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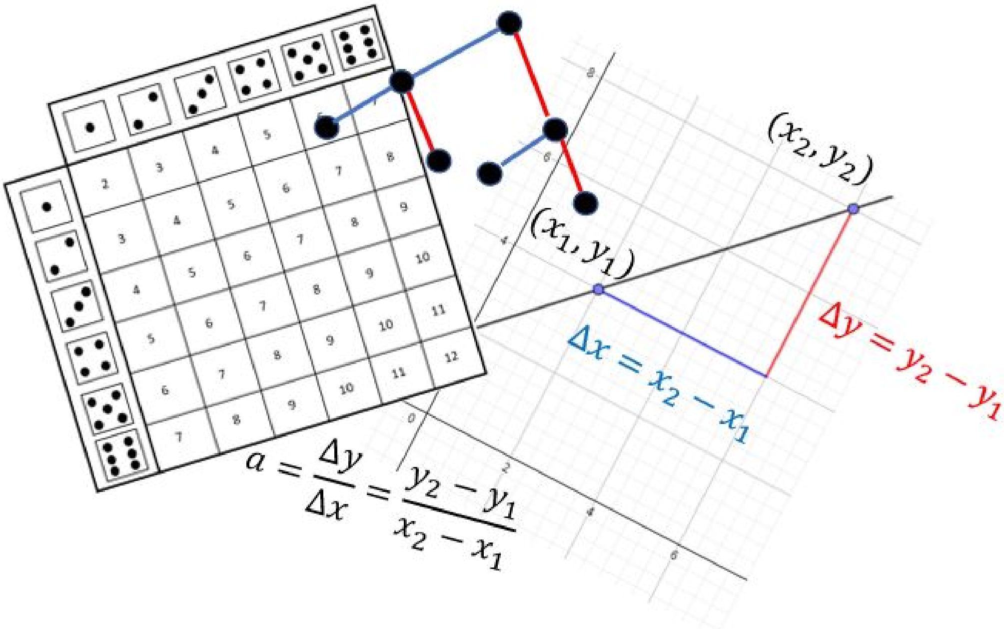

The effects of ICT-based teaching in mathematics

Measuring the effects of ICT-based teaching in the mathematical areas of linear equations and probability

Master's thesis in Learning and Leadership

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CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2022
www.chalmers.se

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Master's Thesis 2022
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Cover: A combined set of figures and equations that are used in the report, created in PowerPoint.

Typeset in LaTeX
Printed by Chalmers Reproservice
Gothenburg, Sweden 2022

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Abstract

The purpose of this study is to examine the effects dynamic mathematics software has on understanding linear functions and theory of probability. The study was limited to investigating GeoGebra's efficacy on the aforementioned mathematical topics. Linear functions and probability present great difficulty for students and educators alike. Two different classes, named 21B ($n = 31$) and 21C ($n = 31$) both studying their first year in the Swedish upper secondary school constituted the examined groups. Class 21B formed the experimental group during the teaching of linear functions and class 21C constituted the control group. When investigating probability the control group and experimental group were switched, making 21C the experimental group and 21B the control group during the second half of the study. Their pre-existing knowledge was measured to be able to compare the end-results fairly and it showed no significant different in level of knowledge before the study. Data on the students' performances were collected by using achievement tests at the end of each lesson and their ordinary final exam was used as an after-test to measure the long term knowledge. The results of this study show GeoGebra having no significant effect on students' understanding of neither linear functions or theory of probability when first being introduced to it.

Keywords: GeoGebra, Teaching, ICT, Linear, Equations, Probability, Students.

Acknowledgements

We would like to acknowledge the great help provided by our supervisor Ola Helenius. He has helped guide us in discussion on which methods would be of appropriate use and helped analyze results. In addition he was always able to provide help on short notice and his insights was a key factor in many of the decisions during the study.

We would also like to thank the Swedish upper secondary school Drottning Blanka which gave us the opportunity to perform this study in their school. A special thank you to Katarina Ekefors who is the students (that the study was conducted on) ordinary teacher for letting us execute the study on her students for a total of 14 lessons. Without you and your help this study would not have been possible.

Jonathan Andersson & Seif Sharif, Gothenburg, May 2022

List of Acronyms

Below is the list of acronyms that have been used throughout this thesis listed in alphabetical order:

ICT	Information and Communication Technology
TAM	Technology Acceptance Model
TPD	Teacher Professional Development

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Introduction

There has long been ambitions within Sweden to integrate digital tools into the educational system. The goal is aimed at promoting students' usage of technological tools:

> Teaching methods are to contain varying working methods, where activities of inquiry constitutes a part. Teaching should, when appropriate, be conducted in relevant and practice close environments with tools used within the subjects. The education will give students opportunities to develop their ability to use digital technologies, digital mediums and other tools that can occur within the taught subjects. (Utbildningsdepartementet, 2010, our translation)

The demands set out by Sweden's Department of Education are clear: the educational system will have to provide students with proper means to meet the demands of an increasingly digitalized society. It has been over a decade since the Department of Education published their demands yet there still seems to exist a knowledge gap among teachers in regards to the usage of ICT. Additionally, teachers commonly appear to oppose adapting their teaching methods to meet the need for change. The Swedish government has raised demands on all municipalities to further strengthen the digital landscape in the educational system (Näringsdepartementet, 2017). It is therefore becoming increasingly difficult to avoid the technological development taking place.

There are certain prerequisites needing to be met in order for schools to successfully incorporate digital tools as part of the education, among which teachers' perception of the benefits of ICT is considered vital (Baya'a and Daher, 2013). Teachers appear reluctant towards the ongoing digitalization of the educational system (Belland, 2009; Somekh, 2008). The teaching profession has long been slow adapters to change (Twining et al., 2013). During the 19^{th} century educators were reluctant to switch out older quills for modernized iron pens (Lavoie, 1994). After the inevitable change took place in the school system there was a noticeable qualitative increase in students' calculating abilities (Guin and Trouche, 1998). The seemingly small change enabled students to perform lengthier calculations by hand, hence enabling some mathematical topics, e.g. arithmetics, to be introduced at an earlier educational stage (Lavoie, 1994). Tools play a vital part in the necessary advancement of knowledge acquisition: “tools wrap up some of the mathematical ontology of the environment and form part of the web of ideas and actions embedded in it” (Noss and Hoyles, 1996, p. 227).

It would be to the detriment of the educational system’s progress to ignore the tools available to us: “the development of mathematics has always been dependent upon the material and symbolic tools available for mathematics computation” (Artigue, 2002, p. 245). In order for the teaching of mathematics to continuously improve, the teaching methods need to regularly adapt to the development of new tools and methods (Dodor et al., 2010; Heider, 2005). It is necessary to meet the increased demands on digital proficiency that exist in major parts of the professional life: “if technology is used to improve the learning of mathematics at all levels, students will be better prepared to use technology appropriately, fluently, and efficiently to do mathematics in technology-rich environments in which they will study and work in the future” (Niess, 2006, p. 202).

Much of the scientific literature suggest that ICT has great educational potential. This interpretation is in line with Sweden’s regulatory documents, which clearly emphasize the importance of moving education towards an increased use of digital tools. There is also widespread consensus in established research that teachers play a vital role in successfully incorporating ICT (Baya’a and Daher, 2013). Teachers need to constantly adapt to both regulatory and technological changes in order to accommodate the demands being placed on the educational system in order to drive forth pedagogical and academic progress. Works of change and development are essential if we are to “obtain and develop [students’] knowledge” (Skollagen 2010:800).

The subjects of interest for this report will be the potential benefits for knowledge acquisition of using dynamic mathematics software, e.g. GeoGebra, in the teaching of mathematics, with specific regard to students’ understanding of linear functions and theory of probability as part of the Swedish national school curricula. This report is limited to investigating only the effects of GeoGebra on the understanding of linear functions and theory of probability, refraining from including different software, digital tools or branches of mathematical topics.

1.1 Background

This study is conducted on Drottning Blankas Gymnasium Göteborg Centrum, which is a upper secondary school in Sweden. The school struggles with educating their students in mathematics; the amount of students that finish upper secondary school and pass introductory mathematics is below the national average of 98.9 % (Skolverket, 2021). The study aims to highlight potential changes in pedagogical approaches in order to increase knowledge attainment among students.

1.2 Purpose

The purpose of this study is to demonstrate potential benefits of ICT-driven teaching in mathematics. The goal is to change current methods of teaching in hopes of improving the overall quality of mathematics education.

1.3 Limitations

The study's experimental and control groups are students in their first year of upper secondary school, only examining students ages 16-17. The experiments are limited to investigating the effects GeoGebra has on understanding mathematics, excluding different software and digital tools. The mathematical topics reviewed are linear functions and theory of probability for students studying introductory mathematics at upper secondary school.

1.4 Questions of study

- Does GeoGebra have an effect on the understanding of linear functions for students in upper secondary school?
- Does GeoGebra have an effect on the understanding of probability for students in upper secondary school?

2

Theory

It is in the interest of improved educational quality to increase knowledge and awareness about the benefits of using ICT (Baya'a & Daher, 2013). Some insight in the scientific literature serves an important role in the process of digitalizing education, mathematics in particular.

Initially this section will broadly present the effect visualization, variation and 'struggle' has on understanding mathematics. Thereafter a description of both opportunities and challenges concerning the integration of digital aid is discussed. To conclude the section a mapping of current literature on GeoGebra, linear functions and theory of probability is outlined.

2.1 Understanding mathematics

For teachers to become effective mediators they need to be educated on how students learn. In this section some important components of mathematical understanding are detailed. These topics, i.e. visualization, variation theory and the concept of struggle are some important facets of mathematical understanding. The subjects are unequivocally linked to the use of ICT in mathematics education.

