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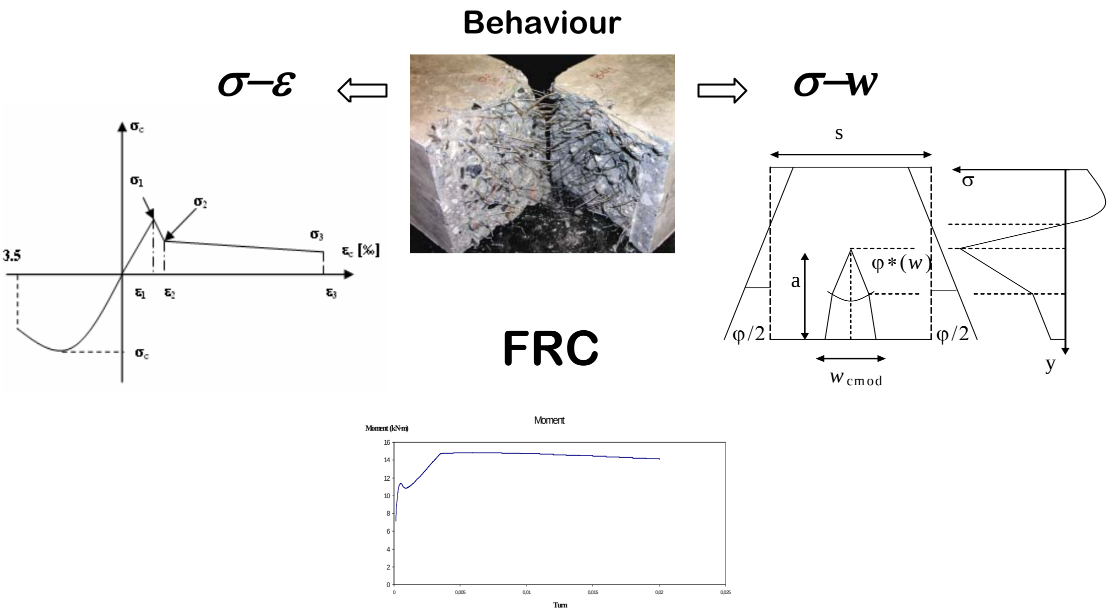

Analysis of Fibre-reinforced Concrete Elements

Bending with σ-ε and σ-w approach

Master's Thesis in the International Master's Programme Structural Engineering

DAVID MARTÍNEZ MARTÍNEZ

Department of Civil and Environmental Engineering
Division of Structural Engineering
Concrete Structures
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2006

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Cover:
Top left: Stress-strain diagram corresponding to the σ–ε approach, see section 3.1
Top centre: Cracked fibre reinforced concrete specimen
Top right: Stress in a FRC cross-section and crack opening profile using Olesen approach, see section 3.2
Bottom: Moment versus turn graph using σ–w approach

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ABSTRACT

The use of fibres has gone through quite a big development in the last 30 years. The advantages of FRC (fibre-reinforced concrete) are proved but the structural behaviour has to be clarified. A common application for FRC are slabs on grade and other non-structural elements but structural elements, like beams, slabs or walls, need an appropriate structural analysis.

RILEM (International Union of Laboratories and Experts in Construction Materials) has developed several recommendations with regard to structural design of FRC members. Many countries may adopt these recommendations due to the lack of other design codes but these recommendations are still being developed and they cannot be considered as a real design code.

In this thesis, the RILEM TC 162-TDF recommendations available for analysing flexural members have been investigated. Two approaches describing the tensional and flexural behaviour of FRC are presented, namely the σ–ε (stress-strain) and the σ–w (stress-crack opening) approach. The work carried out has been focused on (1) analytical non-linear calculations and (2) finite element calculations. All the material properties required to analyse beams and slabs members are presented, as well as a detailed study of the available expressions to calculate the crack-spacing. Some of the material properties have been obtained from laboratory tests while others have been obtained using the conventional reinforced concrete codes like EC2. Different sizes of the elements as well as other characteristics, such as fibre dosage or concrete strength, have also been studied in order to investigate the influences that a change on them causes.

It is concluded that both approaches can be used in the design but some modifications may bee needed in one of them (σ–ε) in order to obtain similar characteristic result. It was found that the σ–ε approach might not be suitable for FEM calculations. In contrast, the result confirm that the σ–w is a very good approach; no size effect is identified comparing the analytical calculations and the FEM calculations and it was found that the for the ultimate limit state the value of the crack-spacing is not so important.

Key words: concrete, fibre-reinforced, crack-spacing, stress-crack opening relationship, non-linear analysis

Análisis de Elementos de Hormigón Reforzado con Fibras
Flexión utilizando los modelos σ–ε y σ–w
Master's Thesis in the International Master's Programme Structural Engineering
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RESUMEN

El uso de fibras ha sufrido un gran desarrollo en los últimos 30 años. Las ventajas del FRC (hormigón reforzado con fibras) han sido demostradas, pero el comportamiento estructural tiene que ser clarificado. Usos comunes para el FRC son losas de cimentación y otros elementos no estructurales, pero los elementos estructurales como vigas, forjados o paredes necesitan un análisis estructural más detallado.

RILEM (Unión Internacional de Laboratorios y Expertos en Materiales de Construcción) ha desarrollado diversas recomendaciones respecto al diseño estructural de elementos de FRC. Muchos países adoptan esas recomendaciones debido a que no existen otros códigos de diseño, pero esas recomendaciones están todavía siendo desarrolladas y no se pueden considerar como un verdadero código de diseño.

Esta tesis analiza las herramientas de diseño disponibles para el estudio de miembros sometidos a flexión pura. Dos modelos que explican el comportamiento en tensión del FRC son estudiados: σ–ε (tensión-deformación) y σ–w (tensión-apertura de grieta). El trabajo llevado a cabo esta centrado en (1) cálculos analíticos utilizando análisis no lineal (2) cálculos usando un modelo de elementos finitos. Todos los datos que son necesarios para analizar vigas y losas son presentados, así como un estudio detallado de las actuales expresiones disponibles para calcular un valor realista de la distancia entre grietas. Algunos de estos valores son extraídos de tests anteriormente realizados mientras que otros son obtenidos usando códigos habituales de diseño como el eurocódigo 2 (EC2).

La tesis concluye que ambas aproximaciones pueden ser utilizadas en diseño, pero son necesarias algunas modificaciones en una de ellas (σ–ε) para obtener un resultado similar en ambas. También es demostrado que el modelo σ–ε no es adecuado para ser utilizado en modelos de elementos finitos y que el modelo σ–w es una aproximación realmente buena. No se ha identificado ningún efecto de forma comparando los resultados analíticos con los de elementos finitos cosa que también demuestra que la influencia de la distancia entre grietas no es tan importante cuando se está analizando el estado límite de servicio.

Key words: concrete, fibre-reinforced, crack-spacing, stress-crack opening relationship, non-linear analysis

Contents

Preface

This master thesis was carried out from November 2005 to May 2006 at the Division of Structural engineering, Department of Civil and Environmental engineering at Chalmers University of Technology in Göteborg, Sweden. The thesis is performed within the European Community exchange program Socrates Erasmus as an outcome of the agreement existing between Chalmers University of Technology, Sweden, and the University of Oviedo, Spain.

First of all I would like to thank my supervisor in Sweden Ingemar Löfgren, PhD, for all the continuous support given these months. I specially appreciate the great ideas about the topic, the constant improvement comments, his useful help when problems appeared and of course his huge patience.

I'm also grateful to Prof. Björn Engström who gave me the great opportunity to work in this department and gave me wise advices in the first days when I was lost in this new country.

Also I would like to thank to my supervisor in Spain Mª Jesus Lamela who guided me before arriving in Sweden which made the first hard moments easier. Thanks to my opponents Qi Huang and T.T Nhan for their helpful comments.

Of course I need to extend thanks to all the Högsbo-family, who have supported me all these fabulous months. I made very good friends from the entire world and I hope this friendship will last forever. Also thanks to my friends from Spain who are very important to me.

Finally all my gratitude goes to my family (dad, mom, Javi and Kyra) for their affection and their blind trust in my decisions. I will always be grateful for their support.

Göteborg, May 2006

David

Notations

Roman upper case letters

A_c	Cross-sectional area of concrete
A_s	Cross-sectional area of conventional reinforcement
$A_{c,eff}$	Effective cross-sectional area of concrete
$D_{dstiffbond}$	Dummy interface stiffness
E_c	Young modulus of concrete
E_s	Young modulus of steel
E_{cRILEM}	Young modulus of the concrete using RILEM approximation
F_L	Maximum load in the interval (δ or $CMOD$) of 0.05mm
F_s	Tensile force supported by steel
F_{cc}	Compressive force supported by concrete
F_{ct}	Tensile force supported by concrete
$F_{R,i}$	Load recorded at $CMOD$ or $\delta_{R,i}$
F_{ctc}	Tensile force supported by concrete in the tensile cracked zone
F_{cte}	Tensile force supported by concrete in the tensile elastic zone
G	Shear modulus
G_F	Fracture energy
K_1	Constant to calculate the crack spacing (Ibrahim and Luxmoore)
$K_2^{'}$	Constant to calculate the crack spacing (Ibrahim and Luxmoore)
K_3	Constant to calculate the crack spacing (Ibrahim and Luxmoore)
L	Span of the specimen
L_{FIB}	Length of the fibres
M_R	Resistant moment
M_s	Moment supported by steel
M_{cc}	Moment supported by concrete in the compression zone
M_{ct}	Moment supported by concrete in the tensile zone
M_{ctc}	Moment supported by concrete in the tensile cracked zone
M_{cte}	Moment supported by concrete in the tensile elastic zone
N	External normal load
$P_{f,pull}$	Pull-out load of fibres per unit area
S_i	Pull-out load of fibres per unit area
V_f	Volume of fibres

Roman lower case letters

a	Crack extension
b	Width of the element
d	Distance of the reinforcement to the top of the section

d_1	Distance of the reinforcement to the top of the section
f_{cd}	Design compressive strength of concrete
f_{ck}	Characteristic compressive strength of concrete
f_{cm}	Mean compressive strength of concrete
f_{ct}	Tensile strength of the concrete
f_{yd}	Design yielding strength of steel conventional reinforcement
f_{yk}	Characteristic yielding strength of steel conventional reinforcement
$f_{R,i}$	Residual flexure strength
$f_{fct,L}$	Residual flexure strength corresponding to F_L
h	Height of the element
h_{sp}	Distance between the tip of the notch and top of cross section
k	Constant to calculate the maximum stress of concrete in compression
k_1	Constant to calculate the crack spacing (EC2)
k_2	Constant to calculate the crack spacing (EC2)
k_3	Constant to calculate the crack spacing fibre factor
k_s	Constant to calculate the maximum strength of steel reinforcement
$l_{t,\max}$	Maximum transfer length
n	Number of reinforcement bars
n_S	Step number
r	Radius of curvature
s	Length of the non-linear hinge or crack spacing
$s_{B\&B}$	Crack spacing calculated by means of Borosnyói and Balázs expression
$s_{I\&L}$	Crack spacing calculated by means of Ibrahim and Luxmoore expression
$s_{m,EC2}$	Crack spacing calculated by means of EC2 expression
$s_{m,\max}$	Maximum value of the crack spacing
$s_{m,EC2F}$	Crack spacing calculated by means of EC2 with fibre factor expression
$s_{m,RILEM}$	Crack spacing calculated by means of RILEM expressionn
$s_{m,VANDE}$	Crack spacing calculated by means of Vandewalle expression
$u_{\mathrm{cov}er}$	Cover of reinforcement bars
u_{spac}	Spacing between reinforcement bars
w	Crack opening
w_c	Critical crack opening for which $\sigma(w)=0$
w/b	Water binder ratio
w_{CMOD}	Crack opening in the bottom of the section (maximum)
y	Distance to the top of the section
y_0	Position of the neutral axis
y_{0ini}	Initial guest value for the position of the neutral axis

Greek lower case letters

α_i	Slopes of the sigma-crack opening curve

α	Angle in the strain diagram of a cross-section
β	Coefficient to calculate bond-stress
β_{crack}	Coefficient to calculate crack width using σ–ε approach
γ	Increase of anchorage of bars due to fibre inclusion
γ_s	Safety coefficient of steel in the design
δ	Deflection
ε_{abs}	Absolute deformation
ε	Strain
ε_c	Strain in the concrete
ε_s	Strain in steel reinforcement
ε_{sm}	Mean steel strain in the reinforcement used to calculate the crack width in σ–ε approach
$\varepsilon_{s.1}$	Strain in steel reinforcement (first layer)
ε_{c1}	Strain when the maximum compressive strength of concrete is reached
ε_{c2}	Strain when the maximum compressive strength of concrete is reached
ε_{abs}	Absolute deformation
ε_{cu1}	Maximum strength of concrete in compression
ε_{cu2}	Maximum strength of concrete in compression
ε_{syd}	Yielding strain of steel in the design
ε_{syk}	Characteristic yielding strain in concrete
ε_{sud}	Ultimate design strain of steel
ε_{suk}	Ultimate characteristic strain of steel
η_s	Ratio of the load carried by the conventional reinforcement relative to the total applied load
κ	Curvature
κ_h	Size factor
ν	Poisson coefficient
ρ	Density
ρ_{eff}	Effective reinforcement ratio
σ	Stress
$\sigma(w)$	Stress as a function of the crack opening
σ_c	Stress in the concrete
σ_s	Tensile stress conventional reinforcement
$\sigma_{s.1}$	Tensile stress conventional reinforcement first layer
σ_w	Stress in transferred by fibres
σ_{cc}	Compressive stress concrete
σ_{ct}	Tensile stress of concrete
σ_{cte}	Tensile stress of concrete in the tensile elastic zone
σ_{ctf}	Tensile stress of concrete in the tensile cracked zone
$\tau(x)$	Bond stress as a function of the length

$\tau_{0.05}$	Bond stress when the relative displacement is 0.05mm
τ_d	Average sliding friction bond stress of fibres
τ_{bm}	Average bond stress
$\tau_{\max}$	Maximum bond stress
τ_{bond}	Average sliding friction bond stress of fibres
τ_{final}	Final bond stress
φ	Absolute turn of the section
φ^*	Opening angle of the crack

Greek upper case letters

$\Delta load$	Basis value for load step
Δr	Extension of the damage region
ϕ_b	Diameter of the reinforcement bar
ϕ_{fib}	Diameter of the fibres

Abbreviations

CMOD	Crack Mouth Opening Displacement
DTU	Technical University of Denmark
EC2	Eurocode 2
FEM	Finite Element Method
FRC	Fibre-Reinforced Concrete
RC-65/60	Specification of Dramix® Fibre (65/60= aspect ratio/length)
RILEM	International Union of Laboratories and Experts in Construction Materials
SLS	Service Limit State
ULS	Ultimate Limit State
WCMOD	Value of the Crack Mouth Opening Displacement
WST	Wedge-Splitting Test
3PBT	Three-Point Bending Test

e.g.	For example (Latin *empli gratia*)
i.e.	That is (Latin *id est*)
vs.	Versus
$\sigma - \varepsilon$	Stress-strain
$\sigma - w$	Stress-crack opening

1 Introduction

1.1 Fibre reinforced concrete.

Fibre reinforced concrete is a composite material that is made of concrete and short fibres. The fibres can be considered as, more ore less, uniformly distributed and their orientation is usually random. Fibre-reinforced concrete can also be combined with conventional reinforcement (steel bars) and post-tensioning or prestressing.

Although fibre-reinforced concrete is a relatively young material, some cultures have used fibres as reinforcement in other materials in different ways. For example, old buildings were made of clay and straw fibres, and the builders made them without any significant technical knowledge.

Currently, the use of fibres has gone through a quite big development in the last 30 years, see e.g. Zollo (1997). There are many applications for fibres e.g. cars, industrial devices, etc. but the application that is going to be treated in the thesis is the fibre reinforced concrete (FRC).

Traditionally steel bars have been used to improve the tensile behaviour of concrete structures. But nowadays, the increased cost of steel has made it necessary to find new materials and ways to design the structural elements. Figure 1 shows the development of steel prices in Europe during the last 11 years, although last year the prices have decreased a bit, they are still high.

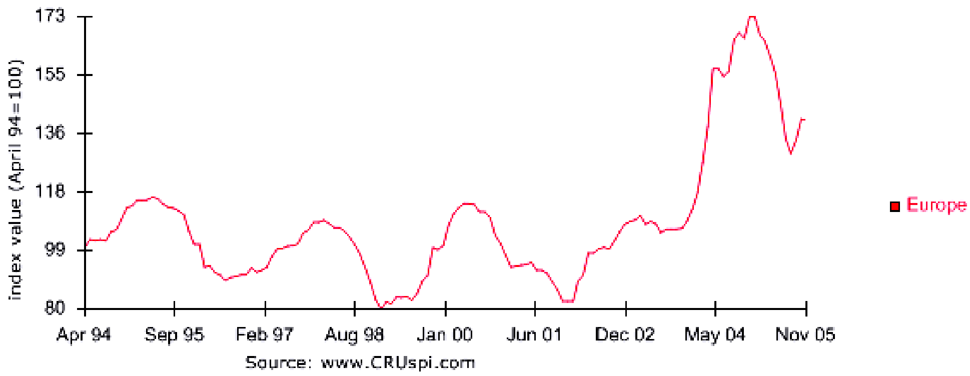


Figure 1.1a Prices of steel in Europe from 1994 to 2005 (from www.CRUspi.com)

There are many kinds of fibres that can be used in FRC: steel, glass, polymeric, carbon, etc. but the most common type is the steel fibres. The other type of fibres could be very important in the future if the cost of the steel continues to increase.

Apart from the issue of the cost, there are many advantages in the use of FRC:

- Improved post-peak response in flexural, which means increased capacity to carry load. The post-peak response usually begins when elastic limit of the concrete in tension is reached, see chapter 2.

- Improved tensile ductility.

- Possible to achieve a 3-D distribution of the fibres, which is favourable for triaxial loads.

- Excellent repair material, e.g. old structures.

- Probably reduced cost of the execution of the work, moreover, the work techniques of manufacturing and distribute fibres are developing quite fast. See Li (2002).

- Advantages in durability

There is, nevertheless, not so much information about the structural behaviour of the FRC structures and if it is possible that the conventional reinforcement can be avoided completely.

Hence, there is a long path to travel until FRC is a commonly used material.

1.2 Applications of FRC

Most of the actual applications of FRC are in non-structural or semistructural elements. This is mostly due to the fact that there is no completely developed code available or a systematic guide to design elements made of FRC.

Examples of applications are: pavements, walls, beams, slabs, tunnel-linings, etc. The use of FRC in these applications also leads to improvements in their behaviour. These depend on the type of load, Li (2002):

- **Flexural members** → Improves the post-peak response and increase the post-peak load.

- **Shear loads** → Increased shear capacity and post-cracking safety.

- **Torsion loads** → Increased torsional capacity and post-cracking safety.

- **Uniaxial tension-members** → increased joint spacing and reduced crack widths.

New application areas may be discovered as more tests and investigations are conducted. For that reason, it is important to define a good and not too complicated code that is useful for all the designers who want to use FRC.

1.3 Background

RILEM (International union of laboratories and experts in construction materials) is an association that has been involved in different studies about FRC among other issues. Several countries may adopt the RILEM TC 162-TDF recommendations due to the lack of other design codes or recommendations.

RILEM has developed two models that try to define the behaviour of FRC, see RILEM TC 162-TDF (2003). It is very important to get a good theoretical model in order to facilitate the designers and extend the use of FRC to structural elements. Another important issue is the importance of knowing the properties of FRC. There are many parameters that are important like w/c-ratio (mix design), class of steel, shape of the fibres, and quantity of fibres. Shape and class of the fibres are obviously very easy to determine, but some properties of the FRC, like tensile strength, and the exact amount of fibres and where are they located, are not so easy to determine. Some tests are being developed in order to obtain flexural parameters of FRC, and a comparison between them has been made, while others are developed to obtain uni-axial properties i.e. $\sigma - w$, see Löfgren et al. (2005). The wedge split test (WST) and three point bending test (3PBT) are two of them.

1.4 Aim and scope of the thesis

The aim of the thesis is to analyse what is the most appropriate model to design and calculate different structures made of fibre reinforced concrete. This is very important because the most appropriate approach or the easiest to apply should be use by the future article of the code about FRC. The thesis will analyse what is the most realistic approach in the ultimate limit state (ULS) and the service limit state (SLS).

Laboratory test have been used to determine the values of the constants according to RILEM specifications. These values have then been introduced in FEM software (DIANA) to simulate the behaviour. Also some equilibrium equations are derived in order to obtain the value that can be obtained by hand calculations. Both ways of analysis are compared in order to decide the validity of the approaches.

Also 3 different sizes and 3 different mixes (concrete with fibres) have been analysed to investigate the effects that can be distinguished by the models if a change of these factors are done. The effect of the bond slip is also considered in the FEM calculations although it is not considered in the analytical analysis.

Finally, in this thesis, it has also been important to use a realistic value for the spacing between cracks when an element is being loaded. Different proposals have been analysed and finally one of those has been chosen as the most appropriate to be used in this thesis.

1.5 Limitations

The thesis is limited to the analysis of these elements:

- behaviour of beams in flexural have been analysed.
- behaviour of slabs in flexural have been analysed.

It is not considered the time-dependent effects in the concrete like creep or shrinkage.

1.6 Outline of the thesis

The thesis consists of 8 chapters that are numbered in chronological order. It allows a good understanding of the research process from the beginning to the end.

In the second chapter, the theoretical basis about the RILEM material models is presented and the approaches are studied in order to get a good understanding of them. It is also shown how to calculate all the values that are needed for the hand calculations and numerical analysis.

In the third chapter analytical expressions to obtain some the variables when the element is being loaded are derived. Also a study of the approaches available for obtaining the crack spacing in flexural elements is made.

The forth chapter explain the FEM models that have been developed for both approaches and the inputs that have to be chosen in order to get the appropriate accuracy in the results.

The fifth chapter explains the numerical values of all the constants required for the hand calculations and the FEM analysis. Furthermore, the values for the crack spacing using the different approaches are compared and one of the approaches is chosen as the most appropriate.

The sixth chapter explains all the results obtained in the analytical calculations as well as in the FEM calculations. Values for the crack spacing are presented and comparisons between the results introducing variations as height of the element, quantity of fibres or concrete strength are made. Also changes in some values of the RILEM constants are introduced in order to obtain a better accuracy of the results.

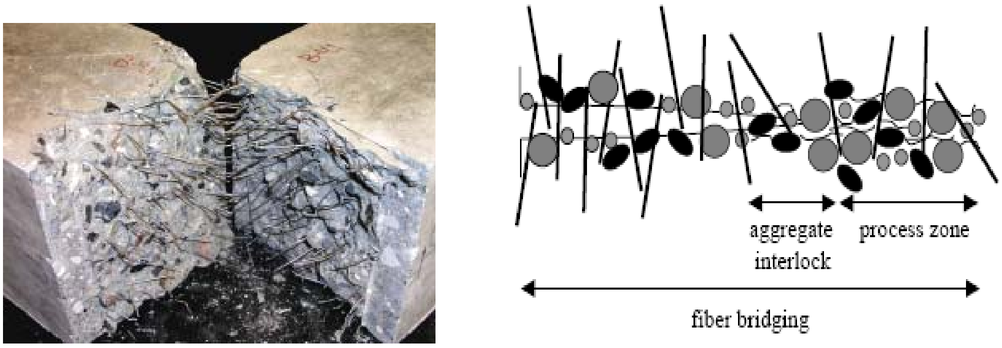
Chapter seven includes the summary, conclusions and suggestion for future research in the FRC design field.

2 Approaches to analyse the flexural behaviour of FRC

In this chapter, a theoretical basis about the two approaches developed by RILEM TC 162-TDF will be explained. The models explain the behaviour of fibre reinforced concrete before and after peak-load.

2.1 The σ–ε approach

This approach is based on more or less the same fundamentals that are used for normal reinforced concrete. However, when normal reinforced concrete is analysed, or used for designing, the σ–ε relationship is very different. This is due to that FRC has a post-cracking resistance, the fibres make a bridge between the cracks and make possible for the concrete to carry a tensile load across the crack, see figure 2.1



(a) (b)

Figure 2.1 Crack process in FRC where fibre bridging occurs, from Löfgren (2005) and RILEM TC 162-TDF (2003).

Figure 2.1a and 2.1b show the real behaviour and the model used for the design of normal reinforced plain concrete elements that are tested in uniaxial tension:

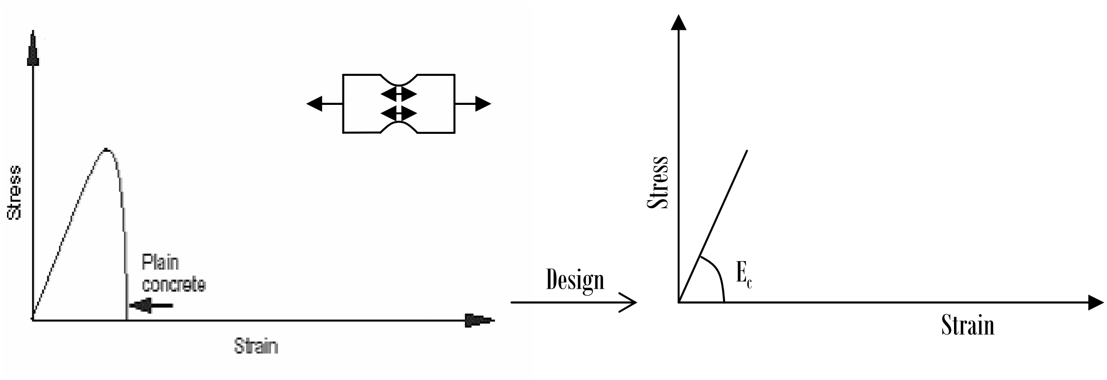


Figure 2.1a σ–ε relationship (Uniaxial tension tests) in plane reinforced concrete

Figure 2.1b model used for the design of plane reinforced concrete elements. The post-peak resistance is neglected

Figure 2.2 shows the simplified behaviour of FRC and plain concrete when loaded in uniaxial tension, it also can be appreciated that a general behaviour is that the load carrying capacity can be improved if more fibres are added (fibre volume).

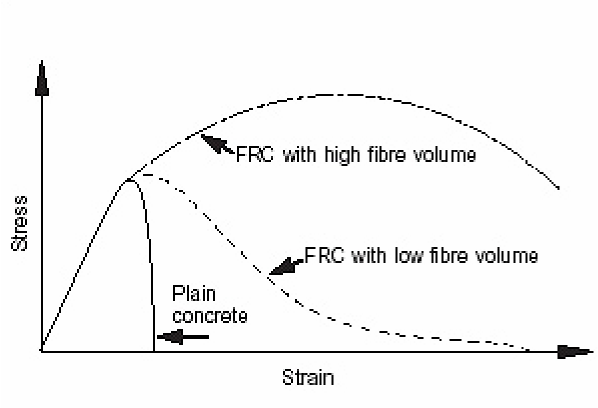


Figure 2.2 Example of the behaviour of FRC and plain concrete. From cement and concrete institute, www.cnci.org.za

To define all the parameters of the RILEM TC 162-TDF σ–ε model, some constants and properties of the concrete have to be known:

f_{cm} is the mean compressive strength of fibre reinforced concrete obtained by concrete cylinder test and $f_{R,i}$ is the residual flexure strength. The residual flexure strength can be determined by conducting a three point bending test and is calculated using the following expression, see RILEM TC 162-TDF (2003):

$$f_{R,i} = \frac{3F_{R,i}L}{2bh^2_{sp}} \quad (\text{N/mm}^2) \tag{2.1}$$

$$f_{fct,L} = \frac{3F_L L}{2bh^2_{sp}} \quad (\text{N/mm}^2) \tag{2.2}$$

Where:

b is the width of the specimen (mm)

h_{sp} is the distance between the tip of the notch and top of cross section (mm). View section 5.1

L is the span of the specimen (mm)

$F_{R,i}$ is the load recorded at $CMOD_i$ or $\delta_{R,i}$ (N)

F_L is the maximum load in the interval (δ or $CMOD$) of 0.05mm

When all the parameters are known, the design model can be defined. Figure 2.3 shows the stress-strain diagram that defines the behaviour of the FRC element. By means of the following expressions it is possible to calculate all the points of the tensile stress-strain diagram and the Young's Modulus in compression and in tension for this approach (the same in both cases).

It is important to underline that test results are needed to calculate most of the parameters, but there are two variables (*h* and *d*) that has to be included from the real element that is going to be analysed. In other words, the test is just used to calculate the $f_{R,i}$ parameters but not the σ_i values if the cross-sectional height of the considered element is different than the tested one.

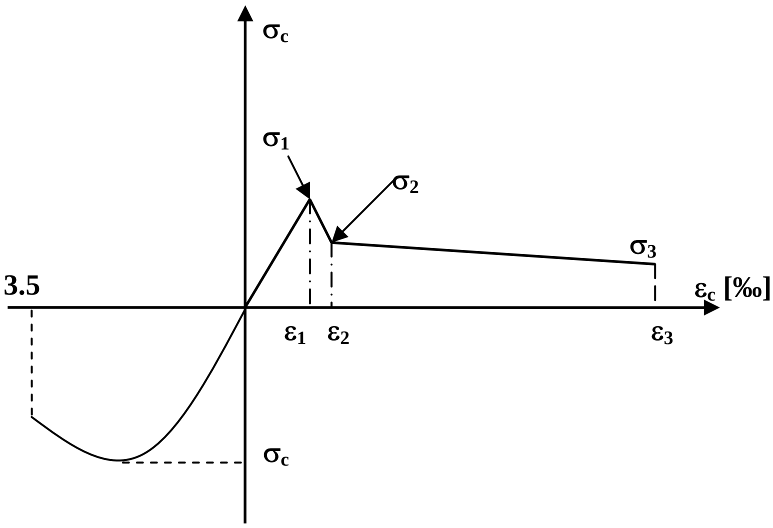


Figure 2.3 Stress-Strain diagram of FRC (σ–ε approach)

Where:

$$\sigma_1 = 0.7 \cdot f_{fctm,l}(1.6 - d) \ \ (\text{d in m}) \qquad \varepsilon_1 = \sigma_1 / E_c \tag{2.3}$$

$$\sigma_2 = 0.45 \cdot f_{R,1} \cdot \kappa_h \qquad \varepsilon_2 = \varepsilon_1 + 0.1‰ \tag{2.4}$$

$$\sigma_3 = 0.37 \cdot f_{R,4} \cdot \kappa_h \qquad \varepsilon_3 = 25‰ \tag{2.5}$$

$$E_c = 9500 \cdot (f_{fcm})^{\frac{1}{3}} \tag{2.6}$$

In the model, a size factor has been introduced, which is necessary to apply if the height of the beam or slab is different from the tested one, as it is in most cases. RILEM TC 162-TDF (2003) explains the use of this factor, stating that when a comparison with experimental results was made, a strong overestimation of the load carrying capacity was found. They also underline that the origin of the size-effect is not fully understood. Nanakorn and Horii (1996), suggests that a size-factor could exist due to that the crack lengths are very different if the size is changed. Therefore, if the length of the crack changes, regarding the stress-crack opening relationship, the load carried will vary. Moreover, it is known that the fibre orientation is influenced by the structural dimensions and this will have an direct effect on the behaviour which may be interpreted as a size-effect. Löfgren (2005) proposed an approach to consider this effect by adjusting for the difference in fibre orientation (fibre efficiency) between material test specimens and structural elements.

$$k_h = 1.0 - 0.6\frac{h[cm] - 12.5}{47.5} \qquad 12.5 \le h \le 60 \text{ [cm]} \tag{2.7}$$

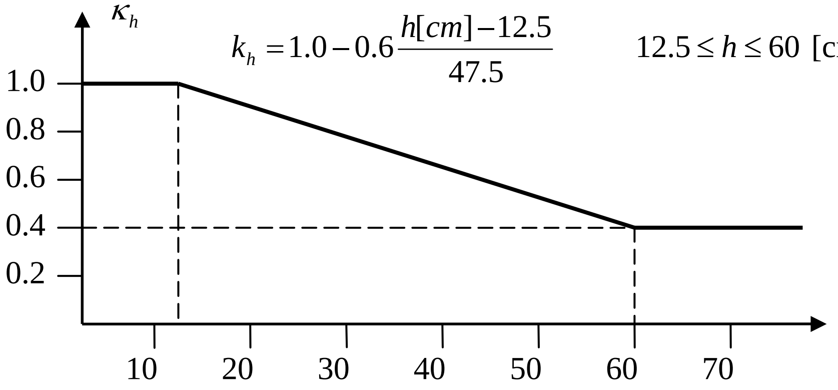


Figure 2.4 Size factor proposed by RILEM TC 162-TDF

The compressive stress-strain relationship is going to be defined according to EC2, which is presented in chapter 3.

Note that $f_{R,4}$ and $f_{R,1}$ factors are calculated considering a linear elastic distribution in the section, figure 2.5a. Although to calculate a more realistic stress in the cracked zone, a constant stress in this zone could be assumed, see figure 2.5b. Furthermore, there is a third approach that was defined before, figure 2.5c. This is the most complicated approach and it is the one proposed by RILEM TC 162-TDF, which is going to be used in the calculations proposed but it is not used to calculate the values of the curve.

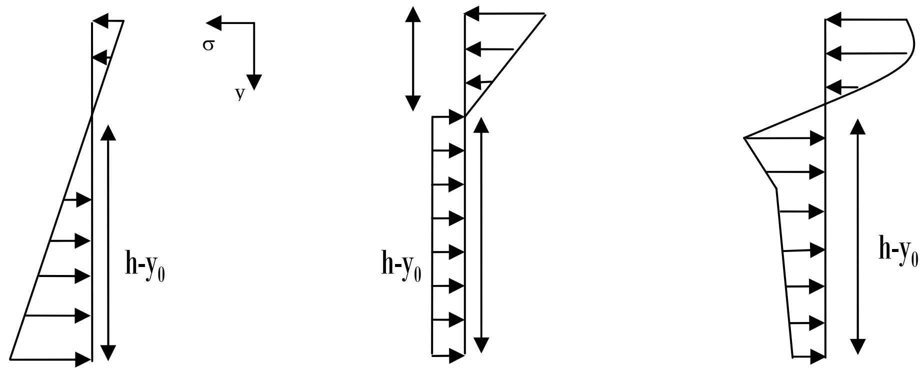


Figure 2.5a Stress distribution. Assumption 1

Figure 2.5b Stress distribution. Assumption 2

Figure 2.5c Stress distribution. Assumption 3

2.2 The σ–*w* approach

This model is based on fracture mechanics and the relationship between stress and crack opening, see Hillerborg (1980). When a concrete specimen, loaded in tensile, is cracking three zones can be distinguished, namely (see Figure 2.6): cracked, a fictitious crack (fracture process zone), and un-cracked.

When no crack is present, the behaviour is assumed as linear elastic. When a crack appears, the fractured zone is modelled as a fictitious crack. Stresses within the fictitious crack are related to the displacement (w), and the stresses outside the fictitious crack are related to the strain (ε). Only if the stress is higher than f_{ct} it is considered as a cracked zone. It is also important to distinguish between a real crack and a fictitious crack. A real crack does not transfer stresses whereas the fictitious crack does. Although a crack is visible, it is only a real crack if the stress on it is zero. Otherwise it is a fictitious crack. For fibre reinforced concrete this definition means that, in the range of crack openings that are of interest, only fictitious cracks are present.

Figure 2.6 explains the fictitious crack model; note the difference between the fracture zone and the non-fracture zone. The maximum stress is in the vertex of the hinge.

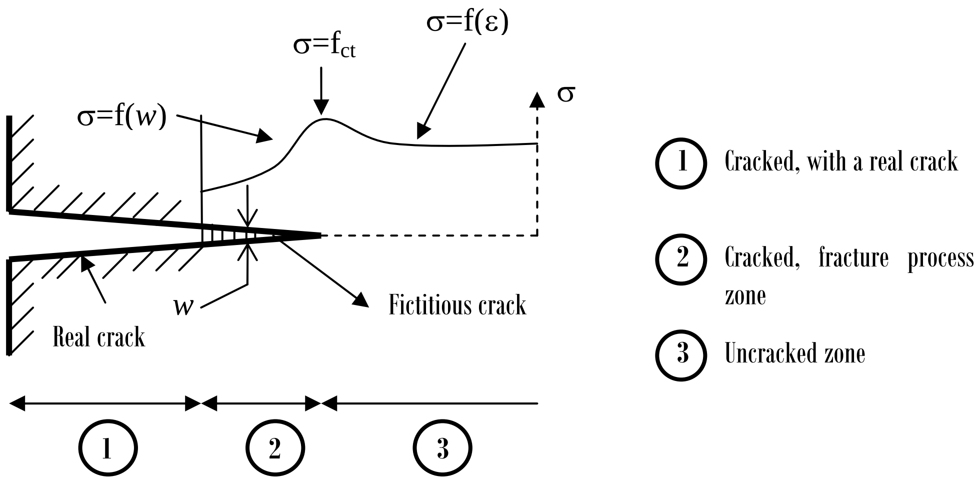


Figure 2.6 Fictitious crack model

For FRC, the σ–*w* relationship can be divided into a contribution from the concrete and from the fibres. However, it is necessary to find a relationship which is not too complicated but that provides a sufficient diagram that can be used as an approximation. The σ–*w* relationship is usually determined by conducting tests and, in some way, analysing the test results. But these often non-linear curves are not practical for design purpose, so it is necessary to obtain an acceptable approximation. An example of a multilinear σ–*w* relationship obtained from test results can be seen in figure 2.7.

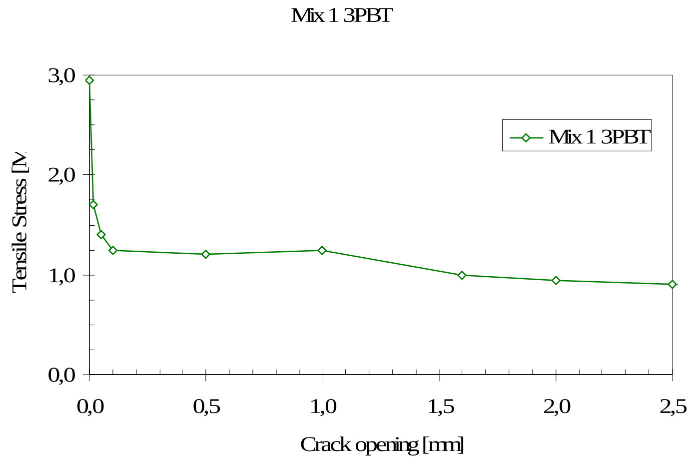


Figure 2.7 Multilinear Approach for the σ–w relationship

A reasonable approximation is the bi-linear relationship, see figure 2.8. The first part of the graph describes the cracking of concrete, which drops quickly, whereas the second part relates to the contribution of the fibres, which decreases slowly.

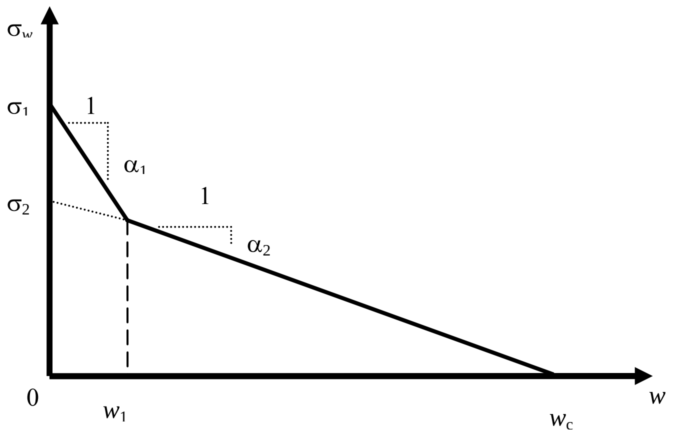


Figure 2.8 Bilinear stress-crack opening relationship

The bi-linear relationship can be described using the following mathematical expression:

$$\sigma_w(w) = \begin{cases} \sigma_1 - \alpha_1 w \quad \text{when} \quad 0 \le w \le w_1 = \dfrac{\sigma_2 - \sigma_1}{\alpha_2 - \alpha_1} \;;\; \alpha_1 > 0 \;;\; \sigma_1 = f_t \\ \sigma_2 - \alpha_2 w \quad \text{when} \quad w_1 \le w \le w_c = \dfrac{\sigma_2}{\alpha_2} \;; \quad \alpha_2 > 0 \end{cases} \tag{2.8}$$

Hence, four parameters are required to completely define the material. Of course it is also needed to know the modulus of elasticity E_c, but this can be determined by means of the expression proposed by EC2, see chapter 5.

If all the expression is divided by f_{ct} it is possible to obtain another expression that also can be used:

$$\frac{\sigma_w(w)}{f_{ct}} = g(w) = \begin{cases} 1 - a_1 w \quad \text{when} \quad 0 \le w \le w_1 = \dfrac{1-b_2}{a_1 - a_2} \;; \\ b_2 - a_2 w \quad \text{when} \quad w'_1 \le w \le w_c = \dfrac{b_2}{a_2} \;; \end{cases} \tag{2.9}$$

This expression can be represented in a graph, see figure 2.9.

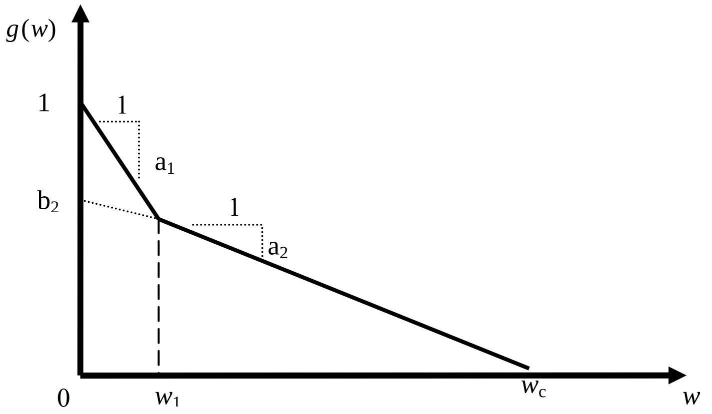


Figure 2.9 Another expression of bilinear stress-crack opening relationship

Figure 2.10 shows the representation of the stress on the element by means of this model (bilinear approach):

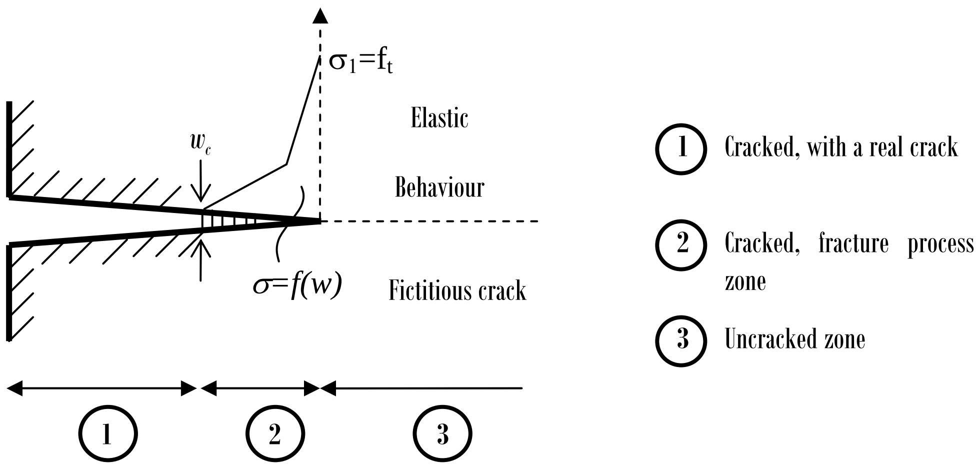


Figure 2.10 Stress on the cracked element with the bilinear approach

3 Analytical analysis of a cross section

In this chapter, general considerations about the behaviour of the elements, which are to be studied, have been written. Furthermore, the analytical models and their equations are presented.

3.1 The σ–ε approach

3.1.1 Material models

3.1.1.1 Concrete in Compression

A continuous equation has been chosen according to the recommendation of EC2 for the use in a non-linear analysis and make easy to solve the model (E_{cm} is replaced by E_c regarding the notation). See figure 3.1.

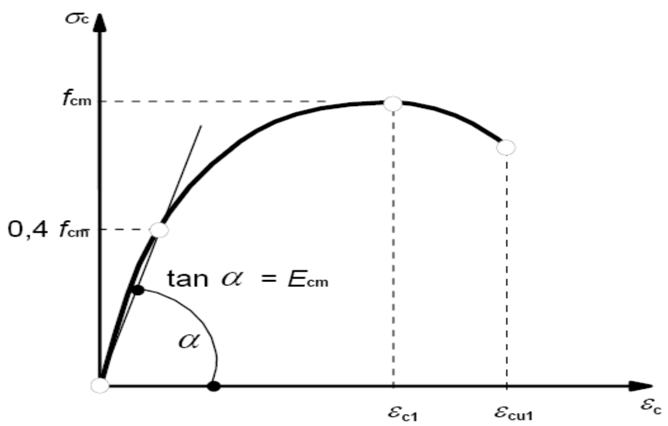


Figure 3.1 Stress-strain diagram of concrete in compression. EC2

This curve has an analytical definition:

$$\sigma_c(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta(\varepsilon(y)) - \eta(\varepsilon(y))^2}{1+(k-2)\cdot\eta(\varepsilon(y))}\right] \tag{3.1}$$

Where $\eta(\varepsilon(y)) = \dfrac{\varepsilon(y)}{\varepsilon_{c1}}$ (3.2)

And $k = 1.05\cdot\dfrac{E_c\cdot\varepsilon_{c1}}{f_{cm}}$ (3.3)

An approximate definition is the secant value between σ_c=0 and σ_c=0.4·f_{cm} (tangent of the angle with the *x* axis). As the strain which yields a stress of 0.4·f_{cm} is unknown and it should be necessary to solve the equation, an alternative equation is proposed:

$E_c = 22\cdot\left(\dfrac{f_{cm}}{10}\right)^{0.3} GPa$ with f_{cm} expressed in MPa (3.4)

3.1.1.2 Concrete in Tension

$\sigma_{ct}(\varepsilon, y)$ is derived from the tri-linear stress-strain relationship proposed by RILEM that was showed in chapter 2 (see appendix C for the whole derivation) and figure 3.2

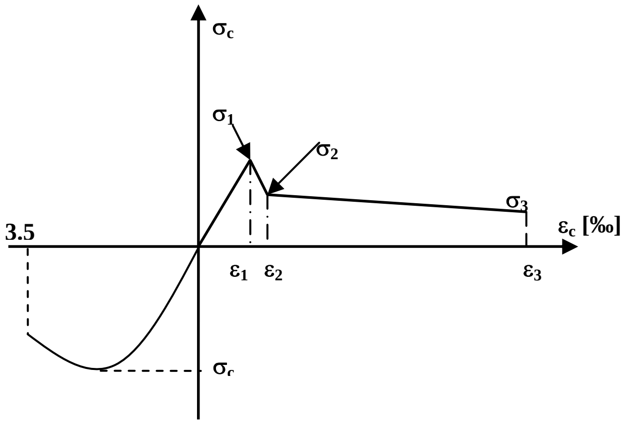


Figure 3.2 Stress and strain diagram (σ–ε approach)

$$\sigma(\varepsilon) = \frac{\sigma_1}{\varepsilon_1} \cdot \varepsilon \qquad \text{if } 0 \le \varepsilon \le \frac{\sigma_1}{E_C} = \varepsilon_1$$

$$\sigma(\varepsilon) = \frac{(\sigma_1 - \sigma_2)}{(\varepsilon_1 - \varepsilon_2)} \cdot [\varepsilon - \varepsilon_1] + \sigma_1 \qquad \text{if } \frac{\sigma_1}{E_C} \le \varepsilon \le \varepsilon_1 + 10^{-4} = \varepsilon_2 \tag{3.5}$$

$$\sigma(\varepsilon) = \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)} \cdot [\varepsilon - \varepsilon_3] + \sigma_3 \quad \text{if } \varepsilon_2 = \varepsilon_1 + 10^{-4} \le \varepsilon \le \varepsilon_3$$

3.1.1.3 Conventional reinforcement

The characteristic stress-strain relationship for the steel is chosen according to EC2 see figure 3.3:

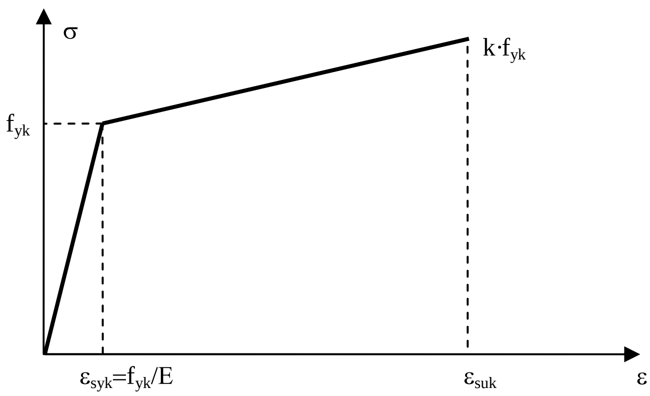


Figure 3.3 Characteristic stress-strain diagram of reinforcement

Due to that, the stress-strain equation is (see appendix C for the whole derivation):

$$\sigma_s(\varepsilon_s) = \begin{cases} E_s \varepsilon_s & \text{if} \quad \varepsilon_s < \dfrac{f_{yk}}{E_s} \\ \dfrac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{uk} - \dfrac{f_{yk}}{E_s}\right)} \cdot \left(\varepsilon_s - \dfrac{f_{yk}}{E_s}\right) + f_{yk} & \text{if} \quad \dfrac{f_{yd}}{E_s} \le \varepsilon_s \le \varepsilon_{uk} \\ 0 & \text{if} \quad \varepsilon_{uk} < \varepsilon_s \end{cases} \tag{3.6}$$

3.1.2 ULS: Flexural and loaded with normal force behaviour

First of all there are some assumptions that are made in the RILEM approach to complete the proposed model, and taken in account in the analytical model:

- Plane sections remain plane (Bernoulli hypothesis)
- The stress-strain diagram is the one showed in the figure 2.3
- The stresses in the reinforcement are derived from a bi-linear stress-strain diagram based on the EC2
- The limit strain in compression is -3.5‰
- The maximum allowable crack opening is 3.5 mm to ensure enough anchorage capacity for the steel fibres.
- For SFRC with conventional reinforcement (bars) the strain at the position of the reinforcement is limited to 25‰
- In some cases the contribution of steel fibres must be reduced, but this is dependent on the exposure class

3.1.2.1 Failure occurs at the same time in concrete as in reinforcement. Equilibrium equations

This is a special case, and not always realistic but it could be a first approximation in order to derive the equations.

The distribution of stress and strain is for a beam in ULS (loaded by a moment and a normal force) if it is supposed that failure of the concrete in compression and the reinforcement occurs at the same time can be seen in figure 3.4. The position of the neutral axis is then predefined.

Then, the equilibrium equation can be like that:

$$0 = F_{fc} + F_{ct} + F_s + N \tag{3.7}$$

Where:

F_{fc} is the compressive force supported by concrete

F_{ct} is the tensile force supported by concrete

F_s is the tensile force supported by steel

N is the possible external normal load

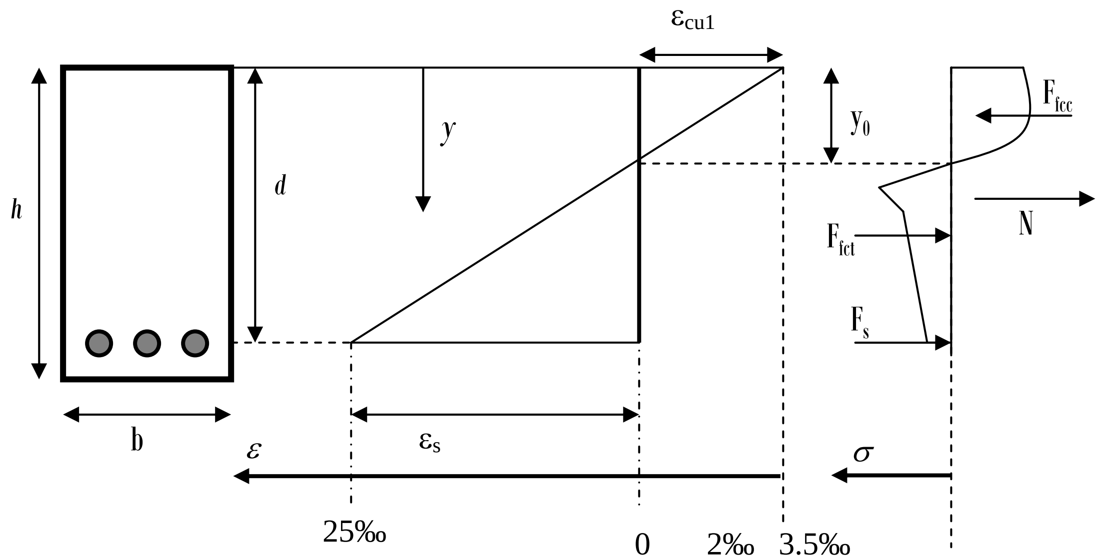


Figure 3.4 Stress and strain diagram (σ–ε approach)

The equilibrium equation can be written by means of stresses like this:

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot b \cdot dy + \int_{y_0}^{d} \sigma_{ct}(\varepsilon, y) \cdot b \cdot dy + \sigma_s \cdot A_s + N \tag{3.8}$$

Where:

$\sigma_{cc}(\varepsilon, y)$ is the compressive stress in concrete (above the neutral axis)

$\sigma_{ct}(\varepsilon, y)$ is the tensile stress in concrete (due to the fibres)

σ_s is the tensile stress in the conventional reinforcement

A_s is the total cross-sectional area for the conventional reinforcement

Concrete in compression. The equations regarding the concrete in compression have to be changed in order to find the relationship between the strain and the position of the neutral axis $\varepsilon(y)$. This has to be done according to figure 3.1 and 3.4. The result is the equation (3.9).The complete derivation can be found in Appendix C.

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{1.05\dfrac{E_c\cdot\varepsilon_{c1}}{f_{cm}}\cdot\left(\dfrac{\varepsilon_{cu1}\cdot\left(1-\dfrac{y}{y_0}\right)}{\varepsilon_{c1}}\right)-\left(\dfrac{\varepsilon_{cu1}\cdot\left(1-\dfrac{y}{y_0}\right)}{\varepsilon_{c1}}\right)^2}{1+\left(1.05\dfrac{E_c\cdot\varepsilon_{c1}}{f_{cm}}-2\right)\cdot\left(\dfrac{\varepsilon_{cu1}\cdot\left(1-\dfrac{y}{y_0}\right)}{\varepsilon_{c1}}\right)}\right] \tag{3.9}$$

Concrete in tension. If equation (3.5) is expressed by means of *y* according to figure 3.2 and 3.4, it yields the equation (3.10). The complete derivation can be found in Appendix C.

$$\sigma_{ct}(y)=\begin{cases} E_c\cdot\varepsilon(y)=E_c\cdot\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0);\ \text{if}\ \ 0\le y\le\dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0=\dfrac{\dfrac{\sigma_1}{E_c}(d-y_0)}{\varepsilon_s}+y_0 \\ \dfrac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_1\right]+\sigma_1;\ \text{if}\ \dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0<y<y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s} \\ \dfrac{\sigma_2-\sigma_3}{(\varepsilon_2-\varepsilon_3)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_3\right]+\sigma_3;\ \text{if}\ \ y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s}\le y\le d \end{cases} \tag{3.10}$$

Where ε_s is the limit of strain of the conventional reinforcement that is the same as the limit proposed for the strain of concrete at the position of the reinforcement (25‰).

Conventional reinforcement. According to figure 3.3 the equation that defines the behaviour of the reinforcement bars is equation (3.6) showed before. As the strain in the reinforcement is supposed to be the maximum permitted in FRC (25‰) the equation can be written as:

$$\sigma_s = \begin{cases} \dfrac{f_{yk}\cdot(k_s-1)}{\left(\varepsilon_{uk}-\dfrac{f_{yk}}{E_s}\right)}\cdot\left(\dfrac{25}{1000}-\dfrac{f_{yk}}{E_s}\right)+f_{yk} \end{cases} \tag{3.11}$$

If these stress terms are added, the final equilibrium equation is obtained. Hence, from this equilibrium equation is possible to obtain the position of the neutral axis y_0. Although its value is predefined because the limits of the strain curve are known, it is necessary that exists a real equilibrium. That means that the amount of reinforcement has to be exactly one defined value to produce the balance failure.

When the position of the neutral axis is known, the moment equilibrium equation can be solved in order to know the maximum moment that can be supported. The equation taking the top concrete point as a reference:

$$0 = M_{cc} + M_{ct} + M_s + M_R \tag{3.12}$$

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot y \cdot b \cdot dy + \int_{y_0}^{d} \sigma_{ct}(\varepsilon, y) \cdot y \cdot b \cdot dy + \sigma_s \cdot A_s \cdot d + M_R \tag{3.13}$$

Where $\sigma_{cc}(\varepsilon, y)$, $\sigma_{ct}(\varepsilon, y)$ and σ_s are the same than defined at (3.9), (3.10) and (3.11) respectively.

3.1.2.2 Failure occurs at different time in reinforcement as in concrete. Equilibrium equations

The assumption that the failure occurs at the same time in the reinforcement and concrete is not so common. Normally concrete elements are designed in the way that yielding in the reinforcement happens before that the concrete failure because the last one is more critical and dangerous than the first one.

Due to that, another diagram is proposed. In this diagram, different values will be given to the strain in the reinforcement position until the allowable limit is reached. It is also necessary to check if the strain in the top is less than the limit for the compressed concrete. If it is not, the assumption of the failure of the FRC in tension is false, and the concrete will fail in compression before. Note that the failure criteria is the same that is used in the RILEM recommendations (strain in the concrete at the level of the reinforcement must be less or equal than 25‰). Figure 3.5 shows the new proposed diagram.

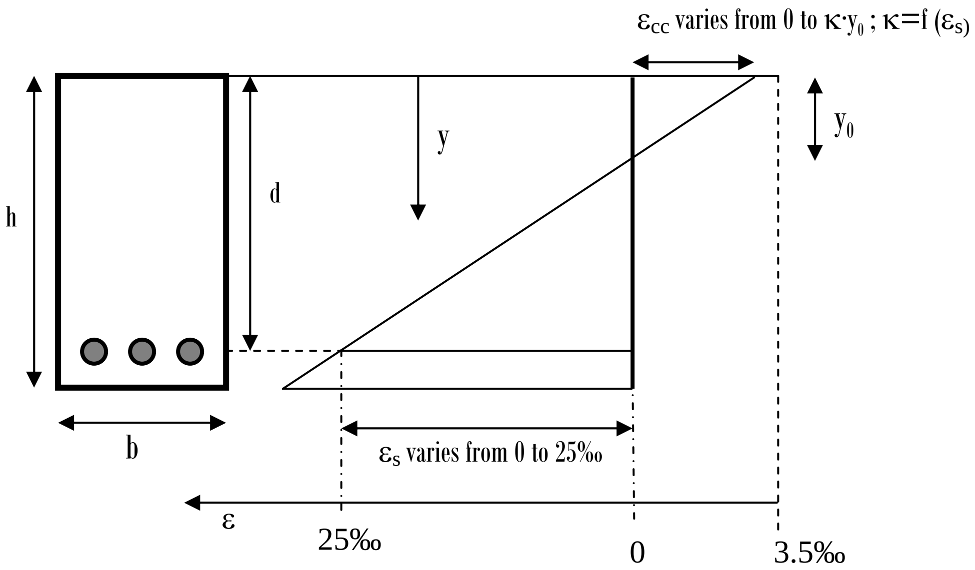


Figure 3.5 alternative strain diagram of a cross-section

Where $\kappa = \dfrac{\varepsilon_s}{d_1 - y_0} = \dfrac{1}{r}$ (3.14)

Where κ is the curvature of the beam and r is the curvature radius.

This new diagram yields new equilibrium equations. It has to be noticed that with this assumptions it is possible to take an advantage of the tensile resistance of FRC between $y=d$ and $y=h$ (RILEM does not take into account this contribution of FRC). This is quite important in order to compare both approaches (σ–ε and σ–w), because the stress-crack opening relationship considers the contribution of the concrete in all the height.

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot b \cdot dy + \int_{y_0}^{h} \sigma_{ct}(\varepsilon, y) \cdot b \cdot dy + \sigma_s \cdot A_s + N \tag{3.15}$$

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot y \cdot b \cdot dy + \int_{y_0}^{h} \sigma_{ct}(\varepsilon, y) \cdot y \cdot b \cdot dy + \sigma_s \cdot A_s \cdot d_1 + M \tag{3.16}$$

Concrete in compression. The new equation for the concrete in compression is (see appendix C for the whole derivation):

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm} \left[\frac{k \cdot \eta\left(\dfrac{\varepsilon_s}{d - y_0} \cdot (y_0 - y) \right) - \eta\left(\dfrac{\varepsilon_s}{d - y_0} \cdot (y_0 - y) \right)^2}{1 + (k-2) \cdot \eta\left(\dfrac{\varepsilon_s}{d - y_0} \cdot (y_0 - y) \right)} \right] \tag{3.17}$$

Concrete in tension. The equation for the concrete in tension is (see appendix C for the complete derivation):

$$\sigma_{ct}(y)=\begin{cases} E_c\cdot\varepsilon(y)=E_c\cdot\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0);\ \text{if}\ \ 0\le y\le\dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0=\dfrac{\dfrac{\sigma_1}{E_c}(d-y_0)}{\varepsilon_s}+y_0 \\ \dfrac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_1\right]+\sigma_1;\ \text{if}\ \ \dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0<y<y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s} \\ \dfrac{\sigma_2-\sigma_3}{(\varepsilon_2-\varepsilon_3)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_3\right]+\sigma_3;\ \text{if}\ \ y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s}\le y\le d \\ 0 \qquad \text{if}\ \ \dfrac{(d-y_0)\cdot\varepsilon_3}{\varepsilon_s}+y_0<\mathrm{y} \end{cases} \tag{3.18}$$

Conventional reinforcement. The equation for the reinforcement (see appendix C for the whole derivation):

$$\sigma_s=\begin{cases} \sigma_s(\varepsilon_s)=E_s\varepsilon_s & \text{if}\ \ \varepsilon_s<\dfrac{f_{yk}}{E_s} \\ \sigma_s(\varepsilon_s)=\dfrac{f_{yk}\cdot(k-1)}{\left(\varepsilon_{uk}-\dfrac{f_{yk}}{E_s}\right)}\cdot\left(\varepsilon_s-\dfrac{f_{yk}}{E_s}\right)+f_{yk} & \text{if}\ \ \dfrac{f_{yd}}{E_s}\le\varepsilon_s\le\varepsilon_{uk} \\ \sigma_s(\varepsilon_s)=0 & \text{if}\ \ \varepsilon_{uk}<\varepsilon_s \end{cases} \tag{3.19}$$

3.1.3 SLS: Cracking

RILEM TC 162-TDF proposes an expression to calculate the crack width based on the same expression that it is used in EC2, see equation 3.18

$$w=\beta_{crack}\cdot s\cdot\varepsilon_{sm} \tag{3.20}$$

Where β_{crack} is a constant which takes into account the size effect, s is is the crack spacing which can be calculated as different ways as it is showed in 3.2.3, and ε_{sm} is the mean steel strain in the reinforcement allowed under relevant combination of loads for the effects of tension stiffening, shrinkage…

The value of the crack width calculated using this expression is a design crack width value. Hence, it not has sense to compare this values with the characteristics ones obtained using the σ–w approach which do not consider other additional effects.

3.2 The σ–*w* approach

3.2.1 The cracked non-linear hinge model

The basis of the cracked hinge model is the concept of the fictitious crack model developed by Hillerborg, see Hillerborg (1980). It basically consists of considering the element as divided in two zones: zone 1 is where a crack is being developed, which is modelled as a non-linear hinge; and zone 2 which is the non cracked part which maintains the elastic behaviour, see Olesen (2001). Of course many cracks and, obviously, many non-linear hinges can occur at the same time in an element such as a beam.

In order to keep the equilibrium it is supposed that the end faces of the non linear hinge remain plane, are connected to the rest of the structural, and are loaded with the generalized stress in the element, see RILEM TC 162-TDF (2003). The length of the non-linear hinge is *s*, and its value is not quite easy to determine. A further study about this value is carried out in chapter 3.3. Figure 3.6 shows the non-linear hinge model.

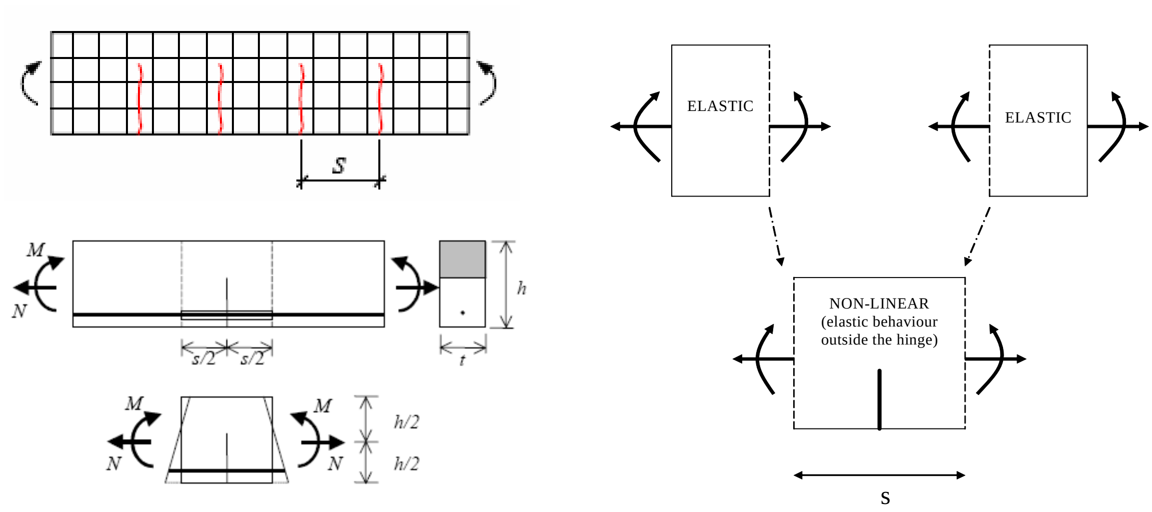


Figure 3.6 Non-linear hinge model, from Löfgren (2005)

3.2.2 ULS: Flexural and loaded with normal force behaviour

There are some assumptions that are important in order to simplify the model and make it suitable to be used easier. The assumptions depend on the model and there are three models proposed by RILEM TC 162-TDF (2003):

- The first one (Pedersen, 1996) assumes that the fictitious crack surfaces remain plane, and that the opening angle is the same as the overall deformation of the non linear hinge. It is the easiest of the three ones. It is also assumed that $\varphi = \varphi*$. See figure 3.7a.

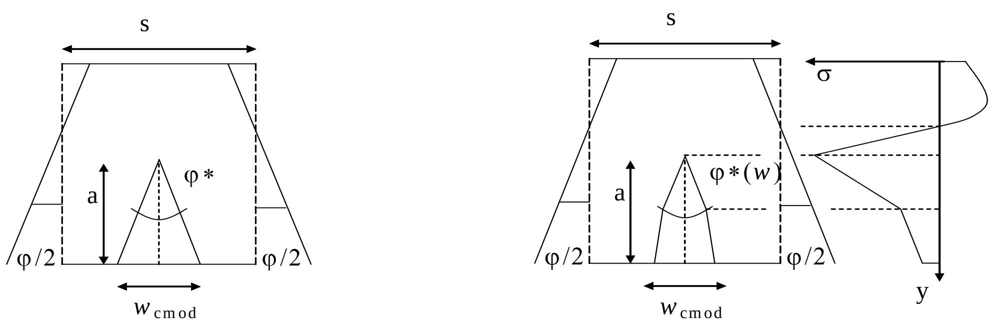


Figure 3.7a Non-linear hinge model 1, first kinematic assumption (Pedersen, 1996)

Figure 3.7b Non-linear hinge model 1, third kinematic assumption (Olesen, 2001)

- The second approach was used by Casanova and Rossi (1996 & 1997). The fictitious crack surface remains plane and the opening angle is the same as the overall deformation of the non linear hinge. But the difference is that the curvature variation is based on an assumption of parabolic variation.

- In the third approach, the fictitious crack surface does not remain plane and the deformation of that is governed by the stress crack opening relationship. This is the most complicated model to solve, even using mathematical software. It was developed by Olesen (2001). See figure 3.7b.

If a comparison between first and third approach is made, it can be seen (studying the moment versus the turn of the section) that there are only small differences between the approaches. Hence, the first approach is a really good approach, at least for FRC with conventional reinforcement. See figure 3.8 and 3.9 (further information about the model one and the derivations can be founded in next chapters).

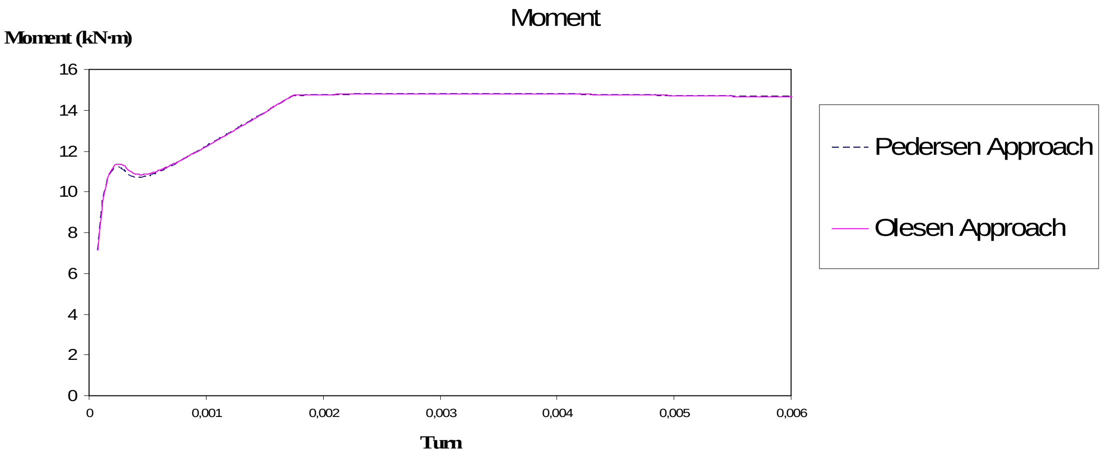


Figure 3.8 Comparison between Pedersen (1996) and Olesen (2001). 125mm beam

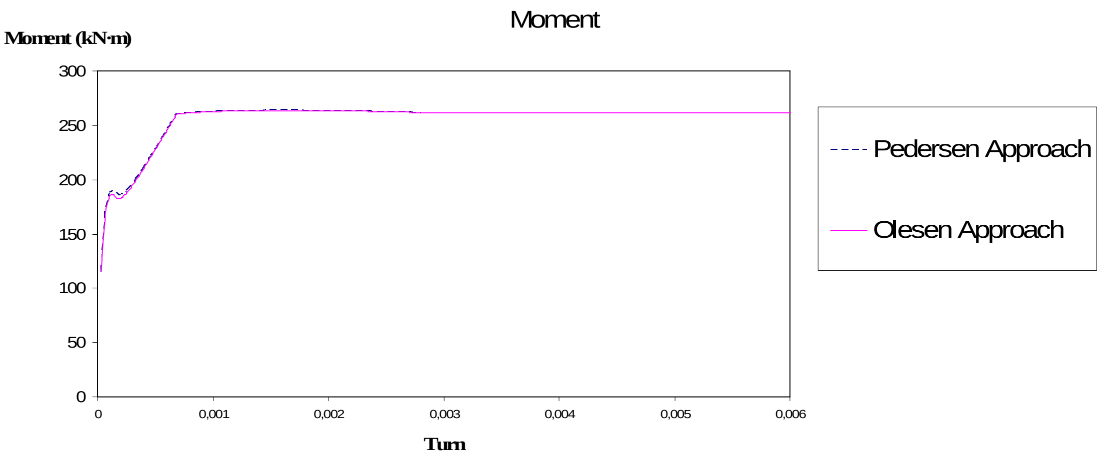


Figure 3.9 Comparison between Pedersen (1996) and Olesen (2001). 500mm beam

Based on this investigation, no significant difference was observed for the two approaches and thus the first model will be used and the equations that govern it are going to be explained.

The curvature of the non-linear hinge is:

$$\kappa = \frac{\varphi}{s} \tag{3.21}$$

And the crack mouth opening displacement (CMOD), as it can be seen in the figure 3.10, is:

$$w_{c\,\mathrm{mod}} = \varphi^* \cdot a \tag{3.22}$$

In the figure 3.10 the stress and the cross section is represented to have a clearer vision of the equilibrium of forces and moments:

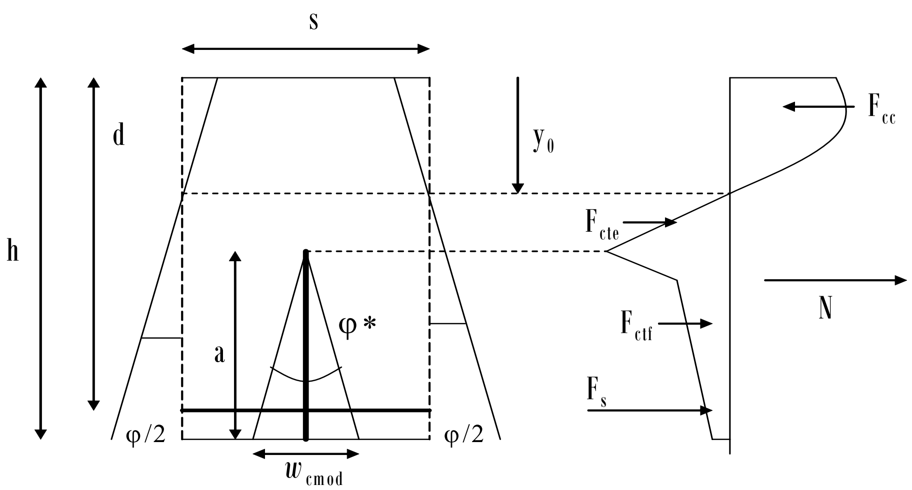


Figure 3.10 Non-linear hinge model 1, stress distribution

To obtain the equations that are needed to define the behaviour of the cross section, the procedure is quite similar to the derivation in the σ–ε approach. The equilibrium of forces are:

$$0 = F_{cc} + F_{cte} + F_{ctf} + F_s + N \tag{3.23}$$

Where:

F_{cc} is the compressive force supported by concrete.

F_{cte} is the tensile force supported by concrete in the elastic part of the curve.

F_{ctf} is the tensile force supported by concrete in the fractured part of the curve.

F_s is the tensile force supported by steel.

From this equation (3.23) the following general equation can be derived:

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot b \cdot dy + \int_{y_0}^{h-a} \sigma_{cte}(\varepsilon, y) \cdot b \cdot dy + \int_{h-a}^{d} \sigma_{ctf}(w, y) \cdot b \cdot dy + \sigma_s \cdot A_s + N \tag{3.24}$$

Where:

a is the length of the crack, which can also be written in function of φ as:

$$tg\varphi \approx \varphi = \frac{\varepsilon_{abs}}{h - a - y_0} \Rightarrow h - a - y_0 = \frac{\varepsilon_{abs}}{\varphi} = \frac{f_t \cdot s}{E_c} \cdot \frac{1}{\varphi} \Rightarrow a = h - \frac{f_t \cdot s}{E_c} \cdot \frac{1}{\varphi} - y_0 \tag{3.25}$$

See figure 3.11.

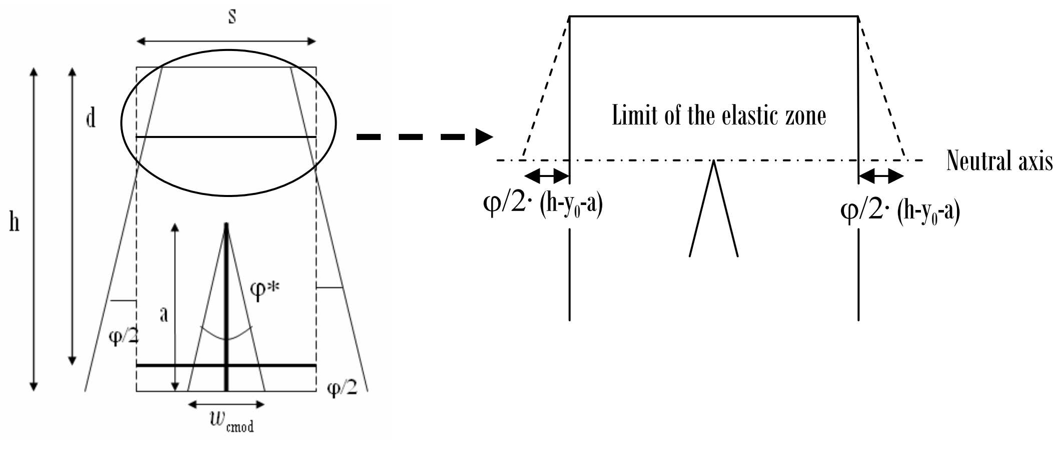


Figure 3.11 Relation between a and φ

$\sigma_{cc}(\varepsilon, y)$ is, like in the σ–ε model, the stress-strain relationship for the concrete in compression zone, see equation (3.25).

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta(\varepsilon(y))-\eta(\varepsilon(y))^2}{1+(k-2)\cdot\eta(\varepsilon(y))}\right] \tag{3.26}$$

It is necessary to transform the equation in another one which depends only on *y* (see Appendix C):

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta\left(\frac{\varphi}{s}\cdot(y_0-y)\right)-\eta\left(\frac{\varphi}{s}\cdot(y_0-y)\right)^2}{1+(k-2)\cdot\eta\left(\frac{\varphi}{s}\cdot(y_0-y)\right)}\right]\frac{\varphi}{s}\cdot(y_0-y) \tag{3.27}$$

The behaviour of concrete in the elastic tensile zone can be obtained by means of this expression which is based on the geometrical assumptions:

$$\sigma_{cte}(\varepsilon, y) = E_c\cdot\varepsilon(y) = \left(\frac{\varphi}{s}\cdot y_0\cdot\frac{y-y_0}{y_o}\right)\cdot E_c \tag{3.28}$$

As it was explained in the section 2.2 the σ–*w* relationship is:

$$\sigma_{ctf}(w,y) = \begin{cases} f_{ct}\cdot(1-a_1\cdot w(y)) & \text{if } 0\le w\le w_1 \\ f_{ct}\cdot(b_2-a_2\cdot w(y)) & \text{if } w_1\le w\le w_c \\ 0 & \text{if } w_c < w \\ \text{where} & \\ w(y)=\varphi\cdot(y-y_0)-\frac{f_{ct}}{E_c}\cdot s-\frac{N}{A\cdot E_c} & \text{if } w\le w_c \end{cases} \tag{3.29}$$

This relationship has to be expressed by means of *y*:

$$\sigma_{ctf}(y) = \begin{cases} f_t\cdot\left[1-a_1\cdot\left(\varphi\cdot(y-y_0)-\frac{f_t}{E_c}\cdot s-\frac{N}{A\cdot E_c}\right)\right] \\ \text{if } \left(\frac{f_t}{E_c}\cdot s+\frac{N}{A\cdot E_c}\right)\cdot\frac{1}{\varphi}+y_0\le y\le\left(w_1+\frac{f_t}{E_c}\cdot s+\frac{N}{A\cdot E_c}\right)\cdot\frac{1}{\varphi}+y_0 \\ f_t\cdot\left[b_2-a_2\cdot\left(\varphi\cdot(y-y_0)-\frac{f_t}{E_c}\cdot s-\frac{N}{A\cdot E_c}\right)\right] \\ \text{if } \left(w_1+\frac{f_t}{E_c}\cdot s+\frac{N}{A\cdot E_c}\right)\cdot\frac{1}{\varphi}+y_0\le y\le\left(w_c+\frac{f_t}{E_c}\cdot s+\frac{N}{A\cdot E_c}\right)\cdot\frac{1}{\varphi}+y_0 \\ 0 \qquad \text{if } y>\left(w_c+\frac{f_t}{E_c}\cdot s+\frac{N}{A\cdot E_c}\right)\cdot\frac{1}{\varphi}+y_0 \end{cases} \tag{3.30}$$

And σ_s is the same than it is showed in the equation (3.6)

Providing values of φ, it is possible to calculate the position of the neutral axis y_0, if the length of the non-linear hinge s is known. Note that the crack opening can be expressed as:

$$w(y) = \varphi \cdot (y - y_0) - \frac{f_t}{E_c} \cdot s - \frac{N}{A \cdot E_c} \quad \text{if } w \le w_c$$

$$w(y) = \varphi \cdot (y - y_0) \quad \text{if } w \ge w_c \tag{3.31}$$

Then it is possible to derive the moment equilibrium equation and to determine the maximum moment that can be supported:

$$0 = M_{cc} + M_{cte} + M_{ctf} + M_s + M_R \qquad \text{that yields} \tag{3.32}$$

$$0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot (y) \cdot b \cdot dy + \int_{y_0}^{h-a} \sigma_{cte}(\varepsilon, y) \cdot (y) \cdot b \cdot dy + \int_{h-a}^{d} \sigma_{ctf}(w, y) \cdot (y) \cdot b \cdot dy + \sigma_s \cdot A_s \cdot (d) + M_R \tag{3.33}$$

3.3 SLS: Crack spacing

3.3.1 Introduction

The values of *s* (crack spacing) depend, in general, on the type of structural element, its dimensions, and the amount, type, and dimension of the reinforcement. The studies that have been published (see e.g. Borosnyói A. and Balázs (2005) and JSCE (1997)) are concentrated mainly on conventional reinforced concrete, and articles dealing with fibre-reinforced concrete and the possibilities to modify the conventional crack spacing models are not that many. Due to this lack of a "universal" formula, it is necessary to choose one of the proposed.

Most of the values proposed for *s* are based on the concept of transfer length. If a load is applied to a reinforced concrete element there is a difference between the strain of the reinforcement and the surrounding concrete. To transfer the load from the reinforcement to the concrete a certain transfer length is required.

The first crack is generally formed at a random place that coincides with a weak section. When a crack is formed in plane concrete, the stress in the concrete instantly becomes zero, and the tensile force is carried by the reinforcement. But at a distance from the crack the concrete starts to carry stresses and the larger the distance the higher the stress is in the concrete. When the distance from the crack is enough the compatibility of strain is recovered and the stress in the concrete approaches the tensile strength. Due to that the crack spacing is mainly governed by these variables:

- The stress in the steel at the crack, which depends on the steel material and geometrical properties
- The bond-slip behaviour, i.e. the bond behaviour of the reinforcement bar

- The concrete cover and concrete strength

When the load is increased, the crack spacing decreases until it reaches the minimum value. See figure 3.12.

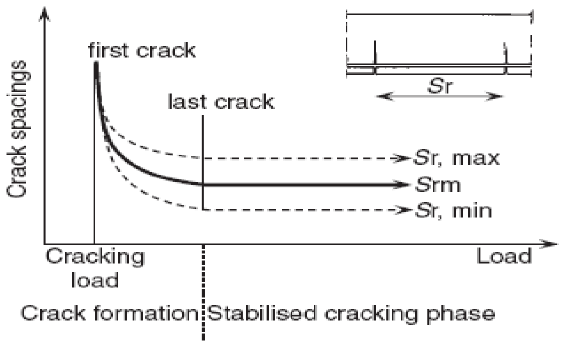


Figure 3.12 Evolution of the crack spacing when load is increased. From Borosnyói and Balázs (2005).

When the minimum value is reached, the previously mentioned transfer lengths in the cracks reach each other due to the fact that the crack spacing is very short.

It is also important to define the so called $A_{c,eff}$. This is the contribution of the concrete in tension, but it is not the same as the tensile area of the concrete. To calculate this value a non-linear expression should be use. However in order to facilitate the calculations in design issues, some simplifications in the expression are available to be used.

As a consequence of the bond-slip, the strain of the reinforcement is not constant along the longitudinal axis of the bar. It also produces a contribution of the concrete increasing the stiffness of the concrete (some force is transferred to the adjacent concrete), see Borosnyói and Balázs (2005). Figure 3.13 shows the mechanism of the strain variation along the crack distance.

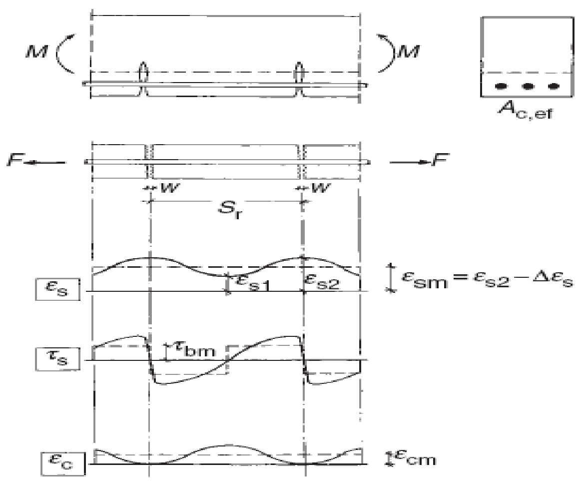


Figure 3.13 Strain and bond stress distribution between cracks. From Borosnyói and Balázs (2005).

For fibre reinforced concrete, the problem is that traditional formulas for plane concrete cannot be used because the crack spacing is different due to the fibre bridging at the crack. Apart from the factors that govern the crack spacing mentioned before, two new factors have to be introduced:

- Diameter and length of the fibres, i.e. fibre slenderness factor
- Volume of fibres

When fibres are including into the plane reinforced concrete, there will be a bridging effect at the cracks. This effect includes a new stress that could be called fibre bridging stress, see figure 3.14. Also in this figure the differences between the stresses of reinforcement bar, plane concrete and fibre reinforced concrete can be seen when a normal load is applied.

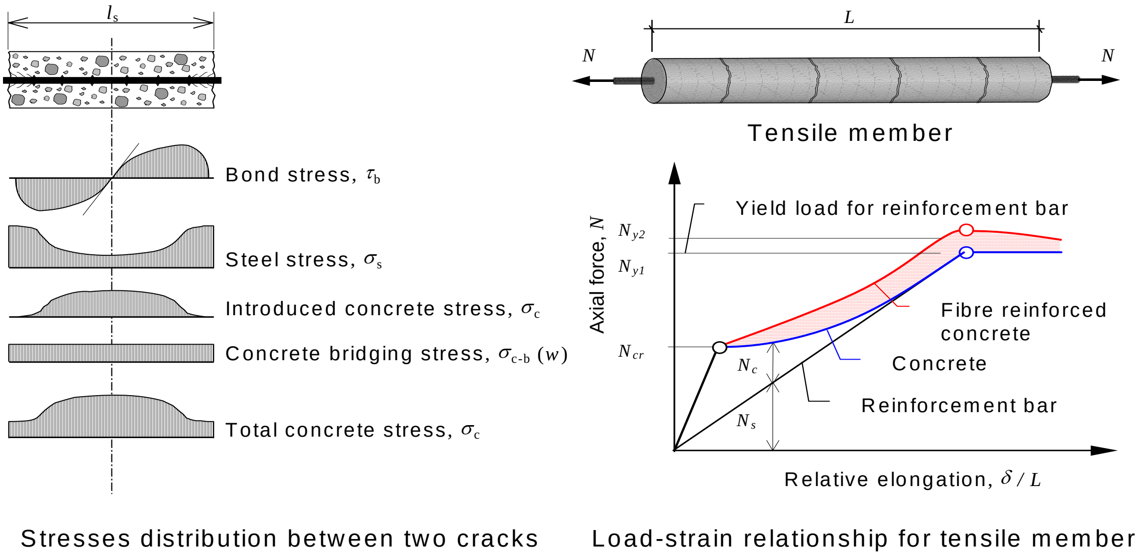


Figure 3.14 Response of reinforced tensile member.

This new effect should be appropriately introduced, e.g. by a new term in the proposed formulations to take into account the presence and the behaviour of the fibres regarding crack spacing.

3.3.2 Analytical approaches of the crack spacing

There are some analytical approaches that are suitable to be used in order to estimate the length of the non-linear hinge.

RILEM rough proposition:

$$s_{m,RILEM} = \frac{h}{2} \tag{3.34}$$

This value is considered an adequate choice and is proposed by RILEM TC-162 TDF. Maybe it is a good choice if there is not conventional reinforcement in the element, but there are not test results which allow to corroborate if the assumption is true.

Eurocode 2 proposition and variations:

Now the assumption that appears in the Eurocode 2 and its variations is analysed. The formula can be derived analysing a reinforced tension rod loaded with a normal force. The rod is reinforced with a reinforced bar (area A_s). For this case, fibres are going to be introduced in order to obtain a formula that takes into account the fibre bridging effect. Figure 3.15 shows the zone that is analysed. The maximum distance between cracks $s_{r,max}$ is equal to $2 \cdot l_{t,max} + 2 \cdot \Delta r$, where $l_{t,max}$ is the maximum transfer length and Δr is the damage region that is considered that does not transfer bond stresses; see Engström (2004).

It can be seen that at the crack, fibre reinforced concrete transfer σ_w. As it was explained before, σ_w depends on the crack opening. Along the rod, the stress increases until it reaches the f_{ct} value because the transfer length mechanism. Also the bond stress τ_b varies along the length, so it can be convenient to take an average value τ_{bm}:

$$\tau_{bm} = \frac{\int_0^{l_{t,max}} \tau_b(x)dx}{l_{t,max}} \tag{3.35}$$

If the equilibrium equations are derived in a $l_{t,max}+\Delta r$ length, see figure 3.11:

$$\tau_{bm} \cdot \pi \cdot \phi_b \cdot (0{,}5 s_{r,max} - \Delta r) + \sigma_w \cdot A_c = f_{ct} \cdot A_c \tag{3.36}$$

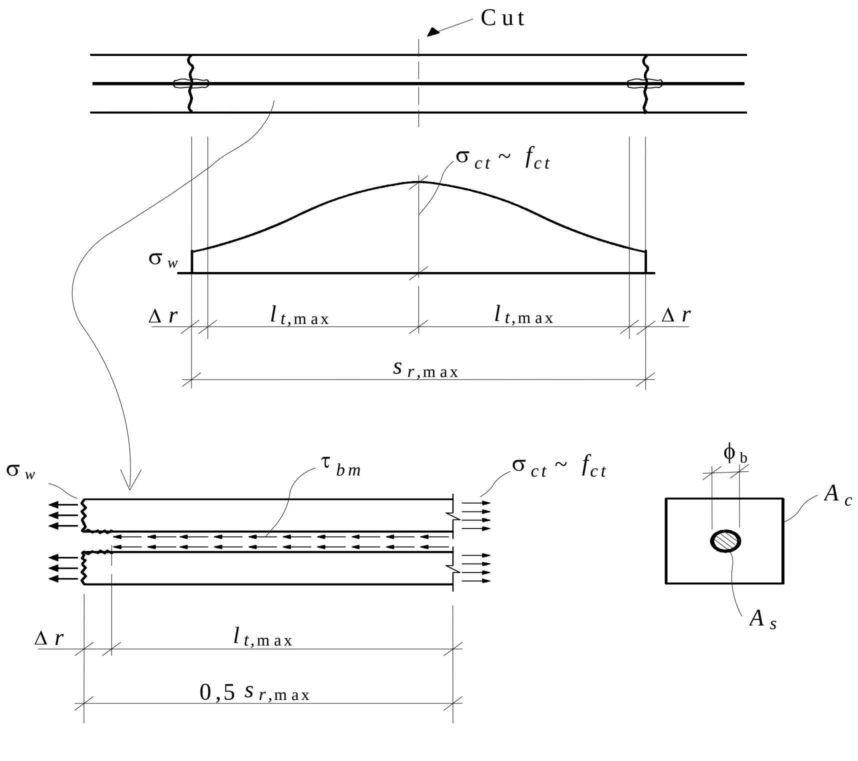


Figure 3.15 Equilibrium forces in a fibre reinforced concrete rod. Based on Engström (2004).

It is better to consider the effective tension area because for a beam or slab not all the concrete area is subjected to the tensile stress. As it was mentioned, this area depends on the distance to the crack. In order to simplify the equation, is assumed as a constant value that is the lesser of $A_{c,eff} = 2.5 \cdot (h-d) \cdot b$, $A_{c,eff} = \frac{h}{2} \cdot b$ or $A_{c,eff} = \frac{(h-y_0)}{3} \cdot b$, where y_0 is the distant to the neutral axis measured from the top (expressions taken from EC2). (3.37)

So, it yields:

$$A_{c,eff} = A_s \cdot \frac{A_{c,eff}}{A_s} = \frac{A_s}{\rho_{eff}} \tag{3.38}$$

Where ρ_{eff} is the effective reinforcement ratio and ϕ_b is the average diameter of the conventional reinforcement bars. If (3.38) is introduced into (3.36), it yields:

$$\tau_{bm} \cdot \pi \cdot \phi_b \cdot (0{,}5 \cdot s_{r,max} - \Delta r) = \frac{\pi \phi_b^{\ 2}}{4\rho_{eff}} \left(f_{ct} - \sigma_w \right) \Rightarrow s_{r,max} = 2\Delta r + \frac{1}{2} \cdot \frac{\left(f_{ct} - \sigma_w \right)}{\tau_{bm}} \cdot \frac{\phi_b}{\rho_{eff}} \tag{3.39}$$

The minimum crack spacing can be defined as half of the maximum crack spacing:

$$s_{r,min} = \Delta r + \frac{1}{4} \cdot \frac{\left(f_{ct} - \sigma_w \right)}{\tau_{bm}} \cdot \frac{\phi_b}{\rho_{eff}} \tag{3.40}$$

So, if the average crack spacing is considering as the average value of (3.38) and (3.39):

$$s_{rm} = 1{,}5\Delta r + \frac{3}{8} \cdot \frac{\left(f_{ct} - \sigma_w \right)}{\tau_{bm}} \cdot \frac{\phi_b}{\rho_{eff}} \tag{3.41}$$

The average bond stress, depends either on the properties of the concrete and of the reinforcement. Based on experimental results, a formula has been derived:

$$\tau_{bm} = \frac{3}{2 \cdot k_1} \cdot f_{ct} \tag{3.42}$$

Where k_1 takes into account the properties of the conventional reinforcement bars and has the value of 0.8 for high bond (ribbed) bars, 1.2 for indented bars and 1.6 for plain (smooth) bars.

If the damage region is considered with a length of $\Delta r \approx 2 \cdot \phi_b + \frac{u_{cover}}{1.5}$, where u_{cover} is the cover of the reinforcement bars, and equation (3.42) is inserted in (3.41):

$$s_{rm} = 1{,}5 \cdot (2 \cdot \phi_b + \frac{u_{cover}}{1.5}) + \frac{3}{8} \cdot \frac{\left(f_{ct} - \sigma_w\right)}{\frac{3 \cdot f_{ct}}{2 \cdot k_1}} \cdot \frac{\phi_b}{\rho_{eff}} = 3 \cdot \phi_b + u_{cover} + \frac{2}{4} \cdot k_1 \cdot \frac{\left(f_{ct} - \sigma_w\right)}{f_{ct}} \cdot \frac{\phi_b}{\rho_{eff}} \Rightarrow$$

$$\Rightarrow s_{rm} = 3 \cdot \phi_b + u_{cover} + 0.25 \cdot k_1 \cdot \left(1 - \frac{\sigma_w}{f_{ct}}\right) \cdot \frac{\phi_b}{\rho_{eff}} \qquad (3.43)$$

So it can be seen that the crack spacing can be reduced using low diameters of reinforcements (using the same reinforcement ratio), using reinforcement with better bond properties, increasing reinforcement ratio and ratio between the bridging stress and concrete tensile stress. Also, a new term has to be introduced in order to consider the effects of strain distribution:

$$s_{rm} = 3 \cdot \phi_b + u_{cover} + 0.25 \cdot k_1 \cdot k_2 \cdot \left(1 - \frac{\sigma_w}{f_{ct}}\right) \cdot \frac{\phi_b}{\rho_{eff}} \qquad (3.44)$$

Where k_2 has the value 0.5 for bending and 1.0 for pure tension.

A new constant can be defined for the fibre contribution:

$$k_3 = \left(1 - \frac{\sigma_w}{f_{ct}}\right) \qquad (3.45)$$

And if expression (3.44) is introduced into (3.43):

$$s_{m,EC2F} = 3 \cdot \phi_b + u_{cover} + 0.25 \cdot k_1 \cdot k_2 \cdot k_3 \cdot \frac{\phi_b}{\rho_{eff}} \qquad (3.46)$$

If there is no fibres in the concrete, the term κ_3 becomes one and also if it is considered that $3 \cdot \phi_b + u_{cover} \approx 50$. The formula that appears in the EC2 is derived:

$$s_{m,EC2} = \left(50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi_b}{\rho_{eff}}\right)(mm) \qquad (3.47)$$

There are studies, see Vandewalle (2000) and Vandewalle and Dupont (2003), which has suggested that the spacing of cracking decrease when more quantity of fibres are included but finally a fibre volume factor is not included in the final expression. This is because the influence of fibre volume is considered by the author as not as important as other factors. Vandewalle (2000) proposed to add a new term to take into account the effect of fibres, but this factor does not depend on fibre volume or any size variable as the same geometry was studied in the tests. The alternative expression by Vandewalle (2000), which is also considered by RILEM TC 162-TDF (2003), is:

$$s_{m.VANDE} = \left(50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi_b}{\rho_{eff}}\right) \cdot \left(\frac{50}{\frac{L_{fib}}{\phi_{fib}}}\right)(mm) \qquad \frac{50}{\frac{L_{fib}}{\phi_{fib}}} \le 1 \tag{3.48}$$

L_{fib} *;* The length of the fibres disposed

ϕ_{fib} *;* The diameter of the fibres

Borosnyói and Balázs proposition:

There are another study carried out by Borosnyói and Balázs (2005) that analyse the spacing and width of the cracks in a loaded reinforced concrete. However, the article is focused on plane concrete. In addition the formulas proposed are simply compiled, but there is not any test to check them. So this is not really a fourth proposition but a high quantity of propositions.
Basically the formulas take into account all the factors that have an influence into crack spacing. The basis expression which is proposed is:

$$s_{B\&B} = f\left(u_{cover}, u_{spac}, \phi_b, \frac{\phi_b}{\rho_{eff}}, \ldots\right) \tag{3.49}$$

Where u_{cover} is the cover of the reinforcement and u_{spac} is the spacing of the reinforcement bars. However these expressions are not further studied in this thesis because they are pointed in conventional reinforced concrete and they are considered as useful.

