed into the pocket in the concrete wall, with the threaded studs embedded vertically into the concrete. The gap between the bearing plate and the embedded plate is filled with grout to accommodate construction tolerances between the timber and concrete. Providing adequate tolerances is particularly important for this type of connection, as timber is sensitive to changes in moisture content and may swell or shrink over time. Without sufficient allowance for movement, restraint from the surrounding concrete could induce additional stresses in the timber or the connection.

This type of connection is capable of carrying high loads, with its capacity primarily governed by the compressive strength of the timber perpendicular to the grain. To ensure long-term durability and performance, it is essential to protect the portions of the timber in contact with the concrete against moisture ingress, thereby reducing the risk of swelling.

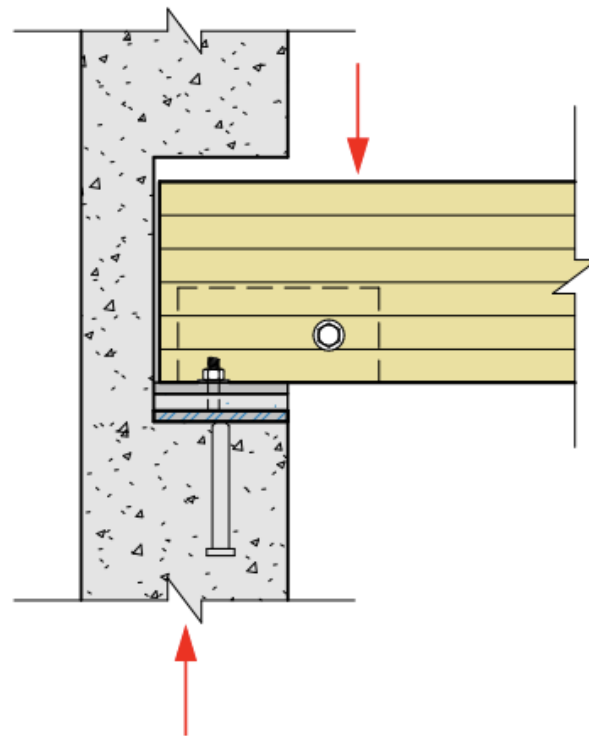


Figure 2.8: Wall notch connection (WoodWorks, 2024)

2.4.3 Embedded steel plate

In an embedded steel plate connection, the timber beam is bolted to a steel plate embedded within the beam, known as an embedded plate. This embedded plate is welded to a steel plate that is anchored to the concrete core with screws (WoodWorks, 2024). Together, these components form a connection capable of transferring the vertical loads from the beam to the concrete. The capacity of the connection depends on the shear strength of the fasteners and the compressive capacity of the timber around the fasteners.

The embedded plate offers the advantage of being protected by the surrounding timber in the event of a fire (Swedish wood, 2022a). The timber delays heat transfer, thereby preventing the steel plate from reaching critical temperatures for a prolonged period. If the embedded plate were replaced by two steel plates located on the exterior of the timber beam, additional fire-protection measures would be required to safeguard the connection. From an aesthetic standpoint, embedding the embedded plate within the beam is also beneficial, as only the timber remains visible.

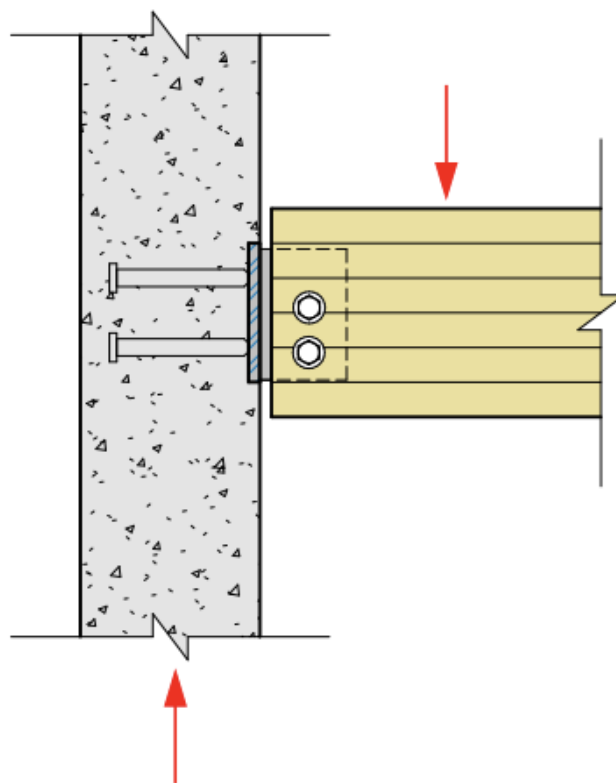


Figure 2.9: embedded plate connection (WoodWorks, 2024)

This study will further investigate this connection. It has been selected as it is a commonly used solution that, in addition to the previously mentioned benefits, also provides an aesthetically favorable appearance in structures with an exposed load-bearing system. Since the steel components are embedded within the timber beam, only the timber is visible in the final construction.

Rothoblaas is a well-established supplier of structural connections and fastening systems, offering a range of different embedded plate solutions. In their product documentation for embedded plate connections, only the shear capacity is specified (Rothblaas, 2022), meaning that these connections are generally considered fully pinned. As a result, the designer must verify that no unintended rotational effects occur in the structure that would generate bending moments in the connection. Otherwise, the connection might reach failure before the designed load. Figure 2.10 illustrates an embedded plate connection between concrete and timber, taken from the product sheet of a Rothoblaas (2022) embedded plate.

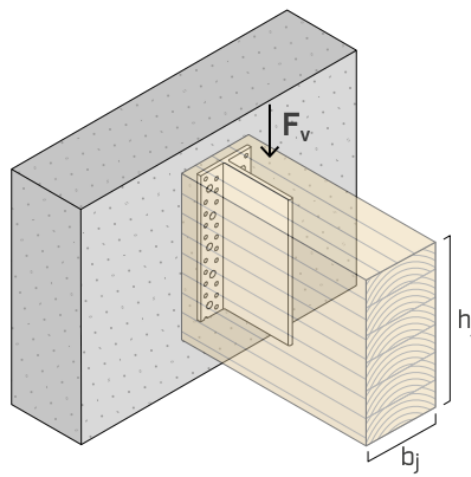


Figure 2.10: Embedded plate connection (Rothblaas, 2022)

2.5 Design of timber-steel connections

Today, in the design of timber connections, Johansen's theory is used (Swedish wood, 2022a). The theory includes three possible types of failure, with each failure type having multiple possible failure modes. The first possible failure mode occurs when the timber member reaches compression failure due to the pressure applied by the dowel, as shown in Figure 2.11 a, c, f, and j. The second failure mode is when the dowel yields and a plastic hinge forms in the stronger material, as illustrated in Figure 2.11 b, d, g, and k. The final failure mode occurs when plastic hinges form in both materials, meaning the dowel develops plastic hinges in both the steel and timber members, as shown in Figure 2.11 e, h, and m.

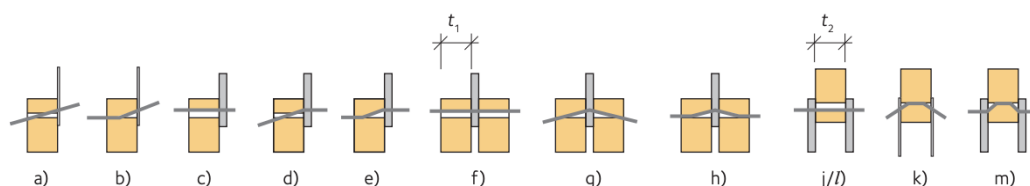


Figure 2.11: Failure modes for steel-timber connections (Swedish wood, 2022b)

Each failure mode is associated with a specific design equation, which depends on the connection configuration (Eurocode 5, 2004a). The capacity is then taken as the weakest of the failure modes. An embedded plate with dowels has the possible failure modes f, g, and h. These design equations are shown in 2.1.

$$F_{v,Rk} = \min \begin{cases} f_{h,1,k} t_1 d & \text{(f)} \\ f_{h,1,k} t_1 d \left[\sqrt{2 + \frac{4M_{y,Rk}}{f_{h,1,k} d t_2}} - 1 \right] + \frac{F_{ax,Rk}}{4} & \text{(g)} \\ 2.3 \sqrt{M_{y,Rk} f_{h,1,k} d + \frac{F_{ax,Rk}^2}{4}} & \text{(h)} \end{cases} \quad (2.1)$$

The design equations based on Johansen's theory provide a shear capacity for the connection from lateral loads. But if a moment is acting in the connection, this needs to be checked separately. In Eurocode 5 (2004a), a timber-steel connection subjected to moment is designed by transforming the applied bending moment into a force couple within the joint. The moment is resisted by tensile forces developing in one part of the fastener group and compressive forces in the opposite zone. The fasteners in tension are verified for axial resistance, including withdrawal capacity, head pull-through, and steel tensile strength where relevant. If shear is present simultaneously, the interaction between axial and shear forces is checked. In addition, the compression stresses in the timber are verified to prevent crushing perpendicular to the grain. The design resistance is governed by the weakest failure mode, with ductile behaviour preferred in accordance with Eurocode principles.

2.6 Previous studies on connection stiffness

2.6.1 Rotational stiffness in Eurocode

Rotational stiffness, according to Eurocode 3 (2005), is defined by the moment–rotation behaviour of a joint, in which the deformable response is described by the relationship between the bending moment acting on the joint and the relative rotation between the connected members. The slope of this relationship in the initial elastic range is defined as the initial rotational stiffness.

Eurocode 3 (2005) provides simplified calculations for the initial rotational stiffness for some types of steel joints using the component method. In this approach, a joint is idealized as an assembly of components, each representing a physical source of deformation within the connection. Each component is assigned a stiffness based on its mechanical behaviour, and the overall rotational stiffness of the joint is obtained by combining the stiffness contributions of the individual components. In this way, the joint stiffness is not assumed, but derived from its material properties and configuration. The initial rotational stiffness can be calculated according to equation 2.2.

$$S_j = \frac{E z^2}{\sum_i \frac{1}{k_i}} \mu \quad (2.2)$$

where,

S_j = rotational stiffness of the joint according to Eurocode

E = Young's modulus of elasticity of the material

z = lever arm of the joint components

k_i = stiffness coefficient for basic joint component i

μ = stiffness ratio $S_{j,ini}/S_j$

The initial rotational stiffness $S_{j,ini}$ is obtained from this expression with $\mu = 1.0$.

This initial rotational stiffness of a joint is used to classify the joint as rigid, semi-rigid, or pinned (Eurocode 3, 2005). The classification is based on the joint's stiffness relative to the bending stiffness of the connected members, rather than on its absolute value. In this comparison, the stiffness is treated as a relative property rather than an absolute one. A joint may be considered rigid in one structural context but semi-rigid in another.

When a joint is classified as rigid, its rotational deformation is considered sufficiently small that it does not significantly influence the distribution of internal forces in the structure (Eurocode 3, 2005). For instance, in the case of a beam-beam joint, the joint may be modeled as fully continuous, and its actual stiffness does not need to be explicitly introduced into the analysis model. When a joint is classified as pinned, its rotational stiffness is considered sufficiently low that it does not transfer significant bending moment, and it may be modeled as a hinge. In both situations, rotational stiffness is evaluated to justify the modeling assumption, but it is not directly inserted into the design. For semi-rigid joints, rotational stiffness plays a direct role in the design.

The calculated stiffness should be incorporated into the structural model, typically by modeling the joint as a rotational spring.

In Eurocode 3 (2005), the limits for a joint to be classified as nominally pinned are:

$$S_{j,ini} \leq \frac{0.5 \cdot EI}{L} \quad (2.3)$$

For the joint to be classified as rigid, it has to meet the requirement of:

$$S_{j,ini} \geq \frac{k_b \cdot EI}{L} \quad (2.4)$$

where, k_b is 8 for frames where bracing reduces horizontal displacement by at least 80%. Otherwise k_b is 25.

If the joint falls in between these requirements, it is to be classified as semi-rigid. Some examples of steel-steel connections and their rigidity (moment-rotation relationship) are shown in Figure 2.12 and Figure 2.13.

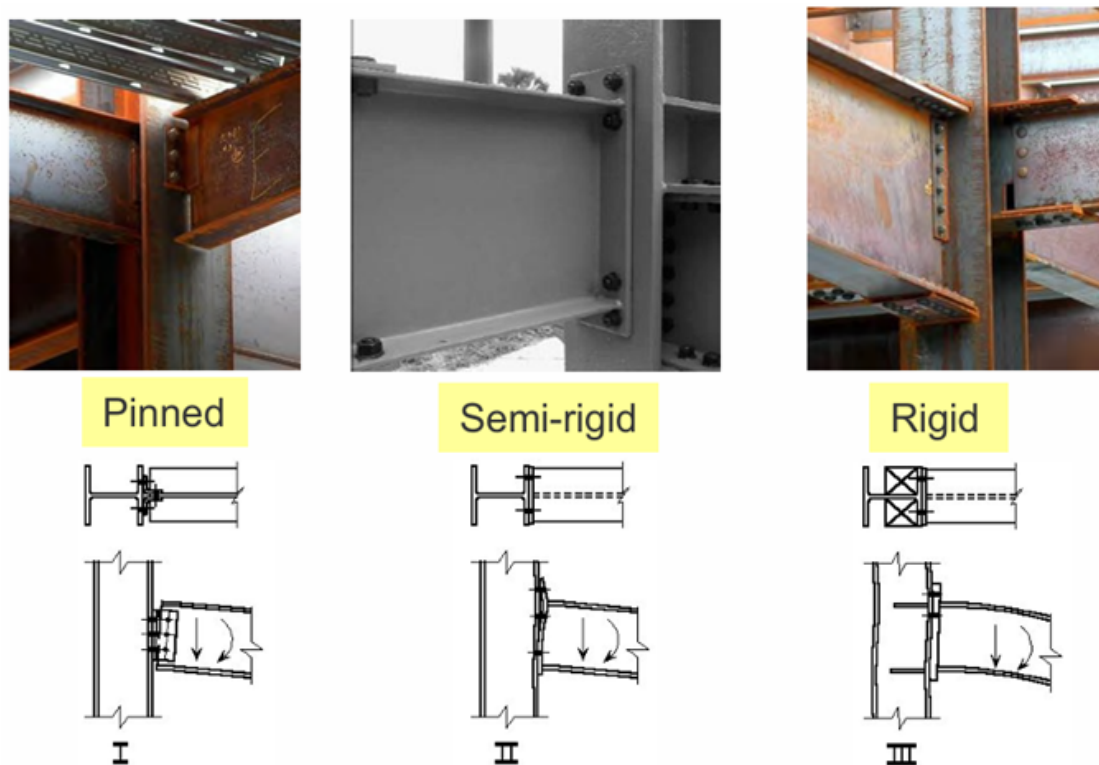


Figure 2.12: Different types of Steel-steel connections and their rigidity classification.

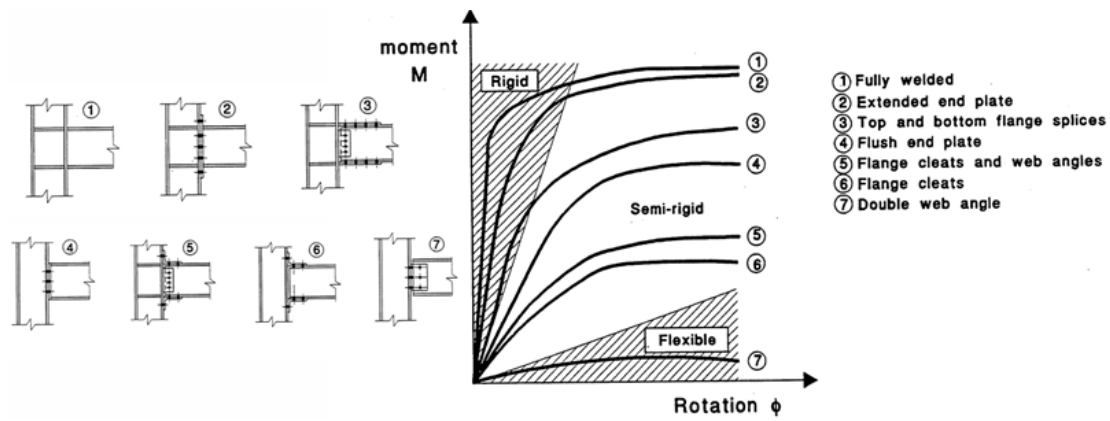


Figure 2.13: Moment-Rotation Relation for Different Types of Steel Connections.

2.6.2 Master Thesis on the effects of deformation differences on connections in hybrid structure, Larsson and Widén, 2025

(Larsson and Widén, 2025) studied embedded steel-plate connection in multistory hybrid structures, where the differential long-term deformations are introduced. Consistent, predefined terms inspired by (Larsson and Widén, 2025) are used throughout this report to improve clarity and readability. Figure 2.14 illustrates the terminology adopted for the connection components.

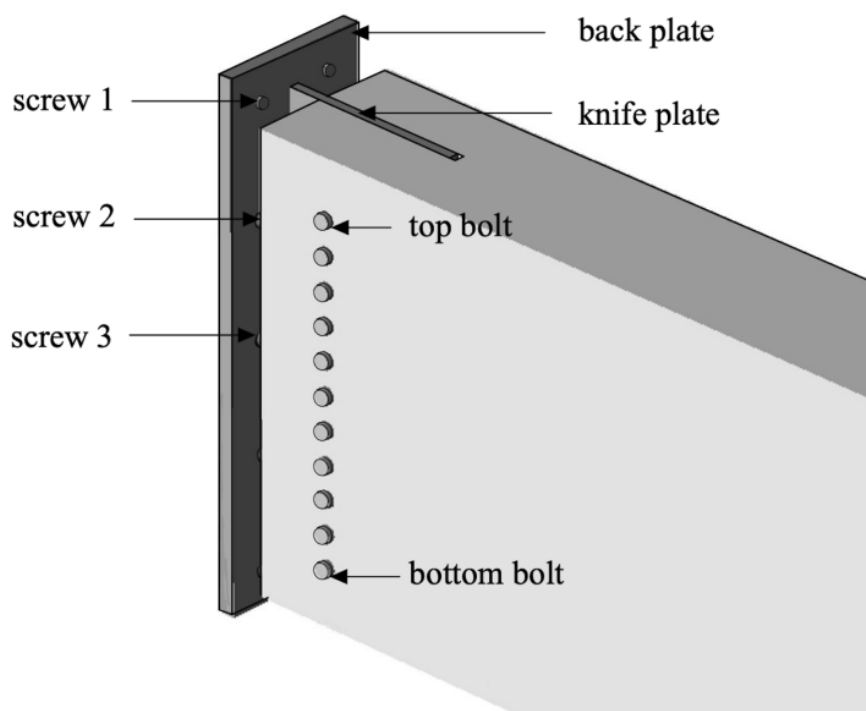


Figure 2.14: Defined terms for the parts in the connection. Larsson & Widén (2025)

The dowels that fasten the timber beam to the embedded plate are referred to as "bolts", while the fasteners that attach the back plate to the concrete core are designated as "screws".

The stiffness of an embedded-plate connection is influenced by several components that govern how moment and rotation are transferred through the joint. Key components that could impact the rotational stiffness are bolts/dowels connecting the timber beam with the embedded plate, the screw-in back plate, and the back plate.

In the Master's thesis by Larsson and Widén (2025), a parametric study was carried out to evaluate the rotational stiffness of an embedded-plate connection using numerical analysis. Although not all components of the connection were examined in detail, the study investigated the influence of bolt configuration, bolt diameter, and the thicknesses of both the embedded plate and the back plate on the overall behaviour of the connection as follows:

2.6.2.1 Bolts in timber beam

The results showed that bolt properties in the timber member were important for stiffness (Larsson and Widén, 2025). As illustrated in Figure 2.15, increasing bolt diameter increased rotational stiffness and rigidity. Larger bolts prevent rotation more effectively and attract higher moments at lower rotations.

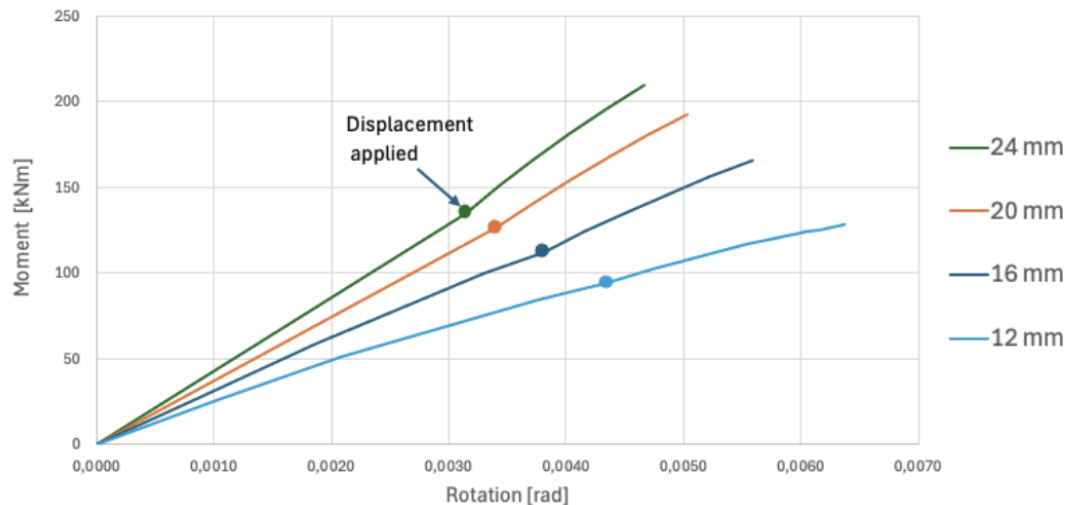


Figure 2.15: Moment-rotation curves for various bolt sizes

Bolt configuration also had a strong influence on the connection behaviour. Figure 2.16 shows that connections with two columns of bolts developed higher moments and were therefore stiffer than single-column arrangements, and that stiffness generally increased with the total number of bolts. However, bolt placement proved more decisive than quantity, as bolts positioned at the top and bottom of the connection contributed most significantly to the overall rotational stiffness.

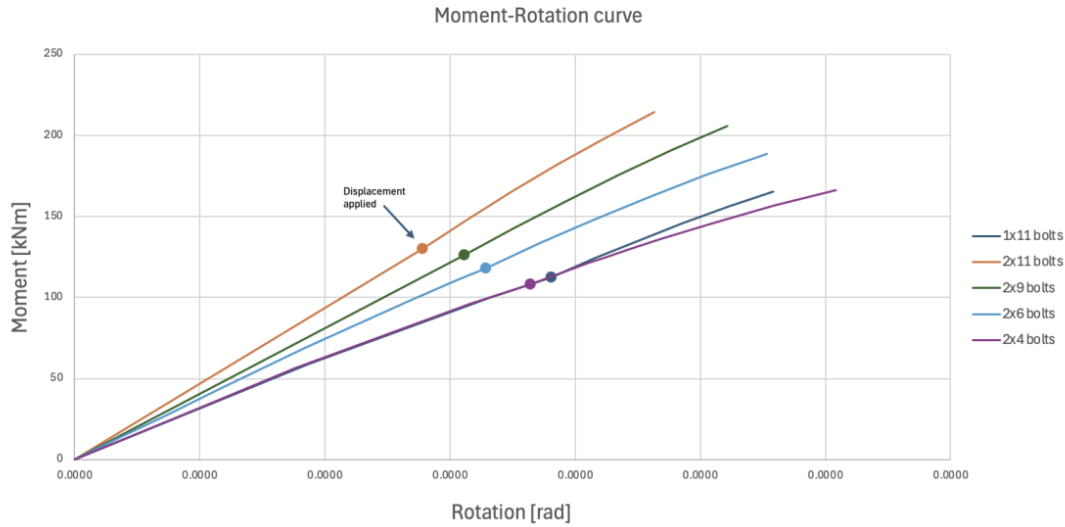


Figure 2.16: Moment-rotation curves for various bolt configurations

2.6.2.2 Back plate

In the parametric study by Larsson and Widén (2025), the back plate thickness was identified as one of the most influential parameters governing rotational stiffness. As shown in Figure 2.17, increasing the back plate thickness resulted in a steeper moment–rotation response with higher moment capacity and greater rotational stiffness. Conversely, thinner back plates led to earlier plastic deformation and a substantial reduction in stiffness. The study further demonstrated that plastic deformation of the back plate largely controls the onset of global ductile behaviour in the connection.

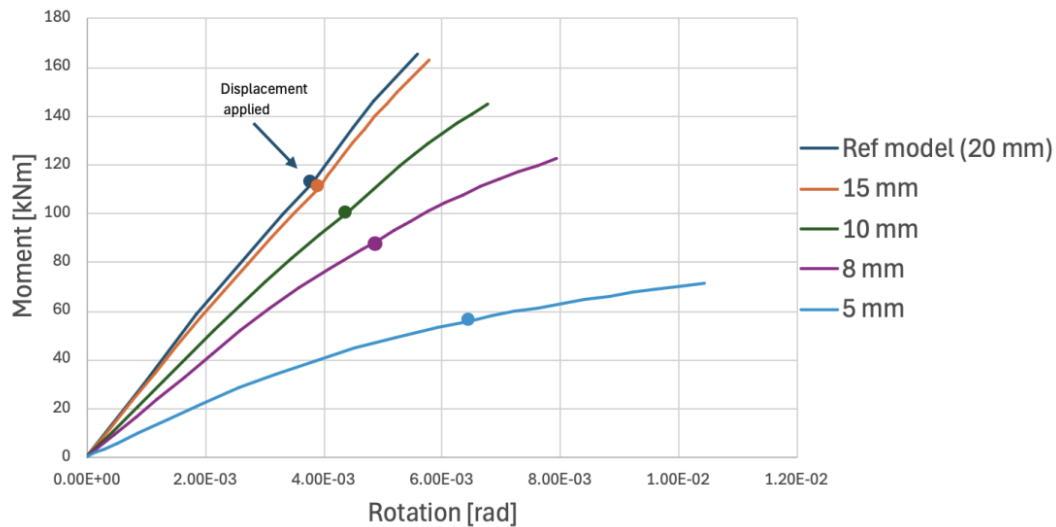


Figure 2.17: Moment-rotation curves for various back plate thicknesses

2.6.2.3 Embedded plate

The embedded-plate thickness had a very limited influence on the connection behaviour. Figure 2.18 illustrates that variations in embedded-plate thickness produced almost identical moment–rotation curves, with negligible differences in stiffness.

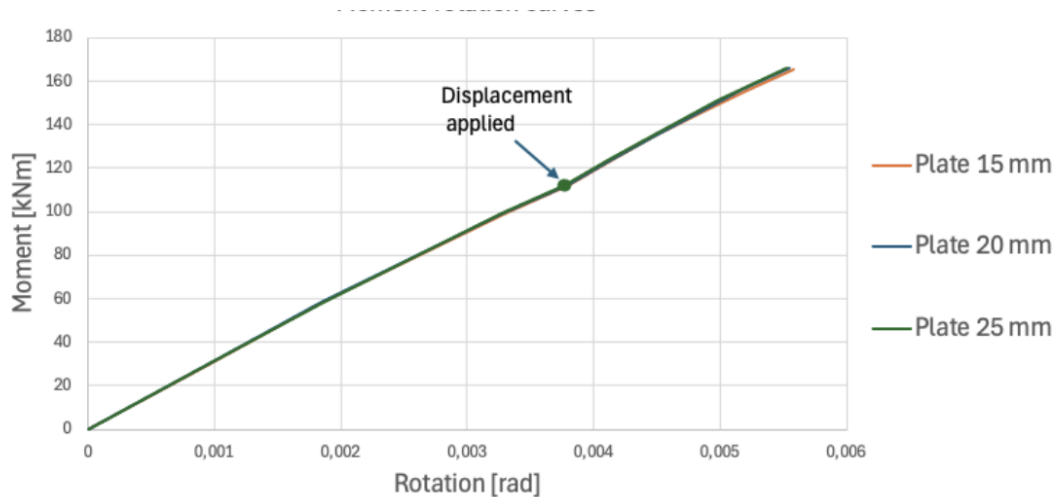


Figure 2.18: Moment-rotation curves for various embedded plate thicknesses

Changes in thickness did not significantly affect the total rotation stiffness of the connection. This minor effect is expected since the embedded plate rotates about its strong axis, and its thickness has a negligible influence on the flexural stiffness compared to its height in that direction.

Overall, the findings of Larsson and Widén (2025) indicate that the thickness of the embedded plate does not significantly influence the rotational stiffness of the connection. In contrast, both bolt diameter and bolt configuration were shown to affect stiffness, while the back plate thickness had the most pronounced impact. However, their study did not investigate the contribution of the timber beam itself or how the fasteners in the back plate influence the rotational stiffness. Furthermore, the numerical model was simplified by representing the concrete as a rigid surface rather than modeling the concrete explicitly. This assumption may affect the results, as the force transfer and stress distribution between the steel components and the concrete were not captured in a fully realistic manner. Because of these simplifications, the results should not be interpreted as exact quantitative predictions. Instead, the overall trends identified in their study are of primary interest and provide insight into the relative influence of connection members.

2.7 Prying action

Prying action is a phenomenon that occurs in bolted steel connections when deformation of the connected plates generates additional tensile forces in the bolts (Kombate, T. J. de la C., & Taskin, K., 2022). The effect typically appears in connections where a plate is subjected to tension or bending, and the bolts are positioned some distance from the free edge of the plate. When a tensile load is applied, the force is initially transferred to the bolts, but the plate may also bend due to its flexibility. As the plate bends, the free edge of the plate may come into contact with the supporting element and act as a secondary support, see Figure 2.19. This contact creates a lever mechanism in which reaction forces develop at the plate edge. These reaction forces act opposite to the bolt force on the plate, but, through the lever effect, increase the tensile force that must be resisted by the bolts. The additional force generated by this mechanism is referred to as the prying force.

Figure 2.19 illustrates a typical example of a T-stub subjected to a tensile force. Due to the bending of the flange plate, reaction forces develop near the plate edges where the flange comes into contact with the supporting element. These reactions generate prying forces that increase the tensile force in the bolts. In such a configuration, the total tensile force in each bolt consists of the direct tensile force from the applied load and the additional force resulting from prying action. Assuming symmetrically placed bolts, the tensile force in each bolt can be expressed according to equation 2.5.

$$F_b = \frac{F_t}{2} + Q \quad (2.5)$$

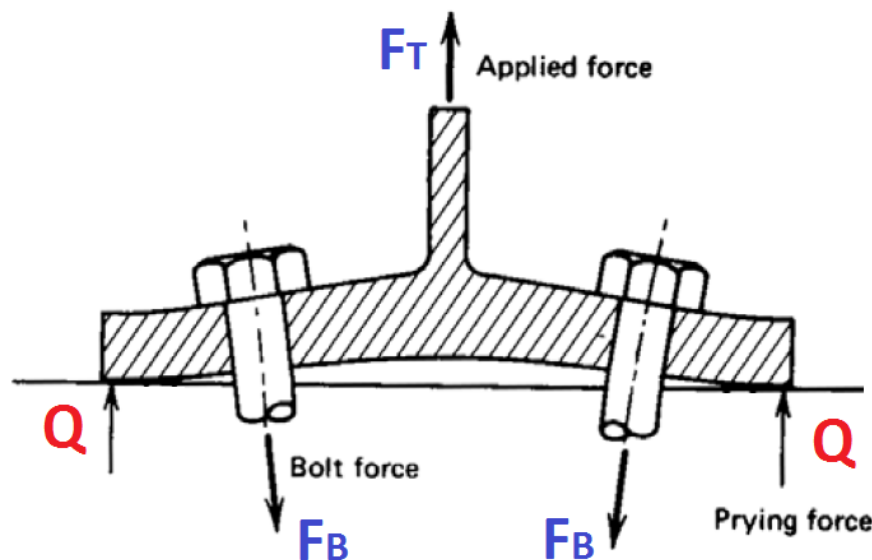


Figure 2.19: Illustration of prying action, (SteelConn, 2026)

In structural design, prying action must be considered because it can significantly increase the tensile force acting in the bolts compared with the directly applied load. In design according to Eurocode 3 (2005), prying action is taken into account through simplified mechanical models that represent the interaction between bolt tension and the bending of the connected plates. A commonly used idealization is the T-stub model, which represents the bending behaviour of the plate together with the tensile resistance of the bolts. This model allows the joint to be analysed using a simplified mechanical representation that captures the key aspects of the load transfer mechanism. The design procedure considers several potential failure modes depending on the relative stiffness and strength of the bolts and the plate.

In some situations, prying action may be neglected if the deformation of the plate is sufficiently small (Eurocode 3, 2005). This is typically evaluated by comparing the calculated bolt elongation length with a reference value. If the elongation length remains below this limit, the plate deformation is considered limited, and the additional prying forces are assumed to be insignificant. The design rules in Eurocode 3 are formulated for verification in the ultimate limit state, where the resistance of both the bolts and the connected plate is checked while accounting for the potential increase in bolt forces caused by prying action. In this study, the prying action applies to the back plate, where it can occur in the connection. As mentioned, prying forces can be neglected if the connection fulfills the requirement in Eurocode 3 (2005). The analytical method proposed in this study is only applicable when prying forces can be neglected, see more under section 4.5.3.

2.8 Beam on elastic foundation

There is no readily available analytical method for calculating the deformation of the bolts embedded in the timber beam. In this study, the beam resting on an elastic bed model is used to simplify the behavior of bolts bearing against timber. Further details are provided in Section 4.8. A beam on an elastic foundation is a structural model used to describe a beam that is supported continuously along its length by a deformable medium rather than by discrete supports (Hetényi, M., 2015). Instead of resting on rigid supports, the beam interacts with a foundation that deforms in response to the applied loads. There are many engineering problems that can be idealized as beams on elastic foundations. Assuming linear behavior of the foundation, the reaction is proportional to the local deflection of the beam. Figure 2.20 illustrates a beam on an elastic foundation.

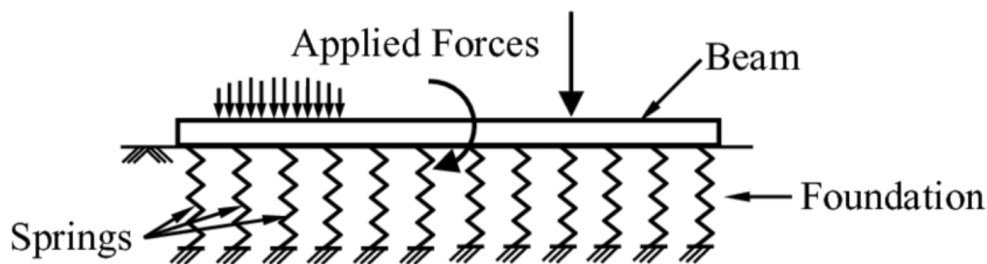


Figure 2.20: Beam on elastic foundation, (Basu et al., 2008)

When designing beams on elastic foundations, one important conceptual distinction is whether the system is treated as having an infinite beam length or a finite beam length (Hetényi, M., 2015). An infinite beam on an elastic foundation is an idealized case where the beam extends indefinitely in both directions. This assumption simplifies the mathematics significantly and is useful for studying local effects of loads without boundary influences. In such cases, the response of the beam is governed purely by the interaction between the applied load and the foundation stiffness.

Engineering problems usually involve finite-length beams, where boundary conditions at the ends of the beam play a crucial role (Hetényi, M., 2015). The beam may be simply supported, fixed, or partially restrained. These constraints significantly influence the bending moments, shear forces, and deflection shapes. Unlike the infinite case, internal forces arising from the boundaries influence the response, and the full system must satisfy both equilibrium and compatibility conditions along the entire beam length.

A key factor in beam-on-elastic-foundation behavior is the stiffness relationship between the beam and the foundation (Hetényi, M., 2015). This is governed by the beam's flexural rigidity (EI) and the foundation stiffness (k_s). Together, they define a parameter often called the relative stiffness or characteristic length, which indicates how the beam and foundation interact and over what distance loads and deflections are distributed.

3

Methods

This thesis aims to develop and verify an analytical model to estimate the rotational stiffness of an embedded steel plate-timber connection. A flowchart illustrating the methodology used in this project to evaluate and verify the analytical solution is presented in Figure 3.1.

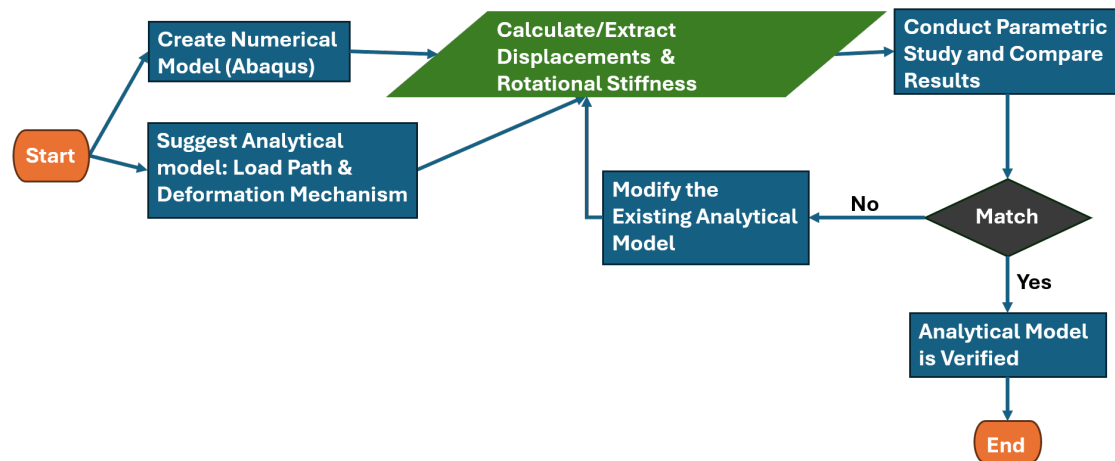


Figure 3.1: Methodology Flowchart.

