

# Evaluation of the Influence of Fatigue in Preliminary Design of Road Bridges

Set-based design of concrete frame bridges

Master's thesis in the Master's Program Structural Engineering and Building Technology

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING

CHALMERS UNIVERSITY OF TECHNOLOGY  
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MASTER'S THESIS ACEX30

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Cover: An illustration of a frame bridge FE-model, where fatigue load model 3 is applied together with the definition of a stress range.

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## Abstract

The set-based design process differs from the traditional point-based design approach as several designs are produced and evaluated simultaneously, which contributes to a more efficient design phase. In order to study the utilization of this modern design procedure, a design tool was developed which included several evaluation criteria and verification steps. As a continuation of a previous project (Löfgren, 2020), where the original design tool was created, a study for the potential of development with respect to implementation of fatigue assessment was conducted. Thus, in order to optimize the designs generated by the design tool even further, fatigue verification was thereby implemented to the FE-analysis to see if this could be beneficial in a preliminary design phase.

A parametric study was conducted in order to identify important aspects in determining the fatigue capacity to the generated bridge designs. Additionally, aspects which are especially important to the fatigue assessment and potentially of interest to choices done prior the design phase was investigated. Furthermore, comparisons between optimized designs based on the initial preliminary designs criteria and the fatigue verification were conducted in order to establish the influence of fatigue to the design tool.

The utilization of the design tool was investigated further by analyzing the selection procedure. Additional measures proved to be of interest in order not to miss potential bridge design in the process. Also, the idea of being able to find standardized bridges which could be suitable for multiple situations was investigated further with the aid of the developed design tool.

The conducted investigations in this project showed that the implementation of fatigue verification can be beneficial in order to increase the quality of the final design proposals from the design tool. Also, comparisons showed great potential for such a design tool, when making decisions regarding various design parameters.

Keywords: frame bridge, set-based design, fatigue, optimization, FE-modelling, parametric design, python, Lambda method, Cumulative Damage method, concrete

Utvärdering av utmattningens inverkan på en preliminär utformning av vägbroar  
*Set-baserad design av plattrambroar i betong*

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## Sammanfattning

Den set-baserade designprocessen skiljer sig från den traditionella punkt-baserade designmetoden då flertalet design-alternativ produceras och utvärderas samtidigt, vilket bidrar till en mer effektiv projekteringsfas. För att studera användningen av denna moderna designprocedur utvecklades ett designverktyg som innehöll flera utvärderingskriterier och verifieringssteg. Som en fortsättning på ett tidigare projekt (Löfgren, 2020), där det ursprungliga designverktyget skapades, genomfördes därför en studie angående dess utvecklingspotential med avseende på en tillförd utmattningsbedömning. För att optimera designen som genereras av designverktyget ännu mer, implementerades utmattningsverifiering i FE-analysen för att se om detta kunde vara fördelaktigt för ett preliminärt designförfarande.

En parametrisk studie genomfördes för att identifiera viktiga aspekter vid bestämning av utmattningskapaciteten för de genererade brokonstruktionerna. Dessutom undersöktes aspekter som är särskilt viktiga för utmattningsbedömningen och potentiellt av intresse för beslut som fattas före designfasen. Dessutom gjordes jämförelser mellan optimerade broar baserat på de initiala preliminära kriterierna och utmattningsverifiering för att fastställa utmattningens påverkan i designverktyget.

Användningen av designverktyget undersöktes ytterligare genom att analysera urvalsförfarandet. Ytterligare åtgärder visade sig vara av intresse för att inte missa potentiell brokonstruktion i processen. Idén att kunna hitta standardiserade broar som kan vara lämpliga för flera situationer undersöktes även med hjälp av det utvecklade designverktyget.

De genomförda undersökningarna i detta projekt visade att implementeringen av utmattningsverifiering kan vara fördelaktig för att öka kvaliteten på de slutliga designförslagen från designverktyget. Jämförelser visade också på en stor potential för ett sådant verktyg, när man fattar beslut om olika designparametrar.

Nyckelord: plattramsbro, set-baserad, utmattning, optimering, FE-modellering, parametrisk design, python, Lambdametoden, Delskademethoden, betong



# Preface

This master's thesis was conducted at Chalmers University of Technology, at the division of Structural Engineering and department of Architecture and Civil Engineering in the spring of 2021. The project was conducted in collaboration and behalf of the division of Structural Engineering at NCC Sverige AB.

Throughout the project, supervision has been carried out by Jesús Armesto Barros, Structural Engineer at NCC and Alexandre Mathern, Doctoral student at Chalmers University of Technology and Structural Engineer at NCC while Rasmus Rempling, Ass. Prof. at Chalmers University of Technology, Division of Structural Engineering has been the examiner of this master's thesis.

We would like to thank everyone that has been involved in this project, for the extensive support and guidance throughout the entire project. The knowledge and feedback given during this thesis has been very appreciated.

Many thanks,

Jon Bohlin & Mattias Johansson, Gothenburg, June 2021





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# Notations

## Latin upper case letters

|                |   |
|----------------|---|
| $A_s$          | Amount of tensile reinforcement                               |
| $A'_s$         | Amount of reinforcement in the compressed zone                |
| $A_{sw}$       | Amount of shear reinforcement                                 |
| $D$            | Fatigue damage  |
| $E_{cd,max,i}$ | Maximum compressive stress level                              |
| $E_{cd,min,i}$ | Minimum compressive stress level                              |
| $G_{k,j}$      | Characteristic value of the permanent load, $j$               |
| $L$            | Critical length of the element influence line                 |
| $M_{Ed}$       | Design bending moment   |
| $N_{Ed}$       | Design normal force   |
| $N_i$          | Ultimate number of loading cycles for a certain load interval |
| $N_{obs}$      | Number of lorries per year according to EN 1991-2             |
| $N_{years}$    | Design life of the bridge                                     |
| $P$            | Value of the prestressing force                               |
| $\bar{Q}$      | Factor for traffic type                                       |
| $Q_{k,1}$      | Characteristic value of the main variable load                |
| $Q_{k,i}$      | Characteristic value of the accompanied variable load, $i$    |
| $Q_{fat}$      | Value of the fatigue load                                     |
| $R_i$          | Stress ratio  |
| $V_{Ed}$       | Design shear force  |
| $V_{Ed,max}$   | Maximum shear force in a loading cycle                        |
| $V_{Ed,min}$   | Minimum shear force in a loading cycle                        |
| $V_{Rd,c}$     | Design shear capacity of the plain concrete                   |

**Latin lower case letters**

|               |   |
|---------------|---|
| $b$           | Width of the surface for the concentrated load                          |
| $d$           | Effective height of the plate   |
| $d'$          | Distance from compressed side to the top reinforcement                  |
| $f_{cd, fat}$ | Design fatigue strength of the concrete                                 |
| $e$           | Eccentricity of the normal force  |
| $k_0$         | Coefficient for earth pressure at rest                                  |
| $k_2$         | Slope of the appropriate S-N curve                                      |
| $n_i$         | Actual number of cycles for a certain load interval                     |
| $m'_{rx}$     | Longitudinal negative resisting moment                                  |
| $m'_{ry}$     | Transverse negative resisting moment                                    |
| $m_{rx}$      | Longitudinal positive resisting moment                                  |
| $m_{ry}$      | Transverse positive resisting moment                                    |
| $m'_x$        | Longitudinal negative bending moment                                    |
| $m'_y$        | Transverse negative bending moment                                      |
| $m_x$         | Longitudinal positive bending moment                                    |
| $m_y$         | Transverse positive bending moment                                      |
| $m_{xy}$      | Torsional moment  |
| $s$           | Spacing between stirrups  |
| $t$           | Thickness of the road surface   |
| $v_0$         | Resultant shear force   |
| $v_x$         | Longitudinal sectional shear force                                      |
| $v_y$         | Transverse sectional shear force  |
| $x$           | Distance from transverse section to the centre of the concentrated load |
| $x_{cc}$      | Height of the compressed zone   |
| $z$           | Distance from the neutral axis to the compressed concrete fibre         |

**Greek upper case letters**

|                         |   |
|-------------------------|---|
| $\Delta\sigma_{S, equ}$ | Equivalent stress range from a fatigue load model |
| $\Delta\sigma_{Rsk}$    | Reference stress range for reinforcement          |
| $\Delta z$              | Depth of the soil                                 |
| $\theta_{fat}$          | Adjusted angle of the compressed strut            |
| $\theta$                | Angle of the compressed strut                     |

### Greek lower case letters

|                     |  |
|---------------------|--|
| $\alpha$            | Ratio between the Young's modulus of the steel and concrete                                |
| $\alpha_{Q1}$       | Adjustment factor dependent on the traffic amounts   |
| $\lambda_{s,1}$     | Factor which takes the element and reinforcement type into account                         |
| $\lambda_{s,3}$     | Factor which takes the design life of the bridge into account                              |
| $\lambda_{s,2}$     | Factor which takes traffic volume into account   |
| $\lambda_{s,4}$     | Factor which takes into account if the structure is loaded from more than one traffic lane |
| $\sigma_{c,max}$    | Maximum compressive stress in a cycle  |
| $\sigma_{c,min}$    | Minimum compressive stress in a cycle  |
| $\sigma_{cd,max,i}$ | The upper stress in a cycle  |
| $\sigma_{cd,min,i}$ | The lower stress in a cycle  |
| $\sigma_c(z_s)$     | Concrete stress at steel level   |
| $\gamma_{F,fat}$    | Partial factor for reinforcement fatigue assessment  |
| $\gamma_{s,fat}$    | Partial factor for uncertainties in the action/model                                       |
| $\phi_{fat}$        | Damage equivalent impact factor controlled by the surface roughness                        |
| $\gamma$            | Weight of the soil   |
| $\psi_{1,1}$        | Frequent value factor for the main variable load   |
| $\psi_{2,i}$        | Quasi-permanent value factor for variable load $i$   |

### Abbreviations

|     |                                    |
|-----|------------------------------------|
| ADT | Annual daily traffic               |
| EPD | Environmental product declaration  |
| LCA | Life-cycle analysis                |
| LCC | Life-cycle cost                    |
| PBD | Point-based design                 |
| SBD | Set-based design                   |
| SLS | Service limit state                |
| STA | Swedish transport administration   |
| SM1 | Sectional longitudinal moment      |
| SM2 | Sectional transverse moment        |
| SM3 | Sectional torsional moment         |
| SF4 | Sectional longitudinal shear force |
| SF5 | Sectional transverse shear force   |
| ULS | Ultimate limit state               |



# 1

## Introduction

The need for optimization and efficiency in a design process is constantly increasing and due to regulations and recommendations within the building industry, parameters such as cost and the desire for a low environmental footprint highly affects the design process and its working procedure (Uppenberg et al., 2017). In order to avoid late and crucial changes to the design and the amount of re-work it entails, a set-based working procedure makes it possible to evaluate several parameters throughout the entire design process, which generates a more satisfactory design solution for everyone involved. Unlike the more common design procedure where one specified design undergo continuous development, the set-based design procedure focuses on keeping a larger set of design possibilities available and under consideration for as long as possible to minimize the need of re-work (Parrish et al., 2007).

In order to reach a satisfactory result by using the set-based design method, the number of designs are reduced successively along with the designing progress. Verification and design checks based on building codes or regulations are implemented to direct the design process towards feasible and valid alternatives. There are several verification and building rules to take into account, including both short and long term perspectives. Verification for bridge constructions with regards to their long term use need to be considered in the design process and fatigue is one of many aspects which can have a large impact on the final design due to the occurrence of cyclic loading from vehicles that passes the carriageway throughout the lifetime of the bridge.

If one wants to make full use of a set-based design process, it is important to be able to establish a design alternative which is satisfactory for the majority of stakeholders involved. In larger projects, it might not be the case that the process has resulted in one single alternative (Parrish et al., 2007). As the civil engineering industry evolves by focusing on sustainability and standardized solutions, a set-based design process may be a viable option which could benefit the development of this area (Mathern et al., 2018).

## 1.1 Purpose and objectives

The main purpose of this thesis is to investigate the influence of when fatigue assessment is applied to frame bridges in reinforced concrete, and how it affects the preliminary design process in terms of evaluation and choice of design. This is achieved through the following defined objectives:

- Determine parameters which can be of significance to fatigue assessment for a frame bridge.
- Implement the corresponding calculations for the fatigue assessment into a parametric design process.
- Study a set-based design method and the possibilities to identify the best design alternatives based on carbon dioxide equivalents (CO<sub>2</sub>-eq), material costs and fatigue verification.
- Evaluate how a set-based design method can be used to group design situations of bridges in a larger road/railway project.

## 1.2 Limitations

The purpose and objectives in this thesis are limited to the following:

- Design is based on the European building regulations in Eurocode and design rules provided by the Swedish Transport Administration (Trafikverket) and the Swedish Transport Agency (Transportstyrelsen) for reinforced concrete frame bridges.
- The contribution to the existing design tool consists of verification of fatigue for reinforced concrete road bridges.
- The included structures are limited to one span frame bridges in reinforced concrete to reduce complexity and because of compatibility with the existing script.

# 2

## Methodology

This project started with a theoretical literal study where knowledge of the theory of fatigue in reinforced concrete was obtained from several sources of acknowledged research and articles. The theory which was derived from both literature and building codes led to an idea of how the implementation could be done using BRIGADE Plus (Scanscot Technology AB, 2018) through a script written in Python 2.7. The focus of the literature study was the implementation of fatigue as the research area of set-based design as a concept was already investigated in previous research (Löfgren, 2020, Mathern et al., 2018, Parrish et al., 2007, Tarazon Ramos and Luis Fernandez, 2014). However, a short introduction to the concept was needed in order to present the principle of this design process.

After investigating how fatigue in reinforced concrete bridges should be assessed based on building codes and regulations by Swedish building rules, the modelling of the load models were introduced to the frame bridge FE-model. As the idea of this model was to be able to run analysis on several bridges, the cyclic loading from a moving load model had to be simplified. This was done by analyzing the load effects from a moving load model, and then locating the positions which generated the worst load effects for each studied section of the bridge. These positions were then translated into static loads applied to the FE-model in order to result in shorter run-time for each bridge analysis.

The original script from the previous master thesis, on which this work is based on, was then further developed by the implementation of a fatigue assessing process. The results from the analysis with the corresponding fatigue load models was then put through the fatigue assessment checks, in order to determine if the design was appropriate with respect to fatigue.

The fatigue assessing script also returned results for the utilization of each bridge design with respect to fatigue. The results were gathered and compiled into a large set of data, on which a parametric study was conducted. The different bridge designs and parameters included in the fatigue assessment were then compared and analyzed in order to find parameters on which the fatigue damage was most influenced by. The information from the parametric study was then utilized to contribute to fulfill the purpose of this thesis and to draw conclusions from the investigation.



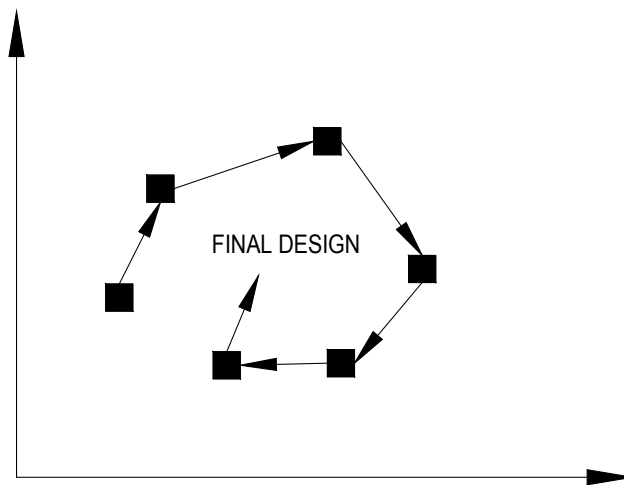
# 3

## Theory

### 3.1 Set-based design

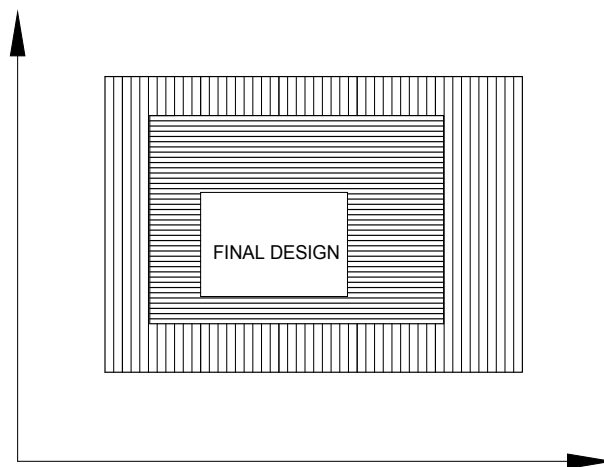
The set-based design concept is well known within the manufacturing industry but in structural engineering, there is an increasing interest to investigate the benefits of applying a method of this kind (Mathern et al., 2018).

The standard method for structural design is the point-design design approach, where the design is based on the prerequisites of the stakeholders and continuously improved and developed to reach the final design (Parrish et al., 2007). This method results in a process which cannot manage several parallel alternatives and can rather manage one alternative at a time, which may end up costing time and eventually money if major changes is needed (Parrish et al., 2007). The starting point in this procedure is an estimated design which the engineer makes on the information that is available at that time, the final design therefore become highly dependent on the initial design parameters. This does not result in a flexible process regarding sustainability because information regarding these question may be insufficient at an early stage (Mathern et al., 2018). Figure 3.1 shows a common way to illustrate a point-based design method.



**Figure 3.1** Illustration of point-based design procedure (Tarazon Ramos & Luis Fernandez, 2014).

In set-based design, the starting point differs from the point-based design. In this method the definite prerequisites from the stakeholders lays a foundation of a large number of alternatives instead of one initial alternative (Parrish et al., 2007). These alternatives may go through the same verification process as the single alternative but generates a large number of parallel designs. The purpose of generating a large number of options is to progressively eliminate solutions and finally find the alternative which is best from different evaluation criteria. If changes are made a in set-based design method the rework is usually less because there may be an alternative which fit the new requirement in the last elimination of options (Tarazon Ramos & Luis Fernandez, 2014). Figure 3.2 shows common way to illustrate a point-based design method.



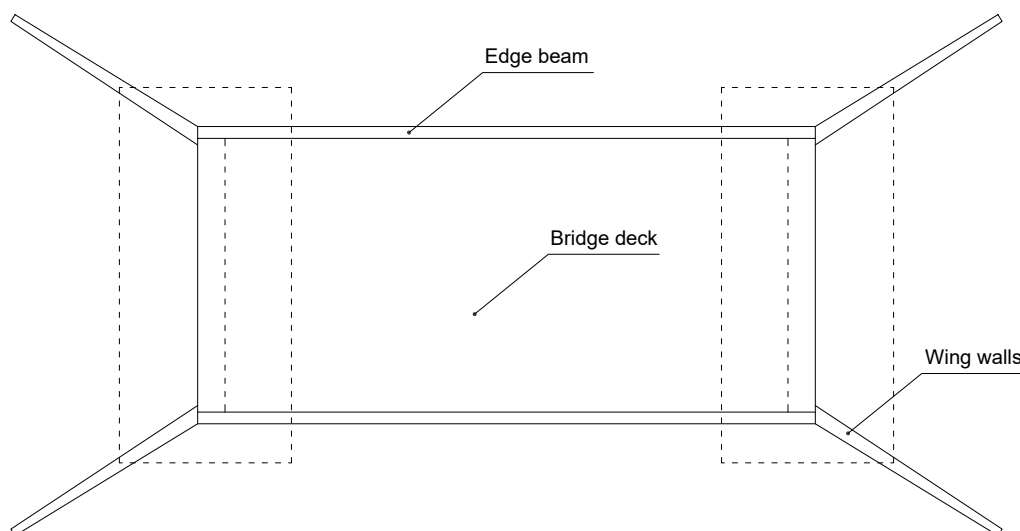
**Figure 3.2** Illustration of set-based design method (Tarazon Ramos & Luis Fernandez, 2014).

When evaluating and investigating the solutions produced in the set-based design method, a set of evaluation criteria is needed to be able to determine which alternative is the most suitable for the specific project. Previous research show that there is two major areas that is identified in a structural engineering project, buildability and sustainability. The term buildability represents the ease of construction for the specific structure and includes many different aspects from planning the project, workplace organization and production method (Löfgren, 2020). A study performed by Löfgren (2020) shows that criteria that correlate to buildability are difficult to quantify to the degree that it is usable in a set-based design tool. Evaluation criteria for sustainability however is easier to quantify. Two criteria that is implemented in the existing set-based design tool is, material cost and material CO<sub>2</sub>-equivalent. These two criteria refer to two of the three pillars in sustainability, economy and environmental impact (Löfgren, 2020).

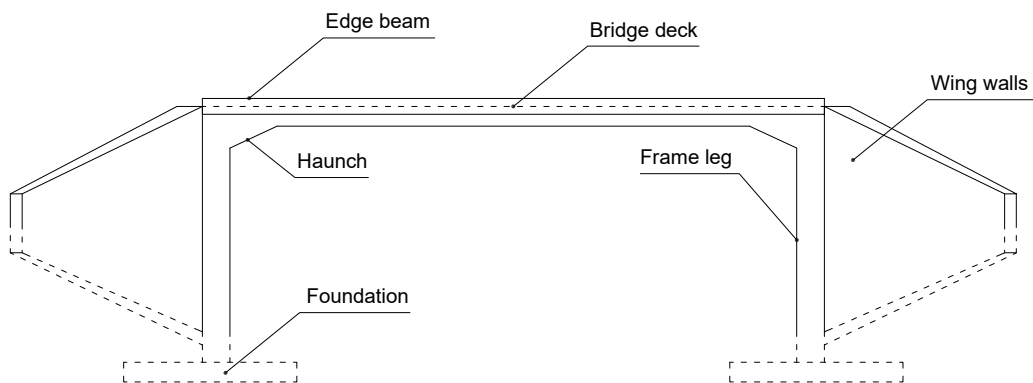
## 3.2 Frame bridge

The slab frame bridge is a commonly used bridge types in Sweden for smaller crossings and is most common for span length shorter than 20 meters. The structural complexity is fairly simple and easy to build and understand (Uppenberg et al., 2017). However, for even longer span lengths, the designer needs to consider if the frame bridge is the suitable option as the bridge type becomes far more complex and expensive if a continuous span is required (Vägverket, 1996).

The bridge deck is constructed as a one-way slab structure which is rigidly connected to the frame legs that transfer the vertical and horizontal loads down to the foundation (Rashem & Swahn, 2018). The pressure from the earth is also carried down to the foundation by the frame legs. In each corner, four wing walls are fixed to the frame legs, see Figure 3.3-3.4. The purpose of the wing walls is to control the slope of the embankment caused by the difference in elevation between the underlying road or railway (Vägverket, 1996).



**Figure 3.3** Principal plan view of a frame bridge.



**Figure 3.4** Principal elevation view of a frame bridge.

A haunch may be located where the bridge deck connects to the frame leg, see Figure 3.4, to transfer the shear force and bending moment between the components (Swedish Transport Administration, 2021a). The size, extent and design of the haunch is dependent of the span length both for technical and aesthetic reasons (Vägverket, 1996). In this stage of the design process and the purpose of this design method, the bridge deck is assumed to have the same thickness along the whole span length.

To adjust and adapt the alignment of the frame bridge according to the connecting road network, it may be required to use a skewed angle of the carriageway, this is a common case for bridges in Sweden (Vägverket, 1996). According to Löfgren (2020) and supported by Vägverket (1996) this results in a more complicated reinforcement arrangement and material-intensive bridge design but that it's possible to perform an alternative design for certain spans where it may be more beneficial to not skew the angle and instead construct the bridge as perpendicular but with a longer span. The recommendations for the angle is depending on the quality of the soil and Vägverket (1996) states that at good condition the angle should not reach below 50 degrees and at worse condition it should not reach below 75 degrees.

## 3.3 Fatigue

### 3.3.1 Background

The concept of fatigue and fracture stretches back to 19th century when this phenomena first was observed in steel structures. Since then, several experiments and tests of this behaviour in steel structures have been made (Gylltoft, 1983). One of the most mentioned investigations regarding fatigue was conducted by August Whöler, which in the 1860's suggested a theory on the limited number of loading cycles a material could withstand. In further research, the Goodman-diagram was established which suggested that a maximum and minimum stress in a loading sequence could be translated into a sinus function in order to describe the cyclic loading on a structure. This could then be utilized in order to determine fatigue life of a structure. With a more realistic approach based on the varying load amplitudes that occurs during the lifetime of a structure, the Swedish engineer Arvid Palmgren defined the Palmgren-Miner's rule which describes failure when the cumulative damage from cyclic loading reaches the fatigue loading capacity in maximum number of load cycles, see Equation 3.1.

$$\sum_{i=1}^{\infty} \frac{n_i}{N_i} = D \quad (3.1)$$

where  $n_i$  is the actual number of cycles for a certain load interval,  
 $N_i$  is the ultimate number of loading cycles for a certain load interval,  
 $D$  is the fatigue damage.

The concept of fatigue in plain concrete is a relatively limited field of research which started in the beginning of the 20th century. With an increasing desire for more slender constructions and the need for constructions in extreme conditions, the knowledge regarding fatigue is especially important. In more slender structures, stress variations increase which contributes to a higher risk of fatigue failure (Gylltoft et al., 1979). Also, structures exposed to conditions such as wind, waves, machinery or temperature and moisture changes have a larger risk of failure due to fatigue damage (Elfgren & Gylltoft, 1997).

The process of fatigue damage can be explained with the three stages of a crack; crack initiation, crack propagation and fracture. Crack initiation starts in discontinuities or stress concentration within a material. As a structure is exposed to cyclic

loading, the crack propagates through the member successively. When the crack has developed to a length where the uncracked section no longer has the load bearing capacity, failure occur (Elfgren & Gylltoft, 1997). As the strength of a material thereby successively decrease with time, it is important to evaluate the load bearing capacity of a structure based on fatigue due to long term cyclic loading and not only the initial loading conditions (Elfgren & Gylltoft, 1997).

Due to the different properties of the involved materials, it is needed to study plain concrete and the reinforcement steel separately in order to determine the fatigue capacity for a structure (SIS, 2005). The verification methods for the separate materials are based on their behaviour and response to cyclic loading, which will be presented in the following chapters.

## **3.3.2 Fatigue in concrete**

### **3.3.2.1 General behaviour**

To establish the fatigue resistance for concrete, several experiments have been conducted to obtain empirical data which could be translated into general formulas for fatigue resistance in concrete (Gylltoft et al., 1979). In an experimental study conducted by H.A.W Cornelissen at Delft University, results from a conducted splitting tension test presents the influence of stress ranges and concrete conditions (Cornelissen, 1984). The study consisted of a concentric tensile test where the stress ranges varied from being just in tension to alternate between tension and compression. The results indicated that the fatigue life is greater for concrete where the stresses varies in just compression compared to the case where the stresses varies between tension and compression. The interest of tensile strength of plain concrete is especially regarded when studying the cracking behaviour and thereby exposure of the embedded reinforcement steel. The study also presents the influence of moisture levels of the concrete, and concluded a clear relation between a higher fatigue life and dry conditions when exposed to cyclic loading (Cornelissen, 1984).

### **3.3.2.2 Fatigue in concrete according to Eurocode**

According to Eurocode, verification of fatigue in concrete can be conducted by several methods. On the basis of Cumulative Damage Method, a more explicit verification can be done by estimating an ultimate number of constant load cycles until failure. This can then be compared to the actual load cycles of which the bridge is being exposed to and confirm sufficient capacity if the "Cumulative Damage Method" rule is satisfied, see Equation 3.3.

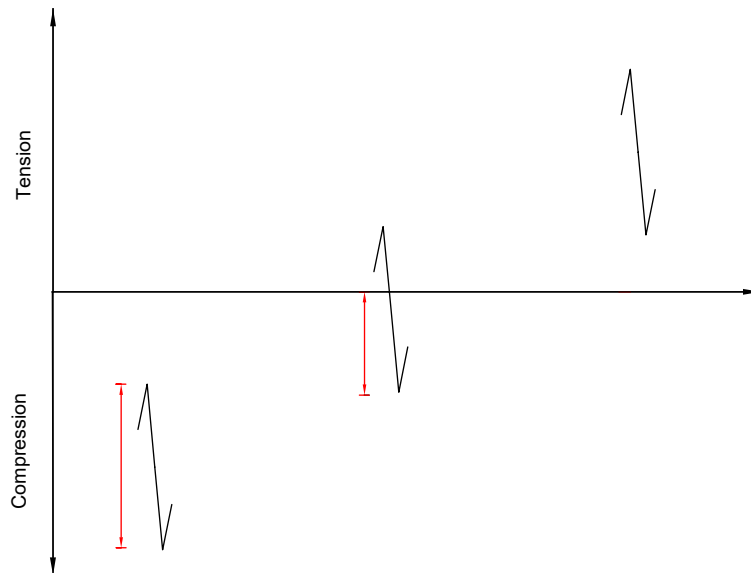
Alternatively, a verification based on a simplified stress comparison is often used to quickly control the fatigue resistance. This implies that a compressive stress range based on a fatigue load model is acting on the same point in a structure, and then compared to the fatigue strength of the material. If the condition stated in the Eurocode is satisfied, fatigue resistance may be assumed sufficient according to

EN1992-1-1 6.8.7. The condition according to Eurocode is stated in Equation 3.2.

$$\frac{\sigma_{c,max}}{f_{cd,fat}} \leq 0.5 + 0.45 \frac{\sigma_{c,min}}{f_{cd,fat}} \quad \left\{ \begin{array}{l} \leq 0.9 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \\ \leq 0.8 \quad \text{for } f_{ck} > 50 \text{ MPa} \end{array} \right. \quad (3.2)$$

where  $\sigma_{c,max}$  is the maximum compressive stress in a cycle [MPa],  
 $\sigma_{c,min}$  is the minimum compressive stress in a cycle [MPa].  
 $f_{cd,fat}$  is the design fatigue strength of the concrete [MPa].

For both methods, the stress range should only takes the compressive stresses into account. If the stress varies between tension and compression, the stress range is only estimated by the stresses that are acting in compression. Thus, if a part of the stress range is acting in tension, it is considered as zero. The stress ranges are defined as presented in Figure 3.5 where the red markings defines the stress range for the corresponding stress variations.



**Figure 3.5** Defined stress ranges for verification of fatigue in concrete.

### 3.3.2.3 The cumulative damage method

In this project, the method used for all fatigue assessment for concrete is the Cumulative Damage Method. The fatigue resistance can thereby be assumed to be sufficient if the condition according to Equation 3.3 is fulfilled.

$$\sum_{i=1}^m \frac{n_i}{N_i} \leq 1 \quad (3.3)$$

When assessing the fatigue resistance for concrete exposed to compression, the absolute number of loading cycles before failure can be calculated according to Equation 3.4 where the maximum and minimum compressive stress is taken from a specified interval, often recommended by the national authorities in EN1992-1-1. However, the results from fatigue assessment in compressed struts due to shear (in rectangular cross sections) has proven to be very rarely governing for the design (Svenska betongföreningen, 2012). This check is therefore neglected in the analysis done in this thesis.

$$N_i = 10^{14 \frac{1-E_{cd,max,i}}{\sqrt{1-R_i}}} \quad (3.4)$$

where

$$R_i = \frac{E_{cd,min,i}}{E_{cd,max,i}}$$

$$E_{cd,min,i} = \frac{\sigma_{cd,min,i}}{f_{cd,fat}}$$

$$E_{cd,max,i} = \frac{\sigma_{cd,max,i}}{f_{cd,fat}}$$

where  $\sigma_{cd,min,i}$  is the lower stress in a cycle [MPa],  
 $\sigma_{cd,max,i}$  is the upper stress in a cycle [MPa],  
 $f_{cd,fat}$  is the design fatigue strength of the concrete [MPa].

### 3.3.2.4 Fatigue resistance due to shear

In order to verify fatigue resistance due to shear forces, a similar method is presented in the Eurocode where the maximum and minimum shear in the same location is compared to the shear capacity of the studied section (EN1992-1-1 (6.8.7)). However, this check is only valid for sections which do not require shear reinforcement. The check is conducted according to Equation 3.5-3.6.

for  $\frac{V_{Ed,min}}{V_{Ed,max}} \geq 0$

$$\frac{|V_{Ed,max}|}{|V_{Rd,c}|} \leq 0.5 + 0.45 \frac{|V_{Ed,min}|}{|V_{Rd,c}|} \begin{cases} \leq 0.9 \text{ up to C50/60} \\ \leq 0.8 \text{ greater than C55/67} \end{cases} \quad (3.5)$$

for  $\frac{V_{Ed,min}}{V_{Ed,max}} < 0$

$$\frac{|V_{Ed,max}|}{|V_{Rd,c}|} \leq 0.5 - \frac{|V_{Ed,min}|}{|V_{Rd,c}|} \quad (3.6)$$

where  $V_{Ed,max}$  is the maximum shear force in a loading cycle [kN],  
 $V_{Ed,min}$  is the minimum shear force in a loading cycle [kN],  
 $V_{Rd,c}$  is the design shear capacity of the plain concrete [kN].

### 3.3.3 Fatigue in reinforcement

#### 3.3.3.1 General behaviour

The general knowledge in fatigue of reinforcement is considerably more recognized than for plain concrete as the research on fatigue for steel constructions has been of interest for a longer period of time. What separates the behaviour in terms of fatigue in reinforcement from concrete is that the steel have a fatigue limit. Unlike for concrete, if the stress variations are kept below a certain limit, no fatigue damage is considered to be applied to the steel. Also, generally the stress range of which the reinforcement is exposed to is considerably larger (Gylltoft et al., 1979).

#### 3.3.3.2 Fatigue in reinforcement according to Eurocode

Due to the fact that the steel does experience a fatigue limit, below which the material does not accumulate fatigue damage, the verification method for reinforcement and prestressed steel varies from concrete. For standard reinforcement bars a simple verification for fatigue can be achieved by comparing the worst applied stress range in a loading cycle to a set value of 70 MPa. This value corresponds to the limit under which the steel does not accumulate any fatigue damage. According to EN 1992-1-1 it can be assumed that the fatigue resistance is sufficient if the stress range is below this value.

For standard cases with known loads, an equivalent stress range verification is often used for road and railway bridges. This method consists of an equivalent stress range based on the corresponding fatigue load model and a number of loading cycles

that is representative for the type of reinforcement. The relationship between number of representative loading cycles and a reference stress range based on the type of reinforcement, are presented in S-N curves and tables in the Eurocode. When the resisting stress range is established for the corresponding reinforcement type, it is compared to the equivalent stress range based of load effects from the appropriate load model, see Equation 3.7. If the condition presented in the Eurocode is fulfilled, fatigue resistance can be assumed to be sufficient according to EN 1992-1-1.

$$\gamma_{F,fat} \cdot \Delta\sigma_{S,equ}(N^*) \leq \frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{s,fat}} \quad (3.7)$$

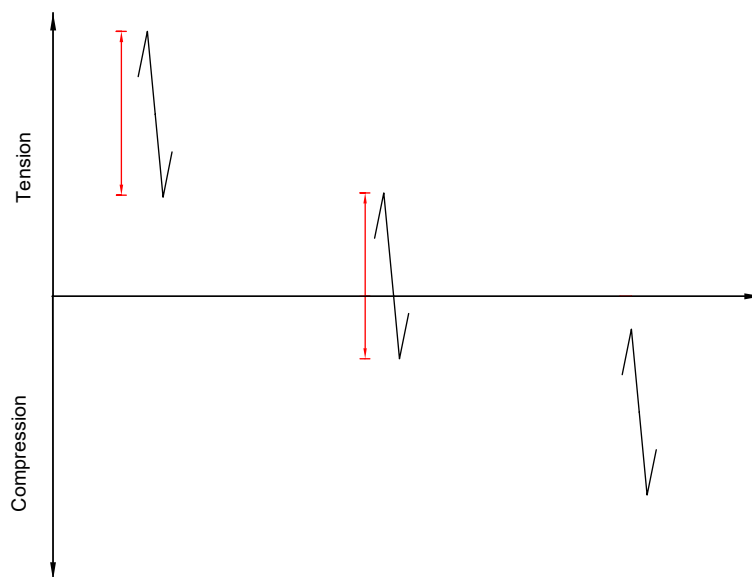
where  $\Delta\sigma_{S,equ}(N^*)$  is the equivalent stress range from a fatigue load model [MPa],  
 $\Delta\sigma_{Rsk}(N^*)$  is the reference stress range for reinforcement determined by the S-N Curves and reinforcement type [MPa],  
 $\gamma_{F,fat}$  partial factor for reinforcement fatigue assessment,  
 $\gamma_{s,fat}$  partial factor for uncertainties in the action/model.

When assessing fatigue for bent reinforcement bars, the reference stress range  $\Delta\sigma_{Rsk}$  should be reduced with a reduction factor  $\zeta$ .

$$\zeta = 0.35 + 0.026 \frac{D}{\phi} \quad (3.8)$$

where  $D$  is the bending diameter for corresponding reinforcement [m],  
 $\phi$  is the reinforcement diameter [m].

Unlike the stress range estimation for concrete, the stress ranges for assessing fatigue damage in reinforcement allows both tensile and compressive stresses to be part of the estimated stress range. However, if the stress variation consists of compressive stresses only, the stress range for fatigue assessment in reinforcement is non-existent and put equal to zero. See Figure 3.6 for an illustrative explanation where the red markings defines the stress ranges for the corresponding stress variations.



**Figure 3.6** Defined stress ranges for verification of fatigue in reinforcement.

### 3.3.3.3 The Lambda method

To establish the equivalent stress range for superstructures of road and railway bridges, Eurocode defines certain simplifications in order to take important conditions into account, also known as the Lambda-method. This means that additional parameters such as span length, traffic volume, design life, number of traffic lanes, traffic type and surface conditions are considered. However, for bridges built according to the regulations and guidelines provided by the Swedish Transport Administration, factors which takes span length and materials into account are adjusted and replaced with established simplifications valid for structures in Sweden.

The Lambda-Method consist of several combined lambda factors which is applied to the equivalent stress range according to Equation 3.9. This final and adjusted stress range is then used in the comparison with the reference stress range according to Equation 3.7. However, the lambda method is not described equally for road and railway bridges. For road bridges, the lambda method is only valid for when assessing fatigue in the reinforcement of the structure and can only be applied for when fatigue load model 3 is utilized in the analysis. For when assessing fatigue in railway bridges, the method can only be utilized for load model LM71 with restrictions according to Eurocode and is also defined for assessment of fatigue in compressed concrete (SIS, 2005).

$$\lambda_s = \phi_{fat} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} \quad (3.9)$$

|       |                 |   |
|-------|-----------------|---|
| where | $\phi_{fat}$    | damage equivalent impact factor controlled by the surface roughness,  |
|       | $\lambda_{s,1}$ | factor which takes the element and reinforcement type into account based on the critical length of the influence line, this factor is presented as adjusted according to the specifications stated in the national authorities, |
|       | $\lambda_{s,2}$ | factor which takes traffic volume into account,   |
|       | $\lambda_{s,3}$ | factor which takes the design life of the bridge into account,  |
|       | $\lambda_{s,4}$ | factor which takes into account if the structure is loaded from more than one traffic lane.   |

The factors which constitutes Equation 3.9 is calculated as below according to Eurocode (SIS, 2010b):

$$\begin{aligned} \phi_{fat} &= 1.2 && \text{for surface of good roughness,} \\ \phi_{fat} &= 1.4 && \text{for surface of medium roughness.} \end{aligned}$$

$$\lambda_{s,1} = \lambda_1 \cdot \left( \alpha_{Q1} - \frac{\alpha_{Q1} - 1}{10} \cdot L \right)$$

|       |               |   |
|-------|---------------|---|
| where | $L$           | is the critical length of the element influence line,   |
|       | $\alpha_{Q1}$ | adjustment factor dependent on the traffic amounts which is stated by the national authorities, |
|       | $\lambda_1$   | factor chosen from EN1992-2 Annex NN, Figure NN.2 for field and carriageway,                    |

$$\lambda_{s,2} = \bar{Q} \sqrt[k_2]{\frac{N_{obs}}{2.0}}$$

|       |           |  |
|-------|-----------|--|
| where | $N_{obs}$ | number of lorries per year according to EN 1991-2, |
|       | $k_2$     | slope of the appropriate S-N curve,                |
|       | $\bar{Q}$ | factor for traffic type,                           |

$$\lambda_{s,3} = \sqrt[k_2]{\frac{N_{years}}{100}}$$

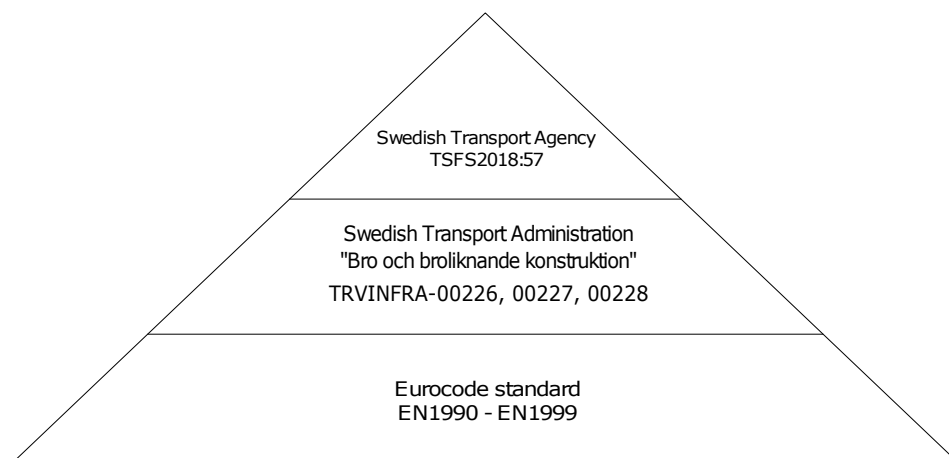
where  $N_{years}$  is the design life of the bridge,

$$\lambda_{s,4} = \sqrt[k_2]{\frac{\sum N_{obs,i}}{N_{obs,1}}}$$

where  $N_{obs,i}$  is the expected lorries per year on lane  $i$ ,  
 $N_{obs,1}$  is the lorries per year on the slow lane.

### 3.3.4 Regulations and requirements

In a case of a new bridge construction on the behalf of the Swedish transport administration, there are rules and guidelines that dictate how a construction product should be designed as a basis for contracts (SIS, 2010a). In the design process these have a specific order in which they should be prioritized, see Figure 3.3. The application of Eurocode is regulated by TSFS 2018:57 (Transportstyrelsens författningssamling) published by the Swedish Transport Agency (Transportstyrelsen). This regulates national parameters that differs from the values in Eurocode (Swedish Transport Agency, 2018). Together with TSFS 2018:57, the Swedish Transport Administration (Trafikverket) provides three documents with requirements and guidelines for the different aspects in general, design and maintenance. These documents give additions to the rules from Eurocode. Within the Eurocode standards there are different rules which apply in general or for specific structures, such as bridges.



**Figure 3.7** Illustration of how the documentation apply.



# 4

## Design

### 4.1 Previous work

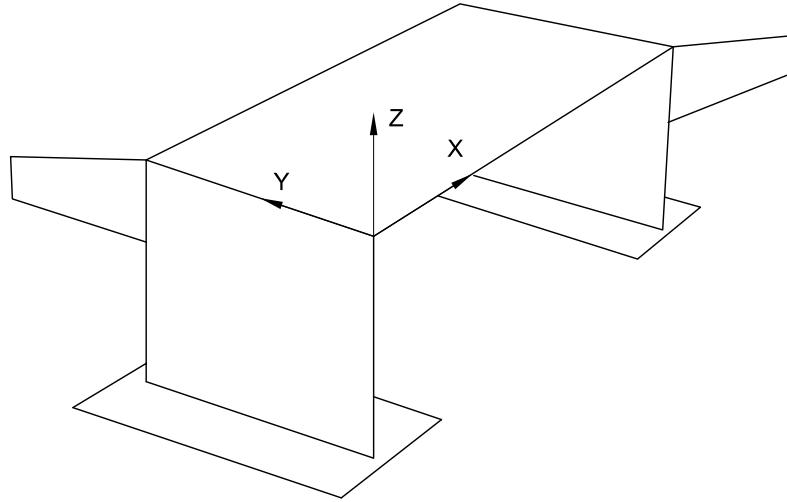
This master's thesis is a continuation on a set-based design tool (Löfgren, 2020). The previous tool generates for several frame bridges a preliminary design of bending and shear reinforcement in ULS according to Eurocode. This includes calculating and distributing reinforcement amounts in the all components of the bridge with some simplification regarding the wing walls and foundation. The tool also verifies the bridge deck for the demand of deflection in SLS (Löfgren, 2020).

To maintain as much compatibility and possibility to use both implementations together, similar simplifications are used for the fatigue verification which is controlling the reinforcement and concrete amounts from the initial tool.

This chapter describes what has been included when implementing fatigue assessment to the existing script (Löfgren, 2020). Therefore, the parts that already existed in the set-based design tool will not be presented further unless these parts is utilized in the implementation of fatigue.

## 4.2 Loads

In the design of the frame bridge with regard to fatigue, not only the cyclic loads need to be taken into account. In addition to these, other variable loads and permanent loads will influence the structure. The coordinate system presented in Figure 4.1 was used when assessing the loads in BRIGADE Plus.



**Figure 4.1** Sketch showing the coordinate system used in the BRIGADE Plus model.

### 4.2.1 Permanent loads

The permanent loads are the loads that constantly act on the frame legs and on the carriageway throughout the whole service life of the structure. In this project, permanent load as self-weight and the earth pressure was included in this analysis. The self-weight correspond to the weight of each material that the bridge consist of, such as reinforcing steel and concrete. A combined value for these were used and determined according to EN-1991-1-1, which states that for normal weight concrete with reinforcing steel, the nominal density is  $25 \text{ kN/m}^3$ . This load was applied on all parts as a gravity load (Löfgren, 2020).

The second permanent load is earth pressure which acts on the frame legs and the wing walls of the bridge. The magnitude of this load was calculated and implemented with Equation 4.1, where the weight of the soil is  $18 \text{ kN/m}^3$  and the earth pressure coefficient was set to 0.34, (Löfgren, 2020).

$$q = \gamma \cdot k_0 \cdot \Delta z \quad (4.1)$$

where  $\gamma$  weight of the soil [ $\text{kN/m}^2$ ],  
 $k_0$  coefficient for earth pressure at rest [-],  
 $\Delta z$  depth of the soil [m]

The magnitude therefore varies depending on the depth. This correspond to an linear influence on the frame legs, see Figure 4.2



**Figure 4.2** Earth pressure acting on the frame legs.

### 4.2.2 Variable loads

The variable loads which should be considered in the combination with the cyclic load are mainly the temperature loads which should be considered at the same time as the wind load. However, according to a study performed by (Löfgren, 2020), both the temperature loads and the wind loads had a low influence to the overall effect on the combination of actions. According to (Swedish Transport Agency, 2018) the snow load can be neglected for this type of structure when the possibility for snow removal is at hand.

Taking the above mentioned consideration into account, it was possible to reduce the number of load cases that were needed to be performed in the analysis, which was beneficial as the analysis that was performed in this thesis was rather extensive.

