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Resource Allocation for Communications and Power Transfer under a Practical RF Energy Harvesting Model

Master of Science Thesis

Xiaowei Xu

Department of Signals and Systems
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Supervisor: Ayca Ozcelikkale, Department of Signals and Systems
Examiner: Tomas McKelvey, Department of Signals and Systems

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Department of Signals and Systems
Division of Signal Processing and Biomedical Engineering
Signal Processing Research Group
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 (0)31 772 1000

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Abstract

Energy harvesting (EH) has been an emerging topic in the recent years. This technology promises to increase the energy efficiency and extend the working time of energy-constrained wireless networks as well as free the devices from wires and batteries. Wireless power transfer (WPT) refers to transmission of electrical power from a source to another device wirelessly. Among different approaches, WPT via radio frequency (RF) is a viable method to implement. On the other hand, RF signal has been widely used for transmitting information, namely wireless information transfer (WIT). Recently, the idea of using RF signals to carry information as well as energy at the same time, i.e., simultaneous wireless information and power transfer (SWIPT) has emerged.

In the area of SWIPT research, it has been typically assumed that energy conversion efficiency is independent from the level of the input power at the energy receiver. On the other end, in practice the energy conversion efficiency exhibits a non-linear behaviour and depends on the input power. This leads to a disperancy between the SWIPT designs made and practical EH circuits. This thesis addresses this issue. We propose a practical quadratic model for the power conversion efficiency using curve fitting tools. Comparisons with the conventional linear models as well as another non-linear model proposed in the literature are made using mean-square based metrics. Using the proposed model, the problem of resource allocation for a multi-user Orthogonal Frequency-Division Multiple Access (OFDMA) system is investigated. In particular, the problem of allocating the power optimally among different sub-channels assigned to information and energy users is considered. Analytic solutions for some illustrative scenarios are provided. Numerical results are presented for the general case. Our investigations show that compared to using the traditional simple linear efficiency model, the proposed model provides more realistic harvested energy estimates at the energy receivers with minimal energy waste at the transmitter.

Keywords: Energy Harvesting, convex optimization.

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Xiaowei Xu, Gothenburg, June 2016

Contents

List of Figures

List of Tables

1
Introduction

This chapter introduces the background and current research state of Simultaneous Wireless Information and Power Transfer (SWIPT) and the problem faced when designing the model representing the efficiency between the received power and output power in receiver. Also, objective as well as methodology are shown.

1.1 Background

Energy harvesting (EH) has been a hot topic in the recent years. EH is a technology that can collect power from external sources (e.g. Radio Frequency (RF), solar power, thermal power, wind power, kinetic energy) and store them for small, wireless autonomous devices. Namely it can capture energy from surroundings and then accumulate, store, transfer the energy into electricity. One reason why EH is so important is that currently most of the infrastructures of electricity are far from energy-efficient, a lot of power are left wasted without being used, for example heat. By energy harvesting technology the wasted heat can be recollected and stored as backup power which can increase the power efficiency. More generally speaking, harvesting those energy from external natural sources which can be regarded as essentially inexhaustible is a huge step for human beings in terms of how to efficiently use energy. Energy harvesting technology, as a promising solution, greatly increase the energy efficiency and extend the working time of energy-constrained wireless networks as well as free the devices from wires and batteries so that they can be more mobile.

1.1.1 Wireless Power Transfer (WPT)

WPT is transmission of electrical power from a source to an electrical load without using man-made conductors such as wires. Among different EH approaches, WPT via radio frequency (RF) is a viable method to implement which is the main focus in this paper. It is worth noting that even though using radiated RF signal as a source of energy harvesting is possible, currently the output of the harvester still can not support the devices with high power demand yet but for those sensors and switches with low consumption it is sufficient to use WPT to support them [2].

1.1.2 Wireless Information Transfer (WIT)

On the other hand, RF signal has been widely used for transmitting information, namely wireless information transfer (WIT). Whenever we connect to WiFi and surf the Internet or make a call we are transmitting information wirelessly to other devices which can also be called wireless communication. Although this technology is quite mature and has bunch of theory and application related, using RF signal to transfer information and energy at the same time is less often studied and discussed until recent years when Varshney raised the topic in his paper [14] for the first time which we will mention later.

1.1.3 Simultaneous wireless information and power transfer (SWIPT)

As mentioned above, considering the fact that the RF signal is able to carry information as well as energy, the interesting idea of integrating WIT and WPT called Simultaneous wireless information and power transfer (SWIPT) has emerged. SWIPT becomes a new interesting field for that it realises both useful utilizations of RF signal at the same time and offers many potential convenience.

1.2 Literature Review and Challenge

Varshney proposed the idea of transmitting information and power simultaneously in [14] for the first time with providing a capacity-energy function to characterise the fundamental performance trade-off for SWIPT. Grover and Sahai [5] expanded the research to frequency-selective channels with additive white Gaussian noise (AWGN) and showed that a non-trivial trade-off exists for information transfer versus energy transfer with power allocation. A lot studies have been conducted in this area from point-to-point transmission to multiple-input multiple-output (MIMO) [9, 16]. In R.Zhang and Ho's [15], a three-node MIMO broadcasting system of one transmitter, one energy receiver and one information receiver was considered and the performance limits in this system was studied. For the simplicity of the analysis, the author made an assumption that the energy conversion efficiency at the energy receiver is independent from the received power. In [12, 11, 8, 6] where antenna design for RF energy harvesting is developed, it is shown that the conversion efficiency is highly dependent on the input power rather than a constant. More precisely speaking, ζ first increases as the input power goes up till a peak and then decreases as the input power goes down. That is to say, harvesting efficiency will be very low if the received power is too small or too big. Thus, power allocation at the transmitter should be considered carefully since the value of power might have a huge effect on the efficient harvested energy at the receiver side. This issue is typically overlooked in the literature.

1.3 Objective

In Boshkovska's work [1], the issue mentioned above was taken into consideration and showed that the conventional linear model may lead to a mismatch between the resource allocation and practical EH circuits. Thus proposing a non-linear model is important to have a better performance in the system. Therefore our objective in this thesis is to see how much effect assuming a traditional linear model will give on the performance of the actual SWIPT system and propose a non-linear model that can capture the characteristics of the EH circuits as well as well as compare and analysis the performance between using linear model and non-linear model through several problem formulation.

1.4 Methodology

Based on the fact that currently we do not hold physical EH hardware, we could not conduct the research with a real device. However, in [12, 11, 8, 6], the nonlinear relationship between the output and input is provided. Therefore we decide to simulate the downlink of the SWIPT in a virtual environment with MATLAB. A mathematical model of relationship that is relatively easy to work with was built and further research was based on this model and found the optimal strategy.

2

Energy Harvesting

In this chapter, the problem regarding energy conversion efficiency is introduced with a simple system of one transmitter (TX) and one Energy Harvesting Receiver (ER). A non-linear model is proposed and comparisons with the typical linear model from the literature and comparison with another non-linear model is presented.

2.1 System Model

The problem focuses on a wireless system consisting of one transmitter (TX) and one Energy Harvesting Receiver (ER). The baseband transmission from the TX to the ER can be modeled by

$$y_{EH} = h_E a x + n_E \tag{2.1}$$

where a is the factor of signal amplification, h_E is the channel coefficient between the TX and the ER, y_{EH} is the signal received by the ER, which is shown in Figure 2.1.