The first stage involves developing a numerical model in ABAQUS and formulating an analytical method for determining the rotational stiffness of the connection. To accurately capture the structural response, all connection components will be modelled as 3D-solid elements in ABAQUS, providing a realistic representation of their behaviour. To accurately determine the rotational stiffness from ABAQUS. The connection was also modeled in FEM-design as a beam structural element to extract a value for the rotational stiffness.

In parallel with the development of the numerical model, an analytical model will be established based on the component method. This approach involves identifying and isolating the individual components that contribute to the connection's rotational stiffness and combining their respective contributions to obtain the overall stiffness.

After establishing the numerical model and developing an analytical model, a parametric study is carried out in which one or more components of the connection are modified to evaluate their influence on the rotational stiffness and to compare the numerical results with the analytical predictions. The investigated parameters comprise:

- Screws in the back plate, (screw diameter and screws configuration)
- back plate thickness
- Bolts in the timber, (bolt diameter and bolt configuration)
- Beam dimensions

If the parametric study shows agreement between the numerical model and the proposed analytical model, the analytical model is considered valid, and its results are numerically verified. If a mismatch is observed between the numerical and analytical solutions, the numerical model is iteratively refined by introducing several adjustments, and the updated outputs are compared again with the analytical model until satisfactory agreement is achieved.

3.1 3D Solid Model

This section describes how the finite element 3D solid model in ABAQUS is developed and implemented. It includes the definition of geometry and material properties, a description of the interactions within the model, as well as the application of mesh, boundary conditions, and loads. The section also covers the verification of the numerical model and the procedures used for data extraction.

3.1.1 Geometry and Structural Model

As noted in Section 2.4.3, the connection examined in this study is the embedded steel plate with an initial geometry that is adopted from Larsson and Widén (2025). The geometry is based on practical, standard connection sizes for this type of joint and is determined using Johansen's theory for a hypothetical multi-story hybrid structure, while the timber beam dimensions are designed in accordance with Eurocode. So these dimensions, edge distances, and center distances are reasonable for connections used in a normal layout of a hybrid structure.

This specific model is called the reference model, and an illustration of the geometry is shown in Figure 3.2. The dimensions of the reference model are explained in Tables 3.1-3.5.

Table 3.1: Screw configuration in the reference model.

Symbol	Value	Parameter
N_{screws}	5	Rows of screws
$V_{end,s}$	50mm	Vertical edge distance
$V_{mid,s}$	168.75mm	Vertical spacing
$H_{end,s}$	53.75mm	Horizontal edge distance
$H_{mid,s}$	122.5mm	Horizontal spacing
d_{screw}	16mm	Screw diameter
d_h	20mm	Screw head diameter

Table 3.2: Bolts configuration in the connection.

Symbol	Value	Parameter
N_{bolts}	11	Rows of bolts
M_{bolts}	1	Columns of bolts
$V_{end,b}$	87.5 mm	Vertical edge distance
$V_{mid,b}$	50 mm	Vertical spacing
$H_{end,b}$	112 mm	Horizontal edge distance
d_{bolt}	16 mm	Bolt diameter

Table 3.3: Back plate dimensions in the reference model

Symbol	Size [mm]	Dimension
H_{back}	775	Height
b_{back}	230	Width
t_p	20	Thickness

Table 3.4: Embedded steel plate dimensions in the reference model

Symbol	Size [mm]	Dimension
$H_{embedded}$	675	Height embedded plate
$t_{embedded}$	15	Thickness embedded plate
$W_{embedded}$	270	Depth of embedded plate
$Z_{embedded}$	50	Distance from glulam to back plate

Table 3.5: Timber beam dimensions in the reference model

Symbol	Size [mm]	Dimension
H_{glulam}	675	Height
b_{glulam}	230	Width
L_{glulam}	1000	Length

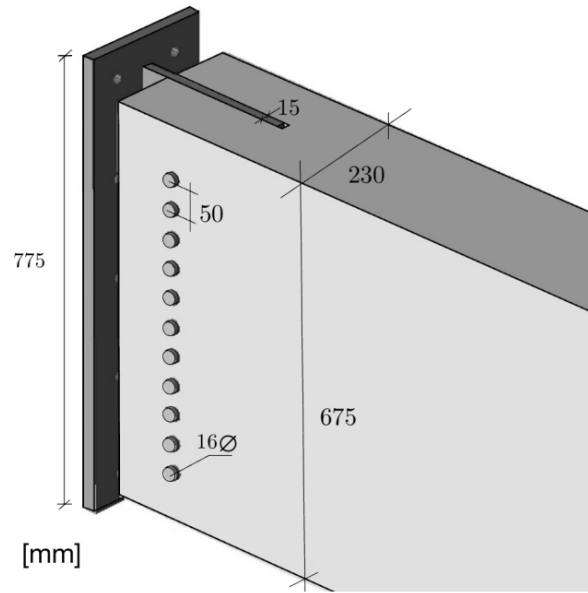


Figure 3.2: Dimensions of the reference model, (Larsson & Widén, 2025).

The structural model of the reference connection is a cantilever with a 1-meter span. This model is structurally determinate and easier to analyze analytically, and improves computational efficiency by avoiding unnecessary beam length, which would otherwise increase the number of elements in the analysis. However, several different structural models, including simple beams and cantilevers with different spans, were used to verify the global calculation of rotational stiffness.

3.1.2 Material properties

To account for the anisotropy of wood, the glulam GL30c material is defined in ABAQUS using engineering constants. This material model was also used by Zhang et al., (2023) and allows different stiffness values to be assigned in each principal direction. In the analysis, the timber is modeled as linear-elastic, meaning that no plasticity or failure mechanisms are included in the model. This is appropriate for analyses where the applied loads are relatively small, and the resulting stresses and deformations are expected to remain within the elastic range. In practice, the timber will likely experience crushing due to compression perpendicular to the grain under loading. Since this study focuses on the initial rotational stiffness rather than the capacity, the timber was modelled as a linear-elastic material in both tension and compression.

The material properties used in the model are presented in Table 3.6, which lists the elastic moduli, Poisson's ratios, and shear moduli in the different principal material directions. In this study, Poisson's ratio is assumed to be identical in all directions. Given the linear-elastic analysis and the focus on the connection's initial rotational stiffness, this simplification is not expected to have a significant influence on the results.

Table 3.6: Engineering constants used in the material model for GL30c.

E1	E2	E3	Nu12	Nu13	Nu23	G12	G13	G23
13000 MPa	300 MPa	300 MPa	0.3	0.3	0.3	650	650	50

Steel is modeled in ABAQUS as an isotropic, elasto-plastic material. Although the study focuses on the elastic state, the steel components were modeled as elasto-plastic to allow for potential future extension of the analysis into the plastic state. The absence of plasticity in the model is verified manually for each model. The connection includes multiple components made of steel, varying in type and strength. Both the embedded plate and the back plate are made of structural steel with a strength grade of S355. The elastic properties of steel are summarized in Table 3.7, while the plastic properties of S355 used in ABAQUS are provided in Table 3.8.

The bolts and screws share the same elastic properties as the embedded plate and back plate. However, these fasteners are made from higher-strength steel of grade 8.8, resulting in different plastic properties. These are presented in Table 3.9.

Table 3.7: Elastic material properties used in ABAQUS, for all parts in steel.

Young's modulus	Possion's ratio
210 GPa	0.3

Table 3.8: Plastic material properties used in ABAQUS, for parts made of structural steel S355.

Stress	Plastic strain
355 MPa	0
530 MPa	0.0176

Table 3.9: Plastic material properties used in ABAQUS, for parts of grade 8.8.

Stress	Plastic strain
640 MPa	0
800 MPa	0.12

For more information about the material properties, check Appendix B.

3.1.3 Boundary conditions

The concrete component, such as a shear wall or column, is not modeled explicitly in Abaqus. Instead, it is represented by a discrete rigid surface positioned behind the back plate. A discrete rigid surface is an undeformable surface that can interact with deformable elements when in contact. Similar to how a concrete wall would behave, assuming the wall is sufficiently stiff, and its deformations are negligible.

In reality, the screws in the back plate are embedded in the concrete item to a certain length. In the ABAQUS model, screws are extended to the rigid surface, where the nodes are constrained in all degrees of freedom. This assumption implies that the concrete provides full restraint to the screws and that the concrete does not crack due to the tensile forces acting in the screws. The boundary conditions are illustrated in Figure 3.3.

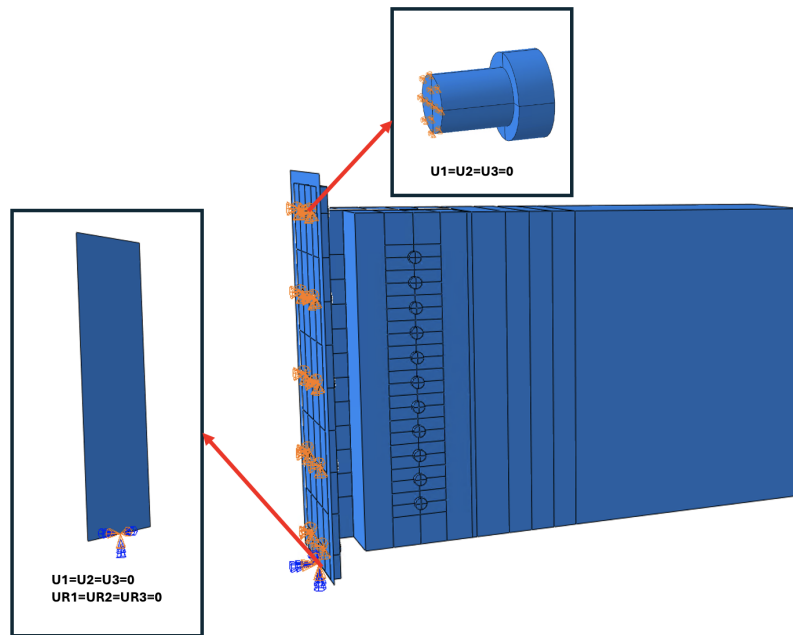


Figure 3.3: BCs applied in ABAQUS.

3.1.4 Load

A pure bending moment is applied to simplify the model and the comparison with the analytical solutions. A moment of 10kN.m is applied, keeping stress levels sufficiently low to ensure the fulfillment of the assumption of linear-elastic behavior. The load is introduced at a reference point (RP) positioned at the centroid of the cross-section at the free end of the beam. This RP is kinematically coupled to the beam's end surface, with all degrees of freedom constrained, as illustrated in Figure 3.4. By constraining the end nodes in this way, the boundary condition enforces Bernoulli's hypothesis, which assumes that plane sections remain plane when deformed.

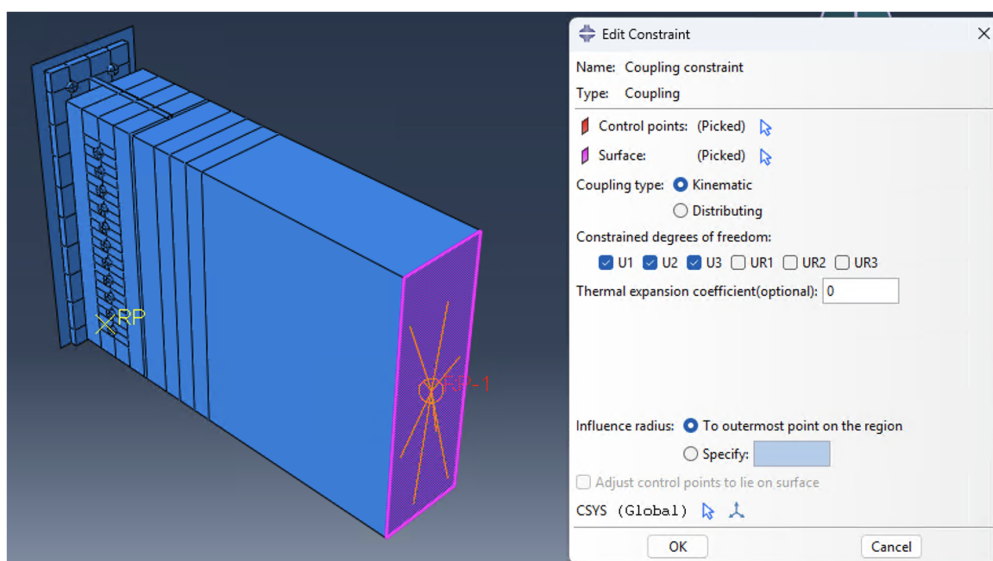


Figure 3.4: Constraint properties, used to couple the RP kinematically to the end of the timber beam.

3.1.5 General interaction

In Abaqus, interactions are used to define the mechanical relationship between different parts of a model during analysis. In the absence of a defined interaction, the parts are treated as independent bodies, which means that they may penetrate each other and do not transfer load. In this analysis, a general contact interaction is employed, with the normal behavior defined as hard contact. See figure 3.5.

However, friction between the components is beyond the scope of this study and is neglected in the analysis, as it is considered negligible and does not affect the results

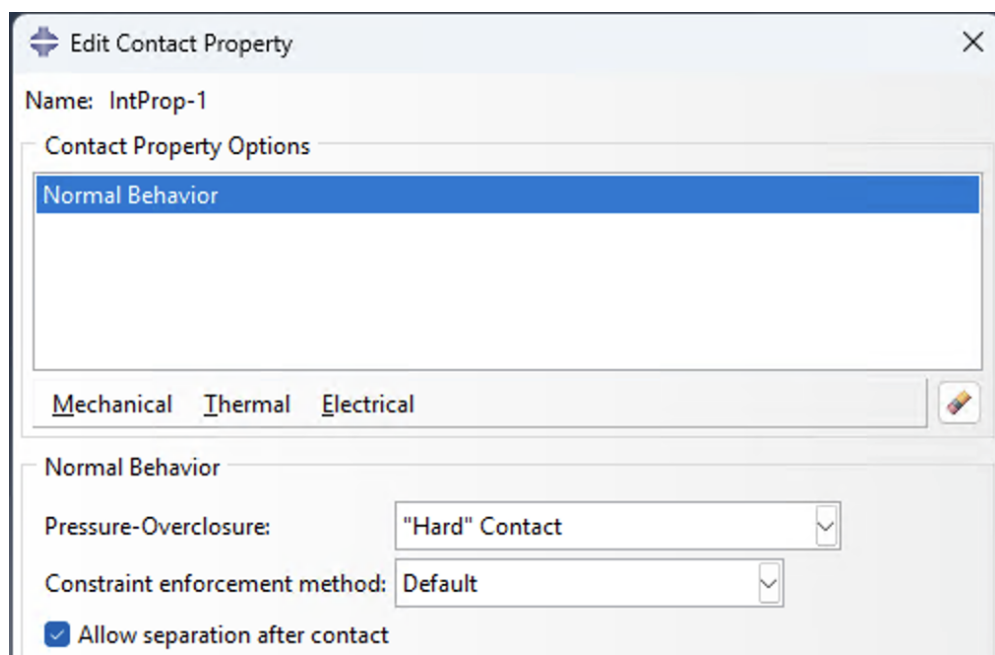


Figure 3.5: General interaction property, used in the analysis.

3.1.6 Parts and mesh

All components in the model are represented as deformable 3D solid items, except for the rigid face, which is modeled as a discrete rigid 3D shell item. The solid parts use C3D8R elements, that is, eight-node linear brick elements with reduced integration and hourglass control. This element type is also adopted by Tako et al., (2023) and is computationally efficient for complex models such as the one used in this study. To achieve a balance between accuracy and computational efficiency, mesh convergence studies were performed for both the steel embedded plate and the timber beam (see Appendix A). To better capture realistic behavior, a finer mesh is applied in regions where stress concentrations or abrupt stress gradients are expected. These regions include areas around bolts and screw holes, as well as the portion of the timber beam adjacent to the embedded plate. Smaller components, such as screws and bolts, are also assigned a refined mesh to ensure a realistic response. The mesh configurations applied to all components are illustrated in Figure 3.6.

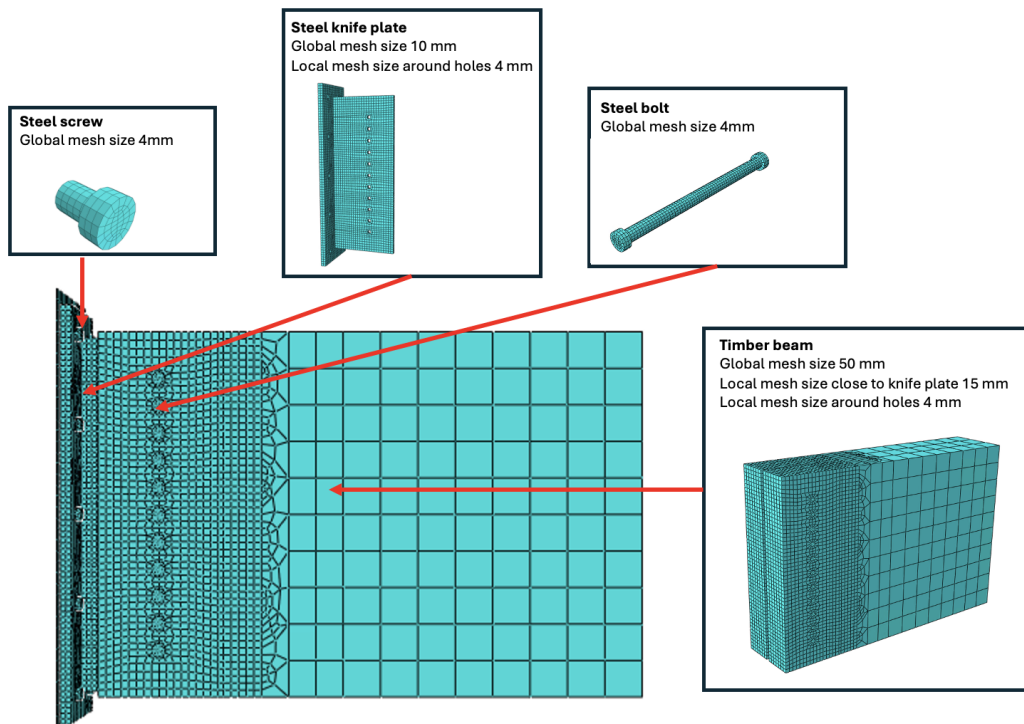


Figure 3.6: Mesh sizes for all the parts modelled in ABAQUS.

3.1.7 Verification of numerical model

Model verification is essential to ensure reliable results. First, the material properties of steel and timber are validated by modeling simply supported beams and comparing the results with hand calculations.

To further verify the structural model in ABAQUS, a timber beam with a span of 7.3m is modelled, and maximum deflection is measured and compared with hand calculations for two idealized cases: a beam simply supported at both ends and a beam simply supported at one end and fixed at the other. Since real connections exhibit semi-rigid behavior, the numerical model is expected to produce results that fall between these two cases. The load applied in this verification is 10 kN/m, and the outcome of this deflection verification is presented in Table 3.10. The results show that the hand calculations and the numerical model produce nearly identical values for the idealized support conditions, indicating that the material properties implemented in the model are appropriate. When the connection is explicitly modeled at one end, the resulting deflection lies between those of the idealized cases, as expected. This confirms the semi-rigid behaviour of the connection.

Table 3.10: Comparison of beam deflection for different support conditions at the end of the embedded plate. The other end is simply supported in all cases.

Support condition	Analytical [mm]	Numerical [mm]
Simply supported	5.62	5.63
Fully fixed	2.92	2.91
Modelled connection	-	4.14

Finally, a mesh convergence study was conducted to verify the selected mesh size and type and ensure correct and accurate results. See Appendix A for a detailed mesh analysis.

3.1.8 Extraction of data

3.1.8.1 Forces

Forces are obtained using the free body cut tool in ABAQUS. For the bolts, the forces are extracted at the section where they are in contact with the embedded plate. For the screws, the forces are taken from the surface at the surface where they connect to the concrete core.

3.1.8.2 Displacements

The displacements are obtained by probing values at selected nodes. For the screws, the node is located at the center of the screw head, where elongation is most accurately represented. For the back plate, the displacement is measured at the center of its top surface, and the elongation of the screws is subtracted to isolate the displacement contribution of the back plate

The maximum vertical displacement is extracted at the end of the cantilever. This value is used to estimate the total rotational stiffness of the connection

3.1.8.3 Neutral axis location

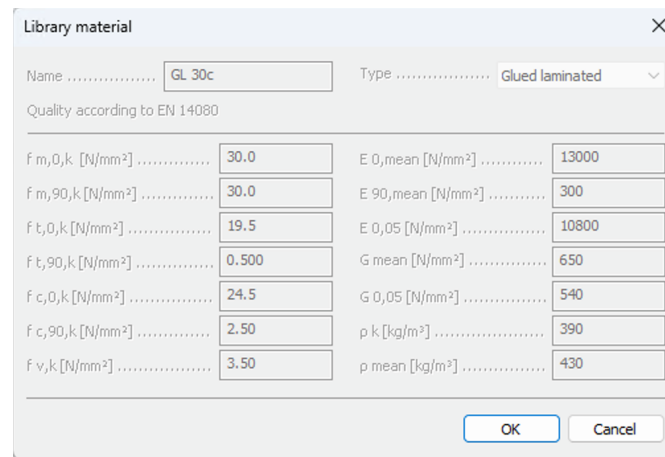
The location of the neutral axis is determined by extracting the CPRESS values from the nodes at the centre of the back plate. Since CPRESS represents the contact pressure, the neutral axis is identified at the point where the pressure value becomes zero.

3.1.8.4 Rotation

Due to the non-linearity of deformations in the cross-section of the deformed shape in the 3D solid model, the rotational stiffness cannot be calculated at the connection directly. Rather, the rotational stiffness is verified by using FEM-design as explained in the following section.

3.2 Structural Beam Model

To extract the rotation of the connection from the ABAQUS model, a simplified model was created in FEM-Design. In this model, the timber beam has a cross-section of 230×675 mm, and material properties for GL30c as seen in figure 3.7).



Library material	
Name	GL 30c
Type	Glued laminated
Quality according to EN 14080	
$f_{m,0,k}$ [N/mm ²]	30.0
$f_{m,90,k}$ [N/mm ²]	30.0
$f_{t,0,k}$ [N/mm ²]	19.5
$f_{t,90,k}$ [N/mm ²]	0.500
$f_{c,0,k}$ [N/mm ²]	24.5
$f_{c,90,k}$ [N/mm ²]	2.50
$f_{v,k}$ [N/mm ²]	3.50
$E_{0,mean}$ [N/mm ²]	13000
$E_{90,mean}$ [N/mm ²]	300
$E_{0,05}$ [N/mm ²]	10800
G_{mean} [N/mm ²]	650
$G_{0,05}$ [N/mm ²]	540
ρ_k [kg/m ³]	390
ρ_{mean} [kg/m ³]	430

Figure 3.7: Material properties for GL30c in FEM-design.

Note that it is the $E_{0,mean}$ and G_{mean} that have an impact on the deflection and need to match with ABAQUS. The beam length is defined as the distance from the free end to the center of gravity of the bolts in the ABAQUS model, as the bolts represent the actual support location for the beam. Consequently, the beam length in FEM-design is 888 mm rather than 1000 mm as in ABAQUS. The connection itself is not explicitly modelled in FEM-Design; instead, it is represented by a point support group. The FEM-Design model is presented in Figure 3.8.

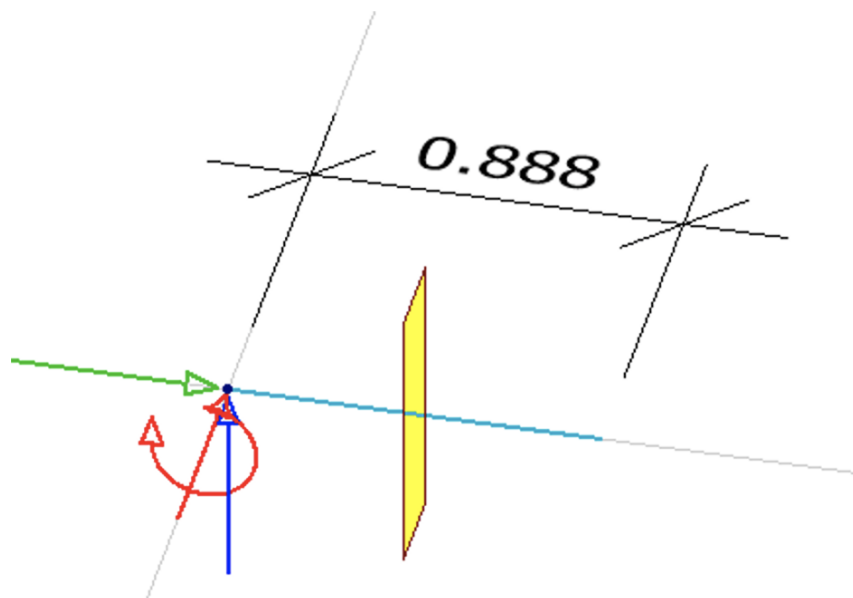


Figure 3.8: FEM-design model of the beam with a point group support.

A point moment of 10 kNm is applied at the free end, consistent with the ABAQUS model. Figure 3.9 illustrates the applied moment in FEM-design.

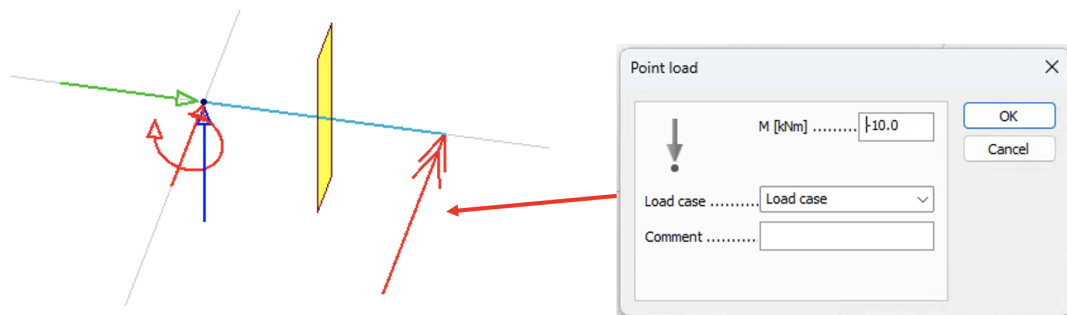


Figure 3.9: Point moment applied at the end of the beam in FEM-design.

To obtain a rotational stiffness equivalent to the ABAQUS model, the rotational stiffness about the y-axis for the support group in FEM-design was adjusted iteratively until the cantilever tip deflection matched the ABAQUS deflection results. Figure 3.10 illustrates the stiffness parameter that was modified for the support group to achieve agreement with the ABAQUS deflection response.

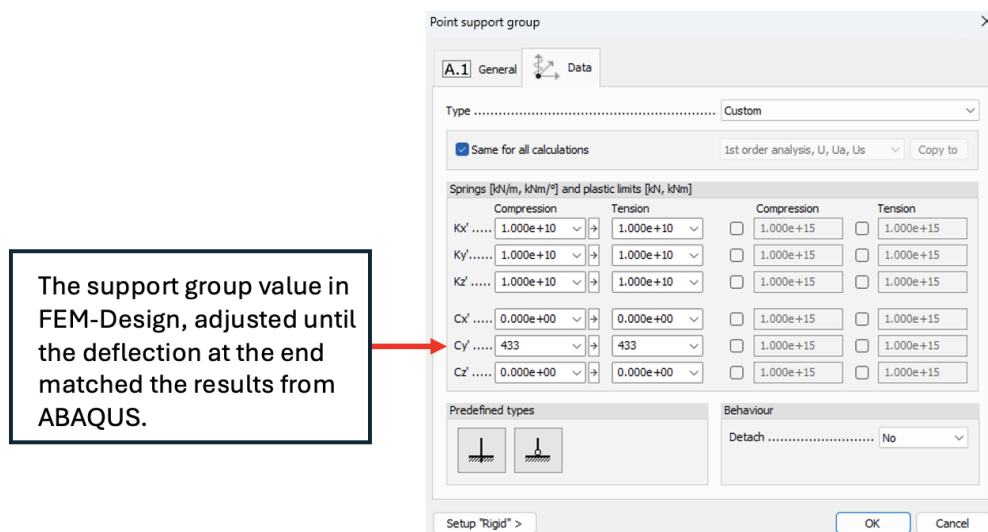


Figure 3.10: Value to iterate for the rotational stiffness of the support point group in FEM-design. The stiffness is adjusted until the deflection corresponds to the ABAQUS results.

It should also be noted that FEM-design expresses rotational stiffness in kNm/°, whereas the analytical method uses MNm/rad. Therefore, unit conversions are required for consistent comparison.

3.3 Foundation modulus of Glulam

Since the bolts are modelled as a beam resting on an elastic foundation, which is the glulam here. Embedment strength or foundation modulus is a material property that is typically determined through extensive experimental testing. However, reliable values for glulam GL30c are not readily available in the literature. Kahn et al., (2026) investigated the embedment strength of several timber species, including radiata pine. In their study, a small timber specimen was fixed in place while a dowel was loaded parallel to the grain under controlled conditions, and the resulting deformation was measured.

To estimate the embedment strength of glulam, the test conducted by Kahn et al., (2026) was replicated in ABAQUS using the same glulam material properties presented in Table 3.6. Determining the embedment strength through ABAQUS simulations provides the advantage that the analytical calculations are based on values consistent with the numerical behaviour observed in the finite element model.

In order to compare the numerical results with those reported by Kahn et al., (2026) and to verify that the simulation produces reasonable values, the same simulation procedure was carried out for radiata pine using the material properties presented in Table 3.11. Since the material properties and strength class of the radiata pine used by Kahn et al., (2026) are not specified, the comparison is intended to assess whether the results are of a similar magnitude rather than to achieve identical embedment strength values.

Table 3.11: Material properties used for radiata pine in ABAQUS.

E1	E2	E3	Nu12	Nu13	Nu23	G12	G13	G23
12000 MPa	450 MPa	450 MPa	0.3	0.3	0.3	750	750	50

In the simulation, both the glulam and radiata pine specimens were modeled with cross-sectional dimensions of 45×100 mm and a length of 120 mm. A load corresponding to a 10 kN point load was applied incrementally to a 19 mm diameter dowel, with the load increments automatically controlled by ABAQUS.

The embedment strength was determined by dividing the force at each increment by the corresponding timber deformation beneath the dowel. The average slope of the load-deformation curve is then taken as the embedment strength of glulam for this study. The resulting embedment strength is presented in section 4.8.2.2. An illustration of the ABAQUS simulation setup is presented in Figure 3.11.

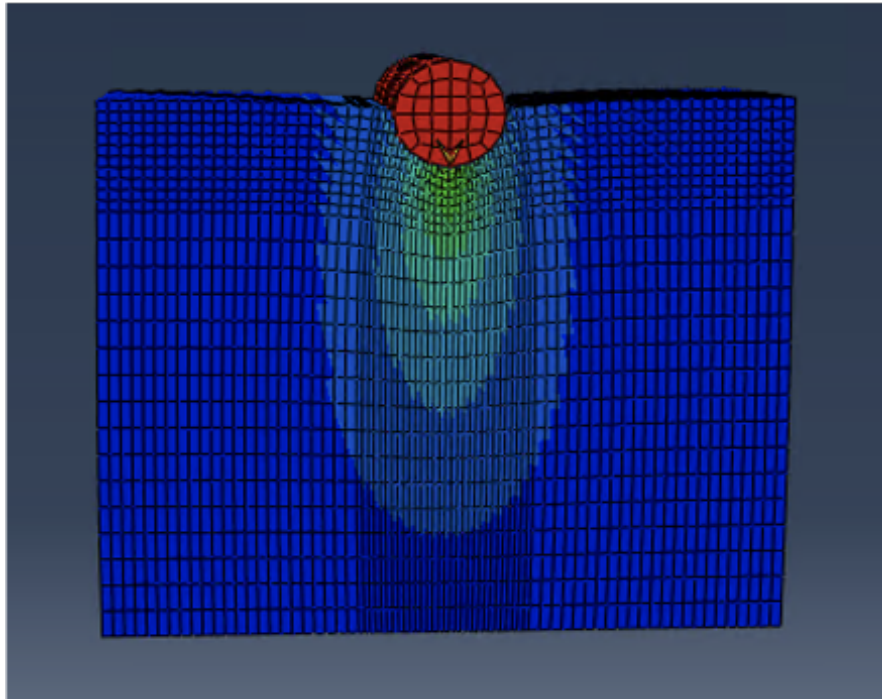


Figure 3.11: Illustration of the embedment test done in ABAQUS.

4

Results

4.1 Analytical approach

The key parameter that governs the response of a connection is the rotational stiffness (rigidity) of the connection. The rotational stiffness defines the relationship between the applied bending moment and the resulting rotation at the connection, and thus controls how the bending moment is redistributed within the structure. Therefore, the first step in the analytical framework is to calculate the connection stiffness, which serves as the basis for determining the induced moments and overall structural response.

In this kind of connection, the rotational stiffness is governed primarily by the mechanical properties of the connection's internal parts and the connected beam material, rather than by the concrete core. Provided that the concrete core can be idealized as a rigid support (undeformable surface), and its rotation can be neglected.

To determine the rotational stiffness analytically, a pure bending moment (M) is applied on the connection within the elastic range. The corresponding rotation θ is then calculated based on that specific applied moment. Then the rotational stiffness can be calculated since it is defined as $k = M/\theta$. Figure 4.1 shows the rotation in a connection subjected to a bending moment.

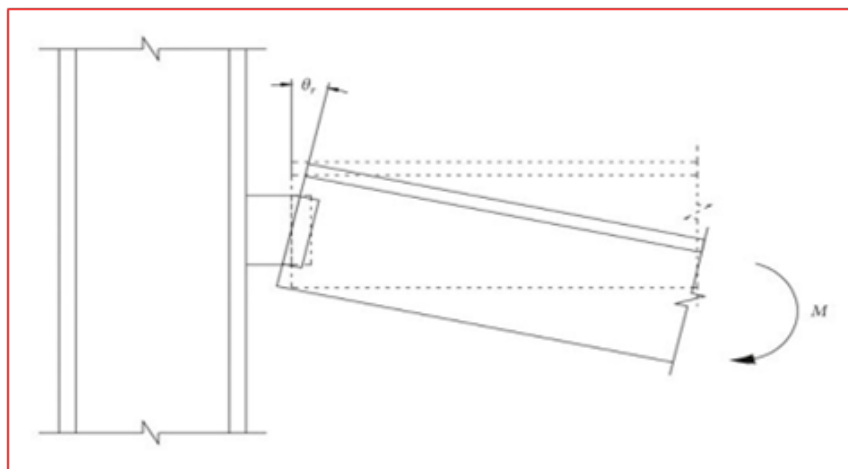


Figure 4.1: Applied bending moment and corresponding rotation at the connection in cantilever structural model

4.1.1 Core principle for calculating rotational stiffness

When the beam is subjected to a bending moment, deformation occurs in all internal parts and joints. Initially, the screws anchored in the concrete, the back plate, and the bolts connecting the timber beam to the embedded steel plate (Figure 4.2).

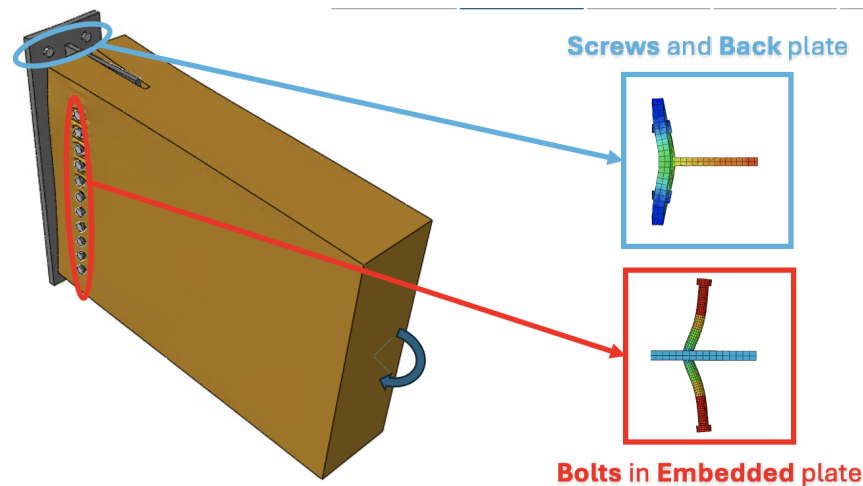


Figure 4.2: Parts of the connection that will deform and contribute to the rotational stiffness, when subjected to a bending moment.

For the screws and the back plate, there will be a compression zone at the bottom of the connection. Due to the restraint provided by the concrete. Movement in this direction is prevented, and consequently, no significant deformation occurs within the compression zone. Deformation is therefore limited to the tension zone. The screws undergo axial deformation, while the back plate deforms in bending between the screw locations. This behaviour is illustrated in Figure 4.3.

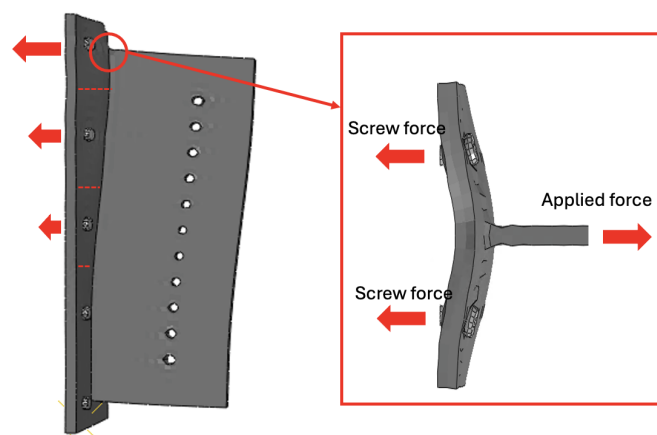


Figure 4.3: Behaviour and structural model of the load for the back plate and screws in the connection.

