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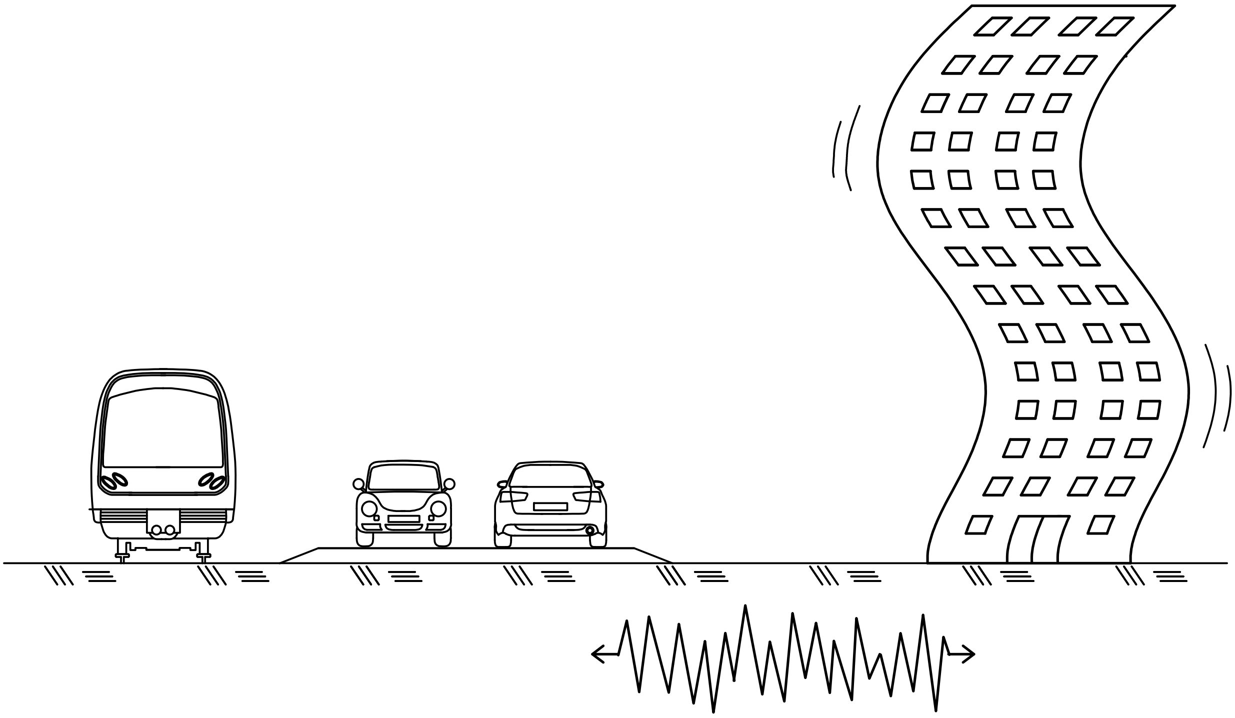

Calculation of Vibrations in Structures in the Early Design Phase

Prediction of comfort disturbing vibrations in buildings

Master's thesis in the Master's Programme Structural Engineering and Building Technology

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Department of Architecture and Civil Engineering
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Master's thesis BOMX02-17-33

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Cover:
Traffic flow causing ground vibration, transferred to building.

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ABSTRACT

Performing dynamic analyses to predict vibrations in buildings, due to e.g. nearby traffic, may require advanced calculations. These type of analyses are therefore seldom made if the purpose only is to study the response with regard to vibrational comfort. This master's thesis aims to investigate how the dynamic response of buildings can be analysed in a simplified manner. In cooperation with *ÅF-Infrastructure AB*, business area *Buildings*, a calculation program, using methods common in earthquake engineering in combination with comfort weighting of vibrations, is developed in Visual Basic for Applications (VBA). Analyses are performed on simple structures and the results are compared to results from the commercial FE program ADINA.

The study shows that when comfort weighting is not considered, the developed program generates results with a maximum of 10% error for the relative response and about 35% error for the total response. Further studies are required in order to find a more reliable approach to include the ground motion and hence reduce the error for the total response.

For the comfort weighted response, the error is between 2 and 87%. The difference in absolute values is however considered as small. It should be noted that the approach of comfort weighting that is used in the study is not an established method but an interpretation developed with help from *Sound & Vibrations* at *ÅF-Infrastructure AB*. Further studies are required in order to find a more reliable method.

Even though the developed program generates results that sometimes differ significantly to results from advanced analyses, it is concluded that the program indicates whether the dynamic responses are potentially critical or not.

Keywords: structural dynamics, vibrations, comfort weighting, response spectrum analysis, time history analysis, ground motion, Newmark

Beräkning av vibrationer i strukturer i det tidiga projekteringsskedet
Uppskattning av komfortstörande vibrationer i byggnader
Examensarbete inom masterprogrammet Konstruktionsteknik och Byggnadsteknologi
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SAMMANFATTNING

Att utföra dynamiska analyser för att uppskatta vibrationer i byggnader, från exempelvis närliggande trafik, kan komma att kräva avancerade beräkningar. Dessa typer av analyser utförs därför sällan om syftet bara är att studera responsen med avseende på komfort. Detta examensarbete syftat till att undersöka hur den dynamiska responsen av byggnader kan analyseras på ett enklare sätt. I samarbete med *ÅF-Infrastructure AB*, affärsområde *Buildings*, har ett beräkningsprogram i Visual Basic for Applications (VBA) utvecklats. Programmet använder sig av tekniker vanliga vid jordbävningsberäkningar tillsammans med komfortvägning av vibrationer. Analyser på enkla strukturer är utförda och resultaten är jämförda med resultat från det kommersiella FE-programmet ADINA.

Studien visar att för den icke komfortvägda responsen ger beräkningsprogrammet en avvikelse på maximalt 10% för den relativa responsen och cirka 35% för den total responsen. Vidare studier krävs för att hitta ett mer tillförlitligt tillvägagångssätt för att inkludera markrörelsen och därmed minska felmarginalen för den total responsen.

För den komfortvägda responsen är avvikelsen mellan 2 och 87%. Skillnaden i absoluta tal anses däremot vara liten. Det bör poängteras att tillvägagångssättet för komfortvägning som används i den här studien inte är någon vedertagen metod utan snarare en tolkning som tagits fram med hjälp av *Sound & Vibrations* på *ÅF-Infrastructure AB*. Vidare studier krävs för att hitta en mer tillförlitlig metod.

Trots att det utvecklade programmet ger resultat som ibland skiljer sig avsevärt mot resultat från avancerade analyser kan slutsatsen dras att programmet ger en fingervisning om den dynamiska responsen är potentiellt kritisk eller inte.

Nyckelord: strukturdynamik, vibrationer, komfortvägning, responsspektrumanalys, tidshistorieanalys, markrörelse, Newmark

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PREFACE

This master's thesis investigates how dynamic responses in structures can be analysed in a simplified manner. Through the work, a calculation program in Visual Basic for Applications (VBA) is developed. The study has been a collaboration between *ÅF-Infrastructure AB*, the Division of Structural Engineering and the Division of Applied Mechanics at *Chalmers University of Technology*. The work was carried out at the office of *ÅF AB* in Gothenburg.

We would like to give a big thank you to our supervisor Mattias Carlsson at *ÅF-Infrastructure AB*, who has been a great help for us. Throughout the work, he continuously gave us support, answered questions, commented on the report and helped out with software hassles. Without you there would not be any thesis at all. Further, we would like to thank our examiners Morgan Johansson and Peter Folkow at *Chalmers University of Technology*. They have contributed with their knowledge and given us helpful feedback throughout the work.

We would also like to thank *ÅF-Infrastructure AB* and especially Joakim Tornberg at *Structural Engineering* for providing us with a place to work and the necessary literature and tools to carry out the study.

Further, we would like to thank Åsa Collet at *ÅF-Infrastructure AB* and Patrik Höstmad at *Chalmers University of Technology* for taking their time to discuss questions with us.

Lastly, we would like to thank the people at *Structural Engineering* at *ÅF-Infrastructutre AB* for welcoming us and letting us be a part of their team.

Gothenburg, May 2017
Linnea Fagerström and Philip Lindorsson

Nomenclature

Acronyms

ABSSUM	Absolute sum
CDM	Central difference method
CQC	Complete quadratic combination
DOF	Degree of freedom
DOFs	Degrees of freedom
FE	Finite element
FFT	Fast Fourier transformation
MDOF	Multi degree of freedom
MPF	Modal participation factor
RMS	Root mean square
RSA	Response spectrum analysis
SDOF	Single degree of freedom
SLS	Serviceability limit state
SRSS	Square root of sum of squares
THA	Time history analysis
ULS	Ultimate limit state
VBA	Visual Basic for Applications

Subscripts

0	Initial
c	Damping
$comf$	Comfort weighted RMS value
g	Ground
h	Horizontal
max	Maximum
min	Minimum
n	Natural/Maximum number
rel	Relative
rms	Root mean square
s	Spring
$SDOF$	Related to an SDOF system
tot	Total
v	Vertical

Greek letters

$[\Phi]$	Eigenmatrix
α	Phase shift of signal
β	Newmark coefficient
Δ	Difference
Γ	Modal participation factor
γ	Newmark coefficient
λ	Eigenvalue
ω	Angular frequency
ζ	Damping ratio
$\{\ddot{\gamma}\}$	Modal acceleration vector
$\{\dot{\gamma}\}$	Modal velocity vector
$\{\gamma\}$	Modal displacement vector
$\{\phi\}$	Eigenvector/natural mode

Roman lower case letters

$\ddot{u}$	Acceleration
$\dot{u}$	Velocity
$\{r\}$	Influence coefficient vector
a	Acceleration
c	Damping coefficient
d	Displacement
F	Force
f	Frequency
k	Stiffness
m	Mass
p	Force
r	Response value
t	Time
u	Displacement
v	Velocity
W_m	Weighting factor

Miscellaneous

$[x]$	Matrix
$\{x\}$	Vector
x	Scalar

Roman capital letters

$[C]$	Modal damping matrix
$[K]$	Modal stiffness matrix
$[M]$	Modal mass matrix
$[R]$	Influence coefficient matrix
S	Spectral value
T	Period

1 Introduction

1.1 Background

In countries where natural hazards such as earthquakes luckily are quite rare, advanced dynamic analyses of buildings are often considered superfluous. However, similar dynamic effects, in a lower scale, might still be originating from other external loads, such as nearby heavy traffic. Whether the demands are regarding capacity in the ultimate limit state (ULS) or regarding comfort in the serviceability limit state (SLS), they need to be taken into account during the design process. Therefore, it might be of interest to analyse not only the effects of static loads, but also the dynamic response of a structure.

Today, performing dynamic analyses to predict future vibrations may require rather advanced calculations, using methods that are both time consuming and expensive. Therefore, these kinds of analyses are seldom made if they only consider comfort, and hence there is a need for a more simplified method. By combining structural dynamics with methods used by acousticians to weight signals regarding comfort, it is possible to quickly approximate the structural response, or determine whether a more thorough analysis is required or not.

1.2 General aim

This master's thesis aims to investigate how the dynamical response of structures, in terms of vibrations caused by nearby heavy traffic, can be analysed in a simplified manner. A simplified method to use in the early design phase, for prediction of the behaviour due to dynamic loads, is developed and evaluated. This method can be used to evaluate if a more thorough investigation is required. It can also be used to make reviewing of advanced analyses more simple. The intention is that the simplified approach will reduce the effort needed, and hence make dynamic analyses more frequently used in the design of buildings. The method is also supposed to bridge the working areas of structural engineers and acousticians in order to utilise expertise from both areas.

1.3 Method

A literature study is performed where the basics in structural dynamics are studied, such as solution methods, time history analysis (THA) and response spectrum analysis (RSA). The information is collected from scientific articles, technical reports and textbooks.

A numerical integration method is studied more thoroughly and a script in Visual Basic for Applications (VBA) is designed. From the script, response spectra showing the maximum accelerations, velocities and displacements, are made. These are generated by studying the response of single degree of freedom (SDOF) systems with various natural frequencies, exposed to a given ground motion. The results are compared to, and verified with, response spectra created in the commercial finite element (FE) program ADINA.

The VBA script is developed such that multi degree of freedom (MDOF) systems also can be analysed. Methods for modal analyses are implemented in the script to translate the responses from the response spectra for SDOF systems to MDOF systems. This is performed by solving the so called modal participation factors (MPFs), which, in combination with the corresponding eigenvector and the responses from the response spectra, give the actual response of a structure. The results are verified with time history analyses performed in ADINA.

Different methods for root mean square (RMS) and weighting of vibrational signals from a comfort perspective are studied and applied to the response spectrum analyses. The results are compared to signals from time history analyses, analysed in a fast fourier transformation (FFT) and comfort weighted.

Through the work process explained above, a verified method to analyse the comfort experience in simple structures affected by dynamic loads is created. This method is used to predict the response due to loads from e.g. traffic to check the demands of comfort in a building in SLS.

1.4 Limitations

The methods used for comfort weighting are not evaluated from an acousticians point of view, only applied to structural engineering. Different methods are compared but the derivations are not studied.

The damping phenomena of a structure is complex and out of the scope of this report; it is therefore only briefly explained and approximately used. Effects of damping are considered, but different damping methods are not investigated or evaluated. In the analyses only modal damping is considered.

The analysed models are limited to simple systems, intended to represent the overall behavior of real structures. The models consist of simple structures, like beams and frames, with varying boundary conditions and loads. Only linear elastic material models are considered, and in those cases where concrete structures are analysed the effect of cracking is disregarded.

Further, geotechnical aspects of vibrations and the effects of foundations, and its transfer of vibrations, are not considered.

1.5 Outline of the report

Chapter 2 is a theory chapter, introducing vibrations and describing some fundamentals of structural dynamics. It covers basic concepts of vibrations, description of comfort weighting, the essential equations of structural dynamics along with the distinction between RSA and THA. The theory described here is used in the development of the calculation program and is referred to throughout the report.

Chapter 3 describes the process of the development of the calculation program. Here, the various analyses and verifications, leading to the final program, are presented. This chapter covers how the response spectra are generated, modal analyses performed on simply supported beams, studies on implementation of comfort weighting and lastly, studies on how to model a multi storey building.

Chapter 4 describes a previous study performed by *ÅF-Infrastructure AB*, with the purpose to predict future vibrations in a residential building. A simplified model of the same building is analysed in the developed program and the obtained results are compared to the previously obtained results.

Chapter 5 and 6 cover discussions and conclusions of the study. Here, a summary of the results is also presented along with comments about recommended future studies. In the appendices, some additional tables of the results are shown along with a description of the calculation sheet for comfort weighting of vibrations and the VBA code to the developed calculation program.

2 Theory

2.1 Introduction to vibrations

When a system is displaced from its equilibrium state, it will react with internal forces to regain equilibrium. Those internal forces will cause the system to move periodically, a movement commonly referred to as a vibration (Britannica Academic, 2017). The vibrations that a system develop can be either forced or free. Free vibrations arise when a system is dislocated from its equilibrium state. The vibration will continue forever, as long as a damping restraint does not exist, and the system will vibrate in its own natural frequency. The natural frequency is independent of the size of the load that induced the dislocation of the system, whereas the amplitude of the vibration is affected by the size of the inducing load.

A mass suspended from a spring is an easy way of illustrating free vibrations, see Figure 2.1. The masses, that all have the mass *m*, will, if dislocated from their equilibrium state, start to move up and down in the system's natural frequency. The three systems in Figure 2.1 will have different natural frequencies due to the different stiffness, k_1, k_2, k_3, of the springs.

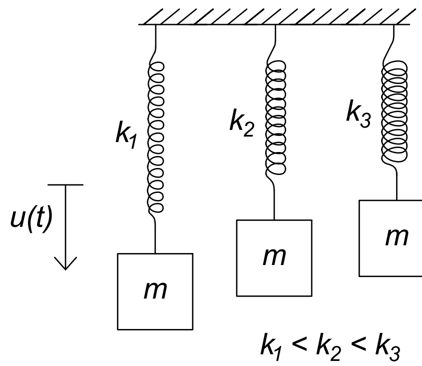


Figure 2.1: Masses suspended from springs with different stiffness.

For the systems in Figure 2.1, the mass will move up and down in a harmonic motion, which can be expressed as:

$$u(t) = \phi \, cos(\omega t - \alpha) \tag{2.1}$$

where $u(t)$ is the vertical displacement dependent on the time t, ϕ is the amplitude of the motion, ω is the angular (natural) frequency of the system and α is the phase shift.

As mentioned above, a free vibration will continue if the movement is not damped. When damped, the amplitude of the vibration will decrease over time, as the damper absorbs the energy in the system, e.g. due to frictional forces (Craig and Kurdila, 2006). A free vibration without damping is a hypothetical case that never exists in reality.

Forced vibrations arise when a system is excited continuously by an external load. If the frequency of the load approaches the natural frequency of the system, it will lead to resonance. Resonance will make the amplitude of the motion grow, and even if damping exists in the system the movements can become considerably large (Britannica Academic, 2017).

2.1.1 Natural vibrational modes

Structures that are more complex than a mass suspended from a spring, i.e. have more than one degree of freedom (DOF), have possibilities to vibrate in several different ways, usually each will have different natural frequencies. A way of vibrating is called a mode and a system will have as many natural frequencies and modes as it has degrees of freedom (DOFs).

A simply supported beam will usually have several natural frequencies; f_1, f_2, ... , f_n, where f_1 is the lowest. The value of the lowest natural frequency is dependent on the stiffness and the mass of the beam; a slender beam has a lower first natural frequency than a more stiff beam. Each natural frequency has a corresponding natural mode, which describes the movement of the structure when vibrating in its natural frequency. For a simply supported beam, the first natural frequency always result in a mode shape as in Figure 2.2b. This mode requires the lowest amount of energy to take place (Chopra, 2007). The second natural frequency, f_2, will always result in the second mode shape, as in Figure 2.2c, and the third natural frequency will result in the third mode shape, and so on.

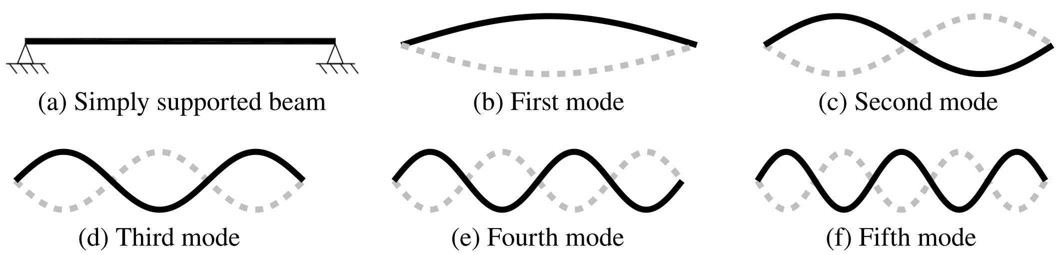


Figure 2.2: Natural modes of a simply supported beam.

If instead studying a cantilever beam, see Figure 2.3a, the mode shapes will look a bit differently due to the different boundary conditions. The first mode shape, corresponding to the first and lowest natural frequency of the beam, will look like Figure 2.3b. The second, third, fourth and fifth mode shape can be seen in Figure 2.3c, 2.3d, 2.3e and 2.3f, respectively.

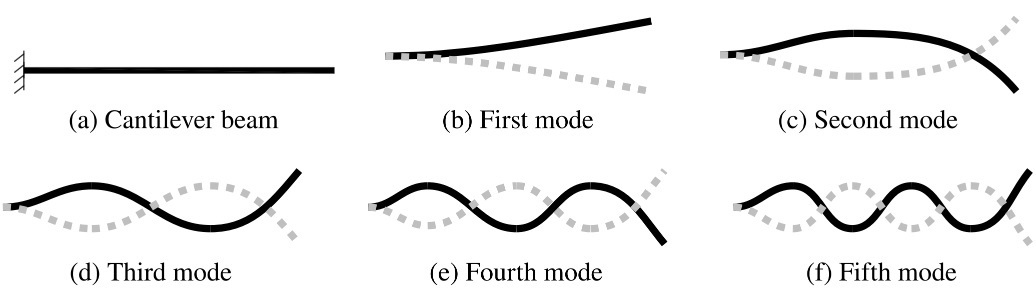


Figure 2.3: Natural modes of a cantilever beam.

Due to the fact that the modes corresponding to the lowest frequencies require least energy to take place, the lowest modes will dominate the total behaviour of the vibrating structure, and hence they are the most important modes to study.

2.1.2 Vibrations in buildings

Different types of buildings will have different design requirements, depending on the specific function, location, etc. For commercial buildings, like office buildings and hospitals, the demands may be flexibility, resulting in e.g. long spans and large column-free floor areas (Naeim, 1991). These are all design demands that might affect the risk of vibrational problems. The response in the middle of a slab will generally be larger than the response at floor areas near supports (SteelConstruction.info, 2016). In the same way as the design demands vary for different buildings, the requirements with regard to vibrations may be different. For residential buildings and schools, the purpose of reducing the vibrational response might be to avoid discomfort, while in a lab or a hospital the demands might be necessary to avoid sensitive equipments getting damaged or stop working properly.

2.1.2.1 Relative and total response

When talking about the vibrational response of a building it is important to distinguish between relative and total response. The relative response is the response of a structure in relation to the ground as a reference. This type of response is often of interest when analysing the structural capacity in ULS, for structures exposed to ground motions from e.g. earthquakes. However, when performing analyses with regard to comfort, it is desired to study the experienced response. In those cases, the ground motion needs to be added to the relative response, which results in what is further referred to as the total response. This describes the response of a structure in relation to the original position as a reference. As an example, Figure 2.4 shows the ground displacement d_g, the relative displacement d_{rel} and the total displacement $d_{tot} = d_g + d_{rel}$.

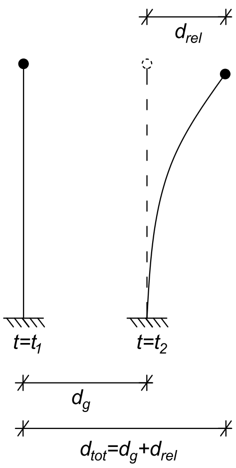


Figure 2.4: Example showing the ground displacement, relative displacement and total displacement.

2.1.2.2 Common vibrational sources

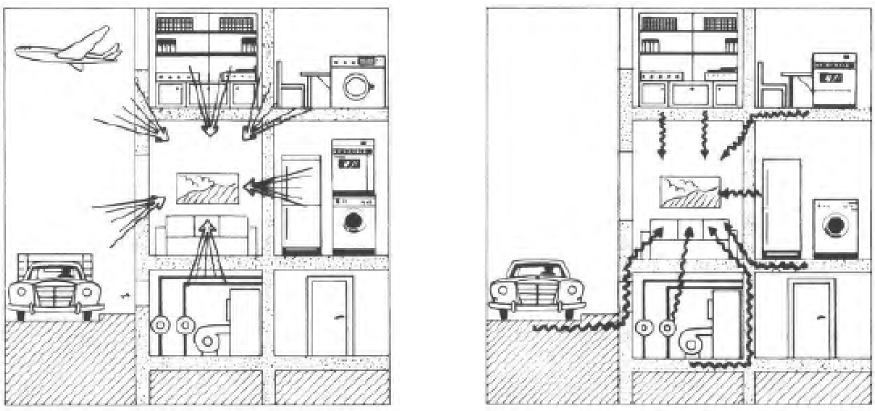
There are many possible sources of vibrations in buildings, both with regard to structural capacity and to comfort. From a ULS perspective the most common sources are earthquakes, which are major concerns in several countries around the world (Chopra, 2007). In Sweden, however, this type of ground motion is very rare and only considered for special designs, such as nuclear power plants. It is more common to concern about vibrational sources generating discomfort. The most common source causing this is human activities within the building, such as walking, jumping or dancing (SteelConstruction.info, 2016). For example, it is common to experience effects of vibration on several levels in buildings having a gymnasium on one of the floors (Naeim, 1991).

Another common problem is vibrations caused by mechanical equipments, like heating, ventilation and air-conditioning systems, not being isolated sufficiently. In industrial facilities, these kinds of problems can also be caused by heavy machinery within the building.

The focus of this report will, however, be vibrations caused by heavy traffic. This affects structures close to roads and train tracks, and as our cities are expanding the problem becomes more current (Hunaidi, 2000). The amplitudes of the vibrations are dependant on factors related to the road and traffic, like surface conditions and weight of the vehicles and their speed. Generally, heavier vehicles cause higher levels of vibration and the rougher the road surface is, the more the speed influence the vibration amplitude. The vibrational level is also dependant on the soil conditions where the building is constructed. According to Hunaidi, 2000, a lower stiffness of the soil causes higher vibrations. The most vulnerable areas are those where the soil consist of a soft clay layer, approximately 7 to 15 meters deep. In those areas there are a higher risk of resonance or amplified vibration occurring, since the natural frequencies of the soil and the building might coincide.

2.1.2.3 Acoustic perspective

Vibrations, from the perspective of an acoustician, generally concern the effects of sounds. According to Ginn, 1978, acoustics in buildings may be described as the propagation and transmission of sounds, either airborne or structure borne, in rooms, dwellings and other buildings. The airborne sounds originates from longitudinal wave motions in the air, created by some vibrating object causing sounds directly to the air, like a speaker or a musical instrument, see Figure 2.5a. The structure borne sounds act directly on the structure of the building, which acts as a medium that the sound waves transmit through, see Figure 2.5b. Common sources are foot steps or vibrating equipments on the floor. Both airborne and structure borne sounds are concerns mainly because of the noise they can cause. Some noises can be dangerous and cause hearing damages, whereas some may just interfere with speech conversations or disturb our concentration.



(a) Airborne sounds (b) Structure borne sounds

Figure 2.5: Sound from different sources, from (Ginn, 1978).

2.1.2.4 Structural perspective

From the structural perspective, the effects of vibrations need to be considered from three main aspects: strength, fatigue and discomfort (SteelConstruction.info, 2016). The strength aspect considers the structural capacity of a building, it need to be able to resist the peak dynamic responses affecting it. In countries with an extremely low probability of earthquakes, this aspect is seldom any concern. However, even if the vibration levels are too low to cause any damage alone, they could contribute to deterioration (Hunaidi, 2000). Even small levels of vibrations may trigger damage by increasing the residual stresses, from e.g. repairs or temperature cycles, in building components. Fatigue damages may occur if the structural components are subjected to repetitive cyclic loading, like vibrations lasting continuously for many years. The last aspect, discomfort, covers how we, as humans, experience vibrations. The response from e.g. heavy traffic is seldom a safety concern, but it can easily disturb our everyday activities, like sleeping or having conversations (SteelConstruction.info, 2016). It has been shown that humans are able to perceive vibrations of very low amplitudes. However, the level of tolerance varies depending on whom you ask, what they are doing and whether they are aware of the vibrational source. This makes it difficult to evaluate the discomfort aspect.

2.1.2.5 Regulations and guidelines

There are guideline values for vibrations in buildings which should be implemented in the design of new buildings. According to the standard SS 460 48 61 (SIS, 1992), the values should be applied strictly on residential buildings, whereas they could be applied less strictly on office buildings. It is not intended that the values should limit vibrations from temporary sources, e.g. construction sites. Table 2.1 shows the guideline values for velocity and acceleration. The values are root mean square (RMS) values and weighted with regard to comfort, explained later in Section 2.1.3.

Table 2.1: Guildeline values for vibrations in buildings, from SS 460 48 61.

	Weighed velocity	**Weighted acceleration**
Moderate disturbance	0.4 - 1.0 mm/s	14.4 - 36.0 mm/s^2
Probable disturbance	>1 mm/s	>36 mm/s^2

The Swedish Transport Administration have developed a guidelines regarding noise and vibration where they state the vibrational limits depending on the type of the premise. For residential buildings, it is stated that the velocity should not exceed 0.4 mm/s, which is equivalent to the lowest values from SS 460 48 61, see Table 2.1. The limit values regard vibrational levels during night (10 p.m.-6 a.m.). The guidelines are developed to concretize what the Swedish Transport Administration regard as acceptable vibration levels and is intended to function as an object when performing actions against high vibration levels.

2.1.3 Comfort weighting of vibrations

Depending on the frequency of a vibration, a human exposed to the vibration can have different experiences. Vibrations in specific frequency ranges are more prone to cause discomfort than others (Brüel and Kjær, 1989). Simply, the human body can be described as a mechanical system where different parts of the body are sensitive to different frequency ranges, see Figure 2.6.

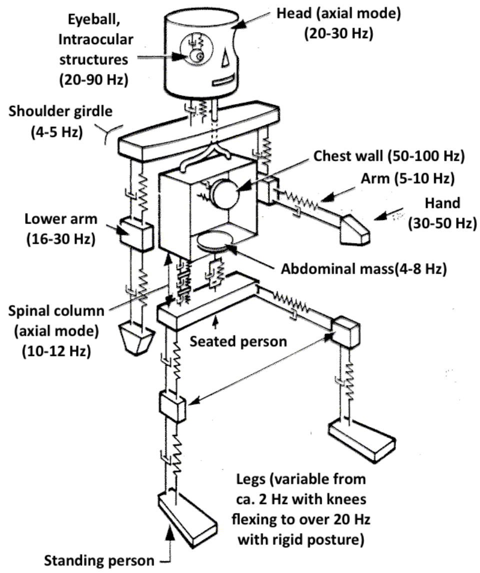


Figure 2.6: Mechanical model showing resonance frequencies, from (Brüel and Kjær, 1989).

As can be seen, the abdomen is most sensitive to frequencies in the range 4-8 Hz, whereas the hands experience most discomfort in the range 30-50 Hz. In buildings, structural vibration can affect the occupants in many ways, for example cause discomfort or reduce their quality of life (SIS, 2003).

2.1.3.1 Purpose of comfort weighting

In order to make it possible to predict if a vibration will cause discomfort or not, frequency weighting can be applied to the response signal (Brüel and Kjær, 1989). Frequencies in ranges where the human body is most sensitive is multiplied with the highest factors and frequencies in other ranges are multiplied with lower factors. By performing this weighting, a correlation between the vibrational response and the subjective feeling of the vibration is obtained.

2.1.3.2 General approach

When evaluating a vibration with regard to comfort it is, according to the standard ISO 2631-2, preferred to use RMS values that are weighted with frequency weighting factors.

RMS is defined as the arithmetic mean of the squares of a set of numbers, as:

$$x_{rms} = \sqrt{\frac{1}{n}({x_1}^2 + {x_2}^2 + ... + {x_n}^2)} \tag{2.2}$$

where x_{rms} is the RMS value of x_1 to x_n. For a sinus shaped signal, the RMS value is always the amplitude divided by $\sqrt{2}$. When calculating the RMS, the signal of interest is divided into a number of so called *windows* and one RMS value is calculated for each window. The size of the window is dependent on the decided time weighting, e.g. fast (125 milliseconds) or slow (1 second). As an example, Figure 2.7 shows a response signal with a corresponding RMS signal. The dots represent the RMS values in each window, which in this example is 1 second long.

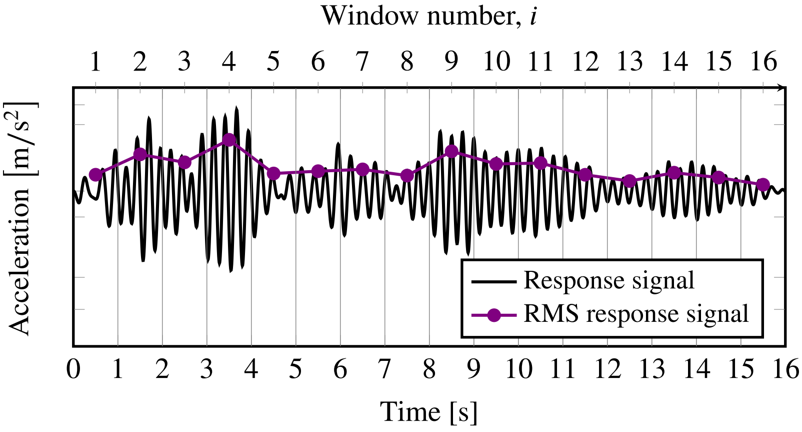


Figure 2.7: The response signal and the corresponding RMS response signal.

The frequency weighting factors should be applied to the RMS response of the signal that is measured inside the building of interest. If it is not possible to get access to the building, for example if the building is not yet built, the concept described in ISO 2631-2 can be applied in the same manner, but trust have to be placed on the prediction of the vibration of the building. The vibrations shall be measured in three directions; x-, y- and z-direction. Values measured from the direction that has the largest frequency weighted vibration magnitude are the values that should be used for the evaluation of the building.

In Figure 2.8, the weighting factors that should be applied to accelerations and velocities are plotted as functions of the frequency. The frequencies are divided into so called *one-third-octave bands* and all the responses of frequencies corresponding to the same one-third-octave band are multiplied with the same weighting factor, see Table 2.2. If the response is in time domain, as in Figure 2.7, the response signal has to be analysed to see what frequencies that are present, to be able to use the correct weighting factor.

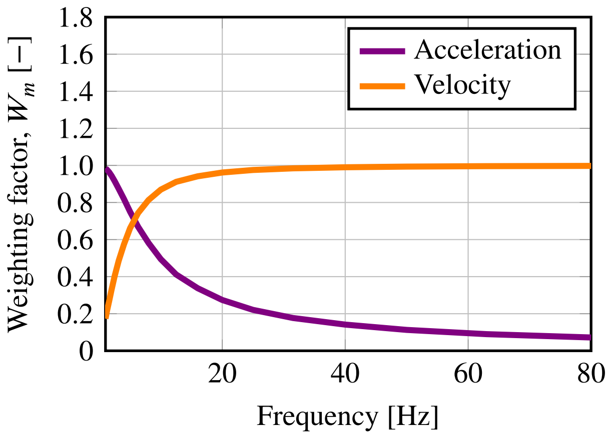


Figure 2.8: Frequency weighting factors for acceleration and velocity (SP, 1993).

According to SP, 1993, to get the weighted value of the acceleration, a_{comf}, or the weighted velocity, v_{comf}, the RMS values of the responses are multiplied with the weighting factors as:

$$a_{comf} = \sqrt{(W_{m.1} \cdot a_1)^2 + (W_{m.1.25} \cdot a_{1.25})^2 + ... + (W_{m.80} \cdot a_{80})^2} \tag{2.3}$$

$$v_{comf} = \sqrt{(W_{m.1} \cdot v_1)^2 + (W_{m.1.25} \cdot v_{1.25})^2 + ... + (W_{m.80} \cdot v_{80})^2} \tag{2.4}$$

where $W_{m.1}$, $W_{m.1.25}$...$W_{m.80}$ are the weighting factors to the corresponding one-third-octave band, according to Table 2.2. a_1, $a_{1.25}$...a_{80} are the acceleration responses in each one-third-octave band and v_1, $v_{1.25}$...v_{80} are the velocity responses.

Table 2.2: One-third-octave bands with corresponding frequency weighting factors.