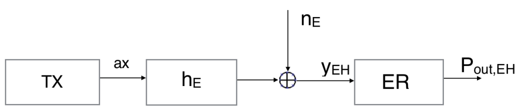

Figure 2.1: Wireless system

Assume that $\mathbb{E}[x] = 0$, $\mathbb{E}[x^2] = 1$, $\mathbb{E}[n_E] = 0$, $\mathbb{E}[n_E^2] = \sigma_{n_E}^2$, we could derive the average power used by the transmitter

$$\mathbb{E}[ax^2] = \mathbb{E}[a^2x^2] = a^2\mathbb{E}[x^2] = a^2 \tag{2.2}$$

Given that $\sigma_{n_E}^2$ is typically small, we ignore the n_E energy havesting process. The average power of the y_{EH} could be derived as follows:

$$P_{in} = a^2 h_E^2 \tag{2.3}$$

Now two models will be considered for the conversion process between P_{in} and $P_{out,EH}$

- Efficiency ζ is constant

$$P_{out,EH} = \zeta P_{in} = \zeta a^2 h_E^2 \tag{2.4}$$

- Efficiency ζ is dependent on P_{in}, namely $\zeta = f(P_{in})$

$$P_{out,EH} = f(P_{in})P_{in} \tag{2.5}$$

(2.5) could also be transformed as follows

$$P_{out,EH} = g(P_{in}) \tag{2.6}$$

where g is a non-linear function of P_{in}

2.2 Energy Harvester

In this section the proposed model will be presented and two comparisons will be shown to better understand the quality of the proposed nonlinear model.

2.2.1 Proposed Model

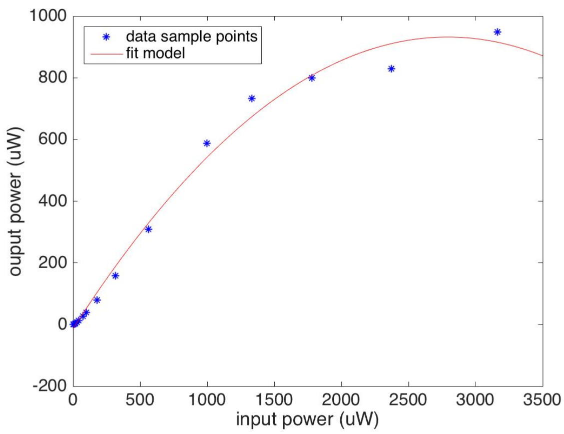

Figure 2.2: Nonlinear model with Polynomial in 2nd order

We have built the mathematical model with curve fitting tool in MATLAB using the nonlinear relationship provided by Song's work. We use a 2nd order polynomial to form the nonlinear model $g(x)$, as shown in Figure 2.2. It is structured in the following form:

$$g(x) = p_1x^2 + p_2x + p_3 \tag{2.7}$$

where parameters p_1, p_2, p_3 can be easily calculated in MATLAB curve fitting toolbox. In general, $g(x)$ is a quadratic function thus it has general properties that a quadratic function have which will be mentioned again when discussed.

Note that the nonlinear model is based on the realistic data in [11], and maximum received power at ER is 5dBm, namely about 3162 μW. Since what will be the subsequent curve after 5dBm was not covered in the literature, we will use 3162 μW as our maximum receiver power in this paper.

2.2.2 Comparison with linear models

To have an intuitive comprehension on the difference between linear model and nonlinear model, we set the ζ equals 0.2, 0.4, 0.6 separately in linear model and compare them to the nonlinear model we just built, which can be shown in Figure 2.3. In 2.3, we could easily observe that the linear models with constant conversion efficiency $20\%, 40\%, 60\%$ poorly match the realistic data set which the proposed non-linear model provides a more accurate match.

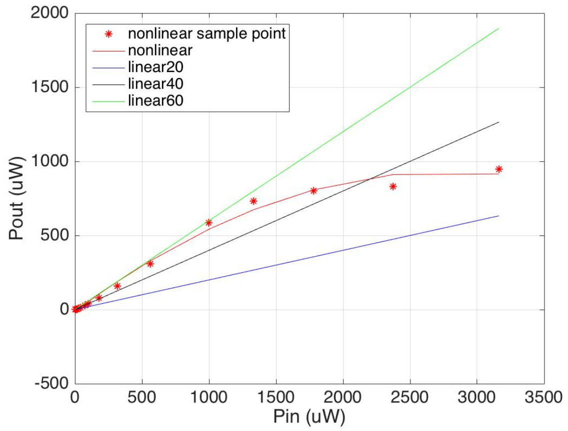

Figure 2.3: Comparison between linear model and nonlinear model. Nonlinear sample points from [11]

2.2.3 Comparison with the nonlinear model from [1]

In [1], a non-linear model is also built to match the actual non-linear relationship between the received power and output power, which is modelled as:

$$\Phi_{ER_j}^{Practical} = \frac{[\Psi_{ER_j}^{Practical} - M_j\Omega_j]}{1-\Omega_j}, \Omega_j = \frac{1}{1+\exp(a_j b_j)} \tag{2.8}$$

$$\Psi_{ER_j}^{Practical} = \frac{M_j}{1+\exp(-a_j(P_{ER_j} - b_j))} \tag{2.9}$$

where $\Phi_{ER_j}^{Practical}$ is the output power, P_{ER_j} is the received power and M_j, a_j, b_j are 3 parameters which can be calculated by curve fitting tool. The logistic function here has non-convexity property [1]. This model is built based on logistic (sigmoidal) function while our proposed model in this paper is based on quadratic function. In Figure 2.6, 2.4 and 2.5, we made a comparison between our model and Boshkovska's model using three different data sets in [6, 11, 8]. We use Root Sum of squares due to error (RSSE) and $R-square$ in TABLE 2.1 to show and compare the performance between Boshovska's model and our proposed model. After modelling data with models there exists several ways to evaluate the goodness of fit (GOF). In MATLAB it provides several GOF statistical methods which include SSE and $R-square$.
$RSSE$ is defined as:

$$RSSE = \sqrt{\sum_{i=1}^{n} w_i(y_i - \hat{y}_i)^2} \tag{2.10}$$

This statistic measures the total deviation of the response values from the fit to the response values. A value closer to 0 implies that the model has a smaller random error component, and that the fit will be more useful for prediction.

$R - square$ is defined as the ratio of the sum of squares of the regression (SSR) and the total sum of squares (SST).

SSR is defined as

$$SSR = \sum_{i=1}^{n} w_i(\hat{y}_i - \overline{y_i})^2 \tag{2.11}$$

SST is defined as:

$$SST = \sum_{i=1}^{n} w_i(y_i - \overline{y_i})^2 \tag{2.12}$$

Given these definitions, R-square is expressed as:

$$R - square = \frac{SSR}{SST} \tag{2.13}$$

R-square is valued between 0 and 1, with a value closer to 1 indicating that a greater proportion of variance is accounted for by the model.

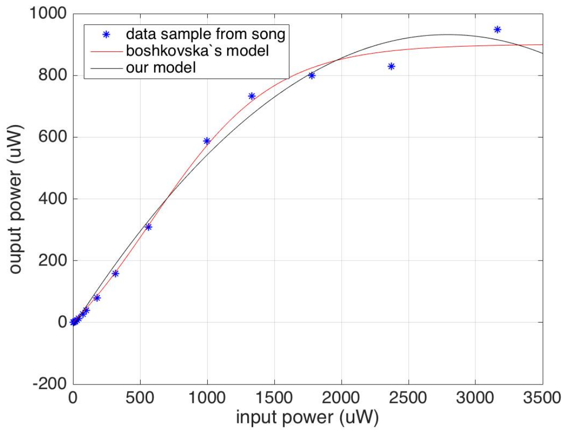

Figure 2.4: Comparison between Boshkovska model and our model with data in [11]

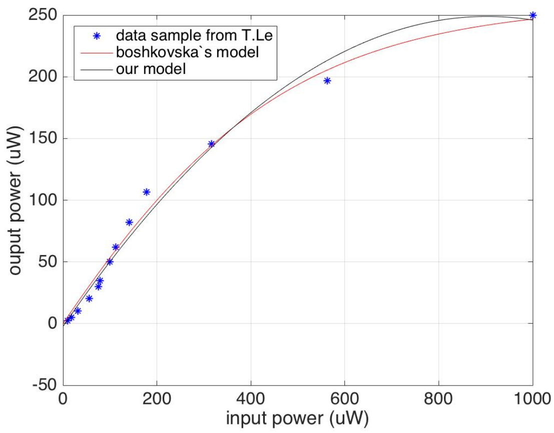

Figure 2.5: Comparison between Boshkovska model and our model with data in [8]