The bolts deform locally under the compressive forces transferred from the embedded steel plate and through flexural bending within the timber member. Assuming linear-elastic behaviour, the timber and bolts are considered to undergo compatible deformations, allowing the timber deformation to be included in the bolt deformation model.

The bolts are therefore modelled as beams on an elastic foundation. Figure 4.4 illustrates the deformation pattern of bolts. Since the timber does not fully restrain deformation, bolts in both the compression and tension zones deform and contribute to the rotational stiffness of the connection.

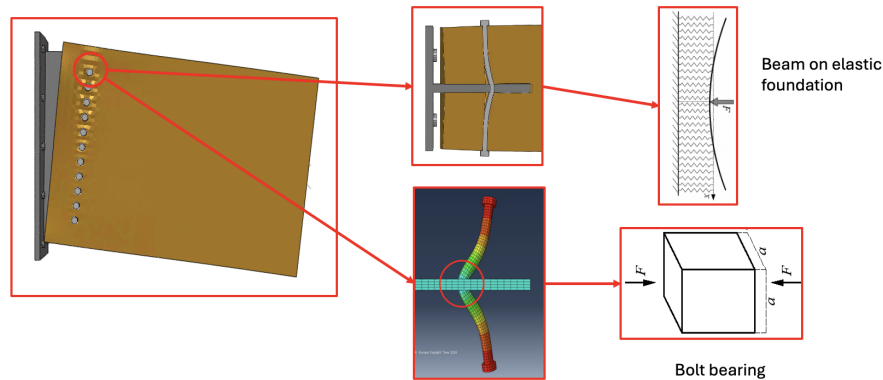


Figure 4.4: Deformation behaviour of bolts in the embedded steel plate.

As a summary, to calculate the rotational stiffness of the connection, some principles and calculation steps must be followed:

- To calculate the rotation resulting from a given bending moment, the applied moment is transferred and distributed through the internal components of the connection by a proper load path.
- After analysing the forces applied to each internal part due to the given moment, the deformation of each part is determined based on the part's structural model and relative stiffness.
- Once the horizontal displacement of each part is determined, the corresponding local rotation is obtained by dividing the horizontal displacement by the vertical distance to the neutral axis.
- The total rotational stiffness is finally obtained by the superposition of each part's rotation. However, in the presence of plasticity, force redistribution occurs, which invalidates the use of the superposition principle. Hence, limits the study to the elastic behaviour only.

4.1.2 Detailed solution steps

The core principle can be broken down into the following objective steps:

- Analyze the connection concerning neutral axis location at the back plate surface (section 4.2).
- Study forces in the screws (section 4.3) and forces in the bolts (section 4.4).
- Calculate the deformation of internal parts, section 4.6 (screws), section 4.5 (back plate), sections 4.7 and 4.8 (bolts),
- Calculate the total rotational stiffness in section 4.9
- Verify the calculated total rotational stiffness in section 4.10
- Extract the final simplified formula of total rotational stiffness in section 4.11

4.2 Neutral Axis

The neutral axis (N.A) referred to in this context is the elastic neutral axis, determined from moment equilibrium under the assumption of linear-elastic behavior.

Location of neutral Axis

The location of a neutral axis is a function of material properties and geometry in the specific cross-section; as a result, the neutral axis location is not fixed in place throughout the connection, rather it varies along the connection depth. In symmetrical sections with the same material properties, such as a glulam beam, N.A is located in the mid-section. Moving towards the back plate, N.A shifts gradually downward, reflecting a response to a change in materials and geometry, where at the back plate N.A reaches its nearest and furthest point from the mid-section, as only the screws are working in tension, while the concrete block is working in compression.

Analysis Approach

At the back plate, the analysis approach is similar to that mentioned in Eurocode 2 (2018) for column-base connections. It assumes a linear stress distribution along a base plate in steel. However, column-base connections consist of a column (typically with an I-section) that is connected to a base plate. Where the tensioned flange is contributing effectively in transferring the bending moment. While in the reference model, the beam is a rectangular section and bolted in regions inside the beam, which makes it less efficient in transferring the bending moment.

Accordingly, a new approach is introduced to determine the neutral axis in back-plate connections referred to as the “Hybrid method”. The main difference between the Hybrid method and the method used in Eurocode 2 (2018) is the location and extent at which the linear stress distribution is transferred to the back plate/base plate. The stress distribution is linear and symmetric in the glulam beam section, but not in the back plate section, where the tension zone is not symmetrical with the compression zone. As a result, the hybrid method underestimates the ability to transfer tension stresses compared to Eurocode 2.

4.2.1 Hybrid method

4.2.1.1 Assumptions

Assumptions made for the hybrid method are:

- Linear strain distribution according to the Bernoulli hypothesis: "Plane sections remain plane and perpendicular to the neutral axis after deformation".
- The tension and compression stress due to the bending moment are extended up to the top and bottom of the embedded plate, while the compression stress is extended up to the back plate bottom fiber, but the tension is limited by the screws that are placed at the level of the embedded plate.
- In the general case, the arbitrary area between two adjacent screw rows is assumed to be equally shared, with each row assigned 0.5 of the area between. However, the top row of screws (maximum in tension) is assigned 0.6 of the arbitrary area,

while the row below is assigned 0.4. The illustration of this assumption is shown in figure 4.5

- The arbitrary area between the screws is limited by the maximum distance between the screws b_{max} . Where $b_{max} = H_{mid,s} - d_{screw}$ and where $H_{mid,s}$ is horizontal spacing between screws. This ensures reasonable stress distribution for a small number of screw rows.

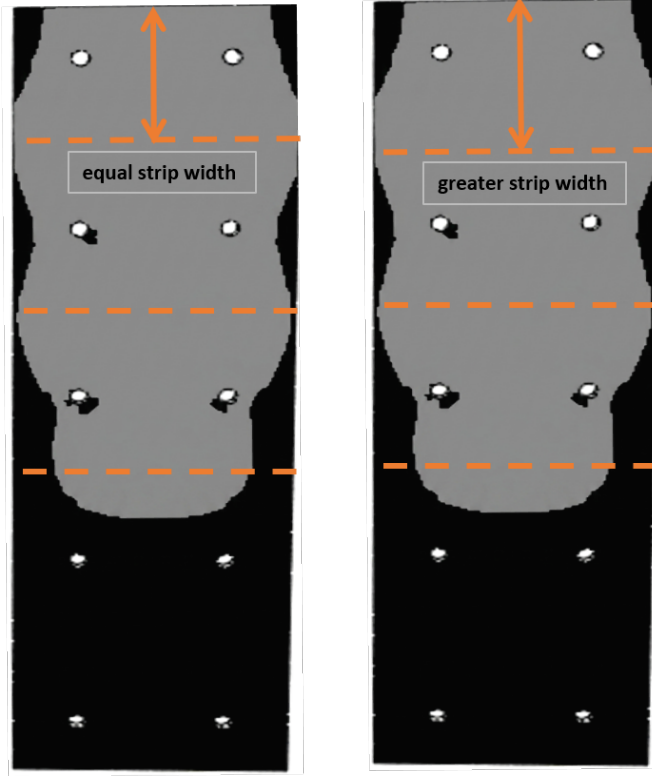


Figure 4.5: The arbitrary area (strip width between screws) used in the general case and in the hybrid method

4.2.1.2 Calculation method

Writing the moment equilibrium equation in the cross-section:

$$M = F \cdot e = \sum F_i \cdot e_i = e \cdot \sum F_i \quad (4.1)$$

Where:

M : applied bending moment

F : sum of internal stresses in cross section

F_i : force in each component (screws in tension or concrete compression zone)

e : eccentricity (distance from the point of application of the resultant axial stress to the neutral axis)

$e_i = (x_i - x)$: eccentricity for each component i in cross section (distance from component's centroid to the N.A)

x_i : distance from component's centroid to the N.A

x : distance to N.A measured from the bottom of the back plate

If the internal forces are expressed in terms of stress and area, then:

$$M = \sum A_i \sigma_i (x_i - x) \quad (4.2)$$

and

$$F = \sum F_i = \sum A_i \sigma_i \quad (4.3)$$

Thus, substituting eq 4.2 and eq 4.1 in eq 4.3 the eccentricity can be expressed as:

$$e = \frac{\sum (\sigma_i A_i x_i)}{\sum (\sigma_i A_i)} \quad (4.4)$$

The cross-section is subjected to pure bending moment, and the resultant internal stresses are resisted by the reaction force on the bottom of the back plate. In this case, the eccentricity (distance between resultant force and bending moment) is the same as the distance to the neutral axis measured from the bottom of the back plate. This outcome is reasonable, as it integrates the stresses in the calculation of the usual elastic N.A that accounts for only geometry. Having a sectional cut at the surface between the back plate and the concrete, and calculating the N.A as a function of stress, area, and distance to the bottom of the back plate for corresponding components, the position of the neutral axis is:

$$x = \frac{\sum [A_{ar,sc,i} \cdot \sigma_{avg,i} \cdot (x_i - x)] \cdot \alpha + \frac{x^2}{3} \cdot b_{back} \cdot \sigma_{avg,c}}{\sum (\sigma_{avg,i} \cdot A_i) + \sigma_{avg,c} \cdot b_{back} \cdot \frac{x}{2}} \quad (4.5)$$

Where:

$A_{ar,sc,i}$: arbitrary area for each screw in tension

$\sigma_{avg,i}$: average stress applied on arbitrary area of screw i

$\sigma_{avg,c}$: maximum compression stress in concrete block

α : modular ratio for steel/concrete

Now the equation is similar to reinforced concrete calculations, idealizing screws as reinforcement bars, and the compression zone in the back plate as a concrete compression zone. In this case, the distinction lies in the fact that the cross-section is assumed to remain in the elastic state, and the stress distribution in the concrete is idealized as a triangle, whereas a rectangular stress block is adopted at the ultimate limit state (ULS). More information about the calculation could be found in Appendix C.

4.2.2 Simplified approach extracted from the hybrid approach

Since stress distribution is linear elastic and thus linearly proportional to the distance from the neutral axis, the previous hybrid approach formula can be simplified by using $(x_i - x)$, the distance to N.A instead of stress σ_i .

$$x = \frac{\sum (A_i \cdot (x_i - x)^2)}{\sum (A_i \cdot (x_i - x))} \quad (4.6)$$

Furthermore, to examine the validity of this equation, consider a cross-section with uniformly sized fasteners only. Where $A_i = A_{ar,sc,i}$ and using equation 4.4 we get:

$$x = \frac{\sum (x_i - x)^2}{\sum (x_i - x)} \quad (4.7)$$

Note the similarity between this method and the vector method used to analyse bolts in bolted connections, where:

$$N_i = M_i \cdot \frac{r_i}{\sum r_i^2} \quad (4.8)$$

Dividing by the screw force N_i :

$$1 = e \cdot \frac{r_i}{\sum r_i^2} \quad (4.9)$$

And isolating the eccentricity:

$$e = \frac{\sum r_i^2}{r_i} \quad (4.10)$$

Observing the relationship between r_i , the screw distance to N.A and e eccentricity of connection or location of neutral axis, a similar pattern is recognized in both equations 4.7 and 4.10, which indicates a reasonable approach and valid simplification.

4.2.3 Eurocode method

The Eurocode method refers to the method used in Eurocode 2 (2018) to analyze base plates in column-base foundations.

4.2.3.1 Assumptions

- Linear strain distribution according to the Bernoulli hypothesis.
- The fasteners' distribution is linear along the base plate face, not only the web plate of the I-section column.
- Screws are stressed according to their strain, which is proportional to their distance from the neutral axis.
- Base plate deformation remains negligible in comparison with the axial displacement of the fasteners.

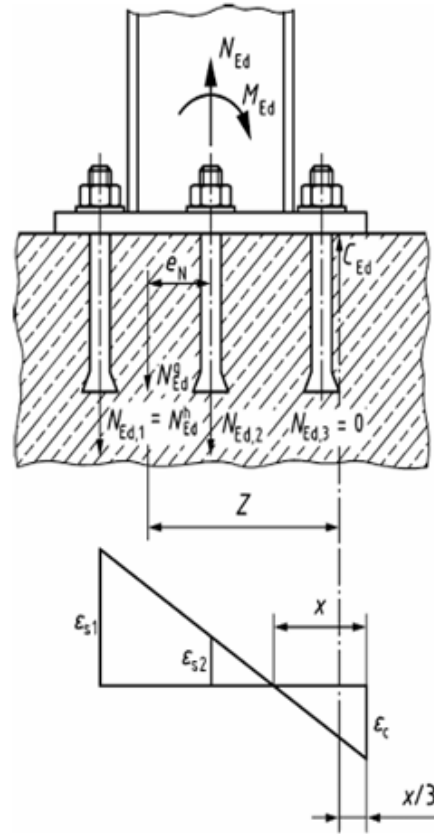
4.2.3.2 Calculation method

According to Eurocode 2 (2018), the moment equilibrium in sections with concrete and steel is:

$$\sum A_{sc,i} \cdot (x_i - x) = \frac{x^2}{2} \frac{b_{back}}{\alpha} \quad (4.11)$$

While this equation is intended to be used for the calculation of the bending moment, it can be used for the determination of N.A since the only unknown is x .

Figure 4.6 illustrates the calculation method and assumptions used in Eurocode 2018.



Key

$$N_{Ed, i} = \varepsilon_{s, i} \cdot E_s \cdot A_s$$

$$C_{Ed} = 0,5 \cdot b_{fix} \cdot x \cdot \varepsilon_c \cdot E_c$$

Figure 4.6: Calculation method and assumptions in Eurocode 2 (2018) for the neutral-axis in the connection.

However, the assumption of negligible deformations in the base plate relative to the fasteners deformation is not valid when for large spacing between fasteners or a thin (flexible) base plate, where the deformations become significant and even larger than the fasteners deformation.

4.2.4 Parametric study on neutral axis using different approaches

In this section, the results of different approaches for calculating the neutral axis are compared by varying key parameters to evaluate the performance of each method.

4.2.4.1 Number of screws in back plate

For a small number of screw rows, the vertical spacing between the screws becomes large, and the back plate deformation increases in those areas. This effect is evident in Figure 4.7 where the EC-approach deviates from the numerical model results for a small number of screws and converges toward them as the number of screws increases.

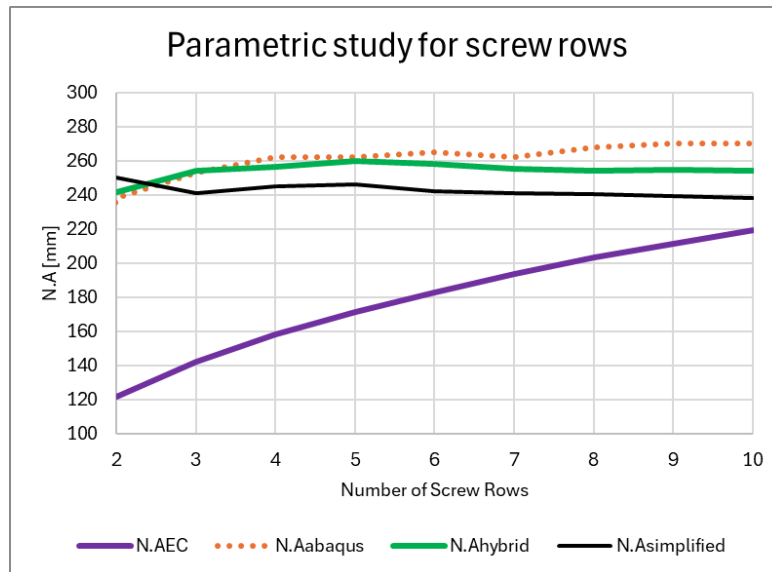


Figure 4.7: Comparison of neutral axis location using different approaches.

4.2.4.2 Thickness of back plate

Back-plate thickness is not considered in either the hybrid or the Eurocode formulation. Meaning the neutral axis remains constant for different thicknesses. But for the numerical model, the thickness of the back plate has an impact on the neutral axis. N.A moves upward for thicker plates, reflecting the increase in the flexural stiffness of the back plate.

However, this parameter does not have a major influence, assuming feasible dimensions; sufficient screw number, such as 5 rows for the reference model, and thickness of the back plate above 15mm. Even in this case, the hybrid method gave better results. The results for the neutral axis for different back plate thicknesses are presented in Figure 4.8.

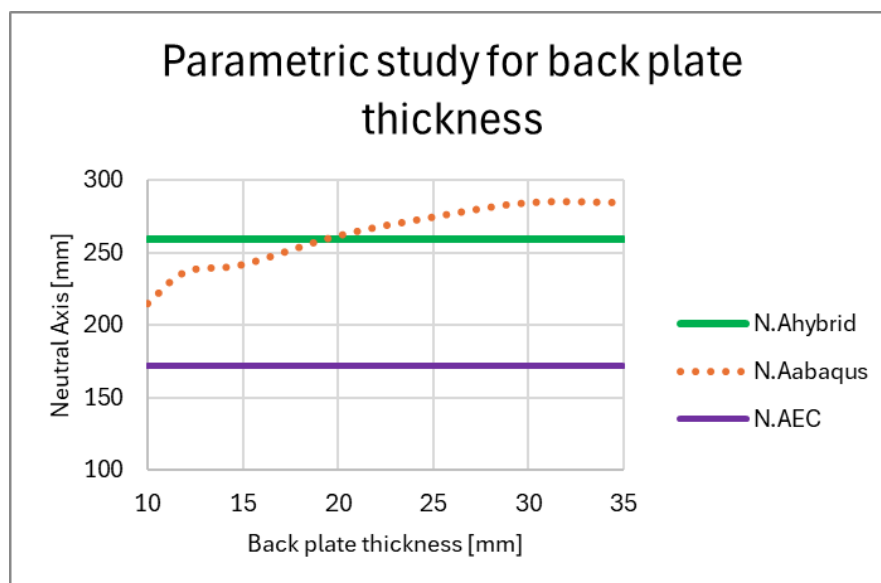


Figure 4.8: Neutral axis location with different back plate thickness.

4.2.4.3 Diameter of screws in the back plate

Numerical results remained unchanged when different screw sizes were used, which is consistent with the hybrid method, as it does not account for screw dimensions. In contrast, this behavior is not aligned with the base-plate method, which explicitly incorporates the screw diameter into its calculations. Again, the hybrid method captures the behavior more realistically in this respect, as illustrated in Figure 4.9.

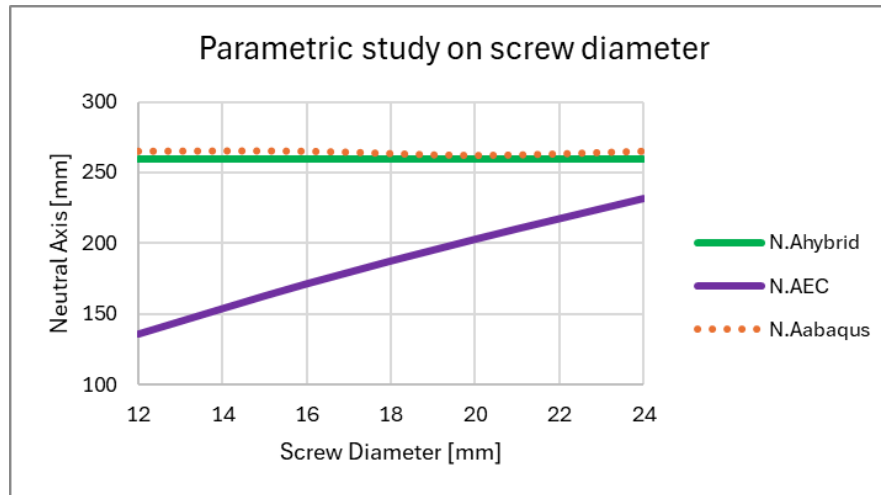


Figure 4.9: Neutral axis location with different screw sizes.

4.2.4.4 Simplification N.A

The parametric study showed that the developed "hybrid method" for determining the neutral axis at the interface between the back plate and the concrete provides a closer agreement with the numerical results than the approach prescribed in Eurocode. However, the hybrid method does not fully capture the behaviour observed in the numerical model, as the influence of the back plate thickness is not explicitly considered in the calculations.

An important finding from the parametric study is that the location of the neutral axis can be simplified and approximated as $y_b = \frac{H_{back}}{3}$. This expression was found to provide a reasonable representation of the neutral axis position across the investigated parameter range.

Where:

y_b : is the position of the neutral axis from the bottom of the back plate

H_{back} : is the height of the back plate

The full analysis of the neutral axis, including other parameters like the configuration of screws (their vertical and horizontal displacement), can be found in Appendix G.

4.3 Analysis of Screws

In contrast to the neutral axis calculation, the screw forces were determined using the design approach for base plate connections given in Eurocode 2 (2018). The hybrid method was found to overestimate the screw forces, whereas the Eurocode-based method produced values that were more consistent and reasonable.

In the ABAQUS model, only the upper five rows are in tension compared to seven rows in the Eurocode model, reflecting differences in the predicted neutral axis position. Despite this variation, the total tension force is consistent between EC and Abaqus models, with some differences observed in the distribution of forces among individual screws. Figure 4.10 shows the tension forces in each row of screws in a connection consisting of 10 rows of screws.

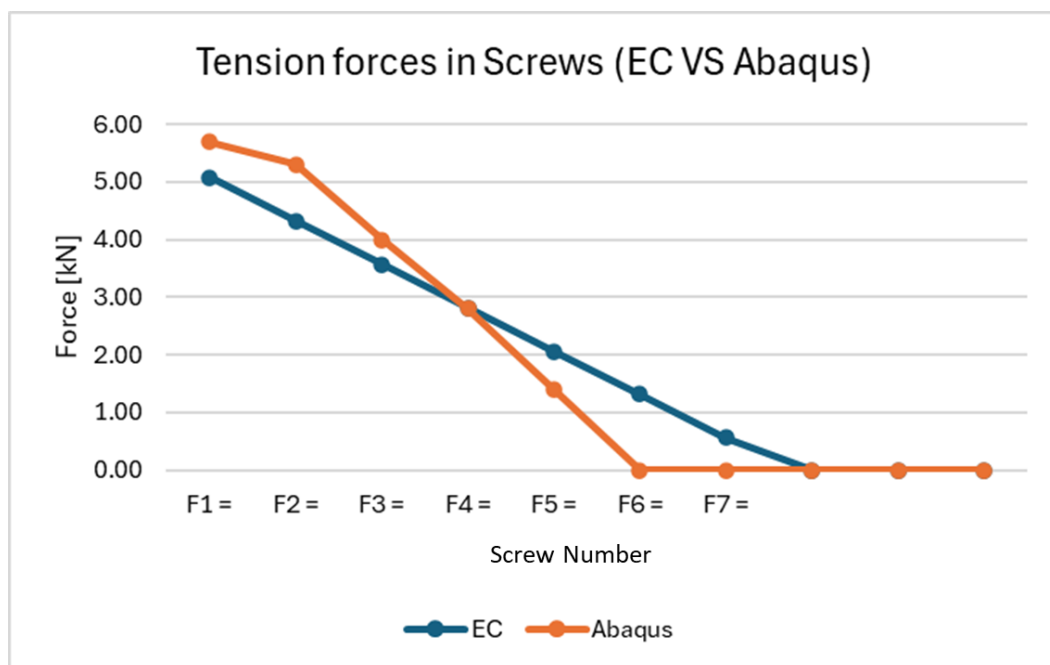


Figure 4.10: Comparison between ABAQUS and EC models in analysing screw forces in connection with 10 rows of screws.

Similarly, Figure 4.11 presents the screw forces for the reference model with the screws arranged in five rows. In this case, the forces in the upper screws are nearly identical between the models. However, some differences are observed for the screws in the middle when comparing the EC model and ABAQUS results. There is once again a discrepancy in the number of screws located within the tension zone between the models. Four screws are considered to be in tension according to the EC model, whereas only three are in tension in the numerical model. As in the case with ten rows of screws, the total tensile force is consistent between the two models, although some variation exists in the force distribution among the individual screws.

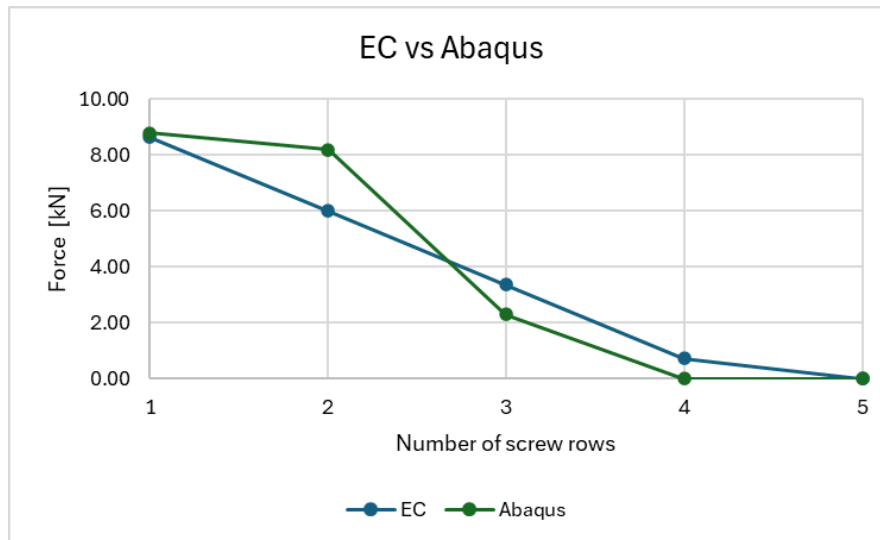


Figure 4.11: Comparison between screw forces in Abaqus and EC for a connection with 5 screw rows.

In this study, the top screw row is of particular importance, as it is used to compute the elongation of the screws and the deformation of the back plate. For this reason, the forces in the top screw row for different models are compared, as shown in Figure 4.12.

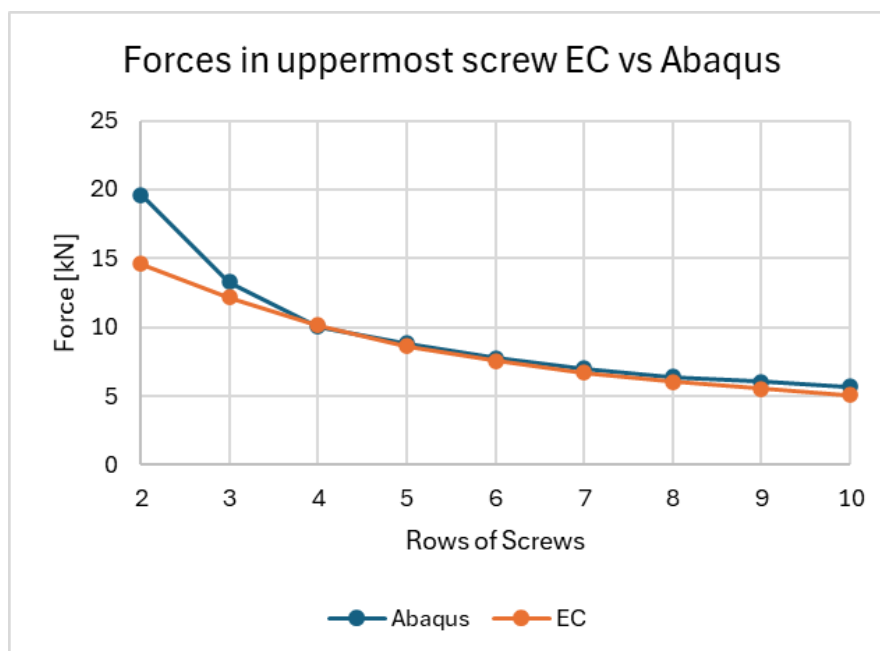


Figure 4.12: Comparison between force in the top row of screws between Abaqus and EC for different numbers of rows in the connection.

For configurations with a sufficient number of screw rows (four or more), the difference in the force in the top row is less than 12%. When the screws are arranged in only two or three rows, the discrepancy increases as the vertical spacing becomes too large and the back plate deformation becomes significant, thereby violating the Eurocode 2 (2018) assumption that the back plate should exhibit negligible deformation. More information regarding the calculations is found in Appendix C.

4.4 Analysis of Bolt forces

Bolt forces are analysed analytically by using the vector method:

$$N_i = \frac{M \cdot r_i}{\sum r_i^2} \quad (4.12)$$

Figures 4.13 and 4.14 present the resulting bolt forces for different bolt configurations. The analysis shows differences between the numerical and analytical models in the force distribution among individual bolts, although the total resultant force is the same in both cases. For full calculations of bolt forces, see Appendix C.

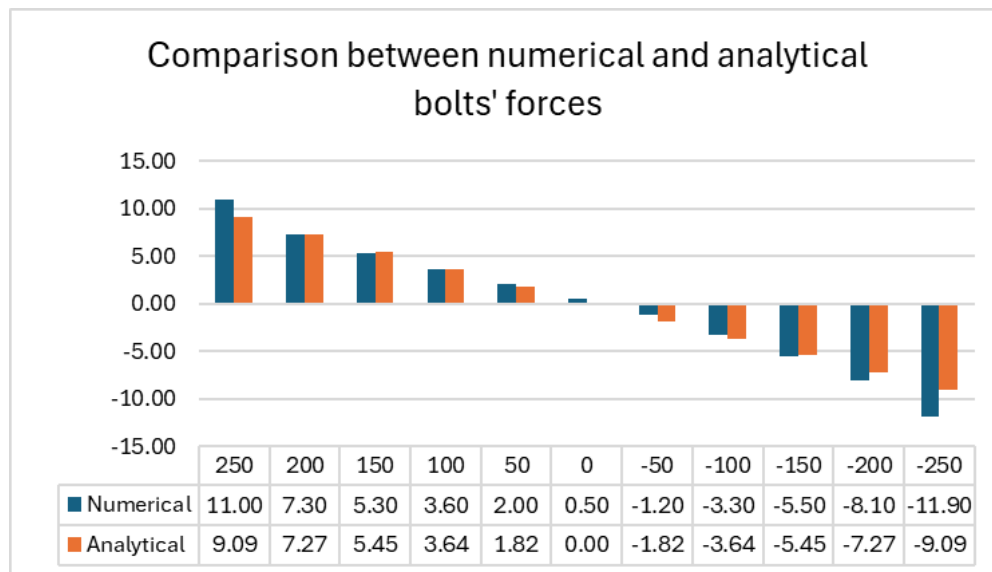


Figure 4.13: Comparison between numerical and analytical results for bolt forces in the reference model, where a configuration of 1x11 is used.

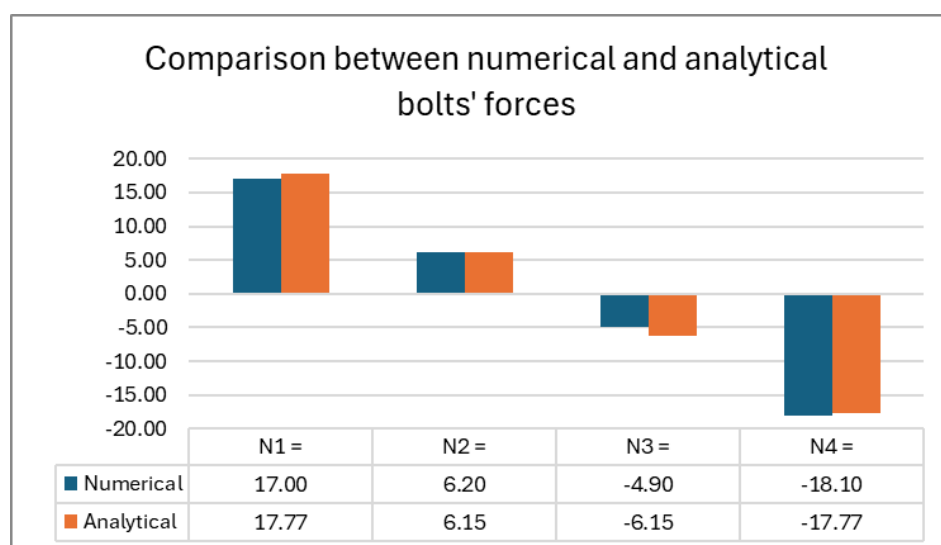


Figure 4.14: Comparison between numerical and analytical results for bolt forces in 2x4 configuration.

4.5 Deformation of Back plate

4.5.1 Deformation mechanism

The back plate is welded to an embedded plate at mid-span and transfers the loads received from that embedded plate to the screws and the concrete element.

When the embedded plate presses against the back plate, the forces are transmitted directly from the embedded plate to the back plate and then into the concrete core. This load path induces negligible deformations in the back plate.

In contrast, when the embedded plate pulls on the back plate, the load is transferred from the embedded plate to the back plate through flexural action of the back plate, and subsequently from the back plate via the screws into the concrete core. In this mechanism, the back plate undergoes more pronounced deformations between the screws due to its relatively low flexural stiffness.

The largest deformation occurs at the top of the embedded plate, where the tensile force due to hogging bending moment is maximum. The deformed shape of the back plate is shown in Figure 4.15.

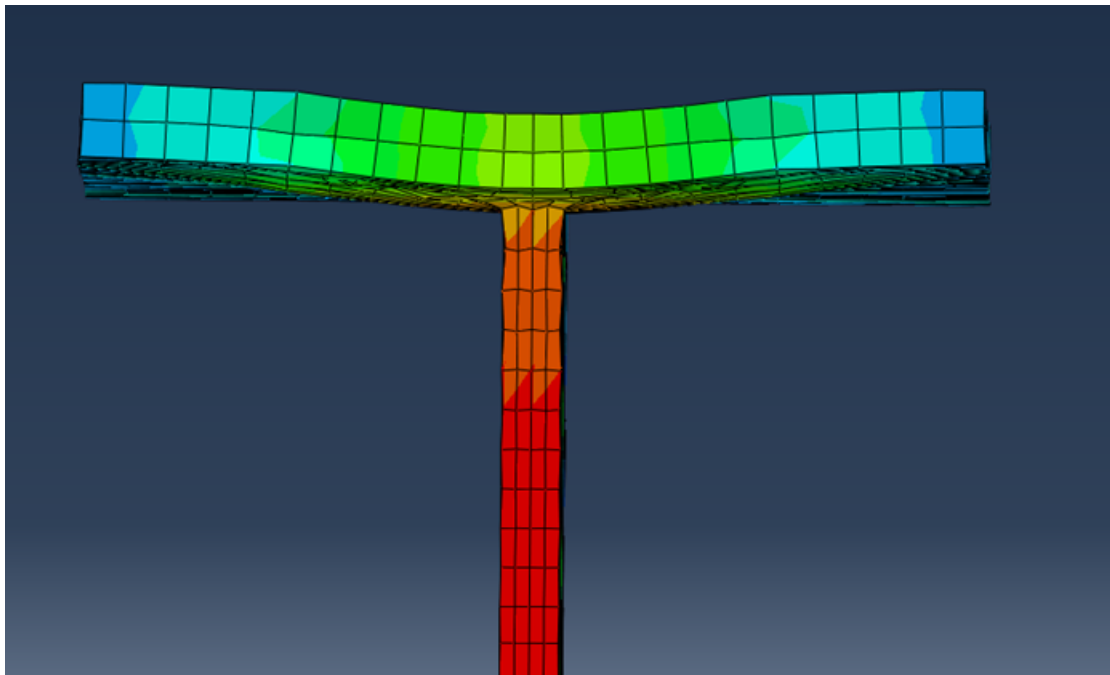


Figure 4.15: ABAQUS picture shows deformation of the back plate in the numerical model.

4.5.2 Calculation method

The deformation of the back plate is determined by idealizing the plate as a series of horizontal strips in the tension zone. Each strip has an effective width supported by the screws and is loaded by the embedded plate. Lemonis and Gantes (2006) proposed an effective width (b_{eff}) for the T-stub analogous to the l_{eff} defined in Eurocode (2005), but specifically adapted for stiffness calculations rather than ultimate resistance strength. These values are derived by equating the numerically computed displacements of the T-stub plates to those of an equivalent simple beam model.

The b_{eff} can be calculated as:

$$b_{eff} = \begin{cases} \frac{b}{0.92 + \frac{0.06}{(m/b)^2}}, & \text{if } m/b < 0.87, \\ b, & \text{if } m/b \geq 0.87. \end{cases} \quad (4.13)$$

Where:

b : is the actual physical width of the T-stub beam

m : is the distance from the screw center to the embedded plate, reduced by $0.8r$ as illustrated in Figure 4.16.

r : is the radius of the weld

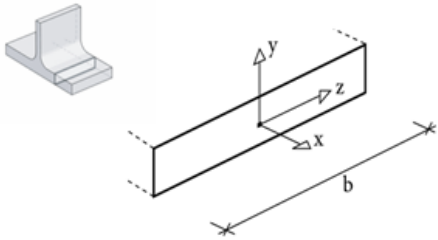


Figure 8. Flange cross section axis definitions.

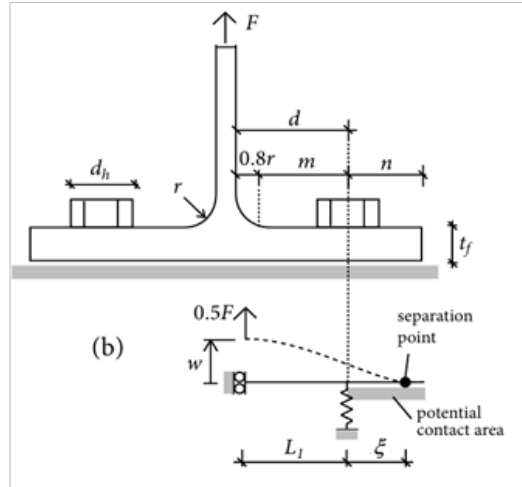
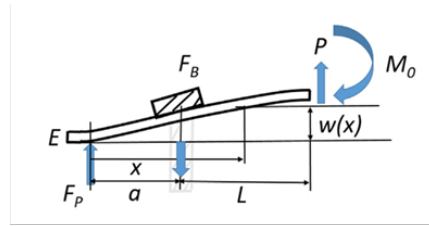


Figure 4.16: Geometry of back plate strips.

