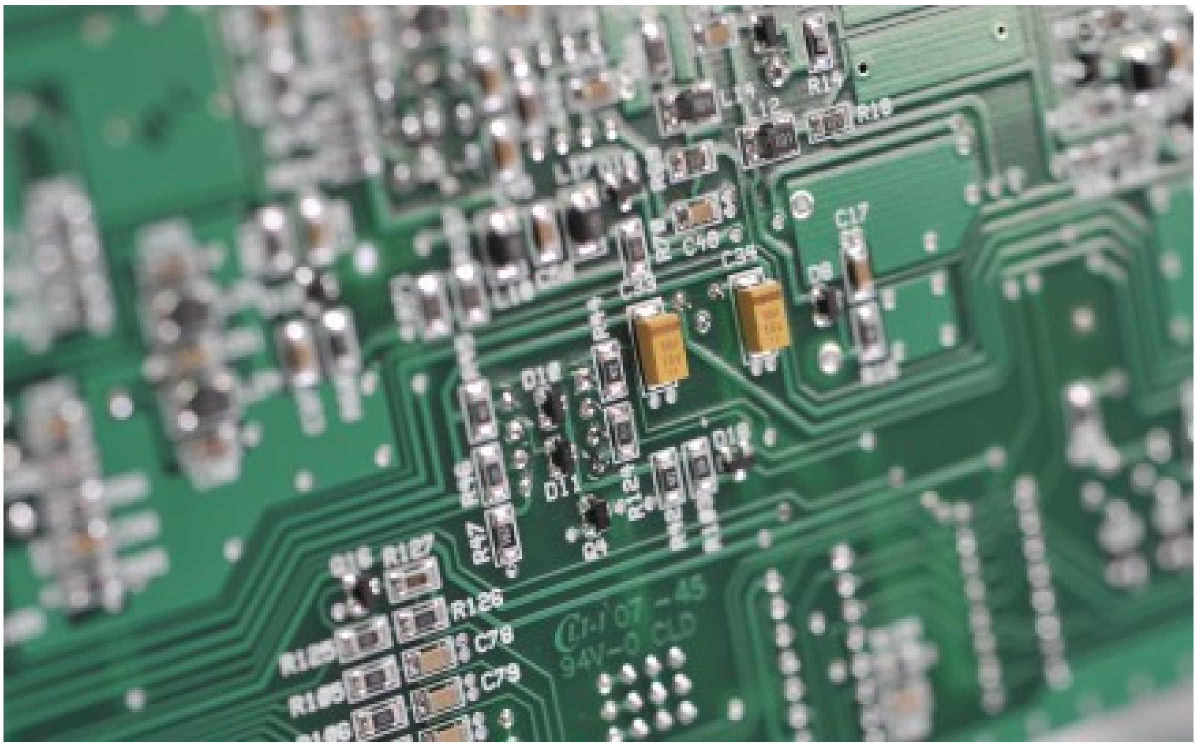

Study On Worst Case Analysis For Safety Electrical and Electronic Circuits

Master's thesis in Embedded Electronic System Design

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Master's thesis 2019

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Abstract

Worst case analysis(WCA) is an analysis method, which provides adequate information about a circuit's robustness at a specific operating point. WCA also provides some statistics about circuit performance in a worst-case scenario. Parameters such as temperature, voltage, current limits, etc. are used to quantify circuits performance. This knowledge aids the designers in developing a robust circuit. This thesis is focused on the integration of WCM into the development process of a company, including finding an appropriate WCA approach. Initial investigation revealed a need for a software tool to perform a smoother and faster worst case analysis. Therefore, the development of an in-house tool in MATLAB with the end user in goal is also addressed. A worst-case analysis was performed on a case study and results were verified with WCM. Hence, demonstrating the integration of WCA tool into the development process.

Keywords: Worst-Case Method(WCM), Worst-Case Analysis(WCA), Sensitivity Analysis, Worst case analyzer, Extreme Value Analysis(EVA), Root Sum Squared(RSS), Monte-Carlo Analysis(MCA), MATLAB, End-of-Life, Beginning-of-life, LT-spice, V-model, circuit analyzer, Equation system solver, Maxima-Minima, Sensitivity analyzer.

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Gothenburg, March 2019

Contents

List of Figures

List of Tables

1
Introduction

Recent developments in the field of technology have made the industries to automate their manufacturing process to reduce the workforce, which in turn leads to cost reduction. However, this cannot completely eradicate human intervention into the process flow. There is a need to focus on the safety of individuals since all machines are not intelligent enough to avoid causalities. Hence, during the time of work, there are possibilities of accidents by machines which causes death or injury to workers.

It is imperative to have safety or protective circuits to avoid accidents. Thus, we need to prioritize safety action inside the industry. Despite the benefits of safety circuits that can reduce accidents and assure the safety of workers, it is impossible to manufacture or develop the components(circuits) which are ideal and can give 100% accuracy on their operation. There are drawbacks, faults and various uncontrollable aspects such as stress on the components, surrounding temperature, material composition and among others, which impact components during their fabrication. As a consequence, we need to concentrate and give more importance to the reliability and robustness of these devices. Hence, to design a reliable and robust safety circuit we need to perform "worst-case analysis"(WCA).

WCA performs a mathematical evaluation considering the parameter variation and attributes such as environmental extremes which affect the desired operation [1]. Hence, WCA is a means to investigate the robustness of the circuit and ensure whether the circuit will perform as required, given that each part is subjected to variations over life [2].

1.1 Purpose

The importance of safety product's robustness and quality has been established above. Hence, it is vital to prioritize the standards of high quality for such products. ABB - Jokab safety at Kungsbacka, is a company which provides solutions for machine safety and residential protection systems. ABB also develop and manufacture safety-related products, which are used extensively in the automation domain across different industries and sectors. ABB wants to increase the robustness and reliability of their products to improve the quality. So, ABB believes WCA is one of the trusted methodologies to improve their quality standards. Hence, ABB wants to perform a scientific study on applying WCM into their development process, with an added goal of having a prototype WCA tool developed and evaluated.

The motivation for developing a WCA tool is that WCA contains a significant amount of calculations which use a large number of parameters, making it impossible for designers to manually consider WCA during design. Therefore, it makes sense to approach this problem methodologically, considering the fundamental properties of the circuit in the framework of a WCA tool. There are several reasons for developing a prototype tool within ABB - Jokab safety:

- Economy and cost-effectiveness:- Purchasing a new tool will be too expensive as it costs for tool installation, maintenance, license, training, etc. It is also important that a WCA tool can be smoothly adapted to the existing development process. Since WCA is an integral part of the process, one must consider the impact the new tool has on the process. A new tool might require changes to the development process, leading to disruptions. From this perspective, developing an in-house tool, which will be more feasible to adopt inside the ABB development process, is better.

- Pre-study suggests that there exist no WCA tools on the market that can satisfy the user requirements. However, there are few WCA techniques, which can be performed using some existing simulation tools. It was found that these commercial software tools do not support certain salient WCA techniques. Here, the purpose of WCA is not to examine the complete behavior of the circuits. It is used to analyze the critical parameters of a circuit and create margin for dimensioning parameter changes suitable for their worst-case scenarios.

- There is a WCA tool developed by RUAG for their internal use, but other companies can not use this tool. However, on some special request ABB - Jokab safety had the opportunity to use it for a while, which helped to gain some knowledge about WCA. However, RUAG's WCA tool has some drawbacks such as it was developed on an old DOS system and it cannot solve the system of mathematical equations. The user must solve the equations himself and input the solution into the tool, which was time-consuming and led to the manual error. These drawbacks are resolved and updated in the WCA tool developed inside ABB, which makes the stream simple, fast and less prone to manual errors.

A scientific problem in this thesis work is to determine verification techniques for WCM and validate the results of the WCA without consuming much development time.

1.2 Objective

The thesis work aims to accomplish below-mentioned goals:

- Identify the state-of-the-art techniques of worst-case methodology.
- Identify the latest platforms suitable for worst-case analysis tool prototype implementation.
- Complete the implementation of the worst-case analysis tool prototype - with a report on the techniques investigated.
- Integrate the worst-case method into product development model(V-model) of the ABB - Jokab safety.
- Validate the results of the tool by experimental setup and comparison with existing RUAG WCA tool.
- Provide a user manual with description of WCA tool to be used by employees at ABB-Jokab safety.

The outcome of this thesis work is to identify the significance of the WCA and its techniques, with complete development of WCA tool and establishing the worst-case method into the development process.

2

Theory

In the following chapter, a brief explanation of relevant concepts, such as the Worst-Case Method(WCM), standard techniques used in Worst Case Analysis(WCA), and guidelines for performing WCA is provided.

2.1 Worst-Case Method(WCM)

WCM is a methodology applied to the development process of safety related products. It constitutes two parts, an analysis part (WCA), in which the circuit is analyzed at an operating point, and a verification part, in which the equations and nominal values for the WCA results are verified [3]. The designer provides the inputs for the analysis part (WCA). The inputs consist of parameter values, tolerance levels, and circuit equations. The output consists of a report, which contains the data or results of the circuit performance at its extreme condition and nominal values. After the WCA, the obtained analysis result is considered into the next level of the WCM, which is verification and validation. Here, nominal circuit values are measured with a practical experiment, which is compared with the WCA results. If the measured and the analyzed nominal values of the circuit match, this validates the derived equations. If the tolerances are correct, then the maximum and minimum worst-case value obtained is trustful. This whole process is monitored and performed in WCM.

2.1.1 Worst-Case Analysis(WCA)

WCA is a circuit analysis, which uses a distinct approach to study the circuit robustness in the worst case scenario, in which the circuit operating condition and variability is considered. In typical circuit analysis, the condition at which the circuit will be operated, and uncertainties of the parameter are not taken into account. These conditions and uncertainties are acknowledged in WCA to examine the circuit robustness at certain operating point [4].

The drift of the parameter value (single component-such as resistor, transistor and etc.) due to the aging, the stress applied to each circuit component (such as humidity, temperature or radiation), and external electrical inputs are considered during the analysis. These variations cannot be neglected as each component will have an impact when subjected to these effects over its lifetime[5]. The knowledge of the changes in components and their impact on the circuit aids the designer to gain a deeper understanding of the circuits robustness. Considering parameters variations into the design process provides the designer confidence that the circuit will meet the desired performance over its lifetime. From this, a designer can identify the issues existing in the new design at the beginning of the design process. As a result, this analysis reduces the risk of redesigning the circuit once it is brought into physical existence. Therefore, this will decrease the cost and time as it avoids to run out the entire development process. The significance of WCA is that it is the only way to determine the circuit's end of life performance under the defined surroundings.

2.2 Approach

The approaches or techniques selected to execute the worst-case analysis on the electronic circuits are explained in the following subsection. The three main approaches opted here were:

- Extreme Value Analysis(EVA)
- Root Sum Squared(RSS)
- Monte Carlo Analysis(MCA)

2.2.1 Extreme Value Analysis(EVA)

EVA is a numerical analysis approach, in which all parameters are evaluated to their extreme value including their initial tolerances. So, by substituting these parameter values into the analytical equation of a circuit and solving them results in the extreme minimum and maximum value of the circuit. Considering the unfavorable conditions of the operating environment, it is possible to estimate the circuits end of life extreme minimum and maximum values [6]. To find out the end of life values, it is important to add the tolerances of parameter drift due to aging, derating factor due to temperature and radiation specified in the parameter data sheet.