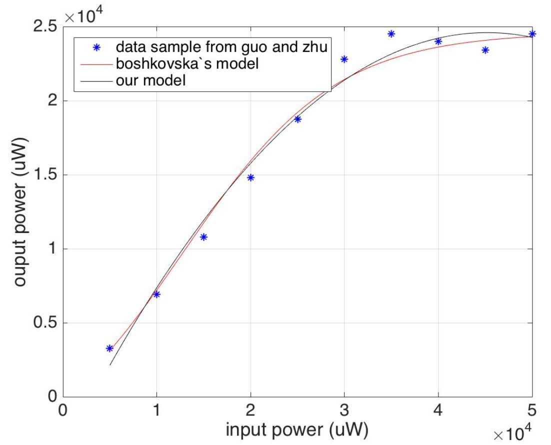

Figure 2.6: Comparison between Boshkovska model and our model with data in [6]

It can be observed that when using our proposed model the values of $rsse$ are slightly larger than the model in [1]. From the definition we can observe that the value of $rsse$ greatly depends on the the value of the data sample and the values in the used data sets are relatively huge which lead to huge $rsse$ values. For $R - square$ we could see that the values for both models however are close. The value of $R - square$ is a ratio and thus less dependent on the specific values of data.Therefore we could conclude that our model is as good as theirs in term of representing the non-linear relationship. Furthermore, a quadratic function is simpler than a logistic model and easier to work with in the computational scene while for the model in [1] a careful initialization should be performed which means that to have a good fit we have to choose a good enough start-point for the 3 parameters. Therefore, we use this quadratic model as our non-linear model in the paper.

Also, since the non-linear model from [1] uses logistic function and it has non-convexity property, it means that the problem based on this model can not be easily

Table 2.1: Comparison of goodness of fit

Data	Model	rsse	rsquare
From [8]	Boshovska et als	28.577	0.9889
	Proposed Model	32	0.9860
From [11]	Boshovska et als	79.850	0.9969
	Proposed Model	125.857	0.9922
From [6]	Boshovska et als	2804	0.9863
	Proposed Model	3007	0.9842

solved by convex optimization algorithms. In this sense, our model is easier to work with than theirs.

Another important issue we need to notice is that the comparison above also shows the proposed model can dynamically capture the non-linear relationship with different level of input power. But in this paper we will use the data from [11], thus the valid range of input is 0 to 3162 μW.

2.3 Case Study

To get a deeper comparison between our proposed nonlinear model and the traditional linear model, a specific system with one transmitter and EH receiver is considered here, which could be written as:

$$y_{ER} = h_E a x + n_E \tag{2.14}$$

Channel h_E is assumed to be 1 in this case. Thus, the average received power P_{in} is:

$$P_{in} = \mathbb{E}[(h_E a x)^2] = a^2 \mathbb{E}[(h_E x)^2] = a^2 \tag{2.15}$$

The relationship between P_{out} and P_{in} would be:

$$P_{out} = f P_{in} \tag{2.16}$$

where f could be a constant parameter or a non-linear function of P_{in}.

We set a range of required power at EH receiver and thus the corresponding proposed power at transmitter for different models can be calculated. With the proposed power at transmitter for different models we could obtain the actual power output by transmitting the proposed power through a specific realistic model that reflects the exact conversion relationship between received power and output power [11]. By realistic model we mean that this model is also based on a realistic data [11] except that it is a high complexity fit and presents the the most accurate over-fit for this exact data set. The Figure 2.7 shows the comparison between our proposed model and realistic model with the data from [11]. It can be observed that the realistic model perfectly captured the sample data points. However since the realistic model uses sine and cosine as base function, even though it gives high accuracy it is hard to work with in a mathematical sense. Thus we only use this model to reflect the exact relationship for the data set in [11]. Note that for all models we use the same valid range as we discussed to make the result fair.

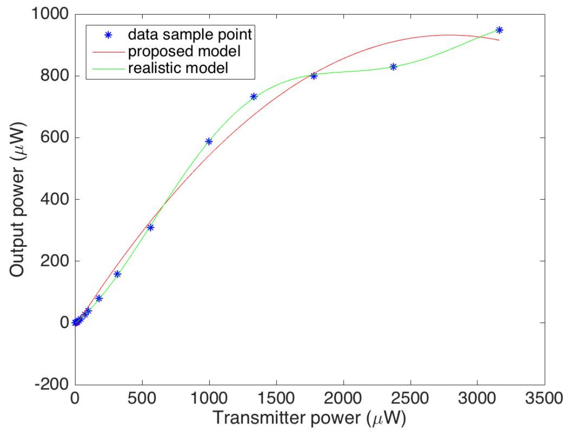

Figure 2.7: Comparison between proposed model and realistic model with data in [11]

From Figure 2.8, 2.9, we could see that, from the transmitter side, the proposed power for li20 and li40 are much higher than the fit design, only the power of li60 is under the fit. And on the receiver side, even though li20 and li40 are able to satisfy the required power the actual power out are too much and become waste. Li60 however is always under the required line (yellow line) thus can not meet the requirement. Only the fit model are always close to the requirement without too much waste. Thus we could conclude that the fit design is the best design we could get among those 3 linear design since it needs relatively the least power from transmitter and and fits requirement the best from the receiver side.

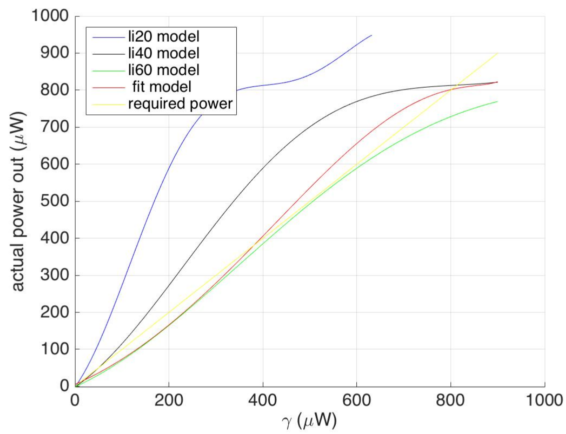

Figure 2.8: Power out for different models

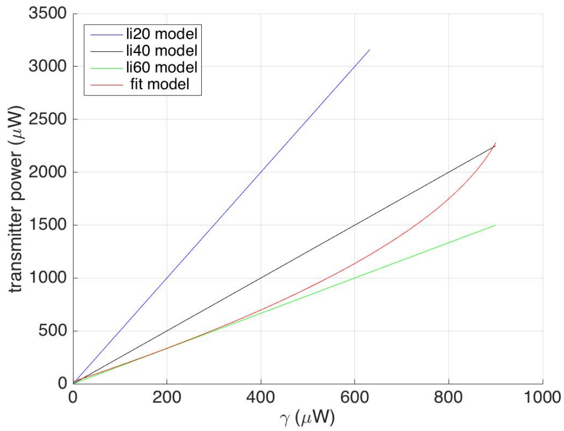

Figure 2.9: Required power at transmitter for different models

3

Resource Allocation for SWIPT OFDMA system

3.1 Background

In this section Orthogonal Frequency Division Multiplexing (OFDM) technology and channel capacity are introduced to get a better understanding for the problem formulation.