Ibrahim and Luxmoore proposition:

Finally there is another article that proposes a formula for fibre reinforced concrete, Ibrahim and Luxmoore (1979). It is based on the Leonhardt's method and it has a quite complicated expression, but at least it takes into account the fibres, so the approximation should be better. The article exposes that the presence of fibres in FRC reduces the crack spacing and the crack width and increases the anchorage strength of the bars by 35-40%. The expression that is proposed is:

$$s_{I\&L} = K_1(a, u_{cover}) + K'_2 \cdot K_3 \cdot \eta_s \frac{\phi_b}{\rho_{eff}} \tag{3.50}$$

The terms of the equation have to be defined:

$$K_1(a,c) = \left| \begin{array}{ll} 1.2 \cdot u_{cover} & \text{if } a \le 2 \cdot u_{cover} \\ 1.2 \cdot \left(u_{cover} + \dfrac{a - 2 \cdot u_{cover}}{4}\right) & \text{if } 14 \cdot \phi_b \ge a > 2 \cdot u_{cover} \end{array} \right. \tag{3.51}$$

$$K'_2 = \frac{K_2}{\gamma} = \frac{f_{ct}/\tau_{bm}}{\gamma} \tag{3.52}$$

Where f_{ct} is the tensile strength of concrete, τ_{bm} is the average bond strength of reinforcement bars embedded in ordinary concrete or fibre concrete, respectively and γ is a factor representing increase of anchorage of bars due to fibre inclusion. Leonhardt stated that K_2 can be considered as a constant and, due to that, independent of concrete quality. The values proposed are 0.4 for standard ribbed bars and 0.74 for smooth hot rolled bars.

The γ value can be only determinate by tests. Using linear interpolation, an analytical formula is proposed to calculate this constant:

$$\gamma = \left(1 + \frac{V_f}{0.01} \cdot 0.4\right) \leq 1.4 \tag{3.53}$$

The K_3 term can be defined as:

0.25 for pure tension

0.125 for pure bending

The value of η_s is based on the ratio of the load carried by the conventional reinforcement relative to the total applied load. The expression proposed is:

$$\eta_s = \frac{200 \cdot A_s}{200 \cdot A_s + P_{f,pull} + A_{c.eff}} \tag{3.54}$$

$P_{f,pull}$ is considering 2-D random position:

$$P_{f,pull} = \frac{2 \cdot V_f \cdot \tau_d \cdot l_{fib}}{\pi \cdot \phi_{fib}} \tag{3.55}$$

Where τ_d is the average sliding friction bond strength of fibres.

Finally $A_{c.eff}$ can be calculated as in equation (3.37)

More information about crack spacing can be found in 5.6 and a comparison between the results is made.

4 Finite element model

This chapter will explain the use of the finite element method to solve the problem that is being discussed.

It is important to get a good model which can be used to make a comparison between the two approaches and the two methods of analysis. It is supposed that the FEM software should give a similar response as this element would have in the reality with the same material properties. The analysis will be done using a finite element software which is called DIANA (see TNO DIANA (2005)).

4.1 Material models. Flexural and loaded with normal

Apart from the different constants of the materials, some models have to be chosen to make possible the comparison between the analytical and the FEM results. It is, therefore, necessary to choose models of the behaviour which are as closer as possible to the models proposed by RILEM and the Eurocode.

4.1.1 The σ–*w* approach

4.1.1.1 Non-Cracked behaviour of concrete

The compressive zone is defined based on the total strain crack model. There are two options in this model: strain relations is fixed or rotating axis. The rotating axis is chosen because it is not necessary to use shear retention parameters.

This total strain model has the advantages that it is not necessary to use complicated functions, it can also be used in concrete for compression and mathematically it does not require complicated iterations. This model has also disadvantages but they are not so important to this analysis. It cannot permit non-orthogonal multidirectional cracking, but in the assumptions this kind of cracks are not taking into account. At last this is not a good model if for example it is necessary to consider creep and shrinkage, see Rots (2002).

Regarding the behaviour in compression it is possible to choose between the models which are presented in the figure 4.1.

The best option consists on inserting points (multilinear model, see figure 4.1b) giving values to the strain and getting the stress value using the EC2 expression which has been explained in 3.1.1.

The behaviour in tension is supposed elastic as in the non linear crack hinge model.

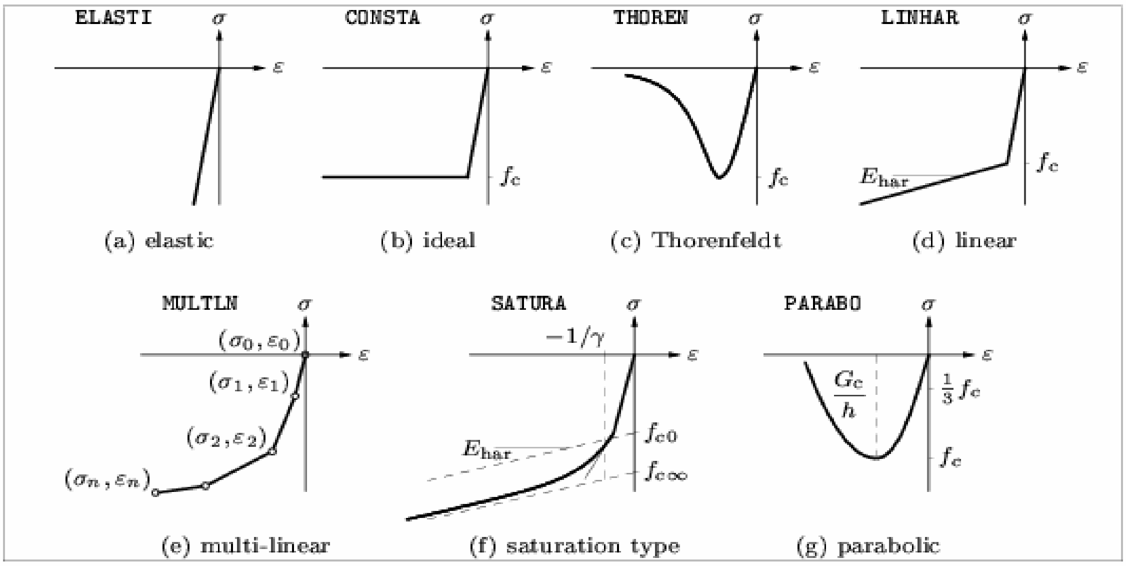


Figure 4.1 Models to define the compressive behaviour. DIANA users manual

4.1.1.2 Cracking behaviour of concrete

The zone that will crack can be defined as an interface zone and it has a special properties. The model is thus based on a discrete crack because the position of the crack is predefined.

To approach the tensile behaviour of concrete, the multilinear tension softening model has been chosen. The main reason is that this model can be adjusted completely to the proposal made by RILEM in its recommendations. In DIANA, this is presented as mode1 (opening mode) 3 (model three – multi-linear).

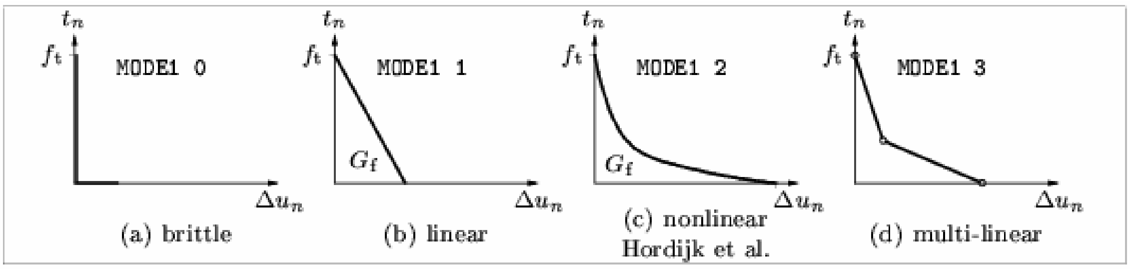


Figure 4.2 Models to define the tension softening behaviour σ–w. DIANA manual

The values have to be the same as the ones used in the analytical model. It has to be noticed that it is necessary to give three values to completely define the curve. Moreover, the values of the crack opening have to be half of the normal values. This is due to the fact that only half of the beam is modelled whereas the analytical model uses the whole element, for further information see 4.2 about geometrical model. These values are showed in figure 4.3.

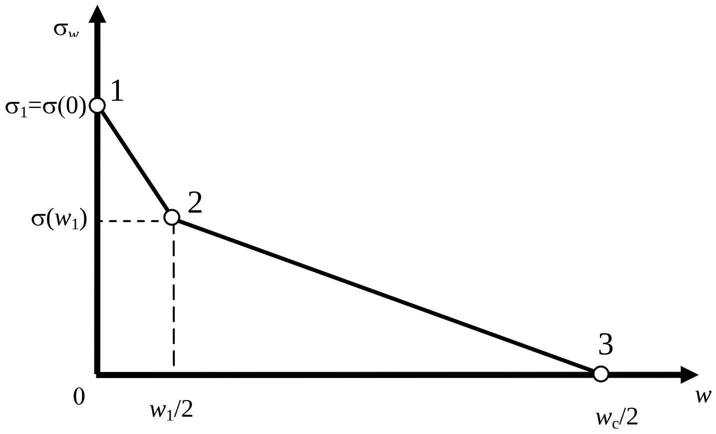


Figure 4.3 Values to introduce in the DIANA model (sigma-opening relationship)

It is also defined that a crack will appear if the tensile stress reaches the limit of the tensile strength f_t. Finally it is supposed that the stress will return directly to the elastic behaviour when an unloading is applied.

I addition a constant shear stiffness modulus is chosen and it has to be an appropriate value in order to obtain realistic results. A very high value for the interface stiffness could give unexpected results. Rots (2002), recommend that this stiffness be chosen approximately according to this expression: $D_{stiffcrack} = 1000 \times \frac{E}{L}$. Where L is the characteristic length of the structure as can be noticed in figure 4.4. The same can be applied to the shear stiffness but changing the young modulus by the shear modulus: $D_{stiffcrack} = 1000 \times \frac{G}{L}$.

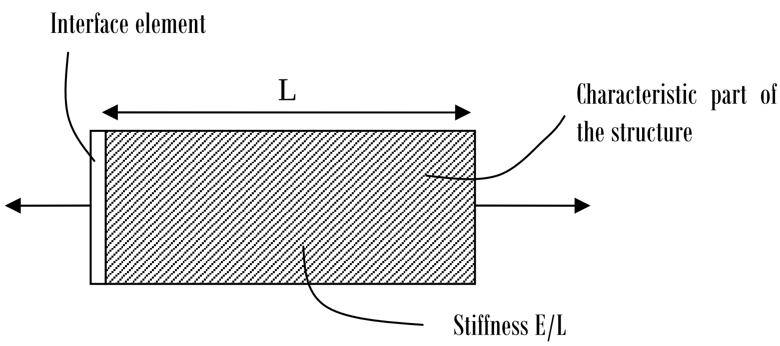


Figure 4.4 Selection of dummy interface stiffness

The value L is calculated as half crack spacing value. Due to that, the final values for the dummy interface stiffness are calculated in Chapter 5.2 where the value for the crack spacing is decided.

4.1.1.3 Behaviour of the reinforcement

The yielding condition that is used is the Von Misses criterion. The behaviour of the steel is showed in the next figure:

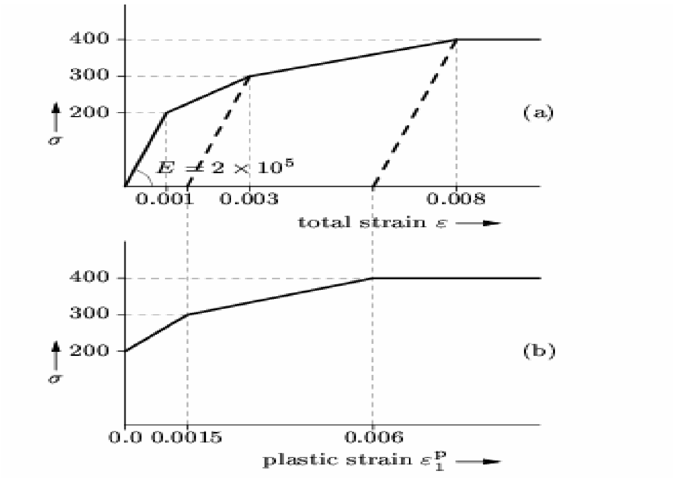


Figure 4.5 Stress-strain relationship steel diagrams. DIANA manual

Hence, the values that are needed to introduce as an input in the program are: (0, f_{yk}), (ε_{uk}- f_{yk}/E, $k_s \cdot f_{yk}$) and (ε_{uk},0). See figure 4.6.

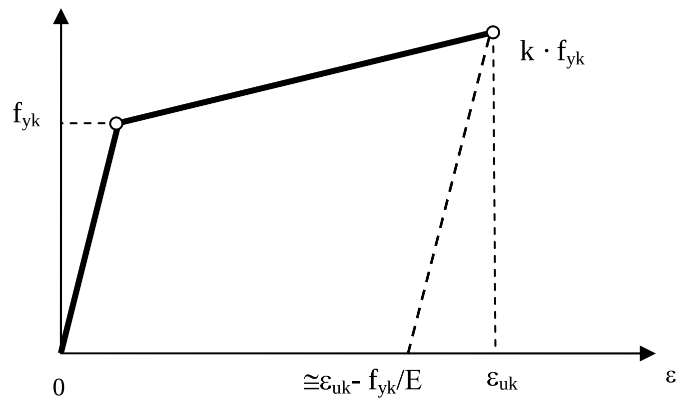


Figure 4.6 Values that define the behaviour of the reinforcement

It can be appreciated the strain hardening after the elastic behaviour and the value ε_{uk}- f_{yk}/E is an approximation.

4.1.1.4 Bond-slip behaviour

The bond mechanism consists of the contact of the concrete and the steel. Some times (like in the analytical calculations) it is usually assumed that the steel and the concrete have the same deformation. But this idealisation is not true, see chapter 3.3 about crack spacing. There are many factors that should be taken into account to obtain the most approximate solution.

DIANA allows considering the bond-slip but it is necessary to create another interface element (like in the cracking zone). The laws which are proposed in DIANA are based on the total deformation theory that consists on obtaining the relationship between the traction force and the relative total displacement. The figure 4.7 shows the options that DIANA gives.

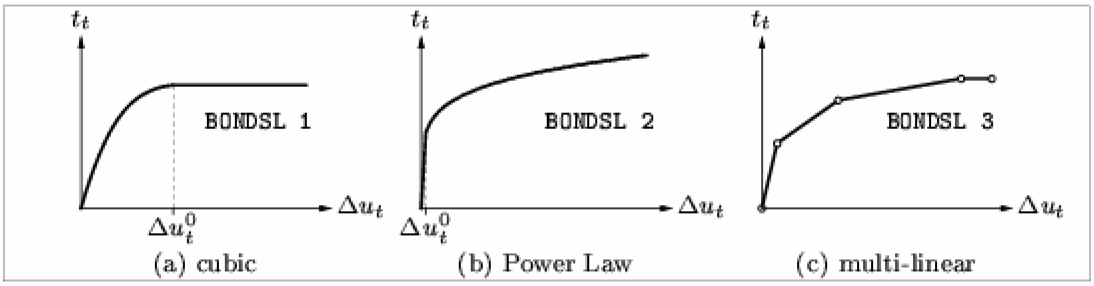


Figure 4.7 Bond-slip models. DIANA manual

The model which is chosen is the last one (c). It is necessary to have an expression describing the relationship between bond and slip, in this thesis the model according to Model Code 1990 was used. The equation that gives the bond stress for each displacement is the equation (4.1).

$$\tau_{bond} = \tau_{\max} \cdot \left(\frac{S_i}{S_1} \right)^{\beta} \tag{4.1}$$

Where:

$$\tau_{\max} = 2.5\sqrt{f_{cm}} \tag{4.2}$$

S_i=relative displacement between reinforcement and concrete

S_1=1(Good conditions)

β=0.4

It is also possible to calculate the final stress (limit when the displacement is very high) as $\tau_{final} = 0.4 \cdot \tau_{\max}$. There is therefore a curve for each compressive strength value and this expression is only valid if the reinforcement is confined. The final curve for a f_{cm}=30MPa concrete is showed in figure 4.8. The rest of the curves and the values can be seen in appendix B.

It is also needed the dummy interface stiffness. It can be calculated following the next expression:

$$D_{stiffbond} = \frac{\tau_{0.05} \cdot 10^6}{0.05 \cdot 10^{-3}}$$

Where $\tau_{0.05}$ is the stress when the relative displacement is 0.05mm.

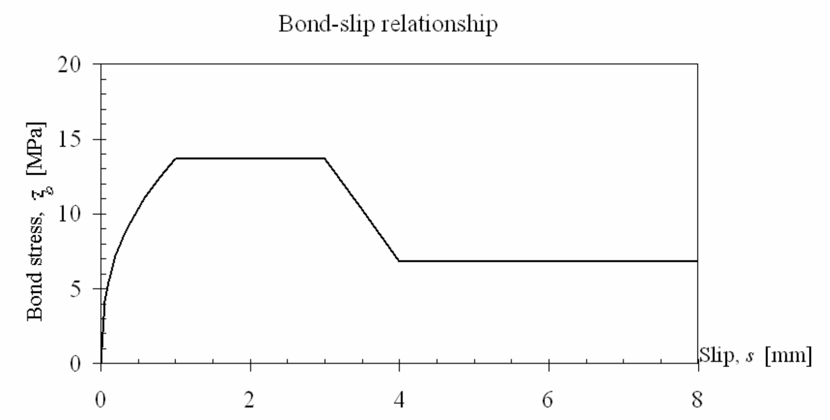


Figure 4.8 Bond-slip curve for f_{cm}=30MPa

4.1.2 The σ–ε approach

4.1.2.1 Behaviour of concrete

This approach will not have a distinction between cracked and non-cracked zone, i.e. all elements are allowed to crack. Furthermore, it is assumed that the deformation of one crack can be smeared out over the element. The compressive behaviour of the concrete is the same as in the other approach, see 4.1.1.1. However, the tensile behaviour is different as it is based on a stress-strain relationship; see figure 4.9.

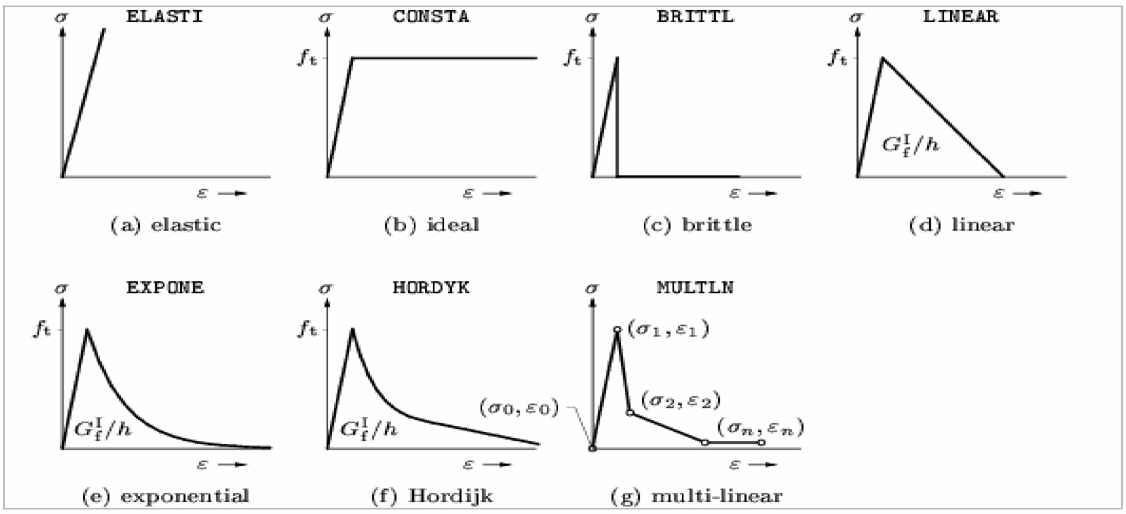


Figure 4.9 Models to define the tension softening behaviour σ–ε. DIANA manual

Obviously the most appropriate model is (g). Following this model it is necessary to define 5 points like is showed in the figure 4.10. Note that it is necessary to add one point (σ=0; ε=0.026) in order to specify that the strength after σ_3 is reached is zero.

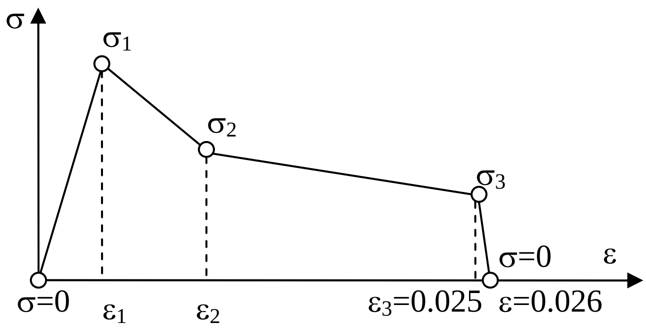


Figure 4.10 Tension softening relationship proposed by RILEM TC 162-TDF adapted to DIANA

4.1.2.2 Behaviour of the reinforcement

For the reinforcement, the same configuration as in the $\sigma - w$ approach can be applied.

4.1.2.3 Bond-slip behaviour

For the bond-slip behaviour, the same configuration as in the $\sigma - w$ approach can be used.

4.2 Geometrical models and element formulation. Flexural tensile load.

It is also important to define a geometrical model with an appropriate mesh. The models for the two approaches are similar but not the same. In both, the load will be applied by means of a incremental rotation because it is important not to create a shear load that could modify the results because of its influence. For this purpose a dummy beam has been created. This dummy beam will be restricted in the gravity direction and also its rotation direction (z). The only rotation that is allowed is the incremental turn that it is applied in each step.

It is necessary to maintain the right side of the specimen rigid (plane section remains plane) and with the same rotation as the dummy beam. Due to that, the right side of the beam will be "tied" to the rotation point by means of a master-slave definition. This definition allows maintaining the same relationship in the displacement for both (eccentric relationship).

4.2.1 The $\sigma - w$ approach

The basis model consists of a simply supported beam that is being rotated at both sides. It generates a pure bending moment. The length of the beam is *s* (non-linear hinge length) because it is the same length analysed in the analytical model. As a crack interface zone has to be defined (with special characteristic), it is placed in the centre of the beam because this is the most probably place to crack first. Figure 4.11 shows the basis model.

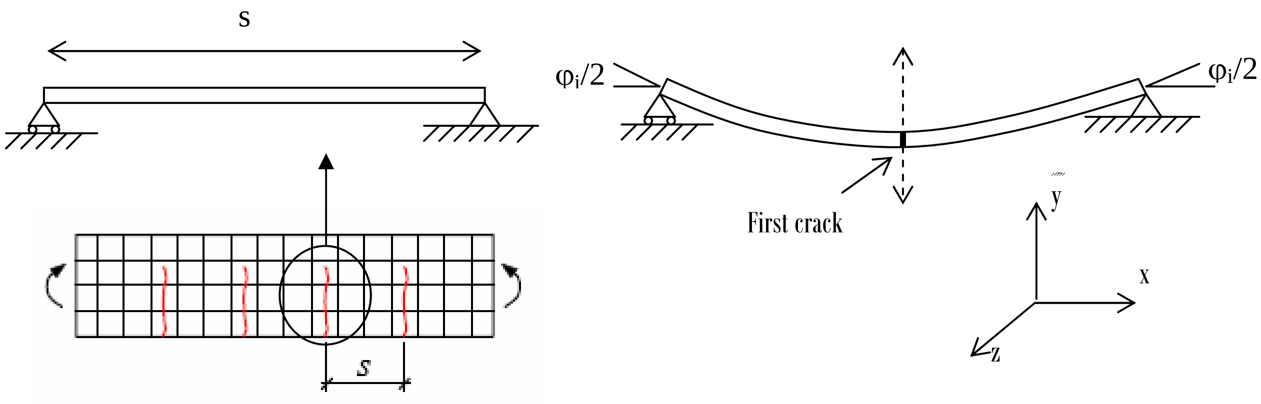


Figure 4.11 Geometrical model σ–w approach

For this approach, only half part of the model is needed because the symmetry. The model has the shape showed on the figure 4.12.

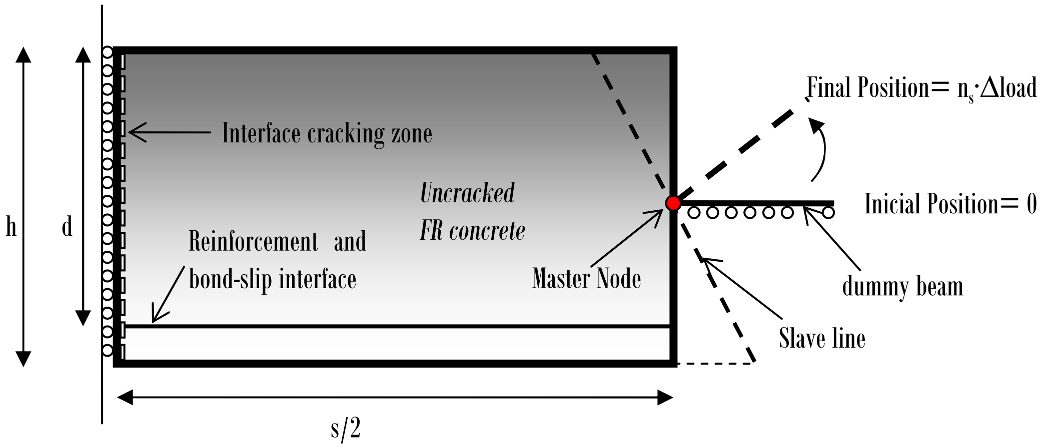


Figure 4.12 Geometrical model σ–w approach

The load (rotation) varies from 0 to *n·Δload*, where *n* is the number of steps that are performed and *Δload* is the length of the step. These values are showed in chapter 6 about Results.

4.2.1.1 Uncracked concrete elements

The uncracked zone is meshed by using a Q8MEM element. It consists of a four-node quadrilateral isoparametric plane stress element, see figure 4.9. Each element is defined to have elastic behaviour.

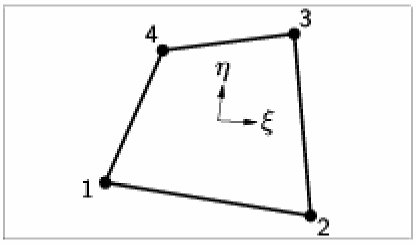


Figure 4.13   Q8MEM element. DIANA manual

4.2.1.2 Interface cracking zone

This zone has to be defined as an interface element, which permits a good and easy measured value of the crack opening. L8IF is a typical interface element which consists on two lines in a two dimensional configuration, see figure 4.10. These elements are defined with the stress crack-opening relationship.

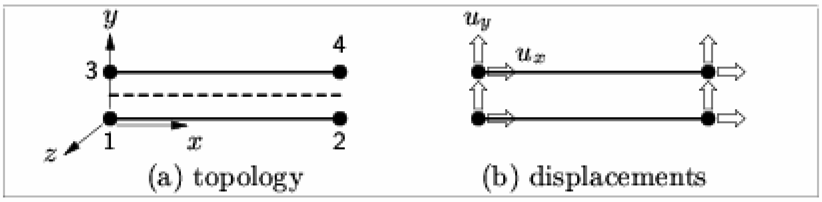


Figure 4.14   L8IF element. DIANA manual

It is very important to define correctly the element following the rule showed on the figure 4.10.a. The correct order of the nodes is 1-2-3-4. The figure 4.11 shows a typical connexion between a Q8MEM and a L8IF element. If this is not correctly defined, the result may not be realistic.

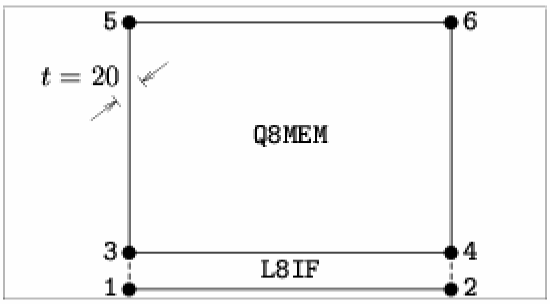


Figure 4.15   Typical connexions between Q8MEM and L8IF element. DIANA manual

4.2.1.3 Reinforcement zone

This zone is meshed by a typical truss element (L2TRU) with the properties of the reinforcement. Each element consists on two nodes only defined in the X axis. See figure 4.16.

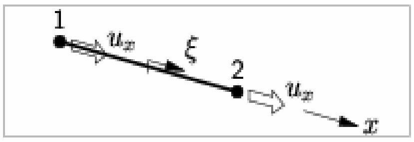


Figure 4.16 L2TRU element. DIANA manual

4.2.1.4 Bond-slip zone

The configuration is the same as in the interface cracking zone. The difference is that this interface element is joined to a L2TRU element and it has different properties like stiffness and relationship force-opening of the interface which represents the bond-slip behaviour of the reinforcement.

4.2.1.5 Final result

This is the final meshing result. Logically the interface elements cannot be appreciated until a load is applied.

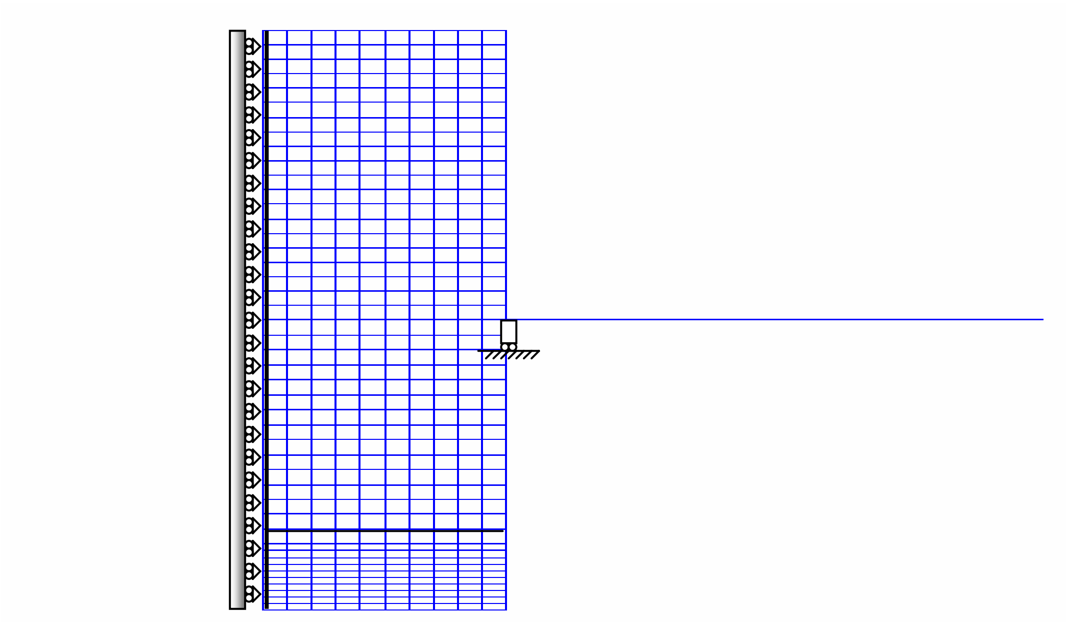


Figure 4.17 DIANA meshing model, sigma-w approach. Beam height 250mm

4.2.2 The $\sigma - \varepsilon$ approach

The model is very similar to the $\sigma - w$ model with the difference that the cracking interface zone does not exist and, therefore, all the concrete elements have the same properties. Due to that the interface element for the cracking zone is not necessary. The length of the beam will also be s/2 due to the fact that it is important to have models with very similar dimensions to compare the results, see figure 4.11 and 4.19. There was found to be a problem in this model due to a concentration of stresses at the right side of the beam. The reason is that there is not a weak element than in the other model (interface element).

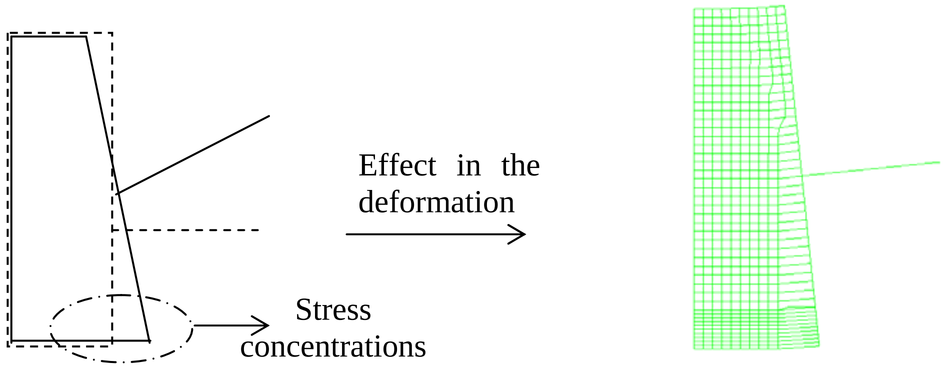


Figure 4.18 Stress concentrations in the element. Effect in the deformation

When the rotation is applied, there is a concentration of stress in the zone that is showed in figure 4.18. Due to that, this part of the beam will crack before the left part and this is not the behaviour desired. To avoid that, it is necessary to introduce a weak element showed in figure 4.19. This element is defined as a 25% of the strength of a normal element.

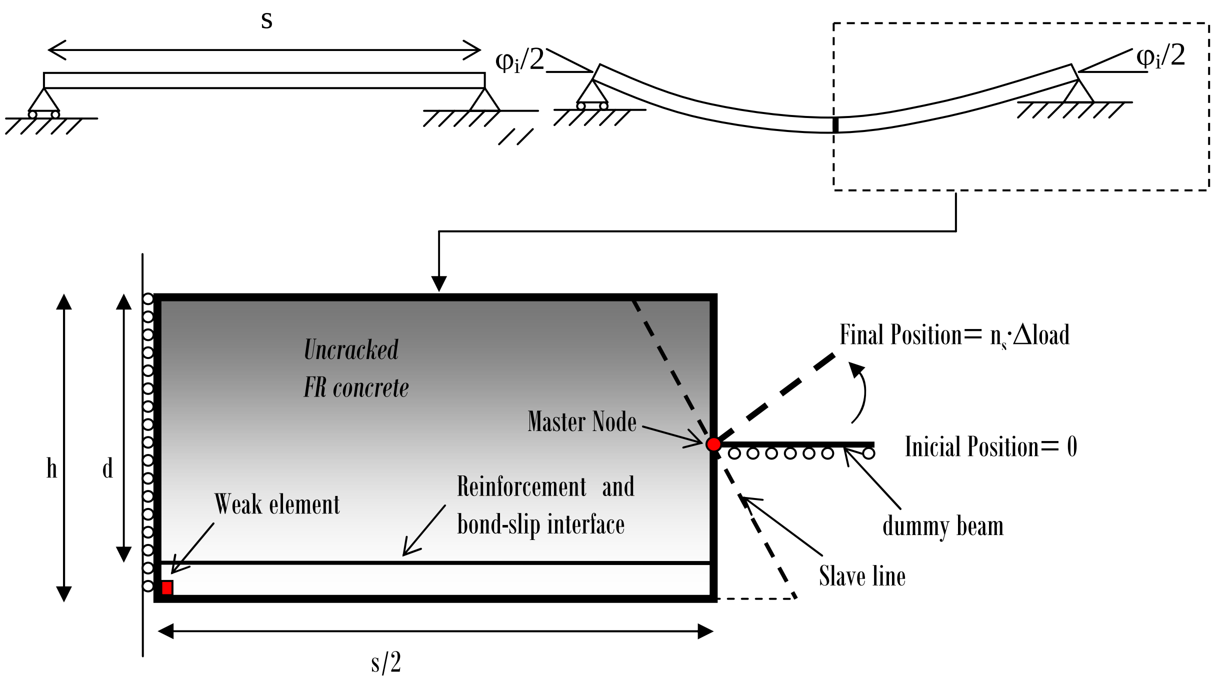


Figure 4.19 Geometrical model $\sigma - \varepsilon$ approach

4.2.2.1 Meshing elements

The elements are the same as in the other approach, so it is not necessary to repeat them again. The only difference is that the interface of the cracking zone does not exist. This is the meshing model that appears the same as the other case:

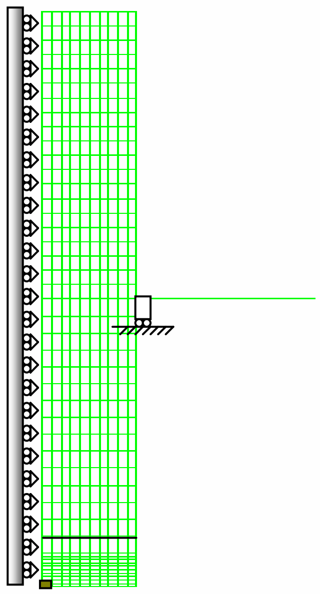


Figure 4.20 DIANA meshing model, sigma-ε approach. Beam height 500mm

5 Material and geometrical values

This chapter presents the different values of the constants, material properties and geometrical dimensions that are needed to solve the different equations.

5.1 FRC, the σ−ε approach

As it was showed in the chapter 2, to know the values of the constants, some tests are necessary.

The test used to determinate the values of the curve is the 3PBT (three-point bending test). Figure 5.1 shows the main approximate dimensions and the shape of this test, figure 5.2 shows the cross-section specimen.

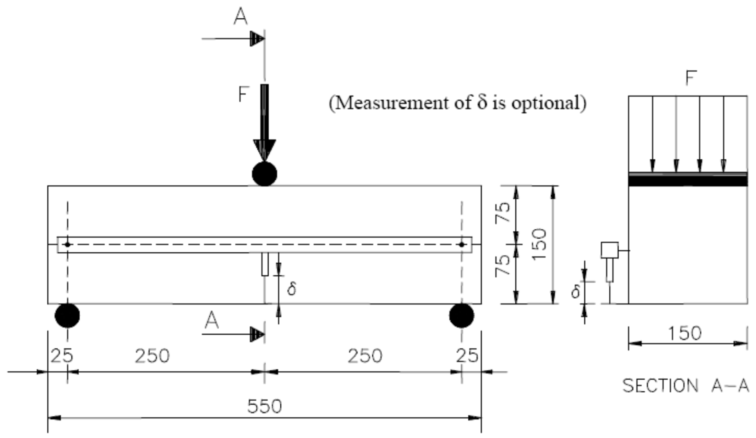


Figure 5.1 Three-point loading test. RILEM, bending tests and interpretation

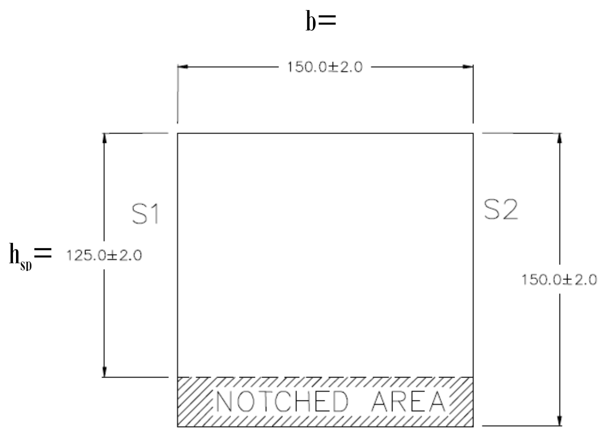


Figure 5.2 Cross-section specimen. RILEM, bending tests and interpretation

When a load is applied, it is possible to obtain the crack mouth opening displacement (CMOD), and the curve which relate them. The curve is essential to get the values of the residual flexure strength $f_{R,i}$.

As it is defined by RILEM, the values of $\boldsymbol{F_{R,L}}$, $\boldsymbol{F_{R,1}}$, and $\boldsymbol{F_{R,4}}$ are the load applied when the CMOD is equal to 0.05mm (or maximum in the interval 0-0.05), 0.5mm and 3.5mm. See the figure 5.2.

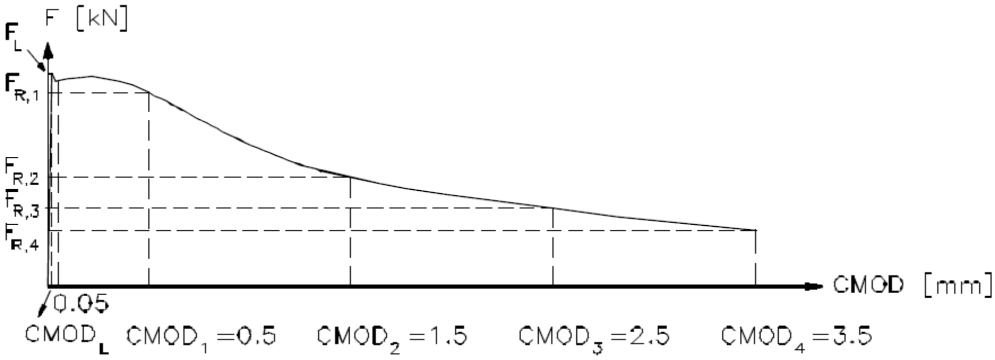


Figure 5.3 Residual tensile strengths. RILEM, bending tests and interpretation

Figure 5.4 shows a real test results from 3PBT tests, made for the 5 mixes, that are studied in the thesis – see Löfgren et al. (2004) and Löfgren (2005). The values of $F_{R,1}$, $F_{R,4}$ and F_L are presented in the analytical calculations and in table 5.1,

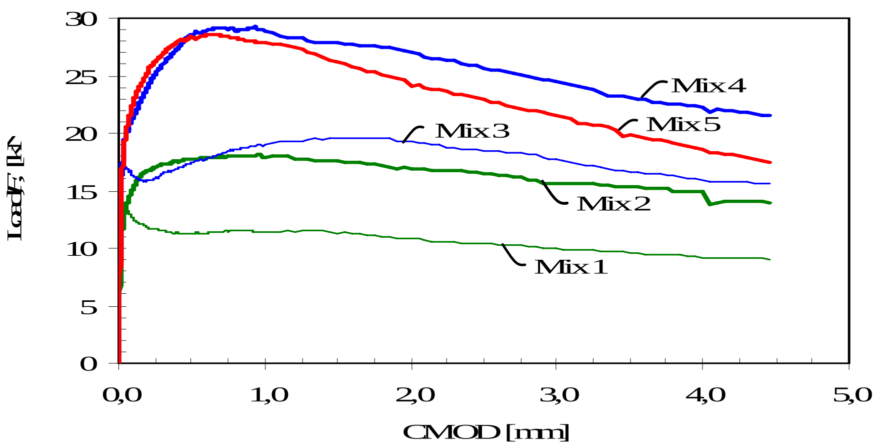


Figure 5.4 3PBT tests for 5 different mixes. From Löfgren et al. (2004), see also Löfgren (2005).

Some experimental results from tests conducted at DTU (technical university of Denmark) have be used to determine the properties, see Löfgren et al. (2004) and Löfgren (2005). Tests were made for five different mixes, table 5.1 shows the value of the most important constants: f_{fcm} (see 2.1), *w/b* ratio (water binder ratio), V_f (volume fraction of fibres), aspect ratio/length of the fibres, *b* (width of the specimen), *L* (length between supports of the beam tested), h_{sp} (distance between the tip of the notch and top of cross section).

Note that the fibres used in the tests were Hooked-end steel fibres (type Dramix™).The typical notation for the fibres are showed in figure 5.5.

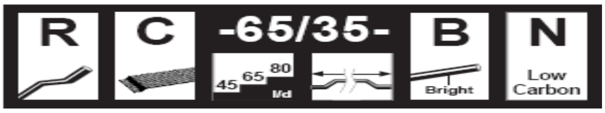


Figure 5.5 Notation of fibres. DRAMIX®

In the first four mixes the type used was RC 65/60-BN, which means that fibre length is 60mm and the slenderness class is 65 (diameter of the fibres 0.9mm). The last mix had RC 65/35 fibres (diameter 0.55mm). The cement used was CEM II/A-LL 52.5 R, with 260 kg/m^3 in mix 1 and 2, while mixes 3, 4 and 5 contained 360 kg/m^3 together with 100 kg/m^3 fly ash (with a k-faktor of 0.5). The tensile strength of the wire is minimum 1100 N/mm^2.

Note that in mix 4 the height is up the tolerance limit. Anyway it is considered that a difference of 3 mm still can be acceptable.

Table 5.1 Different constants of the mixes tested

	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
f_{cm}[MPa]	30	26	49	44	47
w/b ratio	0.58	0.58	0.42	0.42	0.42
V_f [%]	0.5	1.0	0.5	1.0	1.0
Aspect ratio/length	65/60	65/60	65/60	65/60	65/35
Concrete ρ [kg/m^3]	2400	2400	2400	2400	2400
Poisson coefficient (ν) [--]	0.2	0.2	0.2	0.2	0.2
b [mm]	151.3	151.8	151.8	151.4	151.3
L [mm]	500	500	500	500	500
h_{sp} [mm]	124.2	126.13	125.1	128.1	125.7
$F_{R,1}$ [kN]	11.34	17.72	17.48	28.50	28.39
$F_{R,4}$ [kN]	9.62	15.38	16.67	23.15	19.88
F_L [kN]	13.43	13.46	17.27	19.84	20.00

As it is not possible to include all the mixes in the further analysis, two or three of them are chosen. However it is very interesting to show the σ–ε curve for each mix in one height (i.e. for h=125mm) and see the difference in function of the different properties. Figure 5.6 and 5.7 show this curves.

In the appendix C, all the calculations for the rest of the heights can be found included in the MathCAD analytical process file.

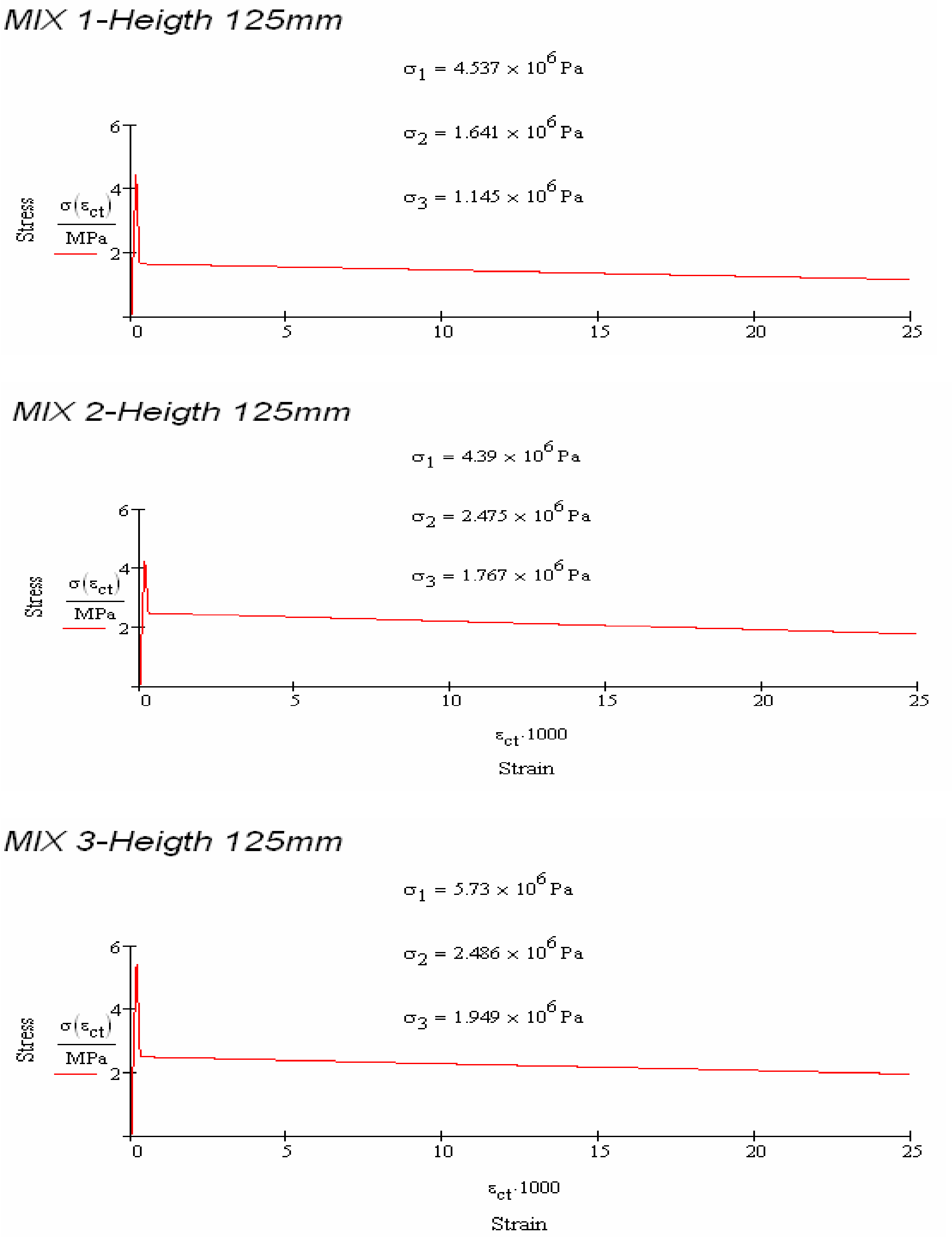
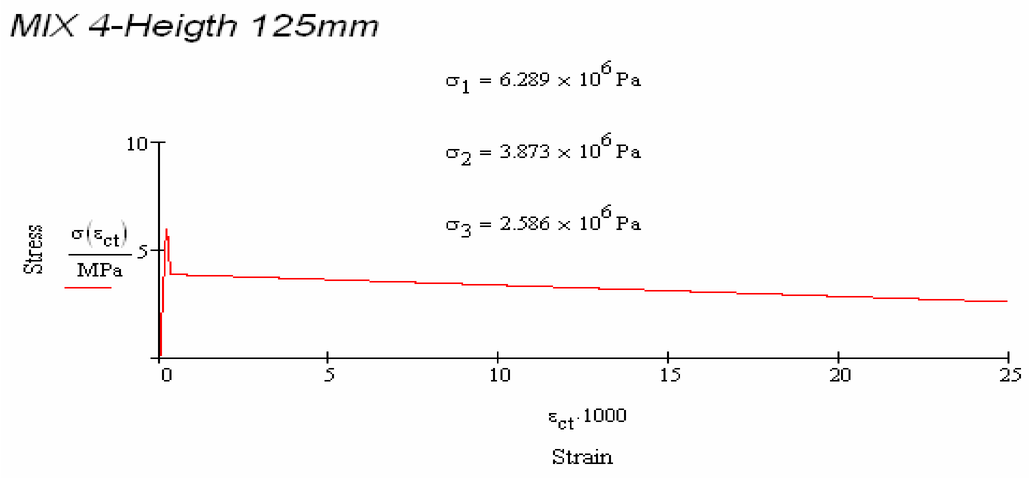


Figure 5.6 σ–ε relationship for mix 1, mix 2 and mix 3



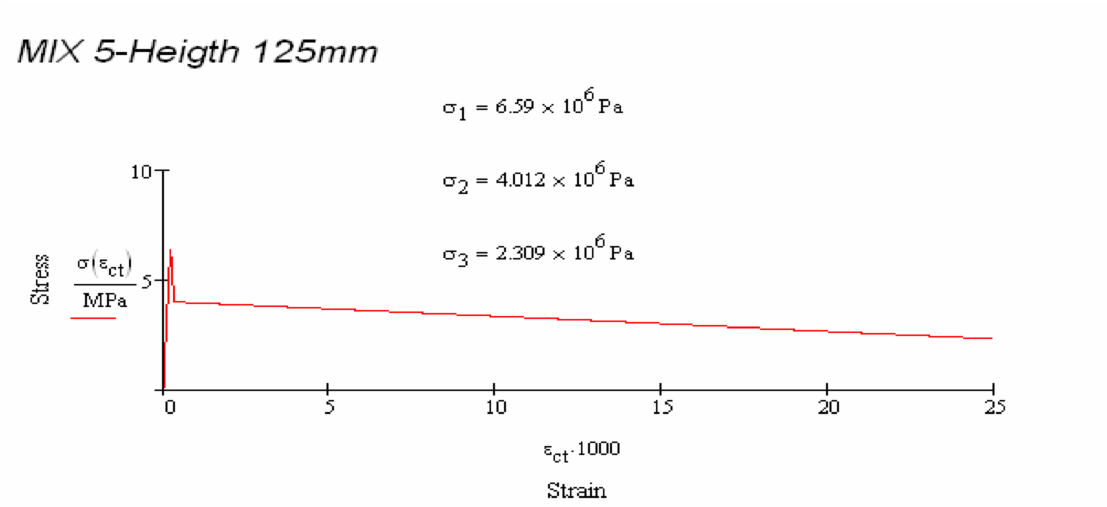


Figure *5.7* *σ–ε* relationship for mix 5

Some comments can be made viewing these results:

- The two first mixes have the same ratio characteristics with the exception of the volume of fibres, which is higher in mix 2. As a consequence of this, a similar elastic limit and compressive resistance are obtained. But there is a considerable difference in the value of σ_2 (residual flexural resistance). This value is higher in the mix 2 because of the higher quantity of fibres. The same effect occurs with σ_3.
- The rest of the mixes have a bigger resistance (in compression) because the concrete had less air content and a lower water-cement ratio.
- Mix 3 and 4 has the same kind of fibres but a different dosage. The effect of this is a bigger residual flexural resistance and elastic limit in the case of the mix 4.
- Mix 5 has fibres with a shorter length and a smaller diameter compared to the other mixes, but has unexpectedly a higher elastic limit and σ_2 value than mixes 3 and 4. It means that the increasing of the length and diameter of the fibres does not mean a better response in flexural behaviour. Nevertheless in the limit of strain (0.025), the stress has a higher value for mix 4. This is a result of the shorter fibre used in this mix.

Finally the mixes chosen to be introduced in the analysis are mix 1, mix 4 and mix 5 to have results with different resistance class of concrete, dosage and length of fibre. The name will be change in order to avoid confusions. Mix 1=Mix A, Mix 2=Mix B, Mix 3=Mix C.

Table 5.2 Mixes chosen to be compared in the analysis

	Mix A	Mix 2	Mix 3	Mix B	Mix C
f_{fcm} [MPa]	30	26	49	44	47
w/b ratio	0.58	0.58	0.42	0.42	0.42
V_f [%]	0.5	1.0	0.5	1.0	1.0
Aspect radio/length	65/60	65/60	65/60	65/60	65/35
Fibre diameter [mm]	0.9	0.9	0.9	0.9	0.55
Fibre length [mm]	60	60	60	60	35

5.2 FRC, the σ–*w* approach

The σ-w relationship is based on inverse analyses of the three-point bending tests (the same that were used for the σ–ε approach) conducted by Löfgren et al. (2004) and (2005). After the tests, inverse analyses were conducted and it was possible to obtain bilinear relationships. See figure 5.6

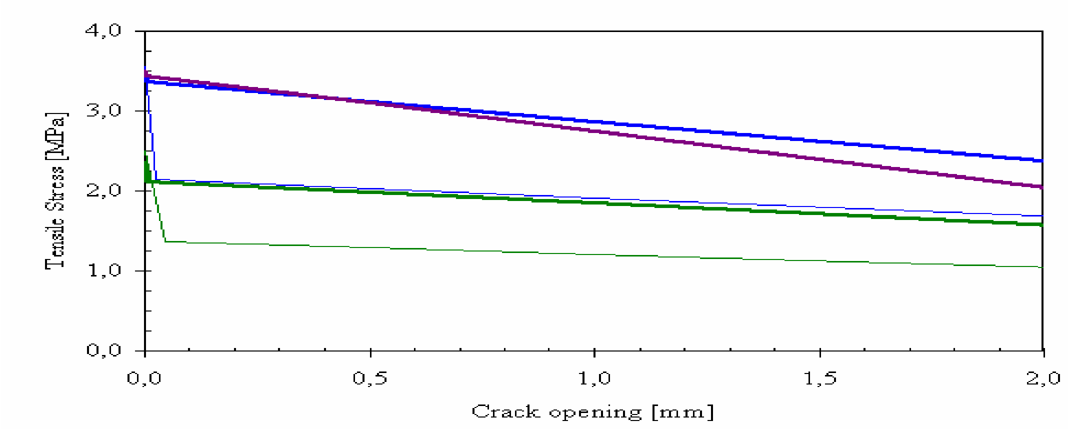


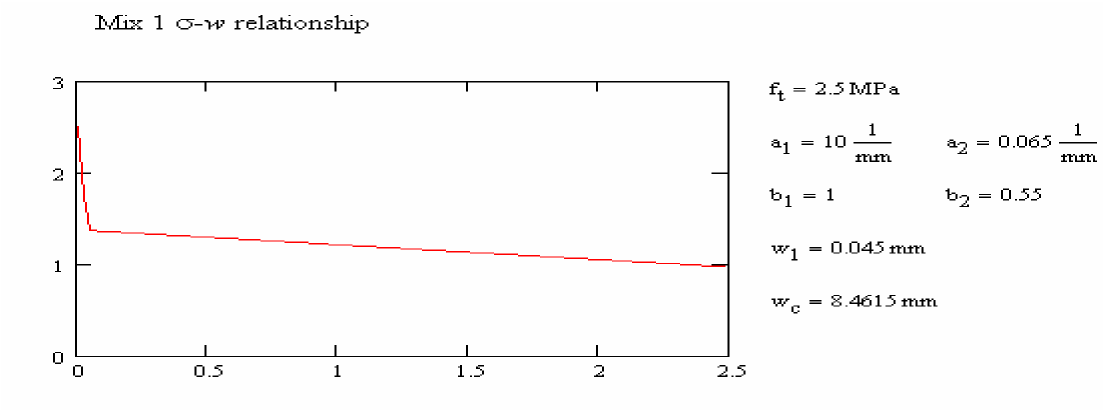
Figure 5.8 *σ–w* relationship for all the mixes

The results of the tests are the bi-linear relationship which can be derived for the different mixes. It is also included the dissipated energy in the crack tip (area under the curve). This energy can be considered the fracture energy G_F, see RILEM TC-162 TDF. As 3 points are needed to define the curve in DIANA, these values are showed under each curve. This approach has not size factor, so the values are the same for each height.

The general expression of the σ–*w* relationship is the equation (2.8):

$$\sigma_w(w) = \begin{cases} f_{ct}(1 - a_1 w) \text{ when } & 0 \le w \le w_1 = \dfrac{1-b_2}{a_1 - a_2} \;; \\ f_{ct}(b_2 - a_2 w) \text{ when } & w'_1 \le w \le w_c = \dfrac{b_2}{a_2} \;; \end{cases}$$

And due to that, the results for every mix can be derived:



$$\begin{cases} w_1 = 4.5 \cdot 10^{-5}\,\text{m}\,; \sigma_{w1}(w_1) = 2.5 \cdot \left(1 - \dfrac{10}{mm} w_1\right) = 1.368 \cdot 10^6\,\text{Pa} \\ w_c = 8.462 \cdot 10^{-3}\,\text{m}\;; \sigma_{w1}(w_c) = 2.5 \cdot \left(0.55 - \dfrac{0.065}{mm} w\right) = 0 \end{cases}$$

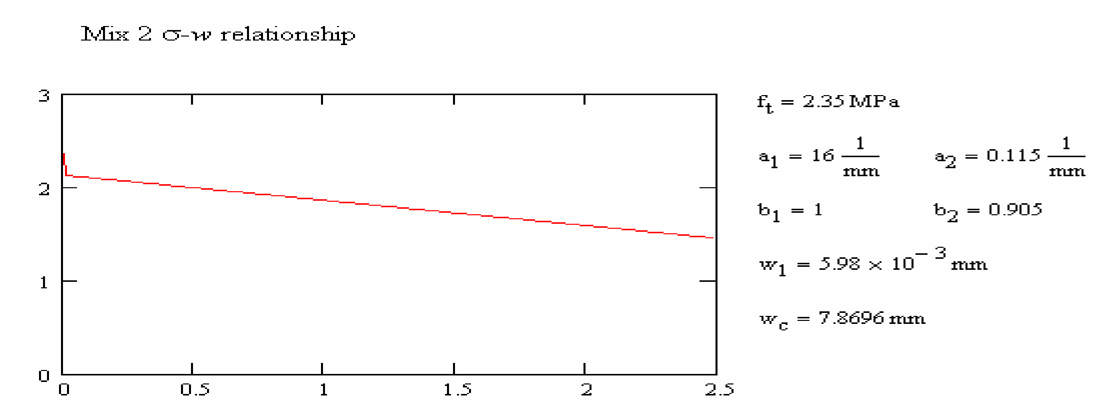
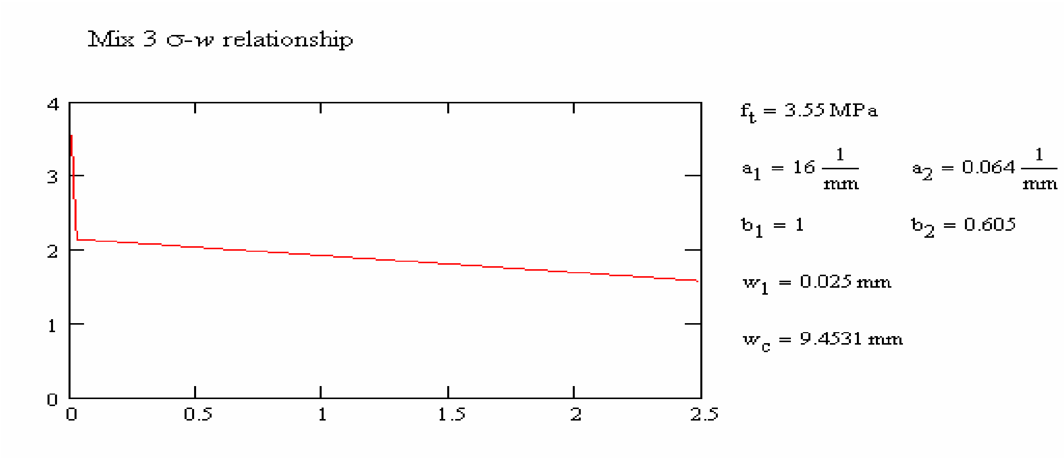
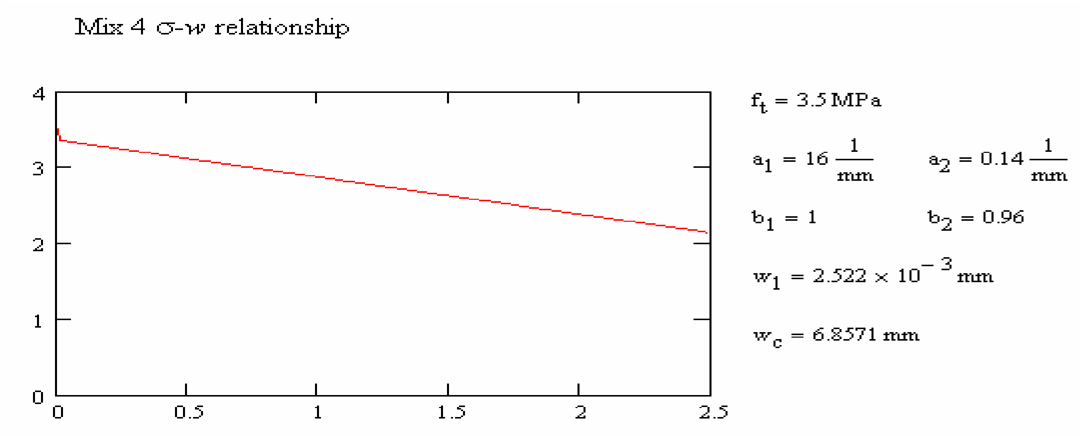


Figure 5.9 σ–w relationship for mix 1 and mix 2

$$\begin{cases} w_1 = 5.98 \cdot 10^{-6}\,\text{m}\,; \sigma_{w1}(w_1) = 2.35 \cdot \left(1 - \dfrac{16}{mm} w_1\right) = 2.125 \cdot 10^6\,\text{Pa} \\ w_c = 7.87 \cdot 10^{-3}\,\text{m}\,; \sigma_{w1}(w_c) = 2.35 \cdot \left(0.905 - \dfrac{0.115}{mm} w\right) = 0 \end{cases}$$

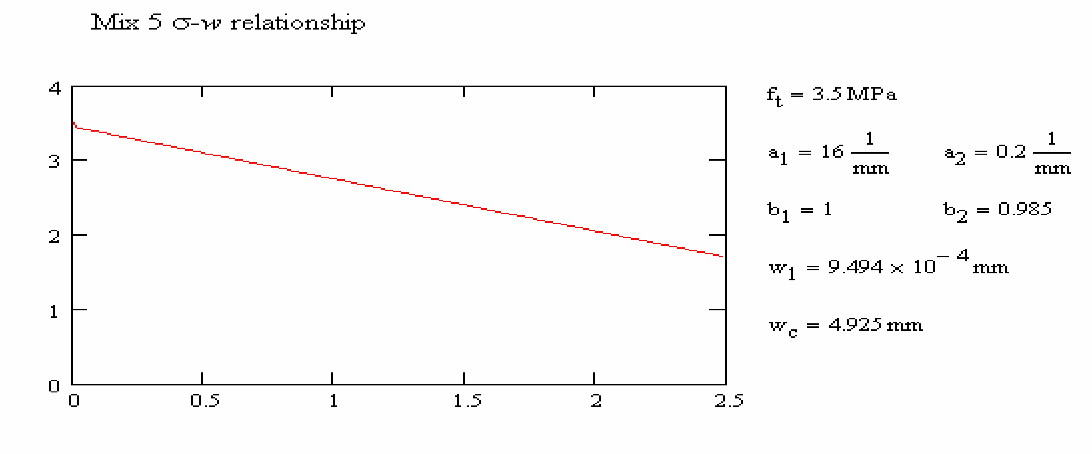


$$\begin{cases} w_1 = 2.5 \cdot 10^{-5}\,\text{m}\,; \sigma_{w1}(w_1) = 3.55 \cdot \left(1 - \dfrac{16}{mm} w_1\right) = 2.142 \cdot 10^6\,\text{Pa} \\ w_c = 9.453 \cdot 10^{-3}\,\text{m}\,; \sigma_{w1}(w_c) = 3.55 \cdot \left(0.605 - \dfrac{0.064}{mm} w\right) = 0 \end{cases}$$



$$\begin{cases} w_1 = 2.522 \cdot 10^{-6}\,\text{m}\,; \sigma_{w1}(w_1) = 3.55 \cdot \left(1 - \dfrac{16}{mm} w_1\right) = 3.359 \cdot 10^6\,\text{Pa} \\ w_c = 6.8571 \cdot 10^{-3}\,\text{m}\,; \sigma_{w1}(w_c) = 3.55 \cdot \left(0.96 - \dfrac{0.14}{mm} w\right) = 0 \end{cases}$$

Figure 5.10 σ–w relationship for mix 3 and mix 4 and crack values for mix2



$$\begin{cases} w_1 = 9.494 \cdot 10^{-7}\,\mathrm{m}\,; \sigma_{w1}(w_1) = 3.5 \cdot \left(1 - \dfrac{16}{mm} w_1\right) = 3.447 \cdot 10^6\,\mathrm{Pa} \\ w_c = 4.925 \cdot 10^{-3}\,\mathrm{m}\ ; \sigma_{w1}(w_c) = 3.5 \cdot \left(0.96 - \dfrac{0.14}{mm} w\right) = 0 \end{cases}$$

Figure 5.11 σ–w relationship for mix 5

5.3 Conventional reinforcement

As it was showed in chapter 3, the model of the steel reinforcement behaviour is taken from the latest version of the Eurocode 2.

There are some values that are needed:

- E_s is the modulus of elasticity. The design value can be assumed to be 200 GPa
- f_{syk} is the yielding strength.
- k_s is a constant.
- ε_{suk} is the characteristic limit of strain.

These values can be found in the annex C of the Eurocode 2. See table 5.4

However, as all the constants and curves used are based on the characteristic values, the curve that is used is like that:

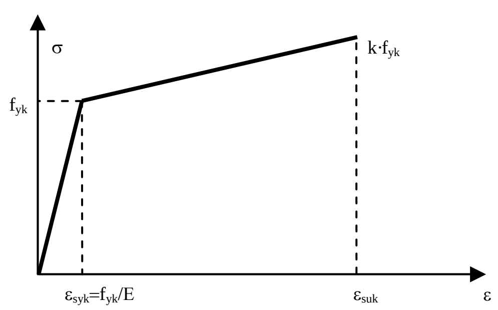


Figure 5.12 Stress-strain diagram of reinforcement to be used in the calculations

Table 5.4 Values to use in the design of reinforcement (II). EC2

Product form	Bars and de-coiled rods			Wire Fabrics			Requirement or quantile value (%)
Class	A	B	C	A	B	C	-
Characteristic yield strength f_{yk} or $f_{0,2k}$ (MPa)	400 to 600						5,0
Minimum value of $k = (f_t/f_y)_k$	≥1,05	≥1,08	≥1,15 <1,35	≥1,05	≥1,08	≥1,15 <1,35	10,0
Characteristic strain at maximum force, ε_{uk} (%)	≥2,5	≥5,0	≥7,5	≥2,5	≥5,0	≥7,5	10,0
Bendability	Bend/Rebend test			-			
Shear strength	-			0,3 A f_{yk} (A is area of wire)			Minimum
Maximum deviation from nominal mass (individual bar or wire) (%) — Nominal bar size (mm): ≤ 8; > 8	± 6,0 ± 4,5						5,0

The steel chosen for the conventional reinforcement is class B with Characteristic yield strength of 500MPa (B500B). Due to this election, the characteristic strain could be the minimum (5%), and the recommendation for the *k* value is 1.08 (minimum).

Finally the ratio reinforcement/concrete approximately is approximately 0.1%, and the initial bar spacing is 150 mm; having these two variables it is possible to obtain the bar spacing and the number of bars required. The cover for the reinforcement is usually 25 mm (according to EC2).

Other values that are needed are showed in the table 5.5.

Table 5.5 Values to use in the design of reinforcement (II).

Constant	Value
Density (ρ)	7850 kg/m^3
Young modulus (E)	200 x 10^9 Pa
Poisson coefficient (ν)	0.3

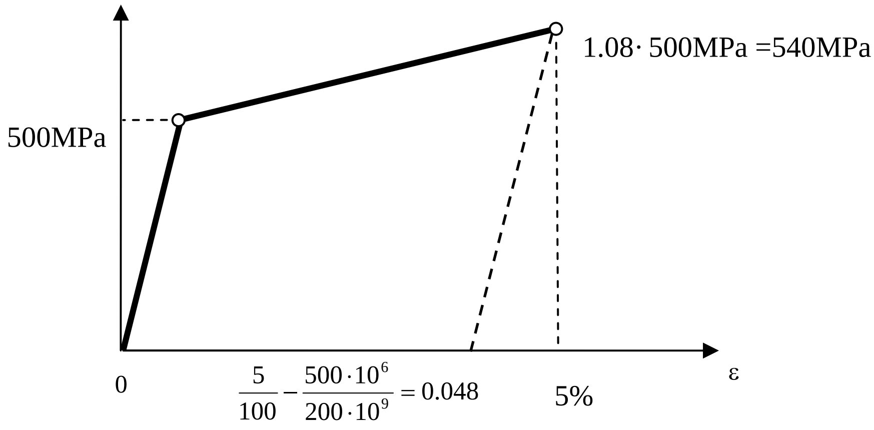


Figure 5.13 Values to be used as input in DIANA

5.4 Concrete in compression

The expression for the concrete in compression according to EC2 is $\sigma_c(\varepsilon(y)) = -f_{cm}\left[\frac{k \cdot \eta(\varepsilon(y)) - \eta(\varepsilon(y))^2}{1 + (k-2) \cdot \eta(\varepsilon(y))}\right]$

$$k = 1.05 \frac{E_{cm} \cdot \varepsilon_{c1}}{f_{cm}}$$

The value of ε_{c1} can be obtained from the next equation taken from the EC2:

$$\varepsilon_{c1}(‰) = 0.7 \cdot f_{cm}(MPa)^{0.31} \leq 2.8$$

And, as each mix has its own compressive strength:

$$\varepsilon_{c1MIXA}(‰) = 0.7 \cdot 30^{0.31} \approx 2.1‰$$
$$\varepsilon_{c1MIXB}(‰) = 0.7 \cdot 44^{0.31} \approx 2.3‰$$
$$\varepsilon_{c1MIXC}(‰) = 0.7 \cdot 47^{0.31} \approx 2.4‰$$

The modulus of elasticity is chosen according to the Eurocode as it was showed in chapter three, see equation (3.6). Numerical values are showed in table 5.6.

Table 5.6 Values to of the young modulus of the concrete

	Young modulus GPa
MIX A	30.6
MIX B	35.4
MIX C	35.0

5.5 Geometrical parameters

Some parameters have to be chosen in order to analyse the different effects if some of the geometrical values are changed.

It is considered that a change in the width (*b*) of the element will not produce an important effect and hence it is considered constant and equal to 1m.

The effects of a change in the height of the element will be taken into account. Three different heights will be analysed 125, 250 and 500mm. Hence, the elements studied are showed in table 5.7.

Table 5.7 Different combinations to be analysed

MIX	Height (h)
Mix A	h_1=125mm
	h_2=250mm
	h_3=500mm
Mix B	h_1=125mm
	h_2=250mm
	h_3=500mm
Mix C	h_1=125mm
	h_2=250mm
	h_3=500mm

5.6 Crack distance, non-linear hinge length

5.6.1 Calculation of crack spacing

It is important to define the non-linear hinge length in order to obtain a good FEM model.

In chapter 3.2.2, few manners to calculate an approximate crack distance were showed. Now a comparison between the results using the different propositions is made. However, first it is necessary to define the exact geometry of the reinforcement, i.e. a reinforcement ratio of 0.1 % were assumed but this has to be calculated into a bar diameter and the number of bars.

To obtain the diameter of the bars the following equation can be used:

$$\left.\begin{array}{l} u_{spac} = \dfrac{b - 2 \cdot u_{\text{cover}} - n \cdot \phi_b}{n-1} \Rightarrow \phi_b = \dfrac{b - u_{spac} \cdot (n-1) - 2 \cdot u_{\text{cover}}}{n} \\ n = \dfrac{A_c \cdot \rho}{A_{sbar}} = \dfrac{A_c \cdot \rho}{A_{sbar}} = \dfrac{A_c \cdot \rho}{\pi \cdot \dfrac{\phi_b^{\,2}}{4}} = \dfrac{4 \cdot A_c \cdot \rho}{\pi \cdot \phi_b^{\,2}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \phi_b = \frac{b - u_{spac} \cdot \left(\dfrac{4 \cdot A_c \cdot \rho}{\pi \cdot \phi_b^{\,2}} - 1 \right) - 2 \cdot u_{\text{cover}}}{\dfrac{4 \cdot A_c \cdot \rho}{\pi \cdot \phi_b^{\,2}}}$$

Solving this equation it is possible to obtain a value of ϕ_b. This value has to be rounded (up or down) to the nearest whole number. Having this value it is possible to obtain an approximate number of the number of bars *n* which also has to be rounded to the nearest whole number. It is preferred to use an exact number of bars and diameter in order to make possible to corroborate this data with laboratory this tests in the future. The whole procedure to calculate the crack spacing can be seen in appendix D.

RILEM rough proposition:

The first option is quite simple and it depends only on the height of the element. Due to that, the three crack spacing values are, using (3.34):

$$s_{m,RILEM1} = \frac{h_1}{2} = \frac{125mm}{2} = 61.5mm$$

$$s_{m,RILEM2} = \frac{h_2}{2} = \frac{250mm}{2} = 125mm$$

$$s_{m,RILEM3} = \frac{h_3}{2} = \frac{500mm}{2} = 250mm$$

Eurocode 2 proposition and variations:

<u>*Eurocode 2 proposition without fibre factor*</u>

The version of the EC2 for plane concrete does not consider the effect of the fibres. Due to this reason its value is constant for each mix. However, as the parameters that are used in the approaches is the ratio between steel and effective concrete area (normal reinforcement ratio) and the bar diameter. The equation that has to be used is (3.47):

$$s_{m,EC2} = \left(50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi_b}{\rho_r} \right)(mm)$$

$k_1 = 0.8$ high bond bars

$k_2 = 0.5$ for pure bending

$$\rho_{eff} = \frac{A_s}{A_{c,eff}} = \frac{n \cdot \pi \cdot \frac{\phi_b^{\,2}}{4}}{A_{c,eff}}$$

Note that one of the expressions to evaluate in order to calculate the effective area of the concrete element depends on the position of the neutral axis that is unknown and also depends on the load that is applied. The whole procedure to calculate the crack spacing can be seen in appendix D.

Table 5.8 Results for the crack spacing using EC2 proposition (no fibre effect)

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	315mm	239mm	160mm
MIX B	315mm	239mm	160mm
MIX C	315mm	239mm	160mm

It could be seen in this results that the crack spacing decreases with the height if all the parameters have a constant value (except the numbers of bars that also change due to the change of the concrete area).

Eurocode 2 alternative proposition with fibre factor

Now the EC2 taking into account the fibre bridging effect is analysed. In this proposal the value of f_{ct} and σ_w are needed. The value of the tensile strength of the concrete can be taken from the laboratory tests carried out by Löfgren et al. (2004) that provide f_{ct}, see figures 5.9, 5.10 and 5.11. The value of the bridging stress providing by fibres obviously depends on the crack opening and due to that also depends on the height. However, a realistic value, which can be assumed, is the minimum bridging stress in the crack opening interval $0.2 \leq w \leq 0.4$mm. These values can be taken from the tests and are showed in table 5.9.

Table 5.9 Results for the crack spacing using alternative EC2 proposition (fibre effect)

	σ_w [MPa]
MIX A	1.31
MIX B	3.16
MIX C	3.17

The equation that has to be used is (3.46):

$$s_{m,EC2F} = 3 \cdot \phi + c + 0.25 \cdot k_1 \cdot k_2 \cdot k_3 \cdot \frac{\phi}{\rho_{eff}}$$

Where $k_3 = \left(1 - \frac{\sigma_w}{f_{ct}}\right) = \left(1 - \frac{\sigma_2}{\sigma_1}\right)$

All the details can be seen in appendix D. The results for the crack spacing using this formula are showed in table 5.10

Table 5.10 Results for the crack spacing using alternative EC2 proposition (fibre effect)

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	166 mm	136 mm	104 mm
MIX B	65 mm	64 mm	62 mm
MIX C	65 mm	63 mm	62 mm

If the results are analysed it can be noticed that when quantity of fibres and compressive strength of the concrete is increased, the crack spacing decreases considerably. Also it can be seen that if the fibre length is decreased but the fibre slenderness factor is maintained the crack spacing does not increase so much. At last if the height of the beam is increased the crack spacing decreases as it was showed in table 5.6 using the EC2 proposition without fibre effect.

Eurocode 2 Vandewalle proposition with fibre factor

Finally the Vandewalle (2000) proposition is analysed. As it was explained in chapter 3, the only variation in this approach is changing the k_3 term by other term that takes into account the fibre slenderness factor. The equation to be used is (3.48):

$$s_{m.VANDE} = \left(50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_{eff}}\right) \cdot \left(\frac{50}{\frac{L_{fib}}{\phi_{fib}}}\right) (mm)$$

The results can be seen in table 5.11.

Table 5.11 Results for the crack spacing using Vandewalle EC2 proposition (fibre effect)

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	236 mm	179 mm	120 mm
MIX B	236 mm	179 mm	120 mm
MIX C	247 mm	188 mm	126 mm

Although the mix A and B have the same fibre slenderness factor as mix C, there is a difference in the value of the crack spacing. This is due to the fact that the values of the diameter and length have been approximated to realistic values without too many numbers. Anyway the difference is not so considerable.

It important to underline that as all the mixes has the same slenderness factor, this formula does not predict differences between them. This formula is based on laboratory tests carried out by Vandewalle (2000). The problem is that the size variation was not considered since the tested beams had the same size (height 305mm). Four specimens were tested, see table 5.12

Table 5.12 Mixes tested by Vandewalle (2000)

	V_f [%]	Class
MIX 1	0.38	RC 65/35 BN
MIX 2	0.56	RC 65/35 BN
MIX 3	0.38	RC 80/50 BN
MIX 4	0.56	RC 80/50 BN

The results obtained comparing laboratory tests with the proposed formula are showed in table 5.13.

Table 5.13 Results of laboratory tests and numerical approach. Vandewalle (2000)

	Test	Equation
MIX 1	102.4	93.5
MIX 2	91.2	93.5
MIX 3	73.1	76.2
MIX 4	80.4	76.2

The test results show that the fibre volume as well as the slenderness factor has an influence in the crack spacing. When the size factor is increased the crack spacing is decreased. Also when more fibres are added, crack spacing is generally decreasing. Although mix 3 and 4 does not follow this rule, other tests carried out by RILEM agree with this general rule, see figure 5.14.

Table 1. Details of test specimens.

Beam	Steel fibers		Tensile reinforcement
	V_f (kg/m^3)	Type	
1	-	-	2 ⌀ 20
2	30	RC 65/35 BN(*)	2 ⌀ 20
3	45	RC 65/35 BN	2 ⌀ 20
4	30	RC 80/50 BN	2 ⌀ 20
5	45	RC 80/50 BN	2 ⌀ 20
6	30	RC 80/60 BN	2 ⌀ 20
7	45	RC 80/60 BN	2 ⌀ 20
8	70	RC 80/60 BN	2 ⌀ 20
9	-	-	3 ⌀ 20
10	20	RC 65/60 BN	3 ⌀ 20
11	60	RC 65/60 BN	3 ⌀ 20
12	-	-	3 ⌀ 16
13	20	RC 65/60 BN	3 ⌀ 16
14	60	RC 65/60 BN	3 ⌀ 16
15	-	-	3 ⌀ 16
16	20	RL 45/50 BN	3 ⌀ 16
17	60	RL 45/50 BN	3 ⌀ 16
18	-	-	3 ⌀ 20
19	20	RL 45/50 BN	3 ⌀ 20
20	60	RL 45/50 BN	3 ⌀ 20
21	40	RC 65/60 BN	3 ⌀ 16
22	40	RC 80/35 BN	3 ⌀ 16
23	60	RC 80/35 BN	3 ⌀ 16
24	40	RC 65/60 BN	3 ⌀ 20

Table 3. Average final crack spacing.

Beam	$s_{rm,test}$ (mm)	$s_{rm,Vand.}$ (mm)	$\frac{s_{rm,Vand.}}{s_{rm,test}}$
1	132.5	121.6	0.918
2	102.4	93.5	0.913
3	91.2	93.5	1.025
4	73.1	76.0	1.037
5	80.4	76.0	0.945
6	75.2	76.0	1.011
7	63.4	76.0	1.199
8	68.3	76.0	1.113
9	100.1	92.5	0.924
10	86.8	71.1	0.819
11	70.1	71.1	1.014
12	80.0	103.1	1.289
13	95.6	79.3	0.830
14	64.3	79.3	1.233
15	148.5	103.1	0.694
16	111.0	103.1	0.929
17	101.5	103.1	1.016
18	109.5	92.5	0.845
19	123.0	92.5	0.752
20	89.0	92.5	1.039
21	97.8	79.3	0.811
22	80.7	64.4	0.798
23	69.8	64.4	0.923
24	75.0	71.1	0.948
mean			0.971

Figure 5.14 RILEM tests to check Vandewalle expression, from Vandewalle and Dupont (2003). L.Vandewalle & D.Dupont

Hence, the Vandewalle expression does not take into account the volume of fibres and this approximation does not seem realistic considering the test results.

Ibrahim and Luxmoore proposition:

The Ibrahim and Luxmoore expression is the equation (3.44):

$$s_{I\&L} = K_1(u_{space}, u_{cover}) + K'_2 \cdot K_3 \cdot \eta_s \frac{\phi_b}{\rho_{eff}}$$

Some new values are needed in order to calculate this expression:

u_{cover} is the cover of the concrete and it is 25mm for the analysis carried off in this thesis.

$\tau_{bm} = \frac{3}{2 \cdot \kappa_1} \cdot f_{ct}$; Where κ_1 is 0.8 for high bonded bars

τ_d is the average sliding friction bond strength of fibres and it is assumed to be the same than τ_{bm} because there are not technical specifications for this value.

The results using this approach can be seen in table 5.14. For the whole calculations see appendix D.

Table 5.14 Results for the crack spacing using I&L proposition (no fibre effect)

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	80mm	69mm	96mm
MIX B	46mm	43mm	87mm
MIX C	76mm	55mm	89mm

These results have not been checked against laboratory tests and they are quite different to the rest of the approaches but closer to the alternative formula to EC2 including the fibre effect. Also is quite strange that the effect of the height is not always increasing or decreasing crack spacing. In conclusion this formula is not used for the calculations although it would be good to check if the results are the same in the reality.

5.6.2 Discussion

After analysing all the possible approaches, the conclusion is that the most realistic is the correction in the derivation of the EC2 expression. Although there are no experimental data to corroborate the expression because it was not possible to find the sigma-epsilon curves for the materials used in the conducted tests, the expression could be a good approximation. However, further studies are required to derive a good formula for the crack spacing for structural elements made of FRC. Table 5.15 shows the input data to be used as crack spacing or non linear hinge length. All the values have been rounded of the nearest 5 mm to simplify the process of modelling in FEM.

To conclude, the total number of models is 9 for each approach. A total number of 18 models are made.

Table 5.15 Non linear hinge length

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	165 mm	135 mm	105 mm
MIX B	65 mm	65 mm	65 mm
MIX C	65 mm	65 mm	65 mm

Also the diameter of bars, number of them and stiffness of the dummy interface are showed in tables 5.16 and 5.17 and 5.18

Table 5.16 Diameter of bars

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	5 mm	7 mm	9 mm
MIX B	5 mm	7 mm	9 mm
MIX C	5 mm	7 mm	9 mm

Table 5.17 number of bars

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	6	6	8
MIX B	6	6	8
MIX C	6	6	8

Table 5.14 Values for the stiffness of the interface

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	3.708E+14	4.532E+14	5.826E+14
MIX B	9.412E+14	9.412E+14	9.412E+14
MIX C	9.412E+14	9.412E+14	9.412E+14

6 Results

6.1 Analytical Results

6.1.1 Crack spacing

As it was explained in the last chapter, there are many alternatives to calculate values for the spacing between cracks when a beam/slab element is loaded in flexural.

The model chosen is a variation of the Eurocode 2 formula which tries to take into account the effect of the fibres in the formula proposed based on the concept of transmission length.

To use this formula it is necessary to calculate the effective area as it was explained before. The expression to calculate this value proposed by the EC2 is the lesser of the values:

$$A_{c,eff} = 2.5 \cdot (h-d) \cdot b \; ; \; A_{c,eff} = \frac{h}{2} \cdot b \text{ or } A_{c,eff} = \frac{(h-y_0)}{3} \cdot b$$

Normally when the height is quite high (about 250 mm and higher) it is normally the first one that is governing, this can also be expressed as $A_{c,eff} = 2.5 \cdot (u_{cover}) \cdot b$. More or less the effective concrete area just depends on the cover of the concrete because is considered as the area that surrounds the concrete.

But the problem begins when the height is lesser than these values. For these heights the third expression is the lesser and to determine it is necessary the value of the position of the neutral axis.

The position of the neutral axis is one of the unknowns that are necessary to determine in each approach, and the crack spacing value is an input that is necessary to obtain the position of the neutral axis. The position of the neutral axis also depends on the turn (step of load) and has to be introduce in the formula of the crack spacing and it is necessary to know what its position is just before the reinforcements begins to yield. So it would be necessary to begin an iterative process which gives the correct value of both variables.

The first analysis has been made for a beam with a height 125 mm and the mix chosen was the mix A. The approach chosen is the $\sigma{-}w$ approach because is the only that uses the crack spacing to determine the position of the neutral axis.

The preliminary value of the crack spacing is (for further information see appendix D):

$$s_{mEC2F} = 166.263 \text{ mm}$$

If this value is introduced (rounded to 165mm) in the equilibrium equation the position of the neutral axis has the representation showed in figure 6.1.

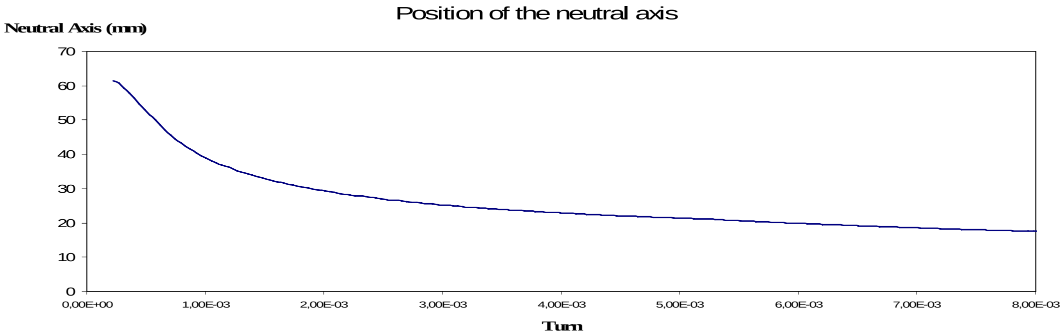


Figure 6.1 Neutral axis when rotation is varied

Then, the next step consists in identifying what turn is equivalent to the stress of the reinforcement just before yielding. It is hence necessary to analyse figure 6.2.

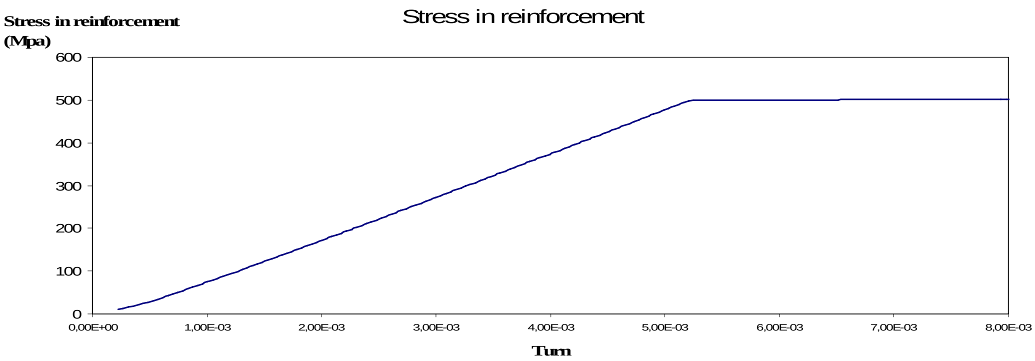


Figure 6.2 Stress in the reinforcement

The yielding process begins just when the elastic limit of the steel is reached. That is for this case when the applied rotation is $5.21 \cdot 10^{-3}$ and the position of the neutral axis for this value is 21.33 mm measured from the top. Then if this figure is introduced in the third expression to calculate the effective area and then the crack spacing are calculated the results are:

$y_0 := 21.133\,mm$ $\qquad S_{mEC2F} = 109.944\,mm$

$$A_{cef2} := \left(\frac{h - y_0}{3}\right) \cdot b \qquad A_{cef2} = 0.035\,m^2$$

If this value of the crack spacing (rounded to 110 mm) is introduced again in the analytical calculations the yielding begins when the applied rotation is $3.47 \cdot 10^{-3}$ and this turn produces the same position of the neutral axis that the previous one. Hence, the final crack spacing for the mix A height 1 is 110 mm.

If the same process is done for the mix B and mix C (same height 1=125 mm) the results are for the mix B:

- The initial value for the crack spacing is 65 mm.
- Yielding of reinforcement occurs when the position of the neutral axis is 26 mm.
- The new crack spacing using the third expression of the effective tension area of the concrete is 53.605 (rounded to 55 mm)
- With this new value of the crack spacing the position of the neutral axis is again 26 mm, hence the result of 55 mm is right.

And for the mix C:

- The initial value of the crack spacing is 65 mm.
- The final value of the crack spacing doing the iterative process is again 55 mm.

So it can be appreciated that this size effect has a strong influence with the mix which has less quantity of fibres (and less compressive resistance). These are the final values for the crack spacing.

Table 6.1New non linear hinge length

	Height-1 125mm	Height-2 250mm	Height-3 500mm
MIX A	110mm	135mm	105mm
MIX B	55mm	65mm	65mm
MIX C	55mm	65mm	65mm

It can be appreciated now that the size effects in the elements are smaller than before.

It is also possible to do a comparison between the results if the crack spacing varies. The moment-turn graph is compared for 5 values of the crack spacing in the $\sigma - w$ approach. The mix chosen is mix A and two heights are studied. The results can be observed in figure 6.3.

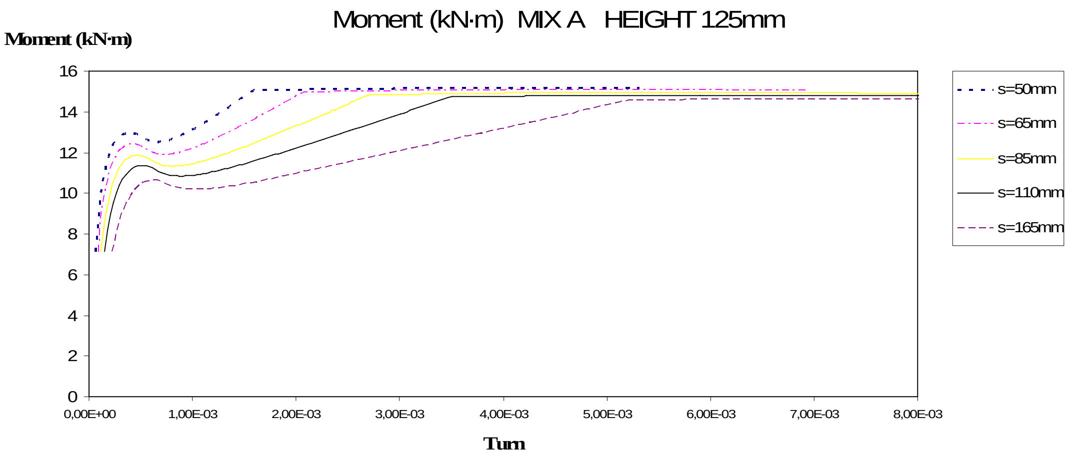


Figure 6.3 Moment versus turn for different values of the crack spacing

In the results it can be seen that the maximum moment is not influenced so much when the crack spacing value changes. However, there are differences regarding the peak moment and the turn that provides the maximum moment (and, hence, the maximum moment curvature). The conclusion is that the crack spacing is not so important when the ultimate limit state is studied but when other factors like cracking or maximum deformation are studied it is important to have a good accuracy in this value. The results for a beam of 250mm high are showed in figure 6.4 where the trend is the same as with the 125mm high element.

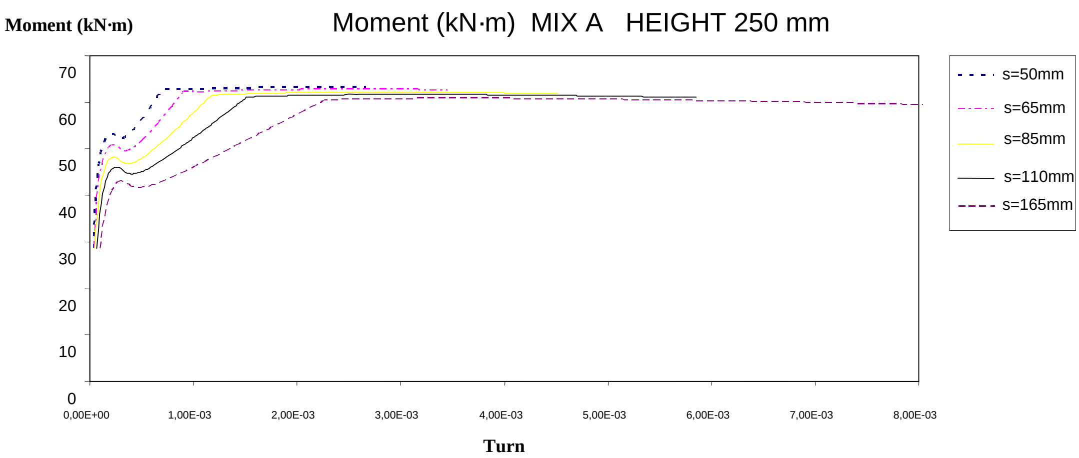


Figure 6.4 Moment versus turn for different values of the crack spacing

6.1.2 σ–ε approach

6.1.2.1 General results

Now the results obtained for mix A height 1 are showed. As it was explained in chapter 3 (analytical approach), once the equilibrium equations are completely defined it is possible to calculate the position of the neutral axis. If the strain in the reinforcement is increased, the next graph showing the position of the neutral axis is obtained. See figure 6.5.

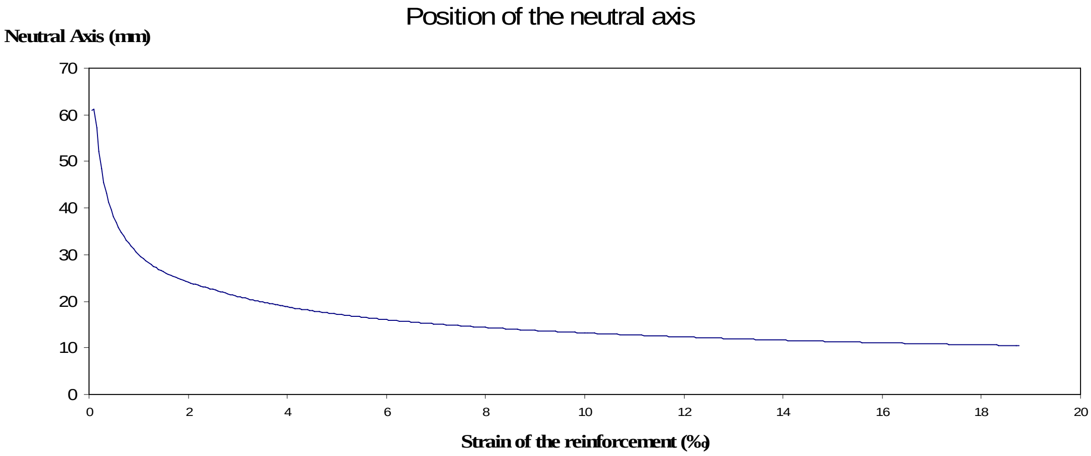


Figure 6.5 Position of the neutral axis versus strain of the reinforcement

The position of the neutral axis decreases with increasing strain in the reinforcement (and therefore with increasing rotation and curvature). Due to this decreasing, the stresses reached in concrete in compression have to be higher in order to maintain the equilibrium because the length of the compressive zone is also decreasing. Figure 6.6 and 6.7 show the stress-strain diagram at the position of the reinforcement of the steel bars and the concrete surrounding the steel.

