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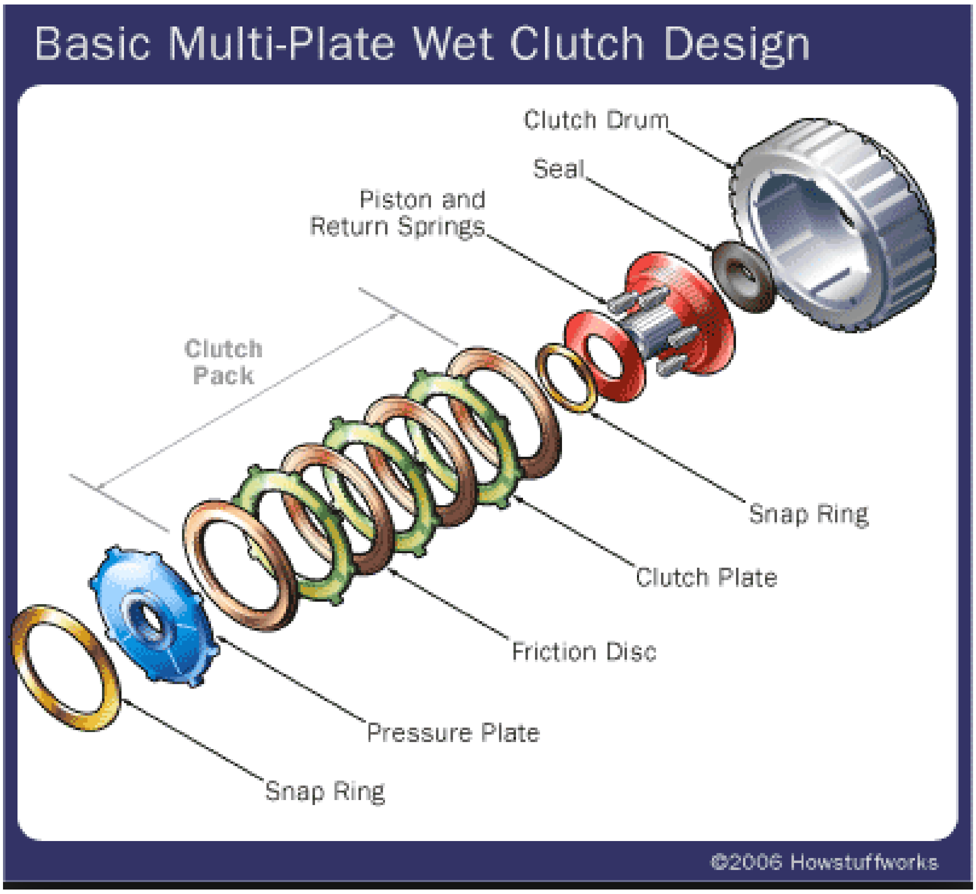

Wet Clutch Modelling Techniques

Design Optimization of Clutches in an Automatic Transmission

Master's Thesis in the Automotive Engineering

MANOJ KUMAR KODAGANTI VENU

Department of Applied Mechanics
Division of Dynamics
CHALMERS UNIVERSITY OF TECHNOLOGY
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Cover:
Figure of a multi-plate wet clutch design

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ABSTRACT

Multi-plate clutches and brakes are commonly used in an automatic transmission for transmitting torque between components or as holding devices. This thesis sheds light on the losses that occur in these clutches when they are disengaged and the design optimization of clutches in a six speed transmission based on these losses. During the disengaged state of a clutch, the transmission fluid offers an internal resistance to the flow in terms of shear stress and this causes loss in power in the form of drag. Two methods to analytically model this clutch drag were compared and evaluated. First, a model based on surface tension effects was used to calculate clutch drag. Second, a model with a new approach to determine the shrink radius of the oil film was evaluated. A comparison between the two models showed that the second model which used a new approach to determine the shrink radius gave much better results. This model was hence chosen in order to find out the losses in a six speed automatic transmission. The speed and torque of all the gears and rotating components are calculated at ideal condition assuming no losses. With the help of the wet clutch model, the losses in the transmission with respect to clutches and brakes are calculated. A comparison between the two cases is made to determine the losses through the entire gearbox due to clutch drag. The effect of various parameters like clearance, grooves, flow rate and temperature on clutch drag was evaluated. This gave a better understanding of clutch drag so as to optimize the design. The optimization was carried out by reducing the number of discs in each clutch and brake and increasing the size of the discs so that there is no compromise in torque capacity. This design change produced lesser losses compared to the original design and showed nearly 46 percent decrease in losses. However, the design of clutches is always a compromise between the engagement and disengagement event because the effect of parameters varies in both the states. This report can be used as a starting point for further investigation of clutch drag using computational fluid dynamics (CFD) which is a very effective tool to analyse the losses. The transient behaviour of the clutches should also be looked into in order to optimize the design of clutches having a right balance for the parameters aiding both the engaged and disengaged state.

Key words: Wet clutch, Drag losses, Design optimization

Contents

Preface

This master thesis is the final part in the Master of Science degree in Automotive Engineering at Chalmers University of Technology. In this thesis, wet clutch modelling techniques by different authors are compared and one of the methods was used in order to determine the drag losses in a six speed automatic transmission. Finally, optimization of the design of clutches has been suggested in order to improve the efficiency of the transmission. The thesis has been carried out from January 2013 to June 2013. The thesis has been carried out at the organisation AVL in Södetälje.

The thesis has been carried out under the supervision of Per Rosander who is the lead Engineer at AVL and Professor Viktor Berbyuk from the department of Applied Mechanics in Chalmers. I would like to thank both Per Rosander and Viktor Berbyuk who have been my supervisors and mentors throughout the entire course of the thesis and provided great encouragement and support. I would also like to thank Alfred Johansson and Niklas Spångberg from AVL for their guidance and technical discussions. They have been an immense support and helped answer many of the doubts that came along during the thesis. I would also like to thank my co-worker Kaushik Desai for his technical and moral support during the entire duration of work.

Finally, I would like to thank my family and friends to have made this thesis possible. I would also like to thank Jagannath Srinivas for his valuable support in Matlab.

Göteborg June 2013

Manoj Kumar Kodaganti Venu

Notations

The symbols and notations used in the report are mentioned below along with what they stand for. This helps the reader to understand all the equations used in the report.

Roman upper case letters

$B1$	Brake B1
$B2$	Brake B2
$B3$	Brake B3
$B4$	Brake B4
$B5$	Brake B5
$F_{totalapplied}$	Total force applied on face of clutch discs in Newton
G_r	Turbulence coefficient factor
G_{Θ}	Turbulence coefficient factor
$K1$	Clutch K1
$K2$	Clutch K2
N	Number of clearances between clutch discs
$N_{friction}$	Number of friction discs
Q	Ideal flow rate in meter cubed per second
Q_i	Actual flow rate in meter cubed per second
Re_h	Characteristic Reynolds number
T	Drag torque in Newton
T_c	Torque of the planetary carrier in Newton
$T_{capacity}$	Torque carrying capacity of the clutch in Newton
T_p	Torque of the planetary gears in Newton
T_r	Torque of the ring gear in Newton
T_s	Torque of the sun gear in Newton
V_r	Radial velocity component in meters per second
V_z	Axial velocity component in meters per second
V_{Θ}	Circumferential velocity component in meters per second

Roman lower case letters

f	Turbulence coefficient factor
n_c	Speed of the planetary carrier in radian per second
n_p	Speed of the planetary gears in radian per second
n_r	Speed of the ring gear in radian per second
n_s	Speed of the sun gear in radian per second
h	Axial clearance in meters

r_1	Inner radius of the clutch discs in meters
r_2	Outer radius of the clutch discs in meters
r_m	Mean radius of the clutch discs in meters
r_o	Equivalent radius of the clutch discs in meters
z	Axial coordinate
z_p	Number of teeth on the planetary gear
z_r	Number of teeth on the ring gear
z_s	Number of teeth on the sun gear

Greek Symbols

μ	Dynamic viscosity Newton seconds per meter squared
$\mu_{friction}$	Coefficient of friction of the friction disc
ω	Relative speed of the clutch discs in radian per second
ρ	Density of the fluid in kilogram per meter cubed
τ	Tangential shear stress in Newton per meter squared

Subscripts 1-5 are used in the report in order to represent the particular set of planetary gears.

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1 Introduction

1.1 Background

The ever diminishing oil reserves and the need for energy efficient automobiles have pushed powertrain developers to tackle all areas of losses in a vehicle. The wet clutches is one such area within transmission where there is a scope for improvement. The wet clutches consists of alternating discs made up of steel and friction material. The friction discs usually have grooves for the flow of transmission fluid and allow for efficient cooling and lubrication. The main function of these clutches is to ensure smooth power transfer thereby transmitting torque from the driving member to the driven member. The parasitic losses account for nearly one-third of the losses in a transmission [2]. Losses in the wet clutches are in the form of viscous drag between two rotating plates having a relative velocity to each other. The optimization of the clutch design is however not simple as both the clutch engagement and disengagement have to be taken into consideration. The optimization of the design should be carried out in such a way that it does not affect the primary purpose of wet clutches which is its torque carrying capacity.

1.2 Scope

The scope of the thesis is limited to modelling of the wet clutches and brakes in MATLAB. The results from the model are then used in order to determine the losses in the six speed gearbox. This report could be used as a starting point for further investigations into wet clutches by performing CFD simulations to validate the results.

1.3 Project plan

"Give me six hours to chop down a tree and I will spend the first four sharpening the axe." — Abraham Lincoln.

This line is a simple way to show the importance of planning and how one can achieve their goals set out by having a proper plan. A project plan was laid out initially in order to achieve the desired goals. This plan was modified accordingly during the course of the thesis as and when any obstacles or problems arose.

This final project plan for the Master thesis which shows a weekly breakup of the goals and sub goals to be achieved is appended in the Appendix I.

2 Background study

2.1 Transmission basics

Transmission is an energy transformation unit that takes the power from the engine to the wheels.

It is basically used to convert the speed and torque of the engine so that it could be used to drive the wheels of the vehicle.

The main task of the transmission is to

- Ensure that the vehicle starts from rest, with the engine running continuously
- Allow the speed/torque ratio to vary between the engine and the wheels
- Transmit the drive torque to the wheels
- Allow the driver/system to run the engine in its optimal point based on the driving situation thereby improving fuel efficiency and/or minimize emissions.

The need for a transmission comes from the physics behind an engine. Vehicles have to produce sufficient power to overcome the road resistance forces in order to start the vehicle from rest or to be in continuous motion. The road resistance forces are in the form of gradient force, aerodynamic force and the rolling resistance of the tires. The engines have a narrow band of revolutions per minute (RPM) where the power and torque are maximum at that particular region. The transmission functions by changing the gear ratio between the engine and wheels of the vehicle so that it tries to stay within this narrow band thereby producing maximum performance and efficiency.

2.2 Transmission types

There are different types of transmission that are available in the market today and these can be classified as follows.

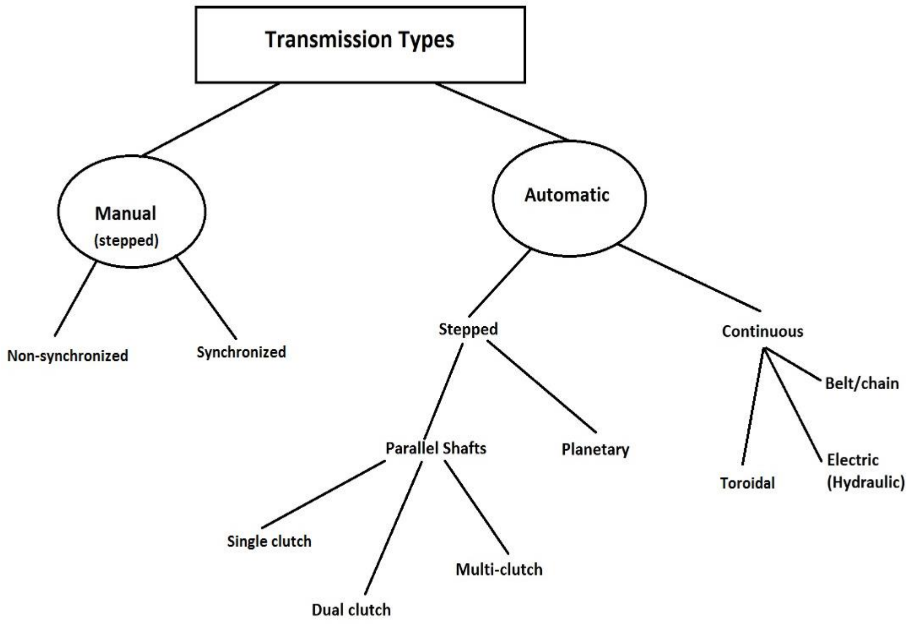

Figure 1: Different Transmission types

The manual transmission as the name suggests involves the driver to regulate torque from the engine to the wheels via the transmission and this is done with the help of user operated clutch pedal, and gearshift fork.

The automatic transmission on the other hand does not require any user intervention and selects the gear ratio automatically based on the appropriate engine speed and torque. The shifting is done with the help of hydraulics and pressurized hydraulic fluids or electric motors.

One similarity between the manual and the automatic transmission is that both of them are stepped gear transmissions. The gear ratios are always fixed for a particular type of transmission. A continuously variable transmission (CVT) on the other hand has an infinite number of gear ratios within a particular range. This enables the CVT to provide a better fuel economy by enabling the engine to always run in its most efficient band. An infinitely variable transmission (IVT) also has an infinite number of gear ratios and also a zero speed ratio in addition.

2.2.1 Automatic transmission parts

The automatic transmission which is focused upon consists of the following main parts.

- Torque converter
- Planetary gear sets
- Clutches and brakes
- Oil pump
- Valve bodies
- Hydraulic system

Torque converter

The principle of torque converter is similar to that of the clutches in manual transmission where in it provides a means for coupling the engine torque to the input shaft of the transmission. The torque convertor essentially has 3 main components-impeller, turbine and the stator.

