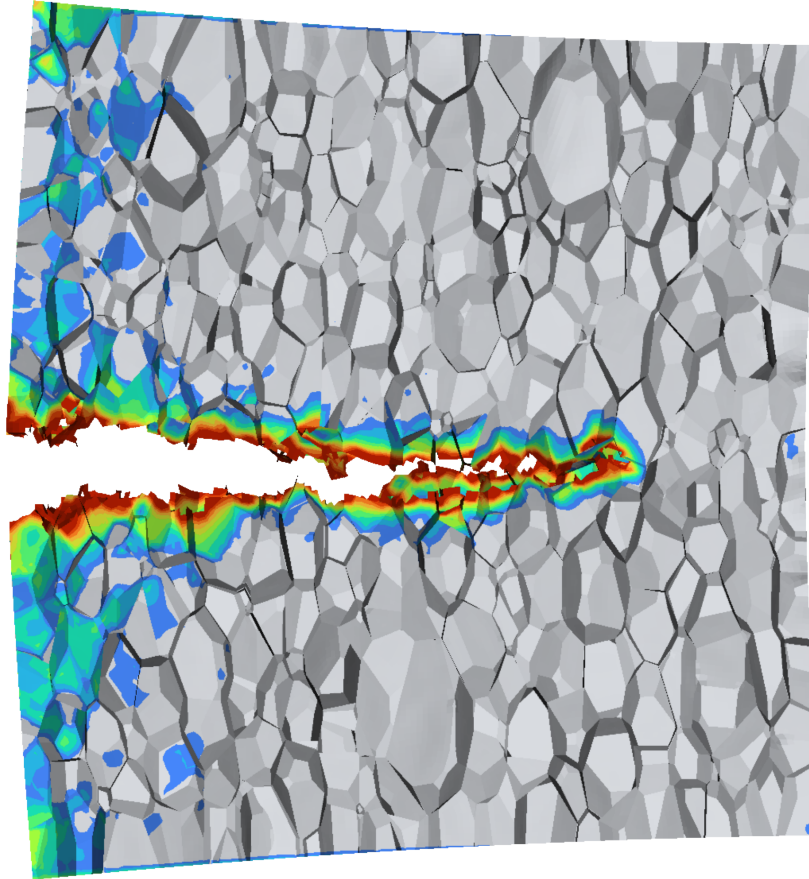




CHALMERS
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Predicting the Mechanical Response of H100 PVC Foam from Mesostructural Finite Element Analysis

Paulo Liborio
Sheyma Salim

DEPARTMENT OF INDUSTRIAL AND MATERIALS SCIENCE

CHALMERS UNIVERSITY OF TECHNOLOGY
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MASTER'S THESIS 2026

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PAULO LIBORIO
SHEYMA SALIM



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Supervisors: Erik Svenning, FS Dynamics
Carl Lindholm, Diab
Examiner: Martin Fagerström, Industrial and Materials Science

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Department of Industrial and Materials Science
Division of Material and Computational Mechanics
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

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Abstract

This thesis presents a mesoscale finite element (FE) framework for predicting the mechanical response and fracture behavior of Divinycell H100 foam used as a core material in sandwich structures. Stochastic multi-cell statistical volume element (SVE)s were generated using tessellation-based approaches informed by morphological data from literature and CT-based characterization studies. The numerical framework was calibrated against experimentally reported tensile and compressive engineering stress–strain responses using an elasto-plastic constitutive model for the foam cell walls, implemented in *LS-DYNA*.

Following calibration, a parametric study was conducted to investigate the influence of key mesostructural parameters, including SVE size, cell shape anisotropy, varying cell equivalent diameter, and varying cell wall thickness distribution. The results showed that these parameters influence both the homogenized mechanical response and the fracture behavior of the foam. Cell shape anisotropy, in particular, showed the largest influence in observed macroscopic mechanical anisotropy.

A loading configuration was implemented to investigate core-dominated fracture behavior in sandwich structures, employing a strain based element erosion criterion, inspired by double cantilever beam (DCB) tests.

The developed framework successfully reproduced several experimentally observed trends and demonstrated the strong relationship between foam morphology and macroscopic response. In future work, this thesis can serve as a reference for investigating how morphological features affect the mechanical response of foam cores and how their implementation in the modified DCB test influences core fracture. It provides a basis for understanding, assessing, and comparing the effects of different morphological characteristics.

Keywords: Divinycell H100 Foam, Statistical Volume Element, Finite Element Method, LS-DYNA, Mesostructural Modeling, Material Calibration.

Acknowledgments

The authors would like to express their sincere gratitude to everyone who contributed to this thesis project.

First and foremost, we would like to thank *FS Dynamics*, and especially Emina Musić, for providing us with the opportunity to conduct this thesis project in collaboration with industry. We are also deeply grateful to *Diab*, particularly Carl-Johan Lindholm, for the valuable discussions, provided material data, and the opportunity to visit the production facilities, which gave us a much deeper understanding of structural foam materials and their complexity.

Special thanks are extended to Erik Svenning for his exceptional supervision, continuous guidance, and encouragement throughout the project. His ability to constantly challenge our understanding and inspire new investigative directions greatly contributed to both the quality of this work and our learning experience.

We would also like to thank Martin Fagerström for his guidance and support during the project, as well as Lei Liu for generously sharing expertise, scripts, and reference data that were highly valuable for the development of the numerical framework.

Finally, we gratefully acknowledge *BETA CAE Systems* for providing academic licenses for *ANSA* and *META*.

Use of Artificial Intelligence Tools

Large language models and language assistance tools, including *ChatGPT*, *Gemini*, and *DeepL*, were used during the preparation of this thesis for language-related support, including proofreading, grammar correction, improvement of readability, and refinement of transitions between sections. These tools were not used to generate technical content, perform analyses, interpret results, develop methodologies, or draw conclusions. All scientific content, engineering judgments, analyses, and conclusions presented in this thesis are the sole responsibility of the authors.

Paulo Liborio and Sheyma Salim

Gothenburg, June 2026

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

ASA	adaptive simulated annealing
CDF	cumulative distribution function
CT	computed tomography
CV	coefficient of variation
DCB	double cantilever beam
DDBC	Direct Displacement Boundary Conditions
FE	finite element
FEA	finite element analysis
FEM	finite element method
IPN	interpenetrating polymer network
KUBC	Kinematic Uniform Boundary Conditions
OMBC	Orthogonal-Mixed Boundary Conditions
PBC	Periodic Boundary Conditions
PDF	probability density function
PVC	polyvinyl chloride
RVE	representative volume element
SEM	scanning electron microscopy
SENB	single-edge notch bending
SRSM	sequential response surface method
SVE	statistical volume element
UTBC	Uniform Traction Boundary Conditions

Nomenclature

Roman Symbols

A_{bb}	Bounding box reference area
a	Initial crack length
b	Specimen width
C	Displacement rate
c	Remaining uncracked length
d_{11}	First component of the bending compliance matrix
d_{cell}	Equivalent cell size / diameter
$E_{1,2}$	In-plane Young's modulus
E_c	Compressive modulus
E_t	Tensile modulus
E_{wall}	Cell wall Young's modulus
E_x	Effective flexural modulus
F	Reaction force
G_{12}	Shear modulus
H	Tangent hardening modulus
h	Half-thickness of the foam and adherends / Characteristic element edge length
k_c	Analytical foundation modulus
L_0	Initial SVE length
L_{SVE}	Characteristic length of the SVE
N_e	Element count
P	Applied opening load
$\mathcal{R}_U$	Toughness ratio
$\mathcal{R}_w$	Cell shape anisotropy ratio
R_i	Rotational degree of freedom in direction i
S	Sphericity
SF	Scaling Factor
t	Time
$t^{(e)}$	Uniform element thickness
t_{wall}	Cell wall thickness
U_i	Translational degree of freedom in direction i
U_T	Bulk toughness
V_{SVE}	Volume of the SVE

$w(x)$ Transverse deflection function

Greek Symbols

α	Reference coordinate for maximum opening displacement
δ	Target prescribed displacement
ΔL	Applied displacement
ε_p	Effective plastic strain
$\langle \varepsilon_p \rangle$	Volume-averaged effective plastic strain
$\hat{\varepsilon}$	Homogenized engineering strain
ε_u^{eff}	Ultimate effective strain
ε_y^{eff}	Yield effective strain
λ	Dimensionless elastic foundation variable
ν_{12}	In-plane Poisson's ratio
ν_{13}	Through-thickness Poisson's ratio
ν_{wall}	Cell wall Poisson's ratio
ρ	Density
ρ_{act}	Actual material density
ρ_{nom}	Nominal density
σ_c	Compressive strength
σ_f	Arbitrary failure stress limit
$\hat{\sigma}$	Homogenized engineering stress
σ_t	Tensile strength
σ_u	Ultimate stress
σ_y	Yield stress
$\sigma_{y,wall}$	Cell wall yield stress
τ_{12}	Shear strength

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1

Introduction

This thesis, conducted in collaboration with FS Dynamics and their Industrial partner Diab, investigates the relationship between the mesostructural morphology and the macroscopic mechanical response of Divinycell H100 foam through a stochastic finite element modeling framework. Multi-cell SVEs were generated based on experimentally reported morphological characteristics and used to simulate the tensile, compressive, and fracture behavior of the foam. After calibration against experimental data, the framework was employed to assess the influence of key morphological parameters on homogenized material properties and damage evolution. The study provides insight into the mechanisms governing the behavior of cellular polymer foams and demonstrates the potential of mesoscale modeling as a predictive tool for the analysis and design of foam-cored sandwich structures.

1.1 Background

Sandwich structures are lightweight, high-stiffness constructions designed to maximize load-bearing capacity and structural rigidity while minimizing mass. A typical sandwich configuration consists of two stiff outer face sheets bonded to a lightweight core. The performance of sandwich structures is governed not only by the face sheets but also by the mechanical behavior of the core material and its interaction with the faces. Specifically, the core dictates the global bending stiffness, shear transfer, and stability of the construction. An example of foam core sandwich structure is illustrated in Figure 1.1.

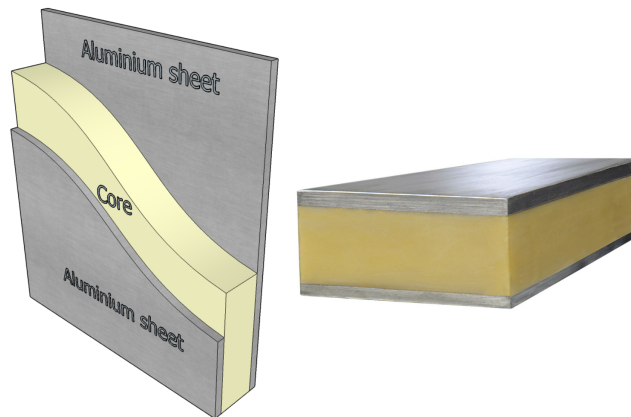


Figure 1.1: Example of a foam core sandwich structure.

The primary function of the foam core is to separate and support the two load-carrying face sheets while maintaining their relative positioning. By providing sufficient out-of-plane stiffness, the core prevents local deformations, such as the inward or outward buckling of the face sheets. Furthermore, the core must exhibit adequate shear rigidity to limit relative sliding between the face sheets. When sufficient shear stiffness is achieved, the face sheets act as a single structural unit; conversely, inadequate shear stiffness compromises this interaction, leading to a significant reduction in overall bending stiffness. This separation of the face sheets dramatically increases the structural stiffness and bending strength of the assembly, while maintaining a low overall weight. Consequently, compared to conventional solid structures like steel plates, sandwich constructions offer superior mechanical performance at a substantially reduced mass [1], [2]. This leads to high stiffness-to-weight and strength-to-weight ratios, making sandwich structures highly attractive for weight-critical engineering applications, including marine vessels, wind turbine blades, aerospace components, and transportation systems [3]. Figure 1.2 shows four different grades of foam core materials, manufactured by Diab.



Figure 1.2: Foams core samples of different grades and categories.

Given their widespread use, it is critical to thoroughly understand the material properties, mechanical behavior, and structural response of foam core sandwich structures under various loading conditions. Equally important is identifying their performance limits during service. These limits are governed by failure modes arising from internal factors, such as manufacturing defects, or external influences, including mechanical loading and environmental exposure. By understanding these failure mechanisms, the mechanical performance of sandwich structures can be optimized and reliably reproduced using modern numerical frameworks.

Under mechanical loading, several deformation and failure mechanisms may develop, including face wrinkling, core shear deformation, delamination (face-core debonding), and core fracture. Delamination, defined as the separation at the interface between the foam core and the face sheets, can be initiated by external factors like impact loading, cyclic fatigue, or environmental degradation, as well as internal factors like imperfect bonding [1].

Historically, sandwich structures relied on honeycomb cores, which provide excellent mechanical performance but are relatively costly and complex to manufacture. The introduction of polymeric foam cores provided a more economical and versatile alternative. Compared to honeycombs, polymeric foams offer easier processing, improved damage tolerance, thermal insulation, and broader compatibility with composite manufacturing methods [4]. However, foam cores also present limitations, including a lower absolute strength than honeycomb structures and a higher sensitivity to failure mechanisms such as crushing, tensile fracture, and delamination [5].

A comprehensive understanding of their benefits and drawbacks is essential for the effective design of foam-cored sandwich structures. This knowledge directly informs the prediction of structural performance and damage resistance. In the present thesis, a DCB-inspired load case is developed to investigate the Mode I fracture response, with a particular emphasis on isolating and analyzing the mechanical behavior of the foam core itself.

The structural foam core investigated in this thesis is Divinycell H100, a PVC-based

foam incorporating an interpenetrating polymer network (IPN) structure, produced by Diab, an international company with over 70 years of experience developing structural foam cores for industrial applications.

1.2 Scope and Limitations

The scope of this Master’s thesis is constrained by available time, computational resources, and the capabilities of the employed modeling tools. The adopted assumptions and simplifications aim to establish a focused numerical framework while retaining the key mechanisms that govern the mechanical response and failure of foam cores of sandwich structures. The principal limitations of the study are listed below:

- **Scope and Methodological Limitations:** The investigation is confined to mesoscale modeling of the foam core using SVEs and does not constitute a fully coupled multiscale approach. Macroscale implications are inferred from mesoscale results rather than obtained through explicit scale-bridging techniques. The software packages mostly used are *ANSA* [6] for pre-processing, *LS-DYNA* [7] for FE solution, *LS-OPT* [8] for material parameter identification, and *META* [9] for post-processing.
- **Constitutive Modeling and Element Formulation:** The foam cell walls are modeled using *LS-DYNA*’s **MAT_024* material law, which represents an elasto-plastic constitutive relationship with strain hardening. The finite element discretization utilizes fully integrated first order elements. The modeling approach is intentionally standard; the primary objective is to capture the homogenized mesoscale response rather than to perform an exhaustive numerical method comparison. While alternative formulations could influence local numerical accuracy, they are not expected to significantly alter the homogenized response targeted in this work.
- **Modeling Assumptions:** The foam material is analyzed independently, without explicitly modeling the face sheets or adhesive layers. Uniaxial virtual testing calibrates the mechanical response of the foam against experimental and literature data. Core fracture is subsequently investigated using a numerical DCB-inspired configuration, restricting the analysis to core-dominated failure under peel-type loading.
- **SVE Geometric Idealization:** SVEs are generated via tessellation-based algorithms driven by morphological descriptions informed by X-ray computed tomography (CT) data, using the *Neper* software package [10]. Although the geometries closely replicate key morphological features of an actual foam, they are statistically representative approximations rather than one-to-one reconstructions.
- **Cell Wall Thickness Representation:** The current approach applies uniform thickness throughout the structure. An investigation of varying thickness distribution along cell walls, without explicitly modeling spatial variations from foam cell to cell, is conducted in the form of a sensitivity analysis.
- **Excluded Mesostructural Features:** Several mesostructural parameters known to influence foam behavior, including local cell wall defects, wall corru-

gation, global wall curvature, and individual cell elongation orientations—are excluded. Their effects under uniaxial and peel-dominated loading conditions require dedicated investigations outside the scope of this thesis.

- **Scale Effects and Representativeness:** Because only a limited number of SVEs were computationally feasible, the homogenized behavior may not capture the absolute limits of the foam’s mesostructural variability. Consequently, some extreme interactions between the mesoscale behavior and the macroscale response may be omitted.
- **Exclusion of Shear Loading:** While tensile properties are highly relevant for core fracture and delamination, shear loading was excluded. Prioritizing the tensile response limited the scope to a manageable level for evaluating a peel-driven load case. Investigating a representative shear-dominated load case would require an expanded numerical setup.
- **Fracture Modeling and Regularization:** The fracture simulations rely on strain-based element erosion without an accompanying regularization framework. Although a few regularization approaches, including nonlocal averaging (MAT_NONLOCAL) and element-size strain scaling (LCREGD), were investigated, their implementation and validation within the SVE framework could not be completed within the scope of this thesis. Furthermore, extending the regularization framework to explicitly account for varying SVE domain sizes was not pursued due to the additional implementation, verification, and validation effort required. Consequently, the adopted erosion criterion is valid only for the specific SVE geometry, mesh density, and element formulation employed in the final fracture analyses. The failure parameters should therefore not be assumed to be directly transferable to different SVE sizes, mesh resolutions, or discretization strategies without further verification and calibration.
- **Time and Resource Limitations:** Emphasis is placed on developing and validating a robust baseline modeling framework. Extensive parametric studies, advanced material descriptions, or enhanced geometric realism are left for future work.

Despite these limitations, the adopted framework provides a basis for identifying key trends in foam core mechanical response upon the variation of fundamental morphological parameters.

1.3 Objectives

The primary objective of this thesis is to develop, calibrate, and evaluate a mesoscale finite element analysis (FEA) framework based on SVEs for predicting the mechanical response and failure behavior of PVC foam cores in sandwich structures.

To achieve this, the following specific objectives are defined:

- Develop a detailed mesoscale SVE model of Divinycell H100 to represent the cellular architecture of the foam core.
- Calibrate the model to accurately capture the macroscopic stress–strain behavior in tension and compression by correlating simulations with experimental data from literature and Diab.

- Introduce and tune an element strain-based erosion criterion to reproduce progressive damage and failure behavior.
- Apply the calibrated framework to a DCB-inspired configuration to investigate core-dominated fracture behavior.
- Conduct a parametric study to evaluate the influence of key mesostructural parameters (e.g., SVE size, cell size distribution, and cell wall varying thickness) on uniaxial stress–strain response and crack onset and progression.

Ultimately, this work aims to establish a validated, modular numerical methodology that deepens the understanding of foam core deformation and fracture mechanisms.

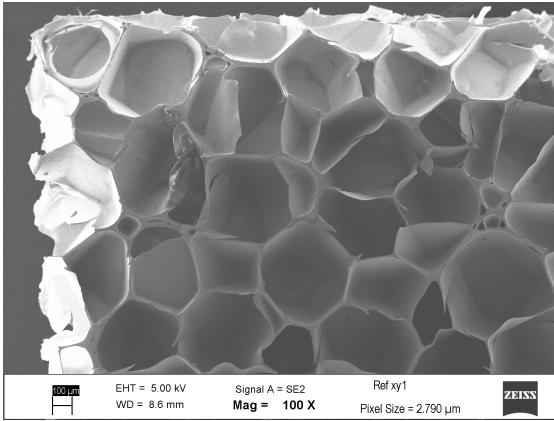
1.4 Literature Review

This literature review establishes the theoretical and experimental background required for the development of the numerical framework presented in this thesis. The review begins with an overview of Divinycell H100 PVC foam, emphasizing its cellular morphology, anisotropic behavior, and key mechanical properties. Subsequently, experimental characterization methods commonly employed to evaluate foam core materials and sandwich structures are discussed, with particular attention given to fracture testing techniques relevant to core-dominated failure. Finally, mesoscale modeling approaches for cellular solids are reviewed, covering homogenization concepts, representative and statistical volume element formulations, and stochastic geometric representations used to capture the influence of foam morphology on macroscopic mechanical response.

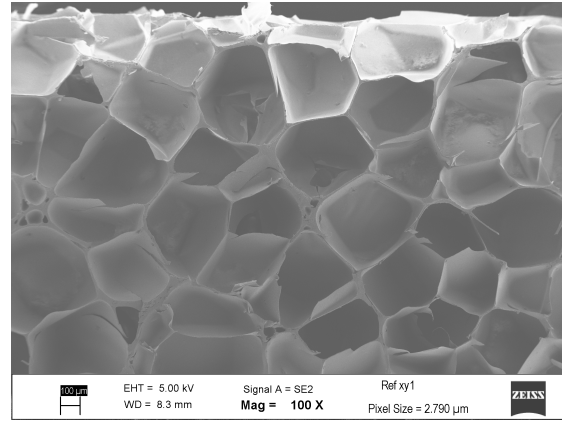
1.4.1 Divinycell H100 Mechanical Properties

Divinycell H100 is a closed-cell structural foam incorporating a physical IPN formed by cross-linked polyurea (or polyamide) and PVC. The material comprises approximately equal proportions of these two polymers, resulting in enhanced toughness and damage resistance compared to standard PVC foams. The controlled manufacturing process, involving forming, heating, and shaping, dictates the final cellular architecture and, consequently, the material properties [11], [12], [13].

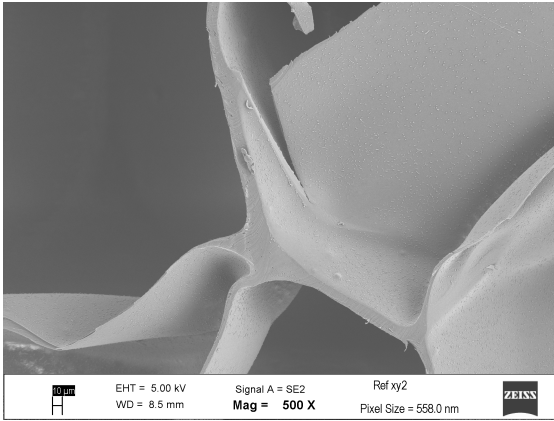
The macroscopic mechanical response is strongly influenced by mesostructural characteristics such as equivalent diameter, apparent density, sphericity, and cell shape anisotropy [14], [15]. Divinycell H100 exhibits pronounced cell shape anisotropy, or elongation, along its rise direction. This geometric anisotropy translates into direction-dependent mechanical behavior: the foam generally exhibits higher stiffness and strength in the foam rise direction compared to the transverse directions, rendering the material transversely isotropic [2]. The cellular structure of this foam is visualized in the SEM obtained image in Figure 1.3 .



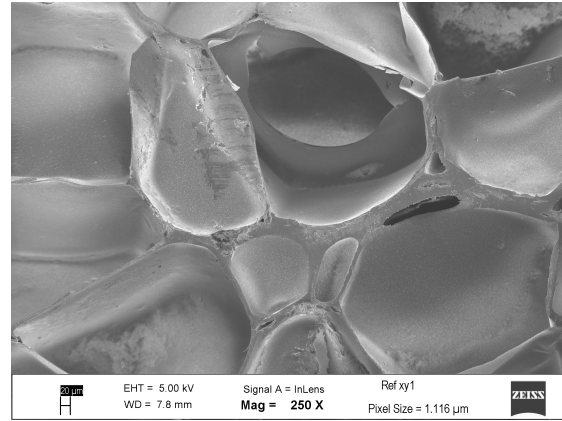
(a) Section perpendicular to the foam rise direction.



(b) Section perpendicular to the foam rise direction.



(c) Close-up image of foam cell walls.



(d) Close-up image of foam cell walls.

Figure 1.3: SEM images illustrating the cellular mesostructure of Divinycell H100.

The foam mesostructure consists of three primary components: the cells (gas-filled voids), the cell walls (membranes separating adjacent cells), and the cell edges or struts (intersections of multiple walls) [11], [12], [16]. Manufacturing effects also introduce local wall corrugation, curvature, and inclination [15], [17]. Tables 1.1 and 1.2 summarize the macroscopic Divinycell H100 parameters reported by Diab and literature, respectively. These values serve as the reference basis for the numerical investigation conducted in this work.

Table 1.1: Manufacturer-reported macroscopic properties for Divinycell H100 PVC foam.

Property	Symbol	Units	Nominal	Minimum
Density	ρ	kg/m ³	100	90
Poisson ratio (through-thickness)	ν_{13}	—	0.40	—
Tensile strength ¹	σ_t	MPa	3.5	2.5
Tensile modulus ¹	E_t	MPa	130	105
Compressive strength ¹	σ_c	MPa	2.0	1.65
Compressive modulus ¹	E_c	MPa	135	115
Shear strength	τ_{12}	MPa	1.6	1.4
Shear modulus	G_{12}	MPa	35	28

Source: Diab manufacturer datasheet [18].

¹ Measured in the through-thickness direction.

Table 1.2: Ranges of reported macroscopic properties for Divinycell H100 PVC foam from the literature.

Property	Symbol	Units	Range
In-plane modulus	$E_{1,2}$	MPa	39–67 ^{a,b,c}
Poisson ratio (in-plane)	ν_{12}	—	0.36–0.40 ^{a,b,c}
Poisson ratio (through-thickness)	ν_{13}	—	0.38–0.42 ^{a,b}
Tensile strength ¹	σ_t	MPa	2.0–4.0 ^{b,c}
Tensile modulus ¹	E_t	MPa	98–141 ^{a,b,c}
Compressive strength ¹	σ_c	MPa	0.99–1.35 ^c
Compressive modulus ¹	E_c	MPa	53–92 ^c
Shear strength	τ_{12}	MPa	1.44 ^b
Shear modulus	G_{12}	MPa	32.5 ^b

^a Funari et al. [19].

^b Taher et al. [20].

^c Shafiq et al. [21].

¹ Measured in the through-thickness direction.

1.4.2 Experimental Characterization Methods

The mechanical behavior of foam core materials is typically characterized using standardized test methods, including uniaxial tension, compression, and shear testing [22], [23], [24]. These techniques extract the elastic properties, strength parameters, and failure characteristics required for modeling sandwich structures [21].

At the macroscale, the mechanical performance of full-scale sandwich panels is evaluated using fracture and delamination tests. Configurations such as the single-edge

notch bending (SENB) [25] and DCB [26] tests are widely utilized to assess crack propagation and fracture toughness [12], [27]. These methods enable the controlled initiation and propagation of cracks either at the face–core interface or within the foam itself.

Among these, the DCB test (see Figure 1.4) is standard for characterizing Mode I fracture and peel-driven delamination. Its ability to isolate core-dominated failure under tensile loading makes it highly suitable for evaluating foam fracture resistance, and it is heavily favored in industrial testing practices [1].

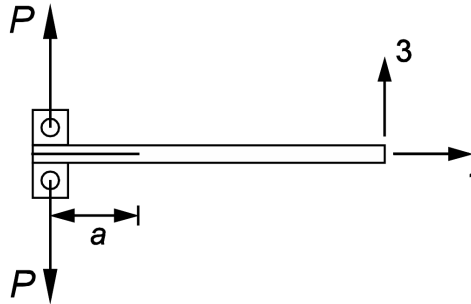


Figure 1.4: Schematic of a DCB test configuration, adapted from Zenkert [1].

1.4.3 Mesoscale Modeling

Multiscale modeling is a computational approach that links material behavior across different length scales. The macroscale views a system as a continuous, homogeneous medium where properties are averaged. The intermediate level is the mesoscale, which groups particles or geometric features into larger clusters, networks or polycrystalline structures to balance detail and efficiency. Finally, at the lowest and most detailed level, there's the microscale, which can simulate individual sub-granular structures, discrete dislocation and atomistic lattice structures. In cellular materials like PVC foams, the macroscopic continuum response is fundamentally governed by the deformation and failure mechanisms of the underlying cellular morphology at the mesoscale. Consequently, the bulk behavior of the foam can be predicted by analyzing its mesostructure and homogenizing the results to obtain equivalent macroscopic properties (Figure 1.5).