4.5.3 Prying action

Prying forces in the T-stub model can influence both the deformation and the force distribution in the screws; it is dependent on the back plate stiffness, configuration of screws in the back plate, and the magnitude of tension force. Lemonis and Gantes (2006) suggested an iterative solution to find the separation point and the response of the t-stub as shown in the Figure 4.17.



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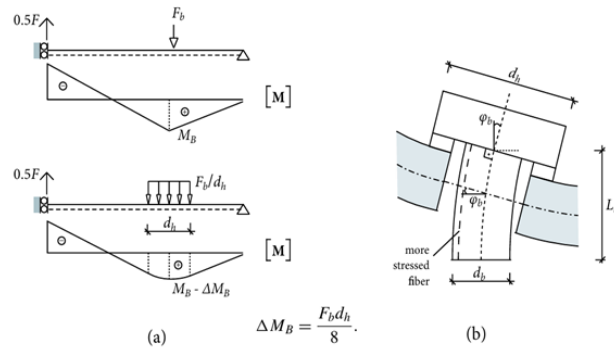


Figure 7. Influence of bolt head size (a) and definitions for the bolt bending treatment (b).

Figure 4.17: Prying action in T-stub models, (Lemonis and Gantes, 2006).

The stress distribution in the back plate is visualized from the results in ABAQUS in Figure 4.18 below. The black regions represent areas of the back plate that are subjected to compression. Black zones on edges are the compression due to prying action.

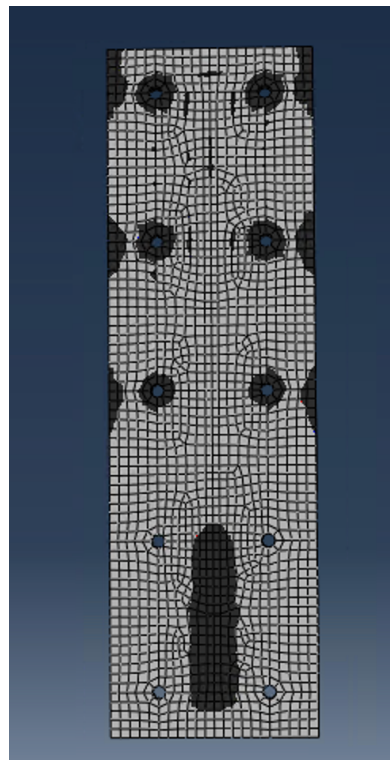


Figure 4.18: Stresses at the back of the back plate in ABAQUS, where the prying force is visible at the edges of the top three rows of screws.

However, since the forces in the top screws are close in magnitude in both the analytical and numerical models, according to Figure 4.12 in section 4.3. The effect of prying action is ignored, and the analytical model can be simplified into a simply supported beam or a fixed-fixed beam as follows:

Fixed-Fixed

$$\delta_i = \frac{F_i L_s^3}{192 b_{eff} E_s I_{back,strip}} \quad (4.14)$$

Pinned-Pinned

$$\delta_i = \frac{F_i L_s^3}{48 b_{eff} E_s I_{back,strip}} \quad (4.15)$$

The two analytical models were compared with the numerical model, and the results are shown in Figure 4.19.

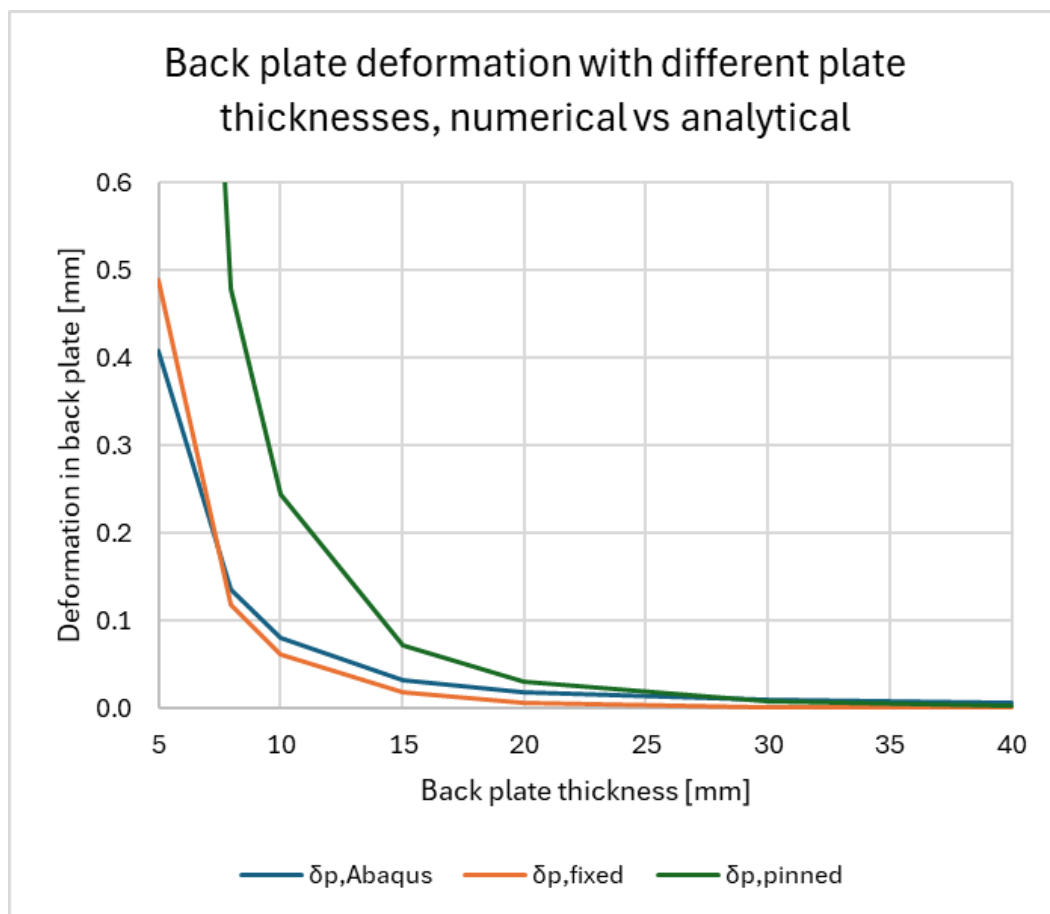


Figure 4.19: Comparison between numerical and analytical models in the calculation of back plate deformation.

As the results indicate, for thicker plates, the deviation is almost negligible, while it increases as the plate thickness decreases. Therefore, the structural model of fixed-fixed beam aligns better with the numerical results.

4.6 Deformation of Screws

The axial tension forces in the screws induce elongation of the screws within the back plate. This deformation then leads to a rotation of the connection. According to Eurocode (2005), L_b is the deformable part of the screw. Where L_b is the bolt elongation length, determined by the connection configuration and the type of fastener used. This includes the embedded anchorage depth of the fastener within the concrete.

Since the concrete is not included in the numerical model, the deformable length of the screw used in the analytical calculations is taken as the sum of the back plate thickness and half the height of the screw head. For the reference model, this corresponds to $20\text{mm} + 5\text{mm} = 25\text{mm}$.

$$L_b = t_{back} + \frac{t_{screw,head}}{2} \quad (4.16)$$

$$\delta_s = F \cdot \frac{L_b}{EA} \quad (4.17)$$

$$\theta_s = \frac{\delta_s}{x_s} \quad (4.18)$$

Figure 4.20 compares the screw deformations obtained from the numerical and analytical models. The results indicate good agreement between the two approaches for screw diameters of 16 mm and larger. However, for smaller screw diameters, the analytical model increasingly deviates from the numerical results.

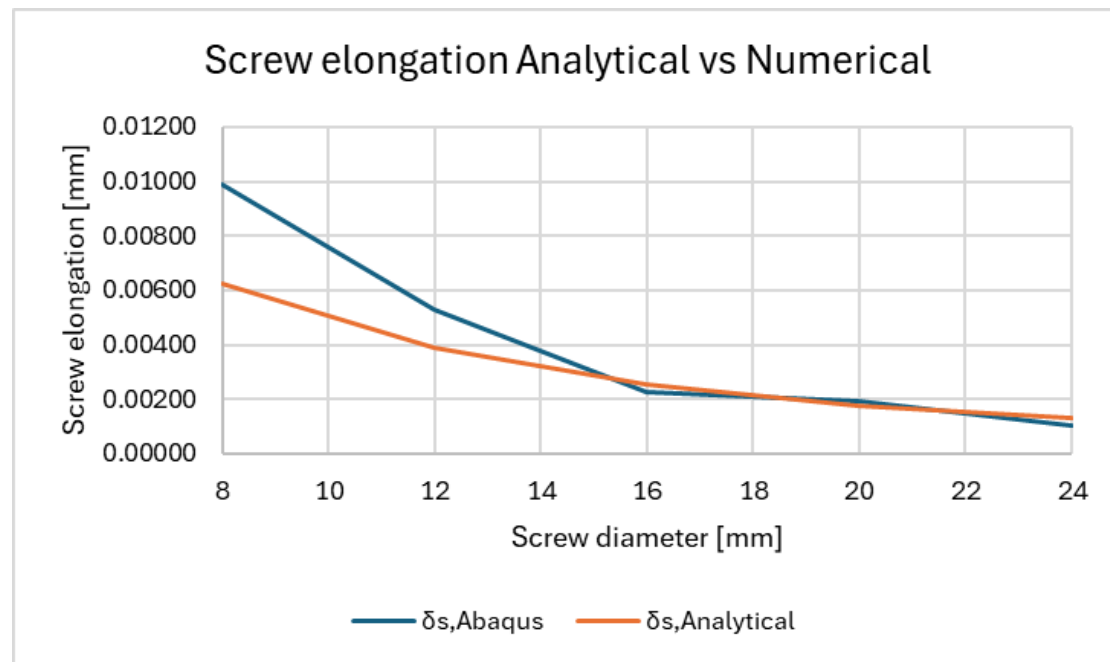


Figure 4.20: Deformation of screws in analytical and numerical models.

The deformation for the screws is very small compared to the other deformations occurring in the connection, and its contribution to the total rotational stiffness is negligible, see section 4.9. For this reason, no further investigation was carried out to improve the accuracy, and the relatively large difference between the analytical and numerical models for bolt diameters of 14mm and less was accepted. See Appendix G for detailed analysis.

4.7 Deformation of Bolts locally

The bolts are subjected to high bearing compressive stresses from the embedded plate because the force transfer occurs over a relatively small contact area between the embedded plate and the bolts. This may lead to local deformation of the bolt cross-section, often described as bearing deformation or local crushing, as shown in Figure 4.21.

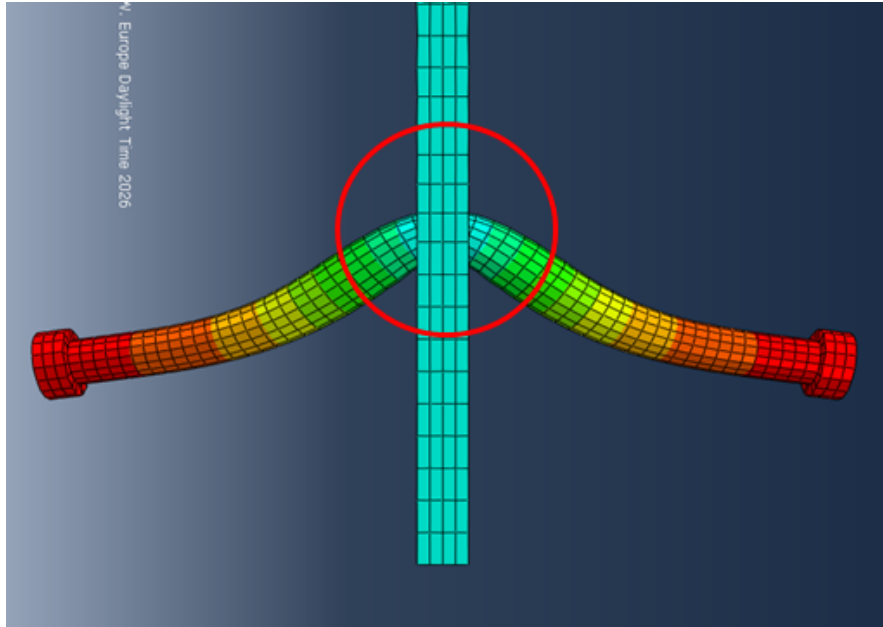


Figure 4.21: Picture from ABAQUS that shows the bearing of bolts under the embedded plate.

The bearing deformation can be calculated as:

$$\delta_{b1} = \frac{W \cdot d_{bolt}}{EA_{bolts}} \quad (4.19)$$

Where A_{bolt} is the area of contact between the embedded steel plate and the bolt, calculated as:

$$A_{bolt} = d_{bolt} \cdot t_{embedded} \quad (4.20)$$

The deflection from the compression can now be expressed as:

$$\delta_{b1} = \frac{W}{Et_{embedded}} \quad (4.21)$$

Where:

W : force applied from the embedded plate on the bolts

d_{bolt} : diameter of the bolt

$t_{embedded}$: thickness of the embedded steel plate

4.8 Deformation of Bolts globally

4.8.1 Structural model

The bolts in the embedded plate connection are subjected to transverse loading, primarily concentrated at their mid-span due to force transfer from the embedded plate. Each bolt is laterally supported by the surrounding glulam member, which provides continuous confinement along its embedded length. When the connection is loaded, the forces introduced through the embedded plate are transferred to the bolts and resisted by bearing and deformation of the surrounding timber. This model captures the load transfer mechanism and the coupled deformation between the bolt and the timber.

This behavior can be idealized as a beam resting on an elastic foundation, where the bolt is modeled as a beam. The surrounding timber acts as the elastic foundation, and the load from the embedded plate is represented as a concentrated load applied at mid-span. The deformed shape of this model is illustrated in Figure 4.22

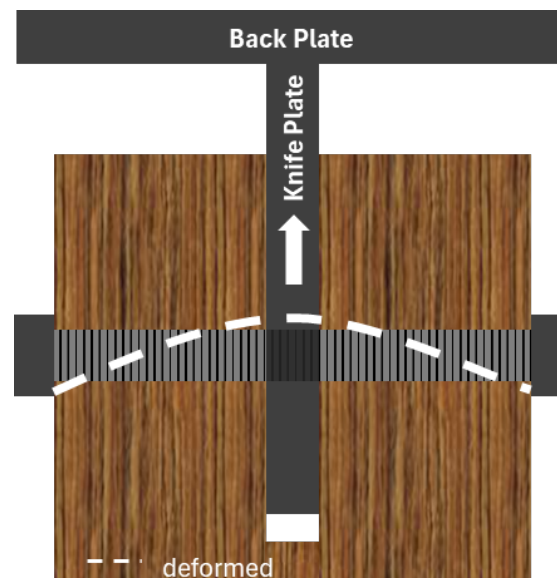


Figure 4.22: Deformed shape of the bolt/glulam model.

4.8.2 Determining foundation modulus for glulam GL30c

Foundation modulus measures the stiffness of the elastic foundation, which is GL30c in our model. Different approaches are used to determine the foundation modulus.

4.8.2.1 Laboratory

According to the laboratory results from Kahn et al., (2026) tests, the foundation modulus for different types of materials are:

Spotted Gum (SG)	$K_{em} = 63.5 \text{ kN/mm}$
Radiata Pine (RP)	$K_{em} = 42.33 \text{ kN/mm}$

For GL30c glulam, an approximate foundation modulus value similar to that of radiata pine can be adopted as a conservative estimate. This is considered reasonable, as the densities are relatively close, $512 \frac{\text{kg}}{\text{m}^3}$ for radiata pine and $450 \frac{\text{kg}}{\text{m}^3}$ for glulam GL30c.

4.8.2.2 Numerically

By modeling the same experimental test from the referenced scientific study as a 3D solid finite element model in Abaqus, numerical values for the foundation modulus of both GL30c and RP can be obtained.

These results can then be directly compared with the experimentally derived values for Radiata pine, enabling quantitative validation of both the numerical model and the adopted results from this model. See, section 3.3 for more information about the numerical model.

The load-deformation curve from the numerical tests is shown in Figure 4.23. The median slope of each pair of points in the load-displacement curve is the foundation modulus used to calculate the deformation of the bolt/glulam model. The numerical simulation yielded an embedment strength of $47.05 \frac{kN}{mm}$ for the GL30c, and this value is used in the analytical calculations. For radiata pine, the numerical simulation resulted in an embedment strength of $48.3 \frac{kN}{mm}$. This aligns reasonably well with the experimental results reported by Kahn et al., (2026), which got $42.33 \frac{kN}{mm}$ from their testing. The small difference in magnitude between the numerical and experimental results indicates that the values obtained from the numerical simulation are reasonable. The test has likely used a weaker class of radiata pine than the one assumed and used in the numerical simulation. More information about the test and the data used is under Appendix E.

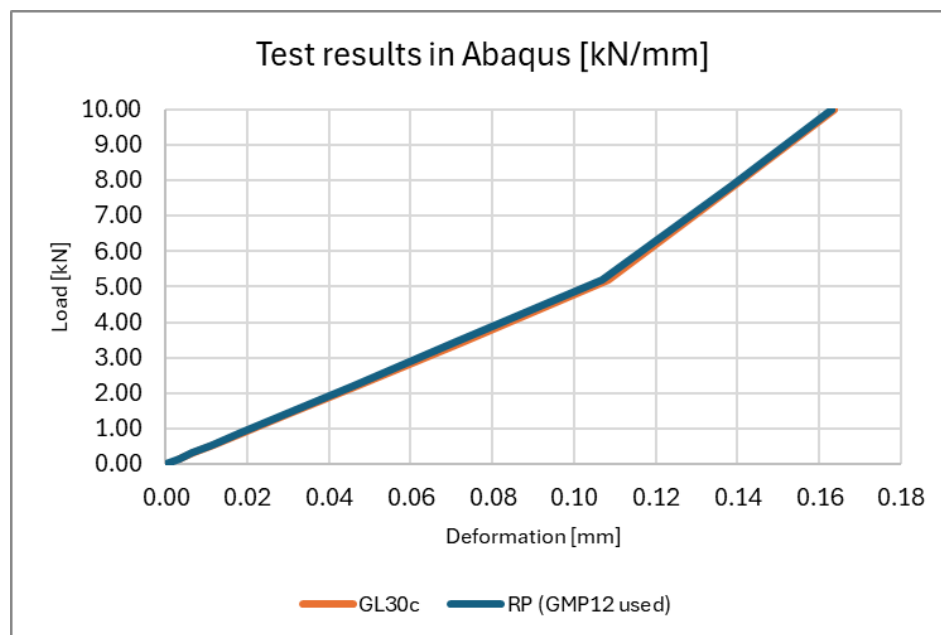


Figure 4.23: Load-deformation relationship for GL30c (glulam) and GMP12 (radiata pine).

4.8.3 Analytical model for bolt/glulam deformation

A widely used reference in structural mechanics and a beam resting on an elastic bed is Roark's Formulas for Stress and Strain by Young and Budynas (2002). According to Roark's formulas, the ratio βl determines which model is used: finite, semi-infinite, or infinite.

Where:

l : is the length of the beam

β : is the relative stiffness and is calculated as:

$$\beta = \sqrt[4]{\frac{K_{em}}{4EI_{bolt}}} \quad (4.22)$$

where:

K_{em} : is the embedment stiffness of glulam

Several models were tested and compared against numerical analysis results to identify the most appropriate approach (see Appendix E for the full calculations). The configurations evaluated included a full-bolt model and a symmetrical half-bolt model. The full-bolt model utilized a span equal to the total bolt length, l . Conversely, the symmetrical half-model used a span length corresponding to the section extending from the embedded plate to the edge of the glulam beam. Boundary conditions were varied across free, pinned, and fixed ends. Additionally, the loading scenarios on the bolt were varied between a concentrated load, representing the force from the embedded plate, and a distributed load. The latter represents the glulam reaction force distributed uniformly across the bolt span, which serves as a conservative assumption.

The most appropriate models were found to be the Symmetric half model with concentrated load at the free end, and the whole bolt model considered as an infinite beam. The Figure 4.24 shows the deformed shape of the symmetrical half model compared to the deformed shape of the numerical model.

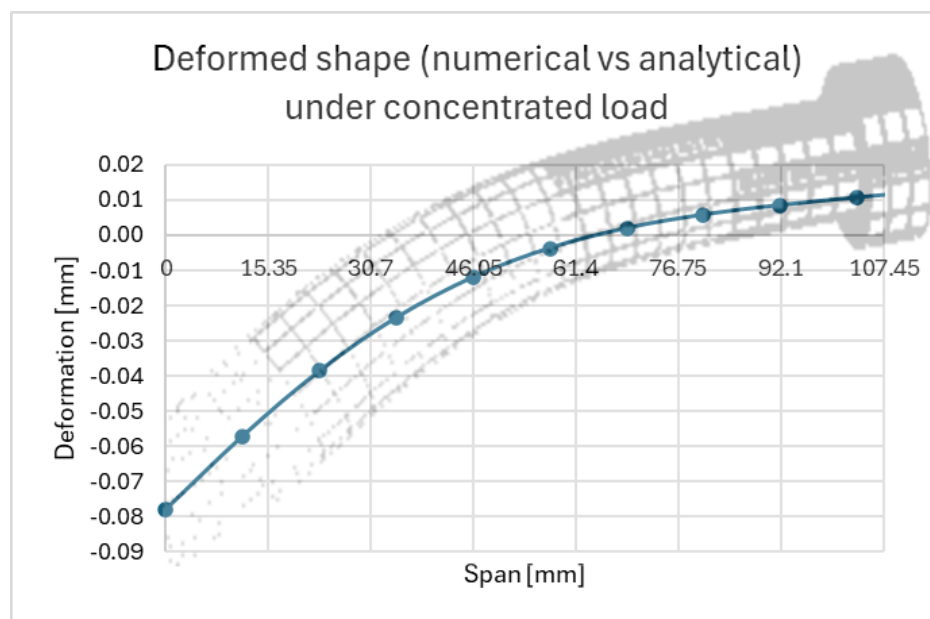


Figure 4.24: Symmetrical half model with Concentrated Load, Free-Free boundary conditions. Comparison between a picture of numerical deformed shape and an analytical deformed curve.

Although the deformed shape shows a match between the numerical and analytical results, the magnitude of deformation is significantly different. Based on that, it is essential to develop a dedicated structural model that accurately captures the real behaviour of bolts in glulam beam connections, and this is precisely the role of the correction factor introduced in the following section.

4.8.4 Correction factor

4.8.4.1 Assumptions for the Correction Factor

As noted, none of the analytical models consistently captured the deformation of the bolts. This discrepancy arises because the analytical models differ from the real system in several respects:

- Circular surface of the bolts, whereas in the models are for rectangular beams.
- The real elastic foundation surrounds the bolts from all directions, whereas in the suggested models, the elastic foundation acts only beneath the beam.
- The combination of two rows of bolts increases the pressure and deformation in reality, but this is not considered in the suggested models.

The analytical models also assume that the entire beam is in contact with the foundation, while in the numerical model, this is not the case. Figure 4.25 shows the regions of gaps between the bolts and the surrounding glulam, where the compression from the elastic foundation is not applicable, as well as the regions with direct contact with the glulam, where the elastic foundation provides support to the bolts.

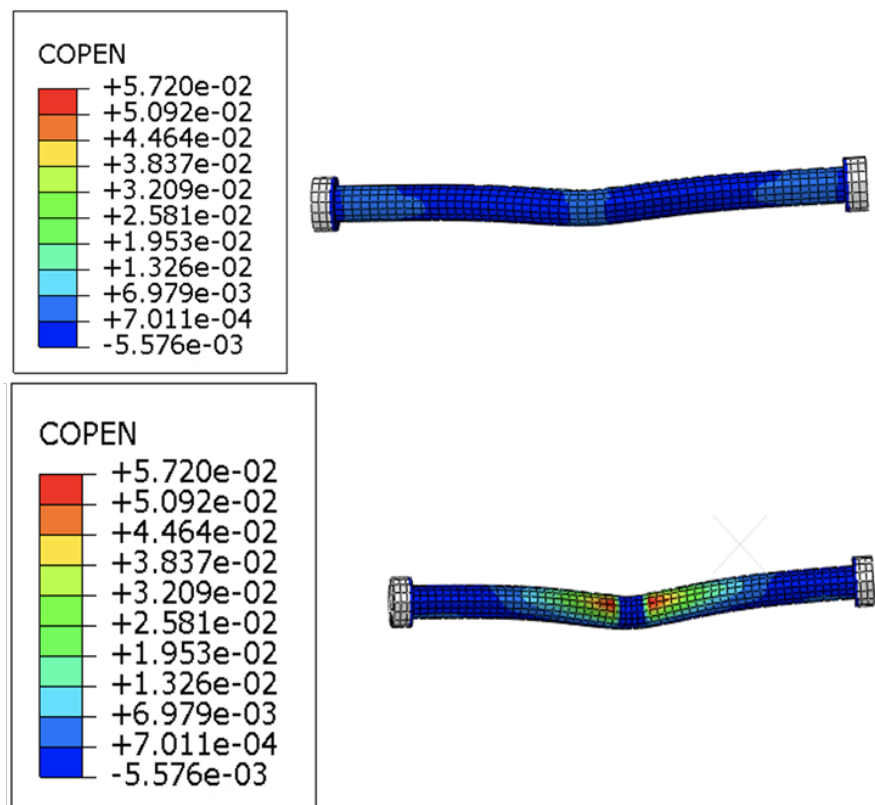


Figure 4.25: ABAQUS picture shows the areas of bolts that are not in contact with the glulam foundation.

4.8.4.2 Calculation method of correction factor

The new analytical model is therefore based on the maximum deformation of an infinite beam, which is the simplest case for evaluating deflection. The deflection is then multiplied by this correction factor to account for the previously mentioned discrepancies. Figure 4.26 shows a model for an infinite beam with a concentrated force in mid span.

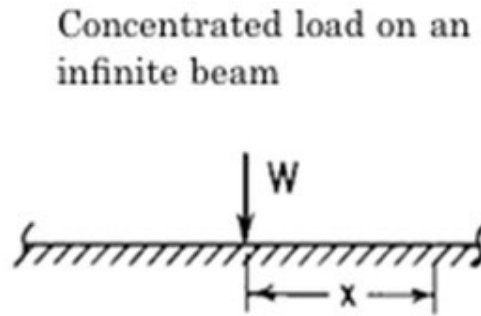


Figure 4.26: Infinite beam model with a concentrated load.

Maximum deflection for an infinite beam subjected to a concentrated load in mid span is calculated as follows:

$$\delta_{max} = \frac{W}{8EI_{bolts}\beta^3} \quad (4.23)$$

Where,

EI_{bolts} : flexural rigidity of one bolt

W : force in the top bolt

β : is the relative stiffness and is calculated as:

$$\beta = \sqrt[4]{\frac{K_{em}}{4EI_{bolt}}} \quad (4.24)$$

where:

K_{em} : is the embedment stiffness of glulam.

By inserting equation 4.24 into equation 4.23, the maximum deflection can be calculated as:

$$\delta_{max} = \frac{W}{2^{1.5} \cdot EI_{bolts}^{0.25} \cdot K_{em}^{0.75}} \quad (4.25)$$

The maximum deflection is then multiplied by a correction factor (F_{corr}) to get the correct bolt deformation.

$$\delta_{bolt} = \delta_{max} \cdot F_{corr} \quad (4.26)$$

The correction factor is determined experimentally and can be both less than or greater than 1. The correction factor is calculated according to equation 4.27:

$$F_{corr} = 0.51 + 0.023 \cdot \frac{d_{bolts} - 16}{d_{bolts}} + M_{bolts} \cdot N_{bolts} \cdot \frac{d_{bolts}}{H_{glulam}} \quad (4.27)$$

Where:

d_{bolts} : bolt diameter [mm]

M_{bolts} : number of bolt rows [-]

N_{bolts} : number of bolt columns [-]

H_{glulam} : height of the glulam beam [mm]

The term $0.023 \cdot \frac{d_{bolts} - 16}{d_{bolts}}$ accounts for the variation in bolt size.

The term $M_{bolts} \cdot N_{bolts} \cdot \frac{d_{bolts}}{H_{glulam}}$, accounts for the configuration of bolts.

4.8.4.3 Parametric study on correction factor

When the correction factor is applied to the bolt deformation, the analytical prediction of the bolt contribution in rotational stiffness shows good agreement with the numerical results, as illustrated in Figure 4.27.

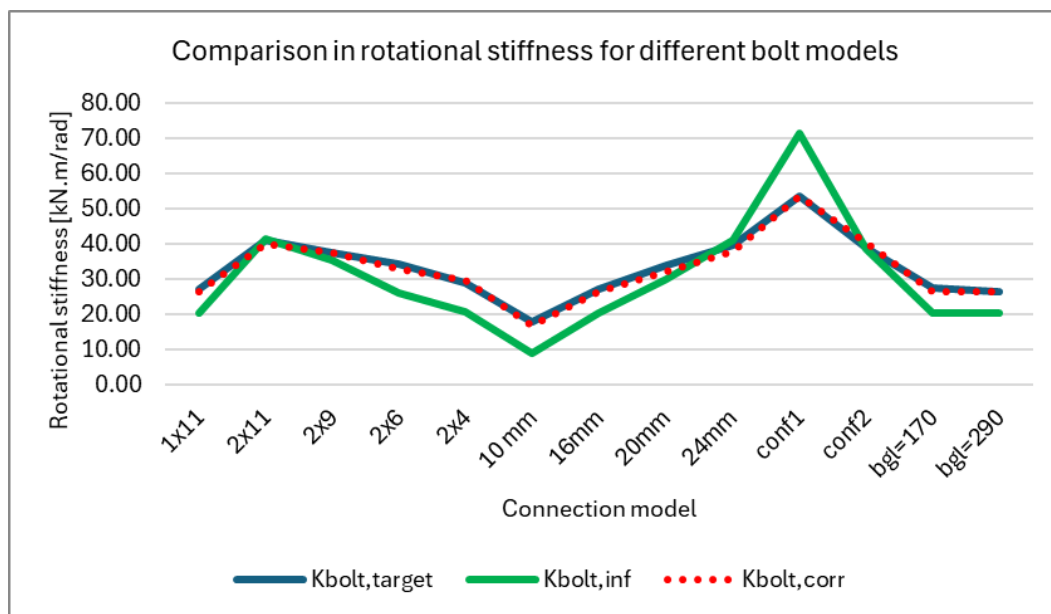


Figure 4.27: Rotational stiffness of the bolts.

Where:

$K_{bolt,target}$: numerical results

$K_{bolt,inf}$: analytical results without the correction factor

$K_{bolt,corr}$: analytical result including the correction factor

The comparison is based on the rotational stiffness of the bolts only, so the contributions from other components, such as the screws, back plate, and local bolt bearing, are excluded. Rotational stiffness is used instead of bolt deformation because the bolt deformation is neither purely axial nor uniform along its span. As a result, different nodes deform in different directions and magnitudes according to local parameters, making it difficult to define a single final deformation measure suitable for global analysis of screw deformation. In such a case, using local nodal deformations is not sufficiently accurate when the global bolt deformation is of interest. This is where the 3D solid model becomes less suitable, so a 2D structural beam model is used to determine the total rotational stiffness corresponding to the 3D solid model.

Figure 4.28 shows the nodal deformation in 3D solid model and how it varies in different locations. With values for the nodal deformation found in Table 4.1. The slope of bolt deformation is maximum at the exposed bolt surface (entry level) and decreases gradually with depth into the glulam beam, eventually reaching a horizontal (zero-slope) profile as shown in the figure.

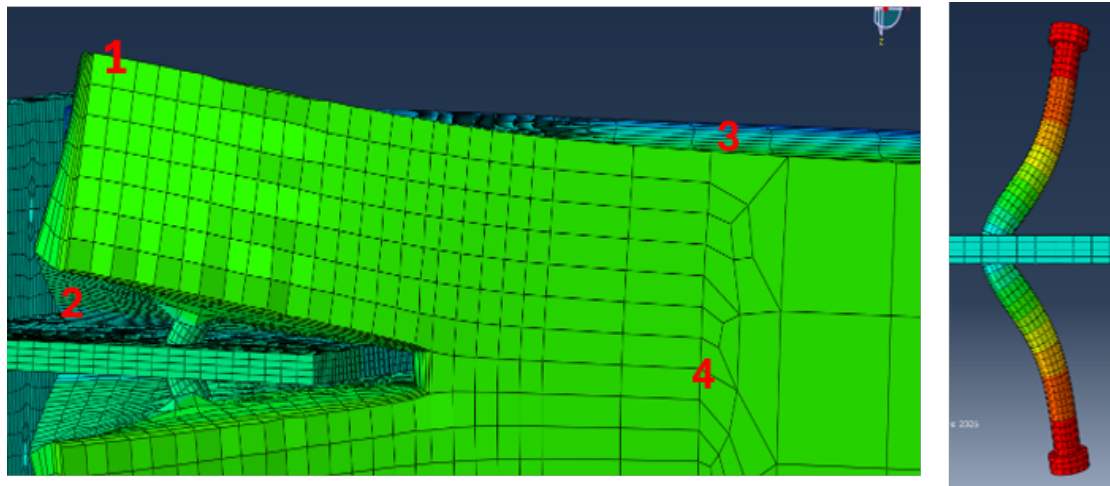


Figure 4.28: Abaqus picture shows the nodal deformation of timber and the bolt deformation

Table 4.1: Nodal deformation of timber and bolts.

Point	1	2	3	4
U_1 [mm]	0.4502	0.3928	0.4311	0.4144
Slope	0.050%		0.015%	

Timber shear stresses between the lamellas in glulam help distribute deformation more uniformly across the beam width. As a result, the observed deformation can be interpreted as an average response of the bolt rather than a purely local deformation. This makes it difficult to measure bolt deformation accurately and use it directly in the parametric study. For this reason, the total rotational stiffness of the connection is used instead of local bolt deformation as per other parts.

4.9 Total Rotational Stiffness calculation

Total rotational stiffness of the connection is obtained by the superposition of the rotational stiffness of each internal part: back plate, screws, bolts, and timber material.

Rotation of each part is calculated from two components: deformation and distance to neutral axis location, as follows:

$$\theta_i = \frac{\delta_i}{x_i} \quad (4.28)$$

The rotational stiffness is then calculated by:

$$K_i = \frac{M_i}{\theta_i} \quad (4.29)$$

And the total rotational stiffness is:

$$\frac{1}{K_{tot}} = \frac{1}{K_b} + \frac{1}{K_p} + \frac{1}{K_{b1}} + \frac{1}{K_s} \quad (4.30)$$

Where:

K_b : Stiffness from screw elongation

K_p : Stiffness from back plate bending

K_{b1} : Stiffness from bolts pressed under embedded plate

K_s : Stiffness from bolts pressing timber

K_{tot} : Total rotational stiffness

The calculation for the reference model and the contribution of each part is presented in Table 4.2.

Table 4.2: Rotational stiffness contributions of connection components.

Component	K [MN·m/Rad]	K ratio
K_s	1820	1.3%
K_p	622	3.9%
K_{b1}	866	2.8%
K_b	26.3	92.0%
Total	24.2	100.0%

The rotational stiffness of the reference model was determined to be 24.2 MNm/rad, corresponding to $(2.25 \cdot EI/L)$. As described in Section 2.6.1, Eurocode (2005) classifies a connection as nominally pinned when $(K \leq 0.5 \cdot EI/L)$, while a connection is considered rigid when $(K \geq 25 \cdot EI/L)$. Since the stiffness of the reference model falls between these two limits, the connection is classified as semi-rigid. Furthermore, its rotational restraint corresponds to approximately 9% of a fully fixed connection.

4.10 Total Rotational Stiffness verification

The rotational stiffness of the connection has been initially evaluated by decomposing the joint into its constituent components and calculating their individual rotational stiffness contributions. To validate this component-based approach, a global rotational stiffness can be determined using 2D structural beam model and compared with the previously obtained local stiffness values calculated using 3D solid model.

The ABAQUS 3D model provides detailed nodal displacement results, which can be used to quantify local deformations of individual components, such as screws elongation and back plate deformation at the connection with the embedded plate. However, assessing cross-sectional rotation directly from the 3D model is challenging. The deformation across the section is generally nonlinear, resulting in a curved profile rather than a linear distribution. This behavior contradicts the assumption of Bernoulli's hypothesis, which presumes plane sections remain plane and therefore makes direct extraction of rotation unreliable.

Moreover, measuring nodal displacements results in local deformations and rotations that are not valid when global/total deformations/rotations are discussed. Check section 4.8.4.3 for more information, where the effect of the local deformations in ABAQUS is discussed.

To overcome this limitation, a beam model representation is employed to evaluate the rotational response at the connection. An equivalent model is constructed in FEM-design, where the rotational stiffness is iteratively calibrated such that the maximum deformation in the cantilever/beam model matches that obtained from the ABAQUS simulation. The resulting stiffness value is thus considered the effective rotational stiffness of the connection, as it produces the same global deformation behavior under identical loading conditions. The models used to calibrate ABAQUS with structural beam models are shown in Table 4.3.

Table 4.3: Beam configurations and span lengths.

Beam ID	Support Condition	Span Length (m)
C1	Cantilever Beam	1.0
C2	Cantilever Beam	2.0
S7.3	Simply Supported Beam	7.3
S4	Simply Supported Beam	4.0

For those models, the total rotational stiffness extracted from Fem-design (2D structural beam model) is compared with the total rotational stiffness used in the analytical solution, and the resulting bending moment for each method is listed in Table 4.4.

Table 4.4: Comparison of hand-calculated and FEM bending moments.

