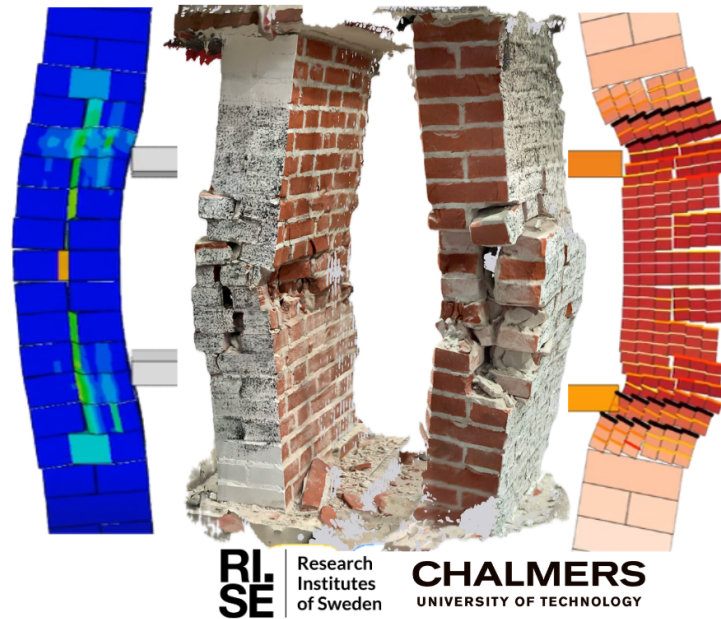




CHALMERS
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HybridFEM and FEM Numerical Framework for the Structural Analysis of Impact-loaded Unreinforced Masonries

Master's thesis in the Master's Program Structural Engineering and Building Technology

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING
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CHALMERS UNIVERSITY OF TECHNOLOGY
Master's thesis

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ABSTRACT

Unreinforced masonry walls constitute a significant portion of Sweden's built environment, yet their resistance to out-of-plane impact loading remains poorly understood and inadequately covered by existing design guidelines. This thesis develops and validates two independent numerical frameworks for simulating the dynamic response of double-wythe URM walls subjected to pendulum impact loading: a continuum finite element model implemented in Abaqus/Explicit, and a Hybrid Discrete-Finite Element Method (HybriDFEM) model. Both adopt a meso-scale modelling strategy and are calibrated against quasi-static four-point bending tests before being validated against three experimental impact campaigns conducted at RISE Research Institutes of Sweden.

The results demonstrate that both frameworks reproduced the global dynamic response, including out-of-plane displacements, impact forces, arching behaviour, wall uplift, and local failure mechanisms. The Abaqus model shows better agreement for low-mass, low-velocity configurations and provides a detailed reference for local stress and damage development, while HybriDFEM performs consistently across a broader range of impact energies at substantially lower computational cost. Together, the two frameworks provide complementary tools for the assessment of URM walls under impulsive loading, and their multi-campaign validation, including a blind prediction, contributes to the ongoing RISE-FORTV programme on design guidelines for masonry structures subjected to blast and impact loading.

Key words: Unreinforced masonry, out-of-plane impact, Abaqus/Explicit, HybriDFEM, meso-scale modelling, dynamic response, arching action, experimental validation, blind prediction

HybridFEM och FEM numeriskt ramverk för strukturanalys av stötbelastade oarmerade murverk

Examensarbete inom masterprogrammet Structural Engineering and Building Technology

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SAMMANFATTNING

Oarmerade murverk utgör en betydande del av Sveriges befintliga byggnadsbestånd, men deras motstånd mot ut-ur-planets impaktbelastning är otillräckligt utforskat och bristfälligt täckt av befintliga dimensioneringsriktlinjer. Denna avhandling utvecklar och validerar två oberoende numeriska ramverk för att simulera det dynamiska beteendet hos dubbel-skiktade oarmerade murverksväggar utsatta för pendelimpakt: en kontinuumsmodell implementerad i Abaqus/Explicit och en Hybrid Diskret–Finit Elementmetod (HybriDFEM). Båda ramverken använder en mesoskalig modelleringsstrategi och kalibreras mot kvasistatiska fyrapunktsböjprov innan de valideras mot tre experimentella impaktkampanjer genomförda vid RISE Research Institutes of Sweden.

Resultaten visar att båda ramverken reproducerar det globala dynamiska beteendet, inklusive ut-ur-planets-förskjutningar, impaktkrafter, välvverkan och lokala brottmekanismer. Abaqus-modellen ger bättre överensstämmelse vid låga energinivåer och ger detaljerad information om lokala spännings- och skadefält, medan HybriDFEM presterar konsekvent över ett bredare energispektrum till betydligt lägre beräkningskostnad. Tillsammans utgör de två ramverken kompletterande verktyg för bedömning av oarmerade murverksväggar under impaktbelastning och bidrar till det pågående RISE–FORTV-programmet för dimensioneringsriktlinjer för murverkskonstruktioner utsatta för blast- och impaktlaster.

Nyckelord: oarmerat murverk, pendelimpakt, HybriDFEM, Abaqus, CDP, välvverkan, brottmekanismer, kalibrering, numerisk modellering, RISE–FORTV

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This thesis also represents the friendship we built along the way, one that neither of us could have anticipated. The odds of our paths crossing were already quite small, and yet from the very first moment we met, something felt familiar. Without even realizing it, we opened the doors of our lives to one another and built a bond that extends far beyond this thesis.

This work has deepened our appreciation for heritage structures, which are more than physical constructions; they carry memory, identity, and the stories of generations. We hope it contributes, in some small way, to their preservation.

Although our journeys have been different, this experience has shown us the value of carrying more than one place in our hearts. It has deepened our appreciation for those who build connections across cultures and communities, and for all those who have the courage to leave what is known in pursuit of growth, opportunity, and a better future. This work is, in many ways, a reflection of that shared resilience, and a dedication to all the women engineers who came before us, whose determination made our paths possible, and to those who will come after us. May we continue to build, to question, and to create.

Notations

CDP	Concrete Damaged Plasticity
DIF	Dynamic Increase Factor
DIC	Digital Image Correlation
EP	ElastoPlastic (contact law)
EP_CD	ElastoPlastic with Contact Deletion
FEM	Finite Element Method
HybridFEM	Hybrid Discrete–Finite Element Method
KE	Kinetic energy (J)
MSF	Mass Scaling Factor
OOP	Out-of-plane
RISE	Research Institutes of Sweden
TSF	Time Scaling Factor
URM	Unreinforced Masonry

E_b	Brick elastic modulus (MPa)
G_c	Compressive fracture energy (N/mm)
G_t	Tensile fracture energy (N/mm)
$G_{f,I}$	Mode I fracture energy at joint (N/mm)
$G_{f,II}$	Mode II fracture energy at joint (N/mm)
K_1	Interface normal stiffness, HybridFEM (GPa/m)
K_{sup}	Top support stiffness (kN/mm)
$K_{sup,x}$	Horizontal top support stiffness (kN/mm)
$K_{sup,y}$	Vertical top support stiffness (kN/mm)

c	Cohesion of mortar joint (MPa)
f_c	Compressive strength of brick (MPa)
f_t	Tensile strength of brick (MPa)
$f_{t,res}$	Residual tensile strength of brick (MPa)
k_n	Interface normal stiffness (MPa/mm)
k_s	Interface shear stiffness (MPa/mm)
k_t	Interface tangential stiffness (MPa/mm)
$k_{n,imp}$	Impactor contact stiffness (GPa/m)
v_0	Pre-impact velocity (m/s)

α_M	Mass-proportional damping coefficient (s^{-1})
$\dot{\varepsilon}$	Strain rate (s^{-1})
μ	Friction coefficient of mortar joint
ν	Poisson's ratio
ψ	Dilation angle ($^\circ$)
σ_i	Compressive stress at yielding initiation (MPa)
σ_p	Compressive peak strength (MPa)
ξ	Damping ratio

1 Introduction

1.1 Background

Unreinforced masonry (URM) buildings form a large part of Europe's built environment, including both heritage and modern structures. Many of these were not designed to resist out-of-plane impacts or blasts, making their dynamic response under such loads uncertain and potentially leading to progressive collapse with severe structural and societal consequences. Due to its limited tensile capacity and brittle behaviour, URM walls are particularly vulnerable in this loading regime, because failure can occur suddenly and with limited energy dissipation, posing significant risks to structural safety and human life. Swedish construction guidelines (Fortifikationsverkets Konstruktionsregler, FKR) provide limited guidance on the blast and impact resistance of masonry structures. This gap is also recognised internationally, where existing standards (Oswald & Zehrt, 2010; U.S. Department of Defense, 2008) rely on simplified analytical assumptions or recommend strengthening measures such as steel reinforcement that are often incompatible with heritage conservation requirements, where structural interventions must remain minimal. Experimental campaigns are therefore essential to improve the understanding of the response of URM walls under impulsive loading, but testing alone cannot represent the full range of configurations found in practice. Validated numerical models are needed to complement these studies, reproduce and extend experimental findings, and support reliable design recommendations for masonry structures under extreme loads.

In recent years, RISE Research Institutes of Sweden, in collaboration with the Swedish Fortifications Agency (FORTV), has undertaken a comprehensive experimental and numerical programme to improve the understanding of masonry behaviour under impulsive loading and support the development of dedicated FKR guidelines. Experimental and numerical methods were first reviewed (Godio et al., 2021). A series of quasi-static four-point bending tests on clay brick walls with varying thicknesses, axial load ratios, and boundary conditions then established the basis for bending and shear failure assessment (Godio et al., 2023b). Two subsequent pendulum impact campaigns examined low-velocity repeated point-load impacts (3–4.5 m/s) (Godio et al., 2023a) and moderate-velocity point-load and line-load impacts (7–8.5 m/s) (Godio & Flansbjer, 2025b), providing key insights into arching action, punching mechanisms, and combined failure under varying boundary and axial load conditions. A third pendulum campaign, conducted at RISE in autumn 2025 and attended by the authors, subsequently extended the programme to high-velocity dual simultaneous line-load impacts, introducing a redesigned impactor with variable mass configurations and eccentric boundary conditions more representative of real floor-wall connections. Together, these campaigns provide a comprehensive experimental dataset for the development and assessment of numerical models.

The numerical modelling of URM walls under impulsive loading is particularly challenging because their response is governed by interface-dominated mechanisms such as joint opening, sliding, separation, and re-contact, which evolve rapidly under dynamic loading and interact with geometric and material nonlinearities. Modelling strategies range from continuum to discrete approaches (Ferrante et al., 2021; Malomo & Pulatsu, 2024), and hybrid formulations have been developed to combine the advantages of both. Building on the experimental foundation described above, the present thesis contributes to the ongoing

RISE–FORTV programme, which aims to develop robust numerical frameworks for blast- and impact-loaded masonry structures. Two independent numerical frameworks are developed and assessed for the dynamic response of double-wythe URM walls under out-of-plane impact loading. The first is a discrete rigid-block model based on HybriDFEM (Bouckaert, 2025), applied here to impact-loaded masonry for the first time. To support this application, a contact deletion and recovery extension of the original elasto-plastic interface law, `ElastoPlastic CD`, is developed and implemented within the framework. The second is a meso-scale three-dimensional continuum model implemented in Abaqus/Explicit, in which brick units are governed by the Concrete Damaged Plasticity model and mortar joints by surface-based cohesive frictional contact formulations. The two frameworks approach the same physical problem from complementary perspectives, providing both a computationally efficient discrete tool and a detailed continuum reference for the assessment of URM walls under impulsive loading.

Funding and project context. This thesis was conducted within the RISE research project “Blast Capacity of Masonry Walls”, part of the research area “CFORT D1: Explosion and Weaponry Effects” coordinated by the Centrum för Fortifikatorisk Kompetens (CFORT). The project is funded by CFORT and encompasses the experimental campaigns at RISE, the academic supervision, and the financial support that made this work possible.

1.1.1 Research Novelty

The main contributions of this work are the following. For the HybriDFEM framework, this is the first study to develop and validate a HybriDFEM model for the out-of-plane impact response of URM walls. The `ElastoPlastic CD` contact law extension, adding contact deletion and recovery for opened joints, is a new development within the framework implemented and assessed here for the first time. For both frameworks, a multi-campaign validation strategy in which parameter set calibrated from Campaigns 1 and 2 is applied without modification to Campaign 3 provides a transparent assessment of predictive capability across a wide range of impact energies, loading configurations, and boundary conditions.

For the Abaqus framework, a three-dimensional continuum finite element model using the Concrete Damaged Plasticity formulation is calibrated from quasi-static four-point bending tests and validated against three independent impact campaigns, spanning a broad range of impact energies, loading configurations, and boundary conditions. Through this validation, the importance of the Dynamic Increase Factor as a key ingredient for reproducing the wall response at higher impact energies is identified and discussed.

For both frameworks, a multi-campaign validation strategy in which the parameter set calibrated from Campaigns 1 and 2 is applied without modification to Campaign 3 provides a transparent assessment of predictive capability across a wide range of impact conditions.

1.2 Aim and Objectives

The overall aim of this thesis is to develop, calibrate, and assess two independent numerical frameworks for predicting the dynamic response of double-wythe unreinforced masonry walls under out-of-plane pendulum impact loading, and to identify the conditions under which each framework is best suited for engineering assessment.

The specific objectives of this project are to:

- Analyse and interpret the experimental data from the four-point bending tests and three pendulum impact campaigns conducted at RISE, with emphasis on the global dynamic response, arching behaviour, and dominant failure mechanisms.
- Develop two numerical modelling frameworks representative of the experimental wall geometry, boundary conditions, and loading configurations: a HybriDFEM model based on rigid block interaction through nonlinear interfaces, including the `ElastoPlastic CD` extension for contact deletion and recovery in opened joints; and a three-dimensional meso-scale Abaqus/Explicit model using Concrete Damaged Plasticity for the brick units and surface-based cohesive-frictional damage contact for the mortar joints.
- Calibrate the material parameters of both frameworks from quasi-static four-point bending tests, cross-validated against experimental modal identification, and calibrate the impact-related parameters against the first two pendulum impact campaigns, covering low-velocity point-load impacts and moderate-velocity point-load and line-load impacts under varying impactor mass, prestress, and boundary conditions.
- Apply the calibrated parameter set of each framework, to the third experimental campaign, comprising high-velocity dual simultaneous line-load impacts, to assess the predictive capability of each framework beyond its calibration range.
- Compare the two frameworks against each other and against the experimental data in terms of out-of-plane displacement, impactor displacement, velocity and force histories, arching behaviour, and local failure mechanisms, identifying the conditions under which each framework is best suited.

1.3 Limitations

This study focuses on double-wythe unreinforced clay brick masonry walls under out-of-plane impact loading; repeated impacts on damaged walls, and other types of impulsive loading are not addressed. The models are calibrated against a finite set of RISE experimental configurations and are primarily valid within the geometric, material, and loading ranges those tests represent. The HybriDFEM framework is two-dimensional and uses a linear elastic impactor contact law without cohesion softening or strain-rate effects; the Abaqus model adopts a meso-scale representation without explicit mortar layers and assumes half-model symmetry. The `ElastoPlastic CD` extension is a proposed contribution, not the primary calibrated model. A detailed discussion of all assumptions and their implications is provided in Sections 3.3.8 and 3.4.8.

1.4 Societal, Ethical, and Ecological Aspects

From a societal perspective, unreinforced masonry buildings represent a significant part of Sweden's built environment, including structures of cultural and historical value. Many of these buildings were not designed to withstand impact or blast-type loads, posing severe safety risks and social consequences. By improving numerical assessment methods for masonry under impulsive loading, the project contributes to safer evaluation, protection, and preservation of existing structures, supporting societal resilience and public safety.

Ethical considerations in the project are limited, as no human participants or personal data

are involved. The work relies on experimental data obtained through established research collaborations and is conducted within institutional research frameworks. Ethical responsibility is addressed primarily through transparent documentation of modelling assumptions, calibration procedures, and limitations, ensuring that results are not misinterpreted or applied beyond their validated scope.

From an ecological standpoint, the project supports sustainable engineering practice by reducing the need for extensive destructive testing using validated numerical simulations. Digital assessment methods help minimise material waste and energy consumption associated with experimental campaigns. In addition, improving assessment techniques for existing masonry structures contributes to extended service life and reduced need for demolition, aligning with broader sustainability and resource-efficiency goals.

1.5 Thesis Outline

The structure of the thesis and the connections between chapters, sections, and appendices are summarised in Figure 1.1, where each label links directly to the corresponding part of the document. In both frameworks, the study follows the same sequential strategy: material and contact parameters are first calibrated from quasi-static four-point bending tests; impact-related parameters are then calibrated against Campaigns 1 and 2 in sequence; and the resulting parameter set is applied without modification to Campaign 3, assessing the predictive capability of each framework beyond its calibration range.

The appendices support this process at each stage: Appendix A derives the interface stiffness parameters and proves stiffness invariance across discretisation levels; Appendix B documents the verification studies; Appendices C, D, and E present the calibration derivations and parameter sweeps for the four-point bending tests and the first two campaigns; and Appendix F presents the implementation and assessment of `ElastoPlastic CD`. The Abaqus appendices document the full modelling procedure, verification studies, calibration results, and campaign assessments. Appendices G–J cover the step-by-step modelling procedure, verification, four-point bending calibration, and the Campaign 1 and 2 impact calibration respectively. Appendix L presents the preliminary assessment of the Dynamic Increase Factor. Appendix K presents the Campaign 3 results for both `HybriDFEM` and Abaqus together.

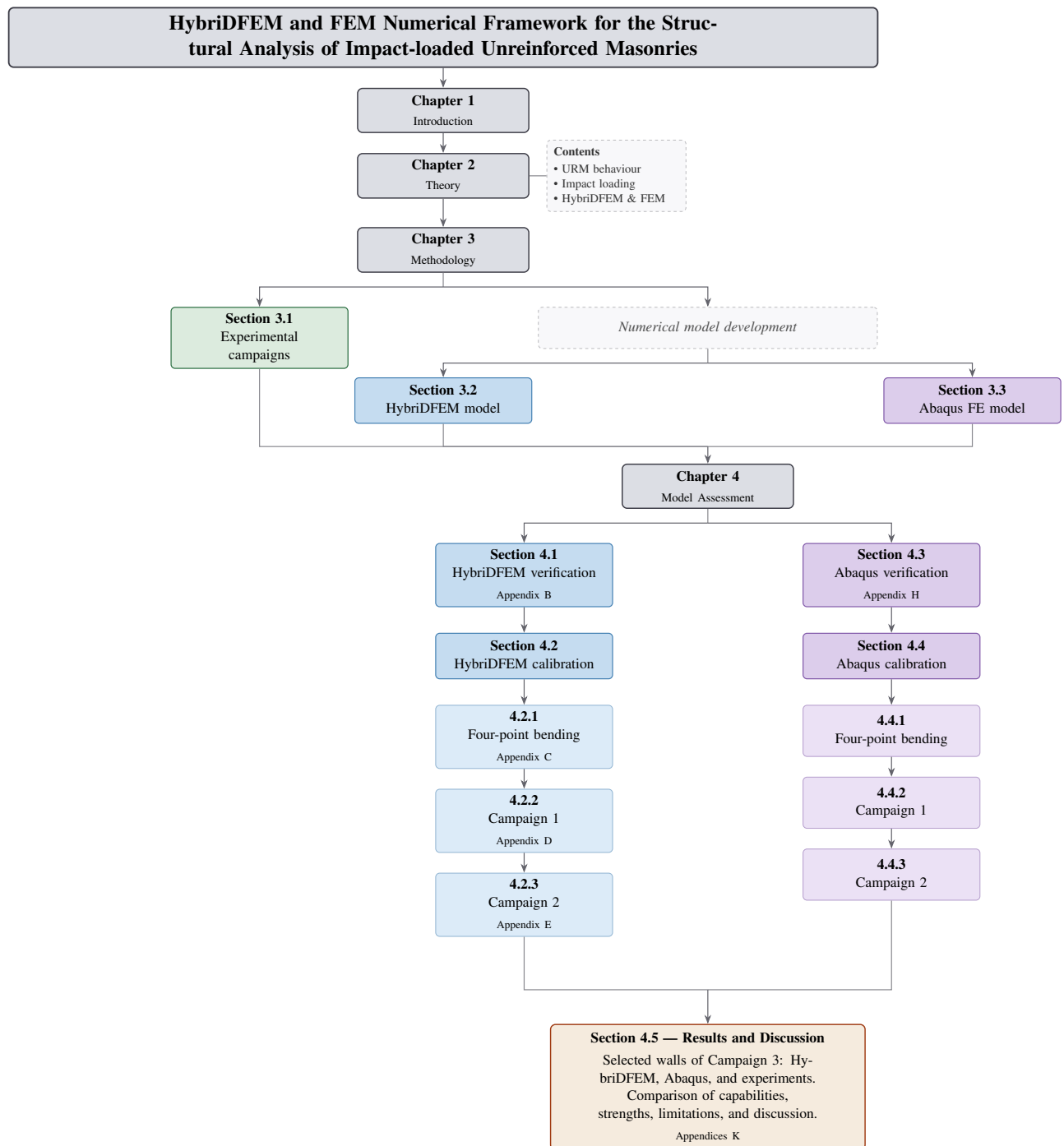


Figure 1.1: Conceptual roadmap of the thesis. Blue boxes denote the HybridFEM branch; purple boxes denote the Abaqus branch. All chapter and section labels are clickable hyper-links.

2 Theory

The response of unreinforced masonry walls under impact loading is governed by the interaction between material behaviour, structural mechanics, and dynamic effects. This study focuses on axially loaded, vertically spanning walls subjected to out-of-plane impulsive loading. Reproducing this response numerically requires a solid understanding of masonry mechanics, the nature of impact loads, and the theoretical foundations of the modelling approaches employed. This chapter establishes that foundation, beginning with the mechanical behaviour of unreinforced masonry and proceeding through impact loading and the available numerical strategies. The theoretical basis of the two frameworks adopted in this work, HybriDFEM and the Finite Element Model (Abaqus), is then presented in turn.

2.1 Unreinforced Masonry Walls

2.1.1 Composition and mechanical behaviour

Unreinforced masonry (URM) is a composite construction material consisting of discrete masonry units, such as clay bricks, concrete blocks, or stone, bonded by mortar joints. The units carry the primary compressive loads while the joints transmit forces between them and accommodate minor differential movements (Lourenço, 1996; Page, 1996). In the absence of steel reinforcement, the structural behaviour is governed entirely by the mechanical interaction between these two phases.

As a consequence of its composition, masonry is both heterogeneous and anisotropic. The bed joints and head joints introduce two planes of weakness with distinct mechanical properties, producing a directional dependence of strength and stiffness (Ahmed & Tanoli, 2024). The mechanical response of the composite depends not only on the properties of the individual constituents but also on the bond pattern, the joint thickness and orientation, and the geometry of the masonry units (Lourenço et al., 1998).

The compressive strength of the assemblage is typically well below that of the individual brick units. The mortar, which is weaker than the bricks, reduces the local strength and induces a bi-axial tensile stress state in the bricks due to the lateral confinement mismatch between the two materials (Lourenço, 1996). Stiffness is characterised by the elastic modulus, conventionally defined as the secant modulus at one-third of the peak compressive stress following EN 1052-1 (EN 1052-1, 1992).

2.1.2 Out-of-plane response and failure mechanisms

Out-of-plane loading acts perpendicular to the wall mid-plane. For vertically spanning walls, it induces bending about horizontal axes. Masonry has limited resistance to such loading due to the low tensile strength of the mortar and brick–mortar interface (Godio et al., 2021, 2023a). Once cracking initiates, the response becomes strongly nonlinear, dominated by joint opening, rocking of cracked segments, and large rigid-body displacements, rather than by elastic material deformation (Lourenço, 1996; Vaculik & Griffith, 2018).

The failure mechanisms that develop in vertically spanning URM walls subjected to quasi-static out-of-plane loading depend on the boundary conditions and the level of axial load, and are discussed in detail in Section 2.1.3. Under quasi-static out-of-plane loading, three failure mechanisms have been identified (Godio et al., 2023b). A *bending failure* forms

through progressive horizontal cracking in the bed joints, which localises into a dominant crack near mid-height and produces a three-hinge rocking mechanism. A *shear failure* develops through diagonal cracking initiating at the head joints and propagating through the bricks. A *combined bending–shear failure* exhibits features of both. For four-sided walls, a two-way bending mechanism consistent with yield-line theory may develop instead (Vaculik & Griffith, 2018).

Under impact loading, two mechanisms develop concurrently in axially loaded URM walls (Godio & Flansbjerg, 2025b; Godio et al., 2023a). A global *bending mechanism* produces a dominant horizontal crack near mid-height, dividing the wall into two regions that undergo rocking. A localised *punching mechanism* develops around the impact point, generating stepped diagonal cracks, out-of-plane brick dislocations, and localised crushing; in severe impacts, bricks have been observed to split and be ejected from the wall (Godio et al., 2023a). Under point-load impacts the punching mechanism tends to dominate, while under line-load impacts the bending mechanism is more pronounced. In both cases, tensile cracking initiates on the rear face directly behind the impact point, while compressive crushing concentrates on the front face in the impact zone (Godio & Flansbjerg, 2025b; Godio et al., 2023a).

2.1.3 Influence of axial load and boundary conditions

The out-of-plane resistance, stiffness, and deformation capacity of vertically spanning URM walls depend strongly on the axial compressive load and on the boundary conditions. These two factors are coupled, the axial load provides the compressive thrust required for arching, and the boundary conditions determine whether and to what degree that thrust can develop.

Role of axial load

Axial compressive load affects the out-of-plane response through three main mechanisms. First, it influences the initial lateral stiffness. The elastic modulus of masonry increases approximately linearly with the axial load ratio σ_0/f_u , following $E \propto f_u(1 + \beta \sigma_0/f_u)$ (Wilding & Beyer, 2018). However, at low axial load ratios this effect is limited. Over the range $\sigma_{ax} = 0.05\text{--}0.20$ MPa, the initial out-of-plane stiffness of the wall specimens considered in this work appears to remain approximately constant, as observed in (Godio et al., 2024).

Second, axial load increases the force capacity through *arching action*. As the wall deflects laterally, a compressive arch develops between the supports. The resulting thrust opposes further out-of-plane displacement and increases the apparent lateral resistance of the wall (Godio et al., 2023a, 2023b).

Third, excessive axial compression can reduce the deformation capacity. At high prestress levels, toe crushing may occur earlier, leading to a more brittle post-peak response. Moderate prestress can therefore increase resistance and energy dissipation, whereas excessive prestress may limit the maximum attainable deflection.

This influence is directly observed in the experimental campaigns of this work, where walls tested at lower axial stress exhibited greater deformability and a larger dynamic amplification of the arch force relative to the initial prestress, confirming that arching becomes a dominant source of resistance in lightly loaded walls (Godio & Flansbjerg, 2025b; Godio et al., 2023a).

Role of boundary conditions

Boundary conditions control whether the axial load can be mobilised into arching action. In a vertically spanning wall, lateral deflection tends to increase the distance between the top and bottom supports. If this vertical movement is restrained, a compressive reaction develops between the wall and the supports. This reaction is the origin of arching. Therefore, the same axial load can lead to very different out-of-plane responses depending on the stiffness of the support system.

Three idealised support conditions are commonly distinguished: *simply supported*, *rigid*, and *non-rigid* (U.S. Department of Defense, 2008). Under *simply supported* conditions, the top support does not restrain the vertical movement associated with wall uplift and elongation. As a result, no significant compressive arch can develop.

Under *rigid* support conditions, the vertical displacement of the top support is prevented. This generates a large compressive arching force, which increases the lateral resistance of the wall. In this case, arching is associated with the restrained elongation of the wall and may continue to increase even after the peak lateral force has been reached.

Under *non-rigid* support conditions, the top restraint has a finite vertical stiffness. The top support is allowed to move, but this movement produces a reaction force proportional to the support displacement. The arching force therefore depends on the stiffness of the support system: a very soft support approaches the simply supported case, whereas a very stiff support approaches the rigid case. This condition is thought to be the most representative of real floor–wall connections, where the surrounding structure provides restraint but is not perfectly rigid (Godio & Flansbjerg, 2025b; Godio et al., 2023b).

2.2 Impact Loading on Masonry Structures

Reliable numerical tools for impact loading on masonry are important for two reasons. First, experimental impact campaigns are destructive and expensive, each test consumes a wall specimen and requires significant laboratory infrastructure. Validated numerical models can reduce the number of physical tests required by allowing boundary conditions, load configurations, and structural parameters to be varied computationally. Beyond wall-level analysis, a validated framework also enables the assessment of adjacent structural elements and their interaction with the impacted wall. Second, a framework calibrated for impact loading can be extended to blast loading, since far-field blast acts as a distributed impulsive pressure over the wall surface (Godio et al., 2021). Progress on impact modelling therefore advances the broader problem of masonry under impulsive loads.

2.2.1 Nature and structural response of impact loads

Impact loading is characterised by a force of high magnitude and short duration applied over a localised area as a result of a collision. Unlike quasi-static loads, impact forces are highly transient and generate significant inertial effects. Because the load duration t_d is typically much shorter than the fundamental natural period T_n of the structure, the response cannot be described by static equilibrium and must be treated as a dynamic problem governed by the equation of motion (Chopra, 2007):

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F(t) \quad (2.1)$$

where m , c , and k are the mass, damping, and stiffness of the system, $u(t)$ is the displacement, and $F(t)$ is the time-varying applied force. In the regime $t_d \ll T_n$, the peak displacement occurs during the subsequent free-vibration phase and the response is governed primarily by the impulse $I = \int F(t) dt$ rather than by the peak force alone (Chopra, 2007). Thus, two impacts with the same peak force can produce different responses if their durations, and therefore their impulses, are different.

Impact and blast loads share this impulsive nature, but differ in their spatial distribution. Blast loading acts approximately as a distributed pressure over the wall surface, whereas impact loading is applied more locally, either at a point or along a line (Godio et al., 2021, 2023a). In laboratory settings, pendulum tests provide a controlled method for generating such loads, where a drop weight released from a prescribed height impacts the wall at a known velocity, and the force history, duration Δt_{imp} , and rebound velocity are the primary observables (Godio & Flansbjerg, 2025b; Godio et al., 2023a).

Under repeated impacts, damage accumulates progressively. Mechanisms initiated during the first impact are amplified by next events. Rocking amplitudes increase, the zone of brick dislocation around the impact point expands, and continued loading may lead to brick ejection and breach formation (Godio et al., 2023a). This progressive damage process is important when assessing residual wall capacity and the ability of masonry walls to resist multiple impact events.

An important aspect of impulsive loading is the strain-rate dependence of material properties. Under impact and blast loading, strain rates in masonry may reach 10^2 s^{-1} , several orders of magnitude above quasi-static rates (10^{-6} – 10^{-4} s^{-1}) (Hao & Tarasov, 2008). At such rates, both stiffness and strength of masonry constituents increase measurably (Godio et al., 2021), with the effect more pronounced in tension than in compression (Hao & Tarasov, 2008; Larcher et al., 2013). This is commonly represented through a Dynamic Increase Factor (DIF), defined as the ratio of dynamic to quasi-static strength.

The DIF has been applied at the macro-scale through continuum damage models with strain-rate effects derived by numerical homogenisation (Wei & Hao, 2009), and at the meso-scale, where the compressive and tensile strengths of individual brick and mortar constituents are scaled by a DIF calibrated from dynamic material tests (Li et al., 2017). Available studies indicate that masonry strength can be amplified by up to six times in compression and ten times in tension under very high strain rates (Hao & Tarasov, 2008; Sielicki & Łodygowski, 2019). The choice of constitutive framework and the method of incorporating rate dependence therefore remain important modelling decisions when simulating masonry structures subjected to impulsive loads.

2.3 Numerical Modelling of Masonry Structures

2.3.1 Modelling scales: macro-, meso- and micro-scale approaches

The response of masonry structures is strongly governed by the interaction between individual units and joints. Mechanisms such as crack opening, sliding, separation, and re-contact control the global behaviour, especially under nonlinear lateral, impact, and blast loading. The modelling strategy must therefore balance physical accuracy, computational cost, and the ability to represent interface-dominated failure mechanisms.

Modelling strategies are broadly classified into continuous and discrete approaches (Ferrante

et al., 2021; Malomo & Pulatsu, 2024). Within continuous approaches, material heterogeneity is represented through homogenised constitutive laws, which is computationally efficient but limits the ability to capture localised phenomena such as crack opening, joint sliding, and block separation (Chen et al., 2023; Fattahi et al., 2024). Discrete approaches represent the structure as an assembly of distinct units interacting through deformable interfaces, naturally capturing joint opening, sliding, and re-contact. Within both classes, three levels of idealisation are distinguished, classically referred to as macro-scale, meso-scale, and detailed micro-scale models (D’Altri et al., 2019; Lourenço et al., 1995).

Both frameworks developed in this thesis operate at the meso-scale, and the remainder of this chapter focuses on the theoretical foundations relevant to this level of idealisation.

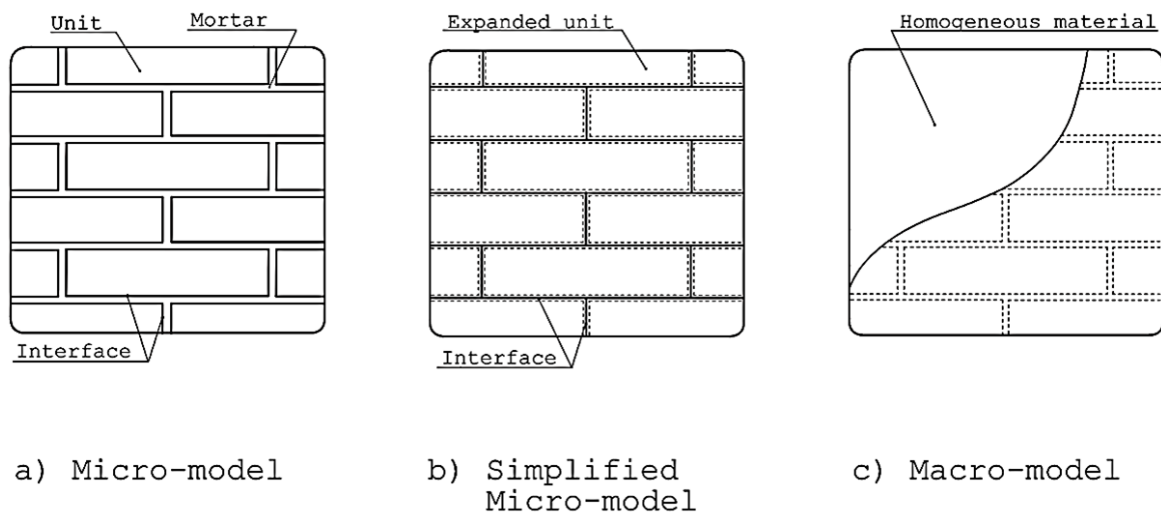


Figure 2.1: Numerical modelling approaches of masonry structures.

2.3.1.1 Macro-scale models

In macro-scale models, bricks and mortar are smeared into an equivalent homogeneous continuum described by a homogenised constitutive law (Cecchi & Tralli, 2012; Roca et al., 2010). The equivalent material properties may be obtained from analytical or numerical homogenisation, or calibrated directly against experimental data (Lourenço, 1996; Lourenço et al., 1998). These models are efficient and suitable for large structures or parametric studies. However, because the heterogeneous microstructure of masonry is averaged out, they cannot explicitly capture local cracking patterns, joint sliding, block separation, or the influence of the bond pattern (D’Altri et al., 2019).

2.3.1.2 Detailed micro-scale models

At the opposite extreme, detailed micro-scale models represent bricks and mortar joints as separate continua with real dimensions and thicknesses. Interface elements may also be introduced to describe debonding at the brick–mortar interface and cracking within the mortar (Lourenço et al., 1995). This provides the most detailed representation of masonry behaviour, including damage evolution in both constituents. However, the large number of elements and the need to resolve contact nonlinearity at every interface make these models computationally very expensive, limiting their applicability to small assemblies (D’Altri et al., 2019; Lourenço et al., 1998).

2.3.1.3 Simplified micro-scale models (Meso-scale models)

Between these two extremes, the meso-scale approach provides a balanced modelling strategy. Masonry units are modelled as deformable continua with expanded dimensions to account for the mortar joint thickness, while the joints are represented through interface formulations or contact laws governing normal compression and tangential frictional sliding between adjacent units (Abdulla et al., 2017; Lourenço et al., 1995). This enables the explicit representation of separation, sliding, and re-contact without the cost of discretising thin mortar layers, and joint opening and sliding emerge naturally from the interface conditions rather than being prescribed through predefined crack paths. The strategy has been widely adopted for masonry walls under lateral, seismic, and dynamic loading, capturing the essential mechanisms of joint opening, frictional resistance, and compressive stress transfer while retaining the computational efficiency required for full-wall dynamic simulations (Abdulla et al., 2017; Lourenço, 1996).

2.4 Hybrid Discrete–Finite Element Method (HybriDFEM)

As discussed in Section 2.3, continuous approaches such as the Finite Element Method represent discontinuities through homogenised constitutive laws (Rust, 2015), which limits their ability to capture localised interface-dominated mechanisms such as crack opening, sliding, and block separation (Chen et al., 2023). Within FEM, micro-modelling approaches explicitly represent discontinuities through refined meshes, providing high accuracy at significant computational cost (Lourenço et al., 1998; Munjiza & Latham, 2002), while macro-modelling averages the microstructural response into an equivalent continuum, improving efficiency at the expense of local accuracy (Cecchi & Tralli, 2012; Roca et al., 2010). To overcome these limitations, discrete approaches such as the Discrete Element Method (DEM) represent the structure as an assembly of rigid blocks interacting through deformable contact interfaces, naturally capturing separation, sliding, and large relative displacements (Lemos, 2017; Pulatsu et al., 2020). However, DEM formulations typically rely on explicit time integration with very small time steps, resulting in high computational costs even for quasi-static problems.

A related class of methods represents the structure as an assembly of rigid units interconnected by deformable interfaces modelled as distributed nonlinear multidirectional springs (Malomo et al., 2018), capable of capturing material behaviour, physical discontinuities, and potential cracking paths (Malomo & Pulatsu, 2024; Meguro & Tagel-Din, 2000; Scattarreggia et al., 2022). Methods in this class include the Body Spring Model (Bertolesi et al., 2017), Rigid Block Models (Casolo, 2009), and the Applied Element Method (Meguro & Tagel-Din, 2000, 2002), all of which enable the simulation of progressive damage and structural collapse with a limited number of degrees of freedom (Malomo et al., 2020). Within the same framework, the Fibre Contact Element Method models the contact through multiple nonlinear springs in series, explicitly accounting for the relative deformability of adjacent blocks of different materials (Estêvão & Carreira, 2015; Estêvão & Oliveira, 2015).

Within this context, HybriDFEM is adopted in the present work as the numerical framework for modelling masonry structures subjected to impulsive loading. The method combines the computational efficiency of finite element formulations with the ability of discrete approaches to explicitly capture interface interactions and discontinuities. By representing the structure as an assembly of rigid blocks connected through deformable interfaces, it enables

the simulation of mechanisms such as joint opening, sliding, and re-contact while retaining a finite element equilibrium and solution framework suitable for large-scale structural analyses. However, for the method to become a reliable engineering tool for the analysis of real structural systems, its predictive capability must be assessed against experimental observations under realistic loading and boundary conditions, extending its application beyond the analytical or idealised examples considered in earlier developments. The present work contributes to this objective by developing and validating a HybriDFEM modelling framework for unreinforced masonry walls subjected to out-of-plane impact loading.

2.4.1 Conceptual basis

In HybriDFEM, the structure is represented as an assembly of rigid blocks interacting through deformable interfaces. Each block behaves as a rigid body characterised by a limited number of degrees of freedom associated with rigid-body motions, while all deformability is concentrated at the *contact faces* (CFs) connecting neighbouring blocks. The CFs are further discretised into *contact pairs* (CPs), where the interaction between blocks is described through nonlinear springs in the normal and tangential directions. The springs are arranged in series, allowing the deformability of each block side to be considered independently and enabling the representation of interfaces between materials with different mechanical properties. The mechanical properties assigned to the interfaces provide an equivalent representation of the combined deformability of mortar joints and masonry units, capturing the dominant mechanisms of joint opening, sliding, and re-contact that govern masonry behaviour. In the present work, masonry walls are modelled using a two-dimensional discretisation, and non-rigid boundary conditions are represented through Timoshenko finite elements connected to the block assembly. A detailed description of the complete formulation is provided in (Bouckaert, 2025), and the following subsections summarise its main components as adopted in this work.

2.4.2 Block discretization

Within the HybriDFEM framework, each block is characterised by three degrees of freedom (DOFs): two in-plane translations and one rotation, defined with respect to a reference point located at the block centroid. Together with the geometric description, the model assigns a mass and rotational inertia to each block. Because blocks are assumed rigid, the global deformability of the structure arises entirely from the relative motion between adjacent blocks, enabling the explicit representation of masonry stereotomy with a relatively small number of degrees of freedom. Figure 2.2 illustrates the discretisation of a typical masonry wall into rigid blocks and the corresponding DOFs.

In dynamic analysis, the mass of each block is represented by a diagonal mass matrix $\mathbf{m}_{block} \in \mathbb{R}^{3 \times 3}$, assembled from the translational mass and rotational inertia of the block:

$$\mathbf{m}_{block} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_\theta \end{bmatrix}, \quad (2.2)$$

where m is the block mass and I_θ its rotational inertia at the reference origin (Bouckaert et al., 2026). The global mass matrix $\mathbf{M}$ is assembled from the block contributions at the corresponding DOFs. Because blocks are rigid, no off-diagonal mass coupling arises between

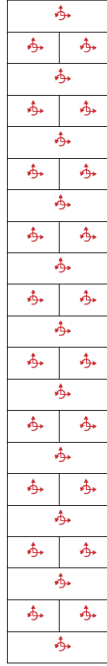


Figure 2.2: Discretization of a typical masonry wall into rigid blocks and corresponding degrees of freedom per block.

blocks and the global mass matrix retains a diagonal structure, which allows the inertial term to be inverted directly at each time step without solving a linear system, a key advantage for explicit time integration. Further details on block kinematics and equilibrium equations can be found in (Bouckaert, 2025; Bouckaert, Godio, & Pacheco de Almeida, 2025).

2.4.3 Contact faces and contact pairs

The mechanical interaction between rigid blocks is represented through contact faces (CFs), along which all relative deformation between blocks is concentrated. Each CF is further discretised into a set of contact pairs (CPs), which are the elementary interaction points where normal and tangential forces are transmitted between blocks. Figure 2.3 illustrates the hierarchical organisation of a HybriDFEM model from the global structure down to blocks, CFs, and CPs.

The number and distribution of CPs along a CF are modelling parameters that control the spatial resolution of the interface response. A refined discretisation allows the interface to approximate continuum-like behaviour, whereas a reduced number of CPs can efficiently capture dominant mechanisms such as rocking or sliding at lower computational cost. This hierarchical organisation mirrors the assembly structure of classical FEM and is central to the computation of the nonlinear resisting forces $\mathbf{P}_r(\mathbf{U})$ that enter the global equation of motion at each time step of the dynamic analysis.

2.4.3.1 Contact pair formulation

Each CP represents a localised interaction between two blocks and constitutes the fundamental deformable entity of the HybriDFEM formulation. A contact pair is composed of two external points attached to the neighbouring blocks and an internal point connecting

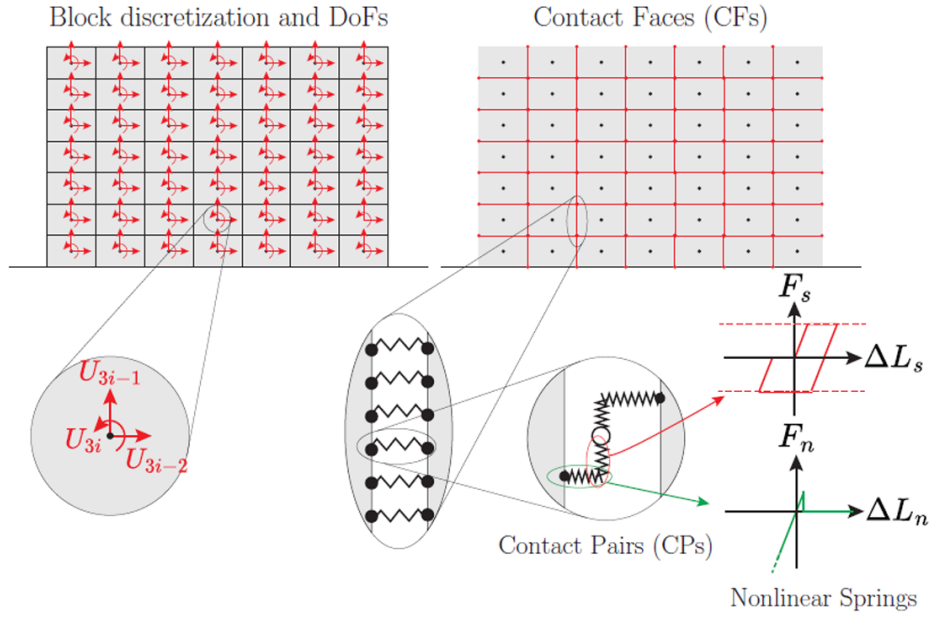


Figure 2.3: Hierarchical organization of a HybriDFEM model from global structure to blocks, contact faces (CFs), and contact pairs (CPs). From Bouckaert, 2025.

the two sides of the interface. The interaction is represented through two nonlinear springs arranged in series, one associated with each interacting block.

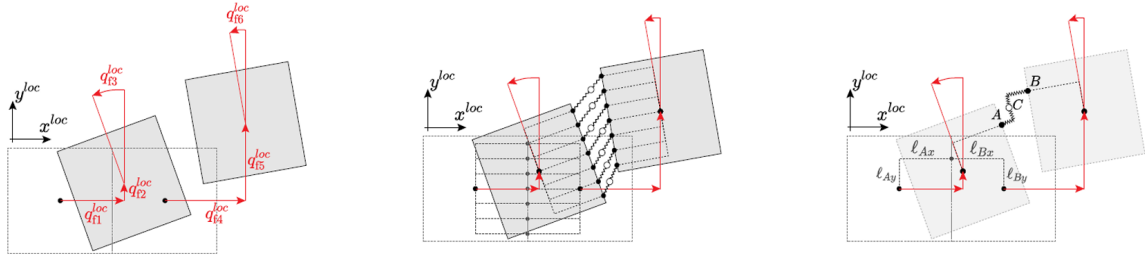


Figure 2.4: Kinematics of a contact pair and two-spring representation used in HybriDFEM. From Bouckaert, 2025.

Compatibility and equilibrium between the two springs require that the relative displacement across the interface equals the sum of the deformations of the two springs, while the transmitted force remains the same in both components:

$$\Delta u = \Delta u^{(A)} + \Delta u^{(B)}, \quad F^{(A)} = F^{(B)} = F_{cp}. \quad (2.3)$$

When the constitutive law is nonlinear, the position of the internal point cannot be determined directly from the global displacements. Instead, an internal iterative solution procedure is required at the CP level to satisfy compatibility and equilibrium simultaneously (Bouckaert et al., 2026). This internal problem is solved locally, and the resulting forces and tangent stiffness are passed upward from spring level to CP, from CP to CF, and from CF to the global resisting force vector $\mathbf{P}_r(\mathbf{U})$.

For two springs in series, each characterised by the same stiffness k , the effective interface stiffness is $K = k/2$. Consequently, to reproduce a target interface stiffness K , each spring must be assigned $k_{nn} = 2K$. This relation is used directly in the interface parameter assignments of the numerical model, as described in Section 3.3.2.4.

2.4.3.2 Contact pair kinematics and equilibrium

The CP kinematics are formulated to handle both geometric and material nonlinearities consistently. Geometric nonlinearities, arising from large displacements and rotations of the blocks, are incorporated through a co-rotational description in which the local normal and tangential directions are updated at each iteration to follow the current deformed configuration. Material nonlinearities are introduced through the nonlinear constitutive laws governing the spring behaviour at each CP (Bouckaert et al., 2026). This ensures that the CP formulation remains physically consistent throughout the response, including during large out-of-plane displacements and joint opening characteristic of impact loading.

By eliminating rigid-body motions, the CP kinematics are expressed through four basic coordinates $\mathbf{q}_{bsc} \in \mathbb{R}^4$, defining the relative displacement and orientation of the interface in the current configuration. The compatibility condition enforces that the total relative displacement between the two blocks equals the sum of the elongations of the two springs:

$$\Delta \mathbf{u} = \mathbf{T}_2(\alpha_A + q_3^{bsc}) \begin{bmatrix} \Delta L_{An} \\ \Delta L_{As} \end{bmatrix} + \mathbf{T}_2(\alpha_B + q_4^{bsc}) \begin{bmatrix} \Delta L_{Bn} \\ \Delta L_{Bs} \end{bmatrix} \quad (2.4)$$

Equilibrium requires that the forces transmitted by the two springs are equal when expressed in a common reference frame:

$$\mathbf{p} = \mathbf{T}_2(\alpha_A + q_3^{bsc}) \begin{bmatrix} F_{An} \\ F_{As} \end{bmatrix} = \mathbf{T}_2(\alpha_B + q_4^{bsc}) \begin{bmatrix} F_{Bn} \\ F_{Bs} \end{bmatrix} \quad (2.5)$$

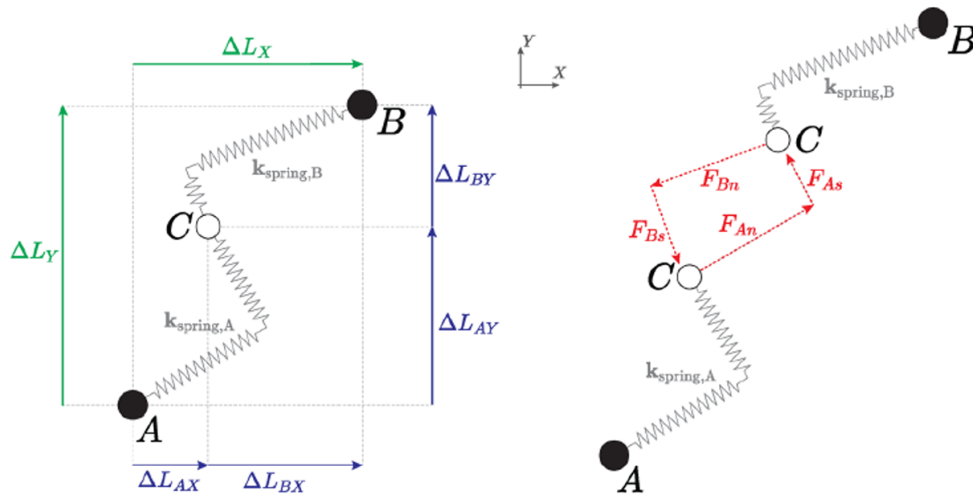


Figure 2.5: Internal contact pair problem: (left) compatibility of the two springs in series; (right) equilibrium in the co-rotational frame. From Bouckaert, 2025.

These relations define a nonlinear system at the CP level, solved iteratively to obtain the spring elongations and contact forces. The outcome is a condensed CP resisting force and a consistent tangent stiffness matrix, transferred to the CF level for assembly into $\mathbf{P}_r(\mathbf{U})$. The constitutive law governing the spring behaviour depends on the contact modelling approach adopted, as described in the following section.

2.4.3.3 Contact modelling

The interaction between adjacent blocks is described at the CP level through constitutive relations acting in the normal and tangential directions. HybriDFEM supports three contact modelling approaches, shown in Figure 2.6, differing in the level at which the constitutive relation is introduced. In the force-displacement approach, the law is defined directly at spring level in terms of forces and elongations, and is particularly suited to point contacts, unilateral interfaces, and rigid block interactions. In the stress-strain approach, the law is introduced at material level and linked to the springs through scaling relations that convert elongations into average strains and material stresses into equivalent spring forces, making it suitable when HybriDFEM is used to approximate continuous media. In the stress-displacement approach, the interface law relates tractions directly to displacement jumps across the contact surface, avoiding the introduction of an equivalent material thickness. In the present work, masonry joints are modelled using the stress-displacement approach, which is inherently suited to zero-thickness interfaces where the mechanical response is governed by traction-separation and frictional slip rather than bulk deformation.

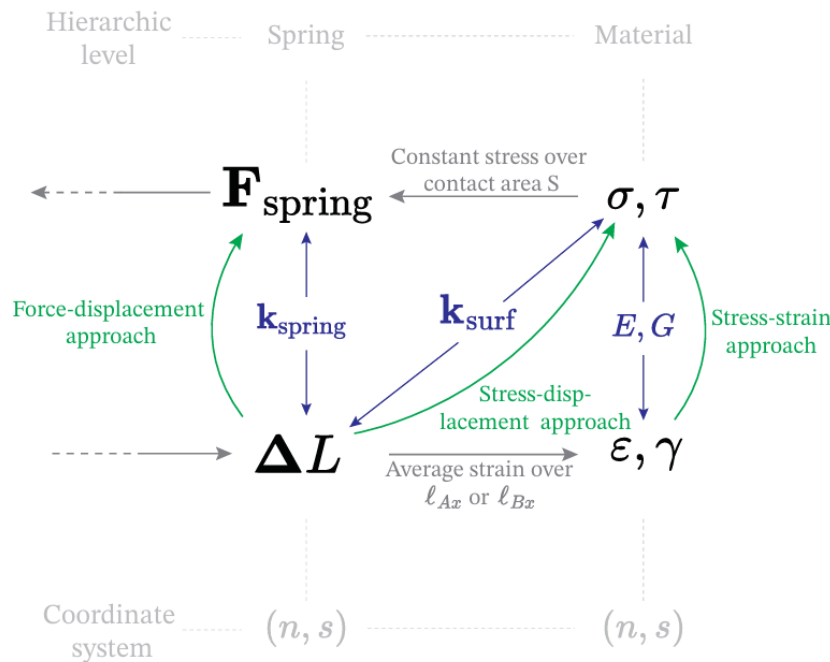


Figure 2.6: Conceptual representation of the different contact modelling approaches available in HybriDFEM. From Bouckaert, 2025.

Typical constitutive laws for masonry interfaces For masonry applications, the most relevant interface behaviours are unilateral normal response, frictional resistance in shear, and compressive inelasticity. These mechanisms are the primary sources of energy dissipation.

pation explicitly captured by $\mathbf{P}_r(\mathbf{U})$ at each time step, a distinction that is important when assigning additional viscous damping, as discussed in Section 2.4.5.4.

No-tension behaviour A no-tension law prevents the transmission of tensile normal stresses across the interface. The tensile strength f_t defines the traction at which the interface loses its tensile capacity, with corresponding opening displacement $\delta_{n,t} = f_t/k_n$. Once this threshold is exceeded, separation occurs and the contact pair is removed from the global assembly, then reactivated if contact is re-established. This allows explicit tracking of joint opening and closure throughout the dynamic response, and is consistent with the low tensile capacity of masonry joints.

Coulomb friction Tangential resistance is described through a Coulomb-type criterion in which the admissible shear stress depends on the compressive normal stress. Frictional dissipation is computed from the contact forces and relative tangential displacements and is included in $\mathbf{P}_r(\mathbf{U})$ rather than in the viscous damping term. The criterion is controlled by the cohesion c and friction coefficient μ , with a non-associated flow rule and dilatancy angle ψ . In the present work, $\psi = 0$ is adopted, implying purely tangential plastic flow with no normal plastic deformation during sliding.

Elasto-plastic law with compression cap The elasto-plastic law extends the Coulomb criterion by bounding the admissible stress domain with four surfaces, as shown in Figure 2.7. A tension cutoff at $\sigma_n = f_t$, two Coulomb friction branches, and an elliptical compression cap in the (σ_n, τ) plane. When the trial stress state lies within this domain the response is elastic. If one or more surfaces are violated, a return-mapping procedure projects the stress back onto the admissible domain. This law represents frictional sliding, compressive inelasticity, and tensile stress limitation within a single constitutive framework, with all inelastic dissipation accounted for in $\mathbf{P}_r(\mathbf{U})$.

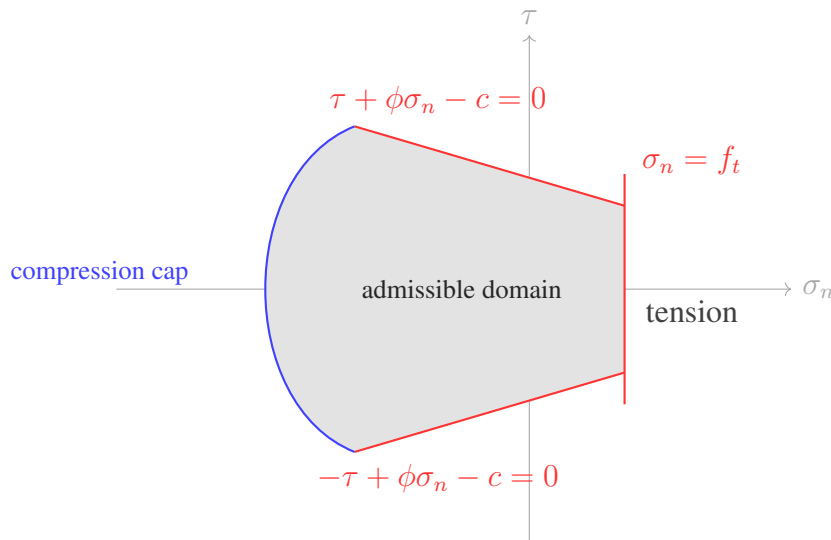


Figure 2.7: Admissible stress domain of the masonry interface law, bounded by a tension cutoff, two Coulomb friction branches, and an elliptical compression cap.

An important distinction in the original `ElastoPlastic` formulation is that tensile failure is treated at the constitutive level, but not as a contact-state change at the assembly level.

The tension cut-off limits the admissible normal stress, and the return mapping projects the trial stress back onto the admissible domain. However, the contact pair itself remains active in the global resisting force and stiffness assembly.

This distinction is relevant for rocking-dominated masonry response, where joint opening and later reclosure are part of the physical mechanism. A joint that has reached its tensile limit may therefore still contribute tangent stiffness in the original formulation, even though the corresponding physical interface would be open.

A contact deletion and recovery extension, `ElastoPlastic_CD`, was therefore developed in this thesis. The extension keeps the original `ElastoPlastic` return mapping unchanged for closed contacts, but omits the contact pair when the total normal contact strain exceeds the tensile threshold $\varepsilon_t = f_t/k_{nn}$. The implementation and verification of this extension are presented in Appendix F.

2.4.4 Global equilibrium equations

At the structural level, equilibrium is enforced by assembling the contributions of all contact faces (CFs) and, when present, classical finite elements. Following the hierarchical assembly procedure described in the previous sections, from spring elongations to CP forces, from CP forces to CF forces, and from CF forces to the global level, the global resisting force vector is obtained as the sum of the local CF contributions mapped to the global coordinate system:

$$\mathbf{P}_r(\mathbf{U}) = \sum_f \mathbf{A}_f^\top \mathbf{p}_f^{\text{glob}}, \quad (2.6)$$

where $\mathbf{A}_f$ denotes the localisation operator of each CF and $\mathbf{p}_f^{\text{glob}}$ the corresponding internal forces in global coordinates. The global equilibrium equation reads:

$$\mathbf{R}(\mathbf{U}) = \mathbf{F}_{\text{ext}} - \mathbf{P}_r(\mathbf{U}) = \mathbf{0}, \quad (2.7)$$

with $\mathbf{F}_{\text{ext}}$ the vector of externally applied loads. The global tangent stiffness matrix is assembled from the condensed CF tangent matrices:

$$\mathbf{K}^{\text{tan}} = \sum_f \mathbf{A}_f^\top \mathbf{k}_f^{\text{glob}} \mathbf{A}_f, \quad (2.8)$$

where $\mathbf{k}_f^{\text{glob}}$ is the tangent stiffness matrix of each CF, itself assembled from the condensed tangent matrices of its constituent CPs. Because the constitutive laws are nonlinear, $\mathbf{K}^{\text{tan}}$ evolves with the structural state and must be recomputed at each iteration or time step. This matrix plays a central role in the implicit solution procedure and in the definition of the stiffness-proportional damping term, as discussed in Section 2.4.5.4.

2.4.5 Solution procedures

Static analyses are performed using implicit iterative procedures, while the dynamic impact problem is solved explicitly in time, as described in the following subsections.

2.4.5.1 Force-controlled static analysis

In force-controlled analysis, the external load vector is applied incrementally and the non-linear equilibrium equation is solved at each load step using a Newton–Raphson iterative procedure:

$$\mathbf{K}^{\text{tan}} \Delta \mathbf{U} = \mathbf{R}(\mathbf{U}), \quad (2.9)$$

where $\mathbf{K}^{\text{tan}}$ is the current tangent stiffness matrix and $\mathbf{R}(\mathbf{U})$ the residual vector. The displacement vector is updated as $\mathbf{U} \leftarrow \mathbf{U} + \Delta \mathbf{U}$ until convergence is achieved. This approach is used in the present work for the application of self-weight and prestress prior to the dynamic impact analysis.

2.4.5.2 Displacement-controlled static analysis

In displacement-controlled analysis, selected degrees of freedom are prescribed and increased incrementally, while equilibrium is enforced for the remaining unknowns. This strategy provides improved robustness in the presence of softening or loss of stiffness, allowing the response to be traced beyond limit points where force-controlled procedures may fail to converge. It is therefore used for the four-point bending tests employed for material parameter identification in the present work (Godio et al., 2023a).

2.4.5.3 Explicit dynamic analysis with the Central Difference Method

The dynamic impact problem is solved using the Central Difference Method (CDM), an explicit time integration scheme in which the displacement at time step t_{i+1} is determined directly from the solutions at previous steps (Chopra, 2007). The governing equation of motion reads:

$$\mathbf{M} \ddot{\mathbf{U}}(t) + \mathbf{C} \dot{\mathbf{U}}(t) + \mathbf{P}_r(\mathbf{U}(t)) = \mathbf{F}_{\text{ext}}(t), \quad (2.10)$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices and $\mathbf{P}_r(\mathbf{U})$ is the nonlinear resisting force vector assembled from all contact faces. Because $\mathbf{M}$ is diagonal, the inertial term can be inverted directly at each time step without solving a linear system, avoiding global matrix factorisation and making the scheme particularly efficient for highly nonlinear problems involving contact and impact. Velocities and accelerations are recovered from the displacement history rather than computed as primary unknowns. However, the scheme is conditionally stable, requiring:

$$\Delta t \leq \frac{T_n}{\pi}, \quad (2.11)$$

where T_n is the smallest vibration period of the system, equivalently expressed as $\Delta t \leq 2/\omega_{\text{max}}$ (Chopra, 2007). This condition is governed by the highest natural frequency of the discretised model, estimated through a preliminary modal analysis. The time step is then selected to satisfy this condition with a sufficient stability margin.

2.4.5.4 Structural damping

Energy dissipation in masonry structures arises from frictional sliding, joint opening, and inelastic deformation at the interfaces, all of which are explicitly captured by $\mathbf{P}_r(\mathbf{U})$ through

the contact constitutive laws. Additional energy losses not represented by the contact model, such as micro-cracking within the units or interaction with non-structural elements, are accounted for through an equivalent viscous damping term at the structural level.

Structural damping is modelled using a Rayleigh formulation:

$$\mathbf{C} = \alpha_M \mathbf{M} + \alpha_K \mathbf{K}, \quad (2.12)$$

where α_M and α_K are the mass- and stiffness-proportional damping coefficients. The mass-proportional term acts even during rigid-body motion, damping global translations independently of deformation. The stiffness-proportional term produces damping forces only when deformation occurs; if the structure moves rigidly, $\mathbf{K}\mathbf{U} = \mathbf{0}$ and no stiffness-proportional forces are introduced. In the present work, only the stiffness-proportional term is retained ($\alpha_M = 0$), so that damping forces arise exclusively from interface deformation and vanish during rigid-body motion or kinematic mechanism phases. The coefficient α_K is determined by prescribing a damping ratio ξ at the dominant natural frequency ω (Chopra, 2007):

$$\alpha_K = \frac{2\xi}{\omega}. \quad (2.13)$$

Stiffness matrix used in the damping term For nonlinearly responding systems, the structural stiffness evolves throughout the response, requiring careful definition of the stiffness-proportional term. Three approaches are available in HybriDFEM. The first, initial stiffness-proportional damping (ISPD), uses the initial stiffness matrix $\mathbf{K}_0$, assembled at the start of the analysis and kept constant thereafter. While computationally inexpensive, this approach can lead to excessive damping forces as the structure softens, since $\mathbf{K}_0$ includes the stiffness of all joints, even those that open during the response, and therefore does not reflect the actual mechanism state (Charney, 2008; Zareian & Medina, 2010). Furthermore, part of the energy dissipation is already accounted for in $\mathbf{P}_r(\mathbf{U})$ when inelastic demands occur, making the use of $\mathbf{K}_0$ physically inconsistent (Bouckaert et al., 2026).

The second approach uses the tangent stiffness matrix $\mathbf{K}^{\text{tan}}$, updated at each time step. In problems involving large displacements, this decomposes as:

$$\mathbf{K}^{\text{tan}} = \mathbf{K}^{\text{mat}} + \mathbf{K}^{\text{geo}}, \quad (2.14)$$

where $\mathbf{K}^{\text{mat}}$ reflects the material and contact stiffness and $\mathbf{K}^{\text{geo}}$ arises from equilibrium under pre-existing axial forces. The geometric stiffness does not represent stored elastic energy and is configuration-dependent; when included in the damping term it may introduce artificial forces and non-physical energy effects (Baena-Urrea & Almeida, 2023).

The third approach, adopted in the present work, uses the linear-geometric tangent stiffness matrix $\mathbf{K}^{\text{tan},lg}$, which excludes the geometric contribution:

$$\mathbf{K}^{\text{tan},lg} \approx \mathbf{K}^{\text{mat}}. \quad (2.15)$$

This ensures that the stiffness-proportional damping reflects only the material and contact state, avoiding the non-physical effects of the geometric stiffness term (Baena-Urrea & Almeida, 2023; Bouckaert et al., 2026).

Behaviour during kinematic mechanisms When joints open and the structure forms a kinematic mechanism, as occurs during out-of-plane rocking, $\mathbf{K}^{\text{tan},lg}$ becomes singular with a zero-energy mode corresponding to the mechanism shape. The stiffness-proportional damping term therefore vanishes entirely during the mechanism phase:

$$\mathbf{K}^{\text{tan},lg} \approx \mathbf{0} \Rightarrow \mathbf{C} = \alpha_K \mathbf{K}^{\text{tan},lg} \approx \mathbf{0}. \quad (2.16)$$

This is physically consistent. During rocking, energy dissipation through friction and sliding is already accounted for in $\mathbf{P}_r(\mathbf{U})$, and additional viscous damping would represent an artificial energy sink. Damping forces are reintroduced only when the structural stiffness temporarily recovers during transitions between mechanisms (Bouckaert et al., 2026). A practical consequence is that the response obtained with $\mathbf{K}^{\text{tan},lg}$ is largely insensitive to the prescribed damping ratio ξ , since the structure spends most of the simulation in a mechanism state where $\mathbf{C} \approx \mathbf{0}$ (Bouckaert et al., 2026). This is consistent with the findings of the damping sensitivity study conducted in this work, discussed in Section 4.2.3.

2.4.5.5 Modal analysis

The natural frequencies and mode shapes are obtained by solving the generalised eigenvalue problem:

$$\mathbf{K} \phi = \omega^2 \mathbf{M} \phi, \quad (2.17)$$

where ω are the natural circular frequencies and ϕ the corresponding mode shapes (Bouckaert, Piedboeuf, et al., 2025). The eigenvalue problem is restricted to the free degrees of freedom and solved using iterative sparse eigensolvers to extract the lowest modes of interest.

The modal analysis serves three purposes in this work. First, the highest natural frequency ω_{max} determines the critical time step for the CDM. Second, the dominant natural frequency in the out-of-plane direction, identified from modal mass participation ratios, is used to calibrate α_K . Third, the computed frequencies provide an independent means of cross-validating the interface stiffness parameters by comparing them against those identified experimentally through dynamic testing of the wall specimens, independently of the quasi-static four-point bending response.

2.5 Applications and developments of HybriDFEM

Since its introduction, HybriDFEM has evolved through a series of methodological developments addressing increasingly complex structural problems. The initial formulation targeted one-dimensional beam-like members, incorporating material and geometric nonlinearities within a hybrid discrete-finite element framework (Bouckaert et al., 2024b). Benchmark studies validated the formulation and confirmed its ability to reproduce elastic behaviour, inelastic strain localisation, geometric stiffness effects, and unilateral contact mechanisms such as rocking and frictional sliding, while demonstrating the coupling between HybriDFEM blocks and conventional finite elements within a unified framework.

The method was subsequently extended to biaxially discretised two-dimensional structures, enabling the representation of frames and masonry assemblies (Bouckaert, Godio, & Pacheco

de Almeida, 2025). These developments introduced contact detection procedures and more versatile constitutive descriptions, allowing HybriDFEM to capture pre- and post-peak behaviour in discontinuous systems and reproduce collapse mechanisms without predefined assumptions on failure geometry. The predictive capability of the method for masonry was further assessed through benchmark studies on the in-plane capacity of unreinforced masonry walls (Bouckaert et al., 2024a), demonstrating that interface-governed mechanisms can be reproduced with a reduced number of degrees of freedom compared to conventional continuum models.

More recently, the framework was extended to dynamic applications. Modal properties such as natural frequencies and mode shapes can now be computed within the discrete-finite element formulation, with the tangent stiffness matrix used in the eigenvalue problem to track the evolution of modal properties as the structure enters nonlinear regimes (Bouckaert, Piedboeuf, et al., 2025). The extension to nonlinear response-history analysis was formalised by Bouckaert et al. (2026), who implemented and validated explicit and implicit time-integration schemes together with several Rayleigh damping formulations. Applications to rocking columns, masonry arches, and mixed reinforced concrete-masonry buildings demonstrated the capability to capture large displacements, mechanism formation, and structural collapse. A key finding concerns the behaviour of tangent stiffness-proportional damping in the presence of kinematic mechanisms, with direct implications for the modelling choices adopted in the present work.

Further developments have expanded the methodological scope of the framework. Legros (Legros, 2026) extended HybriDFEM to enable coupling between rigid block discretisations and two-dimensional finite elements, allowing the representation of hybrid structural systems combining discrete and continuum components within the same computational framework. Cherie-Lignière (Chérie-Lignière, 2026) explored the application to larger-scale masonry structures and developed a graphical pre-processing tool for automated generation of block-based models, validated through static, free-vibration, and dynamic analyses of masonry walls subjected to out-of-plane loading.

Despite these developments, the capability of HybriDFEM to reproduce the dynamic response of masonry structures under impact loading has not yet been systematically investigated. The present work addresses this gap by developing and validating a HybriDFEM framework for unreinforced masonry walls under out-of-plane impact loading, with interface parameters identified from four-point bending tests and modal identification, and the calibrated model applied to simulate impact experiments conducted at RISE.

2.6 Finite Element Method

The Finite Element Method (FEM) is the most widely adopted numerical framework for the analysis of structures in civil and mechanical engineering (Zienkiewicz & Taylor, 2000). The method is based on the discretisation of a continuous structural domain into a finite number of elements, within which the displacement field is approximated using shape functions. Global equilibrium is enforced through the assembly of element stiffness contributions, reducing the governing partial differential equations to a system of algebraic equations in terms of nodal unknowns (Rust, 2015). The method is well suited for resolving stress and strain fields within deformable structural components and is supported by mature, general-purpose software implementations capable of handling material nonlinearity,

geometric nonlinearity, and contact interaction (Belytschko et al., 2000; Dassault Systèmes Simulia Corp., 2024).

A key challenge in applying FEM to masonry structures is the representation of mortar joints and the discontinuous kinematics they produce. Unlike homogeneous structural materials, masonry is a composite assemblage in which the global response is largely governed by the behaviour of the interfaces between individual units, including joint opening, frictional sliding, and re-contact (Lourenço, 1996). Accurately capturing these mechanisms within a continuum framework requires careful choice of modelling scale and constitutive representation. When modelling masonry structures, FEM can therefore be formulated at different levels of idealisation depending on the required accuracy and available computational resources, ranging from homogenised macro-scale models to detailed micro-scale representations of individual bricks and mortar layers (Lourenço et al., 1995; Roca et al., 2010). In this thesis, the meso-scale strategy is adopted within the Abaqus/Explicit framework, in which brick units are represented as deformable continua governed by the Concrete Damaged Plasticity model and the mortar joints are described through surface-based cohesive-frictional contact formulations. This approach provides a balance between physical accuracy and computational efficiency, and is discussed in detail in the following sections.

2.6.1 Element formulations

Within the finite element framework, the structural domain is discretised into elements whose geometry and interpolation functions determine how accurately the displacement field is approximated and at what computational cost. The choice of element type therefore directly influences both the accuracy of the solution and the efficiency of the analysis (Zienkiewicz & Taylor, 2000).

Continuum elements can be formulated in two or three dimensions. Two-dimensional formulations reduce the problem to a single plane and are appropriate when the structural geometry and loading can be idealised as uniform in the out-of-plane direction. Three-dimensional solid elements resolve the full stress and strain state within each element and are required when the geometry, loading, or boundary conditions introduce significant variation in all three spatial directions (Belytschko et al., 2000). For masonry walls subjected to out-of-plane impact loading, where the contact geometry between brick faces, the bond pattern, and the distribution of arching forces all involve inherently three-dimensional behaviour.

Among three-dimensional solid elements, the eight-node hexahedral element uses trilinear shape functions to interpolate the displacement field within each element and is the standard choice for three-dimensional nonlinear structural analyses (Zienkiewicz & Taylor, 2000). In the context of masonry modelling, this element type is widely adopted for the discretisation of brick units in meso-scale finite element models (Abdulla et al., 2017).

2.6.1.1 Full and reduced integration

The stiffness matrix of each element is computed through numerical integration, in which the integral is evaluated at a discrete set of points within the element known as Gauss points. Full integration uses enough Gauss points to evaluate the integral exactly for an undistorted element, while reduced integration uses fewer points, only one for an eight-node hexahedral element. Using fewer integration points reduces the computational cost per element signif-

icantly, which is particularly beneficial for large three-dimensional models (Zienkiewicz & Taylor, 2000).

The drawback of reduced integration is that elements become susceptible to hourglass modes. These are deformation patterns in which the element deforms in a way that produces no strain at the single integration point, and therefore no stress and no resistance to the deformation. Because the element effectively cannot detect these deformations, they cost no energy and can grow freely, eventually corrupting the solution (Belytschko et al., 2000). To prevent this, hourglass control is applied, which introduces a small resistance to these unphysical deformation patterns without affecting the physical response of the element (Abdulla et al., 2017; Belytschko et al., 2000).

2.6.2 Nonlinear solution procedures

The simulation of masonry structures under impact loading involves three main sources of nonlinearity: material nonlinearity arising from inelastic behaviour of the brick units and mortar joints, geometric nonlinearity due to large displacements and rotations, and contact nonlinearity from the opening, closing, and sliding of interfaces between adjacent units (Dassault Systèmes Simulia Corp., 2024). Within the finite element framework, two broad classes of solution procedures are distinguished depending on the nature of the problem: implicit procedures for static and quasi-static analyses, and explicit procedures for transient dynamic analyses.

2.6.2.1 Force-controlled static analysis

The force-controlled static analysis follows the same Newton–Raphson procedure described for HybriDFEM in Section 2.4.5.1, in which the external load is applied incrementally and equilibrium is enforced at each step through iterative corrections to the displacement field.

2.6.2.2 Displacement-controlled static analysis

The displacement-controlled static analysis follows the same procedure described in Section 2.4.5.1, in which selected degrees of freedom are prescribed and increased incrementally, allowing the response to be traced through and beyond the peak resistance without convergence difficulties.

2.6.2.3 Explicit dynamic analysis

Explicit time integration schemes advance the solution directly from known quantities at the current time step, without requiring iteration or assembly of the global tangent stiffness matrix, making them particularly efficient for problems involving short load durations and highly nonlinear contact conditions such as impact simulations (Dhanasekar & Haider, 2008).

The Central Difference Method (CDM) adopted in Abaqus/Explicit follows the same formulation described for HybriDFEM in Section 2.4.5.3. The scheme is conditionally stable, requiring the time step to satisfy (Dassault Systèmes Simulia Corp., 2024; Dhanasekar & Haider, 2008):

$$\Delta t \leq \Delta t_{\text{crit}} = \frac{L_{\min}}{c_d}, \quad c_d = \sqrt{\frac{E}{\rho}} \quad (2.18)$$

where $L_{\min}$ is the smallest element characteristic length, E the elastic modulus, and ρ the material density (Dhanasekar & Haider, 2008). This is equivalent to the modal stability condition $\Delta t \leq 2/\omega_{\max}$ described in Section 2.4.5.3. Since Δt_{crit} decreases with smaller elements or stiffer materials, mass scaling and time scaling could be applied to enhance the computational cost.

Mass scaling. Mass scaling artificially increases the density of elements to raise Δt_{crit} , reducing the number of increments required to reach equilibrium. It is applied during the quasi-static initialisation step, where inertial effects are negligible. The validity of the scaling is verified by confirming that the kinetic energy remains below 10% of the internal energy throughout the majority of the step (Dassault Systèmes Simulia Corp., 2024). Mass scaling could be activated, changed or deactivated from step to another, allowing a flexibility in modeling.

For dynamic analyses, mass scaling must be used with caution, as artificially increasing the element density alters the inertial properties of the model and can degrade the accuracy of the solution. If applied during a dynamic event, it should be limited to a small number of elements rather than the entire model (Dassault Systèmes Simulia Corp., 2024).

Time scaling. Time scaling applies a constant factor to the stable time increment during the dynamic impact step, slightly relaxing the stability constraint without modifying the material densities or the equations of motion. Unlike mass scaling, time scaling does not alter the inertial properties of the model and is therefore applicable during the dynamic phase. The factor is chosen conservatively so that the impact force and displacement histories are insensitive to its value (Dassault Systèmes Simulia Corp., 2024). However, verification that the time scaling factor does not affect the physical response of the model remains essential.

2.6.3 Concrete Damaged Plasticity Model

The Concrete Damaged Plasticity (CDP) model is a continuum constitutive model originally developed for quasi-brittle materials such as concrete (Lubliner et al., 1989), and has been widely applied to masonry units owing to the similarity of their failure behaviour under tensile and compressive loading (Abdulla et al., 2017; Nastri et al., 2023). The model combines isotropic damaged elasticity with isotropic tensile and compressive plasticity to represent the two dominant failure modes of quasi-brittle materials: tensile cracking and compressive crushing. Stiffness degradation is governed by independent scalar damage variables in tension and compression, and plastic strains accumulate separately for each loading mode (Lee & Fenves, 1998; Lubliner et al., 1989). Figure 2.8 provides an overview of the CDP model components and their relationships, as described in the following subsections.

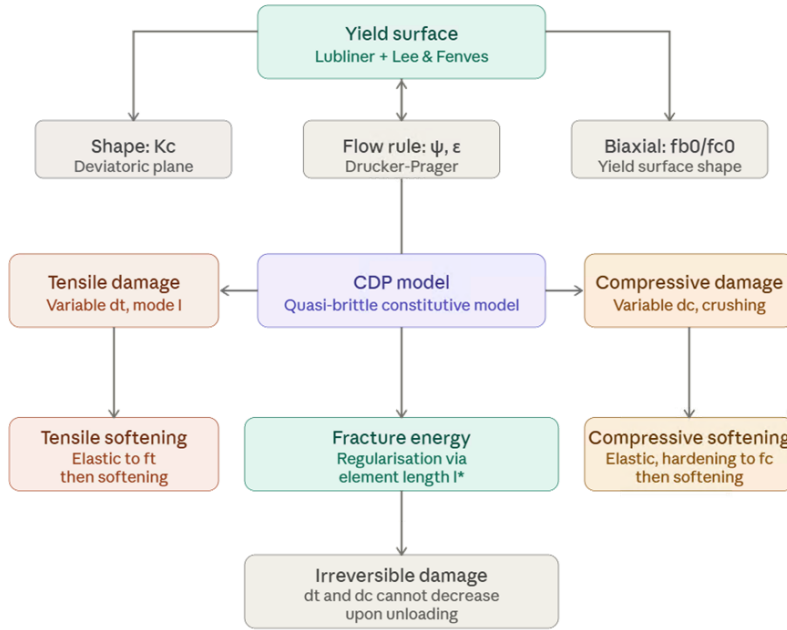


Figure 2.8: Overview of the Concrete Damaged Plasticity model components and their interdependencies. Based on Lubliner et al. (1989), Lee and Fenves (1998), and Nastri et al. (2023).

2.6.3.1 Yield surface and failure criteria

The yield surface of the CDP model is defined in terms of the effective stress tensor, based on the work of Lubliner et al. (1989) with modifications proposed by Lee and Fenves (1998). The shape of the yield surface in the deviatoric plane is controlled by the parameter K_c , defined as the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian. A value of $K_c = 1$ produces a perfectly circular cross-section in the deviatoric plane, while lower values produce a more triangular shape. The default value $K_c = 2/3$, recommended for concrete and cementitious materials such as masonry, gives a triangular cross-section consistent with experimentally observed multiaxial failure envelopes (Lubliner et al., 1989; Nastri et al., 2023). The ratio f_{b0}/f_{c0} of biaxial to uniaxial compressive yield strength is an additional parameter that governs the extent of the yield surface under biaxial stress states, with a typical value of 1.16 for concrete and masonry (Lubliner et al., 1989; Nastri et al., 2023).

The plastic flow is governed by a Drucker–Prager hyperbolic potential function, from which the plastic strain increment direction is derived. A non-associated flow rule is used in the CDP model, meaning the plastic potential differs from the yield surface. This is physically consistent with the behaviour of quasi-brittle materials such as concrete and masonry, which exhibit volumetric plastic expansion during shearing that does not follow the yield surface gradient (Lubliner et al., 1989). The plastic potential is characterised by two parameters: the dilation angle ψ , which controls the rate of plastic volumetric strain developed during shearing, and the eccentricity ε , which defines the rate at which the hyperbolic potential approaches its asymptote in the meridional plane. For masonry, dilation angles in the range up to 30° have been reported in the literature (Nastri et al., 2023).

A viscosity parameter η can be introduced to regularise the constitutive equations in the soft-

ening regime, by allowing the stress state to temporarily exceed the yield surface. This viscoplastic regularisation improves the well-posedness of the boundary value problem in the post-peak regime and is available in Abaqus/Standard analyses (Dassault Systèmes Simulia Corp., 2024). For explicit dynamic analyses, viscoplastic regularisation is not applicable and η is therefore set to zero (Nastri et al., 2023).

2.6.3.2 Damage variables and stiffness degradation

The CDP model captures two concurrent effects of inelastic loading: the accumulation of permanent plastic deformations and the progressive degradation of the elastic stiffness. Stiffness degradation is governed by two independent scalar damage variables, d_t for tension and d_c for compression, each evolving from zero in the undamaged state to one at complete loss of stiffness (Lee & Fenves, 1998; Lubliner et al., 1989). As damage accumulates, the elastic modulus of the material reduces, so that the unloading stiffness becomes progressively lower than the initial value. This degradation is irreversible; once a damage variable increases it cannot decrease upon unloading, reflecting the permanent loss of load-carrying capacity associated with cracking or crushing (Lubliner et al., 1989). The combination of plastic strains and stiffness degradation allows the model to reproduce the experimentally observed behaviour of quasi-brittle materials, where both residual deformations and reduced stiffness are observed after inelastic loading (Nastri et al., 2023).

2.6.3.3 Tension and compression behaviour with softening

Under tensile loading, the material response is linear elastic up to the tensile strength f_t , after which stress decreases as cracking develops. This post-peak softening is governed by the mode I fracture energy $G_{f,I}$, which represents the energy dissipated per unit area of crack surface (Hillerborg et al., 1976). An exponential softening law is adopted, consistent with the brittle post-peak behaviour observed experimentally in clay brick units (Nastri et al., 2023).

Under compressive loading, the response is linear elastic up to an initial yield stress σ_{c0} , followed by a hardening branch up to the peak compressive strength f_c , and a subsequent softening branch governed by the compressive fracture energy $G_{f,c}$ (Lubliner et al., 1989; Nastri et al., 2023).

A practical challenge in implementing softening laws in finite element models is that the energy dissipated during fracture depends on the element size if no special treatment is applied, meaning results change when the mesh is refined (Bažant & Oh, 1983). This is addressed through fracture energy regularisation, in which the softening curve is scaled by the characteristic element length l^* so that the total energy dissipated per unit crack area remains equal to G_f regardless of the mesh size (Bažant & Oh, 1983; Hillerborg et al., 1976).

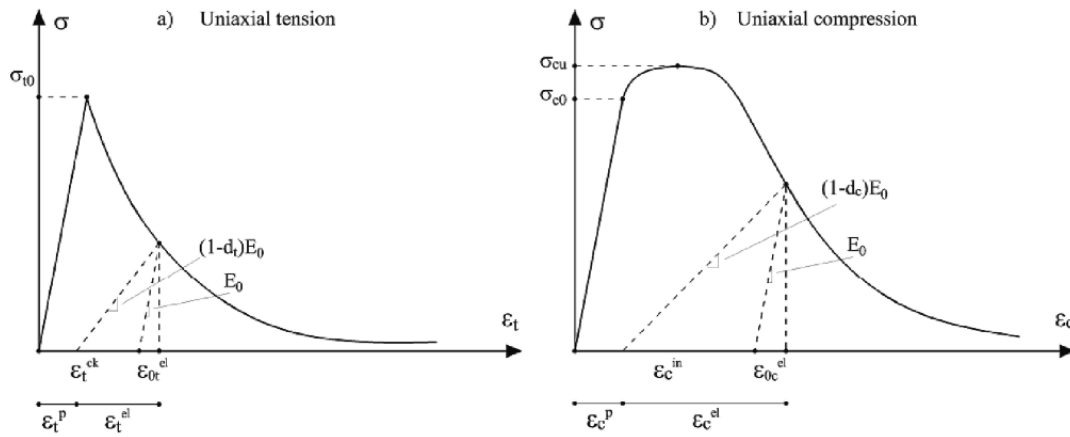


Figure 2.9: Uniaxial response of the CDP model: (a) tensile behaviour with exponential softening; (b) compressive behaviour with hardening followed by softening. Source: Nastri et al. (2023).

2.6.4 Contact Mechanics

The mortar joints between adjacent brick units are represented through surface-based contact formulations rather than discretised continuum elements. The contact formulation governs the interaction between brick surfaces in both the normal and tangential directions, allowing the transmission of compressive forces, cohesive tensile and shear resistance, and frictional resistance, and permitting separation and large relative sliding when the interface strength is exceeded (Abdulla et al., 2017; Dassault Systèmes Simulia Corp., 2024). Figure 2.10 provides an overview of the contact formulation components, as described in the following subsections.

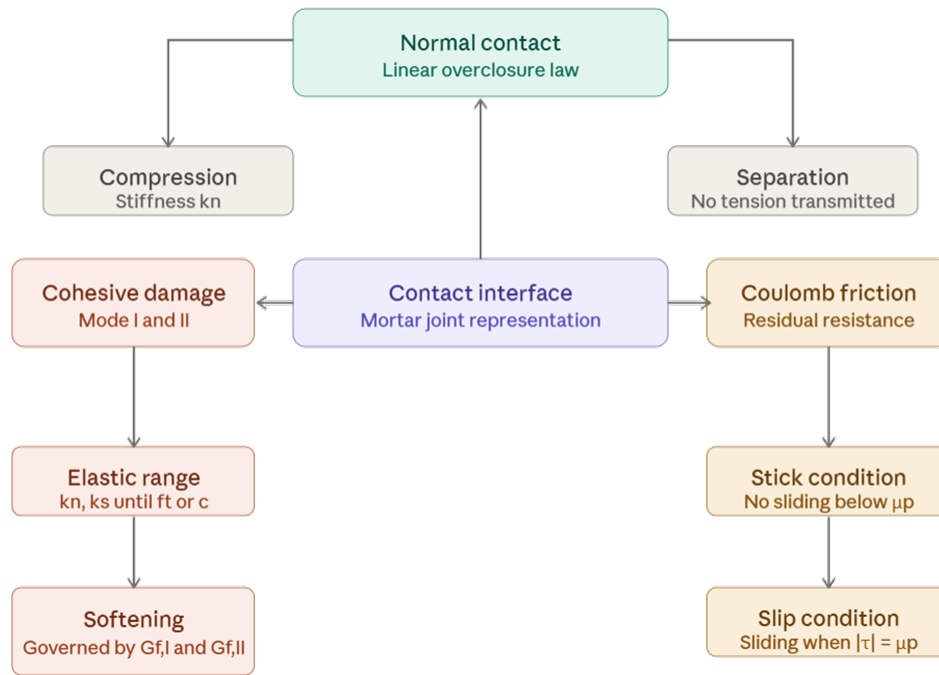


Figure 2.10: Overview of the contact mechanics formulation for mortar joints, comprising normal contact, cohesive damage, and frictional resistance. Based on Dassault Systèmes Simulia Corp. (2024).

2.6.4.1 Normal contact

Normal contact governs the interaction between two surfaces in the direction perpendicular to the interface. In Abaqus, several formulations are available to describe this interaction (Dassault Systèmes Simulia Corp., 2024). The hard contact formulation assumes perfectly rigid surfaces where no interpenetration is permitted and contact pressure increases indefinitely upon contact. The linear pressure–overclosure relationship instead allows a small controlled interpenetration, with the contact pressure increasing linearly with the interpenetration according to a prescribed stiffness k_n . This introduces a finite compliance in the normal direction that can represent the physical deformability of the interface under compression (Dassault Systèmes Simulia Corp., 2024).

When the surfaces separate and the contact pressure vanishes, no compressive stress is transmitted. Any tensile resistance across the interface must therefore be provided by a separate constitutive law, as described in Section 2.6.4.2. This separation of compressive and tensile behaviour is physically consistent with the response of mortar joints, which resist compression through direct contact but have limited tensile capacity (Lourenço, 1996).

2.6.4.2 Cohesive damage frictional interface law

The tensile and shear behaviour of the mortar joint is described by a cohesive damage frictional interface model. This formulation combines two physically distinct mechanisms, cohesive damage and Coulomb friction, within the same interface, capturing the progressive degradation of the joint under both mode I (tensile opening) and mode II (shear sliding) loading, as well as the residual frictional resistance that develops once cohesion is lost (Abdulla et al., 2017; Dassault Systèmes Simulia Corp., 2024).

Elastic range. In the undamaged state, the interface behaves elastically in both the normal and tangential directions, characterised by the normal and tangential stiffnesses k_n and k_s (units of MPa/mm), which relate the interface tractions to the corresponding displacement jumps:

$$\sigma_n = k_n \delta_n, \quad \tau = k_s \delta_s \quad (2.19)$$

where σ_n is the normal traction, τ the shear traction, δ_n the relative opening displacement between the two surfaces, and δ_s the relative sliding displacement along the interface (Abdulla et al., 2017).

Damage initiation and softening. Damage initiates when the normal traction reaches the tensile strength f_t of the joint (mode I), or when the shear traction reaches the cohesion c (mode II) (Abdulla et al., 2017). Beyond these thresholds, the interface enters a softening regime in which the transmitted traction decreases with increasing relative displacement. The softening behaviour is characterised by the fracture energies $G_{f,I}$ and $G_{f,II}$ in the normal and tangential directions respectively, defined as the energy dissipated per unit area of crack surface (Abdulla et al., 2017; Hillerborg et al., 1976). A scalar damage variable d evolving from zero (intact) to one (fully damaged) governs the stiffness degradation of the interface, analogously to the damage variables in the CDP model described in Section 2.6.3 (Abdulla et al., 2017).

Residual frictional resistance. Once the cohesive capacity of the interface is exhausted, the joint can no longer transmit tensile normal stress or cohesive shear stress. Under compressive normal stress, frictional resistance develops according to the Coulomb criterion (Abdulla et al., 2017):

$$|\tau| \leq \mu p \quad (2.20)$$

where μ is the friction coefficient and p the compressive contact pressure. In the Abaqus contact formulation, friction activates only in the damaged portion of the interface, meaning that cohesion governs the undamaged region while friction governs the failed region (Dassault Systèmes Simulia Corp., 2024). For brick–mortar interfaces, friction coefficients in the range $\mu = 0.4$ – 0.8 are commonly reported in literature (Lourenço, 1996).

2.6.4.3 Stick–slip transition and finite sliding

Prior to sliding, surfaces in contact are in a stick condition in which no relative tangential motion occurs and the interface transmits shear stress below the Coulomb limit (Abdulla et al., 2017). When the shear stress reaches μp , the surfaces transition to a slip condition and relative sliding occurs. Two sliding formulations are available in Abaqus: small sliding, which assumes that the relative tangential motion between surfaces remains small throughout the analysis, and finite sliding, which allows arbitrarily large relative tangential motion and continuously updates the contact tracking accordingly (Dassault Systèmes Simulia Corp., 2024). Finite sliding is more computationally expensive than small sliding, as the contact pairing must be recomputed at each increment, but it is necessary when large

relative displacements between surfaces are expected (Dassault Systèmes Simulia Corp., 2024).

3 Methodology

This chapter presents the methodology adopted to assess the response of unreinforced masonry walls subjected to impact loading. The work combines three components: the experimental tests carried out at RISE, as well as the HybriDFEM, and the ABAQUS. Together, these components form an integrated framework in which physical observations drive model construction, calibration, and assessment.

Section 3.1 describes the experimental basis of the study, including quasi-static four-point bending tests and the three impact campaigns conducted at RISE (with a focus on the third campaign) on full-scale double-wythe clay brick masonry walls. These tests provide the reference data used for model calibration, validation, and comparison.

Section 3.3 presents the HybriDFEM model developed in this work. The general formulation is introduced in Chapter 2; therefore, this chapter focuses on problem-specific modelling choices, including wall discretisation, boundary conditions, impactor representation, damping, and simulation workflow.

Section 3.4 presents the ABAQUS finite element model, which provides a continuum-based reference for the same experimental cases. The performance of both numerical approaches are assessed in Chapter 4 and compared in Section 4.5.

3.1 Experimental Tests

The experimental data underlying the numerical investigations presented in this thesis were collected through successive laboratory campaigns conducted at RISE Research Institutes of Sweden, Borås, it consists of a quasi-static four-point bending reference campaign and three successive impact campaigns on double-wythe unreinforced masonry walls. The four-point bending tests are treated separately because their loading protocol and purpose differ from those of the impact campaigns. The three impact campaigns are then described and compared through the variables that directly affect the modelling strategy: axial confinement, impact severity, load topology, boundary conditions, and damage mechanisms.

3.1.1 Common Experimental Basis

Wall typology and construction

The campaigns used vertically spanning double-wythe unreinforced clay brick masonry walls built between a bottom and a top reinforced concrete (RC) slab. The specimens were intended to represent loadbearing masonry walls commonly found in Sweden before the 1940s, with the RC slabs reproducing the contact conditions that can occur between masonry walls and floor slabs in existing buildings (Godio & Flansbjer, 2025b; Godio et al., 2023a, 2023b).

The wall geometry was nominally consistent across the campaigns, but not identical. The wall thickness was approximately 245 mm and the width approximately 780 mm. The wall height was approximately $H_{\text{wall}} = 1.535$ m in Campaign 1, $H_{\text{wall}} = 1.525$ m in Campaign 2, and $H_{\text{wall}} = 1.548$ m in Campaign 3. These small variations are accounted for in the campaign-specific numerical models when required.

All walls were constructed using Swedish standard solid red clay bricks with nominal di-

mensions $245 \times 115 \times 62$ mm. The bricks were laid in a regular Flemish bond pattern, with alternating headers and stretchers. Bed and head joints were filled with hydraulic lime-based mortar of class CS I (D), with maximum aggregate size below 1 mm and an aggregate-to-binder ratio between 1:1 and 4:1. The average bed-joint thickness was approximately 10–11 mm, while the head-joint thickness was approximately 14–15 mm.

Each wall was built between two RC slabs with nominal dimensions $75 \times 500 \times 980$ mm (height $\times$ depth $\times$ width). The slabs were reinforced with two layers of steel mesh and allowed the specimens to be handled and positioned without introducing significant tensile damage into the masonry before testing. In all impact campaigns, the bottom slab rested on the strong floor fully fixed, while the top slab was connected to the reaction frame through a post-tensioned steel-rod system.

Material characterisation

The reference material properties used in this work are mainly taken from the characterisation reported for the earlier RISE campaigns (Godio et al., 2023a, 2023b). The values adopted as baseline properties include a brick compressive strength $f_{c,\text{brick}} = 33.1$ MPa, brick tensile strength $f_{t,\text{brick}} = 6.4$ MPa, mortar compressive strength $f_{c,\text{mortar}} = 0.85$ MPa, masonry elastic modulus $E = 2710$ MPa, and masonry compressive strength $f_{c,\text{masonry}} = 8.2$ MPa. For Campaign 3, an additional campaign-specific compressive strength value of $f_{c,\text{brick}} = 26$ MPa was available from EN 772-1 testing and is reported separately where it is used in the modelling assumptions.

Common response measurements

The impact campaigns were instrumented to capture the main quantities needed for model assessment: impactor kinematics, impact force, wall out-of-plane displacement, and the axial force in the top support system. Impactor displacement and velocity were measured by 2D HS-DIC cameras tracking the impactor surface. The impact force was derived from accelerometers mounted on the impactor. The wall response was measured by HS-DIC on the back face, providing full-field out-of-plane displacement data during the event. The axial force in the top support rods was recorded to quantify the arching force development: through dedicated strain gauges in Campaign 1 (Godio et al., 2023a), force transducers in Campaign 2 (Godio & Flansbjer, 2025b), and force transducers on the four eccentric rods in Campaign 3. Campaign 2 additionally introduced post-impact 3D LiDAR scanning for damage documentation.

3.1.2 Four-Point Bending Reference Tests

A separate series of quasi-static monotonic four-point bending tests was conducted on eleven unreinforced masonry wall strips (Godio et al., 2023b). The purpose of this campaign was to characterise the static out-of-plane response of the walls under controlled loading conditions, with particular focus on the influence of axial load, boundary stiffness, and arching action.

The tests were performed in a bi-axial setup designed for vertically spanning walls. A vertical axial load was applied to the specimen, while a horizontal out-of-plane force was introduced by a 500 kN hydraulic actuator. The horizontal load was transferred through two loading beams positioned at approximately one-third and two-thirds of the wall height, corresponding approximately to the 7th and 15th brick courses from the bottom. Three

support conditions were investigated: simply supported (SS), rigid (R), and non-rigid (NR). Deformation fields and crack patterns were monitored using DIC systems placed in front of and to the side of the walls.

In the present thesis, the force–displacement relationships and mechanical parameters derived from the four-point bending tests are used as reference data for calibration of the numerical models. Their use in the HybriDFEM calibration is described in Section 4.2.1, while the corresponding ABAQUS calibration procedure is presented in Section 4.4. The detailed experimental setup, instrumentation, and results of the four-point bending campaign are reported in Godio et al. (2023b).

3.1.3 Impact Campaigns

All three campaigns share the same wall typology, material composition, and non-rigid vertical support philosophy. They differ in the impact loading system, load topology, boundary conditions, and energy range, and are summarised in Table 3.1.

Campaign 1 (Godio et al., 2023a) used a guided drop-weight system in which the impactor maintained a horizontal trajectory during free fall, striking the wall at mid-height as a point-load through a rounded impactor tip at 3.1–4.4 m/s with masses of 76.5–116 kg. Walls were subjected to repeated impacts until a breach opened, enabling characterisation of progressive damage accumulation. The influence of axial prestress was investigated by varying it between 0.05 and 0.20 MPa. HS-DIC was introduced in this campaign for the first time on masonry walls under impact conditions. The top support was provided by two concentric M16 steel rods with a vertical axial stiffness of $K_{\text{sup},y} = 31.4 \text{ kN/mm}$.

Campaign 2 (Godio & Flansbjerg, 2025b) replaced the guided system with a pendulum in which the 116 kg impactor followed a circular trajectory, reaching velocities of 7.2–8.5 m/s. The higher energies caused significant damage in a single impact, preventing the repeated-loading protocol of Campaign 1. Both point-load (W16, W17) and line-load (W18, W19) configurations were tested, the latter through a spreader beam distributing the force across the full wall width at mid-height. The axial prestress was 0.05 MPa for all walls. A finite horizontal compliance was introduced at the top through a Teflon plate and strapping system, and 3D LiDAR scanning was used to document post-impact damage.

Campaign 3 extended the pendulum system to simultaneous dual line-loads at $H/3$ and $2H/3$, delivered by two contact profiles spanning the full wall width. Three impactor mass configurations were tested (236, 348, and 442 kg) across release heights of 0.03–5.10 m, producing velocities of 0.94–10.10 m/s and kinetic energies from approximately 0.14 to 15.0 kJ. All 13 walls were tested at 0.10 MPa. The top support was redesigned with four steel rods, two on each side, positioned with an eccentricity of 80 mm from the wall centreline toward the back face, giving a vertical axial stiffness of $K_{\text{sup},y} = 14.9 \text{ kN/mm}$. Since this campaign is unpublished, the setup is described in full in Section 3.1.4.



(a) Campaigns 1 and 2.



(b) Campaign 3.

Figure 3.1: Test rigs for Campaigns 1–2 and Campaign 3. The steel rods (highlighted in yellow) provide the non-rigid vertical support: two concentric rods in Campaigns 1–2, four eccentric rods in Campaign 3.

3.1.4 Campaign 3: Experimental Setup

Campaign 3 is described in greater detail because no dedicated experimental paper has yet been published. The description covers the information needed to define the numerical models and interpret the comparison with experimental data. All drawings of the experimental setup for Campaign 3 are provided in Appendix L.

Test rig and boundary conditions

The reaction frame consisted of two braced steel support frames, 2410 mm in height and spaced 1800 mm apart, connected transversally by a transfer beam positioned above the top RC slab, providing the upper anchorage point for the pre-stressing system. The URM wall specimen and its bottom RC slab rested directly on the strong floor independently of the frame, ensuring that the vertical load path through the wall was not affected by frame deformation.

The upper boundary condition was provided by four M16 threaded steel rods, two on each side of the wall, spaced 400 mm apart on each side, with a total left-to-right distance between rod pairs of 2000 mm. The rods connected the top RC slab, via the transfer beam, to the reaction frame above. The transfer beam was positioned with an eccentricity of 80 mm from the wall centreline toward the back face, opposite to the impact direction. This eccentricity introduces a moment arm that influences the axial force redistribution in the rods during impact, and constitutes the main difference in the boundary system relative to Campaigns 1 and 2, where the rods were positioned concentrically (Godio & Flansbjerg, 2025b; Godio et al., 2023a).

An axial compressive prestress of $\sigma_{ax} = 0.10$ MPa was applied by post-tensioning each rod to 5 kN (total 20 kN), monitored through force transducers on each rod. The bottom RC slab was bolted to a steel profile anchored to the support beams, preventing lateral sliding.