EVA is a conservative approach i.e. if the outputs of this approach lie within the desired requirements, then it is assured that the circuit will always meet the design requirements. It is unlikely that all the parameters will reach their extreme values simultaneously. However, the concept is that, if the EVA calculated values give affirmation of the designed circuit output lies within the limit, then it is assured that the circuit will always meet the requirements. If the EVA outcome of the circuit's performance is within the range, this implies the circuit will perform as required in all other scenarios.

2.2.2 Root Sum Squared(RSS)

RSS is a statistical approach, based on the central limit theorem [4], which provides the probability of the circuit performance to be in user defined specification. It is a realistic approach because parameters lie within the interval of values. These parameter values can be assumed as bounded by normal distribution function within which all the parameter values will occur. When there is a large number of parameters it makes the analysis easier by performing analysis statistically using respective distribution function. In RSS, each parameter's standard deviation is combined, and the calculated output will usually be a normal distribution curve of these combined parameter values [5]. Since there are uncertainties in these RSS calculations the limits are multiplied with factor 3 which determine the three-sigma limits which makes the result more precise [5].

To calculate limits of the parameter, their variations are separated as bias and random varaiations. Bias variation means the parameter value will change in the same direction as the change in the environment, e.g., the value will increase if there is an increase in the temperature and decrease if there is a decrease in temperature respectively. In contrast, random variation is unpredictable i.e., the parameter value will change irrespective of changes in the environment. Since the deviation of random terms from their mean values is unpredictable, it is better to determine a standard deviation of these random terms, which is a smaller deviation from mean value and insensitive to any change of location in mean. So, by taking the square of random terms, summing up each term and at last taking the square root of whole term will give the mean standard deviation for the overall random terms. For this reason, in order to calculate the worst-case limit, biased variation is added algebraically and the random terms are used in determining the standard deviation as given in the equation 2.1. Therefore, RSS is a perfect approach for the analysis of random variables [1].

$$\begin{aligned} W.C.MIN &= nominalvalue - \sum negative.bias - \sqrt{\sum (randoms)^2} \\ W.C.MAX &= nominalvalue + \sum positive.bias + \sqrt{\sum (randoms)^2} \end{aligned} \tag{2.1}$$

2.2.3 Monte-Carlo Analysis(MCA)

MCA is a robust statistical analysis technique, which is considered to be the most effective and reliable method for tolerance analysis [7]. The technique randomly chooses the value of the parameter in the circuit and performs multiple runs to converge to the optimum statistical results. The results from MCA depend on the number of runs. The number of runs to be implemented is controlled by the designer. The WCA performed using MCA technique will provide the statistical results of the circuit performance at an operating point. The output of the analysis is a normal distribution curve with tolerance. If this result is well within a certain limit, then it will increase the confidence of circuit performance in the worst-case scenario. In table 2.1, the comparison between all these three approaches is illustrated [4]

2.2.4 Sensitivity Analysis

Sensitivity analysis determines the impact or the effect of each component on the circuit and is an integral part of all the three above approaches mentioned. It is paramount to perform sensitivity analysis to gain a deeper understanding of the effect of each component on the circuit's performance. Worst-case analysis is performed considering the information obtained from sensitivity analysis. The sensitivity of the circuit is its ability to react or respond to the changes in its components [8]. The circuit's sensitivity is determined concerning each of its components, which provides the direction of change and its magnitude of impact. The sign(+/-) obtained by sensitivity analysis identifies whether the fractional change in the components value has a direct or inverse effect on the circuit output. In contrast, the magnitude will determine the amount of change in the circuit's output with respect to the unit change in a component value.

Sensitivity is measured by taking the partial derivative of the circuit equation with respect to its components. Consider X as the output equation and y as one of the components in the equation, then the sensitivity of X w.r.t y is given by 2.2.

$$S_y^X = \frac{\delta X}{\delta y} \times \frac{y}{X} \tag{2.2}$$

Here S_y^X - is the normalized sensitivity value of the X with respect to y. It is more convenient to express the sensitivity in normalized value as it allows comparing the sensitivity of the circuit with respect to each of its parameters [9]. The normalized value obtained conveys that $+1\%$ change in the y will result in $S_y^X\%$ change in X. However, this change either increase or decrease the circuit performance is determined by the sign convention of S_y^X.

The significance of sensitivity analysis is that by looking at the measured sensitivity, it helps the designer to decide about the requirements of components. The designer can select precise components with narrow tolerances for which circuit is more sensitive and wide tolerance components for which circuit is less sensitive. Hence it is feasible to decide upon the components requirements earlier by using the sensitivity result as this has a yield in manufacturing cost savings [10].

Method	Advantages	Disadvantages
Extreme Value Analysis (EVA)	• Most readily obtainable estimate of worst case performance • Does not require statistical inputs for circuit parameters • Database need only supply part parameter variation extremes • If circuit passes EVA, it will always function properly	• Pessimistic estimate of circuit worst case performance • If circuit fails, there is insufficient data to assess risk
Root-Sum-Squared (RSS)	• More realistic estimate of worst-case performance than EVA • Knowledge of part parameter probability density function (pdf) is not required • Provides a limited degree of risk assessment	• Standard deviation (σ) of parameter probability distribution is required • Assumes circuit sensitivities remain constant over range of parameter variability • Assumes circuit performance variability follows a normal distribution
Monte Carlo Analysis (MCA)	• Provides the most realistic estimate of true worst-case performance • Provides additional information in support of circuit/product risk assessment	• Requires use of computer • Consumes a large amount of CPU time • Requires knowledge of part parameter pdf

Table 2.1: This table contrasts EVA, RSS and MCA by looking at the points of interest and hindrances.

3

Methodology

This chapter describes the methodology adopted, and stages(tasks) performed to achieve the objectives of the thesis work. In Section 3.1, the identified state of the art approaches to perform the worst-case method is described. In Section 3.2, describes the adoption of the worst-case method(WCM) into the product development V-model.

3.1 Investigation

The first step in this thesis like any other was to perform a literature survey. The initial steps taken during pre-study is to gain an understanding of the worst case analysis and study the latest WCA approaches. A suitable computing platform was chosen to implement the WCA tool prototype. The reason for selecting the MATLAB as the computing platform is explained in detail in Appendix A.1. The WCA procedures chosen were based on fulfilling the user requirements as provided in Appendix B.1.

According to literature, there are a number of approaches to perform worst case circuit analysis. However, in this thesis, it was decided to consider only three approaches which are extreme value analysis(EVA),root sum squared(RSS) and Monte-carlo simulations(MCA). These approaches are considered to be the standard industrial approaches to the best of the knowledge of the author. In the above Section 2.2, these approaches are explained in detail. However, there are other approaches which have been excluded in Section 2.2. A brief description of these excluded approaches are provided in the below Section 3.1.1 with the reason for not considering them.

3.1.1 Novel Techniques

Novel techniques are those techniques developed lately for evaluation of worst case analysis. During pre-study, there were number of studies, which were identified. In [11], the authors followed the μ - analysis to overcome the uncertainties in the circuit and their component parameters. It is a robust method which tries to minimize the uncertainties. However, to do this analysis, the circuits must be represented using the linear fractional transformation(LFT), which is a difficult and complex task. The study performed in [11] only considered linear circuits which restrict the use of this technique for a large number of applications containing non-linear components. The derivation of LFT is complex, and since it is restrictive to linear circuits, we cannot consider this technique for this work.

Another technique defined in [12], was based on the interval linear equation method, which is used to analyze the uncertainties in the component's parameters. Every component parameter has a specific value and tolerance, which makes the value to lie within a range. Hence it can be expressed using interval linear equations. It is possible to solve these interval equations using existing methods. However, in this technique, the author assume the parameter variation is monotonic[1] with respect to uncertainties, which make it restrictive to analyze the circuits whose parameter variation is not monotonic.

In [13], an approach is based on Taylor models and analysis is enforced in the frequency domain. The worst-case is analyzed in the frequency domain and the performance of the circuit is only verified under some selected frequency. In this thesis, working in the frequency domain will narrow down the analysis as the scope of our thesis work is to analyze the whole circuit's performance at particular operating point (static analysis) and ensure the robustness of it. The complete behavior analysis of the circuit can be performed by tools mentioned in [14]. However, in this thesis, the scope is only limited to WCA and not the complete behavior of a circuit.

The primary reason for not considering these techniques mentioned above is that the industrial standards do not approve of them. Since this is an industrial thesis and the primary objective is to implement a tool prototype to perform WCA, and therefore only techniques which are accepted by the industrial standards are considered. So that in the future, the tool can be validated fulfilling the requirement of the functional safety standards.

3.2 Adoption of WCA

In this section, the product development life cycle model or V-model used in ABB-Jokab safety is explained in brief with the integration of worst-case method into the model. The V-model is shown in Fig. 3.1 and it describes the process flow of product development from the top-left to top-right. The left portion of the model is the processes where the requirements are defined, and significant development of the new product is carried out.The right portion of the figure shows the processes of *verification and validation*, once the product is designed and developed. This whole process arrangement looks like a V shape; hence it is called V-model.

[1]for example: let us consider a parameter 'P' which is a function of temperature t. The parameter $P(t)$ is said to be monotonic if the function is varying in such a way that it either never decreases or never increases. That is if P starts to increase for change in t then it will either increase or be proportional to change in t but never decrease and vice-versa.

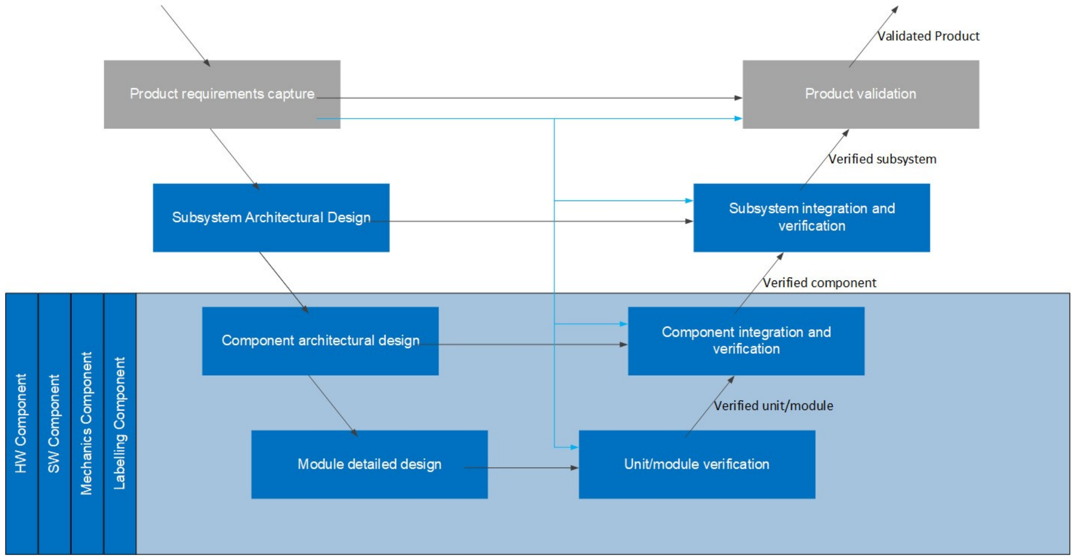

Figure 3.1: The product development process flow based on the V-model followed currently in ABB-Jokab safety is shown here.

