CHALMERS
UNIVERSITY OF TECHNOLOGY

Modeling Interest Rate Risk in Swedish Mortgages Using Survival Analysis

Master's thesis in Engineering Mathematics and Computational Science

Simon Rödén

DEPARTMENT OF MATHEMATICS

CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2026
www.chalmers.se

Master's thesis 2026

Modeling Interest Rate Risk in Swedish Mortgages Using Survival Analysis

Simon Rödén

Department of Mathematical Sciences
Chalmers University of Technology
Gothenburg, Sweden 2026

Modeling Interest Rate Risk in Swedish Mortgages Using Survival Analysis
Simon Rödén

Examiner and supervisor: Holger Rootzén, Department of Mathematical Sciences at Chalmers
Supervisor: Jakob Lavröd, Senior Quantitative Analyst at Handelsbanken

Master's Thesis 2026
Department of Mathematical Science
Engineering Mathematics and Computational Science
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

Cover:

Typeset in LaTeX
Printed by Chalmers Reproservice
Gothenburg, Sweden 2026

Modeling Interest Rate Risk in Swedish Mortgages Using Survival Analysis
Simon Rödén
Department of Mathematical Science
Chalmers University of Technology

Abstract

This thesis investigates interest-rate risk in Swedish adjustable-rate mortgages by combining survival analysis with mortgage valuation techniques. The objective is to estimate the behavioral maturity of mortgage loans and model interest rate sensitivity. Using mortgage data from Handelsbanken, mortgage prepayment behavior is modeled through parametric survival analysis, with particular emphasis on shifted gamma frailty models to capture unobserved heterogeneity among borrowers. A mortgage-rate forecasting model is developed to describe the relationship between mortgage rates and risk-free market rates for a banking industry average.

The estimated survival model is integrated with a valuation framework to determine effective mortgage maturity, duration, and portfolio sensitivity under various interest-rate environments. Results show that Swedish three-month adjustable-rate mortgages exhibit longer behavioral maturities. This is driven by incomplete pass-through of market interest rates to mortgage rates and persistent borrower behavior.

Stress-testing based on Basel-inspired interest-rate shock scenarios demonstrates that mortgage portfolios are particularly vulnerable to rising interest rates. The findings indicate that financing adjustable-rate mortgages exclusively with short-duration liabilities may underestimate the true level of interest-rate risk. This highlights the importance of incorporating behavioral maturity into asset–liability management and provides a practical way for evaluating mortgage-related interest-rate risk in the Swedish banking sector.

Keywords: interest-rate risk, Swedish mortgages, survival analysis, frailty models, behavioral maturity

Acknowledgements

I would like to thank my supervisor, Jakob Lavröd, at Handelsbanken, and everyone at Group Risk Control for their support, helpful discussions, and for making the project an enjoyable experience. I would also like to thank my supervisor and examiner, Holger Rootzén, at Chalmers University of Technology for their guidance and valuable feedback throughout the thesis.

Simon Rödén, Gothenburg, June 2026

Nomenclature

Below is the nomenclature for indices, sets, parameters, and variables used throughout this thesis.

Parameters

t	time
τ	time to maturity
$P(\tau)$	present value
$C(t)$	Cash flow at time t
$r_d(t)$	Discount rate at time t
$r_f(t)$	Risk free rate at time t
D_t	Duration in years at time t
λ	individual baseline hazard rate
a	$\frac{1}{a}$ is the variance of the gamma distribution
ω	Parameter for shiftiness
θ	coefficient for proportional hazard
β_1	Passthrough
β_0	Mortgage spread component
K	The floor of the interest rate
$S(t)$	survival function
$h(t)$	hazard function
$H(t)$	cumulative hazard function
w_t	Weight given time t
$L_f(t)$	Laplace transform
m	Commercial margin
$g(r_f(t))$	Pricing function for mortgage rate given risk free rate
r	risk-free interest rate

Terms

Maturity	Time until full repayment
Duration	Weighted average time of repayments
Present Value	How much the asset or liability is currently worth
floating interest rate	an interest rate that resets periodically
churn	The rate at which customers stop using a product, in this case mortgages

Contents

List of Figures

List of Tables

1

Introduction

This chapter outlines the background, problem statement, purpose, and research questions of the thesis. It also details the limitations, provides an overview of the methodology, and outlines the structure.

1.1 Background

Banks manage a large portfolio of loans whose value fluctuates with changes in the risk-free interest rate. They also obtain financing with varying sensitivities to interest rate movements. The mismatch between the interest rate sensitivities of loans and financing creates interest rate risk. This thesis analyzes interest rate risk using survival analysis applied to mortgage loan data from Handelsbanken.

Approximately 60% of all bank lending in Sweden is extended to households [1], with the remainder going to corporations. Of these, 80% are mortgage loans [2], meaning that almost half of all loans issued by banks in Sweden are mortgage loans. 75% of those are 3-month adjustable-rate mortgage loans [2]. These mortgage loans are important for the entire Swedish financial system. Therefore, understanding the interest rate risk of these loans is crucial.

In 2023, the United States faced a significant banking crisis caused by interest rate hikes intended to curb inflation. The largest collapse was Silicon Valley Bank [3]. In contrast to other banking collapses, the bank didn't hold subprime or other bad debt; instead, it held US Treasury bills, which are loans to the US government and are commonly considered the safest assets. The bank had invested in long US Treasuries with fixed payments. This meant that when interest rates rose, the US Treasuries fell in value on the secondary market because their interest rates were fixed at low levels. However, their deposits were not fixed at the same time frame. Because of rising interest rates and competition, the bank was forced to raise deposit rates but couldn't raise rates on its long-term loans. Facing a crisis, depositors withdrew their money, and to meet its obligations, the bank was forced to sell long-term US Treasuries at a loss. This ultimately led to the bank's collapse, leaving it unable to cover all its depositors. This underscores the importance of managing interest-rate risks.

1.1.1 Mortgages

A mortgage is a loan secured by property [4]. It is a common arrangement, but its implementations vary widely worldwide. Sweden's mortgage market differs from those of other countries in several respects. In particular, it combines adjustable-rate mortgages with strong banking competition between banks. Swedish adjustable-rate mortgages reset every three months, so these loans should track the risk-free three-month rate plus a premium, as banks do not lend without compensation. However, this is not true in practice; changes in the risk-free interest rate are not fully passed on to the mortgage interest rate. The customer's mortgage rate is generally determined by the bank's listed mortgage rate and an individual discount.

The customer has a mandatory contractual amortization schedule determined by the mortgage loan size, the underlying property's value, and income. At the highest contractual amortization rate, the customer must amortize 3% of the mortgage loan; if the mortgage loan is below 50% of the property value, the customer doesn't have to amortize at all. However, the customer can choose to prepay, amortizing more than required for the 3-month adjustable mortgages. The customer can also change the interest rate term; for example, setting it to 12 months will apply a different interest rate. In addition, when a customer has a longer-term mortgage, they must pay the difference in mortgage interest if they prepay the loan. However, they will not be compensated when the mortgage interest rate falls [4].

Most research on behavioral interest rate risks has focused on the deposit side of banking or on mortgage markets in other countries with different regulatory frameworks. A study in the Swedish context could therefore help mitigate future risks in both the mortgage market and the banking sector as a whole.

1.2 Problem Statement

Each mortgage loan doesn't have a maturity date or a remaining term until repayment. Effective duration, by contrast, is the weighted-average time of all payments and measures the loan's sensitivity to interest rate changes. To fund these loans, the bank assumes liabilities, such as deposits, which also have varying effective durations. The difference between the effective durations of assets and liabilities represents the bank's interest rate risk.

Because of the gap between the three-month interest rate and the three-month mortgage rate, there exists an interest rate risk. A bank shouldn't finance these loans solely with 3-month-term liabilities. However, the behavioral maturity of a mortgage is unknown because a customer has no obligation to prepay their loan after a certain period.

1.3 Purpose and Research Questions

The purpose of this thesis is to provide banks with a better understanding of their interest rate risk, particularly that on mortgage loans. Another purpose is to improve the modeling of different scenarios for changes in risk-free interest rates, known as stress tests, by modeling mortgage loans.

The objectives are to develop a model of mortgage prepayments using survival analysis and to investigate how interest rates primarily affect prepayment behavior. In this thesis, the following research questions are to be investigated:

- How can mortgage interest rates be forecasted?
- How can survival analysis be used to model prepayments for deciding the average observed maturity of a mortgage?
- What are the interest rate risks for a portfolio of mortgage loans?
- How do different interest environments affect prepayment behavior?

1.4 Scope and Limitations

This thesis focuses on interest-rate risk in Swedish mortgage loans, using mortgage data from Handelsbanken and publicly available data from all banks. The analysis is restricted to mortgage observations between 2018 and 2026, although the dataset contains loans with origination dates as early as 2010. Consequently, the study is subject to left truncation, as loans that originated and terminated before 2018 are not observed. Many loans are also right-censored because they remain active at the end of the observation period.

The analysis is performed at the portfolio level and does not model individual borrower characteristics such as income, age, geographic location, or loan-to-value ratios. Instead, mortgage behavior is represented through aggregate survival models. The objective is therefore to estimate portfolio-level behavioral maturity rather than predict the behavior of individual borrowers.

1.5 Thesis Structure

The objective of this thesis is to estimate the behavioral maturity of Swedish adjustable-rate mortgages and assess the resulting interest-rate risk. To achieve this, the analysis combines survival analysis with a valuation model for mortgage loans.

First, the mortgage loan data are transformed to be suitable for survival analysis. A parametric survival model is then estimated to describe mortgage prepayment behavior. In addition, a mortgage-rate forecasting model is estimated to describe the relationship between mortgage interest rates and risk-free interest rates.

The behavioral maturity and mortgage-rate models are combined to value the portfolio. This is used to estimate present values, effective durations, and the sensitivity of the mortgage loan portfolio to different interest-rate shock scenarios.

The thesis is organized as follows. Chapter 2 presents the theory, including interest rate risk, survival analysis, frailty models, and stress testing. The data and their handling are described in Chapter 3. Chapter 4 presents the empirical results, while Chapter 5 presents the findings, limitations, and practical implications. Finally, Chapter 6 concludes the thesis with the final remarks.

2

Theory

This chapter introduces the theory used throughout the thesis. First, valuation and measures of interest-rate risk are presented. Survival analysis is then introduced for modeling mortgage prepayments. Finally, frailty and competing-risk models are presented to capture unobserved variability in borrower behavior and distinguish between different mortgage prepayment outcomes.

2.1 Financial theory

A mortgage loan portfolio can be partitioned into different parts. For this reason, all mortgage loans end after a certain period. In a portfolio, the mortgage interest will be the average across all loans, not the individual rate for each customer. Each time-based partition is treated as a monthly interest-bearing loan, with the principal repaid at the end. The aim is not to model the exact amortization schedule, but to capture the interest-rate sensitivity arising from borrower prepayment behavior. Thus, all future payments should be discounted to present value at a discount rate $r_d(t)$ at time t. Discounting is performed using continuous compounding for analytical convenience, meaning that a discounted payment is $e^{-t \cdot r_d(t)}$ instead of nominal discount $(1+r_d(t))^{-t}$. Continuous compounding simplifies analytical derivations and aligns with the European Central Authority's regulatory practice [5]. Cash flows are modeled monthly, while the discount rate is expressed monthly.