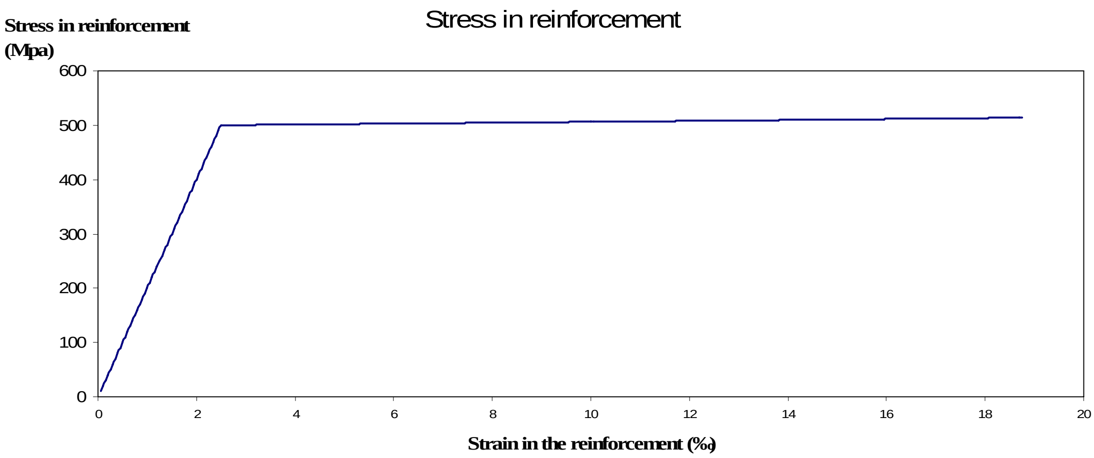


Figure 6.6 Stress of the reinforcement versus strain in the reinforcement

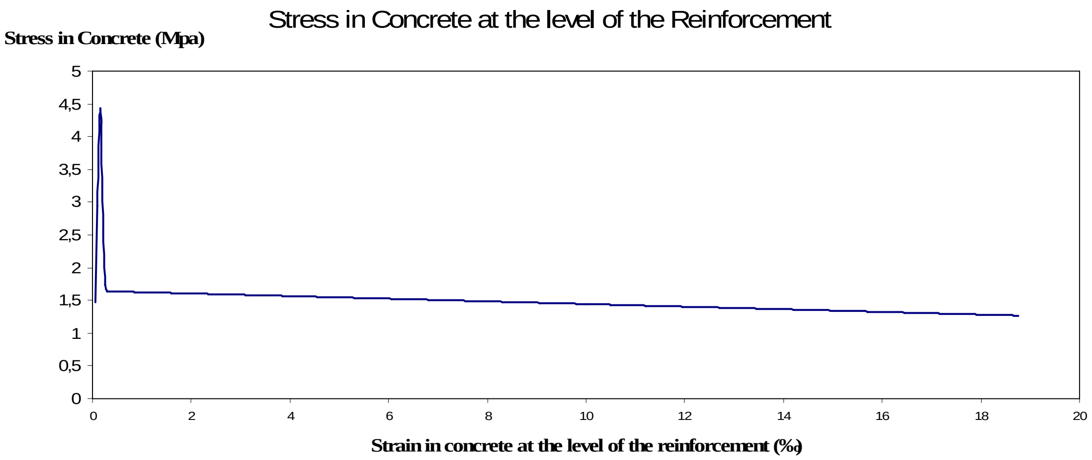


Figure 6.7 Stress of the concrete at the level of reinforcement versus strain in the reinforcement

Observing these pictures it can be seen that the concrete at this level has almost all the time the so called "residual flexural resistance" that is about 1.5 MPa. The reinforcement has exactly the behaviour that has been defined in the corresponding chapter. It is also possible to see that the stress-strain diagram in the top concrete also follows the model described before. This can be seen in figure 6.8.

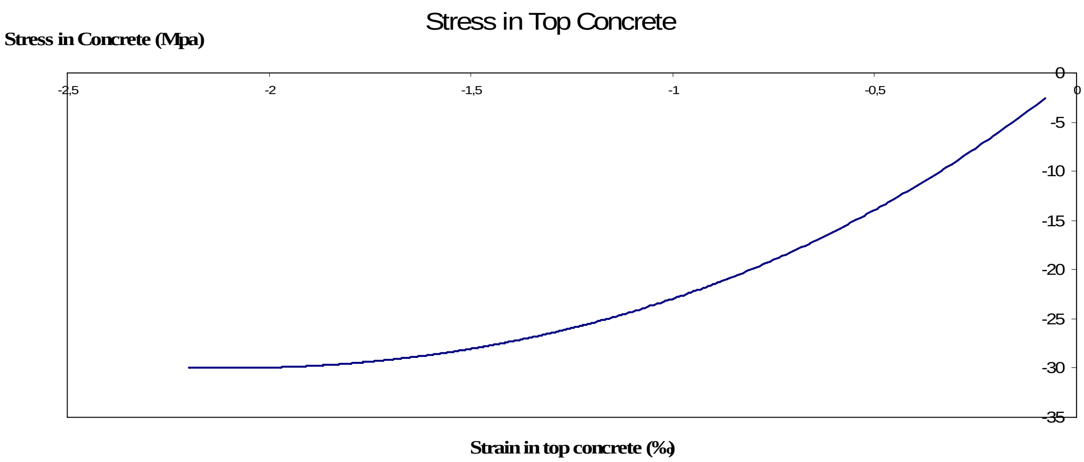


Figure 6.8 Stress of the top concrete versus strain in the reinforcement

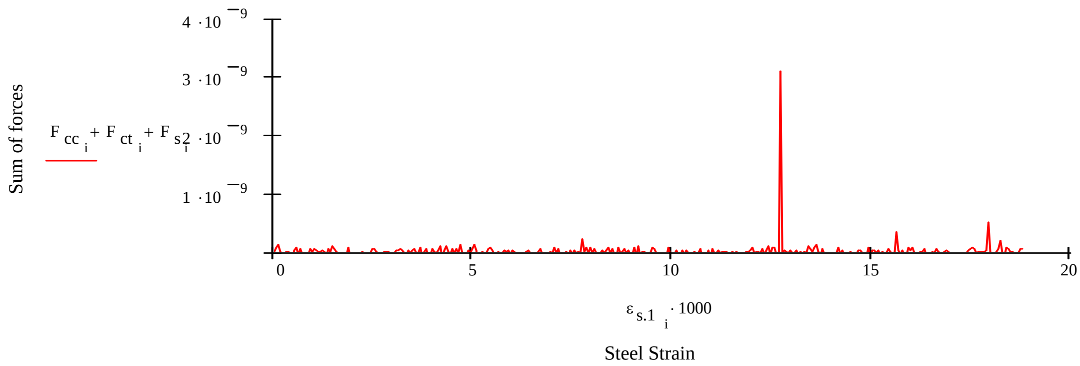


Figure 6.9 Sum of forces to corroborate the equilibrium

Another important issue is to corroborate that the force equilibrium is satisfied. The graph represented in figure 6.9 check it.

When the position of the neutral axis is calculated and checked, it is possible to obtain the moment that is supported by the cross section for each load step. If the moment versus the strain in the reinforcement is represented, the graph represented in figure 6.10 is obtained. In the first part of the graph there is a quick increase because the elastic behaviour of the concrete. Then there is a small drop due to cracking and then the moment starts again to increase due to yielding in the reinforcement until the maximum value is reached. Finally the moment decreases slowly until failure.

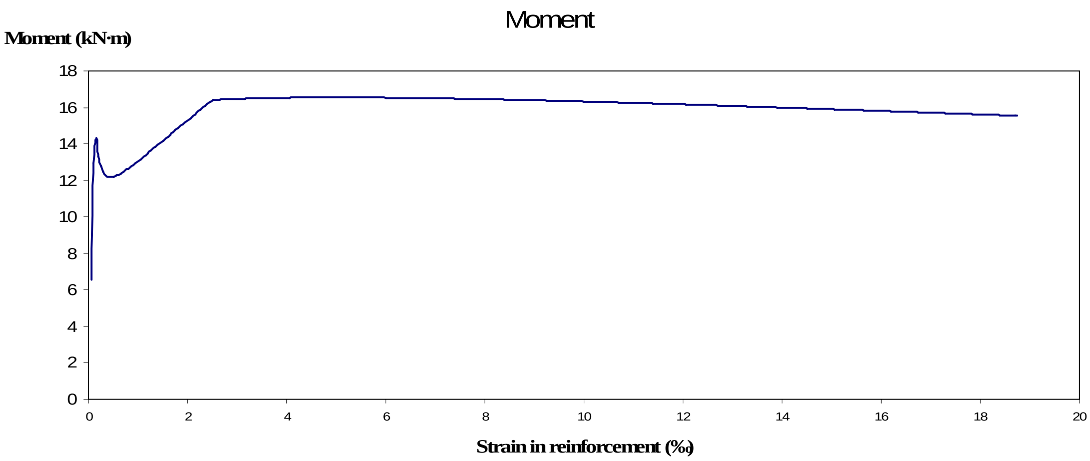


Figure 6.10 Moment versus strain in reinforcement

The maximum moment is about 16.55 kN·m and it is reached when the strain in the reinforcement is about 5%.

It also can be useful to analyse the relationship between the real moment and the moment that exits when the elastic limit at the bottom of the section is reached. To calculate it, it is necessary to calculate the equivalent constants of the section (in state I before cracking):

$$A_{ef} = b \cdot h + \left(\frac{E_s}{E_c} \right) \cdot A_s \tag{6.1}$$

$$y_{ef} = \frac{b \cdot h \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}} \tag{6.2}$$

$$I_{ef} = \frac{b \cdot h^3}{12} + b \cdot h\left(\frac{h}{2} - y_{ef}^2\right) + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot (d_1 - y_{ef}) \tag{6.3}$$

$$M_{cr} = \frac{I_{ef} \cdot f_{ct}}{h - y_{ef}} \tag{6.4}$$

Where A_{ef} is the area of the transformed cross-section (in state I before cracking), y_{ef} is the position of the effective gravity centre, I_{ef} is the effective moment of inertia and M_{cr} is the moment when a crack is initiated. If the moment divided by this new value (normalised moment) is represented versus the strain, the result can be seen in figure 6.11.

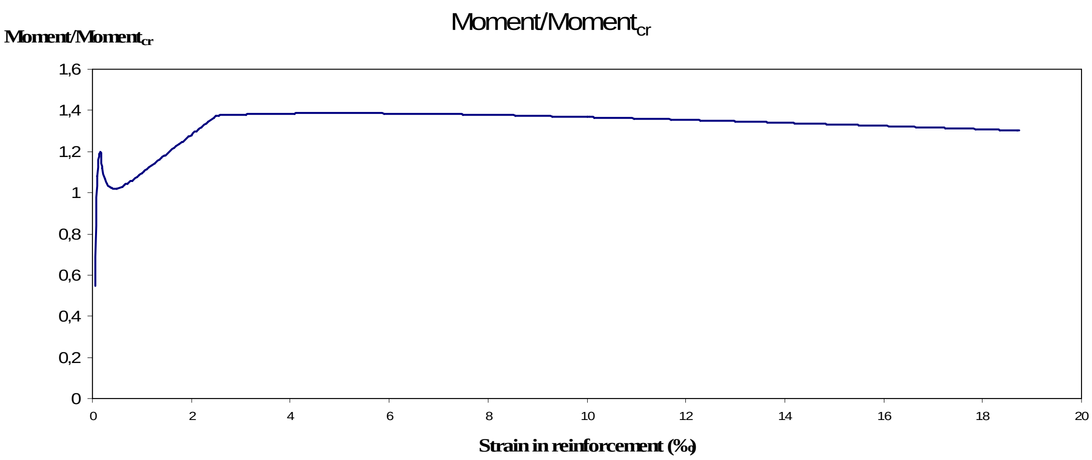


Figure 6.11 Normalised moment versus strain in reinforcement

It is also important to represent the moment versus the curvature and the rotation that exist in the section. The curvature can be calculated by means of equation (3.14). $\kappa = \dfrac{\varepsilon_s}{d - y_0} = \dfrac{1}{r}$; and then the rotation applied is obtained as:

$$\varphi = \kappa \cdot s \tag{6.5}$$

The relationship between the strain in reinforcement and the curvature is linear as it can be seen in figure 6.12. This means that the shape of the curves is the same when Strain in reinforcement is represented in the x-axis as if the curvature is represented.

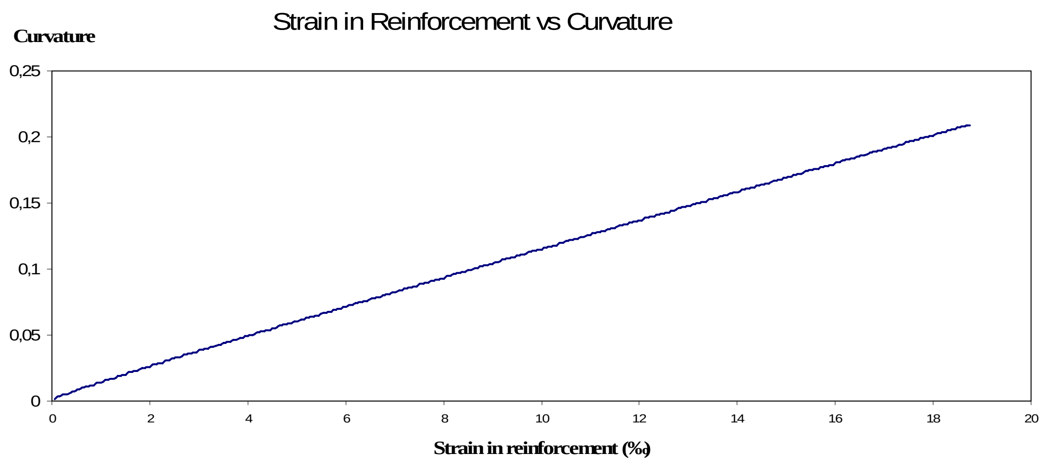


Figure 6.12 Strain in reinforcement versus curvature

And the moment versus curvature and turn is represented in figure 6.13

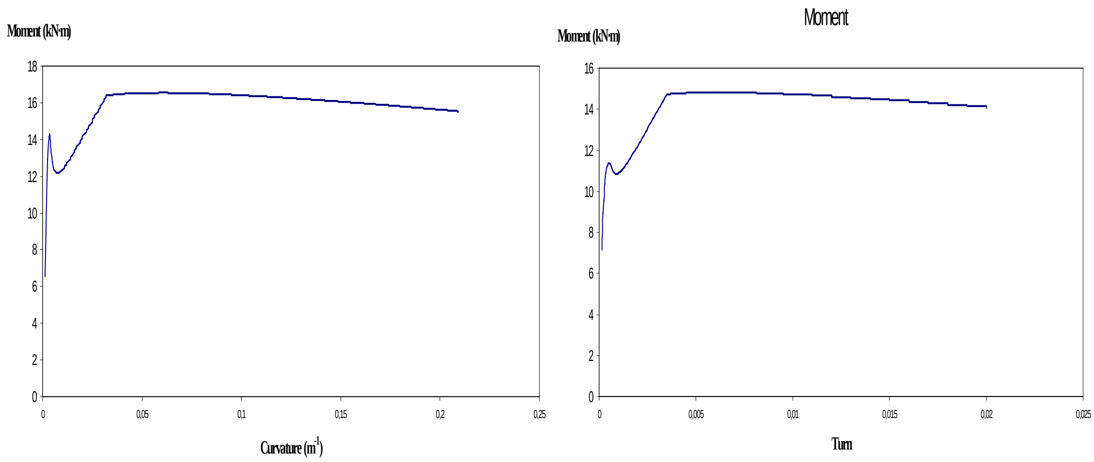


Figure 6.13 Moment versus curvature (a) and turn (b)

Finally there is a representation of the concrete stress in the cross section. Each of these lines represents stress of a load case. It can be appreciated that the length of the compressive zone decreases with the load and the length of the tensile zone increases with it. See figure 6.14

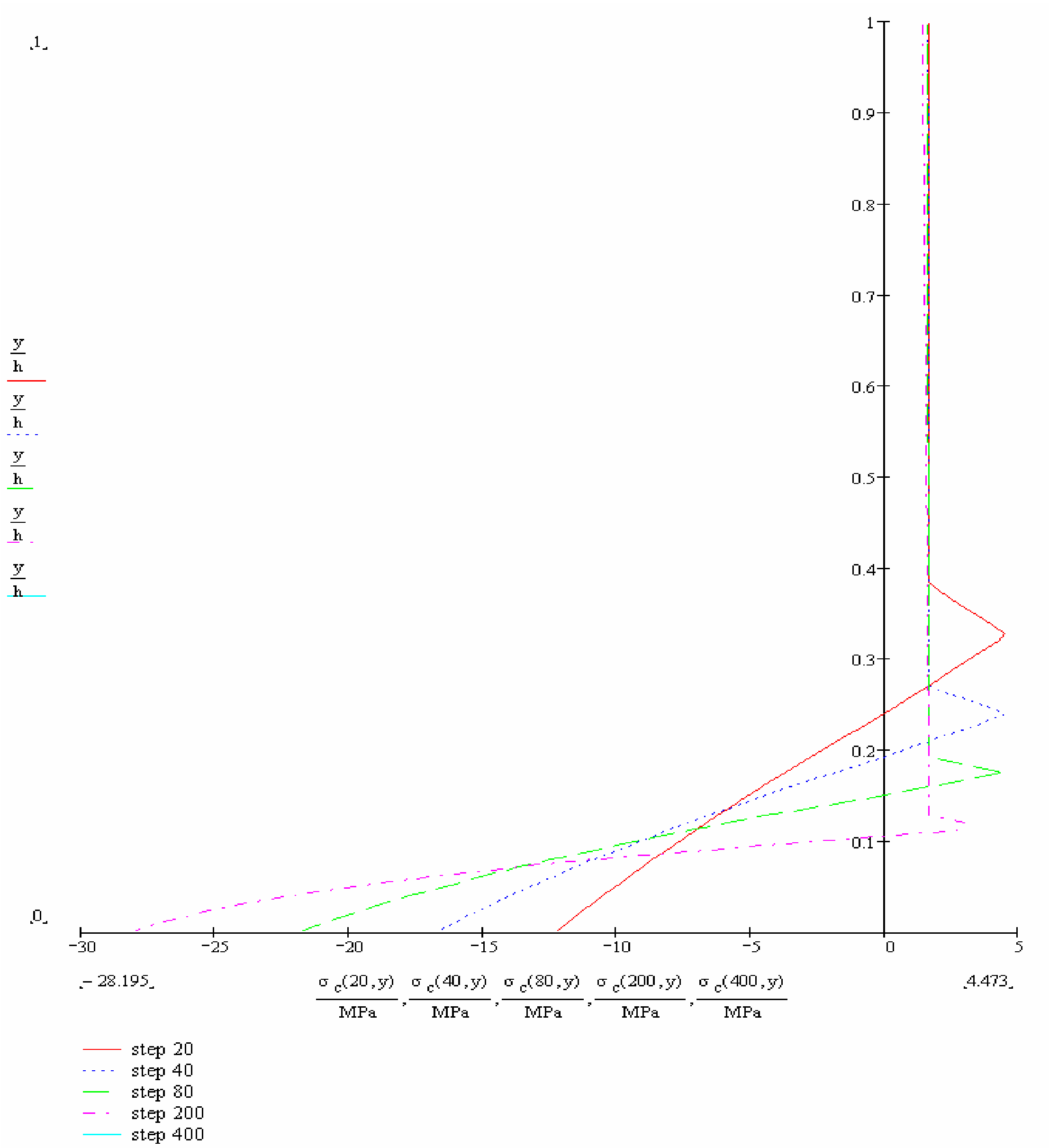
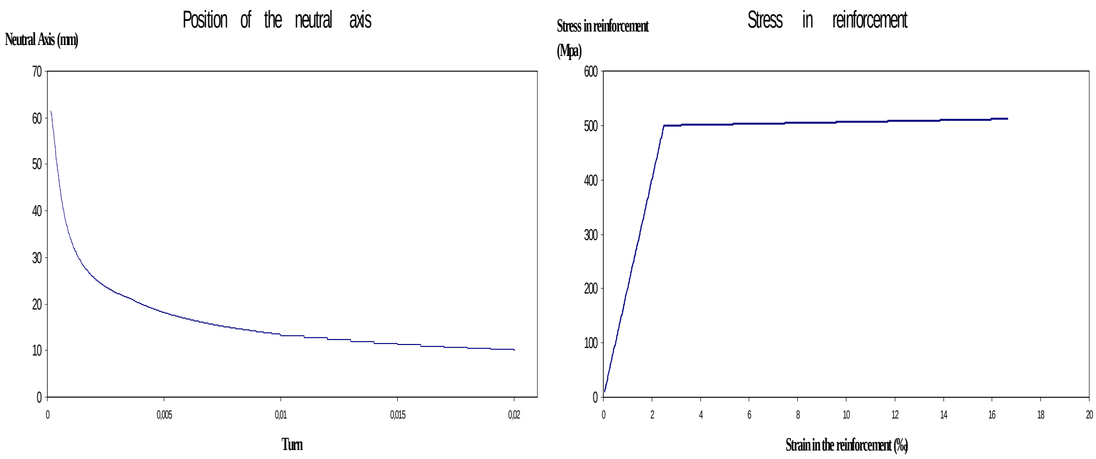


Figure 6.14 Stress in the concrete. Cross sectional analysis

6.1.3 σ–w approach

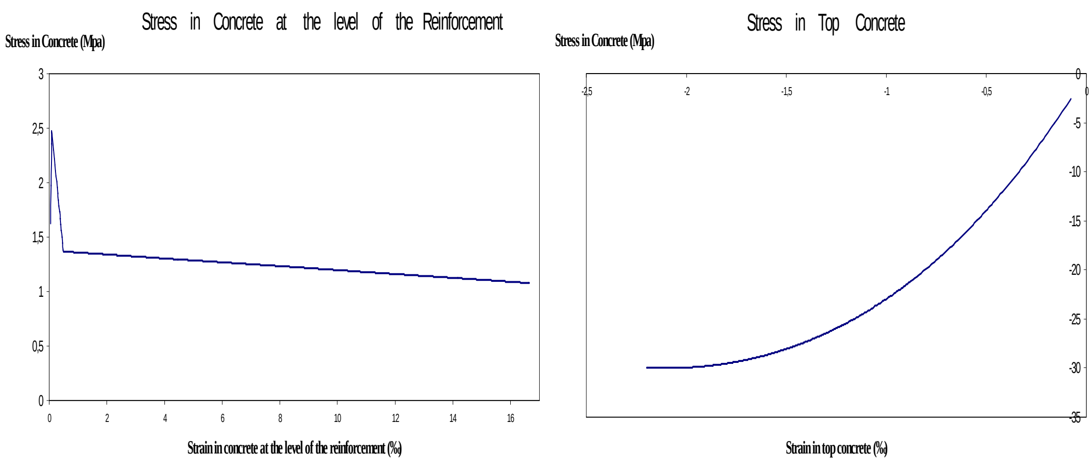
6.1.3.1 General results

The results for the σ–w approach are very similar regarding the shape and the different parts of the curves to the results obtained for the σ–ε approach. In next figures, the curves that were analysed in the σ–ε approach are represented.



(a) *(b)*

Figure 6.15 (a) Position of the neutral axis. (b) Stress-Strain in reinforcement



(a) *(b)*

Figure 6.16 Stress-Strain reinforcement (a) at the level of reinforcement (b) top position

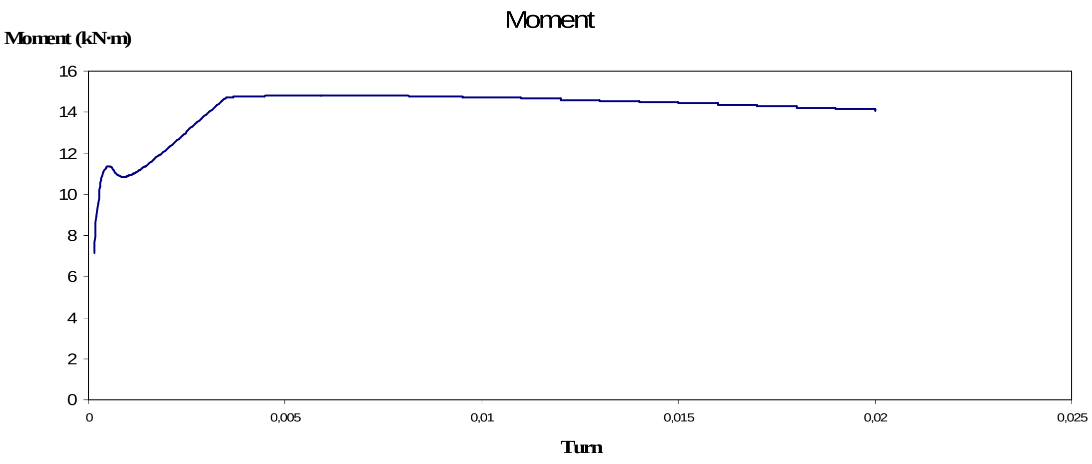


Figure 6.17 Moment versus turn

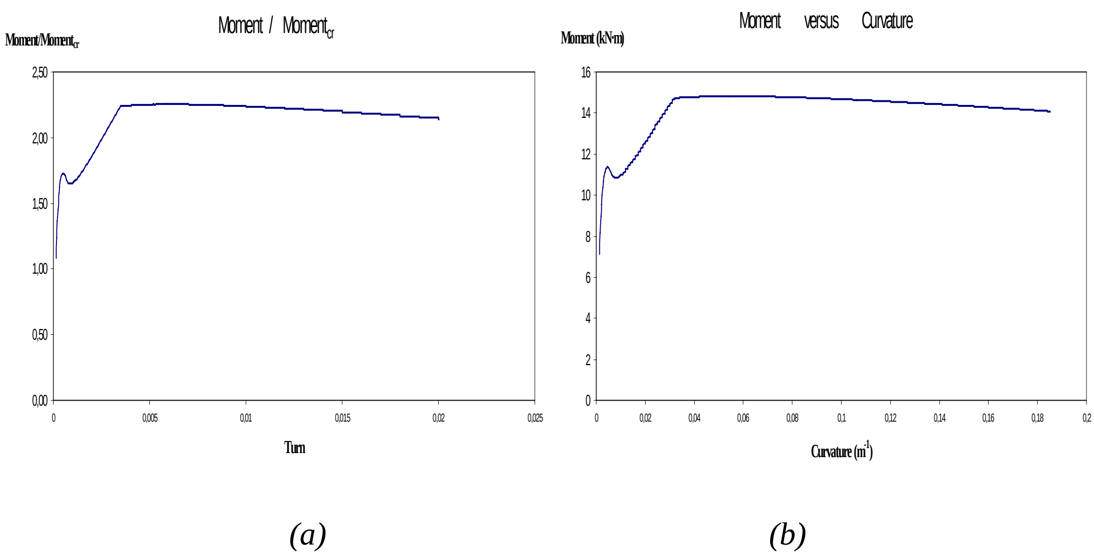
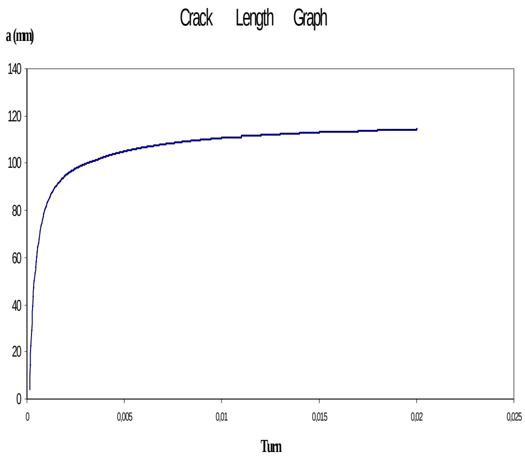
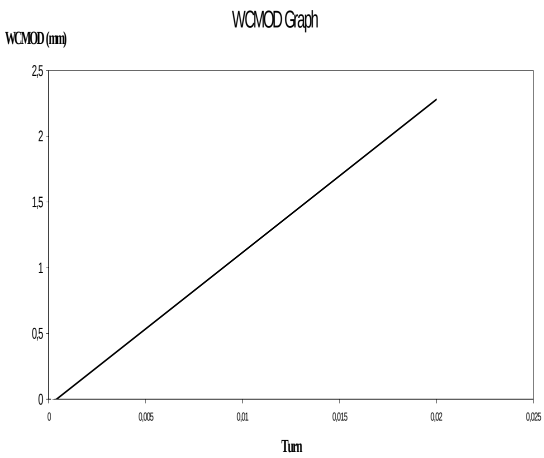


Figure 6.18 (a) Relative moment and (b) Moment versus Curvature

In this approach there are some more variables which are possible to study according to its. The length of the crack opening (or crack extension) *a*, and the maximum crack opening (at the bottom of the section), which is called WCMOD (value of the crack mouth opening displacement), can be studied. Figure 6.18 (a) represents the increases of WCMOD when the rotation of the section is also increased. It can be appreciated that the relationship between both variables is a linear relation. Figure 6.18 (b) represent the increase of the crack length. The growth of the crack is very quick for the first values of the load (rotation) but is quite slow for the last values. Hence, there is a maximum crack length that is not possible to exceed before failure. This is due to the fact that for the last values of the turn the position of the neutral axis is almost constant and the maximum value of the crack length is until the compression zone is reached.





(a) *(b)*

Figure 6.19 (a) Relative moment and (b) Moment versus Curvature

If the moment is represented versus the crack opening in the top, it is obtained a graph with a similar shape as the moment vs. turn graph. However, if the moment is represented versus the crack extension, the graph showed in figure 6.19 is obtained. The first decrease of the curve occurs when the first drop in the moment happens (after a general crack initiation). Then there is a big increase due to yielding of the reinforcement. Hence, is important to underline that the crack extension increases very quickly at the beginning (almost half of the height of the beam for very low load cases).

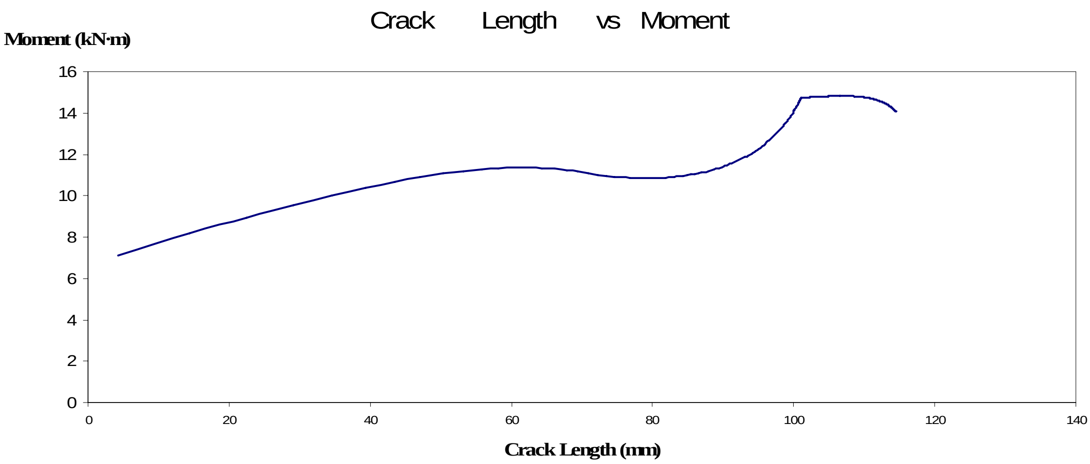


Figure 6.20 Crack length versus moment in the cross section

Finally figure 6.21 represents the stress in the concrete in the whole cross-section.

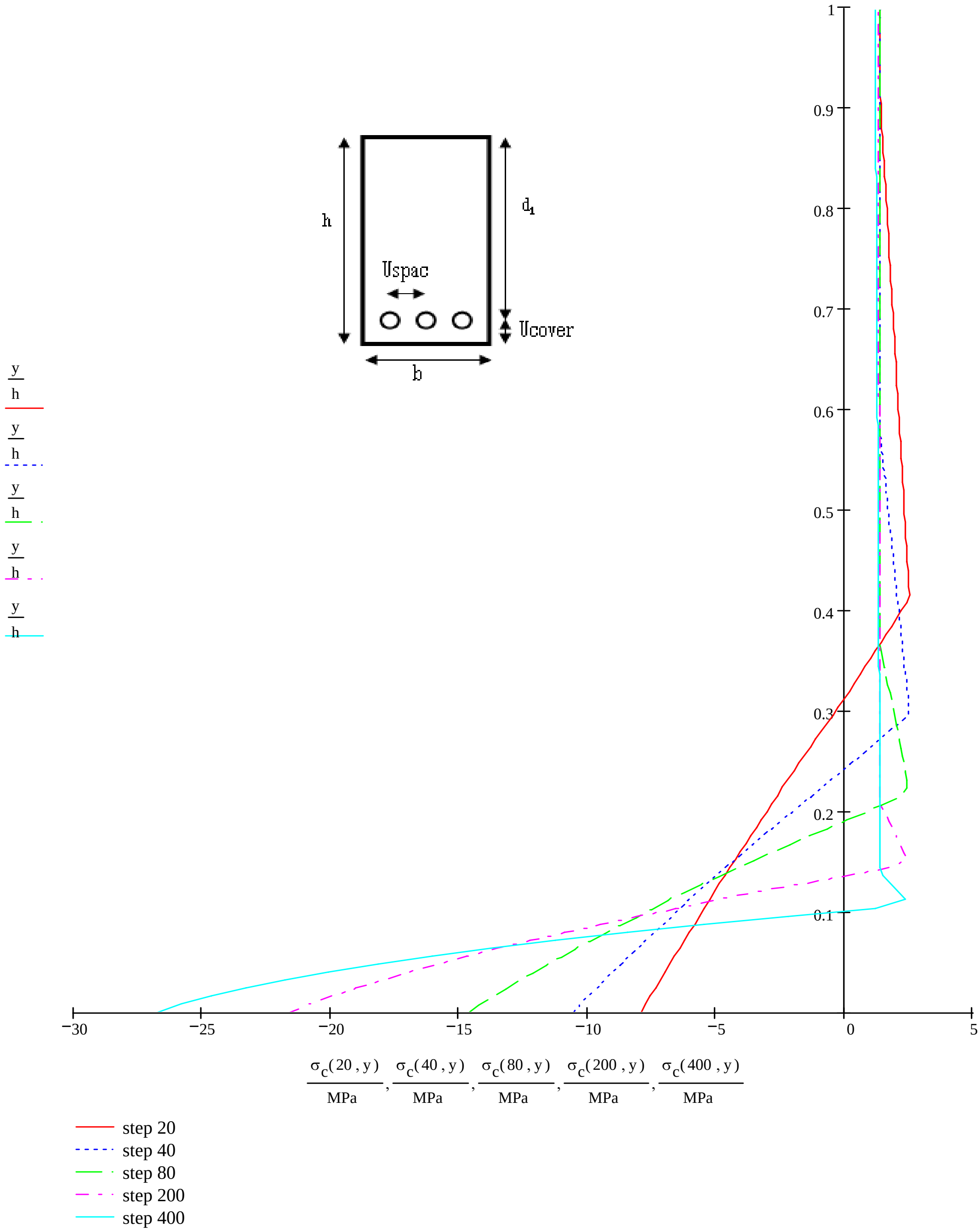


Figure 6.21 Stress in concrete (cross-sectional analysis)

6.1.4 Comparisons

6.1.4.1 Mix A

In this section, a comparison between different approaches using mix A is made. Three different heights are also used to study the possible existence of a size factor or if it exists (as the σ–ε approach), corroborate if is it correct or not.

First of all the position of the neutral axis is studied when the rotation is increased. The rotation can be calculated as $\varphi_i /2$. For further information see figure 4.11 and 4.19 and chapter 3.

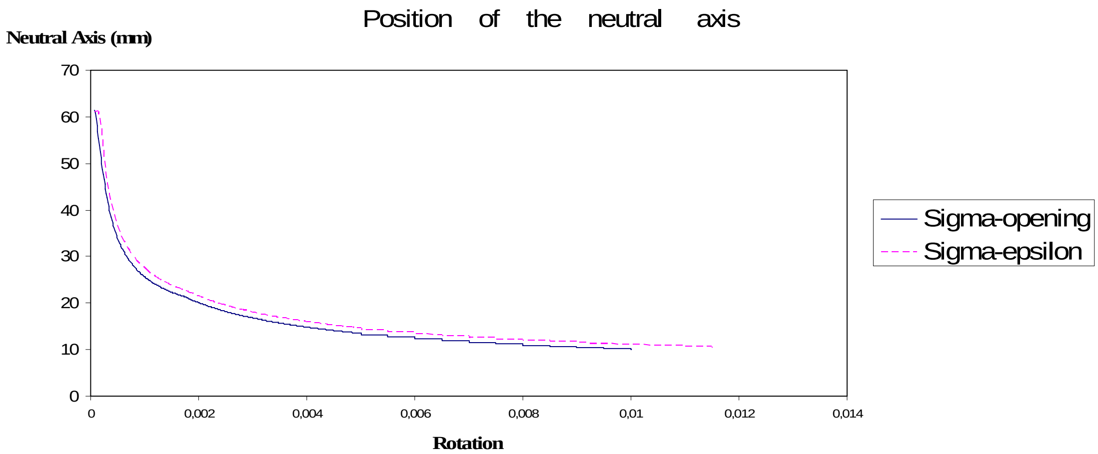
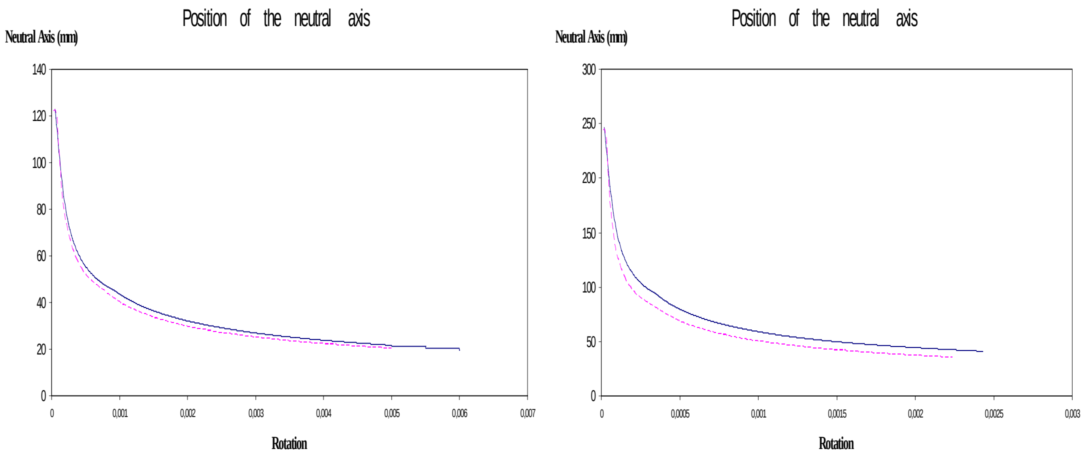


Figure 6.22 Position of the neutral axis. Beam 125mm high



(a) *(b)*

Figure 6.23 Position of the neutral axis, (a) 250 mm high (b) 500 mm high

The position of the neutral axis is very similar in both approaches. It means that, as the compressive zone is defined the same for both approaches; it could be a difference in the tensional zone. However, when the height is increased it can be noticed that the difference, which is minimum in the 250 mm high beam changes and could be considerable for some heights.

If the moment is studied the results are more different between both options.

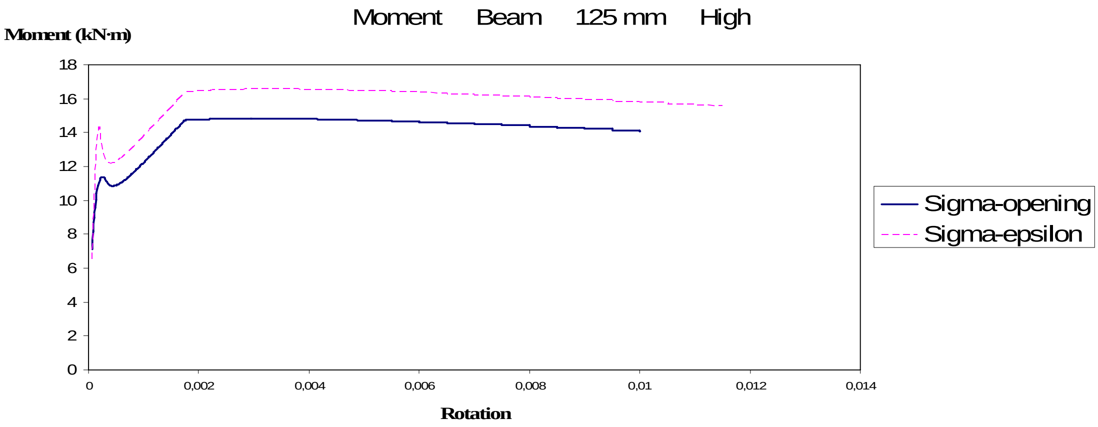


Figure 6.24 Moment versus rotation of the section. Beam 125 mm high

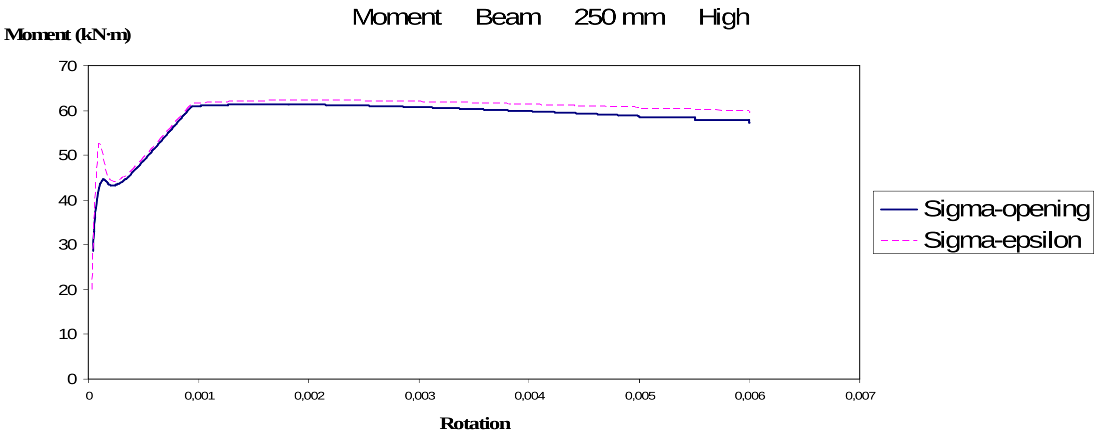


Figure 6.25 Moment versus rotation of the section. Beam 250 mm high

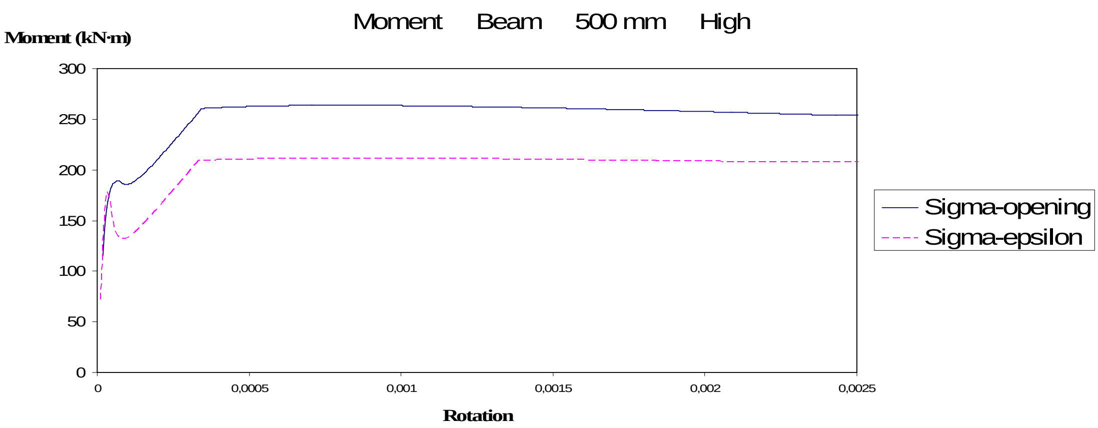


Figure 6.26 Moment versus rotation of the section. Beam 500 mm high

These results are very interesting and some conclusions can be extracted from them. The first peak moment is considerably higher in the sigma-epsilon approach than in the sigma-crack opening approach. This behaviour is because in the RILEM definition of the curve factors, the value of the elastic limit of the concrete in tension is quite higher than the obtained in the tests (σ–w approach) and it means that the behaviour while the elastic limit is not reached in all the section will be different.

Barros et al. (2004) conducted some studies in order to corroborate the validity of the parameters introduced by RILEM in the definitions of the σ–ε approach and the sense of the size factor. Furthermore, Barros et al. (2004) propose new values to calculate these constants. The new constants would be calculated as:

$$\sigma_1 = 0.52 \cdot f_{fctm,l}(1.6 - d) \quad \text{(d in m)} \qquad \varepsilon_1 = \sigma_1 / E_c \tag{6.6}$$

$$\sigma_2 = 0.36 \cdot f_{R,1} \cdot \kappa_h \qquad \varepsilon_2 = \varepsilon_1 + 0.1‰ \tag{6.7}$$

$$\sigma_3 = 0.27 \cdot f_{R,4} \cdot \kappa_h \qquad \varepsilon_3 = 25‰ \tag{6.8}$$

If these constants are used, the result obtained for the first height (not using any size factor) can be observed in next figure:

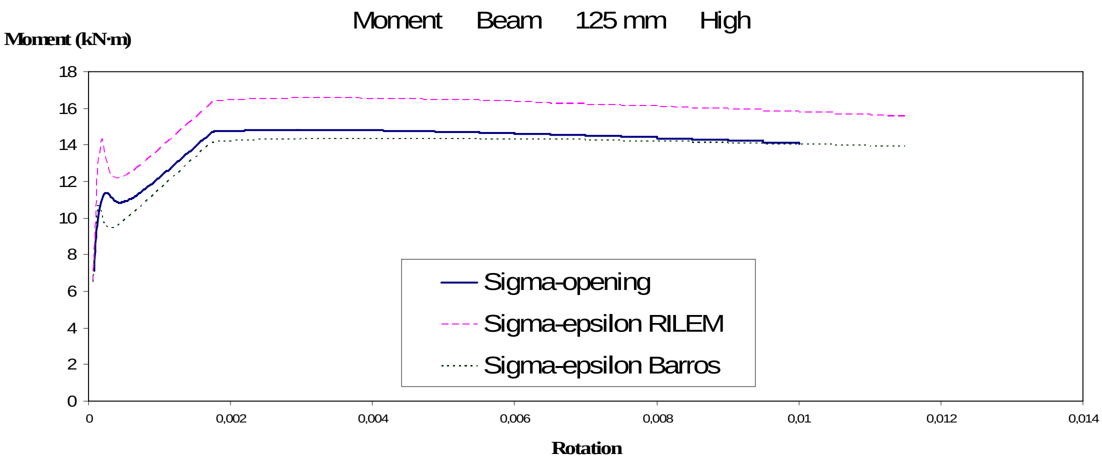
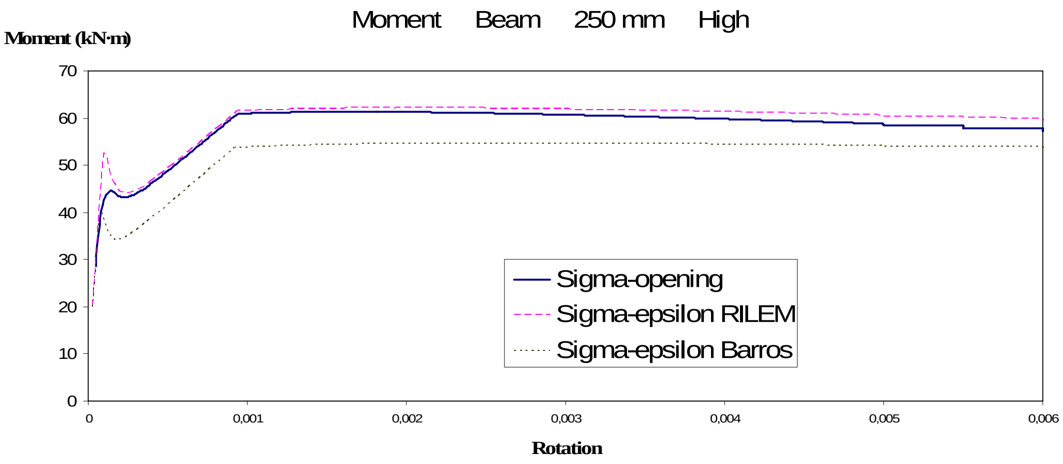


Figure 6.27 Comparison between approaches. Beam 125 mm high

It can be seen that the results for the σ–w approach and σ–ε approach (Barros approach) are closer than using the RILEM constants. Also both peak values (first peak and maximum moment) are very similar. The only different part is regarding the post peak moment resistance that is considerably less in the case of σ–ε approach (both approaches)

For the 125 mm height the size factor is 1, as the height studied is the same as the one that RILEM uses to determine the size factor. Barros et al. (2004) did not study the effect of the size factor and only this height was analysed. If these constants are maintained as well as the size factor *k(h)*. The results for the rest of the heights are as follows (see Figure 6.28):



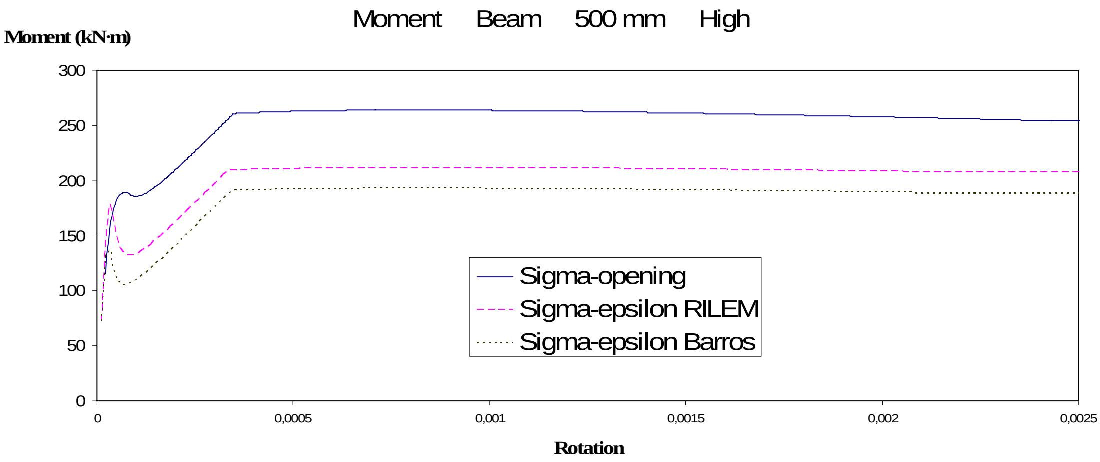
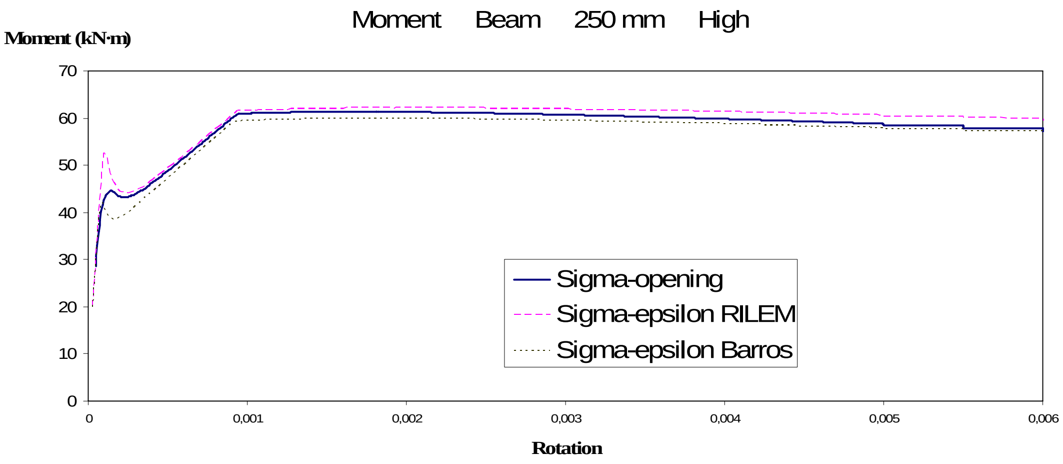


Figure 6.28 Comparison between approaches. Beam 250 mm and 500 mm high

It easy to notice that except in the case (height of 125 mm) when the Barros et al. (2004) approach is in agreement with the stress-crack opening approach, the size factor defined by RILEM TC 162-TDF (2003) cannot be used. Furthermore, if this size factor is eliminated for all heights, the approaches get closer but there is still a disagreement in values around the first peak. It should be pointed out that the size factor is not completely understood by RILEM and these graphs suggest that maybe it is not necessary. However, a comparison between this analytical results and another source (like FEM analysis or laboratory tests) is necessary in order to corroborate that the analytical results using σ–w approach can be used as a reference. It is important to remember that the σ–w approach does not use any size factor.



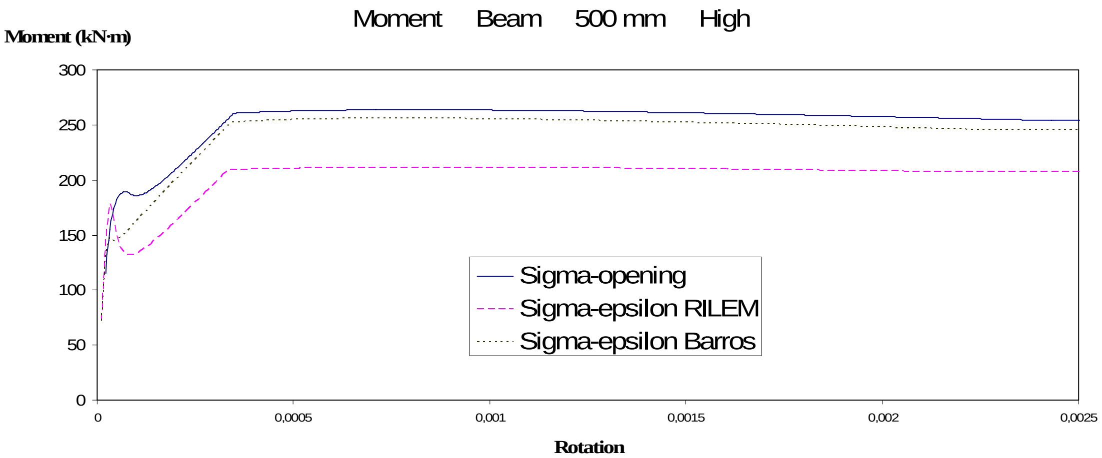


Figure 6.29 Comparison between approaches. Beam 250 mm and 500 mm high without any size factor

6.1.4.2 Mix B

It is also interesting to analyse if this disagreement between both approaches changes if a mix with different properties is analysed. The position of the neutral axis has the same trend as mix A.

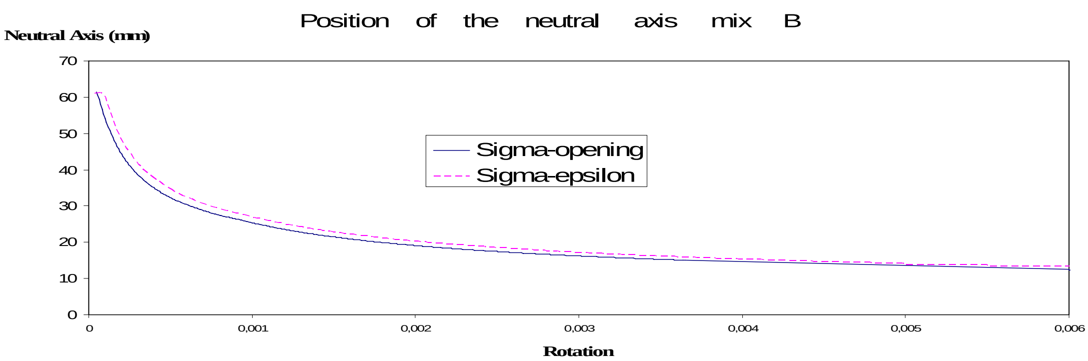


Figure 6.30 Position of neutral axis 125 mm high beam

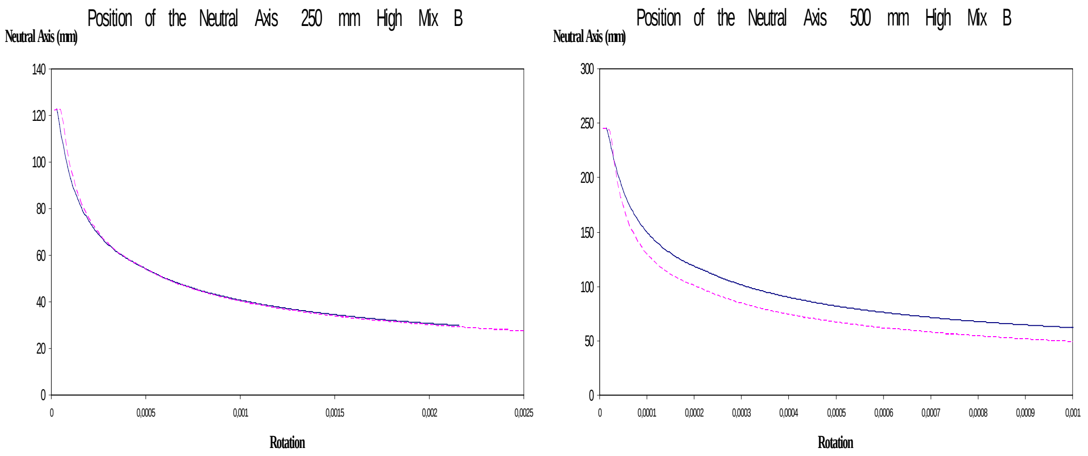


Figure 6.31Position of neutral axis 250 and 500 mm high beam

There is a different behaviour regarding the moment. See figure 6.32.

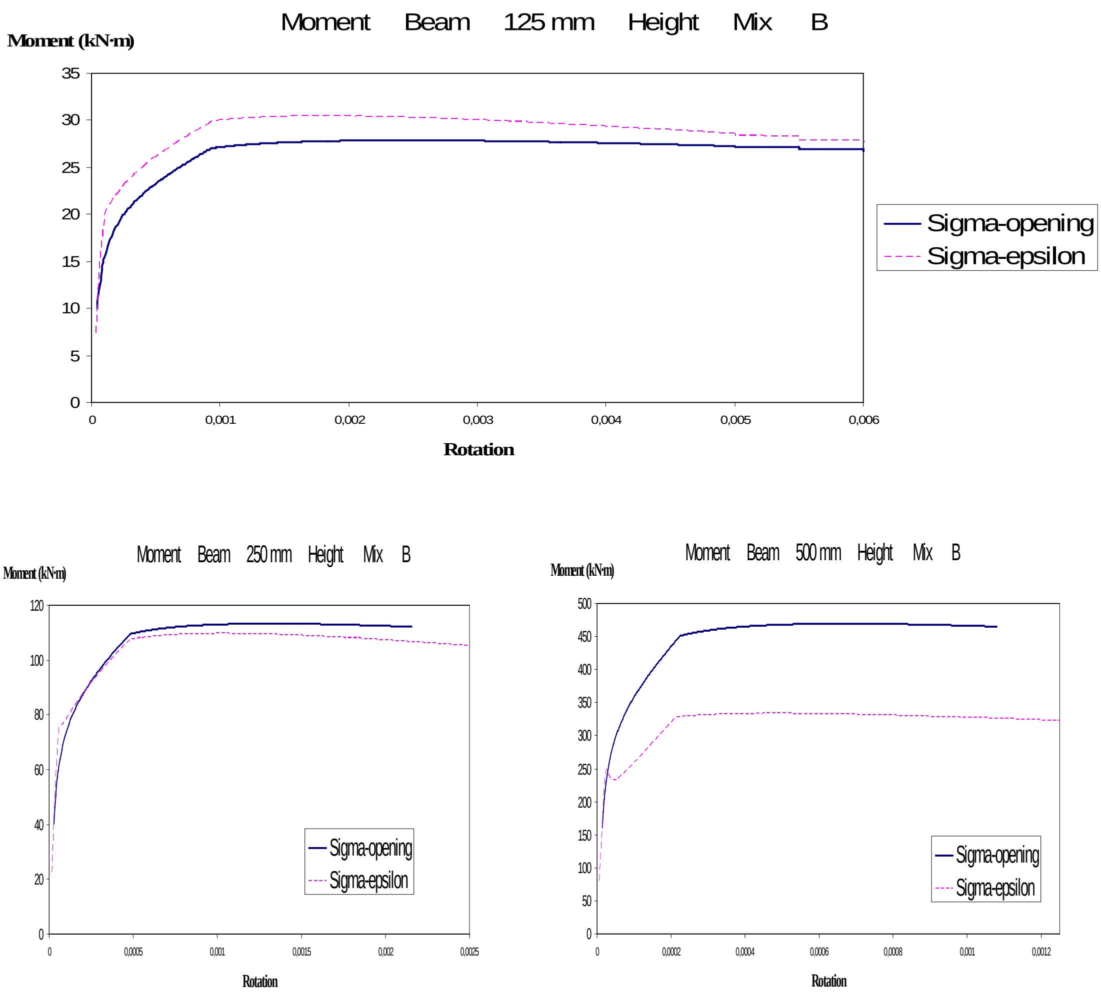


Figure 6.32 Moment versus rotation for mix B beams

If these results for the moment are compared to those obtained in mix A some conclusions can be extracted:

- The post peak response has change due the inclusions of a higher quantity of fibres. In the σ–w approach there is no post peak decreasing but in the σ–ε approach the response depends on the height (there is no decreasing in the moment for the 125 high beam and there is a similar response as the mix A for the 500 mm high beam). It is important to underline that something similar also occurs in mix A (the proportional post-peak drop is deeper in the 500 mm high beam than in the 125 mm high beam). This can be caused by the effect of the size factor.

- After the second peak (maximum moment), the moment decreases more quickly in the σ–ε approach than in the σ–w approach.

Next figures show the results for the moment in the section using Barros et al. (2004) values for the constants in the σ–ε curve.

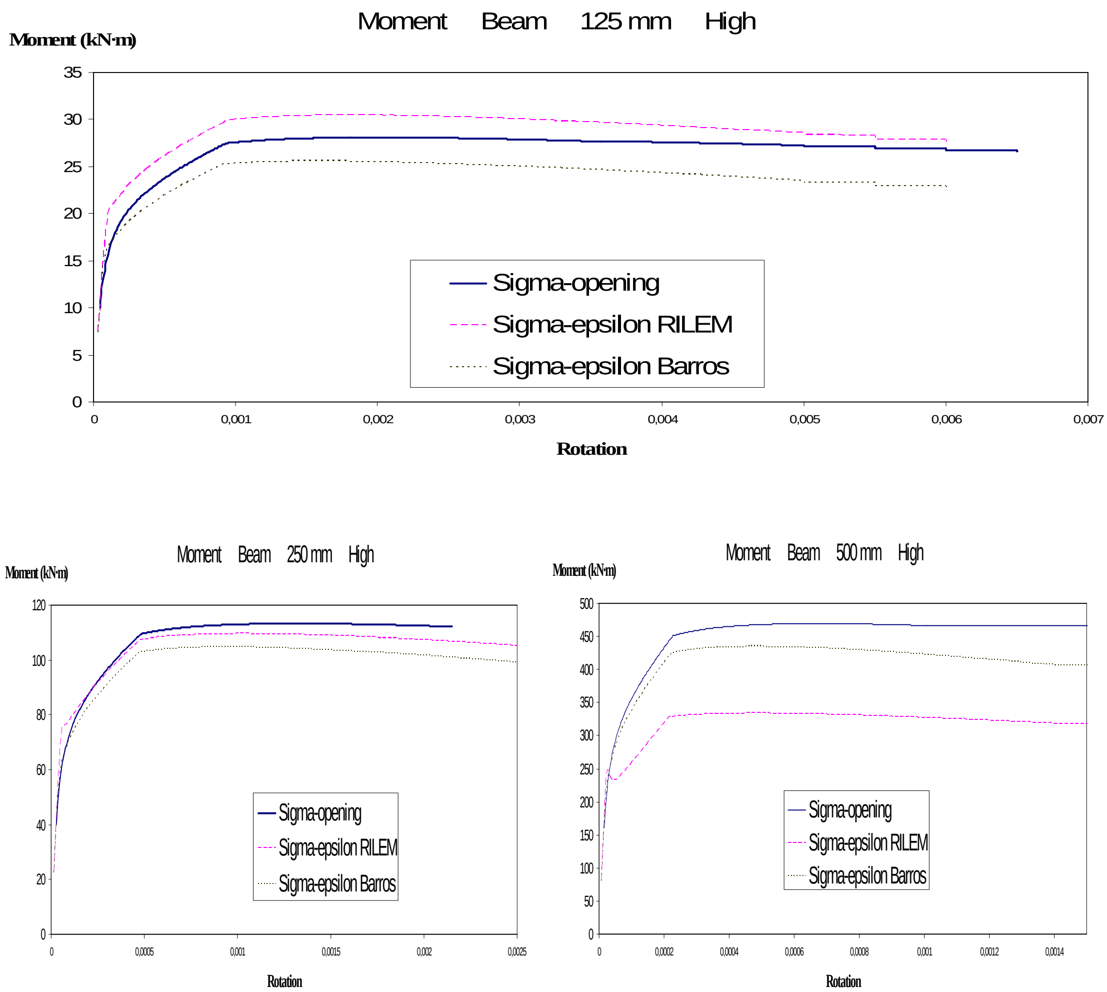


Figure 6.33 Comparison between approaches. Beam 125 mm, 250 mm and 500 mm high without any size factor and Mix B

With these results, the Barros approximation seems not so good if the concrete has a higher compressive strength and a higher quantity of fibres. Anyway as it was said for the results regarding mix A, it is important to compare these results with other sources.

Barros et al. (2004) also has defined other values for the constants based on inverse analysis using the σ–w approach method. The values are defined as follows:

$$\sigma_1 = 0.5 \cdot f_{fctm,l}(1.6 - d) \quad \text{(d in m)} \qquad \varepsilon_1 = \sigma_1 / E_c \tag{6.9}$$

$$\sigma_2 = 0.35 \cdot f_{R,1} \cdot \kappa_h \qquad \varepsilon_2 = \varepsilon_1 + 0.1‰ \tag{6.10}$$

$$\sigma_3 = 0.32 \cdot f_{R,4} \cdot \kappa_h \qquad \varepsilon_3 = 25‰ \tag{6.11}$$

As it can be noticed the values are quite similar as in the previous ones so the results are also expected quite similar. However, it can be appreciated a slightly better fit between σ–ε approach and σ–w approach especially in the post-maximum decreasing part (see figure 6.34). Anyway the conclusions are basically the same.

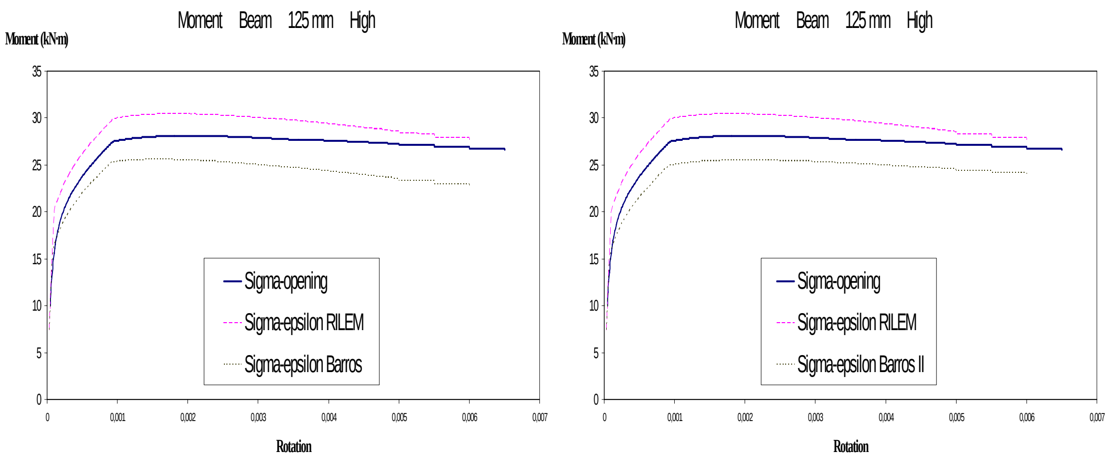
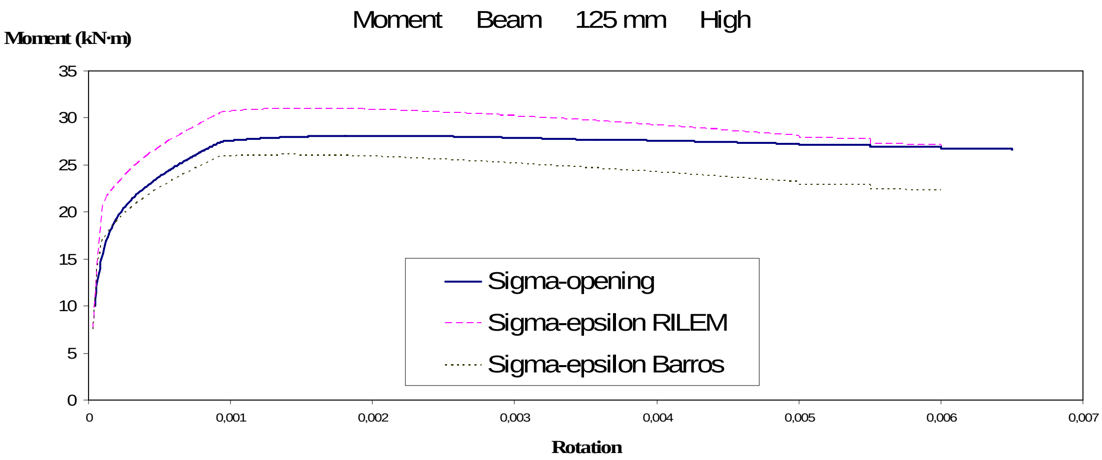
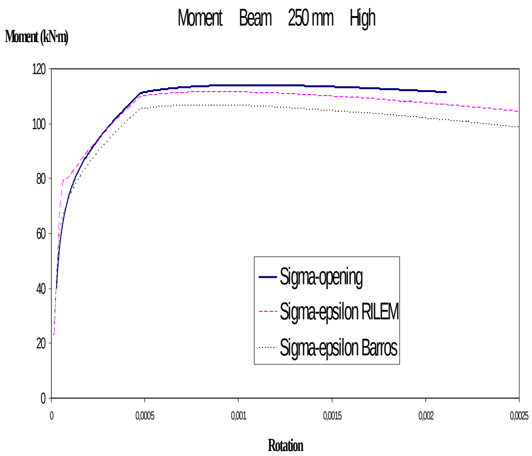


Figure 6.34 Comparison between approaches. Beam 125 mm using both Barros proposals for the value of the RILEM constants

6.1.4.3 Mix C

Mix C has similar properties as mix B so similar results are expected. Only the comparison between the values of the moment of both approaches is made, see figure 6.35.





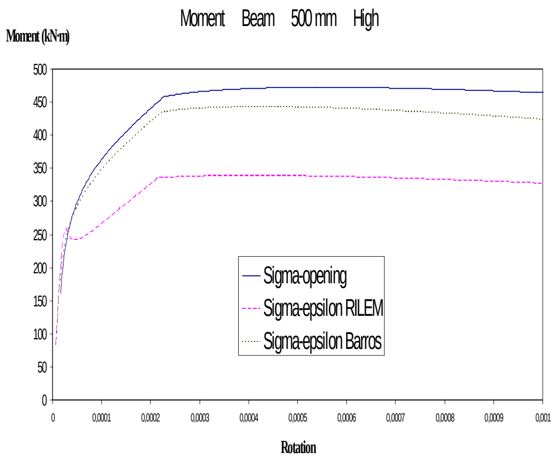


Figure 6.35 Comparison between approaches. Beam 125 mm, 250 mm and 500 mm high without any size factor and Mix C

The results are very similar to those obtained for mix B because the similar properties of the concrete in mix B and C. Hence, the conclusions that can be obtained are the same.

6.2 Finite element method results

Now the results obtained using the finite element method are presented. Firstly, as it was presented in 6.1 dealing analytical analysis, the general results are presented. However, in this chapter all the graphs are presented together with the results belonging to the analytical analysis in order to more easily understand the differences between both methods. Finally a comparison between both approaches is made.

The load applied to get a deformation is a rotation that is defined as $\varphi/2$. In the finite element method software it is necessary to chose a basis value for the load step, then, in each step, the load, can be calculated as $n_s \cdot \Delta load$. Both values are chosen in order to achieve realistic values for the different variables and the values used in the analytical calculations can be used as a guideline. In appendix E the complete procedure to define a model (mix A height 1) in Diana can be found for both approaches.

6.2.1 σ–ε approach

6.2.1.1 General results

This approach does not define a crack surface. As it was explained in chapter 4, it was necessary to introduce a weak element in order to facilitate a realistic behaviour.

The general results showed below belong to a mix A height 125 mm beam element. Some results are not easy to determinate in DIANA, so not all the graphs that were showed for the analytical results can be represented here.

Firstly figure 6.36 shows the deformation of the beam when a load is applied. This deformation is enlarged by multiplying by a factor. Note that the introduction of the weak element allows obtaining this shape.

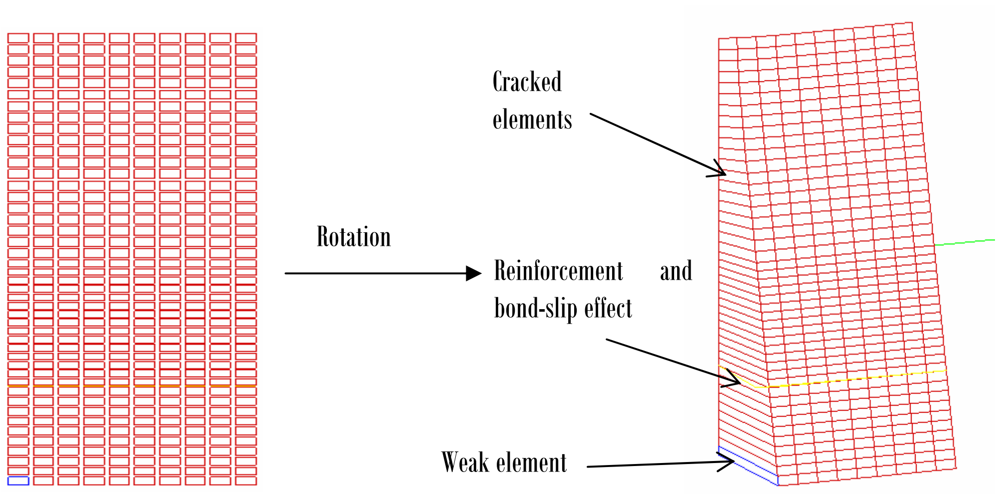


Figure 6.36 Undeformed and deformed beam. σ–ε approach

Figure 6.37 shows the stress-strain relationship in the reinforcement in the cracked cross section. Yielding occurs and the model is following correctly. The differences between both methods are almost negligible.

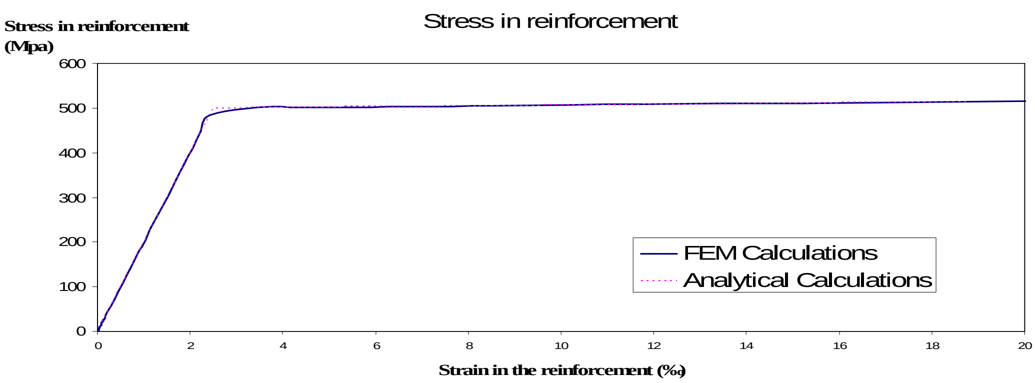


Figure 6.37 Stress-strain reinforcement diagram in σ–ε approach

The stress in the concrete at the level of the reinforcement can also be plotted choosing the cracked element at the height 27.5 mm (element 102 or 112). See figure 6.38. The values does not seem exactly the same as in the analytical results but this is mainly caused by the fact that not all load steps were saved in DIANA. Due to that if the peak is situated between two iteration values the results will be not the same as in MathCAD. This is the cause for the different situation of the peak stress. Note also that it was necessary to introduce an artificial drop because it was not possible to have a vertical drop in the stress.

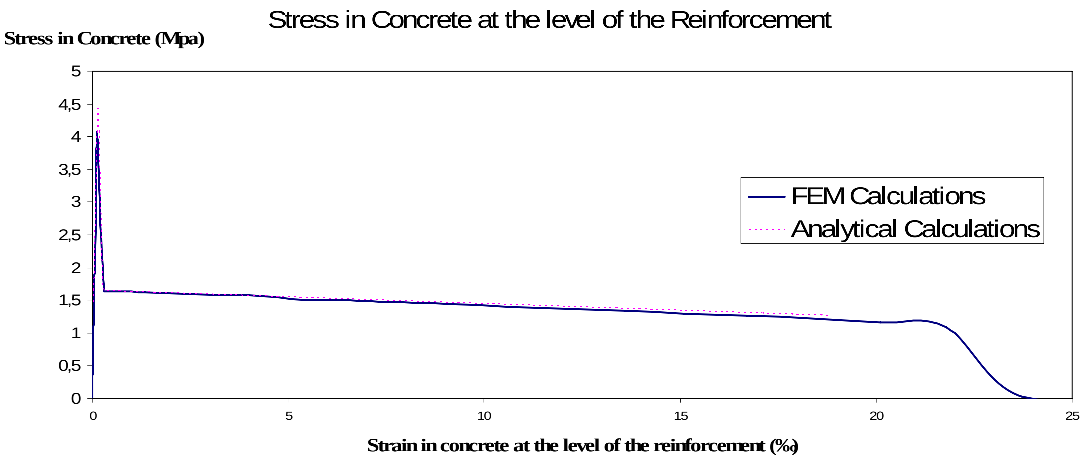


Figure 6.38 Stress-strain concrete at the level of the reinforcement diagram in σ–ε approach

The model used to define the behaviour of the concrete in compression in DIANA is giving points and is based on the same model that is used for the MathCAD (analytical) calculations. The results for the stress-strain relationship in the top of the beam (element 452) are showed in figure 6.39. The results adjust very well to the original graph.

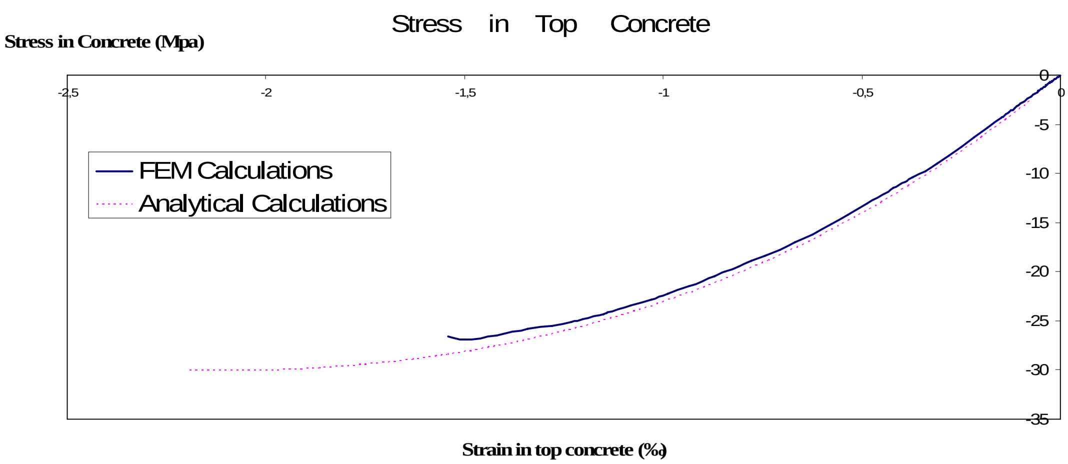


Figure 6.39 Stress-strain in top concrete, σ–ε approach

Figure 6.40 shows the stress in the concrete for the load step 300 (rotation about 0,001). It can be observed that the results are quite realistic.

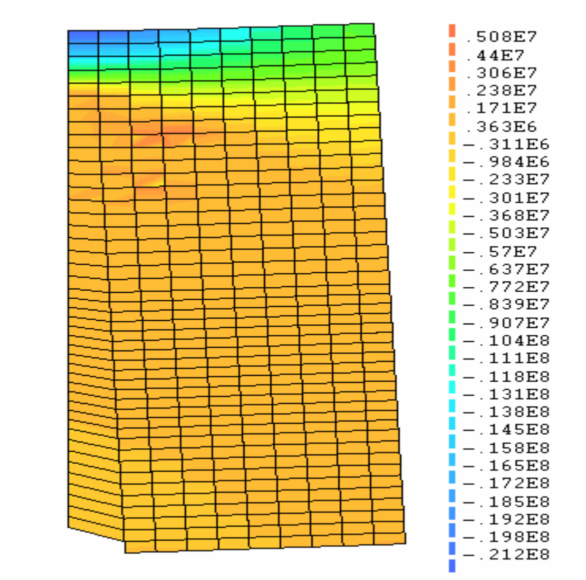


Figure 6.40 Stress in the concrete. Step 300

Now the moment versus rotation is analysed and a big disagreement between both methods is found. It is appreciated that the behaviour regarding the moment has few points in common with the analytical calculations. The only part that is common is the elastic part at the beginning. The crack moment is reached before and is considerably less than the analytical calculations (about 20% less). The behaviour after the cracking point is exceeded shows a very short post-cracking part. Then there is a increasing in the stiffness due to the presence of the fibres but then this stiffness decreases when most of the concrete has exceed its strain limit (25‰) and at this time yielding starts. This behaviour is very different to the obtained in the analytical calculations and it cannot be considered a good approach.

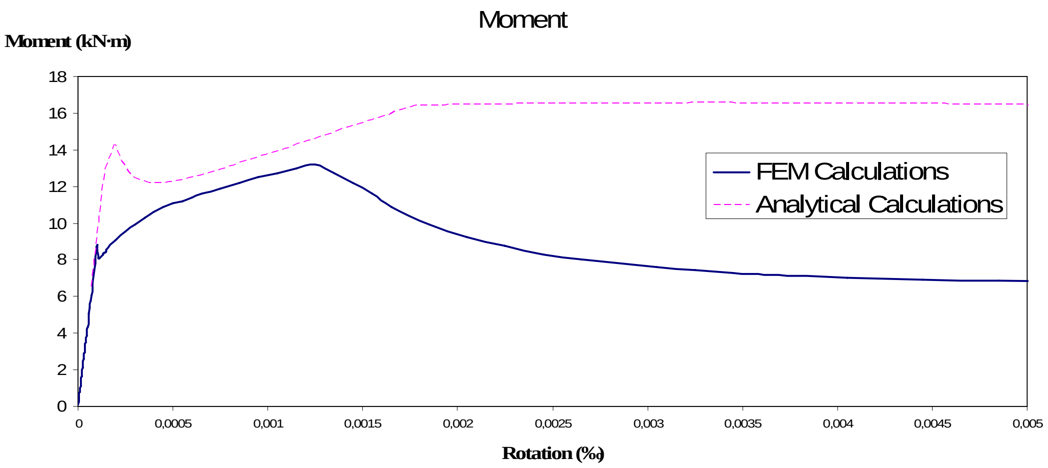


Figure 6.41 Moment versus rotation, σ–ε approach

It is also possible to obtain a graph showing the concrete stress in a vertical line (cross-section) for one load case. This result is showed in figure 6.42 for three load cases.

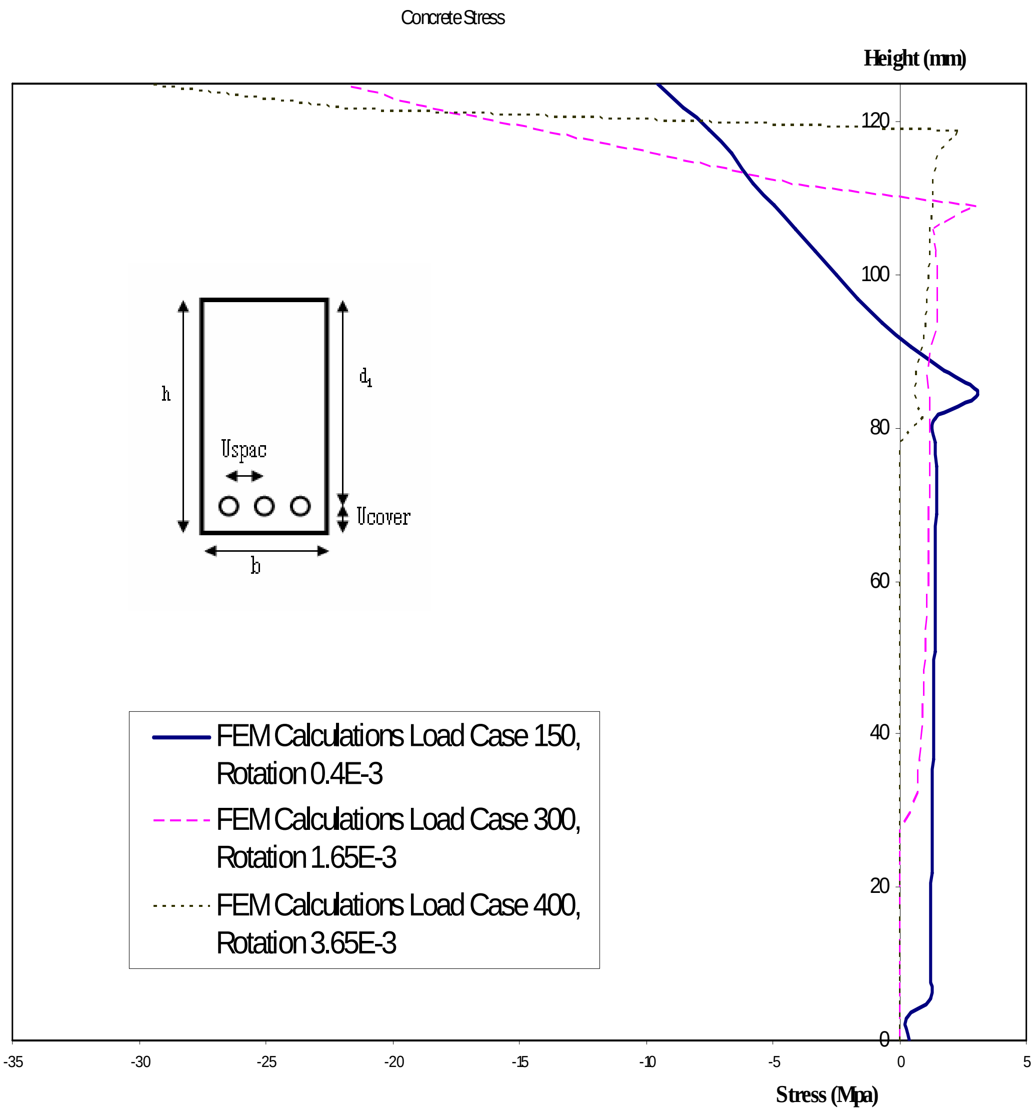


Figure 6.42 Stress in the concrete. Cross sectional analysis

The profile for the stress in the concrete is quite similar to the profile obtained in the analytical calculations for these values of the rotation. In load case 400 almost all the concrete has lost its strength. The reason for this is that almost all the deformation occurs in one element row and which thus gives a result that is mesh –size dependent.

Finally is important to explain that one of the hypothesis that had to be fulfilled (Bernoulli hypothesis) that says that plane sections remain plane, is not fulfilled if the deformed shape is observed. If the bad results regarding the moment are added, it can be concluded that this approach seems not suitable to be used when finite element method analysis are required.

6.2.1.2 Mix A comparisons

As it was seen before, this approach does not seem good to be used in FEM analysis. However the effect of the size variation was also check it and the results regarding the moment are showed in next graphs.

Again the effect for the 250 mm high beam is the same as in the 125 mm high beam. The elastic part is the same in both methods but cracking occurs before in FEM calculations. Then yielding in FEM analysis has a less influence in the behaviour as in the analytical calculations due to the fact that almost all the strength in the concrete is lost when yielding starts.

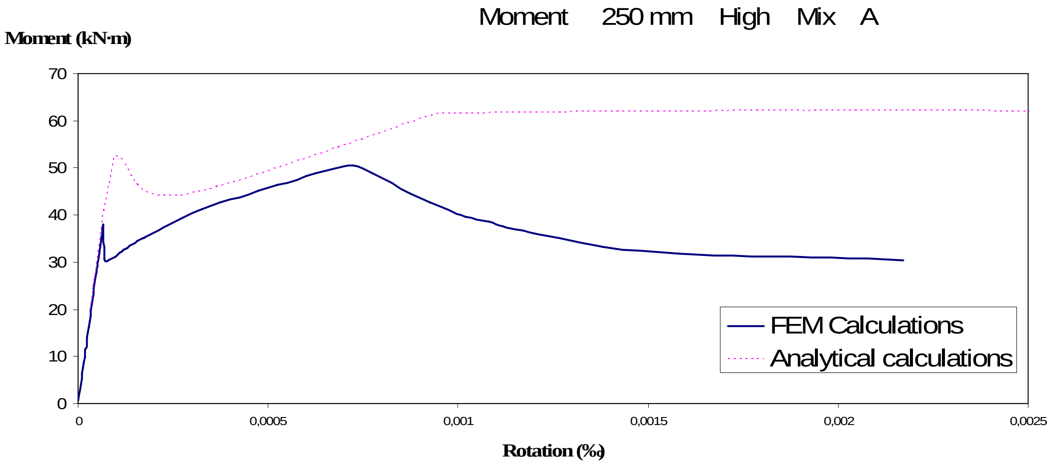


Figure 6.43 Moment versus rotation 250 mm high beam, σ–ε approach

The same can be applied for the 500 mm beam. The percentage of the difference between both methods is more or less constant, so the size effect is not so important. The only problem is that is a little more difficult to obtain the convergence in a high beam than in a short one.

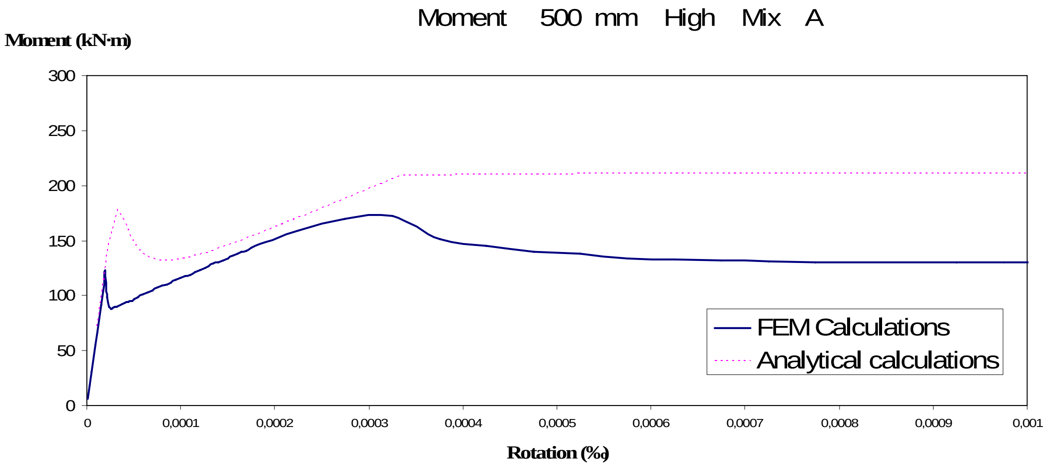


Figure 6.44 Moment versus rotation 500 mm high beam, σ–ε approach

6.2.1.3 Mix B and mix C comparisons

If the mix B is studied, more or less the same effects can be observed regarding the moment. The approximation is also good in the first rotation values. However it is important to underline that the analytical calculations about the moment using the RILEM constants is not the same as using the σ–w approach. Hence, it would be better to use the correct constants in order to have a good approximation as a reference, but as it is seen that the post-cracking behaviour is anyway different in both approaches, that is not considered.

For this mix there are a little more differences between the analytical calculations and FEM calculations, but basically these differences are based on the same origin. One important thing is that at least the post-cracking behaviour in FEM calculations has more or less the same shape as the analytical calculation. In another words, the effect of increasing the amount of fibres is traduces in a no existence of a post-peak decreases as in mix A. This decreasing does not also exist in 500 mm beam but it exists for this beam in the analytical calculations. As it was commented in last chapter this could be caused by the RILEM size factor effect that is too small for a 500 mm beam.

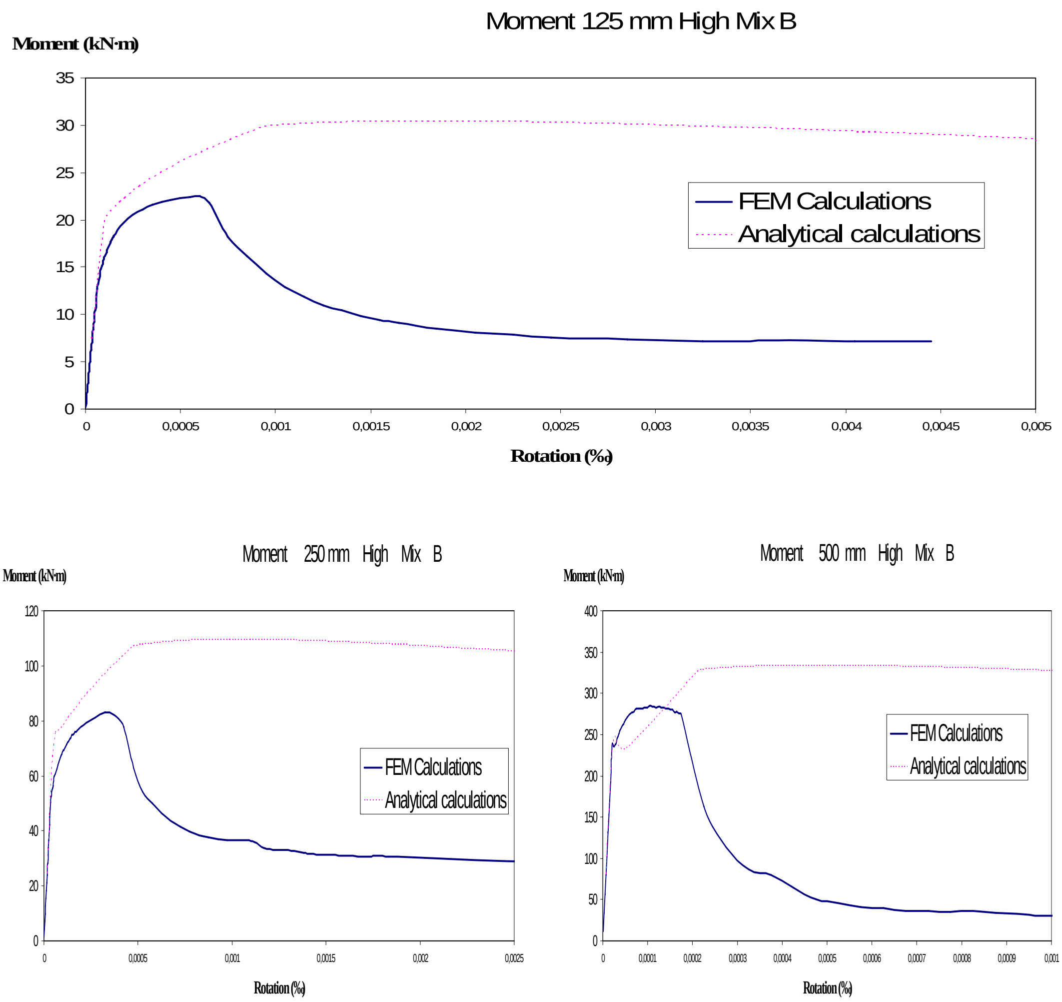


Figure 6.45 Moment versus rotation, mix B

The comparisons for mix C is not represented as the graphs are very similar to the mix B graphs. The conclusions that can be obtained are the same.

6.2.2 σ–w approach

6.2.2.1 General results

Now, following the same procedure that the one used for the σ–ε approach, the analysis of a 125 mm high beam using mix A is made. The deformed shaped obtained when a rotation is applied to the section is showed in figure 6.46.

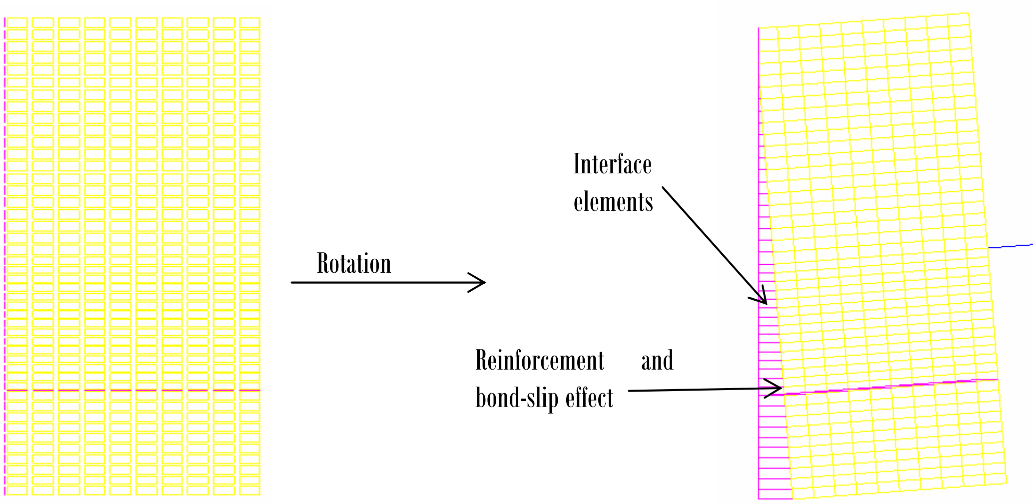


Figure 6.46 Undeformed and deformed beam. σ–w approach

This shape seems more realistic than the one obtained for the previous approach. The first important conclusion that can be obtained is that the crack side seems to remain plane (as one of the Pedersen hypotheses says). This can be appreciated in figure 6.47. The values for the crack opening are very similar in both approaches and the slope of the curve is also more or less the same.

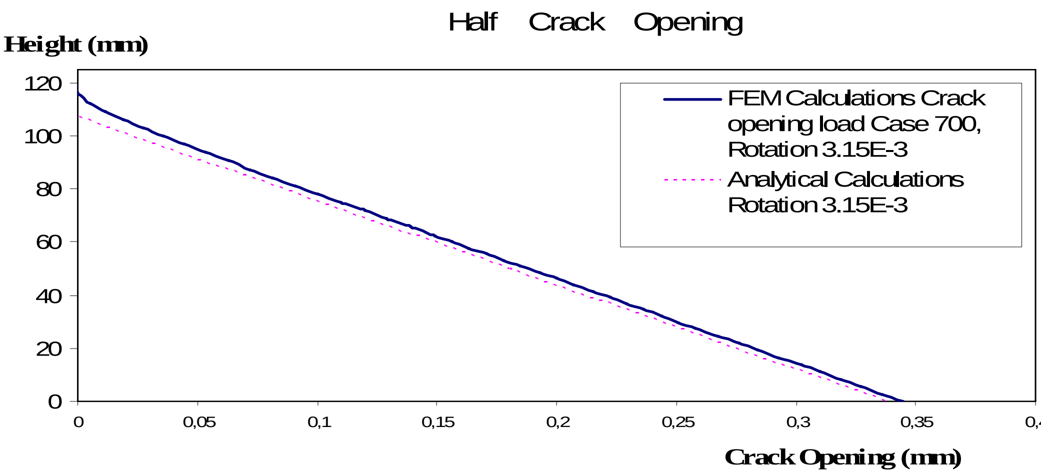


Figure 6.47 Crack opening for one load case in FEM and analytical calculations σ–w approach

The stress-strain diagram of the reinforcement is showed in figure 6.48. As well as in the σ–ε approach the model is following correctly and yielding occurs in a certain load value.