### 4.2.3 Fatigue load

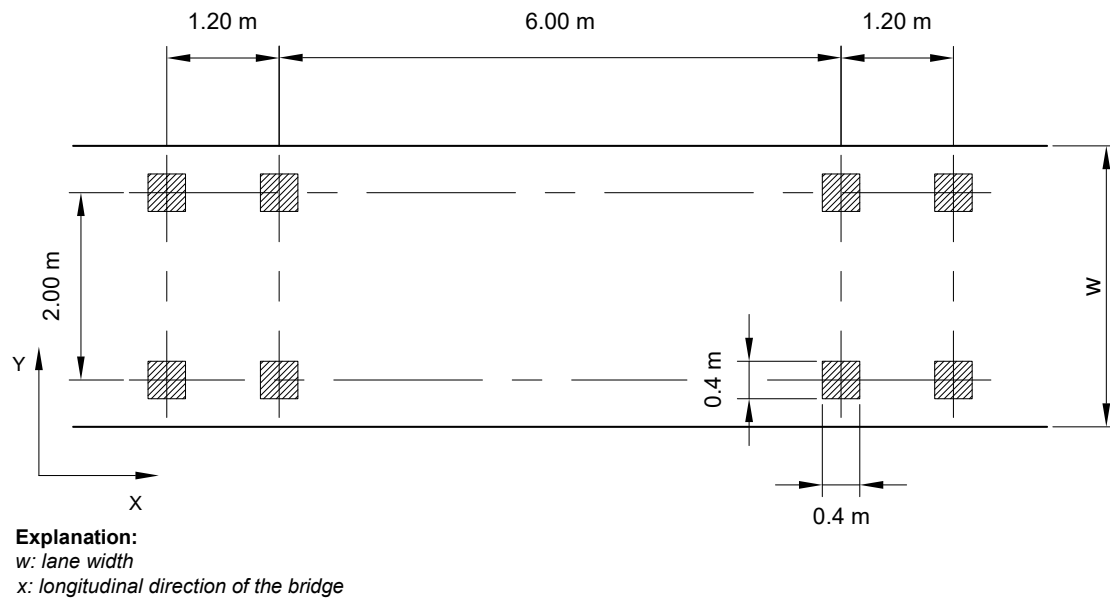
The verification of fatigue requires an analysis from a different set of load models compared to the preliminary static design of the frame bridge. The reason is that these load models would be too conservative and therefore not representative in the case of fatigue verification. Eurocode provides five fatigue load models which may be used depending on the desired verification level and the type of structure that is analyzed (Gulvanessian et al., 2009). For a frame bridge in reinforced concrete, certain fatigue load models are less relevant to use. These are fatigue load model 1 and 2, which are only applicable for steel structures and fatigue load model 5 which is based on real traffic data and therefore more relevant in a case of an existing bridge, where this data is known (SIS, 2010b).

#### 4.2.3.1 Fatigue load model 3

Fatigue load model 3 consist of one vehicle and is suitable for verification of a concrete frame bridge and applicable when performing the different methods for both reinforcement and concrete.

The model is built up by assuming, that instead of many different stress ranges that corresponds to a real traffic situation, a single vehicle produces a stress range that over many cycles builds up to a damage that could be compared to a real life situation (SIS, 2010b).

The model composes of four axle loads of 120 kN in an arrangement according to Figure 4.3. It's recommended to take into account the possibility that there might be several vehicles on the bridge at the same time (SIS, 2010b). However, because of the required distance of 40 meters between the centre of the vehicles, it requires a certain span length to fit both vehicles. As previously mentioned, it's not common to design single span frame bridges with span length over 20 meters. Therefore, this wasn't included in this project at this time.



**Figure 4.3** Fatigue load model 3 vehicle, reproduced from EN1991-2 with permission from SIS.

To take into account that the load model constitutes an estimation of a real traffic situation, a numerous of factors such as annual traffic volume and material dependency needs to be handled with fatigue load model 3 in combination with the Lambda method (SIS, 2008). For an early stage of the design phase, as in this project, fatigue load model 3 was deemed to be sufficient for the purpose of quickly be able to perform fatigue verification of many frame bridges.

#### 4.2.3.2 Fatigue load model 4

Another fatigue load model that is relevant for this bridge type and material is model 4, which is based on five different standard lorries ('equivalent lorries') similar to load model 2. These standard load models occur in different proportion to correspond to a real life traffic situation (SIS, 2010b). This results in a model, where in combination with a more complex fatigue damage calculation, gives a more accurate estimation of the damage caused by the traffic. However, for the purpose of this tool it was deemed sufficient to only implement fatigue load model 3.

#### 4.2.4 Combination of actions

The permanent and variable loads are combined in a basic combination, similar to the frequent load combination. The load which is induced by the axle loads from the fatigue load model is added to this combination (SIS, 2008), see Equation 4.2. According EN1992-2 Annex NN, a factor of 1.40 should be multiplied to the axle loads for the verification of reinforcement when using fatigue load model 3. In the tool the basic combination and the axle loads were created separately in the model and combined in post-processing where the factor also was applied to the relevant bending moments and shear forces induced by the axle loads. By constructing the script in this way, it was possible to reduce the number of steps in the analysis and thereby reducing the computational time.

$$\left( \sum_{j \geq 1} G_{k,j} + P + \psi_{1,1} Q_{k,1} + \sum_{i \geq 1} \psi_{2,i} Q_{k,i} \right) + Q_{fat} \quad (4.2)$$

|       |              |   |
|-------|--------------|---|
| where | $G_{k,j}$    | characteristic value of the permanent load $j$ ,          |
|       | $P$          | value of the prestressing force,                          |
|       | $Q_{k,1}$    | characteristic value of the main variable load,           |
|       | $Q_{k,i}$    | characteristic value of the accompanied variable load $i$ |
|       | $Q_{fat}$    | value of the fatigue load,                                |
|       | $\psi_{1,1}$ | frequent value factor for the main variable load,         |
|       | $\psi_{2,i}$ | quasi-permanent value factor for variable load $i$ .      |

The fatigue load should be combined with the unfavorable basic combination according to EN-1992-1-1, 6.8.3. With the conclusions drawn in section 4.2.2, this resulted in one basic combination containing only two different permanent loads.

## 4.3 Studied sections and sectional design

This chapter contains the framework for the fatigue verification part of the script and presents how the sectional design was performed. It sets the foundation to what information required from the FE-analysis and how this is processed by the script.

### 4.3.1 Sectional forces and reinforcement

The superstructure of the frame bridge consist of a one-way slab which transfers sectional forces and moments in both longitudinal and transverse direction. These sectional forces are obtained from a linear elastic FE-analysis in BRIGADE Plus (Scanscot Technology AB, 2018) and needs to be transformed to design moments and shear force. In a slab element, these sectional forces are calculated as force per unit width (Engström, 2014).

#### 4.3.1.1 Bending moment

The design of resisting moments for the reinforcing steel was calculated according to (Engström, 2014) by combining the bending moments  $m_x$  and  $m_y$  with their counterpart of  $m_{xy}$  separately, according to the directions indicated in Figure 4.3. The sectional moments were obtained from the FE-analysis where a negative moment indicate that the top side of the slab is in tension and the bottom is compressed. It is important to keep this consistent throughout the calculation process to get the correct corresponding resisting moment for the reinforcement design. The same principle was used when calculating the design moment of the cyclic loads, which will produce a maximum and minimum for every cycle, which is another aspect to why it is important to be consistent with the sign conventions.

For the calculations of the support moments, Equation 4.3 - 4.4 were used for x- and y-direction, taking into account the contribution of the torsional moment according to Engström (2014).

$$m'_{rx} = m'_x - |m_{xy}| \quad (4.3)$$

$$m'_{ry} = m'_y - |m_{xy}| \quad (4.4)$$

Similarly, the field moments were calculated using Equation 4.5 - 4.6.

$$m_{rx} = m_x + |m_{xy}| \quad (4.5)$$

$$m_{ry} = m_y + |m_{xy}| \quad (4.6)$$

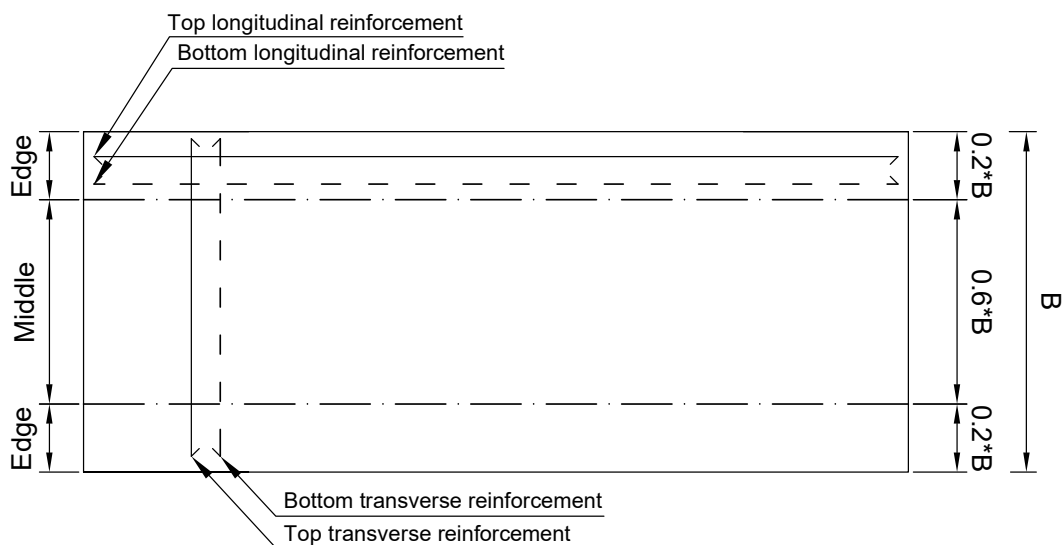
### 4.3.1.2 Bending reinforcement

The process of generating a preliminary reinforcement design for analyzed bridge models was already developed in previous work by Löfgren (2020) where the script utilizes the resisting bending moments obtained from the FE-model. The preliminary reinforcement design was then calculated according to ULS as stated in Equation 4.7 provided by Engström (2014) unless a minimum allowed reinforcement amount was necessary, which was then calculated according to Eurocode 2 (Löfgren, 2020). This was applied for both the slab and for the frame legs. The preliminary reinforcement designs are then verified with respect to fatigue in the parts of the script that was developed for this study.

$$A_s \geq \frac{m_r}{f_{yd}z} \quad (4.7)$$

where  $m_r$       resisting design moment [kNm],  
 $f_{yd}$       design yield stress for the reinforcement [MPa],  
 $z$       internal lever arm in the ultimate limit state [m].

The reinforcement was then arranged as presented in Figure 4.4 in order to estimate the total reinforcement amounts, where the longitudinal reinforcement is distributed in the transverse direction by partitioning the slab into two edge strips of 20% and a middle strip as 60% of the slab width. This arrangement was also implemented to the original script version by Löfgren (2020).



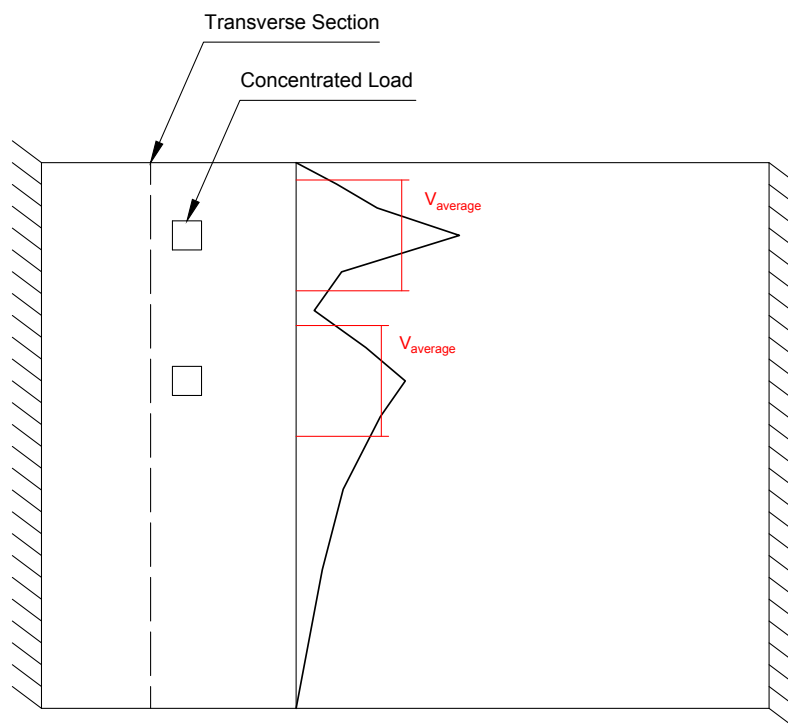
**Figure 4.4** Principle sketch of slab proportions and reinforcement distribution.

### 4.3.1.3 Shear force

The design value for the shear force in a slab is similarly to the bending moment calculated by combining different direction from the FE-analysis, whereas the shear force is performed by calculating the resultant of the force in longitudinal and transverse direction (Pacoste et al., 2012). The procedure for this transformation was performed according to Equation 4.8.

$$v_0 = \sqrt{v_x^2 + v_y^2} \quad (4.8)$$

When performing a FE-analysis on slabs where there is a concentrated load acting on the slab, there is also a risk of obtaining large shear force peaks caused by the load acting on a small area. This was resolved in this project by distributing the force over an area in the perpendicular direction of the force, according to the procedure proposed by Pacoste et al., 2012. This resulted in an average shear force within a width indicated in Figure 4.5.



**Figure 4.5** Distribution of shear force over a transverse section.

Eurocode does not provide any guidelines on how to calculate the width of the distribution and this was therefore performed according to BBK04, see Equation 4.9.

$$b_{ef} = \begin{cases} 7d + b + t \\ 10d + 1.3x \end{cases} \quad (4.9)$$

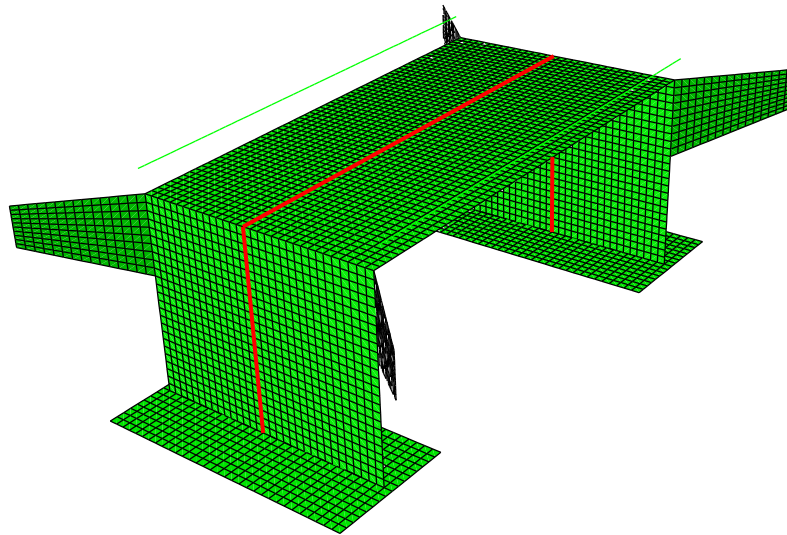
where  $x$  distance from transverse section to the centre of the concentrated load,  
 $d$  effective height of the plate,  
 $t$  thickness of the road surface,  
 $b$  width of the surface for the concentrated load.

### 4.3.1.4 Shear reinforcement

Just like the preliminary bending reinforcement design, the procedure for generating the preliminary shear reinforcement according to ULS was already integrated to the developed script in previous work by Löfgren (2020). The preliminary shear reinforcement is designed according to EN1992-1-1 (SIS, 2008) where the shear capacity of the concrete was compared to the applied shear force. If the shear force exceeds the shear capacity in the studied section, preliminary shear reinforcement was considered necessary and thereby applied to the section. This was applied for both the slab and the frame legs. The shear reinforcement was then verified with respect to fatigue in the parts of the script that was developed for this study.

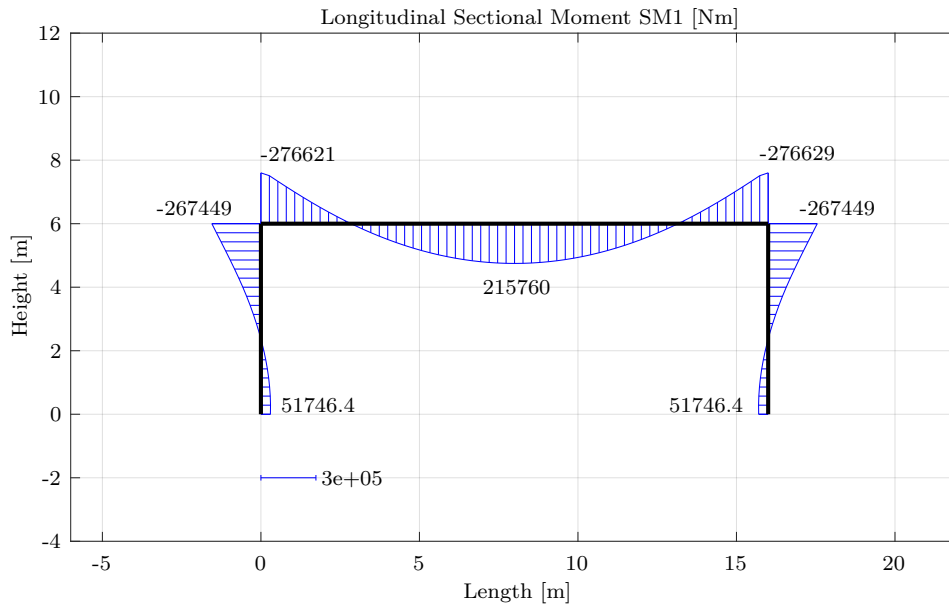
### 4.3.2 Critical sections

An aspect in this project is the computational time for the analysis which is highly affected by the number of steps in the FE-model. To reduce this time some simplification was needed regarding the load positions and the flexibility of checking every sections in the bridge. A study was performed on the sectional forces with the basic combination of actions to determine which sections in the longitudinal direction are critical. The effects was obtained in the middle of the bridge deck, see Figure 4.6



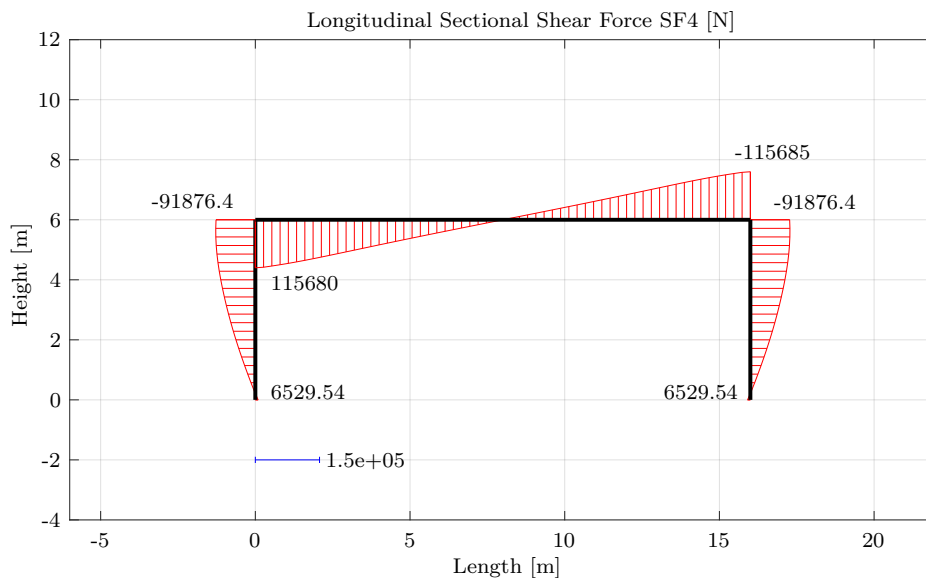
**Figure 4.6** BRIGADE Plus model showing the paths which were used to extract values for studying the critical sections.

The sectional moments in the frame were as expected large in the support sections where the frame legs connects to the bridge deck and in the field section, see Figure 4.7. Therefore, it was of interest to verify the bending moment in these sections of the bridge. In the middle section of the frame legs, the bending moment seemed to be relatively small, which indicated that this section may not be that decisive. However, according to Eurocode EN1992-2, some exceptions can be made regarding where to verify fatigue resistance. Also, Eurocode states that components that are rigidly connected to the superstructure should be controlled. This means that both frame legs and bridge deck are of interest regarding fatigue assessment. Therefore, the middle section of the frame legs was also verified to ensure that it's not decisive.



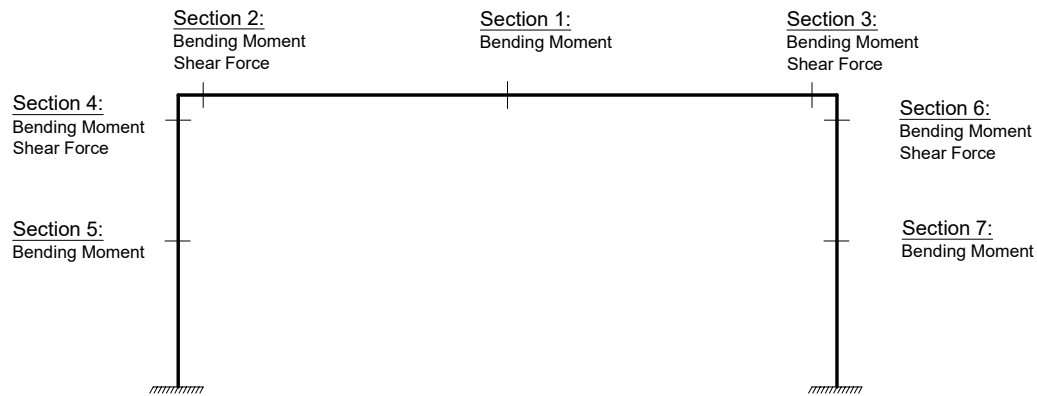
**Figure 4.7** Sectional moment,  $m_x$  and  $m'_x$ , in longitudinal direction, obtained in the middle of the bridge deck and frame legs.

The shear forces were also large in the support sections where the cyclic loads were expected to be large regarding the shear force variations, see Figure 4.8. For the middle sections of the frame legs, the dominate force will be the earth pressure and self-weight. The cyclic load on top of the bridge deck is thereby expected to have a small influence on this section. In this early stage of the design phase it is deemed sufficient that this section only will be checked regarding bending moment.



**Figure 4.8** Sectional shear force,  $v_x$ , in longitudinal direction, obtained in the middle of the bridge deck and frame leg

The critical sections that are of interest to implement and verify with regard to fatigue were therefore deemed to be the sections according to Figure 4.9.



**Figure 4.9** Critical sections and considered effects for each section when verifying fatigue.

The sections closest to the frame leg and bridge deck connection were placed at a distance of  $0.9d$  from the support, where  $d$  is the effective height of the cross-section, because the shear force capacity is not needed to be controlled closer to the support.

### 4.3.3 Influence of creep

The influence of creep in concrete structures is usually included when analyzing structures exposed to long term loading. Thereby, it is suitable to include the long term effect of creep when assessing the fatigue capacity of a structure. However, due to the limitations of this study, simplifications have been applied to the calculations and the influence of creep was neglected. It is therefore important to have in mind that the results would have been different if creep would have been included in the calculations, whereas the Young's modulus is reduced when the influence of creep is included.

### 4.3.4 Sectional design

When assessing fatigue resistance in a reinforced concrete member, the studied section should be assumed to be in a cracked stage (SIS, 2008). As the members are not entirely in pure bending and normal forces are contributing to the sectional forces in the structure, the neutral axis cannot directly be found by equilibrium of sectional areas (El-Emrani et al., 2011). Instead, the sectional constants were established by an iterative procedure where the compressed zone first was assumed and then controlled by calculating the stresses in the neutral axis according to Navier's formula, see Equation 4.10. When the assumption was correct, the stress in this point should be close or equal to zero. The iterative procedure was implemented in the script and performed until the assumption was correct and thereby enabled the establishment of the correct sectional constants, see Equations 4.12-4.14. When the correct sectional constants was determined, the stresses in the reinforcement and in the most compressed fibre could be established by the Navier's formula, see Equation 4.10.

$$\sigma_c(z) = \frac{N_{Ed}}{A_{II}} + \frac{N_{Ed} \cdot e + M_{Ed}}{I_{II}} z \quad (4.10)$$

|       |          |   |
|-------|----------|---|
| where | $N_{Ed}$ | design normal force [N],  |
|       | $M_{Ed}$ | design moment [Nm],   |
|       | $e$      | eccentricity of the normal force [m]                                    |
|       | $z$      | distance from the neutral axis to<br>the compressed concrete fibre [m], |

The stresses in the reinforcement were also calculated according to Navier's formula, except that  $z$  corresponded to the distance from the neutral axis to the reinforcement level. In order to establish the stresses in the reinforcement, the calculated equivalent concrete stress at the level of the steel was multiplied with the modular ratio of concrete and steel, see Equation 4.11.

$$\sigma_s = \alpha \cdot \sigma_c(z_s) \quad (4.11)$$

where  $\alpha$  ratio between the Young's modulus of the steel and concrete,  
 $\sigma_c(z_s)$  concrete stress at steel level [MPa],

and the sectional constants calculated as:

$$A_{II} = b \cdot x_{cc} + (\alpha - 1)A'_s + \alpha A_s \quad (4.12)$$

$$I_{II} = \frac{b \cdot x_{cc}^3}{12} + b \cdot x_{cc} \left( \frac{x_{cc}}{2} - x_{tp} \right)^2 + (\alpha - 1)A'_s (x_{tp} - d')^2 + \alpha A_s (d - x_{tp})^2 \quad (4.13)$$

$$x_{tp} = \frac{bx_{cc} \frac{x_{cc}}{2} + (\alpha - 1)A'_s \cdot d' + \alpha A_s \cdot d}{A_{II}} \quad (4.14)$$

where  $b$  is the width of the considered section,  
 $x_{cc}$  height of the compressed zone,  
 $\alpha$  ratio between the Young's modulus of the steel and concrete,  
 $A'_s$  amount of reinforcement in the compressed zone,  
 $A_s$  amount of reinforcement in tension,  
 $d'$  distance from compressed side to the top reinforcement,  
 $d$  distance from compressed side to the bottom reinforcement.

In the assessment of fatigue in the shear reinforcement due to shear forces, the shear stress was calculated with respect to the identified shear reinforcement amount and the angle of the compressive strut in each studied section, see Equation 4.15.

$$\sigma_{s, shear} = \frac{V_{Ed}}{\frac{A_{sw}}{s} \cdot z \cdot \cot(\theta_{fat})} \quad (4.15)$$

where  $\frac{A_{sw}}{s}$  is the amount of shear reinforcement per bar distance [ $\frac{m^2}{m}$ ],  
 $z$  is the distance between the top and bottom bending reinforcement [m],  
 $\theta_{fat}$  is the adjusted angle of the compressive strut,  
 $V_{Ed}$  is the design shear force[kN].

The angle of the compressed strut was adjusted to be more suitable for calculations regarding fatigue assessment according to (Svenska betongföreningen, 2012), see Equation 4.16.

$$\cot(\theta_{fat}) = \sqrt{\cot(\theta)} \geq 1.0 \quad (4.16)$$

where  $\theta$  is the angle of the compressed strut.

## 4.4 Modelling and script development

This section treats the specifics regarding the implementation in BRIGADE Plus. It includes placements of loads that are the basis for the analysis, how the forces were obtained with respect to the critical sections and how they were applied in the verification of the existing reinforcement arrangement.

### 4.4.1 Longitudinal placement of fatigue load models

In the longitudinal direction, the effects from the fatigue load model was analyzed when the load model was moving along the carriageway. This will give each section of the bridge an amplitudinal variation in bending moment, shear and normal force, which resulted in a maximum and minimum stress for each studied section (SIS, 2010b). With the critical sections located for the basic load combination, which consist of only permanent load, it was possible to estimate which position of the vehicle that produces the largest variation in force and moment when passing the bridge.

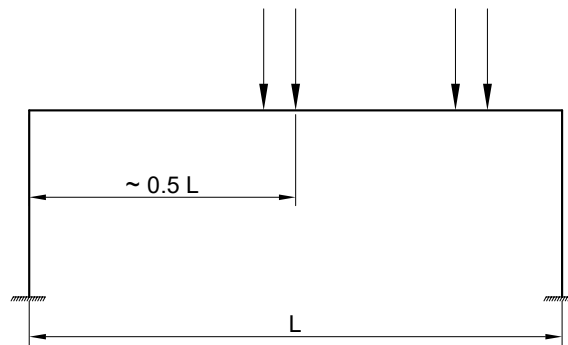
In BRIGADE Plus, a module called Live Loads is included. This module contain the possibility to simulate the passing of a vehicle which is predefined in the design codes. However, because of the fact that this module significantly increases the time it takes to run an analysis, this module was not utilized in the script. However, it was used to estimate simplified positions to be modelled as point loads at specific positions for maximum and minimum forces corresponding to the critical sections.

Several different span lengths were studied with only the live load corresponding to fatigue load model 3 applied, in order to isolate the effect of the vehicle. The different span length were then chosen depending on if the vehicle was completely on the bridge or partially on the bridge. The results were analyzed with the built in BRIGADE Plus tool Live Loads Axle Position (Scanscot Technology AB, 2017), which utilizes influences lines to determine the maximum and minimum position of the axles corresponding to a chosen element (Scanscot Technology AB, 2017). This position was noted with a specific distance from the frame leg corresponding to a proportion of the span length which coincided for the different length with only a small difference.

Analyzing the critical sections with respect to each sectional component generated many different combinations which needed to be simplified. This was done by running all positions and study which positions that gave similar load effects in the critical sections. The positions could then be narrowed down to five positions to utilize in the analysis. To enable different skewed angles and different geometries which results in a non-symmetric frame bridge, the positions presented below were mirrored in the script, which resulted in a total of ten positions.

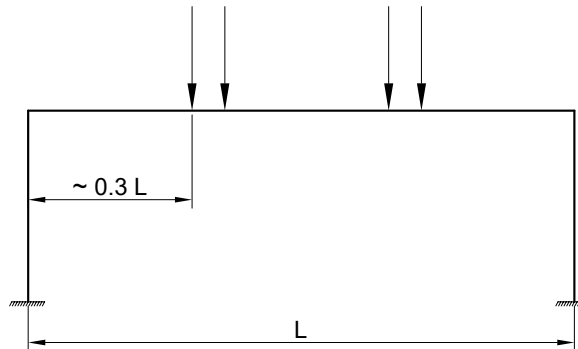
For a one-span frame bridge, many position constituted a variation which only fluctuate between the basic combination, i.e. the vehicle was not placed on the bridge, and the worst position when the vehicle was placed on the bridge.

Figure 4.10 shows the vehicle position for maximum positive sectional moment SM1 and SM2 for critical field section 1. The position for torsional moment did not become governing for any sections and therefore the value for SM3 was the accompanied value corresponding to the same position as to SM1 and SM2. The variation fluctuates between this position and the basic combination.

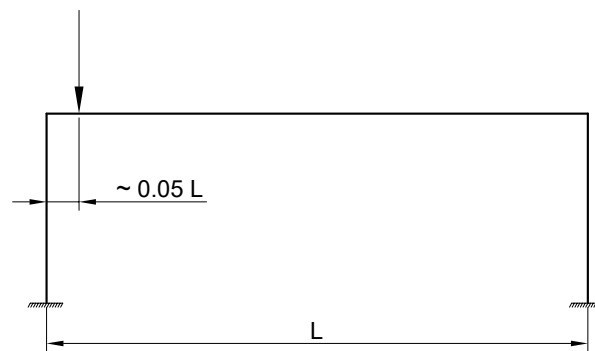


**Figure 4.10** Position of vehicle for maximum positive sectional moment SM1+SM3 and SM2+SM3, critical section 1.

Figure 4.11 - 4.12 shows the vehicle position for maximum and minimum sectional moments. The negative moment produces the largest moment and the variation fluctuate between a large negative moment and a small positive moment. These positions apply for critical support section 2 and 3.

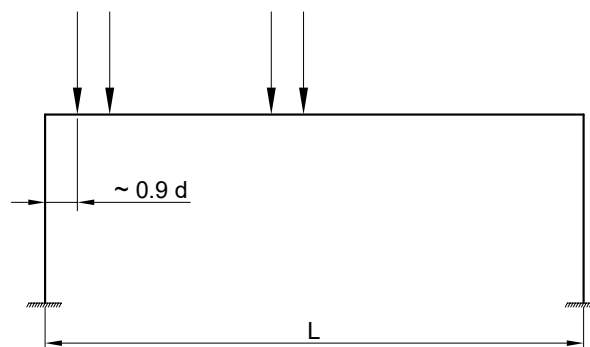


**Figure 4.11** Position of vehicle for maximum negative sectional moment SM1+SM3 and SM2+SM3, section 2 and 3.



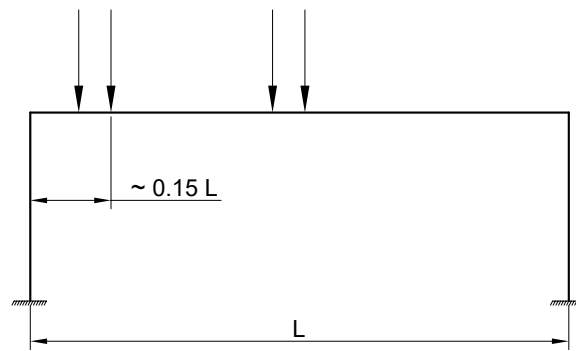
**Figure 4.12** Position of vehicle for minimum positive sectional moment  $SM1+SM3$  and  $SM2+SM3$ , section 2 and 3.

Figure 4.13 shows the maximum position for sectional shear force  $SF4$  and  $SF5$  in critical support section 2 and 3. The shear force fluctuate between this position and the basic combination.



**Figure 4.13** Position of vehicle for maximum sectional shear force  $SF4$  and  $SF5$ , section 2 and 3.

The positions for the frame legs did not differ much compared to each other, see Figure 4.14. These positions were therefore simplified to include all effects which are controlled in the frame legs. The variation for both shear force and sectional moments fluctuate between this position and the basic combination.

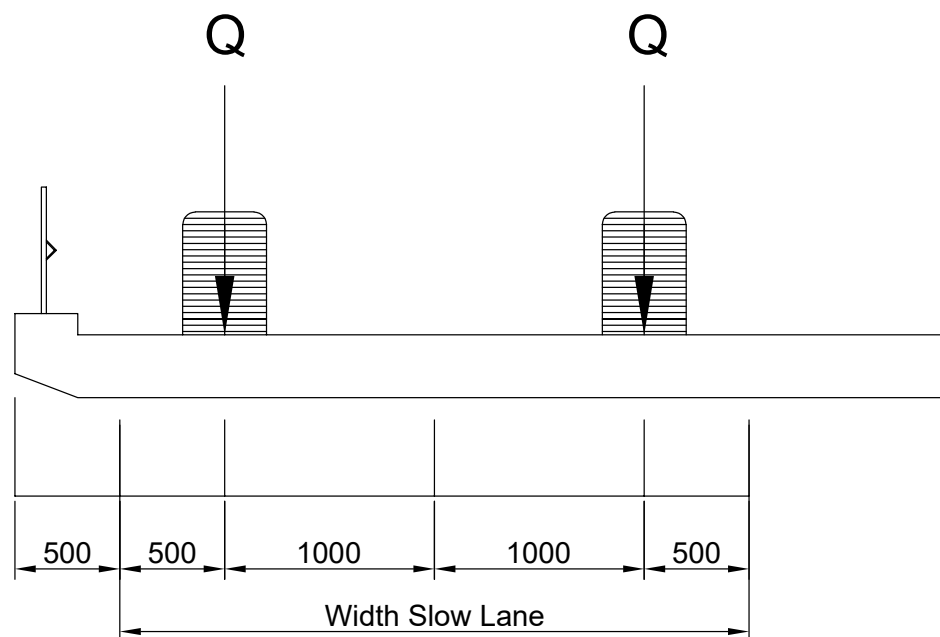


**Figure 4.14** Position of vehicle for maximum negative sectional shear force SF4 and SF5 and sectional moment SM1+SM3 and SM2+SM3, section 4, 5, 6 and 7.

#### 4.4.2 Transverse placement of fatigue load models

For global verification, the general approach is to place the fatigue load model centrally in the notional lanes. However, this may lead to a more conservative result which might not be representative to the fatigue damage. Therefore, the placement of the fatigue loads model may be based on a distribution that corresponds to the physical lanes according to EN1991-2. The physical lanes are defined according to national regulations and are dependent on the allowed speed limits for different road bridge sections (Swedish Transport Administration, 2021b).

When assessing the fatigue capacity of a frame bridge in this project, the fatigue load model was placed centrally in the slow lane. The slow lane was defined as the lane which is predominantly used by heavy traffic such as lorries, which is on the lane to the most right in the direction of travel (SIS, 2010b). The final placement of the fatigue load model in the transverse direction is shown in Figure 4.15.



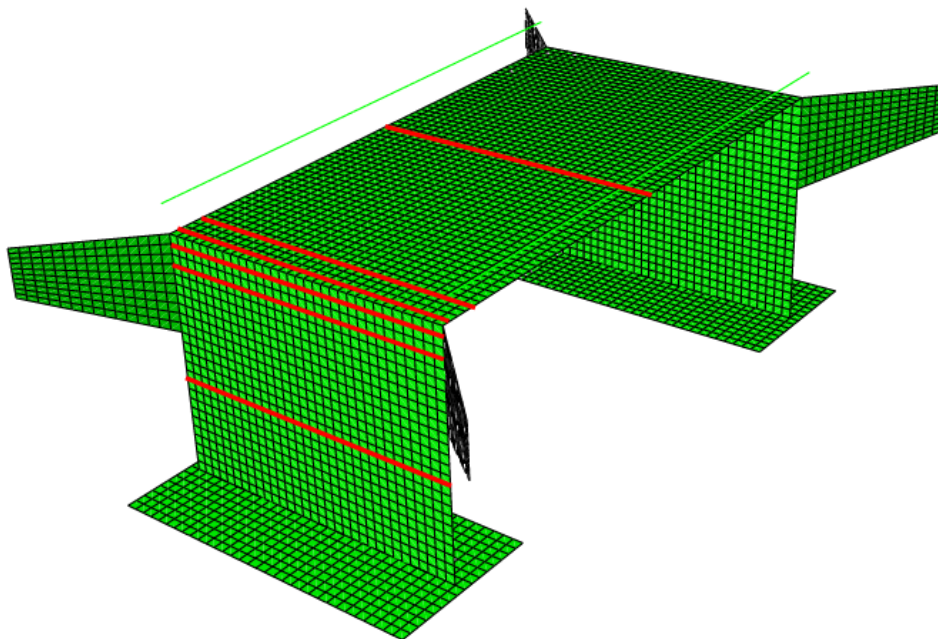
**Figure 4.15** Positioning of fatigue load model 3 centered in a slow lane.

If the carriageway consists of two travelling directions, one slow lane would be placed on each direction with the same distance from the edge as shown in Figure 4.15. This would also affect the number of cycles that the bridge is exposed to. According to EN1991-2 this is managed by calculating the sum of  $N_{obs}$  in one direction and the sum in the other direction. The transverse placement on each side could be symmetrical but they would cause different stress ranges and therefore different fatigue damage depending on where on the bridge the verification is performed. This case would affect the verification which is performed with Palmgren-Miner's principle ac-

According to Chapter 3, which in the script was utilized for the verification of concrete in compression. When using the Lambda-method, this can be incorporated when calculating the factor which takes into account if the structure is loaded from more than one traffic lane. However, due to time limitation this was not included in the script and the analysis was only performed on a single slow lane.

### 4.4.3 Obtaining forces from FE-analysis for fatigue verification

The sectional forces are obtained from transverse paths which correspond to the critical sections presented in Chapter 4.3.2. For section 2 and 3, which refers to the support sections of the slab, multiple effects such as shear force and bending moment were analyzed. However, for some cases these sections do not always match. As mentioned before, the shear force only needs to be controlled at a certain distance from the frame legs which results in that it is dependant on the thickness of the slab. When performing an analysis in a set-based design procedure, many different geometrical combinations occur, even some which may normally be disregarded right away. This creates design alternatives where the path for section 2 and 3 is located far towards the middle of the span. If the bending moment and shear force would be obtained from the same path in this design, the bending moment would result as less representative to the actual behaviour at the support. Therefore, the bending moment was always obtained with a distance of an element size from the frame legs, see Figure 4.16. The same argument applies to the sections in the top of the frame legs.



**Figure 4.16** Placement of transverse paths in BRIGADE Plus where effects are obtained.

In fatigue verification, the effects that need to be obtained always consist of a maximum and minimum force that will generate a stress range. Therefore, the script always needs to find and isolate the two load effects that creates the largest stress amplitude. As the purpose of this fatigue verification was to check the existing geometry and reinforcement in a design, the script utilized the existing arrangement assumptions to determine where to obtain the effects.

As described in Section 4.3.1.2, the deck slab is divided in three longitudinal strips in order to distribute the bending reinforcement, as presented in Figure 4.4. The width of each edge strip is 20% of the total width and is applied for both the plate and for the frame legs. The two edge strips contains the same reinforcement amounts and the middle strip is provided with a different amount depending on the design bending moment from the preliminary design. The effects were therefore obtained in each strip to be able to verify the reinforcement. In post-processing, the script filtered and determine in which strip the greatest difference between maximum and minimum was located and chose to proceed with the worst affected strip.

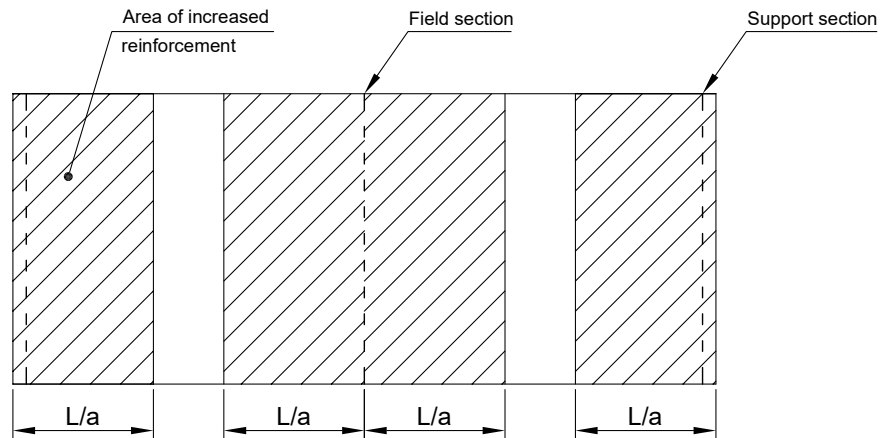
The bending reinforcement that is placed in the longitudinal direction is controlled in a similar manner, but the arrangement in this direction is designed directly from the design value of the bending moment that varies along the bridge. This generates different reinforcement amounts in many sections. Therefore, the reinforcement that was verified for fatigue consisted of the amount that was located at the specific critical section of the current bridge design.

Regarding the shear reinforcement, the frame legs are distributed and managed in a similar manner. However, for the bridge deck, the distribution of the shear reinforcement is based on the spread of the shear force which correlates to the concentrated loads in the FE-model. The shear force for the bridge deck was obtained as a single value for the maximum and minimum shear force, which corresponded to the largest amplitude along the transverse paths in the critical sections.

#### 4.4.4 Modifying reinforcement amounts

As the fatigue verification proved to be sensitive depending on stresses and minor changes to the sectional design, i.e. cross-sectional height and amount of reinforcement, the design tool was designed to either show fatigue damage or utilization ratio for every verification of the included materials. This creates the possibility to analyze options that are close to the fatigue limit and thereby identify alternatives which may become better options with small adjustments in reinforcement quantities. Therefore, the selection process will also include alternatives which exceeds the limit with 10%. By identifying in which verification the fatigue was not fulfilled, it was possible to modify the script in such a way that it provides the option to increase the reinforcement in the relevant part of the structure. As an example, a modification might consist of increasing the bottom transverse tensile reinforcement in the close proximity to the sections that fails. This close proximity was determined by an estimated distance which corresponded to  $1/a$  of the span length and reaches

across the entire width of the frame bridge, where the choice of  $a$  depends on what type of reinforcement should be increased and the dimensions of the structure. For the support section, the distance was estimated away from the frame legs into the slab structure. For the field section, this distance is added on each side of the studied section, see Figure 4.17. The increase of reinforcement was performed in steps of 2.5% until it fulfilled the requirement of the fatigue verification. The same principle was utilized for the reinforcement increase in the frame legs. However, this process was not automated in the script but was conducted for each studied case.



**Figure 4.17** Principle sketch of slab proportions which can be increased.

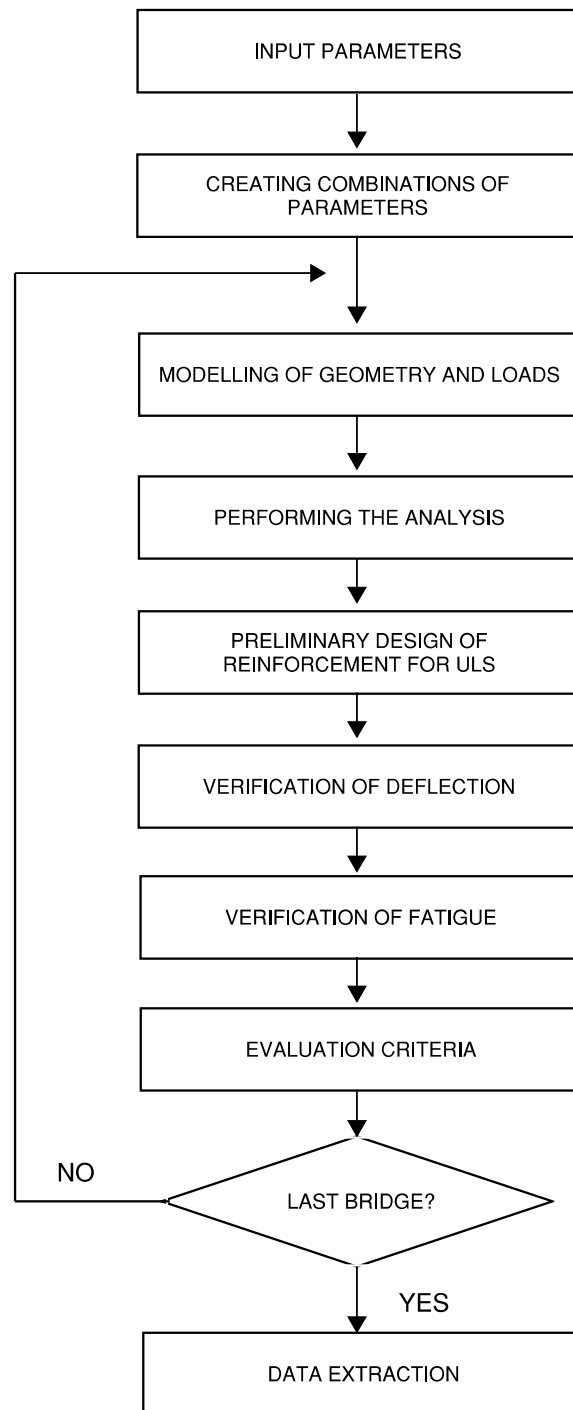
## 4.5 Script structure

In the set-based design method, the purpose is to generate many unique combinations of different parameters which sets the outline of each frame bridge. The script therefore takes parameters such as dimensions of the components in the structure, material properties, type of foundation, traffic situation, service life and combines them into a list of data which will be the basis for modelling and analysis.

The script was constructed by modules with separate assignments to keep the script easy to maintain and make additions. These modules were designed in such a way that they are managing one bridge at a time and then performing the same procedure for every desired combination. Figure 4.18 illustrates the foundation of the script.

Keeping the structure of the script based on modules also eased the transfer of variables and results between the separate modules within the script. This was especially helpful as the implementation of fatigue utilized the reinforcement which was designed in one module and verified for fatigue in another. This also proved to be an efficient way to achieve further control over the calculations and minimize the risk of hidden errors when handling large amounts of data.

Since the script is interacting with BRIGADE Plus, it should be stated that even if the commands are put in different modules, there is still possibility for interference problems. An example of when a problem of this kind could occur was when extracting XY-data from the FE-model in the post-processing. The code that executes this action utilizes a list of steps in BRIGADE Plus where they are sorted in an alphabetical order and the step that should be extracted is determined by a single number. If other modules create additional steps that are included in the same analysis, this number which corresponds to the position in this list changes and there is a substantial risk that the code is extracting and assigning the wrong data to a variable.