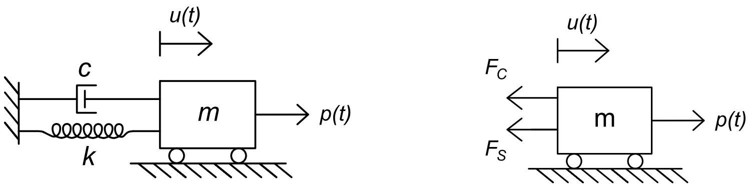
One-third-octave band		Weighting factors, W_m	
Frequency range [Hz]	**Center frequency [Hz]**	**Acceleration**	**Velocity**
- 1.12	1.00	0.9849	0.1733
1.12 - 1.41	1.25	0.9763	0.2162
1.41 - 1.78	1.60	0.9633	0.2686
1.78 - 2.24	2.00	0.9436	0.3312
2.24 - 2.82	2.50	0.9147	0.4042
2.82 - 3.55	3.15	0.8739	0.4861
3.55 - 4.47	4.00	0.8191	0.5737
4.47 - 5.62	5.00	0.7501	0.6614
5.62 - 7.08	6.30	0.6693	0.7430
7.08 - 8.91	8.00	0.5819	0.8132
8.91 - 11.2	10.0	0.4942	0.8694
11.2 - 14.1	12.5	0.4115	0.9114
14.1 - 17.8	16.0	0.3376	0.9413
17.8 - 22.4	20.0	0.2740	0.9617
22.4 - 28.2	25.0	0.2207	0.9753
28.2 - 35.5	31.5	0.1769	0.9842
35.5 - 44.7	40.0	0.1413	0.9900
44.7 - 56.2	50.0	0.1127	0.9936
56.2 - 70.8	63.0	0.08972	0.9960
70.8 - 89.1	80.0	0.07138	0.9974

2.2 Fundamentals of structural dynamics

Structural dynamics is the study of dynamical effects, such as vibrations, in structures subjected to dynamic loads. These are loads that changes with time; in property of magnitude, direction or point of application (Craig and Kurdila, 2006). Structures subjected to dynamic loads will experience time-varying displacements and stresses, which are often referred to as dynamic responses.

2.2.1 Basic equations, SDOF systems

An easy way of describing the basic equations of structural dynamics is with a damped spring-mass model, see Figure 2.9a. The mass is moving horizontally as a rigid body and is prevented from torsion and vertical movement; hence, the system has one degree of freedom. The single degree of freedom (SDOF) system consists of a damper with damping coefficient c, a spring with stiffness k, and a mass with mass m. The mass is excited by the time-dependent force $p(t)$ and the resulting displacement, as a function of time, is given as $u(t)$.



(a) Spring mass system, SDOF (b) Free body diagram

Figure 2.9: SDOF system and corresponding free body diagram.

Figure 2.9b shows the free body diagram of the system in Figure 2.9a, where F_c and F_s represent the damping force and the spring force, respectively. To derive the equation of motion for this system, three fundamental laws of mechanics need to be applied (Craig and Kurdila, 2006). Firstly, Newton's Laws, or other energy principles that are equivalent, must be satisfied. For the system in Figure 2.9b, Newton's 2^{nd} law gives:

$$-F_c(t) - F_s(t) + p(t) = m\ddot{u}(t) \tag{2.5}$$

where $\ddot{u}(t)$ is the acceleration (second time-derivative of the displacement).

Secondly, the force-displacement relation for the spring and the force-velocity relation for the damper need to be stated. These can be expressed respectively, for a linear elastic case, as:

$$F_c(t) = c\dot{u}(t) \tag{2.6}$$

$$F_s(t) = ku(t) \tag{2.7}$$

where $\dot{u}(t)$ is the velocity (first time-derivative of the displacement).

Finally, the kinematics of deformation need to be taken into account. The mass in Figure 2.9 is regarded as a rigid body; hence, no internal deformation of the system is accounted for.
The equations (2.7) and (2.6), inserted in Equation (2.5) gives the equation of motion for a damped SDOF system:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = p(t) \tag{2.8}$$

By defining the natural frequency and the damping ratio the equation of motion can be rewritten as Equation (2.11).

$$\text{Natural frequency:} \quad \omega_n = \sqrt{\frac{k}{m}} \tag{2.9}$$

$$\text{Damping ratio:} \quad \zeta = \frac{c}{2\,m\,\omega_n} = \frac{c}{2\sqrt{km}} \tag{2.10}$$

The equations (2.9) and (2.10) inserted in Equation (2.8), and dividing by m, gives:

$$\ddot{u}(t) + 2\zeta\omega_n\dot{u}(t) + \omega_n^2 u(t) = \frac{p(t)}{m} \tag{2.11}$$

As mentioned in Section 2.1, a system without the damping part would have an infinite motion, for free vibration. The simplest form of damping is linear viscous damping, expressed in Equation (2.11), which also is the most common type to use in analytical studies (Craig and Kurdila, 2006). Generally, there are three cases of linear viscous damping, dependant on the magnitude of the damping ratio: underdamped for $0 < \zeta < 1$, critically damped for $\zeta = 1$ and overdamped for $\zeta > 1$. Figure 2.10 shows a principal response for an SDOF system with the three cases of viscous damping. It is clear that the amplitude decays for all three cases, however, most rapidly for the case with critical damping. It can also be observed that the response only oscillates for the underdamped case, which is the most important case for structural dynamic applications.

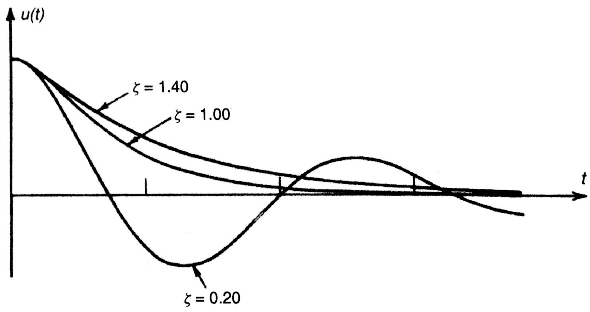
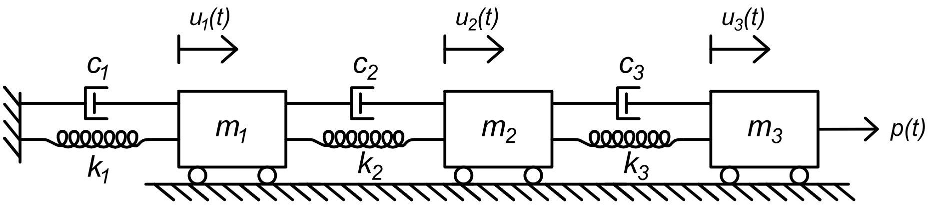


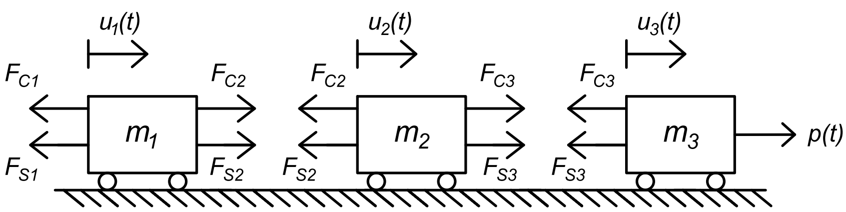
Figure 2.10: Different levels of viscous damping for an SDOF system, from (Craig and Kurdila, 2006).

2.2.2 Basic equations, MDOF systems

In the same manner as described in Section 2.2.1, the equation of motion for a multi degree of freedom (MDOF) system can be derived. Figure 2.11a shows an MDOF system consisting of three different masses, connected with springs and dampers. The system has three degrees of freedom, since the three masses are only moving horizontally. Figure 2.11b shows the free body diagram of the system in Figure 2.11a.



(a) Spring mass system, MDOF



(b) Free body diagram

Figure 2.11: MDOF system and corresponding free body diagram.

The force-displacement relations for the springs and the force-velocity relations for the dampers are respectively stated as:

$$F_{S1} = k_1 u_1 \tag{2.12}$$
$$F_{S2} = k_2(u_2 - u_1) \tag{2.13}$$
$$F_{S3} = k_3(u_3 - u_2) \tag{2.14}$$
$$F_{C1} = c_1 \dot{u}_1 \tag{2.15}$$
$$F_{C2} = c_2(\dot{u}_2 - \dot{u}_1) \tag{2.16}$$
$$F_{C3} = c_3(\dot{u}_3 - \dot{u}_2) \tag{2.17}$$

where every u_i and $\dot{u}_i$ are functions of time t. Newton's 2^{nd} law, together with the force-displacement/velocity relations stated in Equation (2.12) to Equation (2.17), gives:

$$m_1\ddot{u}_1 = -F_{S1} + F_{S2} - F_{C1} + F_{C2} = (-k_1 - k_2)u_1 + k_2u_2 + (-c_1 - c_2)\dot{u}_1 + c_2\dot{u}_2 \tag{2.18}$$
$$m_2\ddot{u}_2 = -F_{S2} + F_{S3} - F_{C2} + F_{C3} = k_2u_1 - (k_2 + k_3)u_2 + k_3u_3 + c_2\dot{u}_1 - (c_2 + c_3)\dot{u}_2 + c_3\dot{u}_3 \tag{2.19}$$
$$m_3\ddot{u}_3 = -F_{S3} - F_{C3} + p(t) = k_3u_2 - k_3u_3 + c_3\dot{u}_2 - c_3\dot{u}_3 + p(t) \tag{2.20}$$

The equations (2.18), (2.19) and (2.20) can be written in matrix-form as:

$$\underbrace{\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}}_{[m]} \underbrace{\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix}}_{\{\ddot{u}\}} + \underbrace{\begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}}_{[c]} \underbrace{\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{bmatrix}}_{\{\dot{u}\}} + \underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & +k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}}_{[k]} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{\{u\}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ p(t) \end{bmatrix}}_{\{p(t)\}}$$

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{p(t)\} \tag{2.21}$$

2.2.2.1 Natural frequencies of an undamped MDOF system

To calculate the natural frequencies of an undamped MDOF system, the free vibration equation of motion need to be stated. A free vibration occurs when the external force is zero, $\{p(t)\} = \{0\}$; hence, Equation (2.21), for an undamped system, becomes:

$$[m]\{\ddot{u}\} + [k]\{u\} = \{0\} \tag{2.22}$$

Inserting the equation of harmonic motion, see Equation (2.1), into Equation (2.22) gives the eigenvalue problem (Craig and Kurdila, 2006):

$$([k] - \omega_i^2[m])\{\phi_i\} = ([k] - \lambda_i[m])\{\phi_i\} = \{0\} \tag{2.23}$$

where ω_i is the different natural frequencies of the system and $\{\phi_i\}$ is the corresponding natural mode, or eigenvector. The variable $\lambda_i = \omega_i^2$ represent the eigenvalues.

In order for a nontrivial solution to Equation (2.23) to exist, it is necessary that the determinant of $([k] - \lambda_i[m])$ is equal to zero. To obtain the natural frequencies of the system, the eigenvalue problem can be solved as:

$$det([k] - \lambda_i[m]) = \{0\}$$

For the system in Figure 2.11, the determinant of $([k] - \lambda_i[m])$ results in a cubic equation; hence, three different eigenvalues, λ_1, λ_2 and λ_3, will be obtained. The number of obtained eigenvalues is always equal to the number of degrees of freedom.

Each eigenvalue has a corresponding eigenvector that satisfies Equation (2.23). The system in Figure 2.11 will generate the eigenvectors: $\{\phi_1\}$, $\{\phi_2\}$ and $\{\phi_3\}$, which can be combined to form an eigenmatrix, $[\Phi] = [\{\phi_1\}\{\phi_2\}\{\phi_2\}]$. The eigenmatrix, in combination with the mass matrix $[m]$ and the stiffness matrix $[k]$ described in Equation (2.21), gives the diagonal modal mass matrix $[M]$ and the diagonal modal stiffness matrix $[K]$. These are defined respectively as:

$$[M] = [\Phi]^T[m][\Phi] \tag{2.24}$$

$$[K] = [\Phi]^T[k][\Phi] \tag{2.25}$$

By letting the displacement vector $\{u\}$ be expressed as $\{u\} = [\Phi]\{\gamma\}$, where $\{\gamma\}$ represent the modal displacement vector, and pre-multiplying with $[\Phi]^T$, Equation (2.22) can be rewritten as:

$$[\Phi]^T[m][\Phi]\{\ddot{\gamma}\} + [\Phi]^T[k][\Phi]\{\gamma\} = [\Phi]^T\{p(t)\} \tag{2.26}$$

Inserting Equation (2.24) and (2.25) in Equation (2.26) gives:

$$[M]\{\ddot{\gamma}\} + [K]\{\gamma\} = [\Phi]^T\{p(t)\} \tag{2.27}$$

2.2.2.2 Damping of MDOF systems

In the same manner as in Section 2.2.2.1, the equation of motion for a damped system can be expressed as:

$$[M]\{\ddot{\gamma}\} + [C]\{\dot{\gamma}\} + [K]\{\gamma\} = [\Phi]^T\{p(t)\} \tag{2.28}$$

where $[C]$ is the modal damping matrix, defined as:

$$[C] = [\Phi]^T[c][\Phi] \tag{2.29}$$

Unlike the modal mass and the modal stiffness matrix, the modal damping matrix is not diagonal and the damping of a structure is seldom expressed in terms of individual element properties. Instead, it is often defined at the system level and there are different ways in which the damping of a system can result in a diagonal modal damping matrix. One of the most common type of damping, which also is the one used in this study, is called modal damping. For this type of damping, orthogonality is assumed, which allows Equation (2.29) to be rewritten as:

$$[C] = [\Phi]^T [c][\Phi] = diag(C_i) = diag(2\zeta_i \omega_i M_i) \tag{2.30}$$

where i represents the different modes of the system. The equation of motion in modal coordinates, Equation (2.27), can then be expressed as:

$$M_i \ddot{\gamma}_i + 2\zeta_i \omega_i M_i \dot{\gamma}_i + \omega_i^2 M_i \gamma_i = \{\phi_i\}^T \{p(t)\} \tag{2.31}$$

2.2.3 Integration methods

In reality, a building is seldom exposed to a continuous excitation, like a sinus wave. Instead, the external force or ground motion will most likely vary randomly (Chopra, 2007). In that case, solving the equation of motion using analytical methods, as in Section 2.2.1, is not possible and instead numerical integrations are required. In methods for numerical integration, the time step for each iteration is defined as $\Delta t_i = t_{i+1} - t_i$, where, $i = 0, 1, 2, ..., n$. The displacement, velocity and acceleration at the time t_i are denoted as u_i, $\dot{u}_i$ and $\ddot{u}_i$, respectively. With the inital conditions $u(0) = u_0$ and $\dot{u}(0) = \dot{u}_0$ the following equation must always be satisfied:

$$m\ddot{u}_i + c\dot{u}_i + (f_s)_i = p_i \tag{2.32}$$

The variable $(f_s)_i$ is the resisting force at the time i. For a linear elastic system it is defined as ku_i but for a general case it is dependant on the prior displacement history. A numerical integration method is applied to solve the equation of motion at the time t_{i+1}:

$$m\ddot{u}_{i+1} + c\dot{u}_{i+1} + (f_s)_{i+1} = p_{i+1} \tag{2.33}$$

In order to perform numerical integrations, a number of different methods can be used to approximate the solution. Figure 2.12 shows a principle graph of these types of time-stepping methods. The approaches of the methods vary, but they all need to fulfil three important requirements: *accuracy*, *stability* and *convergence*. Accuracy means that the difference between the result from the approximation and the exact solution should be less than a defined tolerance. The stability condition requires the numerical solution to be stable due to numerical round-off errors. Lastly, the convergence criterion should be fulfilled so that the numerical solution approaches the exact solution as the time step is decreasing.

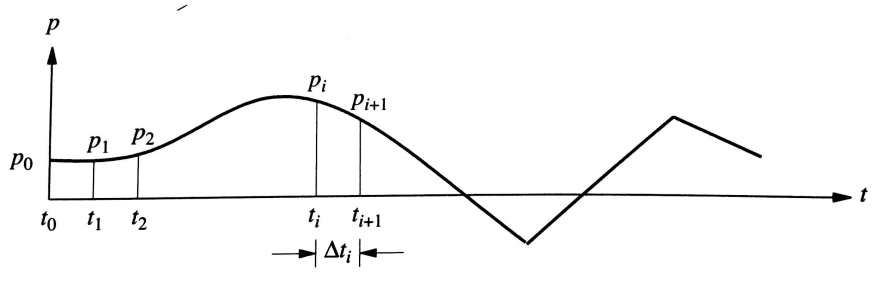


Figure 2.12: Principles for time-stepping, from (Chopra, 2007).

The integration method used exclusively for the studies presented in this report is called the Newmark-β method. This method is a special case of the more common central difference method (CDM). The CDM is convenient in its simplicity, but since it is conditionally stable all natural frequencies of the system need to be known in advance (Craig and Kurdila, 2006). In some cases, that is not possible and it might be necessary to use a method that is stable for any frequency, like the Newmark-β method. It consist of time-stepping methods based on the following equations:

$$u_{i+1} = u_i + \Delta t\dot{u}_i + [(\tfrac{1}{2} - \beta)(\Delta t)^2]\ddot{u}_i + [\beta \Delta t^2]\ddot{u}_{i+1} \tag{2.34}$$

$$\dot{u}_{i+1} = \dot{u}_i + [(1-\gamma)\Delta t]\ddot{u}_i + \gamma \Delta t \ddot{u}_{i+1} \tag{2.35}$$

The parameters γ and β define the variation of acceleration over a time step and may be selected in such manner that the method becomes unconditionally stable. To satisfy the requirement of accuracy, stability and convergence, typical selections of the parameters are; $\gamma = 1/2$ and $1/6 \leq \beta \leq 1/4$. The special case when $\beta = 1/4$ is called *Average acceleration method* and the case when $\beta = 1/6$ is called *Linear acceleration method*. The *Average acceleration method* is unconditionally stable, which can be shown by looking at the stability requirement for the Newmark-β method:

$$\frac{\Delta t}{T_n} \leq \frac{1}{\pi\sqrt{2}\sqrt{\gamma - 2\beta}} \tag{2.36}$$

From definition, when $\gamma = 1/2$ and β is chosen to $1/4$ the condition becomes: $\Delta t/T_n < \infty$, which means the method is stable for any frequency.

The approach of the Newmark-β method is to first select values for γ and β. The initial displacement and velocity are known and defined as: $u(0) = u_0$ and $\dot{u}(0) = \dot{u}_0$. The initial acceleration is defined as:

$$\ddot{u}_0 = \frac{p_0 - c\dot{u}_0 - ku_0}{m} \tag{2.37}$$

For a constant time step, Δt, the variables a, b and $\hat{k}$ are defined as:

$$a = \frac{m}{\beta \Delta t} + \frac{\gamma c}{\beta} \tag{2.38}$$

$$b = \frac{m}{2\beta} + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c \tag{2.39}$$

$$\hat{k} = k + \frac{\gamma c}{\beta \Delta t} + \frac{m}{\beta(\Delta t)^2} \tag{2.40}$$

At the time $t = i$, the variables $\Delta\hat{p}_i$, Δu_i, $\Delta\dot{u}_i$ and $\Delta\ddot{u}_i$ are calculated respectively from the following equations:

$$\Delta u_i = \frac{\Delta\hat{p}_i}{\hat{k}} \tag{2.41}$$

$$\Delta\dot{u}_i = \frac{\gamma\Delta u_i}{\beta\Delta t} - \frac{\gamma\dot{u}_i}{\beta} + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{u}_i \tag{2.42}$$

$$\Delta\ddot{u}_i = \frac{\Delta u_i}{\beta(\Delta t)^2} - \frac{\dot{u}_i}{\beta\Delta t} - \frac{\ddot{u}_i}{2\beta} \tag{2.43}$$

$$\Delta\hat{p}_i = \Delta p_i + a\dot{u}_i + b\ddot{u}_i \tag{2.44}$$

Further, the displacement, velocity and acceleration at the time $t = i + 1$ are defined as:

$$u_{i+1} = u_i + \Delta u_i \tag{2.45}$$

$$\dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i \tag{2.46}$$

$$\ddot{u}_{i+1} = \ddot{u}_i + \Delta\ddot{u}_i \tag{2.47}$$

For the next iteration, i is replaced by $i + 1$.

2.3 Applications of structural dynamics

The equations and principles described in Section 2.2 can be used in various ways to analyses the dynamical response of different systems. The most thorough analysis to perform is a so called time history analysis (THA), where the actual response is calculated at each time step. In some cases, such detailed study is not necessary and only the maximum values are of interest. Then, it is common to perform a response spectrum analysis (RSA), which gives an estimation of the response of an SDOF system. If an MDOF system is to be analysed, the response need to be translated, using methods for modal analysis.

2.3.1 Time history analysis

In order to obtain the total response history record for a system, a THA is required. Such analysis is carried out including all natural frequencies of interest, to get the response as a function of time. Figure 2.13 shows examples of the displacement response of SDOF systems with different natural vibration periods and damping ratios. A THA provides the full response of a system for a specific time period. However, these kinds of analyses are often computationally demanding and time consuming.

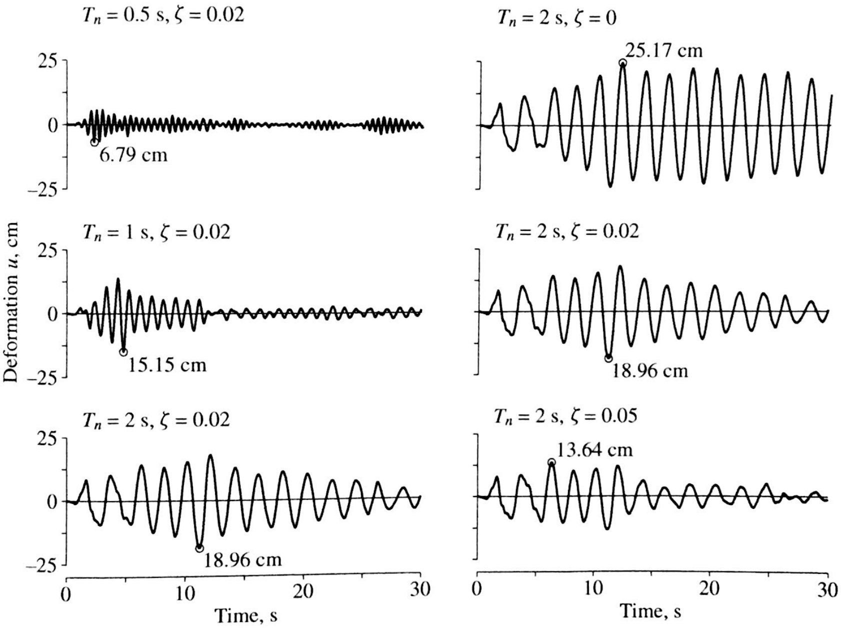


Figure 2.13: Example of time history analysis for displacement response, from (Chopra, 2007).

2.3.2 Response spectrum analysis

Another way to analyse the response of a system is to perform an RSA. Response spectra are plots consisting of peak responses of SDOF systems, excited by a specific load. The spectra describe response quantities (e.g. absolute acceleration or relative displacement) as a function of any of the parameters: natural vibration period, T_n [s], or natural frequency, as either ω_n [rad/s] or as f_n [Hz]. Figure 2.14 shows an example of such plot, where the absolute velocity is plotted as a function of the period of the system. A response spectrum often consists of several plots for different fixed damping ratios, ζ, as can be seen in Figure 2.14.

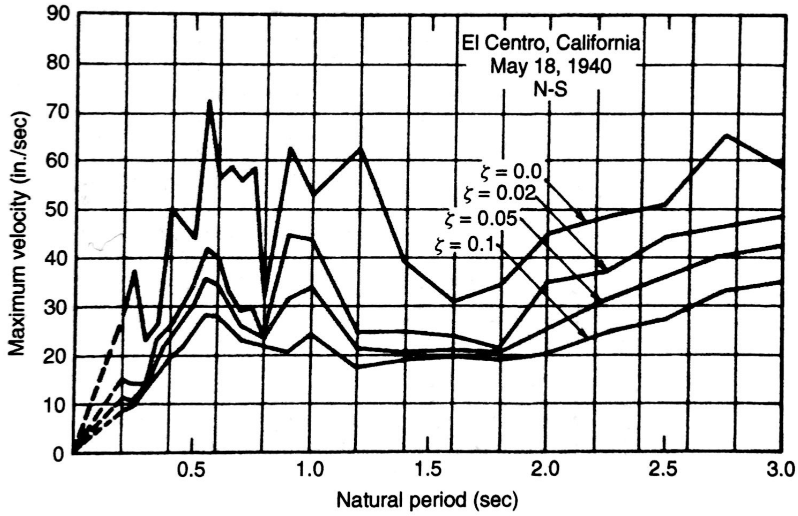


Figure 2.14: Example of velocity response spectrum, from (Chopra, 2007.)

The response spectrum concept is the most common method used in earthquake engineering to estimate the effects of ground motions on structures (Chopra, 2007). The method provides a way to find the peak responses of a linear SDOF system with varying stiffness, exposed to a certain ground motion. The responses are most commonly acceleration, velocity or displacement. The method is less computationally heavy than a THA, since it only considers the peak values and not the total history record. With a response spectrum, the maximum response of a system can be found by simply using the specific natural vibration period to find the corresponding response quantity in the plot, called *the spectral response*. For example, for an SDOF system exposed to the same ground motion as in Figure 2.14, with a natural period $T_n = 1$ s and a damping ratio $\zeta_n = 0.0$, the spectral response is approximately 54 in/s.

Figure 2.15 is a flowchart showing the principles of the design process of response spectra. In this example the response quantity of interest is the total acceleration. At first, a desired frequency range, corresponding to various SDOF systems, is determined, see (1).

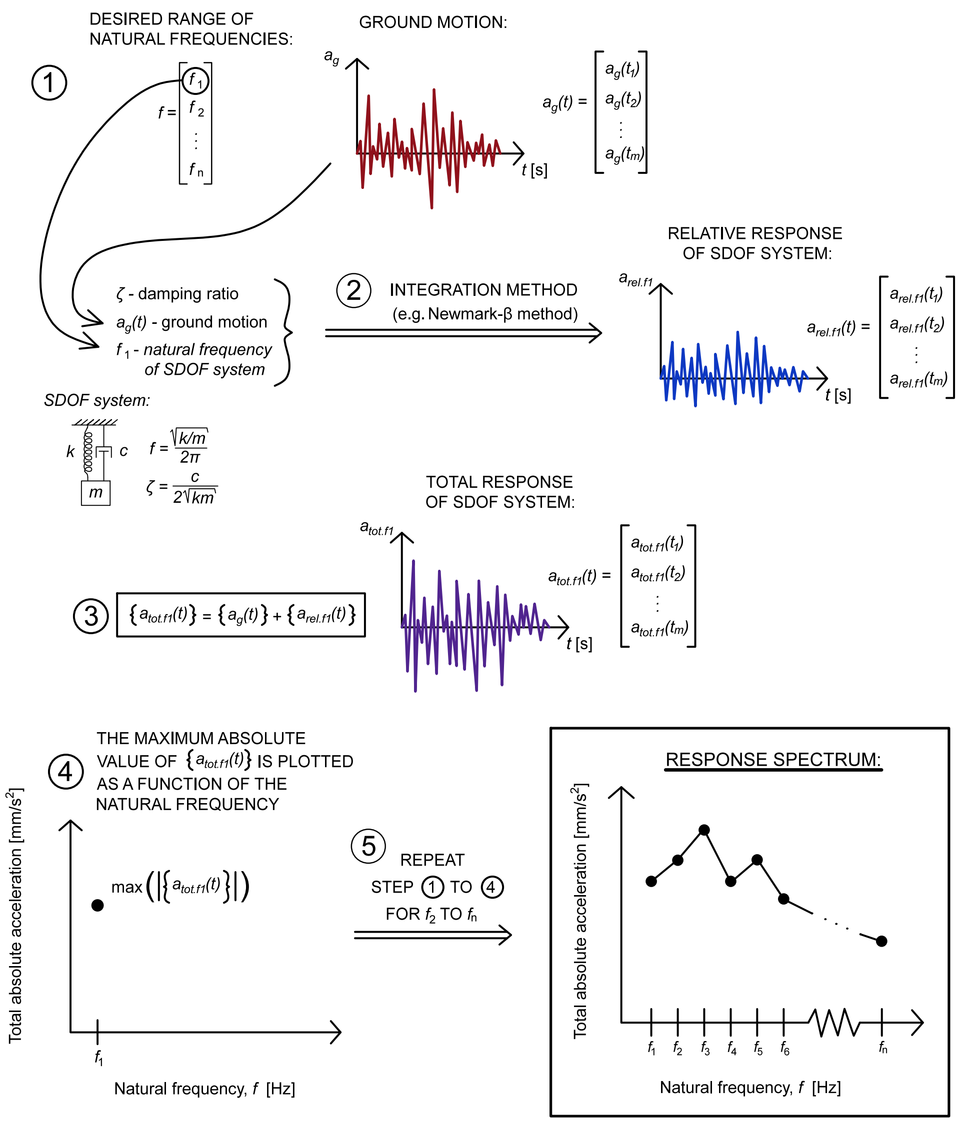


Figure 2.15: Flowchart of the principles when designing a response spectrum.

In (2) an integration method is used, as described in Section 2.2.3, with the damping ratio, ζ, the ground motion, $a_g(t)$, and the first frequency, f_1, as inputs. This results in a vector containing the relative response for each time step. The force exciting the system can be expressed as $F_{SDOF}(t) = m_{SDOF} \cdot a_g(t)$. In (3) the ground motion is added in order to get the total response. Next, the absolute maximum value of the total response vector is found and plotted as a function of the corresponding frequency, see (4). Further, (1) to (4) is repeated for the rest of the frequencies, from f_2 to f_n, until all responses of interest are found and plotted, see (5). This process finally results in a response spectrum showing the total response of various SDOF systems. Further, the spectral values for total displacement, velocity and acceleration can be defined as:

$$
\begin{aligned}
S_{d.tot}(\zeta, f_n) &= |u(t) + u_g(t)|_{max} \\
S_{v.tot}(\zeta, f_n) &= |\dot{u}(t) + \dot{u}_g(t)|_{max} \\
S_{a.tot}(\zeta, f_n) &= |\ddot{u}(t) + \ddot{u}_g(t)|_{max}
\end{aligned}
$$

If only the relative response is of interest the same procedure as in Figure 2.15 is used, but without adding the ground motion in step 3. This results in spectral values for the relative responses described as:

$$
\begin{aligned}
S_d(\zeta, f_n) &= |u(t)|_{max} && \text{for relative displacement} \\
S_v(\zeta, f_n) &= |\dot{u}(t)|_{max} && \text{for relative velocity} \\
S_a(\zeta, f_n) &= |\ddot{u}(t)|_{max} && \text{for relative acceleration}
\end{aligned}
$$

for displacement, velocity and acceleration, respectively.

2.3.3 Modal participation factor

As mentioned in Section 2.3.2, it is possible to calculate the maximum response of an SDOF system through a response spectrum. If an MDOF system is to be analysed, the spectral value from the response spectrum cannot be used directly, since it only represent the response of an SDOF system. Since the DOFs in the MDOF system each will have individual responses, the spectral values (one for each eigenfrequency of the system) need to be combined with the eigenmodes and the so called *modal participation factors* (MPFs), Γ_i, as in Equation (2.48). Every mode *i* of the MDOF system has a corresponding MPF, see Equation (2.49).

$$a_{i.j} = \phi_{i.j} \cdot \Gamma_i \cdot S_{a.i}(\zeta, \omega_i) \tag{2.48}$$

$$\{\Gamma\} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \dots \\ \Gamma_n \end{bmatrix} \tag{2.49}$$

where each Γ_i is an MPF, corresponding to one of the modes of the analysed MDOF system.

To be able to calculate the MPFs, a modal analysis, as described in Section 2.2.2, is performed to obtain the eigenfrequencies and the eigenvectors of the MDOF system. The eigenmatrix can be seen in Equation (2.50), where each column represent an eigenvector corresponding to one natural frequency of the MDOF system.

$$[\Phi] = \begin{bmatrix}\{\phi_1\} & \{\phi_2\} & \dots & \{\phi_n\}\end{bmatrix} = \begin{bmatrix} \phi_{1.1} & \phi_{2.1} & \dots & \phi_{n.1} \\ \phi_{1.2} & \phi_{2.2} & \dots & \phi_{n.2} \\ \dots & \dots & \dots & \dots \\ \phi_{1.n} & \phi_{2.n} & \dots & \phi_{n.n} \end{bmatrix} \tag{2.50}$$

The MPF, Γ_i, can be calculated as:

$$\Gamma_i = \frac{\{\phi_i\}^T\,[M]\,\{r\}}{\{\phi_i\}^T\,[M]\,\{\phi_i\}} \tag{2.51}$$

where $[M]$ is the modal mass matrix described in Section 2.2.2, $\{\phi_i\}$ is the eigenvector of the ith mode and $\{r\}$ is the influence coefficient vector.

The influence coefficient vector $\{r\}$ contains one column and as many rows as the system has DOFs that are not prescribed to zero, n (Datta, 2010). The vector describes how the analysed structure moves in relation to the direction of the load. $\{r\}$ can be calculated using the stiffness matrix $[K]$ and the boundary conditions. First, $[K]$ is rearranged so that the values corresponding to the DOFs that are to be condensed out (apart from the DOFs related to the supports), if any exists, is placed at the end as:

$$[K] = \begin{bmatrix} [K_{dd}] & [K_{dc}] \\ [K_{cd}] & [K_{cc}] \end{bmatrix} \tag{2.52}$$

If the number of DOFs that are condensed out is defined as a and the number of remaining DOFs is defined as b, $[K_{dd}]$ has the dimension $b \times b$, $[K_{dc}]$ is of dimension $b \times a$, $[K_{cd}]$ is $a \times b$ and $[K_{cc}]$ is $a \times a$.

A condensed stiffness matrix, $[K_d]$, corresponding to the remaining DOFs that are not condensed out can be expressed as:

$$[K_d] = [K_{dd}] - [K_{dc}][K_{cc}]^{-1}[K_{cd}] \tag{2.53}$$

The matrix $[K_d]$, which will be of dimension $m \times m$, can be divided into sub matrices as well, where the DOFs related to the supports, and hence prescribed to zero, are placed at the end as:

$$[K_d] = \begin{bmatrix} [K_{nnd}] & [K_{nsd}] \\ [K_{snd}] & [K_{ssd}] \end{bmatrix} \tag{2.54}$$

If the number of DOFs that are related to the supports is defined as c and the number of remaining DOFs is defined as d, $[K_{nnd}]$ has the dimension $d \times d$, $[K_{nsd}]$ is of dimension $d \times c$, $[K_{snd}]$ is $c \times d$ and $[K_{ssd}]$ is $c \times c$.