The flywheel from the engine side is bolted to the convertor housing on to which the impeller is mounted. The turbine on the other hand is splined to the transmission input shaft. A hydraulic fluid fills the torque converter and is responsible for transferring the torque from the engine side to the transmission side through its kinetic energy. The engine crankshaft drives the impeller as well as the fluid in it. As the impeller speed increases, the fluid is thrown outwards on to the turbine because of the centrifugal forces. This causes the turbine to turn and thereby transmit torque from the engine crankshaft to the transmission input. The stator located between the impeller and turbine serves the purpose of redirecting the fluid from the turbine back on to the impeller. This leads to an increase in the overall torque of the system and causes toque multiplication. One of the key features of the torque convertor is that it allows the engine to continue running in gear when the vehicle has come to a complete stop. The torque converter is shown below in Figure 2 [3].

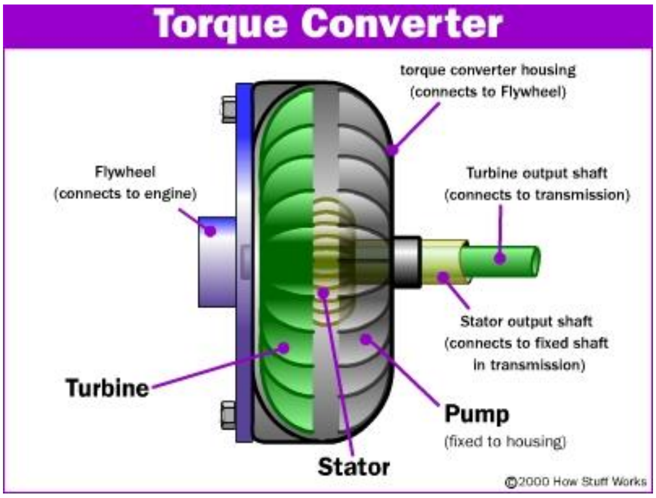

Figure 2: Torque converter

Planetary gear sets

The Planetary gear sets consists of an inner sun gear, an outer ring gear and three or more planetary gears as shown in Figure 3 [4]. These planetary gears are held together by the planetary carrier. They get their name from their resemblance to the arrangement of the planets and the sun in the solar system. The power from the torque convertor is then passed on to these sets of gears. Power transmission is achieved by holding and driving different parts of the planetary gear sets.

One member is used as the input or the driving member, another member is held stationary and the third member is used as the output or driven member. By using different gears as input, stationary and output, various gear ratios can be achieved including that of reverse drive.

With a simple planetary gear set, there are a number of gear ratios which can be achieved as tabulated below.

Table 1: Planetary Gear set Ratios

Input Member	Stationary Member	Output Member	Type of Drive Speed	Type of Drive Torque
Sun Gear	Ring Gear	Planetary carrier	Maximum Forward Reduction	Maximum Forward Increase
Ring Gear	Sun Gear	Planetary carrier	Maximum Forward Increase	Minimum forward Increase
Planetary carrier	Ring Gear	Sun Gear	Maximum Forward increase	Maximum Forward Reduction
Planetary carrier	Sun Gear	Ring Gear	Minimum Forward Increase	Minimum Forward Reduction
Sun Gear	Planetary carrier	Ring Gear	Maximum Reverse Reduction	Maximum Reverse Increase
Ring Gear	Planetary carrier	Sun Gear	Maximum Reverse Increase	Maximum Reverse Reduction
Sun and Ring Gear	-	Planetary carrier	Direct Drive	Direct Drive
Sun Gear	-	-	Neutral	Neutral

There are also compound gear sets such as the ravigneaux which is composed of 2 sets of sun gears- one large and one small, a common ring gear and a planetary carrier with 3 small and 3 large planetary gears. This gear set provides much more combinations compared to the simple gear set and hence produces more number of gear ratios.

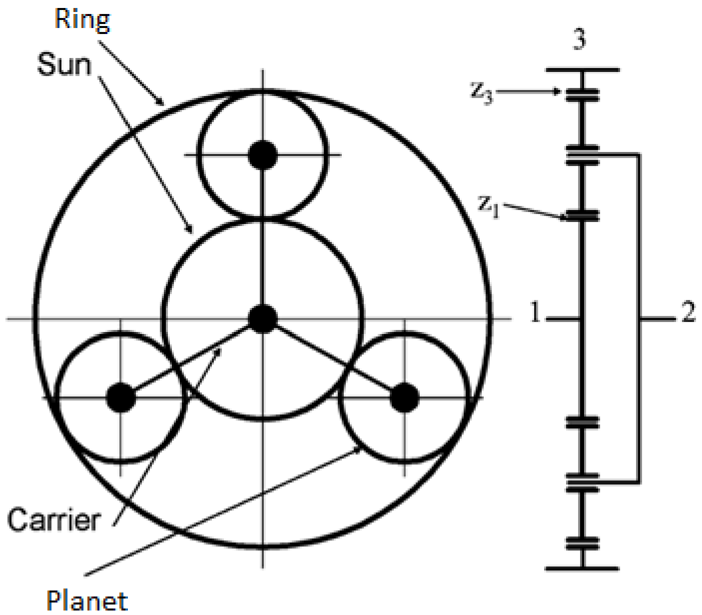

Figure 3: Simple Planetary Gear set

Clutches and brakes

Clutches are used as power transfer devices that either hold or allow the planetary gear set to turn/rotate. Brakes are generally holding devices used for braking purpose by holding the planetary component to the transmission case.

Clutches are of two types- Multiple clutch packs and one way clutches (OWC) and the Brakes are also of two types- multiple plate brakes and brake bands.

Multiple clutch pack basically consists of alternating discs enclosed in a clutch drum. The discs are made of steel. One set of steel plates have friction material bonded to them and are called the friction discs. The friction discs are always located between two alternating steel discs. The steel discs have slots on the outer diameter which fit into the slots of the clutch drum. The friction discs on the other hand are slotted on the inner diameter to fit into the splines of the clutch hub. The clutch drum provides the bore for the clutch piston. The return spring is attached to the clutch piston. The entire component- clutch drum along with the discs and piston is said to rotate. When hydraulic pressure is applied the piston forces the two plates to stick together and rotate as a single component. Once the pressure is released, the return spring forces the clutch piston to get back to its rest position. The clutch pack is also provided with seals so that there are no fluid leakages taking place.

OWC as their name suggests turns freely in one direction but they lock up in the other. They are also called as freewheeling clutches. It mainly consists of spring-loaded rollers mounted in wedge shaped slots or sprags between two races. They work on the principle of wedging action of these sprags between two races.

Multiple plate brakes are similar in construction to that of the multiple clutch pack and only differ in their functioning. Brakes are used as holding devices whereas the clutches are used as power transfer devices.

The brake band is also used as a holding device and it consists of a band that wraps around the outside of a gear train component or clutch drum to hold it in place.

Oil pump

The oil pump is located between the torque convertor and the transmission housing. It is directly driven by the engine. The main purpose of the oil pump is draw transmission fluid from the oil sump and transfers it to all parts of the transmission. It is also used to apply clutches and brakes and supply the torque converters with oil.

Valve bodies

This is the main hydraulic control unit that controls and directs the hydraulic fluid to different parts in the transmission via control valves. It consists of maze of channels through which the fluid flows and based on the signals from the transmission control unit (TCU) drives a particular multiple clutch pack or brake.

Hydraulic system

The hydraulic system is responsible to send the transmission fluid under pressure to all the parts of the transmission and torque converter through its tubes and passages. It plays a very important role since the proper functioning of an automatic transmission depends on the supply of fluid under pressure at the right time.

2.3 Problem definition

2.3.1 Multi plate clutches and brakes

Working

The clutch has 2 modes of operation- engagement and disengagement. During the engagement mode, the plates are squeezed together and the transmission fluid present between the plates is pushed out. Torque is then transmitted through the clutch and to the corresponding gear set. The engagement process takes place in three stages [5] and [6].

- Hydrodynamic stage
- Contact stage
- Locked up stage

During the hydrodynamic stage, the plates are separated from each other and are filled with a film of fluid. As a result of which torque is not developed through frictional contact but in contrast it is developed due to the viscous torque transfer through the automotive transmission fluid (ATF). The plates now start moving closer to each other and the fluid film begins to shrink [5] and [6].

The contact stage is said to occur when the plates are squeezed further towards each other. Few contact points are said to be formed and these are known as asperity points. Torque is slowly transferred through these asperity points which increase gradually and hydrodynamic torque consequently begins to decrease. There is a slip or relative velocity between the plates at this stage due to which there is an increase in the temperature of the plates [5] and [6].

The final stage is the locked up stage where in the torque is entirely transmitted through frictional contact only. The steel and friction plates are completely in contact with each other and hence rotate at the same speed. The transmission fluid in between the plates is completely squeezed out. The presence of grooves in the plates plays a major role in guiding the fluid and hence plays a role in the torque capacity and heat flow [5] and [6].

The disengagement mode also called as the open clutch condition is when the friction disc and the steel plate are separated from each other and are rotating at different speeds. Hence a relative velocity exists between the two. A hydrodynamic torque is developed during such a situation. This is caused because of the shearing effects of the viscous fluid and the torque developed is referred to as drag torque. The drag torque mainly depends on the viscosity of the fluid and also depends on the presence of air bubbles trapped between the plates [5] and [6].

2.3.2 Characteristics of wet clutches

Good Characteristics of Clutches [7] and [8]

- High torque capacity
- Low weight, easy packaging
- No noise or vibrations (Good NVH characteristics)
- Long life
- High energy density
- Low drag Torque to reduce fuel cost

2.3.3 Drag torque physics

Two discs at certain distance apart from each other are considered with a fluid filled between them. One of the discs is stationary while the other is moving. The fluid present within them will offer an internal resistance to the motion of the plate in the form of a shearing force. This shearing force due to the viscous effect of the fluid is termed as drag torque and is also known as drag loss [6].

The drag torque leads to a loss in power in the transmission. For an automatic transmission, it contributes as much as 20 percent to the entire transmission losses [9].

The drag torque in a clutch is said to be influenced by the following parameters [9].

- Drag torque increases with an increase in the number of discs used in a clutch pack
- Drag torque decreases with an increase in clearance between two consecutive discs.
- Drag torque decreases with a decrease in lubrication flow rate.

- Drag Torque increases with a decrease in temperature. Temperature plays a major role and with a change in temperature, the viscosity and density of the fluid also changes.
- Drag torque increases with an increase in the radius of the discs.

A typical drag torque behaviour can be as shown as follows [6].

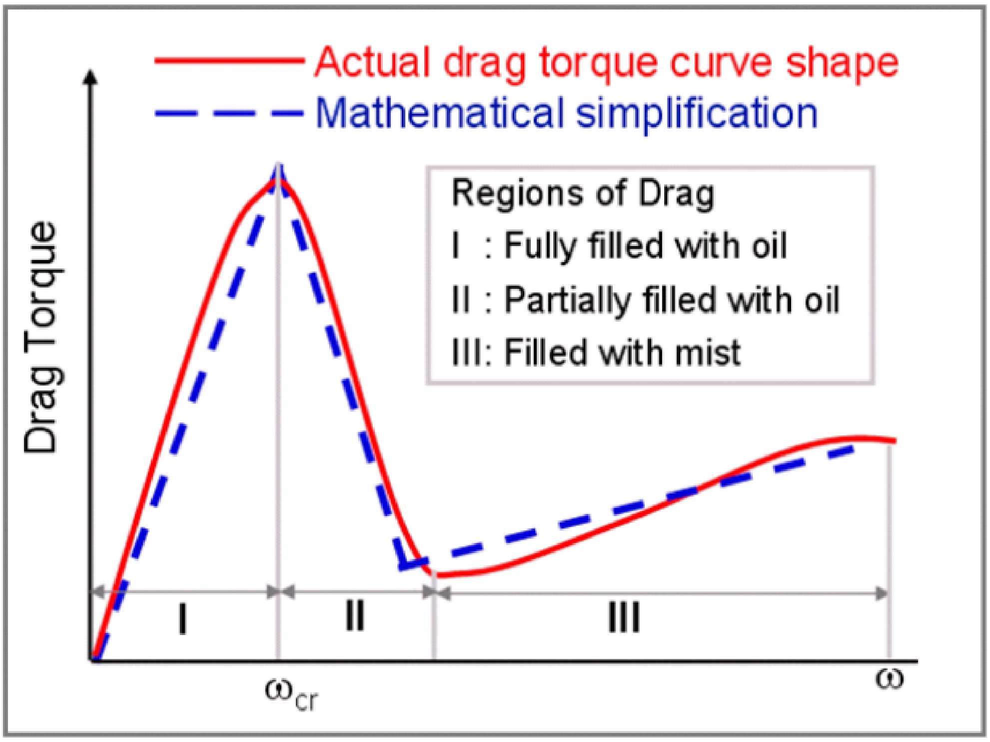

Figure 4: Typical Drag Torque curve

Drag torque in a wet clutch is plotted against the relative speed of the plates. It is divided into three regions- region I, region II and region III.

Initially, there is a linear increase in drag torque with relative speed up to a particular speed (critical speed). This is because drag toque is proportional to the relative speed and there is a full film of oil that exists between the discs. This corresponds to the region I.

The region II can be seen as region of decreasing drag torque and as the speed increased the drag torque decreased further. This is attributed to the fact that earlier in region I, centrifugal effect was quite low and surface tension forces predominated. The surface tension forces were responsible in keeping full film of oil between the plates as much as possible. But as the speed increased, the centrifugal forces dominated and as a result of which small air pockets were formed near the outer diameter of the discs. These air pockets gradually became bigger as the relative speed increased. This consequently led to the oil film diminishing and resulted in a decrease in drag torque.

Finally region III causes an increase in drag torque with increase in relative speed. This is explained by the fact that fluid forms a mist between the clutch plates. This was seen in many visual experiments.