A cornerstone of this transition is the first-order homogenization assumption, also known as the macro-homogeneity or long-wavelength assumption. Under first-order homogenization, the macroscopic strain and stress fields are defined as the volumetric averages of their microscopic counterparts over the mesoscale domain. This approach assumes that the global deformation field varies smoothly across the sample and that any localized fluctuations on the mesoscale are negligible relative to the bulk volume. Thus, the effective macroscopic response becomes a function of the foam's relative density and morphological characteristics, rather than the absolute dimensions of the individual cells.

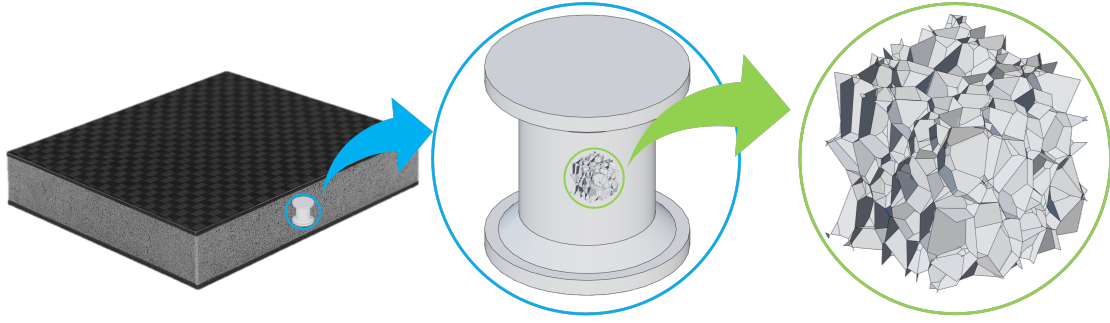


Figure 1.5: Multiscale modeling framework illustrating the transition from macroscale structure to mesoscale cellular representation.

To model the mesoscale behavior, volume element approaches are employed. A representative volume element (RVE) is defined as the smallest material subvolume that statistically captures all essential morphological characteristics, yielding a unique, converged macroscopic response independent of boundary conditions. In contrast, an SVE is a deliberately smaller, stochastic subvolume designed to capture local mesostructural variability and response scatter.

While RVEs are ideal for determining averaged bulk properties, SVEs are essential for investigating stochastic material response, fracture initiation, and the influence of local microstructural heterogeneities on failure behavior in irregular cellular morphologies [28]. Real Divinycell H100 foams exhibit significant randomness in cell geometry and size distribution. Therefore, this study adopts an SVE-based stochastic multi-cell strategy, allowing for the explicit resolution of local stress concentrations and morphological irregularities [29], [30]. Furthermore, SVEs can also be used to obtain representative response, if enough samples are statistically analyzed.

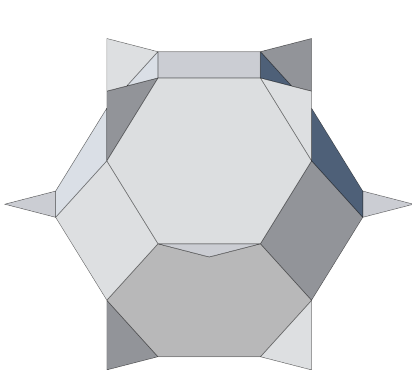
In practice, an SVE is subjected to specific boundary conditions, treating the subvolume as an equivalent homogeneous specimen to extract effective properties. In this study, SVEs are used to reproduce the engineering stress–strain behavior and to capture distinct mesoscale deformation mechanisms, where the underlying constitutive parameters are calibrated to match the observed macroscale response [30]. Mesoscale modeling generally utilizes either single-cell or multi-cell approaches. The single-cell method (or repeating unit cell) assumes a perfectly periodic lattice. While computationally efficient for extracting average elastic-plastic responses, single-cell models inherently suppress statistical variability. They fail to represent interactions between neighboring cells, local stress concentrations, and weakest-link global failure [31].

Conversely, a multi-cell SVE incorporates a statistical distribution of cells. This larger domain simulates localized mechanical interactions and response dispersion. While the absolute size of an RVE dictates full statistical convergence (often requiring 200 to 1400 cells to achieve stability without inertial effects [17], [31]), SVEs require multiple realizations to quantify the response variability arising from this randomness [32].

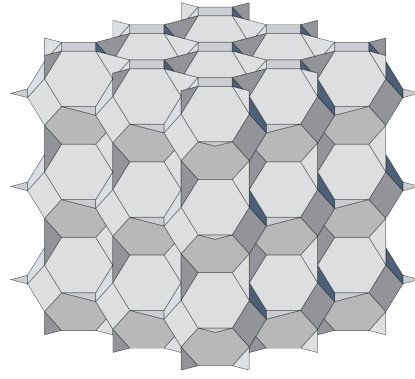
1.4.3.1 Idealized and Stochastic Geometries

Historically, theoretical benchmarks for cellular foams relied on idealized space-filling geometries, such as the Kelvin tetrakaidecahedron and the Weaire–Phelan structure (see Figure 1.6). The Kelvin model is favored for its computational simplicity but introduces artificial anisotropy at low densities and potential stress–strain instability in post-yield regimes [33], [34]. The Weaire–Phelan structure provides a slightly closer resemblance to real foams due to mixed cell types, but its average mechanical response differs only marginally from the Kelvin model [33], [35], [36].

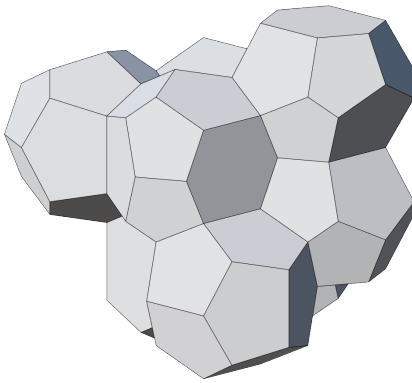
Because real foams possess inherently stochastic morphologies, periodic unit-cells fail to capture vital localization effects. To bridge this gap, stochastically generated models based on Voronoi and Laguerre tessellations are widely utilized [15], [37]. Laguerre tessellations explicitly incorporate randomness in cell geometry, topology, and spatial distribution, providing a superior representation for SVE formulations.



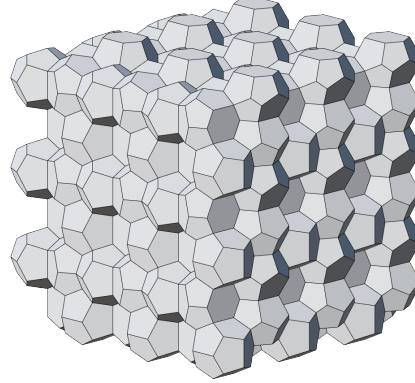
(a) Kelvin unit cell.



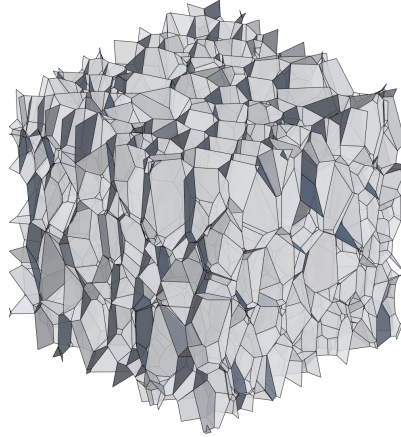
(b) Kelvin structure



(c) Weaire-Phelan cell.



(d) Weaire-Phelan structure.



(e) Laguerre tessellation structure.

Figure 1.6: Comparison of idealized and stochastic foam geometries.

Although tessellations accurately approximate the elastic response, a sufficiently large number of cells is required for statistical representativeness. While thresholds vary by formulation and loading condition [15], [37], [38], studies indicate convergence for basic effective properties typically requires between 85 and 120 cells. The specific morphological parameters used for SVE generation in this work, including topology, size distribution, sphericity, and shape anisotropy, are derived from established literature [12], [14]. The morphology of the PVC foam is characterized us-

ing three-dimensional descriptors obtained from CT-reconstructed microstructures. In particular, sphericity (S) quantifies how closely a cell approximates a spherical shape in three dimensions, while the shape anisotropy ratio ($\mathcal{R}_w$) describes cell elongation as the ratio between the largest and smallest principal cell dimensions. The corresponding mesoscale parameters for the Divinycell H100 foam are summarized in Table 1.3.

Table 1.3: Mesoscale and geometric material properties of Divinycell H100 PVC foam.

Property	Symbol	Value	Units
Cell size	d_{cell}	0.49 ± 0.06^b	mm
Cell wall thickness ¹	t_{wall}	7.47 ± 3.10^b	μm
Cell wall Young's modulus	E_{wall}	2700 ^a	MPa
Cell wall yield stress	$\sigma_{y,wall}$	62 ^a	MPa
Cell wall Poisson's ratio	ν_{wall}	0.38 ^a	–
Wall thickness (2D average) ²	t_{2D}	11.35 ^a	μm
Wall thickness (3D average) ³	t_{3D}	36.30 ^a	μm
Actual density (sample) ⁴	ρ_{act}	95.05 ^a	kg/m^3
Sphericity	S	0.78 ± 0.09^a	–
Shape anisotropy ratio	$\mathcal{R}_w$	1.20 ^a	–
Nominal density ⁴	ρ_{nom}	100 ^a	kg/m^3

^a Derived from Zhou et al. [14]. Reported values correspond to averages where applicable.

^b Data obtained from Saenz et al. [12] utilizing SEM analysis.

² 2D average wall thickness calculated from planar cross-sections (Planes XY/XZ).

³ 3D average wall thickness extracted from volumetric CT data; value is inflated due to boundary blurring introduced by Gaussian denoising filters.

⁴ Nominal density denotes the commercial grade specified by the manufacturer (Diab Group), whereas actual density reflects the empirical measurement obtained by weighing 30 mm cubic specimens.

The parameters used to represent the morphology of the foam core in this thesis, and employed in the numerical modeling, are based on a unpublished study in preparation during the writing of this thesis (Feb-Jun 2026): “Quantitative characterization and consistent generation of cell wall geometrical imperfections in low-density closed-cell foams”. As this work had not been published at the time of writing, the reported values should be regarded as preliminary and may be subject to revision in future versions of the study. The resulting morphological parameters adopted in this work are the following:

- Cell shape anisotropy ratio: $\mathcal{R}_w = 1.29$
- Sphericity: $S = 0.765$
- Bimodal cell size distribution:
 - Small cells: mean 0.145, standard deviation 0.095
 - Large cells: mean 0.495, standard deviation 0.115

2

Theory

2.1 Mechanical Properties of the Foam Core Material

As introduced in Subection 1.4.1, the foam investigated in this work is Divinycell H100. Owing to its manufacturing process, the foam exhibits a transversely isotropic cellular morphology, characterized by elongated cells primarily oriented along the foam rise direction (see Figure 2.1). This morphological anisotropy gives rise to direction-dependent mechanical properties at the macroscale. As such, the stress–strain response depends heavily on both the loading mode and the loading orientation.

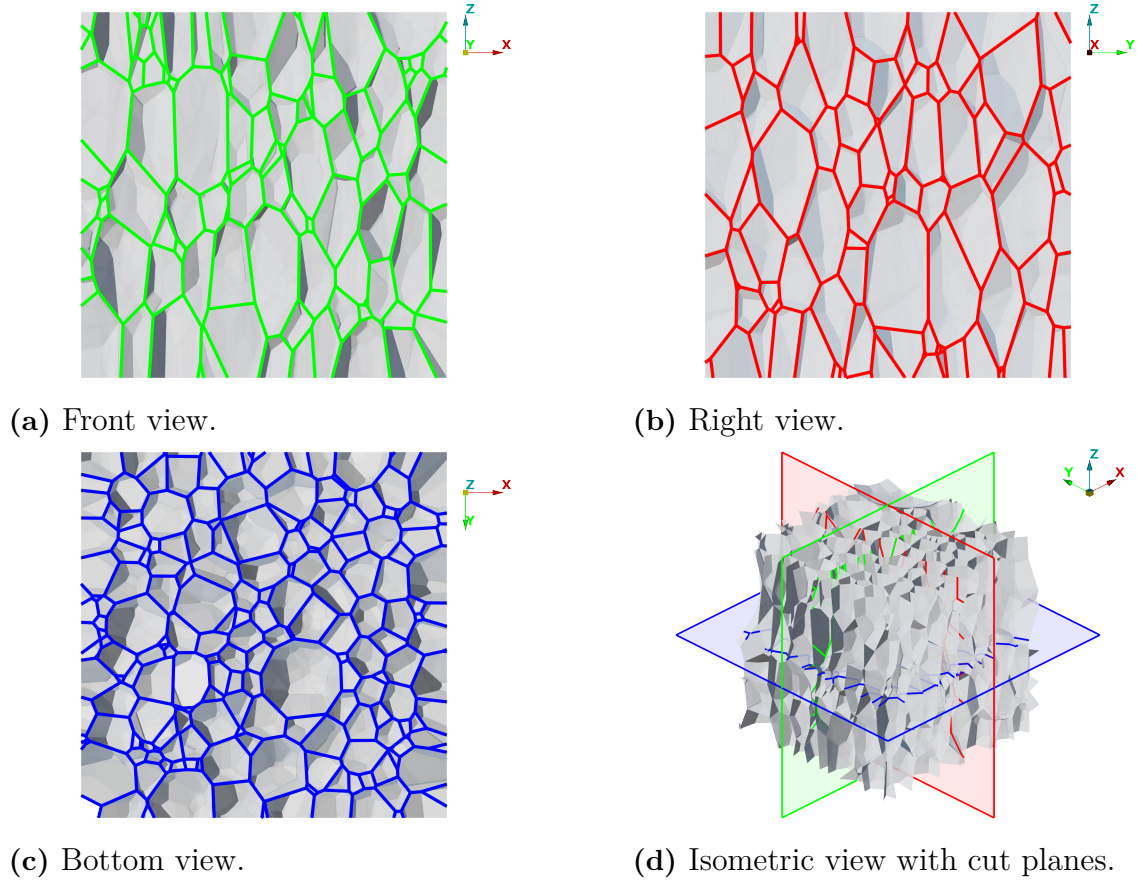


Figure 2.1: Third-angle orthographic cut view of a representative foam SVE.

In Figure 2.1, the Z -axis corresponds to the rise direction, also referred to as the out-of-plane or 33-direction. Owing to the foam's transverse isotropy, the X - and Y -axes exhibit mechanically equivalent behavior and are collectively referred to as the in-plane, transverse, or 11-direction.

Under compression, closed-cell PVC foams typically exhibit three characteristic regimes: an initial increase in stress, a prolonged stress plateau, or crushing, and a final densification stage. The initial region is governed by the membrane stretching and bending of the cell walls. With increasing strain, localized elastic buckling or plastic collapse initiates in the weakest cell walls and progressively propagates throughout the structure, generating a relatively constant stress plateau. At large compressive strains, the cell walls physically compact and the structure densifies, resulting in a steep increase in stiffness and stress (see Figure 2.2) [39].

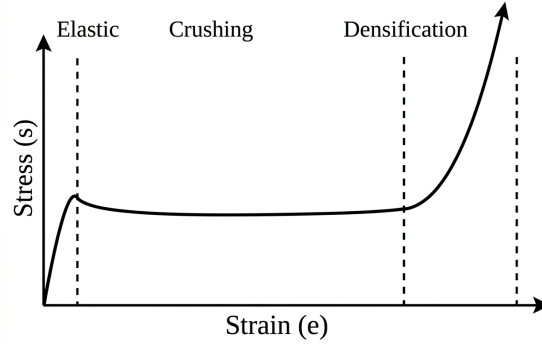


Figure 2.2: Typical compressive engineering stress–strain curve of a polymer foam.

In tension, the macroscopic response differs significantly. PVC foams generally exhibit a nearly linear increase in stress followed by some stiffness loss, then an abrupt failure, behaving in an apparently brittle manner at the macroscale. This contrast between compressive and tensile behavior originates from the underlying mesoscale deformation mechanisms. In compression, cell walls and struts primarily buckle or collapse; in tension, they are stretched, leading to stress concentrations and subsequent early fracture (see Figure 2.3a) [40].

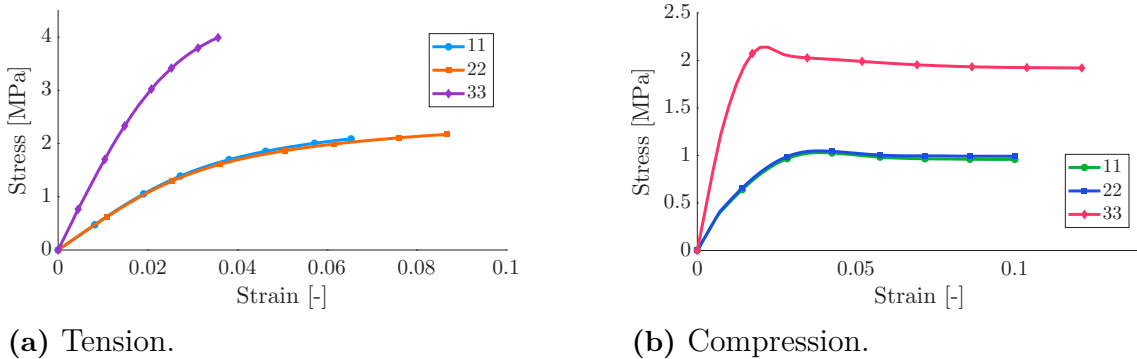


Figure 2.3: Experimental engineering stress–strain responses of PVC H100 foam.

Although PVC foams exhibit an apparently brittle tensile response at the macroscopic scale, the intrinsic solid cell wall material does not behave in brittle manner.

The load-bearing network consists of cell struts and membranes composed of a PVC-based material that is assumed to exhibit elastic–plastic behavior in tension prior to localized fracture [12], [41]. In Divinycell H100, this behavior is further enhanced by a PVC-polyurea IPN, an architecture specifically designed to improve toughness and damage resistance.

However, due to the cellular morphology, stress concentrations arising from geometric irregularities, flaws, or notches trigger localized crack initiation before significant global yielding becomes evident [41]. Consequently, the structural tensile response is governed by this deformation localization and the weakest-link nature of cellular solids, where failure is controlled by the most critically loaded regions rather than the average bulk material properties. This results in limited observable macroscopic plastic deformation prior to failure. Fracture propagation is thus dominated by crack growth through individual struts, with overall fracture resistance decreasing as relative density decreases [41].

2.2 Boundary Conditions

When conducting computational homogenization on RVEs or SVEs, boundary conditions (BCs) are essential to accurately bridge the meso or microscale with the macroscale. They mathematically mimic the physical restraint of the excluded bulk material surrounding the sub-volume [42], [43], [44]. The choice of BCs heavily dictates the mechanical response, the required size of the RVE/SVE necessary to achieve convergence of the quantity of interest, and the overall computational cost [45], [46], [47], [48].

Four common boundary condition formulations utilized in meso/micromechanics are:

- Kinematic Uniform Boundary Conditions (KUBC) / Direct Displacement Boundary Conditions (DDBC) / Dirichlet
- Static Uniform / Uniform Traction Boundary Conditions (UTBC) / Neumann
- Periodic Boundary Conditions (PBC)
- Orthogonal-Mixed Boundary Conditions (OMBC)

KUBC / DDBC / Dirichlet

These boundary conditions are applied by explicitly prescribing displacements on the RVE boundary to match a target macroscopic deformation [43], [45], [48], [49]. The microscopic displacement fluctuation field on the boundary is strictly forced to zero. DDBCs offer several advantages: low computational cost, straightforward implementation, and rapid generation of homogenized properties. However, because they suppress natural displacement fluctuations, they artificially constrain the domain faces to remain perfectly planar. This over-constrains material flow around internal heterogeneities (e.g., pores or stiff inclusions), generally producing an artificially stiff mechanical response.

DDBCs are mathematically proven to generate an upper bound on the effective material response, analogous to the classical Voigt approximation [42], [50], [51].

Consequently, significantly larger domains are required for convergence toward the true bulk response compared to more relaxed boundary conditions [46], [47].

Static Uniform / UTBC / Neumann

Under UTBCs, uniform boundary tractions are prescribed based on a constant macroscopic stress tensor, allowing the boundary displacement field to fluctuate freely [44], [52]. This permits the non-uniform deformation of the boundary to naturally accommodate local microstructural mechanisms, producing the most compliant response among the canonical conditions.

These conditions generate a strict theoretical lower bound on effective material properties, analogous to the classical Reuss approximation [42], [53]. However, UTBCs are computationally demanding and generally exhibit slower convergence with increasing RVE size than PBCs [46], [47]. Furthermore, prescribing pure tractions causes the global stiffness matrix to become singular, introducing severe rigid body motion and numerical instability, especially under compression or shear [54].

PBC

PBCs assume that the RVE or SVE forms part of an infinitely repeating periodic microstructure [43], [49]. They are enforced by constraining opposing boundary faces such that displacement fluctuations remain perfectly periodic and boundary tractions are equal and opposite [43], [48].

PBCs are widely regarded as the most efficient approach for achieving rapid convergence of homogenized properties with respect to domain size [45], [46], [47]. Unlike DDBC, the boundary mesh can deform non-uniformly, substantially reducing artificial boundary stiffening. Despite these advantages, PBCs introduce a massive array of constraint equations that heavily increase computational cost per increment [55]. Additionally, forcing a mathematically repeating boundary may artificially suppress localized failure or progressive collapse modes that are physically incompatible with periodic deformation [56].

OMBC

OMBCs apply a controlled combination of prescribed displacements and prescribed tractions while maintaining energetic orthogonality between the two boundary subsets [57]. This formulation produces an intermediate response safely nested between the stiff DDBC upper bound and the compliant UTBC lower bound.

A major advantage of OMBCs is their ability to reproduce common experimental loading configurations more realistically than purely kinematic or purely traction-based conditions. For example, standard compression tests are efficiently approximated by imposing normal displacements along the loading direction while allowing traction-free lateral boundary expansion. Because of these favorable convergence and realism characteristics, OMBCs are highly attractive for practical multiscale simulations.

2.3 Double Cantilever Beam Test

As introduced in Subsection 1.4.2, the DCB test is commonly used to evaluate the peel strength and Mode I fracture behavior of sandwich structures. It is particularly relevant for assessing delamination and core–face interface performance under tensile opening conditions, where separation occurs normal to the crack plane.

However, unlike many conventional bulk fracture characterization methods, there is no universally adopted standard specifically dedicated to DCB testing of foam core sandwich structures. Existing standards, such as ASTM D5045 [25], provide procedures for determining plane-strain fracture toughness of homogeneous plastics, whereas ASTM D5528 [26] specifies methods for evaluating Mode I interlaminar fracture toughness in fiber-reinforced laminated composites. Because these standards are formulated primarily for bulk isotropic specimens or classical laminates under linear elastic fracture mechanics assumptions, they cannot be directly applied to porous cellular sandwich cores without significant analytical modifications [12], [41].

In sandwich structures, the fracture process is fundamentally complex. The specimen represents a heterogeneous system of stiff face sheets bonded to a compliant cellular core, where crack propagation may occur internally within the foam, cleanly along the interface, or through a combination of both. As a result the stress state, structural compliance, and crack propagation behavior differ significantly from those assumed in conventional standards. This lack of a dedicated framework leads to significant variations in specimen geometry, loading configurations, and data reduction methods across the literature, making direct comparisons of reported fracture toughness values difficult [12], [41], [58], [59].

To address these limitations, the approach proposed by Saenz et al. [60] was identified as the optimal foundation for the present study. The authors developed a tailored sandwich DCB configuration in which a foam core strip is bonded to stiff adherends, loaded in opening Mode I, and mathematically analyzed using a modified Kanninen elastic foundation model [61]. This formulation [61] directly incorporates the mechanical contribution of both the face sheets and the compliant foam core via beam-on-elastic-foundation theory, where the system compliance depends simultaneously on initial crack length, foundation stiffness, and the effective flexural stiffness of the arms. The DCB load case applied in this study utilizes this modified foam core configuration, as illustrated in Figure 2.4.

Modified Kanninen Model

The Modified Kanninen Model [60], [61] analytically describes the displacement field of the specimen via the transverse deflection function $w(x)$, which defines the opening displacement along the longitudinal axis (Equation 2.1). The solution is separated into two domains: the cracked region ($-a \leq x < 0$), where the upper and lower arms deform freely, and the uncracked region ($0 \leq x \leq c$), where the foam core provides continuous elastic restraint. Equation 2.1 describes the deflection field $w(x)$ and how it evolves as a function of the position along the specimen, distinguishing between the cracked and uncracked regions.

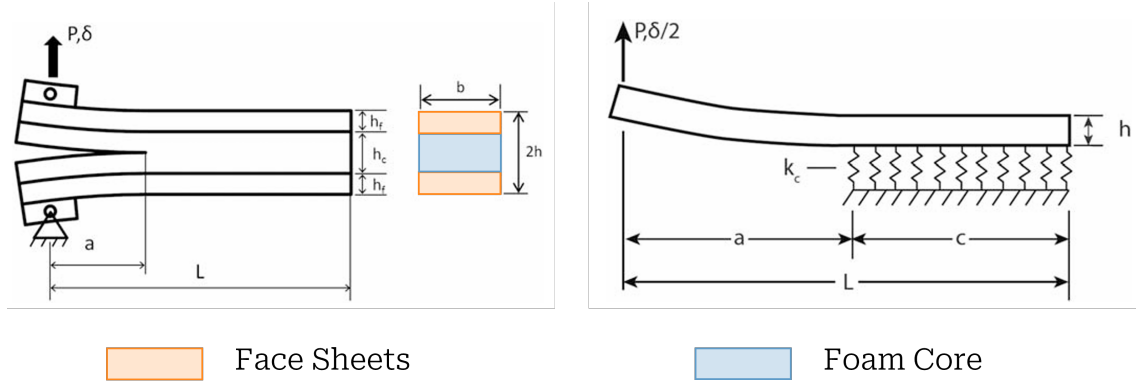


Figure 2.4: Modified Kanninen Model. Image adapted from Saenz et al. [60].

$$w(x) = \frac{6P}{E_x b h^3 \lambda^3} \begin{cases} \frac{\lambda^3 x^3}{3} + a \lambda^3 x^2 - A \lambda x + B, & -a \leq x < 0 \\ a \lambda \sin(\lambda x) \sinh(\lambda x) + B \cos(\lambda x) \cosh(\lambda x) \\ - \left(\frac{A-1}{2} \right) \sin(\lambda x) \cosh(\lambda x) \\ - \left(\frac{A+1}{2} \right) \cos(\lambda x) \sinh(\lambda x), & 0 \leq x \leq c \end{cases} \quad (2.1)$$

Here, E_x is the effective flexural modulus describing the resistance to bending, P is the applied opening load, b is the specimen width, h is the half-thickness of the foam and adherends (stiff face sheets), a is the initial crack length, and c denotes the remaining uncracked length. The integration constants A and B are determined by enforcing continuity conditions between the cracked and uncracked domains; their explicit forms are omitted here for brevity and can be found in [60].