The sub matrices $[K_{nnd}]$ and $[K_{nsd}]$ can then be used to calculate the influence matrix $[R]$, see Equation (2.55), that further can be used to calculate the influence coefficient vector $\{r\}$.

$$[R] = -[K_{nnd}]^{-1}[K_{nsd}] \tag{2.55}$$

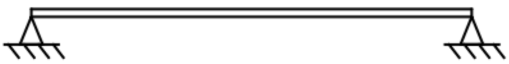
The matrix $[R]$ will be of dimension $d \times c$, where each column corresponds to one of the support DOFs. If the first support DOF is in the horizontal direction, for example as DOF 1 and 7 in Figure 2.16b, the first column in the matrix $[R]$ will also be related to the horizontal direction. To get the vector $\{r\}$ for the horizontal direction (used when the load is applied in the horizontal direction) all the columns that are related to horizontal support DOFs are added as:

$$[r_h] = R_{h1} + R_{h2} + ... + R_{hn} \tag{2.56}$$

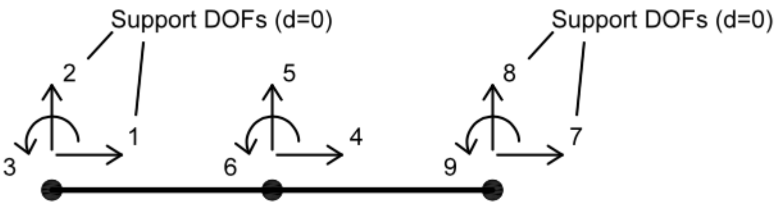
In the same way, to get the vector $\{r\}$ for the vertical direction (used when the load is applied in the vertical direction) all the columns that are related to vertical support DOFs are added as:

$$[r_v] = R_{v1} + R_{v2} + ... + R_{vn} \tag{2.57}$$

For example, if a simply supported beam is to be analysed, as in Figure 2.16a, which is divided into two elements and has 9 DOFs where four of them; 1,2,7 and 8 are support DOFS, see Figure 2.16b. The matrix $[R]$ may look like as in Equation (2.58).



(a) Simply supported beam



(b) Element division with DOF numbering (d is displacement)

Figure 2.16: Simply supported beam with elements and DOFs.

$$[R] = \begin{bmatrix} 0 & -0.067 & 0 & 0.067 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & -0.067 & 0 & 0.067 \\ 0 & -0.067 & 0 & 0.067 \end{bmatrix} \tag{2.58}$$

The columns in $[R]$ correspond to the DOFs 1,2,7 and 8, respectively. The DOFs 1 and 7 are horizontal DOFs, whereas DOF 2 and 8 are vertical DOFs, as can be seen in Figure 2.16b. The vectors $\{r_h\}$ and $\{r_v\}$ can therefore be stated as:

$$\{r_h\} = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \{r_v\} = \begin{bmatrix} -0.067 \\ 0 \\ 0.5 \\ -0.067 \\ -0.067 \end{bmatrix} + \begin{bmatrix} 0.067 \\ 0 \\ 0.5 \\ 0.067 \\ 0.067 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{2.59}$$

Is is notable that the vectors $\{r_h\}$ and $\{r_v\}$ only contains the values 1 and 0. As mentioned earlier, $\{r_{h/v}\}$ describes the movement of the analyzed structure in relation to the load. The rows in $\{r\}$ represent the non-support DOFs 3,4,5,6 and 9. In $\{r_h\}$, the second row, that represent DOF 4, is equal to 1, which is the only horizontal DOF and hence, the only DOF moving in the same direction as the load, if the load is applied in the horizontal direction. $\{r_h\}$ is used if the load is applied in the horizontal direction and $\{r_v\}$ is used if the load is applied in the vertical direction.

When Γ is calculated, the response in each DOF can be found by multiplying each value of the eigenvectors with a spectral value $S_{a.i}(\zeta, \omega_i)$ corresponding to the natural frequency f_i and a corresponding MPF Γ_i, as already described, see Equation (2.48).

Further, to calculate the maximum response in each DOF, responses from all modes need to be taken into account. This is done by combining the responses in each mode with a combination method, explained further in Section 2.3.4.

2.3.4 Modal combination rules

As mentioned in Section 2.2.2, an MDOF system has a number of different eigenfrequencies and eigenmodes. To calculate the response in a specific node, the responses from all modes in that specific node need to be taken into account and combined (Chopra, 2007). In reality, these modes will not occur exactly at the same time instant, and therefore the calculated combined value is an approximation. This is due to the fact that, when using a response spectrum to determine peak responses, information about the time when the peak responses occur is not available.

There are several different methods to combine these responses and the most common are: *Absolute Sum* (ABSSUM), *Square Root of Sum of Squares* (SRSS) and *Complete Quadratic Combination* (CQC). When using ABSSUM, the absolute value of the responses from all modes are added, as in Equation (2.60). This provides an upper bound value, which is usually too conservative and therefore usually not used in structural design applications.

$$a_{max.i} \leq \sum_{j=1}^{n} |a_{i.j}| \tag{2.60}$$

where $a_{max.i}$ is the maximum acceleration in DOF number i, n is representing the total number of modes and $a_{i.j}$ is the acceleration for DOF i for mode number j.

The method that is most straightforward and easy-to-use of the last two is SRSS, but it provides less accuracy than the CQC method (Pozzi and Der Kiureghian, 2015). Nevertheless, if the natural frequencies of the analysed structure are well-separated the method provides very good results (Chopra, 2007). To weight the responses resulting from different modes with the SRSS method, Equation (2.61) is used.

$$a_{max.i} \simeq \sqrt{{a_{i.1}}^2 + {a_{i.2}}^2 + ... + {a_{i.n}}^2} = \sqrt{\sum_{j=1}^{n} {a_{i.j}}^2} \tag{2.61}$$

To overcome the limitations of the SRSS method if, for example, the values in the frequency range are close to each other, the CQC method is recommended. This combination method is described by the following equation:

$$a_{max.i} \simeq \sqrt{\sum_{j=1}^{n} {a_{i.j}}^2 + \underbrace{\sum_{k=1}^{n} \sum_{j=1}^{n}}_{k \neq j} {a_{i.k}}^2 \, {a_{i.j}}^2 \, \rho_{kj}} \tag{2.62}$$

where ρ, called the correlation coefficient, weights the current mode to the previous mode. The value varies between 0 and 1 and there exist many theories on the most correct way to compute it. A common equation, proposed in Chopra, 2007, is:

$$\rho_{kj} = \frac{\zeta^2 (1 + \beta_{kj})^2}{(1 - \beta_{kj})^2 + 4\zeta^2 \beta_{kj}} \tag{2.63}$$

where β describes the ratio between the natural frequency of the current mode and the previous mode as:

$$\beta_{kj} = \frac{f_k}{f_j} \tag{2.64}$$

3 Development of calculation program

3.1 Description of program

The calculation program is written in Visual Basic for Applications (VBA) and will later on be referred to as *the VBA script* or the *the VBA program*. Essential parts of the script can be found in Appendix D. The program consists of two main parts; one that creates response spectra for a specific input load and one that analyse the modal response of a structure of interest. The structure can either be a simply supported beam or a multi-storey building, modelled as a column. The results from the modal response are either nodal accelerations or nodal velocities, comfort weighted or not. These can be compared to current regulations regarding the maximum allowed responses for comfort in a building, see Section 2.1.2.5.

3.1.1 Input values

In the first part of the program, which creates response spectra for a specific load, the user need to insert the ground motion, $a_g(t)$ or $v_g(t)$, for which the response spectra should be generated and set whether the input motion is an acceleration or a velocity. Further, the damping ratio ζ, the desired minimum and maximum natural frequency of the spectra and the frequency percentage increment in each step need to be defined. Further, it is possible to pick the γ and β to decide what type of Newmark-β method that should be used, see Section 2.2.3. With that information, it is possible to use the VBA script to generate response spectra and hence find the maximum responses of SDOF systems with varying natural frequencies.

In order to use the program to analyse MDOF systems, the user need to specify some additional parameters. Two different types of models are possible to analyse: simply supported beams and cantilevers with lumped masses (representing multi-storey buildings). For both models, the material and geometrical parameters need to be defined as well as the number of beam elements the models should consist of. For the simply supported beam, the number of elements controls the accuracy of the results, whereas for the cantilever each element represents one level of a multi-storey building. Additionally, for the cantilever, lumped masses need to be defined at each node to represent the weight of the floors of the multi-storey building to be analysed. For both models, the boundary conditions need to be defined by prescribing concerned DOFs to zero.

When running the analyses of the MDOF systems, the user gets the option whether to include comfort weighting or not, as described in Section 2.1.3.

3.1.2 Output values

Depending on what type of ground motion that is used as input in the VBA program, different results are generated. If the ground motion is a ground acceleration, the program generates response spectra for total acceleration, relative acceleration, relative velocity and relative displacement, using the principles described in Section 2.3.2. If instead the ground motion is a ground velocity, an additional response spectrum, showing the total velocity, is generated as well. This is explained more thoroughly in Section 3.2.1.

With the spectral values in the different generated response spectra, the MDOF analyses give the maximum responses in every DOF of the models. The user gets the option whether these responses should be comfort weighted RMS values or not. The SDOF responses from the spectra are translated to the MDOF systems using the theory described in Section 2.3.3 and the responses of each mode are combined using the SRSS modal combination rule, as described in Section 2.3.4.

3.2 Generation of response spectra

As mentioned, the first part of the program generates response spectra through the VBA script. To compute the responses, the VBA script uses the Newmark-β method, which is described in Section 2.2.3. Through the VBA-script, a vector of natural frequencies is created where the desired minimum and maximum values are defined as inputs, as well as the frequency increment, defined as a percentage. This increment determines the size of the frequency vector, where the total number of frequencies, n, is calculated as:

$$x^n f_{min} = f_{max} \iff n = \frac{\log\left(\frac{f_{max}}{f_{min}}\right)}{\log(x)} \tag{3.1}$$

where x is the frequency increment and f_{min} and f_{max} are the minimum and maximum frequencies. For each natural frequency, the absolute values of acceleration, velocity and displacement are calculated and the maximum values are collected in three vectors, one for each response. These vectors are plotted with the corresponding frequency in order to create the response spectra.

To verify the results from the VBA script, the commercial FE program ADINA is used to generate comparable response spectra for the same ground motion and damping ratio. To generate the response spectra in ADINA, a model is required in order to be able to run the analysis. The model consist of a 1D fixed column, meshed with 10 beam elements. The cross section is rectangular; height and width 0.02 meters. The material is isotropic linear elastic with Young's modulus $E = 30$ GPa and the density $\rho = 2500$ kg/m^2. The ground motion is applied as a mass proportional load. The generated response spectra will be independent of the input parameters of the model, except for the applied ground motion and the damping ratio.

In the VBA script, the input parameters in Table 3.1 are used in the analyses for the verification, described later in Section 3.2.2 and 3.2.3.

Table 3.1: Input data for response spectra of traffic load.

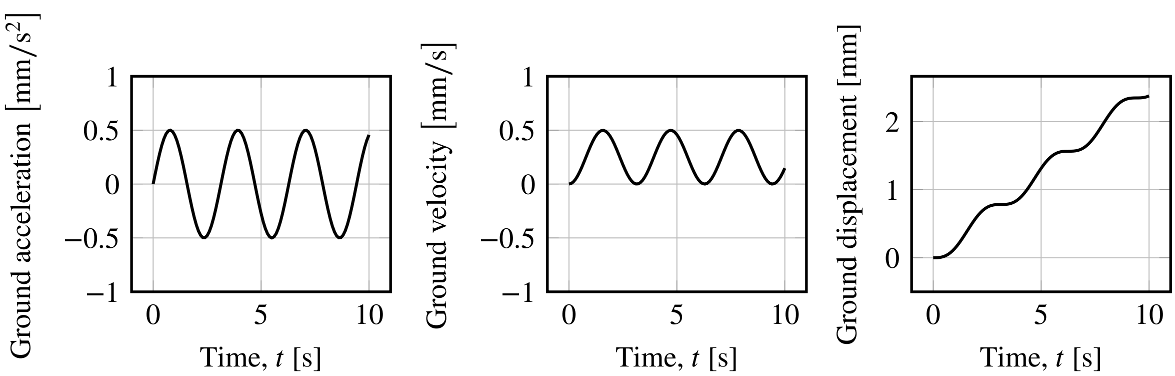
Description	**Parameter**	**Value**	**Unit**
Damping ratio	ζ	2%	-
Initial displacement	d_0	0	m
Initial velocity	v_0	0	m/s
Maximum frequency	f_{max}	50	Hz
Minimum frequency	f_{min}	0.02	Hz
Frequency increment	x	5.012%	-
Newmark coefficient	γ	0.5	-
Newmark coefficient	β	0.25	-

3.2.1 Input ground motion

The Newmark-β method requires the ground acceleration in order to calculate the responses, see Section 2.2.3. If the input ground motion is a ground velocity, and hence the ground acceleration is unknown, a numerical derivation is required. This is performed by calculating the gradient in each time increment as: $a_{g.i} = \frac{\Delta v_g}{\Delta t} = \frac{v_{g.i+1} - v_{g.i}}{t_{i+1} - t_i}$

Numerical integration of a ground motion is not as straightforward as a derivation. It is therefore a problem when the input load is a ground acceleration and it is desired to find the ground velocity, e.g. if the total velocity is of interest. Performing a numerical integration induces an error since the initial velocity, v_0, need to be known in advance. The integration is performed by calculating the area under the graph as: $v_{g.i} = \sum_{i=1}^{n}(\frac{a_{g.i}+a_{g.i+1}}{2} \cdot \Delta t)$

Figure 3.1 illustrates the problem, where the input ground acceleration has the shape of a sinus curve, see Figure 3.1a. If a numerical integration is performed, and the initial velocity is defined as 0, the hypothetical ground velocity becomes as shown in Figure 3.1b. It is clear that the curve does not oscillate about the x-axis, which means the ground velocity is always positive. If an additional numerical integration is performed, the hypothetical ground displacement is obtained, see Figure 3.1c. It is clear that this method is incorrect, since the displacement increases continuously during the whole time period. This means that a structure exposed to this ground displacement would move forward for all eternity. For this example, it is easy to see that the correct value of v_0 should be -0.25. However, for a more complex load, this is not as straightforward and a general approach to find the initial velocity through the VBA script has not been found.



(a) Ground acceleration (b) Integrated ground acceleration (c) Integrated ground velocity

Figure 3.1: Example of numerical integration of a sinus curve.

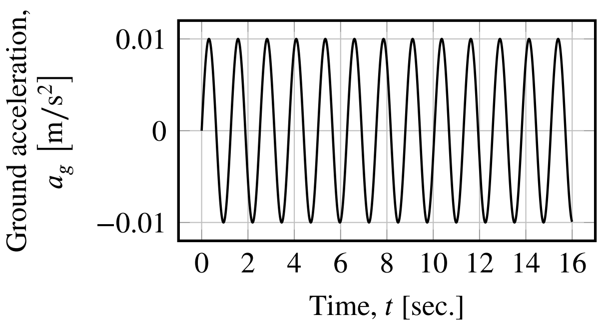
Even though the way of integrating the ground motion as in Figure 3.1 obviously is wrong, it is discovered that ADINA uses this method to obtain the total response spectra from an input ground acceleration, see Section 2.1.2.1 where the distinction between relative and total response is explained. Therefore, to be able to compare the results from VBA with ADINA, this method is used for the analyses in Section 3.2.2, 3.2.3 and 3.3.2. For the further studies in this thesis however, the total velocity is not calculated when the input ground motion is an acceleration. When the ground motion is a velocity, both the total velocity and the total acceleration can be obtained, since numerical derivation is straightforward. Further studies on how to numerically integrate the input ground motion is not performed and it is considered beyond the scope of this report.

3.2.2 Verification - ground motion as sinus shaped accelerations

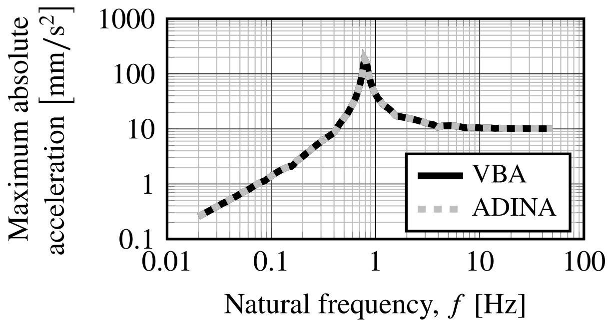
The first ground motion analysed in the VBA script is a sinus shaped ground acceleration, a_g, described as:

$$a_g = A \sin(\omega_1 t)$$

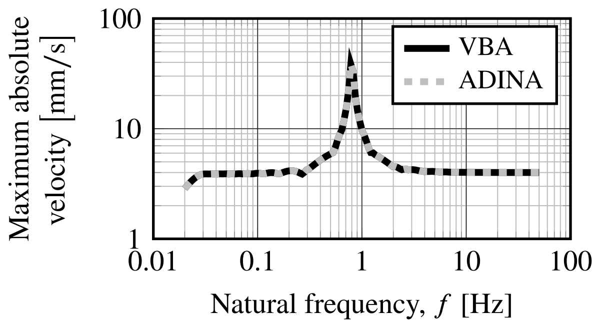
where the amplitude, A, and the angular frequency, ω_1, are arbitrary scalars, defined as 0.01 mm/s^2 and 5 rad/s, respectively, and the variable t represent the time at each time step. The applied ground motion is showed in Figure 3.2a and the results from VBA and ADINA are shown in Figure 3.2b to Figure 3.2d.



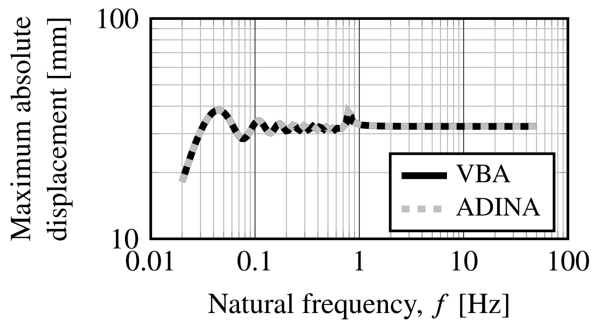
(a) Ground acceleration



(b) Response spectrum, acceleration



(c) Response spectrum, velocity



(d) Response spectrum, displacement

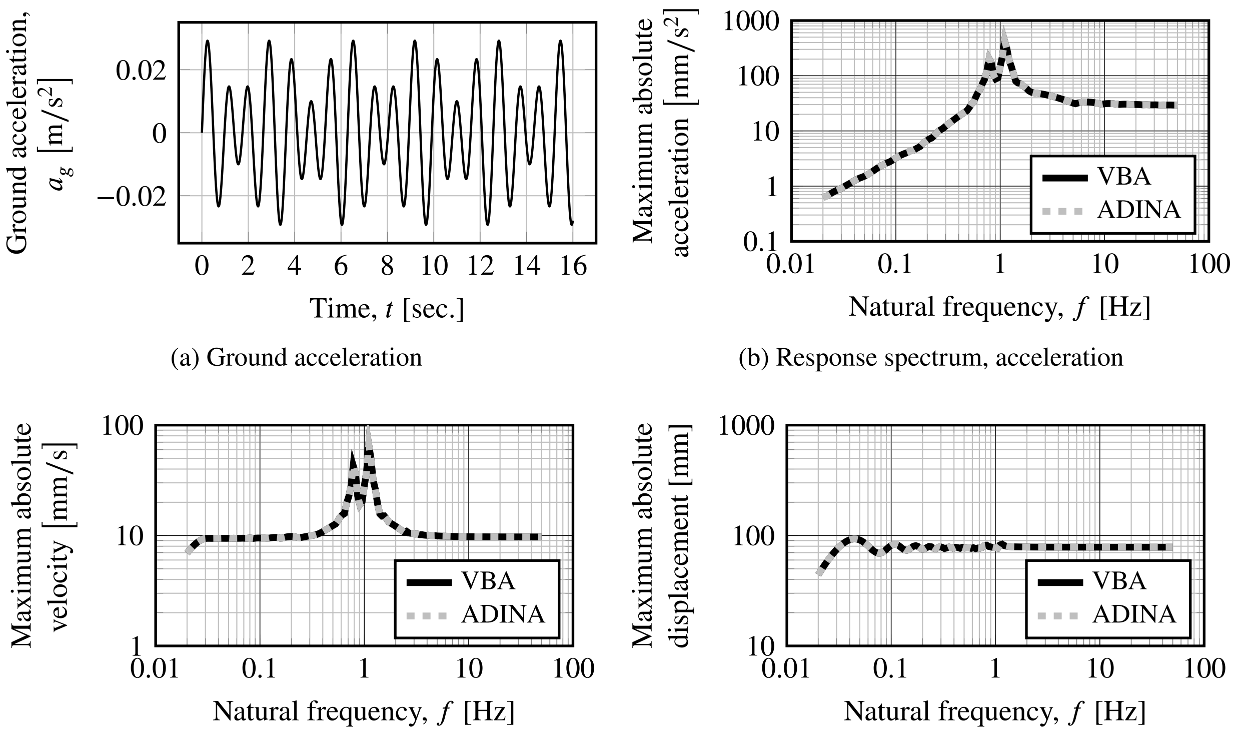
Figure 3.2: Ground acceleration from simple sinus curve and corresponding response spectra.

As can be observed, the results from the VBA script and ADINA are very similar and there is no significant difference. It is notable that in all the response spectra in Figure 3.2b to Figure 3.2d, a peak value can be observed at a natural frequency of 0.79 Hz, which is equal to the frequency of the load: $f = \omega_1/2\pi = 5/2\pi = 0.79$ Hz. Hence, a structure with a natural frequency of 0.79 Hz will experience resonance when the load in Figure 3.2a is applied.

In a similar manner, the VBA script is used to analyse the response of a load with the shape of a double sinus curve, described as:

$$a_g = A \sin(\omega_1 t) + B \sin(\omega_2 t) \tag{3.2}$$

where the amplitude A, and the angular frequency ω_1 are the same as in the previous example, and the amplitude B and angular frequency ω_2 are arbitrarily defined as 0.02 mm/s^2 and 7 rad/s, respectively. The ground motion is described in Figure 3.3a and the results from VBA and ADINA are shown in Figure 3.3b to Figure 3.3d.



(a) Ground acceleration

(b) Response spectrum, acceleration

(c) Response spectrum, velocity

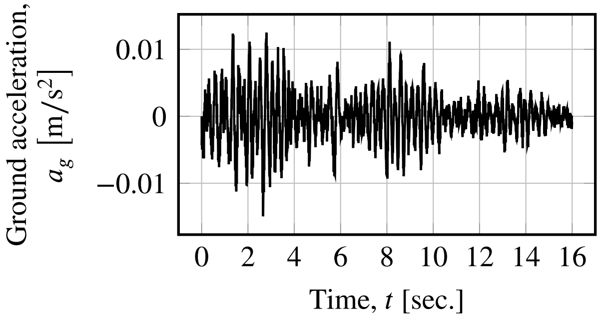
(d) Response spectrum, displacement

Figure 3.3: Ground acceleration from double sinus curve and corresponding response spectra.

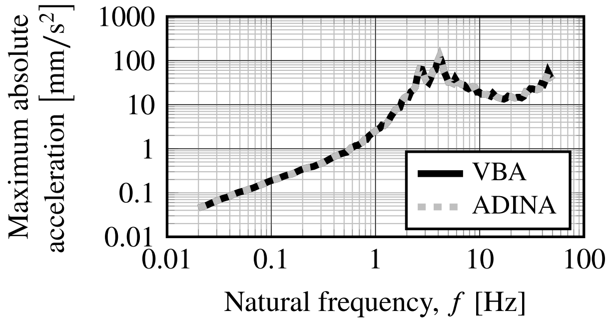
Similarly as in the previous example that described the response from a simple sinus curve, the results from ADINA and the VBA script are very similar. All the response spectra in Figure 3.3b to Figure 3.3d have peak values at the same natural frequencies: 0.79 Hz and 1.11 Hz. These frequencies are equal to the frequencies of the load, due to $f = \omega_1/2\pi = 5/2\pi = 0.79Hz$ and $f = \omega_2/2\pi = 7/2\pi = 1.11$ Hz. Hence, a structure with a natural frequency of 0.79 Hz or 1.11 Hz will experience resonance when the load in Figure 3.3a is applied.

3.2.3 Verification - ground motion as acceleration from traffic flow

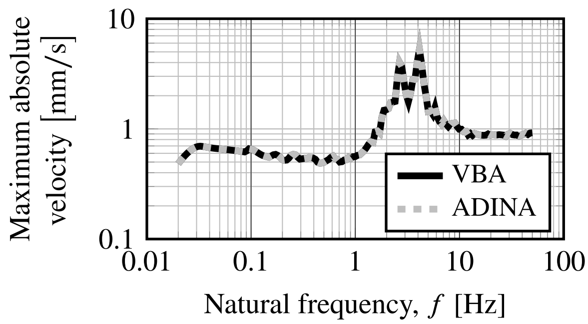
To verify that the VBA script is applicable for a more realistic load case as well, responses due to traffic flow measured at Grafiska Vägen in Gothenburg, close to both a highway and a railway, are evaluated. The measured ground motion is showed in Figure 3.4a. The ground motion has a duration of approximately 16 seconds and the time step between each measurement is about 0.0061 seconds, which results in a load vector with 2622 elements. As can be observed, the results are very similar and there is no significant difference for this load either.



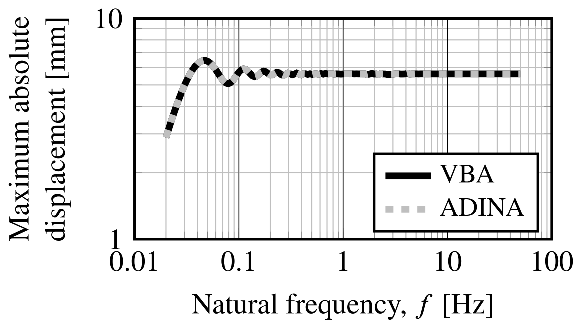
(a) Ground acceleration from traffic flow, measured at Grafiska vägen, Gothenburg



(b) Response spectrum, acceleration



(c) Response spectrum, velocity



(d) Response spectrum, displacement

Figure 3.4: Ground acceleration from traffic flow and corresponding response spectra.

3.3 Calculation of modal response

In the second part of the program, the response in each node of an MDOF system, the modal response, is calculated. First, the natural frequencies and the corresponding modal participation factors (MPFs) for the structure to be analysed need to be calculated. The MPFs, defined in Equation (2.51), are used to translate the SDOF response obtained from the response spectra to the response of an MDOF system, as described in Section 2.3.3. For each natural frequency of the structure, a corresponding spectral value is found in the response spectra which is further multiplied with the corresponding MPF and eigenvector. To verify the MPFs, the responses are compared to responses from ADINA. For the verification, a simply supported beam is analysed. The load is applied in the vertical direction, see Figure 3.5.

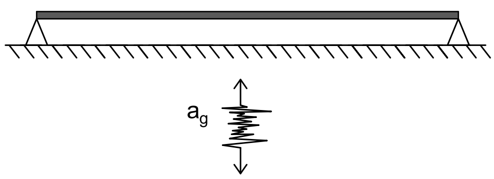
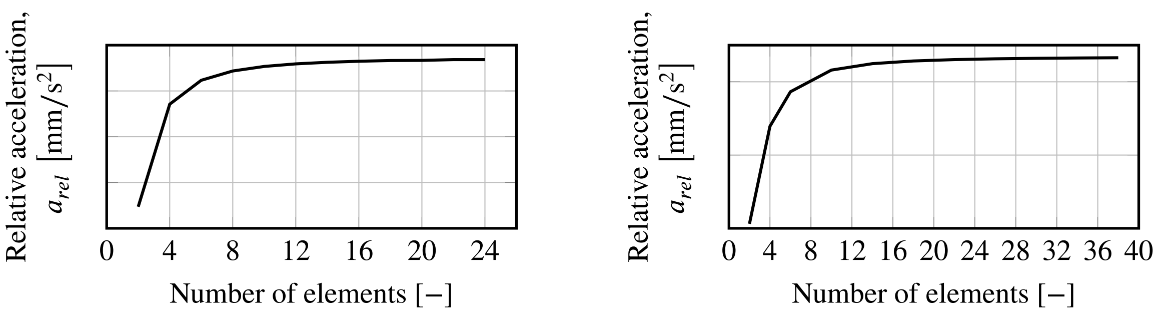


Figure 3.5: Direction of the applied ground acceleration a_g.

In both ADINA and in the VBA script, the beam is divided into an appropriate number of 2D elements, each with two nodes. Each node has three degrees of freedom: horizontal displacement, vertical displacement and rotation about the out-of-plane axis. In the VBA script, the number of non-prescribed DOFs of the structures decide how many natural frequencies that are included. For all analyses in this section, the damping ratio ζ is defined as 5%.

3.3.1 Number of elements used in the analyses

To decide an appropriate number of elements to use in the analyses, the vertical relative acceleration is calculated with both the VBA script and ADINA, for beams divided in varying number of elements, see Figure 3.6. The analysed beam has a length of 10 meters and a rectangular cross section; height 1.2 meter and width 0.6 meter. For the VBA analyses, the relative acceleration starts to converge for about 16 elements, see Figure 3.6a, which is considered to give sufficient reliability in the results and still provide low computational time. When the results from the ADINA analyses are plotted, the curve converges as well, see Figure 3.6b. 30 number of elements in the ADINA model is considered to give sufficient reliability in the results.



(a) Analysis in VBA (b) Analysis in ADINA

Figure 3.6: The impact of number of elements on relative acceleration.

3.3.2 RSA in VBA script verified with RSA in ADINA

The values of the MPFs are highly dependant on how the eigenvectors are scaled, therefore the MPFs from the VBA script can not simply be compared to the ones obtained from ADINA. However, an MPF multiplied with the corresponding eigenvector will give the same result independent of how the eigenvector is scaled. Since the MPF and the corresponding eigenvector should be multiplied with the spectral response, as described in Equation (2.48), the total response is also independent of how the eigenvectors are scaled. Therefore, the total response from the VBA script is verified with the total response from ADINA, instead of comparing the MPF values. In both programs, response spectra are used to obtain the spectral values, which are multiplied with the MPFs to get the total response.

Two simply supported beams with different lengths are analysed, the properties are described in Table 3.2. The applied ground motion is the one in Figure 3.4a in Section 3.2.3. In both analyses, the damping ratio is 5% and the total response (sum of relative response and ground motion) in the mid node is analysed, see Table 3.3 for the results for Beam 1 and Beam 2. As can be observed, the results from the VBA script and ADINA differ with less than 2%, the VBA script is therefore considered reliable for response spectrum analysis (RSA).

Table 3.2: Beam properties for verification with RSA.

Beam	Length [m]	Width [m]	Height [m]	Young's modulus [GPa]	Density [kg/m^3]
1	10	0.25	0.5	30	2500
2	15	0.25	0.5	30	2500

Table 3.3: Comparison of results from ADINA and VBA.

	Beam 1			Beam 2		
	VBA	**ADINA**	**Error**	**VBA**	**ADINA**	**Error**
Total acceleration, a_{tot} [mm/s^2]	0.02553	0.02595	-1.6%	0.04967	0.04995	-0.6%
Total velocity, v_{tot} [mm/s]	0.00138	0.00140	-1.3%	0.00257	0.00259	-0.6%
Total displacement, d_{tot} [mm]	0.00761	0.00773	-1.6%	0.00761	0.00774	-1.6%
First mode frequency, f [Hz]	7.854	7.846	0.1%	3.491	3.489	0.1%

3.3.3 RSA in VBA script verified with THA in ADINA

A number of simply supported beams with varying geometrical properties, see Table 3.4, are analysed with the VBA script using RSA. The model used in the VBA script can be seen in Figure 3.8. For all analyses, the applied ground motion is the one in Figure 3.4a in Section 3.2.3. In order to study the accuracy of the method, a time history analysis (THA) is carried out with mode superposition analysis in ADINA. A THA gives, as described in Section 2.3.1, the response of a structure as a function of time. This means that for every time step a THA gives the actual response, in comparison to an RSA where only the peak values for the natural frequencies of a structure are obtained.

Table 3.4: Properties of the analysed beams, see Figure 3.7 for definitions.

Analysis	Length [m]	Height [m]	Width [m]	Density [kg/m^3]	Young's modulus [GPa]	First mode frequency [Hz]
1	25	0.50	0.25	2500	30	1.26
2	20	0.50	0.25	2500	30	1.96
3	10	0.20	0.10	2500	30	3.14
4	15	0.50	0.25	2500	30	3.49
5	10	0.80	0.40	2500	30	12.57
6	10	1.20	0.60	2500	30	18.85
7	10	1.60	0.80	2500	30	25.13
8	5	0.50	0.25	2500	30	31.41

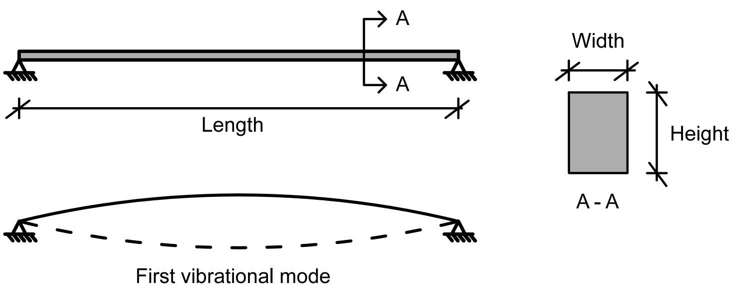


Figure 3.7: Definition of properties in Table 3.4.

To estimate the response of a structure with RSA, the maximum response for every mode in all DOFs is combined using a modal combination rule, described in Section 2.3.4. For the analyses presented in this chapter, the modal combination rule SRSS is used explicitly, the ABSSUM rule is considered too conservative and the CQC rule is, according to Chopra, 2007, superfluous for simple structures. These responses are compared to the absolute maximum responses in the corresponding DOF from a THA, computed with ADINA. Since errors are induced when using a response spectra, along with the approximation when using modal combination rules, the two types of analyses will never give exactly the same results (Chopra, 2007). The errors between the analyses depends on many factors, e.g. the frequency range of the response spectra and the number of modes included, and which method is most conservative may vary from case to case.

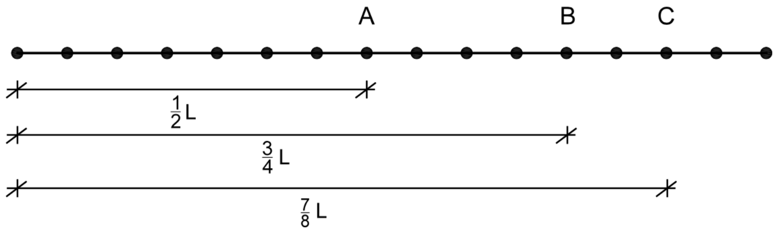
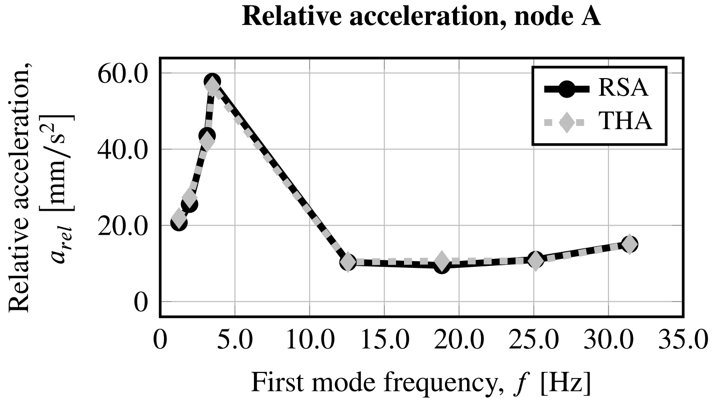


Figure 3.8: The beam model, showing node A, B and C.