2.3.4 Different approaches to determine drag torque

Testing has been the major approach that is incorporated in order to determine the spin losses. Research has been done by various authors such as Lloyd [10] who investigated the various parameters that affect drag torque such as speed, oil viscosity, clearances, oil flow and friction material. Lloyd came to the conclusion that speed was the most important parameter that affects drag torque and this was particularly evident at low speed scenarios [10]. Another author Kitabayashi [11] studied the effect of design factors of the clutch plate affecting drag torque. This was quite similar to the experiments performed by Lloyd and investigated the various parameters such as speed, flow rate, the effect of grooves and plate waviness [11].

Analytical models also exist and have been developed in order to predict the drag torque in clutches. These are however based on empirical relations and simplified flow equations [12]. The main drawback that exists in the current models is that the drag torque is calculated for clutches that have no grooves and also the inability to predict drag decay time [6].

The analytical model by Aphale [13] calculated drag torque during single phase flow when the speed of rotation of the discs is low. This model was verified by performing experiments and also by a CFD model. However, this model does not take into account higher speeds at which drag torque behaves differently due to the formation of mist in the transmission oil [13].

The analytical model by Yiquing Yuan considered a two phase flow and calculated drag torque accordingly. Surface tension has also been accounted for in the model which plays a major role in maintaining the film of oil between the two clutch discs at low speeds [14].

The model in [14] uses an approach where in an equivalent film radius is assumed. The concept behind this is that at low rotating speeds, there is a full film of oil between the two discs. Centrifugal force is quite low and surface tension is the dominant force which helps the oil film to adhere to the space between the clutch discs. As the rotational speed increases, the full film of oil between the discs begins to diminish because centrifugal forces begin to dominate and the fraction of wetted area decreases as the oil is thrown outwards. This equivalent radius is now considered where in the flow rate and drag torque can be evaluated equivalent to that of the original film [14]. However these models fail to determine the drag torque at high speeds effectively.

So another model by author Heyan Li [15] proposed a new drag torque model which uses a new technique to calculate the equivalent oil film radius.

The two models by Yuan and Heyan Li are explained in more detail.

3 Methodology

3.1 Speed and torque relationship

In order to understand the power-flow through any transmission, the speed and torque relationship needs to be understood.

A simple planetary system is considered as shown in the Figure 5

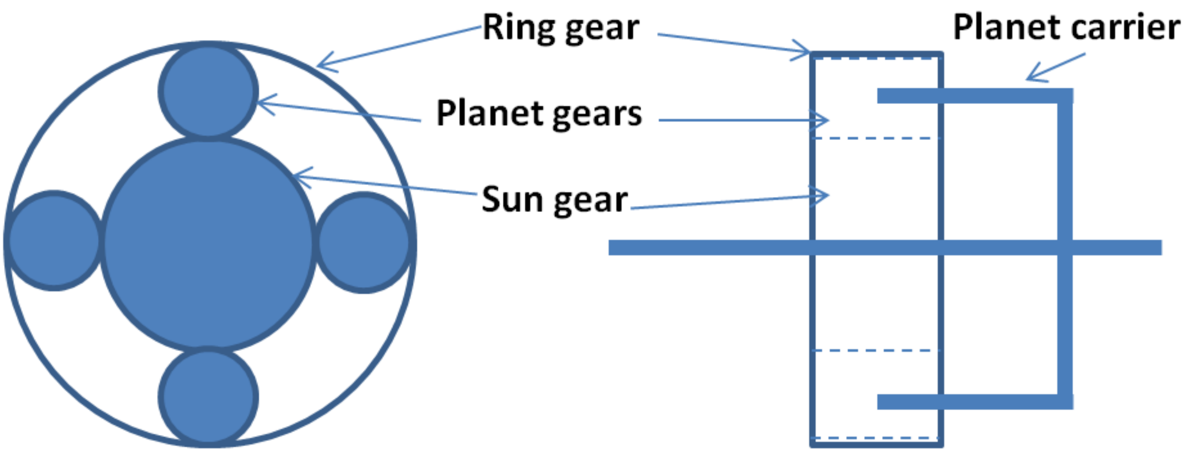

Figure 5: Simple Planetary Gears

The following case is considered where the planetary carrier is kept still. The speed relations are given as

$$n_s = -n_p * \frac{z_p}{z_s} \qquad (1)$$

$$n_r = n_p * \frac{z_p}{z_r} \qquad (2)$$

From equations 1 and 2

$$\frac{n_s}{n_r} = -\frac{z_r}{z_p} \qquad (3)$$

The relative speed of the planetary carrier is also considered now and the speed relation is now given as

$$n_s = -n_p * \frac{z_p}{z_s} + n_c \qquad (4)$$

$$n_r = n_p * \frac{z_p}{z_r} + n_c \qquad (5)$$

From equations 4 and 5

$$\frac{n_s - n_c}{n_r - n_c} = -\frac{z_r}{z_s} \qquad (6)$$

Or

$$n_r z_r + n_s z_s = n_c(z_s + z_r) \qquad (7)$$

Equation 7 is the basic speed relation for a planetary gear.

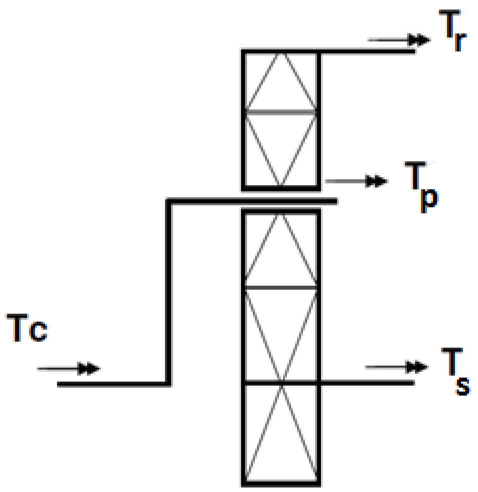

Figure 6: Torque through Planetary Gear set

The torque relation can be deduced from the following Figure 6.

Considering the equilibrium of the total system

$$T_s + T_r + T_c = 0 \qquad (8)$$

Since the planetary gear has no shaft, $T_p = 0$

T_s and T_r must also balance the planetary gear

$$T_r * z_s - T_s * z_r = 0 \qquad (9)$$

Or

$$\frac{T_r}{T_s} = \frac{z_r}{z_s} \qquad (10)$$

Equations (8) and (10) give the basic torque relationship between planetary gears.

3.2 Generic model

The schematic of the gear box and the elements engaged are given in the figures below. The transmission is an automatic type having 6 forward speeds and 2 reverse speeds. The transmission consists of 5 sets of planetary gears. The transmission also has 2 multi-disc clutches and 5 multi-disc brakes.

In order to calculate the power flowing through system, the speed and torque values at all the planetary gears as well as the clutches and brakes needs to be found out.

During an ideal situation when there is no loss in power, there are no drag losses and hence torque through the disengaged clutches and brakes will be zero. However, the speed of the clutches and brakes will be the same irrespective of whether they are engaged or disengaged.

Using the speed relationship and the schematics of the gearbox, the speeds and torques through all the gears, clutches and brakes are determined. The gear ratios for all the gears are also calculated and are tabulated below in Table 3.

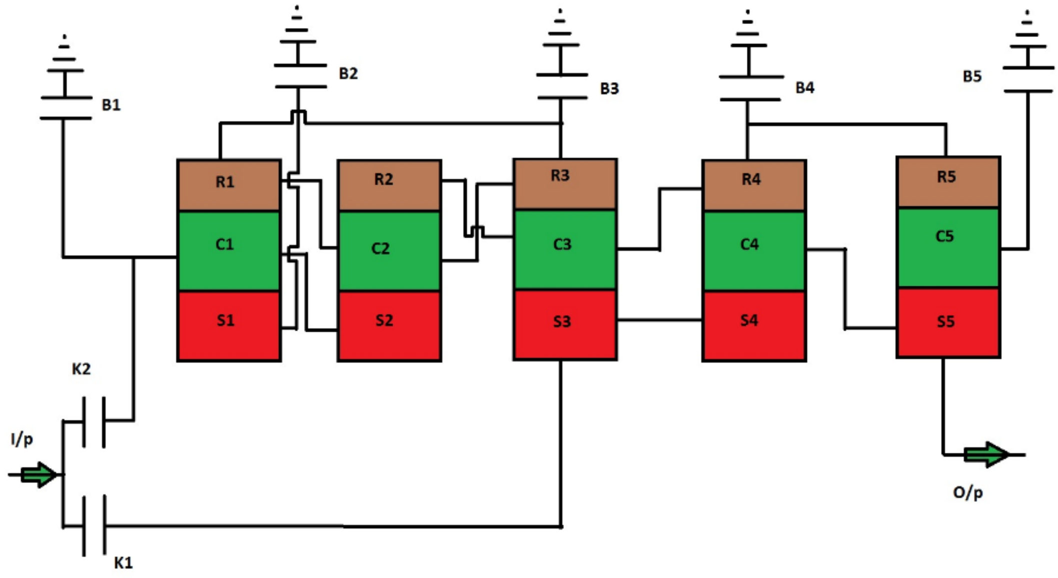

Figure 7: Gear box Layout for 6 speed Transmission

Table 2: Indicator

Symbol	Notation
1	Engaged
0	Disengaged

Table 3: Clutch and Brake position

Gear State	Elements Engaged							Gear Ratio
	K1	K2	B1	B2	B3	B4	B5	
Gear 1	1	0	0	0	0	0	1	6.154
Gear 2	1	0	0	0	0	1	0	4.269

Gear 3	1	0	0	0	1	0	0	2.408
Gear 4	1	0	1	0	0	0	0	1.675
Gear 5	1	0	0	1	0	0	0	1.322
Gear 6	1	1	0	0	0	0	0	1
Reverse Gear 1	0	1	0	0	0	0	1	-6.637

3.3 Surface tension Model

A lot of work has been done to form the governing equations and consequently an analytical model for flow between two flat plates. This has provided a framework to clutch model development because of the similarity in the equations and conditions. The work performed by Hashimoto et al [16] is considered as framework on which many clutch models have been developed [14]. The surface tension model is one such model which uses the equations proposed by Hashimoto.

A theoretical equation for drag torque was proposed by Kitabayashi [11] based on his experimental investigations. The drag torque equation is given as

$$T = \frac{N*\mu*\pi*(r_2^2 - r_1^2)*\omega*r_m^2}{h} \tag{11}$$

Where N refers to the number of friction discs, μ is the dynamic viscosity of the fluid, ω is the relative speed, h is the clearance between the discs, r_1 and r_2 are the inner and outer radius of the discs respectively. This equation (11) however accounts only for the Region I in Figure 4. This is the rising portion of the graph where drag torque is linearly proportional to the relative speed. This does not take into account the shrinking of oil film as the relative speed increases and also the effect of surface tension during low speeds.

The surface tension model has the following assumptions to be considered.

- Fluid is in steady state and is incompressible.
- The clutches are non-grooved.
- The plates operate in turbulent regime.

The governing equations for a fluid flow between two rotating plates based on Navier-Stokes equation is given as

$$-\frac{\rho V_\theta^2}{r} = \frac{\partial p}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} \tag{12}$$

$$0 = -\frac{1}{r}\left(\frac{\partial p}{\partial \theta}\right) + \frac{\partial \tau_{\theta z}}{\partial z} \tag{13}$$

$$\frac{1}{r}\left(\frac{\partial(rV_r)}{\partial r}\right)+\frac{1}{r}\left(\frac{\partial V_\theta}{\partial \theta}\right)+\frac{\partial V_z}{\partial z}=0 \qquad (14)$$

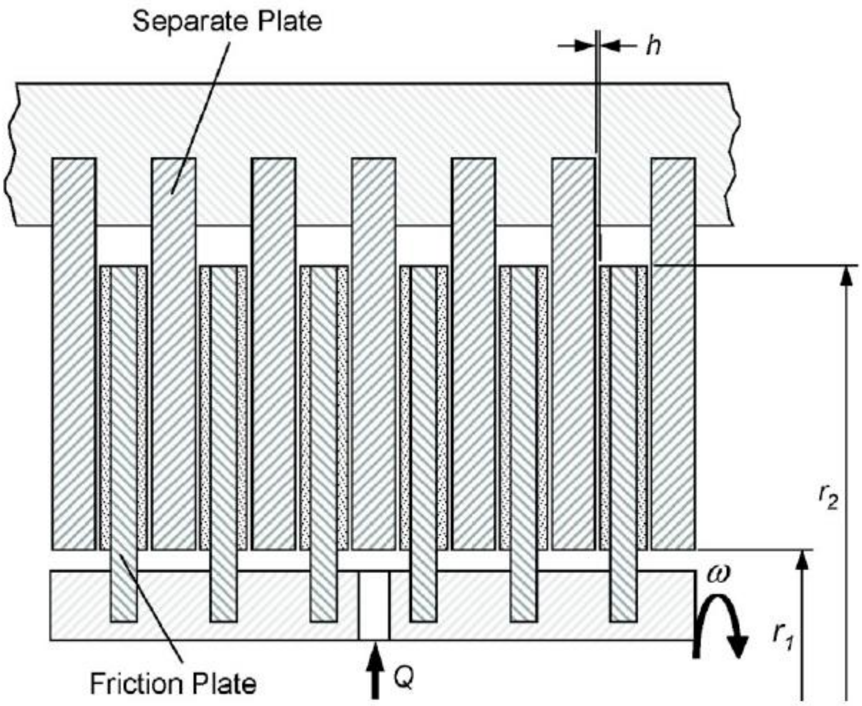

Figure 8: Schematic of Disengaged Clutch

The schematic of a disengaged clutch is as shown in Figure 8. Q here refers to the flow rate, h is the axial clearance, ω is the angular velocity, r_1 and r_2 are the inner and outer radii of the discs.

The boundary condition for the equations are given as

$V_r = V_r$ @ $z = 0$ and $V_r = 0$ @ $z = h$ (15)

$V_\theta = r\omega$ @ $z = 0$ and $V_\theta = 0$ @ $z = h$ (16)

$V_z = 0$ @ $z = 0$ and $V_z = 0$ @ $z = h$ (17)

Where V_r is the radial velocity component, V_z is the axial velocity component and V_θ is the circumferential velocity component.