**Figure 4.18** Flow-chart over the script structure of the set-based design tool.

# 5

## Application of the design tool

### 5.1 Evaluation criteria

Part of this study is to investigate the influence of fatigue to a preliminary design process and how the evaluation and design choices made in early stages of larger projects could be affected. Using a parametric set-based design tool when establishing design alternatives for future projects could result in a simple procedure of distinguishing cost efficient and environmentally friendly options. This could result as beneficial for contractors or designers when submitting design solutions to costumers with ambitions to reduce the environmental impact of the project or material costs.

The evaluation criteria that is studied in this thesis and included in the design tool, consists of estimations of material cost and CO<sub>2</sub>-equivalents for each bridge design produced by the design tool. In order to make comparisons of when fatigue is implemented, the estimations of material cost per m<sup>3</sup> and CO<sub>2</sub>-equivalents are the same as when introduced in the master thesis to which this project originates from (Löfgren, 2020). The CO<sub>2</sub>-eq for both material was obtained from the environmental product declaration (EPD) documentations from Svenska Betong, 2017 and Celsa Steel Service AS, 2015 respectively, which includes the product stage in the LCA of the specific product. The quantification of costs and environmental impact is thereby as stated below:

#### Concrete C35/45

- Estimated cost: 1 985 [SEK/m<sup>3</sup>]
- Estimated CO<sub>2</sub>-eq: 388 [kg CO<sub>2</sub>-eq/m<sup>3</sup>]

#### Reinforcement steel

- Estimated cost: 14 400 [SEK/tn]
- Estimated CO<sub>2</sub>-eq: 370 [kg CO<sub>2</sub>-eq/tn]

## 5.2 Case study

In order to investigate the utilization, benefits of a set-based design method and the aspect of being able to design multiple bridges autonomously, the ongoing project North Bothnia Line was chosen as a reference project where multiple and similar bridges are planned within the same project. The North Bothnia Line consist of a 270km railway connection in the northern part of Sweden and includes over 100 crossings (Swedish Transport Administration, n.d.). This type of project was therefore considered as an inspiration to the idea of being able to utilize a set-based design method, to produce several suitable frame bridge designs for a project where a larger number of similar designs within the project might be needed.

Another aspect which was introduced by the master's thesis on which this work originates from, is the idea of creating sets of standardized frame bridge designs suitable for multiple situations. This could be of great interest in projects like the North Bothnia Line as multiple standardized bridge designs could serve as cost efficient and thereby desirable proposals for tenders in procurement (Löfgren, 2020).

## 5.3 Parametric study

A parametric study was conducted based on a set of input data for potential bridges in the project described in the case study. The results from the design tool was summarized to a large set of data for all of the bridges that were analyzed. Based on this large set of data, parameters could then be isolated for further evaluation and comparison. Also, the influence of when fatigue is implemented to a preliminary design while creating similar and standardized bridge designs was investigated. The results from the parametric investigations are presented in Chapter 6.2.

### 5.3.1 Parameters of interest in fatigue assessment

A parameter which is included in the calculations for the fatigue assessment is the established "Annual daily traffic" (ADT) amount for a specific bridge. The ADT describes the expected yearly traffic amount per day and is chosen based on the expected amount of traffic at the location of the bridge whereas the established traffic category then is determined based on additional regulations from the national authorities (Swedish Transport Administration, 2020). The traffic category is governed by the ADT accordingly:

- Traffic category 1 should be applied if  $6000 < \text{ADT} \leq 24000$
- Traffic category 2 should be applied if  $1500 < \text{ADT} \leq 6000$
- Traffic category 3 should be applied if  $600 < \text{ADT} \leq 1500$
- Traffic category 4 should be applied if  $\text{ADT} \leq 600$

The determined traffic category governs the values for expected passing lorries per year according to Table 5.1 (SIS, 2010b). The value of  $N_{obs}$  is then applied in the calculations for estimating the fatigue capacity.

**Table 5.1:** Number of expected passing lorries per year.

| Traffic category | $N_{obs}$     |
|------------------|---------------|
| 1                | $2e^{+6}$     |
| 2                | $0.5e^{+6}$   |
| 3                | $0.125e^{+6}$ |
| 4                | $0.05e^{+6}$  |

The design life for a structure is decided based on regulations from the national authorities and the responsible designer. It constitutes an important parameter for estimating fatigue capacity in a new bridge design whereas the chosen design life affects the expected loading cycles to the structure.

The design life is decided based on the following (Swedish Transport Administration, 2020):

- If a construction is unable to be repaired or replaced without disrupting nearby railway traffic the design life should be chosen as at least 120 years.
- If a construction is able to be repaired or replaced without disrupting nearby railway traffic the design life should be chosen as at least 40 years.
- For other constructions the design life should be chosen as at least 80 years.

Additionally, a service life of 150 years is added to be able to determine effects from the fatigue in a case that is not a regulated design situation.

## 5.4 Set-based design method

As mentioned in Chapter 3.1, the purpose of the set-based design method is to produce a large number of different design alternatives and narrowing it down to a suitable design. This section presents the prerequisites and input data that were analyzed by the design tool and describes how the set-based design tool is utilized.

### 5.4.1 Set-based design selection procedure

To demonstrate how the tool could be used with the procedure of eliminating alternatives regarding fatigue and deflection, a data-set with a specified annual daily traffic amount and design life, but with several unique combinations of geometries, was created for two predetermined span lengths. The parameters used for the analysis are presented in Table 5.2. In this example the height of the frame legs, the size and type of foundation was set to the same for both sides of the bridge. The intention of choosing these two span lengths was to be able to explore different behaviour in optimization based on deflection and fatigue.

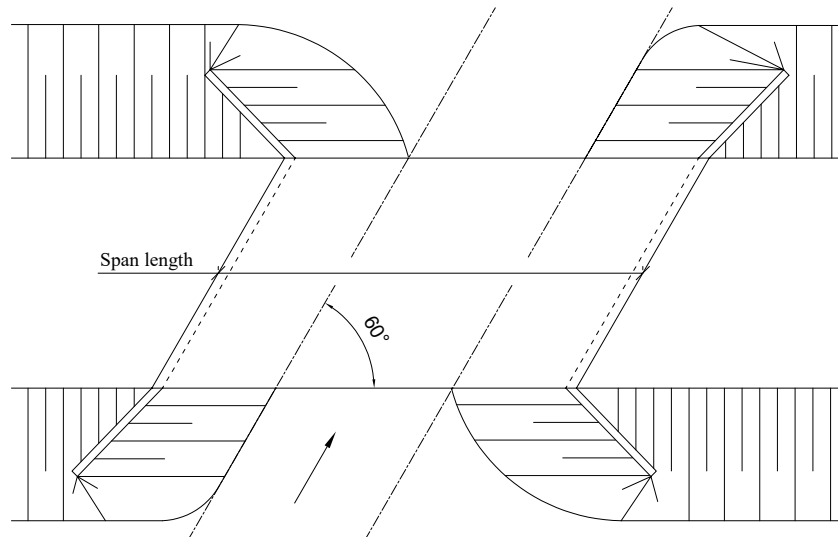
**Table 5.2:** Design parameters used in the analysis to showcase an example of a set-based design procedure.

| Design parameter     | Variation of alternatives | Unit  |
|----------------------|---------------------------|-------|
| Span length          | 8.0, 14.0                 | m     |
| Width                | 7.5                       | m     |
| Thickness slab       | 0.2 - 1.4                 | m     |
| Height frame legs    | 6.0                       | m     |
| Thickness frame legs | 0.2 - 1.0                 | m     |
| Skewed angle         | 60, 70, 80, 90            | °     |
| Foundation           | Slab on ground            | -     |
| Traffic situation    | Traffic category 1        | -     |
| Design life          | 120                       | years |

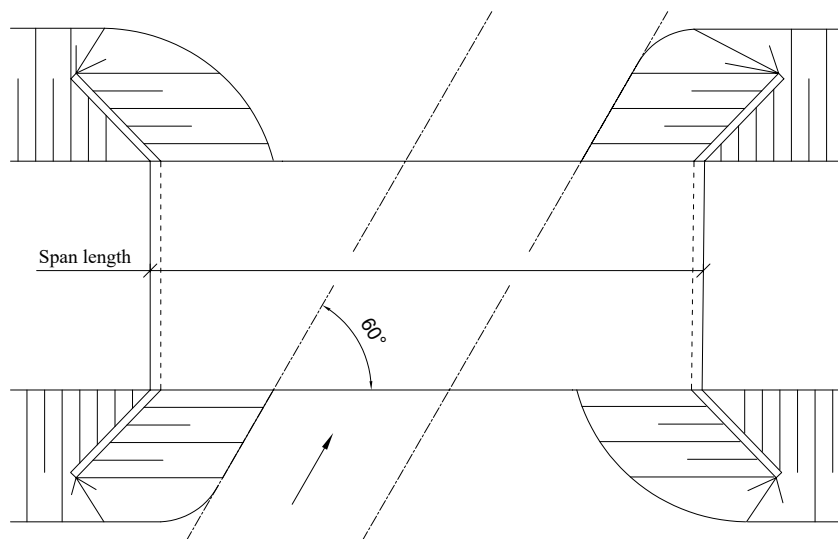
### 5.4.2 Applying set-based design method to evaluate different design alternatives

Another way of utilizing the tool is to find and evaluate potential standardized solution through quick comparison of the evaluation criteria, when producing a set of frame bridges with their specific geometries or by performing the design with fewer unique solutions for the same prerequisite.

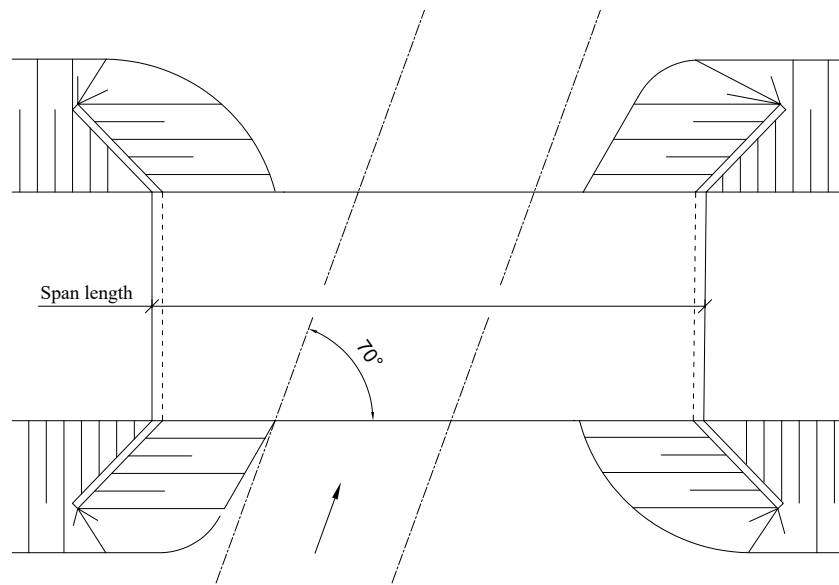
According to Chapter 3.1, Vägverket, 1996 proposes that there may be benefits to increase the span length of the frame bridge to reach the required alignment to the traffic line that connects under the bridge instead of skewing the angle of the bridge, see Figure 5.1 - 5.3 for an example.



**Figure 5.1** Principle sketch of a skewed frame bridge with an alignment of 60° (Vägverket, 1996).



**Figure 5.2** Principle sketch of a perpendicular frame bridge which is adjusted according to an alignment of 60° (Vägverket, 1996).



**Figure 5.3** Principle sketch of the same frame bridge that is adjusted for 60° but which also fulfills alignment for 70°.

To investigate the potential differences in cost and environmental impact in performing the above mentioned design situations, a data-set containing the minimum required increase in span length to allow the same alignment underneath the skewed frame bridge was created for. Because the skewed angle already exist in the previous data-set, two span lengths, 8.0 and 14.0 m was utilized in this analysis. The perpendicular angled span lengths is calculated by adding the length of the adjacent side of the skewed width of the bridge. This produced a number of new span lengths correlated to each skewed angle of 60°, 70° and 80°, see Table 5.3 where the lengths were rounded up to even half-meter lengths. Table 5.4 shows the final parameters in this data-set.

**Table 5.3:** Table showing the alternative design for skewed angled bridges if they were designed as perpendicular.

| Skewed design<br>[Span length, angle] | Perpendicular counterpart<br>[Span length, angle] |
|---------------------------------------|---|
| 8 meter, 60°                          | 12.5 meter, 90°                                   |
| 8 meter, 70°                          | 11.0 meter, 90°                                   |
| 8 meter, 80°                          | 9.5 meter, 90°                                    |
| 14 meter, 60°                         | 18.5 meter, 90°                                   |
| 14 meter, 70°                         | 17.0 meter, 90°                                   |
| 14 meter, 80°                         | 15.5 meter, 90°                                   |

**Table 5.4:** Design parameters for perpendicular skewed frame bridges used in the analysis to compare evaluation criteria with skewed angled frame bridges.

| Design parameter     | Variation of alternatives         | Unit  |
|----------------------|-----------------------------------|-------|
| Span length          | 9.5, 11.0, 12.5, 15.5, 17.0, 18.5 | m     |
| Width                | 7.5                               | m     |
| Thickness slab       | 0.2 - 1.4                         | m     |
| Height frame legs    | 6.0                               | m     |
| Thickness frame legs | 0.2 - 1.0                         | m     |
| Skewed angle         | 90                                | °     |
| Foundation           | Slab on ground                    | -     |
| Traffic situation    | Traffic category 1                | -     |
| Design life          | 120                               | years |



# 6

## Results and Discussion

Results presented in this chapter are generated from running multiple bridge designs through the developed set-based design tool. Several bridges designed by the design tool were controlled by comparing sectional properties with the commercial software Concrete Section 6.5 (StruSoft, 2021) and analytical calculations based on Eurocode in order to validate and ensure the scripts credibility. Examples of analytical comparisons and calculations procedures are presented in the Appendix A-B. Additionally, discussions regarding the results are presented in this chapter.

### 6.1 Executing the script

The script for this project was executed within BRIGADE Plus (Scanscot Technology AB, 2018) with the graphical interface running. The analysis were conducted on a personal computer with an Intel i7 processor which resulted in that the computational time for each analysis including the fatigue verification was around 90 seconds. When only running the part of the script that generates the actual model and preliminary design, the run-time was around 45 seconds. This approximate run-time remained throughout the entire process. This results in the possibility to perform around 50 bridge analysis per hour, which approximately can be expressed in 950 analysis in a day. However, the computational time varied some depending on the parameters and as the bridge was wider, the amount of computational steps increased because of an increased amount of load placements.

Depending on what was of interest, the script was executed with different parameters and intentions. The data-set that was the basis of the parametric study focused mainly on the fatigue parameters; annual daily traffic and design life. This generated 480 unique combinations of geometries, which were combined with the fatigue parameters and thereby produced a total of around 7700 unique combinations of parameters. For the data-set which was used to study the set-based design method, focus was instead set to different geometries which generated around 2000 unique bridge designs.

Combining the data sets for the parametric study and the set-based analysis, a compiled data-set of almost 10 000 combinations of design parameters for these frame bridges took around a week of idle work to produce.

## **6.2 Parametric study**

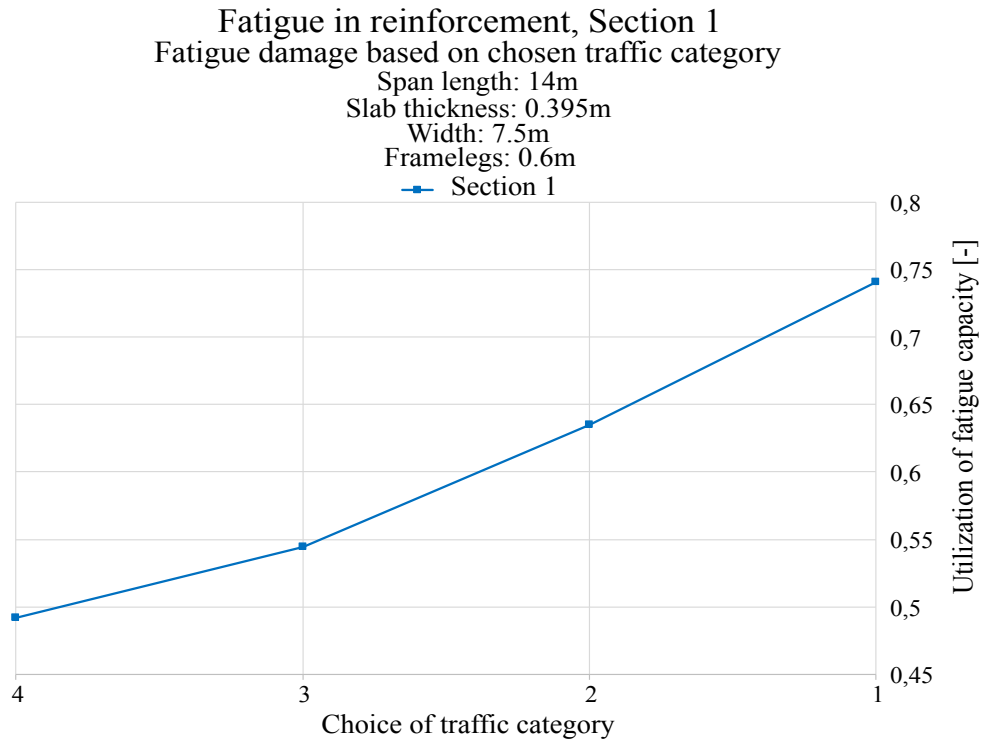
In the parametric study, multiple parameters were analyzed in order to conclude the response to fatigue in frame bridges and identify parameters which could prove to be of larger significance to the results. The results from the parametric analysis refer to different choice of traffic category which is dependent on the expected traffic amounts, as explained in Section 5.3.1. Additionally, results based on chosen design life, also described in Section 5.3.1, are presented in this section to describe differences and similarities to the choice of traffic category. The denoted critical section numbering is based on the explanation in Figure 4.9.

### **6.2.1 Daily traffic amounts and design life**

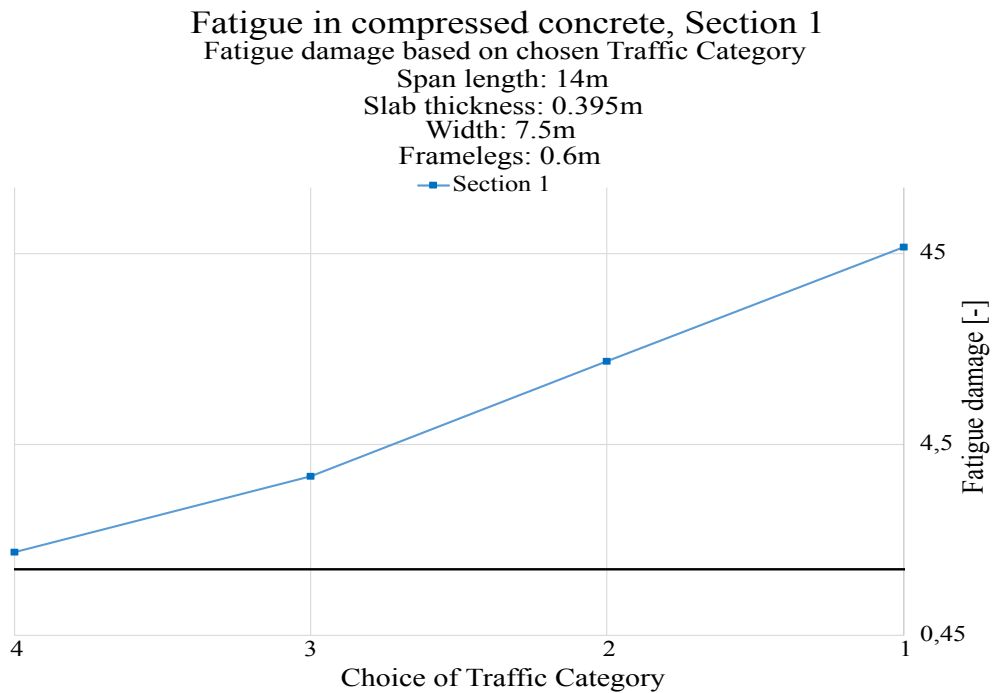
In this section, the influence of the chosen annual daily traffic (ADT) and design life is presented for several bridge designs in different critical sections of the generated frame bridge designs.

#### **6.2.1.1 Influence of traffic category and design life**

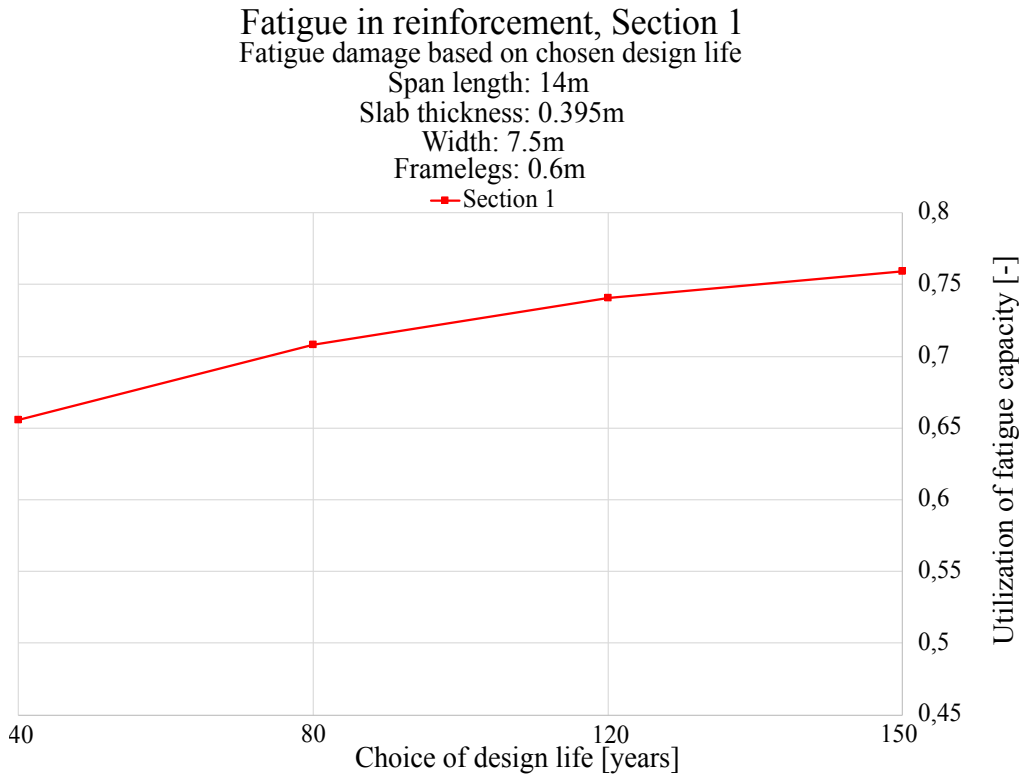
This section describes how fatigue damage in the reinforcement and the compressed concrete is influenced by the different traffic category and design life. Figure 6.1-6.4 show the fatigue damage obtained in the midsection of the carriageway slab for various choice of traffic category and design life. The figures refer to results obtained in the longitudinal direction in a frame bridge with a span length of 14 m with a slab thickness and preliminary reinforcement design that satisfy ULS and deflection requirements. Failure due to fatigue occur if the utilization or fatigue damage exceeds 1.0.



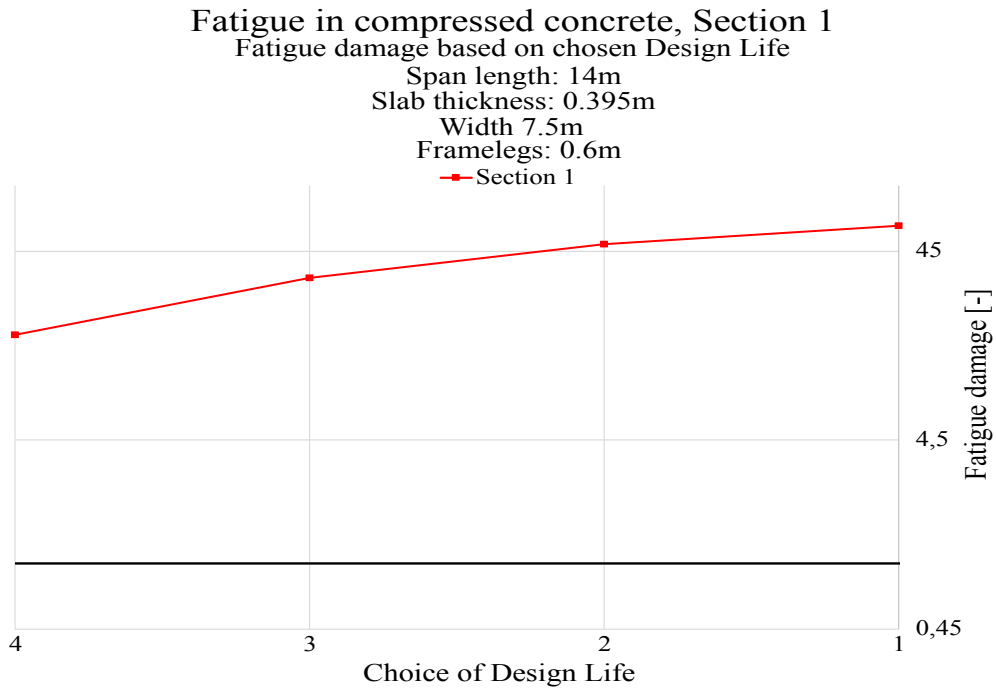
**Figure 6.1** Fatigue damage in the longitudinal tensile reinforcement with respect to the choice of traffic category for a chosen bridge design.



**Figure 6.2** Fatigue damage in compressed concrete with respect to the choice of traffic category for a chosen bridge design.



**Figure 6.3** Fatigue damage in the longitudinal tensile reinforcement with respect to the choice of design life for a chosen bridge design.



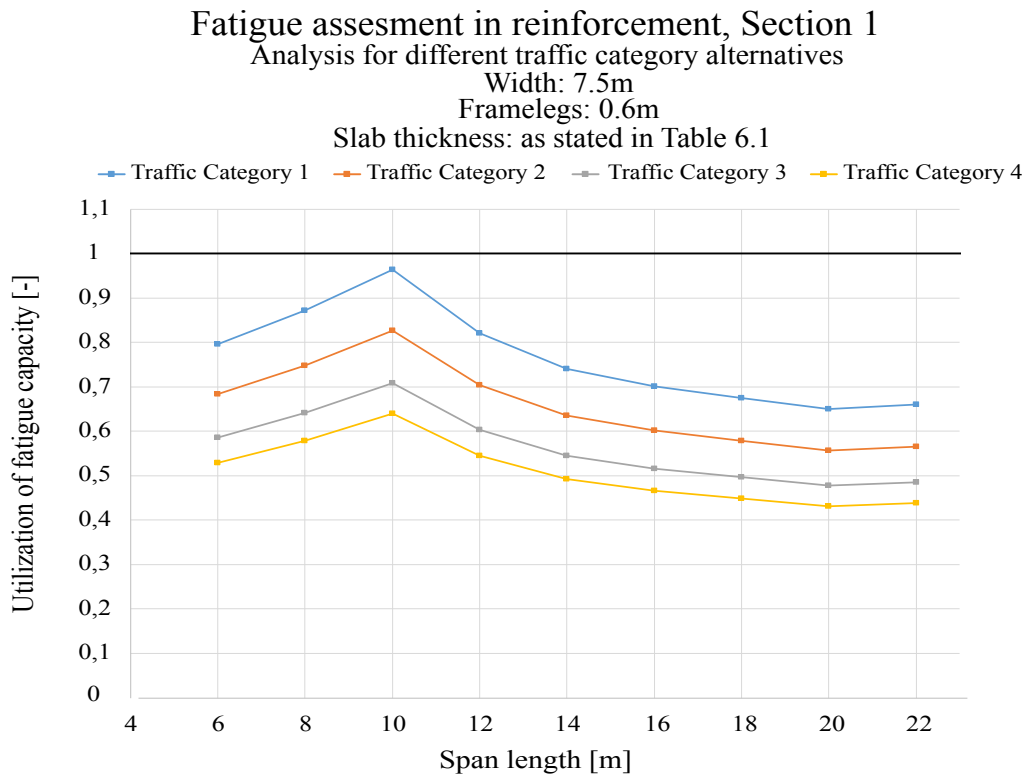
**Figure 6.4** Fatigue damage in compressed concrete with respect to the choice of design life for a chosen bridge design.

### 6.2.1.2 Results of varying traffic category and design life in a set-based design set

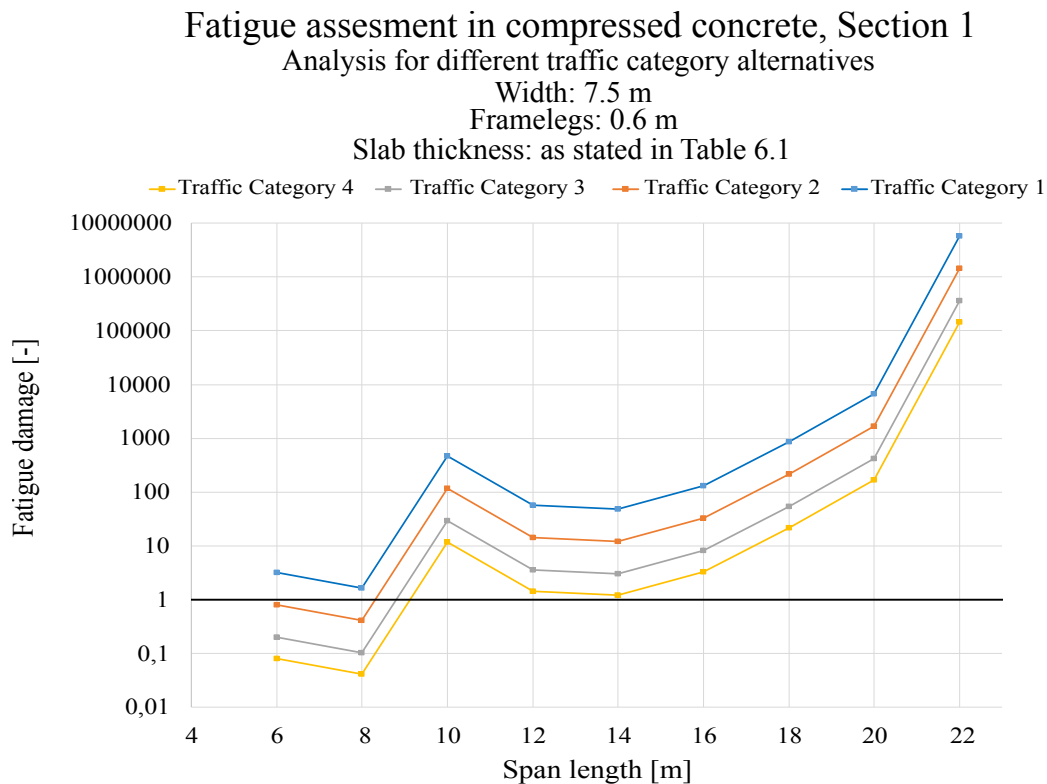
In this section, results from a set-based design procedure for different choices of traffic category and design life are presented for multiple bridge designs with varying span lengths. The design of the slabs, from which the results were obtained, were designed according to ULS and their deflection in comparison to the deflection limits, stated by the national authorities and as implemented in the original script version by (Löfgren, 2020). The slab thicknesses for the studied span lengths are presented in Table 6.1

**Table 6.1:** Table showing the slab thickness for each studied span length in the parametric study. Each slab thickness fulfills the deflection requirement and has a preliminary reinforcement design according to ULS. Additional dimensions for the bridge design are stated in each corresponding figure.

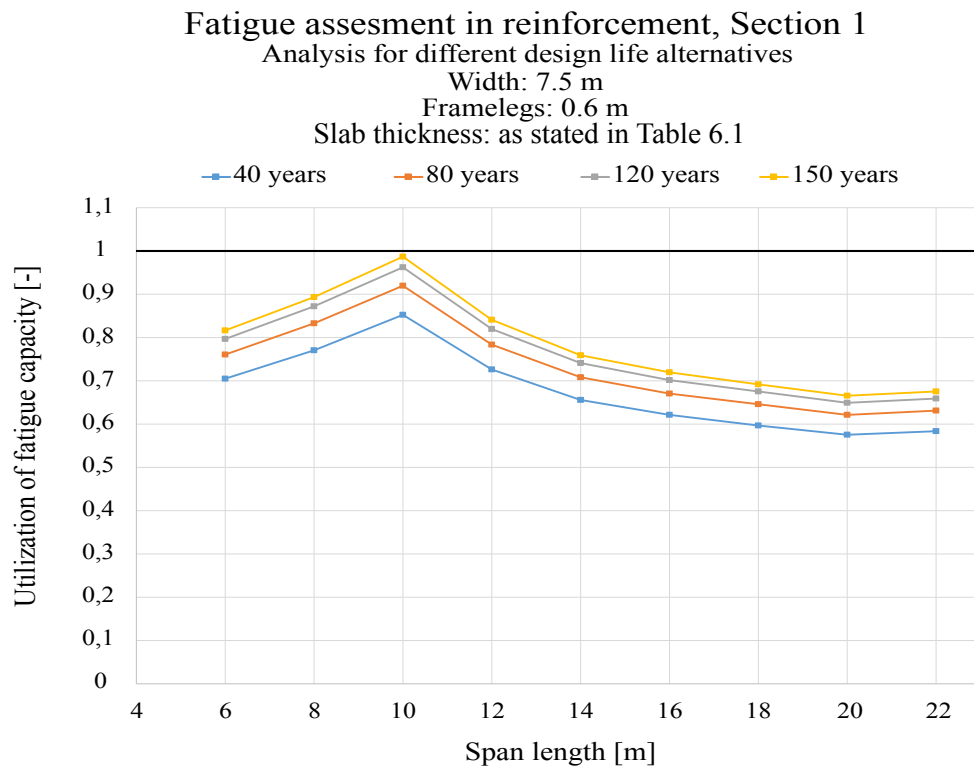
| Span length [m] | Slab thickness [m] |
|-----------------|--------------------|
| 6               | 0.22               |
| 8               | 0.265              |
| 10              | 0.31               |
| 12              | 0.355              |
| 14              | 0.395              |
| 16              | 0.435              |
| 18              | 0.47               |
| 20              | 0.51               |
| 22              | 0.55               |



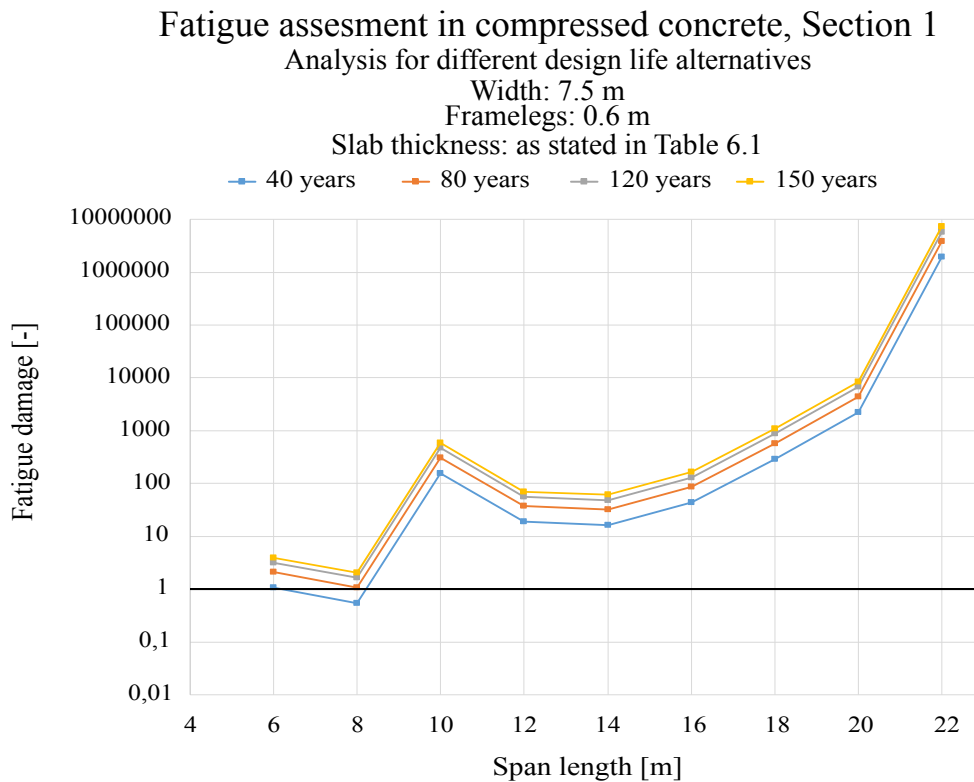
**Figure 6.5** Fatigue in the longitudinal tensile reinforcement for bridges with different span lengths and traffic category.



**Figure 6.6** Fatigue in compressed concrete for bridges with different span lengths and traffic category.



**Figure 6.7** Fatigue in the longitudinal tensile reinforcement for bridges with different span lengths and design life.



**Figure 6.8** Fatigue in compressed concrete for bridges with different span lengths and design life.

### 6.2.1.3 Analysis of results for varying traffic category and design life in a set-based design set

When studying the extracted result, it was possible to distinguish a difference in behaviour between the choice of traffic category and design life. As seen in Figure 6.1 and 6.2, the increase of fatigue damage between the choice of traffic category seems to be larger in comparison to the different choice of design life as in Figure 6.3 and 6.4.

When comparing with a full set of bridge designs with varying span lengths, see Figure 6.5-6.8, the response to the different choice of traffic category or design life remains similar for all studied span lengths, whereas the change in fatigue damage seems to be larger for varying traffic categories than for different choice of design life. Additionally, the increase of fatigue damage due to different traffic categories is larger for traffic situations which correspond to traffic category 2 and 1. On the contrary, the fatigue damage between the different choice of design life seems to reduce with a higher choice of design life.

Looking closer to the behaviour of the tensile reinforcement for different span lengths, see Figure 6.5 and 6.6, it can be observed that the utilization of the fatigue capacity has a peak at a 10 m span length. An explanation to this can be correlated to the Lambda-method where the factor which takes the type of structure into account is, according to EN1992-2 Annex NN, increasing for spans shorter than 10 m. In combination with the adjustment factor, which is interpolated from 0.9 to 1.0 for span lengths up to 10 m according to Swedish Transport Administration, 2020, these two factors may cause the slightly lower utilization of fatigue for the shorter spans. However, because the designs are based on the deflection requirement, this peak may also be explained by the amount of tensile reinforcement from the preliminary design in relation to the generated stresses in the studied section. Additionally, the placement of the fatigue loads may have an impact to the behaviour. Similarly to the tensile reinforcement, the compressed concrete also experience a local peak around 10 m, which could be explained by similar arguments regarding reinforcement amounts and optimized thickness with respect to the deflection limits. Additionally, an increasing fatigue damage was observed for the compressed concrete for longer span lengths, see Figure 6.7-6.8.

The estimation of the fatigue capacity utilizes the Navier's formula in order to calculate the stresses based on the tensile reinforcement amount, normal force and bending moment. As the FE-model could include deviations while calculating the load effects and that the stress calculations were conducted by an automated iterative procedure, the utilization of Navier's formula ended up to be rather sensitive to these deviations which affected the stresses and thereby the results. In the combination with the verification of fatigue in compressed concrete, which is calculated in a logarithmic scale, values for the fatigue damage could vary to a large degree even for minor changes. Therefore, the shifting magnitudes of the resulting fatigue damage for the compressed concrete may be explained by a combination of the logarithmic scale and the sensitivity of the automated stress calculations.

## 6.2.2 Sectional comparisons

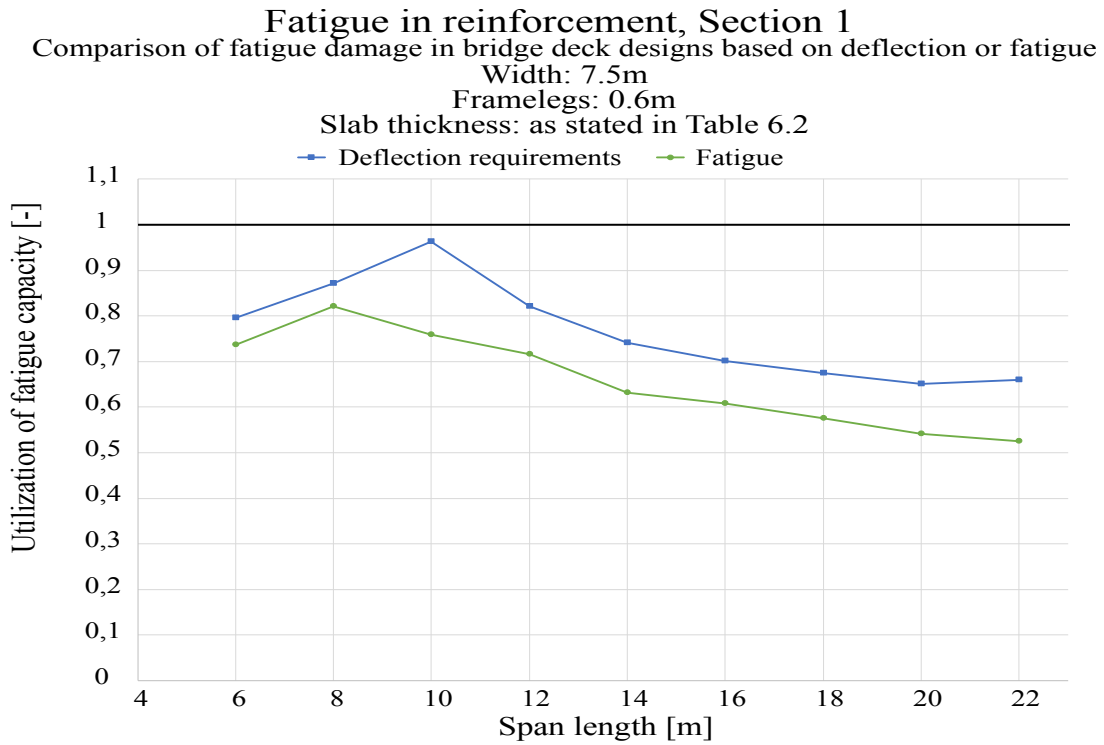
This section presents the comparison of sectional dimensions for generated bridge designs satisfying ULS and that are verified based on deflection and fatigue limits in the bridge deck, respectively. The studied bridge designs were chosen as designs with a bridge deck that just fulfilled the deflection limits and were then compared to a bridge deck design that fulfilled the fatigue verification. For simplicity, the frame legs were ignored in this part and prescribed with a thickness of 0.6 m for all bridges. The traffic category and design life for this comparison are set to Category 1 and 120 years to be able to obtain comparative results.

### 6.2.2.1 Results for fatigue in preliminary bridge designs

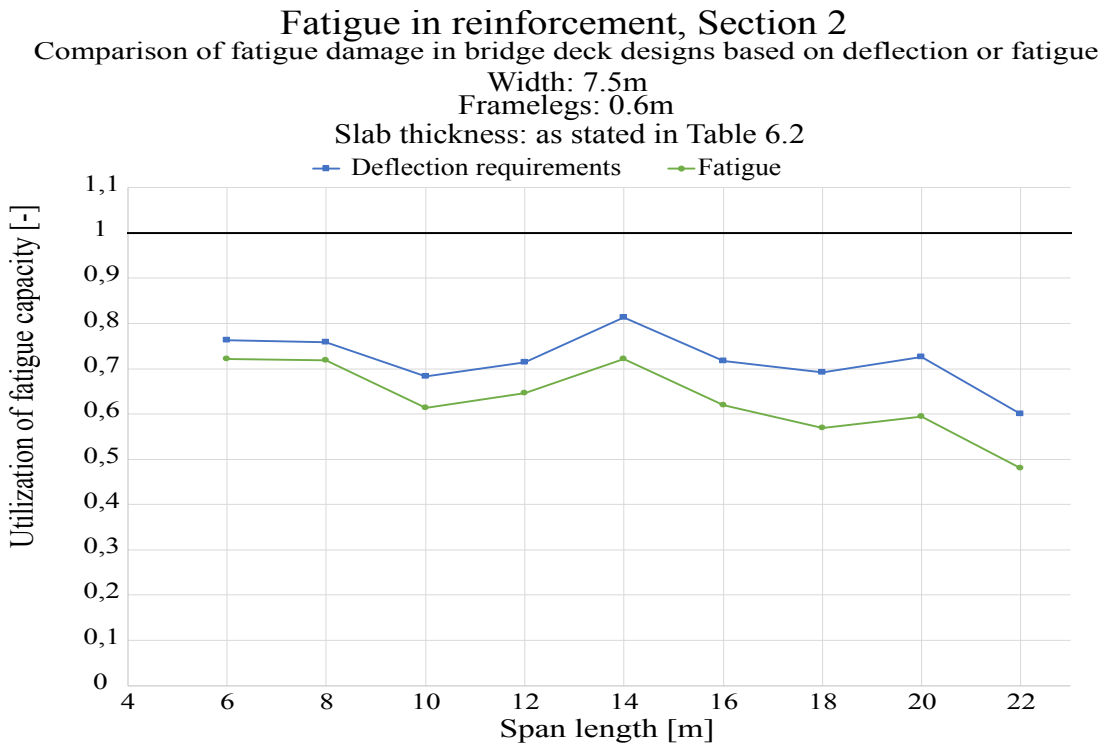
This section presents the resulting fatigue damage for preliminary bridge designs according to ULS, where the deflection and fatigue limits in the bridge deck are just fulfilled. For bridge deck designs, where the fatigue capacity was exceeded, the slab thicknesses were adjusted in order to fulfill the fatigue limit in the the worst affected section in the slab, which was for the support section with respect to fatigue in compressed concrete. The bridge deck dimensions for when based on ULS and verified to deflection or fatigue limits, are presented in Table 6.2. The resulting fatigue damage in the longitudinal reinforcement and the concrete, for both when the bridge deck designs are adjusted with respect to fatigue or if only based on deflection limitations, are shown in Figure 6.9-6.12 for the various critical sections.

**Table 6.2:** Table showing the corresponding slab thickness for each studied span length in this section for when the bridge deck design is based on deflection and fatigue, respectively.