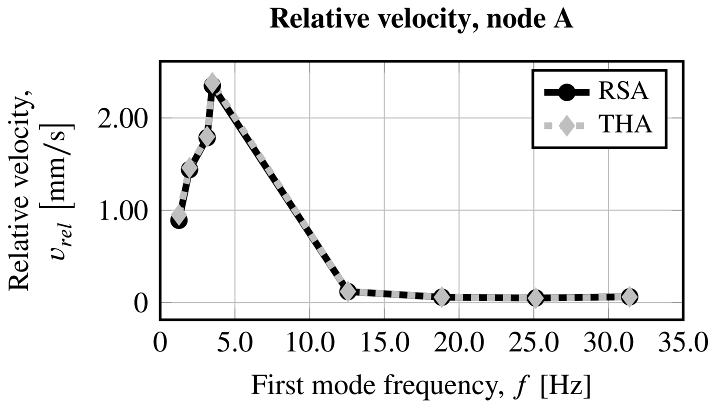
3.3.3.1 Relative response

In the first analyses, the ground motion is not included in the response. The relative response obtained from the VBA script is compared to the relative response from a THA in ADINA. In this way, the actual experienced response is not obtained, since it requires that the ground motion is included as well, but it is possible to study the accuracy of the RSA. As mentioned, the properties of the analysed structures are described in Table 3.4. For each analysis, the vertical relative response in node A, see Figure 3.8, is analysed. The results from the RSA (from the VBA script) and the THA (from ADINA) are shown in Figure 3.9, 3.10 and 3.11, for acceleration, velocity and displacement, respectively. The error term in the tables represents the difference in the results in relation to the THA. The graphs are plotted as a function of the first mode frequency of the beams in Table 3.4, and hence each point represents one analysis.



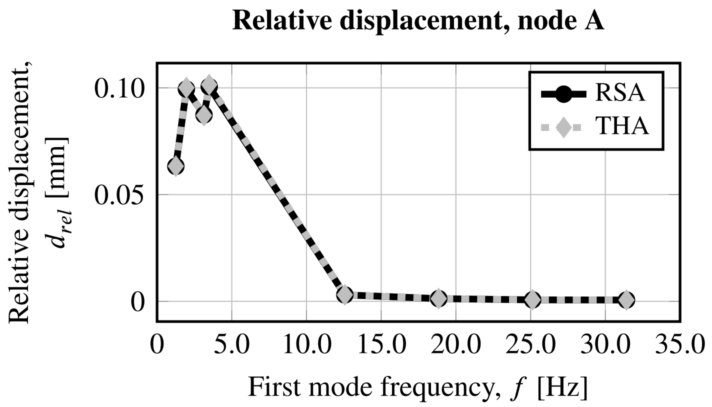
	a_{rel} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	20.73	21.90	-5.4%
2	25.56	27.28	-2.7%
3	43.52	41.98	3.8%
4	57.74	56.44	2.3%
5	10.31	10.40	-0.8%
6	9.48	10.64	-10.9%
7	11.01	10.64	3.5%
8	15.05	15.04	0.1%

Figure 3.9: Comparison of maximum relative acceleration from RSA and THA, node A.



	v_{rel} [mm/s]		
Analysis	**RSA**	**THA**	**Error**
1	0.89	0.95	-6.2%
2	1.44	1.46	-1.3%
3	1.79	1.80	-0.6%
4	2.35	2.38	-1.3%
5	0.119	0.120	-0.6%
6	0.0579	0.0580	-0.2%
7	0.047	0.049	5.0%
8	0.060	0.063	3.5%

Figure 3.10: Comparison of maximum relative velocity from RSA and THA, node A.



	d_{rel} [mm]		
Analysis	**RSA**	**THA**	**Error**
1	0.06325	0.06331	-0.1%
2	0.099	0.100	-0.7%
3	0.0873	0.0870	0.3%
4	0.100	0.102	-0.9%
5	0.00302	0.00304	-0.5%
6	0.00125	0.00127	-1.5%
7	0.00069	0.00070	-2.0%
8	0.00063	0.00061	-0.5%

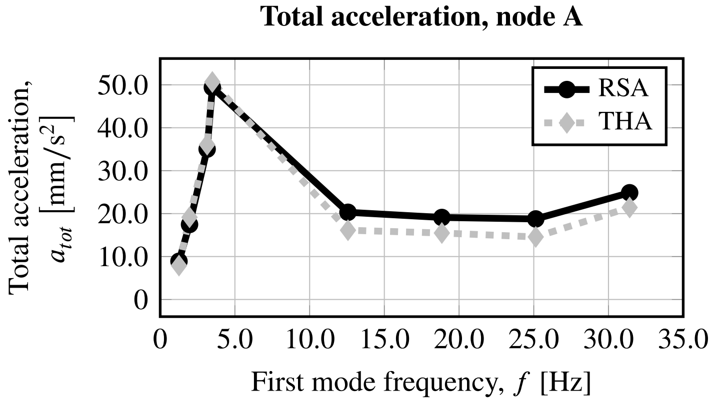
Figure 3.11: Comparison of maximum relative displacement from RSA and THA, node A.

As can be observed, the difference between the RSA and the THA results are less than 11% for all analyses. It is not possible to see any correlation between the size of the error term and the corresponding beam stiffness (first mode frequency), but since the error term is relatively low, the RSA results are considered reliable for the relative response.

3.3.3.2 Studies on how to include ground motion

Solely having the relative response is not sufficient to estimate the vibrational comfort experience of a structure, see Section 2.1.2.1. In addition, the applied ground motion need to be added to get the total response. A question that arises with this is how to include the ground motion in a correct manner. For these studies, only the total acceleration is analysed, since the applied load is an acceleration, see Section 3.2.1. It is assumed that the studied methods are equally applicable to get the total velocity (if the applied load is a velocity instead).

In a first study, the ground motion is included in the response spectrum, as described in Figure 2.15. The ground motion is added to the relative response in every time step for the desired frequency range, giving a total response, and the maximum values are plotted in a response spectrum. As described in Section 2.3.3, the response for each natural frequency of the analysed structure is found and multiplied with the corresponding MPF and eigenvector. However, this induces an error since the total response is multiplied with the MPF, which is only supposed to scale the relative response. As previously described, the MPFs translate the response of an SDOF system to each mode of an MDOF system, but since the ground motion is independent of the modes, it is reasonable to believe that it should not be scaled in the same way as the relative response. In order to study how this influences the results, the structures described in Table 3.4 are analysed. The vertical acceleration of the mid node (node A) is compared and the results are shown in Figure 3.12.



	a_{tot} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	8.89	7.90	12.5%
2	17.47	19.15	-8.8%
3	35.02	36.06	-2.9%
4	49.39	50.66	-2.5%
5	20.31	16.13	25.9%
6	19.10	15.48	23.4%
7	18.76	14.57	28.8%
8	24.86	21.43	16.0%

Figure 3.12: Comparison of maximum total acceleration from RSA and THA, node A.

As could be expected, the results from the RSA are not as close to the results from the THA as for the relative response. The largest error is almost 30%. The explanation to why some of the RSA results are considerably larger than the THA results for the total response is assumed to be that the ground motion is included in the response spectrum, and hence in the modal combination. For some frequencies, the ground motion term is considerably larger than the relative response term and hence has large influence on the total response. It can therefore be assumed that if structures in these frequency ranges are analysed, the difference between the results from the RSA and THA may be significant.

In order to try reducing the error term, a second study is performed. Since the response spectrum analyses are resulting in conservative values for the larger error terms, it is concluded that the responses from these analyses should be lower. In this study, only the maximum relative response for each mode is combined with the SRSS rule and the contribution from the ground motion is added for the first mode only, since this is the most important mode for the response of the mid node, see Section 2.1.1. The equation that is used to calculate the maximum acceleration in each DOF i can be described as:

$$a_{max.i} = \sqrt{a_{i.1}{}^2 + a_{i.2}{}^2 + ... + a_{i.n}{}^2} + \Delta R_1 \tag{3.3}$$

where ΔR_1 represents the difference between the total response and the relative response of the first natural frequency. As an example, the definition of ΔR_1 is graphically explained for Analysis 1 in Figure 3.13 for the natural frequency 1.26 Hz of the first mode.

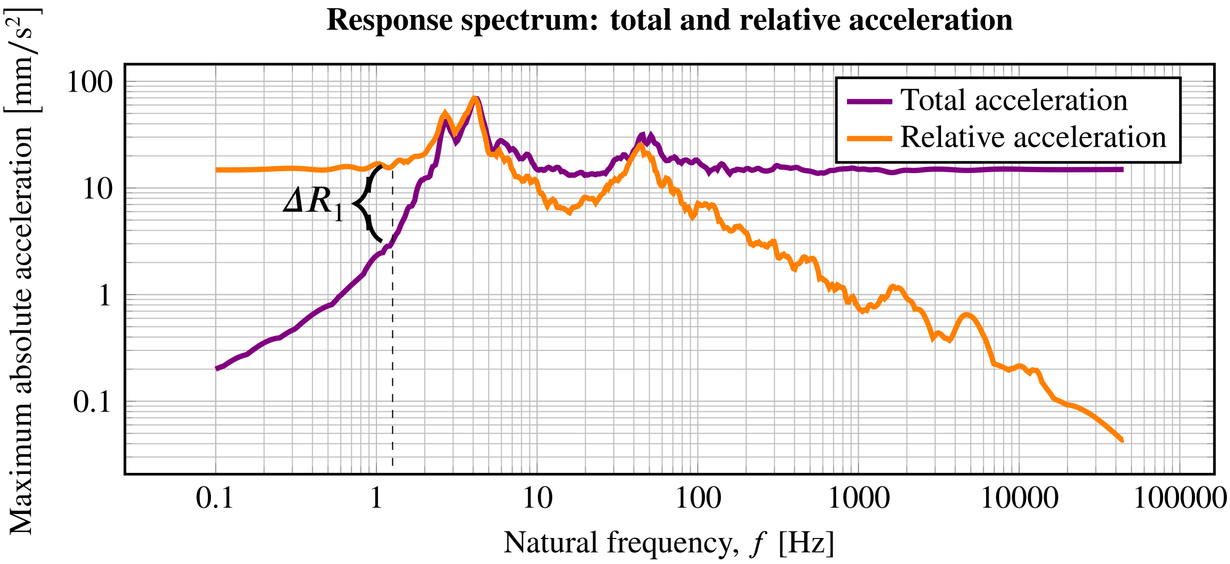
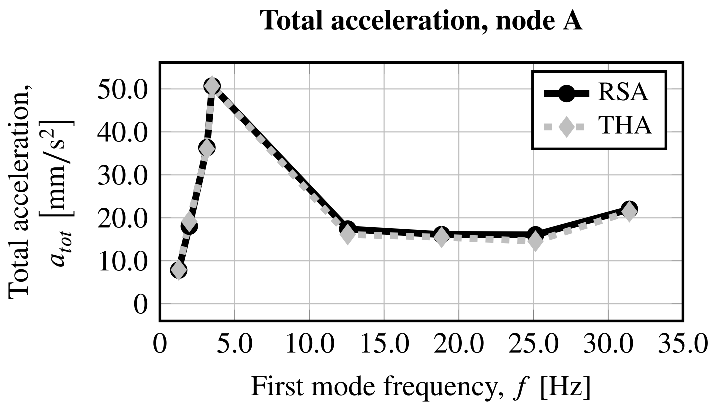


Figure 3.13: Acceleration response spectrum, created in the VBA script.

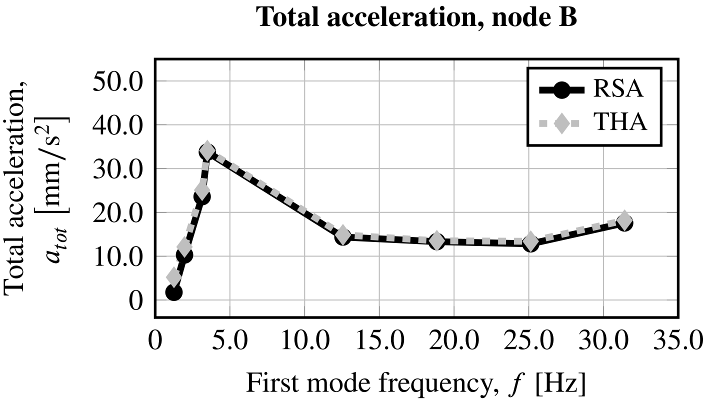
The beams in Table 3.4 are analysed with this method and compared to the same time history analyses from ADINA as in Figure 3.12. Like previously, the total vertical acceleration in the mid node is compared and the results are shown in Figure 3.14.



	a_{tot} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	7.83	7.90	-0.8%
2	18.06	19.15	-5.7%
3	36.34	36.06	0.8%
4	50.68	50.66	0.1%
5	17.39	16.13	7.8%
6	16.10	15.48	4.0%
7	16.05	14.57	10.2%
8	21.99	21.43	2.6%

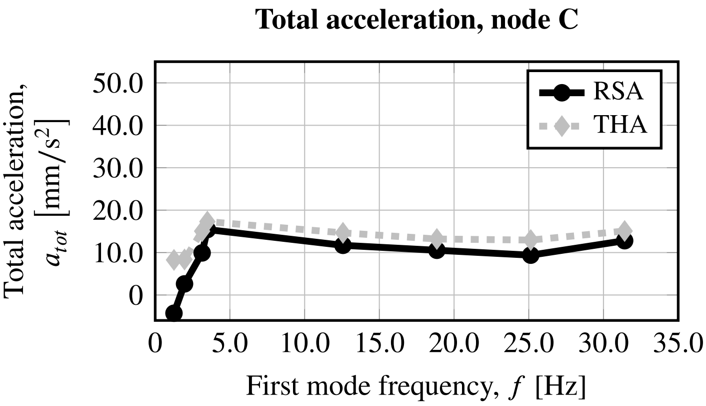
Figure 3.14: Maximum acceleration calculated with Equation (3.3) and maximum acceleration from THA.

The errors, for this second study, are much smaller compared to the analyses in Figure 3.12, but to be sure it is not a coincidence, further studies are performed. Since the contribution from the ground motion is only included for the first mode, the response in a different node (where the first mode necessarily is not decisive) need to be checked. Therefore, the same analyses are performed for node B and C (see Figure 3.8). The results are presented in Figure 3.15 and 3.16. As can be seen in the tables, there are significant differences in the results between the RSA and the THA for some of the analyses. A possible explanation for the large error values could be that for these nodes a ΔR corresponding to one of the other modes should be used instead. As an example, Table 3.5 shows the ΔR values for the five first modes of Analysis 1. As can be seen, the error is smaller if ΔR_4 is added instead of ΔR_1, even though adding ΔR_4 also gives a large error. As a matter of fact, the frequency and mode that gives ΔR_4 is the one generating the third mode, explained in Section 2.1.1, which is the one that most likely will give the largest vertical displacement in node C.



	a_{tot} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	1.76	5.19	-66.1%
2	10.28	12.11	-15.1%
3	23.59	25.08	-5.9%
4	33.77	34.10	-1.0%
5	14.37	14.87	-3.3%
6	13.32	13.52	-1.4%
7	12.83	13.40	-4.3%
8	17.59	18.19	-3.3%

Figure 3.15: Maximum acceleration calculated with Equation (3.3) and maximum acceleration from THA.



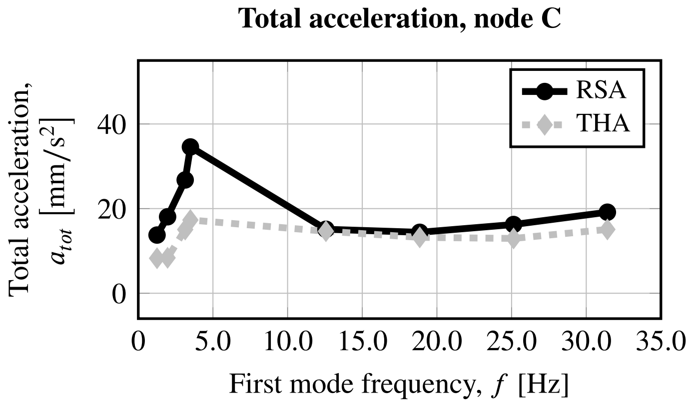
	a_{tot} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	-4.30	8.27	-151.9%
2	2.62	8.37	-68.7%
3	9.92	15.05	-34.1%
4	15.43	17.30	-10.8%
5	11.71	14.65	-20.1%
6	10.54	13.20	-20.2%
7	9.39	12.95	-27.6%
8	12.80	15.08	-15.2%

Figure 3.16: Maximum acceleration calculated with Equation (3.3) and maximum acceleration from THA.

Table 3.5: Comparison of adding different ΔR from Analysis 1 to Equation (3.3).

Mode	ΔR **[mm/s^2]**	**Relative response [mm/s^2]**	**Relative response + ΔR [mm/s^2]**	**Total response from THA [mm/s^2]**	**Error to THA**
1	-12.90	8.60	-4.30	8.27	-152%
2	5.49	8.60	14.09	8.27	70.4%
3	7.65	8.60	16.25	8.27	96.5%
4	5.15	8.60	13.75	8.27	66.3%
5	6.97	8.60	15.57	8.27	88.3%

Further analyses are therefore done for node C, where ΔR_4 is added instead of ΔR_1 in Equation (3.3). The results can be seen in Figure 3.17. It is clear that this method is unreliable for some nodes and it is therefore not further used. Because of this, the method in the first study, where the ground motion is included in every time step, is considered most reliable and is used in further analyses in this report, even though the RSA results and the THA results differ with as much as 30%. However, finding a different method to include the ground motion, and hence reduce the error with respect to the time history analyses, is recommended for future studies.



	a_{tot} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	13.75	8.27	65.8%
2	18.07	8.37	115.9%
3	26.77	15.05	77.9%
4	34.57	17.30	99.8%
5	15.12	14.65	3.2%
6	14.40	13.20	9.1%
7	16.24	12.95	25.4%
8	19.15	15.08	27.0%

Figure 3.17: Maximum acceleration calculated with ΔR_4 instead of ΔR_1 in Equation (3.3).

3.3.3.3 Total response

To further investigate the RSA method where the ground motion is included in every time step, additional simply supported beams are studied, described in Table 3.6. Together with the previously analysed beams from Table 3.4, a wide range of stiffness is covered, the first natural frequency of the beams span from 0.64 Hz to 358 Hz. Additionally, the vertical acceleration in node B and C in Figure 3.8 is analysed. The results for all 16 analyses for node A, B and C are shown in Figure 3.18, 3.19 and 3.20, respectively.

Table 3.6: Properties of the additional analysed beams.

Analysis	**Length [m]**	**Height [m]**	**Width [m]**	**First mode frequency [Hz]**
9	35	0.50	0.25	0.642
10	28	0.50	0.25	1.00
11	17	0.50	0.25	2.72
12	10	0.50	0.4	7.85
13	10	0.65	0.50	10.2
14	6	1.00	1.00	43.1
15	4	1.00	1.00	95.7
16	2	1.00	1.00	358

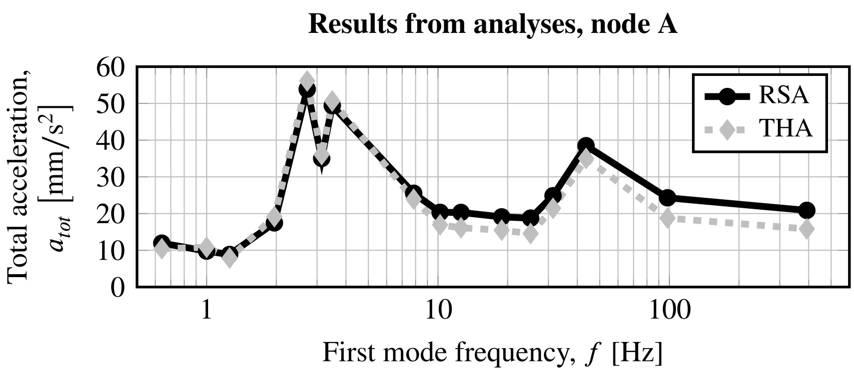


Figure 3.18: Total acceleration in node A for beams in Table 3.4 and Table 3.6. Data from Table B.1.

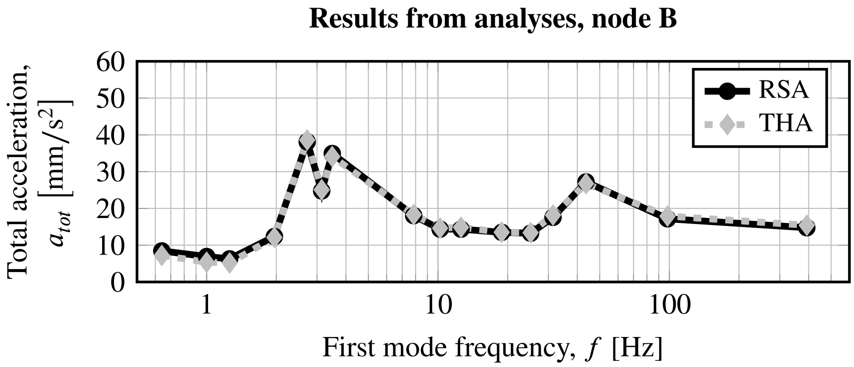


Figure 3.19: Total acceleration in node B for beams in Table 3.4 and Table 3.6. Data from Table B.2.

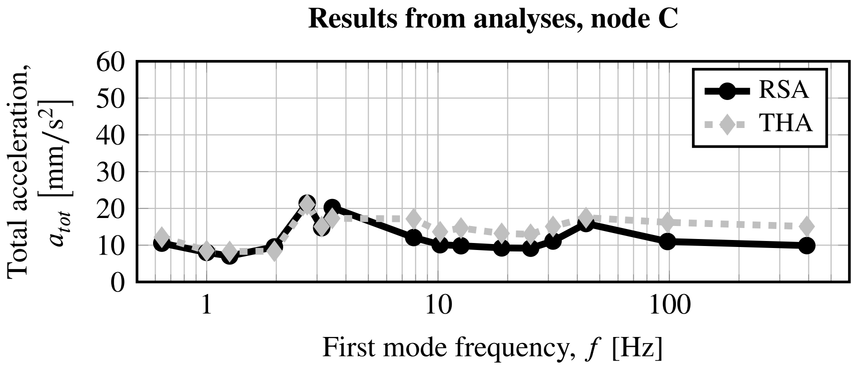


Figure 3.20: Total acceleration in node C for beams in Table 3.4 and Table 3.6. Data from Table B.3.

The largest error value is about 30% for both node A and B and about −30% for node C, see Table B.1, B.2 and B.3 in Appendix B. Even though the response spectrum analyses are not resulting in conservative values for all analyses, the errors obtained are considered to be within an accepted limit for the purpose of this simplified method. To investigate the origin of the errors, the MPF value multiplied with the corresponding eigenvalue for node A, B and C for all 16 analyses is studied, see Table 3.7.

Table 3.7: MPF multiplied with corresponding eigenvalue.

Analysis	**Node A,** $\sqrt{\sum_{i=1}^{n} \Gamma_i \phi_{i.A}}$	**Node B,** $\sqrt{\sum_{i=1}^{n} \Gamma_i \phi_{i.B}}$	**Node C,** $\sqrt{\sum_{i=1}^{n} \Gamma_i \phi_{i.C}}$
1-16	1.36	0.96	0.65

The product for each mode is combined using the SRSS modal combination rule. As mentioned in Section 3.3.3.2, this factor induces an error when multiplied with the total response, since it scales not only the relative response but also the ground motion. From Table 3.7 it is clear that the product of the MPF and the eigenvalue remains the same for all analyses, but it varies for different nodes. It is reasonable to believe that the size of the product is related to the size of the error term between the RSA and the THA results. For example, for node A the product is 1.36, which means the spectral values are multiplied with a factor 1.36, and hence the ground motion contribution is about 36% larger than the actual ground motion. This could explain the largest error term of about 30% between the RSA and THA results for node A, see Figure 3.18. In Table 3.7 it can also be observed that for node B the MPF and eigenvalue product is almost equal to 1 and by looking at Figure 3.19 it is clear that the difference between the RSA and THA results is very small for most frequencies. Lastly, the product for node C is about 0.65 and by looking at Figure 3.20 it can be observed that the largest error term is about −30%, which seems reasonable since the ground motion contribution is about 35% smaller than the actual ground motion.

3.4 Implementation of comfort weighting

As mentioned in Section 3.1, the output values from the VBA script can be compared to response values in current regulations, to evaluate whether the response of a building is within the limits or not. The values in the regulations are root mean square (RMS) values and weighted with frequency weighting factors for comfort, as described in Section 2.1.3. Therefore, the values from the VBA script need to be RMS values and comfort weighted as well, to get values comparable to the regulations. No established method to implement this on response spectrum analyses has been found in literature, and therefore studies are made to find a reliable approach. Two different approaches are studied and compared to each other and to the unweighted response. For both approaches, the damping ratio ζ is defined as 5%.

For the studies in this section, the results from the VBA script are compared to comfort weighted responses from time history analyses. These THA results are comfort weighted using the same principles as described in Section 3.4.2. It should be noted, however, that these values are not confirmed as correctly comfort weighted responses. Instead, the purpose is to illustrate the difference between the RSA and the THA results. For the comparisons, the THA results are considered as the correct responses and the error values in the tables are calculated with regard to them.

3.4.1 Approach 1: weighting factors corresponding to the natural frequencies

As always when designing a response spectrum, the time dependent response for each studied natural frequency is found. To implement RMS on the response spectrum, the time dependent response for each natural frequency is divided into a number of so called *windows*, see the top horizontal axis in Figure 3.21, where each window is defined as one second long, no overlapping. The response values in each window, $r_{f.i.1}$, $r_{f.i.2}$, ... , $r_{f.i.n}$, are inserted in Equation (3.4), which gives a positive mean value of all values in each window, $r_{rms.f.i}$. The reponse values can be either velocities or accelerations.

$$r_{rms.f.i} = \sqrt{\frac{1}{n}({r_{f.i.1}}^2 + {r_{f.i.2}}^2 + ... + {r_{f.i.n}}^2)} \tag{3.4}$$

where $r_{rms.f.i}$ represents the RMS value of the response for a specific SDOF system with the natural frequency f and for each window, i. $r_{f.i.n}$ is the n^{th} response value in the i^{th} window. An example is shown in Figure 3.21, where the dotted line shows the RMS values of the black curve in each window.

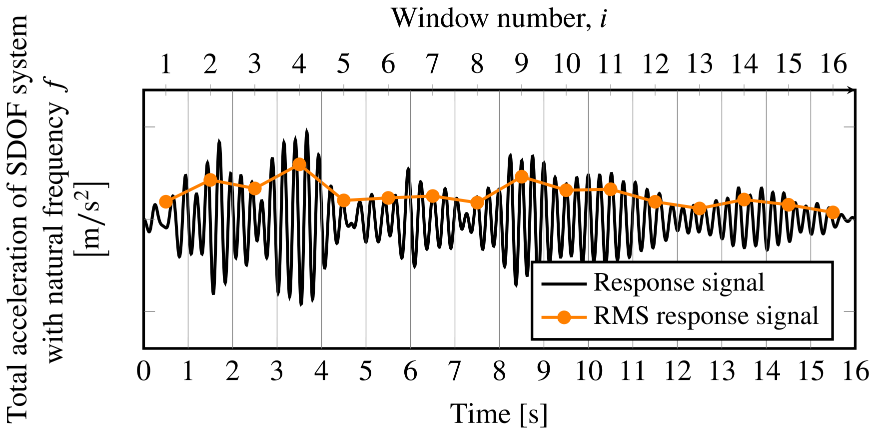


Figure 3.21: The response signal and the corresponding RMS response signal.

The RMS values can be collected in a vector as:

$$\{r_{rms.f}\} = \begin{bmatrix} r_{rms.f.1} \\ r_{rms.f.2} \\ \dots \\ r_{rms.f.n} \end{bmatrix} \tag{3.5}$$

where $\{r_{rms.f}\}$ represent a vector with all the RMS values of the response, for an SDOF system with the natural frequency f. These are shown as the dotted line in Figure 3.21.

All the values of $\{r_{rms.f}\}$ are multiplied with a frequency weighting factor, $W_{m.f}$, see Equation (3.6).

$$\{r_{rms.f}\} \cdot W_{m.f} = \begin{bmatrix} r_{rms.f.1} \cdot W_{m.f} \\ r_{rms.f.2} \cdot W_{m.f} \\ \dots \\ r_{rms.f.n} \cdot W_{m.f} \end{bmatrix} \tag{3.6}$$

The weighting factors correspond to the natural frequency f, see Figure 2.8 in Section 2.1.3. For this first approach, the weighting factors are assumed to correspond to the natural frequencies of the SDOF system that generates the response, and hence all values in the response signal are multiplied with the same weighting factor, even though the total response in fact consist of several frequencies. The reason for this is because the ground motion is added (total response) and that the system is excited with a forced vibration. This approach is therefore regarded as a simple way of including the comfort weighting factors and hence the accuracy need to be checked.

The maximum value of Equation (3.6) is plotted in the response spectrum and the procedure is repeated for every natural frequency to be plotted. A flowchart of the principles of Approach 1 can be seen in Figure 3.22.

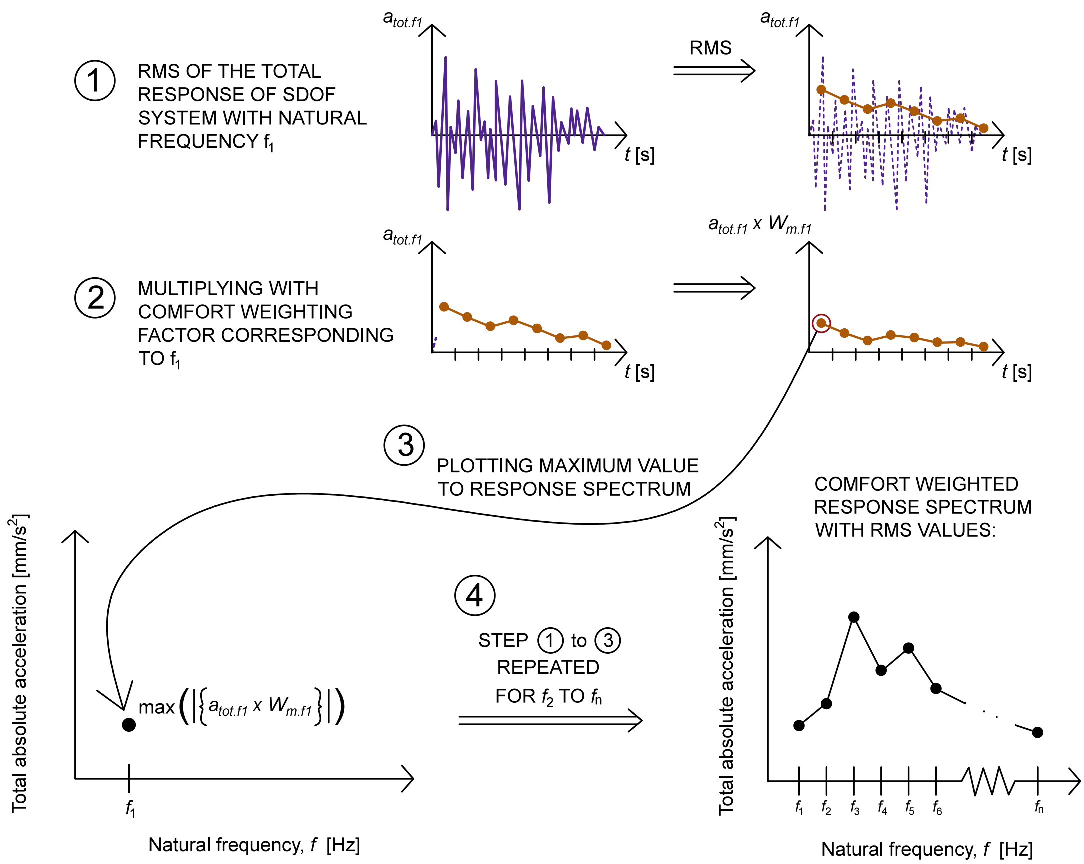


Figure 3.22: Flowchart of the principles when comfort weighting with Approach 1.

The same load as described in Figure 3.4a in Section 3.2.3 is analysed with this approach and the generated response spectrum can be seen in Figure 3.23, showing both the unweighted total acceleration and the comfort weighted RMS acceleration. As can be seen, the comfort weighted curve differ significantly from the unweighted curve for higher frequencies. This is reasonable since the weighting factors for accelerations are lower for high frequencies, see Figure 2.8 in Section 2.1.3.

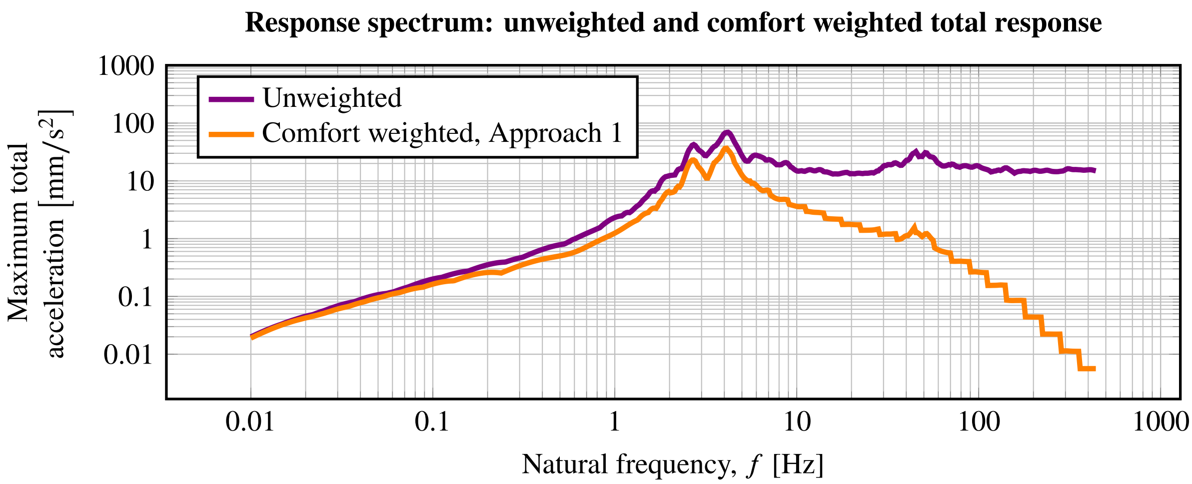
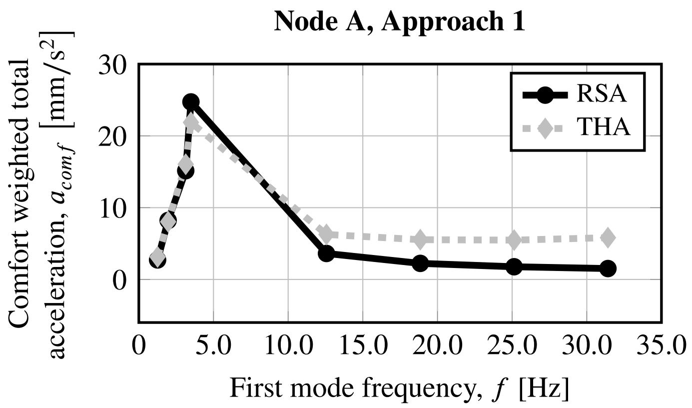


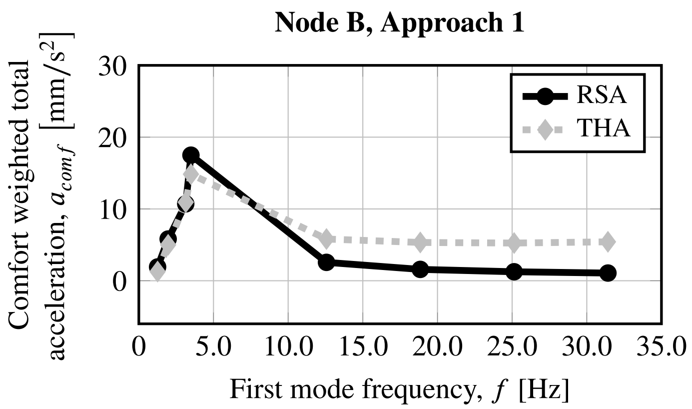
Figure 3.23: Acceleration response spectrum, created in the VBA script.