Integrating the governing equation (12) to (14) in the axial direction and applying boundary conditions

$$-\frac{\rho}{r}\int_0^h V_\theta^2 dz = -\frac{\partial p}{\partial r}h + \tau_{rz}(h) - \tau_{rz}(0) \qquad (18)$$

$$0 = \tau_{\theta z}(h) - \tau_{\theta z}(0) \qquad (19)$$

$$\frac{\partial}{\partial r}\left(r\int_0^h V_r dz\right) + \frac{\partial}{\partial \theta}\left(\int_0^h V_\theta dz\right) = 0 \qquad (20)$$

Approximations are made for the shear stress terms

$$\tau_{rz}(h) - \tau_{rz}(0) = -\frac{\mu}{hG_r}V_{rm} \qquad (21)$$

$$\tau_{\theta z}(h) - \tau_{\theta z}(0) = -\frac{\mu}{hG_\theta}\left(V_{\theta m} - \frac{r\omega}{2}\right) \quad (22)$$

$$\int_o^h V_\theta^2 dz = hV_{\theta m}^2 + fr^2\omega^2 h \quad (23)$$

Where f, G_r and G_θ are turbulence coefficient factors.

A schematic of the clutch and the film of oil are as shown in the Figure 9. In order to calculate the drag torque, the shape or pattern of the oil film must be known. This is the most challenging part of an analytical model for the wet clutches. During low speeds of rotation of the disc, there exists a full film of oil because of surface tension effect. The surface tension forces as well as capillary effect cause the oil film to adhere to the discs. The drag torque can be easily determined considering the oil flow between the inner and outer radii of the discs. As the speed of rotation increases, the centrifugal forces begin to dominate over the surface tension forces and as a result of which the film of oil begins to diminish as shown in the Figure 9(a). The fraction of wetted area decreases because of the centrifugal forces that push the fluid radially outward. At this situation, calculation of drag torque becomes difficult because of the fluid behaviour and the rivulets developed within it. In order to compensate for this and also to include surface tension which was previously omitted by other models, an equivalent radius r_o is assumed according to the Figure 9(b). The resultant flow and drag torque values using this equivalent radius assumption equals to that of the original film and is also validated experimentally by Yuan.

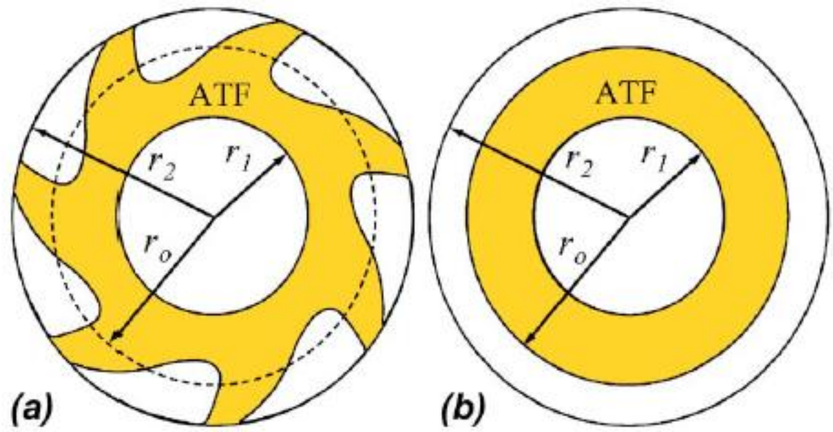

Figure 9: Schematic of clutch along with oil film

Substituting equations (21) to (23) into equations (18), the equation for the outer radius of the film is obtained as

$$\frac{\rho\omega^2}{2}\left(f + \frac{1}{4}\right)r_o^2 - \frac{\mu Q}{2\pi r_m h^3 G_r} r_o + \frac{\mu Q}{2\pi r_m h^3 G_r} r_1 - \frac{2\sigma cos\theta}{h} - \frac{\rho\omega^2}{2}\left(f + \frac{1}{4}\right)r_1^2 = 0 \quad (24)$$

Once the outer radius is known, the drag torque acting on each of the rotating plate can be obtained from equation (25)

$$T = 2\pi \int_{r1}^{r_o} \frac{\mu\omega r^3}{h} (1 + 0.0012\ Re_h^{0.94})\, dr \qquad (25)$$

Where T is the drag torque and Re_h is the characteristic Reynolds number.

3.4 Model based on a new method to determine the shrinking oil film

The main problem with the surface tension model is that it fails to predict the drag torque accurately at high rotating speeds. This model solves the problem by predicting the drag torque accurately even at high rotating speeds [15].

This model also has certain assumptions that are followed

- Oil is incompressible and in steady state
- Clutch discs have no grooves
- Gravity is neglected
- Flow between the clearances are laminar

The schematic of the clutch disengagement event is shown in the Figure 10

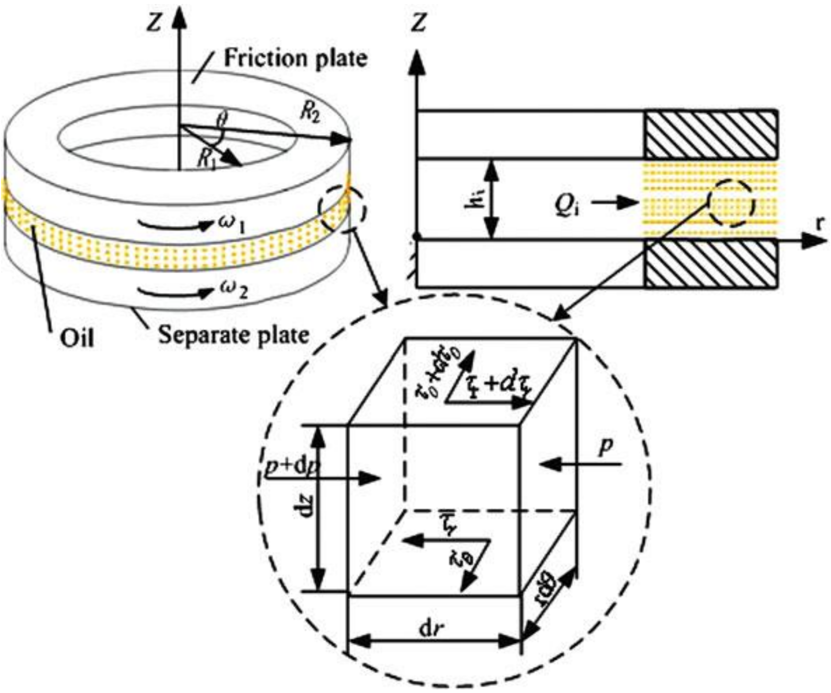

Figure 10: Clutch Disengagement event

The governing equations for fluid between the discs in cylindrical coordinates is given according to the Navier-Stokes equation as

$$-\frac{\partial p}{\partial r} + \frac{\mu\partial^2 v_r}{\partial z^2} = \rho(\frac{v_r\partial v_r}{\partial r} - \frac{v_\theta^2}{r}) \qquad (26)$$

$$\frac{\mu\partial^2 v_\theta}{\partial z^2} = \rho(\frac{v_r\partial v_\theta}{\partial r} + \frac{v_\theta v_r}{r}) \qquad (27)$$

$$\frac{\partial p}{\partial z} = 0 \qquad (28)$$

Integrating the governing equations in the radial direction and applying the boundary conditions, the flow rate equation for a full film of oil is obtained

$$Q = \frac{\frac{6\mu}{\pi h^3}\frac{lnR_1}{R_2}}{\frac{27\rho}{70\pi^2 h^2}(R_2^{-2}-R_1^{-2})} + \sqrt{\frac{(\frac{6\mu}{\pi h^3}ln\frac{R1}{R2})^2 - \frac{81\rho^2\omega^2(R_2^{-2}-R_1^{-2})(R_1^2-R_2^2)-540\rho(R_2^{-2}-R_1^{-2})\Delta p}{700\pi^2 h^2}}{\frac{27\rho}{70\pi^2 h^2}(R_2^{-2}-R_1^{-2})}}$$

(29)

The equation (29) represents the ideal flow rate equation or the needed flow rate that is required in order to maintain a full film of oil between the clutch discs. As long as this equation is satisfied, a full film of oil is said to exist. The ideal flow rate is directly dependent on the rotating speed. As the rotating speed increases, the amount of flow needs to increase to maintain a full film. On the other hand, actual flow is a constant and as a result of which the film of oil begins to shrink when the ideal flow rate exceeds the actual feed flow rate during high speeds of rotation.

The concept of equivalent radius is once again used in this model as well. It is defined according to the following two conditions- when the ideal flow rate is greater than the actual flow rate and vice versa.

For the 1st condition when $Q_i > Q, R_o = R_2$ (30)

In this condition, since the actual flow rate is greater, there will be a full film of oil always and hence the drag torque can be evaluated up till the outer radius of the disc.

However, when $Q_i < Q, R_o = \sqrt{\frac{Q_i}{Q}R_2^2 + R_1^2(1-\frac{Q_i}{Q})}$ (31)

During this condition, the amount of needed flow is greater than the actual flow itself and as a result of which the film of oil is no longer complete. The film begins to shrink and the equivalent radius R_o is evaluated using equation (31).

Once, the equivalent radius is known, the drag torque between each friction pair can be calculated using equation (32)

$$T = \frac{N\pi\mu\omega}{2h}(R_o^4 - R_1^4) \qquad (32)$$

3.5 Gearbox model with losses

The drag losses in the clutches and brakes within the gearbox are now found by using the clutch model. The model based on new method to determine the shrink film radius is used since the surface tension model is not so accurate at high speeds according to [15].

In order to determine the drag torque, the speed at which the clutches and brakes are rotating must be known. This is calculated for the ideal case and since the speeds remain the same, those speed values are used to determine the drag torque at each clutch and brake for a particular gear state.

During the gear state 1, clutches K1 and B5 are engaged and these two components do not have any drag torque. The remaining clutches and brakes have losses due to drag torque since they are disengaged and these losses are calculated using the clutch model according to the speeds at which they are rotating using equation (32).

As for the ideal case, torque equations are once again deduced from the basic torque relationship.

The equations are similar to that of the ideal case except that the losses in the disengaged clutches and brakes must also be included. This will give an indication of the losses through the entire gearbox due to disengaged clutches and brakes.

$$T_{s1} + T_{r1} + T_{c1} = 0 \tag{33}$$

Equation (33) is obtained considering the equilibrium of the total system for the 1st gear set. Similar equations can be written for the remaining set of gears as follows.

$$T_{s2} + T_{r2} + T_{c2} = 0 \tag{34}$$

$$T_{s3} + T_{r3} + T_{c3} = 0 \tag{35}$$

$$T_{s4} + T_{r4} + T_{c4} = 0 \tag{36}$$

$$T_{s5} + T_{r5} + T_{c5} = 0 \tag{37}$$

Equations (38) to (43) given below are obtained from the basic relationship of simple planetary gear set. The sun gear is considered to be the input, the ring gear is held stationary and the planetary carrier is considered to be the output.

$$\frac{T_{c1}}{T_{s1}} = 1 + z_{r1}/z_{s1} \tag{38}$$

Rearranging equation (38)

$$T_{c1} + T_{s1}\left(1 + \frac{z_{r1}}{z_{s1}}\right) = 0 \tag{39}$$

Similarly, equations can be written for the other planetary gear sets as follows.

$$T_{c2} + T_{s2}\left(1 + \frac{z_{r2}}{z_{s2}}\right) = 0 \tag{40}$$

$$T_{c3} + T_{s3}\left(1 + \frac{z_{r3}}{z_{s3}}\right) = 0 \tag{41}$$

$$T_{c4} + T_{s4}\left(1 + \frac{z_{r4}}{z_{s4}}\right) = 0 \tag{42}$$

$$T_{c5} + T_{s5}\left(1 + \frac{z_{r5}}{z_{s5}}\right) = 0 \tag{43}$$

Considering the equilibrium of torque along the ring gears R_1 and R_3 and the carrier C_2 .The Drag torque from the disengaged brake B_3 must also be included in the equation (44) as it is part of the system.

$$T_{r1} + T_{c2} + T_{r3} + T_{B3} = 0 \tag{44}$$

Considering the equilibrium of torque along the ring gears R_2, R_3 and R_5 and the carrier C_3 . Drag torque from Brake B_4 is also included into the equation (45)

$$T_{r2} + T_{c3} + T_{r4} + T_{r5} + T_{B4} = 0 \tag{45}$$

Considering the equilibrium of torque along the carrier C_1 and sun gear S_2. The system also consists of the disengaged components K_2 and B_1.

$$T_{c1} + T_{s2} + T_{K2} + T_{B1} = 0 \quad (46)$$

Considering the equilibrium of torque along the sun gears S_3 and S_4 and the clutch K1.

$$T_{s3} + T_{s4} - T_{K1} = 0 \quad (47)$$

There is a negative sign considered for the clutch K1 since the reaction torque of the clutch is considered here.

Considering the equilibrium of torque along the sun gear S_5, carrier C_4 and the output shaft.

$$T_{c4} + T_{s5} - T_{op} = 0 \quad (48)$$

The output shaft will always rotate in the opposite direction as that of the input shaft and hence the torque is considered negative.

The equilibrium of torque between the sun gear S_1 and the disengaged brake B_2 is considered in equation (49).

$$T_{s1} + T_{B2} = 0 \quad (49)$$

The drag torques calculated at all the disengaged clutches and brakes using the clutch model are represented by equations (50) to (54).

$$T_{B1} = a \quad (50)$$

$$T_{B2} = b \quad (51)$$

$$T_{B3} = c \quad (52)$$

$$T_{B4} = d \quad (53)$$

$$T_{K2} = e \quad (54)$$

Since the brake B5 is engaged, there is a torque through the brake and also the planetary carrier C_5 to which it is connected.

$$T_{c5} + T_{B5} = 0 \quad (55)$$

Finally, considering the equilibrium of torque between the clutches K1, K2 and the input shaft

$$T_{K1} - T_{K2} = -T_{ip} \quad (56)$$

Using equations (33) to (56), the torque at the output shaft of the gearbox T_{op} is determined based on an input torque of 1500 Nm.

