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Crack comparison in radon proof concrete slab

Comparison of crack widths for different compositions of concrete and reinforcement

Master's thesis in Structural engineering and building technology

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Sammanfattning

Om människor utsätts för radioaktiv gas, t.ex. radon, kan hälsoproblem uppstå. Därför måste grundläggningar i betong vara utformade på ett sätt som förhindrar att radonet tränger igenom till miljön innomhus. Syftet med den här rapporten var att undersöka olika sammansättningar av en platta på mark för att studera vad som kan vara den mest effektiva lösningen med hänsyn till sprickbredder i betong.

En analytisk analys utfördes där olika sammansättningar av betong och armeringsmängder undersöktes. En ökad hållfastighet av betongen eller av dimensionen på armeringsstången visade sig kräva en större mängd armering för att uppfylla kravet på sprickbredden. Sprickbredden minskar dock med ökad hållfasthet om samma stångdiameter och stål spänning kan användas.

När krympning och tvång togs i beaktande ökade sprickbredden med en ökad stångdiameter och armerings area. Detta kan förklaras utifrån krympningkraften som uppstår i stålet som sedan överförs till betongen via vidhäftning. Krympningskraften ökar med ökad armeringsmängd vilket orsakar en större sprickbredd.

Nyckelord: radon, betong, grundläggning, sprickbredd, armering, krympning, mothållande kraft

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Abstract

Health problems may occur if humans are exposed to radioactive gas e.g. radon. Therefore, concrete foundations have to be designed in a way to prevent radon to penetrate to the environment inside. The purpose of this thesis was to investigate different compositions of a concrete slab, to find the most efficient solution when designing with respect to crack widths.

An analytical analysis was performed where various compositions of concrete and reinforcement layouts were investigated. When increasing the concrete strength or the dimension of the reinforcing bar, a larger reinforcement amount is required to meet the crack limitations. However, the crack width is reduced with increased concrete strength if the same reinforcement diameter and steel stress can be used.

When considering shrinkage and restraint, the crack width increase with increased bar diameter and reinforcing area. This is explained by the shrinkage force that occur in the steel which is transferred to the concrete by bond. The shrinkage force increase when the amount of reinforcement is increased, causing an expansion of the crack width.

Keywords: radon, concrete, foundation, crack width, reinforcement, shrinkage, restraint force

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Isabelle Persson, Gothenburg, June 2019

Legend

Latin uppercase

A_{c}	Area of concrete cross section
$A_{\mathrm{c,ef}}$	Effective area of concrete cross section
A_{s}	Area of bottom steel cross section
A'_{s}	Area of top steel cross section
E_{c}	Elastic modulus for concrete
$E_{\mathrm{c,ef}}$	Effective elastic modulus for concrete
E_{cm}	Mean value of the effective elastic modulus for concrete
E_{s}	Elastic modulus for steel
F_{cs}	Force caused by restraint of shrinkage
N	Restraint force
N_{cr}	Cracking force
R	Factor of restraint
RH	Relative humidity

Greek uppercase

ϕ	Bar diameter

Latin lowercase

b	Width of cross-section
c	Thickness of concrete cover
c_{min}	Smallest thickness of concrete cover
d	Distance from the top of the cross-section to the center of gravity of the bottom layer of the reinforcement
f_{cd}	Dimension value of the concrete compressive strength
f_{cm}	Mean value of the concrete compressive strength
f_{ctm}	Mean value of the concrete tensile strength
f_{yk}	Yielding capacity of the reinforcing steel
n_{cr}	Number of cracks
h	Height of cross-section
h_0	Notional size of cross-section
h_{ef}	Effective height of cross-section
k	Factor that considers the inner stress distribution caused by restraint
k_1	Factor that considers the surface structure of the reinforcement
k_2	Factor that considers the strain distribution
k_3	National parameter
k_4	National parameter
k_{c}	Factor that considers the stress distribution before cracking
k_{h}	Coefficient that depends on the size of the section
n_{cr}	Number of cracks
$s_{\mathrm{r,max}}$	Maximum crack separation
u	Perimeter of the part of the cross-section exposed to drying
w	Crack width
w_{k}	Characteristic crack width
w_{max}	Maximum crack width
x	Distance from slab edge to the center of gravity

Greek lowercase

α	Modular ratio between steel and concrete
α_{ef}	Effective modular ratio between steel and concrete
γ	Partial coefficent
γ_{c}	Partial coefficient for concrete
γ_{d}	
γ_{s}	Partial coefficient for steel
$\varepsilon_{\mathrm{ca}}(\infty)$	Autogenous shrinkage strain
$\varepsilon_{\mathrm{c,creep}}(\infty)$	Final creep coefficient
$\varepsilon_{\mathrm{cm}}$	Mean strain between cracks in concrete
$\varepsilon_{\mathrm{cd}}(\infty)$	Drying shrinkage strain
$\varepsilon_{\mathrm{cdi}}(\infty)$	Starting value to determine drying shrinkage strain
$\varepsilon_{\mathrm{cs}}$	Shrinkage strain of concrete
ε_{s}	Steel strain
$\varepsilon_{\mathrm{sm}}$	Mean strain of the steel, including effects of forced deformation
ρ	Densiy
σ_{c}	Concrete stress
σ_{s}	Steel stress
$\varphi(\infty, t_0)$	Final creep coefficient
φ_{RH}	Factor that considers the relative humidity of the surroundings
ϕ	Diameter of reinforcing bar

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1 Introduction

This chapter is an introduction to the project and will explain the background and the aim with the study.

1.1 Background

Radon is an odorless and invisible gas that exists naturally in nature. When the gas decomposes it forms radioactive atoms which are harmful to the health if the body is exposed to the substance for a longer period of time. Radon in the residence can cause damage to the lungs that further can be developed to lung cancer. The most common radon source to buildings is in the ground but the substance can also occur in building materials and in the water that is used in the household (Boverket, 2018).

One of the most significant reasons to the radon leakage into a building is because of cracks in the concrete foundation. Due to differences in air pressure outside and inside a building, where it most commonly is higher on the outside than on the inside, the radon can be transferred through the cracks in the foundation by suction. To achieve an indoor environment that fulfills the recommended upper limit of radon content in the indoor air it is necessary to produce a foundation that does not let the substance pass through (Boverket, 2018).

To prevent/minimize radon penetrating through the foundation into the building, the requirements on cracking (allowable crack width and through cracks) in the concrete is high unless other measures are taken. The subject has become more important due to stricter requirements and thus a need for constructions that does not allow the radon into the indoor environment.

1.2 Aim and objectives

The purpose of this project was to investigate and compare different design solutions and cracking in concrete elements with different compositions of concrete and reinforcement amounts and dimensions. The aim was to find the most optimal design solution with respect to crack widths (radon emission). The project was intended to gain knowledge about how to project a foundation to prevent radon penetration, and provide a safe environment especially in schools.

The objectives were:

- To find the acceptable level of radon contamination in the environment inside a school.
- To clarify and provide limitations and demands on crack widths in radon proof concrete foundations.
- To investigate and compare the gained effect of various reinforcement amount and sizes.

- To examine different water cement ratios to gain knowledge about how it affects the crack width when designing to meet the set limitations.

1.3 Limitations

The radon content in already existing structures can be decreased in different ways where various methods are used for different sources of radon leakage. One example of action that could be performed to minimize the contamination of the indoor air is to fill in eventual cracks or other defects that could be reason to the leakage (Boverket, 2018). However, this study will not go further into these solutions but will only examine the radon penetration through newly produced foundations.

Besides the radon penetration through cracks in the foundation, the radon can also get into a building through other entrances integrated in the concrete slab. Pipe penetrations to different service departments, and drainpipes are some examples of other possible ways for the substance to get to the environment inside (Boverket, 2018). This project will not consider these other sources but will only consider elements based on plain reinforced concrete.

An other parameter that affects the radon penetration through the concrete floor is the radon concentration in the ground. Different soils and materials contains different amounts of radon which makes the resulting radon impact dependent of the placement of the building (Boverket, 2018). This project is limited to not investigate the geotechnical aspect but will assume common conditions in Sweden.

The air pressure inside and outside a building is also affecting the ratio of how much radon that is penetrating through the foundation (Boverket, 2018). In this case the ventilation in the building is of great importance and could be adapted to different circumstances. This project is limited to only investigate different concrete elements exposed to the same indoor conditions and will not consider possible improvements of the ventilation in the construction. The environment inside and outside the building is also influencing the cracking process where different conditions, including temperature and relative humidity, effect the foundation in different ways. This project is limited to only investigate concrete elements to a school building with typical indoor conditions for a building in Sweden.

Finally, the building materials are also effecting the radon contamination inside. Even though all stone based materials contains different amounts of radon the ratio are mostly not that high that it has to be considered. It is though important to know that there are exceptions depending on were the material is extracted from and that especially one material, alum shale, should be avoided in this kinds of compositions (Boverket, 2018). The concrete composition used in the calculations in this project will be based on standard materials that are approved to use in this kind of structures.

1.4 Method

A literature study was be performed to assess requirements regarding radon emissions as well as crack width requirements. This was followed by analytic analysis including crack widths, for different concepts of concrete elements according to Eurocode 2. For this, the water cement ratio (compressive strength class and shrinkage) was varied together with the amount and dimension of reinforcement. The result for the different designs was then compared to each other and evaluated based on the different aspects.

2 Radon in the environment

Chemical elements exists in different amounts naturally in nature. They consist of different amounts of protons, neutrons and electrons which define the properties of the element. The placement of the element in the periodic system describes, among other, the number of protons in the atom core i.e. atomic number and the number of electrons in the outer shell. Most elements have variants, so called isotopes, where the amount of protons in the atomic core are the same but the neutron number varies. Isotopes are represented with the same letters as the origin element but are complemented with a number at the end. The number describes, in addition to the proton number, the amount of neutrons in the atomic core. The variation in amount of neutrons is directly connected to the mass but does generally not influence the characteristics of the isotope much. An isotope can either be stable or unstable i.e. radio active (Nationalencyklopedin, 2019a). Some of the most significant characteristics of radio active atoms are their half-life and that they decay during time without outer impact, emitting α-, β or γ- radiation i.e. ionizing radiation in the process.

Radium, **Ra**, is a chemical element that is formed in the decay process in two of the most radio active elements on earth, uranium, **U**, and thorium, **Th**. All of its isotopes are highly radio active and forms mainly α- radiation in their decay processes. The most stable isotope, radium-226 is, when it decays, forming the noble gas radon, **Ra** (Nationalencyklopedin, 2019c). The noble gas is, among other, containing the isotope radon-222 which has a half- life of 3.82 days. Figure 2.1 shows a simplified model of the decay chain of uranium of which in its decay process generates radon isotopes. Despite the short half-life of the isotope the substance will still be present in the future because of its origin. Since the half-life of both uranium and thorium are reaching over several billion years they will, as they decay, continuously generate new radon to the environment (Nationalencyklopedin, 2019d).

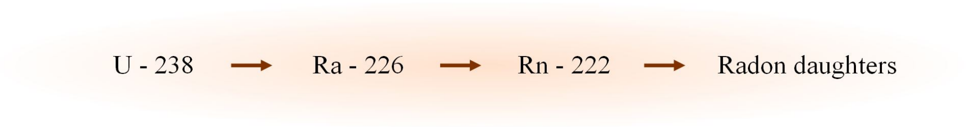

*Figure 2.1: Simplified decay chain of uranium, **U**, of which the noble gas radon, **Rn**, is generated. Ionizing radiation is emitted in the process.*

2.1 Radon's impact on the health

By further decomposition of the radon gas, radon daughters are generated and ionizing radiation is formed in the process. Since the radon daughters are in gas form they blend with the air and are then inhaled by humans where they, to some extent, get stuck in the airways. The radon daughters are by further decay, inside of the body, transmitting α- radiation which is harmful to the health due to the damage they cause on both lungs and bronchus. The damage develops and becomes more severe if the body is exposed to the substance during a longer period of time and can result in lung cancer. The risk of developing lung cancer is drastically increased if the body frequently also is exposed to tobacco smoke which are the most common cause to lung cancer on its own (Boverket, 2018).

2.2 Regulations of radon content in indoor air

Due to the risk of injury caused by the radon contamination in indoor air there are regulations to follow to decrease the risk of damage. According to the Swedish authority for community planning, construction and housing i.e. *Boverket*, the reference value is set to 200 Bq/m^3. The value is a yearly average and applies to all residential and public buildings, including schools and preschools. It also applies to newly produced buildings but as low value as possible is desirable and should be aimed for if possible (Boverket, 2018).

2.3 Protection against radon leakage

The ionizing radiation from the described process is mainly consisting of α- radiation. Different kinds of radiation absorbs or transfers through matters in different amounts. Figure 2.2 shows how α- radiation is blocked to pass through thin layers, like paper, while β- and γ- radiation are blocked by more dense and thicker materials like acrylic glass and led respectively (Nationalencyklopedin, 2019b).

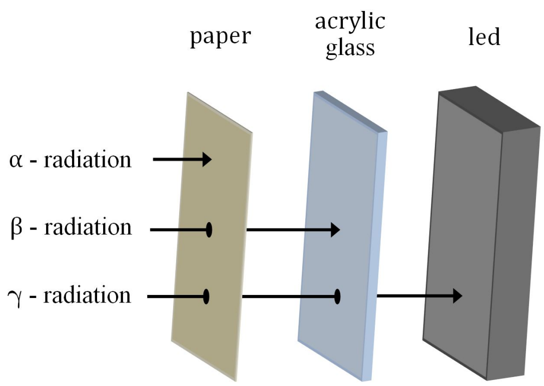

Figure 2.2: Penetration strength of α-, β- and γ- radiation. Different types and thicknesses of material can be used to stop the radiation from penetrating to the other side.

Since the α- radiation is blocked by thinner layers is it possible to create a barrier to prevent the radiation to get inside. As the major source of radon is in the ground the most critical entrance is through the foundation. It is therefore important to create a foundation that does not contain any leakages since it significantly will reduce the efficiency of the barrier.

3 Concrete as a material

Concrete is a material that has been used for several hundred years and is today used in all kind of constructions. It is, due to its durability and strength, one of the most used building materials in load-carrying structures (Burström, 2007). In this chapter the composition of the material is treated, as well as its properties and the internal and external influence from the surrounding environment. The chapter is mainly based on; Burström (2007) and Al-Emrani et al. (2013) .

3.1 Concrete composition

Concrete mainly consists of aggregate, cement, sand and water which, after mixed together and casted in desired shape, forms a durable and high load carrying element. By adding admixtures or supplementary cementing materials the properties of both the fresh and the hardened concrete can be modified to achieve desired result (Burström, 2007).

3.1.1 Aggregate

Aggregate consists of rocks in different fraction sizes which are divided into groups of stones, gravel, sand and filler. According to Burström (2007) the main rock that is used in concrete production is macadam which is a crushed material with sharp edges. To achieve an as dense concrete as possible is it important that the proportions of the different sizes are correct. Big fractions should be surrounded by smaller fractions that should be surrounded by even smaller fractions and so on, to fill as much of the concrete volume as possible. The remaining volume that is not covered by the aggregate is then filled with cement and water.

3.1.2 Cement

Cement is the ”glue” that binds the ingredients in the concrete together into one element. It is a hydraulic binder which means that it, in reaction with water, creates a product which is resistant to water. The main ingredients in Portland cement are limestone and clay which first are heated up to very high temperatures and then cooled down. In the cooling process the material is shaped into small beads i.e. cement clinker, which when ground together with gypsum creates the final cement product.

The velocity of the reaction between the water and cement is dependent on the composition and fineness of the cement and the rate of reaction is divided into three classes. The first class, of which the cement has a slow reaction and hardening time, is called **class S**, (S for Slow) while the cement class with moderate drying time is called **class N**, (N for Normal). The final class for cement that gains its properties the quickest is called **class R**, (R for Rapid) (Burström, 2007).

3.1.3 Water

The quality of the water used in the concrete mixture might effect the final durability and strength of the concrete. Water with high concentrations of salt are not sufficient, nor allowed to use, especially not in structures with desired high strength, due to the increased risk of corrosion. Normally, if the water is naturally and potable, there are no further requirements for the quality of the water to be used the process.

3.1.4 Admixtures

When designing concrete different characteristics are desirable depending on the area of use. To modify the properties of both the fresh concrete as well as the hardened concrete, admixtures are added to the recipe (Burström, 2007). Some of the most common admixtures are listed down below.

Superplasticizers

The most common property that is modified is the consistency of the concrete. Superplasticizers gives the concrete a more loose consistency which reduces the needed quantity of water in the concrete. Smaller proportions of water will eventually result in an increased strength and a decreased shrinkage. With this kind of admixture is it possible to retain desired cohesion and strength while making the concrete more easy to process (Burström, 2007).

Water-reducing admixture

Water-reducing admixtures do give similar effects on the concrete as the superplasticizers but is not as powerful. The advantage of using this admixture is that the concrete gets easier to process because of the reduced friction between the particles in the mixture. Because of the increased processability of the concrete can the water amount decrease, resulting in a higher strength of the final product (Burström, 2007).

Air-entraining admixture

To modify the resistance against frost damages, the pore system is of great importance. The water stored in the pore system freeze, causing an expansion, when the concrete is exposed to cold environments. Large stresses might occur if the volume of the frozen water becomes larger than the existing pore volume, causing internal damage to the structure. To prevent this air-entraining admixtures are used to increase the pore volume, they also decreases the risk of separation between the components (Burström, 2007).

Accelerating and retarding admixtures

The time it takes for the concrete to harden and to achieve it strength can be modified with acceleration or retarding admixtures. This is advantageous e.g. during long transportation where additional time is needed before the concrete can be cast.

3.1.5 Supplementary cementing materials

Supplementary cementing materials are used to modify the structure of the concrete by replacing some of the cement and contribute to the hydration process. Two of the most used materials in Sweden are *silica fume*, *fly ash* and *ground granulated blast furnace slag* which among others improve the cohesion and stability of the concrete (Burström, 2007).

3.2 Fresh concrete

When all the components of the concrete are mixed together the first phase in the hardening process has started. The concrete is then called *fresh concrete* of which the characteristics are of great importance as they influence the properties of the final product. Due to the different density of the materials included in the mixture the heavier particles tend to naturally sink to the bottom while the lighter materials stay by the surface. To create balance and prevent this to happen it is important with good *stability* of the concrete. This means that the components in the concrete do not separate from each other but stays together, as a homogeneous material, during the whole process. Good stability is one of the most important parameters of the fresh concrete. Another important parameter is the ability to process the concrete during the casting, where it is important that the process is carried out correctly to meet the set requirements of the final construction (Burström, 2007).

3.3 Hardening process of the concrete

The hardening process, i.e hydration process starts as soon as the water is added to the cement, creating a cement paste that binds the aggregate together. During the first couple of hours the fresh concrete will slowly thicken to a viscous paste before it enters phase two. In phase two the concrete, now called *young concrete*, is developing new properties as it begins to harden. The velocity of the reaction is higher during the first couple of days and decreases with time until it finally stops when the entire amount of cement has reacted with the water. Figure 3.1 shows the strength development during the different phases in the hardening process (Burström, 2007).

3.3.1 Effect of temperature

Temperature is an important parameter affecting the concrete. Chemical reactions that occur during the hardening process generates a considerable amount of heat. Both the quantity of the cement as well as the cement type do effect how much heat that is released in the process. The temperature increase can cause the element to swell which, when cooled down again, results in contraction which could cause cracks both internal and on the surface if the movement somehow is prevented.

The surrounding temperature of the casted concrete effect the consumed time of the hardening process as well as the properties of the hardened element. High

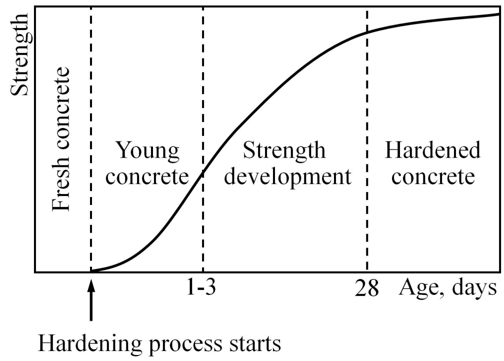

Figure 3.1: Strength development in concrete during the hardening process

temperatures accelerate both the hydration process and the development of the concrete properties but, as a consequence, the strength of the concrete can be effected negatively. This can cause problems if the temperature of the environment becomes too high due to the decreased strength of the element (Burström, 2007).

3.3.2 Effect of relative humidity

One important parameter that affects the drying time is the relative humidity of the surroundings. During the hardening process is the concrete sensitive to environmental changes where the drying time can be affected in an adverse way. A moist environment causes a slower hardening process than a dry environment due to the difference in velocity of moisture exchange between the concrete and the air. Early drying out of the concrete might generate cracks over the surface, i.e map cracking which could be avoided if both concrete mixture and curing are designed in a proper way (Engström, 2014).

3.4 Hardened concrete

When the concrete has cured a significant high compressive strength has been developed, which is about ten times higher than the tensile strength. The compressive strength is measured according to standardized methods where specimens, with an age of 28 days, in the shape of either cylinders or cubes are tested. In Sweden these tests are based on cubes which gives about 20 percent higher strength than the cylinders.

3.4.1 Concrete strength classes

Eurocode 2 define different classes depending on the strength of the concrete where the first number specifies the strength measured on cylinders and the second number specifies the strength of cubes. Different characteristics are used depending on the purpose of the calculation. Table 3.1 and 3.2 displays the compressive and tensile strength for the concrete strength classes that correspond to normal-density concrete. The different fractile in Table 3.2 is used in different ways, depending on what to be evaluated. For determining if cracking occurs the 5% fractile is used while the 95% fractile is more critical when designing the reinforcement. However, the mean tensile strength is usually used in design (Al-Emrani, Engström, Johansson, & Johansson, 2013).

Table 3.1: Compressive strength for different concrete strength classes.

Class	12/15	16/20	20/25	25/30	30/37	35/45	40/50	45/55	50/60
f_{ck} **[MPa]**	12	16	20	25	30	35	40	45	50
f_{cm} **[MPa]**	20	24	28	33	38	43	48	53	58

Table 3.2: Tensile strength for different concrete strength classes.

Class	12/15	16/20	20/25	25/30	30/37	35/45	40/50	45/55	50/60
$f_{ctk0,05}$ **[MPa]**	1.1	1.3	1.5	1.8	2.0	2.2	2.5	2.7	2.9
f_{ctm} **[MPa]**	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.1
$f_{ctk0,95}$ **[MPa]**	2.0	2.5	2.9	3.3	3.8	4.2	4.6	4.9	5.3

The concrete strength classes are also applied to the concrete modulus of elasticity which increases with increased strength. Table 3.3 shows values for the modulus of elasticity for the concrete classes that correspond to normal-density concrete (Al-Emrani et al., 2013).

Table 3.3: Modulus of elasticity for different concrete strength classes.

Class	12/15	16/20	20/25	25/30	30/37	35/45	40/50	45/55	50/60
E_{cm} **[GPa]**	27	29	30	31	33	34	35	36	37

3.4.2 Influence of water cement ratio

When choosing strength class multiple different parameters are influencing the final choice. Normally, the most vital aspect of the concrete composition is the obtained strength but other parameters do have a great importance as well. The durability, casting properties and drying time are some of the characteristics that are affected by the composition and needs to be considered in the process. The relation between the amount of water and cement is called water/cement ratio and is crucial to the final strength of the concrete (Al-Emrani et al., 2013). It is calculated as:

$$w/c - ratio = \frac{W}{C} \tag{3.1}$$

where:

W = amount of water
C = amount of cement

The water/cement ratio is increasing with increased water amount, which results in a lower strength of the concrete. A high water/cement ratio is favorable with respect to crack limitations and crack width calculations but also results in longer drying time and less strength. Figure 3.2 displays the approximate relation between the water/cement ratio and the compressive strength for concrete (Al-Emrani et al., 2013).

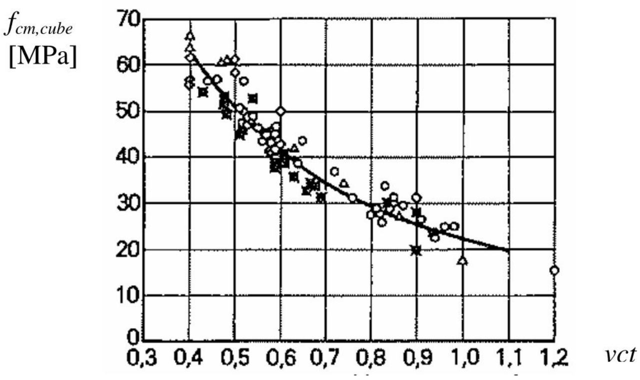

Figure 3.2: Approximate relation between the water/cement ratio and the compressive strength of 28 days cured concrete (Al-Emrani, Engström, Johansson, & Johansson, 2013)

3.5 Time dependent deformations

During the lifetime of the structural member it will be affected by various factors, both externally and internally. Consequently, deformations will form in the element which either are caused by stress dependent strains or by strains independent of stresses (Engström, 2014). This section describes some of the typical time dependent aspects that might affect a concrete element during its lifetime.

3.5.1 Shrinkage

Shrinkage is a time dependent deformation that occur due to volume decrease during the hardening process of the concrete. It is dependent on multiple different aspects including the composition of the concrete as well as the size of the element and the properties of the surroundings. The final total shrinkage is divided into two separate components, drying shrinkage and autogenous shrinkage, and is according to Eurocode 2 calculated as

$$\varepsilon_{\mathrm{cs}}(\infty) = \varepsilon_{\mathrm{cd}}(\infty) + \varepsilon_{\mathrm{ca}}(\infty) \tag{3.2}$$

where:

$\varepsilon_{\mathrm{cs}}(\infty)$ = final total shrinkage
$\varepsilon_{\mathrm{cd}}(\infty)$ = final drying shrinkage
$\varepsilon_{\mathrm{ca}}(\infty)$ = final autogenous shrinkage

Drying shrinkage

Drying shrinkage is dependent on the water/cement ratio and the amount of free water i.e. evaporable water, that did not bond to the cement during the hardening process. The free water is instead being stored in the generated pore system and will evaporate during time if the surroundings of the casted concrete has a lower relative humidity than the concrete itself. The eventual loss of stored water causes a decrease of the total concrete volume, so called drying shrinkage, $\varepsilon_{\mathrm{cd}}$, which could result in cracks in the concrete if it is unable to deform freely (Al-Emrani et al., 2013).

The rate of the drying shrinkage is dependent on the notional size which is the thickness of a, to the element, equivalent wall that has both sides exposed for drying. It has big impact on the final shrinkage and depends on the size of the element where the area of the cross section, together with the exposed circumference of the cross section, are included. An increased exposed circumference will decrease the notional thickness, contributing to a higher value of the drying shrinkage.

Autogenous shrinkage

Another form of shrinkage is the autogenous shrinkage, $\varepsilon_{\mathrm{ca}}$, which unlike the drying shrinkage does not depend on the properties of the surroundings. The deformation is caused by chemical reactions between the water and cement in the hydration process and has the greatest impact during the first 24 hours. This kind of shrinkage is of greater importance to the higher concrete strength classes, of which the water/cement ratio is lower. Because of the high cement content, in relation to the water amount, will a higher proportion of the water become chemically bonded in the structure (Al-Emrani et al., 2013).

3.5.2 Plastic shrinkage

Unlike drying shrinkage and autogenous shrinkage, plastic shrinkage is developed during a shorter period of time when the concrete still is in its plastic form i.e. before final setting and hardening. This kind of shrinkage is mainly affecting the upper layer of the concrete element where the water evaporation from the surface is high. The larger area of which the water are able to evaporate from, the more critical the plastic shrinkage will be. Hence, when constructing concrete slabs the plastic shrinkage is more critical and has to be considered (Narin & Wiklund, 2012).

The first six hours after casting are most critical regarding plastic shrinkage. During this time the concrete still is in liquid state where the water is able to move freely in the concrete. During this time the phenomenon mentioned in Subsection 3.2 can occur, where the different sizes and densities of the particles in the concrete causes separation. Hence, a larger amount of water accumulate close to the surface while bigger particles sink to the bottom. If the surface water evaporate fast during this first period of time, the separation will continue, causing more water to separate from the mix. Eventually, the process causes the top layer to dry out which generates problems in the layer underneath. There, tensile stresses arise between the particles which, when reached tensile capacity of the concrete, result in evenly distributed cracks across the surface. These cracks can achieve crack widths larger than 0.1 mm and might be through cracks. Figure 3.3 (right) shows a schematic illustration of cracks on the concrete surface, caused by plastic shrinkage. With respect to radon leakage, plastic shrinkage cracks should be avoided.

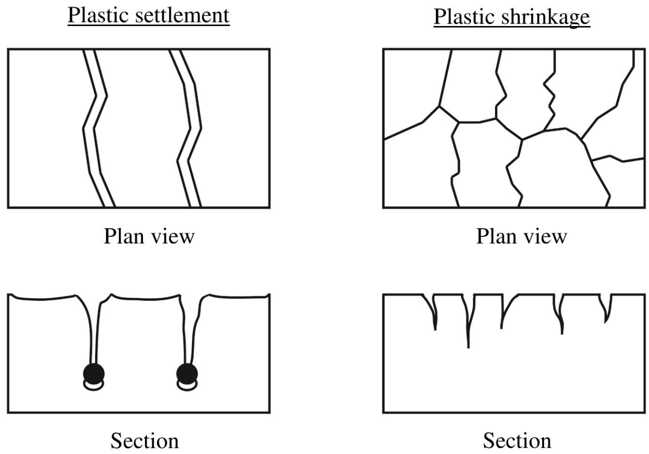

Figure 3.3: Plan- and section illustraions of plastic settlement (right) and plastic shrinkage (left) (Narin & Wiklund, 2012)

Plastic settlement

Another effect that could occur during the first hours is plastic settlement. This is caused when movements in the fresh concrete are prevented by the horizontal reinforcement. Figure 3.3 shows how the upcoming cracks, caused by plastic settlements, are evenly distributed along the reinforcing bar.

3.5.3 Creep

Once an element has been loaded an internal strain is formed in the concrete, resulting in an initial elastic deformation. The deformation increases over time as long as the loading continues but will reach an assumed final value about 70 years after the concrete has been cast. This kind of strain dependent deformation is called *creep* and is one of the possible causes to cracks to appear in loaded concrete structures (Al-Emrani et al., 2013).

The creep coefficient $\varphi(\infty, t_0)$ is according to Eurocode 2 calculated as:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_c(t, t_0) \tag{3.3}$$

where:

$\beta_c(t, t_0)$ = coefficient that describes the development of creep after loading

The notional creep coefficient, φ_0 is estmiated accoring to:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \tag{3.4}$$

where:

φ_{RH} = factor that considers the relative humidity of the surroundings
$\beta(f_{cm})$ = factor that considers the strength of the concrete
$\beta(t_0)$ = factor that considers the age of the concrete once loaded

The factor, $\beta(t_0)$, is dependent on the age of the concrete at the time when the element first is loaded. Figure 3.4 shows how the factor decreases and becomes lower the older the concrete gets before it is exposed to the initial loading (Al-Emrani et al., 2013).

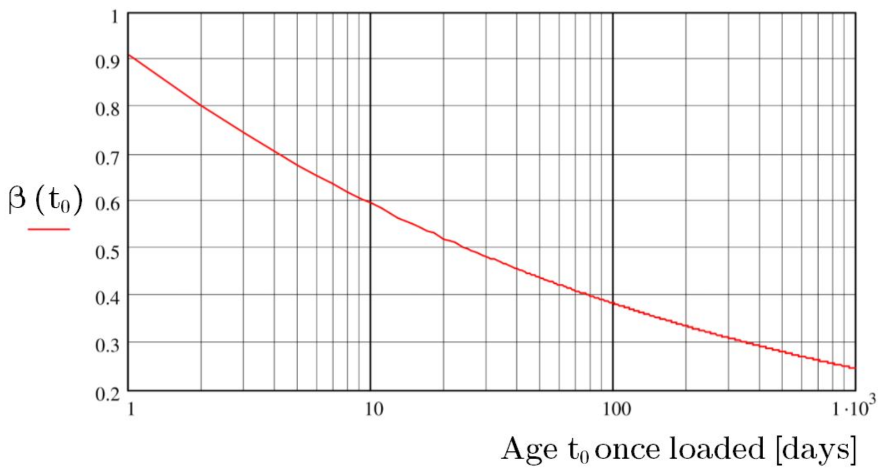

Figure 3.4: Factor $\beta(t_0)$ that considers the age of the concrete once loaded (Al-Emrani, Engström, Johansson, & Johansson, 2013)

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4 Reinforcing steel

The most efficient way to improve the tensile strength of a concrete element is by implementing reinforcing bars. The reinforcement consists normally of steel but other materials, like FRP, can be used in the same purpose. The reinforcing steel has a much higher tensile strength compared to the concrete and by cooperating they create a more versatile system. A plain concrete element loaded in tension will eventually crack but by adding reinforcement the system will be balanced and more durable to withstand tension. By using reinforcement in different layouts it is possible to customize the handling of forces in the concrete. This is done by regulating the stiffness and moment capacity for different parts in a construction (Al-Emrani et al., 2013).

4.1 Common types of reinforcement

Standardized reinforcement products exist in different configurations of reinforcement bars; decoiled wires and rods, welded wire fabric and lattice girders. They are available in different shapes and sizes and the surface is either intended, deformed with ribs or plain which affects the strength of bond to the concrete. The dimensions of the diameter of the bars normally exists in 6, 8, 10, 16, 20, 25, 32 mm but a diameter of 40 mm is also available for ribbed bars (Al-Emrani et al., 2013).

4.2 Response to loading

A general working curve of reinforcing steel is displayed in figure 4.1 where four clear stages can be read; elastic, plastic, yield and collapse. Hence the material acts linearly until it reaches its yielding capacity after which it starts to behave non-linearly (Al-Emrani et al., 2013).

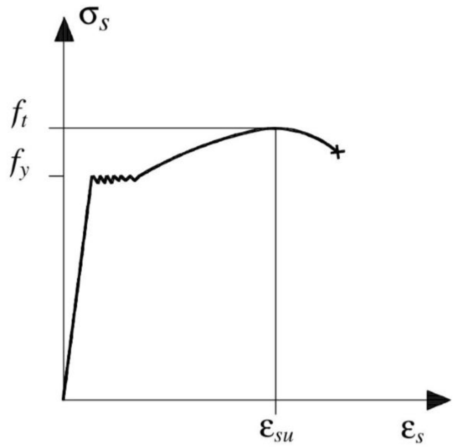

Figure 4.1: General working curve of reinforcing steel (Al-Emrani, Engström, Johansson, & Johansson, 2013).

5 Cracking in concrete

Cracks that appear in concrete can be caused by many different reasons depending on what the element is exposed to. Normally cracking is expected and the implemented reinforcement designed in a way to prevent the appearance of large cracks, but with respect to plastic shrinkage cracks and settlement cracks the reinforcement has no effect on the crack width. Instead the upcoming cracks are smaller and distributed over a larger area which are more favourable to the load carrying system. The cracks in concrete foundations are mainly generated during the hardening process and appears if the concrete is unable to deform freely. This chapter explains the causes to the appearance of cracks in concrete foundations and is mainly based on Al-Emrani et.al. (2011) and Engström (2014).

5.1 Cracks caused by restraint

Restraint in the concrete occurs if the free movement somehow is prevented. It causes stresses to arise in the concrete that might result in the appearance of cracks. The originated movement can both be affected by internal or external restraint where the difference between the two are explained below.

5.1.1 External restraint

If the movement of the element is prevented by supports or by other external boundaries external restraint appears. The degree of external restraint depend on the boundary conditions to the structural member and can vary in different directions, being either partial or fully restraint. An element with few number of fixed boundaries is more capable to deform than an element with more fixed points. Hence, less fixed boundaries are advantageous with respect to external restraint. To allow movements as much as possible the fixed boundaries should favourably be designed in a way that movement in each direction is possible (Engström, 2014).

Engström (2014) writes further that a concrete slab, subjected to movements caused by temperature or shrinkage, is prevented to move freely if it is cast directly on the ground. The restraint is caused by frictional forces between the concrete and the soil that occur due to the oriented movement. Tensile forces are initiated in the concrete since the element will obtain a length that does not correspond to the actual required deformation. Figure 5.1 shows a concrete slab, casted directly on soil, partially prevented to move due to the generated frictional forces.

Larger foundations should, according to Engström (2014) be divided into separate parts to avid or decrease the appearance of cracks in the concrete. But with respect to radon leakage these joints then have to be sealed so that they are gas tight.

A consequence that could appear due to external restraint is bending of the element. This is caused when the structural member is subjected to an eccentrically restraint

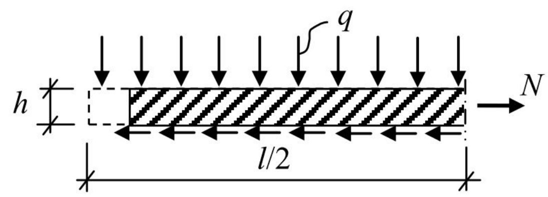

Figure 5.1: Concrete slab casted directly on ground where frictional forces between the slab and the soil prevents the generated movements (Engström, 2014).

force and the element is unable to deform uniformly. The bending can also be a result of a linearly varying strain that e.g. arise when a slab is subjected to solar radiation. The heated top surface of the element will expand due to the increased temperature while the bottom surface area nearly will remain unaffected. Hence, the slab tend to bend upwards resulting in tensile stresses in the bottom edge, caused by the restraint by the dead weight. Figure 5.2 shows the possible deformations of a slab subjected to linearly varying strain.

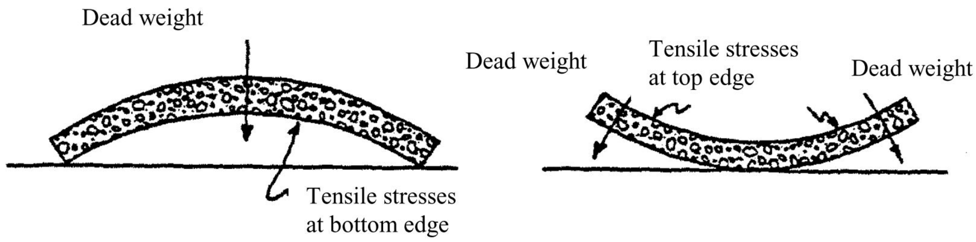

Figure 5.2: Deformed concrete slab casted directly on soil, subjected to external restraint by its dead weight (Engström, 2014).

Similarly, bending of the structural member can be caused by uneven shrinkage. If the top surface of the concrete slab dries out faster than the bottom surface, a difference between the dimensions of the two areas will arise. Hence, the top surface area will decrease in relation to the bottom surface, causing the edges of the slab to bend upwards. If the movement is prevented by external boundaries high tensile stresses will arise in the concrete edges, which eventually might crack or break completely. Figure 5.3 shows three stages from when the concrete starts to bend until collapse (Engström, 2014).

The final stresses and deformations in the concrete element depend, to a great extent, of the boundary conditions of the structural element. More fixed boundaries tend to result in higher stresses.