- **Product requirements** - The first block(top-left) of the model collects all the data from the market. This data is collected for the development team to have a better understanding of the performance of the products. The output of this stage is the defined product requirements or specification. The output is sent to the next stage and also to the parallel validation stage to the top-right of the model to keep a copy of it. The copy saved in the validation stage is used in the later stages to verify the product, after the development is completed. It is necessary to make sure that the product specifications are met.

- **Subsystem architectural design** - In mid-to-large sized systems, it is a preferred idea to break down the whole system into smaller subsystems. For ABB-Jokab safety, a product constitutes a subsystem in a safety system, i.e., several subsystems make up a complete safety system. The system is defined outside V-model. The V-model is used to develop system components(subsystems/products). The output of this stage is specifications for the subsystem or product technical requirements.

- **Component architectural design** - An input to this stage is the subsystem specifications, and it will break the subsystems into components. Here, the components are divided into hardware, software, mechanics and product information components. This stage provides the complete information of the subsystem and each of its components. It follows a top-down approach where one big subsystem is broken-down into small components which makes the work feasible. Hence, the focus can be given to every finite component and its performance.

- **Module detailed design** - Every component specification is taken into account for the complete module design. When the module is done, it is analyzed, simulated and physically realized. Later it is moved into the verification stage with the detailed report of the module for the performance validation.

- **Verification and validation process** - the processes of verifying and integrating all the modules and components into one large subsystem or product are performed here. It is a bottom-up approach where before integrating each element it will be verified and validated for the proper functionality as per the product specification. In Fig. 3.1, it can be seen that the line is moving into all the stages of the verification process this will provide the functionality and method of verification to be followed. Each stage will provide the output with the verification results documented for future use.

Therefore, by understanding the development flow, it is decided that to unify the worst-case method into the module detailed design stage under the hardware components. It is better to perform worst case circuit analysis before the circuits are brought into existence. Once the design is completed by performing the WCA, the parameter sensitivity can be determined. Based on the results of WCA, the precise circuit components are purchased which make the product cost lower and can also bring an immediate change in the design if needed.

4

Prototype Tool's Architecture and Implementation

Section 4.1 explains the basic blocks used in the tool and the process used during the stage of implementation. The tool implementation method is described in Section 4.2.

4.1 WCA tool Architectural Design

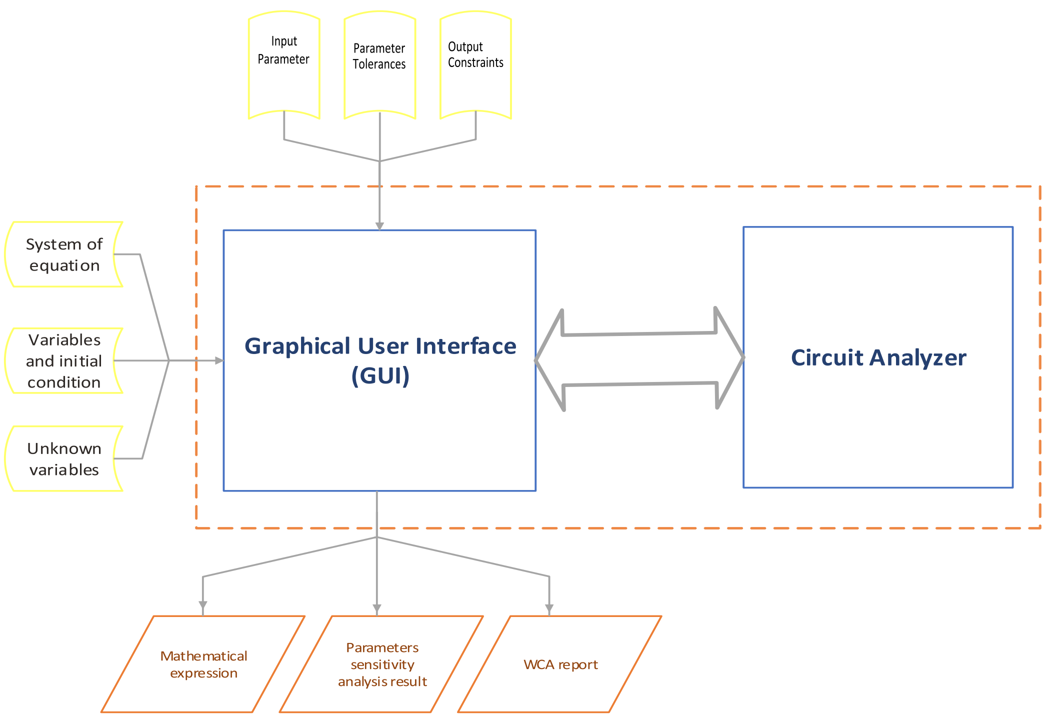

Figure 4.1: User defined top level design of the worst case tool

The complete top level architectural design of the tool is shown in Fig. 4.1. It also illustrates the data flow inside the tool from inputs to circuit analyzer and then to the output. It also illustrates the blocks which are not considered to be part of this thesis work. This thesis is limited to the analyzer block shown in Fig. 4.1. The input data format to be used is defined in this thesis. In Fig. 4.1, the blocks to the left side and on the top of the figure are the inputs provided by the user. The GUI takes the user input and converts it to a specific format accepted by the analyzer. Similar to this, the bottom blocks shown in the figure contain outputs of WCA result.

4.1.1 Design of Circuit Analyzer

In this section, the circuit analyzer block and its components are introduced. Block diagram of the circuit analyzer with its components is shown in Fig. 4.2. The three main components are equation system solver, sensitivity analyzer and worst case analyzer. The information is flowing sequentially from left side to right side of the components:

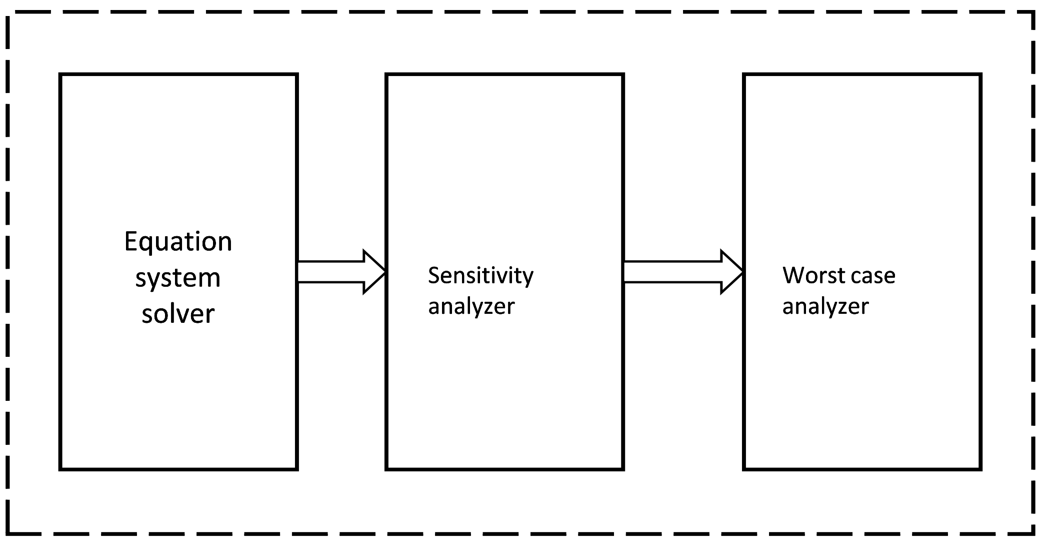

Figure 4.2: Block diagram of circuit analyzer with its contents or components which mainly performs the worst case analysis of the desired circuit

- ***Equation System Solver(ESS)***- The system of equations which describes the circuit is the input to the tool from the input block called **system of equation**. These systems of equations are a direct form of algebraic, differential or differential algebraic equations with known and unknown variables. They contain the desired number of unknown parameters which need to be determined. Hence, this is fulfilled by ESS, by solving the equations for the user defined unknown parameter.

- ***Sensitivity Analyzer(SA)***- In section 2.2.4, the importance of sensitivity analysis is explained. The sensitivity analysis is performed in this block for the derived equation from ESS block. It substitutes the value and its tolerances into the equation. Also, it defines the influence of the desired parameter on the output. It also calculates the effect on circuit output for a relative change in an individual parameter.The operating points are also defined in this block.

- ***Worst Case Analyzer***- The techniques described in section 2.2 are implemented in this block. It considers the result from the two previous blocks and uses it to determine the worst-case values of the circuit. The result of this block is the worst-case analysis report, which contains the worst-case values for the circuit at an operating point defined in the sensitivity analyzer block.

4.2 Implementation

In this section, the different parts of the tool are explained. Fig. 4.3 shows an overview of the parts included in the circuit analyzer. Choices made in the above design section are realized here.

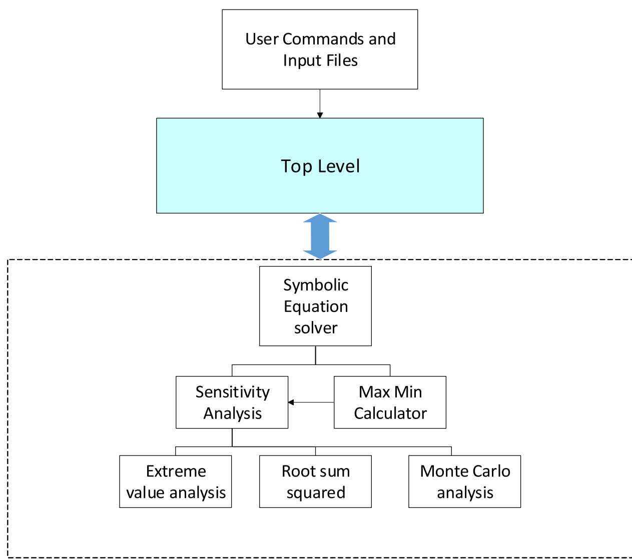

Figure 4.3: Overview of the implemented worst case tool - circuit analyzer and its sub-blocks. The double headed arrow represents the flow of inputs and commands passed into the analyzer parts.

The prototype tool, implemented in MATLAB need commands and input data files, which are provided by the user through the graphical user interface(GUI). The GUI arrange these data in the accepted format and start the analyzer. Based on these inputs, the analysis is carried out, and the results are displayed in the GUI and also stored in the log files.

4.2.1 User Commands and Input Files

During the initial development of the tool, it is essential to decide the formats of the input files of the tool. This will help the designer to develop the tool according to the user requirements. The lack of knowledge on the input files makes tool development a difficult task. There are some inputs and constraints, which are provided through the GUI as the command by the user.

The circuit, which needs to be analyzed is defined with two files:

- The first file contains all the system of equations, variables which are defined as known and unknown, as shown in Fig 4.5. It is divided into seven sections depending upon types of equations and variables to differentiate between one another so that the tool can handle it.
- The second file contains data required by the tool to perform numerical calculations. The known circuit parameter values are tabulated with their nominal, weights and tolerance values as shown in Fig 4.4.