The beams from Table 3.4 are studied to check the accuracy of the approach. The results can be seen in Figure 3.24, 3.25 and 3.26, where also comfort weighted values obtained with THA in ADINA can be seen.



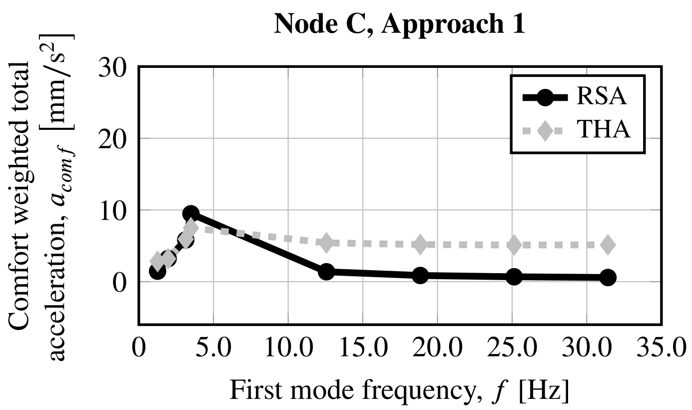
	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	2.72	3.14	-13.3%
2	8.22	8.04	2.3%
3	15.25	16.05	-5.6%
4	24.73	21.87	13.1%
5	3.62	6.27	-42.3%
6	2.24	5.54	-59.1%
7	1.77	5.48	-67.6%
8	1.52	5.82	-73.9%

Figure 3.24: Comparison of comfort weighted total acceleration, node A.



	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	1.92	1.28	50.8%
2	5.81	5.00	16.2%
3	10.71	10.94	-2.1%
4	17.49	14.84	17.8%
5	2.56	5.81	-56.0%
6	1.58	5.33	-70.3%
7	1.25	5.25	-76.1%
8	1.07	5.42	-80.2%

Figure 3.25: Comparison of comfort weighted total acceleration, node B.



	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	1.46	2.82	-48.2%
2	3.22	3.19	1.1%
3	5.82	5.93	-2.0%
4	9.47	7.50	26.3%
5	1.38	5.41	-74.4%
6	0.86	5.18	-83.5%
7	0.68	5.11	-86.7%
8	0.58	5.14	-88.7%

Figure 3.26: Comparison of comfort weighted total acceleration, node C.

3.4.2 Approach 2: FFT of responses

For Approach 1, the weighting factors are assumed to correspond to the natural frequencies of the SDOF systems that gives the response. As mentioned, this induces an error since the responses in fact consist of several frequencies. In order to get a more accurate response value, multiple weighting factors are used for each response. To do this, the SDOF responses need to be analysed with regard to which frequencies they consist of, in order to be able to multiply the response values with the correct weighting factors. This is done with a fast Fourier transformation (FFT), which converts the response signal from the time domain to the frequency domain, see (1) in Figure 3.27.

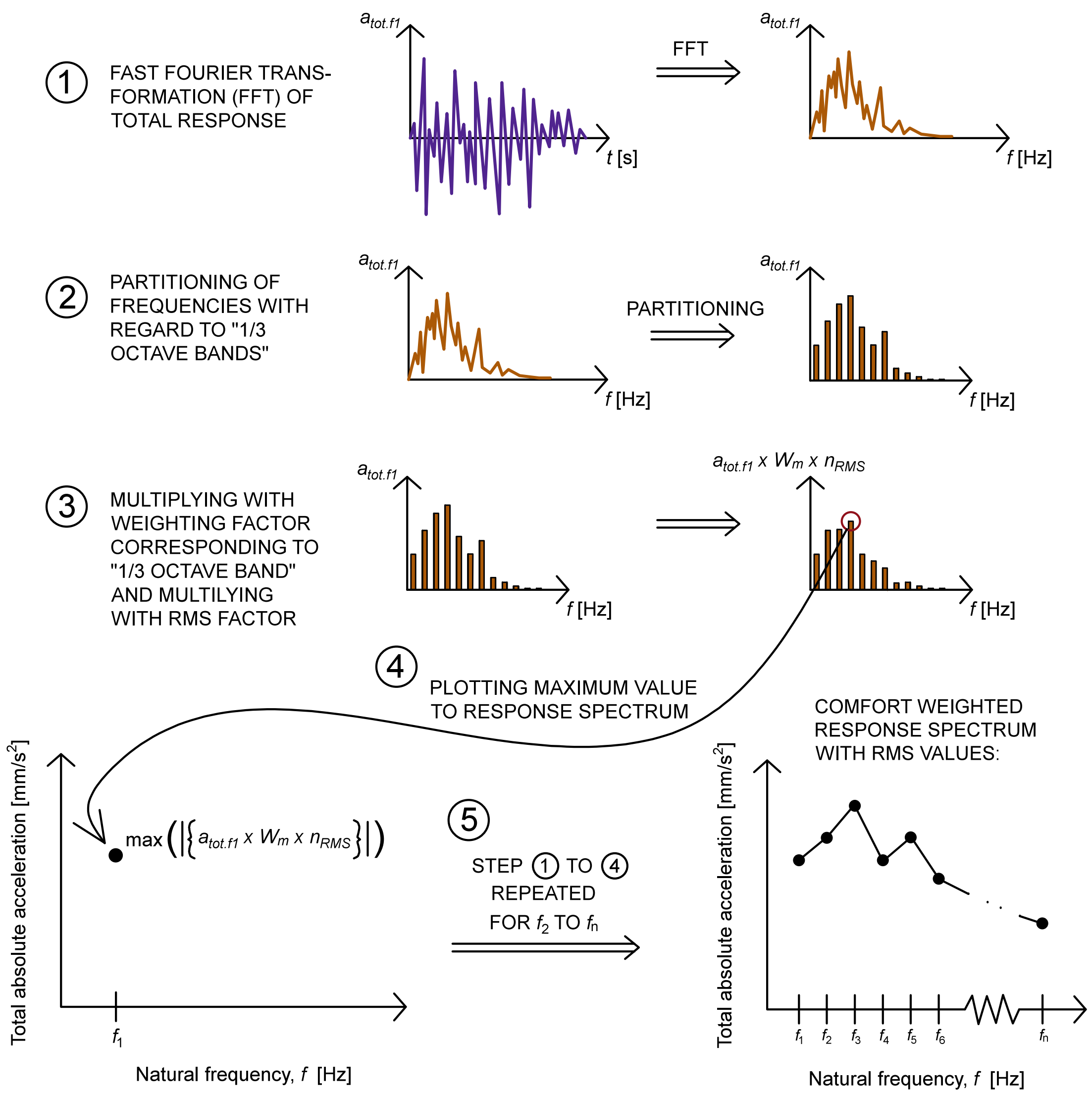


Figure 3.27: Flowchart of the principles when comfort weighting with Approach 2.

Further, this converted response is analysed in a calculation sheet created by *Sound & Vibrations* at *ÅF-Infrastructure AB*. The calculation sheet uses techniques for comfort weighting and RMS, as described in Section 2.1.3. A description of the calculation sheet can be found in Appendix C. The calculation sheet partitions the response signal (see (2) in Figure 3.27) from the FFT with regard to so called *one-third-octave bands*, see Table 2.2 in Section 2.1.3.2. This partitioning is commonly used by acousticians and is a method of dividing a frequency range into sub-ranges. The responses are multiplied with the corresponding weighting factor and an RMS factor, see (3). The RMS signal is calculated within the VBA script with the same method used in Approach 1. A ratio between the RMS signal and the original response signal is calculated and this factor is used in the calculation sheet as the RMS factor. In (4), the maximum value is plotted in the response spectrum and the procedure is repeated for all frequencies of interest (5). This approach is assumed to result in a more correctly comfort weighted response than Approach 1, however it is more computational heavy. Response spectrum created with Approach 1 and Approach 2 is shown in Figure 3.28, together with the unweighted response. Due to the longer computational time that Approach 2 requires, values for only eight randomly picked natural frequencies (2.32 Hz, 3.45 Hz, 4.85 Hz, 6.01 Hz, 7.85 Hz, 70.7 Hz, 196 Hz and 386 Hz) are plotted.

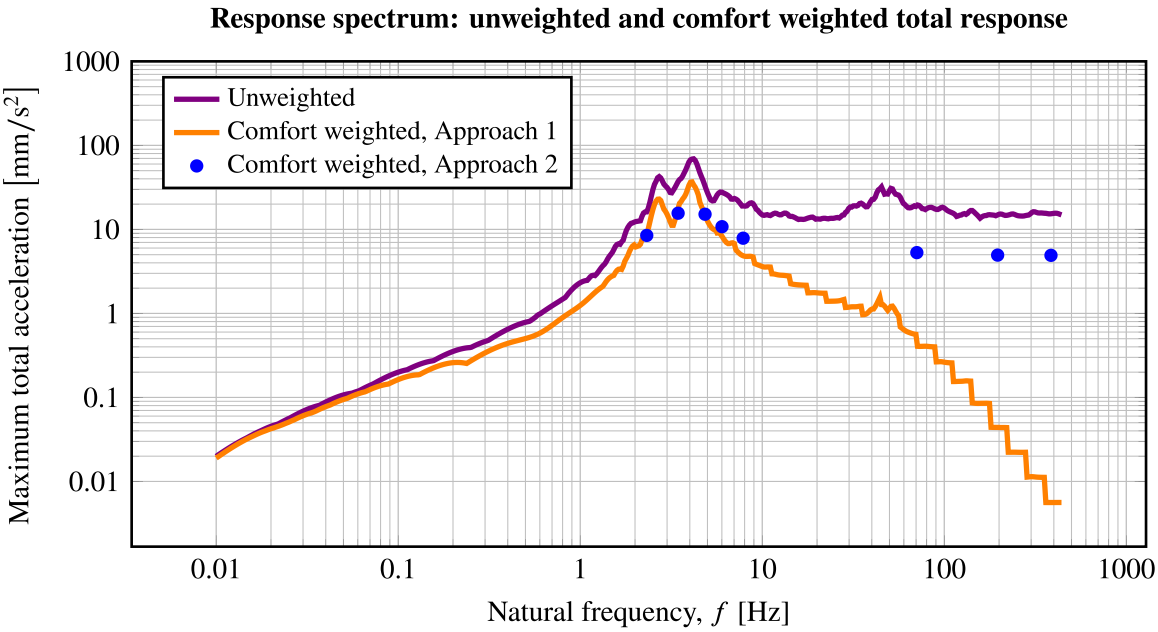
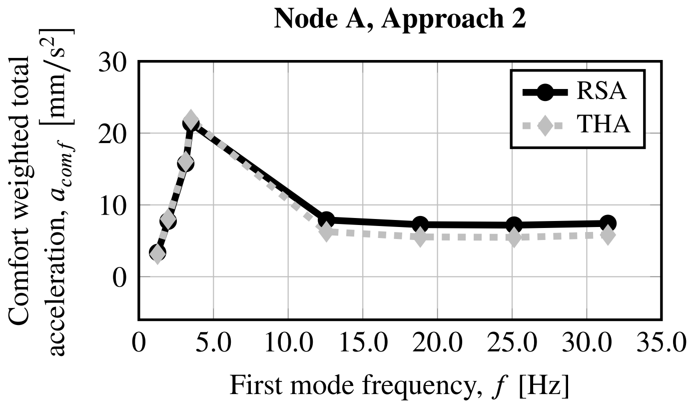


Figure 3.28: Acceleration response spectrum, created in the VBA script.

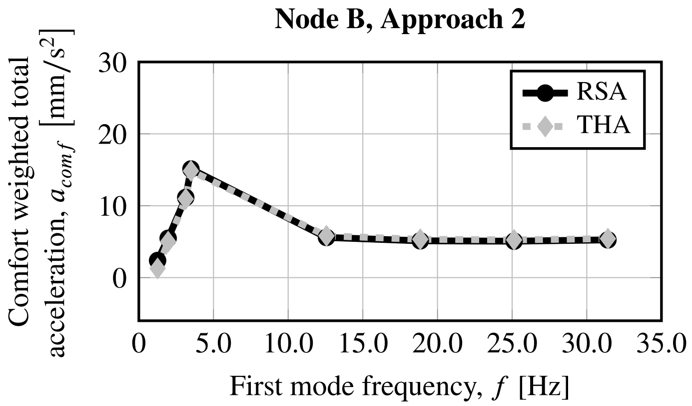
It is obvious that Approach 1 and 2 does not give the same results for higher natural frequencies. One explanation to this, for this specific ground motion, may be that for higher frequencies the ground motion dominates the total response, rather than the relative response of the SDOF systems. This means that the natural frequencies of the SDOF systems does not always dominate the contents of the frequency domains of the responses. Hence, the weighting factors corresponding to the natural frequencies can not simply be used for every response value, as in Approach 1. Since Approach 2 analyse exactly which frequencies the responses consist of and match them with the corresponding weighting factors, it is considered most reliable and is assumed to comfort weight the responses in a correct way. Due to the large difference between the approaches, Approach 1 is considered unusable. Instead, Approach 2 is implemented in the final VBA script to be able to analyse structures with various parameters with regard to comfort weighting.

The beams from Table 3.4 are studied and the same ground motion as in Figure 3.4a in Section 3.2.3 is used for the analyses. The results are compared to comfort weighted values, obtained with THA in ADINA, see Figure 3.29, 3.30 and 3.31.



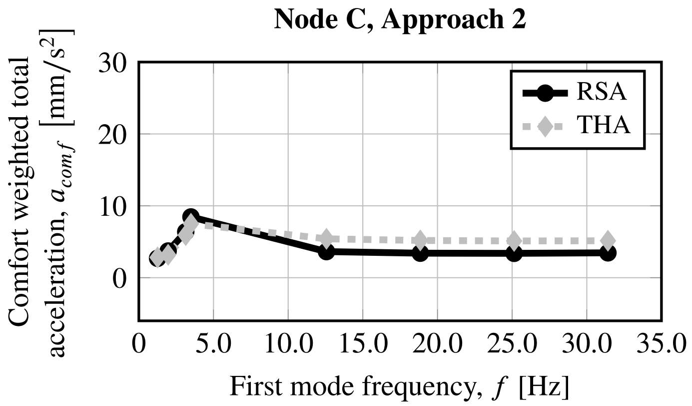
	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	3.37	3.14	7.3%
2	7.74	8.04	-3.7%
3	15.77	16.05	-1.7%
4	21.38	21.87	-2.3%
5	7.90	6.27	26.1%
6	7.25	5.54	30.9%
7	7.18	5.48	31.0%
8	7.42	5.82	27.4%

Figure 3.29: Comparison of comfort weighted total acceleration, node A.



	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	2.38	1.28	86.6%
2	5.47	5.00	9.4%
3	11.15	10.94	1.9%
4	15.11	14.84	1.8%
5	5.59	5.81	-3.8%
6	5.12	5.33	-3.8%
7	5.07	5.25	-3.4%
8	5.25	5.42	-3.2%

Figure 3.30: Comparison of comfort weighted total acceleration, node B.



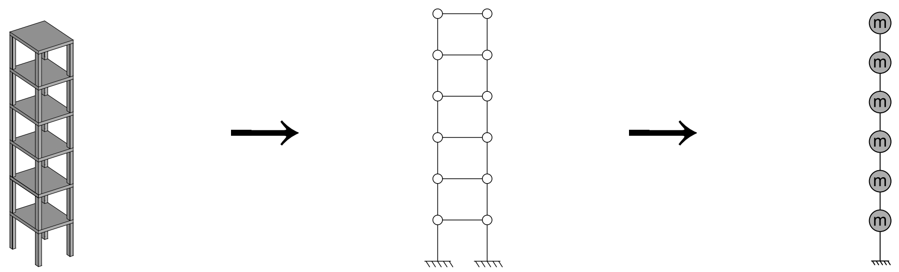
	a_{comf} [mm/s^2]		
Analysis	**RSA**	**THA**	**Error**
1	2.70	2.82	-4.3%
2	3.71	3.19	16.5%
3	6.39	5.93	7.8%
4	8.45	7.50	12.7%
5	3.63	5.41	-33.0%
6	3.41	5.18	-34%
7	3.39	5.11	-33.7%
8	3.46	5.14	-32,7%

Figure 3.31: Comparison of comfort weighted total acceleration, node C.

As can be seen, the error terms are much smaller than the ones obtained with Approach 1 (see Figure 3.24, 3.25 and 3.26). Approach 2 is therefore regarded as a sufficiently reliable method to use to implement the comfort weighting factors into the VBA script.

3.5 Verification of simple building model

In order to use the VBA script to analyse not only simply supported beams but also multi-storey buildings, representative models are designed and verified with ADINA. The structural system of the buildings to be analysed for the verification consist of squared, concrete floors simply supported by concrete columns at each level, see Figure 3.32a. All columns are assumed to have a squared cross section and the dimensions are assumed to be the same along the heights of the buildings. Additionally, each floor is assumed to have the same mass. The structures can be simplified to 2D frame models, which further can be simplified to 2D columns, see Figure 3.32b and 3.32c. The model to be used in the VBA script is a 2D column, fixed at the support, with as many beam elements as levels of the structure to be analysed. Each beam element represents the stiffness and weight of the total number of columns on each level. Lumped masses are added to the nodes between the elements, see Figure 3.32c, to represent the weight of the floors, which are assumed to be rigid.



(a) Analysed building (b) 2D frame model (c) 2D column model

Figure 3.32: Analysed building and building models.

To verify the model in the VBA script, a 2D frame model, as the one in Figure 3.32b, is analysed in ADINA, which is assumed to give a more accurate representation of reality than the 2D column model. The natural frequencies of the 2D column model in the VBA script and the 2D frame model in ADINA are compared in order to see if the models are equivalent. In the ADINA model, each column represents the stiffness and weight of all underlying columns. The floors connecting the columns are given a stiffness 100 times larger than the stiffness of the columns, in order to regard them as rigid.

The equivalence of the 2D frame model in ADINA and the 2D column model in the VBA script is studied with regard to the influence of the floor spans and the influence of the number of columns per level in the models.

3.5.1 Influence of floor span

In order to study how the floor span influence the equivalence of the ADINA models and the VBA models, three analyses are performed for structures with the spans: 4, 8 and 12 meters. All three structures consist of six floors, supported by four columns at each level with a height of 3.5 meters. In the VBA script, the same model is used for all three analyses, whereas in ADINA three different models are used, see Figure 3.33. For the comparison, it is desired to use the same model in the VBA program for all three analyses. Therefore, it is assumed that the floor spans does not influence the mass of the floors, i.e. the densities of the floors in the ADINA models are different for Analysis 1, 2 and 3, see Table 3.8 where the properties of the analysed structures are described.

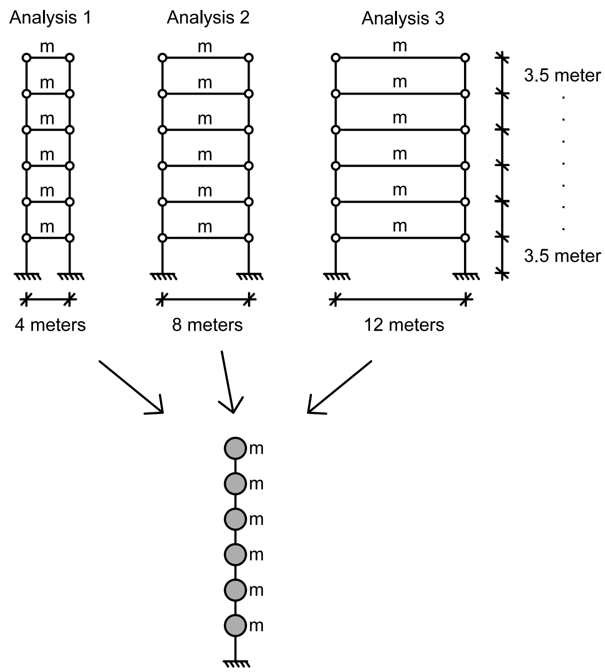
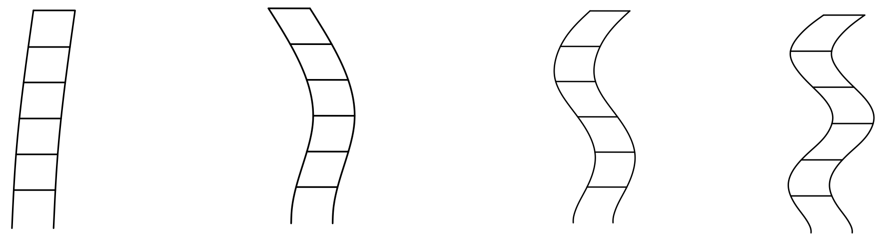


Figure 3.33: The analysed models when studying the influence of floor spans.

Table 3.8: Analysed structures, four columns on each level.

	Floors				Columns				
Analysis	**Width [m]**	**Length [m]**	**Height [m]**	**Density [kg/m^2]**	**Width [m]**	**Depth [m]**	**Height [m]**	**Density [kg/m^2]**	**Young's modulus [GPa]**
1	4	4	0.2	2500	0.3	0.3	3.5	2500	30
2	8	8	0.2	625	0.3	0.3	3.5	2500	30
3	12	12	0.2	278	0.3	0.3	3.5	2500	30

The natural frequencies of the first four modes, shown in Figure 3.34 and 3.35 for the span of 4 meters, are compared. The results can be seen in Table 3.9 to 3.11. The error term is calculated in relation to the natural frequencies obtained from ADINA.



(a) First mode (b) Second mode (c) Third mode (d) Fourth mode

Figure 3.34: Mode shapes from analysis in ADINA.

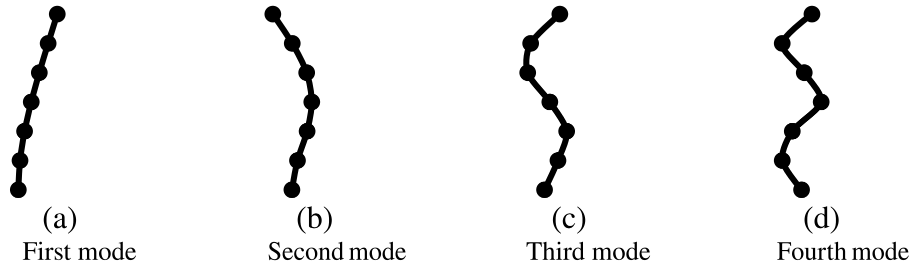


Figure 3.35: Mode shapes from analysis in VBA program (plotted with smooth curves).

Table 3.9: Natural frequencies, Analysis 1.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.181	0.180	-0.19%
2	1.136	1.134	-0.18%
3	3.191	3.185	-0.16%
4	6.247	6.239	-0.12%

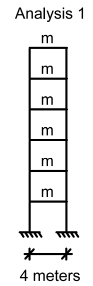


Figure 3.36: Analysed structure.

Table 3.10: Natural frequencies, Analysis 2.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.180	0.180	-0.10%
2	1.135	1.134	-0.09%
3	3.188	3.185	-0.07%
4	6.241	6.239	-0.03%

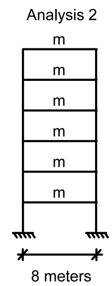


Figure 3.37: Analysed structure.

Table 3.11: Natural frequencies, Analysis 3.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.180	0.180	0.01%
2	1.133	1.134	0.02%
3	3.184	3.185	0.04%
4	6.234	6.239	0.08%

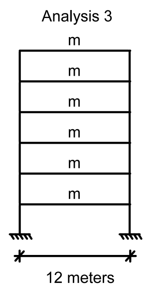


Figure 3.38: Analysed structure.

It is clear that the VBA model and the ADINA model are compatible, independent on the floor spans. The natural frequencies for the first modes are almost the same for the two models and it is therefore assumed that the VBA model can be used in further studies, as long as the floors are assumed to be rigid.

3.5.2 Influence of number of columns

To be able to analyse a wider variation of buildings, it is necessary to study how the size of the building, and hence the number of columns on each level, influence the models in the VBA script and ADINA. In the analyses described in Section 3.5.1, the structures are limited to consist of four columns on each level, whereas this section covers additional analyses with 9, 16 and 25 columns at each level. For all analyses, the span between the columns is 4 meters and the dimensions of the columns and the floor segments is the same as for Analysis 1 in Table 3.8. Figure 3.39 to 3.41 show the analysed structures and the models used in ADINA and VBA. The results are shown in Table 3.12, 3.13 and 3.14 for Analysis 4, 5 and 6, respectively.

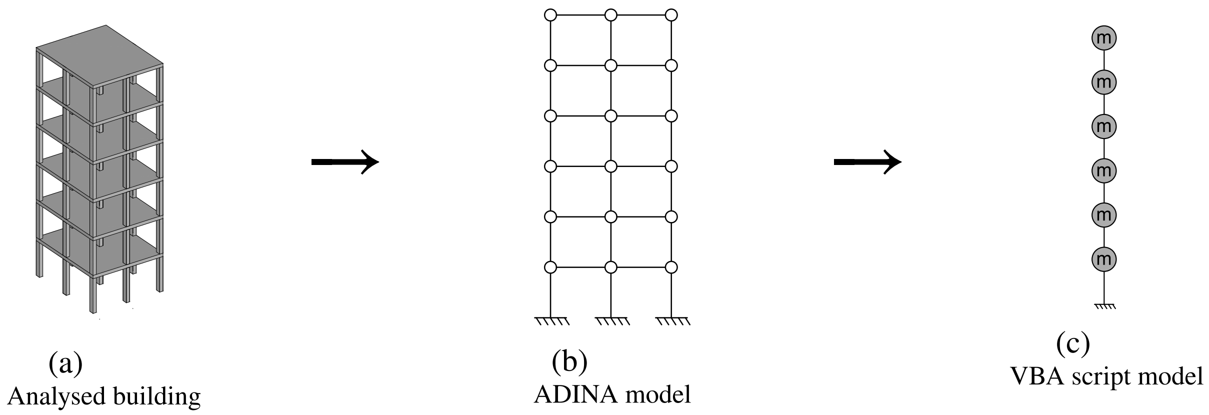


Figure 3.39: Analysis 4: analysed building and building models, 3x3 columns.

Table 3.12: Natural frequencies from ADINA and VBA, Analysis 4.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.143	0.142	-0.2%
2	0.900	0.898	-0.2%
3	2.534	2.539	-0.2%
4	4.966	4.957	-0.2%

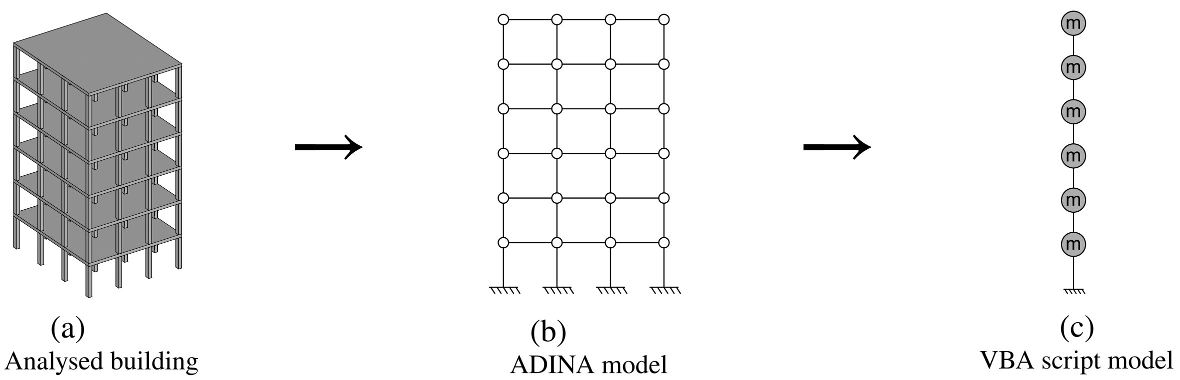
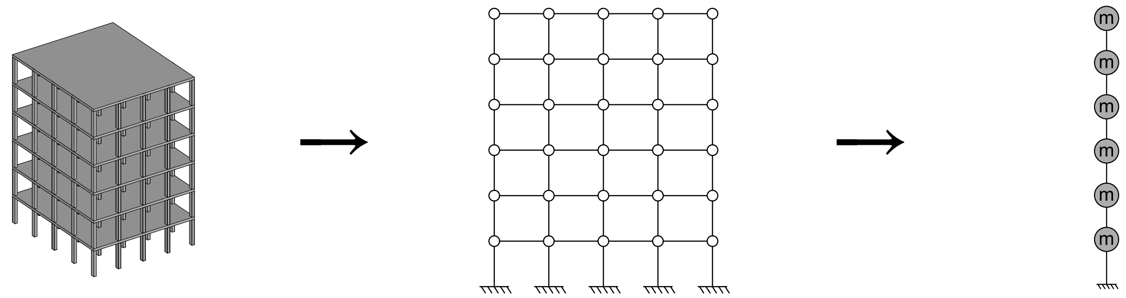


Figure 3.40: Analysis 5: analysed building and building models, 4x4 columns.

Table 3.13: Natural frequencies from ADINA and VBA, Analysis 5.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.129	0.128	-0.2%
2	0.813	0.811	-0.2%
3	2.291	2.286	-0.2%
4	4.490	4.482	-0.2%



(a) Analysed building   (b) ADINA model   (c) VBA script model

Figure 3.41: Analysis 6: analysed building and building models, 5x5 columns.

Table 3.14: Natural frequencies from ADINA and VBA, Analysis 6.

Mode	Natural frequency, ADINA [Hz]	Natural frequency, VBA [Hz]	Error
1	0.122	0.121	-0.2%
2	0.768	0.766	-0.2%
3	2.165	2.160	-0.2%
4	4.245	4.235	-0.2%

The results show that the number of columns on each floor does not influence the difference between the natural frequency of the VBA model and the ADINA model. It can therefore be assumed that the VBA model is fair to use to model buildings of various sizes.

3.6 Summary and conclusions regarding developed program

Through the various analyses and verifications described previously in Chapter 3, a reliable program in VBA is developed. It is concluded that it is possible to use the script to generate response spectra for acceleration, velocity and displacement and obtain results equal to results from a commercial FE program. With these spectra, responses of various SDOF system can be obtained.

With the VBA script, it is also possible to analyse the response of simple structures, as simply supported beams and columns with lumped masses, with various material and geometrical properties. In Section 3.3.3, it is shown that for relative responses the script generates results very similar to results from time history analyses in ADINA. However, when calculating the total response, the difference between the RSA and THA results are almost 30% for some of the analyses. This error is assumed to depend on the fact that the ground motion is included in the response spectra, and hence it is scaled in the same manner as the relative response when translating the SDOF response to an MDOF response.

A method for comfort weighting the response of various structures is implemented in the VBA script, see Section 3.4.2 . The method is referred to as Approach 2 and the principles are that an FFT is performed on the responses corresponding to the natural frequencies of a specific structure, transforming the time domain to the frequency domain. Further, the RMS values of the responses are found and multiplied with comfort weighting factors corresponding to the various frequencies the responses consist of. Comparing the comfort weighted responses obtained with this method to comfort weighted responses from time history analyses in ADINA show that there is a difference of as much as 88% for one of the analyses. However, the difference in absolute values is relatively small (about 1 mm/s) and for the rest of the analyses the difference is maximum 33%.

After discussions with Patrik Höstmad, associate Professor in technical acoustics at *Chalmers University of Technology*, the method of calculating the RMS factor in Approach 2 might not be entirely correct. However, the discussion took place in a late stage of the study and there was no time to investigate it further. For future studies, it is recommended to analyse this more thoroughly and study how this factor affects the comfort weighted response.

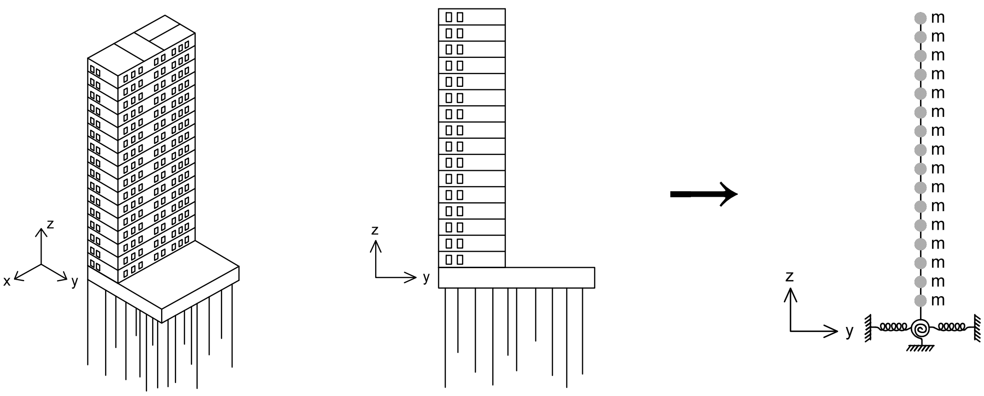
4 Study of residential building in Tändstickan

4.1 Problem description

A previous study has been made by *ÅF-Infrastructure AB* with the purpose to predict future vibrations in the planned residential buildings in the neighbourhood Tändstickan in Gothenburg (Carlsson, 2016). Analyses were made on a building called T41, which was assumed to experience the largest vibrations in the neighbourhood. The study was made with regard to vibrations from nearby train traffic and comfort weighted responses were calculated using time history analyses. Further, the results were compared to current regulations, limiting the comfort weighted RMS response to 0.4 mm/s. Figure 4.1 shows a visualisation of the intended neighbourhood, the location is the same as pointed out in Figure A.1 in Section A.1. Information from the study are collected from a technical report produced by *ÅF-Infrastructure AB* in December 2016.

Figure 4.1: Graphical image of Tändstickan and the analysed building T41, from (Carlsson, 2016).