2.1.1 Visualization

After Piaget (1923, 1926) published two comprehensive studies, there has been a widespread consensus on the importance of representations and illustrations in the learning process. Since then there has been an influx of theories but Piaget's work is still considered to play a central part in our understanding of knowledge acquisition. Piaget (1961) asserts a strong link between understanding and visual representations and Duval (1999) writes that "representation and visualization are at the core of mathematics" (p. 3). The use of semiotics in teaching is, supported by Duval and Piaget, fundamental for the learning process, for mathematics in particular.

The subject of mathematics differs from many other realms of science as it is not readily observable. In physics telescopes are used to study astronomy, while biologists can use microscopes to study bacteria. Mathematics, on the other hand, is absent of any such tool. Hence, the use of semiotics in mathematics education plays a vital role (Duval, 2006). Applying elements of semiotics is made easier when using digital tools. Tutors need to view these tools as necessary in order to, as Duval points out, reach the core of mathematical understanding.

Learning with dynamic visual representations has significant effect on learning (Plass et al., 2009). The authors suggest that visually changeable illustrations supports students' interaction with the learning material, further strengthening their grasp of mathematical phenomena. In addition, Plass et al. conclude that the ability to manipulate certain parameters and observing the dynamic changes that follow promotes stronger understanding between symbolic representation and theoretically rigid definitions. The tutor, making full use of these available methods of teaching, will further promote a more holistic mathematical understanding.

2.1.2 Variation theory

Attorps et al. (2016) defines variation theory as "a theory of learning which can be seen as an expansion of phenomenography, which explores the qualitatively different ways of experiencing or understanding a phenomenon, especially in an educational context" (p. 46). The authors also state that variation theory and the use of ICT in mathematics are inherently linked. Applying variation theory without any digital assistance inhibits learning (Marton et al., 2004) but, furthermore, is a process which is commonly used consciously or unconsciously by the educator. Attorps et al. assert that in order to fully utilize the potential of both ICT and variation theory in education, teachers need to employ deliberate and knowledge-based strategies in order to access the full extent of the benefits that follow.

Variation theory explores a variety of ways to both understand and experience learning content (Ling Lo, 2012). The theory, as stated by Ling Lo, offers pedagogical tools that, when utilized, improves informational retrieving and understanding. A fundamental part of variation theory is the focus on the subject matter being studied, which further suggests that the subject matter's representation plays a vital role in the learning process (Marton et al., 2004). The authors conclude that learning occurs when the pupil internally separates between the different critical aspects constituted within the subject of study.

Variation theory includes four different patterns of variation (Marton et al., 2004): (1) *contrast* is distinguishing whether or not a condition is being fulfilled; (2) *separation* is the awareness of the learning subject's existing critical aspects and is obtained through a cognizant variation of different parts within the learning content; (3) *fusion* continuously integrates main principles of variation; and (4) *generalization* enables an understanding of phenomena beyond specifically contextualized patterns. Working through these patterns of variation allows pupils to visualize and differentiate key elements of the learning content and thereby increase the level of understanding (Attorps et al., 2016). ICT is, according to Leung (2003), necessary in order to fully exploit the benefits of variation theory. Incorporating digital tools makes the visualization of mathematical concepts readily accessible.

2.1.3 Struggle

The phenomenological concept of 'struggle' is vital in mathematics (Hiebert and Grouws, 2007). There is a lengthy history in established literature that emphasize the necessity of struggle to make academic progress. Dewey (1933), for instance, concluded that intellectual dissonance is paramount for students' mathematical endeavors and Piaget (1966) derived that struggle deepened conceptual understanding. Furthermore, some cognitive researchers argue that struggle is a catalyst for cognitive development (Festinger, 1962). Polya (1945), however, points towards experimentation as a stimulant of struggle while Handa (2003) attributes it mainly to "sense-making" (Warshauer, 2015, p. 376).

One could make the argument that the support of ICT alleviates part of the struggle when attempting to conceptualize subconstructs of mathematical topics (Hiebert and Grouws, 2007). Awareness of this potential outcome can therefore prove beneficial when educators attempt to employ integration of ICT in the classroom; likely reducing the risk of negatively impacting students' learning.

2.2 Opportunities and limitations regarding ICT

The implementation of ICT in education is multifaceted. The existing benefits can potentially enhance students' learning. There are, however, inhibiting factors that hinder teachers from making use of the potential benefits ICT can offer.

2.2.1 The student and the learning process

Digital tools can, among a multitude of things, strengthen active learning (Huffaker, 2003; Suryani, 2010). Active learning occurs when students are engaged and able to interact in their own learning process (Rodrigues, 2002; Vygotsky and Cole, 1978). Interactive teaching comes with different benefits. For instance, collaborative discussions is an interactive process which enhances learning (Smith et al., 2009) and they occur more frequently when ICT is effectively used (Keong et al., 2005; Neurath and Stephens, 2006). ICT also seems to increase sharing of information among students (Keong et al., 2005). This approach moves the schooling towards a more student-centered direction (Condie and Munro, 2007; Penglase and Arnold, 1996), in accordance with Sweden's educational control documents (SKOLFS 2011:144). In this method of teaching the educator only acts as support, making it easier for the student to become more active in his/her own learning process. These features are typically attributed to constructivist learning (Duval, 1999), within which students can use digital tools to explore and attain understanding for mathematical concepts (Keong et al., 2005).

According to Slavin (2019) a learner acquires new knowledge and understanding by taking an active role in the learning process. Utilizing technological tools in teaching mathematics would support constructivist learning theories (Ogbonnaya and Mushipe, 2020). Incorporating ICT in both the learning process and the methods of teaching mathematics facilitates the active learner described by Slavin, hence promoting conceptual and phenomenological understanding. When using digital software, e.g. GeoGebra, students have shown to improve upon previous abilities to hypothesize the outcome of experimenting with variables (Disbudak and Akyuz, 2019).

ICT can increase motivation and interest among students in all subjects, mathematics in particular (Keong et al., 2005; Neurath and Stephens, 2006). Higgins et al. (2005), however, caution educators of what Thorndike (1920) labels 'the halo effect'; contextually inferring that an increase in motivation does not necessarily increase learning. Keong et al. and Neurath and Stephens do, nonetheless, conclude that the increase in interest for mathematics contributed to the learning process becoming of greater substantial value for the students. Other research has also proven ICT to be efficient in bolstering mathematical conceptualization (Bester and Brand, 2013; Bray and Tangney, 2017; Ogbonnaya, 2010)

2.2.2 Teachers' pedagogical improvement and collaboration

Digital tools can improve a teacher's pedagogical abilities (Waxman et al., 2003). Current control documents set forth by the Swedish government and department of education emphasize the active implementation of ICT in the educational system. This emphasis on digital tools is aimed to both elevate student learning and the quality of the provided education.

ICT enhances pedagogical development but other aspects of the profession seem to restrain it. In previous studies teachers have been shown to be reluctant to explore new methods of communicating and teaching (Belland, 2009; Somekh, 2008). Despite the fact that the line of work offers a wide range of both personal and pedagogical development teachers seem disinclined to gather new information, within ICT in particular (Twining et al., 2013). As a consequence teachers will not make full use of existing pedagogical tools, thereby limiting students' potential knowledge acquisition.

The most influential aspects in students' learning and academic development are educated and pedagogically skilled tutors (Rivkin et al., 2005). Raymond and Leinenbach (2000) characterizes these teachers as voluntarily examining their own process of development in an attempt to improve upon their current teaching practice. The authors point out that a teacher's ambition to constantly seek to elevate the quality of lectures cultivates into a teaching environment benefiting not only students' learning but also the teacher's personal and professional development. When, however, teachers show animosity towards works of development, students risk missing out on their full potential.

Schulz-Zander and Eickelman (2010) highlight that collaboration between teachers is an integral part of pedagogical advancement. The British Department of Education (2010), based on substantial amount of scientific research available, listed different measures teachers can and should take in order to develop new and improved pedagogical skills; among which are auscultations and a collaborative work environment among tutors. Collegial support among teachers scarcely occurs (Karlberg and Bezzina, 2020; Patrick et al., 2010). More so, teachers commonly appear to work in isolation from each other, not exploiting the collective knowledge within the school (Dodor et al., 2010; Heider, 2005; Karlberg and Bezzina, 2020). Importantly, ICT has been shown to strengthen collegial collaboration (Keong et al., 2005; Penglase and Arnold, 1996).

2.2.3 TAM and limitations

The technology acceptance model (TAM) created by Davis (1989) is a theoretical framework that aims to predict a user's susceptibility for new technology. The model is built upon three key factors: (1) *perceived usefulness*, (2) *perceived ease of use* and (3) *intention to use*. Davis hypothesized that the user's intention to use a technological system plays a central role in the model. The intent is dependent on both perceived usefulness and perceived ease of use. However, perceived ease of use directly impacts the user's perceived usefulness yet not vice versa (see fig. 2.1). This model has been extended and elaborated, involving more behavioral elements that affect a user's probability of using a technological system, yet Davis' original model is still widely used (Lala et al., 2014).