The present value $P(\tau)$ of the mortgage loan with maturity τ is obtained by discounting all future payments $C(t)$ at times t back to the present [6]. The equation for present value is

$$P(\tau) = \sum_{t=1}^{\tau} C(t) \cdot e^{-t \cdot r_d(t)}.$$

Normalizing by the size of the initial loan, and considering a loan with monthly mortgage interest payments and repayment of the initial loan at maturity τ. The mortgage rate gets transformed with $\exp(r(t)) - 1)$, with a mortgage interest rate of $r(t)$ in months, giving a cash flow of

$$C(t) = \begin{cases} r(t) & \text{if } t \neq \tau \\ (1 + r(\tau)) & \text{if } t = \tau \end{cases}$$

results in the present value of a partition. This gives the normalized present value equation for a mortgage loan if the customer pays back their entire loan after τ months:

$$P(\tau) = \sum_{t=1}^{\tau} r(t) \cdot e^{-t \cdot r_d(t)} + e^{-\tau \cdot r_d(\tau)}.$$

This framework can be extended to model a portfolio of mortgage loans with different final maturities τ and amortization schedules. $P(\tau)$ is the present-value component of the portfolio, which matures at τ. For the entire portfolio, all different $P(\tau)$ values need to be summed to obtain P_{total}. However, the volume that ends at τ is determined by a weight w_T which are $1 = \sum_{\tau=1}^{\infty} w_\tau$. This gives the

$$P_{total} = \sum_{\tau=1}^{\infty} P(\tau) \cdot w_\tau.$$

The present value for all loans that end at different times is multiplied by the probability of ending at that time. The weights w_τ are controlled by amortizations. In the method, there will be a discussion on how amortization on parts of the loan works; however, this framework can still be used. It also requires knowledge of the future mortgage interest rate, $r(t)$, and the weights w_τ.

2.1.1 Risk-free interest rate

To ensure a consistent valuation framework, a risk-free interest rate is used. Government bond yields are commonly used as a proxy for the risk-free rate, as government debt is generally considered to have very low credit risk. The valuation of mortgage portfolios depends on future interest rates because floating-rate mortgage cash flows are linked to short-term market rates. In Sweden, adjustable-rate mortgages reset every three months and are therefore closely connected to short-term risk-free interest rates. However, mortgage rates do not move one-to-one with market rates, creating behavioral and valuation effects that contribute to interest-rate risk.

An interest rate swap is an agreement between two parties to exchange future interest payments. This gives the forward interest rate, meaning the implied future interest rate. The forward interest rates given by the swap rates are used as market-consistent estimates of future short-term interest rates $r_f(t)$ at time t. To discount payments, the discount rate $r_d(t)$ should be used rather than the forward rate. The discounting process involves taking the payments and discounting them using all the forward rates up to that point

$$e^{t \cdot r_d(t)} = e^{\int_0^t r_f(u) du},$$

which corresponds to

$$r_d(t) = \frac{1}{t} \int_0^t r_f(u) du.$$

The forward curve is used to project future mortgage interest rates, while discount rates are used to convert future cash flows into present value. Although both are derived from the same underlying interest rate curve, they serve different purposes in valuation.

2.1.2 Discounting margin

Discounting mortgage cash flows using only the risk-free interest rate would result in a present value greater than the original loan amount. This is because mortgage rates are usually higher than risk-free rates, as banks charge extra interest to compensate for the risks and costs of lending. Since the objective of this thesis is to study interest-rate sensitivity rather than excess profitability, a constant discounting margin m is introduced into the discount curve:

$$P(\tau) = \sum_{t=1}^{\tau} C(t) \cdot e^{-t\cdot(m+r_d(t))}.$$

The margin m is calibrated so that the mortgage portfolio has a present value of 1 under the initial interest-rate environment, so that $P(\tau) = 1$. This isolates interest-rate risk, ensures comparability across scenarios, and is consistent with accounting principles. The m is constant at the initial time, but the present value can change depending on the risk-free interest rate after initialization.

2.1.3 Effective Duration

Interest-rate risk arises because the value of future mortgage cash flows depends on market interest rates. A common measure of this sensitivity is effective duration. For fixed-rate instruments with deterministic cash flows, effective duration can be interpreted as the weighted-average timing of payments. However, Swedish adjustable-rate mortgages have behavioral uncertainty because borrowers may prepay, refinance, or change interest-rate terms, which will affect the weights in the portfolio's total present value. The timing and size of future cash flows are therefore dependent on the interest-rate environment itself.

Effective duration D measures the sensitivity of the portfolio value to changes in the entire interest-rate curve while allowing cash flows and expected maturities to change as interest rates change [6]. ΔP_{total} is the change of the present value P_{total} given a small change Δr in risk free interest. This expression approximates the proportional change in present value for a small parallel shift in interest rates:

$$\frac{\Delta P_{total}}{P_{total}} \approx -D \cdot \Delta r.$$

A higher effective duration implies greater sensitivity to interest-rate changes. For example, an effective duration of five years implies that a one-percentage-point increase in risk-free interest rates approximately reduces the portfolio value by five per cent, assuming a sufficiently small, parallel shift in the risk-free interest rate curve.

2.1.4 Interest Rate Shock Scenarios

Interest rate risk is regulated under the Basel IV, which establishes international standards for banking regulation and risk management. In particular guidelines for measuring and managing interest rate risk in the banking book. Basel IV is the latest iteration of the regulations; in Basel III, the stress test came. The Basel III framework specifies that interest rate risk should be assessed using different scenarios [7]. The scenarios are constructed by specifying shocks to the discount rates; however, these can be reverse-engineered into forward rates at different maturities (e.g., 1-month, 12-month, and 24-month). Basel III defines different scenarios for how risk-free interest rates change. These scenarios are used to identify the plausible worst-case scenario and determine the amount of capital reserve needed to remain solvent. The shock scenarios use specific constants for each currency, which differ from one currency to the next. For SEK, see Table 2.1.

The six scenarios to be tested are parallel up and down: all interest rates move up or down by the same amount, with short rates moving up or down while longer rates don't receive the same shock. Then there is steepening, which means that long-term rates increase while short-term rates decrease. And finally, flattening, which is an increase in the short term and a decrease in the long term, makes the curve flatter.

Type	percentage unit change
$\pm C_{\text{parallel}}$	$\pm 2\%$
$\pm C_{\text{short}}$	$\pm 3\%$
$\pm C_{\text{long}}$	$\pm 1.5\%$

Table 2.1: Shock amplitudes used in the interest-rate stress scenarios. The table reports the sizes of the parallel, short-rate, and long-rate shocks applied to SEK interest rates.

The base case for the discount interest rate $r_d(t)$ is defined as $\Delta R(t)$. The different $\Delta R(t)$ is how much the discount interest rate at time t changes. The interest-rate shock scenarios are defined as follows, with a complementary plot in Figure 2.1.

Parallel shock:

$$\Delta R_{\text{parallel}}(t) = \pm C_{\text{parallel}}$$

Short rate shock:

$$\Delta R_{\text{short}}(t) = \pm C_{\text{short}} \cdot e^{-t/x}, \quad \text{with } x = 4 \text{ years}$$

Long rate shock:

$$\Delta R_{\text{long}}(t) = \pm C_{\text{long}} \cdot \left(1 - e^{-t/x}\right), \quad \text{with } x = 4 \text{ years}$$

Steepener shock:

$$\Delta R_{\text{steepener}}(t) = 0.9 \cdot |\Delta R_{long}(t)| - 0.65 \cdot |\Delta R_{short}(t)|$$

Flattener shock:

$$\Delta R_{\text{flattener}}(t) = -0.6 \cdot |\Delta R_{long}(t)| + 0.8 \cdot |\Delta R_{short}(t)|$$

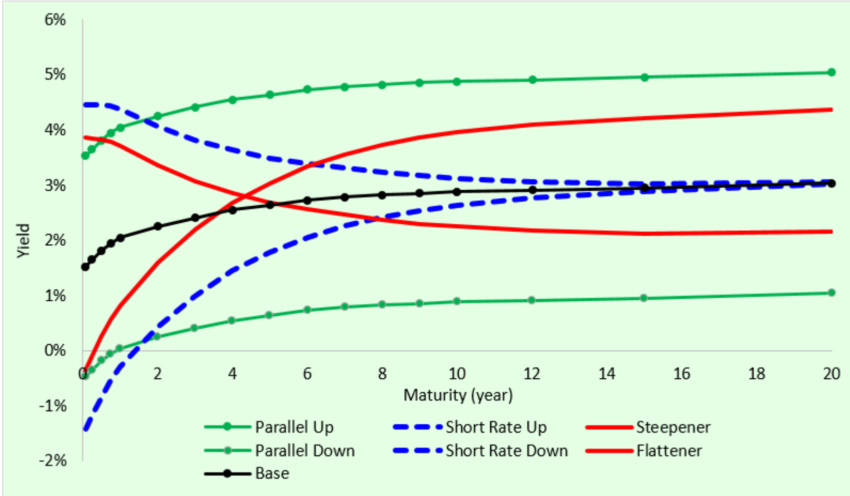

Figure 2.1: Interest-rate shock scenarios used in the stress-testing. The figure illustrates how parallel, short-rate, long-rate, steepener, and flattener shocks affect the interest rate across maturities. The x-axis is the term maturity, and the y-axis is the interest rate.

2.2 Survival analysis

Mortgage loans have no contractual maturity; the customer can prepay the entire loan or any portion of it at any time. For a portfolio of loans, the maturities should be modeled as a statistical distribution. For this thesis, survival analysis is used. Survival analysis is a branch of statistics that focuses on the time until an event occurs. The event can be a machine failure, a default, or death. For A portfolio of mortgage loans the survival is how many mortgage loans remains per time step.

To model mortgage maturities, a survival function [8] is introduced. The survival function represents the probability that a mortgage remains active after time t. Let T denote the random time until prepayment. For a cumulative density function $F(t)$, the survival function is defined as

$$S(t) = P(T > t) = \int_t^\infty f(x)dx = 1 - F(t)$$

where $f(x)$ is a probability density function for the prepayment time with support in $[0, \infty)$.

The hazard function is defined as the probability of prepayment at a maturity t conditional on survival up to t. It takes the form

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t < T \leq t + \Delta t | T > t)}{\Delta t} = \frac{f(t)}{S(t)} = -\frac{d}{dt} \ln(S(t)).$$

The integral of the hazard function is called the cumulative hazard function $H(t)$, given as

$$H(t) = \int_0^t h(u)du = -\ln(S(t)).$$

and conversely

$$S(t) = \exp(-H(t)) = \exp(-\int_0^t h(u)du).$$

Given a survival function, it is valuable to know the expected time until the event. The expected survival time is given by $\int_0^\infty t \cdot f(t)dt = \int_0^\infty S(t)dt$.

2.2.1 Censoring and truncation

Not all mortgage loans will be prepaid before the observation time ends. Therefore, there can be a censoring effect. This can also include not knowing when the mortgage loans are prepaid, as they are only known for an interval during which they are prepaid.

Left truncation occurs when mortgage loans originate before the observation period, because loans that terminate before the observation period begins are not observed. A typical example is the time after the mortgage loan inception. The mortgage loan might have started before the observation time began. This introduces bias because some mortgage loans might have prepaid before the observation period began, and the remaining loans might be less likely to prepay.

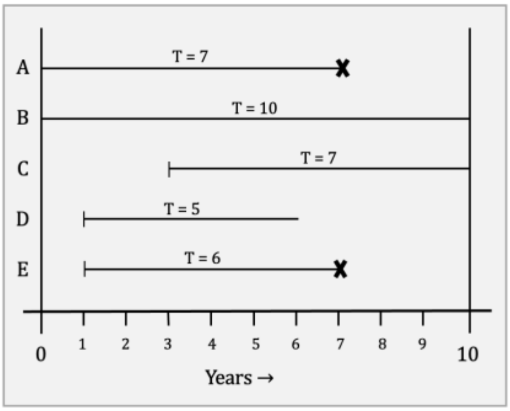

Figure 2.2: The figure shows what censoring and truncation look like for different loans $A, B, ..., E$. The X represents an event, and the line represents an entrance.

2.2.2 Kaplan Meier estimator

The Kaplan-Meier estimator is a widely used nonparametric estimator [9].
It computes survival as the product of conditional survival probabilities at each observed event time. The probability of prepayment at time t is

$$\frac{\text{prepayments at time } t}{\text{loans existing at time } t}.$$

However, this does not include the probability of surviving up to time t. The unconditional probability of prepayment at time t is therefore the probability of no prepayment before t, multiplied by the conditional probability of prepayment at time t.

$$S(t) = \prod_{t_i \leq t} \left(1 - \frac{d_i}{n_i}\right)$$

where:

- $0 < t_1 < t_2 < \cdots < t_k$ are the ordered event times,
- d_i is the number of events (e.g., prepayments) at time t_i,
- n_i denotes the number at risk immediately prior to t_i excluding observations censored at t_i.