The torque output calculated here is compared to that of the torque output during the ideal case. The difference between the two will give the torque losses in the gearbox due to clutches and brakes.

The power losses are then calculated by multiplying the torque losses with the input speed according to equation (57).

$$P_{losses} = T_{losses} * N_{engine} \quad (57)$$

4 Results and Discussion

4.1 Comparison between the two models

A multi disc clutch and a typical ATF were considered. The properties of the disc and fluid are tabulated below. Using this data, the clutch drag was calculated for both the models- surface tension model and the model with new method for calculating shrink radius. The results are shown below.

Table 4: Clutch Disc Properties

Radius of Inner disc	178 mm
Radius of Outer disc	218 mm
Clearance between discs	0.2 mm
Number of discs	10

Table 5: Automotive Transmission Fluid Properties

Density	835 kg/m^3	820 kg/m^3	810 kg/m^3
Dynamic Viscosity	0.0167 Ns/m^2	0.0082 Ns/m^2	0.0045 Ns/m^2
Temperature	60°C	80°C	100°C

The Figure 11 shows a plot of clutch drag v/s relative speed for the surface tension Model. The clutch drag increases linearly from 71 Nm up to a value of 99 Nm. This represents the region I of a typical drag torque curve as shown in Figure 4. The maximum drag torque is attained at 900 rpm. This is the critical speed above which there is a decrease in drag torque. It is at the critical speed where a transition takes place and the full film of oil is no longer formed and begins to diminish. It is due to this shrinking radius that the drag torque also decreases. However, beyond 1100 rpm the drag torque approaches nearly zero which is not the case in real life situation. The model therefore cannot be considered to be completely accurate above 1000 rpm relative speed.

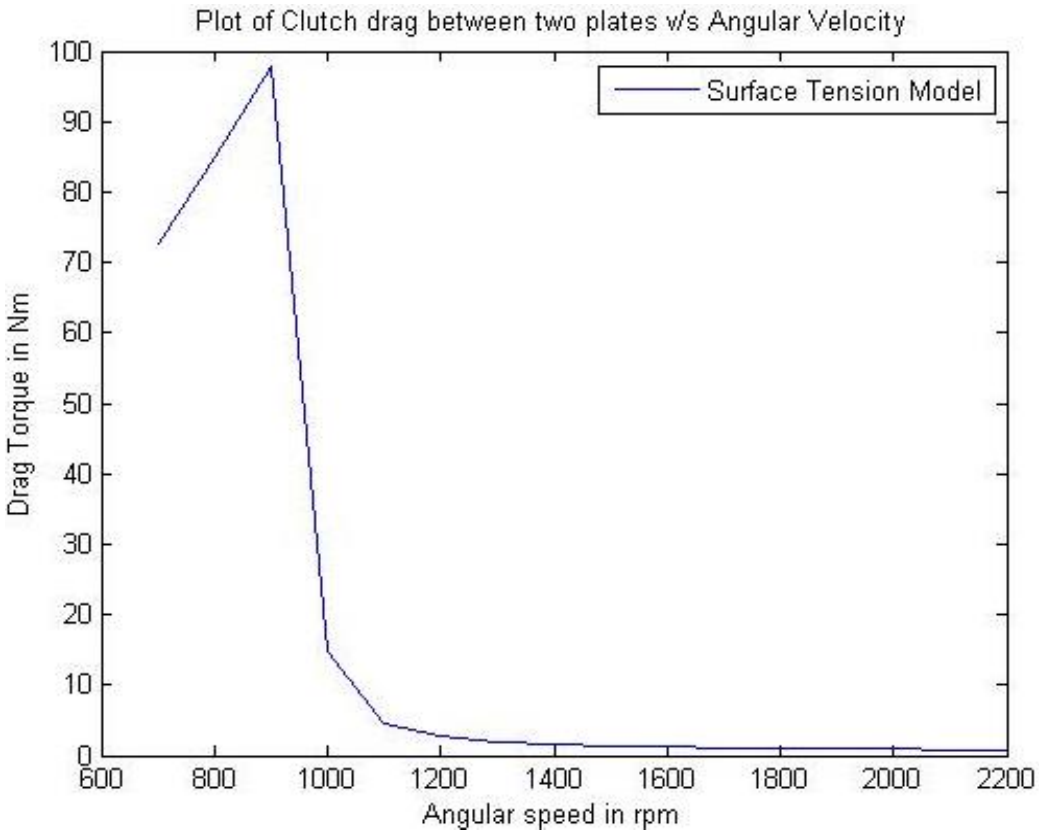

Figure 11: Plot of clutch Drag v/s Relative Speed for Surface Tension Model

The Figure 12 shows the plot of equivalent radius v/s angular velocity for the surface tension model. This Figure 12 gives a better understanding of how the oil film behaves as the speed is increased. Initially there is no change in the equivalent radius of the film. This is because the flow rate is adequate to maintain a complete film in between the discs. At 900 rpm, the equivalent radius begins to decrease because the flow rate is not sufficient to maintain a full film and hence the film begins to shrink. This co-relates well with the previous Figure 11 where in the clutch drag also begins to decrease at 900 rpm. The decrease in equivalent radius leads to a decrease in the drag torque which is similar to region II in a typical drag torque curve shown in Figure 4.

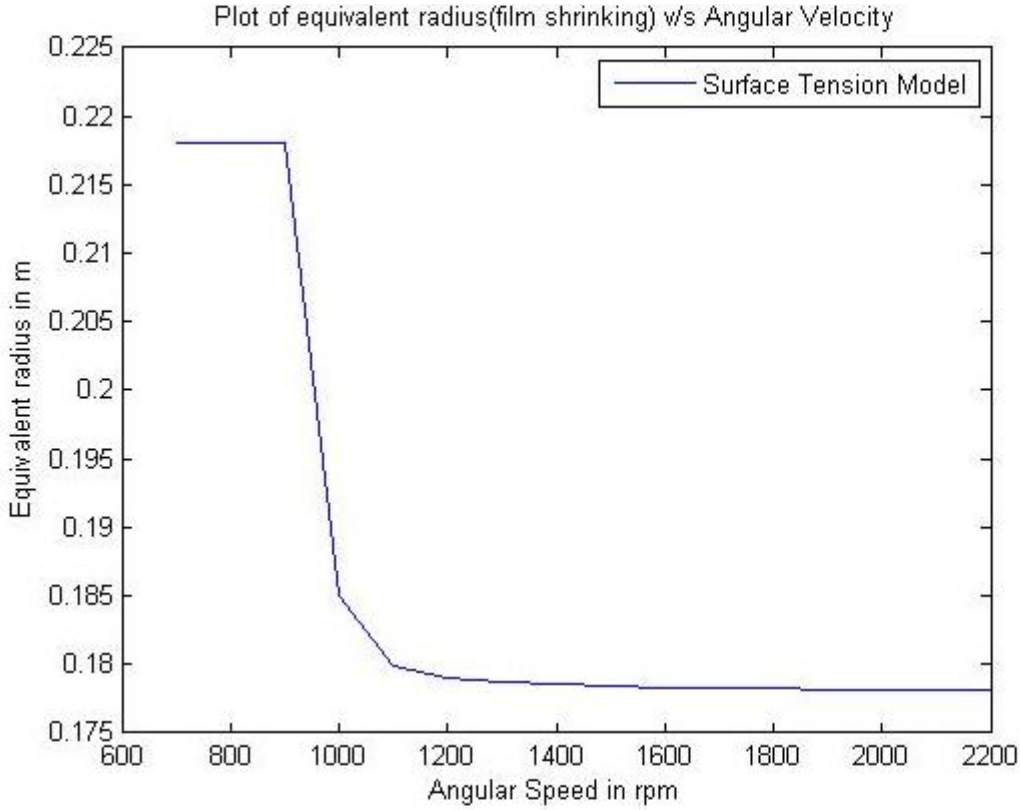

Figure 12: Plot of equivalent radius v/s Angular Velocity for Surface Tension Model

Similar plots are made for the model with a new method to calculate shrink film radius. As seen from Figure 13 and Figure 14, it can be seen that the critical speed here is at 1000 rpm. The clutch drag obtained from this method shows more resemblance to the typical drag torque curve as shown in Figure 4. Also, this model predicts the drag torque more accurately at higher speeds beyond 1000 rpm as the values do not approach zero in comparison to the previous model.

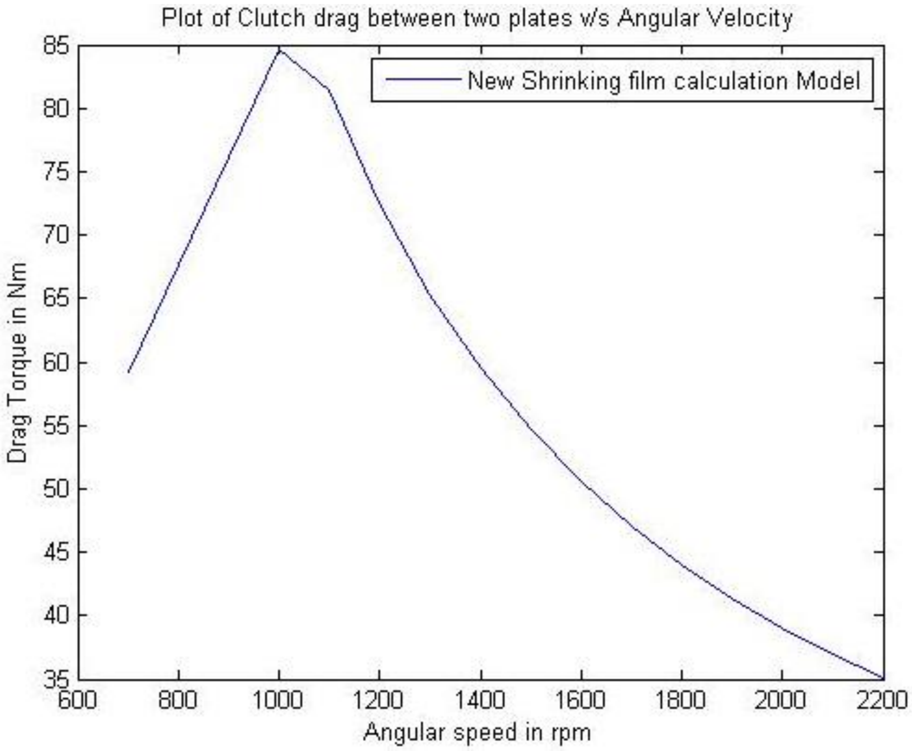

Figure 13: Plot of clutch Drag v/s Relative Speed for New Shrink film Model

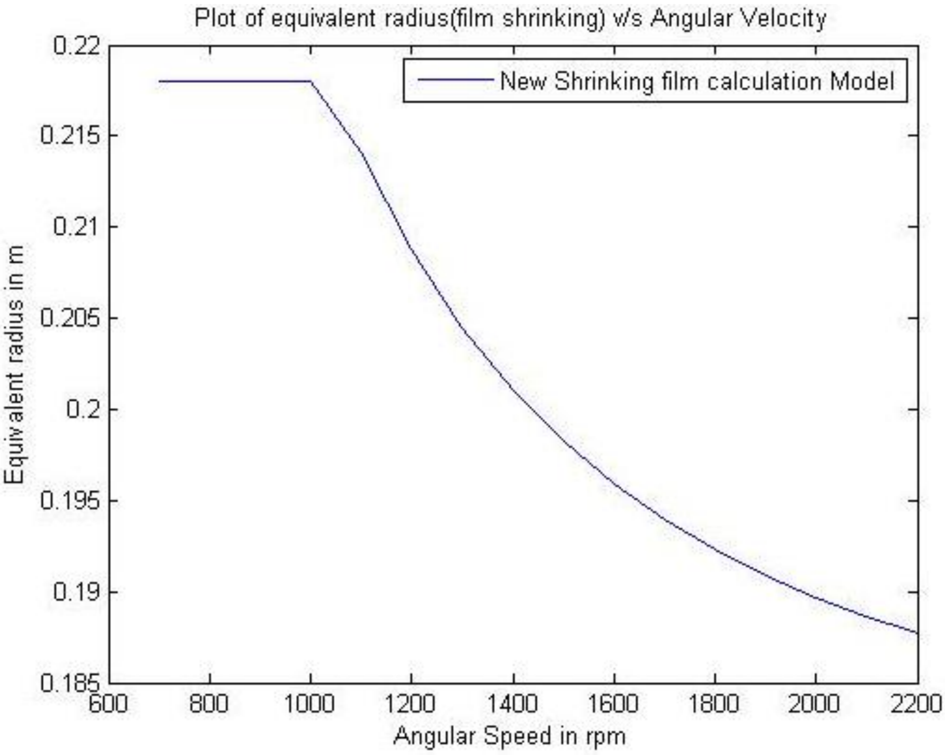

Figure 14: Plot of equivalent radius v/s Angular Velocity for New Shrink film Model

The Figure 15 shows a comparison between the actual and ideal flow rate for a clutch disc. The actual flow rate is always a constant used in this model. It is represented by a dashed line. The ideal flow rate or the needed flow rate is the amount of flow that is required in order to maintain a full film of oil between the discs. It is strongly dependent on the relative speed. As the relative speed increases, the flow rate required also increases. At the point where they coincide in the Figure 15, the film begins to diminish due to the lack of flow. This means that the oil is being drained out from the clutch and as a result of which clutch drag begins to decrease.