5.1.2 Internal restraint

Internal restraint is caused when movements in one part of the cross section of an element are prevented by another part of the same cross-section. An example of the phenomenon is the interaction between concrete and reinforcement. Deformations due to shrinkage will cause movements in the concrete that, due to the bond to

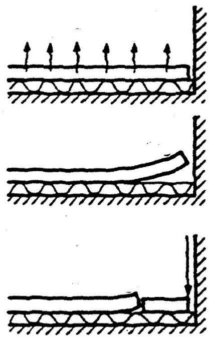

Figure 5.3: Uneven shrinkage causing the concrete to bend. If the movement is prevented by external force the element might break (Engström, 2014).

the reinforcement bar, are partially prevented. The internal restraint causes tensile stresses to appear in the concrete that might lead to cracking (Engström, 2014).

Engström (2014) writes that the oriented deformations depend on how the reinforcement bars are arranged and if the shrinkage is uniform or linearly varying. The interaction between the reinforcement and the surrounding concrete is also important to consider.

5.1.2.1 Interaction between reinforcement and surrounding concrete

Reinforcement is implemented in the concrete to increase the tensile strength of the structural element. This is performed by transmission of forces between the two materials. The interaction is called *bond* of which the strength depend on the surface structure of the reinforcing bar. The more surface irregularities, the stronger the bond gets. Hence, intentionally deformed bars entails a stronger bond than plain reinforcing bars (Al-Emrani, Engström, Johansson, & Johansson, 2011).

El- Emrani et al. (2011) writes that the type of bond is depending on the size of the tensile force that affect the system. Small tensile forces are transferred through adhesion between the two materials while larger tensile forces are transferred via shear key effect. Due to the shear that occur between the two elements diagonal stresses are initiated in the concrete. If the stresses exceeds the tensile capacity of the concrete, cracks will appear in the same direction as the gained stresses. Once the concrete has cracked, the reinforcing bar is anchored by contact forces that occur in the interface between the elements. Figure 5.4 illustrates how diagonal contact forces counteracts the tensile force from the reinforcing bar.

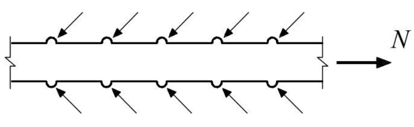

Figure 5.4: Diagonal contact forces caused by shear key effect (Al-Emrani, Engström, Johansson, & Johansson, 2011).

The magnitude of the steel stress is biggest at the edge of the structural element, or at a crack, where load is applied. This entails a difference between the stresses in the concrete and the reinforcing bar which generates a need of deformation between the two components, called *slip*. It is only possible within the *transmission length*, l_t, which is the length of which the steel is able to elongate separate from the concrete. The stronger the bond between the reinforcement and the concrete is, the smaller slip will occur. Farther into the element, where the stress difference is smaller or non existing, the slip decreases and eventually becomes zero. It is after that point no longer necessary with force transmission, hence the two materials act together like one rigid element (Al-Emrani et al., 2011).

The thickness of the concrete cover in relation to the dimension of the reinforcing bar influence the risk of splitting cracks to appear through the cover. Thinner concrete cover increases the risk of cracks. Figure 5.5 illustrates how the stresses around the bar redistribute after a splitting crack appear which eventually might lead to a new splitting crack in another direction.

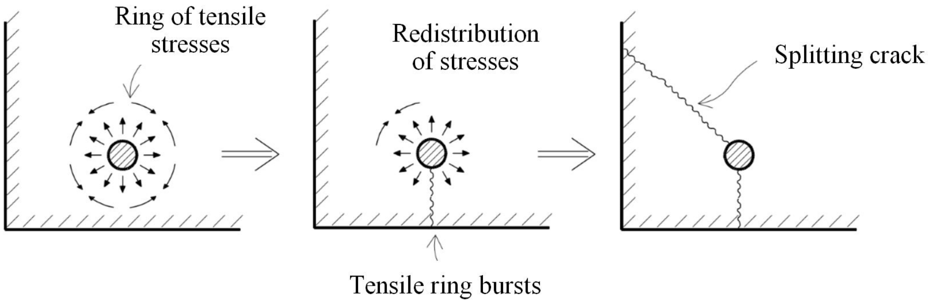

Figure 5.5: Splitting crack caused by too thin concrete cover. Redistribution of stresses causes a new crack to appear in another direction (Al-Emrani et.al, 2011).

When the surrounding concrete of the reinforcing bar has cracked splitting failure might occur causing both the concrete cover and the reinforcing bar to detach from the element (Al-Emrani et al., 2011). Figure 5.6 shows two illustrations of how splitting failure might look.

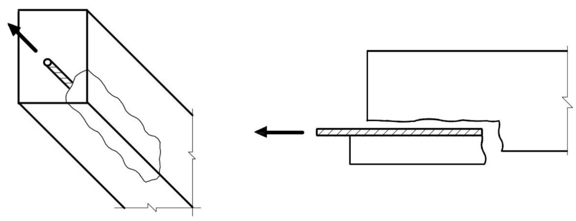

Figure 5.6: Splitting failure due to too thin concrete cover (Al-Emrani et.al, 2011).

5.2 Cracking process

The cracking process in reinforced concrete elements are initiated when the tensile capacity in the concrete is exceeded. As mentioned earlier the stresses in the reinforcing steel is transferred to the concrete through the interaction between the two elements. Naturally, the steel stress decreases while the stresses in the concrete increase. The additional stresses that are generated in the process are called *bond stresses*, τ_b, and only occur within the transmission length, l_t. Figure 5.7 illustrates how the stresses in a tensile loaded element are distributed across the length (Engström, 2014).

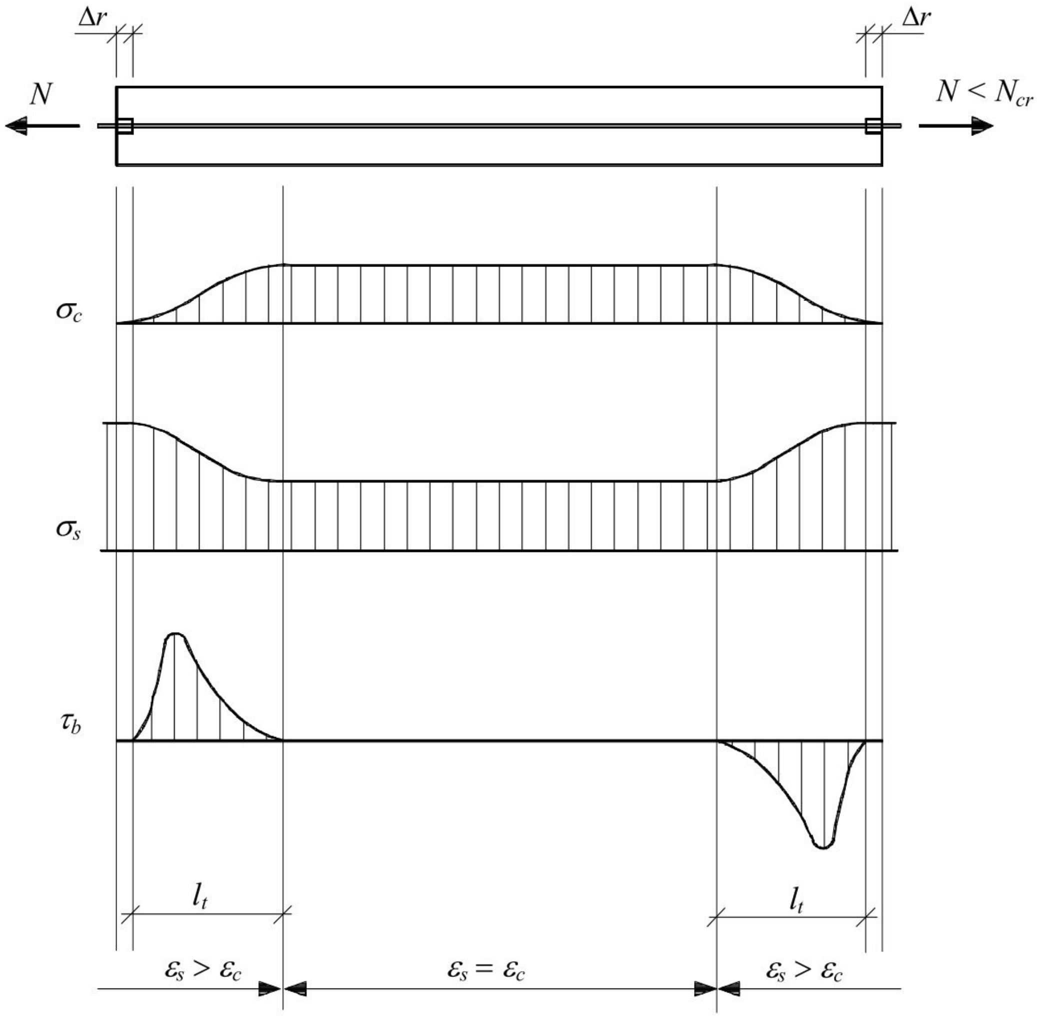

Figure 5.7: Distribution of stresses in a reinforced concrete element subjected to tension. The stresses in the concrete, σ_c, and the steel, σ_s, are presented, as well as the generated bond stresses, τ_b (Engström, 2014).

Engström (2014) writes further that the stresses and the transmission length increases with increased loading until maximum capacity has been reached. However, when cracking load is applied to the system, maximum transmission length is attained and a first crack is initiated. This causes the stresses in the element to redistribute around the damaged area to find new equilibrium to the system. If the tensile stress in an adjacent section remains close to maximum capacity, a new crack might occur. Figure 5.8 displays how the tensile stresses in the concrete redistribute once maximum capacity is reached. It shows how the stresses in the sections between the appeared cracks decreases, reducing the risk of further cracks to occur. The distance between the cracks cannot be smaller than the maximum transmission length and thus, affects the possibility of new cracks to form. The tensile capacity of the concrete cannot be reached if the spacing between the cracks is less than $2 \cdot l_t$. Hence, cracks are not able to form in those regions.

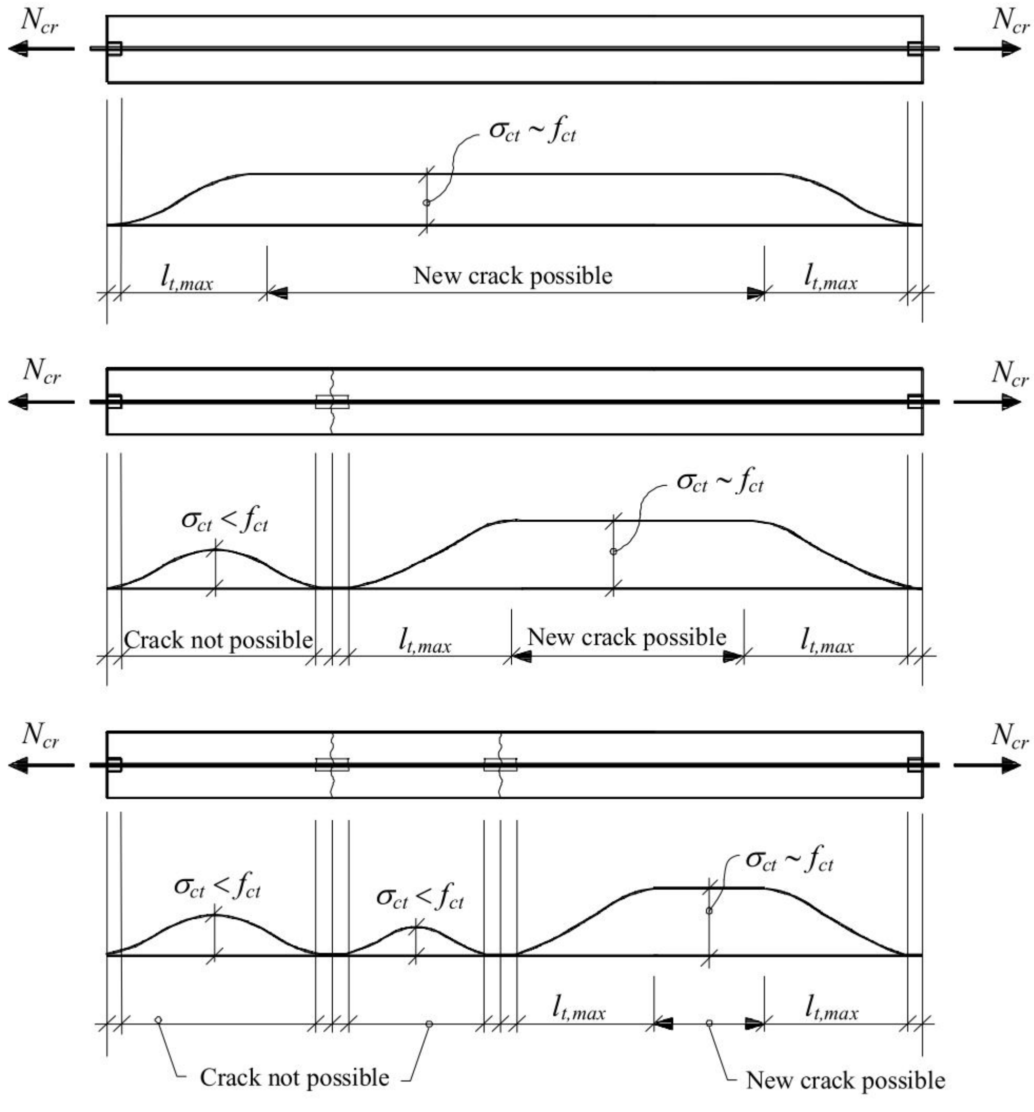

Figure 5.8: Stress distribution in reinforced concrete element. Top figure displays the stress distribution right before the first crack has occurred. Middle figure illustrates how the stresses redistribute after the crack is initiated. Bottom figure shows how another region crack, if its capacity still is exceeded (Engström, 2014).

5.3 Effect of cracking

Despite the cracking, the load-bearing capacity of the structural member is generally not affected significantly. However, other characteristics of the construction might be influenced negatively. Once the element has cracked the waterproof, airtight and sound insulation aspects decrease. Further, the durability of the member can also be abbreviated since both the degradation of the concrete and the reinforcing steel is accelerated once the concrete cover break (Al-Emrani et al., 2011).

5.3.1 Reinforcement corrosion

Al-Emrani et.al. (2011) writes that the concrete covering the reinforcing bars have a passivizing effect on the steel, protecting it form corrosion. However, once the cover cracks and reveals the steel to the surrounding environment, the effect vanish. One of the two main causes for corrosion is carbonation of the concrete, where the passivizing effect of the surrounding concrete decreases with time. Another possible reason for corrosion to start is if the reinforcement bar is subjected to chlorides.

During corrosion, rust is generated on the reinforcing bars, causing an increase of the volume. Consequently, the surrounding concrete crack since the increased steel volume entails a pressure which generates tensile stresses in the concrete. The cracks are similar to the splitting cracks caused by the increased tension in the bars mentioned in Section (5.1.2.1). Figure 5.9 illustrates how splitting cracks might appear due to reinforcement corrosion and how they, if connected over a larger area, could cause large concrete surfaces to detach from the member (Al-Emrani et al., 2011).

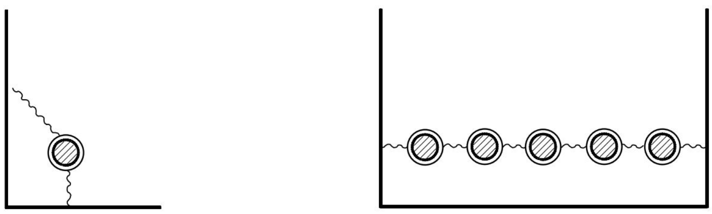

Figure 5.9: Splitting cracks due to reinforcement corrosion (Al-Emrani, Engström, Johansson, & Johansson, 2011).

5.3.2 Minimum reinforcement

Once a crack appear in the concrete it loses its tensile load-carrying capacity, hence the forces in the structure redistribute to find alternative ways to the ground. Before a new equilibrium has been reached, dynamic effects are entailed which could effect the reinforcement negatively. To reduce the risk of damage, the tensile capacity of the concrete has to be considered when designing the reinforcement. This is done with *minimum reinforcement* which is the smallest amount of reinforcement needed to distribute the cracks over a larger area, preventing uncontrolled growth of single cracks. In other words, the amount of reinforcement required to avoid yielding of the steel before a second crack occurs when the cracking process has started. It is important that the reinforcing steel holds enough capacity to limit the crack width as much as required. Hence, the stricter crack width limitations, the larger amount of minimum reinforcement is required in the system. Minimum reinforcement only considers the dimensions of the cross-section as well as the tensile strength of the concrete and does not include any influence of bending or shear (Engström, 2014).

Al-Emrani et.al. (2011) writes that the minimum reinforcement in a slab on ground could be reduced due to the friction that occur between the soil and the concrete. Figure 5.10 shows an illustration of a concrete slab where the minimum reinforcement is influenced by the boundaries of the element. The factor that is used in the reduction is 0.7 which is multiplied with the calculated minimum reinforcement area.

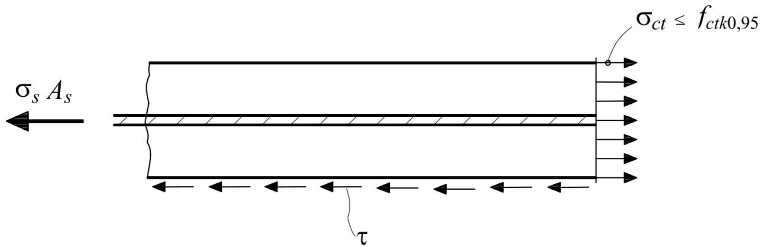

Figure 5.10: Minimum reinforcement in a concrete slab, influenced by the the friction between the concrete and the soil (Al-Emrani, Engström, Johansson, & Johansson, 2011).

5.4 Possible approach to minimize or prevent cracking

Cracks are oriented from when the stresses in the concrete has reached maximum tensile capacity. To minimize the crack widths reinforcement is used to distribute the stresses, resulting in multiple finer cracks instead of few large ones. The method is successful, however, there are alternative methods that could limit or prevent cracking completely (Al-Emrani et al., 2011).

Al-Emrani et.al. (2011) writes that since cracks are initiated when maximum tensile capacity is reached, the process could be delayed and partially or fully prevented if the stresses in the concrete somehow are modified. This could be done by *pre-stressing* the structural element which introduces compressive forces to the system in the production stage. The method uses high strength steel and can either be performed before, i.e. *pre-tensioning* or after, i.e. *post-tensioning*, the concrete has been cast. In both approaches the high strength steel is tensioned in a way that creates compression in the concrete element. The forces are then, when the system is loaded, reduced due to the tensile forces that occur inside the structural member.

The more compressive forces that initially are applied to the system, the less tensile forces will effect the cracking. Hence, with enough prestressing the cracking process can be fully prevented. Furthermore, a prestressed member will entail more strength than a reinforced element.

However, the method is normally not used in concrete foundations due to the large dimensions and the decreased requirement on the load-carrying capacity. A prestressed slab would end up with unnecessary strength and costs.

6 Concrete foundation

When designing concrete foundations the risk of cracking has to be included. Since cracking cannot be prevented completely the implemented reinforcement in the final product has to be designed in a way that fulfill the set requirement.

6.1 Choice of concrete strength class

Generally, when designing structural members, focus lies on creating an element with enough strength to fulfill the set requirements. However, when designing concrete foundations the procedure differ slightly. The strength of the concrete in a foundation can often, to some extent, be put aside since other characteristics e.g. performance due to cracking might be considered as more important. Since interest, in this project, is to create a foundation that stand against radon penetration it is important to choose a concrete where the risk of cracking is smaller. As mentioned in Section (3.4) the strength class of the concrete is influenced by the water/cement ratio. A high water/cement ratio entails a lower strength of the concrete but also a lower resulting shrinkage. Hence, less risk of initiation of cracks (Narin & Wiklund, 2012).

6.2 Regulations of crack widths in foundations

The crack width in concrete members is according to Eurocode 2 limited with respect to the durability and appearance. Maximum characteristic crack width, $w_{\max}$, depend on what the structure is going to be exposed to during its life time and varies between different countries. Tougher exposure conditions requires stricter demands on the crack width which then is reduced.

In Eurocode 2 different environmental conditions are divided into various *exposure classes*. Depending on the estimated environmental conditions an exposure class can be determined from which applicable requirements are recommended. For concrete which are exposed to indoor environments, where the humidity of the air generally is lower exposure class XC1 should be chosen. The same class could also be used if the concrete permanently was going to be submerged in water. Many foundations is according to Eurocode 2 classified within exposure class XC2 where the concrete is exposed to wet to rarely dry environment.

6.2.1 Regulations to prevent gas penetration

The amount of gas that penetrates through a concrete slab depend on the pore system of the concrete. Hence, the water/cement ratio and the development of the hardening process influence the final result since they affect the formation of pores in the system. A decreased water/cement ratio, i.e. increased concrete strength, entails a reduced gas permeability (*Betonghandboken - Material*, 1997).

Gas penetration through a concrete element increase significantly if it contains through cracks. Another parameter that also has to be fulfilled is a presence of air pressure difference. The gas is only enable to pass through the slab if there is a higher air pressure on the outside compared to the inside of the building, which normally is the case.

According to Betonghandboken, when ignoring the actual geometry and structure of the crack, the ratio between the gas permeability and crack width can theoretically be explained as followed: A 10 times decreased crack width entails a 1 000 times smaller gas penetration. Furthermore, the gas permeability of the concrete is also affected by the thickness of the casted slab where an increased thickness contributes to a decreased amount of gas leakage.

In design, a radon proof slab should be modelled as relatively stiff. This is done similarly as when constructing water proof concrete elements (Beyer, 2018). According to Betonghandboken, the maximum crack width in water proof concrete structures should be limited to 0.2 mm.

7 Method

This chapter presents the calculation procedure done for this project. The first part of the calculations was performed according to Eurocode 2 and the second part uses the method presented in Engström (2014) where shrinkage and restraint are considered. Complete calculations can be found in Appendix A and Appendix B.

7.1 Dimensions and characteristics of concrete slab

Two slabs were designed for analyze, both having the same 2D-dimensions 10 000 x 10 000 millimeter. They were then assigned different thicknesses where one of the slabs was designed with a thickness of 120 mm and the other with a thickness of 200 mm. The two slabs were then assigned four different strength classes; 20/25, 25/30, 30/37 and 35/45. The water/cement ratio used for each element was estimated using Figure 3.2 and are displayed in Table 7.1. Exposure class XC2 was assumed in the calculations.

Table 7.1: Dimensions and properties of concrete foundation. Water/cement ratios are estimated according to Figure 3.2.

Element	Thickness [mm]	Strength class	water/cement ratio
S-120_{25}	120	20/25	0.72
S-120_{25}	120	25/30	0.65
S-120_{30}	120	30/37	0.58
S-120_{35}	120	35/45	0.47
S-200_{25}	200	20/25	0.72
S-200_{25}	200	25/30	0.65
S-200_{30}	200	30/37	0.58
S-200_{35}	200	35/45	0.47

7.1.1 Concrete cover

The thickness of the concrete cover was estimated according to Eurocode 2 where it is described as the sum of the minimum concrete cover, c_{min}, and the deviation, Δc_{dev}. The minimum concrete cover was determined as the maximum value of; the diameter of the bar, the minimum cover due to environmental conditions, $c_{\text{min.dur}}$ and 10 mm. The recommended value of the deviation varies between different countries where it, in this project, was set to 10 mm. The minimum cover required regarding the environmental conditions was established using Table 4:3N and 4:4N in Eurocode 2. Recommended Structural Class S4 was initially used but was reduced to S3 since slab geometry was assumed in the element. As the exposure class was set to XC2 the value of $c_{\text{min.dur}}$ could be read to be 20 mm. The thickness of the concrete cover was then calculated as:

$$c = c_{\text{min}} + \Delta c_{\text{dev}} \tag{7.1}$$

where:

$$\begin{aligned} \Delta c_{\text{dev}} &= 10 \text{ mm} \\ c_{\text{min}} &= \max(\phi, c_{\text{min.dur}}, 10 \text{ mm}) \end{aligned}$$

7.2 Characteristics of reinforcing steel

The reinforcement for each element are designed with various dimensions of reinforcement. The reinforcement used in the calculations was **B500B** with a yield capacity, f_y, of 500 MPa. The dimensions of the reinforcing bar used in the calculations was; 6 mm, 8 mm, 12 mm, 16 mm and 25 mm. Table 7.2 displays the properties of the reinforcement.

Table 7.2: Properties and dimensions of reinforcing steel.

Reinforcement	Yielding capacity f_{yk} [MPa]	Modulus of elasticity E_s [GPa]
B500B	500	200

7.3 Creep coefficient

To consider long term effects the creep coefficient had to be calculated. The calculations described below is based on the derivation found in Eurocode 2, Annex B.

The notional size is dependent on the area of the cross-section and on the circumference of the cross-section in contact with the atmosphere. It was assumed that the slab only was covered on the side in contact with the ground, thus, the perimeter in contact with the air was determined according to Figure 7.1. The notional size was then calculated as:

$$h_0 = \frac{2 \cdot A_c}{u} \tag{7.2}$$

where:

$$\begin{aligned} A_c &= \text{cross-sectional area of the concrete} \\ u &= \text{circumference of exposed cross-section} \\ &\quad 2h + b \end{aligned}$$

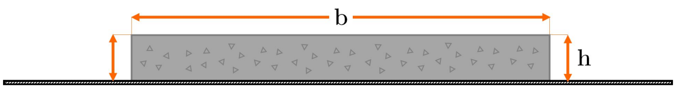

Figure 7.1: Circumference of exposed cross-section, illustrated with orange arrows.

Using the obtained notional size, the factor, φ_{RH}, that considers the effects of relative humidity could be calculated. The relative humidity of the surroundings used in the calculations was set to 40 %. However, later in the process this value was changed to obtain how it affect the different parameters. The factor was calculated according to:

$$\varphi_{\mathrm{RH}} = 1 + \frac{1 - \dfrac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \qquad \text{for} f_{\mathrm{cm}} \leq 35 \text{ MPa} \tag{7.3a}$$

$$\varphi_{\mathrm{RH}} = \left[1 + \frac{1 - \dfrac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \left[\frac{35}{f_{\mathrm{cm}}}\right]^{0.7}\right] \left[\frac{35}{f_{\mathrm{cm}}}\right]^{0.2} \qquad \text{for} f_{\mathrm{cm}} > 35 \text{ MPa} \tag{7.3b}$$

where:

RH = relative humidity of the surroundings [%]
h_0 = notional size of cross section [mm]
f_{cm} = concrete average compressive strength at an age of 28 days [MPa]

Then, the coefficient, $\beta(f_{\mathrm{cm}})$, considering the strength of the concrete and the coefficient, $\beta(t_0)$, considering the age of the concrete at loading could be calculated according to:

$$\beta(f_{\mathrm{cm}}) = \frac{16.8}{\sqrt{f_{\mathrm{cm}}}} \tag{7.4}$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}} \tag{7.5}$$

Where t_0 is taking the type of cement into account for by modifying the age of the concrete when loaded. The cement type used in the calculations was Class N and no adjustment was made due to the temperature,. Hence, $t_{0.\mathrm{T}}$ was set to be equal to the age of the concrete at the beginning of drying shrinkage, which, in this this project the was assumed to start when the concrete had reached an age of 7 days. The modified age of loading was calculated as:

$$t_0 = \max\left(t_{0.\mathrm{T}} \cdot \left(\frac{9}{2 + t_{0.\mathrm{T}}^{1.2}} + 1\right)^{\alpha}, 0.5\right) \tag{7.6}$$

where:

$t_{0.\mathrm{T}}$ = the temperature adjusted age of the concrete at loading
α = depends on the type of cement
-1 for Class S
0 for Class N
1 for Class R

To evaluate the development of creep, another coefficient first had to be estimated. β_{H} is depending on the relative humidity and the notional size and was calculated as:

$$\beta_{\mathrm{H}} = \min\Big(1500, 1.5 \cdot \Big(1 + (0.012 \cdot RH)^{18}\Big) \cdot h_0 + 250\Big) \qquad \text{for} f_{\mathrm{cm}} \leq 35 \text{ MPa} \tag{7.7a}$$

$$\beta_{\mathrm{H}} = \min\Big(1500 \cdot \alpha_3, 1.5 \cdot \Big(1 + (0.012 \cdot RH)^{18}\Big) \cdot h_0 + 250 \cdot \alpha_3\Big) \quad \text{for} f_{\mathrm{cm}} > 35 \text{ MPa} \tag{7.7b}$$

where:

RH = relative humidity of the surroundings
h_0 = notional size
$\alpha_3 = \left[\dfrac{35}{f_{\mathrm{cm}}}\right]^{0.5}$

Consequently, the coefficient that describes the development of creep after loading could be calculated accoriding to:

$$\beta_{\mathrm{c}}(t, t_0) = \left(\frac{t - t_0}{\beta + t - t_0}\right)^{0.3} \tag{7.8}$$

Once all coefficients above was established, the notional creep coefficient φ_0 could be estimated. It was then used to determine the final creep coefficient,$\varphi(t, t_0)$. The two coefficients were calculated according to:

$$\varphi_0 = \varphi_{\mathrm{RH}} \cdot \beta(f_{\mathrm{cm}}) \cdot \beta(t_0) \tag{7.9}$$

$$\varphi(t, t_0) = \beta_{\mathrm{c}}(t, t_0) \cdot \varphi_0 \tag{7.10}$$

7.4 Shrinkage strain

The shrinkage strain does, as mentioned in Section 3.5.1, consist of two separate components; drying shrinkage and autogenous shrinkage. Since the drying shrinkage is dependent of the time elapsed from casting, the coefficient, $\beta_{\mathrm{ds}}(t, t_{\mathrm{s}})$, first had to be evaluated. It was estimated according to:

$$\beta_{\mathrm{ds}}(t, t_{\mathrm{s}}) = \frac{t - t_{\mathrm{s}}}{t - t_{\mathrm{s}} + 0.04 \cdot \sqrt{h_0^3}} \tag{7.11}$$

where:

t = age of concrete
t_s = age of concrete at the beginning of drying shrinkage
h_0 = notional size

As mentioned earlier, the age of when drying shrinkage started, t_s was set to 7 days. Furthermore, the age of the concrete when the study was evaluated, t, was set to ∞ days to achieve the final shrinkage strain.

Next, the factor considering the size of the cross-section, k_h, was estimated. The value was govern by interpolation using the notional size and Table 7.3.

Table 7.3: Factor k_h that accounts for the size of the cross-section (Eurocode 2, 2008).

h_0 [mm]	k_h
100	1.0
200	0.85
300	0.75
≥500	0.70

The last factor that had to be established before the drying shrinkage could be calculated was the *basic drying shrinkage*, $\varepsilon_{cd.0}$ which was calculated according to:

$$\varepsilon_{cd.0} = 0.85\left((220 + 110 \cdot \alpha_{ds1} \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} \tag{7.12}$$

where:

f_{cmo} = 10 MPa
α_{ds1} = coefficient which depends on the type of cement
3 for cement Class S
4 for cement Class N
6 for cement Class R
α_{ds2} = coefficient which depends on the type of cement
0.13 for cement Class S
0.12 for cement Class N
0.11 for cement Class R
$\beta_{RH} = 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right)$
RH = relative humidity of the surroundings
RH_0 = 100 %

The drying shrinkage was then estimated according to:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd.0} \tag{7.13}$$

The autogenous shrinkage is unlike the drying shrinkage dependent on the concrete strength. It was calculated according to:

$$\varepsilon_{\mathrm{ca}}(t) = \beta_{\mathrm{as}}(t) \cdot \varepsilon_{\mathrm{ca}}(\infty) \tag{7.14}$$

where:

$$\beta_{\mathrm{as}}(t) \quad = 1 - \exp(-0.2 \cdot t^{0.5})$$

$$\varepsilon_{\mathrm{ca}}(\infty) = 2.5 \cdot (f_{\mathrm{ck}} - 10) \cdot 10^{-6}$$

Once both the drying shrinkage and the autogenous shrinkage was established the total shrinkage strain could be calculated. This was done according to:

$$\varepsilon_{\mathrm{cs}}(t) = \varepsilon_{\mathrm{cd}}(t) + \varepsilon_{\mathrm{ca}}(t) \tag{7.15}$$

7.5 Crack width calculation according to EC 2

To maintain a structure that does not obtain issues with its durability or function, crack widths should be limited. Depending on the exposure class, Eurocode 2 gives recommended values of maximum allowed crack width, w_{max}. The Exposure class used in this project was, as mentioned earlier, decided to XC2 which, according to Table 7.1.N in Eurocode 2, limit the crack width to maximum 0.3 mm. However, to manage the restriction of radon penetration, the maximum crack width was decreased to 0.2 mm. Furthermore, the following calculations in this section is based on Eurocode 2.

7.5.1 Minimum reinforcement

To control the cracking and to minimize the appearance of large cracks, minimum reinforcement, $A_{\mathrm{s,min}}$, has to be estimated. It is implemented in areas where cracking is expected, i.e. in areas exposed to tension. In this project, tension was assumed in the whole slab, that is why the area of concrete within the tensile zone was estimated to be equal to the total area of the concrete cross-section. Minimum reinforcement was calculated according to:

$$A_{\mathrm{s.min}} \geq k_{\mathrm{c}} \cdot k \cdot A_{\mathrm{ct}} \cdot \frac{f_{\mathrm{ctm}}}{\sigma_{\mathrm{s}}} \tag{7.16}$$

where:

k_{c} = factor that consider the stress distribution before cracking
$k_{\mathrm{c}} = 1.0$ for pure tension
k = factor that considers the inner stress distribution caused by restraint
$k = 1.0$ for $h \leq 300$ mm
A_{ct} = area of concrete within the tensile zone
f_{ctm} = mean value of the concrete tensile strength
σ_{s} = maximum stress in the reinforcement, depend on crack limitation
$\sigma_{\mathrm{s}} \leq f_{\mathrm{yk}}$

With respect to the limitation of the crack width, the steel stress was chosen dependent of the dimension of the reinforcing bar. Table 7.4 displays the recommended steel stress to be used when determining minimum reinforcement and includes the bar diameters used in this project for a maximum crack width limit at 0.2 mm.

Table 7.4: Steel stress used to determine minimum reinforcement (Eurocode 2, 2008)

σ_s [MPa]	Dimension ϕ [mm]
	w_{max}= 0.2 mm
160	25
200	16
240	12
280	8
320	6

7.5.2 Crack spacing

In the method to evaluate crack width according to Eurocode 2 it is assumed that stabilized cracking is reached i.e. that no new cracks can occur. The approach is valid for load induced cracks but, since no restraint is considered, cannot be applied when evaluating restraint cracking.

When minimum reinforcement was established the crack spacing could be estimated. First, the effective cross-section, $A_{c,ef}$, had to be calculated which correspond to the area of the concrete surrounding the reinforcing bars that is subjected to tension. Figure 7.2 illustrates the mentioned area within in the hatched regions.

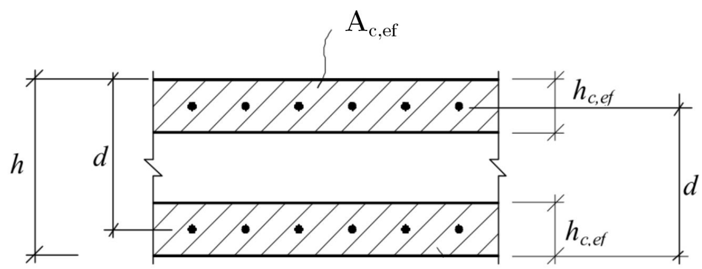

Figure 7.2: Effective concrete cross-section, illustrated within the hatched areas (Al-Emrani, Engström, Johansson, & Johansson, 2011)

The effective height was chosen according to:

$$h_{c,ef} = min\left\{2.5(h-d), \frac{h-x}{3}, \frac{h}{2}\right\} \tag{7.17}$$

where:

x = height of the center of gravity of the cross-section

Consequently, the effective area of the concrete was calculated as:

$$A_{\mathrm{c,ef}} = 2 \cdot h_{\mathrm{c,ef}} \cdot b \tag{7.18}$$

where:

b = width of cross section
$h_{\mathrm{c,ef}}$ = height of effective cross-section

The spacing between the cracks was then estimated according to:

$$s_{\mathrm{r,max}} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{\mathrm{p.ef}}} \tag{7.19}$$

where:

c = thickness of concrete cover
k_1 = coefficient which considers the bond properties of the bonded reinforcement
= 0.8 for high bond bars
k_2 = coefficient which considers the distribution of strain
= 1 for pure tension
k_3 = 3.4
k_4 = 0.425
ϕ = diameter of reinforcing bar
$\rho_{\mathrm{p.ef}} = \dfrac{A_{\mathrm{s}}}{A_{\mathrm{c.ef}}}$

7.5.3 Crack width

The crack width is, according to the method in Eurocode 2, estimated by initially assuming a cracked section. Therefore, the steel stress used when determining minimum reinforcement was used to calculate the difference between the mean strain in the steel and the concrete. This was done according to:

$$\Delta\varepsilon = \max\left(\frac{\sigma_{\mathrm{s}} - k_{\mathrm{t}} \cdot \dfrac{f_{\mathrm{ctm}}}{\rho_{\mathrm{p.ef}}}(1 + \alpha \cdot \rho_{\mathrm{p.ef}})}{E_{\mathrm{s}}}, 0.6 \cdot \frac{\sigma_{\mathrm{s}}}{E_{\mathrm{s}}}\right) \tag{7.20}$$

where:

σ_{s} = steel stress in assumed cracked section
α = modular ratio $\dfrac{E_{\mathrm{s}}}{E_{\mathrm{cm}}}$
k_{t} = factor dependent on the duration of the load
0.6 for short term loading
0.4 for long term loading
f_{ctm} = mean value of tensile strength of the concrete

Finally, the characteristic crack width could be determined according to:

$$w_{\mathrm{k}} = s_{\mathrm{r.max}} \cdot \Delta\varepsilon \tag{7.21}$$

7.6 Crack evaluation according to Engström (2014)

The crack width calculated according to the method presented in Eurocode 2 is simplified and does not include the actual effect of shrinkage and restraint. Therefore, Engström did develop a method that considers the two parameters. This section is bases on the method described in Engström (2014).

7.6.1 Shrinkage force

According to Engström (2014) the need of deformation that occurs in the system due to shrinkage, will generate a shrinkage force, F_{cs} in the steel which is illustrated in Figure 7.3.