The Kaplan–Meier estimator cannot extrapolate beyond the observation period and requires a large number of parameters. Moreover, the estimates cannot be used to make predictions beyond the observation period. These limitations motivate the use of parametric survival models.

2.2.3 Common distributions

Some common parametric distributions in survival analysis include the exponential and Pareto distributions [8]. The exponential distribution is used to model prepayments that occur at a constant rate and are independent of previous events. A Pareto distribution decays more quickly at the beginning, then decreases more slowly. These distributions and their corresponding probability distribution function, cumulative distribution function, survival function, hazard function, and cumulative hazard function are provided in Appendix A.3.

2.2.4 Cox proportional hazards model

In survival analysis, it can be useful to model the survival rate with covariates. This can be the current interest rate or the income level. For covariates, a commonly used model is the Cox proportional hazards model [8]. This model assumes that the baseline hazard rate $h_0(t)$ is proportional conditional on some covariates x and coefficients θ. The conditional probability hazard $h(t|\mathbf{x})$ is expressed as

$$h(t|\mathbf{x}) = h_0(t) \cdot e^{\boldsymbol{\theta}\mathbf{x}}.$$

Comparing different hazards corresponds to

$$\frac{h(t|\mathbf{x}_1)}{h(t|\mathbf{x}_2)} = \frac{h_0(t) \cdot e^{\theta \cdot \mathbf{x}_1}}{h_0(t) \cdot e^{\theta \cdot \mathbf{x}_2}} = e^{\theta \cdot (\mathbf{x}_1 - \mathbf{x}_2)}.$$

The model is semi-parametric. The baseline hazard can be approximately non-parametric, and the θ is approximately parametric. In this case, because we want a parametric model, we will specify the baseline hazard function $h_0(t)$.

2.3 Frailty models

As stated previously, a parametric model is needed to enable prediction beyond the observation period. In a mortgage portfolio, customers do not behave identically; some are more likely to prepay than others due to unobserved factors such as refinancing preferences or moving behavior. Ignoring this heterogeneity may lead to misleading survival estimates. Frailty models address this by introducing an unobserved random effect that scales the individual hazard rate. Customers with higher frailty are more likely to prepay early, while those with lower frailty tend to remain in the portfolio longer.

For each mortgage loan, one can assume that the hazard varies with different covariates. However, there might be multiple unobservable effects that lead to higher or lower hazard. All these effects might be thought of as a random variable $\mathbf{Z}$. The hazard rate is then conditional on $\mathbf{Z}$. For interpretability, $\mathbf{Z}$ is normalized to have a mean of 1 and a finite variance. If frailty is greater than 1, they are more likely to repay than the baseline hazard; if it is less than 1, they are less likely to repay than the baseline hazard. This is frailty $h(t|\mathbf{Z})$ [10] and is conditional on $\mathbf{Z}$

$$h(t|\mathbf{Z}) = \mathbf{Z} \cdot h(t),$$

where $h(t) = h(t|\mathbf{Z} = 1)$ is the baseline hazard and could, for example, be set to a constant value λ. Recalling the Cox proportional hazards model, there is a parallel in how $\mathbf{Z}$ and $e^{\beta X}$ scale the hazard. The cumulative hazard $H(t)$ could be used to give the survival function

$$S(t) = \exp(-H(t) \cdot \mathbf{Z}).$$

However, for the entire portfolio of loans, we are interested in the expected value $\bar{S}(t)$ of $S(t)$. To estimate it, take the expected value of the survival function.

$$\bar{S}(t) = E[e^{-H(t)\cdot\mathbf{Z}}] = \int_0^\infty e^{-H(t)\cdot z} \cdot f(z)dz.$$

For a function $f(x)$, the Laplace transform $L_f(t)$ can be used to express the expected survival function as $\bar{S}(t) = L_f(H(t))$. This provides a convenient way for calculating the expected survival function using established analytical methods.
Commonly used distributions for $\mathbf{Z}$ are the gamma distribution or the inverse Gaussian. The gamma distribution is commonly used because it has an analytical solution via the Laplace transform. For a gamma distribution [11] with mean 1 and a variance of $\frac{1}{a}$, where a is a constant, the probability density function is the following

$$f(x) = \frac{a^a}{\Gamma(a)} \cdot x^{a-1} \cdot e^{-a\cdot x},$$

and using the Laplace transform to get the survival function

$$\bar{S}(t) = L_f(H(t)) = (1 + \frac{H(t)}{a})^{-a}.$$

The frailty-induced survival distribution has a decreasing hazard but an infinite expected value for $a < 1$. The cumulative hazard $H(t)$ could be set to λt for a constant hazard rate per mortgage loan. Another common frailty model is the inverse Gaussian distribution $f(x)$ [12], with parameters $\lambda, \mu > 0$.

$$f(x) = \left(\frac{\lambda}{2 \cdot \pi \cdot x^3}\right)^{1/2} \exp\left(-\frac{\lambda \cdot (z-\mu)^2}{2 \cdot \mu^2 \cdot x}\right), \qquad z > 0,$$

with the frailty

$$\bar{S}(t) = L_f(H(t)) = e^{\frac{\lambda}{\mu} \cdot (1 - \sqrt{1 + 2 \cdot \frac{\mu^2}{\lambda} \cdot H(t)})}.$$

2.3.1 Shifted frailty model

A common probability density function for frailty is a gamma distribution with mean one. However, this could pose a small hazard when $a < 1$, meaning the expected value is infinite. To avoid small hazards, a shifted frailty is introduced $\mathbf{Z'} = \omega + (1-\omega) \cdot \mathbf{Z}$ where $\mathbf{Z}$ is a random variable and $\omega \in [0, 1]$. This will ensure that the mean is still 1, because $E[Z'] = (1-\omega) \cdot E[Z] + \omega = 1$, but as long as ω is greater than zero, the hazard will be strictly larger than zero.

$$\bar{S}(t) = E[e^{-H(t) \cdot (\omega + (1-\omega) \cdot \mathbf{Z})}] = \int_0^\infty e^{-H(t) \cdot (\omega + (1-\omega) \cdot z)} \cdot f(z) dz =$$

$$e^{-H(t) \cdot \omega} \cdot \int_0^\infty e^{-H(t) \cdot (1-\omega) \cdot z} f(z) dz = e^{-\omega \cdot H(t)} \cdot L_f((1-\omega) \cdot H(t)).$$

For $\omega = 1$, it equals $e^{-H(t)}$, and for $\omega = 0$, it equals $L_f(H(t))$. The exponential term ensures sufficiently fast decay under $\omega > 0$, yielding a finite expectation.

2.4 Survival function

Using a gamma distribution with mean 1, a shifted frailty model, and a constant hazard $H(t) = \lambda \cdot t$ yields the following survival function

$$S(t) = e^{-\omega \cdot \lambda \cdot t} \cdot L_f((1-\omega) \cdot \lambda \cdot t) = e^{-\omega \cdot \lambda \cdot t} \cdot (1 + \frac{(1-\omega) \cdot \lambda \cdot t}{a})^{-a}.$$

The survival function details are provided in the appendix. Here, λ is the constant for the baseline hazard, and a represents the inverse variance of the frailty. $\omega \in [0, 1]$ represents how much it behaves like an exponential or a Pareto distribution. It has a finite expected value and a decreasing hazard, meaning that long-standing mortgage loans are less likely to prepay than newer ones. The hazard rate is

$$h(t) = \omega \cdot \lambda + \frac{(1-\omega) \cdot \lambda}{1 + \frac{(1-\omega) \cdot \lambda \cdot t}{a}}$$

The hazard will converge to $\omega \cdot \lambda$ when $t \to \infty$. The decreasing hazard captures mortgage stickiness, and the non-zero asymptotic hazard reflects eventual prepayment; the finite expectation avoids an infinite expected value, which is unrealistic for a portfolio of mortgage loans.

2.5 Likelihood

To estimate the parameters of the survival model, maximum likelihood estimation is used [8]. The likelihood depends on the type of observation available for each mortgage loan. Some mortgage loans terminate during the observation period, while others are censored or truncated.
For a right-censored observation at time C_r, the only information available is that the mortgage loan survived beyond C_r. The contribution to the likelihood is

$$S(C_r).$$

For interval-censored observations, where the event is only known to occur between t_i and t_{i+1}, as is the case for prepayments observed during a given month, the contribution to the likelihood becomes

$$S(t_i) - S(t_i + 1).$$

Assuming independent observations, where I, R denote the sets of interval-censored observations, and right-censored, respectively. The full likelihood for the parameters θ is obtained by multiplying all contributions:

$$L(\theta) = \prod_{i \in R} S(C_{r,i}) \prod_{i \in I} \left(S(t_i) - S(t_i + 1) \right),$$

The model parameters are estimated by numerically maximizing the log-likelihood.

2.5.1 Goodness of Fit

The goodness of fit of the survival models was evaluated using Akaike Information Criterion (AIC) [13]. AIC assesses model quality by penalizing model complexity. It uses the estimated log-likelihood $\ell(\hat{\theta})$ and the number of parameters k. It is defined as

$$\mathrm{AIC} = -2 \cdot \ell(\hat{\theta}) + 2 \cdot k,$$

where $\ell(\hat{\theta})$ denotes the maximized log-likelihood of the model and k is the number of estimated parameters. Lower AIC values indicate a better trade-off between goodness of fit and model complexity.

2.6 Competing hazard

Competing risks arise when multiple prepayment events are possible [8]. For instance, a customer can prepay and leave the bank, or change their loan term. Leaving the bank is commonly called churn and means that a mortgage has been completely terminated. Both of these scenarios are prepayments, but they are inherently different. One could treat the other hazard as a censoring event, but that violates the assumption of non-informative censoring. Instead, a cause-specific hazard function is introduced

$$h_i(t) = \lim_{\Delta t \to 0} \frac{P(t < T \leq +\Delta t, \delta = i | T \geq t)}{\Delta t}.$$

Here is $T = \min(X_1, ..., X_K)$ and X_i is the event i and δ indicate which hazard. All K hazards can be added together to

$$h_T(t) = \sum_{i=1}^{K} h_i(t)$$

Which corresponds to the hazard of any event occurring. Let

$$S(t_i, ..., t_K) = P(X_1 \geq t_1, ..., X_K \geq t_K)$$

which is used to give the cause-specific hazard as

$$h_i(t) = \frac{-\frac{\partial S(t_i, ..., t_K)}{\partial t_i}|_{t_1 = ... = t_k = t}}{S(t, ..., t)}.$$

To get a better interpretation of competing risk, one could look at the Cumulative Incidence Function (CIF), which is defined as

$$F_i(t) = P(T \leq t, \delta = i) = \int_0^t h_i(u) \cdot e^{-H_T(u)} du$$

where $H_T(t)$ is the cumulative distribution for all cause-specific hazards, which is derived by integrating and summing all cause-specific hazards. This is not a cumulative hazard or a true distribution. This is because $F_i(t)$ is the cumulative risk that event i happens before t while all hazards are active, but another event could occur, so $F_i(t)$ cannot go to one [8].

3
Methods

The method consists of four steps. First, the raw mortgage balance data are transformed to be suitable for survival analysis. Second, a mortgage-rate forecasting function is estimated from historical mortgage interest rates and risk-free rates. Third, a parametric survival model is used to estimate the distribution of mortgage loan prepayment. Finally, the forecasting and survival models are combined to value the mortgage portfolio under different interest-rate scenarios.

3.1 Data

The mortgage data were provided by Handelsbanken and included all mortgage loans for Handelsbanken customers between 2018 and 2026. The mortgage portfolio was observed monthly using end-of-month balances. Each observation contains a customer identifier, a mortgage balance, a mortgage type, an interest-rate term, and a loan start date and term start date. Due to confidentiality restrictions, some observations cannot be reported. However, the full modeling framework and estimation procedure are reproducible. Some mortgages originated as early as 2010.