3.1.1 OFDM Technology

Orthogonal Frequency Division Multiplexing (OFDM) is currently a popular method of digital multicarrier modulation. The basic concept of multicarrier modulation is to split a total bitstream into different substreams and send these over many different subchannels. In order to describe the advantages of OFDM, another concept should also be mentioned which is called single-carrier modulation. It literally means that a single frequency carrier is used to transmit a single symbol at a time which is widely used in applications such as Bluetooth, GSM, CDMA. To achieve a high data speed in single-carrier modulation a fast symbol rate is required which will make the system weaker to multi-path distortion and consequently increase the Inter Symbol Interference (ISI). Therefore to keep a high data rate as well as eliminate the ISI, a new approach is designed based on the concept of multicarrier modulation, that break a wideband channel into multiple parallel narrowband channels by means of an orthogonal partition. The number of substream is chosen in a way that the bandwidth of each subchannel should be much less than the coherence bandwidth of the channel to keep a relatively flat-fading characteristics in each subchannel. Due to its advantages the OFDM technology has been widely adopted in the latest wireless telecommunication field (e.g. 4G, WiFi and related standards). Orthogonal Frequency-Division Multiple Access (OFDMA) is a multi-user version of the OFDM scheme.

3.1.2 Channel Capacity

Channel capacity, as one of the most important concept in information theory, defines the maximum data rate that the information can be reliably transmitted over wireless channels with asymptotically small error probability,which is underlain in The mathematical theory of communication written by Claude Shannon [10]. A fundamental theory of the information theory is the Shannon-Hartley theory which makes the maximum rate computable in a channel of a specified bandwidth in the presence of noise:

$$C = B\log_2(1 + \frac{S}{N}) \tag{3.1}$$

where C is the channel capacity in bits per second, B is the bandwidth of the channel, S is the average received power over bandwidth, N is the average power of the interfering Gaussian noise over the bandwidth. S/N is the signal-to-noise ratio, thus the formula could also be written as:

$$C = B\log_2(1 + SNR) \tag{3.2}$$

3.1.3 Convex optimization problem

Convex optimization is a special class of mathematical optimization problems which includes least-squares and linear programming problems. It studies the problem of minimizing convex functions over convex sets which will be introduced later. Convex optimization will be used in the subsequent problem formulation and the introduction of convex optimization will give a better understanding for the problems.[13]

3.1.3.1 Convex function

Before convex function is introduced, some important concepts should be mentioned briefly first. Suppose $x_1 \neq x_2$ are two points in R^n. Points of the form

$$y = \theta x_1 + (1 - \theta)x_2 \tag{3.3}$$

where $\theta \in R$, form the line passing through x_1 and x_2. Values of the parameters θ between 0 and 1 correspond to the line segment between x_1 and x_2.

A set C is convex if the line segment between any two points in C lies in C, *i.e.*, if for any x_1, $x_2 \in C$ and any θ with $0 \leq \theta \leq 1$, we have

$$\theta x_1 + (1 - \theta)x_2 \in C \tag{3.4}$$

Intuitively speaking, this means that the set is connected so that you can pass between any two points without leaving the set and has no dents in its perimeter. Sections of the perimeter may be straight lines. Two examples of convex and nonconvex sets are shown in Figure 3.1 and Figure 3.2.

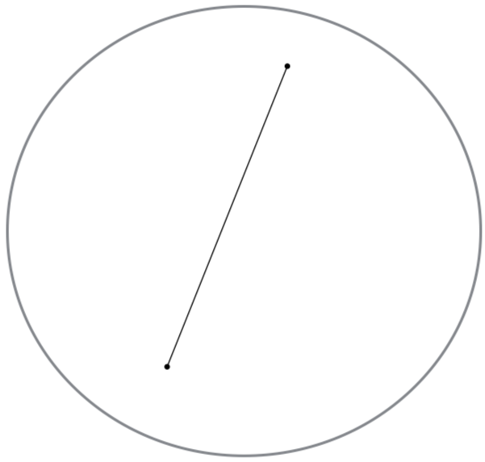

Figure 3.1: Example of a convex set

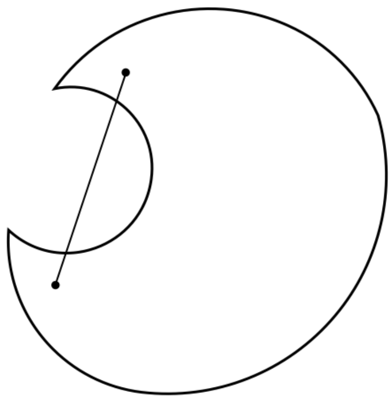

Figure 3.2: Example of a nonconvex set

A function $f : R^n \rightarrow R$ is convex if $dom f$ is a convex set and if for all x, $y \in dom f$, and θ with $0 \leq \theta \leq 1$, we have

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y) \tag{3.5}$$

Geometrically, the inequality indicates that the line segment between $(x, f(x))$ and $(y, f(y))$, which is the chord from x to y, lies above the graph of f,as can be shown in Figure 3.3. A function is strictly convex if strict inequality holds in 3.5 whenever $x \neq y$ and $0 < \theta < 1$. We say f is concave if $-f$ is convex.

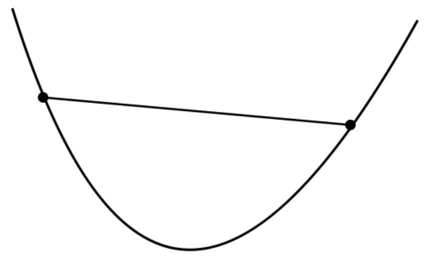

Figure 3.3: Graph of a convex function. The line segment between any two points on the graph lies above the graph

3.1.3.2 Convex optimization problem

We use the notation

$$\begin{aligned} \text{minimize} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \leq 0,\ i = 1, \ldots, m \\ & h_i(x) = 0,\ i = 1, \ldots, p \end{aligned}$$

to describe the problem of finding an x that minimizes $f_0(x)$ among all x that satisfy the conditions $f_i(x) \leq 0, i = 1, \ldots, m$, and $h_i(x) = 0, i = 1, \ldots, p$. $x \in R^n$ is the optimization variable and the function $f_0 : R^n \rightarrow R$ is the objective function or cost function. The inequalities are called inequality constraints and the equations are called the equality constraints. The problem is unconstrained if there exists no constrains ($i.e., m = p = 0$). Similarly, we can solve the maximization problem

$$\begin{aligned} \text{maximize} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \leq 0,\ i = 1, \ldots, m \\ & h_i(x) = 0,\ i = 1, \ldots, p \end{aligned}$$

by minimizing the function $-f_0$ subject to the constraints.

In chapter 3 of [13], it has been proved that : logarithm $\log x$ is concave on R_{++} and quadratic function $ax^2 + bx + c$ is convex if and only if $a > 0$ (concave if and only if $a < 0$). These two functions will be used in the later problem.

3.1.4 CVX

CVX is a Matlab-based modeling system for convex optimization[3, 4]. CVX turns Matlab into a modeling language, allowing constraints and objectives to be specified using standard Matlab expression syntax

3.2 Resource Allocation for SWIPT

3.2.1 Individual performance constraints

Now that we have seen the difference of performance between linear model and non-linear model, we will formulate the problem in a practical sense. In [7] the author introduced OFDM technology into SWIPT and provided a multi-user system where each sub-channel based on OFDMA is assigned to each user.

We consider a wireless system similar consisting one transmitter and in total n receivers, including n_{IR} information receivers (IR) and n_{ER} energy harvesting receivers (ER). Each receiver will receive power from each sub-channel denoted by P_i. In this system we want each of IR can at least reach a threshold denoted by R and each of ER can at least reach a threshold denoted by γ. $P_{threshold}$ is the valid boundary of received power at each ER. Thus the problem can be formulated as following:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} P_i \\ \text{subject to} \quad & \log_2(1 + \frac{P_i {h_i}^2}{\sigma_{n_E}^2}) \geq R_i, 1 \leq i \leq n_{IR} \\ & g(P_j|h_j|^2) \geq \gamma_j, n_{IR} < j \leq n \\ & P_j|h_j|^2 \leq P_{threshold}, n_{IR} < j \leq n \end{aligned}$$

Here we consider the problem of minimizing the total proposed power at the transmitter while satisfying that the information rate of each IR should meet threshold R_i and the harvested energy of each ER should meet the requirement γ_i at the receiver sides. The signals are Gaussian.