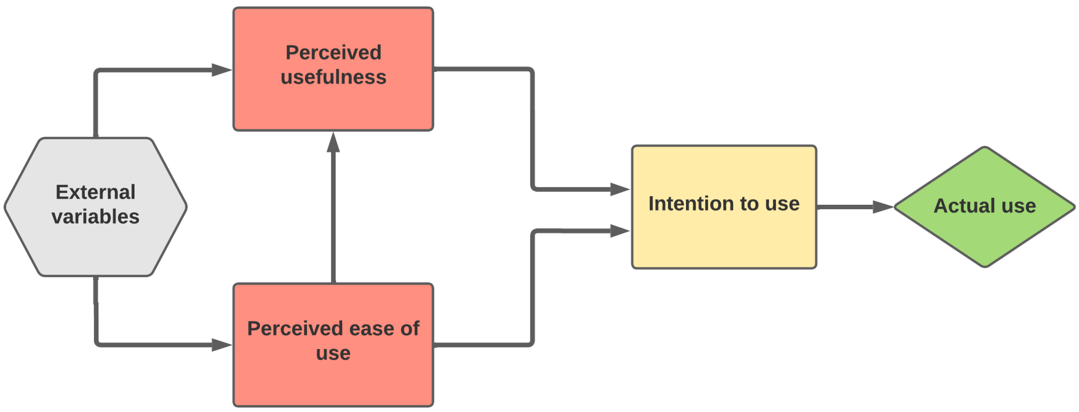

Figure 2.1: Schematic figure for TAM

Perienen (2020) states that TAM is suitable for use in schools, e.g. showing that younger teachers that have greater digital fluency and were more prone to use new and unexplored digital tools in their classes. Teachers, in general, appear to be aware of the potential benefits of ICT in education yet still tend to refrain from employing strategies to integrate digital tools in their methods of teaching (Balanskat et al., 2006). Balanskat, in line with Davis's model, concludes that the reason for this is teachers' perceived ease of use, or lack thereof, for the technological system in question.

2.3 GeoGebra

GeoGebra was created in 2002 by Markus Hohenwarter. The program is aimed at making mathematics interactive, visually connecting algebraic systems with software that dynamically represents geometrical illustrations (Hohenwarter and Jones, 2007). Using this software "enables its users to create mathematical objects and interact with them. GeoGebra users, mostly teachers or students, can use this environment to explain, to explore, and to model mathematical concepts and the relationships between them, or mathematics in general" (Zengin et al., 2012, p. 184). The program is free of charge and according to Phan-Yamad and Man (2018) it supports students' ability to examine their hypotheses and link mathematically rigorous definitions to phenomena encountered in real-life.

GeoGebra has, through various different research, shown promising results internationally (Arbain and Shukor, 2015; Aydos, 2015; Takači et al., 2015; Wassie and Zergaw, 2018). In Sweden, Granberg and Olsson (2015) showed GoeGebra contributing to students' creative reasoning and significantly increasing their collaboration, hence strengthening their problem-solving abilities.

2.4 Linear functions

The definition of a linear function, according to Wijayanti (2018) is "a function f on the real numbers that is given by $f(x) = ax + b$, where a, b are real numbers and $a \neq 0$" (p. 475). Functions are an important area of study in mathematics; providing pupils their first understanding of dependency between two different variables (Pierce, 2005). Pierce further emphasizes the importance of understanding linear functions by stating that it is critical for pupils' trajectory in mathematics. It also offers students a grasp of real-life situations; for instance, functions can be applied to understand the relation between two depending variables, e.g. amount of gas and the cost of said amount. In addition, it is fundamental for understanding some statistical data, enabling students to interpret graphical representations of linear functions.

Various scientific literature promotes the use of digital aid to enhance understanding of linear functions (Ogbonnaya and Mushipe, 2020). Ogbonnaya and Mushipe also conclude that students face great difficulties in the attempt to understand the mathematical concepts regarding linear functions. Sweden's Department of Education (2010), in line with most prevalent research, advocate for increased integration of digital tools in order to elevate students' mathematical abilities. ICT has been shown to reinforce the learning process of understanding linear functions (Ogbonnaya, 2010). Additionally, Granberg and Olsson (2015) showed GeoGebra improving students' understanding of functions.

2.5 Theory of probability

Theory of probability is a topic in mathematics that is commonly used in real-life situations, e.g. medical (evaluating risk of certain medical treatment) or economical (understanding risk). The topic, however, is regarded as being difficult for students (Hirsch and O'Donnell, 2001), while some, e.g. (Abrahamson and Wilensky, 2007; Van Dooren et al., 2003), claim that probability is the most challenging subject in mathematics for both students and teachers. Harradine (2008) concluded that there has been an inadequate development in pedagogy of probability education and Pratt et al. (2011) states that "pedagogic developments have not kept pace with those in software design" (p. 97). This slow adaptation to developmental change in digital accessibility appears to be in line with Lavoie's (1994) analysis of teacher reluctance, previously discussed in the introduction.

A problem in teaching arises when traditional teaching methods are used for abstract conceptual topics (Kuzu, 2021), e.g. probability. According to Kuzu "teachers need to go beyond calculating in the teaching of probability and do activities that will help students make sense of abstract probabilities with the help of real-life situations" (p. 46). The measures that are needed to accomplish this comprehensive goal is to utilize digital tools in order to expand students' conceptual understanding of probability (Franklin et al., 2007). Commonly cited benefits of integrating ICT in teaching probability, according to Abrahamson and Wilson (2007) are "high-speed errorless data processing, dynamic-visualization capabilities, and interactive facilities that can support exploration and the testing of conjecture " (p. 34).

3

Methods

The goal of this study is to measure the effect of ICT-based teaching in two different areas of mathematics. To obtain this goal, access to two different upper secondary school classes was granted and it was allowed to carry out their mathematics classes for seven lessons each. Several tests were conducted which mapped the students pre-existing knowledge, re-tell knowledge (which will be explained in this section) and long term knowledge to be able to measure the effects of ICT-based teaching. This chapter contains a complete description on how the study was carried out and how the results were gathered and analyzed.

3.1 Participants

This study takes place in Gothenburg, Sweden where the authors have been granted the opportunity to fully conduct two different classes' mathematics lessons. These students study their first year in the same community program (which mainly focuses on societal subjects, such as civics) at the Swedish upper secondary school, easily compared to senior high school. The first class referred to as 21B ($n = 31$, 9 males and 22 females all born in 2005) will be compared to the second class referred to as 21C ($n = 31$, 12 males and 19 females all born in 2005). They study the same math course as one another. Ordinarily they share the same math teacher and classroom for their math course which means they have close to identical conditions during their math lessons. Given these similar environmental factors the classes are an eminent group to conduct this study on and is a reason to why they have been chosen.

A number of different factors were needed to be taken into account before conducting lessons with the different classes. Since this study was performed on real students the authors can not alter their usual arrangement in a manner that affects the students negatively. The students will be tested on the areas taught in the study and will be given grades which they later on will use to get their degree and apply to universities. Therefore the authors decided to keep their ordinary arrangements when conducting lessons which implies that the structure of the lessons was kept the same or close to the same. The students are used to a review in the beginning of each class which contains what new mathematical methods and concepts should be focused on during the lesson followed by calculating problems from their math books on their own. This variant of lesson was kept for all 14 lessons conducted.

A method used throughout the lessons is the think-pair-share model (Kaddoura, 2013) which is an established model for helping students form individual ideas to then discuss them with their peers. The method is based on the teacher asking a question and then has the students think about it for a short period of time to then discuss it with a partner sitting nearby to exchange different ideas. It has been shown that this helps the students see different perspectives on the topic and is proven to enhance their learning (2013). The authors decided that this method should be used throughout the lessons so that the lessons follow the same theme and is comfortable for the students.

3.2 Intervention

The strategy for this study is to have close to the same lessons with both classes apart from using ICT tools for teaching in one of them. The first mathematical area is introduction to linear equations. Each class will have four lessons where this area is taught, the difference being one class will experience ICT-based teaching in their classroom. After four lessons the students will begin learning a different area of mathematics; introduction to probability. When the area of mathematics shifts to another so does the class that experience ICT-based teaching. This implies that both classes will experience ICT-based teaching but in the first area of mathematics, linear equations, only one of them (class 21B) while in the second area to be taught the other class will experience it (class 21C).

There are multiple factors that needs to be taking into consideration when comparing the different classes but by using this method it becomes very easy to compare raw results especially if the same questions are given to the students in both classes and the lessons conducted are similar between the classes. By swapping the classes it provides the opportunity to examine two different areas of mathematics when applying ICT-based teaching. It also gives a clearer picture of the general differences between the classes which is important to take into account when comparing the results.

The different areas of mathematics taught might seem unrelated and it is hard to establish a correlation. The reasons for the incoherent areas of mathematics is that the lessons had to follow the students normal curriculum and since this project took place during a limited time frame the areas of mathematics were not changeable. The authors had to adapt to the areas of mathematics that would have been taught if the study did not take place.

3.2.1 The teacher factor

To provide circumstances and a teaching experience as equal as possible for the two classes one can discuss the impact different teachers can have on the students. Research shows that a good teacher does make a difference (Rivkin et al., 2005).

Given this information it is vital to even out the teacher experience for the students since there are a total of two teachers that will conduct lessons in this study. This is done by having each class once a week per person. By dividing the lessons between the authors so that each author has both of the classes once per week their teacher experience is evened out and the classroom experience stays close to identical for the students throughout the study.

3.2.2 Detailed description of the lessons

This section of the report describes the lessons that were held with the students. The content of the students lessons had already been decided since they follow a given curriculum. The goal was to have near identical lessons with the different classes with the difference being one of them experiencing ICT-based teaching. This may lead to differences that needs to be presented. Each lesson is presented by a goal and an execution (one for the class with digital aid and one for the class without it). Each lesson lasted for 1 hour and 15 minutes for both classes.

During lessons 1-4 the focus was introduction to linear equations and during lesson 5-7 the mathematical area that was taught was probability. Class 21C experienced lesson 1-4 without the use of ICT and class 21B were taught with digital aid hence class 21C was taught with ICT during lesson 5-7 while class 21B were not. At the end of each lesson there were quizzes to test how well the students' understood what had been taught during the lesson. The quizzes were answered anonymously in line with Denscombe's (2017) research guide in order for the study to attain legitimacy and reliability. Data collection was conducted in accordance with Matthews' and Ross' (2010) research guide. A collaborative review of presented results were continuously made after each lesson, following Bjørndahl and Nilsson's 2005 book *Det värderade ögat*, stating that sharing interpretations strengthens understanding of given results.