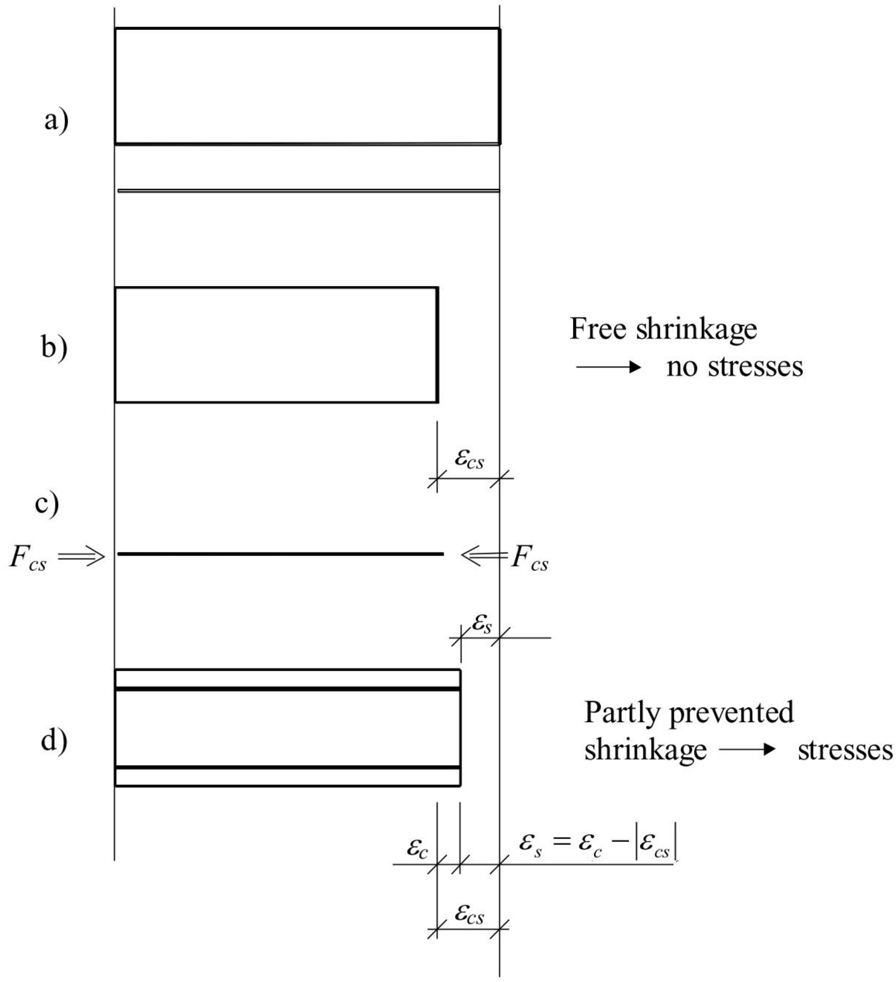

Figure 7.3: Reinforced concrete element subjected to uniform shrinkage (Engström, 2014).

Basically, the volume decrease of the concrete causes movements that are restraint by the reinforcing bar. Consequently, a compressive force are originated in the steel, forcing it to withdraw as well. Once the final shrinkage strain is reached, no further increase of the shrinkage force is possible. Hence, the gained shortening of the reinforcing bar slowly decrease as the steel strives to regain its original shape. This causes a tensile force to arise in the concrete which, eventually, attain the same magnitude as the maximum shrinkage force. It is calculated according to:

$$F_{\mathrm{cs}}(t) = E_{\mathrm{s}} \cdot \varepsilon_{\mathrm{cs}}(t) \cdot A_{\mathrm{s}} \tag{7.22}$$

7.6.2 Calculation of cracking load

To evaluate the risk of cracking, the cracking load, N_{cr}, had do be calculated. If the applied load i.e. the restraint force, exceeded the cracking load cracks was to be expected. Depending on the duration of the load the cracking load was calculated according to:

$$N_{\mathrm{cr}} = f_{\mathrm{ct}} \cdot A_{\mathrm{I}} \qquad \text{(for short term response)} \tag{7.23a}$$

$$N_{\mathrm{cr.\infty}} = f_{\mathrm{ct.sus}} \cdot A_{\mathrm{I.ef}} \qquad \text{(for long term response)} \tag{7.23b}$$

where:

$$\begin{aligned} f_{\mathrm{ct}} &= f_{\mathrm{ct0.05}} \text{ - unfavorable regarding crack limitation} \\ f_{\mathrm{ct.sus}} &= \alpha \cdot f_{\mathrm{ct0.05}} \\ &\quad \alpha = 0.6 \ \text{ for normal strength concrete} \\ A_{\mathrm{I}} &= A_{\mathrm{c}} + A_{\mathrm{s}}(\alpha - 1) \\ A_{\mathrm{I.ef}} &= A_{\mathrm{c}} + A_{\mathrm{s}}(\alpha_{\mathrm{ef}} - 1) \end{aligned}$$

7.6.3 Crack risk evaluation

The risk of cracking was then evaluated. Due to the large area of the element full restraint was considered. The restraint force could then be estimated according to:

$$N = R_{\mathrm{tot}} \cdot (\varepsilon_{\mathrm{cs}} \cdot E_{\mathrm{c.ef}} \cdot A_{\mathrm{I.ef}} - F_{\mathrm{cs}}) \tag{7.24}$$

where:

$$\begin{aligned} R_{\mathrm{tot}} &= \text{restraint degree from internal and external restraint} \\ &= 1 \text{ for full restraint} \\ \varepsilon_{\mathrm{cs}} &= \text{shrinkage coefficient} \\ E_{\mathrm{c.ef}} &= \frac{E_{\mathrm{s}}}{E_{\mathrm{cm}}} \cdot (1 + \varphi(\infty, t_0)) \\ F_{\mathrm{cs}} &= \text{shrinkage force} \end{aligned}$$

The restraint force was then used together with the shrinkage force to calculate the stress originated in the concrete. This was done according to:

$$\sigma_{\mathrm{c}} = \frac{N + F_{\mathrm{cs}}}{A_{\mathrm{I.ef}}} \tag{7.25}$$

where:

N = restraint force
F_{cs} = shrinkage force

The concrete stress was then compared to the tensile capacity corresponding to long term loading, to evaluate the risk of cracking. No risk of cracking was assumed if the obtained concrete stress was lower than the tensile capacity of the 5% fractile, $f_{\mathrm{ctk0.05.sus}}$. A higher concrete stress was interpreted to cause either, *"risk"*, *"high risk"* or *"very high risk of cracking"* where the last mentioned was valid if the concrete stress exceeded the tensile capacity of the 95% fractile, $f_{\mathrm{ctk0.95.sus}}$.

7.6.4 Response during cracking

To estimate the crack width the steel stress, when stabilized cracking was reached, first had to be calculated. Because of the considered affect of shrinkage, long term response including creep had to be included in the calculation. The steel stress was solved by an iterative process where both the steel stress, σ_{s}, and the number of cracks, n_{cr} were modified. First, $n_{\mathrm{cr}} = 1$ was assumed from which the steel stress was adjusted until equilibrium was reached and the deformation condition was fulfilled. The equation used in the process was:

$$\frac{\sigma_{\mathrm{s}} \cdot A_{\mathrm{s}} + F_{\mathrm{cs}}}{E_{\mathrm{c.ef}} \cdot A_{\mathrm{I.ef}}} \cdot L + n_{\mathrm{cr}} \cdot w_{\mathrm{m.sus}} + \varepsilon_{\mathrm{cs}} \cdot L = 0 \tag{7.26}$$

where:

$w_{\mathrm{m.sus}}$ = crack width affected by long term loading

The crack width arisen from long term effect was calculated according to:

$$w_{\mathrm{m.sus}} = 1.24 \cdot w_{\mathrm{net}} + \frac{\sigma_{\mathrm{s}}}{E_{\mathrm{s}}} \cdot 4\phi \tag{7.27}$$

where:

$$w_{\mathrm{net}} = 0.420 \cdot \left(\frac{\phi \cdot \sigma_{\mathrm{s}}^2}{0.22 f_{\mathrm{cm}} \cdot E_{\mathrm{s}} \cdot \left(1 + \frac{E_{\mathrm{s}}}{E_{\mathrm{cm}}} \cdot \frac{A_{\mathrm{s}}}{A_{\mathrm{c.ef}}}\right)} \right)^{0.826} \tag{7.28}$$

When equilibrium was obtained the current steel stress was used to calculate the corresponding restraint force accodring to:

$$N = \sigma_s \cdot A_s \tag{7.29}$$

If the result exceeded the cracking force a new crack would be initiated. Consequently, the iteration process was performed again where n_{cr} was increased by 1. This was repeated until the govern restraint force was estimated to be lower than the cracking force at which the cracking process stopped.

7.6.5 Transmission length

When stabilized cracking was reached the obtained steel stress was used to estimate the transmission length according to:

$$l_t = 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22 f_{cm} \cdot w_{net}^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s}{A_{c.ef}}\right)} + 2 \cdot \phi \tag{7.30}$$

The transmission length that would occur due to long term loading could then be calculated according to:

$$l_{t.sus} = 1.3 \cdot l_t \tag{7.31}$$

7.6.6 Mean crack width

The steel stress established from the iterative process was also used when determining the mean crack width. It was estimated according to:

$$w_m = 0.420 \cdot \left(\frac{\phi \cdot \sigma_s^2}{0.22 f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s}{A_{c.ef}}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\phi \tag{7.32}$$

To compare the governed result the allowable mean crack width for restraint loading was calculated according to:

$$w_{m.all} = \frac{w_{lim}}{1.3} \tag{7.33}$$

where:

w_{lim} = specified limit of characteristic crack width
0.2 mm

The estimated mean crack width was then compared to the allowable mean crack width to determine the sufficiency of the implemented reinforcement. If the obtained mean crack width exceeded the allowable mean crack width an increased reinforcement area was required. Opposite result was interpreted that either sufficient amount of reinforcement was provided or, if the crack width was much lower than the allowed, that unnecessarily large dimension of the reinforcement area was implemented. The condition investigated was expressed as:

$$w_{\mathrm{m}}(\sigma_{\mathrm{s}}) \leq w_{\mathrm{m.all}} \tag{7.34}$$

Furthermore, to achieve the characteristic crack width for the concrete affected by restraint loading the mean crack width was increased by 30 % according to:

$$w_{\mathrm{k}} = 1.3 \cdot w_{\mathrm{m}} \tag{7.35}$$

The estimated value was then compared to the specified limit value of the characteristic crack width which, as mentioned earlier, according to Eurocode 2 was determined to 0.2 mm. The condition was expressed according to:

$$w_{\mathrm{k}} \leq w_{\mathrm{lim}} \tag{7.36}$$

8 Results

This chapter present the results gained using the methods described in Chapter 7. The gathered information was compiled into graphs to ease the understanding of how each parameter is affected by the dimension of the reinforcing bar and the strength of the concrete. The analysis was performed for two different thicknesses of the concrete slab, however, since the achieved results did not differ markedly from one another, mainly the results for one of the slabs (120 mm) will be presented in this chapter. The graphs corresponding to the other slab (200 mm) is presented in Appendix C, where all the graphs are put together.

8.1 Creep and shrinkage

The shrinkage and creep of the concrete does not depend on the properties of the reinforcement, thus, the graphs only contains the investigated concrete strength classes. Figure 8.2 and 8.1 displays how the shrinkage strain and the final creep coefficient varies with the strength of the concrete. It can from the graphs be seen that they both decrease with increased concrete strength.

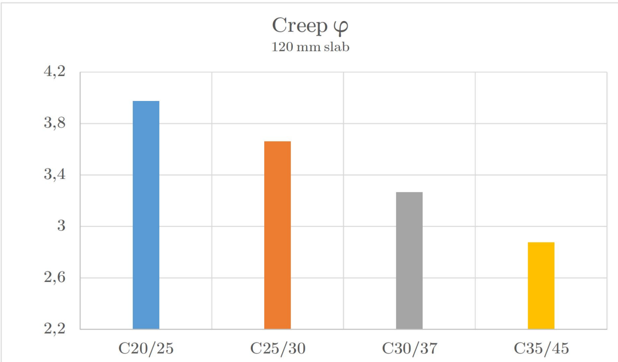

Figure 8.1: Final creep coefficient in a concrete slab with a thickness of 120 mm, for four different concrete strength classes.

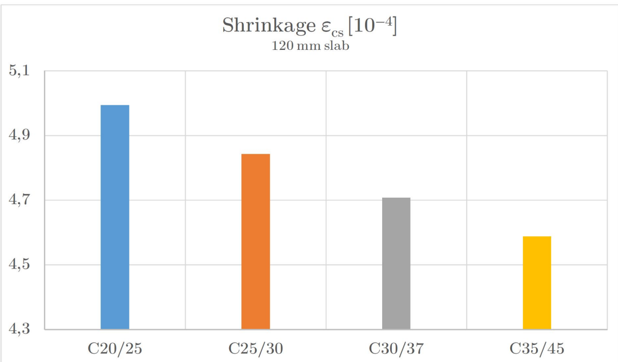

Figure 8.2: Shrinkage strain in a concrete slab with a thickness of 120 mm, for four different concrete strength classes.

8.2 Minimum amount of reinforcement

The minimum amount of reinforcement was calculated according to Eurocode 2. Figure 8.3 displays how the minimum amount of reinforcement varies with the dimension of the reinforcing bar and how the strength of the concrete influence the required reinforcement amount.

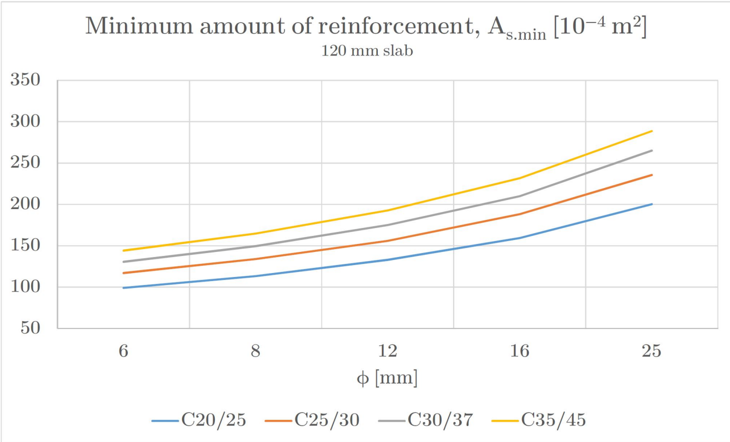

Figure 8.3: Minimum amount of reinforcement as a function of the bar diameter, calculated for four different concrete strength classes.

8.3 Crack width according to Eurocode 2

Depending on the properties of the concrete, the reinforcing bar and the bond between the two materials, the maximum spacing between the resulting cracks was calculated. Figure 8.4 shows how the spacing increase with increased bar diameter. It also displays that by increasing the strength of the concrete the maximum spacing between the cracks is reduced, if the same bar diameter can be used.

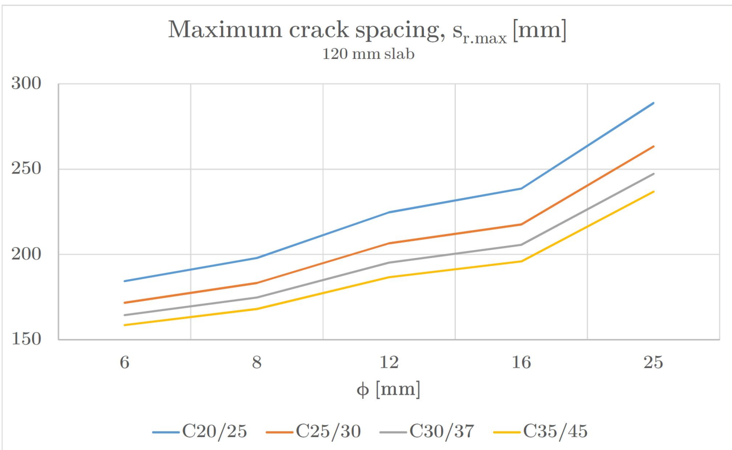

Figure 8.4: Maximum spacing between cracks varying with the dimension of the reinforcing bar and the concrete strength.

From the resulting maximum spacing could then the mean crack width be calculated. Figure 8.5 displays how the crack width decrease with increased bar diameter. The concrete strength affect the crack width in the same way as the crack spacing was influenced. Thus, an increased concrete strength decrease the resulting crack width, when using the same bar diameter.

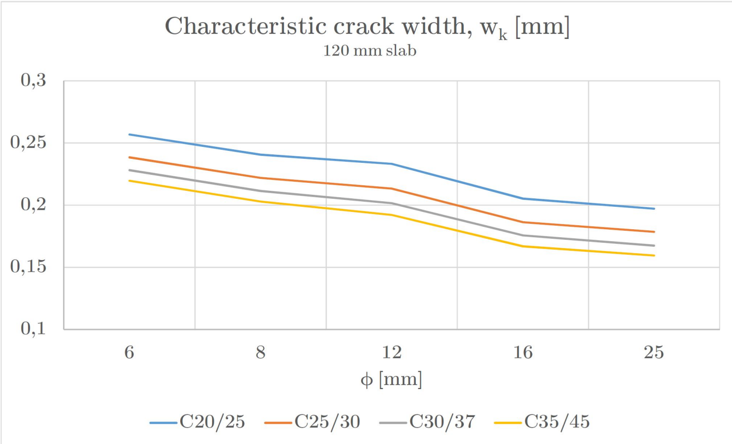

Figure 8.5: Characteristic crack width for varying bar diameter and concrete strength

8.4 Crack width according to Engström (2014)

Since the method to calculate the crack width in Eurocode 2 not consider any influence of shrinkage or restraint, Engström's method was used to evaluate the crack width where those parameters were considered.

8.4.1 Shrinkage force

Figure 8.6 displays the shrinkage force calculated for the four different concrete classes and how it increase with increased bar diameter.

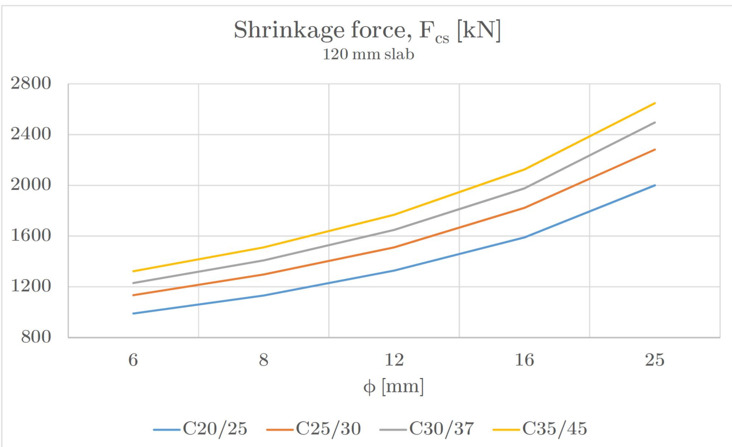

Figure 8.6: Shrinkage force for varying dimension of the reinforcing bar and concrete strength.

8.4.2 Cracking force

The cracking force in the concrete was calculated for each concrete class. Since it depend on the tensile strength of the concrete and of the area of both the concrete and the steel the result varies with both parameters. Figure 8.7 displays how the cracking force is affected by the bar diameter and the strength of the concrete.

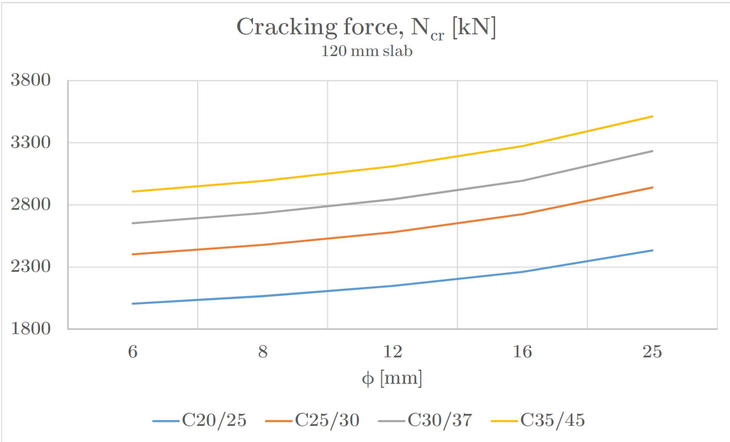

Figure 8.7: Cracking force for varying dimension of the reinforcing bar and concrete strength.

8.4.3 Iterative process

To estimate the crack width an iterative process was performed to find the steel stress of when stabilized cracking is reached. As long as the stress contributes to a restraint force that exceeds the cracking force a new crack will occur in the concrete. Figure 8.8 shows how the steel stress, when stabilized cracking is reached, decrease with increased bar diameter.

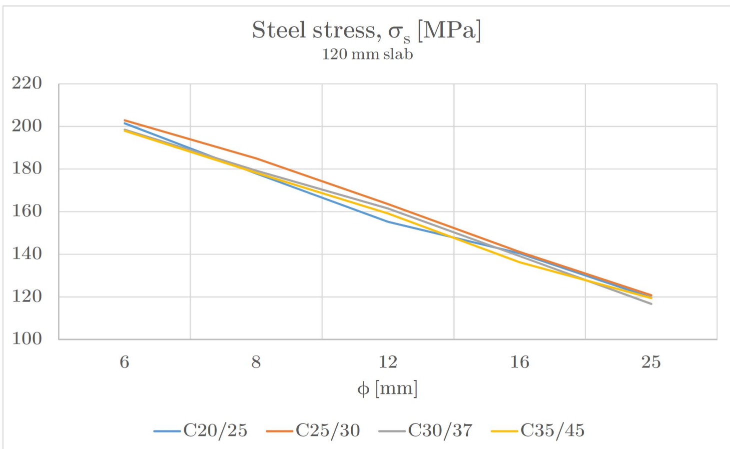

Figure 8.8: Steel stress when stabilized cracking is reached, varying with the dimension of the reinforcing bar.

The corresponding number of cracks is displayed in Figure 8.9 where it can be seen that the number of cracks decrease with increased bar diameter.

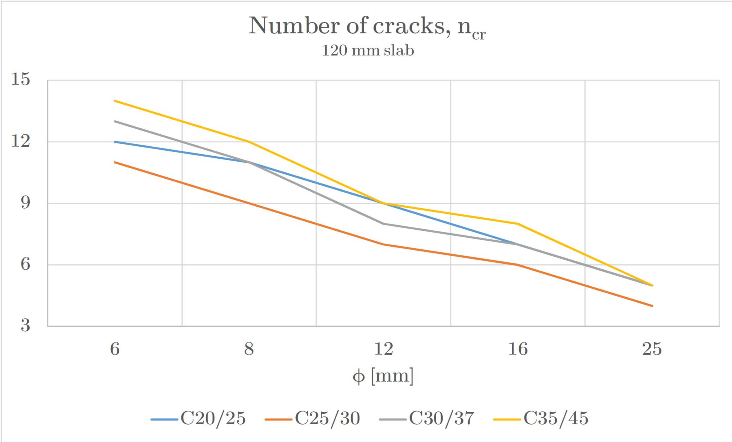

Figure 8.9: Number of cracks for varying dimension of the reinforcing bar and concrete strength.

8.4.4 Transmission length

The steel stress from when stabilized cracking has been reached was then used for calculating the maximum spacing between the cracks. In this method is the distance called transmission length which was evaluated for both short term and long term response. As described in the method can the transmission length corresponding to long term response be predicted to be 30 percent larger than the results gained for short term response. The results from the calculated transmission length for long term response is displayed in Figure 8.10.

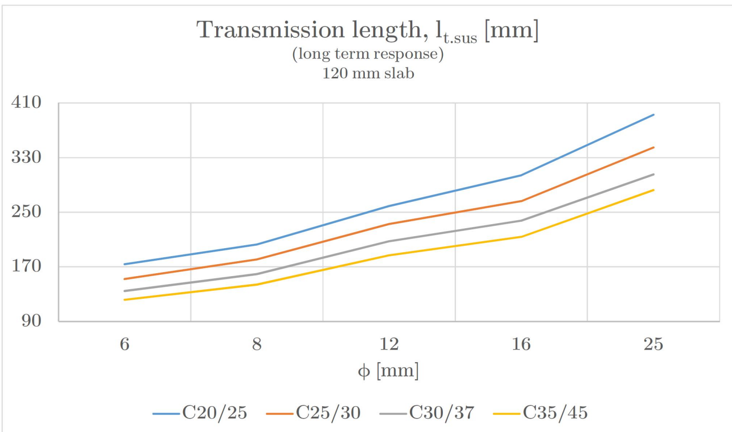

Figure 8.10: Transmission length for varying dimension of the reinforcing bar and concrete strength, long term response.

8.4.5 Characteristic crack width

The mean crack width was calculated using the steel stress gained from the iteration process. Then, the characteristic crack with was achieved by increasing the mean crack width with 30 percent. Figure 8.11 shows how the characteristic crack width increase with increased bar diameter but decrease with increased concrete strength when the same bar diameter is used.

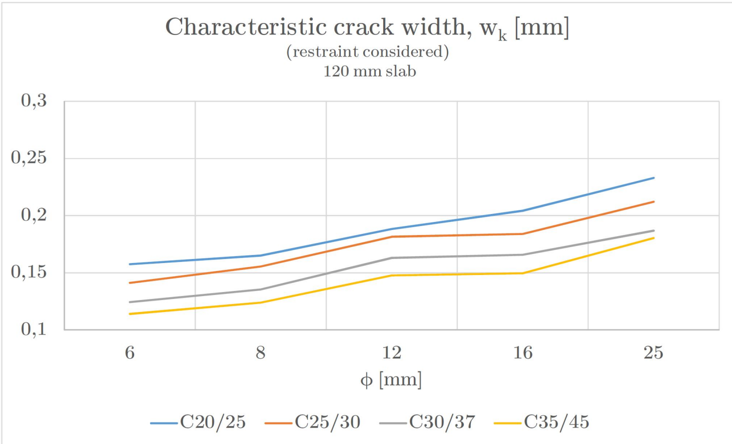

Figure 8.11: Characteristic crack width, considering shrinkage and restraint, for varying dimension of the reinforcing bar and concrete strength.

8.4.6 Influence of relative humidity of the surroundings

The influence of the relative humidity of the surroundings was investigated in a separate study to get an overview of how it affects the characteristic crack width of the concrete. The study was done for one concrete class only, C30/37, where the relative humidity was increased from 40 % to 80 % in steps of 10 %. Figure 8.12 displays how the creep and shrinkage coefficients decrease with increased relative humidity of the surroundings.

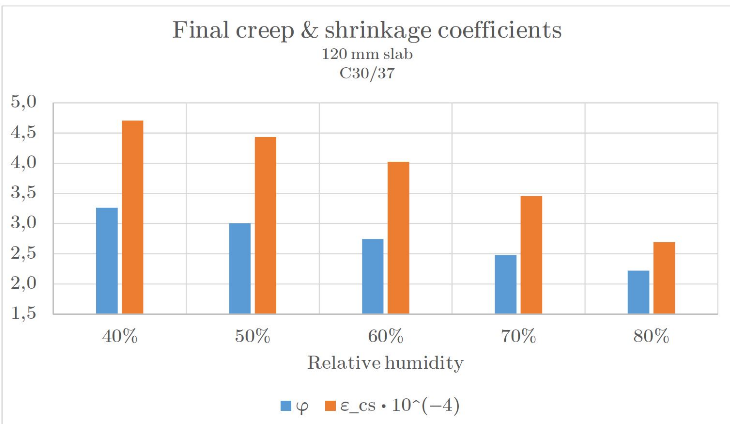

Figure 8.12: Final creep and shrinkage coefficients for various relative humidity for a 120 mm thick concrete slab with the strength class C30/37.

The originated shrinkage force was then evaluated for each relative humidity do see how it was affected. The result is shown in figure 8.13 where it can be seen that the shrinkage force becomes lower for increased relative humidity. This is caused by the decreased shrinkage that occur due to the reduced exchange of humidity between the air and the slab.

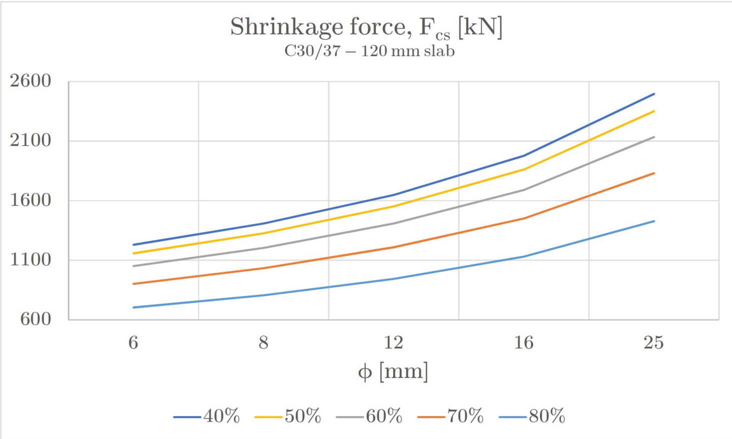

Figure 8.13: Shrinkage force for various dimension of the reinforcing bar and different relative humidity. Calculated for a 120 mm thick concrete slab with the strength class C30/37.

Consequently, the characteristic crack width decrease as the relative humidity of the surroundings is increased which is displayed in Figure 8.14.

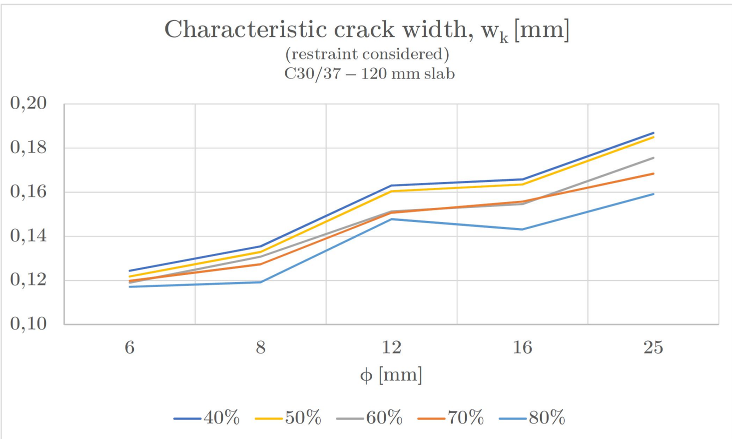

Figure 8.14: Characteristic crack width for various dimension of the reinforcing bar and different relative humidity. Calculated for a 120 mm thick concrete slab with the strength class C30/37.

9 Discussion

The obtained results for each concrete class and reinforcement dimension was compared to each other to evaluate which composition that could be concluded to be the most favourable. The analysis is presented and discussed in this chapter.

9.1 Limitations on crack widths

The crack width limitation in concrete elements depend on what the concrete will be exposed to during its life time. It is the recommended maximum allowable crack width that should be designed for with respect to durability and performance of the element. The exposure class used in this project was XC2 which according to Eurocode 2 limited the crack width to a maximum value of 0.3 millimeters. However, with respect to radon transport, the maximum allowed crack width was reduced to 0.2 millimeters which is similar to the restrictions on water proof concrete elements. For the gas to be able to reach the indoor environment, through cracks have to be present in the slab, together with an air pressure difference between the outside and inside of the building.

9.2 Minimum amount of reinforcement

The minimum amount of reinforcement was estimated according to Eurocode 2. In the method it is presented how it is influenced by valid exposure class where the allowable steel stress is limited according to the recommended maximum crack width. The method is based on the assumption that stabilized cracking is reached from which sufficient amount of reinforcement could be calculated.

9.2.1 Influence of bar diameter

When examined one parameter at a time, it could from Figure 8.3 be seen that an increased bar diameter requires a larger amount of minimum reinforcement. Hence, a larger dimension of the reinforcing bars contributes to an increased need of reinforcement amount to fulfill the given limitations on crack width. This can be explained by that as the bar diameter increase the bond properties/stiffness are reduced as the ratio of the cross-sectional area to circumferential length decrease. Hence, a larger amount of reinforcement is needed if a bar diameter of 20 mm is used compared to a diameter of 8 mm to fulfill the requirement. However, more bars are necessary to cover the minimum reinforcement area if a smaller bar dimension is used. Consequently, the increased number of bars will be placed closer together to fit in the casted slab, contributing to a more even distribution of the oriented stresses and also give a better bond, thus reducing the need of minimum amount of reinforcement.

9.2.2 Influence of water/cement ratio

The result displayed in Figure 8.3 shows that the required minimum amount of reinforcement increases as the strength of the concrete is increased i.e when the water/cement ratio is decreased. Even though no actual shrinkage or restraint is included in the equation to estimate the minimum reinforcement it is compensated by the restricted steel stress. The result shown in Figure 8.3 was compared with the graph in Figure 8.2 to evaluate any connections. Then, it could be seen that the final shrinkage decrease with increased concrete strength. This was interpreted as that despite the the increased need of deformation caused by shrinkage, entailed by the lower concrete strength, less amount of reinforcement is required to meet the set crack width limitation.

9.3 Impact of shrinkage and restraint

Figure 8.6 displays how the shrinkage force increase with increased bar diameter. It is also shown how an increased strength of the concrete contributes to a slightly larger shrinkage force than obtained for a lower concrete strength. As mentioned above, the increased bar diameter contributes to an increased need of minimum reinforcement. Therefore, as the same shrinkage strain occur in the concrete, the implemented reinforcement will obtain a larger shrinkage force if a bar dimension of 20 millimeters is used compared to 8 millimeter bars. The effect of an increased shrinkage force was interpreted to increase the crack width, which is displayed in Figure 8.11.

Figure 8.7 shows how the cracking force increase with increased bar diameter i.e amount of minimum reinforcement. Similar result is obtained when increasing the concrete strength. Thus, a larger restraint force can be withstand before cracks appear in the concrete. With the increased tolerance the number of cracks, when stabilized cracking is reached, is decreased which is displayed in Figure 8.9. Consequently, when fewer cracks appear, the spacing between the cracks increase which can be seen in Figure 8.10.

9.4 Impact on crack width

When estimating the crack width according to the method described in Eurocode 2, no actual affect of shrinkage or restraint was considered. Hence, the obtained result is only valid for load induced cracks and for stabilized cracking. Figure 8.5 displays how the calculated crack width decrease with increased bar diameter i.e. reinforcement amount. This could be explained by the dependence of the steel stress that was used to evaluate the difference between the mean strain in the concrete and the steel. As the steel stress was limited, depending on the dimension of the reinforcing bar, and reduced with increased bar diameter, the resulting strain difference decreased as well. Consequently, the estimated crack width became smaller for an increased dimension of the reinforcing bars.

Unlike the result of crack width evaluated according to Eurocode 2, the crack width estimated with Engström's method increase with increased bar diameter i.e. reinforcement amount. This can be explained by the increased shrinkage force that is caused by the larger amount of reinforcement that is needed for an increased bar dimension. The result of the calculated crack width is displayed in Figure 8.11 where it also can be seen that an increased concrete strength entails smaller crack widths when same bar diameter is used. This could be explained by the reduced need of deformation that occur due to the decreased shrinkage strain that an increased concrete strength contributes to, which is displayed in Figure 8.2.

When comparing the result from the two methods it was observed that the crack width estimated according Eurocode 2 generally is larger than the result obtained from Engström's method. Since the approach described in Eurocode 2 was based on the assumption that stabilized cracking initially was reached, the steel stress used to evaluate the crack width was limited with respect to the bar diameter. However, according to Engström's method, the steel stress obtained at stabilized cracking was estimated in an iterative process where the effect of shrinkage and restraint were included. Figure 8.8 displays how the evaluated steel stress for each bar diameter is lower than the corresponding values used according to Eurocode 2, which is presented in Table 7.4. Therefore, the crack width established by Engström's method becomes smaller than the one estimated according to Eurocode 2. This means that the actual result is lower than the one adopted. Thus, the result obtained using Eurocode 2 can be considered to be on the safe side. Consequently, to avoid to design a slab with unnecessarily large amount of reinforcement, Engström's method should be included in the evaluation.

9.5 Impact of relative humidity

As can be seen in Figure 8.14 the crack width, calculated according do Engström's method, decrease width increased relative humidity of the surroundings. This could be explained by the reduced need of deformation caused by creep and shrinkage that is displayed in Figure 8.12. As the relative humidity of the surroundings increase a decreased exchange of water will occur between the air and the concrete slab, contributing to a smaller final shrinkage strain and also smaller crack widths.

10 Conclusion

From the result presented, an analysis and discussion was performed to reflect how the investigated different parameters affect the final product. The final conclusion is described in this chapter.

The acceptable level of radon contamination in the indoor environment was found to be determined to 200 Bq/m^3. The restriction should be fulfilled otherwise actions to improve the situation is necessary. When designing a concrete foundation this is considered by limiting the allowable crack width to prevent radon to penetrate the slab. Furthermore, in situation with high ground radon, additional measures such as a diffusion tight barrier may be needed. The recommended limit depend on the characteristics of the environment which in Eurocode 2 is converted to an exposure class. From the exposure class the valid maximum crack width is given. However, with respect to radon penetration, the limit may need modification. As a gas proof concrete element should be designed with similar restrictions as if it was water proof, the through crack width should be limited to a maximum value of at least 0.2 mm, but this will also depend on the ground radon concentration, slab thickness and total number of cracks in the slab. Cracks that partially penetrate the concrete were adopted to not contribute to radon leakage since gas transport through the slab then is limited.

It was found that the result obtained by the method described in Eurocode 2, to estimate the minimum amount of reinforcement, was simplified and based on the assumption that stabilized cracking has been reached. However, when considering shrinkage and restraint this is not always the case. The steel stress evaluated, where shrinkage and restraint was considered, was observed to become lower than the one limited depending on the dimension of the reinforcing bar. Therefore, the actual obtained crack width was estimated smaller than the one evaluated based on an adoption. The approach to estimate crack width according to Eurocode 2 could then be concluded to be conservative, thus Engströms method could advantageously be included in the crack width evaluation to prevent unnecessarily large dimension of the reinforcement.

By comparing the gained result it was observed that an increased bar diameter entails a bigger need of minimum reinforcement to fulfill the given requirement of the crack width. It was also concluded that, to not exceed the limit value, more reinforcement was needed when increasing the strength of the concrete i.e decreasing the water/cement ratio.

Lastly, an increased water/cement ratio was concluded to contribute to larger cracks. This was assumed to be the result of the entailed increased shrinkage caused by the high water content in the lower strength concrete, based on the evaluated shrinkage according to Eurocode 2. When increasing the bar diameter, more reinforcement is needed to manage the restrictions on crack width, thus the shrinkage force increase and the resulting crack widths becomes larger.

10.1 Future studies

To attain a result that matches the reality to a greater extent a numerical analysis could be performed. It would include more parameters that actually affects the concrete that was not considered in the analytically analysis performed in this project. In such case, the relative humidity in the slab could be investigated to determine the development of the drying process. The effect of changes in temperature or relative humidity of the surrounding environment could then also be studied.

Furthermore, the effect of gas transport through an uncracked concrete slab and with different crack widths is also of interest.