The effective flexural modulus E_x is defined as

$$E_x = \frac{12}{h^3 d_{11}}, \quad (2.2)$$

where d_{11} is the first component of the bending compliance matrix and represents the effective flexural compliance in the principal bending direction. This formulation is derived from classical laminate theory, where the extensional $\mathbf{A}$, coupling $\mathbf{B}$, and bending $\mathbf{D}$ stiffness matrices collectively govern the global response of layered composite structures. The bending compliance matrix is given by

$$\mathbf{d} = (\mathbf{D} - \mathbf{B} \mathbf{A}^{-1} \mathbf{B})^{-1}. \quad (2.3)$$

The parameter λ is a dimensionless elastic foundation variable governing the interaction between the bending stiffness of the sandwich arms and the elastic restraint provided by the foam core

$$\lambda = \left(\frac{3k_c}{E_c b h^3} \right)^{1/4} \quad (2.4)$$

where k_c is the analytical foundation modulus of the foam core, given by

$$k_c = \frac{2E_c \cdot b}{h_c} \quad (2.5)$$

2.4 Erosion and Regularization

In experimental core fracture testing, specimens are mechanically loaded until physical separation occurs. To successfully capture this phenomenon of crack propagation and material separation within a continuous FEA framework, a numerical technique known as element erosion is introduced.

In *LS-DYNA* [7], element erosion can be included using the `MAT_ADD_EROSION` card, which systematically deletes finite elements from the model once prescribed failure criteria are met. These criteria thresholds can include, for example, limits on effective plastic strain, principal stress, or volumetric pressure, among other available criteria. Once an element satisfies these conditions, its stiffness contribution is permanently removed, mimicking the progression of a physical crack [7].

However, the introduction of element erosion inevitably triggers a well-documented numerical issue known as mesh dependency. Upon initial failure, strain energy tends to localize into an infinitesimally small mathematical region. In an uncorrected FE model, this damage band will artificially collapse into a single row of the smallest available elements, causing the predicted macroscopic energy dissipation to become a function of the arbitrary mesh size rather than the true material fracture toughness. To mitigate this non-physical behavior, regularization schemes are required [62].

Regularization is a stabilization technique that ensures material failure simulations are physically objective and independent of spatial discretization. Its primary function is to prevent pathological strain localization by forcing the failure process to spread over a finite, controlled region instead of collapsing into a zero-volume singularity. One way to achieve this is by embedding a characteristic length scale into the constitutive damage formulation. This length scale explicitly governs the physical width of the actively degrading damage band. By regulating this geometry, the total energy dissipated during numerical fracture becomes strictly tied to the intrinsic material behavior, ensuring objective, mesh-independent results [62]. The investigated stabilization methods, their underlying theory, and the specific *LS-DYNA* [8] implementation challenges encountered are discussed in detail in Appendix C.

In the specific context of SVE modeling, additional forms of regularization are required for maintaining consistency across variable domain sizes. Without it, the predicted post-peak softening regime will drift depending on the absolute size of the SVE.

3

Methods

A FEA framework was developed to systematically generate, pre-process, solve, and evaluate the SVE numerical models. First, the SVE geometries replicating the foam core mesostructure were generated using *Neper* [10]. These geometries were subsequently imported into *ANSA* [6] for automated pre-processing via *Python* scripting, which included re-meshing, material and property assignment, and the definition of boundary conditions. The models were then solved using the explicit FE solver *LS-DYNA* [7]. Finally, post-processing and result evaluation were performed using *META* [9] and *Python*. This overall workflow is illustrated in Figure 3.1.

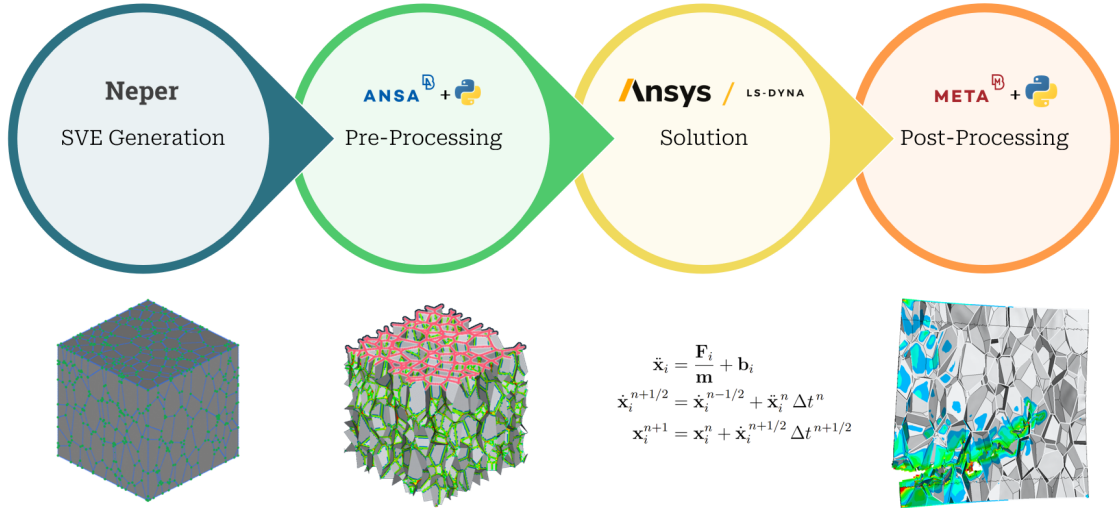


Figure 3.1: Illustration of FEA framework workflow.

3.1 Modeling Strategy

The overall modeling strategy is based on a mesoscale SVE approach designed to link the foam core mesostructure to its effective macroscopic mechanical response under quasi-static loading. The analysis includes five distinct load cases: uniaxial tension and compression in the foam rise direction ($\pm Z$), uniaxial tension and compression in one in-plane direction ($\pm X$), and a DCB-inspired configuration.

The cell walls are represented using an isotropic elasto-plastic continuum material model. This fundamental approach enables the numerical simulation to capture local yielding and stress redistribution within individual struts prior to the onset of global structural failure.

The numerical framework was built upon a reference SVE geometry, termed the baseline configuration, calibrated and supplied by Liu et al. [15]. Using this baseline, the necessary mesh resolution, boundary conditions, domain size for statistical representativeness, and constitutive model parameters were identified and verified. Building upon the calibrated reference configuration, key morphological parameters were varied to assess their influence on the homogenized mechanical response. Specifically, the effects of SVE size, stochastic realization, cell wall thickness distribution, and equivalent cell diameter were investigated. To isolate these geometric effects, the SVEs relative density was maintained at the nominal specification value of ivinycell H100 ($\rho_{nom} = 100 \text{ kg/m}^3$). Assuming uniform shell thickness, element thickness $t^{(e)}$ was universally scaled based on the accumulated surface area of the individual realization

$$t^{(e)} = \frac{V_{SVE}}{\sum_{e=1}^{N_e} A^{(e)}} \cdot \frac{\rho_{nom}}{\rho}, \quad (3.1)$$

where $A^{(e)}$ is the area of element e , N_e is the element count, and ρ is the material density of the solid PVC phase.

Finally, the homogenized mesoscale simulation results were compared with macroscopic experimental data provided by Diab to evaluate the extent to which the SVE models captured the effective structural response of the physical foam.

3.2 SVE Generation

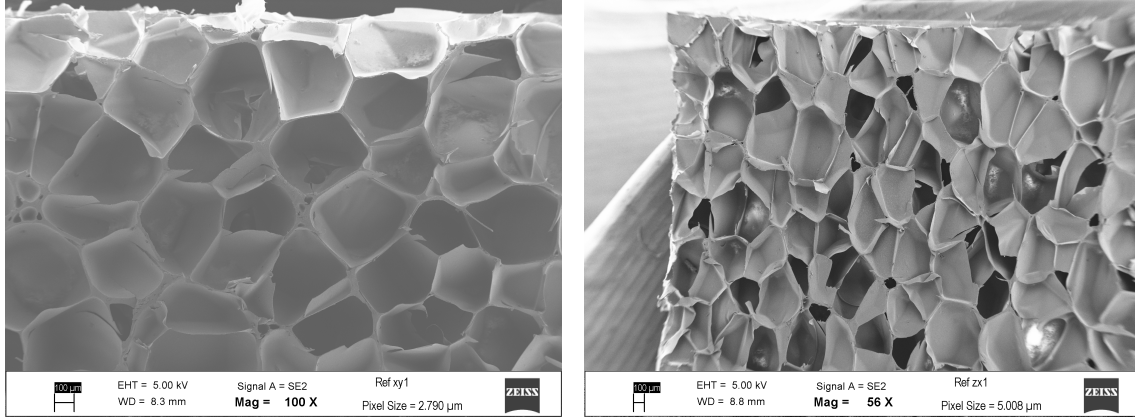
The Laguerre polyhedral tessellation formulation was implemented in *Neper* [10] to generate SVEs. The Laguerre tessellation enables direct control over the equivalent cell diameter distribution, cell sphericity and cell shape anisotropy, or elongation, providing a statistically representative yet computationally tractable approximation of the foam's internal architecture [10]. The corresponding parameters assigned to the finite element model are established based on three main sources. First, matching the morphological characterization and elastic material properties from Liu et al. [15] and the mesoscale properties summarized in Table 1.3, the assigned cell wall parameters are:

- Cell wall Young's modulus: $E_{wall} = 2700 \text{ MPa}$
- Cell wall yield stress: $\sigma_{y,wall} = 62 \text{ MPa}$
- Cell wall Poisson's ratio: $\nu_{wall} = 0.38$
- Cell shape aspect ratio: $\mathcal{R}_w = 1.29$

Next, utilizing the morphological distributions determined in the unpublished study introduced in Subsubsection 1.4.3.1, the baseline foam topology was defined using the following expression in *Neper*:

$$\begin{aligned} \text{MP} = & \text{"diameq:0.47*lognormal(0.145,0.095) + ...} \\ & \text{0.53*lognormal(0.495,0.115),} \\ & \text{1-sphericity:lognormal(0.235,0.085),} \\ & \text{aspratio(1,1,1.29)} \text{"} \end{aligned} \quad (3.2)$$

This expression specifies a bimodal lognormal distribution for the equivalent cell diameter, d_{eq} , as shown in Figure 3.3, defined by a combination of different fractions of small and large cells. d_{eq} represents the diameter of a perfect sphere with the same volume as the respective foam cell. The sphericity of a cell corresponds to the ratio of the surface area of the sphere of equivalent volume to the surface area of the cell. Finally, the aspect ratio of $\mathcal{R}_w = 1.29 \equiv (1 : 1 : 1.29)$ represents the cell shape anisotropy, forcing elongation along the foam rise direction (denoted as the 33 or Z direction throughout this work).



(a) Top view, perpendicular to foam rise direction, 33. (b) Front view, perpendicular to in-plane direction, 11.

Figure 3.2: SEM images of a Divinycell H100 foam sample.

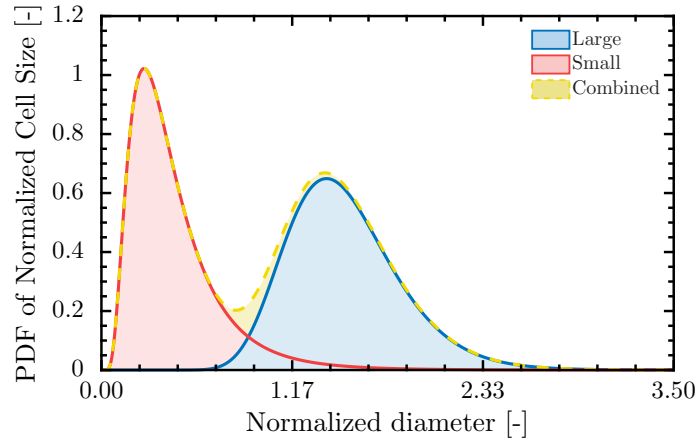


Figure 3.3: Statistical distribution of normalized cell equivalent diameter of Divinycell H100.

3.2.1 Post-Generation Acceptance Criteria

The generated SVEs were subjected to an automated post-processing evaluation to ensure statistical representativeness and numerical robustness prior to FEA. Because the foam mesostructure is generated stochastically, certain realizations may contain

cells with extreme dimensions corresponding to the tails of the probability distributions. Although these extreme geometric features could theoretically exist, they are classified as defects because they often compromise mesh quality and can artificially skew the mechanical response, failing to represent the required average behavior. To prevent such pathological cases, each generated SVE was evaluated against a strict set of statistical and geometric criteria. Any SVE violating these conditions was automatically rejected. A detailed discussion of these filtering criteria is provided in Appendix B.

3.3 Analyses Setup

The uniaxial simulations were conducted under a homogenized uniaxial stress assumption. Consequently, each loading configuration employed a set of OMBC, designed to allow the macroscopic response to develop naturally from local cell interactions while ensuring numerical stability. Across all five investigated load cases (T11, T33, C11, C33, and the DCB-inspired), the underlying topology and elasto-plastic material definitions remained identical.

The common numerical assumptions and modeling parameters adopted for the mesoscale simulations are summarized in Table 3.1.

Table 3.1: Common modeling assumptions and numerical settings for the mesoscale simulations.

Category	Description
Load cases	T11, T33, C11, C33, and DCB-inspired
Material Law	Trilinear elasto-plastic constitutive formulation *MAT_024
Discretization	First order Mix of quadrilateral and triangular shell elements Average element edge length $h = 30 \mu\text{m}$ Uniform thickness
Element Formulation	Fully integrated (ELFORM = 16) Hourglass control (IHQ = 8) Shear correction factor (SHRF = 5/6)
Boundary Conditions	OMBC, enforcing a uniaxial stress state Velocity-controlled prescribed motion Quasi-static loading rates (see Appendix A)
Contact Interaction	Automatic single-surface contact encompassing the entire SVE

Common node sets were defined to consistently prescribe boundary conditions. Let $\Gamma_{i\pm}$, with $i \in \{X, Z\}$, denote the node sets located on the positive (top) and negative (bottom) boundary faces of the SVE along the global coordinate axis i . Here $\mathbf{x} = (x, y, z)^T \in \mathbb{R}^3$ represents the spatial position vector, $\Omega \subset \mathbb{R}^3$ is the SVE domain, and $\partial\Omega$ denotes its boundary. These boundaries are visualized in Figure 3.4.

$$\Gamma_{Z+} = \{\mathbf{x} \in \partial\Omega \mid z = L_z\}, \quad (3.3)$$

$$\Gamma_{Z-} = \{\mathbf{x} \in \partial\Omega \mid z = 0\}, \quad (3.4)$$

$$\Gamma_{X+} = \{\mathbf{x} \in \partial\Omega \mid x = L_x\}, \quad (3.5)$$

$$\Gamma_{X-} = \{\mathbf{x} \in \partial\Omega \mid x = 0\}. \quad (3.6)$$

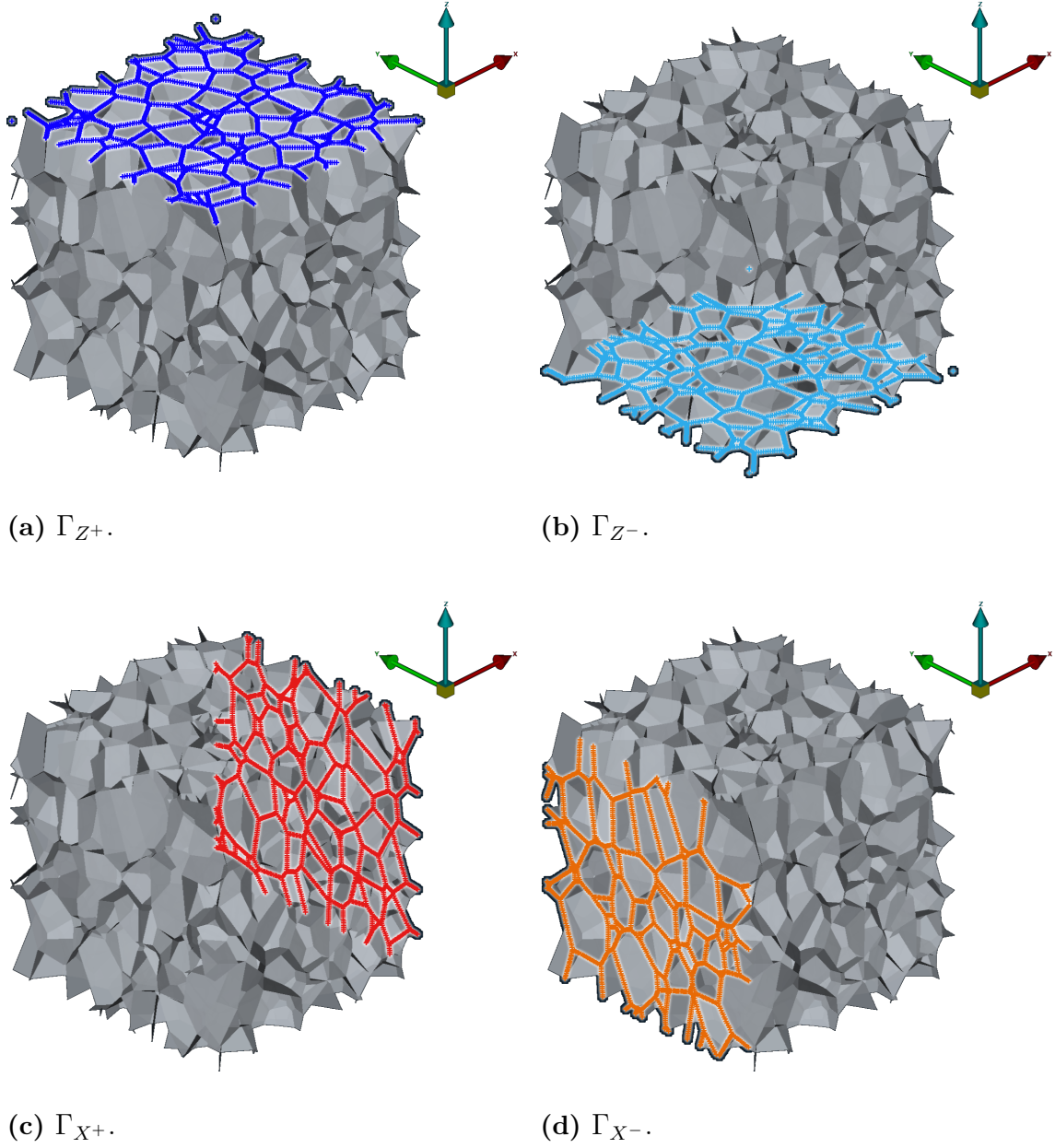


Figure 3.4: Highlighted boundary node sets on a 3.6 mm SVE.

Lateral boundary node sets were further introduced. The notation $\Gamma_{\text{lat},ij}$ follows the conventional tensor indexing of the loading direction, such that $\Gamma_{\text{lat},11}$ and $\Gamma_{\text{lat},33}$ denote the lateral faces associated with loading in the global X - and Z -directions, respectively:

$$\Gamma_{\text{lat},11} = \partial\Omega \setminus (\Gamma_{X+} \cup \Gamma_{X-}), \quad (3.7)$$

$$\Gamma_{\text{lat},33} = \partial\Omega \setminus (\Gamma_{Z+} \cup \Gamma_{Z-}). \quad (3.8)$$

Solution control parameters, including energy tracking, mass scaling, contact enforcement, and shell element constraints, were assigned in accordance with best-

practice industry standards. The solution control include file, *CONTROL.k, used in all simulations is detailed in Appendix E.

The response of interest for the uniaxial load cases is the homogenized engineering stress–strain curve, obtained by summing the reaction forces (F) at all nodes subjected to the prescribed displacement and dividing the resulting force by the reference cross-sectional area (A_{bb}) of the SVE bounding box. Likewise, the homogenized engineering strain is computed as the ratio between the applied displacement (ΔL) and the initial length (L_0) of the SVE in the loading direction. Figure 3.5 illustrates the quantities used in the homogenization procedure.

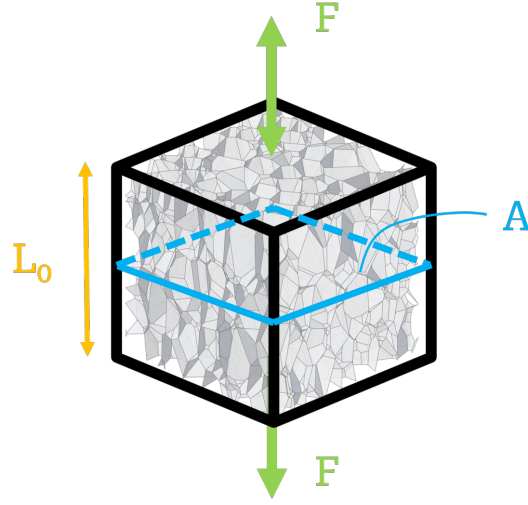


Figure 3.5: Schematic of engineering stress–strain quantities.

For a cubic SVE with side length (L_{SVE}), the reference area is given by ($A_{bb} = L_{\text{SVE}}^2$). The homogenized engineering stress and strain are therefore calculated as

$$\hat{\sigma} = \frac{F}{A_{bb}}, \quad \hat{\varepsilon} = \frac{\Delta L}{L_0}. \quad (3.9)$$

3.3.1 Uniaxial Tension

Uniaxial tension tests were simulated in two orthogonal directions: the foam rise direction (Z or 33) and one in-plane direction (X or 11). Consistent with the OMBC approach outlined in Section 2.2, a prescribed velocity boundary condition was applied to the boundary node set, while the opposing face was fixed against out-of-plane translation and rotation to emulate a symmetry boundary condition.

Uniaxial tensile loading in the foam rise direction was applied by prescribing displacement-controlled kinematic constraints on the top (Γ_{Z+}) and bottom (Γ_{Z-}) boundaries, while the lateral sides $\Gamma_{\text{lat},33}$ are treated as free, thus mimicking but not guaranteeing zero tractions macroscopically. In the following, U_i and R_i , with $i \in \mathbf{x}$, denote the global translational and rotational degrees of freedom, respectively.

$$\left. \begin{array}{l} U_Z = C \cdot t \\ R_X = 0, \\ R_Y = 0 \end{array} \right\} \text{ on } \Gamma_{Z+}, \quad \left. \begin{array}{l} U_Z = 0, \\ R_X = 0, \\ R_Y = 0 \end{array} \right\} \text{ on } \Gamma_{Z-} \quad (3.10)$$

For uniaxial tensile loading in the in-plane direction 11, the lateral sides $\Gamma_{\text{lat},11}$ are free and equivalent kinematic constraints were imposed on the node sets $\Gamma_{X\pm}$:

$$\left. \begin{array}{l} U_X = C \cdot t \\ R_Y = 0, \\ R_Z = 0 \end{array} \right\} \text{ on } \Gamma_{X+}, \quad \left. \begin{array}{l} U_X = 0, \\ R_Y = 0, \\ R_Z = 0 \end{array} \right\} \text{ on } \Gamma_{X-} \quad (3.11)$$

Here, t is time and C represents the displacement rate, derived by integrating the prescribed velocity curve (detailed in Appendix A). A schematic representation of these constraints alongside representative deformation contour plots is shown in Figure 3.6.

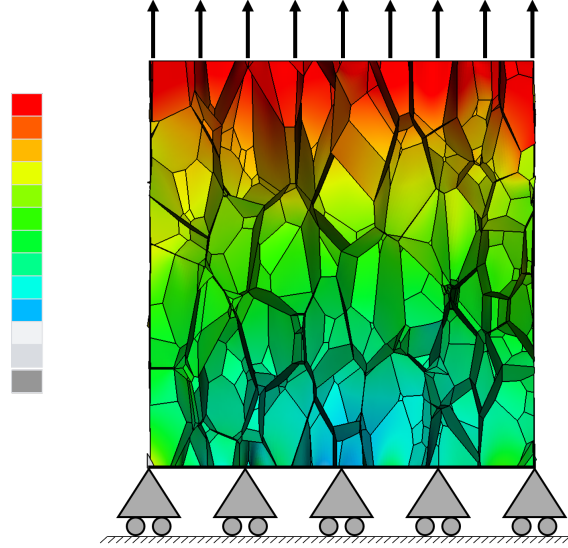


Figure 3.6: Representation of a uniaxial tension load cases.

3.3.2 Uniaxial Compression

Uniaxial compression was analogously evaluated in both the Z and X directions. However, compression load cases utilized rigid mathematical plates adjacent to the boundaries $\Gamma_{i\pm}$ to represent the crossheads of a uniaxial compression testing machine. The corresponding plate sets are denoted by $P_{i\pm}$. The bottom plate was fully constrained, whereas the top plate was prescribed a constant downward velocity to compress the SVE.

For compression in the foam rise direction, the constraints imposed on the rigid top and bottom plates, $P_{Z\pm}$ were:

$$\left. \begin{array}{l} U_X = 0, \\ U_Y = 0, \\ U_Z = -C \cdot t, \\ R_X = 0, \\ R_Y = 0, \\ R_Z = 0, \end{array} \right\} \text{ on } P_{Z+}, \quad \left. \begin{array}{l} U_Z = 0, \\ U_Y = 0, \\ U_X = 0, \\ R_X = 0, \\ R_Y = 0, \\ R_Z = 0 \end{array} \right\} \text{ on } P_{Z-} \quad (3.12)$$

For compression in the in-plane direction 11, the constraints were similarly applied to P_{X+} :

$$\left. \begin{array}{l} U_X = -C \cdot t, \\ U_Y = 0, \\ U_Z = 0, \\ R_X = 0, \\ R_Y = 0, \\ R_Z = 0 \end{array} \right\} \text{ on } P_{X+}, \quad \left. \begin{array}{l} U_Z = 0, \\ U_Y = 0, \\ U_X = 0, \\ R_X = 0, \\ R_Y = 0, \\ R_Z = 0 \end{array} \right\} \text{ on } P_{X-} \quad (3.13)$$

To transmit the compressive loads, a node-to-surface frictional contact interaction was defined between the deformable SVE (the secondary surface) and the rigid plates (the main surface), as illustrated in Figure 3.7.

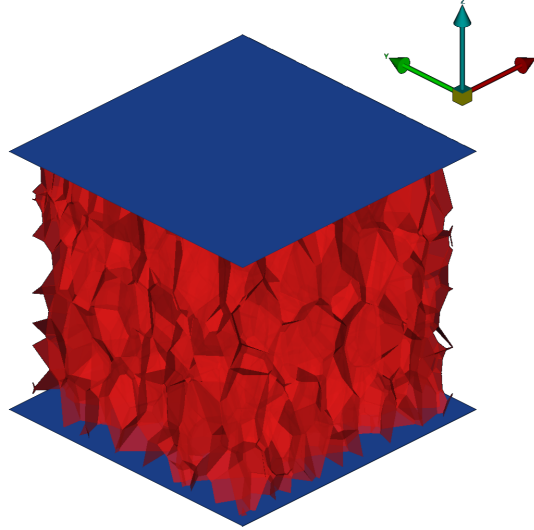


Figure 3.7: Contact interaction configuration between rigid plates and the SVE.

The resulting compressive boundary assumptions and representative deformation fields are shown in Figure 3.8.

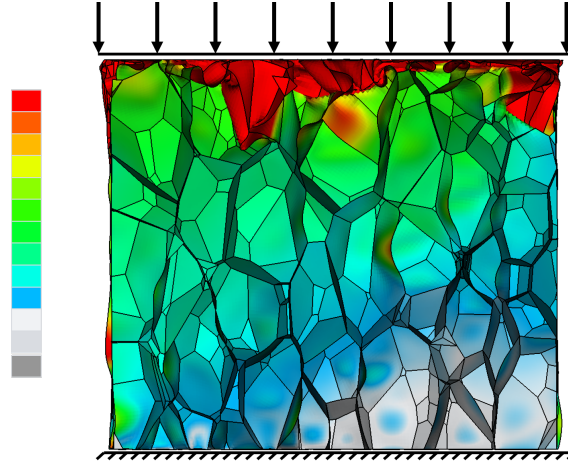


Figure 3.8: Representation of a uniaxial compression load cases.