The same building as in the technical report is analysed in the VBA script, in order to study the applications of the program and see how the results differ. A simplified 2D model is used with the purpose to simulate the building. Figure 4.2 and Figure 4.3 show the x-z elevation and the x-y elevation of the building and the representative 2D models.

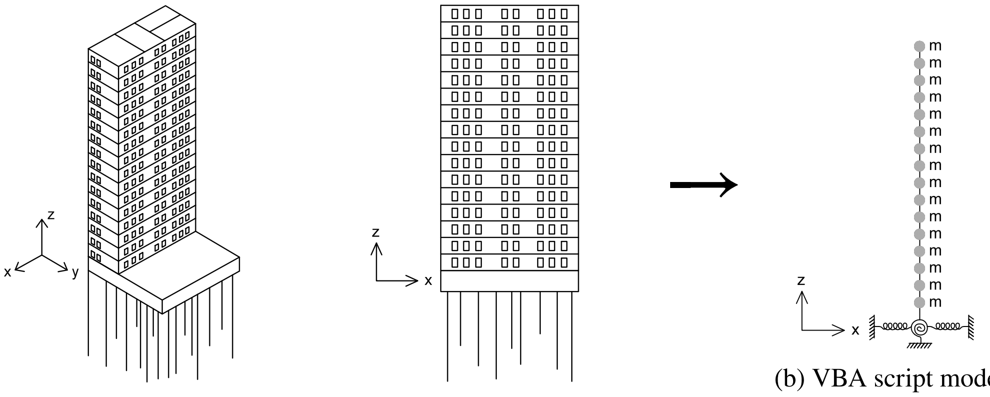


(a) 3D view

(a) Elevation, z-y view

(b) VBA script model

Figure 4.2: The model of T41 when ground motion in the y-direction is applied.



(a) 3D view (a) Elevation, z-x view (b) VBA script model

Figure 4.3: The model of T41 when ground motion in the x-direction is applied.

In order to model the foundation and the connection to the ground, rotational and horizontal springs with varying stiffness are applied to the bottom node of the VBA model. Since only horizontal movement is considered, no vertical springs are implemented to model the foundation properties in the vertical direction.

4.1.1 Geometry

The analysed building is located south west in the neighbourhood Tändstickan. It has 16 levels and the total building height is approximately 45 meters. The building consists of two separate parts, however, analyses are only made on the smaller part. The analysed model, intended to represent the plan view, is shown in Figure 4.4. The areas represent supporting walls, the outer walls consisting of sandwich elements of concrete and insulation. The inner supporting walls are made of concrete. All floors have the same thickness, about 0.3 meters, and consist of concrete as well.

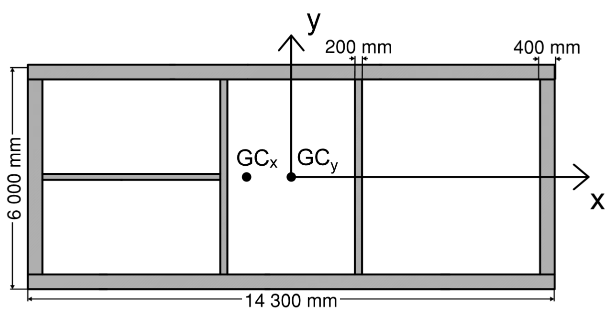


Figure 4.4: Plan view of analysed building T41.

The total area, as well as the moment of inertia in the x- and y-direction, are calculated and used as inputs in the VBA script. Additionally, lumped masses, with the weight of each floor, are added to the nodes.

4.1.2 Loads

In the technical report by *ÅF-Infrastructure AB*, a number of different ground motions were collected, measured at various locations around the planned neighbourhood. In order to find the most critical load, response spectra were carried out for each ground motion in every direction. Further, the spectra were combined and the maximum amplitudes in each frequency were found in order to create enveloped response spectra. Figure 4.5 and Figure 4.6 shows the enveloped total velocity response spectra in the y-direction and the x-direction, respectively.

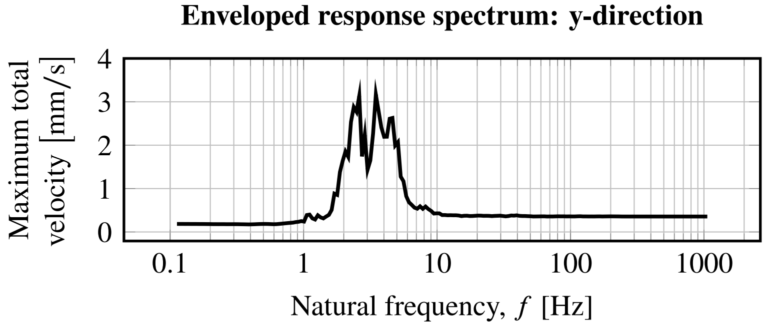


Figure 4.5: Velocity response spectrum, y-direction, from (Carlsson, 2016).

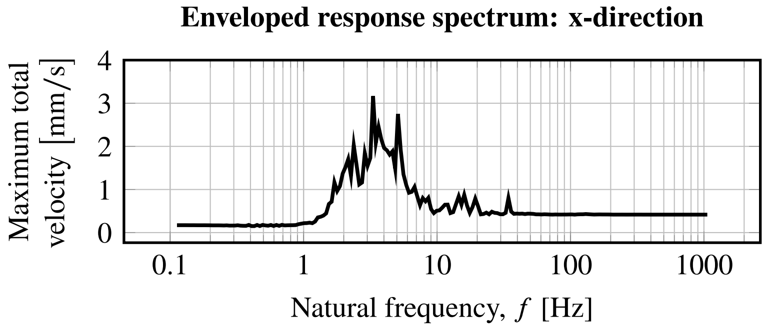
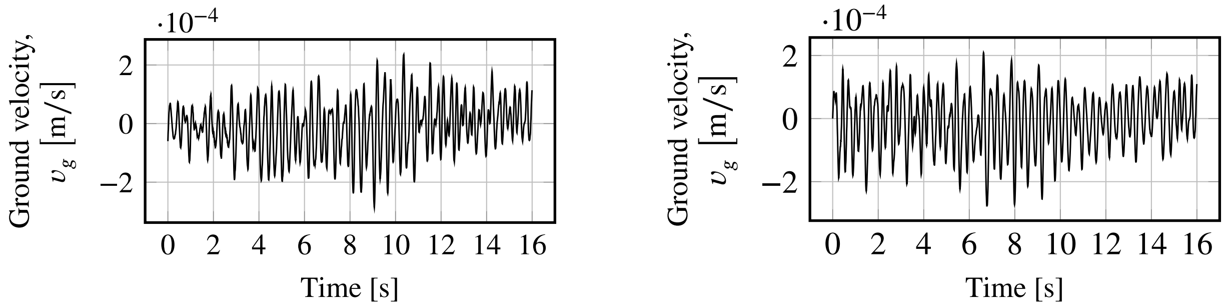


Figure 4.6: Velocity response spectrum, x-direction, from (Carlsson, 2016).

For the analysis performed with the VBA script, no enveloped velocity response spectra are calculated. Instead, the structure is analysed with regard to the ground velocities having the largest amplitudes in the two directions. Figure 4.7 shows the ground motions plotted as a function of time. For the y-direction, the ground motion is the same as in Figure A.2, repeated here for convenience.



(a) Ground velocity, y-direction

(b) Ground velocity, x-direction

Figure 4.7: Ground motions, measured near the planned neighborhood Tändstickan in Gothenburg.

To illustrate the difference between the input loads in the VBA script and the previous analysis, response spectra are carried out for these ground velocities. Figure 4.8 and Figure 4.9 show the total velocity response spectra in the y-direction and the x-direction, respectively. It can be observed that the maximum amplitudes are the same as the enveloped response spectra in Figure 4.5 and Figure 4.6, but for the rest of the frequencies it varies.

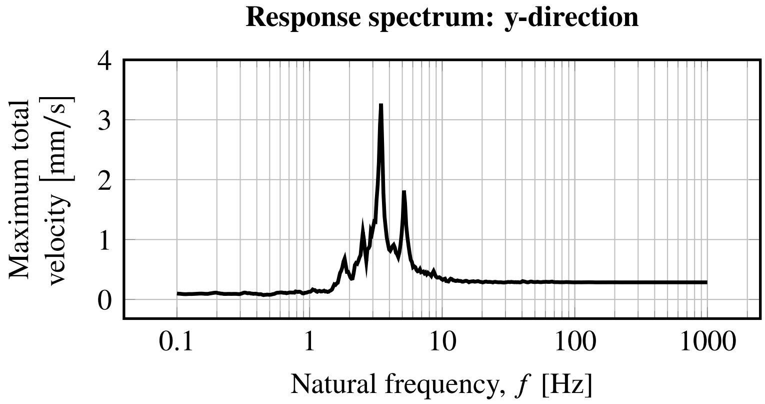


Figure 4.8: Velocity response spectrum, y-direction. Created in the VBA script.

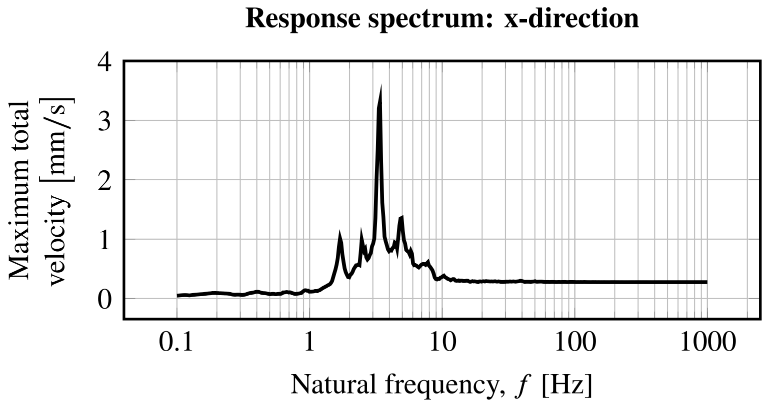
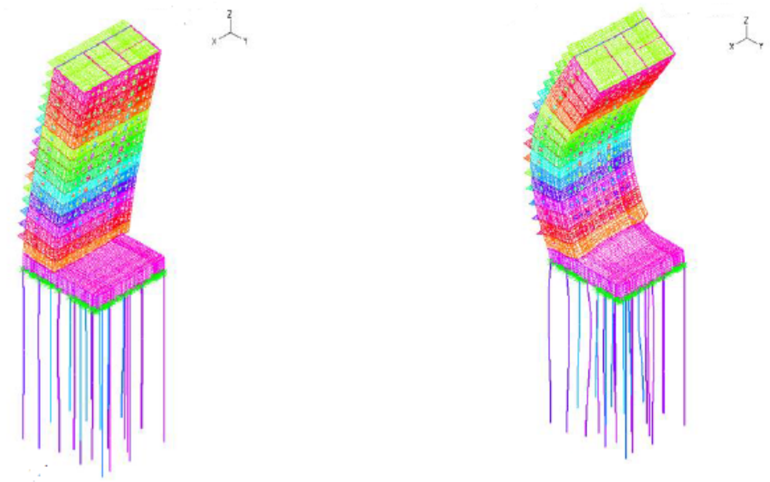


Figure 4.9: Velocity response spectrum, x-direction. Created in the VBA script.

4.2 Results

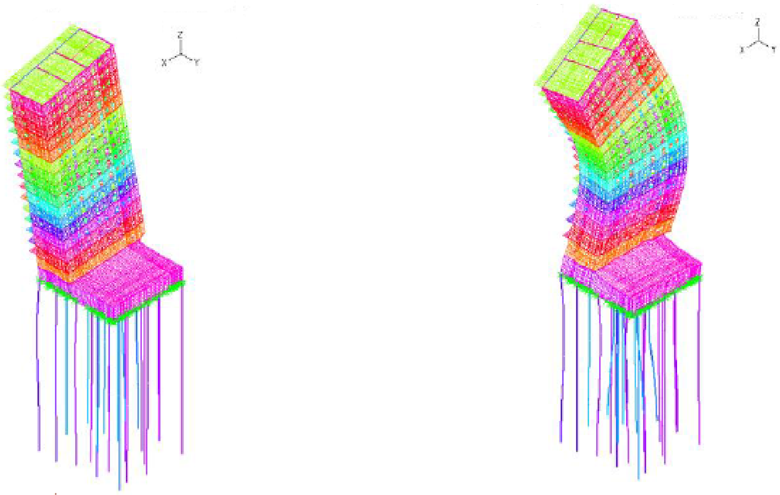
In order to find a VBA model that corresponds to the FE model that was used in the analysis performed by *ÅF-Infrastructure AB*, the natural frequencies of the first two bending modes of both models are compared. The modes and the corresponding natural frequencies, for bending in both directions, for the FE model are shown in Figure 4.10 and Figure 4.11. To get the same results for the VBA model, the stiffness of the horizontal and the rotational spring are iterated until the frequencies are considered close enough to the natural frequencies of the FE model. This is done for bending in both the y-direction and the x-direction. For further studies, it is recommended to find a method to calculate these values for a specific foundation. However, this is beyond the scope of this report and for this analysis the iterated values are used to be able to compare the two analyses. The final stiffness of the springs, along with the natural frequencies of the VBA model, are shown in Table 4.1.



(a) First bending mode,
$f = 1.52$ Hz

(b) Second bending mode,
$f = 8.43$ Hz

Figure 4.10: The two first bending modes in the y-direction, from (Carlsson, 2016).



(a) First bending mode,
$f = 2.61$ Hz

(b) Second bending mode,
$f = 11.34$ Hz

Figure 4.11: The two first bending modes in the x-direction, from (Carlsson, 2016).

Table 4.1: Stiffness and frequencies of the VBA model.

Bending in:	**Stiffness horizontal spring [N/m]**	**Stiffness rotational spring [Nm/rad]**	**Frequency First bending mode [Hz]**	**Frequency Second bending mode [Hz]**
y-direction	$5.22 \cdot 10^9$	$4.07 \cdot 10^{11}$	1.51	8.67
x-direction	$6.43 \cdot 10^9$	$14.84 \cdot 10^{11}$	2.61	11.4

The maximum values of the total comfort weighted RMS velocity in both directions are found. These are compared to the responses presented in the report by *ÅF-Infrastructure AB*, see Table 4.2. The difference term is calculated with regard to the previous analysis.

Table 4.2: Results from analysis of building T41.

Response	Results from VBA script	Results from *ÅF-Infrastructure AB*	Difference
Total y-velocity $v_{comf.y}$ **[mm/s]**	0.15	0.30	-51.1%
Total x-velocity $v_{comf.x}$ **[mm/s]**	0.29	0.35	-16.8%

It is obviously a difference between the results, but with regard to the various simplifications of the VBA model, the results are considered reasonable. In the FE analysis, the foundation and the piles are included in the model, which most likely has an influence on the response. In the VBA model, this is taken into account by adding springs to the bottom node, which only affects the stiffness of the structure. It is also shown in Section 3.4.2 that there is a difference both between comfort weighted responses from a THA and an RSA. Getting the same results from the two analyses is therefore not expected. Additionally, the RMS factor used in the two analyses might not be the same, which also affects the comfort weighted responses. Another factor that affects the results is the ground motion. In the analysis performed by *ÅF-Infrastructure AB*, an enveloped load combination is used, which means the worst case is found for every frequency. In the VBA script, only one specific load is used, which may contribute to the fact that those results are smaller. Comparing the response spectra in Figure 4.5 and 4.6 with the response spectra in Figure 4.8 and 4.9, it is clear that the peak values are the same for both loads. However, the shape of the curves are not the same and it can be seen that the enveloped load is larger for the rest of the frequencies.

The geometry implemented in the VBA script is another rough simplification of the reality. As described in Section 4.1.1, the structural system of each the level is described as a box cross section in the VBA script. From this cross section, the moment of inertia and the cross section area are calculated, without any consideration to the openings, like windows and doors. This might have an influence on the VBA results since it provides a stiffer and less conservative structure.

4.3 Computational time

As mentioned in Section 1.2, the general aim of this master's thesis is to develop a simplified calculation program, which can be used to estimate the dynamical response of structures in the early design phase. Since the intention is that the program should be used to get a rough estimation if a more thorough analysis is required or not, the computational time is of great importance. For analyses of structures like T41, the computational time consist of mainly two parts: the time it takes to design the model and the actual running time of the analysis.

4.3.1 Design of model

The model used in the analysis performed by *ÅF-Infrastructure AB* is a 3D FE model, intended to simulate the overall behaviour of the real building as good as possible. The walls and floors are modelled with about 40,000 shell elements, each with four nodes, and the piles and supports consist of about 7000 beam elements, two nodes each. Additionally, about 400 spring elements are used to simulate the connection between the building and the ground. According to *ÅF-Infrastructure AB*, the time it took to design and adjust the model was more than five days.

In comparison to the FE model, the VBA model is rather simple. It consists exclusively of 2D beam elements, one for each floor, which makes a total of seven elements for this structure. As described in Section 4.1, the building is modelled as a column with horizontal and rotational spring elements at the bottom node with the purpose to simulate the foundation and the connection to the ground. The floors are assumed to be rigid and these are only included in the model as lumped masses on each node. As an estimation, it takes less than one hour to calculate the input parameters and model the structure.

4.3.2 Running time of analysis

The time it takes to run an analysis is not always a reliable indicator, since it is highly dependant on factors like the capacity of the computer and how many programs that are running simultaneously. Therefore, the running times presented here should not be taken as definitive values. Instead, they should provide a rough approximation of the computational effort of the two types of analyses.

According to *ÅF-Infrastructure AB*, the running time of the FE analysis was about ten hours. Additionally, the results were multiplied with an RMS factor and comfort weighted before the maximum values could be found. In comparison, the running time of the VBA analysis is about two minutes for each direction, which makes a total of four minutes to analyse the response in the x and y-direction. The results obtained from the VBA analysis are maximum values of comfort weighted RMS responses in each DOF of the structure.

5 Discussion

5.1 Applications of VBA program

Through the various analyses presented in this thesis, it is shown that it is possible to use the developed calculation program to get a rough estimation of the dynamical response of simple structures. Even though the results may have a significant difference to results from more advanced analyses, the program is considered to fulfill its desired purpose. It is proved that, for the studied structures, the results generated by the VBA script are in the same range as results from time history analyses, generated by ADINA. For these cases, it can therefore be assumed that the program may be used as a complement to commercial FE programs, to tell if the dynamical responses will be close to current regulations. However, it is not concluded whether the VBA script generates conservative results or not. This varies throughout the analyses presented in this thesis and no relation to e.g. stiffness is found.

In Chapter 4, the VBA script is used to analyse a real building, for which a full FE analysis previously has been performed. Owing to the results, it is shown that even for this, relatively complex structure, the VBA script generates reasonable results. The largest difference between the VBA results and the results obtained by *ÅF-Infrastructure AB* is only about 50%. With regard to how simple the analysis in the VBA script is compared to the FE analysis, the difference between the results is very small, especially considering the time aspect. It take less than one hour to design the VBA model and run the analysis, in comparison to the FE analysis which took more than six working days in total. However, it is important to remember that these are the results from one specific analysis. It is possible that the difference would be significantly larger for a different structure and a lot more studies are required in order to draw a general conclusion.

5.2 Discussion of RSA

In Section 3.2 it is shown that the VBA script and ADINA generate identical response spectra for three different loads. Therefore, it is safe to assume that the principles of the method are correct. However, it is important to remember that response spectrum analyses only provides estimations of the responses and the results can never be expected to be exactly the same as from time history analyses. From a THA, responses are generated for the whole time span, whereas an RSA only provides the peak values for a specific frequency range. Further, when performing an RSA on an MDOF system, additional errors are induced since the SDOF response need to be translated to an MDOF response. With this in mind, the results shown in Section 3.3.3 are considered reasonable. It can however be noticed that the errors are larger for the total response than the relative response, which shows that the ground motion might not be correctly included.

5.3 Discussion of comfort weighting

As mentioned in Section 3.4 and Section 3.6, there are still some questions regarding the principles of RMS and comfort weighting of vibrations. More research and studies are required in this subject in order to find a more reliable approach to implement into the VBA script. Unfortunately, the discussions with Patrik Höstmad at *Chalmers University of Technology* were initiated too late and there was no time to make any further investigations on the information he provided. If the discussions would have started earlier, maybe a more reliable method to calculate the RMS values would have been found.

Another problem that arise with comfort weighting is how to perform this on responses in the time domain. The regulations only describes how to perform comfort weighting on responses in the frequency domain. Therefore, the method used in the VBA script, where an FFT is performed on the response signal, is only an interpreted approach and not a confirmed method. It is therefore difficult to compare the obtained results to current regulation, since these might not be referring to the same type of response.

Additionally, it is important to remember that the comfort weighted responses calculated with the VBA script, in this thesis, are compared to responses from time history analyses that are comfort weighted in the same manner. These responses are also translated to the frequency domain using an FFT, and further comfort weighted using the same calculation sheet provided by *Sound & Vibrations* at *ÅF-Infrastructure AB*.

In the studies presented in this thesis, the comfort weighted THA results are regarded as the correct response values. However, these results actually only show the accuracy of the RSA results in comparison to the THA results, for this specific approach of comfort weighting. The same uncertainties with the RMS factor and the implementation of comfort weighting still apply to the THA results. The most correct study to perform would have been to compare the calculated responses to actual measured comfort weighted signals for the same load. Unfortunately, this was not possible since no comparable data could be found. However, since the approach of comfort weighting is a separate part of the VBA script, it can easily be substituted when a more reliable method is found.

5.4 Simplifications and limitations

When studying the results of the analyses in the thesis, it is important to take into account the various simplifications and limitations of the VBA program. Some of which are already mentioned in Section 1.4. One important limitation is the fact that the foundations and its transfer of vibrations is not considered. For example, for the building analysed in Chapter 4, the foundation and the piles are assumed to have a significant influence on the responses. As mentioned, the stiffness contribution is considered by adding rotational and horizontal springs, but the fact that the ground motion actually is transferred through the piles is not taken into account in the VBA script. This is assumed to induce some errors in the results, which need to be kept in mind when analysing structures with various foundation conditions.

Since the purpose of the thesis is to develop a simplified calculation program, where dynamical analyses can be performed rather easy, there are lots of simplifications, which of course induces errors. Some of which are related to the models, for example the column model with the purpose to simulate multi-storey buildings. This model is designed in such way that only one beam element is used to describe the response of each floor. In Section 3.3.1 it was proved that in order to get reliable results, a number of 16 elements would be required. It is therefore obvious that parts of the errors between the RSA and the THA results for the building analyses can be related to this. However, having more than one element for each level would require a lot more computational effort, and parts of the purpose of the program would get lost.

Another simplification of the analyses of the multi-story buildings is that only responses to bending modes are considered. This is of interest when analysing a building with regard to vibrational comfort, but when studying the structural capacity, the shear modes are often more important. Finding methods to include these modes in the responses is recommended in order to improve the program further.

6 Final remarks

6.1 Summary of Results

This study aims to investigate how to predict comfort disturbing vibrations in buildings. Throughout the work, a calculation program is developed in Visual Basic for Applications (VBA). During the development of the program, several analyses and verifications are made and the results are compared to results from the commercial finite element (FE) program ADINA. In this section, a summary of the most essential analyses are presented.

6.1.1 Response spectra

Response spectra are plots consisting of peak responses for a wide range of single degree of freedom (SDOF) systems. These types of plots can be generated by the developed calculation program and the results are verified. When comparing response spectra created with the VBA program and response spectra created with ADINA, only small differences can be seen. The equivalence is independent of the complexity of the load, i.e. if the load has a sinus shape or a more irregular distribution. The results are found in Section 3.2.2 and Section 3.2.3.

6.1.2 Response spectrum analysis in VBA script and ADINA

In order to study the accuracy of the program, response spectrum analyses are performed in both the VBA program and ADINA. Two simply supported beams, with different lengths, are studied and the total responses (total acceleration, total velocity and total displacement) are calculated with response spectrum analysis (RSA) with both the VBA script and with ADINA. The responses calculated with the programs differ with 0.6% – 1.6%. To check the equivalence of the created models, the first mode frequencies of the beams are studied as well, which from the two programs differ with 0.1%. The results are found in Section 3.3.2.

6.1.3 Response spectrum analysis compared to time history analysis

Time history analysis (THA) is the most common method to use today when analysing the dynamic response of buildings. Such analysis provides a full response history and therefore describes the exact behaviour of a system. To study RSA in comparison to THA a number of simply supported beams with varying stiffness and mass are studied and the maximum acceleration (relative and total) in three different positions along the beams are calculated with RSA in the VBA script and with THA in ADINA.

The relative accelerations calculated with the VBA script is very similar to the accelerations from ADINA. The acceleration of the mid node differs with 0.1% – 10.9% and the accelerations of the two other studied nodes differ with 0.6% – 6.2% and 0.1% – 2.0%, respectively. The results are found in Section 3.3.3.1.

Different ways of including the ground motion (to get total acceleration) in the VBA script are studied and the results are compared to time history analyses from ADINA, for the final method. The total acceleration in the mid node differs from the ADINA results with 2.5% – 31.9%. For the two other studied nodes, the results differ with 0.2% – 28.7% and 1.8% – 34.4%, respectively. The results are found in Section 3.3.3.2.

6.1.4 Implementation of comfort weighting

Eight simply supported beams are studied with two different approaches to weight the signal in the VBA script with regard to comfort weighting. The total acceleration in three different positions along the beam are studied. The approach that generates the results considered most reliable gives results that differs with 1.7% – 31.0% in comparison to a THA from ADINA, for the mid node. The acceleration in the other studied nodes differ with 1.9% – 86.6% and 4.3% – 33.7%, respectively, compared to the results generated with ADINA. The results are found in Section 3.4.2.

6.1.5 Simple building model

The resemblance between a 2D frame model, modelled in ADINA, and a cantilever beam with lumped masses, modelled in the VBA script, is studied with regard to the natural frequencies. First, the influence of floor span is studied, where three different ADINA models are analysed. The natural frequencies of the models in ADINA and the VBA script differ with 0.19%, at most.The results are found in Section 3.5.1.

Further, natural frequencies of three ADINA models, representing buildings of different sizes, are compared to the natural frequencies of corresponding VBA models (cantilever with lumped masses). The natural frequencies of all models differ with 0.2%. The results are found in Section 3.5.2.

6.1.6 Analysis of simple MDOF system

A simple MDOF system, a six storey building with two spans, is analysed with the final VBA program and the results (total velocities) are compared to results from a THA in ADINA. Both comfort weighted responses and unweighted responses are studied. The differences between the analyses of the weighted response are maximum 49.4% and for the unweighted response the differences are maximum 22.3%. The results are found in Section A.2.

6.1.7 Study of residential building

The VBA script is used to analyse a residential building that is to be built in Gothenburg, which has already been analysed by *ÅF-Infrastructure AB*, using advanced FE analyses. The difference in the results generated from the two methods is 51.1% for the weak direction of the building and 16.8% for the stiffer direction. The results are found in Section 4.2.

6.2 Conclusion

Throughout the analyses it is shown that it is possible, through the VBA script, to get a rough estimation of the dynamical response of a simple building subjected to a ground motion. However, the program generates results that are not consistently conservative. Therefore, it can not be concluded that the results are always on the safe side. Because of this, the results should be regarded as indicators only and not as response values to be designed for. The script should be used to roughly estimate if a building is at risk of evolving vibrational problems with regard to comfort. If this is the case, further, and more advanced, analyses are needed.

The script uses RSA to estimate the responses of structures subjected to a ground motion, a method commonly used in earthquake engineering. Throughout the work, problems on how to include the ground motion in the response spectra, to get the total response, were discovered. The way the ground motion is included in the final version of the program need to be evaluated further. However, since it is concluded that the program generates results that can be used to estimate the dynamical response, it can also be concluded that RSA is an applicable method for prediction of comfort disturbing vibrations in buildings.

As can be read in Section 6.1, the results from the VBA script differ slightly from the results generated from the time history analyses in ADINA. Therefore, additional studies are needed to further verify the method of the VBA program, before it can be used in practice.

As discussed in Section 5 there are uncertainties regarding the way the program handles comfort weighting. In order to be able to compare the results to current regulations, it is essential to be able to weight the generated signal with regard to comfort in a correct way. It can therefore be concluded that further studies on how to include comfort weighting are needed.

6.3 Further studies

As mentioned in Section 3.3.3.2, further studies is needed to investigate whether there is a better and more correct way of including the ground motion in the response spectrum or not. The way the ground motion is included in the final version of the VBA script induces errors that will, unavoidably, affect the result. Additional literature studies in the subject are required in order to find a solution that gives less errors.

In the final VBA script, it is possible to set the input ground motion as a ground acceleration or a ground velocity. Due to the problems described in Section 3.2.1, it is not possible to get a response spectrum of the total velocity if the ground motion is an acceleration. Further studies how to integrate the acceleration in a correct manner and calculate the initial velocity in a general way, need to be performed.

When measuring a ground motion it is possible, through a software, to get comfort weighted values at once. In further studies these comfort weighted values could be compared to the values weighted with Approach 2, described in Section 3.4.2, to see the resemblance. If the results differ, Approach 2 need to be developed further.

In the script, the signals are converted from the time domain to the frequency domain to be able to use the correct weighting factors. Investigation on how to comfort weight a signal in the time domain, if possible, should be done in further studies. Additionally, it may be possible to change the RMS factor that is multiplied with the converted FFT signal to $1/\sqrt{2}$, as discussed in Section 3.6. The differences in the results when implementing this factor to the VBA script instead of the current implemented factor need to be studied.

Finally, a wider variation of beams and structures need to be analysed and compared to a commercial FE program to check the resemblance of the programs. The results described in this report show large errors for some of the analysed structures, and a correlation between these errors and the properties of the structures need to be found. A wider range of structures need to be covered to find when the program is not consistent.

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A Analysis of a simple MDOF system

A.1 Problem description

To study the applications of the complete VBA script, a full analysis is performed for one of the structures described in Section 3.5.2, see Figure 3.39. For repetition, the geometry is described in Table A.1. For this analysis, the damping ratio is defined as 5% and the applied load is a velocity, applied as a ground motion in the horizontal direction. The motion is measured at Almedalsvägen in Gothenburg, Figure A.1 shows the location. The graph in Figure A.2 shows the velocity plotted as a function of time. The total horizontal velocity (ground velocity and relative velocity) is calculated with RSA in the VBA script and with THA in ADINA. To see how the response vary along the height of the structure, the total velocity in node 2 to 7 in Figure A.3c is studied.

Table A.1: Geometry of analysed building.

Floors				**Columns**					
Width [m]	**Length [m]**	**Height [m]**	**Density [kg/m^3]**	**Number per level**	**Width [m]**	**Depth [m]**	**Height [m]**	**Density [kg/m^3]**	**Young's modulus [GPa]**
8	8	0.2	2500	9	0.3	0.3	3.5	2500	30

Figure A.1: Location where the velocity ground motion is measured. Image from Apple Maps.

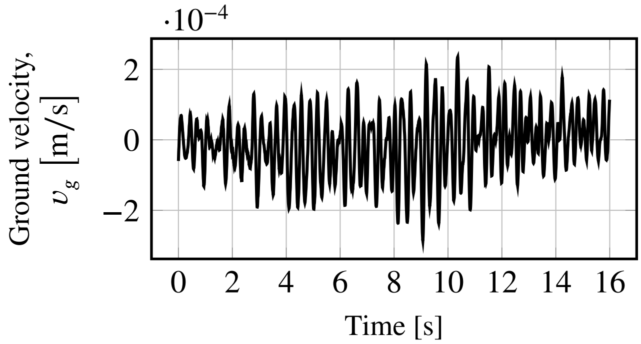
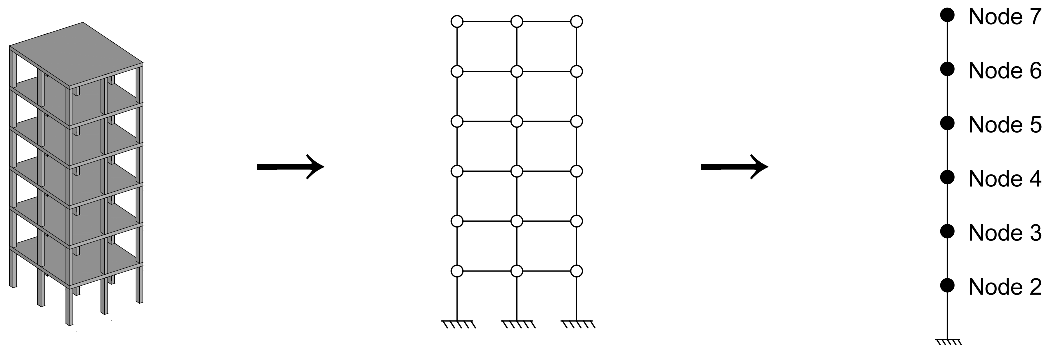


Figure A.2: Ground velocity measured at Almedalsvägen in Gothenburg.



(a) Analysed building (b) ADINA model (c) VBA script model

Figure A.3: Analysed building and building models.

A.2 Results

Studies are performed to obtain both the unweighted total response and the comfort weighted total RMS response in each node. For both analyses, the error term in the tables represents the difference between the responses with respect to the time history analyses. The results for the unweighted response are shown in Table A.2. The difference between the analyses seems reasonable owing to the results shown in Section 3.3.3.3.

Table A.2: Total velocity of analysed MDOF system. Unweighted response.

Node	Total velocity, RSA v_{RSA} [mm/s]	Total velocity, THA v_{THA} [mm/s]	Error
2	0.263	0.339	-22.3%
3	0.283	0.355	-20.3%
4	0.221	0.248	-11.2%
5	0.227	0.193	17.6%
6	0.276	0.317	-12.8%
7	0.256	0.260	-1.5%

The comfort weighted response is shown in Table A.3. It is clear that ratios between the RSA and THA results are not the same as in Table A.2, and hence it can be concluded that the approach of comfort weighting does not work exactly the same for the two types of analyses. This is also proved if comparing the results in Section 3.3.3.3 with the results in Section 3.4.2. Therefore, it seems reasonable that there is a considerable difference between the RSA and the THA results.

Table A.3: Total velocity of analysed MDOF system. Comfort weighted response.

Node	Total velocity, RSA v_{RSA} [mm/s]	Total velocity, THA v_{THA} [mm/s]	Error
2	0.0966	0.140	-31.0%
3	0.0811	0.160	-49.4%
4	0.0861	0.114	-24.6%
5	0.0611	0.112	-45.5%
6	0.0954	0.140	-32.0%
7	0.0702	0.123	-42.8%

Even though the VBA script generates results that are almost 50% off, for this specific building example, the difference in absolute values is relatively small. With respect to the simplifications of the VBA model and the fact that an RSA is an estimation which induces some errors (as shown in Chapter 3), the results are considered reasonable. Additionally, there are uncertainties in the calculation of RMS values, as described in Section 3.4.2, which contributes to the error between the comfort weighted responses.

B Results of total response - RSA compared to THA

This appendix includes the data from which the graphs in Figure 3.18, 3.19 and 3.20 in Section 3.3.3.3 are plotted.

Table B.1: Comparison of total acceleration from RSA and THA, node A.