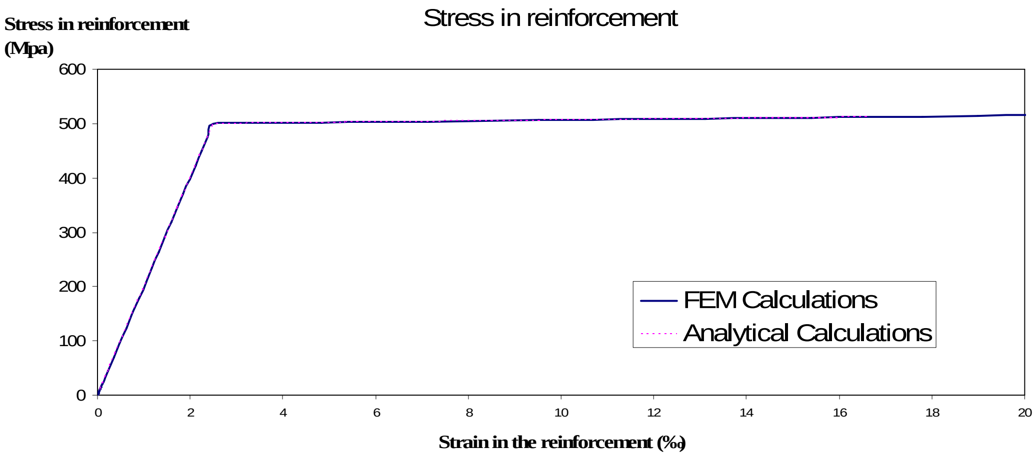


Figure 6.48 Stress-strain reinforcement diagram in σ–w approach

The stress-strain relationship at the level of the reinforcement is not possible to be obtained because DIANA does not permit this feature for the interface elements. Anyway no problems are expected due to the definition of the relationship crack-opening stress is well supported by the software.

Regarding the concrete in compression the behaviour of the element (452) situated in the top of the cracked section is showed in figure 6.49. The behaviour here is also different from the analytical calculations that do not predict the failure of the concrete in compression (compressive concrete strain reached) before the crack opening limit is reached. However there is no other way to define the material without entering in convergence problems.

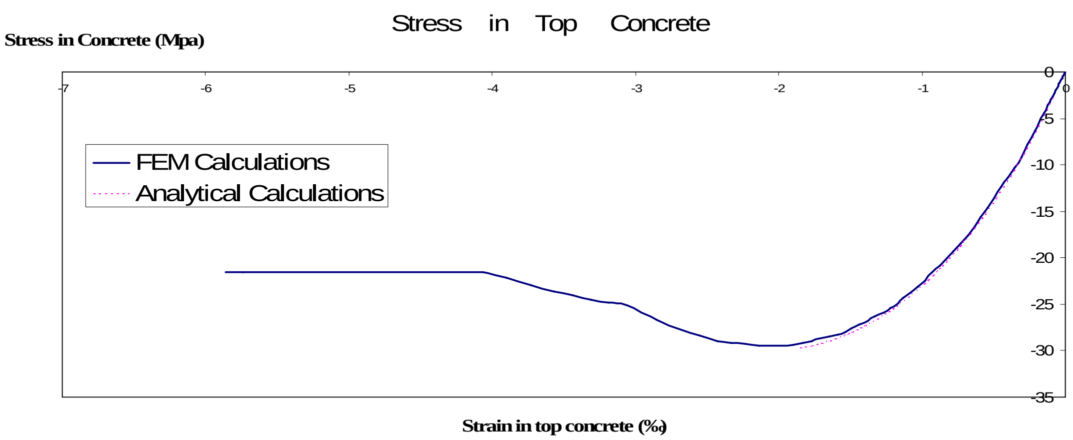


Figure 6.49 Stress-strain in top concrete, σ–w approach

The moment graph is now represented in figure 6.50. The behaviour is very similar using both calculation methods. The maximum moment is 15.6 kN·m for the FEM calculations and 14.8 kN·m for the analytical results so the proportional difference is quite low. Also the crack initiation phase is the same for both. The yielding phase is a little different but it can be caused by some behaviours like bond slip that are not taken into account in the analytical calculations.

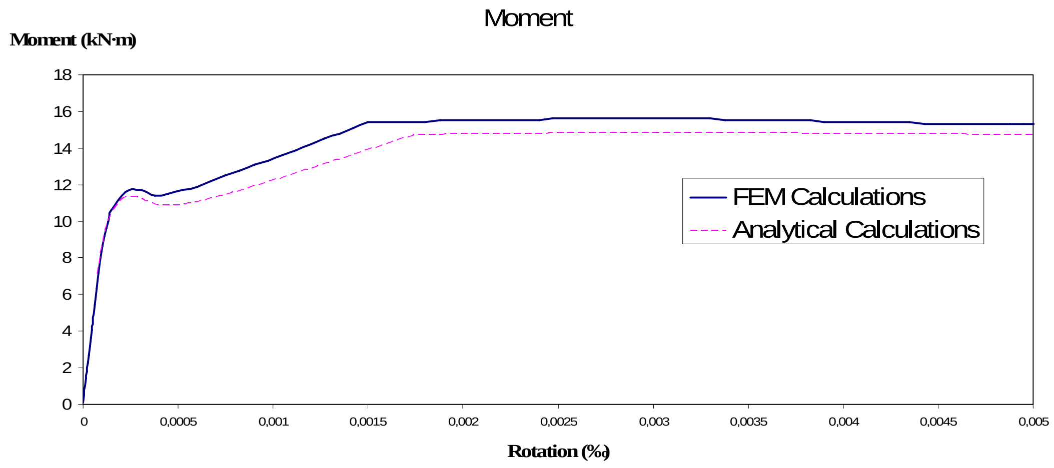


Figure 6.50 Moment versus rotation, σ–w approach

It is not possible to represent the variation of the stress in the concrete when the distance to the top of the section is varied. The reason is the same as the concrete in tension. Interface elements do not allow stress-strain diagrams and tabulation in DIANA. This feature is only possible in the elastic zone but it is not so much interesting.

At last it is possible to represent the crack length (*a*) representing the displacement of the crack interface and choosing the first element that is different than zero in each step. The graph resulting is represented in figure 6.51. The accuracy of the FEM results are quite good but is necessary to realise that there are only 45 elements in the height of the cross section, so only 45 values of the crack length are available.

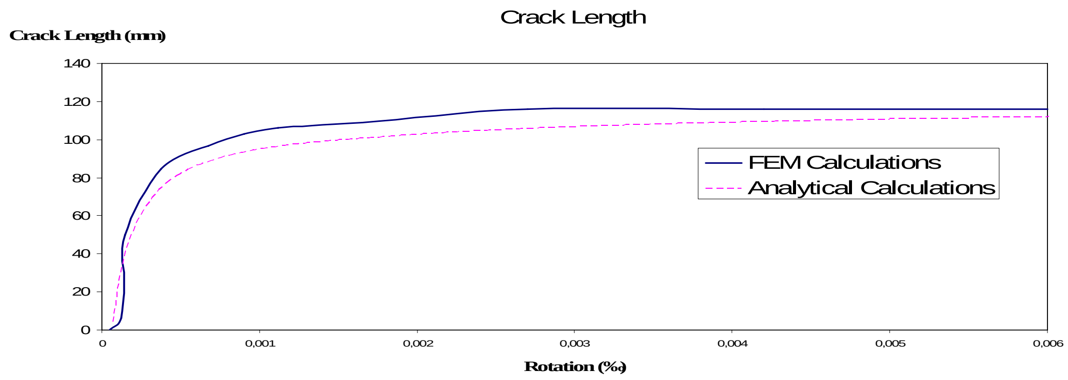


Figure 6.51 Crack length

6.2.2.2 Mix A comparisons

The effect of increasing the height of the beam in the maximum moment supported is studied now. Figure 6.52 shows the moment versus rotation in a 250 mm beam. The conclusions are the same as for the 125 mm beam. There is a good crack initiation and the maximum moment is very similar although it is a little bigger. This can be caused by the assumptions that are not taken into account in the analytical calculations.

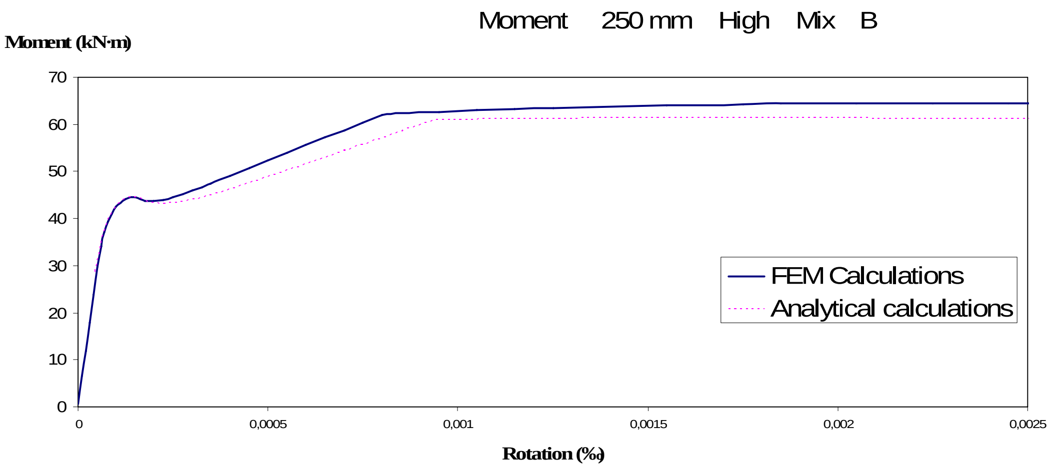


Figure 6.52 Moment versus rotation 250 mm high beam, σ–w approach

It is found when the beam has a considerably height the analytical calculations and the FEM calculations fit better than for the other heights. The approach is really very good and because the moment is so high, the differences between the methods is unnoticeable.

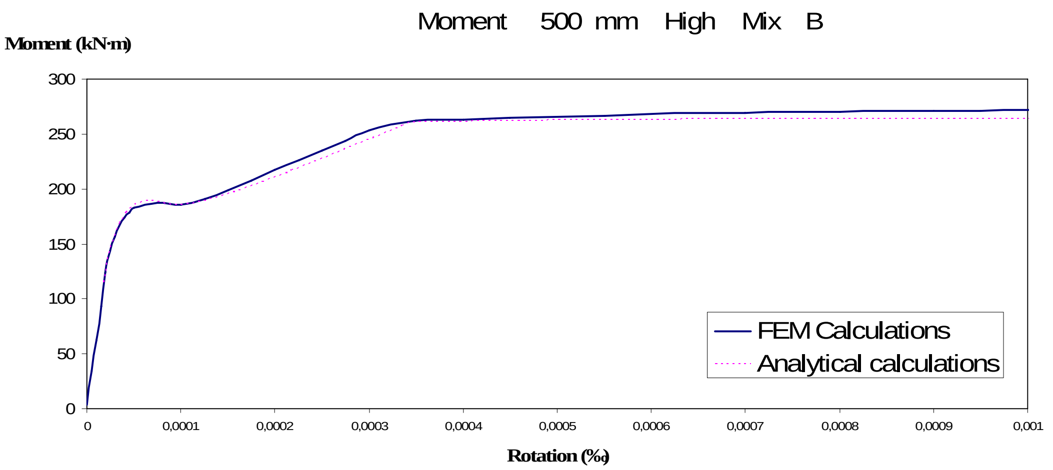


Figure 6.53 Moment versus rotation 500 mm high beam, σ–w approach

6.2.2.3 Mix B and mix C comparisons

Figure 6.54 shows the moment versus rotation when mix B is used. The differences are very small and the behaviour (even crack and yielding phases) occurs at the same time for both methods.

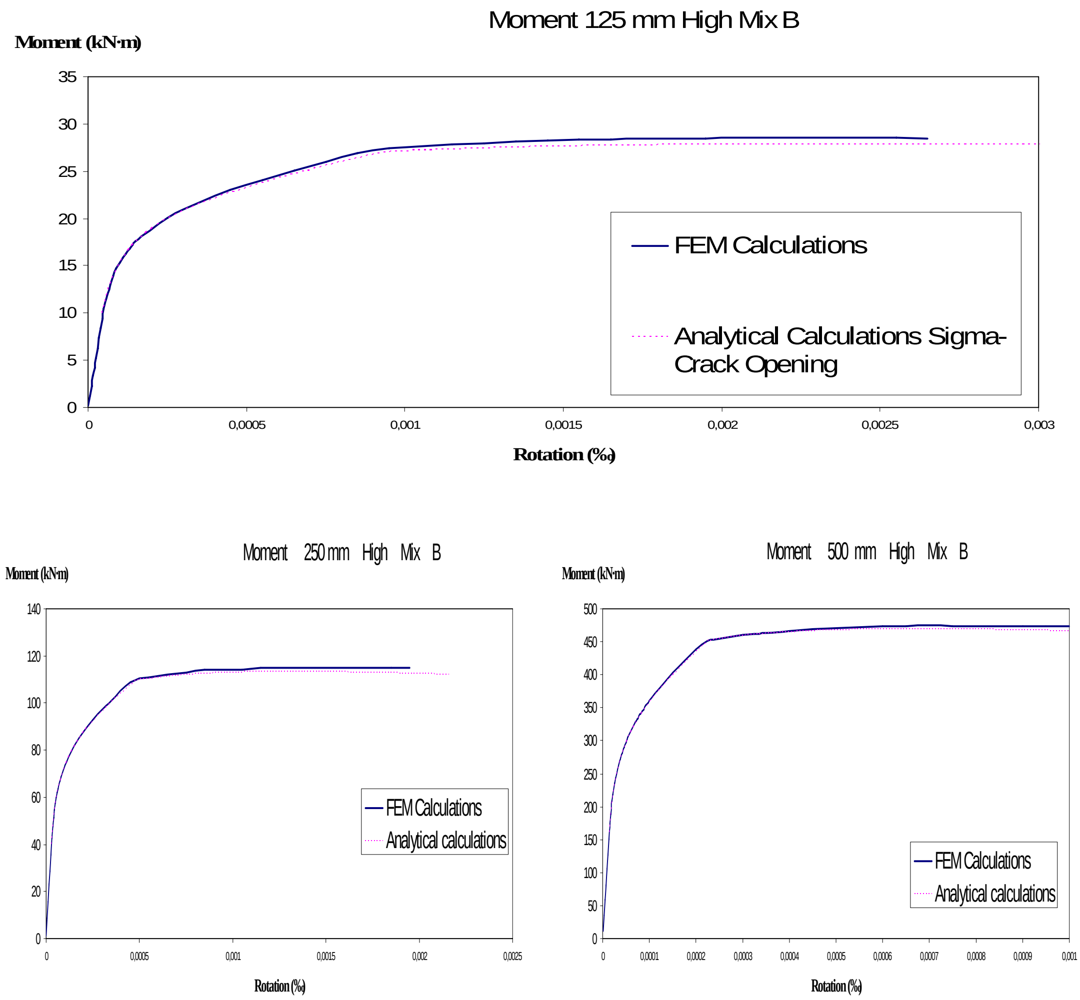


Figure 6.54 Moment versus rotation, mix B, σ–w approach

With these diagrams it can be concluded that the *σ–w* approach is a really good approach and no size or mix differences are appreciated. The basic model is the same for all the variations and it works more or less similar whatever the characteristics of the section are.

Mix C results are quite similar to mix B results. Hence the graphs are not analysed because no extra conclusions can be extracted.

7 Conclusions

7.1 Summary

Design of FRC elements introduces a change into the conventional design processes. As the concrete response in tension is improving it is necessary to include this effect in the design procedure or corresponding code.

RILEM TC 162-TDF has proposed two approaches to explain the behaviour of a FRC element when subjected to a tensional or flexural load. The σ–ε approach is based on the classic concept of the stress-strain relationship. When a FRC element is loaded, the tensile strength causes a change in the traditional diagram, which just considers the elastic behaviour of plane concrete and neglects the tension-softening. This change is taken into account developing new equilibrium equations. The σ–w approach develops the existent relationship between the crack opening and the stress that can be carried by the concrete in the cracked zone. To use this method it is necessary to know what the average crack spacing is. The crack spacing governs the number of cracks (non-linear hinges) that exists when a general structure is loaded. The analytical equations using this approach are also presented.

Some approaches have been studied to obtain the most realistic crack spacing value and a new suggestion based on the classic EC2 crack spacing formula has been presented.

To check the validity of the assumptions and the simplifications made in the analytical calculations FEM analyses have also been conducted. Two models (one for each approach) were developed in order to obtain the most possible similarities between the simulated model and the real behaviour.

Also as one of the approaches considers the existence of a size factor (high depending coefficient) that decreases the RILEM TC 162-TDF stress values. To analyse the validity of this factor three different heights were studied. Also to check the effect in the results of a change in some FRC variables (e.g. dosage of fibres or concrete strength), three different mixes were also considered.

7.2 General conclusions

The first conclusion that can be extracted from this thesis is the difficulty to obtain the correct values that are needed in the design. Most of these values have to be taken from a very specific laboratory tests. When a design of a conventional concrete element is being carried out it is not normally possible to have the exact values for the material constants. However, this issue is more important in the case of FRC. FRC has special characteristics, which makes it quite complicated to know the exact amount of fibres that will be in a generic cross section and the material properties resulting from their presence. It is also essential to improve the fibre performance to ensure a minimum quantity of fibres in a section. Then it would be very interesting to tabulate these values in order to provide the designers with good tools to facilitate their work.

With regard to the analytical approaches, the σ–ε approach probably is easier to understand by a designer as it is closer to the traditional plane concrete approach. The σ–w approach, on the other hand, needs a deeper theoretical basis and it has the main problem on the crack spacing value although, with it, it is possible to obtain directly the values for the crack length and crack opening.

The crack spacing value is difficult to obtain and most of the propositions that exists at the moment are concentrated on plane concrete. It very is important that to facilitate the use of the σ–w approach a valid formula to calculate the crack spacing is provided. This thesis shows that most of the expressions available to calculate the crack spacing yield very different values. It is necessary that an expression take into account all the parameters that are proved to have an influence on the crack spacing value. The expression proposed in this thesis is an extension of the derivation that is also used in the EC2 formula (based on the transfer length concept).

When the analytical results are compared it is notice an almost total disagreement between σ–ε and σ–w approach. However, in this thesis it is shown that it can be fixed if the RILEM TC 162-TDF constants used to calculate the stress values are replaced with new suitable values. Barros et al. (2004) proposes new values for these constants and they seems to give better agreement with the σ–w approach analytical results. Barros et al. (2004) only consider one height (125 mm) in their studies. However, if their values are used for the other heights it can be appreciated that no size factor is really need. This suggests that the RILEM TC 162-TDF constants should be replaced and the origin of the size factor has to be studied. The Barros et al. (2004) values also show a much better behaviour in comparison with the RILEM TC 162-TDF ones if the quantity of fibres and the concrete strength is increasing although the accuracy of this approach decreases.

Finally, if the FEM results are analysed it is noticed that σ–ε approach is not suitable to be used in this kind of analysis. The results obtained are very different compared with the analytical ones and the assumptions were not fulfilled. However, the σ–w approach gives very good results for all of the cases studied, that prove that it is a good approach to be used in FEM analysis. Also, it gives good agreement between the FEM analysis and the analytical calculations, even though the effect of bond-slip is neglected in the analytical calculations. It is necessary to emphasize the good behaviour regarding crack initiation that was obtained. Therefore it is recommended that σ–w approach is to be used in the design if it is necessary to make a verification of some parts using FEM software (DIANA in the case of this thesis).

In conclusion, it seems that both approaches can be used in the design if the required changes in the values of the σ–ε approach are made. The σ–ε approach seems easier to apply but σ–w approach has some advantages regarding the cracking study and the use in FEM analysis. Moreover, if the value of the crack spacing is known only analytical calculations are needed in order to obtain a good approximation and if only ULS is studied the influence of the crack spacing is not so important.

7.3 Further investigations

This thesis reveals the necessity of more research investigations about different issues.

- More laboratory tests are needed to corroborate the validity of the crack spacing formulas when a fibre reinforced concrete element is analysed. Also it could be interesting to check the validity of the expression used in this thesis

- As it was commented it would be interesting to create a document including the characteristic values to use in the design for the most common fibre dosage. This is a basic issue in order to facilitate the introduction of FRC as a conventional construction material.

- It is important to check the results obtained in analytical analysis as well as in FEM analysis by means of laboratory tests. These test would prove the validity of the assumptions that both method uses and maybe it would be possible to investigate if there is any size effect.

- It could be interesting to introduce time dependent effects in the analytical calculations as well as other effects like bond-slip that FEM calculations consider.

- More studies regarding the service limit state needs to be carried out. Cracking behaviour using FRC elements is improved so it is important to have accurate values of the crack width.

- Structures loaded also by a normal force (as well as by flexural moment) can be studied. Walls are a good example of this kind of load case.

- Shear behaviour have also to be studied in order to corroborate the RILEM assumptions regarding that.

- General design expressions have to be developed to facilitate the work of the designers who wants to use FRC in their designs.

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Appendix A

A.1 Derivation of the cross-sectional analysis equations in σ–ε approach

This appendix shows the derivation of the equilibrium equations step by step in order to make easier to follow the process.

A.1.1 σ–ε approach. Failure occurs at the same time in concrete as in reinforcement

Figure C.1 shows the cross-section and the diagrams of stress and strain which RILEM TC 162-TDF proposes for the design if both limits are reached as the same time.

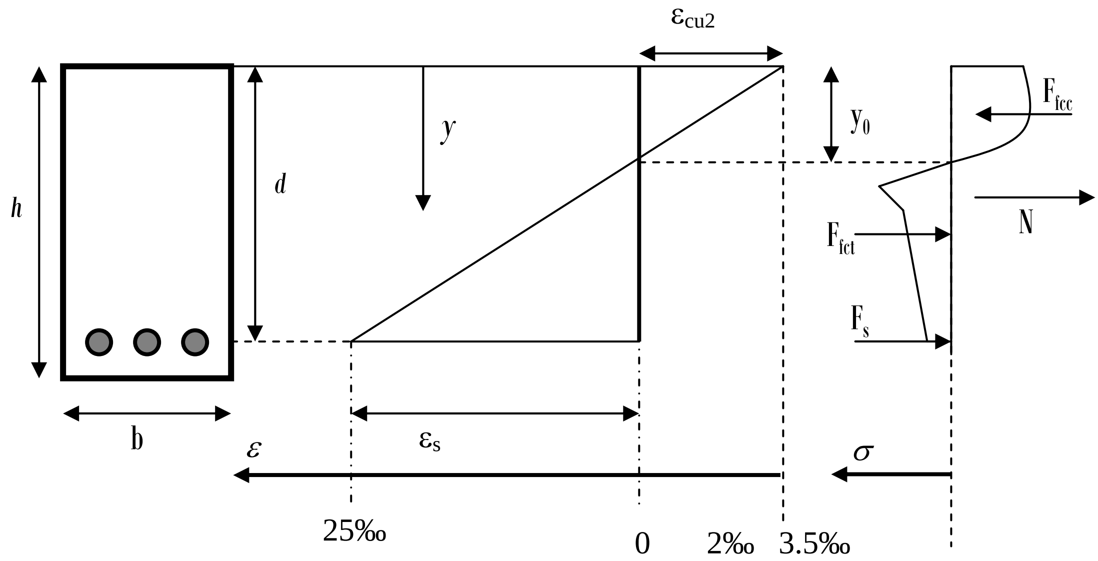


Figure C.1 Stress and strain diagram (σ–ε approach)

From this diagram it is possible to write the equilibrium equation:

$$0 = F_{fc} + F_{ct} + F_s + N \Rightarrow 0 = \int_0^{y_0} \sigma_{cc}(\varepsilon, y) \cdot b \cdot dy + \int_{y_0}^{d} \sigma_{ct}(\varepsilon, y) \cdot b \cdot dy + \sigma_s \cdot A_s + N \quad \text{(C.1)}$$

From this point, it is necessary to obtain the different stress equations, which depend on the strain and the y dimension.

The first stress relationship is $\sigma_{cc}(\varepsilon, y)$.The Eurocode proposes to use the next equation to analyse non-linear problems, see 3.1. Note that the strain has to be taken positive to be introduced in the equation.

$$\sigma_c(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta(\varepsilon(y)) - \eta(\varepsilon(y))^2}{1+(k-2)\cdot\eta(\varepsilon(y))}\right] \qquad \text{(C.2)}$$

It is needed to obtain the relationship between ε and y. This relationship is based on the strain diagram:

$$\left.\begin{aligned}\varepsilon(y) &= a\cdot y + b\\ \varepsilon(y_0) &= a\cdot y_0 + b = 0\\ \varepsilon(0) &= b = \varepsilon_{cu1}\end{aligned}\right\} \Rightarrow 0 = a\cdot y_0 + \varepsilon_{cu1} \Rightarrow a = -\frac{\varepsilon_{cu1}}{y_0}$$

$$\varepsilon(y) = \frac{-\varepsilon_{cu1}}{y_0}\cdot y + \varepsilon_{cu1} = \varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right) \qquad \text{(C.3)}$$

And if (C.3) is introduced in (C.2), the result is:

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta\left(\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)\right) - \eta\left(\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)\right)^2}{1+(k-2)\cdot\eta\left(\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)\right)}\right] \qquad \text{(C.4)}$$

Or if it is included the value of $k = 1.05\frac{E_c\cdot\varepsilon_{c1}}{f_{cm}}$ and $\eta(\varepsilon(y)) = \frac{\varepsilon(y)}{\varepsilon_{c1}}$, the complete equation is:

$$\boxed{\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{1.05\frac{E_c\cdot\varepsilon_{c1}}{f_{cm}}\cdot\left(\frac{\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)}{\varepsilon_{c1}}\right) - \left(\frac{\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)}{\varepsilon_{c1}}\right)^2}{1+\left(1.05\frac{E_c\cdot\varepsilon_{c1}}{f_{cm}}-2\right)\cdot\left(\frac{\varepsilon_{cu1}\cdot\left(1-\frac{y}{y_0}\right)}{\varepsilon_{c1}}\right)}\right]} \qquad \text{(C.5)}$$

The second relationship is $\sigma_{fct}(\varepsilon, y)$. The stress-strain relationship proposed by RILEM TC 162-TDF is base don the curve which is showed in the figure C.2.

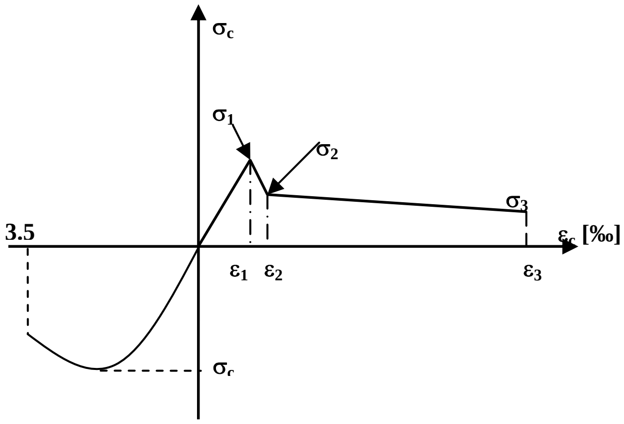


Figure C.2 Stress and strain diagram (σ–ε approach)

This relationship consists of three parts. The first part is:

$$\left.\begin{array}{l}\sigma(\varepsilon)=a\cdot\varepsilon+b\\ \sigma(0)=0+b=0\Rightarrow b=0\\ \sigma(\varepsilon_1)=a\cdot\varepsilon_1+b=\sigma_1\end{array}\right\}\Rightarrow a\cdot\varepsilon_1=\sigma_1\Rightarrow a=\frac{\sigma_1}{\varepsilon_1}$$

$$\sigma(\varepsilon)=a\cdot\varepsilon+b\Rightarrow\sigma(\varepsilon)=\frac{\sigma_1}{\varepsilon_1}\cdot\varepsilon\qquad\text{if}\quad 0\le\varepsilon\le\varepsilon_1=\frac{\sigma_1}{E_C}$$

The second part:

$$\left.\begin{array}{l}\sigma(\varepsilon)=a\cdot\varepsilon+b\\ \sigma(\varepsilon_1)=a\cdot\varepsilon_1+b=\sigma_1\\ \sigma(\varepsilon_2)=a\cdot\varepsilon_2+b=\sigma_2\end{array}\right\}\Rightarrow\sigma_1-\sigma_2=a(\varepsilon_1-\varepsilon_2)\Rightarrow$$

$$\Rightarrow a=\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}$$

$$\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\varepsilon_1+b=\sigma_1\Rightarrow b=\sigma_1-\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\varepsilon_1$$

$$\sigma(\varepsilon)=\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\varepsilon+\sigma_1-\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\varepsilon_1=\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot[\varepsilon-\varepsilon_1]+\sigma_1$$

$$\text{if}\quad\varepsilon_1=\frac{\sigma_1}{E_C}\le\varepsilon\le\varepsilon_2=\varepsilon_1+10^{-4}$$

And at last the third part:

$$\left.\begin{aligned}&\sigma(\varepsilon) = a\cdot\varepsilon + b\\&\sigma(\varepsilon_2) = a\cdot\varepsilon_2 + b = a\cdot(\varepsilon_2) + b = \sigma_2\\&\sigma(\varepsilon_3) = a\cdot\varepsilon_3 + b = a\cdot(\varepsilon_3) + b = \sigma_3\end{aligned}\right\} \Rightarrow \sigma_2 - \sigma_3 = a(\varepsilon_2 - \varepsilon_3) \Rightarrow$$

$$\Rightarrow a = \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}$$

$$\frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot\varepsilon_3 + b = \sigma_3 \Rightarrow b = \sigma_3 - \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot\varepsilon_3$$

$$\sigma(\varepsilon) = \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot\varepsilon + \sigma_3 - \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot\varepsilon_3 = \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot[\varepsilon - \varepsilon_3] + \sigma_3$$
$$\text{if} \quad \varepsilon_1 + 10^{-4} = \varepsilon_2 \le \varepsilon \le \varepsilon_3$$

Hence, the global equation is:

$$\sigma(\varepsilon) = \frac{\sigma_1}{\varepsilon_1}\cdot\varepsilon \qquad \text{if} \quad 0 \le \varepsilon \le \frac{\sigma_1}{E_C}$$

$$\sigma(\varepsilon) = \frac{(\sigma_1 - \sigma_2)}{(\varepsilon_1 - \varepsilon_2)}\cdot[\varepsilon - \varepsilon_1] + \sigma_1 \qquad \text{if } \frac{\sigma_1}{E_C} \le \varepsilon \le \varepsilon_1 + 10^{-4} \tag{C.6}$$

$$\sigma(\varepsilon) = \frac{\sigma_2 - \sigma_3}{(\varepsilon_2 - \varepsilon_3)}\cdot[\varepsilon - \varepsilon_3] + \sigma_3 \quad \text{if} \quad \varepsilon_1 + 10^{-4} \le \varepsilon \le \varepsilon_3$$

But, as the compressive equation, it has to be changed by means of y. The equation of $\varepsilon(y)$ below the neutral axis is:

$$\left.\begin{aligned}&\varepsilon(y) = a\cdot y + b\\&\varepsilon(y_0) = a\cdot y_0 + b = 0\\&\varepsilon(d) = a\cdot d + b = \varepsilon_s\end{aligned}\right\} \Rightarrow \varepsilon_s = a\cdot(d - y_0) \Rightarrow a = \frac{\varepsilon_s}{(d - y_0)}$$

$$a\cdot y_0 + b = 0 \Rightarrow \frac{\varepsilon_s}{(d - y_0)}\cdot y_0 + b = 0 \Rightarrow b = -\frac{\varepsilon_s}{(d - y_0)}\cdot y_0$$

$$\varepsilon(y) = \frac{\varepsilon_s}{(d - y_0)}\cdot(y - y_0) \tag{C.7}$$

And if (C.7) is introduced in (C.6):

$$\sigma(\varepsilon) = \frac{\sigma_1}{\varepsilon_1}\cdot\frac{\varepsilon_s}{(d - y_0)}\cdot(y - y_0) \qquad \text{if} \quad 0 \le \varepsilon \le \frac{\sigma_1}{E_C}$$

$$\sigma(\varepsilon)=\frac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\left[\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_1\right]+\sigma_1 \quad \text{if } \frac{\sigma_1}{E_C}\leq\varepsilon\leq\varepsilon_2 \tag{C.8}$$

$$\sigma(\varepsilon)=\frac{\sigma_2-\sigma_3}{(\varepsilon_2-\varepsilon_3)}\cdot\left[\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_3\right]+\sigma_3 \quad \text{if } \varepsilon_2\leq\varepsilon\leq\varepsilon_3$$

And the limits have also to be changed:

$$\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)=0\Rightarrow y=y_0$$

$$\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)=\frac{\sigma_1}{E_C}\Rightarrow\frac{\sigma_1\cdot(d-y_0)}{E_C\cdot\varepsilon_s}=(y-y_0)\Rightarrow y=y_0+\frac{\sigma_1\cdot(d-y_0)}{E_C\cdot\varepsilon_s}$$

$$\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)=\varepsilon_2\Rightarrow y=y_0+\frac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s}$$

$$\frac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)=\varepsilon_s\Rightarrow y=d$$

That finally yields the final equation which depends on y:

$$\boxed{\sigma_{ct}(y)=\begin{cases} E_c\cdot\varepsilon(y)=E_c\cdot\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0); \text{ if } 0\leq y\leq\dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0=\dfrac{\frac{\sigma_1}{E_c}(d-y_0)}{\varepsilon_s}+y_0 \\ \dfrac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_1\right]+\sigma_1; \text{ if } \dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0<y<y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s} \\ \dfrac{\sigma_2-\sigma_3}{(\varepsilon_2-\varepsilon_3)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_3\right]+\sigma_3; \text{ if } y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s}\leq y\leq d \end{cases}} \tag{C.9}$$

Finally the equation of the steel is based in the diagram of the figure C.3:

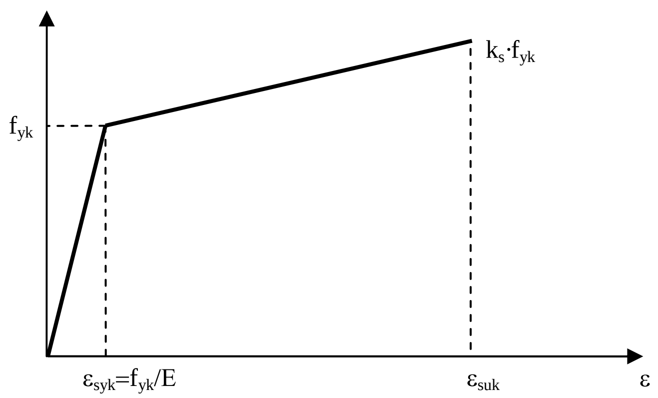


Figure C.3 Stress-strain diagram of reinforcement

The firs part is obvious:

$$\sigma_s(\varepsilon) = E_s\varepsilon \qquad \text{if} \quad \varepsilon \le \frac{f_{yk}}{E_s} \tag{C.10}$$

And the second one is derived from the curve:

$$\sigma_s(\varepsilon) = a \cdot \varepsilon + b$$

$$\left.\begin{aligned} \sigma_s\left(\frac{f_{yk}}{E_s}\right) &= a \cdot \frac{f_{yk}}{E_s} + b = f_{yk} \\ \sigma_s(\varepsilon_{suk}) &= a \cdot \varepsilon_{suk} + b = k_s \cdot f_{yk} \end{aligned}\right\} \Rightarrow f_{yk} \cdot (k_s - 1) = a \cdot \left(\varepsilon_{suk} - \frac{f_{yk}}{E_s}\right) \Rightarrow a = \frac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{suk} - \frac{f_{yk}}{E_s}\right)}$$

$$\sigma_s\left(\frac{f_{yk}}{E_s}\right) = a \cdot \frac{f_{yk}}{E_s} + b = f_{yk} \Rightarrow \frac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{suk} - \frac{f_{kd}}{E_s}\right)} \cdot \frac{f_{yk}}{E_s} + b = f_{yk} \Rightarrow b = f_{yk} - \frac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{suk} - \frac{f_{yk}}{E_s}\right)} \cdot \frac{f_{yk}}{E_s}$$

$$\sigma_s(\varepsilon) = \frac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{suk} - \frac{f_{yk}}{E_s}\right)} \cdot \left(\varepsilon - \frac{f_{yk}}{E_s}\right) + f_{yk} \qquad \text{if} \quad \frac{f_{yk}}{E_s} \le \varepsilon \le \varepsilon_{suk} \tag{C.11}$$

And of course if the stress is higher than ε_{suk}, the stress will be zero. The final equation is, due to the derivation showed above:

$$\boxed{\begin{aligned} &\sigma_s(\varepsilon_s) = E_s\varepsilon_s && \text{if} \quad \varepsilon_s < \frac{f_{yk}}{E_s} \\ &\sigma_s(\varepsilon_s) = \frac{f_{yk} \cdot (k_s - 1)}{\left(\varepsilon_{suk} - \frac{f_{yk}}{E_s}\right)} \cdot \left(\varepsilon_s - \frac{f_{yk}}{E_s}\right) + f_{yk} && \text{if} \quad \frac{f_{yd}}{E_s} \le \varepsilon_s \le \varepsilon_{suk} \\ &\sigma_s(\varepsilon_s) = 0 && \text{if} \quad \varepsilon_{suk} < \varepsilon_s \end{aligned}} \tag{C.12}$$

The value of ε_s is always known. For this reason it is not necessary to transform the equation by means of y.

A.1.2 σ−ε approach. Alternative diagram

The alternative diagram is very similar but the difference of the neutral axis is not predefined. Giving values to the strain at the reinforcement height, it is possible to follow the process from the beginning until the failure of the section. See figure C.4.

ε_{ccm} is the compressive strain in the top concrete when the strain limit at the position of the reinforcement is reached, $\varepsilon_{\max}$ is the maximum tensile strain at the level of the reinforcement, and $\kappa = \dfrac{\varepsilon_s}{d - y_0}$. (C.13)

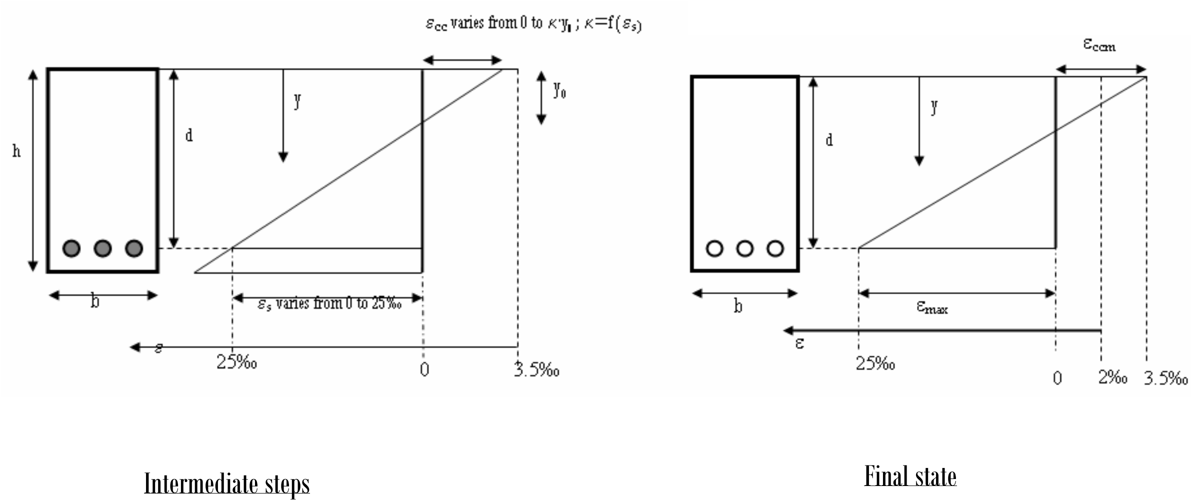


Figure C.4 cross-sectional stress diagram. Alternative model from the beginning until failure

Due to this variation, the equations change. The relationship between stress and strain in FRC in compression will be the same as in the RILEM TC 162-TDF:

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k \cdot \eta(\varepsilon(y)) - \eta(\varepsilon(y))^2}{1 + (k-2) \cdot \eta(\varepsilon(y))}\right] \tag{C.14}$$

But the new relation between strain and y is:

$$\left.\begin{array}{l} \varepsilon(y) = a \cdot y + b \\ \varepsilon(y_0) = a \cdot y_0 + b = 0 \\ a = \kappa = -\dfrac{\varepsilon_s}{d - y_0} \end{array}\right\} \Rightarrow 0 = -\frac{\varepsilon_s}{d - y_0} \cdot y_0 + b \Rightarrow b = \frac{\varepsilon_s}{d - y_0} y_0 \tag{C.15}$$

$$\varepsilon(y) = -\frac{\varepsilon_s}{d - y_0} \cdot y + \frac{\varepsilon_s}{d - y_0} y_0 = \frac{\varepsilon_s}{d - y_0} \cdot (y_0 - y)$$

Hence, the new equation is:

$$\boxed{\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k\cdot\eta\left(\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)\right)-\eta\left(\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)\right)^2}{1+(k-2)\cdot\eta\left(\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)\right)}\right]} \tag{C.16}$$

Or:

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{1.05\frac{E_c\cdot\varepsilon_{c1}}{f_{cm}}\cdot\left(\frac{\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)}{\varepsilon_{c1}}\right)-\left(\frac{\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)}{\varepsilon_{c1}}\right)^2}{1+\left(1.05\frac{E_c\cdot\varepsilon_{c1}}{f_{cm}}-2\right)\cdot\left(\frac{\frac{\varepsilon_s}{d-y_0}\cdot(y_0-y)}{\varepsilon_{c1}}\right)}\right] \tag{C.17}$$

The equation for FRC in tension is basically the same, but it is necessary to change the last limit. This is due to the fact that tensile strength of the concrete is different from when $y > d$ while the strain is less than $\varepsilon_3 = \varepsilon_{ct\max}$. So the maximum height which can carry tensile load is:

$$\varepsilon(y) = \frac{\varepsilon_s}{(d-y_0)}(y_{\max}-y_0) = \varepsilon_3 \Rightarrow \mathrm{y}_{\max} = \frac{(d-y_0)\cdot\varepsilon_3}{\varepsilon_s}+y_0 \tag{C.18}$$

As it can be observed, this height depends on ε_s. When $\varepsilon_s = \varepsilon_3$, the height is logically d. Finally, when $\mathrm{y} > \mathrm{y}_{\max}$, the tensile load is zero.

$$\sigma_{ct}(y)=\begin{cases} E_c\cdot\varepsilon(y)=E_c\cdot\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0); \text{ if } 0\le y\le\dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0=\dfrac{\frac{\sigma_1}{E_c}(d-y_0)}{\varepsilon_s}+y_0 \\ \dfrac{(\sigma_1-\sigma_2)}{(\varepsilon_1-\varepsilon_2)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_1\right]+\sigma_1; \text{ if } \dfrac{\varepsilon_1(d-y_0)}{\varepsilon_s}+y_0<y<y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s} \\ \dfrac{\sigma_2-\sigma_3}{(\varepsilon_2-\varepsilon_3)}\cdot\left[\dfrac{\varepsilon_s}{(d-y_0)}\cdot(y-y_0)-\varepsilon_3\right]+\sigma_3; \text{ if } y_0+\dfrac{\varepsilon_2\cdot(d-y_0)}{\varepsilon_s}\le y\le d \\ 0 \quad \text{if } \dfrac{(d-y_0)\cdot\varepsilon_3}{\varepsilon_s}+y_0<\text{y} \end{cases} \tag{C.19}$$

The equation to explain the behaviour of the steel is (C.13) as in the first option.

A.2 Derivation of the cross-sectional analysis equations in σ–*w* approach

Figure C.5 shows the diagram which is necessary to implement in order to obtain the equations:

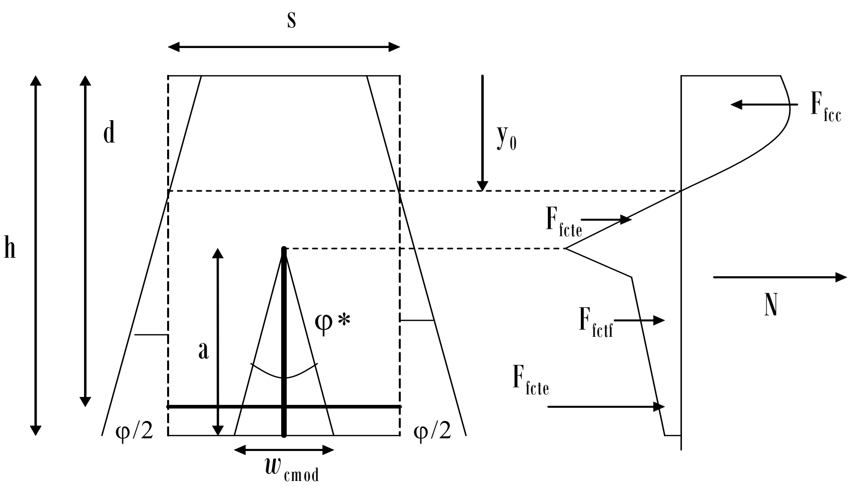


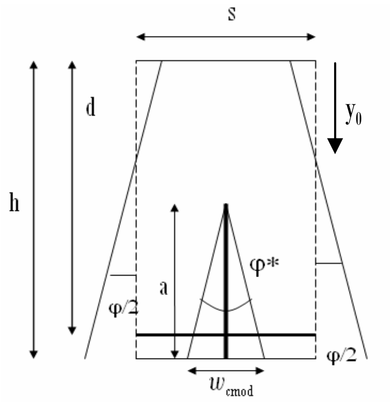
Figure C.5 Non-linear hinge model 1, stress distribution

The forces equilibrium equation is:

$$0=F_{fc}+F_{cte}+F_{ctf}+F_s+N \tag{C.20}$$

$$0=\int_0^{y_0}\sigma_{cc}(\varepsilon,y)\cdot b\cdot dy+\int_{y_0}^{h-a}\sigma_{cte}(\varepsilon,y)\cdot b\cdot dy+\int_{h-a}^{d}\sigma_{ctf}(w,y)\cdot b\cdot dy+\sigma_s\cdot A_s+N \tag{C.21}$$

The relationship between the strain and the height is:



$$\varphi \cong \frac{\varepsilon_{abs}}{(y-y_o)} \Rightarrow \varepsilon_{abs} = \varphi \cdot (y-y_o) \Rightarrow \qquad \text{(C.22)}$$

$$\Rightarrow \varepsilon = \frac{\varphi}{s} \cdot (y-y_0)$$

Figure C.6 strain-height relationship

The relationship between stress and strain in concrete in compression is the same than in the σ–ε approach:

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k \cdot \eta(\varepsilon(y)) - \eta(\varepsilon(y))^2}{1+(k-2)\cdot \eta(\varepsilon(y))}\right] \qquad \text{(C.23)}$$

And if it is changed ε by ε(y), with the condition that the strain must be positive to be introduced into the formula:

$$\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{k \cdot \eta\left(\frac{\varphi}{s}\cdot(y-y_0)\right) - \eta\left(\frac{\varphi}{s}\cdot(y-y_0)\right)^2}{1+(k-2)\cdot \eta\left(\frac{\varphi}{s}\cdot(y-y_0)\right)}\right] \qquad \text{(C.24)}$$

Or:

$$\boxed{\sigma_{cc}(\varepsilon(y)) = -f_{cm}\left[\frac{1.05\frac{E_{cm}\cdot\varepsilon_{c1}}{f_{cm}}\cdot\left(\frac{\frac{\varphi}{s}\cdot(y-y_0)}{\varepsilon_{c1}}\right) - \left(\frac{\frac{\varphi}{s}\cdot(y-y_0)}{\varepsilon_{c1}}\right)^2}{1+\left(1.05\frac{E_{cm}\cdot\varepsilon_{c1}}{f_{cm}}-2\right)\cdot\left(\frac{\frac{\varphi}{s}\cdot(y-y_0)}{\varepsilon_{c1}}\right)}\right]} \qquad \text{(C.25)}$$

The elastic zone has a very simple expression for the stress-height relationship.

$$\sigma_{cte}(\varepsilon, y) = E_c \cdot \varepsilon(y) = \left(\frac{\varphi}{s}\cdot y_0 \cdot \frac{y-y_0}{y_o}\right)\cdot E_c \quad \text{if} \qquad y_0 \le y \le h - y_0 - a \qquad \text{(C.26)}$$

Where *a* is the height of the crack:

$$tg\varphi \approx \varphi = \frac{\varepsilon_{abs}}{h-a-y_0} \Rightarrow h-a-y_0 = \frac{\varepsilon_{abs}}{\varphi} = \frac{f_{ct}\cdot s}{E_c}\cdot\frac{1}{\varphi} \Rightarrow a = h - \frac{f_{ct}\cdot s}{E_c}\cdot\frac{1}{\varphi} - y_0 \quad \text{(C.27)}$$

So it yields:

$$\boxed{\sigma_{cte}(\varepsilon, y) = E_c \cdot \varepsilon(y) = \left(\frac{\varphi}{s}\cdot y_0 \cdot \frac{y-y_0}{y_o}\right)\cdot E_c \quad \text{if} \quad y_0 \le y \le \frac{f_t \cdot s}{E_c}\cdot\frac{1}{\varphi}} \quad \text{(C.28)}$$

The equations of the cracked zone are:

$$\sigma_{ctf}(w,y) = \begin{cases} f_{ct}\cdot\left(1 - a_1 \cdot w(y)\right) & \text{if } 0 \le w \le w_1 \\ f_{ct}\cdot\left(b_2 - a_2 \cdot w(y)\right) & \text{if } w_1 \le w \le w_c \\ 0 & \text{if } w_c < w_{\ \ c} \end{cases} \quad \text{(C.29)}$$

The relationship between w and y can be easily determined. The opening of the crack is the absolute strain in a height y reduced by the elastic absolute strain (deformation which already exists before the tip of the crack) and strain due to the normal load:

$$w(y) = \varphi\cdot\left(y - y_0\right) - \frac{f_{ct}}{E_c}\cdot s - \frac{N}{A\cdot E} \quad \text{if } w \le w_c$$

$$w(y) = \varphi\cdot\left(y - y_0\right) \quad \text{if } w \ge w_c \quad \text{(C.30)}$$

The limits are:

$$w(y) = 0 \Rightarrow \varphi\cdot\left(y-y_0\right) - \frac{f_{ct}}{E_c}\cdot s - \frac{N}{A\cdot E} = 0 \Rightarrow \left(\frac{f_{ct\,t}}{E_c}\cdot s + \frac{N}{A\cdot E}\right)\cdot\frac{1}{\varphi} + y_0 = y$$

$$w(y) = w_1 \Rightarrow \varphi\cdot\left(y-y_0\right) - \frac{f_{ct\,t}}{E_c}\cdot s - \frac{N}{A\cdot E} = 0 \Rightarrow \left(w_1 + \frac{f_{ct}}{E_c}\cdot s + \frac{N}{A\cdot E}\right)\cdot\frac{1}{\varphi} + y_0 = y$$

$$w(y) = w_1 \Rightarrow \varphi\cdot\left(y-y_0\right) - \frac{f_{ct\,t}}{E_c}\cdot s - \frac{N}{A\cdot E} = 0 \Rightarrow \left(w_c + \frac{f_{ct}}{E_c}\cdot s + \frac{N}{A\cdot E}\right)\cdot\frac{1}{\varphi} + y_0 = y$$

So the complete equation is:

$$\sigma_{fctf}(y)=\begin{cases} f_{ct}\cdot\left[1-a_1\cdot\left(\varphi\cdot(y-y_0)-\dfrac{f_{ct}}{E_c}\cdot s-\dfrac{N}{A\cdot E}\right)\right] \\ \text{if } \left(\dfrac{f_{ct}}{E_c}\cdot s+\dfrac{N}{A\cdot E}\right)\cdot\dfrac{1}{\varphi}+y_0\le y\le\left(w_1+\dfrac{f_{ct}}{E_c}\cdot s+\dfrac{N}{A\cdot E}\right)\cdot\dfrac{1}{\varphi}+y_0 \\ f_{ct}\cdot\left[b_2-a_2\cdot\left(\varphi\cdot(y-y_0)-\dfrac{f_{ct}}{E_c}\cdot s-\dfrac{N}{A\cdot E}\right)\right] \\ \text{if } \left(w_1+\dfrac{f_{ct}}{E_c}\cdot s+\dfrac{N}{A\cdot E}\right)\cdot\dfrac{1}{\varphi}+y_0\le y\le\left(w_c+\dfrac{f_{ct}}{E_c}\cdot s+\dfrac{N}{A\cdot E}\right)\cdot\dfrac{1}{\varphi}+y_0 \\ 0 \qquad \text{if } y>\left(w_c+\dfrac{f_{ct}}{E_c}\cdot s+\dfrac{N}{A\cdot E}\right)\cdot\dfrac{1}{\varphi}+y_0 \end{cases} \tag{C.31}$$

The equation of the steel stress can be considered the same than in the σ–ε approach, but the difference of the strain of the steel is not an input. As it was explained in C.1, the steel stress equation is:

$$\begin{aligned} &\sigma_s(\varepsilon_s)=E_s\varepsilon_s && \text{if } \varepsilon_s<\frac{f_{yd}}{E_s} \\ &\sigma_s(\varepsilon_s)=\frac{f_{yd}\cdot(k-1)}{\left(\varepsilon_{suk}-\dfrac{f_{yd}}{E_s}\right)}\cdot\left(\varepsilon_s-\frac{f_{yd}}{E_s}\right)+f_{yd} && \text{if } \frac{f_{yd}}{E_s}\le\varepsilon_s\le\varepsilon_{sud} \\ &\sigma_s(\varepsilon_s)=0 && \text{if } \varepsilon_{sud}<\varepsilon_s \end{aligned} \tag{C.32}$$

When ε_s is calculated using the equation (C.23)

$$\varepsilon(y)=\frac{\varphi}{s}\cdot(y-y_0)\Rightarrow\varepsilon_s=\varepsilon(d)=\frac{\varphi}{s}\cdot(d-y_0) \tag{C.33}$$

Hence the equation is:

$$\begin{aligned} &\sigma_s(\varphi,y_0)=E_s\left(\frac{\varphi}{s}\cdot(d-y_0)\right) && \text{if } \frac{\varphi}{s}\cdot(d-y_0)<\frac{f_{yd}}{E_s} \\ &\sigma_s(\varphi,y_0)=\frac{f_{yd}\cdot(k-1)}{\left(\varepsilon_{suk}-\dfrac{f_{yd}}{E_s}\right)}\cdot\left(\left(\frac{\varphi}{s}\cdot(d-y_0)\right)-\frac{f_{yd}}{E_s}\right)+f_{yd} && \text{if } \frac{f_{yd}}{E_s}\le\frac{\varphi}{s}\cdot(d-y_0)\le\varepsilon_{suk} \\ &\sigma_s(\varphi,y_0)=0 && \text{if } \varepsilon_{suk}<\frac{\varphi}{s}\cdot(d-y_0) \end{aligned} \tag{C.34}$$

Appendix B

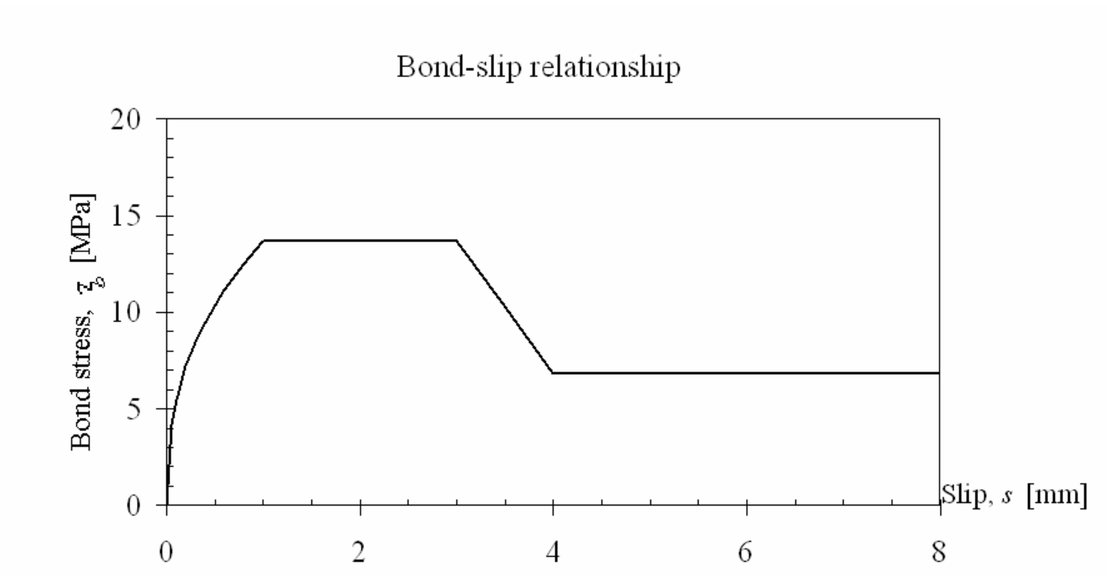
B.1 Bond-Slip Curves

MIX A

D_{stiff} = **8,26E+10**

Concrete Strength:	**30**
Confined Concrete Bond Conditions	Good
S_1	**1,00**
β	**0,40**
τ_{max}	**13,69**
τ_f	**5,48**

s	τ
0,00	**0,00**
0,0500	**4,13**
0,10	**5,45**
0,20	**7,19**
0,30	**8,46**
0,40	**9,49**
0,50	**10,38**
0,60	**11,16**
0,80	**12,52**
1,00	**13,69**
3,00	**13,69**
3,50	**9,59**
4,00	**5,48**
10,00	**5,48**

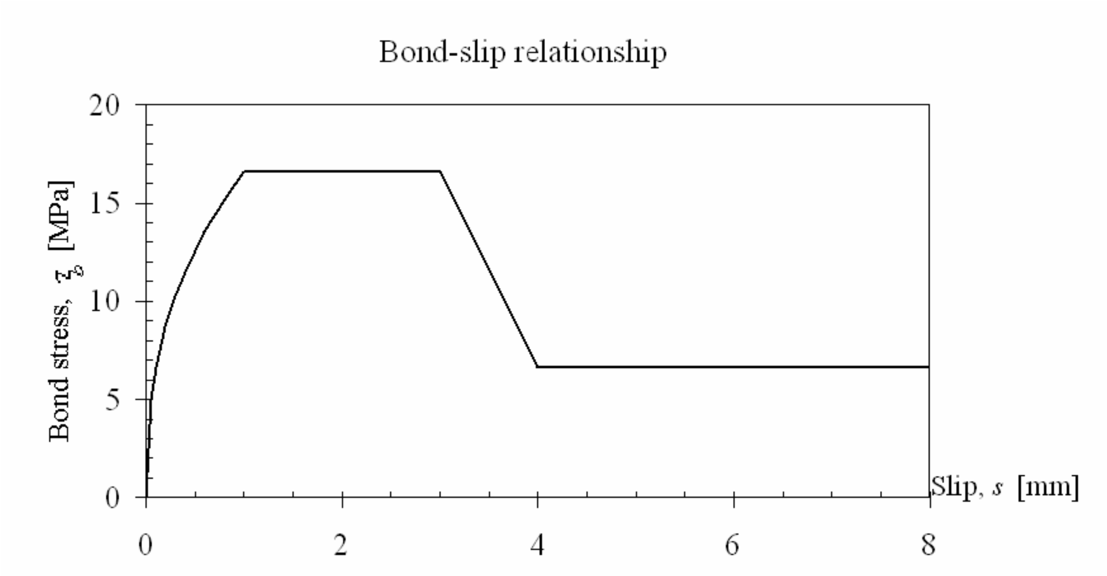


MIX B

D_{stiff} = **1E+11**

Concrete Strength:	**44**
Confined Concrete Bond Conditions	Good
S_1	**1,00**
β	**0,40**
τ_{max}	**16,58**
τ_f	**6,63**

s	τ
0,00	**0,00**
0,0500	**5,00**
0,10	**6,60**
0,20	**8,71**
0,30	**10,25**
0,40	**11,49**
0,50	**12,57**
0,60	**13,52**
0,80	**15,17**
1,00	**16,58**
3,00	**16,58**
3,50	**11,61**
4,00	**6,63**
10,00	**6,63**

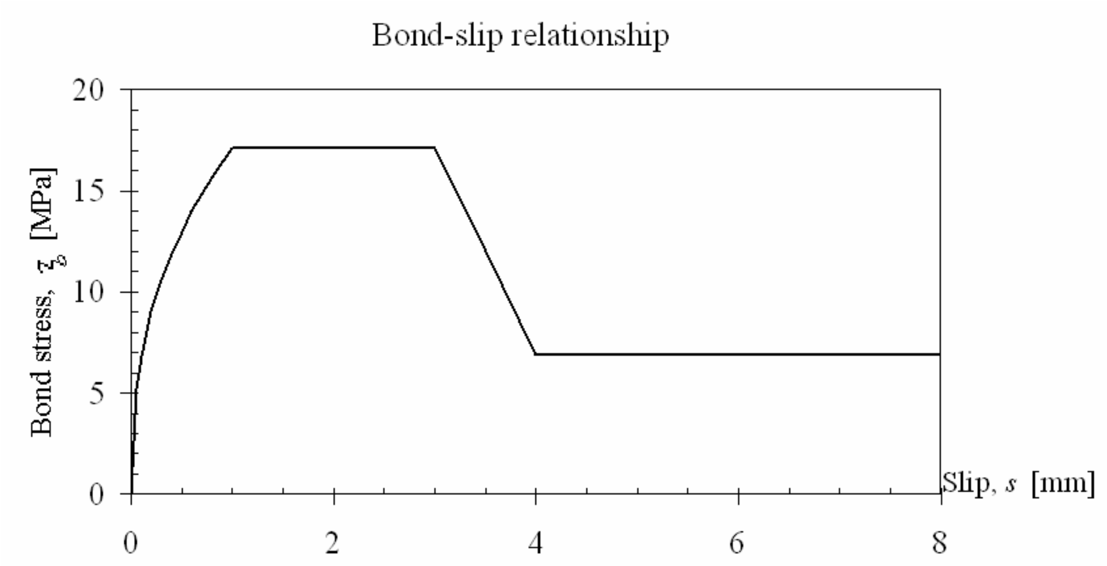


MIX C

D_{stiff} =**1.03E+11**

Concrete Strength:	**47**
Confined Concrete Bond Conditions	Good
S_1	**1,00**
β	**0,40**
τ_{max}	**17,14**
τ_f	**6,86**

s	τ
0,00	**0,00**
0,0500	**5,17**
0,10	**6,82**
0,20	**9,00**
0,30	**10,59**
0,40	**11,88**
0,50	**12,99**
0,60	**13,97**
0,80	**15,68**
1,00	**17,14**
3,00	**17,14**
3,50	**12,00**
4,00	**6,86**
10,00	**6,86**



Appendix C

C.1.1 Sigma-crack opening relationship, analytical analysis. Mix A

MATERIAL PROPERTIES

Reinforcing steel:

Young modulus Steel: $E_s := 200 \cdot GPa$

Ultimate strain: $\varepsilon_{suk} := \frac{50}{1000}$

Yielding strength: $f_{yk} := 500 \cdot MPa$

$k_s := 1.08$

Yielding strain: $\varepsilon_{syk} := \frac{f_{yk}}{E_s}$

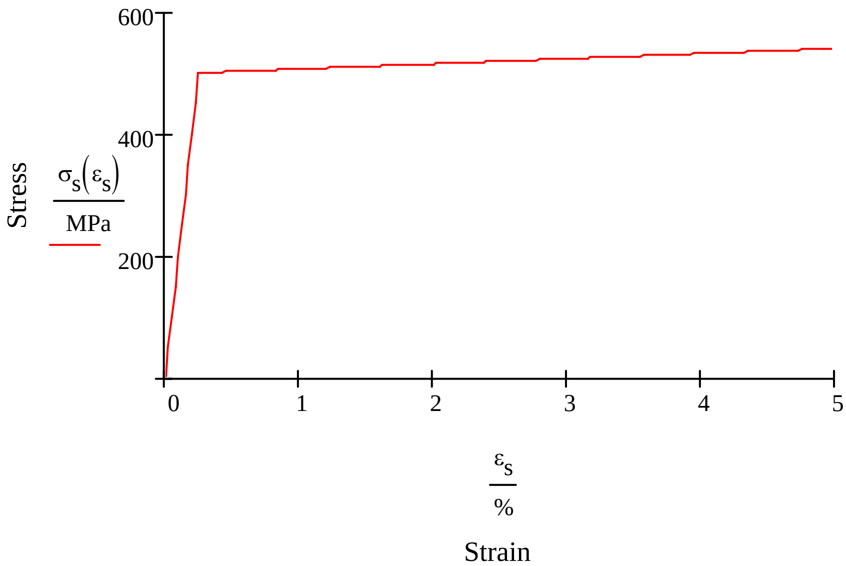
$\varepsilon_{syk} = 2.5 \times 10^{-3}$

Ultimate strength: $f_{uk} := k_s \cdot f_{yk}$

$f_{uk} = 540 MPa$

Reinforcement stress:

$$\sigma_s(\varepsilon_s) := \begin{cases} E_s \cdot \varepsilon_s & \text{if } \varepsilon_s \le \varepsilon_{syk} \\ \dfrac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_s - \dfrac{f_{yk}}{E_s} \right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_s \le \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \varepsilon_s > \varepsilon_{suk} \end{cases}$$



DIANA input strain: $\varepsilon_{diana} := \left(\frac{5}{100} \right) - \frac{f_{yk}}{E_s}$ $\varepsilon_{diana} = 0.048$

Concrete in compression:

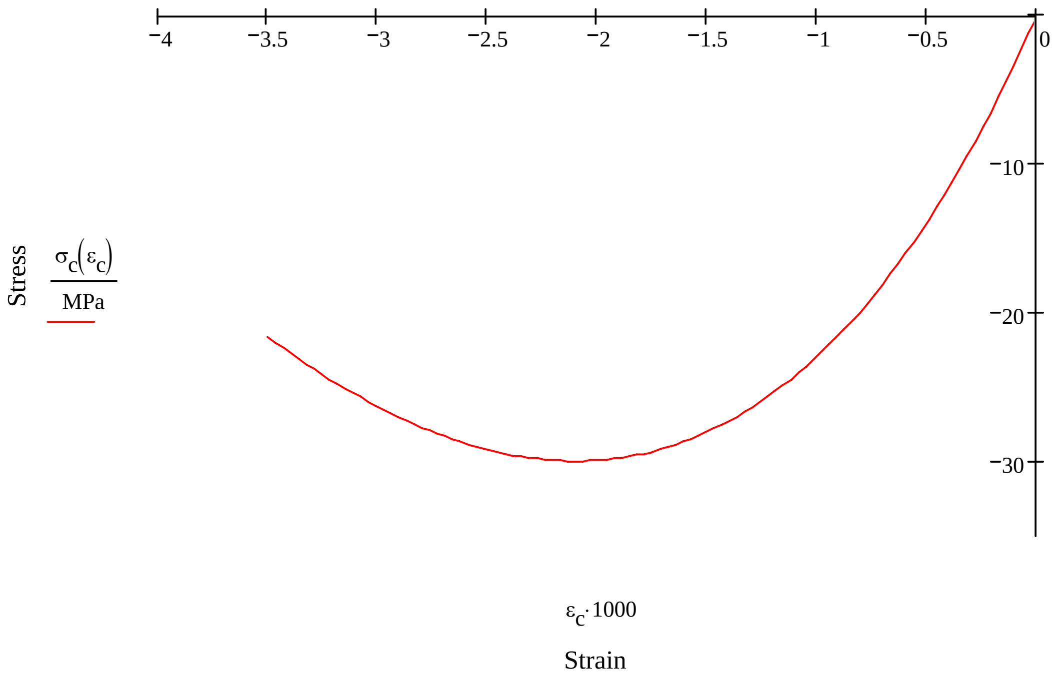
Mean compressive strength: $f_{cm} := 30MPa$

Modulus of Elasticity: $E_c := 22\left(\dfrac{\dfrac{f_{cm}}{MPa}}{10}\right)^{0.3}\cdot GPa$ $\quad E_c = 30.589GPa$

Ultimate strain $\varepsilon_{cu1} := \dfrac{3.5}{1000}$

Stress block factors: $\varepsilon_{c1} := 0.21\cdot\%$ $\quad \eta(\varepsilon_c) := \dfrac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\quad k := 1.1\cdot\dfrac{E_c\cdot|\varepsilon_{c1}|}{f_{cm}}$

Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm}\cdot\dfrac{k\cdot\eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2)\cdot\eta(\varepsilon_c)}$ $\quad \varepsilon_c := 0, \dfrac{-\varepsilon_{cu1}}{100} .. -\varepsilon_{cu1}$



Concrete in tension:

Bi-linear Stress-Crack Opening Relationship MIX A:

Tensional strength $f_{ct} := 2.5 \cdot MPa$

Cracking strain $\varepsilon_{ct.cr} := \frac{f_{ct}}{E_c}$ $\varepsilon_{ct.cr} = 8.173 \times 10^{-5}$

Curve constants:

$a_1 := 10 \cdot \frac{1}{mm}$ $a_2 := 0.065 \frac{1}{mm}$

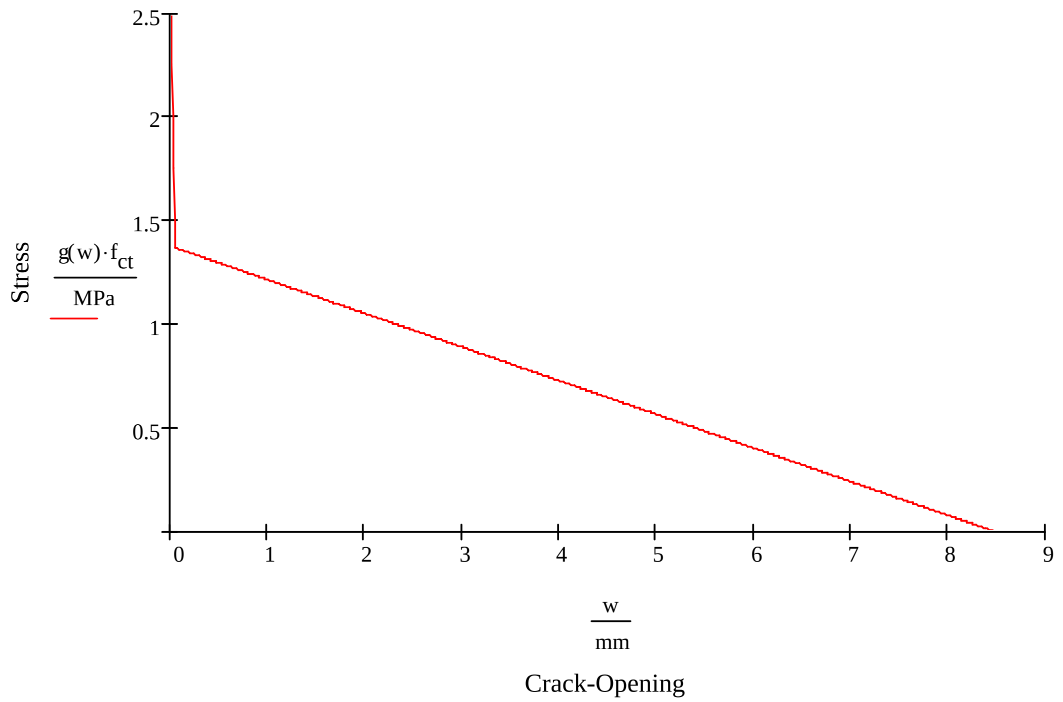
$b_1 := 1$ $b_2 := 0.55$

$w_1 := \frac{b_1 - b_2}{a_1 - a_2}$ $w_1 = 0.045mm$ $w_c := \frac{b_2}{a_2}$

$w_c = 8.4615mm$

$$g(w) := \begin{vmatrix} b_1 - a_1 \cdot w & \text{if } 0 \le w < w_1 \\ b_2 - a_2 \cdot (w) & \text{if } w_1 \le w \le w_c \end{vmatrix}$$

Fracture energy: $G_F := \int_{0 \cdot mm}^{w_c} f_{ct} \cdot g(w)\, dw$ $G_F = 5843 \frac{N \cdot m}{m^2}$



SECTIONAL ANALYSIS

HEIGHT 1.- 125 mm

Height of beam: $h := 125\cdot mm$

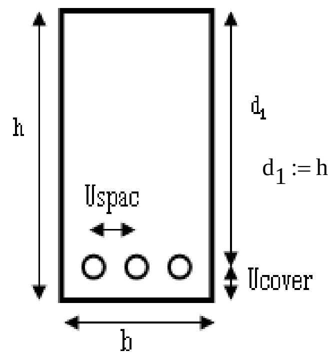
Width of beam: $b := 1000\,mm$

Depth of concrete cover: $u_{cover} := 25\cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150\,mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b\cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$$

Final bar diameter: $\phi_b = 5mm$

Steel one bar Area: $A_{s.i} := \pi\frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n-1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n\cdot A_{s.i}$ $A_s = 1.178\times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2\cdot\pi\cdot\left(\frac{\phi_b}{2}\right)\cdot n$ $perim = 0.094m$

Effective area: $A_{ef} := b\cdot h + \left(\frac{E_s}{E_c}\right)\cdot A_s$ $A_{ef} = 0.126m^2$

Position of effective gravity centre: $x_{ef} := \frac{b\cdot h\cdot\frac{h}{2} + \left(\frac{E_s}{E_c}\right)\cdot A_s\cdot d_1}{A_{ef}}$ $x_{ef} = 62.73mm$

Inertia Moment: $I_{ef} := \frac{b\cdot h^3}{12} + b\cdot h\cdot\left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right)\cdot A_s\cdot\left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63837\times 10^8\,mm^4$

Critical moment (moment just before cracking) $M_{cr} := \frac{I_{ef}\cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 6.578kN\cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 110\,mm$

Critical turn: $\gamma_{cr} := \frac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 1.444 \times 10^{-4}$ Critical curvature: $\kappa_{cr} := \frac{\gamma_{cr}}{s}$

$\kappa_{cr} = 1.3125014 \times 10^{-3} \frac{1}{m}$

Number of steps: $n := 700$ $i := 0..n$

Values of the turn: $\gamma_i := \left(\gamma_{cr} + \frac{\gamma_{cr}}{20}\right) + \frac{\gamma_{cr}}{5} \cdot i$

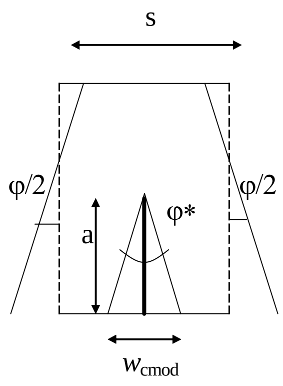
Initial value position of neutral axis: $y_{0ini} := \frac{h}{20}$

Equilibrium equation to find the position of the neutral axis:

$$Y_{0_i} := \mathrm{root}\left[\int_0^{y_{0ini}} \left[-f_{cm} \cdot \frac{k \cdot \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right] - \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]^2}{1 + (k-2) \cdot \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]}\right] \cdot b\, dy \ldots \right.$$
$$+ \int_{y_{0ini}}^{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}} \left[\frac{\gamma_i}{s} \cdot \left[(y - y_{0ini}) \cdot E_c\right]\right] \cdot b\, dy \ldots$$
$$+ \int_{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}}^{h} \left[f_{ct} \cdot \left| \begin{array}{l} b_1 - a_1 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] \text{ if } \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ b_2 - a_2 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] \text{ if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ 0 \text{ if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \end{array}\right.\right] \cdot b\, dy \ldots$$
$$\left. + A_s \cdot \left| \begin{array}{l} E_s \cdot \frac{\gamma_i}{s} \cdot (d_1 - y_{0ini}) \text{ if } \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} - \frac{f_{yk}}{E_s}\right] + f_{yk} \text{ if } \varepsilon_{syk} < \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} < \varepsilon_{suk} \\ 0 \cdot MPa \text{ if } \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} > \varepsilon_{suk} \end{array}\right. , y_{0ini}\right]$$

Crack extension:

$$a_i := h - \left(\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i} + Y_{0_i}\right)$$



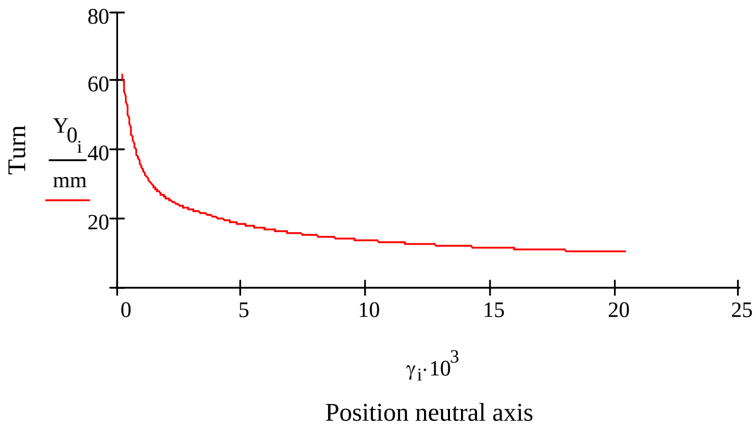
Maximum crack opening:

$$w_{CMOD_i} := \begin{cases} \gamma_i\cdot\left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } 0 \le \gamma_i\cdot\left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s \le w_c \\ \frac{\left(h - Y_{0_i}\right)\cdot\gamma_i}{1} & \text{if } \frac{\left(h - Y_{0_i}\right)\cdot\gamma_i}{1} > w_c \end{cases}$$

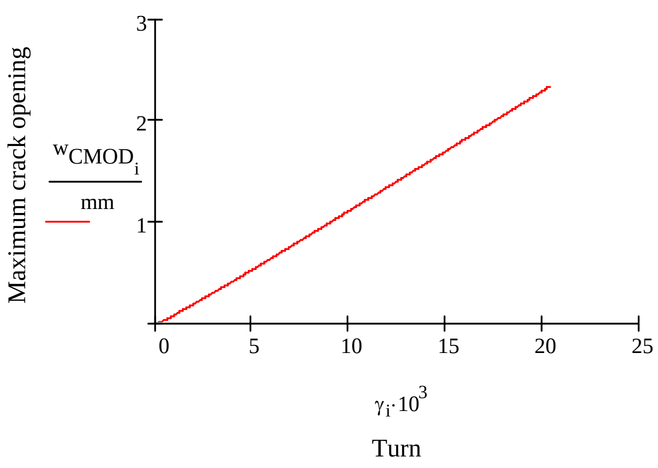
General expression for the crack opening:

$$w(i,y) := \begin{cases} \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s < 0\text{mm} \\ \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } 0\text{mm} \le \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s < w_c \\ \frac{\left(y - Y_{0_i}\right)\cdot\gamma_i}{1} & \text{if } \frac{\left(y - Y_{0_i}\right)\cdot\gamma_i}{1} > w_c \end{cases}$$

Position of the neutral axis when turn is increasing:



Maximum crack opening when turn is increasing:



Stress and Strain STEEL:

Strain in reinforcement steel:

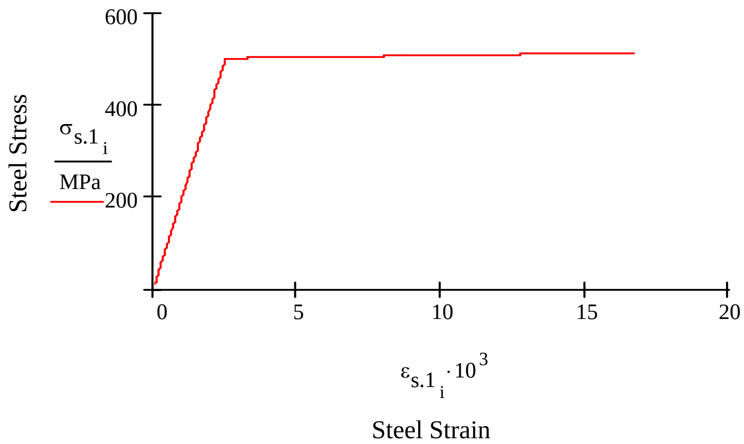
Bottom steel

$$\varepsilon_{s.1_i} := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - d_1}{Y_{0_i}}$$

Stress in reinforcement steel :

Bottom steel

$$\sigma_{s.1_i} := \left| \begin{array}{ll} E_s \cdot \frac{\gamma_i}{s} \cdot \left(d_1 - Y_{0_i}\right) & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} - \frac{f_{yk}}{E_s} \right] + f_{yk} & \text{if } \varepsilon_{syk} < \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} < \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} > \varepsilon_{suk} \end{array} \right.$$



Stress and Strain CONCRETE:

Concrete strain:

$$\varepsilon_{cc}(i, y) := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - y}{Y_{0_i}}$$

Concrete stress:

Concrete in compresion:

$$\sigma_{cc}(i, y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right] - \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right]^2}{1 + (k - 2) \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right]}$$

Concrete in elastic behaviour:

$$\sigma_{ct}(i,y) := \left[\frac{\gamma_i}{s}\cdot\left[\left(y - Y_{0_i}\right)\cdot E_c\right]\right]$$

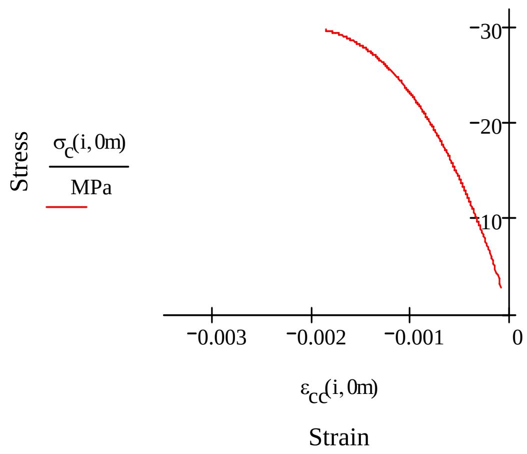
Cracked concrete:

$$\sigma_{cf}(i,y) := \left[f_{ct}\cdot \left| \begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 \text{ if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{array} \right. \right]$$

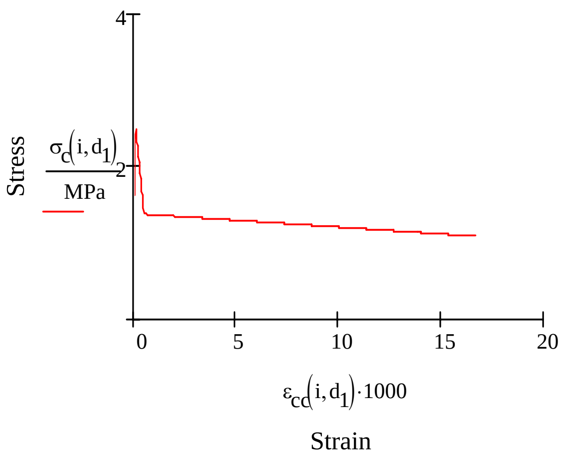
Final expression:

$$\sigma_c(i,y) := \left| \begin{array}{l} \sigma_{cc}(i,y) \text{ if } 0mm \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) \text{ if } Y_{0_i} < y \le \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} \\ \sigma_{cf}(i,y) \text{ if } \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} < y \le h \end{array} \right.$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium: $F_{cc} + F_{ft} + F_{ct} + F_s = 0$

Steel force: $F_{s_i} := A_s\cdot\sigma_{s.1_i}$

Concrete in compression force:

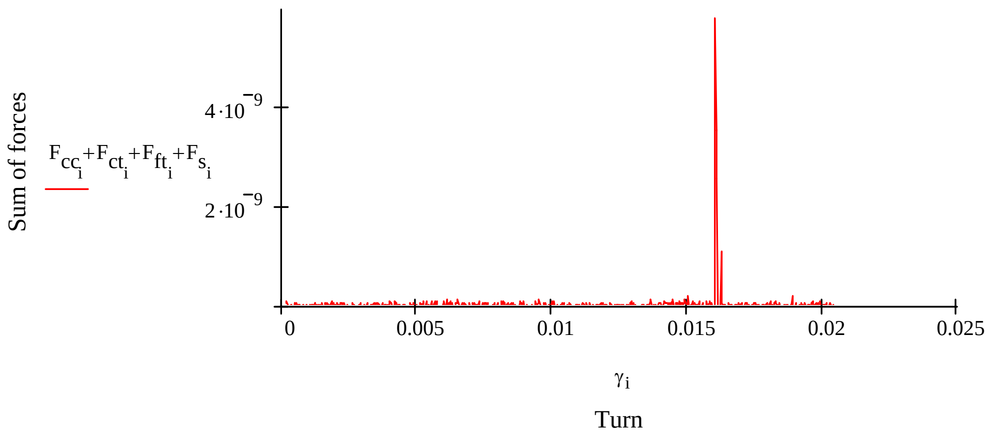
$$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right] - \eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]^2}{1 + (k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]} \right]\cdot b\,dy$$

Concrete in tension force (elastic zone):

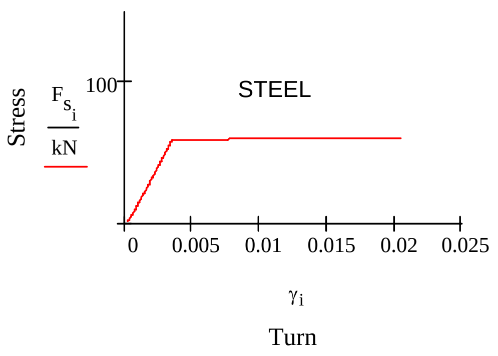
$$F_{ct_i} := \int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}} \left[\frac{\gamma_i}{s}\cdot\left(y - Y_{0_i}\right)\cdot E_c\right]\cdot b\,dy$$

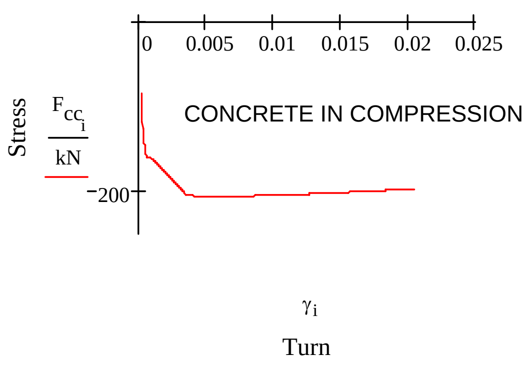
Cracked concrete force:

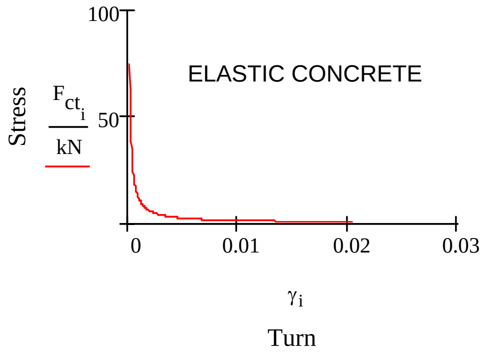
$$F_{ft_i} := \int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h} \left[f_{ct}\cdot \begin{cases} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 & \text{if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{cases} \right]\cdot b\,dy$$

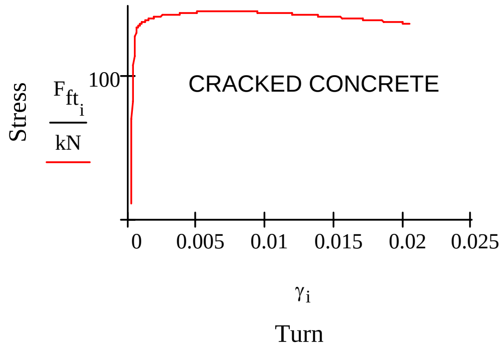


Graphs of forces:









Moment :

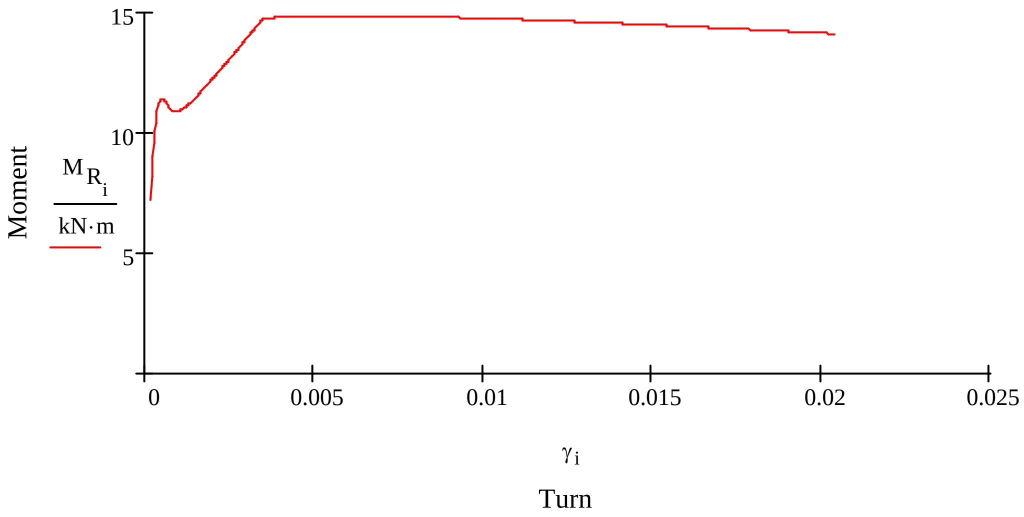
$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right] - \eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]^2}{1+(k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]}\right]\cdot b\cdot(y)\,dy\ ...$$

$$+ \int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}} \left[\frac{\gamma_i}{s}\cdot\left(y-Y_{0_i}\right)\cdot E_c\right]\cdot b\cdot y\,dy\ ...$$

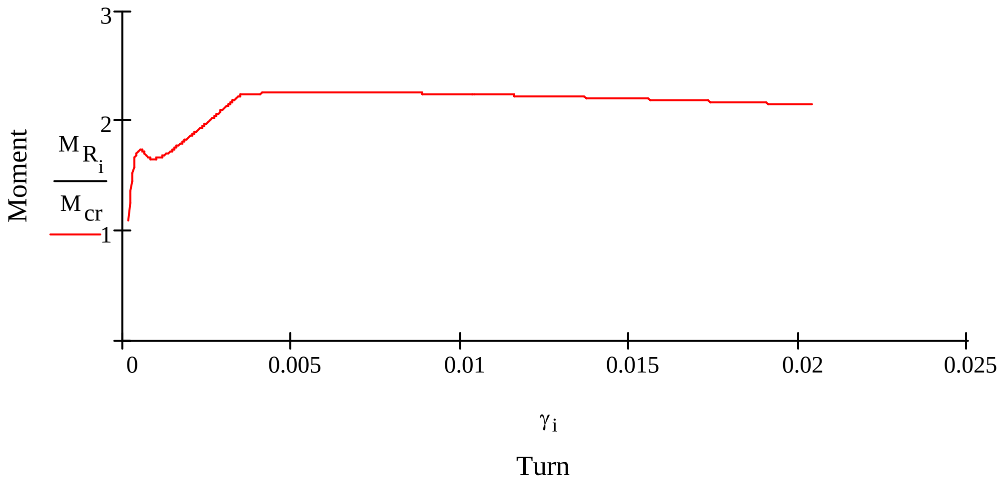
$$+ \int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h} \left[f_{ct}\cdot \left| \begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] \ \text{ if } \ \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] \ \text{ if } \ \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 \ \text{ if } \ y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{array}\right.\right]\cdot b\cdot y\,dy\ ...$$

$$+ F_{s_i}\cdot\left(d_1\right)$$

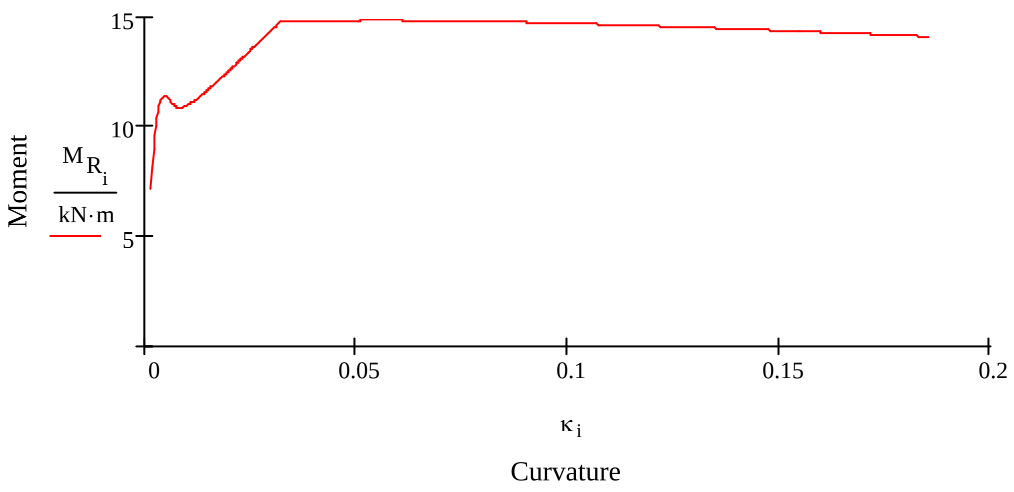
MOMENT-TURN GRAPH



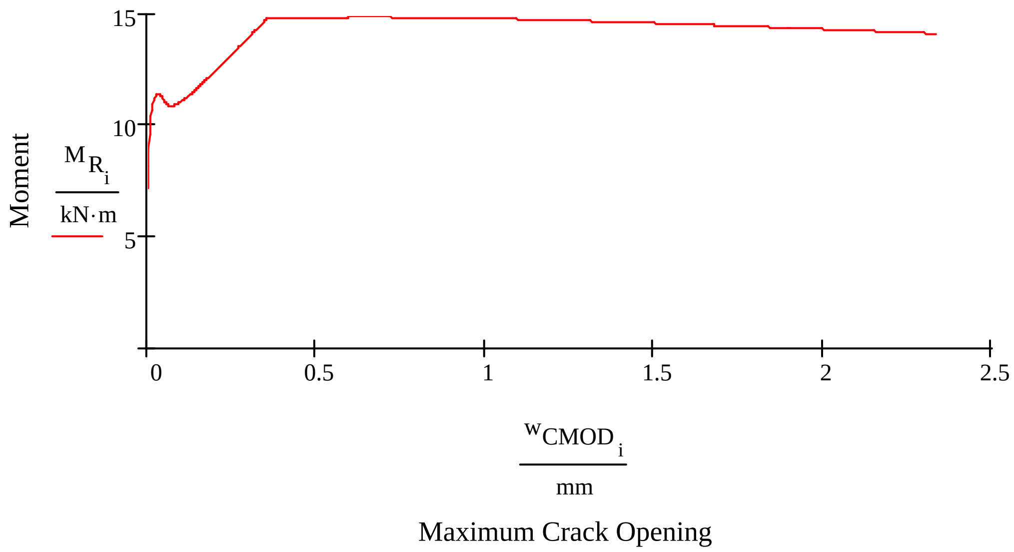
NORMALISED MOMENT-TURN GRAPH



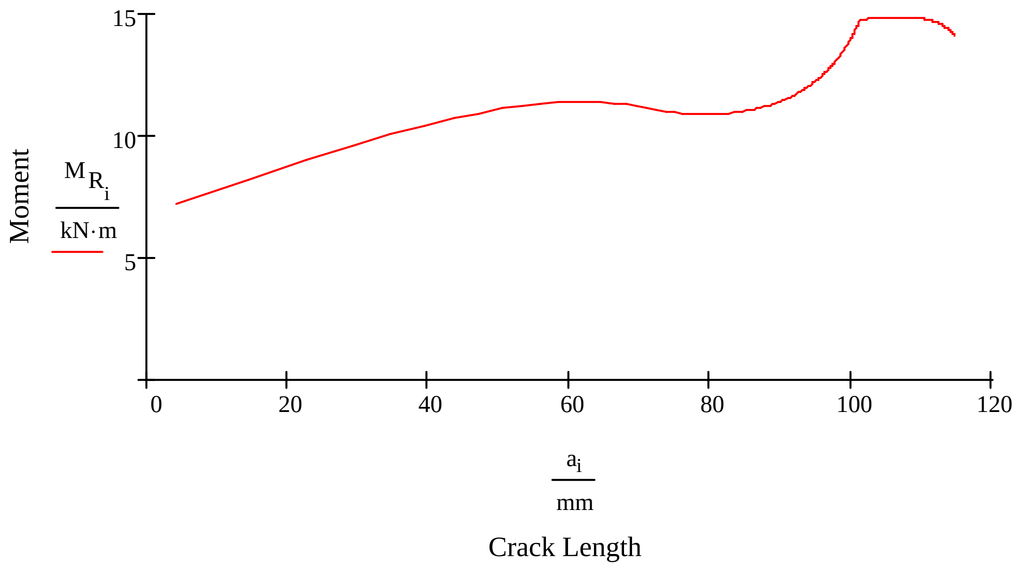
MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\gamma_i}{s}$



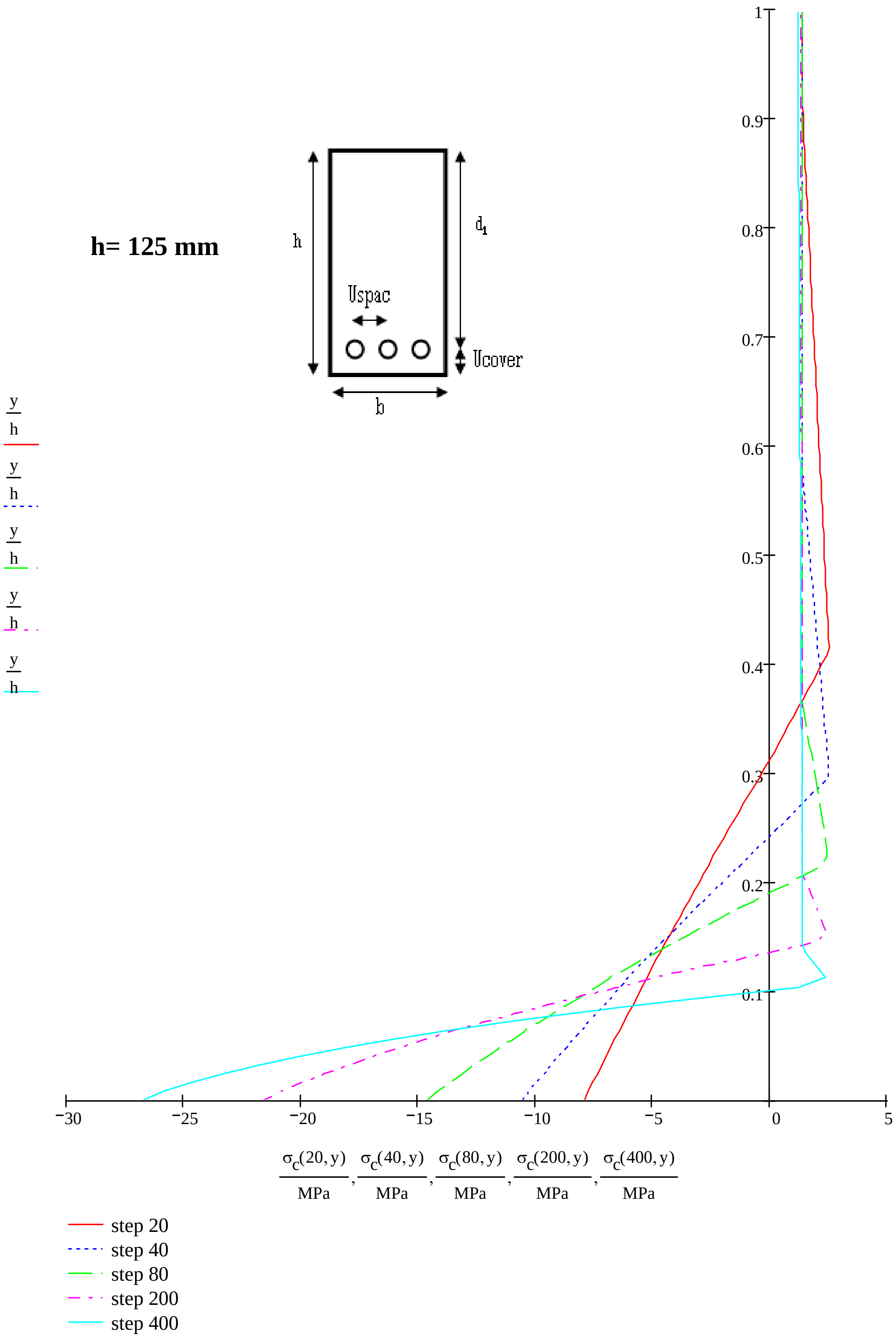
MOMENT-MAXIMUM OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