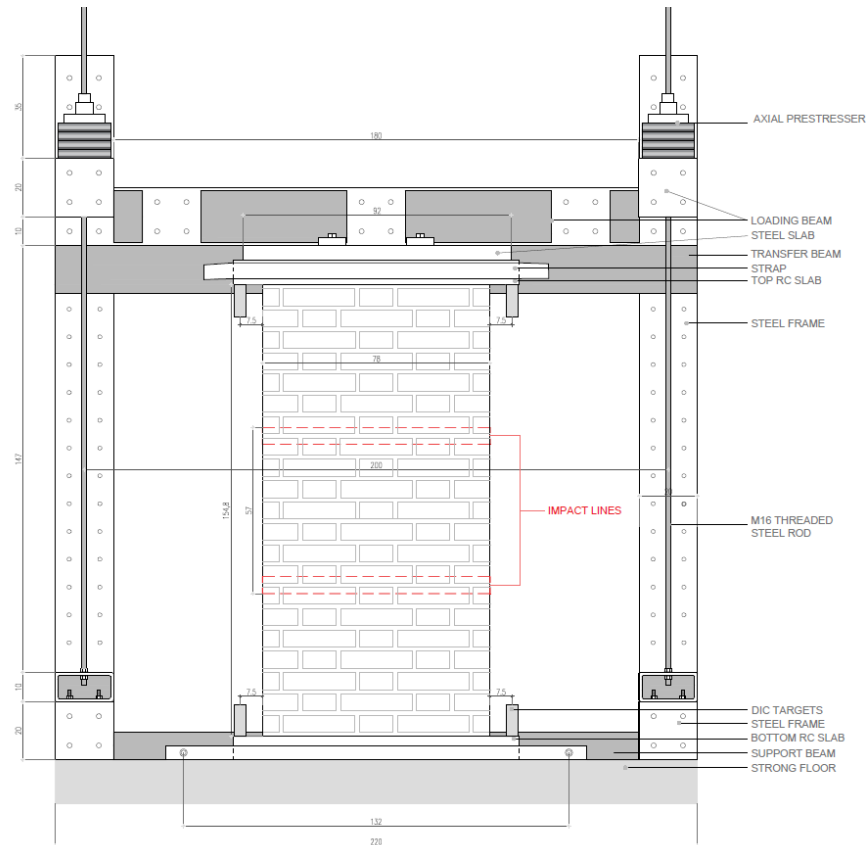


Figure 3.2: Front view of the Campaign 3 test rig showing the steel frame, RC slabs, eccentric transfer beam, and steel rods.

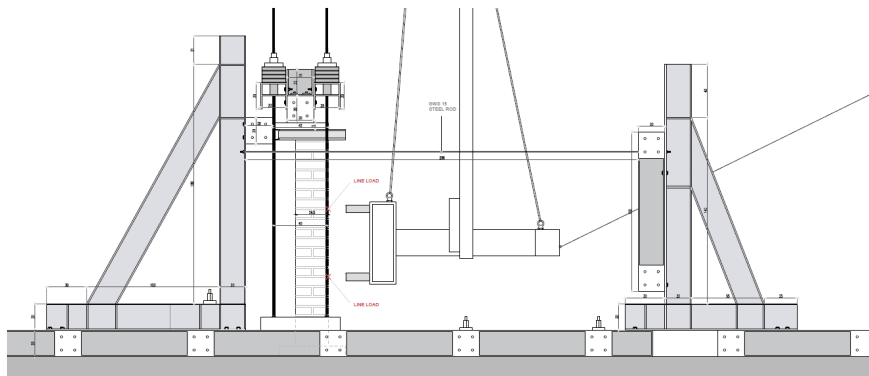


Figure 3.3: Side view of the Campaign 3 test rig showing the 80 mm eccentricity of the transfer beam relative to the wall centreline.

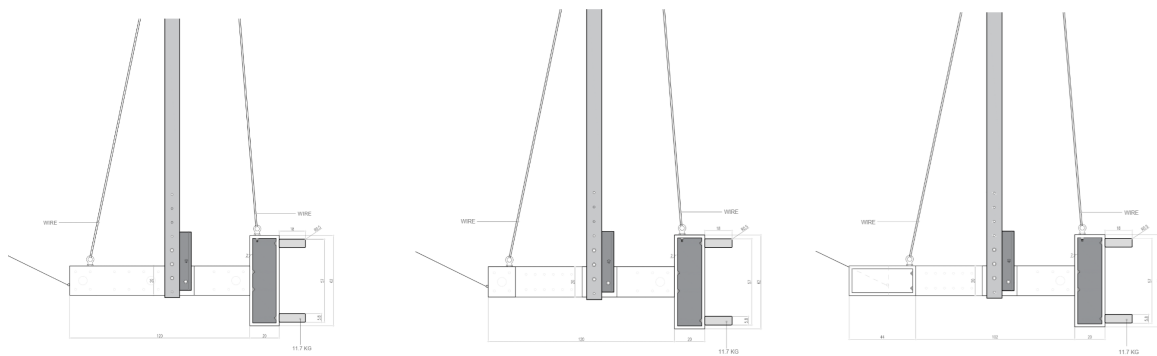
Impact loading system

The impact loading in Campaign 3 was applied by a pendulum steel impactor released from prescribed heights to strike the front face of the wall. The assembly was suspended by steel wires and reached the wall near the lowest point of its circular trajectory. The configuration was designed to generate two simultaneous line loads at $H/3$ and $2H/3$; however, because of the curved path, a small time offset may occur, with the lower contact profile reaching the

wall slightly before the upper one. The release height was varied between 0.03 and 5.10 m, producing pre-impact velocities from 0.94 to 10.10 m/s (Table 3.1).

The impactor assembly had a total length of approximately 1400 mm and was used in three mass configurations: 236, 348, and 442 kg. The mass was varied by adding steel weights to the base assembly, with Configuration 1 corresponding to the base impactor and Configurations 2 and 3 including progressively larger added masses, as shown in Figure 3.4. In all configurations, the load was transferred to the wall through two rectangular steel contact profiles with filleted corners. Each profile was 810 mm long, had a cross-section of 180×58 mm, and had a mass of 11.7 kg. The 58 mm dimension defined the contact depth against the wall, while the 810 mm length exceeded the wall width and ensured full-width contact.

The two contact profiles were positioned at approximately $H/3$ and $2H/3$ of the wall height, corresponding to about 512 mm and 1023 mm above the top of the bottom RC slab. Campaign 3 therefore replaced the single mid-height point or line impact used in the previous campaigns with two simultaneous full-width line loads. the full impactor-wall configuration is presented in Figure 3.5.



(a) Config. 1: $m = 236$ kg. (b) Config. 2: $m = 348$ kg. (c) Config. 3: $m = 442$ kg.

Figure 3.4: Side views of the three Campaign 3 impactor mass configurations.

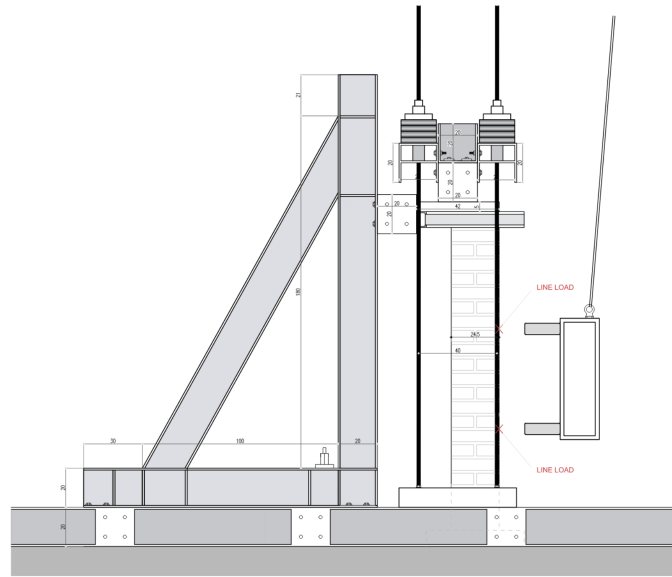


Figure 3.5: Campaign 3 impactor–wall configuration showing the two full-width line loads applied at approximately $H/3$ and $2H/3$ of the wall height.

Instrumentation and test protocol

Campaign 3 measurements were obtained from synchronised optical and analogue records for each impact. The optical records provide the impactor centre displacement and velocity, the wall out-of-plane displacement and velocity at mid-height on the back face, and the relative gaps between the lower and upper contact profiles and the wall. The corrected impactor centre quantities were used in the analysis, accounting for the scale correction applied to the exported DIC data.

The analogue records contain the impact force histories derived from accelerometers mounted on the impactor assembly, with separate force signals associated with the upper and lower contact lines and a filtered average force used for the main calculations. The same records include the axial force channels from the top support rods, which were used to estimate the total axial force at the top support and the corresponding arching-force increment relative to the pre-impact baseline. The main experimental observables used for model assessment are therefore the impactor displacement and velocity, impact force, wall out-of-plane displacement, and arching-force response.

Each wall was subjected to one or more impacts according to the prescribed mass and release-height configuration, with the number of impacts depending on the observed damage progression and remaining load-carrying capacity. In this thesis, the numerical models simulate the first impact on the initially undamaged wall, as discussed in Section 3.3.8.

3.1.5 Test Programme

The test programme is summarised in Table 3.1. σ_{ax} denotes the applied axial stress, m the impactor mass, v_0 the pre-impact velocity, and $KE = \frac{1}{2}mv_0^2$ the kinetic energy at impact. The axial stress values exclude the self-weight of the RC slab and the mass of the steel loading assembly attached to the top of the wall. The slab self-weight was evaluated from its dimensions, including reinforcement. For Campaigns 1 and 2, the steel plate and loading

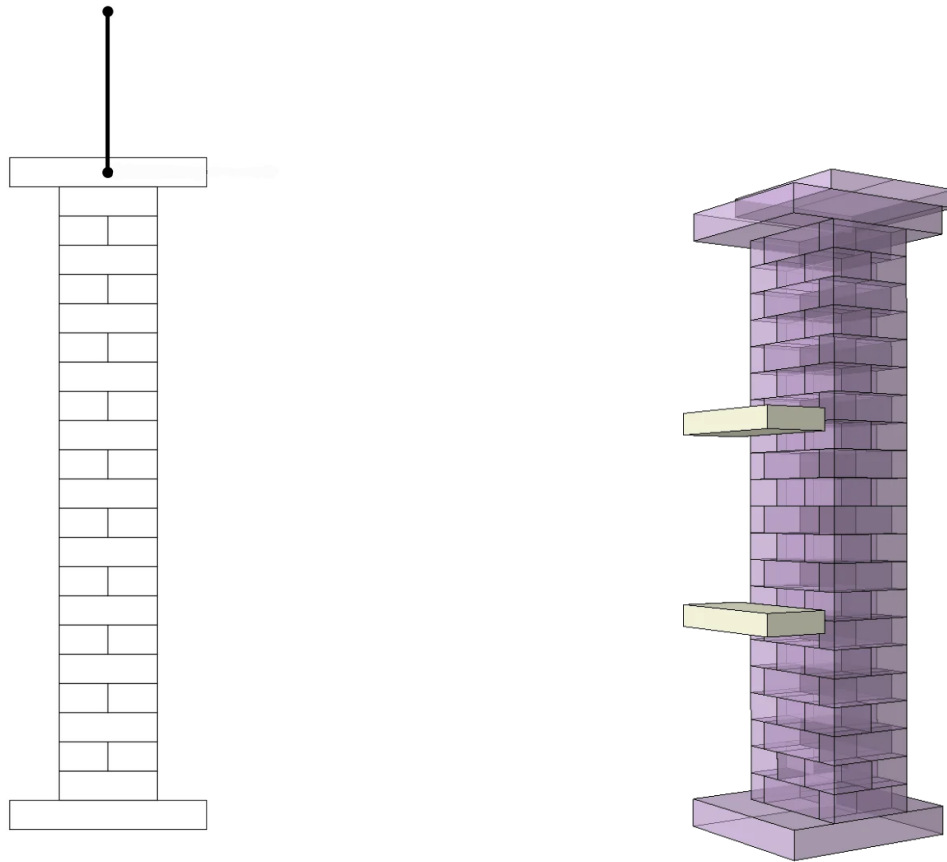
beam had a combined mass of 262 kg, while for Campaign 3 the steel loading beam assembly had a total mass of 460 kg. The vertical axial support stiffness adopted in the numerical models is $K_{\text{sup},y} = 31.4 \text{ kN/mm}$ for Campaigns 1 and 2 and $K_{\text{sup},y} = 14.9 \text{ kN/mm}$ for Campaign 3.

Table 3.1: Test programme summary.

Test	σ_{ax} (MPa)	Load type	m (kg)	v_0 (m/s)	KE (kJ)
<i>Campaign 1 — Low-velocity repeated point-load impacts</i>					
W9–1	0.20	Point	76.5	3.13	0.37
W10–1	0.20	Point	116.0	4.43	1.14
W11–1	0.05	Point	116.0	4.43	1.14
<i>Campaign 2 — Moderate-velocity point- and line-load impacts</i>					
W16	0.05	Point	116	7.23	3.03
W17	0.05	Point	116	8.49	4.18
W18	0.05	Line	116	7.17	2.98
W19	0.05	Line	116	8.35	4.05
<i>Campaign 3 — Dual line-load impacts with variable mass and velocity</i>					
W23	0.10	Dual line	236	1.07	0.14
W24	0.10	Dual line	236	6.16	4.48
W25	0.10	Dual line	236	8.10	7.74
W26	0.10	Dual line	348	0.94	0.15
W27	0.10	Dual line	348	5.20	4.70
W28	0.10	Dual line	348	6.90	8.28
W29	0.10	Dual line	348	8.45	12.42
W30	0.10	Dual line	442	1.10	0.27
W31	0.10	Dual line	442	4.50	4.48
W32	0.10	Dual line	442	6.08	8.17
W33	0.10	Dual line	442	8.25	15.04
W34	0.10	Dual line	236	10.10	12.04
W35	0.10	Dual line	348	9.03	14.19

3.2 Numerical Modelling

The experimental data described in Section 3.1 form the basis for the development and assessment of two independent numerical models: a two-dimensional discrete rigid-block model implemented in HybriDFEM, and a three-dimensional continuum finite element model implemented in ABAQUS. The two frameworks are illustrated in Figure 3.6 and described in Sections 3.3 and 3.4 respectively.



(a) HybriDFEM: two-dimensional rigid-block model.

(b) ABAQUS: three-dimensional continuum model.

Figure 3.6: The two numerical models developed in this work.

3.3 HybriDFEM Numerical Model

3.3.1 Overview

The HybriDFEM model represents each masonry wall as a two-dimensional assembly of rigid blocks interacting through deformable interfaces, solved using the explicit Central Difference Method under transient impact loading. The same modelling framework is applied to all three experimental campaigns, with campaign-specific adaptations to the wall geometry, mass, prestress, boundary conditions, and impactor configuration. The mathematical formulation of the method is presented in Chapter 2; the present section focuses on the modelling decisions specific to the impact problem studied here.

All impact simulations follow a two-step procedure, illustrated in Figure 3.7. First, a static prestress analysis is performed to establish the pre-impact equilibrium state of the wall. Second, the impactor is assigned its experimentally measured pre-impact velocity and the explicit dynamic analysis is carried out. The detailed implementation of this workflow is described in Section 3.3.7.

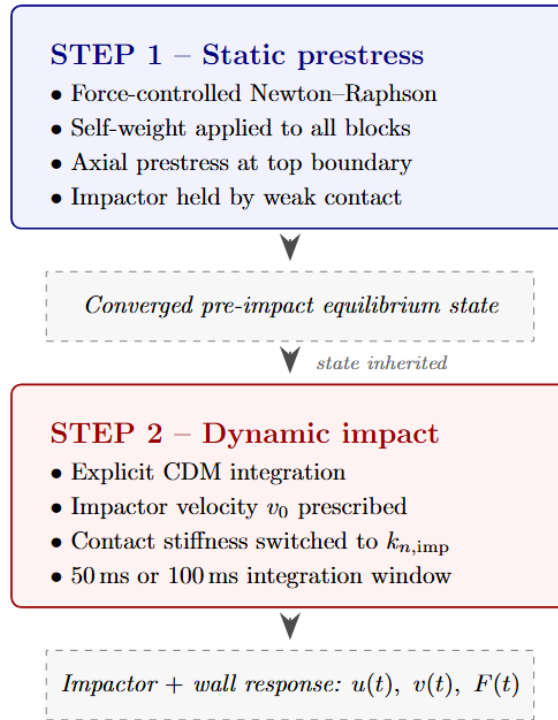


Figure 3.7: Two-step impact simulation procedure.

The model parameters are identified through the sequential calibration strategy described in Chapter 4. The effective masonry interface stiffness K_1 is first calibrated from quasi-static four-point bending tests (Godio et al., 2023b) and independently cross-validated against experimentally identified modal frequencies (Godio et al., 2024). Once fixed, this stiffness is transferred to the impact simulations, where the impact-contact stiffness, friction coefficient, and damping ratio are identified from Campaigns 1 and 2. The resulting parameter set is then applied to Campaign 3 without recalibration.

The framework is implemented in Python 3.8 as an in-house object-oriented codebase developed at UCLouvain (Bouckaert, 2025), organised around core objects that encapsulate the geometry, material state, and mechanical response of each component. All simulations were executed on a Linux HPC cluster at RISE using SLURM job scheduling, which allowed the calibration parameter sweeps to run in parallel across compute nodes. Post-processing was performed in Python using `h5py`, `NumPy`, and `Matplotlib`.

The following subsections describe the main modelling decisions: wall discretisation (Section 3.3.2), boundary conditions (Section 3.3.3), impactor modelling (Section 3.3.4), damping formulation (Section 3.3.5), and simulation workflow (Section 3.3.7). The main assumptions and limitations are summarised in Section 3.3.8.

3.3.2 Wall Discretisation

3.3.2.1 Geometry and block pattern

The masonry wall is represented as a two-dimensional assembly of rigid blocks, it spans vertically between the two RC slabs and deforms predominantly in a single plane, making the 2D idealisation an efficient representation. The out-of-plane width of all blocks is set equal to the nominal wall width around $b = 780$ mm.

The wall is discretised into 21 horizontal rows of blocks, with the block height given by:

$$h_{\text{block}} = \frac{H_{\text{wall}}}{21}, \quad (3.1)$$

where $H_{\text{wall}} = 1.535$ m for Campaign 1, $H_{\text{wall}} = 1.525$ m for Campaign 2, and $H_{\text{wall}} = 1.548$ m for Campaign 3, reflecting the slight difference in specimen height between series. The bond pattern follows a simplified Flemish bond: even-numbered rows are represented by a single full-width block, while odd-numbered rows are represented by two half-blocks. This discretisation captures the staggered joint geometry that governs the crack propagation and rocking mechanisms observed experimentally.

3.3.2.2 Reinforced concrete (RC) slabs, steel plate, and loading beam

The bottom and top RC slabs are represented as rigid blocks in all three campaigns. An effective density is assigned to each slab so that the total block mass matches the experimentally measured slab mass including reinforcement.

In addition to the RC slabs, the experimental setup includes a steel plate and loading beam assembly positioned above the top slab and connected to the reaction frame through the pre-stressing rods. The representation of this assembly differs between campaigns.

Campaigns 1 and 2. The steel plate and loading beam, with a combined mass of 262 kg, are not modelled as explicit blocks. Their weight is instead introduced as a static concentrated force at the top slab node during the prestress step. This simplification is physically acceptable, with moderate out-of-plane displacements and a stiff vertical boundary ($K_{\text{sup},y} = 31.4$ kN/mm), the top-boundary acceleration remains small and the inertial contribution of the assembly is negligible relative to the wall and impactor dynamics.

Campaign 3. The steel loading beam assembly, with a total mass of 460 kg, is modelled as an explicit rigid block placed immediately above the top RC slab (Figure 3.8(c)). Two considerations motivate this choice. First, the transfer beam is positioned with an eccentricity of +80 mm from the wall centreline; representing it as an explicit block with its centroid at the correct eccentric position captures this geometry naturally through the block kinematics. Second, the higher impact energies in Campaign 3 combined with the lower vertical boundary stiffness ($K_{\text{sup},y} = 14.9$ kN/mm) produce larger top-boundary accelerations, making the inertial contribution of the assembly non-negligible. A rigid continuity bond is assigned between the top RC slab and the steel plate block so that they move as a coupled unit. The axial prestress and boundary springs are applied at the centroid of the steel plate block, consistently with the eccentric rod connection point.

3.3.2.3 Coarse and refined discretisations

Two levels of spatial discretisation are used, selected based on the campaign-specific damage regime and computational requirements, as illustrated in Figure 3.8.

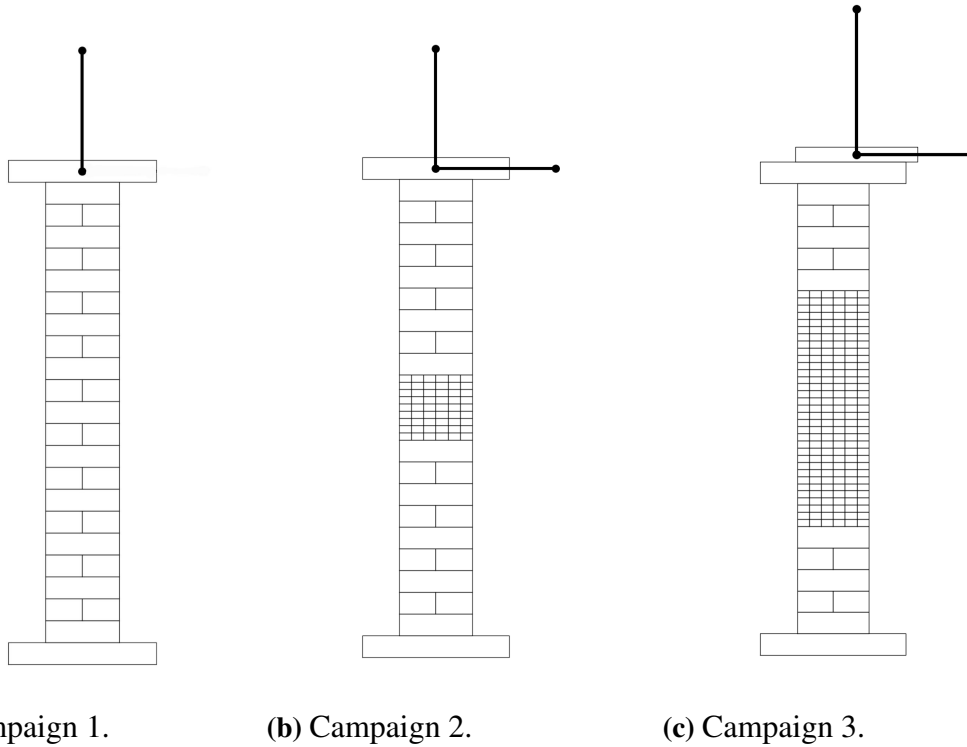


Figure 3.8: HybriDFEM model discretisations for the three campaigns: (a) coarse model; (b) refined band at mid-height; (c) extended refined band between $H/3$ and $2H/3$, with the explicit steel plate block above the top RC slab.

In the *coarse* model, each contact face carries 8 uniformly distributed contact pairs throughout the wall. This resolution captures the dominant out-of-plane mechanisms, such as block rocking, joint opening, and sliding, and is used for all Campaign 1 simulations and for the parameter sweep studies in Campaign 2. The adopted value $NCP = 8$ is selected on the basis of a convergence study on wall W10; the supporting results are presented in Section 4.1, and details in Appendix B.

In the *refined* model, a localised band of rows in the vicinity of the impact line is subdivided into a finer sub-block mesh while the remaining rows retain the coarse discretisation. Within the refined band, each parent block is replaced by a $n_y \times n_x$ grid of sub-blocks with $n_y = 3$ and $n_x = 2n_y = 6$, and each contact face within the band carries 2 contact pairs. This refinement resolves the contact and deformation field near the impact line, where stress gradients are highest, and enables the representation of more severe damage patterns observed in Campaigns 2 and 3. The 3×6 discretisation is adopted as the production level. The mesh sensitivity study supporting this choice is presented in Section 4.1, and details in Appendix B. The extent of the refined band is campaign-specific: for Campaign 2, rows 9–11 are refined, centred on the single mid-height impact line; for Campaign 3, rows 5–14 span the region between the two impact lines at $H/3$ and $2H/3$.

3.3.2.4 Interface stiffness and constitutive law

All masonry interfaces in the calibrated HybriDFEM model use `ElastoPlastic` law with compression cap described in Section 2.4.3.3. This law represents joint opening, fric-

tional sliding, and compressive inelasticity within a single constitutive framework, and is used as the primary formulation because the interface and impact parameters were identified with it.

Five interface types are defined for the refined model in Appendix A, each assigned stiffness and strength parameters consistent with the physical nature of the contact it represents. Their parameters are summarised in Table 3.2 and illustrated in Figure 3.9.

A limitation of the original `ElastoPlastic` (Section 2.4.3.3) law is that contact pairs are not removed from the global assembly after reaching the tensile cut-off. Opened joints may therefore continue contributing stiffness, which can restrain post-impact wall rebound and maintain the arching force above the pre-impact baseline. This limitation is discussed in Section 3.3.8 and addressed by the `ElastoPlastic_CD` extension in Appendix F. However, `ElastoPlastic_CD` is not used as the primary calibrated model; it is included only as an additional assessment of the effect of contact deletion and recovery on post-impact rebound and arching-force unloading.

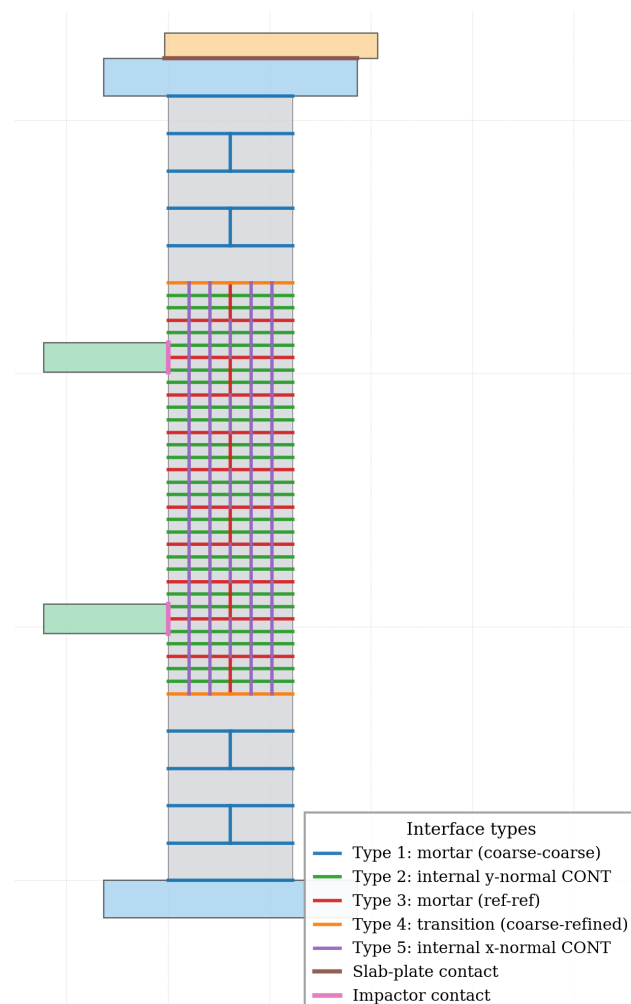


Figure 3.9: Interface type map for a representative Campaign 3 model.

Type 1 (blue) — composite mortar joints. Type 1 interfaces represent bed joints between full-height blocks in the coarse region. Their normal stiffness $K1$ is calibrated from the initial slope of the four-point bending force–displacement curve and independently cross-validated

against experimentally identified modal frequencies, as presented in Section 4.2.3. The calibrated value is:

$$K_1 = 17 \text{ GPa/m.} \quad (3.2)$$

This value is adopted without modification across all three campaigns.

Types 2 (green) and 5 (purple) — internal continuity cuts. Within the refined band, each parent block is subdivided into sub-blocks, introducing internal interfaces through the brick material where no mortar joint is physically present. These continuity cuts must transmit the full elastic stiffness of the brick. Their stiffnesses are derived from the same series-spring formula using the sub-block dimensions, ensuring that the total axial stiffness of the assembly is exactly preserved across the coarse-to-refined transition. The complete derivation and analytical proof of invariance are provided in Appendix A.

Types 3 (red) and 4 (orange) — mortar joints in the refined band. Types 3 and 4 represent real mortar joints within and at the boundary of the refined band, respectively. Their stiffnesses are derived from the same series-spring analogy in Appendix A, accounting for the different tributary heights of the interacting sub-blocks at each side of the interface. Their strength parameters are the same of those used in Type 1, reflecting the mortar-dominant character of these interfaces.

The friction coefficient μ applies to all interface types and is calibrated from the impact simulations rather than from the four-point bending tests, since the calibration target in the latter (the initial elastic slope of the force–displacement curve) lies in the pre-sliding regime where only the interface stiffness governs the response and μ has no influence (Appendix C, Section C.1). The dilatancy angle is set to $\psi = 0$ in all cases, implying purely tangential plastic flow with no normal expansion during sliding.

Numerical regularisation. A small cohesion $c = 0.01$ MPa is assigned to mortar interfaces (Types 1, 3, and 4) as a numerical regularisation. This value prevents near-zero-stress head joints from producing singular or ill-conditioned tangent stiffness behaviour after Step 1, without affecting the mechanical response. This is documented in Section 4.1.3.2.

Table 3.2: Interface constitutive parameters across the three campaigns.

Type	Physical meaning	k_{nn}	f_c (MPa)	f_t (MPa)
<i>Coarse region</i>				
1	Composite mortar joint	$2K_1$	8.2	0.4
<i>Refined band — internal continuity cuts</i>				
2	Horizontal continuity (brick)	$2K_2$	$f_{c,\text{brick}}^*$	6.4
5	Vertical continuity (brick)	$2K_5$	$f_{c,\text{brick}}^*$	6.4
<i>Refined band — real mortar joints</i>				
3	Mortar joint (ref–ref)	$2K_3$	4.0	0.4
4	Transition (coarse–ref)	$2K_4$	4.0	0.4

* $f_{c,\text{brick}} = 33$ MPa (C1–C2, literature (Godio et al., 2023b)); 26.0 MPa (C3, measured EN 772-1).

Contact geometry linearisation. All simulations are performed with contact face orientations updated at each time step to follow the current deformed configuration, rather than fixed at their initial orientation (`lin_geom = False`). For Campaign 1, where block rotations reach $\theta \approx 0.015$ rad, the response is practically insensitive to this setting. For Campaigns 2 and 3, where rotations reach $\theta \approx 0.05$ – 0.08 rad, the geometric update captures the P - δ restoring effect of the axial prestress acting on laterally displaced blocks and is therefore the more physically consistent choice.

3.3.3 Boundary Conditions and Support Modelling

The boundary conditions applied to the wall model reflect the physical restraints provided by the test rig. Both the bottom and top boundaries evolve across campaigns to match the changes in the experimental setup. The numerical model for Campaign 3 with all boundary conditions explicitly shown is presented in Figure 3.10.

3.3.3.1 Bottom boundary

The bottom RC slab rests directly on the strong floor and is bolted to the support structure through anchor profiles. In all three campaigns, the bottom slab is modelled as fully fixed ($U_x = U_y = R_z = 0$).

3.3.3.2 Top boundary — vertical spring

The axial stiffness of the pre-stressing rod system is represented by a Timoshenko finite element acting as a vertical spring between the top slab and a fixed dummy node above, providing a non-rigid top boundary consistent with the experimental configuration (Godio et al., 2023b). The element is used only to provide axial stiffness in the vertical direction; its flexural stiffness is suppressed so that it does not introduce additional bending restraint at the top boundary. The stiffness values reflect the rod geometry and configuration of each campaign: $K_{\text{sup},y} = 31.4$ kN/mm for Campaigns 1 and 2, and $K_{\text{sup},y} = 14.9$ kN/mm for Campaign 3. The axial prestress is applied as a concentrated force at the top slab node in Campaigns 1 and 2, and at the steel plate centroid node in Campaign 3, consistently with the physical rod connection point.

3.3.3.3 Top boundary — horizontal restraint

The horizontal restraint at the top of the wall differs between campaigns and constitutes one of the key modelling decisions of this work.

Campaign 1. The Campaign 1 rig was designed to provide a stiff clamping condition at the top slab. The horizontal displacement of the top slab is therefore fixed ($U_x = 0$) and the rotation is suppressed ($R_z = 0$).

Campaign 2. The Campaign 2 rig introduced a finite horizontal compliance through a Teflon plate and strapping system at the top, allowing limited lateral sliding of the top slab. The horizontal restraint is modelled as a Timoshenko spring of stiffness $K_{\text{sup},x}$, with the top slab rotation suppressed ($R_z = 0$). The spring stiffness was identified by matching the numerically computed cantilever frequency to the value $f_1 = 14.07$ Hz identified experimentally in the SIII dynamic identification campaign (Godio et al., 2024), yielding $K_{\text{sup},x} = 1$ kN/mm.

This cross-validation is presented in detail in Appendix B.3, Section B.3.5. The quantitative improvement in DIC agreement relative to the rigid boundary condition is presented in Section 4.1.

Campaign 3. The Campaign 3 rig uses a stiffer connection between the transfer beam and the reaction frame. In the absence of a dedicated dynamic identification campaign equivalent to SIII (Godio et al., 2024), the horizontal stiffness cannot be independently measured. A value of $K_{\text{sup},x} = 35 \text{ kN/mm}$ is therefore adopted as a near-rigid modelling assumption and retained as an open question for future characterisation. Both the horizontal and vertical springs are attached to the centroid of the eccentric steel plate block, so the +80 mm eccentricity of the transfer beam is represented directly through the block kinematics.

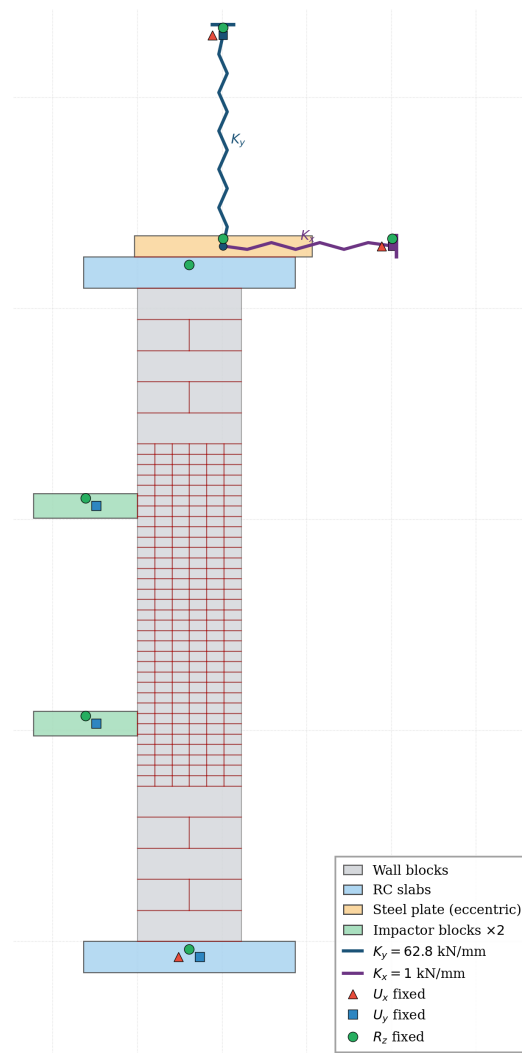


Figure 3.10: Numerical model showing boundary conditions, Campaign 3. Vertical spring ($K_y = 14.9 \text{ kN/mm}$, navy) and horizontal spring ($K_x = 35 \text{ kN/mm}$, purple) attached to the eccentric steel plate (orange).

3.3.4 Impactor Modelling

3.3.4.1 Impactor block geometry

The impactor is represented as a rigid block placed immediately to the left of the wall face, with its initial position offset by -1 mm from the wall surface to avoid initial contact overlap. The geometry and number of impactor blocks differ between campaigns, reflecting the evolution of the experimental loading configuration illustrated in Figure 3.11.

In Campaign 1, a single impactor block is placed at mid-height (row 10) representing the point-load configuration. In Campaign 2, the same block geometry is used for both the point-load and line-load walls, with a height equal to h_{block} . The block mass is set to match the experimentally measured impactor mass for each test, and its width spans the full model depth in the out-of-plane direction.

In Campaign 3, two impactor blocks are placed at $H/3$ and $2H/3$, centred on the mortar joints, with a height of 58 mm consistent with the steel contact profiles described in Section 3.1. Each impactor block carries half the total impactor mass, since the two physical beams are assumed to receive equal load in the symmetric pendulum configuration.

Since the model is two-dimensional, point-load and line-load configurations are modelled identically as rectangular blocks in contact with the wall face. The physical distinction between the two is accounted for indirectly through the calibrated contact stiffness $k_{n,\text{imp}}$ and friction coefficient μ , as discussed in Chapter 4. The consequences of this geometric simplification for force prediction are discussed in Section 3.3.8.

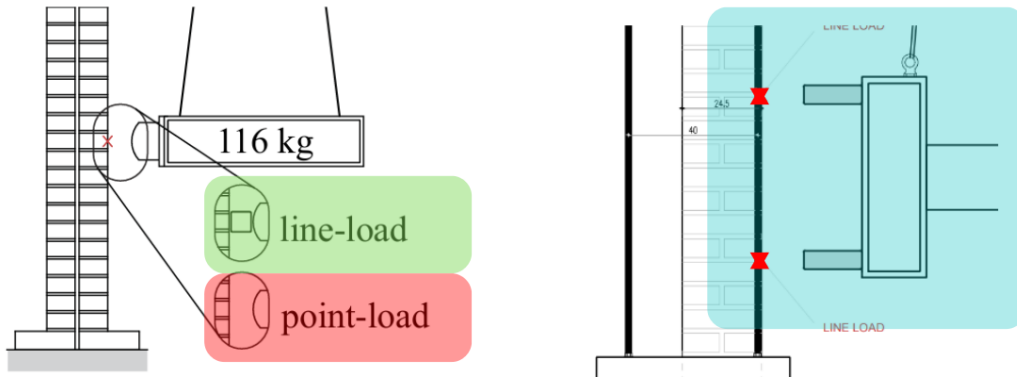


Figure 3.11: Contact topology across campaigns.

3.3.4.2 Contact law and simulation staging

The interaction between the impactor and the wall is governed by a no-tension contact law with contact deletion and recovery (NoTension_CD), which prevents tensile force transmission and allows the contact pair to be removed from the global assembly when the impactor separates from the wall and reactivated upon re-engagement. This formulation is physically appropriate for the impactor–wall interface, where no tensile bond exists and separation and re-contact occur naturally during and after impact. The contact force is computed as $F = k_{n,\text{imp}} \delta$, where δ is the normal penetration and $k_{n,\text{imp}}$ is a calibrated effective stiffness that absorbs the combined effect of impactor geometry, local compliance, and impact velocity. The consequences of this linear elastic simplification for the force prediction are discussed in Section 3.3.8.

The simulation follows the two-step procedure described in Section 3.3.7. In Step 1, the impactor is held in place by a weak contact with negligible stiffness ($k_n = k_s = 2 \text{ Pa/m}$), preventing any force transfer to the wall during the static prestress phase. At the start of Step 2, the contact stiffness is switched to the calibrated values $k_{n,\text{imp}}$ and $k_{s,\text{imp}} = k_{n,\text{imp}}/5$, and the experimentally measured pre-impact velocity v_0 is prescribed as an initial condition on the impactor block.

3.3.5 Damping Formulation

Structural damping is introduced through a stiffness-proportional Rayleigh formulation with $\alpha_M = 0$, using the linear-geometric tangent stiffness $\mathbf{K}^{\text{tan},lg}$ as the stiffness operator. The theoretical basis for this choice, including the physical motivation for preferring $\mathbf{K}^{\text{tan},lg}$ over the initial stiffness matrix $\mathbf{K}_0$, the behaviour of the damping term during kinematic mechanisms, and the formula $\alpha_K = 2\xi/\omega$ is presented in Section 2.4.5.4. The quantitative comparison between the two operators is presented in Section 4.2.3.

The damping ratio ξ is identified separately for each campaign. For Campaign 1, $\xi = 2\%$ is adopted; the response is weakly sensitive to this value at the low impact energies of that campaign, consistent with the near-zero contribution of $\mathbf{K}^{\text{tan},lg}$ during rocking phases. For Campaigns 2 and 3, ξ is identified from a dedicated sensitivity study presented in Section 4.2.3.

3.3.6 Modal Analysis of the Wall Model

After Step 1, an eigenvalue analysis is performed on the converged pre-stressed state using both the initial stiffness matrix $\mathbf{K}_0$ and the tangent stiffness matrix $\mathbf{K}^{\text{tan}}$. In this work, the modal analysis serves three purposes: to determine the critical time step for the explicit Central Difference Method, to define the reference frequency used in the stiffness-proportional damping formulation, and to cross-validate the calibrated interface stiffness K_1 against the SIII dynamic identification campaign (Godio et al., 2024). The theoretical formulation is presented in Section 2.4.5.5, while the spectral structure, stability limits, and campaign-specific time step selection are documented in Appendix B.3.

3.3.7 Simulation Workflow

All impact simulations follow the two-step procedure introduced in Figure 3.7. The converged geometry, stress state, and contact conditions obtained at the end of Step 1 are inherited directly by the dynamic impact analysis in Step 2.

3.3.7.1 Step 1 — Static prestress

Step 1 is a quasi-static force-controlled analysis solved with the Newton–Raphson procedure over 20 load increments. Self-weight is applied to all blocks, excluding the impactor, and the axial prestress is applied as a concentrated force at the top slab node in Campaigns 1 and 2, or at the steel plate centroid node in Campaign 3. During this step, the impactor is held in its initial position by a weak contact with negligible stiffness, as described in Section 3.3.4. The converged Step 1 state defines the initial geometry, stress state, and contact conditions for the dynamic analysis. Mass assignment and static equilibrium are verified in Appendix B.

3.3.7.2 Step 2 — Dynamic impact

Step 2 is an explicit dynamic analysis solved with the Central Difference Method. At the beginning of this step, the impactor contact stiffness is switched to the calibrated values $k_{n,\text{imp}}$ and $k_{s,\text{imp}}$, the experimental pre-impact velocity v_0 is prescribed to the impactor block, and the damping matrix $\mathbf{C}$ is assembled from the modal frequency identified after Step 1. The analysis duration is 50 ms for Campaign 1 and 100 ms for Campaigns 2 and 3.

Two time-step sizes are used:

$$\Delta t = \begin{cases} 5 \times 10^{-5} \text{ s} & \text{coarse model,} \\ 1 \times 10^{-5} \text{ s} & \text{refined model.} \end{cases} \quad (3.3)$$

Both values satisfy the CDM stability condition, as verified by the modal analysis in Appendix B.3 and the time-step sensitivity study in Appendix B, Section B.3.7.

3.3.7.3 Four-point bending simulations

The four-point bending simulations use the same static solution procedure, but without the impactor block. The dynamic impact step is replaced by a displacement-controlled loading phase, with two horizontal point loads applied at the 7th and 15th brick courses from the bottom, consistent with the experimental configuration (Godio et al., 2023b). These simulations are used only to identify the initial elastic interface stiffness K_1 , as detailed in Appendix C.

3.3.8 Modelling Assumptions

The HybriDFEM model developed in this work rests on a set of modelling assumptions that define its scope and predictive capability. These are stated explicitly here so that the results presented in Chapter 4 can be interpreted within the correct physical framework.

Two-dimensional idealisation. For point-load walls, the 2D model distributes the impactor contact over the full wall depth, whereas the physical contact is localised at the rounded tip. This produces a systematic underestimation of the contact pressure and consequently of the impact force for point-load configurations. For line-load walls, where the spreader beam covers the full wall width, this limitation does not apply. This distinction governs the interpretation of all force comparisons presented in Chapter 4.

Linear elastic contact law. The impactor–wall contact force is computed as $F = k_{n,\text{imp}} \delta$, where δ is the normal penetration. This does not capture the nonlinear Hertz-type contact mechanics of a rounded impactor tip, nor the local energy dissipation through mortar micro-cracking at the contact area. At low impact energies, where the response is contact-governed, this produces systematic force overestimation. At higher energies, where global wall dynamics dominate, the influence diminishes. This regime-dependent behaviour governs the interpretation of the force calibration results in Chapter 4.

Absence of cohesion softening. The elasto-plastic contact law does not include a post-peak softening branch for cohesion. Cohesion is either active at its prescribed value or lost instantaneously when the yield surface is reached, with no fracture energy governing the

transition. As a consequence, the cohesion assigned to mortar interfaces is kept at a low regularising value ($c = 0.01$ MPa) rather than the physical mortar cohesion, since using the true value would maintain an unrealistically high shear resistance after joint cracking. This also limits the four-point bending simulations to the pre-crack elastic regime.

Post-impact wall rebound and arching force. As noted in Section 2.4.3.3, the original `ElastoPlastic` law does not omit contact pairs after they reach the tensile cut-off. Opened joints may therefore continue contributing stiffness to the global assembly, restraining post-impact wall motion and keeping the arching force above the pre-impact baseline. The `ElastoPlastic_CD` extension developed in this thesis addresses this by adding contact deletion and recovery, while retaining the original return-mapping procedure for closed contacts. Re-runs with this extension indicate improved post-impact rebound and arching-force unloading. However, since the primary calibration was performed with the original `ElastoPlastic` law, a full recalibration of `ElastoPlastic_CD` lies outside the present scope; it is presented as a proposed extension rather than the primary calibrated model. Its implementation, verification against the Griffith wall benchmark, and Campaign 3 re-runs are reported in Appendix F.

Horizontal boundary stiffness in Campaign 3. The value $K_{\text{sup},x} = 35$ kN/mm is a modelling assumption that has not been independently validated through dynamic identification, in contrast to the Campaign 2 value of $K_{\text{sup},x} = 1$ kN/mm, which was confirmed against the SIII modal campaign (Godio et al., 2024). The Campaign 3 rig introduced eccentrically positioned steel rods, which are expected to provide a substantially stiffer lateral restraint than the Campaign 2 configuration. The adopted value reflects this increased stiffness while retaining some lateral flexibility, consistent with the observed wall kinematics. Independent validation of $K_{\text{sup},x}$ through dynamic identification remains a subject for future work.

First-impact modelling only. The model represents each wall in its undamaged initial state and simulates only the first impact event. Some walls in the experimental campaigns were subjected to multiple consecutive impacts; the accumulated damage from subsequent impacts is not represented.

3.4 ABAQUS Finite Element Model

3.4.1 Overview

The Abaqus model represents each masonry wall as a three-dimensional assembly of deformable continuum brick elements interacting through surface-based contact formulations. In contrast to the HybriDFEM model described in Section 3.3, which treats masonry units as rigid blocks in a two-dimensional framework, the Abaqus model resolves stress and strain fields within individual brick units and represents the three-dimensional contact geometry between brick surfaces explicitly. The two frameworks therefore approach the same physical problem from complementary perspectives, and their comparison provides insight into the capabilities and trade-offs of discrete versus continuum modelling strategies for masonry under impact loading. All simulations were performed using Abaqus/Explicit 2024 or 2024-HF2.

The same framework is applied to all three experimental campaigns, with campaign-specific

adaptations to geometry, mass, prestress, boundary conditions, and impactor configuration. Two simulation types are defined, corresponding to the two experimental configurations: the four-point bending test and the impact test. The four-point bending simulations are used to calibrate the material and interface parameters, which are subsequently applied without modification to the impact simulations. In the impact simulations, only the interface damping is refined against the experimental response.

The modelling decisions are described in the following sections, covering geometry and mesh, material parameters, contact interface parameters, boundary conditions, impactor modelling, and simulation workflow. Assumptions and limitations are discussed in Section 3.4.8. All modelling choices, parameter definitions, and verification studies are documented in detail in Appendices G–J.

3.4.2 Geometry and Mesh Discretisation

3.4.2.1 Wall geometry

The masonry wall is represented as a three-dimensional assembly of deformable solid bricks arranged in a Flemish bond pattern. Each brick is modelled with expanded dimensions that account for half the thickness of the adjacent mortar joints on each face, consistent with the meso-scale modelling convention described in Section 2.3.1. For the double-wythe walls used in this study, two brick types are defined:

- **Stretcher:** dimensions $260 \times 73 \times 122.5$ mm ($L \times H \times t$)
- **Header:** dimensions $130 \times 73 \times 245$ mm ($L \times H \times t$)

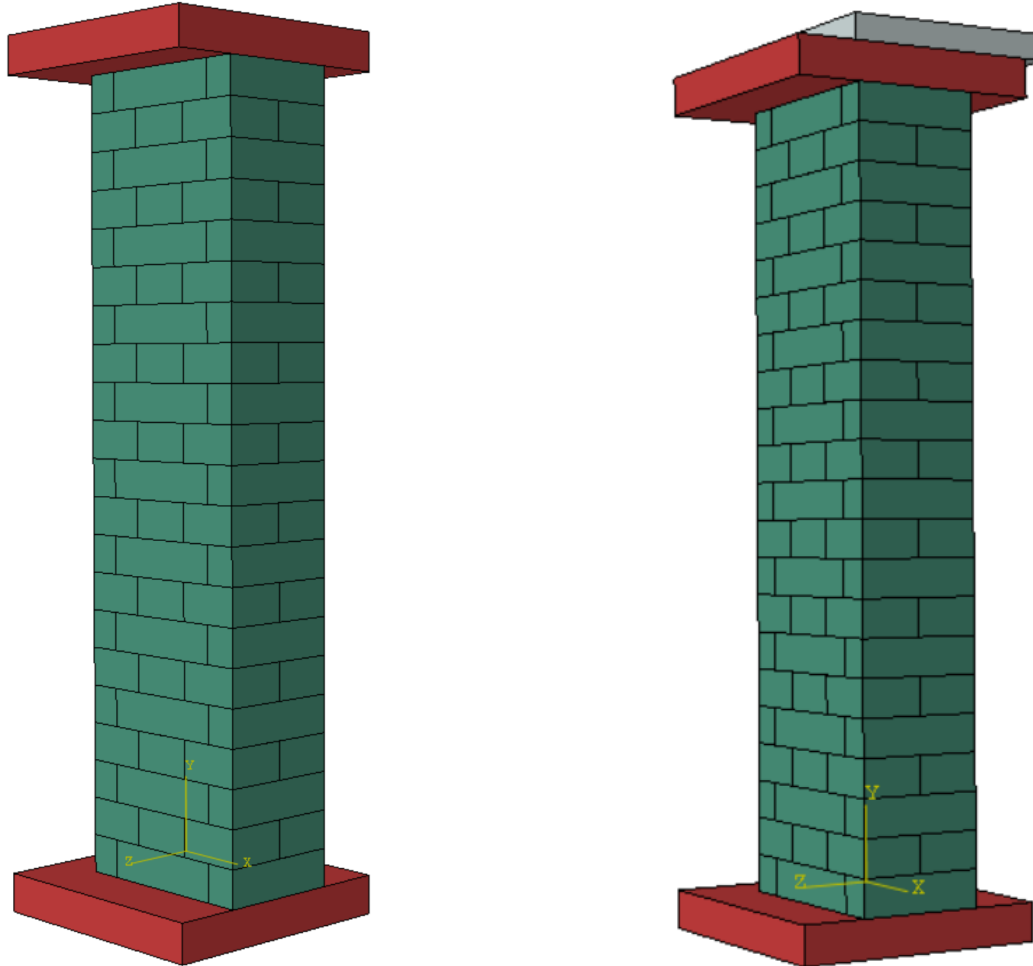
At the lateral extremes of the wall, bricks half Stretchers and Headers are assembled to accommodate the Flemish bond pattern within the wall boundaries. As described in Section 3.1.1, the wall height differs slightly between campaigns, ranging from 1.525 m to 1.548 m. Since these differences are small and creating separate geometric assemblies for each campaign is particularly demanding in a three-dimensional continuum model, a uniform wall height of 1.535 m is adopted for all simulations. This simplification is considered acceptable given the small magnitude of the variation. Due to the symmetry of the wall geometry and loading about the vertical mid-plane parallel to the wall thickness, only half the wall width is modelled ($L_w/2 = 390$ mm). Symmetry boundary conditions are applied at the cut plane, reducing the total number of elements and the associated computational cost significantly.

3.4.2.2 RC slabs and steel assembly

The top and bottom reinforced concrete slabs are modelled as deformable solid parts with dimensions $490 \times 75 \times 500$ mm, corresponding to half the full slab width. For the four-point bending tests and Campaign 1 and 2 impact simulations, the steel plate and loading beam assembly is not modelled as a separate part. Its mass is instead incorporated into the adjusted density of the top RC slab, so that the total slab mass accounts for both the reinforced concrete slab and the steel assembly above it.

For Campaign 3, the top RC slab density accounts for the slab and reinforcement only. The steel plate is modelled as a separate deformable solid part placed immediately above the top RC slab, and its density is adjusted to account for the combined mass of the loading beam and all other components of the assembly above the slab. The eccentricity of 80 mm from

the wall centreline is captured through the geometric position of the steel plate part in the assembly. The steel plate is connected to the top RC slab through a tie constraint, ensuring that the two parts move as a coupled unit.



(a) 4PB and Campaign 1 and 2: Wall and RC Slab assembly.

(b) Campaign 3: Wall, RC Slab and Steel plate assembly.

Figure 3.12: Full wall assembly in the Abaqus model for the two modelling configurations.

3.4.2.3 Mesh strategy

All brick units, RC slabs, and the steel plate are meshed using eight-node hexahedral elements with reduced integration and hourglass control (C3D8R), following indications in the literature in Section 2.6.1. The choice of element size is governed by two competing requirements: sufficient spatial resolution to capture the dominant physical mechanisms, and a manageable computational cost given the three-dimensional nature of the model and the large number of explicit time increments required for impact simulations.

Four-point bending simulations. For the four-point bending simulations, a uniform mesh with an element size of 40 mm is applied throughout the wall. With a double-wythe wall thickness of 245 mm, this gives approximately six elements through the thickness, which is

sufficient to resolve the bending stress distribution and the formation of the plastic hinge at the compressive face of the wall. The same element size is applied in the in-plane directions, giving a regular brick-shaped mesh that ensures consistent integration accuracy across the model. The adequacy of this mesh size is discussed in Section 4.3.

Impact simulations. For the impact simulations, a non-uniform mesh strategy is adopted to balance accuracy in the critical region with computational efficiency in the rest of the model. Away from the impact zone, a coarse mesh with an element size of 80 mm is applied. This coarser discretisation is appropriate in regions where the response is dominated by global bending and rocking kinematics rather than localised stress concentrations, and it significantly reduces the total element count and the number of explicit time increments required to complete the simulation.

In the region surrounding the impact point, the mesh is refined to an element size of 15 mm. This finer discretisation is necessary to resolve the steep stress and strain gradients that develop at the contact zone during impact, the localised brick damage and crushing in the vicinity of the impactor, and the contact force pulse which evolves over a very short time window. The extent of the refined region is defined to cover the area of the wall directly exposed to the impactor.

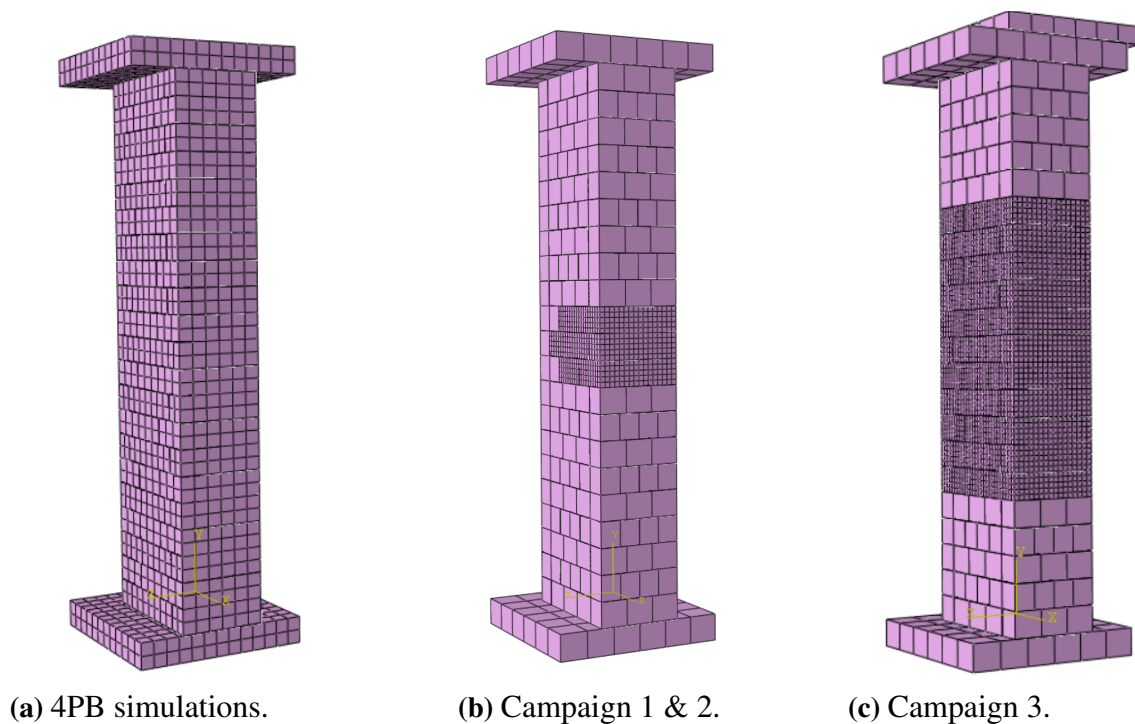


Figure 3.13: Mesh discretisation for the three simulation configurations. The refined region is visible as the denser element zone at the impact location.

3.4.3 Material Parameters

3.4.3.1 Brick units — Concrete Damaged Plasticity

The brick units are assigned the Concrete Damaged Plasticity (CDP) material model, as described in Section 2.6.3. The model is used to represent the nonlinear behaviour of the

brick–mortar system at the mesoscale, including tensile cracking, compressive crushing, and the associated stiffness degradation. The detailed strategy and choice of parameters is described in Appendix H.

Elastic properties. The elastic modulus of the brick units is set to $E_b = 7,000$ MPa, within the range of experimentally reported and literature-suggested values for clay brick units of this type (Bomben et al., 2025; Tomassetti et al., 2019). The modelling strategy adopted here fixes the brick modulus and to allow for the calibration of interface stiffness against the four-point bending tests to recover the correct global wall stiffness, consistent with the method internally adapted at RISE (Godio & Flansbjerg, 2025a). The Poisson’s ratio is set $\nu = 0.1$, within the interval of proposed ratios for hydraulic lime mortar (Drougkas et al., 2019).

CDP plasticity parameters. The shape of the yield surface and the flow rule are controlled by the following parameters:

- Dilation angle: $\psi = 15^\circ$
- Eccentricity: $e = 0.1$
- Biaxial to uniaxial compressive strength ratio: $f_{b0}/f_{c0} = 1.16$
- Deviatoric plane shape factor: $K_c = 0.667$ (Produces a triangular cross-section of the yield surface in the deviatoric plane)
- Viscosity parameter: $\mu = 0$ (not required for explicit dynamic analyses)

Uniaxial tensile behaviour. The tensile response follows the exponential softening law described in Section 2.6.3.3. The fracture energy G_t represents the energy dissipated per unit area of crack surface, corresponding to the area under the stress–crack displacement curve. The values applied in this work are:

- Tensile strength: $f_t = 0.90$ MPa
- Residual tensile strength: $f_{t,\text{res}} = 0.009$ MPa
- Tensile fracture energy: $G_t = 0.001$ N/mm

The tensile strength is derived from the measured compressive strength of the brick units following the proportionality $f_t = f_c/5$, and the near-zero fracture energy reflects the extremely brittle post-peak behaviour of the brick material under tension. The parameter values are consistent with those established in Godio and Flansbjerg (2025a) for masonry walls of identical construction and material composition, where material characterisation tests on the same brick type were used to justify these choices.

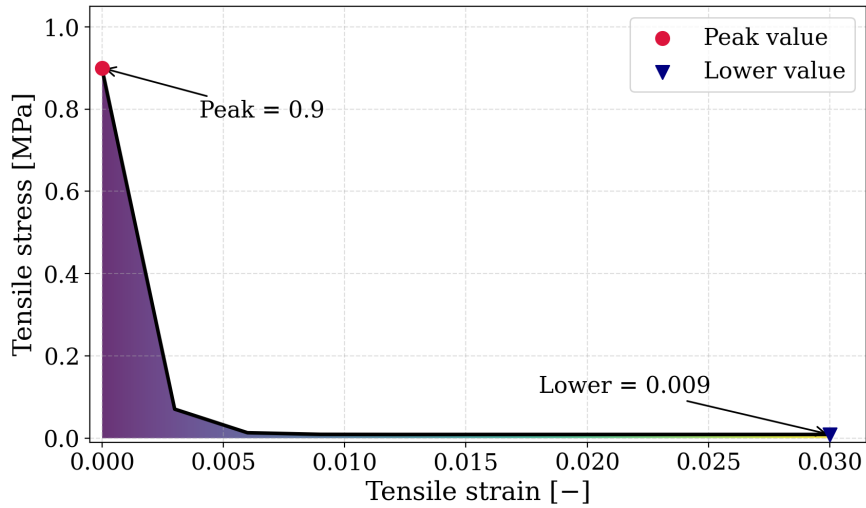


Figure 3.14: Uniaxial tensile stress–crack displacement curve adopted for the brick units, showing the exponential softening from f_t to $f_{t,res}$ governed by the fracture energy G_t .

Uniaxial compressive behaviour. The compressive response follows a parabolic-exponential law that captures the linear elastic range, a hardening branch to the peak compressive strength, and a subsequent softening branch. The values applied in this work are:

- Compressive yield initiation stress: $\sigma_i = 3.0$ MPa
- Peak compressive strength: $f_c = 4.5$ MPa
- Compressive fracture energy: $G_c = 0.116$ N/mm

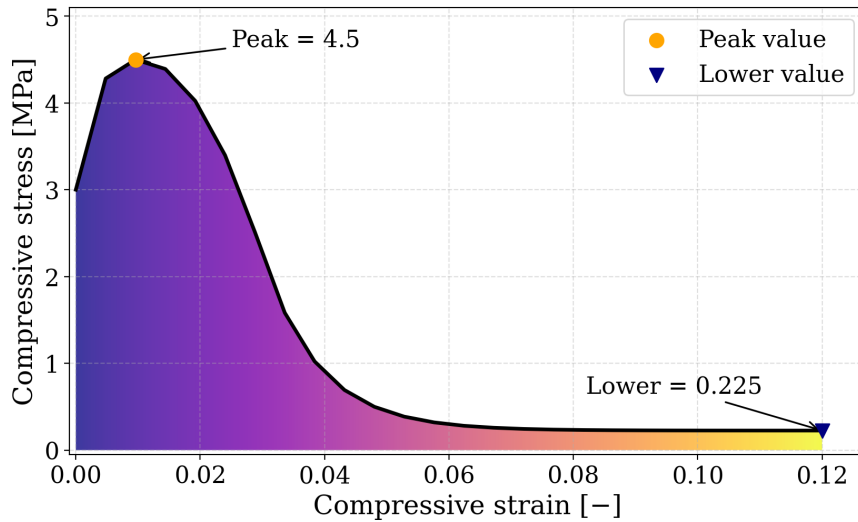


Figure 3.15: Uniaxial compressive stress–strain curve adopted for the brick units, showing the parabolic hardening branch to f_c and the subsequent exponential softening governed by G_c .

Brick damping. In an ideal formulation, no damping would be assigned to the brick material. However, the abrupt energy release associated with localised brick fracture excites high-frequency oscillations in the explicit solver that can cause numerical instabilities and

force the time increment below practical limits. A small mass-proportional damping coefficient $\alpha_M = 6.28 \times 10^{-4} \text{ s}^{-1}$ is therefore assigned to the brick material as a numerical stabilisation measure. This value has no physical correspondence to the damping properties of the brick material, and was chosen as the lowest value that maintained numerical stability across all wall configurations considered in this study.

3.4.3.2 RC slabs and steel assembly

The RC slabs are modelled as linear elastic with an elastic modulus of $E = 25,000 \text{ MPa}$ and a Poisson's ratio of $\nu = 0.15$. The reinforcement is not modelled explicitly. Instead, the slab density is adjusted to account for the total mass of the slab including reinforcement and, for the four-point bending tests and Campaign 1 and 2, the steel assembly above the top slab. The adjusted density of the top RC slab is $\rho = 9.714 \times 10^{-9} \text{ tonne/mm}^3$.

For Campaign 3, the steel plate is modelled as a separate linear elastic part with an elastic modulus of $E = 210,000 \text{ MPa}$ and a Poisson's ratio of $\nu = 0.3$. Its density is adjusted to $\rho = 2.381 \times 10^{-8} \text{ tonne/mm}^3$ to account for the combined mass of the loading beam and all other components of the assembly above the top slab.

The steel loading beam is modelled as an elasto-plastic material with an elastic modulus of $E = 210,000 \text{ MPa}$, a Poisson's ratio of $\nu = 0.3$, a yield strength of $f_y = 355 \text{ MPa}$, and a density of $\rho = 2.71 \times 10^{-9} \text{ tonne/mm}^3$. A perfectly plastic post-yield behaviour is assumed, meaning no strain hardening is introduced beyond the yield point.

3.4.4 Contact Formulation

3.4.4.1 General contact framework

All contact interactions in the model are defined within the General Contact framework in Abaqus/Explicit. This framework detects and tracks contact between all surfaces in the model at each time step, which is well suited to the complex three-dimensional geometry of the Flemish bond pattern where many brick faces interact simultaneously. The contact surfaces at brick–brick interfaces are automatically identified using the Find Contact Pairs feature in the Interaction module, and all detected pairs are assigned within the general contact domain.

All interfaces use a finite sliding formulation, which allows arbitrarily large relative tangential motion between surfaces. This is essential for impact simulations, where bricks in the vicinity of the impact zone may undergo significant relative displacements with respect to adjacent units.

3.4.4.2 Normal behaviour: linear pressure–overclosure

The normal behaviour at all brick–brick interfaces is defined using a linear pressure–overclosure relationship, in which the contact pressure increases linearly with the interpenetration between the surfaces according to a prescribed contact stiffness. This formulation introduces a controlled compliance in the normal direction, serving two purposes: it provides a physically motivated representation of the deformability of the mortar layer under compression, and it avoids the numerical interlocking artefacts associated with a perfectly rigid hard-contact formulation in out-of-plane bending simulations.

3.4.4.3 Tangential behaviour: cohesive damage frictional law

The tangential and tensile behaviour at brick–brick interfaces is described by the cohesive damage frictional contact model, as introduced in Section 2.6.4.2. The model combines cohesive damage with Coulomb friction and governs the complete lifecycle of a mortar joint from the intact elastic state through progressive damage, loss of cohesion, and residual frictional sliding.

The interface parameters entering this law are:

- Normal and tangential stiffnesses: $k_n = 12$, $k_s = 5.5$, $k_t = 5.5$
- Tensile strength: $f_t = 0$
- Cohesion: $c = 0.7$
- Mode I and Mode II fracture energies: $G_{f,I} \approx G_{f,II} \approx 0$ (elastic-brittle behaviour)
- Friction coefficient: $\mu = 0.8$

The fracture energies are set close to zero, modelling elastic-brittle behaviour at the joint in both tension and shear. This is physically motivated by the observed behaviour of the mortar joints, which fail in a sudden and brittle manner. For the tangential direction, this assumption is justified by the small influence of the mode II fracture energy on the overall wall response, which is dominated by the friction coefficient once cohesion is lost.

Interface damping. A contact damping coefficient is assigned to all mortar joint interfaces to prevent high-frequency oscillations that arise when damage propagates suddenly across the interface:

- Quasi-static simulations: damping coefficient = 0.4
- Dynamic impact simulations: damping coefficient = 0.01

Near-zero interface damping during the dynamic phase is consistent with standard practice for impact simulations, where the early oscillations are physically meaningful and should not be artificially suppressed. The small non-zero value of 0.01 is retained to prevent numerical penetration through the contact interfaces and is the lowest value feasible value allowed by the simulation, all values below this threshold failed to complete the analysis.

3.4.4.4 Impact–wall contact

The contact between the impactor and the wall front face is defined as a separate surface interaction within the general contact framework, with different properties from the brick–brick interfaces:

- Normal behaviour: hard contact (no penetration, no tensile force transmission)
- Tangential behaviour: Low friction

3.4.5 Boundary Conditions

The boundary conditions applied to the wall model reflect the physical restraints provided by the test rig at RISE. Both the bottom and top boundaries are adapted for each campaign to match the successive changes in the experimental setup.

3.4.5.1 Bottom boundary

The bottom RC slab rests directly on the strong floor and is bolted to the support structure. In all three campaigns, the bottom face of the lower RC slab is fully fixed in all translational and rotational degrees of freedom.

3.4.5.2 Top boundary — axial spring

The top RC slab is connected to the reaction frame through pre-tensioned steel rods, which provide a non-rigid axial support condition. In the model, the axial stiffness of this system is represented by a connector element (CONN3D2) acting as a linear elastic spring in the vertical direction between a reference point coupled to the top slab and a fixed node above. This formulation reproduces the non-rigid boundary condition defined in the experimental programme (Godio et al., 2023b). The spring stiffness is set to:

- Campaigns 1 and 2 (two M16 rods): $K_{\text{sup,y,half}} = 15.7 \text{ kN/mm}$
- Campaign 3 (four M16 rods): $K_{\text{sup,y,half}} = 7.45 \text{ kN/mm}$

3.4.5.3 Top boundary — horizontal restraint

The horizontal restraint at the top of the wall differs between campaigns, following the same reasoning described for HybriDFEM in Section 3.3.3. For Campaign 1, the top slab is fully fixed in the horizontal direction ($U_x = 0$), consistent with the stiff clamping condition of the Campaign 1 rig. For Campaign 2, a horizontal connector spring with stiffness $K_{\text{sup,x}} = 1 \text{ kN/mm}$ is introduced to represent the finite lateral compliance of the Teflon plate and strapping system. This value was identified through modal identification in the HybriDFEM framework and adopted directly for the Abaqus model, as described in Section 4.3. For Campaign 3, a stiffer horizontal spring with $K_{\text{sup,x}} = 35 \text{ kN/mm}$ is adopted as a modelling assumption representing the near-rigid connection of the Campaign 3 rig, consistent with the value used in HybriDFEM.

3.4.6 Impactor Modelling

The impactor is modelled as a rigid body whose geometry reproduces the physical impactor used in each experimental campaign. The impactor geometry is assigned a rigid body constraint via a reference point at its centroid (located at the back surface of the impactor). Rather than applying the mass directly at the reference point, the impactor density is adjusted to match the total impactor mass for each test configuration, allowing the different mass configurations used across the test programme to be reproduced within the same geometric model.

For the point-load configuration in Campaign 1 and the point-load walls in Campaign 2, the impactor is represented with a hemispherical head consistent with the physical impactor geometry.

For the line-load walls in Campaign 2, the same hemispherical impactor is used, but a steel spreader beam is additionally modelled in front of the wall. The impactor strikes the spreader beam, which in turn distributes the load as a line load across the full width of the wall. The spreader beam is modelled as a deformable part to capture any local yielding that may occur during impact.

For Campaign 3, the impactor is represented as a rectangular steel beam spanning the full

width of the wall at the two impact lines at $H/3$ and $2H/3$, consistent with the experimental configuration described in Section 3.1.3.

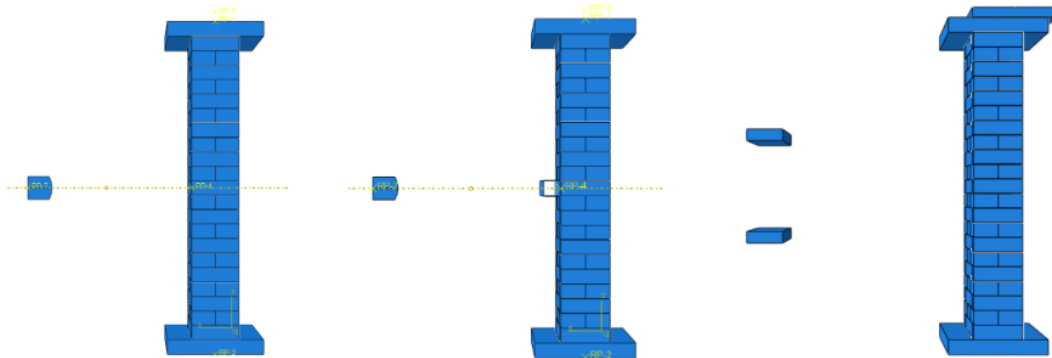


Figure 3.16: Impactor configurations across the three experimental campaigns: (a) Campaign 1, single hemispherical point-load impactor at mid-height; (b) Campaign 2, hemispherical impactor with steel spreader beam for the line-load configuration, both at mid-height; (c) Campaign 3, two rectangular steel beam impactors at $H/3$ and $2H/3$ of the wall height.

3.4.6.1 Initial conditions

The impactor is positioned at a distance from the wall surface determined by the required pre-impact velocity and the available acceleration distance for each test. At the beginning of Step 3, the experimentally measured pre-impact velocity v_0 is prescribed as gravity load scaled to the quantity needed for the impactor to reach a specific velocity during a step of the length 0.04-0.06s, this is further explained in Appendix G.

3.4.7 Simulation Workflow

All simulations follow a sequential multi-step procedure in which each step inherits the full deformed state and stress conditions from the previous one. Two simulation types are defined: the four-point bending simulation, comprising three steps, and the impact simulation, comprising four steps. The procedure is illustrated in Figure 3.17 and the individual steps are described in the following subsections.

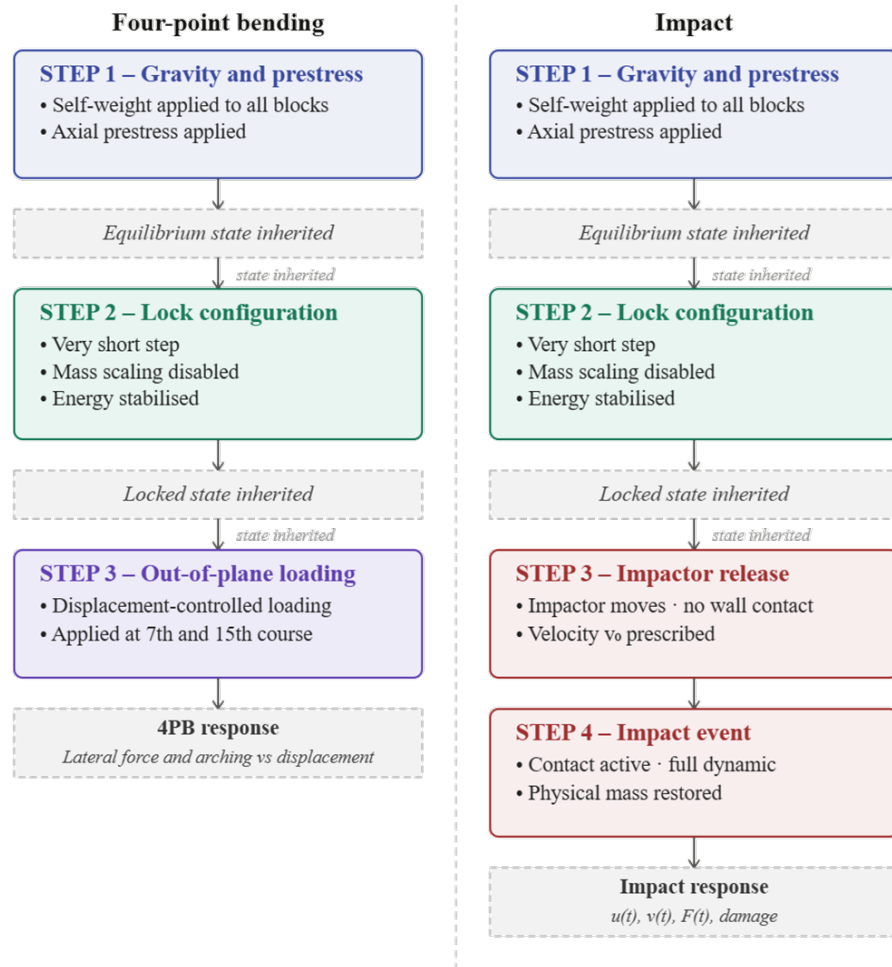


Figure 3.17: Sequential simulation procedure for the four-point bending (left) and impact (right) simulations. Shared steps are shown in teal, four-point bending specific steps in purple, and impact specific steps in coral.

3.4.7.1 Step 1 — Gravity and prestress

Step 1 applies self-weight to all structural components and the axial prestress force at the top boundary. Mass scaling of 10^5 is applied throughout this step to reduce the number of explicit increments required to reach static equilibrium. The step runs over a duration of 10-20 s. The converged state at the end of Step 1 is inherited directly by Step 2.

3.4.7.2 Step 2 — Lock configuration

Step 2 is a very short step of duration 0.0004 s whose purpose is to transition from the mass-scaled quasi-static state of Step 1 to the physical dynamic state required for the subsequent loading steps. Mass scaling is disabled and the physical material densities are restored. No new loads or boundary conditions are introduced in this step.

3.4.7.3 Step 3 — Out of plane load application (4PBT)

Step 3 applies the out-of-plane displacement-controlled load at the 7th and 15th courses of the wall, corresponding to the two loading lines used in the four-point bending tests (Godio et al., 2023b). The load is applied incrementally over a step time of approximately 150 s,

which is sufficient to reach the maximum prescribed displacement while maintaining quasi-static conditions throughout the analysis.

3.4.7.4 Step 3 — Impactor release (Impact simulations)

Step 3 for the impact simulations drives the impactor toward the wall by applying a scaled gravity load to the impactor in the horizontal direction. The magnitude of the gravity scaling factor and the duration of the step are chosen together so that the impactor reaches the experimentally measured pre-impact velocity v_0 at the wall face. No contact is activated between the impactor and the wall surface during this step. A time scaling factor of 50 is applied to reduce the computational cost. The step duration varies from 0.06 s for Campaign 1 and 2, and 0.04 s for Campaign 3. This has no physical effect on the simulation, as the gravity scaling factor is adapted to reach the prescribed velocity within the prescribed step time.

3.4.7.5 Step 4 — Impact event

Step 4 activates the contact between the impactor and the wall and integrates the equations of motion over the full impact event, including the contact phase, rebound, and post-impact oscillation. A time scaling factor of 50 is used to reduce the computational cost. The integration window is 0.05–0.09 s depending on the length needed for each specific wall in order to be compared with the experimental results. As Abaqus simulations take a long time to run, approximately 6 hours per 0.01 s (for the third Campaign), some analyses were terminated earlier, including only the required observation window.

3.4.8 Modelling Assumptions

The Abaqus model rests on a set of modelling assumptions that define its scope and determine the boundaries of its predictive capability. These are stated here so that the comparisons presented in Chapter 4 can be interpreted within the correct physical framework.

Three-dimensional continuum idealisation. The use of three-dimensional deformable solid elements for the brick units allows the model to resolve stress and strain fields within individual bricks, capture three-dimensional contact geometry, and reproduce brick splitting through the CDP damage model. However, the three-dimensional model is significantly more computationally expensive than the two-dimensional HybriDFEM model. A single impact simulation requires 5–40 hours on a single CPU core, compared to minutes for HybriDFEM, which limits the number of parametric variations that can be explored.

Mortar not explicitly modelled. In the meso-scale approach adopted here, the mortar joints are not discretised as separate continuum elements. Their mechanical contribution is represented entirely through the contact interface law. This means that mortar crushing cannot be captured explicitly: compressive inelastic behaviour at the joint is instead represented through the CDP model of the adjacent brick elements, which is a simplification.

Strain-rate effects. The material properties assigned to the brick units and mortar joints are based on quasi-static experimental data and do not include strain-rate dependent strengthening. Under impact loading, strain rates in masonry structures can reach 10^2 s^{-1} (Hao & Tarasov, 2008), at which both brick and mortar exhibit measurably higher strengths due to

dynamic increase factors (DIF). The omission of strain-rate effects means that the model may underestimate the peak impact force and resistance, particularly at the higher impact velocities of Campaigns 2 and 3.

Half-model symmetry. The use of a half-model exploits the geometric and loading symmetry of the wall. This is valid as long as the deformation mode and failure pattern are symmetric about the mid-plane. For point-load impacts where the contact is centred on the wall, this assumption is well justified. For configurations where asymmetric damage develops, the symmetry assumption may limit the model's ability to capture the full failure pattern. In the experimental campaigns considered in this study, the observed deformation profiles and damage patterns were predominantly symmetric about the wall centreline, supporting the validity of the half-model assumption for the configurations tested.

4 Model Assessment and Results

This chapter presents the assessment of the two numerical frameworks developed in this work: the HybriDFEM and Abaqus model. Both frameworks are assessed independently against the same experimental dataset from the three RISE impact campaigns, following a common structure organised into three sequential stages.

The first stage, *model verification*, establishes that each numerical model behaves in a physically coherent manner before any parameter identification is performed. This includes checks on geometry, mass, prestress, boundary conditions, discretisation, solver stability, and static response. Both models are verified independently, as illustrated in Figure 4.1.

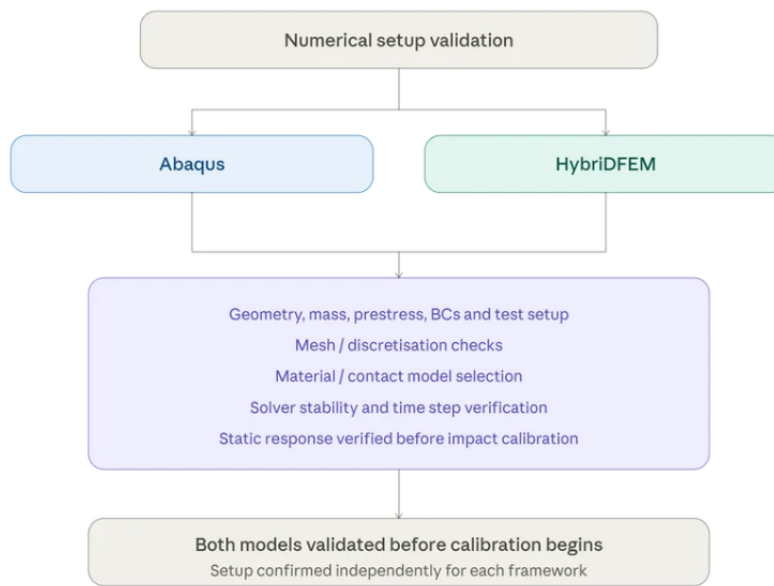


Figure 4.1: Numerical setup verification strategy.

The second stage, *calibration*, identifies the model parameters by comparison with experimental observations. The two frameworks follow different calibration paths, as illustrated in Figure 4.2. For HybriDFEM, interface stiffness is first identified from quasi-static four-point bending tests and independently cross-validated against experimentally identified modal frequencies; impact-specific parameters are subsequently calibrated from the Campaign 1 and 2 impact data. For Abaqus, a richer set of material parameters (including elastic moduli, interface stiffnesses, fracture energies, and friction) is calibrated from the four-point bending tests, and damping is subsequently refined from the impact campaigns. In both cases, the calibration follows a sequential strategy in which each parameter is identified from the observable it governs, and the fixed parameters from one stage are not re-adjusted in subsequent ones.

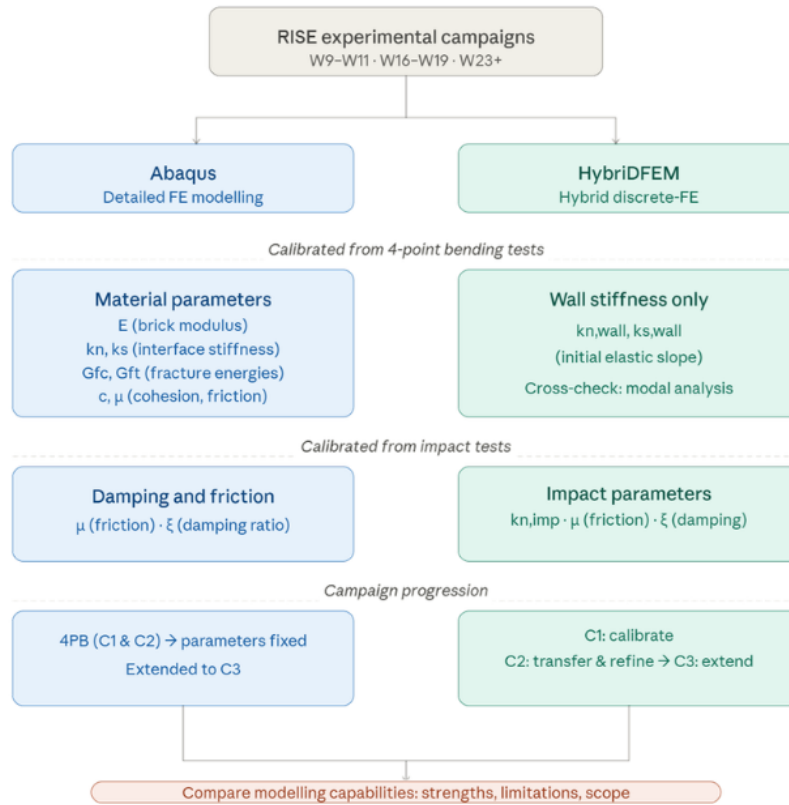


Figure 4.2: Calibration strategy for HybriDFEM and Abaqus.

The third stage, *parameter transfer*, applies the calibrated parameters to Campaign 3 without re-fitting. Campaign 3-specific changes are limited to the experimental differences in geometry, mass, prestress, impactor configuration, and support representation. The comparison is therefore treated as a predictive assessment of parameter transferability rather than as a new calibration stage.

The assessment of each framework concludes with a discussion of parameter transferability, identified scope boundaries, and modelling recommendations. A comparative analysis of the two frameworks against the same experimental dataset is presented in Section 4.5.

4.1 HybriDFEM — Model Verification

Model verification establishes that the HybriDFEM model correctly represents the intended physical system before any parameter identification is performed. The checks are organised into six areas: mass and static equilibrium, contact pair discretisation, modal analysis and stability limits, time step sensitivity, mesh sensitivity, and boundary condition selection. Unless otherwise stated, results are shown for wall W18 (Campaign 2, refined 3×6 model), which is representative of the full model complexity. Equivalent checks for Campaign 1 (W10) and Campaign 3 (W27) are reported in Appendix B.

4.1.1 Mass and Static Equilibrium

After Step 1, the computed base reaction is compared against the expected value $F_{y,\text{base}} = W_{\text{self}} + F_{\text{axial}}$ for representative walls from each campaign.

Table 4.1: Mass and static equilibrium check after Step 1.

	W10 (C1)	W18 (C2)	W27 (C3)
m_{wall} (kg)	499.9	497.2	538.4
m_{slabs} (kg)	190.0	230.0	174.9
$m_{\text{plate+beam}}$ (kg)	262*	262*	460 [†]
W_{self} (kN)	6.767	7.134	11.510
F_{axial} (kN)	40.545	12.076	19.836
$F_{y,\text{expected}}$ (kN)	47.313	19.210	31.346
$F_{y,\text{computed}}$ (kN)	47.313	19.210	31.346
$\ \mathbf{R}_{\text{free}}\ $ (N)	3.3×10^{-10}	4.3×10^{-10}	1.4×10^{-9}

*Applied as static concentrated force; not modelled as explicit block.

[†]Modelled as explicit rigid block at eccentric position (+80 mm); weight included in W_{self} .

The computed reaction matches the expected value exactly in all three cases and residual norms are below 2×10^{-9} N, confirming correct mass assignment, load application, and solver convergence. Full details are reported in Appendix B.

4.1.2 Contact Pair Sensitivity

The sensitivity of the global response to the number of contact pairs per contact face (N_{CP}) is assessed on wall W10 using pre-calibration working parameters. Five values are tested: $N_{\text{CP}} \in \{2, 4, 8, 12, 16\}$.

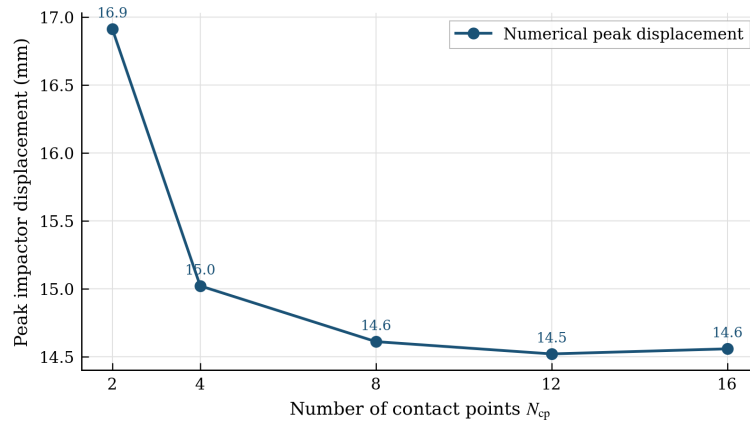


Figure 4.3: Peak impactor displacement as a function of N_{CP} , wall W10 (Campaign 1, coarse model). The response is converged for $N_{\text{CP}} \geq 8$.

The response is converged for $N_{\text{CP}} \geq 8$, with a change of less than 0.3% in peak displacement between $N_{\text{CP}} = 8$ and 16. The value $N_{\text{CP}} = 8$ is therefore adopted for all coarse-model interfaces. Force, velocity, and displacement histories showing the full superposition of curves for $N_{\text{CP}} \geq 8$ are provided in Appendix B. For refined interfaces, $N_{\text{CP}} = 2$ is used without a dedicated study, since the sub-block faces are geometrically small and the spatial resolution near the impact zone is provided by the band refinement itself.

4.1.3 Modal Analysis, Stability Limits, and Stiffness Regularisation

The modal analysis after Step 1 serves three purposes: determining $\Delta t_{\text{crit}} = 2/\omega_{\text{max}}$, computing $\alpha_K = 2\xi/\omega_1$ for the damping formulation (Section 2.4.5.4), and cross-validating the interface stiffness and boundary condition parameters against experimentally identified frequencies.

4.1.3.1 Spectral structure and stability limits

Table 4.2 reports system sizes and CDM stability limits for wall W18 at three discretisation levels.

Table 4.2: System sizes and CDM stability limits, wall W18. $\omega_{\text{max}}(\mathbf{K}_0) = \omega_{\text{max}}(\mathbf{K}^{\text{tan}})$ at all levels after regularisation (Section 4.1.3.2).

Model	Free DOFs	ω_{max} (rad/s)	Δt_{crit} (s)	$\Delta t_{\text{adopted}}$ (s)
Coarse	95	2.38×10^4	8.40×10^{-5}	5×10^{-5}
Refined 3×6	242	1.60×10^5	1.25×10^{-5}	1×10^{-5}
Refined 5×10	530	2.87×10^5	6.97×10^{-6}	—

The adopted time steps carry safety factors of $1.68\times$ (coarse) and $1.25\times$ (refined 3×6) relative to Δt_{crit} . The decrease with refinement is super-linear because ω_{max} is governed by localised sub-block interface modes at the impact zone rather than by global wall dynamics. This comes directly from $K_5 \propto E/\Delta x_{\text{sub}}$ (Appendix A), finer sub-blocks are narrower and therefore stiffer, raising ω_{max} independently of the global response. The full scaling across levels is documented in Appendix B, Table B.3.

Figure 4.4 illustrates the contrast between global and stability-critical modes. Mode 1 is the cantilever mode and Mode 2 is the first vertical bending mode; both govern the cross-validation in Section B.3.5. The ω_{max} mode is a highly localised deformation confined to the sub-blocks at the impact row.

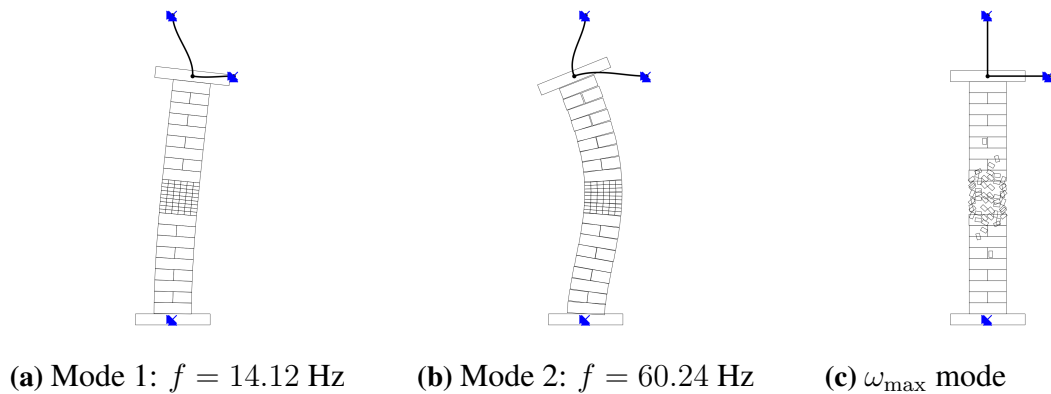


Figure 4.4: Mode shapes, refined 3×6 model, $\mathbf{K}_0$, wall W18.

Force-based participation analysis at the impact DOF confirms that 90% of the impact participation is concentrated in modes below ω_{max} , with the stability-critical mode carrying negligible energy content (Appendix B, Section B.3.4). This does not imply that a larger

time step could be used safely. The CDM integrates the full system at every step without modal truncation, so exponential growth in the $\omega_{\max}$ component occurs regardless of its energy content once Δt_{crit} is exceeded. The stability limit is a hard mathematical constraint. This is demonstrated in Section 4.1.5, and the choice of 3×6 over 5×10 is justified in Section 4.1.6.

4.1.3.2 The $\mathbf{K}^{\text{tan}}$ pathology and its resolution

With zero cohesion ($c = 0$), the tangent stiffness matrix $\mathbf{K}^{\text{tan}}$ becomes numerically ill-conditioned after Step 1. The problem originates at vertical head joints in alternating Flemish bond courses, where the normal stress is close to zero under gravity. The Mohr–Coulomb threshold then reduces to $|\tau| \leq \mu\sigma_N \approx 0$, so small shear stresses place these contacts on the yield surface, which produces a semi-definite $\mathbf{K}^{\text{tan}}$ and corrupts the modal participation results, as shown in Figure 4.5a. This issue had not been previously documented in the HybridFEM literature and was identified in the present work as a consequence of the Flemish bond geometry combined with zero cohesion.

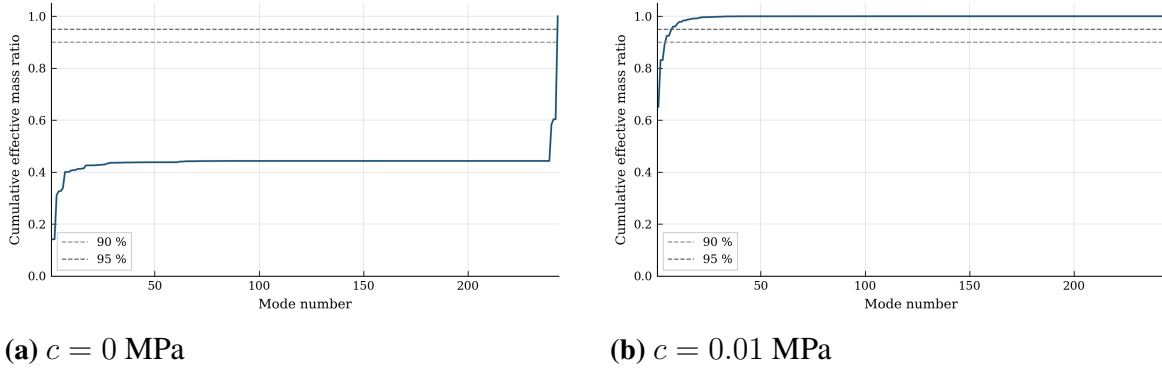


Figure 4.5: Global U_x cumulative mass participation under $\mathbf{K}^{\text{tan}}$, refined 3×6 model, wall W18.

A small regularising cohesion, $c = 0.01$ MPa, removes the numerical pathology without affecting the mechanical response. The modal participation curve becomes smooth, the Step 1 residual norm remains unchanged, and $\omega_{\max}(\mathbf{K}_0) = \omega_{\max}(\mathbf{K}^{\text{tan}})$ at all discretisation levels. Full details are provided in Appendix B, Section B.3.3.

4.1.3.3 Cross-validation against experimental frequencies

The first two out-of-plane natural frequencies are compared against those identified in the SIII dynamic identification campaign at RISE on wall W20 (Godio et al., 2024), which is geometrically identical to the Campaign 2 impact specimens and was tested at the same axial level ($\sigma_{\text{ax}} = 0.05$ MPa). This provides an independent cross-validation of the interface stiffness $K_1 = 17$ GPa/m, calibrated from four-point bending tests (Section 4.2.1), and the horizontal spring stiffness $K_{\text{sup},x} = 1$ kN/mm, using a different experimental campaign, different specimens, and a different measurement system. Details about the experimental campaign and results are provided in Appendix B, Section B.3.5.

Table 4.3: First two out-of-plane natural frequencies at $\sigma_{ax} = 0.05$ MPa, HybridFEM refined 3×6 model vs. SIII experimental values (Godio et al., 2024).

Mode	Source	f (Hz)
Mode 1 (cantilever)	Exp: OMA	14.07
	Exp: LVDT ODS	13.5
	Num: $\mathbf{K}_0$	14.12
	Num: $\mathbf{K}^{\tan}$	12.57
Mode 2 (vertical bending)	Exp: LVDT ODS	57.0
	Exp: LDV+LVDT ODS	50.6
	Num: $\mathbf{K}_0$	60.24
	Num: $\mathbf{K}^{\tan}$	58.71

As presented in Table 4.3, the $\mathbf{K}_0$ prediction $f_1 = 14.12$ Hz matches the OMA value of 14.07 Hz to within 0.4% and lies within the experimental scatter of 13.5–15.1 Hz. The agreement is meaningful because the cantilever mode is jointly governed by K_1 and $K_{sup,x}$. Without the horizontal spring, no cantilever mode exists below the first bending frequency and the 14.07 Hz target cannot be reached by adjusting K_1 alone. The simultaneous agreement with an independent dataset therefore cross-validates both the calibrated interface stiffness and the Campaign 2 horizontal boundary condition. The $\mathbf{K}^{\tan}$ prediction of 12.57 Hz falls below the range as expected, since partial yielding of vertical head joints under self-weight softens the tangent stiffness, making $\mathbf{K}^{\tan} \leq \mathbf{K}_0$ and its frequencies a lower bound.

For Mode 2, the $\mathbf{K}_0$ prediction overestimates the experimental range by 3–19%. This mode could not be confirmed as a pure eigenmode by OMA (Godio et al., 2024) and the two extraction methods differ by approximately 12%, so the experimental uncertainty is substantial and the overestimation is not treated as a calibration discrepancy. Full spectral results are provided in Appendix B, Section B.3.5.

4.1.4 Campaign 3 Modal Analysis

The modal analysis is repeated for wall W23 (Campaign 3, refined 3×6), whose geometry differs from Campaign 2 through the eccentric steel plate block, lower vertical spring ($K_{sup,y} = 14.9$ vs 31.4 kN/mm), and stiffer horizontal boundary ($K_{sup,x} = 35$ vs 1 kN/mm).

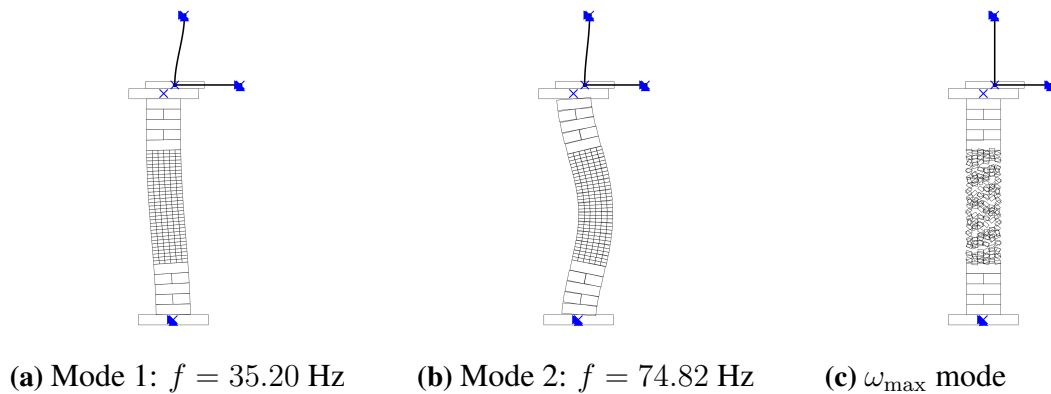


Figure 4.6: Mode shapes, refined 3×6 model, $\mathbf{K}_0$, wall W23.

The fundamental frequency, $f_1 = 35.20$ Hz, is approximately $2.5\times$ higher than the Campaign 2 value, mainly because the stiffer horizontal boundary constrains the cantilever mode. The $\omega_{\max}$ mode retains the same localised sub-block character observed in Campaign 2, giving $\Delta t_{\text{crit}} = 1.305 \times 10^{-5}$ s and a safety factor of $1.31\times$ for the adopted $\Delta t = 1 \times 10^{-5}$ s. No experimental modal cross-validation is available for Campaign 3. Full spectral results are provided in Appendix B, Section B.3.6.

4.1.5 Time Step Sensitivity

The stability and accuracy of the CDM integrator are verified for both the coarse and refined 3×6 models on wall W18 using the calibrated parameter set, testing time steps spanning from well below to above Δt_{crit} . Figure 4.7 shows the impactor displacement histories; full metrics for both models are reported in Appendix B, Tables B.10 and B.11.

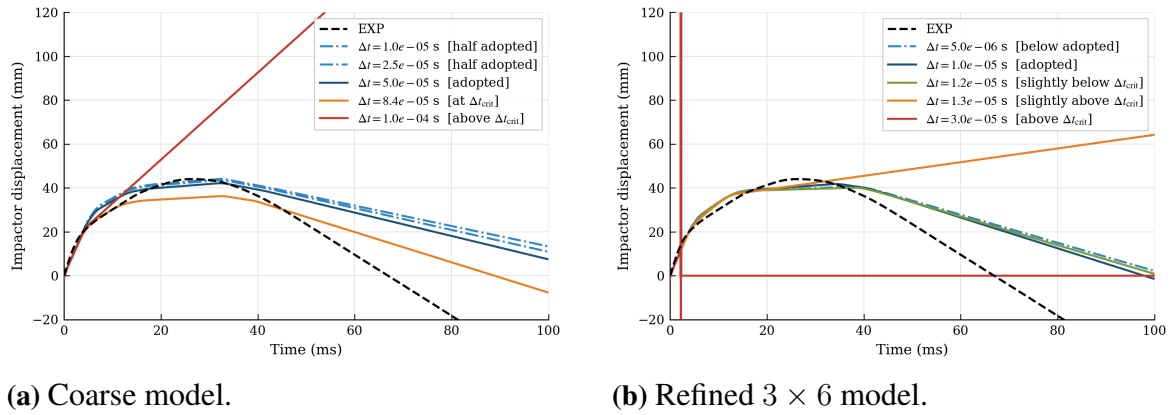


Figure 4.7: Impactor displacement histories for a range of time steps, wall W18.