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Appendices

A Mathcad calculations for 120 mm slab

A.1 Concrete class C20/25

$$Concrete_{class} := \text{“20/25”}$$
$$h := 120\ mm$$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$$Concrete_{class} = \text{“20/25”}$$

$$\alpha_{sus} := 0.6$$

$$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 0.9\ MPa$$

$$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.32\ MPa$$

$$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 1.74\ MPa$$

$$f_{cmo} := 10\ MPa$$

$$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$$

$$w_{max} := 0.2\ mm$$

$$Cement_{class} := \text{“N”}$$

Dimensions

$h = 120\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 1.2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$$E_s := 200\ GPa$$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.24\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.234\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.973$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 3.175$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| 1 \\ \text{else} \\ \quad \| \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 1.118$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| min\left(1500, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 601.563$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.976$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.976$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.667$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 33.17$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 6.029\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 234.375\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix}100\ mm\\200\ mm\\300\ mm\\500\ mm\end{bmatrix}, \begin{bmatrix}1\\0.85\\0.75\\0.7\end{bmatrix}, h_0\right) = 0.816$$

$$\beta_{ds}(t) := \frac{(t-t_s)\ day^{-1}}{(t-t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.992$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85\left((220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.816 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 4.707 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 4.744 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2\left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 2.5 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 2.5 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.957 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.994 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\; \| \; 1 \;\; \text{else} \;\; \| \; 0.65 \quad = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 1.2\ m^2$

$$\sigma_s(\phi) := \begin{array}{l} \text{if } \phi = 6\ mm \\ \quad \| \; 320 \\ \text{else if } \phi = 8\ mm \\ \quad \| \; 280 \\ \text{else if } \phi = 12\ mm \\ \quad \| \; 240 \\ \text{else if } \phi = 16\ mm \\ \quad \| \; 200 \\ \text{else if } \phi = 25\ mm \\ \quad \| \; 160 \\ \text{else} \\ \quad \| \; \text{"Invalid bar diameter"} \end{array} \; MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2\ A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 60\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h - x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

Crack spacing

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

Crack width

$$k_t(t) := \text{if } t < 1\ yr$$
$$\quad 0.6 \qquad \text{(short term loading)}$$
$$\text{else}$$
$$\quad 0.4 \qquad \text{(long term loading)}$$

$$\Delta\varepsilon(t, \phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\ (1 + \alpha \cdot \rho_{p.ef}(\phi))}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t, \phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t, \phi)$$

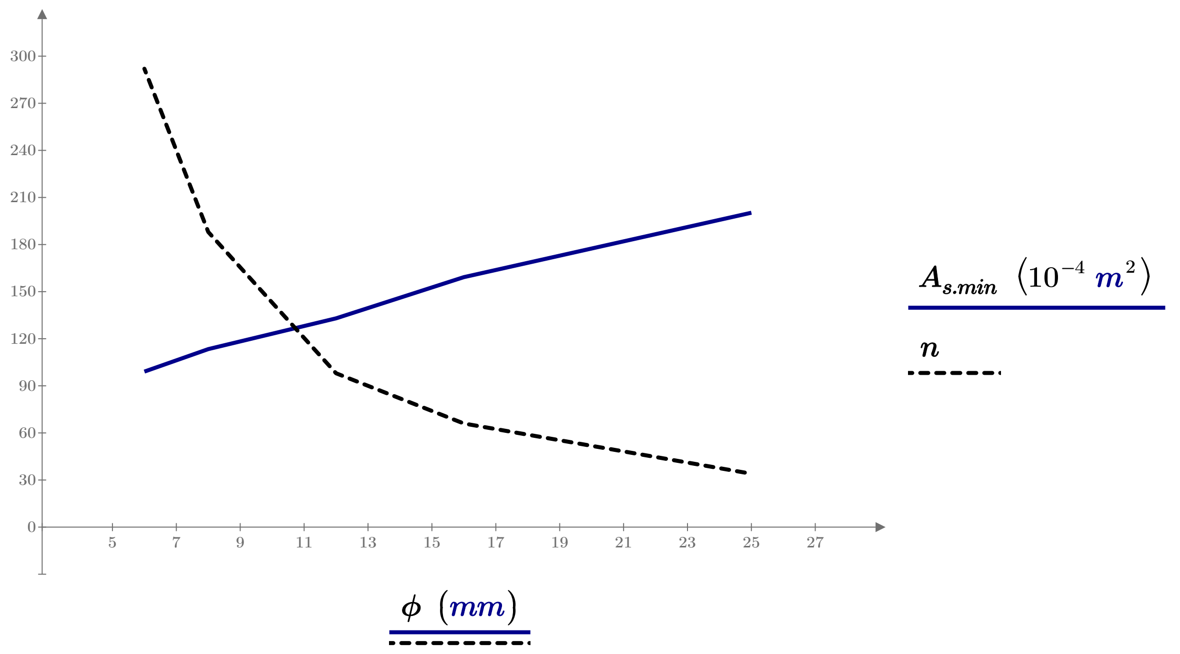
$$w_{max} = 0.2\ mm$$

if $w_k(\phi) < w_{max}$
  "OK!"
else
  "Not OK! – modify reinforcement amount"

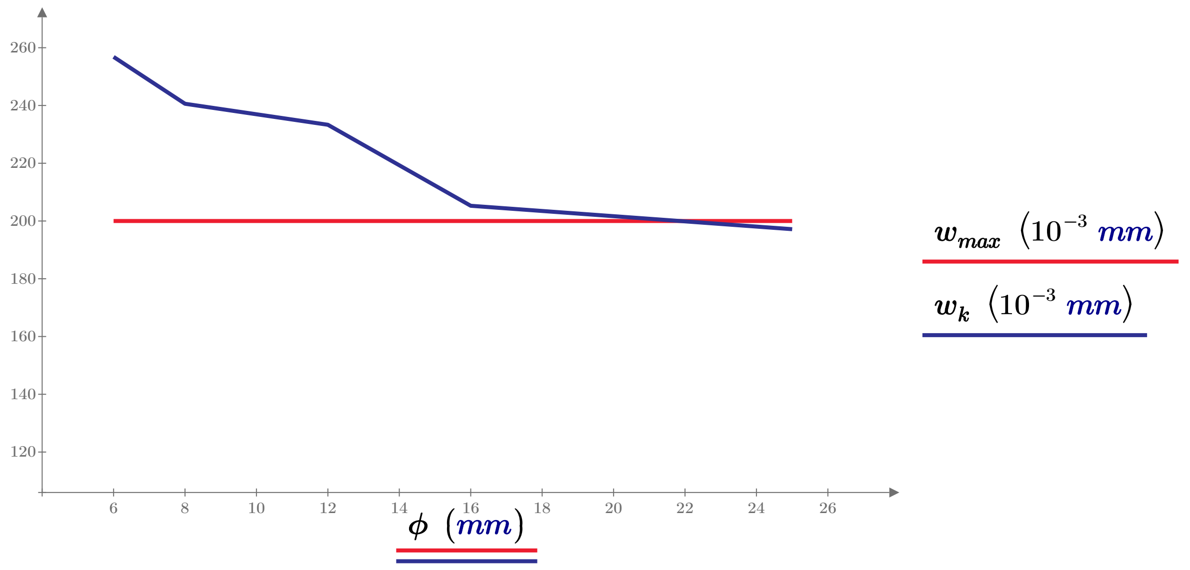
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t, \phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t, \phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t, \phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t, \phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t, \phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$$\varepsilon_{cs}(t) = 4.994 \cdot 10^{-4}$$

$$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.994 \cdot 10^{-4}$$

$$F_{cs}(t, \phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$$

Cracking force

$$f_{ct} := f_{ctm} = 2.2\ MPa$$

Short term response

$$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$$

Long term response

$$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.994 \cdot 10^{-4}$$ full restraint

$$R_{tot} := 1$$

$$N(t, \phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t, \phi)\right)$$

$$\sigma_c(t, \phi) := \frac{N(t, \phi) + F_{cs}(t, \phi)}{A_{I.ef}(\phi)}$$

$$risk(t, \phi) := \begin{array}{l} \text{if } \sigma_c(t, \phi) < f_{ctk0.05.sus} \\ \quad \text{"No risk of cracking"} \\ \text{else if } f_{ctk0.05.sus} \le \sigma_c(t, \phi) < f_{ctm.sus} \\ \quad \text{"Risk risk of cracking"} \\ \text{else if } f_{ctm.sus} \le \sigma_c(t, \phi) \le f_{ctk0.95.sus} \\ \quad \text{"High risk risk of cracking"} \\ \text{else} \\ \quad \text{"Very high risk risk of cracking"} \end{array}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot \sigma_s^{\ 2}}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot \sigma_s^{\ 2}}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + (-\varepsilon_{cs}(t)) \cdot L$$

$$\begin{aligned}
\sigma_{s.it}(t,\phi) := \ & \text{for } i \in 1..100 \\
& \quad n_{cr} \leftarrow i \\
& \quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\
& \quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
& \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
& \quad \text{if } N \geq N_{cr.\infty}(\phi) \\
& \quad\quad \text{continue} \\
& \quad \text{else} \\
& \quad\quad \text{return } \sigma_s \\
& \quad\quad \text{break}
\end{aligned}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$n_{cr}(t,\phi) := \text{for } i \in 1..100$$
$$\quad n_{cr} \leftarrow i$$
$$\quad \text{while } iteration\,(t,\phi,n_{cr},\sigma_s) > 0\ mm$$
$$\quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa$$
$$\quad N \leftarrow \sigma_s \cdot A_s(\phi)$$
$$\quad \text{if } N \geq N_{cr.\infty}(\phi)$$
$$\quad\quad \text{continue}$$
$$\quad \text{else}$$
$$\quad\quad \text{return } n_{cr}$$
$$\quad\quad \text{break}$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

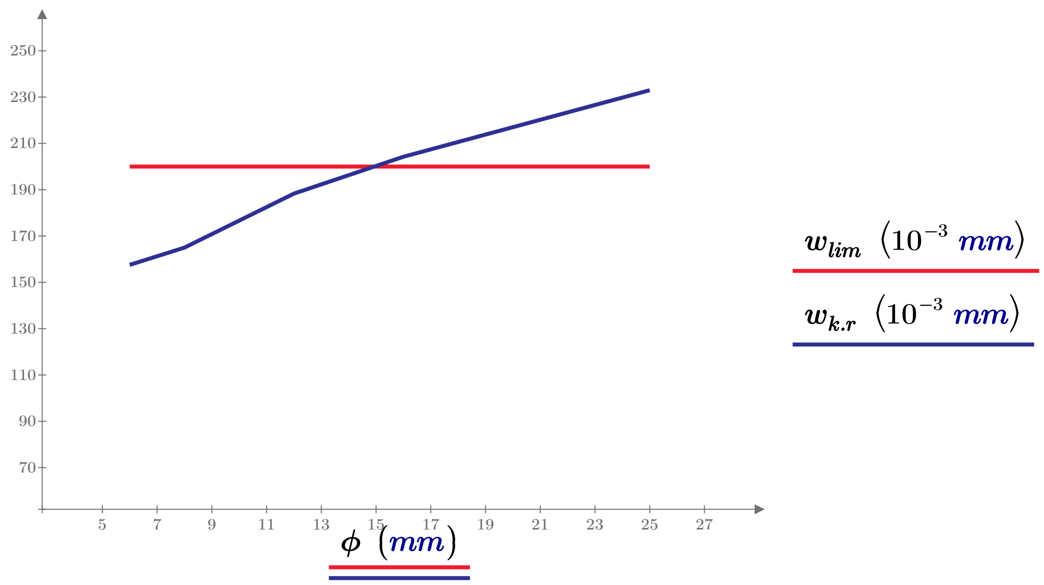
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}(t,\phi_1)$	$n_{cr}(t,\phi_1)$	$N(t,\phi_1)$	$F_{cs}(t,\phi_1)$	$N_{cr.\infty}(\phi_1)$
8	$\sigma_{s.it}(t,\phi_2)$	$n_{cr}(t,\phi_2)$	$N(t,\phi_2)$	$F_{cs}(t,\phi_2)$	$N_{cr.\infty}(\phi_2)$
12	$\sigma_{s.it}(t,\phi_3)$	$n_{cr}(t,\phi_3)$	$N(t,\phi_3)$	$F_{cs}(t,\phi_3)$	$N_{cr.\infty}(\phi_3)$
16	$\sigma_{s.it}(t,\phi_4)$	$n_{cr}(t,\phi_4)$	$N(t,\phi_4)$	$F_{cs}(t,\phi_4)$	$N_{cr.\infty}(\phi_4)$
25	$\sigma_{s.it}(t,\phi_5)$	$n_{cr}(t,\phi_5)$	$N(t,\phi_5)$	$F_{cs}(t,\phi_5)$	$N_{cr.\infty}(\phi_5)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m(\phi_1,\sigma_{s.it_1})$	$1.3\ w_{m_1}$	$l_t(\phi_1,\sigma_{s.it_1})$	$l_{t.sus}(\phi_1,\sigma_{s.it_1})$
8	$w_m(\phi_2,\sigma_{s.it_2})$	$1.3\ w_{m_2}$	$l_t(\phi_2,\sigma_{s.it_2})$	$l_{t.sus}(\phi_2,\sigma_{s.it_2})$
12	$w_m(\phi_3,\sigma_{s.it_3})$	$1.3\ w_{m_3}$	$l_t(\phi_3,\sigma_{s.it_3})$	$l_{t.sus}(\phi_3,\sigma_{s.it_3})$
16	$w_m(\phi_4,\sigma_{s.it_4})$	$1.3\ w_{m_4}$	$l_t(\phi_4,\sigma_{s.it_4})$	$l_{t.sus}(\phi_4,\sigma_{s.it_4})$
25	$w_m(\phi_5,\sigma_{s.it_5})$	$1.3\ w_{m_5}$	$l_t(\phi_5,\sigma_{s.it_5})$	$l_{t.sus}(\phi_5,\sigma_{s.it_5})$

Plot of crack width and crack width limit



A.2 Concrete class C25/30

$Concrete_{class} := \text{“25/30”}$
$h := 120\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“25/30”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.08\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.56\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 1.98\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 120\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 1.2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.24\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.234\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{cases} 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} & \text{if } f_{cm} \le 35\ MPa \\ \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}}\right)^{0.7}\right) \cdot \left(\dfrac{35\ MPa}{f_{cm}}\right)^{0.2} & \text{else} \end{cases} = 1.973$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.925$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{cases} -1 & \text{if } Cement_{class} = \text{“S”} \\ 0 & \text{else if } Cement_{class} = \text{“N”} \\ 1 & \text{else if } Cement_{class} = \text{“R”} \\ \text{“invalid cement class”} & \text{else} \end{cases} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day}\right)^{1.2}} + 1\right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 1.03$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| min\left(1500, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 601.563$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.662$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.662$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.452$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 30.077$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 6.649\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 234.375\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.816$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.992$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85\left(\left(220 + 110 \cdot \alpha_{ds1}\right) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.478 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 4.433 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 4.468 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2 \left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 3.75 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 3.75 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.808 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.843 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \ \Big\|\ 1 \ \text{else}\ \Big\|\ 0.65 \quad = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 1.2\ m^2$

$$\sigma_s(\phi) := \begin{array}{l} \text{if } \phi = 6\ mm \\ \quad \| 320 \\ \text{else if } \phi = 8\ mm \\ \quad \| 280 \\ \text{else if } \phi = 12\ mm \\ \quad \| 240 \\ \text{else if } \phi = 16\ mm \\ \quad \| 200 \\ \text{else if } \phi = 25\ mm \\ \quad \| 160 \\ \text{else} \\ \quad \| \text{"Invalid bar diameter"} \end{array} \Bigg|\ MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2\ A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 60\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h-x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

<u>Crack spacing</u>

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

<u>Crack width</u>

$k_t(t) :=$ if $t < 1\ yr$
‖ 0.6 (short term loading)
else
‖ 0.4 (long term loading)

$$\Delta\varepsilon(t, \phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\left(1 + \alpha \cdot \rho_{p.ef}(\phi)\right)}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t, \phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t, \phi)$$

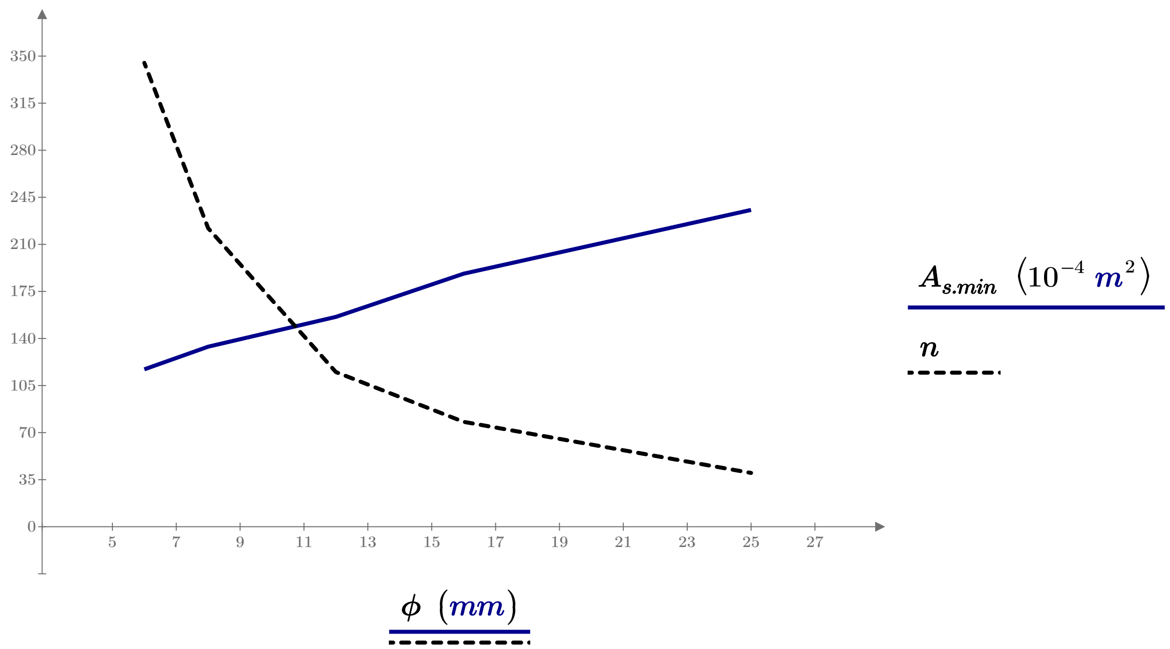
$$w_{max} = 0.2\ mm$$

if $w_k(\phi) < w_{max}$
‖ "OK!"
else
‖ "Not OK! – modify reinforcement amount"

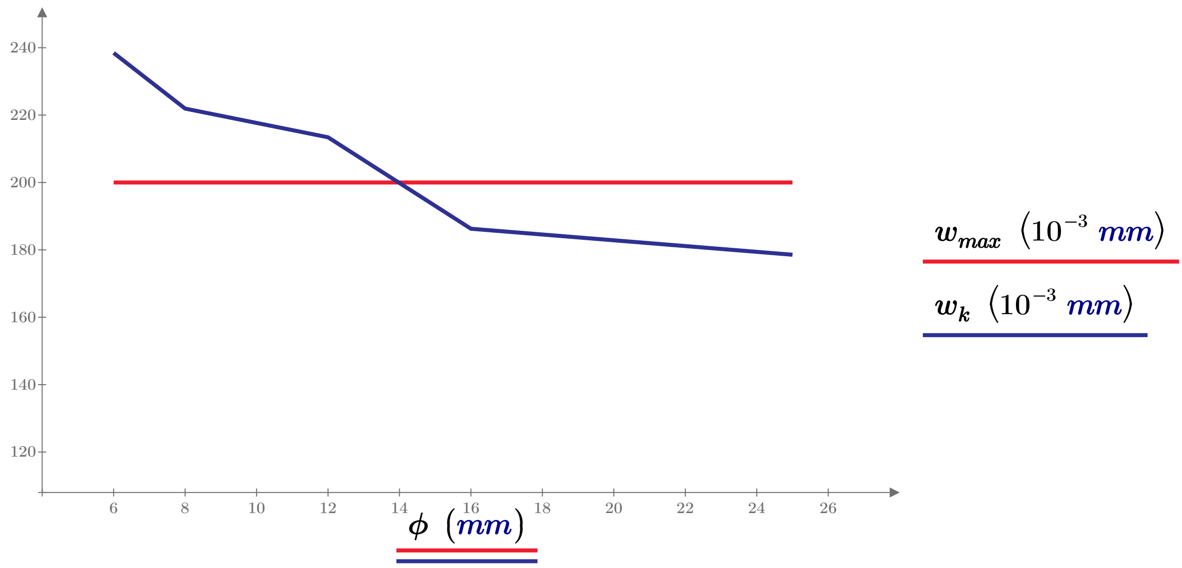
$$t := \infty$$

ϕ	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
(mm)					
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t, \phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t, \phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t, \phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t, \phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t, \phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$\varepsilon_{cs}(t) = 4.843 \cdot 10^{-4}$

$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.843 \cdot 10^{-4}$

$F_{cs}(t, \phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$

Cracking force

$f_{ct} := f_{ctm} = 2.6\ MPa$

Short term response

$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$

Long term response

$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.843 \cdot 10^{-4}$ full restraint

$R_{tot} := 1$

$N(t, \phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t, \phi)\right)$

$$\sigma_c(t, \phi) := \frac{N(t, \phi) + F_{cs}(t, \phi)}{A_{I.ef}(\phi)}$$

$$\begin{aligned}
risk(t, \phi) := \ & \text{if } \sigma_c(t, \phi) < f_{ctk0.05.sus} \\
& \quad \text{"No risk of cracking"} \\
& \text{else if } f_{ctk0.05.sus} \le \sigma_c(t, \phi) < f_{ctm.sus} \\
& \quad \text{"Risk risk of cracking"} \\
& \text{else if } f_{ctm.sus} \le \sigma_c(t, \phi) \le f_{ctk0.95.sus} \\
& \quad \text{"High risk risk of cracking"} \\
& \text{else} \\
& \quad \text{"Very high risk risk of cracking"}
\end{aligned}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\dfrac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \dfrac{E_s}{E_{cm}} \cdot \dfrac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\dfrac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \dfrac{E_s}{E_{cm}} \cdot \dfrac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\dfrac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \dfrac{E_s}{E_{cm}} \cdot \dfrac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + (-\varepsilon_{cs}(t)) \cdot L$$

$$\begin{aligned}
\sigma_{s.it}(t,\phi) := \ & \text{for } i \in 1..100 \\
& \quad n_{cr} \leftarrow i \\
& \quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\
& \quad \quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
& \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
& \quad \text{if } N \geq N_{cr.\infty}(\phi) \\
& \quad \quad \text{continue} \\
& \quad \text{else} \\
& \quad \quad \text{return } \sigma_s \\
& \quad \quad \text{break}
\end{aligned}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$
\begin{aligned}
n_{cr}(t,\phi) := \ & \text{for } i \in 1..100 \\
& \quad n_{cr} \leftarrow i \\
& \quad \text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\
& \quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
& \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
& \quad \text{if } N \geq N_{cr.\infty}(\phi) \\
& \quad\quad \text{continue} \\
& \quad \text{else} \\
& \quad\quad \text{return } n_{cr} \\
& \quad\quad \text{break}
\end{aligned}
$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

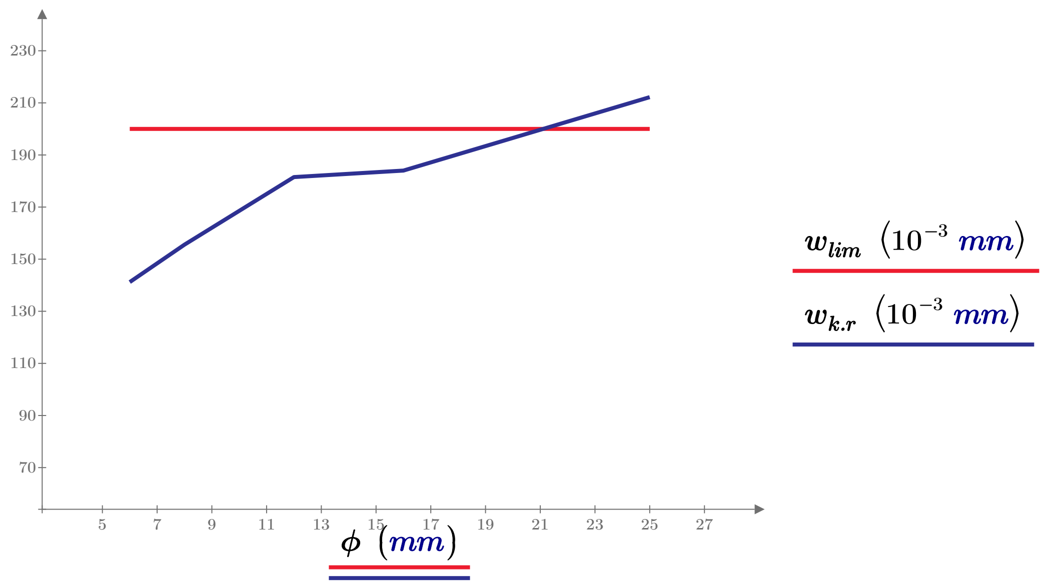
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

Plot of crack width and crack width limit



A.3 Concrete class C30/37

$Concrete_{class} := \text{“30/37”}$
$h := 120\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“30/37”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.2\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.74\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 2.28\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 120\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab

$A_c := b \cdot h = 1.2\ m^2$ area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max\left(\phi, c_{min.dur}, 10\ mm\right)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.24\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.234\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.887$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.725$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| 0 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| 1 \\ \text{else} \\ \quad \| \text{"invalid cement class"} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 0.96$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| min\left(1500, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 591.491$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.264$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.264$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.061$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 25.844$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 7.739\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 234.375\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.816$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.992$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{|l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{|l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85\left(\left(220 + 110 \cdot \alpha_{ds1}\right) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.159 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 4.175 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 4.207 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2 \left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 5 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 5 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.675 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.707 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\; \| \; 1 \;\; \text{else} \;\; \| \; 0.65 \quad = 1$$

$k_c := 1$ — pure tension is assumed

$$A_{ct} := A_c = 1.2\ m^2$$

$$\sigma_s(\phi) := \begin{cases} 320 & \text{if } \phi = 6\ mm \\ 280 & \text{else if } \phi = 8\ mm \\ 240 & \text{else if } \phi = 12\ mm \\ 200 & \text{else if } \phi = 16\ mm \\ 160 & \text{else if } \phi = 25\ mm \\ \text{“Invalid bar diameter”} & \text{else} \end{cases} \quad MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$$n(\phi) := \operatorname{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$$ number of reinforcement bars

$$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$$ area of reinforcement

$$A_s(\phi) := 1.2\ A_s(\phi)$$ increasing the reinforcement area to meet requirement on crack width

$$A_{net}(\phi) := A_c - A_s(\phi)$$

$$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$$ short time response

$$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ — pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$x := \frac{h}{2} = 60\ mm$

$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h - x}{3}, \frac{h}{2}\right)$

$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$

$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$

Crack spacing

$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$

Crack width

$k_t(t) :=$ if $t < 1\ yr$
  || 0.6 (short term loading)
else
  || 0.4 (long term loading)

$\Delta\varepsilon(t, \phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\ (1 + \alpha \cdot \rho_{p.ef}(\phi))}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$

$w_k(t, \phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t, \phi)$

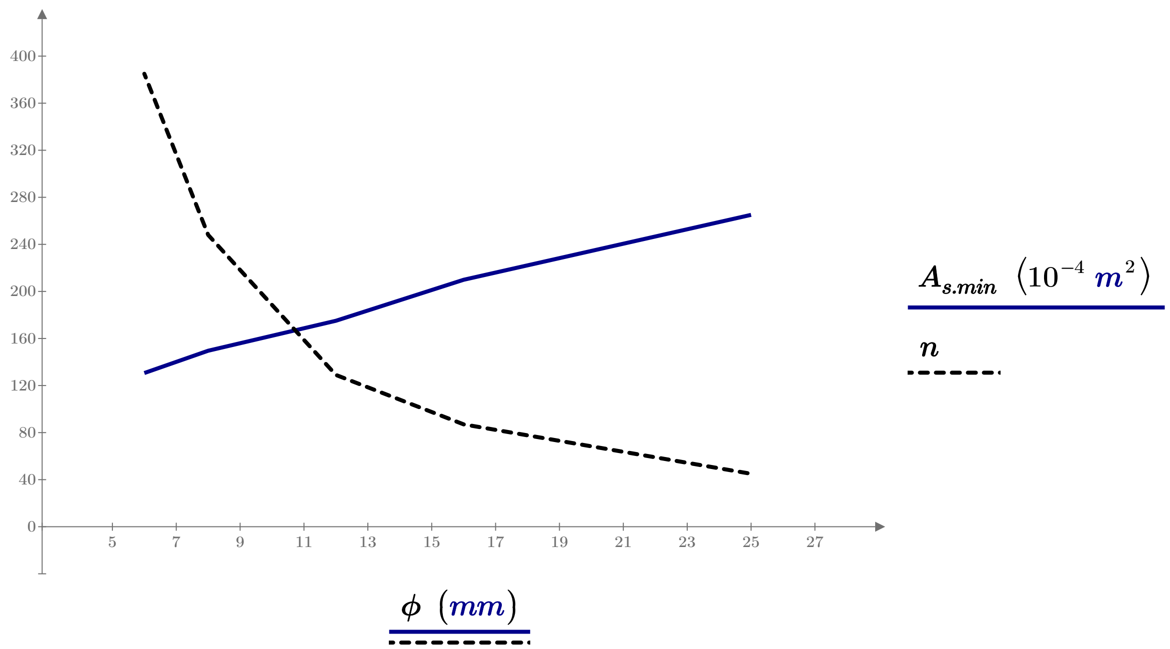
$w_{max} = 0.2\ mm$

if $w_k(\phi) < w_{max}$
  || "OK!"
else
  || "Not OK! – modify reinforcement amount"

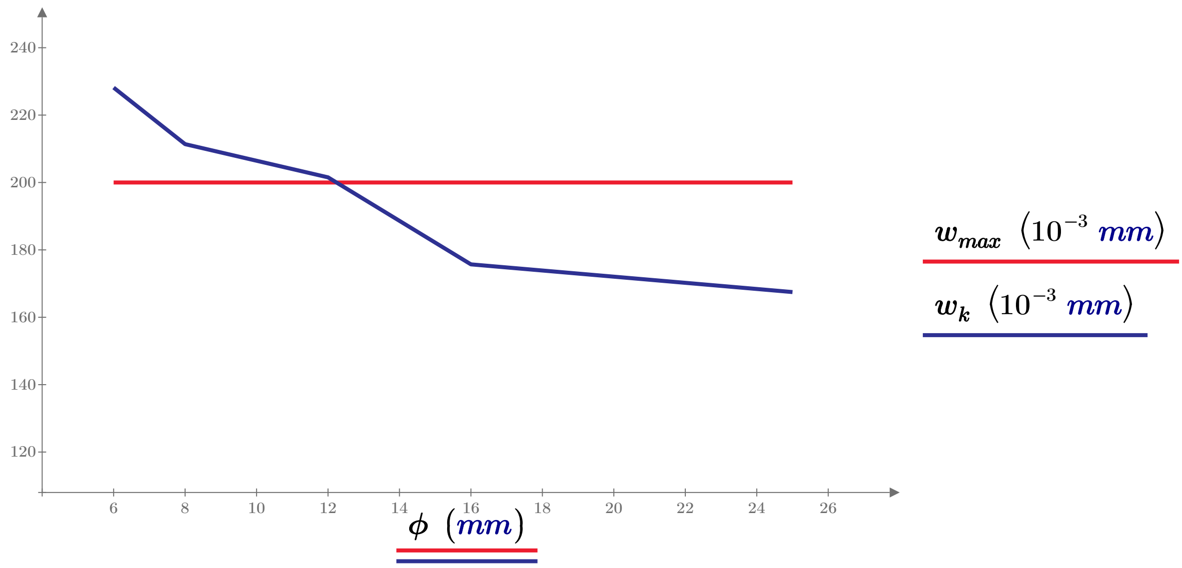
$t := \infty$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t, \phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t, \phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t, \phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t, \phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t, \phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$$\varepsilon_{cs}(t) = 4.707 \cdot 10^{-4}$$

$$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.707 \cdot 10^{-4}$$

$$F_{cs}(t, \phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$$

Cracking force

$$f_{ct} := f_{ctm} = 2.9\ MPa$$

Short term response

$$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$$

Long term response

$$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.707 \cdot 10^{-4}$$ full restraint

$$R_{tot} := 1$$

$$N(t, \phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t, \phi)\right)$$

$$\sigma_c(t, \phi) := \frac{N(t, \phi) + F_{cs}(t, \phi)}{A_{I.ef}(\phi)}$$

$$\begin{aligned} risk(t, \phi) := & \text{ if } \sigma_c(t, \phi) < f_{ctk0.05.sus} \\ & \quad \text{"No risk of cracking"} \\ & \text{else if } f_{ctk0.05.sus} \le \sigma_c(t, \phi) < f_{ctm.sus} \\ & \quad \text{"Risk risk of cracking"} \\ & \text{else if } f_{ctm.sus} \le \sigma_c(t, \phi) \le f_{ctk0.95.sus} \\ & \quad \text{"High risk risk of cracking"} \\ & \text{else} \\ & \quad \text{"Very high risk risk of cracking"} \end{aligned}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + \left(-\varepsilon_{cs}(t)\right) \cdot L$$

```
σs.it(t, φ) := for i ∈ 1..100
                   n_cr ← i
                   while iteration(t, φ, n_cr, σs) > 0 mm
                       σs ← σs − 1 kPa
                   N ← σs · As(φ)
                   if N ≥ N_cr.∞(φ)
                       continue
                   else
                       return σs
                       break
```

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$n_{cr}(t,\phi) := \begin{array}{l} \text{for } i \in 1..100 \\ \quad n_{cr} \leftarrow i \\ \quad \text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\ \quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\ \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\ \quad \text{if } N \geq N_{cr.\infty}(\phi) \\ \quad\quad \text{continue} \\ \quad \text{else} \\ \quad\quad \text{return } n_{cr} \\ \quad\quad \text{break} \end{array}$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

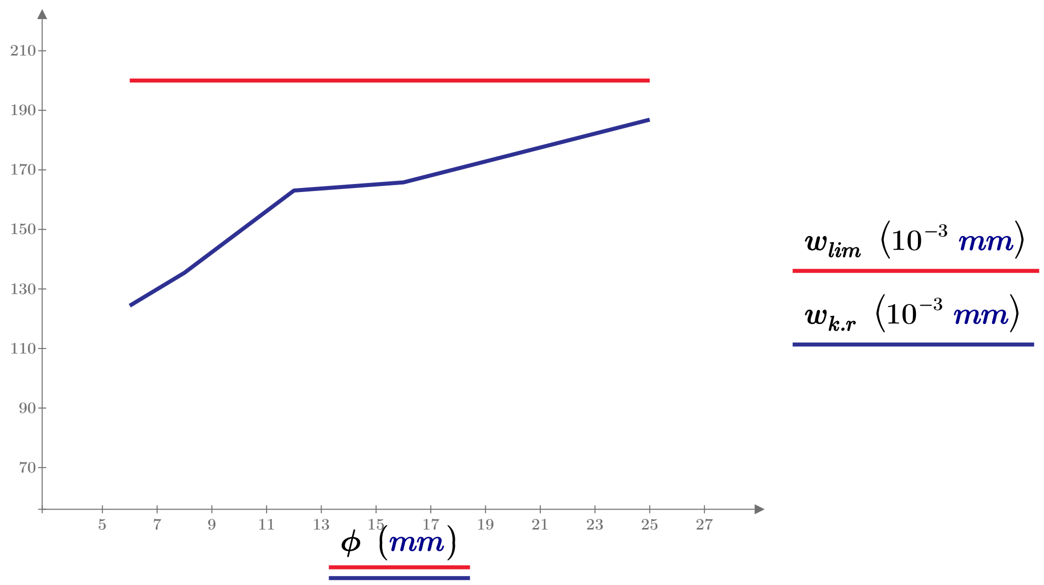
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

Plot of crack width and crack width limit



A.4 Concrete class C35/45

$Concrete_{class} := \text{"35/45"}$
$h := 120\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck}	f_{cm}	$f_{ctk0.05}$	f_{ctm}	$f_{ctk0.95}$	E_{cm}
	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(GPa)
"20/25"	20	28	1.5	2.2	2.9	30
"25/30"	25	33	1.8	2.6	3.3	31
"30/37"	30	38	2.0	2.9	3.8	33
"35/45"	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{"35/45"}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.32\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.92\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 2.52\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{"N"}$

Dimensions

$h = 120\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 1.2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.24\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.234\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \ 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \ \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.768$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.562$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 0.902$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \ min\left(1500, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| \ min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 577.111$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 2.875$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 2.875$$

$$\alpha := \frac{E_s}{E_{cm}} = 5.882$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 22.794$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 8.774\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 234.375\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.816$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.992$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85 \left(\left(220 + 110 \cdot \alpha_{ds1}\right) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 4.858 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 3.932 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 3.962 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2\left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 6.25 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 6.25 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.557 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.587 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\middle\|\; 1 \;\text{else}\; \middle\|\; 0.65 = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 1.2\ m^2$

$$\sigma_s(\phi) := \begin{cases} 320 & \text{if } \phi = 6\ mm \\ 280 & \text{else if } \phi = 8\ mm \\ 240 & \text{else if } \phi = 12\ mm \\ 200 & \text{else if } \phi = 16\ mm \\ 160 & \text{else if } \phi = 25\ mm \\ \text{“Invalid bar diameter”} & \text{else} \end{cases} \ MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$$ number of reinforcement bars

$$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$$ area of reinforcement

$$A_s(\phi) := 1.2\ A_s(\phi)$$ increasing the reinforcement area to meet requirement on crack width

$$A_{net}(\phi) := A_c - A_s(\phi)$$

$$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$$ short time response

$$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$
$k_2 := 1$ pure tension
$k_3 := 3.4$
$k_4 := 0.425$

$$x := \frac{h}{2} = 60\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h-x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

Crack spacing

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

Crack width

$$k_t(t) := \begin{array}{l} \text{if } t < 1\ yr \\ \quad \| 0.6 \\ \text{else} \\ \quad \| 0.4 \end{array}$$

(short term loading)

(long term loading)

$$\Delta\varepsilon(t,\phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)} \left(1 + \alpha \cdot \rho_{p.ef}(\phi)\right)}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t,\phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t,\phi)$$

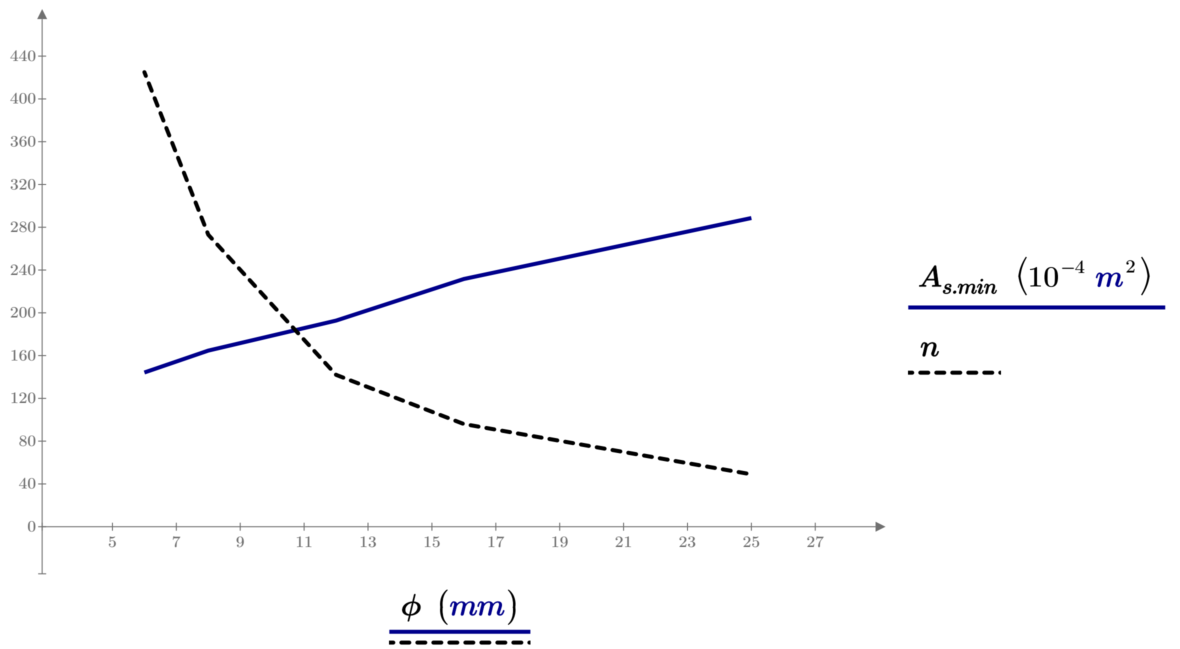
$$w_{max} = 0.2\ mm$$

$$\begin{array}{l} \text{if } w_k(\phi) < w_{max} \\ \quad \| \text{“OK!”} \\ \text{else} \\ \quad \| \text{“Not OK! – modify reinforcement amount”} \end{array}$$