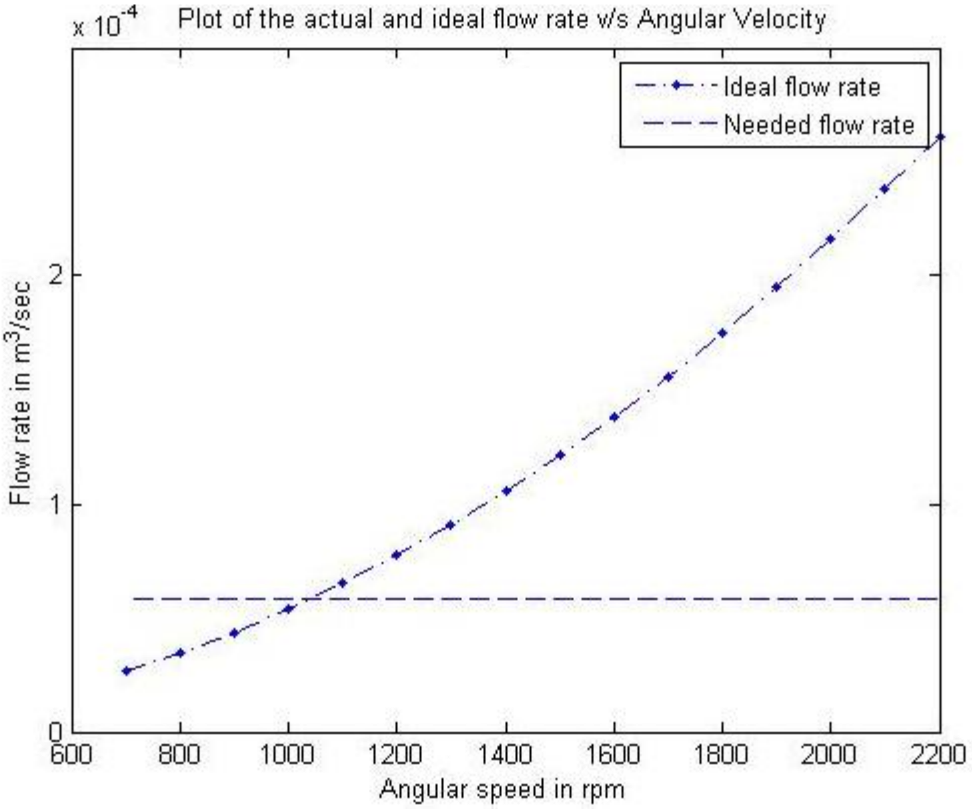

Figure 15: Comparison between Actual and Ideal flow rate for a clutch disc

A comparison between drag torque obtained using the surface tension model, the model with a new method to calculate the shrink film radius and the theoretical drag torque is plotted against relative speed as shown in Figure 16. The theoretical drag torque increases linearly with relative speed and represent the region I of a typical drag torque curve. From the Figure 16, it can be seen that the region I for the theoretical drag toque and the new shrinking film calculation model coincides. This is a very good evidence to consider that the new shrink film model is accurate in order to represent the losses due to clutch drag.

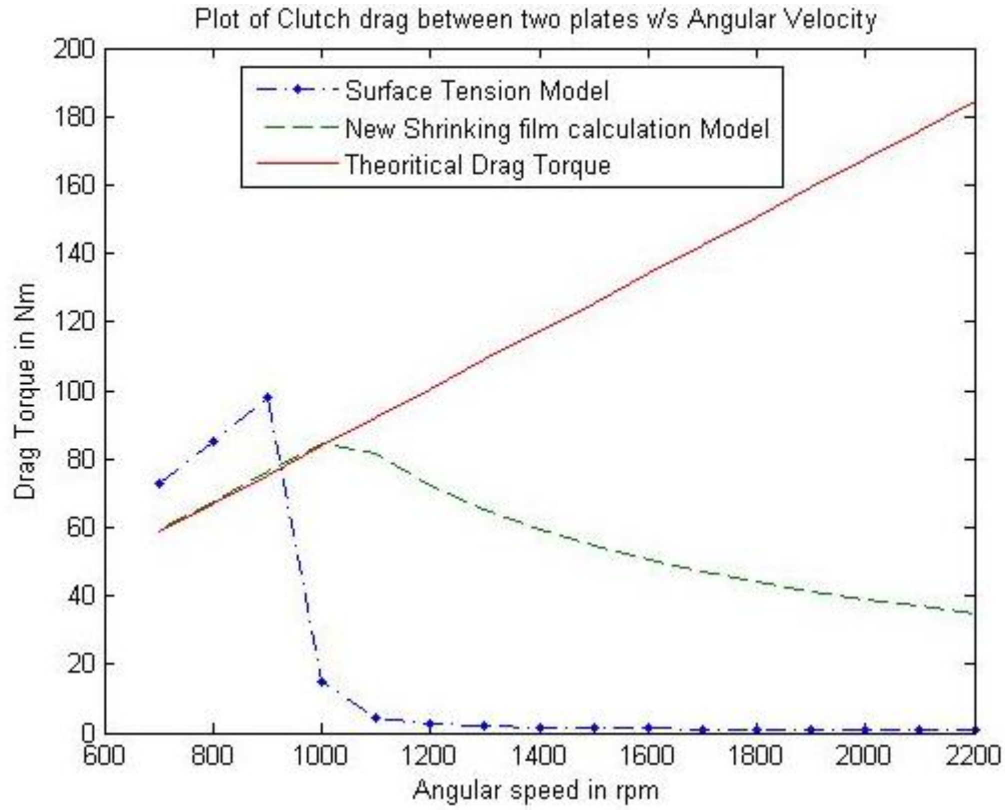

Figure 16: Comparison of Drag torque between two models and theoretical equation

4.2 Gearbox model with losses

The torque through all the planetary components as well as the multi disc clutches and brakes are calculated and the drag losses are also considered. The torque through the disengaged components are not considered zero unlike in the ideal case but are calculated according to the relative speed at which it rotates using the wet clutch model.

A range of input speed is considered from 700 RPM to 1500 RPM. The torques through the entire gearbox is calculated using equations (33) to (56). The ideal torque through the gear box is also previously calculated. The difference between the two gives the losses in the entire gearbox due to clutch drag.

4.2.1 Standard settings for parameters

The torque losses for Gear 1 are plotted below in Figure 17. A temperature of 80 degrees is considered for the ATF. The flow rate is varied linearly with the engine speed. The flow rate is varied from 1-5 litres/minute. A mean clearance is considered between the discs. Also, the plates are considered to be grooved and a factor of 0.95 is used. The factor 0.95 is an engineering approximation made in order to account for the grooves. The presence of grooves in reality leads to a decrease in area of the discs and in order to account for this reduction in clutch area, this area factor is used. This relates to modelling a clutch with grooves to a certain extent and leads to decrease in the drag torque. These are used as standard settings for calculating the losses.

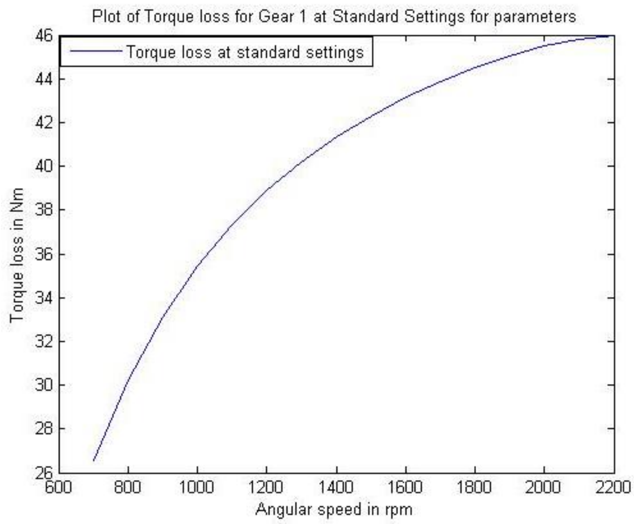

Figure 17: Plot of Torque loss at Standard settings for Gear 1

4.2.2 Effect of clearances on losses

The losses are now calculated by varying the clearances. The losses are calculated at maximum and minimum clearance play in addition to the mean clearance. This is done to see the effect of clearance on losses.

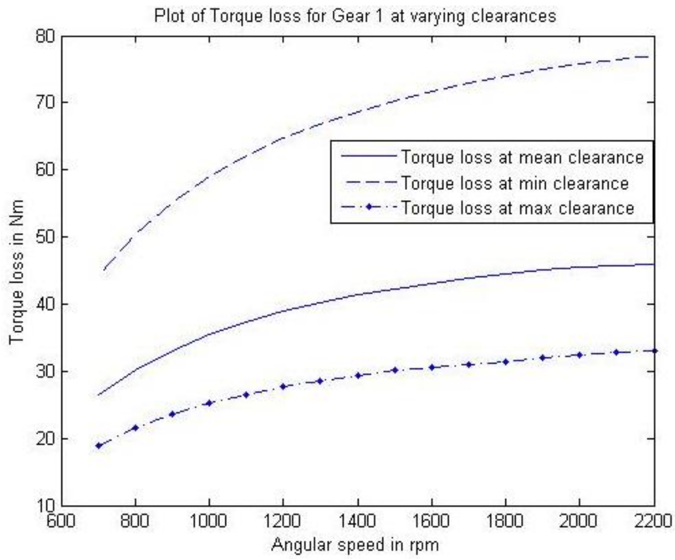

Figure 18: Plot of Torque losses for Gear 1 at varying clearances

It can be clearly seen from Figure 18 that the losses are considerably lesser when maximum play is used. At minimum play the drag losses are highest and as the clearance is increased, the drag losses decrease gradually and are the least at maximum clearance play. The main reason for high drag torque at minimum clearance play is because when the discs are closer to each other, there is greater coverage of oil layer over the clutch discs due to wall effects. This causes the oil to be retained for longer time but for losses to be minimized the oil needs to be drained out as quickly as possible which happens at much faster rate when there is maximum clearance.

4.2.3 Effect of flow rate

The flow rate is varied and its effect on drag losses is tabulated below.

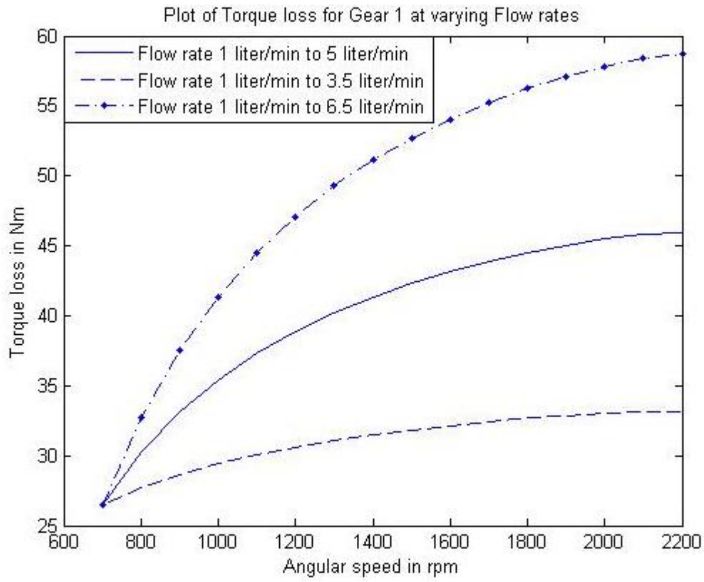

Figure 19: Plot of Torque loss for Gear 1 at varying flow rates

From the Figure 19 it can be seen that initially the losses are same since the flow rate is 1 litre/min for both the cases. For minimum flow rate (1-3.5 l/min), the losses are lesser as the engine speed increases. This is because, for disengagement there should be minimum amount of flow and the oil should be drained out of the clutch packs as fast as possible. So having lesser amount of oil flow rate leads to lesser losses. When the flow rate is higher (1-6.5 l/min), the losses are much higher at higher speeds. The large flow rate helps by maintaining the oil film layer for a longer period of time by replacing readily any oil that has been lost. This leads to higher drag torque.

4.2.4 Effect of grooves

The effect of grooves is considered and compared in Figure 20. The losses are higher when no grooves are considered. This is because when grooves are considered, it acts as a flow path for the fluid and tends to flow in these grooves because it offers a lesser flow resistance. This helps to drain the fluid from the clutches as fast as possible thereby ensuring lesser losses. The grooves also cause the shear stresses generated by the fluid against the rotating plates to reduce which also is a reason for lesser losses.

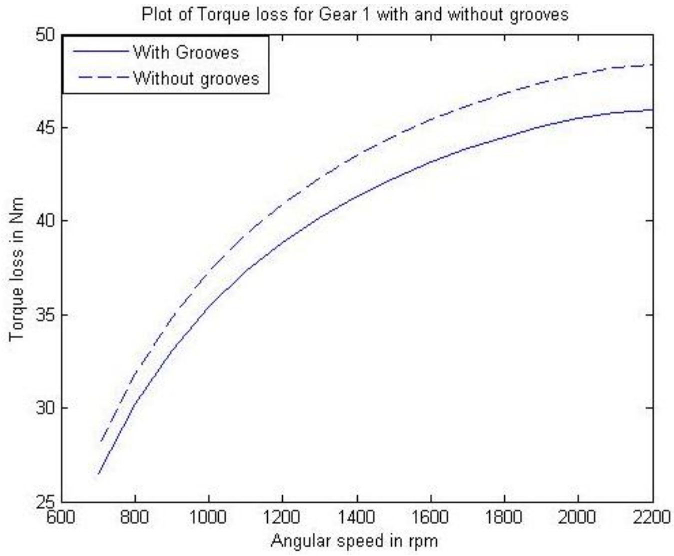

Figure 20: Plot of Torque losses for Gear 1 with and without grooves

4.2.5 Effect of temperature

Temperature plays a very important role in the losses that occur in a gear box. The effect of temperature is shown below.