Beam ID	C1	C2	S4	S7.3
M_{Hand} [kN·m]	10	10	5.58	28.21
M_{FEM} [kN·m]	10	10	5.3	27.60

While C1 and C2 are structurally determined models, the resulting analysis was independent of the connection stiffness and the same for both numerical and analytical methods. The undetermined structural models S4 and S7.3 produced only small, acceptable differences in the bending-moment results, which indicates a correspondingly minor variation in the calculated rotational stiffness. For more details about the analysis approach for undetermined structures, check Appendix F.

4.11 Simplified Formula

The results of the parametric study indicate that the contribution of the screws to the overall rotational stiffness is negligible. Since the objective of this thesis is to provide an estimate of the rotational stiffness rather than an exact prediction, the screw contribution has been omitted from equation 4.30 in section 4.9. This assumption allows for a more concise analytical expression while maintaining a satisfactory level of accuracy.

4.11.1 Calculation method for simplified formula

The simplified formula is a compact final equation used to describe the total rotational stiffness of the connection. It is simplified, accurate, and practical. The development of the simplified equation is described below:

Total Rotational Stiffness K_{tot}

In the simplified formula, the rotational stiffness is assumed to be provided by the back plate and the bolts. The back plate contributes through bending deformation between the screws located in the tension zone. The bolt contribution consists of two components: deformation caused by compression between the bolts and the embedded steel plate and flexural bending of the bolts within the timber beam.

The total rotational stiffness can be calculated as:

$$K_{tot} = \frac{1}{\left(\frac{1}{K_p} + \frac{1}{K_{b1}} + \frac{1}{K_b}\right)} \quad (4.31)$$

Where:

K_p : Stiffness from back plate bending

K_{b1} : Stiffness from bolts pressed under embedded plate

K_s : Stiffness from bolts pressing timber

K_{tot} : Total rotational stiffness

As previously explained in section 4.9, the rotational stiffness is calculated as $K = \frac{M}{\theta}$ and the rotation is calculated as $\theta = \frac{\delta}{x}$. Where x is the distance to the neutral axis. By combining these equations, the expression for the rotational stiffness becomes:

$$K_i = \frac{M}{\frac{\delta}{x}} \quad (4.32)$$

Or written as:

$$K_i = \frac{M \cdot x}{\delta} \quad (4.33)$$

This is the expression used to calculate the rotational stiffness of each internal part of the connection.

Rotational stiffness of back plate K_p

For the back plate, the distance to the neutral axis is expressed as:

$$x_{back} = H_{back} - y \quad (4.34)$$

The deformation of the back plate is calculated according to section 4.5, where the back plate is viewed as a fixed beam between the screws:

$$\delta_i = \frac{F_i L_s^3}{192 b_{eff} E_s I_{back,strip}} \quad (4.35)$$

By inserting this into equation 4.33, the rotational stiffness contribution of the back plate can be calculated as:

$$K_p = \frac{M \cdot (H_{back} - y)}{\frac{F_i \cdot L_s^3}{192 \cdot E I_{back,strip} \cdot b_{eff}}} \quad (4.36)$$

Or expressed as:

$$K_p = \frac{M \cdot (H_{back} - y) \cdot 192 \cdot E I_{back,strip} \cdot b_{eff}}{F_i \cdot L_s^3} \quad (4.37)$$

As stated in section 4.2.4.4, the neutral axis at the back plate level can be approximated by $y = \frac{H_{back}}{3}$. The rotational stiffness of the back plate can then be expressed as:

$$K_p = \frac{128 \cdot M \cdot H_{back} \cdot E I_{back,strip} \cdot b_{eff}}{F_i \cdot L_s^3} \quad (4.38)$$

Where,

$$L_s = \frac{H_{mid}}{2} - 0.8r - \frac{t_{embedded}}{2} \quad (4.39)$$

F_i : force in the top row of screws

M : applied bending moment

H_{back} : height of back plate

H_{mid} : horizontal center distance between screws

r : is the radius of the weld

$t_{embedded}$: thickness of the embedded steel plate

b_{eff} : calculated as follows

$$b_{eff} = \begin{cases} \frac{b}{0.06 + \frac{1}{(m/b)^2}}, & \text{if } m/b < 0.87, \\ b, & \text{if } m/b \geq 0.87. \end{cases} \quad (4.40)$$

Where, b : actual physical width of the T-stub beam

m : s the distance from the screw center to the embedded plate, reduced by 0.8r

Rotational stiffness of bolts bearing K_{b1}

For the bolts in the embedded steel plate, the local axial deformation of the bolts is calculated as explained in section 4.7 as:

$$\delta_{b1} = \frac{W}{Et_{embedded}} \quad (4.41)$$

Since the neutral axis of the bolt group is located at the mid-depth of the beam, the contribution of bolt bearing to the rotational stiffness can be calculated as:

$$K_{b1} = \frac{M \cdot y_b \cdot E \cdot t_{embedded}}{W} \quad (4.42)$$

Where,

M : applied bending moment

y_b : distance from neutral axis in center of beam to last bolt

$t_{embedded}$: thickness of the embedded steel plate

W : force in the top bolt

Rotational stiffness of bolts bending K_b

The deflection for the bolts bending in the timber is explained in section 4.8.4.2, and it could be calculated as:

$$\delta_{bolt} = \frac{W \cdot F_{corr}}{2^{1.5} \cdot EI_{bolts}^{0.25} \cdot K_{em}^{0.75}} \quad (4.43)$$

As previously noted, the neutral axis of the bolt group is located at the mid-depth of the beam, and the contribution of bolt bending to the rotational stiffness is calculated as:

$$K_b = \frac{M \cdot y_b \cdot 2^{1.5} \cdot EI_{bolts}^{0.25} \cdot K_{em}^{0.75}}{W \cdot F_{corr}} \quad (4.44)$$

Where,

M : applied bending moment

y_b : distance from neutral axis in center of beam to last bolt

W : force in the top bolt

EI_{bolts} : flexural rigidity of one bolt

K_{em} : embedment strength of beam material

F_{corr} : correction factor, calculated according to section 4.8.4.2 as:

$$F_{corr} = 0.51 + 0.023 \cdot \frac{d_{bolts} - 16}{d_{bolts}} + M_{bolts} \cdot N_{bolts} \cdot \frac{d_{bolts}}{H_{glulam}} \quad (4.45)$$

Where:

d_{bolts} : bolt diameter [mm]

M_{bolts} : number of bolt rows [-]

N_{bolts} : number of bolt columns [-]

H_{glulam} : height of the glulam beam [mm]

When having one column of bolts, or two columns with a relatively small horizontal spacing between them, the distance for the top bolt to the center of gravity of the bolt group can be approximated as the distance from the bolt centroid to the neutral axis, $y_b \approx r_i$. So by replacing y_b with r_i in equation 4.44 and replacing the force (W) with:

$$W = \frac{M \cdot r_i}{\sum r_i^2} \quad (4.46)$$

The rotational stiffness contribution resulting from bolt bending in the timber beam can now be determined directly, without the need to calculate the forces acting on the bolts. It is given by:

$$K_b = 2^{1.5} * EI_{bolts}^{0.25} * K_{em}^{0.75} * \frac{\sum r_i^2}{F_{corr}} \quad (4.47)$$

Where,

$\sum r_i^2$: is the sum of the squared distances from the centroid of the bolt group to the center of each bolt.

4.11.2 Result Simplified Formula

A final parametric study was conducted to investigate the variation in rotational stiffness, both analytically and numerically, as a combination of parameters in the connection is varied. The purpose is to assess the validity of the analytical approach by comparing its results with the numerical model. Figure 4.29 shows the results of three approaches: numerical model, analytical model that includes the screws' contribution, and simplified formula that neglects the contribution of the screws.

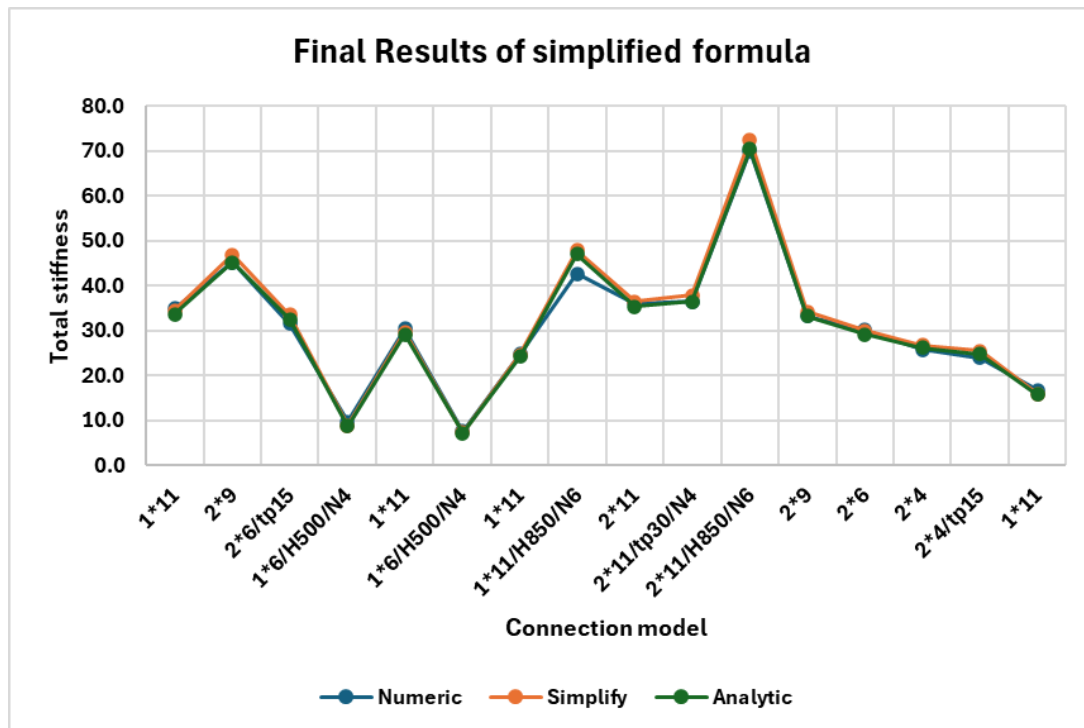


Figure 4.29: Comparison among numerical solution, analytical solution, and final simplified formula in the calculation of rotational stiffness for different connections.

Where:

N : number of screw rows

H : Height of glulam beam

t_p : back plate thickness

$x*xx$: bolts configuration (bolt column * bolt rows)

As observed in Figure 4.29, the simplified formula and the analytical model produce nearly identical results for all tested connection configurations. This suggests that neglecting the contribution of the screws to the rotational stiffness is a reasonable assumption. The maximum deviation between the numerical and analytical methods was 10%, whereas the simplified equation showed a maximum deviation of about 12%. These deviations were induced when increasing the height of the beam to 850mm, with 6 rows of screws in the back plate, and a 1*11 bolt configuration. This configuration is an outlier within the study. This deviation may be attributable to a modelling error in ABAQUS, although it could also reflect a variation in the proposed model.

These results indicate that the simplified formula proposed in this thesis provides a reasonable estimate of the rotational stiffness for an embedded steel plate–timber connection. The model does not yield precise values suitable for direct design use. Instead, it can serve as a useful tool to assess whether a given configuration behaves sufficiently flexibly to be considered pinned or whether it should be classified as semi-rigid.

5

Conclusion

The aim of this thesis was to develop an analytical model capable of evaluating the elastic rotational stiffness of an embedded steel plate–timber connection and to enhance the understanding of the mechanisms governing connection behaviour in hybrid timber–concrete structures.

A component-based analytical approach was developed in which the overall rotational stiffness was determined by combining the stiffness contributions of the back plate, screws, and bolts within the timber beam. As the deformation behaviour of the bolts could not be accurately represented using existing simplified models, a correction factor was introduced. The proposed model and correction factor were validated through comparison with numerical simulations. The predicted rotational stiffness showed good agreement with the numerical results across the investigated parameter range. These findings indicate that the proposed methodology successfully captures the governing deformation mechanisms of the connection.

The results showed that the bolts embedded in the timber beam are the dominant source of deformation, accounting for approximately 92% of the total flexibility in the reference model. The second most significant contribution comes from the bending of the back plate. However, for typical back plate thicknesses exceeding 15 mm, this contribution remains below 9%. In contrast, screw elongation was found to have a negligible influence on the overall rotational stiffness. Based on these findings, a simplified analytical expression was developed in which the contribution of screw elongation was neglected. Consequently, the rotational stiffness of the connection is governed primarily by the timber properties, bolt diameter, bolt configuration, and back plate thickness.

The study found that the reference model is classified as semi-rigid according to Eurocode (2005). This finding supports previous observations that embedded steel plate connections cannot be accurately idealized as perfectly pinned connections.

Overall, the thesis demonstrates that a relatively simple analytical approach can provide a reasonable estimation of rotational stiffness for embedded steel plate–timber connections. The developed methodology offers a practical alternative to computationally demanding numerical analyses. Although the proposed model is limited to the linear-elastic range and should not be used directly for detailed design, it provides valuable insight into connection behaviour and can assist engineers in determining whether a connection should be treated as pinned, semi-rigid, or rigid in structural analysis.

5.1 Future studies

This study was limited to the linear-elastic behaviour of the connection. Future research could therefore investigate the nonlinear response and ultimate failure behaviour of embedded steel plate–timber connections. Such studies could identify the governing failure modes and assess how steel yielding, timber crushing, and other nonlinear effects influence both rotational stiffness and load-carrying capacity.

Further research could also extend the proposed methodology to other types of timber–steel and steel–steel connections. This would help determine the applicability of the component-based approach to a wider range of structural details.

In addition, the correction factor introduced in this study was derived from numerical simulations and does not fully explain the underlying physical behaviour of the bolts. Future studies could focus on developing an analytical model capable of describing the deformation mechanism of the bolts inside the timber directly. Such a model would provide a stronger theoretical basis for the prediction of rotational stiffness and reduce the reliance on empirically calibrated correction factors.

6

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Appendix A - Mesh Analysis

A mesh sensitivity analysis was carried out in ABAQUS to determine appropriate global mesh sizes for both the timber beam and the steel connection. The results of the analysis are presented in Figure A.1 and Figure A.2.

The analysis showed that the global mesh size had no significant influence on the timber beam results. This is expected, as the beam is subjected to pure bending and exhibits linear-elastic behaviour. In addition, the mesh was locally refined around the bolts and near the steel plate, ensuring that local stress concentrations and interaction effects were adequately captured. Based on these findings, a global mesh size of 50 mm was selected for the timber beam to achieve good computational efficiency.

For the steel connection, a global mesh size of 10 mm was chosen, as the mesh sensitivity analysis indicated that the results had converged at this level of refinement.

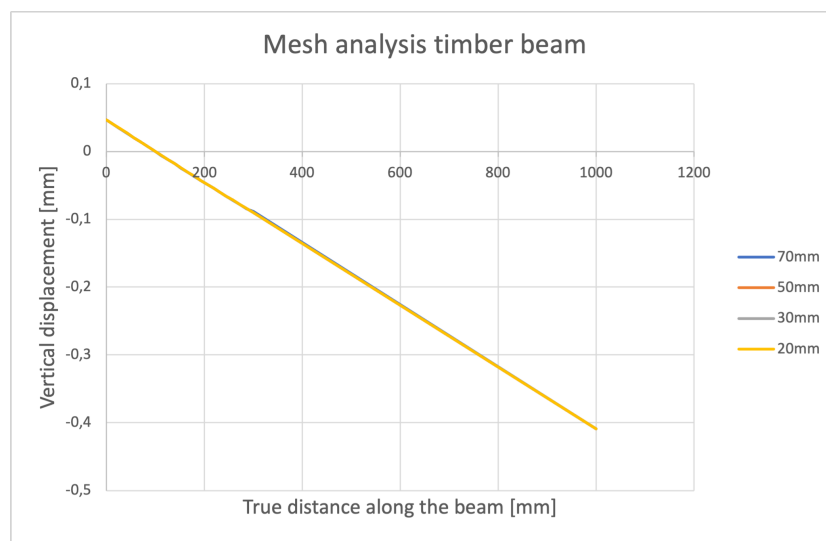


Figure A.1: Global mesh analysis for the timber beam.

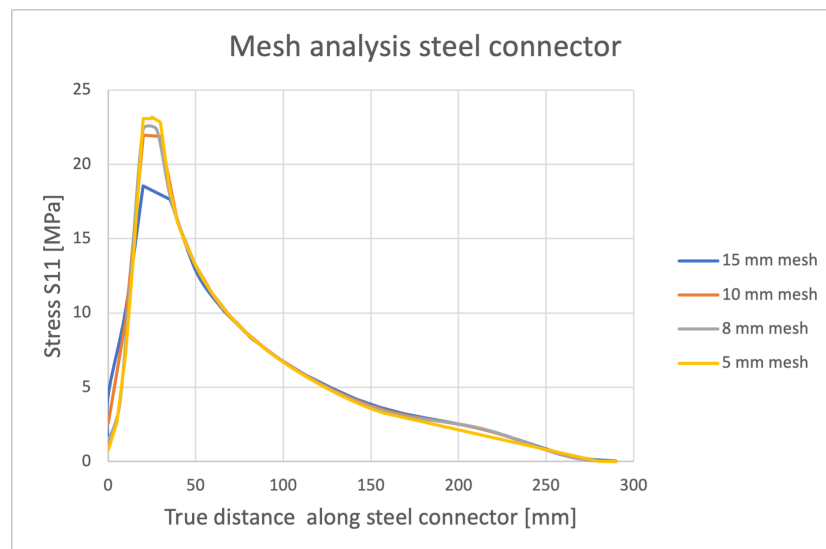


Figure A.2: Global mesh analysis for the steel connection.

Appendix B: Material Properties and Sectional Constants

Material Properties



Timber:

Material	Glulam GL30c
k_{mod}	0.8
$\rho_k =$	390 kg/m ³
$\rho_m =$	430 kg/m ³
$E_{0,mean} =$	13 Gpa

Bolts & Screws:

$f_y =$	640 Mpa
$f_u =$	800 Mpa
$E_b =$	210 Gpa

Steel Plates:

S355	
$f_y =$	355 Mpa
$f_u =$	530 Mpa
$E_s =$	210 Gpa

Concrete:

$E_c =$	30 Gpa
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Sectional Constants



Moment of inertial

$I_{knife} =$	3.8E+08 mm ⁴	
$I_{back,strip} =$	666.667 mm ⁴ /mm	
$I_{glulam} =$	5.9E+09 mm ⁴	
$I_{back} =$	8.9E+09 mm ⁴	
$I_{bolts} =$	3216.99 mm ⁴	
$I_{screws} =$	3216.99 mm ⁴	
$A_{screw} =$	201.1 mm ²	
$\alpha = E_b/E_c =$	7	modular ratio

Appendix C: Connection Analysis

Neutral Axis



Apply bending moment within elasticity range

$$M = 10 \text{ kN.m}$$

Approach 1: Hybrid method

Assumptions:

stress distribution is linear along knife plate face

stress distribution is not linear in screws

linear strain distribution

Principle:

$$M = \Sigma F * y \quad \text{moment equilibrium in cross section}$$

$$y = M / \Sigma F \quad y \text{ is the eccentricity}$$

$$\text{where } M = \Sigma (A_i * \sigma_i * y_i)$$

$$F = \Sigma (A_i * \sigma_i)$$

$$\Sigma (\sigma_i * A_i * y_i) = \Sigma (\sigma_i * A_i) * y$$

$$y = \Sigma (\sigma_i * A_i * y_i) / \Sigma (\sigma_i * A_i)$$

Distance of screws from the bottom of back plate

$x_1 =$	725.0 mm
$x_2 =$	556.3 mm
$x_3 =$	387.5 mm
$x_4 =$	218.8 mm
$x_5 =$	50.0 mm
$x_6 =$	0.0 mm
$x_7 =$	0.0 mm
$x_8 =$	0.0 mm
$x_9 =$	0.0 mm
$x_{10} =$	0.0 mm
$x_c =$	-259.6 mm

Distances to N.A at different points

Take $x =$ 259.6 mm From bottom of back plate (Initial guess of N.A)

$y_1 =$	465.40 mm	At level of first row screws
$y_{12} =$	364.15 mm	At 60% of distance between y_1 & y_2 from y_1
$y_2 =$	296.65 mm	At level of 2nd row screws
$y_{23} =$	212.28 mm	Between 2nd and 3rd row of screws
$y_3 =$	127.90 mm	At level of 3rd row screws
$y_{34} =$	43.53 mm	Between 3rd and 4th row of screws
$y_4 =$	-40.85 mm	At level of 4th row screws
$y_{45} =$	-125.23 mm	Between 4th and 5th row of screws
$y_5 =$	-209.60 mm	At level of 5th row screws
$y_{56} =$	-234.60 mm	Between 5th and 6th row of screws
$y_6 =$	-259.60 mm	At level of 6th row screws
$y_{67} =$	-259.60 mm	Between 6th and 7th row of screws

$y_7 =$	-259.60 mm	At level of 7th row screws
$y_{78} =$	-259.60 mm	Between 7th and 8th row of screws
$y_8 =$	-259.60 mm	At level of 8th row screws
$y_{89} =$	-259.60 mm	Between 8th and 9th row of screws
$y_9 =$	-259.60 mm	At level of 9th row screws
$y_{910} =$	-259.60 mm	Between 9th and 10th row of screws
$y_{10} =$	-259.60 mm	At level of 10th row screws
$y_c =$	-259.60 mm	At the bottom of compression zone

Stress Distribution in back plate due to its moment of inertia

$$\sigma = M / I_{\text{knife}} * y$$

$\sigma_1 =$	12.11 Mpa	$\sigma_{\text{avg},1} =$	10.79 Mpa
$\sigma_{12} =$	9.47 Mpa		
$\sigma_2 =$	7.72 Mpa	$\sigma_{\text{avg},2} =$	7.50 Mpa
$\sigma_{23} =$	5.52 Mpa		
$\sigma_3 =$	3.33 Mpa	$\sigma_{\text{avg},3} =$	3.33 Mpa
$\sigma_{34} =$	1.13 Mpa		
$\sigma_4 =$	-1.06 Mpa	$\sigma_{\text{avg},4} =$	0.57 Mpa
$\sigma_{45} =$	-3.26 Mpa		
$\sigma_5 =$	-5.45 Mpa	$\sigma_{\text{avg},5} =$	0.00 Mpa
$\sigma_{56} =$	-6.10 Mpa		
$\sigma_6 =$	-6.75 Mpa	$\sigma_{\text{avg},6} =$	0.00 Mpa
$\sigma_{67} =$	-6.75 Mpa		
$\sigma_7 =$	-6.75 Mpa	$\sigma_{\text{avg},7} =$	0.00 Mpa
$\sigma_{78} =$	-6.75 Mpa		
$\sigma_8 =$	-6.75 Mpa	$\sigma_{\text{avg},8} =$	0.00 Mpa
$\sigma_{89} =$	-6.75 Mpa		
$\sigma_9 =$	-6.75 Mpa	$\sigma_{\text{avg},9} =$	0.00 Mpa
$\sigma_{910} =$	-6.75 Mpa		
$\sigma_{10} =$	-6.75 Mpa	$\sigma_{\text{avg},10} =$	0.00 Mpa
$\sigma_c =$	-6.75 Mpa	$\sigma_{\text{avg},c} =$	-3.38 Mpa

Arbitrary Area for each row of screws and compression zone

$b_{\text{max}} =$	106.5 mm	max width for arbitrary area for each bolt
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$$\text{Experimentally: } b_{\text{max}} = H_{\text{mid},s} - d_{\text{screw}}$$

$$A_{\text{ar},i} = b_i * b_{\text{back}}$$

Strip Width (Between Screws)

$A_{\text{ar},\text{sc}1} =$	1519 mm ²	$b_{b,1} =$	101.25 mm
$A_{\text{ar},\text{sc}2} =$	1598 mm ²	$b_{b,2} =$	106.50 mm
$A_{\text{ar},\text{sc}3} =$	1598 mm ²	$b_{b,3} =$	106.50 mm
$A_{\text{ar},\text{sc}4} =$	1598 mm ²	$b_{b,4} =$	106.50 mm
$A_{\text{ar},\text{sc}5} =$	1598 mm ²	$b_{b,5} =$	106.50 mm
$A_{\text{ar},\text{sc}6} =$	653 mm ²	$b_{b,6} =$	43.53 mm

$$\begin{aligned}
A_{ar,sc7} &= 0 \text{ mm}^2 & b_{b,7} &= 0.00 \text{ mm} \\
A_{ar,sc8} &= 0 \text{ mm}^2 & b_{b,8} &= 0.00 \text{ mm} \\
A_{ar,sc9} &= 0 \text{ mm}^2 & b_{b,9} &= 0.00 \text{ mm} \\
A_{ar,sc10} &= 0 \text{ mm}^2 & b_{b,10} &= 0.00 \text{ mm} \\
A_{ar,c} &= 3894 \text{ mm}^2 & b_c = b_{back}, A_{ar,c} &= b_c * (x)
\end{aligned}$$

Neutral Axis (Weighted by the stresses and arbitrary areas on front surface) (moment equilibrium)

$$\begin{aligned}
x &= \sum A_i * \sigma_{avg,i} * (x_i - x) / \sum \sigma_{avg} * A_i \\
x &= (A_{ar,sc,1} * \sigma_{avg,1} * (x_1 - x) + A_{ar,sc,2} * \sigma_{avg,2} * (x_2 - x) + A_{ar,sc,3} * \sigma_{avg,3} * (x_3 - x) + \\
&A_{ar,sc,4} * \sigma_{avg,4} * (x_4 - x) + \dots - (x)^2 / 3 * b_{back} * \sigma_{avg,c}) / \sum (\sigma_{avg} * A_i)
\end{aligned}$$

assumed 259.6 mm

x = 259.6 mm ok From bottom of back plate

Simplified approach extracted from the hybrid approach

Simplified: $\sum (A_i * d_i^2) / (A_i * d_i)$ where σ_i is linearly proportional to d_i

Note the similarity between this method and vector method used to analyse bolts in steel connections

$$\begin{aligned}
N_i &= M * r_i / \sum r_i^2 & \rightarrow & N_i * e_i = N_i / (r_i / \sum r_i^2) \\
e_i &= \sum r_i^2 / r_i & & \text{where } A_i \text{ is equal for all bolts}
\end{aligned}$$

Take x = 246.2 mm

x = 246.2 mm ok

Approach 2: First moment of area Around Neutral Axis (EC)

Assumptions

According to Eurocode: SS-EN 1992-4:2018 €

The fixture is sufficiently rigid such that linear strain distribution will be valid (analogous to Bernoulli hypothesis)

If this requirement for the deformation is not fulfilled the elastic deformation behaviour of the fixture shall be taken into account adequately to determine the design value of tension loads acting on each fastener.

Principle

$$A_{sc,1} * (x_1 - y) + A_{sc,2} * (x_2 - y) + A_{sc,3} * (x_3 - y) + A_{sc,4} * (x_4 - y) = y^2 / 2 * b_{back} / \alpha$$

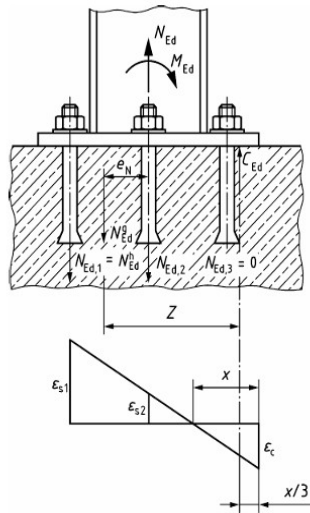
y = 171.5 mm from bottom of back plate

$N_{sc,comp}$ = 1 screws in compression

$N_{sc,ten}$ = 4 screws in tension

Equilibrium equation = -49 N.mm ≈ 0 ok

Analysis of Screws



Key

$$N_{Ed, i} = \varepsilon_{s, i} \cdot E_s \cdot A_s$$

$$C_{Ed} = 0,5 \cdot b_{fix} \cdot x \cdot \varepsilon_c \cdot E_c$$

$$I = 1.8E+09 \text{ mm}^4$$

moment of inertial of cross section

$$\sigma_i = M / I * (x_i - y) * \alpha$$

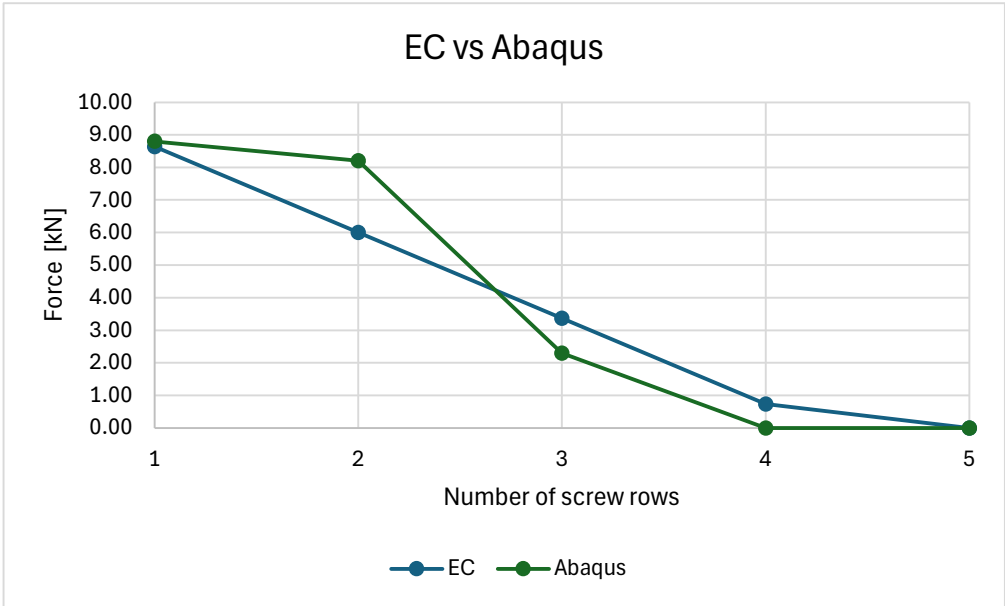
$\sigma_1 =$	21.48 Mpa	$F_1 =$	8.64 kN	For two screws
$\sigma_2 =$	14.93 Mpa	$F_2 =$	6.01 kN	For two screws
$\sigma_3 =$	8.38 Mpa	$F_3 =$	3.37 kN	For two screws
$\sigma_4 =$	1.83 Mpa	$F_4 =$	0.74 kN	For two screws
$\sigma_5 =$	-4.72 Mpa	$F_5 =$	-1.90 kN	For two screws
$\sigma_6 =$	-6.66 Mpa	$F_6 =$	-2.68 kN	For two screws
$\sigma_7 =$	-6.66 Mpa	$F_7 =$	-2.68 kN	For two screws
$\sigma_8 =$	-6.66 Mpa	$F_8 =$	-2.68 kN	For two screws
$\sigma_9 =$	-6.66 Mpa	$F_9 =$	-2.68 kN	For two screws
$\sigma_{10} =$	-6.66 Mpa	$F_{10} =$	-2.68 kN	For two screws

Data from Abaqus (Forces in Screws [kN])

Row	2	3	4	5	6	7	8	9	10
$F_1 =$	19.6	13.3	10.1	8.8	7.7	7	6.4	6	5.7
$F_2 =$		5.7	8.3	8.2	7.7	7	6.4	5.8	5.3
$F_3 =$			0.1	2.3	3.6	4.1	4.2	4.1	4
$F_4 =$						1.1	2.2	2.6	2.8
$F_5 =$								0.7	1.4
$F_6 =$									
$F_7 =$									
$F_8 =$									
$F_9 =$									
$F_{10} =$									
Sum	19.6	19	18.5	19.3	19	19.2	19.2	19.2	19.2

Forces in Screws

Force	EC	Hybrid	Abaqus
$F_1 =$	8.64	16.4	8.8 kN
$F_2 =$	6.01	12.0	8.2 kN
$F_3 =$	3.37	5.3	2.3 kN
$F_4 =$	0.74	0.9	0 kN
$F_5 =$	0.00	0.0	0 kN
$F_6 =$	0.00	0.0	0 kN
$F_7 =$	0.00	0.0	0 kN
$F_8 =$	0.00	0.0	0 kN
$F_9 =$	0.00	0.0	0 kN
$F_{10} =$	0.00	0.0	0 kN
Sum	18.75	34.58	19.3 kN



Analysis of bolts



Distances to N.A at different points

Take x = 337.5 mm

from bottom of knife plate

Analysis of bolts: Vector method

$$N_i = M * r_i / \sum r_i^2$$

$y_1 =$ 250 mm

$y_2 =$ 200 mm

$y_3 =$ 150 mm

$y_4 =$ 100 mm

$y_5 =$ 50 mm

$y_6 =$ 0 mm

$y_7 =$ -50 mm

$y_8 =$ -100 mm

$y_9 =$ -150 mm

$y_{10} =$ -200 mm

$y_{11} =$ -250 mm

$x =$ 0 mm

$r_1 =$ 250.00 mm

$r_2 =$ 200.00 mm

$r_3 =$ 150.00 mm

$r_4 =$ 100.00 mm

$r_5 =$ 50.00 mm

$r_6 =$ 0.00 mm

$r_7 =$ -50.00 mm

$r_8 =$ -100.00 mm

$r_9 =$ -150.00 mm

$r_{10} =$ -200.00 mm

$r_{11} =$ -250.00 mm

$\sum r_i^2 =$ 275000 mm²

Bolt	Force
$N_1 =$	9.1 kN
$N_2 =$	7.3 kN
$N_3 =$	5.5 kN
$N_4 =$	3.6 kN
$N_5 =$	1.8 kN
$N_6 =$	0.0 kN
$N_7 =$	-1.8 kN
$N_8 =$	-3.6 kN
$N_9 =$	-5.5 kN
$N_{10} =$	-7.3 kN
$N_{11} =$	-9.1 kN
sum	27.3 kN

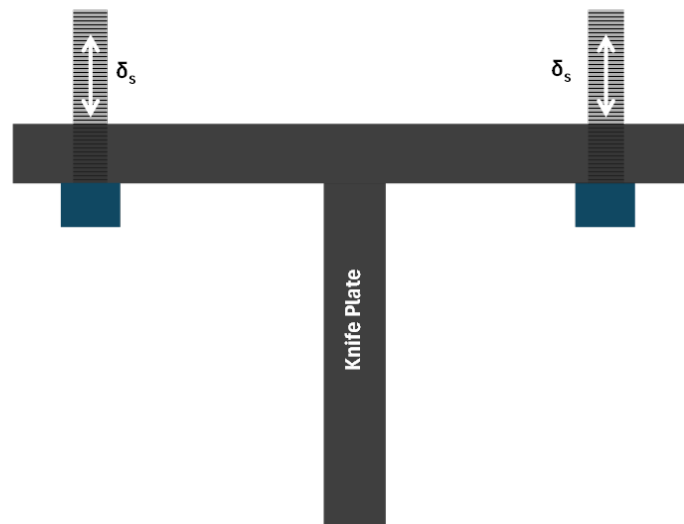
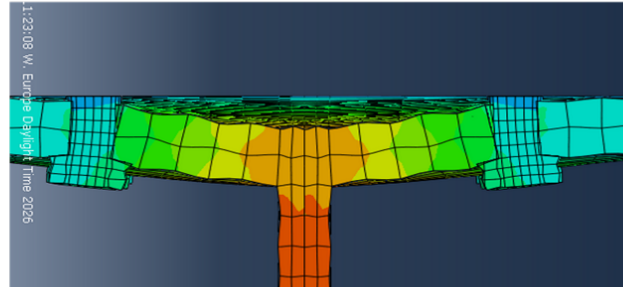
For single bolt in case of 2 rows of bolts

Appendix D: Deformation of screws, back plate, and bolts bearing

Deformation of Screws ↔



Axial tension forces in screws cause elongation of screws in back plate



According to EN 1993-1-8:2005 L_b is the deformable part of the screw where :

- L_b The bolt elongation length, taken equal to the grip length (total thickness of material and washers), plus half the sum of the height of the bolt head and the height of the nut
- Or the anchor bolt elongation length, taken equal 10 the sum of 8 times the nominal bolt diameter, the grout layer, the plate thickness, the washer and half the height of the nut
- For simplification and to compare with numerical solution:
- $L_b = t_{back} + 5 \text{ mm}$

$$\delta_s = F * L_b / EA$$

forces from EC

$$L_b = 25 \text{ mm}$$

$$\delta_1 = 0.00256 \text{ mm}$$

$$\delta_2 = 0.00178 \text{ mm}$$

$$\delta_3 = 0.001 \text{ mm}$$

$$\delta_4 = 0.00022 \text{ mm}$$

$$\delta_5 = 0 \text{ mm}$$

$$\delta_6 = 0 \text{ mm}$$

$$\delta_7 = 0 \text{ mm}$$

$$\delta_8 = 0 \text{ mm}$$

$$\delta_9 = 0 \text{ mm}$$

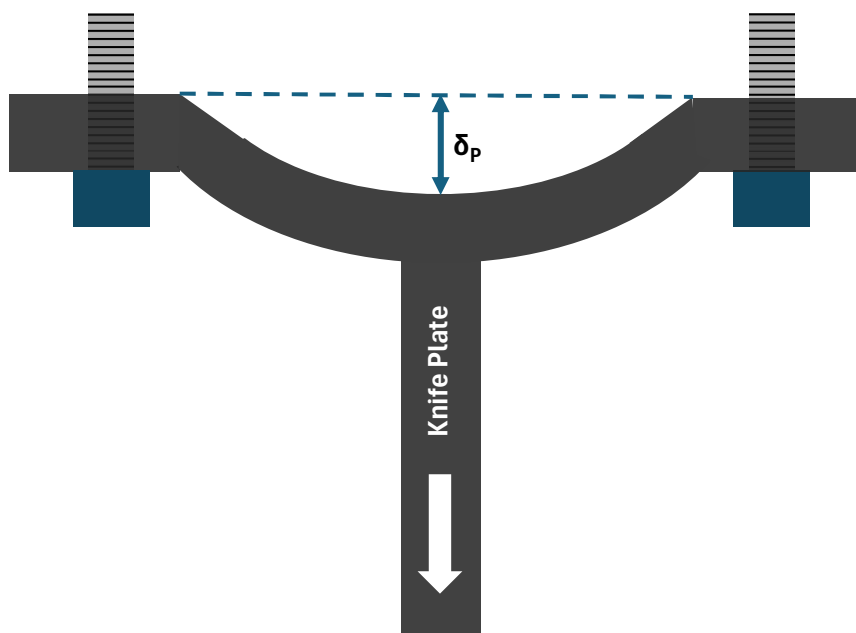
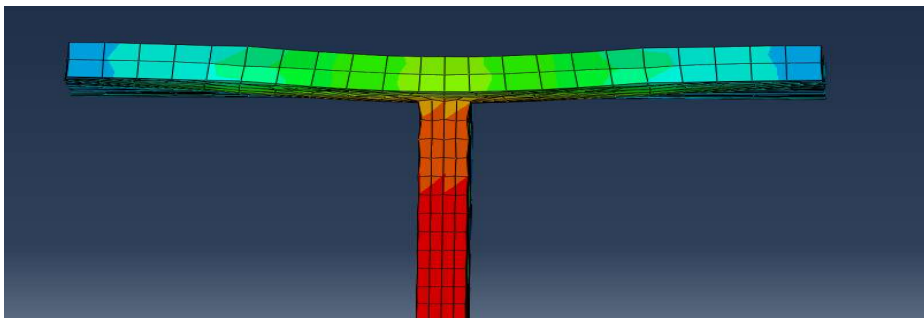
$$\delta_{10} = 0 \text{ mm}$$

$$\theta_s = \delta_s / x_s$$

$$\theta_s = 5.5E-06 \text{ Rad}$$

$$k_s = 1819.6 \text{ MNm/Rad}$$

Deformation of Back Plate 10



Lemonis and Gantes, in their paper "Incremental Modeling of T-Stub Connections," proposed an effective width (b_{eff}) for the T-stub analogous to the b_{eff} defined in Eurocode 3, but specifically adapted for stiffness calculations rather than ultimate resistance strength. These values are derived by equating the numerically computed displacements of the T-stub plates to those of an equivalent simple beam model.

$$\frac{b}{b_{\text{eff}}} = \begin{cases} 0.92 + \frac{0.06}{(m/b)^2}, & m/b < 0.87, \\ 1, & m/b \geq 0.87. \end{cases}$$

where: b is the actual physical width of the t-stub beam.
 m is shown in following figure

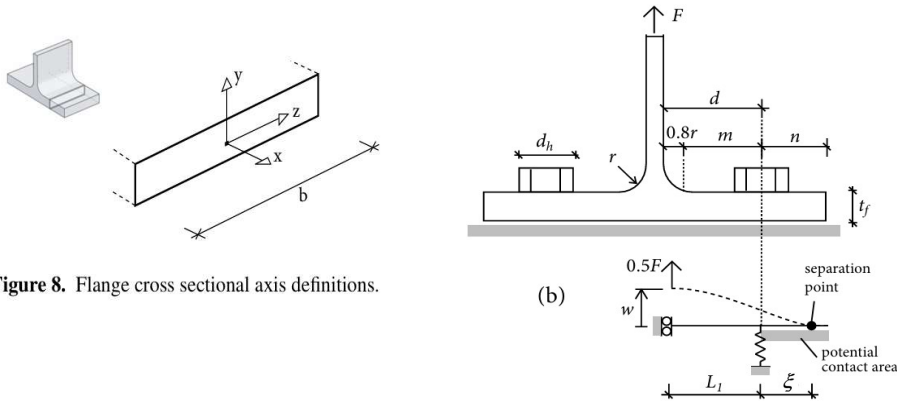


Figure 8. Flange cross sectional axis definitions.