3.2.3 Lesson 1

The goal of this lesson is to get familiar with the basics of linear equations and learn how to calculate the slope of a straight line using a given formula.

Execution Class 21C (without ICT): Since this is the first time many students encounter linear equations the lesson began with a real life example of two cars going different speeds as shown in Figure 3.1. Note that this is a digital made replica and the one the students were shown was hand drawn on the white board in the classroom. The students were given the assignment to figure out which car is moving faster and why. This exercise was conducted according to the think-pair-share model (Kaddoura, 2013) where the students first had to think about the task then share it with a friend sitting nearby and finally discuss it with everyone in the classroom.

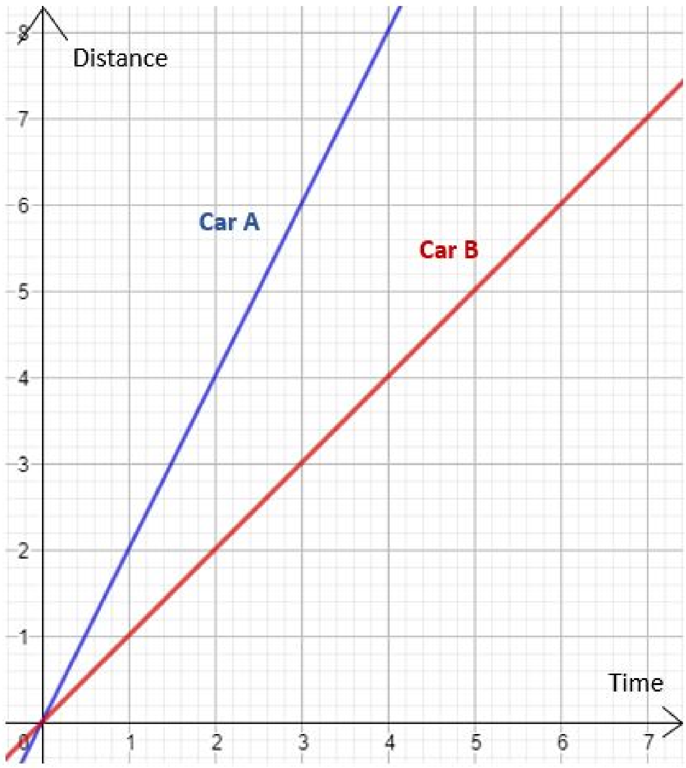

Figure 3.1: The example the students were given

The students were then told that it is the slope of the line that determines which car moves the fastest and that all straight lines can be written in the form $y = ax + b$. The students then got a lecture on how to calculate the slope by using the equation where two points need to be known. A few points for the different cars in Figure 3.1 were then given and they got the opportunity to test their newly acquainted equation. The remainding time was spent calculating problems from the students math books.

Execution Class 21B (with ICT): Similarly to the execution for class 21C, students were presented with a real-life situation regarding two cars driving with different speeds. These linear functions, however, were presented using GeoGebra. Using the dynamic mathematics software made the visual representation of said functions readily available. Students were also asked to reflect on how a faster or slower car than the other two would be represented, supporting "exploration and the testing of conjecture" (Abrahamson & Wilensky, 2007, p. 34).

The students were then told, step-by-step, how to construct a general linear equation in GeoGebra. With the help of a slider the students were able to change the value of the constant a from the general equation and see a dynamic visualization of how the constant changes the slope of the line (Attorps et al., 2016). Figure 3.2 shows two still pictures that describes what the students were able to do and what they saw.

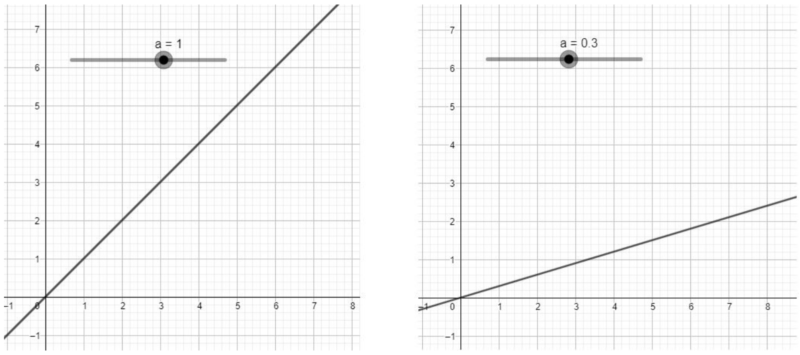

Figure 3.2: Two still pictures that describes what the students were able to do in GeoGebra

The students were then given the same equation as the other class for calculating the slope with two different points given. The rest of the lesson was spent calculating relevant problems from the students math books.

3.2.4 Lesson 2

This lesson focuses on learning how to draw linear functions, given the funcion.

Execution Class 21C (without ICT): To be able to draw a linear equation on a piece of paper one must understand how the constants a and b affects the depiction of the line in the given equation from lesson 1. The letter a represents the slope and determines how great it is. This was taught in lesson 1 and the students got a brief summary of that in the beginning of the class to then move on to be taught what the letter b represents in the general equation.

The students were told that the constant b in the equation is where the line meets the y-axis in the coordinate system. They were also told that it is the point where $x = 0$. They were then given an exercise to be performed according to the listen-think-pair-share model (Kaddoura, 2013) where they were given an a and a b value and try to draw the graph by themselves. The remaining time was spent calculating problems from the students math books.

Execution Class 21B (with ICT): The students in this class were given a short summary of lesson 1 in the beginning of the class to then be asked open up GeoGebra on their own computers. As in lesson 1 the students were told to draw a general linear equation with sliders but this time the equations also used the constant b so that they could visualize how different equations appear depending on the constants. Figure 3.3 illustrates what the students saw.

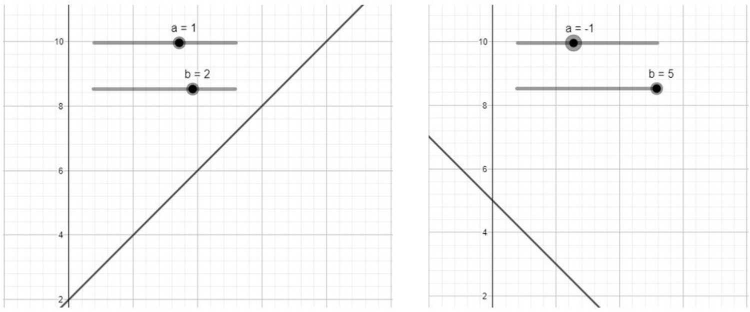

Figure 3.3: Two still pictures that describes what the students were able to do in GeoGebra

The students were then given the same task as the other class where they had the assignment of drawing an equation given two constants. The rest of the class was spent calculating problems from the students own math books.

3.2.5 Lesson 3

This lesson goes in depth on how to calculate the a constant from the equation $y = ax + b$.

Execution Class 21C (without ICT): This lesson started by repeating the equation given in lesson 1, namely $a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. The students had a hand-drawn linear equation written on the white board and it was explained what exactly was meant with the difference in y divided by the difference in x. It was also mention that it does not matter for a linear equation where we decide to measure these differences, the a constant will remain the same throughout the entire line. Figure 3.4 presents what was drawn on the white board.

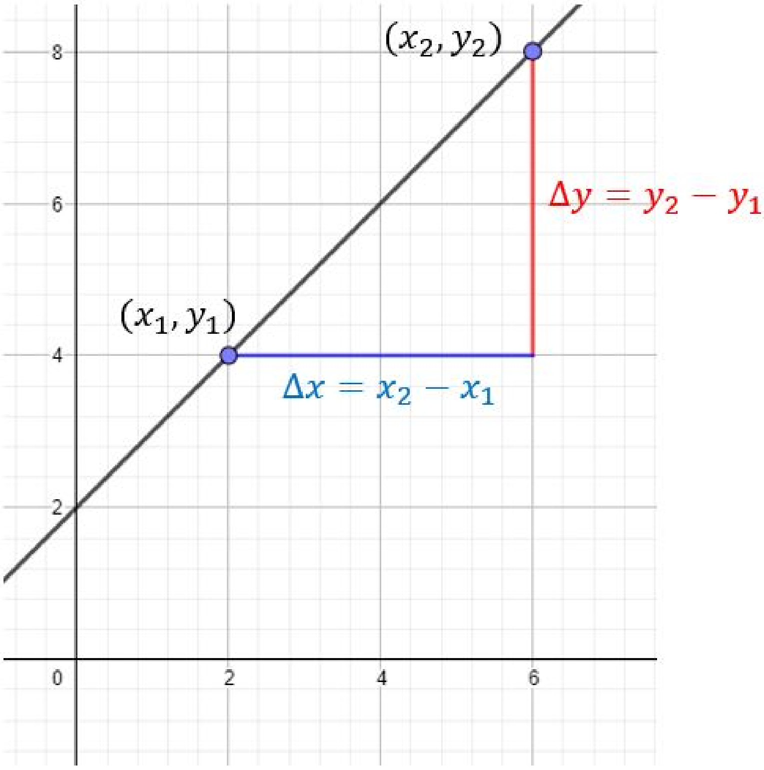

Figure 3.4: A visual representation of what the students were shown on the white board

It was explained that given any two point on the line the constant a was able to be calculated. The students were then given two different points and told that they were on the same line. They were then asked to determine the a value according to the think-pair-share model (2013). The rest of the lesson was spent calculating problems from the students own math books.