Stress Diagram of the cross section:



SECTIONAL ANALYSIS

HEIGHT 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

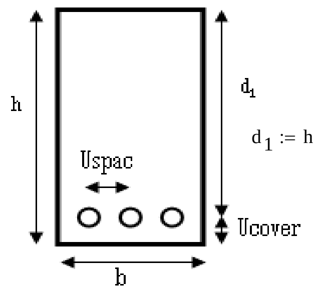
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \quad \phi_{bap} = 6.588\ mm \quad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 7\,mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 181.6\ mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 2.309 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.132\ m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.252\ m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 125.6\ mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.31709 \times 10^9\ mm^4$

Critical moment (moment just before cracking) $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 26.469\ kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 135mm$

Critical turn: $\gamma_{cr} := \frac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 8.869 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \frac{\gamma_{cr}}{s}$

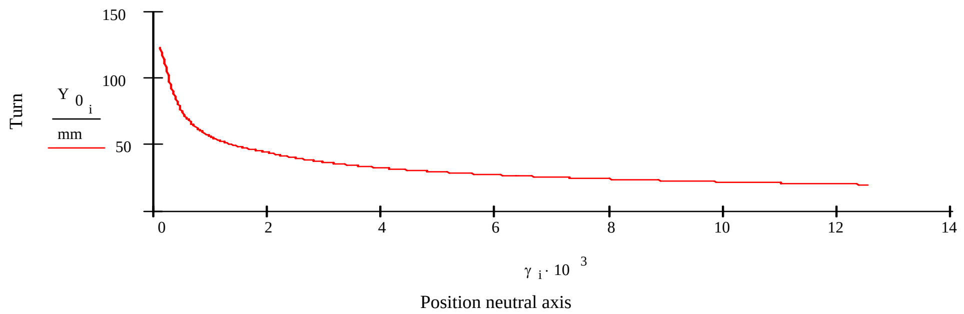
$\kappa_{cr} = 6.5699422 \times 10^{-4}\ \frac{1}{m}$

Number of steps: $n := 700$ $i := 0 .. n$

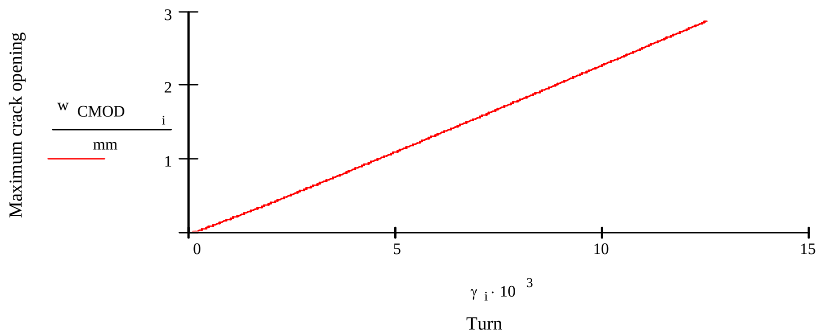
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \frac{\gamma_{cr}}{20}\right) + \frac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{20}$

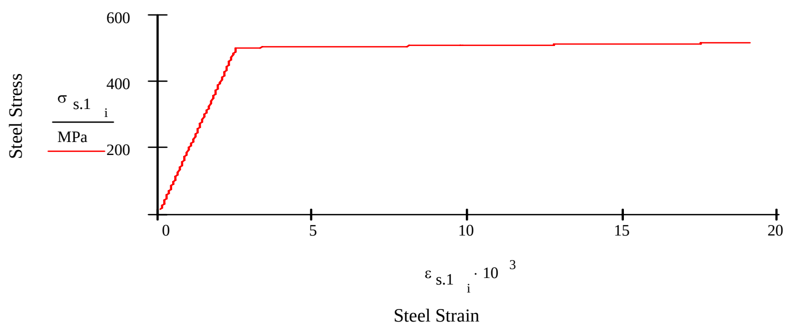
Position of the neutral axis when turn is increasing:



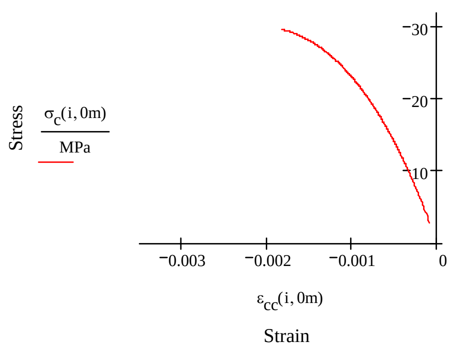
Maximum crack opening when turn is increasing:



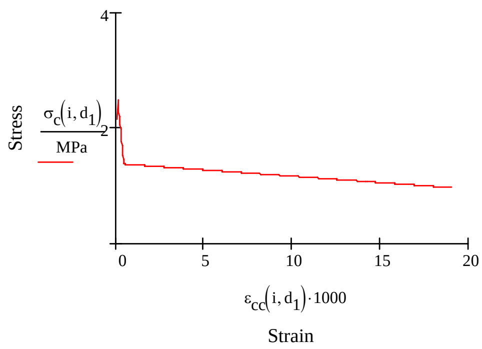
Stress Strain Reinforcement Diagram:



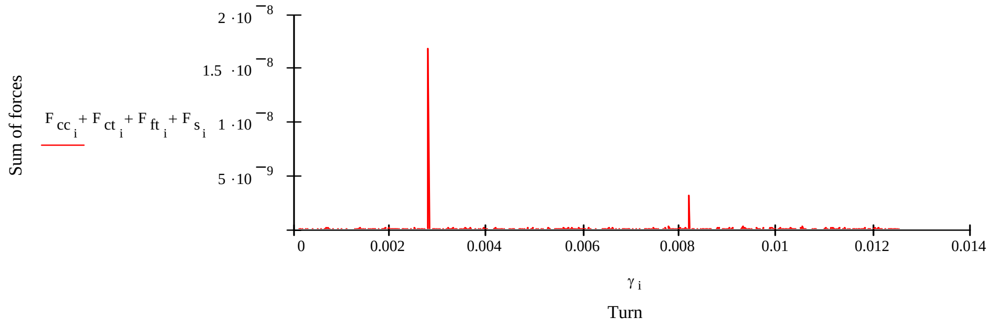
Stress-Strain diagram of the top concrete:



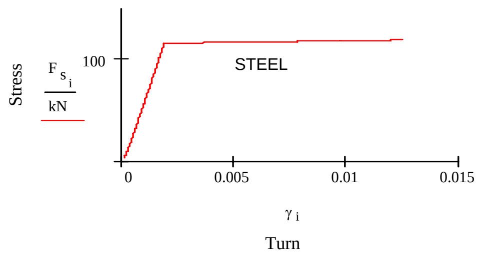
Stress-Strain relationship at the level of reinforcement:

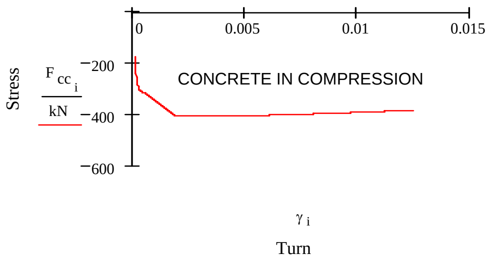


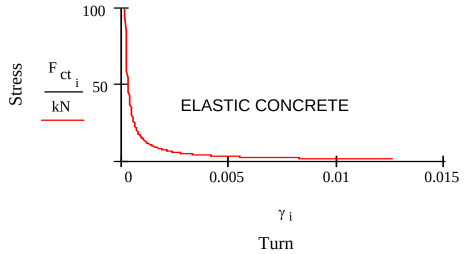
SUM OF FORCES=0 GRAPH

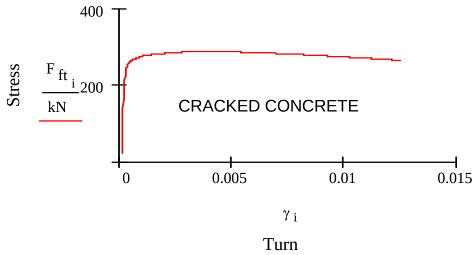


GRAPHS OF FORCES

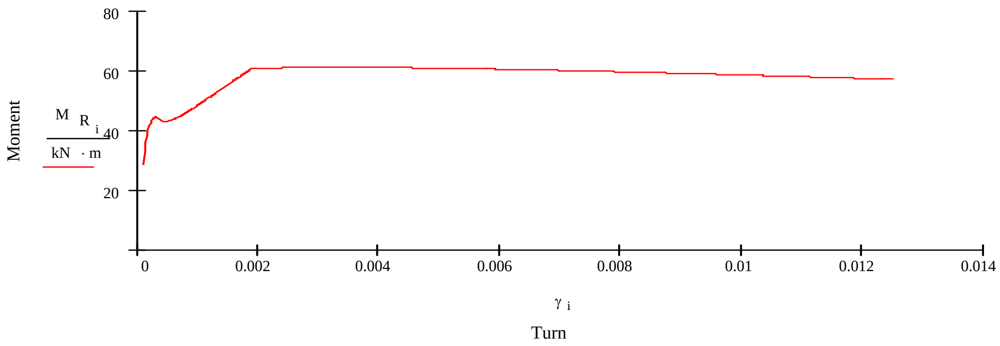




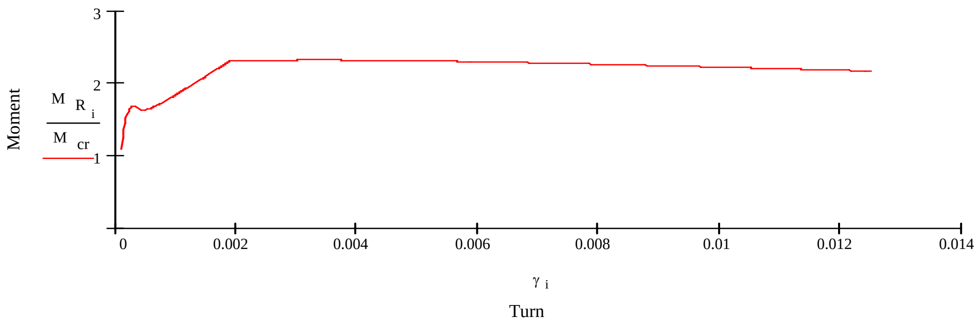




MOMENT-TURN GRAPH

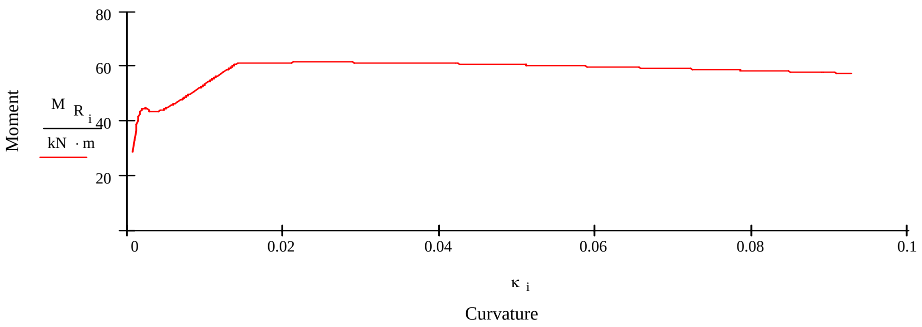


NORMALISED MOMENT-TURN GRAPH

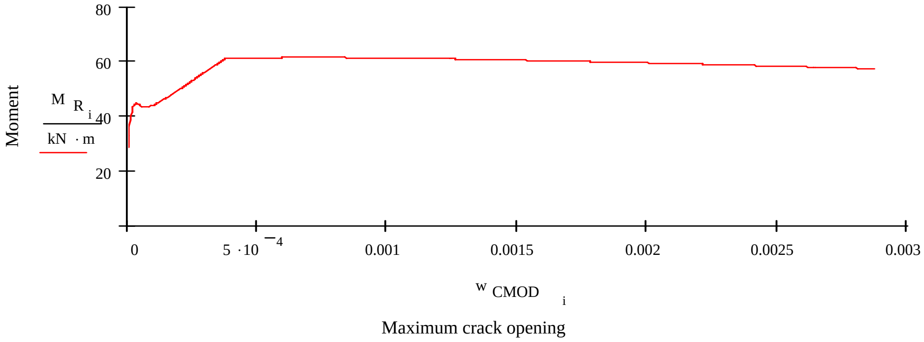


MOMENT-CURVATURE GRAPH

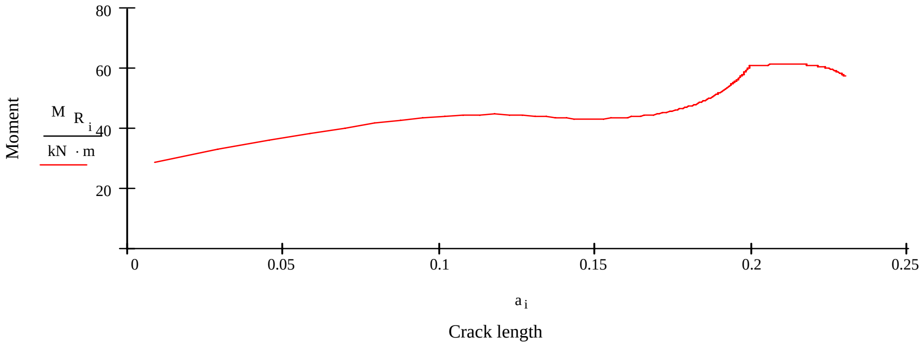
$$\kappa_i := \frac{\gamma_i}{s}$$



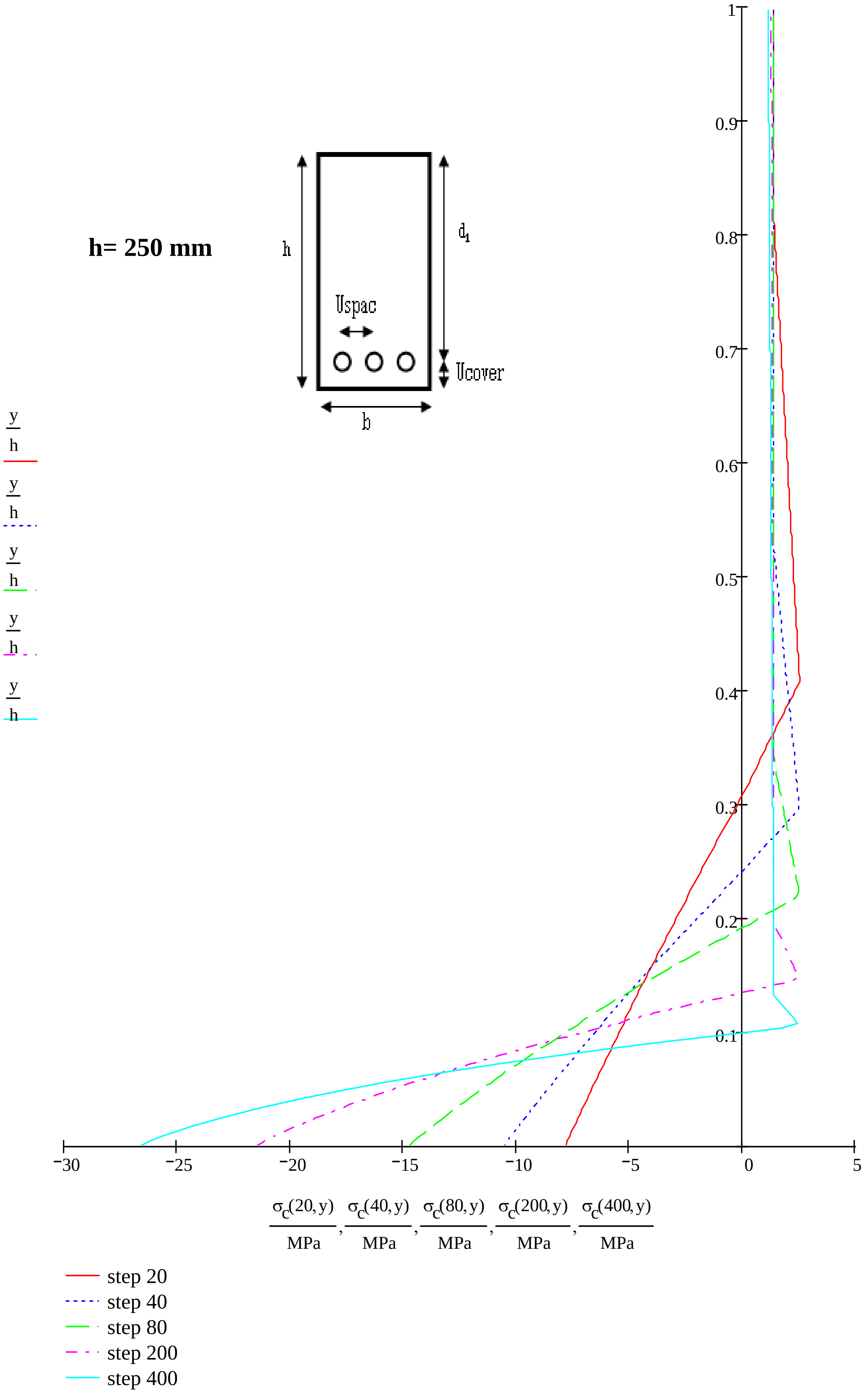
MOMENT-MAXIMUM OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS HEIGHT 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

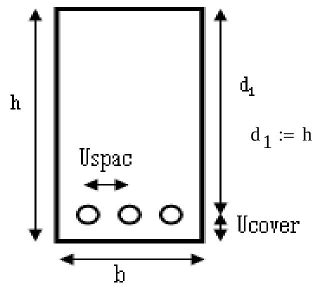
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317 \text{ mm} \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 9\,mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 8$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 125.429 \text{ mm}$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4} \, m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.226 \text{ m}$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.503 \, m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 251.488 \text{ mm}$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.0584 \times 10^{10} \, mm^4$

Critical moment (moment just before cracking) $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 106.474 \, kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 105mm$

Critical turn: $\gamma_{cr} := \dfrac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 3.453 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \dfrac{\gamma_{cr}}{s}$

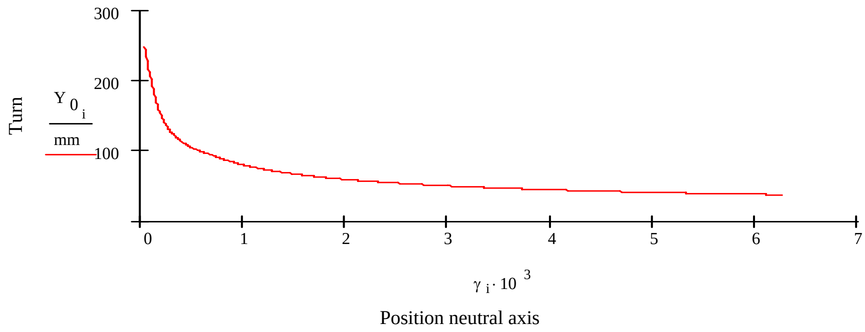
$\kappa_{cr} = 3.2887645 \times 10^{-4} \, \dfrac{1}{m}$

Number of steps: $n := 700$ $i := 0 .. n$

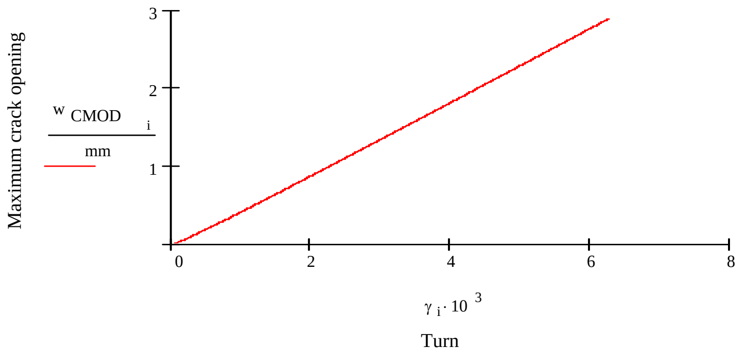
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \dfrac{\gamma_{cr}}{20}\right) + \dfrac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{20}$

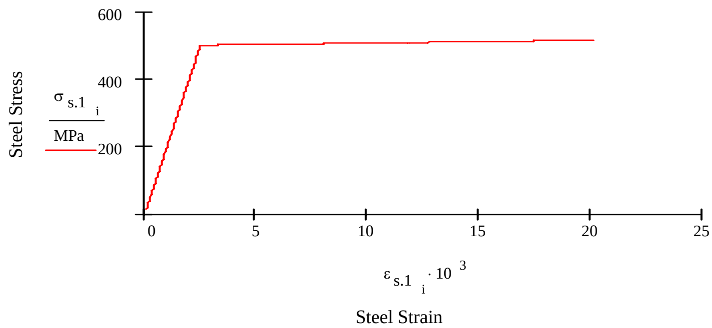
Position of the neutral axis when turn is increasing:



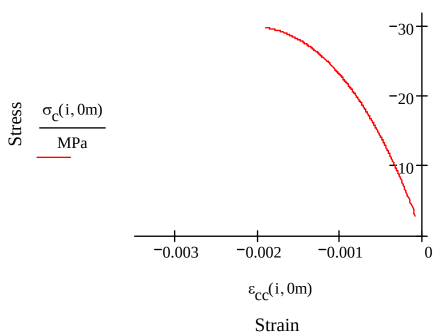
Maximum crack opening when turn is increasing:



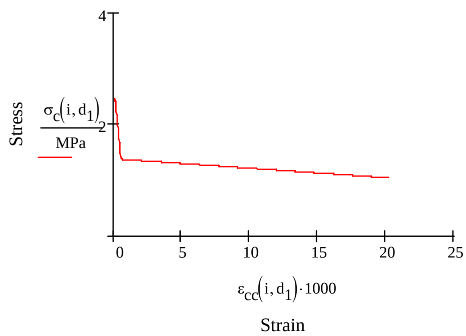
Stress Strain Reinforcement Diagram:



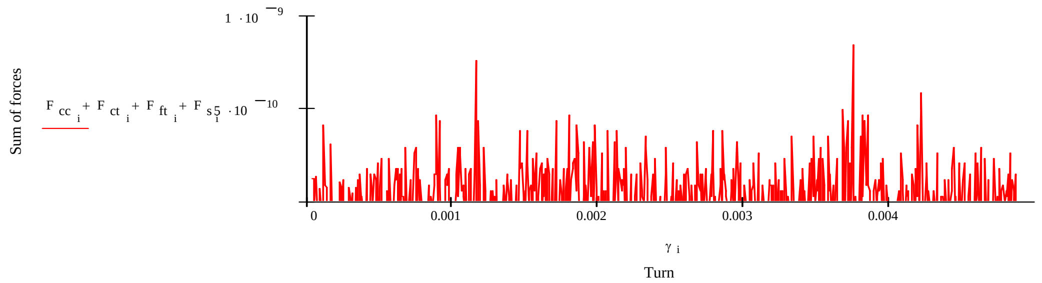
Stress-Strain diagram of the top concrete:



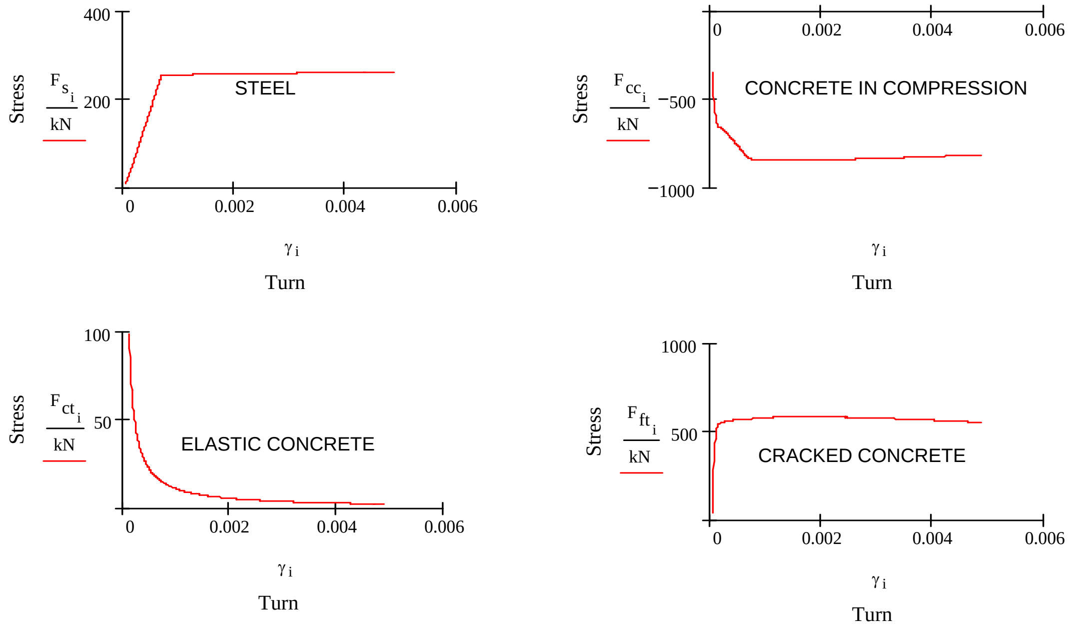
Stress-Strain relationship at the level of reinforcement:



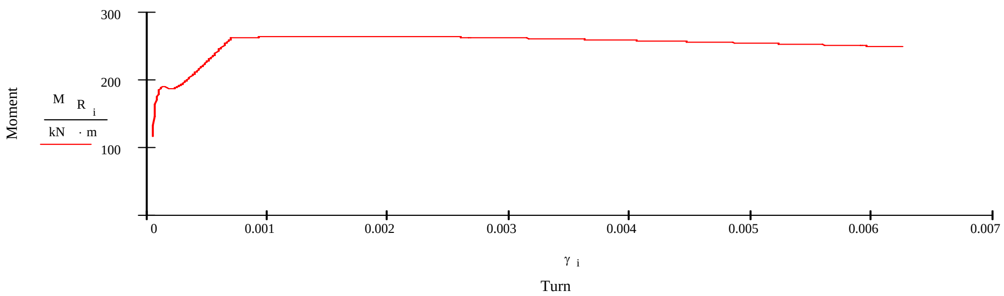
SUM OF FORCES=0 GRAPH



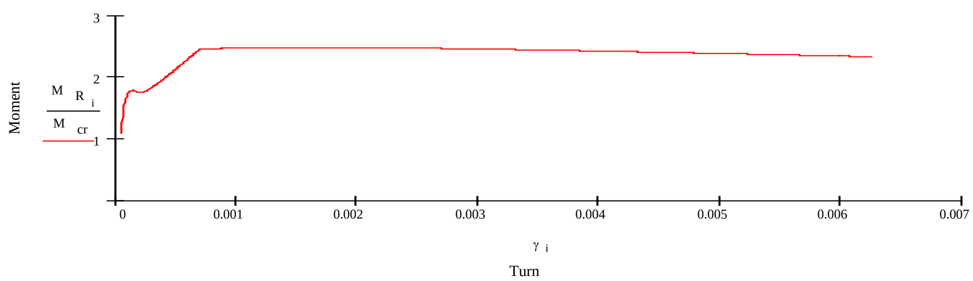
GRAPHS OF FORCES



MOMENT-TURN GRAPH

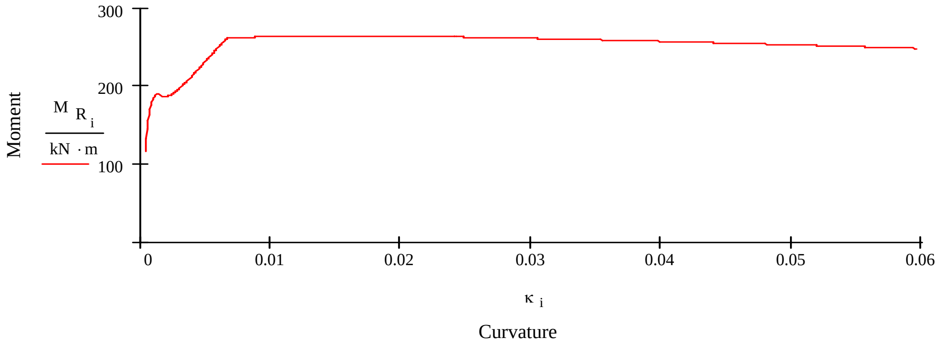


NORMALISED MOMENT-TURN GRAPH

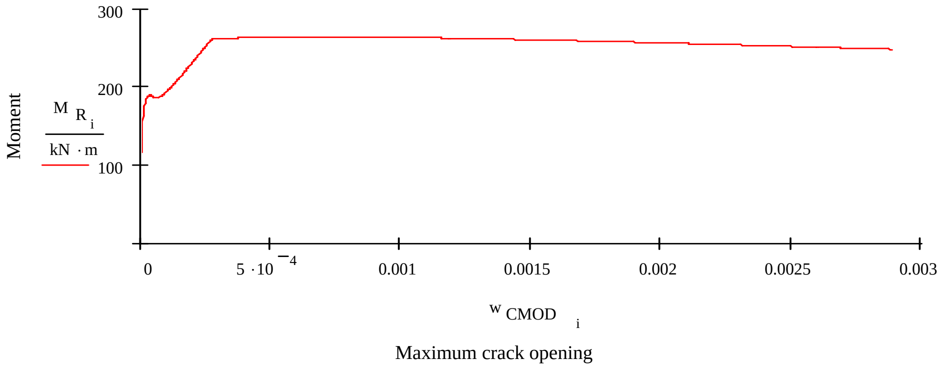


MOMENT-CURVATURE GRAPH

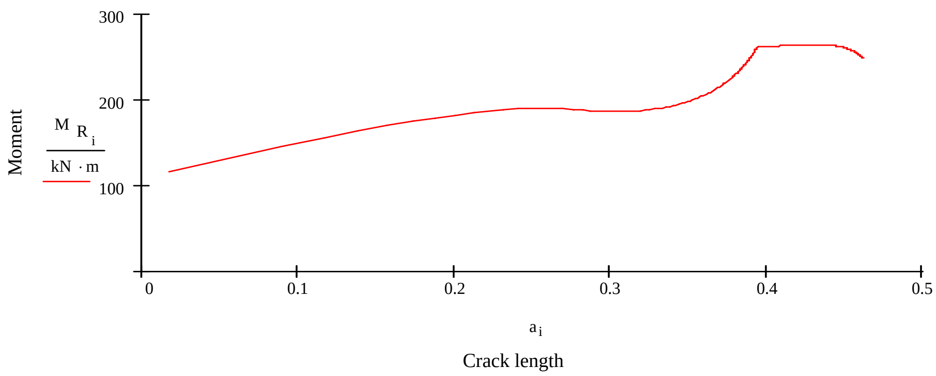
$$\kappa_i := \frac{\gamma_i}{s}$$



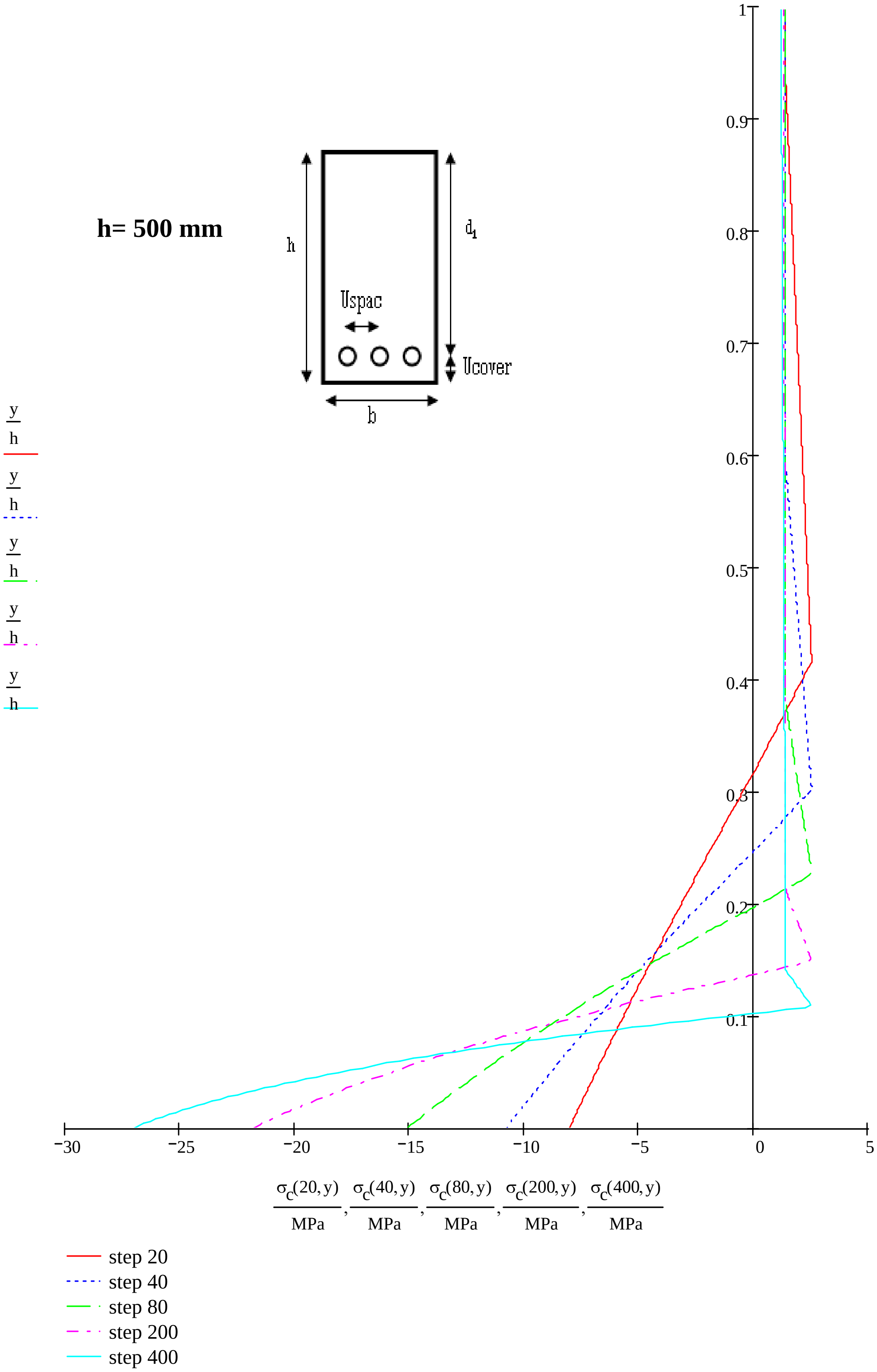
MOMENT-MAXIMUM OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



C.1.2 Sigma-crack opening relationship, analytical analysis. Mix B

MATERIAL PROPERTIES

Concrete in compression:

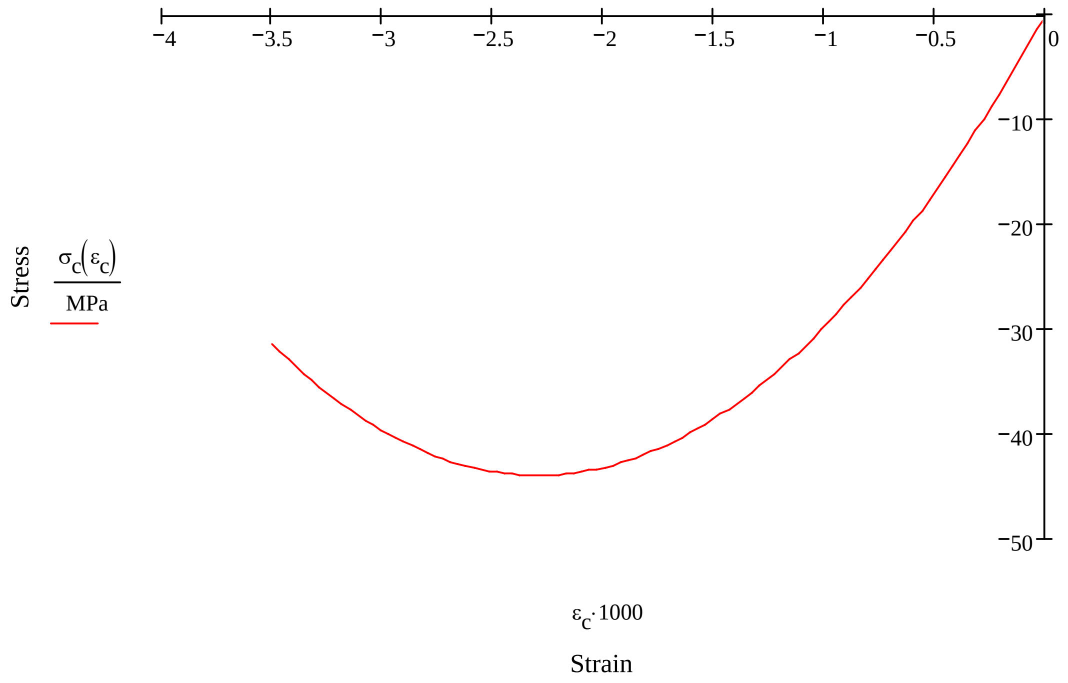
Mean compressive strength: $f_{cm} := 44\text{MPa}$

Modulus of Elasticity: $E_c := 22\left(\dfrac{\frac{f_{cm}}{\text{MPa}}}{10}\right)^{0.3}\cdot\text{GPa}$ $\quad E_c = 34.313\text{GPa}$

Ultimate strain $\varepsilon_{cu} := \dfrac{3.5}{1000}$

Stress block factors: $\varepsilon_{c1} := 0.23\%$ $\quad \eta(\varepsilon_c) := \dfrac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\quad k := 1.1\cdot\dfrac{E_c\cdot|\varepsilon_{c1}|}{f_{cm}}$

Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm}\cdot\dfrac{k\cdot\eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2)\cdot\eta(\varepsilon_c)}$ $\quad \varepsilon_c := 0, \dfrac{-\varepsilon_{cu}}{100} .. -\varepsilon_{cu}$



Concrete in tension:

Bi-linear Stress-Crack Opening Relationship MIX B:

Tensional strength $f_{ct} := 3.5 \cdot MPa$

Cracking strain $\varepsilon_{ct.cr} := \frac{f_{ct}}{E_c}$ $\varepsilon_{ct.cr} = 1.02 \times 10^{-4}$

Curve constants:

$a_1 := 16 \cdot \frac{1}{mm}$ $a_2 := 0.14 \cdot \frac{1}{mm}$

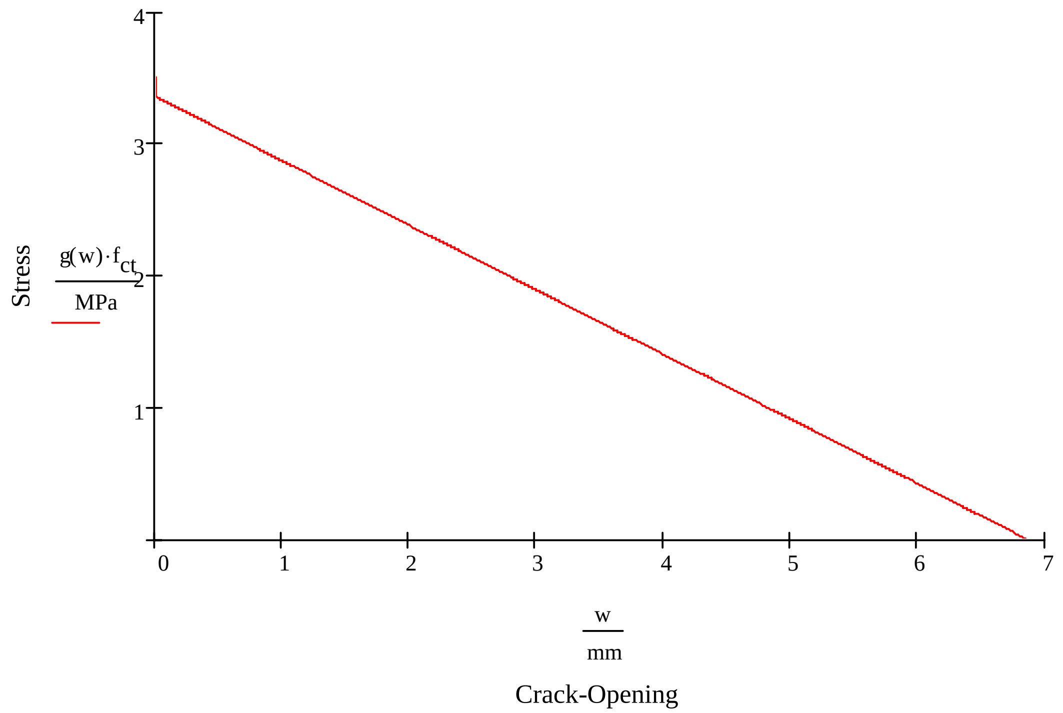
$b_1 := 1$ $b_2 := 0.96$

$w_1 := \frac{b_1 - b_2}{a_1 - a_2}$ $w_1 = 2.522 \times 10^{-3} mm$ $w_c := \frac{b_2}{a_2}$

$w_c = 6.8571mm$

$$g(w) := \begin{cases} b_1 - a_1 \cdot w & \text{if } 0 \le w < w_1 \\ b_2 - a_2 \cdot (w) & \text{if } w_1 \le w \le w_c \end{cases}$$

Fracture energy: $G_F := \int_{0 \cdot mm}^{w_c} f_{ct} \cdot g(w)\, dw$ $G_F = 11520 \frac{N \cdot m}{m^2}$



SECTIONAL ANALYSIS

HEIGHT 1.- 125 mm

Height of beam: $h := 125 \cdot mm$

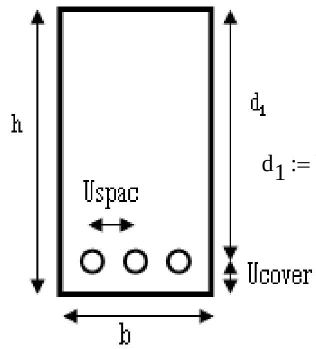
Width of beam: $b := 1000\,mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 5mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 1.178 \times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.094\,m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.126\,m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 62.705mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63721 \times 10^8\,mm^4$

Critical moment (moment just before cracking) $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 9.199kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 55mm$

Critical turn: $\gamma_{cr} := \frac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 9.006 \times 10^{-5}$

Critical curvature: $\kappa_{cr} := \frac{\gamma_{cr}}{s}$

$\kappa_{cr} = 1.6374028 \times 10^{-3} \frac{1}{m}$

Number of steps: $n := 700$ $i := 0..n$

Values of the turn: $\gamma_i := \left(\gamma_{cr} + \frac{\gamma_{cr}}{20}\right) + \frac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{20}$

Equilibrium equation to find the position of the neutral axis:

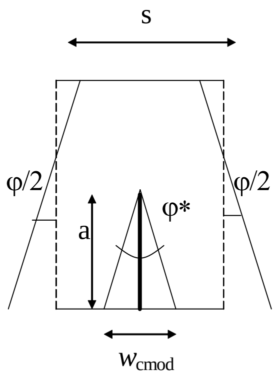
$$Y_{0_i} := \mathrm{root}\left[\int_0^{y_{0ini}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right] - \eta \left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]^2}{1 + (k-2) \cdot \eta \left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]} \right] \cdot b\, dy \ldots \right.$$

$$+ \int_{y_{0ini}}^{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}} \left[\frac{\gamma_i}{s} \cdot \left[(y - y_{0ini}) \cdot E_c\right]\right] \cdot b\, dy \ldots$$

$$+ \int_{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}}^{h} \left[f_{ct} \cdot \begin{cases} b_1 - a_1 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] & \text{if } \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ b_2 - a_2 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] & \text{if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ 0 & \text{if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \end{cases} \right] \cdot b\, dy \ldots$$

$$\left. + A_s \cdot \begin{cases} E_s \cdot \frac{\gamma_i}{s} \cdot (d_1 - y_{0ini}) & \text{if } \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} - \frac{f_{yk}}{E_s}\right] + f_{yk} & \text{if } \varepsilon_{syk} < \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} < \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} > \varepsilon_{suk} \end{cases} \right., y_{0ini}\right]$$

Crack extension: $a_i := h - \left(\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + Y_{0_i} \right)$



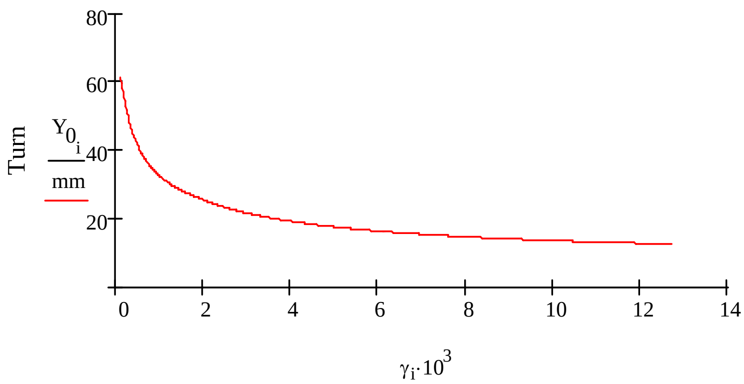
Maximum crack opening:

$$w_{CMOD_i} := \begin{cases} \gamma_i \cdot \left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s & \text{if } 0 \le \gamma_i \cdot \left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s \le w_c \\ \frac{\left(h - Y_{0_i}\right) \cdot \gamma_i}{1} & \text{if } \frac{\left(h - Y_{0_i}\right) \cdot \gamma_i}{1} > w_c \end{cases}$$

General expression for the crack opening:

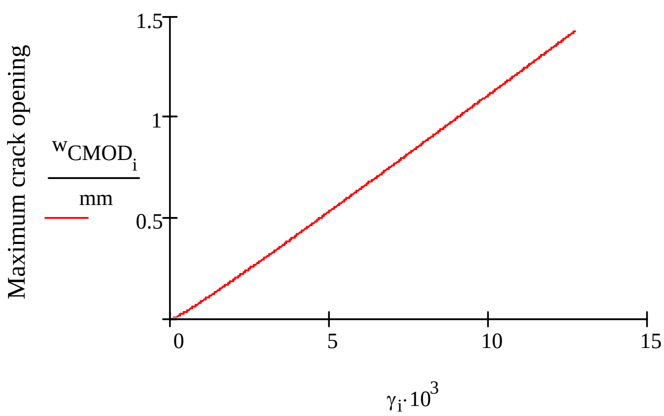
$$w(i,y) := \begin{cases} \gamma_i \cdot \left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s & \text{if } \gamma_i \cdot \left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s < 0\text{mm} \\ \gamma_i \cdot \left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s & \text{if } 0\text{mm} \le \gamma_i \cdot \left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c} \cdot s < w_c \\ \frac{\left(y - Y_{0_i}\right) \cdot \gamma_i}{1} & \text{if } \frac{\left(y - Y_{0_i}\right) \cdot \gamma_i}{1} > w_c \end{cases}$$

Position of the neutral axis when turn is increasing:



Position neutral axis

Maximum crack opening when turn is increasing:



Turn

Stress and Strain STEEL:

Strain in reinforcement steel:

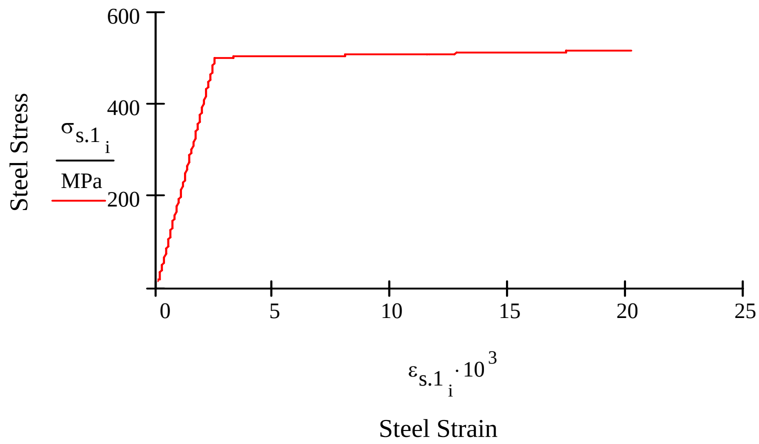
Bottom steel

$$\varepsilon_{s.1_i} := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - d_1}{Y_{0_i}}$$

Stress in reinforcement steel :

Bottom steel

$$\sigma_{s.1_i} := \begin{cases} E_s \cdot \frac{\gamma_i}{s} \cdot \left(d_1 - Y_{0_i}\right) & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} - \frac{f_{yk}}{E_s}\right] + f_{yk} & \text{if } \varepsilon_{syk} < \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} < \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} > \varepsilon_{suk} \end{cases}$$



Stress and Strain CONCRETE:

Concrete strain:

$$\varepsilon_{cc}(i, y) := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - y}{Y_{0_i}}$$

Concrete stress:

Concrete in compresion:

$$\sigma_{cc}(i, y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right)\right] - \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right)\right]^2}{1 + (k - 2) \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right)\right]}$$

Concrete in elastic behaviour: $\sigma_{ct}(i,y) := \frac{\gamma_i}{s}\cdot\left[\left(y - Y_{0_i}\right)\cdot E_c\right]$

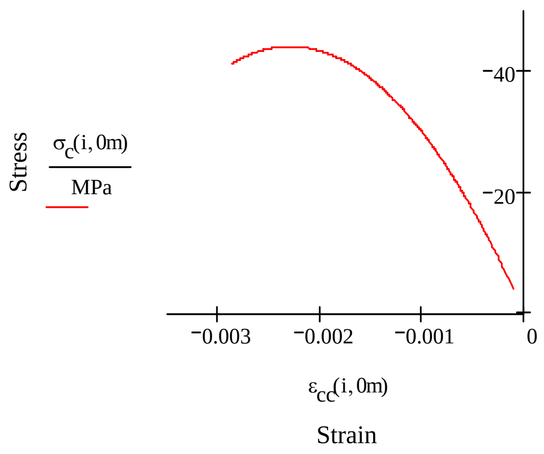
Cracked concrete:

$$\sigma_{cf}(i,y) := \left[f_{ct}\cdot \begin{cases} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 & \text{if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{cases} \right]$$

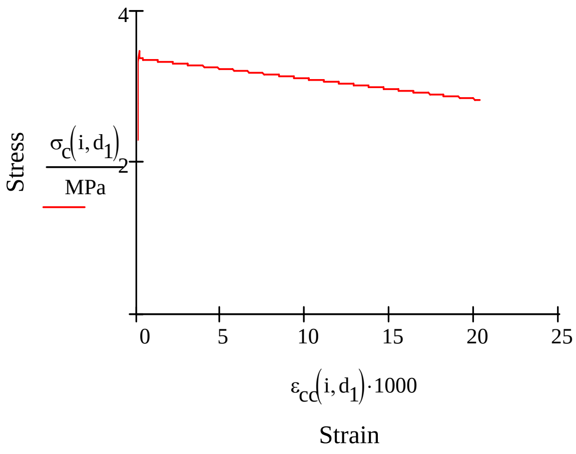
Final expression:

$$\sigma_c(i,y) := \begin{cases} \sigma_{cc}(i,y) & \text{if } 0\text{mm} \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) & \text{if } Y_{0_i} < y \le \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} \\ \sigma_{cf}(i,y) & \text{if } \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} < y \le h \end{cases}$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium: $F_{cc} + F_{ft} + F_{ct} + F_s = 0$

Steel force: $F_{s_i} := A_s\cdot\sigma_{s.1_i}$

Concrete in compression force:

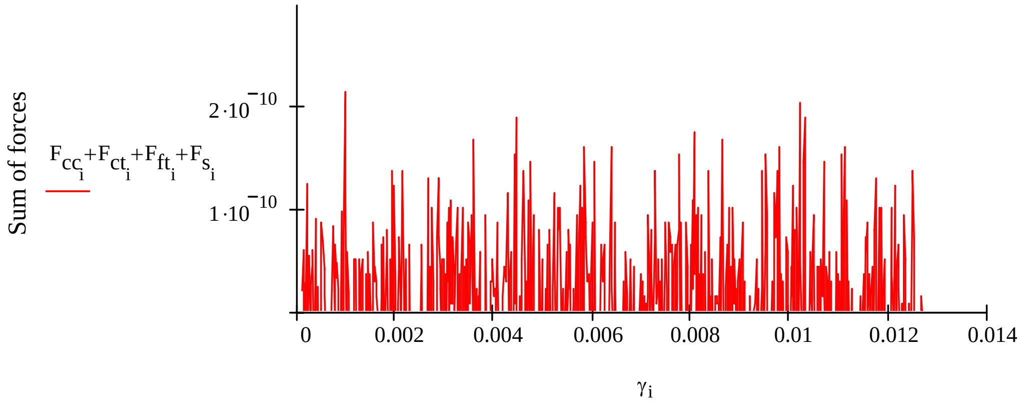
$$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right] - \eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]^2}{1 + (k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]} \right]\cdot b\,dy$$

Concrete in tension force (elastic zone):

$$F_{ct_i} := \int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}} \left[\frac{\gamma_i}{s}\cdot\left(y - Y_{0_i}\right)\cdot E_c\right]\cdot b\,dy$$

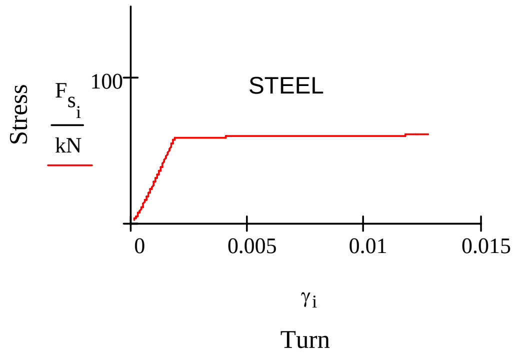
Cracked concrete force:

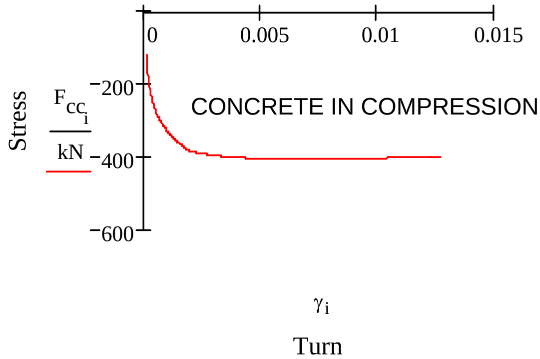
$$F_{ft_i} := \int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h} \left[f_{ct}\cdot \left| \begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \ \text{ if } \ \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \ \text{ if } \ \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 \ \text{ if } \ y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{array} \right. \right]\cdot b\,dy$$

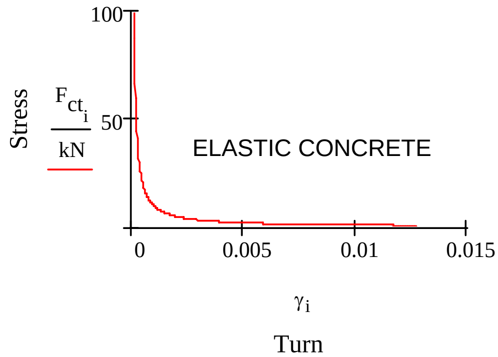


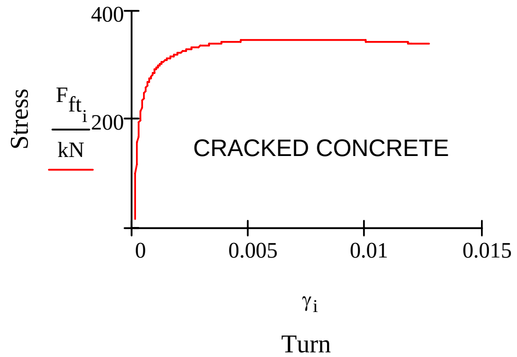
Turn

Graphs of forces:









Moment :

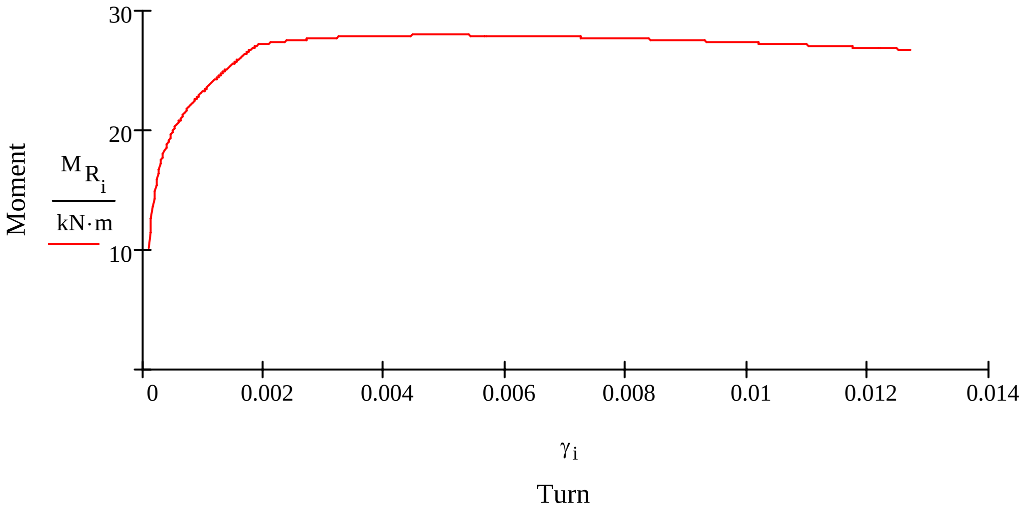
$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm}\cdot \frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]-\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]^2}{1+(k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]} \right]\cdot b\cdot(y)\,dy \ldots$$

$$+\int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}} \left[\frac{\gamma_i}{s}\cdot\left(y-Y_{0_i}\right)\cdot E_c\right]\cdot b\cdot y\,dy \ldots$$

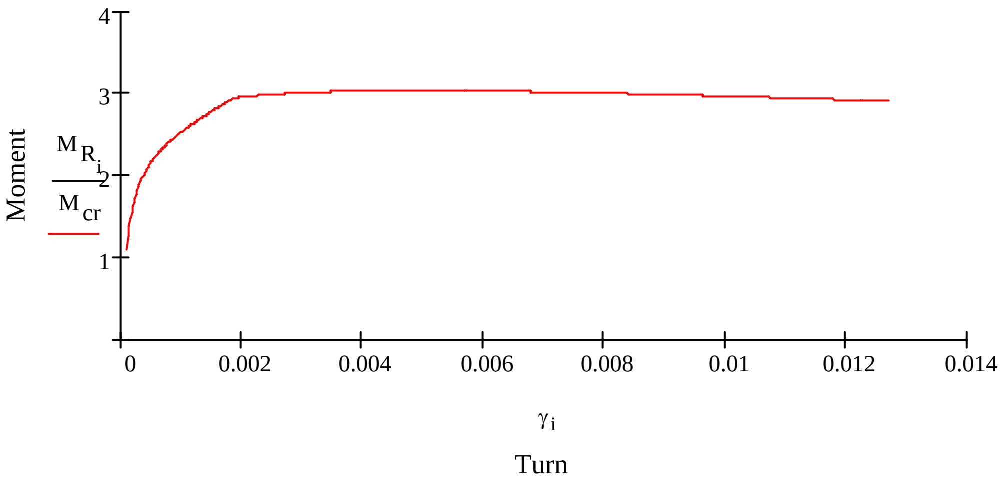
$$+\int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h} \left[f_{ct}\cdot \left| \begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i}\le y<\frac{w_1}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{w_1}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i}\le y\le\frac{w_c}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} \\ 0 \text{ if } y>\frac{w_c}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} \end{array}\right. \right]\cdot b\cdot y\,dy \ldots$$

$$+F_{s_i}\cdot\left(d_1\right)$$

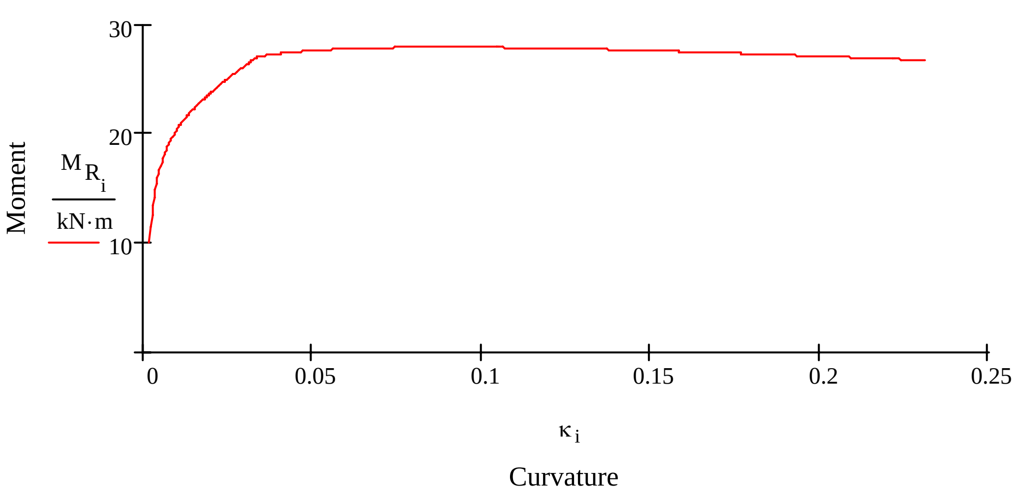
MOMENT-TURN GRAPH



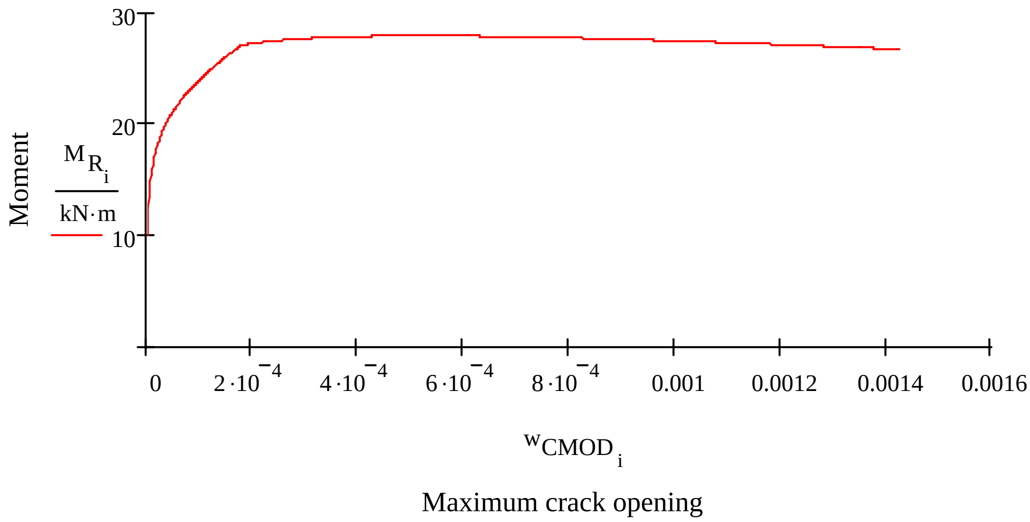
NORMALISED MOMENT-TURN GRAPH



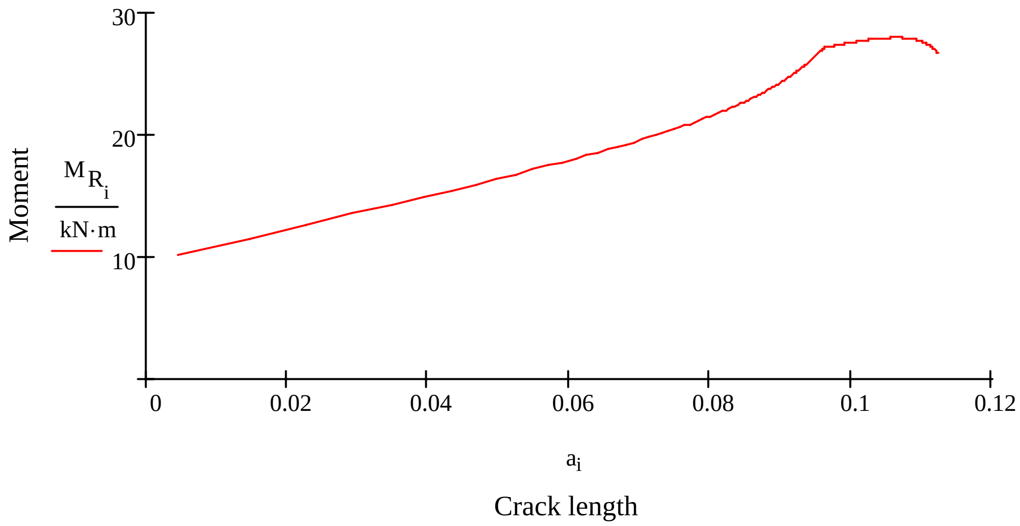
MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\gamma_i}{s}$



MOMENT-MAXIMUM OPENING GRAPH

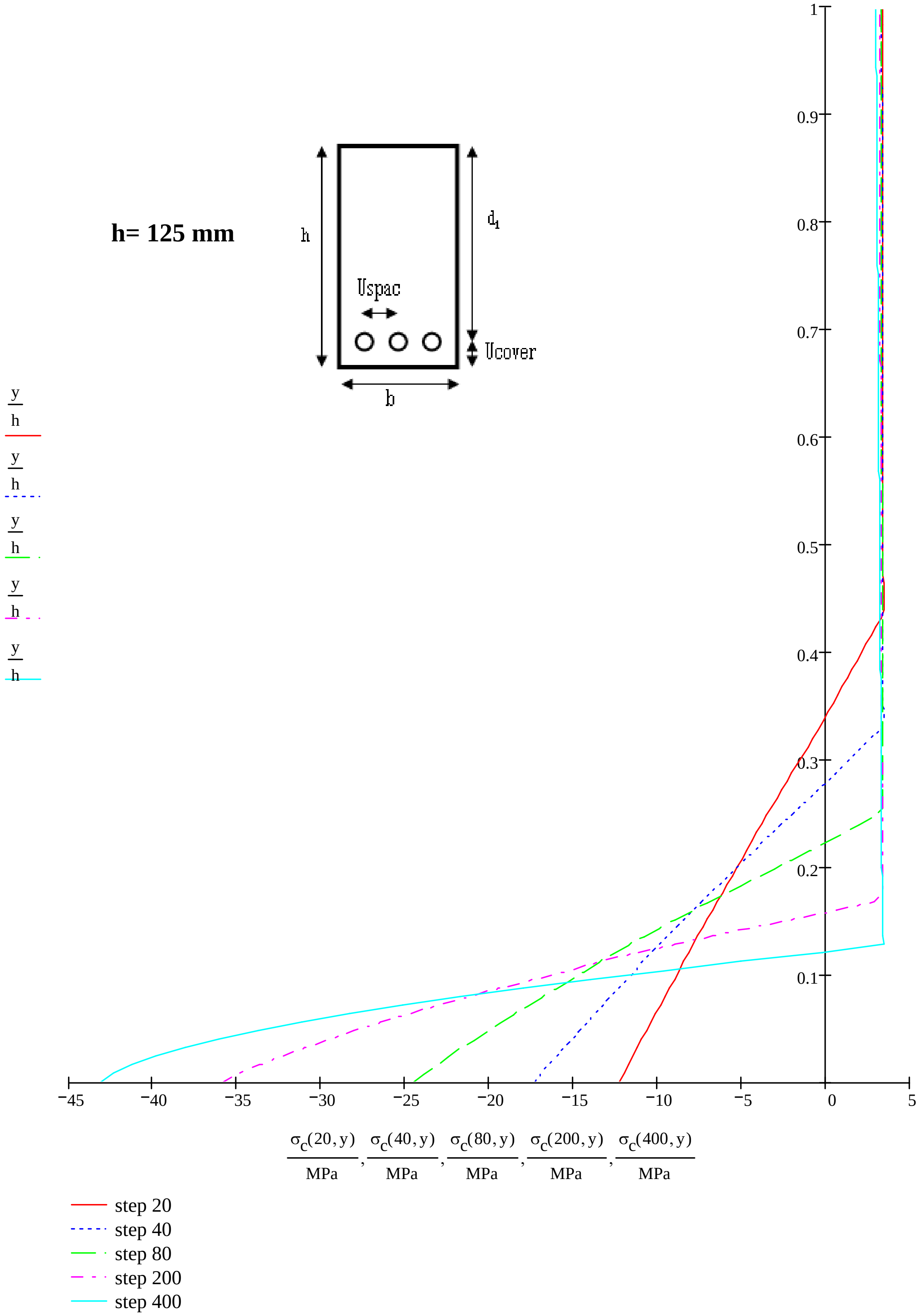


MOMENT-CRACK EXTENSION GRAPH



Stress Diagram of the cross section:

h= 125 mm



SECTIONAL ANALYSIS

HEIGHT 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

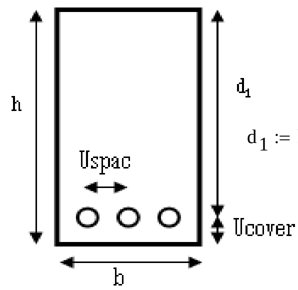
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \text{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588 \text{ mm} \qquad \phi_b := \text{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 7 \text{ mm}$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \text{round}\left(n_{ap}, 0\right)$ $n = 6$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 181.6 \text{ mm}$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 2.309 \times 10^{-4} \text{ m}^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.132 \text{ m}$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.251 \text{ m}^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 125.535 \text{ mm}$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.31547 \times 10^9 \text{ mm}^4$

Critical moment (moment just before cracking) $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 36.992 \text{ kN} \cdot \text{m}$

Width of non-linear zone (crack spacing), see appendix D: $s := 65mm$

Critical turn: $\gamma_{cr} := \dfrac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 5.327 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \dfrac{\gamma_{cr}}{s}$

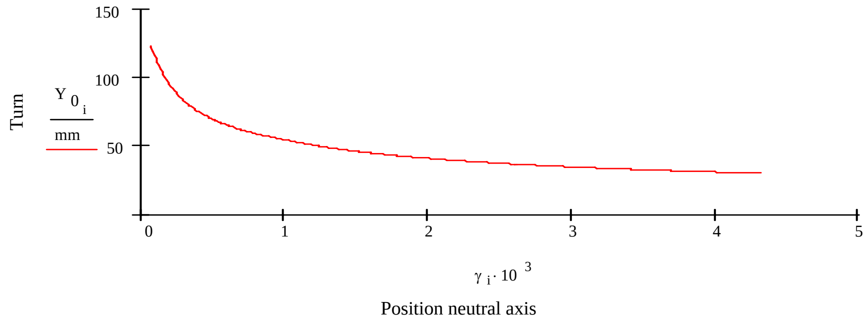
$\kappa_{cr} = 8.1952832 \times 10^{-4} \dfrac{1}{m}$

Number of steps: $n := 400$ $i := 0 .. n$

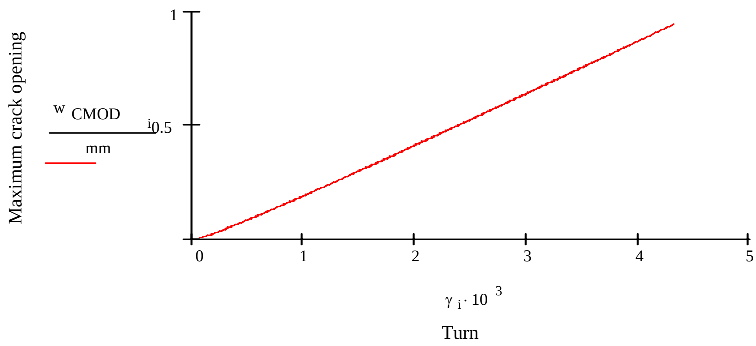
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \dfrac{\gamma_{cr}}{20}\right) + \dfrac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{20}$

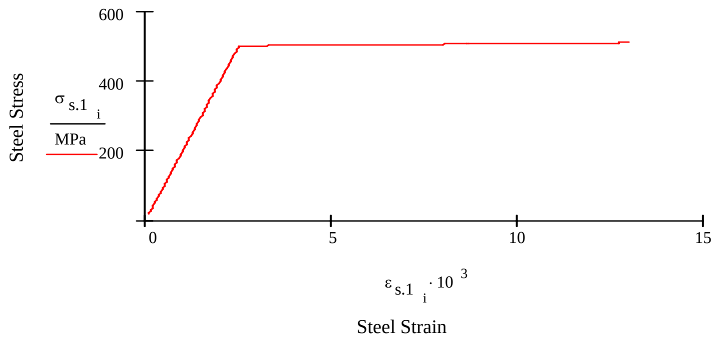
Position of the neutral axis when turn is increasing:



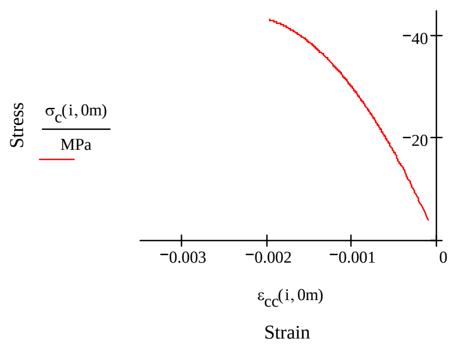
Maximum crack opening when turn is increasing:



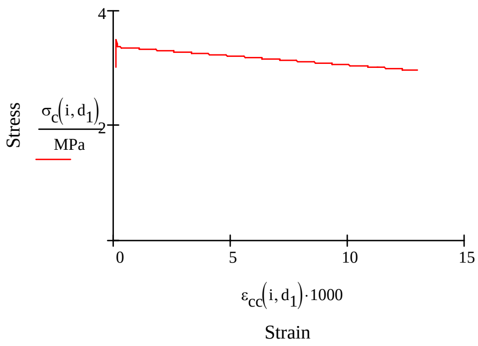
Stress Strain Reinforcement Diagram:



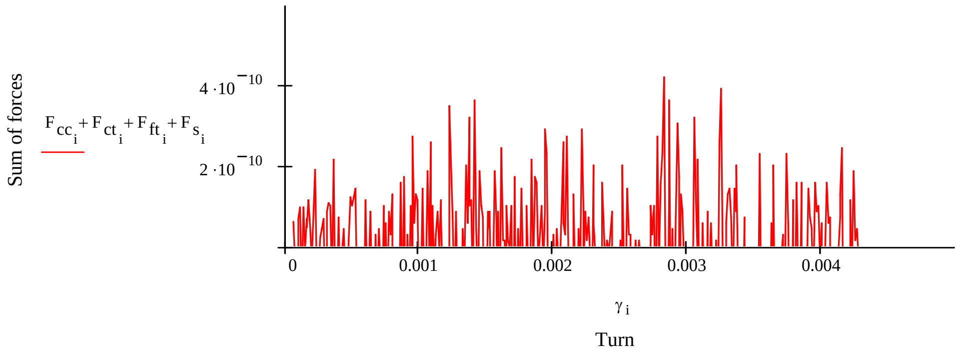
Stress-Strain diagram of the top concrete:



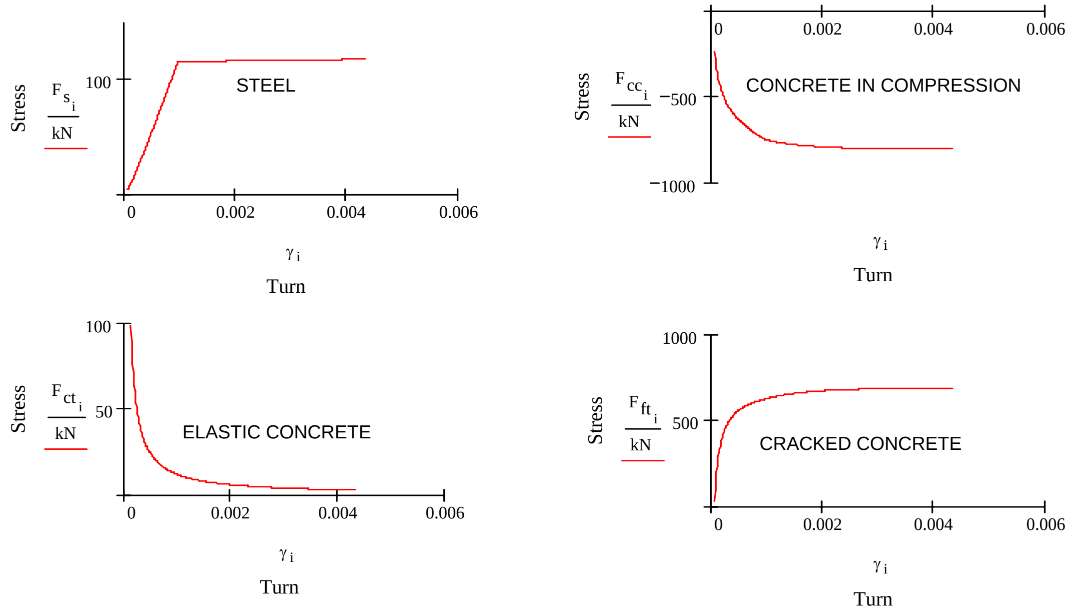
Stress-Strain relationship at the level of reinforcement:



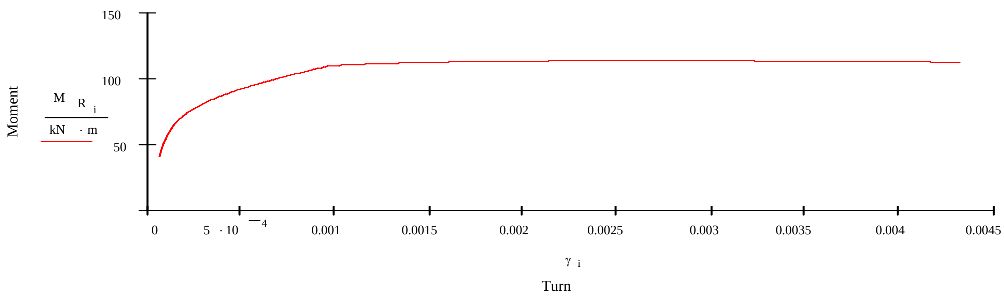
SUM OF FORCES=0 GRAPH



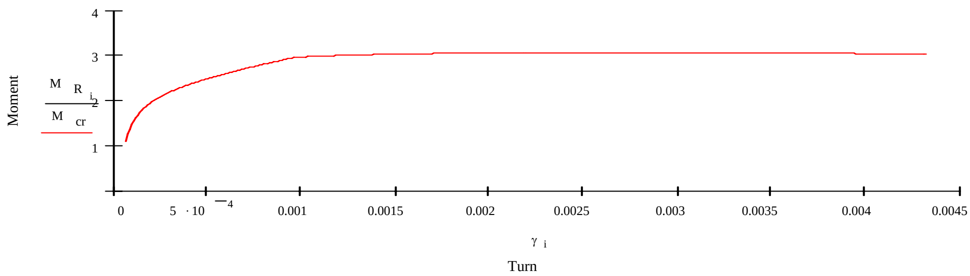
GRAPHS OF FORCES



MOMENT-TURN GRAPH

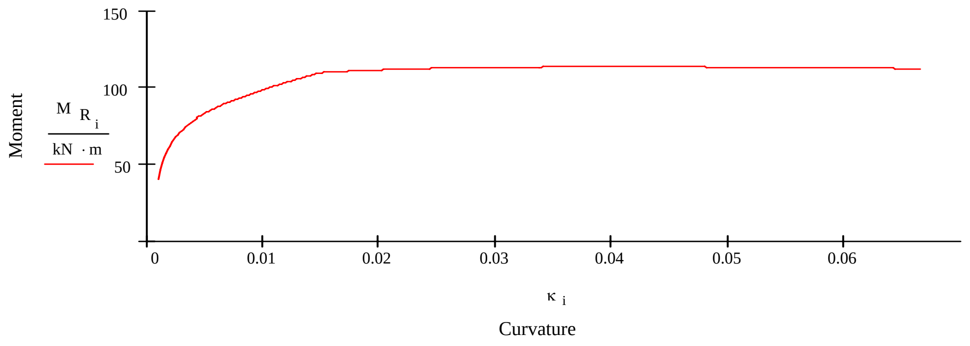


NORMALISED MOMENT-TURN GRAPH

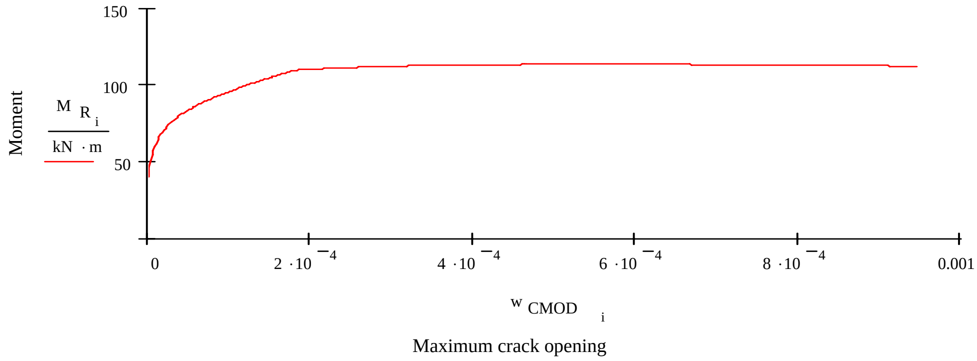


MOMENT-CURVATURE GRAPH

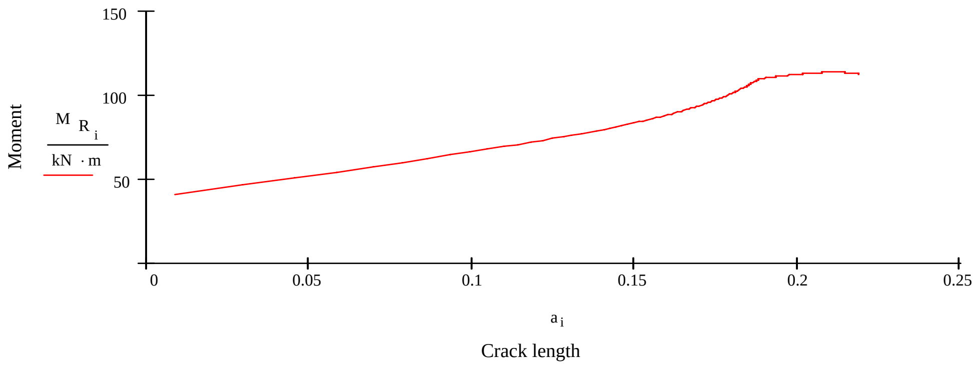
$$\kappa_i := \frac{\gamma_i}{s}$$



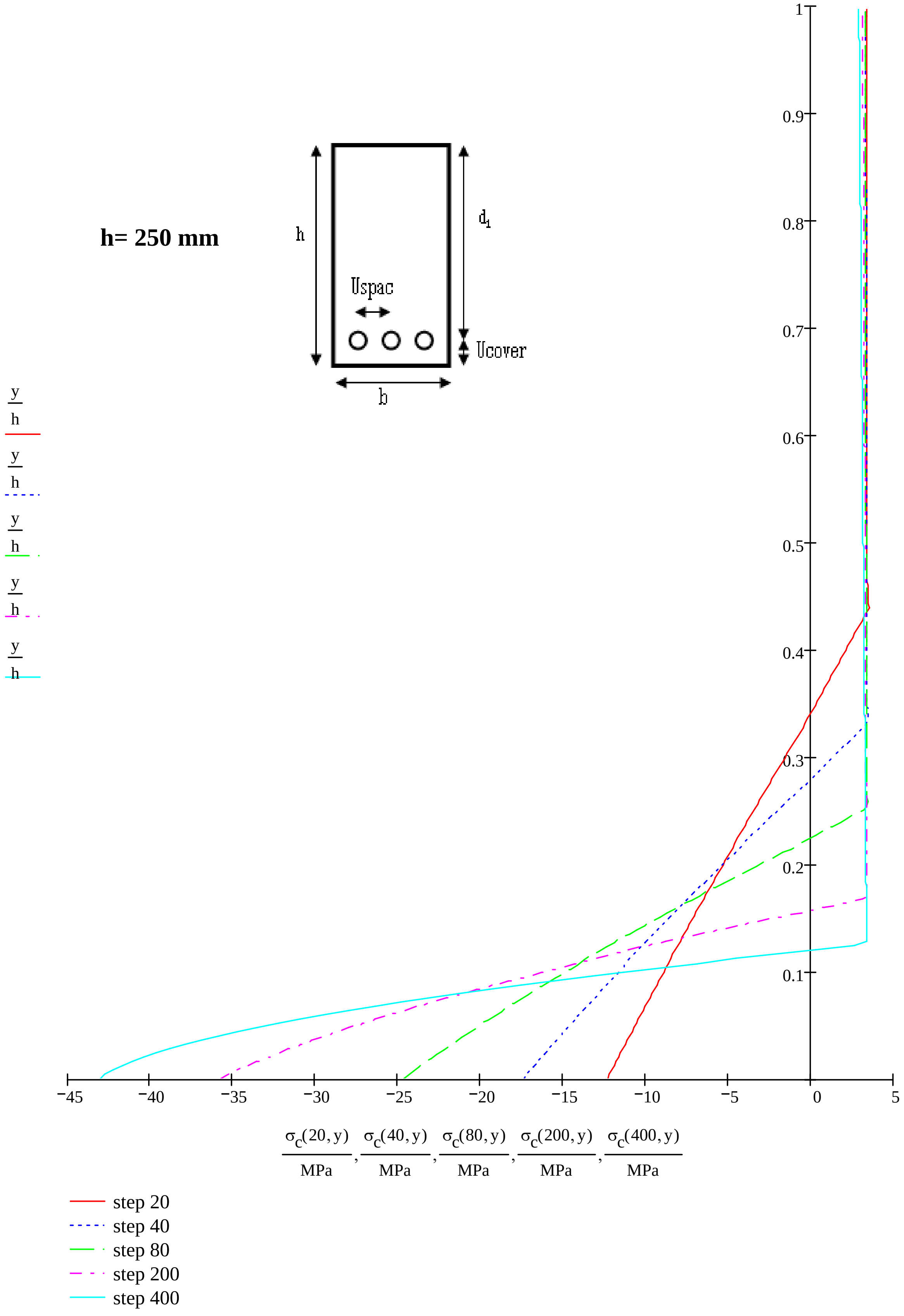
MOMENT-MAXIMUM CRACK OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS

HEIGHT 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

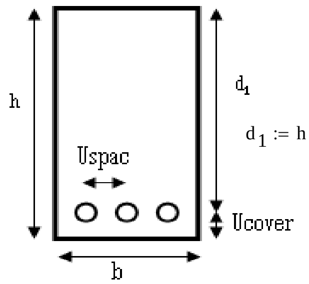
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci}\cdot\left(\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317\ mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$$

Final bar diameter: $\phi_b = 9\ mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \mathrm{round}\left(n_{ap}, 0\right)$ $n = 8$ Final bar spacing: $u_{spac} := \dfrac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n - 1}$ $u_{spac} = 125.429\ mm$

Total steel area: $A_s := n\cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2\cdot\pi\cdot\left(\dfrac{\phi_b}{2}\right)\cdot n$ $perim = 0.226\ m$

Effective area: $A_{ef} := b\cdot h + \left(\dfrac{E_s}{E_c}\right)\cdot A_s$ $A_{ef} = 0.503\ m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b\cdot h\cdot\dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right)\cdot A_s\cdot d_1}{A_{ef}}$ $x_{ef} = 251.327\ mm$

Inertia Moment: $I_{ef} := \dfrac{b\cdot h^3}{12} + b\cdot h\cdot\left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right)\cdot A_s\cdot\left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.0566 \times 10^{10}\ mm^4$

Critical moment (moment just before cracking) $M_{cr} := \dfrac{I_{ef}\cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 148.713\ kN\cdot m$

<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Critical turn: $\gamma_{cr} := \dfrac{s}{h - x_{ef}}\cdot\varepsilon_{ct.cr}$ $\gamma_{cr} = 2.666 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \dfrac{\gamma_{cr}}{s}$

E

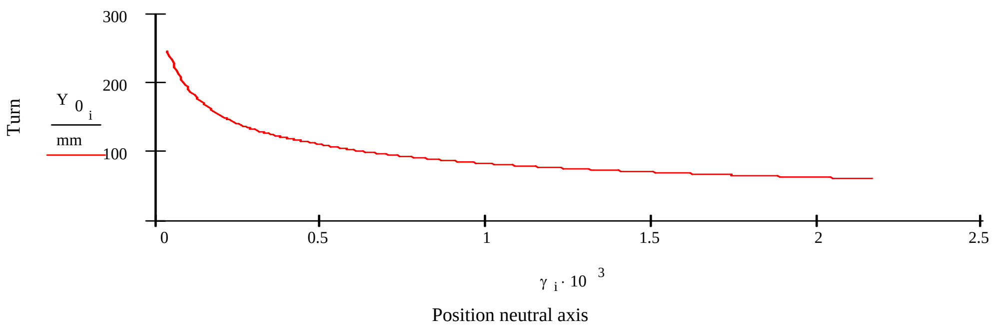
$\kappa_{cr} = 4.1018613 \times 10^{-4}\ \dfrac{1}{m}$

Number of steps: $n := 400$ $i := 0..\ n$

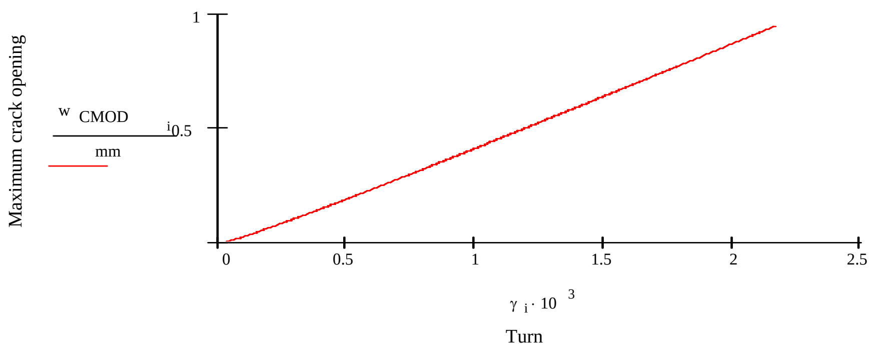
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \dfrac{\gamma_{cr}}{20}\right) + \dfrac{\gamma_{cr}}{5}\cdot i$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{10}$

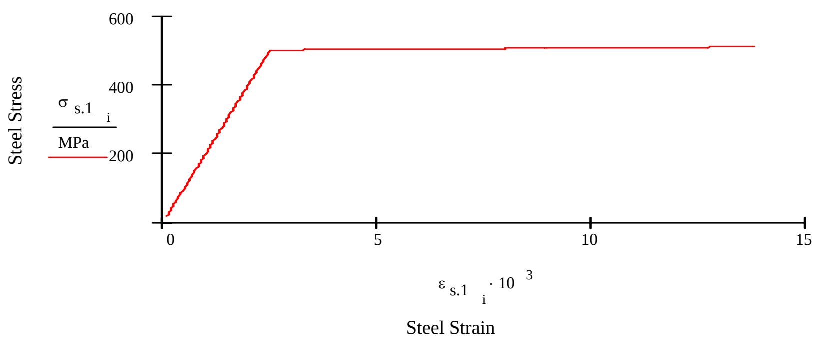
Position of the neutral axis when turn is increasing:



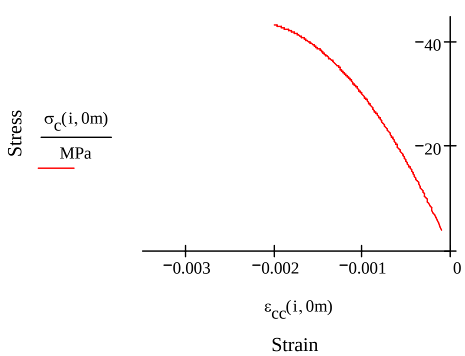
Maximum crack opening when turn is increasing:



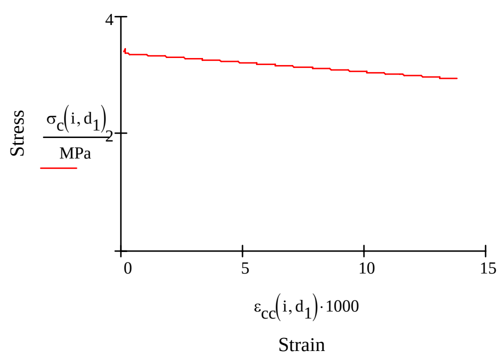
Stress Strain Reinforcement Diagram:



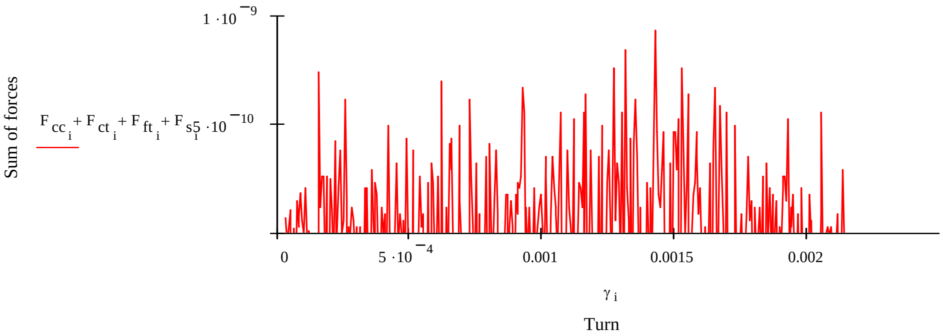
Stress-Strain diagram of the top concrete:



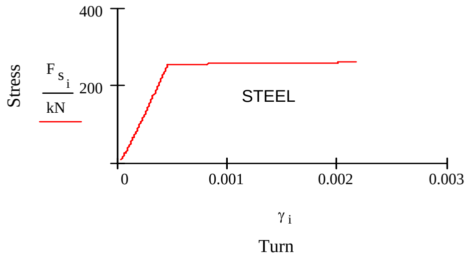
Stress-Strain relationship at the level of reinforcement:

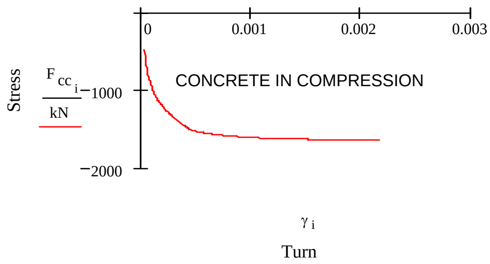


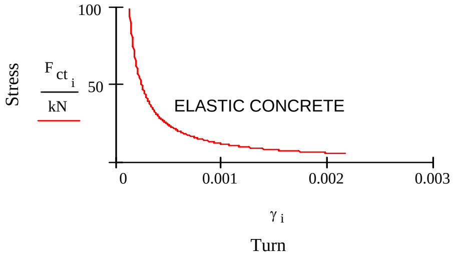
SUM OF FORCES=0 GRAPH

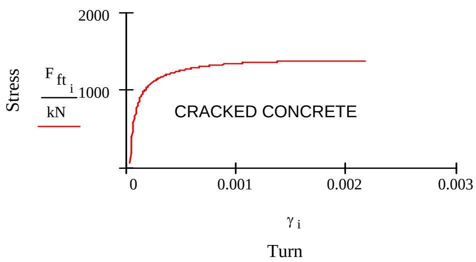


GRAPHS OF FORCES

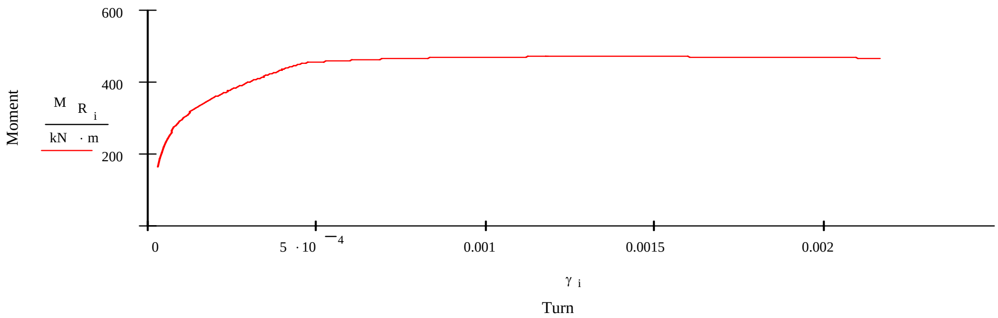




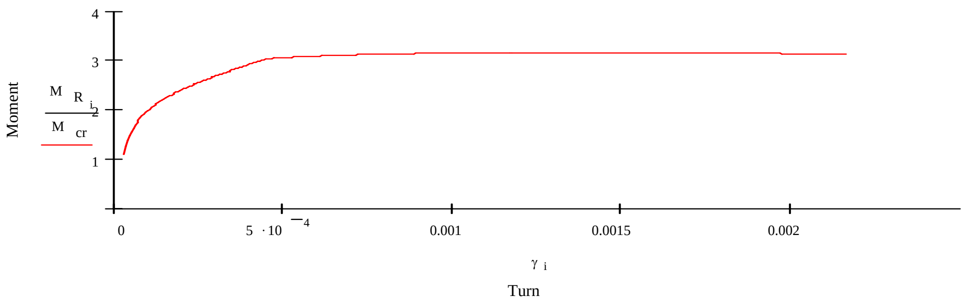




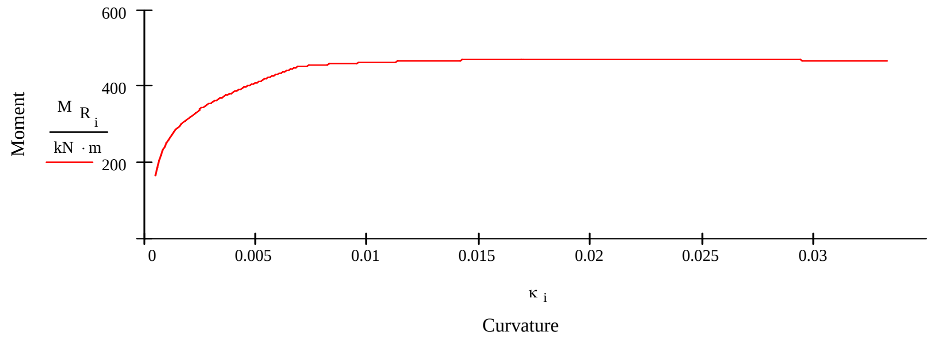
MOMENT-TURN GRAPH



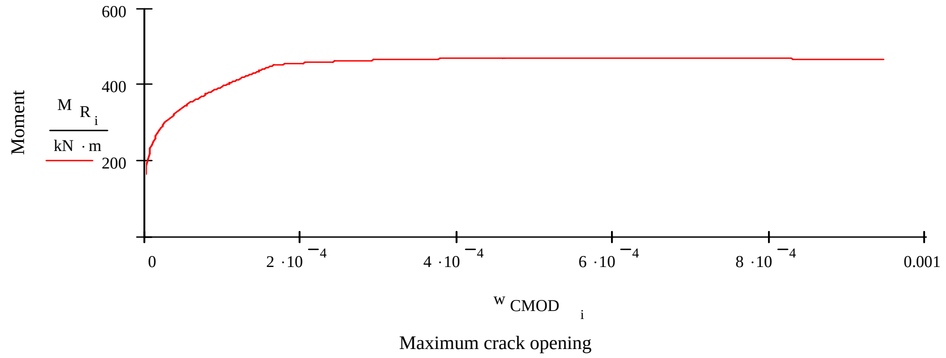
NORMALISED MOMENT-TURN GRAPH



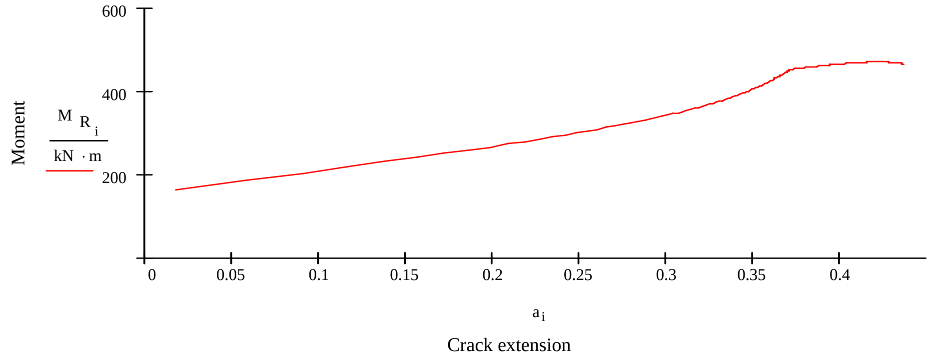
MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\gamma_i}{s}$