Variable	Description
Customer ID	Anonymous borrower identifier
Date	Monthly observation date
Balance	Outstanding mortgage balance
Mortgage Type	Interest-rate fixation period
Start date	Start date of the total loan

Table 3.1: Variables included in the Handelsbanken mortgage-loan dataset. The table summarizes the mortgage loan fields in the dataset.

The forward interest rate curve was used to forecast monthly mortgage interest rates based on publicly available interest rate data aggregated across all banks. This mortgage interest data stretched from 2013 to 2026. In addition, the forward interest rate data cover all forward interest rates from 1 to 360 months, month by month, for the dates 2013 to 2026.

3.1.1 Data limitations

Only mortgage loan data from 2018 to 2026 were included in the analysis. However, the dataset contains mortgage loans with origination dates as early as 2010. Consequently, the data are subject to left truncation, since loans that originated and terminated before 2018 are unobserved. Many mortgage loans are also right-censored because they remained active at the end of the observation period. In addition, some loans may, in practice, be older than their recorded origination dates indicate. The mortgage interest-rate dataset did not exhibit the same limitations and was available continuously from 2013 to 2026.

3.2 Data preparation

Mortgage balances are observed as dynamic time series rather than fixed contractual loans. Customers may increase balances, make partial prepayments, or change terms. Consequently, the raw data cannot directly be interpreted as standard survival observations.

A customer can increase or decrease their mortgage loan and pay it back whenever they want. To model these mortgage loans, one proposed approach is to break payments into parts using a method similar to that used for deposits [14]. This is done by splitting each customer's total loan amount into inflows and outflows. When a customer's total mortgage loans increase or are added, it is modeled as if a new mortgage loan has been initiated, and it decreases from the top down as payments are made according to Last In, First Out (LIFO). This is a modeling assumption, but it can be motivated on behavioral grounds: if a customer increases their mortgage balance and subsequently makes a large repayment, it is reasonable to assume that the newly added borrowing is repaid first.

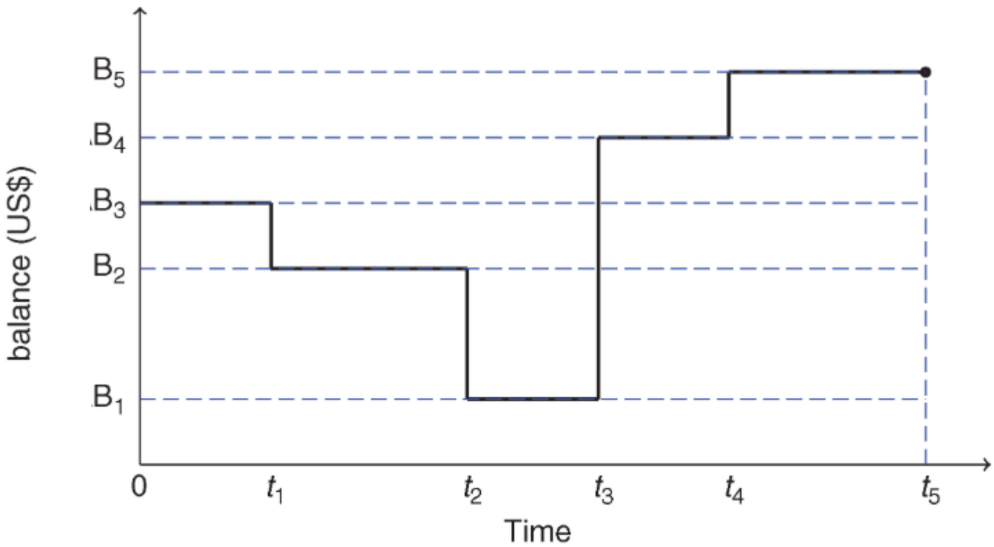

Figure 3.1: Illustration of the mortgage loan transformation procedure. Increases in a borrower's total mortgage balance are treated as new loan components, while subsequent repayments are allocated according to a last-in-first-out structure.

For mortgages, this can be a change of term, a change of bank, or excess amortization. The loan can also increase if the customer increases their mortgage. This transformation makes the mortgage loan data suitable for survival analysis. This meant that if a customer increased their mortgage for a renovation or a similar purpose, a new mortgage loan was initiated, and all additional amortization was applied to the new loan until it was depleted.

Amortizations smaller than 0.25% per month were excluded to avoid capturing contractual amortization rather than behavioral prepayments. This ensured that amortizations did not exceed 3% per year, the maximum contractual amortization requirement. Only behavioral maturity is modeled because contractual amortization requirements changed over the observation period and cannot be consistently identified from the available data. Also, large mortgage loans that existed for only 1 to 3 months were filtered out (sizes over 50,000 SEK). These observations likely represent bridge financing and were therefore excluded.

The mortgage loans were then classified either as a mixture of loans or by term (e.g., 3-month, 12-month, etc.). If a customer had multiple mortgage loans, they were merged if they were identical. Otherwise, it was classified as a mixed mortgage loan. This is because the data structure doesn't allow for clear analysis of different loan types; that's why we need this modeling approach.

3.3 Mortgage interest rate forecasting model

Swedish adjustable-rate mortgages are repriced using short-term market interest rates. However, mortgage rates do not move one-for-one with market rates. This non-full pass-through creates interest-rate risk because mortgage cash flows don't fully adjust to changes in the discount rate. A pricing function is introduced to estimate future mortgage interest rates from forward risk-free interest rates. A function for the mortgage interest rate is proposed to estimate mortgage interest rates $g(r_f(t))$ using the forward risk-free interest rate $r_f(t)$ by

$$g(r_f(t)) = r_{min} + \beta_1 \cdot \max(r_f(t) - K, 0)$$

with the parameters:

- $r_f(t)$: forward risk-free three months interest rate for given time t
- β_1: passthrough coefficient
- r_{min}: Minimum interest, which is equal to $\beta_0 + K \cdot \beta_1$
- K: floor threshold
- β_0: mortgage spread component

The passthrough coefficient determines how much of the change in market rates is passed on to borrowers. A passthrough below one implies that mortgage interest rates don't fully adjust to the risk-free interest rate. The floor reflects the empirical observation that mortgage rates do not decrease linearly as market rates fall to near-zero or negative levels.

The parameters were estimated using historical observations of the average three-month mortgage interest rate and the corresponding three-month forward risk-free interest rate. r_i is the mortgage interest rate and $r_{f,i}$ at index i is the risk-free interest rate, n is the number of data points, and the β_i are the coefficients that should be estimated

$$(\hat{\beta}_0, \hat{\beta}_1, \hat{K}) = \arg\ \min_{\beta_0,\beta_1} \sum_i^n (r_i - g(r_{f,i}))^2.$$

For each chosen threshold value $\hat{K}$, the coefficients $(\hat{\beta}_0, \hat{\beta}_1)$ were re-estimated conditional on $\hat{K}$. The observations were divided into two regions depending on whether the forward risk-free interest rate was above or below $\hat{K}$. For observations with $r_{f,i} < \hat{K}$, the mortgage rate was assumed to remain constant at the floor level. For observations with $r_{f,i} \geq \hat{K}$, a linear regression model was estimated without the intercept. The optimal threshold $\hat{K}$ was then obtained numerically with $\hat{K}$, as the value that minimized the total squared error across both regions.

The function assumes constant pass-through behavior over time and does not account for customer-specific pricing, competitive dynamics, or changing funding conditions. The model is intended to capture average portfolio-level repricing behavior rather than individual mortgage pricing. To evaluate R^2 was chosen

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum_i^n (r_i - g_i(r_{f,i}))^2}{\sum_i^n (r_i - \hat{r}_i)^2}$$

3.4 Survival model

Survival analysis is used to estimate the distribution of behavioral maturity for mortgage exposures. Because mortgage prepayment is borrower-driven rather than contractually fixed, maturity must be estimated statistically. The survival function is the remaining balance per month, weighted by the mortgage loan amount. A prepayment is classified as an amortization of the mortgage loan or a change in term. Model fit was evaluated using Kaplan-Meier and parametric models. The parametric models were the shifted gamma and inverse Gaussian frailty models. The mortgage loan data were processed, and the parameters were estimated using maximum likelihood. Delayed entry was incorporated into the likelihood by conditioning on survival up to the entry time. For comparisons, AIC was used to compare different models. Competing risks are modeled separately because these events have different implications for valuation and repricing behavior.

3.5 Portfolio valuation and stress testing

The mortgage portfolio is interpreted as a collection of uncertain cash-flow streams whose maturities are determined behaviorally rather than contractually. Using the pricing function $g(r_f(t))$, which computes the mortgage interest rate from the forward risk-free interest rate curve $r_f(t)$ for a given time t, can predict the mortgage interest rate and calculate the present value. Given that we have a mortgage loan with a behavioral maturity at time τ, the present value given maturity $P(\tau)$ is

$$P(\tau) = \sum_{t=1}^{\tau} g(r_f(t)) \cdot e^{-t\cdot(r_d(t)+m)} + e^{-\tau\cdot(r_d(t)+m)}.$$

The monthly payments are the three-month mortgage rate (divided by 12), and at the final time τ, the mortgage loan is repaid. The commercial margin can then be calculated by setting $P(\tau)$ to one. However, the maturity of the mortgage loan is unknown; a survival function can be used to estimate the number of mortgage loans that mature at each time step. This will make the total P_{total} be

$$P_{total} = \sum_{t=1}^{\infty} w_t \cdot P(t).$$

This will be the average present value for all maturities. The weight w_τ is $w_\tau = S(\tau - 1) - S(\tau)$. This represents the probability of prepayment occurring during month t. Mortgage loans are not unlimited, so an upper limit should be set. It is set at 30 years or 360 months; all remaining mortgage loans are paid back ($w_{360} = S(360)$). 30 years is chosen as a modeling assumption and is reasonable in this context because a mortgage loan can't outlive the borrower, and the risk-free forward interest rate data is constrained by 30 years. The equations are connected in Figure 3.2.

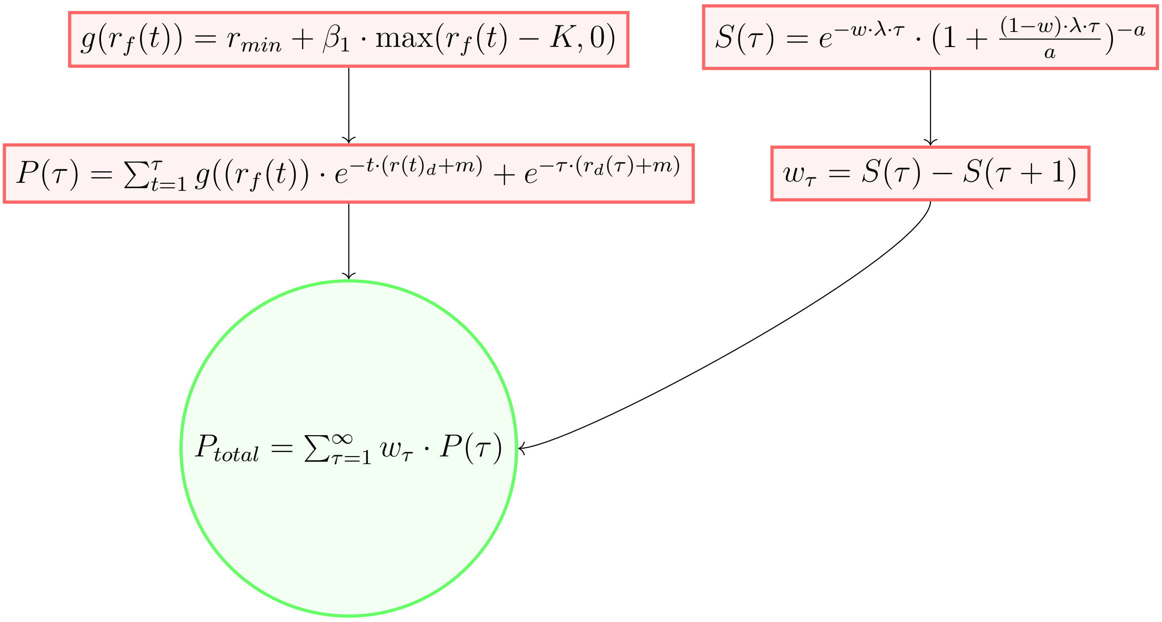

Figure 3.2: The diagram shows how the mortgage-rate forecasting model, survival function, maturity weights, and present-value equation are combined to estimate portfolio value.