	Total acceleration (Node A), a_{tot} **[mm/s²]**		
Analysis	**RSA (from VBA script)**	**THA (from ADINA)**	**Error**
1	8.86	7.90	12.2%
2	17.47	19.15	-8.8%
3	35.02	36.06	-2.9%
4	49.39	50.66	-2.5%
5	20.31	16.13	25.9%
6	19.10	15.48	23.4%
7	18.76	14.57	28.8%
8	24.86	21.43	16.0%
9	11.94	10.43	14.5%
10	9.78	10.83	-9.7%
11	53.88	56.12	-3.4%
12	25.53	23.91	6.8%
13	20.33	16.93	20.1%
14	38.48	35.02	9.9%
15	24.30	18.77	29.5%
16	20.87	15.82	31.9%

Table B.2: Comparison of total acceleration from RSA and THA, node B.

	Total acceleration (Node B), a_{tot} **[mm/s²]**		
Analysis	**RSA (from VBA script)**	**THA (from ADINA)**	**Error**
1	6.28	5.19	21.0%
2	12.36	12.11	2.1%
3	24.76	25.08	-1.3%
4	34.92	34.10	2.4%
5	14.36	14.87	-3.4%
6	13.50	13.52	-0.2%
7	13.26	13.40	-1.0%
8	17.58	18.19	-3.4%
9	8.44	7.21	17.0%
10	6.92	5.38	28.7%
11	38.10	38.61	-1.3%
12	18.05	18.33	–1.5%
13	14.38	14.64	-1.8%
14	27.21	26.75	1.7%
15	17.18	17.92	-4.1%
16	14.76	15.43	-4.3%

Table B.3: Comparison of total acceleration from RSA and THA, node C.

	Total acceleration (Node C), a_{tot} **[mm/s^2]**		
Analysis	**RSA (from VBA script)**	**THA (from ADINA)**	**Error**
1	7.07	8.27	-14.5%
2	9.53	8.37	13.9%
3	14.74	15.05	-2.1%
4	20.23	17.30	16.9%
5	9.85	14.65	-32.8%
6	9.26	13.20	-29.9%
7	9.18	12.95	-29.1%
8	11.12	15.08	-26.3%
9	10.55	12.17	-13.3%
10	8.03	8.49	-5.4%
11	21.43	21.04	1.8%
12	12.08	17.16	-29.6%
13	10.13	13.64	-25.7%
14	15.89	17.47	-9.1%
15	11.00	16.21	-32.1%
16	9.89	15.09	-34.4%

C Description of calculation sheet

In the calculation sheet, originally written by *Sound & Vibrations* at *ÅF-Infrastructure AB*, a signal in the frequency domain need to be inserted. The signal is inserted into two columns, one for frequency and one for the magnitude of the response corresponding to each frequency, see Table C.1. Further, the signal is plotted, see Figure C.1.

Table C.1: Example of input signal.

Frequency [Hz]	**Magnitude**
0	$3.25 \cdot 10^{-5}$
0.04	$2.00 \cdot 10^{-5}$
0.08	$2.55 \cdot 10^{-5}$
...	...

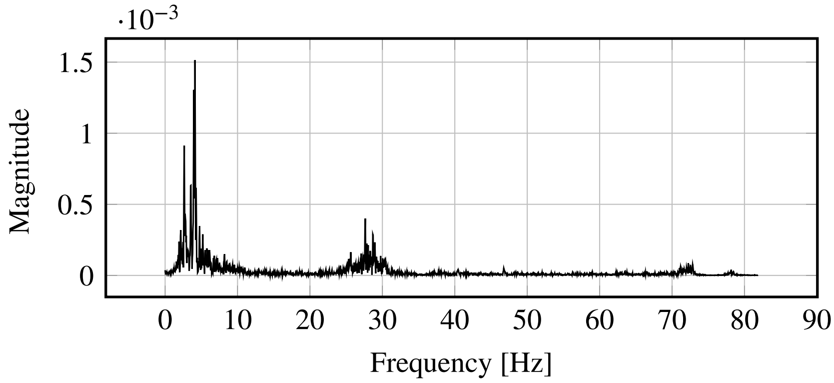


Figure C.1: Example of input signal, plotted as a function of frequency.

If the signal to be analysed is in time domain, an FFT of the signal need to be done first. When inserted, the signal is divided with regard to one-third-octave bands, where each band corresponds to a specific frequency range. The magnitudes of the responses belonging to each one-third-octave band are summed up, and a table with the magnitude in each one-third-octave band is automatically written, see Table C.2

Table C.2: The signal divided into one-third-octave bands.

Center frequency [Hz]	**Total magnitude**
1	$1.48 \cdot 10^{-4}$
1.25	$2.28 \cdot 10^{-4}$
...	...
63	$33.1 \cdot 10^{-4}$
80	0

The desired RMS factor also need to be an input in the sheet. The factor is multiplied with all the magnitude values in Table C.2. Further, the magnitude values are multiplied with weighting factors corresponding to each frequency. The weighting factors can be found in Table 2.2. Finally, the response is calculated using Equation (2.3) or Equation (2.4) from Section 2.1.3.2, repeated here for convenience:

$$a_{comf} = \sqrt{(W_{m.1} \cdot a_1)^2 + (W_{m.1.25} \cdot a_{1.25})^2 + ... + (W_{m.80} \cdot a_{80})^2}$$

$$v_{comf} = \sqrt{(W_{m.1} \cdot v_1)^2 + (W_{m.1.25} \cdot v_{1.25})^2 + ... + (W_{m.80} \cdot v_{80})^2}$$

where $W_{m.1}$, $W_{m.1.25}...W_{m.80}$ are the weighting factors to the corresponding one-third-octave band, according to Table 2.2. a_1, $a_{1.25}...a_{80}$ are the acceleration responses in each one-third-octave band and v_1, $v_{1.25}...v_{80}$ are the velocity responses.