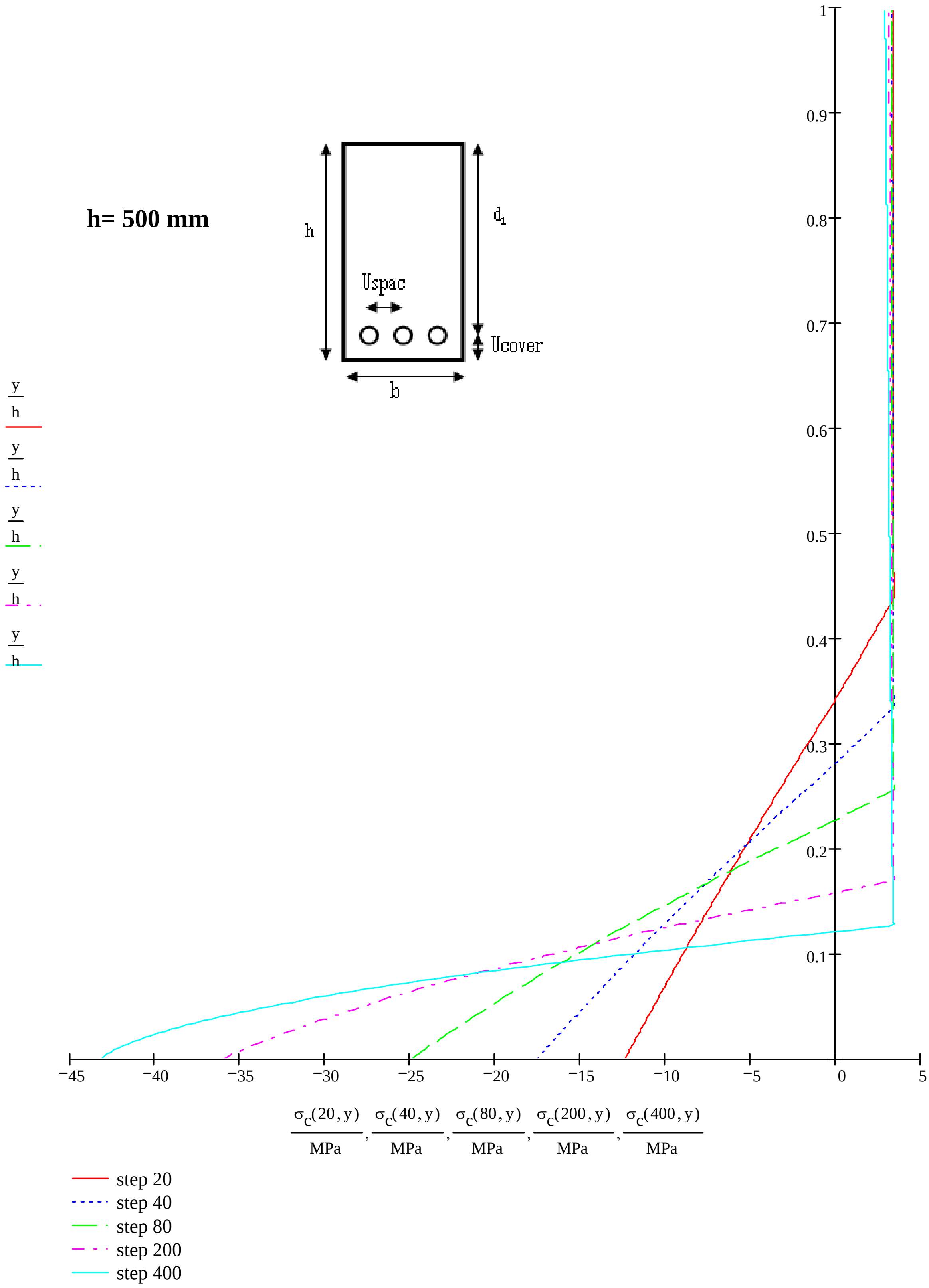
MOMENT-MAXIMUM CRACK OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



C.1.3 Sigma-crack opening relationship, analytical analysis. Mix C

MATERIAL PROPERTIES

Concrete in compression:

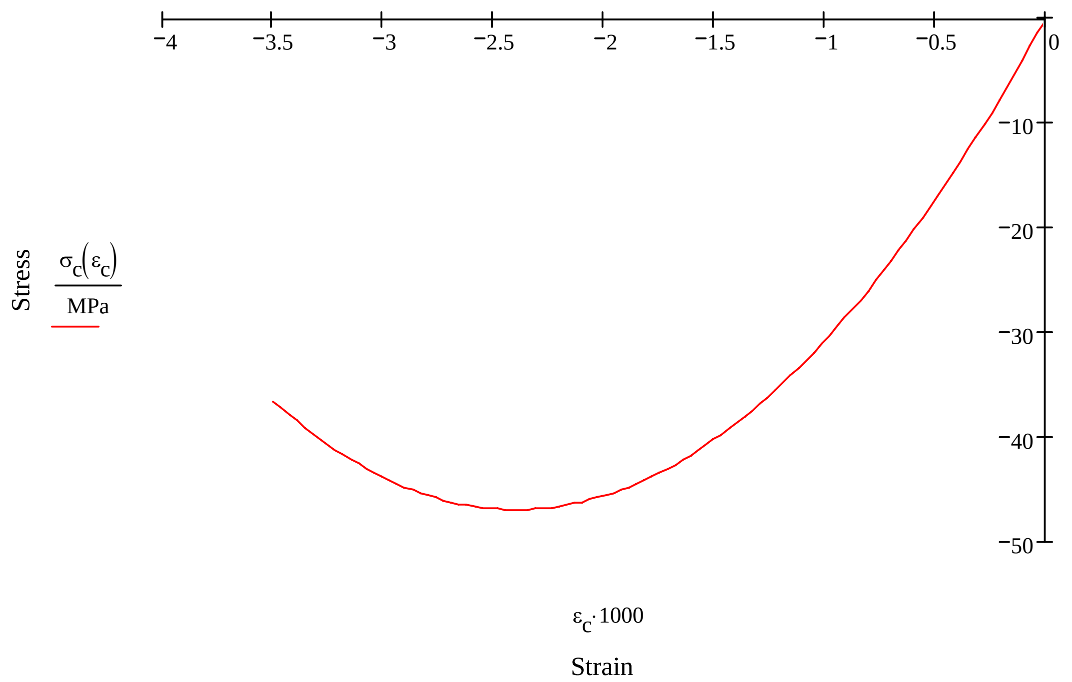
Mean compressive strength: $f_{cm} := 47\text{MPa}$

Modulus of Elasticity: $E_c := 22\left(\dfrac{\frac{f_{cm}}{\text{MPa}}}{10}\right)^{0.3}\cdot\text{GPa}$ $\qquad E_c = 34.999\text{GPa}$

Ultimate strain $\varepsilon_{cu} := \dfrac{3.5}{1000}$

Stress block factors: $\varepsilon_{c1} := 0.24\%$ $\qquad \eta(\varepsilon_c) := \dfrac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\qquad k := 1.1\cdot\dfrac{E_c\cdot|\varepsilon_{c1}|}{f_{cm}}$

Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm}\cdot\dfrac{k\cdot\eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2)\cdot\eta(\varepsilon_c)}$ $\qquad \varepsilon_c := 0, \dfrac{-\varepsilon_{cu}}{100} .. -\varepsilon_{cu}$



Concrete in tension:

Bi-linear Stress-Crack Opening Relationship MIX C:

Tensional strength $f_{ct} := 3.5 \cdot MPa$

Cracking strain $\varepsilon_{ct.cr} := \frac{f_{ct}}{E_c}$ $\varepsilon_{ct.cr} = 1 \times 10^{-4}$

Curve constants:

$a_1 := 16 \cdot \frac{1}{mm}$ $a_2 := 0.2 \cdot \frac{1}{mm}$

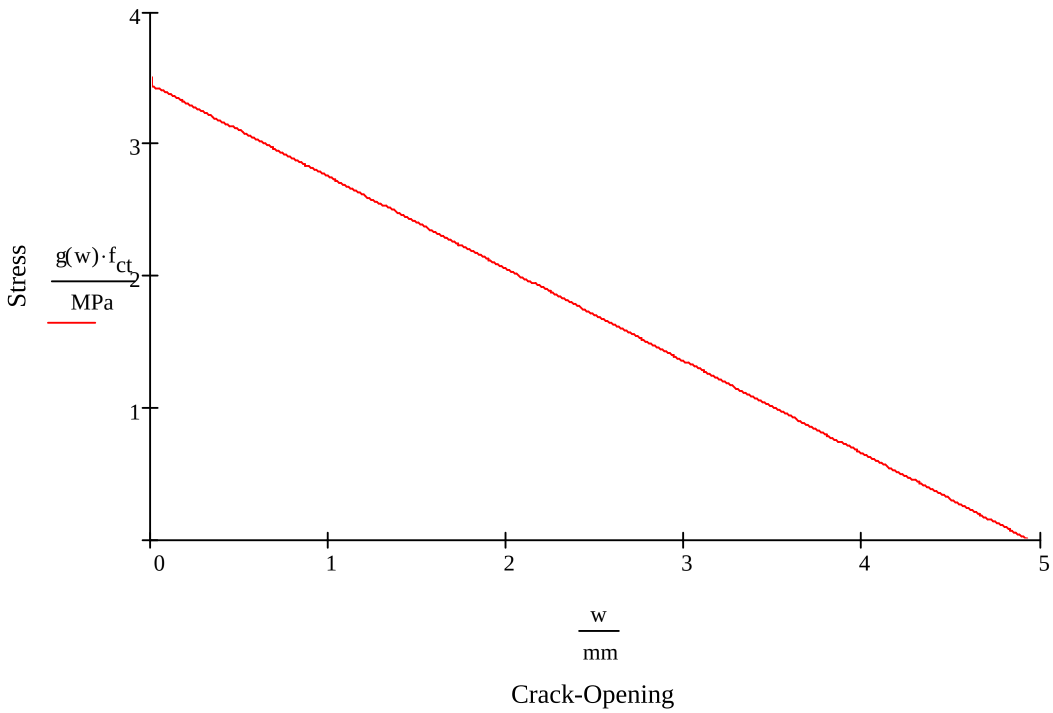
$b_1 := 1$ $b_2 := 0.985$

$w_1 := \frac{b_1 - b_2}{a_1 - a_2}$ $w_1 = 9.494 \times 10^{-4} mm$ $w_c := \frac{b_2}{a_2}$

$w_c = 4.925 mm$

$$g(w) := \begin{cases} b_1 - a_1 \cdot w & \text{if } 0 \le w < w_1 \\ b_2 - a_2 \cdot (w) & \text{if } w_1 \le w \le w_c \end{cases}$$

Fracture energy: $G_F := \int_{0 \cdot mm}^{w_c} f_{ct} \cdot g(w)\, dw$ $G_F = 8489 \frac{N \cdot m}{m^2}$



SECTIONAL ANALYSIS

HEIGHT 1.- 125 mm

Height of beam: $h := 125 \cdot mm$

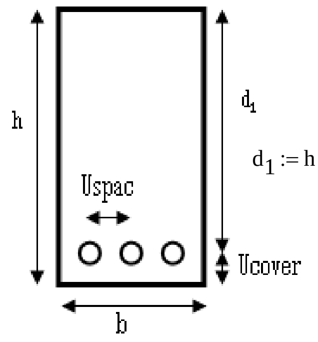
Width of beam: $b := 1000\,mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 5mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 1.178 \times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.094m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.126m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 62.701mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63702 \times 10^8\,mm^4$

Critical moment (moment just before cracking) $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 9.197kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 55mm$

Critical turn: $\gamma_{cr} := \frac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 8.829 \times 10^{-5}$

Critical curvature: $\kappa_{cr} := \frac{\gamma_{cr}}{s}$

$\kappa_{cr} = 1.6052184 \times 10^{-3} \frac{1}{m}$

Number of steps: $n := 400$ $i := 0..n$

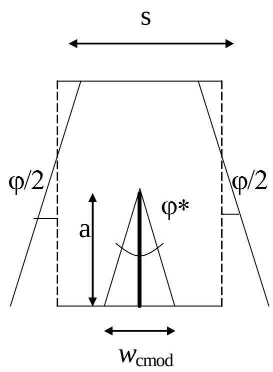
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \frac{\gamma_{cr}}{20}\right) + \frac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{20}$

Equilibrium equation to find the position of the neutral axis:

$$
\begin{aligned}
Y_{0_i} := \mathrm{root}\Bigg[& \int_0^{y_{0ini}} \left[-f_{cm} \cdot \frac{k \cdot \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right] - \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]^2}{1 + (k-2) \cdot \eta\left[\frac{\gamma_i}{s} \cdot (y_{0ini} - y)\right]} \right] \cdot b \, dy \ \ldots \\
& + \int_{y_{0ini}}^{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}} \left[\frac{\gamma_i}{s} \cdot \left[(y - y_{0ini}) \cdot E_c\right]\right] \cdot b \, dy \ \ldots \\
& + \int_{\frac{f_{ct}}{E_c} \cdot \frac{s}{\gamma_i} + y_{0ini}}^{h} \left[f_{ct} \cdot \left| \begin{array}{l} b_1 - a_1 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] \ \text{if} \ \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ b_2 - a_2 \cdot \left[\gamma_i \cdot (y - y_{0ini}) + \frac{-f_{ct}}{E_c} \cdot s\right] \ \text{if} \ \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \\ 0 \ \text{if} \ y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c \cdot \gamma_i} \cdot s + y_{0ini} \end{array} \right. \right] \cdot b \, dy \ \ldots \\
& + A_s \cdot \left| \begin{array}{l} E_s \cdot \frac{\gamma_i}{s} \cdot (d_1 - y_{0ini}) \ \text{if} \ \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} - \frac{f_{yk}}{E_s}\right] + f_{yk} \ \text{if} \ \varepsilon_{syk} < \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} < \varepsilon_{suk} \\ 0 \cdot MPa \ \text{if} \ \frac{\gamma_i \cdot (d_1 - y_{0ini})}{s} > \varepsilon_{suk} \end{array} \right. \quad , y_{0ini} \Bigg]
\end{aligned}
$$

Crack extension: $$a_i := h - \left(\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i} + Y_{0_i}\right)$$



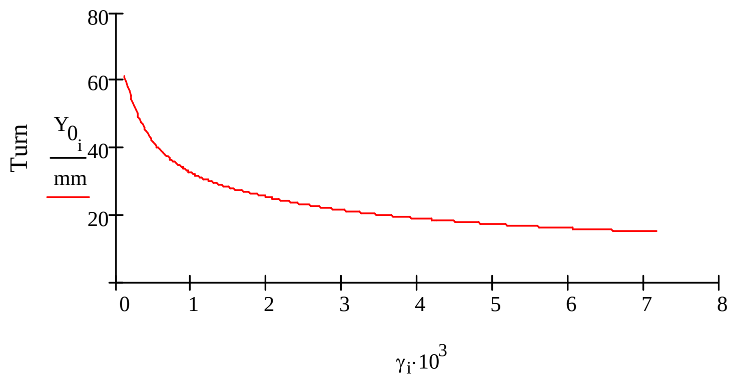
Maximum crack opening:

$$w_{CMOD_i} := \begin{cases} \gamma_i\cdot\left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } 0 \le \gamma_i\cdot\left(h - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s \le w_c \\ \frac{\left(h - Y_{0_i}\right)\cdot\gamma_i}{1} & \text{if } \frac{\left(h - Y_{0_i}\right)\cdot\gamma_i}{1} > w_c \end{cases}$$

General expression for the crack opening:

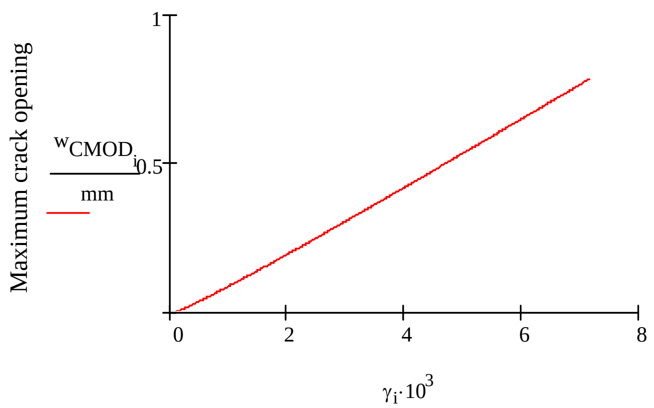
$$w(i,y) := \begin{cases} \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s < 0\text{mm} \\ \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s & \text{if } 0\text{mm} \le \gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s < w_c \\ \frac{\left(y - Y_{0_i}\right)\cdot\gamma_i}{1} & \text{if } \frac{\left(y - Y_{0_i}\right)\cdot\gamma_i}{1} > w_c \end{cases}$$

Position of the neutral axis when turn is increasing:



Position neutral axis

Maximum crack opening when turn is increasing:



Turn

Stress and Strain STEEL:

Strain in reinforcement steel:

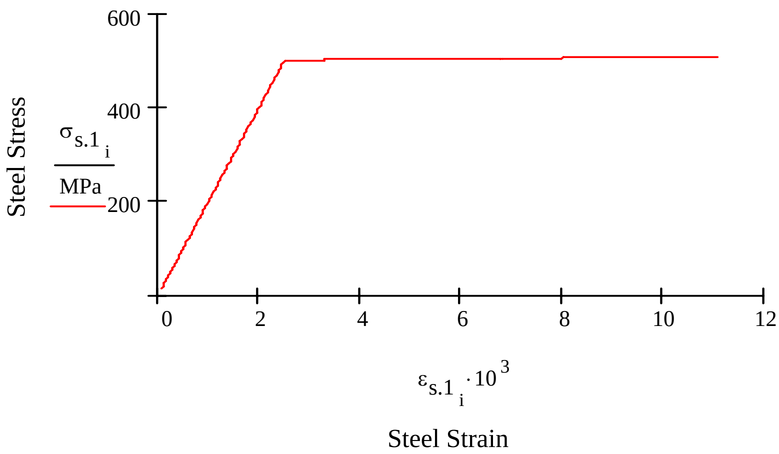
Bottom steel

$$\varepsilon_{s.1_i} := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - d_1}{Y_{0_i}}$$

Stress in reinforcement steel :

Bottom steel

$$\sigma_{s.1_i} := \left| \begin{array}{ll} E_s \cdot \frac{\gamma_i}{s} \cdot \left(d_1 - Y_{0_i}\right) & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} \le \varepsilon_{syk} \\ \frac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \frac{f_{yk}}{E_s}} \left[\frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} - \frac{f_{yk}}{E_s} \right] + f_{yk} & \text{if } \varepsilon_{syk} < \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} < \varepsilon_{suk} \\ 0 \cdot \text{MPa} & \text{if } \frac{\gamma_i \cdot \left(d_1 - Y_{0_i}\right)}{s} > \varepsilon_{suk} \end{array} \right.$$



Stress and Strain CONCRETE:

Concrete strain:

$$\varepsilon_{cc}(i, y) := \frac{-\gamma_i}{s} \cdot Y_{0_i} \cdot \frac{Y_{0_i} - y}{Y_{0_i}}$$

Concrete stress:

Concrete in compresion:

$$\sigma_{cc}(i, y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right] - \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right]^2}{1 + (k - 2) \cdot \eta \left[\frac{\gamma_i}{s} \cdot \left(Y_{0_i} - y\right) \right]}$$

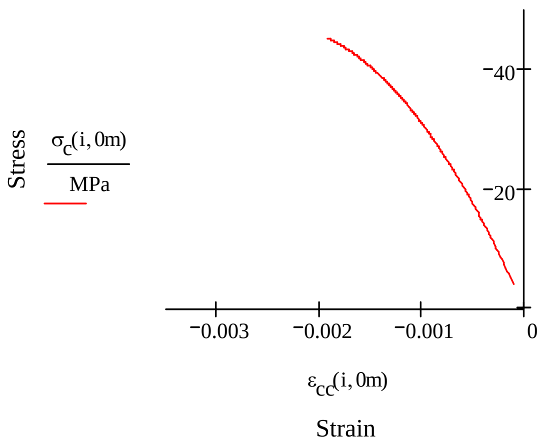
Concrete in elastic behaviour: $\sigma_{ct}(i,y) := \frac{\gamma_i}{s}\cdot\left[\left(y - Y_{0_i}\right)\cdot E_c\right]$

Cracked concrete: $$\sigma_{cf}(i,y) := \left[f_{ct}\cdot \left| \begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 \text{ if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{array} \right. \right]$$

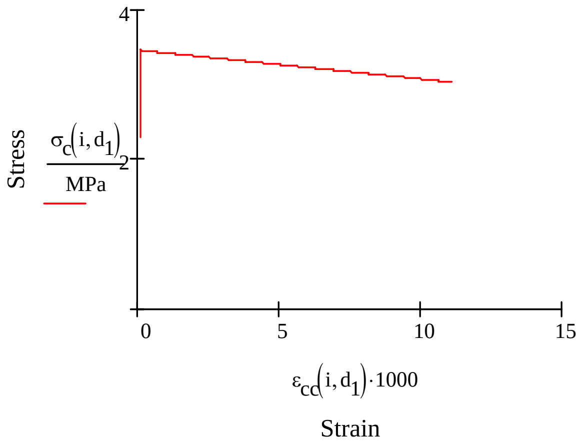
Final expression:

$$\sigma_c(i,y) := \left| \begin{array}{l} \sigma_{cc}(i,y) \text{ if } 0\text{mm} \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) \text{ if } Y_{0_i} < y \le \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} \\ \sigma_{cf}(i,y) \text{ if } \frac{f_{ct}}{E_c}\cdot\left(\frac{s}{\gamma_i}\right) + Y_{0_i} < y \le h \end{array} \right.$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium: $F_{cc} + F_{ft} + F_{ct} + F_s = 0$

Steel force: $F_{s_i} := A_s\cdot\sigma_{s.1_i}$

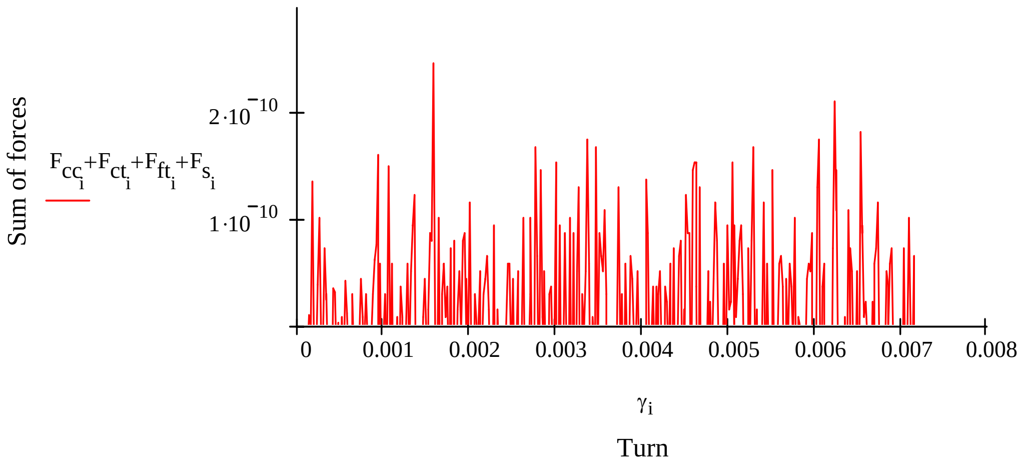
Concrete in compression force: $$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right] - \eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]^2}{1 + (k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i} - y\right)\right]} \right]\cdot b\, dy$$

Concrete in tension force (elastic zone):

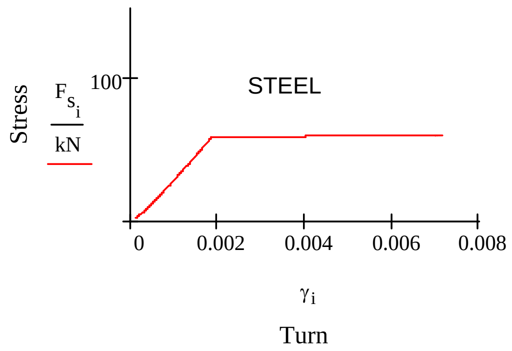
$$F_{ct_i} := \int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}} \left[\frac{\gamma_i}{s}\cdot\left(y - Y_{0_i}\right)\cdot E_c\right]\cdot b\,dy$$

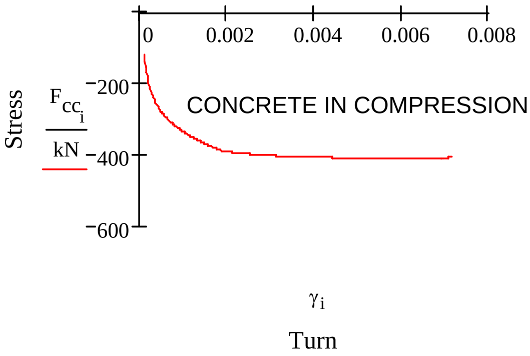
Cracked concrete force:

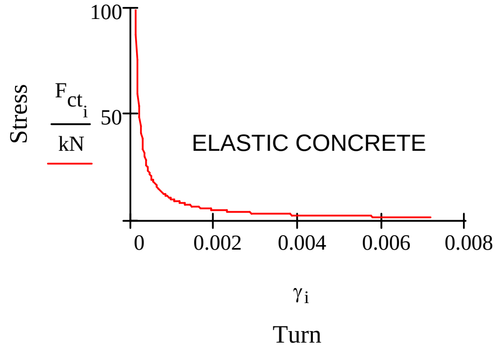
$$F_{ft_i} := \int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h} \left[f_{ct}\cdot\left|\begin{array}{l} b_1 - a_1\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y < \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ b_2 - a_2\cdot\left[\gamma_i\cdot\left(y - Y_{0_i}\right) + \frac{-f_{ct}}{E_c}\cdot s\right] \text{ if } \frac{w_1}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \le y \le \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \\ 0 \text{ if } y > \frac{w_c}{\gamma_i} + \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s + Y_{0_i} \end{array}\right.\right]\cdot b\,dy$$

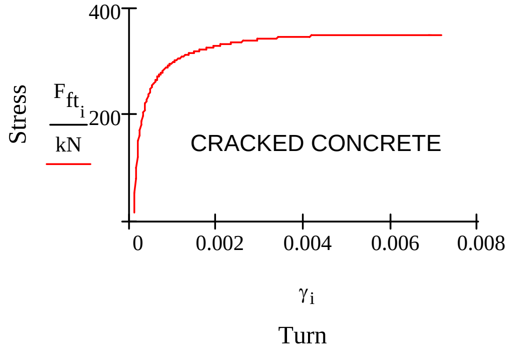


Graphs of forces:









Moment :

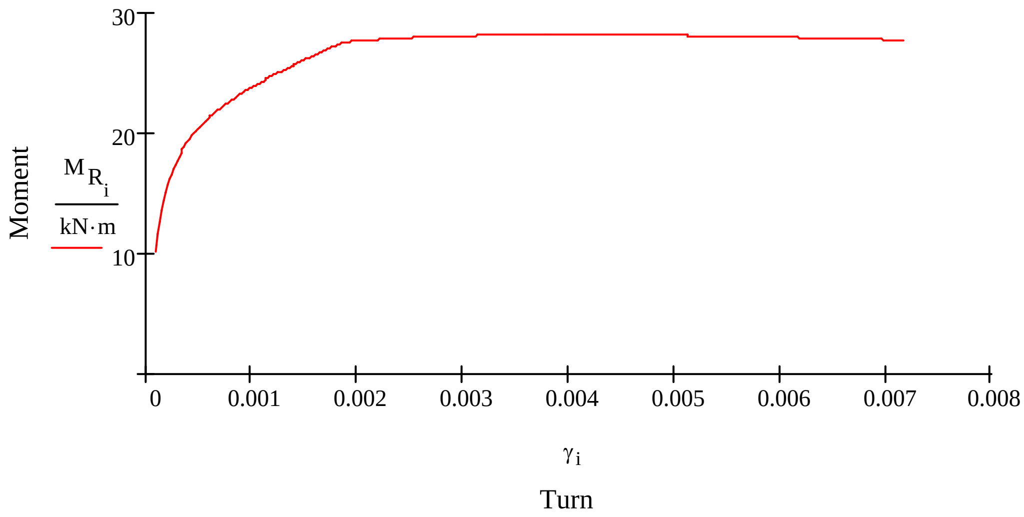
$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]-\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]^2}{1+(k-2)\cdot\eta\left[\frac{\gamma_i}{s}\cdot\left(Y_{0_i}-y\right)\right]}\right]\cdot b\cdot(y)\,dy\ \ldots$$

$$+\int_{Y_{0_i}}^{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}\left[\frac{\gamma_i}{s}\cdot\left(y-Y_{0_i}\right)\cdot E_c\right]\cdot b\cdot y\,dy\ \ldots$$

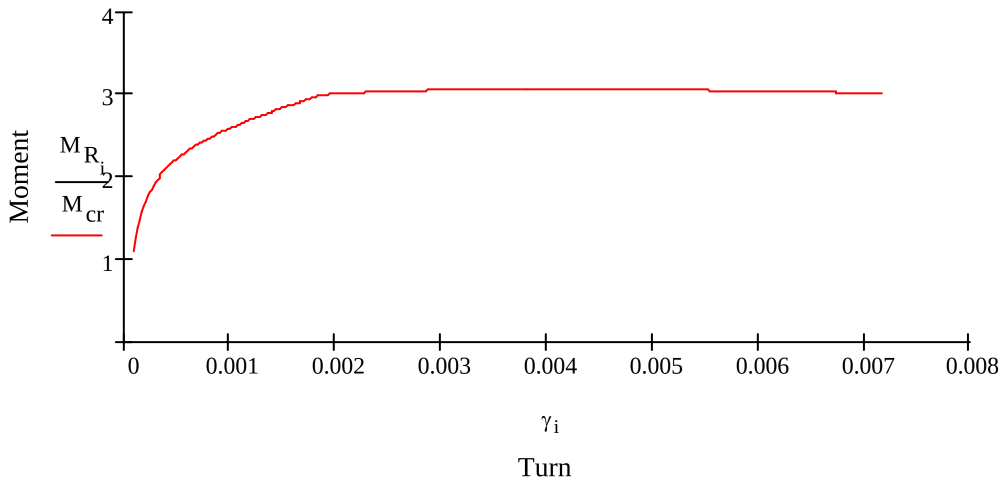
$$+\int_{\frac{f_{ct}}{E_c}\cdot\frac{s}{\gamma_i}+Y_{0_i}}^{h}\left[f_{ct}\cdot\left|\begin{array}{ll} b_1-a_1\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i}\le y<\frac{w_1}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} \\ b_2-a_2\cdot\left[\gamma_i\cdot\left(y-Y_{0_i}\right)+\frac{-f_{ct}}{E_c}\cdot s\right] & \text{if } \frac{w_1}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i}\le y\le\frac{w_c}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} \\ 0 \ \text{ if } \ y>\frac{w_c}{\gamma_i}+\frac{f_{ct}}{E_c\cdot\gamma_i}\cdot s+Y_{0_i} & \end{array}\right.\right]\cdot b\cdot y\,dy\ \ldots$$

$$+F_{s_i}\cdot\left(d_1\right)$$

MOMENT-TURN GRAPH

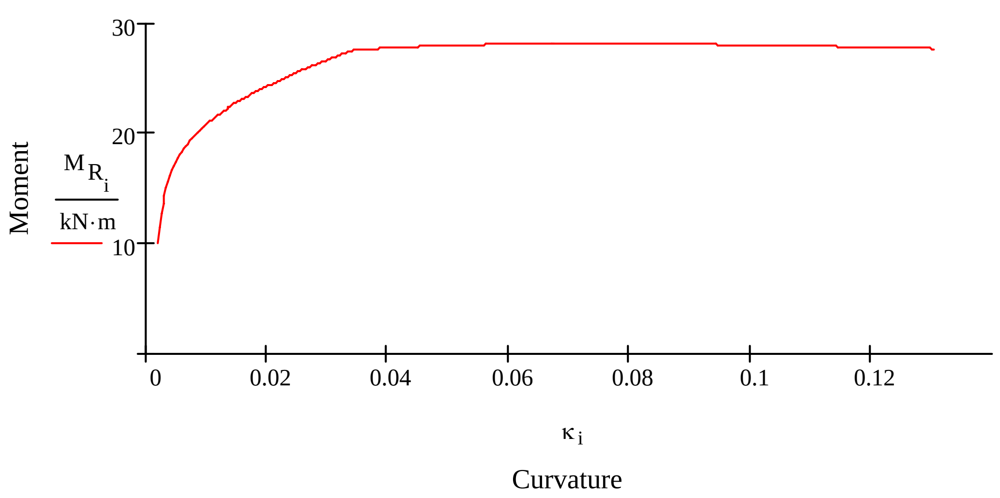


NORMALISED MOMENT-TURN GRAPH

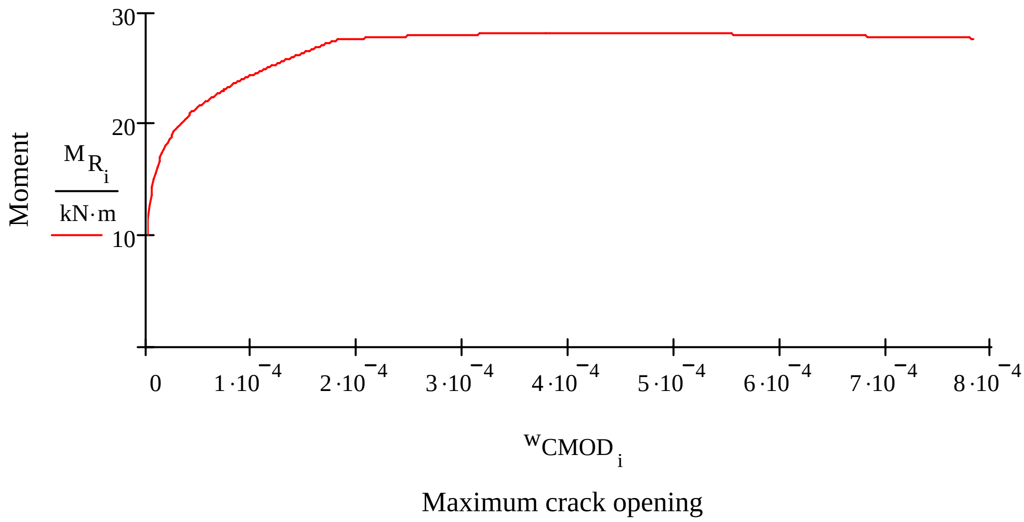


MOMENT-CURVATURE GRAPH

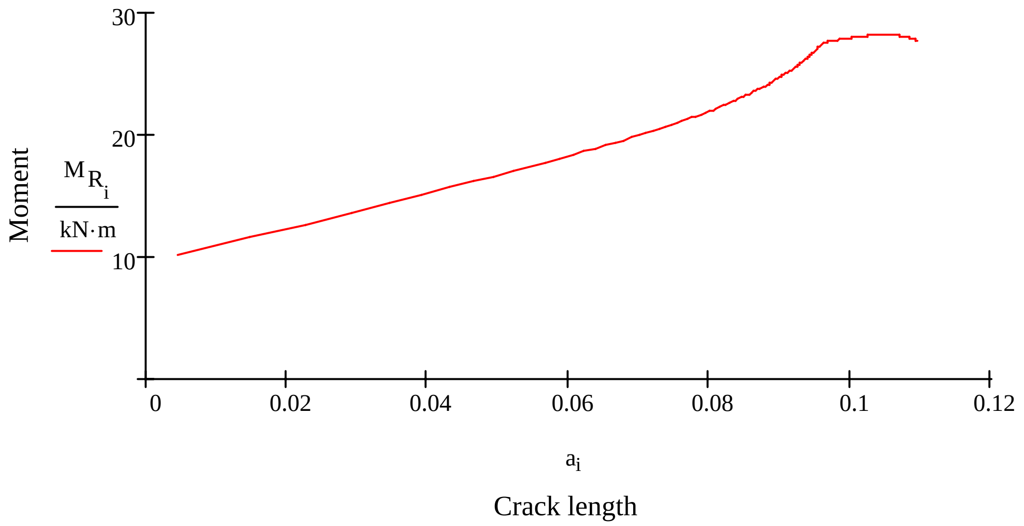
$$\kappa_i := \frac{\gamma_i}{s}$$



MOMENT-MAXIMUM CRACK OPENING GRAPH

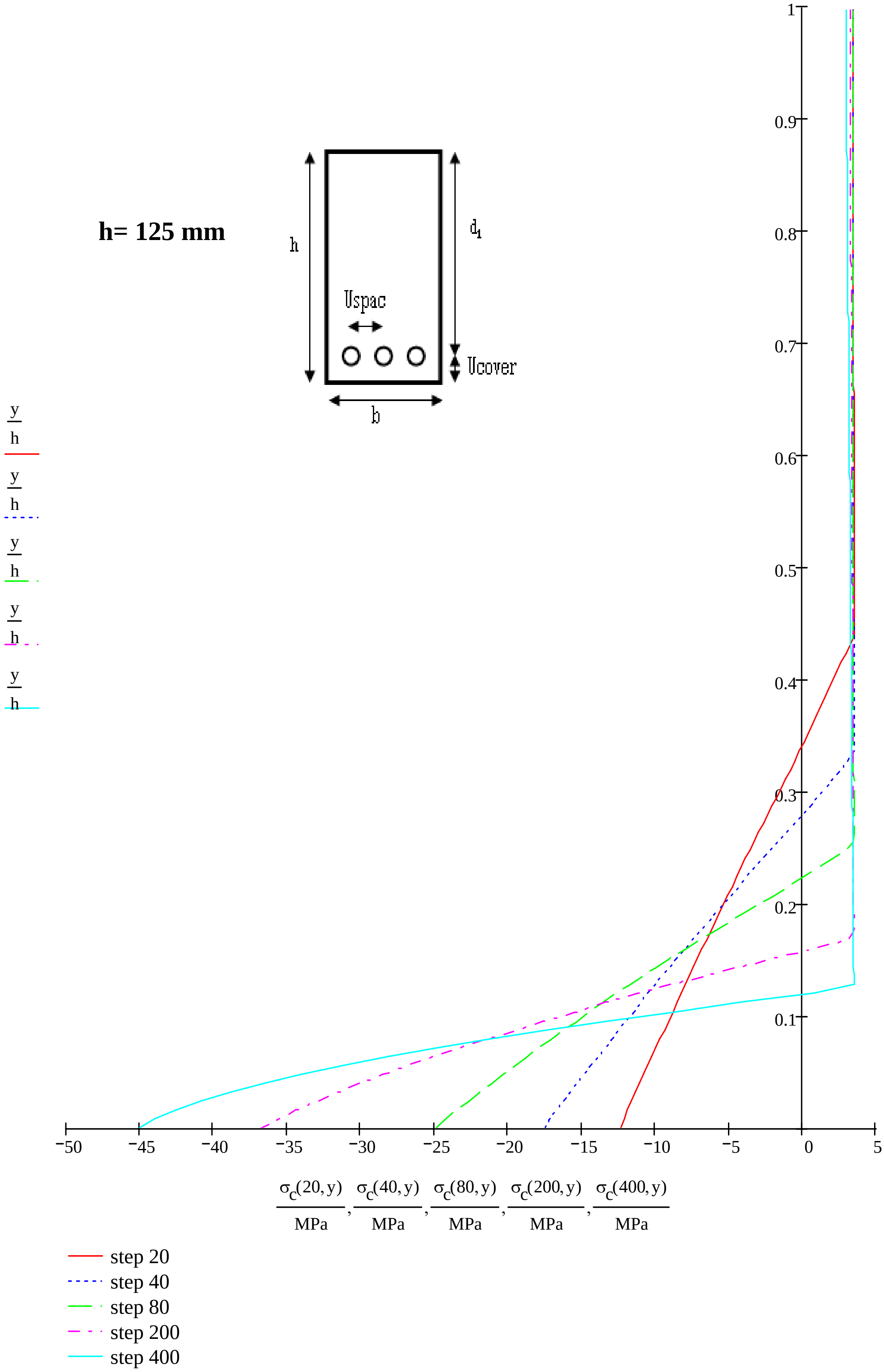


MOMENT-CRACK EXTENSION GRAPH



Stress Diagram of the cross section:

h= 125 mm



SECTIONAL ANALYSIS

HEIGHT 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

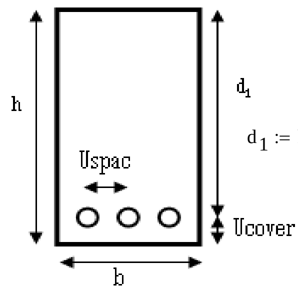
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588\ mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 7\ mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^{\ 2}}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 181.6\ mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 2.309 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.132\ m$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.251\ m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 125.525\ mm$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.31521 \times 10^9\ mm^4$

Critical moment (moment just before cracking) $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 36.981\ kN \cdot m$

<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Critical turn: $\gamma_{cr} := \dfrac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 5.222 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \dfrac{\gamma_{cr}}{s}$

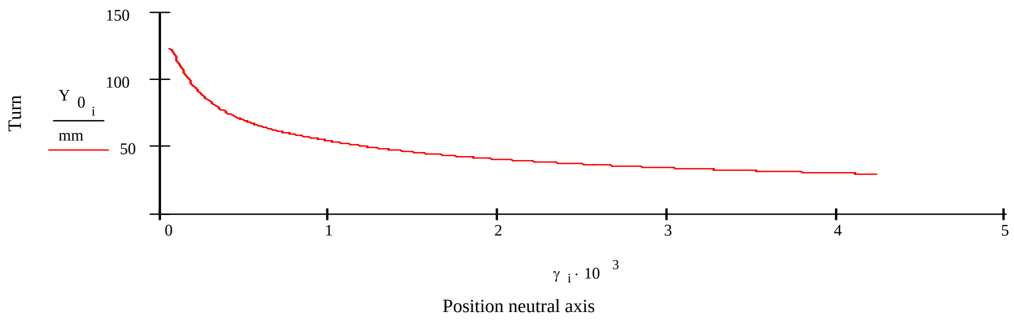
$\kappa_{cr} = 8.0340401 \times 10^{-4}\ \dfrac{1}{m}$

Number of steps: $n := 400$ $i := 0..n$

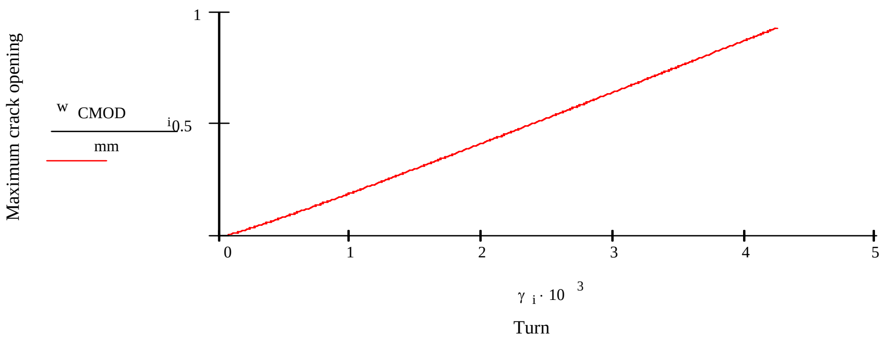
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \dfrac{\gamma_{cr}}{20}\right) + \dfrac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{20}$

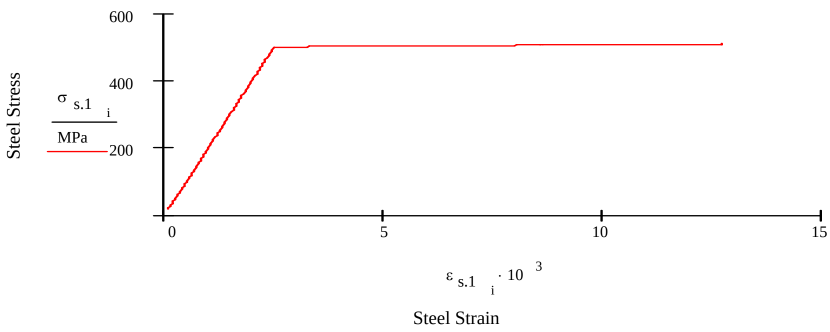
Position of the neutral axis when turn is increasing:



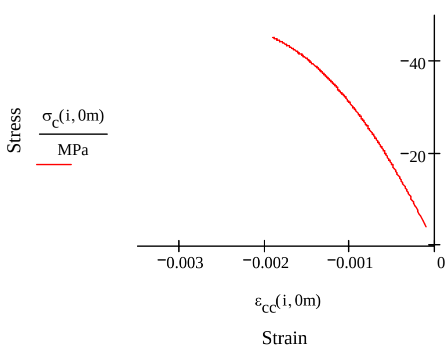
Maximum crack opening when turn is increasing:



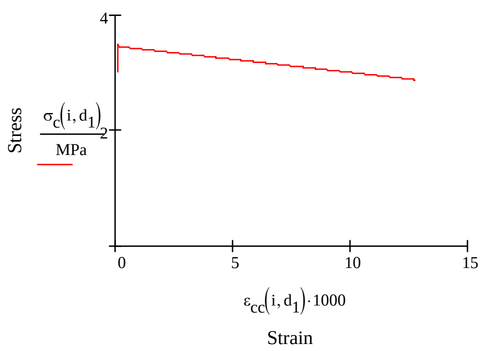
Stress Strain Reinforcement Diagram:



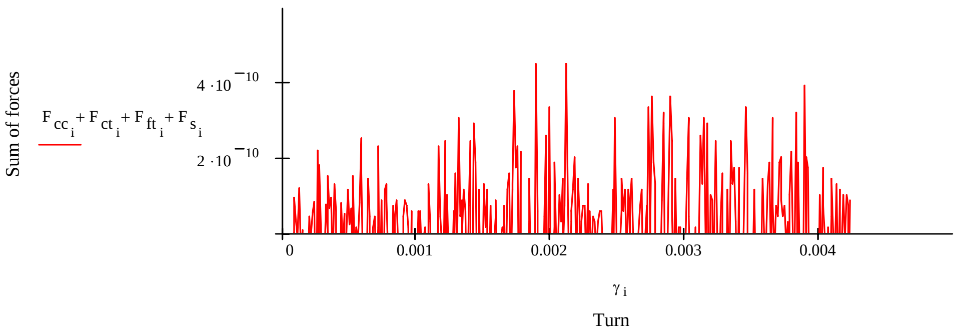
Stress-Strain diagram of the top concrete:



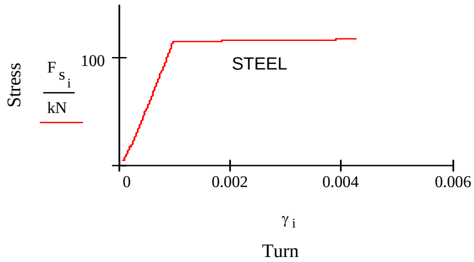
Stress-Strain relationship at the level of reinforcement:

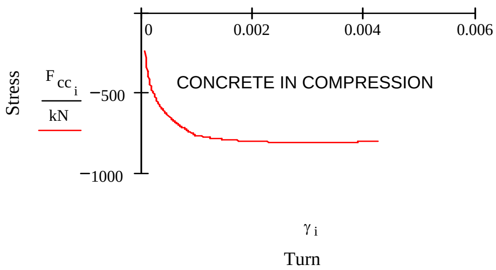


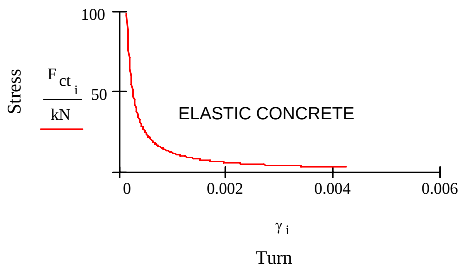
SUM OF FORCES=0 GRAPH

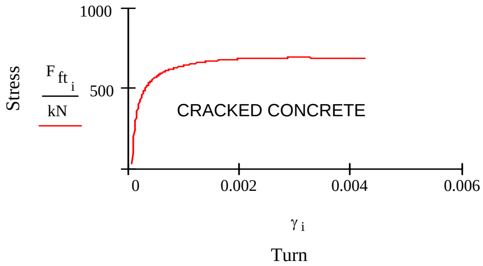


GRAPHS OF FORCES

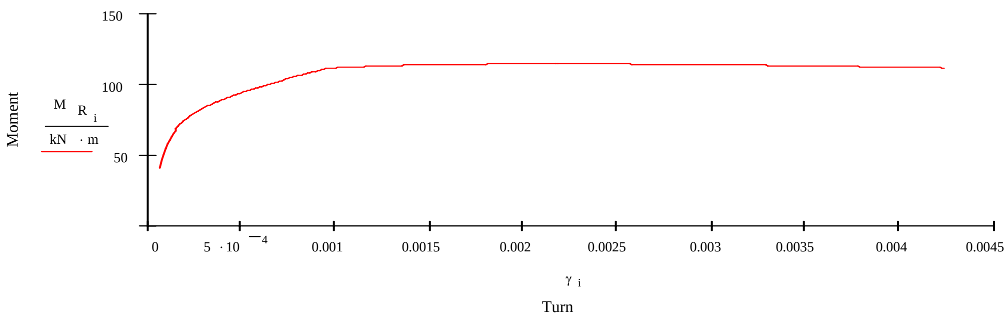




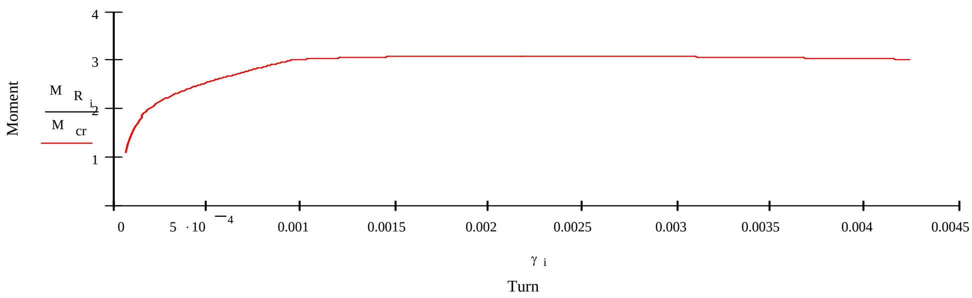




MOMENT-TURN GRAPH

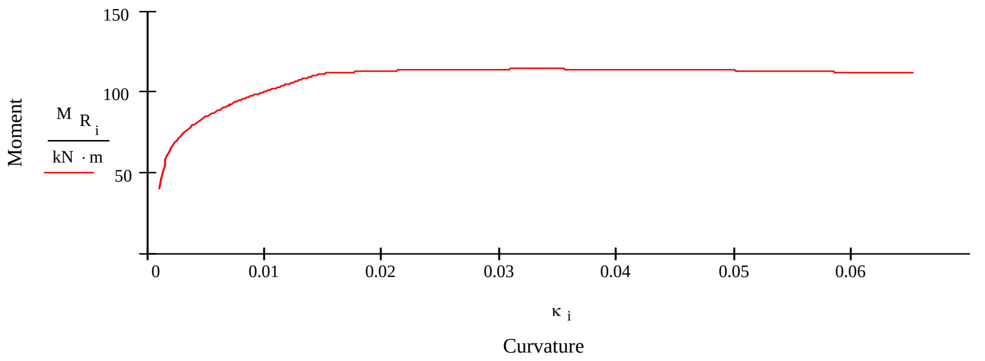


NORMALISED MOMENT-TURN GRAPH

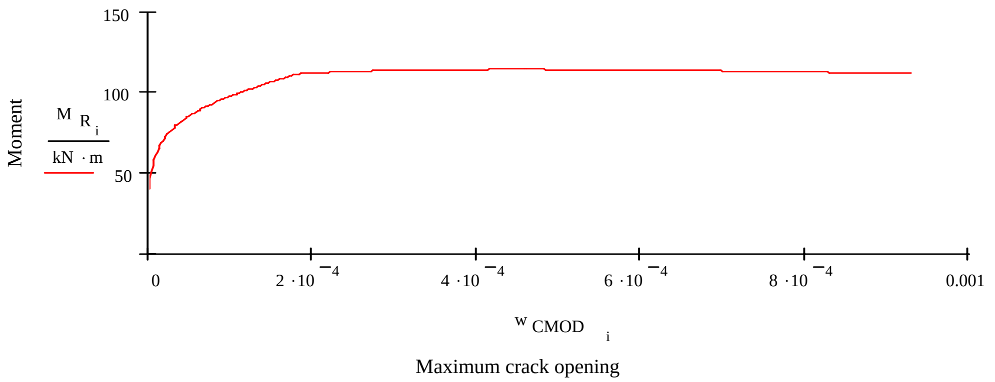


MOMENT-CURVATURE GRAPH

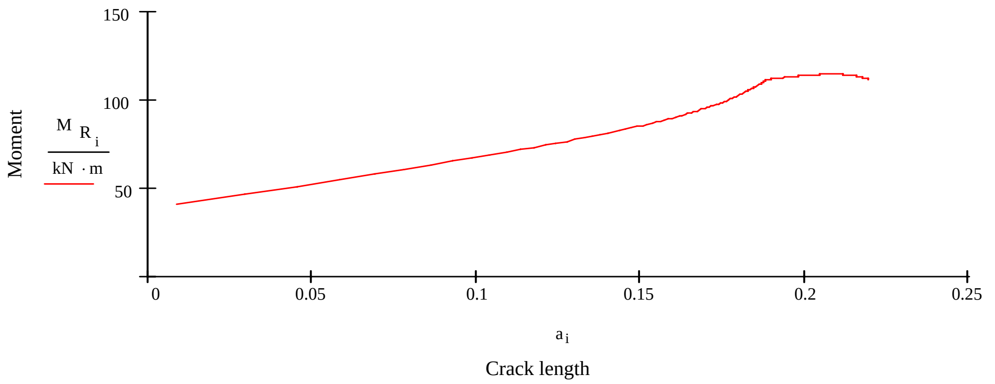
$$\kappa_i := \frac{\gamma_i}{s}$$



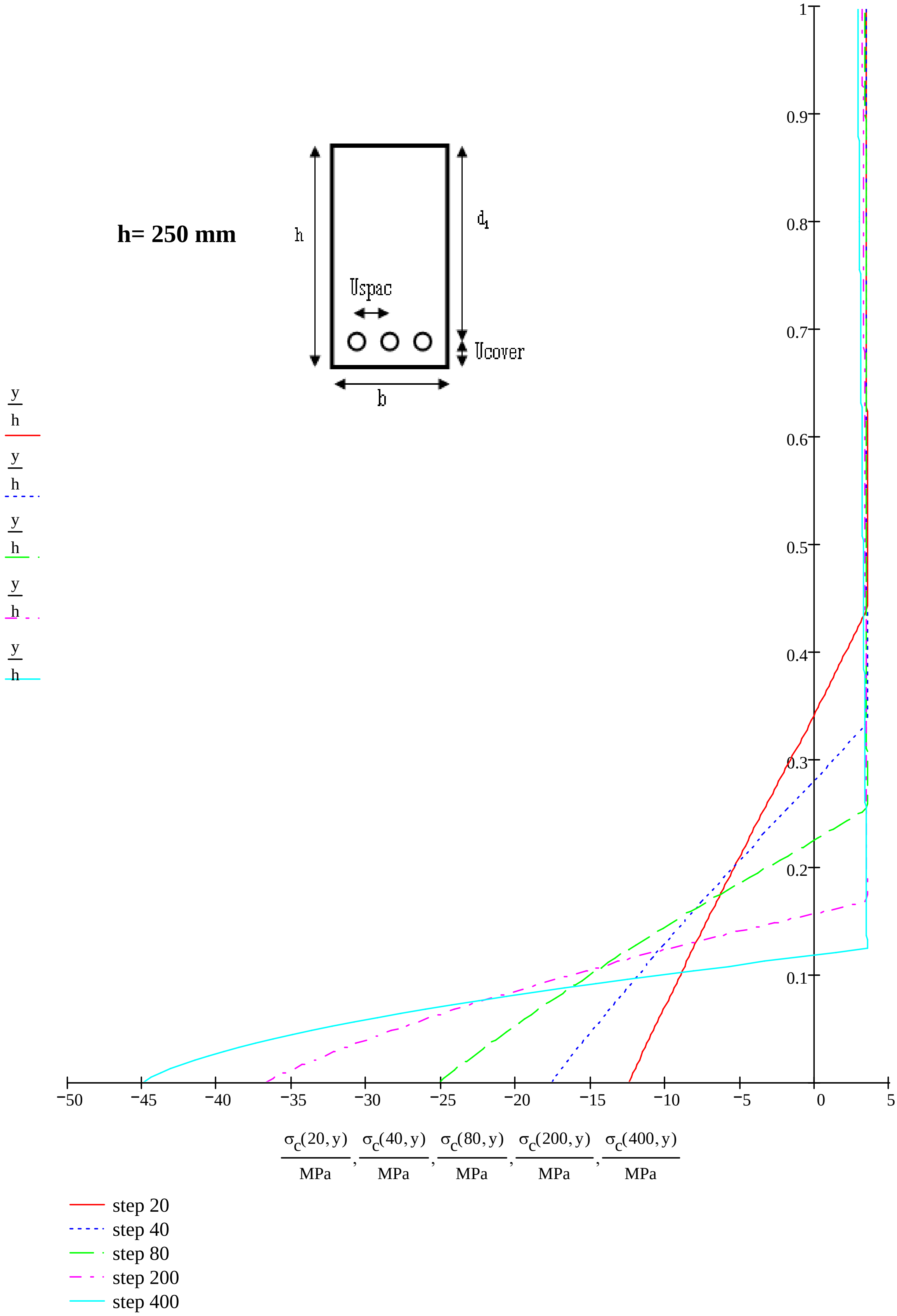
MOMENT-MAXIMUM CRACK OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS HEIGHT 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

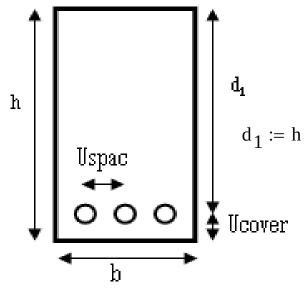
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \text{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317\,mm \qquad \phi_b := \text{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 9\,mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \text{round}\left(n_{ap}, 0\right)$ $n = 8$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 125.429\,mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.226\,m$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.503\,m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 251.301\,mm$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.0563 \times 10^{10}\,mm^4$

Critical moment (moment just before cracking) $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 148.656\,kN \cdot m$

<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Critical turn: $\gamma_{cr} := \dfrac{s}{h - x_{ef}} \cdot \varepsilon_{ct.cr}$ $\gamma_{cr} = 2.614 \times 10^{-5}$ Critical curvature: $\kappa_{cr} := \dfrac{\gamma_{cr}}{s}$

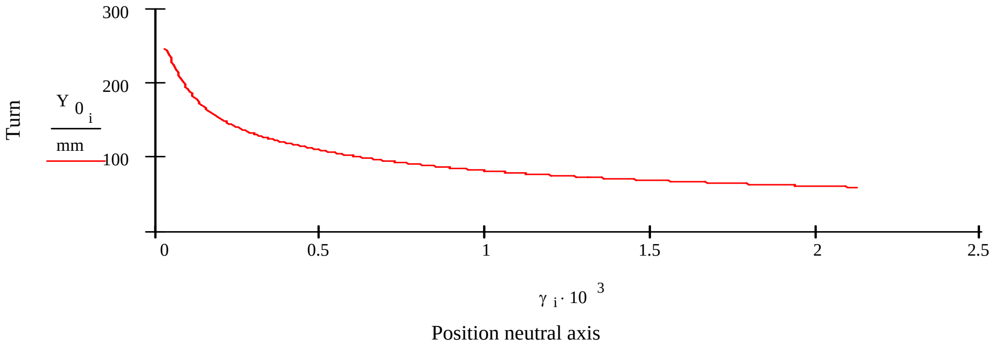
$\kappa_{cr} = 4.021076 \times 10^{-4}\,\dfrac{1}{m}$

Number of steps: $n := 400$ $i := 0..n$

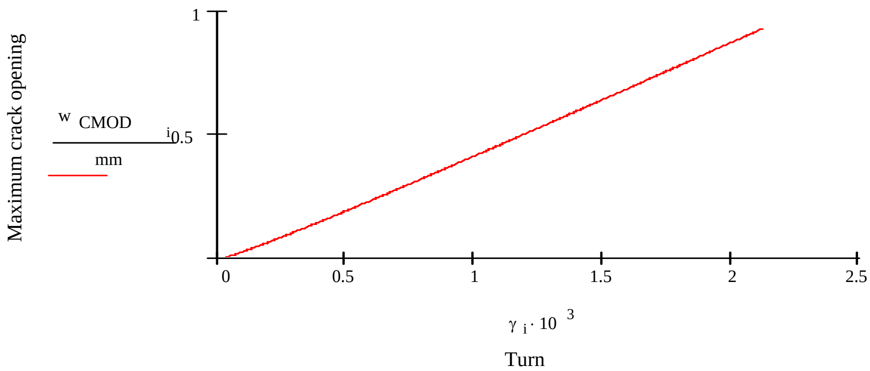
Values of the turn: $\gamma_i := \left(\gamma_{cr} + \dfrac{\gamma_{cr}}{20}\right) + \dfrac{\gamma_{cr}}{5} \cdot i$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{20}$

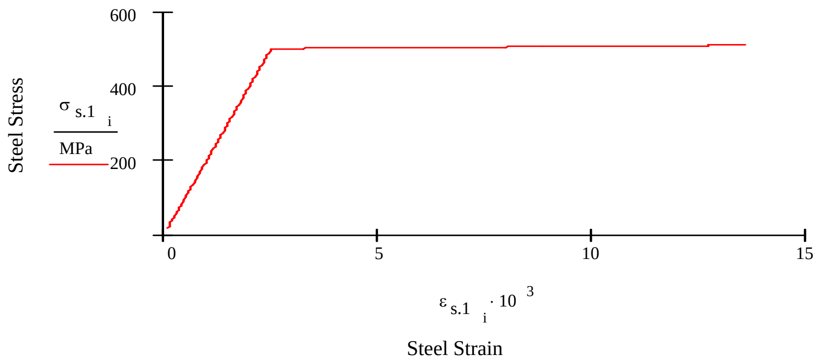
Position of the neutral axis when turn is increasing:



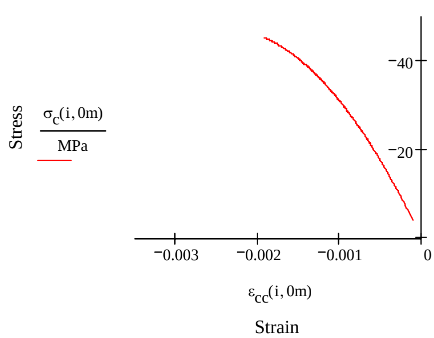
Maximum crack opening when turn is increasing:



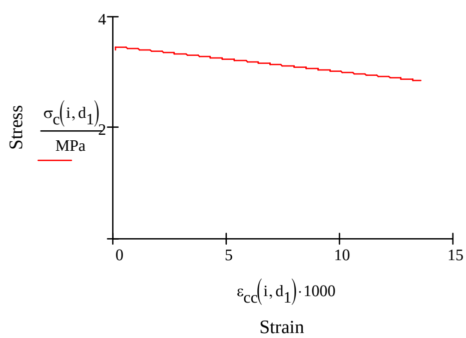
Stress Strain Reinforcement Diagram:



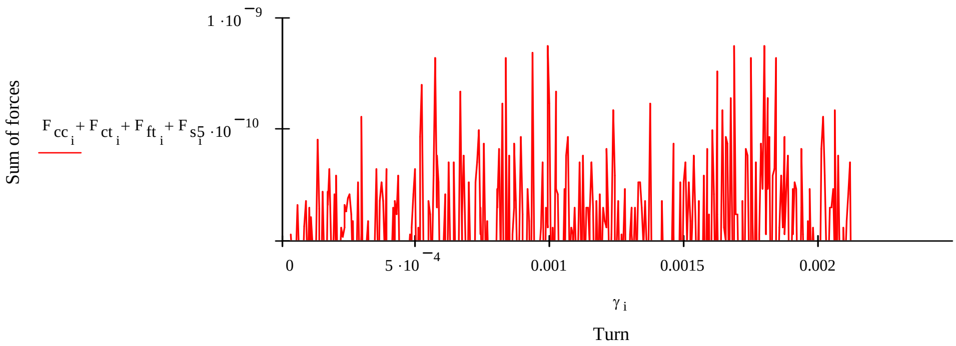
Stress-Strain diagram of the top concrete:



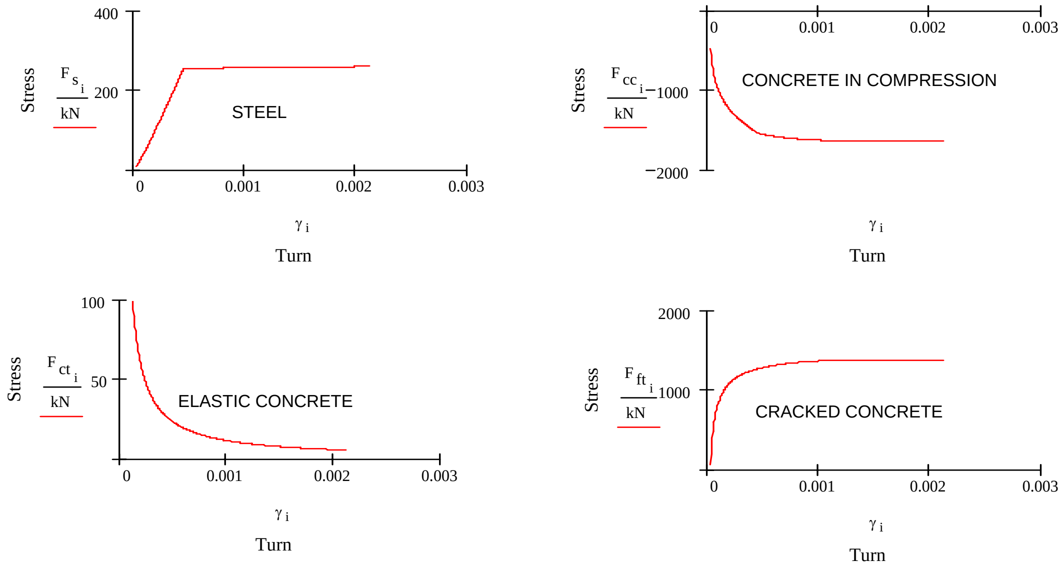
Stress-Strain relationship at the level of reinforcement:



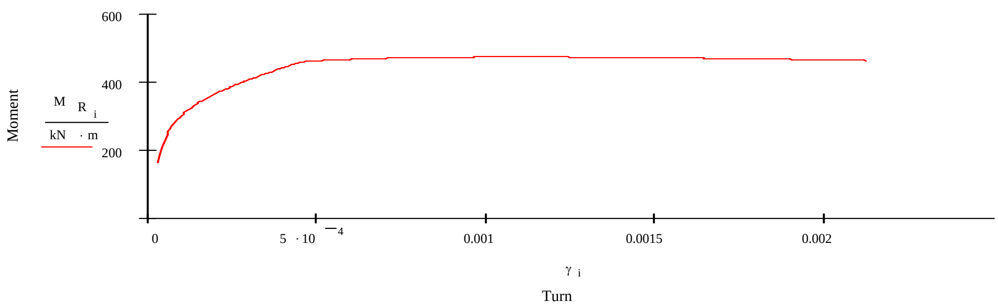
SUM OF FORCES=0 GRAPH



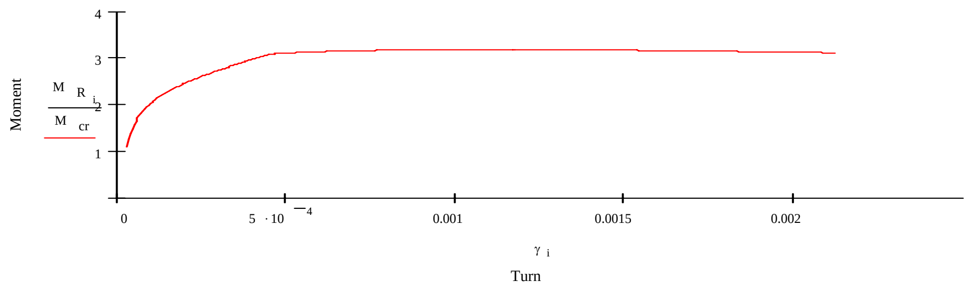
GRAPHS OF FORCES



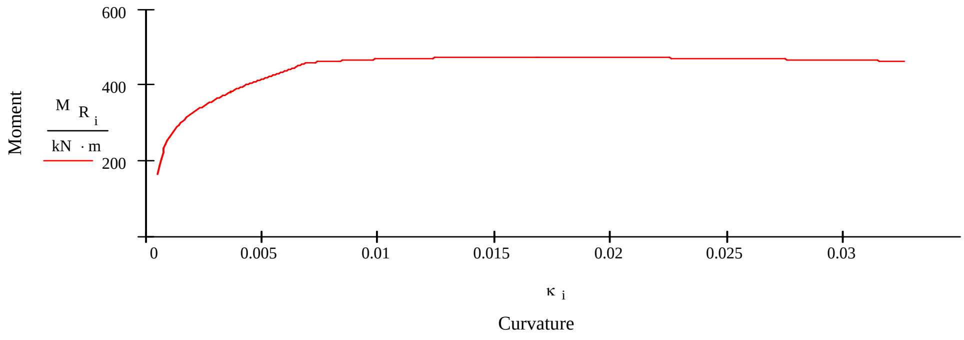
MOMENT-TURN GRAPH



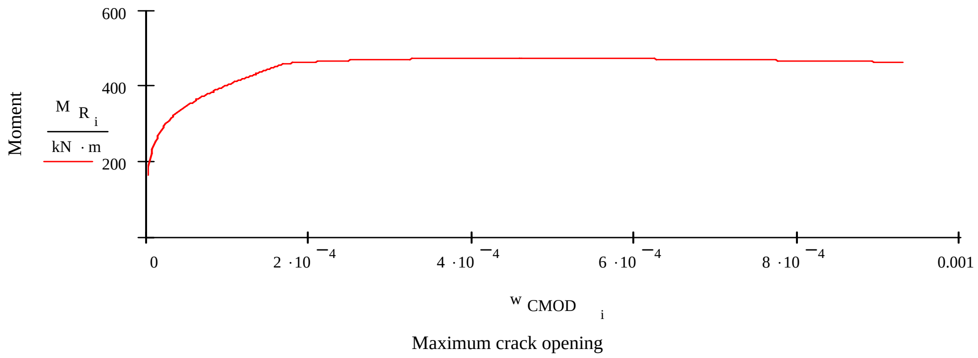
NORMALISED MOMENT-TURN GRAPH



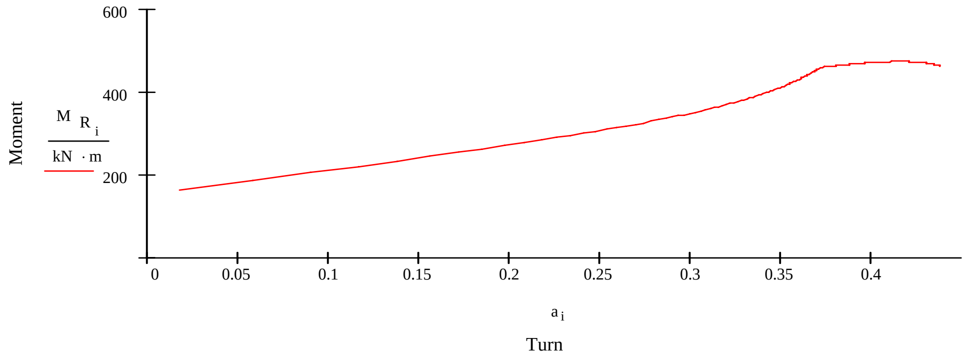
MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\gamma_i}{s}$



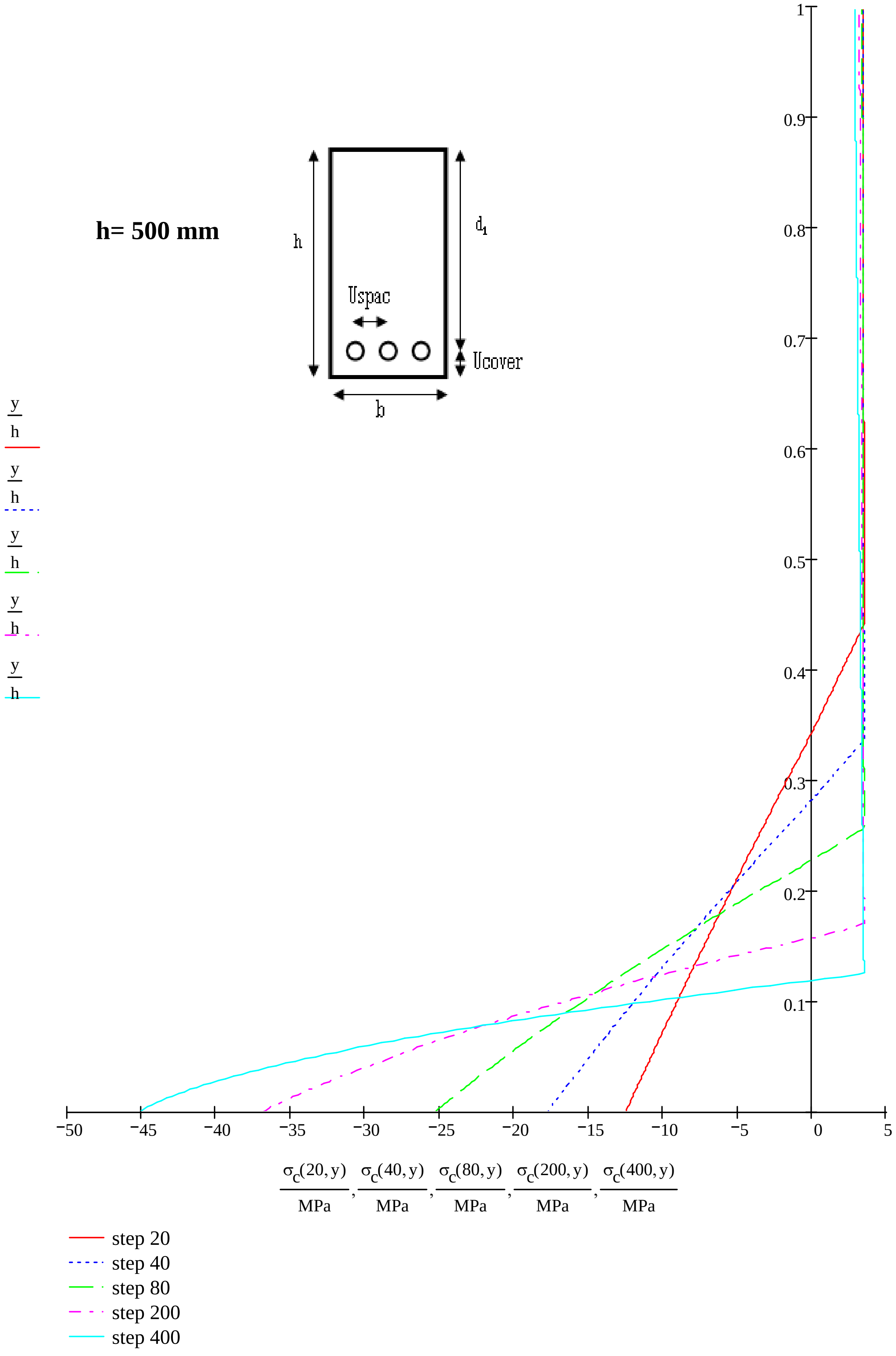
MOMENT-MAXIMUM CRACK OPENING GRAPH



MOMENT-CRACK EXTENSION GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



C.2.1 Sigma-epsilon relationship, analytical analysis. Mix A

MATERIAL PROPERTIES

Reinforcing steel:

Young modulus Steel: $E_s := 200 \cdot GPa$

Yielding strength: $f_{yk} := 500 \cdot MPa$

Yielding strain: $\varepsilon_{syk} := \frac{f_{yk}}{E_s}$

$\varepsilon_{syk} = 2.5 \times 10^{-3}$

Ultimate strain: $\varepsilon_{suk} := \frac{50}{1000}$

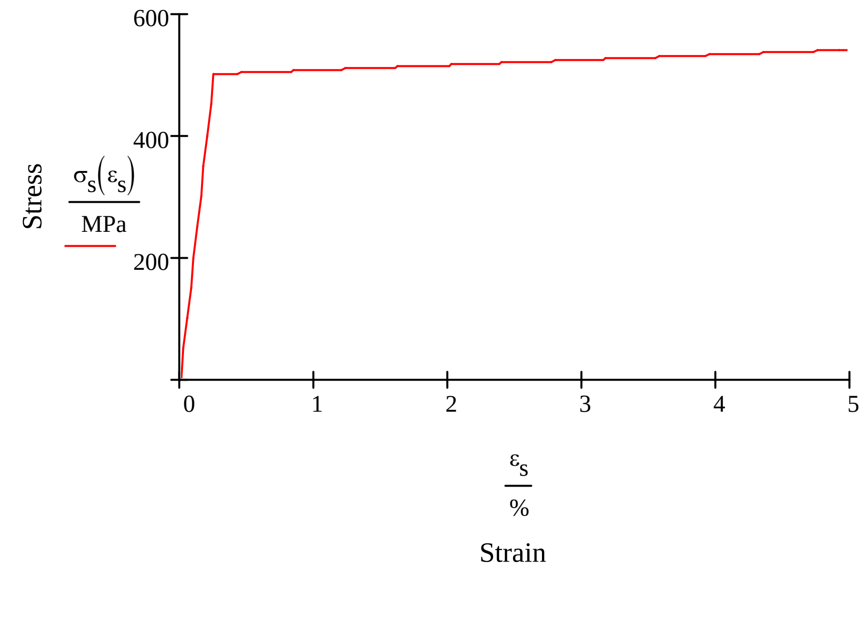
$k_s := 1.08$

Ultimate strength: $f_{uk} := k_s \cdot f_{yk}$

$f_{uk} = 540 MPa$

Reinforcement stress:

$$\sigma_s(\varepsilon_s) := \begin{cases} E_s \cdot \varepsilon_s & \text{if } \varepsilon_s \le \varepsilon_{syk} \\ \dfrac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_s - \dfrac{f_{yk}}{E_s} \right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_s \le \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \varepsilon_s > \varepsilon_{suk} \end{cases}$$



DIANA input strain: $\varepsilon_{diana} := \left(\frac{5}{100} \right) - \frac{f_{yk}}{E_s}$ $\varepsilon_{diana} = 0.048$

Concrete in compression:

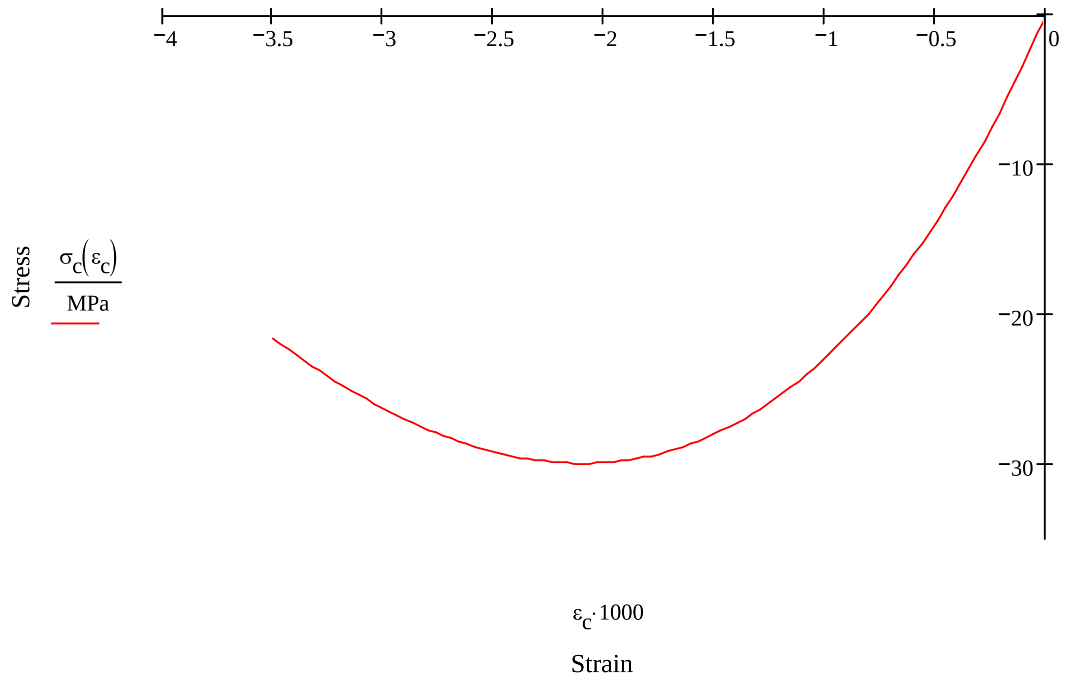
Mean compressive strength: $f_{cm} := 30\text{MPa}$

Modulus of Elasticity: $E_c := 22\left(\dfrac{\frac{f_{cm}}{\text{MPa}}}{10}\right)^{0.3}\cdot\text{GPa}$ $\qquad E_c = 30.589\text{GPa}$

Ultimate strain $\varepsilon_{cu} := \dfrac{3.5}{1000}$

Stress block factors: $\varepsilon_{c1} := 0.21\cdot\%$ $\qquad \eta(\varepsilon_c) := \dfrac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\qquad k := 1.1\cdot\dfrac{E_c\cdot|\varepsilon_{c1}|}{f_{cm}}$

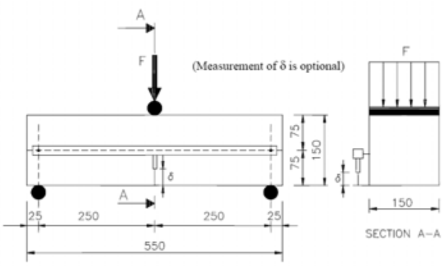
Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm}\cdot\dfrac{k\cdot\eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2)\cdot\eta(\varepsilon_c)}$ $\qquad \varepsilon_c := 0, \dfrac{-\varepsilon_{cu}}{100} .. -\varepsilon_{cu}$



Concrete in tension:

Tri-linear Stress-Crack Opening Relationship:

MIX A



TEXT SPECIMEN VALUES:

$h_{sp} := 124.15mm$ $\quad Leng := 500mm$ $\quad b := 151.26mm$

$f_{cm} := 30MPa$ $\quad F_{R1} := 11.335kN$ $\quad F_{R4} := 9.619kN$ $\quad F_L := 13.431kN$

REAL BEAM VALUES:

$u_{cover} := 25mm$

$h := 125mm$

$d_1 := h - u_{cover}$

Values of the RILEM constants:

$$f_{r1} := \frac{3 \cdot F_{R1} \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}} \qquad f_{r4} := \frac{3 \cdot F_{R4} \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}} \qquad f_{fctL} := \frac{3 \cdot F_L \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}}$$

$f_{r1} = 3.646MPa$ $\quad f_{r4} = 3.094MPa$ $\quad f_{fctL} = 4.321MPa$

$$k_h(h) := 1 - 0.6 \cdot \frac{\frac{h}{mm \cdot 10} - 12.5}{47.5} \qquad k_h(h) = 1$$

$$E_{cRILEM} := 9500 \left(\frac{f_{cm}}{MPa} \right)^{\frac{1}{3}} MPa \qquad E_{cRILEM} = 29.519GPa$$

Final Values for the curve:

$$\sigma_1 := \left[0.7 \cdot f_{fctL} \cdot \left(1.6 - \frac{d_1}{mm \cdot 1000} \right) \right] \qquad \varepsilon_1 := \frac{\sigma_1}{E_{cRILEM}} \qquad \sigma_1 = 4.537 \times 10^6 \ Pa \qquad \varepsilon_1 = 1.537 \times 10^{-4}$$

$$\sigma_2 := 0.45 f_{r1} \cdot k_h(h) \qquad \varepsilon_2 := \varepsilon_1 + \frac{0.1}{1000} \qquad \sigma_2 = 1.641 \times 10^6 \ Pa \qquad \varepsilon_2 = 2.537 \times 10^{-4}$$

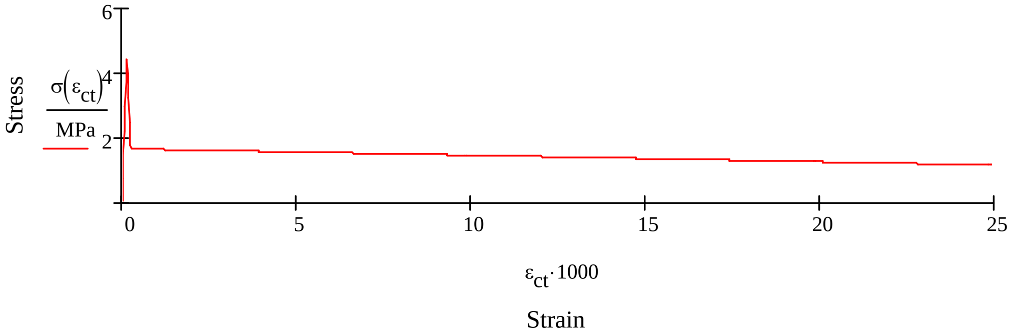
$$\sigma_3 := 0.37 \cdot f_{r4} \cdot k_h(h) \qquad \varepsilon_3 := \frac{25}{1000} \qquad \sigma_3 = 1.145 \times 10^6 \ Pa \qquad \varepsilon_3 = 0.025$$

Final expression for the curve:

$f_{ct} := \sigma_1$

$$\sigma(\varepsilon_{ct}) := \begin{cases} \frac{\sigma_1}{\varepsilon_1} \cdot \varepsilon_{ct} & \text{if } 0 \le \varepsilon_{ct} \le \varepsilon_1 \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot (\varepsilon_{ct} - \varepsilon_1) + \sigma_1 & \text{if } \varepsilon_1 < \varepsilon_{ct} \le \varepsilon_2 \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot (\varepsilon_{ct} - \varepsilon_3) + \sigma_3 & \text{if } \varepsilon_2 \le \varepsilon_{ct} \le \varepsilon_3 \end{cases}$$

Sigma-Epsilon relationship (Stress-Strain)



SECTIONAL ANALYSIS

Height of beam: $h := 125 \cdot mm$

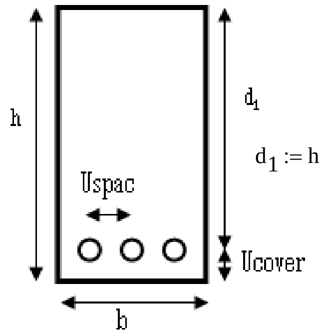
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spac} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spac} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 5\,mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 1.178 \times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.094\,m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.126\ m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 62.73\,mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63837 \times 10^8\,mm^4$

Critical moment (moment just before cracking) $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 11.936\,kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 110mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{20}$

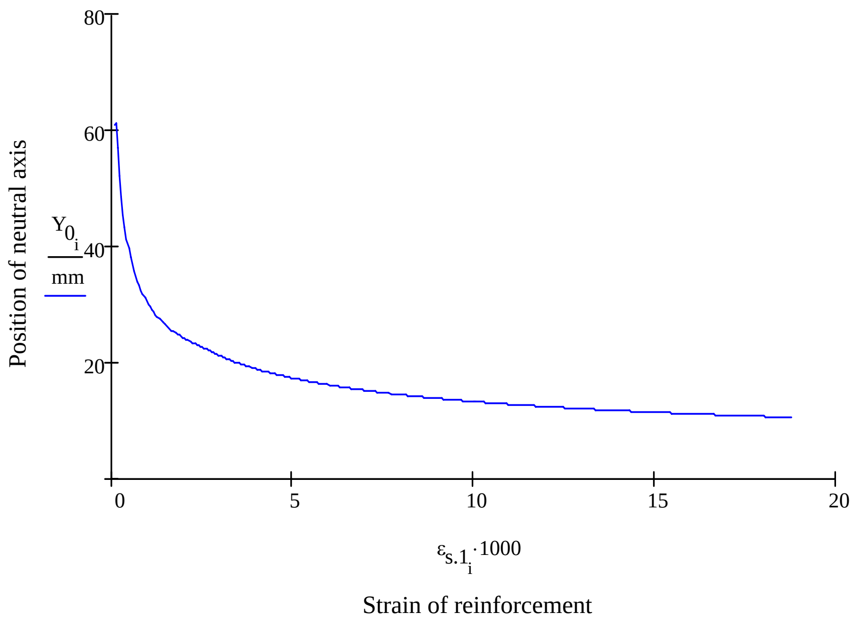
Equilibrium equation to find the position of the neutral axis:

$$Y_{0_i} := \operatorname{root}\Bigg[\int_0^{y_{0ini}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\dfrac{\varepsilon_{s.1_i}\cdot(y_{0ini}-y)}{d_1-y_{0ini}}\right]-\eta\left[\dfrac{\varepsilon_{s.1_i}\cdot(y_{0ini}-y)}{d_1-y_{0ini}}\right]^2}{1+(k-2)\cdot\eta\left[\dfrac{\varepsilon_{s.1_i}\cdot(y_{0ini}-y)}{d_1-y_{0ini}}\right]}\right]\cdot b\,dy \;\ldots$$

$$+\int_{y_{0ini}}^{h}\left[\begin{array}{ll} \dfrac{\sigma_1}{\varepsilon_1}\cdot\dfrac{\varepsilon_{s.1_i}\cdot(y-y_{0ini})}{d_1-y_{0ini}} & \text{if } y_{0ini}\le y\le \dfrac{\varepsilon_1\cdot(d_1-y_{0ini})}{\varepsilon_{s.1_i}}+y_{0ini} \\ \dfrac{\sigma_1-\sigma_2}{\varepsilon_1-\varepsilon_2}\cdot\left[\dfrac{\varepsilon_{s.1_i}\cdot(y-y_{0ini})}{d_1-y_{0ini}}-\varepsilon_1\right]+\sigma_1 & \text{if } \dfrac{\varepsilon_1\cdot(d_1-y_{0ini})}{\varepsilon_{s.1_i}}+y_{0ini}<y\le\dfrac{\varepsilon_2\cdot(d_1-y_{0ini})}{\varepsilon_{s.1_i}}+y_{0ini} \\ \dfrac{\sigma_2-\sigma_3}{\varepsilon_2-\varepsilon_3}\cdot\left[\dfrac{\varepsilon_{s.1_i}\cdot(y-y_{0ini})}{d_1-y_{0ini}}-\varepsilon_3\right]+\sigma_3 & \text{if } \dfrac{\varepsilon_2\cdot(d_1-y_{0ini})}{\varepsilon_{s.1_i}}+y_{0ini}<y\le\dfrac{(d_1-y_{0ini})\cdot\varepsilon_3}{\varepsilon_{s.1_i}}+y_{0ini} \\ 0\cdot\text{MPa} & \text{if } \dfrac{(d_1-y_{0ini})\cdot\varepsilon_3}{\varepsilon_{s.1_i}}+y_{0ini}<y \end{array}\right]\cdot b\,dy \;\ldots$$

$$+A_s\cdot\left|\begin{array}{ll} E_s\cdot\varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i}\le\varepsilon_{syk} \\ \dfrac{f_{yk}\cdot(k_s-1)}{\varepsilon_{suk}-\dfrac{f_{yk}}{E_s}}\left(\varepsilon_{s.1_i}-\dfrac{f_{yk}}{E_s}\right)+f_{yk} & \text{if } \varepsilon_{syk}<\varepsilon_{s.1_i}\le\varepsilon_{suk} \\ 0\cdot\text{MPa} & \text{if } \varepsilon_{s.1_i}>\varepsilon_{suk} \end{array}\right.\;,\;y_{0ini}\Bigg]$$

Position of the neutral axis when steel strain is increasing:



Stress and Strain STEEL:

Strain in reinforcement steel:

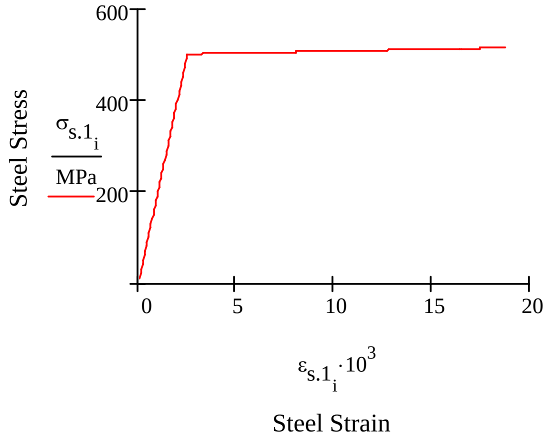
Bottom steel

$\varepsilon_{s.1_i}$

Stress in reinforcement steel:

Bottom steel

$$\sigma_{s.1_i} := \begin{cases} E_s \cdot \varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i} \le \varepsilon_{syk} \\ \dfrac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_{s.1_i} - \dfrac{f_{yk}}{E_s} \right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_{s.1_i} \le \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \varepsilon_{s.1_i} > \varepsilon_{suk} \end{cases}$$



Stress and Strain CONCRETE:

Concrete strain:

$$\varepsilon_{cc}(i,y) := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}} \cdot \left(y - Y_{0_i} \right)$$

Concrete stress:

Concrete in compresion:

$$\sigma_{cc}(i,y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right] - \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k-2) \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]}$$

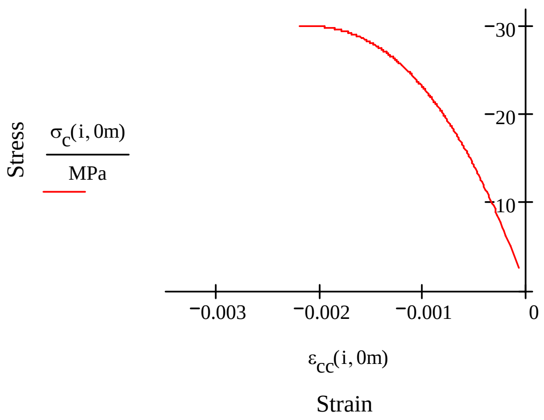
Concrete in tension:

$$\sigma_{ct}(i,y) := \begin{cases} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot MPa & \text{if } \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases}$$

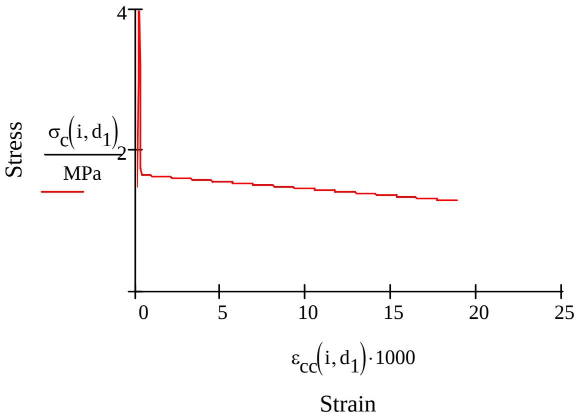
Final expression:

$$\sigma_c(i,y) := \begin{cases} \sigma_{cc}(i,y) & \text{if } 0\text{mm} \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) & \text{if } Y_{0_i} < y \le h \end{cases}$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium:

$F_{cc}+F_{ft}+F_{ct}+F_s=0$

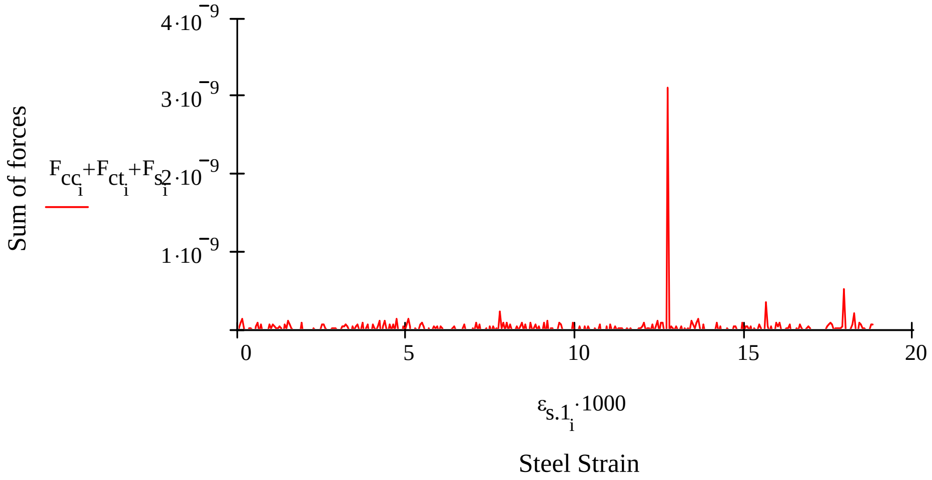
Steel force: $F_{s_i} := A_s \cdot \sigma_{s.1_i}$