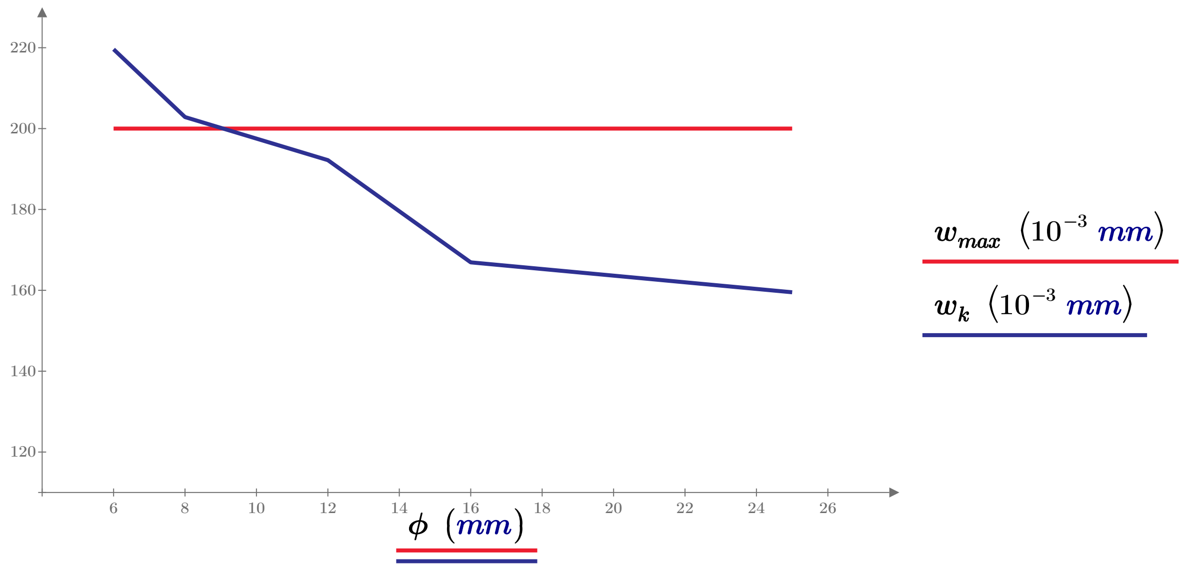
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t,\phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t,\phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t,\phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t,\phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t,\phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$\varepsilon_{cs}(t) = 4.587 \cdot 10^{-4}$

$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.587 \cdot 10^{-4}$

$F_{cs}(t,\phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$

Cracking force

$f_{ct} := f_{ctm} = 3.2\ MPa$

Short term response

$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$

Long term response

$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.587 \cdot 10^{-4}$ full restraint

$R_{tot} := 1$

$N(t,\phi) := R_{tot} \cdot (\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t,\phi))$

$$\sigma_c(t,\phi) := \frac{N(t,\phi) + F_{cs}(t,\phi)}{A_{I.ef}(\phi)}$$

$$risk(t,\phi) := \begin{cases} \text{"No risk of cracking"} & \text{if } \sigma_c(t,\phi) < f_{ctk0.05.sus} \\ \text{"Risk risk of cracking"} & \text{else if } f_{ctk0.05.sus} \le \sigma_c(t,\phi) < f_{ctm.sus} \\ \text{"High risk risk of cracking"} & \text{else if } f_{ctm.sus} \le \sigma_c(t,\phi) \le f_{ctk0.95.sus} \\ \text{"Very high risk risk of cracking"} & \text{else} \end{cases}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + \left(-\varepsilon_{cs}(t)\right) \cdot L$$

$$\begin{array}{l} \sigma_{s.it}(t,\phi) := \text{for } i \in 1..100 \\ \quad n_{cr} \leftarrow i \\ \quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\ \quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\ \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\ \quad \text{if } N \geq N_{cr.\infty}(\phi) \\ \quad\quad \text{continue} \\ \quad \text{else} \\ \quad\quad \text{return } \sigma_s \\ \quad\quad \text{break} \end{array}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$
n_{cr}(t,\phi) := \text{for } i \in 1..100 \\
\quad n_{cr} \leftarrow i \\
\quad \text{while } iteration\,(t,\phi,n_{cr},\sigma_s) > 0\ mm \\
\quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
\quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
\quad \text{if } N \geq N_{cr.\infty}(\phi) \\
\quad\quad \text{continue} \\
\quad \text{else} \\
\quad\quad \text{return } n_{cr} \\
\quad\quad \text{break}
$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

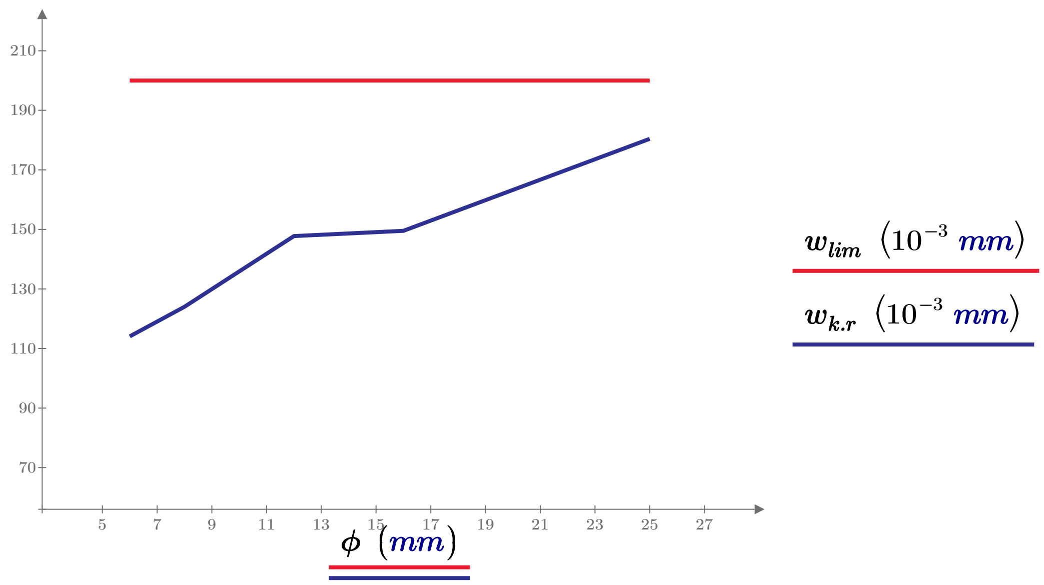
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}(t,\phi_1)$	$n_{cr}(t,\phi_1)$	$N(t,\phi_1)$	$F_{cs}(t,\phi_1)$	$N_{cr.\infty}(\phi_1)$
8	$\sigma_{s.it}(t,\phi_2)$	$n_{cr}(t,\phi_2)$	$N(t,\phi_2)$	$F_{cs}(t,\phi_2)$	$N_{cr.\infty}(\phi_2)$
12	$\sigma_{s.it}(t,\phi_3)$	$n_{cr}(t,\phi_3)$	$N(t,\phi_3)$	$F_{cs}(t,\phi_3)$	$N_{cr.\infty}(\phi_3)$
16	$\sigma_{s.it}(t,\phi_4)$	$n_{cr}(t,\phi_4)$	$N(t,\phi_4)$	$F_{cs}(t,\phi_4)$	$N_{cr.\infty}(\phi_4)$
25	$\sigma_{s.it}(t,\phi_5)$	$n_{cr}(t,\phi_5)$	$N(t,\phi_5)$	$F_{cs}(t,\phi_5)$	$N_{cr.\infty}(\phi_5)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m(\phi_1,\sigma_{s.it_1})$	$1.3\ w_{m_1}$	$l_t(\phi_1,\sigma_{s.it_1})$	$l_{t.sus}(\phi_1,\sigma_{s.it_1})$
8	$w_m(\phi_2,\sigma_{s.it_2})$	$1.3\ w_{m_2}$	$l_t(\phi_2,\sigma_{s.it_2})$	$l_{t.sus}(\phi_2,\sigma_{s.it_2})$
12	$w_m(\phi_3,\sigma_{s.it_3})$	$1.3\ w_{m_3}$	$l_t(\phi_3,\sigma_{s.it_3})$	$l_{t.sus}(\phi_3,\sigma_{s.it_3})$
16	$w_m(\phi_4,\sigma_{s.it_4})$	$1.3\ w_{m_4}$	$l_t(\phi_4,\sigma_{s.it_4})$	$l_{t.sus}(\phi_4,\sigma_{s.it_4})$
25	$w_m(\phi_5,\sigma_{s.it_5})$	$1.3\ w_{m_5}$	$l_t(\phi_5,\sigma_{s.it_5})$	$l_{t.sus}(\phi_5,\sigma_{s.it_5})$

Plot of crack width and crack width limit



B Mathcad calculations for 200 mm slab

B.1 Concrete class C20/25

$Concrete_{class} := \text{“20/25”}$
$h := 200\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“20/25”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 0.9\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.32\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 1.74\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 200\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.4\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.385\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \; 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \; \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}}\right)^{0.7}\right) \cdot \left(\dfrac{35\ MPa}{f_{cm}}\right)^{0.2} \right. \end{array} = 1.825$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 3.175$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \; 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \; 1 \\ \text{else} \\ \quad \| \; \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day}\right)^{1.2}} + 1\right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 1.118$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \min\left(1500, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| \min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 826.923$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.677$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.677$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.667$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 31.181$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 6.414\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 384.615\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.729$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.984$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85\left(\left(220 + 110 \cdot \alpha_{ds1}\right) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.816 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 4.17 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 4.239 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2\left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 2.5 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 2.5 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.42 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.489 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\|\; 1 \;\text{else}\; \|\; 0.65 \quad = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 2\ m^2$

$$\sigma_s(\phi) := \begin{cases} 320 & \text{if } \phi = 6\ mm \\ 280 & \text{else if } \phi = 8\ mm \\ 240 & \text{else if } \phi = 12\ mm \\ 200 & \text{else if } \phi = 16\ mm \\ 160 & \text{else if } \phi = 25\ mm \\ \text{"Invalid bar diameter"} & \text{else} \end{cases} \quad MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2\ A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 100\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h - x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

Crack spacing

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

Crack width

$$k_t(t) := \begin{array}{l} \text{if } t < 1\ yr \\ \quad \| 0.6 \\ \text{else} \\ \quad \| 0.4 \end{array}$$

(short term loading)

(long term loading)

$$\Delta\varepsilon(t, \phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\ (1 + \alpha \cdot \rho_{p.ef}(\phi))}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t, \phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t, \phi)$$

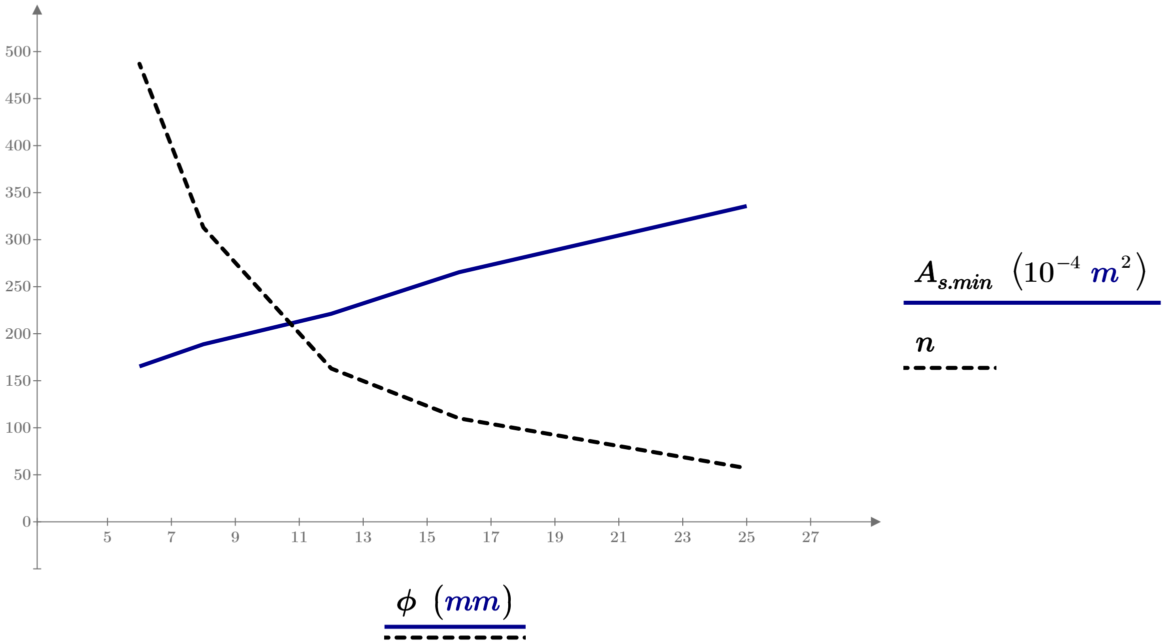
$$w_{max} = 0.2\ mm$$

$$\begin{array}{l} \text{if } w_k(\phi) < w_{max} \\ \quad \| \text{“OK!”} \\ \text{else} \\ \quad \| \text{“Not OK! – modify reinforcement amount”} \end{array}$$

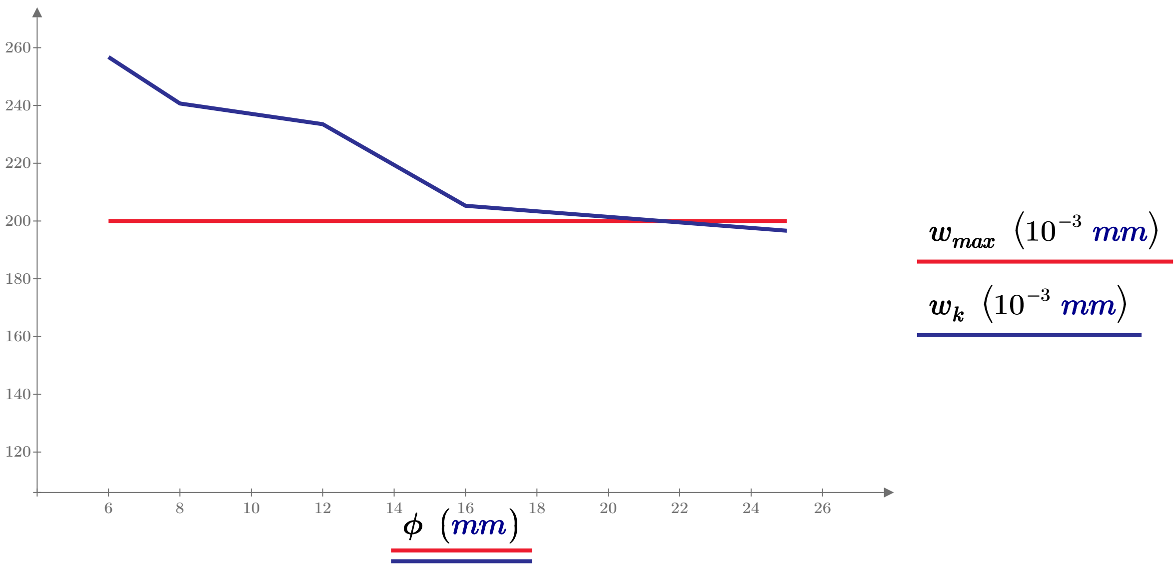
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t, \phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t, \phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t, \phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t, \phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t, \phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$$\varepsilon_{cs}(t) = 4.489 \cdot 10^{-4}$$

$$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.489 \cdot 10^{-4}$$

$$F_{cs}(t, \phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$$

Cracking force

$$f_{ct} := f_{ctm} = 2.2\ MPa$$

Short term response

$$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$$

Long term response

$$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.489 \cdot 10^{-4}$$ full restraint

$$R_{tot} := 1$$

$$N(t, \phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t, \phi)\right)$$

$$\sigma_c(t, \phi) := \frac{N(t, \phi) + F_{cs}(t, \phi)}{A_{I.ef}(\phi)}$$

$$
\begin{aligned}
risk(t, \phi) := \ & \text{if } \sigma_c(t, \phi) < f_{ctk0.05.sus} \\
& \quad \| \text{“No risk of cracking”} \\
& \text{else if } f_{ctk0.05.sus} \le \sigma_c(t, \phi) < f_{ctm.sus} \\
& \quad \| \text{“Risk risk of cracking”} \\
& \text{else if } f_{ctm.sus} \le \sigma_c(t, \phi) \le f_{ctk0.95.sus} \\
& \quad \| \text{“High risk risk of cracking”} \\
& \text{else} \\
& \quad \| \text{“Very high risk risk of cracking”}
\end{aligned}
$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + (-\varepsilon_{cs}(t)) \cdot L$$

$$\begin{array}{l}
\sigma_{s.it}(t,\phi) := \text{for } i \in 1..100 \\
\quad n_{cr} \leftarrow i \\
\quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\
\quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
\quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
\quad \text{if } N \geq N_{cr.\infty}(\phi) \\
\quad\quad \text{continue} \\
\quad \text{else} \\
\quad\quad \text{return } \sigma_s \\
\quad\quad \text{break}
\end{array}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$
\begin{aligned}
n_{cr}(t,\phi) := \text{for } i \in 1..100 \\
&\quad n_{cr} \leftarrow i \\
&\quad \text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\
&\quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
&\quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
&\quad \text{if } N \geq N_{cr.\infty}(\phi) \\
&\quad\quad \text{continue} \\
&\quad \text{else} \\
&\quad\quad \text{return } n_{cr} \\
&\quad\quad \text{break}
\end{aligned}
$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

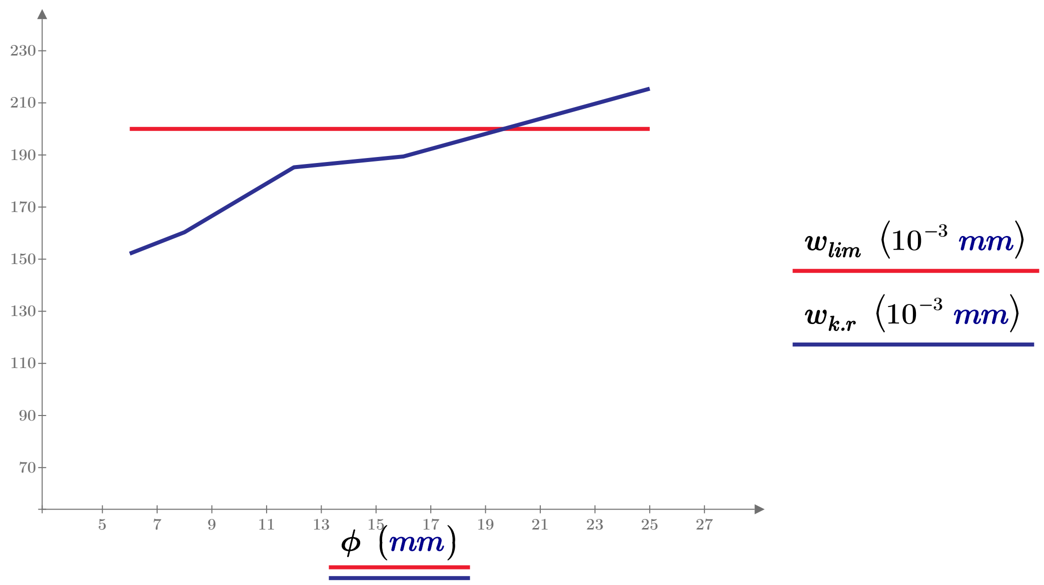
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

Plot of crack width and crack width limit



B.2 Concrete class C25/30

$Concrete_{class} := \text{“25/30”}$
$h := 200\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“25/30”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.08\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.56\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 1.98\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 200\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.4\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.385\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.825$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.925$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| 1 \\ \text{else} \\ \quad \| \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 1.03$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \ min\left(1500, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| \ min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 826.923$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.387$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.387$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.452$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 28.304$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 7.066\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 384.615\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix}100\ mm\\200\ mm\\300\ mm\\500\ mm\end{bmatrix}, \begin{bmatrix}1\\0.85\\0.75\\0.7\end{bmatrix}, h_0\right) = 0.729$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.984$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85 \left((220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.478 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 3.927 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 3.992 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2 \left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 3.75 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 3.75 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.302 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.367 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \begin{cases} \text{if } h \le 300\ mm & 1 \\ \text{else} & 0.65 \end{cases} = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 2\ m^2$

$$\sigma_s(\phi) := \begin{cases} \text{if } \phi = 6\ mm & 320 \\ \text{else if } \phi = 8\ mm & 280 \\ \text{else if } \phi = 12\ mm & 240 \\ \text{else if } \phi = 16\ mm & 200 \\ \text{else if } \phi = 25\ mm & 160 \\ \text{else} & \text{"Invalid bar diameter"} \end{cases} \quad MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2 \cdot A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 100\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h-x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

<u>Crack spacing</u>

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

<u>Crack width</u>

$$k_t(t) := \begin{array}{l} \text{if } t < 1\ yr \\ \quad \| 0.6 \\ \text{else} \\ \quad \| 0.4 \end{array}$$

(short term loading)

(long term loading)

$$\Delta\varepsilon(t, \phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)} \left(1 + \alpha \cdot \rho_{p.ef}(\phi)\right)}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t, \phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t, \phi)$$

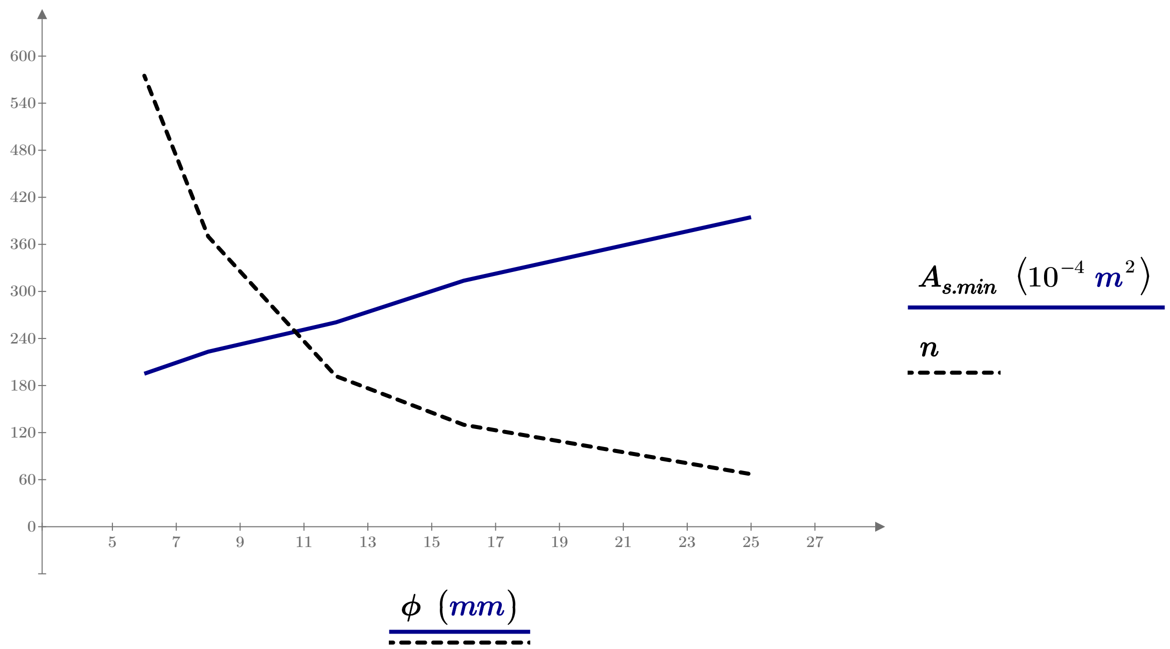
$$w_{max} = 0.2\ mm$$

$$\begin{array}{l} \text{if } w_k(\phi) < w_{max} \\ \quad \| \text{"OK!"} \\ \text{else} \\ \quad \| \text{"Not OK! – modify reinforcement amount"} \end{array}$$

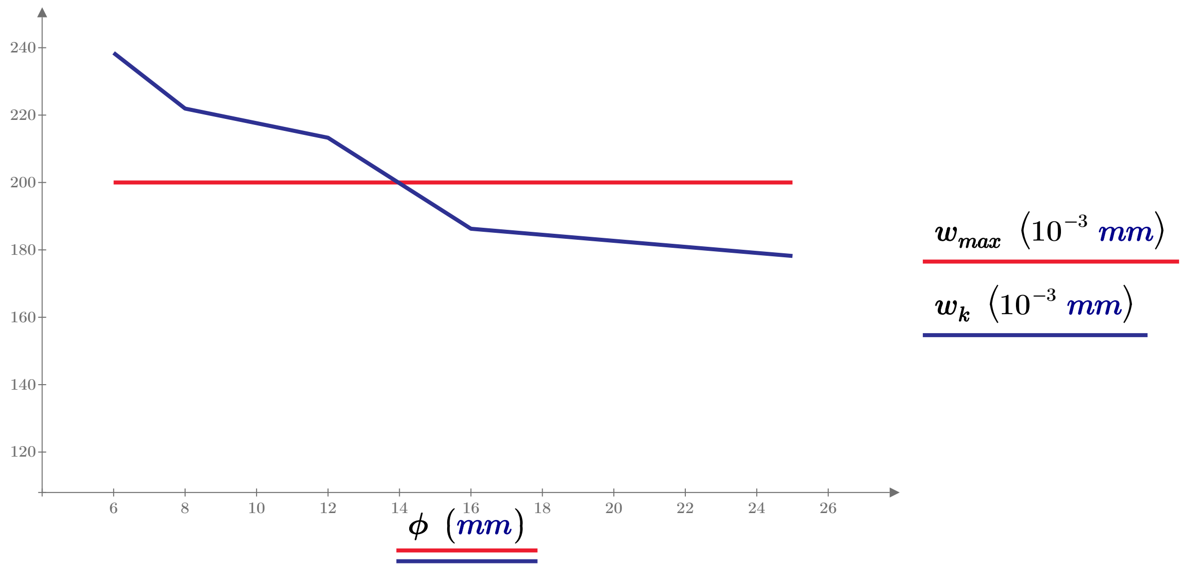
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t, \phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t, \phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t, \phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t, \phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t, \phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$$\varepsilon_{cs}(t) = 4.367 \cdot 10^{-4}$$

$$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.367 \cdot 10^{-4}$$

$$F_{cs}(t,\phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$$

Cracking force

$$f_{ct} := f_{ctm} = 2.6\ MPa$$

Short term response

$$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$$

Long term response

$$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.367 \cdot 10^{-4}$$ full restraint

$$R_{tot} := 1$$

$$N(t,\phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t,\phi)\right)$$

$$\sigma_c(t,\phi) := \frac{N(t,\phi) + F_{cs}(t,\phi)}{A_{I.ef}(\phi)}$$

$$\begin{aligned} risk(t,\phi) := \ & \text{if } \sigma_c(t,\phi) < f_{ctk0.05.sus} \\ & \quad \text{“No risk of cracking”} \\ & \text{else if } f_{ctk0.05.sus} \le \sigma_c(t,\phi) < f_{ctm.sus} \\ & \quad \text{“Risk risk of cracking”} \\ & \text{else if } f_{ctm.sus} \le \sigma_c(t,\phi) \le f_{ctk0.95.sus} \\ & \quad \text{“High risk risk of cracking”} \\ & \text{else} \\ & \quad \text{“Very high risk risk of cracking”} \end{aligned}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot \sigma_s^{\ 2}}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot \sigma_s^{\ 2}}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + (-\varepsilon_{cs}(t)) \cdot L$$

```
σs.it(t,ϕ) := for i ∈ 1..100
                  n_cr ← i
                  while iteration(t,ϕ,n_cr,σs) > 0 mm
                      σs ← σs − 1 kPa
                  N ← σs·As(ϕ)
                  if N ≥ N_cr.∞(ϕ)
                      continue
                  else
                      return σs
                      break
```

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$
\begin{aligned}
n_{cr}(t,\phi) := \text{for } & i \in 1..100 \\
& n_{cr} \leftarrow i \\
& \text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\
& \quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
& N \leftarrow \sigma_s \cdot A_s(\phi) \\
& \text{if } N \geq N_{cr.\infty}(\phi) \\
& \quad \text{continue} \\
& \text{else} \\
& \quad \text{return } n_{cr} \\
& \quad \text{break}
\end{aligned}
$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$

$w_{lim} := w_{max}$

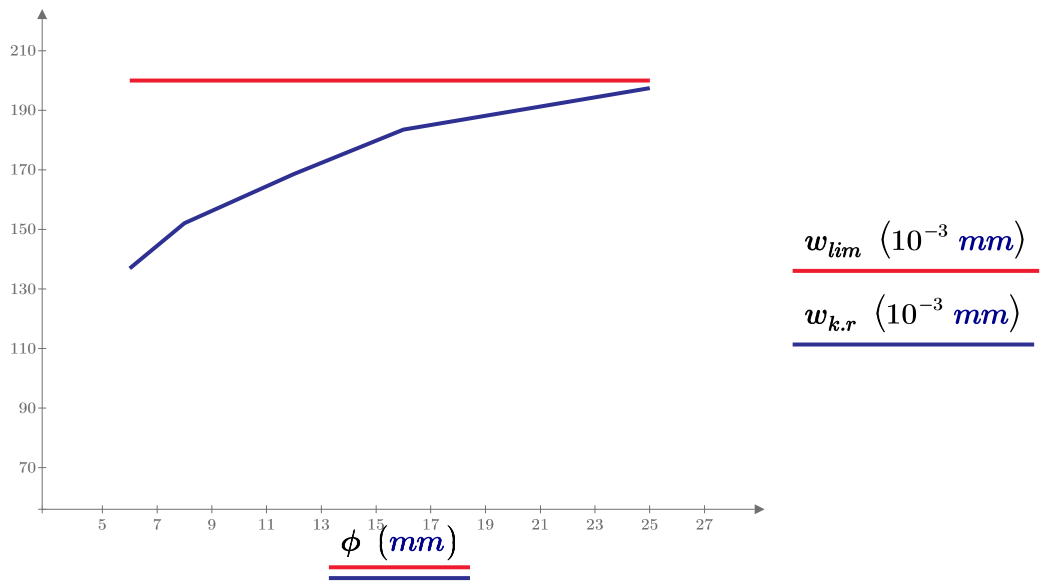
$w_{m.all} := \frac{w_{lim}}{1.3}$

$t := \infty$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

Plot of crack width and crack width limit



B.3 Concrete class C30/37

$Concrete_{class} := \text{“30/37”}$
$h := 200\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“30/37”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.2\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.74\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 2.28\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 200\ mm$	thickness
$L := 10\ m$	length of slab
$b := 10\ m$	width of slab
$A_c := b \cdot h = 2\ m^2$	area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.4\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.385\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.75$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.725$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| 1 \\ \text{else} \\ \quad \| \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 0.96$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \ min\left(1500, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| \ min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + \left(0.012 \cdot RH\right)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 816.852$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 3.026$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 3.026$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.061$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 24.403$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 8.196\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 384.615\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.729$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.984$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{|l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{|l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85\left((220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 5.159 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 3.699 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 3.76 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2\left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 5 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 5 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.199 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.26 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\middle|\; = 1$$
$$\quad \| \; 1$$
$$\quad \text{else}$$
$$\quad \| \; 0.65$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 2\ m^2$

$$\sigma_s(\phi) := \text{if } \phi = 6\ mm \;\middle|\; MPa$$
$$\quad \| \; 320$$
$$\quad \text{else if } \phi = 8\ mm$$
$$\quad \| \; 280$$
$$\quad \text{else if } \phi = 12\ mm$$
$$\quad \| \; 240$$
$$\quad \text{else if } \phi = 16\ mm$$
$$\quad \| \; 200$$
$$\quad \text{else if } \phi = 25\ mm$$
$$\quad \| \; 160$$
$$\quad \text{else}$$
$$\quad \| \; \text{"Invalid bar diameter"}$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2\ A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 100\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h-x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

Crack spacing

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

Crack width

$$k_t(t) := \begin{array}{l} \text{if } t < 1\ yr \\ \quad \| 0.6 \\ \text{else} \\ \quad \| 0.4 \end{array}$$

(short term loading)

(long term loading)

$$\Delta\varepsilon(t,\phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\ (1 + \alpha \cdot \rho_{p.ef}(\phi))}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t,\phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t,\phi)$$

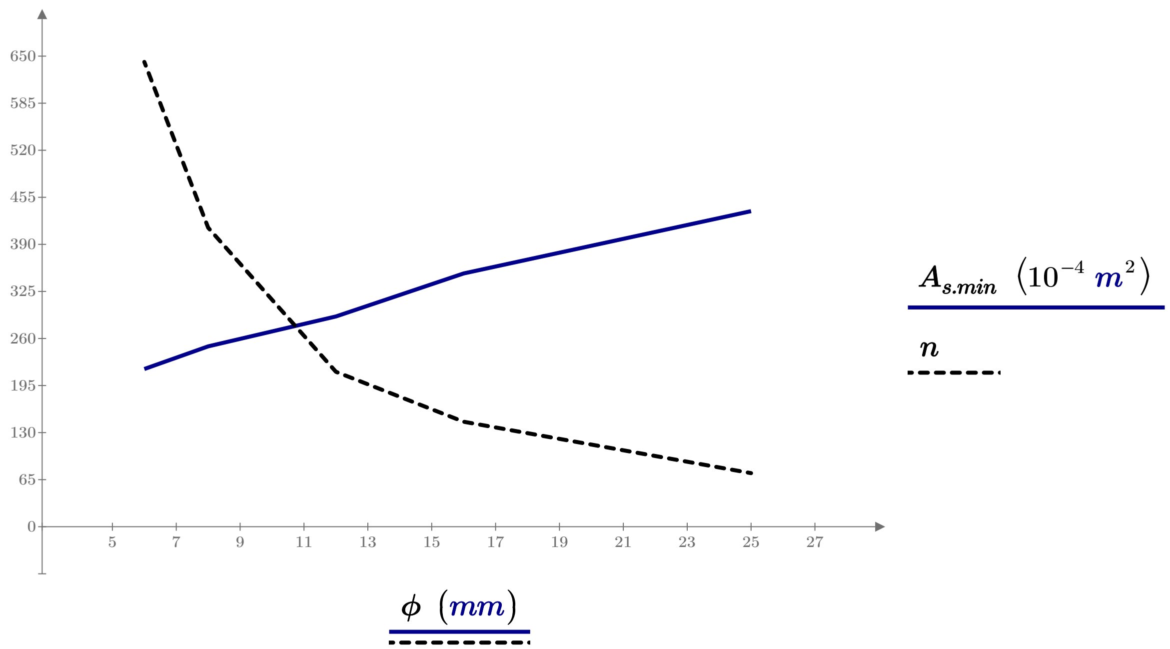
$$w_{max} = 0.2\ mm$$

$$\begin{array}{l} \text{if } w_k(\phi) < w_{max} \\ \quad \| \text{“OK!”} \\ \text{else} \\ \quad \| \text{“Not OK! – modify reinforcement amount”} \end{array}$$

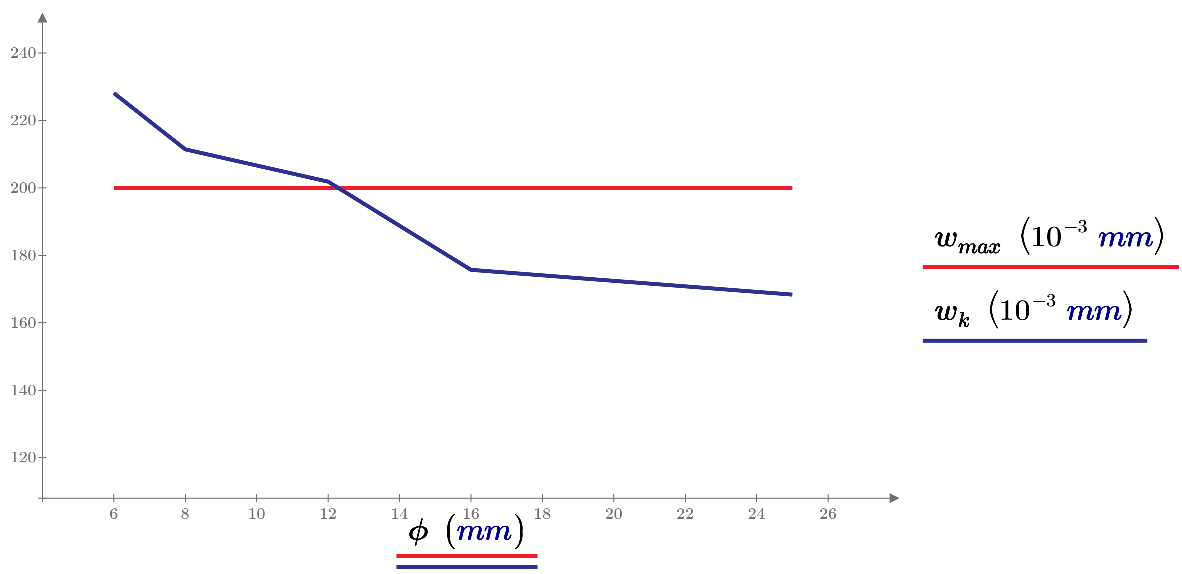
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t,\phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t,\phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t,\phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t,\phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t,\phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$$\varepsilon_{cs}(t) = 4.26 \cdot 10^{-4}$$

$$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.26 \cdot 10^{-4}$$

$$F_{cs}(t,\phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$$

Cracking force

$$f_{ct} := f_{ctm} = 2.9\ MPa$$

Short term response

$$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$$

Long term response

$$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.26 \cdot 10^{-4}$$ full restraint

$$R_{tot} := 1$$

$$N(t,\phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t,\phi)\right)$$

$$\sigma_c(t,\phi) := \frac{N(t,\phi) + F_{cs}(t,\phi)}{A_{I.ef}(\phi)}$$

$$
\begin{aligned}
risk(t,\phi) := &\ \text{if } \sigma_c(t,\phi) < f_{ctk0.05.sus} \\
&\quad \text{"No risk of cracking"} \\
&\ \text{else if } f_{ctk0.05.sus} \le \sigma_c(t,\phi) < f_{ctm.sus} \\
&\quad \text{"Risk risk of cracking"} \\
&\ \text{else if } f_{ctm.sus} \le \sigma_c(t,\phi) \le f_{ctk0.95.sus} \\
&\quad \text{"High risk risk of cracking"} \\
&\ \text{else} \\
&\quad \text{"Very high risk risk of cracking"}
\end{aligned}
$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + \left(-\varepsilon_{cs}(t)\right) \cdot L$$

$$\begin{array}{l}
\sigma_{s.it}(t,\phi) := \text{for } i \in 1..100 \\
\quad n_{cr} \leftarrow i \\
\quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\
\quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\
\quad N \leftarrow \sigma_s \cdot A_s(\phi) \\
\quad \text{if } N \geq N_{cr.\infty}(\phi) \\
\quad\quad \text{continue} \\
\quad \text{else} \\
\quad\quad \text{return } \sigma_s \\
\quad\quad \text{break}
\end{array}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$n_{cr}(t,\phi) := \begin{array}{l} \text{for } i \in 1..100 \\ \quad \left\| \begin{array}{l} n_{cr} \leftarrow i \\ \text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\ \quad \| \sigma_s \leftarrow \sigma_s - 1\ kPa \\ N \leftarrow \sigma_s \cdot A_s(\phi) \\ \text{if } N \geq N_{cr.\infty}(\phi) \\ \quad \| \text{continue} \\ \text{else} \\ \quad \left\| \begin{array}{l} \text{return } n_{cr} \\ \text{break} \end{array} \right. \end{array} \right. \end{array}$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