Parameter |	Nom |	Weight |	Min |	Max |	Max_eol |	Min_eol |
V_in	7.105	N	-0.234	0.24	0	0
Q1	0.3	N	-0.30	0.3	0	0
D1	0.6	N	-0.15	0.15	0	0
Q2	0.3	N	-0.3	0.5	0	0
kr	180	N	-18	18	0	0
kl	1.3	N	-0.8	0.8	0	0
D2	0.6	N	-0.15	0.15	0	0
c1	0.2	u	-9.60E-07	8.00E-07	0	0
b2	0.036	N	0	0	0	0

Figure 4.4: Format of one of the input files, which contains the tabulated data of the parameter nominal, initial and end of life tolerance values with their weights such as N - No weight, k - kilo, u - micro, etc.

There are some commands to be specified by the user such as:

- WCA techniques - it helps the tool to select any one technique among EVA, RSS and MCA.
- Input files - equations and data file.
- Time range.
- Number of runs for Monte Carlo if MCA is used.

4.2.2 Equation System Solver

The equation solver reads the set of mathematical equations, and it solves for the user-defined unknown variables. So, the output is also a number of solved equations of the unknowns. It solves the equation with a symbolic solution, which only contains known variables. This part of the tool is implemented using the symbolic toolbox of MATLAB [15]. The user must define the known, unknown and desired variables (unknown variables on which the user is interested in performing analysis) for the tool. Hence, the format of the input file is designed in such a way that this can be accessed easily by the solver. Fig 4.5 shows the example format designed for the tool with the various sections allocated for the types of variables and equations.

```
variables
q1
d1
c1
q2
kr
kl
d2
v_in
b2
unknownvariables
bc(t)
diff_variables
b(t)
uc(t)
equations
bc(t)+b(t)==0
diff_equations
bc(t)+c1*diff(uc(t),t)==0
-b(t)*kr+uc(t)-d2-diff(b(t),t)*kl==0
conditions
b(0)==b2
uc(0)==v_in-q1*b(0)-d1
desired_variables
uc(t)
end
```

Figure 4.5: Example of an input file format designed for the WCA tool prototype.

Once the file is set up by placing all the inputs under correct heading, it is read by the solver, and the variables and equations are converted into a symbolic format suitable for the WCA tool. The solver is capable of distinguishing the equations and variables and solve for the desired variables symbolically. The circuit equations contain several unknown variables, but the user might not be interested in all the unknowns. The user will select the variables that should be analyzed. It can handle a maximum 20 number of equations at a time, and it is capable of solving algebraic equations, second order differential equations and first order system of differential algebraic equations. For solver, to solve for the unknown variables, the number of equations must be equal to the number of unknown variables. If the system of derived circuit equations is of higher order, the tool might crash or output wrong results. In that case, the user should preferably break the circuit into a smaller pieces or reduce the order of the equation.

4.2.3 Sensitivity Analyzer

The need for sensitivity analysis is mentioned in section 4.1.1. As the output generated from this analysis is used further in different WCA techniques, a separate function has been developed to perform sensitivity analysis. It contains three input arguments: the equations of the unknown variable, a file containing the tabulated parameter values (data file) of components and an output file. These components parameter values are defined as known variables in the equation file. The tool reads this data and calculates the parameter limits, i.e., maximum, minimum and nominal values. After this step, the function substitutes previously defined known symbolic variables with their respective nominal values to find the circuit's nominal output performing the numerical calculation. Sensitivity is determined by substituting these nominal parameter values into the first derivative of the derived equation with respect to each parameter. The output, magnitude, and change of direction represent the impact on the circuit output due to the change in the parameter.

Even though the component tolerances and their values are selected precisely based on the sensitivity result, sometimes there are possibilities of facing difficulty to maintain the output value within the desired limit. This difficulty is due to many factors and uncertainties in the circuit's output. One such factor is the relative change of the component with respect to the circuit. To explain the importance of the relative change, we will consider a voltage divider circuit as an example shown in Fig 4.6. The equation of the voltage divider circuit is given in 4.2. The sensitivity of the voltage divider circuit is calculated, and it is found that $R1$ has a significant impact on the output. Hence, after the consideration of the sensitivity output, a precise $R1$ with narrow tolerance is used for the design. However, in reality, it was difficult to maintain the output voltage within the desired limit. The parameters with wide tolerance could affect the circuit performance due to their wide range of operating values.

$$RelativeChange(V_{inmax}) = \frac{X}{X_{ref}} - 1 \tag{4.1}$$

Here X is the circuit output calculated by keeping the input voltage at its maximum value, and the other parameters at their nominal values. X_{ref} is the circuit output calculated when all the circuit parameters values are kept at their nominal values.

$$V_{out} = \frac{R2}{R1 + R2} \times V_{in} \tag{4.2}$$

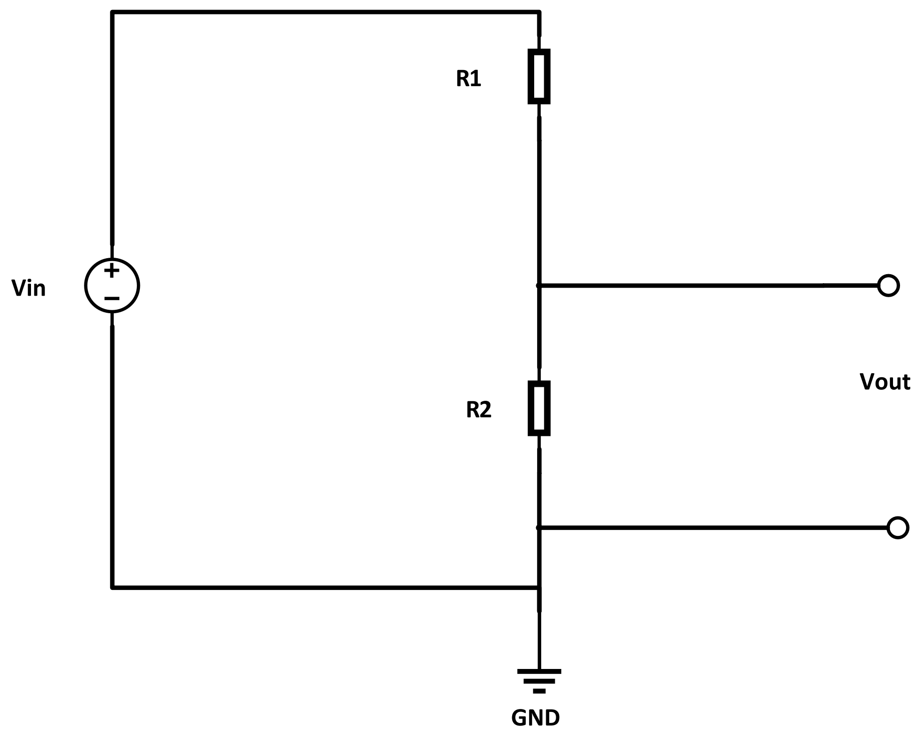

Figure 4.6: Voltage divider example circuit used to explain importance of relative change of the components with respect to the circuit output.

Table 4.1 shows the impact of the input voltage on the voltage divider circuit. The relative change in the output voltage for a specific change in input V_{in} is very high. This voltage divider circuit is an example of the high impact of the parameter which has a wide tolerance. Sensitivity analyzer function is developed such that it can also calculate the relative change values of all the parameters and their impact on the output.

Parameters	Values	Sensitivity	Relative Change
V_{in}	12±0.5	0.666	-0.0415
$R1$	2±0.1	-1.333	0.0163
$R2$	4±0.2	0.666	-0.0161

Table 4.1: This table contains the nominal values of the voltage divider circuit and the calculated sensitivity and relative change of the output.

The results from this analyzer have both sensitivity and relative change of the parameters. The user can utilize the sensitivity result once the draft design is done so that the components impact on the circuit can be analyzed and changed. Later, if the results are not satisfactory, the user can consider the relative change value for further optimization of the design parameters.

4.2.4 Maxima-Minima Calculator

The equations for a circuit containing capacitors and inductors are generally in differential form and time-dependent. Hence, it is important to define the operating point or time at which the user is interested in analyzing the circuit. The defined operating point will narrow down the analysis into the static analysis. The user is interested in the maxima and minima values of the circuits since they are the possible extreme values. Within a given operating point, the time is determined at which the local maximum or minimum appear. A MATLAB-function is implemented to find the maxima and minima of the function using the first derivatives principle based on calculus.

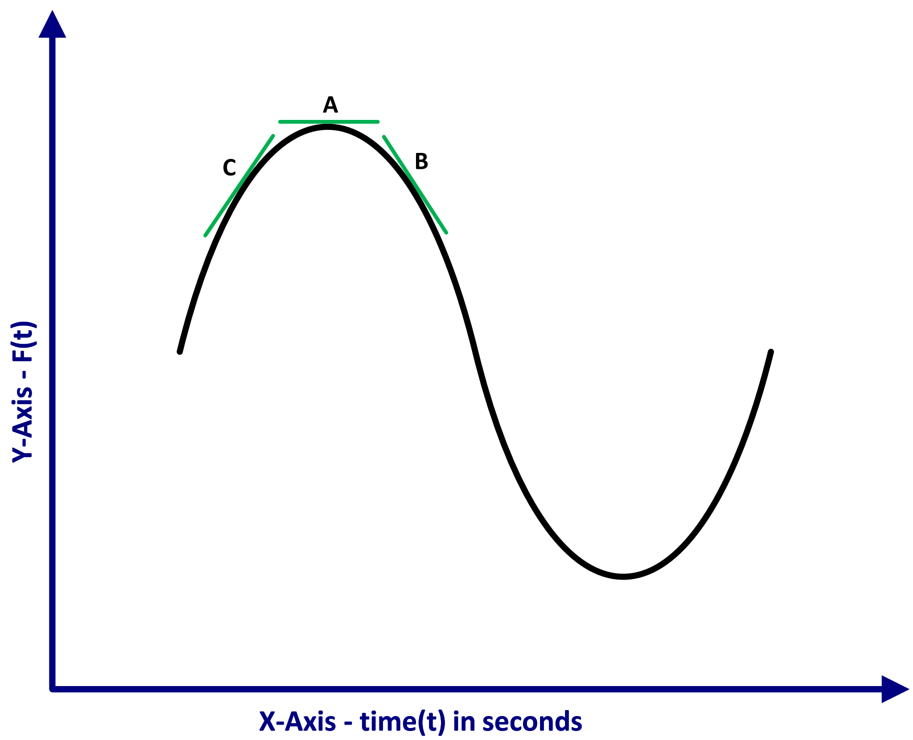

Figure 4.7: The figure represents the function $F(t)$, which varies with time and reaches its maxima and minima value in a certain time. The tangent C has a positive slope, tangent A has zero slope value, and tangent B has a negative slope value drawn to the curve.

In calculus, to find the maxima or minima of a function $F(t)$ there is a method, which accomplishes to find the slope of a tangent drawn to the function as shown in Fig 4.7. The slope of the tangents drawn to function can be determined by the first derivative of the function's equation. As shown in the Fig 4.7, the tangent A drawn to the curve at its maximum value is parallel to the x-axis, which means the slope is equal to zero, i.e., the value of the first derivative is zero. However, to determine whether the time at which the slope is equal to zero, is at the maximum or minimum value of the circuit. The tool will find the slopes around the time at which the slope is equal to zero. If the values of the slope change from plus(+) sign to minus(-) sign, as shown in the figure tangent C and B then the obtained value is a maximum and similarly if the change is from minus(-) to plus(+) sign then it is at minimum.

4.2.5 WCA Techniques Realization

The techniques chosen to perform WCA in this thesis work is already explained in Chapter 2. Now, in this section the implementation and its significance is described.