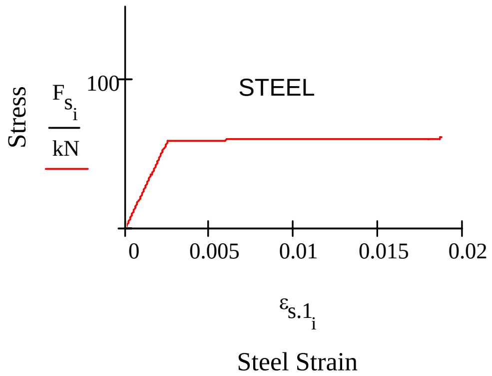
Concrete in compression force:

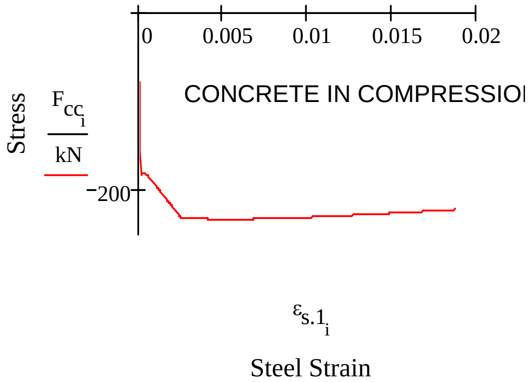
$$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right] - \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k-2) \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]} \right] \cdot b \, dy$$

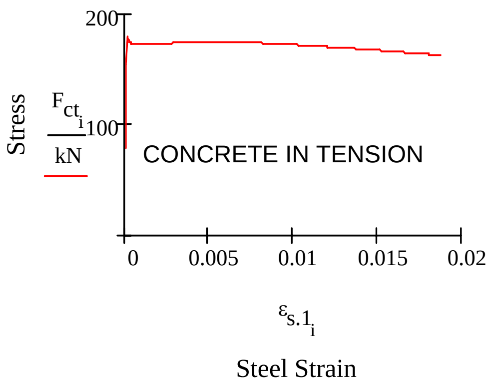
Concrete in tension:

$$F_{ct_i} := \int_{Y_{0_i}}^{h} \left[\begin{cases} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot \text{MPa} & \text{if } \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases} \right] \cdot b \, dy$$


Sum of forces
$F_{cc_i}+F_{ct_i}+F_{s_i}$
$4\cdot10^{-9}$
$3\cdot10^{-9}$
$2\cdot10^{-9}$
$1\cdot10^{-9}$
0
5
10
15
20
$\varepsilon_{s.1_i}\cdot1000$
Steel Strain


Stress
$\frac{F_{s_i}}{kN}$
100
STEEL
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Steel Strain


0
0.005
0.01
0.015
0.02
CONCRETE IN COMPRESSIO
Stress
$\frac{F_{cc_i}}{kN}$
−200
$\varepsilon_{s.1_i}$
Steel Strain

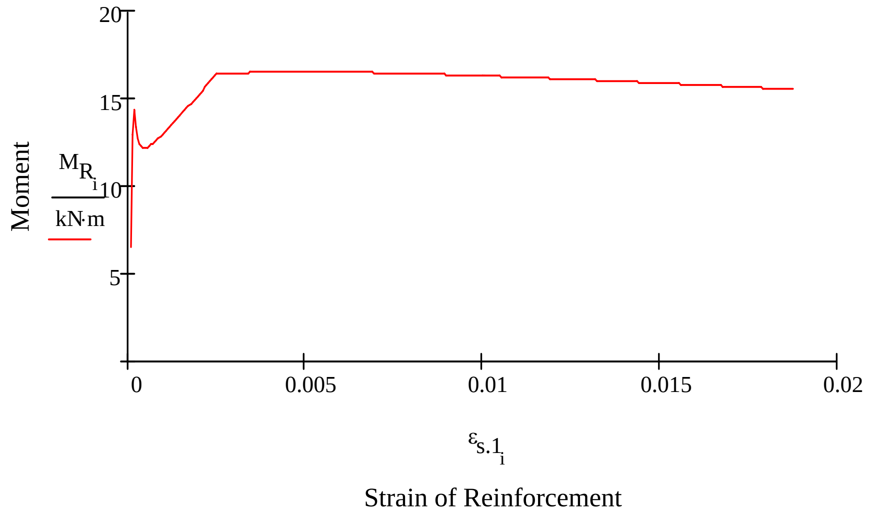

200
Stress
$\frac{F_{ct_i}}{kN}$
100
CONCRETE IN TENSION
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Steel Strain

Moment :

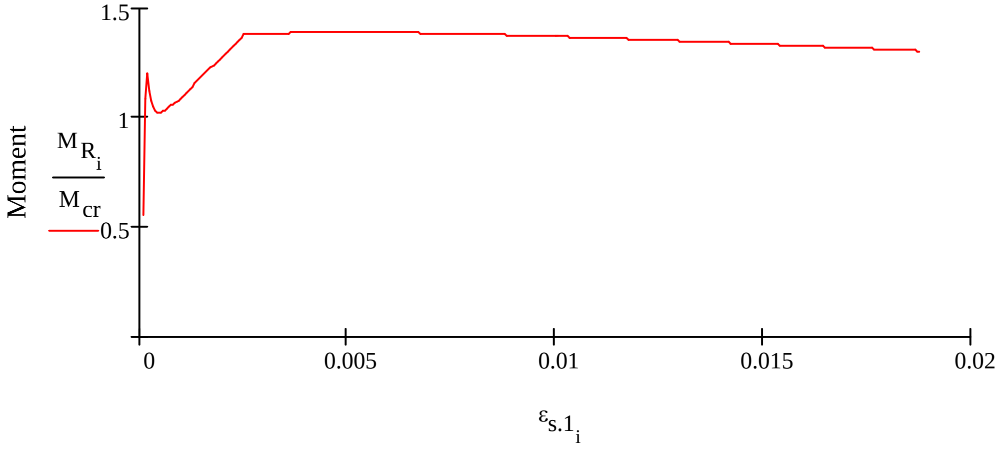
$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1-Y_{0_i}}\right] - \eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1-Y_{0_i}}\right]^2}{1+(k-2)\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1-Y_{0_i}}\right]} \right]\cdot b\cdot y\,dy\ \ldots$$

$$+ \int_{Y_{0_i}}^{h} \left[\begin{array}{ll} \frac{\sigma_1}{\varepsilon_1}\cdot\frac{\varepsilon_{s.1_i}\cdot\left(y-Y_{0_i}\right)}{d_1-Y_{0_i}} & \text{if } Y_{0_i} \le y < \frac{\varepsilon_1\cdot\left(d_1-Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_1-\sigma_2}{\varepsilon_1-\varepsilon_2}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y-Y_{0_i}\right)}{d_1-Y_{0_i}} - \varepsilon_1\right] + \sigma_1 & \text{if } \frac{\varepsilon_1\cdot\left(d_1-Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\varepsilon_2\cdot\left(d_1-Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_2-\sigma_3}{\varepsilon_2-\varepsilon_3}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y-Y_{0_i}\right)}{d_1-Y_{0_i}} - \varepsilon_3\right] + \sigma_3 & \text{if } \frac{\varepsilon_2\cdot\left(d_1-Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\left(d_1-Y_{0_i}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0\cdot MPa & \text{if } \frac{\left(d_1-Y_{0_i}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{array} \right]\cdot b\cdot y\,dy + F_{s_i}\cdot\left(d_1\right)$$

MOMENT-REINFORCEMENT STRAIN GRAPH

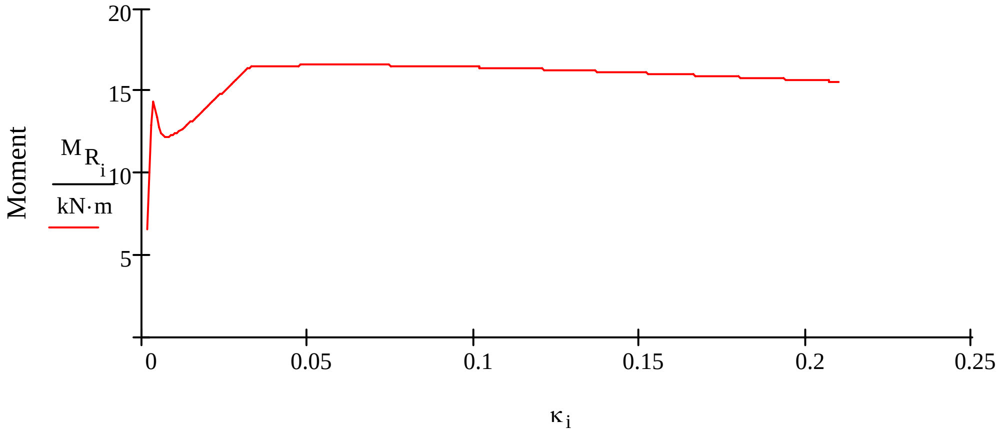


NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH



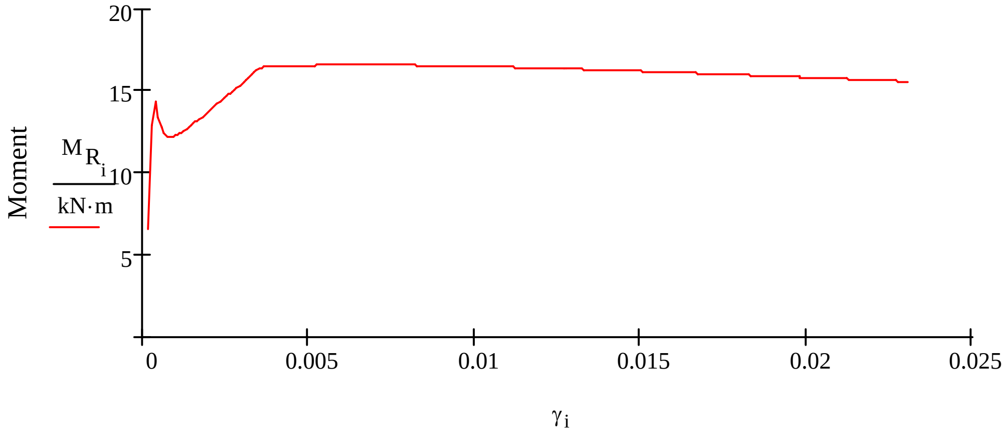
Strain of reinforcement

MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}}$ $\gamma_i := \kappa_i \cdot s$



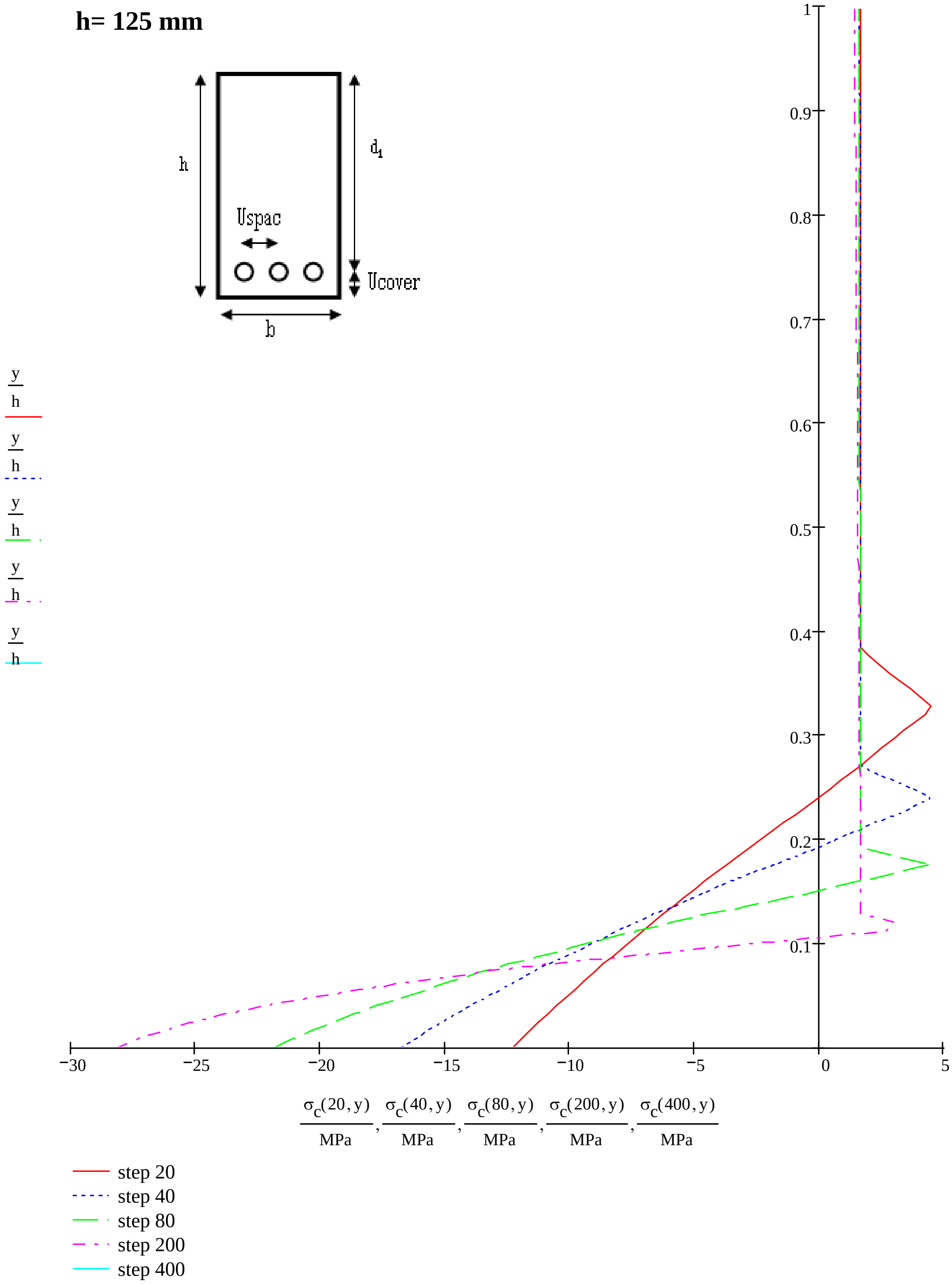
Curvature

MOMENT-TURN GRAPH



Turn

STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS

HEIGHT 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

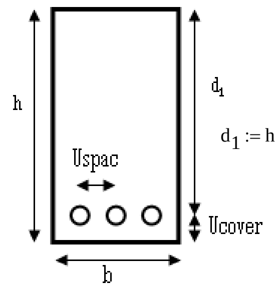
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci}\cdot\left(\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$$

Final bar diameter: $\phi_b = 7\,mm$

Steel one bar Area: $A_{s.i} := \pi\dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \dfrac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n-1}$ $u_{spac} = 181.6mm$

Total steel area: $A_s := n\cdot A_{s.i}$ $A_s = 2.309\times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2\cdot\pi\cdot\left(\dfrac{\phi_b}{2}\right)\cdot n$ $perim = 0.132\,m$

Effective area: $A_{ef} := b\cdot h + \left(\dfrac{E_s}{E_c}\right)\cdot A_s$ $A_{ef} = 0.252\,m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b\cdot h\cdot\dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right)\cdot A_s\cdot d_1}{A_{ef}}$ $x_{ef} = 125.6mm$

Inertia Moment: $I_{ef} := \dfrac{b\cdot h^3}{12} + b\cdot h\cdot\left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right)\cdot A_s\cdot(d_1 - x_{ef})^2$ $I_{ef} = 1.31709\times 10^9\,mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \dfrac{I_{ef}\cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 44.03kN\cdot m$

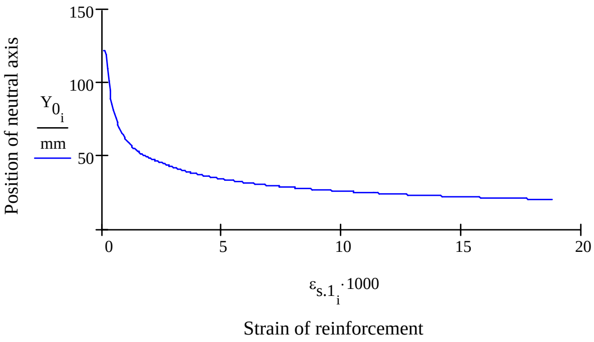
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 135mm$

Number of steps: $n_{step} := 375$ $i := 1..n_{step}$

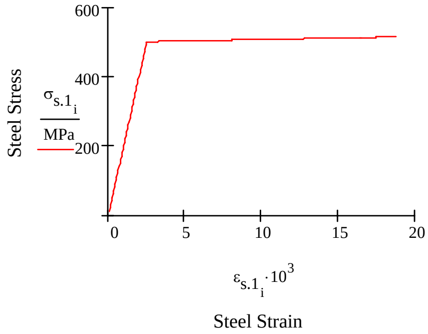
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \dfrac{0.05\cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{10}$

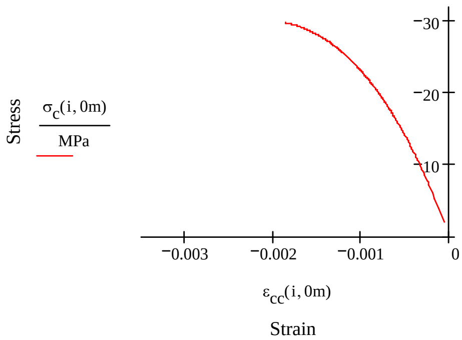
Position of the neutral axis when steel strain is increasing:



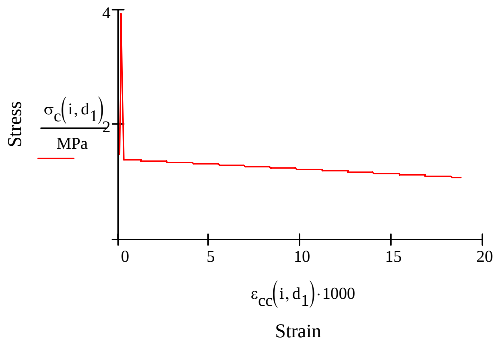
Stress Strain Reinforcement Diagram:



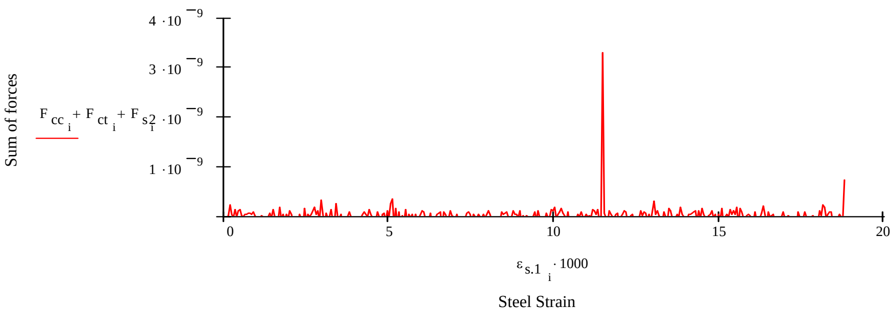
Stress-Strain diagram of the top concrete:



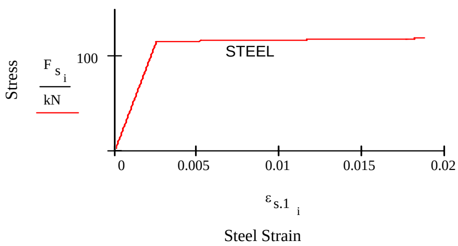
Stress-Strain diagram at the level of reinforcement:

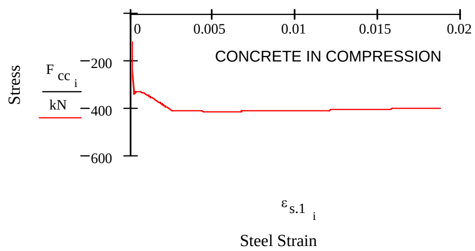


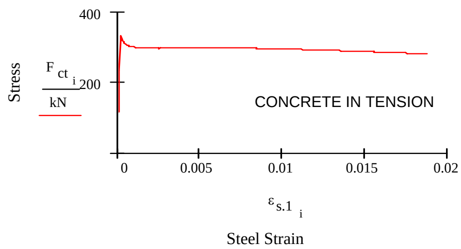
SUM OF FORCES=0 GRAPH



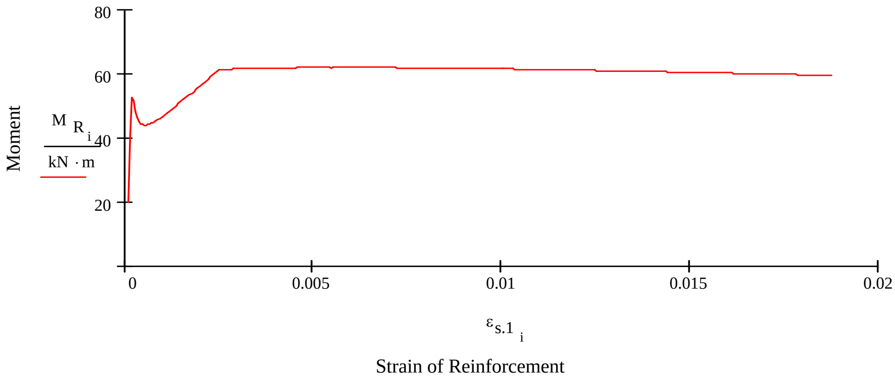
GRAPHS OF FORCES



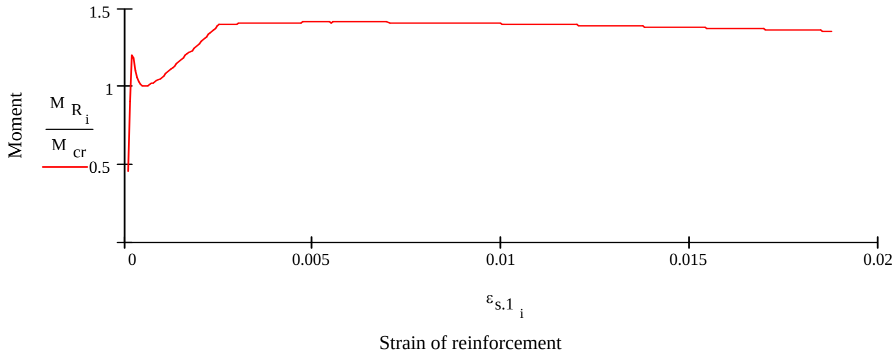




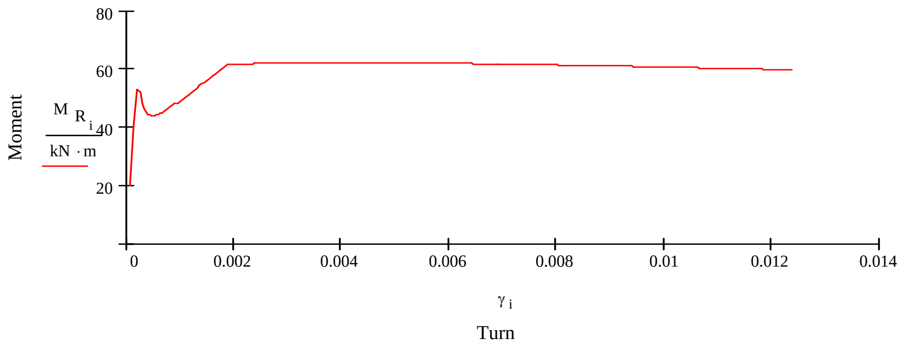
MOMENT-REINFORCEMENT STRAIN GRAPH



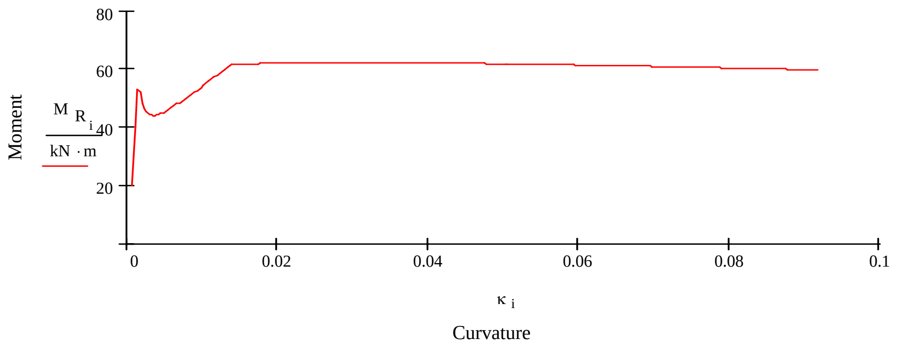
NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH



MOMENT-TURN GRAPH $\kappa_i := \dfrac{\varepsilon_{s.1_i}}{d_1 - Y0_i}$ $\gamma_i := \kappa_i \cdot s$

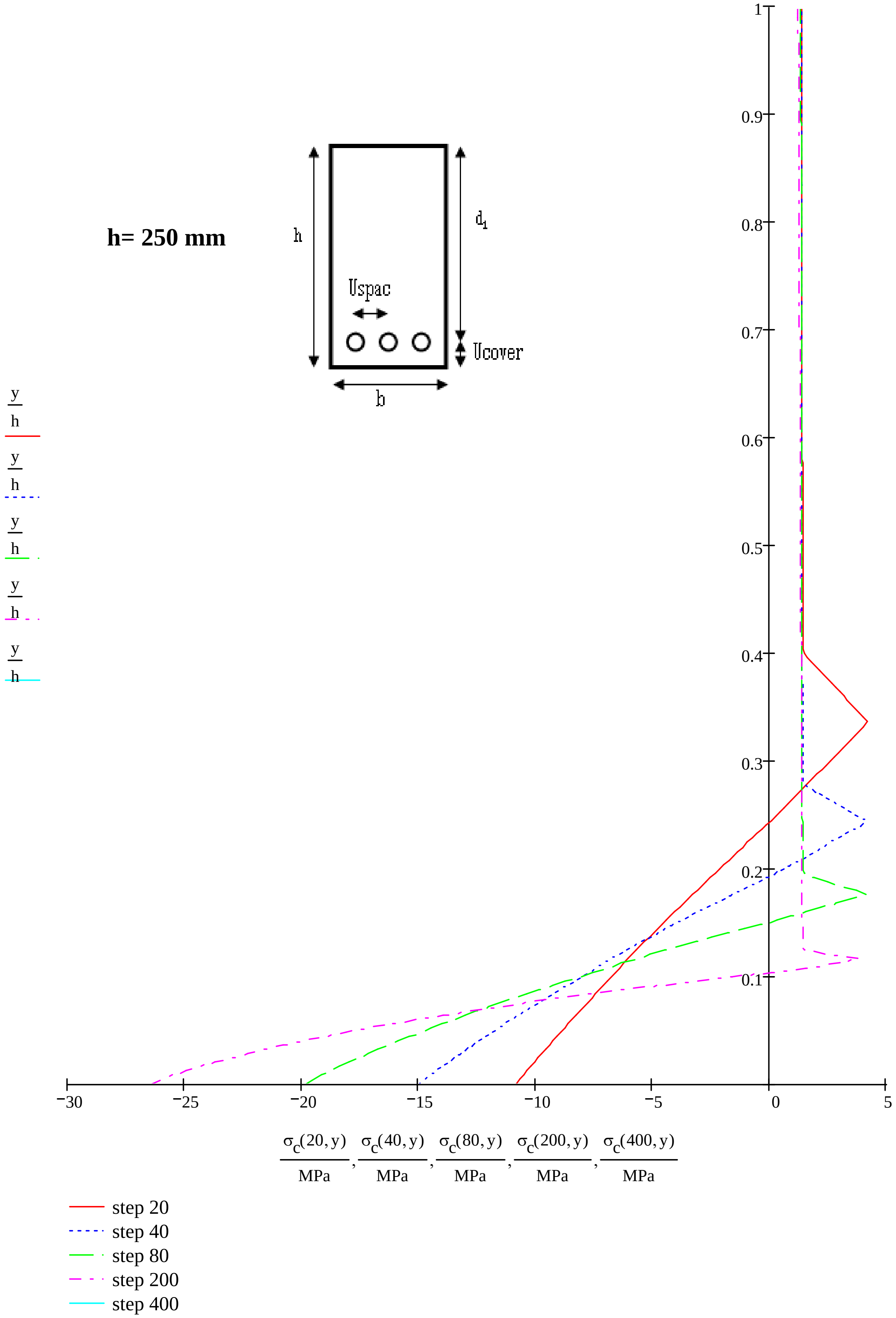


MOMENT-CURVATURE GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:

h= 250 mm



SECTIONAL ANALYSIS

HEIGHT 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

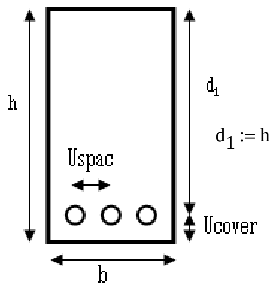
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 9mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 8$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 125.429mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4} \, m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.226 \, m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.503 \, m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 251.488mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot (d_1 - x_{ef})^2$ $I_{ef} = 1.0584 \times 10^{10} \, mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 144.912kN \cdot m$

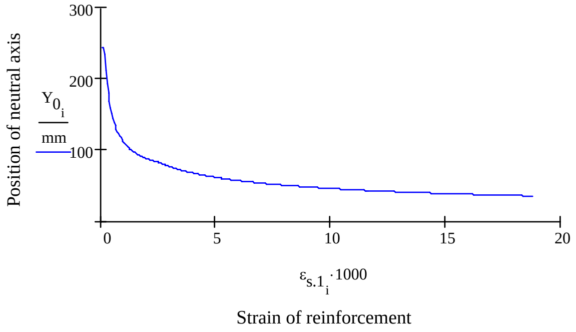
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 105mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

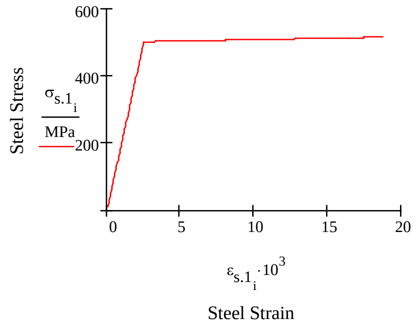
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{15}$

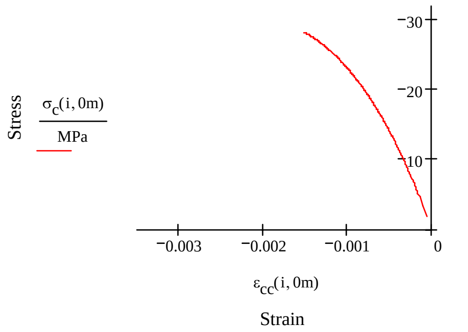
Position of the neutral axis when steel strain is increasing:



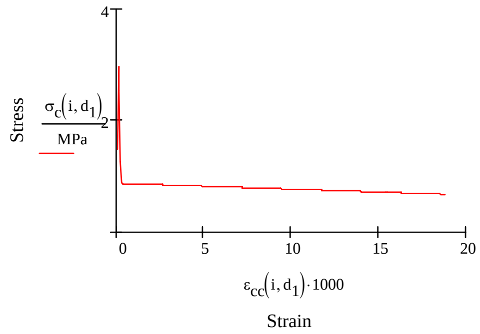
Stress Strain Reinforcement Diagram:



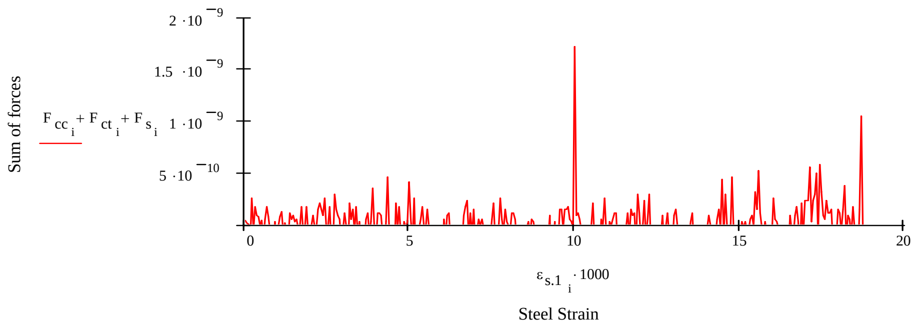
Stress-Strain diagram of the top concrete:



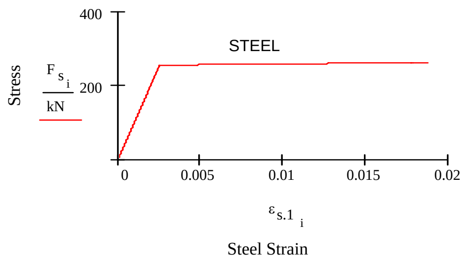
Stress-Strain diagram at the level of reinforcement:

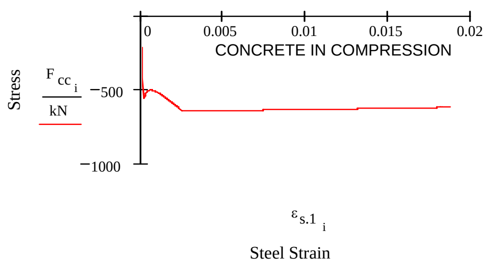


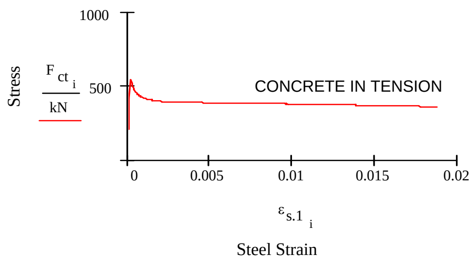
SUM OF FORCES=0 GRAPH



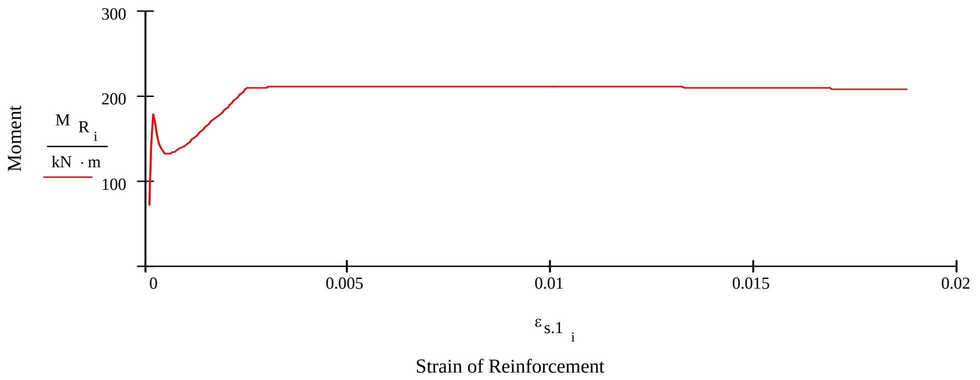
GRAPHS OF FORCES



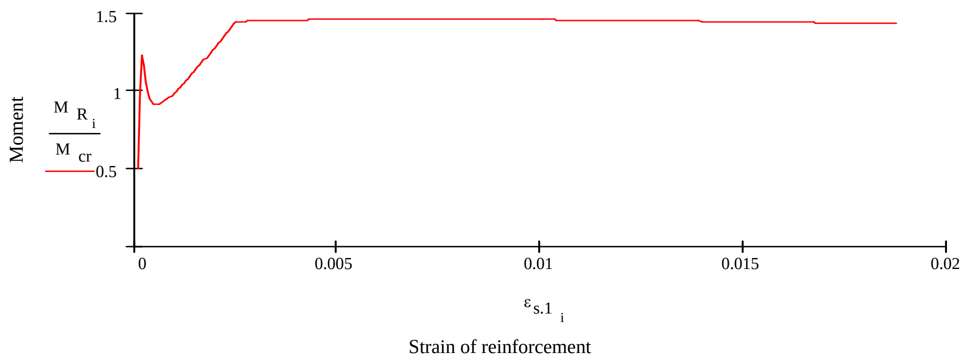




MOMENT-REINFORCEMENT STRAIN GRAPH

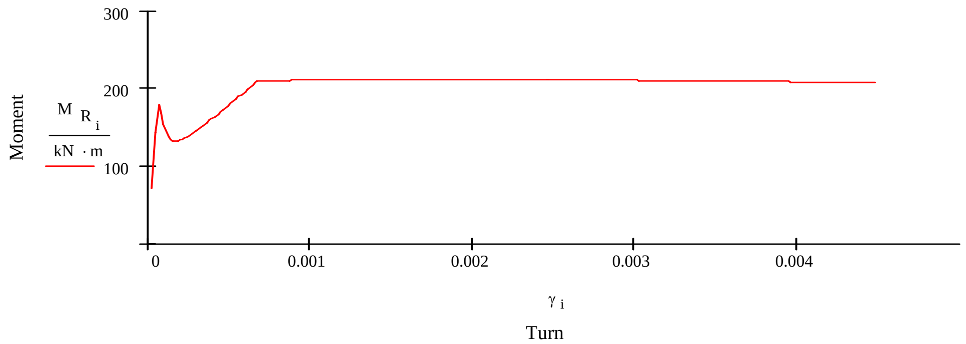


NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH

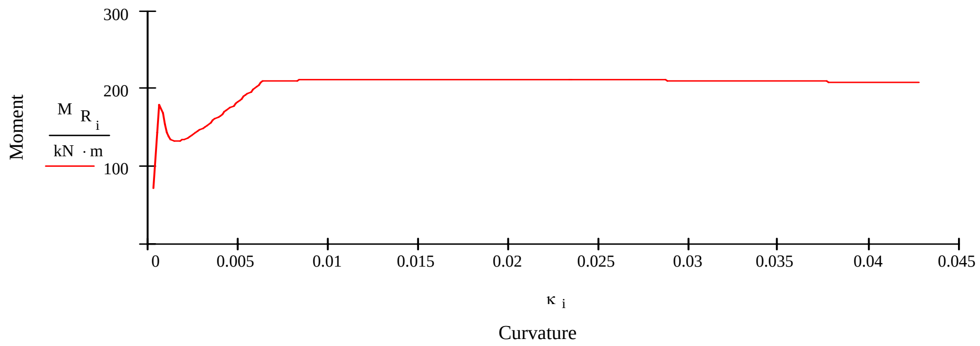


MOMENT-TURN GRAPH

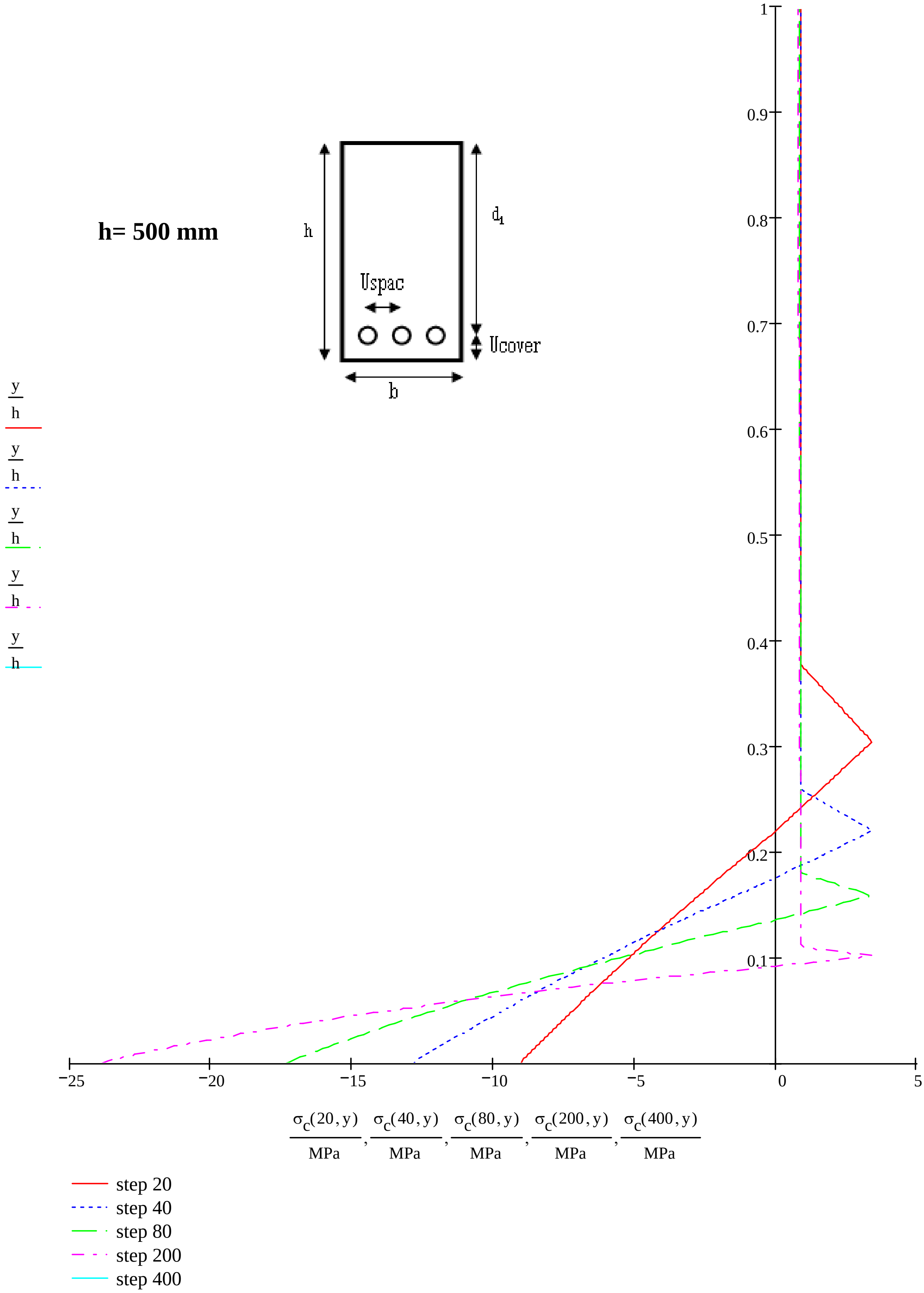
$$\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}} \qquad \gamma_i := \kappa_i \cdot s$$



MOMENT-CURVATURE GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



C.2.2 Sigma-epsilon relationship, analytical analysis. Mix B

MATERIAL PROPERTIES

Concrete in compression:

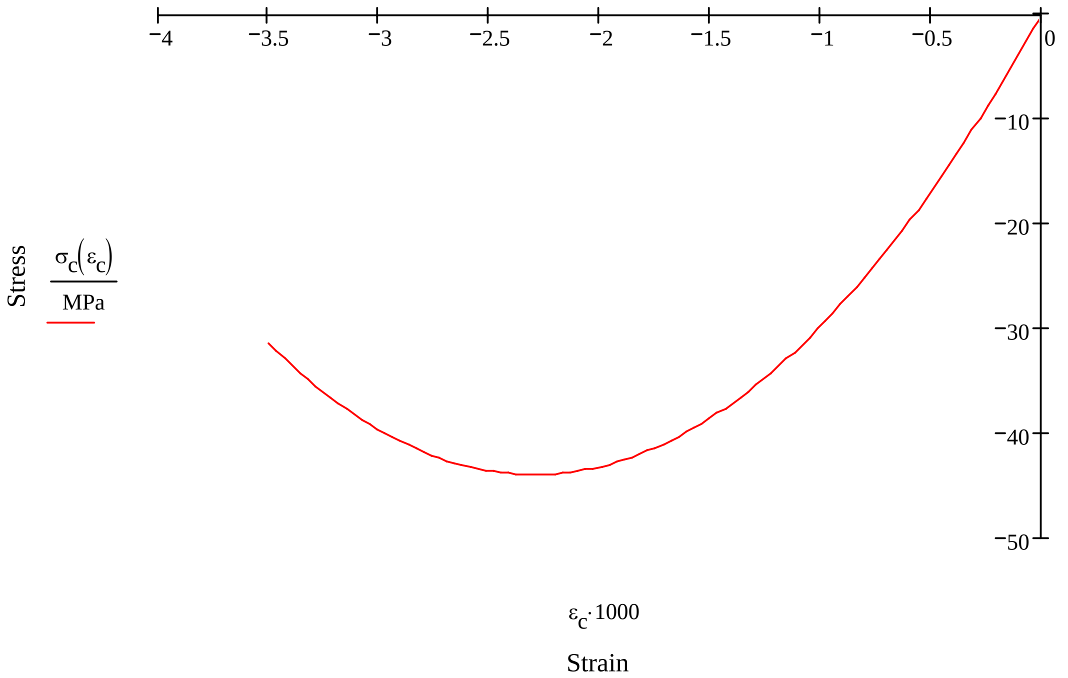
Mean compressive strength: $f_{cm} := 44MPa$

Modulus of Elasticity: $E_c := 22\left(\frac{\frac{f_{cm}}{MPa}}{10}\right)^{0.3} \cdot GPa$ $\qquad E_c = 34.313GPa$

Ultimate strain $\varepsilon_{cu} := \frac{3.5}{1000}$

Stress block factors: $\varepsilon_{c1} := 0.23\%$ $\qquad \eta(\varepsilon_c) := \frac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\qquad k := 1.1 \cdot \frac{E_c \cdot |\varepsilon_{c1}|}{f_{cm}}$

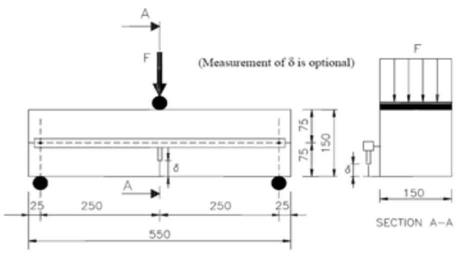
Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm} \cdot \frac{k \cdot \eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2) \cdot \eta(\varepsilon_c)}$ $\qquad \varepsilon_c := 0, \frac{-\varepsilon_{cu}}{100} .. -\varepsilon_{cu}$



Concrete in tension:

Tri-linear Stress-Crack Opening Relationship:

MIX B



TEXT SPECIMEN VALUES:

$h_{sp} := 128.07mm$ $\quad Leng := 500mm$ $\quad b := 151.44mm$

$f_{cm} := 44MPa$ $\quad F_{R1} := 28.502kN$ $\quad F_{R4} := 23.145kN$ $\quad F_L := 19.837kN$

REAL BEAM VALUES:

$u_{cover} := 25mm$

$h := 125mm$

$d_1 := h - u_{cover}$

Values of the RILEM constants:

$$f_{r1} := \frac{3 \cdot F_{R1} \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}} \qquad f_{r4} := \frac{3 \cdot F_{R4} \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}} \qquad f_{fctL} := \frac{3 \cdot F_L \cdot Leng}{2 \cdot b \cdot h_{sp}^{\ 2}}$$

$f_{r1} = 8.606\,MPa$ $\quad f_{r4} = 6.988\,MPa$ $\quad f_{fctL} = 5.99\,MPa$

$$k_h(h) := 1 - 0.6 \cdot \frac{\frac{h}{mm \cdot 10} - 12.5}{47.5} \qquad k_h(h) = 1 \qquad E_{cRILEM} := 9500 \cdot \left(\frac{f_{cm}}{MPa}\right)^{\frac{1}{3}} MPa \qquad E_{cRILEM} = 33.538\,GPa$$

Final Values for the curve:

$$\sigma_1 := \left[0.7 \cdot f_{fctL} \cdot \left(1.6 - \frac{d_1}{mm \cdot 1000}\right)\right] \qquad \varepsilon_1 := \frac{\sigma_1}{E_{cRILEM}} \qquad \sigma_1 = 6.289 \times 10^6\ Pa \qquad \varepsilon_1 = 1.875 \times 10^{-4}$$

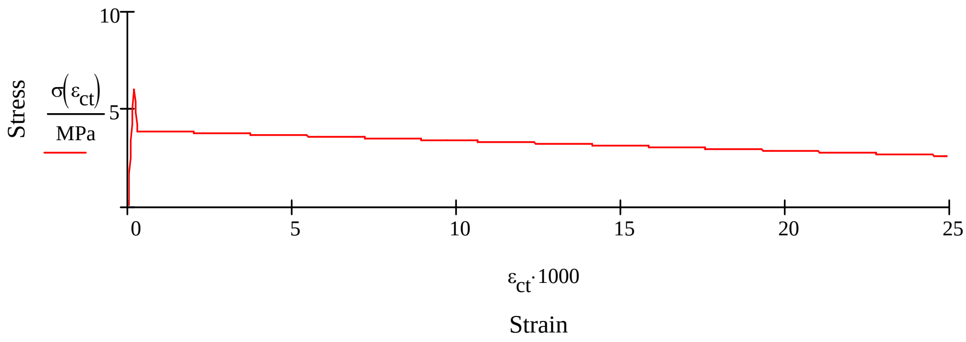
$$\sigma_2 := 0.45 \cdot f_{r1} \cdot k_h(h) \qquad \varepsilon_2 := \varepsilon_1 + \frac{0.1}{1000} \qquad \sigma_2 = 3.873 \times 10^6\ Pa \qquad \varepsilon_2 = 2.875 \times 10^{-4}$$

$$\sigma_3 := 0.37 \cdot f_{r4} \cdot k_h(h) \qquad \varepsilon_3 := \frac{25}{1000} \qquad \sigma_3 = 2.586 \times 10^6\ Pa \qquad \varepsilon_3 = 0.025$$

Final expression for the curve:

$$\sigma(\varepsilon_{ct}) := \left| \begin{array}{l} \frac{\sigma_1}{\varepsilon_1} \cdot \varepsilon_{ct} \ \text{ if } \ 0 \le \varepsilon_{ct} \le \varepsilon_1 \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot (\varepsilon_{ct} - \varepsilon_1) + \sigma_1 \ \text{ if } \ \varepsilon_1 < \varepsilon_{ct} \le \varepsilon_2 \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot (\varepsilon_{ct} - \varepsilon_3) + \sigma_3 \ \text{ if } \ \varepsilon_2 \le \varepsilon_{ct} \le \varepsilon_3 \end{array} \right.$$

Sigma-Epsilon relationship (Stress-Strain)



SECTIONAL ANALYSIS

Height of beam: $h := 125mm$

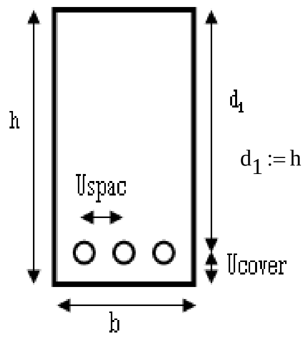
Width of beam: $b := 1000mm$

Depth of concrete cover: $u_{cover} := 25mm$

Initial spacing of reinforcement: $u_{spac} := 150mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spac} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right]$$

$\phi_{bap} = 4.659mm$ $\phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$ Final bar diameter: $\phi_b = 5mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 1.178 \times 10^{-4} m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.094m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.126\,m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 62.705mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63721 \times 10^8\,mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 16.529kN \cdot m$

<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 55mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{10}$

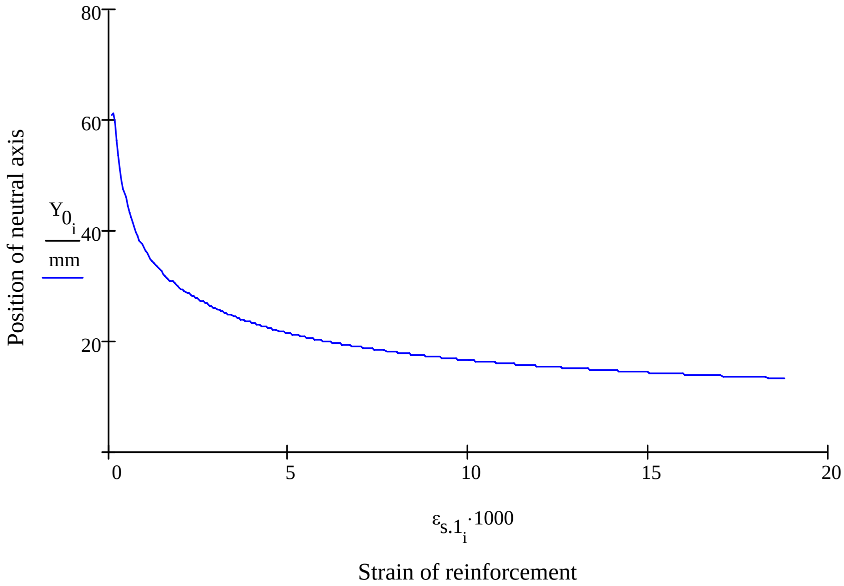
Equilibrium equation to find the position of the neutral axis:

$$Y_{0_i} := \mathrm{root}\left[\int_0^{y_{0ini}} \left[-f_{cm}\cdot\frac{k\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(y_{0ini}-y\right)}{d_1-y_{0ini}}\right]-\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(y_{0ini}-y\right)}{d_1-y_{0ini}}\right]^2}{1+(k-2)\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(y_{0ini}-y\right)}{d_1-y_{0ini}}\right]}\right]\cdot b\,dy\ \ldots \right.$$

$$+\int_{y_{0ini}}^{h} \left[\begin{cases} \frac{\sigma_1}{\varepsilon_1}\cdot\frac{\varepsilon_{s.1_i}\cdot\left(y-y_{0ini}\right)}{d_1-y_{0ini}} & \text{if } y_{0ini}\le y\le\frac{\varepsilon_1\cdot\left(d_1-y_{0ini}\right)}{\varepsilon_{s.1_i}}+y_{0ini} \\ \frac{\sigma_1-\sigma_2}{\varepsilon_1-\varepsilon_2}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y-y_{0ini}\right)}{d_1-y_{0ini}}-\varepsilon_1\right]+\sigma_1 & \text{if } \frac{\varepsilon_1\cdot\left(d_1-y_{0ini}\right)}{\varepsilon_{s.1_i}}+y_{0ini}<y\le\frac{\varepsilon_2\cdot\left(d_1-y_{0ini}\right)}{\varepsilon_{s.1_i}}+y_{0ini} \\ \frac{\sigma_2-\sigma_3}{\varepsilon_2-\varepsilon_3}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y-y_{0ini}\right)}{d_1-y_{0ini}}-\varepsilon_3\right]+\sigma_3 & \text{if } \frac{\varepsilon_2\cdot\left(d_1-y_{0ini}\right)}{\varepsilon_{s.1_i}}+y_{0ini}<y\le\frac{\left(d_1-y_{0ini}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}}+y_{0ini} \\ 0\cdot\mathrm{MPa} & \text{if } \frac{\left(d_1-y_{0ini}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}}+y_{0ini}<y \end{cases}\right]\cdot b\,dy\ \ldots$$

$$\left. +A_s\cdot\begin{cases} E_s\cdot\varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i}\le\varepsilon_{syk} \\ \frac{f_{yk}\cdot\left(k_s-1\right)}{\varepsilon_{suk}-\frac{f_{yk}}{E_s}}\left(\varepsilon_{s.1_i}-\frac{f_{yk}}{E_s}\right)+f_{yk} & \text{if } \varepsilon_{syk}<\varepsilon_{s.1_i}\le\varepsilon_{suk} \\ 0\cdot\mathrm{MPa} & \text{if } \varepsilon_{s.1_i}>\varepsilon_{suk} \end{cases}\ ,\ y_{0ini}\right]$$

Position of the neutral axis when steel strain is increasing:



Stress and Strain STEEL:

<u>Strain in reinforcement steel:</u>

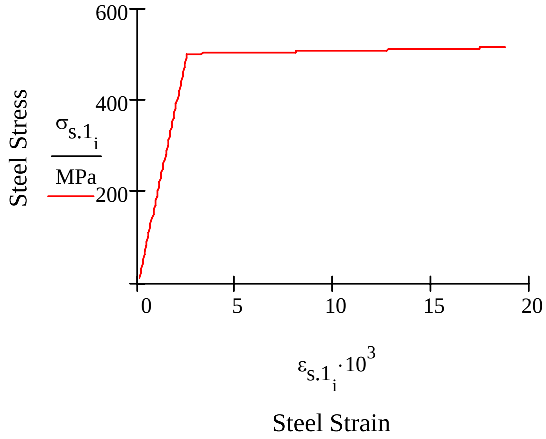
Bottom steel

$$\varepsilon_{s.1_i}$$

<u>Stress in reinforcement steel:</u>

Bottom steel

$$\sigma_{s.1_i} := \begin{cases} E_s \cdot \varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i} \le \varepsilon_{syk} \\ \dfrac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_{s.1_i} - \dfrac{f_{yk}}{E_s} \right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_{s.1_i} \le \varepsilon_{suk} \\ 0 \cdot MPa & \text{if } \varepsilon_{s.1_i} > \varepsilon_{suk} \end{cases}$$



Stress and Strain CONCRETE:

<u>Concrete strain:</u>

$$\varepsilon_{cc}(i, y) := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}} \cdot \left(y - Y_{0_i} \right)$$

<u>Concrete stress:</u>

Concrete in compresion:

$$\sigma_{cc}(i, y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right] - \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k - 2) \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]}$$

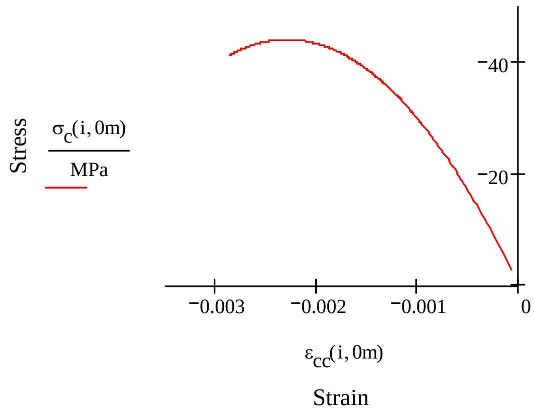
Concrete in tension:

$$\sigma_{ct}(i, y) := \begin{cases} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot MPa & \text{if } \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases}$$

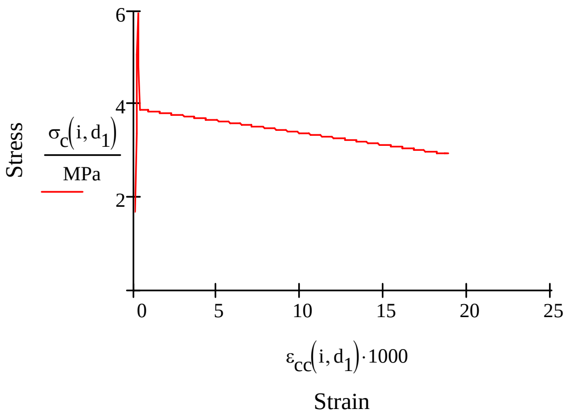
Final expression:

$$\sigma_c(i,y) := \begin{cases} \sigma_{cc}(i,y) & \text{if } 0\text{mm} \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) & \text{if } Y_{0_i} < y \le h \end{cases}$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium:

$F_{cc} + F_{ft} + F_{ct} + F_s = 0$

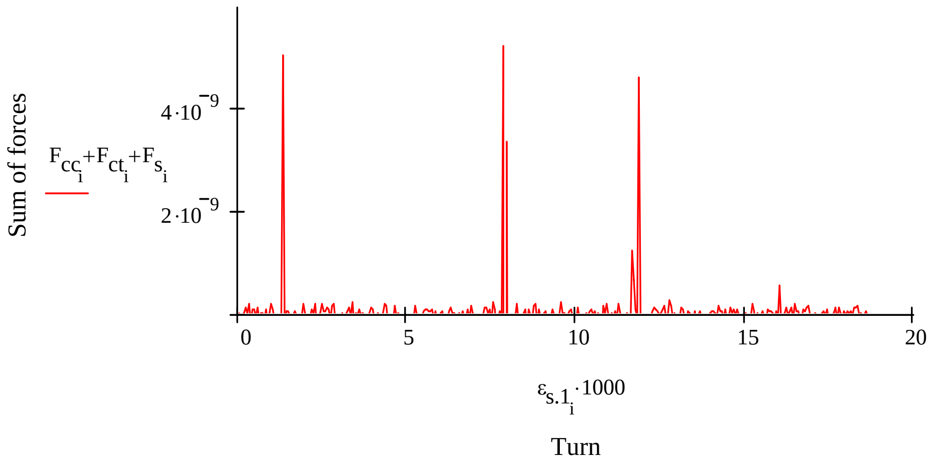
Steel force: $F_{s_i} := A_s \cdot \sigma_{s.1_i}$

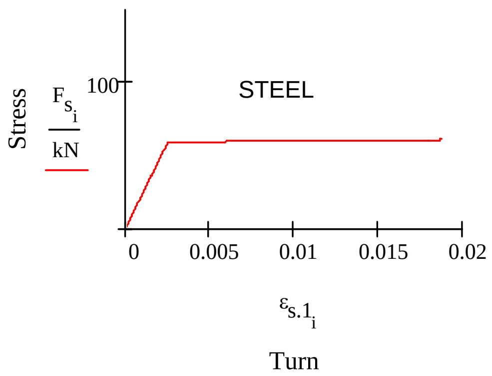
Concrete in compression force:

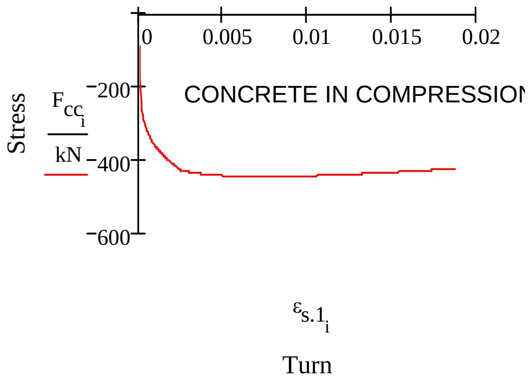
$$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1 - Y_{0_i}}\right] - \eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1 - Y_{0_i}}\right]^2}{1 + (k-2)\cdot\eta\left[\frac{\varepsilon_{s.1_i}\cdot\left(Y_{0_i}-y\right)}{d_1 - Y_{0_i}}\right]} \right] \cdot b\, dy$$

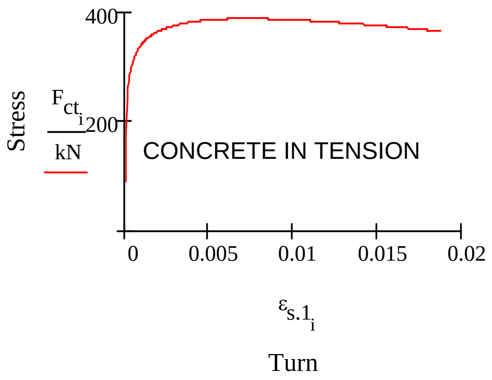
Concrete in tension:

$$F_{ct_i} := \int_{Y_{0_i}}^{h} \left[\begin{cases} \frac{\sigma_1}{\varepsilon_1}\cdot\frac{\varepsilon_{s.1_i}\cdot\left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \frac{\varepsilon_1\cdot\left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_1\right] + \sigma_1 & \text{if } \frac{\varepsilon_1\cdot\left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\varepsilon_2\cdot\left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3}\cdot\left[\frac{\varepsilon_{s.1_i}\cdot\left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_3\right] + \sigma_3 & \text{if } \frac{\varepsilon_2\cdot\left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\left(d_1 - Y_{0_i}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0\cdot MPa & \text{if } \frac{\left(d_1 - Y_{0_i}\right)\cdot\varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases} \right] \cdot b\, dy$$


Sum of forces
$F_{cc_i}+F_{ct_i}+F_{s_i}$
$4\cdot 10^{-9}$
$2\cdot 10^{-9}$
0
5
10
15
20
$\varepsilon_{s.1_i}\cdot 1000$
Turn


Stress
$\frac{F_{s_i}}{kN}$
100
STEEL
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Turn


Stress
$\frac{F_{cc_i}}{kN}$
0
0.005
0.01
0.015
0.02
−200
−400
−600
CONCRETE IN COMPRESSION
$\varepsilon_{s.1_i}$
Turn

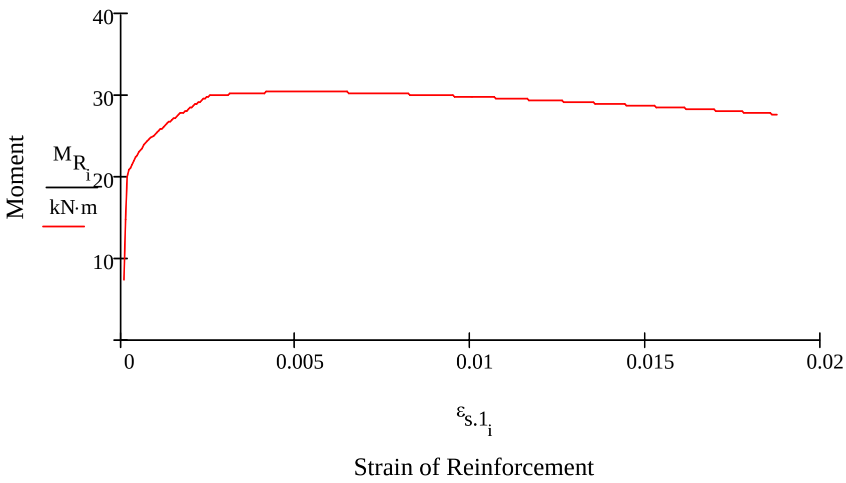

Stress
$\frac{F_{ct_i}}{kN}$
400
200
CONCRETE IN TENSION
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Turn

Moment :

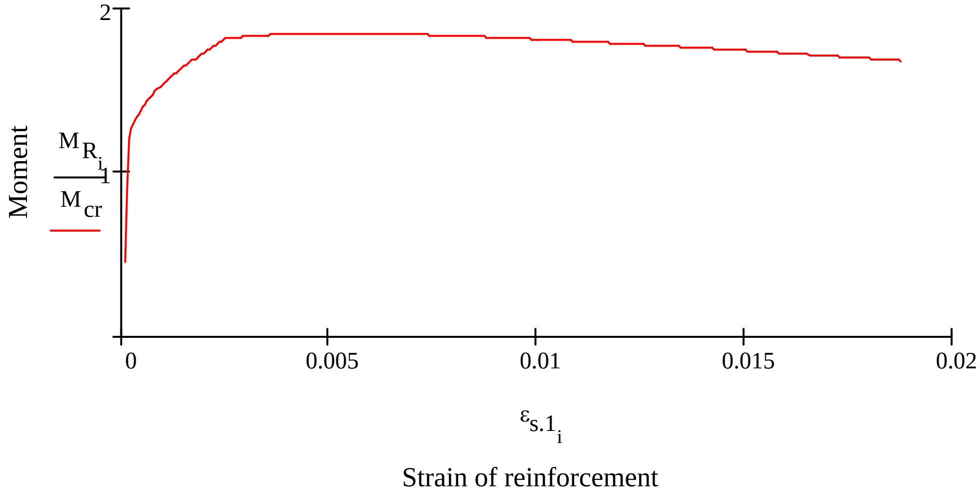
$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right] - \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k-2) \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y \right)}{d_1 - Y_{0_i}} \right]} \right] \cdot b \cdot y \, dy \; \ldots$$

$$+ \int_{Y_{0_i}}^{h} \left[\begin{array}{ll} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i} \right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i} \right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot \text{MPa} & \text{if } \dfrac{\left(d_1 - Y_{0_i} \right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{array} \right] \cdot b \cdot y \, dy + F_{s_i} \cdot \left(d_1 \right)$$

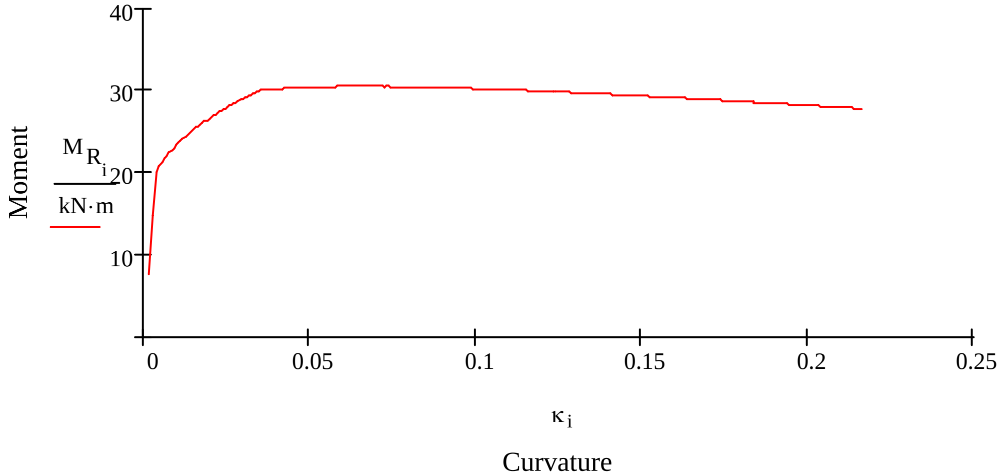
MOMENT-REINFORCEMENT STRAIN GRAPH



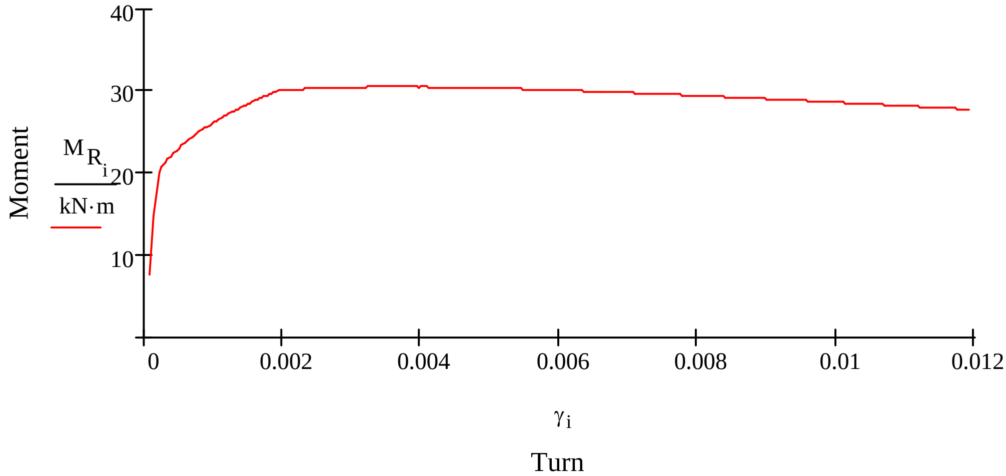
NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH



MOMENT-CURVATURE GRAPH $\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}}$ $\gamma_i := \kappa_i \cdot s$

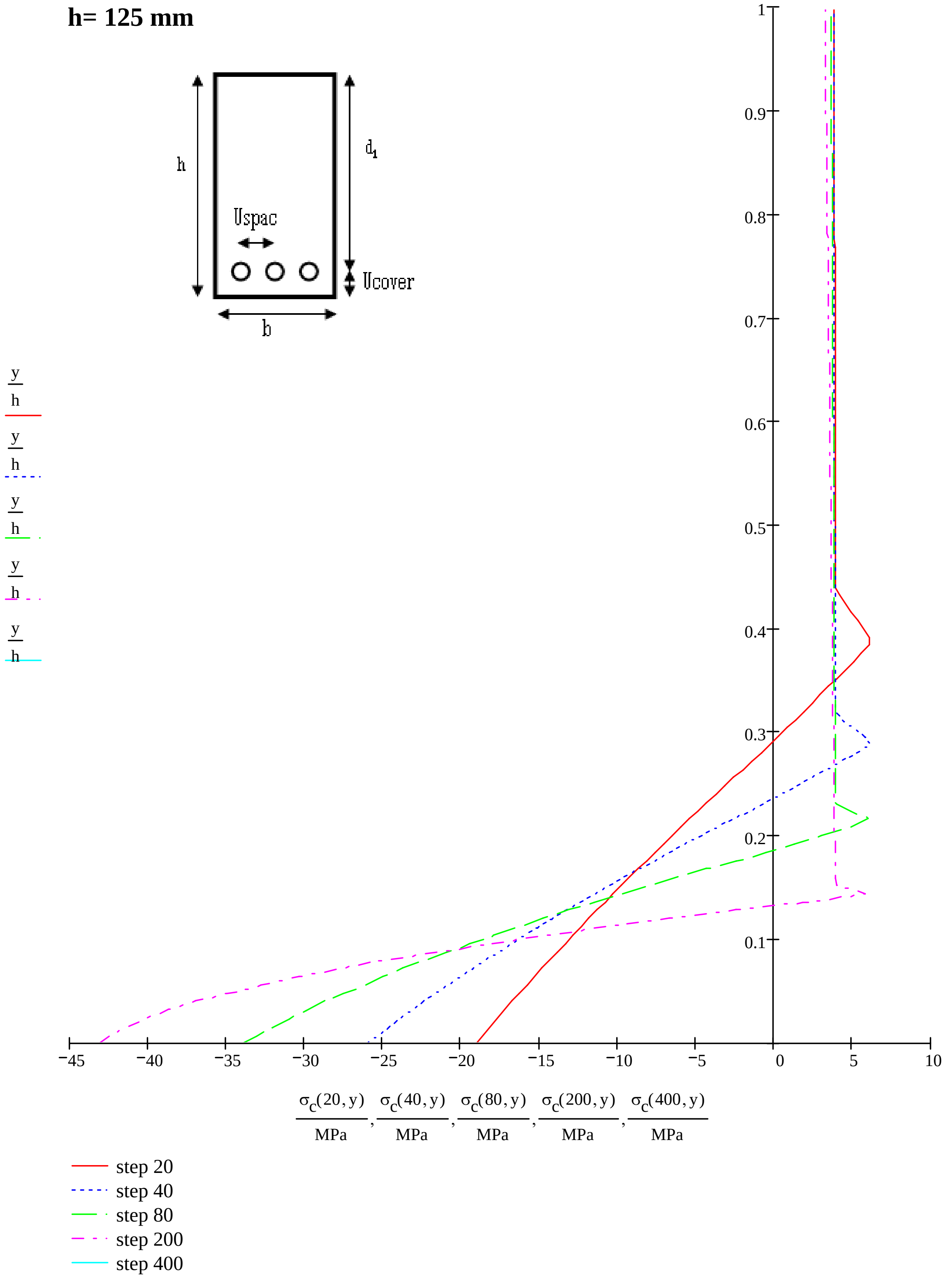


MOMENT-TURN GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:

h= 125 mm



SECTIONAL ANALYSIS

Height 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

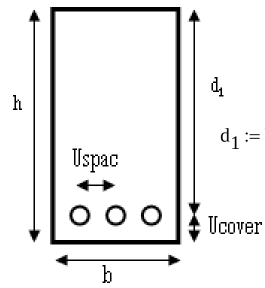
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \text{root}\left[\left[\frac{b - u_{spaci}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588mm \qquad \phi_b := \text{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$$

Final bar diameter: $\phi_b = 7\,mm$

Steel one bar Area: $A_{s.i} := \pi\frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \text{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n - 1}$ $u_{spac} = 181.6mm$

Total steel area: $A_s := n\cdot A_{s.i}$ $A_s = 2.309\times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2\cdot\pi\cdot\left(\frac{\phi_b}{2}\right)\cdot n$ $perim = 0.132\,m$

Effective area: $A_{ef} := b\cdot h + \left(\frac{E_s}{E_c}\right)\cdot A_s$ $A_{ef} = 0.251\,m^2$

Position of effective gravity centre: $x_{ef} := \frac{b\cdot h\cdot\frac{h}{2} + \left(\frac{E_s}{E_c}\right)\cdot A_s\cdot d_1}{A_{ef}}$ $x_{ef} = 125.535mm$

Inertia Moment: $I_{ef} := \frac{b\cdot h^3}{12} + b\cdot h\cdot\left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right)\cdot A_s\cdot\left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.31547\times 10^9\,mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \frac{I_{ef}\cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 60.931kN\cdot m$

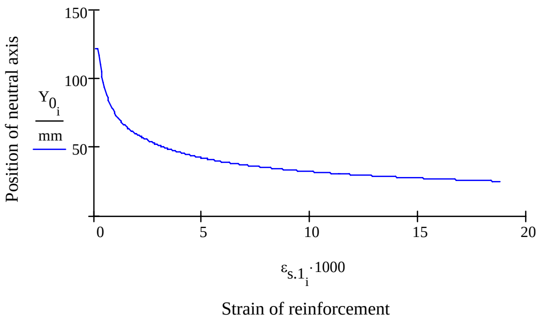
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Number of steps: $n_{step} := 375$ $i := 1..\,n_{step}$

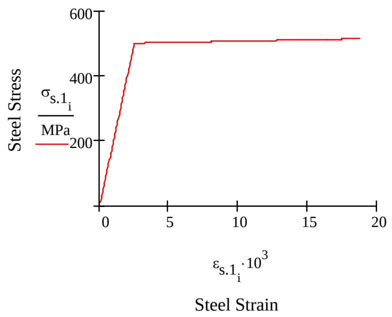
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05\cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{10}$

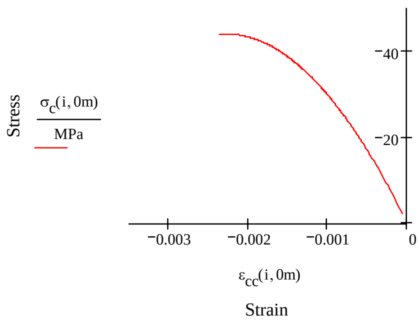
Position of the neutral axis when steel strain is increasing:



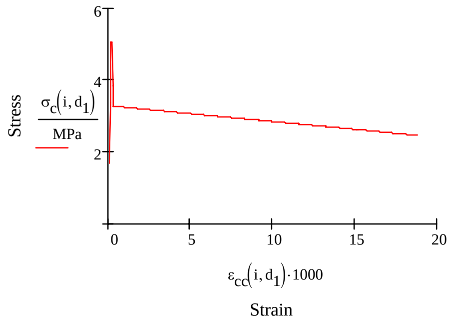
Stress Strain Reinforcement Diagram:



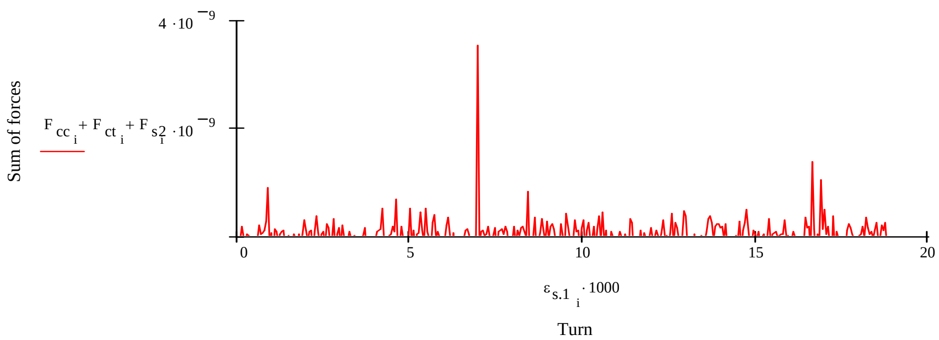
Stress-Strain diagram of the top concrete:



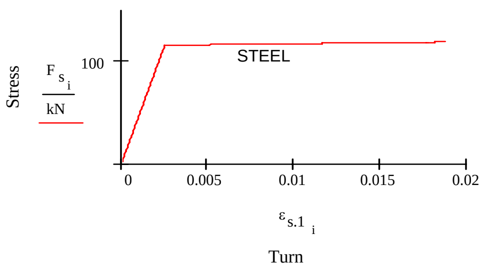
Stress-Strain diagram at the level of reinforcement:

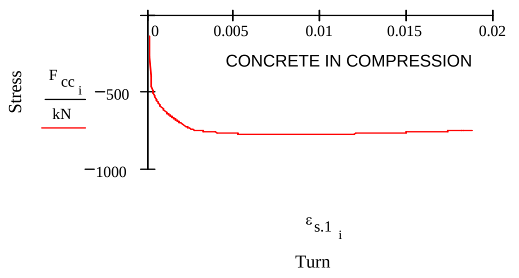


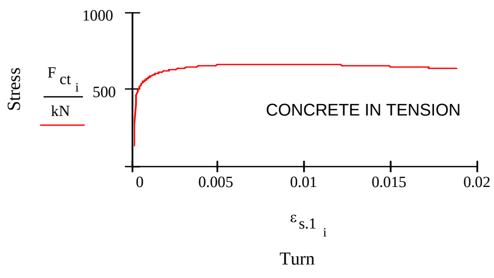
SUM OF FORCES=0 GRAPH



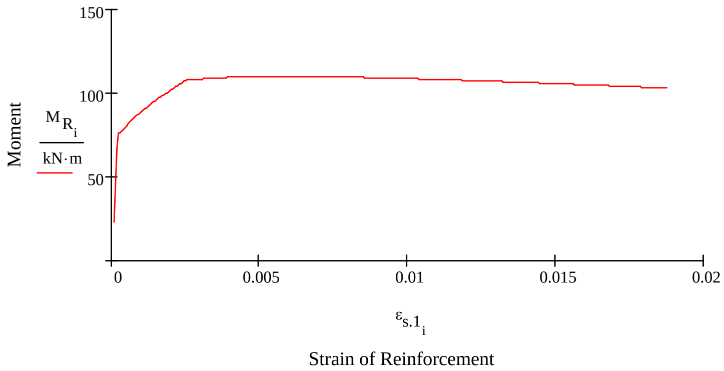
GRAPHS OF FORCES



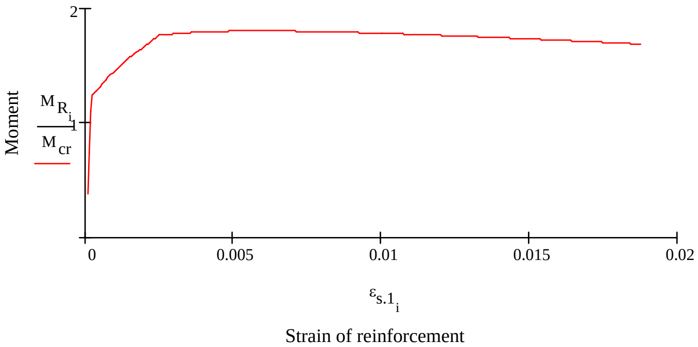




MOMENT-REINFORCEMENT STRAIN GRAPH

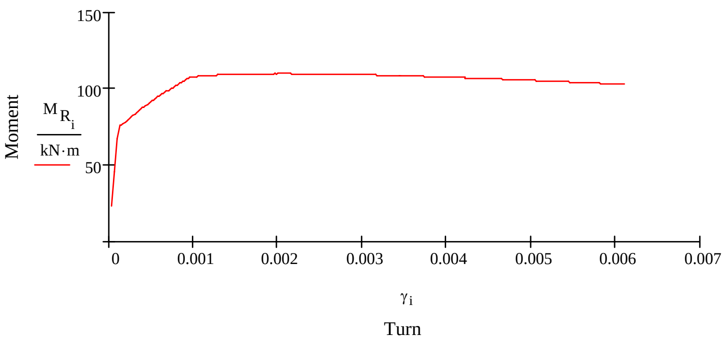


NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH

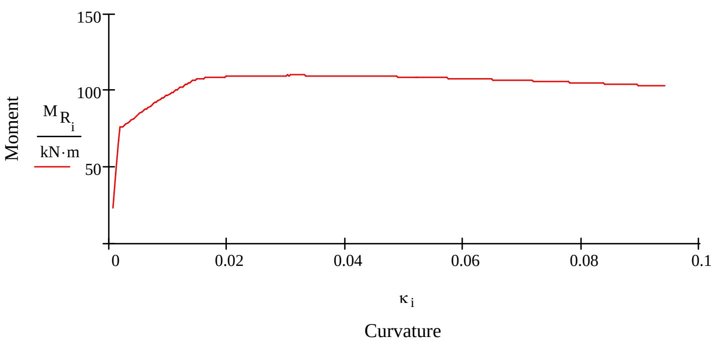


MOMENT-TURN GRAPH

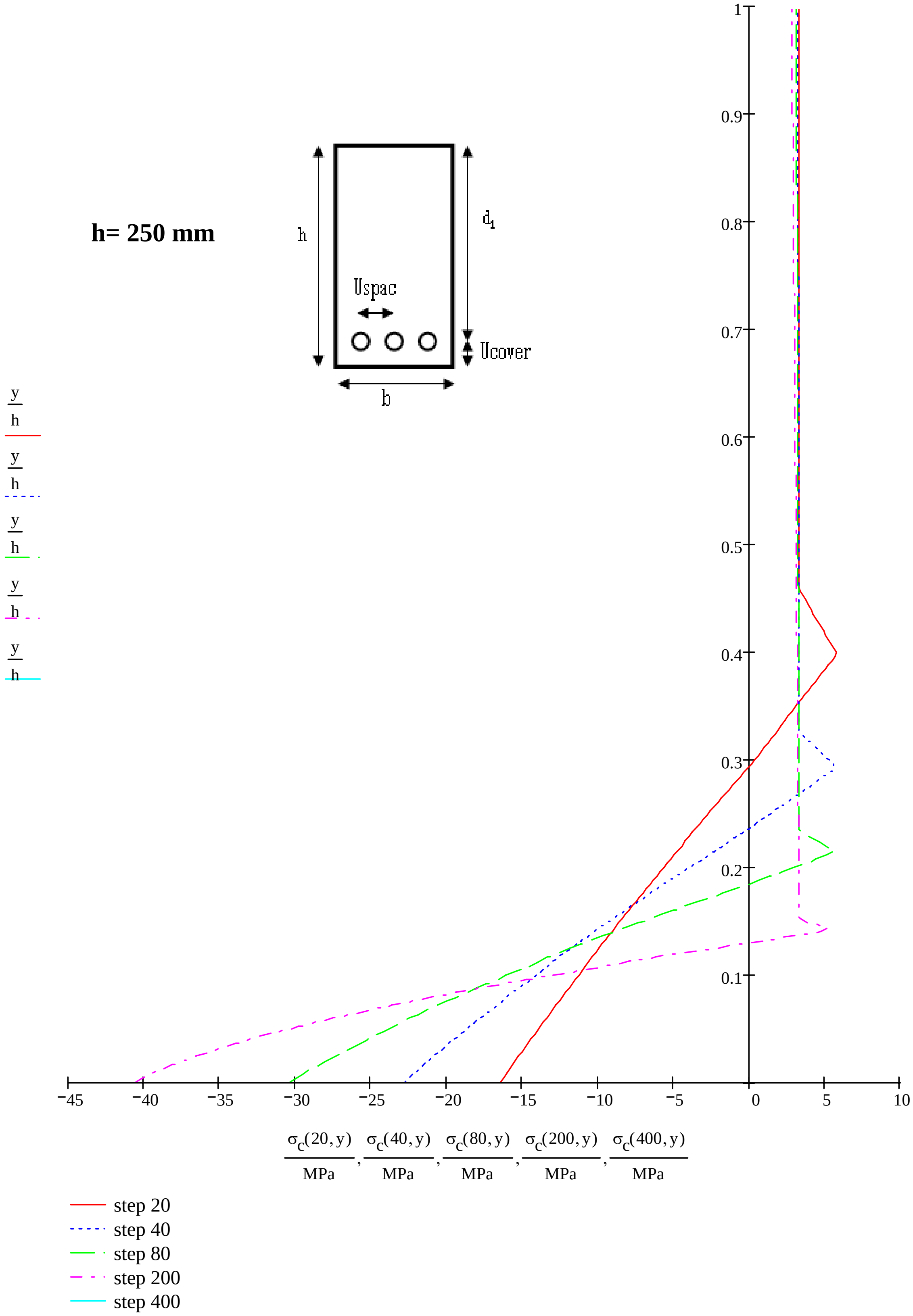
$\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}}$ $\gamma_i := \kappa_i \cdot s$



MOMENT-CURVATURE GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS Height 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

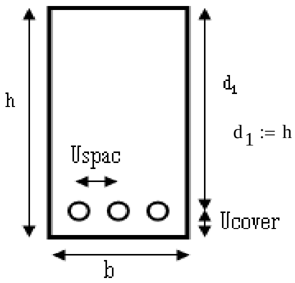
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317\ mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 9\ mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \mathrm{round}\left(n_{ap}, 0\right)$ $n = 8$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 125.429\ mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.226\ m$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.503\ m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 251.327\ mm$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.0566 \times 10^{10}\ mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 200.416\ kN \cdot m$

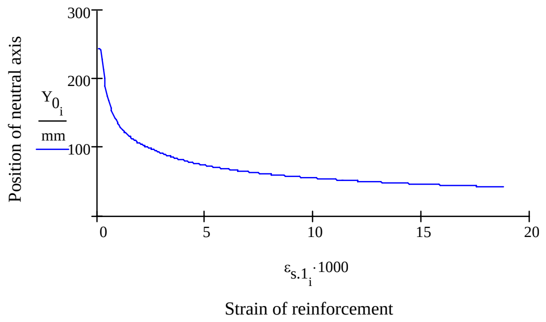
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

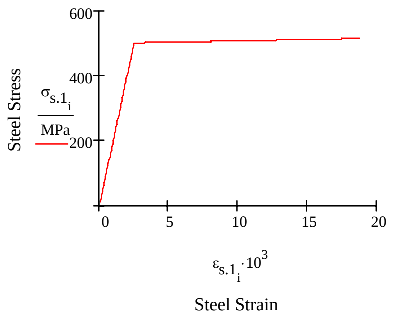
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \dfrac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{10}$

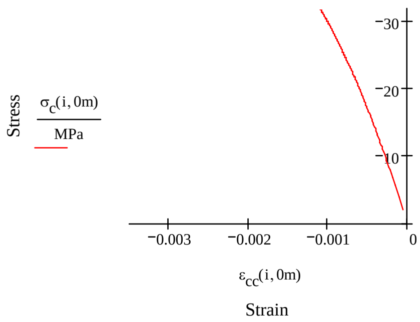
Position of the neutral axis when steel strain is increasing:



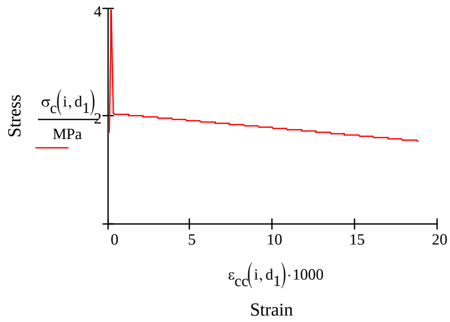
Stress Strain Reinforcement Diagram:



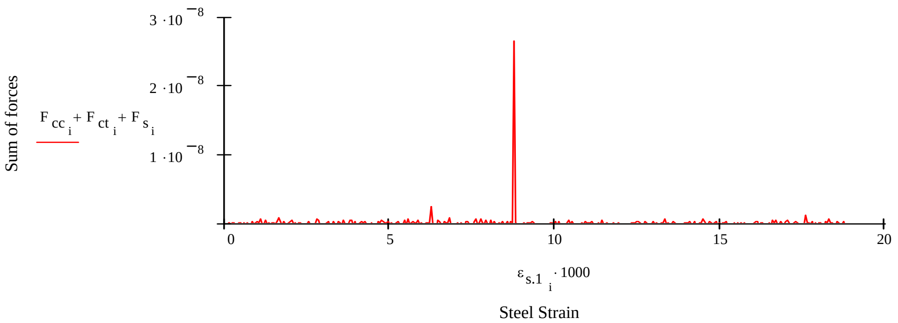
Stress-Strain diagram of the top concrete:



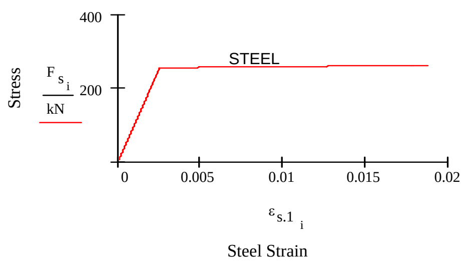
Stress-Strain diagram at the level of reinforcement:

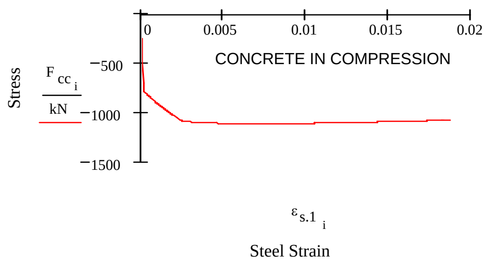


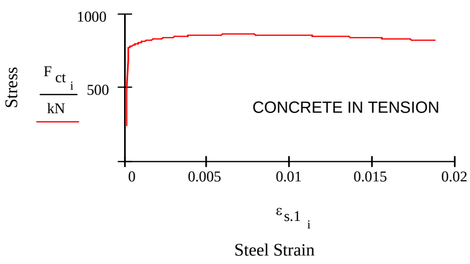
SUM OF FORCES=0 GRAPH



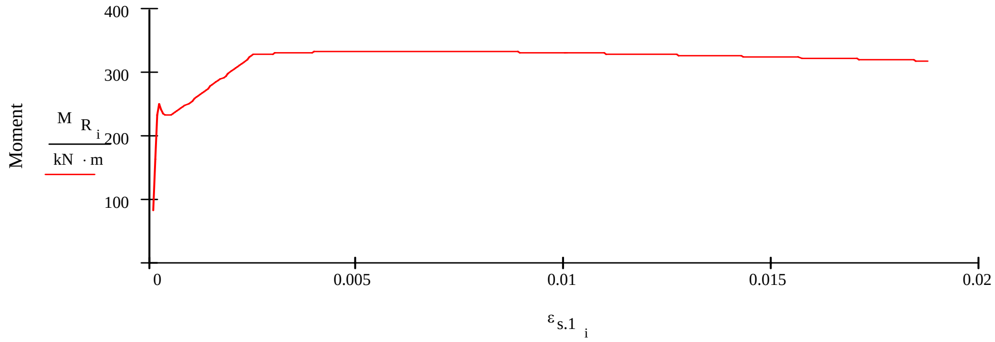
GRAPHS OF FORCES





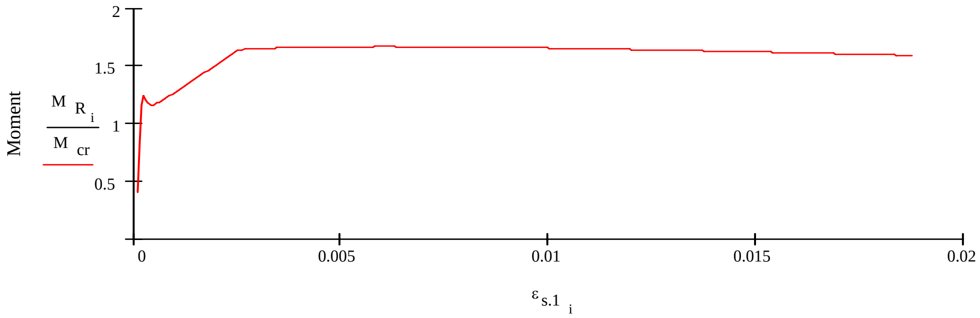


MOMENT-REINFORCEMENT STRAIN GRAPH



Strain of Reinforcement

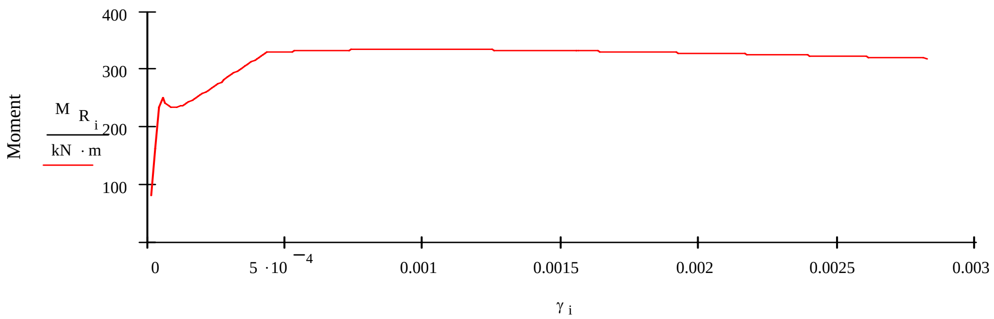
NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH



Strain of reinforcement

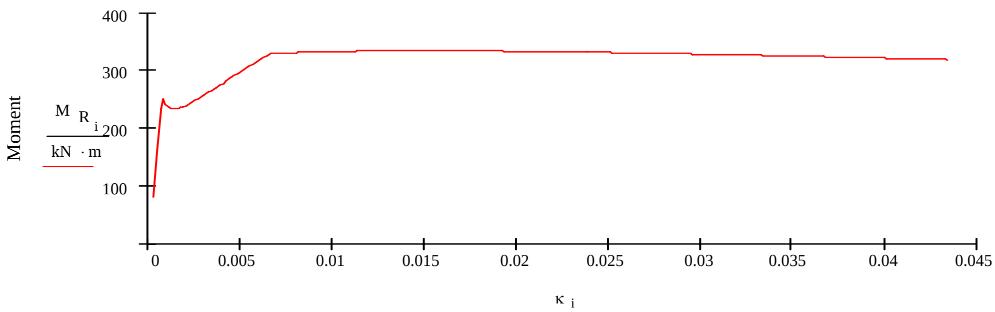
MOMENT-TURN GRAPH

$$\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}} \qquad \gamma_i := \kappa_i \cdot s$$