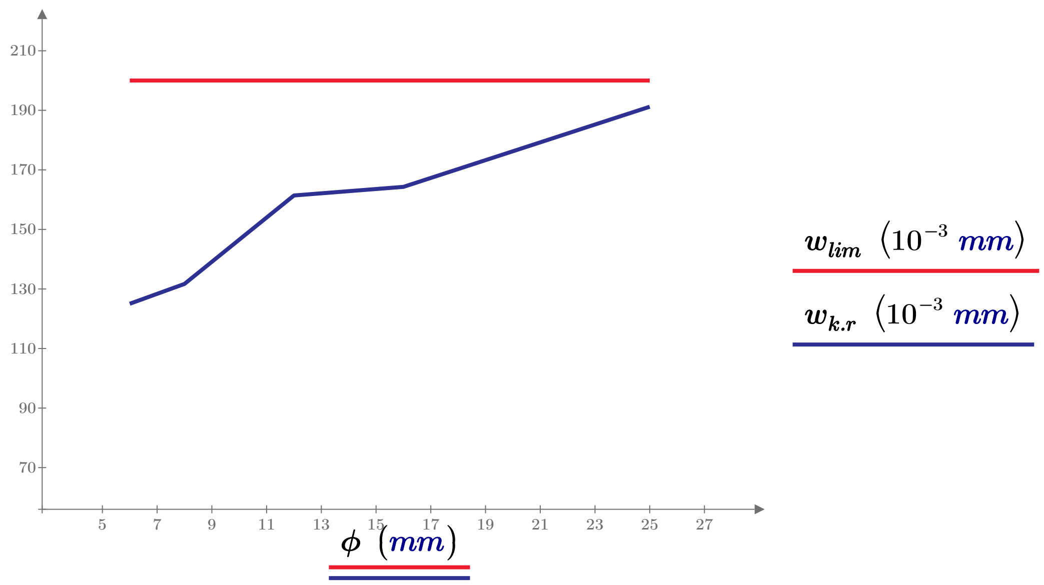
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

Plot of crack width and crack width limit



B.4 Concrete class C35/45

$Concrete_{class} := \text{“35/45”}$
$h := 200\ mm$

Dimensions and characteristics

Concrete

$Concrete$	f_{ck} (MPa)	f_{cm} (MPa)	$f_{ctk0.05}$ (MPa)	f_{ctm} (MPa)	$f_{ctk0.95}$ (MPa)	E_{cm} (GPa)
“20/25”	20	28	1.5	2.2	2.9	30
“25/30”	25	33	1.8	2.6	3.3	31
“30/37”	30	38	2.0	2.9	3.8	33
“35/45”	35	43	2.2	3.2	4.2	34

$Concrete_{class} = \text{“35/45”}$

$\alpha_{sus} := 0.6$

$f_{ctk0.05.sus} := \alpha_{sus} \cdot f_{ctk0.05} = 1.32\ MPa$

$f_{ctm.sus} := \alpha_{sus} \cdot f_{ctm} = 1.92\ MPa$

$f_{ctk0.95.sus} := \alpha_{sus} \cdot f_{ctk0.95} = 2.52\ MPa$

$f_{cmo} := 10\ MPa$

$\varepsilon_{cu} := 3.5 \cdot 10^{-3}$

$w_{max} := 0.2\ mm$

$Cement_{class} := \text{“N”}$

Dimensions

$h = 200\ mm$ thickness
$L := 10\ m$ length of slab
$b := 10\ m$ width of slab

$A_c := b \cdot h = 2\ m^2$ area of concrete slab, disregarding reinforcing area

Reinforcing steel B500B

$E_s := 200\ GPa$

$f_{yk} := 500\ MPa$

$A_{s'}(\phi) := \frac{\pi}{4} \cdot \phi^2$ area of one reinforcement bar

Prerequisites

$c_{min.dur} := 20\ mm$ (EC2 - Table 4:3N- 4:4N) Member with slab geometry - Structural class S4 is reduced to S3

$c_{min}(\phi) := \max(\phi, c_{min.dur}, 10\ mm)$ minimum concrete cover

$c(\phi) := c_{min}(\phi) + 10\ mm$ concrete cover

$d(\phi) := h - c(\phi) - \frac{\phi}{2}$ effective height

Environmental conditions

$RH := 40\%$ relative humidity of the surroundings

$RH_0 := 100\%$

$t := 50\ yr$ age of concrete at the moment considered [days]

$t_s := 7\ day$ age of concrete at the beginning of drying shrinkage [days]

$\infty := 1 \cdot 10^{10}\ yr$

Creep

Notional size of one unit lenght of the slab

$l_{h0} := L = 10\ m$

$u_{h0} := L + 2\ h = 10.4\ m$

$$h_0 := \frac{2 \cdot l_{h0} \cdot h}{u_{h0}} = 0.385\ m$$

Notional creep coefficient

$$\varphi_{RH} := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| 1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \right. \\ \text{else} \\ \quad \left\| \left(1 + \dfrac{1 - RH}{0.1 \cdot \sqrt[3]{\dfrac{h_0 \cdot 1000}{m}}} \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.7} \right) \cdot \left(\dfrac{35\ MPa}{f_{cm}} \right)^{0.2} \right. \end{array} = 1.645$$

$$\beta_{f.cm} := \frac{16.8}{\sqrt{\dfrac{f_{cm}}{MPa}}} = 2.562$$

$t_{0.T} := t_s = 7\ day$ (not adjusted due to temperature)

$$\alpha_{t0} := \begin{array}{l} \text{if } Cement_{class} = \text{“S”} \\ \quad \| -1 \\ \text{else if } Cement_{class} = \text{“N”} \\ \quad \| 0 \\ \text{else if } Cement_{class} = \text{“R”} \\ \quad \| 1 \\ \text{else} \\ \quad \| \text{“invalid cement class”} \end{array} = 0$$

$$t_0 := t_{0.T} \cdot \left(\frac{9}{2 + \left(\dfrac{t_{0.T}}{day} \right)^{1.2}} + 1 \right)^{\alpha_{t0}} = 7\ day$$

$t_0 := \max\left(0.5\ day, t_0\right) = 7\ day$

$$\beta_{t0} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}} = 0.635$$

$$\alpha_3 := \left(\frac{35\ MPa}{f_{cm}}\right)^{0.5} = 0.902$$

$$\beta_H := \begin{array}{l} \text{if } f_{cm} \le 35\ MPa \\ \quad \left\| \min\left(1500, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250\right) \right. \\ \text{else} \\ \quad \left\| \min\left(1500 \cdot \alpha_3, 1.5 \cdot \left(1 + (0.012 \cdot RH)^{18}\right) \cdot h_0 \cdot \frac{1000}{m} + 250 \cdot \alpha_3\right) \right. \end{array} = 802.472$$

$$\beta_c(t) := \left(\frac{(t - t_0)\ day^{-1}}{\beta_H + (t - t_0)\ day^{-1}}\right)^{0.3}$$

$$\varphi_0 := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t0} = 2.675$$

$$\varphi(t) := \beta_c(t) \cdot \varphi_0$$

$$\varphi_{\infty.t0} := \varphi(\infty) = 2.675$$

$$\alpha := \frac{E_s}{E_{cm}} = 5.882$$

$$\alpha_{ef} := \frac{E_s}{E_{cm}} \cdot \left(1 + \varphi_{\infty.t0}\right) = 21.617$$

$$E_{c.ef} := \frac{E_{cm}}{1 + \varphi_{\infty.t0}} = 9.252\ GPa$$

Shrinkage

Drying shrinkage

$h_0 = 384.615\ mm$

$$k_h := \text{linterp}\left(\begin{bmatrix} 100\ mm \\ 200\ mm \\ 300\ mm \\ 500\ mm \end{bmatrix}, \begin{bmatrix} 1 \\ 0.85 \\ 0.75 \\ 0.7 \end{bmatrix}, h_0\right) = 0.729$$

$$\beta_{ds}(t) := \frac{(t - t_s)\ day^{-1}}{(t - t_s)\ day^{-1} + 0.04 \cdot \sqrt{\left(\frac{h_0}{mm}\right)^3}}$$

$\beta_{ds}(t) = 0.984$
$\beta_{ds}(\infty) = 1$

$$\alpha_{ds1} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 3 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 4 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 6 \end{array} = 4$$

$$\alpha_{ds2} := \begin{array}{l} \text{if } Cement_{class} = \text{"S"} \\ \quad \| \ 0.13 \\ \text{else if } Cement_{class} = \text{"N"} \\ \quad \| \ 0.12 \\ \text{else if } Cement_{class} = \text{"R"} \\ \quad \| \ 0.11 \end{array} = 0.12$$

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{RH_0}\right)^3\right) = 1.451$$

$$\varepsilon_{cd.0} := 0.85 \left((220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right) \cdot 10^{-6} \cdot \beta_{RH} = 4.858 \cdot 10^{-4}$$

$$\varepsilon_{cd}(t) := \beta_{ds}(t) \cdot k_h \cdot \varepsilon_{cd.0}$$

$t = 50\ yr$

$\varepsilon_{cd}(t) = 3.483 \cdot 10^{-4}$

$\varepsilon_{cd}(\infty) = 3.541 \cdot 10^{-4}$

Autogenous shrinkage

$$\beta_{as}(t) := 1 - \exp\left(-0.2\left(\frac{t}{day}\right)^{0.5}\right)$$

$$\beta_{as}(t) = 1$$

$$\varepsilon_{ca.\infty} := 2.5 \cdot \left(\frac{f_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 6.25 \cdot 10^{-5}$$

$$\varepsilon_{ca}(t) := \beta_{as}(t) \cdot \varepsilon_{ca.\infty}$$

$$\varepsilon_{ca}(t) = 6.25 \cdot 10^{-5}$$

Total shrinkage

$$\varepsilon_{cs}(t) := \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$t = 50\ yr$$

$$\varepsilon_{cs}(t) = 4.108 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 4.166 \cdot 10^{-4}$$

Crack width evaluation according to Eurocode 2

Minimum reinforcement area

$$k := \text{if } h \le 300\ mm \;\; \begin{array}{l} \| \; 1 \\ \text{else} \\ \| \; 0.65 \end{array} \quad = 1$$

$k_c := 1$ pure tension is assumed

$A_{ct} := A_c = 2\ m^2$

$$\sigma_s(\phi) := \begin{array}{l} \text{if } \phi = 6\ mm \\ \quad \| \; 320 \\ \text{else if } \phi = 8\ mm \\ \quad \| \; 280 \\ \text{else if } \phi = 12\ mm \\ \quad \| \; 240 \\ \text{else if } \phi = 16\ mm \\ \quad \| \; 200 \\ \text{else if } \phi = 25\ mm \\ \quad \| \; 160 \\ \text{else} \\ \quad \| \; \text{“Invalid bar diameter”} \end{array} \quad MPa$$

$$A_{s.min}(\phi) := k_c \cdot k \cdot A_{ct} \cdot \frac{f_{ctm}}{\sigma_s(\phi)}$$

$n(\phi) := \text{ceil}\left(\frac{A_{s.min}(\phi)}{A_{s'}(\phi)}\right)$ number of reinforcement bars

$A_s(\phi) := n(\phi) \cdot A_{s'}(\phi)$ area of reinforcement

$A_s(\phi) := 1.2\ A_s(\phi)$ increasing the reinforcement area to meet requirement on crack width

$A_{net}(\phi) := A_c - A_s(\phi)$

$A_I(\phi) := A_c + A_s(\phi) \cdot (\alpha - 1)$ short time response

$A_{I.ef}(\phi) := A_c + A_s(\phi) \cdot (\alpha_{ef} - 1)$ long time response

Maximum crack spacing at the surface

$k_1 := 0.8$

$k_2 := 1$ pure tension

$k_3 := 3.4$

$k_4 := 0.425$

$$x := \frac{h}{2} = 100\ mm$$

$$h_{c.ef}(\phi) := min\left(2.5\ (h - d(\phi)), \frac{h-x}{3}, \frac{h}{2}\right)$$

$$A_{c.ef}(\phi) := 2\ h_{c.ef}(\phi) \cdot b$$

$$\rho_{p.ef}(\phi) := \frac{A_s(\phi)}{A_{c.ef}(\phi)}$$

Crack spacing

$$s_{r.max}(\phi) := k_3 \cdot c(\phi) + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p.ef}(\phi)}$$

Crack width

$$k_t(t) := \text{if } t < 1\ yr \; \| \; 0.6 \; \text{else} \; \| \; 0.4$$

(short term loading)

(long term loading)

$$\Delta\varepsilon(t,\phi) := \max\left(\frac{\sigma_s(\phi) - k_t(t) \cdot \frac{f_{ctm}}{\rho_{p.ef}(\phi)}\ (1 + \alpha \cdot \rho_{p.ef}(\phi))}{E_s}, 0.6 \cdot \frac{\sigma_s(\phi)}{E_s}\right)$$

$$w_k(t,\phi) := s_{r.max}(\phi) \cdot \Delta\varepsilon(t,\phi)$$

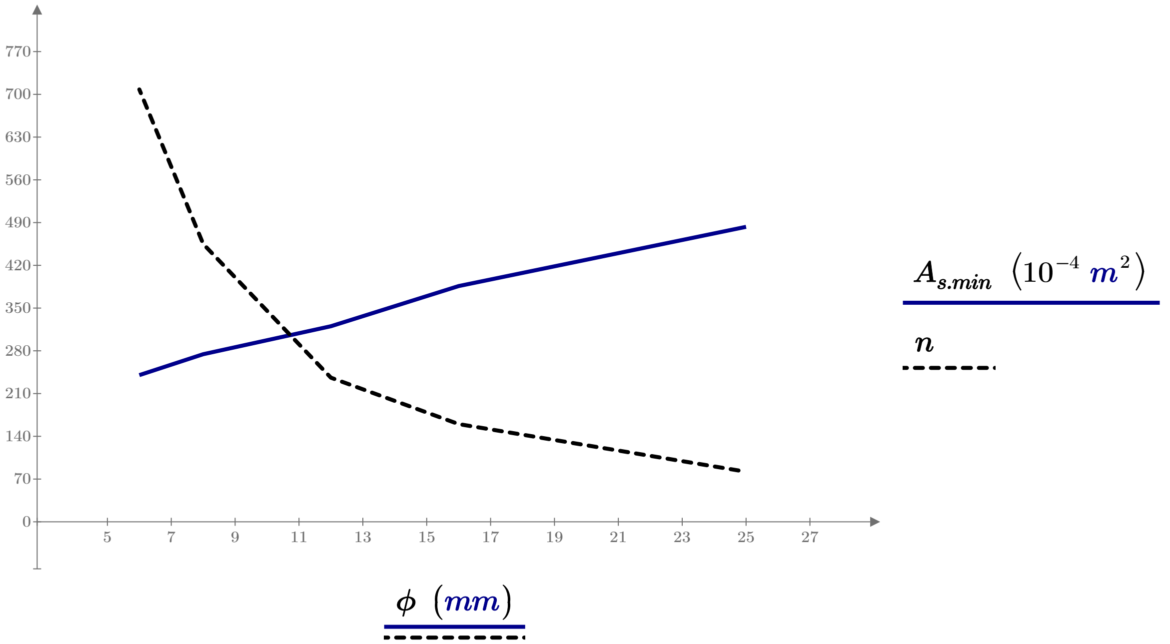
$$w_{max} = 0.2\ mm$$

if $w_k(\phi) < w_{max}$
  ‖ "OK!"
else
  ‖ "Not OK! – modify reinforcement amount"

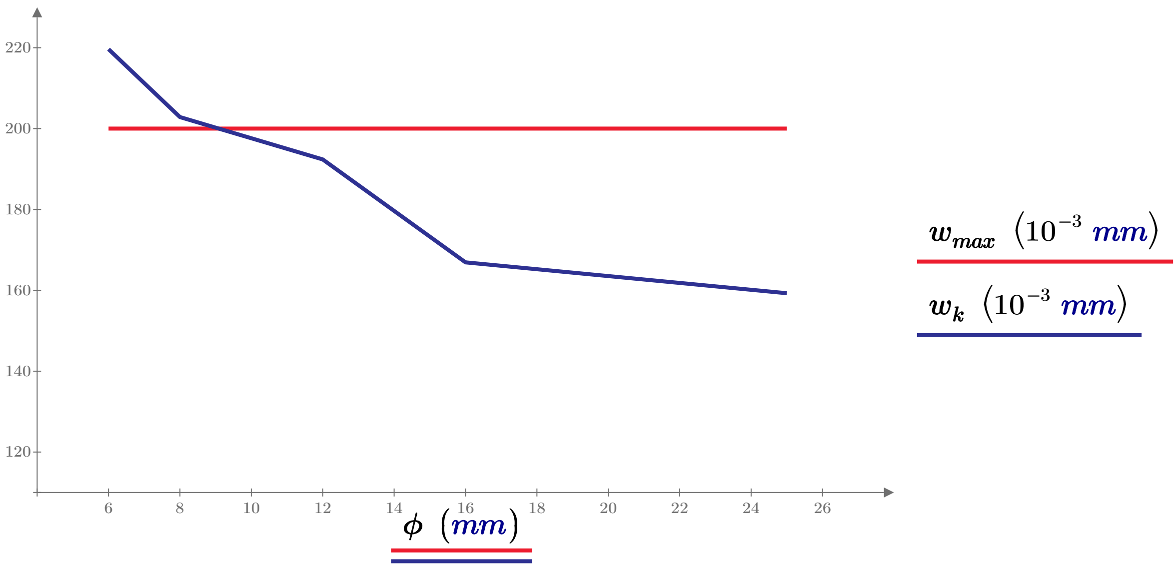
$$t := \infty$$

ϕ (mm)	$A_{s.min}$	n	$s_{r.max}$	w_k	w_{max}
6	$A_s(\phi_1)$	$n(\phi_1)$	$s_{r.max}(\phi_1)$	$w_k(t,\phi_1)$	w_{max}
8	$A_s(\phi_2)$	$n(\phi_2)$	$s_{r.max}(\phi_2)$	$w_k(t,\phi_2)$	w_{max}
12	$A_s(\phi_3)$	$n(\phi_3)$	$s_{r.max}(\phi_3)$	$w_k(t,\phi_3)$	w_{max}
16	$A_s(\phi_4)$	$n(\phi_4)$	$s_{r.max}(\phi_4)$	$w_k(t,\phi_4)$	w_{max}
25	$A_s(\phi_5)$	$n(\phi_5)$	$s_{r.max}(\phi_5)$	$w_k(t,\phi_5)$	w_{max}

Plot of minimum amount of reinforcement, considering the dimension of the reinforcing bar. (Total steel area and number of bars)



Plot of characteristic crack width and maximum allowed crack width



Crack width evaluation according to Engström (2014)

Shrinkage force

$\varepsilon_{cs}(t) = 4.166 \cdot 10^{-4}$

$\varepsilon_{cs.\infty} := \varepsilon_{cs}(\infty) = 4.166 \cdot 10^{-4}$

$F_{cs}(t,\phi) := E_s \cdot \varepsilon_{cs}(t) \cdot A_s(\phi)$

Cracking force

$f_{ct} := f_{ctm} = 3.2\ MPa$

Short term response

$N_{cr}(\phi) := f_{ct} \cdot A_I(\phi)$

Long term response

$N_{cr.\infty}(\phi) := f_{ctm.sus} \cdot A_{I.ef}(\phi)$

Restraint

Combination of internal and external restraint - fully fixed along the bottom edge

$\varepsilon_c := -\varepsilon_{cs}(\infty) = -4.166 \cdot 10^{-4}$ full restraint

$R_{tot} := 1$

$N(t,\phi) := R_{tot} \cdot \left(\varepsilon_{cs}(t) \cdot E_{c.ef} \cdot A_{I.ef}(\phi) - F_{cs}(t,\phi)\right)$

$$\sigma_c(t,\phi) := \frac{N(t,\phi) + F_{cs}(t,\phi)}{A_{I.ef}(\phi)}$$

$$risk(t,\phi) := \begin{array}{l} \text{if } \sigma_c(t,\phi) < f_{ctk0.05.sus} \\ \quad \text{"No risk of cracking"} \\ \text{else if } f_{ctk0.05.sus} \le \sigma_c(t,\phi) < f_{ctm.sus} \\ \quad \text{"Risk risk of cracking"} \\ \text{else if } f_{ctm.sus} \le \sigma_c(t,\phi) \le f_{ctk0.95.sus} \\ \quad \text{"High risk risk of cracking"} \\ \text{else} \\ \quad \text{"Very high risk risk of cracking"} \end{array}$$

Mean crack width in cracked section

$$w_m(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$w_{net}(\phi,\sigma_s) := 0.420\ mm \cdot \left(\frac{\frac{\phi}{mm} \cdot {\sigma_s}^2}{0.22\ f_{cm} \cdot E_s \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} \right)^{0.826}$$

$$w_{m.sus}(\phi,\sigma_s) := 1.24 \cdot w_{net}(\phi,\sigma_s) + \frac{\sigma_s}{E_s} \cdot 4\ \phi$$

$$l_t(\phi,\sigma_s) := 0.443 \cdot \frac{\phi \cdot \sigma_s}{0.22\ f_{cm} \cdot \left(\frac{w_{net}(\phi,\sigma_s)}{mm}\right)^{0.21} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \frac{A_s(\phi)}{A_{c.ef}(\phi)}\right)} + 2\ \phi$$

$$l_{t.sus}(\phi,\sigma_s) := 1.3\ l_t(\phi,\sigma_s)$$

Response during cracking - iteration

$\sigma_s := f_{yk} = 500\ MPa$ start value for iteration process

$$iteration(t,\phi,n_{cr},\sigma_s) := \frac{\sigma_s \cdot A_s(\phi) + F_{cs}(t,\phi)}{E_{c.ef} \cdot A_{I.ef}(\phi)} \cdot L + n_{cr} \cdot w_{m.sus}(\phi,\sigma_s) + (-\varepsilon_{cs}(t)) \cdot L$$

$$\begin{array}{l} \sigma_{s.it}(t,\phi) := \text{for } i \in 1..100 \\ \quad n_{cr} \leftarrow i \\ \quad \text{while } iteration(t,\phi,n_{cr},\sigma_s) > 0\ mm \\ \quad\quad \sigma_s \leftarrow \sigma_s - 1\ kPa \\ \quad N \leftarrow \sigma_s \cdot A_s(\phi) \\ \quad \text{if } N \geq N_{cr.\infty}(\phi) \\ \quad\quad \text{continue} \\ \quad \text{else} \\ \quad\quad \text{return } \sigma_s \\ \quad\quad \text{break} \end{array}$$

iterate to find σ_s where $N < N_{cr.\infty}$
- long term response due to shrinkage

$$
n_{cr}(t,\phi) := \begin{array}{l}
\text{for } i \in 1..100 \\
\quad \left\| \begin{array}{l}
n_{cr} \leftarrow i \\
\text{while } iteration\left(t,\phi,n_{cr},\sigma_s\right) > 0\ mm \\
\quad \left\| \ \sigma_s \leftarrow \sigma_s - 1\ kPa \right. \\
N \leftarrow \sigma_s \cdot A_s(\phi) \\
\text{if } N \geq N_{cr.\infty}(\phi) \\
\quad \left\| \ \text{continue} \right. \\
\text{else} \\
\quad \left\| \begin{array}{l} \text{return } n_{cr} \\ \text{break} \end{array} \right.
\end{array} \right.
\end{array}
$$

iterate to find number of cracks, n_{cr} when stabilised cracking is reached

$$N(t,\phi) := \sigma_{s.it}(t,\phi) \cdot A_s(\phi)$$

$$w_{lim} := w_{max}$$

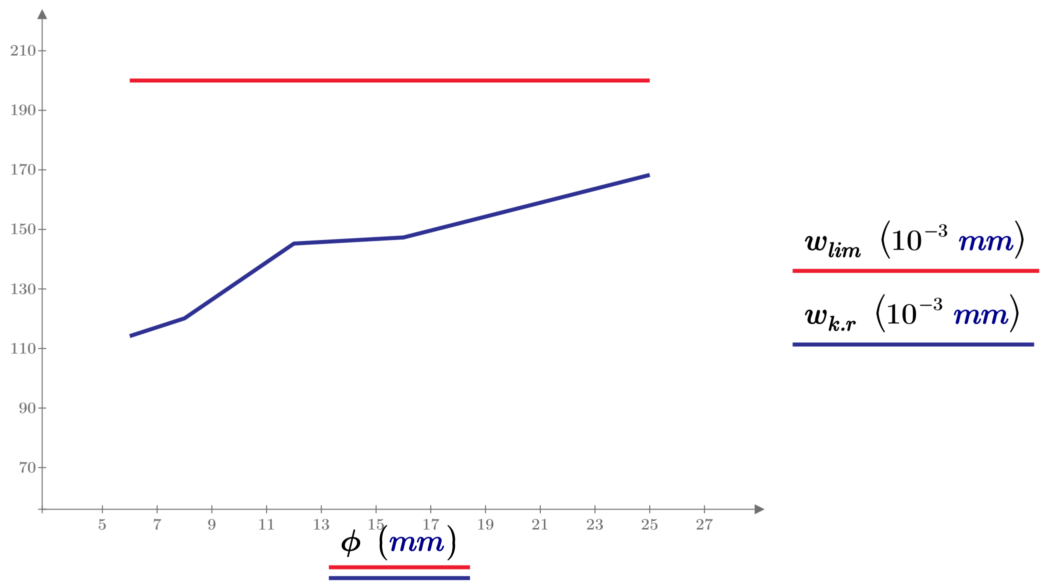
$$w_{m.all} := \frac{w_{lim}}{1.3}$$

$$t := \infty$$

ϕ (mm)	$\sigma_{s.it}$	n_{cr}	N	F_{cs}	$N_{cr.\infty}$
6	$\sigma_{s.it}\left(t,\phi_1\right)$	$n_{cr}\left(t,\phi_1\right)$	$N\left(t,\phi_1\right)$	$F_{cs}\left(t,\phi_1\right)$	$N_{cr.\infty}\left(\phi_1\right)$
8	$\sigma_{s.it}\left(t,\phi_2\right)$	$n_{cr}\left(t,\phi_2\right)$	$N\left(t,\phi_2\right)$	$F_{cs}\left(t,\phi_2\right)$	$N_{cr.\infty}\left(\phi_2\right)$
12	$\sigma_{s.it}\left(t,\phi_3\right)$	$n_{cr}\left(t,\phi_3\right)$	$N\left(t,\phi_3\right)$	$F_{cs}\left(t,\phi_3\right)$	$N_{cr.\infty}\left(\phi_3\right)$
16	$\sigma_{s.it}\left(t,\phi_4\right)$	$n_{cr}\left(t,\phi_4\right)$	$N\left(t,\phi_4\right)$	$F_{cs}\left(t,\phi_4\right)$	$N_{cr.\infty}\left(\phi_4\right)$
25	$\sigma_{s.it}\left(t,\phi_5\right)$	$n_{cr}\left(t,\phi_5\right)$	$N\left(t,\phi_5\right)$	$F_{cs}\left(t,\phi_5\right)$	$N_{cr.\infty}\left(\phi_5\right)$

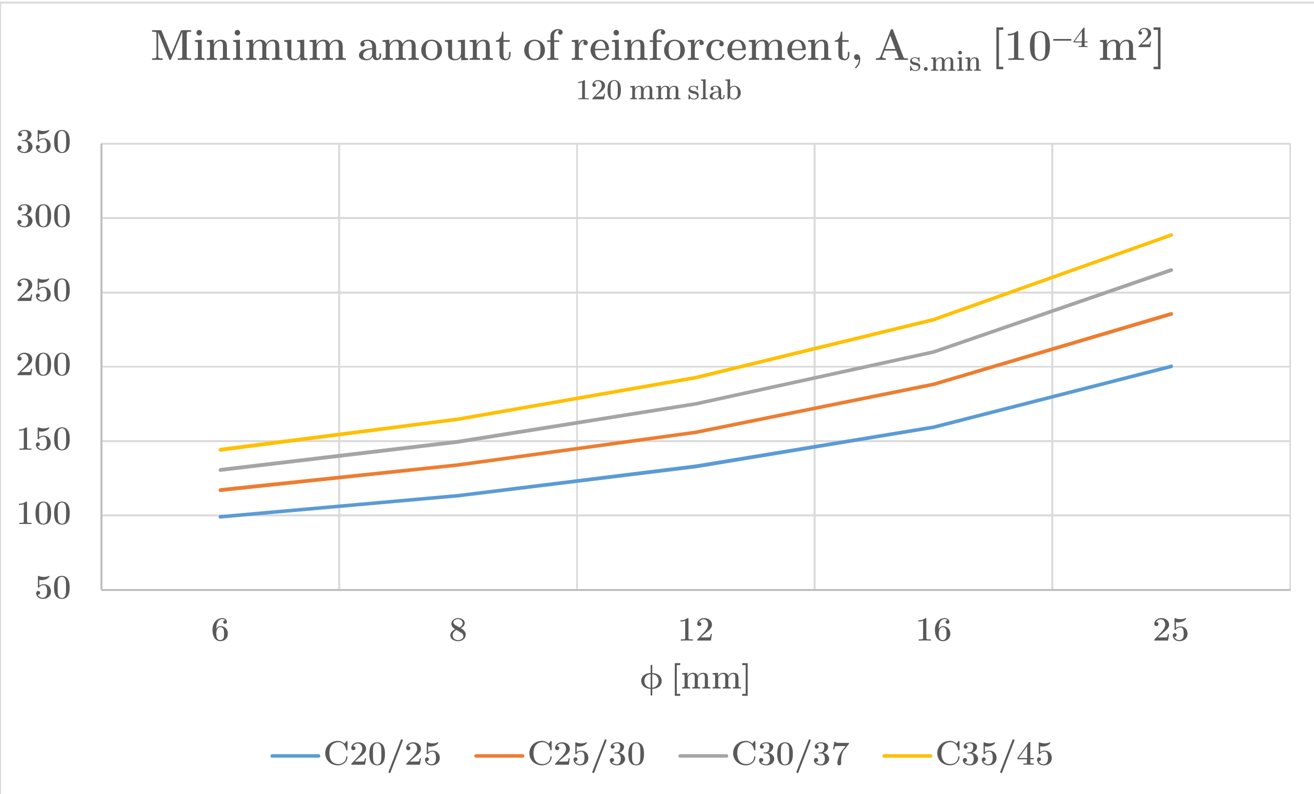
ϕ (mm)	w_m	$w_{k.r}$	l_t	$l_{t.sus}$
6	$w_m\left(\phi_1,\sigma_{s.it_1}\right)$	$1.3\ w_{m_1}$	$l_t\left(\phi_1,\sigma_{s.it_1}\right)$	$l_{t.sus}\left(\phi_1,\sigma_{s.it_1}\right)$
8	$w_m\left(\phi_2,\sigma_{s.it_2}\right)$	$1.3\ w_{m_2}$	$l_t\left(\phi_2,\sigma_{s.it_2}\right)$	$l_{t.sus}\left(\phi_2,\sigma_{s.it_2}\right)$
12	$w_m\left(\phi_3,\sigma_{s.it_3}\right)$	$1.3\ w_{m_3}$	$l_t\left(\phi_3,\sigma_{s.it_3}\right)$	$l_{t.sus}\left(\phi_3,\sigma_{s.it_3}\right)$
16	$w_m\left(\phi_4,\sigma_{s.it_4}\right)$	$1.3\ w_{m_4}$	$l_t\left(\phi_4,\sigma_{s.it_4}\right)$	$l_{t.sus}\left(\phi_4,\sigma_{s.it_4}\right)$
25	$w_m\left(\phi_5,\sigma_{s.it_5}\right)$	$1.3\ w_{m_5}$	$l_t\left(\phi_5,\sigma_{s.it_5}\right)$	$l_{t.sus}\left(\phi_5,\sigma_{s.it_5}\right)$

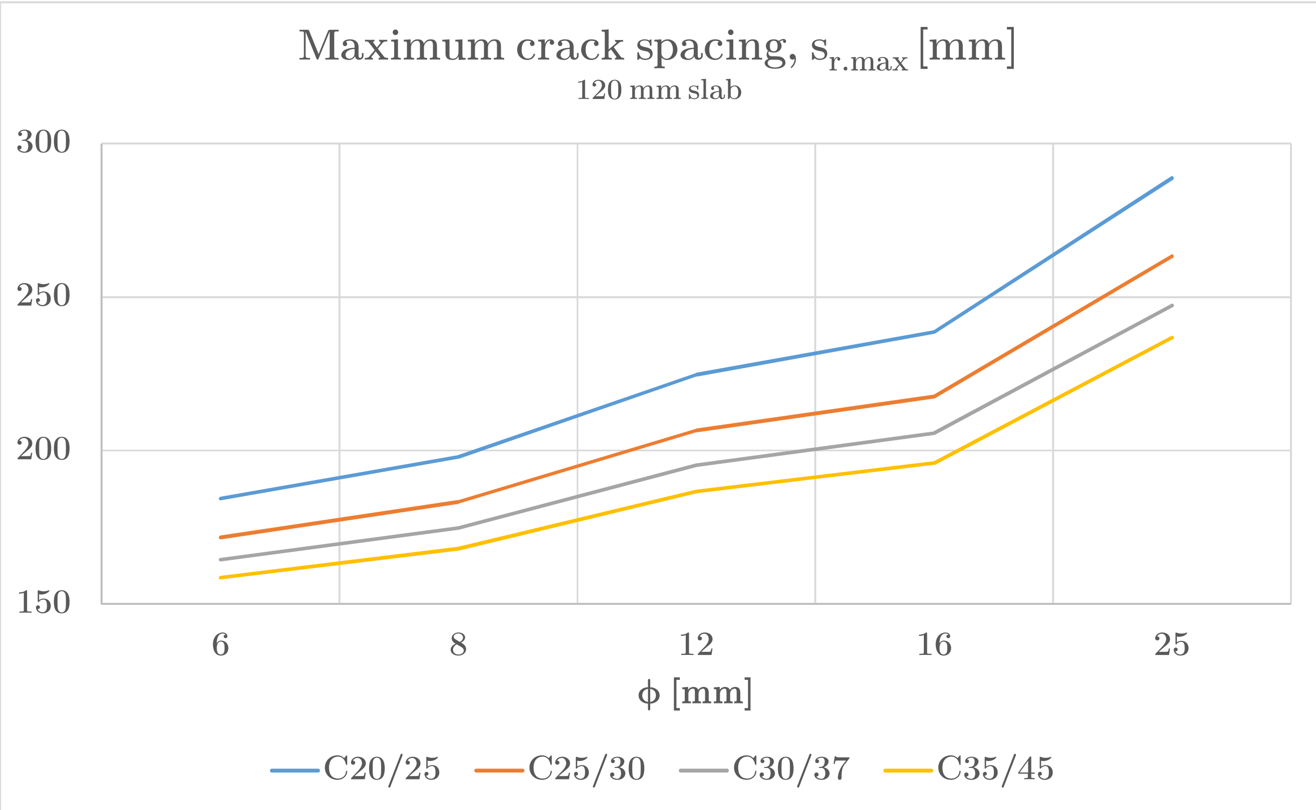
Plot of crack width and crack width limit