Execution Class 21B (with ICT): This lesson also began by repeating the equation for determining the a constant. I was also explained in depth of what exactly is meant with the difference in y divided by the difference in x. This class was also given exercises where two different points on a line is given and they have to determine the a constant. The main thing that differed this lesson from the one not using ICT was that the students were able to confirm their answer in GeoGebra. By drawing a line with their calculated slope and two points given they were able to see if their answer was reasonable or not.

3.2.6 Lesson 4

The goal of this lesson is to determine both the parameters in the $y = ax + b$ form for a linear equation when different factors, such as two points or the a value and one point in which the linear equation crosses, are provided.

Execution Class 21C (without ICT): During this lesson the students were presented with four different scenarios where the goal was to determine the a and b constant for a linear equation and present it in the form $y = ax + b$. The first scenario, being the easiest one, the students were given both constants. For example if they were given that $a = 2$ and $b = 3$ the answer would be $y = 2x + 3$.

In the second scenario the students were given the a constant and a point in which the linear equation would go through. From here the students were asked to determine the b constant.

The third scenario is very similar to the second. The students were presented with the constant b and a point on the line and had from here on determine the constant a. Both the second and the third scenario requires the students to use algebra to figure out the constants.

In the fourth scenario the students were given two points on a line and were told to determine both constants. Each of the scenarios were given as a think-pair-share exercise (2013). The rest of the lesson was spent calculating problems from the students own math books.

Execution Class 21B (with ICT): The class followed the same execution as the class not experiencing ICT with the difference being that they were told to confirm their answers in GeoGebra. When using GeoGebra they could be certain that they had calculated the constants correctly by drawing a line that fulfills all of the criteria given in the exercise. The rest of the lesson was spent calculating problems from the students own math books.

3.2.7 Lesson 5

This is the students first lesson to introduction to probability. Therefore this lesson focuses on the basic equations used in the mathematical area and using coordinate systems in probability.

Execution Class 21B (without ICT): The lesson began with asking the students what they already knew about probability. After the definition was laid down on them it was possible to move on to how to calculate probability and they were told that the number of favorable outcomes divided by the number of total outcomes gives the probability for one of the favorable outcomes to happen. They then got a few assignment where in the first one they were told that they have a bag of pellets, 5 red ones and 3 blue ones. They were suppose to calculate the probability of picking a red one at random from the bag. The second assignment was similar to the first one but this time they had three different colors (red, blue and green) of pellets in the bag and they had to calculate the probability of not picking a green one. These exercises were conducted according to the think-pair-share model (2013).

The students were then introduced to coordinate systems used in probability. The example they were given contained two dice with the question what is the probability of rolling a total number of 6 with two ordinary dice. They got some time to think about this problem on their own and their thoughts were discussed. A method of using coordinate systems to figure out how many favorable outcomes and total outcomes was then introduced, illustrated in figure 3.5. The Figure presents a coordinate system where all the possible outcomes when rolling two dice are shown and their added value. With this method it is easier to see how many different favorable and total outcomes there are. The lesson ended with the students calculating problems from their own math books.

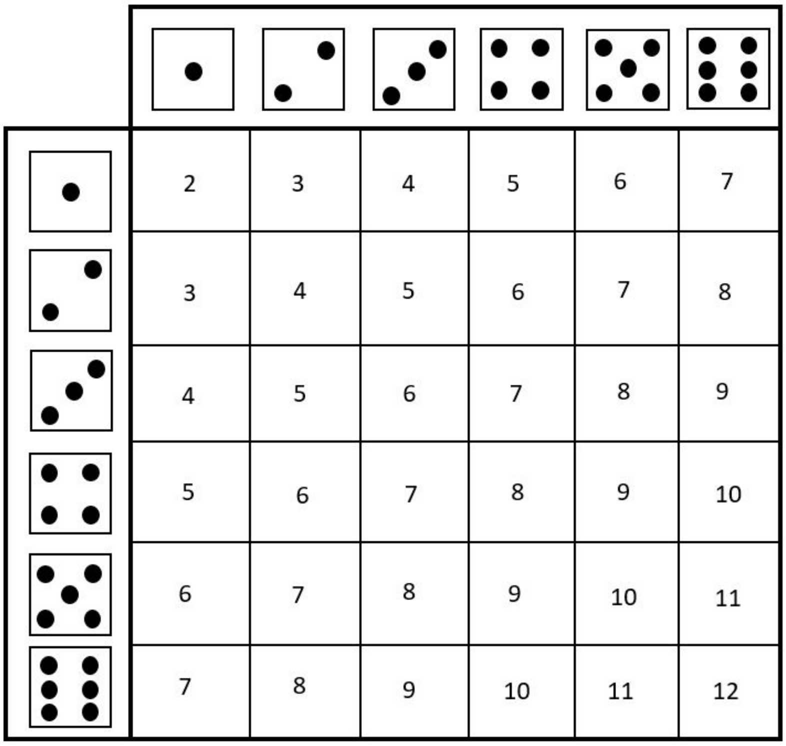

Figure 3.5: A visual representation of what the coordinate system the students were shown looked like

Execution Class 21C (with ICT): This lesson taught the students the same concepts and method as the one not using ICT-based teaching the difference being they used Geogebra for their assignments. For the example with the pellets the students were introduced to an already prepared GeoGebra lesson called *Sannolikhet: Påsen med kulorna* where a picture of a bag with pellets in different colors is introduced made by Mattias Börjesson and Robert Fant (2018a). The program asks the probability of picking a given color from the bag. The students then have to count how many favorable and total outcomes there are. By entering their results the program will tell them if they were correct or not and then produce another problem with a different amount of pellets in different colors.

The coordinate method for calculating probability was also conducted using GeoGebra. By using an already existing lesson where the students roll two dice on the computer the program visualizes how many total and favorable outcomes there are and calculates the probability automatically. This lesson was is called *Sannolikhet: summan av två tärningar* and is made by Mattias Börjesson and Robert Fant

(2018b). This exercise is not as much of a gamification of the mathematical concept as the one with the pellets but gives an overall view on how this method works. For both lessons used in GeoGebra questions were then asked according to the think-pair-share model (2013). The remaining time was spent calculating problems from the students own math books.

3.2.8 Lesson 6

The second lesson with probability focuses on tree diagrams and how to use them to calculate probability in several steps.

Execution Class 21B (without ICT): The lesson began by asking the students if they knew a method for calculating probability in several steps. A question that was asked was what is the probability of flipping a coin and have it land on tails two times in a row. After a brief discussion tree diagram was introduced to the students which is a method used for calculating the probability with several steps. A tree diagram was drawn on the white board and the method for calculating each branch was taught. The students then got their own assignment with different pellets in a bag similar to the example for lesson 5. This time they were asked to calculate the probability of picking two red pellets in a row without putting the first one back in the bag. This was then explained and drawn on the whiteboard. An illustrative picture as to what the students saw is shown in figure 3.6. All the examples the students were given applied the think-pair-share model (2013). The rest of the lesson was spent calculating problems from the students own math books.

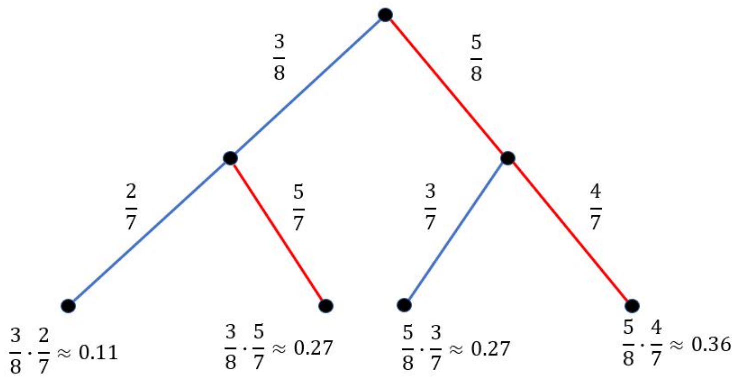

Figure 3.6: A visual representation of what the tree diagram the students were shown looked like

Execution Class 21C (with ICT): After discussing if anyone had an idea on how to calculate probability in different steps the students were introduced to an already existing lesson in GeoGebra where a tree diagram is presented. This lesson in GeoGebra is called *Ma1b och Ma1c Sannolikhet beroende händelser svarta och röda* and is made by Mattias Börjesson and Robert Fant (2015). In this GeoGebra lesson it gives an example with pellets in a bag. The user has the power to change how many different pellets there are of a specific color and are able to see the tree diagram change values for the newly set number of pellets. This was used to explain the concept and a few different combinations were tested. By using GeoGebra several different combinations were able to be tested and the students got specific questions according to the think-pair-share model (2013) as to why the diagram looked the way it did. An example of a question asked was why is the probability 3.6% for picking two black colored pellets in a row with these given parameters.

3.2.9 Lesson 7

The last lesson on probability builds on the tree diagram concept but this time goes more in depth and brings up complementary events such as what is the probability of pick at least one blue pellet when picking three pellets from a bag with 5 red and 4 blue pellets.

Execution Class 21B (without ICT): The students were as in lesson 6 shown a hand drawn tree diagram on the white board with all the numbers correctly placed. A brief recapitulation of what a tree diagram is and what information it contains was given. It was then taught that all of the last branches of the tree always adds up to 100% since there are no other possible outcomes. Knowing this information the students were asked what the probability of picking at least one red pellet when picking a total of two. This exercise followed the pair-share model (2013). It was then explained that instead of adding the probability of all the branches containing at least red pellet it is possible to add all of the branches and only eliminating the one which does not contain a red pellet. Another similar question was asked with a different number of pellets and the students calculated problems from their own math book for the remaining time of the lesson.

Execution Class 21C (with ICT): This lesson follows the same steps as the one not using ICT-based teaching but when explaining the concept of complementary events it uses the lesson from GeoGebra from lesson 6 (Börjesson and Fant, 2015). This visualizes the tree diagram for the students and helps explain the new concept.