4.2.5.1 EVA

EVA is a pessimistic approach, which tends to calculate the worst possible value of the circuit. To calculate the worst case values, EVA considers the results of the sensitivity analyzer and maximum-minimum values of a parameter as an input. Here, the sensitivity result is used in such a way that, if the sensitivity of a parameter of the circuit is directly proportional(+) to the output, then to find its worst case maximum value, the maximum value of a parameter is considered. In case of inversely proportional (-) the minimum value of a parameter is considered to find the worst case maximum value of the circuit. Similarly, the above method is followed while calculating the worst case minimum value of the circuit.

Consider an example equation with two variables and its sensitivity as $variable_1$ being inversely proportional(-) and $variable_2$ being directly proportional(+) respectively then:

$$Y = \frac{variable_1}{variable_2} \tag{4.3}$$

$$Y(WCA)_(max) = \frac{variable_{1(}min)}{variable_{2(}max)} \tag{4.4}$$

$$Y(WCA)_(min) = \frac{variable_{1(}max)}{variable_{2(}min)} \tag{4.5}$$

When it comes to the circuits which is defined using differential equations, the user will define the time range of their interest to determine maximum or minimum value. EVA function will consider the time at which the maximum or minimum is reached, then calculate the worst-case maximum and minimum values are calculated at an operating point.

4.2.5.2 RSS

Described in section 2.2.2, RSS is based on central limit theorem, which states that given a distribution with mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with mean μ and variance σ^2/n, where n is the number of samples. Accordingly, this approach considers that the parameters are uniformly distributed around its nominal values, and lie within their tolerances as shown in Fig 4.8. To calculate the standard deviation of any parameter which follows uniform distribution, the maximum value of the parameter must be divided by $\sqrt{3}$. Substituting the parameter's standard deviation and their sensitivity result in Equation 4.6, the circuit's standard deviation can be estimated.

$$\sigma(ciruit) = \sqrt{(\frac{\delta X}{\delta p_1} \times \sigma(p_1))^2 + ... + (\frac{\delta X}{\delta p_n} \times \sigma(p_n))^2} \tag{4.6}$$

Here, X - is the circuit's nominal value, p_1 is one of the circuit parameter, $\sigma(circuit)$-standard deviation of the circuit, and $\sigma(p_n)$ - standard deviation of the nth parameter.

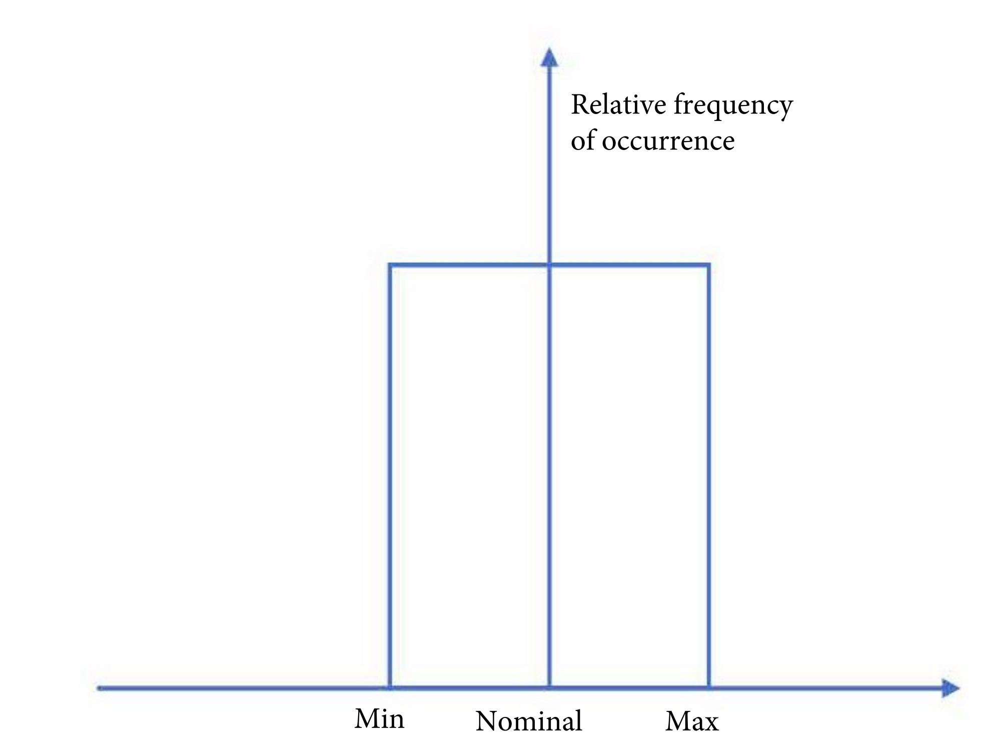

Figure 4.8: Uniform distribution of the parameter values around its nominal values based on their tolerances.

RSS function is implemented using the Statistics toolbox of MATLAB. If the tolerances of the component's parameters are known, then the worst case limits of a circuit can be determined using the RSS technique. However, RSS is commonly used in the manufacturing industries to evaluate their production processes. The normal distribution of the circuits manufactured is determined using their standard deviations calculated by the RSS technique. This circuit's normal distribution will give the worst case values of the circuit. However, this work is labor intensive and most of the parameters data is not available to do the calculations. Hence, RSS technique is limited by data availability.

4.2.5.3 MCA

MCA function for the WCA tool is also implemented using statistics toolbox of MATLAB. MCA uses the random values of the circuit parameters to determine the circuit's performance. The random values of the parameters used by MCA for numerical calculations lie within their tolerances and these random values are generated using the inbuilt MATLAB function called *random*.

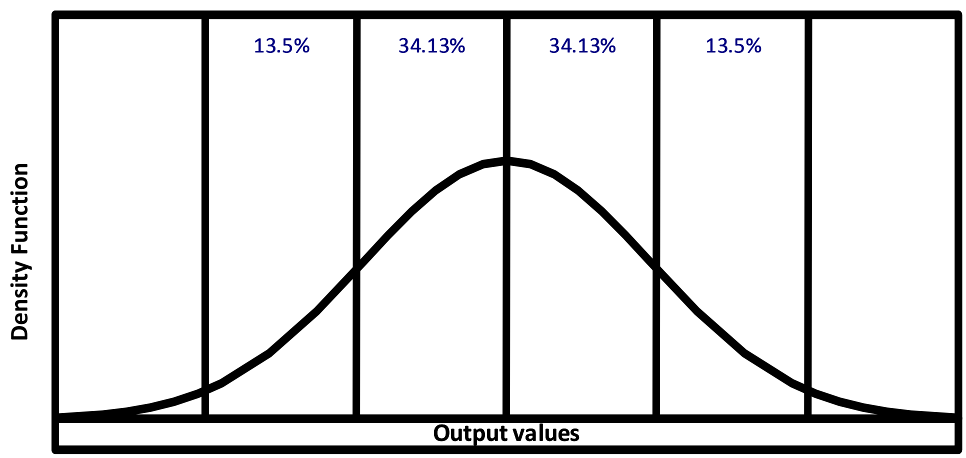

Figure 4.9: Normal distribution curve (Bell curve), where x-axis represents the output values of the circuit and y-axis represents the probability density function. The vertical lines on the curve are drawn at the standard deviations 1σ, 2σ, 3σ of the curve.

The parameter's random value is substituted into their respective variables of the circuit equations to calculate their results. This MCA calculation is made for N number of times for a circuit to determine the statistical result of it. Here, N is the number of runs the Monte Carlo analysis has been performed for a circuit, and the value of N is defined by the user. After the Monte Carlo calculations each circuit will have N number of results, from which the maximum and minimum value of the circuit is determined. The mean and standard deviation of a circuit is calculated from these results, which is used to produce the normal distribution curve of the circuit as shown in Fig 4.9. The normal distribution curve will determine the limits and the probability density of the circuit's performance. The accuracy of MCA depends on the number of runs, the more runs, the higher the accuracy.

4.2.5.4 Top-Level

A top level is developed to control the data flow and user commands into the WCA tool prototype. The number of inputs required to perform full scale WCA is nine. These inputs are divided into 3 commands, which are provided by the user. The first command includes the system of equation file which is shown in Fig 4.5. The second command deals with parameter data file which is shown in Fig 4.4. The third command deals with the technique to perform worst case calculation, the number of runs for MCA and the time range. These inputs are provided through the GUI, and accordingly, WCA is performed. The results generated are displayed in the GUI and also stored in log files.

5

Results

In this chapter, the verification of the WCA tool and test methods are described. The comparison between the experimental results with the results obtained from the developed worst case tool prototype is also described. In Section 5.1, the verification and test methods are explained. In Section 5.2, the nominal value calculations and the symbolic equation solver is examined. In Section 5.3 and 5.3.1, worst case EVA and MCA results of $circuit-1$ and $circuit-2$ are compared respectively.

As described in Section 2.1.1 the worst case values of the circuit are calculated only at a specific operating point. It is essential to mention that, in general, the output results from the WCA tool prototype are static values at a specific operating point. However, for the better understanding of the tool's performance and its limitation here the results are shown in plots, which helps the author to explain the results and their differences.

5.1 Method of Verification

This section describes the method used to validate the circuit analyzer of the WCA tool prototype. It is paramount to verify the WCA tool prototype performance, so that the output can be verified. The validation of the WCA tool prototype is made considering the example circuit's explained in section 5.1.1 and section 5.1.2.

5.1.1 Circuit-1: LCR circuit

Electronic $circuit-1$, shown in Fig 5.1, is selected because the characteristic of this $circuit-1$ is described using second order differential equations. Each parameter value of this $circuit-1$ physical prototype is measured in order to eliminate the BOL tolerances (initial tolerances) of the components used in it. These measured values are used in WCA tool prototype for the worst case calculations.

The EOL tolerances were determined only for the capacitor $C1$ by adding a known capacitor value in series and parallel respectively. The delta change in the capacitance of $C1$ is treated as EOL tolerance for the capacitor in the WCA tool prototype. The circuit parameters and their tolerances used for calculations are showed in Table 5.1.

There are two cases when analyzing the $circuit - 1$:

- Case 1 - when the switch is closed - in which circuit components are charged.
- Case 2 - when the switch is open - in which the circuit components starts discharging.

In this validation process, the circuit is analyzed for the case when the switch opens, i.e. case 2.

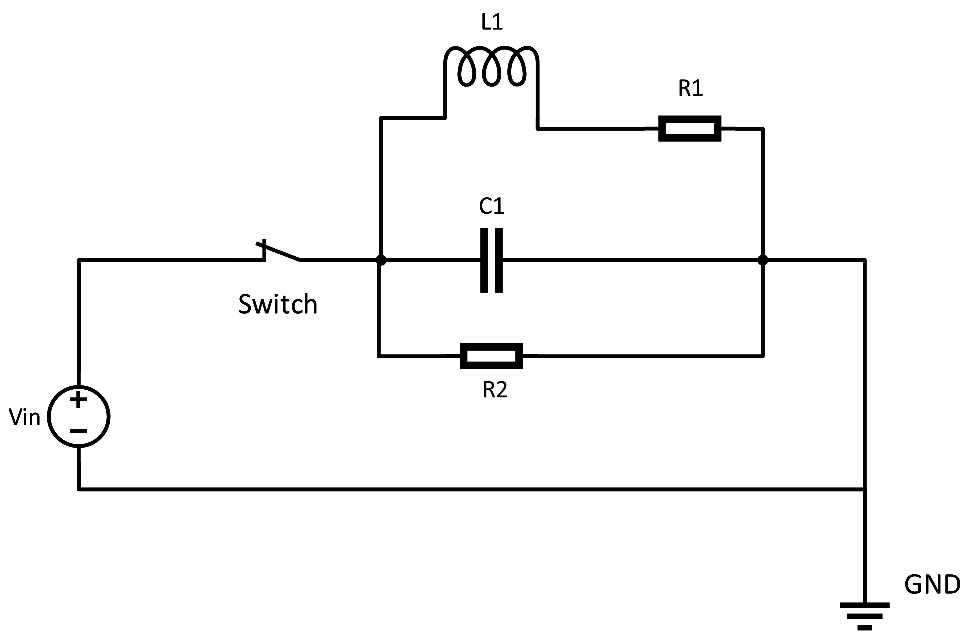

Figure 5.1: The example circuit used to verify the results calculated from the WCA tool prototype. The parallel LCR circuit with a switch connected in series to control the circuit's input.