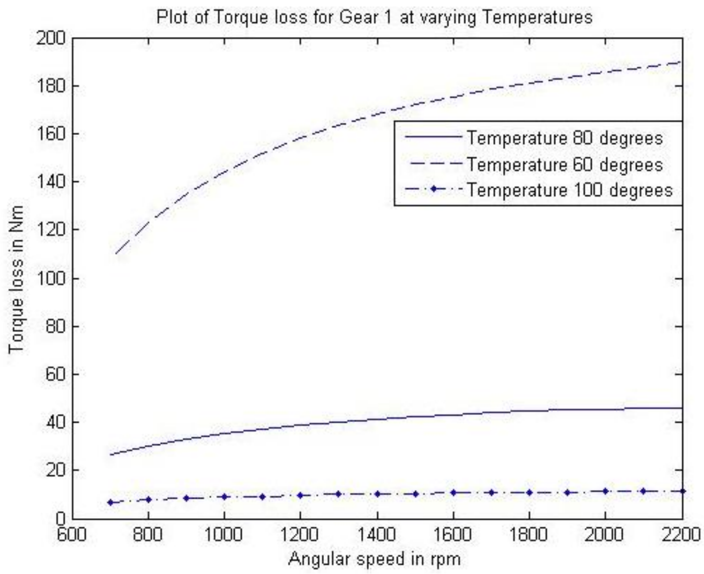

Figure 21: Plot of Torque losses for Gear 1 at varying Temperatures

From Figure 21, it can be clearly seen that the drag losses decrease considerably with an increase in temperature. Higher temperatures are favourable to decrease the losses. This is because with an increase in temperature, the viscosity of the fluid decreases. This enables the fluid to drain out of the discs at a much faster rate during disengagement event.

4.3 Design optimization

In order to perform any design changes, the initial torque carrying capacity of the clutches and brakes needs to be determined. The design optimization should not compromise on the torque carrying capacity since torque transfer is the main purpose of these devices. The torque carrying capacity of the clutches and brakes are calculated using the equation (58) and are tabulated below.

$$T_{capacity} = F_{totalapplied} * r_m * N_{friction} * \mu_{friction} \qquad (58)$$

Where $T_{capacity}$ is the torque capacity, $F_{totalapplied}$ is the total force applied on the face of the clutch discs, r_m is the mean radius of the discs, $N_{friction}$ is the number of friction discs and $\mu_{friction}$ is the coefficient of friction of the friction material.

Table 6: Torque carrying Capacity of existing Clutches and Brakes

Component	K1	K2	B1	B2	B3	B4	B5
Torque Capacity in N	5395	5395	3565	2883	7338	15929	10618

The values shown in

*Table **6*** are the bench mark values to be maintained and any design changes made should ensure that the Torque capacity still remains within these values.

The drag torque in clutches and brakes are affected by different parameters and their effects are seen in 4.2.2 to 4.2.5.

The number of clutch discs and size of the discs also play a very important role in drag losses. The number of discs has a major impact on losses since the losses are directly proportional to the number of discs.

One pair of discs is removed in all the clutches and brakes and the drag losses were evaluated. The losses decreased considerably. However, the torque capacity of the clutches and brakes was compromised. The torque capacity is increased by increasing the radius of the discs and clearances between the plates were also increased.

The new design for the clutches and brakes with a decrease in number of plates and an increase in size of the discs showed an improvement in efficiency when compared to the original standard settings. A comparison of the losses is made below.

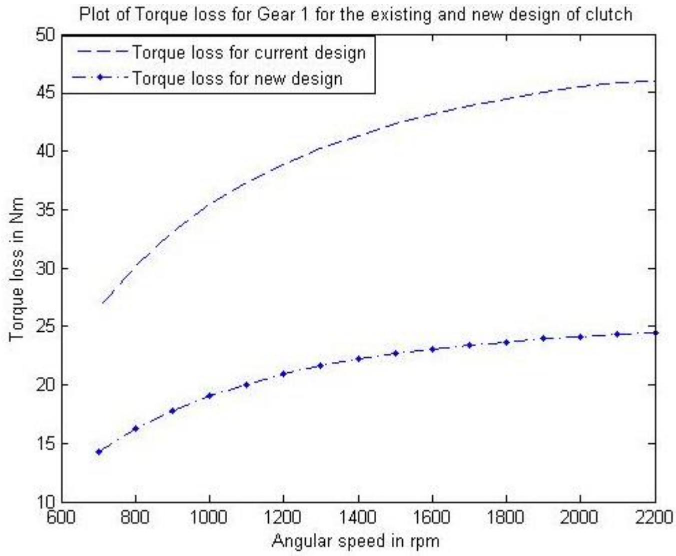

Figure 22: Comparison of Torque losses between the current and new design

The torque capacity of the new design also is not compromised and is found to be within limits. The torque capacity of the new design is also tabulated.

Table 7: Torque carrying Capacity of new Clutches and Brakes

Component	K1	K2	B1	B2	B3	B4	B5
Torque Capacity in N	5406	5406	3161	2248	6868	15442	9264

The losses are reduced by nearly 46 % with the new design. This could be one of the ways to improve the transmission efficiency with respect to clutches and brakes.

5 Conclusion

Two methods to analytically model this clutch drag were compared and evaluated. First, a model based on surface tension effects was used to calculate clutch drag. Second, a model with a new approach to determine the shrink radius of the oil film was evaluated. A comparison between the two models showed that the second model which used a new approach to determine the shrink radius gave much better results. This model was hence chosen in order to find out the losses in a six speed automatic transmission. The speed and torque of all the gears and rotating components are calculated at ideal condition assuming no losses. With the help of the wet clutch model, the losses in the transmission with respect to clutches and brakes are calculated. A comparison between the two cases is made to determine the losses through the entire gearbox due to clutch drag. The effect of various parameters like clearance, grooves, flow rate and temperature on clutch drag was evaluated. This gave a better understanding of clutch drag so as to optimize the design. The optimization was carried out by reducing the number of discs in each clutch and brake and increasing the size of the discs so that there is no compromise in torque capacity. This design change produced lesser losses compared to the original design and showed nearly 46 percent decrease in losses. However, the design of clutches is always a compromise between the engagement and disengagement event because the effect of parameters varies in both the states.

The transient operation of multi-plate clutch between the engagement and disengagement modes is a very tricky phenomenon and there is always a trade-off for choosing the right parameters. This is because certain parameters like flow rate for instance needs to be high for engagement event. This aids in sufficient cooling and lubrication of the clutches during high temperature scenarios. On the other hand, the flow rate is supposed to be low during disengagement because oil should be drained out completely as quickly as possible in order to reduce the drag losses. The engineers need to arrive at a compromise in such a situation so as to have both sufficient cooling and lubrication and also lesser drag losses.

The following can be concluded from the thesis:

- Clutch design is very challenging and a lot of effort needs to be put into determining the right geometry. The number of discs plays the most important role in that aspect and maintaining a minimum number would help reduce drag torque considerably provided the torque carrying capacity is not compromised.
- The increase in clearance between discs also leads to a reduction in drag losses. But there is always a critical limit up to which this clearance can be increased. The tolerances on the clearances should be within this critical limit to ensure minimized losses.
- The size of the discs can be minimized in order to minimize losses. However, the improvement gained by reducing the size of the discs is not as much when compared to reducing the number of plates. A balance has to be found when considering the geometry of the plates.
- Better clutch groove designs and better friction material for the friction discs helps to minimize drag losses.

- The flow rate of transmission fluid as mentioned above also has to be compromised for good engagement as well as disengagement characteristics.
- The automotive transmission fluid also plays a very important role in drag losses. The viscosity of the fluid should be as minimum possible especially at low temperatures where the losses are found to be maximum.

6 Future work

These are some of the possible areas of future work that can be carried out

- With the rapid advancement in Computational Fluid Dynamics, it is a viable tool that can be used to model the wet clutch and determine losses. CFD enables one to include grooves which are major advantage when compared to the many of the theoretical models present today which fails to include grooves or has a radial groove assumption. The CFD model can be a very efficient tool to validate the theoretical results or can also be coupled with experiments.
- The temperature dependence of the losses can be investigated further. The present model accurately predicts losses at working temperature of the fluids ranging from 40 degrees and above. The losses predicted are very high and inaccurate at temperatures below 40 degrees. This could be looked into.
- The model right now predicts the losses for the particular six speed transmission data used. This could be expanded further into a generic model where in the model could determine losses for any given transmission.

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8 Appendix

8.1 Appendix I

8.1.1 Project Plan

Autum 2013			Week				
Activities	Start	Finish	days	4	5	6	7
Introduction	21/1/2013	21/1/2013	1				
Background analysis	22/1/2013	15/2/2013	25				
GATE 1 meeting	18/2/2013	18/2/2013	1				

Activities	Start	Finish	days	8	9	10	11	12	13	14
Basic Speed and Torque Calculation	18/2/2013	1/3/2013	12							
Numerical model for the drag losses	4/3/2013	29/3/2013	25							
Calculation of the losses in Transmission due to clutches	1/4/2013	12/4/2013	12							
GATE 2 meeting	15/4/2013	15/4/2013	1							

Activities	Start	Finish	days	16	17	18	19	20	21
Estimating the standard settings to compare losses with Experimental data	15/4/2013	3/5/2013	25						
Optimization of the Design to improve the Efficiency	6/5/2013	10/5/2013							
GATE 3 meeting	13/5/2013	13/5/2013	1						
Report writing Rough Draft	14/5/2013	24/5/2013	12						

Activities	Start	Finish	days	22	23	24	25	26	27
Report writing Final Draft	26/5/2013	31/5/2013	6						
Presentation	2/6/2013	8/6/2013	7						

8.2 Appendix II

8.2.1 Clutch Drag Models (Matlab codes)

Numerical calculations for clutch drag %%

Input parameters%

```
clear all
clc
r_1=178*10^-3;                    % inner radius of disk in m
r_2=218*10^-3;                    % outer radius of the disc in m
h=0.2*10^-3;                      % clearance between the two discs
Q_o=5.83*10^-5;                     % Actual flow rate in m^3 per second
Q_otest=[5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.83,5.
83]*10^-5;
rho=820;                        % density of ATF in kg/m^3
mu=0.0082;                         % Viscosity in Ns/m^2
sigma=0.030;                      % Surface tension in N/m
% omega=[300:100:2200]*(pi/30);     % Angular velocity in rad/second
% omega_rpm=[300:100:2200];           % Angular velocity in RPM
omega=[700:100:2200]*(2*pi/60);     % Angular velocity in rad/second
omega_rpm=[700:100:2200];      % Angular velocity in rpm
theta=0*(2*pi/60);                % Contact angle in rad
r_m=(r_1+r_2)/2;                  % Mean radius in m
N=10;                   % Number of plates
```

Method 1 based on surface tension %%%

```
Re_h=(rho.*omega*r_1*h)/mu;             %Calculate characteristic Reynold's number
```

Defining a cell array %%%

```
m=zeros(16,1);
G_r=m;
f=m;
syms r_o_1
syms r
s=cell(16,1);
r_o_final_1=m;
T_1=m;
T_final_1=m;

for i=1:16
```

```
    if Re_h(i)>=500
        f(i)=0.885.*Re_h(i).^-0.367;         % Turbulence coefficient number 1
    else
        f(i)=0.09;
    end
    G_r(i)=1/12*(1+(0.00069.*Re_h(i).^0.88));    % Turbulence coefficient number 2
```

Calculate Effective radius %%%

```
    s{i}= vpasolve(((rho.*(omega(i).^2/2)).*(f(i)+(1/4)).*(r_o_1^2))-
((mu*Q_o.*r_o_1)/(2*pi*r_m*h^3.*G_r(i)))-((rho.*(omega(i).^2/2)).*(f(i)+(1/4))*r_1^2)-
((2*sigma*cos(theta))/h)+((mu*Q_o*r_1)/(2*pi*r_m*h^3.*G_r(i)))==0,r_o_1);
```

Choose one effective radius between r1 and r2 %%

```
    a=double(s{i}(1)>=r_1) && double(s{i}(1)<=r_2);
    b=double(s{i}(2)>=r_1) && double(s{i}(2)<=r_2);

        if a
        r_o_final_1(i)=s{i}(1);
        elseif b
        r_o_final_1(i)=s{i}(2);
        else
            r_o_final_1(i)=r_2;
        end


T_1(i)=int(((mu.*omega(i)*r^3)/h)*(1+(0.0012.*Re_h(i)^0.94)),r_1,r_o_final_1(i));
%Clutch drag Torque for single rotating plate in Nm
        T_final_1(i)=2*N*pi.*T_1(i);    % Clutch Drag Torque for the entire clutch
pack in Nm

end

figure(1)
plot (omega_rpm,r_o_final_1);
title('Plot of equivalent radius(film shrinking) v/s Angular Velocity');
xlabel('Angular Speed in rpm');
ylabel('Equivalent radius in m');
legend('Surface Tension Model');
figure(2)
plot (omega_rpm,T_final_1);
title('Plot of Clutch drag between two plates v/s Angular Velocity');
xlabel('Angular speed in rpm');
ylabel('Drag Torque in Nm');
legend('Surface Tension Model');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%% Method 2 based on centrifugal effect %%%%%%

for i=1:16
```

Flow rate equation when there is full film (Needed flow rate equation based on rotating speed)

```
Q_i(i)=((((6*mu)/(pi*h^3))*(log(r_1/r_2)))/(((27*rho)*(r_2^-2-r_1^-
2))/(70*pi^2*h^2)))+((sqrt((((6*mu)/(pi*h^3))*(log(r_1/r_2)))^2-
(((81*rho^2.*omega(i).^2)*(r_2^-2-r_1^-2)*(r_1^2-
r_2^2))/(700*pi^2*h^2)))))/((27*rho*(r_2^-2-r_1^-2))/(70*pi^2*h^2)));
```

Determine the equivalent radius based on the flow rate %%%

```
  if Q_o>=Q_i(i)
      r_o_2(i)=r_2;
  else
      r_o_2(i)=sqrt(((Q_o./Q_i(i))*r_2^2)+(r_1^2*(1-(Q_o./Q_i(i)))));
  end
```

Drag Torque of clutch/brake %%%

```
T_2(i)=((N*pi*mu.*omega(i))/(2*h))*(r_o_2(i).^4-r_1^4);

%%%% Theoritical Drag Torque %%%%

 T_theoritical(i)=(N*mu*pi*(r_2^2-r_1^2)*r_m^2.*omega(i))/h;

end


figure(3)
plot(omega_rpm,T_2);
title('Plot of Clutch drag between two plates v/s Angular Velocity');
xlabel('Angular speed in rpm');
ylabel('Drag Torque in Nm');
legend('New Shrinking film calculation Model');
figure(4)
plot(omega_rpm,r_o_2);
title('Plot of equivalent radius(film shrinking) v/s Angular Velocity');
xlabel('Angular Speed in rpm');
ylabel('Equivalent radius in m');
legend('New Shrinking film calculation Model');
figure(5)
plot(omega_rpm,Q_i,'.-.');
hold on
plot(omega_rpm,Q_otest,'--');
title('Plot of the actual and ideal flow rate v/s Angular Velocity');
xlabel('Angular speed in rpm');
ylabel('Flow rate in m^3/sec');
legend('Ideal flow rate','Needed flow rate');
```