Solution: We can start thinking at the point where $n_{IR} = n_{ER} = 1$. The problem focuses on a wireless system consisting of one transmitter (TX) and two Energy Harvesting Receivers (ER). The baseband transmission from the TX to the ER can be modeled by

$$\begin{bmatrix} y_{EH_1} \\ y_{EH_2} \end{bmatrix} = \begin{bmatrix} a_1 h_{E_1} \\ a_2 h_{E_2} \end{bmatrix} x + \begin{bmatrix} n_{e_1} \\ n_{e_2} \end{bmatrix}$$

where a_1, a_2 are the factor of signal amplification, h_{E_1}, h_{E_2} are the channel coefficient between the TX and the ERs, y_{EH_1}, y_{EH_2} are the signal received by the ER, which is shown in Fig.1. Assume that $\mathbb{E}[x] = 0$, $\mathbb{E}[x^2] = 1$, $\mathbb{E}[n_E] = 0$, $\mathbb{E}[n_{E_1}^2] = \mathbb{E}[n_{E_2}^2] = \sigma_{n_E}^2$, we could derive the average power used by the transmitter

$$\mathbb{E}[a_1 x^2] = \mathbb{E}[a_1^2 x^2] = a_1^2 \mathbb{E}[x^2] = a_1^2 \tag{3.6}$$

$$\mathbb{E}[a_2 x^2] = \mathbb{E}[a_2^2 x^2] = a_2^2 \mathbb{E}[x^2] = a_2^2 \tag{3.7}$$

Given that $\sigma_{n_E}^2$ is small, we ignore the n_{E_1} and n_{E_2} in this problem. Assume the total average power of the TX is no more than ρ,we could obtain the following relationship:

$$\mathbb{E}[a_1^2 x^2] + \mathbb{E}[a_2^2 x^2] = a_1^2 + a_2^2 \leq \rho \tag{3.8}$$

The average power of the y_{EH_1} and y_{EH_2} could be derived as follows:

$$P_{in_1} = a_1^2 h_{E_1}^2 \tag{3.9}$$

$$P_{in_2} = a_2^2 h_{E_2}^2 \tag{3.10}$$

We assume that the total average power of transmitter is denoted by ρ , the ER_1 and ER_2 should respectively harvest more than γ_1 and γ_2. The first constraint formula about IR can be transformed into:

$$\begin{aligned} 1 + \frac{P_1 {h_1}^2}{\sigma_{n_E}^2} &\geq 2^{R_1} \\ \frac{P_1 {h_1}^2}{\sigma_{n_E}^2} &\geq 2^{R_1} - 1 \end{aligned} \tag{3.11}$$

We take $\frac{h_1}{\sigma_{n_E}^2}$ as β since it is a constant,and $2^{R_1} - 1$ as $\tilde{R}$, therefore it becomes:

$$P_1 \beta \geq \tilde{R} \tag{3.12}$$

For the second formula constraint about ER, we have already known that $g(P_2)$ is a quadratic function and therefore can be written as:

$$p_1 (P_2 {h_2}^2)^2 + p_2 (P_2 {h_2}^2) + p_3 \geq \gamma_2 \tag{3.13}$$

Here we will use a property that belongs to the quadratic function.Whenever we have a quadratic function as follows:

$$p_1 x^2 + p_2 x + p_3 = 0 \tag{3.14}$$

we have two roots that can satisfy the equation when ${p_2}^2 - 4p_1p_3 > 0$,they are $\frac{-p_2 - \sqrt{{p_2}^2 - 4p_1p_3}}{2p_1}$ and $\frac{-p_2 + \sqrt{{p_2}^2 - 4p_1p_3}}{2p_1}$ separately. Further more, if $a < 0$ is satisfied then we have

$$p_1 x^2 + p_2 x + p_3 > 0 \tag{3.15}$$

when $x \in (\frac{-p_2 - \sqrt{{p_2}^2 - 4p_1p_3}}{2p_1}, \frac{-p_2 + \sqrt{{p_2}^2 - 4p_1p_3}}{2p_1})$. Thus apparently we could see the problem will be solved when we set P_1 as $\frac{\tilde{R}}{\beta}$ for (3.12) and set P_2 as the left root of (3.15) according to the nature of the two functions. When increasing the number of the receivers we could easily observe that no matter how many receivers we have in the system, as long as each receiver use power based on the solution of 2-receiver case discussed above, we could also always get minimum power in total for the transmitter, namely solve the problem.

3.2.2 Minimize transmitter power

Above we discussed about the problem where the harvested power of each ER should meet the requirement, it would be also meaningful to look at the problem where the total harvested power of all ER should satisfy the required power. Thus we would have a similar problem formulation as:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} P_i \\ \text{subject to} \quad & \sum_{i=1}^{n_{IR}} \log_2(1 + \frac{P_i {h_i}^2}{\sigma_{n_E}^2}) \geq R \\ & \sum_{j=1}^{n_{ER}} g(P_j|h_j|^2) \geq \gamma \\ & P_j|h_j|^2 \leq P_{threshold}, 1 \leq j \leq n_{ER} \end{aligned}$$

where P_i is the power of n-th sub-stream from the transmitter, h_i is the channel coefficient of the n-th sub-stream, $\sigma_{n_E}^2$ is the noise variance of the channel, R is the required transmission rate, n is the total number of IR and ER, γ is the required harvested power, $P_{threshold}$ is the valid boundary that the received power at each ER can not exceed.

The problem here we are considering is different from the individual performance constraints that to minimize the total transmitted power at the transmitter, the total information rate of all IRs must meet R and the total harvested power of all ERs must meet the requirement γ. Also, the the proposed power for all ERs should not exceed 3162 μW as we discussed before. The total transmitter power is allowed to exceed the valid boundary however since it also includes the power for IRs. Since the problem is a convex problem and it can be solved numerically by standard algorithms for convex optimization. We use CVX to solve this problem to get a numerical result. This problem was studied in [7] but with linear model.

We will first start with 2-user case,namely one IR and one ER, and then extend the number of user to get a general result. From Figure 3.4 to Figure 3.9, we show the proposed power at the transmitter and actual total output power of the ERs for different models in different channel states. For different channel state we could observe the same patterns for proposed power and actual power. From the transmitter side we could observe that the linear models with conversion efficiency 20% and 40% have more proposed power than the fit model has, only the linear model with 60% has a bit lower proposed power than fit model. From the receivers' point of view, we could see the yellow line which represents the required power, if the curve lies above the curve it means it meets the requirement otherwise it does not. The output power for the 20% and 40% linear models are too much for the requirement for most of the time and the curve for the 60% linear model is always under the requirement. Only the fit model is always close to the requirement. Therefore, from the observations for the proposed power and actual output power for different models we could conclude that the fit model is the best model since it does not need the transmitter provide too much power and could still harvest relatively close power to the EH requirement from the receiver side.