3.3.3 DCB-Inspired

To simulate the localized material behavior during a Mode I fracture test, a DCB-inspired loading condition was developed. The macroscopic DCB is represented analytically by the Kanninen model, which treats the specimen as an Euler-Bernoulli beam resting on an elastic Winkler foundation, as described in Section 2.3. By solving this model, a continuous symmetric, with respect to the beam's midsurface, deflection profile along the beam's length is obtained.

A subregion of the foam core at the crack front is then explicitly modeled using an SVE with characteristic length of $L = 3.6$ mm. The macroscopic beam domain is defined as $-a \leq x \leq c$, where a is the pre-cracked length, c is the uncracked length, and the crack tip is located at the origin ($x = 0$). The SVE is virtually positioned at the crack tip, centered vertically with respect to the DCB mid-plane, and spans the longitudinal interval $-1 \leq x \leq 2$. Finally, the analytically derived deflection profile corresponding to this local interval is extracted and applied as a prescribed velocity boundary condition to the top and bottom faces of the SVE. This multiscale placement is visualized in Figure 3.9.

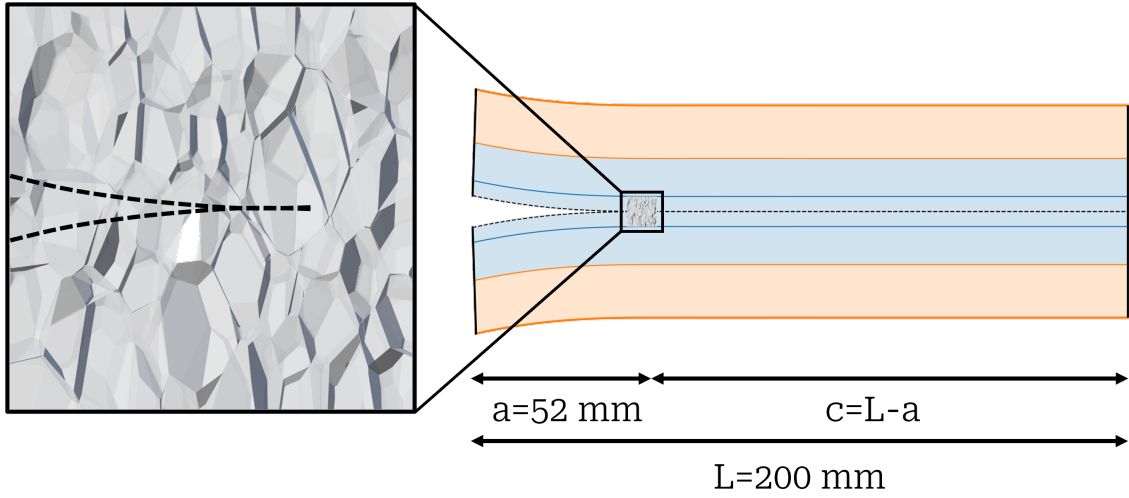


Figure 3.9: Virtual placement of the SVE at the crack tip within the analytical Kanninen Model.

Thin sheets, matching the uniform thickness of the SVE cell walls, were introduced via nodal equivalence at the $\Gamma_{Z^\pm}$ boundaries. These sheets ensure a continuous transfer of the prescribed non-uniform displacements while artificially stiffening the boundary regions to mimic the constraining effects of the bulk core material surrounding the evaluated volume in a physical DCB specimen.

Sheets boundary node sets were additionally defined to prescribe boundary conditions on the sheets' edges. Let $\Gamma_{S,Y}$ and $\Gamma_{S,X}$ denote the node sets located on the sheets boundaries at $y = L_y$ and $x = L_x$, respectively, where $\mathcal{S} \subset \Omega$ represents the sheets domain. These sets are illustrated in Figure 3.10 and defined as

$$\Gamma_{S,Y} = \{\mathbf{x} \in \partial\mathcal{S} \mid y = L_y\}, \quad (3.14)$$

$$\Gamma_{S,X} = \{\mathbf{x} \in \partial\mathcal{S} \mid x = L_x\}. \quad (3.15)$$

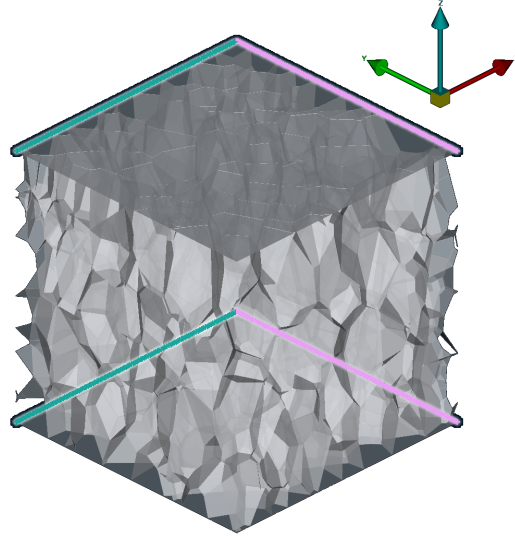


Figure 3.10: Visualization of added sheets and $\Gamma_{S,X}$ and $\Gamma_{S,Y}$ sets.

The spatial deflection obtained from the analytical model was fitted by a second order polynomial function, and adapted to be applied symmetrically as a velocity field at $\Gamma_{Z\pm}$. The profile is visualized in Figure 3.11 and defined as

$$\dot{w}(x, t) = \pm \left(3x^2 + 19x + 28 \right) \cdot \frac{k}{t_f} \quad (3.16)$$

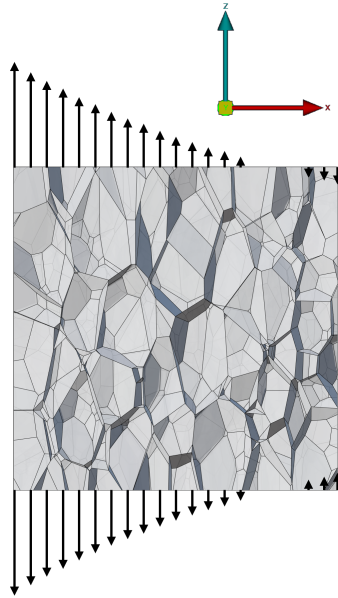


Figure 3.11: Symmetric deflection profile.

Here, t_f is the termination time. The scaling factor k is determined by enforcing a prescribed displacement δ at a reference coordinate α over the integration interval

$$\int_0^{t_f} \dot{w}(\alpha, t) dt = \delta \quad (3.17)$$

The coordinate α is chosen as the left-most point ($x = -1$ mm) of the extracted interval, corresponding to the location of maximum opening displacement in the applied deflection field. The target displacement δ is defined as the elongation experienced by a 3.6 mm SVE subjected to the experimentally observed macroscopic tensile failure strain in the foam rise direction 33, shown in Figure 2.3a. Consequently, the imposed DCB-inspired loading reaches a displacement level comparable to that required to induce tensile failure under homogeneous uniaxial loading. Rigid body motion prevention is enforced by applying kinematic constraints to $\Gamma_{S,Y}$ and $\Gamma_{S,X}$ as

$$U_X = 0 \quad \text{on } \Gamma_{S,X}, \quad U_Y = 0 \quad \text{on } \Gamma_{S,Y} \quad (3.18)$$

Material element erosion was introduced exclusively in this load case to replicate macroscopic core fracture. To prevent premature numerical erosion near the artificially stiffened loading boundaries, the outermost 10% of the material thickness at both the top and bottom faces was restricted to strict elasto-plasticity (erosion disabled). Consequently, the progression of damage was isolated within the center of the core domain. The assignment of these material properties is detailed in Figure 3.12.

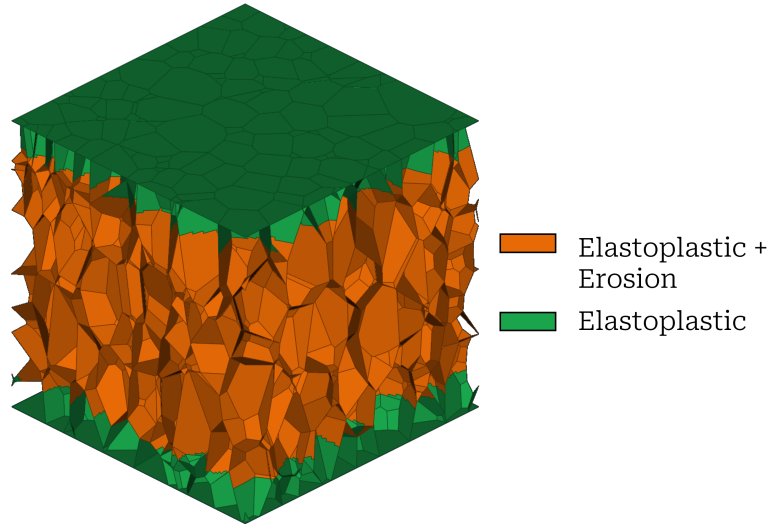


Figure 3.12: Spatial partitioning of material properties within the DCB-inspired model.

3.4 Mesh Sensitivity Analysis

Given the approximate nature of the finite element method (FEM), a comprehensive mesh sensitivity analysis was conducted to guarantee a robust, mesh-independent

mechanical response. The primary objective was to determine the maximum element size capable of fully resolving the physical behavior of the cellular walls without incurring unnecessary computational expense. Although the DCB load case incorporates element deletion, element erosion was disabled during the mesh convergence study. This isolation decoupled constitutive convergence (elasto-plasticity) from the severe mesh-dependencies typically associated with failure and element deletion, discussed in Section 2.4. Fully integrated, first-order shell elements utilizing five through-thickness integration points and no thinning effects were utilized for this assessment.

The global mesh density was parameterized using characteristic element edge lengths ranging from 40 μm down to 15 μm :

$$h = [40, 35, 30, 25, 20, 15] \text{ } \mu\text{m}.$$

3.4.1 Global stress–strain response

The volume-homogenized macroscopic tensile stress–strain response was the primary indicator of mesh independence. Because this response acts as the foundation for the overarching material parameter calibration, it is imperative to verify its stability across varying discretizations.

3.4.2 Volume-averaged plastic strain

To capture changes in localized mechanics, the volume-averaged effective plastic strain was continuously evaluated. Because it is a volume-weighted integral measure, it evaluates total accumulated plasticity independent of simple geometric partitioning. It is computed as:

$$\langle \varepsilon_p \rangle = \frac{1}{V} \int_V \varepsilon_p(\mathbf{x}) \, dV \approx \frac{\sum_{e=1}^{N_e} \varepsilon_p^{(e)} V^{(e)}}{\sum_{e=1}^{N_e} V^{(e)}}, \quad (3.19)$$

where $\varepsilon_p^{(e)}$ and $V^{(e)}$ denote the maximum effective plastic strain and volume of element e , respectively, and N_e is the total number of elements contributing to the volume average.

3.4.3 95th percentile plastic strain

Relying on absolute peak stress or peak plastic strain to evaluate convergence is systematically flawed, as localized point-values will mathematically trend toward infinity at stress concentrators upon continued mesh refinement. To bypass this pathological dependence while still characterizing extreme localization, the 95th percentile of the total effective plastic strain distribution was utilized. This measure tracks the strain magnitudes endured by the 5% most deformed elements.

3.4.4 Spatial convergence

Lastly, a qualitative spatial assessment was executed. A numerical solution was only considered successfully converged once successive mesh refinements produced visibly

identical structural load paths and equivalent localization zones.

3.5 SVE Size and Realization Invariance

To evaluate geometric representativeness and assess size-independent mechanical response, four SVEs were considered. The models had characteristic lengths $L_{SVE} = [1.8, 2.7, 3.6, 4.5]$ mm, corresponding to $[140, 473, 1122, 2191]$ enclosed cells, respectively. For each characteristic length, three samples were generated and their responses averaged.

The size invariance study was restricted to uniaxial tension in the foam rise direction. This specific load case is of primary interest because it most closely mimics the loading conditions experienced in the DCB-inspired setup. Furthermore, expanding this study to encompass the remaining load cases, particularly those under compression, would be computationally prohibitive. Compressive responses are more sensitive to random morphological realizations. To accurately decouple geometric size effects from stochastic topological variations in these cases, a significantly larger number of sample realizations (well beyond the three generated per size) would be required, rendering the scope of the study unfeasible.

While sufficiently large RVEs inherently exhibit macroscopic invariance, smaller SVEs will predictably scatter due to varying statistical distributions. Following the identification of a baseline characteristic length, an SVE realization sensitivity study was conducted across $n = 5$ unique topological instances. A 95% statistical confidence interval for the structural response was mapped for each load case utilizing Student's t -distribution [63] ($t_{n-1, \alpha/2} \approx 2.776$ for $\alpha = 0.05$):

$$\sigma_{\text{pred}}(\varepsilon) = \bar{\sigma}(\varepsilon) \pm \left(t_{n-1, \alpha/2} \cdot s(\varepsilon) \cdot \sqrt{1 + \frac{1}{n}} \right) \quad (3.20)$$

To extrapolate the dispersion characteristics observed to future SVE realizations, the coefficient of variation (CV), was evaluated. The CV provides a normalized measure of scatter, computed across the strain history as

$$CoV(\varepsilon) = \frac{s(\varepsilon)}{\bar{\sigma}(\varepsilon)} \quad (3.21)$$

This formulation assumes that the underlying stochastic parameters governing the cell morphology dictate a proportional variance in the mechanical response. Consequently, for a newly generated SVE realization drawn from the same statistical distribution, the previously derived $CoV(\varepsilon)$ can be used to estimate the expected standard deviation, $s_{\text{est}}(\varepsilon)$. Treating the macroscopic stress-strain response of the new realization, $\sigma^{\text{new}}(\varepsilon)$, as the expected mean for that specific topology, an empirical 2σ confidence interval for the new envelope is formulated as:

$$\sigma_{\text{pred}}^{\text{new}}(\varepsilon) = \sigma^{\text{new}}(\varepsilon) \pm 2 \cdot s_{\text{est}}(\varepsilon), \quad \text{where} \quad s_{\text{est}}(\varepsilon) = CoV(\varepsilon) \cdot \sigma^{\text{new}}(\varepsilon) \quad (3.22)$$

3.6 Cell Shape Aspect Ratio Calibration

Initial simulations of the baseline morphology defined in Section 3.2 indicated underestimated structural anisotropy between the rise and in-plane directions, in comparison to documented experimental values. To resolve this discrepancy, the parameter controlling cell elongation in the rise direction was evaluated.

The parameter was manipulated by altering the ‘aspratio’ variable within Equation 3.2 for the $L_{SVE} = 3.6$ mm SVE. The iterative investigation proceeded as follows:

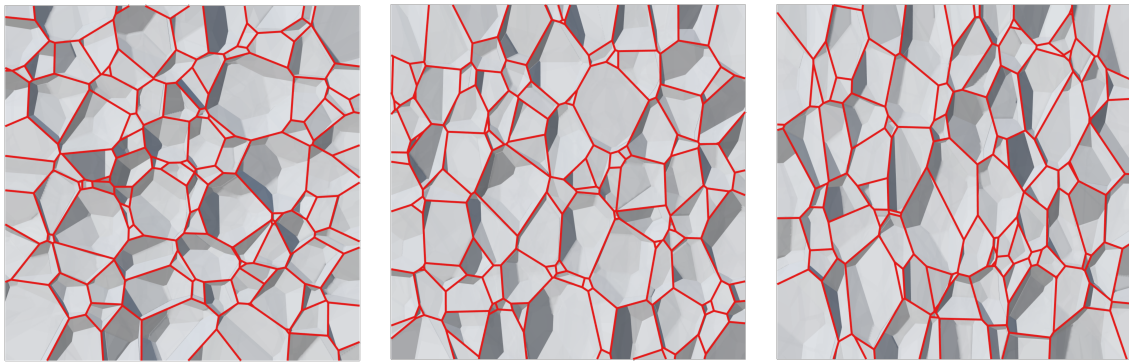
1. **Scope Definition:** Uniaxial tension tests were mapped across three progressive aspect ratios: $\mathcal{R}_w = [1.00, 1.29, 1.58]$. Resulting geometries are highlighted in Figure 3.13.
2. **Evaluation Metric:** mechanical anisotropy was directly quantified using the ratio of structural toughness between directions 33 and 11, for the tensile load cases:

$$\mathcal{R}_U = \frac{U_{T,33}}{U_{T,11}} \quad (3.23)$$

where bulk toughness U_T represents the integral area beneath the engineering stress–strain curve:

$$U_T = \int \sigma d\varepsilon \quad (3.24)$$

3. **Fitted Model:** The discrete toughness evaluations were mathematically correlated to the input aspect ratio utilizing a continuous power-law regression, allowing the framework to explicitly interpolate the necessary structural elongation needed to achieve the target physical anisotropy.
4. **Validation:** An updated geometry was generated matching this target aspect ratio and validated against the original experimental stress-strain curves.
5. **Integration:** Once validated, this morphologically adjusted generation logic was permanently adopted as the structural reference for all subsequent simulations.



(a) $\mathcal{R}_w = 1.00$

(b) $\mathcal{R}_w = 1.29$

(c) $\mathcal{R}_w = 1.58$

Figure 3.13: Visualized progression of cell elongation in the foam rise direction, upwards, with increasing aspect ratio $\mathcal{R}_w$.

3.7 Material Calibration

Because reliable experimental characterization of individual microscopic cell walls is exceptionally difficult, the foundational solid material parameters were identified via numerical calibration. This inverse engineering was driven by *LS-OPT* [8], framing the correlation as a mathematical optimization task. The primary objective function penalized the Euclidean distance discrepancy between the macroscale experimental curve data provided by Diab and the homogenized engineering stress–strain numerical response output by the FE model across all tension and compression load cases, for an SVE with characteristic length of 3.6 mm and calibrated cell aspect ratio $\mathcal{R}_w = 1.74$.

The optimization utilized the sequential response surface method (SRSM), the standard *LS-OPT* approach for parameter identification [8]. SRSM constructs computationally efficient surrogate mathematical models from initial variable sampling. While dramatically faster than executing direct FE simulations at every iteration step, SRSM algorithms are susceptible to converging upon local minima, which must be carefully monitored.

Consistent with Divinycell specifications, isotropic solid PVC (detailed in Table 1.3) formed the baseline for the linear elastic regime. As the plastic response was approximated using isotropic trilinear strain-hardening, two additional unknown parameters had to be assumed and calibrated: the tangent hardening modulus $H = d\sigma/d\varepsilon$ and the ultimate effective strain ε_u^{eff} .

The optimization solver sought to tune the hardening modulus, ultimate effective strain, elastic modulus, and yield stress. This trilinear material curve was defined by four geometric coordinates: $P_1 = [0.0, 0.0]$, $P_2 = [\varepsilon_y^{eff}, \sigma_y]$, $P_3 = [\varepsilon_u^{eff}, \sigma_u]$, and an arbitrary extreme failure point $P_4 = [\varepsilon_f^{eff} = 0.5, \sigma_f]$. The "ultimate" domain signifies structural collapse or complete cell-wall folding, after which the analytical stress–strain trajectory transitions to a nearly horizontal plateau. Perfect plasticity was mathematically avoided to bypass severe explicit numerical instability; thus, the plateau maintained a positive 10% fraction of the lowest-bound hardening modulus, resulting in a tangent modulus of 10 MPa. The derived parameters that constitute the material curve points were algebraically defined as

$$\begin{aligned}\varepsilon_y^{eff} &= \frac{E}{\sigma_y} \\ \sigma_u &= H \cdot (\varepsilon_u^{eff} - \varepsilon_y^{eff}) + \sigma_y \\ \sigma_f &= 10 \cdot (\varepsilon_f^{eff} - \varepsilon_u^{eff}) + \sigma_u\end{aligned}$$

This inverse problem was mapped utilizing a D-Optimal polynomial sampling scheme, directed by a hybrid adaptive simulated annealing (ASA) framework to initially scan the global design space (Table 3.2), followed by a localized gradient refinement algorithm. The schematic architecture of this framework is depicted in Figure 3.14.

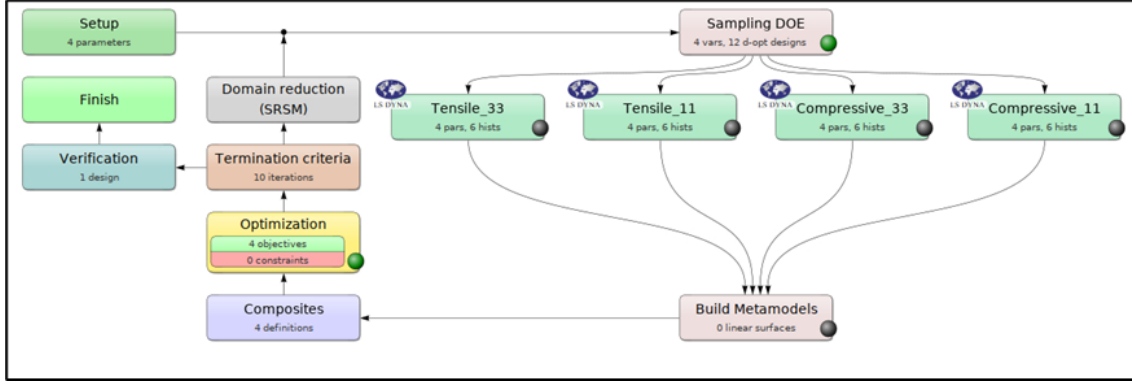


Figure 3.14: Conceptual architecture of the *LS-OPT* multi-objective material parameter identification framework.

The limits governing the surrogate bounds for the optimization variables are detailed in Table 3.2.

Table 3.2: Design of Experiments bounded variable parameter ranges.

Parameter	Lower Bound	Nominal	Upper Bound	Units
E	2400	2700	3000	MPa
H	100	400	1000	MPa
σ_y	48	56	64	MPa
$\varepsilon_{u,p}^{eff}$	0.05	0.10	0.15	—

The bounds adopted for the design variables reflected the differing levels of confidence associated with each parameter. The elastic modulus E and yield stress σ_y were constrained to a relatively narrow $\pm 10\%$ interval around nominal values reported in the literature for solid PVC [14], [15]. Although these references correspond to homogeneous PVC rather than the interpenetrating PVC–PU network present in Divinycell H100, see Subsection 1.4.1, they provide a justified baseline while still allowing sufficient flexibility for the optimizer to account for microstructural and manufacturing effects. In contrast, no experimental data describing the hardening modulus H or the ultimate effective strain ε_u^{eff} at the cell-wall scale could be identified. Consequently, their nominal values were selected based on engineering judgment and assigned substantially wider bounds to ensure that the optimization could explore a broad range of post-yield responses. The selected limits were verified to generate trilinear constitutive curves while avoiding numerical instabilities associated with excessively low hardening modulus.

The resulting limits for the trilinear curve accessible to the solver are visualized in Figure 3.15.

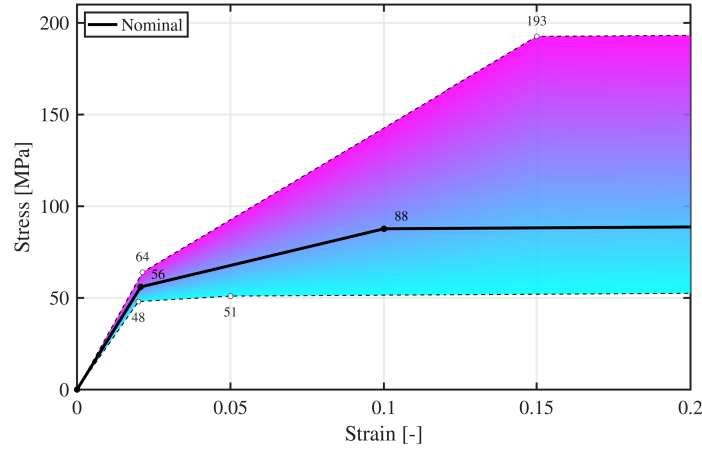


Figure 3.15: Envelope of allowable trilinear stress-strain elastoplastic curves.

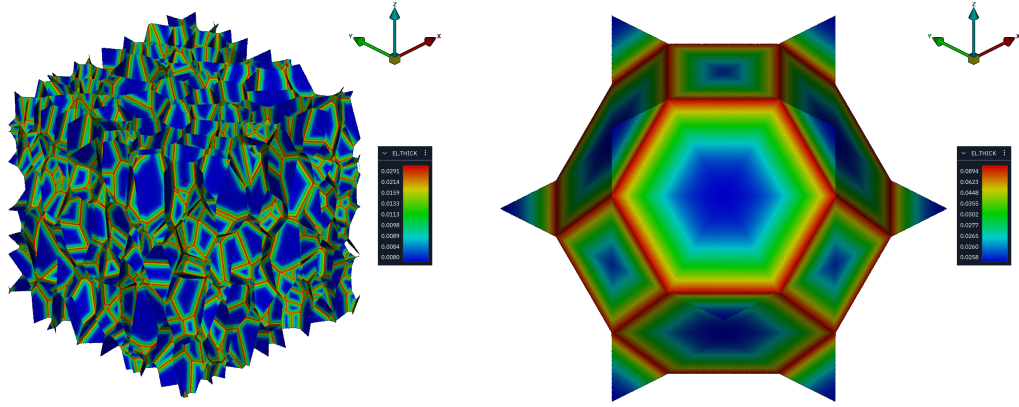
The optimization was multi-objective, combining the curve matching cost function of all four uniaxial load cases with equal weights.

3.8 Parametric Study of Morphological Parameters

Following the establishment of the calibrated properties, a parametric study of the morphological features intrinsic to the Laguerre tessellation was executed to assess their specific influence on bulk mechanical response.

3.8.1 Varying Thickness

The fundamental SVE models described above assumed identical, uniform cell-wall thickness globally. However, actual foam cores exhibit significant mass accumulations, or varying thicknesses, near intersecting strut borders and relatively thin central membranes (see Figure 1.3c). To evaluate the macroscopic sensitivity to this structural feature, an idealized exponential decay law (derived in Appendix D) was applied to assign node-variable shell thicknesses across cell-wall faces. This complex material pooling is visualized in Figure 3.16.



(a) Laguerre SVE.

(b) Kelvin unit cell.

Figure 3.16: Implementation of mathematically idealized varying cell wall thicknesses representing mass accumulations at boundary struts.

All foundational load cases defined in Section 3.3 were evaluated against both Uniform Thickness (UT) and Varying Thickness (VT) topological variants to track emergent deviations in mechanical response.

3.8.2 Equivalent Diameter Distribution

The final parametric investigation consisted of two parts. The first approach investigated the effects of scaling down the diameter of both large and small cells equally, essentially isolating the scaling mechanics of the primary morphological structure. This tracked how variations in the average physical size of individual cells altered the overall structural characteristics.

The second approach isolated how the volumetric proportion of large versus small cells affects the mechanical response. By maintaining a constant domain size and fixed cell dimensions, this approach shifted the distribution by varying the fraction between smaller and larger cells. The objective of both parts was to determine how these specific morphological ratios dictate the resulting mechanical behavior.