The system of equations shown in 5.1 to 5.6 were used in the WCA tool to analyze the $circuit - 1$. The WCA tool solves these equations to determine the solution for the desired outputs.

Equation system:

$$i_{L1} = -i_{C1} - i_{R2} \tag{5.1}$$

$$i_{C1} = C1 * \frac{dV_{C1}}{dt} \tag{5.2}$$

$$i_{R2} = \frac{V_{R2}}{R2} \tag{5.3}$$

$$V_L - L1 * \frac{di_{L1}}{dt} - i_{L1} * R1 = 0 \tag{5.4}$$

Initial conditions:

$$i_{L1} = \frac{Vin}{R1} \tag{5.5}$$

$$V_{L1} = Vin \tag{5.6}$$

Here, i_{L1}, i_{C1} and i_{R2} - are current through inductor $L1$, capacitor $C1$ and resistor $R2$ respectively. V_{C1}, V_{R2} and V_{L} - voltage across capacitor $C1$, resistor $R2$ and inductor $L1$ with resistor $R1$ respectively and the desired output is V_{L}.

Parameters	Nominal	BOL-Min	BOL-Max	EOL-Min	EOL-Max	Comment
Vin	0.492 V	0	0	0	0	-
$L1$	1.06 H	0	0	0	0	Measured in room temperature at 1V and 100Hz
$C1$	$1.05\mu F$	0	0	-0.54μ	1μ	Measured in room temperature biased at 10 V
$R1$	40.8Ω	0	0	0	0	Measured in room temperature
$R2$	$1.993\ k\Omega$	0	0	0	0	Measured in room temperature

Table 5.1: This table shows the parameter values and their tolerances of the components, which are used in $circuit-1$. The components that are used to develop the physical prototype of the LCR circuit have been measured in room temperature to get their parameter values.

5.1.2 Circuit-2: Power supply circuit

A power supply circuit, $circuit - 2$, shown in Fig 5.2, is the second example circuit. This contains two voltage regulators $IC - 1$ and $IC - 2$ along with resistors and a transistor. The power supply circuit has an input v_buf and two outputs v_int and v_ctrl respectively.

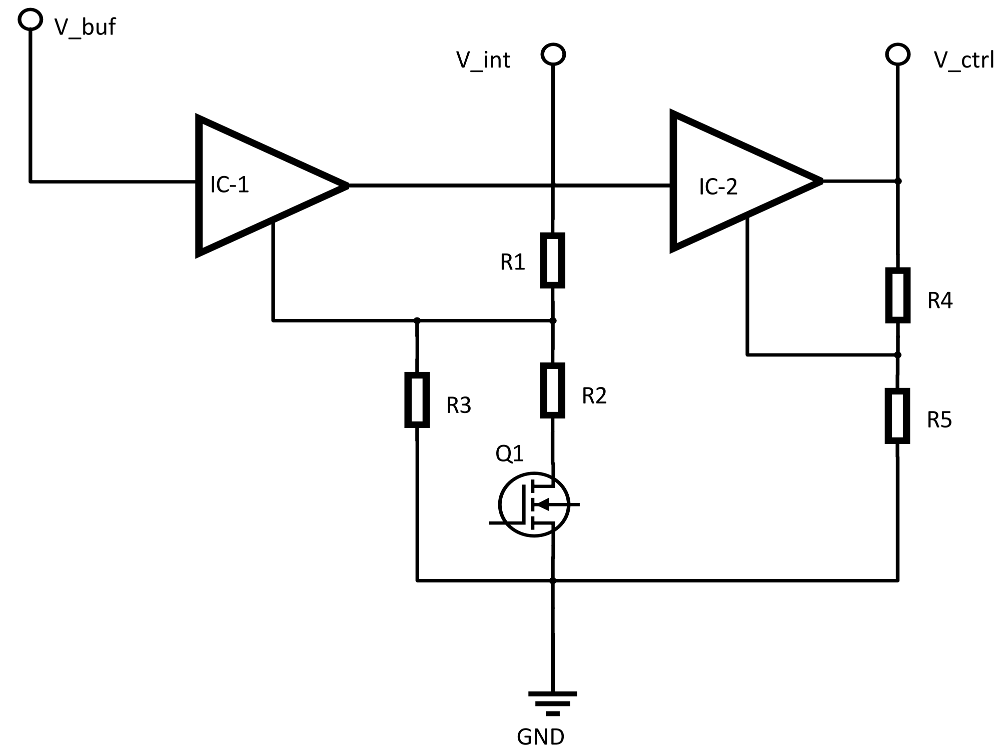

Figure 5.2: This figure shows the power supply circuit, which regulates the input voltage and outputs the constant voltages.

$Circuit - 2$ has been analyzed and verified before by ABB Jokab Safety using the RUAG WCA tool (old), and the result obtained from the RUAG WCA tool calculations has been verified experimentally. Hence, considering $circuit - 2$ as an example circuit for verifying and validating the ABB's WCA tool performance will provide a reliable method for testing. The ABB WCA tool is used to analyze $circuit - 2$, and the results were compared with the results of the RUAG WCA tool. The worst case analysis is performed using the EVA and MCA techniques to calculate the worst case results of $circuit - 2$.

The system of equations shown in 5.7 to 5.9 were used to analyze the worst case performance of $circuit - 2$. The WCA tool solves these equations to determine the solution for the desired outputs. The circuit parameters and their tolerances used for calculating $circuit - 2$ worst case values are showed in Table 5.2

Parameters	Nominal	BOL-Min	BOL-Max	EOL-Min	EOL-Max	Comment
$R1$	86.7 $k\Omega$	-86.7	86.7	650.25	650.25	-
$R2$	11 $k\Omega$	-11	11	-82.5	82.5	-
$R3$	11 $k\Omega$	-11	11	-82.5	82.5	-
$R4$	131Ω	-1.31	1.31	-3.93	3.93	-
$R5$	392Ω	-3.92	3.92	-11.76	11.76	-
$R5$	392Ω	-3.92	3.92	-11.76	11.76	-
$q1\_rds$	4Ω	-2	4	-4	6	FET transistor on state resistance
ic_vfb	0.8 V	-16e-3	16e-3	-16e-3	16e-3	-
$ic2\_vref$	1.25 V	-42e-3	50e-3	-42e-3	50e-3	-
$ic2\_iadj$	50μA	-50e-6	50e-6	-50e-6	50e-6	-

Table 5.2: This table shows the parameter values and their tolerances of the components, which are used in $circuit-2$.

Equation system:

$$v\_int\_l - ic\_vfb * (\frac{R1}{R3+1}) = 0 \tag{5.7}$$

$$-v\_ctrl + ic2\_vref * (\frac{1+R5}{R4}) + ic2\_iadj * R5 = 0 \tag{5.8}$$

$$-v\_int\_h + ic\_vfb * (\frac{R1}{\frac{R3*(R2+q1\_rds)}{R3+R2+q1\_rds}} + 1) = 0 \tag{5.9}$$

Here,

- ic_vfb - IC1 forward voltage(voltage regulator)
- $ic2\_iadj$ - IC2 forward voltage(voltage regulator)
- $ic2\_vref$ - IC2 reference voltage(voltage regulator)
- $q1\_rds$ - FET on state resistance of transistor $Q1$
- $R1$, $R2$, $R3$, $R4$ and $R5$ - Resistance value of each resistor respectively
- v_int_h and v_int_l- internal reference voltage high and low respectively
- v_ctrl - control voltage

Here, v_int_h, v_int_l and v_ctrl are the unknown variables, which are desired outputs.

5.1.3 Validation Of the WCA Tool Prototype

The validation of the WCA tool prototype was performed in the following sequence:

- Validation of the equation system solver.
- Validation of the worst-case analyzer, EVA part.
- Validation of the worst-case analyzer, MCA part.

The steps listed below were followed to perform a validation of the WCA tool prototype.

- Analyze $circuit - 1$ which is defined using the differential equations.
  Purpose: To validate the differential equation solver.
- Analyze $circuit - 2$ which is defined using the algebraic equations.
  Purpose: To validate algebraic equation solver.
- For $circuit - 1$, consider the nominal values of the component and compare the WCA tool result plotted from MATLAB with LT spice simulation plot.
  Purpose: To verify that the WCA tool solves the system of equations correctly.
  Pass criteria: The difference between the WCA tool value at the first minima and the corresponding LT spice value shall be lower than 0.01%.
- For $circuit - 1$, consider the nominal values of the component and compare the WCA tool plot with the measured experimental values, i.e,. oscilloscope plot.
  Purpose: To show that the physical circuit can be used to perform the validation step.
  Pass criteria: The difference between the WCA tool value and the measured value at the first minima shall be lower than 0.1%.
- For $circuit1$, include the components EOL tolerances and compare the WCA tool worst case EVA results with the measured experimental values.
  Purpose: To verify the correctness of EVA technique implemented in the WCA tool prototype.
  Pass criteria: The difference between the WCA tool worst case EVA value and the measured value from $circuit - 1$ with EOL tolerances, at the first minima, shall be lower than 0.1%.
- For $circuit - 2$, include the components EOL tolerances and compare the WCA

tool worst case EVA results with RUAG WCA tool worst case EVA results.
Purpose: To verify that the WCA tool worst case EVA result is correct when comparing with a proven WCA tool.
Pass criteria: The difference between the WCA tool values and the RUAG WCA tool values shall be lower than 0.1%.

- For $circuit-2$, include the components EOL tolerances and compare the WCA tool worst case MCA results with RUAG tool worst case MCA results.
Purpose: To verify that the WCA tool worst case MCA result is correct when comparing with a proven RUAG WCA tool.
Pass criteria: The difference between the WCA tool values and the RUAG WCA tool values shall be lower than 0.1%.

5.2 Symbolic equation system solver validation

In this section, the symbolic equation solver and its capability of solving the system of equations derived for the circuit is examined. The example *circuit* − 1 (LCR circuit) has been considered for the verification as it dealt with first order DAE's, which is the highest possible symbolic equation the solver can handle. Since, the solution of the DAE's is a function of time, it is plotted for certain time limit and this plot is used to verify the solution. The example *circuit* − 1 is simulated in LT-spice using the exact parameter values and the time limits which were used in the WCA calculations.