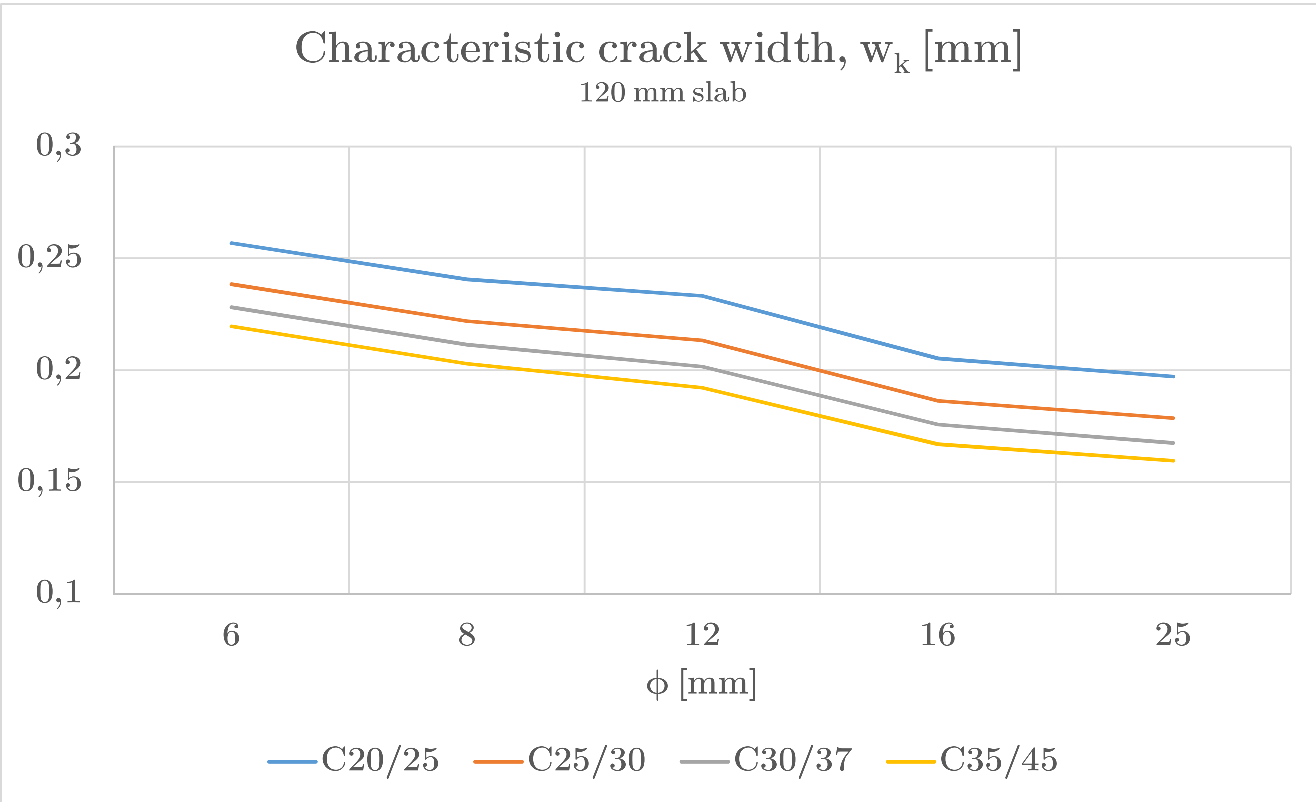
C Graphs - Summary results

C.1 Compiled results from calculations for a 120 mm slab

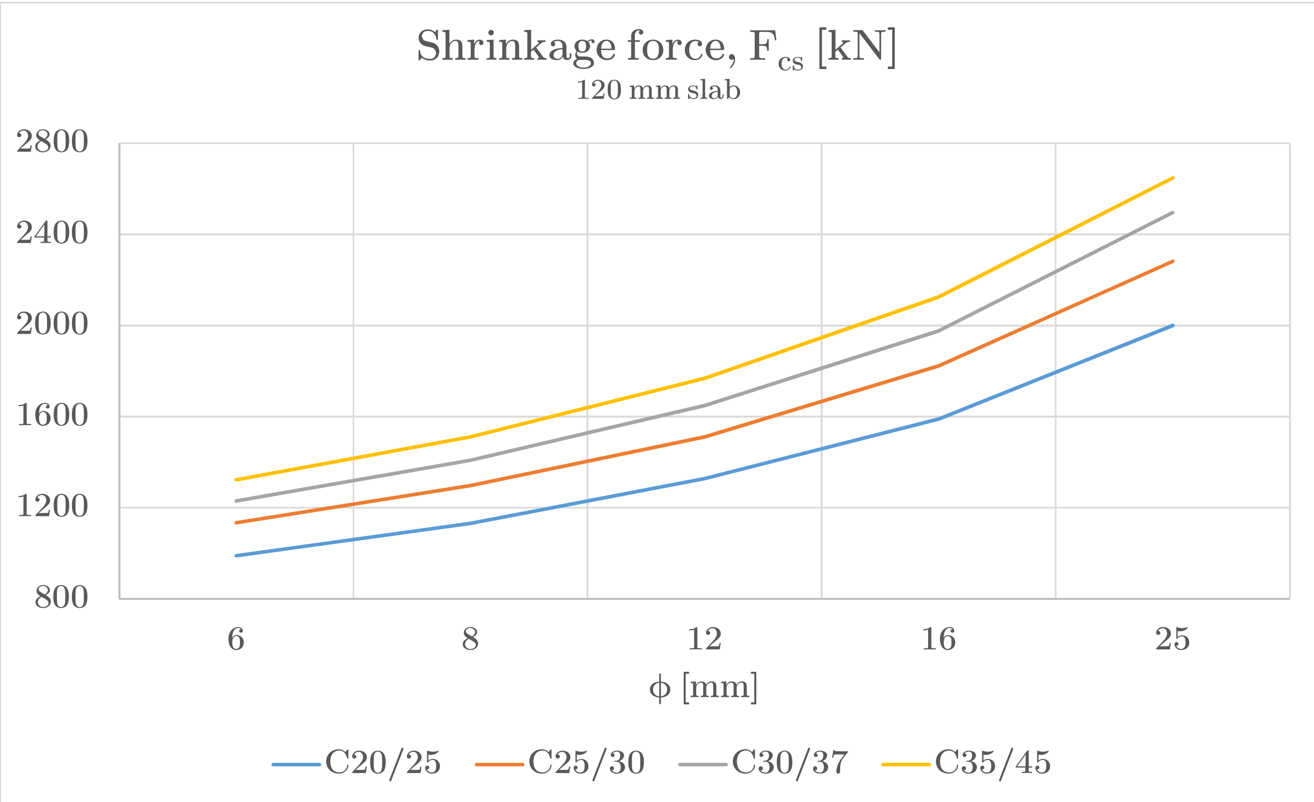

Minimum amount of reinforcement, $A_{s.min}$ [10^{-4} m^2]
120 mm slab
350
300
250
200
150
100
50
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



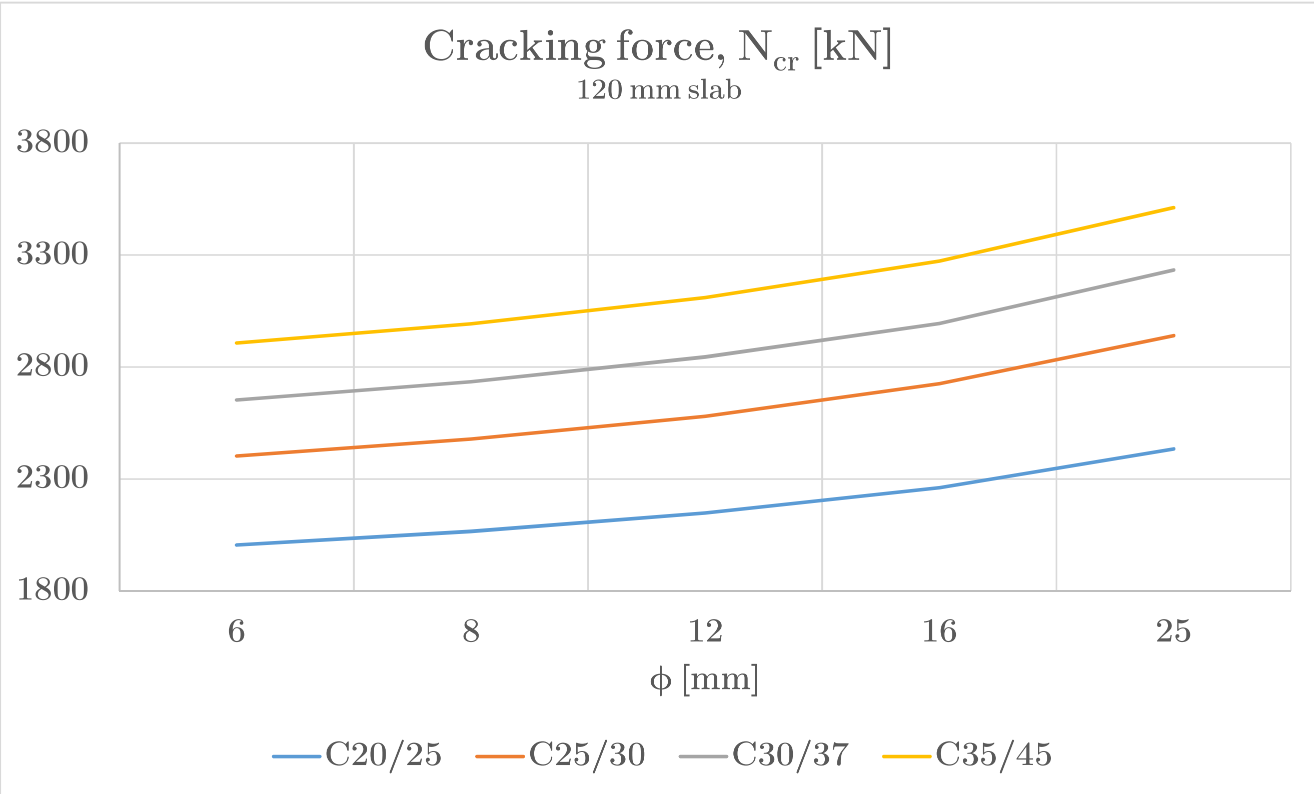
Maximum crack spacing, s r.max [mm]
120 mm slab
300
250
200
150
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



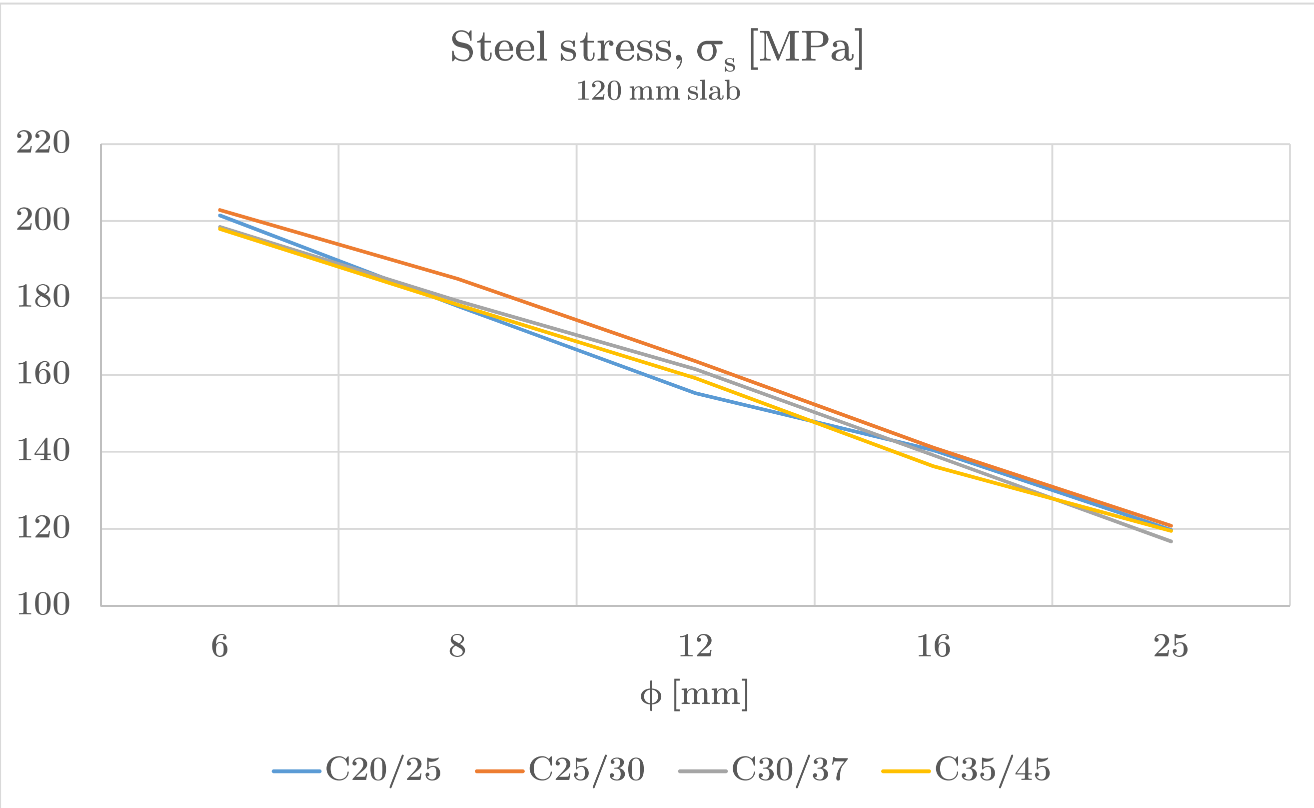
Characteristic crack width, w_k [mm]
120 mm slab
0,3
0,25
0,2
0,15
0,1
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



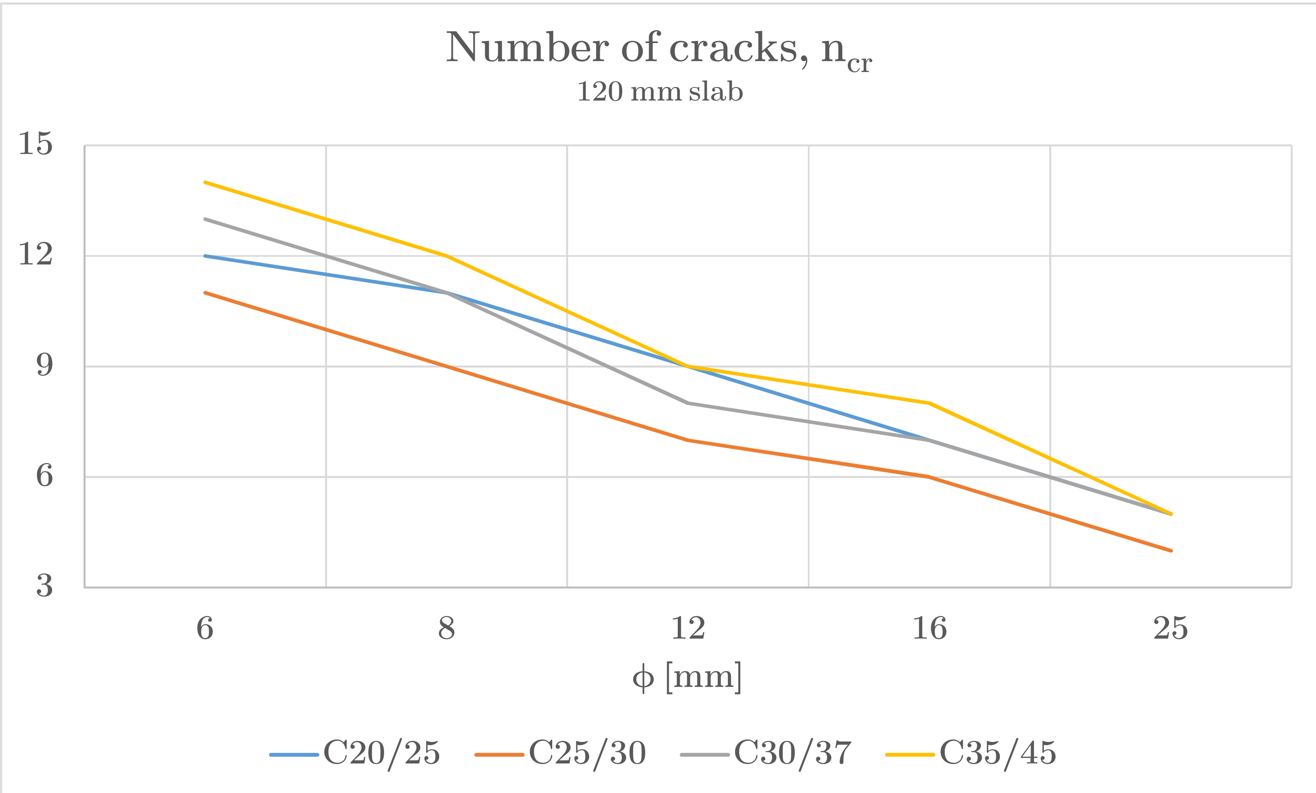
Shrinkage force, F_{cs} [kN]
120 mm slab
2800
2400
2000
1600
1200
800
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



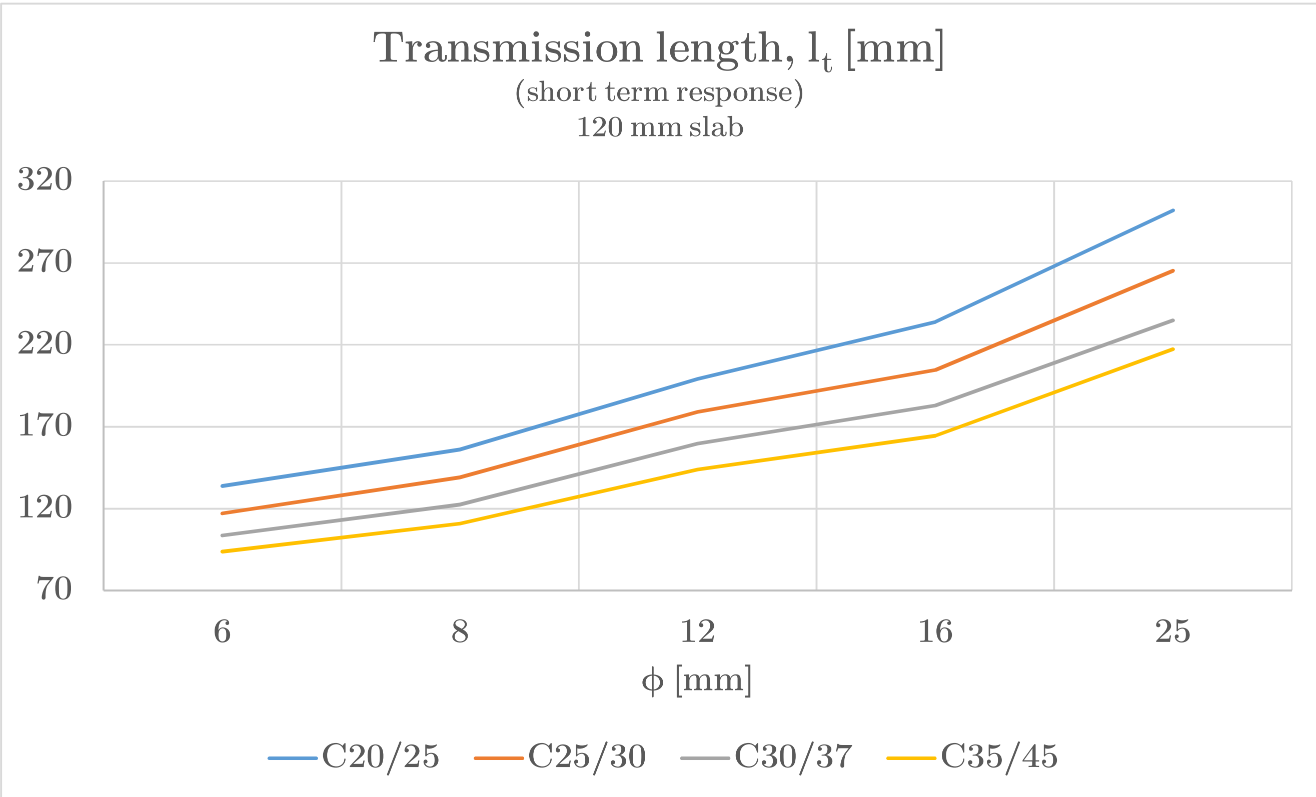
Cracking force, N_{cr} [kN]
120 mm slab
3800
3300
2800
2300
1800
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



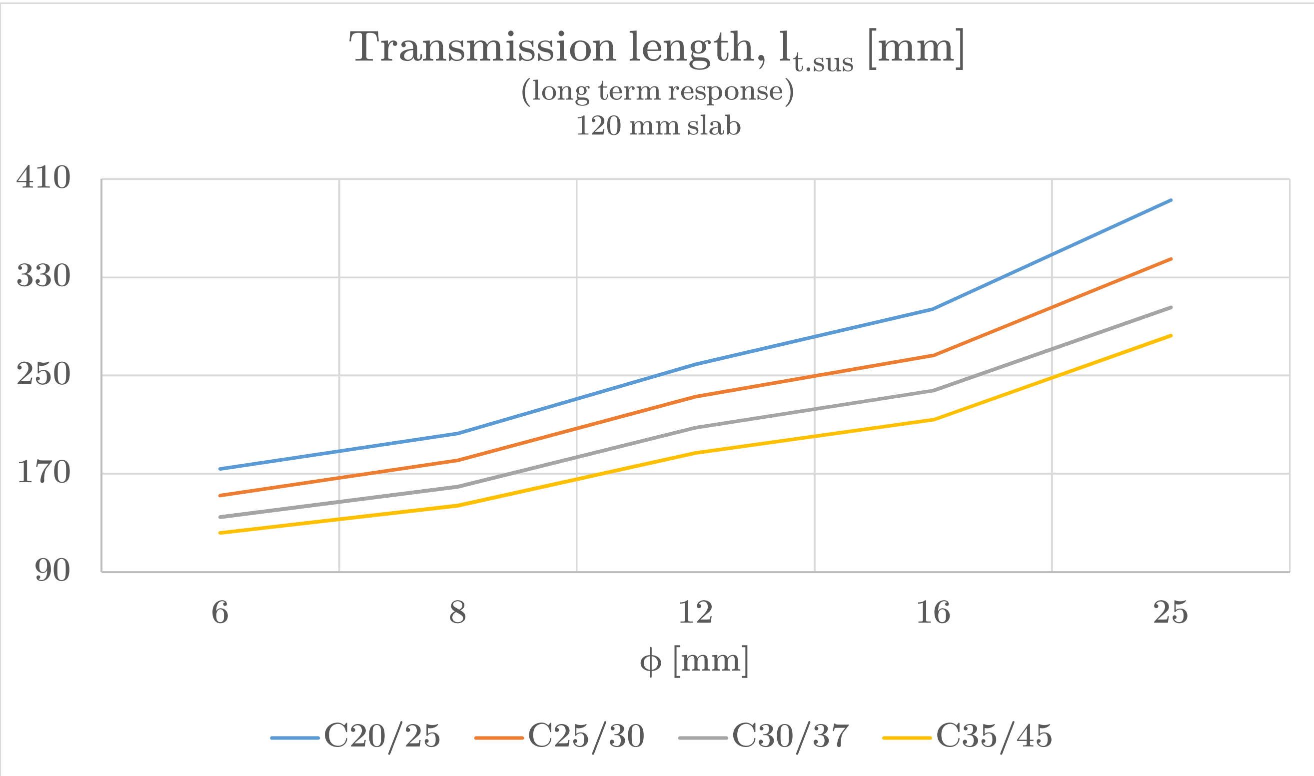
Steel stress, σs [MPa]
120 mm slab
220
200
180
160
140
120
100
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



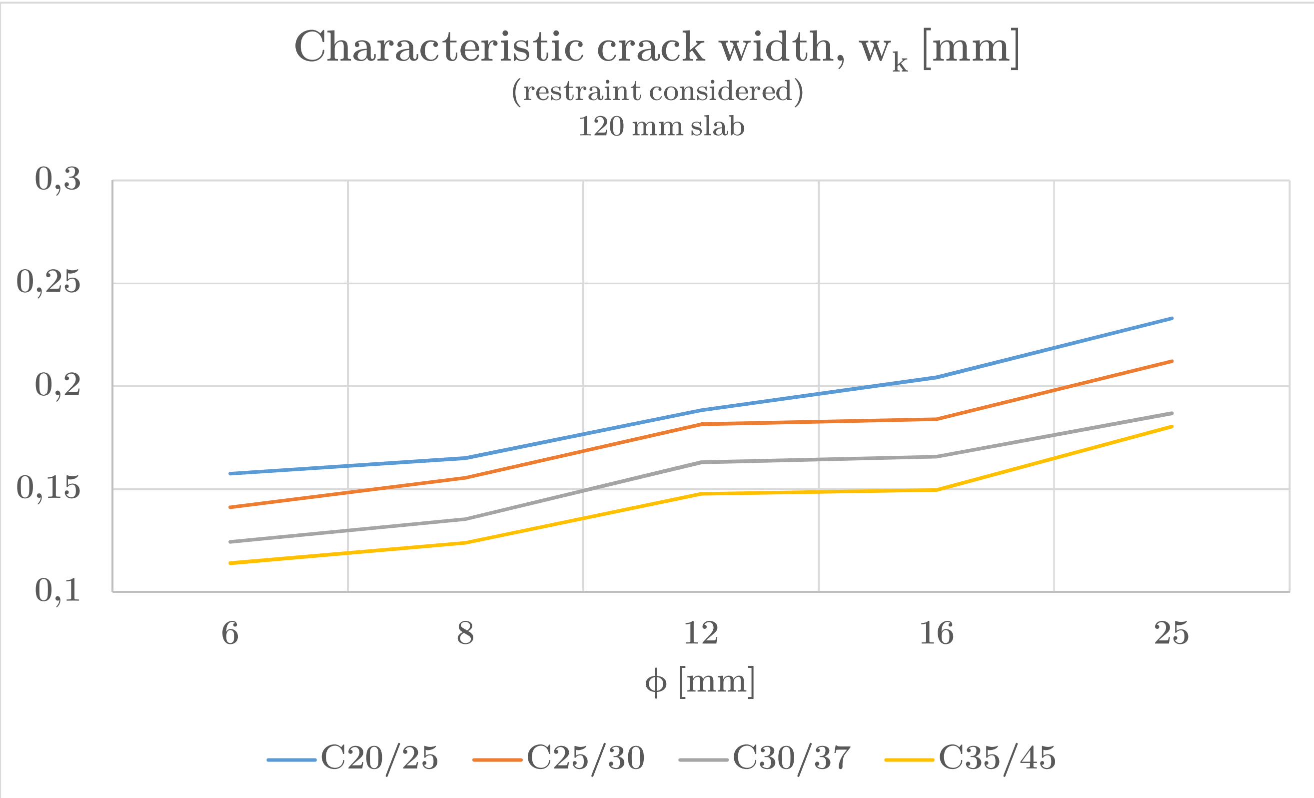
Number of cracks, n_{cr}
120 mm slab
15
12
9
6
3
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



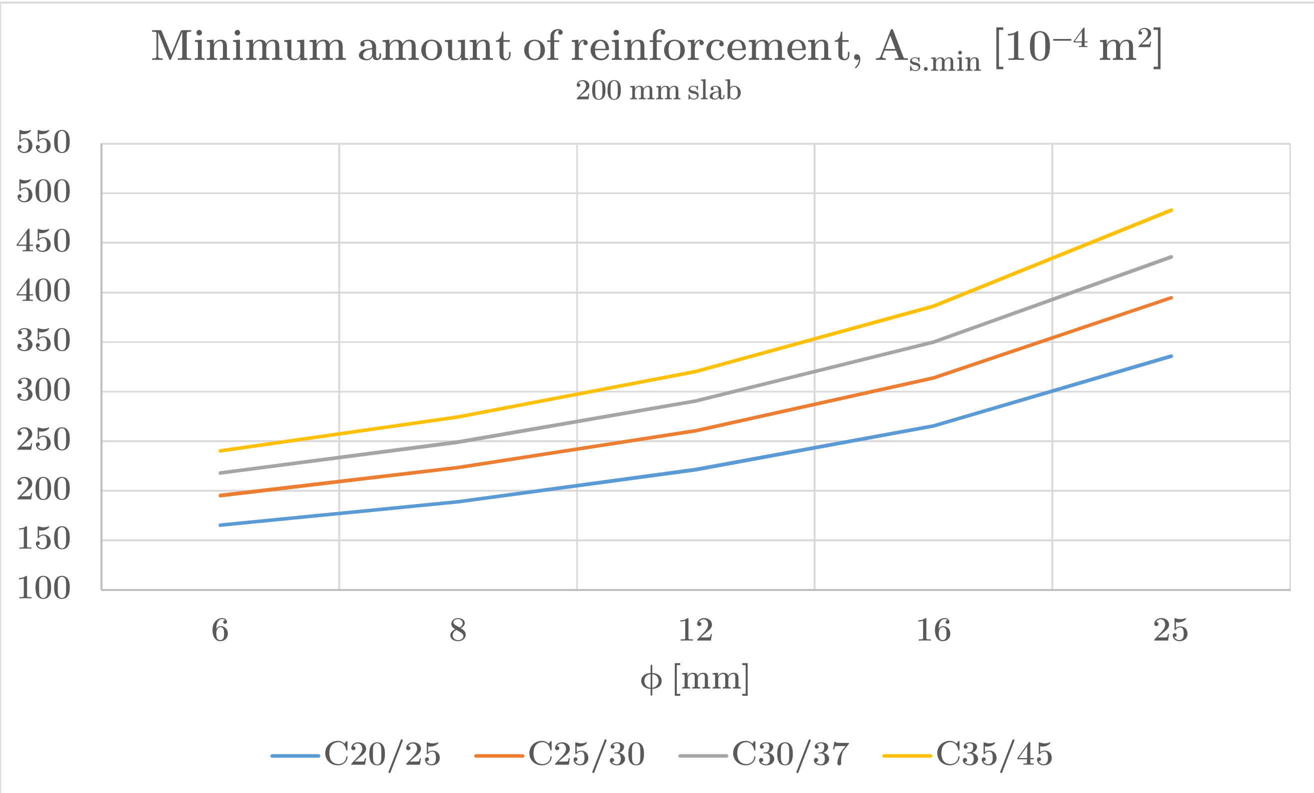
Transmission length, l_t [mm]
(short term response)
120 mm slab
320
270
220
170
120
70
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

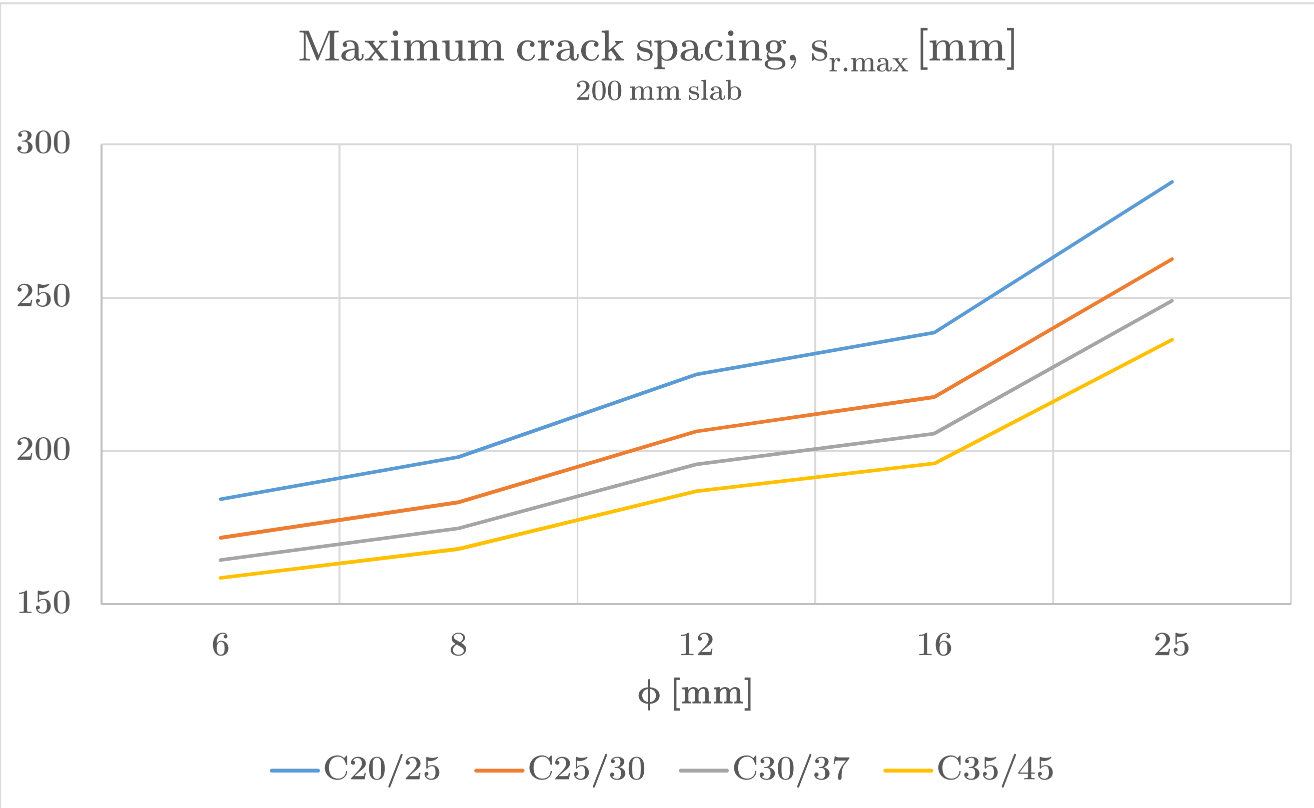


Transmission length, $l_{t.sus}$ [mm]
(long term response)
120 mm slab
410
330
250
170
90
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

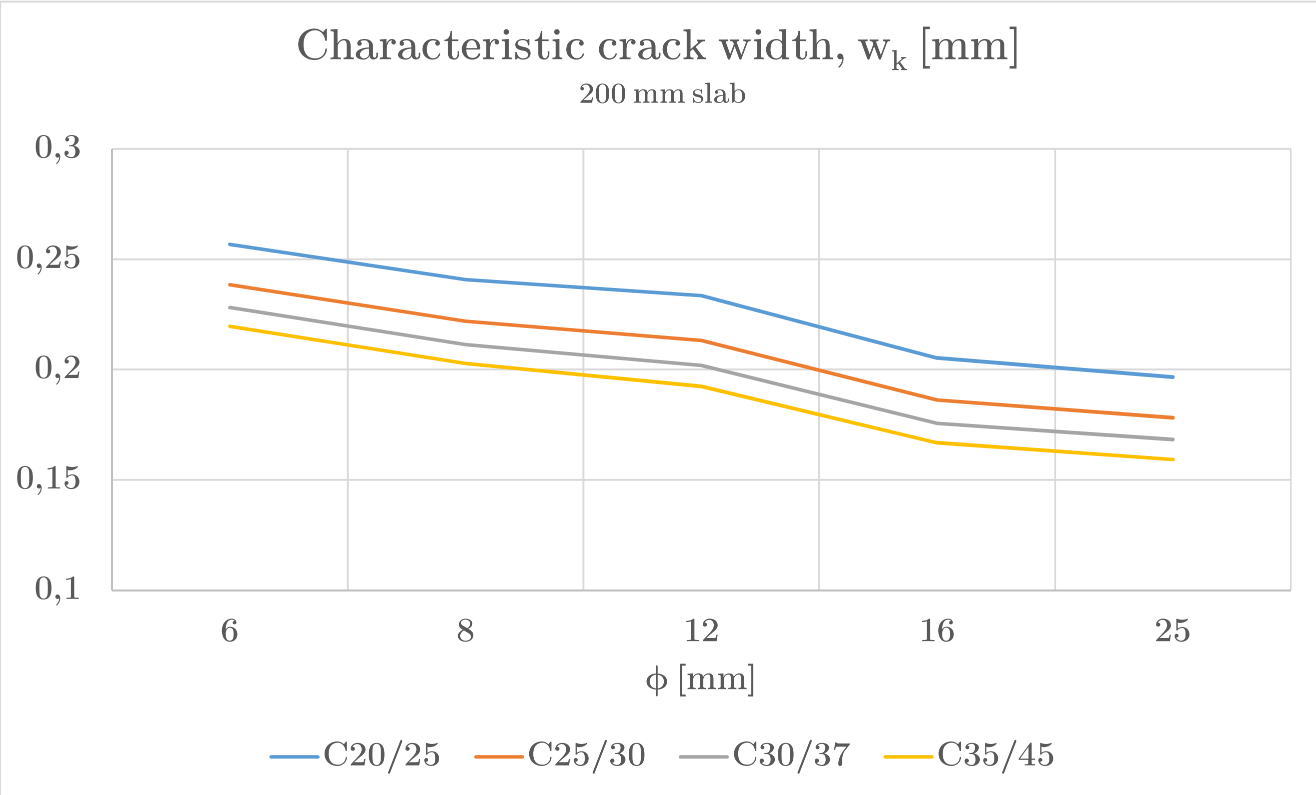

Characteristic crack width, w_k [mm]
(restraint considered)
120 mm slab
0,3
0,25
0,2
0,15
0,1
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

C.2 Compiled results from calculations for a 200 mm slab

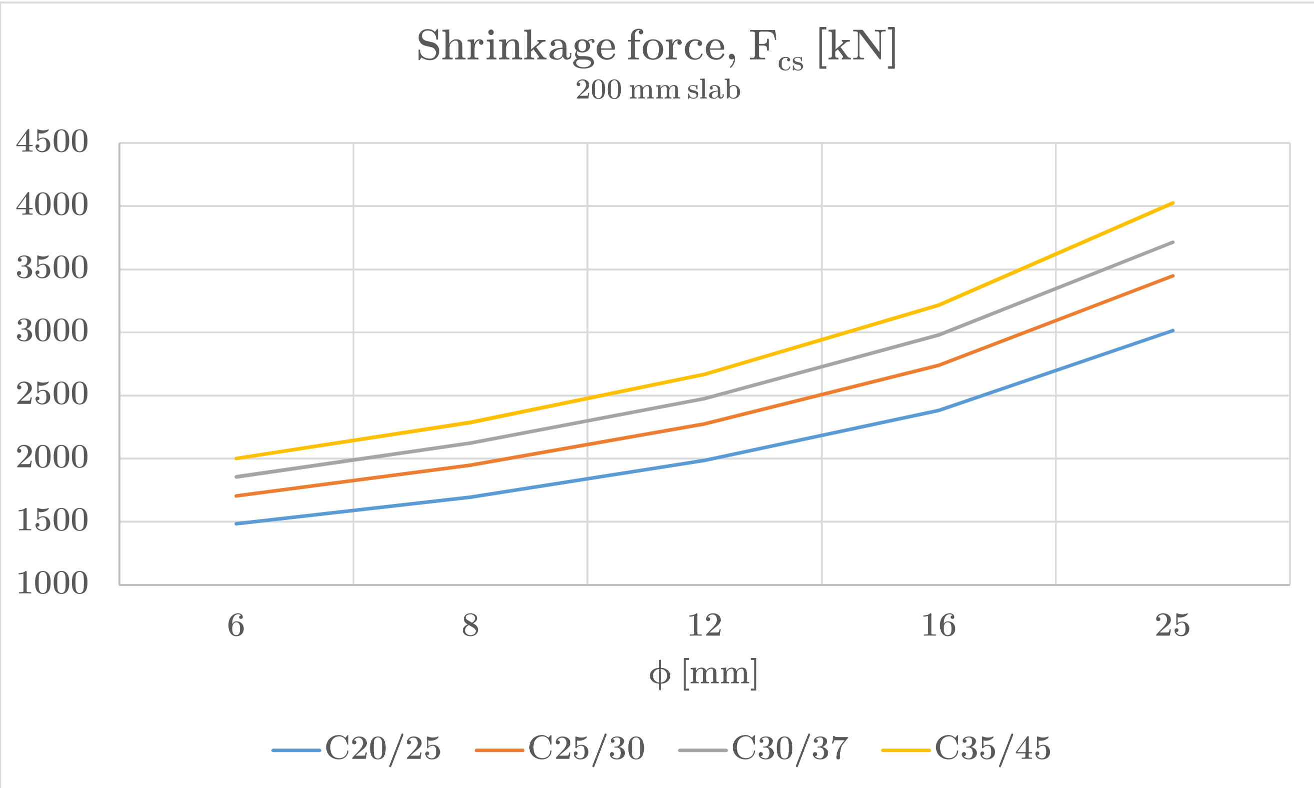

Minimum amount of reinforcement, $A_{s.min}$ [10^{-4} m^2]
200 mm slab
550
500
450
400
350
300
250
200
150
100
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

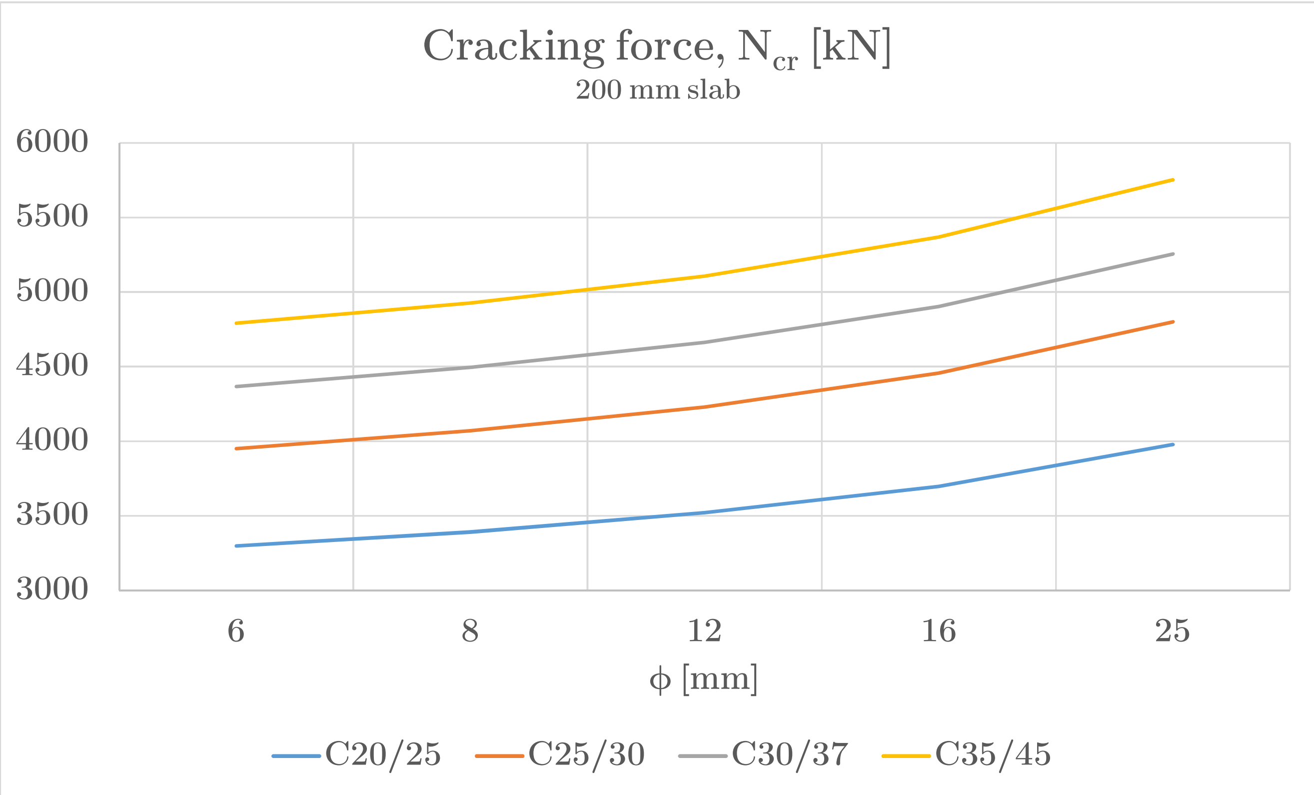


Maximum crack spacing, s_r.max [mm]
200 mm slab
300
250
200
150
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