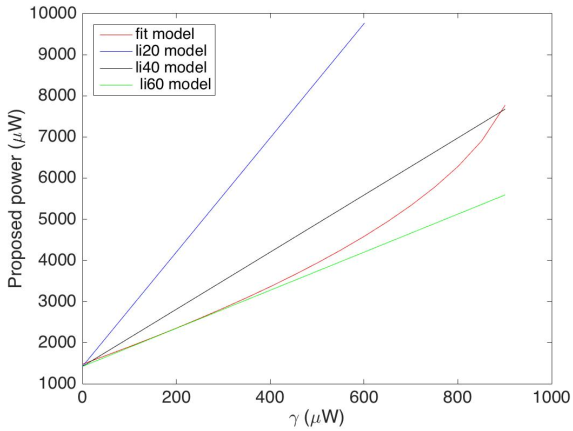

Figure 3.4: Power at transmitter for different models when channel coefficients $h_{IR_1} = 0.6, h_{ER_1} = 0.6$ (2-user)

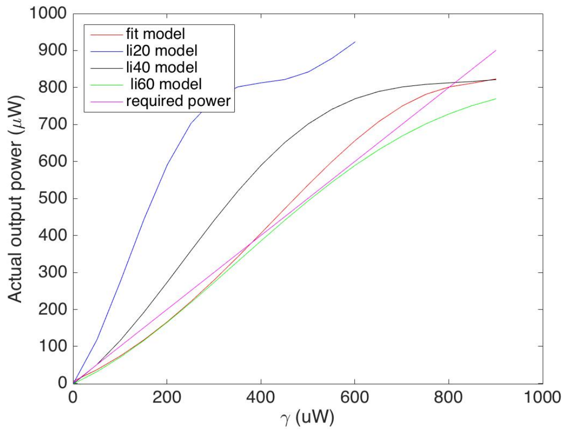

Figure 3.5: Actual total output power of ERs for different models when channel coefficients $h_{IR_1} = 0.6, h_{ER_1} = 0.6$ (2-user)

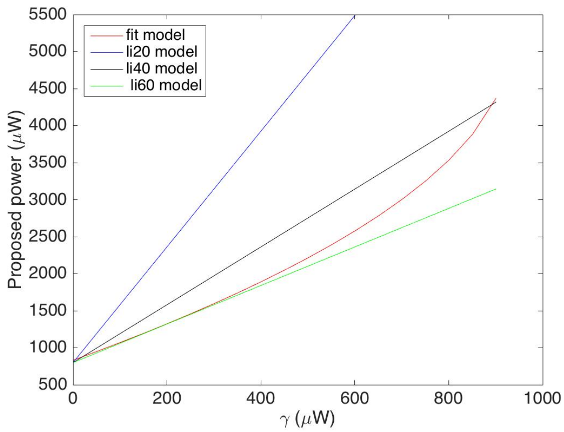

Figure 3.6: Proposed power at transmitter for different models when channel coefficients are $h_{IR_1} = 0.8, h_{ER_1} = 0.8$ (2-user)

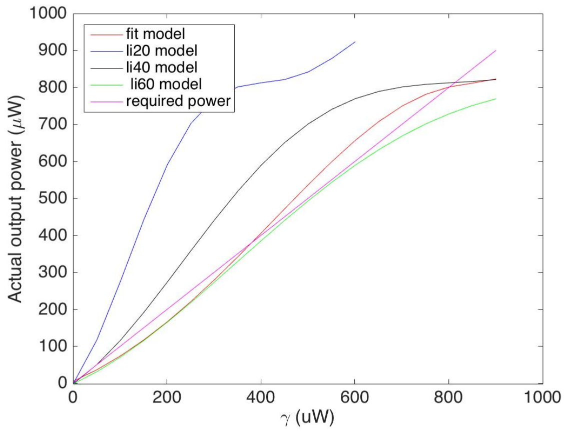

Figure 3.7: Actual total output power of ERs for different models when channel coefficients are $h_{IR_1} = 0.8, h_{ER_1} = 0.8$ (2-user)

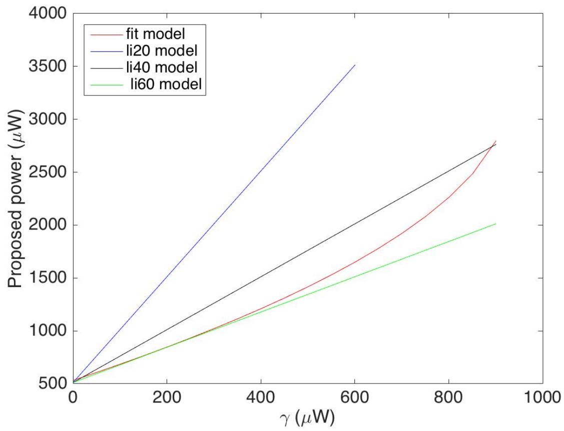

Figure 3.8: Proposed power at transmitter for different models when channel coefficients are $h_{IR_1} = 1, h_{ER_1} = 1$ (2-user)

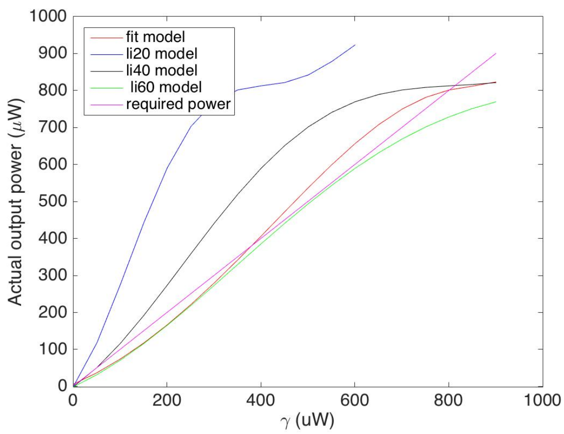

Figure 3.9: Actual total output power of ERs for different models when channel coefficients are $h_{IR_1} = 1, h_{ER_1} = 1$ (2-user)

Now that we have discussed about 2-user case. We will increase the user to see a more general pattern. We use 6 user to represent multi-user case in this paper. From Figure 3.10 to Figure 3.17, the proposed power and actual output power for different models are shown with different channel coefficients set. Note that since our main focus is ER, so we only change the channel coefficients of ERs.

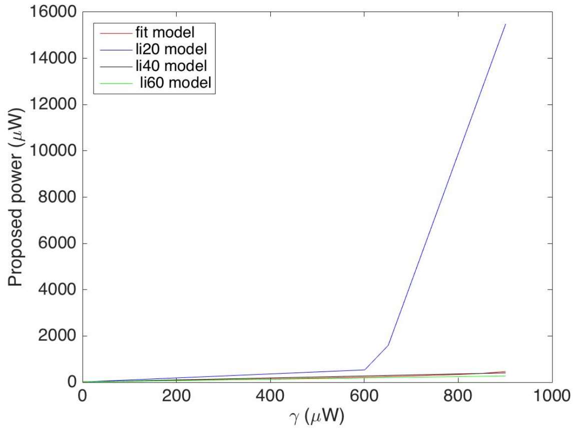

Figure 3.10: Proposed power at transmitter for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$ (6-user)

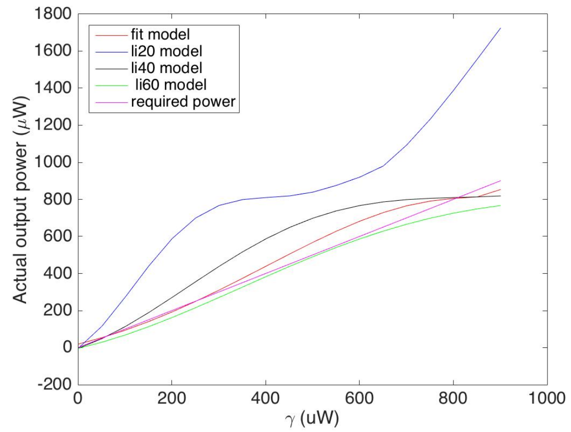

Figure 3.11: Actual output power of ERs for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$ (6-user)