Three observations follow. First, divergence takes two forms depending on how far Δt_{crit} is exceeded, far above it ($\Delta t = 3 \times 10^{-5}$ s, safety $0.42\times$ in the refined model) the solution overflows within 2.3 ms; marginally above it ($\Delta t = 1.3 \times 10^{-5}$ s, safety $0.96\times$) the instability grows slowly, reaching a peak displacement of 64.2 mm by $t = 100$ ms, this is a case that would not be detected by visual inspection at early times. Second, the stability boundary is sharp, an 8% reduction in Δt , from 1.3×10^{-5} to 1.2×10^{-5} s, separates slow divergence from a fully converged result with peak displacement of 40.1 mm. Third, all stable cases produce peak displacements within 4% of one another in both models, confirming that the adopted time steps lie well within the converged regime. Detailed metrics for all tested values and discussion are provided in Appendix B, Tables B.10 and B.11.

4.1.6 Mesh Sensitivity

The sensitivity of the impact response to the rigid block discretisation is assessed for wall W18 by comparing the coarse model, used for all parameter sweep studies, and the refined 3×6 and 5×10 models, all levels under the calibrated parameter set. Table 4.4 reports the key metrics.

Table 4.4: Mesh convergence metrics, wall W18.

Model	$\hat{u}_{\text{imp}}$ (mm)	$\hat{t}_u$ (ms)	$\hat{F}_{\text{imp}}$ (kN)	CPU cost (rel.)
EXP	44.0	26.4	341.8	—
Coarse	42.1	32.3	342.7	$0.5\times$
Refined 3×6	37.2	35.9	318.1	$1\times$
Refined 5×10	39.2	37.7	315.3	$\approx 2\times$

The coarse model reproduces the peak force to within 0.3% of the experimental value at half the computational cost of the refined 3×6 model, making it suitable for the large parameter sweeps in Sections 4.2.2 and 4.2.3. Between the two refined levels, force and velocity histories are visually indistinguishable, with a peak force difference of only 0.9%, as shown in Figure B.20. The 5×10 model is approximately twice as expensive, and the sub-block band of the 3×6 model already provides sufficient spatial resolution to represent localised mechanisms near the impact zone, including joint opening, crushing, and block separation, and finer refinement adds no meaningful new information. The 3×6 discretisation is therefore adopted for all production simulations. Full time histories figures are provided in Appendix B, Section B.3.8.

4.1.7 Horizontal Boundary Condition Selection

The horizontal boundary condition at the top slab differs between campaigns. In Campaign 1, a rigid condition ($U_x = 0$) is applied, consistent with the stiff clamping frame. In Campaign 2, the rig introduced finite horizontal compliance; the stiffness $K_{\text{sup},x} = 1$ kN/mm is identified from modal cross-validation (Section B.3.5). In Campaign 3, a near-rigid value $K_{\text{sup},x} = 35$ kN/mm is adopted as a modelling assumption in the absence of equivalent dynamic identification (Section 3.3.8).

The quantitative effect is assessed for Campaign 2 by comparing the mean DIC displacement RMSE at four monitored heights ($H/8$, $H/4$, $3H/4$, $7H/8$).

Table 4.5: Mean DIC height RMSE (mm) under both horizontal boundary conditions, Campaign 2.

Boundary condition	W16	W17	W18	W19
Spring ($K_{\text{sup},x} = 1$ kN/mm)	2.29	3.65	3.41	4.06
Rigid ($U_x = 0$)	4.21	5.35	5.22	6.54

The spring condition reduces the DIC RMSE by 1.7–2.5 mm across all four walls, with the largest improvement at the upper heights ($3H/4$ and $7H/8$) where the rigid restraint suppresses the global tilting motion. DIC height displacement histories for both conditions are shown in Appendix E, Figure E.1. This result is consistent with the modal cross-validation of Section B.3.5, both independently support the spring boundary condition.

4.2 HybriDFEM — Model Calibration

The calibration proceeds sequentially. The interface stiffness K_1 is identified from quasi-static four-point bending tests (Section 4.2.1). With K_1 fixed, impact-specific parameters

$(k_{n,\text{imp}}, \mu, \xi)$ are calibrated from Campaign 1 (Section 4.2.2) and subsequently refined for Campaign 2 (Section 4.2.3). The resulting parameter set is applied without modification to Campaign 3.

4.2.1 Interface Stiffness Calibration from Four-Point Bending Tests

4.2.1.1 Calibration objective and parameter isolation

The normal interface stiffness K_1 is calibrated from the initial slope of the lateral force–displacement curve measured in the 4PB campaign (Godio et al., 2023b), described in Section 3.1.2. In the elastic regime, this slope is governed exclusively by K_1 and the boundary conditions. μ is inactive since no joint sliding occurs, and ξ is irrelevant for monotonic quasi-static loading. The 4PB initial slope therefore provides an isolated observable for K_1 , independently of the parameters identified in the impact calibration.

4.2.1.2 Wall selection

Seven walls were tested under three support conditions and two axial stress levels. The calibration set is restricted to walls at $\sigma_{\text{ax}} = 0.20$ MPa. As discussed in Section 2.1.3, the initial lateral stiffness of URM walls increases with axial load ratio (Wilding & Beyer, 2018), but this effect is limited at low axial levels. The SIII dynamic identification campaign confirms that the lateral stiffness remains approximately constant between $\sigma_{\text{ax}} = 0.05$ and 0.20 MPa (Section C.3, Figure C.1), justifying the use of the 0.20 MPa walls to represent the 0.05 MPa impact conditions. Three walls are excluded, W7 shows an anomalously low experimental stiffness that diverges from all numerical predictions regardless of K_1 ; W8 and W12 were tested at $\sigma_{\text{ax}} = 0.40$ MPa, where the response is measurably stiffer and not representative of the impact regime. The calibration set is $\mathcal{C} = \{\text{W5, W6, W13, W14}\}$. Full exclusion justification is provided in Appendix C, Section C.3.

4.2.1.3 Stiffness evaluation method

Contact settling at very small displacements makes direct linear regression unreliable. A degree-3 polynomial constrained through the origin is fitted over the full control window $[0, u_{\text{ctrl}}]$, and the Half-1 stiffness is taken as the slope at the centre of Half-1, i.e. at $u_{\text{ctrl}}/4$. Full details are given in Appendix C, Section C.2.

4.2.1.4 Parametric sweep and result

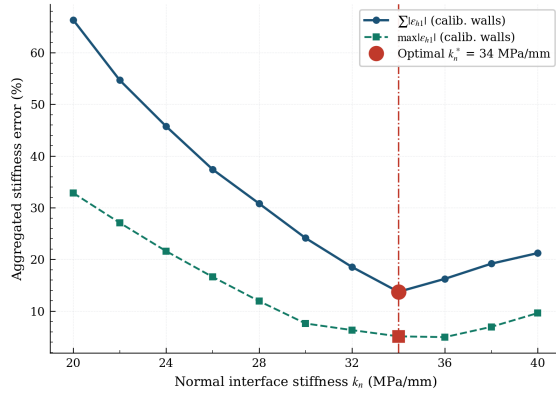
The input stiffness $k_{nn} = 2K_1$ is swept over eleven values from 20 to 40 MPa/mm, following the HybriDFEM side-spring convention (Appendix A). The optimal value minimises the sum of absolute Half 1 errors:

$$k_{nn}^* = \arg \min_{k_{nn}} \sum_{w \in \mathcal{C}} |\varepsilon_{h1}(w, k_{nn})|. \quad (4.1)$$

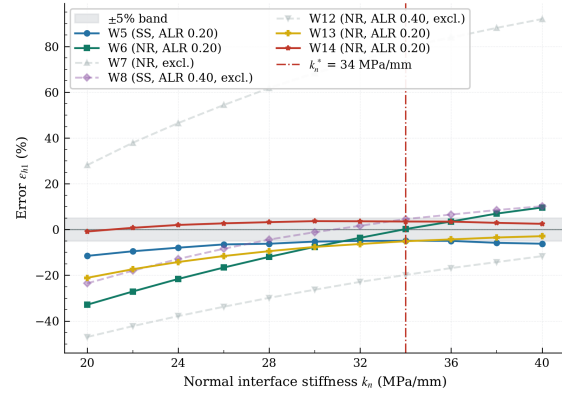
Both the sum and maximum criteria reach their minimum at $k_{nn}^* = 34$ MPa/mm, and the individual wall errors remain within the selected tolerance for all calibration walls (Figure 4.8), giving:

$$K_1 = \frac{k_{nn}^*}{2} = 17 \text{ GPa/m}. \quad (4.2)$$

All four calibration walls satisfy $|\varepsilon_{h1}| \leq 5.1\%$ at this value; the full results table is provided in Appendix C, Table C.2.



(a) Calibration criteria.

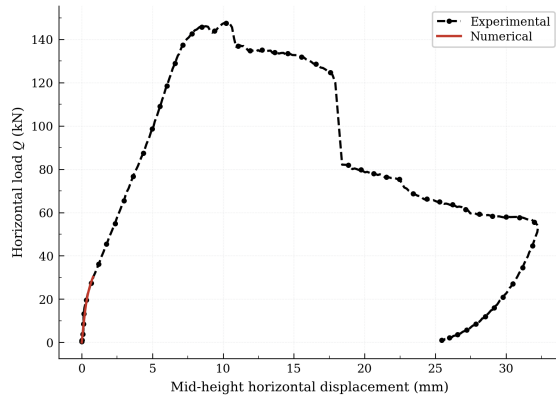


(b) Individual Half 1 errors.

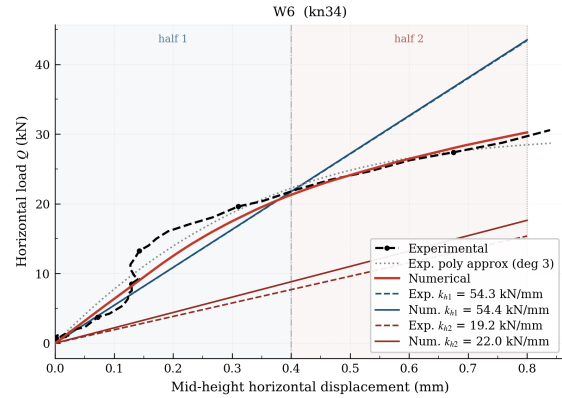
Figure 4.8: Selection of the calibrated interface stiffness. Both the aggregate criteria and the individual wall errors support $k_{nn}^* = 34$ MPa/mm.

Figure 4.9 shows the force–displacement comparison for wall W6, which has a stiff non-rigid top support ($K_{\text{sup}} = 59.9$ kN/mm) and gives the best individual agreement in the calibration set ($\varepsilon_{h1} = +0.2\%$). The numerical model captures the initial elastic stiffness very closely, confirming the selected value $K_1 = 17$ GPa/m.

The corresponding kinematic evolution is shown in Figure 4.10. The deformed shapes show a progressive out-of-plane bending mechanism, with rotations concentrated around the main bending zone and a more restrained response near the supports than in the simply supported wall W5. The softer non-rigid supports W13 and W14 produce intermediate or near-simply-supported kinematic profiles, respectively, as shown in Appendix C. The W6 result therefore confirms that the calibrated model not only reproduces the initial stiffness, but also gives boundary-condition-dependent deformation mechanisms consistent with the test configuration.



(a) Full range.



(b) Zoom $[0, 0.8]$ mm.

Figure 4.9: Force–displacement comparison, wall W6, $K_1 = 17$ GPa/m.

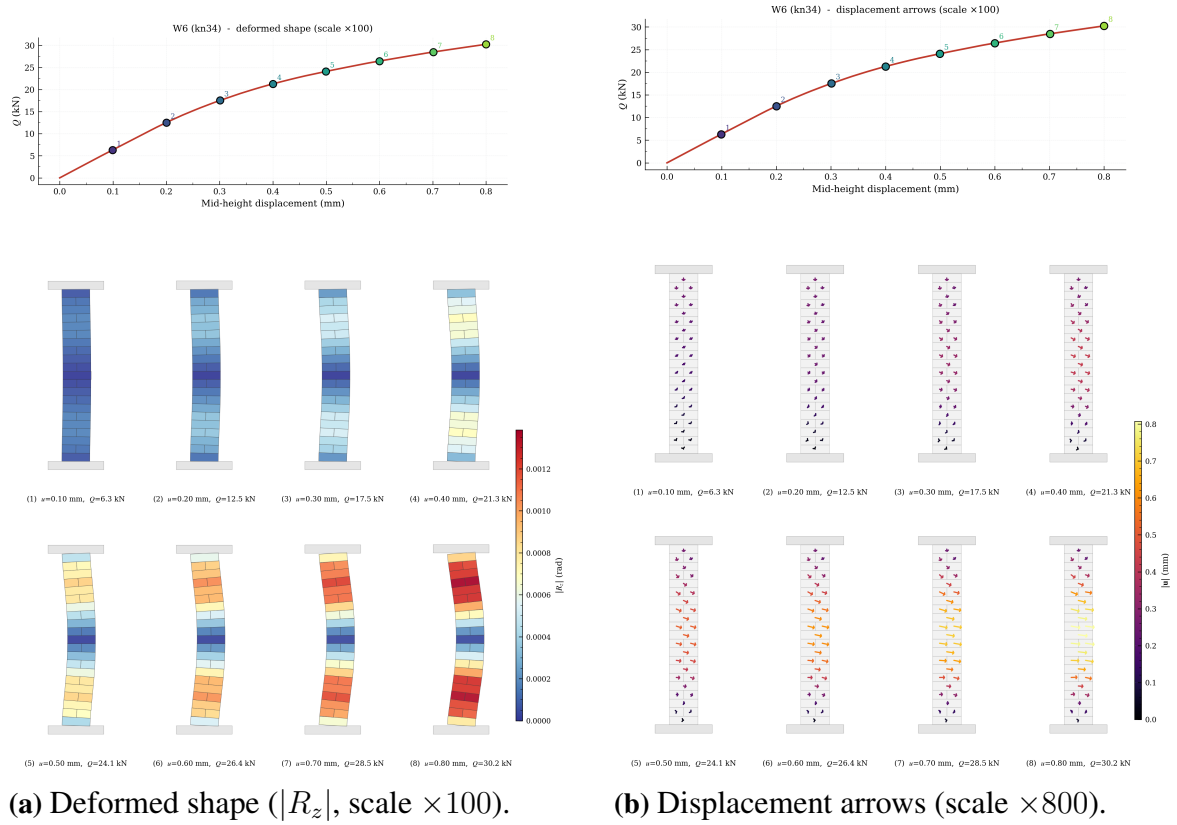


Figure 4.10: Kinematic evolution, wall W6, $K_1 = 17$ GPa/m.

At larger displacements (Half-2), the numerical response is systematically softer than the experimental one. This is a direct consequence of the $c = 0.01$ MPa regularising cohesion, which causes the model to be governed by friction alone once sliding begins, while the physical wall retains some cohesive contribution. Since the calibration target is the pre-crack elastic stiffness, this mismatch does not affect the identified value of K_1 and is accepted as a known limitation (Section 3.3.8). Full Half-2 results are in Appendix C, Section C.6.

4.2.1.5 Cross-validation

The value $K_1 = 17$ GPa/m is independently confirmed by the modal cross-validation against SIII experimental frequencies (Section 4.1.3.3), using a different experimental campaign, different specimens, and a different measurement system.

4.2.2 Campaign 1 — Impact Parameter Calibration

4.2.2.1 Calibration strategy and parameter roles

Three parameters govern the impact response and are identified sequentially. The impactor contact stiffness $k_{n,\text{imp}}$ controls the early contact phase (first ~ 5 ms), governing the force peak, force timing, and rate of momentum transfer to the wall. The friction coefficient μ becomes active once joint sliding develops from approximately $t = 5$ ms onward, controlling whether deformation remains localised near the impact row or distributes over the wall height. The damping ratio ξ affects the post-contact velocity and displacement decay for $t \geq 20$ ms. With $K_1 = 17$ GPa/m fixed from Section 4.2.1, the three parameters are identified from the impactor force, velocity, and displacement histories over the full 50 ms

observation window. DIC wall deformation profiles are used as a secondary consistency check. Combinations are ranked using a composite score aggregating displacement, velocity, and force sub-scores evaluated at $t = 10, 20, 30, 40, 50$ ms; full details of the scoring framework are given in Appendix D, Section D.3.

4.2.2.2 Experimental reference values

Table 4.6 summarises the experimental reference values used for calibration.

Table 4.6: Experimental reference values, Campaign 1.

Wall	v_0 (m/s)	F_{exp} (kN)	t_F (ms)	u_{exp} (mm)	t_u (ms)	I_{exp} (kN·s)
W9	3.13	86.2	0.90	8.14	12.88	0.302
W10	4.43	155.4	1.18	19.90	16.13	0.674
W11	4.43	163.0	0.08	22.89	20.15	0.662

W10 and W11 share the same pre-impact velocity but W11 develops a larger peak displacement due to its lower axial prestress ($\sigma_{\text{ax}} = 0.05$ MPa vs 0.20 MPa), which reduces the arching resistance and allows greater wall deflection.

4.2.2.3 Contact-controlled vs system-controlled regime

A systematic trend must be noted before presenting the sweep results. As shown in Table 4.7, the force peak overestimation decreases progressively from W9 to W11, independent of $k_{n,\text{imp}}$. This is not a calibration failure but reflects a change in the mechanism governing the contact force.

Table 4.7: Force peak ratio $F_{\text{num}}/F_{\text{exp}}$ at $\mu = 0.65$, $\xi = 0.020$ for selected $k_{n,\text{imp}}$ values.

Wall	F_{exp} (kN)	$k_{n,\text{imp}} = 2.5$ GPa/m	$k_{n,\text{imp}} = 5.0$ GPa/m
W9	86.2	1.58	2.03
W10	155.4	1.22	1.58
W11	163.0	1.02	1.35

In the *contact-controlled regime* (W9), the wall barely moves during contact and the force peak is determined almost entirely by the local contact law. The linear elastic model $F = k_{n,\text{imp}}\delta$ assigns a constant stiffness for all penetration depths, whereas in reality a spherical-tipped impactor follows a nonlinear Hertz-type response (Johnson, 1985), where stiffness increases with penetration. At the small penetration depths of W9, the real contact stiffness is low and the linear model overestimates it. Additional dissipative mechanisms at the contact region, like mortar micro-cracking, crushing, and geometric irregularities further reduce the effective stiffness in the experiment (Godio et al., 2023a), but are not present in the linear elastic formulation.

In the *system-controlled regime* (W11), the higher impact energy and lower axial prestress produce significant wall motion during contact. The force peak is then governed by the coupled interaction between impactor inertia, wall inertia, and global wall deformation, and the influence of the simplified contact law decreases. The overestimation essentially

vanishes for W11 since the wall participates fully in the dynamic response. W10 lies in a transitional regime between the two.

As a consequence, the force metric is not a reliable discriminator for W9; displacement and velocity are the primary calibration observables for that wall. For W10 and W11, where wall participation is greater, the force ratio remains a meaningful criterion. Full discussion is provided in Appendix D, Section D.4.1.

4.2.2.4 Parameter identification

An exploratory sweep of 60 combinations was performed:

$$k_{n,\text{imp}} \in \{1.5, 2.0, 3.0, 5.0, 8.0\} \text{ GPa/m}, \quad \mu \in \{0.65, 0.70, 0.75, 0.80\}, \\ \xi \in \{0.005, 0.010, 0.020\}.$$

Values $k_{n,\text{imp}} \geq 5.0$ GPa/m clearly overshoot the experimental force peak and are excluded, identifying the admissible range as $k_{n,\text{imp}} \approx 2.0\text{--}3.0$ GPa/m. Full sweep histories, scatter maps, and discussion are provided in Appendix D, Section D.4.2.

Figure 4.11 illustrates the distinct influence of each parameter on the wall deformation profiles at three time instants. At $t = 1$ ms the profiles separate by $k_{n,\text{imp}}$, confirming that contact stiffness governs the early local response before friction becomes active. At $t = 10$ ms the spread is instead governed by μ , lower friction concentrates deformation near the impact row while higher friction distributes it over the wall height. At $t = 40$ ms, the experimental profile shows clear rebound toward the initial position while all numerical profiles remain displaced. This is consistent with the arching force overestimation discussed in Section 4.2.2.7, without contact deletion, opened joints continue contributing stiffness to the global assembly, preventing the rocking recovery that gravity and arching should drive after impactor separation.

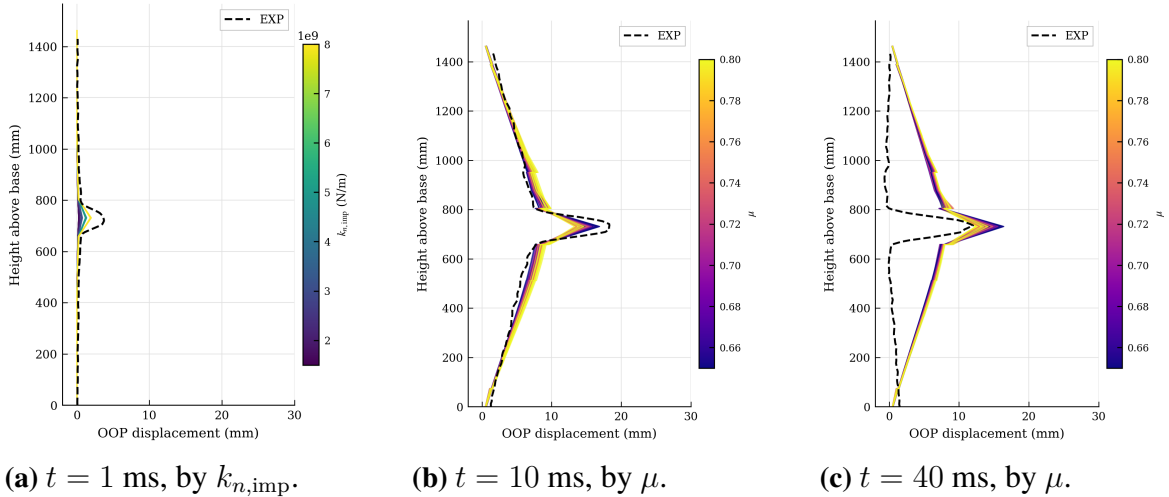


Figure 4.11: Wall W10 deformation profiles from the exploratory sweep.

A refined sweep of 27 combinations was then run jointly on W9, W10, and W11:

$$k_{n,\text{imp}} \in \{2.5, 2.8, 3.0\} \text{ GPa/m}, \quad \mu \in \{0.60, 0.65, 0.70\}, \quad \xi \in \{0.010, 0.015, 0.020\}.$$

The impactor force, velocity, and displacement histories coloured by μ are shown in Appendix D, Figures D.10–D.12. The preferred friction value reveals a cross-wall conflict. W9 and W10 ($\sigma_{ax} = 0.20$ MPa) favour $\mu = 0.60$, where larger normal compression already provides significant frictional resistance. W11 ($\sigma_{ax} = 0.05$ MPa) favours $\mu = 0.70$, since the lower confinement allows the joints to slide more freely and higher friction is needed to restrain the response. The detailed wall-by-wall metrics are reported in Appendix D, Table D.3. No single combination ranks in the top-5 for all three walls simultaneously (Appendix D, Table D.4), so $\mu = 0.65$ is selected as the most robust value.

The response is weakly sensitive to ξ within the 50 ms observation window, as shown in Figure 4.12, where all damping curves remain close throughout the record with only minor differences in post-peak decay. This is consistent with the near-zero contribution of $\mathbf{K}^{\tan,lg}$ during rocking phases (Section 2.4.5.4). The value $\xi = 2\%$ is retained as a working value; a dedicated damping calibration is deferred to Campaign 2 where the observation window is longer and impact energies higher (Section 4.2.3). The full ξ sensitivity table is provided in Appendix D, Table D.5.

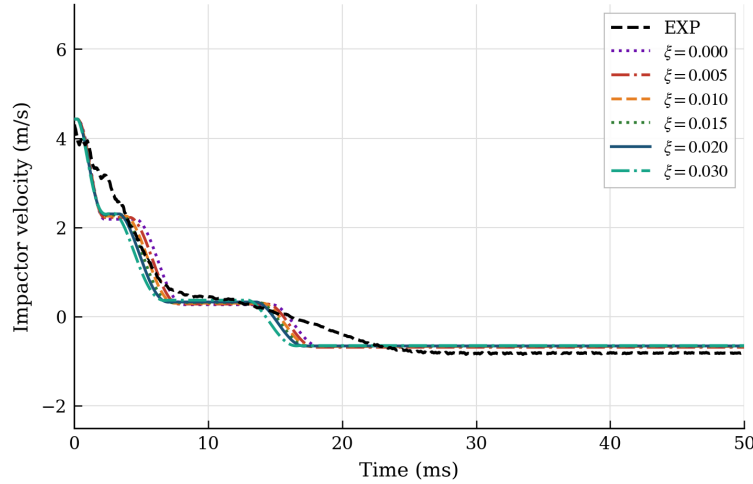


Figure 4.12: W10 impactor velocity histories for $\xi \in \{0, 0.5, 1, 1.5, 2, 3\}\%$.

4.2.2.5 Selected parameter set and cross-wall metrics

The selected Campaign 1 parameter set is:

$$k_{n,imp} = 2.5 \text{ GPa/m}, \quad \mu = 0.65, \quad \xi = 0.020 \text{ (working).}$$

Table 4.8 reports the impactor metrics for the three walls under this set. The force ratio decreases from W9 to W11 ($1.58 \rightarrow 1.22 \rightarrow 1.02$), consistent with the transition from contact-controlled to system-controlled regime. The impulse ratio remains stable at 0.88–0.89, confirming that total momentum transfer is reproduced consistently across all three walls.

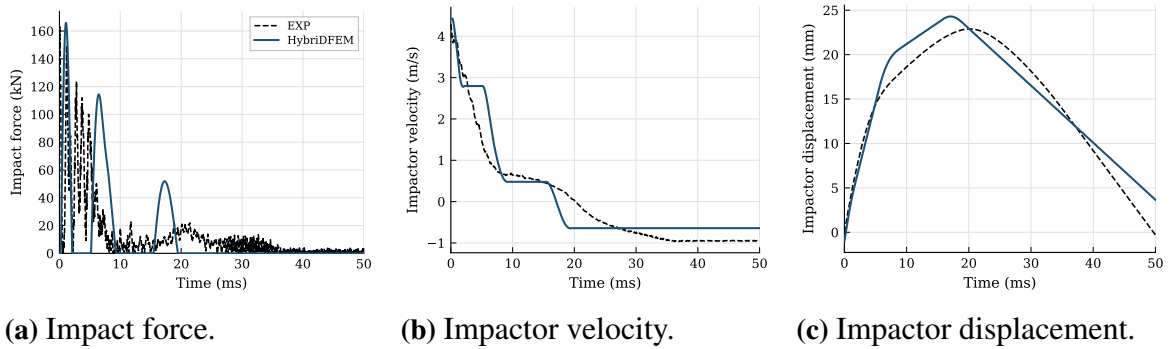
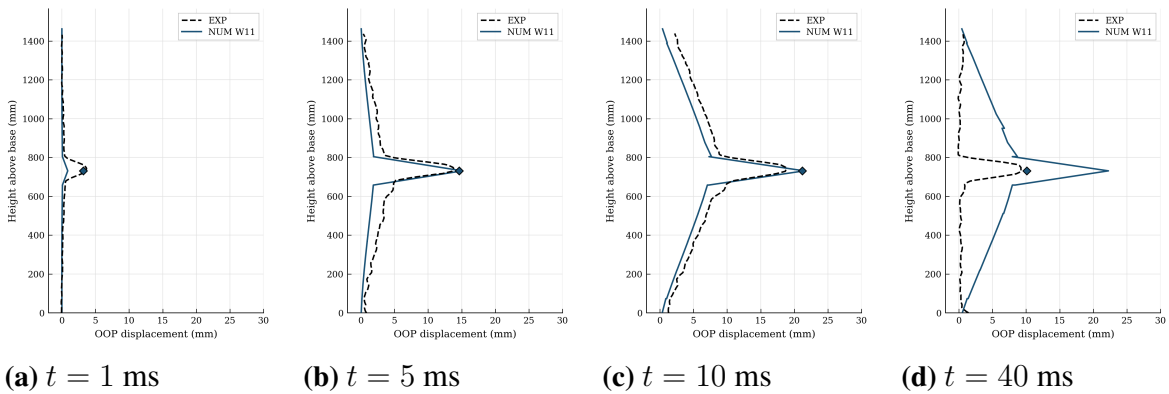
Table 4.8: Impactor metrics for the selected Campaign 1 parameter set.

Wall	u/u_{exp}	Δt_u (ms)	uRMSE (mm)	F/F_{exp}	Δt_F (ms)	I/I_{exp}
W9	0.83	-2.88	2.22	1.58	+0.40	0.88
W10	0.80	-1.28	4.45	1.22	+0.03	0.88
W11	1.06	-3.05	2.27	1.02	+1.02	0.89

4.2.2.6 Wall-level results

Impactor histories and wall deformation profiles for W9 and W10 are presented in Appendix D, Section D.6. Wall W11 is shown here as the decisive case; it shares the same impact velocity as W10 but operates at lower axial prestress, producing a more system-controlled response. The selected set reproduces the force peak accurately ($F/F_{\text{exp}} = 1.02$) and gives the best force agreement of the campaign; the displacement amplitude is slightly overpredicted ($u/u_{\text{exp}} = 1.06$), which remains acceptable. As presented in Figure 4.13 the overall dynamic trend is well captured; the timing of the displacement peak, the velocity plateau during contact, the initial slope of the displacement history in the first milliseconds, and the post-contact impactor velocity are all reproduced with good agreement.

The deformation profiles in Figure 4.14 confirm that the global bending shape and the localised penetration depth at the impact row are both well reproduced, supporting the selected value of $\mu = 0.65$ as physically consistent with the observed wall response.

**Figure 4.13:** W11 ($v_0 = 4.43$ m/s, $\sigma_{\text{ax}} = 0.05$ MPa) — impactor response.**Figure 4.14:** W11 — wall deformation profiles at four DIC snapshot times.

4.2.2.7 Arching force

The arching force is reconstructed as $F_{\text{arch}}^{\text{num}} = P_0 + K_{\text{sup}} \Delta u_y$ and compared against the experimental rod load cell measurements. Table 4.9 reports the peak ratios for all three walls; Figure 4.15 shows the W11 history. Full histories for W9 and W10 are provided in Appendix D, Figure D.22.

Table 4.9: Arching force peak ratios, Campaign 1.

Wall	$F_{\text{arch,peak}}^{\text{exp}}$ (kN)	$F_{\text{arch,peak}}^{\text{num}}$ (kN)	Ratio
W9	57.7	79.6	1.38
W10	91.1	142.1	1.56
W11	87.6	125.4	1.43

Two systematic deviations are observed. The peak arching force is overestimated by factors of 1.38–1.56, attributed primarily to the 2D representation of the point impact because the model mobilises the full wall depth, whereas the physical contact activates a more localised arching mechanism. The arching force also does not return to the prestress baseline after impact, consistent with the DIC profiles at $t = 40$ ms in Figure 4.14(d), where the wall remains displaced. This is a direct consequence of the `ElastoPlastic` assembly-level limitation identified in Section 3.3.8 and addressed by the `ElastoPlastic_CD` extension in Appendix F. Both deviations are treated as known modelling limitations rather than calibration errors.

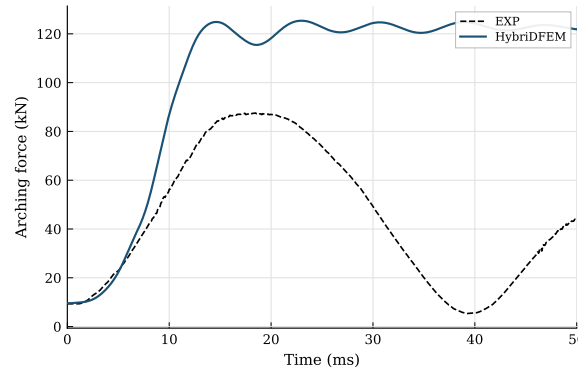


Figure 4.15: W11 — arching force history for the selected Campaign 1 parameter set.

4.2.3 Campaign 2 — Impact Parameter Calibration Review

4.2.3.1 Overview and calibration strategy

Campaign 2 extends the parameter identification to higher impact velocities ($v_0 = 7.2$ – 8.5 m/s), a longer observation window (100 ms), and two loading configurations: point-load (W16–W17) and line-load (W18–W19). The calibration is anchored on W18 and W19. For line-load walls, the spreader beam covers the full wall width consistently with the 2D model, making force and impulse meaningful calibration quantities. For point-load walls this is not the case, as established in Section 3.3.8. Since Campaign 3 also uses line-load configurations, the W18–W19 calibration transfers directly.

A parameter sweep of 168 combinations is performed using the coarse model for efficiency; the selected combination from Campaign 1 is subsequently re-run with the refined 3×6 model:

$$k_{n,\text{imp}} \in \{2.0, 2.5, 3.0, 4.0, 5.0, 6.0, 8.0\} \text{ GPa/m}, \quad \mu \in \{0.55, 0.60, 0.65, 0.70, 0.75, 0.80\},$$

$$\xi \in \{1, 1.5, 2, 3\}\%.$$

The Campaign 1 set ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 2\%$) is included as one of the 168 cases, allowing its performance to be ranked directly within the sweep.

4.2.3.2 Experimental reference values

Table 4.10 summarises the experimental reference values for the four walls.

Table 4.10: Experimental reference values, Campaign 2.

Wall	Load	v_0 (m/s)	F_{exp} (kN)	t_F (ms)	u_{exp} (mm)	t_u (ms)	I_{exp} (kN·s)
W16	point	7.23	484.3	0.46	53.6	27.6	4.015
W17	point	8.49	498.3	0.43	80.9	32.2	3.915
W18	line	7.17	341.8	2.06	44.0	26.4	1.238
W19	line	8.35	468.0	1.47	51.3	30.1	1.489

4.2.3.3 Transfer of Campaign 1 parameters

Table 4.11 reports the performance of the Campaign 1 set within the 168-combination sweep.

Table 4.11: Impactor metrics for the transferred Campaign 1 parameter set.

Wall	Load	u/u_{exp}	uRMSE (mm)	F/F_{exp}	I/I_{exp}	Rank
W16	point	0.79	16.74	0.41	0.23	125/168
W17	point	0.65	18.72	0.46	0.27	100/168
W18	line	0.95	19.67	0.58	0.73	101/168
W19	line	1.01	18.23	0.48	0.71	88/168

All four walls rank in the lower half of the 168-combination space. The force peak is strongly underestimated and the displacement histories show a systematic timing delay, a direct consequence of $k_{n,\text{imp}} = 2.5 \text{ GPa/m}$ being too soft for the higher-velocity Campaign 2. Full impactor histories for all four walls under the transferred set are provided in Appendix E, Section E.4, Figures E.2.

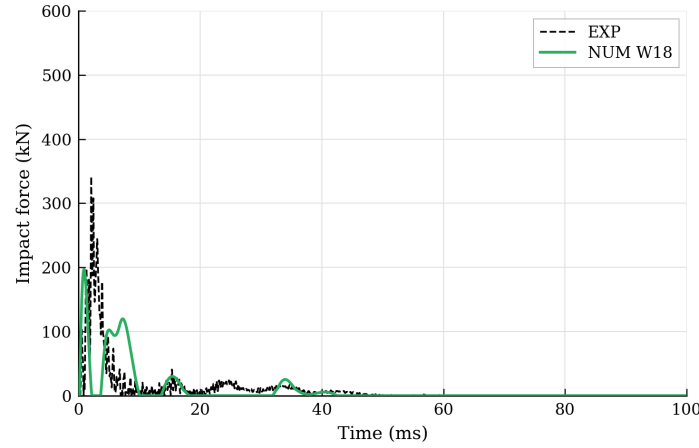


Figure 4.16: W18 — impact force, transferred Campaign 1 parameter set ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 2\%$). Rank 101/168.

4.2.3.4 Contact stiffness: $k_{n,\text{imp}} = 6.0 \text{ GPa/m}$

Figure 4.17 shows the force peak ratio vs impulse ratio for all W18 combinations, coloured by $k_{n,\text{imp}}$. The impulse ratio is approximately constant across the full stiffness range ($\approx 0.69\text{--}0.74$), confirming that $k_{n,\text{imp}}$ governs the shape and timing of the force pulse rather than the total transferred impulse, which is determined by the global impact dynamics. The force ratio increases systematically with $k_{n,\text{imp}}$, approaching the experimental level at 6.0 GPa/m. At 8.0 GPa/m the ratio exceeds unity. The numerical values for all stiffness levels are tabulated in Appendix E, Table E.4.

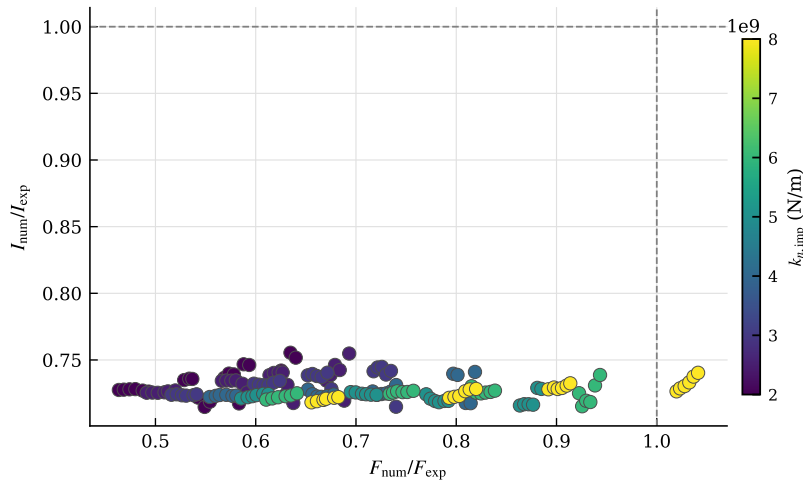


Figure 4.17: W18 — force peak ratio vs impulse ratio for all 168 combinations, coloured by $k_{n,\text{imp}}$.

A slight remaining underestimation at 6.0 GPa/m is physically expected. In the experiment, the spreader beam is not a perfectly rigid body and contacts the wall progressively rather than instantaneously across its full width, producing a higher measured peak force than a rigid simultaneous contact can reproduce. The increase from 2.5 to 6.0 GPa/m is physically consistent with the higher Campaign 2 velocities, since larger local penetrations activate a stiffer part of the nonlinear contact response, requiring a higher effective linear stiffness.

4.2.3.5 Friction coefficient: physical basis of the μ split

With $k_{n,\text{imp}} = 6.0$ GPa/m fixed, the effect of μ differs fundamentally between loading configurations.

Under *point-load impact* (W16–W17), the physical contact is localised at the impactor tip, but the 2D model distributes it over the full wall depth. This geometric overestimation of the contact area artificially increases the effective frictional resistance relative to the real wall. Retaining $\mu = 0.65$ from Campaign 1 compensates for this artefact and reproduces the measured displacement amplitude.

Under *line-load impact* (W18–W19), the spreader beam distributes the load uniformly across the full wall width, activating a global rocking mechanism in which multiple bed joints slide simultaneously. This distributed sliding requires a higher friction value to be adequately restrained. Figure 4.18 shows the W18 displacement histories coloured by μ , it is clear that the peak decreases with increasing the friction, with $\mu = 0.75$ providing the best agreement. Full sweep metrics are tabulated in Appendix E.

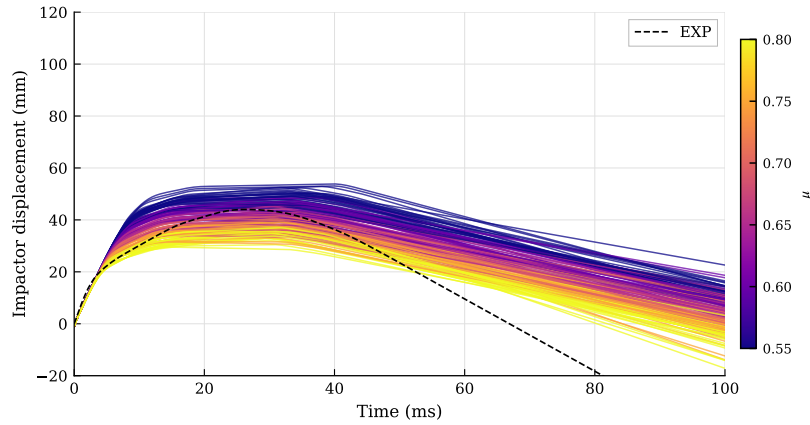


Figure 4.18: W18 — impactor displacement histories coloured by μ ($k_{n,\text{imp}} = 6.0$ GPa/m, $\xi = 1\%$).

The adopted values are:

$$\mu = 0.65 \text{ (W16, W17, point-load)}, \quad \mu = 0.75 \text{ (W18, W19, line-load)}.$$

Full μ sweep results and discussion for both configurations are provided in Appendix E, Sections E.5.3 and E.5.4.

4.2.3.6 Damping sensitivity study

A dedicated damping study is performed on W18 and W19 with $k_{n,\text{imp}} = 6.0$ GPa/m and $\mu = 0.75$, sweeping $\xi \in \{0.25, 0.5, 1, 2.5, 5\}\%$ under two stiffness operators: initial stiffness (ISPD, $\mathbf{K}_0$) and linear-geometric tangent stiffness (TAN_LG, $\mathbf{K}^{\text{tan},lg}$). The theoretical basis for this choice is discussed in Section 2.4.5.4.

Figures 4.19 and 4.20 show the impactor velocity and force histories for all cases. The ISPD formulation must be rejected. $\mathbf{K}_0$ retains the stiffness of joints that have opened during rocking, introducing viscous forces through physically inactive contacts. This suppresses the displacement peak ($u/u_{\text{exp}} = 0.27\text{--}0.58$ across all ISPD cases) and overestimates the force

peak by up to 53%, regardless of the chosen ξ value. The TAN_LG formulation reproduces the experimental response qualitatively across the full observation window. Differences between ξ values are visible over $t = 10\text{--}50$ ms and diminish after $t = 60$ ms, as $\mathbf{K}^{\text{tan},lg} \rightarrow \mathbf{0}$ during the kinematic mechanism phase and damping forces vanish (Section 2.4.5.4). The value $\xi = 1\%$ provides the best overall balance across velocity decay, force level, and displacement amplitude, and is adopted for Campaigns 2 and 3. Full displacement histories and the equivalent analysis for W19 are provided in Appendix E, Section E.5.5, Figures E.5a–E.5c.

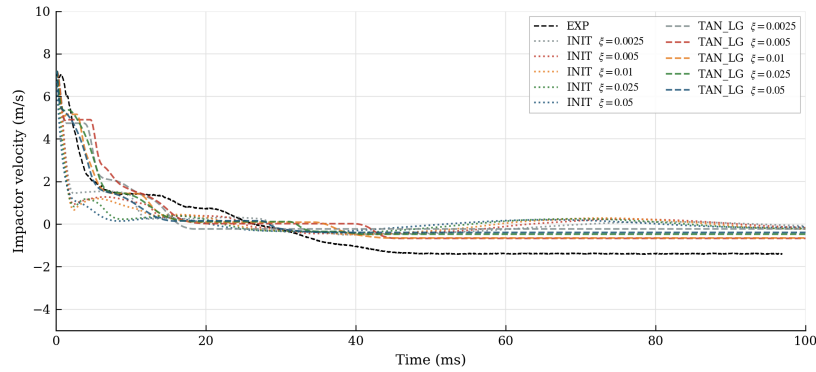


Figure 4.19: W18 — impactor velocity histories for all ISPD and TAN_LG damping cases.

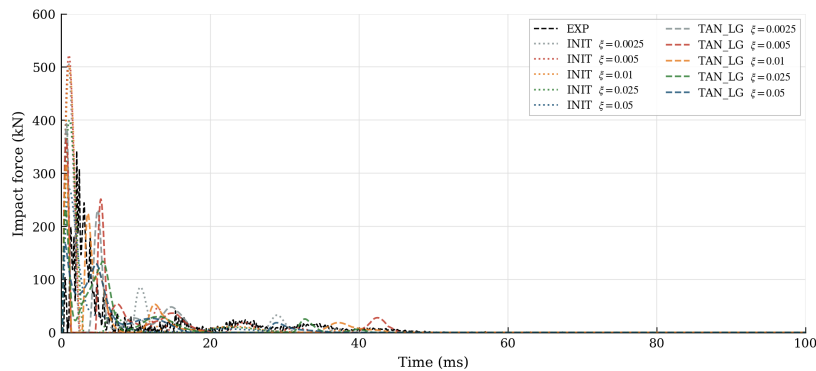


Figure 4.20: W18 — impact force histories for all damping cases.

4.2.3.7 Selected parameter sets and cross-wall metrics

Table 4.12 summarises the selected parameter sets and Table 4.13 reports the impactor metrics under the refined 3×6 model.

Table 4.12: Selected parameter sets, Campaign 2.

Parameter	W16/W17 (point)	W18/W19 (line)
$k_{n,\text{imp}}$ (GPa/m)	6.0	6.0
$k_{s,\text{imp}}$ (GPa/m)	1.2	1.2
μ	0.65	0.75
ξ	1%	1%

Table 4.13: Impactor metrics for the selected Campaign 2 parameter sets, refined 3×6 model. [†]Force and impulse are not calibration targets for point-load walls (Section 3.3.8).

Wall	Load	u/u_{exp}	uRMSE (mm)	F/F_{exp}	I/I_{exp}	Rank
W16	point	0.87	22.5	0.66 [†]	0.23 [†]	11/168
W17	point	0.71	15.1	0.74 [†]	0.27 [†]	19/168
W18	line	0.85	17.8	0.93	0.74	8/168
W19	line	0.96	16.9	0.78	0.72	5/168

All four walls improve from the lower half of the parameter space (ranks 88–125/168) to the top 12% (ranks 5–19/168), with the line-load walls reaching ranks 8/168 and 5/168.

4.2.3.8 Wall-level results

W16 and W19 are presented as representative cases for the point-load and line-load configurations respectively. Results for W17 and W18, including impactor histories and DIC height displacement comparisons at $H/8$, $H/4$, $3H/4$, and $7H/8$, are provided in Appendix E, Section E.7, Figures E.8–E.11.

W16 — point load ($v_0 = 7.23$ m/s) and W19 — line load ($v_0 = 8.35$ m/s). W19 achieves the best overall ranking of the campaign (5/168) with the displacement peak reproduced within 4% and the force within 22%; W16 gives a reasonable displacement agreement ($u/u_{\text{exp}} = 0.87$) while force and impulse are underestimated due to the two-dimensional point-load geometry limitation and are not calibration targets. Across both walls, the initial displacement slope, the velocity decrease during contact, and the post-contact velocity are well reproduced, with the final velocity slightly underestimated.

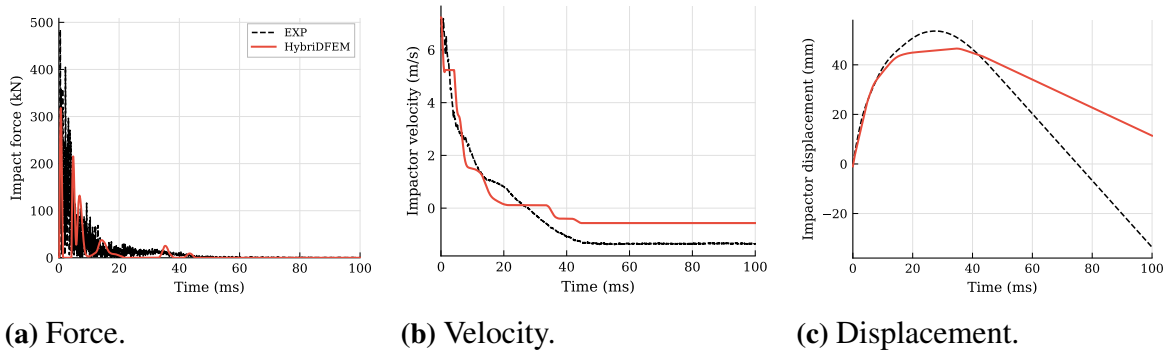


Figure 4.21: W16 ($v_0 = 7.23$ m/s, point load) — impactor response for the selected Campaign 2 parameter set. Rank 11/168.

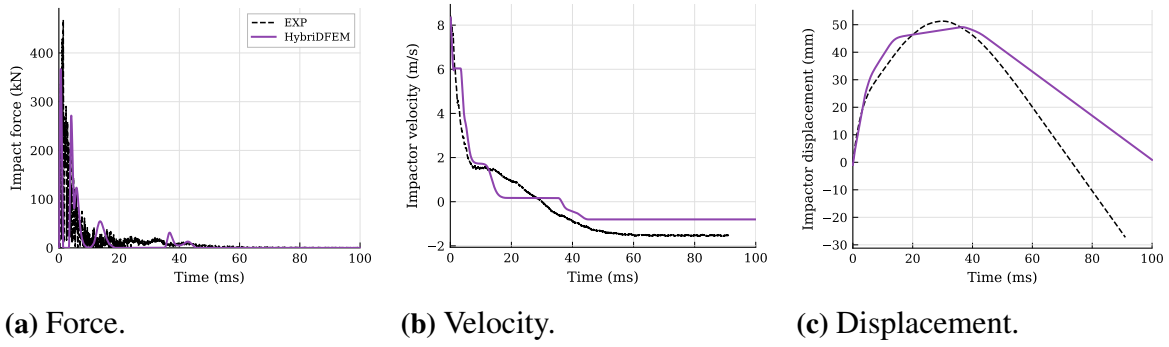


Figure 4.22: W19 ($v_0 = 8.35$ m/s, line load) — impactor response for the selected Campaign 2 parameter set. Rank 5/168.

4.2.3.9 Arching force

Table 4.14 reports the peak arching force ratios for all four walls. Figures 4.23a and 4.23b show the histories for the representative walls; full histories for W17 and W18 are provided in Appendix E, Figure E.14.

Table 4.14: Arching force peak ratios, Campaign 2.

Wall	Load	$F_{\text{arch,peak}}^{\text{exp}}$ (kN)	$F_{\text{arch,peak}}^{\text{num}}$ (kN)	Ratio
W16	point	128.3	175.0	1.36
W17	point	134.9	198.3	1.47
W18	line	128.8	194.5	1.51
W19	line	138.6	213.8	1.54

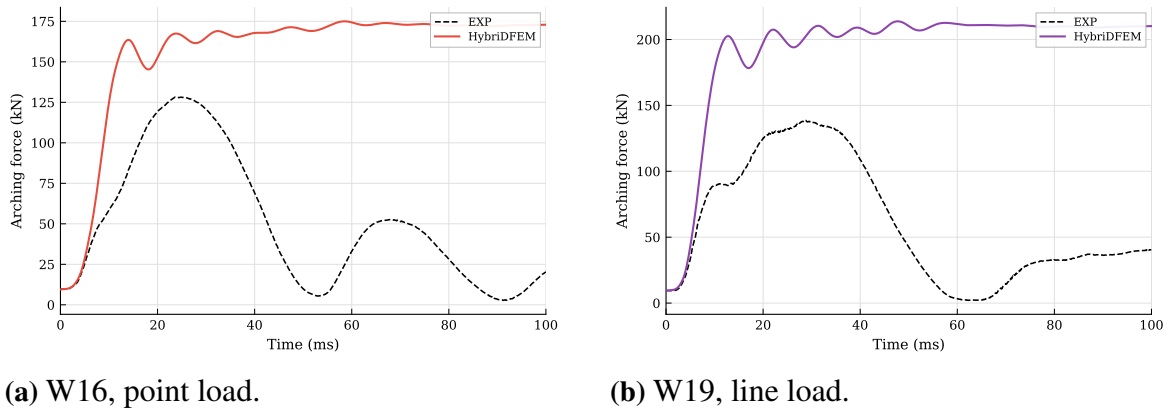


Figure 4.23: Arching force histories for the representative Campaign 2 walls under the selected parameter sets.

The peak arching force is overestimated by 1.36–1.54 across all walls, including the line-load walls where the 2D geometry limitation does not apply. The primary cause is the *ElastoPlastic* assembly-level limitation that prevents post-impact wall rebound, keeping the arching force artificially elevated above the pre-impact baseline (Section 3.3.8). The free-vibration benchmark in Appendix F, Section F.3 confirms that contact deletion restores this behaviour.

4.3 Abaqus — Model Verification

Before any calibration is performed, the ABAQUS model is verified to ensure that it behaves as physically intended. This covers five areas: geometry and mass, boundary conditions, mesh sensitivity, mass scaling, and time scaling. The specific walls and campaigns used for each check are stated within the respective subsections.

4.3.1 Mass and static equilibrium

Following the same verification approach described for HybriDFEM in Section 4.1.1, the total model mass and the vertical base reaction are checked against the expected values from the experimental setup. The check is performed at the end of the quasi-static initialisation step, in which gravity and axial load are applied, before proceeding to the out-of-plane loading phase in both the four point bending (4PB) and impact simulations. The results are summarised in Table 4.15.

Table 4.15: Mass and static equilibrium check after Step 1. All masses in kg, reactions in kN.

Component	4PBT (W5)/(W12)	Campaign 1 (W10)	Campaign 2 (W18)	Campaign 3 (W27)
m_{wall} (kg)	538.0	499.9	497.2	538.4
m_{slabs} (kg)	190.0	190.0	230.0	174.9
$m_{\text{plate+beam}}$ (kg)	-	262*	262*	460†
m_{total} (kg)	728.0	852	981	1173
W_{self} (kN)	7.14	6.76	7.13	11.51
F_{axial} (kN)	38.2/75.5	40.54	12.07	19.83
$F_{y,\text{expected}}$ (kN)	45.6/82.6	47.31	19.21	31.34
$F_{y,\text{computed}}$ (kN)	45.8/82.6	46.4	19.2	30.6
$\ \mathbf{R}_{\text{free}}\ $ (N)	0.2/0	1.9	0.01	0.74

* Not modeled explicitly. Added by adapting the density of the top RC-slab.

† Not modeled explicitly. Added by adapting the density of the steel plate.

The base reaction errors obtained with the Abaqus model are larger than those reported for HybriDFEM in Section 4.1.1, where residual norms of the order of 10^{-10} N were achieved. Two factors contribute to this difference. First, the axial prestress in the Abaqus model is applied as a prescribed velocity ramped over the duration of Step 1, rather than as a directly applied force. The velocity magnitude is chosen to produce the target prestress level, but small deviations in the final force are unavoidable with this approach. Second, the use of an explicit time integration scheme introduces a small degree of dynamic oscillation even during the quasi-static initialisation step, which prevents the solution from reaching the same level of static equilibrium as an implicit solver. The resulting errors in the base reaction remain within acceptable limits for engineering purposes and do not affect the subsequent calibration or impact simulations.

4.3.2 Mesh sensitivity

A uniform mesh size of 40 mm was initially selected for the four-point bending simulations, as it provides a sufficient number of integration points through the wall thickness and yielded a well-behaved force–displacement response. To verify this choice, a mesh sensitivity study was carried out on a representative wall, W5, comparing four uniform mesh densities: 40 mm, 30 mm, 20 mm, and 15 mm, as shown in Figure 4.24. The comparison targets are the lateral force–displacement curve and the peak lateral force relative to the experimental value, which are the primary calibration observables for the four-point bending simulations.

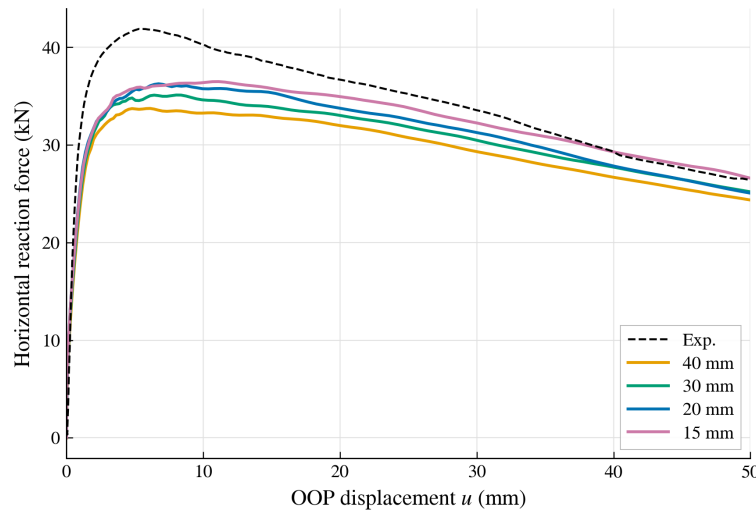


Figure 4.24: Mesh sensitivity study for W5: lateral force versus out-of-plane displacement for mesh densities of 40 mm, 30 mm, 20 mm, and 15 mm.

The results, shown in Figure 4.24 and summarised in Table 4.16, indicate that the differences in peak lateral force between the four mesh densities are within 3 kN and 7%, confirming that the global wall response is slightly sensitive to further refinement beyond 40 mm for the four-point bending configuration. In contrast, the computational time increases significantly with mesh refinement, ranging from approximately 30 minutes for the 40 mm mesh to approximately 10 hours for the 15 mm mesh, as illustrated in Figure 4.25. On the basis of these results, the 40 mm mesh is retained for all four-point bending simulations, as it provides sufficient accuracy at a fraction of the computational cost of finer discretisations. It should nevertheless be noted that this sensitivity study was performed on a single wall, and the conclusions may not generalise to all wall configurations considered in this study.

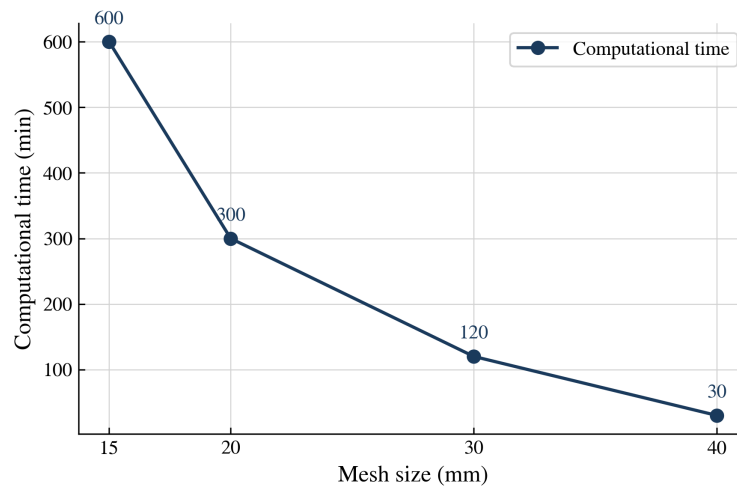


Figure 4.25: Computational time versus mesh size for the mesh sensitivity study on W5.

Table 4.16: Mesh sensitivity study for W5 — peak lateral force compared against the experimental value for each mesh density.

Mesh size (mm)	$Q_{\text{peak,exp}}$ (kN)	$Q_{\text{peak,num}}$ (kN)	Error (%)	Computational time (min)
40	42.30	34.25	19.0 %	30
30	42.30	36.53	13.6 %	120
20	42.30	36.84	12.9 %	300
15	42.30	37.32	11.8 %	600

For the impact simulations, a dedicated mesh sensitivity study was not performed. Instead, the mesh strategy is motivated by the physical requirements of the problem. A coarse mesh with an element size of 80 mm is applied throughout most of the wall, where the response is governed by global bending and rocking kinematics and a detailed spatial resolution is not required. In the region surrounding the impact zone, the mesh is refined to 15 mm, as this size was found to yield the best agreement with the experimental results. Unlike the four-point bending simulations, the increased computational cost associated with the finer mesh in the impact zone was accepted, given the need to accurately resolve the steep stress and strain gradients, the localised brick damage, and the contact force pulse that develop during impact. This non-uniform strategy limits the total element count while concentrating computational effort where it is physically needed.

4.3.3 Mass Scaling and Time Scaling

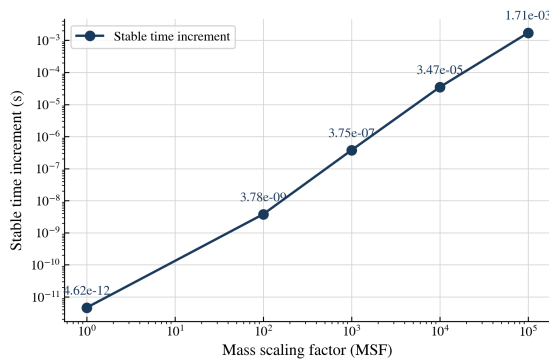
Two techniques are employed to improve the computational efficiency of the explicit analyses without compromising physical accuracy: mass scaling during the quasi-static initialisation step, which was verified during the 4 Point Bending analysis, and time scaling during the dynamic impact analysis. The validity of both is assessed in this section.

4.3.3.1 Mass scaling

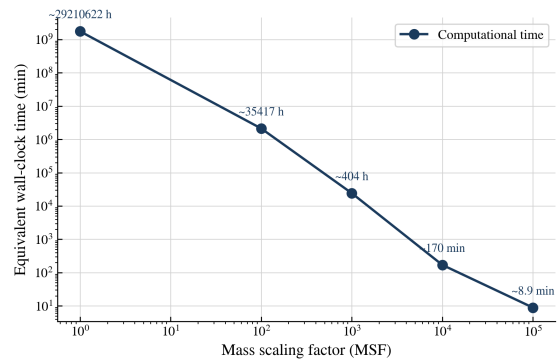
Mass scaling is applied during the quasi-static initialisation step to reduce the number of increments required to reach static equilibrium, as described in Section 3.4.7. Table 4.17 summarises the effect of the mass scaling factor on the stable time increment and on the equivalent computational rate. Without mass scaling, a mass scaling factor of 1 yields a stable time increment of 4.62×10^{-12} s, corresponding to approximately 9.70×10^{-8} analysis seconds per minute of computation, making the quasi-static initialisation entirely intractable. Increasing the mass scaling factor to 10^5 raises the stable time increment to 1.71×10^{-3} s, allowing 19 analysis seconds to be computed per minute of wall-clock time, reducing the initialisation step to a manageable duration. The effect of the mass scaling factor on the stable time increment is further illustrated in Figure 4.26.

Table 4.17: Effect of the mass scaling factor (MSF) on the stable time increment and computational rate for a representative wall.

MSF	Stable time increment (s)	Analysis time per min (s/min)	Equivalent wall-clock time 170s analysis time
10^5	1.71×10^{-3}	19	~8.9 min
10^4	3.47×10^{-5}	1	~170 min
10^3	3.75×10^{-7}	0.007	~404 h
10^2	3.78×10^{-9}	0.00008	~35417 h
1	4.62×10^{-12}	9.70×10^{-8}	~29210622 h



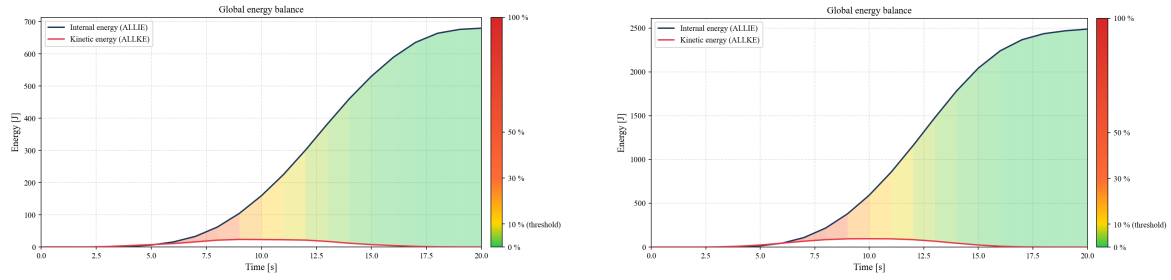
(a) Mass scaling factor versus stable time increment.



(b) Mass scaling factor versus computational time.

Figure 4.26: Effect of the mass scaling factor on the stable time increment and computational time for a representative wall.

The physical validity of the mass scaling is assessed by verifying that the kinetic energy remains small compared to the internal energy (<10% of the Internal energy) throughout the majority of the initialisation step. The energy histories for representative walls are shown in Figure 4.27.



(a) 4 Point bending - 0.2 MPa axial load.

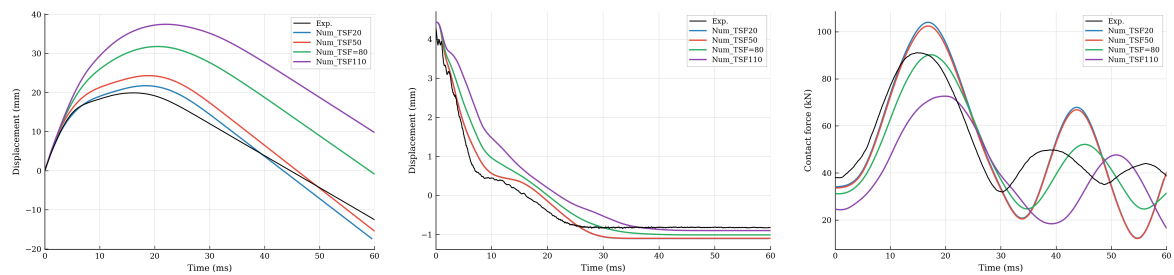
(b) 4 Point bending - 0.4 MPa axial load.

Figure 4.27: Kinetic and internal energy histories during the quasi-static initialisation step for the 4PB representative walls.

4.3.3.2 Time scaling

During the dynamic impact step, mass scaling is deactivated and the physical material densities are restored. A time scaling factor is applied to increase the stable time increment, reducing the number of increments required to complete the simulation without modifying the material densities or the equations of motion. In practice, this means the simulation runs faster while the physical behaviour is preserved, provided the factor remains within acceptable limits. The sensitivity of the simulated response to this factor is assessed by running the same simulation with four values: 20, 50, 80 and 110. The comparison targets are the impactor displacement history, impactor velocity and wall arching.

The results of the representative wall (W10) show that a time scaling factor of 50 produces response histories in close agreement with those obtained at a factor of 20, while reducing the computational time significantly. A factor of 80 and 110 introduces visible deviations from the reference response and is therefore rejected. On this basis, a time scaling factor of 50 is adopted for all production impact simulations as the most efficient value that preserves the physical accuracy of the solution.

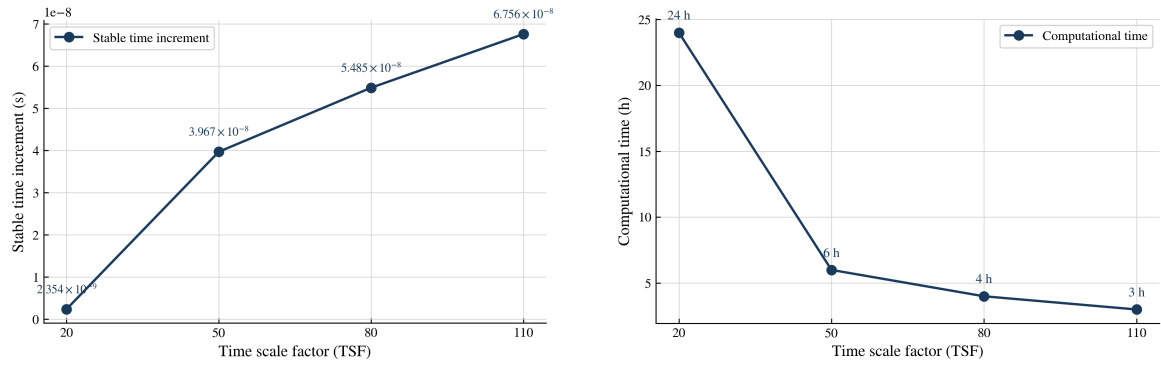


(a) Impactor displacement

(b) Impactor velocity.

(c) Wall arching.

Figure 4.28: Time scaling factor sensitivity study: impactor displacement and velocity, and wall arching for four values of the time scaling factor for a representative wall.



(a) Time scaling factor versus stable time increment.

(b) Time scaling factor versus computational time.

Figure 4.29: Effect of the time scaling factor on the stable time increment and computational time for a representative wall.

4.4 ABAQUS - Model Calibration

4.4.1 4PBT — Calibration and Wall Results

The calibration followed a sequential, multi-stage strategy summarised in Table 4.18 and described in full in Appendix H. Wall W5 was used to identify the elastic parameters, W12 to calibrate the non-linear evolution laws and joint properties, and W13–W14 to verify transferability to non-rigid boundary conditions.

Table 4.18: Abaqus 4PBT calibration strategy.

Wall	Configuration	Calibrated parameters	Role
W5	Double wythe, simply supported	$E_b, k_n, k_s, G_{f,I}$	Calibration
W8	Double wythe, simply supported	—	Validation
W12	Double wythe, rigid	Evolution laws, $c, \mu, G_{f,II}$	Calibration
W6, W7	Double wythe, rigid	—	Validation
W13	Double wythe, non-rigid	$K_{\text{sup}} = 19.0 \text{ kN/mm}$	Validation
W14	Double wythe, non-rigid	$K_{\text{sup}} = 9.5 \text{ kN/mm}$	Validation

4.4.1.1 Wall W5 — elastic parameter calibration

Wall W5 was tested under simply supported conditions with an axial compression of 37.7 kN. The brick elastic modulus was fixed at $E_b = 7000 \text{ MPa}$ (Godio & Flansbjer, 2025a), and a parametric sweep of $k_n \in \{20, 16, 14, 12, 10\} \text{ MPa/mm}$ was carried out with $k_s = k_n / (2 \times 1.1)$. As shown in Figure 4.30, the force–displacement curves are insensitive to k_n within this range, and the failure mechanism was used as the discriminating criterion. Values of $k_n \geq 14 \text{ MPa/mm}$ produced non-uniform edge separation, while $k_n = 12 \text{ MPa/mm}$ produced a uniform crack opening along a single horizontal course, consistent with the experimentally observed three-hinge bending failure (Godio et al., 2023b), as shown in Figure 4.31. The values $k_n = 12 \text{ MPa/mm}$ and $k_s = 5.5 \text{ MPa/mm}$ were therefore adopted.

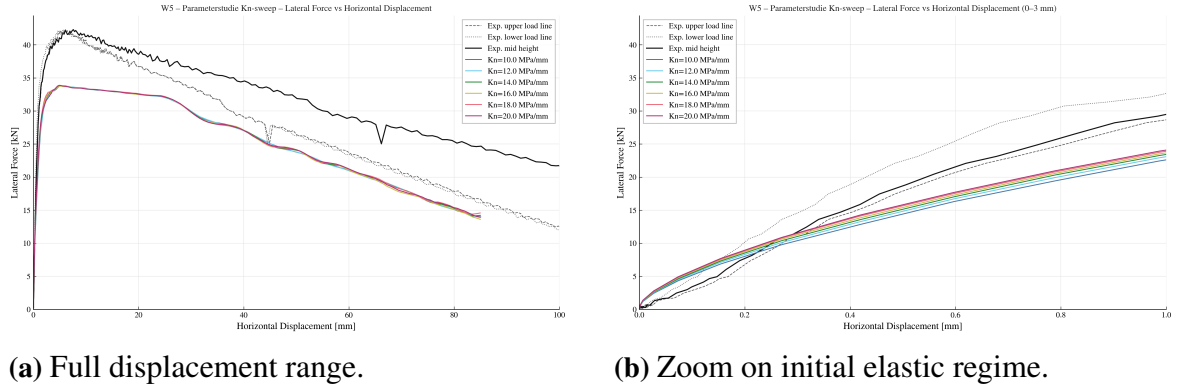


Figure 4.30: W5 — lateral force versus out-of-plane displacement for the k_n parametric sweep (Godio et al., 2023b).

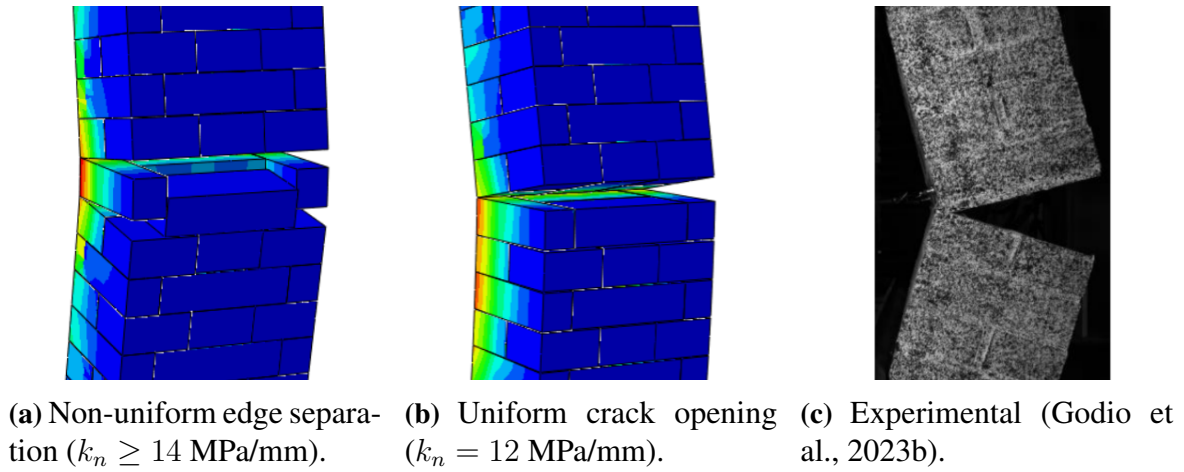
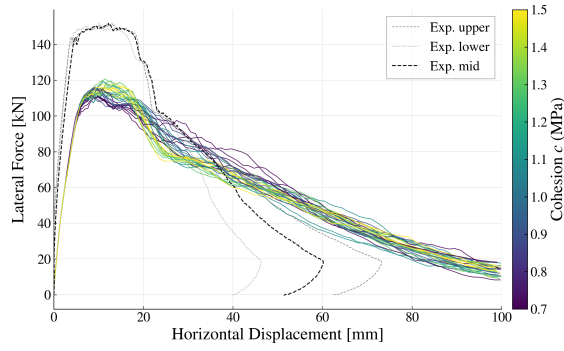


Figure 4.31: W5 — failure mechanism comparison for the k_n parametric sweep.

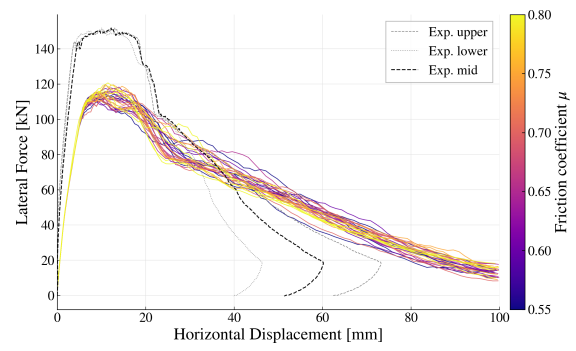
4.4.1.2 Wall W12 — non-linear parameter calibration

Wall W12 was tested under rigid top support conditions with an axial compression of 75.5 kN, failing through a punching shear mechanism. The simplified perfectly plastic law from Stage 1 produced a bending-dominated response with underdeveloped arching, and was replaced by the RISE softening laws with $G_t = 0.001$ N/mm and $f_{t,res} = 0.009$ MPa (Godio & Flansbjerg, 2025a), which activated the correct failure mechanism.

With the evolution laws fixed, the cohesion c and friction coefficient μ were calibrated through a sweep of 30 combinations, shown in Figures 4.32 and 4.33. Higher cohesion increases the peak lateral force but degrades the post-peak arching fit, while lower cohesion better reproduces the arching discharge. Friction increases lateral force capacity with no clear effect on arching. The values $c = 0.7$ MPa and $\mu = 0.8$ were selected as the most robust combination across all walls in Abaqus/Explicit 2024-HF2. The final results and failure mechanism are shown in Figures 4.34 and 4.35.

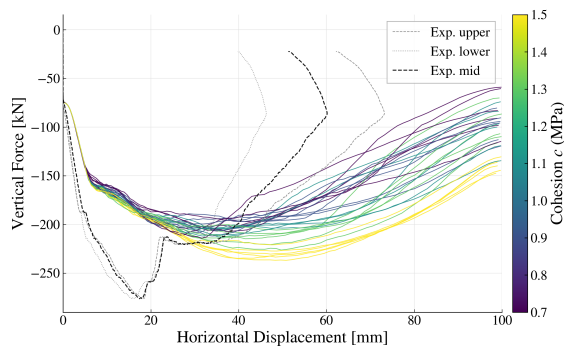


(a) Coloured by cohesion c .

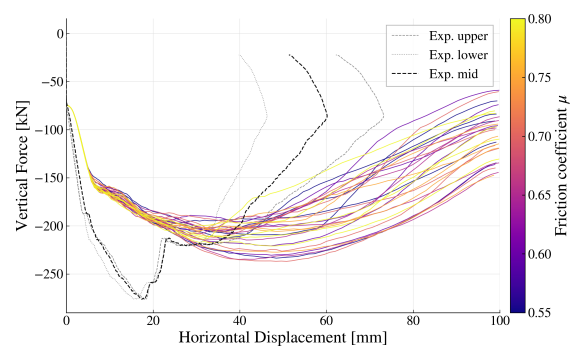


(b) Coloured by friction coefficient μ .

Figure 4.32: W12 — lateral force parametric sweep of c and μ .

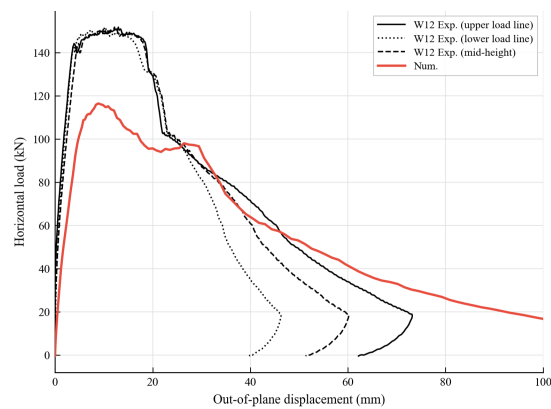


(a) Coloured by cohesion c .

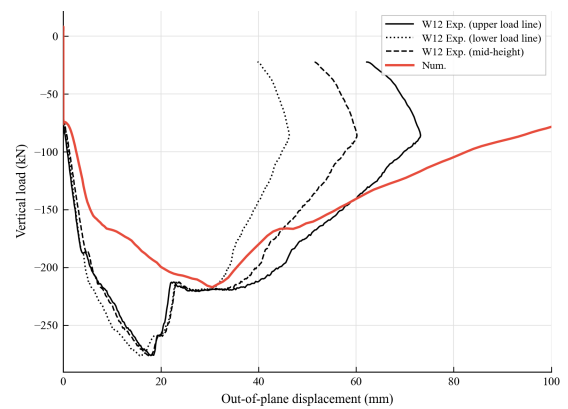


(b) Coloured by friction coefficient μ .

Figure 4.33: W12 — arching force parametric sweep of c and μ .



(a) Lateral force.



(b) Vertical reaction (arching force).

Figure 4.34: W12 — final results under the adopted parameter set ($c = 0.7$ MPa, $\mu = 0.8$) (Godio et al., 2023b).

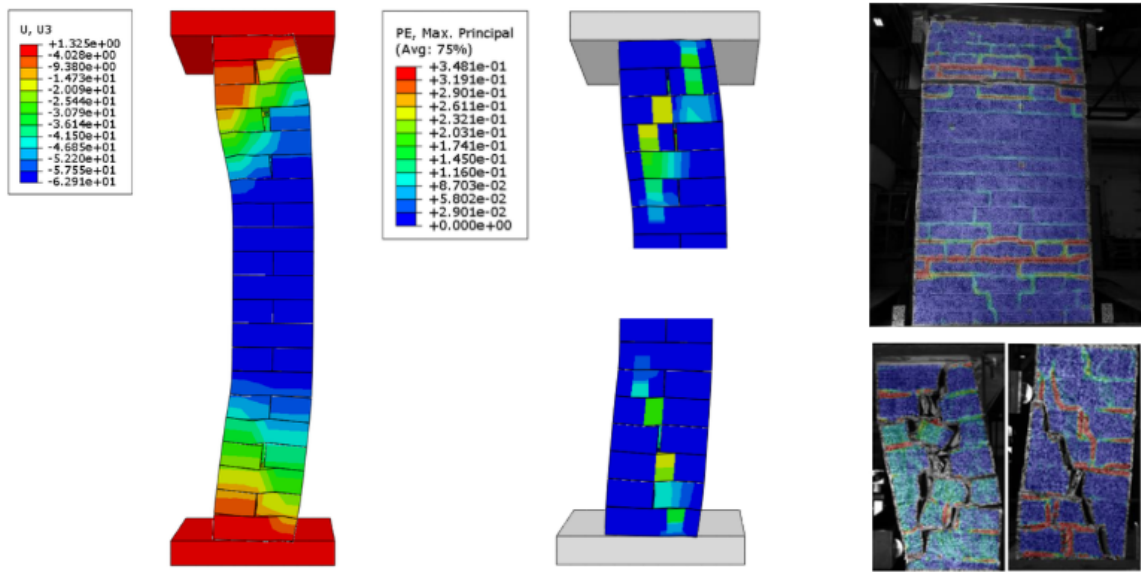


Figure 4.35: W12 — failure mode: Abaqus U3 field (left), plastic strain PE field (centre), and experimental crack pattern (right) (Godio et al., 2023b).

4.4.1.3 Wall W14 — non-rigid boundary condition

Wall W14 was tested under non-rigid top support conditions with $K_{\text{sup}} = 9.5 \text{ kN/mm}$ and an axial compression of 38.2 kN, exhibiting a hybrid bending–shear failure (Godio et al., 2023b). No material parameters were re-calibrated; only the top boundary condition was updated to the measured spring stiffness. The model reproduces the pre-peak stiffness and arching response satisfactorily, as shown in Figure 4.36. The abrupt load drop at approximately 40 mm associated with diagonal crack onset is not reproduced, as the CDP formulation smears damage over the continuum rather than localising it along discrete crack surfaces. The failure mechanism is shown in Figure 4.37, confirming that the hybrid bending–shear pattern is captured qualitatively. Results for W13 are provided in Appendix H.

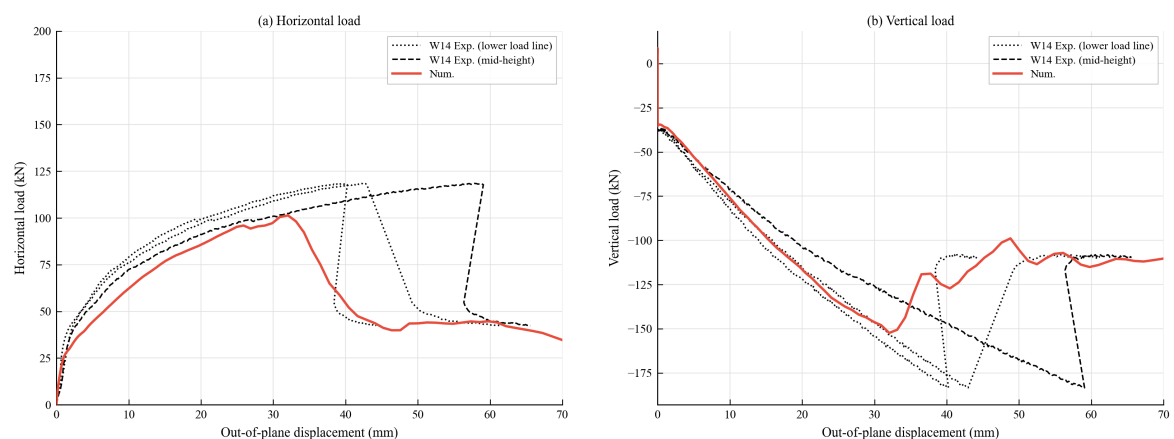


Figure 4.36: W14 — lateral force (a) and vertical reaction (b) versus out-of-plane displacement under the transferred parameter set ($K_{\text{sup}} = 9.5 \text{ kN/mm}$) compared against the experimental measurements (Godio et al., 2023b).

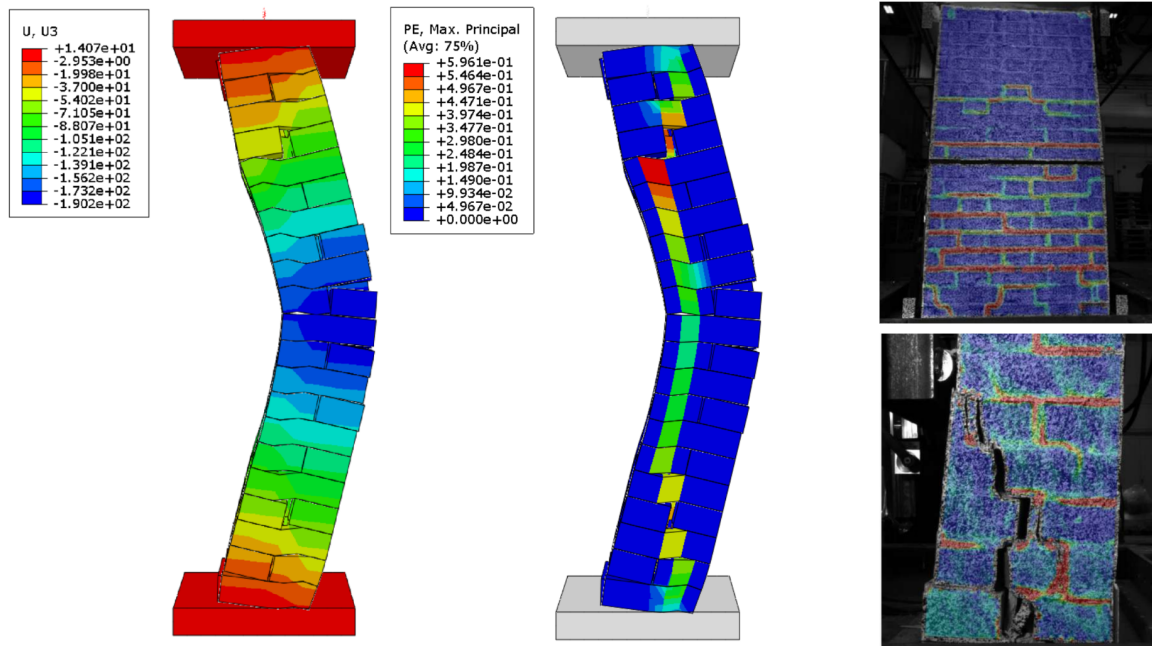


Figure 4.37: W14 — failure mode: Abaqus U3 field (left), plastic strain PE field (centre), and experimental crack pattern (right) (Godio et al., 2023b).

4.4.2 Campaign 1 & 2 — Impact Parameter Calibration and Wall Results

4.4.2.1 Damping calibration

The only impact-specific parameter calibrated for Campaigns 1 and 2 in the Abaqus model is the interface damping coefficient applied at the impactor–wall contact during the dynamic step. A sensitivity study was carried out on walls W9, W10, and W11 from Campaign 1 and W16 from Campaign 2, testing damping values ranging from 0.03 down to 0.005. Values below 0.01 caused the analysis to terminate prematurely before the full impact event could be captured and were therefore rejected. The value of 0.01 produced stable results in agreement with the experimental response across all tested walls and was adopted as the final value for Campaigns 1, 2, and 3.

Figure 4.38 shows the impactor displacement, velocity, and arching force histories for the tested damping values on the representative wall W10, illustrating the agreement obtained with the adopted value.

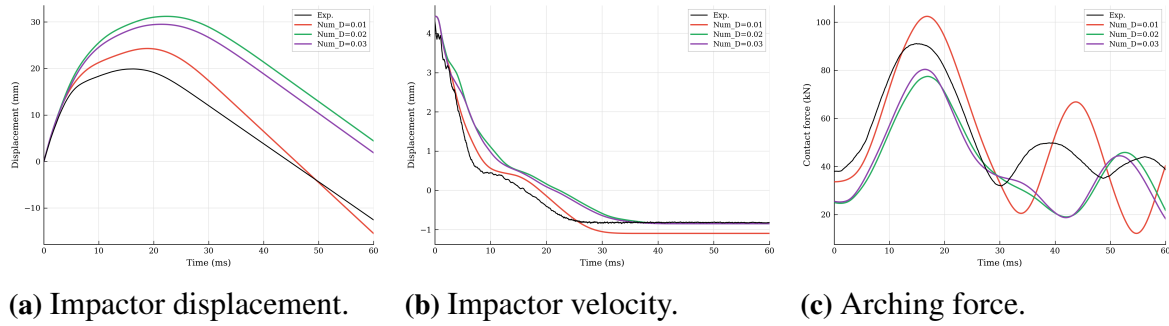


Figure 4.38: Damping sensitivity study on W10 — impactor displacement, velocity, and arching force histories for the tested damping values. The adopted value of 0.01 is highlighted.

4.4.2.2 Wall W10 — low-velocity point-load impact ($v_0 = 4.43$ m/s)

Wall W10 is a double-wythe specimen subjected to a point-load impact at a pre-impact velocity of 4.43 m/s under an axial compression stress of 0.20 MPa. Figure 4.39 shows the impactor response histories and Figure 4.40 shows the arching force. The model reproduces the initial displacement slope and the general shape of the velocity history satisfactorily. The peak arching force is overestimated, which is attributed to the low-velocity contact-controlled regime: at this impact energy level the wall deformation is small and the model overestimates the lateral stiffness mobilised during the short contact phase, producing a higher arching reaction than measured experimentally.

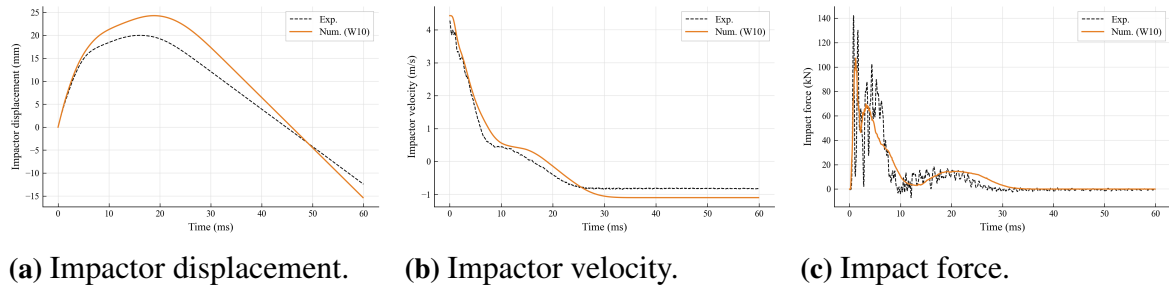


Figure 4.39: W10 — impactor response histories ($v_0 = 4.43$ m/s, point-load) Godio et al., 2023a.

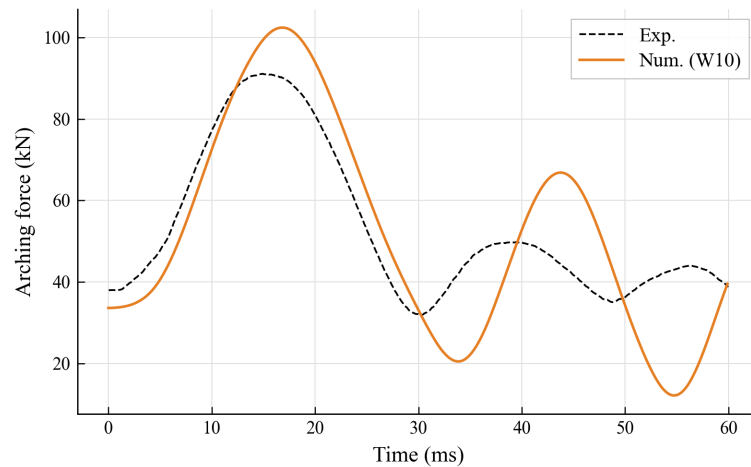
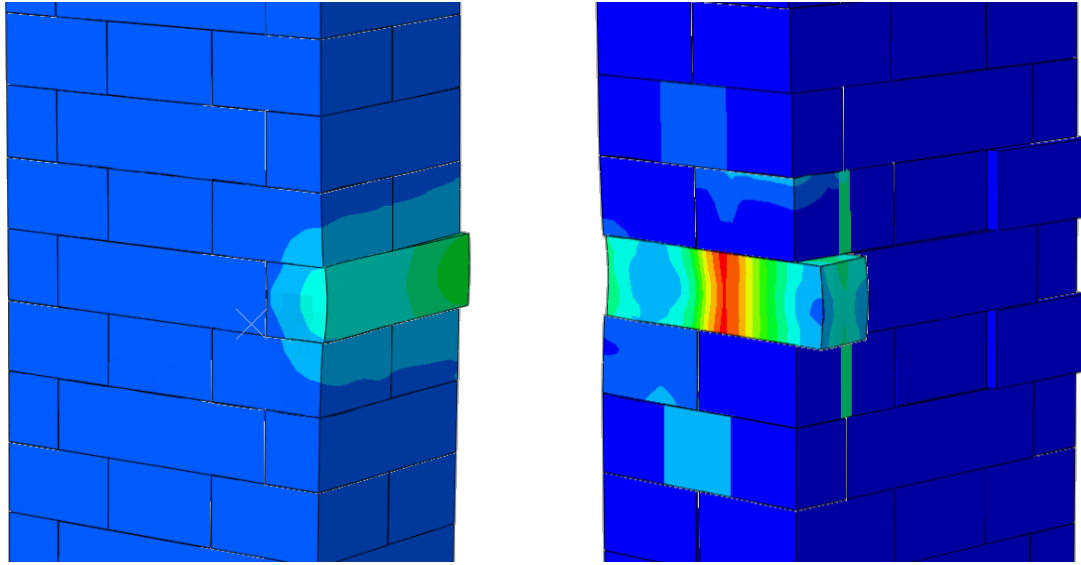


Figure 4.40: W10 — arching force history.

The out-of-plane displacement and plastic strain fields at peak displacement are shown in Figure 4.41. The displacement field confirms the localised punching deformation at the impact zone, with the surrounding wall regions remaining largely undisturbed, clearly illustrating the concentrated penetration of the impactor into the wall surface. The plastic strain field reveals two distinct zones of inelastic deformation: a primary concentration at the impact zone propagating diagonally outward from the contact point, consistent with the punching-type localisation expected under point-load impact, and a secondary concentration located at mid-height of the wall, within the brick body, indicating the onset of crack formation through the brick at that course. The CDP formulation distributes this damage as smeared plastic straining rather than discrete brick fracture, so the brick splitting and mortar crushing observed experimentally at the contact zone are not explicitly reproduced. Nevertheless, the location and extent of both inelastic zones are physically consistent with the experimental damage pattern.



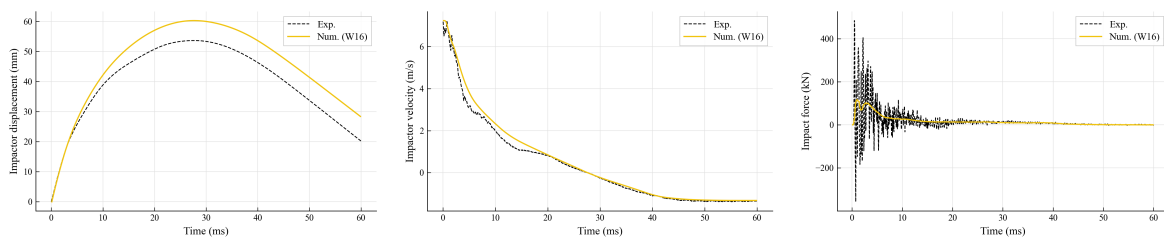
(a) Out-of-plane displacement (U3) field.

(b) Plastic strain (PE) field.

Figure 4.41: W10 — out-of-plane displacement and plastic strain fields at peak impactor displacement, showing the localised punching deformation at the impact zone.

4.4.2.3 Wall W16 — moderate-velocity point-load impact ($v_0 = 7.23 \text{ m/s}$)

Wall W16 is subjected to a higher-energy point-load impact at 7.23 m/s under an axial compression stress of 0.05 MPa . Figure 4.42 shows the impactor response histories and Figure 4.43 shows the arching force. The displacement amplitude is reasonably reproduced, while the impact force is underestimated, which is consistent with the three-dimensional Abaqus model distributing the contact over the full wall depth rather than concentrating it at a single point as in the experiment. The arching force is underestimated, consistent with the higher impact energy producing large wall deformations that cause the continuum model to lose stiffness more rapidly than observed experimentally, reducing the predicted arching reaction below the measured level.



(a) Impactor displacement.

(b) Impactor velocity.

(c) Impact force.

Figure 4.42: W16 — impactor response histories ($v_0 = 7.23 \text{ m/s}$, point-load) Godio and Flansbjerg, 2025b.

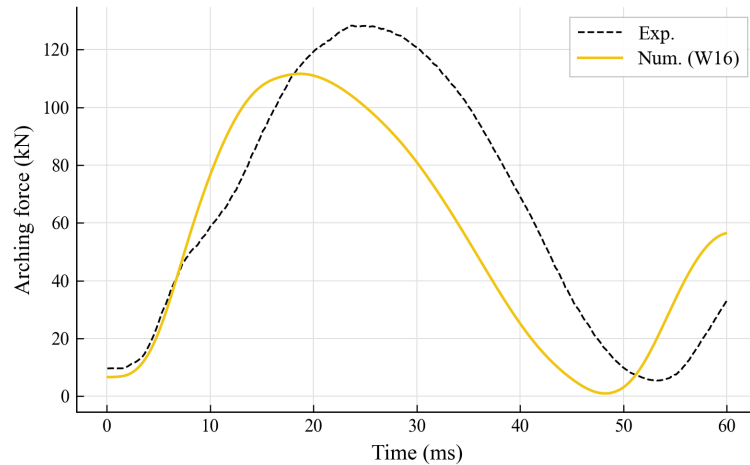
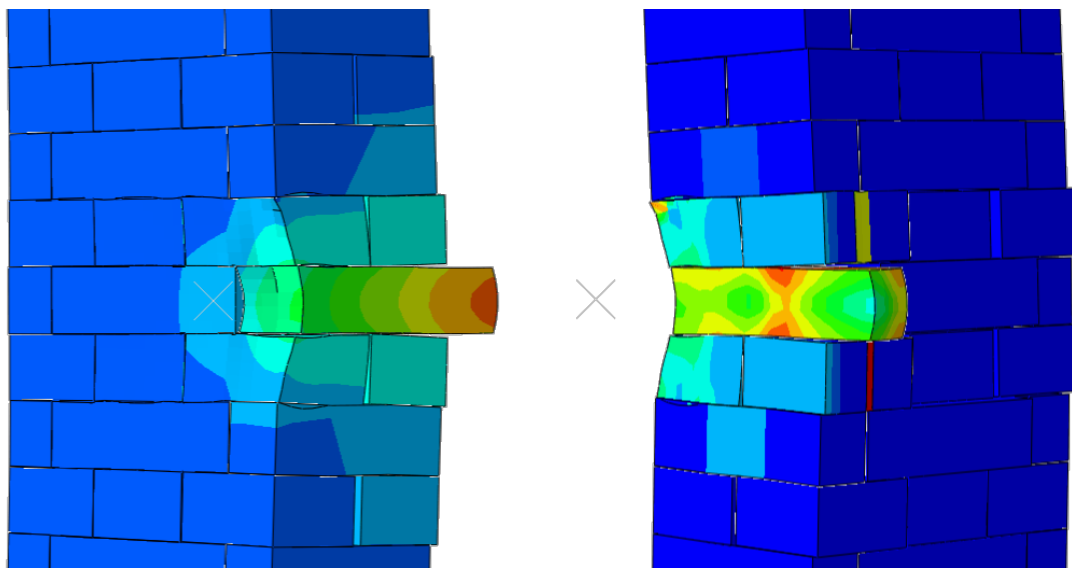


Figure 4.43: W16 — arching force history.

The out-of-plane displacement and plastic strain fields at peak displacement are shown in Figure 4.44. The displacement field shows more extensive wall deformation compared to W10, consistent with the higher impact energy, with the punching effect at the contact zone clearly visible. The plastic strain field indicates damage concentrating at the impact zone and extending toward the wall ends, with additional plastic straining visible at the adjacent bricks to the loading brick. Experimentally, W16 exhibited central brick dislocation and vertical splitting of bricks at the back face of the wall at the impact location, along with crushing of bricks and mortar on the front face Godio and Flansbjerg, 2025b. The model does not reproduce the discrete brick fracture and dislocation explicitly, but the plastic strain field indicates the location and extent of the damaged region, with the concentrated plasticity at the middle of the directly impacted brick, serving as an indicator of where fracture would be expected to occur.



(a) Out-of-plane displacement (U3) field.

(b) Plastic strain (PE) field.

Figure 4.44: W16 — out-of-plane displacement and plastic strain fields at peak impactor displacement.

4.4.2.4 Wall W18 — moderate-velocity line-load impact ($v_0 = 7.17$ m/s)

Wall W18 is subjected to a line-load impact at 7.17 m/s under an axial compression stress of 0.05 MPa. Figure 4.45 shows the impactor response histories and Figure 4.46 shows the arching force. For W18, the model diverges more notably from the experimental response compared to the point-load walls. The impactor penetrates further into the wall than observed in the laboratory, and the arching force does not fully develop over the observation window, remaining below the experimental level throughout. This reflects the same stiffness degradation under large deformation that limits the arching prediction for the higher-energy Campaign 2 walls, and is further amplified by the line-load configuration. The analysis of Campaign 3 is nevertheless carried forward using the adopted parameters, as the overall kinematic trend and damage location remain physically consistent.

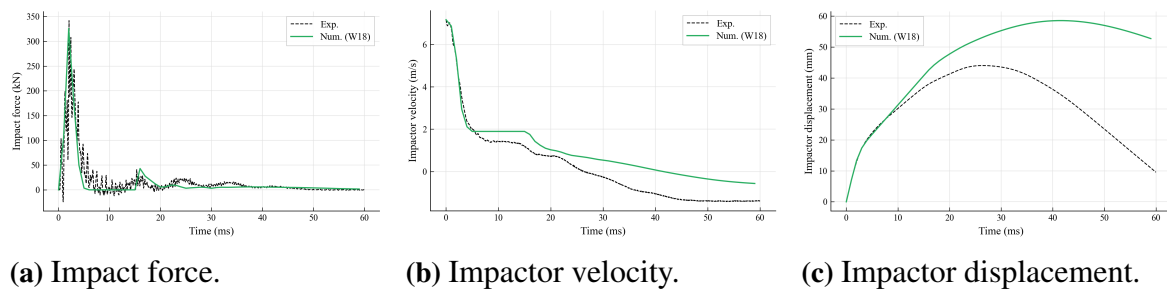


Figure 4.45: W18 — impactor response histories ($v_0 = 7.17$ m/s, line-load) Godio and Flansbjerg, 2025b.