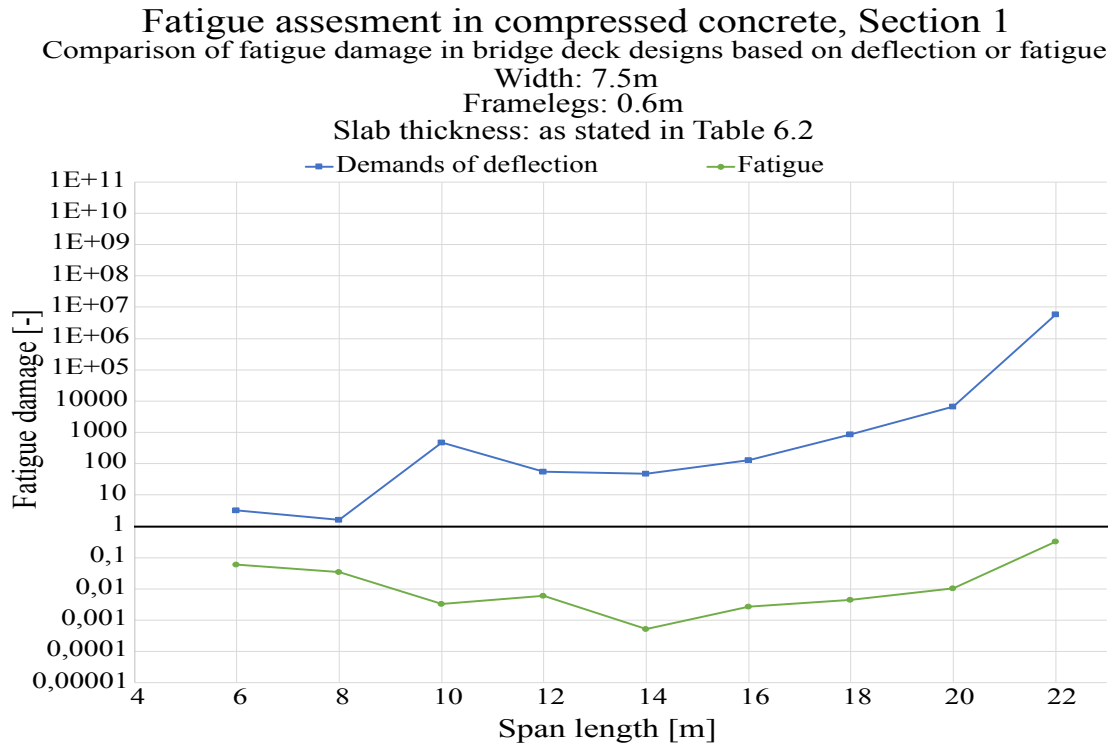
| Span length [m] | Slab thickness based on ULS and deflection [m] | Slab thickness based on ULS and fatigue limits [m] |
|-----------------|--|--|
| 6               | 0.22   | 0.25   |
| 8               | 0.265  | 0.3  |
| 10              | 0.31   | 0.38   |
| 12              | 0.355  | 0.45   |
| 14              | 0.395  | 0.54   |
| 16              | 0.435  | 0.58   |
| 18              | 0.47   | 0.65   |
| 20              | 0.51   | 0.73   |
| 22              | 0.55   | 0.76   |



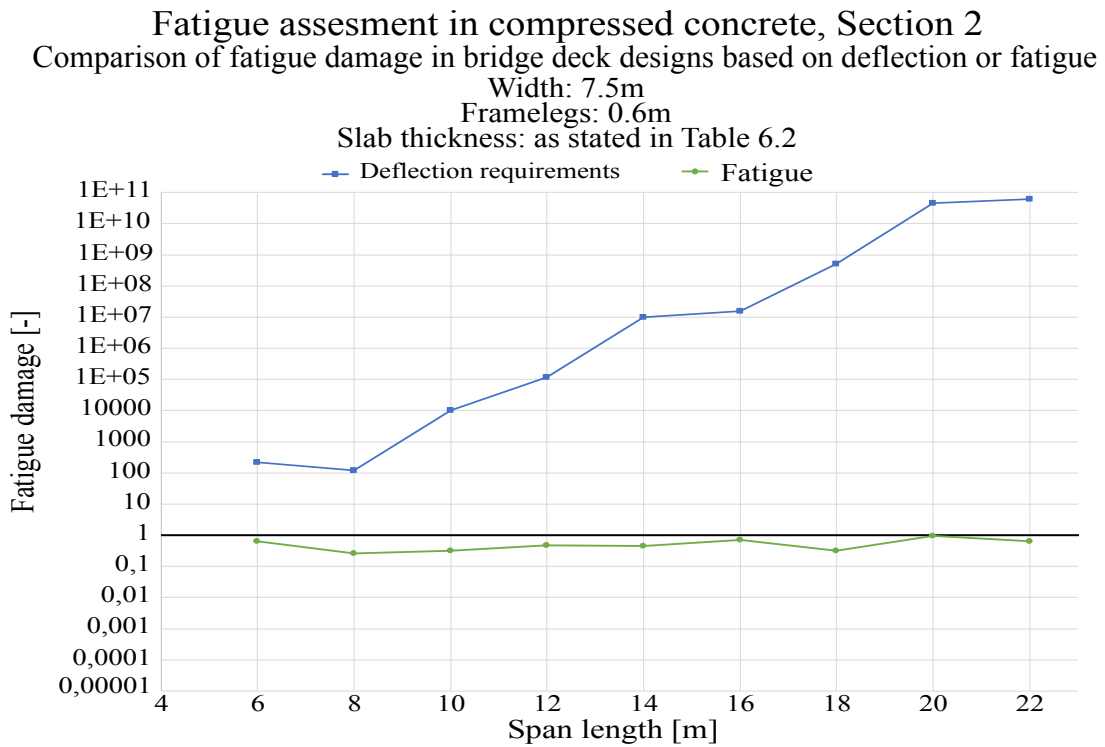
**Figure 6.9** Fatigue damage to the longitudinal tensile reinforcement, in the midsection of bridges with bridge deck dimensions according to ULS and verified for deflection limits and fatigue, respectively.



**Figure 6.10** Fatigue damage to the longitudinal tensile reinforcement, in the support section of bridges with bridge deck dimensions according to ULS and verified for deflection limits and fatigue, respectively.



**Figure 6.11** Fatigue damage in the compressed concrete, in the midsection of bridges with bridge deck dimensions according to ULS and verified for deflection limits and fatigue, respectively.

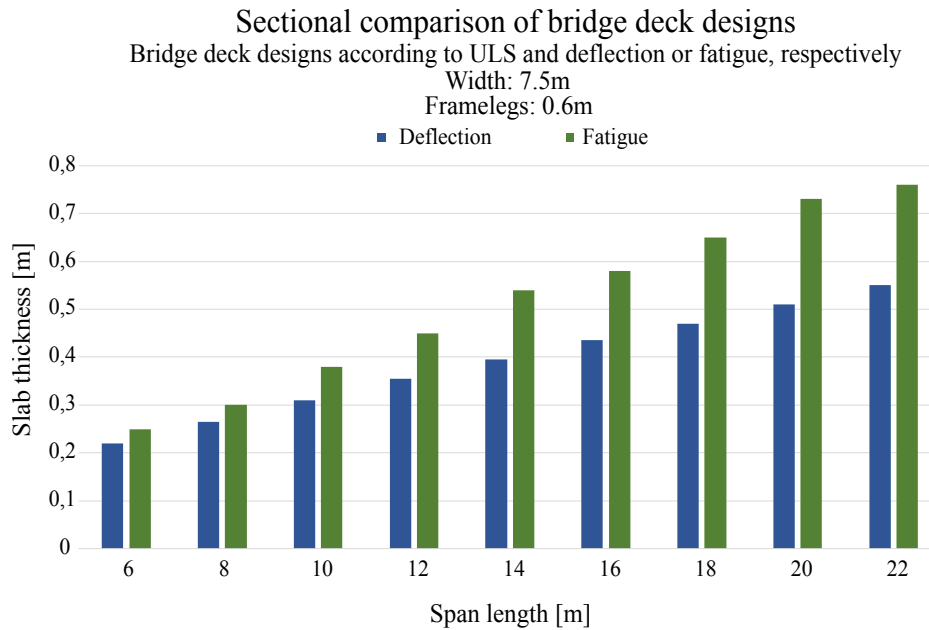


**Figure 6.12** Fatigue damage in the compressed concrete, in the support section of bridges with bridge deck dimensions according to ULS and verified for deflection limits and fatigue, respectively.

The modified bridge deck dimensions and their corresponding utilization ratios of the deflection limits are presented in Table 6.3, for when the bridge deck designs satisfy ULS and are verified for fatigue in the most critical section in the carriageway slab. This was done to describe the influence of the fatigue verification to the resulting deflections, in comparison to if the bridge deck only was verified to the deflection requirements. The resulting slab thicknesses, adjusted to fulfill the fatigue verification, can also be observed in Figure 6.13 in a comparison to if only the deflection limits were considered.

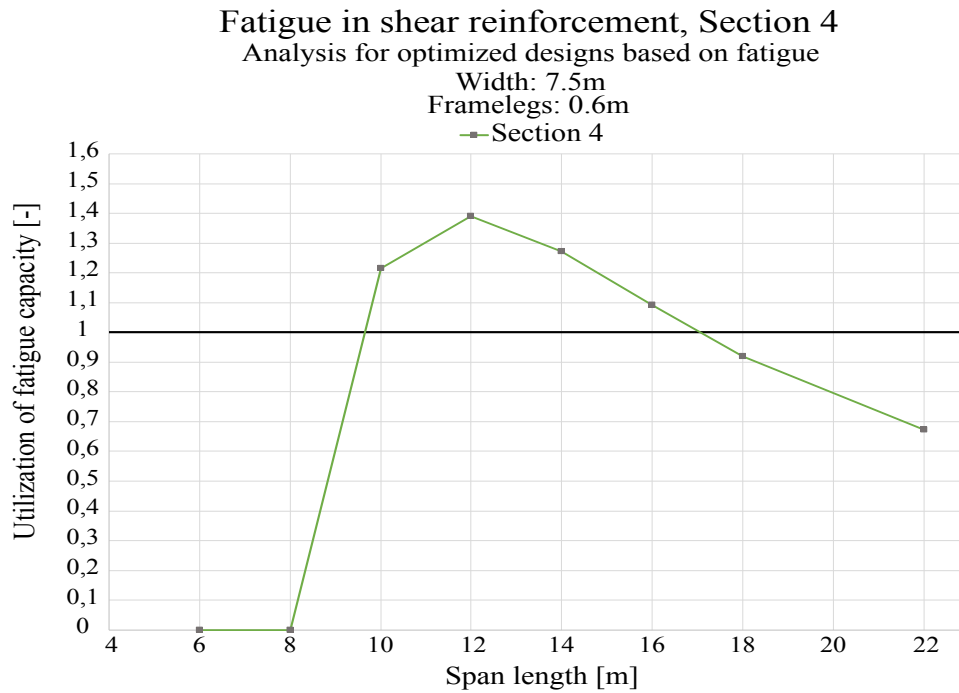
**Table 6.3:** Table showing the final utilization of the deflection limits for when bridge deck designs are based on fatigue requirements.

| Span length [m] | Slab thickness [m] | Deflection [m] | Deflection limits [m] | Utilization ratio of deflection limits [%] |
|-----------------|--------------------|----------------|-----------------------|--|
| 6               | 0.25               | 0.01005        | 0.015                 | 67.0                                       |
| 8               | 0.3                | 0.01387        | 0.02                  | 69.4                                       |
| 10              | 0.38               | 0.0147         | 0.025                 | 58.8                                       |
| 12              | 0.45               | 0.01612        | 0.03                  | 53.7                                       |
| 14              | 0.54               | 0.01628        | 0.035                 | 46.5                                       |
| 16              | 0.58               | 0.01976        | 0.04                  | 49.4                                       |
| 18              | 0.65               | 0.02107        | 0.045                 | 46.8                                       |
| 20              | 0.73               | 0.02186        | 0.05                  | 43.7                                       |
| 22              | 0.76               | 0.02662        | 0.055                 | 48.4                                       |



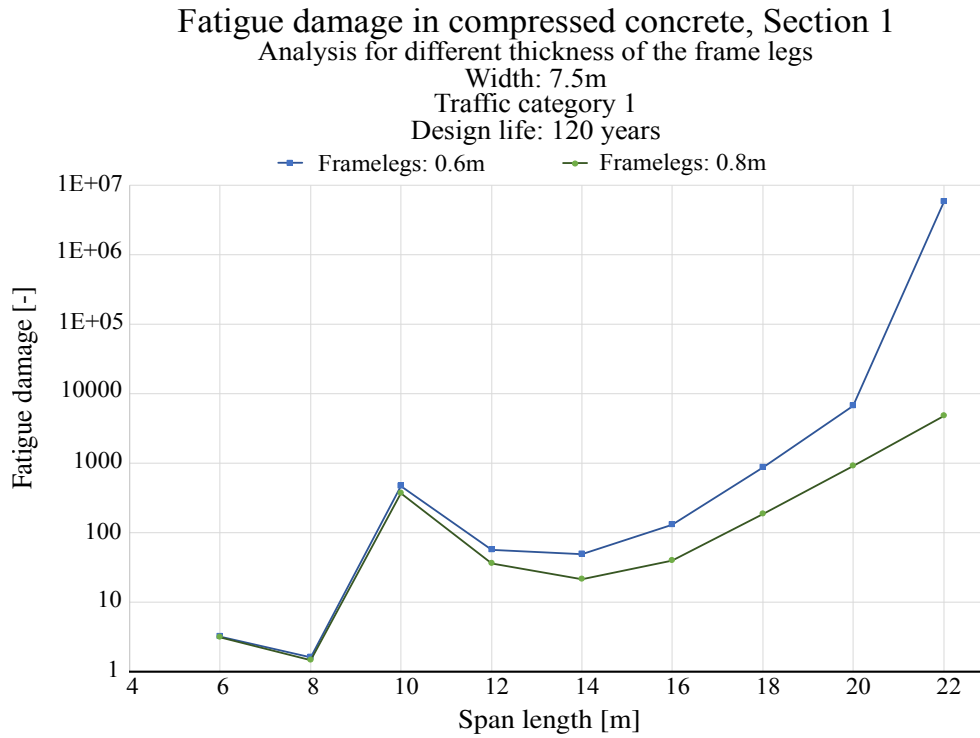
**Figure 6.13** Comparison of required slab thickness for bridge deck designed according to ULS and verified for deflection limits and fatigue, respectively.

Along with the adjusted slab thickness presented in Table 6.3, the shear reinforcement in the frame legs is influenced as in Figure 6.14 where the fatigue damage is presented for a section in the top of the frame leg. The studied bridge designs satisfies ULS and the bridge decks are verified for both deflection and fatigue.



**Figure 6.14** Fatigue damage in the frame legs shear reinforcement for when a bridge design, according to ULS and deflection, is verified for fatigue in the slab.

Figure 6.15 presents the fatigue damage for the compressed concrete in the midsection of the bridge deck and how the fatigue damage is influenced by an increased frame leg thickness. The studied bridge designs have the dimensions required to satisfy ULS and deflection requirements and presents how the different frame leg thickness influences the fatigue damage in the bridge deck.



**Figure 6.15** Fatigue in compressed concrete in the midsection of the slab for different thickness of the frame legs.

### 6.2.2.2 Analysis of fatigue in preliminary bridge designs

In the results for the preliminary bridge designs it was noticeable that the fatigue limit was exceeded for the compressed concrete and not for the reinforcement, see Figure 6.9-6.10 and Figure 6.12. This can be explained by the relatively low slab thickness required to fulfill the demands of deflection for each bridge design. This will lead to larger stresses in the carriageway slab which generates higher fatigue damage in the concrete. Therefore, the need for larger dimensions, compared to the ones obtained based on the deflection limits in the script, was therefore necessary in order to fulfill the requirements for fatigue in the compressed concrete.

As mentioned, it was possible to observe that the fatigue in the reinforcement rarely exceeded the fatigue limit and thereby can be interpreted as seldom governing in the slab of perpendicular bridge designs. When studying the adjusted bridge deck designs, the overall fatigue damage in the reinforcement is reduced as seen in Figure 6.9 and Figure 6.10. This is reasonable as the amount of reinforcement is increased along with the slab thickness. This leads to lower stresses in the reinforcements which contributes to a lower fatigue damage. Observing the sections in the slab, the behaviour of the tensile reinforcement is quite different when the sections are compared to each other. The fatigue damage in the support section follows a general descending trend against the increased span lengths, see Figure 6.10, which corresponds to a similar behaviour as in midsection of the slab. However, as the span length exceeds 14 m, another axle load from the load model is applied on the carriageway which might explain the peak in fatigue damage that can be seen in Figure 6.10.

The comparison of the fatigue damage in concrete subjected to compression for different frame leg thickness proved to be especially interesting as a thicker frame leg resulted in a lowered fatigue damage for longer span lengths. The comparison presented in Figure 6.15 implies that thicker frame legs enables a bridge design to have a thinner slab thickness. Furthermore, this supports that an inclusion of the bridges every component would also increase the quality in the optimization with the set-based design tool.

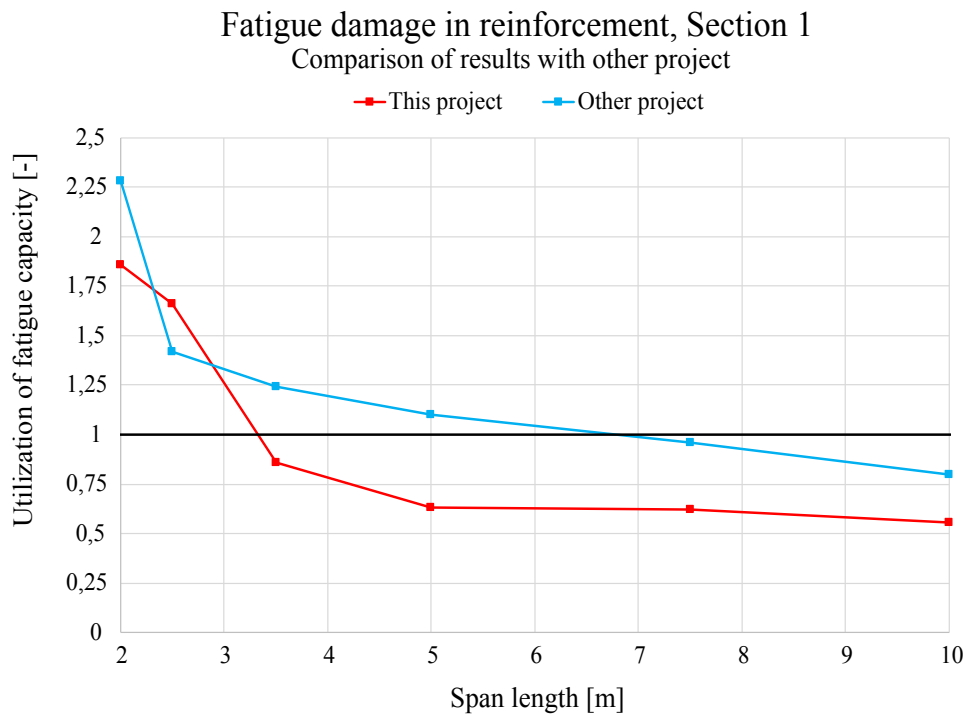
As seen in Figure 6.14, the fatigue limits were exceeded in verification of the shear reinforcement in the frame legs for bridge designs with span lengths between 10 m and 18 m. This suggests that in order to optimize a bridge design, all of its components and multiple parameters need to be evaluated and compared in order to fulfill the satisfactory requirement for fatigue. This is further described in section 6.3 regarding set-based design selection.

### 6.2.2.3 Comparison to previous research

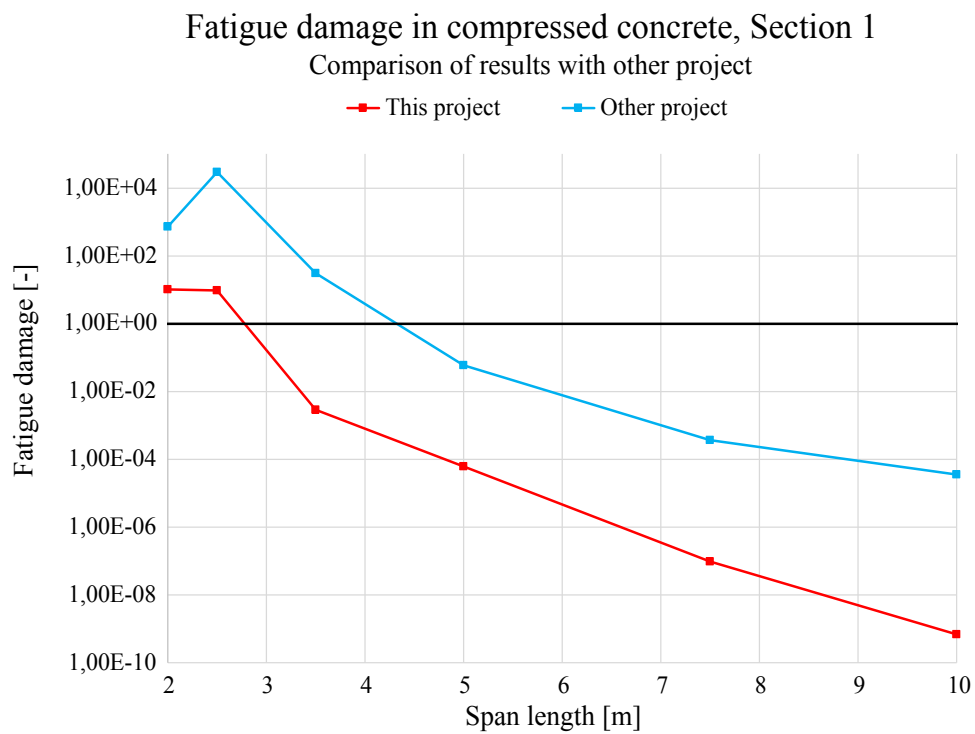
In order to support the arguments for the behaviour of fatigue damage for different span lengths, other similar research were studied and compared to the results obtained for this thesis. The particular results which were put in comparison to are gathered from a master's thesis written by Karin Olsson and Josef Pettersson at the Chalmers University of Technology in 2010 (Olsson & Pettersson, 2010). This thesis presents a comparison between the different fatigue assessment methods which includes the lambda method and cumulative damage method, which also are used in this project. Therefore, a comparison of the fatigue behaviour was conducted for the same type of frame bridges and methods. In order to obtain comparative values and to see if the developed script for this master thesis worked as intended, the same dimensions as in (Olsson & Pettersson, 2010) was used for analyzing the fatigue capacity estimated by the design tool developed in this project. However, it is important to have in mind that the thesis by Olsson and Pettersson (2010) is based on railway bridge designs while this thesis focuses on analysing road bridges. Thus, this means that the loads are considerably smaller while analyzing road bridges in comparison with railway bridges. Also, as the script is taking the full bridge structure into account by FE-analysis, this could have an impact on the comparison. Additionally, simplifications such as load placement and long term effects differ between the two theses. The comparison of the fatigue capacity for reinforcement and concrete are presented in Figure 6.16 and 6.17, respectively. The span lengths and corresponding slab thickness for the frame bridge designs that were used in this comparison is presented in Table 6.4. The traffic category was set to Category 1 and the design life to 120 years for the bridge designs analyzed by the design tool developed in this project.

**Table 6.4:** Table showing the dimensions for the bridge designs used in the comparison to previous research.

| Span length [m] | Slab thickness [m] |
|-----------------|--------------------|
| 2               | 0.2                |
| 2.5             | 0.2                |
| 3.5             | 0.27               |
| 5               | 0.39               |
| 7.5             | 0.58               |
| 10              | 0.77               |



**Figure 6.16** Fatigue damage for longitudinal tensile reinforcement for the midsection of a frame bridge analyzed in this project and compared to other research.



**Figure 6.17** Fatigue damage for compressed for the midsection of a frame bridge analyzed in this project and compared to other research.

## 6. Results and Discussion

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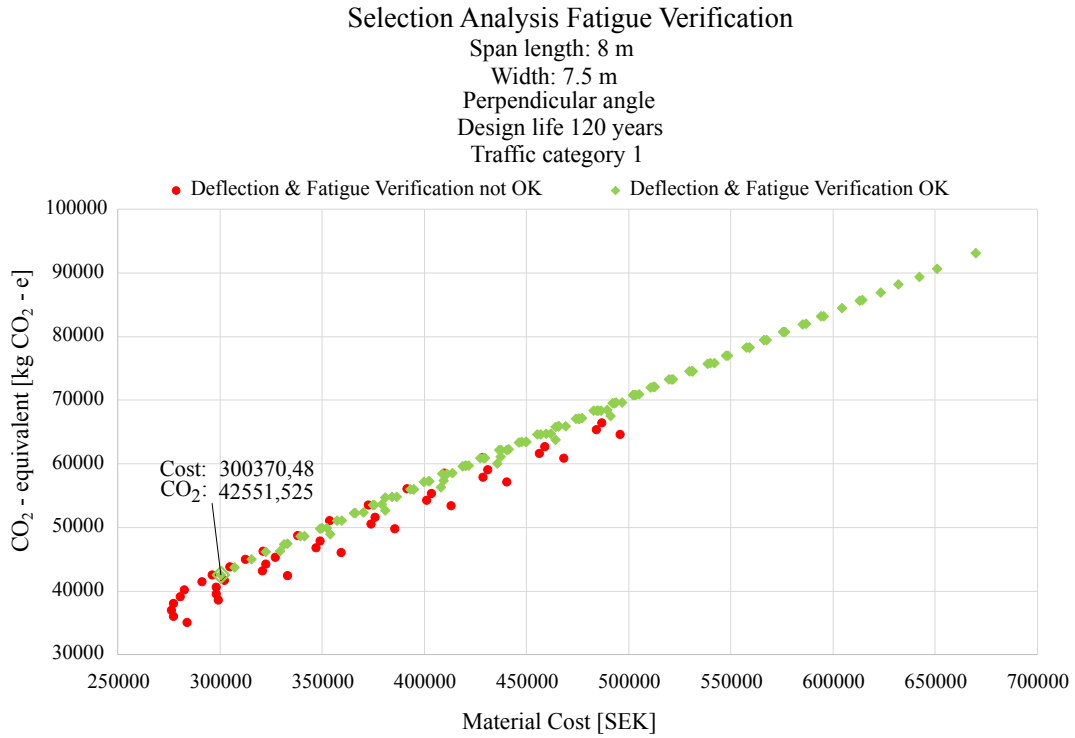
The comparison shows that the fatigue behaviour correlates well between the two theses, but with an obvious difference in magnitude of the fatigue damage. This is reasonable as the train load model consists of larger loads which would inflict more fatigue damage on the bridge structure.

## 6.3 Set-based design method

In this section, an example of the selection procedure is presented for two chosen span lengths. This method is used to evaluate different design alternatives to further investigate how the design tool can be used as an aid in selecting an appropriate design solution. While the previous section focused on analyzing the behaviour of fatigue regarding the slab, this section will take a holistic approach by including additional dimensions for the design and the possibility of different skewed angles. Additionally, this process also includes an extensive fatigue verification, i.e. tensile reinforcement in longitudinal and transverse direction, concrete in compression, shear in concrete and shear reinforcement. These were verified in every critical section.

### 6.3.1 Results for set-based design selection procedure

The results presented in this section are based on the data-set presented in Section 5.4. The selection is illustrated as a scatter analysis where each dot represents a unique frame bridge. Figure 6.18 shows the elimination of alternatives designed according to ULS and then verified for deflection and fatigue. This was done for bridges designs with a span length of 8 m and a skewed angle of 90°. The analysis is performed on 159 different combinations of varying slab and frame leg thickness, see Table 5.2. The alternative which has a sufficient design for every verification and has the lowest material cost and total CO<sub>2</sub>-eq is indicated in Figure 6.18. This alternative represents the options which is most suitable in this data-set.



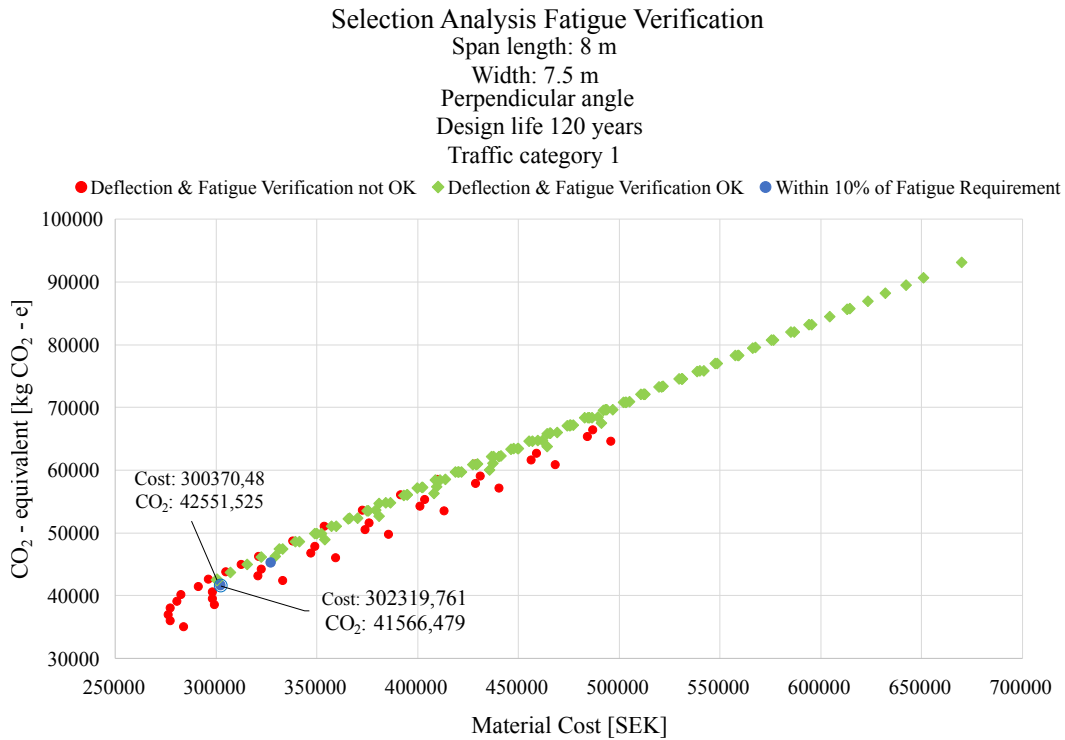
**Figure 6.18** Scatter plot showing the elimination of frame bridges which doesn't fulfill the deflection and fatigue requirement for a 8 m span length.

The elimination in the selection procedure was performed according to the order defined in Table 6.5. The verification is performed in separate steps in order to illustrate the accumulated number of eliminated alternatives after each step.

**Table 6.5:** Table showing the elimination order, the accumulated number of eliminated bridges and the corresponding percentage in relation to the total number of 159 bridges. Table contains values for 8 m span length.

| Verification order                                  | Accumulated number of eliminated alternatives | Percentage |
|---|---|------------|
| 1. Deflection<br><i>Fatigue</i>                     | 21  | 13%        |
| 2. Tensile reinforcement<br>- longitudinal          | 21  | 13%        |
| 3. Tensile reinforcement<br>- transverse            | 30  | 19%        |
| 4. Concrete in compression                          | 43  | 27%        |
| 5. Shear reinforcement                              | 43  | 27%        |
| Number of alternatives satisfying all verifications | 116   |            |

Figure 6.19 illustrate the same elimination of alternatives as Figure 6.18 but with the inclusion of the alternatives that just exceeded the fatigue limit. These alternatives were analyzed further to identify in which critical section the fatigue limit was exceeded. Then this critical section was reinforced according to the procedure explained in section 4.4.4, see Table 6.6.



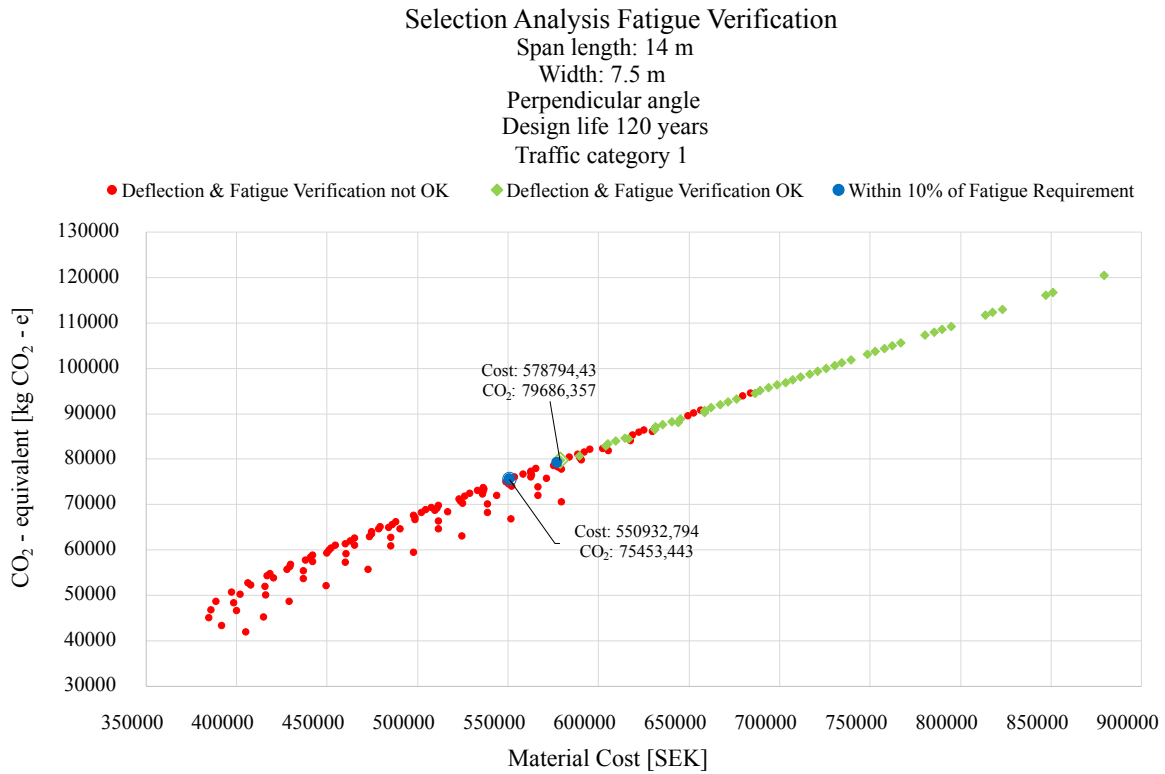
**Figure 6.19** Scatter plot showing the elimination of frame bridges which do not fulfill the deflection and fatigue requirement but also indicating interesting alternatives where the fatigue limit is exceeded by just under 10%, for a 8 m span length.

**Table 6.6:** Table showing the alternative within the 10% fatigue limit before and after modification in comparison to the selected option which fulfilled the fatigue verification. Shows alternatives correlated to the 8 m span length.

| Frame bridge         | Modification  | Price [SEK] | CO <sub>2</sub> -eq [kg CO <sub>2</sub> -e] |
|----------------------|---|-------------|---|
| <i>Alternative A</i> |   |             |   |
| Selected             | -   | 300370      | 42551                                       |
| <i>Alternative B</i> |   |             |   |
| Original             | -   | 302319      | 41566                                       |
| Modified             | Section 1, Transverse<br>Tensile reinforcement<br>5% Increase | 302379      | 41567                                       |

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Utilizing the same data-set as for the 8 m span length, the same procedure was performed on a frame bridge with a 14 m span length. See Figure 6.20 for the selection procedure with indication of alternatives that just exceeded the fatigue limit. Table 6.7 - 6.8 shows the elimination percentages and modifications for the frame bridge closest to the fatigue limit.



**Figure 6.20** Scatter plot showing the elimination of frame bridges which do not fulfill the deflection and fatigue requirement but also indicating interesting alternatives where the fatigue limit is exceeded by just under 10%, for a 14 meter span length.

**Table 6.7:** Table showing the elimination order, the accumulated number of eliminated bridges and the corresponding percentage in relation to the total number of 162 bridges. Table contains values for 14 m span length.

| Verification order                                  | Accumulated number of eliminated alternatives | Percentage |
|---|---|------------|
| 1. Deflection<br><i>Fatigue</i>                     | 43  | 27%        |
| 2. Tensile reinforcement<br>- longitudinal          | 43  | 27%        |
| 3. Tensile reinforcement<br>- transverse            | 56  | 35%        |
| 4. Concrete in compression                          | 101   | 62%        |
| 5. Shear reinforcement                              | 115   | 71%        |
| Number of alternatives satisfying all verifications | 47  |            |

**Table 6.8:** Table showing the alternative within the 10% fatigue limit before and after modification in comparison to the selected option which fulfilled the fatigue verification. Shows alternatives correlated to the 14 m span length.

| Frame bridge         | Modification  | Price [SEK] | CO <sub>2</sub> -eq [kg CO <sub>2</sub> -e] |
|----------------------|---|-------------|---|
| <i>Alternative A</i> |   |             |   |
| Selected             | -   | 578794      | 79686                                       |
| <i>Alternative B</i> |   |             |   |
| Original             | -   | 550932      | 75453                                       |
| Modified             | Section 4 and 6<br>Shear reinforcement<br>5% Increase | 550986      | 75454                                       |

### 6.3.2 Analysis of set-based design selection procedure

Analyzing the elimination procedure further indicates that the preliminary design were sufficient for fatigue regarding the tensile reinforcement in longitudinal direction, which correlates to the results from the parametric study. This was a behaviour that also could be observed for the frame legs when investigating thinner dimensions. However, when studying the transverse direction, several alternatives indicated an insufficient design regarding the tensile reinforcement, but with a fulfilled deflection requirement which occurred more often for the slab. Investigating this further showed that the cases that failed were those that just fulfilled the requirement of deflection. When studying the reinforcement arrangement of these bridge designs, they revealed that it was often only minimum reinforcement applied to the critical section that failed. This indicates that the preliminary design sometimes produces an insufficient design for the tensile reinforcement in the transverse direction. By applying the possibility to detect the cases where the fatigue limit was exceeded by a small percentage made it possible to identify and increase the reinforcement amount of such alternatives where the tensile reinforcement in transverse direction failed. As an example, the studied frame bridge with the 8 meter span only exceeded the fatigue limit limit by 4% in the critical section 1. The adjustment of the reinforcement amounts resulted in an insignificant increase in environmental impact or material cost and can therefore be identified as a better design alternative, see Table 6.6. Therefore, one could suggest that if a fatigue verification were to be implemented in a set-based design environment, it would be necessary to include a procedure to identify and adjust the preliminary design proposals which satisfy ULS and the verification for deflection, but just exceeds the limit for fatigue. This, in order not to mistakenly eliminate suitable alternatives.

Using the same data-set in both analysis resulted in, as expected, several additional alternatives that were eliminated for a studied bridge with 14 m span length. However, the behaviour of tensile reinforcement was similar for both span lengths, which again indicate that the preliminary design that the script generates is sufficient regarding the fatigue verification for this type of reinforcement. The largest difference in number of eliminated bridges between the two span lengths was found in the fatigue verification of concrete in compression. The reason for this is that the reinforcement is designed according to the bending moment which results in a higher reinforcement amount for longer spans because of larger load effects. The concrete in compression is more dependent on the cross-sectional height which is predetermined correlated to which design alternatives the user wants to analyze. Investigating which critical sections that failed due to fatigue for concrete in compression showed that it was often the compressed zones, as shown in the parametric study, that failed. When including additional options for the frame leg thickness, this behaviour was also observed in the compressed zone located in the thinner frame legs.

By investigating the frame legs closer, it was noticeable that the design alternatives that failed for longer span lengths were those with thinner frame legs and the fatigue failed mainly in the critical section 4 and 6. Failure in the midsections of the frame

legs, critical section 5 and 7, proved to be very rare. It could therefore be assumed that the fatigue capacity will not be governing in these sections. However, if these sections would fail, the probability is high that other sections would already have failed. For the design alternative of 14 m, it could be observed that the frame legs had an important role as an increased span length demanded further capacity in the frame legs because of the increased load from both self-weight and the placement of the fatigue load. This enabled the possibility for additional design alternatives to be close to the fatigue limit and thereby require a minor change to fulfill the fatigue verification. By investigating the cases where the fatigue limit was exceeded with less than 10%, it was possible to find a design which had a significantly lower material cost and environmental impact compared to the selected case, as seen in Table 6.8. This particular design only exceeded the limit by 4% for the shear reinforcement in critical section 4 and 6, i.e. the top of the frame legs. As the amount of shear reinforcement in this section proved to be very small, an increase of only 5% of the reinforcement amount in the close proximity of the section was required to fulfill the fatigue verification.

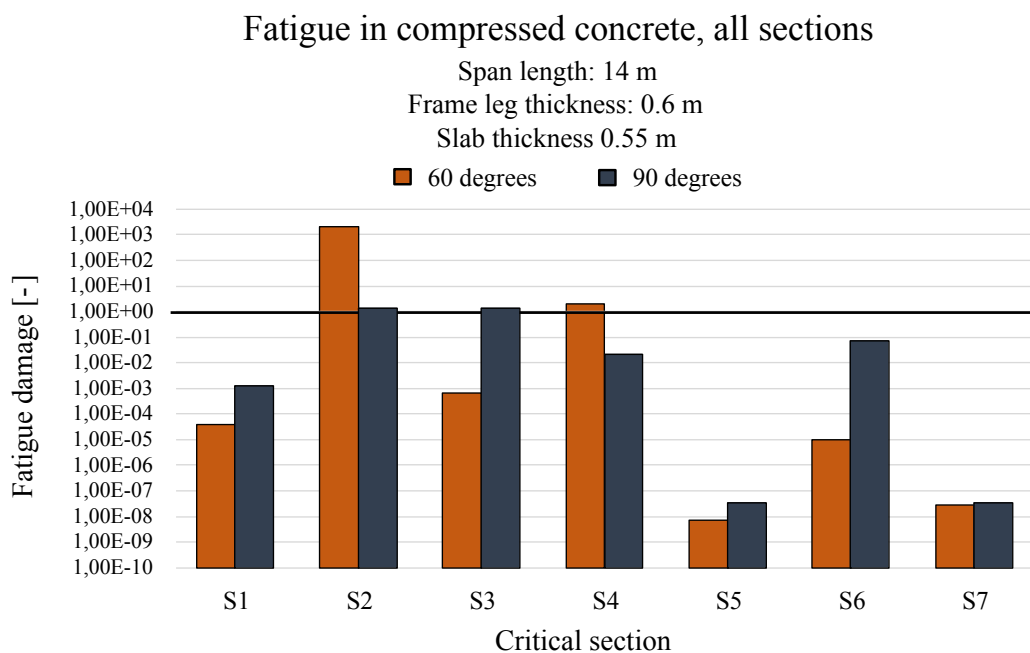
In general, the fatigue verification for the shear reinforcement had no influence for the shorter span length compared to the longer span lengths where several alternatives were eliminated by insufficient shear reinforcement amounts. The sections that failed regarding the shear reinforcement was located both in the slab and in the frame legs. This implies that a longer span will put more demand on these parts regarding the fatigue strength in the shear reinforcement.

By implementing the fatigue verification, the design tool includes and analyzes more parts of the bridge. This is important in a set-based design method because the implementation creates the possibility to eliminate combinations that fulfill the requirement for deflection but not for fatigue. As the verification based on deflection do not affect the frame legs as much as for the slab, the fatigue verification includes the frame legs in the selection process. Therefore, by implementing fatigue verification for both the frame legs and the slab makes the optimization of an entire bridge design even more accurate and realistic.

In order to obtain a bridge design that is optimized through a set-based design procedure, it is therefore needed to specify as many design combinations as possible for the tool to analyze. In this way, the accuracy of the optimization is increased and also the quality of the set-based design procedure. In the examples investigated in this section, it showed that only two cases of alternatives where the fatigue limit exceeded within the 10% for each span lengths existed. If the parameters would have been refined to smaller intervals, the probability for additional cases would be higher which might result in better options.

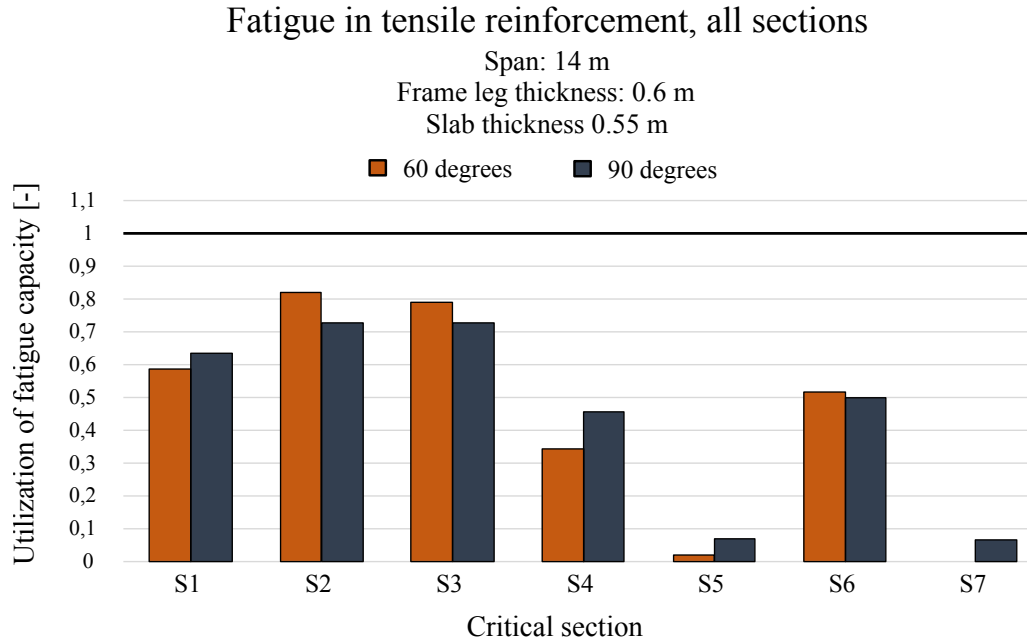
### 6.3.3 Results of design alternatives for skewed angled frame bridges

This section presents results with the purpose of comparing the consequences of choosing different design alternatives for the same design situation. Therefore, this part focuses on the comparison of a skewed angled and a perpendicular frame bridge. With the objective to investigate the influence of fatigue, a perpendicular frame bridge, where the slab thickness was optimized based on the fatigue in compressed concrete, was initially analyzed. The same dimensions were then analyzed but with the skewed angle of  $60^\circ$  instead. The comparison of the fatigue damage due to the skewed design is presented in 6.21.



**Figure 6.21** Fatigue damage in the compressed concrete for a bridge with a 14 m span when perpendicular and when skewed by an angle of  $60^\circ$ . The diagram shows the fatigue damage in every critical section of the frame bridge.

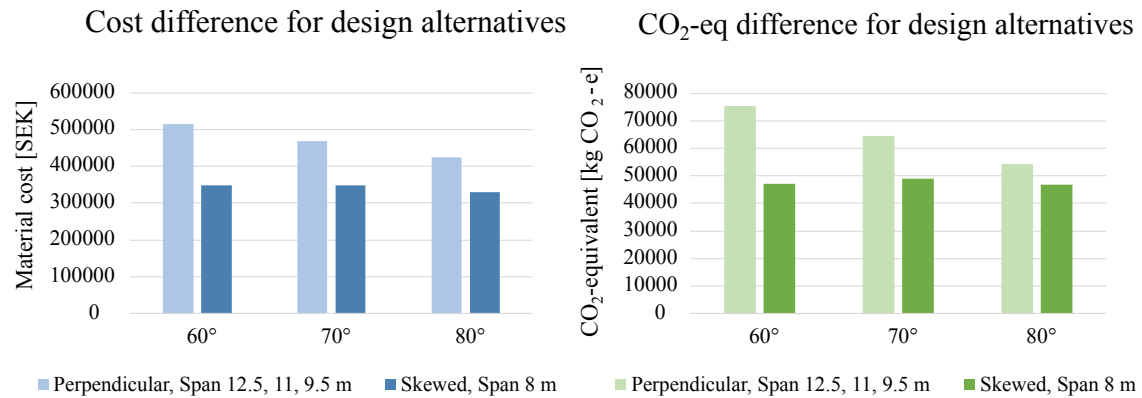
Even if the tensile reinforcement seldom proved to be governing in the parametric study, it was of interest to investigate if the skewed angle had an effect on the utilization of fatigue in the tensile reinforcement, see Figure 6.22.



**Figure 6.22** Utilization of the fatigue capacity in the tensile reinforcement for a bridge with a 14 m span when perpendicular and when skewed by an angle of  $60^\circ$ . The diagram shows the utilization in every critical section of the frame bridge.