A portfolio of mortgage loans can be constructed from two parts: a 3-month, fully interest-rate-resetting part and a core part. The core represents the fraction of the mortgage portfolio that behaves like a long-duration fixed-interest-rate asset rather than a short-duration floating-rate asset. The total present value P_{total} of the mortgage portfolio is therefore represented as a weighted combination of a short-duration present value P_{short} and a fixed-interest-rate long-duration present value P_{fixed}:

$$P_{total} = c \cdot P_{fixed} + (1 - c) \cdot P_{short}.$$

The $c \in [0, 1]$ is the core percentage, indicating how much is allocated to the fixed part; c can then be modeled over time. The fixed-part percentage is important because, if it remains consistent over time, it can serve as a baseline for financing, thereby reducing interest-rate risk. However, if it increases or decreases rapidly across different interest periods, it poses a risk that warrants monitoring.

The short-term component P_{short} is calculated using a pricing function based on the three-month risk-free rate. For the fixed term part P_{fixed} a fixed interest rate, r_a, is needed and it can be calculated by

$$1 = r_a \cdot \sum_{t=1}^{\tau} e^{-r_d(t)\cdot t} + e^{-r_d(\tau)\cdot\tau},$$

which gives r_a closed form solution

$$r_a = \frac{1 - e^{-r_d(\tau)\cdot\tau}}{\sum_{t=1}^{\tau} e^{-r_d(t)\cdot t}}.$$

We can then calculate the core percentage using a small change in present values $\Delta P_{total}, \Delta P_{short}, \Delta P_{fixed}$, which gives

$$c = \frac{\Delta P_{total} - \Delta P_{short}}{\Delta P_{fixed} - \Delta P_{short}}.$$

Otherwise, the present value will be sensitive to changes in the shape of the risk-free interest rate curve.

3.6 Implementation

All empirical analyses were implemented in Python. Data preparation was performed using *pandas* and *numpy*. Survival models were estimated using $lifelines$, with custom likelihood functions implemented for delayed entry, censoring, and frailty specifications. Numerical optimization was performed using *scipy.optimize*. Figures were produced using *matplotlib*.

4

Results

This chapter presents the empirical results obtained from the mortgage rate forecasting model, the survival analysis, and the valuation framework. The findings include estimates of mortgage rate pass-through, behavioral maturity, portfolio duration, stress-test outcomes, and additional behavioral analyses.

4.1 Mortgage interest rate forecasting model

For the mortgage interest rate forecasting model, an average of all banks' interest rates was used and compared to the 3-month forward interest rate. A comparison between the 3-month mortgage interest rate and the 3-month forward interest rate shows that they are approximately linearly related; however, the interest rate appears to have a minimum (or floor) at negative interest rates. Optimizations with a minimum mortgage interest rate in mind are shown in Figure 4.1. The optimized values are presented in Table 4.1. It shows that the mortgage rate increases, but at a slower rate than the risk-free rate, meaning the spread, the difference between the mortgage and risk-free interest rate, shrinks when risk-free interest rates rise and widens when they fall. An observation is that the slope appears different when rates are increasing than when they are decreasing. This, however, has not been accounted for in the regression.

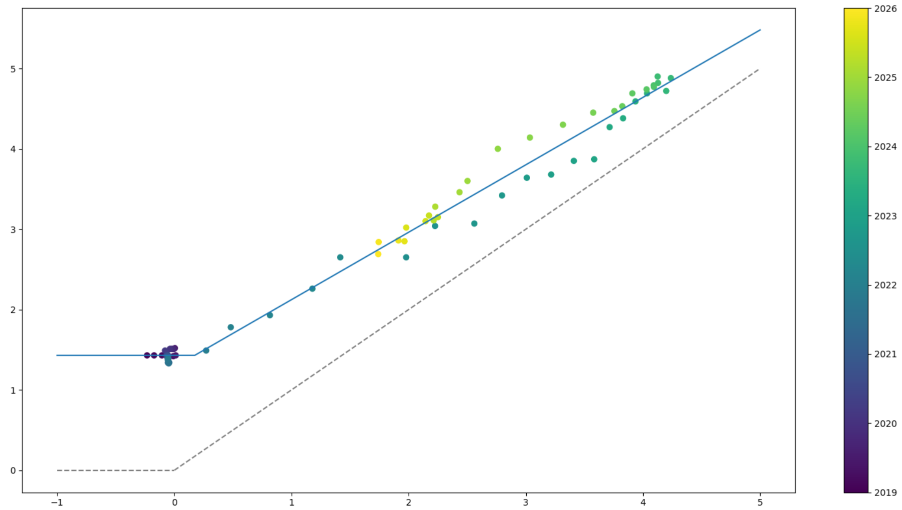

Figure 4.1: Relationship between the three-month risk-free forward rate on the x-axis and the observed three-month mortgage rate on the y-axis. The blue line is the estimated pricing function that captures the mortgage-rate floor and pass-through from risk-free interest rates to mortgage rates. The dotted line is a floor at 0 and a one-to-one increase.

	mean square
Passthrough	0.84
Floor	1.43
Floor interest rate	0.17
R^2	0.9891

Table 4.1: Estimated parameters of the mortgage-rate forecasting model. The table reports the estimated pass-through coefficient, mortgage-rate floor, the floor threshold, and the model R^2.

4.2 Mortgage loan survival behavior

For the behavioral maturity modeling, survival analysis was used with a Kaplan–Meier estimator to establish a nonparametric baseline. Afterward, a parametric survival model, specifically the shifted-gamma frailty model, was estimated and compared. This analysis was conducted using a mortgage loan dataset comprising approximately 90 million observations.

4.2.1 Kaplan-Meier estimation

The Kaplan–Meier estimate of the survival function for Handelsbanken mortgage loans is shown in Figure 4.2. The survival curve shows the proportion of the initial mortgage loans that remain in the portfolio over time. The observation period extends to approximately 175 months, though the oldest loans are subject to left truncation because they originated before 2018. Three survival curves are presented: one for churn, one for borrowers changing interest-rate terms (e.g., from a three-month term to a longer term), and one for either event occurring.

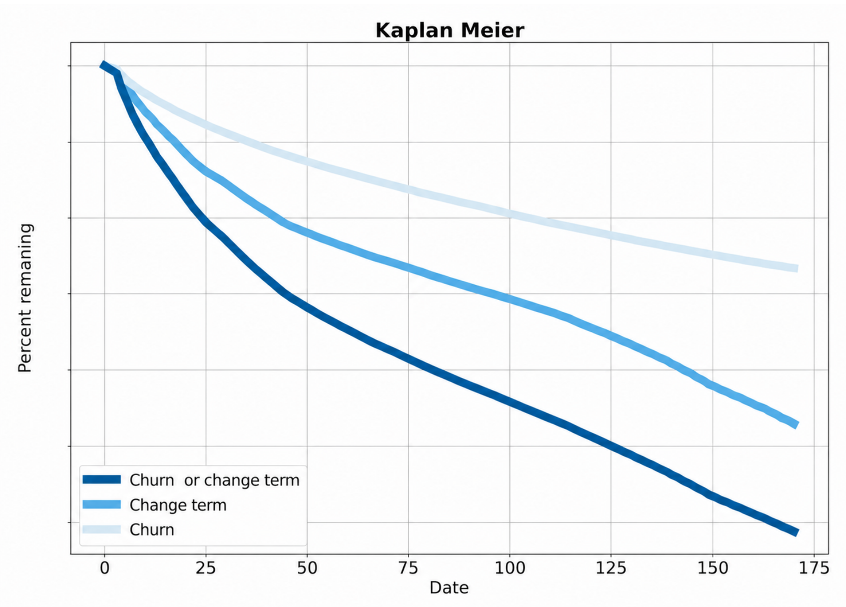

Figure 4.2: Kaplan–Meier survival estimates for mortgage-loan termination events by month on the x-axis. The curves show the share of the initial mortgage exposure remaining over time before churn, change in interest-rate terms, or experience either event.

4.2.2 Shifted gamma frailty model

The estimated shifted gamma frailty model closely follows the Kaplan–Meier estimator throughout the observation period. The estimated shifted gamma frailty model closely matches the Kaplan–Meier estimate over most of the observation period, suggesting it provides a reasonable description of the observed prepayment behavior.

Estimating the parametric model discussed in the theory gives Figure 4.3. The expected survival time for the parametric model, defined as the time until the mortgage loan terminates, either by switching to another term or by churn, is 19.2 years. However, when using the survival function for valuation, a restriction is imposed: all active loans will be terminated after 30 years. This effectively reduces the expected lifetime to 13.5 years instead.

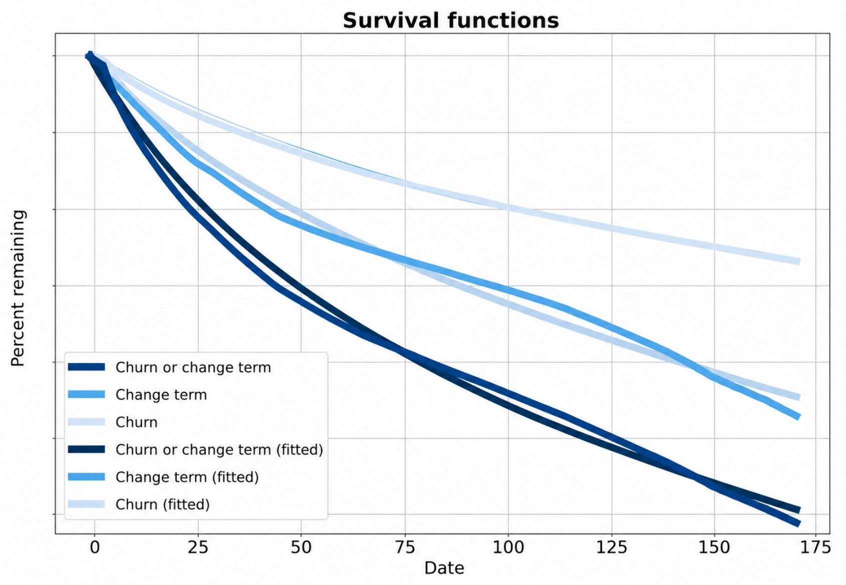

Figure 4.3: Comparison between the Kaplan–Meier survival estimate and estimated shifted-gamma frailty models. The estimated curves show how the parametric survival model approximates the observed mortgage prepayment behavior.

4.2.3 Inverse Gaussian vs shifted gamma frailty model

For model comparison, two frailty models were used: shifted gamma and inverse Gaussian. Table 4.2 shows the goodness of fit for the models. Both models exhibit similar goodness-of-fit metrics, although the shifted gamma frailty model achieves a marginally lower AIC. The likelihood scale depends on the SEK volume, which is large in this case.

Metric	Shifted gamma	Inverse gaussian	Difference
Log-Likelihood	$-9.639 \cdot 10^{11}$	$-9.643 \cdot 10^{11}$	$3.568 \cdot 10^{8}$
AIC	$1.928 \cdot 10^{12}$	$1.929 \cdot 10^{12}$	$-7.137 \cdot 10^{8}$

Table 4.2: Goodness-of-fit of the shifted-gamma and inverse-Gaussian frailty models. The table reports log-likelihood and AIC values for the two parametric survival specifications.

4.3 Portfolio valuation and duration

In the final step, the mortgage interest rate model and the mortgage loan survival model are combined to calculate the present value. The present value is normalized to 1 with a commercial margin, meaning it is normalized to the nominal value at the initial time. A 1 percentage point shock to the risk-free interest rate curve is shown in Figure 4.4. The change depends on the mortgage loan date and is ca −3% on average.