3.3 Measures

To be able to compare the effects of the ICT-based teaching different types of tests will be executed and analyzed. Three types of tests will be conducted to measure different types of knowledge. The different types are pre-existing level of knowledge, re-tell knowledge (which is explained as what it refers to in this section) and long term knowledge. This chapter describes why and how these types of knowledge were measured.

3.3.1 Mapping the pre-existing level of knowledge

To be able to fairly compare the students knowledge in the different areas of mathematics after the study has taken place it is of great importance to map their pre-existing level of knowledge. If only the end-results are taking into account it might be misleading if one of the classes has an overall lower knowledge level than the other at the beginning of the study. It could also be that one of the classes, generally, has an overall higher performance level than the other which is important to acknowledge since the end-result could be misleading if not taking into account.

The school where the study is taking place has since the beginning of the semester hired a third-party consultant company which aims to map the student level of knowledge in mathematics. This company is called MyStudyWeb and is hired by different schools to measure the students knowledge. They conduct six different tests with the students which is then summarized and compared to their collected average. The company conducts around 150 000 tests per year and measures the students knowledge in addition/subtraction, multiplication/division, calculation methods, the number system, application in real life, fractions and percentages, geometry, statistics and equations.

These mathematical areas that are being tested gives an overall view to which level of prior knowledge the students possesses. Unfortunately they do not test the students knowledge in linear equations and probability but it is fair to assume that this test gives a general guideline as to how well the students know mathematics and can be used as a foundation for their pre-existing level of knowledge.

The authors decided to use the consulting companies' results for a number of reasons. It saves time and gives the opportunity to focus on formatting the different lessons, they have conducted these tests for several years and have developed a reliable method for mapping students knowledge and it does not take up time from the students ordinary curriculum, they would have conducted these tests if the study was taking place or not.

3.3.2 Measuring the re-tell knowledge

It is of interest to measure how well the students understand different concept just after hearing about them and if ICT-based teaching helped increase their initial understanding. Therefore a multiple choice quiz was conducted after each lesson containing math problems from what had been taught that lesson. It might not measure how well the information is carried on in the long term but it is a way of measuring how well the students were able to understand the concept and re-tell it shortly after they first learned it which is what this study refers to with the term "re-tell knowledge". It is also of interest to see if there was a difference between the class using ICT-based teaching and the one which did not when first learning concepts related to linear equations and probability. The same questions was asked in the quiz with both classes to ensure that the conditions were equal. The authors uses an online survey service called Pollev. Pollev allows several questions and figures associated with the questions. It has a limit of 25 answers per quiz which is almost an entire class.

The main reason for using this method of an online multiple choice quiz is so that it did not take up too much time from the students lesson. With the conditions given it would not be fair to spend an excessive amount of time conducting tests each lesson but by doing this at the end of each lesson a pattern might present itself to see if ICT-based teaching had an impact on understanding concepts related to linear equations and probability.

3.3.3 Measuring the long term knowledge

Knowledge can be measured in several different ways and one of the most vital factors is determining how well the information sticks. The method used for measuring the re-tell knowledge is of value when examining how well the students understood concepts when first hearing about them but it does not measure how well they learned it in the long term. Therefore, after a period of time has elapsed from when they had their lessons, a test is conducted to measure the long term knowledge.

To not take up any unnecessary time from the students it was decided that their ordinary test would be used as the test to measure their long term knowledge. Their test only contained questions regarding linear equations and probability. The difficulty of the questions on the test varied and each question had different types of points correlating to it. The types of points that were able to be obtained on the test follows the Swedish grading system A-F which means a question could be worth a number of A-, C- or E-points. This implies that the questions worth A-points are deemed harder than the questions worth E-points.

The test was divided into two parts where on the first part only an answer had to be given to the question. On the second part the students had to show their solution process and reasoning as to how they calculated the answer. There was a total of 23 questions on the test where 13 of them tested the students knowledge on linear equations and 10 of them tested their knowledge on probability. Both areas of mathematics tested the students knowledge on an A-, C- and E level according to the Swedish grading system.

It was deemed reasonable to use this exam to test the students long term knowledge. The test measures different levels of knowledge which can be used to analyze the students understanding of the mathematical concepts. The total number of questions is enough so that a pattern of difference in understanding could present itself. This test counts towards the students final grade and they would have conducted the test regardless of this study. This implies that the students will try their best at showing how much they understand these concepts. It also does not take up any unnecessary time from the students ordinary curriculum.

3.4 Data analysis

The results from the described tests will be used in the analysis of how well ICT-based teaching works for learning linear equations and probability. The pre-test is used to set a foundation on the different classes general knowledge and performance level. For example if one class generally performs at a much higher level than the other class or one class possesses a much greater knowledge level then the other it could be misleading only looking at the end-results. This means the pre-test will be used to fairly analyze the end-results so that the relative knowledge development can be measured.

The measuring of the re-tell knowledge and the long term knowledge will be analyzed in different ways. When looking at the results from the post lesson tests it gives an perception on how well the students understood the mathematical concepts when first hearing about them. This will answer the question if ICT-based learning can aid students understand certain concepts in mathematics but it does not answer the question if it helps their ability to remember it for a long period of time. That is why the after-test is conducted. This fairly measures the students long-term knowledge since it is executed three weeks from having the last lesson.

This study uses two different classes and the goal is to see if there is a general difference between the classes when using ICT-based teaching. Therefore the results from the tests are presented as a mean percentage of correctness for the whole class. This way of presenting the results shows how well the classes as a whole understood the mathematical concepts and it is possible to compare the two. It could also be of interest to present the results from the more difficult questions from the post lesson tests and the after-test. One could assume that the easier questions students would have understood with or without ICT-based teaching which would result in similar levels of correctness but by only looking at questions which only a certain number of students got correct the results between the classes could vary at a much higher level.

To determine if there is a significant difference between the students results for the re-tell and long term knowledge a Mann-Whitney U-test will be applied for the results. The Mann-Whitney U-test is used to compare the two averages between two different groups. The reason for using this method to determine a significant difference is that for the Mann-Whitney U-test the results do not need to be normally or symmetrically distributed. The Mann-Whitney U-test is preferred when the results are ordinal but not interval scaled which is expected for this studys' results. The null hypothesis is that there is no difference between the groups and if this hypothesis is rejected the difference can be considered significant. The significant level for the hypothesis is usually determined at a p-value of 0.05 or lower and will be in this study as well (McKnight and Najab, 2010).

4

Results

In this section the results of the report is presented. As described in chapter 2 the study focuses on three different types of results which all aim to map the students level of knowledge. The first type of results, which in this report is called expected knowledge level, is the prior knowledge the students possesses and what kind of knowledge level should be expected from each class. The second category is called re-tell knowledge and is based of the result of a number of small multiple choice questions the students had to do after each class. The last type of results focuses on the long term knowledge and is based of a real test the students do which is graded. The results of this kind is called long term knowledge.

4.1 Pre-test information

To be able to compare the level of learning in the different classes' in these areas of mathematics it is important to look at what prior knowledge level the students possesses and what to expect from them. If only the end-result is analyzed it could lead to a deficient conclusion.

The school at which the studied was performed has since the beginning of the school-year been conducting a project of their own in which the goal is to map the students knowledge in mathematics. This study uses their previous results from other schools as a base to be able to compare class 21B and class 21C to the average. In figure 4.1-4.2 the results of their study is presented. The base or the average is the red line where $y = 100$. The subject areas in the figures have been translated into English by us.

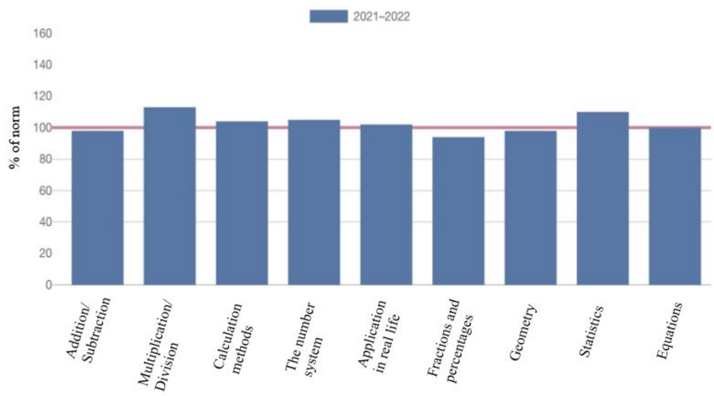

Figure 4.1: The results from the schools' mapping of the students prior knowledge, class 21B compared to the average ($y = 100$).

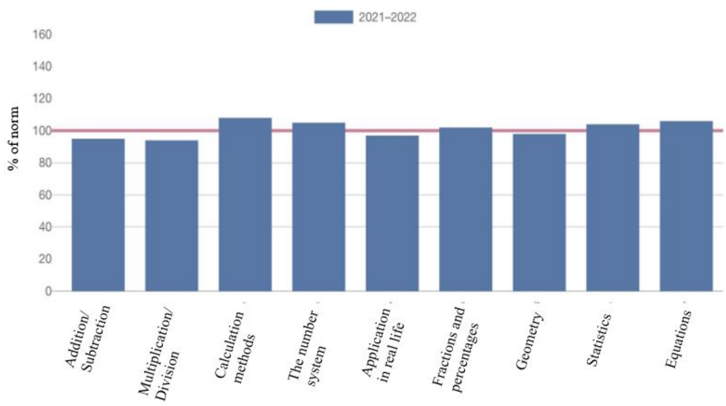

Figure 4.2: The results from the schools' mapping of the students prior knowledge, class 21C compared to the average ($y = 100$).