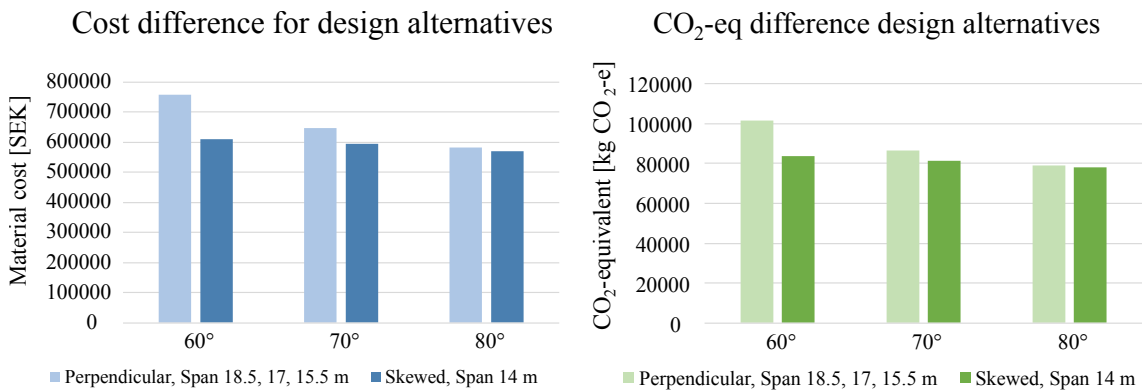
## 6. Results and Discussion

To further determine the consequence of choosing different design alternatives regarding the different evaluation criteria, the same set-based design selection procedure as in section 6.3.1 was studied to find the most suitable frame bridge in the data-set. Two span lengths of a skewed angled frame bridge were included in this analysis to see potential differences in behaviour between a shorter and longer span. Figure 6.23 compares the evaluation criteria for a 8 m frame bridge of the different design alternatives explained in section 5.4.2.



**Figure 6.23** Comparison of evaluation criteria for different skewed angled of a 8 meter span length and its corresponding minimum required span length designed with a perpendicular angle.

The same comparison of evaluation criteria is performed for a frame bridge with a span length of 14 m, see Figure 6.24.



**Figure 6.24** Comparison of evaluation criteria for different skewed angled of a 14 m span length and its corresponding minimum required span length designed with a perpendicular angle.

### 6.3.4 Analysis of design alternatives for skewed angled frame bridges

Analyzing the results closer, a descending trend can be observed when comparing the material cost between the different span lengths, a trend that also can be found in the comparison of CO<sub>2</sub>-eq for the longer span. However, for the shorter span lengths, the differences in material cost were only minor and the CO<sub>2</sub>-eq for a skewed frame bridge of 70° was greater than for the other studied angles. Investigating this further, the material cost for concrete was cheaper compared to reinforcement steel which indicated that the frame bridge with the skewed angle of 70° utilizes more concrete than the other options while the dimensions remains very similar. This situation is an ideal example of where a refinement of the input parameters may be needed to achieve an even more accurate comparison.

Comparing the differences between the adjusted and the original design, it is clear that the effect of designing a frame bridge with a greater angle as perpendicular had a larger impact than the other skewed angles. This behaviour was expected because the amount of material is increased for alternatives with a skewed design. It can also be observed that the difference was smaller for the longer span option which could be an indication that the frame legs is affected more regarding the material increase than the slab in a skewed design. For the longer span length, the slab constitutes a larger part of the structure compared to the frame legs. This resulted in a small effect to the evaluation criteria regarding changes in material in the frame legs.

In further comparison between skewed and perpendicular bridge designs, the fatigue in compressed concrete was exceeded when a perpendicular bridge design with a certain span length was studied as skewed. Figure 6.21 shows the damage for all section and indicates that the fatigue damage was increased in sections on one side of the bridge. The reason for this is because of the asymmetrical load case when assuming one slow lane on the bridge. If the frame bridge would be designed with respect to fatigue, the load effects would have been mirrored to handle all load situations, which for a skewed bridge means that it's mirrored diagonally to the opposite side. Observing Figure 6.21 closer, it can be seen that there was a small difference in results for section 6 compared to section 4 for a perpendicular bridge design, even if this was analyzed for a symmetrical design. This may be explained by the sensitivity of the fatigue verification in combination with the sensitive stress calculation mentioned in the parametric study, where small difference in the normal force and bending moment from the BRIGADE Plus model and small differences in the reinforcement amounts, ends up making a noticeable impact to the result.

The fatigue behaviour of the tensile reinforcement was as expected in line with previous results, where the fatigue verification was not governing. However, a slight increase in fatigue damage could be observed for the two critical sections in the slab closest to the supports as seen in Figure 6.22.

## 6.4 Applying set-based design on North Bothnia line

One purpose of this master thesis is to investigate how a set-based design method may lead to an easier way of reviewing and overlooking different design alternatives. A large project where such method can be utilized is as previously mentioned the North Bothnia line with over 100 crossings with similar design situations. This creates the possibility to evaluate if there may be benefits of grouping different design alternatives. In order to present an example of how to apply the method, a scenario is set up which consists in 10 frame bridges with different prerequisites, where every frame bridge has the same width of 7.5 m. The purpose was to evaluate the total effect regarding the criteria when these 10 frame bridges are designed within different groups, instead of individually. Each frame bridge in this analysis was selected according to the same set-based design method as presented in previous sections. The designs in Table 6.9 represent the bridges that needs to be designed, with the dimensions that are the most suitable for their situation.

**Table 6.9:** Table showing the most suitable designs for the initial design scenario including span length, skewed angle, material cost and CO<sub>2</sub>-equivalent.

| Frame bridge   | Span length [m] | Angle | Price [SEK] | CO <sub>2</sub> -eq [kg CO <sub>2</sub> -e] |
|----------------|-----------------|-------|-------------|---|
| Bridge 1       | 8.0             | 60°   | 347900      | 47229                                       |
| Bridge 2       | 8.0             | 70°   | 347114      | 48882                                       |
| Bridge 3       | 8.0             | 80°   | 330022      | 46835                                       |
| Bridge 4       | 9.5             | 90°   | 383915      | 54209                                       |
| Bridge 5       | 11.0            | 90°   | 459555      | 64401                                       |
| Bridge 6       | 14.0            | 60°   | 610071      | 83562                                       |
| Bridge 7       | 14.0            | 70°   | 595238      | 81497                                       |
| Bridge 8       | 14.0            | 80°   | 569112      | 78054                                       |
| Bridge 9       | 14.0            | 90°   | 550986      | 75454                                       |
| Bridge 10      | 15.5            | 90°   | 583103      | 79129                                       |
| Total          |                 |       | 4777020     | 659256                                      |
| Nr. of designs | 10              |       |             |   |

Taking a closer look at the bridges, there are many bridges with the same span length but with different alignments. If these bridges could be grouped in such way that the skewed angled bridges are replaced with perpendicular angled frame bridge, it may save time designing all the alternatives. A conservative option would be to set up a design group based on perpendicular designs corresponding to the angle of 60°, see Table 6.11. If the group is compared to the original execution in terms of material cost and environmental impact, the adjusted design group resulted in a significant increase of almost 30% for both criteria. Grouping them in this manner would result in the need to produce only 3 different frame bridge designs. However, the large increase in both material costs and CO<sub>2</sub>-e might constitute the argument to reconsider a different design group.

**Table 6.10:** Table showing group alternative 1, which results in 3 different designs for all 10 design situations and presents the material cost and environmental impact compared to the initial execution.

| Frame bridge   | Span length [m] | Angle | Price [SEK] | CO <sub>2</sub> -eq [kg CO <sub>2</sub> -e] |
|----------------|-----------------|-------|-------------|---|
| Bridge 1       | 12.5            | 90°   | 542153      | 75522                                       |
| Bridge 2       | 12.5            | 90°   | 542153      | 75522                                       |
| Bridge 3       | 12.5            | 90°   | 542153      | 75522                                       |
| Bridge 4       | 12.5            | 90°   | 542153      | 75522                                       |
| Bridge 5       | 12.5            | 90°   | 542153      | 75522                                       |
| Bridge 6       | 18.5            | 90°   | 759676      | 101630                                      |
| Bridge 7       | 18.5            | 90°   | 759676      | 101630                                      |
| Bridge 8       | 18.5            | 90°   | 759676      | 101630                                      |
| Bridge 9       | 15.5            | 90°   | 583103      | 79129                                       |
| Bridge 10      | 15.5            | 90°   | 583103      | 79129                                       |
| Total          |                 |       | 6156006     | 840765                                      |
| Increase       |                 |       | 29%         | 28%   |
| Nr. of designs |                 |       | 3           |   |

The increase for group alternative 1 is quite extensive. Therefore, other options might be more suitable to adapt. As shown in section 6.3.3, the largest difference between the evaluation criteria when executing the bridge in a perpendicular design is found when the skewed angle is 60°. Keeping these alternatives according to the original execution may therefore be more beneficial. Another proposal for a design group could therefore be that the perpendicular option could instead be adapted based on an angle of 70° which would result in shorter span lengths for the other frame bridges. Changing these alternatives accordingly results in a total increase of 10% for material cost and environmental impact. This group alternative resulted in the need to design 5 different frame bridges.

**Table 6.11:** Table showing group alternative 2, which results in 5 different designs for all 10 design situations and presents the material cost and environmental impact compared to the initial execution.

| Frame bridge   | Span length [m] | Angle | Price [SEK] | CO <sub>2</sub> -eq [kg CO <sub>2</sub> -e] |
|----------------|-----------------|-------|-------------|---|
| Bridge 1       | 8.0             | 60°   | 347900      | 47229                                       |
| Bridge 2       | 11.0            | 90°   | 459555      | 64401                                       |
| Bridge 3       | 11.0            | 90°   | 459555      | 64401                                       |
| Bridge 4       | 11.0            | 90°   | 459555      | 64401                                       |
| Bridge 5       | 11.0            | 90°   | 459555      | 64401                                       |
| Bridge 6       | 14.0            | 60°   | 610071      | 83562                                       |
| Bridge 7       | 17.0            | 90°   | 647240      | 86617                                       |
| Bridge 8       | 17.0            | 90°   | 647240      | 86617                                       |
| Bridge 9       | 15.5            | 90°   | 583103      | 79129                                       |
| Bridge 10      | 15.5            | 90°   | 583103      | 79129                                       |
| Total          |                 |       | 5256881     | 719891                                      |
| Increase       |                 |       | 10%         | 9%  |
| Nr. of designs | 5               |       |             |   |

Even if it is clear that the group alternatives will result in an increase in both material cost and environmental impact, the knowledge of how much a whole group of bridges is affecting the final result may be valuable in decision regarding production methods. According to a study performed by Rashem and Swahn, 2018, which presented two case studies with the purpose to compare prefabricated and in-situ cast frame bridges, the results indicated that prefabricating a frame bridge would result in both savings and improvements regarding material cost, transport, working environment and environmental impact but could also result in reduced construction time and traffic delay. This could possibly be put in comparison to the total cost and CO<sub>2</sub>-eq of the group alternatives to determine if the savings in time and material are possible to achieve by producing groups of similar prefabricated frame bridges and reduce the number of designs. If the savings could outbalance the 10% increase it would be beneficial to design these frame bridges as groups. It is valuable to be able to do this assessment as early in the design phase as possible and as the tool can quickly create a range of designs and evaluate these, this method can be done with ease and be of aid in the decision making. A set-based design tool may therefore be a valuable asset in a large project where standardizing several bridge designs might be beneficial.

As shown above, by utilizing the method in a large project to create a data-set of several bridge designs, it is easy to analyze and compare not only individual design but rather whole groups. This gives the possibility, in an early design phase, to get an overview of which alternatives can benefit from different production methods. However, further research is needed to be able to conclude possible consequences in term of savings when utilizing different production methods.

## 6.5 General observations

Throughout the generated results of fatigue damage in the critical sections studied in this analysis, a general behaviour in fatigue damage for concrete frame road bridges, with different span lengths, can be observed. It is noticeable that the fatigue damage in the midsection increases along with an increasing span length until a point where it starts to decrease. This response is especially noticeable for designs with shorter span lengths. At the support sections, the response for different span lengths seem to differ from in the midsections where the fatigue damage can be assumed as more constant. This constant increase of fatigue damage can be due to the fact that the reinforcement amounts remains constant as a minimum allowed reinforcement amount. In combination with an increasing load and amplification of the lambda method, this would generate an increasing fatigue damage in designs with different span lengths. For support sections, the moments are larger and the reinforcement amounts may change even for bridges with shorter spans. This would explain why the fatigue damage remains constant, as loads and reinforcement amounts both increase.

Additionally, a noticeable change in the results for the reinforcement occurs for bridges with span lengths shorter than 10 m. A possible explanation could be that the Lambda-method that is used in the calculations for the reinforcement contains factors which differs between shorter and longer span lengths. This suggests, in line with the inadequate instructions in (SIS, 2005) for spans shorter than 10 m, that the lambda method would be more suitable for assessing bridges with spans longer than 10 m.

## 6.6 Sources of error and deviations

In order to reduce run-time in the analysis, simplifications were made for load placements as described in chapter 4. These simplifications could result in an approximation of the load effects that governs the maximum and minimum limits in a stress cycle, which thereby affects the estimated fatigue damage. If the load placements could be developed further for each bridge design analyzed in the set-based design procedure, the quality of the fatigue assessment would be improved.

The purpose of set-based design is to analyze large amount of different geometrical combinations. To get the script working for all possible combination has been one of the most challenging aspects of the project. One simplification that reaches across the modules in the script that designs the shear reinforcement and verifies it for fatigue, was regarding the connection between slab and the frame legs. In the current state of the script, the arrangement of the shear reinforcement close to the this connection is generated in such a way that the large differences in thickness of the slab and the frame leg becomes problematic regarding how BRIGADE plus analyzes the load effects. This part needs to be further developed so that the arrangement can

be more accurate and generate the shear reinforcement in a more realistic way. This problem translates to the verification of fatigue in the shear reinforcement. Solving this would increase the accuracy for extreme combinations.

As the calculation of sectional stresses is conducted through an automated iterative process, the quality of the results could be affected by the accuracy of the calculations. In order to reduce the run-time for the analysis, the increments in the iterative process in the script was kept to a certain length. However, if the iterative process would be even further developed by increasing the number of iterations, the accuracy of the stress calculations might be improved.

# 7

## Conclusions

### 7.1 Concluding remarks

As the traffic category and choice of design life proved to have an influence to the final fatigue damage in the generated bridge designs from the set-based design tool, the actual choice of traffic category could result in beneficial consequences for the bridge design. Not only would the design choices affect how the bridge performs when built, but also enable the designer to prevent future problems if the traffic conditions of the bridge would change. Therefore, the design would benefit from a choice of traffic category which could withstand this possible change of traffic situation.

During this project, several comparisons of bridge designs generated by the design tool were conducted in order to determine the influence of an inclusion of fatigue to the preliminary designs according to ULS generated by the design tool. This proved to be of significance as several of the initially proposed designs, which fulfilled the deflection requirement, failed when assessing the fatigue capacity, especially for the concrete subjected to compression. The fatigue limit was rarely exceeded for the bending reinforcement in the majority of the designs.

Implementation of the fatigue verification resulted in that several more parts of the frame bridge was included in the design verification process and thereby to the selection procedure, which in the end influenced the final results for the material costs and CO<sub>2</sub>-eq. By investigating cases for which the fatigue limit was exceeded by a small degree, it was possible to identify and reinforce these bridge designs. This resulted in additional frame bridges that were even better regarding the evaluation criteria. Thus, the result suggested that it is important to optimize geometry and reinforcement based on fatigue rather than just verifying if the fatigue is fulfilled.

Applying the principle of a set-based design method to a large infrastructure project, it was possible to quickly compare and evaluate different alternative designs in an early phase. It was shown that the tool can be used as an aid to the decision making of production methods in the projects to determine and standardize designs which then can be produced as groups. The evaluation based on material costs and environmental impact suggested that standardization could be beneficial in terms of costs related to design work. However, in order to conclude if a standardized bridge design could be beneficial with respect to environmental impact, further research

with a wider perspective in LCA and how the choice of production method may affect the evaluation criteria need to be explored.

### 7.2 Further studies

This section presents parts that may need to be further studied in order to generate more reliable results and develop the design method.

The effect from the traffic situation proved to have an influence to the fatigue verification. Because this project only included one slow lane and only considers road bridges, it may be interesting to further study additional traffic conditions. This may include investigating the effects of additional lanes and railway bridges. By investigating this, the width of the frame bridge could be of more importance when analyzing the possibility of standardizing different alternatives.

As mentioned in the report, the design is affected by the alignment from the passing underneath and the connection to the network that crosses over the frame bridge. What was not considered in the project however, was that in Sweden there is a hierarchy regarding priority between waterways, railway traffic and road traffic. The waterway is the highest prioritized traffic type and road traffic is the least prioritized, which means that in a case where railway traffic and road traffic overlap, the road is adapted according to the design of the railway. Therefore, it might be necessary to include this in the set-based design method to be able to consider what traffic type the frame bridge is designed for.

An automated adjustment of the reinforcement amount for bridge designs which fatigue limit is exceeded by just a smaller percentage could be implemented in order to identify additional design alternatives which could be perceived as suitable design proposals, as described in Section 6.3. A possible solution to achieve this could be to implement a parametrization of the reinforcement which takes the fatigue assessment into account.

In this project, the selection procedure was based on evaluation criteria where material costs and CO<sub>2</sub>-equivalents were quantified. Further research regarding LCC and LCA analysis could be implemented in order to achieve an even more accurate evaluation of the bridge designs with respect to aspects beyond the product stage. This would contribute to an even more realistic evaluation in the set-based design tool.

If further development of the existing evaluation criteria but for additional concrete classes and reinforcement types would be implemented in the tool, the possibility to draw conclusions regarding different alternatives would increase.

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# A

## Result of analytical comparison

This appendix presents the analytical comparison between hand calculations and the extracted results produced by the script of the calculation process for fatigue capacity utilized for this project. The calculations are based on sectional forces and dimensions obtained from the FE-model produced by the developed Python script. The reinforcement amounts are also extracted from the preliminary design generated by the same script.

# Analytical verification of fatigue calculations according to Eurocode

## Carriageway

### Compressive stresses

| Section 1                          |                        |                        | Section 2                 |                        |                         | Section 3                 |                        |                         |
|------------------------------------|------------------------|------------------------|---------------------------|------------------------|-------------------------|---------------------------|------------------------|-------------------------|
| Sectional forces                   |                        |                        | Sectional forces          |                        |                         | Sectional forces          |                        |                         |
| M_Ed [kNm]                         | max                    | min                    | M_Ed [kNm]                | max                    | min                     | M_Ed [kNm]                | max                    | min                     |
|                                    | 423.255                | 312.168                |                           | 382.646                | 293.581                 |                           | 382.631                | 293.552                 |
| N_Ed [kN]                          | 117.485                | 98.53                  |                           | 200.485                | 164.127                 |                           | 200.685                | 164.338                 |
|                                    |                        |                        |                           | 135.077                | 124.744                 |                           | 135.071                | 124.744                 |
| <b>Reinforcement</b>               |                        |                        |                           |                        |                         |                           |                        |                         |
| As_top [m <sup>2</sup> ]           | 0.002139               |                        | As_top [m <sup>2</sup> ]  | 0.003219               |                         | As_top [m <sup>2</sup> ]  | 0.003219               |                         |
| As_bot [m <sup>2</sup> ]           | 0.004590               |                        | As_bot [m <sup>2</sup> ]  | 0.002139               |                         | As_bot [m <sup>2</sup> ]  | 0.002139               |                         |
| <b>Stress/sectional Comparison</b> |                        |                        |                           |                        |                         |                           |                        |                         |
| MathCad                            | Script                 |                        | MathCad                   | Script                 |                         | MathCad                   | Script                 |                         |
| x_cc_max [mm]                      | 178.32                 | 178.2                  | x_cc_max [mm]             | 169.5                  | 168.2                   | x_cc_max [mm]             | 169.55                 | 168.3                   |
| x_cc_min [mm]                      | 180.3                  | 180.1                  | x_cc_min [mm]             | 171.35                 | 170.1                   | x_cc_min [mm]             | 171.4                  | 170.1                   |
| I_y_max [m <sup>4</sup> ]          | 9.587*10 <sup>-3</sup> | 9.589*10 <sup>-3</sup> | I_y_max [m <sup>4</sup> ] | 7.809*10 <sup>-3</sup> | 7.825*10 <sup>-3</sup>  | I_y_max [m <sup>4</sup> ] | 7.809*10 <sup>-3</sup> | 7.825*10 <sup>-3</sup>  |
| I_y_min [m <sup>4</sup> ]          | 9.588*10 <sup>-3</sup> | 9.589*10 <sup>-3</sup> | I_y_min [m <sup>4</sup> ] | 7.81*10 <sup>-3</sup>  | 7.826*10 <sup>-3</sup>  | I_y_min [m <sup>4</sup> ] | 7.81*10 <sup>-3</sup>  | 7.826*10 <sup>-3</sup>  |
| sigma_tens_max [MPa]               | 7.359                  | 7.358                  | sigma_tens_max [MPa]      | 7.2                    | 7.133                   | sigma_tens_max [MPa]      | 7.199                  | 7.1325                  |
| sigma_tens_min [MPa]               | 5.435                  | 5.434                  | sigma_tens_min [MPa]      | 5.526                  | 5.47                    | sigma_tens_min [MPa]      | 5.526                  | 5.474                   |
| <b>Fatigue damage</b>              |                        |                        |                           |                        |                         |                           |                        |                         |
| Top                                | 5.458*10 <sup>-8</sup> | 6.535*10 <sup>-8</sup> | Top                       | -                      | -                       | Top                       | -                      | -                       |
| Bottom                             | -                      | -                      | Bottom                    | 3.294*10 <sup>-9</sup> | 3.0411*10 <sup>-9</sup> | Bottom                    | 3.304*10 <sup>-9</sup> | 3.0509*10 <sup>-9</sup> |

### Input Data

|                       |         |
|-----------------------|---------|
| h [m]                 | 0.8     |
| L [m]                 | 15      |
| b [m]                 | 7.5     |
| H_framelegs [m]       | 6       |
| t_framelegs [m]       | 0.6     |
| angle of bridge [deg] | 90      |
| ADT                   | 10000   |
| Design Life           | 100 Yrs |
| Concrete class        | C35/45  |

### Tensile stresses

| Section 1                           |                        |                        | Section 2                 |                        |                        | Section 3                 |                        |                        |
|-------------------------------------|------------------------|------------------------|---------------------------|------------------------|------------------------|---------------------------|------------------------|------------------------|
| Sectional forces                    |                        |                        | Sectional forces          |                        |                        | Sectional forces          |                        |                        |
| M_Ed [kNm]                          | max                    | min                    | M_Ed [kNm]                | max                    | min                    | M_Ed [kNm]                | max                    | min                    |
|                                     | 467.69                 | 312.168                |                           | 419.558                | 294.868                |                           | 419.55                 | 294.839                |
| N_Ed [kN]                           | -125.067               | -98.53                 |                           | -216.392               | -165.562               |                           | -216.66                | -165.775               |
|                                     |                        |                        |                           | 219.453                | 188.174                |                           | 219.454                | 188.163                |
| <b>Reinforcement</b>                |                        |                        |                           |                        |                        |                           |                        |                        |
| As_top [m <sup>2</sup> ]            | 0.0021399              |                        | As_top [m <sup>2</sup> ]  | 0.0032197              |                        | As_top [m <sup>2</sup> ]  | 0.0032197              |                        |
| As_bot [m <sup>2</sup> ]            | 0.004590               |                        | As_bot [m <sup>2</sup> ]  | 0.0021399              |                        | As_bot [m <sup>2</sup> ]  | 0.0021399              |                        |
| Asw [m <sup>2</sup> /s]             |                        |                        | Asw [m <sup>2</sup> /s]   | 0.00107                |                        | Asw [m <sup>2</sup> /s]   | 0.00107                |                        |
| <b>Stress/sectional Comparison</b>  |                        |                        |                           |                        |                        |                           |                        |                        |
| MathCad                             | Script                 |                        | MathCad                   | Script                 |                        | MathCad                   | Script                 |                        |
| x_cc_max [mm]                       | 177.8                  | 177.7                  | x_cc_max [mm]             | 169.1                  | 167.8                  | x_cc_max [mm]             | 169.12                 | 167.9                  |
| x_cc_min [mm]                       | 180.2                  | 180.1                  | x_cc_min [mm]             | 171.5                  | 170.2                  | x_cc_min [mm]             | 171.55                 | 170.3                  |
| I_y_max [m <sup>4</sup> ]           | 9.587*10 <sup>-3</sup> | 9.589*10 <sup>-3</sup> | I_y_max [m <sup>4</sup> ] | 7.81*10 <sup>-3</sup>  | 7.827*10 <sup>-3</sup> | I_y_max [m <sup>4</sup> ] | 7.81*10 <sup>-3</sup>  | 7.825*10 <sup>-3</sup> |
| I_y_min [m <sup>4</sup> ]           | 9.588*10 <sup>-3</sup> | 9.589*10 <sup>-3</sup> | I_y_min [m <sup>4</sup> ] | 7.812*10 <sup>-3</sup> | 7.826*10 <sup>-3</sup> | I_y_min [m <sup>4</sup> ] | 7.812*10 <sup>-3</sup> | 7.827*10 <sup>-3</sup> |
| sigma_tens_max [MPa]                | 142.529                | 142.507                | sigma_tens_max [MPa]      | 156.665                | 156.563                | sigma_tens_max [MPa]      | 156.623                | 156.521                |
| sigma_tens_min [MPa]                | 93.549                 | 93.53                  | sigma_tens_min [MPa]      | 108.177                | 108.10                 | sigma_tens_min [MPa]      | 108.133                | 108.057                |
|                                     |                        |                        | shear_max [MPa]           | 107.295                | 106.139                | shear_max [MPa]           | 107.296                | 106.138                |
|                                     |                        |                        | shear_min [MPa]           | 92.002                 | 91.11                  | shear_min [MPa]           | 91.997                 | 91.00                  |
| <b>Utilization of fatigue limit</b> |                        |                        |                           |                        |                        |                           |                        |                        |
| Top                                 | -                      | -                      | Top                       | 0.52                   | 0.5249                 | Top                       | 0.52                   | 0.5249                 |
| Bottom                              | 0.525                  | 0.53                   | Bottom                    | -                      | -                      | Bottom                    | -                      | -                      |
|                                     |                        |                        | Shear                     | 0.422                  | 0.421                  | Shear                     | 0.422                  | 0.421                  |

# Framelegs

## Compressive stresses

| Section 4                          |            |            | Section 5                          |             |             | Section 6                          |            |            | Section 7                          |             |             |
|------------------------------------|------------|------------|------------------------------------|-------------|-------------|------------------------------------|------------|------------|------------------------------------|-------------|-------------|
| Sectional forces                   |            |            | Sectional forces                   |             |             | Sectional forces                   |            |            | Sectional forces                   |             |             |
| M_Ed [kNm]                         | max        | min        | M_Ed [kNm]                         | max         | min         | M_Ed [kNm]                         | max        | min        | M_Ed [kNm]                         | max         | min         |
| M_Ed [kNm]                         | 399.045    | 308.683    | 75.218                             | 43.962      | 308.634     | 398.985                            | 308.634    | 308.634    | 75.194                             | 43.944      | 43.944      |
| N_Ed [kN]                          | 370.927    | 244.512    | 295.446                            | 214.415     | 244.117     | 370.428                            | 244.117    | 244.117    | 295.504                            | 214.458     | 214.458     |
| <b>Reinforcement</b>               |            |            | <b>Reinforcement</b>               |             |             | <b>Reinforcement</b>               |            |            | <b>Reinforcement</b>               |             |             |
| As_top [m²]                        | 0.003858   |            | 0.001604                           |             | 0.003307    | 0.003307                           |            | 0.003307   | 0.001604                           |             | 0.001604    |
| As_bot [m²]                        | 0.001604   |            | 0.001604                           |             | 0.001604    | 0.001604                           |            | 0.001604   | 0.001604                           |             | 0.001604    |
| <b>Stress/sectional Comparison</b> |            |            | <b>Stress/sectional Comparison</b> |             |             | <b>Stress/sectional Comparison</b> |            |            | <b>Stress/sectional Comparison</b> |             |             |
| MathCad                            | Script     |            | MathCad                            | Script      |             | MathCad                            | Script     |            | MathCad                            | Script      |             |
| x_cc_max [mm]                      | 160.5      | 160.4      | 255.95                             | 255.9       | 150.65      | 150.65                             | 150.5      | 150.5      | 256.1                              | 256.1       | 256.1       |
| x_cc_min [mm]                      | 155.95     | 155.8      | 335.2                              | 335.1       | 146.25      | 146.25                             | 146.1      | 146.1      | 335.3                              | 335.3       | 335.3       |
| I_y_max [m⁴]                       | 4.603*10⁻³ | 4.603*10⁻⁴ | 2.999*10⁻³                         | 3.0*10⁻⁴    | 4.049*10⁻³  | 4.049*10⁻³                         | 4.049*10⁻⁴ | 4.049*10⁻⁴ | 3.001*10⁻³                         | 3.003*10⁻⁴  | 3.003*10⁻⁴  |
| I_y_min [m⁴]                       | 4.6*10⁻³   | 4.601*10⁻⁴ | 4.523*10⁻³                         | 4.523*10⁻⁴  | 4.047*10⁻³  | 4.047*10⁻³                         | 4.047*10⁻⁴ | 4.047*10⁻⁴ | 4.526*10⁻³                         | 4.529*10⁻⁴  | 4.529*10⁻⁴  |
| sigma_comp_max [MPa]               | 11.768     | 11.767     | 2.391                              | 2.392       | 12.437      | 12.437                             | 12.436     | 12.436     | 2.39                               | 2.389       | 2.389       |
| sigma_comp_min [MPa]               | 9.084      | 9.0825     | 1.274                              | 1.275       | 9.609       | 9.609                              | 9.607      | 9.607      | 1.274                              | 1.273       | 1.273       |
| <b>Fatigue damage</b>              |            |            | <b>Fatigue damage</b>              |             |             | <b>Fatigue damage</b>              |            |            | <b>Fatigue damage</b>              |             |             |
| Earthside                          | -          | -          | Earthside                          | -           | -           | Earthside                          | -          | -          | Earthside                          | -           | -           |
| Airside                            | 0.164      | 0.195      | Airside                            | 4.888*10⁻¹¹ | 5.852*10⁻¹⁰ | Airside                            | 2.242      | 2.681      | Airside                            | 4.876*10⁻¹¹ | 5.841*10⁻¹⁰ |

## Tensile stresses

| Section 4                           |            |          | Section 5                           |            |            | Section 6                           |            |            | Section 7                           |            |            |
|-------------------------------------|------------|----------|-------------------------------------|------------|------------|-------------------------------------|------------|------------|-------------------------------------|------------|------------|
| Sectional forces                    |            |          | Sectional forces                    |            |            | Sectional forces                    |            |            | Sectional forces                    |            |            |
| M_Ed [kNm]                          | max        | min      | M_Ed [kNm]                          | max        | min        | M_Ed [kNm]                          | max        | min        | M_Ed [kNm]                          | max        | min        |
| M_Ed [kNm]                          | 435.19     | 308.683  | 87.72                               | 43.962     | 308.634    | 435.126                             | 308.634    | 308.634    | 87.695                              | 43.944     | 43.944     |
| N_Ed [kN]                           | 421.493    | 244.512  | 327.858                             | 214.415    | 244.117    | 420.953                             | 244.117    | 244.117    | 327.922                             | 214.458    | 214.458    |
| V_Ed [kN]                           | 339.293    | 251.066  |                                     |            | 251.795    | 340.245                             |            |            |                                     |            |            |
| <b>Reinforcement</b>                |            |          | <b>Reinforcement</b>                |            |            | <b>Reinforcement</b>                |            |            | <b>Reinforcement</b>                |            |            |
| As_top [m²]                         | 0.003858   |          | 0.001604                            |            | 0.003307   | 0.003307                            |            | 0.003307   | 0.001604                            |            | 0.001604   |
| As_bot [m²]                         | 0.001604   |          | 0.001604                            |            | 0.001604   | 0.001604                            |            | 0.001604   | 0.001604                            |            | 0.001604   |
| Asw [m²/s]                          | 0.001721   |          |                                     |            | 0.001727   |                                     |            | 0.001727   |                                     |            |            |
| <b>Stress/sectional Comparison</b>  |            |          | <b>Stress/sectional Comparison</b>  |            |            | <b>Stress/sectional Comparison</b>  |            |            | <b>Stress/sectional Comparison</b>  |            |            |
| MathCad                             | Script     |          | MathCad                             | Script     |            | MathCad                             | Script     |            | MathCad                             | Script     |            |
| x_cc_max [mm]                       | 161.82     | 161.7    | 241.3                               | 241.2      | 151.92     | 151.92                              | 151.8      | 151.8      | 241.5                               | 241.3      | 241.3      |
| x_cc_min [mm]                       | 155.95     | 155.8    | 335.1                               | 335.1      | 146.25     | 146.25                              | 146.1      | 146.1      | 335.3                               | 335.3      | 335.3      |
| I_y_max [m⁴]                        | 4.604*10⁻³ | 4.6*10⁻³ | 2.817*10⁻³                          | 2.818*10⁻³ | 4.05*10⁻³  | 4.05*10⁻³                           | 4.0*10⁻³   | 4.0*10⁻³   | 2.819*10⁻³                          | 2.819*10⁻³ | 2.819*10⁻³ |
| I_y_min [m⁴]                        | 4.6*10⁻³   | 4.6*10⁻³ | 4.52*10⁻³                           | 4.524*10⁻³ | 4.047*10⁻³ | 4.047*10⁻³                          | 4.047*10⁻³ | 4.047*10⁻³ | 4.526*10⁻³                          | 4.529*10⁻³ | 4.529*10⁻³ |
| sigma_tens_max [MPa]                | 176.578    | 176.539  | 20.808                              | 20.8       | 203.931    | 203.931                             | 203.906    | 203.906    | 18.206                              | 207.687    | 207.687    |
| sigma_tens_min [MPa]                | 131.636    | 131.6    | 4.592                               | 4.591      | 152.199    | 152.199                             | 152.18     | 152.18     | 4.58                                | 45.793     | 45.793     |
| shear_max [MPa]                     | 146.247    | 144.616  |                                     |            | 146.148    | 146.148                             |            |            |                                     |            |            |
| shear_min [MPa]                     | 108.218    | 107.011  |                                     |            | 108.155    | 108.155                             |            |            |                                     |            |            |
| <b>Utilization of fatigue limit</b> |            |          | <b>Utilization of fatigue limit</b> |            |            | <b>Utilization of fatigue limit</b> |            |            | <b>Utilization of fatigue limit</b> |            |            |
| Earthside                           | 0.386      | 0.389    | Earthside                           | 0.139      | 0.14       | Earthside                           | 0.444      | 0.448      | Earthside                           | 0.117      | 0.14       |
| Airside                             | -          | -        | Airside                             | -          | -          | Airside                             | -          | -          | Airside                             | -          | -          |
| Shear                               | 0.839      | 0.8377   | Shear                               | -          | -          | Shear                               | 0.838      | 0.8370     | Shear                               | -          | -          |



# B

## Analytical calculation procedure for fatigue

This appendix includes the hand calculations used in order to validate the results generated from the script with respect to the calculation process for fatigue according to Eurocode. The calculations are based on sectional forces and dimensions obtained from the FE-model produced by the developed Python script. The reinforcement amounts are also extracted from the preliminary design generated by the same script. The structure of the analytical calculation is set up in a way which makes it possible to control specific variables and easily compare it to specific variables in the script.

## Fatigue verification for compressed concrete and shear in carriageway slab

Calculation is performed according to:

TSFS 2018:57  
 TRV00226 Allmänna  
 TRV00227 Byggande  
 SS-EN1990  
 SS-EN1991-1-1  
 SS-EN1991-2  
 SS-EN1992-1-1  
 SS-EN1992-2

Results extracted from analysis in Brigade Plus:

Snitt 1:  $M_{Ed,max1} := 423.255 \cdot \text{kN} \cdot \text{m}$   $M_{Ed,min1} := 312.168 \text{kN} \cdot \text{m}$

$N_{Ed,max1} := -117.485 \text{kN}$   $N_{Ed,min1} := -98.53 \text{kN}$

Snitt 2:  $M_{Ed,max2} := -382.646 \cdot \text{kN} \cdot \text{m}$   $M_{Ed,min2} := -293.581 \text{kN} \cdot \text{m}$

$N_{Ed,max2} := -200.435 \text{kN}$   $N_{Ed,min2} := -164.127 \text{kN}$

Snitt 3:  $M_{Ed,max3} := -382.631 \cdot \text{kN} \cdot \text{m}$   $M_{Ed,min3} := -293.552 \text{kN} \cdot \text{m}$

$N_{Ed,max3} := -200.685 \text{kN}$   $N_{Ed,min3} := -164.338 \text{kN}$

$$M_{Ed,max} := \begin{pmatrix} M_{Ed,max1} \\ M_{Ed,max2} \\ M_{Ed,max3} \end{pmatrix} \quad M_{Ed,min} := \begin{pmatrix} M_{Ed,min1} \\ M_{Ed,min2} \\ M_{Ed,min3} \end{pmatrix}$$

$$N_{Ed,max} := \begin{pmatrix} N_{Ed,max1} \\ N_{Ed,max2} \\ N_{Ed,max3} \end{pmatrix} \quad N_{Ed,min} := \begin{pmatrix} N_{Ed,min1} \\ N_{Ed,min2} \\ N_{Ed,min3} \end{pmatrix}$$

Sectional geometry and constants:

Calculated in unit width of the bridge deck.

$b := 1 \text{m}$   $L_{\text{span}} := 15 \text{m}$

$h := 0.8 \text{m}$

Reinforcement amount extracted from script:

Snitt 1:  $A_{s,bot1} := 0.004590 \text{m}^2$   $A_{s,top1} := 0.002139 \text{m}^2$

$d_{bot1} := 709 \text{mm}$   $d_{top1} := 59 \text{mm}$

Snitt 2:  $A_{s,bot2} := 0.002139 \cdot \text{m}^2$   $A_{s,top2} := 0.003219 \cdot \text{m}^2$

$$d_{\text{bot}2} := 741 \text{ mm}$$

$$d_{\text{top}2} := 59 \text{ mm}$$

Snitt 3:

$$A_{\text{s.bot}3} := 0.002139 \cdot \text{m}^2$$

$$A_{\text{s.top}3} := 0.003219 \cdot \text{m}^2$$

$$d_{\text{bot}3} := 741 \text{ mm}$$

$$d_{\text{top}3} := 59 \text{ mm}$$

$$A_{\text{s.bot}} := \begin{pmatrix} A_{\text{s.bot}1} \\ A_{\text{s.bot}2} \\ A_{\text{s.bot}3} \end{pmatrix} = \begin{pmatrix} 4.59 \times 10^{-3} \\ 2.139 \times 10^{-3} \\ 2.139 \times 10^{-3} \end{pmatrix} \text{ m}^2$$

$$A_{\text{s.top}} := \begin{pmatrix} A_{\text{s.top}1} \\ A_{\text{s.top}2} \\ A_{\text{s.top}3} \end{pmatrix} = \begin{pmatrix} 2.139 \times 10^{-3} \\ 3.219 \times 10^{-3} \\ 3.219 \times 10^{-3} \end{pmatrix} \text{ m}^2$$

$$d_{\text{bot}} := \begin{pmatrix} d_{\text{bot}1} \\ d_{\text{bot}2} \\ d_{\text{bot}3} \end{pmatrix} = \begin{pmatrix} 709 \\ 741 \\ 741 \end{pmatrix} \cdot \text{mm}$$

$$d_{\text{top}} := \begin{pmatrix} d_{\text{top}1} \\ d_{\text{top}2} \\ d_{\text{top}3} \end{pmatrix} = \begin{pmatrix} 59 \\ 59 \\ 59 \end{pmatrix} \cdot \text{mm}$$

$$E_{\text{cm}} := 34077146199.2 \text{ Pa}$$

$$E_{\text{s}} := 200 \text{ GPa}$$

$$A_{\text{c}} := b \cdot h = 0.8 \text{ m}^2 \quad \alpha := \frac{E_{\text{s}}}{E_{\text{cm}}} = 5.869$$

Calculating compressed zone for each studied section in stage II:

According to 'Bärande konstruktioner Del 2' Al-Emrani et al. 2011

Modular ratio assigned to the reinforcement area depending on compressed top or bottom.

$$\alpha_{\text{top}.1} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.max}_0} > 0 \\ \alpha & \text{if } M_{\text{Ed.max}_0} < 0 \end{cases} = 4.869$$

$$\alpha_{\text{bot}.1} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top}.1} = \alpha \\ \alpha & \text{if } \alpha_{\text{top}.1} = (\alpha - 1) \end{cases} = 5.869$$

$$d_{\text{top}.1} := \begin{cases} d_{\text{top}0} & \text{if } M_{\text{Ed.max}_0} > 0 \\ d_{\text{bot}0} & \text{if } M_{\text{Ed.max}_0} < 0 \end{cases}$$

$$d_{\text{bot}.1} := \begin{cases} d_{\text{bot}0} & \text{if } M_{\text{Ed.max}_0} > 0 \\ d_{\text{top}0} & \text{if } M_{\text{Ed.max}_0} < 0 \end{cases}$$

$$\alpha_{\text{top}.2} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.max}_1} > 0 \\ \alpha & \text{if } M_{\text{Ed.max}_1} < 0 \end{cases} = 5.869$$

$$\alpha_{\text{bot}.2} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top}.2} = \alpha \\ \alpha & \text{if } \alpha_{\text{top}.2} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{\text{top}.2} := \begin{cases} d_{\text{top}1} & \text{if } M_{\text{Ed.max}_1} > 0 \\ d_{\text{bot}1} & \text{if } M_{\text{Ed.max}_1} < 0 \end{cases}$$

$$d_{\text{bot}.2} := \begin{cases} d_{\text{bot}1} & \text{if } M_{\text{Ed.max}_1} > 0 \\ d_{\text{top}1} & \text{if } M_{\text{Ed.max}_1} < 0 \end{cases}$$

$$\alpha_{\text{top}.3} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.max}_2} > 0 \\ \alpha & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases} = 5.869$$

$$\alpha_{\text{bot}.3} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top}.3} = \alpha \\ \alpha & \text{if } \alpha_{\text{top}.3} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{\text{top}.3} := \begin{cases} d_{\text{top}2} & \text{if } M_{\text{Ed.max}_2} > 0 \\ d_{\text{bot}2} & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases}$$

$$d_{\text{bot}.3} := \begin{cases} d_{\text{bot}2} & \text{if } M_{\text{Ed.max}_2} > 0 \\ d_{\text{top}2} & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases}$$

$$\alpha_{\text{top.1.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_0} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases} = 4.869 \quad \alpha_{\text{bot.1.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.1.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.1.min}} = (\alpha - 1) \end{cases} = 5.869$$

$$d_{\text{top.1.min}} := \begin{cases} d_{\text{top}_0} & \text{if } M_{\text{Ed.min}_0} > 0 \\ d_{\text{bot}_0} & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases} \quad d_{\text{bot.1.min}} := \begin{cases} d_{\text{bot}_0} & \text{if } M_{\text{Ed.min}_0} > 0 \\ d_{\text{top}_0} & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases}$$

$$\alpha_{\text{top.2.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_1} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases} = 5.869 \quad \alpha_{\text{bot.2.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.2.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.2.min}} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{\text{top.2.min}} := \begin{cases} d_{\text{top}_1} & \text{if } M_{\text{Ed.min}_1} > 0 \\ d_{\text{bot}_1} & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases} \quad d_{\text{bot.2.min}} := \begin{cases} d_{\text{bot}_1} & \text{if } M_{\text{Ed.min}_1} > 0 \\ d_{\text{top}_1} & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases}$$

$$\alpha_{\text{top.3.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_2} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases} = 5.869 \quad \alpha_{\text{bot.3.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.3.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.3.min}} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{\text{top.3.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.min}_2} > 0 \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases} \quad d_{\text{bot.3.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.min}_2} > 0 \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases}$$

Sectional constants for maximum moments:

### Section 1: Midsection

Guess  $x_{\text{cc}}$  until sigma is close to or zero:

$$x_{\text{cc}1} := 178.32 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{\text{cc.1}} := x_{\text{cc}1} \cdot b = 1.783 \times 10^5 \cdot \text{mm}^2$$

$$A_{\text{II.1}} := A_{\text{cc.1}} + \alpha_{\text{top.1}} \cdot A_{\text{s.top}_0} + \alpha_{\text{bot.1}} \cdot A_{\text{s.bot}_0} = 0.216 \text{ m}^2$$

$$x_{\text{tp}1} := \frac{A_{\text{cc.1}} \cdot \frac{x_{\text{cc}1}}{2} + \alpha_{\text{top.1}} \cdot A_{\text{s.top}_0} \cdot d_{\text{top.1}} + \alpha_{\text{bot.1}} \cdot A_{\text{s.bot}_0} \cdot d_{\text{bot.1}}}{A_{\text{II.1}}} = 165.125 \cdot \text{mm}$$

$$I_{\text{II.1}} := \frac{b \cdot x_{\text{cc}1}^3}{12} + b \cdot x_{\text{cc}1} \cdot \left( \frac{x_{\text{cc}1}}{2} - x_{\text{tp}1} \right)^2 + \alpha_{\text{top.1}} \cdot A_{\text{s.top}_0} \cdot (x_{\text{tp}1} - d_{\text{top.1}})^2 + \alpha_{\text{bot.1}} \cdot A_{\text{s.bot}_0} \cdot (d_{\text{bot.1}} - x_{\text{tp}1})^2$$

$$I_{\text{II.1}} = 9.587 \times 10^{-3} \text{ m}^4$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_1 := x_{\text{cc}1} - x_{\text{tp}1} = 13.195 \cdot \text{mm}$$

$$e_{N.1} := \left| x_{tp1} - \frac{h}{2} \right| = 234.875 \cdot \text{mm}$$

$$\sigma_{cc.max1} := \frac{N_{Ed.max0}}{A_{II.1}} + \frac{N_{Ed.max0} \cdot e_{N.1} + |M_{Ed.max0}|}{I_{II.1}} \cdot z_1 = -1.953 \times 10^{-4} \cdot \text{MPa}$$

## Section 2: Edgesection

Guess x.cc:

$$x_{cc2} := 169.5 \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.2} := x_{cc2} \cdot b = 0.17 \text{m}^2$$

$$A_{II.2} := A_{cc.2} + \alpha_{top.2} \cdot A_{s.top_1} + \alpha_{bot.2} \cdot A_{s.bot_1} = 0.199 \text{m}^2$$

$$x_{tp2} := \frac{A_{cc.2} \cdot \frac{x_{cc2}}{2} + \alpha_{top.2} \cdot A_{s.top_1} \cdot d_{top.2} + \alpha_{bot.2} \cdot A_{s.bot_1} \cdot d_{bot.2}}{A_{II.2}} = 0.146 \text{m}$$

$$I_{II.2} := \frac{b \cdot x_{cc2}^3}{12} + b \cdot x_{cc2} \cdot \left( \frac{x_{cc2}}{2} - x_{tp2} \right)^2 + \alpha_{top.2} \cdot A_{s.top_1} \cdot (x_{tp2} - d_{top.2})^2 + \alpha_{bot.2} \cdot A_{s.bot_1} \cdot (d_{bot.2} - x_{tp2})^2$$

$$I_{II.2} = 7.809 \times 10^{-3} \text{m}^4$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_2 := x_{cc2} - x_{tp2} = 0.024 \text{m}$$

$$e_{N.2} := \left| x_{tp2} - \frac{h}{2} \right| = 0.254 \text{m}$$

$$\sigma_{cc.max2} := \frac{N_{Ed.max1}}{A_{II.2}} + \frac{N_{Ed.max1} \cdot e_{N.2} + |M_{Ed.max1}|}{I_{II.2}} \cdot z_2 = 2.481 \times 10^{-5} \cdot \text{MPa}$$