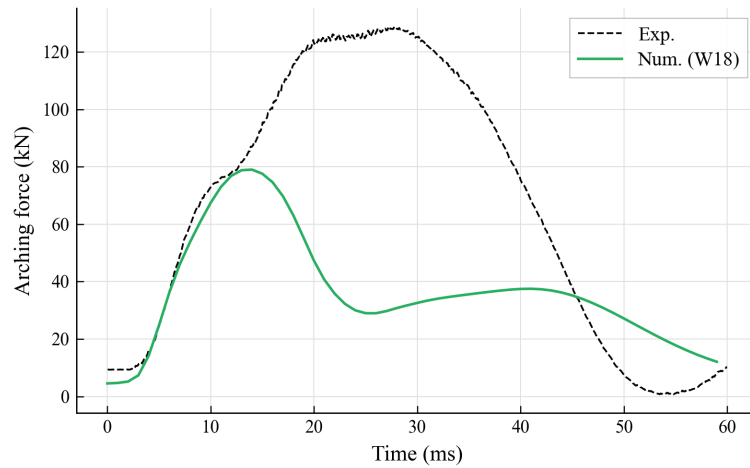
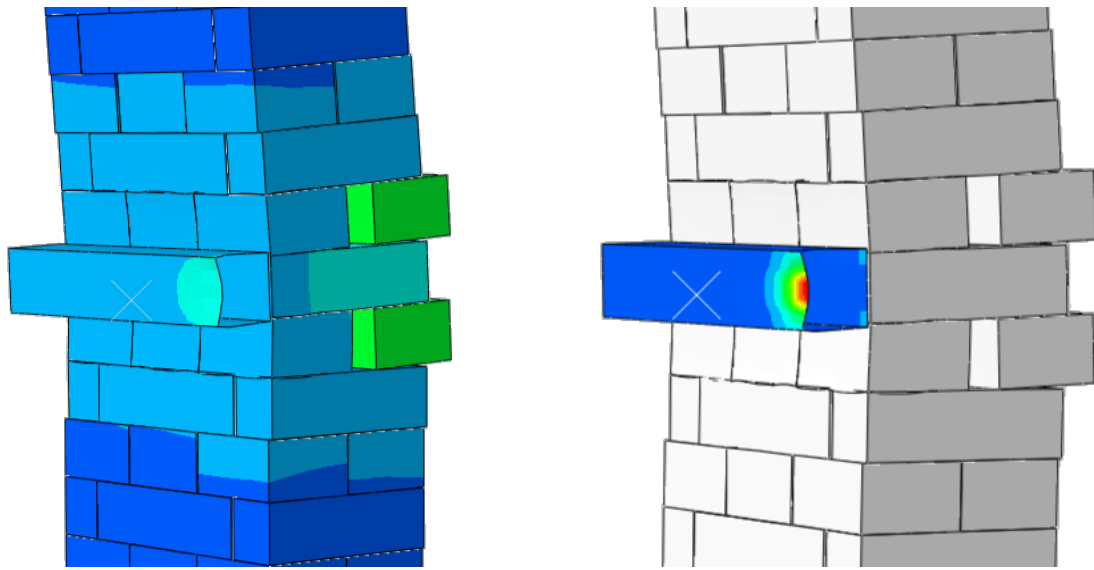


Figure 4.46: W18 — arching force history.

The out-of-plane displacement and plastic strain fields at peak displacement are shown in Figure 4.47. The displacement field reflects the line-load configuration, with deformation distributed more uniformly across the central courses of the wall rather than concentrating at a single contact point. The plastic strain field shows that the inelastic deformation is concentrated primarily in the steel loading beam.



(a) Out-of-plane displacement (U3) field.

(b) Plastic strain (PE) field.

Figure 4.47: W18 — out-of-plane displacement and plastic strain fields at peak impactor displacement.

4.5 Campaign 3: Comparative Assessment of HybriDFEM and Abaqus

The parameter set identified from Campaign 2 for HybriDFEM and Abaqus is applied respectively without modification to all 13 walls of Campaign 3. No recalibration is performed; the objective is to assess the predictive capability of each framework beyond its calibration range.

For the HybriDFEM model, results are presented for both the original `ElastoPlastic` law and the `ElastoPlastic_CD` extension developed in this thesis. As demonstrated in the Griffith wall benchmark (Appendix F, Section F.3), the absence of contact deletion in `ElastoPlastic` prevents post-impact wall rebound and produces a systematic overestimation of the residual arching force. Since `ElastoPlastic_CD` addresses this without introducing new parameters, Campaign 3 is the first campaign in which both formulations are compared directly against experimental data and the Abaqus model. Each figure shows four traces: experimental measurements (EXP, black dashed), HybriDFEM with `ElastoPlastic` (EP), HybriDFEM with `ElastoPlastic_CD` (EP_CD), and Abaqus.

Table 4.19 summarises the prediction metrics for all 13 walls organised by impact energy level. One representative wall per energy group is discussed in detail to illustrate the behaviour observed across the group; the remaining walls in each group are discussed using the tabulated metrics. Complete time-history results for all walls are provided in Appendix K.

Table 4.19: Campaign 3 prediction metrics organised by impact energy level. Quantities are numerical-to-experimental peak ratios. EP: ElastoPlastic; CD: ElastoPlastic_CD; Abq: Abaqus. *: representative wall

Wall	m (kg)	v_0 (m/s)	KE (J)	u/u_{exp}			F/F_{exp}			$F_{\text{arch}}/F_{\text{arch,exp}}$			w/w_{exp}		
				EP	CD	Abq	EP	CD	Abq	EP	CD	Abq	EP	CD	Abq
<i>Low energy (136–262 J)</i>															
W23*	236	1.07	137	0.55	0.81	1.18	1.05	1.07	1.15	1.41	1.28	1.17	1.04	0.84	0.99
W26	348	0.94	159	0.48	0.46	0.55	1.66	1.61	0.45	1.36	1.26	1.13	1.18	0.99	1.28
W30	442	1.10	262	0.98	0.93	1.09	0.98	0.96	0.89	1.27	1.22	1.11	1.58	1.34	1.65
<i>Moderate energy (4107–4548 J)</i>															
W31	442	4.50	4107	1.23	1.23	1.49	0.33	0.37	0.14	1.19	1.02	0.77	1.19	0.96	1.69
W24*	236	6.16	4273	1.08	1.18	1.41	0.71	0.80	0.58	1.16	1.02	0.73	1.04	0.92	1.70
W27	348	5.20	4548	1.29	1.34	1.19	0.54	0.64	0.20	1.20	1.01	0.78	1.11	0.95	1.02
<i>Moderate-high energy (7909–8173 J)</i>															
W25	236	8.10	7909	1.19	1.22	1.76	0.72	0.88	0.32	1.12	1.02	0.57	1.11	1.03	2.36
W32	442	6.08	8015	1.36	1.44	2.90	0.45	0.55	0.22	1.11	0.89	0.53	1.21	1.09	3.59
W28*	348	6.90	8173	1.34	1.34	2.34	0.42	0.51	0.24	1.10	0.95	0.54	1.18	1.01	3.89
<i>High energy (12086–14399 J)</i>															
W29	348	8.45	12086	1.38	1.61	2.33	0.51	0.61	0.25	1.09	0.85	0.46	1.20	1.44	3.35
W34	236	10.10	12288	1.61	1.86	3.22	0.59	0.70	0.26	1.06	0.85	0.43	1.21	1.79	6.60
W35	348	9.03	13515	1.54	1.83	4.07	0.37	0.43	0.17	1.08	0.86	0.42	1.22	1.57	6.36
W33*	442	8.25	14399	1.49	1.73	3.26	0.34	0.40	0.14	1.07	0.84	0.44	1.18	1.50	4.16

4.5.1 Low energy — W23, W26, W30 ($KE = 137\text{--}262\text{ J}$)

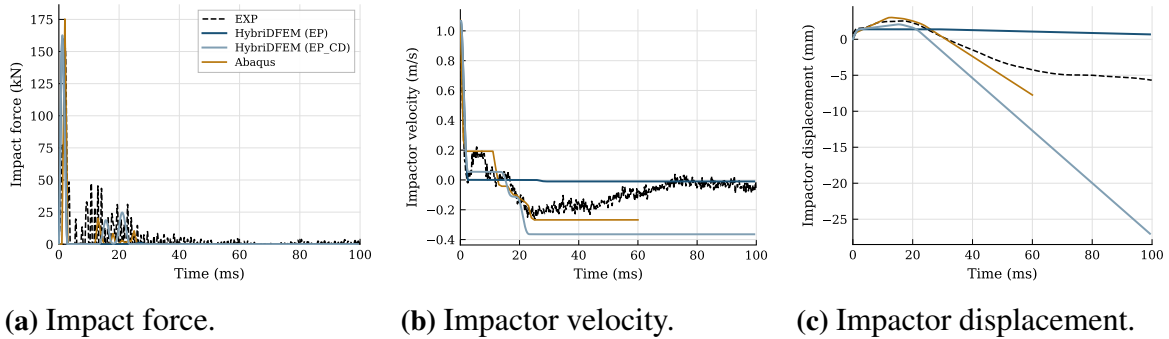
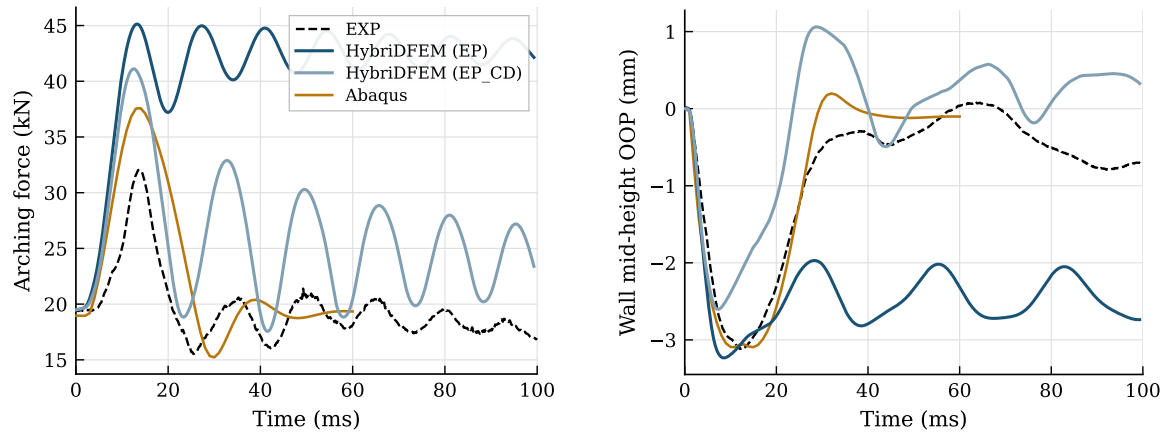


Figure 4.48: W23 — impactor response ($m = 236\text{ kg}$, $v_0 = 1.07\text{ m/s}$), representative of the low-energy group.



(a) Arching force.

(b) Wall mid-height OOP displacement.

Figure 4.49: W23 — arching force and wall response.

The three low-energy walls exhibit two distinct sub-regimes governed by the impactor mass. W23 ($m = 236$ kg) and W26 ($m = 348$ kg) are contact-controlled: the impact duration is short and the wall barely deflects. W30 ($m = 442$ kg) behaves differently because the larger impactor inertia mobilises a global rocking response even at $v_0 = 1.10$ m/s.

Impactor response. For W23 and W26, both HybriDFEM variants underestimate the peak impactor displacement while overestimating the force. This is the expected under contact-controlled regime, where $k_{n,imp}$ was calibrated at moderate energy, and is too stiff for the brief contact event at these energy levels, causing premature impactor rebound and overestimating the contact pressure. The experimental peak displacements are below 3 mm and 7 mm respectively, so small absolute errors produce large relative differences. W26 is the most challenging wall for all models, the impactor displacement is underestimated by EP, CD, and Abaqus, reflecting a contact event that no framework calibrated at higher energies reproduces reliably. W30 yields the best impactor prediction of the campaign for both HybriDFEM variants, confirming that the larger impactor mass activates the global rocking regime for which the parameters were calibrated.

The velocity and displacement histories for W23 (Figure 4.48b–c) show that EP stays flat near the peak with no rebound, CD reverses to a small negative velocity consistent with the experiment, and Abaqus slightly overpredicts the peak but follows the rebound slope well within the portion of the simulation that runs.

Wall response. The arching force is overestimated by all models across the group, with the overestimation decreasing from W23 toward W30 as the rocking mechanism progressively activates. For W23 (Figure 4.49a), EP oscillates persistently above baseline without returning to the pre-impact level. This is consistent with all Campaign 1 and 2 walls, where *ElastoPlastic* produced arching ratios of 1.38–1.56 and 1.36–1.54 respectively, confirming that the joint-locking artefact is a limitation of the law. CD oscillates with decaying amplitude and returns toward baseline; Abaqus returns closest to the experimental results.

The wall OOP history (Figure 4.49b) shows the consequence of contact deletion at low energy. The experimental wall rebounds to near zero; EP oscillates without recovery; CD rebounds above zero, closely following the experimental shape. Abaqus captures the peak well and follows the initial rebound slope with good agreement. For W30, all models over-

predict the wall OOP displacement, possibly related to the unvalidated $K_{\text{sup},x} = 35 \text{ kN/mm}$ (Section 3.3.8).

4.5.2 Moderate energy — W24, W27, W31 ($KE = 4107\text{--}4548 \text{ J}$)

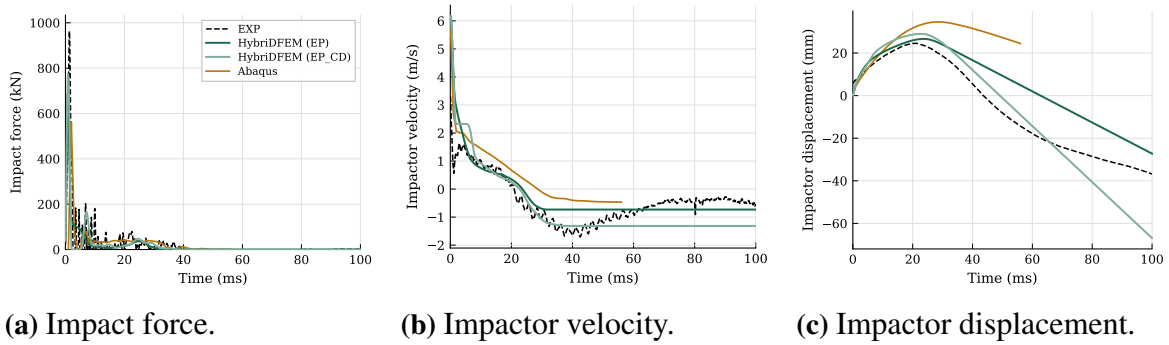


Figure 4.50: W24 — impactor response ($m = 236 \text{ kg}$, $v_0 = 6.16 \text{ m/s}$), representative of the moderate-energy group.

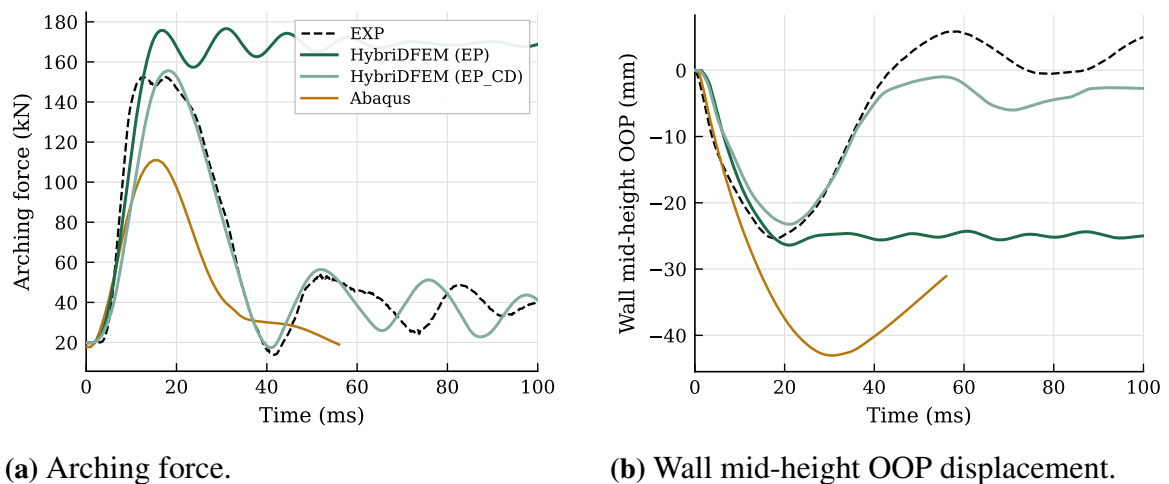


Figure 4.51: W24 — arching force and wall response.

At moderate energy the global rocking mechanism is fully activated across all three walls, and the response is dominated by wall inertia.

Impactor response. Force is underestimated by all models across the group (EP: 0.33–0.71; CD: 0.37–0.80; Abaqus: 0.14–0.58), with Abaqus producing the most severe. Part of this underestimation is attributable to the modelling assumption that the two impactor beams contact the wall simultaneously and uniformly, whereas the experimental records indicate a slight sequential activation of the two contact lines. Absent strain-rate strengthening in both frameworks further contributes to the discrepancy, as discussed in limitations. The post-peak secondary oscillations in the experimental force signal are reproduced qualitatively by both HybriDFEM variants (Figure 4.50a) but not by Abaqus, which produces only a smooth decaying pulse.

The impactor displacement is overestimated by all models. EP and CD give similar peak ratios (1.08–1.29 and 1.18–1.34 respectively) closer to the experimental. Abaqus overesti-

mates more and does not reproduce the experimental rebound rate (Figure 4.50c). Both EP and CD capture the velocity reversal timing and the late-time plateau consistently with the experiment (Figure 4.50b); Abaqus decelerates more slowly and does not settle within the window.

Wall response The arching history for W24 (Figure 4.51a) illustrates the contact deletion effect directly. EP fails to discharge after peak and remains elevated through 100 ms. CD restores the post-impact arching discharge, oscillating with frequency and amplitude comparable to the experimental record. Since the parameters were calibrated under `ElastoPlastic`, the good CD agreement at moderate energy reflects that the energy balance shift from contact deletion is not yet significant at this level. In Campaign 2, EP arching ratios for line-load walls W18 and W19 were 1.51 and 1.54; the present CD results confirm that this overestimation was attributable to the joint-locking artefact rather than to the parameter values. Abaqus underestimates the arching peak and does not reproduce the post-peak oscillatory unloading. This Abaqus arching underestimation for line-load walls was already present in Campaign 2 (W18: 0.61, W19: 0.52) and is reproduced here in the same direction.

The wall OOP displacement shows the same pattern. EP locks at peak with no recovery; CD rebounds partially, following the experimental direction of motion (Figure 4.51b). For Abaqus, the walls where the OOP displacement is most overpredicted also show the lowest arching force ratios. This is physically coherent since a larger lateral wall displacement reduces the effective eccentricity of the compressive arching strut, geometrically softening the arching action. This coupling is consistent across both the moderate and moderate-high energy groups.

4.5.3 Moderate-high energy — W25, W28, W32 ($KE = 7909\text{--}8173\text{ J}$)

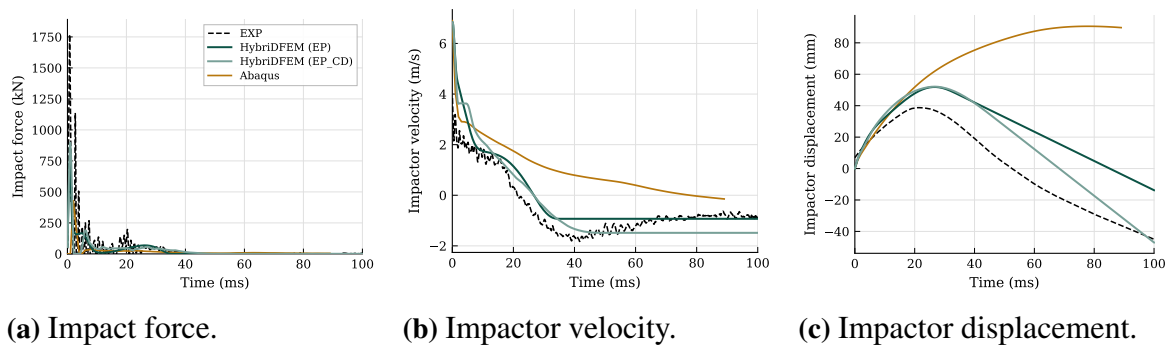


Figure 4.52: W28 — impactor response ($m = 348\text{ kg}$, $v_0 = 6.90\text{ m/s}$), representative of the moderate-high energy group.

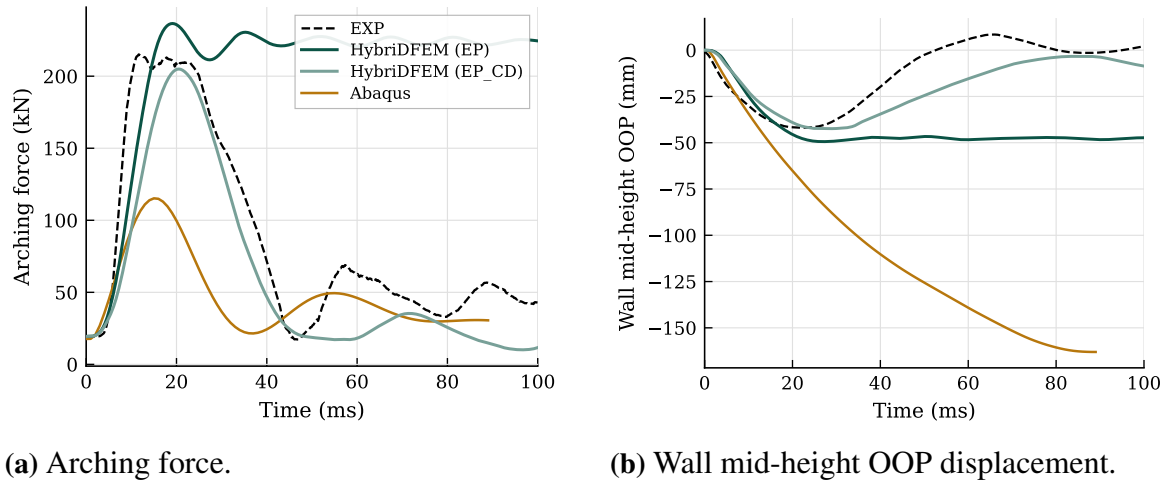


Figure 4.53: W28 — arching force and wall response.

At moderate-high energy the rocking mechanism is fully developed across all three walls. This is the energy group where the divergence between HybriDFEM and Abaqus global predictions is most apparent.

Impactor response. Force is underestimated by all models, with the same physical arguments as the previous group. The post-peak secondary force structure visible experimentally is reproduced qualitatively by both HybriDFEM variants but not by Abaqus, which produces only a smooth decaying pulse (Figure 4.52a).

The impactor displacement is overestimated by all models, with EP and CD giving nearly identical peak ratios closer to the experimental one. Abaqus overpredicts substantially more. The displacement history for W28 (Figure 4.52c) shows that after the peak both HybriDFEM variants follow the experimental rebound trend at a comparable rate, though the absolute values diverge as the models overestimated the peak. Abaqus does not reverse within the simulation window.

The velocity history (Figure 4.52b) reveals an important distinction between EP and CD. EP plateaus at a late-time velocity close to the experimental level, while CD overshoots to a more negative value, reflecting the additional kinetic energy released when joints are allowed to recover under EP-calibrated parameters. Abaqus decelerates far more slowly and barely approaches zero at ~90 ms.

Wall response EP arching ratios are 1.10–1.12 across all three walls, this is consistent with the joint-locking artefact producing the same overestimation regardless of impactor mass. The arching history for W28 (Figure 4.53a) shows EP staying at peak level with no discharge. CD restores the post-impact arching discharge, qualitatively following the experimental unloading shape, though the discharged level is not perfectly tuned since the parameters were not recalibrated for the CD law. A clear trend is visible, as impactor mass increases from W25 to W32, the CD arching ratio decreases progressively, suggesting that heavier impactors drive more joint opening and the subsequent recovery releases slightly more arching force than the experiment shows. The same inverse OOP–arching coupling observed in the moderate group is present in Abaqus.

The wall OOP displacement (Figure 4.53b) provides the clearest illustration of what CD was developed to demonstrate. EP locks at peak with no recovery. CD rebounds in the

correct direction after peak. Recalibration of CD lies outside the scope of this work. Abaqus continues monotonically past the experimental peak with no reversal.

4.5.4 High energy — W29, W33, W34, W35 ($KE = 12086\text{--}14399\text{ J}$)

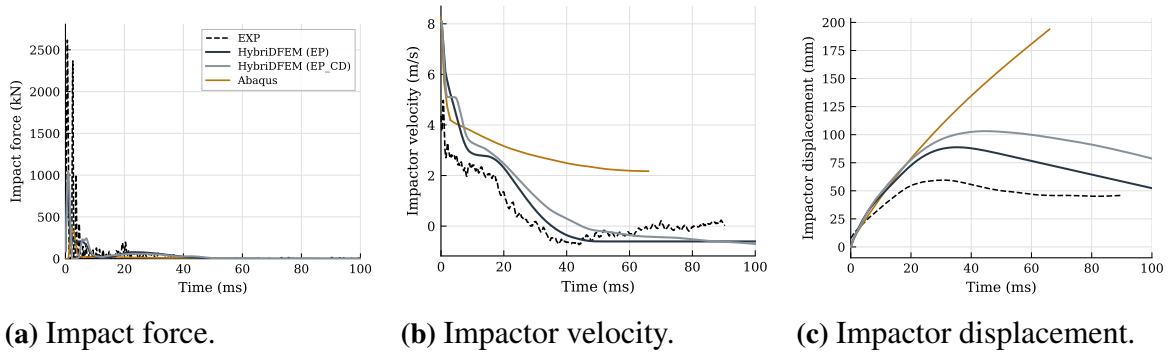


Figure 4.54: W33 — impactor response ($m = 442\text{ kg}$, $v_0 = 8.25\text{ m/s}$), representative of the high-energy group.

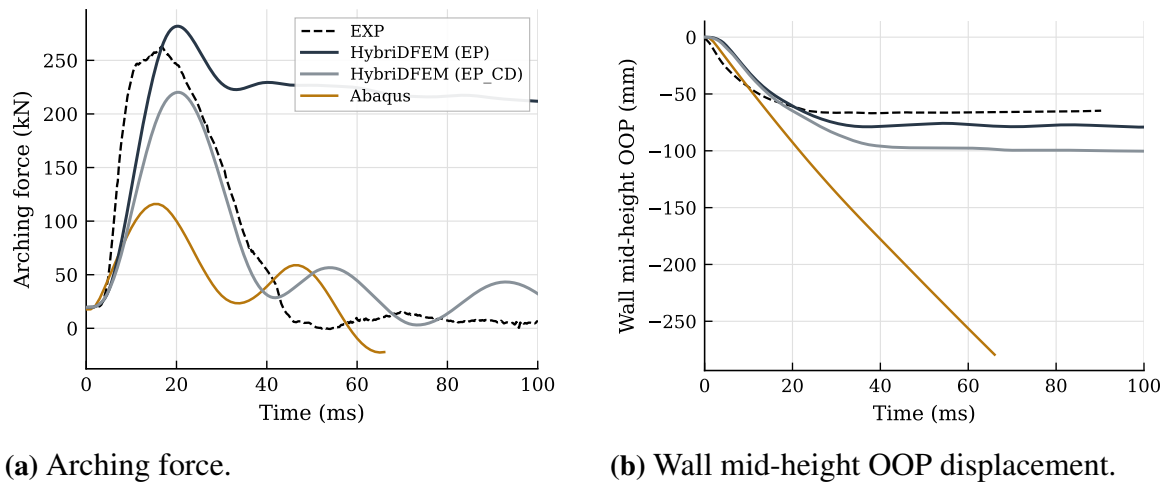


Figure 4.55: W33 — arching force and wall response.

The four high-energy walls (W29, W33, W34, W35) span $KE = 12086\text{--}14399\text{ J}$ with impactor masses of 236–442 kg, and represent the most energetic tests of the campaign.

Impactor response. Force underestimation is the most severe of any group, consistent across all three models and all four walls. The simultaneous-contact assumption and absent strain-rate strengthening are the dominant contributing factors, as discussed for the previous groups. EP and CD give nearly identical force ratios since the contact law is unchanged between the two formulations; the post-peak secondary force structure is reproduced qualitatively by both HybridFEM variants.

Impactor displacement is overestimated by all models. EP and CD both show a slow post-peak descent (Figure 4.54c), qualitatively consistent with the experimental trajectory: neither model nor experiment returns to zero within 100 ms, confirming that the wall and

impactor remain at a large out-of-plane position after impact. Abaqus does not reverse direction within the simulation window, meaning the impactor continues penetrating at an approximately constant rate until termination, reaching displacements far beyond the experimental peak. The velocity history (Figure 4.54b) confirms this, EP and CD both decelerate and plateau at a small negative velocity, broadly consistent with the experiment, while Abaqus remains at large positive velocity at termination.

Post-impact wall behaviour and arching. The most physically important observation of the campaign comes from the wall OOP displacement (Figure 4.55b). For moderate and moderate-high energy walls the experimental wall rebounded after peak within 100 ms. For all four high-energy walls this no longer occurs, the experimental wall reaches its peak and then remains approximately flat, with no elastic recovery within the observation window. At this energy level the plastic joint deformation accumulated during impact is large enough that the arching restoring force is insufficient to drive wall recovery.

Both EP and CD lateral displacements stabilise near a flat plateau after peak, consistent with the experiment, though both overestimate the magnitude. The arching history for W33 (Figure 4.55a) shows the key distinction, EP peaks close to the experimental value but remains elevated through 100 ms; CD underestimates the peak but correctly discharges toward zero, reproducing the experimental post-peak behaviour. Abaqus does not settle, the wall OOP continues increasing monotonically ($w/w_{\text{exp}} = 3.35\text{--}6.60$) and the arching force is severely underestimated, reflecting the same inverse OOP–arching coupling observed in the previous groups.

The CD quantitative discrepancies at high energy are a consequence of EP-calibrated parameters; recalibration lies outside the scope of this work. The CD results confirm that the no-rebound behaviour of `ElastoPlastic` was caused by the joint-locking artefact, not by a fundamental limitation of the rigid-block rocking model.

4.5.5 Failure mechanisms

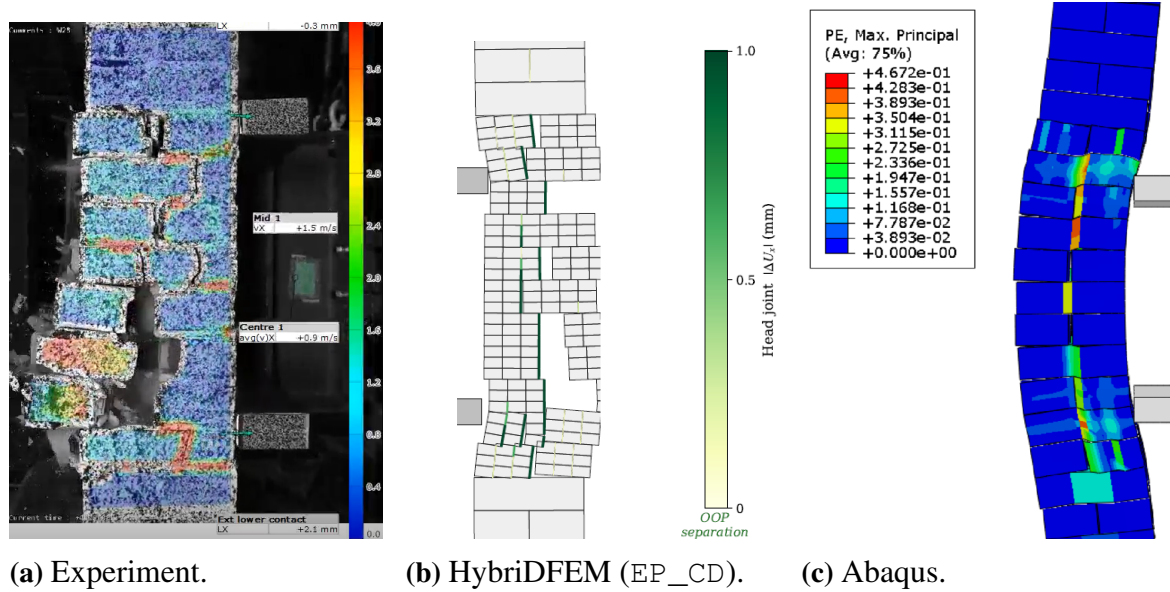


Figure 4.56: W28 — failure mode comparison at peak displacement ($m = 348 \text{ kg}$, $v_0 = 6.90 \text{ m/s}$): experimental damage pattern, HybriDFEM (ElastoPlastic_CD) joint separation and deformed shape, and Abaqus tensile damage field.

Both the experimental DIC field (Figure 4.56a) and the HybriDFEM head-joint separation field (Figure 4.56b) show OOP displacement concentrating at approximately $H/3$ and $2H/3$, consistent with the expected three-block rocking mechanism and the hinge formation.

Within the hinge bands, the refined 3×6 sub-block discretisation allows HybriDFEM to capture two distinct failure mechanisms visible in the experimental DIC. The first is block dislocation, where individual brick units slide out of their original position along the mortar joints, which act as preferential failure planes due to their lower tensile strength. The second is intra-brick splitting, also known in the literature as spalling, where the compressive wave generated at impact propagates through the wall thickness and reflects as a tensile wave at free surfaces and joint discontinuities, producing internal fracture perpendicular to the wave direction. In HybriDFEM, both mechanisms are represented through the differential OOP displacement of adjacent sub-blocks; a coarser single-block model would not provide the internal joints needed to capture either.

The Abaqus tensile damage field (Figure 4.56c) shows damage concentrating in the same hinge regions at approximately $H/3$ and $2H/3$, qualitatively consistent with both the experimental and HybriDFEM patterns. Although the Abaqus model does not accurately predict the global kinematic response at higher impact energies, as discussed in Section 4.5.3, the failure mechanism is still well captured. The damage field indicates localised sliding between brick courses and a concentration of plastic strain in the hinge zones, consistent with the experimentally observed spalling and brick dislocation at the impact location. The CDP formulation distributes damage as smeared plastic straining over the continuum rather than as discrete crack opening or block separation, so the localised brick dislocation and intra-brick splitting are not explicitly reproduced, but the location and extent of the damaged region are physically consistent with the experimental observations.

Both the experimental and numerical patterns are asymmetric about the wall centreline, with separation concentrating toward the impacted face. This is consistent with the geometry of the compressive arching strut, which is inclined rather than purely vertical, concentrating compressive stresses and the associated tensile wave reflections toward the loaded face.

Neither framework models progressive damage from repeated impacts, limiting the comparison to first-impact conditions (Section 3.3.8). Abaqus damage field comparisons for all Campaign 3 walls are provided in Appendix K.

4.5.6 Discussion

Conditions where each framework is best suited.

- **HybridFEM** is the preferred framework for global kinematic predictions at moderate to moderate-high energy ($4000 \lesssim KE \lesssim 10000$ J): arching force within $\pm 5\text{--}10\%$, wall OOP within $1\text{--}10\%$, and runs completed in a couple of hours. These quantities are directly relevant for designing adjacent structural elements and assessing boundary uplift forces. The framework also captures local failure mechanisms including block dislocation and intra-brick fracture through the sub-block discretisation. The `ElastoPlastic_CD` extension demonstrates that post-impact wall rebound and arching discharge are physically achievable within the framework, the absence of contact deletion in `ElastoPlastic` was the cause of the no-rebound behaviour, not a fundamental limitation of the rigid-block model. At high energy, joint recovery releases more kinetic energy than the EP-calibrated parameters can dissipate, producing excessive rebound; a dedicated recalibration of $k_{n,\text{imp}}$ and ξ under the CD law is required to extend predictive accuracy into this regime. Where only peak metrics at maximum displacement are needed and post-impact recovery is not of interest, `ElastoPlastic` provides a conservative bound on arching force across all energy levels.
- **Abaqus** performs best at low energy ($KE \lesssim 1000$ J) and provides more detailed local stress and damage fields — intra-brick tensile damage, compressive strut visualisation, and continuum strain localisation — that complement the HybridFEM joint separation maps for failure pattern interpretation. At moderate to high energy its global kinematic predictions deteriorate substantially and runs become computationally prohibitive without reduction of the time-scaling factor and calibration of dynamic material properties.

5 Conclusions

This thesis developed, calibrated, and assessed two parallel numerical frameworks for modelling the response of unreinforced masonry walls to pendulum impact loading. The two frameworks, HybriDFEM, a two-dimensional discrete rigid-block model, and Abaqus/Explicit, a three-dimensional continuum finite element model using the Concrete Damaged Plasticity formulation, were assessed independently and comparatively against the same experimental dataset from three impact campaigns conducted at RISE Research Institutes of Sweden (Section 4.5), covering a wide range of impact energies (137–14399 J), impactor masses (236–442 kg), and loading configurations. The following conclusions are drawn from the work.

5.1 Calibration strategy and parameter identification

The two frameworks followed fundamentally different calibration paths. For HybriDFEM, the only parameter identified from the quasi-static four-point bending tests was the normal interface stiffness $K_1 = 17$ GPa/m. This value was independently confirmed by modal cross-validation against experimentally identified frequencies from the SIII dynamic identification campaign. The remaining impact-specific parameters, such as impactor contact stiffness, friction coefficient, and damping ratio were calibrated from Campaign 1 and 2 impact data and transferred without modification to Campaign 3.

For Abaqus, the majority of the material parameters, such as elastic moduli, tensile and compressive fracture energies, interface stiffness, and friction coefficient were calibrated from the four-point bending tests. The only parameter subsequently refined from the impact campaigns was the interface damping coefficient. Both frameworks confirmed that the calibrated parameters transfer to Campaign 3 without re-fitting, establishing that quasi-static tests combined with a limited set of dynamic data provide a sufficient calibration basis for both frameworks.

5.2 HybriDFEM

HybriDFEM accurately captures the impactor and wall response across all energy levels tested in this study, with the quality of agreement depending on the governing regime. At low impact energies, the response is contact-dominated: the wall barely moves during impact and the force peak is governed almost entirely by how the impactor-wall contact stiffness is defined. Any simplification in the contact law introduces larger relative errors in this regime, as the linear elastic contact model does not reproduce the nonlinear Hertz-type response of the physical contact. As energy increases, the response transitions to a system-dominated regime in which wall inertia and global rocking dynamics take over, the influence of the contact law decreases, and the agreement with the experimental response improves substantially.

Within this system-dominated regime, the framework captures the impactor force, velocity, and displacement histories well, and the wall kinematics and out-of-plane displacement profiles are reproduced with good accuracy. For the arching force, the original `ElastoPlastic` formulation provides a conservative upper bound across all energy levels, while the `ElastoPlastic_CD` extension developed in this thesis restores the physically correct post-impact arching discharge and wall rebound. Failure mechanisms, including block dislocation and intra-brick

fracture, are captured through the sub-block discretisation and are qualitatively consistent with the experimental DIC observations. Campaign 3 confirms that the calibrated parameter set transfers without modification across a wide range of impactor masses and impact energies, demonstrating the predictive capability of the framework beyond its calibration range.

Beyond predictive accuracy, HybriDFEM is fast, reliable, and computationally affordable, completing impact simulations in minutes to hours compared to the days required by Abaqus. The framework also supports connection to adjacent structural elements, beams and columns via Timoshenko beam elements, enabling full structural assessment of wall-structure systems in ways that are not feasible at this scale with a three-dimensional continuum model.

The framework carries inherent limitations that define the boundaries of its current applicability and motivate future development. The two-dimensional formulation does not capture three-dimensional effects such as out-of-plane torsion, spatial variability of boundary conditions, or localised three-dimensional cracking phenomena. The impactor contact law is linear elastic and does not account for cohesion softening or strain-rate effects on the joint parameters. The `ElastoPlastic_CD` extension was developed and assessed in this thesis but was not adopted as the primary calibrated model; a full recalibration at high impact energies remains as future work. Extension to repeated impacts on pre-damaged walls, to blast loading scenarios, and ultimately to a three-dimensional HybriDFEM formulation are identified as the primary directions for future development.

5.3 Abaqus

The Abaqus framework provides a three-dimensional continuum representation of the wall response, capturing intra-brick stress fields, tensile damage propagation, compressive strut geometry, and localised failure patterns that are not accessible from a rigid-block model. At low impact energies and in the four-point bending configuration, the framework reproduced the wall response satisfactorily and the failure mechanisms were consistent with the experimental observations. The parameter transfer to Campaign 3 was confirmed, with the model capturing the overall wall behaviour qualitatively across the tested configurations.

At moderate to high impact energies, the global kinematic predictions deteriorated, with the impactor penetrating further into the wall than observed experimentally and the wall out-of-plane displacement being substantially overestimated. Three contributing factors were identified. The time scaling factor was calibrated for lower-energy walls and may introduce inaccuracies at higher velocities. The brick elastic modulus adopted in the model is lower than the value of the physical material, making the wall more compliant than the real specimen under rapid loading. The most likely dominant reason, however, is the absence of strain-rate dependent material strengthening. Under impact loading, masonry strain rates can reach 10^2 s^{-1} , at which both brick strength and stiffness are measurably higher than their quasi-static values. The omission of the Dynamic Increase Factor means that the model systematically underestimates the wall resistance as impact energy increases.

A preliminary test of the DIF approach was carried out on one representative wall, applying a constant strength amplification factor to the tensile and compressive properties, available in Appendix L. The available results indicate that this correction improves the predicted peak resistance and brings the response closer to the experimental measurements, suggesting that the DIF is physically the most relevant missing ingredient. However, this was tested

on a single wall only, and a broader confirmation across multiple walls and energy levels is needed before any definitive conclusion can be drawn. Furthermore, the DIF is expected to vary between walls depending on the local strain rate distribution, which is a function of the impact velocity and wall configuration, meaning that a wall-specific calibration would be required. The experimental strain rates were not available in this study, so the DIF could not be computed rigorously and had to be assumed. A systematic DIF calibration based on measured strain rates and dedicated dynamic material testing is identified as the most important direction for future work to extend the predictive capability of the Abaqus framework.

Despite its limitations at higher energies, Abaqus remains a valuable tool for detailed three-dimensional damage assessment, providing physical insight into local failure patterns that complements the global kinematic output of HybriDFEM.

5.4 Overall assessment and recommendations

The two frameworks are complementary rather than competing. HybriDFEM delivers better predictive accuracy at moderate to high impact energies, where the system-dominated regime aligns with the assumptions of the rigid-block model, and does so at a fraction of the computational cost. Abaqus provides richer physical detail at the local level and performs reliably at low energies, but currently falls short at higher energies due to the missing dynamic material effects. If the DIF is properly incorporated and calibrated, the Abaqus framework has the potential to match HybriDFEM in accuracy across a broader energy range while retaining its unique advantage in three-dimensional damage visualisation.

Together, the two frameworks provide a more complete picture of URM wall behaviour under impact than either could alone. Both confirmed that arching action is the central mechanism governing the lateral resistance of confined walls under impact loading, and that its correct representation requires physically consistent boundary conditions, contact formulations, and material descriptions. HybriDFEM is recommended as a general-purpose tool for the structural assessment of masonry walls under impact loading: it captures global kinematics, joint opening, block dislocation, and failure mechanisms, and its native compatibility with classical finite elements makes it straightforward to connect walls to surrounding structural components such as beams, columns, and floor systems. This connectivity enables the assessment of critical structural elements beyond the wall itself, extending the framework to realistic building configurations subjected to impulsive loads. The limitation of HybriDFEM relative to Abaqus lies in the absence of an internal stress field within the rigid blocks: intra-brick stress distributions, tensile damage propagation, and spalling cannot be resolved. Abaqus is therefore recommended as a complementary tool where three-dimensional local damage patterns and internal stress fields are required, or where the impact energy is sufficiently low that the contact regime governs and the continuum representation of the brick material is needed. The combined use of the two frameworks provides a practical and scalable modelling strategy for the structural safety assessment of masonry buildings exposed to accidental impact loading.

6 References

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A Appendix A: Interface Stiffness Derivations and Mesh Refinement Invariance

This appendix derives the interface stiffnesses used in the HybriDFEM model and verifies that local mesh refinement does not introduce artificial compliance. The derivation is based on a one-dimensional series-spring representation of the brick and mortar layers in the interface-normal direction.

A.1 Physical basis: rigid blocks and deformable interfaces

In HybriDFEM, masonry units are represented as rigid blocks. All deformability is therefore concentrated at the contact pairs between adjacent block faces. Each contact pair contains two side springs in series, one associated with each block face (Figure A.1).

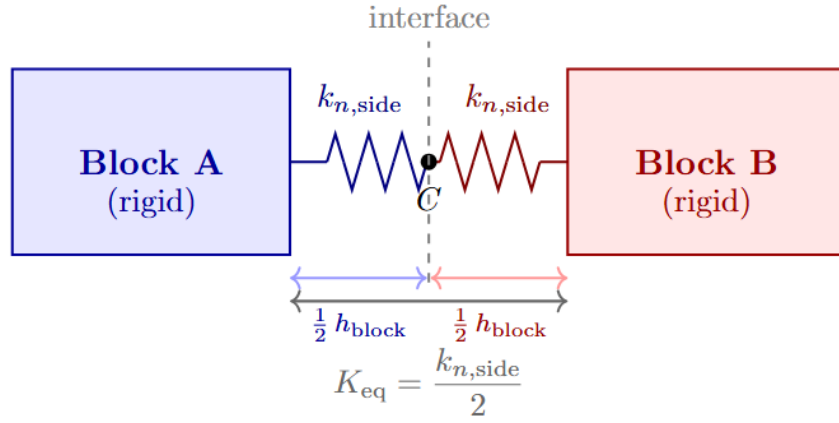


Figure A.1: Contact pair between two rigid blocks. The two side springs act in series, so the effective interface stiffness is $K_{eq} = k_{n,side}/2$.

Since the blocks are rigid, each side spring represents the elastic deformability of one half of the interacting block, or one half of the corresponding sub-block in the refined model. The spring stiffness is therefore derived from the elastic modulus of the block material and the relevant tributary half-height.

For two springs of equal stiffness $k_{n,side}$ in series, the effective interface stiffness is

$$K_{eq} = \frac{k_{n,side}}{2} \iff k_{n,side} = 2 K_{eq}. \quad (A.1)$$

Thus, the stiffness calibrated from experiments or derived analytically is always the *effective interface stiffness* K_{eq} , while the value assigned to each HybriDFEM side spring is twice this value. All derivations in this appendix follow this convention.

All stiffnesses are defined per unit area, with units of Pa/m ($\equiv$ N/m³).

A.2 Effective stiffness from the four-point bending calibration

The four-point bending calibration (Appendix C) provides the effective coarse bed-joint stiffness used in the HybriDFEM model:

$$K_1 = 17 \text{ GPa/m.}$$

This value represents the composite stiffness of a real bed joint, including the mortar layer and the adjacent brick deformability.

In the series-spring idealisation, a Type 1 bed joint consists of three elements in series: half of the upper block, the mortar joint, and half of the lower block, as shown in Figure A.2. For a layer of thickness t and elastic modulus E , the stiffness per unit area is $K = E/t$. Thus, for a full block height h_{block} , the half-block stiffness is

$$K_{\text{half}} = \frac{E}{h_{\text{block}}/2} = \frac{2E}{h_{\text{block}}}. \quad (\text{A.2})$$

The effective Type 1 stiffness is therefore

$$\frac{1}{K_1} = \frac{2}{K_{\text{half}}} + \frac{1}{K_m}, \quad (\text{A.3})$$

where K_m is the mortar stiffness per unit area.

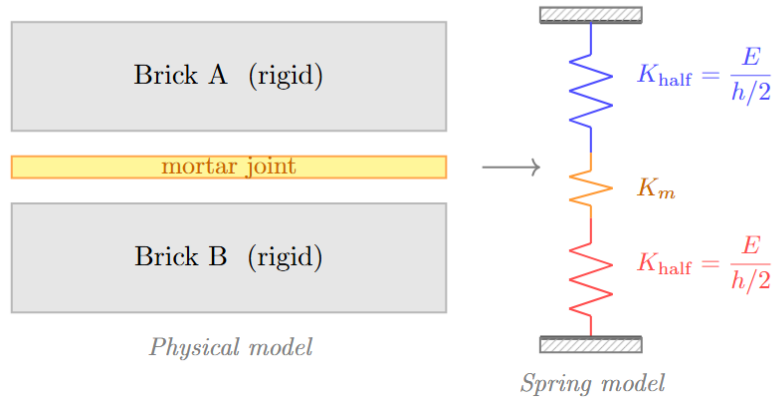


Figure A.2: Series-spring model for a Type 1 bed joint between two full-height blocks.

The mortar stiffness used in the subsequent interface derivations is obtained by rearranging Equation (A.3):

$$K_m = \left(\frac{1}{K_1} - \frac{2}{K_{\text{half}}} \right)^{-1}. \quad (\text{A.4})$$

Finally, using the HybriDFEM side-spring convention (Equation (A.1)), the Type 1 input stiffness is

$$k_{n,\text{side}}^{(1)} = 2K_1 = 34 \text{ GPa/m.} \quad (\text{A.5})$$

See Type 1 in blue, in Figure A.3.

A.3 Refinement geometry

When a parent block of height h_{block} is subdivided into n_y equal sub-rows, each sub-block has height:

$$h_{\text{sub}} = \frac{h_{\text{block}}}{n_y}, \quad (\text{A.6})$$

and the elastic stiffness of one sub-block over its full height is:

$$K_{\text{sub}} = \frac{E}{h_{\text{sub}}} = \frac{n_y E}{h_{\text{block}}} = \frac{n_y}{2} K_{\text{half}}. \quad (\text{A.7})$$

Each spring in a contact pair involving a sub-block represents the elastic deformability of *half* of that sub-block, i.e., a layer of thickness $h_{\text{sub}}/2$, with stiffness $2K_{\text{sub}}$.

Similarly, when a parent block of width L_{block} is subdivided horizontally into n_x segments, each sub-block has width:

$$\Delta x = \frac{L_{\text{block}}}{n_x}, \quad (\text{A.8})$$

and the stiffness per unit area in the horizontal normal direction is:

$$K = \frac{E}{\Delta x} = \frac{n_x E}{L_{\text{block}}}. \quad (\text{A.9})$$

In the present work, $n_y = 3$ and $n_x = 2n_y = 6$ (Appendix B.3.8). See Figure A.3 below and the derivation of each type of interface in the following sub sections.

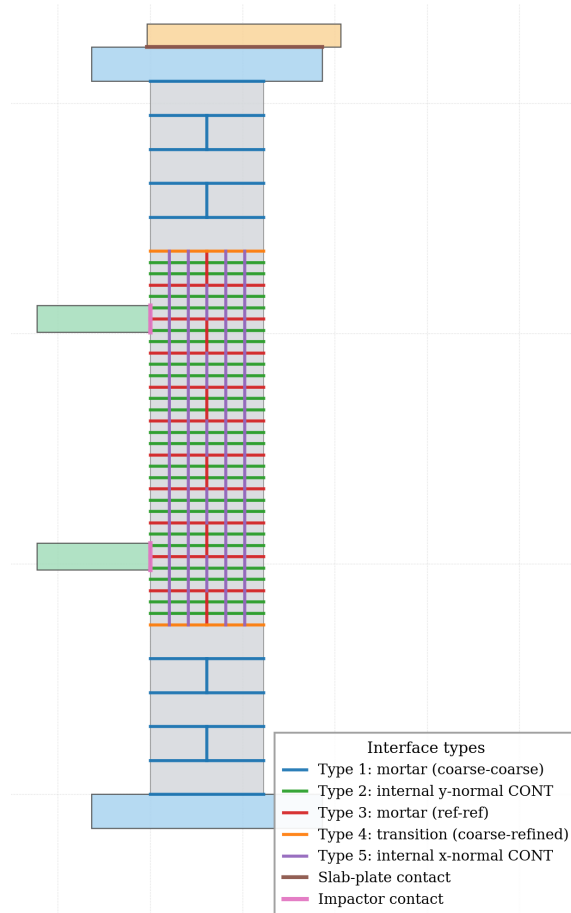


Figure A.3: Refined model, interfaces types.

A.4 Type 2 — Internal horizontal continuity cut

Type 2 interfaces arise between adjacent sub-blocks within the *same parent block* when that block is subdivided vertically (n_y sub-rows). No mortar joint is present; the interface represents elastic continuity through the brick material. See Type 2 in green, in Figure A.3. Each side spring represents half of the adjacent sub-block:

$$\underbrace{\left(\frac{h_{\text{sub}}}{2}, E \right)}_{\text{spring A}} + \underbrace{\left(\frac{h_{\text{sub}}}{2}, E \right)}_{\text{spring B}}.$$

The effective stiffness is therefore equal to the stiffness of a full sub-block layer:

$$K_2 = \frac{E}{h_{\text{sub}}} = K_{\text{sub}} = \frac{n_y}{2} K_{\text{half}}, \quad (\text{A.10})$$

and the HybriDFEM input is:

$$k_{n,\text{side}}^{(2)} = 2K_2 = n_y K_{\text{half}}. \quad (\text{A.11})$$

A.5 Type 3 — Refined mortar joint (refined–refined, different parents)

Type 3 interfaces represent real mortar joints between sub-blocks belonging to *different parent blocks* within the refined band. See Type 3 in red, in Figure A.3. Each side spring represents half of the adjacent sub-block plus the mortar joint is shared between the two:

$$\underbrace{\left(\frac{h_{\text{sub}}}{2}, E \right)}_{\text{Spring A (half sub-block)}} + \underbrace{K_m}_{\text{mortar}} + \underbrace{\left(\frac{h_{\text{sub}}}{2}, E \right)}_{\text{Spring B (half sub-block)}}.$$

In HybriDFEM, the mortar stiffness is incorporated into the side springs through the series-spring formula. Each side spring therefore represents:

$$\frac{1}{k_{\text{side}}^{(3)}} = \frac{1}{2K_{\text{sub}}} + \frac{1}{2K_m},$$

giving:

$$\frac{1}{K_3} = \frac{1}{K_{\text{sub}}} + \frac{1}{K_m} = \frac{2}{n_y K_{\text{half}}} + \frac{1}{K_m}, \quad (\text{A.12})$$

and:

$$k_{n,\text{side}}^{(3)} = 2K_3. \quad (\text{A.13})$$

A.6 Type 4 — Coarse-to-refined transition joint

Type 4 interfaces occur at the boundaries of the refined band, where a full-height coarse block meets a sub-block across a real mortar joint. See Type 4 in orange, in Figure A.3.

$$\underbrace{\left(\frac{h_{\text{block}}}{2}, E \right)}_{\text{spring A (coarse half)}} + \underbrace{K_m}_{\text{mortar}} + \underbrace{\left(\frac{h_{\text{sub}}}{2}, E \right)}_{\text{spring B (sub-block half)}}.$$

The two sides have different tributary heights, so the two side springs are not equal. Following the same series logic:

$$\frac{1}{K_4} = \frac{1}{K_{\text{half}}} + \frac{1}{K_m} + \frac{1}{2K_{\text{sub}}} = \left(1 + \frac{1}{n_y}\right) \frac{1}{K_{\text{half}}} + \frac{1}{K_m}, \quad (\text{A.14})$$

and:

$$k_{n,\text{side}}^{(4)} = 2K_4. \quad (\text{A.15})$$

A.7 Type 5 — Internal vertical continuity cut

Type 5 interfaces arise when a parent block is subdivided horizontally into n_x segments, introducing vertical cuts in the x -direction. See Type 5 in purple, in Figure A.3. The interface normal is horizontal, and no mortar is present. Each side spring represents half of the adjacent sub-block in the horizontal direction:

$$K_5 = \frac{E}{\Delta x} = \frac{n_x E}{L_{\text{block}}}, \quad k_{n,\text{side}}^{(5)} = 2K_5. \quad (\text{A.16})$$

A.8 Tangential stiffnesses

Tangential stiffnesses are derived by replacing E with $G = E/[2(1 + \nu)]$ throughout all expressions above:

$$K_i^t = \frac{G}{E} K_i \quad (i = 1, 2, 3, 4), \quad K_5^t = \frac{G}{\Delta x}, \quad k_{t,\text{side}}^{(i)} = 2K_i^t. \quad (\text{A.17})$$

The mortar tangential stiffness K_m^t is back-calculated analogously to Equation (A.4), using the consistent tangential target $K_1^t = (G/E) K_1$.

A summary of all five interface types, their physical meaning, and the corresponding stiffness expressions is provided in Table A.1.

Table A.1: Summary of interface types.

Type	Physical meaning	Location	K_i expression
1	Composite mortar joint	Coarse region	$\left(\frac{2}{K_{\text{half}}} + \frac{1}{K_m}\right)^{-1} = 17 \text{ GPa/m}$
2	Horizontal continuity cut	Within refined band	$K_{\text{sub}} = \frac{n_y}{2} K_{\text{half}}$
3	Mortar joint (ref–ref)	Within refined band	$\left(\frac{1}{K_{\text{sub}}} + \frac{1}{K_m}\right)^{-1}$
4	Transition (coarse–ref)	Band boundary	$\left(\frac{1}{K_{\text{half}}} + \frac{1}{K_m} + \frac{1}{2K_{\text{sub}}}\right)^{-1}$
5	Vertical continuity cut	Within refined band	$\frac{E}{\Delta x} = \frac{n_x E}{L_{\text{block}}}$

A.9 Analytical proof of axial invariance

The following demonstrates that the axial stiffness of the interface chain is preserved exactly when blocks within the assembly are replaced by refined sub-blocks, the stiffness assignments of Sections A.4–A.6 are applied.

Consider a coarse assembly where three bed joints connect four blocks in series. The total axial compliance is:

$$\frac{1}{K_{\text{tot,coarse}}} = \frac{3}{K_1} \implies K_{\text{tot,coarse}} = \frac{K_1}{3}. \quad (\text{A.18})$$

Now replace two adjacent blocks with refined sub-blocks (n_y sub-rows each). The interface chain between the remaining coarse blocks becomes:

$$K_4 - \underbrace{K_2 - \dots - K_2}_{(n_y-1) \text{ times}} - K_3 - \underbrace{K_2 - \dots - K_2}_{(n_y-1) \text{ times}} - K_4,$$

with total compliance:

$$\frac{1}{K_{\text{tot,ref}}} = \frac{2}{K_4} + \frac{2(n_y-1)}{K_2} + \frac{1}{K_3}. \quad (\text{A.19})$$

Substituting Equations (A.7), (A.12), and (A.14):

$$\begin{aligned} \frac{1}{K_{\text{tot,ref}}} &= 2 \left[\left(1 + \frac{1}{n_y} \right) \frac{1}{K_{\text{half}}} + \frac{1}{K_m} \right] + 2(n_y-1) \left[\frac{2}{n_y K_{\text{half}}} \right] + \left[\frac{2}{n_y K_{\text{half}}} + \frac{1}{K_m} \right] \\ &= \frac{2}{K_{\text{half}}} + \frac{2}{n_y K_{\text{half}}} + \frac{4(n_y-1)}{n_y K_{\text{half}}} + \frac{2}{n_y K_{\text{half}}} + \frac{2}{K_m} + \frac{1}{K_m} \\ &= \frac{2}{K_{\text{half}}} + \frac{4n_y - 4 + 2 + 2}{n_y K_{\text{half}}} + \frac{3}{K_m} \\ &= \frac{2}{K_{\text{half}}} + \frac{4(n_y-1) + 4}{n_y K_{\text{half}}} + \frac{3}{K_m} \\ &= \frac{2}{K_{\text{half}}} + \frac{4}{K_{\text{half}}} + \frac{3}{K_m} = \frac{6}{K_{\text{half}}} + \frac{3}{K_m} = 3 \left(\frac{2}{K_{\text{half}}} + \frac{1}{K_m} \right) = \frac{3}{K_1}. \end{aligned} \quad (\text{A.20})$$

Therefore:

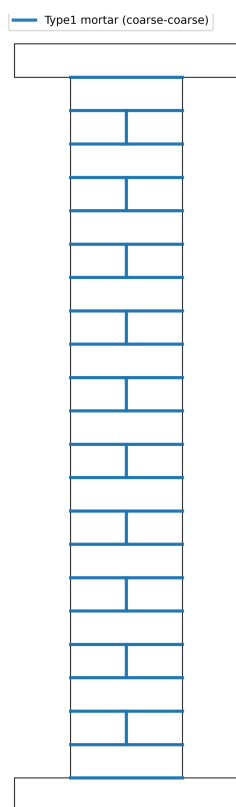
$$\boxed{K_{\text{tot,ref}} = \frac{K_1}{3} = K_{\text{tot,coarse}}.} \quad (\text{A.21})$$

The axial stiffness of the assembly is exactly preserved for any value of $n_y \geq 2$. The same invariance holds in the tangential direction under purely elastic response.

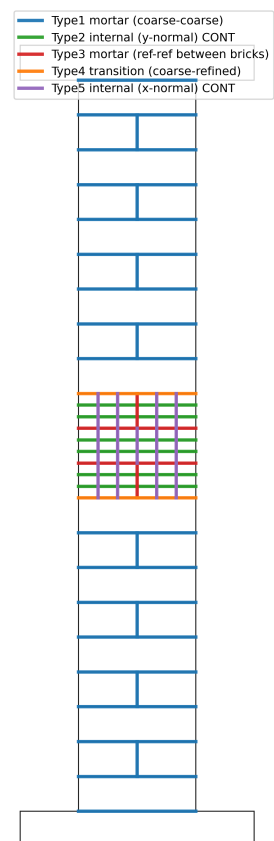
A.10 Numerical verification — wall model Step 1

The analytical result of Section B.9 was verified numerically using a full wall model. Two HybriDFEM models of wall W10 were constructed: a coarse model, shown in Figure A.4 (a) with Type 1 interfaces throughout, and a refined model, shown in Figure A.4 (b) with a local band of $n_y = 3$, $n_x = 6$ sub-blocks centred on the impact row (rows 9–11). Both models were subjected to identical Step 1 loading: self-weight applied to all blocks and an axial prestress of $\sigma_{ax} = 0.20$ MPa applied at the top slab node.

The base reaction F_y is plotted against the top slab vertical displacement $U_{y,\text{top}}$ in Figure A.5. The curves from the coarse and refined models are practically coincident across the full



(a) Coarse model: all interfaces Type 1.



(b) Refined model: Types 1–5 in refinement band.

Figure A.4: Interface classification before Step 1 solve.

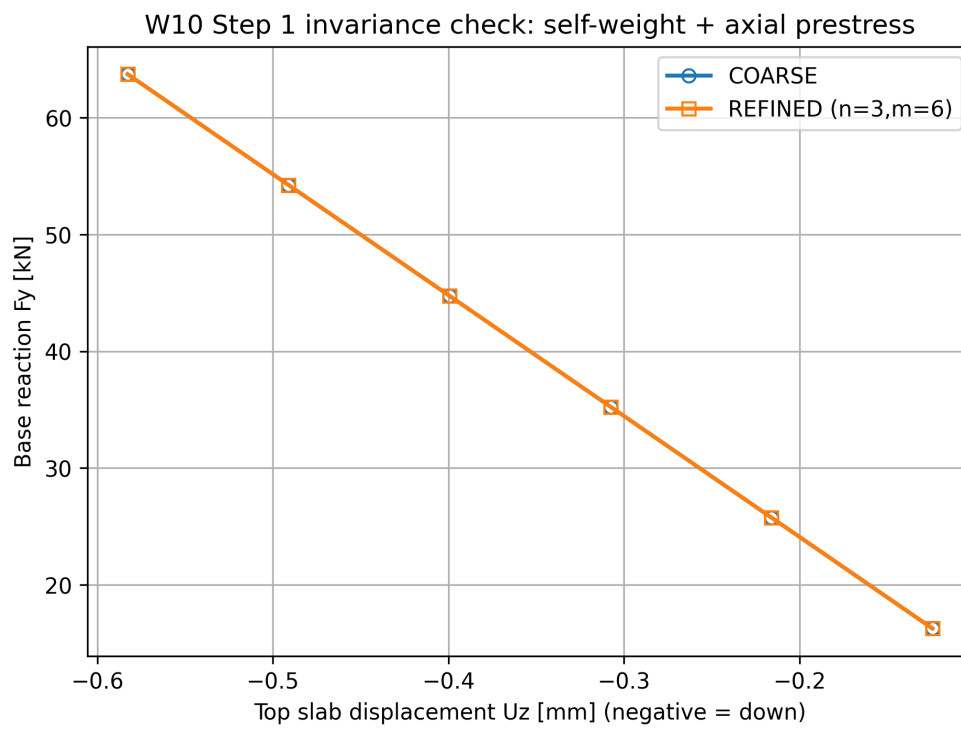


Figure A.5: Step 1 axial invariance check.

loading range, confirming that the refinement strategy and interface stiffness assignments do not alter the global pre-impact equilibrium state.

B Appendix B: HybriDFEM Model Verification

This appendix documents the verification checks performed on the HybriDFEM model prior to parameter calibration. The checks confirm that mass assignment, static equilibrium, spatial discretisation, modal response, and time integration are correctly implemented. Representative walls are selected for each campaign: W10 (Campaign 1, coarse model), W18 (Campaign 2, refined model), and W27 (Campaign 3, refined model, dual impactor).

B.1 Mass and Static Equilibrium

After Step 1, the total structural mass and base reaction are compared against analytically computed reference values. The expected base reaction is

$$F_{y,\text{base}} = W_{\text{self}} + F_{\text{axial}}, \quad (\text{B.1})$$

where W_{self} is the total self-weight of all structural components and F_{axial} is the applied axial prestress. The residual norm $\|\mathbf{R}_{\text{free}}\|$ and the top slab horizontal displacement $U_{x,\text{top}}$ are also checked after Newton–Raphson convergence.

Table B.1 summarises the results for the three representative walls. The impactor mass is listed for reference but excluded from W_{self} .

Table B.1: Mass breakdown and static equilibrium verification after Step 1 for representative walls from each campaign.

	W10 (C1)	W18 (C2)	W27 (C3)
<i>Mass breakdown</i>			
m_{wall} (kg)	499.9	497.2	538.4
m_{slabs} (kg)	190.0	230.0	174.9
$m_{\text{plate+beam}}$ (kg)	—*	—*	460.0
m_{impactor} (kg)	116.0	116.0	2×174
$m_{\text{total,structural}}$ (kg)	689.9	727.2	1173.3
<i>Loading</i>			
W_{self} (kN)	6.767	7.134	11.510
F_{σ} (kN)	37.975	9.506	19.836
F_{steel} (kN)	2.570	2.570	—†
F_{axial} (kN)	40.545	12.076	19.836
σ_{ax} (MPa)	0.20	0.05	0.10
<i>Equilibrium check</i>			
$F_{y,\text{expected}}$ (kN)	47.313	19.210	31.346
R_{F_y} computed (kN)	47.313	19.210	31.346
$\ \mathbf{R}_{\text{free}}\ $ (N)	3.25×10^{-10}	4.33×10^{-10}	1.40×10^{-9}
$U_{x,\text{top}}$ (mm)	$< 10^{-3}$	$< 10^{-3}$	$< 10^{-3}$
$U_{y,\text{top}}$ (mm)	−0.299	−0.107	−0.189

*Steel assembly modelled as static load; weight included in $F_{\text{steel}} = 2.570$ kN.

†Steel plate and beam (460 kg) incorporated into block density; weight included automatically in W_{self} .

The computed base reaction matches the expected value exactly in all three cases and the residual norms are below 2×10^{-9} N, confirming correct mass assignment, load application, and solver convergence. The negligible top slab horizontal displacement ($U_{x,\text{top}} < 10^{-3}$ mm) is consistent with symmetric gravity loading. The vertical displacement $U_{y,\text{top}}$ reflects the elastic compression of the wall and vertical spring under the combined self-weight and prestress, and increases consistently with the applied axial stress level across the three campaigns.

B.2 Contact Pair Sensitivity

The number of contact pairs per contact face (N_{CP}) controls the integration resolution of the traction field across each interface. A sensitivity study is performed on wall W10 (Campaign 1, coarse model) to identify the minimum value beyond which the global response becomes insensitive to further refinement.

Five values are tested: $N_{\text{CP}} \in \{2, 4, 8, 12, 16\}$. All simulations are run with the same pre-calibration working parameter set, including the W10-1 impact velocity, preliminary masonry and impact-contact stiffnesses, $\mu = 0.80$, $\xi = 3\%$, $\sigma_{ax} = 0.20$ MPa, and $\Delta t = 1 \times 10^{-4}$ s. These values are not used as calibrated model parameters; they are only kept fixed to isolate the effect of N_{CP} . The objective is therefore to assess discretisation convergence, not to reproduce the experimental record.

Figure B.1 shows the convergence of the peak impactor displacement, while Figure B.2 compares the corresponding displacement, velocity, and force histories.

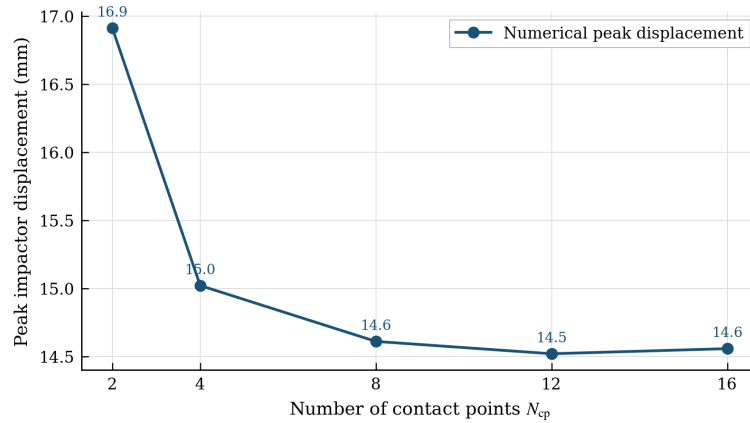


Figure B.1: Peak impactor displacement as a function of N_{CP} , wall W10.

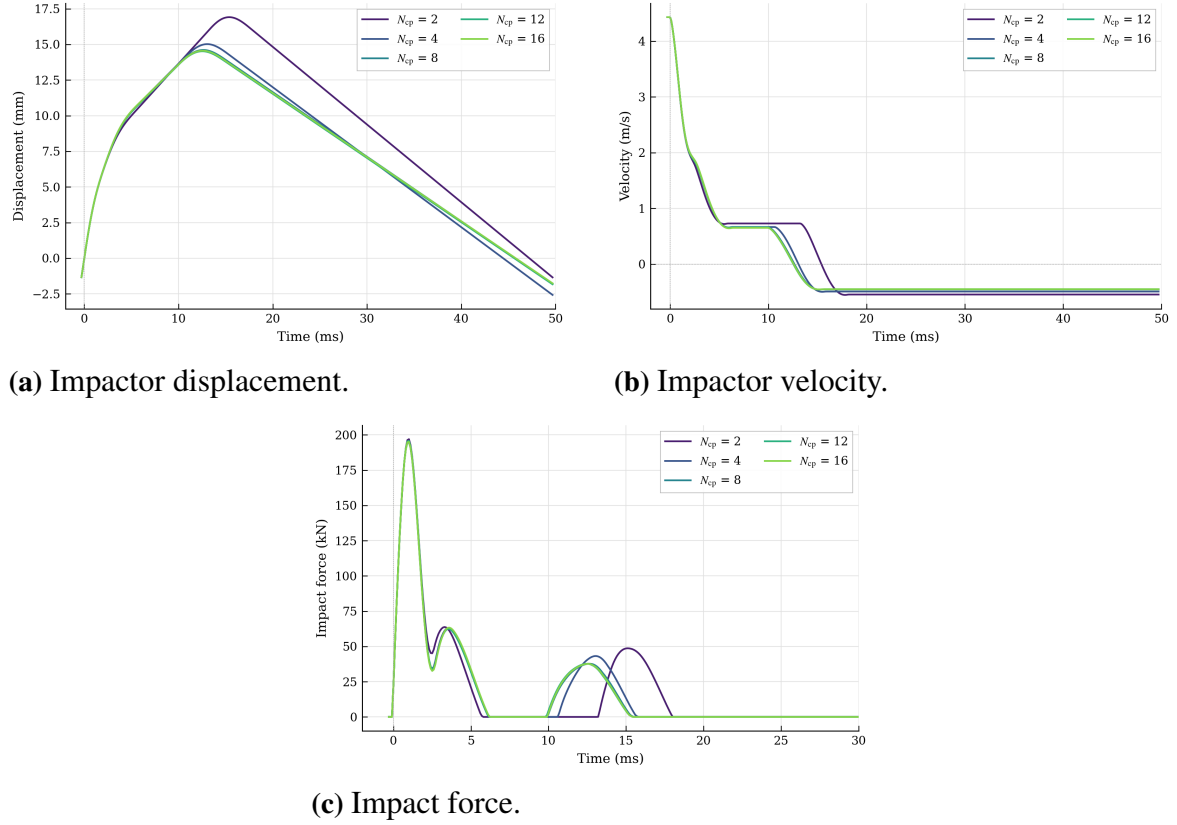


Figure B.2: Impactor response histories for $N_{CP} \in \{2, 4, 8, 12, 16\}$, wall W10 (Campaign 1, coarse model).

The peak displacement decreases from 16.91 mm at $N_{CP} = 2$ to 14.52 mm at $N_{CP} = 12$, with a change of less than 0.3% between $N_{CP} = 12$ and $N_{CP} = 16$. The full time histories are visually indistinguishable for $N_{CP} \geq 8$. On this basis, $N_{CP} = 8$ is adopted for all coarse-model interfaces. For refined interfaces, $N_{CP} = 2$ is used without a dedicated study, since the sub-block faces are geometrically small and the spatial resolution near the impact zone is provided by the band refinement itself.

B.3 Modal Analysis — Spectral Structure, Stability Limits, and Stiffness Matrix Regularisation

The modal analysis is performed on the HybriDFEM wall models after Step 1 for three purposes: (i) to determine the critical time step Δt_{crit} for the CDM integrator; (ii) to characterise the spectral structure and identify which modes are excited by the impact; and (iii) to diagnose and resolve a numerical pathology in $\mathbf{K}^{tan}$ arising from zero cohesion and the Flemish bond geometry.

The analysis uses wall W18 as representative of all Campaign 2 walls, which share identical geometry, boundary conditions, and prestress state. The coarse W18 model also provides Δt_{crit} for Campaign 1, whose geometry is nearly identical –except for the horizontal spring of top slab; no independent modal study is performed for Campaign 1 since experimental frequencies are unavailable. Campaign 3 results follow the same structure at the end of this section, cross-validation against experimental frequencies is also not possible for that campaign.

B.3.1 Theoretical background

The CDM integrator is conditionally stable with critical time step

$$\Delta t_{\text{crit}} = \frac{2}{\omega_{\text{max}}} = \frac{T_{\text{min}}}{\pi}, \quad (\text{B.2})$$

where ω_{max} is the highest angular frequency of the system. The analysis is performed for both the initial stiffness $\mathbf{K}_0$ (all interfaces elastic) and the tangent stiffness $\mathbf{K}^{\text{tan}}$ (at the end of Step 1); the impactor block is excluded in both cases.

Two participation metrics are used. The **global mass participation ratio**

$$\rho_i = \frac{(\boldsymbol{\varphi}_i^\top \mathbf{M} \mathbf{r})^2}{(\boldsymbol{\varphi}_i^\top \mathbf{M} \boldsymbol{\varphi}_i)(\mathbf{r}^\top \mathbf{M} \mathbf{r})}, \quad \sum_i \rho_i = 1, \quad (\text{B.3})$$

where $\mathbf{r}$ is the influence vector with unit entries at all U_x DOFs. This metric is familiar from seismic analysis, where the load is a distributed base excitation; it is less appropriate for an impact problem since the load is localised at a single DOF. It is retained here primarily as a diagnostic tool for the $\mathbf{K}^{\text{tan}}$ pathology described in Section B.3.3.

The physically appropriate metric for an impact problem is the **force-based participation factor** $\hat{\alpha}_i$:

$$\hat{\alpha}_i = \frac{|\boldsymbol{\varphi}_i^\top \mathbf{f}|/(\boldsymbol{\varphi}_i^\top \mathbf{M} \boldsymbol{\varphi}_i)}{\sum_j |\boldsymbol{\varphi}_j^\top \mathbf{f}|/(\boldsymbol{\varphi}_j^\top \mathbf{M} \boldsymbol{\varphi}_j)}, \quad (\text{B.4})$$

where $\mathbf{f}$ has a single non-zero entry at the impact DOF.

B.3.2 System sizes and stability limits

Four discretisation levels are considered. The coarse model is used for sweep across large sets of combinations for Campaigns 1 and 2 (See Appendix D and E); the refined models (3×6 , 5×10 , 7×14) subdivide the three rows nearest the impact point using the interface types derived in Appendix A. The Step 1 equilibrium state is identical across all levels and all Campaign 2 walls, and is reported in Table B.1. At Step 2, the base slab is fully fixed, the top slab is restrained in rotation only with U_x and U_y controlled by the vertical and horizontal springs, and two dummy spring-anchor nodes are fully fixed, giving a total of 10 constrained DOFs and hence $N_{\text{free}} = N_{\text{total}} - 10$. Table B.2 summarises the system sizes and stability limits.

Table B.2: System sizes and CDM stability limits for wall W18 ($c = 0.01$ MPa, $K_{\text{sup},x} = 1$ kN/mm). $\omega_{\text{max}}(\mathbf{K}_0) = \omega_{\text{max}}(\mathbf{K}^{\text{tan}})$ to within solver tolerance at all levels.

Model	Total DOFs	Free DOFs	ω_{max} (rad/s)	Δt_{crit} (s)	$\Delta t_{\text{adopted}}$ (s)
Coarse	105	95	2.381×10^4	8.40×10^{-5}	5×10^{-5}
Refined 3×6	252	242	1.597×10^5	1.252×10^{-5}	1×10^{-5}
Refined 5×10	540	530	2.870×10^5	6.969×10^{-6}	—
Refined 7×14	966	956	4.103×10^5	4.874×10^{-6}	—

Refined 5×10 and 7×14 not used in production; see Appendix B.3.8.

The reduction in Δt_{crit} with refinement level is super-linear, as shown in Table B.3. This follows from the vertical continuity interface stiffness scaling as $K_5 \propto E/\Delta x_{\text{sub}}$ (Appendix A): finer sub-blocks are narrower and therefore stiffer, raising ω_{max} through localised interface modes at the impact zone rather than through global wall dynamics. This is confirmed by the ω_{max} mode shape in Figure B.9.

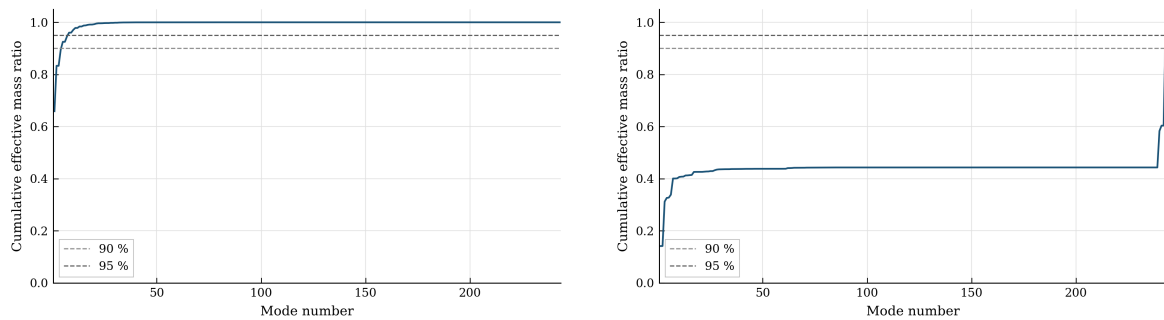
Table B.3: Scaling of Δt_{crit} with refinement level relative to the coarse model.

Model	Δt_{crit} (s)	Ratio vs. coarse	n_y
Coarse	8.40×10^{-5}	$1.0\times$	—
Refined 3×6	1.252×10^{-5}	$\approx 7\times$	3
Refined 5×10	6.969×10^{-6}	$\approx 12\times$	5
Refined 7×14	4.874×10^{-6}	$\approx 17\times$	7

B.3.3 The $\mathbf{K}^{\text{tan}}$ pathology and its resolution

With $c = 0$ MPa, the $\mathbf{K}_0$ cumulative mass participation is smooth and physically correct, while the $\mathbf{K}^{\text{tan}}$ curve stagnates at approximately 30–33% through nearly all modes and then jumps discontinuously to 100% in the final four to six modes (Figure B.3).

Root cause. At the vertical head joints in alternating Flemish bond courses, $\sigma_N \approx 0$ since these interfaces carry no compressive load from gravity. With $c = 0$ MPa, the Mohr–Coulomb threshold reduces to $|\tau| \leq \mu \sigma_N \approx 0$, so the infinitesimal shear from the staggered geometry places these contacts on the yield surface at the end of Step 1. The tangent stiffness of yielded contacts is $k_{\text{tan}} = 0$, making $\mathbf{K}^{\text{tan}}$ positive semi-definite. The eigensolver then returns slightly negative eigenvalues for the degenerate modes, and the completeness identity forces the unresolved mass into NaN modes — the apparent jump to 100% is a numerical artefact. Note that the global U_x participation metric is used here as a diagnostic only; the physically appropriate metric for an impact problem is the force-based factor of Section B.3.4.



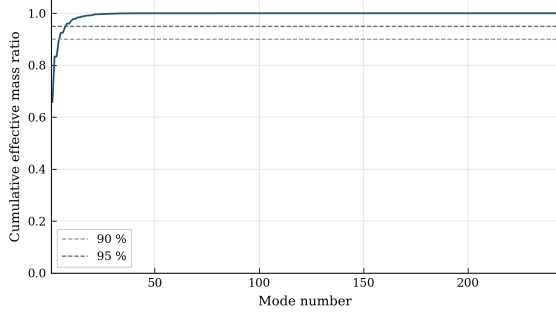
(a) $\mathbf{K}_0$, $c = 0$ MPa.

(b) $\mathbf{K}^{\text{tan}}$, $c = 0$ MPa.

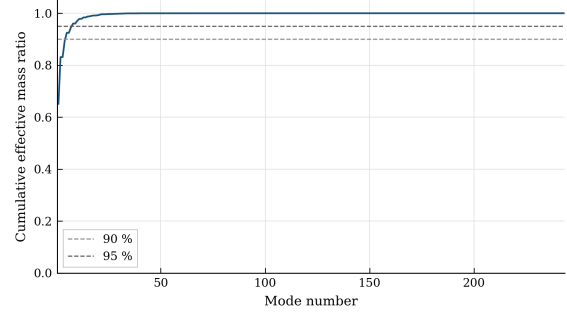
Figure B.3: Global U_x cumulative mass participation, refined 3×6 model, $c = 0$ MPa.

Fix and verification. A numerical cohesion $c = 0.01$ MPa is assigned to all mortar interfaces. Mortar cohesion is a real material property, but the elastoplastic contact law has no softening branch, so assigning the true value would produce an unrealistic permanent

tensile capacity at all joints. The value remains physically correct for the impact phase; the regulariser is introduced solely to remove the Step 1 pathology and is small enough to have no mechanical consequences during the dynamic response. The fix is verified by three criteria: (i) the Step 1 residual norm is unchanged; (ii) both participation curves are smooth and mutually consistent after regularisation (Figure B.4); and (iii) $\omega_{\max}(\mathbf{K}_0) = \omega_{\max}(\mathbf{K}^{\tan})$ to within solver tolerance at all discretisation levels (Table B.2).



(a) $\mathbf{K}_0$.

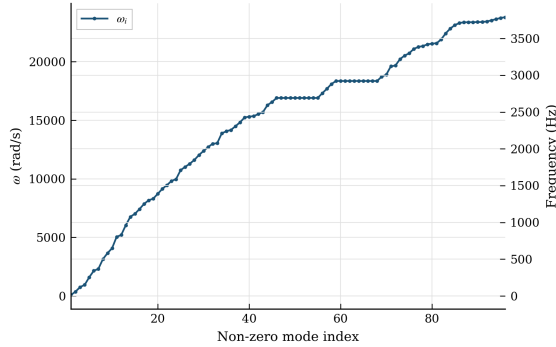


(b) $\mathbf{K}^{\tan}$.

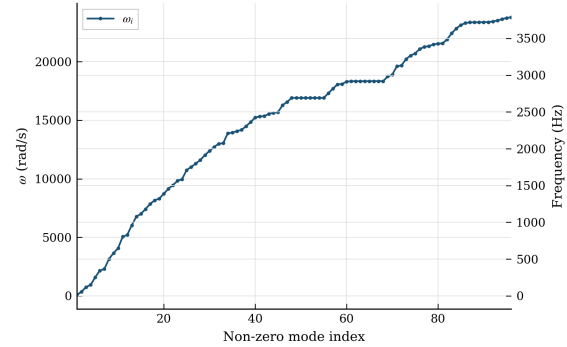
Figure B.4: Global U_x cumulative mass participation after regularisation ($c = 0.01$ MPa), refined 3×6 model.

B.3.4 Spectral structure and participation

Modal spectra. Figures B.5–B.8 show the modal spectra under $\mathbf{K}_0$ and $\mathbf{K}^{\tan}$ for each discretisation level. In all cases the two spectra are in close agreement, confirming that Step 1 prestress does not significantly alter the stiffness of the system.

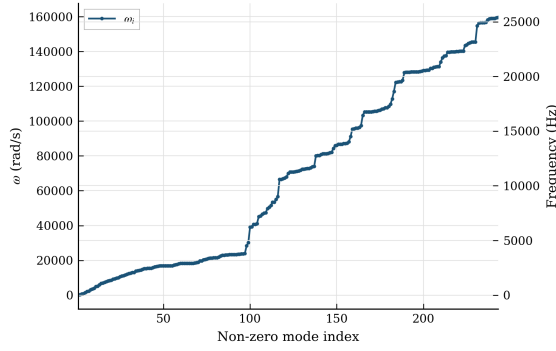


(a) $\mathbf{K}_0$.

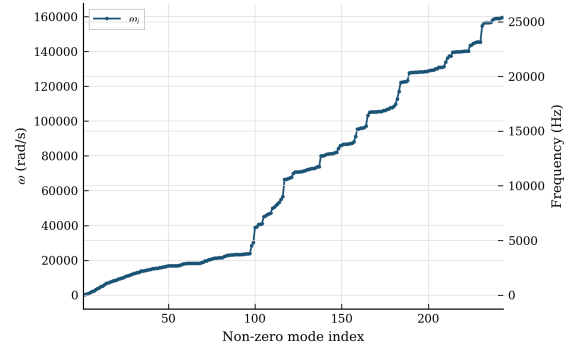


(b) $\mathbf{K}^{\tan}$.

Figure B.5: Modal spectra, coarse model.

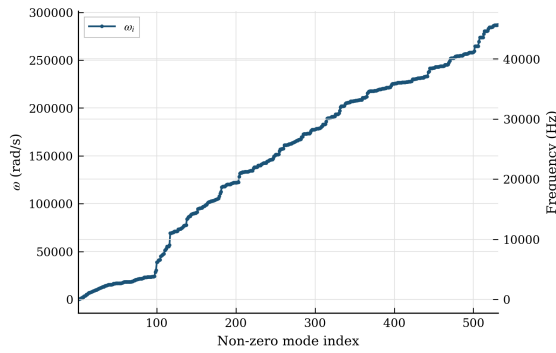


(a) K_0 .

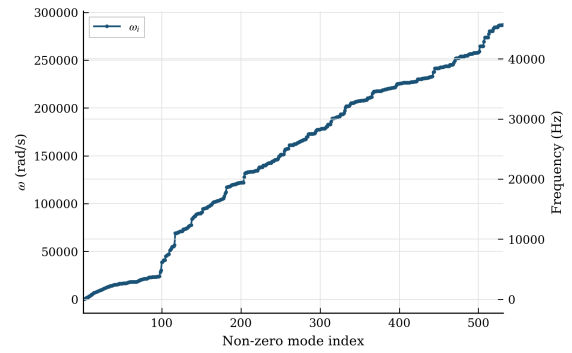


(b) $K^{\tan}$.

Figure B.6: Modal spectra, refined 3×6 model.

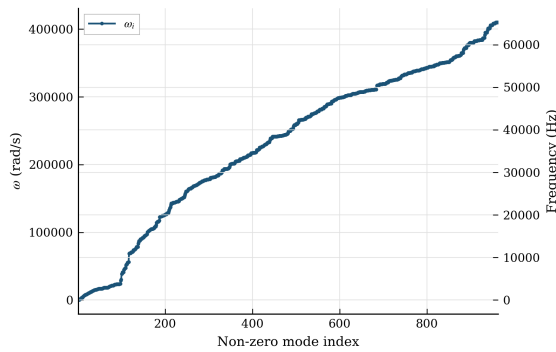


(a) K_0 .

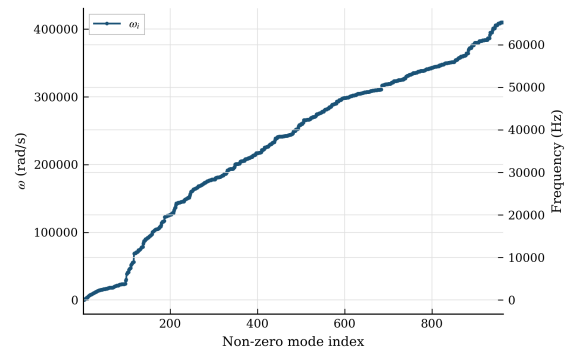


(b) $K^{\tan}$.

Figure B.7: Modal spectra, refined 5×10 model.



(a) K_0 .



(b) $K^{\tan}$.

Figure B.8: Modal spectra, refined 7×14 model.

Natural frequencies. Tables B.4–B.7 report the first five natural frequencies under K_0 and $K^{\tan}$ for each level. The K_0 frequencies are practically identical across all four models, confirming that band refinement does not alter the global dynamic character of the wall. Mode 1 is the fundamental out-of-plane cantilever bending mode. The $K^{\tan}$ frequencies are systematically lower due to softening of yielded mortar joints under axial prestress.

Table B.4: First five natural frequencies, coarse model.

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
$\mathbf{K}_0$	14.13	60.28	121.36	151.46	255.31
$\mathbf{K}^{\text{tan}}$	12.99	58.78	121.36	150.40	254.52

Table B.5: First five natural frequencies, refined 3×6 model.

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
$\mathbf{K}_0$	14.12	60.24	121.36	151.00	254.75
$\mathbf{K}^{\text{tan}}$	12.57	58.71	121.36	149.38	253.69

Table B.6: First five natural frequencies, refined 5×10 model.

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
$\mathbf{K}_0$	14.12	60.27	121.36	151.01	254.80
$\mathbf{K}^{\text{tan}}$	12.13	58.72	121.36	148.83	253.47

Table B.7: First five natural frequencies, refined 7×14 model.

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
$\mathbf{K}_0$	14.12	60.28	121.36	151.01	254.81
$\mathbf{K}^{\text{tan}}$	11.65	58.71	121.36	148.25	253.21

Mode shapes. Figure B.9 shows the first five mode shapes and the ω_{max} mode for the refined 3×6 model under $\mathbf{K}^{\text{tan}}$. Modes 1–5 represent the global patterns of the wall; the ω_{max} mode is a highly localised sub-block deformation at the impact row.

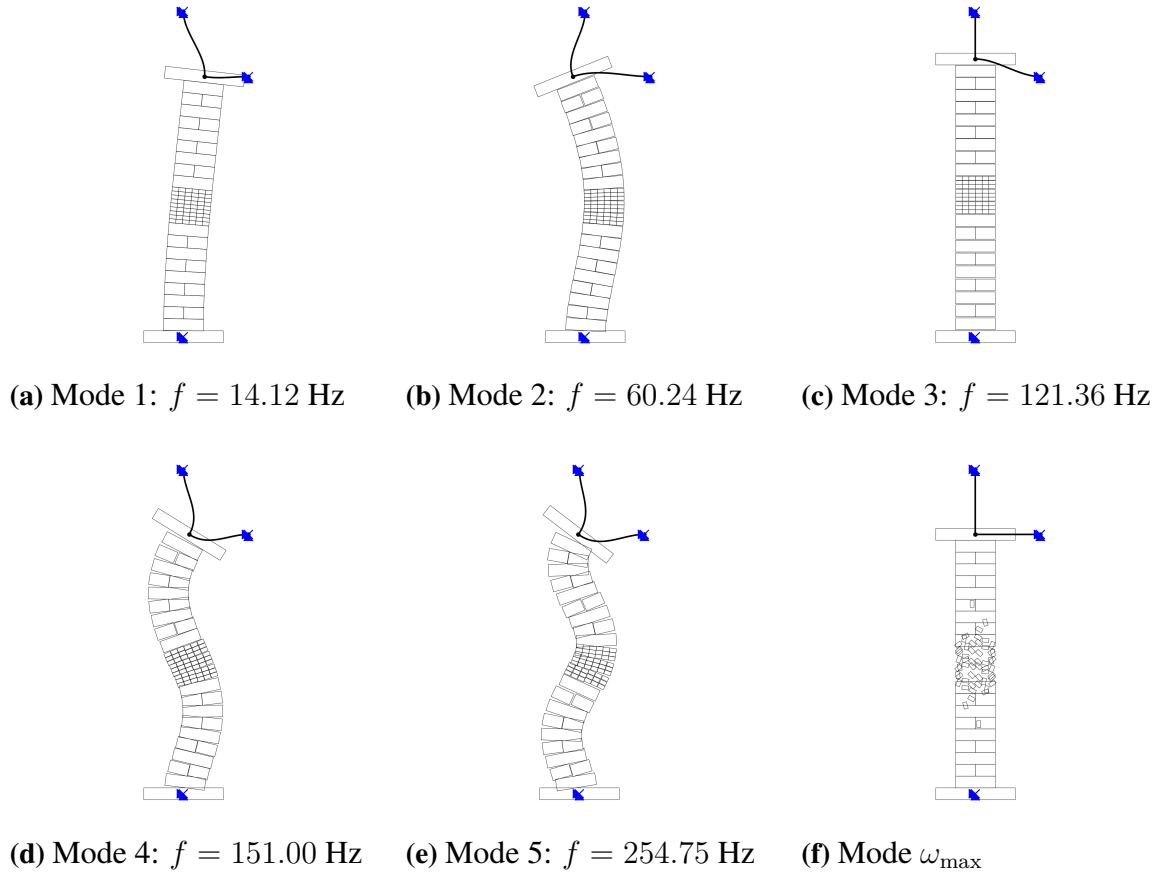
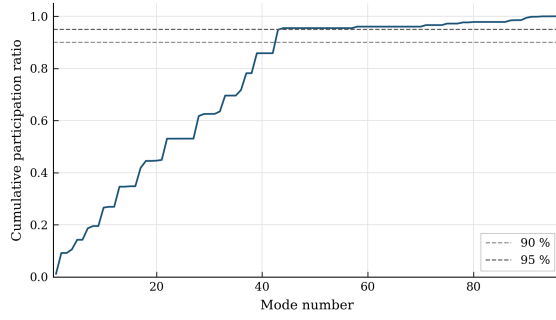


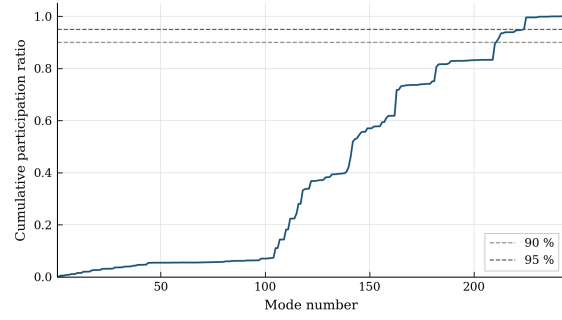
Figure B.9: Mode shapes, refined 3×6 model, $\mathbf{K}^{\text{tan}}$.

The localised character of the $\omega_{\max}$ mode is consistent with the super-linear scaling of Δt_{crit} discussed in Section B.3.4: $\omega_{\max}$ is governed by the sub-block interface stiffness at the impact zone, not by the global wall dynamics.

Force-based participation at the impact DOF. Figure B.10 shows the cumulative force-based participation for the coarse and refined 3×6 models. In the coarse model, participation is broadly distributed and 90% is reached within ≈ 40 modes, with all contributing modes lying below $\omega \approx 14,000$ rad/s — well below $\omega_{\max} = 2.381 \times 10^4$ rad/s. In the refined model, participation is negligible through the first ≈ 100 modes and then rises sharply, driven by the local sub-block modes at the impact zone (Figure B.9), with the 90% threshold not reached until well beyond mode 200.



(a) Coarse model.



(b) Refined 3×6 model.

Figure B.10: Cumulative force-based participation at the impact DOF, $K^{\tan}$, $c = 0.01$ MPa.

On the relationship between Δt_{crit} and modal participation. The participation curves raise a natural question: since the stability-critical modes carry negligible impact energy, could a larger time step be used without meaningful loss of accuracy? This reasoning is physically intuitive but mathematically incorrect. Unlike modal superposition, where only the modes contributing meaningfully to the response need to be retained (Chopra, 2007), the CDM integrates the full discretised system at every time step without modal truncation. A time step exceeding $2/\omega_{\text{max}}$ therefore produces exponential growth in the ω_{max} component, regardless of how little energy is initially projected onto that mode (Belytschko et al., 2000). The force-based participation curves thus quantify the *accuracy* requirements of the time step, but cannot be used to relax the *stability* requirement. The numerical evidence for this is presented in Section B.3.7.

B.3.5 Cross-validation with experimental frequencies

This section compares the two lowest numerical frequencies against those identified in the SIII dynamic identification campaign conducted at RISE on wall W20 (Godio et al., 2024), providing an independent cross-validation of both the interface stiffness $K_1 = 17$ GPa/m and the horizontal spring stiffness $K_{\text{sup},x}$.

Experimental campaign. The SIII campaign conducted non-destructive free-vibration tests on six natural-scale double-wythe URM walls (W15–W20) of identical geometry and construction to the Campaign 2 impact specimens. Wall W20 was subjected to drop-mass excitations at five axial compression levels (0–57.3 kN), with out-of-plane response monitored by a Laser Doppler Vibrometer (LDV), nine LVDTs, and a 3D HS-DIC system. Operational Modal Analysis (OMA) was additionally performed on the LVDT dataset. The test setup is shown in Figure B.11.

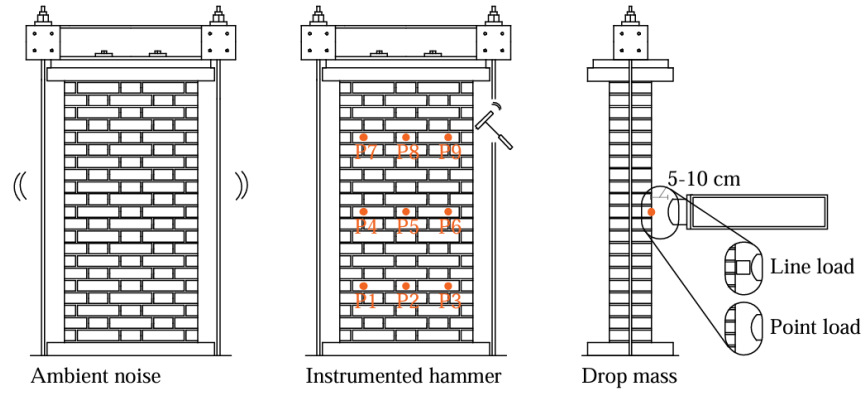


Figure B.11: Test setup of the SIII dynamic identification campaign. Figure taken from (Godio et al., 2024).

Comparison. The HybriDFEM model includes a horizontal Timoshenko spring $K_{\text{sup},x} = 1 \text{ kN/mm}$ at the top slab, selected to reproduce the SIII cantilever frequency at $\sigma_{ax} = 0.05 \text{ MPa}$ ($N = 9.6 \text{ kN}$), which is the same axial level used in all Campaign 2 impact tests. Table B.8 reports the first two out-of-plane natural frequencies at $\sigma_{ax} = 0.05 \text{ MPa}$, comparing the HybriDFEM refined 3×6 model and SIII experimental values for wall W20 (Godio et al., 2024). Numerical values from $\mathbf{K}_0$ and $\mathbf{K}^{\text{tan}}$ at the end of Step 1. The OMA result gives $f = 14.07 \text{ Hz}$ for Mode 1; the LVDT-based ODS extraction yields a range of 13.5–15.1 Hz across the two methods.

Table B.8: First two out-of-plane natural frequencies at $\sigma_{ax} = 0.05 \text{ MPa}$: HybriDFEM refined 3×6 model vs. SIII experimental values (wall W20) (Godio et al., 2024).

Mode	Source	f (Hz)
Mode 1		
Cantilever	Exp: OMA (Godio et al., 2024)	14.07
	Exp: LVDT ODS	13.5
	Exp: LDV+LVDT ODS	15.1
	Num: $\mathbf{K}_0$	14.12
	Num: $\mathbf{K}^{\text{tan}}$	12.57
Mode 2		
Vert. bending		
(high uncert. [†])	Exp: LVDT ODS	57.0
	Exp: LDV+LVDT ODS	50.6
	Num: $\mathbf{K}_0$	60.24
	Num: $\mathbf{K}^{\text{tan}}$	58.71

[†]Mode 2 could not be confirmed as a pure eigenmode by OMA; see text.

Cantilever mode. The $\mathbf{K}_0$ prediction of $f_1 = 14.12 \text{ Hz}$ matches the OMA value of 14.07 Hz (Figure B.12) to within 0.4% and lies within the experimental scatter of 13.5–15.1 Hz. The ODS at $N = 9.6 \text{ kN}$ (Figure B.13) shows the wall tilting as a near-rigid body, consistent with numerical Mode 1 (Figure B.9 (a)). The agreement is meaningful because the cantilever mode is jointly governed by two independent parameters: $K_1 = 17 \text{ GPa/m}$, which controls the wall bending stiffness, and $K_{\text{sup},x} = 1 \text{ kN/mm}$, which introduces the

low-frequency rigid-body tilting mode. A model without the horizontal spring produces no cantilever mode below the first bending frequency, so the 14.07 Hz target cannot be reached by adjusting K_1 alone. The simultaneous agreement of both parameters with a dataset from a different campaign, different specimens, and a different measurement system constitutes a genuine independent cross-validation of the Campaign 2 boundary condition and interface stiffness assignments. The K^{tan} prediction of 12.57 Hz falls slightly below the experimental range, as expected: partial yielding of the vertical head joints under self-weight softens the tangent stiffness, so $K^{\text{tan}} \leq K_0$ and the corresponding frequency is a lower bound.

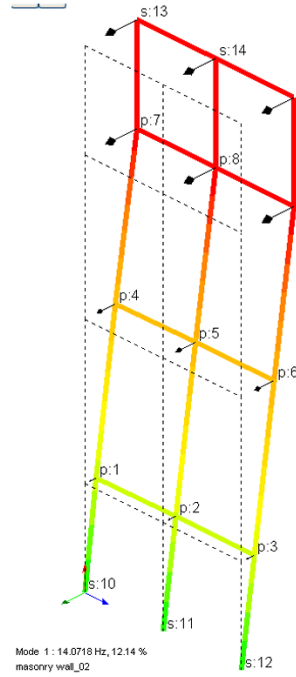


Figure B.12: Cantilever mode identified by OMA at $N = 9.6$ kN, wall W20: $f = 14.07$ Hz. Figure taken from (Godio et al., 2024).