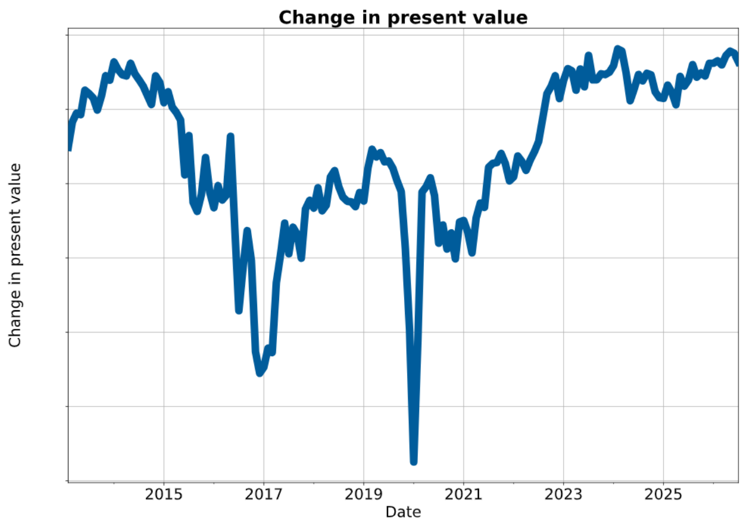

Figure 4.4: Percentage change in mortgage-portfolio present value following a one-percentage-point parallel interest-rate shock. The estimates are calculated separately for each monthly valuation date from 2013 to 2026.

To measure the interest rate sensitivity, the effective duration was calculated. The effective duration was at least ca 2 years, but during periods of low interest rates, it could extend to ca 6 years. As seen in Figure 4.5. The estimated effective duration ranged from 2 to 4 years for non-negative interest periods.

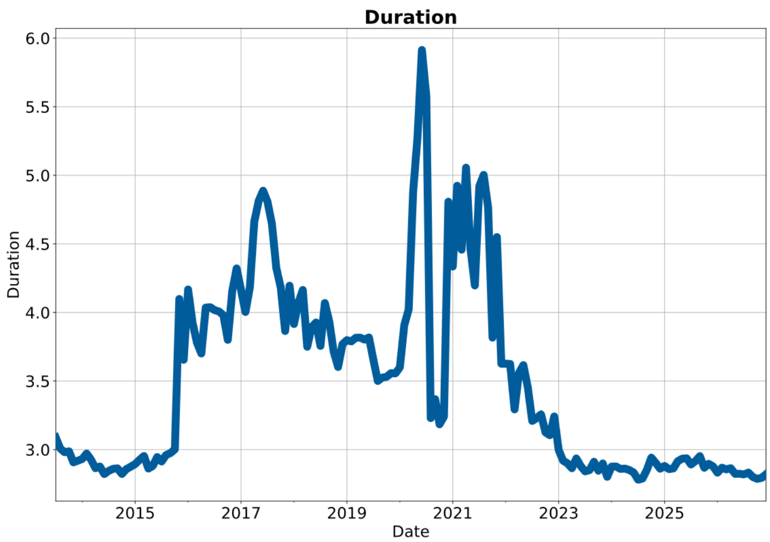

Figure 4.5: Estimated effective duration of the mortgage portfolio over time. The figure shows how the portfolio's interest-rate sensitivity varies across the valuation period.

4.4 Stress Testing and Funding Risk

The Basel interest-rate shock scenarios are illustrated in Figure 4.6. The portfolio is relatively insensitive to short-rate shocks, indicating that changes in short-term rates alone have a limited impact on present value. In contrast, the present value increases under both the parallel-down and flattener scenarios, reflecting the effect of lower interest rates. The largest declines in present value occur under the parallel-up, steepener, and long-rate-up scenarios. It should be noted that the analysis does not impose a floor on negative interest rates; there is literature on capping the risk-free interest rate at -1.5%. Consequently, these estimated gains under falling-rate scenarios may be somewhat larger than those observed in practice.

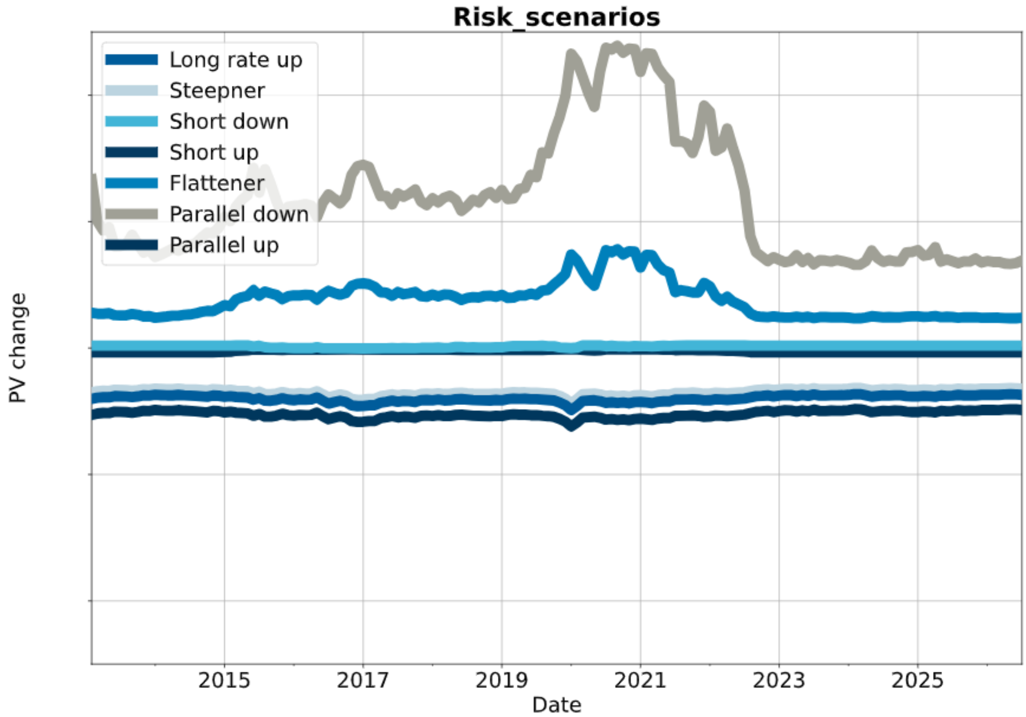

Figure 4.6: Change in mortgage-portfolio present value under the interest-rate shock scenarios. The impact of parallel, short-rate, long-rate, steepener, and flattener shocks from Basel III is shown.

4.4.1 Funding

Figure 4.7 compares the change in present value following a 2 percentage-point interest-rate shock under two funding strategies: three-month duration financing and two-year duration financing. The results show that two-year-duration financing produces smaller losses than three-month-duration financing throughout the sample period. Indicating that longer-duration funding reduces the portfolio's exposure to interest rate risk.

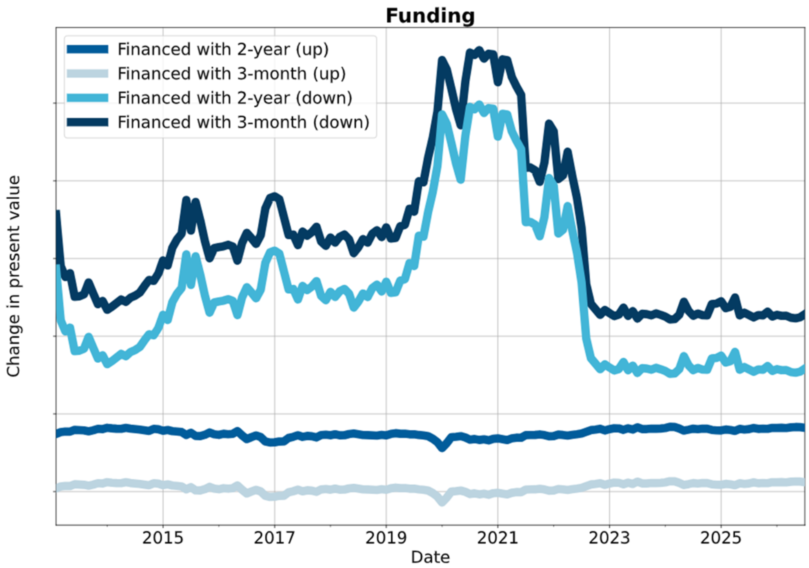

Figure 4.7: Comparison of portfolio value changes under two-year financing and three-month financing following a two-percentage-point interest-rate shock up and down.

4.4.2 Core

Figure 4.8 shows the estimated proportion of the mortgage portfolio held in the fixed-rate portfolio, called core, from 2012 to 2026. However, the pricing function could be implemented with or without a max or floor function, and each scenario yields different effects. The core falls to approximately $10-30\%$ and doubles during periods of low interest rates. Consequently, as interest rates decline and the mortgage rate floor takes effect, a larger share of the portfolio behaves like a long-term fixed-rate asset. However, the core will not be 100% because of the length of the mortgage loans; the long-term risk-free interest rate will be positive. This increases the portfolio's effective duration from around three years to more than six years in the most extreme cases.

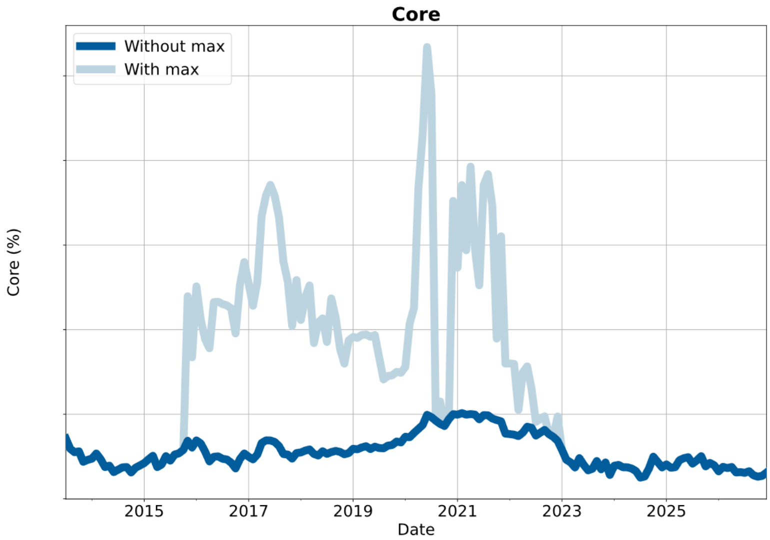

Figure 4.8: Estimated core share of the mortgage portfolio over time. The figure compares how the stable long-term component changes when the mortgage pricing function includes or excludes an interest-rate floor compared with a floor and no floor.

4.5 Behavioral extension

The baseline survival model provides an aggregate description of mortgage prepayment behavior, but additional factors may influence the observed maturities. To explore these effects, the analysis was extended to include competing-risk and Cox proportional hazards models. These approaches allow the underlying behavioral mechanisms behind mortgage prepayments to be examined in greater detail.

4.5.1 Competing risk

The competing-risk estimates are shown in Figure 4.9, with the estimated parametric model presented in Figure 4.10. The parametric model allows future mortgage loan transitions to be forecast.

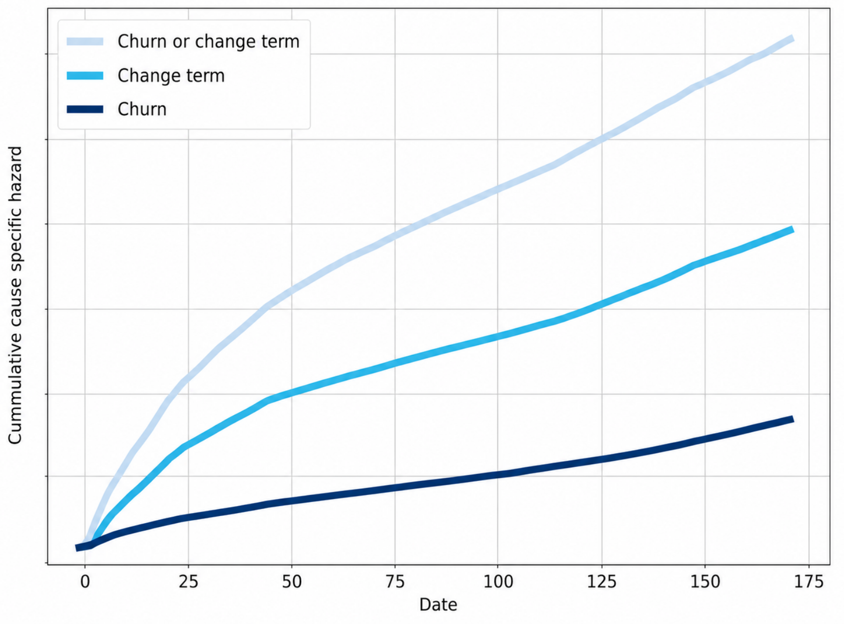

Figure 4.9: Cumulative incidence functions for competing mortgage termination events over time. The figure shows the probability of each event type occurring while accounting for the presence of other competing events.