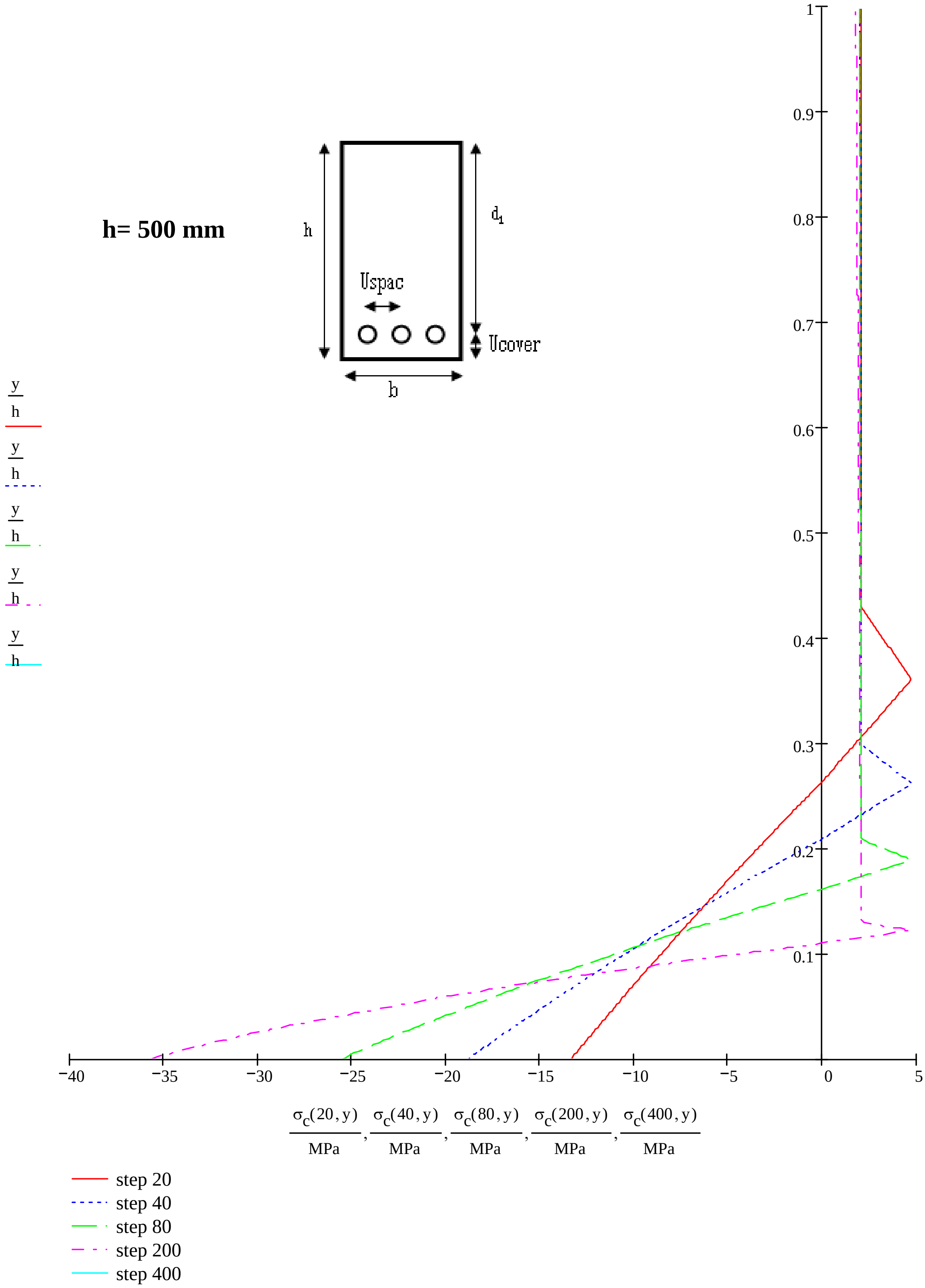
Turn

MOMENT-CURVATURE GRAPH



Curvature

STRESS DIAGRAM OF THE CROSS SECTION:



C.2.3 Sigma-epsilon relationship, analytical analysis. Mix C

MATERIAL PROPERTIES

Concrete in compression:

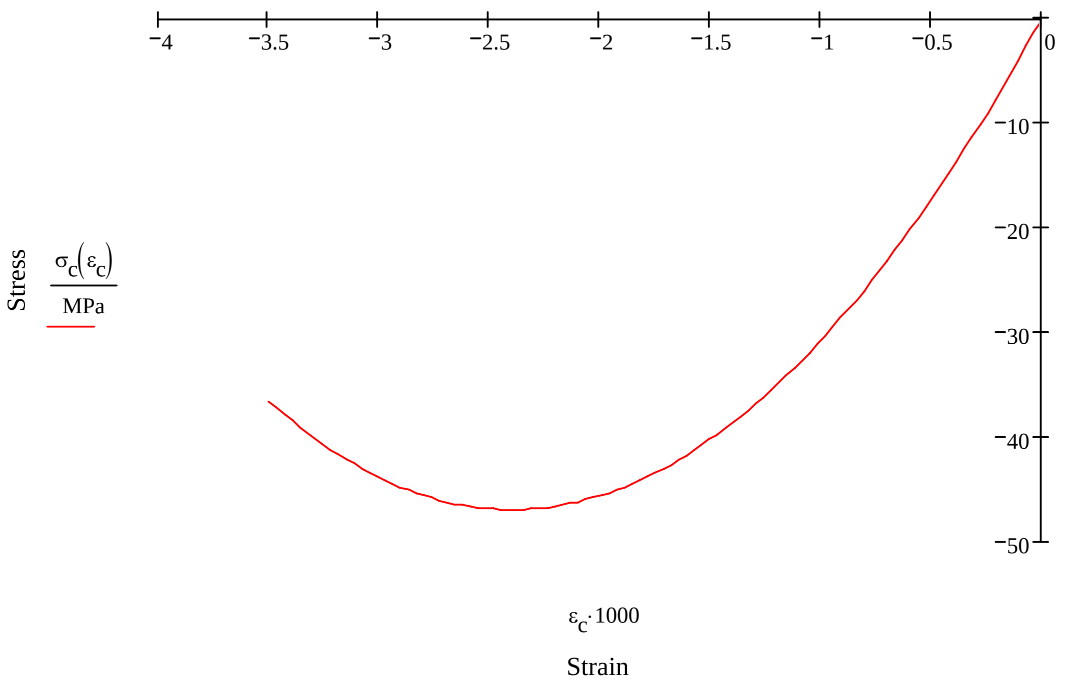
Mean compressive strength: $f_{cm} := 47MPa$

Modulus of Elasticity: $E_c := 22\left(\frac{\frac{f_{cm}}{MPa}}{10}\right)^{0.3}\cdot GPa$ $\quad E_c = 34.999GPa$

Ultimate strain $\varepsilon_{cu} := \frac{3.5}{1000}$

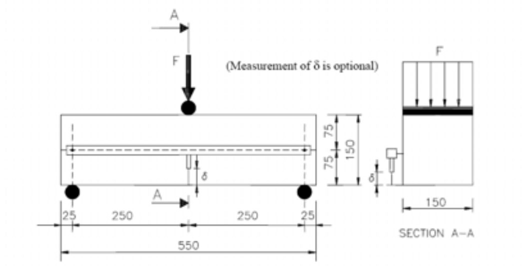
Stress block factors: $\varepsilon_{c1} := 0.24\%$ $\quad \eta(\varepsilon_c) := \frac{|\varepsilon_c|}{\varepsilon_{c1}}$ $\quad k := 1.1\cdot\frac{E_c\cdot|\varepsilon_{c1}|}{f_{cm}}$

Concrete stress: $\sigma_c(\varepsilon_c) := -f_{cm}\cdot\frac{k\cdot\eta(\varepsilon_c) - \eta(\varepsilon_c)^2}{1 + (k-2)\cdot\eta(\varepsilon_c)}$ $\quad \varepsilon_c := 0, \frac{-\varepsilon_{cu}}{100} .. -\varepsilon_{cu}$



Concrete in tension:

Tri-linear Stress-Crack Opening Relationship:



MIX C

TEXT SPECIMEN VALUES:

REAL BEAM VALUES:

$h_{sp} := 125.65mm$ $\quad Leng := 500mm$ $\quad b := 151.28mm$

$u_{cover} := 25mm$

$f_{cm} := 47MPa$ $\quad F_{R1} := 28.392kN$ $\quad F_{R4} := 19.876kN$ $\quad F_L := 19.998kN$

$h := 125mm$

$d_1 := h - u_{cover}$

Values of the RILEM constants:

$$f_{r1} := \frac{3 \cdot F_{R1} \cdot Leng}{2 \cdot b \cdot h_{sp}^2} \qquad f_{r4} := \frac{3 \cdot F_{R4} \cdot Leng}{2 \cdot b \cdot h_{sp}^2} \qquad f_{fctL} := \frac{3 \cdot F_L \cdot Leng}{2 \cdot b \cdot h_{sp}^2}$$

$$f_{r1} = 8.916MPa \qquad f_{r4} = 6.241MPa \qquad f_{fctL} = 6.28MPa$$

$$k_h(h) := 1 - 0.6 \cdot \frac{\frac{h}{mm \cdot 10} - 12.5}{47.5} \qquad k_h(h) = 1$$

$$E_{cRILEM} := 9500 \left(\frac{f_{cm}}{MPa}\right)^{\frac{1}{3}} MPa \qquad E_{cRILEM} = 34.284GPa$$

Final Values for the curve:

$$\sigma_1 := \left[0.7 \cdot f_{fctL} \cdot \left(1.6 - \frac{d_1}{mm \cdot 1000}\right)\right] \qquad \varepsilon_1 := \frac{\sigma_1}{E_{cRILEM}} \qquad \sigma_1 = 6.594 \times 10^6 \ Pa \qquad \varepsilon_1 = 1.923 \times 10^{-4}$$

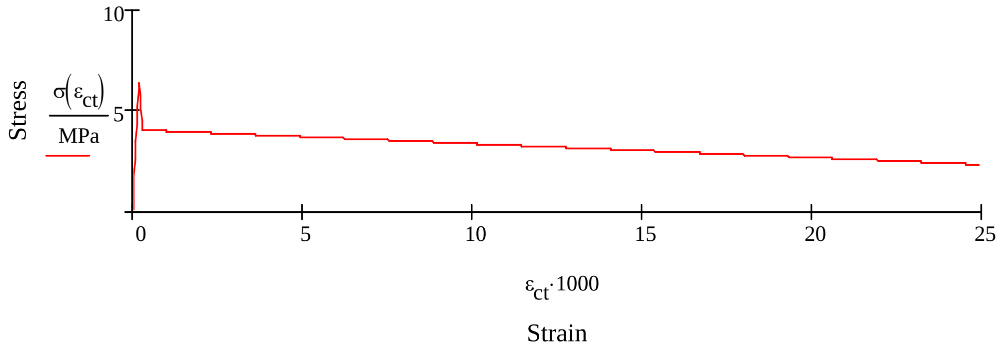
$$\sigma_2 := 0.45 f_{r1} \cdot k_h(h) \qquad \varepsilon_2 := \varepsilon_1 + \frac{0.1}{1000} \qquad \sigma_2 = 4.012 \times 10^6 \ Pa \qquad \varepsilon_2 = 2.923 \times 10^{-4}$$

$$\sigma_3 := 0.37 \cdot f_{r4} \cdot k_h(h) \qquad \varepsilon_3 := \frac{25}{1000} \qquad \sigma_3 = 2.309 \times 10^6 \ Pa \qquad \varepsilon_3 = 0.025$$

Final expression for the curve:

$$\sigma(\varepsilon_{ct}) := \left| \begin{array}{l} \frac{\sigma_1}{\varepsilon_1} \cdot \varepsilon_{ct} \ \text{ if } \ 0 \le \varepsilon_{ct} \le \varepsilon_1 \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot (\varepsilon_{ct} - \varepsilon_1) + \sigma_1 \ \text{ if } \ \varepsilon_1 < \varepsilon_{ct} \le \varepsilon_2 \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot (\varepsilon_{ct} - \varepsilon_3) + \sigma_3 \ \text{ if } \ \varepsilon_2 \le \varepsilon_{ct} \le \varepsilon_3 \end{array} \right.$$

Sigma-Epsilon relationship (Stress-Strain)



SECTIONAL ANALYSIS

Height of beam: $h := 125\,mm$

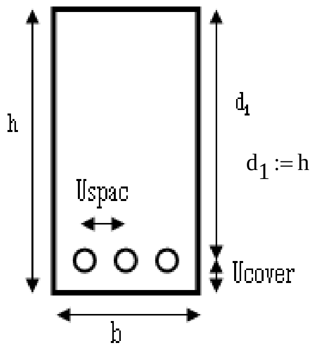
Width of beam: $b := 1000\,mm$

Depth of concrete cover: $u_{cover} := 25\,mm$

Initial spacing of reinforcement: $u_{spac} := 150\,mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spac}\cdot\left(\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\dfrac{A_c\cdot\rho}{\pi\dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$$

Final bar diameter: $\phi_b = 5\,mm$

Steel one bar Area: $A_{s.i} := \pi\dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 6.366$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \dfrac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n - 1}$ $u_{spac} = 184mm$

Total steel area: $A_s := n\cdot A_{s.i}$ $A_s = 1.178\times 10^{-4}\,m^2$ Total perimeter of bars: $perim := 2\cdot\pi\cdot\left(\dfrac{\phi_b}{2}\right)\cdot n$ $perim = 0.094m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.126\,m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 62.701\,mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.63702 \times 10^8\,mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 17.326\,kN \cdot m$

Width of non-linear zone (crack spacing), see appendix D: $s := 55mm$

Number of steps: $n_{step} := 375$ $i := 1..\,n_{step}$

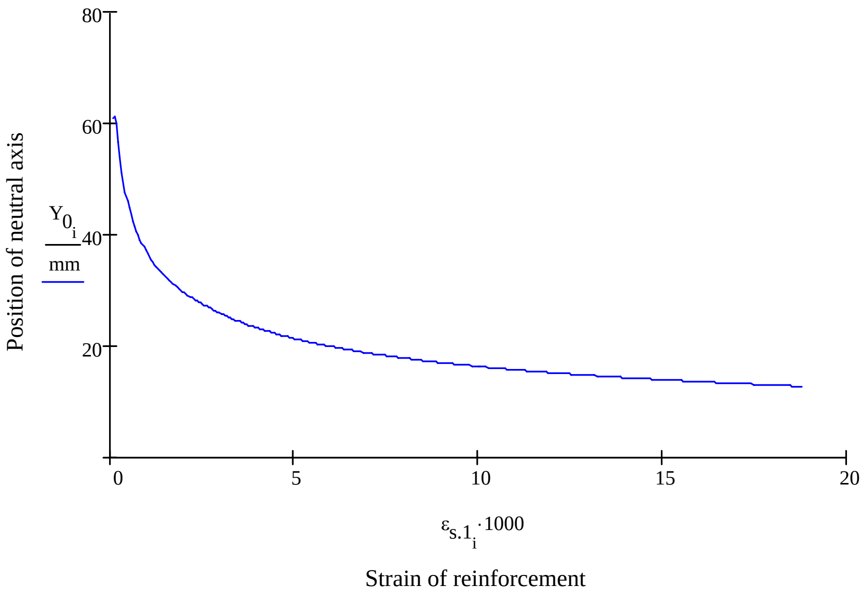
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05\,i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{10}$

Equilibrium equation to find the position of the neutral axis:

$$
\begin{aligned}
Y_{0_i} := \mathrm{root}\Bigg[& \int_0^{y_{0ini}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot (y_{0ini} - y)}{d_1 - y_{0ini}} \right] - \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot (y_{0ini} - y)}{d_1 - y_{0ini}} \right]^2}{1 + (k-2) \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot (y_{0ini} - y)}{d_1 - y_{0ini}} \right]} \right] \cdot b\, dy \ldots \\
& + \int_{y_{0ini}}^{h} \left[\begin{array}{ll} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \cdot (y - y_{0ini})}{d_1 - y_{0ini}} & \text{if } y_{0ini} \le y \le \dfrac{\varepsilon_1 \cdot (d_1 - y_{0ini})}{\varepsilon_{s.1_i}} + y_{0ini} \\
\dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot (y - y_{0ini})}{d_1 - y_{0ini}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot (d_1 - y_{0ini})}{\varepsilon_{s.1_i}} + y_{0ini} < y \le \dfrac{\varepsilon_2 \cdot (d_1 - y_{0ini})}{\varepsilon_{s.1_i}} + y_{0ini} \\
\dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot (y - y_{0ini})}{d_1 - y_{0ini}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot (d_1 - y_{0ini})}{\varepsilon_{s.1_i}} + y_{0ini} < y \le \dfrac{(d_1 - y_{0ini}) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + y_{0ini} \\
0 \cdot \mathrm{MPa} & \text{if } \dfrac{(d_1 - y_{0ini}) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + y_{0ini} < y
\end{array} \right] \cdot b\, dy \ldots \\
& + A_s \cdot \left| \begin{array}{ll} E_s \cdot \varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i} \le \varepsilon_{syk} \\
\dfrac{f_{yk} \cdot (k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_{s.1_i} - \dfrac{f_{yk}}{E_s} \right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_{s.1_i} \le \varepsilon_{suk} \\
0 \cdot \mathrm{MPa} & \text{if } \varepsilon_{s.1_i} > \varepsilon_{suk}
\end{array} \right. , y_{0ini} \Bigg]
\end{aligned}
$$

Position of the neutral axis when steel strain is increasing:



Stress and Strain STEEL:

Strain in reinforcement steel:

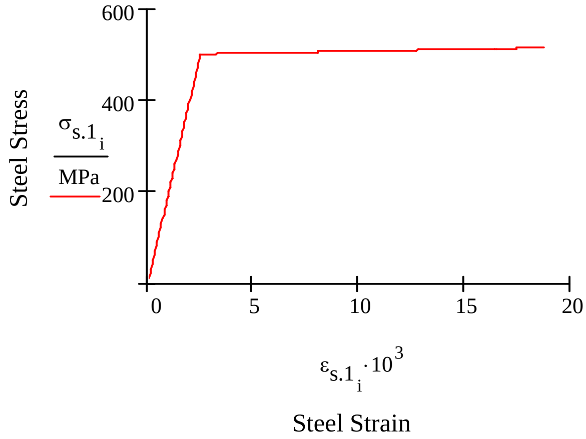
Bottom steel

$\varepsilon_{s.1_i}$

Stress in reinforcement steel :

Bottom steel

$$\sigma_{s.1_i} := \begin{cases} E_s \cdot \varepsilon_{s.1_i} & \text{if } \varepsilon_{s.1_i} \le \varepsilon_{syk} \\ \dfrac{f_{yk}\cdot(k_s - 1)}{\varepsilon_{suk} - \dfrac{f_{yk}}{E_s}} \left(\varepsilon_{s.1_i} - \dfrac{f_{yk}}{E_s}\right) + f_{yk} & \text{if } \varepsilon_{syk} < \varepsilon_{s.1_i} \le \varepsilon_{suk} \\ 0 \cdot \text{MPa} & \text{if } \varepsilon_{s.1_i} > \varepsilon_{suk} \end{cases}$$



Stress and Strain CONCRETE:

Concrete strain:

$$\varepsilon_{cc}(i,y) := \frac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}} \cdot \left(y - Y_{0_i}\right)$$

Concrete stress:

Concrete in compresion:

$$\sigma_{cc}(i,y) := -f_{cm} \cdot \frac{k \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}}\right] - \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}}\right]^2}{1 + (k-2) \cdot \eta \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}}\right]}$$

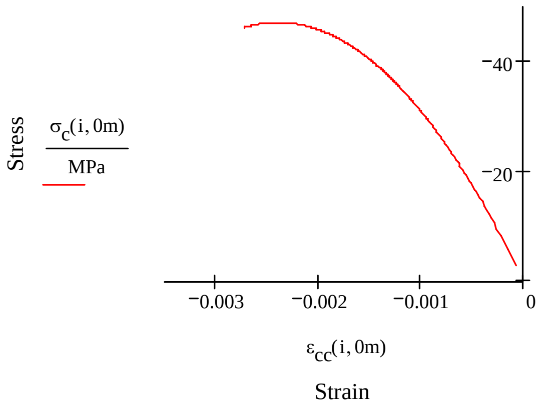
Concrete in tension:

$$\sigma_{ct}(i,y) := \begin{cases} \dfrac{\sigma_1}{\varepsilon_1} \cdot \dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_1\right] + \sigma_1 & \text{if } \dfrac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \dfrac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\dfrac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_3\right] + \sigma_3 & \text{if } \dfrac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \dfrac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot \text{MPa} & \text{if } \dfrac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases}$$

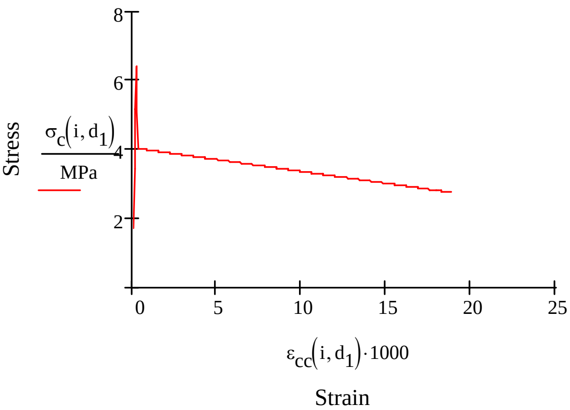
Final expression:

$$\sigma_c(i,y) := \begin{cases} \sigma_{cc}(i,y) & \text{if } 0\text{mm} \le y \le Y_{0_i} \\ \sigma_{ct}(i,y) & \text{if } Y_{0_i} < y \le h \end{cases}$$

Stress-Strain relationship in the top concrete:



Stress-Strain relationship at the level of reinforcement



Check force equilibrium:

$F_{cc} + F_{ft} + F_{ct} + F_s = 0$

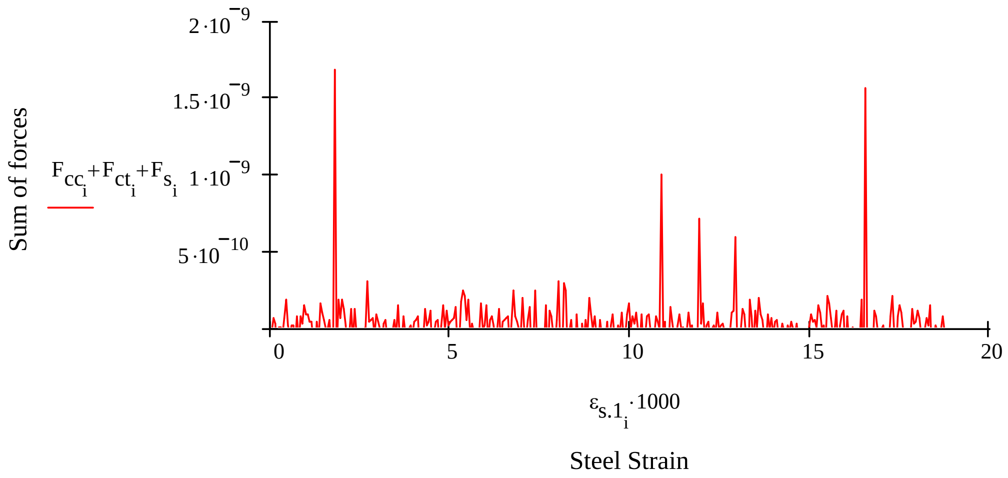
Steel force: $F_{s_i} := A_s \cdot \sigma_{s.1_i}$

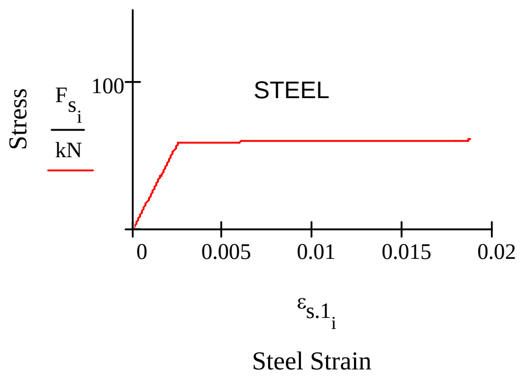
Concrete in compression force:

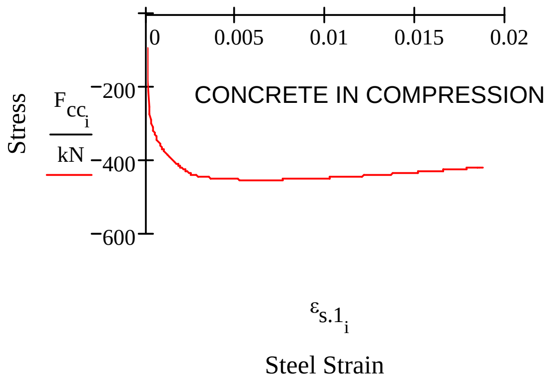
$$F_{cc_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right] - \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k-2) \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right]} \right] \cdot b \, dy$$

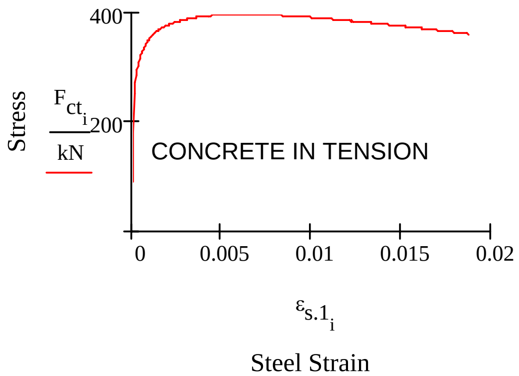
Concrete in tension:

$$F_{ct_i} := \int_{Y_{0_i}}^{h} \left[\begin{cases} \frac{\sigma_1}{\varepsilon_1} \cdot \frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \frac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \frac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \frac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot \text{MPa} & \text{if } \frac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{cases} \right] \cdot b \, dy$$


Sum of forces
$\frac{F_{cc_i}+F_{ct_i}+F_{s_i}}{}$
$2\cdot10^{-9}$
$1.5\cdot10^{-9}$
$1\cdot10^{-9}$
$5\cdot10^{-10}$
0
5
10
15
20
$\varepsilon_{s.1_i}\cdot1000$
Steel Strain


Stress
$\frac{F_{s_i}}{kN}$
100
STEEL
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Steel Strain


Stress
$\frac{F_{cc_i}}{kN}$
0
0.005
0.01
0.015
0.02
−200
−400
−600
CONCRETE IN COMPRESSION
$\varepsilon_{s.1_i}$
Steel Strain

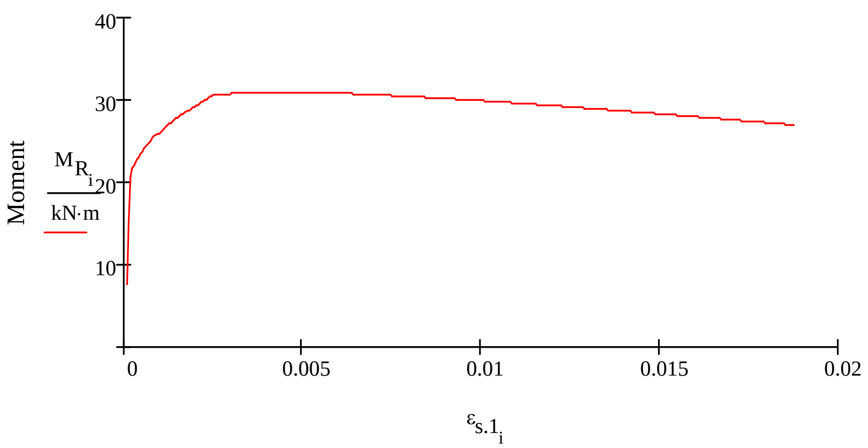

Stress
$\frac{F_{ct_i}}{kN}$
400
200
CONCRETE IN TENSION
0
0.005
0.01
0.015
0.02
$\varepsilon_{s.1_i}$
Steel Strain

Moment :

$$M_{R_i} := \int_0^{Y_{0_i}} \left[-f_{cm} \cdot \frac{k \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right] - \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right]^2}{1 + (k-2) \cdot \eta \left[\frac{\varepsilon_{s.1_i} \cdot \left(Y_{0_i} - y\right)}{d_1 - Y_{0_i}} \right]} \right] \cdot b \cdot y \, dy \; \ldots$$

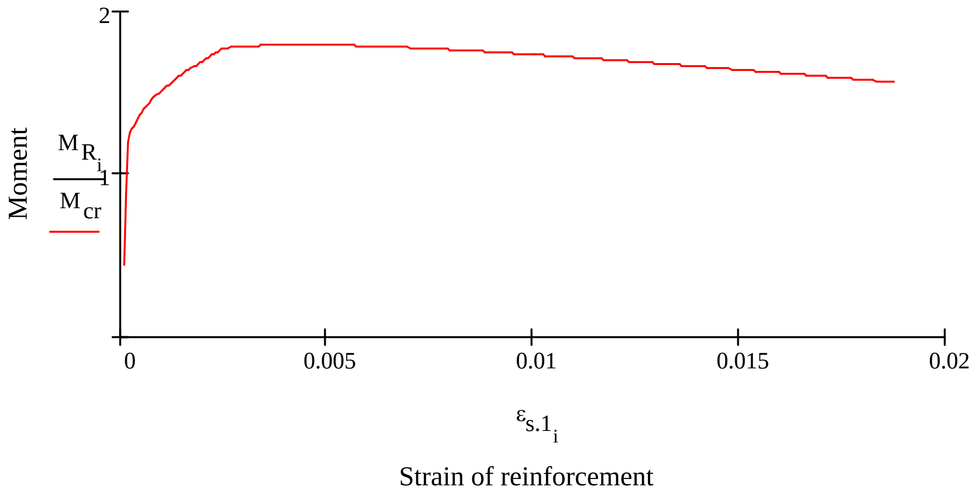
$$+ \int_{Y_{0_i}}^{h} \left[\begin{array}{ll} \frac{\sigma_1}{\varepsilon_1} \cdot \frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} & \text{if } Y_{0_i} \le y < \frac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \cdot \left[\frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_1 \right] + \sigma_1 & \text{if } \frac{\varepsilon_1 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} \\ \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} \cdot \left[\frac{\varepsilon_{s.1_i} \cdot \left(y - Y_{0_i}\right)}{d_1 - Y_{0_i}} - \varepsilon_3 \right] + \sigma_3 & \text{if } \frac{\varepsilon_2 \cdot \left(d_1 - Y_{0_i}\right)}{\varepsilon_{s.1_i}} + Y_{0_i} < y \le \frac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} \\ 0 \cdot \text{MPa} & \text{if } \frac{\left(d_1 - Y_{0_i}\right) \cdot \varepsilon_3}{\varepsilon_{s.1_i}} + Y_{0_i} < y \end{array} \right] \cdot b \cdot y \, dy + F_{s_i} \cdot \left(d_1\right)$$

MOMENT-REINFORCEMENT STRAIN GRAPH

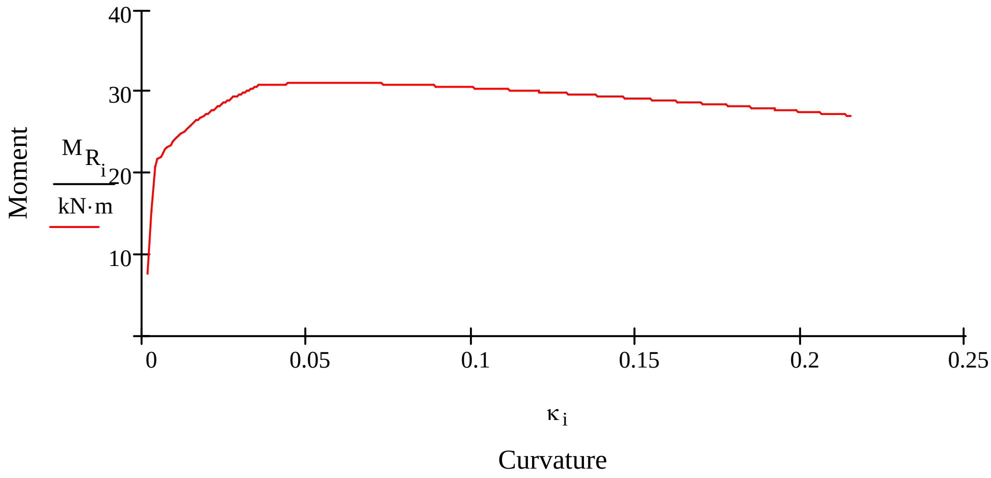


Strain of reinforcement

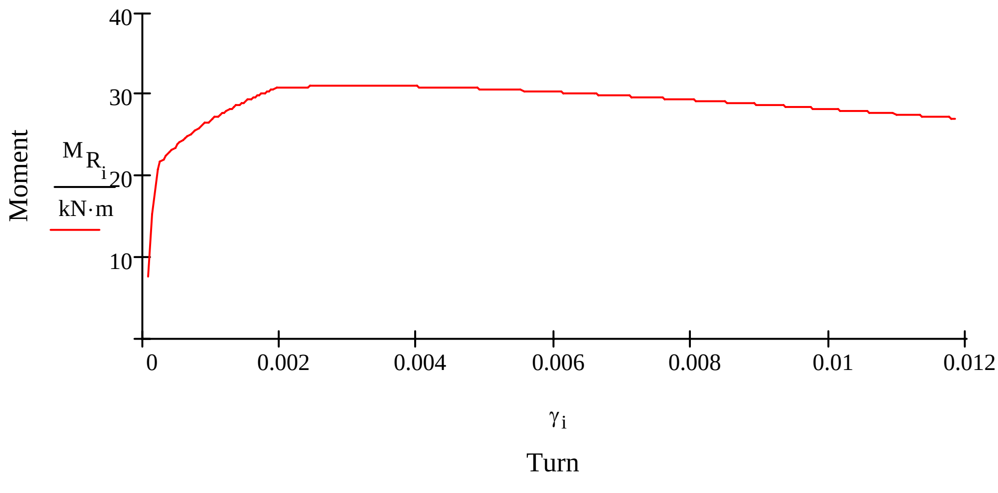
NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH



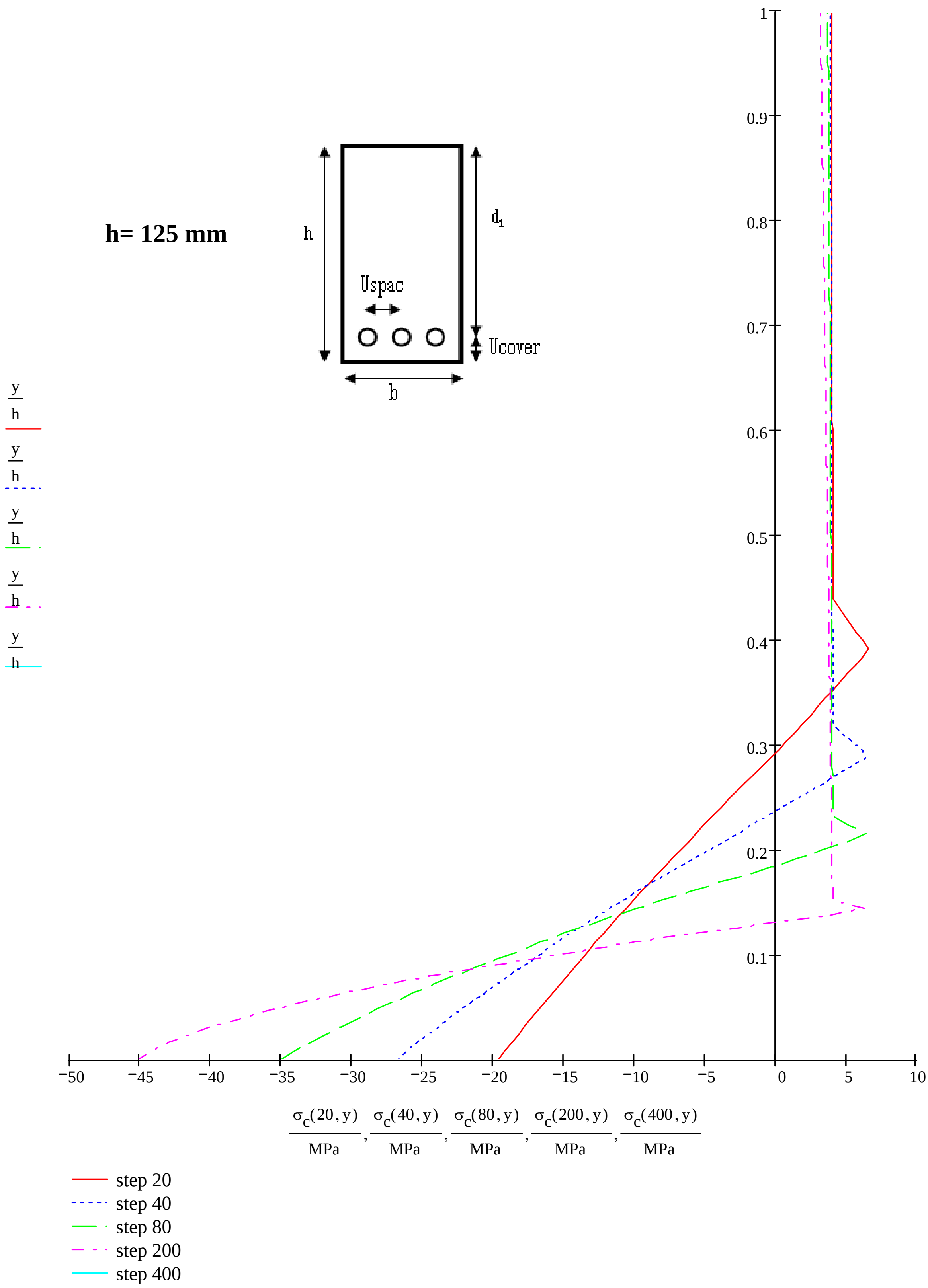
MOMENT-CURVATURE GRAPH $\kappa_i := \dfrac{\varepsilon_{s.1_i}}{d_1 - Y_{0_i}}$ $\gamma_i := \kappa_i \cdot s$



MOMENT-TURN GRAPH



Stress Diagram of the cross section:



SECTIONAL ANALYSIS

Height 2.- 250 mm

Height of beam: $h := 250 \cdot mm$

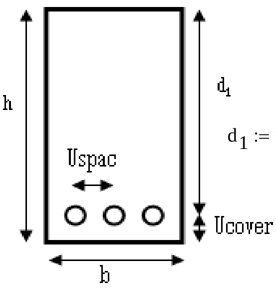
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\dfrac{A_c \cdot \rho}{\pi \dfrac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588\,mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 7\,mm$

Steel one bar Area: $A_{s.i} := \pi \dfrac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \dfrac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.496$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 6$ Final bar spacing: $u_{spac} := \dfrac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 181.6\,mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 2.309 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\dfrac{\phi_b}{2}\right) \cdot n$ $perim = 0.132\ m$

Effective area: $A_{ef} := b \cdot h + \left(\dfrac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.251\ m^2$

Position of effective gravity centre: $x_{ef} := \dfrac{b \cdot h \cdot \dfrac{h}{2} + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 125.525\,mm$

Inertia Moment: $I_{ef} := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - x_{ef}\right)^2 + \left(\dfrac{E_s}{E_c}\right) \cdot A_s \cdot (d_1 - x_{ef})^2$ $I_{ef} = 1.31521 \times 10^9\ mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \dfrac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 63.864\,kN \cdot m$

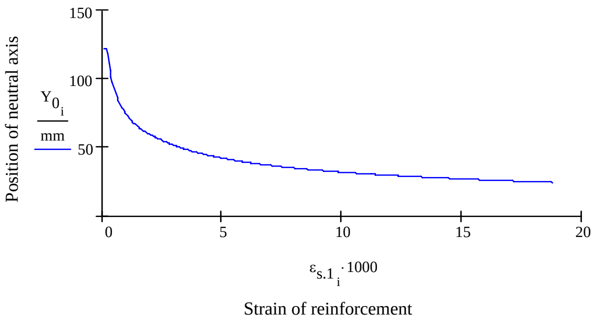
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

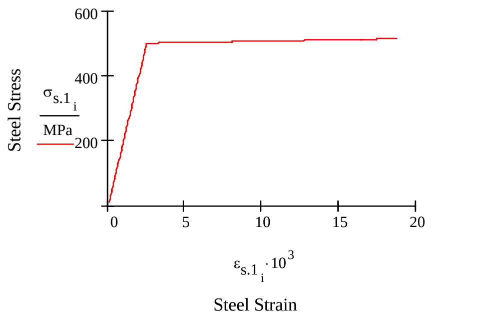
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \dfrac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \dfrac{h}{10}$

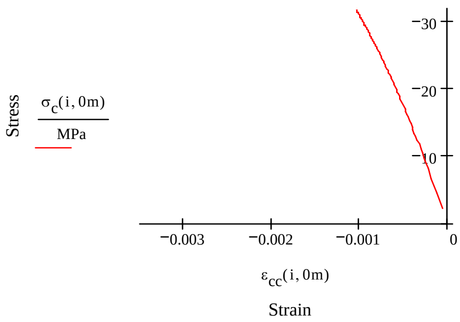
Position of the neutral axis when steel strain is increasing:



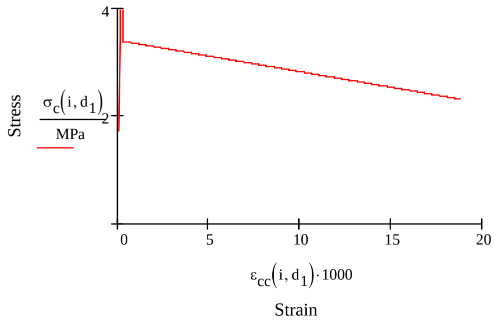
Stress Strain Reinforcement Diagram:



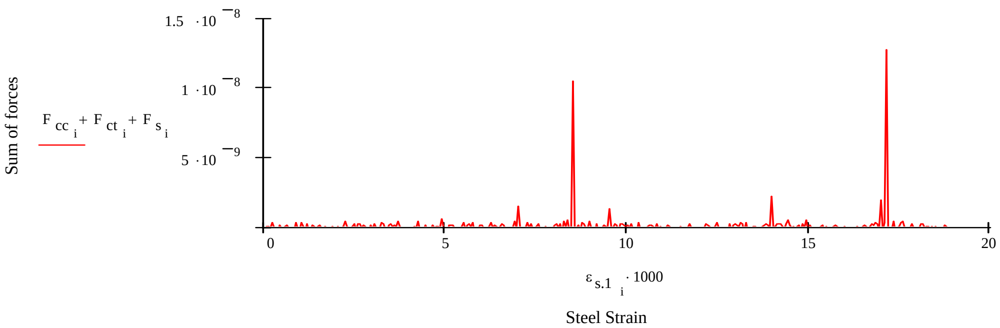
Stress-Strain diagram of the top concrete:



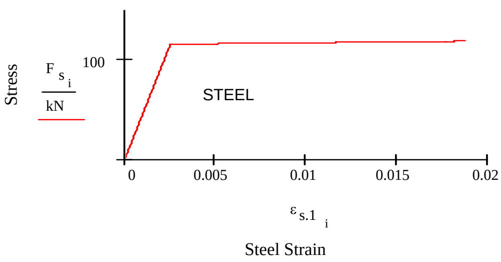
Stress-Strain diagram at the level of reinforcement:

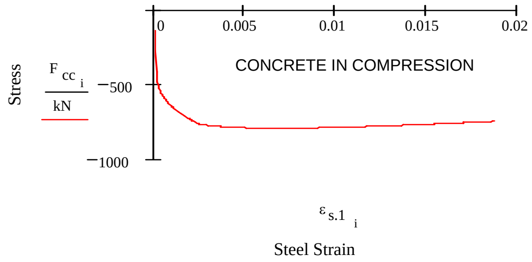


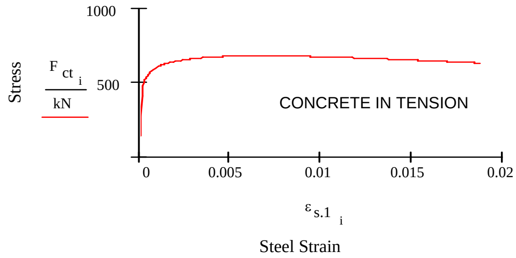
SUM OF FORCES=0 GRAPH



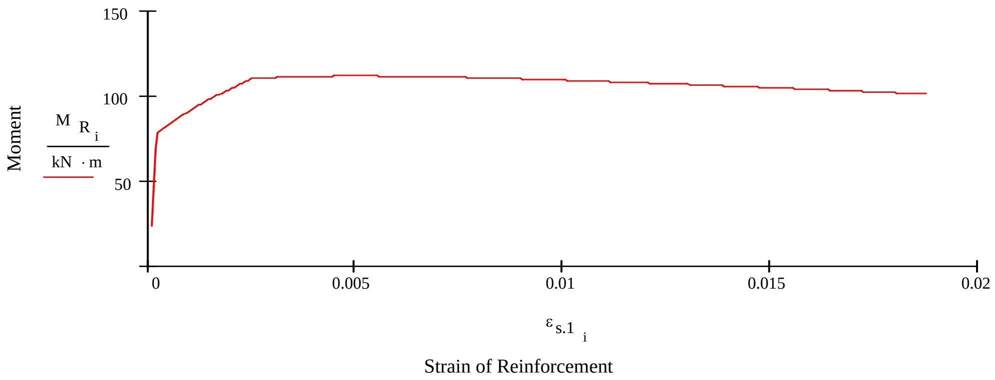
GRAPHS OF FORCES



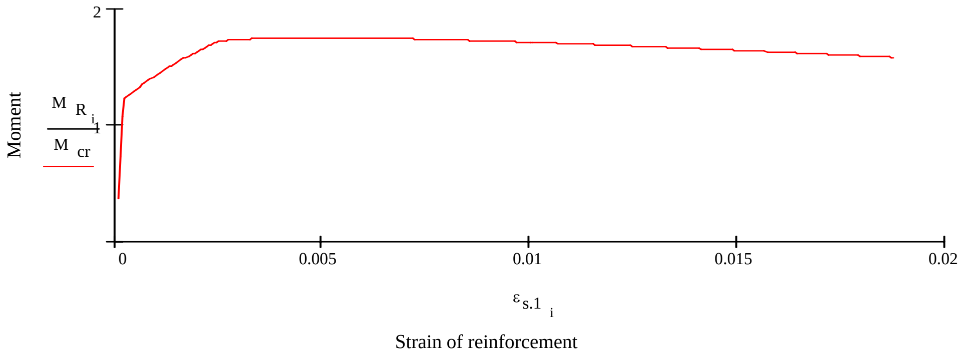




MOMENT-REINFORCEMENT STRAIN GRAPH

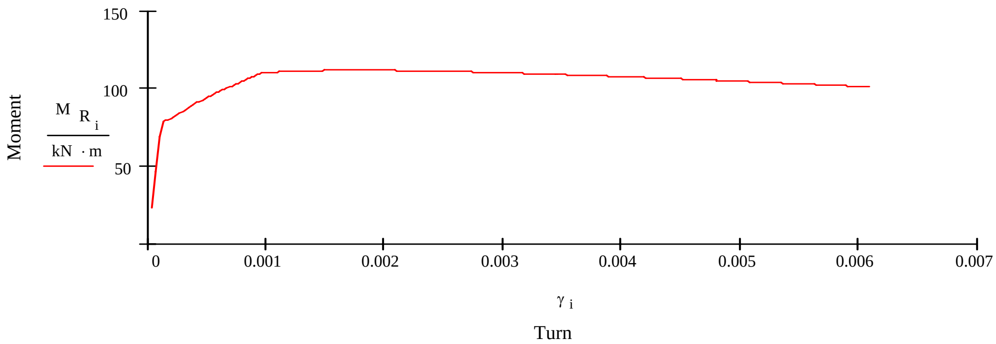


NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH

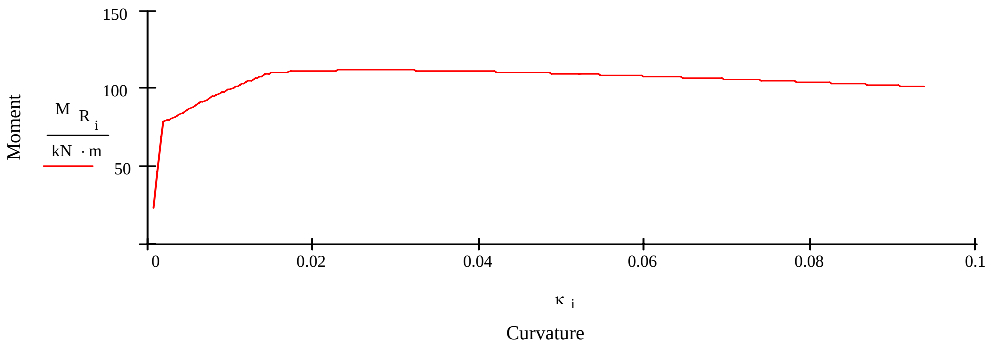


MOMENT-TURN GRAPH

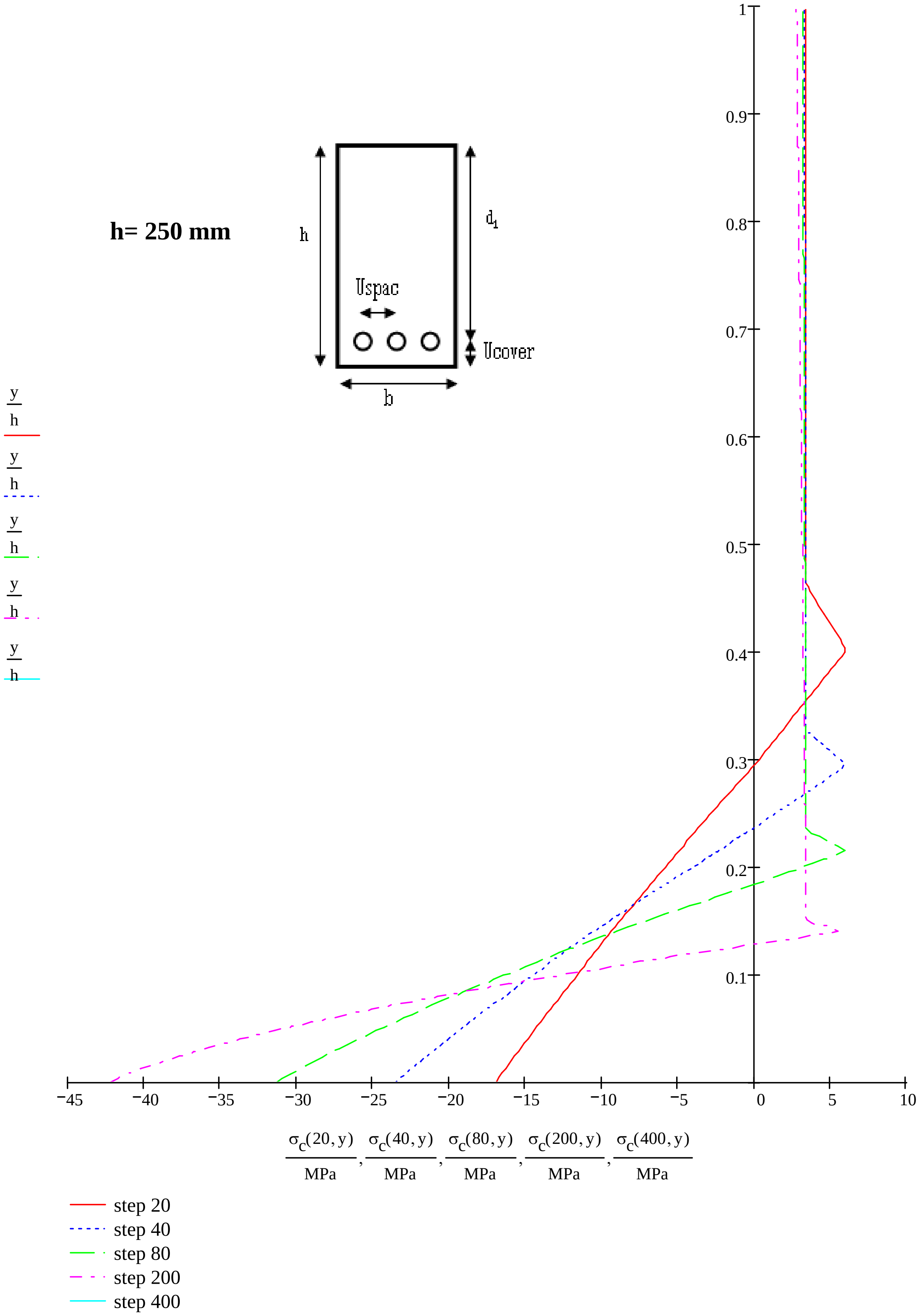
$$\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y0_i} \qquad \gamma_i := \kappa_i \cdot s$$



MOMENT-CURVATURE GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



SECTIONAL ANALYSIS

Height 3.- 500 mm

Height of beam: $h := 500 \cdot mm$

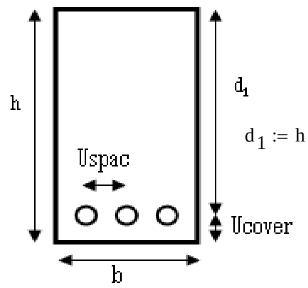
Width of beam: $b := 1000 \cdot mm$

Depth of concrete cover: $u_{cover} := 25 \cdot mm$

Initial spacing of reinforcement: $u_{spaci} := 150 \cdot mm$

Initial spacing reinforcement ratio: $\rho := 0.1\%$

Inicial diameter: $\phi_{bi} := 7mm$



$d_1 := h - u_{cover}$

Concrete Area: $A_c := b \cdot h$

Approximate bar diameter (withour rounding):

$$\phi_{bap} := \mathrm{root}\left[\left[\frac{b - u_{spaci} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 9.317\ mm \qquad \phi_b := \mathrm{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm$$

Final bar diameter: $\phi_b = 9\,mm$

Steel one bar Area: $A_{s.i} := \pi \frac{\phi_b^2}{4}$ Approximate number of bars: $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 7.86$

Final number of bars: $n := \mathrm{round}(n_{ap}, 0)$ $n = 8$ Final bar spacing: $u_{spac} := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $u_{spac} = 125.429\ mm$

Total steel area: $A_s := n \cdot A_{s.i}$ $A_s = 5.089 \times 10^{-4}\ m^2$ Total perimeter of bars: $perim := 2 \cdot \pi \cdot \left(\frac{\phi_b}{2}\right) \cdot n$ $perim = 0.226\ m$

Effective area: $A_{ef} := b \cdot h + \left(\frac{E_s}{E_c}\right) \cdot A_s$ $A_{ef} = 0.503\ m^2$

Position of effective gravity centre: $x_{ef} := \frac{b \cdot h \cdot \frac{h}{2} + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot d_1}{A_{ef}}$ $x_{ef} = 251.301\ mm$

Inertia Moment: $I_{ef} := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_{ef}\right)^2 + \left(\frac{E_s}{E_c}\right) \cdot A_s \cdot \left(d_1 - x_{ef}\right)^2$ $I_{ef} = 1.0563 \times 10^{10}\ mm^4$

Critical moment (moment just before cracking) $f_{ct} := \sigma_1$ $M_{cr} := \frac{I_{ef} \cdot f_{ct}}{h - x_{ef}}$ $M_{cr} = 210.043\ kN \cdot m$

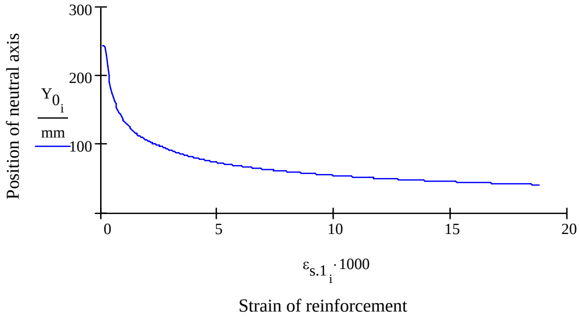
<u>Width of non-linear zone (crack spacing), see appendix D:</u> $s := 65mm$

Number of steps: $n_{step} := 375$ $i := 1 .. n_{step}$

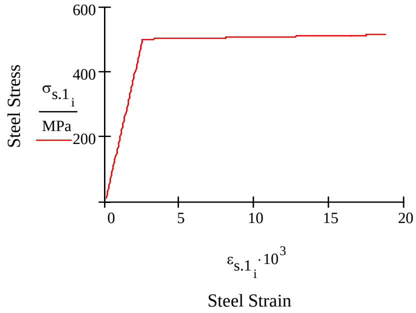
Values of the strain in reinforcement: $\varepsilon_{s.1_i} := \frac{0.05 \cdot i}{1000}$

Initial value position of neutral axis: $y_{0ini} := \frac{h}{10}$

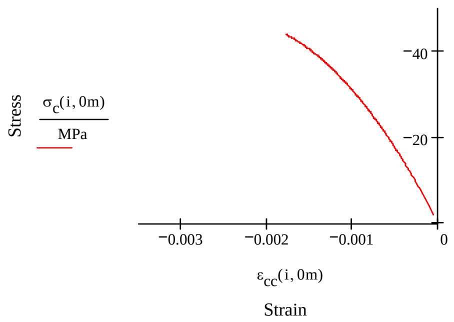
Position of the neutral axis when steel strain is increasing:



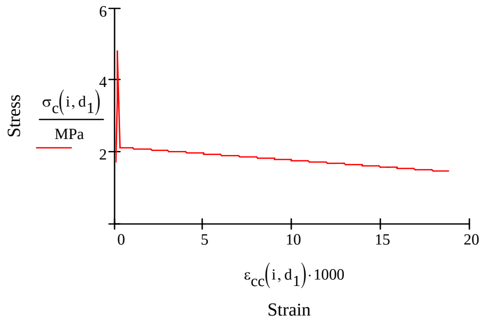
Stress Strain Reinforcement Diagram:



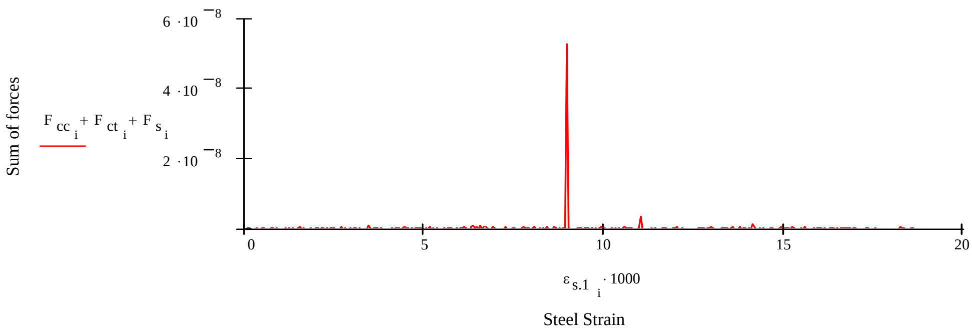
Stress-Strain diagram of the top concrete:



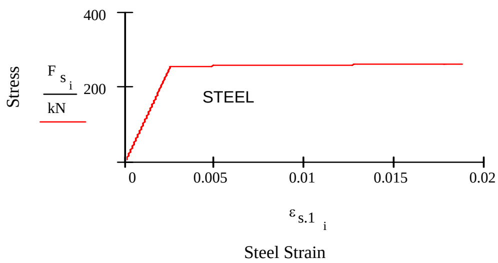
Stress-Strain diagram at the level of reinforcement:

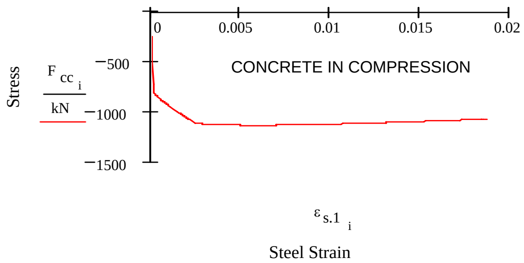


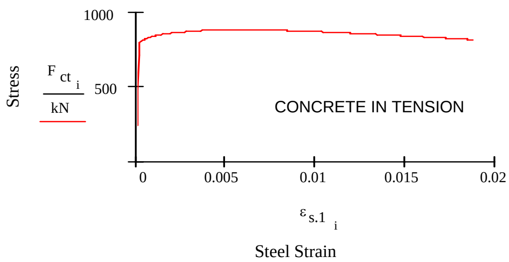
SUM OF FORCES=0 GRAPH



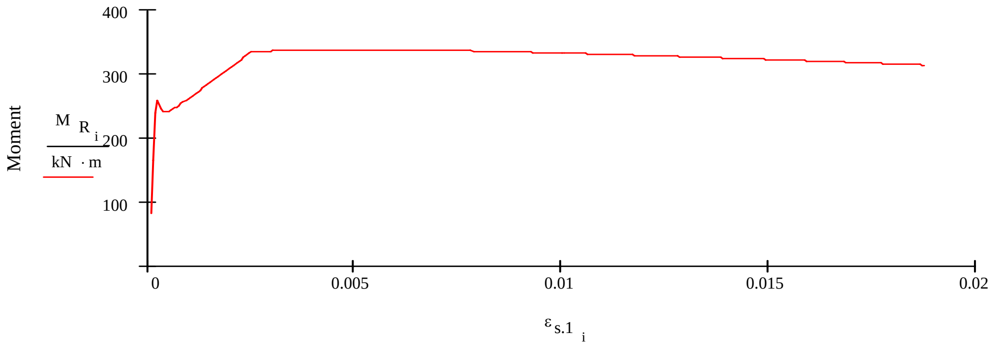
GRAPHS OF FORCES



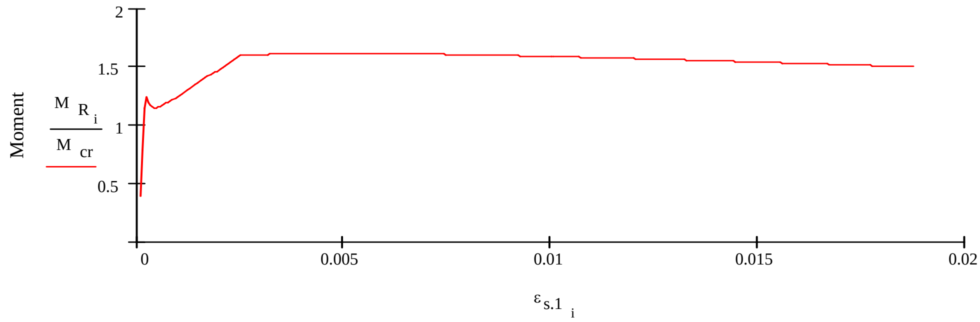




MOMENT-REINFORCEMENT STRAIN GRAPH

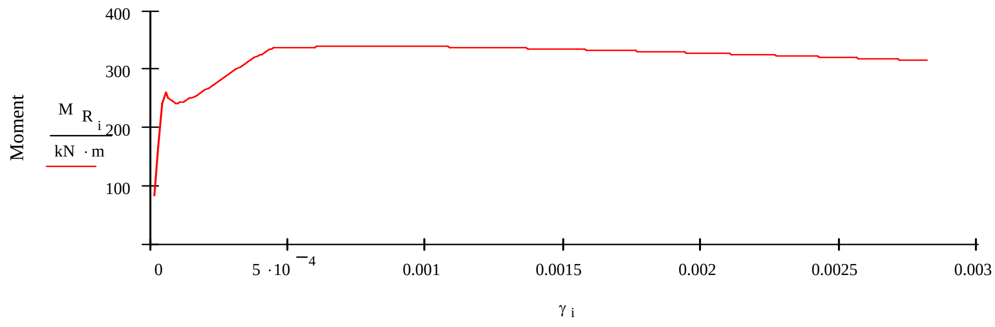


NORMALISED MOMENT-REINFORCEMENT STRAIN GRAPH

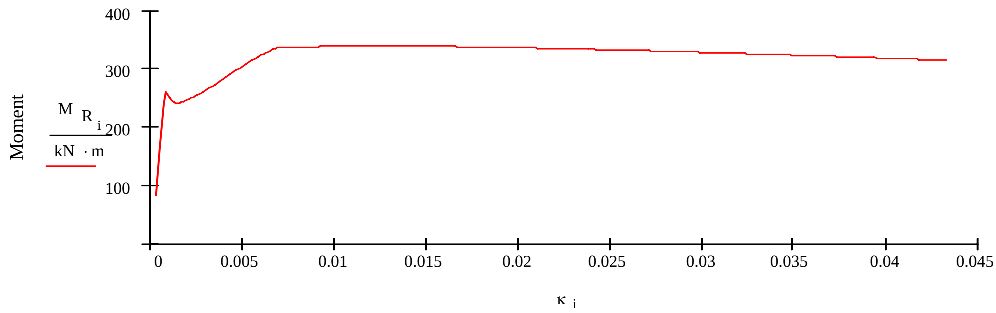


MOMENT-TURN GRAPH

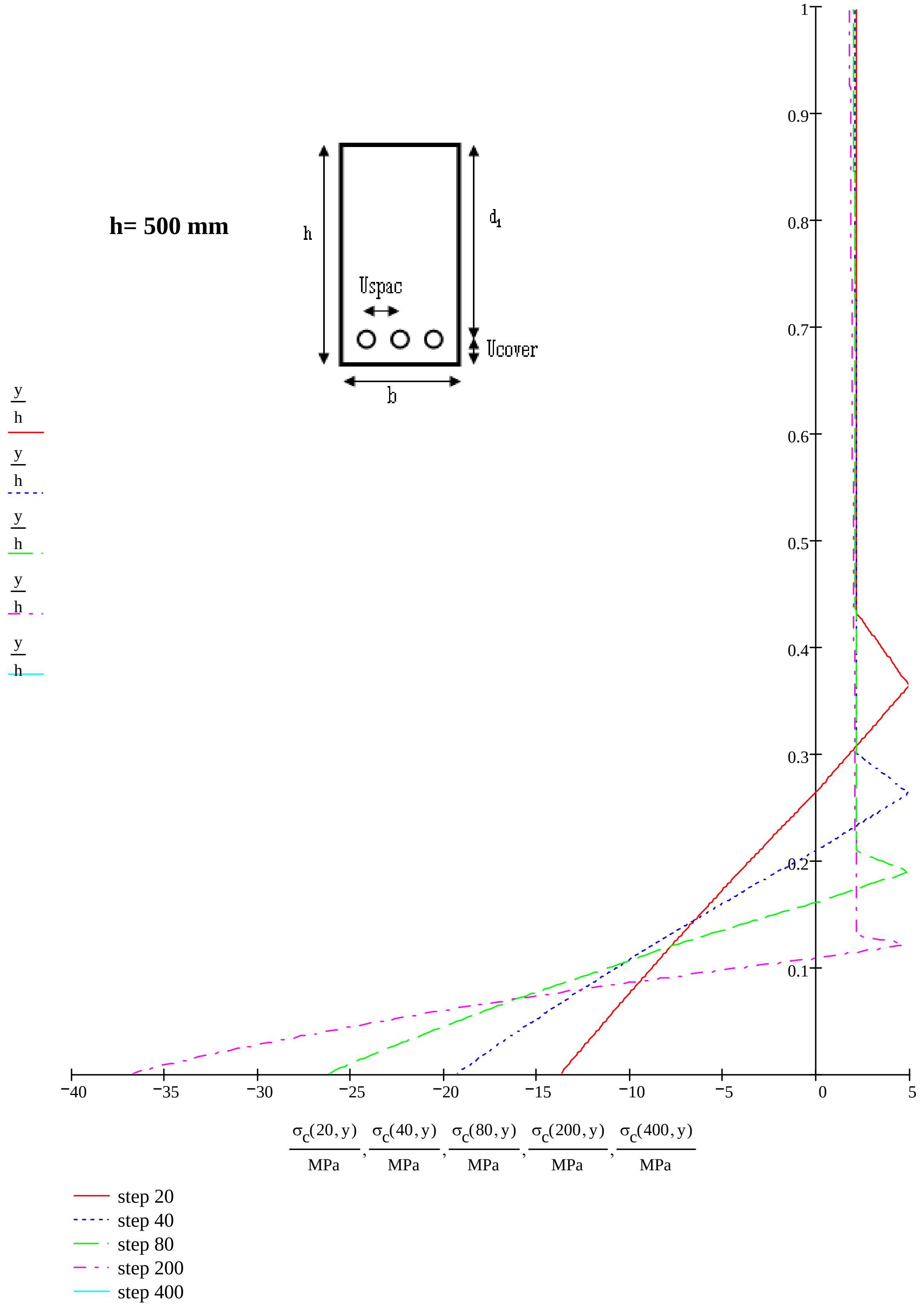
$$\kappa_i := \frac{\varepsilon_{s.1_i}}{d_1 - Y0_i} \qquad \gamma_i := \kappa_i \cdot s$$



MOMENT-CURVATURE GRAPH



STRESS DIAGRAM OF THE CROSS SECTION:



C.2.4 Sigma-epsilon relationship, analytical analysis. RILEM CONSTANTS

Now the different values for the RILEM constants (σ_i and ε_i) are presented as well as the value for the size factor:

MIX A	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	ε_1(‰)	ε_2(‰)	ε_3(‰)	k(h)
HEIGHT 1	4.537	1.641	1.145	0.15	0.25	25	1
HEIGHT 2	4.159	1.382	0.964	0.14	0.24	25	0.842
HEIGHT 3	3.403	0.863	0.602	0.115	0.215	25	0.526

MIX B	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	ε_1(‰)	ε_2(‰)	ε_3(‰)	k(h)
HEIGHT 1	6.289	3.873	2.586	0.19	0.29	25	1
HEIGHT 2	5.765	3.261	2.177	0.17	0.27	25	0.842
HEIGHT 3	4.717	2.038	1.361	0.14	0.24	25	0.526

MIX C	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	ε_1(‰)	ε_2(‰)	ε_3(‰)	k(h)
HEIGHT 1	6.594	4.012	2.309	0.19	0.29	25	1
HEIGHT 2	6.044	3.379	1.945	0.18	0.28	25	0.842
HEIGHT 3	4.945	2.112	1.215	0.14	0.24	25	0.526

Appendix D Crack spacing. Calculations with different approaches

CRACK SPACING

MATERIAL PROPERTIES HEIGHT 1-MIX A

$h := 125mm$ $\quad \rho := 0.1\%$ $\quad L_{fib} := 60mm$ $\quad \phi_{fib} := 0.9mm$ $\quad V_f := 0.5\%$ $\quad \sigma_w := 1.31\cdot 10^6 Pa$ $\quad f_{ct} := 2.5\cdot 10^6 Pa$

$k_1 := 0.8$ $\quad u_{cover} := 25mm$ $\quad d := h - u_{cover}$ $\quad \phi_{bi} := 7mm$

$k_2 := 0.5$ $\quad b := 1000mm$ $\quad A_c := b\cdot h$ $\quad a_{ap} := 150mm$

$$\phi_{bap} := \text{root}\left[\left[\frac{b - a_{ap}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \text{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm \qquad \phi_b = 5mm$$

$A_{s.i} := \pi\frac{\phi_b^2}{4}$ $\quad n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}}$ $\quad n_{ap} = 6.366$ $\quad n := \text{round}(n_{ap}, 0)$ $\quad a := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n - 1}$ $\quad a = 184mm$

$n = 6$

$A_s := n\cdot A_{s.i}$

$$A_{cef} := \begin{cases} 2.5\cdot(h - d)\cdot b & \text{if } 2.5\cdot(h - d)\cdot b \le \frac{h\cdot b}{2} \\ \frac{h\cdot b}{2} & \text{if } 2.5\cdot(h - d)\cdot b > \frac{h\cdot b}{2} \end{cases}$$

$A_{cef} = 6.25\times 10^4 mm^2$ $\quad \rho_{ef} := \frac{A_s}{A_{cef}}$ $\quad \rho_{ef} = 1.885\times 10^{-3}$

$A_{cef2} := \left(\frac{h - 0.063m}{3}\right)\cdot b$ $\quad A_{cef2} = 0.021m^2$

$\rho_{ef2} := \frac{A_s}{A_{cef2}}$ $\quad \rho_{ef2} = 5.7\times 10^{-3}$

RILEM ROUGH PROPOSITION

$S_{mRILEM} := \frac{h}{2}$ $\quad S_{mRILEM} = 62.5mm$

EUROCODE 2 PROPOSITION

$$S_{mEC2} := \left(50 + 0.25 k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad S_{mEC2} = 315.258mm$$

EUROCODE 2 ALTERNATIVE WITH FIBRES

$$k_3 := \left(1 - \frac{\sigma_w}{f_{ct}}\right)$$

$$S_{mEC2F} := \left(\frac{u_{cover}}{mm} + 3\cdot\phi_b\cdot\frac{1}{mm} + 0.25\cdot k_1\cdot k_2\cdot k_3\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad S_{mEC2F} = 166.263mm$$

EUROCODE 2 ALTERNATIVE VANDEWALLE AND RILEM

$$S_{mVANDE} := \left(50 + 0.25\cdot k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot\left(\frac{50}{\frac{L_{fib}}{\phi_{fib}}}\right)\cdot mm \qquad S_{mVANDE} = 236.444mm$$

IBRAHIM AND LUXMOORE PROPOSITION

$$\tau_{bm} := \left(\frac{3}{2\cdot k_1}\right)\cdot f_{ct}$$

$$K_1 := \begin{cases} 1.2\cdot u_{cover} & \text{if } a \le 2\cdot u_{cover} \\ 1.2\cdot\left(u_{cover} + \frac{a - 2\cdot u_{cover}}{4}\right) & \text{if } 14\cdot\phi_b \ge a > 2\cdot u_{cover} \end{cases} \qquad K_1 = 0\ m \qquad K_2 := 0.4$$

$$\gamma := \left(\left(1 + \frac{V_f}{0.01}\cdot 0.4\right)\right) \qquad K_{2f} := \frac{K_2}{\gamma} \qquad \gamma = 1.2 \qquad P_{fpull} := \frac{V_f\cdot\tau_{bm}\cdot L_{fib}}{2\cdot\phi_b}$$

$$P_{fpull} = 1.406\times 10^5\ Pa$$

$$K_3 := 0.125$$

$$\eta_s := \frac{200\cdot\frac{N}{mm^2}\cdot A_s}{200\cdot A_s\cdot\frac{N}{mm^2} + P_{fpull}\cdot A_{cef}}$$

$$S_{I\xi L} := K_1 + K_{2f}\cdot K_3\cdot\eta_s\cdot\frac{\phi_b}{\rho_{ef}} \qquad S_{I\xi L} = 80.497mm$$

MATERIAL PROPERTIES HEIGHT 1-MIX B

$h := 125mm$ $\rho := 0.1\%$ $L_{fib} := 60mm$ $\phi_{fib} := 0.9mm$ $V_f := 1\%$ $\sigma_w := 3.16\cdot 10^6 Pa$ $f_{ct} := 3.5\cdot 10^6 Pa$

$k_1 := 0.8$ $u_{cover} := 25mm$ $d := h - u_{cover}$ $\phi_{bi} := 7mm$

$k_2 := 0.5$ $b := 1000mm$ $A_c := b\cdot h$ $a_{ap} := 150mm$

$$\phi_{bap} := root\left[\left[\frac{b - a_{ap}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := round\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm \qquad \phi_b = 5mm$$

$$A_{s.i} := \pi\frac{\phi_b^2}{4} \qquad n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}} \qquad n_{ap} = 6.366 \qquad n := round(n_{ap}, 0) \qquad a := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n - 1} \qquad a = 184mm$$

$A_s := n\cdot A_{s.i}$

$$A_{cef} := \begin{cases} 2.5(h - d)\cdot b & \text{if } 2.5(h - d)\cdot b \le \frac{h\cdot b}{2} \\ \frac{h\cdot b}{2} & \text{if } 2.5(h - d)\cdot b > \frac{h\cdot b}{2} \end{cases} \qquad A_{cef} = 6.25\times 10^4 mm^2 \qquad \rho_{ef} := \frac{A_s}{A_{cef}}$$

RILEM ROUGH PROPOSITION

$$S_{mRILEM} := \frac{h}{2} \qquad S_{mRILEM} = 62.5mm$$

EUROCODE 2 PROPOSITION

$$S_{mEC2} := \left(50 + 0.25k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad S_{mEC2} = 315.258mm$$

EUROCODE 2 ALTERNATIVE WITH FIBRES

$$k_3 := \left(1 - \frac{\sigma_w}{f_{ct}}\right)$$

$$s_{mEC2F} := \left(\frac{u_{cover}}{mm} + 3\cdot\phi_b\cdot\frac{1}{mm} + 0.25\cdot k_1\cdot k_2\cdot k_3\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad s_{mEC2F} = 65.768\,mm$$

EUROCODE 2 ALTERNATIVE VANDEWALLE AND RILEM

$$s_{mVANDE} := \left(50 + 0.25\cdot k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot\left(\frac{50}{\frac{L_{fib}}{\phi_{fib}}}\right)\cdot mm \qquad s_{mVANDE} = 236.444\,mm$$

IBRAHIM AND LUXMOORE PROPOSITION

$$\tau_{bm} := \left(\frac{3}{2\cdot k_1}\right)\cdot f_{ct}$$

$$K_1 := \begin{cases} 1.2\cdot u_{cover} & \text{if } a \le 2\cdot u_{cover} \\ 1.2\cdot\left(u_{cover} + \frac{a - 2\cdot u_{cover}}{4}\right) & \text{if } 14\cdot\phi_b \ge a > 2\cdot u_{cover} \end{cases} \qquad K_1 = 0\ m \qquad K_2 := 0.4$$

$$\gamma := \left(\left(1 + \frac{V_f}{0.01}\cdot 0.4\right)\right) \qquad K_{2f} := \frac{K_2}{\gamma} \qquad \gamma = 1.4 \qquad P_{fpull} := \frac{V_f\cdot\tau_{bm}\cdot L_{fib}}{2\cdot\phi_b}$$

$$P_{fpull} = 3.938\times 10^5\ Pa$$

$$K_3 := 0.125$$

$$\eta_s := \frac{200\cdot\frac{N}{mm^2}\cdot A_s}{200\cdot A_s\cdot\frac{N}{mm^2} + P_{fpull}\cdot A_{cef}}$$

$$s_{I\xi L} := K_1 + K_{2f}\cdot K_3\cdot\eta_s\cdot\frac{\phi_b}{\rho_{ef}} \qquad s_{I\xi L} = 46.338\,mm$$

MATERIAL PROPERTIES HEIGHT 1-MIX C

$h := 125mm$ $\rho := 0.1\%$ $L_{fib} := 35mm$ $\phi_{fib} := 0.55mm$ $V_f := 1\%$ $\sigma_w := 3.17 \cdot 10^6 Pa$ $f_{ct} := 3.5 \cdot 10^6 Pa$

$k_1 := 0.8$ $u_{cover} := 25mm$ $d := h - u_{cover}$ $\phi_{bi} := 7mm$

$k_2 := 0.5$ $b := 1000mm$ $A_c := b \cdot h$ $a_{ap} := 150mm$

$$\phi_{bap} := \text{root}\left[\left[\frac{b - a_{ap}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 4.659mm \qquad \phi_b := \text{round}\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm \qquad \phi_b = 5mm$$

$$A_{s.i} := \pi\frac{\phi_b^2}{4} \qquad n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}} \qquad n_{ap} = 6.366 \qquad n := \text{round}(n_{ap}, 0) \qquad a := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n-1} \qquad a = 184mm$$

$A_s := n \cdot A_{s.i}$

$$A_{cef} := \begin{cases} 2.5\cdot(h-d)\cdot b & \text{if } 2.5\cdot(h-d)\cdot b \le \frac{h\cdot b}{2} \\ \frac{h\cdot b}{2} & \text{if } 2.5\cdot(h-d)\cdot b > \frac{h\cdot b}{2} \end{cases} \qquad A_{cef} = 6.25\times 10^4 mm^2 \qquad \rho_{ef} := \frac{A_s}{A_{cef}}$$

RILEM ROUGH PROPOSITION

$$S_{mRILEM} := \frac{h}{2} \qquad S_{mRILEM} = 62.5mm$$

EUROCODE 2 PROPOSITION

$$S_{mEC2} := \left(50 + 0.25\,k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad S_{mEC2} = 315.258mm$$

EUROCODE 2 ALTERNATIVE WITH FIBRES

$$k_3 := \left(1 - \frac{\sigma_w}{f_{ct}}\right)$$

$$S_{mEC2F} := \left(\frac{u_{cover}}{mm} + 3\cdot\phi_b\cdot\frac{1}{mm} + 0.25\cdot k_1\cdot k_2\cdot k_3\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot mm \qquad S_{mEC2F} = 65.01\ mm$$

EUROCODE 2 ALTERNATIVE VANDEWALLE AND RILEM

$$S_{mVANDE} := \left(50 + 0.25\cdot k_1\cdot k_2\cdot\frac{\phi_b\cdot\frac{1}{mm}}{\rho_{ef}}\right)\cdot\left(\frac{50}{\frac{L_{fib}}{\phi_{fib}}}\right)\cdot mm \qquad S_{mVANDE} = 247.703\ mm$$

IBRAHIM AND LUXMOORE PROPOSITION

$$\tau_{bm} := \left(\frac{3}{2\cdot k_1}\right)\cdot f_{ct}$$

$$K_1 := \begin{cases} 1.2\cdot u_{cover} & \text{if } a \le 2\cdot u_{cover} \\ 1.2\cdot\left(u_{cover} + \frac{a - 2\cdot u_{cover}}{4}\right) & \text{if } 14\cdot\phi_b \ge a > 2\cdot u_{cover} \end{cases} \qquad K_1 = 0\ m \qquad K_2 := 0.4$$

$$\gamma := \left(\left(1 + \frac{V_f}{0.01}\cdot 0.4\right)\right) \qquad K_{2f} := \frac{K_2}{\gamma} \qquad \gamma = 1.4 \qquad P_{fpull} := \frac{V_f\cdot\tau_{bm}\cdot L_{fib}}{2\cdot\phi_b}$$

$$P_{fpull} = 2.297 \times 10^5\ Pa$$

$$K_3 := 0.125$$

$$\eta_s := \frac{200\cdot\frac{N}{mm^2}\cdot A_s}{200\cdot A_s\cdot\frac{N}{mm^2} + P_{fpull}\cdot A_{cef}}$$

$$S_{I\xi L} := K_1 + K_{2f}\cdot K_3\cdot\eta_s\cdot\frac{\phi_b}{\rho_{ef}} \qquad S_{I\xi L} = 58.869\ mm$$

MATERIAL PROPERTIES HEIGHT 2-MIX A

$h := 250mm$ $\rho := 0.1\%$ $L_{fib} := 60mm$ $\phi_{fib} := 0.9mm$ $V_f := 0.5\%$ $\sigma_w := 1.31 \cdot 10^6 Pa$ $f_{ct} := 2.5 \cdot 10^6 Pa$

$k_1 := 0.8$ $u_{cover} := 25mm$ $d := h - u_{cover}$ $\phi_{bi} := 7mm$

$k_2 := 0.5$ $b := 1000mm$ $A_c := b \cdot h$ $a_{ap} := 150mm$

$$\phi_{bap} := root\left[\left[\frac{b - a_{ap} \cdot \left(\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}} - 1\right) - 2 \cdot u_{cover}}{\frac{A_c \cdot \rho}{\pi \frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right] \qquad \phi_{bap} = 6.588mm \qquad \phi_b := round\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right] \cdot mm \qquad \phi_b = 7mm$$

$A_{s.i} := \pi \frac{\phi_b^2}{4}$ $n_{ap} := \frac{A_c \cdot \rho}{A_{s.i}}$ $n_{ap} = 6.496$ $n := round(n_{ap}, 0)$ $a := \frac{b - 2 \cdot u_{cover} - n \cdot \phi_b}{n - 1}$ $a = 181.6mm$

$A_s := n \cdot A_{s.i}$ $n = 6$

$$A_{cef} := \begin{cases} 2.5(h - d) \cdot b & \text{if } 2.5(h - d) \cdot b \le \frac{h \cdot b}{2} \\ \frac{h \cdot b}{2} & \text{if } 2.5(h - d) \cdot b > \frac{h \cdot b}{2} \end{cases} \qquad A_{cef} = 6.25 \times 10^4 mm^2 \qquad \rho_{ef} := \frac{A_s}{A_{cef}}$$

$S_{mRILEM} = 125mm$ $S_{mEC2} = 239.47mm$

$S_{I\xi L} = 69.498mm$ $S_{mEC2F} = 136.188mm$ $S_{mVANDE} = 179.603mm$

MATERIAL PROPERTIES HEIGHT 2-MIX B

$S_{mRILEM} = 125mm$ $S_{mEC2} = 239.47mm$

$S_{I\xi L} = 49.012mm$ $S_{mEC2F} = 64.406mm$ $S_{mVANDE} = 179.603mm$

MATERIAL PROPERTIES HEIGHT 2-MIX C

$S_{mRILEM} = 125mm$ $S_{mEC2} = 239.47mm$

$S_{I\xi L} = 55.373mm$ $S_{mEC2F} = 63.864mm$ $S_{mVANDE} = 188.155mm$

MATERIAL PROPERTIES HEIGHT 3-MIX A

$h := 500mm$ $\rho := 0.1\%$ $L_{fib} := 60mm$ $\phi_{fib} := 0.9mm$ $V_f := 0.5\%$ $\sigma_w := 1.31 \cdot 10^6 Pa$ $f_{ct} := 2.5 \cdot 10^6 Pa$

$k_1 := 0.8$ $u_{cover} := 25mm$ $d := h - u_{cover}$ $\phi_{bi} := 7mm$

$k_2 := 0.5$ $b := 1000mm$ $A_c := b \cdot h$ $a_{ap} := 150mm$

$$\phi_{bap} := root\left[\left[\frac{b - a_{ap}\cdot\left(\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}} - 1\right) - 2\cdot u_{cover}}{\frac{A_c\cdot\rho}{\pi\frac{\phi_{bi}^2}{4}}}\right], \phi_{bi}\right]$$

$\phi_{bap} = 9.317mm$ $\phi_b := round\left[\left(\frac{\phi_{bap}}{mm}\right), 0\right]\cdot mm$ $\phi_b = 9\,mm$

$A_{s.i} := \pi\frac{\phi_b^2}{4}$ $n_{ap} := \frac{A_c\cdot\rho}{A_{s.i}}$ $n_{ap} = 7.86$ $n := round(n_{ap}, 0)$ $a := \frac{b - 2\cdot u_{cover} - n\cdot\phi_b}{n-1}$ $a = 125.429mm$

$n = 8$

$A_s := n\cdot A_{s.i}$

$2.5\cdot(h-d)\cdot b = 6.25\times 10^4\,mm^2$

$$A_{cef} := \begin{cases} 2.5\cdot(h-d)\cdot b & \text{if } 2.5\cdot(h-d)\cdot b \le \frac{h\cdot b}{2} \\ \frac{h\cdot b}{2} & \text{if } 2.5\cdot(h-d)\cdot b > \frac{h\cdot b}{2} \end{cases}$$

$A_{cef} = 6.25\times 10^4\,mm^2$ $\rho_{ef} := \frac{A_s}{A_{cef}}$

$S_{mRILEM} = 250mm$ $S_{mEC2} = 160.524mm$

$S_{I\xi L} = 96.572mm$ $S_{mEC2F} = 104.61mm$ $S_{mVANDE} = 120.393mm$

MATERIAL PROPERTIES HEIGHT 3-MIX B

$S_{mRILEM} = 250mm$ $S_{mEC2} = 160.524mm$

$S_{I\xi L} = 87.427mm$ $S_{mEC2F} = 62.737mm$ $S_{mVANDE} = 120.393mm$

MATERIAL PROPERTIES HEIGHT 3-MIX C

$S_{mRILEM} = 250mm$ $S_{mEC2} = 160.524mm$

$S_{I\xi L} = 89.233mm$ $S_{mEC2F} = 62.421mm$ $S_{mVANDE} = 126.126mm$

Appendix E DIANA data files

In this appendix one example of the DIANA data file in each approach is represented. The comments explain how the data have to be introduced.

E.1.1 Sigma-epsilon relationship Mix A

```
FEMGEN MODEL      : MOD_MIXA_H1_V2
ANALYSIS TYPE     : Structural 2D
MODEL DESCRIPTION : Sigma-opening mix A height 1
'UNITS'
LENGTH   M
TIME     SEC
TEMPER   KELVIN
FORCE    N
'COORDINATES' DI=2 : It defines the geometry of the body
    1     -5.000000E-02     2.750000E-02
    2     -4.450000E-02     2.750000E-02
    3     -3.900000E-02     2.750000E-02
    4     -3.350000E-02     2.750000E-02
................
  561     -5.000000E-02     1.156250E-01
  562     -5.000000E-02     1.187500E-01
  563     -5.000000E-02     1.218750E-01
  564     -5.000000E-02     1.250000E-01
'ELEMENTS'
CONNECTIVITY : It defines the characteristics of the elements
    1 L2TRU  1 2
   11 L6BEN  12 13
   12 Q8MEM  14 15 26 25

..................
  513 L8IF   130 131 7 8
  514 L8IF   131 132 8 9
  515 L8IF   132 133 9 10
  516 L8IF   133 134 10 11

MATERIALS           :Elements
:Concrete elastic
/ 12-461 / 1
:Concrete crack
/ 462-506 / 2
:Reinforcement
/ 1-10 / 3
```

```
:Bond-slip
/ 507-516 / 4
:Dummy beam
/ 11 / 5

GEOMETRY            :Elements
:Concrete elastic
/ 12-461 / 1
:Concrete crack
/ 462-506 / 2
:Reinforcement
/ 1-10 / 3
:Bond-slip
/ 507-516 / 4
:Dummy beam
/ 11 / 5

'GROUPS'
ELEMEN
   1 CONCR / 12-461 /
ELEMEN
   2 CRACK / 462-506 /
ELEMEN
   3 REBAR / 1-10 /
ELEMEN
   4 BONDS / 507-516 /
ELEMEN
   5 DUMMY / 11 /

'MATERIALS'
    1 DENSIT 2.4E+03
:                   Density of the uncracked concrete
      TOTCRK ROTATE
:                   Rotating axis (total strain model)
      POISON 0.2
:                   Poisson coefficient
      YOUNG  30.589E+09
:                   Young modulus
      COMSTR 30E+06
:                   Compressive strength
      COMCRV MULTLN
:                   Multilinear approach in compression and values
      COMPAR 0E+0 0E+0
```

```
            -9.42E+6  -3.15E-4
            -1.74E+7  -6.65E-4
            -2.32E+7  -1.02E-3
            -2.70E+7  -1.37E-3
            -2.92E+7  -1.72E-3
            -3.00E+7  -2.07E-3
            -2.95E+7  -2.42E-3
            -2.80E+7  -2.77E-3
            -2.54E+7  -3.12E-3
            -2.16E+7  -3.50E-3
      TENCRV ELASTI
:                    Elastic behaviour in tension
    2 DSTIF  5.562E+14 5.562E+14
:                    Values of the stiffnes chosen appropiately
      DISCRA 1
:                    Discrete crack initiation criterion of normal traction
      DCRVAL 2.50E+06
:                    Tensile strength
      MODE1  3
:                    Crack-opening  Stress  relationship  (bilinear)  but  half
values of the crack opening are needed
      MO1VAL 2.50E+6 0
             1.368E+6 2.265E-5
             0.00E+6 4.231E-3
      UNLO1  2
:                    Secant unloading: a straight line back to the origin (no
so important here)
      MODE2  1
:                    Constant shear modulus after cracking
      MO2VAL 10.0E+06
:                    Value of the shear modulus
    3 DENSIT 7.85E+03
:                    Density of steel
      YOUNG  200E+09
:                    Young modulus of steel
      POISON 0.3
:                    Poisson coefficient
      YIELD  VMISES
:                    Yielding criteria (Von Mises)
      HARDEN STRAIN
:                    Hardening hypothesis (work hardening) and values
      HARDIA 500.0E+06  0.0
             540.0E+06  0.048
```

```
             200.0E+06  0.05
             0.0E+06    0.1
    4 DSTIF  8.26E+10  8.26E+10
:                    Values of the stiffnes chosen appropiately !
      BONDSL 3
:                    Model chosen for the bond-slip
      SLPVAL 0 0
             4.13E+06 0.050E-3
             5.45E+06 0.1E-3
             7.19E+06 0.200E-3
             8.46E+06 0.300E-3
             9.49E+06 0.400E-3
             10.38E+06 0.500E-3
             11.16E+06 0.600E-3
             12.52E+06 0.800E-3
             13.69E+06 1.000E-3
             13.69E+06 3.000E-3
             9.59E+06 3.500E-3
             5.48E+06  4.00E-3
             5.48E+06  1.00E-2
    5 YOUNG  200E+09
:                    Young modulus of dummy beam

'GEOMETRY'
    1 THICK  1.0
:                    Thickness of the concrete element
    2 THICK  1.0
:                    Thickness of the crack interface
      CONFIG MEMBRA
:                    Configuration of the crack interface (plane stress)
    3 CROSSE 1.178E-04
:                    Total cross area (reinforcement)
    4 CONFIG  BONDSL
:                    Configuration of the interface (bond slip)
      THICK  0.094
:                    Sum of the Perimeter of reinforcement bars
    5 RECTAN 0.125 1
:                    Dimensions of a filled rectangle (dummy beam)



'SUPPORTS'
```

```
 / 1 519-564 /   TR     1
 / 12 /   TR     2
 / 12 /   RO     3

 'TYINGS'
ECCENT TR 1
 / 13 24 35 46 57 68 79 90 101 112 123 134 145 156 167 178
 189 200 211 222 233 244 255 266 277 288 309 320 331 342
 353 364 375 386 397 408 419 430 441 452 463 474 485 496
 507 518 / 12
```
: **The whole rigth side and the dummy beam has the same X displacements as the master node**

```
ECCENT TR 2
 / 11 24 35 46 57 68 79 90 101 112 123 134 145 156 167 178
 189 200 211 222 233 244 255 266 277 288 309 320 331 342
 353 364 375 386 397 408 419 430 441 452 463 474 485 496
 507 518 / 12
```
: **The whole rigth side and the dummy beam has the same Y displacements as the master node**

```
'LOADS'
CASE 1
DEFORM
  12  RO 3 1.0E-03
```
:   **Load applied**

```
'DIRECTIONS'
    1   1.000000E+00   0.000000E+00   0.000000E+00
    2   0.000000E+00   1.000000E+00   0.000000E+00
    3   0.000000E+00   0.000000E+00   1.000000E+00
'END'
```

E.2.1 Sigma-crack opening relationship Mix B

```
FEMGEN MODEL      : MOD_MIXB_H1_V2
ANALYSIS TYPE     : Structural 2D
MODEL DESCRIPTION : Model sigma-opening Mix B height 1
'UNITS'
LENGTH   M
TIME     SEC
TEMPER   KELVIN
FORCE    N
'COORDINATES' DI=2
    1     -5.000000E-02     2.750000E-02
```

```
    2     -4.725000E-02     2.750000E-02
    3     -4.450000E-02     2.750000E-02
    4     -4.175000E-02     2.750000E-02
    5     -3.900000E-02     2.750000E-02
    6     -3.625000E-02     2.750000E-02
....................
  559     -5.000000E-02     1.093750E-01
  560     -5.000000E-02     1.125000E-01
  561     -5.000000E-02     1.156250E-01
  562     -5.000000E-02     1.187500E-01
  563     -5.000000E-02     1.218750E-01
  564     -5.000000E-02     1.250000E-01
'ELEMENTS'
CONNECTIVITY
    1 L2TRU  1 2
   11 L6BEN  12 13
   12 Q8MEM  14 15 26 25
....................
514 L8IF   127 126 4 3
  515 L8IF   126 125 3 2
  516 L8IF   125 124 2 1

MATERIALS            :Elements
:Concrete elastic
/ 12-461 / 1
:Concrete crack
/ 462-506 / 2
:Reinforcement
/ 1-10 / 3
:Bond-slip
/ 507-516 / 4
:Dummy beam
/ 11 / 5

GEOMETRY             :Elements
:Concrete elastic
/ 12-461 / 1
:Concrete crack
/ 462-506 / 2
:Reinforcement
/ 1-10 / 3
:Bond-slip
/ 507-516 / 4
```

```
:Dummy beam
/ 11 / 5

'GROUPS'
ELEMEN
   1 CONCR / 12-461 /
ELEMEN
   2 CRACK / 462-506 /
ELEMEN
   3 BONDS / 1-10 507-516 /
ELEMEN
   4 REBAR / 1-10 /
ELEMEN
   5 DUMMY / 11 /


'MATERIALS'
    1 DENSIT 2.4E+03
:                    Density of the uncracked concrete
      TOTCRK ROTATE
:                    Rotating axis (total strain model)
      POISON 0.2
:                    Poisson coefficient
      YOUNG  34.313E+09
:                    Young modulus
      COMSTR 44E+06
:                    Compressive strength
      COMCRV MULTLN
:                    Multilinear approach in compression and values
      COMPAR 0E+0 0E+0
             -1.11E+7  -3.15E-4
             -2.16E+7  -6.65E-4
             -3.01E+7  -1.02E-3
             -3.66E+7  -1.37E-3
             -4.11E+7  -1.72E-3
             -4.35E+7  -2.07E-3
             -4.39E+7  -2.42E-3
             -4.21E+7  -2.77E-3
             -3.83E+7  -3.12E-3
             -3.15E+7  -3.50E-3
      TENCRV ELASTI
:                    Elastic behaviour in tension
    2 DSTIF  1.112E+15 1.112E+15
```

```
:                    Values of the stiffnes chosen appropiately
      DISCRA 1
:                    Discrete crack initiation criterion of normal traction
      DCRVAL 3.50E+06
:                    Tensile strength
      MODE1  3
:                    Crack-opening  Stress  relationship  (bilinear)  but  half
values of the crack opening are needed
      MO1VAL 3.50E+6 0
             3.359E+6 1.261E-6
             0.00E+6 3.429E-3
      UNLO1  2
:                    Secant unloading: a straight line back to the origin (no
so important here)
      MODE2  1
:                    Constant shear modulus after cracking
      MO2VAL 10.0E+06
:                    Value of the shear modulus
    3 DENSIT 7.85E+03
:                    Density of steel
      YOUNG  200E+09
:                    Young modulus of steel
      POISON 0.3
:                    Poisson coefficient
      YIELD  VMISES
:                    Yielding criteria (Von Mises)
      HARDEN STRAIN
:                    Hardening hypothesis (work hardening) and values
      HARDIA 500.0E+06  0.0
             540.0E+06  0.048
             200.0E+06  0.05
             0.0E+06    0.1
    4 DSTIF  1.06E+11  1.06E+11
:                    Values of the stiffnes chosen appropiately !
      BONDSL 3
:                    Model chosen for the bond-slip
      SLPVAL 0 0
             5.00E+06 0.050E-3
             6.60E+06 0.1E-3
             8.71E+06 0.200E-3
             10.25E+06 0.300E-3
             11.49E+06 0.400E-3
             12.57E+06 0.500E-3
             13.52E+06 0.600E-3
```

```
            15.17E+06 0.800E-3
            16.58E+06 1.000E-3
            16.58E+06 3.000E-3
            11.61E+06 3.500E-3
            6.63E+06  4.00E-3
            6.63E+06  1.00E-2
    5 YOUNG  200E+09
```

: **Young modulus of dummy beam**

```
'GEOMETRY'
    1 THICK  1.0
```

: **Thickness of the concrete element**

```
    2 THICK  1.0
```

: **Thickness of the crack interface**

```
      CONFIG MEMBRA
```

: **Configuration of the crack interface (plane stress)**

```
    3 CROSSE 1.178E-04
```

: **Total cross area (reinforcement)**

```
    4 CONFIG BONDSL
```

: **Configuration of the interface (bond slip)**

```
      THICK  0.094
```

: **Sum of the Perimeter of reinforcement bars**

```
    5 RECTAN 0.125 1
```

: **Dimensions of a filled rectangle (dummy beam)**

```
'SUPPORTS'
 / 1 519-564 /   TR     1
 / 12 /   TR     2
 / 12 /   RO     3
  'TYINGS'
ECCENT TR 1
 / 11 24 35 46 57 68 79 90 101 112 123 134 145 156 167 178
 189 200 211 222 233 244 255 266 277 288 309 320 331 342
 353 364 375 386 397 408 419 430 441 452 463 474 485 496
 507 518 / 12
```

: The whole rigth side and the dummy beam has the same X displacements as the master node

```
ECCENT TR 2
 / 11 24 35 46 57 68 79 90 101 112 123 134 145 156 167 178
 189 200 211 222 233 244 255 266 277 288 309 320 331 342
 353 364 375 386 397 408 419 430 441 452 463 474 485 496
 507 518 / 12
```

```
: The whole rigth side and the dummy beam has the same Y displacements as
the master node

'LOADS'
CASE 1
DEFORM
  12  RO 3 1.0E-03
:                    Load applied
'DIRECTIONS'
    1   1.000000E+00   0.000000E+00   0.000000E+00
    2   0.000000E+00   1.000000E+00   0.000000E+00
    3   0.000000E+00   0.000000E+00   1.000000E+00
'END'
```