$d =$	53.75	mm
$r =$		mm
$n =$	53.75	mm
$L_1 =$	53.75	mm

From surface of knife plate to the screw axis

Flange to web fillet radius

Distance from the screw axis to the outer edge of the end-plate

Distance between the screw axis and the point where the flange bears against the web (compression block).

$$L_1 = H_{\text{mid}}/2 - 0.8r - t_{\text{knife}}/2$$

Effective width of strips

$b_{\text{max}} =$	306.5	mm
--------------------	-------	----

$b_{b,1} =$	84.38	mm	$b_{\text{eff},1} =$	79.01	mm
$b_{b,2} =$	151.88	mm	$b_{\text{eff},2} =$	92.16	mm
$b_{b,3} =$	168.75	mm	$b_{\text{eff},3} =$	92.16	mm
$b_{b,4} =$	168.75	mm	$b_{\text{eff},4} =$	92.16	mm
$b_{b,5} =$	127.90	mm	$b_{\text{eff},5} =$	92.16	mm
$b_{b,6} =$	43.53	mm	$b_{\text{eff},6} =$	43.53	mm
$b_{b,7} =$	0.00	mm	$b_{\text{eff},7} =$	0.00	mm
$b_{b,8} =$	0.00	mm	$b_{\text{eff},8} =$	0.00	mm
$b_{b,9} =$	0.00	mm	$b_{\text{eff},9} =$	0.00	mm
$b_{b,10} =$	0.00	mm	$b_{\text{eff},10} =$	0.00	mm

Simplified model (Ignoring prying forces)

Prying forces in t-stub affects the deformation and the forces in screws as per following pictures

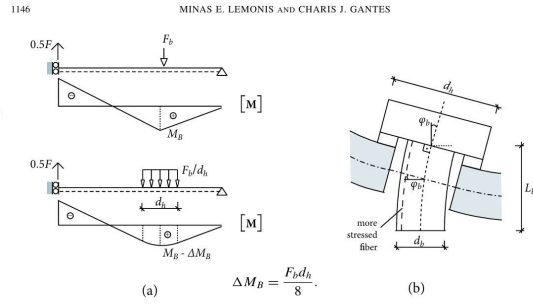
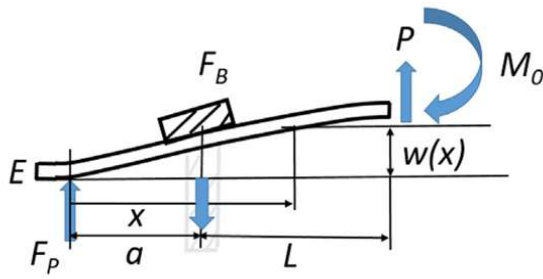
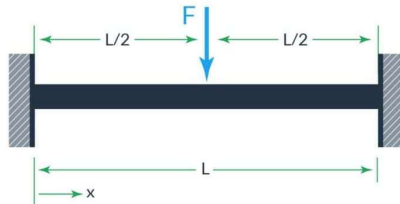


Figure 7. Influence of bolt head size (a) and definitions for the bolt bending treatment (b).

"INCREMENTAL MODELING OF T-STUB CONNECTIONS" suggested an iterational solution to find the separation point and the response of the t-stub

Since the distribution of back plate in total rotational stiffness is small we can ignore the prying action effect and simplify the model into simple beam or fixed-fixed beam as follows

Model Fixed-Fixed



$$L_s = 122.5 \text{ mm}$$

Fixed-Fixed

$$\delta_i = F_i * L_s^3 / (192 * b_{\text{eff},i} * E_s * I_{\text{back,strip},i})$$

Forces from EC

$$\delta_1 = 0.0075 \text{ mm}$$

$$\delta_2 = 0.0045 \text{ mm}$$

$$\delta_3 = 0.0025 \text{ mm}$$

$$\delta_4 = 0.0005 \text{ mm}$$

$$\delta_5 = 0.0000 \text{ mm}$$

$$\delta_6 = 0.0000 \text{ mm}$$

$$\delta_7 = 0.0000 \text{ mm}$$

$$\delta_8 = 0.0000 \text{ mm}$$

$$\delta_9 = 0.0000 \text{ mm}$$

$$\delta_{10} = 0.0000 \text{ mm}$$

$$\theta = \delta_1 / (H_{\text{knife}} - (y - V_{\text{end}}))$$

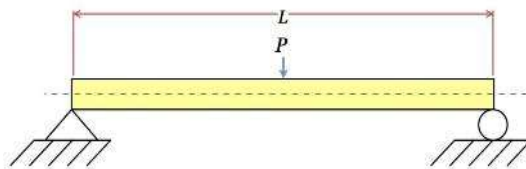
$$y - V_{\text{end}} = 209.6 \text{ mm}$$

$$\theta = 0.00002 \text{ Rad}$$

$$k = M/\theta$$

$$k_{F-F} = 622.4 \text{ MN.m/Rad}$$

Model Pinned-Pinned



$$L_s = 122.5 \text{ mm}$$

Pinned-Pinned

$$\delta_i = F_i * L_s^3 / (48 * b_{\text{eff},i} * E_s * I_{\text{back,strip},i})$$

Forces from EC

$$\delta_1 = 0.0299 \text{ mm}$$

$$\delta_2 = 0.0178 \text{ mm}$$

$$\delta_3 = 0.0100 \text{ mm}$$

$$\delta_4 = 0.0022 \text{ mm}$$

$$\delta_5 = 0.0000 \text{ mm}$$

$$\delta_6 = 0.0000 \text{ mm}$$

$$\delta_7 = 0.0000 \text{ mm}$$

$$\delta_8 = 0.0000 \text{ mm}$$

$$\delta_9 = 0.0000 \text{ mm}$$

$$\delta_{10} = 0.0000 \text{ mm}$$

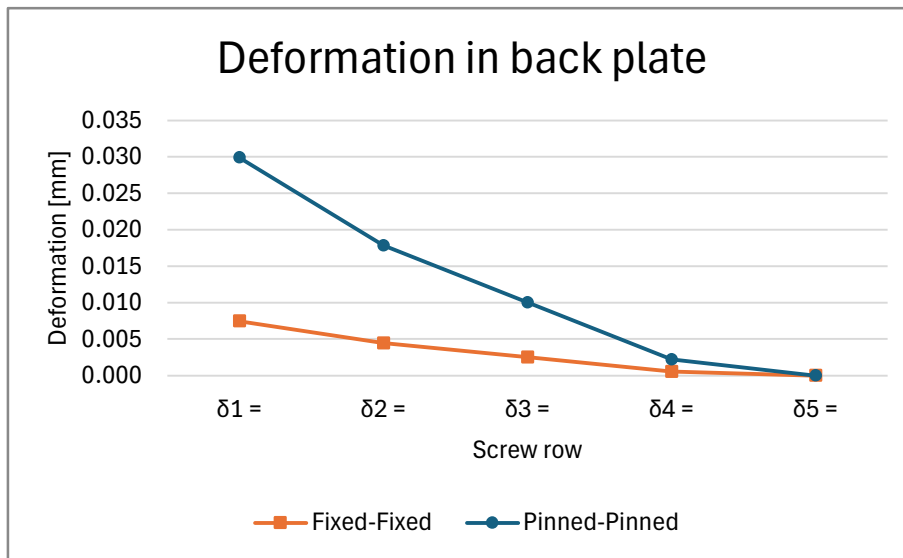
$$\theta = \delta_1 / (H_{\text{knife}} - (y - V_{\text{end}}))$$

$$y - V_{\text{end}} = 209.6 \text{ mm}$$

$$\theta = 0.0001 \text{ Rad}$$

$$k = M/\theta$$

$$k_{P-P} = 155.6 \text{ MN.m/Rad}$$



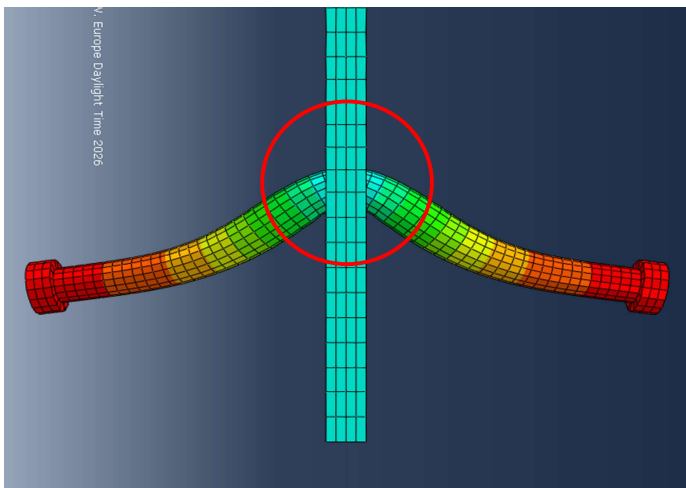
Choose Fixed-Fixed

$k_p =$ 622.4 MN.m/Rad

Deformation in Bolts due to Local bearing

The bolts are subjected to high bearing compressive stresses from the knife plate because the force transfer occurs over a relatively small contact area between the knife plate and the bolts. This may lead to local deformation of the bolt cross-section, often described as bearing deformation or local crushing.

The figure shows local bearing deformation of the bolt under the knife plate, which leads to a reduction in the effective bolt diameter in the loaded region.



$$\delta_{b1} = W * M_{bolts} * d_{bolt} / (EA_{bolts})$$

$$W = 9091 \text{ N} \quad \text{Force on one row of bolts}$$

$$A_{bolt} = 240 \text{ mm}^2$$

$$\delta_{b1} = 0.00289 \text{ mm}$$

$$\theta_{b1} = \delta_{b1} / y_b$$

$$\theta_{b1} = 0.00001 \text{ Rad} \quad 866.25$$

$$k_{b1} = M / \theta_{b1}$$

$$k_{b1} = 866.25 \text{ MN.m/Rad}$$

Appendix E: Deformation of bolts globally

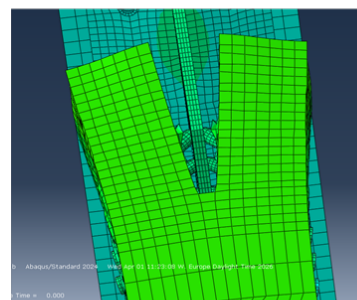
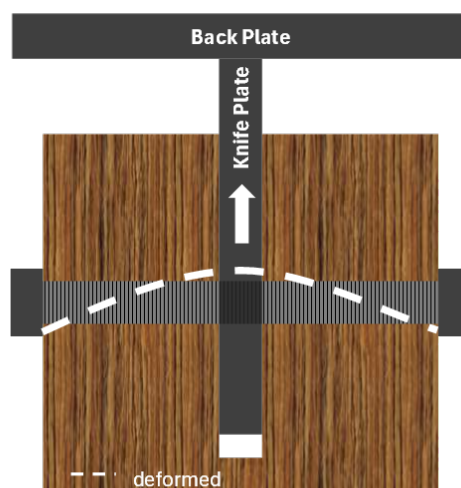
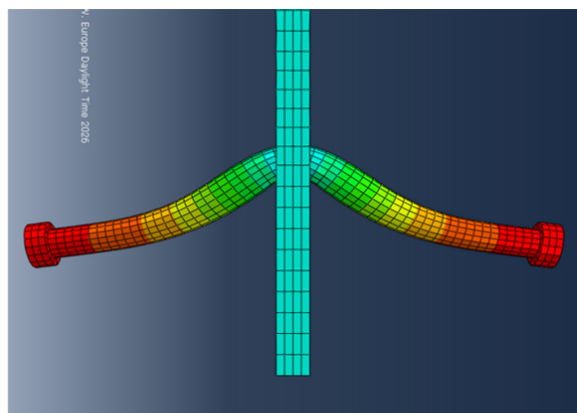
Deformation of Bolts due to Global Bending



The bolts in the knife plate connection are subjected to transverse loading, primarily concentrated at their midspan due to force transfer from the knife plate. Each bolt is laterally supported by the surrounding glulam member, which provides continuous confinement along its embedded length.

When the connection is loaded, the forces introduced through the knife plate are transferred to the bolts and resisted by bearing and deformation of the surrounding timber. This interaction between the bolt and the glulam results in a distributed reaction along the bolt length.

This behavior can be idealized as a beam on an elastic foundation, where the bolt is modeled as a beam, the surrounding timber acts as the elastic foundation, and the load from the knife plate is represented as a concentrated force applied at midspan. This simplification captures the load transfer mechanism and the coupled deformation between the bolt and the timber.



Determining foundation modulus for glulam GL30c

Foundation modulus measures the stiffness of the elastic foundation, which is GL30c in our model
Experimentally

According to: "Experimental evaluation and theoretical prediction of single hardwood dowel connections in softwood and hardwood"

Table 3

Key results from embedment tests using steel and TO dowels in RP and SG timber (coefficient of variation, COV, in parentheses).

Materials ID	n	MC %	L mm	w mm	t mm	f_c MPa	$f_{c,max}$ MPa	K_{em} kN/mm	k_{em} N/mm ³
SGSD	5	10.66 (6.32)	100	120	42	72.48 (4.16)	-	63.50 (11.06)	-
RPSD	5	11.59 (7.23)	100	120	45	37.69 (4.30)	-	42.33 (14.71)	-
SGWD	5	10.83 (5.43)	100	120	42	14.87 (1.91)	18.65 (4.64)	8.82 (3.04)	11.05 (3.04)
RPWD	5	11.82 (5.11)	100	120	45	8.75 (2.28)	10.96 (4.46)	6.05 (7.75)	7.08 (7.75)

The foundation modulus for different types of materials according to the laboratory results are:

Spotted Gum (SG) $K_{em} = 63.5$ kN/mm where $\rho_m = 1040.1$ kg/m³
Radiata Pine (RP) $K_{em} = 42.33$ kN/mm where $\rho_m = 521.53$ kg/m³
Glulam GL30c $K_{em} = ??$ kN/mm where $\rho_m = 430$ kg/m³

Convert to k_{em} [kN/mm³]

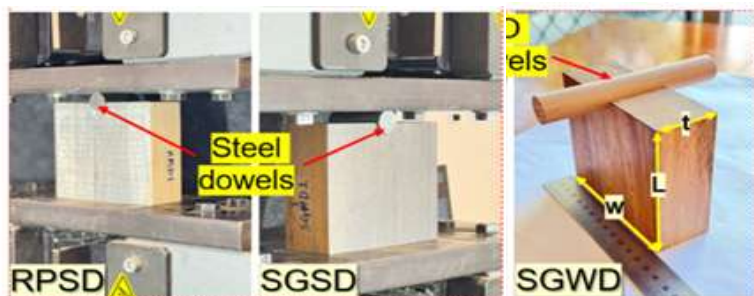
for RP: $k_{em,RP} = K_{em} / (d * t)$ where $d = 19$ mm, $t = 45$ mm
 $k_{em,RP} = 49.51$ N/mm³

For GL30c glulam, an approximate foundation modulus value similar to that of radiata pine can be adopted as a conservative estimate.

Numerically

By modeling the same experimental test from the referenced scientific study as a 3D solid finite element model in Abaqus, numerical values for the foundation modulus of both GL30c and RP can be obtained.

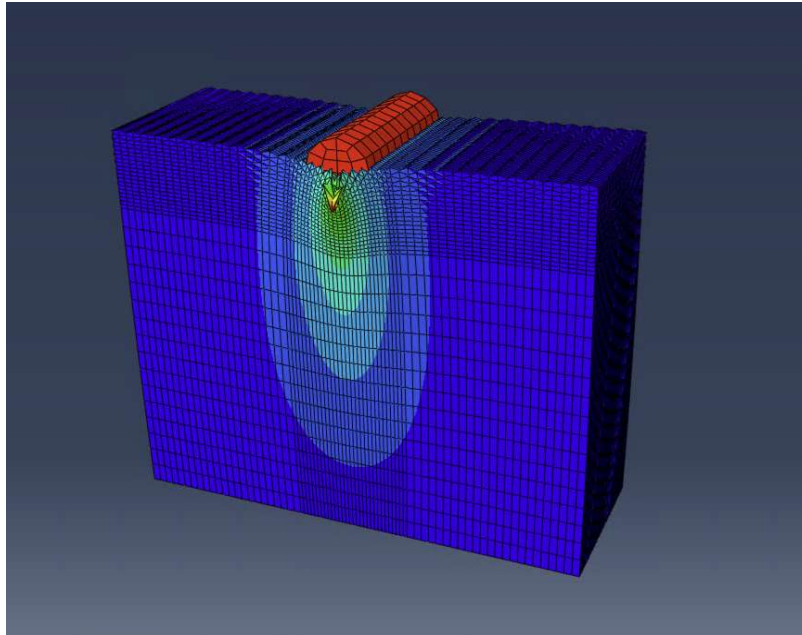
These results can then be directly compared with the experimentally derived values for radiata pine, enabling quantitative validation of both the numerical model and the adopted results from this model.



L mm	w mm	t mm
100	120	42

Model in Abaqus

same dimensions are used as per experimental test, the load applied is 10KN within the elasticity range



Radiata pine = MGP12 (due to same density $521 \approx 520 \text{ kg/m}^3$)

Property	Value	Unit	Source
ρ_{mean}	520	kg/m^3	Experiment; FWPA; CSIRO
$E_{\text{o,mean}}$	12,000	MPa	AS/NZS 1748 (MGP12); AS/NZS 1720.1; FWPA
$E_{\text{90,mean}}$	450	MPa	AS/NZS 1720.1 ratios; Australian research
G_{mean}	750	MPa	AS/NZS 1720.1; FWPA; CSIRO research

Glulam material properties gl30c

E_1	E_2	E_3	Nu_{12}	Nu_{13}	Nu_{23}	G_{12}	G_{13}	G_{23}
13000	600	600	0.3	0.3	0.3	650	650	50
Mpa	Mpa	Mpa				Mpa	Mpa	Mpa

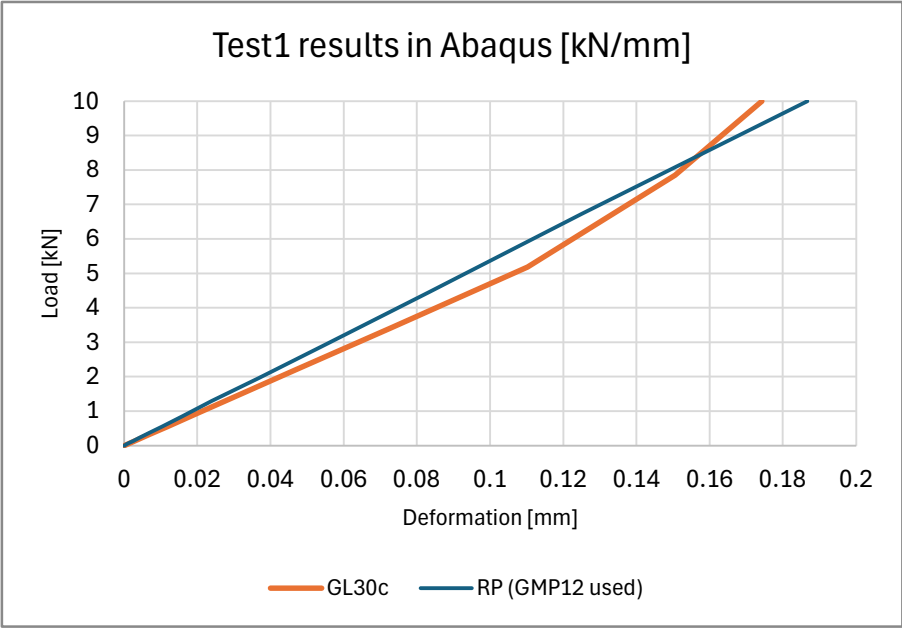
Test results: L = 42mm, d = 16mm

GL30c		
Load[kN]	δ [mm]	K_{em}
0	0	[kN/mm]
0.16	0.003	46.71
0.31	0.007	46.64
0.55	0.012	46.75
0.90	0.019	46.77
1.43	0.030	46.93
2.22	0.047	46.93
3.40	0.073	46.90
5.18	0.110	47.01
7.85	0.151	52.12
10.00	0.174	57.41
$K_{\text{em, GL30c}}$		46.91 kN/mm

RP (GMP12 used)		
Load[kN]	δ [mm]	K_{em}
0	0	[kN/mm]
0.04	0.001	53.13
0.08	0.001	55.71
0.14	0.003	52.92
0.22	0.004	53.08
0.36	0.007	53.13
0.55	0.010	53.27
0.85	0.016	53.46
1.30	0.024	54.17
1.96	0.037	53.26
2.96	0.056	53.24
4.47	0.084	53.42
6.72	0.125	53.76
10.00	0.187	53.56
$K_{\text{em, RP}}$		53.27 kN/mm

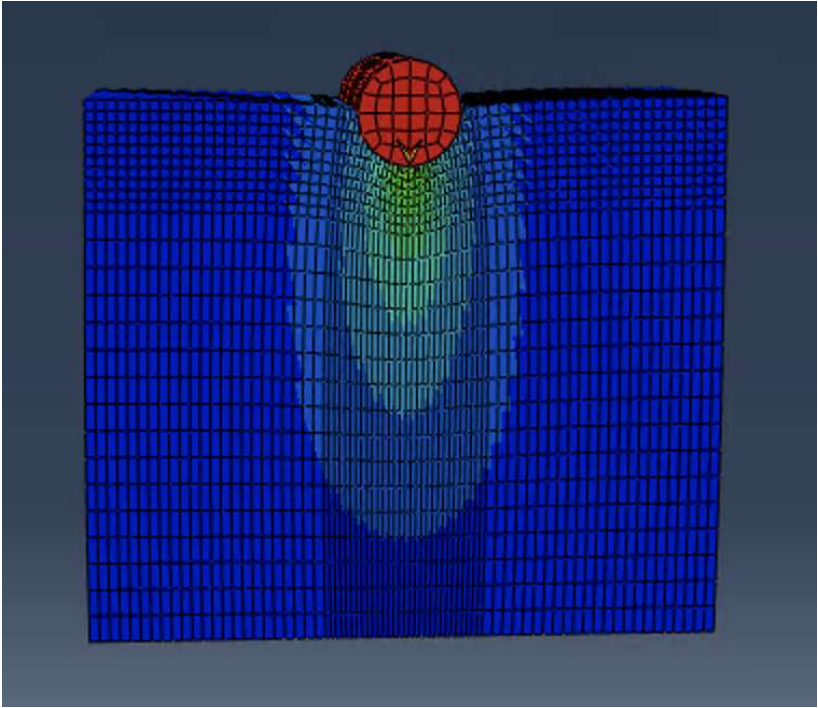
$k_{em, GL30c}$	69.81	N/mm^3
$k_{em, RP}$	79.27	N/mm^3

$K_{em, GL30c}$	88% $K_{em, RP}$
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The slope of the curves is the foundation modulus [kN/mm]

Test2



Test results: L = 45mm, d = 19mm

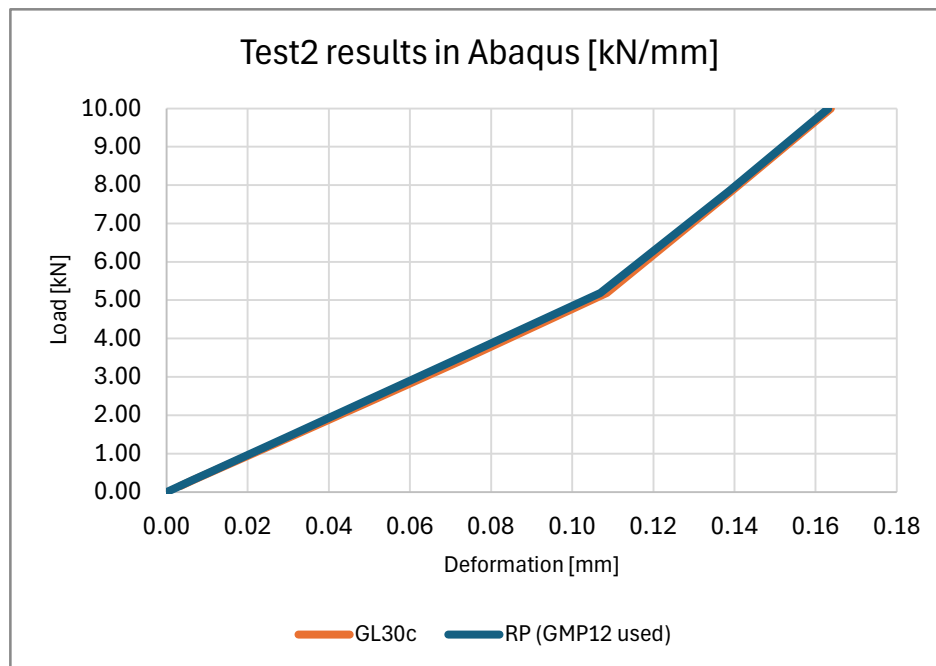
GL30c		
Load[kN]	δ [mm]	K_{em}
		[kN/mm]
0.16	0.0033	46.88
0.31	0.007	46.93
0.55	0.012	46.94
0.90	0.019	46.96
1.43	0.030	47.02
2.22	0.047	47.08
3.40	0.072	47.17
5.18	0.109	47.77
7.85	0.139	56.33
10.00	0.164	61.09
$K_{em, GL30c}$		47.05 kN/mm

RP (GMP12 used)		
Load[kN]	δ [mm]	K_{em}
		[kN/mm]
0	0	0
0.16	0.003	48.18
0.31	0.006	48.20
0.55	0.011	48.23
0.90	0.019	48.23
1.43	0.030	48.27
2.22	0.046	48.32
3.40	0.070	48.39
5.18	0.107	48.48
7.85	0.139	56.62
10.00	0.163	61.31
$K_{em, RP}$		48.30 kN/mm

$k_{em, GL30c}$	55.03	N/mm ³
$k_{em, RP}$	56.49	N/mm ³

$K_{em, GL30c}$	97% $K_{em, RP}$
-----------------	------------------

This ratio is used to calibrate the foundation modulus in experimental results



Take : Numerical Results Test2 Calibration ratio 1

$$K_{em, GL30c} = 55.03 \text{ N/mm}^3$$

Mean density for glulam is 430kg/m³ smaller than mean density for RP with 520 kg/m³

Converting unit from kN/mm³ to N/mm² to use in Winkler equation

$$k_{em, GL30c} = K_{em, GL30c} \cdot d_{bolt}$$

$$k_{em, GL30c} = 880.4 \text{ N/mm}^2$$

This value describes how stiff the wood behaves locally when compressed by the dowel.

Embedement strength:

$$f_h = 0.082 * \rho_k * (1 - 0.01 * D) = 26.86 \text{ Mpa}$$

Stresses in timber shall not be exceed embedement strength to be in elastic range

Calculation of deformation

Models:

Whole bolt models	Using the total length of the bolts
Symmetrical half models	Using the half length of the bolts

Boundary Conditions for Whole bolt models

- Concentrated load in mid span, Free-Free, end- to mid-span deflection
- Concentrated load in mid span, Free-Free, mid-span deflection
- Distributed load, Free-Free, mid-span deflection
- Concentrated Load, infinite beam, max deflection
- Concentrated Load, semi-infinite beam, max deflection

Boundary Conditions for Symmetrical half models

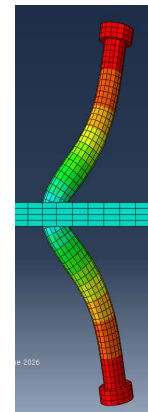
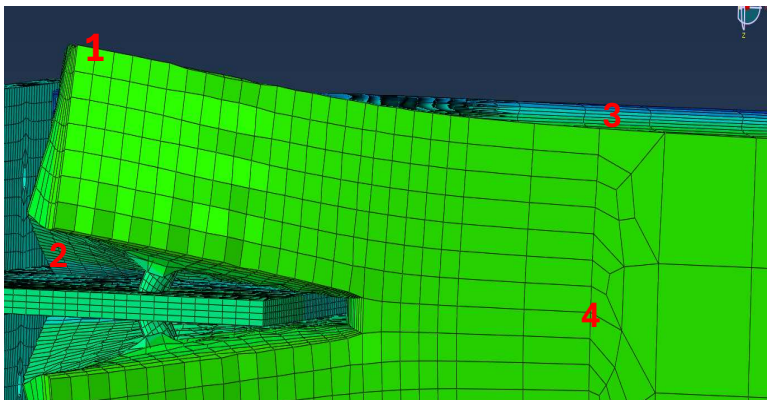
- Concentrated Load, Free-Free, end- to mid-span deflection
- Concentrated Load + bending moment, Free-Free, end- to mid-span deflection
- Concentrated, Free-Free, mid-span deflection
- Distributed load, Free-Fixed, mid- to end-span deflection
- Distributed load, Free-Fixed, end-span deflection

Models' explanation

End- to mid-span deflection: is the integral of deformation curve (area under the deformed shape of bolt) divided by the length of the bolt. By other words, is the average deformation value for the bolt. This happens due to timber shear stresses.

Mid-span deflection: is the deflection at the mid span y_A

Timber shear stresses



Point	1	2	3	4	
U1 [mm]	0.45021	0.39279	0.4311	0.41438	mm
slope	0.050%		0.015%		

Conclusions

The slope of bolt deformation is maximum at the exposed bolt surface (entry level) and decreases gradually with depth into the glulam beam, eventually reaching a horizontal (zero-slope) profile as shown in the figure.

Timber shear stresses between the lamellas in glulam help distribute deformation more uniformly across the beam width. As a result, the observed deformation can be interpreted as an average response of the bolt rather than a purely local deformation.

This makes it difficult to measure bolt deformation accurately and use it directly in the parametric study. For this reason, the total rotational stiffness of the connection is used instead of local bolt deformation as per other parts.