D VBA code

```
\textsmall
Option Base 1
Sub Beam_cantilever()

Application.ScreenUpdating = False
Application.Calculation = xlCalculationManual

If _
MsgBox("Would you like to include comfort weighting?", vbYesNo, "") _
= vbYes Then
RMS_comf = 1
Sheets("Beam Simply Supported").Cells(22, 4) _
= "Responses comfort weighted"
Sheets("Beam Cantilever").Cells(22, 4) =
"Responses comfort weighted"
Else
RMS_comf = 0
Sheets("Beam Cantilever").Cells(22, 4) =
"Responses NOT comfort weighted"
End If
'-----------------------MPF----------------------'
'Read input values
'Young's modulos
E = Sheets("Beam Cantilever").Cells(10, 5).Value
'Cross section area
Area = Sheets("Beam Cantilever").Cells(8, 5).Value
'Moment if inertia
inertia = Sheets("Beam Cantilever").Cells(9, 5).Value
'Density
density = Sheets("Beam Cantilever").Cells(7, 5).Value
'Mass per meter
mass = Area * density

'Stiffness of ground (modelled as a spring)
K_ground_rot = Sheets("Beam Cantilever").Cells(12, 5).Value
K_ground_hor = Sheets("Beam Cantilever").Cells(13, 5).Value

'Defining number of beam elements and length
n_elements = Sheets("Beam Cantilever").Cells(3, 5).Value
'Length of one beam element
length_e = Sheets("Beam Cantilever").Cells(6, 5).Value
Length = length_e * n_elements

'Defining pi
```

```
pi = 4 * Atn(1)

'Clear output cells
Sheets("Beam_Cantilever").Range("E16:G18").ClearContents
Sheets("Beam_Cantilever").Range("V26:EZ600").ClearContents
Sheets("Beam_Cantilever").Range("U33:U174").ClearContents
Sheets("Beam_Cantilever").Range("B26:F500").ClearContents

'Defining number of DOF
n_dof = (Sheets("Beam_Cantilever").Cells(3, 5).Value + 1) * 3

'Defining number of zeros
n_zeros = Sheets("Beam_Cantilever").Cells(4, 5).Value

'Defining number of reduced DOF
n_dof_red = n_dof - n_zeros

'Defining vectors with all DOFS, active DOFS,
'support DOFS and lumped mass
Dim DOFS()
ReDim Preserve DOFS(n_dof, 1)
Dim active()
ReDim Preserve active(n_dof, 1)
Dim support()
ReDim Preserve support(n_dof, 1)
Dim lumped_mass_pos()
ReDim Preserve lumped_mass_pos(n_dof, 1)

For i = 1 To n_dof
BC = Sheets("Beam_Cantilever").Cells(i + 2, 14).Value
    If BC = "" Then
    active(i, 1) = i
    Else
    support(i, 1) = i
    End If
DOFS(i, 1) = Sheets("Beam_Cantilever").Cells(i + 2, 10).Value
Next i

j = 1
Dim active_DOF_long()
ReDim Preserve active_DOF_long(n_dof, 1)
For i = 1 To n_dof
If active(i, 1) <> "" Then
active_DOF_long(j, 1) = active(i, 1)
j = j + 1
End If
Next i
```

```
Dim active_DOF()
ReDim Preserve active_DOF(j - 1, 1)
For i = 1 To j - 1
active_DOF(i, 1) = active_DOF_long(i, 1)
Next i

For i = 1 To UBound(active_DOF)
Sheets("Beam_Cantilever").Cells(32 + i, 21).Value = _
"DOF:" & active_DOF(i, 1)
Next i

j = 1
Dim support_DOF_long()
ReDim Preserve support_DOF_long(n_dof, 1)
For i = 1 To n_dof
If support(i, 1) <> "" Then
support_DOF_long(j, 1) = support(i, 1)
j = j + 1
End If
Next i

Dim support_DOF()
ReDim Preserve support_DOF(j - 1, 1)
For i = 1 To j - 1
support_DOF(i, 1) = support_DOF_long(i, 1)
Next i

'Add titels to modal vectors and modal participation factors
For i = 1 To n_dof_red
Sheets("Beam_Cantilever").Cells(26, 21 + i) = ChrW(915) & i
Sheets("Beam_Cantilever").Cells(29, 21 + i) = "f" & i
Sheets("Beam_Cantilever").Cells(32, 21 + i) = ChrW(966) & i
Next i

'Define matrix for mass
Dim m()
ReDim Preserve m(n_dof, n_dof)
For i = 1 To n_dof
For j = 1 To n_dof
m(i, j) = 0
Next j
Next i

'Define matrix for lumped mass
Dim m_lumped()
ReDim Preserve m_lumped(n_dof, n_dof)
```

```
For i = 1 To n_dof
m_lumped(i, i) = Sheets("Beam_Cantilever").Cells(i + 2, 16).Value
Next i

For i = 1 To n_dof
For j = 1 To n_dof
If m_lumped(i, j) = "" Then
m_lumped(i, j) = 0
Else
End If
Next j
Next i

'Define matrix for stiffness
Dim Ke()
ReDim Preserve Ke(6, 6)
Dim k()
ReDim Preserve k(n_dof, n_dof)
For i = 1 To n_dof
For j = 1 To n_dof
k(i, j) = 0
Next j
Next i

'Define modal matrix and modal vectors
Dim Phi_scaled()
ReDim Preserve Phi_scaled(n_dof_red, n_dof_red)
Dim smallphi_t()
ReDim Preserve smallphi_t(1, n_dof_red)
Dim smallphi()
ReDim Preserve smallphi(n_dof_red, 1)
Dim M_e()
ReDim Preserve M_e(6, 6)

'Define boundary conditions
Dim b()
ReDim Preserve b(n_zeros, 1)
b = support_DOF

'Define edof-matrix
Dim edof()
ReDim Preserve edof(n_elements, 7)
For j = 1 To n_elements
    For i = 2 To 4
    edof(j, 1) = j
    edof(j, i) = DOFS((i - 1) + 3 * (j - 1), 1)
    edof(j, i + 3) = DOFS(i + 2 + 3 * (j - 1), 1)
```

```
    Next i
Next j

'Define geometry

'x-coordinates
Dim ex()
ReDim Preserve ex(n_elements, 2)
For j = 1 To n_elements
ex(j, 1) = (j - 1) * length_e
ex(j, 2) = (j) * length_e
Next j

'y-coordinates
Dim ey()
ReDim Preserve ey(n_elements, 2)
For j = 1 To n_elements
ey(j, 1) = 0
ey(j, 2) = 0
Next j

'Define property vector
Dim epd()
ReDim Preserve epd(6)
epd(1) = E 'E
epd(2) = Area 'A
epd(3) = inertia 'I
epd(4) = mass 'm
epd(5) = 1
epd(6) = 1

'Assembly stiffness and mass matrix
For i = 1 To n_elements
Ke = Beam2d.Beam2d(ex, ey, epd, "Ke", i)
If i = 1 Then
Ke(3, 3) = Ke(3, 3) + K_ground_rot
Ke(2, 2) = Ke(2, 2) + K_ground_hor
End If
k = Assem.Assem(edof, k, Ke, 0, 0, i, 1)
M_e = Beam2d.Beam2d(ex, ey, epd, "Me", i)
m = Assem.Assem(edof, m, M_e, 0, 0, i, 1)
Next i

For i = 1 To n_dof
For j = 1 To n_dof
m(i, j) = m(i, j) + m_lumped(i, j)
Next j
```

```
Next i

'Reducing rows in k and m, associated to BC's
K2 = redb.redb(m, k, b, 1)
M2 = redb.redb(m, k, b, 2)

'Calculate r-vector
'partitionated k-matrix
Dim k_part()
ReDim Preserve k_part(n_dof, n_dof)

'Order of DOFS after partitioning
Dim part_order()
ReDim Preserve part_order(n_dof, 1)
For i = 1 To UBound(active_DOF)
part_order(i, 1) = active_DOF(i, 1)
Next i
For i = (UBound(active_DOF) + 1) To n_dof
part_order(i, 1) = support_DOF(i - UBound(active_DOF), 1)
Next i

For i = 1 To n_dof
For j = 1 To n_dof
k_part(j, i) = k(part_order(j, 1), part_order(i, 1))
Next j
Next i
'----------------R-vector---------------------------'
Dim Knnd()
ReDim Preserve Knnd(UBound(active_DOF), UBound(active_DOF))
For i = 1 To UBound(active_DOF)
For j = 1 To UBound(active_DOF)
Knnd(j, i) = k_part(j, i)
Next j
Next i

Dim Knsd()
ReDim Preserve Knsd(UBound(active_DOF), UBound(support_DOF))
For i = 1 To UBound(support_DOF)
For j = 1 To UBound(active_DOF)
Knsd(j, i) = k_part(j, UBound(active_DOF) + i)
Next j
Next i

Dim Ksnd()
ReDim Preserve Ksnd(UBound(support_DOF), UBound(active_DOF))
For i = 1 To UBound(active_DOF)
For j = 1 To UBound(support_DOF)
```

```
Ksnd(j, i) = k_part(j + UBound(active_DOF), i)
Next j
Next i

Dim Kssd()
ReDim Preserve Kssd(UBound(support_DOF), UBound(support_DOF))
For i = 1 To UBound(support_DOF)
For j = 1 To UBound(support_DOF)
Kssd(j, i) = k_part(j + UBound(active_DOF), i + UBound(active_DOF))
Next j
Next i

Dim Knnd_inv_neg()
Dim r()
ReDim Preserve r(n_dof, UBound(support_DOF))
Knnd_inv = Application.MInverse(Knnd)
Knnd_inv_neg = Matrix.M_PROD_S(Knnd_inv, -1)
r = Application.MMult(Knnd_inv_neg, Knsd)

Dim rx()
ReDim Preserve rx(n_dof_red, 1)
For j = 1 To n_dof_red
rx(j, 1) = r(j, 1)
Next j

Dim ry()
ReDim Preserve ry(n_dof_red, 1)
If n_zeros > 1 Then
For j = 1 To n_dof_red
ry(j, 1) = r(j, 2)
Next j

Else
For n = 1 To UBound(ry)
ry(n, 1) = 0
Next n
For n = 1 To n_elements + 1
ry(1 + 3 * (n - 1), 1) = 1
Next n
End If
'--------------------------------------------------'
'Calculate frequencies and modal matrix
MOutput = ModalAnalysis.ModalAnalysis(M2, K2)
f = MOutput(1)
Phi = MOutput(2)
```

```
'Calculate natural frequencies for the sytem
Dim omega()
ReDim Preserve omega(UBound(f), 1)
For i = 1 To UBound(f)
      omega(i, 1) = f(i, 1) * 2 * pi
Next i

'Calculate sum of each vector in Phi
Dim norm_Phi()
ReDim Preserve norm_Phi(n_dof_red, 1)

Dim phi_sum()
ReDim Preserve phi_sum(n_dof_red, 1)

'Scale eigenvectors, normalize length of each vector=1
For i = 1 To n_dof_red
For j = 1 To n_dof_red
value_squared = (Phi(j, i) ^ 2)
phi_sum(i, 1) = phi_sum(i, 1) + value_squared
Next j
norm_Phi(i, 1) = Sqr(phi_sum(i, 1))
Next i

For i = 1 To n_dof_red
For j = 1 To n_dof_red
Phi_scaled(j, i) = Phi(j, i) / norm_Phi(i, 1)
Sheets("Beam_Cantilever").Cells(32 + j, 21 + i).Value = Phi_scaled(j, i)
Next j
Next i
'-----Check if the equation (K-omega^2 M)Phi=0''----------'
Dim DD()
ReDim Preserve DD(n_dof_red, n_dof_red)

Dim Phi_scaled_vector()
ReDim Preserve Phi_scaled_vector(n_dof_red, 1)

Dim omega2_m2()
ReDim Preserve omega2_m2(n_dof_red, 1)

Dim zero_matrix()
ReDim Preserve zero_matrix(n_dof_red, n_dof_red)

Dim zero_vector()
ReDim Preserve zero_vector(n_dof_red, 1)

'Multiplying M-matrix with omega^2
For nn = 1 To n_dof_red
```

```
 omega2_m2 = Matrix.M_PROD_S(M2, (omega(nn, 1) ^ 2))
    For i = 1 To n_dof_red
    For j = 1 To n_dof_red

'K-omega ^2 M
    DD(i, j) = K2(i, j) - omega2_m2(i, j)
    Phi_scaled_vector(j, 1) =
    Phi_scaled(j, nn)
    Next j
    Next i

zero_vector = Application.MMult(DD, Phi_scaled_vector)

    For n = 1 To n_dof_red
    zero_matrix(n, nn) = zero_vector(n, 1)
    Next n
Next nn


For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    Sheets("Beam_Cantilever").Cells(125 + j, 21 + i).Value = _
    zero_matrix(j, i)
    Next j
Next i

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(zero_matrix(j, i)) < 0.01 Then
        Else
        Sheets("Beam_Cantilever").Cells(126 _
        + UBound(zero_matrix), 21 + i) = "WRONG!"
        End If
    Next j
Next i

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(zero_matrix(j, i)) < 0.001 Then
        Else
        MsgBox "ERROR: (K-omega^2*M)Phi not zero!", vbCritical
        Exit Sub
        End If
    Next j
Next i
'-------------------------------------------------'
'Transpose Phi matrix
```

```
phi_t = WorksheetFunction.Transpose(Phi_scaled)

'Defining vectors for calculating modal participation factor
Dim phit_m_r()
ReDim Preserve phit_m_r(1, 1)

Dim phit_m_phi()
ReDim Preserve phit_m_phi(1, 1)

Dim GAMMA_vector()
ReDim Preserve GAMMA_vector(n_dof_red, 1)

'Modal participation factors
For i = 1 To n_dof_red
For j = 1 To n_dof_red
smallphi_t(1, j) = phi_t(i, j)  'Modal vector, 1 x n_floors
smallphi(j, 1) = phi_t(i, j)    'Modal vector, n_floors x 1

Next j

m_ry = Application.MMult(M2, ry)

'Numerator for MPPF
phit_m_ry = Application.MMult(smallphi_t, m_ry)
m_phi = Application.MMult(M2, smallphi)
'Denominator for MPPF
phit_m_phi = Application.MMult(smallphi_t, m_phi)
phit_m_phi_inv = Application.MInverse(phit_m_phi)
'Modal participation factor
Gamma = Application.MMult(phit_m_ry, phit_m_phi_inv)

GAMMA_vector(i, 1) = Gamma(1)

'Output MPPF and frequencies
Sheets("Beam_Cantilever").Cells(27, 21 + i).Value = Gamma
Sheets("Beam_Cantilever").Cells(30, 21 + i).Value = f(i, 1)
Next i

MsgBox "MPPFs calculated!", vbInformation

Dim a_RS_interpol()
ReDim Preserve a_RS_interpol(n_dof_red, 1)
Dim v_RS_interpol()
ReDim Preserve v_RS_interpol(n_dof_red, 1)

'------- NOT COMFORT WEIGTHED --------------------'
```

```
If RMS_comf = 0 Then

'----------- Response spectrum ---------------------'
'Read if the load is an acceleration or a velocity
choice = Sheets("Input").Cells(5, 12)

'num=1 gives matrix with freq, a_max,
'a_rel, v_rel and d_rel (and if choice=0, also v_max)
output_RS = Response_spectrum.Response_spectrum(1)

Dim freq()
ReDim Preserve freq(UBound(output_RS), 1)
Dim a_max()
ReDim Preserve a_max(UBound(output_RS), 1)
Dim a_rel()
ReDim Preserve a_rel(UBound(output_RS), 1)
Dim v_rel()
ReDim Preserve v_rel(UBound(output_RS), 1)
Dim d_rel()
ReDim Preserve d_rel(UBound(output_RS), 1)

If choice = 0 Then
Dim v_max()
ReDim Preserve v_max(UBound(output_RS), 1)
End If

For i = 1 To UBound(output_RS)
freq(i, 1) = output_RS(i, 1)
a_max(i, 1) = output_RS(i, 2)
a_rel(i, 1) = output_RS(i, 4)
v_rel(i, 1) = output_RS(i, 5)
d_rel(i, 1) = output_RS(i, 6)
If choice = 0 Then
v_max(i, 1) = output_RS(i, 3)
End If
Next i

n_steps = output_RS(1, 7)

If f(1, 1) > freq(1, 1) Then
Else
MsgBox _
"Adjust frequency range for RS. The lower
limit is too high!", vbCritical
End If

If f(UBound(f), 1) < freq(UBound(freq), 1) Then
```

```
Else
MsgBox _
"Adjust␣frequency␣range␣for␣RS.
The␣upper␣limit␣is␣too␣low!", vbCritical
End If

MsgBox "Response␣spectrum␣created!", vbInformation
'————————————————————————————————————————'
'-Reading responses from response specta,
'corresponding to eigenfrequencies--'

'Finding corresponding row in frequency vector to each DOF and mode

'Position in frequency vector from
'RS (x-values), where the eigenfrequencies is found
Dim freq_pos()
ReDim Preserve freq_pos(n_dof_red, 1)
For i = 1 To n_dof_red
j = 0

'Stepping through the frequency vector to find a value that is higher
For j = 1 To n_steps + 1
    If f(i, 1) > freq(j, 1) Then
        Else
        freq_pos(i, 1) = j
        j = n_steps + 1
    End If
    Next j
    Next i

'Interpolate the responses from response spectra
'for the frequencies of the structure

'Total acceleration
Dim grad_a()
ReDim Preserve grad_a(n_dof_red, 1)
Dim y_start_a()
ReDim Preserve y_start_a(n_dof_red, 1)

For i = 1 To n_dof_red
z = freq_pos(i, 1)
 'Calculates the gradient to be able to interpolate
grad_a(i, 1) = (a_max(z, 1) - a_max(z - 1, 1)) / _
(freq(z, 1) - freq(z - 1, 1))
y_start_a(i, 1) = a_max(z, 1) - grad_a(i, 1) * freq(z, 1)
 'Gives the interpolated value for acceleration in the RS
a_RS_interpol(i, 1) = grad_a(i, 1) * f(i, 1) + y_start_a(i, 1)
```

```
Next i

'Relative acceleration
Dim a_RS_interpol_rel()
ReDim Preserve a_RS_interpol_rel(n_dof_red, 1)

For i = 1 To n_dof_red
z = freq_pos(i, 1)
'Calculates the gradient to be able to interpolate
grad_a(i, 1) = (a_rel(z, 1) - a_rel(z - 1, 1)) / _
(freq(z, 1) - freq(z - 1, 1))
y_start_a(i, 1) = a_rel(z, 1) - grad_a(i, 1) * freq(z, 1)
'Gives the interpolated value for acceleration in the RS
a_RS_interpol_rel(i, 1) = grad_a(i, 1) * f(i, 1) + y_start_a(i, 1)
Next i

If choice = 0 Then
'Total velocity
Dim grad_v()
ReDim Preserve grad_v(n_dof_red, 1)
Dim y_start_v()
ReDim Preserve y_start_v(n_dof_red, 1)

For i = 1 To n_dof_red
z = freq_pos(i, 1)
'Calculates the gradient to be able to interpolate
grad_v(i, 1) = (v_max(z, 1) - v_max(z - 1, 1)) / _
(freq(z, 1) - freq(z - 1, 1))
y_start_v(i, 1) = v_max(z, 1) - grad_v(i, 1) * _
freq(z, 1)
'Gives the interpolated value for velocity in the RS
v_RS_interpol(i, 1) = grad_v(i, 1) * f(i, 1) + _
y_start_v(i, 1)
Next i
End If

'Relative velocity
Dim grad_v_rel()
ReDim Preserve grad_v_rel(n_dof_red, 1)
Dim y_start_v_rel()
ReDim Preserve y_start_v_rel(n_dof_red, 1)
Dim v_RS_interpol_rel()
ReDim Preserve v_RS_interpol_rel(n_dof_red, 1)

For i = 1 To n_dof_red
z = freq_pos(i, 1)
'Calculates the gradient to be able to interpolate
```

```
grad_v_rel(i, 1) = (v_rel(z, 1) - v_rel(z - 1, 1)) / _
(freq(z, 1) - freq(z - 1, 1))
y_start_v_rel(i, 1) = v_rel(z, 1) - grad_v_rel(i, 1) * _
freq(z, 1)
'Gives the interpolated value for velocity in the RS
v_RS_interpol_rel(i, 1) = grad_v_rel(i, 1) * f(i, 1) _
+ y_start_v_rel(i, 1)
Next i

'Relative displacement
Dim grad_d_rel()
ReDim Preserve grad_d_rel(n_dof_red, 1)
Dim d_RS_interpol_rel()
ReDim Preserve d_RS_interpol_rel(n_dof_red, 1)
Dim y_start_d_rel()
ReDim Preserve y_start_d_rel(n_dof_red, 1)

For i = 1 To n_dof_red
z = freq_pos(i, 1)
'Calculates the gradient to be able to interpolate
grad_d_rel(i, 1) = (d_rel(z, 1) - d_rel(z - 1, 1)) / _
(freq(z, 1) - freq(z - 1, 1))
y_start_d_rel(i, 1) = d_rel(z, 1) - grad_d_rel(i, 1) * _
freq(z, 1)
'Gives the interpolated value for displacement in the RS
d_RS_interpol_rel(i, 1) = grad_d_rel(i, 1) * f(i, 1) _
+ y_start_d_rel(i, 1)
Next i
'----------------------------------------------------'

'-------- COMFORT WEIGTHED ---------------------------'
Else     ' If RMS_comf=1

RMS_factors = RMS.RMS(RMS_comf)
factor_a = RMS_factors(1, 1)
If choice = 0 Then
factor_v = RMS_factors(1, 2)
End If

Sheets("ComfWeight_Acceleration").Cells(2, 6) = factor_a

If choice = 0 Then
Sheets("ComfWeight_Velocity").Cells(2, 6) = factor_v
End If

FFT_Comfweight_matrix = FFT_Comfweight.FFT_Comfweight(f)
```

```
For i = 1 To n_dof_red
a_RS_interpol(i, 1) = FFT_Comfweight_matrix(i, 1)
        If choice = 0 Then
        v_RS_interpol(i, 1) = FFT_Comfweight_matrix(i, 2)
        Else
        End If
Next i
'------------------------------------------------'
End If

'--Find maximum response in each DOF and for each mode --'
' Calculate the real response by: gamma_i x phi_i x value from RS
'Total acceleration
Dim a_DOF_mode()
ReDim Preserve a_DOF_mode(n_dof_red, n_dof_red)
Dim MPPF()
ReDim Preserve MPPF(n_dof_red, 1)
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    MPPF(i, 1) = GAMMA_vector(i, 1) * Phi_scaled(j, i)
    a_DOF_mode(j, i) = MPPF(i, 1) * a_RS_interpol(i, 1)
    Next j
Next i

a_dof_mode_max = 0
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(a_dof_mode_max) > Abs(a_DOF_mode(j, i)) Then
        Else
        a_dof_mode_max = Abs(a_DOF_mode(j, i))
        dof_a_max_pos = j
        mode_a_max = i
        End If
    Next j
Next i

If RMS_comf = 0 Then

'Relative acceleration
Dim a_DOF_mode_rel()
ReDim Preserve a_DOF_mode_rel(n_dof_red, n_dof_red)
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    a_DOF_mode_rel(j, i) = GAMMA_vector(i, 1) _
    * Phi_scaled(j, i) * a_RS_interpol_rel(i, 1)
    Next j
Next i
```

```
a_dof_mode_max_rel = 0
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(a_dof_mode_max_rel) > Abs(a_DOF_mode_rel(j, i)) Then
        Else
        a_dof_mode_max_rel = Abs(a_DOF_mode_rel(j, i))
        dof_a_max_pos_rel = j
        mode_a_max_rel = i
        End If
    Next j
Next i
End If

If choice = 0 Then

'Total velocity
Dim v_DOF_mode()
ReDim Preserve v_DOF_mode(n_dof_red, n_dof_red)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    v_DOF_mode(j, i) = GAMMA_vector(i, 1) * _
    Phi_scaled(j, i) * v_RS_interpol(i, 1)
    Next j
Next i

v_dof_mode_max = 0
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(v_dof_mode_max) > Abs(v_DOF_mode(j, i)) Then
        Else
        v_dof_mode_max = Abs(v_DOF_mode(j, i))
        dof_v_max_pos = j
        mode_v_max = i
        End If
    Next j
Next i
End If

If RMS_comf = 0 Then

'Relative velocity
Dim v_DOF_mode_rel()
ReDim Preserve v_DOF_mode_rel(n_dof_red, n_dof_red)

For i = 1 To n_dof_red
```

```
    For j = 1 To n_dof_red
    v_DOF_mode_rel(j, i) = GAMMA_vector(i, 1) * _
    Phi_scaled(j, i) * v_RS_interpol_rel(i, 1)
    Next j
Next i

v_dof_mode_max_rel = 0
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(v_dof_mode_max_rel) > Abs(v_DOF_mode_rel(j, i)) Then
        Else
        v_dof_mode_max_rel = Abs(v_DOF_mode_rel(j, i))
        dof_v_max_pos_rel = j
        mode_v_max_rel = i
        End If
    Next j
Next i

'Relative displacement
Dim d_DOF_mode_rel()
ReDim Preserve d_DOF_mode_rel(n_dof_red, n_dof_red)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    d_DOF_mode_rel(j, i) = GAMMA_vector(i, 1) * _
    Phi_scaled(j, i) * d_RS_interpol_rel(i, 1)
    Next j
Next i

d_dof_mode_max_rel = 0
For i = 1 To n_dof_red
    For j = 1 To n_dof_red
        If Abs(d_dof_mode_max_rel) > Abs(d_DOF_mode_rel(j, i)) Then
        Else
        d_dof_mode_max_rel = Abs(d_DOF_mode_rel(j, i))
        dof_d_max_pos_rel = j
        mode_d_max_rel = i
        End If
    Next j
Next i
Else
End If
'---------------------------------------------------'

'------------------Calculate SRSS values-------------'
'Write in sheet
For i = 1 To UBound(active_DOF)
```

```
Sheets("Beam_Cantilever").Cells(25 + i, 2).Value = _
"DOF:" & active_DOF(i, 1)
Next i

'SRSS Acceleration

'Total
Dim a_SRSS()
ReDim Preserve a_SRSS(n_dof_red, 1)

Dim a_squared_sum()
ReDim Preserve a_squared_sum(n_dof_red, 1)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    a_value_squared = (a_DOF_mode(i, j) ^ 2)
    a_squared_sum(i, 1) = a_squared_sum(i, 1) + a_value_squared
    Next j
    a_SRSS(i, 1) = Sqr(a_squared_sum(i, 1))
    Next i

 If RMS_comf = 0 Then

'Relative
Dim a_SRSS_rel()
ReDim Preserve a_SRSS_rel(n_dof_red, 1)

Dim a_squared_sum_rel()
ReDim Preserve a_squared_sum_rel(n_dof_red, 1)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    a_value_squared_rel = (a_DOF_mode_rel(i, j) ^ 2)
    a_squared_sum_rel(i, 1) = a_squared_sum_rel(i, 1) _
    + a_value_squared_rel
    Next j
    a_SRSS_rel(i, 1) = Sqr(a_squared_sum_rel(i, 1))
    Next i
Else
End If

' SRSS Velocity
If choice = 0 Then

' Total
Dim v_SRSS()
ReDim Preserve v_SRSS(n_dof_red, 1)
```

```
Dim v_squared_sum()
ReDim Preserve v_squared_sum(n_dof_red, 1)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    v_value_squared = (v_DOF_mode(i, j) ^ 2)
    v_squared_sum(i, 1) = v_squared_sum(i, 1) + v_value_squared
    Next j
    v_SRSS(i, 1) = Sqr(v_squared_sum(i, 1))
    Next i
End If

If RMS_comf = 0 Then

'Relative
Dim v_SRSS_rel()
ReDim Preserve v_SRSS_rel(n_dof_red, 1)

Dim v_squared_sum_rel()
ReDim Preserve v_squared_sum_rel(n_dof_red, 1)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    v_value_squared_rel = (v_DOF_mode_rel(i, j) ^ 2)
    v_squared_sum_rel(i, 1) = v_squared_sum_rel(i, 1) _
    + v_value_squared_rel
    Next j
    v_SRSS_rel(i, 1) = Sqr(v_squared_sum_rel(i, 1))
    Next i

' SRSS Displacement

'Relative
Dim d_SRSS_rel()
ReDim Preserve d_SRSS_rel(n_dof_red, 1)

Dim d_squared_sum_rel()
ReDim Preserve d_squared_sum_rel(n_dof_red, 1)

For i = 1 To n_dof_red
    For j = 1 To n_dof_red
    d_value_squared_rel = (d_DOF_mode_rel(i, j) ^ 2)
    d_squared_sum_rel(i, 1) = d_squared_sum_rel(i, 1) _
    + d_value_squared_rel
    Next j
    d_SRSS_rel(i, 1) = Sqr(d_squared_sum_rel(i, 1))
```

```
    Next i

Else
End If

'Output SRSS values
For i = 1 To n_dof_red
Sheets("Beam_Cantilever").Cells(25 + i, 3) = a_SRSS(i, 1)
If RMS_comf = 0 Then
Sheets("Beam_Cantilever").Cells(25 + i, 5) = a_SRSS_rel(i, 1)
Sheets("Beam_Cantilever").Cells(25 + i, 6) = v_SRSS_rel(i, 1)
Else
End If

If choice = 0 Then
Sheets("Beam_Cantilever").Cells(25 + i, 4) = v_SRSS(i, 1)
End If
Next i
'————————————————————————————————————————————————'

MsgBox "Operation_completed!", vbInformation

Application.ScreenUpdating = True
Application.Calculation = xlCalculationAutomatic

End Sub

Option Base 1
'————————————————Response spectrum—————————————————'
Function Response_spectrum(num)

Sheets("Input").Range("O2:O5000").ClearContents

'num=1 gives  matrix with freq, a_max,
'a_rel, v_rel and d_rel (and if choice=0, also v_max)
'num=2 gives  n_steps

'Read if the load is an acceleration or a velocity
choice = Sheets("Input").Cells(5, 12)

'Defining pi
pi = 4 * Atn(1)

'Defining gravity
gravity = 9.82    '[m/s^2]

'Desired min and max period in response spectrum
```

```
max_period = Sheets("Input").Cells(14, 5)
min_period = Sheets("Input").Cells(15, 5)

'Desired increase in frequency
inc = Sheets("Input").Cells(16, 5)
inc = 1 + inc

'Calculates min and max frequencies
'and number of steps needed in iteration
freq_min = 1 / max_period
freq_max = 1 / min_period
n_steps = Log(freq_max / freq_min) / Log(inc)
n_steps = Round(n_steps, 0)

'Clearing previous values in
'"Response spectrum"-sheet, and naming the columns
Sheets("Response spectrum").Range("A1:I1000").ClearContents
Sheets("Response spectrum").Cells(1, 1) = "Frequency, f [Hz]"
Sheets("Response spectrum").Cells(1, 2) = "Period, T [s]"
Sheets("Response spectrum").Cells(1, 3) = "a_tot [mm/s^2]"
Sheets("Response spectrum").Cells(1, 5) = "a_rel [mm/s^2]"
Sheets("Response spectrum").Cells(1, 6) = "v_rel [mm/s]"
Sheets("Response spectrum").Cells(1, 7) = "d_rel [mm]"
If choice = 0 Then
Sheets("Response spectrum").Cells(1, 4) = "v_tot [mm/s]"
End If

'Read input values
M_unit = 1                                     'mass [kg]
zeta = Sheets("Input").Cells(3, 5)          'damping ratio [-]
alpha = Sheets("Input").Cells(6, 5)         'alpha [-]
beta = Sheets("Input").Cells(7, 5)          'beta [-]
d_0 = 0                                        'Initial displacement [m]
v_0 = 0                                        'Initial velocity [m/s]

'Read input vectors for load and time
With Sheets("Input")
LastRow = .Range("A" & .Rows.Count).End(xlUp).row
End With
Dim time As Variant
Dim load As Variant
time = Sheets("Input").Range("A2:A" & LastRow).Value
'For constant time step
delta_t = Sheets("Input").Cells(3, 1) - _
Sheets("Input").Cells(2, 1)
load = Sheets("Input").Range("B2:B" & LastRow).Value
```

```
'Check assumption: constant time step
For i = 1 To (UBound(time, 1) - 1)
    Delta = time(i + 1, 1) - time(i, 1) - delta_t
    If Delta < 1 * 10 ^ -4 Then
        delta_t = delta_t
    Else
    W = i + 2

        'Application.Speech.Speak "Time step not constant!"
        MsgBox _
        "ERROR: Time step not constant! Check Input, A" & W, _
        vbCritical
        End
        End If
Next i

'Define load vector dependant on input as
'ground acceleration or ground velocity
Dim a_ground()
ReDim Preserve a_ground(UBound(time), 1)
If choice = 1 Then      'choice=1 gives acceleration as input
a_ground = load
force = M_PROD_S(a_ground, -M_unit)        '-m    a_ground

Else                                    'choice=0 gives force as input
v_ground = load
For i = 1 To UBound(time) - 1
a_ground(i, 1) = (v_ground(i + 1, 1) - v_ground(i, 1)) / delta_t
Sheets("Input").Cells(1 + i, 15).Value = a_ground(i, 1)
Next i
force = M_PROD_S(a_ground, -M_unit)             '-m    a_ground
End If

'Creating zero-vectors
Dim freq()                              'frequency [Hz]
ReDim Preserve freq(n_steps + 1, 1)
Dim period()                            'period [s]
ReDim Preserve period(n_steps + 1, 1)
Dim omega()                            'period [s]
ReDim Preserve omega(n_steps + 1, 1)

'maximum absolut acceleration [m/s^2]
Dim a_max()
ReDim Preserve a_max(n_steps + 1, 1)
 'maximum absolut velocity [m/s]
Dim v_max()
ReDim Preserve v_max(n_steps + 1, 1)
```

```
'______________________________________________________'
'Creating vector for frequencies, periods,
'natural frequencies, stiffnesses and damping
For i = 1 To n_steps + 1
freq(i, 1) = freq_min * inc ^ (i - 1)
period(i, 1) = 1 / freq(i, 1)
omega(i, 1) = 2 * pi * freq(i, 1)
omega_SDOF = 2 * pi * freq(i, 1)
f_SDOF = freq(i, 1)
k_SDOF = M_unit * omega_SDOF ^ 2
c = 2 * zeta * M_unit * omega_SDOF
m_effective = M_unit + c * delta_t * _
alpha + k_SDOF * delta_t ^ 2 * beta
a_0 = force(1, 1) / M_unit - 2 * zeta * _
omega_SDOF * v_0 - omega_SDOF ^ 2 * d_0
a_abs_0 = a_0 - force(1, 1) / M_unit

If choice = 0 Then
v_abs_0 = v_ground(1, 1)
End If

Dim A()                          'define acceleration vector
ReDim Preserve A(UBound(time, 1), 1)
A(1, 1) = a_0                         'first value = a_0

Dim a_abs()                     'define absolute acceleration vector
ReDim Preserve a_abs(UBound(time, 1), 1)
a_abs(1, 1) = a_abs_0                 'first value = a_abs_0

Dim v_abs()                      'define absolute acceleration vector
ReDim Preserve v_abs(UBound(time, 1), 1)
v_abs(1, 1) = v_abs_0                 'first value = v_abs_0

Dim v()                               'define velocity vector
ReDim Preserve v(UBound(time, 1), 1)  'size velocity vector
v(1, 1) = v_0                         'first value = v_0

Dim d()                               'define displacement vector
ReDim Preserve d(UBound(time, 1),  'size displacement vector
d(1, 1) = d_0                         'first value = d_0
                  'first value = d_abs_0
'______________________________________________________'
'Creating vector for maximum accelerations,
'velocities and displacements
For j = 1 To (UBound(time, 1) - 1)
A(j + 1, 1) = (-M_unit * a_ground(j + 1, 1) - _
```

```
c * (delta_t / 2 * A(j, 1) + v(j, 1)) - k_SDOF * (0.5 * delta_t ^ 2 * _
(1 - 2 * beta) * A(j, 1) + delta_t * v(j, 1) + d(j, 1))) / m_effective
v(j + 1, 1) = A(j, 1) * delta_t * (1 - alpha) + A(j + 1, 1) * _
delta_t * alpha + v(j, 1)
d(j + 1, 1) = A(j, 1) * (delta_t ^ 2) / 2 * (1 - 2 * beta) + _
A(j + 1, 1) * delta_t ^ 2 * beta + v(j, 1) * delta_t + d(j, 1)

'Total absolute values
If choice = 1 Then
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
Else
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
v_abs(j + 1, 1) = v(j + 1, 1) + v_ground(j + 1, 1)
End If
Next j
'-----------------------------------------------------'
'-------------Find maximum values-------------------'
'Find the maximum, total absolute values of
'acceleration and velocity
MaxA = 0
MinA = 0
    For j = 1 To UBound(a_abs, 1)
        If a_abs(j, 1) > MaxA Then
            MaxA = a_abs(j, 1)
        End If
        If a_abs(j, 1) < MinA Then
            MinA = a_abs(j, 1)
        End If
    Next j
a_max(i, 1) = Application.Max(MaxA, Abs(MinA))

If choice = 0 Then
MaxV = 0
MinV = 0
    For j = 1 To UBound(v_abs, 1)
        If v_abs(j, 1) > MaxV Then
            MaxV = v_abs(j, 1)
        End If
        If v_abs(j, 1) < MinV Then
            MinV = v_abs(j, 1)
        End If
    Next j
v_max(i, 1) = Application.Max(MaxV, Abs(MinV))
End If

'Find the maximum, absolute values of
'the relative accelerations, velocities, and displacements
```

```
MaxA = 0
MinA = 0
Dim a_rel()
ReDim Preserve a_rel(n_steps + 1, 1)

    For j = 1 To UBound(A, 1)
        If A(j, 1) > MaxA Then
            MaxA = A(j, 1)
        End If
        If A(j, 1) < MinA Then
            MinA = A(j, 1)
        End If
    Next j
a_rel(i, 1) = Application.Max(MaxA, Abs(MinA))

MaxV = 0
MinV = 0
Dim v_rel()
ReDim Preserve v_rel(n_steps + 1, 1)
    For j = 1 To UBound(v, 1)
        If v(j, 1) > MaxV Then
            MaxV = v(j, 1)
        End If
        If v(j, 1) < MinV Then
            MinV = v(j, 1)
        End If
    Next j
v_rel(i, 1) = Application.Max(MaxV, Abs(MinV))

'Displacement
MaxD = 0
MinD = 0
Dim d_rel()
ReDim Preserve d_rel(n_steps + 1, 1)
    For j = 1 To UBound(d, 1)
        If d(j, 1) > MaxD Then
            MaxD = d(j, 1)
        End If
        If d(j, 1) < MinD Then
            MinD = d(j, 1)
        End If
    Next j
d_rel(i, 1) = Application.Max(MaxD, Abs(MinD))
'————————————————————————————————————————————————'
Next i      ' next frequency
```

```
'Multiply vectors with 1000 to get in mm
a_max = M_PROD_S(a_max, 1000)
a_rel = M_PROD_S(a_rel, 1000)
v_rel = M_PROD_S(v_rel, 1000)
d_rel = M_PROD_S(d_rel, 1000)
If choice = 0 Then
v_max = M_PROD_S(v_max, 1000)
End If

'Insert values to spreadsheet to create response spectra
For i = 1 To UBound(freq, 1)
   Sheets("Response_spectrum").Cells(i + 1, 1) = freq(i, 1)
   Sheets("Response_spectrum").Cells(i + 1, 2) = period(i, 1)
   Sheets("Response_spectrum").Cells(i + 1, 3) = a_max(i, 1)
   Sheets("Response_spectrum").Cells(i + 1, 5) = a_rel(i, 1)
   Sheets("Response_spectrum").Cells(i + 1, 6) = v_rel(i, 1)
   Sheets("Response_spectrum").Cells(i + 1, 7) = d_rel(i, 1)
If choice = 0 Then
   Sheets("Response_spectrum").Cells(i + 1, 4) = v_max(i, 1)
End If

Next i

Dim output_RS()
ReDim Preserve output_RS(UBound(freq), 7)
output_RS(1, 7) = n_steps
    For i = 1 To UBound(freq)
    output_RS(i, 1) = freq(i, 1)
    output_RS(i, 2) = a_max(i, 1)
    output_RS(i, 4) = a_rel(i, 1)
    output_RS(i, 5) = v_rel(i, 1)
    output_RS(i, 6) = d_rel(i, 1)
    If choice = 0 Then
    output_RS(i, 3) = v_max(i, 1)
    End If

    Next i

If num = 1 Then
Response_spectrum = output_RS
End If

End Function

Option Base 1

Function RMS(RMS_comf)
```

```
'_______________________________________________'
'Read if the load is an acceleration or a velocity
choice = Sheets("Input").Cells(5, 12)

'Defining pi
pi = 4 * Atn(1)

'Defining gravity
gravity = 9.82    '[m/s^2]

'Desired min and max period in response spectrum
max_period = Sheets("Input").Cells(14, 5)
min_period = Sheets("Input").Cells(15, 5)

'Desired increase in frequency
inc = Sheets("Input").Cells(16, 5)
inc = 1 + inc

'Calculates min and max frequencies and
'number of steps needed in iteration
freq_min = 1 / max_period
freq_max = 1 / min_period
n_steps = Log(freq_max / freq_min) / Log(inc)
n_steps = Round(n_steps, 0)

'Read input values
M_unit = 1                                   'mass [kg]
zeta = Sheets("Input").Cells(3, 5)    'damping ratio [-]
alpha = Sheets("Input").Cells(6, 5)   'alpha [-]
beta = Sheets("Input").Cells(7, 5)     'beta [-]
d_0 = 0                                      'Initial displacement [m]
v_0 = 0                                      'Initial velocity [m/s]

'Read input vectors for load and time
With Sheets("Input")
LastRow = .Range("A" & .Rows.Count).End(xlUp).row
End With
Dim time As Variant
Dim load As Variant
time = Sheets("Input").Range("A2:A" & LastRow).Value
delta_t = Sheets("Input").Cells(3, 1) - _
Sheets("Input").Cells(2, 1)
load = Sheets("Input").Range("B2:B" & LastRow).Value

'Check assumption: constant time step
For i = 1 To (UBound(time, 1) - 1)
    Delta = time(i + 1, 1) - time(i, 1) - delta_t
```

```
    If Delta < 1 * 10 ^ -4 Then
        delta_t = delta_t
    Else
    W = i + 2

        MsgBox _
        "ERROR: Time step not constant! Check Input, A" & W, _
        vbCritical
        End
        End If
Next i

'—————————————————————————————————————————————————'
'Define load vector dependant on input as
'ground acceleration or ground velocity
Dim a_ground()
ReDim Preserve a_ground(UBound(time), 1)
If choice = 1 Then          'choice=1 gives acceleration as input
a_ground = load
force = M_PROD_S(a_ground, -M_unit)      '-m  a_ground

Else                    'choice=0 gives velocity as input
v_ground = load
For i = 1 To UBound(time) - 1
a_ground(i, 1) = (v_ground(i + 1, 1) - v_ground(i, 1)) / _
delta_t
Sheets("Input").Cells(1 + i, 15).Value = a_ground(i, 1)
Next i
force = M_PROD_S(a_ground, -M_unit)       '-m  a_ground
End If

'Creating zero-vectors
Dim freq()                              'frequency [Hz]
ReDim Preserve freq(n_steps + 1, 1)
Dim period()                            'period [s]
ReDim Preserve period(n_steps + 1, 1)
Dim omega()                             'period [s]
ReDim Preserve omega(n_steps + 1, 1)
Dim a_max()
ReDim Preserve a_max(n_steps + 1, 1)
Dim a_max_RMS()
ReDim Preserve a_max_RMS(n_steps + 1, 1)
Dim v_max()
ReDim Preserve v_max(n_steps + 1, 1)
Dim v_max_RMS()
ReDim Preserve v_max_RMS(n_steps + 1, 1)
'—————————————————————————————————————————————————'
```

```
'Creating vector for frequencies, periods, natural frequencies,
'stiffnesses and damping
For i = 1 To n_steps + 1
freq(i, 1) = freq_min * inc ^ (i - 1)
period(i, 1) = 1 / freq(i, 1)
omega(i, 1) = 2 * pi * freq(i, 1)
omega_SDOF = 2 * pi * freq(i, 1)
f_SDOF = freq(i, 1)
k_SDOF = M_unit * omega_SDOF ^ 2
c = 2 * zeta * M_unit * omega_SDOF
m_effective = M_unit + c * delta_t * alpha + _
k_SDOF * delta_t ^ 2 * beta
a_0 = force(1, 1) / M_unit - 2 * zeta * omega_SDOF * _
v_0 - omega_SDOF ^ 2 * d_0
a_abs_0 = a_0 - force(1, 1) / M_unit

If choice = 0 Then
v_abs_0 = v_ground(1, 1)
End If

Dim A()                                'define acceleration vector
ReDim Preserve A(UBound(time, 1), 1)
A(1, 1) = a_0                          'first value = a_0

Dim a_abs()                      'define absolute acceleration vector
ReDim Preserve a_abs(UBound(time, 1), 1)
a_abs(1, 1) = a_abs_0                  'first value = a_abs_0

Dim v_abs()                      'define absolute acceleration vector
ReDim Preserve v_abs(UBound(time, 1), 1)
v_abs(1, 1) = v_abs_0                  'first value = v_abs_0

Dim v()                                'define velocity vector
ReDim Preserve v(UBound(time, 1), 1)
v(1, 1) = v_0                          'first value = v_0

Dim d()                                'define displacement vector
ReDim Preserve d(UBound(time, 1), 1)
d(1, 1) = d_0                          'first value = d_0
                    'first value = d_abs_0
'————————————————————————————————————————————————————'
'Creating vector for maximum accelerations,
'velocities and displacements
For j = 1 To (UBound(time, 1) - 1)
A(j + 1, 1) = (-M_unit * a_ground(j + 1, 1) - c * _
(delta_t / 2 * A(j, 1) + v(j, 1)) - k_SDOF * (0.5 * delta_t ^ 2 * _
(1 - 2 * beta) * A(j, 1) + delta_t * v(j, 1) + d(j, 1))) / m_effective
```

```
v(j + 1, 1) = A(j, 1) * delta_t * (1 - alpha) + A(j + 1, 1) * _
delta_t * alpha + v(j, 1)
d(j + 1, 1) = A(j, 1) * (delta_t ^ 2) / 2 * (1 - 2 * beta) + _
A(j + 1, 1) * delta_t ^ 2 * beta + v(j, 1) * delta_t + d(j, 1)

'Total absolute values
If choice = 1 Then
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
Else
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
v_abs(j + 1, 1) = v(j + 1, 1) + v_ground(j + 1, 1)
End If
Next j
'---------------------------------------------------'
'-------------RMS, window=1 s----------------------'
With Sheets("Input")
LastValue = .Range("A" & .Rows.Count).End(xlUp).Value
End With
LastValue_rounded = Round(LastValue, 0)
Window = 1 '[s] Desired window
n_window = LastValue_rounded / Window
n_steps_per_window = Round(UBound(time, 1) / _
n_window + 0.5, 0)

'Total acceleration
Dim a_abs_rms()
ReDim Preserve a_abs_rms(n_steps_per_window * _
n_window, 1)

For k = 1 To UBound(a_abs)
a_abs_rms(k, 1) = a_abs(k, 1)
Next k

For k = 1 To UBound(a_abs_rms)
If a_abs_rms(k, 1) = "" Then
a_abs_rms(k, 1) = 0
Else
End If
Next k

Dim a_rms()
ReDim Preserve a_rms(n_window, 1)

For k = 1 To n_window
a_rms_n = 0
For kk = 1 To n_steps_per_window
a_rms_temp = a_abs_rms(n_steps_per_window * _
```

```
(k - 1) + kk, 1)
a_rms_n = a_rms_n + (a_rms_temp ^ 2)
Next kk
a_rms(k, 1) = Sqr(1 / n_steps_per_window * a_rms_n)
Next k

' If input as velocity
' Total velocity
If choice = 0 Then
Dim v_abs_rms()
ReDim Preserve v_abs_rms(n_steps_per_window * _
n_window, 1)

For k = 1 To UBound(v_abs)
v_abs_rms(k, 1) = v_abs(k, 1)
Next k

For k = 1 To UBound(v_abs_rms)
If v_abs_rms(k, 1) = "" Then
v_abs_rms(k, 1) = 0
Else
End If
Next k

Dim v_rms()
ReDim Preserve v_rms(n_window, 1)

For k = 1 To n_window
v_rms_n = 0
For kk = 1 To n_steps_per_window
v_rms_temp = v_abs_rms(n_steps_per_window * _
(k - 1) + kk, 1)
v_rms_n = v_rms_n + (v_rms_temp ^ 2)
Next kk
v_rms(k, 1) = Sqr(1 / n_steps_per_window * v_rms_n)
Next k
End If
'--------------------------------------------------------'
'-------------Find maximum values------------------'

'Find the maximum, absolute values of RMS accelerations
MaxA = 0
MinA = 0
    For j = 1 To UBound(a_rms, 1)
        If a_rms(j, 1) > MaxA Then
            MaxA = a_rms(j, 1)
        End If
```

```
            If  a_rms(j, 1) < MinA Then
                MinA = a_rms(j, 1)
            End If
        Next j
a_max_RMS(i, 1) = Application.Max(MaxA, Abs(MinA))


MaxA = 0
MinA = 0
        For j = 1 To UBound(a_abs, 1)
            If  a_abs(j, 1) > MaxA Then
                MaxA = a_abs(j, 1)
            End If
            If  a_abs(j, 1) < MinA Then
                MinA = a_abs(j, 1)
            End If
        Next j
a_max(i, 1) = Application.Max(MaxA, Abs(MinA))


If choice = 0 Then
MaxV = 0
MinV = 0
        For j = 1 To UBound(v_rms, 1)
            If  v_rms(j, 1) > MaxV Then
                MaxV = v_rms(j, 1)
            End If
            If  v_rms(j, 1) < MinV Then
                MinV = v_rms(j, 1)
            End If
        Next j
v_max_RMS(i, 1) = Application.Max(MaxV, Abs(MinV))


MaxV = 0
MinV = 0
        For j = 1 To UBound(v_abs, 1)
            If  v_abs(j, 1) > MaxV Then
                MaxV = v_abs(j, 1)
            End If
            If  v_abs(j, 1) < MinV Then
                MinV = v_abs(j, 1)
            End If
        Next j
v_max(i, 1) = Application.Max(MaxV, Abs(MinV))
End If
```

```
'———————————————————————————————————————————————'
Next i     ' next frequency

'Multiply vectors with 1000 to get in mm
a_max = M_PROD_S(a_max, 1000)
a_max_RMS = M_PROD_S(a_max_RMS, 1000)
If choice = 0 Then
v_max = M_PROD_S(v_max, 1000)
v_max_RMS = M_PROD_S(v_max_RMS, 1000)
End If

' Decide at which row the average RMS value
'starts calculating from
start_factor = 250
Dim factor_vector_a()
ReDim Preserve factor_vector_a(UBound(freq) - _
(start_factor - 1), 1)
Dim factor_vector_v()
ReDim Preserve factor_vector_v(UBound(freq) - _
(start_factor - 1), 1)

factor_save_a = 0
For i = start_factor To UBound(freq)
factor_vector_a(i - (start_factor - 1), 1) = _
a_max_RMS(i, 1) / a_max(i, 1)
values_a_sum = factor_vector_a(i - _
(start_factor - 1), 1) + factor_save_a
factor_save_a = values_a_sum
Next i

If choice = 0 Then
factor_save_v = 0
For i = start_factor To UBound(freq)
factor_vector_v(i - (start_factor - 1), 1) = _
v_max_RMS(i, 1) / v_max(i, 1)
values_v_sum = factor_vector_v(i - _
(start_factor - 1), 1) + factor_save_v
factor_save_v = values_v_sum
Next i
End If

factor_a = values_a_sum / UBound(factor_vector_a)

If choice = 0 Then
factor_v = values_v_sum / UBound(factor_vector_v)
End If
```

```
Dim RMS_factors()
ReDim Preserve RMS_factors(1, 2)

RMS_factors(1, 1) = factor_a
RMS_factors(1, 2) = factor_v

RMS = RMS_factors


End Function

Option Base 1
'-------------------------------------------------------------'
'The function calculates the response for every frequency
'with Newmark-beta method, FFT and Comfort Weighting
Function FFT_Comfweight(f)
Sheets("Response_spectrum").Range("A2:O5000").ClearContents

'Read if the load is an acceleration or a velocity
choice = Sheets("Input").Cells(5, 12)

'Defining pi
pi = 4 * Atn(1)

'Defining gravity
gravity = 9.82    '[m/s^2]

'Defining number of steps for frequencies
n_steps = UBound(f) - 1

'Read input values
M_unit = 1                                   'mass [kg]
zeta = Sheets("Input").Cells(3, 5)   'damping ratio [-]
alpha = Sheets("Input").Cells(6, 5)  'alpha [-]
beta = Sheets("Input").Cells(7, 5) 'beta [-]
d_0 = 0                                      'Initial displacement [m]
v_0 = 0                                      'Initial velocity [m/s]

'-------------------------------------------------------------'
'Read input vectors for load and time
With Sheets("Input")
LastRow = .Range("A" & .Rows.Count).End(xlUp).row
End With
Dim time As Variant
Dim load As Variant
time = Sheets("Input").Range("A2:A" & LastRow).Value
```

```
delta_t = Sheets("Input").Cells(3, 1) - _
Sheets("Input").Cells(2, 1)
load = Sheets("Input").Range("B2:B" & LastRow).Value

'Check assumption: constant time step
For i = 1 To (UBound(time, 1) - 1)
    Delta = time(i + 1, 1) - time(i, 1) - delta_t
    If Delta < 1 * 10 ^ -4 Then
        delta_t = delta_t
    Else
    W = i + 2
        'Application.Speech.Speak "Time step not constant!"
        MsgBox _
        "ERROR: Time step not constant! Check Input, A" & W, _
        vbCritical
        End
        End If
Next i

'Define load vector dependant on input as
'ground acceleration or ground velocity
Dim a_ground()
ReDim Preserve a_ground(UBound(time), 1)
If choice = 1 Then    'choice=1 gives acceleration as input
a_ground = load
force = M_PROD_S(a_ground, -M_unit)  '-m   a_ground

Else                          'choice=0 gives force as input
v_ground = load
For i = 1 To UBound(time) - 1
a_ground(i, 1) = (v_ground(i + 1, 1) - v_ground(i, 1)) / delta_t
Sheets("Input").Cells(1 + i, 15).Value = a_ground(i, 1)
Next i
force = M_PROD_S(a_ground, -M_unit)            '-m   a_ground
End If

'Creating zero-vectors
Dim freq()                                  'frequency [Hz]
ReDim Preserve freq(n_steps + 1, 1)
Dim period()                                'period [s]
ReDim Preserve period(n_steps + 1, 1)
Dim omega()                                 'period [s]
ReDim Preserve omega(n_steps + 1, 1)
Dim a_max()
ReDim Preserve a_max(n_steps + 1, 1)
Dim v_max()
ReDim Preserve v_max(n_steps + 1, 1)
```

```
'————————————————————————————————————————————————————'
'Creating vector for frequencies,
'periods, natural frequencies, stiffnesses and damping
If choice = 0 Then
Dim v_abs()                    'define absolute acceleration vector
ReDim Preserve v_abs(UBound(time, 1), 1)
v_abs_0 = v_ground(1, 1)
v_abs(1, 1) = v_abs_0                 'first value = v_abs_0
End If

Dim A()                               'define acceleration vector
ReDim Preserve A(UBound(time, 1), 1)


Dim a_abs()                    'define absolute acceleration vector
ReDim Preserve a_abs(UBound(time, 1), 1)


Dim v()                               'define velocity vector
ReDim Preserve v(UBound(time, 1), 1)
v(1, 1) = v_0                         'first value = v_0

Dim d()                               'define displacement vector
ReDim Preserve d(UBound(time, 1), 1)
d(1, 1) = d_0                         'first value = d_0
                     'first value = d_abs_0
Dim a_max_FFT()
ReDim Preserve a_max_FFT(UBound(f), 1)

Dim v_max_FFT()
ReDim Preserve v_max_FFT(UBound(f), 1)

For i = 1 To n_steps + 1
Sheets("1FFTInput").Range("A2:O10000").ClearContents
freq(i, 1) = f(i, 1)
period(i, 1) = 1 / freq(i, 1)
omega(i, 1) = 2 * pi * freq(i, 1)
omega_SDOF = 2 * pi * freq(i, 1)
f_SDOF = freq(i, 1)
k_SDOF = M_unit * omega_SDOF ^ 2
c = 2 * zeta * M_unit * omega_SDOF
m_effective = M_unit + c * delta_t * _
alpha + k_SDOF * delta_t ^ 2 * beta
a_0 = force(1, 1) / M_unit - 2 * zeta * _
omega_SDOF * v_0 - omega_SDOF ^ 2 * d_0
a_abs_0 = a_0 - force(1, 1) / M_unit
```

```
a_abs(1, 1) = a_abs_0              'first value = a_abs_0
A(1, 1) = a_0                               'first value = a_0

'_____________________________________________________'
'Creating vector for maximum accelerations, velocities
'and displacements
For j = 1 To (UBound(time, 1) - 1)
A(j + 1, 1) = (-M_unit * a_ground(j + 1, 1) - c * _
(delta_t / 2 * A(j, 1) + v(j, 1)) - k_SDOF * (0.5 * delta_t ^ 2 * _
(1 - 2 * beta) * A(j, 1) + delta_t * v(j, 1) + d(j, 1))) / m_effective
v(j + 1, 1) = A(j, 1) * delta_t * (1 - alpha) + A(j + 1, 1) * _
delta_t * alpha + v(j, 1)
d(j + 1, 1) = A(j, 1) * (delta_t ^ 2) / 2 * (1 - 2 * beta) + _
A(j + 1, 1) * delta_t ^ 2 * beta + v(j, 1) * delta_t + d(j, 1)

'Total values
If choice = 1 Then
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
Else
a_abs(j + 1, 1) = A(j + 1, 1) + a_ground(j + 1, 1)
v_abs(j + 1, 1) = v(j + 1, 1) + v_ground(j + 1, 1)
End If
Next j
'_____________________________________________________'
'__________________FFT_______________________________'
'Find dimensions of input vectors
exp_FFT = Round(Log(UBound(time)) / Log(2) + 0.5)
length_FFT = 2 ^ exp_FFT
Dim time_FFT()
ReDim Preserve time_FFT(length_FFT, 1)

Dim freq_FFT()
ReDim Preserve freq_FFT(length_FFT, 1)
freq_FFT(1, 1) = 0

Dim P1_FFT_acc()
ReDim Preserve P1_FFT_acc(length_FFT, UBound(f))

Dim P1_FFT_vel()
ReDim Preserve P1_FFT_vel(length_FFT, UBound(f))

Dim a_tot_FFT()
ReDim Preserve a_tot_FFT(length_FFT, 1)

'Defining frequency range for FFT analysis output
f_sample = 1 / delta_t                'Sample frequency [Hz]
f_folding = f_sample / length_FFT   'Folding frequency [Hz]
```

```
For j = 2 To length_FFT
freq_FFT(j, 1) = freq_FFT(j - 1, 1) + f_folding
'Sheets("1FFTInput").Cells(j + 1, 3).Value = freq_FFT(j, 1)
Next j

Const FFT As String = "ATPVBAEN.XLAM!Fourier"
' Fast Fourier transformation for total acceleration
If choice = 1 Then            'IF LOAD AS ACCELERATION

        For k = 1 To UBound(time)
                time_FFT(k, 1) = time(k, 1)
                a_tot_FFT(k, 1) = a_abs(k, 1)
        Next k

        For k = 1 To length_FFT
                If time_FFT(k, 1) = "" Then
                time_FFT(k, 1) = 0
                a_tot_FFT(k, 1) = 0
                Else
                End If
        Next k
        For j = 1 To length_FFT
        Sheets("1FFTInput").Cells(j + 1, 1).Value = a_tot_FFT(j, 1)
        Next j
'---------------------------------------------------------'
        Run FFT, Sheets("1FFTInput").Range("A2:A4097"), _
        Sheets("1FFTInput").Cells(2, 2), False, False
        For j = 1 To length_FFT
        P1_FFT_acc(j, i) = (2 / length_FFT) * _
        WorksheetFunction.ImAbs(Sheets("1FFTInput").Cells(j + _
        1, 2).Value)
        'Sheets("1FFTInput").Cells(j + 1, 4) = P1_FFT_acc(j, i)
        Next j

' Fast Fourier transformation for total acceleration
'and total velocity
Else                                'If load as a velicity

        Dim v_tot_FFT()
        ReDim Preserve v_tot_FFT(length_FFT, 1)

        For k = 1 To UBound(time)
                time_FFT(k, 1) = time(k, 1)
                a_tot_FFT(k, 1) = a_abs(k, 1)
                v_tot_FFT(k, 1) = v_abs(k, 1)
        Next k
```

```
        For k = 1 To length_FFT
                If time_FFT(k, 1) = "" Then
                time_FFT(k, 1) = 0
                a_tot_FFT(k, 1) = 0
                v_tot_FFT(k, 1) = 0
                Else
                End If
        Next k

        For j = 1 To length_FFT
        Sheets("1FFTInput").Cells(j + 1, 1).Value = a_tot_FFT(j, 1)
        Sheets("1FFTInput").Cells(j + 1, 5).Value = v_tot_FFT(j, 1)
        Next j

        Run FFT, Sheets("1FFTInput").Range("A2:A4097"), _
        Sheets("1FFTInput").Cells(2, 2), False, False
        Run FFT, Sheets("1FFTInput").Range("E2:E4097"), _
        Sheets("1FFTInput").Cells(2, 6), False, False
        For j = 1 To length_FFT
        P1_FFT_acc(j, i) = (2 / length_FFT) * _
        WorksheetFunction.ImAbs(Sheets("1FFTInput").Cells(j + _
        1, 2).Value)
        P1_FFT_vel(j, i) = (2 / length_FFT) * _
        WorksheetFunction.ImAbs(Sheets("1FFTInput").Cells(j + _
        1, 6).Value)
        Next j

End If
'-----------------------------------------------------'
'---------------Comfort weighting in Excel sheet'----------'

If choice = 1 Then          ' Acceleration
For j = 1 To length_FFT / 2
Sheets("ComfWeight_Acceleration").Cells(25 + _
j, 3) = freq_FFT(j, 1)
Sheets("ComfWeight_Acceleration").Cells(25 + _
j, 4) = P1_FFT_acc(j, i)
Next j

 'Refresh sheet, "F9"
Sheets("ComfWeight_Acceleration").Calculate
a_max_FFT(i, 1) = _
Sheets("ComfWeight_Acceleration").Cells(4, 45).Value

Else                              ' Acceleration and velocity
For j = 1 To length_FFT / 2
```

```
Sheets("ComfWeight_Acceleration").Cells(25 + j, 3) = _
freq_FFT(j, 1)
Sheets("ComfWeight_Acceleration").Cells(25 + j, 4) = _
P1_FFT_acc(j, i)

Sheets("ComfWeight_Velocity").Cells(25 + j, 3) = _
freq_FFT(j, 1)
Sheets("ComfWeight_Velocity").Cells(25 + j, 4) = _
P1_FFT_vel(j, i)
Next j
Sheets("ComfWeight_Acceleration").Calculate
Sheets("ComfWeight_Velocity").Calculate
a_max_FFT(i, 1) = _
Sheets("ComfWeight_Acceleration").Cells(4, 45).Value
v_max_FFT(i, 1) = _
Sheets("ComfWeight_Velocity").Cells(4, 45).Value
Sheets("ComfWeight_Velocity").Range("C26:D10000").ClearContents
End If
Sheets("ComfWeight_Acceleration").Range("C26:D10000").ClearContents

Next i      ' next frequency
'_____________________________________________________'
Dim FFT_Comfweight_matrix()
ReDim Preserve FFT_Comfweight_matrix(UBound(f), 2)

For i = 1 To UBound(f)
FFT_Comfweight_matrix(i, 1) = a_max_FFT(i, 1)
    If choice = 0 Then
    FFT_Comfweight_matrix(i, 2) = v_max_FFT(i, 1)
    End If
Next i
FFT_Comfweight = FFT_Comfweight_matrix
End Function
```