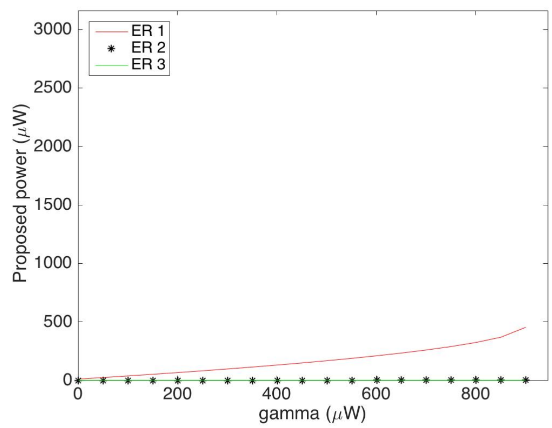

Figure 3.12: Distribution of transmitted power for 3 ERs when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$ (6-user)

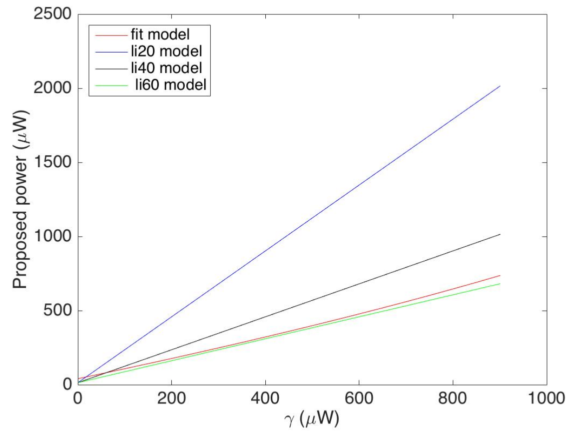

Figure 3.13: Proposed power at transmitter for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 1.5, h_{ER_2} = 1.5, h_{ER_3} = 0$ (6-user)

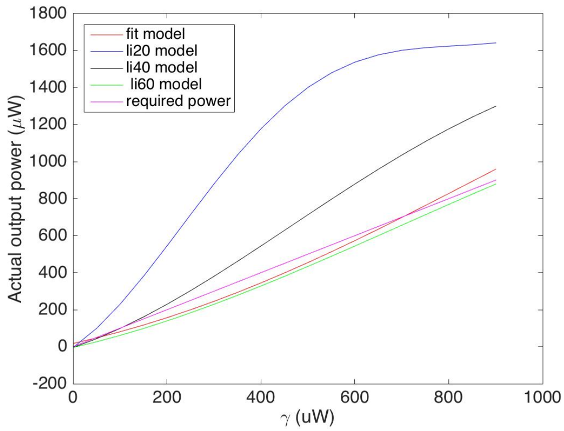

Figure 3.14: Actual output power of ERs for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 1.5, h_{ER_2} = 1.5, h_{ER_3} = 0$ (6-user)

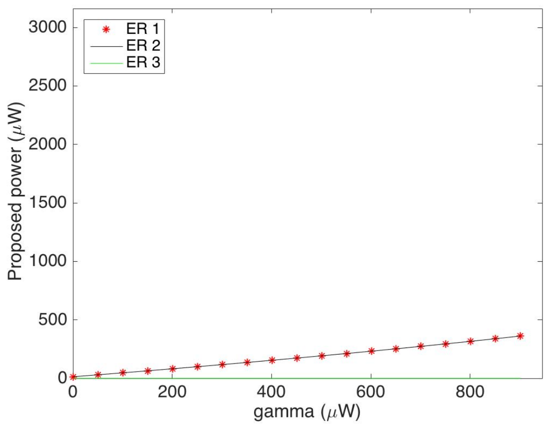

Figure 3.15: Distribution of transmitted power for 3 ERs when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$ (6-user)

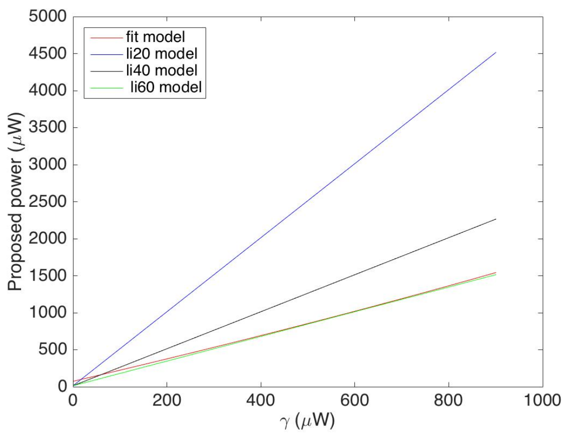

Figure 3.16: Proposed power at transmitter for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 1, h_{ER_2} = 1, h_{ER_3} = 1$ (6-user)

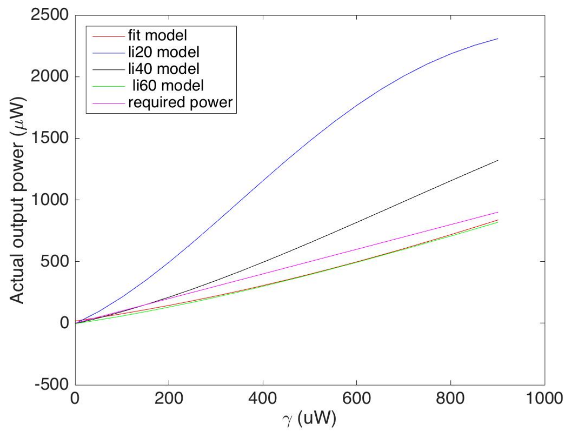

Figure 3.17: Actual output power of ERs for different models when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 1, h_{ER_2} = 1, h_{ER_3} = 1$ (6-user)

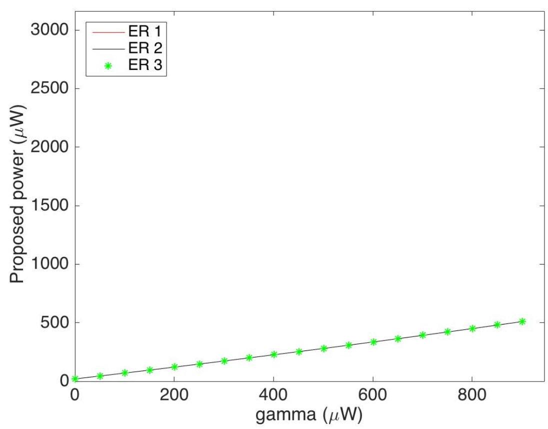

Figure 3.18: Distribution of transmitted power for 3 ERs when channel coefficients are $h_{IR_1} = 1, h_{IR_2} = 1, h_{IR_3} = 1, h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$ (6-user)

The result shows that the distribution of channel gives a huge effect on the actual output power. Generally speaking, the observation we obtained here is similar as what we saw in the 2-user case. It is noticeable that in Figure 3.10, there is a huge gap between the li20 and other models. The reason is that after the ER_1 takes all the power it can handle with the power constraints, the other 2 ERs with low efficients need to much more power to satisfy the requirement.