## Section 3: Edgesection

Guess x.cc:

$$x_{cc3} := 169.55 \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.3} := x_{cc3} \cdot b = 0.17 \text{m}^2$$

$$A_{II.3} := A_{cc.3} + \alpha_{top.3} \cdot A_{s.top_3} + \alpha_{bot.3} \cdot A_{s.bot_3} = 0.199 \text{ m}^2$$

$$x_{tp3} := \frac{A_{cc.3} \cdot \frac{x_{cc3}}{2} + \alpha_{top.3} \cdot A_{s.top_2} \cdot d_{top.3} + \alpha_{bot.3} \cdot A_{s.bot_2} \cdot d_{bot.3}}{A_{II.3}} = 0.146 \text{ m}$$

$$I_{II.3} := \frac{b \cdot x_{cc3}^3}{12} + b \cdot x_{cc3} \cdot \left( \frac{x_{cc3}}{2} - x_{tp3} \right)^2 + \alpha_{top.3} \cdot A_{s.top_2} \cdot (x_{tp3} - d_{top.3})^2 + \alpha_{bot.3} \cdot A_{s.bot_2} \cdot (d_{bot.3} - x_{tp3})^2$$

$$I_{II.3} = 7.809 \times 10^{-3} \text{ m}^4$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_3 := x_{cc3} - x_{tp3} = 0.024 \text{ m}$$

$$e_{N.3} := \left| x_{tp3} - \frac{h}{2} \right| = 0.254 \text{ m}$$

$$\sigma_{cc.max3} := \frac{N_{Ed.max2}}{A_{II.3}} + \frac{N_{Ed.max2} \cdot e_{N.3} + |M_{Ed.max2}|}{I_{II.3}} \cdot z_3 = 6.519 \times 10^{-4} \cdot \text{MPa}$$

### Sectional constants for minimum moments:

#### **Section 1: Midsection, min moment**

Guess x.cc until sigma is close to or zero:

$$x_{cc1.min} := 180.3 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc1.min} := x_{cc1.min} \cdot b = 0.18 \text{ m}^2$$

$$A_{II.1.min} := A_{cc1.min} + \alpha_{top.1.min} \cdot A_{s.top_0} + \alpha_{bot.1.min} \cdot A_{s.bot_0} = 0.218 \text{ m}^2$$

$$x_{tp1.min} := \frac{A_{cc1.min} \cdot \frac{x_{cc1.min}}{2} + \alpha_{top.1.min} \cdot A_{s.top_0} \cdot d_{top.1.min} + \alpha_{bot.1.min} \cdot A_{s.bot_0} \cdot d_{bot.1.min}}{A_{II.1.min}} = 0.165 \text{ m}$$

$$I_{II.1.min} := \frac{b \cdot x_{cc1.min}^3}{12} + b \cdot x_{cc1.min} \cdot \left( \frac{x_{cc1.min}}{2} - x_{tp1.min} \right)^2 + \alpha_{top.1.min} \cdot A_{s.top_0} \cdot (x_{tp1.min} - d_{top.1.min})^2 + \alpha_{bot.1.min} \cdot A_{s.bot_0} \cdot (d_{bot.1.min} - x_{tp1.min})^2 \dots$$

$$I_{II.1.min} = 9.588 \times 10^{-3} \text{ m}^4$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{1.min} := x_{cc1.min} - x_{tp1.min} = 0.015 \text{ m}$$

$$e_{N.1.min} := \left| x_{tp1.min} - \frac{h}{2} \right| = 0.235 \text{ m}$$

$$\sigma_{cc.min1} := \frac{N_{Ed.min0}}{A_{II.1.min}} + \frac{N_{Ed.min0} \cdot e_{N.1.min} + |M_{Ed.min0}|}{I_{II.1.min}} \cdot z_{1.min} = 8.899 \times 10^{-4} \cdot \text{MPa}$$

### **Section 2: Edgesection, min moment**

Guess x.cc until sigma is close to or zero:

$$x_{cc2.min} := 171.35 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.2.min} := x_{cc2.min} \cdot b = 0.171 \text{ m}^2$$

$$A_{II.2.min} := A_{cc.2.min} + \alpha_{top.2.min} \cdot A_{s.top1} + \alpha_{bot.2.min} \cdot A_{s.bot1} = 0.201 \text{ m}^2$$

$$x_{tp2.min} := \frac{A_{cc.2.min} \cdot \frac{x_{cc2.min}}{2} + \alpha_{top.2.min} \cdot A_{s.top1} \cdot d_{top.2.min} + \alpha_{bot.2.min} \cdot A_{s.bot1} \cdot d_{bot.2.min}}{A_{II.2.min}} = 0.146 \text{ m}$$

$$I_{II.2.min} := \frac{b \cdot x_{cc2.min}^3}{12} + b \cdot x_{cc2.min} \cdot \left( \frac{x_{cc2.min}}{2} - x_{tp2.min} \right)^2 + \alpha_{top.2.min} \cdot A_{s.top1} \cdot (x_{tp2.min} - d_{top.2.min})^2 + \alpha_{bot.2.min} \cdot A_{s.bot1} \cdot (d_{bot.2.min} - x_{tp2.min})^2$$

$$I_{II.2.min} = 7.81 \times 10^{-3} \text{ m}^4$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{2.min} := x_{cc2.min} - x_{tp2.min} = 0.025 \text{ m}$$

$$e_{N.2.min} := \left| x_{tp2.min} - \frac{h}{2} \right| = 0.254 \text{ m}$$

$$\sigma_{cc.min2} := \frac{N_{Ed.min1}}{A_{II.2.min}} + \frac{N_{Ed.min1} \cdot e_{N.2.min} + |M_{Ed.min1}|}{I_{II.2.min}} \cdot z_{2.min} = -6.415 \times 10^{-5} \cdot \text{MPa}$$

### **Section 3: Edgesection, min moment**

Guess x.cc until sigma is close to or zero:

$$x_{cc3.min} := 171.4 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.3.min} := x_{cc3.min} \cdot b = 0.171 \text{ m}^2$$

$$A_{II.3.min} := A_{cc.3.min} + \alpha_{top.3.min} \cdot A_{s.top_2} + \alpha_{bot.3.min} \cdot A_{s.bot_2} = 0.201 \text{ m}^2$$

$$x_{tp3.min} := \frac{A_{cc.3.min} \cdot \frac{x_{cc3.min}}{2} + \alpha_{top.3.min} \cdot A_{s.top_2} \cdot d_{top.3.min} + \alpha_{bot.3.min} \cdot A_{s.bot_2} \cdot d_{bot.3.min}}{A_{II.3.min}} = 0.146 \text{ m}$$

$$I_{II.3.min} := \frac{b \cdot x_{cc3.min}^3}{12} + b \cdot x_{cc3.min} \cdot \left( \frac{x_{cc3.min}}{2} - x_{tp3.min} \right)^2 + \alpha_{top.3.min} \cdot A_{s.top_2} \cdot (x_{tp3.min} - d_{top.3.min})^2 + \alpha_{bot.3.min} \cdot A_{s.bot_2} \cdot (d_{bot.3.min} - x_{tp3.min})^2$$

$$I_{II.3.min} = 7.81 \times 10^{-3} \text{ m}^4$$

Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{3.min} := x_{cc3.min} - x_{tp3.min} = 25.403 \cdot \text{mm}$$

$$e_{N.3.min} := \left| x_{tp3.min} - \frac{h}{2} \right| = 254.003 \cdot \text{mm}$$

$$\sigma_{cc.min3} := \frac{N_{Ed.min2}}{A_{II.3.min}} + \frac{N_{Ed.min2} \cdot e_{N.3.min} + |M_{Ed.min2}|}{I_{II.3.min}} \cdot z_{3.min} = 2.283 \times 10^{-4} \cdot \text{MPa}$$

### Summary of sectional constants, maximum and minimum

Final compressed zone height for each section maximum and minimum moment:

$$x_{cc} := \begin{pmatrix} x_{cc1} \\ x_{cc2} \\ x_{cc3} \end{pmatrix} = \begin{pmatrix} 178.32 \\ 169.5 \\ 169.55 \end{pmatrix} \cdot \text{mm} \quad x_{cc.min} := \begin{pmatrix} x_{cc1.min} \\ x_{cc2.min} \\ x_{cc3.min} \end{pmatrix} = \begin{pmatrix} 180.3 \\ 171.35 \\ 171.4 \end{pmatrix} \cdot \text{mm}$$

Cross sectional constants for each studied section maximum moment:

$$x_{tp} := \begin{pmatrix} x_{tp1} \\ x_{tp2} \\ x_{tp3} \end{pmatrix} = \begin{pmatrix} 165.125 \\ 145.764 \\ 145.77 \end{pmatrix} \cdot \text{mm} \quad I_y := \begin{pmatrix} I_{II.1} \\ I_{II.2} \\ I_{II.3} \end{pmatrix} = \begin{pmatrix} 9.587 \times 10^{-3} \\ 7.809 \times 10^{-3} \\ 7.809 \times 10^{-3} \end{pmatrix} \text{ m}^4 \quad A_{II} := \begin{pmatrix} A_{II.1} \\ A_{II.2} \\ A_{II.3} \end{pmatrix} = \begin{pmatrix} 0.216 \\ 0.199 \\ 0.199 \end{pmatrix} \text{ m}^2$$

Cross sectional constants for each studied section minimum moment:

$$x_{tp.min} := \begin{pmatrix} x_{tp1.min} \\ x_{tp2.min} \\ x_{tp3.min} \end{pmatrix} = \begin{pmatrix} 0.165 \\ 0.146 \\ 0.146 \end{pmatrix} \text{ m} \quad A_{II.min} := \begin{pmatrix} A_{II.1.min} \\ A_{II.2.min} \\ A_{II.3.min} \end{pmatrix} = \begin{pmatrix} 0.218 \\ 0.201 \\ 0.201 \end{pmatrix} \text{ m}^2$$

$$I_{y.min} := \begin{pmatrix} I_{II.1.min} \\ I_{II.2.min} \\ I_{II.3.min} \end{pmatrix} = \begin{pmatrix} 9.588 \times 10^{-3} \\ 7.81 \times 10^{-3} \\ 7.81 \times 10^{-3} \end{pmatrix} \text{ m}^4$$

Stresses in most compressed fibre due to bending:

Snitt 1:

$$\sigma_{cd.max1} := \frac{N_{Ed.max0}}{A_{II0}} + \frac{N_{Ed.max0} \cdot e_{N.1} + |M_{Ed.max0}|}{I_{y0}} \cdot x_{tp0} = -7.359 \cdot \text{MPa}$$

$$\sigma_{cd.min1} := \frac{N_{Ed.min0}}{A_{II.min0}} + \frac{N_{Ed.min0} \cdot e_{N.1.min} + |M_{Ed.min0}|}{I_{y.min0}} \cdot x_{tp.min0} = -5.435 \cdot \text{MPa}$$

Snitt 2:

$$\sigma_{cd.max2} := \frac{N_{Ed.max1}}{A_{II1}} + \frac{N_{Ed.max1} \cdot e_{N.2} + |M_{Ed.max1}|}{I_{y1}} \cdot x_{tp1} = -7.2 \cdot \text{MPa}$$

$$\sigma_{cd.min2} := \frac{N_{Ed.min1}}{A_{II.min1}} + \frac{N_{Ed.min1} \cdot e_{N.2.min} + |M_{Ed.min1}|}{I_{y.min1}} \cdot x_{tp.min1} = -5.526 \cdot \text{MPa}$$

Snitt 3:

$$\sigma_{cd.max3} := \frac{N_{Ed.max2}}{A_{II2}} + \frac{N_{Ed.max2} \cdot e_{N.3} + |M_{Ed.max2}|}{I_{y2}} \cdot x_{tp2} = -7.199 \cdot \text{MPa}$$

$$\sigma_{cd.min3} := \frac{N_{Ed.min2}}{A_{II.min2}} + \frac{N_{Ed.min2} \cdot e_{N.3.min} + |M_{Ed.min2}|}{I_{y.min2}} \cdot x_{tp.min2} = -5.526 \cdot \text{MPa}$$

Summarizing stresses:

$$\sigma_{cd.max} := \begin{pmatrix} \sigma_{cd.max1} \\ \sigma_{cd.max2} \\ \sigma_{cd.max3} \end{pmatrix} = \begin{pmatrix} -7.359 \\ -7.2 \\ -7.199 \end{pmatrix} \cdot \text{MPa} \quad \sigma_{cd.min} := \begin{pmatrix} \sigma_{cd.min1} \\ \sigma_{cd.min2} \\ \sigma_{cd.min3} \end{pmatrix} = \begin{pmatrix} -5.435 \\ -5.526 \\ -5.526 \end{pmatrix} \cdot \text{MPa}$$

Stress ranges:

$$\Delta\sigma_{cd1} := |\sigma_{cd.max0} - \sigma_{cd.min0}| = 1.925 \cdot \text{MPa}$$

$$\Delta\sigma_{cd2} := |\sigma_{cd.max1} - \sigma_{cd.min1}| = 1.673 \cdot \text{MPa}$$

$$\Delta\sigma_{cd3} := \left| \sigma_{cd.max_2} - \sigma_{cd.min_2} \right| = 1.673 \cdot \text{MPa}$$

## Fatigue verification, concrete under compression or shear

Verification of concrete under compression:

According to EN-1992-2 6.8.7

Approximated loading cycles during one bridge lifetime:

$$N_{\text{years}} := 100$$

$$\dot{A}DT := 10000$$

$$\text{Category}_{\text{traffic}} := \begin{cases} "1" & \text{if } 6000 < \dot{A}DT \leq 24000 = "1" \\ "2" & \text{if } 1500 < \dot{A}DT \leq 6000 \\ "3" & \text{if } 600 < \dot{A}DT \leq 1500 \\ "4" & \text{if } \dot{A}DT \leq 600 \end{cases}$$

$$N_{\text{obs}} := \begin{cases} 2 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "1" \\ 0.5 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "2" \\ 0.125 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "3" \\ 0.05 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "4" \end{cases} = 2 \times 10^6 \quad \text{number of lorries on slow lane per year}$$

$$n := N_{\text{obs}} \cdot N_{\text{years}} = 2 \times 10^8$$

Concrete:

Class: C35/40

$$\gamma_C := 1.5 \quad \text{Partial factors for concrete, EN1992-1-1 2.4.2.4, Table 2.1N}$$

The characteristic strength of concrete obtained according to EN1992-1-1, 3.1.3, Table 3.1

$$f_{ck} := 35 \text{MPa}$$

$$f_{ctk} := 2 \text{MPa}$$

$$f_{cd} := \frac{f_{ck}}{\gamma_C} = 23.333 \cdot \text{MPa}$$

$$f_{ctd} := \frac{f_{ctk}}{\gamma_C} = 1.333 \cdot \text{MPa}$$

$$t_0 := 28$$

Concrete age when loaded

$$CEM_{\text{class}} := "52.5N"$$

Dependency of cement class, see EN-1992-1-1, 3.1.2 (6)

$$s_{\text{cem}} := 0.2 \quad \text{for } 52.5N$$

$$k_1 := 0.85$$

Recommended value in EC

$$\beta_{cc} := e^{\left[ s_{cem} \cdot \left[ 1 - \left( \frac{28}{t_0} \right)^{0.5} \right] \right]} = 1 \quad \text{According to EN-1992-1-1, 3.1.2 (6)}$$

$$f_{cd.fat} := k_1 \cdot \beta_{cc} \cdot f_{cd} \cdot \left( 1 - \frac{f_{ck}}{250 \text{ MPa}} \right) = 17.057 \cdot \text{MPa} \quad \text{Fatigue strength, EN1992-2, 6.8.7 (6.76)}$$

### Palmgren-Miner verification method, EN1992-2 6.8.7

Relative over and under stresslimits

(Compressive stresses as  
"positive" sign.)

$$E_{cd.min.i} := \frac{-\sigma_{cd.min}}{f_{cd.fat}} = \begin{pmatrix} 0.319 \\ 0.324 \\ 0.324 \end{pmatrix} \quad E_{cd.max.i} := \frac{-\sigma_{cd.max}}{f_{cd.fat}} = \begin{pmatrix} 0.431 \\ 0.422 \\ 0.422 \end{pmatrix} \quad \text{EN1992-2, 6.8.7 (6.108-6.109)}$$

Stress ratio

$$R_{equ} := \frac{E_{cd.min.i}}{E_{cd.max.i}} = \begin{pmatrix} 0.738 \\ 0.768 \\ 0.768 \end{pmatrix} \quad \text{EN1992-2, 6.8.7 (6.107)}$$

Corresponding cycles:

$$N_c := 10^{\left( 14 \cdot \frac{1 - E_{cd.max.i}}{\sqrt{1 - R_{equ}}} \right)} = \begin{pmatrix} 3.664 \times 10^{15} \\ 6.071 \times 10^{16} \\ 6.053 \times 10^{16} \end{pmatrix} \quad \text{EN1992-2, 6.8.7 (6.106)}$$

Final damage for compressed concrete:

$$D := \frac{n}{N_c} = \begin{pmatrix} 5.458 \times 10^{-8} \\ 3.294 \times 10^{-9} \\ 3.304 \times 10^{-9} \end{pmatrix} \quad \text{EN1992-2, 6.8.7 (6.105)}$$

Verification of shear, If no shear reinforcement is needed:

According to EN 1992-1-1 6.8.7 (4).

**NOTE: In this particular case, shear reinforcement is applied in section 2 and 3 which makes the check below un-necessary. The results below are therefore evaluated as if no reinforcement was applied.**

From Brigade:

Snitt 2       $V_{Rd.c2} := 3575.247 \text{ kN}$       Snitt 2:       $V_{Ed.max2} := 135.077 \cdot \text{kN}$        $V_{Ed.min2} := 124.744 \cdot \text{kN}$

Snitt 3

$$V_{Rd.c3} := 3575.247 \text{ kN}$$

Snitt 3:

$$V_{Ed.max3} := 135.071 \cdot \text{kN}$$

$$V_{Ed.min3} := 124.744 \cdot \text{kN}$$

$$V_{Ed.max} := \begin{pmatrix} V_{Ed.max2} \\ V_{Ed.max3} \end{pmatrix} \quad V_{Ed.min} := \begin{pmatrix} V_{Ed.min2} \\ V_{Ed.min3} \end{pmatrix}$$

$$V_{check} := \frac{V_{Ed.min}}{V_{Ed.max}} = \begin{pmatrix} 0.924 \\ 0.924 \end{pmatrix}$$

$$U_{V2} := \begin{cases} 0.5 + 0.45 \cdot \left| \frac{V_{Ed.min0}}{V_{Rd.c2}} \right| & \text{if } V_{check0} \geq 0 \\ 0.5 - \left| \frac{V_{Ed.min0}}{V_{Rd.c2}} \right| & \text{if } V_{check0} < 0 \end{cases} = 0.516$$

$$Check_{V2} := \begin{cases} \text{"Ok"} & \text{if } \left| \frac{V_{Ed.max0}}{V_{Rd.c2}} \right| \leq U_{V2} \\ \text{"Not Ok"} & \text{otherwise} \end{cases} = \text{"Ok"}$$

$$U_{V3} := \begin{cases} 0.5 + 0.45 \cdot \left| \frac{V_{Ed.min1}}{V_{Rd.c3}} \right| & \text{if } V_{check1} \geq 0 \\ 0.5 - \left| \frac{V_{Ed.min1}}{V_{Rd.c3}} \right| & \text{if } V_{check1} < 0 \end{cases} = 0.516$$

$$Check_{V3} := \begin{cases} \text{"Ok"} & \text{if } \left| \frac{V_{Ed.max1}}{V_{Rd.c3}} \right| \leq U_{V3} \\ \text{"Not Ok"} & \text{otherwise} \end{cases} = \text{"Ok"}$$

## Fatigue verification for reinforcement in carriageway slab

Calculation is performed according to:

TSFS 2018:57  
 TRV00226 Allmänna  
 TRV00227 Byggande  
 SS-EN1990  
 SS-EN1991-1-1  
 SS-EN1991-2  
 SS-EN1992-1-1  
 SS-EN1992-2

Results extracted from analysis in Brigade Plus:

The axle loads of fatigue load model 3 is multiplied by factor 1.40 for verification of reinforcement.

|                 |  |                                     |
|-----------------|--|-------------------------------------|
| <b>Snitt 1:</b> | $M_{Ed.max1} := 467.69 \cdot kN \cdot m$ | $M_{Ed.min1} := 312.168 kN \cdot m$ |
|                 | $N_{Ed.max1} := -125.067 kN$             | $N_{Ed.min1} := -98.53 kN$          |

|                 |  |                                      |
|-----------------|--|--------------------------------------|
| <b>Snitt 2:</b> | $M_{Ed.max2} := -419.558 \cdot kN \cdot m$ | $M_{Ed.min2} := -294.868 kN \cdot m$ |
|                 | $V_{Ed.max2} := 219.453 \cdot kN$          | $V_{Ed.min2} := 188.174 \cdot kN$    |
|                 | $N_{Ed.max2} := -216.392 kN$               | $N_{Ed.min2} := -165.562 kN$         |

|                 |   |                                      |
|-----------------|---|--------------------------------------|
| <b>Snitt 3:</b> | $M_{Ed.max3} := -419.55 \cdot kN \cdot m$ | $M_{Ed.min3} := -294.839 kN \cdot m$ |
|                 | $V_{Ed.max3} := 219.454 \cdot kN$         | $V_{Ed.min3} := 188.163 \cdot kN$    |
|                 | $N_{Ed.max3} := -216.66 kN$               | $N_{Ed.min3} := -165.775 kN$         |

$$M_{Ed.max} := \begin{pmatrix} M_{Ed.max1} \\ M_{Ed.max2} \\ M_{Ed.max3} \end{pmatrix} \quad M_{Ed.min} := \begin{pmatrix} M_{Ed.min1} \\ M_{Ed.min2} \\ M_{Ed.min3} \end{pmatrix}$$

$$V_{Ed.max} := \begin{pmatrix} V_{Ed.max2} \\ V_{Ed.max3} \end{pmatrix} \quad V_{Ed.min} := \begin{pmatrix} V_{Ed.min2} \\ V_{Ed.min3} \end{pmatrix}$$

$$N_{Ed.max} := \begin{pmatrix} N_{Ed.max1} \\ N_{Ed.max2} \\ N_{Ed.max3} \end{pmatrix} \quad N_{Ed.min} := \begin{pmatrix} N_{Ed.min1} \\ N_{Ed.min2} \\ N_{Ed.min3} \end{pmatrix}$$

Sectional geometry and constants:

Calculated in unit width of the bridge deck.

$b := 1m$        $L_{spann} := 15m$

$h := 0.8m$

Reinforcement amount extracted from script:

|          |                                     |                                     |
|----------|-------------------------------------|-------------------------------------|
| Snitt 1: | $A_{s.bot1} := 0.004590m^2$         | $A_{s.top1} := 0.0021399m^2$        |
|          | $d_{bot1} := 709mm$                 | $d_{top1} := 59mm$                  |
| Snitt 2: | $A_{s.bot2} := 0.0021399 \cdot m^2$ | $A_{s.top2} := 0.0032197 \cdot m^2$ |
|          | $d_{bot2} := 741mm$                 | $d_{top2} := 59mm$                  |
| Snitt 3: | $A_{s.bot3} := 0.0021399 \cdot m^2$ | $A_{s.top3} := 0.0032197 \cdot m^2$ |
|          | $d_{bot3} := 741mm$                 | $d_{top3} := 59mm$                  |

$$A_{s.bot} := \begin{pmatrix} A_{s.bot1} \\ A_{s.bot2} \\ A_{s.bot3} \end{pmatrix} = \begin{pmatrix} 4.59 \times 10^{-3} \\ 2.14 \times 10^{-3} \\ 2.14 \times 10^{-3} \end{pmatrix} m^2$$

$$A_{s.top} := \begin{pmatrix} A_{s.top1} \\ A_{s.top2} \\ A_{s.top3} \end{pmatrix} = \begin{pmatrix} 2.14 \times 10^{-3} \\ 3.22 \times 10^{-3} \\ 3.22 \times 10^{-3} \end{pmatrix} m^2$$

$$d_{bot} := \begin{pmatrix} d_{bot1} \\ d_{bot2} \\ d_{bot3} \end{pmatrix} = \begin{pmatrix} 709 \\ 741 \\ 741 \end{pmatrix} \cdot mm$$

$$d_{top} := \begin{pmatrix} d_{top1} \\ d_{top2} \\ d_{top3} \end{pmatrix} = \begin{pmatrix} 59 \\ 59 \\ 59 \end{pmatrix} \cdot mm$$

$$E_{cm} := 34077146199.2Pa$$

$$E_s := 200GPa$$

$$A_c := b \cdot h = 0.8 m^2 \quad \alpha := \frac{E_s}{E_{cm}} = 5.869$$

Calculating compressed zone for each studied section in stage II:

According to 'Bärande konstruktioner Del 2' Al-Emrani et al. 2011

Modular ratio assigned to the reinforcement area depending on compressed top or bottom.

$$\alpha_{top.1} := \begin{cases} ((\alpha - 1)) & \text{if } M_{Ed.max_0} > 0 \\ \alpha & \text{if } M_{Ed.max_0} < 0 \end{cases} = 4.869$$

$$\alpha_{bot.1} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{top.1} = \alpha \\ \alpha & \text{if } \alpha_{top.1} = (\alpha - 1) \end{cases} = 5.869$$

$$d_{top.1} := \begin{cases} d_{top_0} & \text{if } M_{Ed.max_0} > 0 \\ d_{bot_0} & \text{if } M_{Ed.max_0} < 0 \end{cases}$$

$$d_{bot.1} := \begin{cases} d_{bot_0} & \text{if } M_{Ed.max_0} > 0 \\ d_{top_0} & \text{if } M_{Ed.max_0} < 0 \end{cases}$$

$$\alpha_{top.2} := \begin{cases} ((\alpha - 1)) & \text{if } M_{Ed.max_1} > 0 \\ \alpha & \text{if } M_{Ed.max_1} < 0 \end{cases} = 5.869$$

$$\alpha_{bot.2} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{top.2} = \alpha \\ \alpha & \text{if } \alpha_{top.2} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{top.2} := \begin{cases} d_{top_1} & \text{if } M_{Ed.max_1} > 0 \\ d_{bot_1} & \text{if } M_{Ed.max_1} < 0 \end{cases}$$

$$d_{bot.2} := \begin{cases} d_{bot_1} & \text{if } M_{Ed.max_1} > 0 \\ d_{top_1} & \text{if } M_{Ed.max_1} < 0 \end{cases}$$

$$\alpha_{\text{top.3}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.max}_2} > 0 \\ \alpha & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases} = 5.869 \quad \alpha_{\text{bot.3}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.3}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.3}} = (\alpha - 1) \end{cases} = 4.869$$

$$d_{\text{top.3}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases} \quad d_{\text{bot.3}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \end{cases}$$

$$\alpha_{\text{top.1.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_0} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases} = 4.869 \quad \alpha_{\text{bot.1.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.1.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.1.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{top.1.min}} := \begin{cases} d_{\text{top}_0} & \text{if } M_{\text{Ed.min}_0} > 0 \\ d_{\text{bot}_0} & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases} \quad d_{\text{bot.1.min}} := \begin{cases} d_{\text{bot}_0} & \text{if } M_{\text{Ed.min}_0} > 0 \\ d_{\text{top}_0} & \text{if } M_{\text{Ed.min}_0} < 0 \end{cases}$$

$$\alpha_{\text{top.2.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_1} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases} = 5.869 \quad \alpha_{\text{bot.2.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.2.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.2.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{top.2.min}} := \begin{cases} d_{\text{top}_1} & \text{if } M_{\text{Ed.min}_1} > 0 \\ d_{\text{bot}_1} & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases} \quad d_{\text{bot.2.min}} := \begin{cases} d_{\text{bot}_1} & \text{if } M_{\text{Ed.min}_1} > 0 \\ d_{\text{top}_1} & \text{if } M_{\text{Ed.min}_1} < 0 \end{cases}$$

$$\alpha_{\text{top.3.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_2} > 0 \\ \alpha & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases} = 5.869 \quad \alpha_{\text{bot.3.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{top.3.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{top.3.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{top.3.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.min}_2} > 0 \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases} \quad d_{\text{bot.3.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.min}_2} > 0 \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.min}_2} < 0 \end{cases}$$

Sectional constants for maximum moments:

### Section 1: Midsection

Guess  $x_{\text{cc}}$  until sigma is close to or zero:

$$x_{\text{cc1}} := 177.8\text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{\text{cc.1}} := x_{\text{cc1}} \cdot b = 1.778 \times 10^5 \cdot \text{mm}^2$$

$$A_{II.1} := A_{cc.1} + \alpha_{top.1} \cdot A_{s.top_0} + \alpha_{bot.1} \cdot A_{s.bot_0} = 0.215 \text{ m}^2$$

$$x_{tp1} := \frac{A_{cc.1} \cdot \frac{x_{cc1}}{2} + \alpha_{top.1} \cdot A_{s.top_0} \cdot d_{top.1} + \alpha_{bot.1} \cdot A_{s.bot_0} \cdot d_{bot.1}}{A_{II.1}} = 165.092 \cdot \text{mm}$$

$$I_{II.1} := \frac{b \cdot x_{cc1}^3}{12} + b \cdot x_{cc1} \cdot \left( \frac{x_{cc1}}{2} - x_{tp1} \right)^2 + \alpha_{top.1} \cdot A_{s.top_0} \cdot (x_{tp1} - d_{top.1})^2 \dots = 9.587 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{bot.1} \cdot A_{s.bot_0} \cdot (d_{bot.1} - x_{tp1})^2$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_1 := x_{cc1} - x_{tp1} = 12.708 \cdot \text{mm}$$

$$e_{N.1} := \left| x_{tp1} - \frac{h}{2} \right| = 234.908 \cdot \text{mm}$$

$$\sigma_{cc.max1} := \frac{N_{Ed.max0}}{A_{II.1}} + \frac{N_{Ed.max0} \cdot e_{N.1} + |M_{Ed.max0}|}{I_{II.1}} \cdot z_1 = -2.844 \times 10^{-4} \cdot \text{MPa}$$

## **Section 2: Edgesection**

Guess x.cc:

$$x_{cc2} := 169.1 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.2} := x_{cc2} \cdot b = 0.169 \text{ m}^2$$

$$A_{II.2} := A_{cc.2} + \alpha_{top.2} \cdot A_{s.top_1} + \alpha_{bot.2} \cdot A_{s.bot_1} = 0.198 \text{ m}^2$$

$$x_{tp2} := \frac{A_{cc.2} \cdot \frac{x_{cc2}}{2} + \alpha_{top.2} \cdot A_{s.top_1} \cdot d_{top.2} + \alpha_{bot.2} \cdot A_{s.bot_1} \cdot d_{bot.2}}{A_{II.2}} = 0.146 \text{ m}$$

$$I_{II.2} := \frac{b \cdot x_{cc2}^3}{12} + b \cdot x_{cc2} \cdot \left( \frac{x_{cc2}}{2} - x_{tp2} \right)^2 + \alpha_{top.2} \cdot A_{s.top_1} \cdot (x_{tp2} - d_{top.2})^2 \dots = 7.81 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{bot.2} \cdot A_{s.bot_1} \cdot (d_{bot.2} - x_{tp2})^2$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_2 := x_{cc2} - x_{tp2} = 0.023 \text{ m}$$

$$e_{N.2} := \left| x_{tp2} - \frac{h}{2} \right| = 0.254 \text{ m}$$

$$\sigma_{cc.max2} := \frac{N_{Ed.max1}}{A_{II.2}} + \frac{N_{Ed.max1} \cdot e_{N.2} + |M_{Ed.max1}|}{I_{II.2}} \cdot z_2 = 3.362 \times 10^{-4} \cdot \text{MPa}$$

### Section 3: Edgesection

Guess x.cc:

$$x_{cc3} := 169.12 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.3} := x_{cc3} \cdot b = 0.169 \text{ m}^2$$

$$A_{II.3} := A_{cc.3} + \alpha_{top.3} \cdot A_{s.top_2} + \alpha_{bot.3} \cdot A_{s.bot_2} = 0.198 \text{ m}^2$$

$$x_{tp3} := \frac{A_{cc.3} \cdot \frac{x_{cc3}}{2} + \alpha_{top.3} \cdot A_{s.top_2} \cdot d_{top.3} + \alpha_{bot.3} \cdot A_{s.bot_2} \cdot d_{bot.3}}{A_{II.3}} = 0.146 \text{ m}$$

$$I_{II.3} := \frac{b \cdot x_{cc3}^3}{12} + b \cdot x_{cc3} \cdot \left( \frac{x_{cc3}}{2} - x_{tp3} \right)^2 + \alpha_{top.3} \cdot A_{s.top_2} \cdot (x_{tp3} - d_{top.3})^2 + \alpha_{bot.3} \cdot A_{s.bot_2} \cdot (d_{bot.3} - x_{tp3})^2 \dots = 7.81 \times 10^{-3} \text{ m}^4$$

#### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_3 := x_{cc3} - x_{tp3} = 0.023 \text{ m}$$

$$e_{N.3} := \left| x_{tp3} - \frac{h}{2} \right| = 0.254 \text{ m}$$

$$\sigma_{cc.max3} := \frac{N_{Ed.max2}}{A_{II.3}} + \frac{N_{Ed.max2} \cdot e_{N.3} + |M_{Ed.max2}|}{I_{II.3}} \cdot z_3 = -3.09 \times 10^{-4} \cdot \text{MPa}$$

### Sectional constants for minimum moments:

#### Section 1: Midsection, min moment

Guess x.cc until sigma is close to or zero:

$$x_{cc1.min} := 180.2 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc1.min} := x_{cc1.min} \cdot b = 0.18 \text{ m}^2$$

$$A_{II.1.min} := A_{cc1.min} + \alpha_{top.1.min} \cdot A_{s.top_0} + \alpha_{bot.1.min} \cdot A_{s.bot_0} = 0.218 \text{ m}^2$$

$$x_{tp1.min} := \frac{A_{cc1.min} \cdot \frac{x_{cc1.min}}{2} + \alpha_{top.1.min} \cdot A_{s.top_0} \cdot d_{top.1.min} + \alpha_{bot.1.min} \cdot A_{s.bot_0} \cdot d_{bot.1.min}}{A_{II.1.min}}$$

$$x_{tp1.min} = 0.165 \text{ m}$$

$$I_{II.1.min} := \frac{b \cdot x_{cc1.min}^3}{12} + b \cdot x_{cc1.min} \cdot \left( \frac{x_{cc1.min}}{2} - x_{tp1.min} \right)^2 \dots = 9.588 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{top.1.min} \cdot A_{s.top_0} \cdot (x_{tp1.min} - d_{top.1.min})^2 \dots$$

$$+ \alpha_{bot.1.min} \cdot A_{s.bot_0} \cdot (d_{bot.1.min} - x_{tp1.min})^2$$

Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{1.min} := x_{cc1.min} - x_{tp1.min} = 0.015 \text{ m}$$

$$e_{N.1.min} := \left| x_{tp1.min} - \frac{h}{2} \right| = 0.235 \text{ m}$$

$$\sigma_{cc.min1} := \frac{N_{Ed.min_0}}{A_{II.1.min}} + \frac{N_{Ed.min_0} \cdot e_{N.1.min} + |M_{Ed.min_0}|}{I_{II.1.min}} \cdot z_{1.min} = -2.054 \times 10^{-3} \cdot \text{MPa}$$

## Section 2: Edgesection, min moment

Guess x.cc until sigma is close to or zero:

$$x_{cc2.min} := 171.5 \cdot \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.2.min} := x_{cc2.min} \cdot b = 0.172 \text{ m}^2$$

$$A_{II.2.min} := A_{cc.2.min} + \alpha_{top.2.min} \cdot A_{s.top_1} + \alpha_{bot.2.min} \cdot A_{s.bot_1} = 0.201 \text{ m}^2$$

$$x_{tp2.min} := \frac{A_{cc.2.min} \cdot \frac{x_{cc2.min}}{2} + \alpha_{top.2.min} \cdot A_{s.top_1} \cdot d_{top.2.min} + \alpha_{bot.2.min} \cdot A_{s.bot_1} \cdot d_{bot.2.min}}{A_{II.2.min}}$$

$$x_{tp2.min} = 0.146 \text{ m}$$

$$I_{II.2.min} := \frac{b \cdot x_{cc2.min}^3}{12} + b \cdot x_{cc2.min} \cdot \left( \frac{x_{cc2.min}}{2} - x_{tp2.min} \right)^2 \dots = 7.812 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{top.2.min} \cdot A_{s.top_1} \cdot (x_{tp2.min} - d_{top.2.min})^2 \dots$$

$$+ \alpha_{bot.2.min} \cdot A_{s.bot_1} \cdot (d_{bot.2.min} - x_{tp2.min})^2$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{2.min} := x_{cc2.min} - x_{tp2.min} = 0.025 \text{ m}$$

$$e_{N.2.min} := \left| x_{tp2.min} - \frac{h}{2} \right| = 0.254 \text{ m}$$

$$\sigma_{cc.min2} := \frac{N_{Ed.min_1}}{A_{II.2.min}} + \frac{N_{Ed.min_1} \cdot e_{N.2.min} + |M_{Ed.min_1}|}{I_{II.2.min}} \cdot z_{2.min} = 1.874 \times 10^{-4} \text{ MPa}$$

### **Section 3: Edgesection, min moment**

Guess x.cc until sigma is close to or zero:

$$x_{cc3.min} := 171.55 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.3.min} := x_{cc3.min} \cdot b = 0.172 \text{ m}^2$$

$$A_{II.3.min} := A_{cc.3.min} + \alpha_{top.3.min} \cdot A_{s.top_2} + \alpha_{bot.3.min} \cdot A_{s.bot_2} = 0.201 \text{ m}^2$$

$$x_{tp3.min} := \frac{A_{cc.3.min} \cdot \frac{x_{cc3.min}}{2} + \alpha_{top.3.min} \cdot A_{s.top_2} \cdot d_{top.3.min} + \alpha_{bot.3.min} \cdot A_{s.bot_2} \cdot d_{bot.3.min}}{A_{II.3.min}}$$

$$x_{tp3.min} = 0.146 \text{ m}$$

$$I_{II.3.min} := \frac{b \cdot x_{cc3.min}^3}{12} + b \cdot x_{cc3.min} \cdot \left( \frac{x_{cc3.min}}{2} - x_{tp3.min} \right)^2 \dots = 7.812 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{top.3.min} \cdot A_{s.top_2} \cdot (x_{tp3.min} - d_{top.3.min})^2 \dots$$

$$+ \alpha_{bot.3.min} \cdot A_{s.bot_2} \cdot (d_{bot.3.min} - x_{tp3.min})^2$$

### Stress from bending at the neutral axis:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{3.min} := x_{cc3.min} - x_{tp3.min} = 25.523 \cdot \text{mm}$$

$$e_{N.3.min} := \left| x_{tp3.min} - \frac{h}{2} \right| = 253.973 \cdot \text{mm}$$

$$\sigma_{cc.min3} := \frac{N_{Ed.min2}}{A_{II.3.min}} + \frac{N_{Ed.min2} \cdot e_{N.3.min} + |M_{Ed.min2}|}{I_{II.3.min}} \cdot z_{3.min} = 4.734 \times 10^{-4} \cdot \text{MPa}$$

### Summary of sectional constants, maximum and minimum

Final compressed zone height for each section maximum and minimum moment:

$$x_{cc} := \begin{pmatrix} x_{cc1} \\ x_{cc2} \\ x_{cc3} \end{pmatrix} = \begin{pmatrix} 177.8 \\ 169.1 \\ 169.12 \end{pmatrix} \cdot \text{mm} \quad x_{cc.min} := \begin{pmatrix} x_{cc1.min} \\ x_{cc2.min} \\ x_{cc3.min} \end{pmatrix} = \begin{pmatrix} 180.2 \\ 171.5 \\ 171.55 \end{pmatrix} \cdot \text{mm}$$

Cross sectional constants for each studied section maximum moment:

$$x_{tp} := \begin{pmatrix} x_{tp1} \\ x_{tp2} \\ x_{tp3} \end{pmatrix} = \begin{pmatrix} 0.165 \\ 0.146 \\ 0.146 \end{pmatrix} \text{ m} \quad I_y := \begin{pmatrix} I_{II.1} \\ I_{II.2} \\ I_{II.3} \end{pmatrix} = \begin{pmatrix} 9.587 \times 10^{-3} \\ 7.81 \times 10^{-3} \\ 7.81 \times 10^{-3} \end{pmatrix} \text{ m}^4 \quad A_{II} := \begin{pmatrix} A_{II.1} \\ A_{II.2} \\ A_{II.3} \end{pmatrix} = \begin{pmatrix} 0.215 \\ 0.198 \\ 0.198 \end{pmatrix} \text{ m}^2$$

Cross sectional constants for each studied section minimum moment:

$$x_{tp.min} := \begin{pmatrix} x_{tp1.min} \\ x_{tp2.min} \\ x_{tp3.min} \end{pmatrix} = \begin{pmatrix} 0.165 \\ 0.146 \\ 0.146 \end{pmatrix} \text{ m} \quad I_{y.min} := \begin{pmatrix} I_{II.1.min} \\ I_{II.2.min} \\ I_{II.3.min} \end{pmatrix} = \begin{pmatrix} 9.588 \times 10^{-3} \\ 7.812 \times 10^{-3} \\ 7.812 \times 10^{-3} \end{pmatrix} \text{ m}^4 \quad A_{II.min} := \begin{pmatrix} A_{II.1.min} \\ A_{II.2.min} \\ A_{II.3.min} \end{pmatrix} = \begin{pmatrix} 0.218 \\ 0.201 \\ 0.201 \end{pmatrix} \text{ m}^2$$

Stresses in reinforcement due to bending:

$$\sigma_{cs.max1} := \frac{N_{Ed.max0}}{A_{II0}} + \frac{N_{Ed.max0} \cdot e_{N.1} + |M_{Ed.max0}|}{I_{y0}} \cdot (d_{bot0} - x_{tp0}) = 24.285 \cdot \text{MPa}$$

$$\sigma_{s.max1} := \alpha \cdot \sigma_{cs.max1} = 142.529 \cdot \text{MPa}$$

$$\sigma_{cs.min1} := \frac{N_{Ed.min0}}{A_{II.min0}} + \frac{N_{Ed.min0} \cdot e_{N.1.min} + |M_{Ed.min0}|}{I_{y.min0}} \cdot (d_{bot0} - x_{tp.min0}) = 15.939 \cdot \text{MPa}$$

$$\sigma_{s.min1} := \alpha \cdot \sigma_{cs.min1} = 93.549 \cdot \text{MPa}$$

$$\sigma_{cs.max2} := \frac{N_{Ed.max1}}{A_{II1}} + \frac{N_{Ed.max1} \cdot e_{N.2} + |M_{Ed.max1}|}{I_{y1}} \cdot (d_{bot1} - x_{tp1}) = 26.693 \cdot \text{MPa}$$

$$\sigma_{s.max2} := \alpha \cdot \sigma_{cs.max2} = 156.665 \cdot \text{MPa}$$

$$\sigma_{cs.min2} := \frac{N_{Ed.min1}}{A_{II.min1}} + \frac{N_{Ed.min1} \cdot e_{N.2.min} + |M_{Ed.min1}|}{I_{y.min1}} \cdot (d_{bot1} - x_{tp.min1}) = 18.432 \cdot \text{MPa}$$

$$\sigma_{s.min2} := \alpha \cdot \sigma_{cs.min2} = 108.177 \cdot \text{MPa}$$

$$\sigma_{cs.max3} := \frac{N_{Ed.max2}}{A_{II2}} + \frac{N_{Ed.max2} \cdot e_{N.3} + |M_{Ed.max2}|}{I_{y2}} \cdot (d_{bot2} - x_{tp2}) = 26.686 \cdot \text{MPa}$$

$$\sigma_{s.max3} := \alpha \cdot \sigma_{cs.max3} = 156.623 \cdot \text{MPa}$$

$$\sigma_{cs.min3} := \frac{N_{Ed.min2}}{A_{II.min2}} + \frac{N_{Ed.min2} \cdot e_{N.3.min} + |M_{Ed.min2}|}{I_{y.min2}} \cdot (d_{bot2} - x_{tp.min2}) = 18.424 \cdot \text{MPa}$$

$$\sigma_{s.min3} := \alpha \cdot \sigma_{cs.min3} = 108.133 \cdot \text{MPa}$$

Summarizing stresses:

$$\sigma_{s.max} := \begin{pmatrix} \sigma_{s.max1} \\ \sigma_{s.max2} \\ \sigma_{s.max3} \end{pmatrix} = \begin{pmatrix} 142.529 \\ 156.665 \\ 156.623 \end{pmatrix} \cdot \text{MPa} \quad \sigma_{s.min} := \begin{pmatrix} \sigma_{s.min1} \\ \sigma_{s.min2} \\ \sigma_{s.min3} \end{pmatrix} = \begin{pmatrix} 93.549 \\ 108.177 \\ 108.133 \end{pmatrix} \cdot \text{MPa}$$

Stress ranges:

$$\Delta\sigma_{s1} := |\sigma_{s.max0} - \sigma_{s.min0}| = 48.981 \cdot \text{MPa}$$

$$\Delta\sigma_{s2} := |\sigma_{s.max1} - \sigma_{s.min1}| = 48.488 \cdot \text{MPa}$$

$$\Delta\sigma_{s3} := |\sigma_{s.max2} - \sigma_{s.min2}| = 48.489 \cdot \text{MPa}$$

## Fatigue verification for reinforcing steel

### Verification of shear reinforcement and moment resisting reinforcement

Approximated loading cycles during one bridge lifetime:

$$N_{years} := 100$$

$$\dot{A}DT := 10000$$

$$\text{Category}_{traffic} := \begin{cases} "1" & \text{if } 6000 < \dot{A}DT \leq 24000 = "1" \\ "2" & \text{if } 1500 < \dot{A}DT \leq 6000 \\ "3" & \text{if } 600 < \dot{A}DT \leq 1500 \\ "4" & \text{if } \dot{A}DT \leq 600 \end{cases}$$

$$N_{\text{obs}} := \begin{cases} 2 \cdot 10^6 & \text{if Category}_{\text{traffic}} = "1" \\ 0.5 \cdot 10^6 & \text{if Category}_{\text{traffic}} = "2" \\ 0.125 \cdot 10^6 & \text{if Category}_{\text{traffic}} = "3" \\ 0.05 \cdot 10^6 & \text{if Category}_{\text{traffic}} = "4" \end{cases} = 2 \times 10^6 \quad \text{number of lorries on slow lane per year}$$

$$n := N_{\text{obs}} \cdot N_{\text{years}} = 2 \times 10^8$$

### Verification procedure for reinforcement and prestressing steel

Calculation of equivalent stress range according to EN1992-2 Annex NN, Road Bridges N2.1

$$\Delta\sigma_{s,\text{Ec}} := \begin{pmatrix} \Delta\sigma_{s1} \\ \Delta\sigma_{s2} \\ \Delta\sigma_{s3} \end{pmatrix} = \begin{pmatrix} 48.981 \\ 48.488 \\ 48.489 \end{pmatrix} \cdot \text{MPa} \quad \text{Stress range caused by fatigue load model 3 according to EN 1992-2, Annex NN}$$

Shape of the fatigue strength curves (S-N-curves)      Found in EN-1992-1-1

**Table 6.3N: Parameters for S-N curves for reinforcing steel**

| Type of reinforcement               | $N^*$  | stress exponent |       | $\Delta\sigma_{\text{Rsk}}$ (MPa)<br>at $N^*$ cycles |
|-------------------------------------|--------|-----------------|-------|--|
|                                     |        | $k_1$           | $k_2$ |  |
| Straight and bent bars <sup>1</sup> | $10^6$ | 5               | 9     | 162,5  |
| Welded bars and wire fabrics        | $10^7$ | 3               | 5     | 58,5   |
| Splicing devices                    | $10^7$ | 3               | 5     | 35   |

**Note 1:** Values for  $\Delta\sigma_{\text{Rsk}}$  are those for straight bars. Values for bent bars should be obtained using a reduction factor  $\zeta = 0,35 + 0,026 D / \phi$ .  
where:  
 $D$     diameter of the mandrel  
 $\phi$     bar diameter

**Reinforcement type** in structure:

straight and bent bars:       $N_{\text{star}} := 10^6$      $k_1 := 5$      $k_2 := 9$

$\Delta\sigma_{\text{Rsk}} := 162.5 \text{MPa}$       EN-1992-1-1, Table 6.3N

Damage equivalent stress range for fatigue verification

Correction factor considering type of structure and the influence of traffic with regard to critical influence length: EN1992-2 Annex NN with the addition of TRVINFRA-00227 table 7.1-12, which states that L is determined by EN1993-2, 9.5.3 (4)a for both shear force and moment and factor  $\alpha$  may vary.