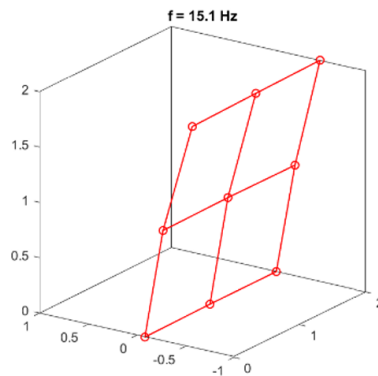


Figure B.13: Cantilever mode ODS at $N = 9.6$ kN, SIII campaign (LDV+LVDT, 10 cm drop mass, wall W20): $f = 15.1$ Hz. Taken from (Godio et al., 2024).

Vertical bending mode. The K_0 prediction of $f_2 = 60.24$ Hz overestimates the experimental range of 50.6–57.0 Hz by 3–19%, with K^{tan} reducing this to 3–16%. This mode

could not be confirmed as a pure eigenmode by OMA (Godio et al., 2024), and the two extraction methods differ by ≈ 6.4 Hz ($\sim 12\%$), so the experimental range itself carries substantial uncertainty. The ODS at $N = 9.6$ kN (Figure B.14) shows a single-curvature bow qualitatively consistent with numerical Mode 2 (Figure B.9 (b)), and the overestimation is not treated as a calibration discrepancy.

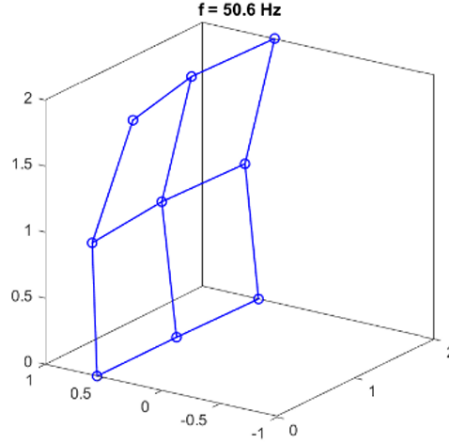


Figure B.14: Vertical bending mode ODS at $N = 9.6$ kN, SIII campaign (LDV+LVDT, 10 cm drop mass, wall W20): $f = 50.6$ Hz. Taken from (Godio et al., 2024).

B.3.6 Campaign 3 modal analysis

The modal analysis for Campaign 3 was performed on wall W23 using the refined 3×6 discretisation. The model geometry differs from Campaign 2 in three respects: (i) the steel plate block ($L = 0.420$ m, $t = 0.050$ m, $m = 460$ kg, eccentric $+80$ mm from the wall centreline) replaces the RC slab as the prestress anchor and spring attachment point; (ii) the vertical spring stiffness is $K_{\text{sup},y} = 14.9$ kN/mm; and (iii) the refined band covers rows 5–15, spanning both impact lines at $H/3$ and $2H/3$. No coarse model is analysed; the production time step is taken directly from the refined model stability limit. Cross-validation against experimental frequencies is not available for Campaign 3. Table B.9 summarises the stability results and first five natural frequencies.

Table B.9: Modal stability results for Campaign 3, wall W23, refined 3×6 model ($c = 0.01$ MPa, $K_{\text{sup},y} = 14.9$ kN/mm, $K_{\text{sup},x} = 35$ kN/mm).

Quantity	K_0	K^{tan}
ω_{max} (rad/s)	1.533×10^5	1.533×10^5
Δt_{crit} (s)	1.305×10^{-5}	1.305×10^{-5}
$\Delta t_{\text{adopted}}$ (s)	1×10^{-5}	
f_1 (Hz)	35.20	34.48
f_2 (Hz)	74.82	74.41
f_3 (Hz)	77.11	74.82
f_4 (Hz)	178.01	174.37
f_5 (Hz)	279.51	279.51

The fundamental frequency ($f_1 = 35.20$ Hz under $\mathbf{K}_0$) is approximately $2.5\times$ higher than the Campaign 2 value of 14.12 Hz, despite the lower vertical spring stiffness ($K_{\text{sup},y} = 14.9$ vs 31.4 kN/mm). This reflects primarily the much stiffer horizontal boundary condition ($K_{\text{sup},x} = 35$ vs 1 kN/mm), which strongly constrains the out-of-plane cantilever mode and shifts the natural frequency upward. The critical time step $\Delta t_{\text{crit}} = 1.305 \times 10^{-5}$ s is governed by local sub-block interface modes at the impact rows, consistent with the mechanism established for Campaign 2 (Section B.3.4), and gives a safety factor of 1.305 for the adopted $\Delta t = 1 \times 10^{-5}$ s. The agreement between $\omega_{\text{max}}(\mathbf{K}_0)$ and $\omega_{\text{max}}(\mathbf{K}^{\text{tan}})$ confirms that the $\mathbf{K}^{\text{tan}}$ pathology is fully resolved by $c = 0.01$ MPa, as in Campaign 2.

Figures B.15 and B.16 show the frequency spectrum and mode shapes.

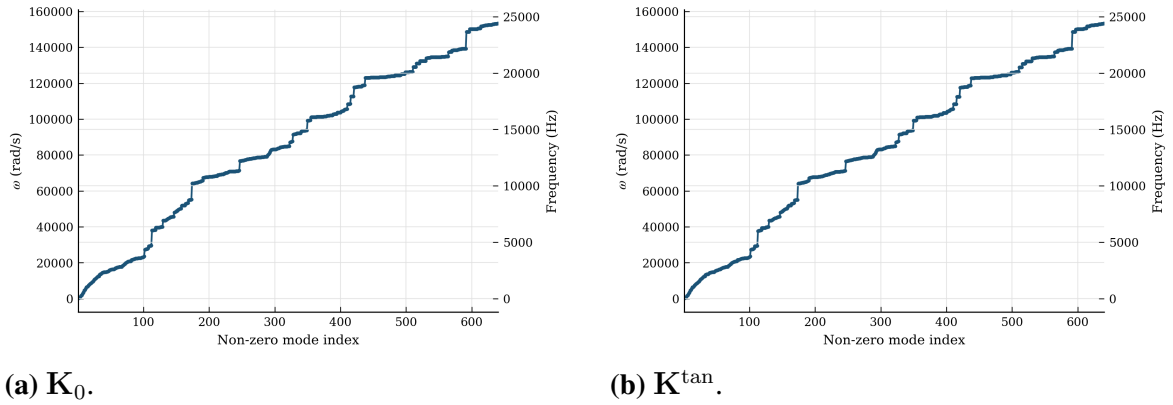


Figure B.15: Modal spectra, Campaign 3 refined 3×6 model.

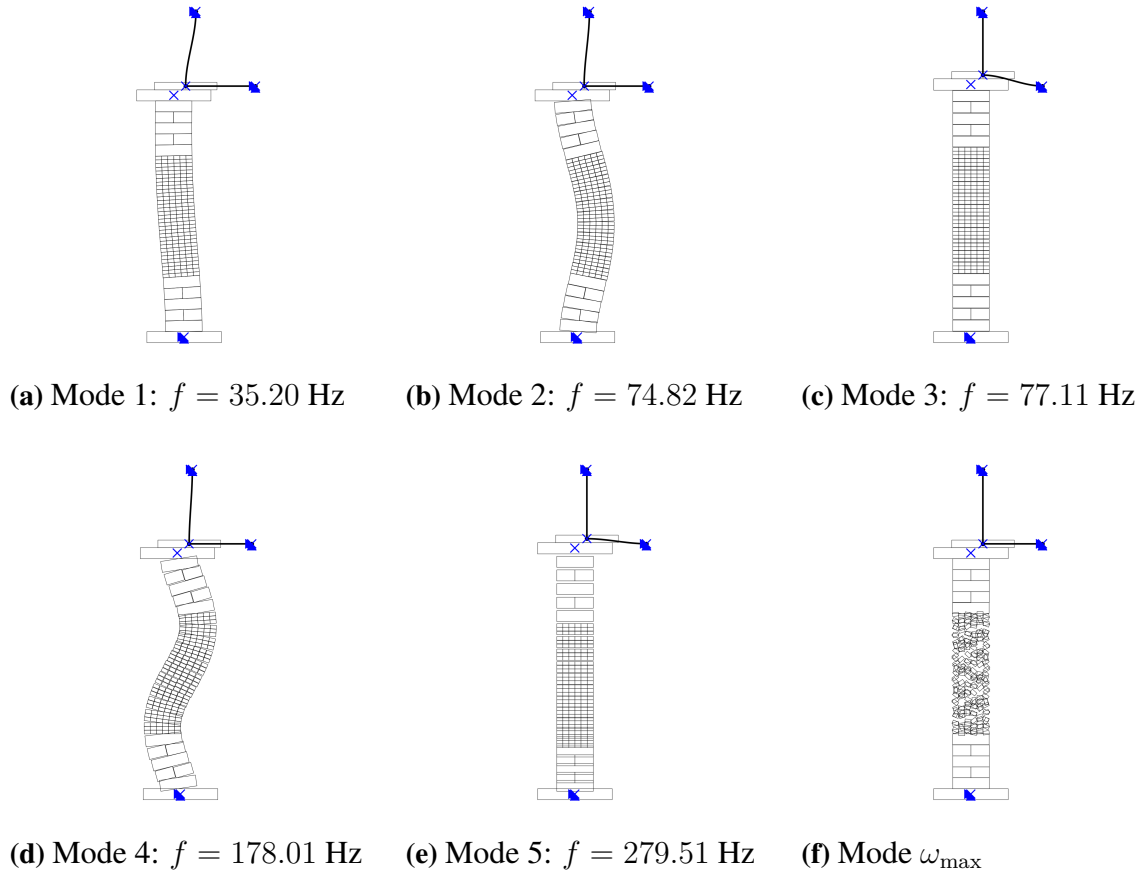


Figure B.16: Mode shapes, Campaign 3 refined 3×6 model, $\mathbf{K}^{\tan}$.

Figure B.17 shows the cumulative force-based participation at the lower ($H/3$) and upper ($2H/3$) impact DOFs. Both curves confirm the pattern established for Campaign 2: participation is negligible through the first ≈ 50 – 100 modes and dominated by local sub-block modes, with the 90% threshold not reached until mode ≈ 450 (lower row) and ≈ 580 (upper row).

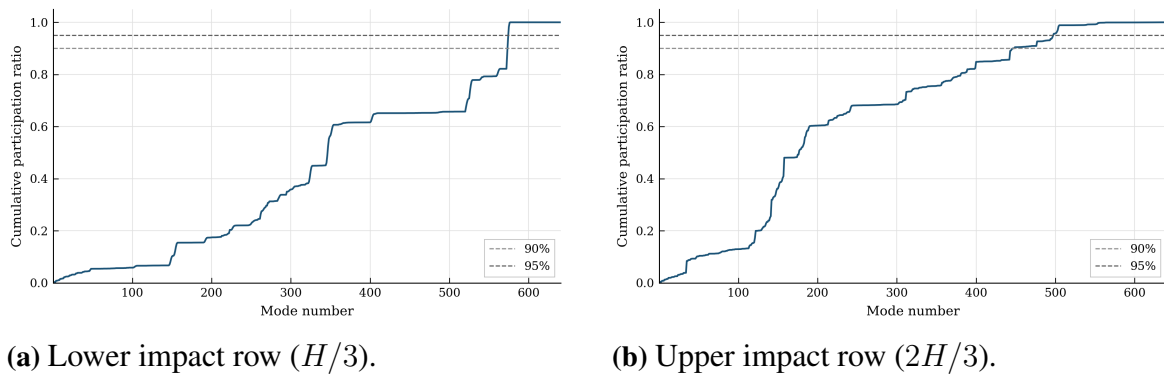


Figure B.17: Cumulative force-based participation at the lower and upper impact DOFs, $\mathbf{K}^{\tan}$, Campaign 3 refined 3×6 model.

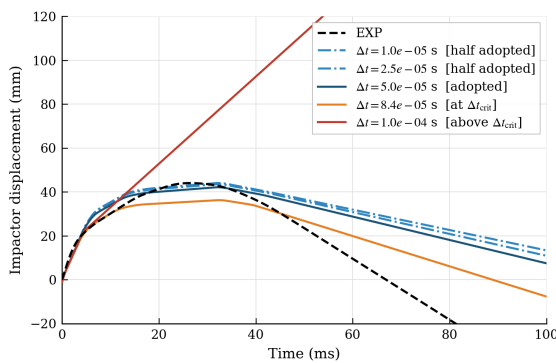
B.3.7 Time Step Sensitivity

This section provides numerical evidence that exceeding Δt_{crit} produces divergence regardless of modal energy content, and verifies that the adopted time steps lie in the converged regime. The study is conducted on wall W18 using the calibrated parameter set ($k_{n,\text{imp}} = 6.0 \text{ GPa/m}$, $\mu = 0.75$, $\xi = 0.010$), monitoring impactor displacement, velocity, and force over the full 100 ms window. The tested values are:

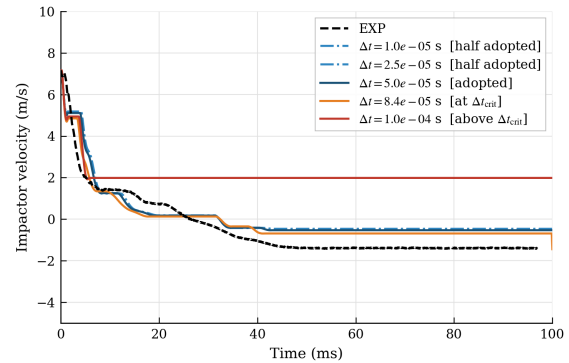
- **Coarse model** ($\Delta t_{\text{crit}} = 8.40 \times 10^{-5} \text{ s}$, adopted $\Delta t = 5 \times 10^{-5} \text{ s}$): $\Delta t \in \{1 \times 10^{-4}, 8.4 \times 10^{-5}, 5 \times 10^{-5}, 2.5 \times 10^{-5}, 1 \times 10^{-5}\} \text{ s}$.
- **Refined 3×6 model** ($\Delta t_{\text{crit}} = 1.252 \times 10^{-5} \text{ s}$, adopted $\Delta t = 1 \times 10^{-5} \text{ s}$): $\Delta t \in \{3 \times 10^{-5}, 1.3 \times 10^{-5}, 1.2 \times 10^{-5}, 1 \times 10^{-5}, 5 \times 10^{-6}\} \text{ s}$.

B.3.7.1 Coarse model

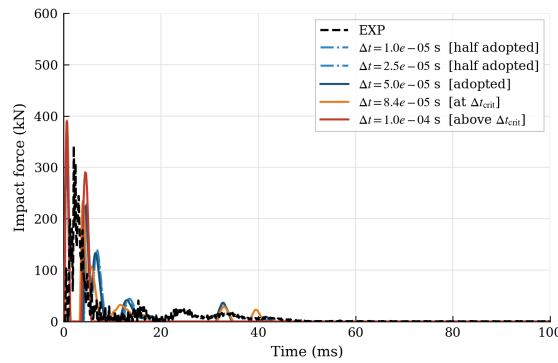
Figure B.18 and Table B.10 show the results. At $\Delta t = 1 \times 10^{-4} \text{ s}$ (safety $0.84\times$), the displacement grows monotonically to 211.7 mm, the velocity locks onto a spurious $+2 \text{ m/s}$ plateau, and the force history is physically meaningless after first contact. At $\Delta t = 8.4 \times 10^{-5} \text{ s}$ (safety $1.00\times$), the simulation remains bounded but the response is degraded: peak displacement drops to 36.2 mm and peak force rises to 391.6 kN, confirming that marginal stability is not sufficient for accuracy. The three stable cases ($\Delta t = 5 \times 10^{-5}, 2.5 \times 10^{-5}, 1 \times 10^{-5} \text{ s}$) produce visually indistinguishable histories, confirming that the adopted value $\Delta t = 5 \times 10^{-5} \text{ s}$ lies in the converged regime.



(a) Impactor displacement.



(b) Impactor velocity.



(c) Impact force.

Figure B.18: Time step sensitivity, coarse model, wall W18.

Table B.10: Time step sensitivity metrics, coarse model, wall W18.

Δt (s)	Regime	Safety	$\hat{u}$ (mm)	$\hat{t}_u$ (ms)	$\hat{F}$ (kN)	RMSE $_{\hat{u}}$ (mm)
1.0×10^{-4}	above Δt_{crit}	$0.84\times$	211.7	100.0	389.4	124.3
8.4×10^{-5}	at Δt_{crit}	$1.00\times$	36.2	32.4	391.6	15.6
5.0×10^{-5}	adopted	$1.68\times$	42.1	32.3	342.7	23.0
2.5×10^{-5}	half adopted	$3.36\times$	43.5	32.3	329.8	24.8
1.0×10^{-5}	one-fifth adopted	$8.40\times$	44.1	32.3	319.1	26.0

B.3.7.2 Refined model

Figure B.19 and Table B.11 show the results.

Far above Δt_{crit} : catastrophic divergence. At $\Delta t = 3 \times 10^{-5}$ s (safety $0.42\times$), the solution overflows within 2.3 ms, with displacement and force reaching $\gg 10^{90}$ mm and $\gg 10^{40}$ kN. This confirms catastrophic instability driven by the high-frequency sub-block modes at the impact zone.

Slightly above Δt_{crit} : slow divergence. At $\Delta t = 1.3 \times 10^{-5}$ s (safety $0.96\times$), the histories appear physically coherent through the contact phase but the instability grows slowly, reaching a peak displacement of 64.2 mm by $t = 100$ ms. A user terminating the simulation at $t = 30$ ms might not detect the problem from visual inspection alone, which underlines the importance of the theoretical stability check.

At and below Δt_{crit} : converged. At $\Delta t = 1.2 \times 10^{-5}$ s (safety $1.04\times$), the solution is fully stable: peak displacement 40.1 mm, peak force 319.1 kN. A safety factor of just 4% is sufficient to cross the stability boundary, confirming that the transition is sharp and that the theoretical Δt_{crit} is a reliable predictor of the actual limit. The three stable cases ($\Delta t = 1.2 \times 10^{-5}$, 1.0×10^{-5} , 5.0×10^{-6} s) produce peak displacements within 1.7 mm (4.2%) and peak forces within 4.8 kN (1.5%) of one another, confirming convergence.

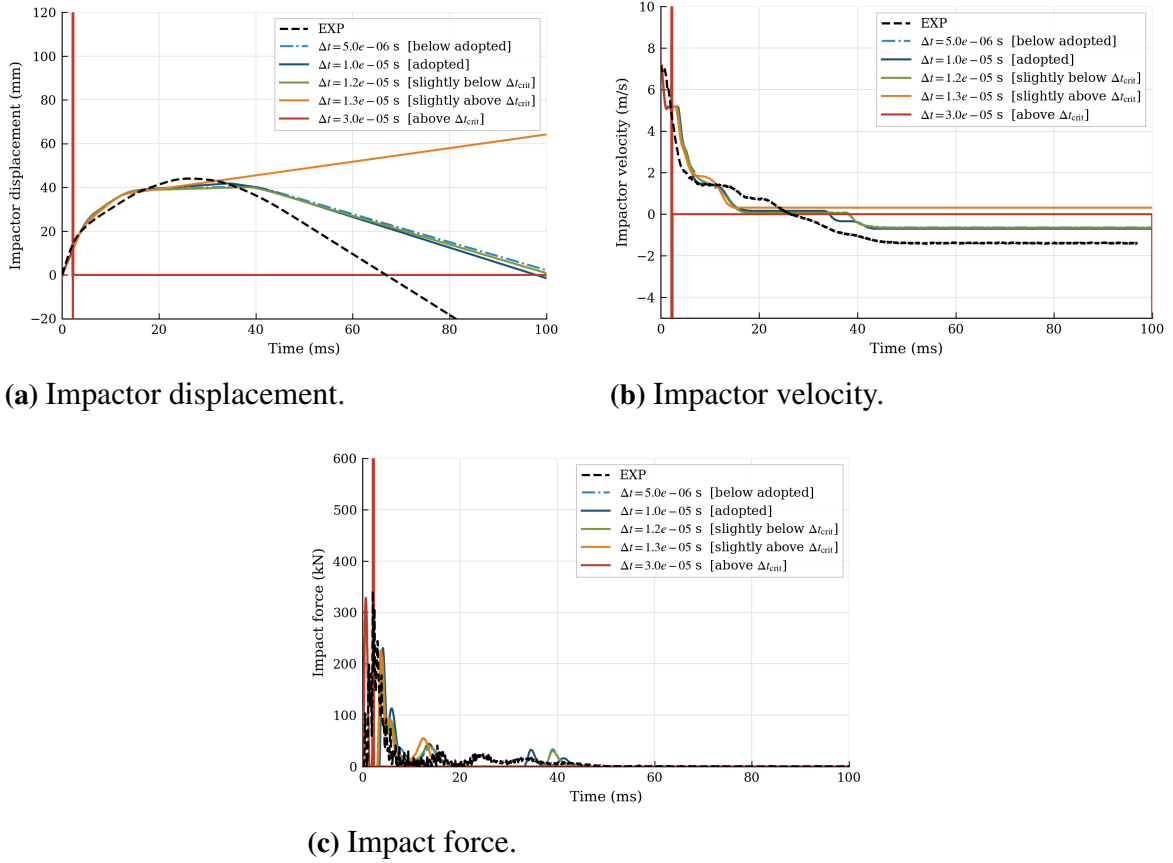


Figure B.19: Time step sensitivity, refined 3×6 model, wall W18.

Table B.11: Time step sensitivity metrics, refined 3×6 model, wall W18.

Δt (s)	Regime	Safety	$\hat{u}$ (mm)	$\hat{t}_u$ (ms)	$\hat{F}$ (kN)	RMSE $_u$ (mm)
3.0×10^{-5}	above Δt_{crit}	$0.42 \times$	$\gg 10^{90}$	2.3	$\gg 10^{40}$	—
1.3×10^{-5}	slightly above Δt_{crit}	$0.96 \times$	64.2	100.0	321.4	48.4
1.2×10^{-5}	slightly below Δt_{crit}	$1.04 \times$	40.1	38.1	319.1	20.4
1.0×10^{-5}	adopted	$1.25 \times$	41.8	34.3	316.5	19.6
5.0×10^{-6}	below adopted	$2.50 \times$	40.6	38.3	314.3	21.2

Taken together, the results confirm three points: Δt_{crit} is a hard limit whose violation by as little as 4% leads to divergence; the stability boundary is sharp, with an 8% change in Δt separating a converged result from a slowly diverging one; and the adopted time steps (5×10^{-5} s coarse, 1×10^{-5} s refined) lie well within the converged regime for all production runs.

B.3.8 Mesh Sensitivity

Increasing the refinement level raises ω_{max} and reduces Δt_{crit} , substantially increasing computational cost. This study determines whether the 3×6 discretisation already provides converged results, or whether a finer band changes the predicted response in a physically meaningful way. Two levels are compared for wall W18 using the calibrated parameter set ($k_{n,\text{imp}} = 6.0$ GPa/m, $\mu = 0.75$, $\xi = 0.010$):

- **Refined 3×6 :** $n_y = 3, n_x = 6$. Free DOFs: 242. Adopted $\Delta t = 1 \times 10^{-5}$ s (safety $1.25\times, \Delta t_{\text{crit}} = 1.252 \times 10^{-5}$ s).
- **Refined 5×10 :** $n_y = 5, n_x = 10$. Free DOFs: 531. Adopted $\Delta t = 5 \times 10^{-6}$ s (safety $1.39\times, \Delta t_{\text{crit}} = 6.969 \times 10^{-6}$ s).

Figure B.20 shows the impactor histories for both levels against the experimental record, and Table B.12 summarises the quantitative comparison.

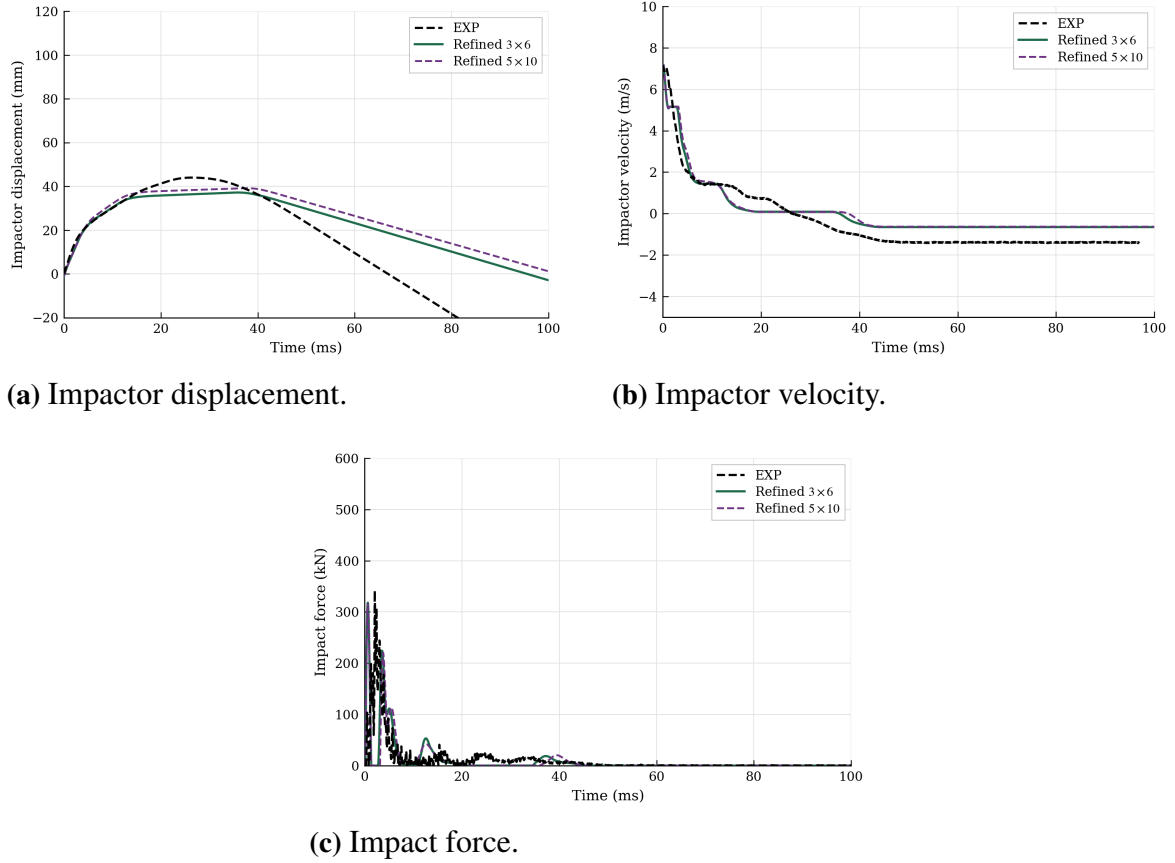


Figure B.20: Mesh sensitivity, wall W18 ($k_{n,\text{imp}} = 6.0$ GPa/m, $\mu = 0.75$, $\xi = 0.010$).

Table B.12: Mesh convergence metrics, wall W18.

Model	$\hat{u}_{\text{imp}}$ (mm)	$\hat{t}_u$ (ms)	RMSE $_u$ (mm)	$\hat{F}_{\text{imp}}$ (kN)
EXP	44.0	26.4	—	341.8
Refined 3×6	37.2	35.9	18.1	318.1
Refined 5×10	39.2	37.7	20.4	315.3
$ \Delta $ (3×6 vs 5×10)	2.0	1.8	—	2.8
Relative diff. (%)	5.3	5.0	—	0.9

The velocity and force histories are visually indistinguishable between the two levels, with a peak force difference of only 2.8 kN (0.9%). The displacement histories agree well during the loading phase but diverge modestly in the post-peak regime, with a peak displacement

difference of 2.0 mm (5.3%). The sub-block band structure of the 3×6 model provides the spatial resolution necessary to represent localised mechanisms such as joint opening, crushing, and block separation near the impact zone. The 3×6 discretisation is therefore adopted as the production level: it provides converged global response and sufficient local resolution at a computational cost approximately $2\times$ lower than the 5×10 model.

C Appendix C: Interface Stiffness Calibration from Four-Point Bending Tests

This appendix documents the calibration of the effective masonry interface stiffness K_1 from the quasi-static four-point bending (4PB) tests described in Section 3.1.2. The calibrated value is used without modification in all HybriDFEM impact simulations across the three experimental campaigns. The corresponding HybriDFEM side-spring input, $k_{nn} = 2K_1$, is derived in Appendix A, while the independent cross-validation against experimental modal frequencies is presented in Appendix B.3.

C.1 Calibration objective

Only the initial slope of the lateral force–displacement curve is used for calibration, since this response range is governed primarily by the effective masonry interface stiffness K_1 and the boundary conditions. This makes the initial slope a suitable observable for isolating the elastic stiffness of the HybriDFEM masonry interfaces.

The 4PB tests are therefore used exclusively to calibrate K_1 . The friction coefficient μ becomes relevant only after joint sliding develops, while the damping ratio ξ is irrelevant for quasi-static monotonic loading. The impact contact stiffness, friction coefficient, and damping ratio are identified separately from the impact simulations in Appendices D and E.

C.2 Stiffness evaluation method

For each wall, a displacement control window $[0, u_{\text{ctrl}}]$ is defined and divided into two equal parts. The first half, $[0, u_{\text{ctrl}}/2]$, represents the initial elastic response and is used as the calibration range. The second half, $[u_{\text{ctrl}}/2, u_{\text{ctrl}}]$, is inspected only as a diagnostic of the post-initial response and is not included in the calibration criterion.

Because the experimental data at very small displacements may be affected by contact settling, the Half 1 stiffness is not obtained directly from a linear regression. Instead, a third-degree polynomial constrained through the origin is fitted over the full control window,

$$Q_{\text{poly}}(u) = a_1 u + a_2 u^2 + a_3 u^3, \quad (\text{C.1})$$

and the experimental Half 1 stiffness is evaluated as the tangent slope at the centre of Half 1,

$$k_{\text{exp},h1} = Q'_{\text{poly}}\left(\frac{u_{\text{ctrl}}}{4}\right). \quad (\text{C.2})$$

The numerical and experimental Half 1 stiffnesses are compared through

$$\varepsilon_{h1} = \frac{k_{\text{num},h1} - k_{\text{exp},h1}}{k_{\text{exp},h1}} \times 100\%. \quad (\text{C.3})$$

For completeness, the Half 2 stiffness was also evaluated by direct linear regression over $[u_{\text{ctrl}}/2, u_{\text{ctrl}}]$. However, it is not used for selecting K_1 , since this range may already be influenced by progressive joint sliding and contact nonlinearity.

C.3 Wall selection and calibration set

Table C.1 summarises the 4PB wall matrix.

Table C.1: Wall matrix — four-point bending campaign.

Wall	Support	K_{sup} (kN/mm)	σ_{ax} (MPa)	u_{ctrl} (mm)	ALR	Status
W5	SS	—	0.20	2.0	low	Calibration
W6	NR	59.9	0.20	0.8	low	Calibration
W7	NR	74.8	0.20	1.0	low	Excluded (anomalous)
W8	SS	—	0.40	2.0	high	Excluded (ALR)
W12	NR	163.2	0.40	1.5	high	Excluded (ALR)
W13	NR	12.8	0.20	1.5	low	Calibration
W14	NR	8.7	0.20	2.5	low	Calibration

Exclusion of W8 and W12 ($\sigma_{\text{ax}} = 0.40$ MPa). The experimental data in Table C.2 shows that the initial stiffness is higher for walls tested at $\sigma_{\text{ax}} = 0.40$ MPa (W8: $k_{\text{exp},h1} = 45.11$ kN/mm; W12: 68.23 kN/mm) than for walls at 0.20 MPa (W5–W6–W13–W14: range 22.21–54.29 kN/mm). This is consistent with the general trend documented by Wilding and Beyer (2018), who showed from 79 shear-compression tests that the initial stiffness of URM walls increases with axial load ratio and proposed an empirical relationship of the form $E(\sigma_0) = \alpha f_u (1 + \beta \sigma_0 / f_u)$, with a non-zero intercept at zero axial load. Because the K – σ_{ax} relationship is non-linear and does not pass through the origin, a linear extrapolation from the 0.20 and 0.40 MPa data points down to the impact test condition ($\sigma_{\text{ax}} = 0.05$ MPa) yields a negative stiffness, which is physically impossible. The stiffness at 0.05 MPa is therefore expected to remain close to the 0.20 MPa value rather than decreasing further, and including W8 and W12 in the calibration would bias the identified K_1 toward a level unrepresentative of the impact test conditions. These walls are therefore excluded.

Confirmation: $K(0.05) \approx K(0.20)$ MPa. This is supported by the SIII dynamic identification campaign (Godio et al., 2024), in which the first cantilever frequency of wall W20 remains nearly constant between $N = 9.6$ kN ($\sigma_{\text{ax}} \approx 0.05$ MPa) and $N = 38.2$ kN ($\sigma_{\text{ax}} \approx 0.20$ MPa, Figure C.1). Since $f_1^2 \propto K/M$ and the wall mass is constant, similar frequencies confirm approximately equal lateral stiffness at these two axial levels.

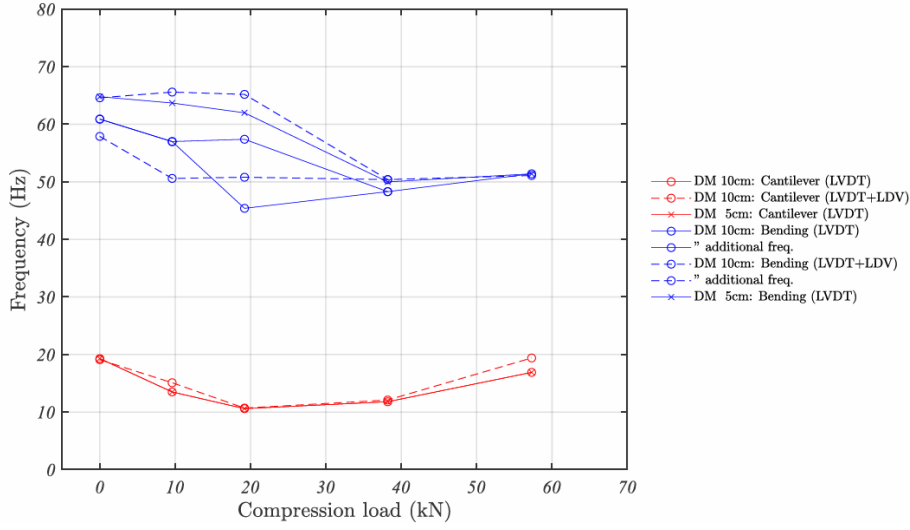


Figure C.1: First bending frequency of wall W20 vs. axial compression load, SIII campaign (Godio et al., 2024).

Exclusion of W7 (anomalous data). W7 shares the same conditions as W6 but the Half 1 error ε_{h1} grows monotonically from +28% at $k_n = 20$ MPa/mm to +92% at $k_n = 40$ MPa/mm with no convergence toward zero, indicating an anomalously low experimental stiffness. W7 is therefore excluded.

The calibration set is $\mathcal{C} = \{W5, W6, W13, W14\}$.

C.4 Parametric sweep and selection of K_1

The normal side-spring stiffness was swept over eleven values,

$$k_{nn} \in \{20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40\} \text{ MPa/mm}, \quad (\text{C.4})$$

corresponding to effective interface stiffnesses $K_1 = k_{nn}/2 \in [10, 20]$ GPa/m, in accordance with the HybriDFEM convention $k_{nn} = 2K_1$ derived in Appendix A. All other parameters were kept fixed during the sweep. A preliminary friction coefficient $\mu = 0.8$, a numerical cohesion $c = 0.01$ MPa, zero tensile capacity, a masonry compressive strength $f_c = 8.2$ MPa, and $N_{CP} = 8$ contact pairs per face were used. These values were not calibration targets in this study; they were kept constant to isolate the influence of K_1 on the initial stiffness.

The optimal stiffness is selected by minimising the sum of absolute Half 1 errors over the calibration set,

$$k_{nn}^* = \arg \min_{k_{nn}} \sum_{w \in \mathcal{C}} |\varepsilon_{h1}(w, k_{nn})|, \quad \mathcal{C} = \{W5, W6, W13, W14\}. \quad (\text{C.5})$$

Figure C.2 shows the calibration criteria and the individual Half 1 errors across the stiffness sweep. Both the sum and the maximum of $|\varepsilon_{h1}|$ reach their minimum at $k_{nn}^* = 34$ MPa/mm, corresponding to

$$K_1 = \frac{k_{nn}^*}{2} = 17 \text{ GPa/m.} \quad (\text{C.6})$$

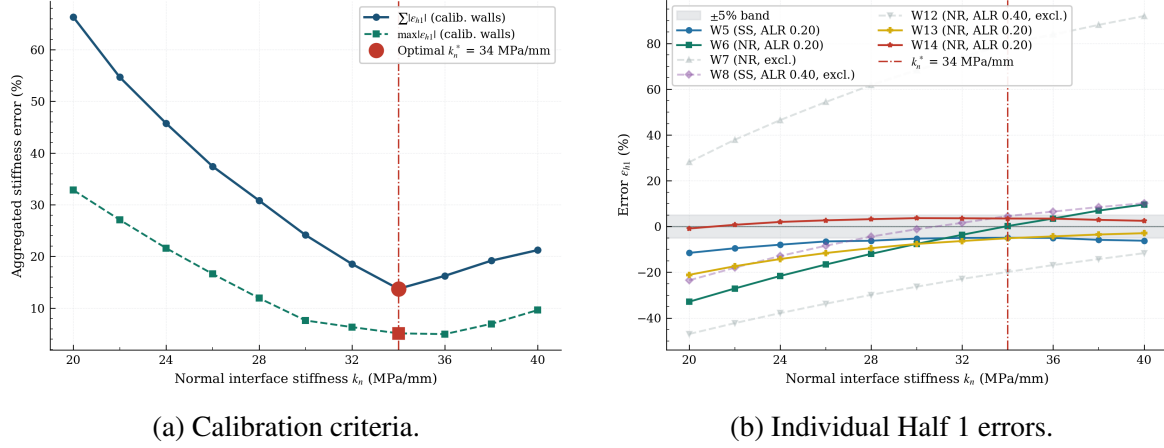


Figure C.2: Selection of the calibrated interface stiffness. The left plot shows the sum and maximum of $|\varepsilon_{h1}|$ over the calibration set. The right plot shows the individual Half 1 errors for all walls; calibration walls are shown with solid lines and excluded walls with faded dashed lines.

At $k_{nn}^* = 34 \text{ MPa/mm}$, all calibration walls satisfy $|\varepsilon_{h1}| \leq 5.1\%$. Wall W7 shows a monotonic divergence from the calibration range, confirming its exclusion due to anomalously low experimental stiffness.

Table C.2 reports the Half 1 stiffness metrics for all walls at the selected value. Half 2 is not included because it is not used in the calibration criterion.

Table C.2: Half 1 stiffness results at $k_{nn}^* = 34 \text{ MPa/mm}$ ($K_1 = 17 \text{ GPa/m}$). Calibration walls are shown in bold. Stiffnesses are in kN/mm and errors in percent.

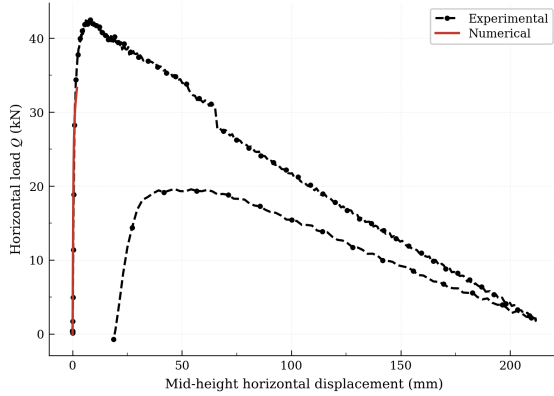
Wall	σ_{ax}	$k_{exp,h1}$	$k_{num,h1}$	ε_{h1}
W5	0.20	28.81	27.39	-4.9
W6	0.20	54.29	54.40	+0.2
W7	0.20	27.39	49.27	+79.9
W8	0.40	45.11	47.17	+4.6
W12	0.40	68.23	54.69	-19.9
W13	0.20	38.69	36.71	-5.1
W14	0.20	22.21	22.99	+3.5

C.5 Individual wall results at $k_{nn}^* = 34 \text{ MPa/mm}$

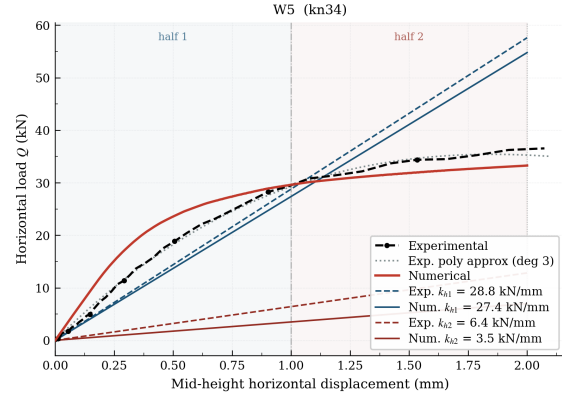
Force–displacement comparisons are presented for the calibration walls at $k_{nn}^* = 34 \text{ MPa/mm}$. The excluded walls are briefly reported for completeness, but their individual figures are omitted since they are not used in the calibration set. Each calibration wall shows the full F–D range and a zoom into the calibration window $[0, u_{ctrl}]$ with the polynomial approximation and both stiffness slope lines. Kinematic visualisations are included for the calibration walls.

C.5.1 W5 — simply supported, $\sigma_{ax} = 0.20$ MPa

W5 is simply supported with no axial confinement. The experimental response shows an immediate load drop after the peak ($Q_{exp} = 42.6$ kN), consistent with the absence of compressive arching. The kinematic evolution (Figure C.4) shows a single-curvature rocking profile with maximum $|R_z|$ at mid-height, consistent with simply supported boundary conditions.

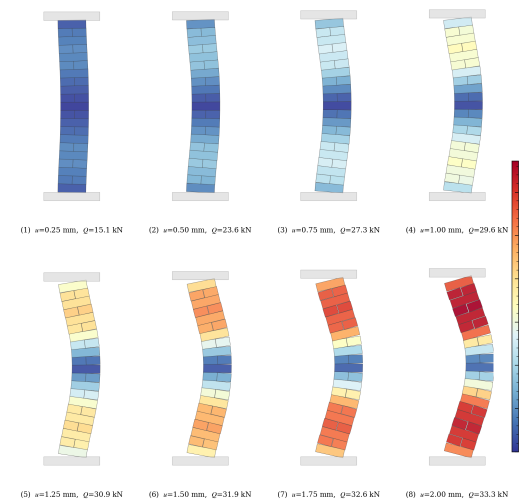
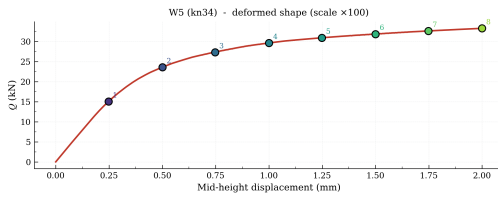


(a) Full F–D range.

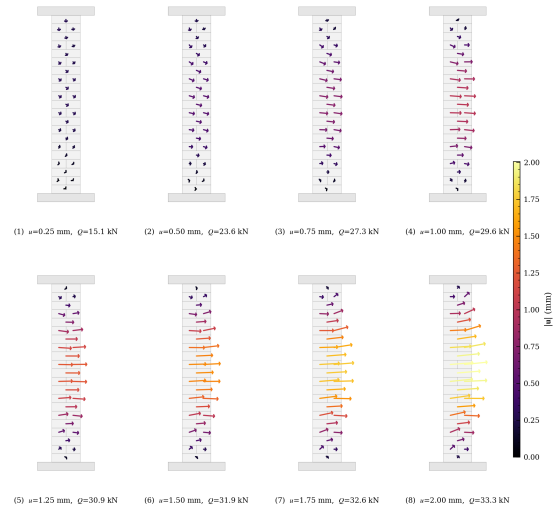
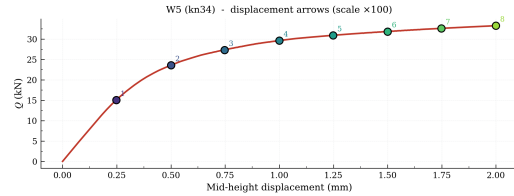


(b) Dual-half zoom ($[0, 2.0]$ mm).

Figure C.3: W5 at $k_{nn}^* = 34$ MPa/mm. $k_{exp,h1} = 28.8$ kN/mm, $k_{num,h1} = 27.4$ kN/mm ($\varepsilon_{h1} = -4.9\%$).



(a) Deformed shape ($|R_z|$, scale $\times 100$).

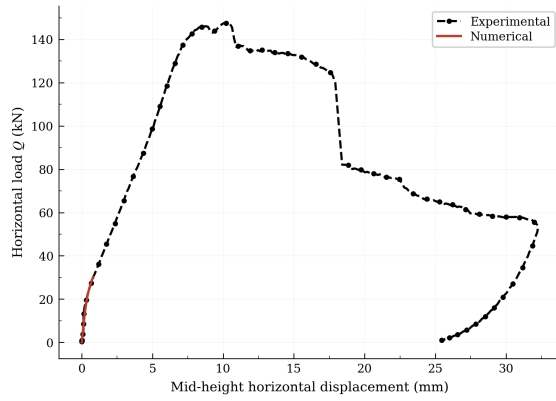


(b) Displacement arrows (scale $\times 800$).

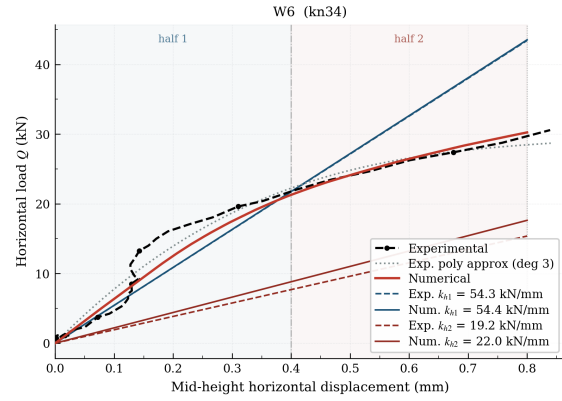
Figure C.4: W5 — kinematic evolution.

C.5.2 W6 — non-rigid support, $\sigma_{ax} = 0.20$ MPa

W6 has a stiff non-rigid top support ($K_{sup} = 59.9$ kN/mm), which generates significant compressive arching and a high experimental peak load ($Q_{exp} = 148.0$ kN). The Half 1 stiffness is very well captured ($\varepsilon_{h1} = +0.2\%$). The kinematic evolution (Figure C.6) shows reduced top-block rotation relative to W5, reflecting the partial rotational restraint introduced by the non-rigid support.

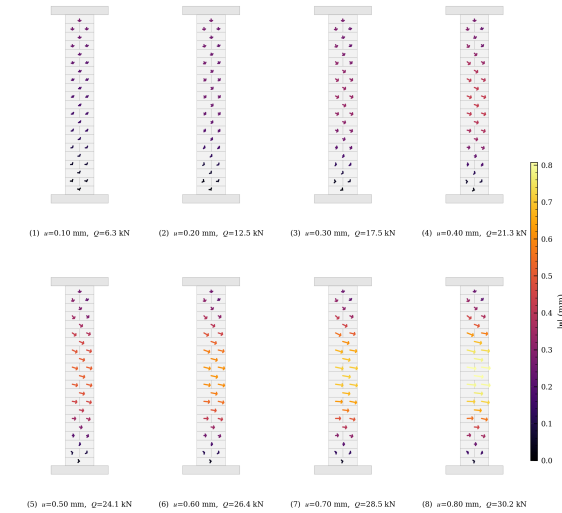
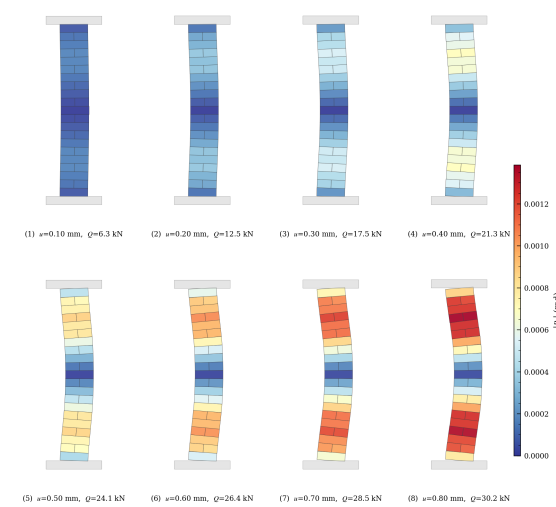
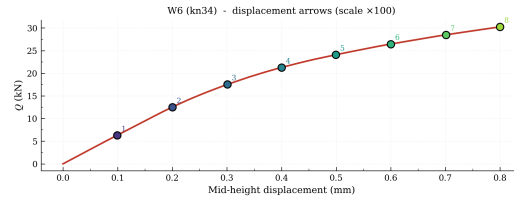
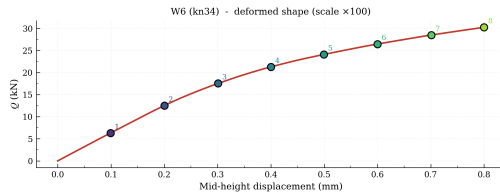


(a) Full F-D range.



(b) Dual-half zoom ($[0, 0.8]$ mm).

Figure C.5: W6 at $k_{nn}^* = 34$ MPa/mm. $k_{exp,h1} = 54.3$ kN/mm, $k_{num,h1} = 54.4$ kN/mm ($\varepsilon_{h1} = +0.2\%$).



(a) Deformed shape ($|R_z|$, scale $\times 100$).

(b) Displacement arrows (scale $\times 800$).

Figure C.6: W6 — kinematic evolution.

C.5.3 W7 — excluded (anomalous data)

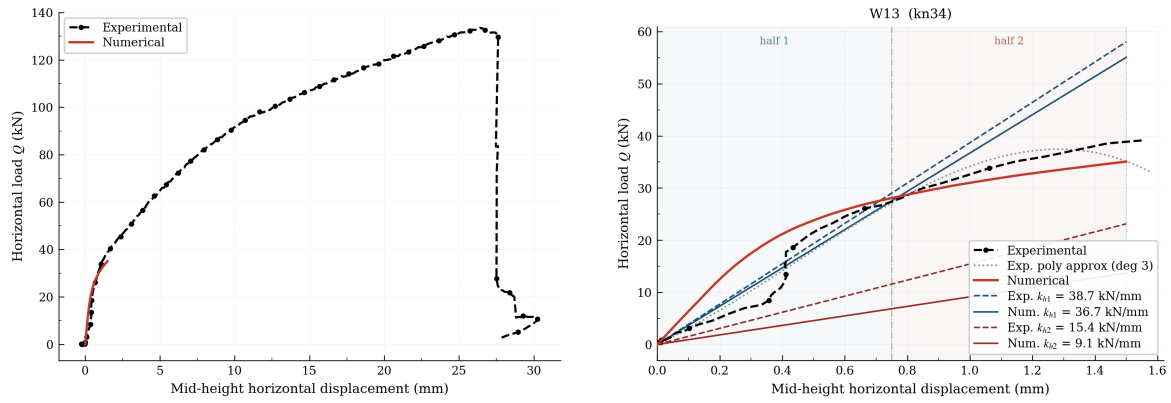
W7 is excluded from the calibration set. The numerical model significantly overestimates the initial stiffness ($\varepsilon_{h1} = +79.9\%$), consistent with an anomalously low experimental stiffness.

C.5.4 W8 and W12— excluded ($\sigma_{ax} = 0.40$ MPa)

W8 and W12 are excluded from the calibration set because they were tested at $\sigma_{ax} = 0.40$ MPa, which is outside the axial stress range selected for the stiffness calibration.

C.5.5 W13 — non-rigid support, $\sigma_{ax} = 0.20$ MPa

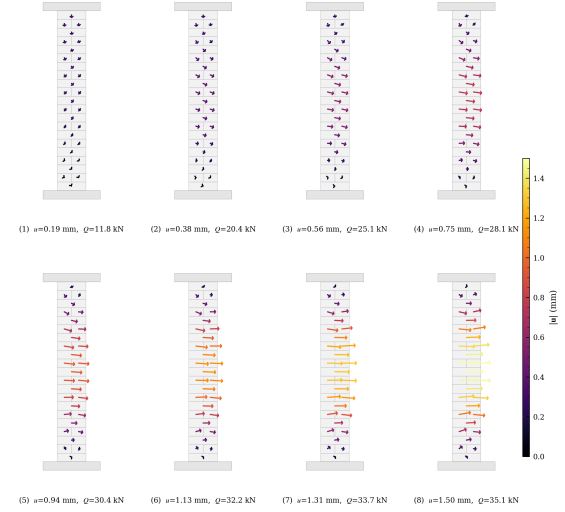
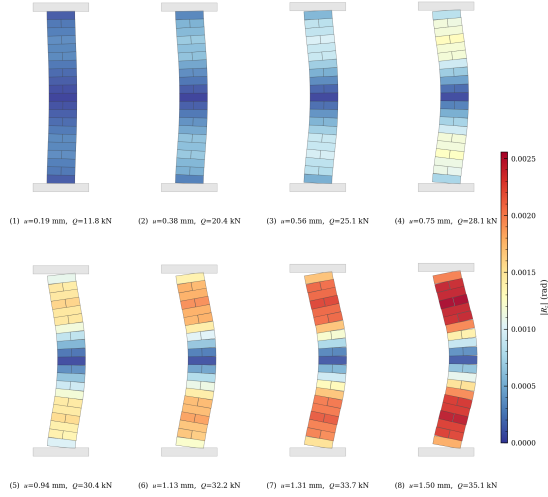
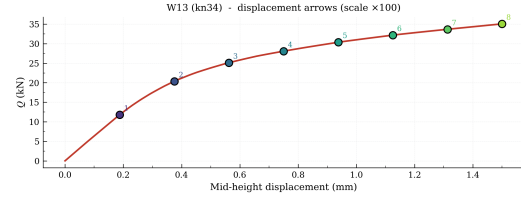
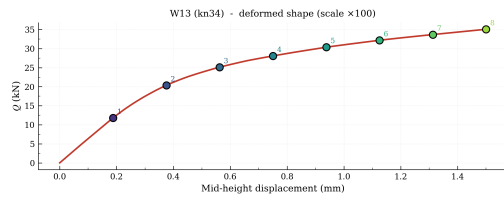
W13 has a soft non-rigid support ($K_{sup} = 12.8$ kN/mm) and the initial stiffness is well captured ($\varepsilon_{h1} = -5.1\%$). The kinematic profile (Figure C.8) is intermediate between the single-curvature rocking of W5 (simply supported) and the rotationally restrained profile of W6, consistent with the intermediate support stiffness.



(a) Full F-D range.

(b) Dual-half zoom ([0, 1.5] mm).

Figure C.7: W13 at $k_{nn}^* = 34$ MPa/mm. $k_{exp,h1} = 38.7$ kN/mm, $k_{num,h1} = 36.7$ kN/mm ($\varepsilon_{h1} = -5.1\%$).



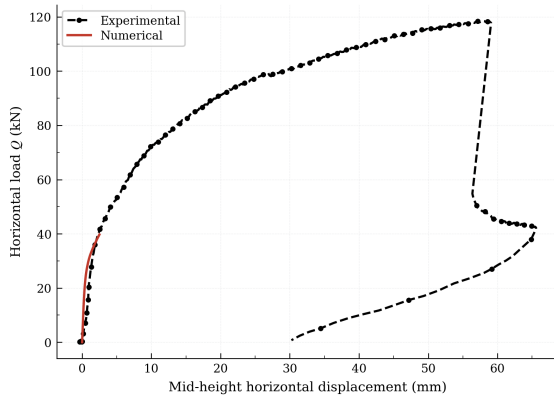
(a) Deformed shape ($|R_z|$, scale $\times 100$).

(b) Displacement arrows (scale $\times 800$).

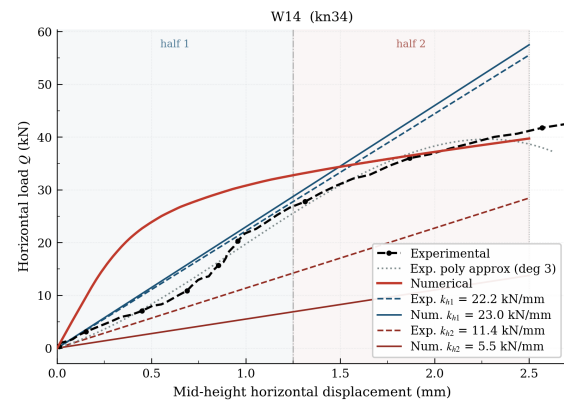
Figure C.8: W13 — kinematic evolution.

C.5.6 W14 — non-rigid support, $\sigma_{ax} = 0.20$ MPa

W14 has the softest support in the campaign ($K_{sup} = 8.7$ kN/mm) and the best individual result ($\varepsilon_{h1} = +3.5\%$). The displacement profile is close to single-curvature rocking, similar to the simply supported W5.

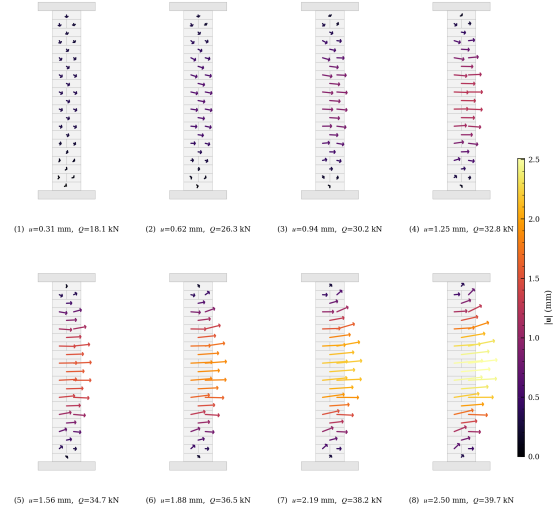
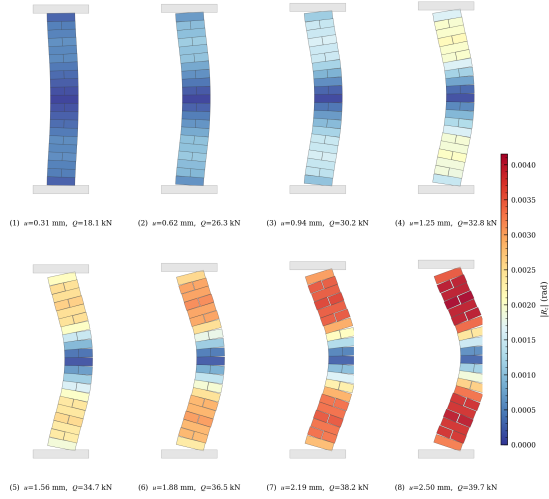
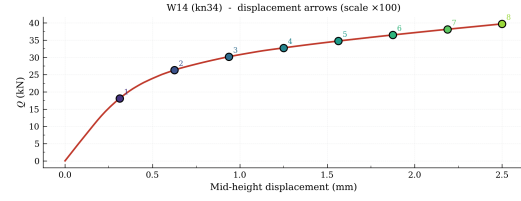
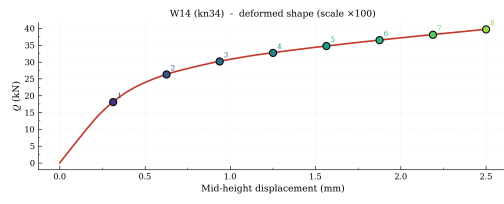


(a) Full F-D range.



(b) Dual-half zoom ($[0, 2.5]$ mm).

Figure C.9: W14 at $k_{nn}^* = 34$ MPa/mm. $k_{exp,h1} = 22.2$ kN/mm, $k_{num,h1} = 23.0$ kN/mm ($\varepsilon_{h1} = +3.5\%$).



(a) Deformed shape ($|R_z|$, scale $\times 100$).

(b) Displacement arrows (scale $\times 800$).

Figure C.10: W14 — kinematic evolution.

C.6 Half-2 diagnostic check

Although the calibration is based exclusively on the Half 1 stiffness, the Half 2 response was inspected as an additional diagnostic. As shown in Figure C.11, the Half 2 errors are large and negative for several walls, particularly W5, W13, and W14, and do not improve systematically with increasing k_{nn} . This indicates that the discrepancy is not controlled by the elastic interface stiffness K_1 .

The observed trend is consistent with the simplified contact assumption used in the calibration model. The near-zero cohesion adopted for the mortar interfaces causes the numerical response to enter a friction-dominated sliding regime earlier than the physical walls, where some residual cohesive resistance may still contribute after the initial elastic range. Since this behaviour occurs beyond the initial stiffness range, it is not used to select K_1 .

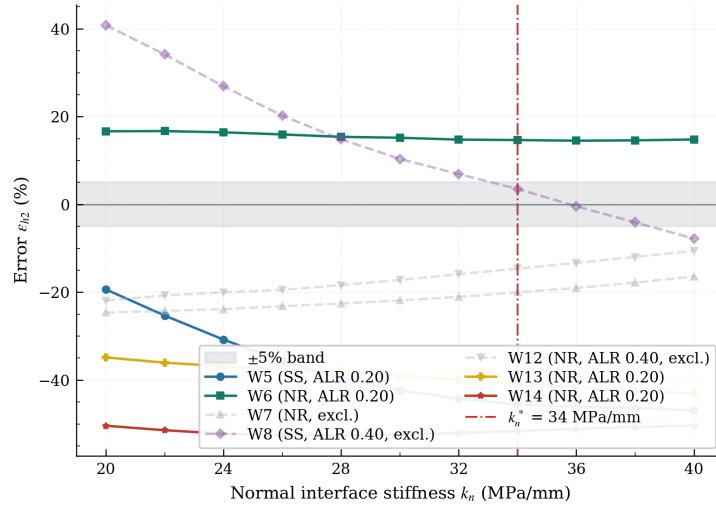


Figure C.11: Half 2 stiffness error across the k_{nn} sweep, inspected as a diagnostic only.

C.7 Cross-validation via modal frequencies

An independent cross-validation of $K_1 = 17$ GPa/m against the experimental cantilever and bending frequencies of wall W20 is presented in Appendix B.3.5. That comparison confirms that the same interface stiffness, without any re-fitting, reproduces the cantilever frequency of 14.07 Hz and the bending frequency of 57.00 Hz identified in the SIII dynamic identification campaign with good agreement, using a completely different experimental campaign, different specimens, and a different measurement system. The SIII frequency data also independently confirms that $K(0.05 \text{ MPa}) \approx K(0.20 \text{ MPa})$ (Section C.3).

On the basis of the 4PB calibration and the modal cross-validation, the interface stiffness is fixed at:

$$\boxed{K_1 = 17 \text{ GPa/m}, \quad k_{nn} = 2K_1 = 34 \text{ GPa/m.}} \quad (\text{C.7})$$

This value is adopted without modification for all HybriDFEM impact simulations across Campaigns 1, 2, and 3. It is not a free parameter in the impact calibration.

D Appendix D: Campaign 1 Impact Calibration HybriDFEM— W9, W10, W11

This appendix documents the calibration of the impact-specific parameters $k_{n,\text{imp}}$, μ , and ξ for Campaign 1 walls W9, W10, and W11. The wall interface stiffness is kept fixed at $K_1 = 17$ GPa/m, as calibrated from four-point bending tests and cross-validated against modal frequencies (Appendices C and B.3.5). The selected Campaign 1 set is

$$k_{n,\text{imp}} = 2.5 \text{ GPa/m}, \quad \mu = 0.65, \quad \xi = 0.020.$$

This set is transferred to Campaign 2 as the starting point for the W16–W19 calibration (Appendix E).

D.1 Overview and calibration strategy

The three calibrated parameters affect different stages of the impact response. The impactor contact stiffness $k_{n,\text{imp}}$ controls the early contact phase, mainly within the first ~ 5 ms. It governs the force peak, force timing, and rate of momentum transfer to the wall.

The friction coefficient μ controls tangential resistance at the mortar joints. Its influence becomes visible once the wall response has developed beyond the contact zone, from approximately $t = 5$ ms onward. Higher μ restrains local sliding and promotes a broader deformation pattern, while lower μ allows more localised sliding near the impact row. Since $k_{n,\text{imp}}$ and μ jointly control how the impulse is transferred into the wall, they are identified together.

The damping ratio ξ controls viscous energy dissipation through the stiffness-proportional damping model

$$\mathbf{C} = 2\xi \mathbf{K}^{\text{tan},lg}.$$

Its effect is mainly visible after contact, especially in the velocity and displacement decay for $t \geq 20$ ms. Campaign 1 has moderate impact velocities and a short observation window of 50 ms, so the response is only weakly sensitive to ξ within the tested range. The value $\xi = 0.020$ is therefore retained as a working value, and a dedicated damping calibration is deferred to Campaign 2.

The calibration is performed in three steps: first, an exploratory sweep over $k_{n,\text{imp}}$ removes inadmissible contact stiffness values; second, a refined cross-wall sweep selects $k_{n,\text{imp}}$ and μ jointly; third, the sensitivity to ξ is assessed. The workflow is shown in Figure D.1.

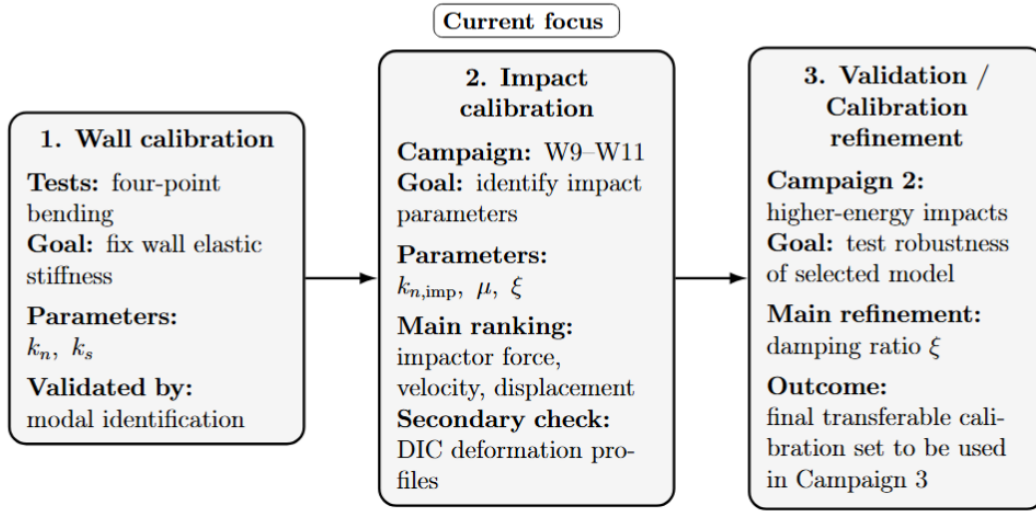


Figure D.1: Sequential calibration workflow. Campaign 1 identifies $k_{n,imp}$ and μ ; the damping calibration is deferred to Campaign 2.

D.2 Experimental reference values

Table D.1 summarises the experimental reference values used for calibration.

Table D.1: Experimental reference values for Campaign 1. v_0 : pre-impact velocity; F_{exp} : peak force; t_F : force peak time; u_{exp} : peak displacement; t_u : displacement peak time; I_{exp} : impulse from numerical integration of the force record.

Wall	v_0 (m/s)	F_{exp} (kN)	t_F (ms)	u_{exp} (mm)	t_u (ms)	I_{exp} (kN·s)
W9	3.13	86.2	0.90	8.14	12.88	0.302
W10	4.43	155.4	1.18	19.90	16.13	0.674
W11	4.43	163.0	0.08	22.89	20.15	0.662

W10 and W11 have the same pre-impact velocity ($v_0 = 4.43$ m/s), but W11 develops a larger peak displacement (22.89 mm compared with 19.90 mm). This is mainly attributed to the lower axial prestress in W11 ($\sigma_{ax} = 0.05$ MPa, compared with 0.20 MPa for W10). As reported by Godio et al. (2023a), the lower prestress led to larger wall deflection and kinetic response, while W10 dissipated a larger fraction of the impact energy through cracking and crushing. W11 also showed the most severe damage, including ejection of the middle brick at the impact row.

D.3 Scoring framework

The parameter combinations are ranked using only the impactor response: displacement, velocity, and force histories. The DIC wall deformation profiles are used as a secondary consistency check, but do not enter the numerical score.

Three sub-scores are computed for each combination:

$$\text{disp_score} = \left| \frac{u_{num}}{u_{exp}} - 1 \right| + \frac{|\Delta t_u|}{10 \text{ ms}} + \frac{\text{RMSE}_u}{u_{exp}}, \quad (\text{D.1})$$

$$\text{vel_score} = \frac{\text{RMSE}_v}{v_{\text{exp}}} + \frac{\max_i |v_{\text{num},i} - v_{\text{exp},i}|}{v_{\text{exp}}}, \quad (\text{D.2})$$

$$\text{force_score} = \left| \frac{F_{\text{num}}}{F_{\text{exp}}} - 1 \right| + \frac{|\Delta t_F|}{5 \text{ ms}} + \left| \frac{I_{\text{num}}}{I_{\text{exp}}} - 1 \right|. \quad (\text{D.3})$$

The total score is

$$\text{impact_score} = \text{disp_score} + \text{vel_score} + \text{force_score}. \quad (\text{D.4})$$

The displacement and velocity RMSE values are evaluated at $t = 10, 20, 30, 40$, and 50 ms. This avoids sensitivity to differences in record length. A shorter timing reference is used for the force peak because the force pulse is much narrower than the displacement response. The impulse term penalises cases that match the force peak but do not reproduce the total transferred impulse. A low impact score means that the response of the wall is better captured. All three sub-scores are given equal weight. The composite score is used as a ranking tool, while the final selection is based on both the score and the physical interpretation of the response across all three walls.

D.4 Parameter sweep and identification

D.4.1 Systematic force overestimation: physical interpretation

Before presenting the sweep results, one systematic trend must be noted. The numerical model generally overestimates the experimental force peak, especially for the lower-energy wall W9. The overestimation decreases as impact energy and wall deformation increase. Table D.2 shows this trend for fixed $\mu = 0.65$ and $\xi = 0.020$.

Table D.2: Force peak ratio $F_{\text{num}}/F_{\text{exp}}$ and numerical peak force in kN, in brackets, for selected $k_{n,\text{imp}}$ values at $\mu = 0.65$, $\xi = 0.020$.

Wall	F_{exp} (kN)	$k_{n,\text{imp}} = 1.5 \text{ GPa/m}$	$k_{n,\text{imp}} = 2.5 \text{ GPa/m}$	$k_{n,\text{imp}} = 5.0 \text{ GPa/m}$
W9	86.2	1.32 [114]	1.58 [136]	2.03 [174]
W10	155.4	1.03 [159]	1.22 [190]	1.58 [246]
W11	163.0	0.83 [135]	1.02 [166]	1.35 [221]

At $k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, the force peak is overestimated by 58% for W9, but only by 2% for W11. This difference reflects a change in the mechanism controlling the contact force.

Contact-controlled regime. At low impact energy the wall barely moves during contact, so the force peak is governed almost entirely by the contact law: the impactor compresses the spring, reaches peak force, and rebounds before wall inertia has time to participate. The linear elastic contact law used in the model,

$$F = k_{n,\text{imp}} \delta,$$

assigns a constant stiffness for all penetration levels. In reality, a spherical-tipped impactor against a flat surface follows a nonlinear Hertz-type response, where $F \propto \delta^{3/2}$ (Johnson,

1985). The stiffness of the contact increases with penetration depth, meaning that the contact becomes progressively stiffer as the impactor pushes further into the wall. At the small penetration depths associated with W9, the real contact stiffness is low, and the linear model (which assigns the same $k_{n,\text{imp}}$ regardless of penetration) overestimates it and therefore the force peak. Second, local dissipative mechanisms at the contact point, such as mortar micro-cracking, crushing, and geometric irregularities reduce the effective contact stiffness in the experiment but are absent in the linear elastic formulation (Godio et al., 2023a).

System-controlled regime. For W10 and W11, the higher impact energy produces significant wall motion during contact. The force peak is then governed by the coupled interaction between impactor inertia, wall inertia, and global wall deformation, rather than by the local contact law alone. As the participation of the wall increases, the influence of the simplified contact law decreases. This effect is stronger for W11, where the lower axial prestress allows larger wall deformation; consequently, the force overestimation is almost eliminated.

Because all combinations overestimate the W9 force peak, the force metric is not a reliable discriminator for that wall. For W9, displacement and velocity are, therefore, the main calibration observables. For W10 and W11, the force ratio remains a meaningful criterion in the refined sweep.

D.4.2 Exploratory sweep — effect of $k_{n,\text{imp}}$

An exploratory sweep of 60 combinations was first performed the three walls:

$$k_{n,\text{imp}} \in \{1.5, 2.0, 3.0, 5.0, 8.0\} \text{ GPa/m}, \quad \mu \in \{0.65, 0.70, 0.75, 0.80\},$$

$$\xi \in \{0.005, 0.010, 0.020\}.$$

W10 is used for this first screening because it remains sensitive to force, velocity, and displacement. In W9, the force response is dominated by the systematic overestimation discussed in Section D.4.1; in W11, the low prestress makes the response more strongly controlled by global wall motion. The aim of this sweep is therefore not to select the final parameters, but to identify the admissible range of $k_{n,\text{imp}}$.

Figures D.2–D.4 show the 60 W10 responses coloured by $k_{n,\text{imp}}$.

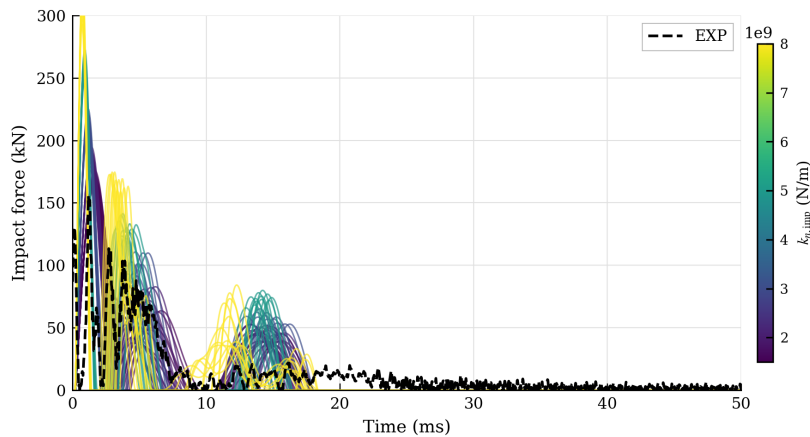


Figure D.2: W10 — impact force histories for the exploratory sweep. Black dashed: experiment.

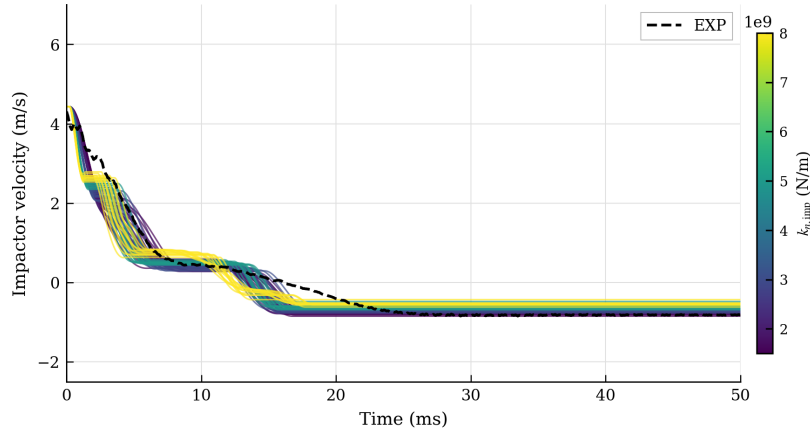


Figure D.3: W10 — impactor velocity histories for the exploratory sweep. Black dashed: experiment.

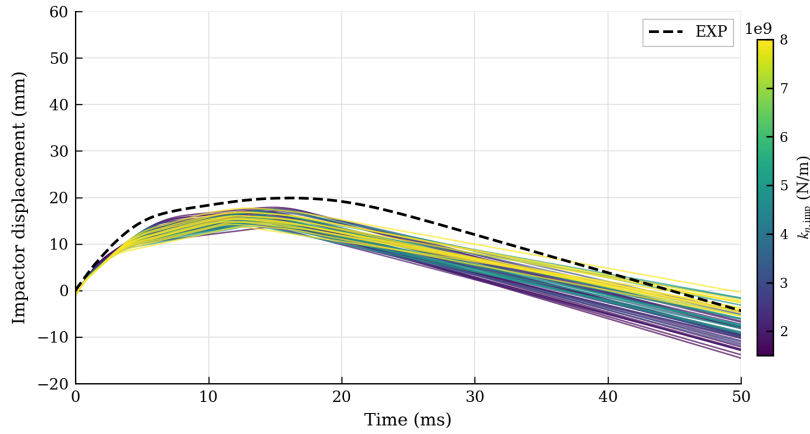


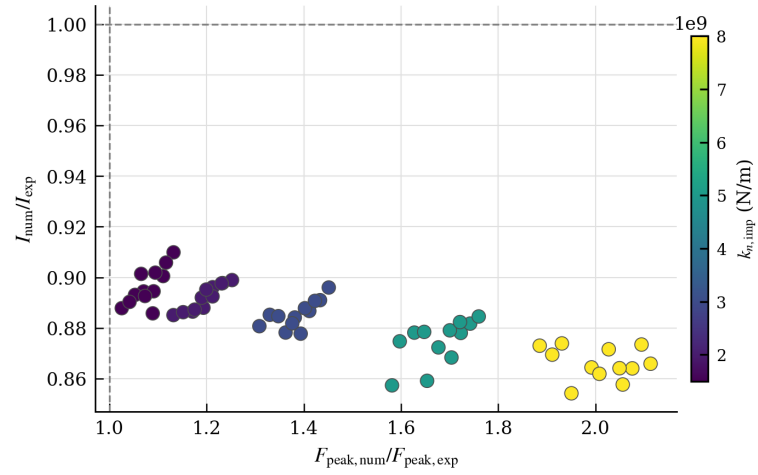
Figure D.4: W10 — impactor displacement histories for the exploratory sweep. Black dashed: experiment.

The force response is mainly controlled by $k_{n,\text{imp}}$. Larger stiffness values produce higher and earlier force peaks, whereas softer contacts produce lower and delayed peaks. Values $k_{n,\text{imp}} \geq 5.0$ GPa/m clearly overestimate the experimental force peak and are therefore excluded.

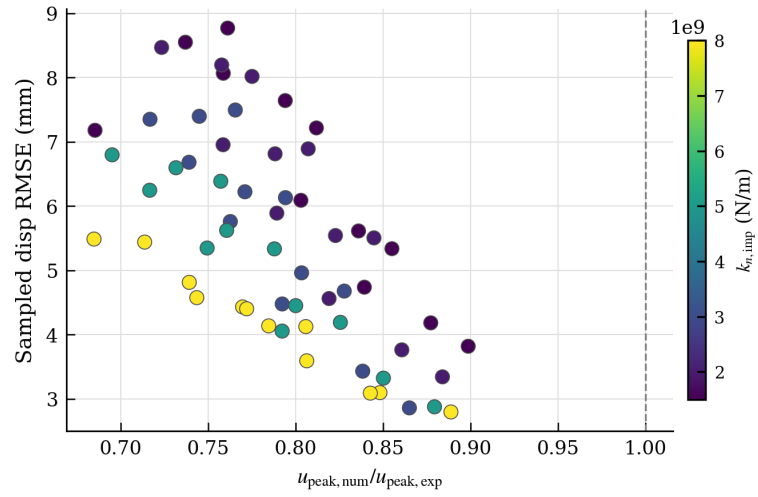
The velocity histories are almost identical during the first ~ 2 ms. Differences appear during the transition phase ($t \approx 2\text{--}15$ ms): stiffer contacts decelerate the impactor more abruptly, while softer contacts give a more gradual decay and track the experimental curve more closely.

The displacement response is less sensitive to the force peak but sensitive to timing. All combinations underestimate the experimental peak displacement, but intermediate stiffness values give the best balance between peak timing, residual displacement, and velocity decay. These trends confirm that $k_{n,\text{imp}}$ should be selected primarily from the force response, while μ should be selected from the deformation pattern and displacement response.

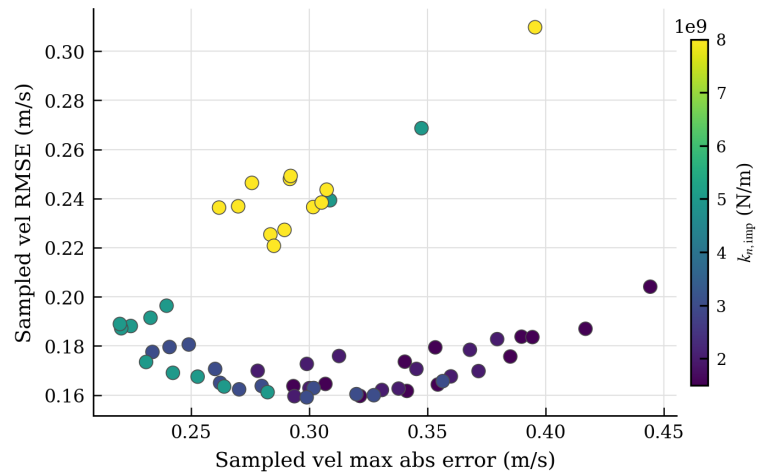
Scatter maps. Figures D.5–D.7 show diagnostic scatter maps for W10, W9, and W11. The colour indicates $k_{n,\text{imp}}$; points with the same colour correspond to different μ – ξ combinations at fixed contact stiffness. The maps are used to identify admissible parameter regions before the refined cross-wall sweep.



(a) Force peak vs. impulse ratio.

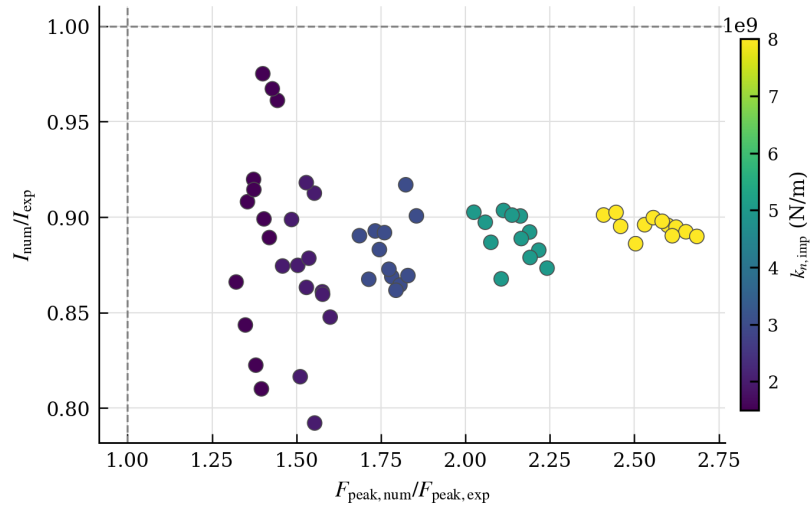


(b) Displacement peak ratio vs. sampled RMSE.

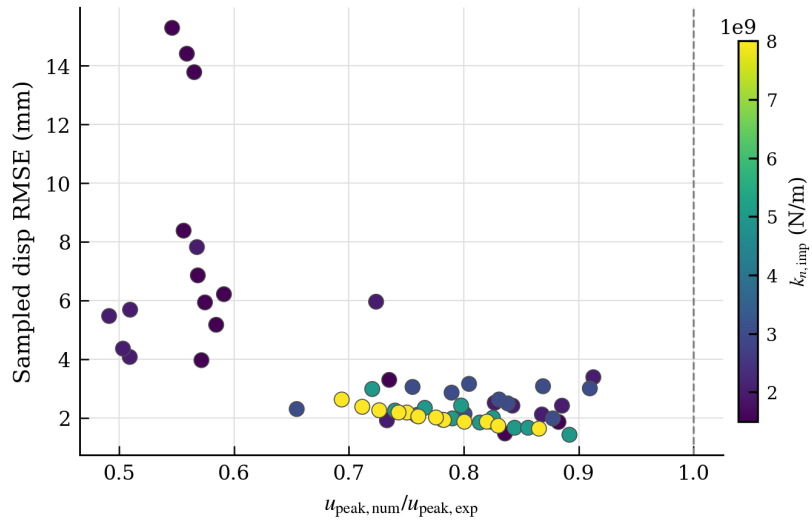


(c) Velocity RMSE vs. maximum velocity error.

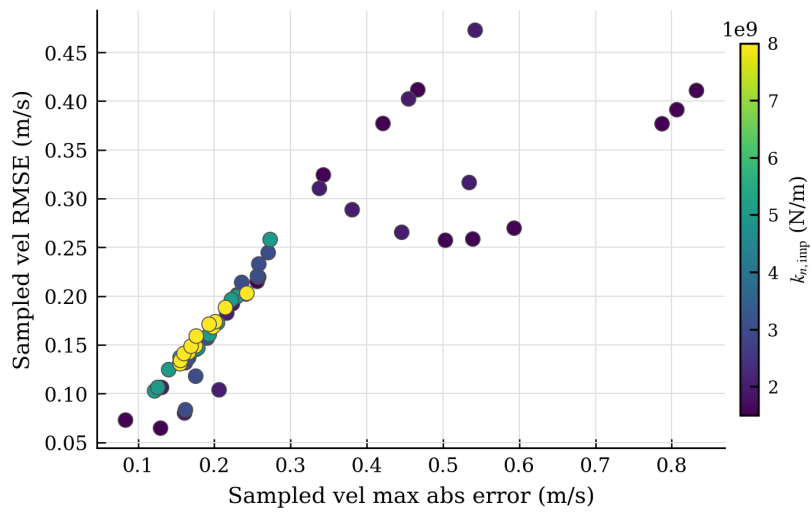
Figure D.5: Scatter maps for W10. Colour indicates $k_{n,\text{imp}}$.



(a) Force peak vs. impulse ratio.

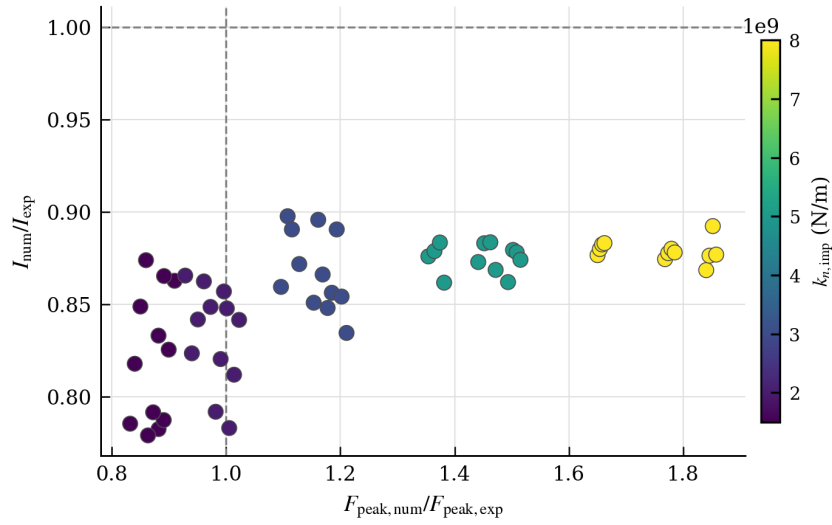


(b) Displacement peak ratio vs. sampled RMSE.

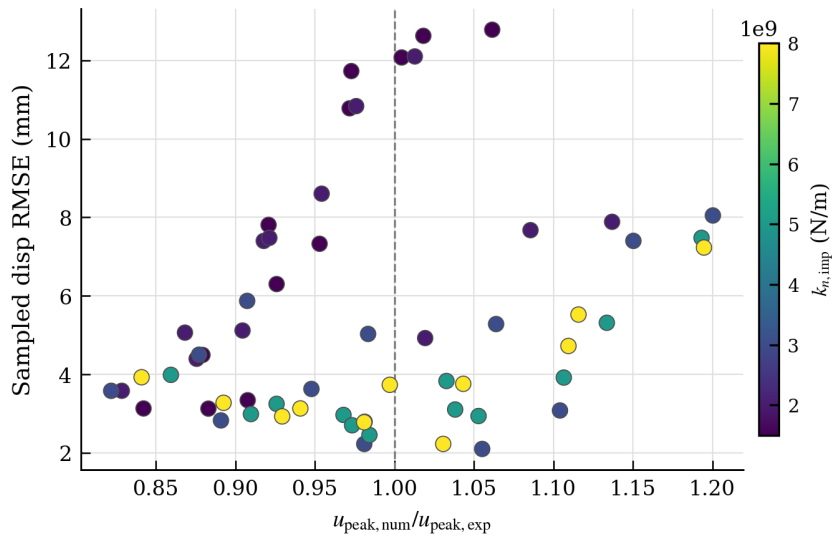


(c) Velocity RMSE vs. maximum velocity error.

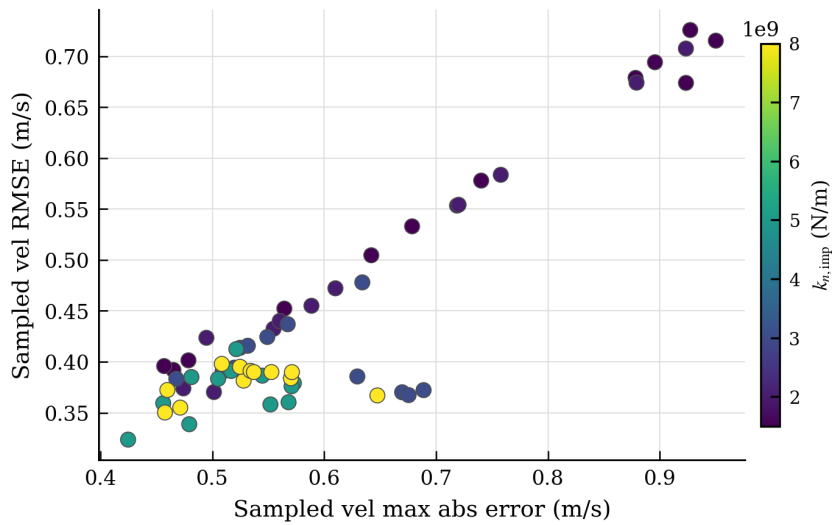
Figure D.6: Scatter maps for W9. Colour indicates $k_{n,imp}$.



(a) Force peak vs. impulse ratio.



(b) Displacement peak ratio vs. sampled RMSE.



(c) Velocity RMSE vs. maximum velocity error.

Figure D.7: Scatter maps for W11. Colour indicates $k_{n,imp}$.

The three walls give complementary information. For W10, the force–impulse map separates clearly by $k_{n,\text{imp}}$: low stiffness values approach the force target, while $k_{n,\text{imp}} \geq 5.0$ GPa/m overestimate the peak force. The impulse ratio varies little, confirming that $k_{n,\text{imp}}$ mainly controls the force pulse shape rather than the total impulse. The displacement and velocity maps favour the intermediate range $k_{n,\text{imp}} \approx 2.0\text{--}3.0$ GPa/m.

For W9, all combinations overestimate the force peak, so the force metric is not useful for selecting $k_{n,\text{imp}}$. The displacement and velocity maps are therefore more informative. For W11, the lowest stiffness values can underpredict the force peak, whereas the highest values overpredict it. The intermediate range again provides the best compromise between force and displacement.

Overall, the exploratory sweep identifies the admissible contact stiffness interval as

$$k_{n,\text{imp}} \approx 2.0\text{--}3.0 \text{ GPa/m.}$$

Values outside this range are excluded from the refined cross-wall sweep.

Deformation profiles. Figures D.8 and D.9 show W10 deformation profiles at five DIC time instants, coloured by $k_{n,\text{imp}}$ and μ , respectively. The two parameters affect different stages of the response.

At $t = 1$ ms, the response is local and contact-dominated. The profiles coloured by $k_{n,\text{imp}}$ already show a visible spread at the impact row, while the profiles coloured by μ are almost identical. This confirms that the early response is governed by impactor contact stiffness, before frictional sliding develops.

From $t = 5$ ms onward, the influence of μ becomes dominant. Lower friction values concentrate deformation near the impact row and produce sharper hinges. Higher friction values couple adjacent rows and distribute deformation more uniformly over the wall height. Thus, μ controls the transition from a localised sliding response to a more global rocking response.

Near the displacement peak ($t = 20$ ms), intermediate values $\mu \approx 0.65\text{--}0.70$ best reproduce the experimental profile. At $t = 40$ ms, no combination fully reproduces the experimental recovery. This is attributed to a contact limitation of the current `ElastoPlastic` formulation, rather than to the calibrated parameter values. In this formulation, tensile opening is handled by a constitutive tension cut-off, but opened contacts remain active in the global force and stiffness assembly. As a result, joints that should be cracked and force-free can still restrain the wall, which limits the post-impact rocking recovery.

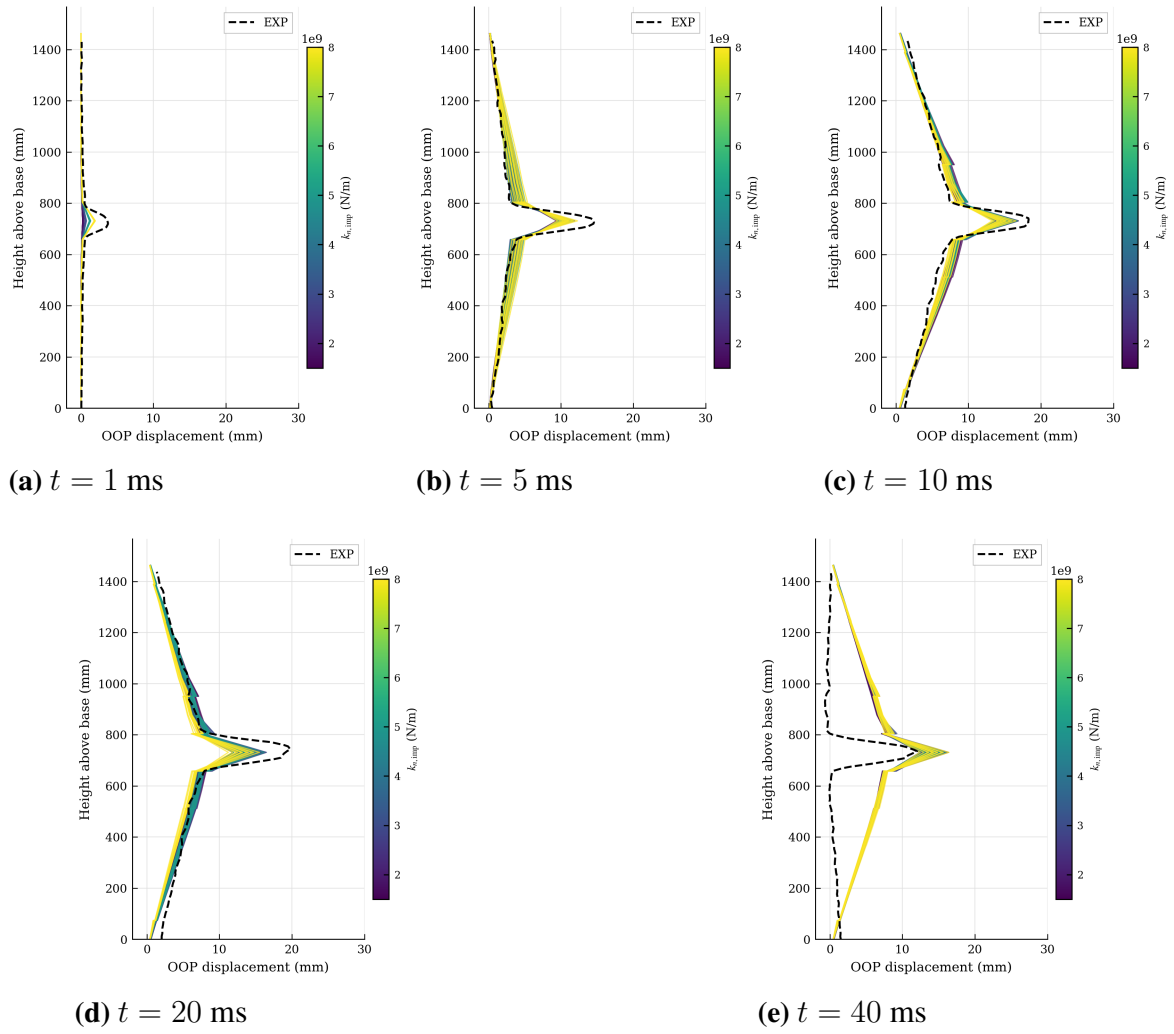


Figure D.8: W10 deformation profiles from the exploratory sweep, coloured by $k_{n,\text{imp}}$.

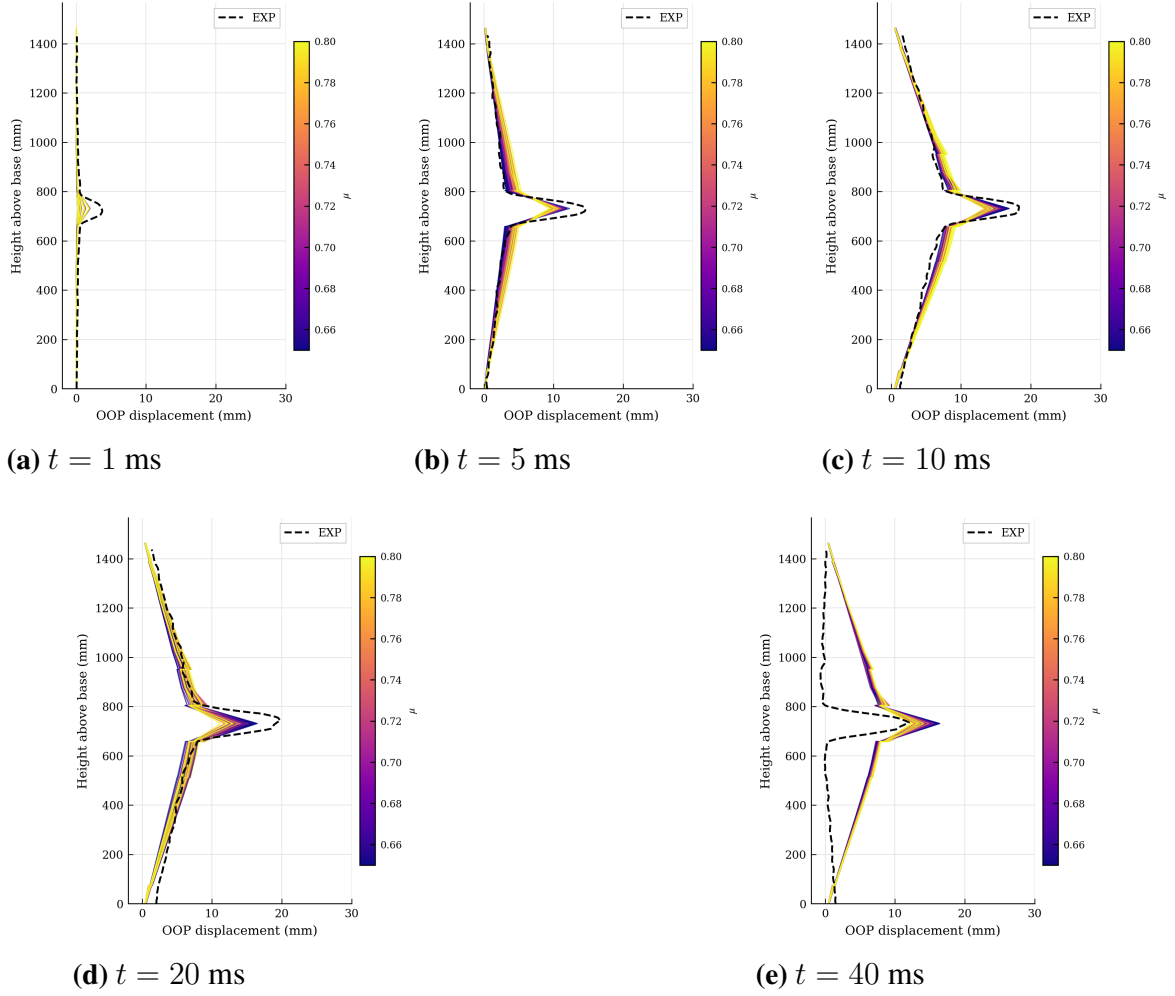


Figure D.9: W10 deformation profiles from the exploratory sweep, coloured by μ .

D.4.3 Refined sweep — effect of μ and cross-wall selection

A refined sweep of 27 combinations was run jointly on W9, W10, and W11:

$$k_{n,\text{imp}} \in \{2.5, 2.8, 3.0\} \text{ GPa/m}, \quad \mu \in \{0.60, 0.65, 0.70\}, \quad \xi \in \{0.010, 0.015, 0.020\}.$$

Impactor histories coloured by μ . Figures D.10–D.12 show the W10 force, velocity, and displacement histories coloured by μ .

The force response is almost insensitive to μ during the primary contact pulse ($t < 5$ ms). At fixed $k_{n,\text{imp}}$, all three friction values give nearly the same force peak, confirming that the initial contact phase is governed mainly by impactor stiffness rather than mortar-joint friction. Differences in μ appear only in the secondary force pulses.

The velocity response shows a clearer friction effect after $t \approx 5$ ms. Higher μ values restrain joint sliding and transfer momentum to a larger effective wall mass, producing earlier impactor deceleration. Lower μ values allow more local sliding and penetration, delaying the velocity decay. At later times, the curves approach a similar negative velocity plateau.

The displacement response is the most sensitive to μ . Higher friction arrests the impactor earlier and gives a steeper post-peak return, while lower friction allows a larger and later

displacement. For W10, $\mu = 0.60$ gives the closest agreement in peak timing and post-peak displacement evolution, although all combinations underestimate the experimental displacement peak.

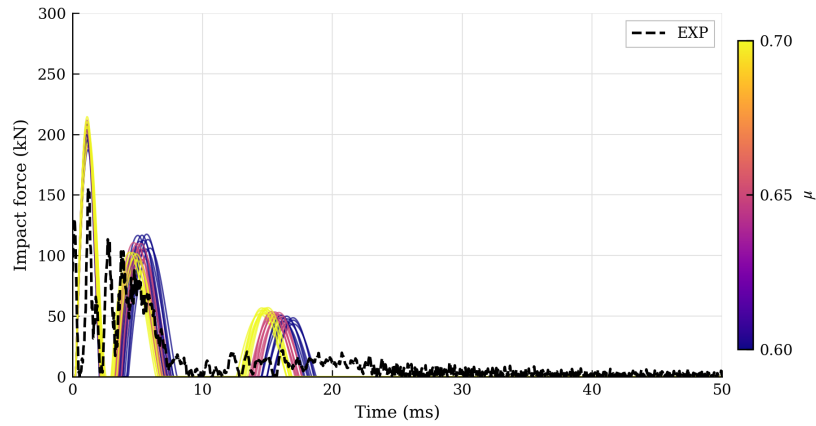


Figure D.10: W10 — impact force histories from the refined sweep, coloured by μ .