Besides, when we look at the distribution of transmitted power for each ER,the distribution of h also gives effect on the transmitted power distribution. In the case where $h_{ER_1} = 2.4, h_{ER_2} = 0.3, h_{ER_3} = 0.3$, the ER with highest coefficient gets the full transmitted power for ERs. And in second case where $h_{ER_1} = 1.5, h_{ER_2} = 1.5, h_{ER_3} = 0$, the first two coefficients get the same value, and the two ERs also get the same amount of power. With distribution of h becomes flat, the power distribution also becomes flat. We observe that the transmitted power goes first to the ER which has the highest coefficient. With multiple ERs which have the same coefficients share equally the total transmitted power. When the total transmitted power increases that the ERs with the highest coefficients can not handle all the power, the left power goes to the ERs with lower coefficient until the requirement is satisfied. Note that the sum of the coefficients remains the same in all cases. To explain this we should go back and look at the ER constraint of the problem formulation again:

$$\sum_{i=1}^{n_{ER}} g(P_i|h_i|^2) \geq \gamma \tag{3.16}$$

Since we are using our proposed non-linear model,namely quadratic function, the equation could be written as:

$$\sum_{i=1}^{n_{ER}} p_1(P_i(h_i)^2)^2 + p_2(P_i(h_i)^2) + p_3 \geq \gamma \tag{3.17}$$

where p_1, p_2, p_3 are coefficients of quadratic function, computed by curve fitting tool box. With h_i same to every channel, the output power for each channel are also same which is consistent with the equation (3.21), thus the problem turn out to be a single ER case again:

$$n_{ER}(p_1(P(h)^2)^2 + p_2(P(h)^2) + p_3) \geq \gamma \tag{3.18}$$

and we could subsequently get:

$$p_1(P(h)^2)^2 + p_2(P(h)^2) + p_3 \geq \frac{\gamma}{n_{ER}} \tag{3.19}$$

In the simulation, we set the range of γ from 1 to 945, after dividing the n_{ER}, 3 in this case, the new range roughly becomes 1 to 315. And from the Figure 3.19, we could observe that there exists difference between this range, and the general gap is about 30. The gap explains the result in the simulation. Although the proposed model is generally fit with the realistic data, however in this multi-user case, the range of input is reduced, and therefore part detail of the curve is enlarged, which could lead to a gap in the simulation.

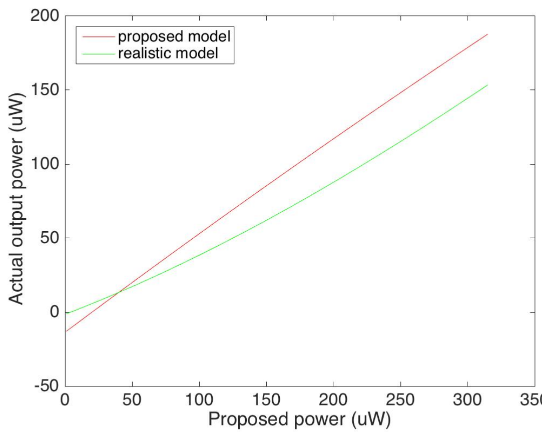

Figure 3.19: Difference between realistic model and our proposed model

3.2.3 Weighted average based formulation

Above we discussed three problem formulations separately, now we will take all the factors into consideration and provide an alternative formulation to get insight of the relationship between parameters. Here we set aside the issues with the range of valid power values for the EH devices and focus on the general structure of the solutions. Hence we omit the input power constraints on EH devices.To pursue the maximization $\sum g(P_m|h_m|^2)$ is equal to pursue the minimization of $-\sum g(P_m|h_m|^2)$. We denote the quadratic function as $-\sum f_{ER}(P_m|h_m|^2)$. Similarly the target function to maximize $\sum \log_2(1+\frac{P_j {h_i}^2}{\sigma^2_{n_E}})$ can be turned to minimize $-\sum \log_2(1+\frac{P_j {h_i}^2}{\sigma^2_{n_E}})$. We denote the logarithm function as $-\sum f_{IR}(P_j|h_j|^2)$. The target function of minimizing the proposed power remains the same, which is denoted as $\sum f_{TX}(P_i|h_i|^2)$. Thus we could get the new problem formulation:

$$\begin{aligned} &\text{minimize} \quad \zeta_A \sum f_{TX}(P_i|h_i|^2) - \zeta_B \sum f_{IR}(P_j|h_j|^2) - \zeta_C \sum f_{ER}(P_m|h_m|^2) \\ &\text{subject to} \quad P_i \geq 0 \end{aligned}$$

Solution: The way of solving the problem is to make the first derivative of the target function and set it to zero, the value that satisfy the equation would be a extremum value. To simplify the problem we first start with 2-user case, which there are 1 IR and 1 ER, correspond to P_1 and P_2 respectively. Thus, we could get:

$$\frac{\partial}{\partial P_1} = \zeta_A - \zeta_B \frac{(h_1)^2}{\sigma^2 + P_1(h_1)^2} = 0 \tag{3.20}$$

$$\frac{\partial}{\partial P_2} = \zeta_A - \zeta_C(2p_1(h_2)^4 P_2 + p_2(h_2)^2) = 0 \tag{3.21}$$

And then we could obtain:

$$P_1 = \frac{\zeta_B}{\zeta_A} - \frac{\sigma^2}{(h_1)^2} \tag{3.22}$$

$$P_2 = \frac{1}{2p_1(h_2)^4}\frac{\zeta_A}{\zeta_C} - \frac{p_2}{2p_1(h_2)^2} \tag{3.23}$$

From 3.22 we could see that with $\frac{\zeta_B}{\zeta_A}$ fixed, the higher the channel coefficient is, the higher the P_1 is. With the channel coefficients fixed, the higher the ratio of $\frac{\zeta_B}{\zeta_A}$ is, the higher the P_1 is. From 3.23, with the channel coefficients fixed, the higher the ratio of $\frac{\zeta_A}{\zeta_C}$ is, the lower the P_2 is.

For multi-user case, we could get similar result:

$$P_i = \frac{\zeta_B}{\zeta_A} - \frac{\sigma^2}{(h_i)^2} \tag{3.24}$$

$$P_j = \frac{1}{2p_1(h_j)^4}\frac{\zeta_A}{\zeta_C} - \frac{p_2}{2p_1(h_j)^2} \tag{3.25}$$

where P_i represents the transmitted power for i-th IR and P_j represents the transmitted power fro j-th ER. Generally we could see that, in (3.24) with ratio fixed, higher h_i will will give a higher P_i, with parameters fixed, if we have more weight on IR constraint, we will obtain higher P_i. In (3.25) with ratio fixed, with parameters fixed, if we have more weight on ER constraint we will obtain higher P_j.

4

Conclusions

In this thesis, a practical RF Energy Harvesting model is proposed to capture the non-linear characteristics of EH circuits in SWIPT systems. A comparison between the proposed model and linear models is made to show that the linear model gives a poor match of the realistic case. Another comparison between the proposed model and another non-linear model proposed in [1] is made to show that the proposed model is as good as the other model in terms of capturing the characteristics and easier to work with in a mathematical sense. A simple case is discussed where a system including one transmitter and one ER is considered. The figures shown in this case provides a observation that the proposed model needs relatively less power from transmitter as well as satisfying the EH requirement without too much waste.

In Chapter 3, the resource allocation for SWIPT in OFDMA system is discussed. Several problems are formulated to show the issue from different perspectives. Here to obtain numerical results, we choose to use CVX to solve the problems since they can be solved be standard algorithms in optimization problem. From the constraints' point of view, the problem formulations can be split into 3 parts. The first part is individual performance constraints where for each receiver there is requirement to be met, that is each IR should meet a threshold of information rate and each ER should meet a threshold of required harvested power. The other part is to look at the total performance for all the receivers. To satisfy the constraints in the second formulation, the total information rate of all the IRs should meet a threshold and the total harvested power of all ERs should meet a threshold. The last part is to combine all the factors together. Each factor is weighted so that we can see the general relationship between each factor.

The solution for the first problem, is to obtain the minimum value of each constraints, which is to obtain the minimum of a logarithm function and a quadratic function. To solve the second problem, two situations are considered. For the case with one IR and one ER, a similar conclusion can be observed with different channel conditions. With multi-user case, the distribution of channel coefficients might give a huge effect on the actual output power since it will enlarge the difference between the proposed model and realistic model. The distribution of h gives influence on the transmitted power distribution. The observation is consistent with the equations we derived in weighted average based formulation. To solve the last problem, a first derivative is made to the target function and set the derivative to zero so that we could get extremum value.

The subsequent direction of research on this field could be finding more specific relationship between distribution of h and performance of resource allocation in OFDMA SWIPT system. Moreover interference between sub-channel would be an interesting factor that influences the performance of the system.

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