3.8.2.1 Scaling Factor

The first approach globally modified the distribution's magnitude by applying a Scaling Factor (**SF**), shifting the mean and standard deviations of the lognormal cell diameter geometries while strictly preserving both relative aspect ratios and bimodal proportionality. This logic is transcribed in Equation 3.25.

$$\begin{aligned} \text{MP} = & \text{"diameq:0.47*lognormal}(\text{SF*0.145}, \text{SF*0.095}) + \dots \\ & 0.53*\text{lognormal}(\text{SF*0.495}, \text{SF*0.115}), \\ & 1\text{-sphericity:lognormal}(0.235, 0.085), \\ & \text{aspratio}(1, 1, 1.74)\text{"} \end{aligned} \quad (3.25)$$

Although applying a decreasing scale factor mathematically increases the amount of cell in a given volumetric domain, it bears no theoretical impact on relative morphology. Because shrinking the cells or increasing the domain volume affects the ratio of internal wall surface area to fixed total volume, the uniform base shell thickness had to be scaled to preserve the critical macroscopic nominal density ($\rho_{nom} = 100 \text{ kg/m}^3$). The configurations and key morphological features are summarized in Table 3.3.

Table 3.3: Configured Scaling Factors controlling global equivalent diameter reduction.

Scaling Factor (SF)	L_{SVE} [mm]	Thickness [mm]	A_{surf}/V_{SVE}	Number of Cells
1.00	3.6	0.0124	6.45	1122
0.75	3.6	0.0090	8.87	2666
0.50	3.6	0.0058	13.79	8973
1.00	7.2	0.0116	6.88	8973

A visual summary showcasing the resulting cell densification of the generated SVEs is provided in Figure 3.17.

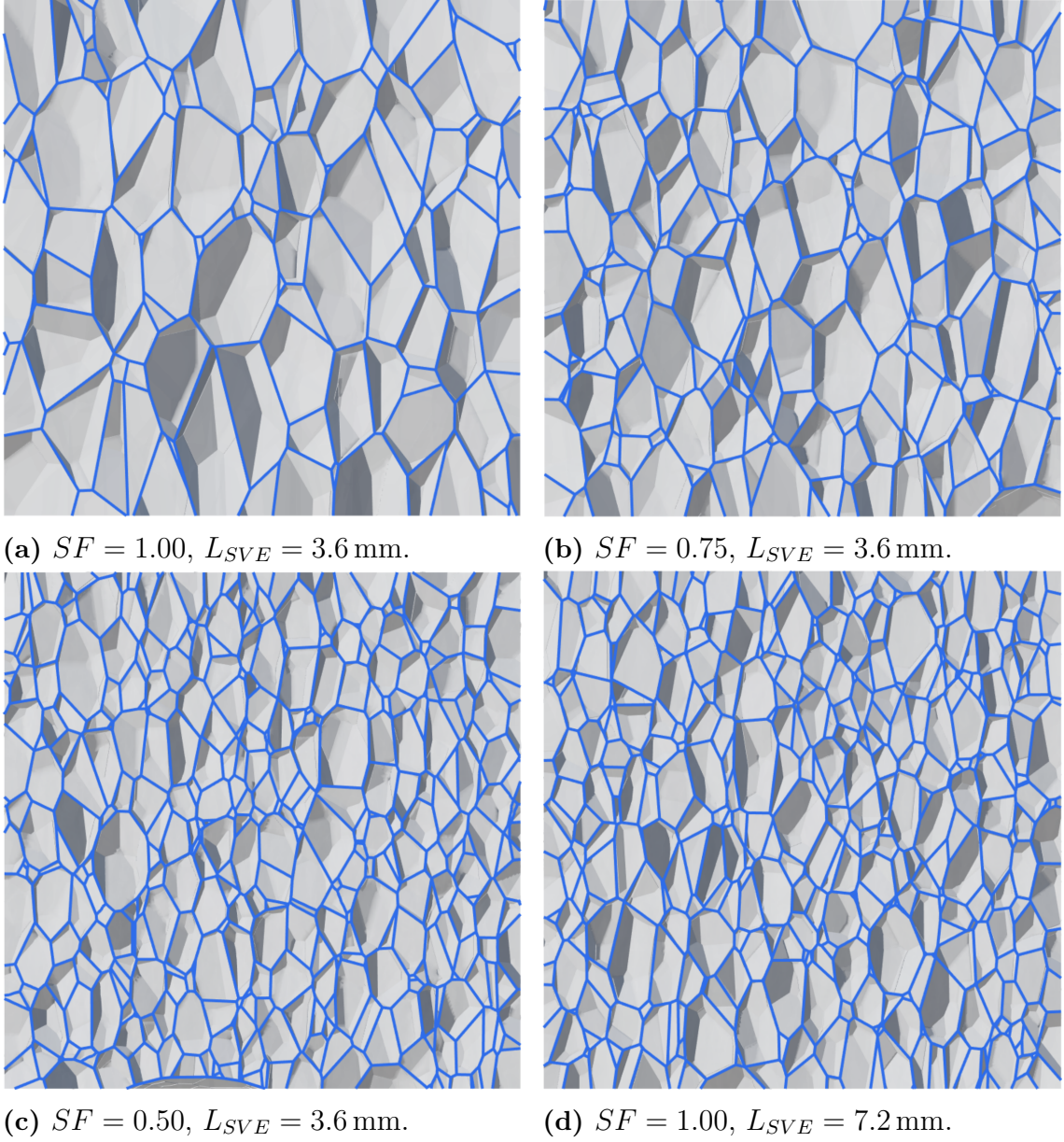


Figure 3.17: Generated SVEs cross-sections', parallel to foam rise direction, with varying scale factors.

As shown in Figure 3.18a, the normalized probability density function (PDF) of the equivalent cell diameter collapse onto a common distribution for all considered values of SF . This indicates that the equivalent cell diameter scales appropriately under uniform geometric scaling. To assess whether this apparent morphological equivalence also results in a comparable mechanical response, an additional SVE with characteristic length $L_{SVE} = 7.2$ mm with $SF = 1.00$ was generated and compared against the SVE with $L_{SVE} = 3.6$ mm with $SF = 0.50$, as both configurations are theoretically equivalent in terms of their scaled morphology.

This invariance of the normalized cell size distribution across different scale factors is also demonstrated by the realized cumulative distribution function (CDF) shown

in Figure 3.18b, where the curves for all scale factors stack on top of each other.

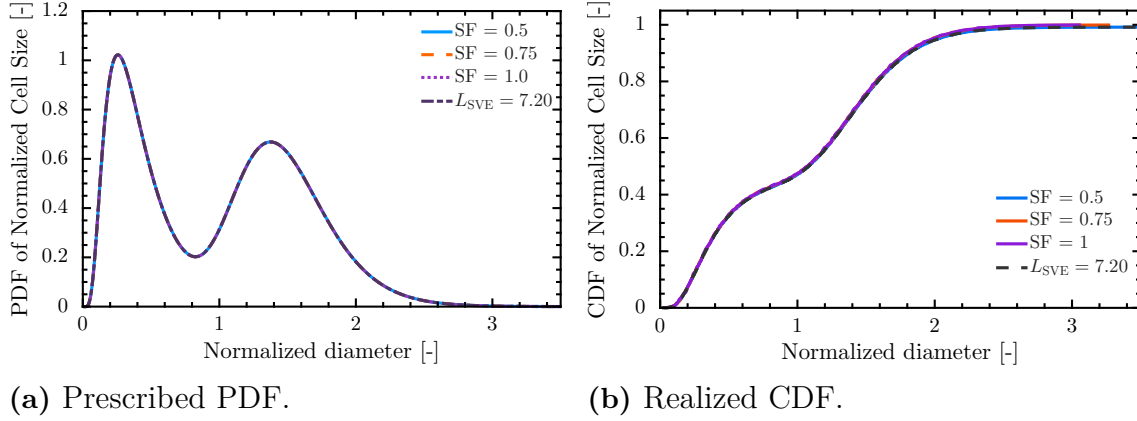


Figure 3.18: Analysis of the statistical distribution of normalized equivalent cell diameter with varying SF .

3.8.2.2 Cell Size Fraction

The second approach investigated the effects of shifting the bimodal ratios that control the probability fractions of large versus small cells. The variable **WS** defined the statistical weight allocated to small cells, while **WL** governed the presence of larger cells. This customized implementation is mapped in Equation 3.26.

$$\begin{aligned}
 \text{MP} = & \text{"diameq: } \mathbf{WS} * \text{lognormal}(0.145, 0.095) + \dots \\
 & \mathbf{WL} * \text{lognormal}(0.495, 0.115), \\
 & \text{1-sphericity: lognormal}(0.235, 0.085), \\
 & \text{aspratio}(1, 1, 1.74) \text{"}
 \end{aligned} \tag{3.26}$$

Modifying these fractional percentages modified bulk surface areas; the global uniform thickness was thus adjusted to protect the target apparent density. The matrix of evaluated bimodal interactions is presented in Table 3.4.

Table 3.4: Distribution weight configurations mapping relative cell size fractions.

Weights (WS / WL)	Thickness [mm]	Number of Cells
80/20	0.0111	2509
60/40	0.0121	1434
47/53	0.0129	1122
0/100	0.0124	627

A visual summary showcasing the resulting morphology SVEs is provided in Figure 3.19.

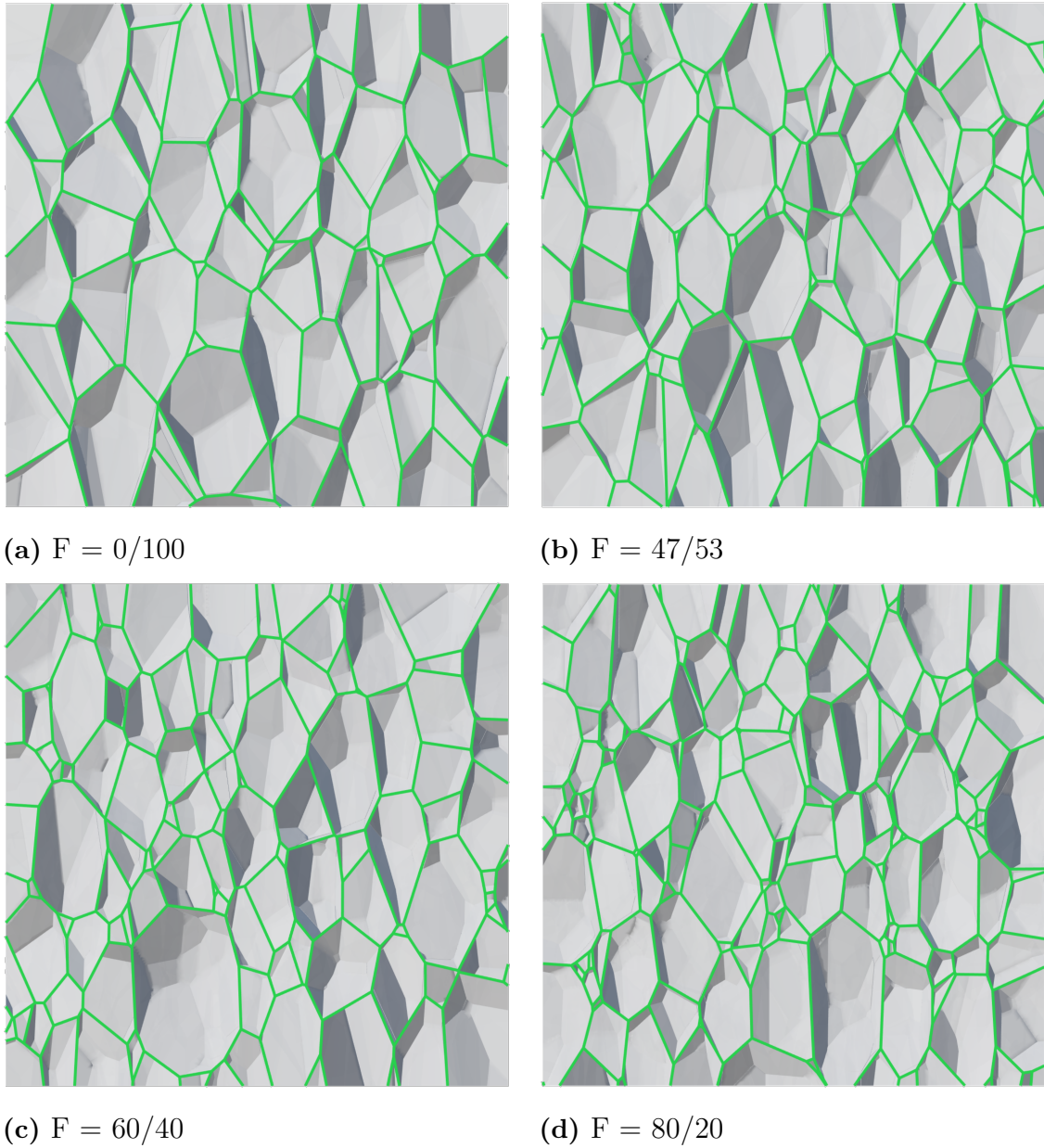


Figure 3.19: Generated SVEs cross-sections' with varying cell size fraction $F = WS/WL$.

Shifting the proportion of smaller to larger cells effectively creates a completely new morphology (see Figure 3.20). Unlike the scaling factor approach shown in Figure 3.18b, the statistical distribution of the equivalent diameter changes when the weights are modified, meaning the distributions do not collapse onto a common curve when normalized. As a consequence these modified morphologies no longer represent the Divinycell H100 foam. This investigation therefore serves as a sensitivity exploration, aiming to isolate how variations in cell size proportions dictate the mechanical behavior of the SVE foam core.

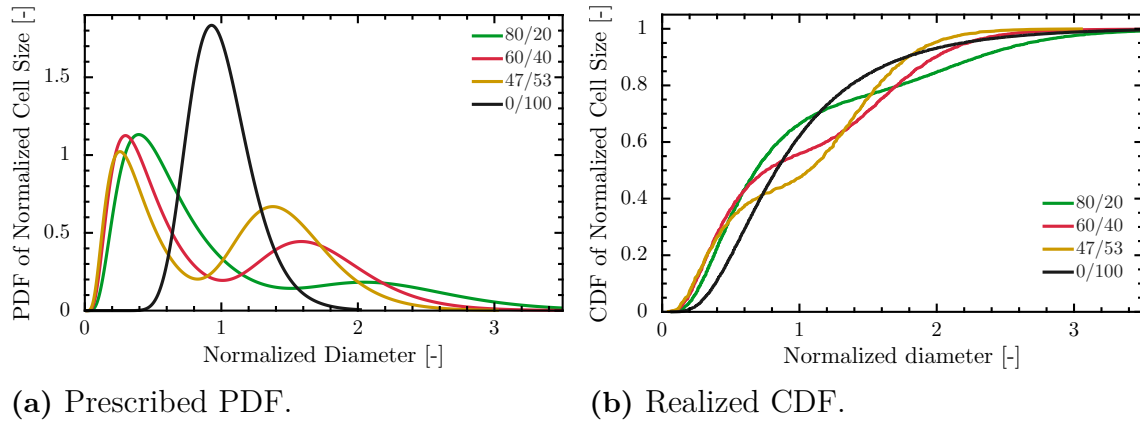


Figure 3.20: Analysis of the statistical distribution of normalized equivalent cell diameter with varying cell size fractions.

4

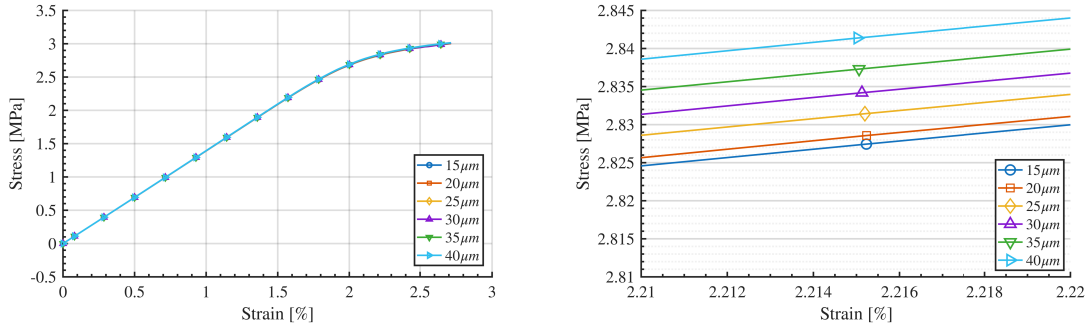
Results

4.1 Mesh Sensitivity

This section presents the results of the mesh sensitivity analysis described in Section 3.4. Both global and local measures are evaluated to assess convergence with respect to mesh refinement.

4.1.1 Global stress–strain response

Figure 4.1 shows the homogenized stress–strain response for all investigated mesh sizes. A high degree of consistency is observed across successive mesh refinements, with only negligible deviations between the curves. This indicates that the macroscopic mechanical response is essentially mesh-independent for the investigated range of element sizes. In particular, both the elastic stiffness and the onset and evolution of plasticity remain unaffected by further mesh refinement.



(a) Full span.

(b) Zoomed in.

Figure 4.1: Macroscopic stress–strain response for different mesh sizes.

4.1.2 Volume-averaged plastic strain

The evolution of the volume-averaged effective plastic strain is shown in Figure 4.2. As expected, the curves exhibit very limited sensitivity to mesh refinement, confirming that this metric is robust with respect to discretization. Only minor differences can be observed at higher strain levels, which may be attributed to small variations in the distribution of localized deformation. However, the overall trend and magnitude remain consistent across all mesh sizes.

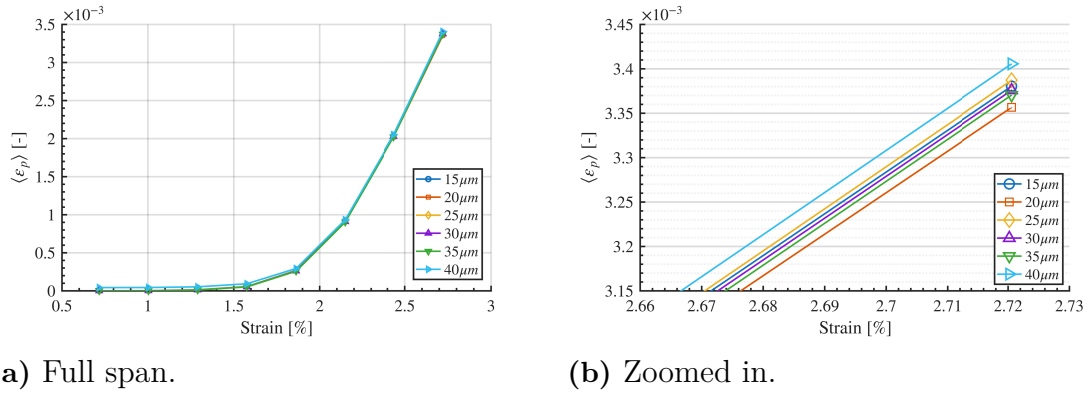


Figure 4.2: Volume-averaged effective plastic strain for different mesh sizes.

4.1.3 95th percentile

Figure 4.3 presents the evolution of the 95th percentile of the effective plastic strain, defined in Section 3.4. Compared to the volume-averaged measure, this metric shows slightly greater sensitivity to mesh refinement, as it captures the upper tail of the deformation distribution. Nevertheless, the results demonstrate clear convergence behavior, with the curves approaching a stable response as the mesh is refined. This indicates that highly localized plastic deformation is adequately resolved for sufficiently fine meshes.

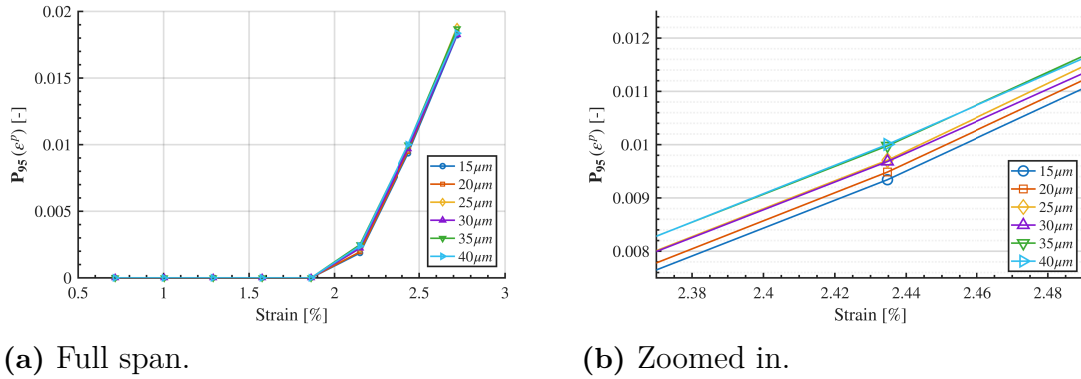


Figure 4.3: 95th percentile of effective plastic strain for different mesh sizes.

4.1.4 Spatial convergence

The spatial distribution of effective plastic strain for selected mesh sizes is shown in Figure 4.4. The deformation patterns exhibit strong qualitative agreement across different mesh resolutions. In particular, consistent load paths, similar localization zones, and identical deformation mechanisms can be observed. This confirms that the governing deformation behavior is not influenced by the spatial discretization.

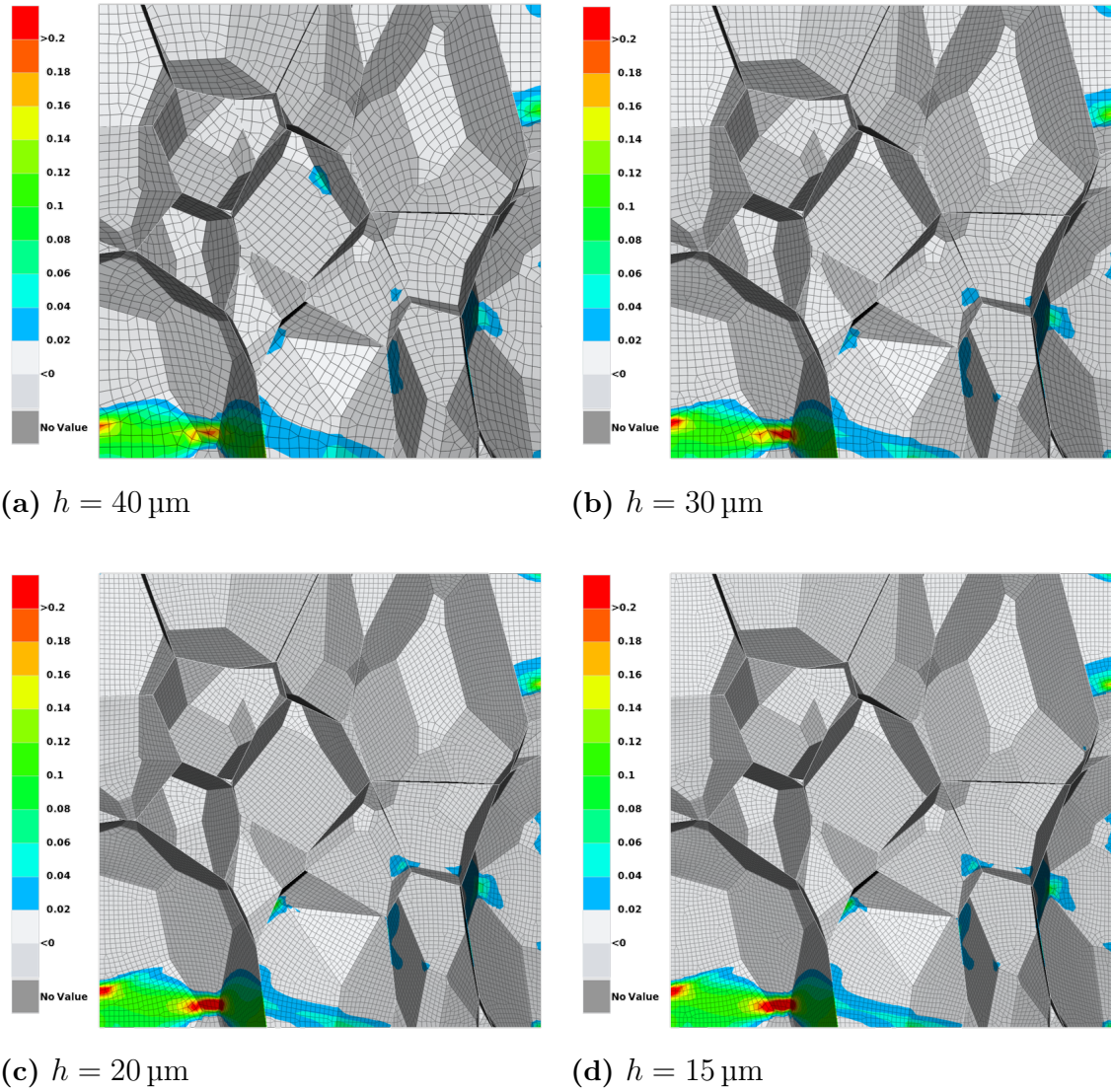


Figure 4.4: Effective plastic strain ε_p^{eff} contour plots for different mesh sizes.

Overall, all considered metrics demonstrate consistent convergence with mesh refinement. The global response is effectively mesh-independent, while local measures show stable behavior for sufficiently fine discretizations. In addition, the spatial distribution of deformation remains unchanged across meshes. Based on these observations, a global mesh size of $h = 30 \mu\text{m}$ is considered sufficient to accurately resolve the mechanical response of the foam mesostructure. This mesh resolution is therefore adopted for all subsequent simulations.

4.2 SVE Size and Realization Invariance

Table 4.1 summarizes key characteristics of the generated SVEs to assess size invariance. Doubling the characteristic length results in an eight-fold increase in the volume, thus correspondingly increasing the total number of cells by roughly eight-fold. The ratio of cell wall area to SVE volume is also dependent on the characteristic

4. Results

length and realization; therefore, the wall thickness must be scaled to maintain the apparent density of Divinycell H100, according to Equation 3.1.

Table 4.1: Geometrical characteristics of the generated foam samples.

Size [mm]	Sample #	Area [mm ²]	Thickness [mm]	Mean D [mm]	# of Cells [-]
1.8	1	31.29	0.0149	0.3309	140
	2	31.66	0.0147		
	3	32.01	0.0146		
2.7	1	116.99	0.0135	0.3306	473
	2	117.19	0.0134		
	3	117.20	0.0134		
3.6	1	289.90	0.0129	0.3309	1122
	2	290.05	0.0129		
	3	290.45	0.0129		
4.5	1	582.98	0.0125	0.3305	2191
	2	582.23	0.0125		
	3	581.05	0.0125		

As shown in Figure 4.5a, the macroscale tensile stress-strain response, averaged across three samples per size, remains invariant to SVE size.

This macroscale invariance is mirrored in the morphological domain by the realized CDF shown in Figure 4.5b. The overlapping curves indicate that even the smallest SVE size ($L_{SVE} = 1.8$ mm) is sufficient to reach statistical convergence regarding the normalized cell size distribution. However, to maintain a conservative safety margin against boundary effects while keeping computational costs manageable, an intermediate SVE size of $L_{SVE} = 3.6$ mm was selected for the remaining analyses.

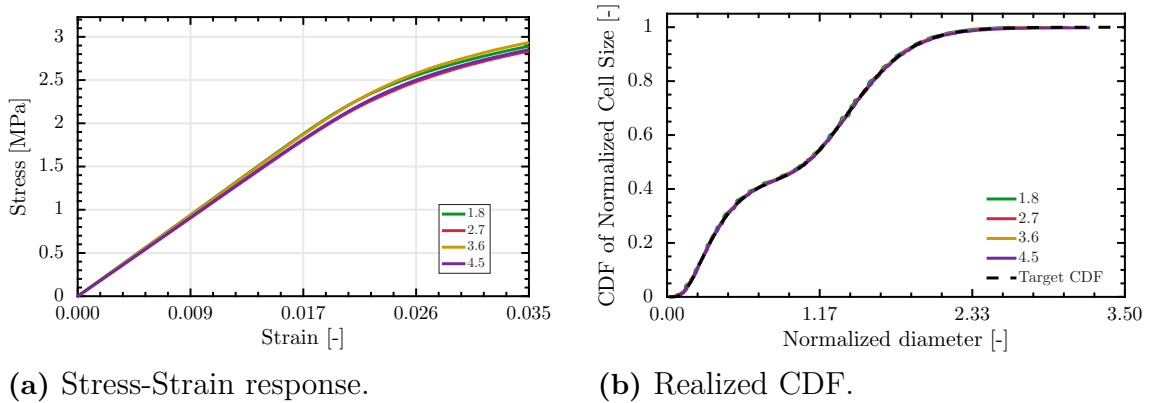


Figure 4.5: Size invariance results.