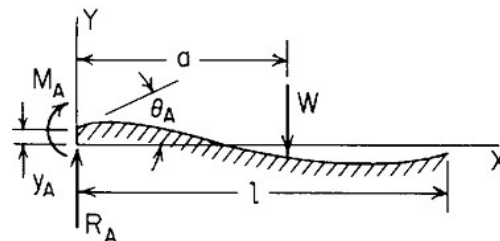
Whole bolt models

Concentrated load in mid span, Free-Free, end- to mid-span deflection

According to: "Roarks fomulas for stress and strain by WARREN C. YOUNG
RICHARD G. BUDYNAS"

Deflection in uppermost screw:

1. Concentrated intermediate load



$W = 9091 \text{ N}$ Force from knife plate (simplified to concentrated)

$$\beta = \sqrt[4]{(k/4EI)} = (k/EI)^{(1/4)}$$

$$\beta = (k_{em}/(4 \cdot E \cdot I))^{0.25} \text{ mm}^{-1}$$

$$\beta = 0.0239 \text{ mm}^{-1}$$

$$E \cdot I_{bolts} = 6.8E+08 \text{ N.mm}^2$$

k_{em} is the foundation modulus (Winkler)

$$l = 230 \text{ mm} \quad \text{beam span}$$

$$l - a = 115 \text{ mm} \quad \text{distance from left support to the concentrated load}$$

$$\beta l = 5.50 < 6 \text{ OK, model is valid}$$

since $\beta l < 6$ we can use Table 8.5 of Roark's Table for Stress and Strain (Warren and Richard, 2002).

$$C_2 = \cosh \beta l \sin \beta l + \sinh \beta l \cos \beta l$$

$$C_2 = \cosh \beta l \cdot \sin \beta l + \sinh \beta l \cdot \cos \beta l$$

$$C_2 = -0.479$$

$$C_3 = \sinh \beta l \cdot \sin \beta l$$

$$C_3 = \sinh \beta l \cdot \sin \beta l$$

$$C_3 = -86.318$$

$$C_4 = \cosh \beta l . \sin \beta l - \sinh \beta l . \cos \beta l$$

$$C_4 = \cosh \beta l . \sin \beta l - \sinh \beta l . \cos \beta l$$

$$C_4 = -172.16$$

$$C_{a1} = \cosh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a1} = \cosh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a1} = -7.2335$$

$$C_{a2} = \cosh \beta (l - a) . \sin \beta (l - a) + \sinh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a2} = \cosh \beta (l - a) . \sin \beta (l - a) + \sinh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a2} = -4.1664$$

$$C_{11} = \sinh^2 \beta l - \sin^2 \beta l$$

$$C_{11} = \sinh^2 \beta l - \sin^2 \beta l$$

$$C_{11} = 14819.3$$

	Right end	Free
Left end	$R_A = 0$	$M_A = 0$
Free	$\theta_A = \frac{W}{2EI\beta^2} \frac{C_2 C_{a2} - 2C_3 C_{a1}}{C_{11}}$	$y_A = \frac{W}{2EI\beta^3} \frac{C_4 C_{a1} - C_3 C_{a2}}{C_{11}}$

$$\theta_A = -0.001 \text{ Rad}$$

$$y_A = 0.02949 \text{ mm}$$

$$F_1 = \cosh \beta x \cos \beta x$$

$$F_2 = \cosh \beta x \sin \beta x + \sinh \beta x \cos \beta x$$

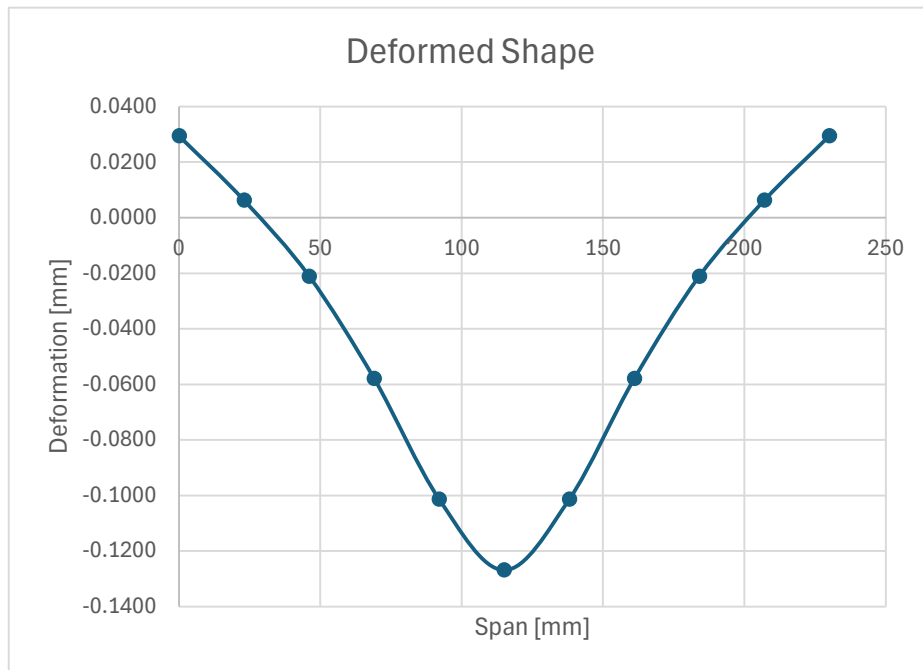
$$F_{a4} = \cosh \beta \langle x - a \rangle \sin \beta \langle x - a \rangle - \sinh \beta \langle x - a \rangle \cos \beta \langle x - a \rangle$$

$$\text{Deflection} = y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 - \frac{W}{4EI\beta^3} F_{a4}$$

x [mm]	F ₁ =	F ₂ =	F _{a4} =	y _(x) -y ₍₀₎	A _i =(x _i *y _i)	Sum
0	1.00	0.00	0.00	0.0295	0.21	0.21
23	0.98	1.10	0.00	0.006	0.12	0.33
46	0.76	2.09	0.00	-0.021	-0.54	-0.21
69	-0.21	2.49	0.00	-0.058	-1.37	-1.58
92	-2.68	1.08	0.00	-0.101	-2.23	-3.81
115	-7.23	-4.17	0.00	-0.127	-2.62	-6.43
138	-13.37	-15.43	0.11	-0.101	-2.23	-8.66
161	-17.84	-33.01	0.88	-0.058	-1.37	-10.03
184	-12.62	-51.17	2.88	-0.021	-0.54	-10.57
207	16.24	-52.14	6.30	0.006	0.12	-10.45
230	85.85	-0.48	10.18	0.029	0.21	-10.24

The deformed shape $y(x)$ has a value $[-,+]$ but in order to do the integral take $y_{(x)} - y_{(0)} \geq 0$ (the curve is ready for integration)

$\delta_b =$	0.0445 mm
--------------	-----------



Double check deformation

Assuming rigid beam (bolts are much stiffer than timber in flexural stiffness)

$$\delta = W [N] / k_{em} [N/mm]$$

$$W = 9091 \text{ N}$$

$$k_{em} = 202499 \text{ N/mm} \quad \text{Axial stiffness}$$

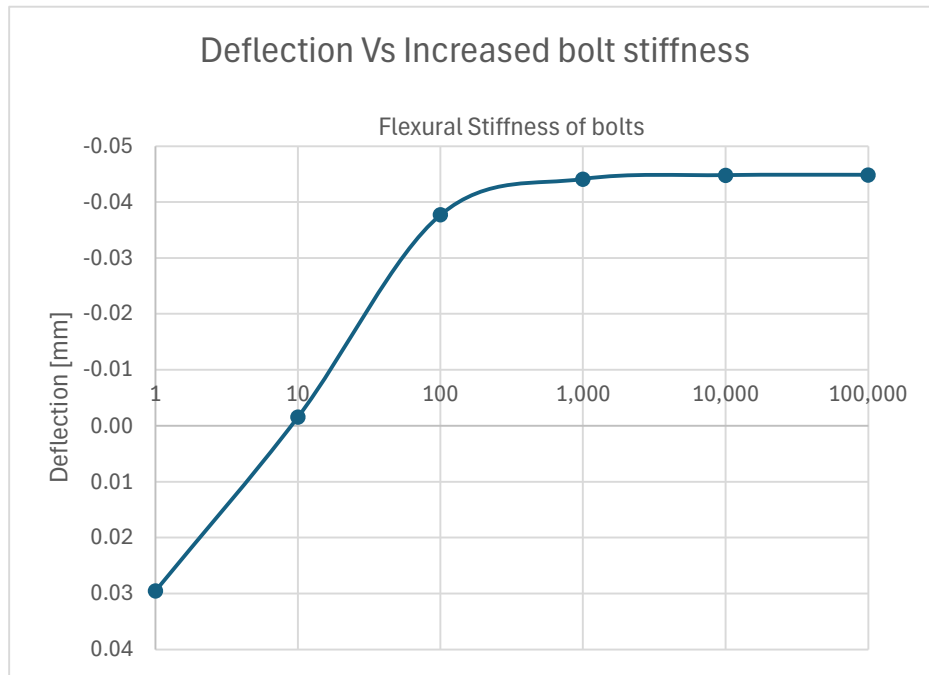
$$\delta = W/K = 1.01 \times \delta_b \quad 0.0449 \text{ mm}$$

Using higher flexural rigidity of bolts and calculating deformation according to winkler equation we get

$x \text{ EI}_{bolt}$	$y_A \text{ [mm]}$
1	0.0295
10	-0.0016
100	-0.0378
1,000	-0.0441
10,000	-0.0448
100000	-0.0449

Ok

Same value using assumption of rigid beam



Concentrated load in mid span, Free-Free, mid-span deflection

$$y_A = 0.02949 \text{ mm}$$

$$\delta_b = 0.12683 \text{ mm}$$

Distributed load, Free-Free, mid-span deflection

	Right end	Free
Left end	$R_A = 0$	$M_A = 0$
	$\theta_A = \frac{w}{2EI\beta^3} \frac{C_2 C_{a3} - C_3 C_{a2}}{C_{11}}$	
Free	$y_A = \frac{w}{4EI\beta^4} \frac{C_4 C_{a2} - 2C_3 C_{a3}}{C_{11}}$	

$$l - a = 230 \text{ mm}$$

$$C_{a2} = -0.4787$$

$$C_{a3} = \sinh \beta(l - a) \sin \beta(l - a)$$

$$C_{a3} = -86.318$$

$$C_3 = -86.318$$

$$C_4 = -172.16$$

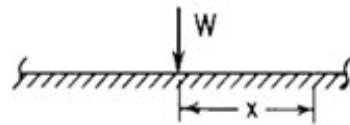
$$C_{11} = 14819.3$$

$$y_A = -0.0449$$

$$\delta_b = \text{not calcul mm}$$

Concentrated Load, infinite beam, max deflection

10. Concentrated load on an infinite beam

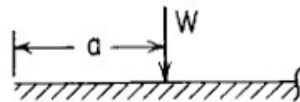


$$\text{Max } y = -\frac{W}{8EI\beta^3}$$

$\delta_b =$ Not valid, mm

Concentrated Load, semi-infinite beam, max deflection

1. Concentrated intermediate load (if $\beta a > 3$, see case 10)



$\beta \cdot a =$ 2.7475 use infinite beam

$A_1 =$ -0.0296

$\delta_b =$ #VALUE! mm

Symmetrical half models

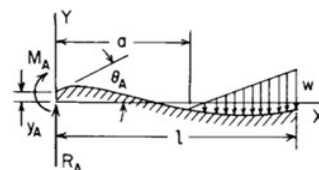
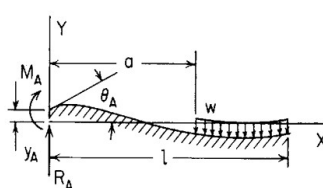
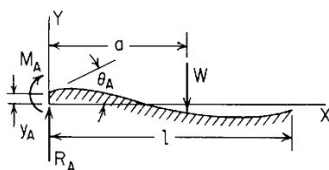
According to "Roark's Formulas for Stress and Strain" by WARREN C. YOUNG
RICHARD G. BUDYNAS

$\beta =$ 0.0239

$l =$ 115

$\beta l =$ 2.75 < 3 OK Model is valid

Load	Point	Uniform
$a =$	115	0 mm
$l - a =$	0	115 mm



$$C_1 = \cosh \beta l \cos \beta l :$$

$$C_1 = \cosh \beta l . \cos \beta l$$

$C_1 =$	-7.2335	-7.2335
---------	---------	---------

$$C_2 = \cosh \beta l \sin \beta l + \sinh \beta l \cos \beta l$$

$$C_2 = \cosh \beta l . \sin \beta l + \sinh \beta l . \cos \beta l$$

$C_2 =$	-4.166	-4.166
---------	--------	--------

$$C_3 = \sinh \beta l . \sin \beta l$$

$C_3 =$	2.9833	2.9833
---------	--------	--------

$$C_4 = \cosh \beta l . \sin \beta l - \sinh \beta l . \cos \beta l$$

$$C_4 = \cosh \beta l . \sin \beta l - \sinh \beta l . \cos \beta l$$

$C_4 =$	10.182	10.182
---------	--------	--------

$$C_{a1} = \cosh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a1} = \cosh \beta (l - a) . \cos \beta (l - a)$$

$C_{a1} =$	1.000	-7.233
------------	-------	--------

$$C_{a2} = \cosh \beta (l - a) . \sin \beta (l - a) + \sinh \beta (l - a) . \cos \beta (l - a)$$

$$C_{a2} = \cosh \beta (l - a) . \sin \beta (l - a) + \sinh \beta (l - a) . \cos \beta (l - a)$$

$C_{a2} =$	0	-4.1664
------------	---	---------

$$C_{a3} = \sinh \beta (l - a) \sin \beta (l - a)$$

$$C_{a3} = \sinh \beta (l - a) . \sin \beta (l - a)$$

$C_{a3} =$	0.000	2.983
------------	-------	-------

$$C_{a4} = \cosh \beta (l - a) \sin \beta (l - a) - \sinh \beta (l - a) \cos \beta (l - a)$$

$$C_{a4} = \cosh \beta (l - a) . \sin \beta (l - a) - \sinh \beta (l - a) . \cos \beta (l - a)$$

$C_{a4} =$	0	10.1822
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$$C_{a5} = 1 - C_{a1}$$

$$C_{a5} = 1 - C_{a1}$$

$C_{a5} =$	0.000	8.233
------------	-------	-------

$$C_{a6} = 2\beta (l - a) - C_{a2}$$

$C_{a6} =$	0.000	9.661
------------	-------	-------

$$C_{11} = \sinh^2 \beta l - \sin^2 \beta l$$

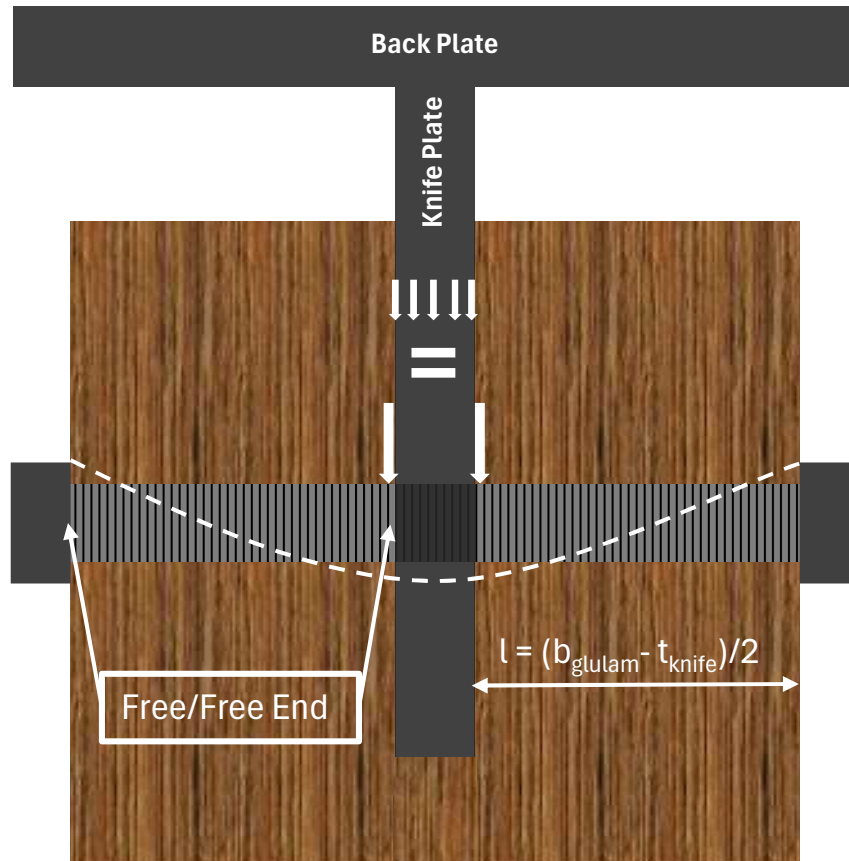
$$C_{11} = \sinh^2 \beta l - \sin^2 \beta l$$

$C_{11} =$	60.22	60.22
------------	-------	-------

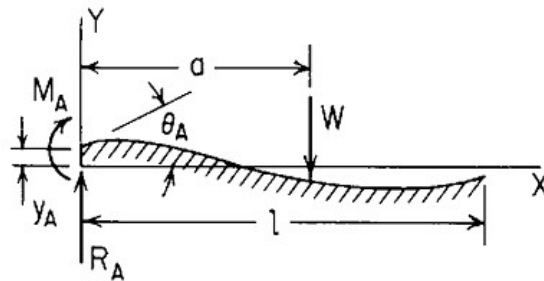
$$C_{13} = \cosh \beta l \cdot \sinh \beta l - \cos \beta l \cdot \sin \beta l$$

$C_{13} =$	61.22	61.22
------------	-------	-------

Concentrated Load, Free-Free, end- to mid-span deflection



1. Concentrated intermediate load



$$\theta_A = \frac{W}{2EI\beta^2} \frac{C_2 C_{a2} - 2C_3 C_{a1}}{C_{11}}$$

$$\theta_A = -0.0006 \text{ Rad}$$

$$y_A = \frac{W}{2EI\beta^3} \frac{C_4 C_{a1} - C_3 C_{a2}}{C_{11}}$$

$$y_A = 0.0417 \text{ mm}$$

$$a = 115 \text{ mm}$$

$$F_1 = \cosh \beta x \cos \beta x$$

$$F_2 = \cosh \beta x \sin \beta x + \sinh \beta x \cos \beta x$$

$$F_{a4} = \cosh \beta(x-a) \sin \beta(x-a) - \sinh \beta(x-a) \cos \beta(x-a)$$

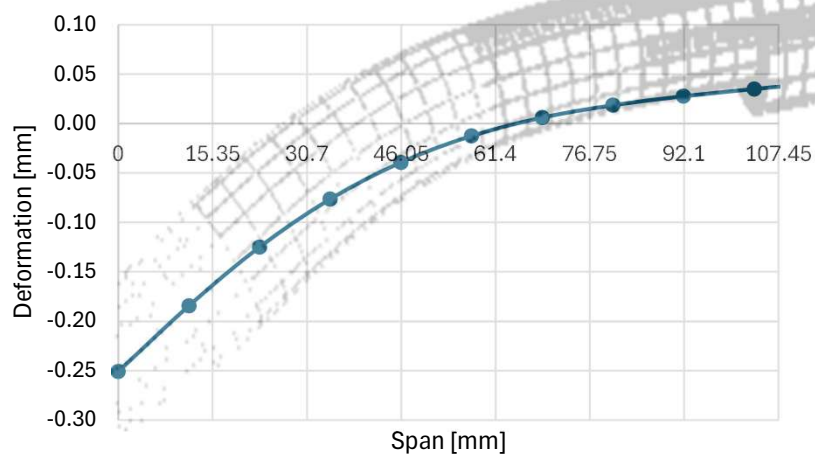
$$\text{Deflection} = y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 - \frac{W}{4EI\beta^3} F_{a4}$$

where $R_A = M_A = 0$

Load deformation

x =	L - x	F ₁ =	F ₂ =	F _{a4} =	y _(x) -y ₍₀₎	A _i =(x _i *y _i)	Sum
0	115	1.0000	0.0000	0.0000	0.04	0.22	0.22
11.5	103.5	0.9991	0.5494	0.0000	0.03	0.40	0.62
23	92	0.9848	1.0957	0.0000	0.03	0.31	0.93
34.5	80.5	0.9232	1.6232	0.0000	0.02	0.20	1.14
46	69	0.7577	2.0913	0.0000	0.01	0.05	1.19
57.5	57.5	0.4114	2.4229	0.0000	-0.01	-0.17	1.02
69	46	-0.2093	2.4933	0.0000	-0.04	-0.48	0.54
80.5	34.5	-1.2064	2.1239	0.0000	-0.08	-0.91	-0.37
92	23	-2.6757	1.0808	0.0000	-0.12	-1.47	-1.83
103.5	11.5	-4.684	-0.916	0.0000	-0.18	-2.14	-3.97
115	0	-7.233	-4.166	0.0000	-0.25	-1.25	-5.22
$\delta_b =$		0.0454 mm					

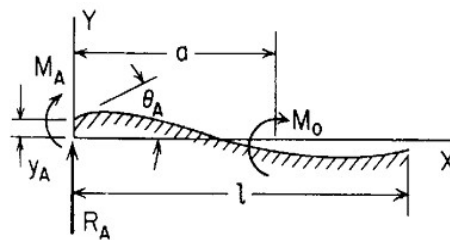
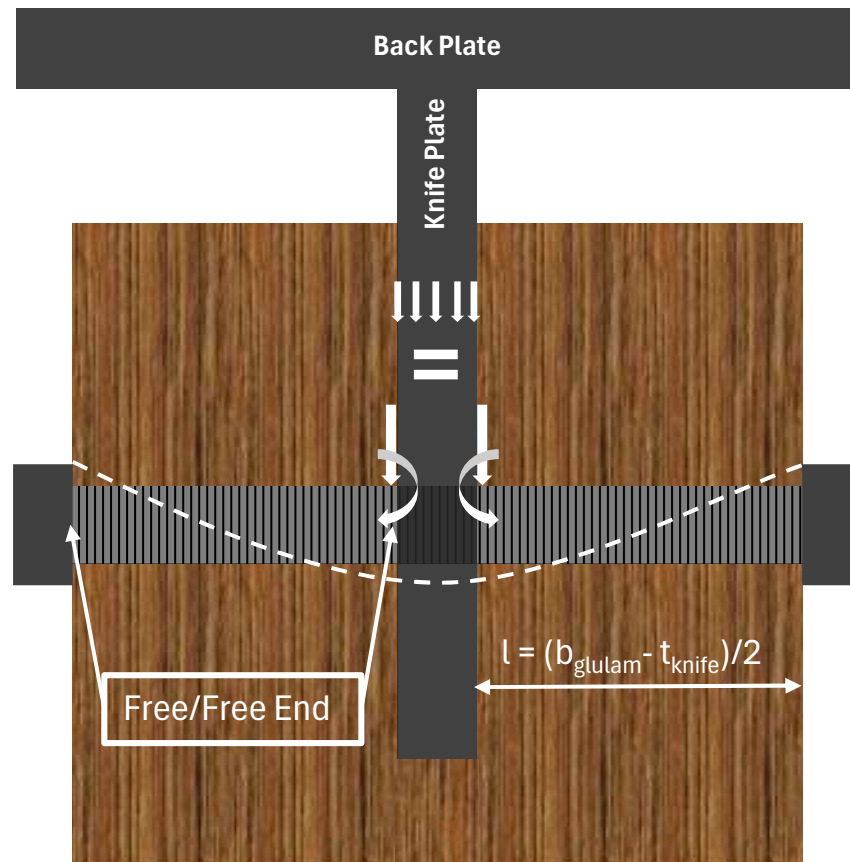
Deformed shape (numerical vs analytical)
under concentrated load



Concentrated Load + bending moment, Free-Free, end- to mid-span deflection

Adding concentrated bending moment due to transfer of concentrated load
with $t_{knife} / 2$

$$M = -34091 \text{ N.mm}$$



	Right end	Free
Left end	$R_A = 0$	$M_A = 0$
Free	$\theta_A = \frac{-M_0 C_3 C_{a4} + C_2 C_{a1}}{EI\beta C_{11}}$	$y_A = \frac{M_0}{2EI\beta^2} \frac{2C_3 C_{a1} + C_4 C_{a4}}{C_{11}}$

$$\theta_A = -0.0001$$

$$\text{Deflection} = y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 + \frac{M_0}{2EI\beta^2} F_{a3}$$

$$y_{A2} = -0.0044$$

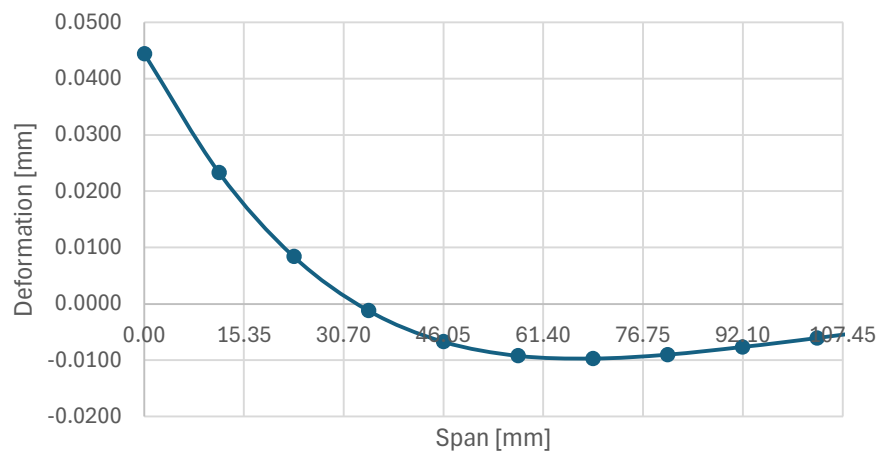
$$a = 115.00$$

$$F_{a3} = \sinh \beta \langle x - a \rangle \sin \beta \langle x - a \rangle$$

Moment deformation

x =	L - x =	F ₁ =	F ₂ =	F _{a3} =	y(x) =	A _i =(x _i *y _i)	Sum
0.00	115.00	1.0000	0.0000	0.0000	-0.0044	-0.03	-0.03
11.5	103.50	0.9991	0.5494	0.0000	-0.0061	-0.07	-0.10
23	92.00	0.9848	1.0957	0.0000	-0.0077	-0.09	-0.19
34.5	80.50	0.9232	1.6232	0.0000	-0.0090	-0.10	-0.29
46	69.00	0.7577	2.0913	0.0000	-0.0097	-0.11	-0.40
57.5	57.50	0.4114	2.4229	0.0000	-0.0092	-0.10	-0.50
69	46.00	-0.2093	2.4933	0.0000	-0.0067	-0.07	-0.57
80.5	34.50	-1.2064	2.1239	0.0000	-0.0012	0.00	-0.57
92	23.00	-2.6757	1.0808	0.0000	0.0084	0.11	-0.46
103.5	11.50	-4.6835	-0.9157	0.0000	0.0233	0.29	-0.17
115.00	0.00	-7.2335	-4.1664	0.0000	0.0444	0.19	0.03

Deformed shape under concentrated moment

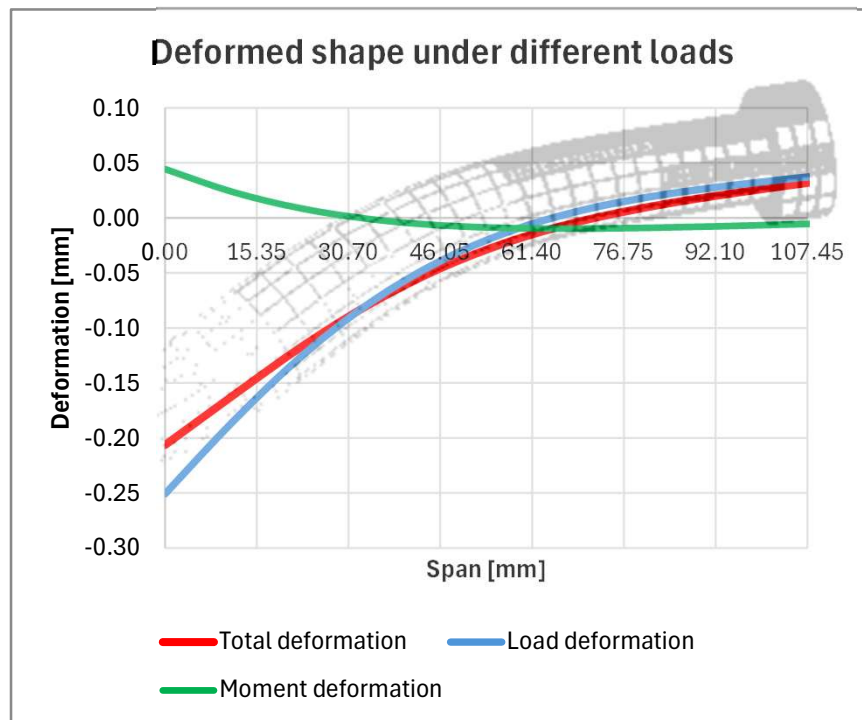


Total deformation

L - x =	y(x) =	A _i =(x _i *y _i)	Sum
115.00	0.04	0.19	0.19
103.50	0.03	0.33	0.52
92.00	0.02	0.23	0.75
80.50	0.01	0.10	0.85
69.00	0.00	-0.06	0.79
57.50	-0.02	-0.27	0.53
46.00	-0.05	-0.55	-0.02
34.50	-0.08	-0.91	-0.93
23.00	-0.12	-1.35	-2.29
11.50	-0.16	-1.85	-4.14
0.00	-0.21	-1.06	-5.20

Total deformation (due to load + eccentricity)

$$\delta_b = 0.0452 \text{ mm}$$



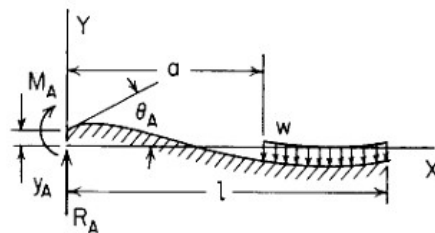
The concentrated load curve (blue) is better matching with the analytical results than the load+moment curve (red)

Concentrated, Free-Free, mid-span deflection

$$\delta_b = 0.2508 \text{ mm}$$

Distributed load, Free-Fixed, mid- to end-span deflection

2. Partial uniformly distributed load



Fixed	
$R_A = 0$	$M_A = 0$
$\theta_A = \frac{w}{2EI\beta^3} \frac{C_1 C_{a4} + C_4 C_{a5}}{2 + C_{11}}$	
$y_A = \frac{w}{4EI\beta^4} \frac{2C_1 C_{a5} - C_2 C_{a4}}{2 + C_{11}}$	

$$w = 39.53 \text{ N/mm}$$

$$a = 0 \text{ mm}$$

$$\theta_A = 0.00035 \text{ Rad}$$

$$y_A = -0.0553 \text{ mm}$$

$$\text{Deflection} = y = y_A F_1 + \frac{\theta_A}{2\beta} F_2 + \frac{M_A}{2EI\beta^2} F_3 + \frac{R_A}{4EI\beta^3} F_4 - \frac{w}{4EI\beta^4} F_{a5}$$

$$F_{a5} = \langle x - a \rangle^0 - F_{a1}$$

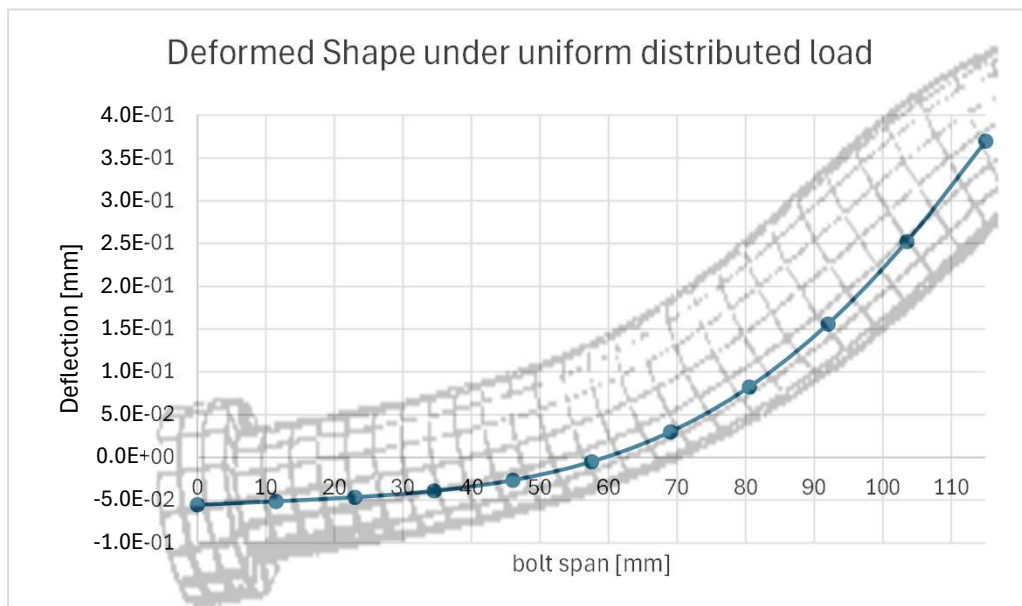
$$F_{a1} = \langle x - a \rangle^0 \cosh \beta \langle x - a \rangle \cos \beta \langle x - a \rangle$$

The unit step function is denoted by $\langle x - a \rangle^0$ where the use of the angle brackets $\langle \rangle$ is defined as follows: If $x < a$, $\langle x - a \rangle^0 = 0$; if $x > a$, $\langle x - a \rangle^0 = 1$. At $x = a$ the unit step function is undefined just as vertical shear is undefined directly beneath a concentrated load. The

x =	F ₁ =	F ₂ =	F _{a1} =	F _{a5} =	y (x) =	A _i =(x _i *y _i)	Sum
0	1	0	1	0.000	-5.5E-02	-0.31	-0.31
11.5	0.999	0.549	1	0.001	-5.1E-02	-0.59	-0.89
23	0.985	1.096	1	0.015	-4.6E-02	-0.53	-1.42
34.5	0.923	1.623	1	0.077	-3.9E-02	-0.44	-1.86
46	0.758	2.091	1	0.242	-2.7E-02	-0.28	-2.14
57.5	0.411	2.423	0	0.589	-5.0E-03	-0.02	-2.15
69	-0.209	2.493	0	1.209	3.0E-02	0.39	-1.76
80.5	-1.206	2.124	-1	2.206	8.2E-02	1.01	-0.75
92	-2.676	1.081	-3	3.676	1.6E-01	1.86	1.11
103.5	-4.684	-0.916	-5	5.684	2.5E-01	2.96	4.07
115	-7.233	-4.166	-7	8.233	3.7E-01	1.79	5.86
Sum					5.86		

Average deformation

$$\delta_b = 0.05 \text{ mm}$$



Unvalid model, since the deformation curve is not matched between numerical and analytical results

Distributed load, Free-Fixed, end-span deflection

$$\delta_b = 0.36963 \text{ mm} = y_{(xmid)}$$

Rotational stiffness due to bolts/timber deformation

Model:	Symetrical half models	
Model:	Concentrated, Free-Free, mid-span deflection	
Elastic bed:	Single	$k = 1 * k$
Negative deformation:	Yes	

Correction factor

$$F_{corr} = 0.51 + 0.023 * (d_{bolts} - 16) / d_{bolts} + M_{bolts} * N_{bolts} * d_{bolts} / H_{glulam}$$

$$F_{corr} = 0.77 \quad \text{multiplied with deformation}$$

Related to circular surface of bolts, two elastic beds on top and bottom of the bolt, the effect of having two columns of bolts, the vertical displacement between bolts (number of rows) and other variations in used model from the real model.

Selected bolts model Without Correction Factor	
$\delta_b =$	0.251 mm
$\theta_b = \delta_b / y_b$	
$y_b = y_1 + \min(25, V_{end,b} / 2)$	
$y_b =$	250 mm
$\theta_b =$	0.0010 rad 0.0575 °
$K_b =$	9.97 MNm/Rad

Infinite beam model with correction factor	
$\delta_b =$	0.095 mm
$y_b =$	250 mm
$\theta_b =$	0.0004 rad 0.0218 °
$K_b =$	26.30 MNm/Rad

Appendix F: Total rotational stiffness calculation and verification

Total Rotational Stiffness Calculation

$$1/k_{\text{tot}} = 1/k_b + 1/k_p + 1/k_{b1} + 1/k_s$$

	δ [mm]	θ Rad	k [MN.m/Rad]	K ratio	
s	0.003	0.00001	1820	1.3%	Screw elongation
p	0.007	0.00002	622	3.9%	Back plate bending
b ₁	0.003	0.00001	866	2.8%	Bolts pressed under knife plate
b	0.0951	0.00038	26.3	92.0%	Bolts pressing timber
Total	0.0951	0.00038	24.2	100.0%	

$$k_{\text{tot}} = 24.19 \text{ MN.m/Rad}$$

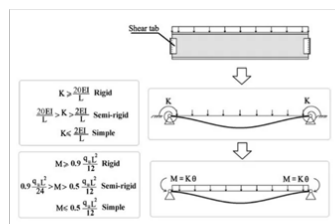
Semi-Rigid

$$R = K_{\text{tot}} / K_{\text{rigid}}$$

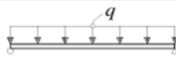
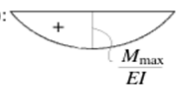
$$R = 11.3\%$$

Connection is Semi-Rigid .Acts 11% as fixed connection and 89% as pinned connection

Classification of connection rigidity			
11.3%	K (Rigid)	∞	MN.m/Rad
	K (Semi-Rigid)	213.22	MN.m/Rad
		21.32	MN.m/Rad
88.7%	K (Simple)	0.00	MN.m/Rad



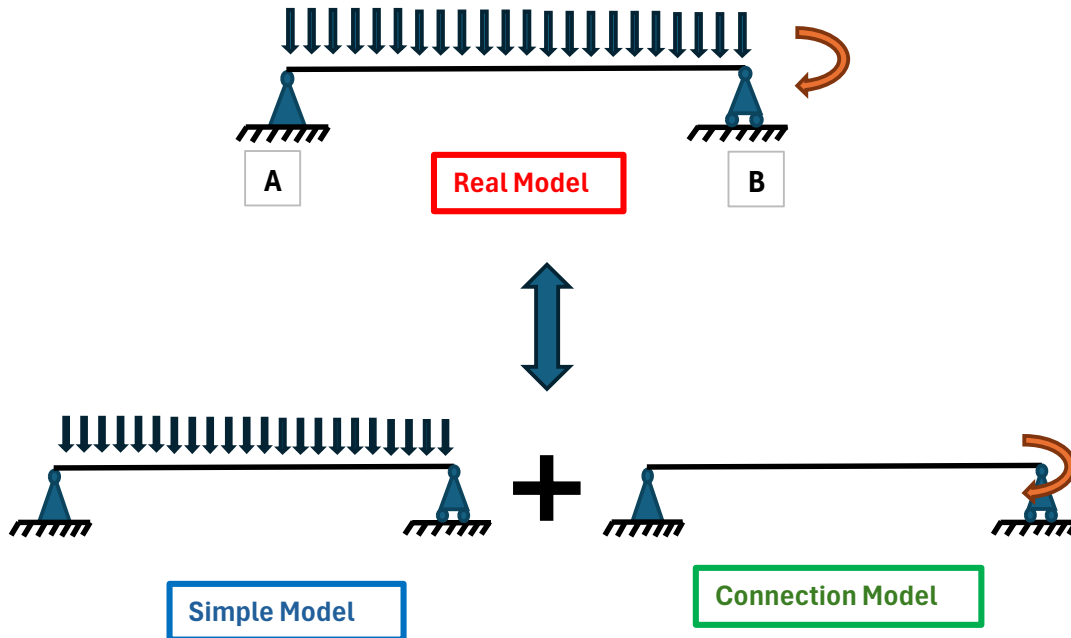
Total Rotational Stiffness Verification

Line	Load case	Derivation	Support rotation
1	 $\frac{1}{r}(x)$: 	$\theta_A = \theta_B = \frac{A}{2}$ $\frac{A}{2} = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{M_{\text{max}}}{EI} \cdot l = \frac{1}{3} \cdot \frac{ql^2}{EI} \cdot l$	$\theta_A = \theta_B = \frac{ql^3}{24EI}$
3	 $\frac{1}{r}(x)$: 	$f_B = \theta_A \cdot l - A(l - \bar{x}) = 0 \Rightarrow$ $\theta_A = \frac{1}{2} \cdot \frac{M}{EI} \cdot l(l - \frac{2}{3}l) = \frac{Ml}{6EI}$ $\theta_B = A - \theta_A =$ $= \frac{1}{2} \cdot \frac{M}{EI} \cdot l - \frac{Ml}{6EI} = \frac{Ml}{3EI}$	$\theta_A = \frac{Ml}{6EI}$ $\theta_B = \frac{Ml}{3EI}$

Beam model (Simple supported beam with uniform distributed loads)

$L =$ 7188 mm Support = bolts' centroid
 $q =$ 10 kN/m load within elasticity limits, used in numerical model
 $EI_g =$ 7.7E+13 N.mm²

we can simplify the model into two models, simple beam with uniform load and simple beam with concentrated moment accounting for rotational stiffness of connection.