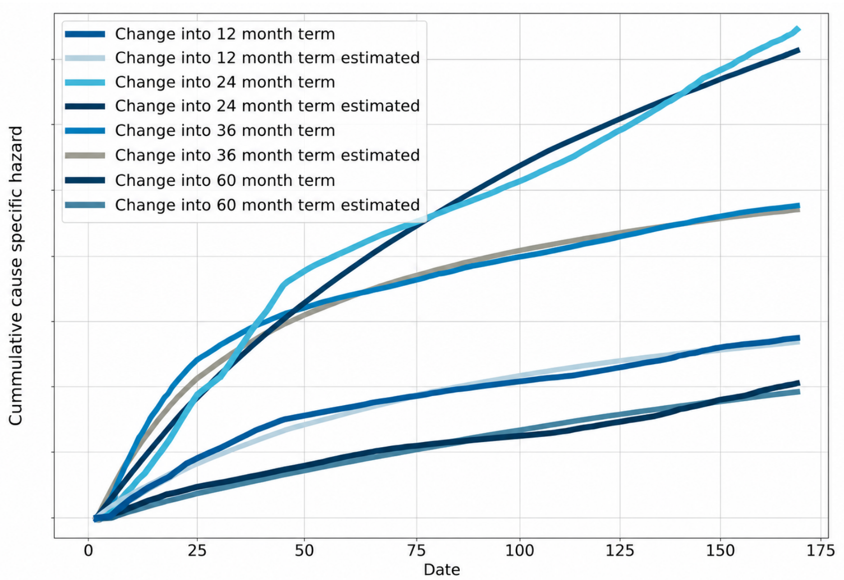

Figure 4.10: Cumulative incidence functions for mortgage termination events by event type. The figure separates the probability of churn from the probability of changing interest-rate terms.

4.5.2 Cox proportional hazards model

To investigate the relationship between interest rates and mortgage prepayments, interest-rate variables were included as covariates in a Cox proportional hazards model with a shifted gamma frailty term. Two covariates were considered: the current interest rate and the change in interest rates between quarters, which are displayed in Figure 4.11. The former shows the current risk free interest rate, while the latter captures the effect of risk free interest rate movements on borrower behavior. These results are in Table 4.3 and Table 4.4, which show that the spread between the three-month floating mortgage interest rate and longer-term rates drives term switching.

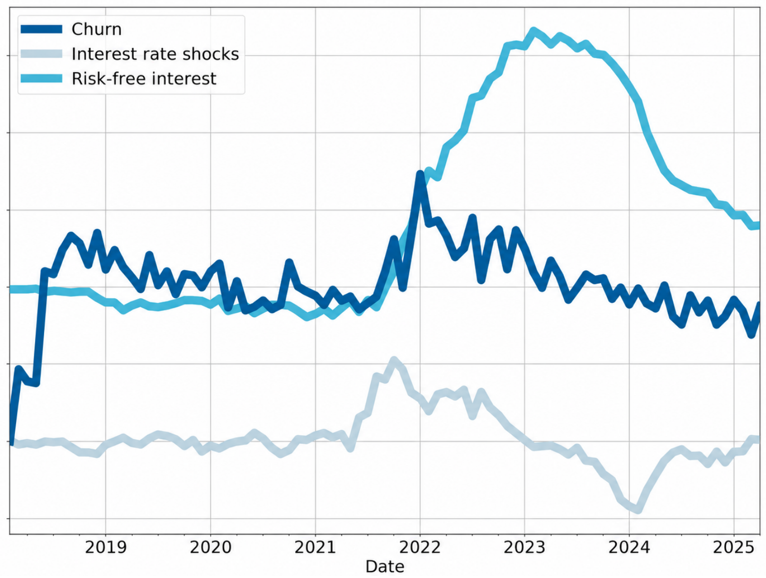

Figure 4.11: Relationship between mortgage termination behavior and risk-free interest-rate conditions. The figure compares the churn rate for the portfolio against both the current risk-free interest rate and the change in the risk-free interest rate.

	3 month interest rate	Shock
Churn or change term	-0.17	0.75
Churn	-0.16	1.05
Change term	-0.09	0.56

Table 4.3: Estimated Cox proportional-hazard model coefficients for mortgage prepayment behavior. The table reports the estimated effects on the event hazard given the covariate.

Changed to term	Spread	Shock of the term rate
1 year	0.08	1.06
2 year	0.49	0.92
3 year	0.92	-0.43
5 year	0.77	-0.91

Table 4.4: Estimated Cox proportional-hazard model coefficients for the mortgage term change. The table reports covariate effects on changes in term, given the difference between the term mortgage interest rate and the three-month mortgage interest rate.

5
Discussion

It has been shown in section 4.3 that 3-month adjustable-rate mortgages have a duration longer than three months. This is because changes in interest rates are not fully passed through to borrowers. Combined with the long behavioral maturity estimated through survival analysis, this results in a portfolio duration longer than the contractual interest-rate reset period.

The mortgage interest rate is the aggregate for the entire portfolio and isn't the individual mortgage interest rate per customer. The estimated mortgage interest rate forecasting function has a pass-through coefficient which is smaller than 1. This indicates that increases in the risk-free interest rates are only partially passed to customers. This implies that the spread between the risk-free interest rate and the mortgage interest rate compresses in rising-rate environments and widens in low-rate environments. When the risk-free interest rate is below 0
Behavioral maturity is modeled using survival analysis. Specifically, a shifted gamma frailty survival function, where the shift ensures finite expected survival time, and the frailty component captures heterogeneity across mortgage loans. This specification is similar to mortgage prepayment models: it allows expected lifetime behavior to remain bounded while also producing "stickiness," meaning that loans that have survived for a long time become less likely to prepay.

5.1 Risks in a mortgage loan portfolio

When constructing a fixed-interest portfolio and a short three-month interest-rate portfolio to approximate the present value of the mortgage loan portfolio, the following results are obtained, shown in Figure 4.8. The fixed-interest portion is roughly $10 - 30\%$ and has a duration of 12.5 years. However, during periods of negative interest rates, it can double. This is because when the risk-free interest rate is below 0%, mortgage loans behave more like fixed-rate loans, which is why the percentage increases. The results suggest that exclusive reliance on short-duration funding may yield a lower present value than longer-duration funding.

The different risk scenarios can be used to draw some conclusions from section section 4.4. The biggest effects come from parallel shifts in risk-free interest rates. The best case is when all risk-free interest rates fall, and the worst case is when all risk-free interest rates rise. This connects to the pass-through because the rising risk-free interest rate isn't fully passed on to the customer; therefore, the present value decreases. However, when the risk-free interest rate falls, the customer doesn't fully benefit from the lower rate, resulting in an increase in present value. This risk is, however, asymmetric: the downside is smaller than the upside gains. For a bank with large mortgage lending, the risk is in rising interest rates.

5.2 Model risk and limitations

The shifted gamma frailty model has been shown to be a useful model for modeling mortgage portfolios. The expected survival time is estimated to be 19.2 years; when all mortgage loans terminate after 30 years, it is 13.5 years. This behavioral maturity could be used alongside the mortgage interest rate forecasting function to reduce the bank's risk; the position's duration could then be calculated. The loans could be financed with the estimated model rather than three-month floating financing. For instance, for February 2013, the estimated duration is around 3.09 years. Taking financing with the same duration can reduce risks. However, this model remains subject to parameter uncertainty; for instance, the parameter estimates could be incorrect. This motivates reliability testing by testing different parameters.

The parameters to be changed are the pass-through and the hazard. The pass-through is the extent to which the mortgage interest rate changes in response to changes in the risk-free interest rate. The hazard affects the time until prepayment for the mortgage. Changing the hazard in the survival function will make the behavioral maturity longer or shorter. Changing parameters from the original estimation gives comparison cases. The change is compared to 3-month financing. This means that 3-month financing is less risky than the financing provided by the model.

Comparison of strategy

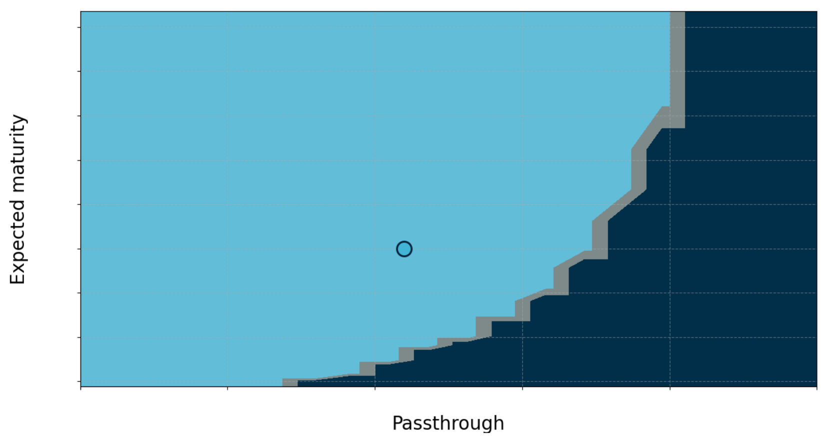

Figure 5.1: Sensitivity of average behavioral maturity to the mortgage-rate pass-through parameter. The light blue region indicates where the modeled mortgage portfolio financing outperforms three-month funding, while the marker shows the estimated pass-through value.

Figure 5.1 shows the model sensitivity for different parameters. This suggests that customer behavior needs to change significantly before the estimated strategy becomes more sensitive to interest rate changes than a three-month financing would be. The pass-through is the more sensitive parameter; if it increases significantly, the three-month floating-rate interest financing becomes less sensitive to interest rates. If the pass-through is 1, maturity doesn't matter because the interest rate will fully reset. An increase in the hazard will shorten the duration of the fixed-interest-rate period. However, it isn't as sensitive to duration, so it can drop significantly before the three-month financing improves.

5.2.1 Model assumptions

The results presented in this thesis depend on several modeling assumptions. These assumptions were necessary due to data limitations and computational constraints, but they also introduce model risk.

Mortgage loan data consist of monthly balances rather than individual loan contracts. To obtain observations suitable for survival analysis, mortgage loans were decomposed using a Last-In-First-Out (LIFO) assumption. New borrowing was treated as a new piece, and reductions in the mortgage loan were allocated first to the most recent. This assumption was chosen because additional borrowing is often associated with specific purposes and may therefore be expected to be repaid before older borrowing. Alternative assumptions, such as first-in, first-out (FIFO) or proportional allocation, could yield different estimates of survival distributions and behavioral maturities.

The analysis focuses on behavioral maturity rather than contractual maturity. Small amortizations were excluded to avoid modeling mandatory amortization requirements. As a result, the estimated survival curves describe customer prepayment behavior rather than the complete contractual repayment process.

Survival models were estimated at the aggregate portfolio level. Individual borrower characteristics, such as income, age, loan-to-value ratio, geographic location, and property type, were not included. This approach was chosen because the objective was to estimate portfolio-level interest-rate sensitivity rather than individual customer behavior.

The pass-through was assumed to be a constant over time. In practice, pass-through behavior may change due to competition, funding conditions, regulatory changes, or shifts in customer behavior. Therefore, the estimated duration should be interpreted conditionally on the historical pass-through relationship remaining approximately unchanged.