So, the plots obtained from LT-spice and WCA tool provides a reliable way for comparison of the solution, which also helps the developer to verify the symbolic solution. The comparison between WCA and LT-spice results are shown in Fig 5.3. The plot of the inductor voltage V_L is drawn for 20 *ms* considering the nominal values of the parameter. From the figure we can observe that both the WCA and LT-spice resulted curves align with each other and fulfills the pass criteria mentioned in section 5.1.3. Upon comparison we can assure that the solution from the solver for the output V_L is correct and hence validates the functionality of the symbolic equation system solver.

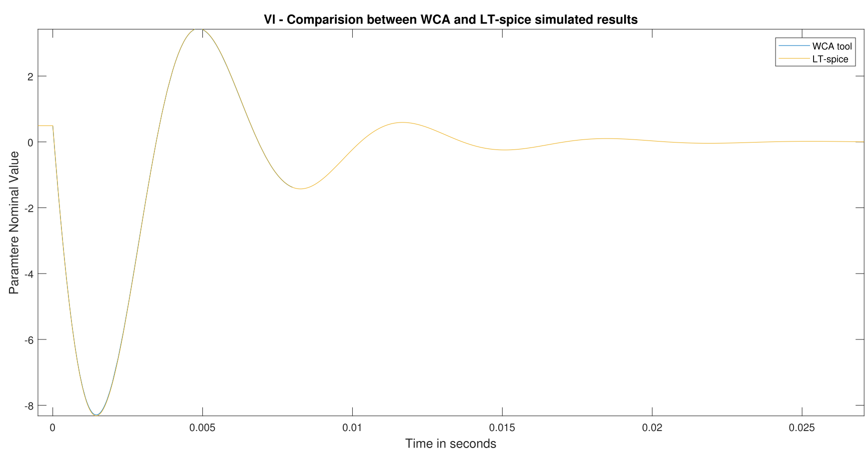

Figure 5.3: Comparison plot between LT-Spice simulated data and the resulted determined by substituting the nominal values to the solved equation obtained by equation solver for voltage across inductor V_L from the example LCR circuit.

5.3 WCA Tool EVA Results Verification

In Fig 5.4, we can see the comparison between the experimental values and the WCA calculated values. The blue curve represents the values calculated from the WCA tool prototype, and the red curve represents the experimental values. Between the time interval from 0 *ms* to 1.8 *ms*, it is clear that both the curve align perfectly with their values starting from 0.5 V. Later, as it decrease gradually to the minimum value around 2 ms and as it starts increasing the curves do not align together but start deviating.

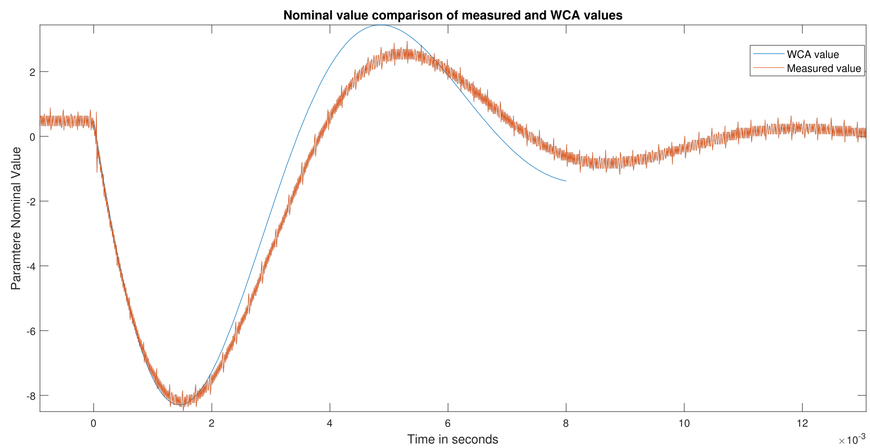

Figure 5.4: Both the nominal value calculated from tool and the captured value from the experimental setup is plotted together for comparison. The red line represents the tool calculated values and blue line is of experimental values.

The reason for this deviation is that the experimental curve is damping faster due to the change in the capacitance value of the ceramic capacitor due to its high dielectric constant. Due to the high dielectric constant the capacitance of the capacitor varies with respect to change in the voltage, due to which there is a more damping in the experimental values as the voltage starts increasing. Troubleshooting the issues such as dependence of the components performance on temperature, voltage and etc. are not considered in this thesis. Hence, the variation of capacitance value due to the change in voltage is not analyzed.

The impact of each of the component on the circuit's output (inductor voltage V_{L}) is also analyzed using the sensitivity analyzer. From the sensitivity results shown in Table 5.3, we can observe that the circuit's output V_{L} is very sensitive to the changes in the capacitance of $C1$ and it is also evident that capacitor $C1$ has the highest impact on the output V_{L}. So, using the sensitivity result the designers can understand the influences of each of the components used in the circuit, and select the component which is more stable (narrow tolerances) and have high efficiency to design a robust circuit.

Parameters	Sensitivity at operating point
$L1$	-2.89
$C1$	2.87e+06
$R2$	0.0065
$R1$	-0.0012

Table 5.3: This table contains the sensitivity result of the LCR example circuit calculated from the WCA tool.

WCA is a static and analytical analysis, it is not a simulation tool which studies the behavior of the circuit. Hence, to perform the WCA for $circuit-1$ (LCR circuit) an operating point is to be determined within the provided time limit. This operating time has been determined by the maximum-minimum function at which the desired output reaches its minimum value. The operating time determined by the function is at time 1.8 ms for the $case$ 2 around which the inductor voltage V_{L} reaches the minimum value which is shown in Fig 5.5.

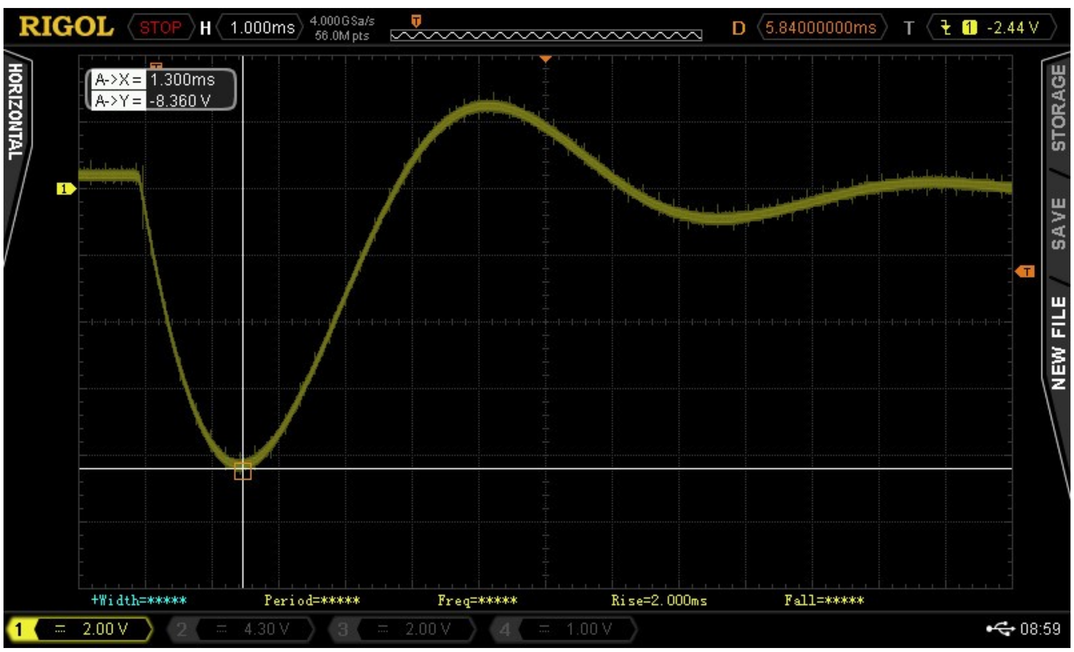

Figure 5.5: Capture of the inductor voltage(V_{L}) measured in an oscilloscope, where x-axis represents time and y-axis represents voltage.

5.3.1 Circuit 1 - Worst Case Value Comparison

An experimental setup of $circuit-1$ has been developed and the experimental measurements were recorded, which are used for the comparison. Using the parameter values and the sensitivity results shown in Table 5.1 and Table 5.3, the worst case values of the inductor voltage V_L at time 1.8 *ms* is calculated by the WCA tool using the EVA technique.

Each of the component's nominal values were determined experimentally and these measured experimental values were considered for the nominal value calculations of the circuit. In contrast, for the end of life value calculations, an additive tolerance has been created by changing a parameter as mentioned in section 5.1. In other words, capacitance value is changed by adding an additional capacitor in parallel and series connection to the capacitor used in the circuit. Then the capacitance values are measured to obtain the experimental value of the capacitor. This measured experimental values of the capacitor is considered for the EOL minimum and maximum tolerances respectively. Thus, Table 5.4 shows only the nominal and EOL worst-case results calculated by the ABB WCA tool prototype in comparison with measured experimental result. If the calculated nominal value and the experimental value of the circuit matches, then we can assure that the WCA calculated end of life worst case maximum and minimum values are also correct as nominal value matches or at least it assures that these calculated WCA values envelopes the circuit's EOL maximum and minimum limits.

Parameters	Beginning of life worst case minimum value	Nominal Value	Beginning of life worst case maximum value	End of life worst case minimum value	End of life worst case maximum value
V_L by EVA	-	-8.29	-	-10.62	-6.39
V_L by Measured	-	-8.32	-	-10.74	-6.22

Table 5.4: This table contains the beginning and end of life worst case values of V_L of the $circuit - 1$, which is calculated using EVA techniques at time 1.8 *ms* and it also shows the experimental values.

However, the comparison results do not satisfy the pass criteria mention in section 5.1.3, but it was understood that the mentioned pass criteria is highly conservative as it is difficult to maintain the experimental values, and WCA calculated values within the mentioned limits due to the components variations and measurement deviations, which is explained in section 5.3. However, with the existence of the minute difference in the result, we can be assured that the purpose of the WCA tool prototype is fulfilled.

5.3.2 Worst Case Value Comparison Of Power Supply Circuit

The worst case values of $circuit-2$ (power supply circuit) were calculated using the EVA technique. The system of algebraic equations shown in equation 5.7 to 5.9, defines $circuit-2$, which is used to calculate the worst case values. These worst case values calculated by ABB WCA tool prototype were compared with worst case values calculated by RUAG WCA tool to validate the WCA tool prototype's results. The worst case results and their differences are tabulated in Table 5.5.

From the results provided in Table 5.5, we can infer that the worst case analysis results from ABB WCA tool prototype fulfills the strict pass criteria of 0.01% as mentioned in the Section 5.1. Hence, we can say that the ABB WCA tool prototype calculations using the EVA technique is verified and validated.

Tool type	Parameter	BOL-minimum value	Nominal Value	BOL-maximum value	EOL-minimum value	EOL-maximum value
ABB	v_ctrl	4.751	5.0101	5.308	4.581	5.471
RUAG	v_ctrl	4.751	5.0101	5.309	4.582	5.471
Difference	-	0	0	0.001	0.001	0
ABB	v_int_L	6.951	7.1055	7.250	6.873	7.344
RUAG	v_int_L	6.951	7.1055	7.260	6.871	7.345
Difference	-	0	0	0.01	0.002	0.001
ABB	v_int_H	13.113	13.409	13.708	12.9532	13.873
RUAG	v_int_H	13.113	13.409	13.704	12.9532	13.874
Difference	-	0	0	0.004	0	0.001

Table 5.5: This table contains the beginning and end of life worst case values of power supply circuit calculated using EVA technique. The tabulated values have two sets of data for the same parameters calculated from WCA tool of ABB(new) and RUAG(old)

5.4 MCA

WCA results of $circuit-2$ (power supply circuit) are calculated by ABB WCA tool prototype using the MCA technique and these calculated worst case values of the output v_ctrl are shown in Table 5.6. The normal distribution curve for the output v_ctrl is shown in Fig 5.6, it was determined by performing 500 runs of the MCA algorithm. The BOL and EOL mean, maximum and minimum values of v_ctrl are also marked in the figure.