Furthermore, by solving multiple random SVE realizations of size $L_{SVE} = 3.6$ mm, the confidence interval was computed according to Equation 3.20 and is shown in Figure 4.6.

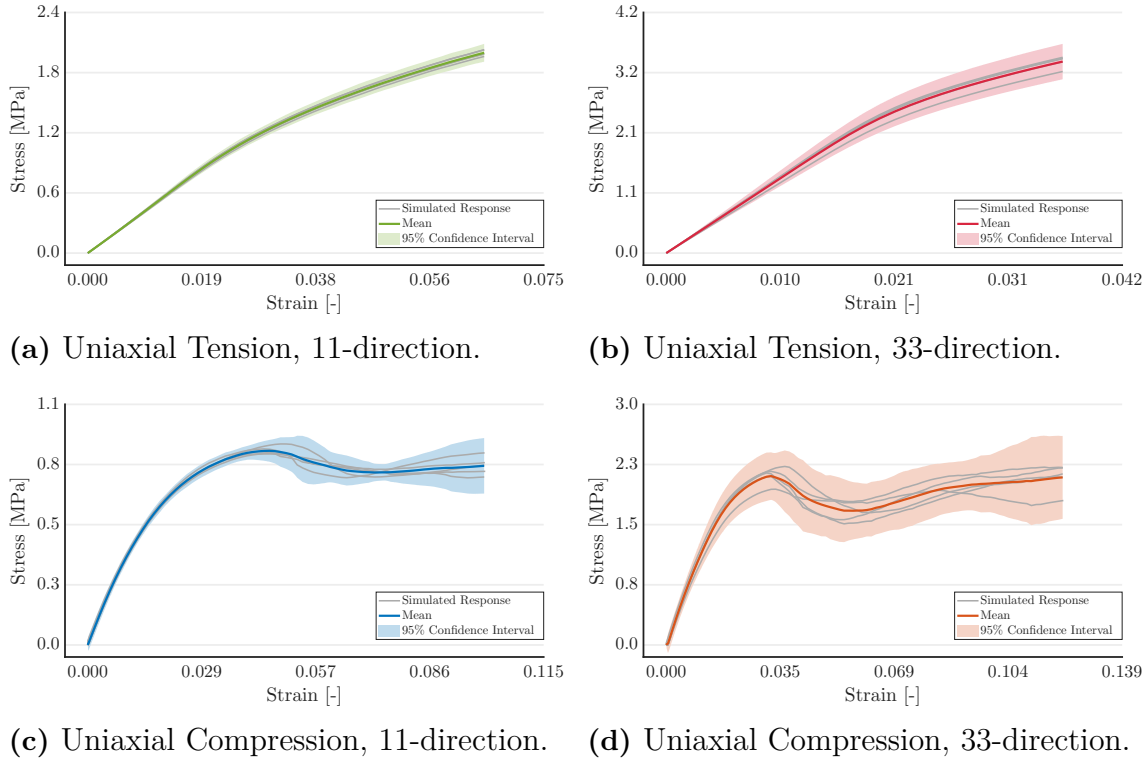


Figure 4.6: Confidence interval results computed from $n = 5$ samples.

The tensile load cases (Figures 4.6a and 4.6b) exhibit narrower bands than their compressive counterparts (Figures 4.6c and 4.6d). A similar behavior is also observed for the different directions: in both tension and compression, 11-direction exhibits a narrower band than the 33-direction.

4.3 Cell Shape Anisotropy

The response of the SVE generated with the initial aspect ratio, $\mathcal{R}_w = 1.29$, detailed in Section 1.4.3.1 (Table 1.3), is presented in Figure 4.7. This aspect ratio yielded a significantly less anisotropic response than that observed experimentally. Note that the term *Baseline* in Figure 4.7 refers to the 3.6 mm SVE generated using the initial aspect ratio of $\mathcal{R}_w = 1.29$.

4. Results

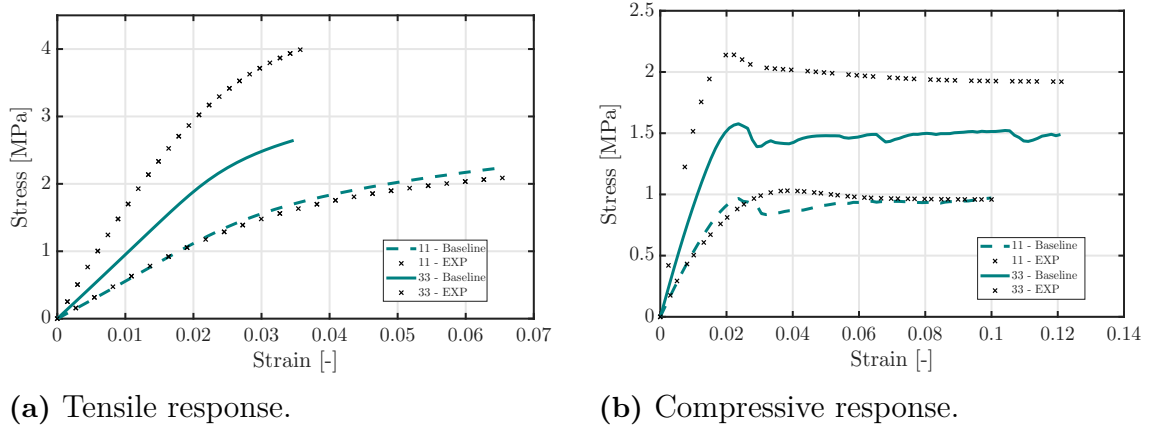


Figure 4.7: Uniaxial response comparison of $\mathcal{R}_w = 1.29$ vs. experimental data.

Following the procedure described in Section 3.6, the influence of the cell aspect ratio on the tensile anisotropy was evaluated for the $L_{SVE} = 3.6$ mm SVE. The simulated tensile responses in the foam rise-direction (33) and in-plane direction (11) are shown in Figure 4.8- 4.9.

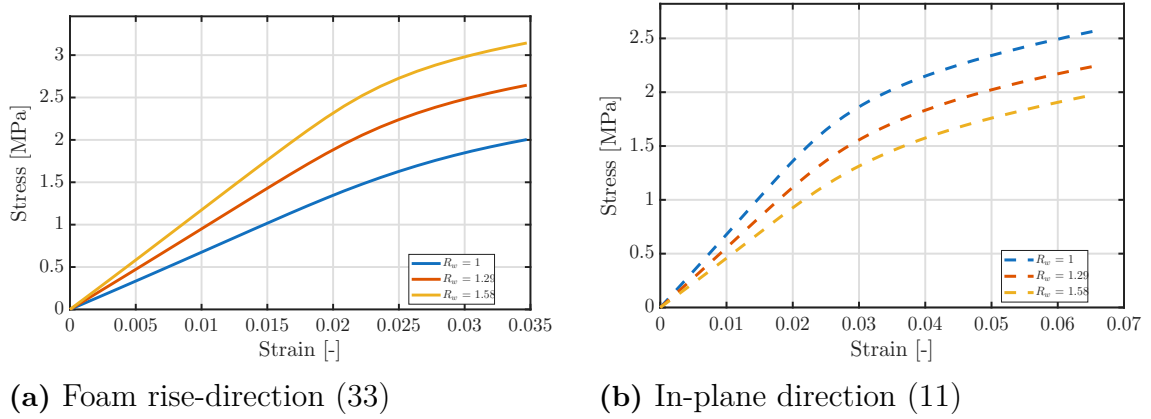


Figure 4.8: Tensile response for the $L_{SVE} = 3.6$ mm SVE for varying aspect ratios.

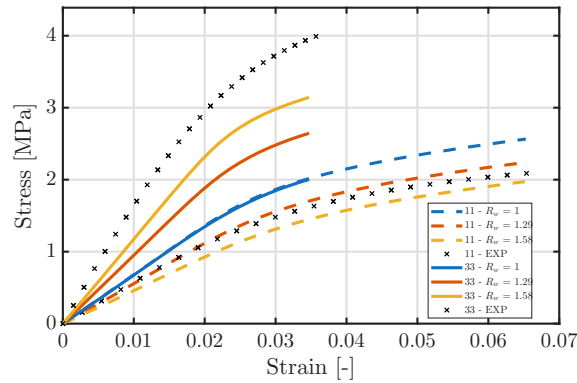


Figure 4.9: Tensile response for the $L_{SVE} = 3.6$ mm SVE for varying aspect ratios compared with the experimental data.

The obtained toughness ratios are summarized in Table 4.2.

Table 4.2: Aspect ratio study for the $L_{SVE} = 3.6$ mm SVE.

Aspect Ratio	Toughness Ratio $\mathcal{R}_U = \frac{U_{T,33}}{U_{T,11}}$
1.00	0.35
1.29	0.57
1.58	0.81

The obtained toughness ratios were fitted through a power-law regression as a function of the aspect ratio. The resulting fitted model is shown in Figure 4.10.

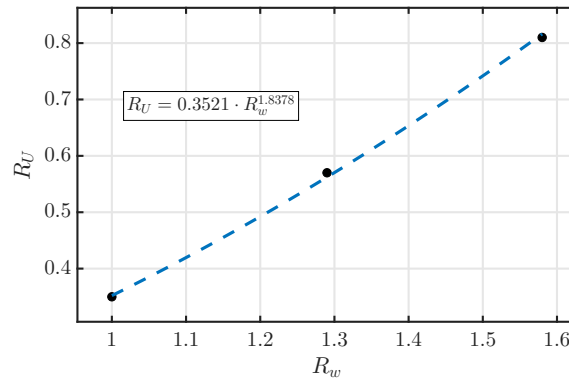


Figure 4.10: Power-law regression fit of the toughness ratio as a function of the aspect ratio.

Rearranging the regression model to solve for the aspect ratio yields:

$$\begin{aligned} \mathcal{R}_U &= 0.3529 \cdot \mathcal{R}_w^{1.8181} \\ \Rightarrow \mathcal{R}_{w_{fitted}} &= \left(\frac{\mathcal{R}_U}{0.3529} \right)^{\frac{1}{1.8181}} \end{aligned} \quad (4.1)$$

Using the fitted expression above, the aspect ratio corresponding to the experimental anisotropy was calculated as:

$$\begin{aligned} \mathcal{R}_{U_{Exp}} &= 0.975 \\ \mathcal{R}_{w_{fitted}} &= \left(\frac{\mathcal{R}_{U_{Exp}}}{0.3529} \right)^{\frac{1}{1.8181}} \\ \mathcal{R}_{w_{fitted}} &\approx 1.746 \end{aligned} \quad (4.2)$$

Using the fitted aspect ratio ($\mathcal{R}_{w_{fitted}} \approx 1.74$), new tensile simulations were performed. The resulting toughness ratio from the fitted anisotropy is $R_U = 0.928$, compared to the experimental value of $R_U = 0.975$, corresponding to a relative difference of approximately -4.84% .

Comparison of fitted responses

A comparison between the experimental results and the fitted responses for the $L_{SVE} = 3.6$ mm SVE is shown in Figure 4.11.

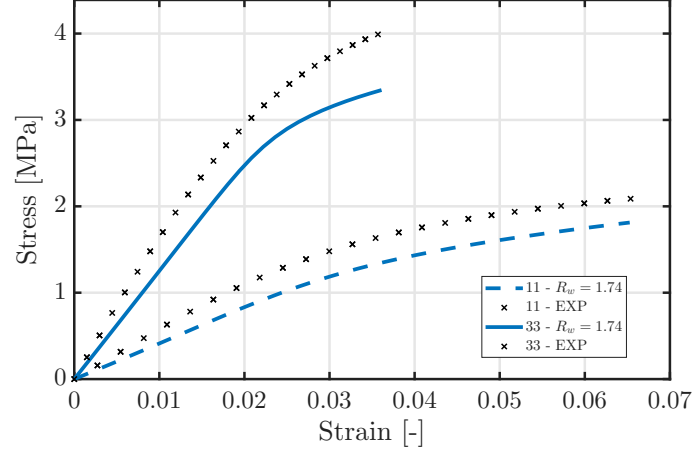


Figure 4.11: Comparison between experimental anisotropy ratios and fitted responses for the $L_{SVE} = 3.6$ mm SVE.

Figures 4.8–4.11 demonstrate that the cell shape aspect ratio is positively correlated with stiffness in the 33-direction and negatively correlated with stiffness in the 11-direction. The aspect ratio obtained using Equation 4.1, $\mathcal{R}_{w_{fitted}} = 1.74$, resulted in an anisotropy level that better matches the behavior observed in the experimental data. Compared to the literature values summarized in Section 1.4.3.1 (Table 1.3), the fitted aspect ratio obtained in this study is notably higher than the values obtained via CT-scan analysis.

4.4 Material Calibration

The nominal and identified material parameters are presented in Table 4.3, and the comparison between the baseline and calibrated material curves is shown in Figure 4.12. The optimization converged within three iterations, suggesting that the nominal parameters already provided a strong correlation with the experimental response.

Table 4.3: Identified Material Parameters.

Parameter	Units	Nominal	Identified	% Difference
E	MPa	2700	3000	+11.11
H	MPa	400	1000	+150.00
σ_y	MPa	56	64	+18.52
$\varepsilon_{u,p}^{eff}$	-	0.1	0.05	-50.00

Both the baseline and calibrated material curves, for the 3.6 mm SVE size with the calibrated aspect ratio of 1.74 from Section 4.3, are plotted in Figure 4.12. In

comparison with the baseline model, the material becomes stiffer in both the linear and plastic regions, exhibiting an earlier total loss of stiffness (i.e., it is stiffer and more brittle).

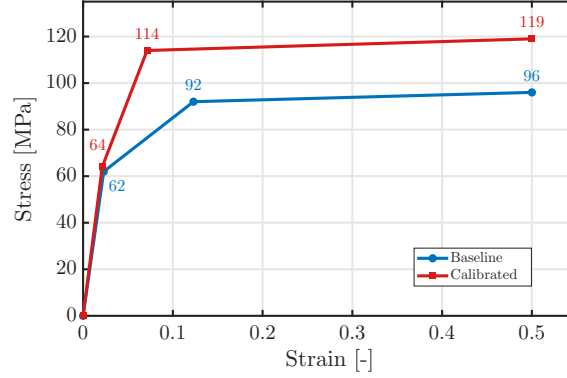


Figure 4.12: Baseline vs. calibrated material curve for the cell wall material.

The uniaxial responses of the 3.6 mm SVE, with an aspect ratio of 1.74, showing a comparison between the baseline, calibrated, and experimental curves, are presented in Figure 4.13.

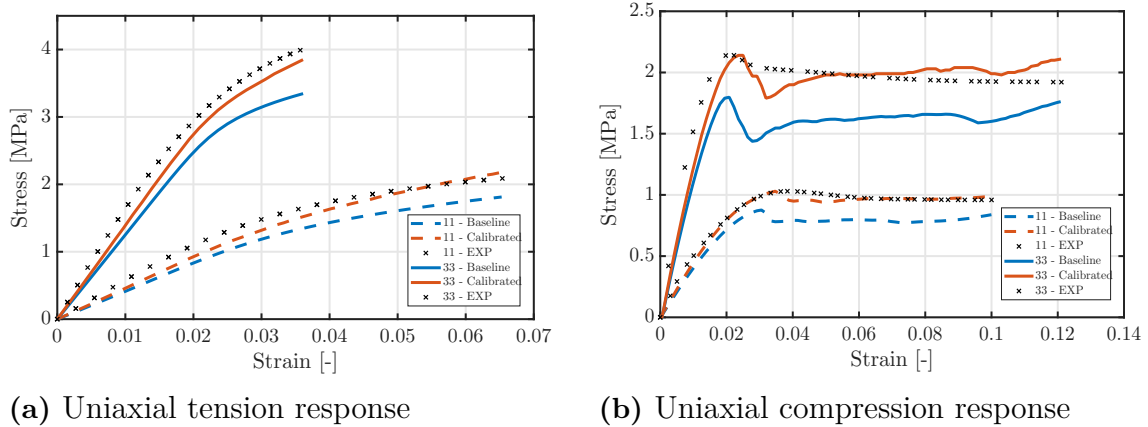


Figure 4.13: Comparison of uniaxial tensile and compressive responses.

4.5 Parametric Study of Morphological Features

4.5.1 Varying Thickness

The stress–strain curves for tension and compression in the rise and in-plane directions, along with external work and eroded energy results from the DCB-inspired test, can be seen in Figures 4.14-4.15.

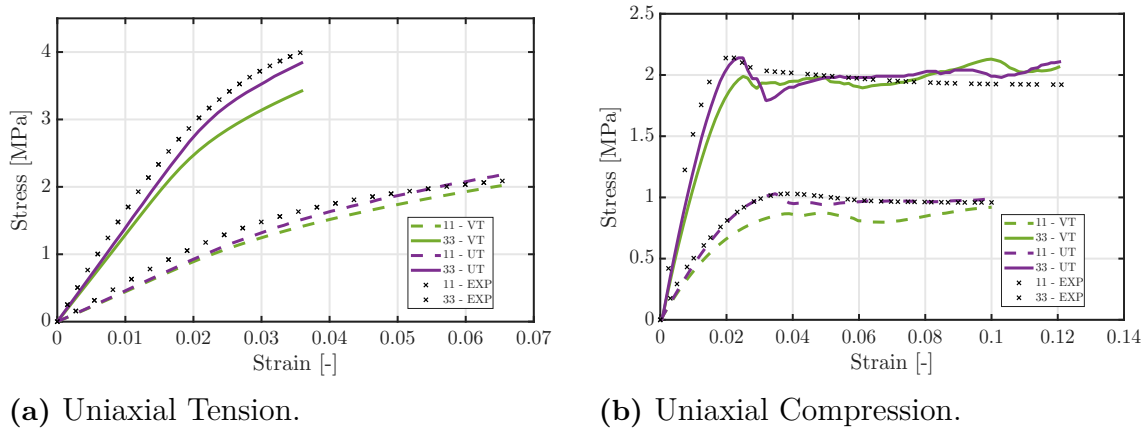


Figure 4.14: Mechanical response from the varying thickness study: (a) uniaxial tension and (b) uniaxial compression. Varying thickness (VT), Uniform thickness (UT).

The stress-strain response when varying thickness is applied to the SVE is softer than the uniform thickness distribution case for uniaxial tension and compression.

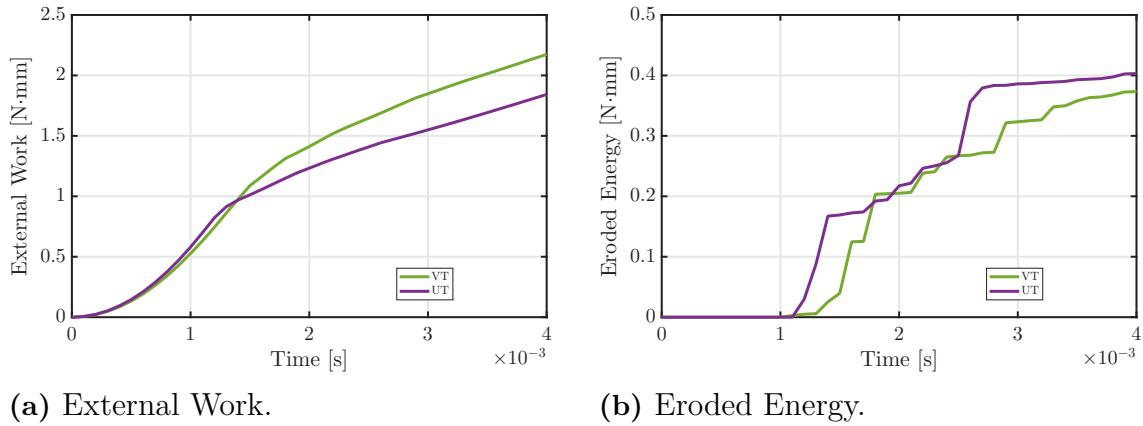


Figure 4.15: Energy metrics from the varying thickness study: (a) external work and (b) eroded energy. Varying thickness (VT), Uniform thickness (UT).

The external work curves obtained from the DCB load case indicate that the varying thickness approach is actually stiffer under this mode of loading. Furthermore, the eroded energy curves suggest higher ductility for the varying thickness configuration.

4.5.2 Equivalent Diameter Distribution

Following the methodology described in Section 3.8.2, the influence of the equivalent diameter distribution was investigated using two separate approaches: scaling the entire cell morphology through a scaling factor (SF), and modifying the bimodal distribution weights between smaller and larger cells.

4.5.2.1 Scaling Factor

Figure 4.16 presents the macroscopic stress-strain responses under uniaxial tension and compression for the various scaling factor configurations. For moderate densification ($SF = 1.00$ and $SF = 0.75$) within the 3.6 mm SVE, the mechanical responses remain largely consistent in both the foam's rise direction (33) and the transverse in-plane direction (11). This consistency suggests that minor uniform scaling adequately preserves the foundational mechanical behavior of the geometric model.

However, a distinct shift in behavior is observed for the $SF = 0.50$ configuration. Under tension, this highly densified sample exhibits a more isotropic mechanical response, characterized by softening in the 33-direction and stiffening in the 11-direction. Under compression, the $SF = 0.50$ configuration yields a stiffer response across both directions.

Finally, a direct comparison is made between the theoretically equivalent scaled morphologies: the 3.6 mm SVE with $SF = 0.50$ and the 7.2 mm SVE with $SF = 1.00$. Both configurations encompass exactly 8973 cells. As shown in Figure 4.16, the tensile responses in the transverse 11-direction align well between the two SVEs. However, the larger 7.2 mm SVE exhibits a stiffer response in the 33-direction. While the compressive response in both directions is stiffer for the smaller densified SVE (3.6 mm with $SF = 0.50$).

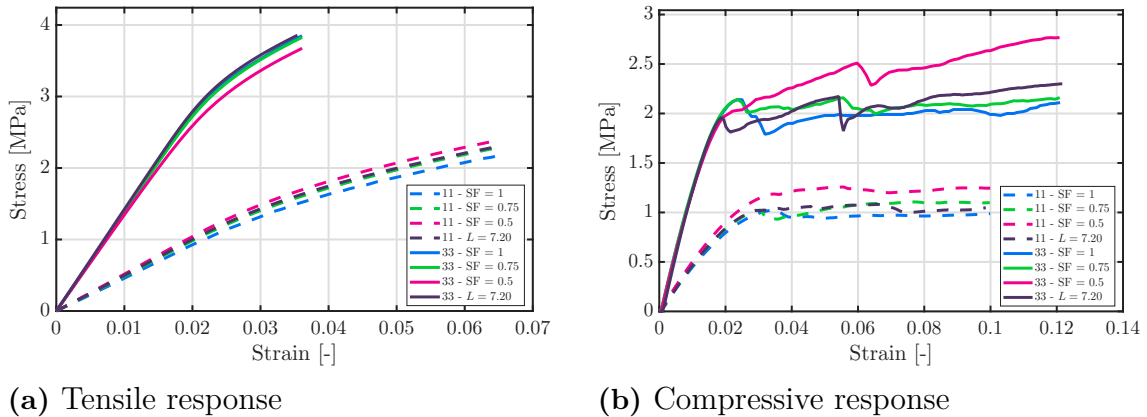


Figure 4.16: Comparison of tensile and compressive responses for different scaling factor configurations.

4.5.2.2 Cell Size Fraction

The same evaluation procedure was carried out for the modified bimodal distribution weights. The resulting tensile and compressive responses are shown in Figure 4.17. Overall, the tensile responses exhibit only minor sensitivity to the weight distribution, while the compressive responses shows decreasing stiffness and the fraction of larger cells increase.

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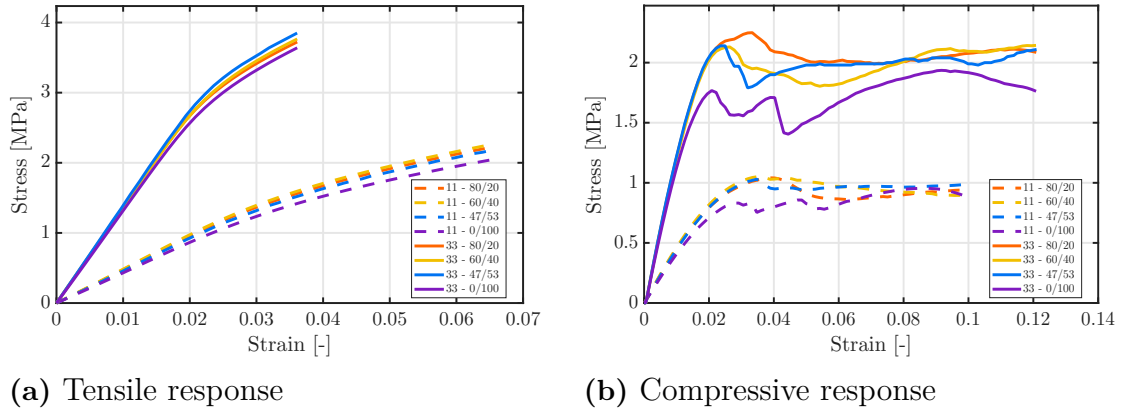


Figure 4.17: Mechanical response for different equivalent diameter weight configurations.

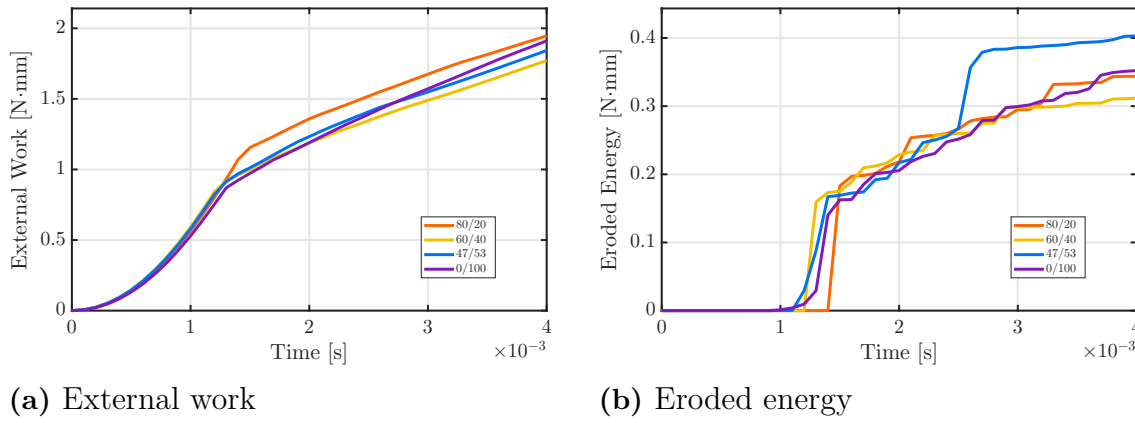


Figure 4.18: DCB-inspired load case results for different equivalent diameter weight configurations.