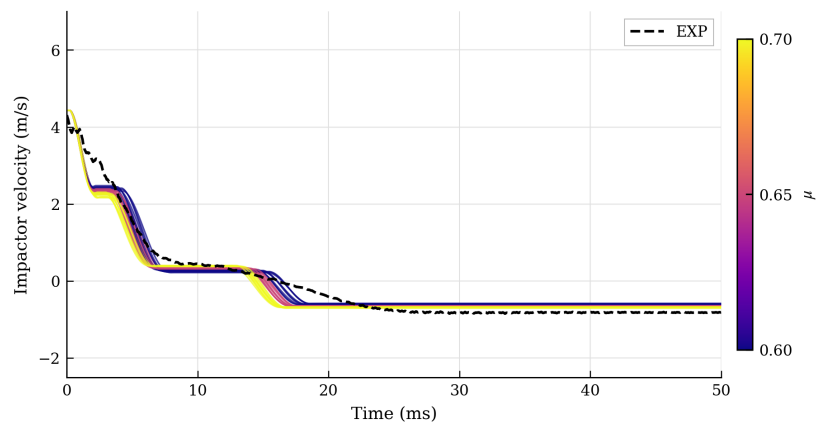


Figure D.11: W10 — impactor velocity histories from the refined sweep, coloured by μ .

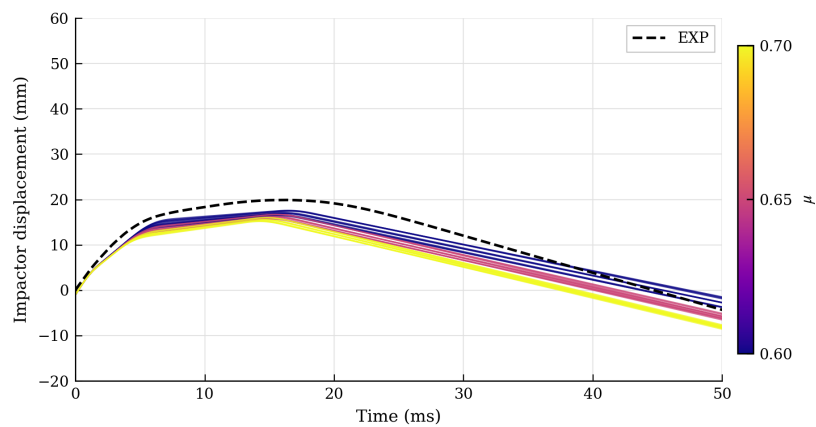


Figure D.12: W10 — impactor displacement histories from the refined sweep, coloured by μ .

Wall-by-wall μ analysis. Table D.3 reports the main metrics at $k_{n,\text{imp}} = 2.5$ GPa/m and $\xi = 0.020$. The preferred friction value differs between walls: W9 and W10 favour $\mu = 0.60$, while W11 favours $\mu = 0.70$. Therefore, the final choice cannot be based on a single wall, but must be selected as a cross-wall compromise.

Table D.3: Impactor metrics at $k_{n,\text{imp}} = 2.5$ GPa/m, $\xi = 0.020$ for $\mu \in \{0.60, 0.65, 0.70\}$. Bold marks the best value per metric per wall; $\star$ marks the selected value.

Wall	μ	u/u_{exp}	Δt_u (ms)	uRMSE (mm)	F/F_{exp}	ImpRat	ImpactSc
W9	0.60	0.87	−2.33	1.64	1.56	0.89	1.398
	0.65 $\star$	0.83	−2.88	2.22	1.58	0.88	1.644
	0.70	0.80	−2.82	2.08	1.60	0.88	1.660
W10	0.60	0.84	−0.43	3.01	1.20	0.87	0.789
	0.65 $\star$	0.80	−1.28	4.45	1.22	0.88	0.992
	0.70	0.77	−2.07	5.81	1.25	0.89	1.195
W11	0.60	1.15	−3.65	6.65	1.01	0.86	1.359
	0.65 $\star$	1.06	−3.05	2.27	1.02	0.89	1.036
	0.70	0.95	−1.55	2.49	1.03	0.88	0.875

For W9 and W10, $\mu = 0.60$ gives the lowest impact score, mainly because it improves displacement timing and reduces the sampled RMSE. These two walls have higher axial prestress ($\sigma_{\text{ax}} = 0.20$ MPa), so the larger normal compression already provides significant frictional resistance.

W11 gives the opposite trend. Because of its lower axial prestress ($\sigma_{\text{ax}} = 0.05$ MPa), the joints are less confined and the wall slides more freely. At $\mu = 0.60$, the displacement is overestimated ($u/u_{\text{exp}} = 1.15$) and the RMSE is high (6.65 mm). Increasing the friction to $\mu = 0.70$ improves W11 and gives its best impact score.

The selected value is therefore $\mu = 0.65$. It is not the best value for every individual wall, but it provides the most robust compromise between the higher-prestress walls W9–W10 and the lower-prestress wall W11.

Selection of $k_{n,\text{imp}}$. Within the refined range, $k_{n,\text{imp}} = 2.5$ GPa/m gives the most consistent cross-wall response. Increasing the stiffness to 2.8 or 3.0 GPa/m raises the force peak without improving the displacement or velocity histories. For W11, the selected stiffness already reproduces the force peak accurately ($F_{\text{num}}/F_{\text{exp}} = 1.02$ at $\mu = 0.65$ and $\xi = 0.020$), while remaining compatible with W10.

The selected pair is therefore

$$k_{n,\text{imp}} = 2.5 \text{ GPa/m}, \quad \mu = 0.65.$$

Table D.4 lists the five best-ranked combinations for each wall. The table confirms the cross-wall conflict: the W10 top-ranked cases are dominated by $\mu = 0.60$, whereas the W11 top-ranked cases are dominated by $\mu = 0.70$. No single combination ranks among the best cases for all three walls, which justifies selecting a robust compromise rather than a wall-specific optimum.

Table D.4: Top-5 combinations by overall impact score for each wall in the refined 27-combination sweep. Lower score is better.

Rank	Combination	u/u_{exp}	uRMSE (mm)	F/F_{exp}	ImpactSc
<i>Wall W9</i>					
1	$k_n=2.5, \mu=0.60, \xi=0.015$	0.90	1.68	1.58	1.397
2	$k_n=2.5, \mu=0.60, \xi=0.020$	0.87	1.64	1.56	1.398
3	$k_n=2.8, \mu=0.60, \xi=0.015$	0.89	1.55	1.65	1.403
4	$k_n=2.8, \mu=0.60, \xi=0.020$	0.86	1.59	1.62	1.405
5	$k_n=3.0, \mu=0.60, \xi=0.020$	0.86	1.60	1.66	1.423
<i>Wall W10</i>					
1	$k_n=2.5, \mu=0.60, \xi=0.015$	0.86	2.54	1.23	0.738
2	$k_n=2.5, \mu=0.60, \xi=0.010$	0.88	2.21	1.25	0.753
3	$k_n=2.5, \mu=0.60, \xi=0.020$	0.84	3.01	1.20	0.789
4	$k_n=2.8, \mu=0.60, \xi=0.015$	0.85	2.67	1.28	0.816
5	$k_n=2.8, \mu=0.60, \xi=0.010$	0.88	2.31	1.31	0.835
<i>Wall W11</i>					
1	$k_n=2.5, \mu=0.70, \xi=0.015$	0.98	3.64	1.05	0.831
2	$k_n=2.5, \mu=0.70, \xi=0.020$	0.95	2.49	1.03	0.875
3	$k_n=2.5, \mu=0.70, \xi=0.010$	1.01	5.40	1.08	0.897
4	$k_n=2.8, \mu=0.70, \xi=0.015$	1.01	1.82	1.10	0.964
5	$k_n=2.8, \mu=0.70, \xi=0.010$	1.04	2.31	1.13	0.970

D.4.4 Working value of ξ and deferral

The damping ratio was varied at the selected contact parameters ($k_{n,\text{imp}} = 2.5$ GPa/m, $\mu = 0.65$):

$$\xi \in \{0.000, 0.005, 0.010, 0.015, 0.020, 0.030\}.$$

The objective was to check sensitivity within the Campaign 1 observation window, not to perform a final damping calibration. Table D.5 reports the main metrics, and Figures D.13–D.15 show the W10 histories.

Table D.5: Sensitivity of the selected contact parameters ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$) to ξ . $\star$ marks the adopted working value.

Wall	ξ	F/F_{exp}	u/u_{exp}	Δt_u (ms)	uRMSE (mm)
W9	0.000	1.70	0.94	−2.33	2.15
	0.005	1.66	0.91	−2.48	2.96
	0.010	1.63	0.88	−2.63	2.24
	0.015	1.61	0.86	−2.73	2.03
	0.020 $\star$	1.58	0.83	−2.88	2.22
	0.030	1.54	0.79	−3.03	2.57
W10	0.000	1.33	0.90	−0.18	2.24
	0.005	1.30	0.87	−0.53	3.07
	0.010	1.27	0.85	−0.83	3.53
	0.015	1.25	0.82	−1.08	4.04
	0.020 $\star$	1.22	0.80	−1.28	4.45
	0.030	1.18	0.78	−1.83	5.19
W11	0.000	1.13	1.25	−1.90	6.74
	0.005	1.10	1.19	−2.25	5.44
	0.010	1.07	1.14	−2.35	3.91
	0.015	1.04	1.10	−2.70	2.92
	0.020 $\star$	1.02	1.06	−3.05	2.27
	0.030	0.98	0.99	−3.70	2.15

Increasing ξ produces small and mostly monotonic changes. The force and displacement peak ratios decrease as damping increases. For W11 this improves the displacement RMSE, while for W9 and W10 it moves the already underestimated displacement peaks further from the experimental values. The W10 histories confirm that all damping curves remain close during the 50 ms record; the main visible effect is a faster post-peak displacement decay at larger ξ .

Campaign 1 is therefore not sufficient to identify ξ uniquely. The impact velocities are moderate and the observation window is short, so damping effects are partly masked by contact stiffness, inertia, and joint friction. The value $\xi = 0.020$ is retained as a working value, and the damping calibration is deferred to Campaign 2, where the impact energy and post-contact observation window are larger.

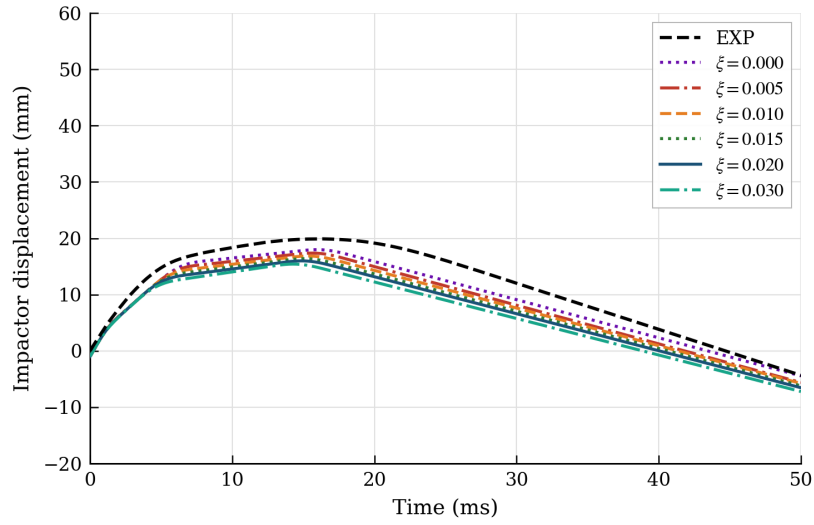


Figure D.13: W10 displacement histories for different values of ξ ($k_{n,\text{imp}} = 2.5$ GPa/m, $\mu = 0.65$).

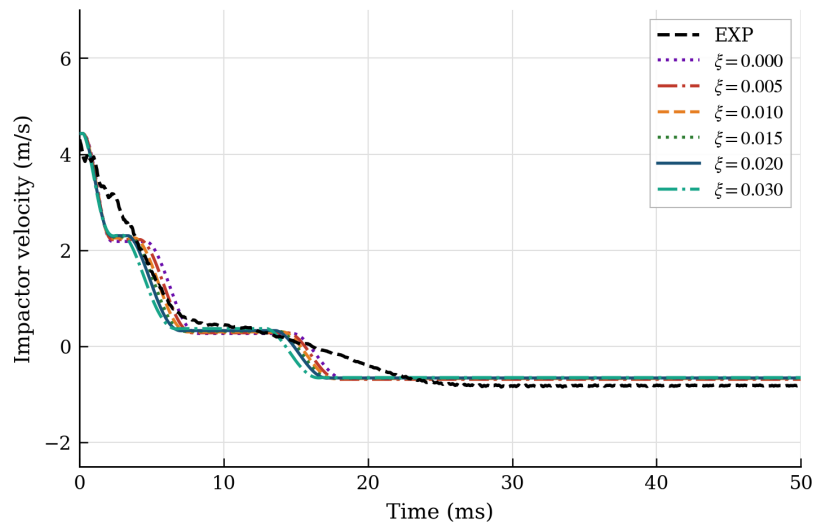


Figure D.14: W10 velocity histories for different values of ξ .

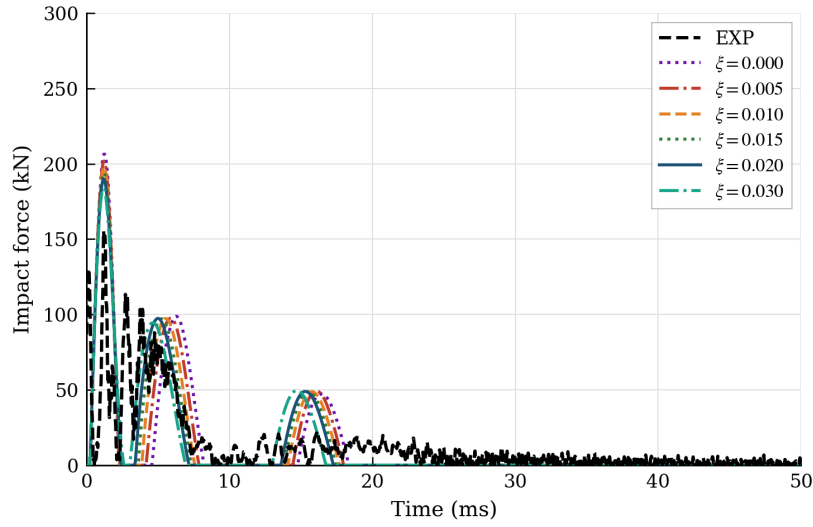


Figure D.15: W10 force histories for different values of ξ .

The magnitude of these changes is small compared with the modelling uncertainties associated with contact stiffness, local damage, and the short experimental record, so no unique optimum can be justified from Campaign 1 alone.

D.5 Selected combination and cross-wall validation

The final combination adopted for Campaign 1 is

$$k_{n,\text{imp}} = 2.5 \text{ GPa/m}, \quad \mu = 0.65, \quad \xi = 0.020 \text{ (working)}.$$

Table D.6 reports the corresponding impactor metrics for W9, W10, and W11.

Table D.6: Impactor metrics for the selected Campaign 1 combination across W9, W10, and W11.

Wall	u/u_{exp}	Δt_u (ms)	uRMSE (mm)	F/F_{exp}	Δt_F (ms)	I/I_{exp}
W9	0.83	−2.88	2.22	1.58	+0.40	0.88
W10	0.80	−1.28	4.45	1.22	+0.03	0.88
W11	1.06	−3.05	2.27	1.02	+1.02	0.89

The selected set gives a consistent cross-wall response. The force ratio decreases from W9 to W11 ($1.58 \rightarrow 1.22 \rightarrow 1.02$), which is consistent with the transition from a contact-controlled response at low impact energy to a more system-controlled response at higher deformation demand. The impulse ratio remains stable around 0.88–0.89, indicating that the total momentum transfer is reproduced consistently across the three walls. The remaining differences are mainly associated with local contact effects in W9 and the larger deformation demand in W11.

D.6 Wall-by-wall results

For each wall, the impactor force, velocity, and displacement histories are shown first, followed by the wall deformation profiles at five DIC snapshot times. The impactor response

is used as the primary calibration criterion, while the deformation profiles are used to verify the spatial consistency of the wall response. In all cases, the numerical wall retains a residual out-of-plane displacement after impact instead of rebounding fully. This limitation is discussed with the arching-force response in Section D.6.4.

D.6.1 W9 — low energy ($v_0 = 3.13$ m/s)

W9 is the most contact-controlled case. The selected combination captures the response timing and the localised deformation mechanism around the impact row, but overestimates the force peak by 58%. This is consistent with the contact-controlled regime discussed in Section D.4.1. The displacement peak is underpredicted ($u/u_{\text{exp}} = 0.83$), but the wall profile shape is reproduced with acceptable agreement.

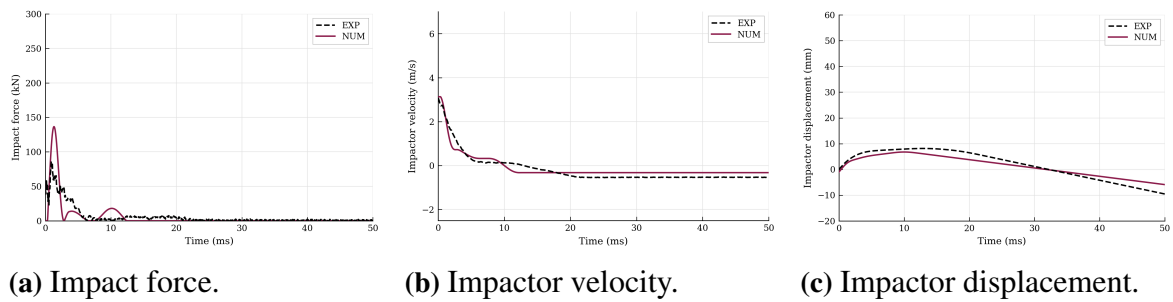


Figure D.16: W9 — impactor response histories for the selected Campaign 1 calibration set.

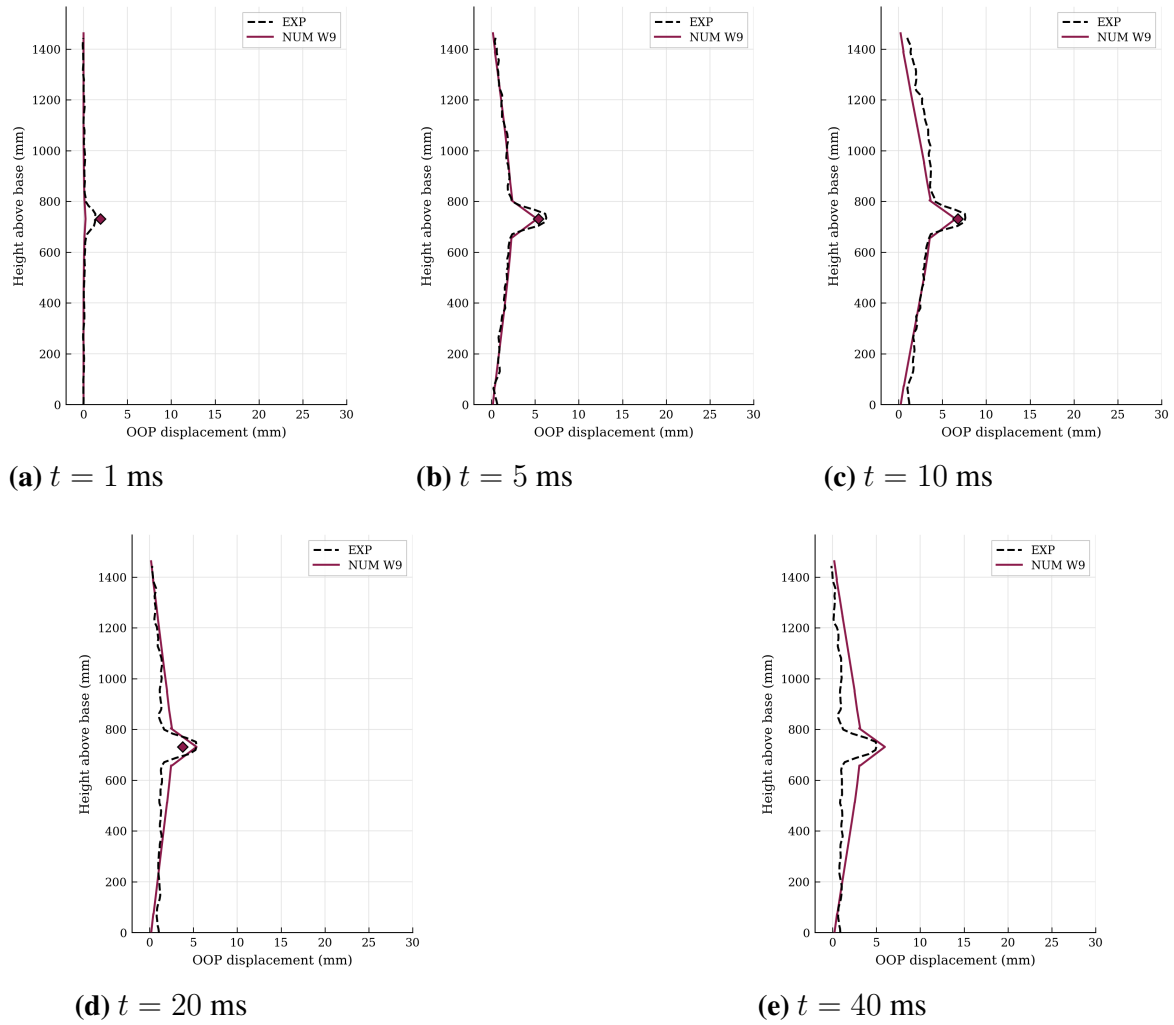


Figure D.17: W9 — wall deformation profiles at five DIC snapshot times for the selected Campaign 1 calibration set.

D.6.2 W10 — intermediate energy ($v_0 = 4.43$ m/s)

W10 provides the most balanced response among the three walls. The force overestimation decreases to 22%, and the impulse ratio remains close to that obtained for W9 and W11. The displacement peak is still underpredicted ($u/u_{\text{exp}} = 0.80$), but the selected combination reproduces the timing and deformation pattern reasonably well.

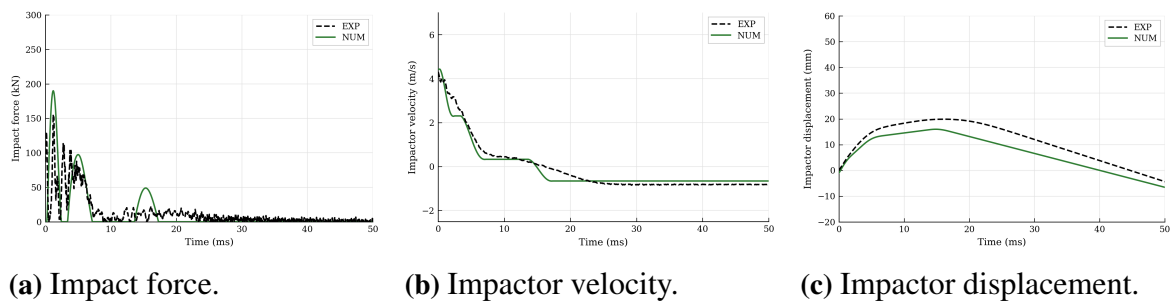


Figure D.18: W10 — impactor response histories for the selected Campaign 1 calibration set.

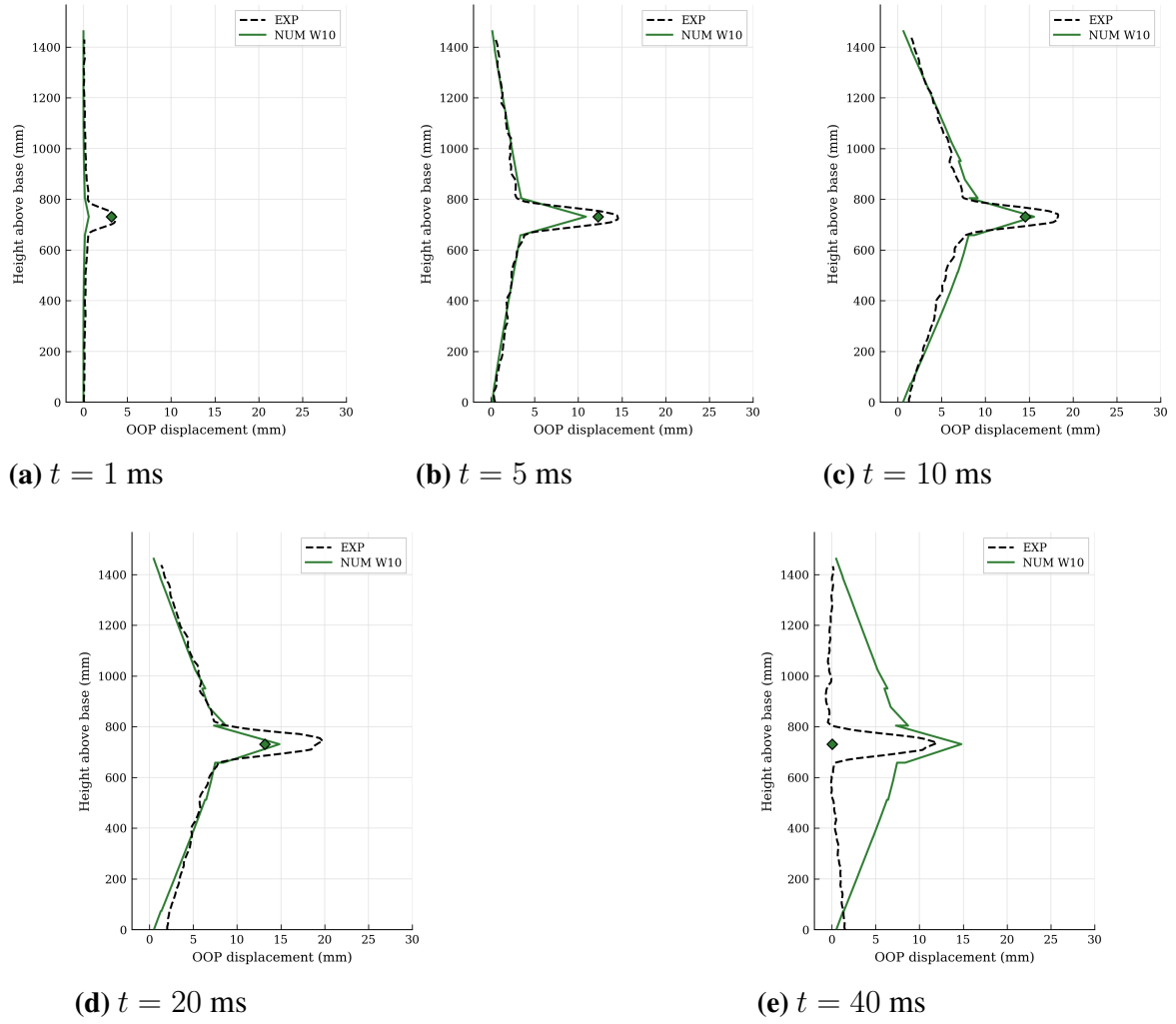


Figure D.19: W10 — wall deformation profiles at five DIC snapshot times for the selected Campaign 1 calibration set.

D.6.3 W11 — high energy ($v_0 = 4.43$ m/s, $\sigma_{ax} = 0.05$ MPa)

W11 is the decisive wall for the cross-wall calibration. It has the same impact velocity as W10 but lower axial prestress, which produces larger wall deformation and a more system-controlled response. The selected combination reproduces the force peak accurately ($F/F_{\text{exp}} = 1.02$) and gives the best force agreement of the campaign. The displacement amplitude is slightly overpredicted ($u/u_{\text{exp}} = 1.06$), but the result remains acceptable and supports the selection of $k_{n,\text{imp}} = 2.5$ GPa/m and $\mu = 0.65$ as a robust cross-wall compromise.

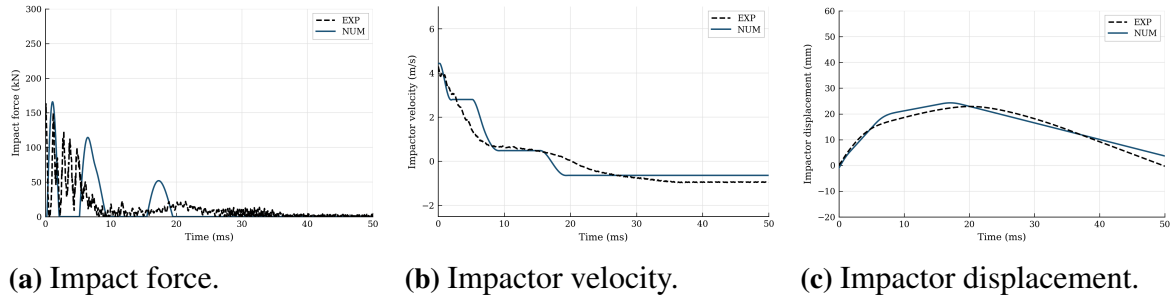


Figure D.20: W11 — impactor response histories for the selected Campaign 1 calibration set.

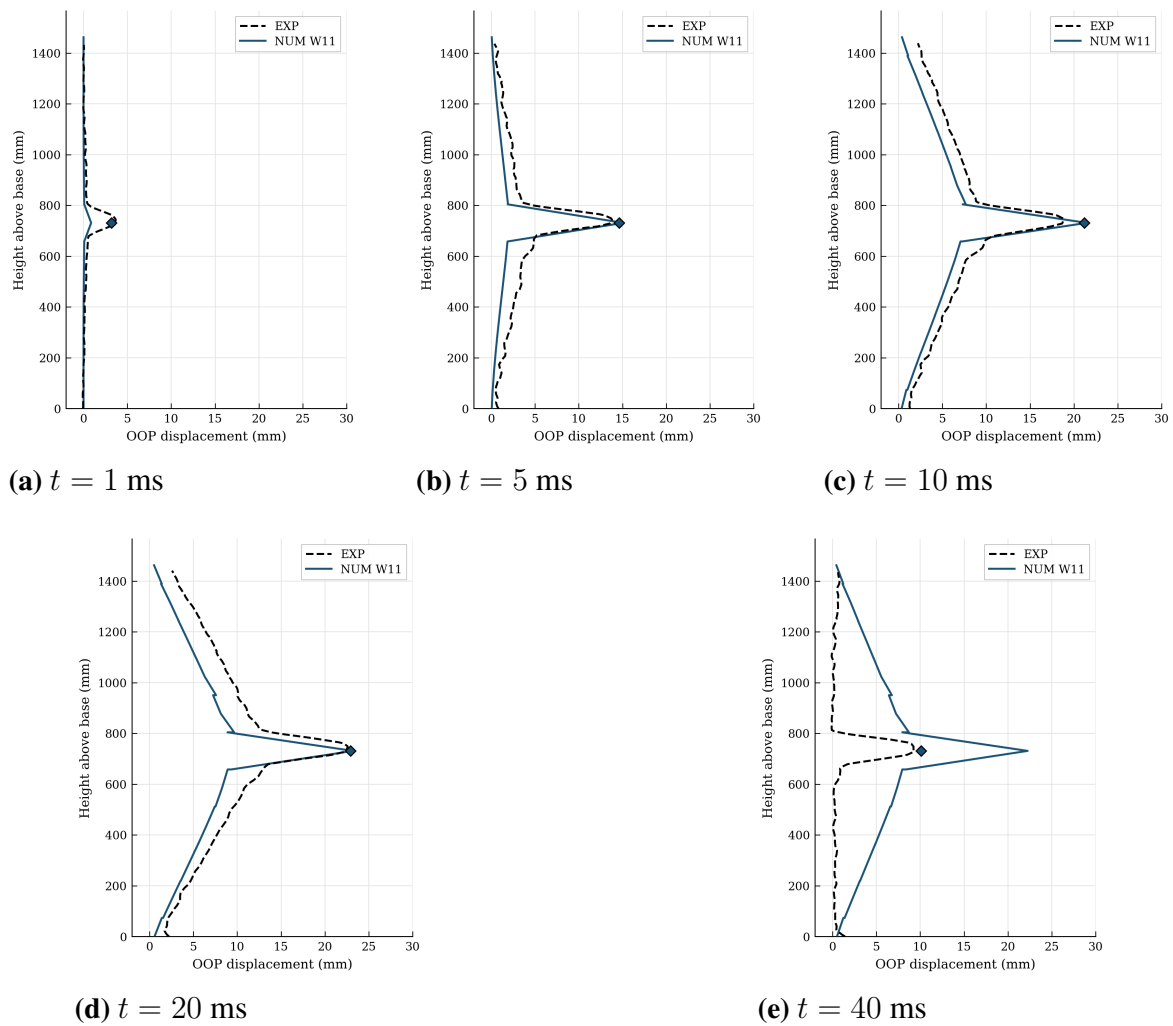


Figure D.21: W11 — wall deformation profiles at five DIC snapshot times for the selected Campaign 1 calibration set.

D.6.4 Arching force response

Under out-of-plane impact, the wall develops a compressive arch between the top and bottom supports as it bows out of plane. The arching force is used here as a secondary validation observable, complementary to the primary impactor-based metrics.

The experimental arching force is obtained from the two prestress rod load cells,

$$F_{\text{arch}}^{\text{exp}}(t) = F_R(t) + F_L(t).$$

The numerical arching force is reconstructed from the vertical spring deformation as

$$F_{\text{arch}}^{\text{num}}(t) = P_0 + K_{\text{sup}} \Delta u_y^{N_1}(t), \quad (\text{D.5})$$

where $P_0 = \sigma_{\text{axial}} Lb$ is the applied prestress and $\Delta u_y^{N_1}$ is the vertical displacement of the top slab node from the start of the dynamic step.

Table D.7 reports the peak arching force for the selected Campaign 1 combination ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 0.020$). The corresponding time histories are shown in Figure D.22.

Table D.7: Arching force peak comparison for Campaign 1. The ratio is $F_{\text{arch,peak}}^{\text{num}}/F_{\text{arch,peak}}^{\text{exp}}$.

Wall	$F_{\text{arch,peak}}^{\text{exp}}$ (kN)	$F_{\text{arch,peak}}^{\text{num}}$ (kN)	Ratio
W9	57.7	79.6	1.38
W10	91.1	142.1	1.56
W11	87.6	125.4	1.43

Two systematic deviations are observed. First, the peak arching force is overestimated by factors of 1.38–1.56. This is mainly attributed to the 2D representation of the point impact: the numerical model mobilises the full wall depth, whereas the experimental point impact activates a more local arching mechanism around the contact zone.

Second, the numerical arching force does not return to the prestress baseline after impact. This is consistent with the DIC profiles at $t = 40 \text{ ms}$, where the numerical wall remains displaced in all three cases (Figures D.17, D.19, and D.21). The non-return is linked to a contact-activation limitation of the current `ElastoPlastic` formulation. In the physical wall, joints that open during rocking should become force-free and close again under gravity and arching action. In the present formulation, tensile opening is limited by the constitutive law, but the opened contact pairs remain active in the global force and stiffness assembly. The wall is therefore artificially restrained in its displaced configuration, keeping the vertical spring compressed and the arching force above the prestress level.

These deviations are treated as known modelling limitations rather than calibration errors. The arching force is therefore used only to verify that the arching mechanism is activated with the correct sign and order of magnitude.

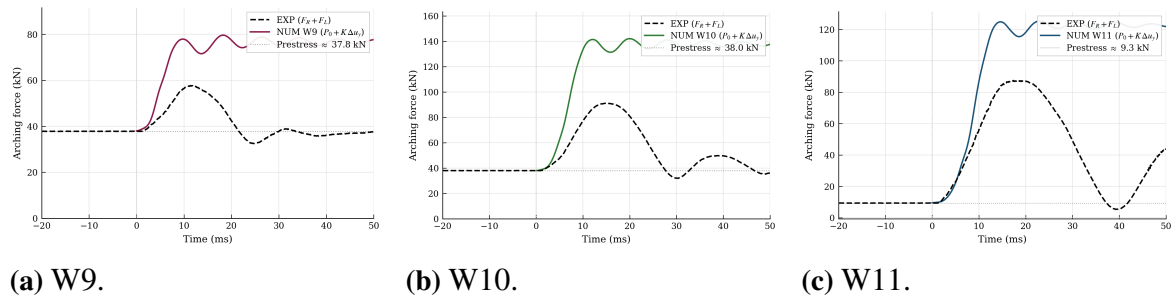


Figure D.22: Arching force histories for Campaign 1 using the selected calibration set.

D.7 Contact geometry linearisation

All Campaign 1 simulations were performed with full geometric nonlinearity at the contact faces (`lin_geom = False`), in which joint orientations are updated at each time step using the current block positions. The sensitivity to this choice was verified by running the selected combination with the linearised assumption (`lin_geom = True`). Differences in all key metrics — force ratio, displacement ratio, timing errors, and sampled RMSE — are negligible across W9, W10, and W11.

The geometric explanation is straightforward: the maximum impactor displacement in Campaign 1 is ≈ 23 mm on a wall of height $H = 1525$ mm, giving block rotations near the impact row of order $\theta \approx 0.015$ rad. At such small angles, $\cos \theta \approx 1$ and $\sin \theta \approx \theta$, so the updated contact face orientation deviates from the reference by a second-order quantity. The Campaign 1 results are therefore valid independently of the linearisation choice, and the selected parameter combination transfers directly to Campaign 2 without modification.

E Appendix E: Campaign 2 Impact Calibration Review — W16, W17, W18, W19 - HybriDFEM

This appendix documents the Campaign 2 impact calibration for walls W16–W19, tested by RISE and reported in Godio and Flansbjerg, 2025b. The Campaign 1 parameter set ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 0.020$) is first transferred to Campaign 2 to assess its limitations. The effective masonry interface stiffness is kept fixed at $K_1 = 17 \text{ GPa/m}$, as calibrated from the four-point bending tests in Appendix C. Based on the Campaign 2 response, revised impact-contact, friction, and damping parameters are then selected and used as the reference set for the Campaign 3 parameter-transfer assessment.

E.1 Overview and calibration strategy

Campaign 2 differs from Campaign 1 by higher impact velocities ($v_0 = 7.2\text{--}8.5 \text{ m/s}$), a longer observation window of 100 ms, and two loading configurations: point-load impacts (W16–W17) and line-load impacts (W18–W19). The direct transfer of the Campaign 1 parameters is not satisfactory, the impact contact stiffness is too low for the higher-velocity impacts, and $\mu = 0.65$ gives excessive displacement for the line-load walls.

The calibration is therefore anchored on W18 and W19. These walls are most consistent with the two-dimensional HybriDFEM idealisation because the spreader beam distributes the impact load across the wall width, making force and impulse meaningful calibration quantities. In contrast, W16 and W17 are point-load tests, where the experimental load is highly localised while the 2D model distributes the response through the wall depth. Since Campaign 3 also uses line-load configurations, the W18–W19 calibration provides the most relevant basis for parameter transfer.

E.2 Boundary condition selection

The Campaign 1 simulations used a rigid horizontal restraint at the top slab, $U_x = 0$, which was consistent with the stiffer clamping frame of W9–W11. When transferred to W16–W19, this boundary condition gave stable and apparently reasonable impactor force, velocity, and displacement histories. However, comparison with the 3D DIC measurements showed that the upper part of the wall was too stiff because the numerical displacements at $3H/4$ and $7H/8$ remained below the experimental amplitudes, indicating that the top boundary could not be treated as horizontally rigid.

This conclusion is supported independently by the modal analysis in Appendix B.3. The SIII dynamic identification campaign measured a near-rigid-body cantilever mode at approximately 14.07 Hz for $\sigma_{\text{ax}} = 0.05 \text{ MPa}$. A model with $U_x = 0$ cannot reproduce this low-frequency mode, whereas a horizontal Timoshenko spring with $K_{\text{sup},x} = 1 \text{ kN/mm}$ matches the experimental frequency within 0.4% (Section B.3.5). The rigid horizontal restraint is therefore replaced by this spring boundary condition for Campaign 2.

The spring boundary condition improves the upper-height displacement amplitudes and oscillation pattern, as shown in Figure E.1. A residual positive displacement remains at lower heights after the impact peak, but this is not attributed to the boundary condition. It is caused by the original `ElastoPlastic` contact formulation, in which contact pairs remain active in the global assembly after reaching the tensile cut-off. This residual displacement is

therefore interpreted as a contact-law limitation related to the absence of contact deletion and recovery.

E.2.1 Quantitative comparison

Table E.1 compares the mean DIC height RMSE for the transferred Campaign 1 parameter set under the rigid and spring top boundary conditions. The spring boundary reduces the error for all four walls, with improvements of 1.7–2.5 mm.

Table E.1: Mean DIC height RMSE (mm) for the transferred Campaign 1 parameter set ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 0.020$).

Boundary condition	W16	W17	W18	W19
Spring BC, $K_{\text{sup},x} = 1 \text{ kN/mm}$	2.29	3.65	3.41	4.06
Rigid BC, $U_x = 0$	4.21	5.35	5.22	6.54

The effect is illustrated for W18 in Figure E.1. The rigid boundary underestimates the upper-height displacement amplitudes, whereas the spring boundary reproduces the measured response more consistently. All subsequent W16–W19 calibration results therefore use $K_{\text{sup},x} = 1 \text{ kN/mm}$.

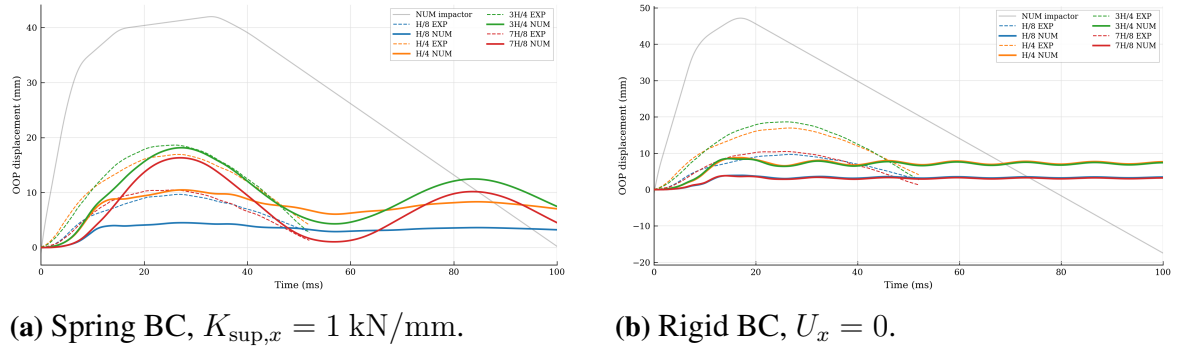


Figure E.1: W18 DIC height displacement histories for the transferred Campaign 1 parameter set.

E.3 Experimental reference values

Table E.2 summarises the experimental quantities used for the Campaign 2 calibration. Impactor displacement, velocity, and force histories are used as the primary ranking criteria, while the 3D DIC displacement histories at $H/8$, $H/4$, $H/2$, $3H/4$, and $7H/8$ are used as a secondary check of the wall deformation shape.

Table E.2: Experimental reference values for W16–W19.

Wall	Load	v_0 (m/s)	F_{exp} (kN)	t_F (ms)	u_{exp} (mm)	t_u (ms)	I_{exp} (kN·s)
W16	point	7.23	484.3	0.46	53.6	27.6	4.015
W17	point	8.49	498.3	0.43	80.9	32.2	3.915
W18	line	7.17	341.8	2.06	44.0	26.4	1.238
W19	line	8.35	468.0	1.47	51.3	30.1	1.489

E.4 Response of the transferred Campaign 1 parameter set

The Campaign 1 parameter set ($k_{n,\text{imp}} = 2.5 \text{ GPa/m}$, $\mu = 0.65$, $\xi = 0.020$) was first evaluated for all four Campaign 2 walls. This case was part of a coarse-model sweep of 168 combinations, with $k_{n,\text{imp}} = 2.0\text{--}8.0 \text{ GPa/m}$, $\mu = 0.55\text{--}0.80$, and $\xi = 0.010\text{--}0.030$. The coarse model was used to reduce computational cost and to rank the global impact response before rerunning the selected parameter sets with the refined model.

Table E.3 shows that the transferred Campaign 1 set ranks in the lower half of the sweep for all walls. The response is therefore not directly transferable to Campaign 2.

Table E.3: Impactor metrics for the transferred Campaign 1 parameter set. Displacement RMSE is sampled at $t = 10, 20, \dots, 100 \text{ ms}$.

Wall	Load	u/u_{exp}	$\Delta t_u \text{ (ms)}$	uRMSE (mm)	F/F_{exp}	$\Delta t_F \text{ (ms)}$	I/I_{exp}	Rank
W16	point	0.79	+5.43	16.74	0.41	+0.42	0.23	125/168
W17	point	0.65	+1.21	18.72	0.46	+0.42	0.27	100/168
W18	line	0.95	+6.62	19.67	0.58	−1.18	0.73	101/168
W19	line	1.01	+3.29	18.23	0.48	−0.62	0.71	88/168

The point-load walls W16–W17 show strong underestimation of force and impulse, reflecting the incompatibility between the localised experimental impact and the two-dimensional load representation. The line-load walls W18–W19 show better impulse agreement because the spreader beam is more consistent with the 2D model, but the force peak remains too low and the displacement response is delayed. These trends indicate that $k_{n,\text{imp}} = 2.5 \text{ GPa/m}$ is too soft for the higher-velocity Campaign 2 impacts.

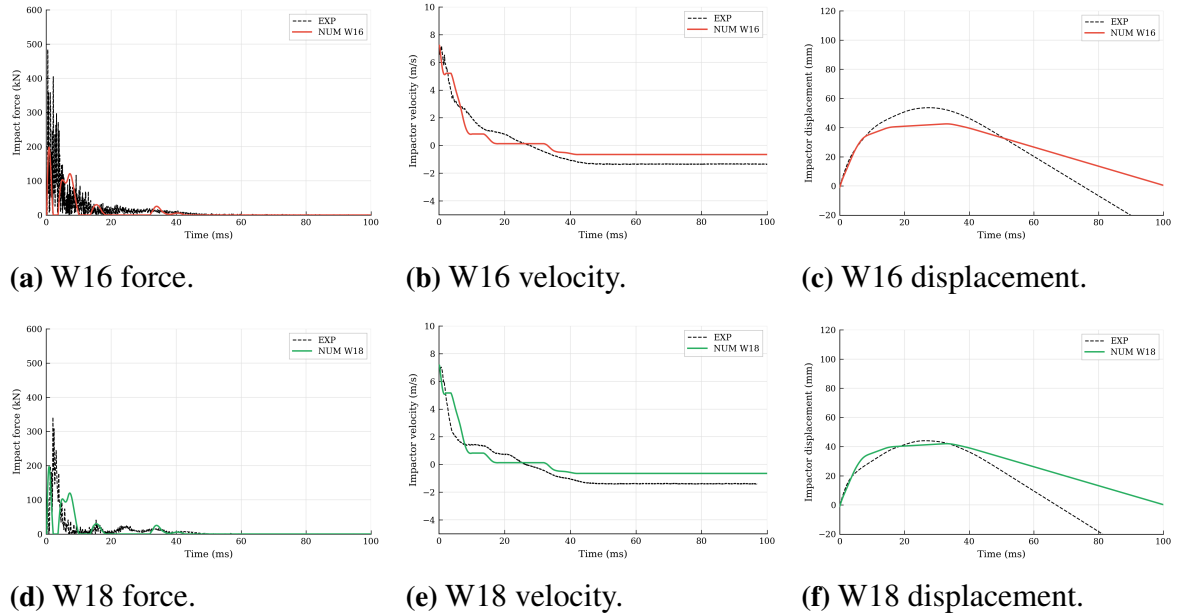


Figure E.2: Representative transferred responses for the Campaign 1 parameter set. W16 represents the point-load configuration and W18 the line-load configuration.

Both representative cases confirm the same limitation. The transferred impact contact stiff-

ness is too low for Campaign 2 and must be increased in the revised calibration.

E.5 Parameter sweep and identification

E.5.1 Calibration strategy

The identification is performed sequentially. First, $k_{n,\text{imp}}$ is selected from the W18–W19 (line load) force peak ratio and displacement timing. Then, the friction coefficient is selected from the W18–W19 displacement and early post-impact velocity response. Finally, the selected impact stiffness is checked on the point-load walls W16–W17, for which a separate effective friction value is retained.

E.5.2 Contact stiffness selection

Table E.4 shows the effect of $k_{n,\text{imp}}$ on W18 and W19 at fixed $\mu = 0.70$ and $\xi = 0.010$. Increasing $k_{n,\text{imp}}$ improves the force peak ratio and reduces the displacement timing error. At 6.0 GPa/m, the force ratio reaches 0.93 for W18 and 0.79 for W19, while the displacement timing errors are reduced to +4.2 ms and +0.8 ms. Increasing the stiffness to 8.0 GPa/m slightly improves the force level, but begins to overshoot W18 and worsens the displacement timing.

Table E.4: Effect of $k_{n,\text{imp}}$ on the line-load response.

$k_{n,\text{imp}}$	W18				W19			
	F/F_{exp}	Δt_u	u/u_{exp}	I/I_{exp}	F/F_{exp}	Δt_u	u/u_{exp}	I/I_{exp}
2.5	0.68	+7.1	0.98	0.74	0.57	+4.7	1.02	0.72
3.0	0.73	+6.4	0.97	0.74	0.61	+2.7	1.02	0.71
4.0	0.81	+4.5	0.97	0.72	0.68	+1.3	1.02	0.69
5.0	0.88	+4.4	0.96	0.72	0.74	+1.0	1.01	0.69
6.0	0.93	+4.2	0.96	0.72	0.79	+0.8	1.00	0.69
8.0	1.03	+5.0	0.94	0.73	0.87	+1.7	0.99	0.71

The impulse ratio remains almost unchanged over the full stiffness range, approximately 0.69–0.74. This indicates that $k_{n,\text{imp}}$ controls the shape and timing of the force pulse rather than the total transferred impulse, which is governed mainly by the global impact dynamics, impactor mass, initial velocity, and wall compliance.

The remaining force underestimation is attributed to the simplified impactor representation. In the experiment, the spreader beam engages the wall progressively, whereas in the model the rigid impactor applies the load instantaneously and uniformly. This also explains the negative force timing error observed for the line-load walls.

The increase from 2.5 to 6.0 GPa/m is treated as an effective contact-stiffness adjustment for the higher Campaign 2 velocities. The parameter is not only a masonry material property, but a representation of the combined effect of impactor geometry, local compliance, and brick deformation at the contact zone.

$$k_{n,\text{imp}} = 6.0 \text{ GPa/m}, \quad k_{s,\text{imp}} = k_{n,\text{imp}}/5 = 1.2 \text{ GPa/m}.$$

Figure E.3 confirms this trend for the full W18 sweep. The force ratio increases with $k_{n,\text{imp}}$, while the impulse ratio remains nearly constant.

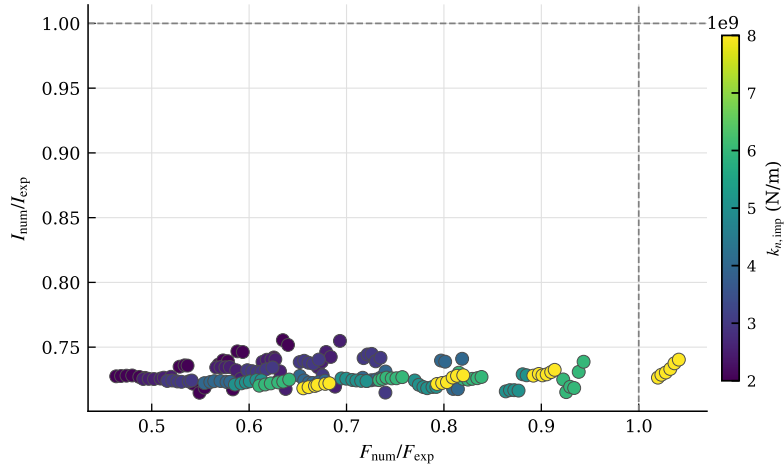


Figure E.3: W18 force peak ratio versus impulse ratio for the full sweep, grouped by $k_{n,\text{imp}}$.

E.5.3 Effective friction for line-load walls

With $k_{n,\text{imp}} = 6.0 \text{ GPa/m}$ fixed, the influence of μ is assessed for W18 and W19. The early force response is almost insensitive to μ , since the initial contact phase is governed mainly by the impact contact stiffness. The velocity and displacement responses are more sensitive, because friction controls the joint sliding and rocking mobilised after the contact.

Figure E.4 shows the effect for W18. Higher friction values produce earlier velocity decay and smaller displacement demand by restraining sliding. Lower values allow larger relative sliding and penetration, giving a delayed rebound. After approximately 40–50 ms, the velocity curves become closer, indicating that μ mainly controls the early sliding–rocking response rather than the late-time velocity plateau.

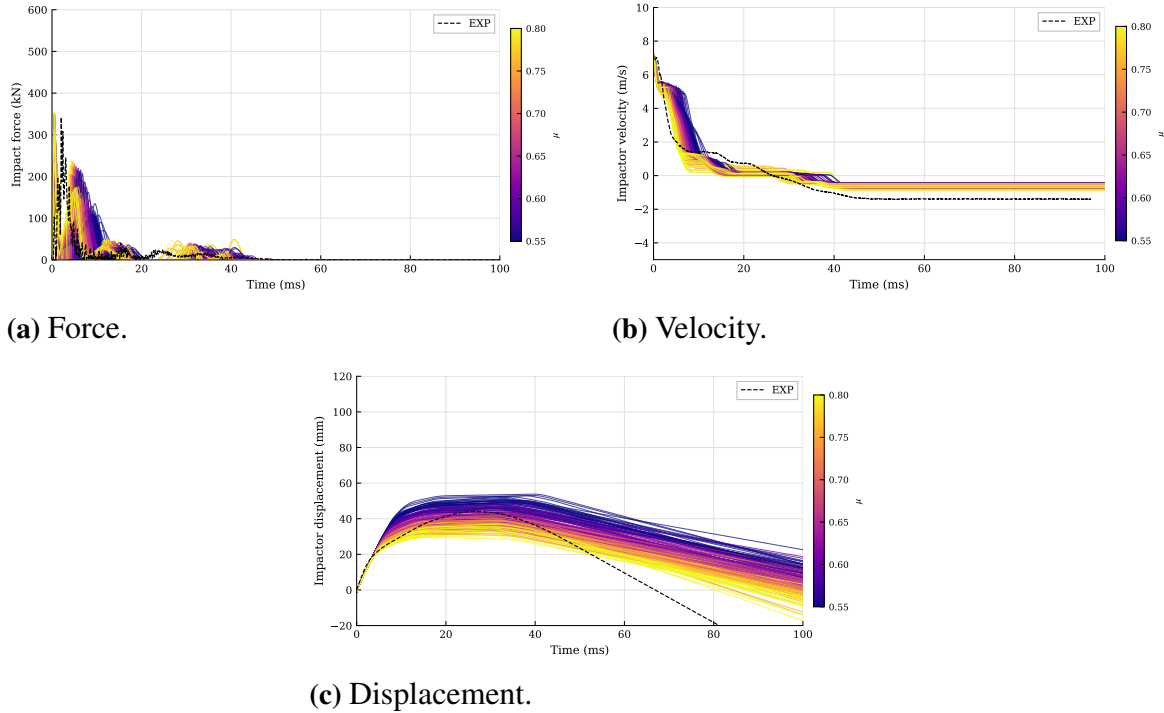


Figure E.4: Influence of μ on the W18 impactor response at $k_{n,\text{imp}} = 6.0 \text{ GPa/m}$.

The transferred value $\mu = 0.65$ allows slightly excessive sliding for the line-load walls, with displacement ratios of 1.02 for W18 and 1.07 for W19. Values around 0.70–0.75 reduce this demand and give a more distributed-rocking response. The value $\mu = 0.75$ is therefore adopted as an effective numerical friction coefficient for the line-load configuration. It is not interpreted as a direct material friction measurement, but as a calibrated parameter that compensates for distributed sliding in the two-dimensional representation of the spreader-beam impact.

$$\mu = 0.75 \quad \text{for W18 and W19, line-load configuration.}$$

E.5.4 Effective friction for point-load walls

For the point-load walls W16 and W17, the friction sweep at $k_{n,\text{imp}} = 6.0 \text{ GPa/m}$ shows that the Campaign 1 value $\mu = 0.65$ remains the most appropriate effective value. This lower friction compensates for the two-dimensional modelling assumption. In the experiments, the point-load impact is highly localised and produces stronger local penetration and sliding near the impact zone, whereas in the model the load is distributed through the wall depth.

Reducing μ to 0.55 improves the W17 displacement peak but degrades W16. Increasing μ to 0.70 suppresses the displacement amplitude, especially for W17. A single effective value of $\mu = 0.65$ is therefore retained for both point-load walls.

$$\mu = 0.65 \quad \text{for W16 and W17, point-load configuration.}$$

E.5.5 Damping sensitivity study

A damping sensitivity study was performed for the line-load walls W18 and W19 using the selected line-load parameters

$$k_{n,\text{imp}} = 6.0 \text{ GPa/m}, \quad \mu = 0.75.$$

Only stiffness-proportional damping was considered ($\alpha_M = 0$), following Section 2.4.5.4. Two damping operators were compared: the initial stiffness matrix $\mathbf{K}_0$ (ISPD, denoted `INIT`) and the linear-geometric tangent stiffness matrix $\mathbf{K}^{\text{tan},lg}$ (TSPD, denoted `TAN_LG`). The full tangent matrix was not considered, since including geometric stiffness in the damping operator may introduce non-physical energy effects in large-displacement rocking problems (Baena-Urrea & Almeida, 2023; Bouckaert et al., 2026).

The damping ratio was varied as

$$\xi \in \{0.0025, 0.005, 0.010, 0.025, 0.050\}.$$

The study was carried out for W18 and W19. Since both walls led to the same conclusions, W18 is presented as the representative case.

The assessment is based on the post-contact impactor velocity. After approximately $t = 20$ ms, the impactor has separated from the wall, so the response is no longer controlled by the contact stiffness or the force pulse. Two windows are considered: a primary window from 20 to 50 ms, where the velocity decay is most sensitive to ξ , and a secondary window from 60 to 100 ms, used to check the late-time velocity plateau. An acceptable damping model must reproduce both windows without artificially suppressing the force peak or displacement amplitude.

Figure E.5a shows that the `INIT` cases decay too rapidly, reaching near-zero velocity by $t \approx 10$ ms. The displacement histories in Figure E.5c show the same limitation, all `INIT` cases strongly underpredict the displacement peak ($u/u_{\text{exp}} = 0.27\text{--}0.58$). This occurs because $\mathbf{K}_0$ retains the stiffness of joints that have opened during rocking, so viscous forces continue to act through contacts that are no longer active in the physical mechanism. In addition, Figure E.5b shows that the `INIT` cases can overestimate the force peak (F/F_{exp} up to 1.53), because excessive damping restrains wall motion during contact and makes the wall artificially stiff. The `INIT` formulation is therefore rejected for all values of ξ .

The `TAN_LG` cases reproduce the experimental velocity decay qualitatively over the full observation window. Differences between damping ratios are most visible over 10–50 ms, but become small after $t = 60$ ms.

Within the `TAN_LG` family, $\xi = 0.010$ gives the best overall balance between velocity decay, displacement amplitude, and force level. Although $\xi = 0.005$ gives a displacement peak closer to the experimental value ($u/u_{\text{exp}} = 1.01$), it produces a large timing error (+13.9 ms) and a higher post-contact velocity RMSE than $\xi = 0.010$. Larger damping ratios suppress the response: at $\xi = 0.025$ and 0.050 , the displacement peak decreases to $u/u_{\text{exp}} = 0.87$ and 0.75 , while the force peak decreases to $F/F_{\text{exp}} = 0.68$ and 0.49 . Thus, $\xi = 0.010$ is selected as the lowest stable damping level that provides consistent post-contact velocity decay without artificially reducing force or displacement.

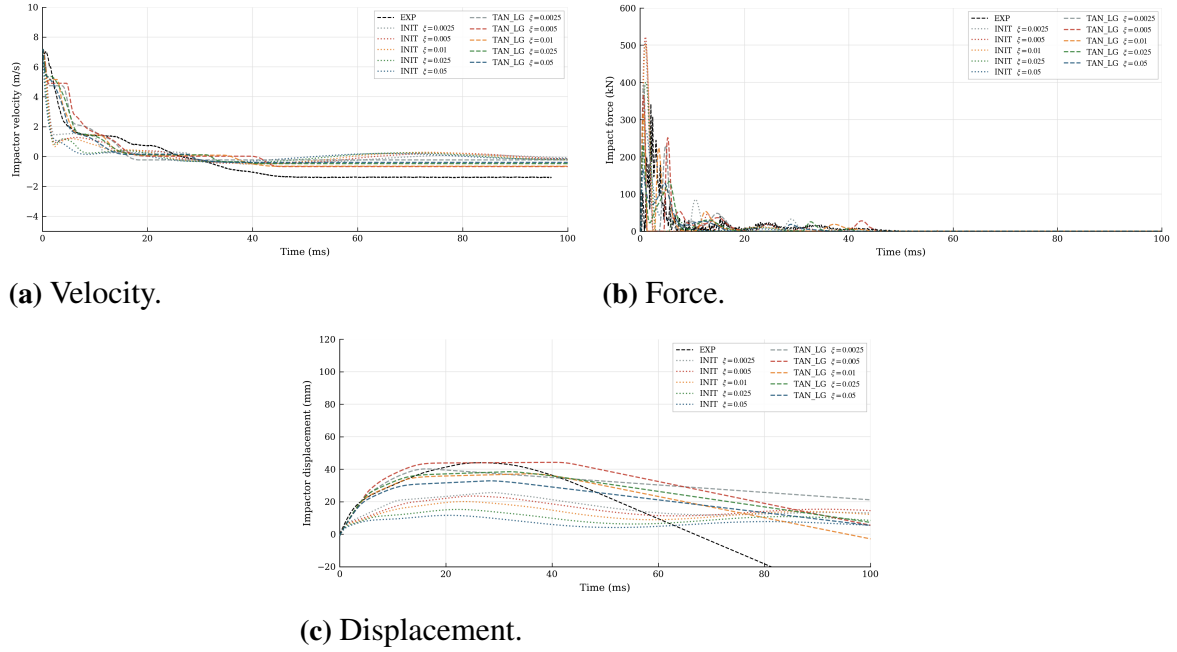


Figure E.5: W18 damping sensitivity study for INIT and TAN_LG damping operators.

The adopted damping model is therefore

$$C = 2\xi K^{\tan,lg}, \quad \xi = 0.010.$$

This value is not selected from a single scalar metric, but from the combined agreement in post-contact velocity decay, force level, and displacement amplitude. The same conclusion was obtained for W19.

E.6 Selected combinations and cross-wall metrics

The selected Campaign 2 parameter combinations are summarised in Table E.5. The same impact contact stiffness and damping ratio are used for all four walls, while the effective friction coefficient differs between point-load and line-load cases.

Table E.5: Selected parameter combinations for Campaign 2.

Parameter	Symbol	W16–W17	W18–W19	Source
Contact stiffness	$k_{n,imp}$	6.0 GPa/m	6.0 GPa/m	W18–W19 force calibration
Shear stiffness	$k_{s,imp}$	$k_n/5$	$k_n/5$	Fixed proportion
Friction coefficient	μ	0.65	0.75	Cross-wall sweep
Damping ratio	ξ	0.010	0.010	Damping study

The refined-model impactor metrics are reported in Table E.6. The selected combinations rank in the top 5% of the sweep for the line-load walls and in the top 7–13% for the point-load walls, compared with the lower-half ranking of the transferred Campaign 1 parameter set (Table E.3).

Table E.6: Impactor metrics for the selected Campaign 2 combinations, evaluated with the refined model.

Wall Load	u/u_{exp}	Δt_u (ms)	uRMSE (mm)	F/F_{exp}	Δt_F (ms)	I/I_{exp}	Rank
W16 point	0.87	+7.0	22.5	0.66 [†]	+0.17	0.23 [†]	11/168
W17 point	0.71	+6.0	15.1	0.74 [†]	+0.18	0.27 [†]	19/168
W18 line	0.85	+9.5	17.8	0.93	-1.43	0.74	8/168
W19 line	0.96	+6.4	16.9	0.78	-0.86	0.72	5/168

[†] Force and impulse ratios for point-load walls are reported as indicators only, due to the 2D point-load geometry limitation.

E.7 Wall-by-wall results

For each wall, the impactor force, velocity, and displacement histories are shown together with the out-of-plane DIC displacement profiles at $H/8$, $H/4$, $3H/4$, and $7H/8$. The numerical walls generally retain a residual out-of-plane displacement after impact instead of fully rebounding. This limitation is discussed together with the arching-force response in Section E.8.

E.7.1 W16 — point load, $v_0 = 7.23$ m/s

W16 is the lower-energy point-load case. The impactor displacement is reasonably reproduced, while force and impulse are underestimated due to the two-dimensional point-load representation. The DIC profiles show good amplitude agreement, although a residual post-impact displacement remains.

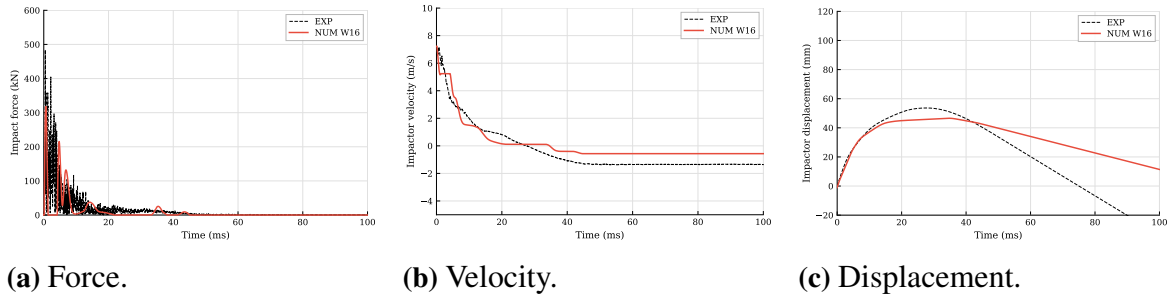


Figure E.6: W16 impactor response for the selected combination, $k_{n,\text{imp}} = 6.0$ GPa/m, $\mu = 0.65$, $\xi = 0.010$. Rank 11/168.

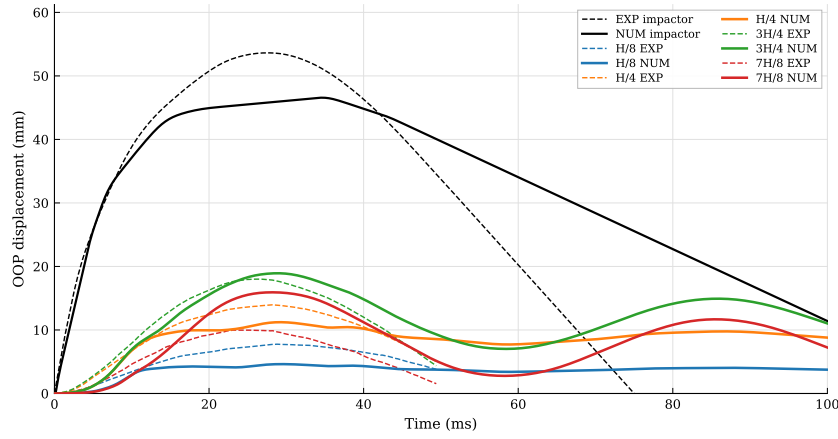


Figure E.7: W16 out-of-plane DIC displacement histories.

E.7.2 W17 — point load, $v_0 = 8.49$ m/s

W17 is the most demanding point-load case. The displacement amplitude is underestimated by approximately 30%, a deficit that persists across the friction sweep and indicates an identifiability limit of the two-dimensional rigid-block model for high-energy point impacts. The DIC profiles nevertheless reproduce the correct deformation trend at the monitored heights.

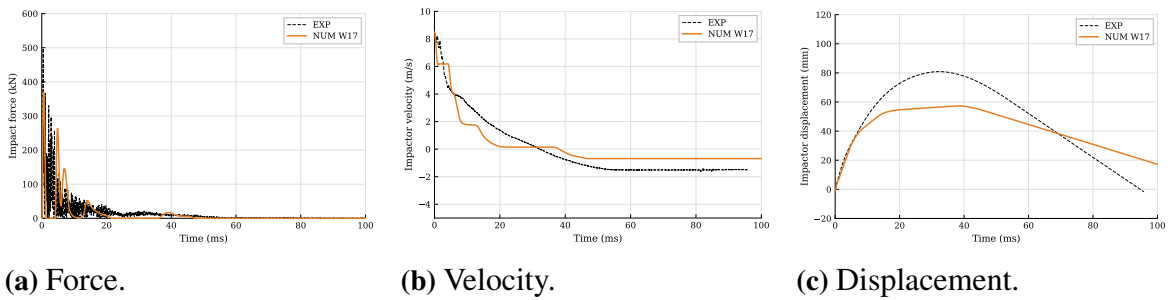


Figure E.8: W17 impactor response for the selected combination, $k_{n,\text{imp}} = 6.0$ GPa/m, $\mu = 0.65$, $\xi = 0.010$. Rank 19/168.

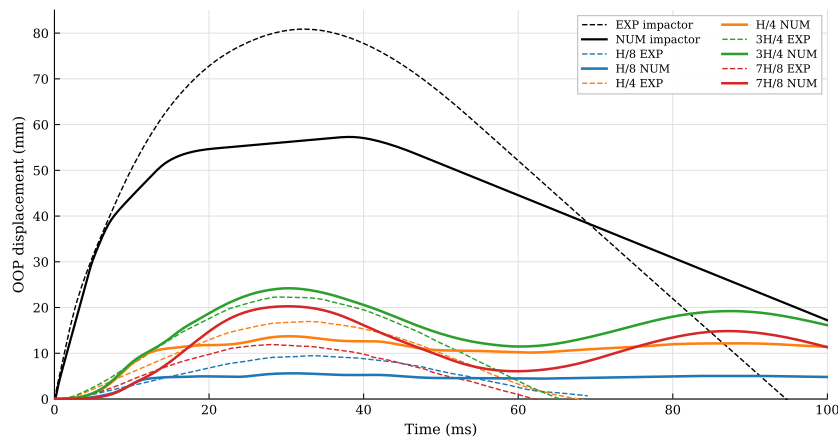


Figure E.9: W17 out-of-plane DIC displacement histories.

E.7.3 W18 — line load, $v_0 = 7.17$ m/s

W18 is the main line-load calibration case. The force peak is reproduced within 7%, and the displacement amplitude within 15%. The DIC profiles capture the correct trend and order of magnitude, with remaining differences consistent with the simplified two-dimensional rigid-block representation.

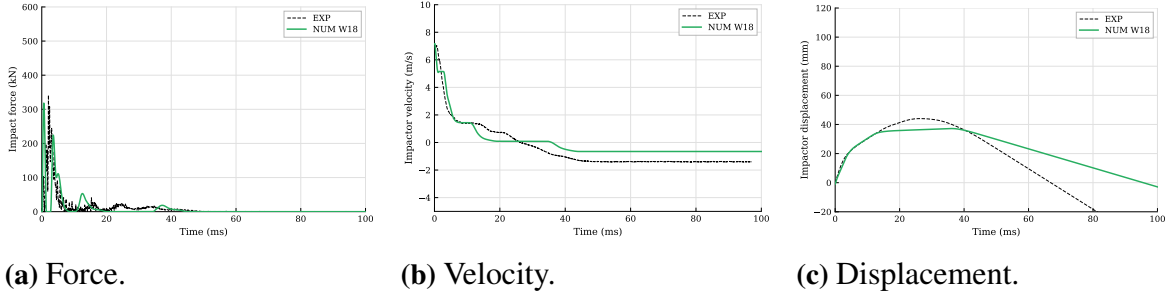


Figure E.10: W18 impactor response for the selected combination, $k_{n,imp} = 6.0$ GPa/m, $\mu = 0.75$, $\xi = 0.010$. Rank 8/168.

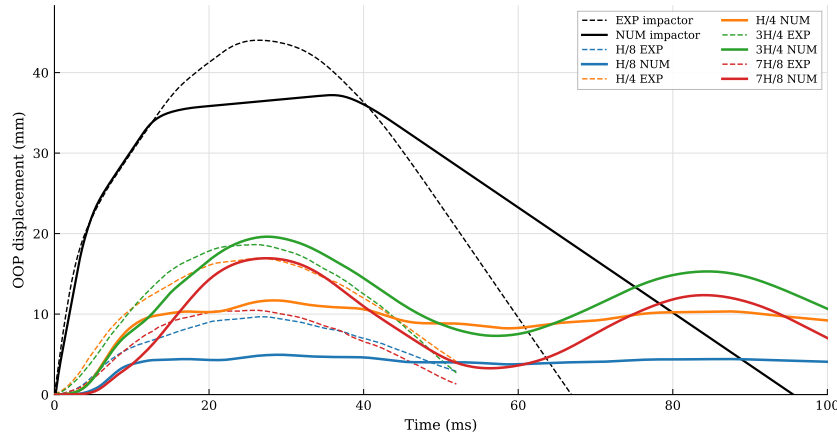


Figure E.11: W18 out-of-plane DIC displacement histories.

E.7.4 W19 — line load, $v_0 = 8.35$ m/s

W19 gives the best overall ranking of the campaign. The displacement peak is reproduced within 4%, while the remaining force underestimation is attributed to the simplified spreader-beam representation, which becomes more influential at the higher impact velocity. The DIC profiles follow the experimental trend at all heights, with some amplitude differences in the upper part of the wall.

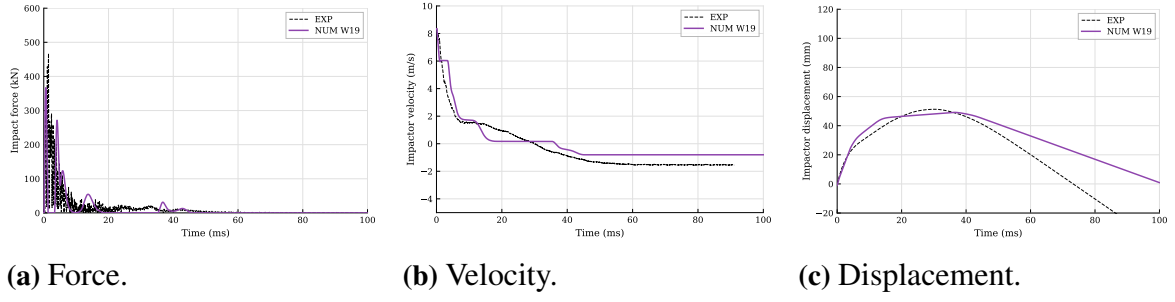


Figure E.12: W19 impactor response for the selected combination, $k_{n,imp} = 6.0 \text{ GPa/m}$, $\mu = 0.75$, $\xi = 0.010$. Rank 5/168.

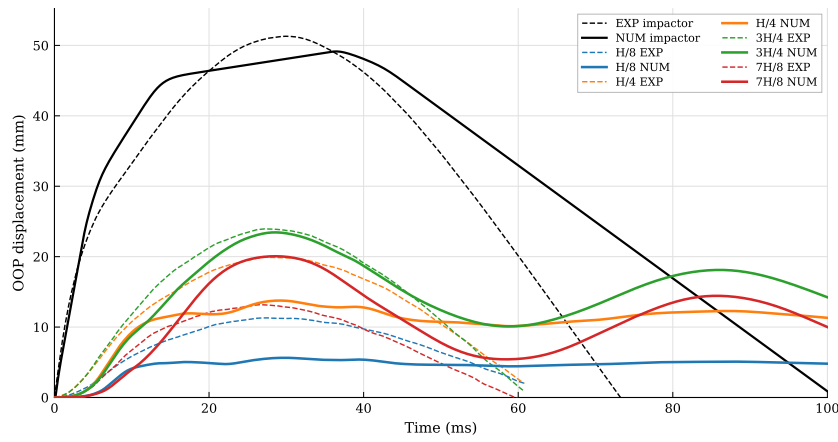


Figure E.13: W19 out-of-plane DIC displacement histories.

E.8 Arching force response

The arching force comparison follows the same methodology as Campaign 1 (Section D.6.4). The experimental signal is obtained from the analogue load cells as $F_{arch}^{exp}(t) = F_R(t) + F_L(t)$. The numerical signal is reconstructed from the vertical spring deformation as

$$F_{arch}^{num}(t) = P_0 + K_{sup} \Delta u_y^{N_1}(t), \quad (\text{E.1})$$

with $P_0 = 9.51 \text{ kN}$ and $K_{sup} = 31.4 \text{ MN/m}$ for all four walls. Table E.7 reports the peak arching force ratios.

Table E.7: Arching force peak comparison for W16–W19.

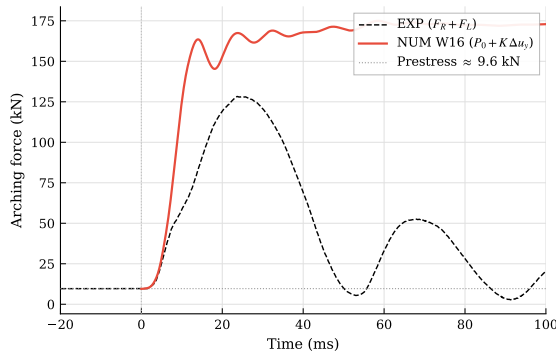
Wall	Load	$F_{arch,peak}^{exp} \text{ (kN)}$	$F_{arch,peak}^{num} \text{ (kN)}$	Ratio
W16	point	128.3	175.0	1.36
W17	point	134.9	198.3	1.47
W18	line	128.8	194.5	1.51
W19	line	138.6	213.8	1.54

Two systematic deviations are observed. First, the peak arching force is overestimated by

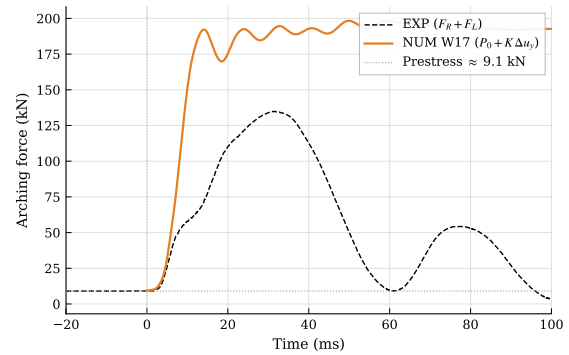
factors of 1.36–1.54. Second, the numerical arching force does not return to the prestress level after impact, whereas the experimental signal recovers within approximately 40–60 ms.

Both deviations are consistent with the current `ElastoPlastic` contact formulation. In the physical wall, opened joints become force-free and may close again under gravity and arching action after impactor separation. In the numerical model, tensile opening is limited by a constitutive cut-off, but the opened contact pair remains active in the global force and stiffness assembly. The cracked joint therefore continues to restrain rocking recovery, leaving the wall in a displaced configuration and maintaining an elevated arching force.

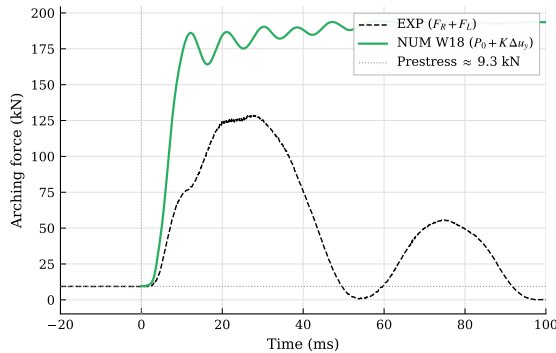
A free-vibration benchmark confirmed that this missing rebound is mainly caused by the absence of contact deletion and recovery. The original `ElastoPlastic` law recovered only 20% of an imposed 11 mm displacement, whereas the `ElastoPlastic_CD` extension recovered 99%. The detailed benchmark is discussed in Section F.2.



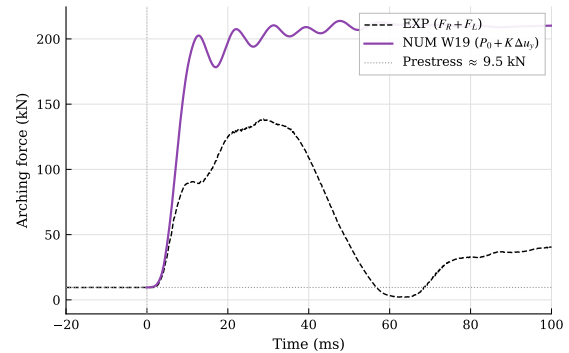
(a) W16, point load.



(b) W17, point load.



(c) W18, line load.



(d) W19, line load.

Figure E.14: Arching force histories for W16–W19.

F Appendix F: The `ElastoPlastic_CD` Contact Law: Diagnosis, Implementation, and Preliminary Assessment

The systematic arching force overestimation and absence of post-impact wall rebound observed across all three campaigns (Sections D.6.4, E.8) was attributed to a limitation of the `ElastoPlastic` contact law rather than to parameter calibration. This appendix identifies the root cause, documents the `ElastoPlastic_CD` extension developed in this thesis to address it, and presents a free-vibration verification benchmark.

F.1 Root cause: absence of contact deletion

Under out-of-plane impact loading, bed joints on the tension side of the wall open as the wall deflects, forming the kinematic hinges of the rocking mechanism. After the impactor separates, gravity and arching forces drive the wall back toward its initial position. For this recovery to occur in the model, opened joints must carry zero force and stiffness and be excluded from the global assembly while open.

The `ElastoPlastic` law correctly limits the normal stress at the constitutive level through the tension cut-off $F_0 = \sigma_n - f_t$, but does not remove the contact pair from the global assembly when the joint opens, because `to_ommit()` is not overridden and always returns `False`. Opened joints therefore continue contributing a residual $\mathbf{D}_{ep}$, holding the wall artificially in its displaced position. The `Coulomb` law produces correct rebound because it overrides `to_ommit()`, removing the contact pair when $\varepsilon_n^{tot} > \varepsilon_t = f_t/k_{nn}$. `ElastoPlastic_CD` applies this same assembly-level mechanism to the elasto-plastic law.

F.2 Implementation of `ElastoPlastic_CD`

`ElastoPlastic_CD` inherits the complete `ElastoPlastic` constitutive formulation without modification: the four yield surfaces (F_0 – F_3), the Newton–Raphson return mapping, the plastic strain evolution, the compression cap, and the consistent tangent $\mathbf{D}_{ep}$ are unchanged, and no new constitutive parameters are introduced. The only addition is a contact-state layer controlling whether the contact pair is active in or excluded from the global assembly at each time step.

One notational point before proceeding: the quantities `self.disps['n']` and `self.disps['s']` are accumulated total contact strains $\boldsymbol{\varepsilon}^{tot} = [\varepsilon_n^{tot}, \varepsilon_s^{tot}]^T$, not geometric displacements; the name `disps` is inherited from the base class `Surface`. All contact deletion and reclosure logic is formulated in terms of ε_n^{tot} . Table F.1 summarises the complete contact state cycle before the individual components are described

Table F.1: Contact state cycle in `ElastoPlastic_CD`.

State	<code>to_ommit()</code>	σ	$\mathbf{D}_{ep}$	<code>is_open</code>
Closed, elastic	False	EP normal	$\mathbf{D}_{el}$	False
Closed, plastic	False	EP normal	$\mathbf{D}_{ep} \neq \mathbf{D}_{el}$	False
Opening ($\varepsilon_n^{tot} > \varepsilon_t$)	True	$\mathbf{0}$	$\mathbf{0}$	True
Open	True	$\mathbf{0}$	$\mathbf{0}$	True
First step after closure	False	EP restarted	$\mathbf{D}_{el}$	False

F.2.1 Additional state variables

Three variables are added in the constructor:

```

1 self.disps['n_t'] = self.ft / self.stiff0['kn']
2 self.disps['s_ref'] = 0.0
3 self.is_open = False

```

The tensile strain threshold defines the value of ε_n^{tot} at which the contact pair is deleted:

$$\varepsilon_t = \frac{f_t}{k_{nn}}. \quad (\text{F.1})$$

When $f_t = 0$ any positive normal strain activates deletion; when $f_t > 0$ the contact can sustain elastic tensile stress up to f_t before being removed.

The tangential strain reference $\varepsilon_{s,\text{ref}}$ stores the value of ε_s^{tot} at the moment of opening. While the contact is open the two blocks can slide freely and ε_s^{tot} continues to accumulate. Without this reference, the return mapping would interpret that accumulated tangential strain as shear deformation upon reclosure and generate artificial shear forces. The effective tangential strain passed to the parent law after reclosure is therefore:

$$\varepsilon_s^{eff} = \varepsilon_s^{tot} - \varepsilon_{s,\text{ref}}, \quad (\text{F.2})$$

so that only strain accumulated after reclosure enters the constitutive routine.

The boolean `is_open` records whether the contact was open in the last committed state, allowing `update()` to detect the transition from open to closed.

F.2.2 Contact deletion criterion: `to_ommit()`

```

1 def to_ommit(self):
2     if (self.disps['n'] - self.disps['n_t']) > self.tol_disp:
3         return True
4     return False

```

The contact pair is excluded from the global assembly when $\varepsilon_n^{tot} > \varepsilon_t$. This criterion is structurally identical to the existing `Coulomb.to_ommit()`.

F.2.3 Open-contact state: `_set_open_state()`

When the trial normal strain exceeds the threshold, the following method is called:

```
1 def _set_open_state(self, eps_trial):
2     self.is_open = True
3     self.disps['n'] = eps_trial[0]
4     self.disps['s'] = eps_trial[1]
5     self.disps['s_ref'] = eps_trial[1]
6     self.sigma = np.zeros(2)
7     self.eps_tot = np.zeros(2)
8     self.eps_tot_el = np.zeros(2)
9     self.eps_tot_pl = np.zeros(2)
10    self.D_ep = np.zeros((2, 2))
11    self.stiff['kn'] = 0.0
12    self.stiff['ks'] = 0.0
13    self.stiff['kns'] = 0.0
14    self.stiff['ksn'] = 0.0
15    self.Nact = 0
16    self.error = 0
17    self.commit()
```

While open, the contact pair transmits no stress and contributes no stiffness to the global assembly:

$$\boldsymbol{\sigma} = \mathbf{0}, \quad \mathbf{D}_{ep} = \mathbf{0}. \quad (\text{F.3})$$

The constitutive strain variables are reset to zero so that no plasticity accumulates while the joint is open. The tangential reference $\varepsilon_{s,\text{ref}}$ is set to the current $\varepsilon_s^{\text{tot}}$, recording the tangential position at the moment of opening. The state is immediately committed so that `revert_commit()` restores the open state correctly after a rejected time step.

F.2.4 Reclosure state: `_reset_state_on_reclosure()`

When the contact closes after being open, the elastoplastic state is completely restarted:

```
1 def _reset_state_on_reclosure(self):
2     self.is_open = False
3     self.sigma = np.zeros(2)
4     self.sigma_prev = np.zeros(2)
5     self.eps_tot = np.zeros(2)
6     self.eps_tot_el = np.zeros(2)
7     self.eps_tot_pl = np.zeros(2)
8     self.eps_tot_prev = np.zeros(2)
9     self.eps_el_prev = np.zeros(2)
10    self.eps_pl_prev = np.zeros(2)
11    self.D_ep = self.D_el.copy()
12    self.stiff['kn'] = self.D_ep[0, 0]
13    self.stiff['ks'] = self.D_ep[1, 1]
14    self.stiff['ksn'] = self.D_ep[1, 0]
15    self.stiff['kns'] = self.D_ep[0, 1]
```

All stress and strain variables are reset to zero and the tangent stiffness is restored to the initial elastic value $\mathbf{D}_{ep} = \mathbf{D}_{el}$. Once a joint has fully opened, carrying forward the plastic history from before opening would generate artificial shear forces at re-contact; a clean restart avoids this.

F.2.5 Modified update sequence: `update()`

```
1 def update(self, d_eps_tot, N_elpl=1):
2     d_eps      = np.array(d_eps_tot, dtype=float)
3     eps_old    = np.array([self.disps['n'],
4                             self.disps['s']], dtype=float)
5     eps_trial  = eps_old + d_eps
6     eps_t      = self.disps['n_t']
7     if (eps_trial[0] - eps_t) > self.tol_disp:
8         self._set_open_state(eps_trial)
9         return
10    was_open = getattr(self, 'is_open', False)
11    if was_open:
12        self._reset_state_on_reclosure()
13    eps_s_ref = self.disps.get('s_ref', 0.0)
14    eps_eff   = np.array([eps_trial[0],
15                            eps_trial[1] - eps_s_ref],
16                            dtype=float)
17    d_eps_eff = eps_eff - self.eps_tot_prev
18    super().update(d_eps_eff, N_elpl=N_elpl)
19    self.disps['n'] = eps_trial[0]
20    self.disps['s'] = eps_trial[1]
21    self.commit()
```

At each time step, the method first checks whether the trial normal strain exceeds ε_t . If it does, the joint is set as open and the update returns immediately, meaning no return mapping is called and no plasticity accumulates while the joint is open. If the joint is closed, the method checks whether it was open in the previous step and resets the elastoplastic state if so. The tangential reference $\varepsilon_{s,\text{ref}}$ is then subtracted before calling the original `ElastoPlastic.update()`, so that sliding during the open phase is not interpreted as shear deformation upon reclosure. Finally, the true total contact strains are restored so that the deletion criterion operates on the physical opening.

F.2.6 State management: `commit()` and `revert_commit()`

```
1 def commit(self):
2     super().commit()
3     self.disps_conv = self.disps.copy()
4     self.stiff_conv  = self.stiff.copy()
5     self.is_open_conv = getattr(self, 'is_open', False)
6
7 def revert_commit(self):
8     super().revert_commit()
9     if hasattr(self, 'disps_conv'):
10         self.disps = self.disps_conv.copy()
11     if hasattr(self, 'stiff_conv'):
12         self.stiff = self.stiff_conv.copy()
13     if hasattr(self, 'is_open_conv'):
14         self.is_open = self.is_open_conv
```

At the end of each accepted time step, `commit()` saves the contact strain state, the stiffness, and the open/closed flag alongside the original elastoplastic variables. If the solver rejects a step and calls `revert_commit()`, all three are restored to the last accepted state. Without this, the open/closed flag could become inconsistent after a rejected step and corrupt the contact-state logic in subsequent increments.

F.3 Griffith column free-vibration benchmark

F.3.1 Setup

A free-vibration test was designed to isolate the effect of contact deletion from all other modelling choices. A slender masonry column (Griffith wall) is discretised with the coarse HybriDFEM mesh with the following parameters: unit weight $\gamma = 1800 \text{ kg/m}^3$, friction coefficient $\mu = \tan 35^\circ$, cohesion $c = 2.0 \text{ MPa}$, and tensile strength $f_t = 0.7 \text{ MPa}$ on all standard interfaces. The three interfaces at the base, mid-height, and top (interfaces 0, 9, and 18) are assigned a reduced tensile strength $f_t = 0.05 \text{ MPa}$, representing the pre-cracked condition observed in the physical tests. This value was chosen deliberately to test the contact deletion mechanism under a non-zero tensile threshold: the contact pair should remain active while the normal stress is below 0.05 MPa , and be removed from the global assembly only once that threshold is exceeded. Rayleigh damping $\xi = 5\%$ is applied with the TAN_LG operator and an axial compression $\sigma_{ax} = 0.10 \text{ MPa}$ is applied at the top.

The column is displaced quasi-statically to a mid-height out-of-plane displacement of $u_0 = -11 \text{ mm}$ and then released under self-weight. The free vibration is simulated for 3.5 s under three laws: Coulomb, ElastoPlastic, and ElastoPlastic_CD. All other modelling choices are identical, so any difference in rebound behaviour is unambiguously attributed to the contact law.

F.3.2 Results and interpretation

Figure F.1 shows the mid-height displacement time history and Table F.2 summarises the key metrics.

Coulomb and ElastoPlastic_CD produce virtually identical responses, both recovering more than 99.9% of the imposed displacement and decaying to a near-zero residual within 3.5 s . ElastoPlastic recovers only 57% and settles at approximately -4.68 mm , because the opened joints at interfaces 0, 9, and 18 remain active in the global assembly and hold the wall in its displaced position.

Table F.2: Free-vibration metrics. u_∞ is the mean displacement over the final 10% of the simulation. Recovery = $1 - |u_\infty|/|u_0|$.

Law	$u_0 \text{ (mm)}$	$u_\infty \text{ (mm)}$	$u_{\max}^+ \text{ (mm)}$	Recovery (%)
Coulomb	-11.0	-0.000	+5.44	99.99
ElastoPlastic	-11.0	-4.680	-0.05	57.45
ElastoPlastic_CD	-11.0	-0.006	+5.41	99.94

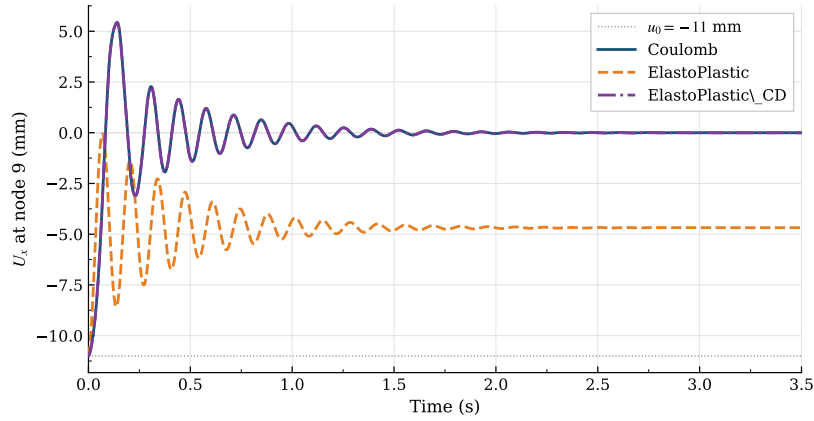


Figure F.1: Mid-height OOP displacement for the free-vibration benchmark. Interfaces 0, 9, and 18: $f_t = 0.05$ MPa; all others: $f_t = 0.7$ MPa.

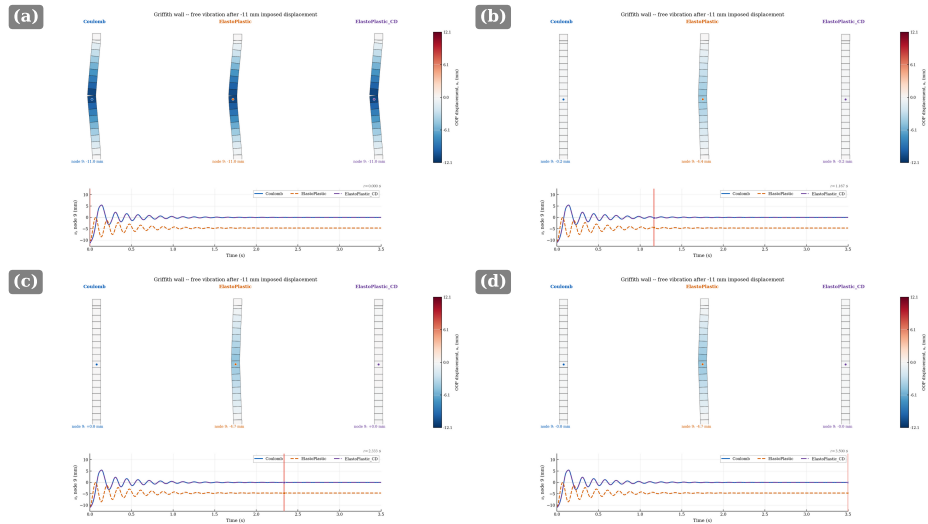


Figure F.2: Deformed shape at $t = 0.000, 0.015, 0.079$, and 0.114 s.

The 0.05% difference in recovery between Coulomb and ElastoPlastic_CD is physically expected: ElastoPlastic_CD accumulates a small irreversible plastic slip during the initial loading cycle, which is correctly retained after reclosure. The benchmark confirms two things: first, that the missing rebound in ElastoPlastic is caused exclusively by the absent `to_ommit()` override and not by the plastic slip formulation; second, that ElastoPlastic_CD correctly activates the deletion criterion for a non-zero tensile threshold, removing the contact pair only once the normal stress exceeds $f_t = 0.05$ MPa.

G Appendix G: Abaqus Modelling Procedure

This appendix documents in full detail the steps undertaken to construct, configure, and run the Abaqus finite element models used in this study. The procedure applies to both the quasi-static four-point bending simulations and the dynamic impact simulations. All models were built and run in Abaqus/Explicit 2024 and 2024-HF2 (Dassault Systèmes Simulia Corp., 2024). It should be noted that a slight difference in numerical behaviour was discovered between the two software versions at a later stage of the work. To ensure consistency of results across all walls and campaigns, it is important to run all analyses within the same version.

G.1 Unit System

All models were constructed using the SI-mm unit system consistently throughout, as summarised in Table G.1. All material properties, geometric dimensions, and load magnitudes must be expressed in this unit system.

Table G.1: Consistent unit system adopted in all Abaqus models.

Quantity	Unit
Length	mm
Force	N
Mass	tonne (10^3 kg)
Time	s
Stress	MPa (N/mm^2)
Energy	mJ (10^{-3} J)
Density	tonne/ mm^3
Acceleration	mm/s^2

G.2 Part Creation and Assembly

G.2.1 Brick units and RC slabs

Each brick unit was created as a deformable solid part using an extruded solid feature. Two brick types were defined to accommodate the Flemish bond pattern: stretchers and headers, each with expanded dimensions absorbing half the mortar joint thickness on each face, as described in Section 3.4.2. Half-bricks at the lateral wall boundaries were created as separate parts to maintain the correct bond pattern within the symmetry half-model.

The top and bottom reinforced concrete slabs and, for Campaign 3, the steel plate were likewise created as deformable solid extruded parts. The impactor was created as an analytical rigid body, with its geometry reproducing the physical impactor used in each campaign, as described in Section 3.4.6.

G.2.2 Wall assembly

The full wall was assembled in the Assembly module by positioning and instancing each brick type according to the Flemish bond arrangement, course by course. The RC slabs

were placed immediately above and below the brickwork. For Campaign 3, the steel plate was positioned above the top RC slab with the prescribed eccentricity of 80 mm from the wall centreline, and connected to the slab via a tie constraint. The impactor was positioned at the required distance from the wall front face, as detailed in Section G.6.2.

G.2.3 Reference points and sets

Reference points were created at the following locations to enable result extraction and load application:

- Top slab: a reference point kinematically coupled to the full top face of the top RC slab, used to apply the axial prestress and extract the vertical reaction and displacement.
- Bottom slab: a reference point kinematically coupled to the full bottom face of the bottom RC slab, used to extract the base reaction.
- Impactor: the rigid body reference point at the centroid of the impactor, used to prescribe motion and extract displacement, velocity, and contact force.
- Wall mid-height: a node set at the mid-height of the wall, used to extract the out-of-plane displacement time history.

G.3 Material Assignment and Contact Definition

G.3.1 Material assignment

Materials were assigned to each part in the Property module. Brick units were assigned the Concrete Damaged Plasticity model with the parameters described in Section 3.4.3.1. RC slabs were assigned linear elastic properties, and the steel plate and loading beam were assigned elasto-plastic properties as described in Section 3.4.3.2. The impactor was assigned a density adjusted to match the total impactor mass for each test configuration.

G.3.2 Contact definition

All contact interactions were defined using the General Contact framework in the Interaction module (Dassault Systèmes Simulia Corp., 2024). The Find Contact Pairs feature was used to automatically identify all brick–brick surface pairs based on geometric proximity, and all detected pairs were assigned within the general contact domain. This approach is necessary given the large number of contact surfaces in the Flemish bond assembly and ensures that no interface is missed during the simulation.

The normal and tangential contact properties were then assigned to the general contact domain as described in Section 3.4.4. The impactor–wall contact was defined as a separate interaction with hard normal contact and low friction.

G.4 Axial Prestress Application

G.4.1 Simply supported boundary condition

For the simply supported walls (four-point bending tests with simply supported top conditions), the axial prestress was applied as a concentrated force at the reference point coupled to the top face of the top RC slab. The force magnitude corresponds to the experimentally

applied axial load (Godio et al., 2023b). The load is applied incrementally using a smooth step amplitude to avoid exciting dynamic oscillations during the quasi-static initialisation. The step duration must be sufficiently long for the wall to reach static equilibrium; if the step is too short, residual oscillations persist and contaminate the subsequent loading response.

G.4.2 Rigid and non-rigid boundary conditions

For walls with rigid or non-rigid top boundary conditions, the axial prestress was applied through a velocity boundary condition at the reference point coupled to the top slab, rather than as a concentrated force. This approach avoids the need to modify the boundary condition between steps, which is essential for the arching mechanism to remain active throughout the simulation. After the prestress step is complete, the velocity boundary condition remains active with a value of zero and propagates into subsequent steps, locking the top slab at the achieved displacement level and ensuring that the axial force is maintained as the wall deflects out-of-plane.

The procedure to identify the correct velocity magnitude is iterative. A target axial force $F_{\text{ax,target}}$ is defined from the experimental setup. An initial velocity $v_{\text{ax}}^{(0)}$ is selected as a starting guess and applied as a smooth step amplitude over the Step 1 duration T_1 . The analysis is run and the resulting axial force $F_{\text{ax}}^{(0)}$ at the top reference point is extracted at the end of Step 1. A corrected velocity is then estimated by linear interpolation:

$$v_{\text{ax}}^{(1)} = v_{\text{ax}}^{(0)} \cdot \frac{F_{\text{ax,target}}}{F_{\text{ax}}^{(0)}} \quad (\text{G.1})$$

The analysis is rerun with $v_{\text{ax}}^{(1)}$ and the resulting axial force is again compared against the target. This process is repeated until the axial force at the end of Step 1 matches the target value to within an acceptable tolerance, typically within 1%. In practice, convergence is achieved within two to three iterations due to the near-linear relationship between the applied velocity and the resulting prestress force within the relevant range.

G.5 Four-Point Bending Simulation Steps

G.5.1 Step 1 — Gravity and prestress

Gravity is applied to all structural components and the axial prestress is introduced as described in Section G.4. Mass scaling of 10^5 is active throughout this step. The step runs for 10–20 s depending on the wall configuration, which is sufficient to reach a quasi-static equilibrium state with negligible kinetic energy relative to the internal energy.

G.5.2 Step 2 — Lock configuration

Step 2 is a short transitional step of 4×10^{-4} s in which mass scaling is disabled and the physical material densities are restored. No new loads or boundary conditions are introduced. This step allows the explicit solver to transition smoothly from the mass-scaled quasi-static state to the physical dynamic configuration before out-of-plane loading begins.

G.5.3 Step 3 — Out-of-plane displacement loading

The out-of-plane displacement-controlled load is applied at the 7th and 15th courses of the wall through velocity boundary conditions imposed at the corresponding node sets. The

velocity is applied slowly using a smooth step amplitude to maintain quasi-static conditions. The step runs until the target out-of-plane displacement is reached or until numerical failure occurs.

G.6 Impact Simulation Steps

G.6.1 Step 1 and Step 2

Steps 1 and 2 follow the same procedure as described for the four-point bending simulations in Section G.5.

G.6.2 Step 3 — Impactor release

The impactor must reach the experimentally measured pre-impact velocity v_0 at the wall face at the end of Step 3. A predefined velocity field could in principle be used to assign the initial velocity, but this can only be applied in the Initial Step, before the prestress and gravity are introduced. This would require the impactor to be placed far from the wall so that it travels throughout Steps 1 and 2 and only reaches the wall at the start of Step 4, which is impractical given the large distances involved and the duration of the prestress step.