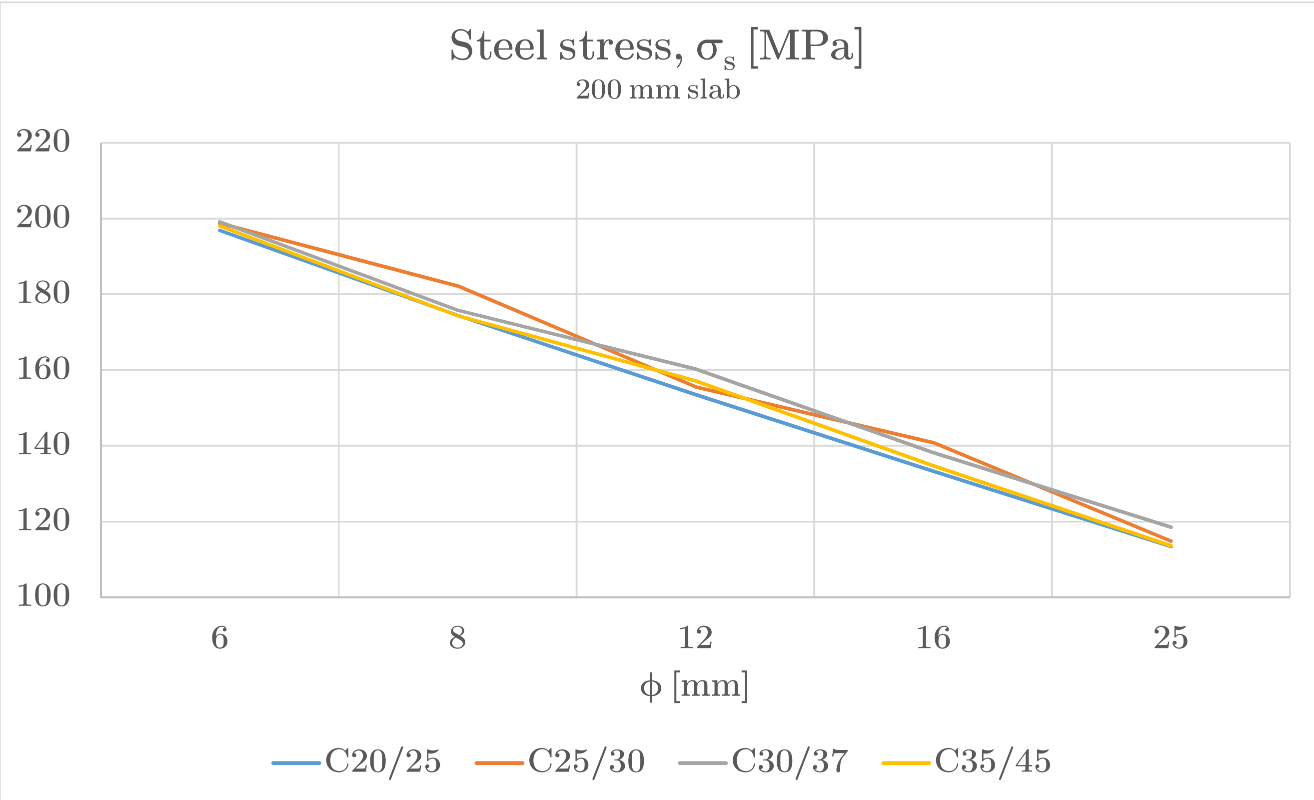


Characteristic crack width, w_k [mm]
200 mm slab
0,3
0,25
0,2
0,15
0,1
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

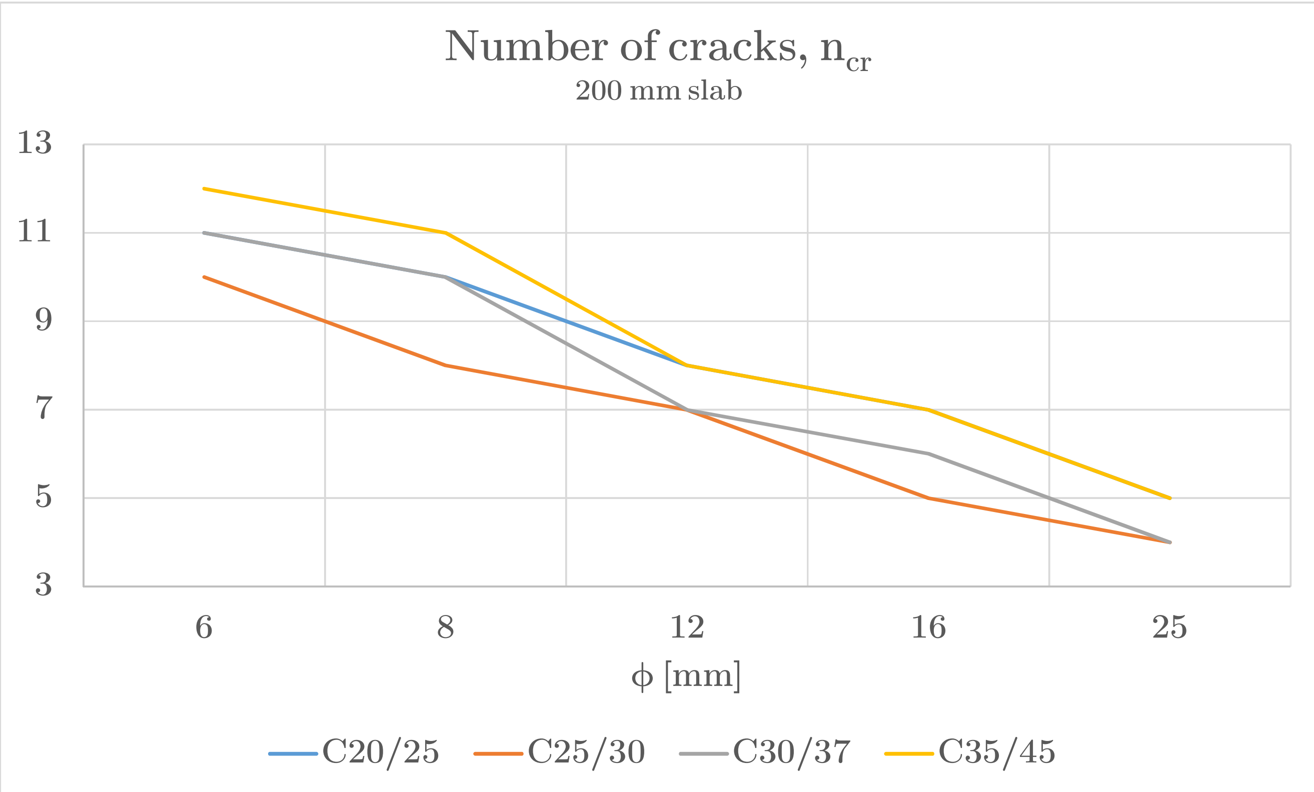

Shrinkage force, F_{cs} [kN]
200 mm slab
4500
4000
3500
3000
2500
2000
1500
1000
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

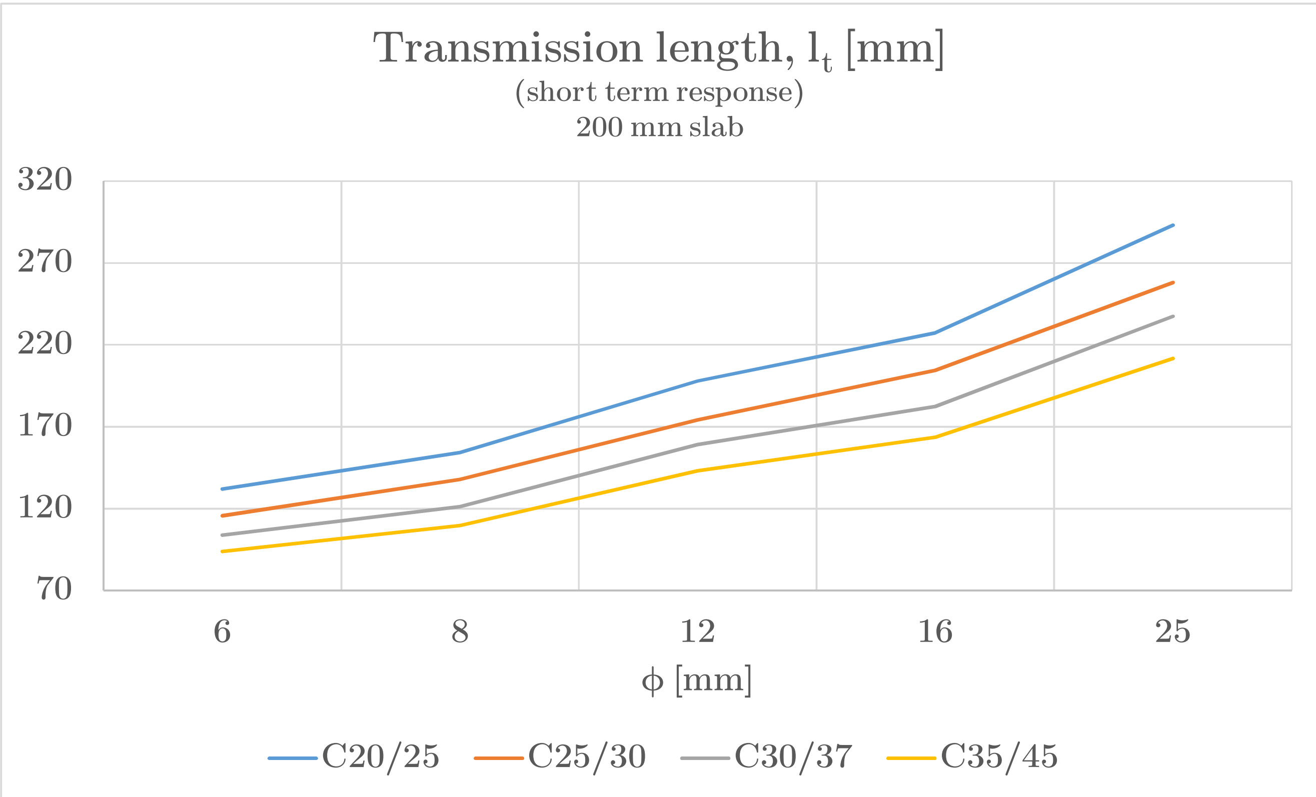


Cracking force, N_{cr} [kN]
200 mm slab
6000
5500
5000
4500
4000
3500
3000
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

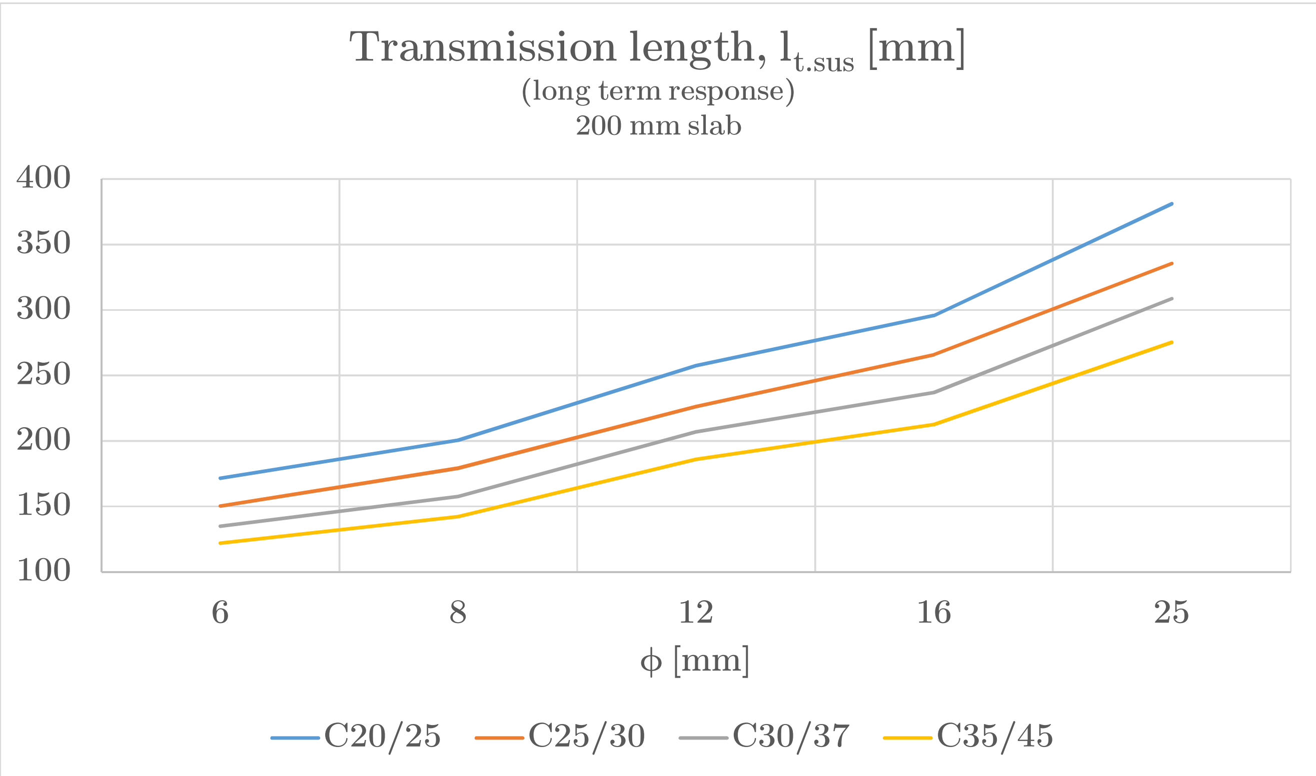


Steel stress, σ_s [MPa]
200 mm slab
220
200
180
160
140
120
100
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

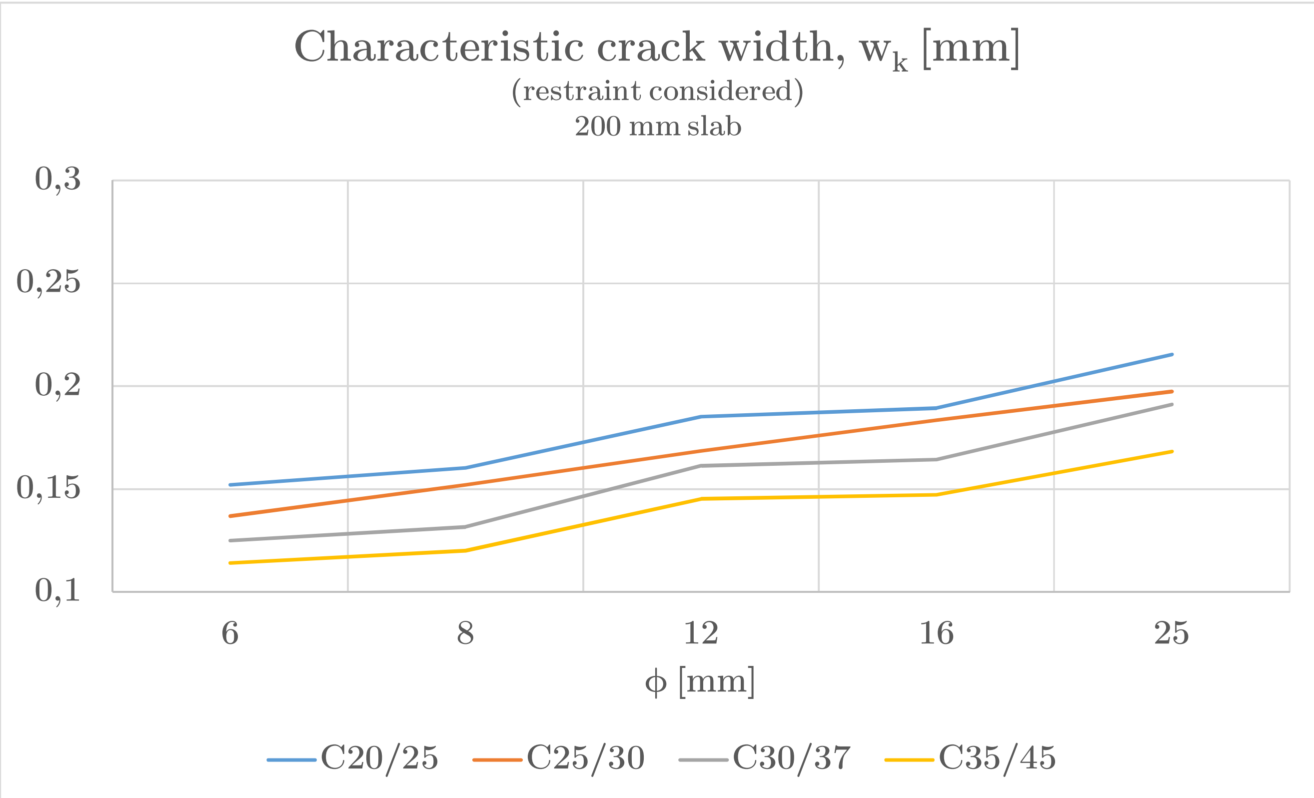

Number of cracks, n_cr
200 mm slab
13
11
9
7
5
3
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



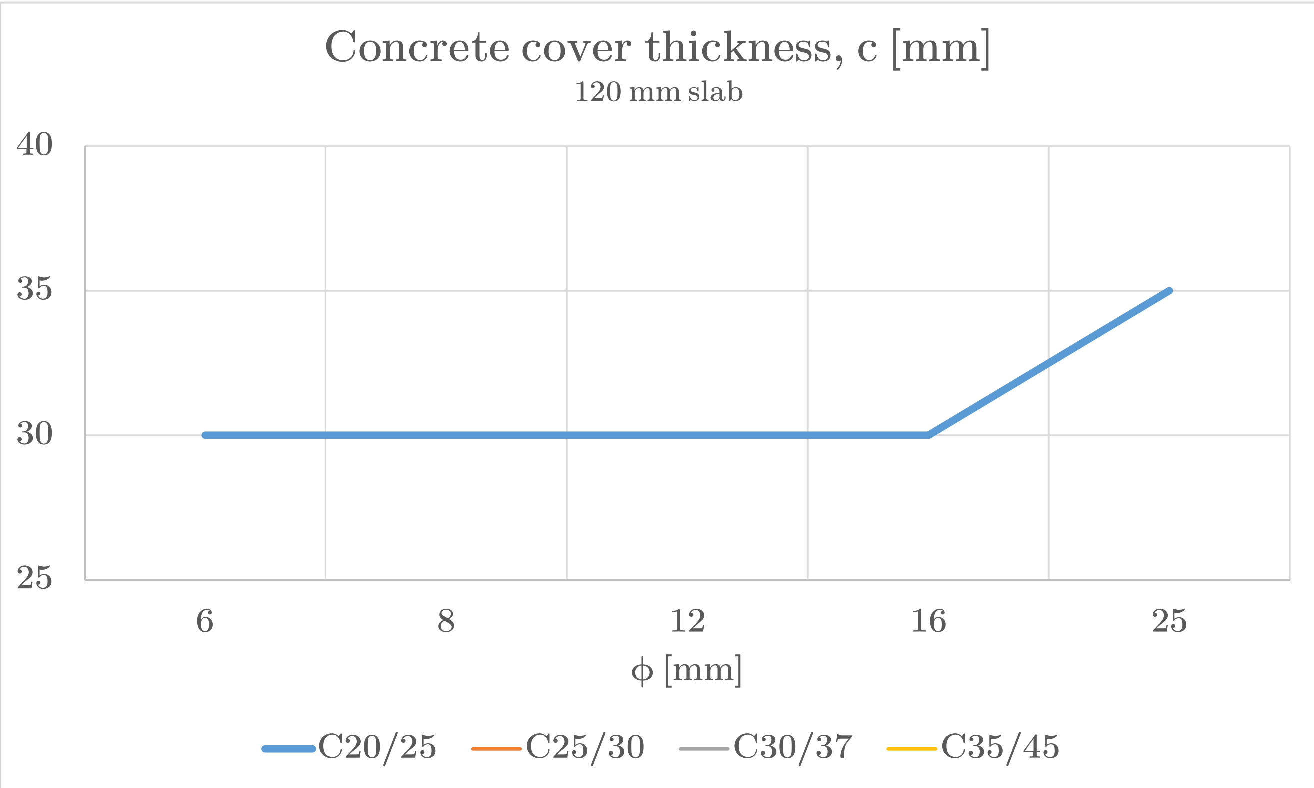
Transmission length, l_t [mm]
(short term response)
200 mm slab
320
270
220
170
120
70
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



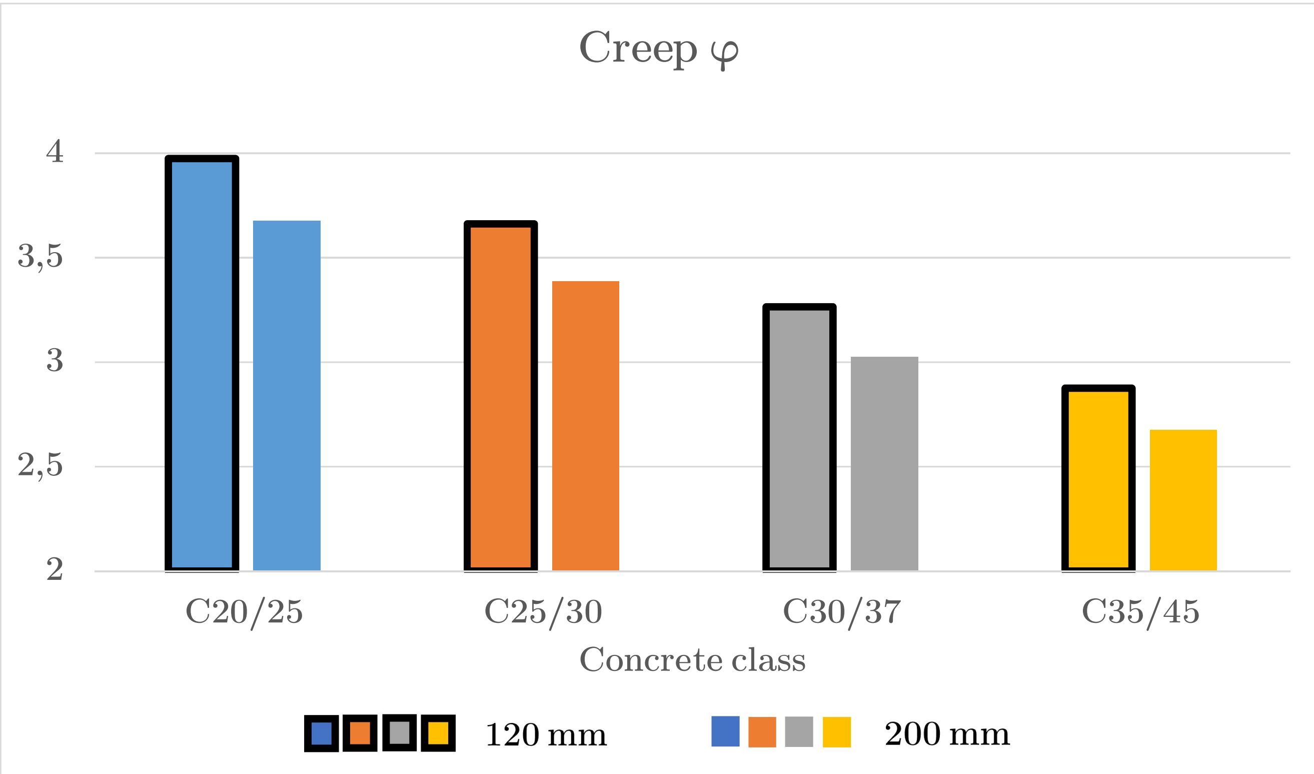
Transmission length, $l_{t.sus}$ [mm]
(long term response)
200 mm slab
400
350
300
250
200
150
100
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45


Characteristic crack width, w_k [mm]
(restraint considered)
200 mm slab
0,3
0,25
0,2
0,15
0,1
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45

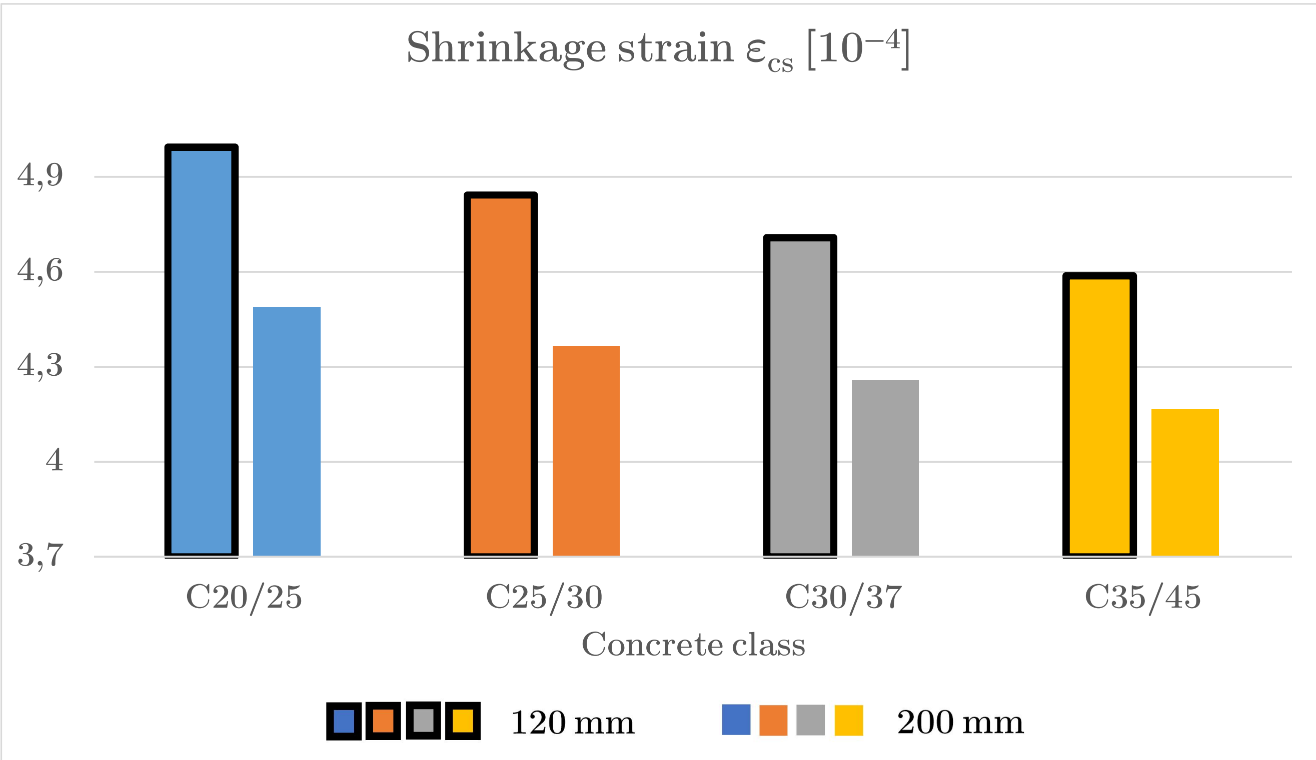
C.3 Compiled results from calculations for both 120 mm and 200 mm slab



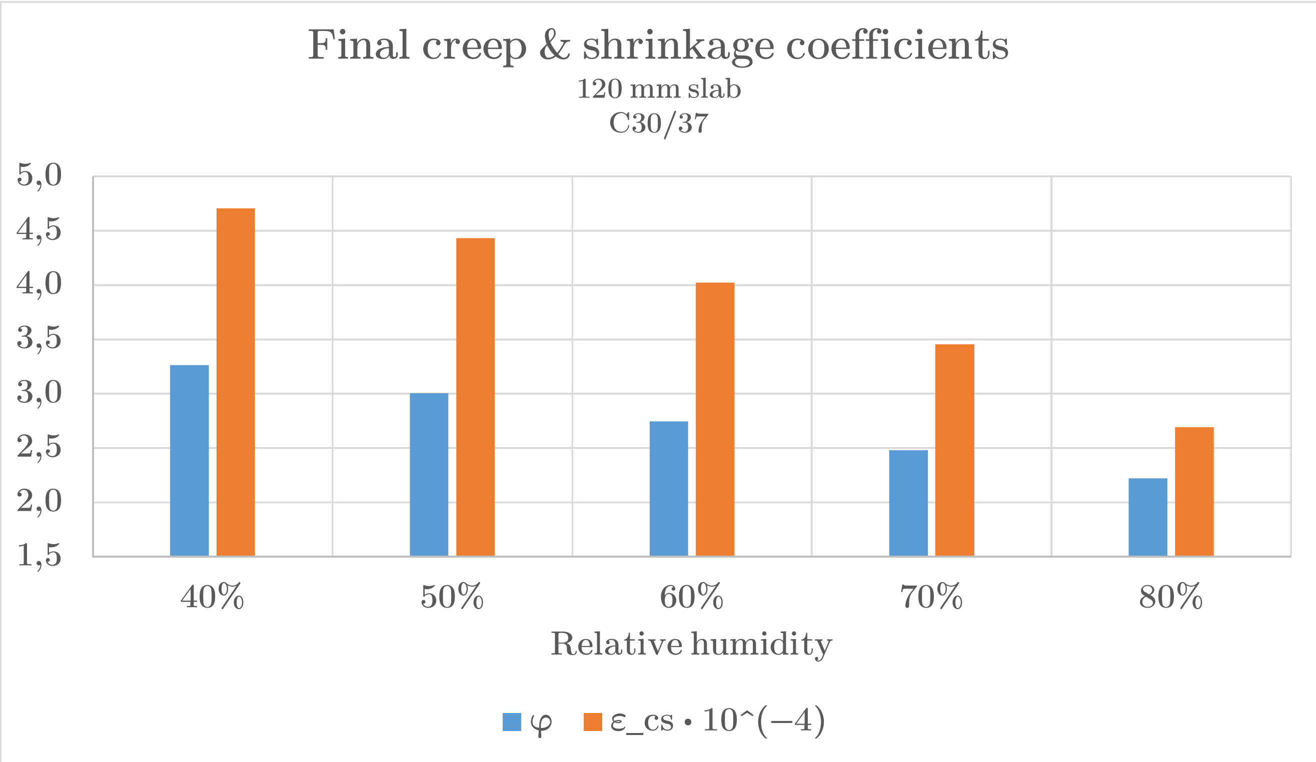
Concrete cover thickness, c [mm]
120 mm slab
40
35
30
25
6
8
12
16
25
ϕ [mm]
C20/25
C25/30
C30/37
C35/45



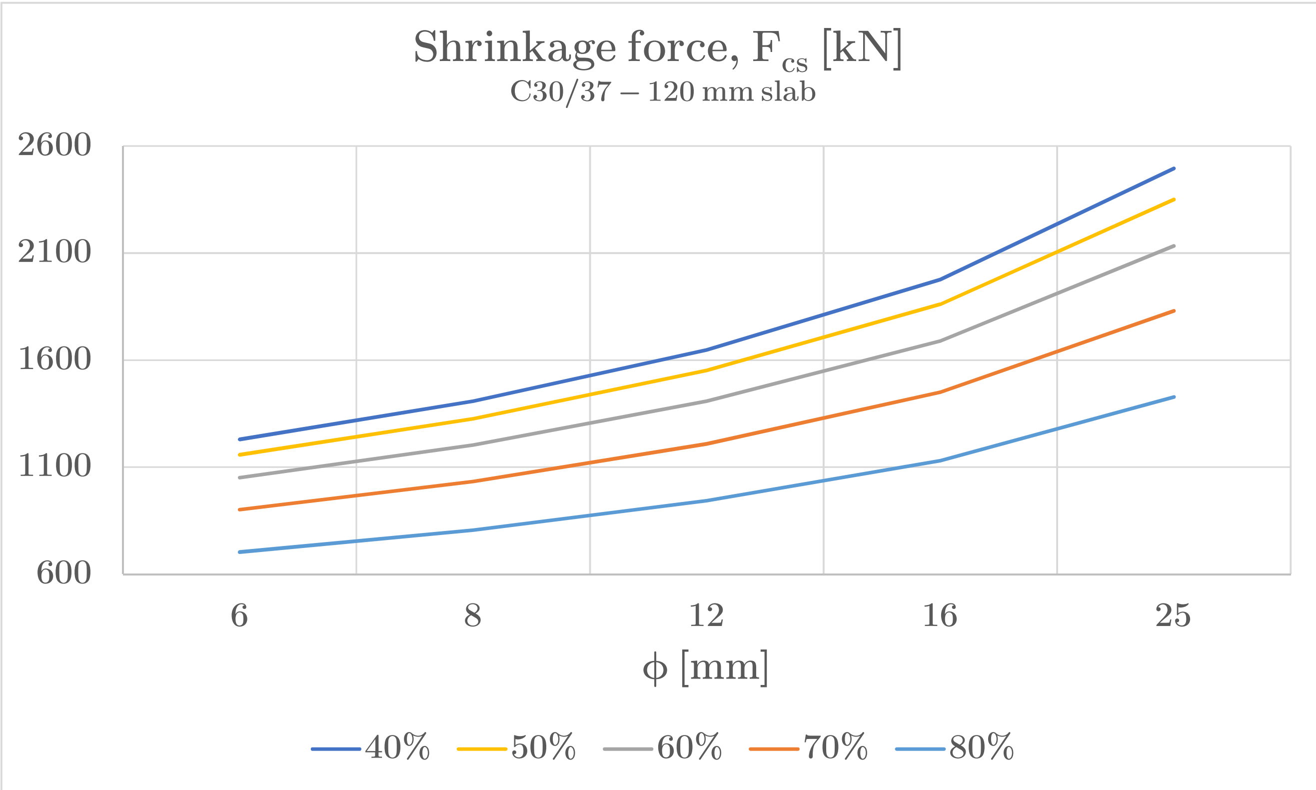
Creep φ
4
3,5
3
2,5
2
C20/25
C25/30
C30/37
C35/45
Concrete class
120 mm
200 mm

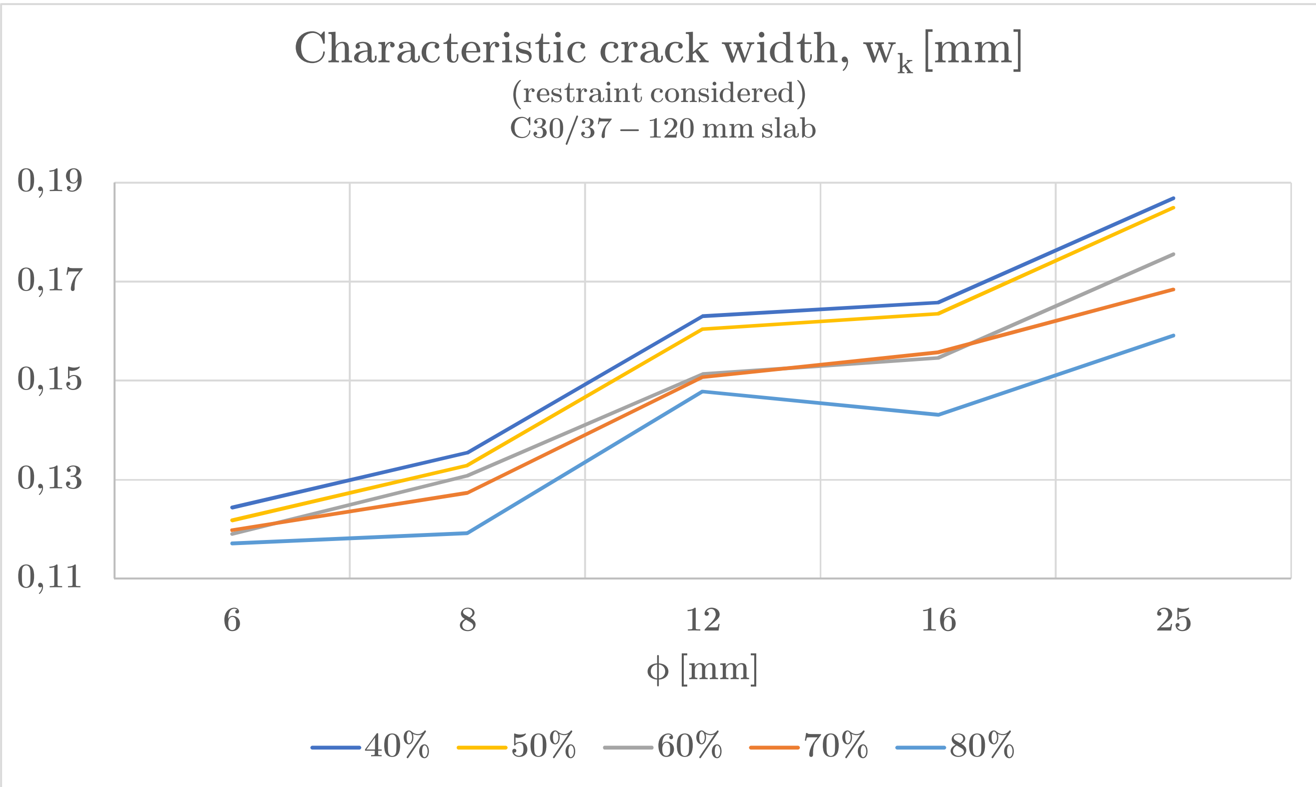


Shrinkage strain ε_{cs} [10^{-4}]
4,9
4,6
4,3
4
3,7
C20/25
C25/30
C30/37
C35/45
Concrete class
120 mm
200 mm



Final creep & shrinkage coefficients
120 mm slab
C30/37
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
40%
50%
60%
70%
80%
Relative humidity
φ
ε_cs • 10^(−4)


Shrinkage force, F_{cs} [kN]
C30/37 – 120 mm slab
2600
2100
1600
1100
600
6
8
12
16
25
ϕ [mm]
40%
50%
60%
70%
80%


Characteristic crack width, w_k [mm]
(restraint considered)
C30/37 – 120 mm slab
0,19
0,17
0,15
0,13
0,11
6
8
12
16
25
ϕ [mm]
40%
50%
60%
70%
80%