It can be observed from the eroded energy response in Figure 4.18b that, as the proportion of larger cells decreases, the response becomes more ductile. However, aside from the calibrated SVE response, the remaining configurations appear to be relatively similar. From the external work response shown in Figure 4.18a, the stiffest behavior is found for the weight distribution configured with $WS = 80$ and $WL = 20$.

5

Conclusion and Future Work

Despite the stochastic nature of the proposed numerical framework, material uncertainties, and other modeling limitations, meaningful conclusions can be drawn from the obtained results.

5.1 Mesh Sensitivity

Although the macroscopic stress–strain responses exhibit negligible differences across the investigated mesh sizes (as shown in Section 4.1 and Figure 4.1), the present study does not establish mesh convergence in the strict numerical sense. A formal convergence study requires systematic mesh refinement by a constant ratio (typically a factor of two) to mathematically quantify the reduction in error between successive solutions. Nevertheless, the results indicate that deviations between the curves decrease with further refinement, suggesting practical asymptotic convergence of the homogenized response. Therefore, for the purposes of capturing the homogenized effective properties, the selected mesh resolution proved sufficiently stable and robust.

For an average equivalent cell diameter of $\mu_s = 0.145$ mm, a mesh size of $h = 30$ μm is sufficient to properly resolve the cellular morphology. Therefore, a relative mesh size limit of $h/\mu_s \leq 0.21$ is proposed for assessing homogenized stress–strain responses.

5.2 SVE Size and Realization Invariance

Size invariance was achieved for relatively small SVEs with a characteristic length of $L_{SVE} = 1.8$ mm (see Section 4.2, Figure 4.5a). It must be noted, however, that while size invariance was confirmed, this validation was exclusively conducted under uniaxial tension in the foam-rise direction.

Because SVEs are inherently realization-dependent, a confidence interval was established (Figure 4.6). The tensile responses in both orthogonal directions exhibit a relatively narrow confidence interval with a low coefficient of variation, indicating consistent behavior and low scatter. In contrast, the compressive responses display a significantly broader spread. This increased variance is attributed to the more complex deformation mechanisms involved in compression, namely geometric instability, cell wall buckling, and self-contact, all of which are highly sensitive to the specific localized geometry of a given SVE realization.

The rationale for adopting the CV to predict the structural response envelopes of new SVE realizations stems from the underlying geometric characteristics and generation

process. Because the mesostructural cells in new realizations are populated using identical or similar parent statistical distributions for parameters such as equivalent diameter and sphericity, for example, the relative macroscopic dispersion is expected to be conserved.

While the absolute stress response may shift depending on the specific topological instance, the proportional scatter remains tied to the generation parameters. By operating under the assumption that the new SVE response effectively acts as the central tendency for its specific mesostructural configuration, scaling it by the pre-calculated CV bypasses the computationally expensive requirement of running a full sensitivity study for every new topology. The resulting $\pm 2s_{\text{est}}$ envelope provides a computationally efficient and statistically grounded estimate of the expected variance, enabling the evaluation of new SVEs without redundant simulation overhead. It is expected that as the SVE size increases toward the macroscale, these scatter effects will diminish, and the mechanical behavior will converge strictly to the homogenized bulk response. Likewise, averaging over a larger population of random SVE realizations would smooth the variance and yield a converged mean response.

5.2.1 Cell Shape Anisotropy

The cell shape anisotropy study was motivated by the observation that the initial baseline model ($\mathcal{R}_w = 1.29$) significantly underestimated the transverse anisotropy observed experimentally in both tension and compression.

To address this, the aspect ratio was actively calibrated to match the toughness ratio extracted from experimental data (Section 4.3, Equation 4.2). The resulting fitted $\mathcal{R}_w$ value differs from those typically reported in the literature. Consequently, the calibrated aspect ratio diverges from the original diameter-equivalent formulation proposed by Liu et al. [15] (Section 3.2, Equation 3.2). It is worth noting that more recent CT-based analyses by Liu et al. [15] suggest a revised aspect ratio of approximately 1.39, indicating that earlier morphological estimates were likely conservative.

The results demonstrate that for an SVE size of $L_{\text{SVE}} = 3.6$ mm, a high aspect ratio of approximately 1.74 (a 35% increase) was required to reproduce the experimentally observed macroscopic transverse isotropy. This confirms that the aspect ratio is a highly dominant parameter governing the transversely isotropic behavior of foam core materials.

5.3 Material Calibration

Section 4.4 (Figure 4.13) compares the baseline, optimized, and experimental stress–strain responses for the 3.6 mm SVE size with the calibrated aspect ratio of 1.74. Following the inverse calibration procedure, the optimized material parameters resulted in a uniaxial response that closely mimics the experimental behavior in both tension and compression.

In tension, the numerical model mildly underestimates the stiffness in the 33-direction. In the 11-direction, it slightly underestimates the response at lower strains before

slightly overshooting at higher strains. In compression, the 11-direction shows excellent overall agreement, whereas the 33-direction initially underestimates the response before slightly overestimating it at higher strains.

These minor discrepancies can be attributed to the stochastic nature of the SVE realization, as the generated geometry directly dictates the local mechanical response. Combined with the confidence intervals shown in Figure 4.6, this implies that alternative SVE realizations could yield responses that align even better with the target experimental curves.

Furthermore, due to the limited availability of data for the Divinycell H100 cell wall material, nominal solid PVC material properties were assumed for the base cell wall material elastic constants. Due to lack of data, the calibrated parameters are specific to this modeling framework and should be interpreted as effective numerical properties rather than true physical parameters. If direct experimental characterization of the isolated cell wall material becomes available, the calibrated framework could achieve an even closer match without relying on inverse optimization, or relying on more conservative bounds. As mentioned in Section 1.4.1, IPN foams are stronger and stiffer than regular PVC foams cores, which corroborates the stiffer and stronger identified material in the material calibration procedure.

5.4 Parametric Study

5.4.1 Varying Thickness

Section 4.5.1 (Figures 4.14- 4.15) highlights distinct mechanical differences between the Uniform Thickness (UT) assumption and the applied Varying Thickness (VT) distribution. Specifically, for the DCB-inspired load case, the VT framework exhibited a stiffer response in external work (Figure 4.15a) while demonstrating a more ductile eroded energy profile compared to the UT case (Figure 4.15b).

Even under the UT assumption, structural intersections where multiple cell walls meet inherently accumulate more mass. These regions act as the stiff strut-like features observed in real CT-based foam reconstructions (see Section 1.4.1, Figure 1.3). Because mechanical loads naturally follow the stiffest available path, these struts become the primary load-bearing members. When the VT distribution is introduced, the struts become mathematically thicker and stiffer, while the central cell walls become thinner, thus representing better the foam's morphology.

Despite these behavioral differences, neither approach can be definitively labeled as superior based solely on homogenized uniaxial load cases. The VT model represents a deliberate step toward greater morphological realism; however, it remains an idealization and does not constitute a one-to-one representation of physical foam geometries, which would require direct 3D CT-scan integration.

5.4.2 Equivalent Diameter Distribution

Scale Factor

The scaling factor study demonstrates that modifying the equivalent diameter distribution influences the mechanical response beyond the effect of simply changing the

characteristic cell size. Under the assumptions of first-order homogenization, uniformly scaling all geometric dimensions should preserve the normalized morphology and therefore result in identical macroscopic behavior. Consequently, the 7.2 mm SVE with $SF = 1.00$ and the 3.6 mm SVE with $SF = 0.50$, which represent theoretically equivalent scaled morphologies and contain the same number of cells, would be expected to exhibit the same response. Likewise, reducing the scaling factor within a fixed 3.6 mm domain should primarily increase the number of cells while preserving the underlying morphology, resulting in an unchanged homogenized response once sufficient statistical representativeness has been achieved.

However, neither of these expectations is fully observed. Differences remain between the responses of the scaled configurations in both tension and compression, indicating that the applied equivalent diameter scaling is accompanied by additional morphological changes that are not captured by the normalized diameter distribution alone. Furthermore, particularly under compression, localized deformation mechanisms may develop, causing the assumptions of first-order homogenization to become less applicable.

Therefore, the present results suggest that changing the equivalent diameter distribution affects not only the characteristic cell size but also other aspects of the foam morphology. The resulting mechanical response should consequently be interpreted as the combined effect of cell-size scaling and accompanying morphological variations, rather than as the isolated influence of equivalent diameter alone.

Cell Size Fraction

Varying the statistical weight ratio between small and large cells generated entirely new topologies (Figure 3.20). As observed in Section 4.5.2.2 (Figure 4.17), the tensile and compressive responses were largely insensitive to the cell size fraction, with most simulations falling within the established confidence interval.

A greater spread was observed in the compressive load cases, where the 0/100 cell size fraction consistently exhibited the softest response across all loading directions. This behavior may be attributed to the dominance of larger, more slender cells, which are inherently more susceptible to geometric instabilities and localized crushing under compression. Furthermore, the DCB-inspired results (Figure 4.18) showed no clear monotonic trend or distinct improvement in fracture resistance among the investigated configurations.

5.5 Summary

In conclusion, this thesis successfully established a robust, SVE-based FE-framework capable of predicting the macroscopic mechanical response and fracture behavior of Divinycell H100 PVC foam directly from its underlying cellular mesostructure. By integrating stochastic geometric generation, rigorous mesh sensitivity analyses, and inverse material calibration, the framework demonstrated a high degree of fidelity in replicating experimental behaviors under both tensile and compressive loading. Furthermore, the systematic parametric investigations provided deep theoretical insights into how isolated morphological features, such as cell wall thickness varia-

tions, geometric anisotropy, and size distributions, fundamentally dictate structural performance and failure modes. While certain idealizations regarding boundary conditions and material constitutive law remain, the developed methodology provides a powerful, modular foundation for the multiscale analysis of polymeric foams, ultimately offering a pathway toward the optimized, microstructurally informed design of advanced lightweight sandwich structures.

5.6 Future Work

The following points outline potential areas for future research and model refinement:

- **Inclusion of shear loading conditions:** Shear loading should be implemented to evaluate its influence on localized cell-wall bending and to assess its impact on the multi-axial material calibration procedure.
- **Cell-wise thickness variability:** While the current model incorporates intra-cell thickness variation, the thickness distribution is applied uniformly across all cells through a global rule implemented in the VT script (Appendix D). As a result, the natural cell-to-cell variability observed in real foam mesostructures is not captured. A valuable direction for future work would therefore be to introduce a stochastic cell-to-cell thickness distribution, allowing each cell to exhibit its own unique thickness characteristics. This would better represent the inherent geometric heterogeneity of actual foam structures and move the model closer to true geometric realism.
- **Increased SVE sampling:** The statistical reliability of the homogenized response could be improved by evaluating a larger population of random SVE realizations, as literature indicates that complex property extraction typically requires significantly more than five discrete samples.
- **Constitutive Modeling:** A trilinear elasto-plastic material law with isotropic hardening was utilized in this study. Future work may explore different and more complex material models, including for example strain rate dependence and/or temperature effects.
- **Extension to alternative foam densities:** Running the calibrated framework against higher and lower density grades (e.g., H60, H80, and H130) would provide direct validation of the structure–property relationships and improve predictive confidence.
- **Direct experimental characterization:** Conducting micro-mechanical testing on the isolated solid cell-wall material could eliminate the need for inverse numerical calibration and ground the framework in direct physical reality.
- **High-fidelity mesoscale imaging:** Utilizing advanced, high-resolution X-ray CT scans to directly drive the FE model would bypass some of the limitations of stochastic Laguerre tessellations and resolve existing discrepancies regarding true cell representation. Although, multiple samples would still be required to obtain an averaged response.
- **Alternative boundary conditions:** Future studies should explore the implementation of other boundary conditions to evaluate the upper and lower bounds of the effective stiffness envelope.

- **Cell Size Fraction:** Because the DCB-inspired load case yielded results with no apparent trend, future work could investigate multiple realizations for each cell size fraction. The responses could then be averaged and compared with the averaged responses of other cell size fractions. This approach may reveal an underlying trend, or alternatively, provide stronger evidence that the response is insensitive to cell size fraction for this particular load case.

- **Cell Shape Aspect Ratio:**

Future investigations could expand the SVE size invariance study to include the cell shape aspect ratio, exploring whether alternative SVE sizes yield an aspect ratio that better reflects the physical CT-scan measurements. Furthermore, evaluating this parameter within the broader parametric study under all five load cases would help quantify its influence on the foam's mechanical response. This is especially pertinent for the DCB-inspired load case, where the material behavior is inherently more complex and less predictable than under standard uniaxial loading. Given that Diab can tailor this specific morphological parameter during manufacturing to a certain degree, a deeper computational understanding of its macroscopic effects would be highly beneficial.

- **Investigation into Boundary Conditions effects:** Future work should further investigate the role of boundary conditions, as they are known to strongly influence the macroscopic response of the representative volume element, as discussed in Section 2.2. In particular, the use of strict PBC for stochastic tessellation-based models remains challenging, as enforcing perfect periodicity on highly irregular cellular morphologies may artificially constrain natural deformation mechanisms.

In this study, a mixed Neumann and Dirichlet-like boundary formulation (OMBC) was employed. This approach is known to produce an effective stiffness response bounded by the Voigt and Reuss limits, with a tendency toward the lower end of the spectrum. Future investigations could explore the systematic use of alternative boundary conditions, including more kinematically constrained formulations such as DDBC, in order to quantify the resulting upper and lower bounds of the effective stiffness envelope.

Such studies would also help clarify how boundary-induced stiffness bias interacts with inverse calibration procedures. In particular, more restrictive boundary conditions may artificially stiffen the structural response, thereby requiring compensatory adjustments in the identified material parameters. A more comprehensive comparison of boundary formulations would therefore improve the robustness and physical interpretability of the calibration framework, especially when relating numerical predictions to experimentally observed stress-strain behavior.

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A

Quasi-Static Assumption

Explicit dynamic formulations are inherently time dependent, which makes the simulation of quasi-static events nontrivial. The loading must be applied sufficiently slowly to avoid inertia effects, yet not so slowly that the computational cost becomes unnecessarily large. An appropriate balance between physical accuracy and computational efficiency is therefore required.

In a classical sense, a process can be considered quasi-static when inertial forces are negligible compared to internal forces. In explicit dynamic finite element analyses, this condition is commonly assessed by verifying that the kinetic energy remains much smaller than the internal (strain) energy throughout the simulation, i.e.,

$$E_{\text{kin}} \ll E_{\text{int}}. \quad (\text{A.1})$$

A frequently adopted practical criterion is that the kinetic energy should remain below approximately 5–10% of the internal energy. When this condition is satisfied, the structural response is dominated by material deformation rather than inertia effects. For this project, a strict ratio $E_{\text{kin}}/E_{\text{int}}$ of 1% was adopted.

All analyses were performed under velocity-controlled loading. A prescribed velocity curve was applied and integrated in time to obtain the corresponding displacement. In explicit dynamic solvers, prescribing velocity is generally preferred over displacement because it results in smoother force transmission, reduces the likelihood of numerical instabilities associated with displacement constraints, and allows better control of kinetic energy levels during loading.

To determine the maximum admissible loading rate consistent with the quasi-static assumption, a parametric study on applied velocity was conducted, using the uniaxial load case in the foam rise direction. The same constitutive model, including erosion, used in the analyses defined in Section 3.3 was employed. At the time of this assessment, however, the material parameters had not yet been fully calibrated; therefore, the absolute response differs from the final calibrated model, although the inertia-related trends remain representative.

Applying a constant velocity instantaneously at the beginning of the simulation would introduce artificial velocity shocks, potentially generating stress waves and numerical oscillations. To avoid this, the prescribed velocity was defined via a polynomial of the fifth order approximation, ensuring:

- C^2 continuity
- zero slope at start and end
- zero acceleration at start and end

The curve is visualized in Figure A.1

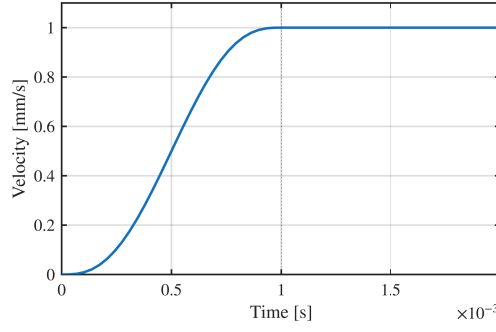


Figure A.1: Prescribed quintic smooth step velocity curve.

The curve was scaled by a constant factor $SF = [5, 10, 20, 40, 100, 200]$ for different simulation runs, while all other analysis parameters were kept unchanged.

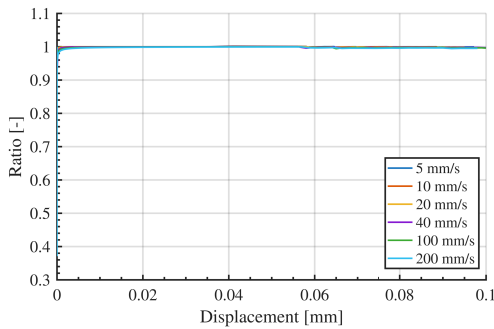
To evaluate the quasi-static condition, the ratios between internal energy and total energy E_{int}/E_{tot} and between kinetic energy and internal energy E_{kin}/E_{int} were monitored as functions of the applied displacement. Since the total energy can be expressed as

$$E_{tot} = E_{int} + E_{kin} + E_{other}, \quad (\text{A.2})$$

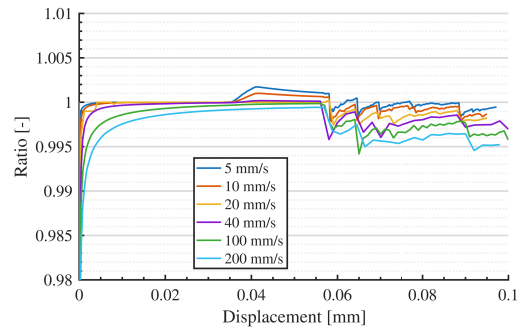
where E_{other} includes contact and hourglass contributions, maintaining a high ratio E_{int}/E_{tot} ensures that kinetic effects remain small.

At the beginning of each simulation, a kink, proportional to the velocity magnitude, appears in the energy ratio curves due to the initial rapid ramp-up of the velocity profile. This behavior can be observed in the full-span plots in Figures A.2a and A.3a. Following this short transient phase, both ratios quickly stabilize, as highlighted in the corresponding zoomed-in views (Figures A.2b and A.3b).

As material erosion progresses, fragments detach from the SVE and acquire kinetic energy, leading to small fluctuations (below 0.6%) in the energy ratios. These oscillations are visible in the zoomed regions and are attributed to inertia effects associated with the sudden release of strain energy in eroded elements.



(a) Full span.



(b) Zoomed in.

Figure A.2: E_{int}/E_{tot} Ratio.

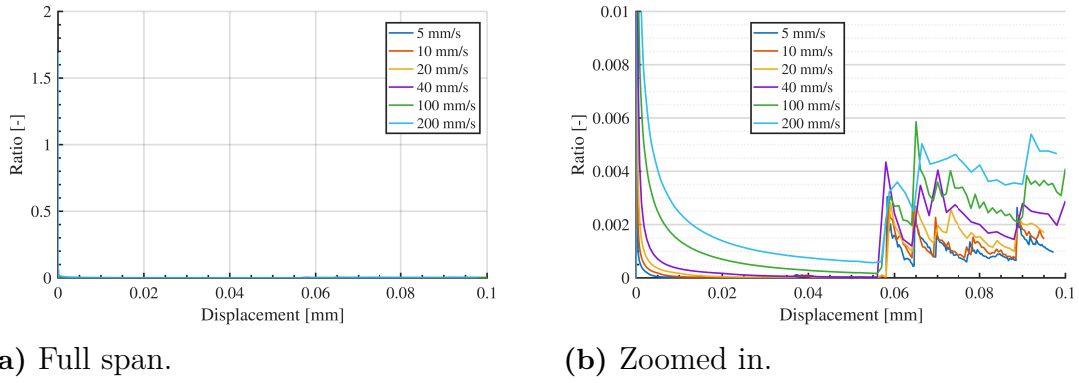


Figure A.3: E_{kin}/E_{int} Ratio.

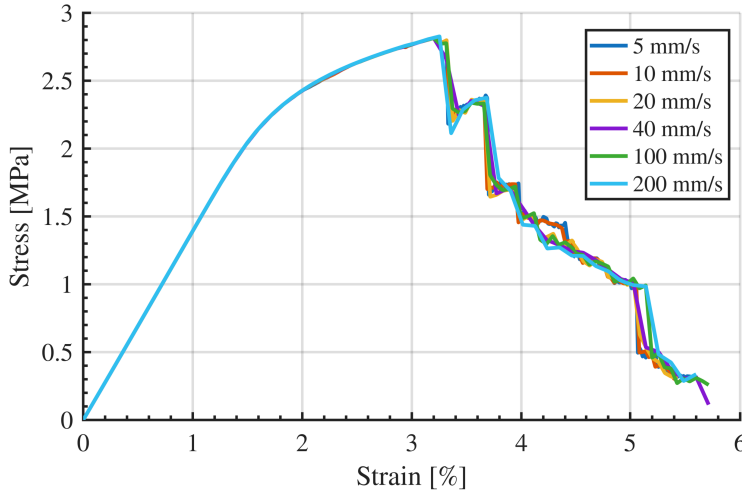


Figure A.4: Macroscopic stress-strain response

For all investigated velocity scale factors, the macroscopic mechanical response of the SVE remains virtually identical, as shown in Figure A.4. At the same time, the kinetic energy remains well below the commonly accepted threshold relative to the internal energy, as evidenced by the stabilized regions of Figures A.2 and A.3.

For values beyond $SF = 200$ (corresponding to peak velocities exceeding 200 mm/s), inertia effects remain negligible. However, for values beyond $SF = 100$, the displacement accumulated during the ramp-up phase becomes comparable to the macroscopic strain required to trigger element erosion, which should ideally occur under constant velocity conditions. Furthermore, in the compression load cases, the contacting plates are not initially in contact with the SVE. If the velocity is too high when contact first occurs, inertial and stress-wave effects might occur, causing localized damage like shearing, spalling, or perforation. Therefore, the velocity scale factor should be limited to 50.

Based on these observations, a scale factor of 50 was selected for all velocity-controlled analyses, as it provides a suitable balance between computational efficiency and adherence to the quasi-static assumption.

B

Statistical Distribution of Equivalent Diameters

Due to the stochastic nature of the equivalent diameter distribution, coupled with the sphericity distribution, very small or very large foam cells may occasionally be generated, although with low probability. While this may represent physical reality, it poses practical challenges in the present study. Generating multiple SVEs with sufficiently large characteristic lengths and hundreds of random seeds to fully capture the stochastic mechanical response would require computational resources beyond what is available.

In this work, extremely small and extremely large cells are treated as defects and are therefore undesirable in the analyzed SVEs. In particular, very small cells introduce significant pre-processing difficulties: meshing such features leads to excessively small elements, which in turn reduce the stable time increment in explicit dynamic simulations. The following criteria were applied to limit SVE realization with highly irregular features.

Distribution Tail Control and Numerical Stability

To ensure the realization adheres to the prescribed bimodal distribution while maintaining numerical stability, the equivalent cell diameters were compared against 5–95% percentile truncation bounds: $d_{eq}^{\min, \text{dist}} = 0.0454$ and $d_{eq}^{\max, \text{dist}} = 0.7030$. The percentage of cells falling below the lower threshold was computed as:

$$\text{perc}_{\text{small}} = \frac{N(d_{eq} < d_{eq}^{\min, \text{dist}})}{N_{\text{cells}}} \times 100 \quad (\text{B.1})$$

The SVE was rejected if $\text{perc}_{\text{small}} > 3\%$. This condition limits the occurrence of excessively small cells, which would otherwise necessitate severe mesh refinement and result in prohibitively small explicit time steps.

Geometric Representativity and Size Constraints

To prevent the mesostructure from being dominated by a small number of over-sized cells, which could distort spatial heterogeneity and compromise homogenization, three constraints were enforced relative to the SVE edge length L_{SVE} :

1. **Large Cell Frequency:** The percentage of cells exceeding a size threshold of $L_{SVE}/3$ must not exceed 3% of the total population:

$$\text{perc}_{L/3} = \frac{N(d_{eq} > L_{SVE}/3)}{N_{\text{cells}}} \times 100 \leq 3\% \quad (\text{B.2})$$

2. **Diameter Severity:** To prevent "near-spanning" cells from localizing deformation or dominating the response, an absolute limit was imposed on the maximum equivalent diameter:

$$d_{eq}^{\max} \leq 0.4L_{SVE}$$

3. **Volume Fraction:** The volume of the largest cell, $V_{\max}$, must occupy less than 4% of the total SVE volume $V_{SVE} = L_{SVE}^3$:

$$f_{\max} = \frac{V_{\max}}{L_{SVE}^3} \leq 0.04 \quad (\text{B.3})$$

By enforcing these multi-level controls, the study ensures that every analyzed SVE is both a statistically valid representation of the Divinycell H100 foam and a numerically robust model for explicit dynamic analysis.

Acceptance Logic

The filtering procedure follows a conservative logic, meaning the violation of any single criterion leads to rejection of the SVE. This multi-level control ensures the following:

- statistical consistency with the prescribed bimodal distribution,
- geometric admissibility relative to the SVE size,
- prevention of extreme outliers,
- numerical robustness during meshing and explicit dynamic simulations.

In addition to the post-generation rejection strategy described above, two alternative statistically based approaches were investigated: (i) shifting the parameters of the underlying distribution to reduce tail intensity, and (ii) directly truncating the distribution prior to SVE generation. To attempt to mitigate the occurrence of outlier cells, two percentile-based cutoff strategies were evaluated. The lower and upper tails of the distribution were limited by removing the lowest and highest $X\%$ of the data, with $X = [1, 5]$. It is important to note that Neper generates morphologies iteratively by minimizing a cost function. Even when limits and constraints are imposed, the optimizer may relax them slightly if strictly enforcing the diameter bounds compromises other target metrics, such as sphericity or aspect ratio. Consequently, small deviations and occasional outliers may still appear.

Shifted Distribution Approach

The first strategy consists of shifting (rescaling) the equivalent diameter distribution so that cell generation occurs strictly within a prescribed minimum and maximum diameter. This approach is generally more efficient for Neper's optimization algorithm, as the PDF is mathematically rescaled to fit within the specified bounds. For the 5%–95% percentile limits, the Neper command is:

```
DMIN=0.0454
```

```
DMAX=0.7030
```

```
MP="diameq:0.47*lognormal(0.145,0.095,from=$DMIN,to=$DMAX)
```



```
+0.53*lognormal(0.495,0.115,from=$DMIN,to=$DMAX),
1-sphericity:lognormal(0.235,0.085),
aspratio(1,1,1.29)"
```

Figure B.1 compares the original equivalent diameter distribution with the shifted distributions obtained using the 1–99 and 5–95 percentile limits, for two SVE sizes $L_{RVE} = [1.8, 3.6]$.