The worst-case values of the parameters such as v_int_L and v_int_H are also calculated and tabulated for the comparison, which is shown in Table 5.6. Upon

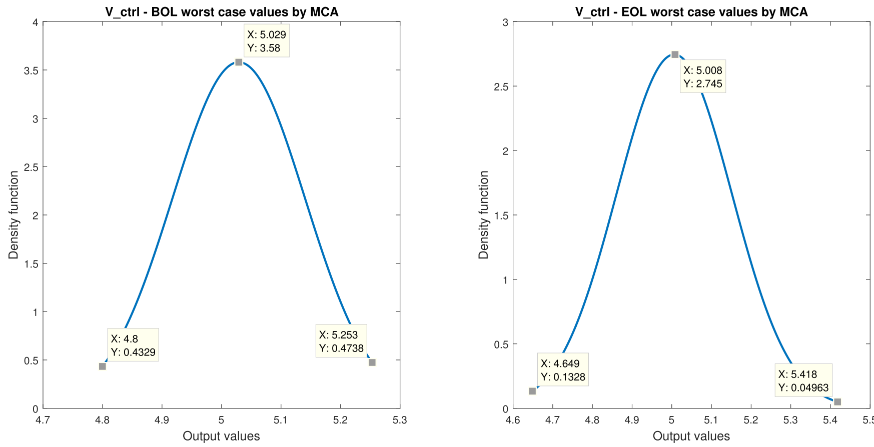

Figure 5.6: *v_ctrl* worst case value determined using MCA technique at operating point.

comparison, we can assure that the difference between the worst case values calculated using two different WCA tools are very minute and fulfills the pass criteria, hence the MCA technique is verified and validated.

Tool type	Parameter	BOL-minimum value	Nominal Value	BOL-maximum value	EOL-minimum value	EOL-maximum value
ABB	*v_ctrl*	4.752	5.011	5.257	4.645	5.400
RUAG	*v_ctrl*	4.763	5.010	5.297	4.607	5.459
Difference	-	0.011	0.001	0.040	0.042	0.059
ABB	*v_int_L*	6.952	7.101	7.243	6.889	7.304
RUAG	*v_int_L*	6.952	7.105	7.258	6.878	7.334
Difference	-	0	0.004	0.015	0.011	0.030
ABB	*v_int_H*	13.120	13.390	13.668	13.026	13.742
RUAG	*v_int_H*	13.118	13.408	13.699	12.968	13.849
Difference	-	0.002	0.018	0.032	0.058	0.107

Table 5.6: This table contains the beginning and end of life worst case values of power supply circuit calculated using MCA technique. The tabulated values have two sets of data for the same parameters calculated from WCA tool of ABB(new) and RUAG(old)

Lastly we can concluded that with the help of the RUAG WCA tool and the two example circuits the power suppply circuit and LCR circuit the ABB WCA tool prototype and its components have been validated and verified.

6

Discussion and Future Improvements

In this chapter, we look back into the methods carried in this thesis work and discuss about their outcome. Section 6.2, contains the highlights of the future improvements that can be carried out further to improve the prototype tool's performance.

6.1 Design Process and Techniques

Tool architectural design explained in Section 4.1, follows with two main structure GUI and the circuit analyzer. In this thesis work, the circuit analyzer is designed and implemented completely, for which the investigation has been carried out to choose the appropriate WCA techniques to perform WCA. To implement these selected techniques a suitable mathematical platform was selected. The iterative method is used to implement the components of the circuit analyzer in accordance with the parallel verification.

While developing the symbolic equation solver of the circuit analyzer, it has been found out that for the DAEs the MATLAB's default results are the numerical solutions, but we were in need of the symbolic solutions, because this symbolic solution of the equations will be used in the other components of the circuit analyzer to determine the worst case values of the circuit. Hence, a new MATLAB function called as symbolic equation solver was developed, which is capable of solving algebraic, differential and first order system of differential equations altogether. The function has been developed such that it solves the system of equations only for the desired variables, which are defined by the user. However, this symbolic equation solver function can be optimized in a better way to reduce process time for solving the equations and memory utilization by proper handling of the data, and the type of equations.

The output of the sensitivity analyzer will have a major impact on the results of the time-varying equations, so it is very important to determine the sensitivity results at the exact operating point. Hence, it is needed that the user of the WCA tool must have sufficient knowledge about the circuit's behaviour in order to define the proper time limit for the worst case calculations.

To calculate the worst case values using the RSS technique explained in Section 2.2, it is understood that to calculate the correct results of a circuit, the minimum data size required is 30, i.e., a circuit must contain a minimum of 30 components. Then for the use of the RSS calculations, we need to determine whether each of these components vary randomly or in bias with respect to the environmental changes, which is done manually leading to human intensive work. In some cases, it is even difficult to determine those variations as it is not provided in the data sheets of the components. Hence, considering the previously mentioned aspects the decision to use RSS technique has to be made by the user based on the information available.

MCA technique and its approach towards the worst case calculations is explained in Section 2.2, from which we can understand that MCA is an iterative process in which the numerical calculations are carried out for larger number of iterations to obtain the circuit's performance results. So, the accuracy of MCA results depends largely on number of iterations, higher the number of iterations better the results. Hence, this technique has a trade off between computing time and the precision of the results.

6.2 Future Improvements

In this section we summarize the future improvements for the WCA tool. The most prominent improvement is to implement the GUI. Besides that, the tool can be improved and optimized by implementing few ideas mentioned below:

- Implement the tool to such an extent that, provided the netlist of the circuit's drawn in the simulation tool can be used to extract the system of equations of the circuit and then perform WCA.

- Develop the libraries containing the parameter values and tolerances of the components used in the circuit's. Defining libraries will reduce the amount of work to find the components parameter every time a new circuit is analyzed.

- Optimize the tool with better algorithm to handle the circuit data in a smarter way, which perform the WCA calculations in quicker way.

- Another improvisation is regarding maximum and minimum value calculations as discussed in Section 6.1. A new method such as the Newton-Raphson method can be implemented.

7

Conclusion

The outcome of this thesis is a study on worst-case analysis and complete implementation of circuit analyzer part of the worst-case analysis tool prototype for safety electronic and electrical circuits. However, the GUI implementation part remains excluded from the scope of this thesis work. The main goal of the thesis was achieved. A circuit analyzer with its components such as equation system solver, sensitivity analyzer and the worst case analyzer was developed and verified. We tested the circuit analyzer on the specific known circuits, then the results determined by WCA tool prototype were verified experimentally, and were also verified against the RUAG WCA tool. Techniques such as MCA and EVA techniques were also verified, but the RSS technique is not verified due to lack of time. We did a comparative study to understand the trade-offs between various mathematical platforms and to choose a suitable platform among them to implement the circuit analyzer of the WCA tool prototype according to the user requirements. Initially, a mock prototype tool was developed and as knowledge about WCA increased, it provided the opportunities for us to improve this mock prototype tool with additional functionalities leading to complete development of the WCA tool prototype. We were performing the parallel verification of the tool, which helped us to keep optimizing the tool's performance and detect bugs during the development. However, the combined influence of the various components (equation system solver, sensitivity analyzer and worst case analyzer) on the performance of the circuit analyzer was validated during the verification process. The issues due to combined effects are highlighted in chapter 6, along with suggestions to resolve them. Throughout the project, we followed an agile methodology to develop the tool.

To summarize, we gained knowledge about WCA methodology. In addition, we defined and realized techniques for worst-case analysis and developed a software tool prototype for worst-case analysis of the circuits. Finally, we compared the results of the ABB WCA tool prototype with the RUAG WCA tool and also the results were verified experimentally.

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A

Appendix 1

A.1 Computing Platform

WCA is an analytical approach which deals with equation solving and numerical computing. So, do develop the WCA tool prototype a suitable platform was needed which can handle huge number of data and perform mathematical calculations. Hence, we narrow down the search to the softwares, which are capable of solving the engineering problems. Based on pre-study we came across the few profound software's such as MATLAB, MATHCAD and MATHEMATICA and one of these softwares has to be selected for the WCA tool implementation. These softwares have their own pros and cons based on the application it is used for. Hence, the criteria to select the suitable platform is to check whether the software can fulfill the user requirements mentioned in Appendix B.1.

MATHCAD, a computer algebraic system agile in solving the symbolic and numerical problems and capable to handle the SI units [16]. The "what you see is what you get(WYSIWYG)" user interface provide the easy working and editable script, but the user interface cannot be altered. If the user is developing a own customized tool, then it is difficult to make their own user interface using the MATHCAD. Whereas, MATLAB provides a flexible environment for programming and it has a intuitive programming language capable of solving complex program-oriented problems. It also provides a option to generate customized user interface model and can handle large number of data which helps the user to develop the customized tools [17][18].

MATHEMATICA, is a software capable of solving all types of numerical problems, but the concern is that it is not an user friendly software. So, user must spend some time to use and understand the tool. As this is a thesis work with main scope of implementing a WCA tool prototype which is to be accomplished in the limited time, spending time to learn a new software is not feasible. Hence, it is decided to use MATLAB as a tool since author is familiar with the tool and its programming language.

During this pre-study, the author came across some simulation tools such as LT-spice, Pspice and others, which can perform simulation and circuit analysis. Among them some can only preform circuit analysis and fail to perform worst-case analysis. Whereas, there are simulation tools which can do circuit analysis but relatively expensive than developing the own tool [19].

MATLAB	MATHCAD
Matrix laboratory - Numerical computing environment	MATHCAD is a computer algebra system primarily for verifying calculations
Can handle complex problem	Not so compatible to handle complex problem
Cannot handle SI units	Can handle SI units
Facilitate creation of user interface	Fixed user interface
Allows implementation of algorithm	Cannot implement our own algorithm
Intuitive programming language	Just provides live editing worksheet
Easy to find documentation	Poor documentation

Table A.1: Few major difference between MATLAB and MATHCAD explained in above section is noted in this table.

B

Appendix 2

B.1 User Requirements

The main requirements to be accomplished in this thesis work has defined by the user are defined here:

- Investigate the techniques to perform worst case analysis
- Perform literature survey on the existing mathematical tools and select suitable computing platform for the WCA tool
- Integrate the WCA into the product development model(V-model)
- Define the format of the inputs into the tool
- Complete implementation of the WCA tool prototype
- Verify the tool performance for the example circuits

The below mentioned points are in specific to the WCA tool prototype, which are the requirements needed in the tool according to user to perform the analysis:

- Tool must symbolically solve the system of algebraic equations, differential equations and also first order system of DAE's.
- Must perform worst case analysis using any one of the techniques based on user command and output the minimum, nominal and maximum values of the circuit.
- Must perform an sensitivity analysis of the circuit's with respect to its parameters.
- Output the results in a log file.