4.2 Lesson post-test information

It is of great importance to the project that the students knowledge level is documented. Therefore after each class that was conducted a multiple choice quiz was given to the students. This section of the report presents the results from these quizzes. The number of questions in the quizzes are four except for lesson number 3 where three questions were given. The correctness in percent that is shown on the y-axis is the classes combined number of correct answers divided by the total number of answers for all of the questions in that lesson.

Figure number 4.3 presents the results from the multiple choice quizzes from the first part of the study. In the first part of the study linear equations were taught to the students. The class named 21B experienced teaching with digital aid while the class named 21C were not. The sample size of the questions are $N = 15$.

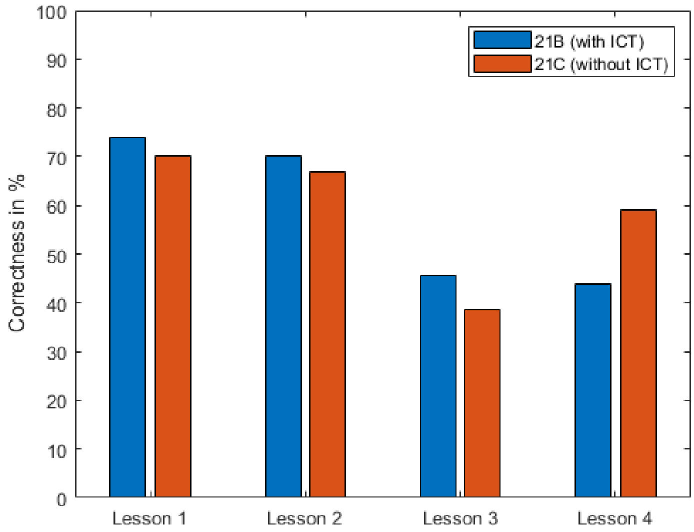

Figure 4.3: The results from the multiple choice quizzes from the first part of the study

In Figure number 4.4 the results from the second part of the study is shown. In the second part of the study the students were taught probability and Figure 4.4 shows the results from the multiple choice quizzes after these lessons. The sample size of the questions are $N = 12$

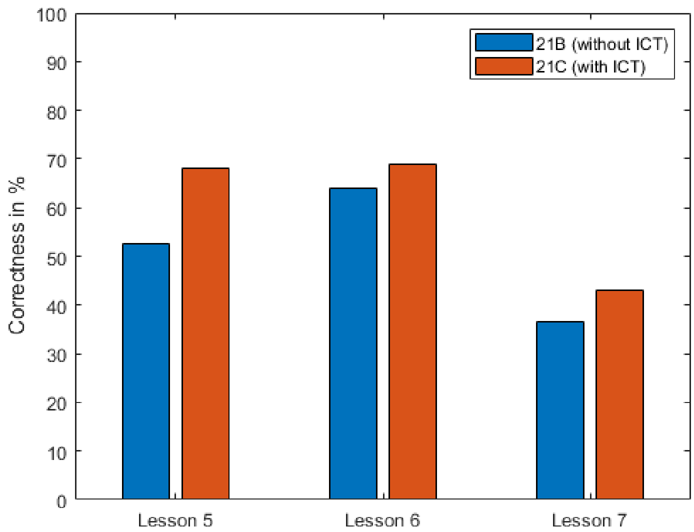

Figure 4.4: The results from the multiple choice quizzes from the second part of the study

To give an in depth picture of how the students performed in the multiple choice quizzes it may be helpful to look at the results from the more difficult questions. Figure 4.5-4.6 presents the difference in correctness between the classes when only looking at questions where at least on of the classes had a correctness rate lower than a given percentage. The x-axis in Figure 4.5-4.6 shows which percentage that is examined and the y-axis shows the mean percentage difference in correctness. This implies that when x is close to 0 few or none of the questions are being taken into consideration and when looking at when x is close to 100 most or all questions are being taken into consideration. Hence when x is close to 0 the more difficult questions are being examined. When the line is above $y = 0$ it means that class 21B had a better correctness rate than class 21C when the line is below $y = 0$ it means the opposite.

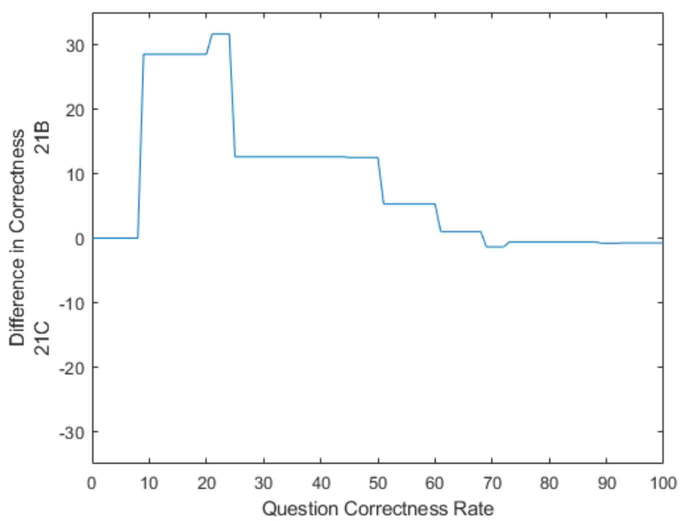

Figure 4.5: The difference in correctness between the classes when only looking at questions where at least one of the classes had a correctness rate of below a certain percentage. This Figure shows the results from the first part of the study.

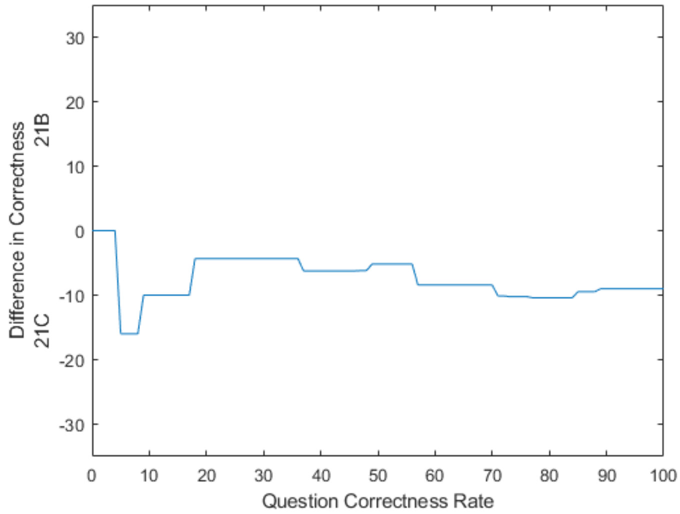

Figure 4.6: The difference in correctness between the classes when only looking at questions where at least one of the classes had a correctness rate of below a certain percentage. This Figure shows the results from the second part of the study.

The Results shows that there is a difference when looking at more difficult questions for the first part of the study. It is not shown as clearly for the second part of the study. To see if there is a significant difference between the two groups the Mann-Whitney U-test was conducted on all percentages and are presented in Table 4.1-4.2.

Linear Equations	
Question difficulty level [%]	Significance level (p-value)
9 – 20	0.33
21 – 24	0.1
25 – 44	0.21
44 – 50	0.22
51 – 60	0.36
61 – 68	0.78
69 – 72	0.84
73 – 88	0.77
89 – 92	0.71
93 – 100	0.72

Table 4.1: The significant levels from the post-lesson tests for different levels of difficult questions with the questions regarding linear equations.

When looking at the results for questions with different difficulties no significant level was below 0.05 for either part of the test.

Probability	
Question difficulty level [%]	Significance level (p-value)
$5 - 8$	1
$9 - 17$	0.33
$18 - 36$	0.7
$37 - 48$	0.69
$47 - 48$	0.55
$49 - 56$	0.48
$57 - 70$	0.53
$71 - 72$	0.50
$73 - 76$	0.44
$77 - 84$	0.34
$85 - 88$	0.32
$89 - 100$	0.33

Table 4.2: The significant levels from the post-lesson tests for different levels of difficult questions with the questions regarding probability.

4.3 Intervention post test information

To measure the long term knowledge the students ordinary test is analyzed. The test took place three weeks after the conducted lessons and the questions have been divided into two groups. One group analyzes questions on the exam relating to linear equations and the other group studies questions regarding probability. Figure 4.7 presents the mean correctness in % for all of the questions on the test relating to linear equations while Figure 4.8 shows the same results but for the questions regarding probability. Note that the y-axis is limited to 50% to be able to present the difference between the classes more clearly.

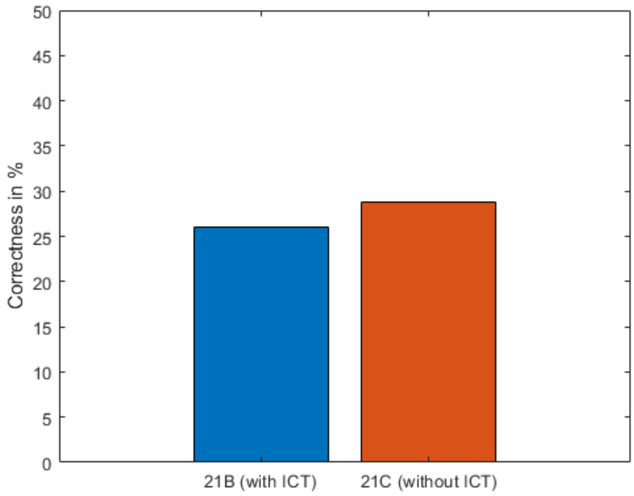

Figure 4.7: The results from the classes ordinary test with all of the questions regarding linear equations

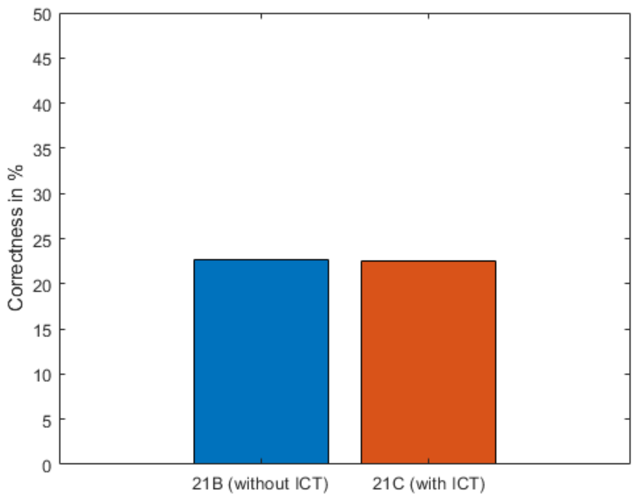

Figure 4.8: The results from the classes ordinary test with all of the questions regarding probability

The test was divided into two different parts where on the first part students only had to give an answer to the question and on the second part their reasoning and solution process had to be shown. Figure 4.9 presents the results from the questions regarding linear equation when looking at the two different parts of the test separately while Figure 4.10 shows the same results but when looking at the questions regarding probability.