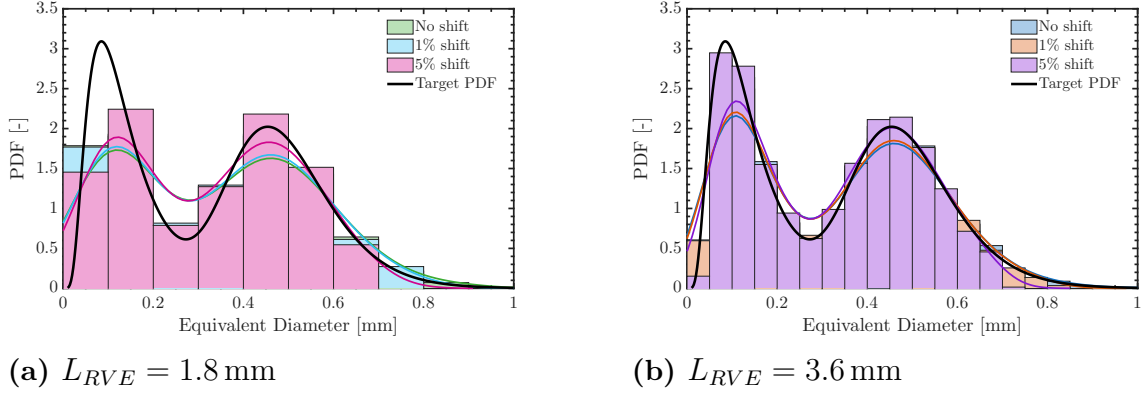


Figure B.1: Equivalent diameter distribution comparison for shifting method.

Due to the smaller size of the SVE in Figure B.1a, the number of generated cells (samples) is limited, and the distribution is therefore not well represented. This issue is mitigated by increasing the SVE size, which increases the number of samples. The larger SVE shown in Figure B.1b exhibits better agreement with the target PDF. Overall, Figure B.1 shows that the shifted distribution remains close to the analytical target PDF, regardless of sample size, thus preserving statistical representativeness. However, trimming the distribution tails increases the probability of generating cells with equivalent diameters near the mean values of the original bimodal distribution. Because the smaller-diameter peak already has higher probability, this leads to a larger number of cells within the same SVE volume.

An increased cell count results in greater total cell wall surface area, a larger number of finite elements, and potentially increased geometric complexity. These factors negatively impact meshing robustness and computational efficiency.

Truncated Distribution Approach

The second strategy explicitly constrains the equivalent diameter to lie within strict minimum and maximum bounds. Cells generated outside the valid interval are rejected or truncated. This method does not operate as a rigid post-processing filter; otherwise, excessive rejection could significantly slow down the polycrystal generation process. For the 5%–95% percentile truncation, the Neper expression is:

```
DMIN=0.0454
DMAX=0.7030
MP="diameq:0.47*lognormal(0.145,0.095)
+0.53*lognormal(0.495,0.115),
```

```
diameq: interval($DMIN,$DMAX),
1-sphericity: lognormal(0.235,0.085),
aspratio(1,1,1.29)"
```

Figure B.2 presents the comparison between the original and truncated distributions for the same percentile limits and SVE sizes.

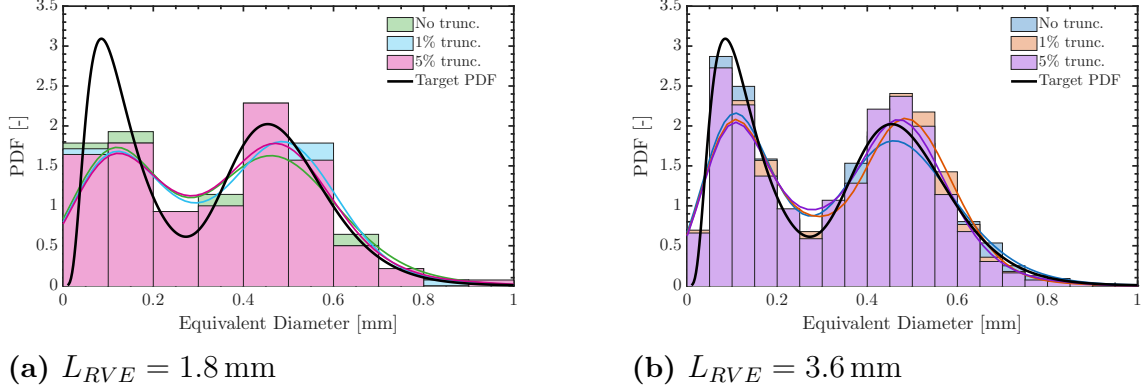


Figure B.2: Equivalent diameter distribution comparison for truncation method.

As in the shifted case, the smaller SVE does not adequately capture the distribution due to the limited number of samples. For the larger SVE (Figure B.2b), the effect of truncation becomes more apparent: the mean of the larger-diameter population shifts slightly toward higher values, altering the original bimodal characteristics of the distribution.

Table B.1 summarizes the total number of generated cells for each cutoff strategy.

Table B.1: Comparison of cutoff strategies for equivalent diameter.

RVE Size	Method	Percentile	$d_{\min}$ [mm]	$d_{\max}$ [mm]	Resulting Cells
1.8^3	Shift	-	-	-	140
		1-99	0.0302	0.8219	147
		5-95	0.0454	0.0730	165
3.6^3	Truncate	-	-	-	1122
		1-99	0.0302	0.8219	1122
		5-95	0.0454	0.0730	1122

Truncation preserves the total number of cells relative to the original distribution but modifies its statistical shape. In contrast, the shifting method better preserves the analytical PDF but increases the number of small cells and, consequently, the total number of cells within the SVE.

Therefore, neither approach fully achieves the objective of simplifying SVE generation in terms of geometric complexity and computational cost. For this reason, the original equivalent diameter distribution is retained. To control outliers, each generated mesostructure is evaluated using Neper output files and visual inspection. If significant outliers are identified, the mesostructure is regenerated using a different initial random seed until acceptable statistical characteristics are obtained.

C

Regularization Methods

In this study, two regularization approaches available in *LS-DYNA* were considered. The first method was utilizing the variable LCREGD (Element Size Scaling) within the MAT_ADD_EROSION. This method regularizes the response by scaling the failure strain based on the size of each finite element. Specifically smaller elements are assigned a higher failure strain, making them effectively more ductile while larger elements are assigned a lower failure strain, making them relatively more brittle. The underlying goal is to ensure that each element dissipates approximately the same amount of energy during failure, regardless of its size. By doing so, the overall response becomes less sensitive to mesh discretization. However, this approach operates at the local (element) level and does not explicitly control how deformation localizes spatially. As a result, while it can improve consistency in energy dissipation, it does not fully prevent the formation of narrow or mesh-dependent localization bands. An example of the setup made in the material card and its load curve can be observed below:

*MAT_ADD_EROSION					
Foam_Erosion					
1000001	0.	-	-0.2	-50.	
					12

*DEFINE_CURVE	
12	Scale failure by localized volume
0.01	3.0
0.02	1.5
0.03	1.0
0.04	0.75
0.06	0.5
0.10	0.3
0.30	0.1
1.00	0.03

The implementation of this approach is based on a mesh-dependent scaling of the failure criterion rather than a fixed material threshold. The failure strain is adjusted according to the characteristic size of each element, such that smaller elements

require larger deformation to fail, while larger elements fail at reduced strain levels. The scaling function is anchored at a reference element size (approximately 0.03 mm), where no modification is applied.

In this way, the formulation aims to preserve a consistent energy dissipation per element during fracture, which reduces mesh sensitivity in terms of global force—displacement response. However, since the correction is applied locally at the element level, it does not introduce a length scale that governs how damage spreads across multiple elements. Consequently, strain localization can still occur, even though the overall energy balance becomes more mesh-consistent.

The other method employs the MAT_NONLOCAL (Nonlocal Averaging) card which introduces a nonlocal formulation. In this method, failure-related variables (such as plastic strain) are not evaluated at a single element alone. Instead, they are averaged over a surrounding neighborhood defined by a characteristic length scale. This length typically spans multiple elements. As a result the response at a point depends on the behavior of its neighbors, see Figure C.1. Failure is no longer triggered by a single element reaching a critical value. Instead, a group of elements must collectively reach the failure condition. This enforces deformation to spread over a finite region, creating a deformation band (or crack zone) with a physically meaningful width.

Compared to local approaches, nonlocal regularization is generally considered more robust and physically representative, as it directly controls strain localization and reduces mesh dependency more effectively.

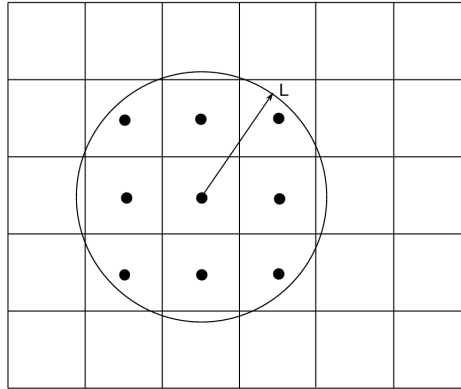


Figure C.1: Nonlocal averaging concept used in MAT_NONLOCAL, where the response at a point depends on surrounding elements within a defined neighborhood, rather than being evaluated locally at a single element. Image from the *LS-DYNA* user manual [8].

In practice, both regularization methods presented challenges. For the LCREGD approach, the results showed only minor differences compared to the unregularized stress–strain response. One possible explanation is that the variation in element sizes within the SVE was relatively small, limiting the effectiveness of the scaling. For the MAT_NONLOCAL approach, the behavior was more complex. While the method is theoretically more robust, the simulations exhibited irregular deformation patterns under tension, suggesting that further calibration of parameters, such as

the characteristic length, may be required. Additionally, the computational cost associated with the nonlocal formulation was significantly higher. For example, the simulation of the 3.6 mm RVE required more than 12 hours to complete. This made it impractical to perform a comprehensive parametric study within the available time-frame.

Due to these limitations, a full investigation of regularization strategies for achieving SVE size-independent results was not feasible within this study and was thus not included when implementing the mesh refinement and SVE size-invariance study.

An important consequence of strain localization is that, once a deformation band forms, the concept of a homogenized (global) stress–strain response becomes less meaningful. In the post-peak regime the response becomes dominated by the localized damage zone. Global averages depend on the size of the SVE and the extent of the undamaged material. Therefore, meaningful comparisons must instead be based on quantities that describe the local fracture process, such as normalized strain or Traction versus opening displacement (traction–separation response). These measures focus on the behavior within the deformation band and provide a more objective basis for comparison across different models [64].

D

Thickness Distribution Algorithm

The algorithm assigns a spatially varying shell thickness to the elements representing foam cell walls. The thickness field is determined from the relative position of each element node with respect to the effective wall boundary, while excluding artificial boundaries generated by cutting the foam morphology.

The procedure is summarized as follows.

1. Identify shell macro-regions corresponding to individual cell walls or wall patches.

For each wall region, compute:

- the total wall face area,
- and the geometric centroid (center of gravity),

$$A_f,$$

$$\mathbf{x}_{\text{CoG}}.$$

2. Detect global free edges of the mesh.

Free edges are edges connected to only one shell element, i.e.,

$$n_e = 1,$$

where n_e is the number of adjacent shell elements sharing a given edge.

These free edges correspond to artificial cut surfaces generated during extraction of the foam morphology and therefore do not represent physical cell-wall boundaries.

Consequently, nodes belonging to global free edges are excluded from the boundary definition used in the thickness computation.

3. Determine the effective wall boundary.

Let

$$\mathcal{B}$$

denote the set of local perimeter nodes of a wall face. The effective boundary set is defined as

$$\mathcal{B}_{\text{eff}} = \mathcal{B} \setminus \mathcal{B}_{\text{free}},$$

where

$$\mathcal{B}_{\text{free}}$$

contains nodes located on global free edges.

Thus, only physically connected wall boundaries contribute to the thickness distribution.

4. For each element node located at position $\mathbf{x}$, compute the minimum distance to the effective boundary,

$$d_c(\mathbf{x}) = \min_{\mathbf{x}_b \in \mathcal{B}_{\text{eff}}} \|\mathbf{x} - \mathbf{x}_b\|.$$

Let $\mathbf{x}_b^*$ denote the closest boundary point.

5. Compute a normalized parametric distance,

$$\xi(\mathbf{x}) = \frac{\|\mathbf{x} - \mathbf{x}_b^*\|}{\|\mathbf{x}_{\text{CoG}} - \mathbf{x}_b^*\|}.$$

This definition gives

$$\xi = 0 \quad \text{at the wall boundary,}$$

and

$$\xi = 1 \quad \text{at the wall centroid.}$$

6. Define a characteristic wall length from the wall face area,

$$d_w = \sqrt{A_f}.$$

The characteristic decay length controlling the thickness gradient is then defined as

$$l_c = \frac{d_w}{p_1 \left(\frac{d_w}{t_m} \right)^{p_2}},$$

where:

- t_m is the target average wall thickness,
 - p_1 and p_2 are empirical parameters.
7. The shell thickness is prescribed using an exponential decay law,

$$t(d_c) = (t_0 - t_e) \exp\left(-\frac{d_c}{l_c}\right) + t_e,$$

where:

- t_0 is the wall thickness at the effective boundary,
- t_e is the asymptotic interior thickness,
- d_c is the minimum distance to the effective boundary.

The interior thickness is defined as

$$t_e = \frac{t_0}{c_1},$$

where c_1 controls the ratio between edge and interior thickness.

8. The computed thickness values are assigned directly to the shell nodal thickness fields,

$$\text{THIC}_i,$$

for each shell node i .

To ensure that the resulting foam morphology attains the prescribed average wall thickness, an area-weighted average thickness is computed as

$$\bar{t} = \frac{\sum_e A_e t_e}{\sum_e A_e},$$

where:

- A_e is the shell element area,
- t_e is the mean nodal thickness of the element.

The boundary thickness parameter is then updated iteratively according to

$$t_0^{(n+1)} = t_0^{(n)} \frac{t_m}{\bar{t}},$$

and the thickness assignment procedure is repeated until the resulting average thickness approaches the target value t_m .

E

LS-DYNA Input Files

This appendix presents the *LS-DYNA* input files used in the DCB-inspired simulations. The complete solver deck is composed of a main keyword file and a set of supporting include files that define the material model, analysis setup, and solution controls. When combined, these files form the complete input provided to the solver. The main keyword file shown in Listing E.1 serves as the entry point for the simulation. It references all supporting include files and defines fundamental analysis settings, including output requests, termination criteria, and timestep control parameters.

Listing E.1: Main Key File

```
*KEYWORD
*TITLE
$ =====
$ SVE FOAM MICROSTRUCTURE, DCB
$ Solver: LS-DYNA (Explicit)
$ =====
$
$ =====
$ MATERIAL PARAMETERS
$ These parameters make up the effective stress-strain curve for *MAT_24
$ =====
*PARAMETER
$   PRMR1|      VAL1|
R Emod          3000.
R sigy           64.
R Hmod          1000.
R effpu          0.05
R Ttime          0.0040
$
*PARAMETER_EXPRESSION
$   PRMR1|                                     EXPRESSION1|
R eMax      effpu + sigy/Emod
$
$ =====
$ RUN SETTINGS
$ =====
*CONTROL_TERMINATION
$   ENDTIM|   ENDCYC|   DTMIN|   ENDENG|   ENDMAS|   NOSOL|
&Ttime          0.0001          100.
*CONTROL_TIMESTEP
$   TDINT|   TSSFAC|   ISDO|   TSLIMT|   DT2MS|   LCTM|   ERODE|   MSIST
$       0.     0.9     0       0.     -1.E-8     11     1       0
$   DT2MSF|   DT2MSLC|   IMSCL|           |           |   RMSCL|   EMSCL|   IHDO
$       0
*DATABASE_BINARY_D3PLOT
$   DT|   LCDT|   BEAM|   NPLTC|   PSETID|
$   0.0001     0       0       0
*DEFINE_CURVE_TITLE
Max dt
$   LCID|   SIDR|   SFA|   SFO|   OFFA|   OFFO|   DATTYP|   LCINT|
$   11     0     1.     1.E-7     0.     0.     0       0
```

```

$           Ai |           Oi |
$           0.           1.
$         0.001         1.
$         10.         1.
$
$ =====
$ FUNDAMENTAL OUTPUT REQUEST
$ =====
$ *DATABASE_NODAL_FORCE_GROUP
$   NSID |   CID |
$   10000015
$
$ *DATABASE_NODAL_FORCE_GROUP
$   NSID |   CID |
$   10000016
$
$ *DATABASE_HISTORY_NODE
$   ID1 |   ID2 |
$   354991   15174
$
$ =====
$ INCLUDE FILES
$ =====
$ --- MESH & TOPOLOGY (Variable) ---
$ Contains *NODE, *ELEMENT, *SHELL_SECTION (and dependents) and *SET definition.
$
$ *INCLUDE
$ Topology_DCB.k
$
$ =====
$ --- MATERIALS (Static) ---
$ Contains *MAT information, including *MAT_ADD_EROSION.
$
$ *INCLUDE
$ MAT.k
$
$ =====
$ --- SETUP (Static) ---
$ Contains *BOUNDARY_PRESCRIBED_MOTION_SET.
$ Contains *CONTACT_AUTOMATIC_SINGLE_SURFACE for self-contact of cell walls.
$
$ *INCLUDE
$ SETUP_DCB.k
$
$ =====
$ --- CONTROL & DATABASE (Static) ---
$ Contains *CONTROL_ and *DATABASE_ cards
$
$ *INCLUDE
$ CONTROL.k
$
$ =====
$ *END

```

Listing E.2 contains the analysis-specific setup for the DCB-inspired load case. This file defines the prescribed boundary conditions, loading configuration, and the displacement-time function used to impose the prescribed deflection.

Listing E.2: Analyses Setup

```

$ =====
$ MODEL SETUP
$ =====
*KEYWORD
$
$ -----
$ SVE SELF CONTACT
$ -----
*CONTACT_AUTOMATIC_SINGLE_SURFACE_ID
$ ID| NAME|
1 *CONTACT_AUTOMATIC_SINGLE_SURFACE
$ SSID| MSID| STYP| MSTYPE| SBOXID| MBOXID| SPR| MPR|
1001 2 0 0
$ FS| FD| DC| VC| VDC| PENCHK| BT| DT|
0.3 0.3 20. 0
$ SFS| SFM| SST| MST| SFST| SFMT| FSF| VSF|
1. 1.
$ SOFT| SOFSCL| LCIDAB| MAXPAR| SBOPT| DEPTH| BSORT| FRCFRQ|
2 0.1 1.025 5. 35 0 0
$ PENMAX| THKOPT| SHLTHK| SNLOG| ISYM| I2D3D| SLDTHK| SLDSTF|
0. 0 0 0 0 0. 0.
$ IGAP| IGNORE| DPRFAC| DTSTIF| | | FLANGL| CID_RCF|
1 0. 0.
$ Q2TRI| DTPCHK| SFNBR| FNLSCL| DNLSCS| TCSO| TIEDID| SHLEDG|
0 -0.5 0 0 0
$ -----
$ BOUNDARY CONDITIONS
$ -----
*BOUNDARY_PRESCRIBED_MOTION_SET_ID
$ ID| HEADING|
1YT_EDGES_TY
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
1000 2 0 1 1. 0 0.
2XB_EDGES_TX
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
1002 1 0 1 1. 0 0.
$ ID| HEADING|
3ZT_TZ
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000016 3 0 2 1. 0 0.
$ ID| HEADING|
4ZT_RX
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000016 5 0 1 1. 0 0.
$ ID| HEADING|
5ZT_RY
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000016 6 0 1 1. 0 0.
$ ID| HEADING|
9ZB_TZ
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000015 3 0 2 -1. 0 0.
$ ID| HEADING|
10ZB_RX
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000015 5 0 1 1. 0 0.
$ ID| HEADING|
11ZB_RY
$ TYPEID| DOF| VAD| LCID| SF| VID| DEATH| BIRTH|
10000015 6 0 1 1. 0 0.
*DEFINE_CURVE_TITLE
Constant 0.0
$ LCID| SIDR| SFA| SFO| OFFA| OFFO| DATTYP| LCINT|
1 0 1. 1. 0. 0. 0

```

E. *LS-DYNA* Input Files

```
$              Ai |              Oi |
              0.
              0.001
              10.
              0.
              0.
              0.

*DEFINE_FUNCTION
$      FID |              HEADING |
      2DCB_Deflection
$              FUNCTION |
w(t,x) = 0.004 * (3*x**2 - 19*x + 28) / &Ttime
$
*END
```

The material definitions employed in the simulations are provided in Listing E.3. The file contains the calibrated trilinear elastoplastic constitutive model together with the erosion criteria adopted for element deletion.

Listing E.3: Material Data

```

$ =====
$ MATERIAL DATA
$ =====
*KEYWORD
*MAT_PIECEWISE_LINEAR_PLASTICITY_TITLE
RVE_Trilinear_Plastic_EROSION
$      MID|      R0|      E|      PR|      SIGY|      ETAN|      FAIL|      TDEL|
      1000001  1.25E-9  &Emod  0.38  &sigy  &Hmod  1.E20      0.
$      C|      P|      LCSS|      LCSR|      VP|
      0.      0.
$      EPS1|      EPS2|      EPS3|      EPS4|      EPS5|      EPS6|      EPS7|      EPS8|
      0.      0.      0.      0.      0.      0.      0.      0.
$      ES1|      ES2|      ES3|      ES4|      ES5|      ES6|      ES7|      ES8|
      0.      0.      0.      0.      0.      0.      0.      0.
*MAT_PIECEWISE_LINEAR_PLASTICITY_TITLE
RVE_Trilinear_Plastic
$      MID|      R0|      E|      PR|      SIGY|      ETAN|      FAIL|      TDEL|
      1000003  1.25E-9  &Emod  0.38  &sigy  &Hmod  1.E20      0.
$      C|      P|      LCSS|      LCSR|      VP|
      0.      0.
$      EPS1|      EPS2|      EPS3|      EPS4|      EPS5|      EPS6|      EPS7|      EPS8|
      0.      0.      0.      0.      0.      0.      0.      0.
$      ES1|      ES2|      ES3|      ES4|      ES5|      ES6|      ES7|      ES8|
      0.      0.      0.      0.      0.      0.      0.      0.
$
*MAT_ADD_EROSION_TITLE
FOAM_EROSION
$      MID|      EXCL|      MXPRES|      MNEPS|      EFFEPS|      VOLEPS|      NUMFIP|      NCS|
      1000001      0.
$      MNPRES|      SIGP1|      SIGVM|      MXEPS|      EPSSH|      SIGTH|      IMPULSE|      FAILTM|
      &eMax
*END

```

Finally, Listing E.4 contains the remaining solution control and database cards required for the analyses. These settings govern numerical solution procedures, contact treatment, and additional output requests not defined in the main keyword file.

Listing E.4: Solution Control

```

$ =====
$ SOLUTION CONTROL CARDS + DATABASE (OUTPUT) REQUEST
$ =====

*KEYWORD
*CONTROL_ACCURACY
$   OSU|      INN|      PIDOSU|      IACC|
      1        4          0
*CONTROL_BULK_VISCOSITY
$   Q1|      Q2|      TYPE|      BTYPE|
      1.5      0.06      -2        0
*CONTROL_CONTACT
$   SLSFAC|  RWPNAL|  ISLCHK|  SHLTHK|  PENOPT|  THKCHG|  ORIEN|  ENMASS
      0.1      1.      2        0        1        1        1        0
$   USRSTR|  USRFRC|  NSBCS|  INTERM|  XPENE|  SSTHK|  ECDT|  TIEDPRJ
      0        0        0        0        4.      1        0        0
$   SFRIC|  DFRIC|  EDC|  VFC|  TH|  TH_SF|  PEN_SF|  PTSCL
      0.      0.      0.      0.      0.      0.      0.
$   IGNORE|  FRCENG|  SKIPRWG|  OUTSEG|  SPOTSTP|  SPOTDEL|  SPOTHIN|
      0        1        0        0        2        1        0.
$   ISYM|  NSEROD|  RWGAPS|  RWGDTH|  RWKSF|  ICOV|  SWRADF|  ITHOFF
      0        0        1        0.      1.      0        0.      0
$   SHLEDG|  PSTIFF|  ITHCNT|  TDCNOF|  FTALL|  |  SHLTRW|  IGACTC
      0        0        0        0        1        |  0.      0

*CONTROL_ENERGY
$   HGEN|  RWEN|  SLNTEN|  RYLEN|
      2        2        2        2
*CONTROL_OUTPUT
$   NPOPT|  NEECHO|  NREFUP|  IACCOP|  OPIFS|  IPNINT|  IKEDIT|  IFLUSH
      1        3        1        0        0.      0        100      5000
$   IPRTF|  IERODE|  TET10|  MSGMAX|  IPCURV|  GMDT|  IP1DBLT|  EOCS
      0        1        2        50      0        0.      0        0
$   TOLEV|  NEWLEG|  FRFREQ|  MINFO|  SOLSIG|  MSGFLG|  CDETOL|
      2        0      10000      0        0        0        10.
$   PHSCHNG|  DEMDEN|  ICRFILE|  SPC2BND|  PENOUT|  SHLSIG|  HISNOUT|  ENGOUT
      0        0        0        0        0        0        1        0
$   INSF|  ISOLSF|  IBSF|  ISSF|  MLKBAG|
      0        0        0        0        0

*CONTROL_SHELL
$   WRPANG|  ESORT|  IRNXX|  ISTUPD|  THEORY|  BWC|  MITER|  PROJ
      30.      1      -1        0        1        2        1        1
$   ROTASCL|  INTGRD|  LAMSHT|  CSTYP6|  TSHELL|
      1.      0        5        1        0
$   PSTUPD|  SIDT4TU|  CNTCO|  ITSFLG|  IRQUAD|  W-MODE|  STRETCH|  ICRQ
      0        1        0        0        3        0.      0.      0
$   NFAIL1|  NFAIL4|  PSNFAIL|  KEEPCS|  DELFR|  DRCPSID|  DRCPRM|  INTPERR
      0        1        0        0        0        1.      1.      0

*CONTROL_SOLUTION
$   SOLN|  NLQ|  ISNAN|  LCINT|
      0        0        1      101
*CONTROL_SOLID
$   ESORT|  FMATRIX|  NIPTETS|  SWLOCL|  PSFAIL|  T10JTOL|  ICOH|  TET13K
      1        0        4        1      0.00
      1        0        0
*CONTROL_MPP_DECOMPOSITION_AUTOMATIC
*CONTROL_MPP_IO_NODUMP
*CONTROL_MPP_IO_LSTC_REDUCE
*DATABASE_RWFORC
$   DT|  BINARY|  LCUR|  IOOPT|
      0.0001      3        1
*DATABASE_NODOUT
$   DT|  BINARY|  LCUR|  IOOPT|  DTHF|  BINHF|
      0.0001      3        1        0.      0

```



```

*DATABASE_ELOUT
$      DT|    BINARY|    LCUR|    IOOPT|  OPTION1|  OPTION2|  OPTION3|  OPTION4
    0.0001        3        1        0        0        0        0
*DATABASE_GLSTAT
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_MATSUM
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_RCFORC
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_NODFOR
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_BNDOUT
$      DT|    BINARY|    LCUR|    IOOPT|  OPTION1|  OPTION2|  OPTION3|  OPTION4
    0.0001        3        1
*DATABASE_RBDOUT
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_SLEOUT
$      DT|    BINARY|    LCUR|    IOOPT|
    0.0001        3        1
*DATABASE_EXTENT_BINARY
$      NEIPH|    NEIPS|  MAXINT|  STRFLG|  SIGFLG|  EPSFLG|  RLTF LG|  ENGFLG
        0        1        10        1        1        1        1        1
$      CMPFLG|  IEVERP|  BEAMIP|  DCOMP|  SHGE|  STSSZ|  N3THDT|  IALEMAT
        0        1        0        1        2        1        2        1
$      NINTSLD|  PKP_SEN|  SCLP|  HYDRO|  MSSCL|  THERM|  INTOUT|  NODOUT
        0        0        1.        0        1        0
$      DTD T|  RESPLT|  NEIPB|  QUADSLD|  CUBSLD|
        0        0        0        0        0
*END

```

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CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden
www.chalmers.se



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