The valuation framework assumes that all remaining mortgages are repaid after 30 years. This assumption was introduced because forward interest-rate data were only available up to a 30-year horizon and because mortgage loans cannot realistically remain active indefinitely. Although relatively few mortgage exposures survive to very long horizons, the assumption affects the tail of the maturity distribution and therefore influences estimated expected maturities and durations.

5.3 Behavioral interpretation

The thesis has mainly focused on interest rate risk in a mortgage loan portfolio. However, certain factors may lead to prepayment behavior. The main driver that was studied was mortgage interest rates. A comparison was made between leaving the bank and changing the interest rate term. This, in addition to how the spread between mortgage interest rates between terms affected switching behavior. 30% churn and 70% switch mortgage terms. A customer is therefore more likely to change their terms on their mortgage loan than to churn.

The spread between three-month and longer-term interest rates drives outflows into the respective longer-term instruments. Implying that customers are more likely to change mortgage loan terms if it is likely to improve their interest rate. On the other hand, customers are less likely to change their interest rates when doing so would increase their mortgage interest rate.

6
Conclusion

The core finding of the thesis is that for a portfolio of mortgage loans with three-month floating interest rates, the estimated duration exceeds 3 months. The results suggest that the pass-through effect is significant. This means that three-month mortgage interest rate risk may not be minimized with three-month financing. The maturity of mortgages, therefore, needs to be modeled, because otherwise the interest rate risk is neglected.

Mortgage maturity has been modeled using survival analysis. The shifted gamma frailty model performed well in the empirical analysis and may be useful for future studies of mortgage-prepayment behavior. Different approaches were examined. The cause-specific effects of leaving the bank versus moving into different-term interest rates. Additionally, different spreads were considered to model prepayment behavior.

The models were then used to conduct different stress tests. The results suggest that the present value will decrease under parallel up shocks and steepening scenarios. The present value will increase during parallel down shocks and flattening. The short-rate scenarios don't meaningfully affect the present value. The risk is asymmetric: positive scenarios increase in value more than negative scenarios decrease during periods when the risk-free interest rate is negative.

6.1 Implications for funding and regulation

The results suggest that financing a portfolio of three-month mortgages with liabilities of two- to three-year durations may reduce potential losses by protecting against the theoretical worst-case scenarios. For a bank to minimize interest rate risk, a 2-year duration financing is suggested, structured as part 3-month duration financing and the additional 12.5 years, split roughly $70-90\%$ short duration and $10-30\%$ long duration. The ratio of fixed-rate, long-duration financing to short-duration financing shifted over time, especially under negative interest rates, when the mortgage loan portfolio behaved more like a fixed-interest-rate portfolio. For further improvement, the model can be used in full, but the 2-year $70-90\%$ 3-month and $10-30\%$ 12.5-year financing may serve as practical benchmarks derived from the estimated model.

Under current regulatory guidelines, three-month mortgage loans are considered to have a duration of three months. Banks that hold short-term financing are considered less interest-rate sensitive than banks with longer-duration funding. The results suggest that contractual repricing may underestimate behavioral sensitivity to interest rates in Swedish adjustable-rate mortgages. The thesis results suggest that behavioral maturity should be considered when assessing the interest-rate sensitivity of mortgage loan portfolios, because the maturity is unknown and therefore needs to be modeled. The current regulatory framework makes banks sensitive to interest rate changes. With non-maturing deposit modeling becoming increasingly prevalent in the industry, the asset side of banking needs to undergo similar modeling and regulatory scrutiny.

6.2 Limitations and future research

The work relies on a number of modeling assumptions. In addition to flaws in LIFO decomposition, there are assumptions. The forecasting function is static; however, it may behave differently during periods of rising versus falling interest rates. The model could be strengthened by adding covariates, with the survival function capturing macroeconomic conditions and the pass-through component modeled as a stochastic interest-rate process. However, in the Basel scenarios, the macroeconomic variables may be correlated, so this must be kept in mind.

For future research, it is important to combine a mortgage and financing model. This could be useful in modeling changes in equity under different shocks. There might even be a relationship between them. Additional parameters, such as age and income, could be used to obtain a better survival function.

6.3 Use of AI

AI tools were primarily used to support the writing process, particularly to improve language, clarity, grammar, and phrasing. AI was not used to generate ideas, develop the methodology, perform the analysis, or write the code used in this work. The underlying reasoning, modeling choices, implementation, analysis, and conclusions were carried out independently.

Bibliography

[1] *Utlåning till allmänheten*. URL: https://www.ekonomifakta.se/sakomraden/finansiell-ekonomi/lan-och-finansiering/utlaning-till-allmanheten_1213865.html.

[2] *Räntebindning för hushåll*. URL: https://www.ekonomifakta.se/sakomraden/finansiell-ekonomi/lan-och-finansiering/rantebindning-for-hushall_1213870.html.

[3] Board of Governors of the Federal Reserve System. *Review of the Federal Reserve's Supervision and Regulation of Silicon Valley Bank*. Tech. rep. 2023.

[4] *Bolåneguiden: Vad är bolån & hur funkar det? | Handelsbanken*. URL: https://www.handelsbanken.se/sv/ekonomi-i-livet/privatekonomi/boendeekonomi/kopa-bostad/bolaneguiden.

[5] "Guidelines on IRRBB and CSRBB (1)". In: ().

[6] Rahul Ghosh. *Basel Interest Rate Risk in the Banking Book: IRRBB Measurement and Management*. Independently published, Apr. 2025.

[7] Basel Committee on Banking Supervision. *Basel Committee on Banking Supervision Standards Interest rate risk in the banking book*. 2016. ISBN: 978-92-9197-498-6. URL: https://www.bis.org/basel_framework/.

[8] John P. Klein and Melvin L. Moeschberger. "Survival Analysis". In: Statistics for Biology and Health (2003). DOI: 10.1007/b97377. URL: http://link.springer.com/10.1007/b97377.

[9] Richard Van Noorden, Brendan Maher, and Regina Nuzzo. "The top 100 papers". In: *Nature News* 514.7524 (Oct. 2014), p. 550. ISSN: 14764687. DOI: 10.1038/514550a. URL: http://www.nature.com/news/the-top-100-papers-1.16224.

[10] Theodor A. Balan and Hein Putter. "A tutorial on frailty models". In: *Statistical Methods in Medical Research* 29.11 (Nov. 2020), p. 3424. ISSN: 14770334. DOI: 10.1177/0962280220921889. URL: https://pmc.ncbi.nlm.nih.gov/articles/PMC7534210/.

[11] *Gamma distribution - Wikipedia*. URL: https://en.wikipedia.org/wiki/Gamma_distribution.

[12] *Inverse Gaussian distribution - Wikipedia*. URL: https://en.wikipedia.org/wiki/Inverse_Gaussian_distribution.

[13] Kenneth P. Burnham and David R. Anderson, eds. *Model Selection and Multimodel Inference*. New York, NY: Springer New York, 2004. ISBN: 978-0-387-95364-9. DOI: 10.1007/b97636.

[14] Fidelis Musakwa and Eric Schaling. "Estimating maturity profiles of nonmaturing deposits". In: *Journal of Risk* 21.6 (Aug. 2019), pp. 101–121. ISSN: 17552842. DOI: 10.21314/JOR.2019.414.

A

Appendix 1

A.1 Censoring and truncation

Censoring and truncation for the different scenarios are presented in Table A.1. This is useful, for instance, for maximum-likelihood estimation.

Exact lifetime	$f(t)$
Right censoring	$S(t)$
Left censoring	$1 - S(t)$
Intervall censoring	$S(t_1) - S(t_2)$
Left truncation	$\frac{f(t)}{S(t)}$
Right truncation	$\frac{f(t)}{1-S(t)}$
Interval truncation	$\frac{f(t)}{S(t_1)-S(t_2)}$

Table A.1: Likelihood contributions under different censoring and truncation schemes. The table summarizes how exact, censored, and truncated observations are incorporated into the survival likelihood.

A.2 Expected lifetimes

In Table A.2, we present the expected values of the estimated survival functions. It is worth noting that cause-specific hazards don't indicate the time until a specific event. The leave asset scenario, however, shows the time until the event, assuming no other events occur. This is why the expected time to leave is so long: a customer is more likely to change terms than leave the bank.

	Expected value
Leave or fix	19.2 years
Leave	102.4 years
fix	29.8 years

Table A.2: Expected lifetimes implied by the estimated survival models. The table reports the estimated average time until termination for the combined event and for the separate cause-specific event models.

A.3 Survival functions

Some survival functions have been discussed throughout the thesis. This section has a detailed list of all of them.

A.3.1 Exponential

Probability distribution function (PDF):

$$f(t) = \lambda e^{-\lambda t}, \qquad t \geq 0, \quad \lambda > 0.$$

Cumulative distribution function (CDF):

$$F(t) = 1 - e^{-\lambda t}, \qquad t \geq 0, \quad \lambda > 0.$$

Survival function:

$$S(t) = P(T > t) = e^{-\lambda t}, \qquad t \geq 0, \quad \lambda > 0.$$

Hazard function:

$$h(t) = \frac{f(t)}{S(t)} = \lambda, \qquad t \geq 0, \quad \lambda > 0.$$

A.3.2 Pareto

Probability distribution function (PDF):

$$f(t) = \frac{\theta \lambda^{\theta}}{(t+\lambda)^{\theta+1}}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Cumulative distribution function (CDF):

$$F(t) = 1 - \left(\frac{1}{\frac{t}{\lambda} + 1}\right)^{\theta}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Survival function:

$$S(t) = P(T > t) = \left(\frac{1}{\frac{t}{\lambda} + 1}\right)^{\theta}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Hazard function:

$$h(t) = \frac{f(t)}{S(t)} = \frac{\theta}{t+\lambda}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

A.3.3 Shifted Gamma Frailty

Probability distribution function (PDF):

$$f(t) = \left[\omega\lambda + \frac{(1-\omega)\lambda}{1+\frac{(1-\omega)\lambda t}{a}}\right] e^{-\omega\lambda t}\left(1+\frac{(1-\omega)\lambda t}{a}\right)^{-a}, \qquad t \geq 0, \quad \lambda > 0, \quad a > 0, \quad 0 \leq \omega \leq 1.$$

Cumulative distribution function (CDF):

$$F(t) = 1 - e^{-\omega\lambda t}\left(1+\frac{(1-\omega)\lambda t}{a}\right)^{-a}, \qquad t \geq 0, \quad \lambda > 0, \quad a > 0, \quad 0 \leq \omega \leq 1.$$

Survival function:

$$S(t) = P(T > t) = e^{-\omega\lambda t}\left(1+\frac{(1-\omega)\lambda t}{a}\right)^{-a}, \qquad t \geq 0, \quad \lambda > 0, \quad a > 0, \quad 0 \leq \omega \leq 1.$$

Hazard function:

$$h(t) = \frac{f(t)}{S(t)} = \omega\lambda + \frac{(1-\omega)\lambda}{1+\frac{(1-\omega)\lambda t}{a}}, \qquad t \geq 0, \quad \lambda > 0, \quad a > 0, \quad 0 \leq \omega \leq 1.$$

A.3.4 Inverse Gaussian Frailty

Probability distribution function (PDF):

$$f(t) = \frac{\lambda}{\sqrt{1+2\theta\lambda t}} \exp\left\{\frac{1}{\theta}\left[1-\sqrt{1+2\theta\lambda t}\right]\right\}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Cumulative distribution function (CDF):

$$F(t) = 1 - \exp\left\{\frac{1}{\theta}\left[1-\sqrt{1+2\theta\lambda t}\right]\right\}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Survival function:

$$S(t) = P(T > t) = \exp\left\{\frac{1}{\theta}\left[1-\sqrt{1+2\theta\lambda t}\right]\right\}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

Hazard function:

$$h(t) = \frac{f(t)}{S(t)} = \frac{\lambda}{\sqrt{1+2\theta\lambda t}}, \qquad t \geq 0, \quad \lambda > 0, \quad \theta > 0.$$

DEPARTMENT OF MATHEMATICS
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden
www.chalmers.se

AVANCEZ
1829
CHALMERS
UNIVERSITY OF TECHNOLOGY