Assuming constant flexural rigidity, we can use the principle of superposition as follows:

$$\theta_{B,s} + \theta_{B,c} = \theta_{B,r}$$

Analysis of Simple Model

$$\theta_A = \theta_B = \frac{ql^3}{24EI}$$

$$\theta_{B,s} = 0.00202 \text{ Rad} \quad 0.12^\circ$$

$$\delta_{\max,s} = 5P \cdot L^4 / 384EI$$

$$\delta_{\max,s} = 4.54 \text{ mm}$$

$$x = 3913 \text{ mm}$$

$$\delta(x) = 4.49 \text{ mm}$$

$$M_{\max,s} = 64.58 \text{ kN.m} \quad \text{in mid span}$$

$$R_{A,s} = R_{B,s} = 35.94 \text{ kN}$$

Real Model Analytically (by superposition)

$$M_{B,r} = M_{B,c} + M_{B,s} \quad : M_{B,s} = 0 \quad \text{pinned connection in simple model}$$

$$M_{B,c} = M_{B,r}$$

$$\theta_{B,r} = \theta_{B,s} - \theta_{B,c} \quad \text{Compatibility}$$

$$M_{B,r} / K = ql^3 / 24EI_g - M_{B,r} l / 3EI_g$$

$$M_{B,r} = ql^3 / (24 * EI_g / k + 8 * l)$$

$$M_{B,r} = 28.21 \text{ kN.m}$$

$$\theta_{B,c} = M_{B,c} / k$$

$$\theta_{B,c} = 0.0011 \text{ Rad} \quad 0.07^\circ$$

$$\theta_{B,r} = \theta_{B,s} - \theta_{B,c}$$

$$\theta_{B,r} = 0.00088 \text{ Rad} \quad 0.05^\circ$$

Analysis of Connection Model

$$k_{B,c} = 24.80 \text{ MN.m/Rad} \quad \text{Calculated analytically}$$

$$\theta_B = \frac{Ml}{3EI}$$

$$M_{B,c} = 28.21 \text{ kN.m}$$

$$\theta_{B,c} = 0.00114 \text{ Rad}$$

$$R_{A,c} = 3.92 \text{ kN}$$

$$R_{B,c} = -3.92 \text{ kN}$$

In case of fixed connection

$$M_{\text{fixed}} = q * L^2 / 8$$

$$M_{\text{fixed}} = 64.58 \text{ kN.m}$$

Deflection:

$$x = 3913 \text{ mm}$$

$$M_{(x)} = 12.85 \text{ kN.m}$$

$$\delta_{(x)} = 6.76 \text{ mm}$$

Comparison between different models in FEM-design and analytical solution

	C1	C2	S4	S7.3	
$M_{\text{Hand}} =$	10	10	5.58	28.21	kN.m
$M_{\text{FEM}} =$	10	10	5.3	27.60	kN.m

C1 = cantiliver 1m span

C2 = cantiliver 2m span

S7.3 = simple supported beam, 7.3m span

S4 = simple supported beam, 4m span

Appendix G: Parametric study on connection analysis

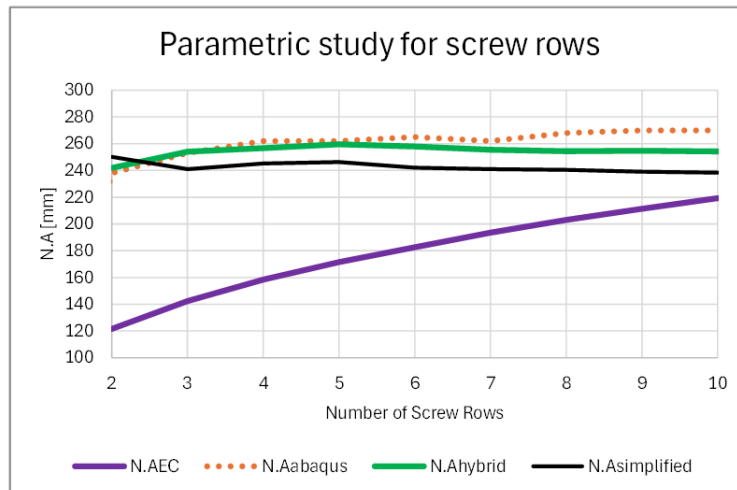
Parametric Study on Neutral Axis



Number of screw rows

Using concrete block in abaqus

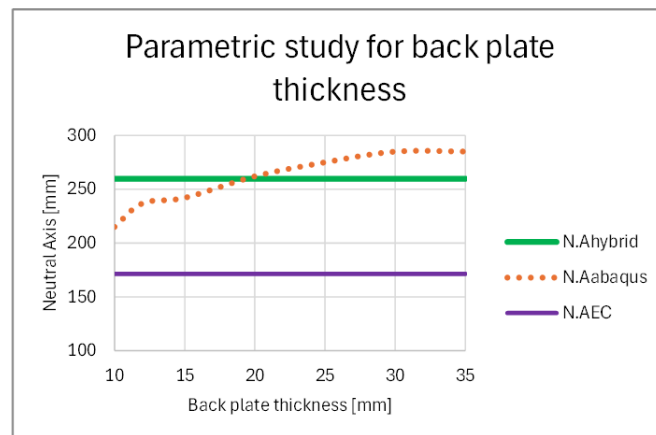
R_{Screw}	2	3	4	5	6	7	8	9	10
$N.A_{\text{hybrid}}$	241.8	254	256.7	259.6	258	255.4	254.4	254.7	254.2
$N.A_{\text{simplified}}$	250.2	241	245.2	246.2	242	241	240.4	239	238.4
$N.A_{\text{EC}}$	121.5	142.4	158.4	171.5	182.6	193.6	203.1	211.3	219.3
$N.A_{\text{abaqus}}$	238	253	262	262	265	262	268	270	270



Thickness of back plate

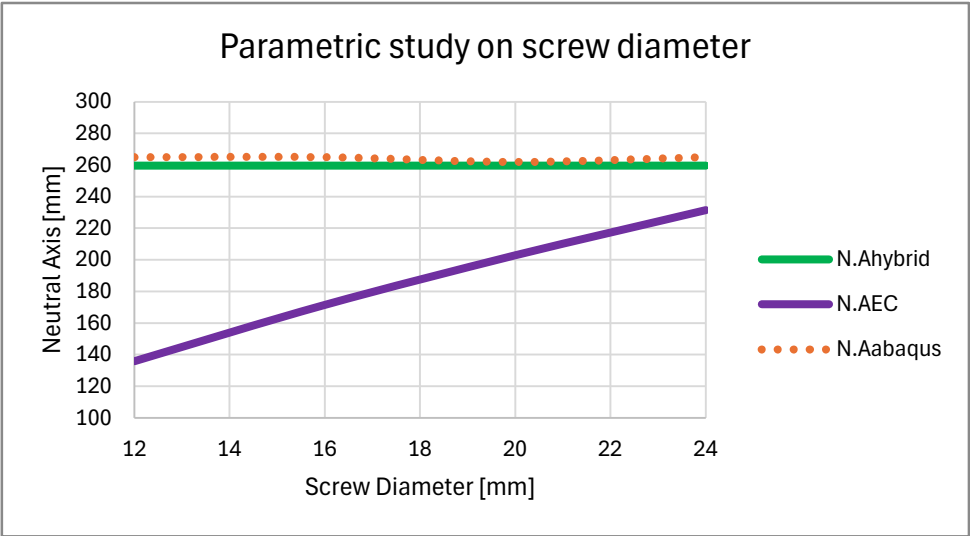
Using reference model. Concrete block in abaqus.

t_{back}	10	12	15	20	25	30	35
$N.A_{\text{hybrid}}$	259.6	259.6	259.6	259.6	259.6	259.6	259.6
$N.A_{\text{EC}}$	171.5	171.5	171.5	171.5	171.5	171.5	171.5
$N.A_{\text{abaqus}}$	215	237	242	262	275.00	285	285

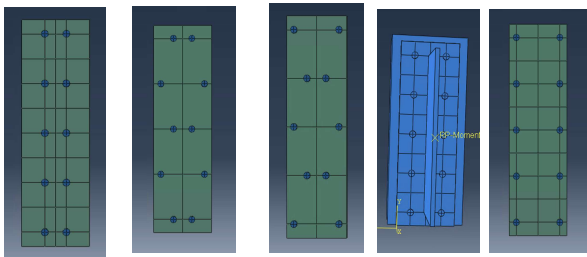


Screws diameter

d [mm]	12	16	20	24
N.A _{hybrid}	259.6	259.6	259.6	259.6
N.A _{EC}	136	171.5	202.9	231.6
N.A _{abaqus}	265	265	262	265

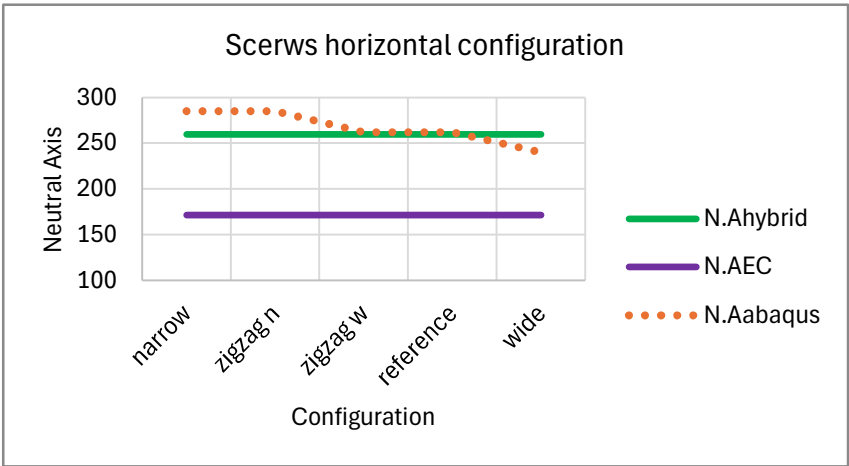


horizontal Spacing of screws

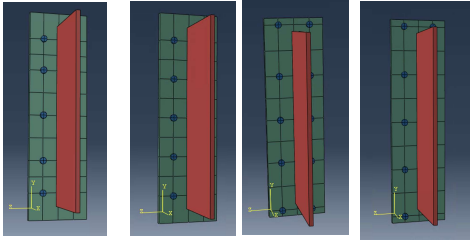


conf.	narrow	zigzag n	zigzag w	reference	wide
N.A _{hybrid}	259.6	259.6	259.6	259.6	259.6
N.A _{EC}	171.5	171.5	171.5	171.5	171.5
N.A _{abaqus}	285	285	262	262	240

zigzag w = uppermost screws are displaced 25mm wider
zigzag n = uppermost screws are displaced 25mm narrower



Vertical Spacing of screws



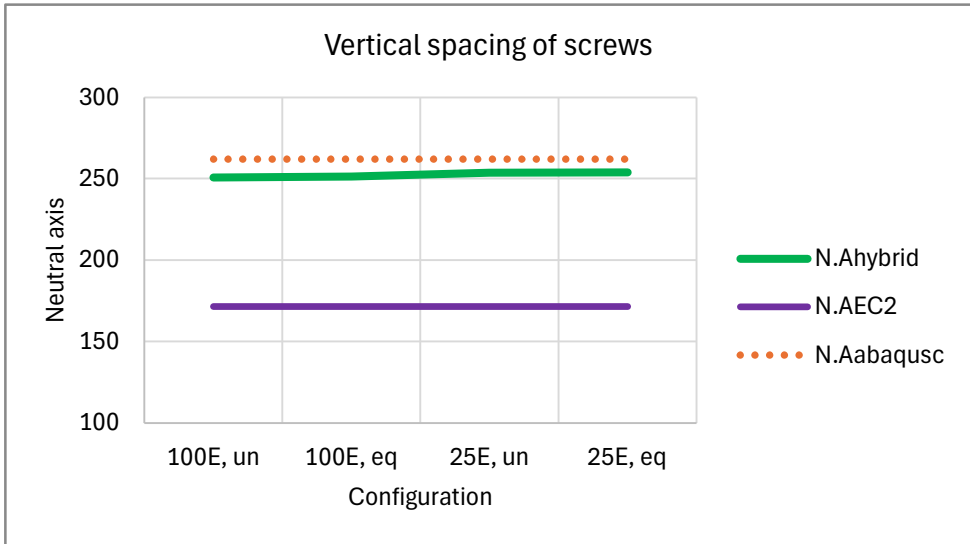
conf.	100E, un	100E, eq	25E, un	25E, eq
N.A _{hybrid}	250.8	251.2	253.7	254
N.A _{EC2}	171.5	171.5	171.5	171.5
N.A _{abaqusC}	262	262	262	262

In case of 25E, outer screws show reduced arbitrary area compared to 100E and reference model.

E = edge distance

un = unequal distance between screws

eq = equal distance between screws

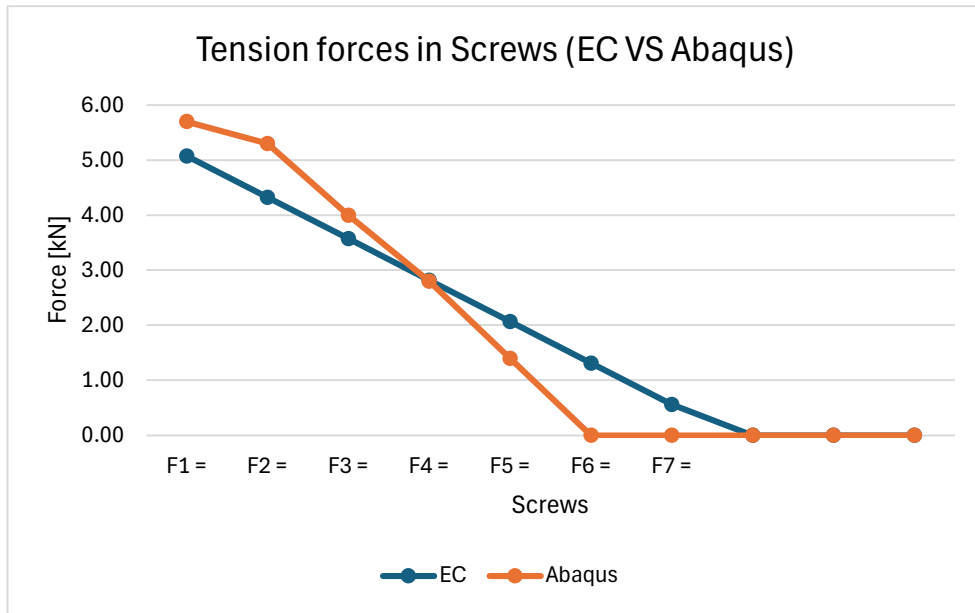


Parametric Study on Screw Forces



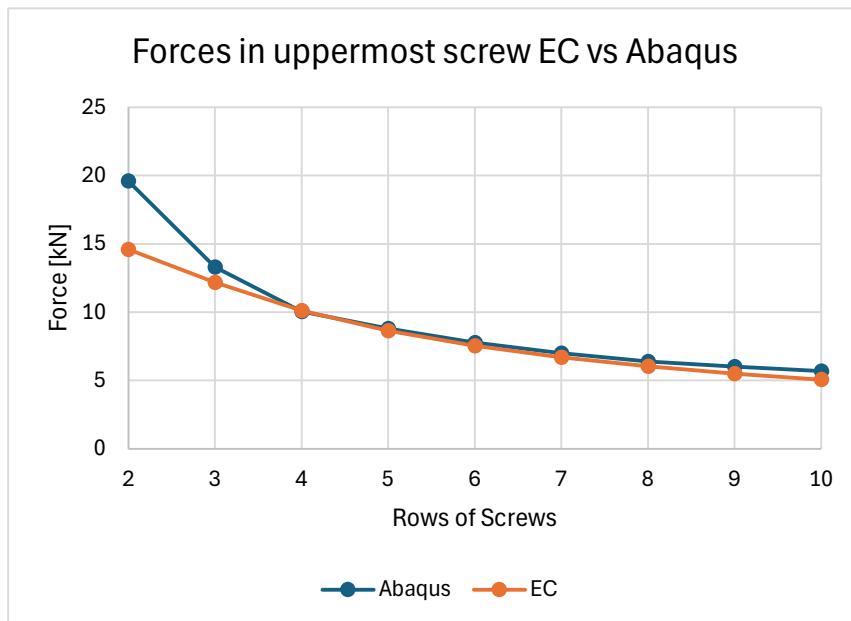
10 Rows of screws

Force	EC	Hybrid	Abaqus	
F ₁ =	5.08	3.61	5.7	kN
F ₂ =	4.32	4.73	5.3	kN
F ₃ =	3.57	4.28	4	kN
F ₄ =	2.82	3.27	2.8	kN
F ₅ =	2.06	2.25	1.4	kN
F ₆ =	1.31	1.23	0	kN
F ₇ =	0.56	0.36	0	kN
F ₈ =	0.00	0	0	kN
F ₉ =	0.00	0	0	kN
F ₁₀ =	0.00	0	0	kN
Sum	19.72	19.72	19.20	kN



2 to 10 rows (comparison of force in upper most screw)

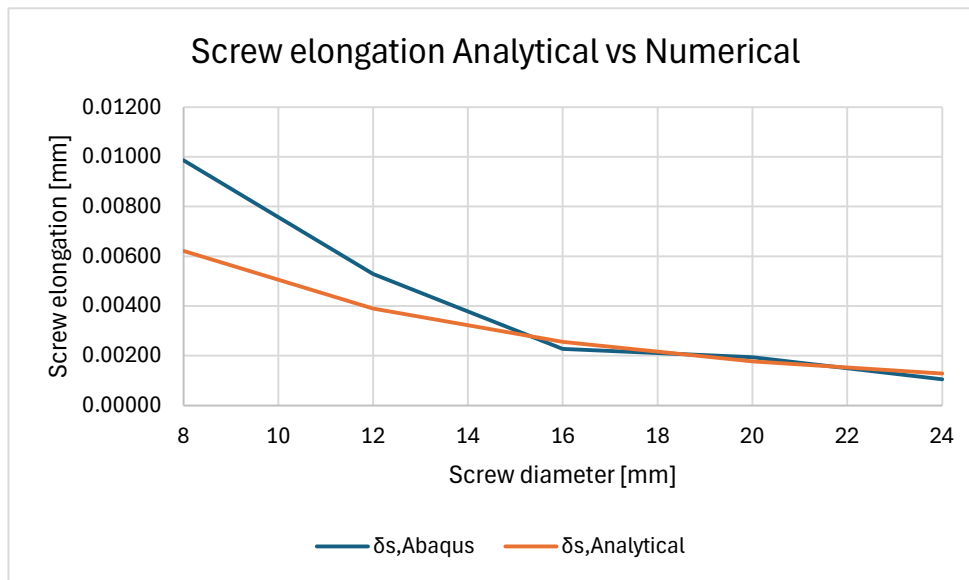
Rows	2	3	4	5	6	7	8	9	10
Abaqus	19.6	13.3	10.05	8.8	7.77	7	6.4	6.03	5.7
EC	14.6	12.18	10.13	8.64	7.54	6.7	6.04	5.51	5.08
Divergenc	34%	9%	-1%	2%	3%	4%	6%	9%	12%



Parametric Study on Screws Elongation



d_s	$\delta_{s,Abaqus}$	$\delta_{s,Analytical}$	
8	0.00987	0.00623	-37% mm
12	0.00529	0.00390	-26% mm
16	0.00227	0.00256	13% mm
20	0.00194	0.00177	-9% mm
24	0.00106	0.00129	22% mm



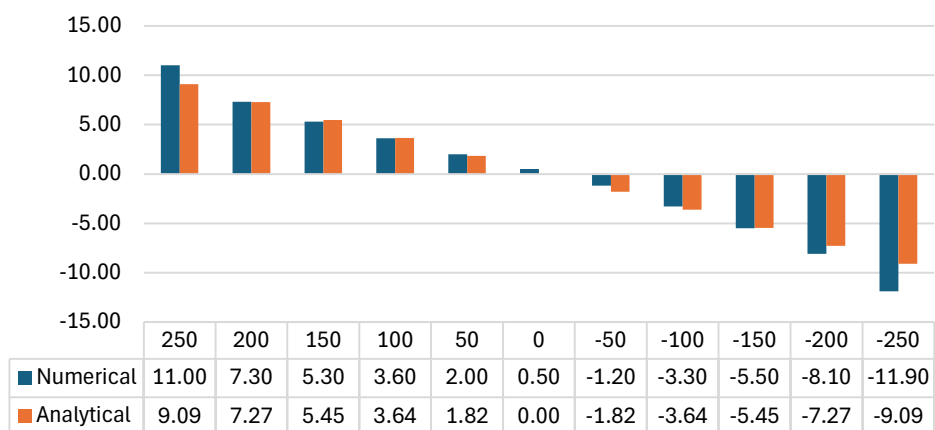
Parametric Study on Bolts Forces



1x11 screw configuration, using M = 10 kN.m

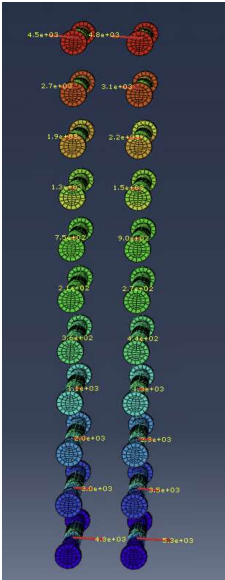
Distance	Forces	Numerical	Analytical	Ratio	
250	$N_1 =$	11.00	9.09	17%	kN
200	$N_2 =$	7.30	7.27	0%	kN
150	$N_3 =$	5.30	5.45	-3%	kN
100	$N_4 =$	3.60	3.64	-1%	kN
50	$N_5 =$	2.00	1.82	9%	kN
0	$N_6 =$	0.50	0.00		kN
-50	$N_7 =$	-1.20	-1.82	-52%	kN
-100	$N_8 =$	-3.30	-3.64	-10%	kN
-150	$N_9 =$	-5.50	-5.45	1%	kN
-200	$N_{10} =$	-8.10	-7.27	10%	kN
-250	$N_{11} =$	-11.90	-9.09	24%	kN
	sum	-0.30	0.00		kN

Comparison between numerical and analytical bolts' forces

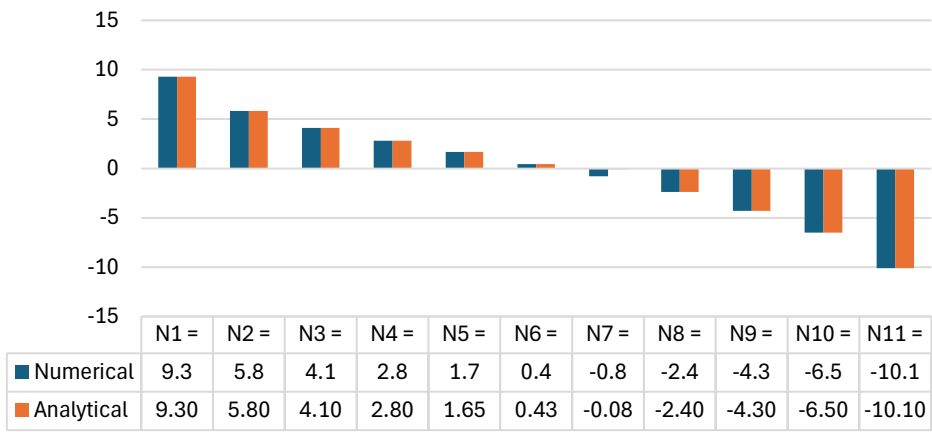


Configuration 2*11 diameter 16mm comparison

Forces	Abaqus	EC
N ₁ =	9.3	9.30
N ₂ =	5.8	5.80
N ₃ =	4.1	4.10
N ₄ =	2.8	2.80
N ₅ =	1.7	1.65
N ₆ =	0.4	0.43
N ₇ =	-0.8	-0.08
N ₈ =	-2.4	-2.40
N ₉ =	-4.3	-4.30
N ₁₀ =	-6.5	-6.50
N ₁₁ =	-10.1	-10.10
Sum	0.0	0.7



Comparison between numerical and analytical bolts' forces



Configuration 2*4 diameter 16mm comparison

Forces	Abaqus	EC
N ₁ =	17.00	17.77
N ₂ =	6.20	6.15
N ₃ =	-4.90	-6.15
N ₄ =	-18.10	-17.77

Appendix H: Parametric study on deformations and rotations

Parametric Study on Bolts Deformation



Since different parameters govern the deformation of bolts including timber shear stresses, configuration and size of bolts, elastic beds surrounding the bolts. It is difficult to measure the deformation directly as per back plate or screws. Here we compare the rotational stiffness of bolts that is calculated by substituting the rotational stiffness of other parts (back plate and screws) from the total rotational stiffness. The resultant rotational stiffness is the rotational stiffness of bolts only.

Calculation of bolts rotational stiffness			
Conf	$K_{tot,abaq}$	$K_{dif,hand}$	$K_{bolt,target}$
1x11	24.8	302.1	27.0183
2x11	36.1	304.1	40.9623
2x9	33.2	286.9	37.5448
2x6	30.1	253.7	34.1513
2x4	25.8	226.6	29.1147
10 mm	16.8	302.1	17.7894
16mm	24.8	302.05	27.0183
20mm	30.4	302.05	33.802
24mm	35	302.05	39.5871
conf1	45.2	286.89	53.653
conf2	31.6	165.76	39.0433
$b_{gl}=170$	25.1	302.05	27.3748
$b_{gl}=290$	24.4	302.05	26.5442

$K_{bolt,target}$ Bolts rotational stiffness numerically

$K_{tot,abaq}$ Total rotational stiffness numerically

$K_{dif,hand}$ Rotational stiffness of other parts analytically

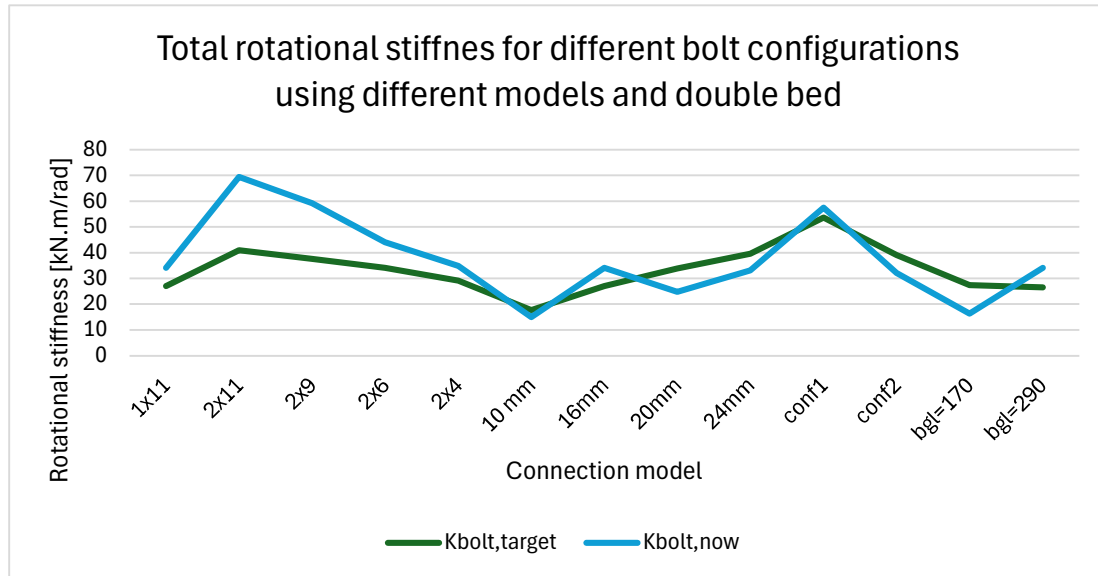
conf1: 2x9, $t_p=20\text{mm}$, $d_{bolt}=24$

conf2: 2x6, $t_p=15\text{mm}$, $d_{bolt}=20$

Different bolt configurations using doubled bed 2*k and infinite beam model + concentrated load in symmetrical half model (mid span deflection)

Using double bed and different models				
Conf	$K_{bolt,now}$	$F_{bolt,need}$	Model	$K_{tot,hand}$
1x11	34.1	0.79	inf	30.6304
2x11	69.5	0.59	inf	56.5929
2x9	59.2	0.63	inf	49.0752
2x6	44.1	0.77	inf	37.5653
2x4	34.9	0.83	inf	30.2254
10 mm	15.0	1.19	inf	14.268
16mm	34.1	0.79	inf	30.6304
20mm	24.8	1.36	sym	22.9153
24mm	33.1	1.20	sym	29.8066
conf1	57.4	0.93	sym	47.8551
conf2	32.1	1.22	sym	26.8749
$b_{gl}=170$	16.3	1.68	sym	15.4429
$b_{gl}=290$	34.1	0.78	inf	30.6304

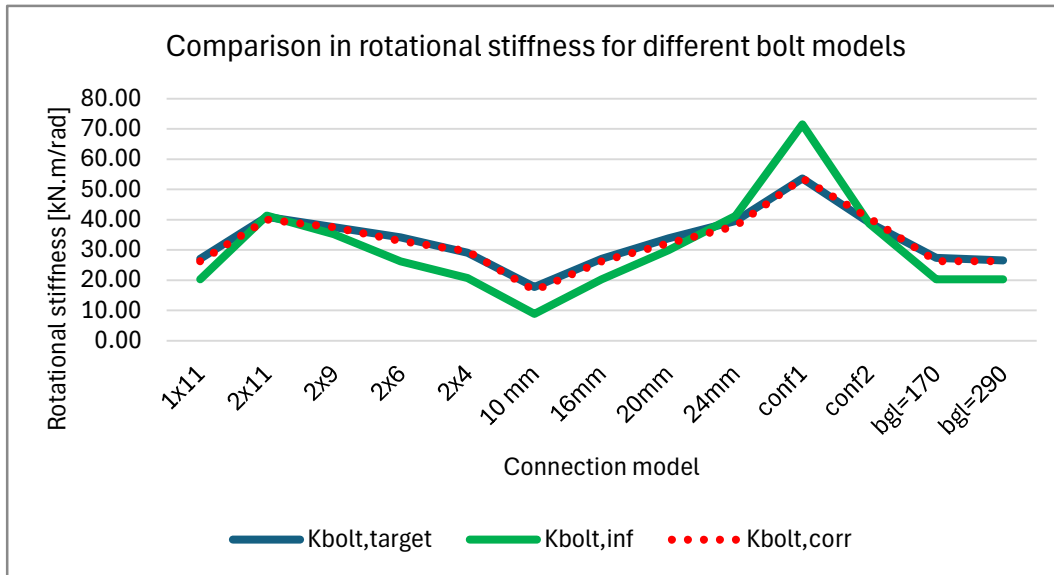
$K_{\text{bolt,now}}$ Bolts rotational stiffness analytically
 $F_{\text{bolt,need}}$ Correction factor needed for bolts deformation
 $K_{\text{tot,hand}}$ Total rotational stiffness analytically
 Model: model is chosen based on the criteria $\beta \cdot l$
 inf: using infinite beam model
 sym: using Concentrated, Free-Free, mid-span deflection model



Using infinite beam model and using correction factor extracted experimentally

Infinite beam with correction factor					
Conf	$K_{\text{bolt,target}}$	$K_{\text{bolt,inf}}$	$F_{\text{bolt,need}}$	$F_{\text{bolt,now}}$	$K_{\text{bolt,corr}}$
1x11	27.02	20.27	0.750	0.771	26.30
2x11	40.96	41.34	1.009	1.031	40.08
2x9	37.54	35.20	0.938	0.937	37.58
2x6	34.15	26.22	0.768	0.794	33.00
2x4	29.11	20.74	0.712	0.700	29.64
10 mm	17.79	8.90	0.501	0.535	16.64
16mm	27.02	20.27	0.750	0.771	26.30
20mm	33.80	29.95	0.886	0.928	32.28
24mm	39.59	41.21	1.041	1.085	37.98
conf1	53.65	71.57	1.334	1.334	53.65
conf2	39.04	38.74	0.992	0.958	40.46
$b_{gl}=170$	27.37	20.27	0.740	0.771	26.30
$b_{gl}=290$	26.54	20.27	0.764	0.771	26.30

$K_{\text{bolt,inf}}$ bolt rotational stiffness using infinite beam model

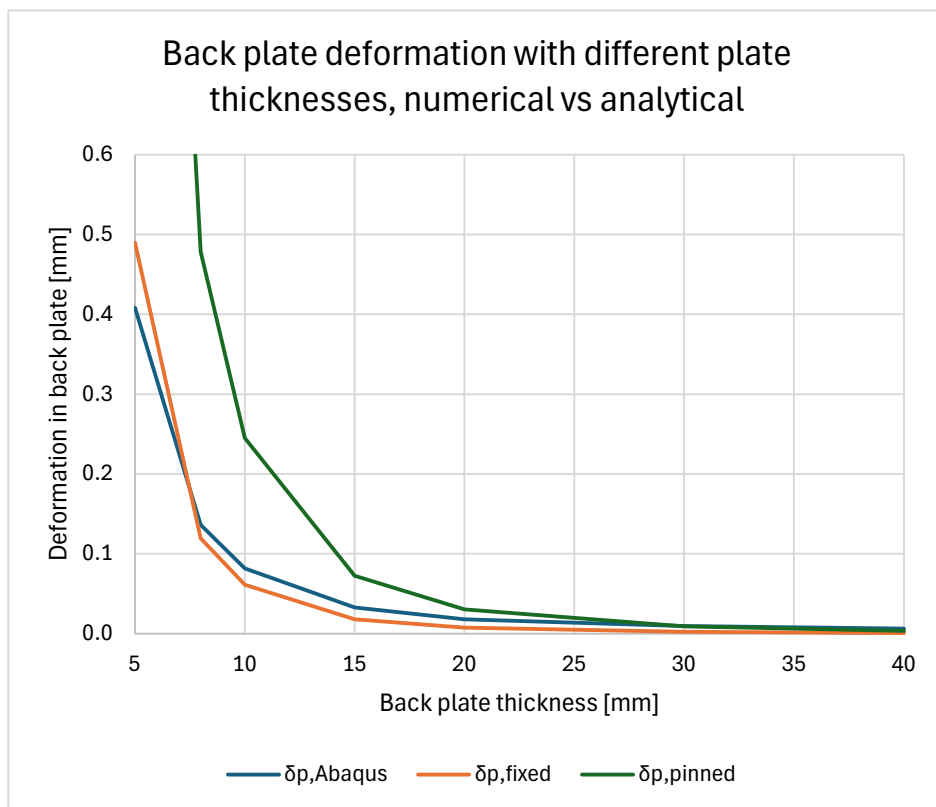


Parametric Study on Back Plate Thickness



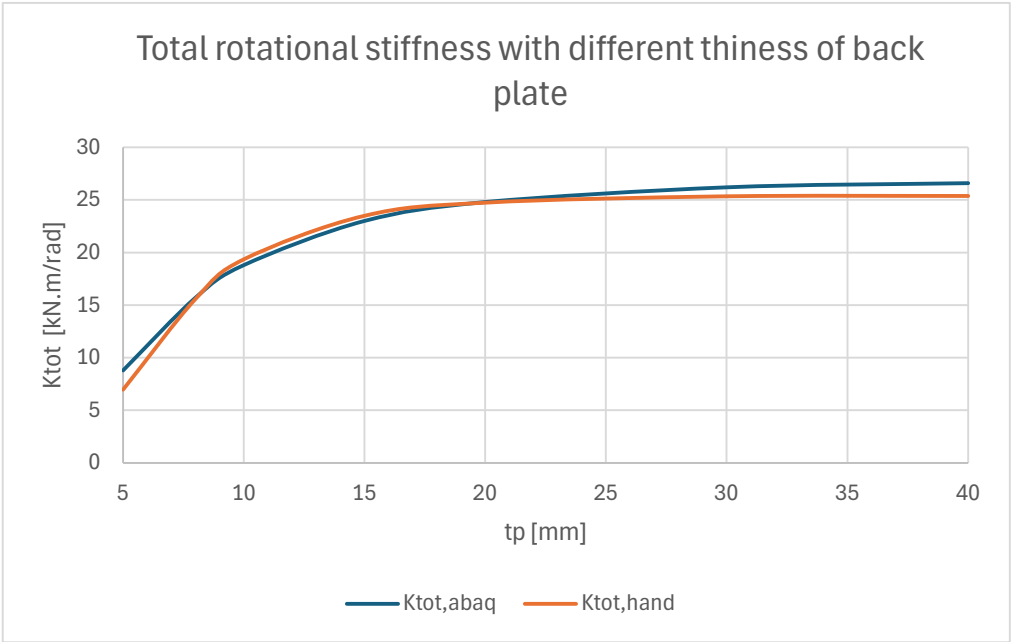
Different thicknesses of backplate

t_p	$\delta_{p,Abaqus}$	$\delta_{p,fixed}$	$\delta_{p,pinned}$	fixed	pinned
5	0.4082	0.4896	1.9586	20%	380%
8	0.1363	0.1195	0.4782	-12%	251%
10	0.0817	0.0612	0.2448	-25%	200%
15	0.0329	0.0181	0.0725	-45%	120%
20	0.0182	0.0077	0.0306	-58%	68%
30	0.0097	0.0023	0.0091	-77%	-6%
40	0.0065	0.0010	0.0038	-85%	-41%



Total rotational stiffness for different thicknesses of back plate

t_p	5	8	10	15	20	30	40	mm
$K_{tot,abaq}$	8.8	15.7	18.8	23	24.8	26.2	26.6	kN.m
$K_{tot,hand}$	7.0	15.6	19.4	23.5	24.8	25.3	25.4	kN.m



Final parametric study



d_{bolt}	Conf	Numeric	Analytic	Ratio	Simplify	Ratio
24mm	1*11	35.0	33.7	0.0	34.6	-1%
	2*9	45.2	45.2	0.0	46.9	4%
20mm	2*6/ t_p 15	31.6	32.5	0.0	33.5	6%
	1*6/H500/N4	9.8	8.8	-10%	8.9	-9%
	1*11	30.4	29.2	0.0	29.8	-2%
16mm	1*6/H500/N4	7.7	7.2	-6%	7.3	-5%
	1*11	24.8	24.2	-2%	24.6	-1%
	1*11/H850/N6	42.7	47.1	10%	47.9	12%
	2*11	36.1	35.4	-2%	36.5	1%
	2*11/ t_p 30/N4	36.5	36.4	0%	37.9	4%
	2*11/H850/N6	70.0	70.4	1%	72.6	4%
	2*9	33.2	33.2	0%	34.2	3%
	2*6	30.1	29.2	-3%	30.0	0%
	2*4	25.8	26.2	0.0	26.8	4%
	2*4/ t_p 15	24.0	24.9	0.0	25.5	6%
10mm	1*11	16.8	15.8	-0.1	16.0	-5%
MSE =			0.22%			0.26%

N number of screws
H Height of glulam beam
 t_p back plate thickness