8.2.2 Model to calculate Drag losses and Torque capacity in the 6 Speed Transmission

Calculation method 3 (Modelling and parametric study journal) %%%

```
clear all
clc

       %   K2     B1      B2      B3      B4    B5
ratios=[2.0785,1.0785,2.4356,0.4322,0.0937,0;            % Speed of Clutchs and Brakes
in Gear 1(Ratio multiplied by Engine Speed)
        1.9004,0.9004,2.1413,0.3095,0,-0.0857;           % Speed of Clutchs and Brakes
in Gear 2(Ratio multiplied by Engine Speed)
        1.4512,0.4512,1.3988,0,-0.2364,-0.3018;          % Speed of Clutchs and Brakes
in Gear 3(Ratio multiplied by Engine Speed)
        1,0,0.653,-0.3109,-0.4738,-0.5189;               % Speed of Clutchs and Brakes
in Gear 4(Ratio multiplied by Engine Speed)
```

```
        0.605,-0.395,0,-0.5831,-0.6817,-0.7089;          % Speed of Clutchs and Brakes
in Gear 5(Ratio multiplied by Engine Speed)
        1,-1,-1,-1,-1,-1];                               % Speed of Clutchs and Brakes
in Gear 6(Ratio multiplied by Engine Speed)

             %   K1    B1    B2     B3     B4    B5
    ratios_rev=[1.9272,-1,-2.2584,-0.4008,-0.0869,0];    % Speed of Clutchs and Brakes
in Reverse Gear (Ratio multiplied by Engine Speed)
    omega_ip=[700:100:2200]*(2*pi/60);                   % Engine Speed in RPM
    range=1:6;
```

Determine which gear to calculate the losses for %%

```
gearnumber=input('Enter the gear number:');

        %%%% Gear 1-6 %%%%
    if ismember(gearnumber,range(:))

        omega_k1_final=omega_ip;
        for m=1:6
       omega_final(m,:)=ratios(gearnumber,(m))*omega_ip;
        end


    else

        %%%% Gear RG %%%%%
        for m=1:6
        omega_final(m,:)=ratios_rev(m)*omega_ip;
        end

    end

%%%%%Input Parameters%%%%%
Q_o=linspace(1.5*10^-5,8.33*10^-5,16);    % Actual flow rate in m^3/sec
rho=[860,845,835,820,810,795];            % density of ATF in kg/m^3
nu=[60,25,20,10,5,2.5]*1*10^-6;           % Absolute Viscosiy in m^2/s
mu=[];                                    % Dynamic Viscosity in Ns/m^2
mu=rho.*nu;



        %k2   b1  b2   b3   b4  b5
h_min=[0.19,0.3,0.37,0.43,0.4,0.58]*10^-3;     % min clearance play for clutches and
brakes in m
h_mean=[0.22,0.33,0.40,0.45,0.42,0.62]*10^-3;% mean clearance play for clutches and
brakes in m
h_max=[0.24,0.36,0.45,0.48,0.44,0.65]*10^-3;   % max clearance play for clutches and
brakes in m
% h_mean=[0.26,0.38,0.5,0.52,0.5,0.7]*10^-3;
 N=[10,8,6,10,12,8];                          % Number of plates for clutches and
brakes
 N_friction=[5,4,3,5,6,4];                    % Number of friction discs for clutches
and brakes
%  N=[8,6,4,8,10,6];
%  N_friction=[4,3,2,4,5,3];
```

```
r_1=[178,254,272,272,284,284]*10^-3;         % Inner radius of friction discs for all
clutches and brakes in m
r_2=[218,294.5,316,316,328,328]*10^-3;       % Outer radius of friction discs for all
clutches and brakes in m
%  r_1=[228,304,322,322,334,334]*10^-3;
%  r_2=[268,344.5,366,366,378,378]*10^-3;

for k=1:6
     for j=1:6
         for i=1:16
```

Determine the needed flow rate %%%

```
Q_min(j,i,k)=((((6*mu(k))/(pi*h_min(j)^3))*(log(r_1(j)/r_2(j))))/(((27*rho(k))*(r_2(j)
^-2-r_1(j)^-
2))/(70*pi^2*h_min(j)^2)))+((sqrt((((6*mu(k))/(pi*h_min(j)^3))*(log(r_1(j)/r_2(j))))^2
-(((81*rho(k)^2.*omega_final(j,i).^2)*(r_2(j)^-2-r_1(j)^-2)*(r_1(j)^2-
r_2(j)^2))/(700*pi^2*h_min(j)^2))))/((27*rho(k)*(r_2(j)^-2-r_1(j)^-
2))/(70*pi^2*h_min(j)^2)));

Q_mean(j,i,k)=((((6*mu(k))/(pi*h_mean(j)^3))*(log(r_1(j)/r_2(j))))/(((27*rho(k))*(r_2(
j)^-2-r_1(j)^-
2))/(70*pi^2*h_mean(j)^2)))+((sqrt((((6*mu(k))/(pi*h_mean(j)^3))*(log(r_1(j)/r_2(j))))
^2-(((81*rho(k)^2.*omega_final(j,i).^2)*(r_2(j)^-2-r_1(j)^-2)*(r_1(j)^2-
r_2(j)^2))/(700*pi^2*h_mean(j)^2))))/((27*rho(k)*(r_2(j)^-2-r_1(j)^-
2))/(70*pi^2*h_mean(j)^2)));

Q_max(j,i,k)=((((6*mu(k))/(pi*h_max(j)^3))*(log(r_1(j)/r_2(j))))/(((27*rho(k))*(r_2(j)
^-2-r_1(j)^-
2))/(70*pi^2*h_max(j)^2)))+((sqrt((((6*mu(k))/(pi*h_max(j)^3))*(log(r_1(j)/r_2(j))))^2
-(((81*rho(k)^2.*omega_final(j,i).^2)*(r_2(j)^-2-r_1(j)^-2)*(r_1(j)^2-
r_2(j)^2))/(700*pi^2*h_max(j)^2))))/((27*rho(k)*(r_2(j)^-2-r_1(j)^-
2))/(70*pi^2*h_max(j)^2)));
```

Determine the equivalent radius based on the flow rate %%%

```
  if Q_o(i)>=Q_min(j,i,k)
      r_o_min(j,i,k)=r_2(j);      % Equivalent radius of the film in meters at minimum
clearance play
  else
      r_o_min(j,i,k)=sqrt(((Q_o(i)./Q_min(j,i,k))*r_2(j)^2)+(r_1(j)^2*(1-
(Q_o(i)./Q_min(j,i,k)))));
  end

  if Q_o(i)>=Q_mean(j,i,k)
      r_o_mean(j,i,k)=r_2(j);     % Equivalent radius of the film in meters at mean
clearance play
  else
      r_o_mean(j,i,k)=sqrt(((Q_o(i)./Q_mean(j,i,k))*r_2(j)^2)+(r_1(j)^2*(1-
(Q_o(i)./Q_mean(j,i,k)))));
  end

  if Q_o(i)>=Q_max(j,i,k)
      r_o_max(j,i,k)=r_2(j);      % Equivalent radius of the film in meters at maximum
```

```
clearance play
  else
      r_o_max(j,i,k)=sqrt(((Q_o(i)./Q_max(j,i,k))*r_2(j)^2)+(r_1(j)^2*(1-
(Q_o(i)./Q_max(j,i,k)))));
  end
```

Drag Torque of each friction pair %%%

```
T_temp_min(j,i,k)=((N(j)*0.95*pi*mu(k).*omega_final(j,i))/(2*h_min(j)))*(r_o_min(j,i,k
).^4-r_1(j)^4);     % Drag Torque of the clutch/brake in N at minimum clearnce play
T_min=real(T_temp_min);

T_temp_mean(j,i,k)=((N(j)*0.95*pi*mu(k).*omega_final(j,i))/(2*h_mean(j)))*(r_o_mean(j,
i,k).^4-r_1(j)^4);  % Drag Torque of the clutch/brake in N at mean clearnce play
T_mean=real(T_temp_mean);

T_temp_max(j,i,k)=((N(j)*0.95*pi*mu(k).*omega_final(j,i))/(2*h_max(j)))*(r_o_max(j,i,k
).^4-r_1(j)^4);     % Drag Torque of the clutch/brake in N at maximum clearnce play
T_max=real(T_temp_max);
```

Torque carrying Capacity of clutches and brakes%%%

```
r_m(j)=(r_1(j)+r_2(j))/2;                                   % Mean radius of the
discs in m
A_clutchorbrake(j)=pi*((r_2(j)).^2-(r_1(j)).^2);            % Area of clutch or
brke in m^2
mu_friction=0.1;                                            % Coefficient of
friction
P_maxapplied=2*10^6;                                        % Max applied Pressure
by the piston in Pascals(N/m^2)
P_maxfaceallowed=6.4*10^6;                                  % Max face pressure
allowed at the friction disc
A_piston=[0.029,0.017,0.017,0.026,0.044,0.044];             % Area of Piston for
clutch k1 in m^2
F_springreturn=[3500,1500,1312,2080,1240,1250];             % Return Spring force
for clutch k1 in N
F_maxapplied(j)=P_maxapplied*A_piston(j);                   % Max applied force
applied in N
F_totalapplied(j)=F_maxapplied(j)-F_springreturn(j);        % Total force applied
in N taking into account the Spring return force
P_maxfacecalc(j)=F_totalapplied(j)/(A_clutchorbrake(j)*0.95);   % Max face pressure
calculated on the friction disc in Pascals
if P_maxfacecalc(j)<P_maxfaceallowed
    T_capacity(j)=F_totalapplied(j)*r_m(j)*N_friction(j)*mu_friction;  % Torque
capacity of the clutch or brake in N
else
    input('Clutch cannot be designed due to lack of torque carrying capacity');
end


         end
     end
end
```

8.2.3 Model to calculate final losses in the six speed transmission

Model with losses %%%

```
for k=1:6
for i=1:16
gearnumber==1
%%%% Gear 1 losses %%%%

%  ts1 tr1 tc1 ts2 tr2 tc2 ts3 tr3 tc3 ts4 tr4 tc4 ts5tr5 tc5 K1 K2 B1 B2 B3 B4 B5
trqout
A1=[1   1   1   0    0   0   0   0   0   0   0   0   0   0   0  0  0  0  0  0  0  0   0
    0   0   0   1    1   1   0   0   0   0   0   0   0   0   0  0  0  0  0  0  0  0   0
    0   0   0   0    0   0   1   1   1   0   0   0   0   0   0  0  0  0  0  0  0  0   0
    0   0   0   0    0   0   0   0   0   1   1   1   0   0   0  0  0  0  0  0  0  0   0
    0   0   0   0    0   0   0   0   0   0   0   0   1   1   1  0  0  0  0  0  0  0   0
   3.1  0   1   0    0   0   0   0   0   0   0   0   0   0   0  0  0  0  0  0  0  0   0
   0    0   0 2.90    0  1   0   0   0   0   0   0   0   0   0  0  0  0  0  0  0  0   0
   0    0   0   0    0   0 4.23 0    1  0   0   0   0   0   0  0  0  0 0  0   0  0   0
   0    0   0   0    0   0   0   0   0 4.26  0  1   0   0   0  0  0  0  0  0  0  0   0
   0    0   0   0    0   0   0   0   0 0    0   0 2.73  0   1  0  0  0   0  0  0  0   0
   0    1   0   0    0   1   0   1   0 0    0   0   0   0   0  0  0  0   0  1  0  0   0
   0    0   0   0    1   0   0   0   1 0    1   0   0   1   0  0  0  0  0  0  1  0   0
   0    0   1   1    0   0   0   0   0 0    0   0   0   0   0  0  1  1  0  0  0  0   0
   0    0   0   0    0   0   1   0   0 1    0   0   0   0   0 -1  0  0   0  0  0  0   0
   0    0   0   0    0   0   0   0   0 0    0   1   1   0   0  0  0  0  0  0  0  0   -1
   1    0   0   0    0   0   0   0   0 0    0   0   0   0   0  0  0  0   1  0  0  0   0
   0    0   0   0    0   0   0   0   0 0    0   0   0   0   0  0  0  1   0  0 0  0   0
   0    0   0   0    0   0   0   0   0 0    0   0   0   0   0  0  0  0   1  0  0  0   0
   0    0   0   0    0   0   0   0   0 0    0   0   0   0   0  0  0  0   0  1  0  0   0
   0    0   0   0    0   0   0   0   0 0    0   0   0   0   0  0  0  0   0  0 1  0   0
   0    0   0   0    0 0     0   0   0 0    0   0   0   0   0  0  1  0   0  0  0  0   0
   0    0   0   0    0 0     0   0   0 0    0   0   0   0   1  0  0  0   0  0  0  1   0
   0    0   0   0    0 0     0   0   0 0    0   0   0   0   0  1  -1 0   0  0  0  0   0];

B1(:,i)=[
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
T_mean(2,i,k)                %B1
T_mean(3,i,k)                %B2
T_mean(4,i,k)                %B3
```

```
T_mean(5,i,k)              %B4
T_mean(1,i,k)              %K2
0                        %B5
-1500];

X1(:,i,k)=A1\B1(:,i);
T_1_ideal=9.2308*10^3;                       % Ideal Torque in N at Gear 1 (No
losses)
T_1_loss(23,i,k)=(T_1_ideal-X1(23,i,k))/6.154; % Torque in N at Gear 1 with losses
P_loss(23,i,k)=T_1_loss(23,i,k)*omega_ip(i);   % Power in Watts at Gear 1 with losses


end
end
```

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