For a one-span frame bridge the critical influence length is equal to the span length.

$L_\lambda := L_{\text{span}} = 15 \text{ m}$      $\alpha_\lambda := 0.9$       Factor according to TSFS2018:57

$$\alpha_{\text{mod},\lambda} := \min \left[ \alpha_{\lambda} + \frac{(1 - \alpha_{\lambda})}{10\text{m}} \cdot L_{\lambda}, 1 \right] = 1 \quad \text{Factor according to TSFS2018:57}$$

$$\lambda_{\text{s},1.0} := 1.25 \quad \text{According to EN1992-2, Annex NN, fig. NN.2}$$

$$\lambda_{\text{s},1} := \lambda_{\text{s},1.0} \cdot \alpha_{\text{mod},\lambda} = 1.25$$

Correctionfactor considering yearly traffic volume and type: EN1992-2 Annex NN (105)

**Table NN.1 — Factors for traffic type**

| $\bar{Q}$ -factor for | Traffic type (see EN 1991-2 Table 4.7) |                 |               |
|-----------------------|--|-----------------|---------------|
|                       | Long distance                          | Medium distance | Local traffic |
| $k_2 = 5$             | 1,0                                    | 0,90            | 0,73          |
| $k_2 = 7$             | 1,0                                    | 0,92            | 0,78          |
| $k_2 = 9$             | 1,0                                    | 0,94            | 0,82          |

$Q := 1$  Chosen as 1, TRV recommendation to choose "Long distance"

$$N_{\text{obs.lamb}} := \frac{N_{\text{obs}}}{1 \cdot 10^6} \quad \text{N.obs in millions}$$

$$\lambda_{\text{s},2} := Q \cdot \sqrt{\frac{k_2 \cdot N_{\text{obs.lamb}}}{2.0}} = 1$$

Correctionfactor considering service life: EN1992-2 Annex NN (106)

$$\lambda_{\text{s},3} := \sqrt{\frac{k_2 \cdot N_{\text{years}}}{100}} = 1$$

Correctionfactor considering several lanes: EN1992-2 Annex NN (107)

Depending on the yearly day volume according to TRV.

If several lanes,  $\hat{A}DT$  is reduced with factor 0.1 due to load spread to other lanes corresponding to adjacent fast lane.

If bridge is one-way,  $\hat{A}DT$  is doubled.

$$n_{\text{lanes}} := 1$$

$$N_{\text{obs.sum}} := N_{\text{obs}} + N_{\text{obs}} \cdot 0.1 = 2.2 \times 10^6 \quad \text{sum of expected lorries in all lanes per year}$$

$$\lambda_{\text{s},4} := \sqrt{\frac{k_2 \cdot N_{\text{obs.sum}}}{N_{\text{obs}}}} = 1.011$$

Damage equivalent impact factor controlled by surface roughness

$$\varphi_{\text{quality}} := 1.2 \quad \text{EN1991-2 Bilaga B, "good quality"}$$

$$\varphi_{\text{fat}} := \varphi_{\text{quality}} = 1.2$$

Summarizing the correction factors: EN1992-2 Annex NN (103)

$$\lambda_s := \varphi_{\text{fat}} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} = 1.516$$

Final damage equivalent stress range: EN1992-2 Annex NN (102)

$$\Delta\sigma_{s,\text{equ1}} := \Delta\sigma_{s,\text{Ec}_0} \cdot \lambda_s = 74.253 \cdot \text{MPa}$$

$$\Delta\sigma_{s,\text{equ2}} := \Delta\sigma_{s,\text{Ec}_1} \cdot \lambda_s = 73.506 \cdot \text{MPa}$$

$$\Delta\sigma_{s,\text{equ3}} := \Delta\sigma_{s,\text{Ec}_2} \cdot \lambda_s = 73.508 \cdot \text{MPa}$$

Equivalent stress range Check: EN 1992-1-1, 6.8.5 (3)

Partial factors

$$\gamma_{s,\text{fat}} := 1.15 \quad \text{Partial factors for steel, EN1992-1-1 2.4.2.4, Table 2.1N}$$

$$\gamma_{F,\text{fat}} := 1.0 \quad \text{Partial factors for fatigue loads, EN1992-1-1 2.4.2.3, (1)}$$

$$\text{Check}_{\text{fatigue,steel}} = \left( \begin{array}{l} \text{"Not OK"} \quad \text{if } \gamma_{F,\text{fat}} \cdot \Delta\sigma_{s,\text{equ}} > \frac{\Delta\sigma_{\text{Rsk}}}{\gamma_{s,\text{fat}}} \\ \text{"OK"} \quad \text{if } \gamma_{F,\text{fat}} \cdot \Delta\sigma_{s,\text{equ}} < \frac{\Delta\sigma_{\text{Rsk}}}{\gamma_{s,\text{fat}}} \end{array} \right)$$

Utilization of fatigue limit for reinforcement

$$D_{\text{steel1}} := \left| \frac{\gamma_{F,\text{fat}} \cdot \Delta\sigma_{s,\text{equ1}}}{\frac{\Delta\sigma_{\text{Rsk}}}{\gamma_{s,\text{fat}}}} \right| = 0.525$$

$$D_{\text{steel2}} := \left| \frac{\gamma_{F,\text{fat}} \cdot \Delta\sigma_{s,\text{equ2}}}{\frac{\Delta\sigma_{\text{Rsk}}}{\gamma_{s,\text{fat}}}} \right| = 0.52$$

$$D_{\text{steel3}} := \left| \frac{\gamma_{F,\text{fat}} \cdot \Delta\sigma_{s,\text{equ3}}}{\frac{\Delta\sigma_{\text{Rsk}}}{\gamma_{s,\text{fat}}}} \right| = 0.52$$

## Verification of shear reinforcement:

Performed according to 1992-2 Annex NN2.1.

Reinforcement area obtained from script:

Reinforcement areas are expressed in  $A_{sw}/s$  in the script.

$$A_{sw2} := 0.0010723 \frac{m^2}{m} \quad A_{sw3} := 0.0010723 \frac{m^2}{m} \quad \text{Shear reinforcement are for section 2 and 3}$$

The angle of the shear reinforcement compressive strut for fatigue design,

$\theta := 22.5 \cdot \text{deg}$  Angle of compressive strut according to regular shear reinforcement design

$$\tan \theta_{fat} := \sqrt{\tan(\theta)} = 0.644 \quad \text{EN1992-1-1 6.8.3 (3)}$$

$$\cot \theta_{fat} := \frac{1}{\tan \theta_{fat}} = 1.554$$

Internal lever arm

$$z_{sw} := 0.9 \cdot \left[ h - \left( h - d_{bot1} \right) - d_{top1} \right] = 0.614 \text{ m}$$

$\beta_{reduction} := 0.5$  according to EN-1992-1-1, 6.2.3 (8)

Section 2: stress range

$$\Delta \sigma_{sw2} := \frac{\left( V_{Ed,max0} - V_{Ed,min0} \right) \cdot \beta_{reduction}}{A_{sw2} \cdot z_{sw} \cdot \cot \theta_{fat}} = 15.293 \cdot \text{MPa}$$

Section 3: stress range

$$\Delta \sigma_{sw3} := \frac{\left( V_{Ed,max1} - V_{Ed,min1} \right) \cdot \beta_{reduction}}{A_{sw3} \cdot z_{sw} \cdot \cot \theta_{fat}} = 15.299 \cdot \text{MPa}$$

$$\Delta \sigma_{sw.Ec} := \begin{pmatrix} \Delta \sigma_{sw2} \\ \Delta \sigma_{sw3} \end{pmatrix} = \begin{pmatrix} 15.293 \\ 15.299 \end{pmatrix} \cdot \text{MPa} \quad \text{Stressrange caused by fatigue load model 3 according to EN 1992-2, Annex NN}$$

Calculation of equivalent stress range according to EN1992-2 Annex NN

The axle loads of fatigue load model 3 is multiplied by factor 1.40 for the shear force.

Shape of the fatigue strength curves (S-N-curves) Found in EN-1992-1-1

**Reinforcement type** in structure:

For stirrups, bent bars  $\phi 16$  with D 24 mm is used.

where D is the radius of the bent bar. Reduction factor  $\zeta$  may be used.

$D_{radius} := 24 \text{ mm}$  Recommended by "Armeringshandboken" - BE group

$\phi_{stir} := 16 \text{ mm}$

$$\zeta := 0.35 + 0.026 \cdot \frac{D_{\text{radius}}}{\phi_{\text{stir}}} = 0.389$$

$$\Delta\sigma_{\text{Rsk}} := 162.5 \text{ MPa}$$

Straight and bent bars EN1992-1-1 6.8.4, Table 6.3N

$$\Delta\sigma_{\text{Rsk.red}} := \zeta \cdot \Delta\sigma_{\text{Rsk}} = 63.212 \text{ MPa}$$

straight and bent bars:  $N_{\text{stir}} := 10^6$   $k_1 := 5$   $k_2 := 9$

Damage equivalent stress range for fatigue verification

Correction factor considering type of structure and the influence of traffic with regard to critical influence length: EN1992-2 Annex NN with the addition of TRVINFRA-00227 table 7.1-12 which states that L is determined by EN1993-2, 9.5.3 (4)a for both shear force and moment and factor  $\alpha$  may vary.

For a onspan frame bridge the critical influence length is equal to the span length.

$$L_{\lambda} := L_{\text{span}} = 15 \text{ m} \quad \alpha_{\lambda} := 0.9 \quad \text{Factor according to TSFS2018:57}$$

$$\alpha_{\text{mod},\lambda} := \min \left[ \alpha_{\lambda} + \frac{(1 - \alpha_{\lambda})}{10 \text{ m}} \cdot L_{\lambda}, 1 \right] = 1 \quad \text{Factor according to TSFS2018:57}$$

$$\lambda_{s,1.0} := 1.25 \quad \text{According to EN1992-2, Annex NN, fig. NN.2}$$

$$\lambda_{s,\lambda} := \lambda_{s,1.0} \cdot \alpha_{\text{mod},\lambda} = 1.25$$

Correction factor considering yearly traffic volume and type: EN1992-2 Annex NN (105)

$$Q := 1 \quad \text{Chosen as 1, TRV recommendation to choose "Long distance"}$$

$$N_{\text{obs.lamb}} := \frac{N_{\text{obs}}}{1 \cdot 10^6} \quad \text{N.obs in millions}$$

$$\lambda_{s,2} := Q \cdot \sqrt{\frac{k_2 \cdot N_{\text{obs.lamb}}}{2.0}} = 1$$

Correction factor considering service life: EN1992-2 Annex NN (106)

$$\lambda_{s,3} := \sqrt{\frac{k_2 \cdot N_{\text{years}}}{100}} = 1$$

Correction factor considering several lanes: EN1992-2 Annex NN (107)

Depending on the yearly day volume according to TRV.

If several lanes,  $\hat{A}DT$  is reduced with factor 0.1 due to load spread to other lanes corresponding to adjacent fast lane.

If bridge is one-way,  $\hat{A}DT$  is doubled.

$$n_{\text{lanes}} := 1$$

$$N_{\text{obs.sum}} := N_{\text{obs}} + N_{\text{obs}} \cdot 0.1 = 2.2 \times 10^6 \quad \text{sum of expected lorries in all lanes per year}$$

$$\lambda_{sw4} := \sqrt[k_2]{\frac{N_{obs.sum}}{N_{obs}}} = 1.011$$

Damage equivalent impact factor controlled by surface roughness

$$\varphi_{quality} := 1.2 \quad \text{EN1991-2 Bilaga B, "good quality"}$$

$$\varphi_{fat} := \varphi_{quality} = 1.2$$

Summarizing the correction factors: EN1992-2 Annex NN (103)

$$\lambda_{sw2} := \varphi_{fat} \cdot \lambda_{s.1} \cdot \lambda_{s.2} \cdot \lambda_{s.3} \cdot \lambda_{s.4} = 1.516 \quad \text{Section 2}$$

$$\lambda_{sw3} := \varphi_{fat} \cdot \lambda_{s.1} \cdot \lambda_{s.2} \cdot \lambda_{s.3} \cdot \lambda_{s.4} = 1.516 \quad \text{Section 3}$$

Final damage equivalent stress range: EN1992-2 Annex NN (102)

$$\Delta\sigma_{sw.equ2} := \Delta\sigma_{sw2} \cdot \lambda_{sw2} = 23.184 \cdot \text{MPa}$$

$$\Delta\sigma_{sw.equ3} := \Delta\sigma_{sw3} \cdot \lambda_{sw3} = 23.193 \cdot \text{MPa}$$

Equivalent stress range Check: EN 1992-1-1, 6.8.5 (3)

Partial factors

$$\gamma_{swfat} := 1.15 \quad \text{Partial factors for steel, EN1992-1-1 2.4.2.4, Table 2.1N}$$

$$\gamma_{F.fat} := 1.0 \quad \text{Partial factors for fatigue loads, EN1992-1-1 2.4.2.3, (1)}$$

$$\text{Check}_{fatigue.steel} = \left( \begin{array}{l} \text{"Not OK"} \quad \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{s.equ} > \frac{\Delta\sigma_{Rsk.red}}{\gamma_{s.fat}} \\ \text{"OK"} \quad \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{s.equ} < \frac{\Delta\sigma_{Rsk.red}}{\gamma_{s.fat}} \end{array} \right)$$

Utilization of fatigue limit for shear for reinforcement

$$D_{shear2} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{sw.equ2}}{\frac{\Delta\sigma_{Rsk.red}}{\gamma_{s.fat}}} \right| = 0.422$$

$$D_{shear3} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{sw.equ3}}{\frac{\Delta\sigma_{Rsk.red}}{\gamma_{s.fat}}} \right| = 0.422$$

## Fatigue verification for compression and shear in frame legs

Calculation is performed according to:

TSFS 2018:57  
 TRV00226 Allmänna  
 TRV00227 Byggande  
 SS-EN1990  
 SS-EN1991-1-1  
 SS-EN1991-2  
 SS-EN1992-1-1  
 SS-EN1992-2

Results extracted from analysis in Brigade Plus:

|                 |  |  |
|-----------------|--|--|
| <b>Snitt 4:</b> | $M_{Ed,max4} := -399.045 \cdot \text{kN} \cdot \text{m}$ | $M_{Ed,min4} := -308.683 \text{kN} \cdot \text{m}$ |
|                 | $V_{Ed,max4} := 314.059 \cdot \text{kN}$                 | $V_{Ed,min4} := 251.066 \cdot \text{kN}$           |
|                 | $N_{Ed,max4} := -370.927 \text{kN}$                      | $N_{Ed,min4} := -244.512 \text{kN}$                |

|                 |   |   |
|-----------------|---|---|
| <b>Snitt 5:</b> | $M_{Ed,max5} := -75.218 \cdot \text{kN} \cdot \text{m}$ | $M_{Ed,min5} := -43.962 \text{kN} \cdot \text{m}$ |
|                 | $N_{Ed,max5} := -295.446 \text{kN}$                     | $N_{Ed,min5} := -214.415 \text{kN}$               |

|                 |  |  |
|-----------------|--|--|
| <b>Snitt 6:</b> | $M_{Ed,max6} := -398.985 \text{kN} \cdot \text{m}$ | $M_{Ed,min6} := -308.634 \text{kN} \cdot \text{m}$ |
|                 | $V_{Ed,max6} := 314.948 \cdot \text{kN}$           | $V_{Ed,min6} := 251.795 \cdot \text{kN}$           |
|                 | $N_{Ed,max6} := -370.428 \text{kN}$                | $N_{Ed,min6} := -244.117 \text{kN}$                |

|                 |   |   |
|-----------------|---|---|
| <b>Snitt 7:</b> | $M_{Ed,max7} := -75.194 \cdot \text{kN} \cdot \text{m}$ | $M_{Ed,min7} := -43.944 \text{kN} \cdot \text{m}$ |
|                 | $N_{Ed,max7} := -295.504 \text{kN}$                     | $N_{Ed,min7} := -214.458 \text{kN}$               |

$$M_{Ed,max} := \begin{pmatrix} M_{Ed,max4} \\ M_{Ed,max5} \\ M_{Ed,max6} \\ M_{Ed,max7} \end{pmatrix} \quad M_{Ed,min} := \begin{pmatrix} M_{Ed,min4} \\ M_{Ed,min5} \\ M_{Ed,min6} \\ M_{Ed,min7} \end{pmatrix}$$

$$V_{Ed,max} := \begin{pmatrix} V_{Ed,max4} \\ V_{Ed,max6} \end{pmatrix} \quad V_{Ed,min} := \begin{pmatrix} V_{Ed,min4} \\ V_{Ed,min6} \end{pmatrix}$$

$$N_{Ed,max} := \begin{pmatrix} N_{Ed,max4} \\ N_{Ed,max5} \\ N_{Ed,max6} \\ N_{Ed,max7} \end{pmatrix} \quad N_{Ed,min} := \begin{pmatrix} N_{Ed,min4} \\ N_{Ed,min5} \\ N_{Ed,min6} \\ N_{Ed,min7} \end{pmatrix}$$

Sectional geometry and constants:

$$\begin{aligned} b &:= 1\text{m} & H_{\text{ramben}} &:= 6\text{m} \\ h &:= 0.6\text{m} & L_{\text{spann}} &:= 15\text{m} \end{aligned}$$

Reinforcement amount extracted from script:

$$\begin{aligned} \text{Snitt 4:} & \quad A_{s,\text{luftsida4}} := 0.001604\text{m}^2 & A_{s,\text{jordsida4}} &:= 0.003858\text{m}^2 \\ & \quad d_{\text{luftsida4}} := 541\text{mm} & d_{\text{jordsida4}} &:= 59\text{mm} \end{aligned}$$

$$\begin{aligned} \text{Snitt 5:} & \quad A_{s,\text{luftsida5}} := 0.001604 \cdot \text{m}^2 & A_{s,\text{jordsida5}} &:= 0.001604 \cdot \text{m}^2 \\ & \quad d_{\text{luftsida5}} := 541\text{mm} & d_{\text{jordsida5}} &:= 59\text{mm} \end{aligned}$$

$$\begin{aligned} \text{Snitt 6:} & \quad A_{s,\text{luftsida6}} := 0.001604 \cdot \text{m}^2 & A_{s,\text{jordsida6}} &:= 0.003307 \cdot \text{m}^2 \\ & \quad d_{\text{luftsida6}} := 541\text{mm} & d_{\text{jordsida6}} &:= 59\text{mm} \end{aligned}$$

$$\begin{aligned} \text{Snitt 7:} & \quad A_{s,\text{luftsida7}} := 0.001604 \cdot \text{m}^2 & A_{s,\text{jordsida7}} &:= 0.001604 \cdot \text{m}^2 \\ & \quad d_{\text{luftsida7}} := 541\text{mm} & d_{\text{jordsida7}} &:= 59\text{mm} \end{aligned}$$

$$A_{s,\text{luftsida}} := \begin{pmatrix} A_{s,\text{luftsida4}} \\ A_{s,\text{luftsida5}} \\ A_{s,\text{luftsida6}} \\ A_{s,\text{luftsida7}} \end{pmatrix} = \begin{pmatrix} 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \end{pmatrix} \text{m}^2 \quad d_{\text{bot}} := \begin{pmatrix} d_{\text{luftsida4}} \\ d_{\text{luftsida5}} \\ d_{\text{luftsida6}} \\ d_{\text{luftsida7}} \end{pmatrix}$$

$$A_{s,\text{jordsida}} := \begin{pmatrix} A_{s,\text{jordsida4}} \\ A_{s,\text{jordsida5}} \\ A_{s,\text{jordsida6}} \\ A_{s,\text{jordsida7}} \end{pmatrix} = \begin{pmatrix} 3.858 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 3.307 \times 10^{-3} \\ 1.604 \times 10^{-3} \end{pmatrix} \text{m}^2 \quad d_{\text{top}} := \begin{pmatrix} d_{\text{jordsida4}} \\ d_{\text{jordsida5}} \\ d_{\text{jordsida6}} \\ d_{\text{jordsida7}} \end{pmatrix}$$

$$E_{\text{cm}} := 34077146199.2\text{Pa}$$

$$E_s := 200\text{GPa}$$

$$A_c := b \cdot h = 0.6\text{m}^2 \quad \alpha := \frac{E_s}{E_{\text{cm}}} = 5.869$$

Calculating compressed zone for each studied section in stage II:

According to 'Bärande konstruktioner Del 2' Al-Emrani et al. 2011

Takes into account if bottom or top reinforcement is in the compressed zone or not:



$$\alpha_{\text{jordsida.6.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_2} > 0 \text{MPa} \\ \alpha & \text{if } M_{\text{Ed.min}_2} < 0 \text{MPa} \end{cases} = 5.869 \quad \alpha_{\text{luftsida.6.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{jordsida.6.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{jordsida.6.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{jordsida.6.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases} \quad d_{\text{luftsida.6.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases}$$

$$\alpha_{\text{jordsida.7.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_3} > 0 \text{MPa} \\ \alpha & \text{if } M_{\text{Ed.min}_3} < 0 \text{MPa} \end{cases} = 5.869 \quad \alpha_{\text{luftsida.7.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{jordsida.7.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{jordsida.7.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{jordsida.7.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases} \quad d_{\text{luftsida.7.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases}$$

Sectional constants for maximum moments

#### Section 4: Top frameleg, max moment

Guess x.cc until sigma is close to or zero:

$$x_{\text{cc4}} := 160.5 \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{\text{cc.4}} := x_{\text{cc4}} \cdot b = 0.161 \text{m}^2$$

$$A_{\text{II.4}} := A_{\text{cc.4}} + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} + \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} = 0.191 \text{m}^2$$

$$x_{\text{tp4}} := \frac{A_{\text{cc.4}} \cdot \frac{x_{\text{cc4}}}{2} + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} \cdot d_{\text{jordsida.4}} + \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} \cdot d_{\text{luftsida.4}}}{A_{\text{II.4}}} = 0.134 \text{m}$$

$$I_{\text{II.4}} := \frac{b \cdot x_{\text{cc4}}^3}{12} + b \cdot x_{\text{cc4}} \cdot \left( \frac{x_{\text{cc4}}}{2} - x_{\text{tp4}} \right)^2 + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} \cdot (x_{\text{tp4}} - d_{\text{jordsida.4}})^2 + \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} \cdot (d_{\text{luftsida.4}} - x_{\text{tp4}})^2 \dots = 4.603 \times 10^{-3} \text{m}^4$$

#### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_4 := x_{\text{cc4}} - x_{\text{tp4}} = 0.026 \text{m}$$

$$e_{\text{N.4}} := \left| x_{\text{tp4}} - \frac{h}{2} \right| = 165.984 \cdot \text{mm}$$

$$\sigma_{\text{cc.max4}} := \frac{N_{\text{Ed.max}_0}}{A_{\text{II.4}}} + \frac{N_{\text{Ed.max}_0} \cdot e_{\text{N.4}} + |M_{\text{Ed.max}_0}|}{I_{\text{II.4}}} \cdot z_4 = -7.213 \times 10^{-4} \cdot \text{MPa}$$

## Section 5: Mid framlegs

Guess  $x_{cc}$ :

$$x_{cc5} := 255.95 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.5} := x_{cc5} \cdot b = 0.256 \text{ m}^2$$

$$A_{II.5} := A_{cc.5} + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} + \alpha_{luftsida.5} \cdot A_{s.luftsida_1} = 0.273 \text{ m}^2$$

$$x_{tp5} := \frac{A_{cc.5} \cdot \frac{x_{cc5}}{2} + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} \cdot d_{jordsida.5} + \alpha_{luftsida.5} \cdot A_{s.luftsida_1} \cdot d_{luftsida.5}}{A_{II.5}} = 0.14 \text{ m}$$

$$I_{II.5} := \frac{b \cdot x_{cc5}^3}{12} + b \cdot x_{cc5} \cdot \left( \frac{x_{cc5}}{2} - x_{tp5} \right)^2 + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} \cdot (x_{tp5} - d_{jordsida.5})^2 \dots = 2.999 \times 10^{-3} \text{ m}^4$$
$$+ \alpha_{luftsida.5} \cdot A_{s.luftsida_1} \cdot (d_{luftsida.5} - x_{tp5})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_5 := x_{cc5} - x_{tp5} = 0.116 \text{ m}$$

$$e_{N.5} := \left| x_{tp5} - \frac{h}{2} \right| = 159.764 \cdot \text{mm}$$

$$\sigma_{cc.max5} := \frac{N_{Ed.max1}}{A_{II.5}} + \frac{N_{Ed.max1} \cdot e_{N.5} + |M_{Ed.max1}|}{I_{II.5}} \cdot z_5 = -6.452 \times 10^{-4} \cdot \text{MPa}$$

## Section 6: Top frameleg

Guess  $x_{cc}$ :

$$x_{cc6} := 150.65 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.6} := x_{cc6} \cdot b = 0.151 \text{ m}^2$$

$$A_{II.6} := A_{cc.6} + \alpha_{jordsida.6} \cdot A_{s.jordsida_2} + \alpha_{luftsida.6} \cdot A_{s.luftsida_2} = 0.178 \text{ m}^2$$

$$x_{tp6} := \frac{A_{cc.6} \cdot \frac{x_{cc6}}{2} + \alpha_{jordsida.6} \cdot A_{s.jordsida_2} \cdot d_{jordsida.6} + \alpha_{luftsida.6} \cdot A_{s.luftsida_2} \cdot d_{luftsida.6}}{A_{II.6}} = 0.125 \text{ m}$$

$$I_{II.6} := \frac{b \cdot x_{cc6}^3}{12} + b \cdot x_{cc6} \cdot \left( \frac{x_{cc6}}{2} - x_{tp6} \right)^2 + \alpha_{jordsida.6} \cdot A_{s.jordsida.2} \cdot (x_{tp6} - d_{jordsida.6})^2 \dots = 4.049 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{luftsida.6} \cdot A_{s.luftsida.2} \cdot (d_{luftsida.6} - x_{tp6})^2$$

#### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_6 := x_{cc6} - x_{tp6} = 0.025 \text{ m}$$

$$e_{N.6} := \left| x_{tp6} - \frac{h}{2} \right| = 174.578 \cdot \text{mm}$$

$$\sigma_{cc.max6} := \frac{N_{Ed.max2}}{A_{II.6}} + \frac{N_{Ed.max2} \cdot e_{N.6} + |M_{Ed.max2}|}{I_{II.6}} \cdot z_6 = 1.561 \times 10^{-4} \cdot \text{MPa}$$

### **Section 7: Mid framlegs**

Guess x.cc:

$$x_{cc7} := 256.1 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.7} := x_{cc7} \cdot b = 0.256 \text{ m}^2$$

$$A_{II.7} := A_{cc.7} + \alpha_{jordsida.7} \cdot A_{s.jordsida.3} + \alpha_{luftsida.7} \cdot A_{s.luftsida.3} = 0.273 \text{ m}^2$$

$$x_{tp7} := \frac{A_{cc.7} \cdot \frac{x_{cc7}}{2} + \alpha_{jordsida.7} \cdot A_{s.jordsida.3} \cdot d_{jordsida.7} + \alpha_{luftsida.7} \cdot A_{s.luftsida.3} \cdot d_{luftsida.7}}{A_{II.7}} = 0.14 \text{ m}$$

$$I_{II.7} := \frac{b \cdot x_{cc7}^3}{12} + b \cdot x_{cc7} \cdot \left( \frac{x_{cc7}}{2} - x_{tp7} \right)^2 + \alpha_{jordsida.7} \cdot A_{s.jordsida.3} \cdot (x_{tp7} - d_{jordsida.7})^2 \dots = 3.001 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{luftsida.7} \cdot A_{s.luftsida.3} \cdot (d_{luftsida.7} - x_{tp7})^2$$

#### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_7 := x_{cc7} - x_{tp7} = 0.116 \text{ m}$$

$$e_{N.7} := \left| x_{tp7} - \frac{h}{2} \right| = 159.7 \cdot \text{mm}$$

$$\sigma_{cc.max7} := \frac{N_{Ed.max3}}{A_{II.7}} + \frac{N_{Ed.max3} \cdot e_{N.7} + |M_{Ed.max3}|}{I_{II.7}} \cdot z_7 = -7.397 \times 10^{-4} \cdot \text{MPa}$$

## Sectional constants for minimum moments

### Section 4: Top frameleg, min moment

Guess  $x_{cc}$  until sigma is close to or zero:

$$x_{cc4.min} := 155.95 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.4.min} := x_{cc4.min} \cdot b = 0.156 \text{ m}^2$$

$$A_{II.4.min} := A_{cc.4.min} + \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} + \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} = 0.186 \text{ m}^2$$

$$x_{tp4.min} := \frac{A_{cc.4.min} \cdot \frac{x_{cc4.min}}{2} + \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} \cdot d_{jordsida.4.min} + \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} \cdot d_{luftsida.4.min}}{A_{II.4.min}}$$

$$x_{tp4.min} = 0.133 \text{ m}$$

$$I_{II.4.min} := \frac{b \cdot x_{cc4.min}^3}{12} + b \cdot x_{cc4.min} \cdot \left( \frac{x_{cc4.min}}{2} - x_{tp4.min} \right)^2 \dots = 4.6 \times 10^{-3} \text{ m}^4$$
$$+ \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} \cdot (x_{tp4.min} - d_{jordsida.4.min})^2 \dots$$
$$+ \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} \cdot (d_{luftsida.4.min} - x_{tp4.min})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{4.min} := x_{cc4.min} - x_{tp4.min} = 0.023 \text{ m}$$

$$e_{N.4.min} := \left| x_{tp4.min} - \frac{h}{2} \right| = 166.575 \cdot \text{mm}$$

$$\sigma_{cc.min4} := \frac{N_{Ed.min_0}}{A_{II.4.min}} + \frac{N_{Ed.min_0} \cdot e_{N.4.min} + |M_{Ed.min_0}|}{I_{II.4.min}} \cdot z_{4.min} = 3.272 \times 10^{-4} \cdot \text{MPa}$$

### Section 5: Mid framlegs, min moment

Guess  $x_{cc}$ :

$$x_{cc5.min} := 335.2 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.5.min} := x_{cc5.min} \cdot b = 0.335 \text{ m}^2$$

$$A_{II.5.min} := A_{cc.5.min} + \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} + \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} = 0.352 \text{ m}^2$$

$$x_{tp5.min} := \frac{A_{cc.5.min} \cdot \frac{x_{cc5.min}}{2} + \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} \cdot d_{jordsida.5.min} + \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} \cdot d_{luftsida.5.min}}{A_{II.5.min}}$$

$$x_{tp5.min} = 0.175 \text{ m}$$

$$I_{II.5.min} := \frac{b \cdot x_{cc5.min}^3}{12} + b \cdot x_{cc5.min} \cdot \left( \frac{x_{cc5.min}}{2} - x_{tp5.min} \right)^2 \dots = 4.523 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} \cdot (x_{tp5.min} - d_{jordsida.5.min})^2 \dots$$

$$+ \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} \cdot (d_{luftsida.5.min} - x_{tp5.min})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{5.min} := x_{cc5.min} - x_{tp5.min} = 0.16 \text{ m}$$

$$e_{N.5.min} := \left| x_{tp5.min} - \frac{h}{2} \right| = 124.832 \cdot \text{mm}$$

$$\sigma_{cc.min5} := \frac{N_{Ed.min_1}}{A_{II.5.min}} + \frac{N_{Ed.min_1} \cdot e_{N.5.min} + |M_{Ed.min_1}|}{I_{II.5.min}} \cdot z_{5.min} = 2.173 \times 10^{-5} \cdot \text{MPa}$$

### **Section 6: Top frameleg, min moment**

Guess x.cc:

$$x_{cc6.min} := 146.25 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.6.min} := x_{cc6.min} \cdot b = 0.146 \text{ m}^2$$

$$A_{II.6.min} := A_{cc.6.min} + \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} + \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} = 0.173 \text{ m}^2$$

$$x_{tp6.min} := \frac{A_{cc.6.min} \cdot \frac{x_{cc6.min}}{2} + \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} \cdot d_{jordsida.6.min} + \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} \cdot d_{luftsida.6.min}}{A_{II.6.min}}$$

$$x_{tp6.min} = 0.125 \text{ m}$$

$$I_{II.6.min} := \frac{b \cdot x_{cc6.min}^3}{12} + b \cdot x_{cc6.min} \cdot \left( \frac{x_{cc6.min}}{2} - x_{tp6.min} \right)^2 \dots = 4.047 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} \cdot (x_{tp6.min} - d_{jordsida.6.min})^2 \dots$$

$$+ \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} \cdot (d_{luftsida.6.min} - x_{tp6.min})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{6.min} := x_{cc6.min} - x_{tp6.min} = 0.021 \text{ m}$$

$$e_{N.6.min} := \left| x_{tp6.min} - \frac{h}{2} \right| = 175.162 \cdot \text{mm}$$

$$\sigma_{cc.min6} := \frac{N_{Ed.min2}}{A_{II.6.min}} + \frac{N_{Ed.min2} \cdot e_{N.6.min} + |M_{Ed.min2}|}{I_{II.6.min}} \cdot z_{6.min} = -6.127 \times 10^{-4} \cdot \text{MPa}$$

## Section 7: Mid framlegs, min moment

Guess x.cc:

$$x_{cc7.min} := 335.3 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.7.min} := x_{cc7.min} \cdot b = 0.335 \text{ m}^2$$

$$A_{II.7.min} := A_{cc.7.min} + \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} + \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} = 0.353 \text{ m}^2$$

$$x_{tp7.min} := \frac{A_{cc.7.min} \cdot \frac{x_{cc7.min}}{2} + \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} \cdot d_{jordsida.7.min} + \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} \cdot d_{luftsida.7.min}}{A_{II.7.min}}$$

$$x_{tp7.min} = 0.175 \text{ m}$$

$$I_{II.7.min} := \frac{b \cdot x_{cc7.min}^3}{12} + b \cdot x_{cc7.min} \cdot \left( \frac{x_{cc7.min}}{2} - x_{tp7.min} \right)^2 \dots = 4.526 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} \cdot (x_{tp7.min} - d_{jordsida.7.min})^2 \dots$$

$$+ \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} \cdot (d_{luftsida.7.min} - x_{tp7.min})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{7.min} := x_{cc7.min} - x_{tp7.min} = 0.16 \text{ m}$$

$$e_{N.7.min} := \left| x_{tp7.min} - \frac{h}{2} \right| = 124.787 \cdot \text{mm}$$

$$\sigma_{cc.min7} := \frac{N_{Ed.min3}}{A_{II.7.min}} + \frac{N_{Ed.min3} \cdot e_{N.7.min} + |M_{Ed.min3}|}{I_{II.7.min}} \cdot z_{7.min} = -5.468 \times 10^{-4} \cdot \text{MPa}$$

Summarize of compressed zone height for each section and moment

$$x_{cc} := \begin{pmatrix} x_{cc4} \\ x_{cc5} \\ x_{cc6} \\ x_{cc7} \end{pmatrix} = \begin{pmatrix} 160.5 \\ 255.95 \\ 150.65 \\ 256.1 \end{pmatrix} \cdot \text{mm} \quad x_{cc.min} := \begin{pmatrix} x_{cc4.min} \\ x_{cc5.min} \\ x_{cc6.min} \\ x_{cc7.min} \end{pmatrix} = \begin{pmatrix} 155.95 \\ 335.2 \\ 146.25 \\ 335.3 \end{pmatrix} \cdot \text{mm}$$

Cross sectional constants for each studied section

$$x_{tp} := \begin{pmatrix} x_{tp4} \\ x_{tp5} \\ x_{tp6} \\ x_{tp7} \end{pmatrix} = \begin{pmatrix} 0.134 \\ 0.14 \\ 0.125 \\ 0.14 \end{pmatrix} \text{m} \quad I_y := \begin{pmatrix} I_{II.4} \\ I_{II.5} \\ I_{II.6} \\ I_{II.7} \end{pmatrix} = \begin{pmatrix} 4.603 \times 10^{-3} \\ 2.999 \times 10^{-3} \\ 4.049 \times 10^{-3} \\ 3.001 \times 10^{-3} \end{pmatrix} \text{m}^4 \quad A_{II} := \begin{pmatrix} A_{II.4} \\ A_{II.5} \\ A_{II.6} \\ A_{II.7} \end{pmatrix} = \begin{pmatrix} 0.191 \\ 0.273 \\ 0.178 \\ 0.273 \end{pmatrix} \text{m}^2$$

$$x_{tp.min} := \begin{pmatrix} x_{tp4.min} \\ x_{tp5.min} \\ x_{tp6.min} \\ x_{tp7.min} \end{pmatrix} = \begin{pmatrix} 0.133 \\ 0.175 \\ 0.125 \\ 0.175 \end{pmatrix} \text{m} \quad I_{y.min} := \begin{pmatrix} I_{II.4.min} \\ I_{II.5.min} \\ I_{II.6.min} \\ I_{II.7.min} \end{pmatrix} = \begin{pmatrix} 4.6 \times 10^{-3} \\ 4.523 \times 10^{-3} \\ 4.047 \times 10^{-3} \\ 4.526 \times 10^{-3} \end{pmatrix} \text{m}^4 \quad A_{II.min} := \begin{pmatrix} A_{II.4.min} \\ A_{II.5.min} \\ A_{II.6.min} \\ A_{II.7.min} \end{pmatrix} = \begin{pmatrix} 0.186 \\ 0.352 \\ 0.173 \\ 0.353 \end{pmatrix} \text{m}^2$$

Stresses in most compressed fibre due to bending:

$$\sigma_{cd.max4} := \frac{N_{Ed.max0}}{A_{II0}} + \frac{N_{Ed.max0} \cdot e_{N.4} + |M_{Ed.max0}|}{I_{y0}} \cdot -x_{tp0} = -11.768 \cdot \text{MPa}$$

$$\sigma_{cd.min4} := \frac{N_{Ed.min0}}{A_{II.min0}} + \frac{N_{Ed.min0} \cdot e_{N.4.min} + |M_{Ed.min0}|}{I_{y.min0}} \cdot -x_{tp.min0} = -9.084 \cdot \text{MPa}$$

$$\sigma_{cd.max5} := \frac{N_{Ed.max1}}{A_{II1}} + \frac{N_{Ed.max1} \cdot e_{N.5} + |M_{Ed.max1}|}{I_{y1}} \cdot -x_{tp1} = -2.391 \cdot \text{MPa}$$

$$\sigma_{cd.min5} := \frac{N_{Ed.min1}}{A_{II.min1}} + \frac{N_{Ed.min1} \cdot e_{N.5.min} + |M_{Ed.min1}|}{I_{y.min1}} \cdot -x_{tp.min1} = -1.274 \cdot \text{MPa}$$

$$\sigma_{cd.max6} := \frac{N_{Ed.max2}}{A_{II2}} + \frac{N_{Ed.max2} \cdot e_{N.6} + |M_{Ed.max2}|}{I_{y2}} \cdot -x_{tp2} = -12.437 \cdot \text{MPa}$$

$$\sigma_{cd.min6} := \frac{N_{Ed.min2}}{A_{II.min2}} + \frac{N_{Ed.min2} \cdot e_{N.6.min} + |M_{Ed.min2}|}{I_{y.min2}} \cdot -x_{tp.min2} = -9.609 \cdot \text{MPa}$$

$$\sigma_{cd.max7} := \frac{N_{Ed.max3}}{A_{II3}} + \frac{N_{Ed.max3} \cdot e_{N.7} + |M_{Ed.max3}|}{I_{y3}} \cdot -x_{tp3} = -2.39 \cdot \text{MPa}$$

$$\sigma_{cd.min7} := \frac{N_{Ed.min3}}{A_{II.min3}} + \frac{N_{Ed.min3} \cdot e_{N.7.min} + |M_{Ed.min3}|}{I_{y.min3}} \cdot -x_{tp.min3} = -1.274 \cdot \text{MPa}$$

Summarize of stresses

$$\sigma_{cd.max} := \begin{pmatrix} \sigma_{cd.max4} \\ \sigma_{cd.max5} \\ \sigma_{cd.max6} \\ \sigma_{cd.max7} \end{pmatrix} = \begin{pmatrix} -11.768 \\ -2.391 \\ -12.437 \\ -2.39 \end{pmatrix} \cdot \text{MPa} \quad \sigma_{cd.min} := \begin{pmatrix} \sigma_{cd.min4} \\ \sigma_{cd.min5} \\ \sigma_{cd.min6} \\ \sigma_{cd.min7} \end{pmatrix} = \begin{pmatrix} -9.084 \\ -1.274 \\ -9.609 \\ -1.274 \end{pmatrix} \cdot \text{MPa}$$

Stress ranges:

$$\Delta\sigma_{cd1} := |\sigma_{cd.max0} - \sigma_{cd.min0}| = 2.685 \cdot \text{MPa}$$

$$\Delta\sigma_{cd2} := |\sigma_{cd.max1} - \sigma_{cd.min1}| = 1.117 \cdot \text{MPa}$$

$$\Delta\sigma_{cd3} := |\sigma_{cd.max2} - \sigma_{cd.min2}| = 2.829 \cdot \text{MPa}$$

$$\Delta\sigma_{cd4} := |\sigma_{cd.max3} - \sigma_{cd.min3}| = 1.117 \cdot \text{MPa}$$

## Fatigue verification, concrete under compression or shear

Verification of concrete under compression:

According to EN-1992-2 6.8.7

Approximated loading cycles during one bridge lifetime:

$$N_{years} := 100$$

$$\dot{A}DT := 10000$$

$$\text{Category}_{traffic} := \begin{cases} "1" & \text{if } 6000 < \dot{A}DT \leq 24000 \\ "2" & \text{if } 1500 < \dot{A}DT \leq 6000 \\ "3" & \text{if } 600 < \dot{A}DT \leq 1500 \\ "4" & \text{if } \dot{A}DT \leq 600 \end{cases}$$

$$N_{obs} := \begin{cases} 2 \cdot 10^6 & \text{if } \text{Category}_{traffic} = "1" \\ 0.5 \cdot 10^6 & \text{if } \text{Category}_{traffic} = "2" \\ 0.125 \cdot 10^6 & \text{if } \text{Category}_{traffic} = "3" \\ 0.05 \cdot 10^6 & \text{if } \text{Category}_{traffic} = "4" \end{cases} = 2 \times 10^6 \quad \text{number of lorries on slow lane per year}$$

$$n := N_{obs} \cdot N_{years} = 2 \times 10^8$$

Concrete:

Class: C35/40

$\gamma_C := 1.5$  Partial factor, EN1992-1-1 2.4.2.4, Table 2.1N

The characteristic strength of concrete obtained according to EN1992-1-1, 3.1.3, Table 3.1

$f_{ck} := 35\text{MPa}$

$$f_{cd} := \frac{f_{ck}}{\gamma_C} = 23.333 \cdot \text{MPa}$$

$t_0 := 28$  Age when loaded

$CEM_{class} := "52.5N"$  Dependency of cement class, see EN-1992-1-1, 3.1.2 (6)

$s_{cem} := 0.2$  for 52.5N

$k_1 := 0.85$  Recommended value in Eurocode

$$\beta_{cc} := e^{-\left[ s_{cem} \left[ 1 - \left( \frac{28}{t_0} \right)^{0.5} \right] \right]} = 1 \quad \text{According to EN-1992-1-1, 3.1.2 (6)}$$

$$f_{cd.fat} := k_1 \cdot \beta_{cc} \cdot f_{cd} \cdot \left( 1 - \frac{f_{ck}}{250\text{MPa}} \right) = 17.057 \cdot \text{MPa} \quad \text{Fatigue strength}$$

Palmgren-Miner Cumulative damage verification method:

$$E_{cd.min.i} := \frac{-\sigma_{cd.min}}{f_{cd.fat}} = \begin{pmatrix} 0.533 \\ 0.075 \\ 0.563 \\ 0.075 \end{pmatrix} \quad E_{cd.max.i} := \frac{-\sigma_{cd.max}}{f_{cd.fat}} = \begin{pmatrix} 0.69 \\ 0.14 \\ 0.729 \\ 0.14 \end{pmatrix}$$

$$R_{equ} := \frac{E_{cd.min.i}}{E_{cd.max.i}} = \begin{pmatrix} 0.772 \\ 0.533 \\ 0.773 \\ 0.533 \end{pmatrix}$$

Corresponding cycles:

$$N_c := 10^{14} \left( \frac{1 - E_{cd.max.i}}{\sqrt{1 - R_{equ}}} \right) = \begin{pmatrix} 1.224 \times 10^9 \\ 4.091 \times 10^{17} \\ 8.925 \times 10^7 \\ 4.102 \times 10^{17} \end{pmatrix}$$

Final damage for compressed concrete:

$$D := \frac{n}{N_c} = \begin{pmatrix} 0.163 \\ 4.888 \times 10^{-10} \\ 2.241 \\ 4.876 \times 10^{-10} \end{pmatrix}$$

Verification for shear If no shear reinforcement is needed:  
According to EN 1992-1-1 6.8.7 (4).

**NOTE:** In this particular case, shear reinforcement is applied in section 2 and 3 which makes the check below un-necessary. The results below are therefore evaluated as if no reinforcement was applied.