Instead, a scaled gravity load is applied to the impactor element set in the horizontal (wall-normal) direction during Step 3. This drives the impactor from rest over a short distance, allowing it to be positioned close to the wall at the start of Step 3 and reach v_0 by the end of the step. The scaled gravity is then deactivated at the start of Step 4 so that it does not further accelerate the impactor after contact.

The magnitude of the scaled gravity g_{fake} and the required starting distance z_{impactor} from the wall face are computed as follows. Let T_3 be the Step 3 duration, t_{ramp} the ramp duration of the smooth step amplitude, and $t_{\text{fall}} = T_3 - t_{\text{ramp}}$ the remaining free-acceleration time.

Step 1 — Compute g_{fake} . During the ramp phase, the mean acceleration is $g_{\text{fake}}/2$ and the velocity at the end of the ramp is:

$$v_{\text{ramp}} = \frac{g_{\text{fake}}}{2} \cdot t_{\text{ramp}} \quad (\text{G.2})$$

During the free-fall phase, the impactor accelerates at g_{fake} from v_{ramp} , reaching v_0 at the end of Step 3:

$$v_0 = v_{\text{ramp}} + g_{\text{fake}} \cdot t_{\text{fall}} = g_{\text{fake}} \left(\frac{t_{\text{ramp}}}{2} + t_{\text{fall}} \right) \quad (\text{G.3})$$

Solving for g_{fake} :

$$g_{\text{fake}} = \frac{v_0}{\frac{t_{\text{ramp}}}{2} + t_{\text{fall}}} \quad (\text{G.4})$$

Step 2 — Compute starting distance. The distance travelled during the ramp phase is:

$$s_{\text{ramp}} = \frac{1}{2} \cdot \frac{g_{\text{fake}}}{2} \cdot t_{\text{ramp}}^2 \quad (\text{G.5})$$

The distance travelled during the free-fall phase is:

$$s_{\text{fall}} = v_{\text{ramp}} \cdot t_{\text{fall}} + \frac{1}{2} \cdot g_{\text{fake}} \cdot t_{\text{fall}}^2 \quad (\text{G.6})$$

The required starting position of the impactor front face from the wall front face is:

$$z_{\text{impactor}} = s_{\text{ramp}} + s_{\text{fall}} \quad (\text{G.7})$$

Example — Wall W16. For $v_0 = 7230$ mm/s, $t_{\text{ramp}} = 0.01$ s, and $t_{\text{fall}} = 0.05$ s:

$$g_{\text{fake}} = \frac{7230}{0.005 + 0.05} = 131,454 \text{ mm/s}^2 \quad (\text{G.8})$$

$$v_{\text{ramp}} = \frac{131,454}{2} \cdot 0.01 = 657 \text{ mm/s} \quad (\text{G.9})$$

$$s_{\text{ramp}} = \frac{1}{2} \cdot \frac{131,454}{2} \cdot 0.01^2 = 3.29 \text{ mm} \quad (\text{G.10})$$

$$s_{\text{fall}} = 657 \cdot 0.05 + \frac{1}{2} \cdot 131,454 \cdot 0.05^2 = 197.2 \text{ mm} \quad (\text{G.11})$$

$$z_{\text{impactor}} = 3.29 + 197.2 = 200.5 \text{ mm} \quad (\text{G.12})$$

The impactor front face is therefore placed 200.5 mm from the wall front face at the start of Step 3. A time scaling factor of 50 is applied during Step 3 to reduce the computational cost. This has no physical effect on the outcome, as the scaled gravity magnitude is derived from the physical target velocity and step time independently of the time scaling.

G.6.3 Step 4 — Impact event

At the start of Step 4, the scaled gravity on the impactor is deactivated and contact between the impactor and the wall is activated. The equations of motion are integrated over the full impact event, including the contact phase, rebound, and post-impact oscillation. A time scaling factor of 50 is maintained during this step. The integration window is 0.05–0.09 s depending on the duration required to capture the relevant response quantities for each wall configuration.

H Appendix H: 4PBT - Calibration and results

H.1 Overall Calibration strategy

The calibration of the Abaqus model from the four-point bending tests followed a sequential, multi-stage strategy of increasing complexity. The double-wythe simply supported walls W5–W8 were used as the starting point, as this configuration isolates the bending response without arching action, providing the cleanest observable for identifying the elastic material parameters of the brick and the interface. At this stage, the brick elastic modulus E_b , the interface normal and shear stiffnesses k_n and k_s , and the mode-I fracture energy $G_{f,I}$ were identified. A simplified bilinear compression–tension behaviour was adopted for the brick material at this initial stage to limit the number of free parameters.

With the elastic parameters fixed, the calibration was extended to the rigid boundary condition wall W12. This configuration activates strong arching action and a punching shear failure mechanism, requiring a more physically representative post-peak description of the brick material. The simplified evolution laws from the first stage were therefore replaced by the fuller parabolic-exponential compressive softening and exponential tensile softening curves. At this stage, the cohesion c , friction coefficient μ , and mode-II fracture energy $G_{f,II}$ at the mortar joints were also calibrated, as the punching shear mechanism activated significant joint sliding not present in the simply supported configuration. The parameter set identified from W12 was subsequently validated against the rigid boundary condition walls W6 and W7 without further adjustment.

Finally, the non-rigid boundary condition walls W13 and W14 were used to verify the transferability of the calibrated parameter set to configurations with finite top support stiffness. The axial support stiffness was set directly from the experimentally measured values of $K_{\text{sup}} = 19.0$ kN/mm for W13 and $K_{\text{sup}} = 9.5$ kN/mm for W14 (Godio et al., 2023b). At this stage, additional parametric testing was carried out varying E_b and the full set of material parameters to assess the sensitivity of the hybrid bending–shear response to each quantity. The overall calibration strategy is outlined in Table H.1.

Table H.1: Abaqus 4PBT calibration strategy for double-wythe walls.

Wall	Configuration	Calibrated parameters	Role
W5	Double wythe, simply supported	$E_b, f_c, f_t, k_n, k_s, G_{f,I}$	Calibration
W8	Double wythe, simply supported	—	Validation
W12	Double wythe, rigid	Evolution laws ($G_{f,c}, G_{f,t}$), $c, \mu, G_{f,II}$	Calibration
W6	Double wythe, rigid	—	Validation
W7	Double wythe, rigid	—	Validation
W13	Double wythe, non-rigid	$K_{\text{sup}} = 19.0$ kN/mm	Validation
W14	Double wythe, non-rigid	$K_{\text{sup}} = 9.5$ kN/mm	Validation

H.2 Elastic parameter calibration

Wall W5, tested under simply supported top conditions with an axial compression of 37.7 kN and failing experimentally through a three-hinge bending mechanism, was selected as the calibration wall for the elastic parameters. The absence of lateral confinement eliminates arching action, isolating the bending response and providing the cleanest observable for identifying the elastic material parameters of the brick and the interface without interference from non-linear joint sliding or compressive arching effects.

The calibration followed a strategy developed internally at RISE (Godio & Flansbjerg, 2025a), in which E_b is fixed first from experimentally confirmed values for the masonry type, and the interface stiffnesses k_n and k_s are subsequently calibrated from the force–displacement response. This decoupled approach isolates the contribution of the joint stiffness to the initial lateral response, reducing the risk of compensating errors between parameters. The brick elastic modulus was set to $E_b = 7000$ MPa, consistent with experimentally measured values (Bomben et al., 2025; Tomassetti et al., 2019). A high cohesion value of $c = 88$ MPa was assigned to the mortar joints at this stage as a numerical device to suppress joint sliding entirely, ensuring that the force–displacement response is governed exclusively by the elastic stiffness of the brick and the interface, without premature activation of the non-linear joint behaviour. The non-linear joint properties are introduced in Stage 2.

With E_b fixed and joint sliding suppressed, a parametric sweep of the interface normal stiffness k_n was carried out over the values $k_n \in \{20, 16, 14, 12, 10\}$ MPa/mm, with the shear stiffness defined as $k_s = k_n / (2 \times 1.1)$ in all cases. The resulting lateral force–displacement curves, shown in Figures H.1a and H.1b, indicate that all tested values produce closely similar global responses, with no major differences in the initial stiffness, peak force, or post-peak descent, and the force curve alone does not provide a sufficient discriminator for selecting k_n .

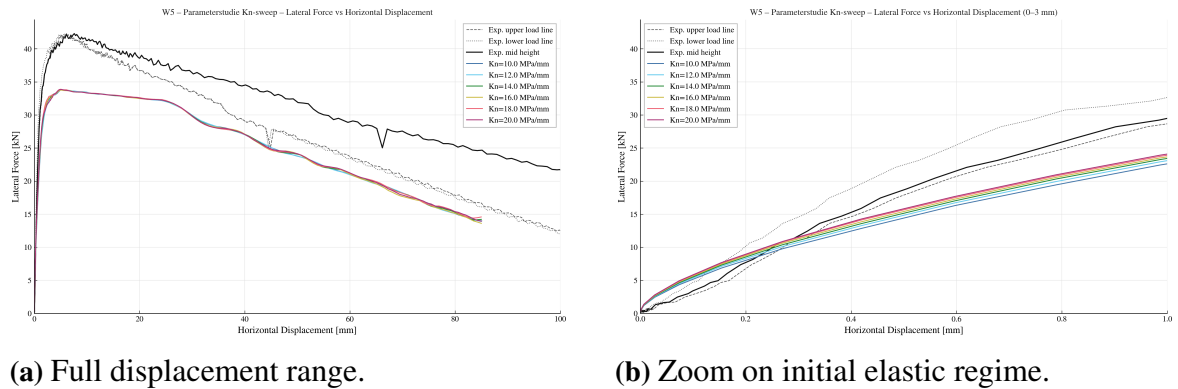
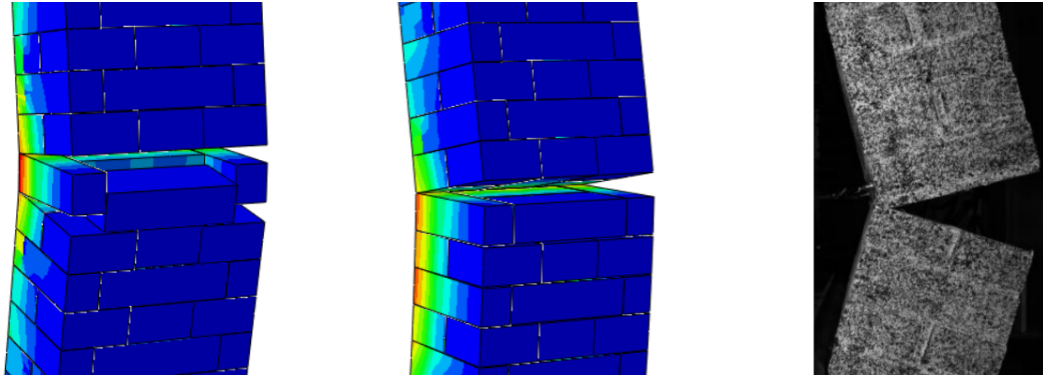


Figure H.1: Lateral force versus out-of-plane displacement for the k_n parametric sweep on W5.

The distinguishing criterion was instead found in the failure mechanism. For values of k_n ranging from 20 down to 14 MPa/mm, the model produced a non-uniform crack opening pattern in which the bricks at the two lateral extremes of the wall detached preferentially from the rest, while the central bricks remained more intact. This localised edge separation is not consistent with the experimentally observed failure pattern. For $k_n = 12$ MPa/mm, the crack opening became uniform along the full length of a single horizontal course, with all

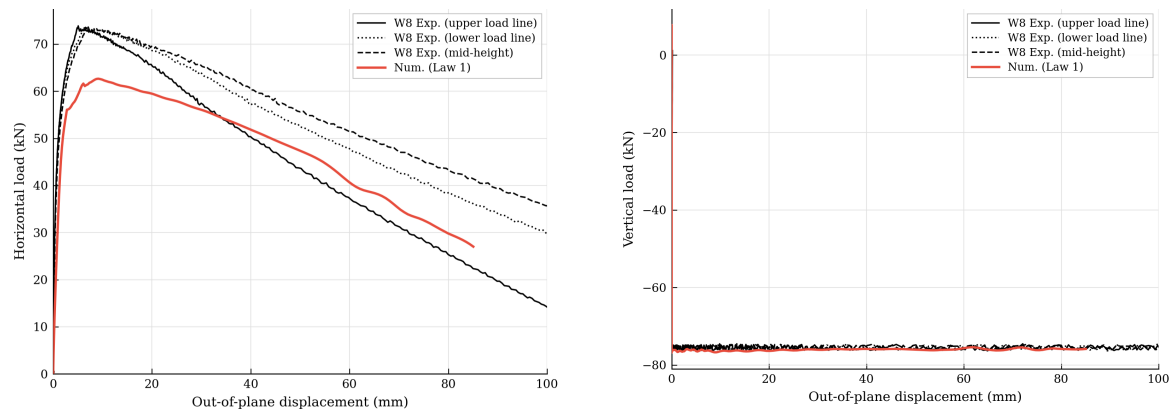
bricks along that line opening simultaneously and to a similar extent. This uniform opening is physically consistent with the experimentally observed failure mode and is the expected response for a wall loaded symmetrically under four-point bending. The two failure patterns are compared in Figure H.2.



(a) Non-uniform edge separation ($k_n \geq 14$ MPa/mm). (b) Uniform crack opening ($k_n = 12$ MPa/mm). (c) Experimental crack pattern (Godio et al., 2023b).

Figure H.2: Comparison of failure mechanisms for the k_n parametric sweep on W5.

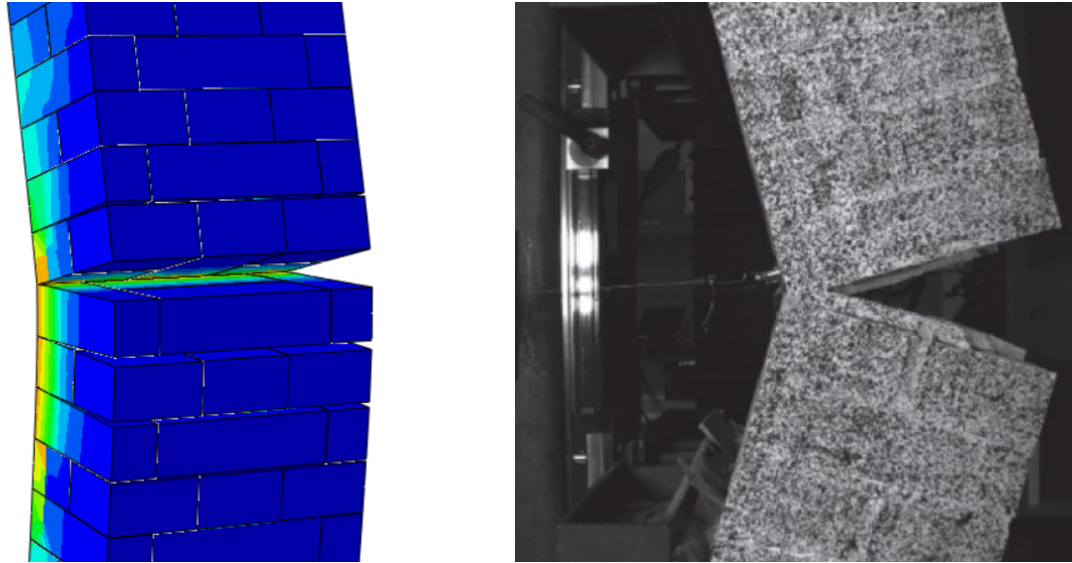
The selected values $k_n = 12$ MPa/mm and $k_s = 5.5$ MPa/mm were verified against wall W8, which was tested under simply supported top conditions with a higher axial compression of 76.0 kN. The force–displacement response and failure mechanism for W8 under the identified elastic parameter set are shown in Figures H.3 and H.4. The results confirm that the selected values produce a consistent response for W8, with the initial stiffness and the three-hinge failure pattern in qualitative agreement with the experimental observations (Godio et al., 2023b). The elastic parameters were therefore adopted as fixed values for all subsequent calibration stages.



(a) Lateral force versus out-of-plane displacement.

(b) Vertical reaction versus out-of-plane displacement.

Figure H.3: W8 — force–displacement response under the identified elastic parameter set ($k_n = 12$ MPa/mm, $k_s = 5.5$ MPa/mm), compared against the experimental measurements (Godio et al., 2023b).



(a) Uniform crack opening ($k_n = 12 \text{ MPa/mm}$). (b) Experimental crack pattern (Godio et al., 2023b).

Figure H.4: W8 — failure mode comparison: Abaqus uniform crack opening at $k_n = 12 \text{ MPa/mm}$ and experimental crack pattern, confirming consistency of the identified elastic parameters.

Confirming that the correct behaviour and failure mechanism were achieved, the elastic parameters were fixed for all subsequent calibration stages. The complete elastic parameter set, including the artificially high cohesion used to suppress joint sliding during this stage, is summarised in Table H.2.

Table H.2: Elastic parameter set adopted after Stage 1 calibration.

Bricks		Joints	
E_b, ν (MPa, —)	k_n, k_s, k_t (MPa/mm)	$c, \mu, G_{f,II}$ (MPa, —, N/mm)	$f_t, G_{f,I}$ (MPa, N/mm)
7000, 0.1	12, 5.5, 5.5	88, 0.80, ≈ 0	0., ≈ 0

The brick CDP behaviour at this stage was defined using a simplified perfectly plastic law, with no post-peak softening in either tension or compression, as summarised in Table H.3.

Table H.3: Initial simplified brick CDP evolution law used in Stage 1.

Compression		Tension	
Stress (MPa)	Inelastic strain	Stress (MPa)	Cracking strain
3.97	0.000	1.59	0.000
4.50	0.001		

H.3 Non-linear brick evolution laws and joint parameter calibration

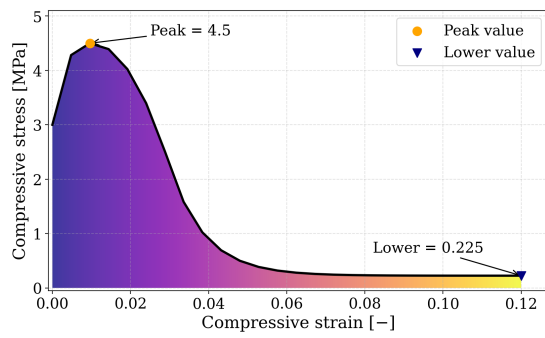
With the elastic parameters fixed, the calibration was extended to the non-linear regime using wall W12, which was tested under rigid top support conditions with an axial compression of 75.5 kN and failed through a punching shear mechanism. This configuration required a physically representative description of the post-peak brick behaviour and the non-linear joint properties, neither of which were active in the simply supported configuration.

H.3.1 Evolution laws.

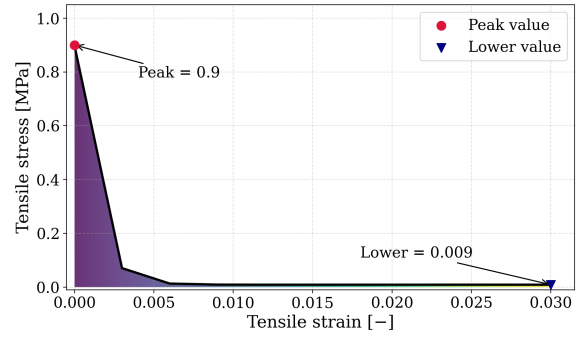
The calibration of the non-linear brick behaviour proceeded in two steps. Initially, a simplified perfectly plastic approximation was adopted as a starting point, with no post-peak softening in either tension or compression. The uniaxial behaviour was defined by two points only, as shown in Table H.3, representing an elastic response up to the peak strength followed by a constant plateau with no degradation.

With this simplified law, the model produced a predominantly bending-dominated failure response, with deformation concentrating in a three-hinge pattern rather than the experimentally observed punching shear mechanism. The arching force was also not well developed, indicating that the absence of post-peak softening prevented the model from activating the compressive strut geometry associated with the confined wall behaviour under rigid support conditions. The failure mechanism and the arching response were therefore not satisfactorily reproduced.

The evolution laws were consequently updated to the physically motivated softening laws investigated at RISE (Godio & Flansbjer, 2025a), which describe an exponential tensile softening and a parabolic-exponential compressive softening, with $G_t = 0.001$ N/mm and $f_{t,res} = 0.009$ MPa. The physical justification for these values is rooted in three-point bending tests performed on the bricks during this project, which indicated a highly brittle behaviour ideally tending toward $G_t \approx 0$ and $f_{t,res} \approx 0$ MPa (Godio & Flansbjer, 2025a). The full tabular definition of the adopted law is given in Table H.4, and the corresponding plots are shown in Figure H.5. The updated law produced significantly better agreement with the experimental response in terms of both the lateral force–displacement curve and the arching force, and the failure mechanism transitioned from a bending-dominated pattern to the experimentally observed punching shear localisation. Figure H.6 shows the lateral force and arching force responses compared against the experimental measurements.



(a) Uniaxial compressive stress–strain curve, showing the parabolic hardening branch to f_c and the subsequent exponential softening governed by G_c .

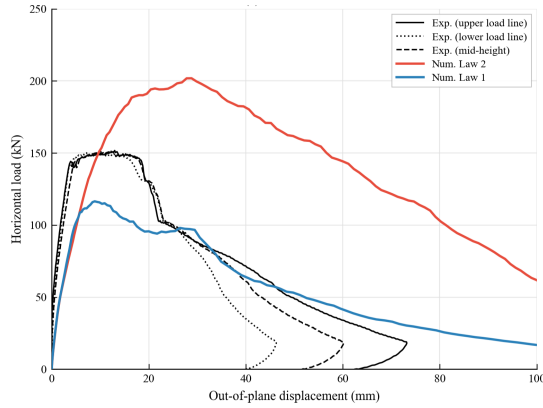


(b) Uniaxial tensile stress–crack opening curve, showing the exponential softening governed by G_t and the residual strength $f_{t,res}$.

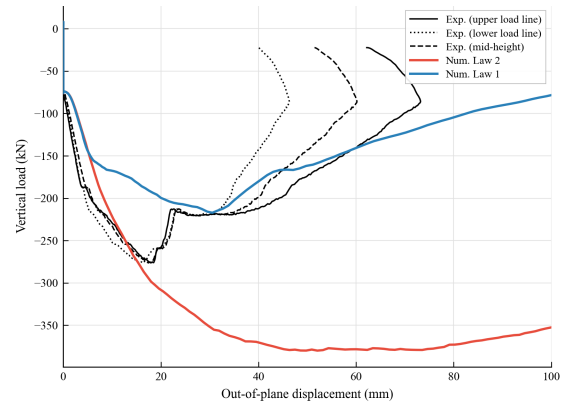
Figure H.5: Brick CDP evolution laws adopted from Stage 2 onward: uniaxial compressive and tensile behaviour (Godio & Flansbjerg, 2025a).

Table H.4: Final brick CDP evolution laws adopted from Stage 2 onward.

Compression		Tension	
Stress (MPa)	Inelastic strain	Stress (MPa)	Cracking strain
3.0000	0.0000	0.9000	0.0000
4.2813	0.0048	0.0706	0.0030
4.4988	0.0096	0.0132	0.0060
4.3911	0.0144	0.0093	0.0090
4.0239	0.0192	0.0090	0.0120
3.3975	0.0240	0.0090	0.0150
2.5119	0.0288	0.0090	0.0180
1.5824	0.0336	0.0090	0.0210
1.0213	0.0384	0.0090	0.0240
0.6922	0.0432	0.0090	0.0270
0.4991	0.0480	0.0090	0.0300
0.3858	0.0528		
0.3193	0.0576		
0.2803	0.0624		
0.2575	0.0672		
0.2440	0.0720		
0.2362	0.0768		
0.2316	0.0816		
0.2289	0.0864		
0.2273	0.0912		
0.2263	0.0960		
0.2258	0.1008		
0.2255	0.1056		
0.2253	0.1104		
0.2252	0.1152		
0.2251	0.1200		



(a) Lateral force versus out-of-plane displacement.



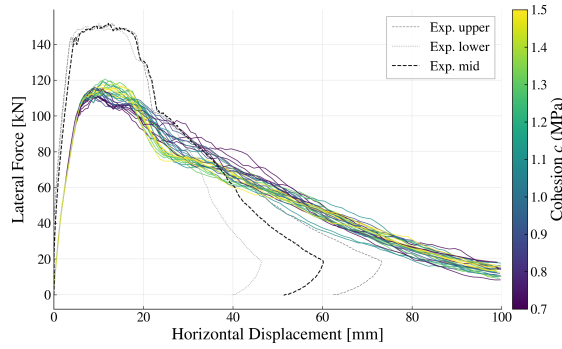
(b) Vertical reaction (arching force) versus out-of-plane displacement.

Figure H.6: W12 — comparison of the two evolution laws against the experimental measurements: lateral force and arching force versus out-of-plane displacement.

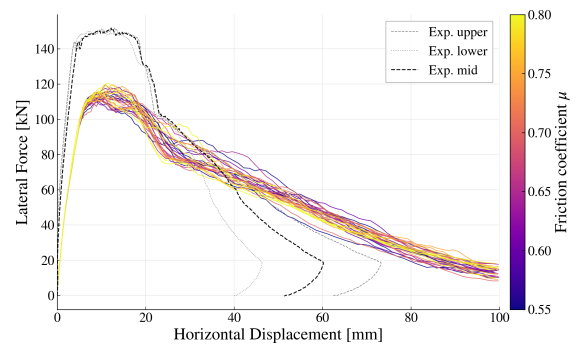
H.3.2 Cohesion and friction coefficient.

With the evolution laws fixed, the cohesion c and friction coefficient μ of the mortar joints were calibrated. The mode-II fracture energy $G_{f,II}$ was set close to zero in both normal and tangential directions, modelling elastic-brittle interface behaviour consistent with the assumption used for mode I (Godio & Flansbjerg, 2025a). A combined parametric sweep of c and μ was carried out, testing 30 combinations with $c \in \{0.7, 0.8, 0.9, 1.0, 1.1, 1.2\}$ MPa and $\mu \in \{0.55, 0.60, 0.65, 0.70, 0.75, 0.80\}$, to identify how each parameter influences the lateral force and arching response.

The results are shown in Figures H.7a–H.8b, where the force–displacement curves are coloured by cohesion and friction respectively for both the lateral force and the vertical reaction. Several patterns can be identified from these plots. Higher cohesion values produce higher peak lateral force capacity, as the increased joint cohesion delays sliding and allows greater stress to develop in the mortar joints before failure. However, the post-peak arching response is better reproduced by lower cohesion values, where the more compliant joint behaviour allows the arching strut to develop and degrade in a manner closer to the experimental observations. Physically, this reflects the competition between the cohesive resistance that governs the pre-peak regime and the frictional sliding that governs the post-peak redistribution of compressive forces through the wall. For the friction coefficient, higher values of μ also increase the peak lateral force capacity through greater resistance to joint sliding, but no clear pattern emerges in the arching response, as the frictional contribution to the compressive strut geometry is less direct than that of the cohesion.

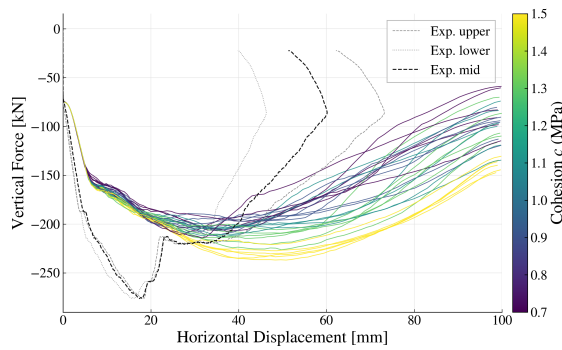


(a) Lateral force (OOP), coloured by cohesion c .

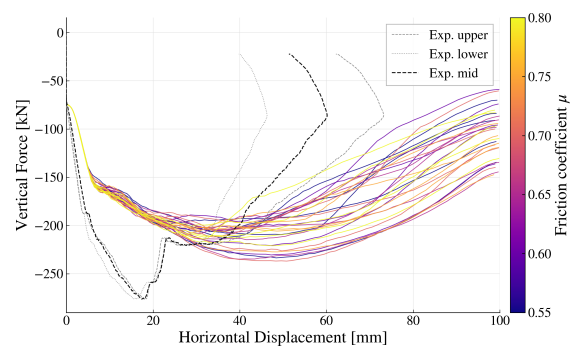


(b) Lateral force (OOP), coloured by friction coefficient μ .

Figure H.7: W12 — lateral force versus out-of-plane displacement for the combined parametric sweep of c and μ .



(a) Vertical reaction, coloured by cohesion c .



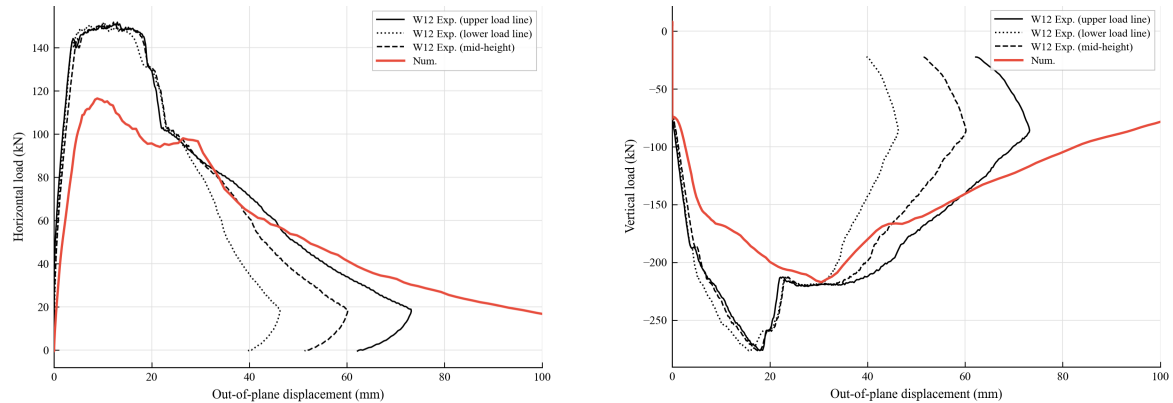
(b) Vertical reaction, coloured by friction coefficient μ .

Figure H.8: W12 — vertical reaction (arching force) versus out-of-plane displacement for the combined parametric sweep of c and μ .

Based on these observations, a lower cohesion value is preferred to better capture the arching degradation, while a higher friction coefficient compensates for the reduced lateral force capacity. The selected combination was further tested on the other walls in the campaign to assess stability and consistency across different boundary conditions. When run in Abaqus/Explicit 2024-HF2, certain combinations with higher cohesion values caused the analysis to fail prematurely for a subset of wall configurations due to numerical instabilities. The final values $c = 0.7$ MPa and $\mu = 0.8$ were the most robust combination that reproduced the correct failure mechanism, provided satisfactory agreement with both the lateral force and the arching response, and produced stable analyses across all walls without premature termination.

The final results for W12 under the adopted parameter set ($c = 0.7$ MPa, $\mu = 0.8$) are shown in Figure H.9. The model reproduces the initial lateral stiffness and the general shape of the force–displacement response reasonably well. The peak lateral force is underestimated, with the numerical peak of approximately 115 kN falling below the experimental value of approximately 145 kN. In the post-peak regime, the numerical response descends more gradually than the experimental curve, which exhibits a steeper drop consistent with brittle

punching failure. The arching response is captured satisfactorily, with the vertical reaction becoming increasingly compressive with growing out-of-plane displacement.



(a) Lateral force versus out-of-plane displacement.

(b) Vertical reaction (arching force) versus out-of-plane displacement.

Figure H.9: W12 — final results under the adopted parameter set ($c = 0.7$ MPa, $\mu = 0.8$) compared against the experimental measurements (Godio et al., 2023b).

Regarding the failure mechanism, Figure H.10 compares the experimentally observed crack pattern with the plastic strain field obtained from the simulation. The CDP formulation represents damage through distributed plastic straining rather than discrete crack opening and block separation, so the model does not reproduce the brittle, localised cracking and brick dislocation observed in the laboratory. Nevertheless, the location of the damage zone and the overall deformed shape are qualitatively consistent with the experimentally observed punching shear failure pattern (Godio et al., 2023b).

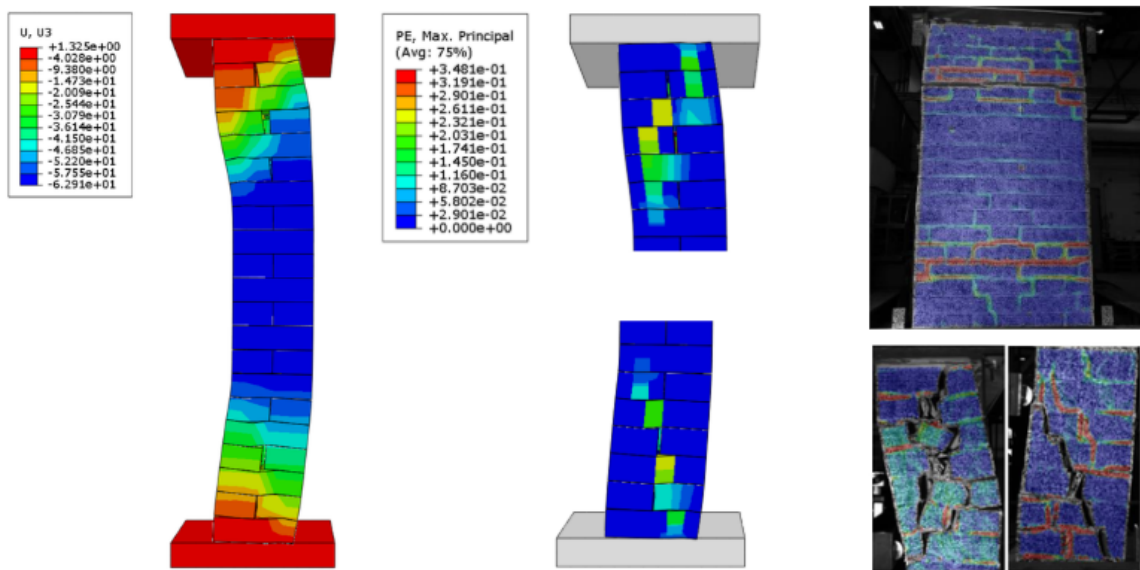
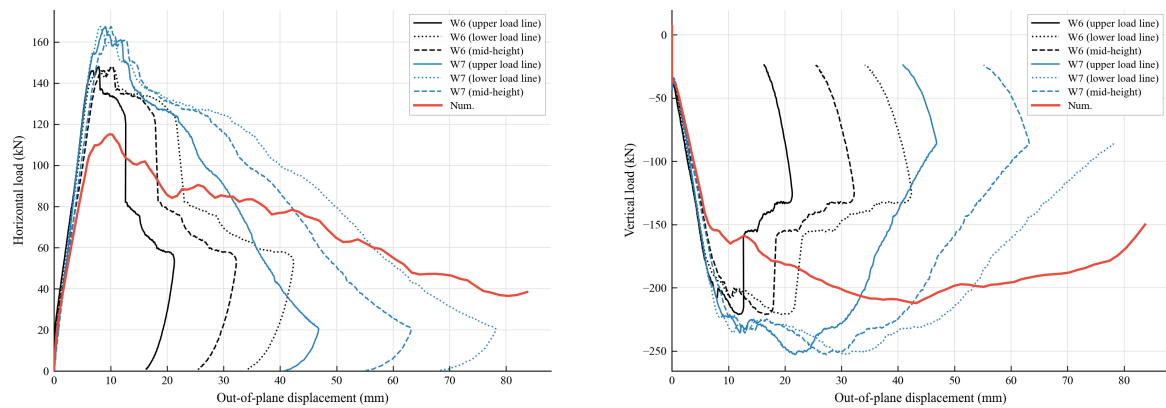


Figure H.10: W12 — failure mode: Abaqus out-of-plane displacement (U3) field (left), Abaqus plastic strain (PE) field (centre), and experimental crack pattern (right) (Godio et al., 2023b).

The adopted parameter set was subsequently transferred to walls W6 and W7 without modification to confirm its validity for the rigid boundary condition at lower axial compression of 38.0 kN. The lateral force and arching force responses for both walls are shown together in Figure H.11. For both walls, the model reproduces the initial lateral stiffness and the general shape of the force–displacement response satisfactorily. The peak lateral force is underestimated, with the numerical peak of approximately 115 kN falling around 24% below the experimental average of 152 kN. The arching response is captured satisfactorily in both cases, with the vertical reaction becoming increasingly compressive with growing out-of-plane displacement, consistent with the experimental measurements (Godio et al., 2023b).



(a) Lateral force versus out-of-plane displacement.

(b) Vertical reaction (arching force) versus out-of-plane displacement.

Figure H.11: W6 and W7 — validation results under the adopted parameter set ($c = 0.7$ MPa, $\mu = 0.8$) compared against the experimental measurements (Godio et al., 2023b).

The failure mechanism for both walls is shown in Figure H.12. The correct punching shear failure pattern was obtained, with the plastic strain concentrating diagonally toward the wall ends, consistent with the experimentally observed diagonal cracking (Godio et al., 2023b). The parameter set was thereby confirmed and the same values were carried forward to Stage 3, where the axial support springs representing the non-rigid boundary conditions are introduced for walls W13 and W14.

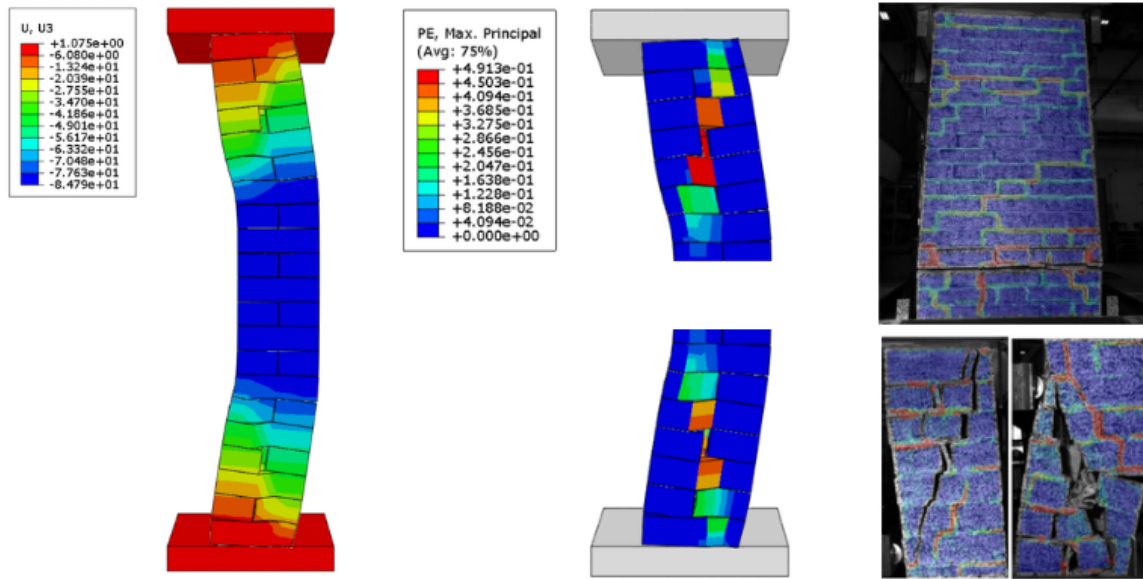


Figure H.12: W6–W7 — failure mode: Abaqus out-of-plane displacement (U3) field (left), Abaqus plastic strain (PE) field (centre), and experimental crack pattern (right) (Godio et al., 2023b).

The complete non-linear parameter set adopted from this stage onward is summarised in Table H.5.

Table H.5: Non-linear parameter set adopted after Stage 2 calibration.

Bricks		Joints	
E_b, ν (MPa, —)	k_n, k_s, k_t (MPa/mm)	$c, \mu, G_{f,II}$ (MPa, —, N/mm)	$f_t, G_{f,I}$ (MPa, N/mm)
7000, 0.1	12, 5.5, 5.5	0.70, 0.80, ≈ 0	0.90, 0.001

H.4 Stage 3 — Non-rigid boundary condition walls W13 and W14

Walls W13 and W14 are double-wythe specimens tested under non-rigid top support conditions with initial axial compression forces of 38.0 kN and 38.2 kN, respectively. The two walls differ in the stiffness of the top support: W13 was tested with a support stiffness of 19.0 kN/mm, while W14 had a lower stiffness of 9.5 kN/mm. Despite this difference, both walls exhibited the same combined bending–shear failure mechanism, consisting of a three-hinge mechanism followed by sudden diagonal cracking in the wall regions, referred to as a hybrid failure in (Godio et al., 2023b).

No additional material parameter calibration was performed at this stage. The complete parameter set identified in Stages 1 and 2, including the elastic brick and interface properties and the non-linear evolution laws, cohesion, and friction coefficient, was transferred without modification. The only change introduced was the replacement of the rigid top boundary condition with an axial spring of stiffness K_{sup} , set directly from the experimentally mea-

sured values of 19.0 kN/mm for W13 and 9.5 kN/mm for W14 (Godio et al., 2023b). This stage therefore serves as a transferability check of the calibrated parameter set under a different boundary condition rather than a new calibration.

The model reproduces the initial lateral stiffness and the pre-peak shape of the force-displacement response reasonably well for both walls, as shown in Figures H.13 and H.14. The peak lateral force is underestimated in both cases: for W13, the numerical peak of approximately 110 kN falls below the experimental value of approximately 134 kN, while for W14 the model reaches approximately 103 kN against an experimental value of approximately 120 kN. The arching response is captured satisfactorily, with the vertical reaction increasing progressively in compression with growing out-of-plane displacement, consistent with the partial lateral restraint provided by the non-rigid support.

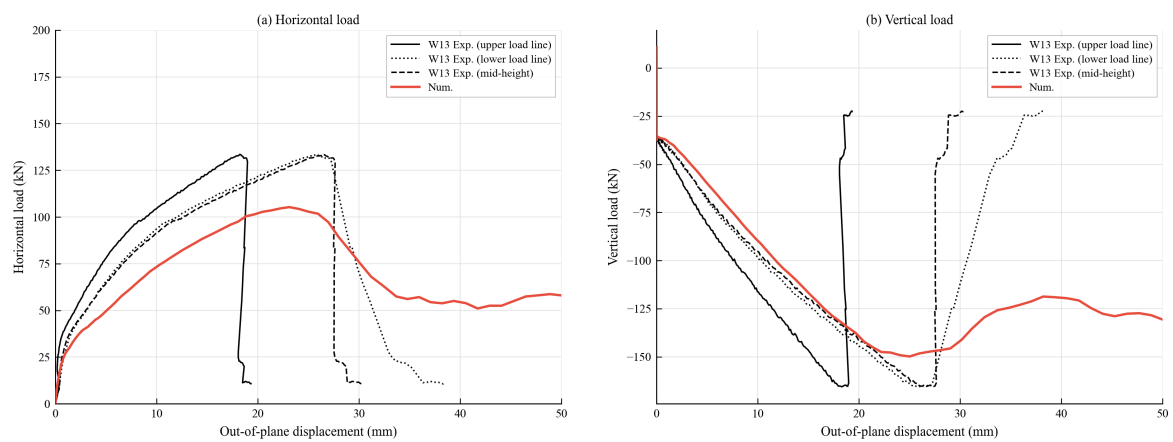


Figure H.13: W13 — lateral force (a) and vertical reaction (b) versus out-of-plane displacement under non-rigid top support conditions ($K_{sup} = 19.0$ kN/mm). Experimental measurements at the upper load line, lower load line, and mid-height are compared against the numerical result.

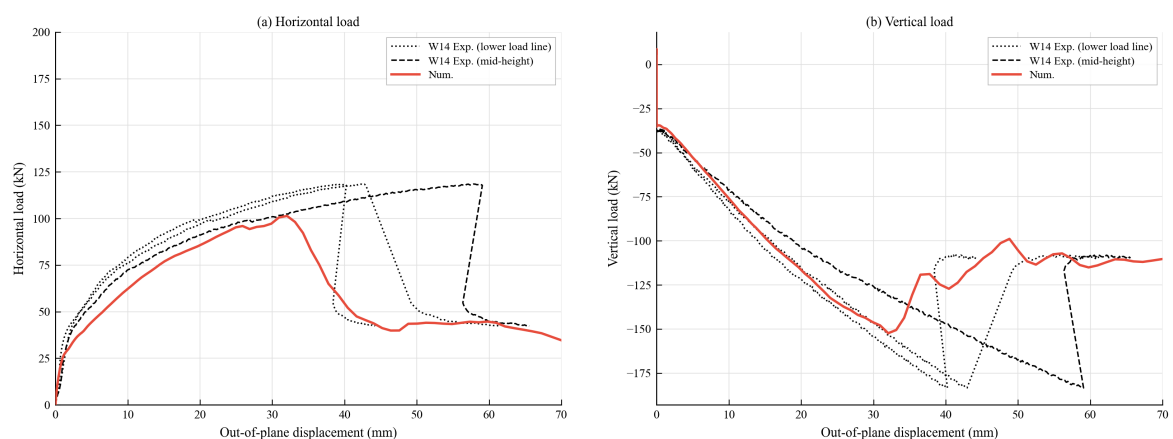


Figure H.14: W14 — lateral force (a) and vertical reaction (b) versus out-of-plane displacement under non-rigid top support conditions ($K_{sup} = 9.5$ kN/mm). Experimental measurements at the upper load line, lower load line, and mid-height are compared against the numerical result.

A key feature of the experimental response is the abrupt load drop associated with the onset of diagonal cracking, occurring at approximately 19 mm for W13 and 40 mm for W14. The model does not reproduce this sudden drop; instead, the numerical post-peak descent is gradual, which is a consequence of the CDP formulation smearing damage over the continuum rather than localising it along discrete crack surfaces. The deformed shape and plastic strain field at the end of the simulation, shown in Figures H.15 and H.16, indicate damage concentrating diagonally in the wall regions consistent with the experimentally observed hybrid mechanism, though without the explicit block separation and crack opening visible in the laboratory.

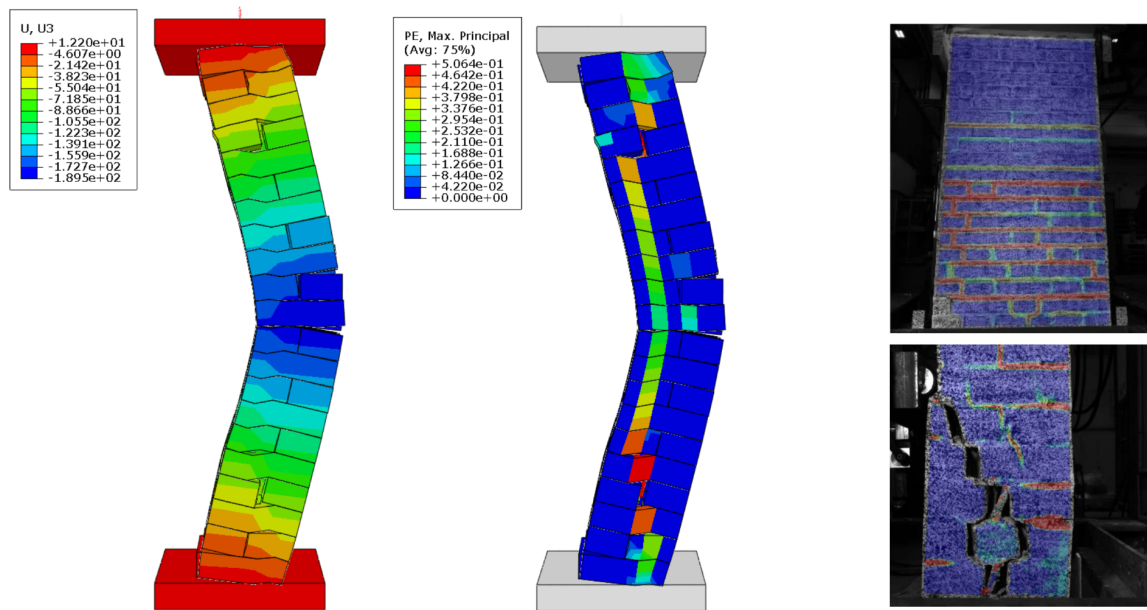


Figure H.15: W13 — failure mode: Abaqus out-of-plane displacement (U3) field (left), Abaqus plastic strain (PE) field (centre), and experimental crack pattern (right) at the end of the simulation (Godio et al., 2023b).

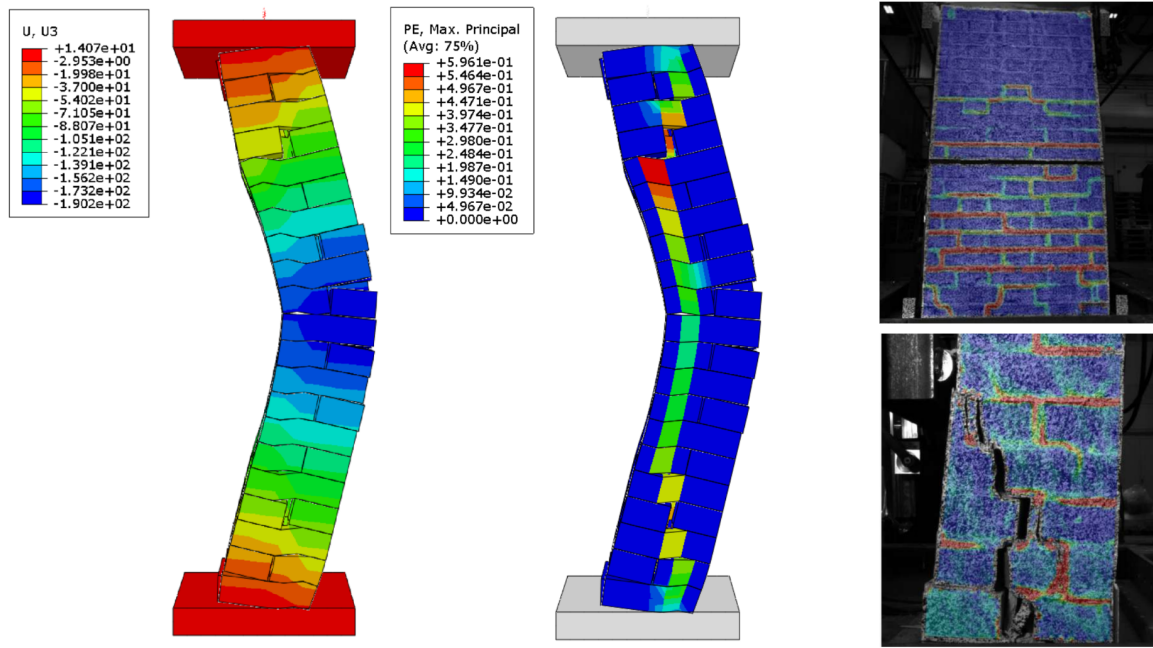


Figure H.16: W14 — failure mode: Abaqus out-of-plane displacement (U3) field (left), Abaqus plastic strain (PE) field (centre), and experimental crack pattern (right) at the end of the simulation (Godio et al., 2023b).

H.5 Final 4PB Results

This section presents the results obtained using the final set of calibrated parameters applied uniformly across all walls, including the non-linear evolution laws, cohesion, and friction coefficient identified from W12 in the preceding calibration stage. Walls W5 and W8 were therefore reanalysed with this complete parameter set to verify that the overall behaviour and failure mechanism remained consistent with the earlier calibration.

H.5.1 Simply supported boundary condition — W5 and W8

Walls W5 and W8 are double-wythe specimens tested under simply supported top conditions with initial axial compression forces of 37.7 kN and 76.0 kN, respectively. Both walls failed experimentally through a three-hinge bending mechanism, with crack opening concentrating at mid-height and at the bed joints adjacent to the loading lines, without apparent brick or mortar crushing (Godio et al., 2023b).

The reanalysed model with the final parameter set produces an overall force–displacement response that does not differ significantly from the earlier calibration, as shown in Figures H.17 and H.18. The peak lateral force remains underestimated in both cases: for W5, the numerical peak of approximately 33 kN falls below the experimental value of approximately 42 kN, while for W8 a similar underestimation is observed. In the post-peak regime, the numerical response follows a descent of similar slope to the experimental curves, though offset downward as a consequence of the lower predicted peak force. Under simply supported conditions, no arching action is generated, and the vertical reaction remains approximately constant throughout the loading history.

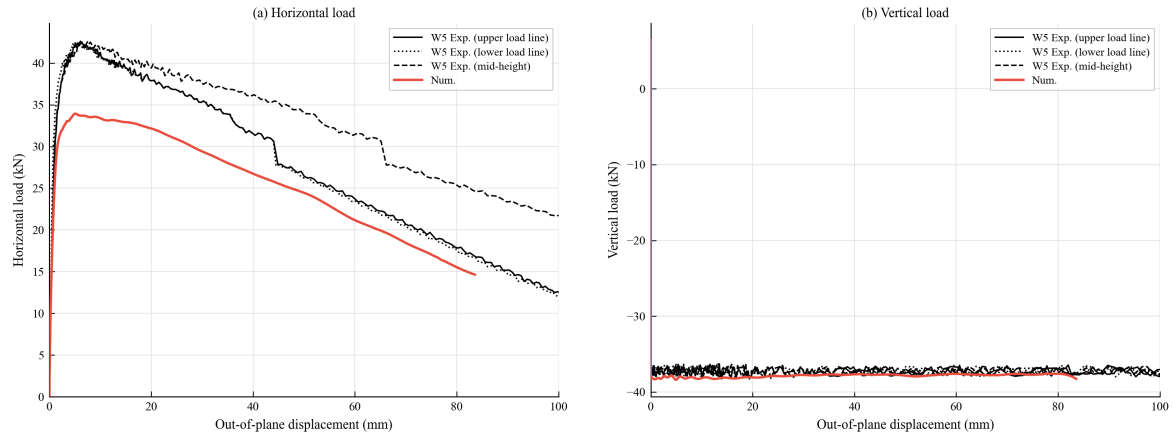


Figure H.17: W5 — lateral force (a) and vertical reaction (b) versus out-of-plane displacement under simply supported top conditions. Experimental measurements at the upper load line, lower load line, and mid-height are compared against the numerical result.

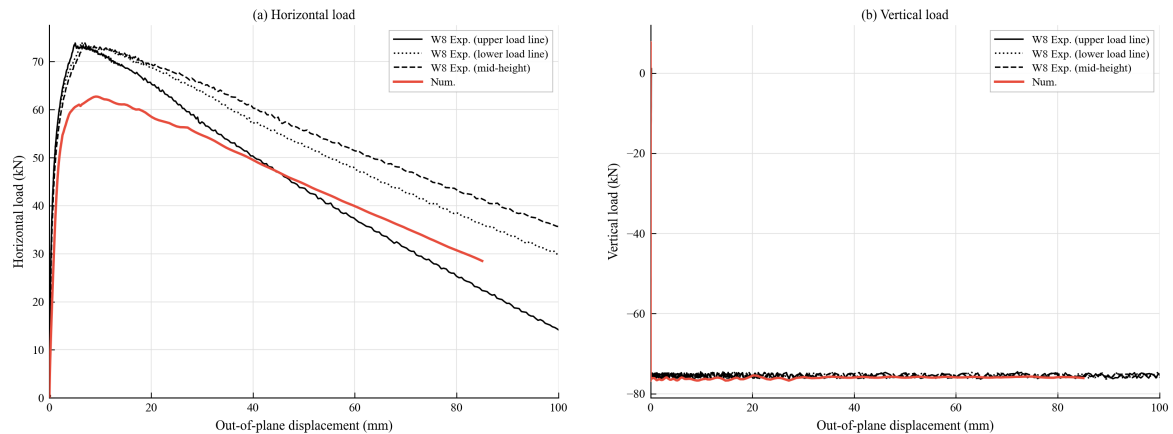


Figure H.18: W8 — lateral force (a) and vertical reaction (b) versus out-of-plane displacement under simply supported top conditions. Experimental measurements at the upper load line, lower load line, and mid-height are compared against the numerical result.

A slight change in the failure mechanism was observed when comparing the reanalysed model against the earlier calibration. For W5, the model now exhibits multiple crack openings distributed across the 10th, 11th, and 12th courses, with the most pronounced opening at the 11th course, in closer agreement with the experimentally observed location at the 10th course. For W8, the crack openings are distributed more symmetrically across the 10th and 11th courses. Despite these minor differences, both walls retain the three-hinge bending failure mechanism consistent with the experimental observations (Godio et al., 2023b), and the results were deemed acceptable to proceed with the final parameter set.

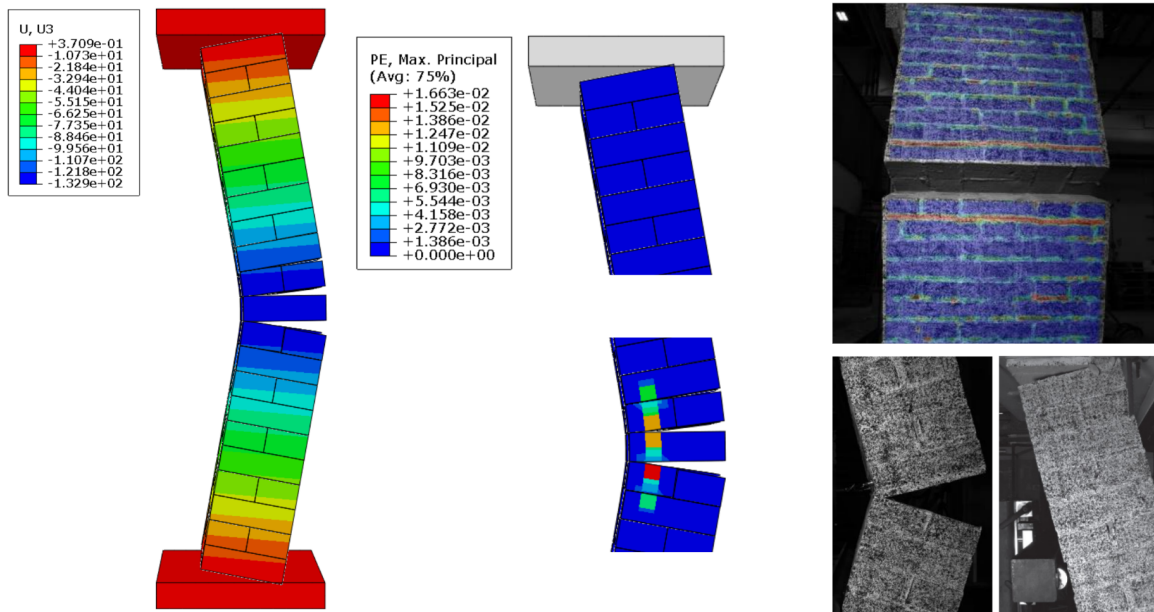


Figure H.19: W5 — failure mode: Abaqus three-hinge deformed shape (left), mid-wall brick opening (centre), and experimental crack pattern (right) (Godio et al., 2023b).

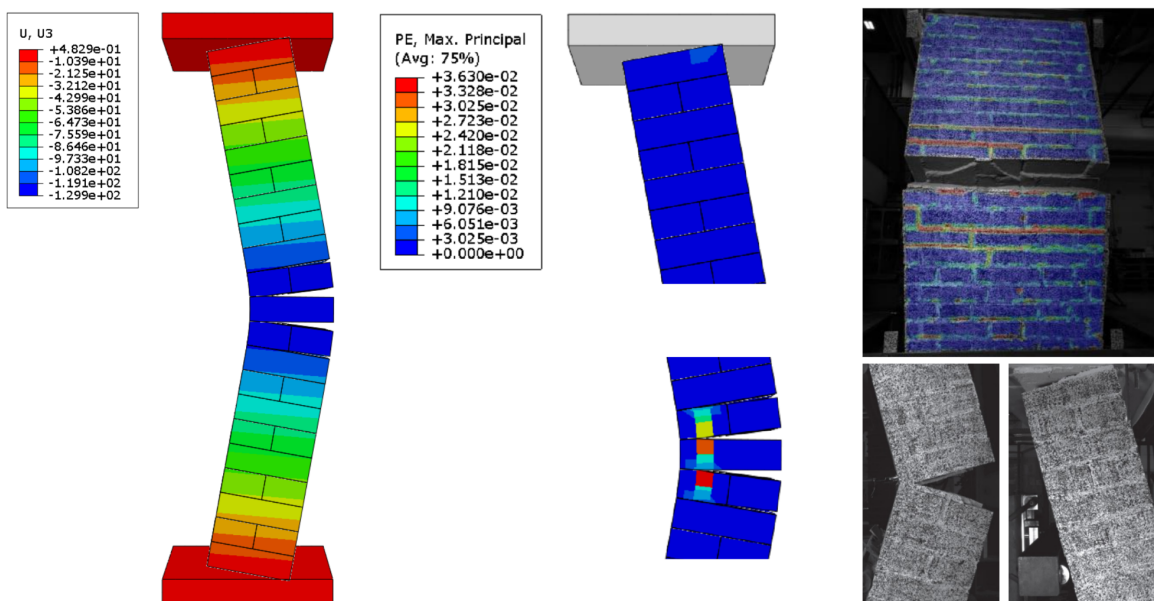
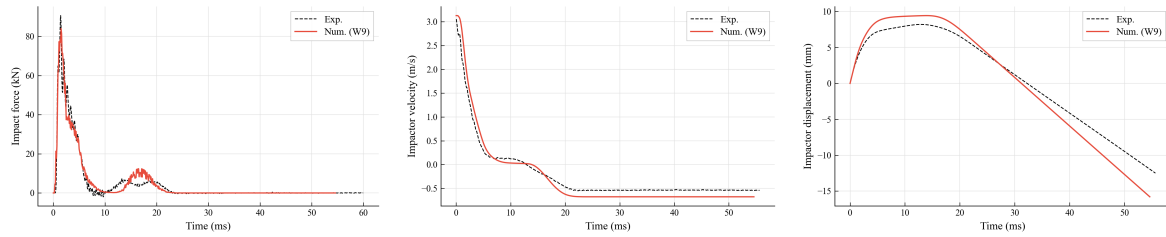


Figure H.20: W8 — failure mode: Abaqus three-hinge deformed shape (left), mid-wall brick opening (centre), and experimental crack pattern (right) (Godio et al., 2023b).

I Appendix I: Campaign 1 — Low-velocity point-load impacts

The Campaign 1 parameters are transferred from the four-point bending calibration. The following results show the impactor response and wall behaviour for walls W9–W11.

Wall W9 — 76.5 kg, $v_0 = 3.13$ m/s



(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure I.1: W9 — impactor response histories.

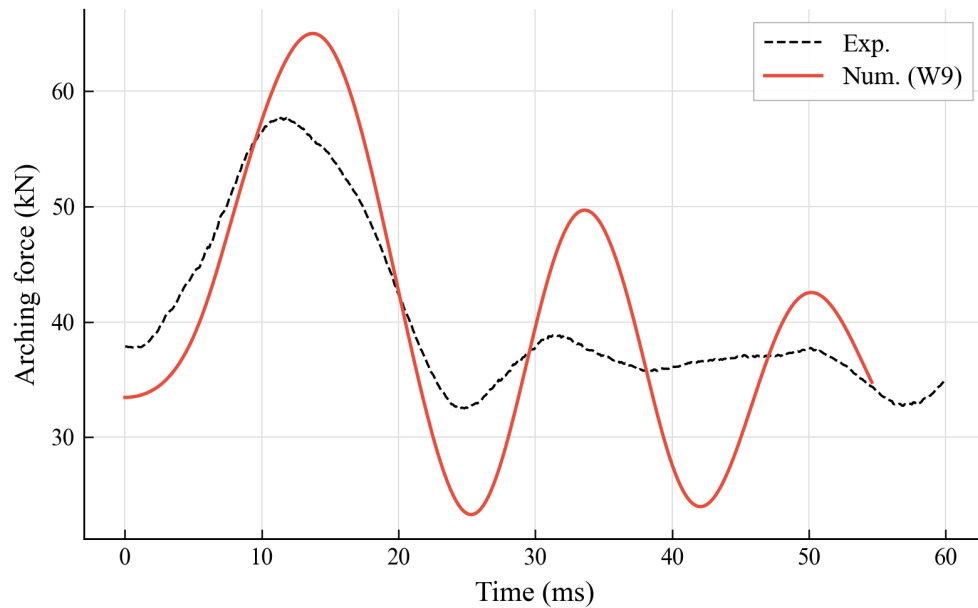


Figure I.2: W9 — arching force.

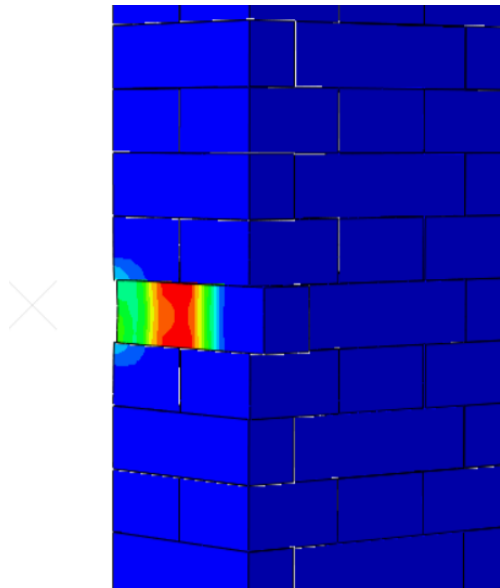
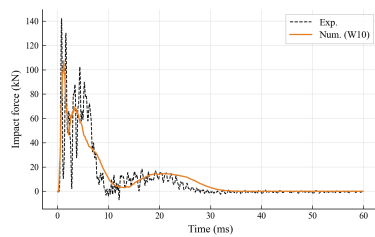
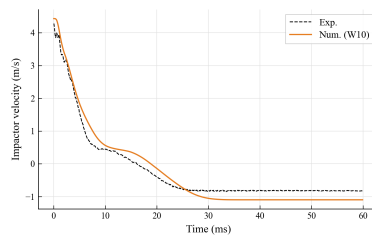


Figure I.3: W9 — plastic strain PE at peak displacement.

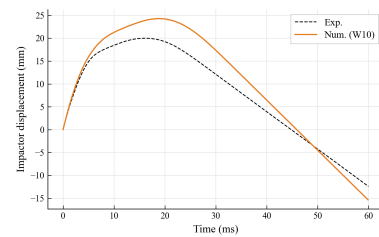
Wall W10 — 116 kg, $v_0 = 4.43$ m/s



(a) Impact force.



(b) Impact velocity.



(c) Impact displacement.

Figure I.4: W10 — impactor response histories.

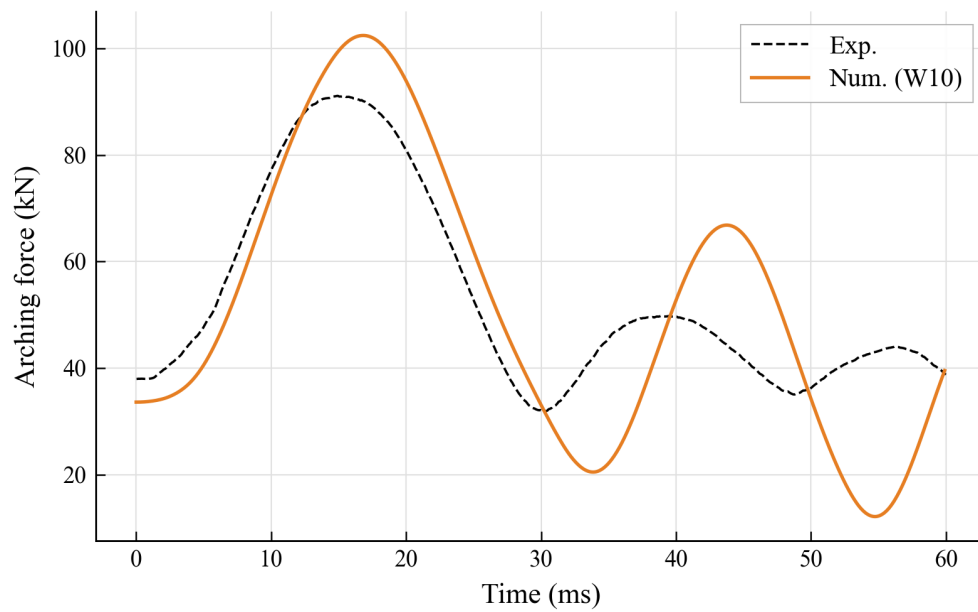


Figure I.5: W10 — arching force.

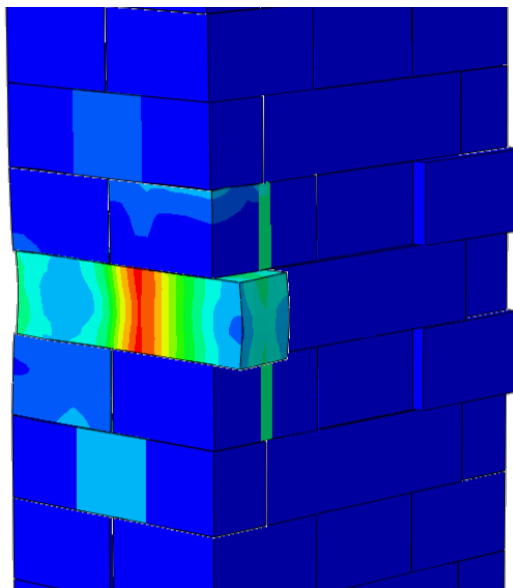
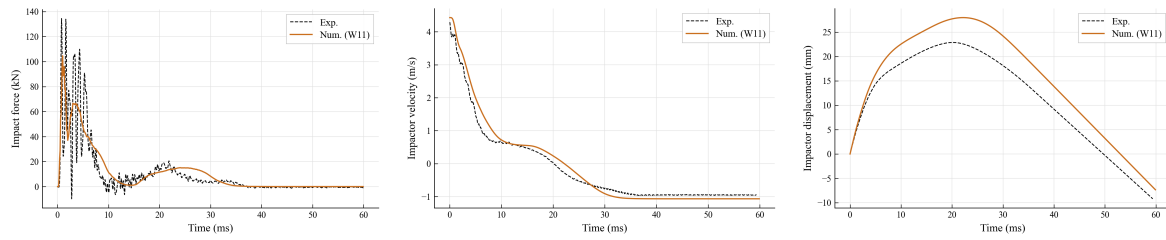


Figure I.6: W10 — plastic strain PE at peak displacement.

Wall W11 — 116 kg, $v_0 = 4.43$ m/s



(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure I.7: W11 — impactor response histories.

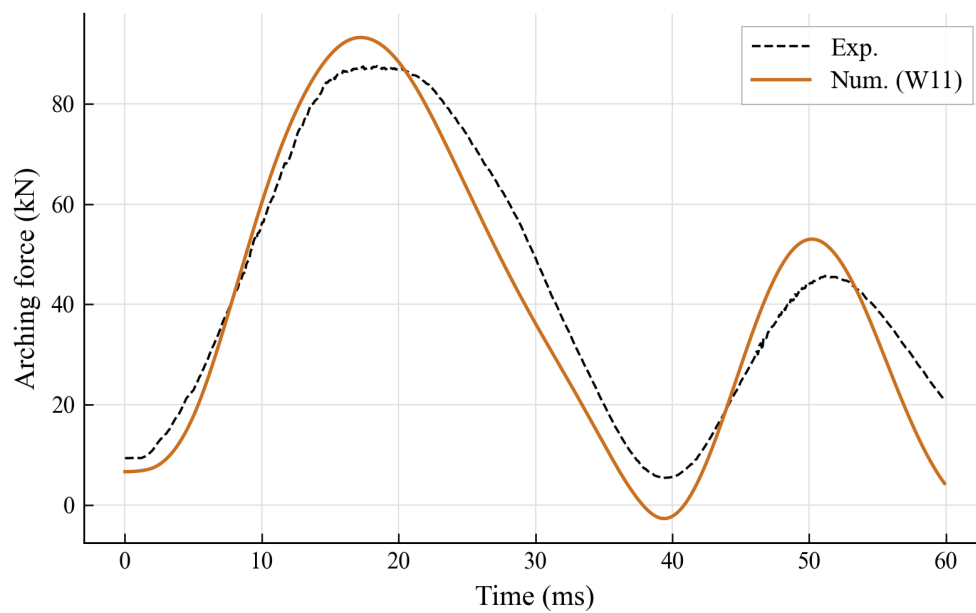


Figure I.8: W11 — arching force.

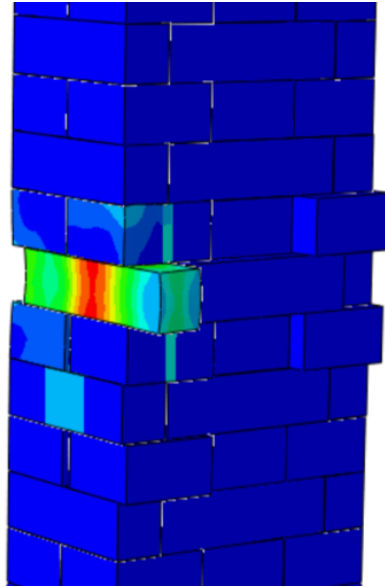


Figure I.9: W11 — plastic strain PE at peak displacement.

J Appendix J: Campaign 2 — Moderate-velocity point- and line-load impacts

Wall W16 — point-load, 116 kg, $v_0 = 7.23$ m/s

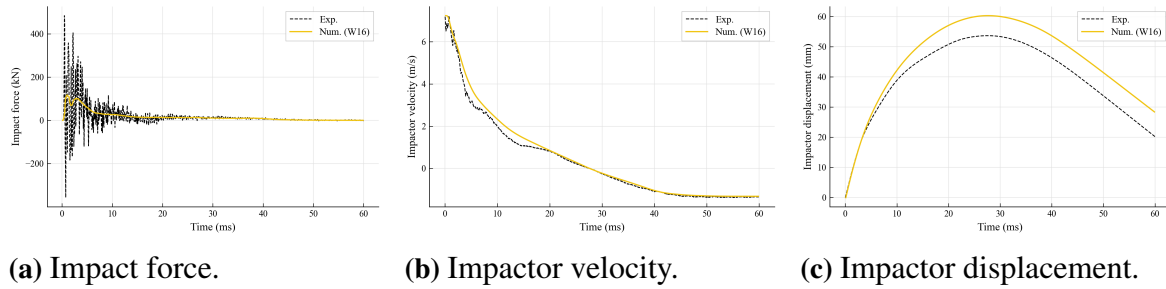


Figure J.1: W16 — impactor response histories.

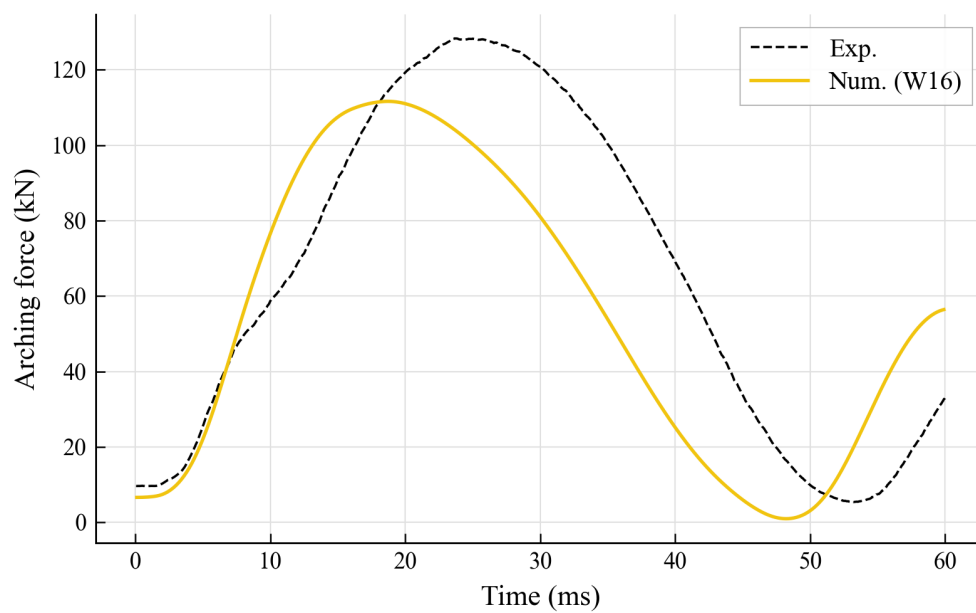


Figure J.2: W16 — arching force.

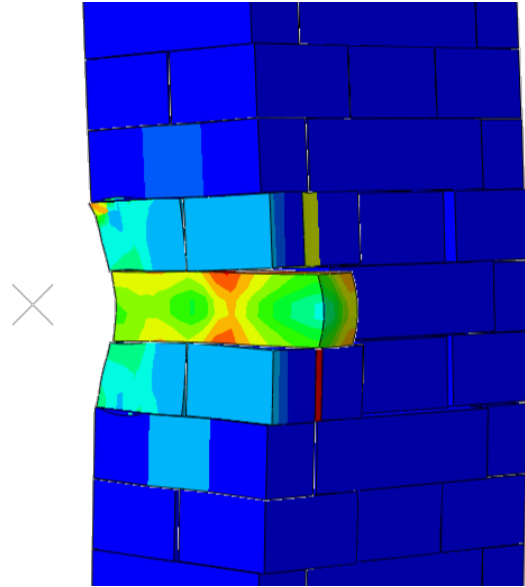
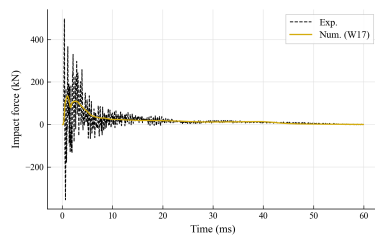
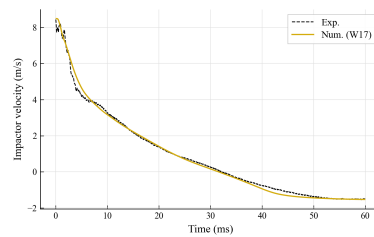


Figure J.3: W16 — plastic strain PE at peak displacement.

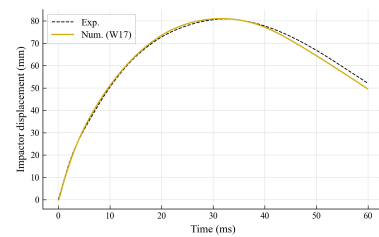
Wall W17 — point-load, 116 kg, $v_0 = 8.49$ m/s



(a) Impact force.



(b) Impactor velocity.



(c) Impactor displacement.

Figure J.4: W17 — impactor response histories.

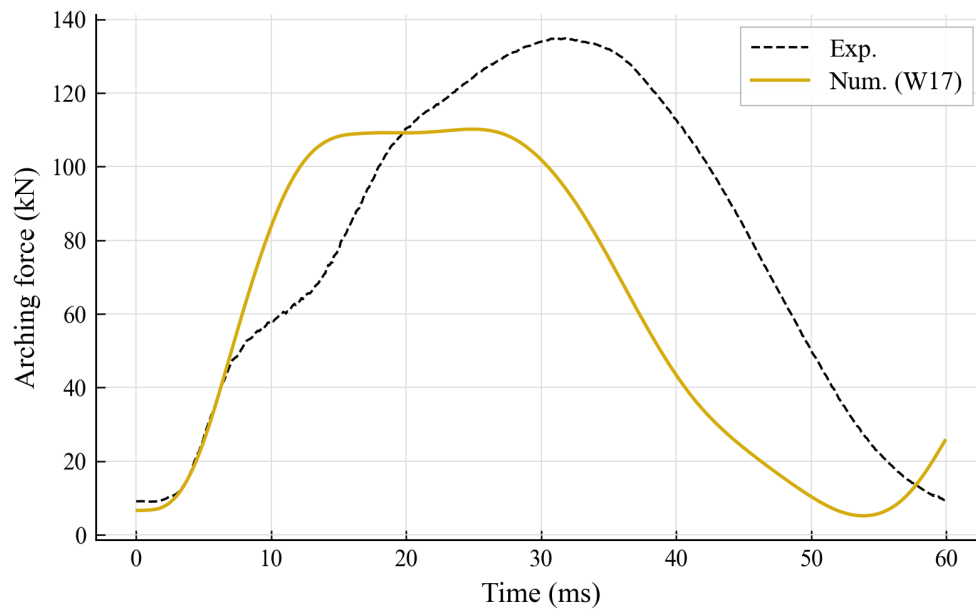


Figure J.5: W17 — arching force.

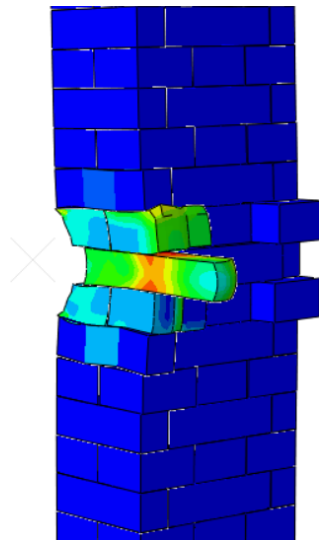
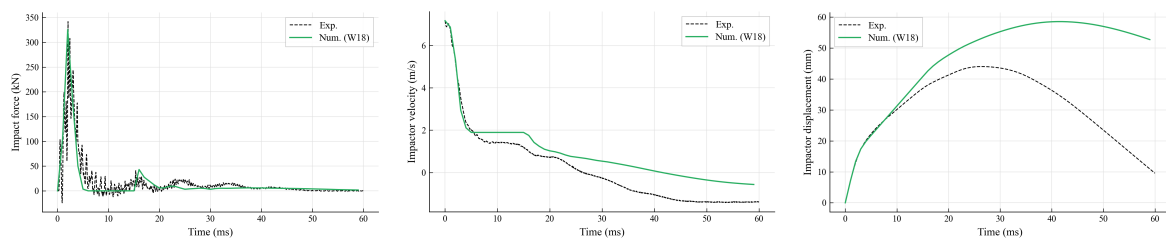


Figure J.6: W17 — plastic strain PE at peak displacement.

Wall W18 — line-load, 116 kg, $v_0 = 7.17$ m/s



(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure J.7: W18 — impactor response histories.

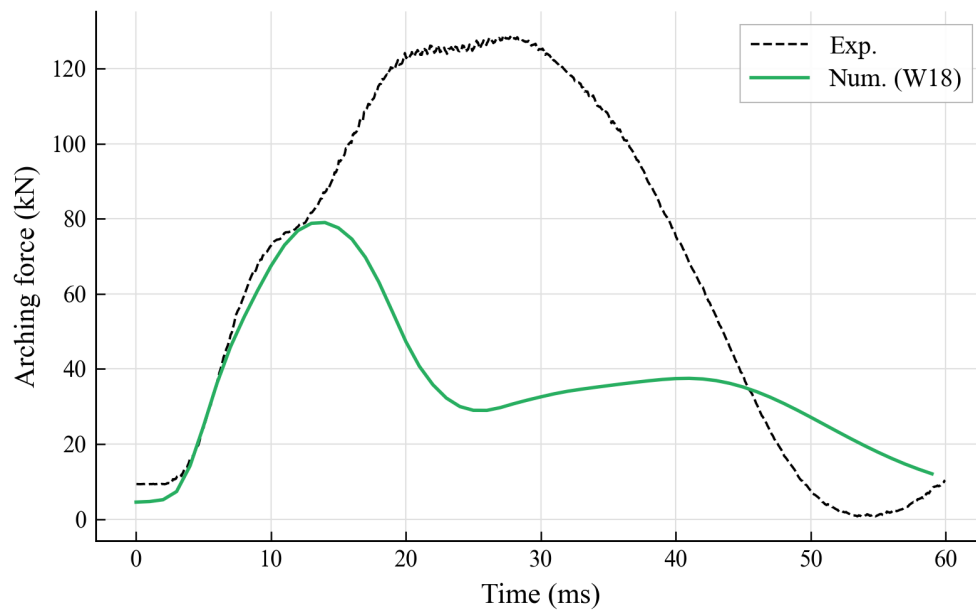


Figure J.8: W18 — arching force.

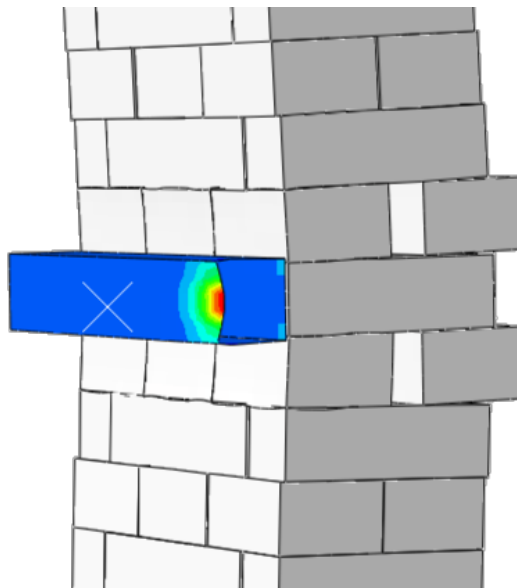


Figure J.9: W18 — plastic strain PE at peak displacement.

Wall W19 — line-load, 116 kg, $v_0 = 8.35$ m/s

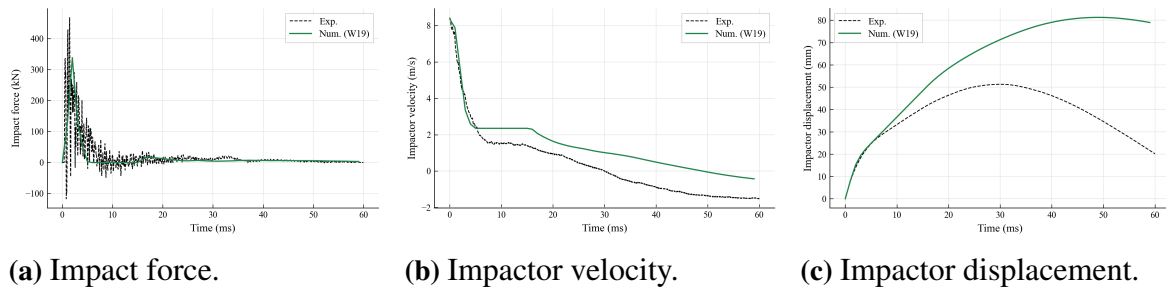


Figure J.10: W19 — impactor response histories.

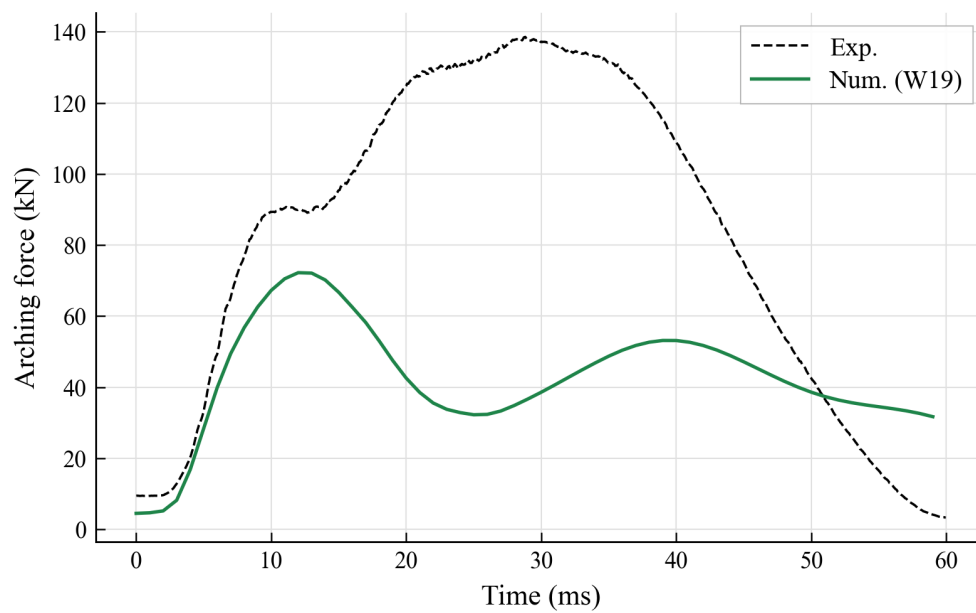


Figure J.11: W19 — arching force.

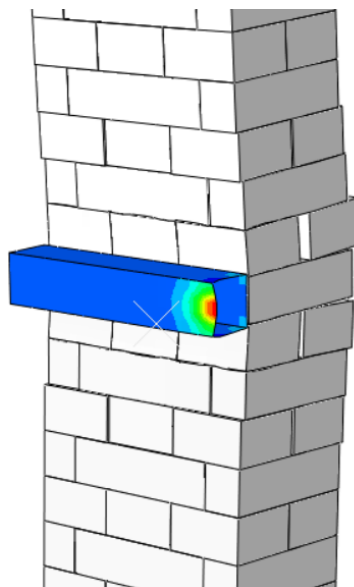


Figure J.12: W19 — plastic strain PE at peak displacement.

K Appendix M: HybriDFEM / Abaqus Campaign 3 — W23–W35

This appendix provides the complete wall-by-wall results for Campaign 3 (W23–W35). The model configuration and experimental reference values are given in Sections K.1 and K.2. Cross-wall prediction metrics are reported in Table 4.19 in the main text. Three numerical models are overlaid on each figure: HybriDFEM with `ElastoPlastic` (EP, blue), HybriDFEM with `ElastoPlastic_CD` (CD, orange), and Abaqus (AB, green). The experimental record is shown in black.

K.1 Model configuration

Campaign 3 differs from Campaign 2 in four main aspects. First, the impact is applied through two simultaneous line loads at $H/3$ and $2H/3$. Second, the top boundary uses an eccentric steel plate block (length 0.420 m, thickness 0.050 m, mass 460 kg, eccentricity +80 mm from the wall centreline) as the prestress anchor and spring attachment point. The support stiffnesses are

$$K_{\text{sup},y} = 14.9 \text{ MN/m}, \quad K_{\text{sup},x} = 35 \text{ kN/mm}.$$

Third, the axial prestress is $\sigma_{\text{ax}} = 0.10 \text{ MPa}$ for all walls. Fourth, the impactor mass is 236, 348, or 442 kg, with pre-impact velocities between 0.94 and 10.10 m/s. All HybriDFEM analyses use the refined 3×6 mesh, $\Delta t = 1 \times 10^{-5} \text{ s}$, and `TAN_LG` damping with $\xi = 0.010$. The Abaqus model uses `TSF` = 50 and $\xi = 0.010$.

K.2 Experimental reference values

Table K.1: Experimental reference values, Campaign 3.

Wall	m_{imp} (kg)	v_0 (m/s)	u_{exp} (mm)	F_{exp} (kN)	$F_{\text{arch,exp}}$ (kN)	w_{exp} (mm)
W23	236	1.07	2.6	152.5	32.1	−3.1
W24	236	6.16	24.6	963.7	152.6	−25.4
W25	236	8.10	38.7	1111.9	192.6	−36.2
W26	348	0.94	6.7	102.1	36.0	−3.2
W27	348	5.20	25.5	1074.1	164.2	−29.2
W28	348	6.90	38.8	1768.4	215.1	−41.9
W29	348	8.45	52.2	1702.3	242.2	−52.4
W30	442	1.10	5.1	198.2	45.8	−3.8
W31	442	4.50	26.3	1702.1	161.4	−27.6
W32	442	6.08	39.0	1494.8	223.7	−43.1
W33	442	8.25	59.5	2616.5	262.8	−67.1
W34	236	10.10	41.9	1634.6	230.4	−43.7
W35	348	9.03	52.4	2557.1	252.4	−56.8

Table K.2: Campaign 3 prediction metrics organised by impact energy level. Quantities are numerical-to-experimental peak ratios. EP: ElastoPlastic; CD: ElastoPlastic_CD; AB: Abaqus. †: low-energy impact.

Wall	m (kg)	v_0 (m/s)	KE (J)	u/u_{exp}			F/F_{exp}			$F_{\text{arch}}/F_{\text{arch,exp}}$			w/w_{exp}		
				EP	CD	AB	EP	CD	AB	EP	CD	AB	EP	CD	AB
Low energy (136–262 J)															
W23 [†]	236	1.07	137	0.55	0.81	1.18	1.05	1.07	1.15	1.41	1.28	1.17	1.04	0.84	0.99
W26 [†]	348	0.94	159	0.48	0.46	0.55	1.66	1.61	0.45	1.36	1.26	1.13	1.18	0.99	1.28
W30 [†]	442	1.10	262	0.98	0.93	1.09	0.98	0.96	0.89	1.27	1.22	1.11	1.58	1.34	1.65
Moderate energy (4107–4548 J)															
W31	442	4.50	4107	1.23	1.23	1.49	0.33	0.37	0.14	1.19	1.02	0.77	1.19	0.96	1.69
W24	236	6.16	4273	1.08	1.18	1.41	0.71	0.80	0.58	1.16	1.02	0.73	1.04	0.92	1.70
W27	348	5.20	4548	1.29	1.34	1.19	0.54	0.64	0.20	1.20	1.01	0.78	1.11	0.95	1.02
Moderate-high energy (7909–8173 J)															
W25	236	8.10	7909	1.19	1.22	1.76	0.72	0.88	0.32	1.12	1.02	0.57	1.11	1.03	2.36
W32	442	6.08	8015	1.36	1.44	2.90	0.45	0.55	0.22	1.11	0.89	0.53	1.21	1.09	3.59
W28	348	6.90	8173	1.34	1.34	2.34	0.42	0.51	0.24	1.10	0.95	0.54	1.18	1.01	3.89
High energy (12086–14399 J)															
W29	348	8.45	12086	1.38	1.61	2.33	0.51	0.61	0.25	1.09	0.85	0.46	1.20	1.44	3.35
W34	236	10.10	12288	1.61	1.86	3.22	0.59	0.70	0.26	1.06	0.85	0.43	1.21	1.79	6.60
W35	348	9.03	13515	1.54	1.83	4.07	0.37	0.43	0.17	1.08	0.86	0.42	1.22	1.57	6.36
W33	442	8.25	14399	1.49	1.73	3.26	0.34	0.40	0.14	1.07	0.84	0.44	1.18	1.50	4.16

K.3 Wall-by-wall impactor response

K.3.1 W23 — 236 kg, $v_0 = 1.07$ m/s

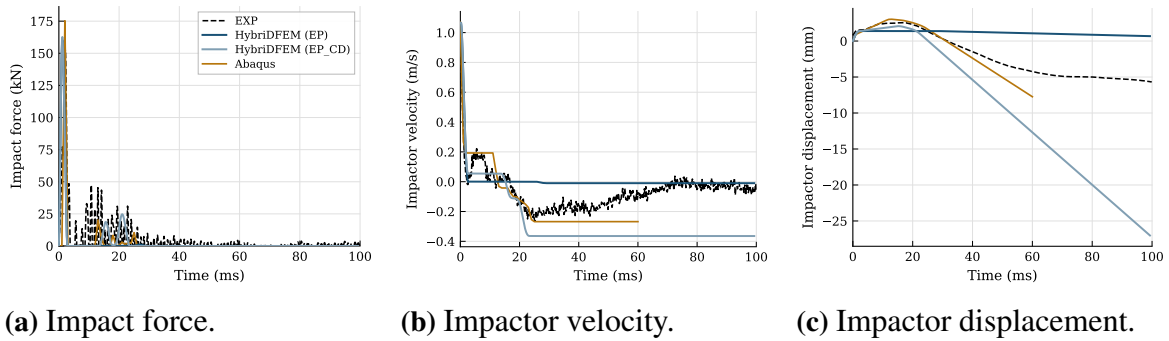


Figure K.1: W23 — impactor response ($m_{\text{imp}} = 236$ kg, $v_0 = 1.07$ m/s).

K.3.2 W24 — 236 kg, $v_0 = 6.16$ m/s

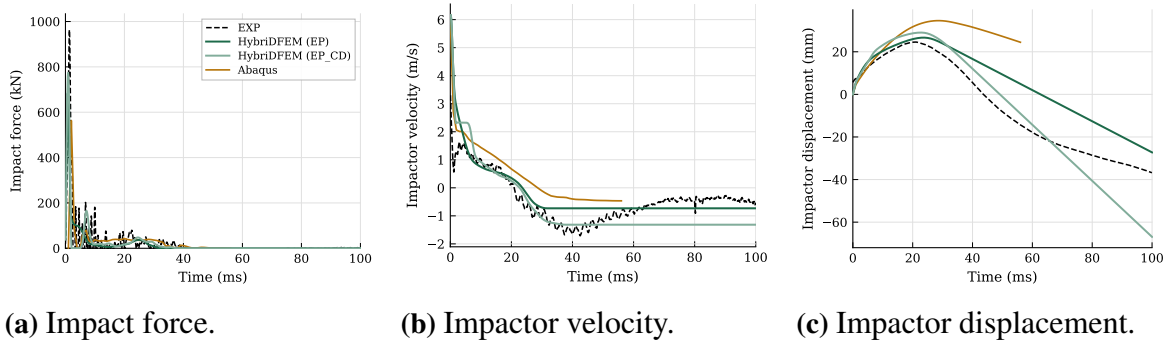


Figure K.2: W24 — impactor response ($m_{\text{imp}} = 236$ kg, $v_0 = 6.16$ m/s).

K.3.3 W25 — 236 kg, $v_0 = 8.10$ m/s

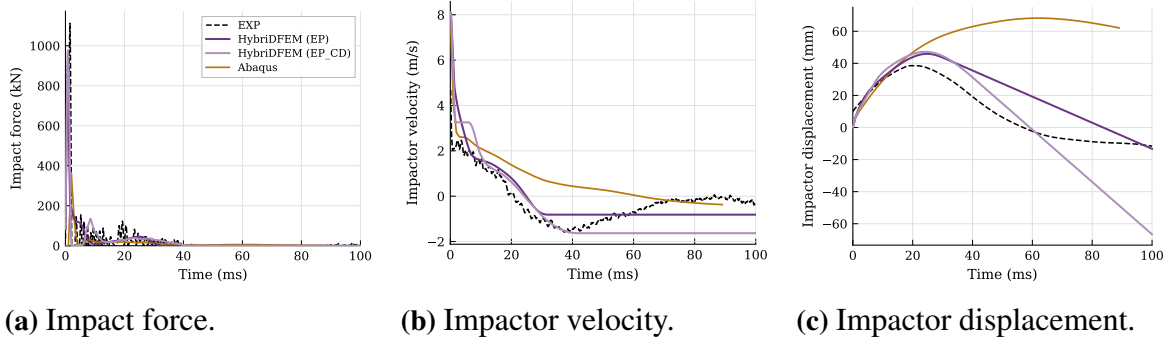


Figure K.3: W25 — impactor response ($m_{\text{imp}} = 236$ kg, $v_0 = 8.10$ m/s).

K.3.4 W26 — 348 kg, $v_0 = 0.94$ m/s

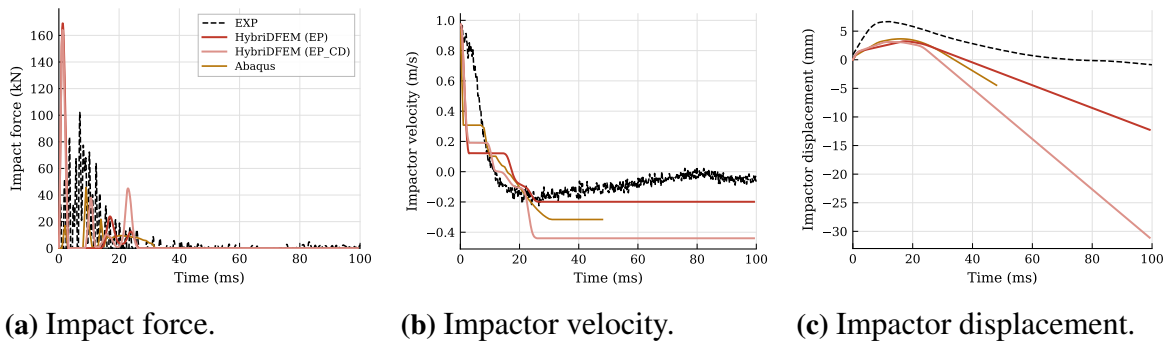


Figure K.4: W26 — impactor response ($m_{\text{imp}} = 348$ kg, $v_0 = 0.94$ m/s).

K.3.5 W27 — 348 kg, $v_0 = 5.20$ m/s

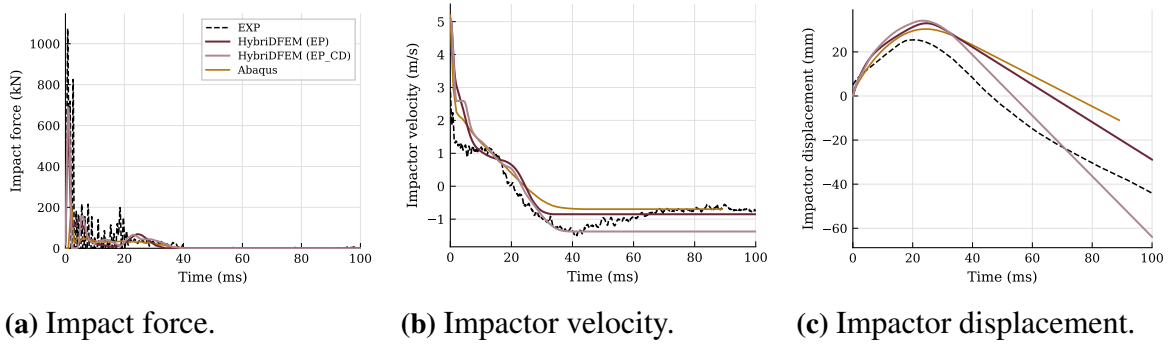


Figure K.5: W27 — impactor response ($m_{\text{imp}} = 348$ kg, $v_0 = 5.20$ m/s).

K.3.6 W28 — 348 kg, $v_0 = 6.90$ m/s

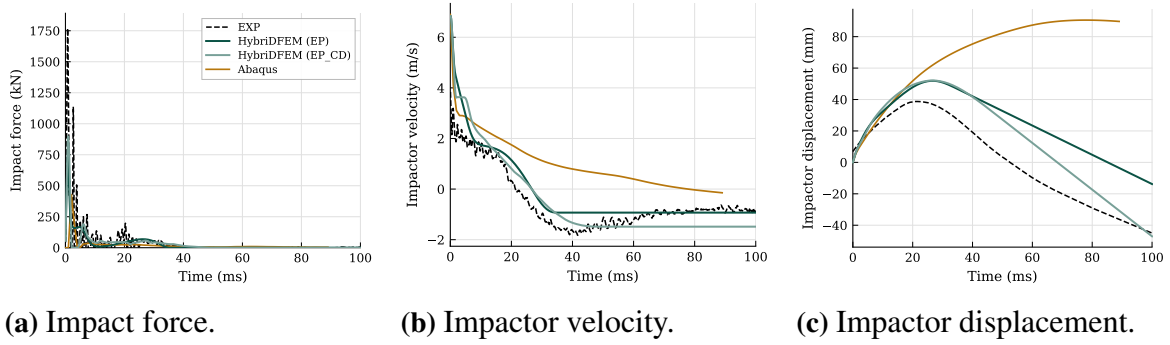


Figure K.6: W28 — impactor response ($m_{\text{imp}} = 348$ kg, $v_0 = 6.90$ m/s).

K.3.7 W29 — 348 kg, $v_0 = 8.45$ m/s

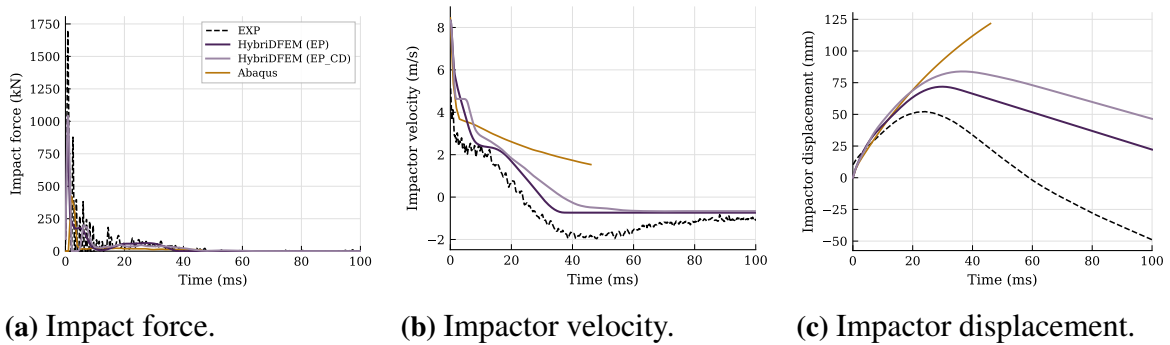
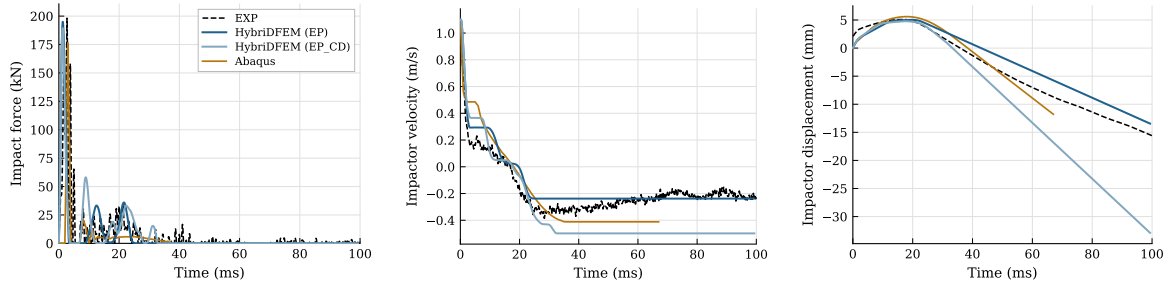


Figure K.7: W29 — impactor response ($m_{\text{imp}} = 348$ kg, $v_0 = 8.45$ m/s).

K.3.8 W30 — 442 kg, $v_0 = 1.10$ m/s



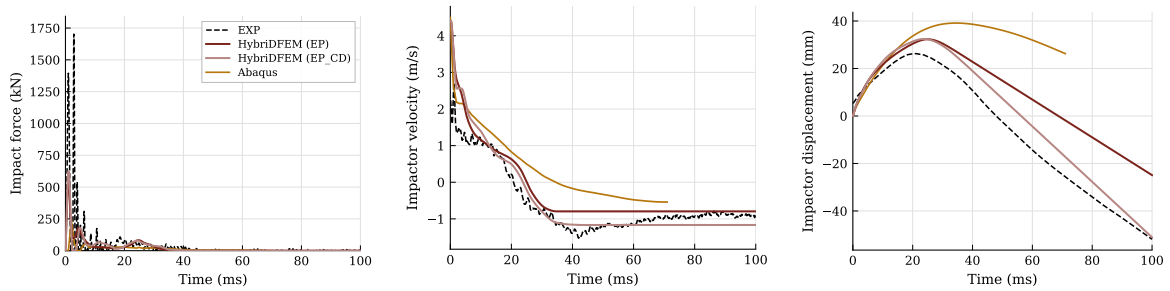
(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure K.8: W30 — impactor response ($m_{\text{imp}} = 442$ kg, $v_0 = 1.10$ m/s).

K.3.9 W31 — 442 kg, $v_0 = 4.50$ m/s



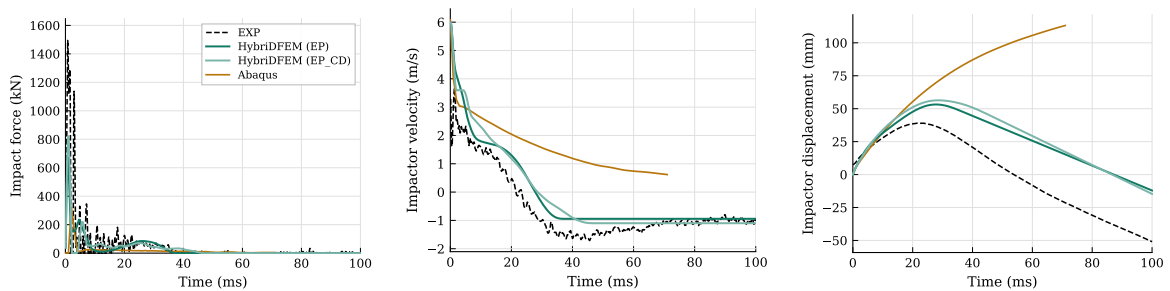
(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure K.9: W31 — impactor response ($m_{\text{imp}} = 442$ kg, $v_0 = 4.50$ m/s).

K.3.10 W32 — 442 kg, $v_0 = 6.08$ m/s



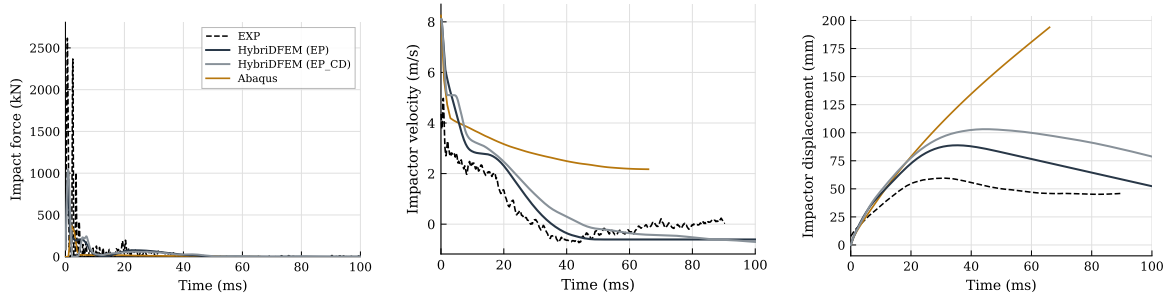
(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure K.10: W32 — impactor response ($m_{\text{imp}} = 442$ kg, $v_0 = 6.08$ m/s).

K.3.11 W33 — 442 kg, $v_0 = 8.25$ m/s



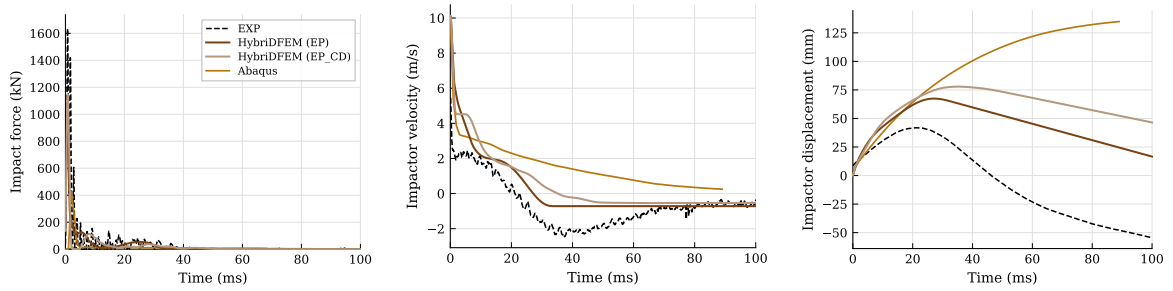
(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure K.11: W33 — impactor response ($m_{\text{imp}} = 442$ kg, $v_0 = 8.25$ m/s).

K.3.12 W34 — 236 kg, $v_0 = 10.10$ m/s



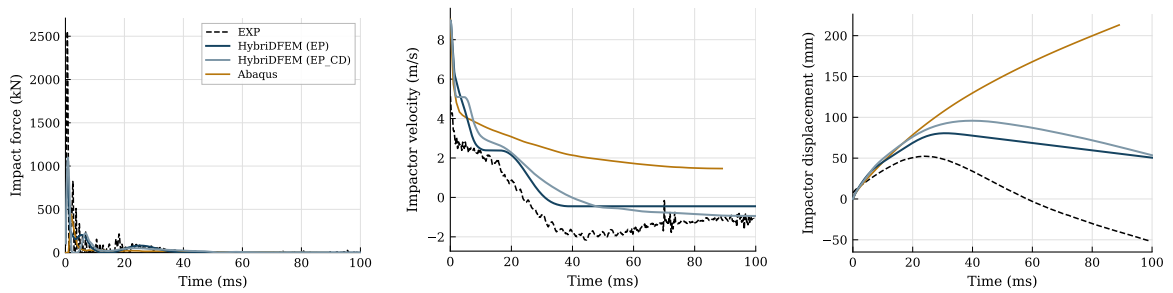
(a) Impact force.

(b) Impactor velocity.

(c) Impactor displacement.

Figure K.12: W34 — impactor response ($m_{\text{imp}} = 236$ kg, $v_0 = 10.10$ m/s).

K.3.13 W35 — 348 kg, $v_0 = 9.03$ m/s



(a) Impact force.

(b) Impactor velocity.

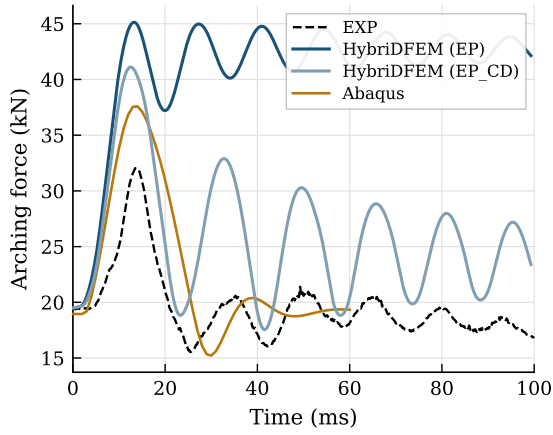
(c) Impactor displacement.

Figure K.13: W35 — impactor response ($m_{\text{imp}} = 348$ kg, $v_0 = 9.03$ m/s).

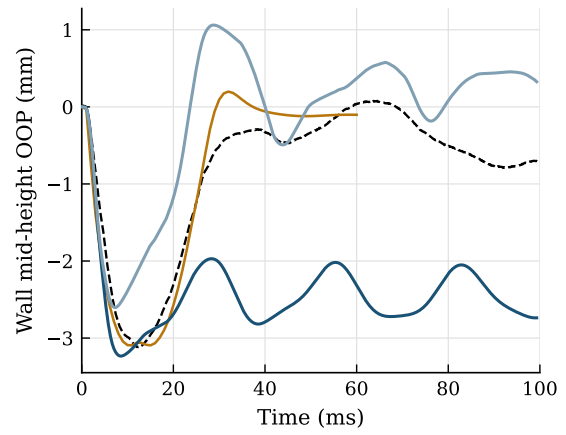
K.4 Wall-by-wall arching force and OOP displacement

The numerical arching force is computed as $F_{\text{arch}}^{\text{num}}(t) = P_0 + K_{\text{sup},y} \Delta u_y^{N_1}(t)$. Under `ElastoPlastic` the arching force does not return to the pre-impact level, because opened

bed joints remain active in the global assembly (Appendix F); `ElastoPlastic_CD` restores post-impact rebound through contact deletion.

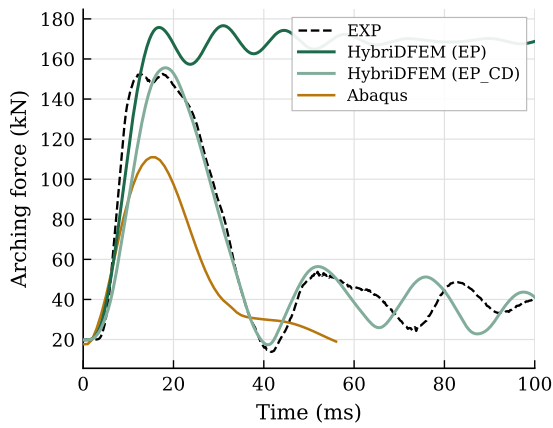


(a) Arching force.

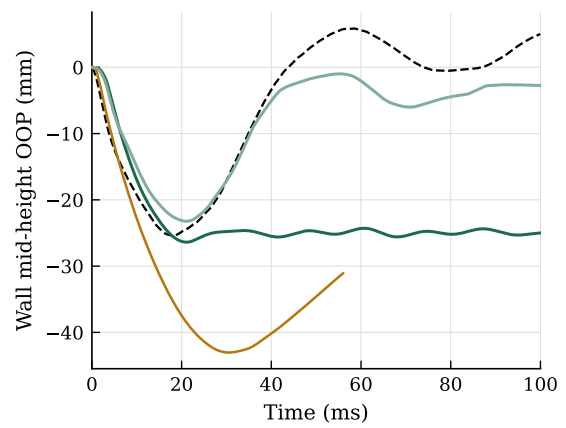


(b) Wall mid-height OOP.

Figure K.14: W23 ($m_{\text{imp}} = 236 \text{ kg}$, $v_0 = 1.07 \text{ m/s}$).

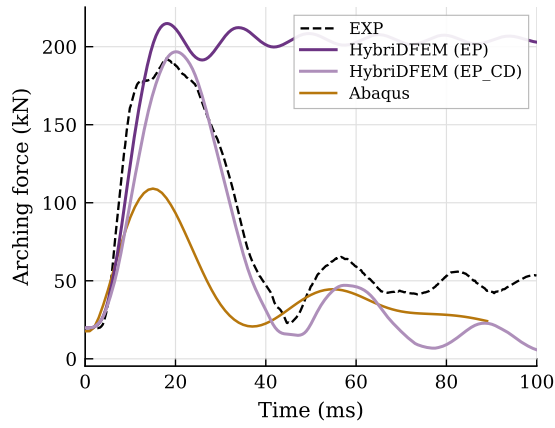


(a) Arching force.

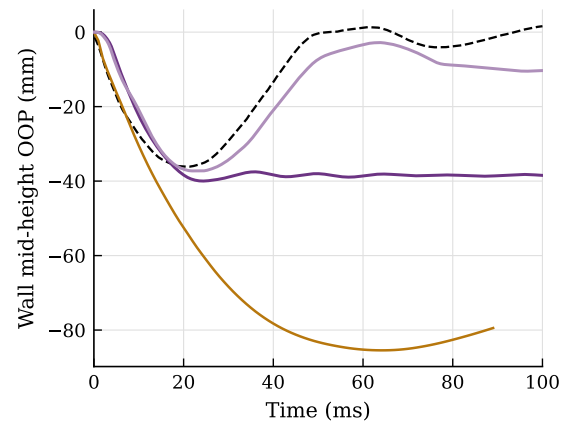


(b) Wall mid-height OOP.

Figure K.15: W24 ($m_{\text{imp}} = 236 \text{ kg}$, $v_0 = 6.16 \text{ m/s}$).

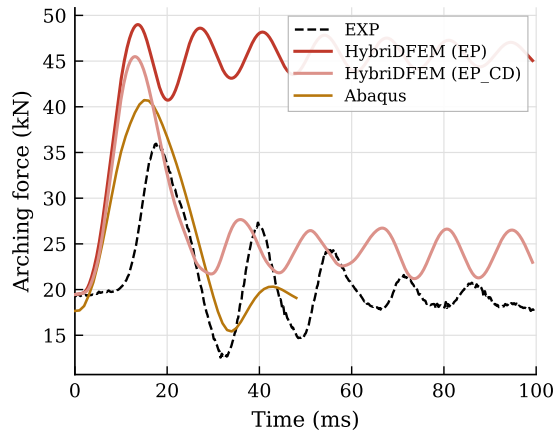


(a) Arching force.

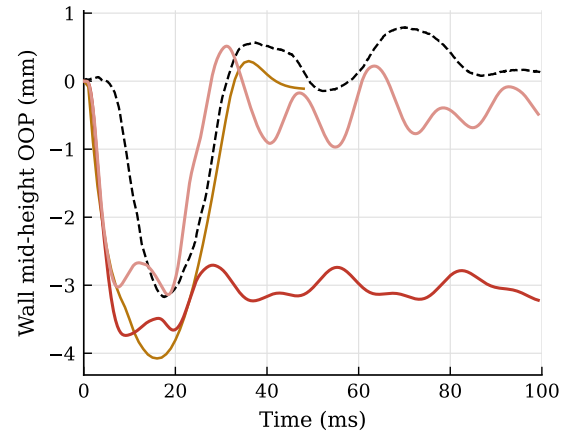


(b) Wall mid-height OOP.

Figure K.16: W25 ($m_{\text{imp}} = 236 \text{ kg}$, $v_0 = 8.10 \text{ m/s}$).

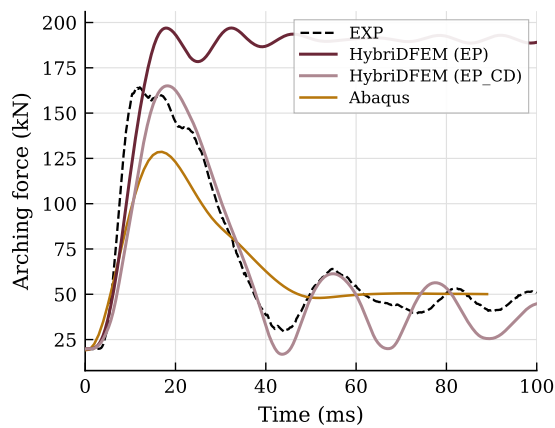


(a) Arching force.

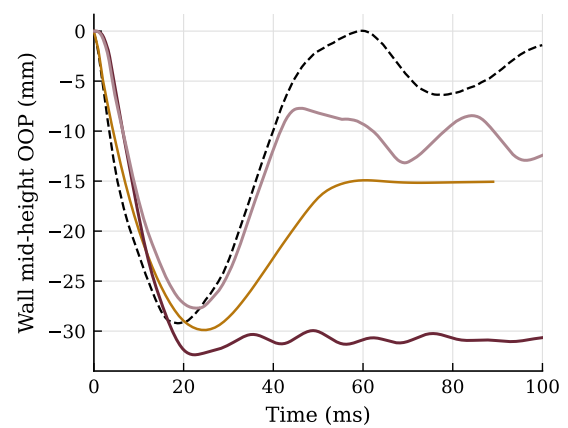


(b) Wall mid-height OOP.

Figure K.17: W26 ($m_{\text{imp}} = 348 \text{ kg}$, $v_0 = 0.94 \text{ m/s}$).

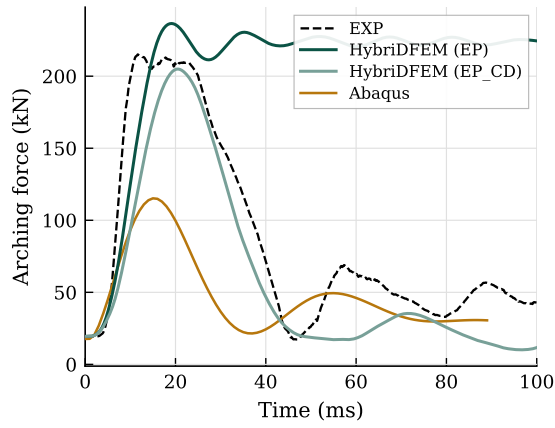


(a) Arching force.

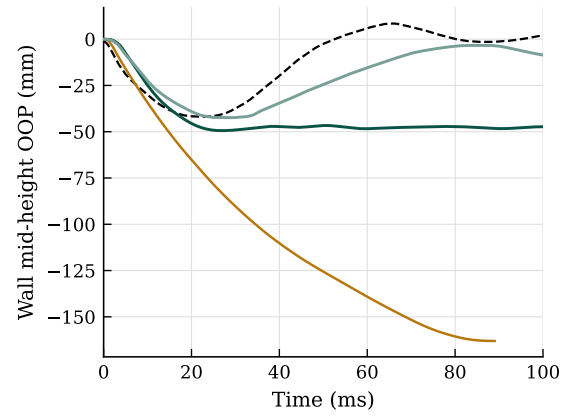


(b) Wall mid-height OOP.

Figure K.18: W27 ($m_{\text{imp}} = 348 \text{ kg}$, $v_0 = 5.20 \text{ m/s}$).

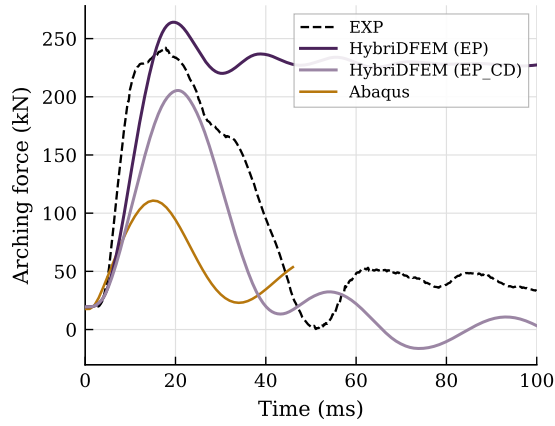


(a) Arching force.

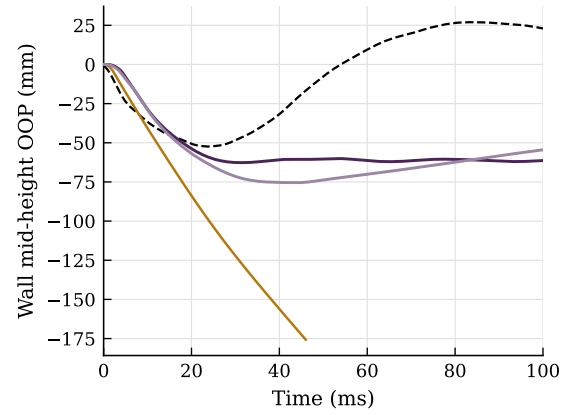


(b) Wall mid-height OOP.

Figure K.19: W28 ($m_{\text{imp}} = 348 \text{ kg}$, $v_0 = 6.90 \text{ m/s}$).

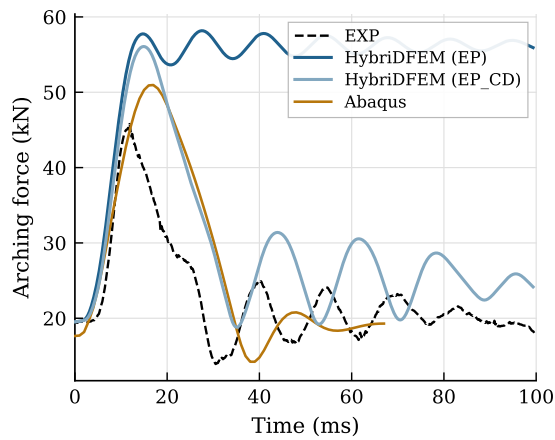


(a) Arching force.

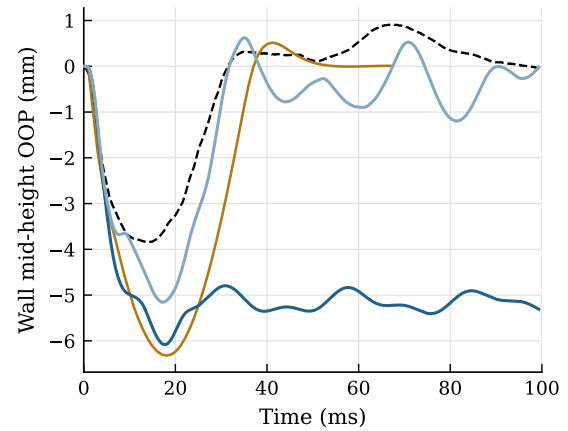


(b) Wall mid-height OOP.

Figure K.20: W29 ($m_{\text{imp}} = 348 \text{ kg}$, $v_0 = 8.45 \text{ m/s}$).

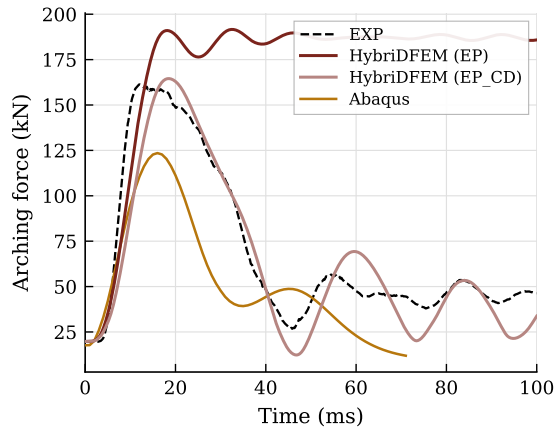


(a) Arching force.

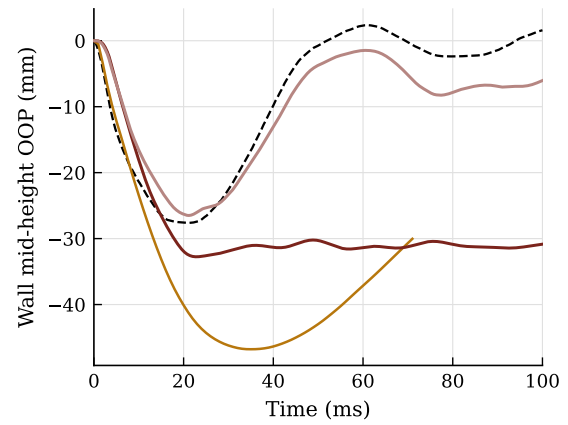


(b) Wall mid-height OOP.

Figure K.21: W30 ($m_{\text{imp}} = 442 \text{ kg}$, $v_0 = 1.10 \text{ m/s}$).

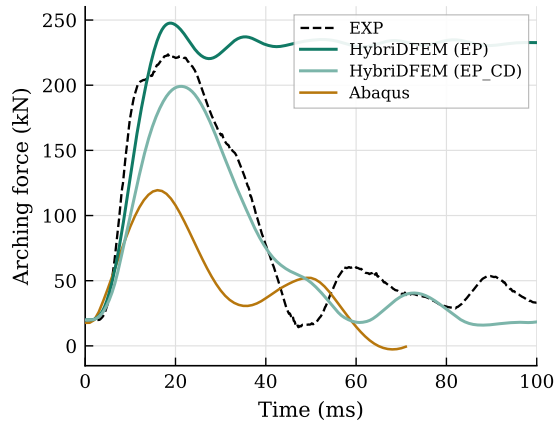


(a) Arching force.

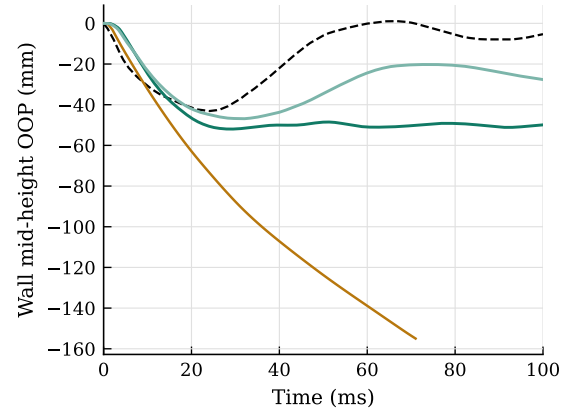


(b) Wall mid-height OOP.

Figure K.22: W31 ($m_{\text{imp}} = 442 \text{ kg}$, $v_0 = 4.50 \text{ m/s}$).

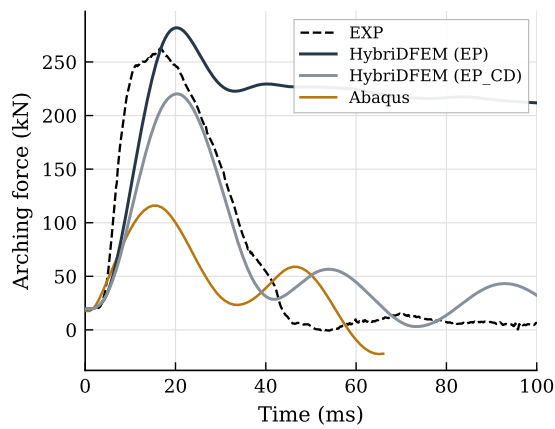


(a) Arching force.

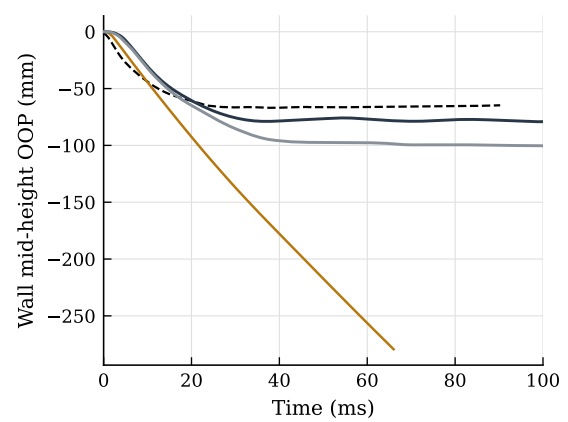


(b) Wall mid-height OOP.

Figure K.23: W32 ($m_{\text{imp}} = 442 \text{ kg}$, $v_0 = 6.08 \text{ m/s}$).

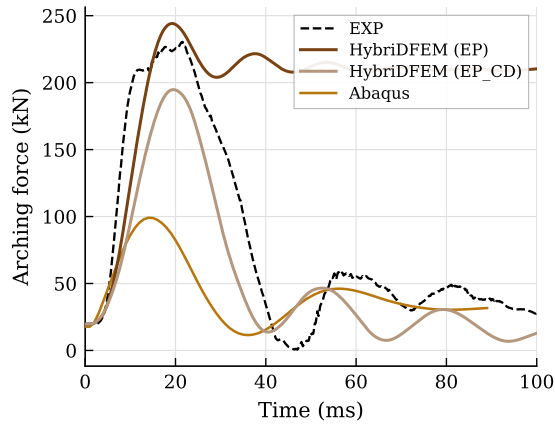


(a) Arching force.

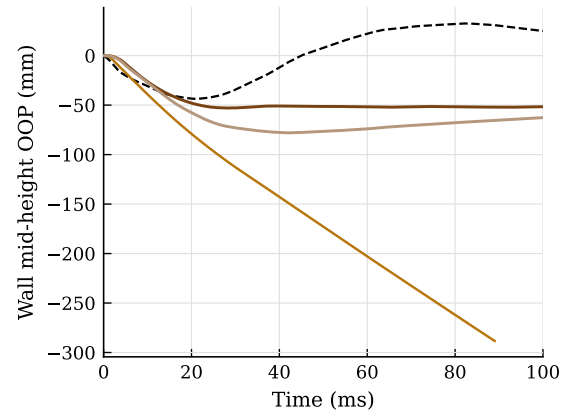


(b) Wall mid-height OOP.

Figure K.24: W33 ($m_{\text{imp}} = 442 \text{ kg}$, $v_0 = 8.25 \text{ m/s}$).

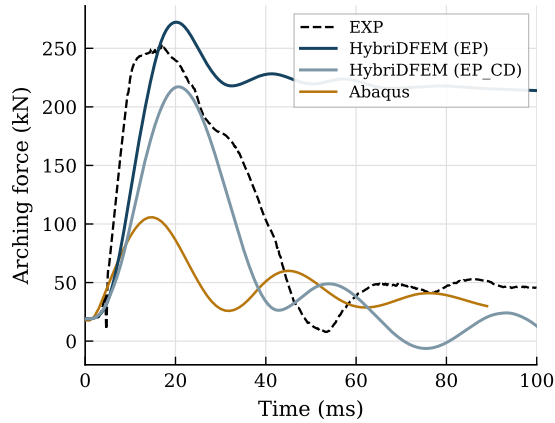


(a) Arching force.

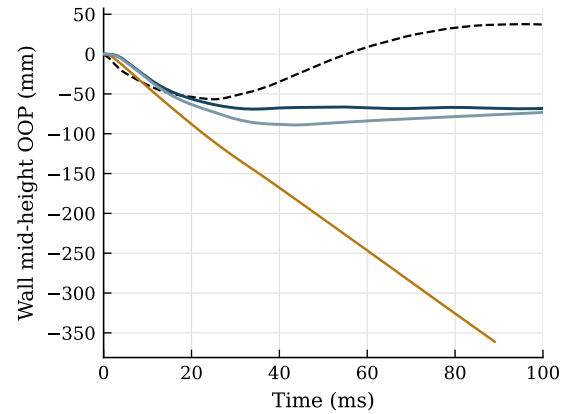


(b) Wall mid-height OOP.

Figure K.25: W34 ($m_{\text{imp}} = 236 \text{ kg}$, $v_0 = 10.10 \text{ m/s}$).



(a) Arching force.



(b) Wall mid-height OOP.

Figure K.26: W35 ($m_{\text{imp}} = 348 \text{ kg}$, $v_0 = 9.03 \text{ m/s}$).

K.5 Deformation snapshots

Snapshots are shown for four representative walls at $t = 1, 5, 10, 20, 40, 70, 100 \text{ ms}$. The walls span the low-energy (W23), moderate-energy (W31), moderate-high energy (W28), and high-energy (W33) regimes. For each wall, one strip is shown per model (EP, CD, AB). HybriDFEM blocks are coloured by out-of-plane displacement U_x [mm]; deformation is amplified by a factor of 4.

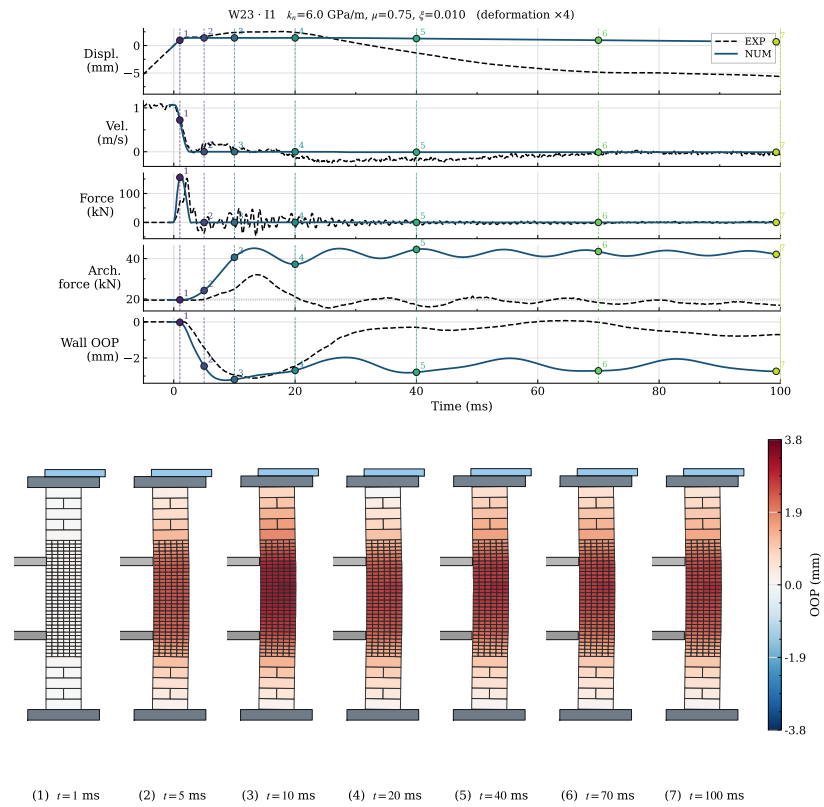


Figure K.27: W23 (EP) — deformation snapshots ($m = 236$ kg, $v_0 = 1.07$ m/s).

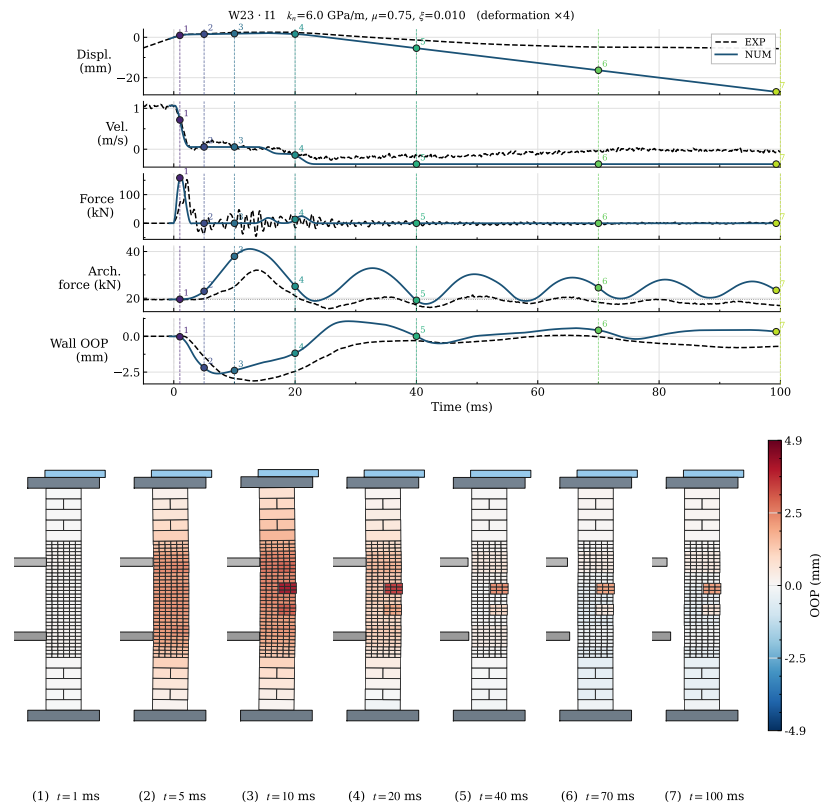


Figure K.28: W23 (EPCD) — deformation snapshots ($m = 236$ kg, $v_0 = 1.07$ m/s).

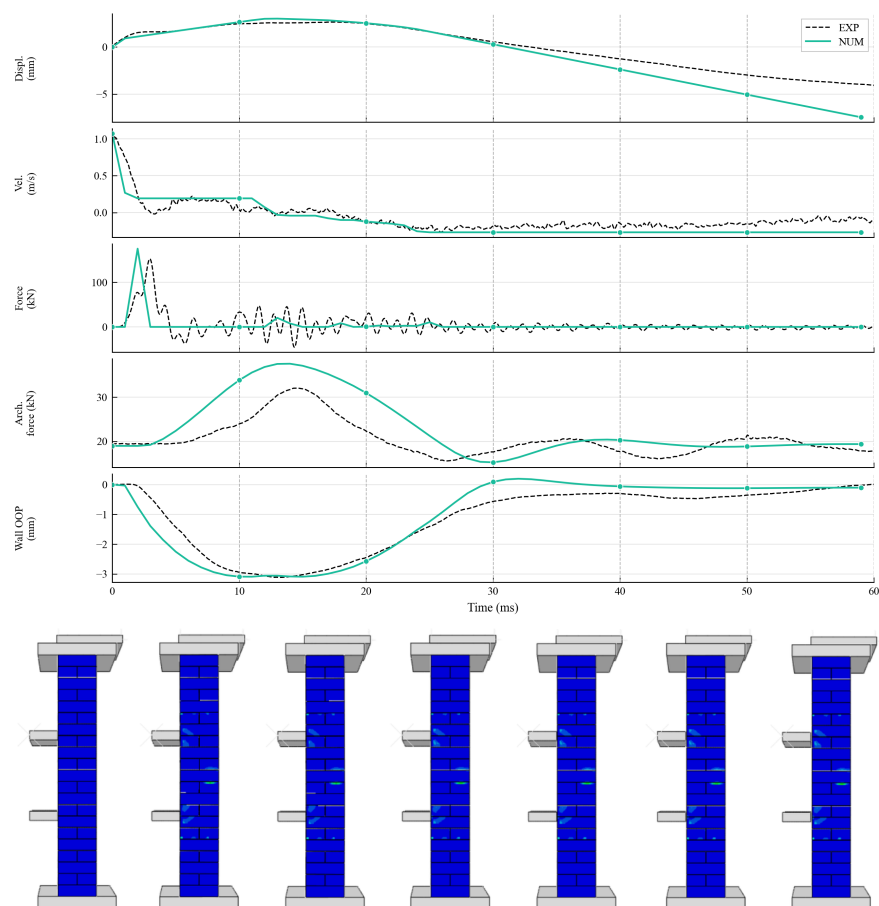


Figure K.29: W23 (AB) — Abaqus deformation response ($m = 236$ kg, $v_0 = 1.07$ m/s).

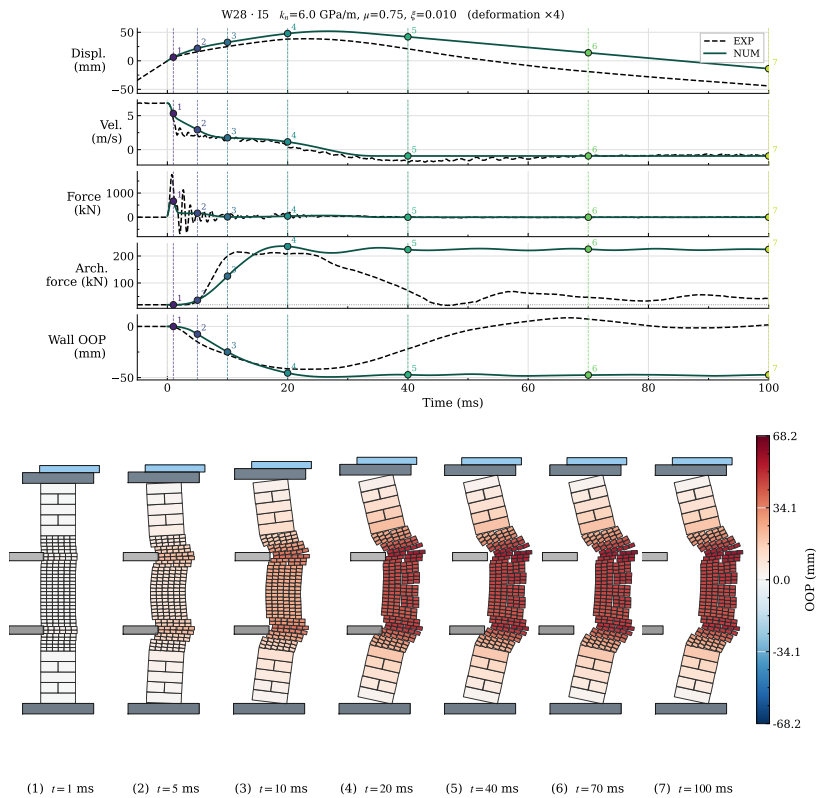


Figure K.30: W28 (EP) — deformation snapshots ($m = 348$ kg, $v_0 = 6.90$ m/s).

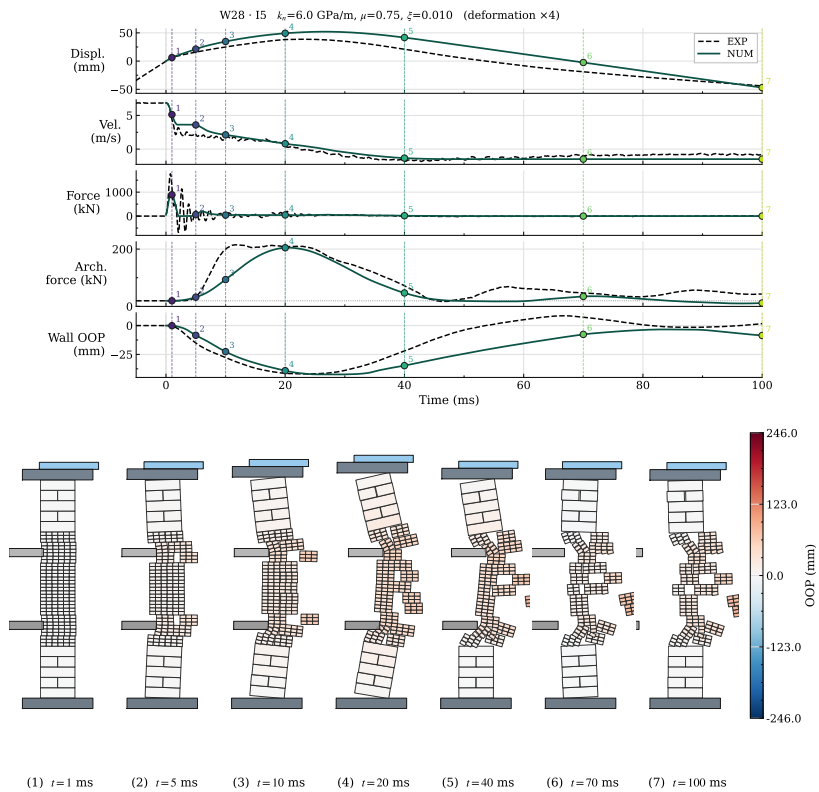


Figure K.31: W28 (EPCD) — deformation snapshots ($m = 348$ kg, $v_0 = 6.90$ m/s).

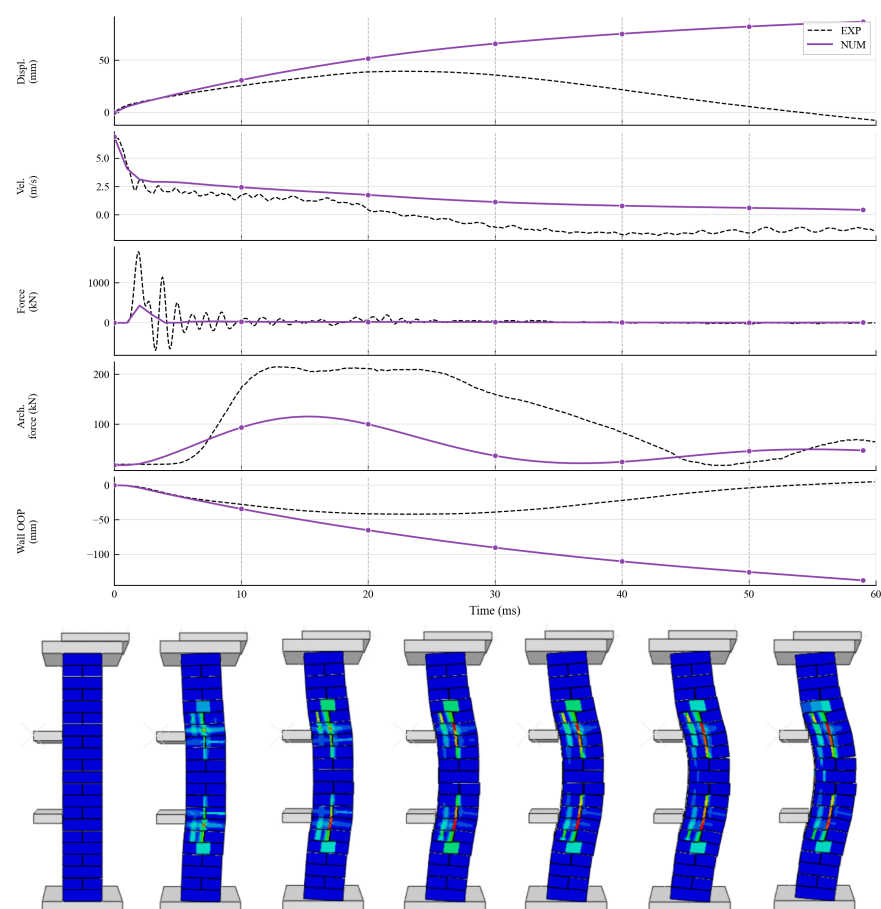


Figure K.32: W28 (AB) — Abaqus deformation response ($m = 348$ kg, $v_0 = 6.90$ m/s).

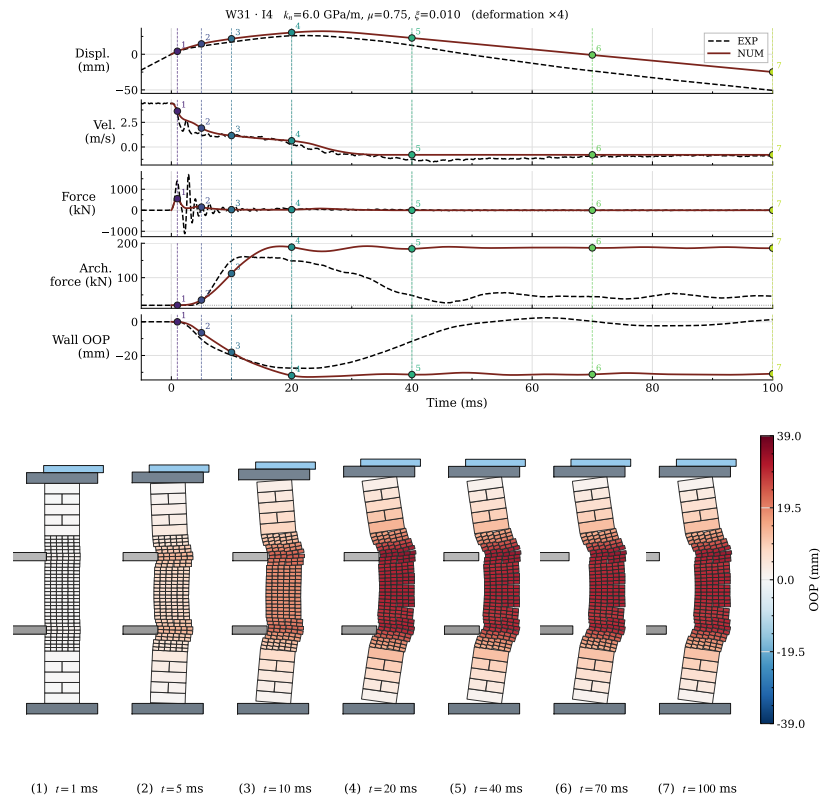


Figure K.33: W31 (EP) — deformation snapshots ($m = 442$ kg, $v_0 = 4.50$ m/s).

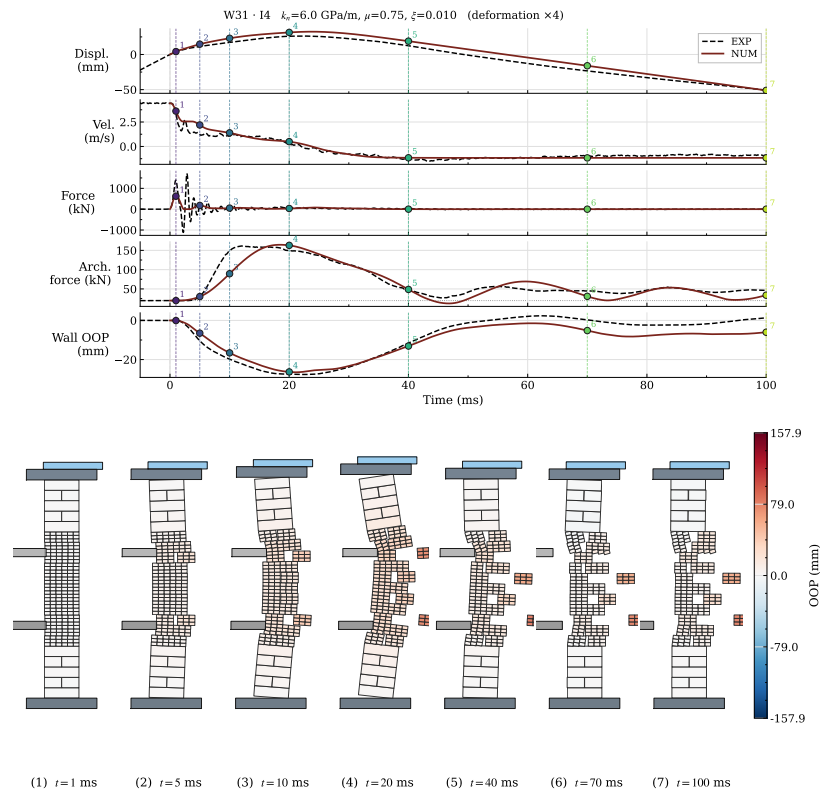


Figure K.34: W31 (EPCD) — deformation snapshots ($m = 442$ kg, $v_0 = 4.50$ m/s).

W31

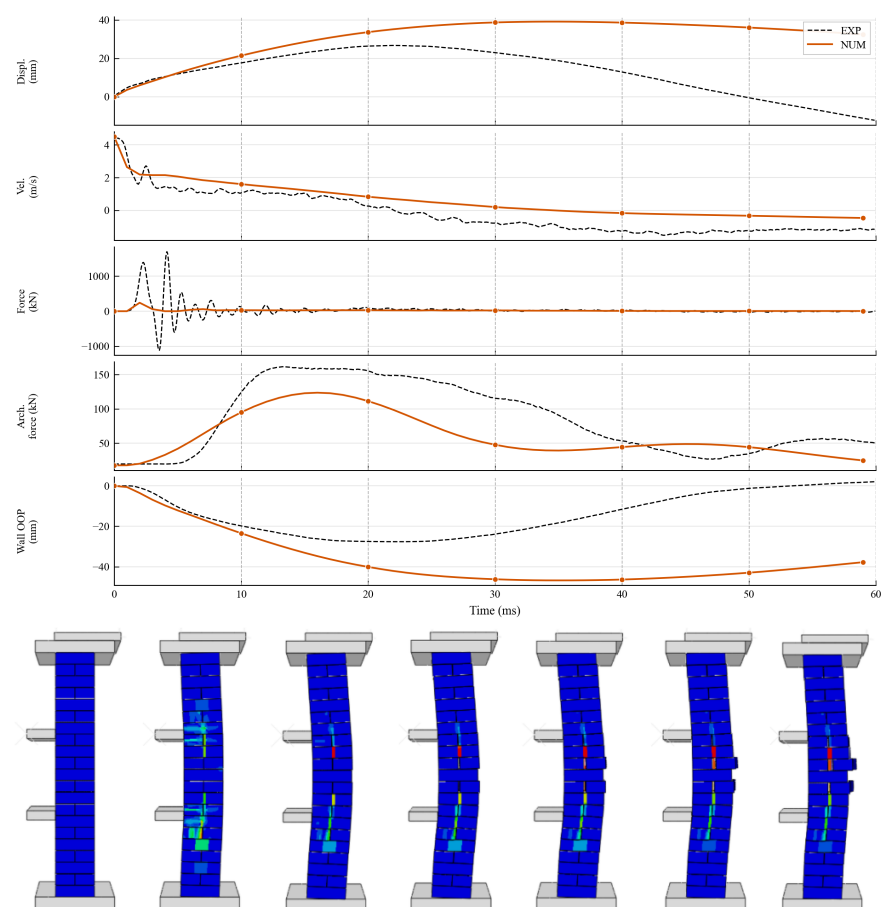


Figure K.35: W31 (AB) — Abaqus deformation response ($m = 442 \text{ kg}$, $v_0 = 4.50 \text{ m/s}$).

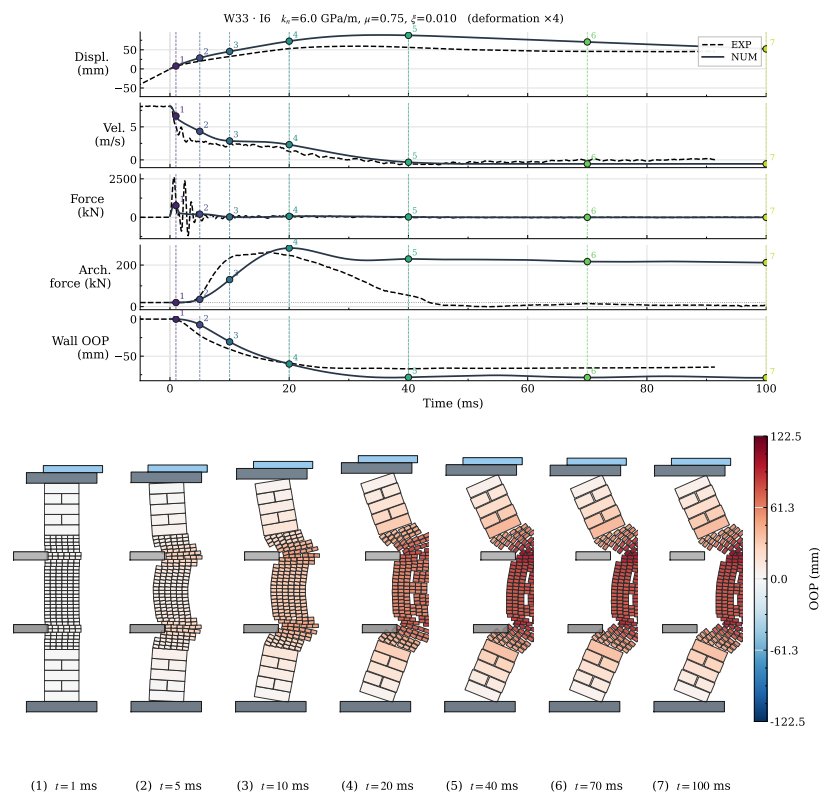


Figure K.36: W33 (EP) — deformation snapshots ($m = 442$ kg, $v_0 = 8.25$ m/s).

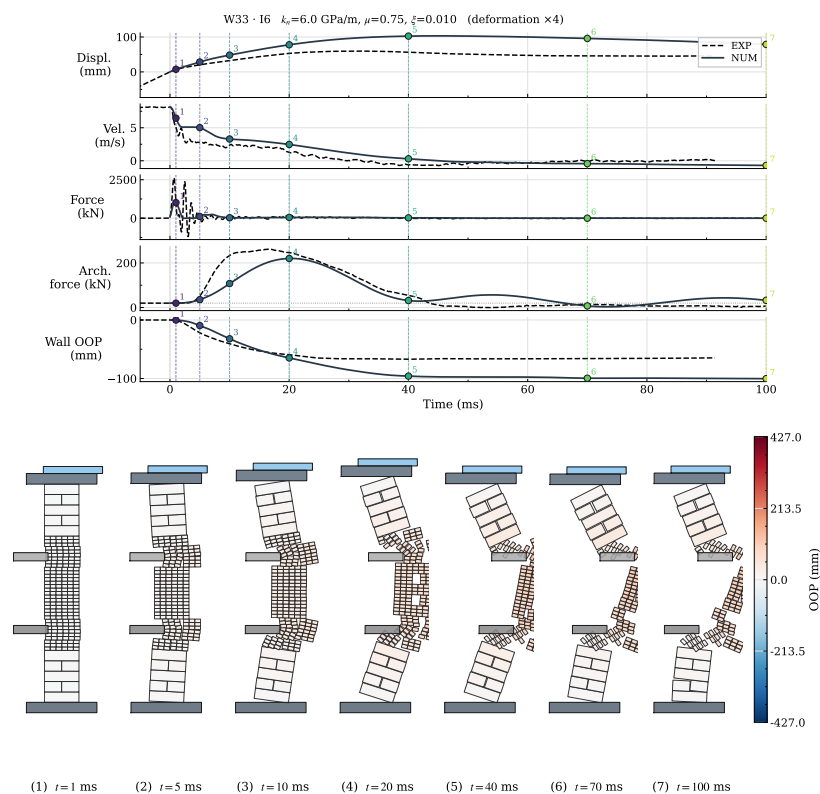


Figure K.37: W33 (EPCD) — deformation snapshots ($m = 442$ kg, $v_0 = 8.25$ m/s).

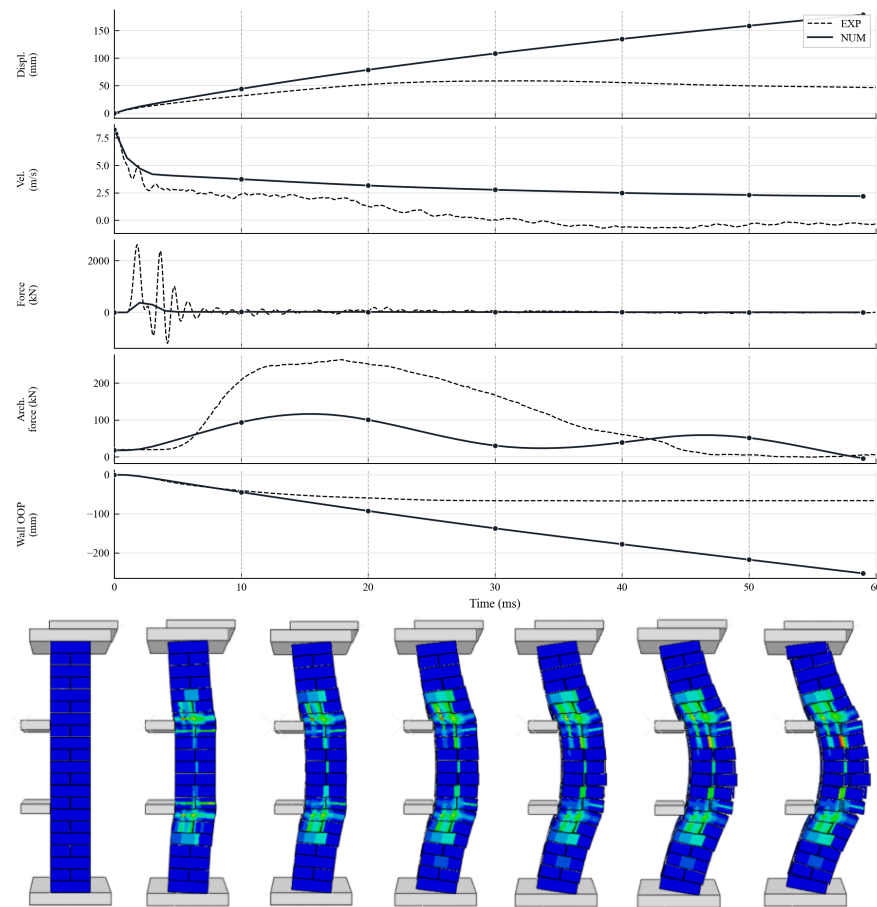


Figure K.38: W33 (AB) — Abaqus deformation response ($m = 442 \text{ kg}$, $v_0 = 8.25 \text{ m/s}$).

K.6 Damage and joint separation

Head-joint out-of-plane separation fields and Abaqus damage fields are shown at the time of peak impactor displacement for four representative walls: W23 (low energy), W28 (moderate-high energy), W31 (moderate energy), and W33 (high energy). HybriDFEM joints are coloured by $|\Delta U_x|$ [mm] from white (negligible) to dark green (maximum); bed joints are shown in grey.

HybriDFEM — ElastoPlastic

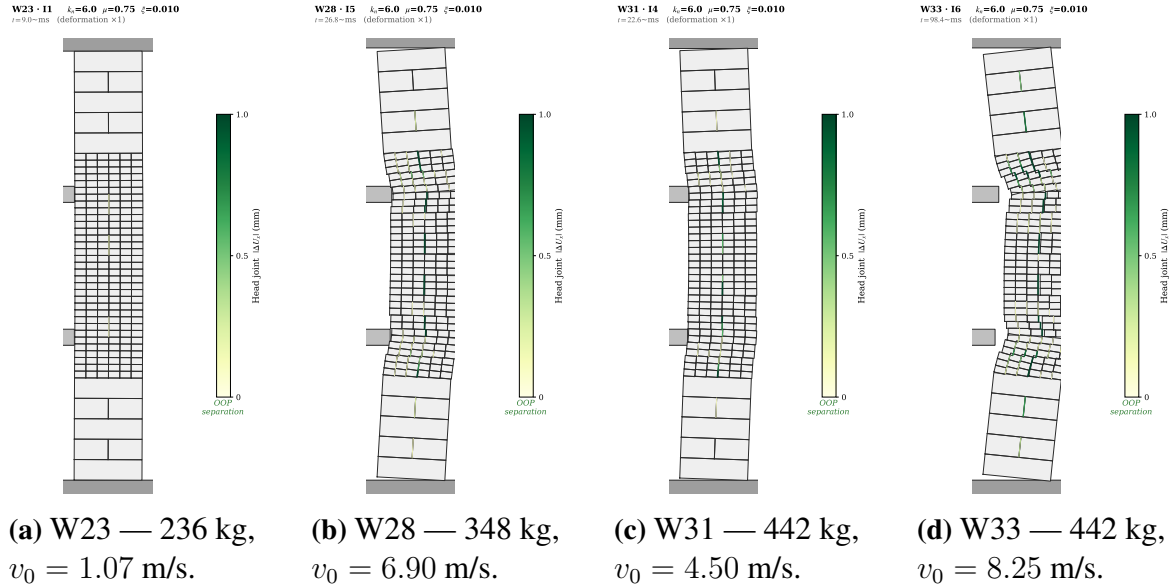


Figure K.39: HybriDFEM (ElastoPlastic) — head-joint out-of-plane separation $|\Delta U_x|$ at peak impactor displacement.

HybriDFEM — ElastoPlastic_CD

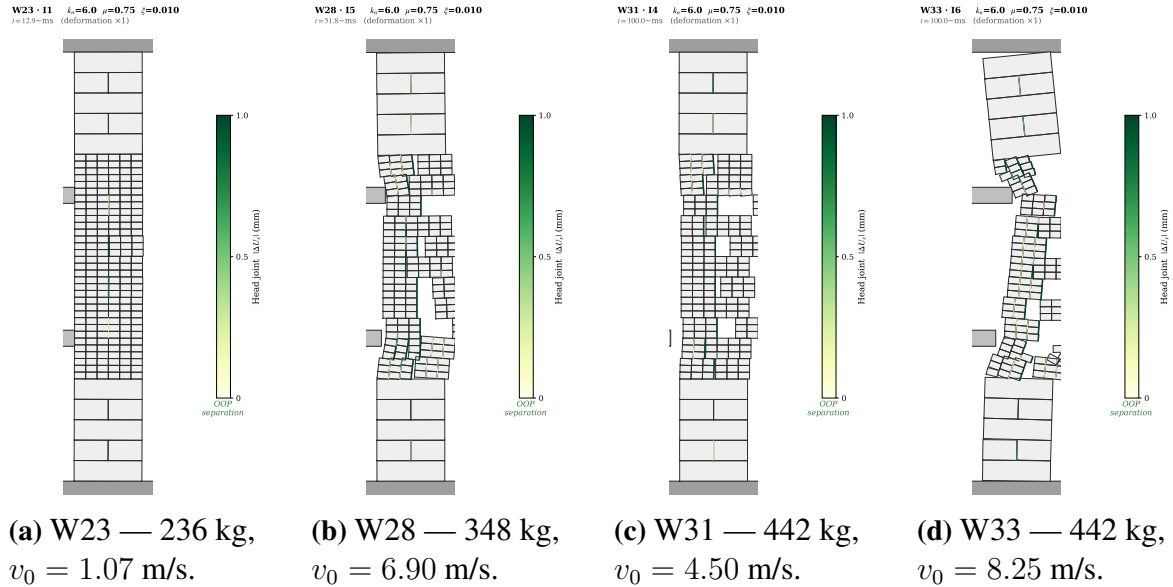


Figure K.40: HybriDFEM (ElastoPlastic_CD) — head-joint out-of-plane separation $|\Delta U_x|$ at peak impactor displacement.

Abaqus — Plastic strain

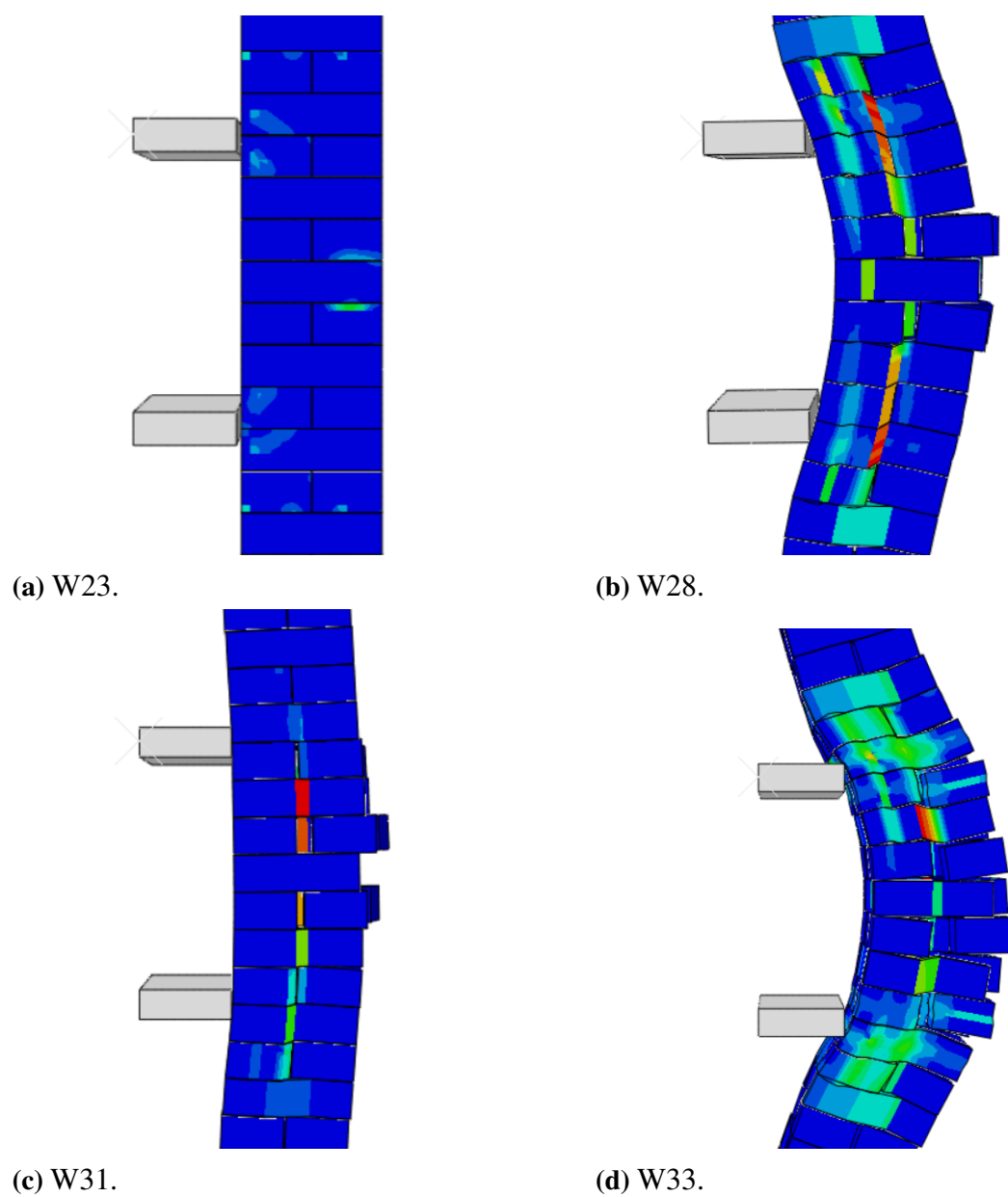
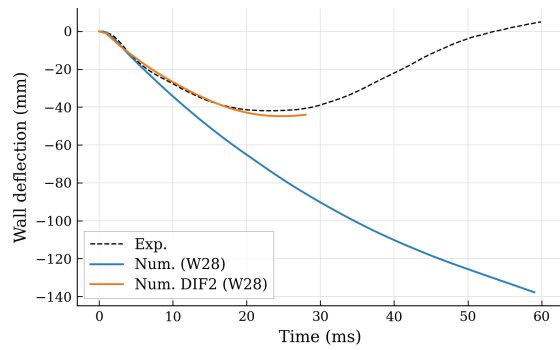


Figure K.41: Abaqus — plastic strain PE at peak impactor displacement for representative Campaign 3 walls.

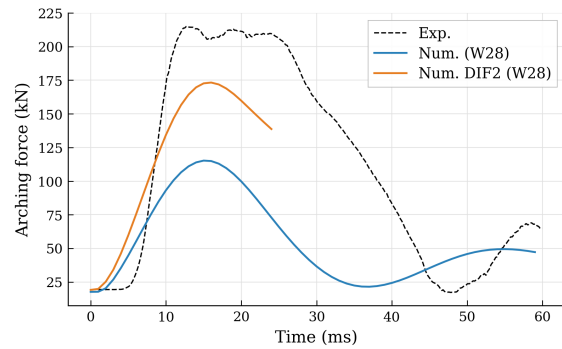
L Appendix L: Importance of dynamic increase factor

As discussed in Section 5.3, the absence of strain-rate dependent material strengthening is identified as the most likely dominant reason for the deterioration of Abaqus predictions at moderate to high impact energies. Under dynamic loading, both brick and mortar exhibit higher strength and stiffness than under quasi-static conditions, an effect quantified through the Dynamic Increase Factor (Hao & Tarasov, 2008). To assess the potential impact of this correction, a preliminary test was carried out on wall W28 ($m = 348$ kg, $v_0 = 6.90$ m/s) by applying a constant DIF = 2 to the tensile and compressive strengths of the brick material.

The DIF value was selected by trial, as the experimental strain rates for the impact campaigns are not available and the factor could not be derived rigorously from first principles. A value that produced results close to the experimental behaviour was identified, and the analysis was run over a shortened observation window sufficient to capture the key response quantities and verify the hypothesis. The results are shown in Figures L.1 and L.2.

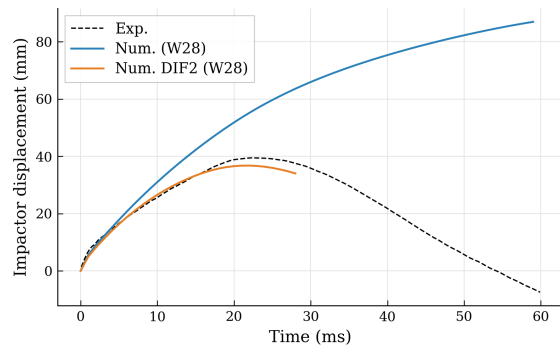


(a) Wall mid-height out-of-plane displacement.

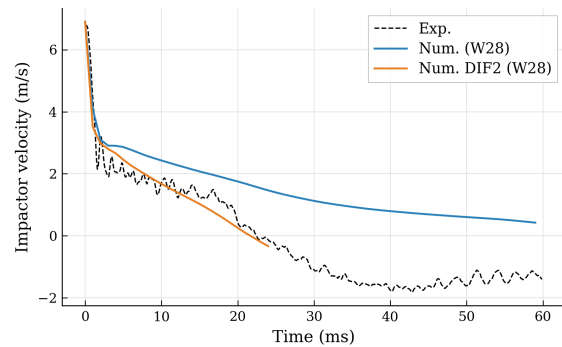


(b) Arching force.

Figure L.1: W28 — wall OOP displacement and arching force with and without DIF, compared against the experimental measurements (Godio & Flansbjerg, 2025b).



(a) Impactor displacement.



(b) Impactor velocity.

Figure L.2: W28 — impactor displacement and velocity with and without DIF, compared against the experimental measurements (Godio & Flansbjerg, 2025b).

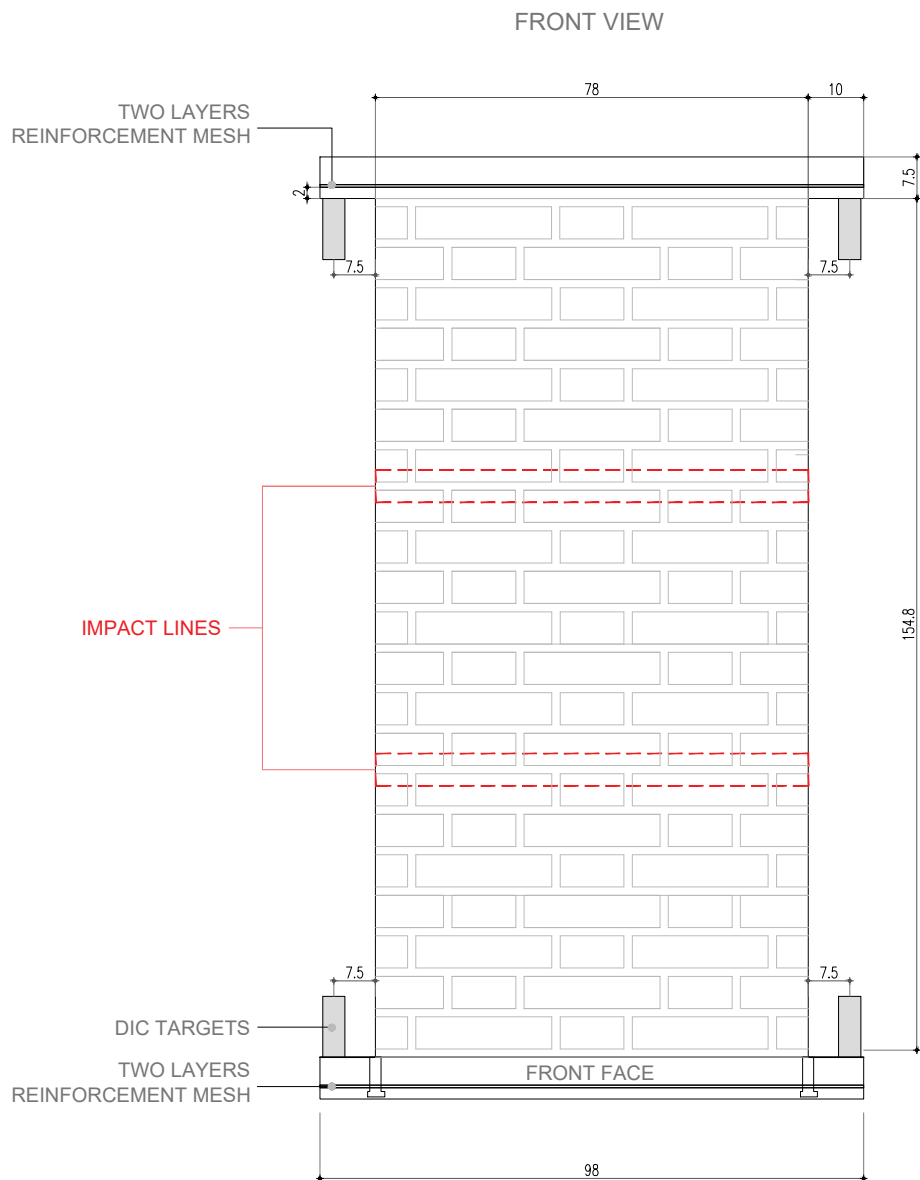
The results indicate a substantial improvement in the predicted response when the DIF is applied. The wall out-of-plane displacement, the arching force, and the impactor kinematics are all brought considerably closer to the experimental measurements, with the correct kinematic behaviour reproduced over the available observation window. This confirms that strain-rate strengthening is a physically relevant and likely dominant correction for the Abaqus framework at higher impact energies.

It should be noted, however, that the DIF value used here was identified by trial rather than computed from measured strain rates. The experimental strain rate histories for the impact campaigns were not available, which means the DIF cannot be derived rigorously from the empirical relations reported in the literature (Hao & Tarasov, 2008). Furthermore, the appropriate DIF is expected to vary between walls depending on the local strain rate distribution, which is a function of impact velocity, impactor mass, and wall geometry, so a single constant factor cannot be reliably applied across all configurations without dedicated calibration. If strain rate measurements were available, the DIF could in principle be computed directly from the empirical relations of (Hao & Tarasov, 2008) and applied to each wall configuration individually. A systematic DIF calibration based on measured strain rates and dedicated dynamic material testing is therefore identified as the most important direction for future work to extend the predictive capability of the Abaqus framework to higher energy impacts.

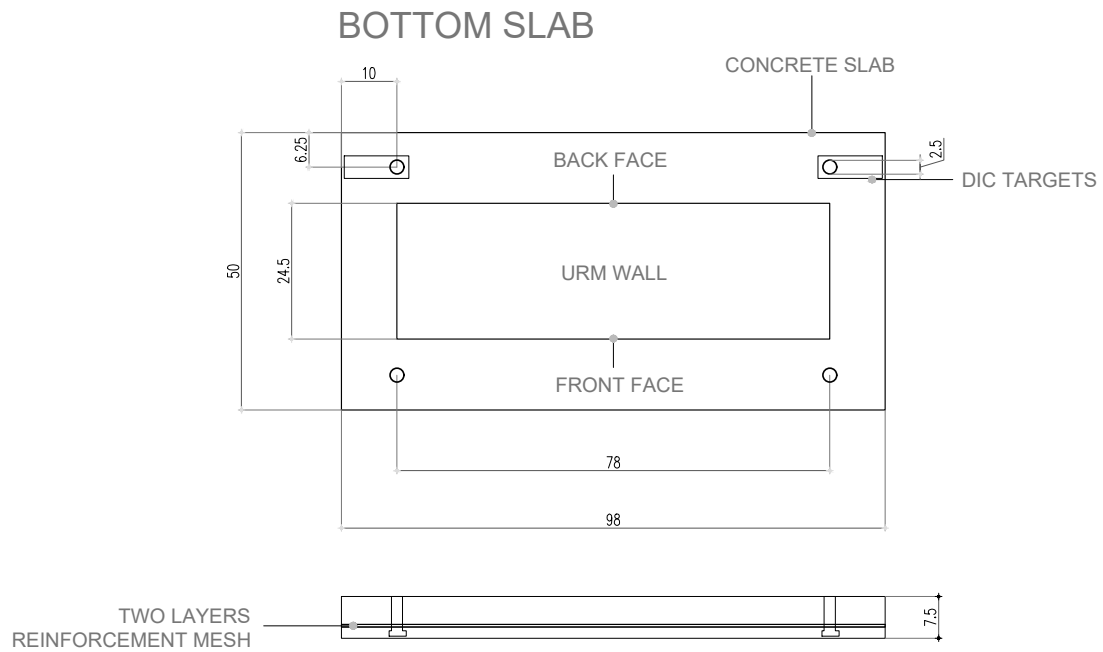
Appendix M: Experimental Setup Drawings

The following appendix contains the experimental setup drawings used to document the wall, reinforced concrete slabs, impactor configurations, and complete test setup.

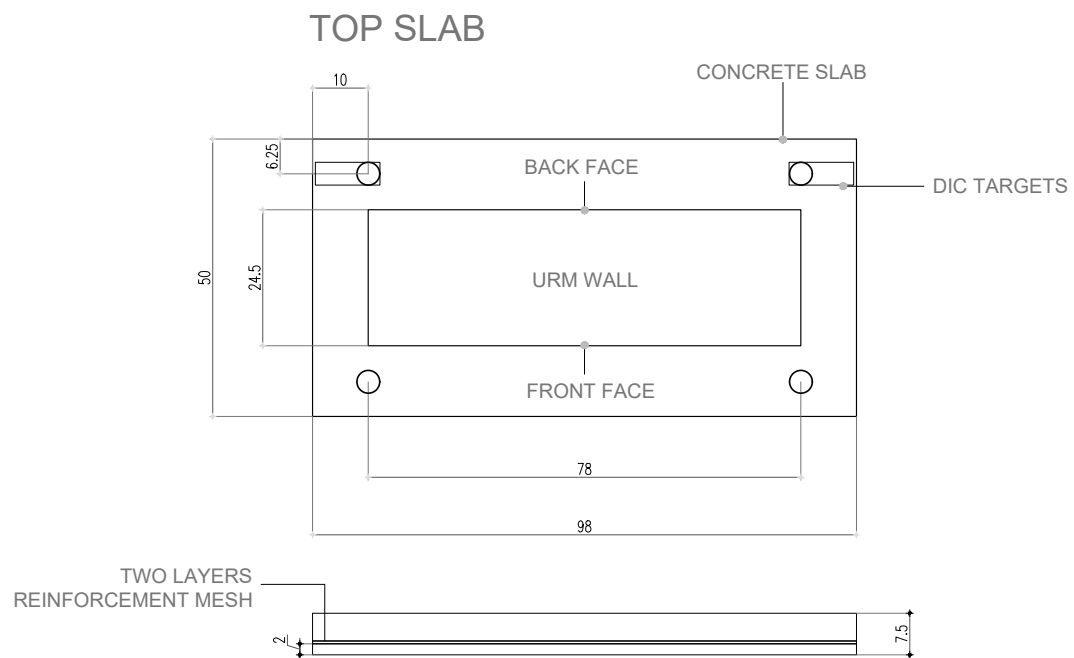
1. FRONT VIEW OF WALL



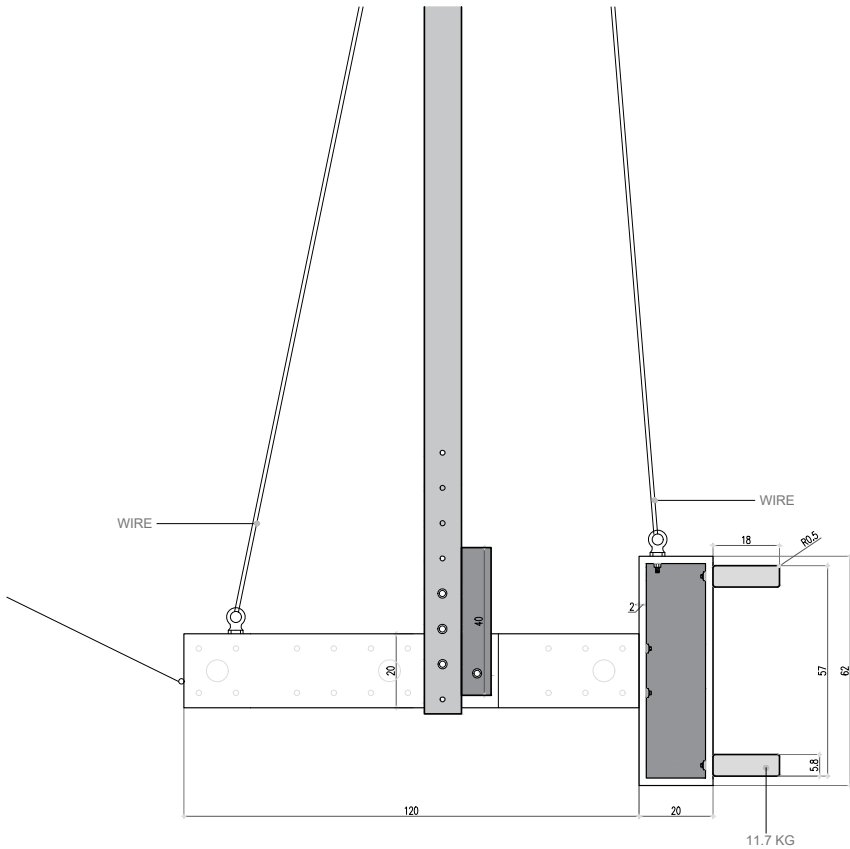
2. TOP AND SIDE VIEW OF BOTTOM RC SLAB



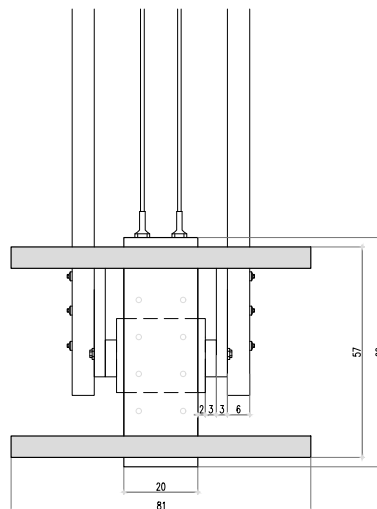
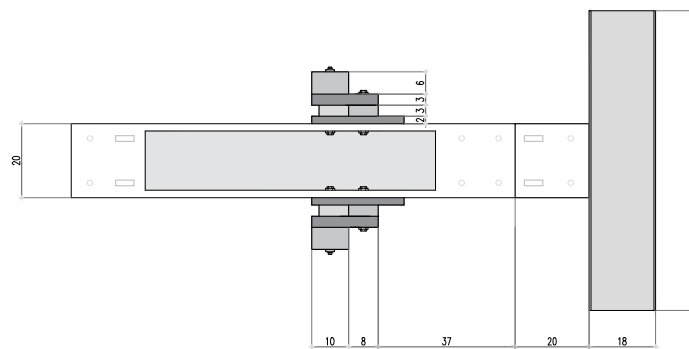
2. TOP AND SIDE VIEW OF TOP RC SLAB



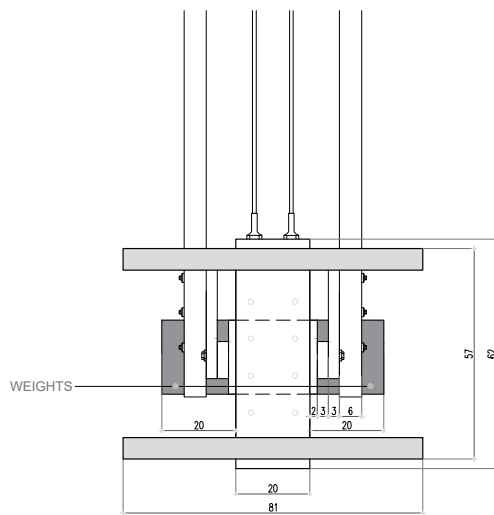
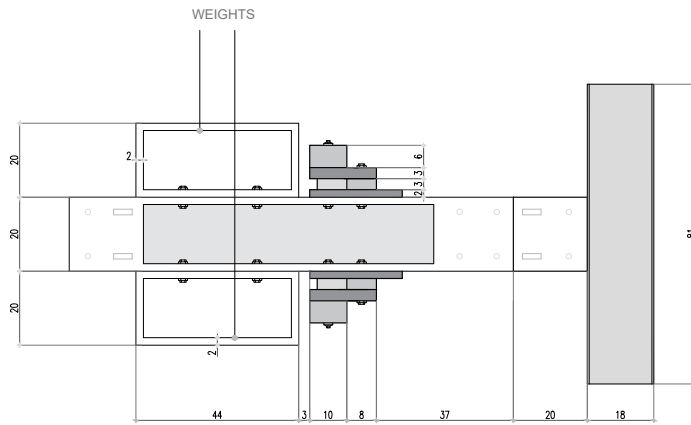
3. CONFIGURATION 1: SIDE VIEW OF IMPACTOR (NO WEIGHTS)



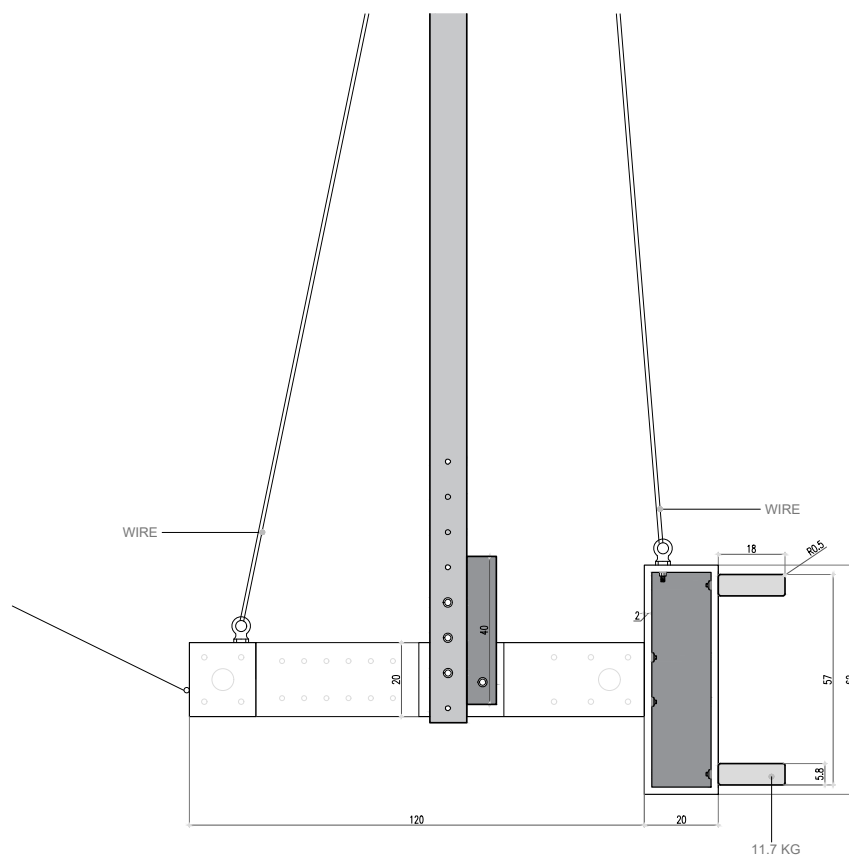
3. CONFIGURATION 1: FRONT AND TOP VIEW OF IMPACTOR (NO WEIGHTS)



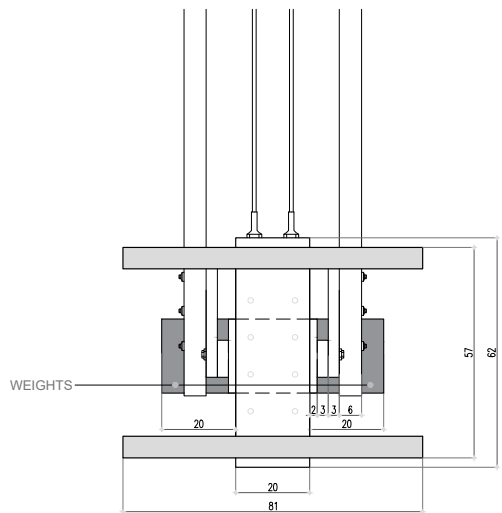
3. CONFIGURATION 2: FRONT AND TOP VIEW OF IMPACTOR



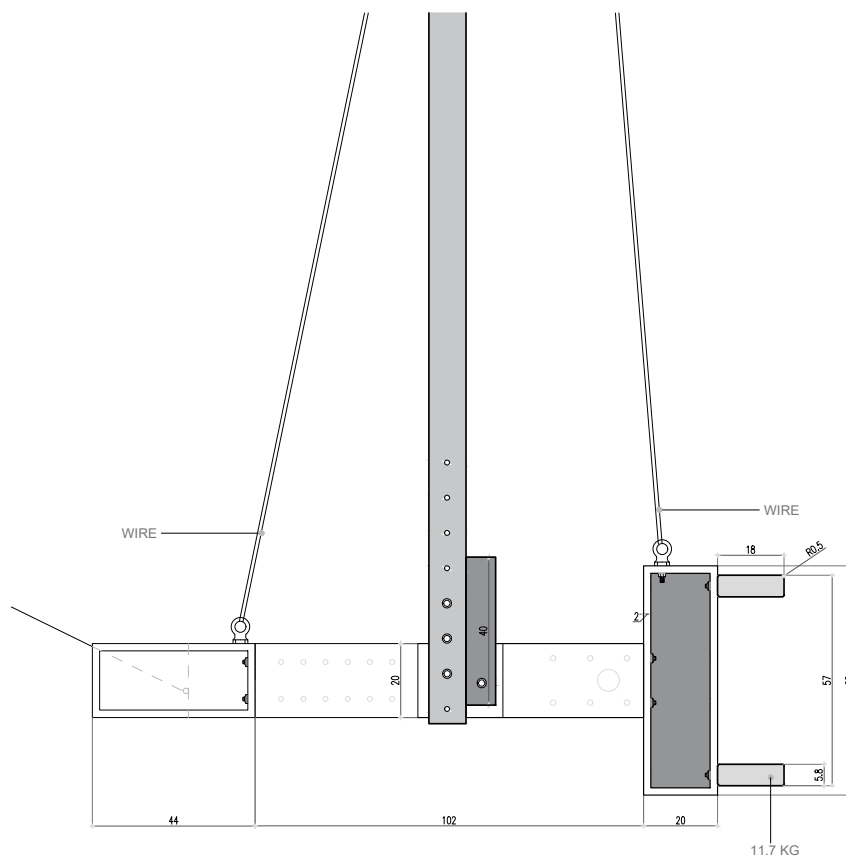
3. CONFIGURATION 2: SIDE VIEW OF IMPACTOR



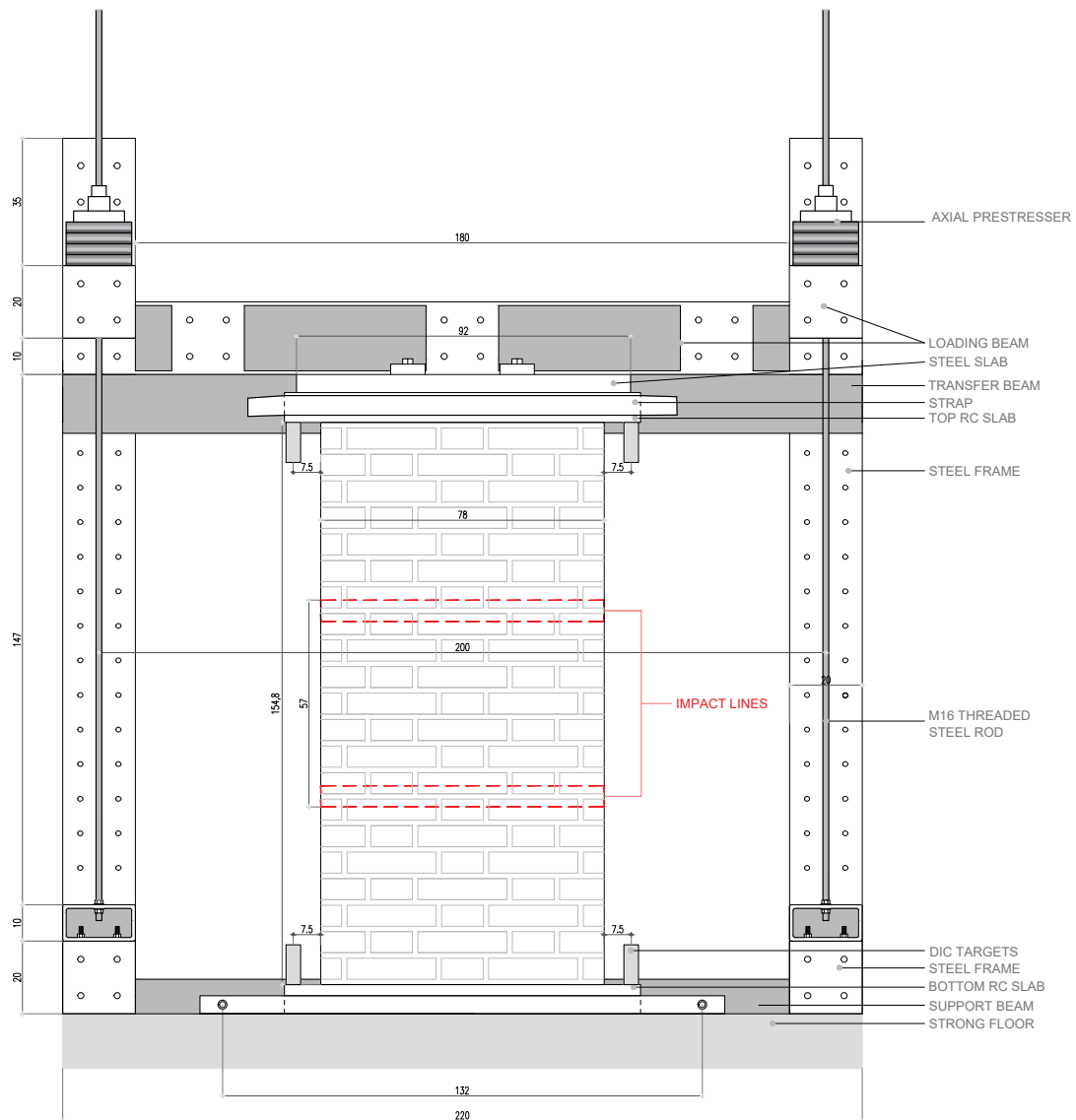
3. CONFIGURATION 3: FRONT AND TOP VIEW OF IMPACTOR



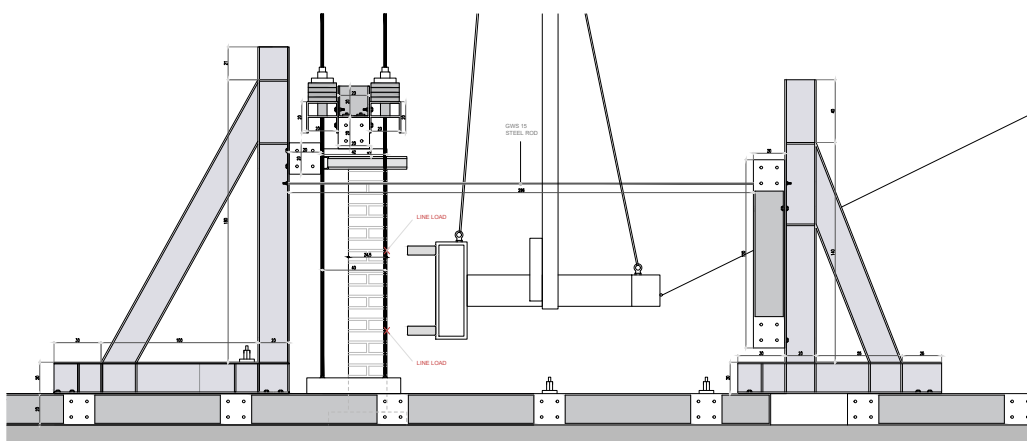
3. CONFIGURATION 3: SIDE VIEW OF IMPACTOR



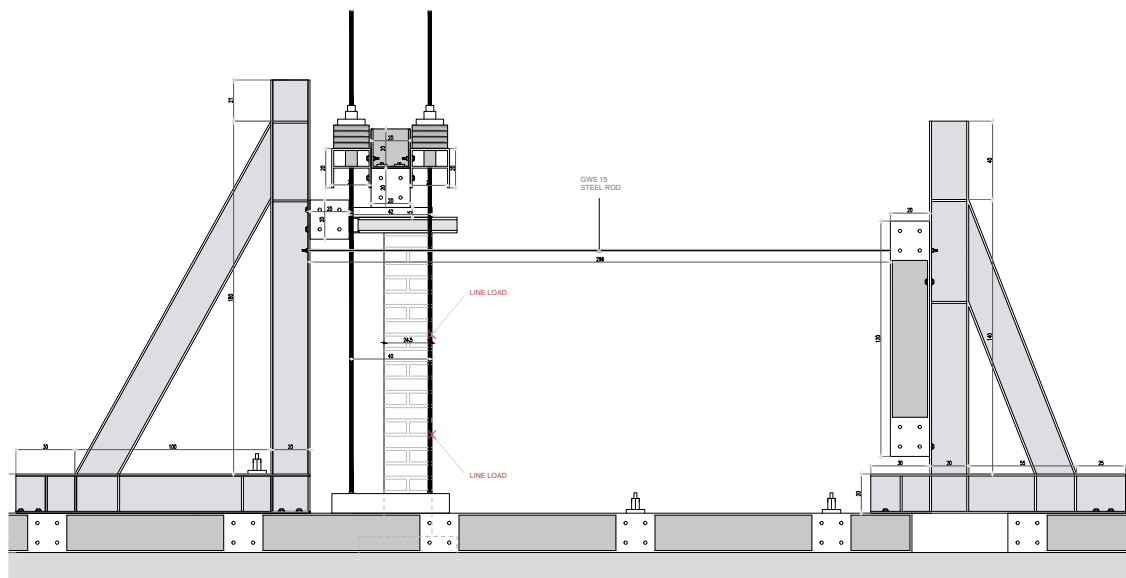
4. FRONT VIEW OF ENTIRE SETUP



4. SIDE VIEW OF ENTIRE SETUP



5. FULL SETUP (WITHOUT IMPACTOR)



IMPACTOR X WALL (IMPACT CONFIGURATIC