From Brigade:

**Snitt 4**       $V_{Rd.c4} := 1230\text{kN}$

**Snitt 6**       $V_{Rd.c6} := 1230\text{kN}$

$$V_{check} := \frac{V_{Ed.min}}{V_{Ed.max}} = \begin{pmatrix} 0.799 \\ 0.799 \end{pmatrix}$$

$$U_{v4} := \begin{cases} 0.5 + 0.45 \cdot \left| \frac{V_{Ed.min_0}}{V_{Rd.c4}} \right| & \text{if } V_{check_0} \geq 0 \\ 0.5 - \left| \frac{V_{Ed.min_0}}{V_{Rd.c4}} \right| & \text{if } V_{check_0} < 0 \end{cases} = 0.592 \quad \text{Check}_{v4} := \begin{cases} \text{"Ok"} & \text{if } \left| \frac{V_{Ed.max_0}}{V_{Rd.c4}} \right| \leq U_{v4} \\ \text{"Not Ok"} & \text{otherwise} \end{cases} = \text{"Ok"}$$

$$U_{v6} := \begin{cases} 0.5 + 0.45 \cdot \left| \frac{V_{Ed.min_1}}{V_{Rd.c6}} \right| & \text{if } V_{check_1} \geq 0 \\ 0.5 - \left| \frac{V_{Ed.min_1}}{V_{Rd.c6}} \right| & \text{if } V_{check_1} < 0 \end{cases} = 0.592 \quad \text{Check}_{v6} := \begin{cases} \text{"Ok"} & \text{if } \left| \frac{V_{Ed.max_1}}{V_{Rd.c6}} \right| \leq U_{v6} \\ \text{"Not Ok"} & \text{otherwise} \end{cases} = \text{"Ok"}$$

## Fatigue verification for reinforcement in frame legs

Calculation is performed according to:

TSFS 2018:57  
 TRV00226 Allmänna  
 TRV00227 Byggande  
 SS-EN1990  
 SS-EN1991-1-1  
 SS-EN1991-2  
 SS-EN1992-1-1  
 SS-EN1992-2

Results extracted from analysis in Brigade Plus:

|                 |  |   |
|-----------------|--|---|
| <b>Snitt 4:</b> | $M_{Ed,max4} := -435.19 \text{ kN} \cdot \text{m}$       | $M_{Ed,min4} := -308.683 \text{ kN} \cdot \text{m}$ |
|                 | $V_{Ed,max4} := 339.293 \cdot \text{kN}$                 | $V_{Ed,min4} := 251.066 \cdot \text{kN}$            |
|                 | $N_{Ed,max4} := -421.493 \text{ kN}$                     | $N_{Ed,min4} := -244.512 \text{ kN}$                |
| <b>Snitt 5:</b> | $M_{Ed,max5} := -87.72 \cdot \text{kN} \cdot \text{m}$   | $M_{Ed,min5} := -43.962 \text{ kN} \cdot \text{m}$  |
|                 | $N_{Ed,max5} := -327.858 \text{ kN}$                     | $N_{Ed,min5} := -214.415 \text{ kN}$                |
| <b>Snitt 6:</b> | $M_{Ed,max6} := -435.126 \cdot \text{kN} \cdot \text{m}$ | $M_{Ed,min6} := -308.634 \text{ kN} \cdot \text{m}$ |
|                 | $V_{Ed,max6} := 340.245 \cdot \text{kN}$                 | $V_{Ed,min6} := 251.795 \cdot \text{kN}$            |
|                 | $N_{Ed,max6} := -420.953 \text{ kN}$                     | $N_{Ed,min6} := -244.117 \text{ kN}$                |
| <b>Snitt 7:</b> | $M_{Ed,max7} := -87.695 \cdot \text{kN} \cdot \text{m}$  | $M_{Ed,min7} := -43.944 \text{ kN} \cdot \text{m}$  |
|                 | $N_{Ed,max7} := -327.922 \text{ kN}$                     | $N_{Ed,min7} := -214.458 \text{ kN}$                |

$$M_{Ed,max} := \begin{pmatrix} M_{Ed,max4} \\ M_{Ed,max5} \\ M_{Ed,max6} \\ M_{Ed,max7} \end{pmatrix} \quad M_{Ed,min} := \begin{pmatrix} M_{Ed,min4} \\ M_{Ed,min5} \\ M_{Ed,min6} \\ M_{Ed,min7} \end{pmatrix}$$

$$V_{Ed,max} := \begin{pmatrix} V_{Ed,max4} \\ V_{Ed,max6} \end{pmatrix} \quad V_{Ed,min} := \begin{pmatrix} V_{Ed,min4} \\ V_{Ed,min6} \end{pmatrix}$$

$$N_{Ed,max} := \begin{pmatrix} N_{Ed,max4} \\ N_{Ed,max5} \\ N_{Ed,max6} \\ N_{Ed,max7} \end{pmatrix} \quad N_{Ed,min} := \begin{pmatrix} N_{Ed,min4} \\ N_{Ed,min5} \\ N_{Ed,min6} \\ N_{Ed,min7} \end{pmatrix}$$

Sectional geometry and constants:

$$b := 1 \text{ m}$$

$$H_{ramben} := 6 \text{ m}$$

$$h := 0.6\text{m}$$

$$L_{\text{spann}} := 15\text{m}$$

Reinforcement amount extracted from script:

Snitt 4:

$$A_{s,\text{luftsida4}} := 0.001604\text{m}^2$$

$$A_{s,\text{jordsida4}} := 0.003858\text{m}^2$$

$$d_{\text{luftsida4}} := 541\text{mm}$$

$$d_{\text{jordsida4}} := 59\text{mm}$$

Snitt 5:

$$A_{s,\text{luftsida5}} := 0.001604 \cdot \text{m}^2$$

$$A_{s,\text{jordsida5}} := 0.001604 \cdot \text{m}^2$$

$$d_{\text{luftsida5}} := 541\text{mm}$$

$$d_{\text{jordsida5}} := 59\text{mm}$$

Snitt 6:

$$A_{s,\text{luftsida6}} := 0.001604 \cdot \text{m}^2$$

$$A_{s,\text{jordsida6}} := 0.003307 \cdot \text{m}^2$$

$$d_{\text{luftsida6}} := 541\text{mm}$$

$$d_{\text{jordsida6}} := 59\text{mm}$$

Snitt 7:

$$A_{s,\text{luftsida7}} := 0.001604 \cdot \text{m}^2$$

$$A_{s,\text{jordsida7}} := 0.001604 \cdot \text{m}^2$$

$$d_{\text{luftsida7}} := 541\text{mm}$$

$$d_{\text{jordsida7}} := 59\text{mm}$$

$$A_{s,\text{luftsida}} := \begin{pmatrix} A_{s,\text{luftsida4}} \\ A_{s,\text{luftsida5}} \\ A_{s,\text{luftsida6}} \\ A_{s,\text{luftsida7}} \end{pmatrix} = \begin{pmatrix} 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 1.604 \times 10^{-3} \end{pmatrix} \text{m}^2$$

$$d_{\text{bot}} := \begin{pmatrix} d_{\text{luftsida4}} \\ d_{\text{luftsida5}} \\ d_{\text{luftsida6}} \\ d_{\text{luftsida7}} \end{pmatrix}$$

$$A_{s,\text{jordsida}} := \begin{pmatrix} A_{s,\text{jordsida4}} \\ A_{s,\text{jordsida5}} \\ A_{s,\text{jordsida6}} \\ A_{s,\text{jordsida7}} \end{pmatrix} = \begin{pmatrix} 3.858 \times 10^{-3} \\ 1.604 \times 10^{-3} \\ 3.307 \times 10^{-3} \\ 1.604 \times 10^{-3} \end{pmatrix} \text{m}^2$$

$$d_{\text{top}} := \begin{pmatrix} d_{\text{jordsida4}} \\ d_{\text{jordsida5}} \\ d_{\text{jordsida6}} \\ d_{\text{jordsida7}} \end{pmatrix}$$

$$E_{\text{cm}} := 34077146199.2\text{Pa}$$

$$E_s := 200\text{GPa}$$

$$A_c := b \cdot h = 0.6\text{m}^2 \quad \alpha := \frac{E_s}{E_{\text{cm}}} = 5.869$$

Calculating compressed zone for each studied section in stage II:

According to 'Bärande konstruktioner Del 2' Al-Emrani et al. 2011

Takes into account if bottom or top reinforcement is in the compressed zone or not.



$$\alpha_{\text{jordsida.6.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_2} > 0 \text{MPa} \\ \alpha & \text{if } M_{\text{Ed.min}_2} < 0 \text{MPa} \end{cases} = 5.869 \quad \alpha_{\text{luftsida.6.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{jordsida.6.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{jordsida.6.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{jordsida.6.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases} \quad d_{\text{luftsida.6.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases}$$

$$\alpha_{\text{jordsida.7.min}} := \begin{cases} ((\alpha - 1)) & \text{if } M_{\text{Ed.min}_3} > 0 \text{MPa} \\ \alpha & \text{if } M_{\text{Ed.min}_3} < 0 \text{MPa} \end{cases} = 5.869 \quad \alpha_{\text{luftsida.7.min}} := \begin{cases} ((\alpha - 1)) & \text{if } \alpha_{\text{jordsida.7.min}} = \alpha \\ \alpha & \text{if } \alpha_{\text{jordsida.7.min}} = (\alpha - 1) \end{cases}$$

$$d_{\text{jordsida.7.min}} := \begin{cases} d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases} \quad d_{\text{luftsida.7.min}} := \begin{cases} d_{\text{bot}_2} & \text{if } M_{\text{Ed.max}_2} > 0 \text{MPa} \\ d_{\text{top}_2} & \text{if } M_{\text{Ed.max}_2} < 0 \text{MPa} \end{cases}$$

## Sectional constants for maximum moments

### Section 4: Top frameleg, max moment

Guess x.cc until sigma is close to or zero:

$$x_{\text{cc4}} := 161.82 \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{\text{cc.4}} := x_{\text{cc4}} \cdot b = 0.162 \text{m}^2$$

$$A_{\text{II.4}} := A_{\text{cc.4}} + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} + \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} = 0.192 \text{m}^2$$

$$x_{\text{tp4}} := \frac{A_{\text{cc.4}} \cdot \frac{x_{\text{cc4}}}{2} + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} \cdot d_{\text{jordsida.4}} + \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} \cdot d_{\text{luftsida.4}}}{A_{\text{II.4}}} = 0.134 \text{m}$$

$$I_{\text{II.4}} := \frac{b \cdot x_{\text{cc4}}^3}{12} + b \cdot x_{\text{cc4}} \cdot \left( \frac{x_{\text{cc4}}}{2} - x_{\text{tp4}} \right)^2 + \alpha_{\text{jordsida.4}} \cdot A_{\text{s.jordsida}_0} \cdot (x_{\text{tp4}} - d_{\text{jordsida.4}})^2 \dots = 4.604 \times 10^{-3} \text{m}^4$$

$$+ \alpha_{\text{luftsida.4}} \cdot A_{\text{s.luftsida}_0} \cdot (d_{\text{luftsida.4}} - x_{\text{tp4}})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_4 := x_{\text{cc4}} - x_{\text{tp4}} = 0.028 \text{m}$$

$$e_{\text{N.4}} := \left| x_{\text{tp4}} - \frac{h}{2} \right| = 165.798 \cdot \text{mm}$$

$$\sigma_{\text{cc.max4}} := \frac{N_{\text{Ed.max}_0}}{A_{\text{II.4}}} + \frac{N_{\text{Ed.max}_0} \cdot e_{\text{N.4}} + |M_{\text{Ed.max}_0}|}{I_{\text{II.4}}} \cdot z_4 = -7.341 \times 10^{-4} \cdot \text{MPa}$$

## Section 5: Mid framlegs

Guess  $x_{cc}$ :

$$x_{cc5} := 241.3 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.5} := x_{cc5} \cdot b = 0.241 \text{ m}^2$$

$$A_{II.5} := A_{cc.5} + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} + \alpha_{luftsida.5} \cdot A_{s.luftsida_1} = 0.259 \text{ m}^2$$

$$x_{tp5} := \frac{A_{cc.5} \cdot \frac{x_{cc5}}{2} + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} \cdot d_{jordsida.5} + \alpha_{luftsida.5} \cdot A_{s.luftsida_1} \cdot d_{luftsida.5}}{A_{II.5}} = 0.134 \text{ m}$$

$$I_{II.5} := \frac{b \cdot x_{cc5}^3}{12} + b \cdot x_{cc5} \cdot \left( \frac{x_{cc5}}{2} - x_{tp5} \right)^2 + \alpha_{jordsida.5} \cdot A_{s.jordsida_1} \cdot (x_{tp5} - d_{jordsida.5})^2 \dots = 2.817 \times 10^{-3} \text{ m}^4$$
$$+ \alpha_{luftsida.5} \cdot A_{s.luftsida_1} \cdot (d_{luftsida.5} - x_{tp5})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_5 := x_{cc5} - x_{tp5} = 0.107 \text{ m}$$

$$e_{N.5} := \left| x_{tp5} - \frac{h}{2} \right| = 165.906 \cdot \text{mm}$$

$$\sigma_{cc.max5} := \frac{N_{Ed.max1}}{A_{II.5}} + \frac{N_{Ed.max1} \cdot e_{N.5} + |M_{Ed.max1}|}{I_{II.5}} \cdot z_5 = 2.968 \times 10^{-5} \cdot \text{MPa}$$

## Section 6: Top frameleg

Guess  $x_{cc}$ :

$$x_{cc6} := 151.92 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.6} := x_{cc6} \cdot b = 0.152 \text{ m}^2$$

$$A_{II.6} := A_{cc.6} + \alpha_{jordsida.6} \cdot A_{s.jordsida_2} + \alpha_{luftsida.6} \cdot A_{s.luftsida_2} = 0.179 \text{ m}^2$$

$$x_{tp6} := \frac{A_{cc.6} \cdot \frac{x_{cc6}}{2} + \alpha_{jordsida.6} \cdot A_{s.jordsida_2} \cdot d_{jordsida.6} + \alpha_{luftsida.6} \cdot A_{s.luftsida_2} \cdot d_{luftsida.6}}{A_{II.6}} = 0.126 \text{ m}$$

$$I_{II.6} := \frac{b \cdot x_{cc6}^3}{12} + b \cdot x_{cc6} \cdot \left( \frac{x_{cc6}}{2} - x_{tp6} \right)^2 + \alpha_{jordsida.6} \cdot A_{s.jordsida_2} \cdot (x_{tp6} - d_{jordsida.6})^2 \dots = 4.05 \times 10^{-3} \text{ m}^4$$
$$+ \alpha_{luftsida.6} \cdot A_{s.luftsida_2} \cdot (d_{luftsida.6} - x_{tp6})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.  
 Calculated with Naviers formula according  
 Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_6 := x_{cc6} - x_{tp6} = 0.026 \text{ m}$$

$$e_{N,6} := \left| x_{tp6} - \frac{h}{2} \right| = 174.394 \cdot \text{mm}$$

$$\sigma_{cc,max6} := \frac{N_{Ed,max2}}{A_{II,6}} + \frac{N_{Ed,max2} \cdot e_{N,6} + |M_{Ed,max2}|}{I_{II,6}} \cdot z_6 = 1.339 \times 10^{-4} \cdot \text{MPa}$$

## Section 7: Mid framlegs

Guess x.cc:

$$x_{cc7} := 241.5 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc,7} := x_{cc7} \cdot b = 0.242 \text{ m}^2$$

$$A_{II,7} := A_{cc,7} + \alpha_{jordsida,7} \cdot A_{s,jordsida,3} + \alpha_{luftsida,7} \cdot A_{s,luftsida,3} = 0.259 \text{ m}^2$$

$$x_{tp7} := \frac{A_{cc,7} \cdot \frac{x_{cc7}}{2} + \alpha_{jordsida,7} \cdot A_{s,jordsida,3} \cdot d_{jordsida,7} + \alpha_{luftsida,7} \cdot A_{s,luftsida,3} \cdot d_{luftsida,7}}{A_{II,7}} = 0.134 \text{ m}$$

$$I_{II,7} := \frac{b \cdot x_{cc7}^3}{12} + b \cdot x_{cc7} \cdot \left( \frac{x_{cc7}}{2} - x_{tp7} \right)^2 + \alpha_{jordsida,7} \cdot A_{s,jordsida,3} \cdot (x_{tp7} - d_{jordsida,7})^2 + \alpha_{luftsida,7} \cdot A_{s,luftsida,3} \cdot (d_{luftsida,7} - x_{tp7})^2 \dots = 2.819 \times 10^{-3} \text{ m}^4$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.  
 Calculated with Naviers formula according  
 Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_7 := x_{cc7} - x_{tp7} = 0.107 \text{ m}$$

$$e_{N,7} := \left| x_{tp7} - \frac{h}{2} \right| = 165.823 \cdot \text{mm}$$

$$\sigma_{cc,max7} := \frac{N_{Ed,max3}}{A_{II,7}} + \frac{N_{Ed,max3} \cdot e_{N,7} + |M_{Ed,max3}|}{I_{II,7}} \cdot z_7 = 7.908 \times 10^{-4} \cdot \text{MPa}$$

## Sectional constants for minimum moments

### Section 4: Top frameleg, min moment

Guess x.cc until sigma is close to or zero:

$$x_{cc4,min} := 155.95 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.4.min} := x_{cc4.min} \cdot b = 0.156 \text{ m}^2$$

$$A_{II.4.min} := A_{cc.4.min} + \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} + \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} = 0.186 \text{ m}^2$$

$$x_{tp4.min} := \frac{A_{cc.4.min} \cdot \frac{x_{cc4.min}}{2} + \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} \cdot d_{jordsida.4.min} + \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} \cdot d_{luftsida.4.min}}{A_{II.4.min}}$$

$$x_{tp4.min} = 0.133 \text{ m}$$

$$I_{II.4.min} := \frac{b \cdot x_{cc4.min}^3}{12} + b \cdot x_{cc4.min} \cdot \left( \frac{x_{cc4.min}}{2} - x_{tp4.min} \right)^2 \dots = 4.6 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.4.min} \cdot A_{s.jordsida_0} \cdot (x_{tp4.min} - d_{jordsida.4.min})^2 \dots$$

$$+ \alpha_{luftsida.4.min} \cdot A_{s.luftsida_0} \cdot (d_{luftsida.4.min} - x_{tp4.min})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{4.min} := x_{cc4.min} - x_{tp4.min} = 0.023 \text{ m}$$

$$e_{N.4.min} := \left| x_{tp4.min} - \frac{h}{2} \right| = 166.575 \cdot \text{mm}$$

$$\sigma_{cc.min4} := \frac{N_{Ed.min_0}}{A_{II.4.min}} + \frac{N_{Ed.min_0} \cdot e_{N.4.min} + |M_{Ed.min_0}|}{I_{II.4.min}} \cdot z_{4.min} = 3.272 \times 10^{-4} \cdot \text{MPa}$$

## Section 5: Mid framlegs, min moment

Guess x.cc:

$$x_{cc5.min} := 335.1 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.5.min} := x_{cc5.min} \cdot b = 0.335 \text{ m}^2$$

$$A_{II.5.min} := A_{cc.5.min} + \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} + \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} = 0.352 \text{ m}^2$$

$$x_{tp5.min} := \frac{A_{cc.5.min} \cdot \frac{x_{cc5.min}}{2} + \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} \cdot d_{jordsida.5.min} + \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} \cdot d_{luftsida.5.min}}{A_{II.5.min}}$$

$$x_{tp5.min} = 0.175 \text{ m}$$

$$I_{II.5.min} := \frac{b \cdot x_{cc5.min}^3}{12} + b \cdot x_{cc5.min} \cdot \left( \frac{x_{cc5.min}}{2} - x_{tp5.min} \right)^2 \dots = 4.52 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.5.min} \cdot A_{s.jordsida_1} \cdot (x_{tp5.min} - d_{jordsida.5.min})^2 \dots$$

$$+ \alpha_{luftsida.5.min} \cdot A_{s.luftsida_1} \cdot (d_{luftsida.5.min} - x_{tp5.min})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{5.min} := x_{cc5.min} - x_{tp5.min} = 0.16 \text{ m}$$

$$\epsilon_{N.5.min} := \left| x_{tp5.min} - \frac{h}{2} \right| = 124.878 \cdot \text{mm}$$

$$\sigma_{cc.min5} := \frac{N_{Ed.min_1}}{A_{II.5.min}} + \frac{N_{Ed.min_1} \cdot \epsilon_{N.5.min} + |M_{Ed.min_1}|}{I_{II.5.min}} \cdot z_{5.min} = -3.586 \times 10^{-4} \cdot \text{MPa}$$

## Section 6: Top frameleg, min moment

Guess x.cc:

$$x_{cc6.min} := 146.25 \text{ mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.6.min} := x_{cc6.min} \cdot b = 0.146 \text{ m}^2$$

$$A_{II.6.min} := A_{cc.6.min} + \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} + \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} = 0.173 \text{ m}^2$$

$$x_{tp6.min} := \frac{A_{cc.6.min} \cdot \frac{x_{cc6.min}}{2} + \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} \cdot d_{jordsida.6.min} + \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} \cdot d_{luftsida.6.min}}{A_{II.6.min}}$$

$$x_{tp6.min} = 0.125 \text{ m}$$

$$I_{II.6.min} := \frac{b \cdot x_{cc6.min}^3}{12} + b \cdot x_{cc6.min} \cdot \left( \frac{x_{cc6.min}}{2} - x_{tp6.min} \right)^2 \dots = 4.047 \times 10^{-3} \text{ m}^4$$

$$+ \alpha_{jordsida.6.min} \cdot A_{s.jordsida_2} \cdot (x_{tp6.min} - d_{jordsida.6.min})^2 \dots$$

$$+ \alpha_{luftsida.6.min} \cdot A_{s.luftsida_2} \cdot (d_{luftsida.6.min} - x_{tp6.min})^2$$

Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{6.min} := x_{cc6.min} - x_{tp6.min} = 0.021 \text{ m}$$

$$e_{N.6.min} := \left| x_{tp6.min} - \frac{h}{2} \right| = 175.162 \cdot \text{mm}$$

$$\sigma_{cc.min6} := \frac{N_{Ed.min2}}{A_{II.6.min}} + \frac{N_{Ed.min2} \cdot e_{N.6.min} + |M_{Ed.min2}|}{I_{II.6.min}} \cdot z_{6.min} = -6.127 \times 10^{-4} \cdot \text{MPa}$$

## Section 7: Mid framlegs, min moments

Guess  $x_{cc}$ :

$$x_{cc7.min} := 335.3 \text{mm}$$

Calculate cross sectional constants with guessed value:

$$A_{cc.7.min} := x_{cc7.min} \cdot b = 0.335 \text{m}^2$$

$$A_{II.7.min} := A_{cc.7.min} + \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} + \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} = 0.353 \text{m}^2$$

$$x_{tp7.min} := \frac{A_{cc.7.min} \cdot \frac{x_{cc7.min}}{2} + \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} \cdot d_{jordsida.7.min} + \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} \cdot d_{luftsida.7.min}}{A_{II.7.min}}$$

$$x_{tp7.min} = 0.175 \text{m}$$

$$I_{II.7.min} := \frac{b \cdot x_{cc7.min}^3}{12} + b \cdot x_{cc7.min} \cdot \left( \frac{x_{cc7.min}}{2} - x_{tp7.min} \right)^2 \dots = 4.526 \times 10^{-3} \text{m}^4$$

$$+ \alpha_{jordsida.7.min} \cdot A_{s.jordsida_3} \cdot (x_{tp7.min} - d_{jordsida.7.min})^2 \dots$$

$$+ \alpha_{luftsida.7.min} \cdot A_{s.luftsida_3} \cdot (d_{luftsida.7.min} - x_{tp7.min})^2$$

### Stress range from bending:

Calculate stresses at neutral axis with guess, repeat with another guess until stress is close to zero.

Calculated with Naviers formula according

Performed in cracked stage according to EN1992-1-1 6.8.2 (1).

$$z_{7.min} := x_{cc7.min} - x_{tp7.min} = 0.16 \text{m}$$

$$e_{N.7.min} := \left| x_{tp7.min} - \frac{h}{2} \right| = 124.787 \cdot \text{mm}$$

$$\sigma_{cc.min7} := \frac{N_{Ed.min3}}{A_{II.7.min}} + \frac{N_{Ed.min3} \cdot e_{N.7.min} + |M_{Ed.min3}|}{I_{II.7.min}} \cdot z_{7.min} = -5.468 \times 10^{-4} \cdot \text{MPa}$$

Summarize of compressed zone height for each section and moment

$$x_{cc} := \begin{pmatrix} x_{cc4} \\ x_{cc5} \\ x_{cc6} \\ x_{cc7} \end{pmatrix} = \begin{pmatrix} 161.82 \\ 241.3 \\ 151.92 \\ 241.5 \end{pmatrix} \cdot \text{mm}$$

$$x_{cc.min} := \begin{pmatrix} x_{cc4.min} \\ x_{cc5.min} \\ x_{cc6.min} \\ x_{cc7.min} \end{pmatrix} = \begin{pmatrix} 155.95 \\ 335.1 \\ 146.25 \\ 335.3 \end{pmatrix} \cdot \text{mm}$$

Cross sectional constants for each studied section

$$x_{tp} := \begin{pmatrix} x_{tp4} \\ x_{tp5} \\ x_{tp6} \\ x_{tp7} \end{pmatrix} = \begin{pmatrix} 0.134 \\ 0.134 \\ 0.126 \\ 0.134 \end{pmatrix} \text{ m} \quad I_y := \begin{pmatrix} I_{II.4} \\ I_{II.5} \\ I_{II.6} \\ I_{II.7} \end{pmatrix} = \begin{pmatrix} 4.604 \times 10^{-3} \\ 2.817 \times 10^{-3} \\ 4.05 \times 10^{-3} \\ 2.819 \times 10^{-3} \end{pmatrix} \text{ m}^4 \quad A_{II} := \begin{pmatrix} A_{II.4} \\ A_{II.5} \\ A_{II.6} \\ A_{II.7} \end{pmatrix} = \begin{pmatrix} 0.192 \\ 0.259 \\ 0.179 \\ 0.259 \end{pmatrix} \text{ m}^2$$

$$x_{tp.min} := \begin{pmatrix} x_{tp4.min} \\ x_{tp5.min} \\ x_{tp6.min} \\ x_{tp7.min} \end{pmatrix} = \begin{pmatrix} 0.133 \\ 0.175 \\ 0.125 \\ 0.175 \end{pmatrix} \text{ m} \quad I_{y.min} := \begin{pmatrix} I_{II.4.min} \\ I_{II.5.min} \\ I_{II.6.min} \\ I_{II.7.min} \end{pmatrix} = \begin{pmatrix} 4.6 \times 10^{-3} \\ 4.52 \times 10^{-3} \\ 4.047 \times 10^{-3} \\ 4.526 \times 10^{-3} \end{pmatrix} \text{ m}^4 \quad A_{II.min} := \begin{pmatrix} A_{II.4.min} \\ A_{II.5.min} \\ A_{II.6.min} \\ A_{II.7.min} \end{pmatrix} = \begin{pmatrix} 0.186 \\ 0.352 \\ 0.173 \\ 0.353 \end{pmatrix} \text{ m}^2$$

Stresses in reinforcement due to bending:

$$\sigma_{cs.max4} := \frac{N_{Ed.max0}}{A_{II0}} + \frac{N_{Ed.max0} \cdot e_{N.4} + |M_{Ed.max0}|}{I_{y0}} \cdot (d_{bot0} - x_{tp0}) = 30.086 \cdot \text{MPa}$$

$$\sigma_{s.max4} := \alpha \cdot \sigma_{cs.max4} = 176.578 \cdot \text{MPa}$$

$$\sigma_{cs.min4} := \frac{N_{Ed.min0}}{A_{II.min0}} + \frac{N_{Ed.min0} \cdot e_{N.4.min} + |M_{Ed.min0}|}{I_{y.min0}} \cdot (d_{bot0} - x_{tp.min0}) = 22.429 \cdot \text{MPa}$$

$$\sigma_{s.min4} := \alpha \cdot \sigma_{cs.min4} = 131.636 \cdot \text{MPa}$$

$$\sigma_{cs.max5} := \frac{N_{Ed.max1}}{A_{II1}} + \frac{N_{Ed.max1} \cdot e_{N.5} + |M_{Ed.max1}|}{I_{y1}} \cdot (d_{bot1} - x_{tp1}) = 3.545 \cdot \text{MPa}$$

$$\sigma_{s.max5} := \alpha \cdot \sigma_{cs.max5} = 20.808 \cdot \text{MPa}$$

$$\sigma_{cs.min5} := \frac{N_{Ed.min1}}{A_{II.min1}} + \frac{N_{Ed.min1} \cdot e_{N.5.min} + |M_{Ed.min1}|}{I_{y.min1}} \cdot (d_{bot1} - x_{tp.min1}) = 0.782 \cdot \text{MPa}$$

$$\sigma_{s.min5} := \alpha \cdot \sigma_{cs.min5} = 4.592 \cdot \text{MPa}$$

$$\sigma_{cs.max6} := \frac{N_{Ed.max2}}{A_{II2}} + \frac{N_{Ed.max2} \cdot e_{N.6} + |M_{Ed.max2}|}{I_{y2}} \cdot (d_{bot2} - x_{tp2}) = 34.747 \cdot \text{MPa}$$

$$\sigma_{s.max6} := \alpha \cdot \sigma_{cs.max6} = 203.931 \cdot \text{MPa}$$

$$\sigma_{cs.min6} := \frac{N_{Ed.min2}}{A_{II.min2}} + \frac{N_{Ed.min2} \cdot e_{N.6.min} + |M_{Ed.min2}|}{I_{y.min2}} \cdot (d_{bot2} - x_{tp.min2}) = 25.933 \cdot \text{MPa}$$

$$\sigma_{s.min6} := \alpha \cdot \sigma_{cs.min6} = 152.199 \cdot \text{MPa}$$

$$\sigma_{cs.max7} := \frac{N_{Ed.max3}}{A_{II_0}} + \frac{N_{Ed.max3} \cdot e_{N.7} + |M_{Ed.max3}|}{I_{y3}} \cdot (d_{bot3} - x_{tp3}) = 3.102 \cdot \text{MPa}$$

$$\sigma_{s.max7} := \alpha \cdot \sigma_{cs.max7} = 18.206 \cdot \text{MPa}$$

$$\sigma_{cs.min7} := \frac{N_{Ed.min3}}{A_{II.min3}} + \frac{N_{Ed.min3} \cdot e_{N.7.min} + |M_{Ed.min3}|}{I_{y.min3}} \cdot (d_{bot3} - x_{tp.min3}) = 0.78 \cdot \text{MPa}$$

$$\sigma_{s.min7} := \alpha \cdot \sigma_{cs.min7} = 4.58 \cdot \text{MPa}$$

Summarize of stresses

$$\sigma_{s.max} := \begin{pmatrix} \sigma_{s.max4} \\ \sigma_{s.max5} \\ \sigma_{s.max6} \\ \sigma_{s.max7} \end{pmatrix} = \begin{pmatrix} 176.578 \\ 20.808 \\ 203.931 \\ 18.206 \end{pmatrix} \cdot \text{MPa} \quad \sigma_{s.min} := \begin{pmatrix} \sigma_{s.min4} \\ \sigma_{s.min5} \\ \sigma_{s.min6} \\ \sigma_{s.min7} \end{pmatrix} = \begin{pmatrix} 131.636 \\ 4.592 \\ 152.199 \\ 4.58 \end{pmatrix} \cdot \text{MPa}$$

Stress ranges:

$$\Delta\sigma_{s4} := |\sigma_{s.max0} - \sigma_{s.min0}| = 44.942 \cdot \text{MPa}$$

$$\Delta\sigma_{s5} := |\sigma_{s.max1} - \sigma_{s.min1}| = 16.216 \cdot \text{MPa}$$

$$\Delta\sigma_{s6} := |\sigma_{s.max2} - \sigma_{s.min2}| = 51.732 \cdot \text{MPa}$$

$$\Delta\sigma_{s7} := |\sigma_{s.max3} - \sigma_{s.min3}| = 13.625 \cdot \text{MPa}$$

## Fatigue verification for reinforcing steel

### Verification of shear reinforcement and moment resisting reinforcement

Approximated loading cycles during one bridge lifetime:

$$N_{years} := 100$$

$$\dot{A}DT := 10000$$

$$\text{Category}_{\text{traffic}} := \begin{cases} "1" & \text{if } 6000 < \text{ÅDT} \leq 24000 = "1" \\ "2" & \text{if } 1500 < \text{ÅDT} \leq 6000 \\ "3" & \text{if } 600 < \text{ÅDT} \leq 1500 \\ "4" & \text{if } \text{ÅDT} \leq 600 \end{cases}$$

$$N_{\text{obs}} := \begin{cases} 2 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "1" \\ 0.5 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "2" \\ 0.125 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "3" \\ 0.05 \cdot 10^6 & \text{if } \text{Category}_{\text{traffic}} = "4" \end{cases} = 2 \times 10^6 \quad \text{number of lorries on slow lane per year}$$

$$n := N_{\text{obs}} \cdot N_{\text{years}} = 2 \times 10^8$$

### Verification procedure for reinforcement and prestressing steel

Calculation of equivalent stress range according to EN1992-2 Annex NN, Road Bridges N2.1

$$\Delta\sigma_{s, Ec} := \begin{pmatrix} \Delta\sigma_{s4} \\ \Delta\sigma_{s5} \\ \Delta\sigma_{s6} \\ \Delta\sigma_{s7} \end{pmatrix} = \begin{pmatrix} 44.942 \\ 16.216 \\ 51.732 \\ 13.625 \end{pmatrix} \cdot \text{MPa} \quad \text{Stress range caused by fatigue load model 3 according to EN 1992-2, Annex NN}$$

Shape of the fatigue strength curves (S-N-curves) Found in EN-1992-1-1

**Table 6.3N: Parameters for S-N curves for reinforcing steel**

| Type of reinforcement               | $N^*$  | stress exponent |       | $\Delta\sigma_{Rsk}$ (MPa)<br>at $N^*$ cycles |
|-------------------------------------|--------|-----------------|-------|---|
|                                     |        | $k_1$           | $k_2$ |   |
| Straight and bent bars <sup>1</sup> | $10^6$ | 5               | 9     | 162,5   |
| Welded bars and wire fabrics        | $10^7$ | 3               | 5     | 58,5  |
| Splicing devices                    | $10^7$ | 3               | 5     | 35  |

**Note 1:** Values for  $\Delta\sigma_{Rsk}$  are those for straight bars. Values for bent bars should be obtained using a reduction factor  $\zeta = 0,35 + 0,026 D / \phi$ .  
where:  
 $D$  diameter of the mandrel  
 $\phi$  bar diameter

**Reinforcement type** in structure:

straight and bent bars:  $N_{\text{star}} := 10^6 \quad k_1 := 5 \quad k_2 := 9$

$$\Delta\sigma_{Rsk} := 162.5 \text{MPa} \quad \text{EN-1992-1-1, Table 6.3N}$$

Damage equivalent stress range for fatigue verification

Verification procedure for reinforcement and prestressing steel

For road bridges: According to Annex nn in EN 1992-2

For the calculation of damage equivalent stress ranges for steel verification, the axle loads of fatigue load model 3 shall be multiplied by the following factors:

- 1,75 for verification at intermediate supports in continuous bridges
- 1,40 for verification in other areas.

Found in EN-1992-2, Annex NN

Shape of the fatigue strength curves (S-N-curves) Found in EN-1992-1-1

**Table 6.3N: Parameters for S-N curves for reinforcing steel**

| Type of reinforcement               | $N^*$  | stress exponent |       | $\Delta\sigma_{Rsk}$ (MPa)<br>at $N^*$ cycles |
|-------------------------------------|--------|-----------------|-------|---|
|                                     |        | $k_1$           | $k_2$ |   |
| Straight and bent bars <sup>1</sup> | $10^6$ | 5               | 9     | 162,5   |
| Welded bars and wire fabrics        | $10^7$ | 3               | 5     | 58,5  |
| Splicing devices                    | $10^7$ | 3               | 5     | 35  |

**Note 1:** Values for  $\Delta\sigma_{Rsk}$  are those for straight bars. Values for bent bars should be obtained using a reduction factor  $\zeta = 0,35 + 0,026 D / \phi$ .  
 where:  
 $D$  diameter of the mandrel  
 $\phi$  bar diameter

Correctionsfactor considering type of structure and the influence of traffic with regard to critical influence length: EN1992-2 Annex NN with the addition of TRVINFRA-00227 table 7.1-12 which states that L is determined by EN1993-2, 9.5.3 (4) a for both shear force and moment and factor  $\alpha$  may vary.

For a onespan frame bridge the critical influence length is equal to the span length.

$$L_\lambda := L_{\text{span}} = 15 \text{ m} \quad \alpha_\lambda := 0.9 \quad \text{Factor according to TSFS2018:57}$$

$$\alpha_{\text{mod},\lambda} := \min \left[ \alpha_\lambda + \frac{(1 - \alpha_\lambda)}{10\text{m}} \cdot L_\lambda, 1 \right] = 1$$

$$\lambda_{s,1.0} := 1.0 \quad \text{According to EN1992-2, Annex NN, fig. NN.2}$$

$$\lambda_{s,1} := \alpha_{\text{mod},\lambda} \cdot \lambda_{s,1.0} = 1$$

Correctionfactor considering yearly traffic volume and type: EN1992-2 Annex NN (105)

Table NN.1 — Factors for traffic type

| $\bar{Q}$ -factor for | Traffic type (see EN 1991-2 Table 4.7) |                 |               |
|-----------------------|--|-----------------|---------------|
|                       | Long distance                          | Medium distance | Local traffic |
| $k_2 = 5$             | 1,0                                    | 0,90            | 0,73          |
| $k_2 = 7$             | 1,0                                    | 0,92            | 0,78          |
| $k_2 = 9$             | 1,0                                    | 0,94            | 0,82          |

$Q := 1$  Chosen as 1, TRV recommendation to choose "Long distance"

$$N_{\text{obs.lamb}} := \frac{N_{\text{obs}}}{1 \cdot 10^6} \quad \text{N.obs in millions}$$

$$\lambda_{s,2} := Q \cdot \sqrt{\frac{k_2 N_{\text{obs.lamb}}}{2.0}} = 1$$

Correctionfactor considering service life: EN1992-2 Annex NN (106)

$$\lambda_{s,3} := \sqrt{\frac{k_2 N_{\text{years}}}{100}} = 1$$

Correctionfactor considering several lanes: EN1992-2 Annex NN (107)

Depending on the yearly day volume according to TRV.

If several lanes,  $\dot{A}DT$  is reduced with factor 0.1 due to load spread to other lanes corresponding to adjacent fast lane.

If bridge is one-way,  $\dot{A}DT$  is doubled.

$$n_{\text{lanes}} := 1$$

$$N_{\text{obs.sum}} := N_{\text{obs}} + N_{\text{obs}} \cdot 0.1 = 2.2 \times 10^6 \quad \text{sum of expected lorries in all lanes per year}$$

$$\lambda_{s,4} := \sqrt{\frac{k_2 N_{\text{obs.sum}}}{N_{\text{obs}}}} = 1.011$$

Damage equivalent impact factor controlled by surface roughness

$$\varphi_{\text{quality}} := 1.2 \quad \text{EN1991-2 Bilaga B, "good quality"}$$

$$\varphi_{\text{fat}} := \varphi_{\text{quality}} = 1.2 \quad \text{EN1991-2 Bilaga B (3)}$$

Summarizing the correctionfactors: EN1992-2 Annex NN (103)

$$\lambda_{\text{sr}4} := \varphi_{\text{fat}} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} = 1.213 \quad \text{Section 4}$$

$$\lambda_{\text{sr}5} := \varphi_{\text{fat}} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} = 1.213 \quad \text{Section 5}$$

$$\lambda_{\text{sr}6} := \varphi_{\text{fat}} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} = 1.213 \quad \text{Section 6}$$

$$\lambda_{sr7} := \varphi_{fat} \cdot \lambda_{s.1} \cdot \lambda_{s.2} \cdot \lambda_{s.3} \cdot \lambda_{s.4} = 1.213 \quad \text{Section 7}$$

Final damage equivalent stress range: EN1992-2 Annex NN (102)

$$\Delta\sigma_{s.equ4} := \Delta\sigma_{s.Ec_0} \cdot \lambda_{sr4} = 54.504 \cdot \text{MPa} \quad \text{Section 4}$$

$$\Delta\sigma_{s.equ5} := \Delta\sigma_{s.Ec_1} \cdot \lambda_{sr5} = 19.666 \cdot \text{MPa} \quad \text{Section 5}$$

$$\Delta\sigma_{s.equ6} := \Delta\sigma_{s.Ec_2} \cdot \lambda_{sr6} = 62.739 \cdot \text{MPa} \quad \text{Section 6}$$

$$\Delta\sigma_{s.equ7} := \Delta\sigma_{s.Ec_3} \cdot \lambda_{sr7} = 16.524 \cdot \text{MPa} \quad \text{Section 7}$$

Equivalent stress range Check: EN 1992-1-1, 6.8.5 (3)

Partial factors

$$\gamma_{s.fat} := 1.15 \quad \text{Partial factors for steel, EN1992-1-1 2.4.2.4, Table 2.1N}$$

$$\gamma_{F.fat} := 1.0 \quad \text{Partial factors for fatigue loads, EN1992-1-1 2.4.2.3, (1)}$$

$$\text{Check}_{\text{fatigue.steel}} = \left( \begin{array}{l} \text{"Not OK"} \quad \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{s.equ} > \frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}} \\ \text{"OK"} \quad \text{if } \gamma_{F.fat} \cdot \Delta\sigma_{s.equ} < \frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}} \end{array} \right)$$

Utilization level for reinforcement

$$D_{\text{steel4}} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{s.equ4}}{\frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}}} \right| = 0.386$$

$$D_{\text{steel5}} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{s.equ5}}{\frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}}} \right| = 0.139$$

$$D_{\text{steel6}} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{s.equ6}}{\frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}}} \right| = 0.444$$

$$D_{\text{steel7}} := \left| \frac{\gamma_{F.fat} \cdot \Delta\sigma_{s.equ7}}{\frac{\Delta\sigma_{Rsk}}{\gamma_{s.fat}}} \right| = 0.117$$

Verification of shear reinforcement:

Performed according to 1992-2 Annex NN2.1.

Reinforcement area obtained from script:  
Reinforcement areas are expressed in Asw/s in the script.

$$A_{sw4} := 0.001721 \frac{m^2}{m}$$

$$A_{sw6} := 0.001727 \frac{m^2}{m}$$

Shear reinforcement are for section 4 and 6

The angle of the shear reinforcement compressive strut for fatigue design,

$$\theta := 22.5 \cdot \text{deg} \quad \text{Angle of compressive strut according to regular shear reinforcement design}$$

EN1992-1-1 6.8.3 (3)

$$\tan\theta_{fat} := \sqrt{\tan(\theta)} = 0.644$$

$$\cot\theta_{fat} := \frac{1}{\tan\theta_{fat}} = 1.554$$

Internal lever arm

$$z_{sw} := 0.9 \cdot [h - (h - d_{bot1}) - d_{top1}] = 0.434 \text{ m}$$

$$\beta_{reduction} := 0.5 \quad \text{according to EN-1992-1-1, 6.2.3 (8)}$$

Section 4: stress range

$$\Delta\sigma_{sw4} := \frac{(V_{Ed,max0} - V_{Ed,min0}) \cdot \beta_{reduction}}{A_{sw4} \cdot z_{sw} \cdot \cot\theta_{fat}} = 38.029 \cdot \text{MPa}$$

Section 6: stress range

$$\Delta\sigma_{sw6} := \frac{(V_{Ed,max1} - V_{Ed,min1}) \cdot \beta_{reduction}}{A_{sw6} \cdot z_{sw} \cdot \cot\theta_{fat}} = 37.993 \cdot \text{MPa}$$

$$\Delta\sigma_{sw.Ec} := \begin{pmatrix} \Delta\sigma_{sw4} \\ \Delta\sigma_{sw6} \end{pmatrix} = \begin{pmatrix} 38.029 \\ 37.993 \end{pmatrix} \cdot \text{MPa}$$

Stressrange caused by fatigue load model 3 according to EN 1992-2, Annex NN

Calculation of equivalent stress range according to EN1992-2 Annex NN

The axle loads of fatigue load model 3 is multiplied by factor 1.40 for the shear force.

Shape of the fatigue strength curves (S-N-curves) Found in EN-1992-1-1

**Reinforcement type** in structure:

For stirrups, bent bars  $\phi 16$  with D 24 mm is used.

where D is the radius of the bent bar. Reduction factor  $\zeta$  may be used.

$$D_{radius} := 24\text{mm} \quad \text{Recommended by "Armeringshandboken" - BE group}$$

$$\phi_{stir} := 16\text{mm}$$

$$\zeta := 0.35 + 0.026 \cdot \frac{D_{radius}}{\phi_{stir}} = 0.389$$

$$\Delta\sigma_{Rsk} := 162.5\text{MPa}$$

Straight and bent bars EN1992-1-1 6.8.4, Table 6.3N

$$\Delta\sigma_{Rsk.red} := \zeta \cdot \Delta\sigma_{Rsk} = 63.212 \cdot \text{MPa}$$

straight and bent bars:  $N_{star} := 10^6$   $k_1 := 5$   $k_2 := 9$

Damage equivalent stress range for fatigue verification

Corrections factor considering type of structure and the influence of traffic with regard to critical influence length: EN1992-2 Annex NN with the addition of TRVINFR-00227 table 7.1-12 which states that L is determined by EN1993-2, 9.5.3 (4) a for both shear force and moment and factor  $\alpha$  may vary.

For a onespan frame bridge the critical influence length is equal to the span length.

$L_{\lambda} := L_{span} = 15 \text{ m}$   $\alpha_{\lambda} := 0.9$  Factor according to TSFS2018:57

$$\alpha_{mod,\lambda} := \min \left[ \alpha_{\lambda} + \frac{(1 - \alpha_{\lambda})}{10 \text{ m}} \cdot L_{\lambda}, 1 \right] = 1$$

$\lambda_{s,1.0} := 1.0$  According to EN1992-2, Annex NN, fig. NN.2

$$\lambda_{s,i} := \alpha_{mod,\lambda} \cdot \lambda_{s,1.0} = 1$$

Correction factor considering yearly traffic volume and type: EN1992-2 Annex NN (105)

$Q := 1$  Chosen as 1, TRV recommendation to choose "Long distance"

$$N_{obs,lamb} := \frac{N_{obs}}{1 \cdot 10^6} \quad \text{N.obs in millions}$$

$$\lambda_{s,2} := Q \cdot \sqrt{\frac{k_2 N_{obs,lamb}}{2.0}} = 1$$

Correction factor considering service life: EN1992-2 Annex NN (106)

$$\lambda_{s,3} := \sqrt{\frac{k_2 N_{years}}{100}} = 1$$

Correction factor considering several lanes: EN1992-2 Annex NN (107)

Depending on the yearly day volume according to TRV.

If several lanes,  $\hat{A}DT$  is reduced with factor 0.1 due to load spread to other lanes corresponding to adjacent fast lane.

If bridge is one-way,  $\hat{A}DT$  is doubled.

$$n_{lanes} := 1$$

$$N_{obs,sum} := N_{obs} + N_{obs} \cdot 0.1 = 2.2 \times 10^6 \quad \text{sum of expected lorries in all lanes per year}$$

$$\lambda_{s,4} := \sqrt{\frac{k_2 N_{obs,sum}}{N_{obs}}} = 1.011$$

### Damage equivalent impact factor controlled by surface roughness

$$\varphi_{\text{fat}} := 1.2$$

Summarizing the correction factors: EN1992-2 Annex NN (103)

$$\lambda_{\text{sw}4} := \varphi_{\text{fat}} \cdot \lambda_{\text{s},1} \cdot \lambda_{\text{s},2} \cdot \lambda_{\text{s},3} \cdot \lambda_{\text{s},4} = 1.213 \quad \text{Section 4}$$

$$\lambda_{\text{sw}6} := \varphi_{\text{fat}} \cdot \lambda_{\text{s},1} \cdot \lambda_{\text{s},2} \cdot \lambda_{\text{s},3} \cdot \lambda_{\text{s},4} = 1.213 \quad \text{Section 6}$$

Final damage equivalent stress range: EN1992-2 Annex NN (102)

$$\Delta\sigma_{\text{sw},\text{equ}4} := \Delta\sigma_{\text{sw}4} \cdot \lambda_{\text{sw}4} = 46.12 \cdot \text{MPa} \quad \text{Section 4}$$

$$\Delta\sigma_{\text{sw},\text{equ}6} := \Delta\sigma_{\text{sw}6} \cdot \lambda_{\text{sw}6} = 46.076 \cdot \text{MPa} \quad \text{Section 6}$$

Equivalent stress range Check: EN 1992-1-1, 6.8.5 (3)

Partial factors

$$\gamma_{\text{s},\text{fat}} := 1.15 \quad \text{Partial factors for steel, EN1992-1-1 2.4.2.4, Table 2.1N}$$

$$\gamma_{\text{F},\text{fat}} := 1.0 \quad \text{Partial factors for fatigue loads, EN1992-1-1 2.4.2.3, (1)}$$

$$\text{Check}_{\text{fatigue,steel}} = \left( \begin{array}{l} \text{"Not OK"} \quad \text{if } \gamma_{\text{F},\text{fat}} \cdot \Delta\sigma_{\text{s},\text{equ}} > \frac{\Delta\sigma_{\text{Rsk},\text{red}}}{\gamma_{\text{s},\text{fat}}} \\ \text{"OK"} \quad \text{if } \gamma_{\text{F},\text{fat}} \cdot \Delta\sigma_{\text{s},\text{equ}} < \frac{\Delta\sigma_{\text{Rsk},\text{red}}}{\gamma_{\text{s},\text{fat}}} \end{array} \right)$$

### Utilization level for shear for reinforcement

$$D_{\text{shear}4} := \left| \frac{\gamma_{\text{F},\text{fat}} \cdot \Delta\sigma_{\text{sw},\text{equ}4}}{\frac{\Delta\sigma_{\text{Rsk},\text{red}}}{\gamma_{\text{s},\text{fat}}}} \right| = 0.839$$

$$D_{\text{shear}6} := \left| \frac{\gamma_{\text{F},\text{fat}} \cdot \Delta\sigma_{\text{sw},\text{equ}6}}{\frac{\Delta\sigma_{\text{Rsk},\text{red}}}{\gamma_{\text{s},\text{fat}}}} \right| = 0.838$$

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