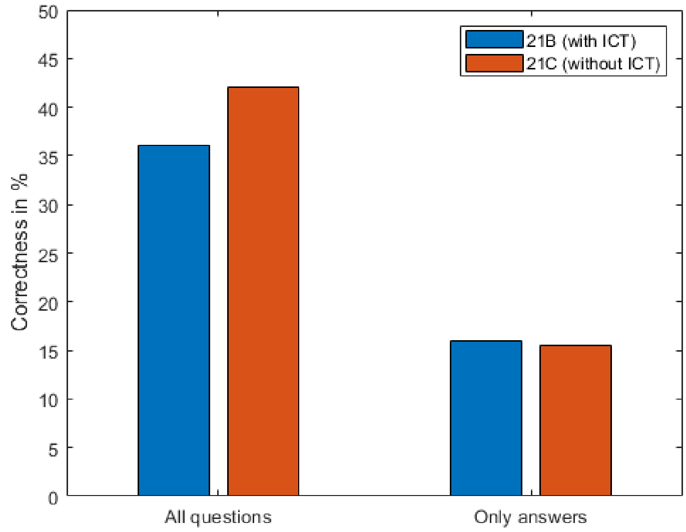

Figure 4.9: The results from the classes ordinary test with all of the questions regarding linear equations when looking at the two parts separately

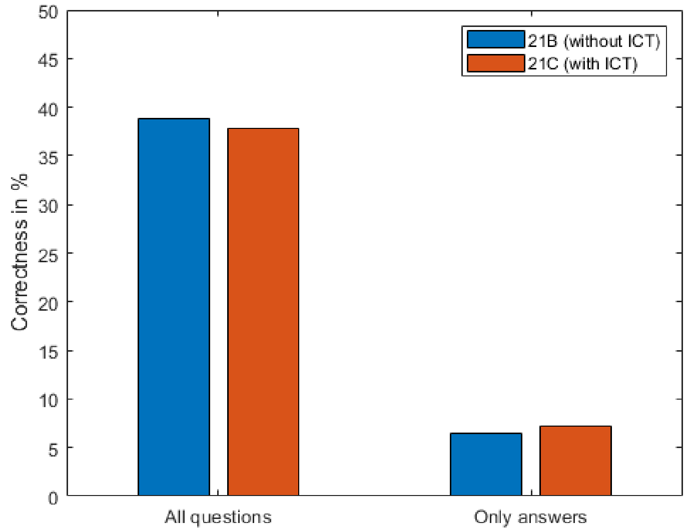

Figure 4.10: The results from the classes ordinary test with all of the questions regarding probability when looking at the two parts separately

When applying the Mann-Whitney U-test for the post lesson test and looking at the questions where at least one of the class had a lower correctness rate of a given percentage the results presents itself in Table 4.3 for linear equations and Table 4.4 for questions regarding probability. Only the results where the significant level is the lowest is presented.

Linear Equations	
Question difficulty level [%]	Significance Level
$1-3$	0.0476
$4-10$	0.0541

Table 4.3: The significant levels from the post test for where the significance level is the smallest. Questions regarding linear equations.

Probability	
Question difficulty level [%]	Significance Level
$1-3$	0.1818
$4-6$	0.3798

Table 4.4: The significant levels from the post test for where the significance level is the smallest. Questions regarding probability.

When looking at the significant level for all of the questions on the test there was a significance level below 0.05 for the harder questions regarding linear equations but not for the questions regarding probability. When doing the Mann-Whitney U-test for the different parts of the post test no significant level below 0.05 was detected. The p-value showed to increase when looking at questions with a higher correctness rate.

4.4 Result summary

Given the results from the pre-test no significant difference could be seen between the classes at an overall level. They do vary in some areas of mathematics for example multiplication/division but not so much that their pre-existing knowledge could be considered significantly different. Class 21B had a mean value of 103 while class 21C had a mean value of 101 when all of the mathematical areas that were tested were taken into consideration.

The lesson post-test information shows a trend when looking at the mean percentage of correctness. When looking at the first area that was taught, linear equations, class 21B (which experienced ICT-based teaching) answer at a higher rate of mean correctness than class 21C (which did not experience ICT-based teaching) for lesson 1-3. For lesson 4 class 21C answered at a higher correctness rate then 21B. When looking at all the questions during the tests correlating to lesson 1-4 class 21B had a mean correctness percentage of 58.42% while class 21C had a mean correctness percentage of 58.67%. When looking at lesson 1-3 class 21B had a correctness rate of 63.22% while 21C had a rate of 58.56%. Although no significant results were recorded when looking at all the questions and applying the Mann-Whitney U-test.

When taking into account the questions deemed more difficult class 21B had a better correctness rate than class 21C. When looking only looking at questions where at least one of the classes answered at a correctness rate below 24% the significant level was the lowest at 0.1.

When comparing the same tests for the second part of the study, lessons 5-7, class 21B did not experience ICT-based teaching while class 21C did. Class 21B answered at a correctness rate of 51.00% while class 21C's results were 60.00% when measuring the results from all questions. When looking at the questions deemed more difficult the lowest significant level was recorded when looking at questions with a correctness rate of 9-17% for at least one of the classes. The significant level was then 0.33.

Looking at the results from the measuring of the long term knowledge no significant difference in the students performance could be noticeable when looking at all of the questions. When looking at the questions the students found more difficult a significant level of 0.0476 was recorded for linear equation when examining the questions where at least one of the classes correctness level was below 3%. When doing the same for linear equations the significant level was 0.18 for questions with a correctness rate below 3%.

5

Discussion

5.1 Interpreting the results

This section's aim is to discuss the results from both the quizzes given out at the end of each lesson and the exam held three weeks after the last lesson.

5.1.1 Pre-tests

The pre-tests showed similar results for both classes. The two groups are therefore considered to be close to identical in their mathematical abilities. This, according to Denscombe's (2017) research guide, strengthens a study's reliability when comparing two different groups.

5.1.2 Lessons' post test

Linear functions

In the teaching of linear functions class 21B constituted the experimental group, i.e. taught with use of ICT, while class 21C constituted the control group, i.e. taught using traditional teaching methods without ICT. In all lessons there were introductory questions that were less challenging. About 80-95 % of students answered these questions correctly in both classes. These questions were considered being too simple, enabling students to answer correctly independently of being in the control group or experimental group. Removing these results made the distinction between the experimental group and the control group clearer (see Table 4.1).

During lessons 1-3 there is a difference between the students' knowledge acquisition, favoring pupils in the experimental group, suggesting that ICT promoted learning and knowledge acquisition. The difference in the understanding of linear functions might be attributed to a multitude of things. The first lesson targeted the introduction of linear functions. Students were shown two different straight lines, each representing a car's speed. The control group were presented with these functions on a whiteboard while the experimental group were asked to draw the functions on GeoGebra, simultaneously being demonstrated how on a projector controlled by the assigned tutor. Students using GeoGebra were able to experiment with different "car speeds", promoting instantaneous testing of conjecture (Abrahamson & Wilensky, 2007). This may have been a contributing factor to the experimental group outperforming the control group in the quizzes.

The sliders that were used by the experimental group in the first lesson may also have assisted the students' conceptual understanding of linear functions. According to Plass et al. (2009) dynamic visual representations elicit stronger mathematical understanding. When using a slider, the value of the slope visibly varies, which according to Attorps et al. (2016) can enhance learning. The students themselves were also able to change the value of the slope by moving the slider (see fig. 3.2), making the learner an active participant in his/her own learning process (Rodrigues, 2002; Vygotsky and Cole, 1978).

The specific topic of the second lesson presented the greatest difference between the two groups, where the experimental group scored higher than the control group. The lesson's specific topic of study was drawing linear functions. These results also seem to align with previously conducted studies pertaining the specific subject matter of drawing linear functions (Kushwaha et al., 2014; Ogbonnaya and Mushipe, 2020; Seloraji and Eu, 2017; Shadaan and Leong, 2013). GeoGebra makes drawing accurate graphs easier, in turn enabling students to efficiently draw correct conclusions from the illustrations created by the dynamic software (Zulnaidi et al., 2020). In contrast, manually drawing graphs increases the likelihood of incorrect and inaccurate representations of the studied function, raising the risk of misunderstanding and faulty conjecture.

What also may explain the discrepancy between the results on lessons 1-3 is how enthused younger students are about technology (Bester and Brand, 2013). This increase in interest creates a more meaningful and engaging learning environment for the pupils (Huffaker, 2003; Suryani, 2010). The effects of this learning environment has previously been shown to promote learning and positively impact mathematical achievement (Mthethwa, 2015; Ogbonnaya and Mushipe, 2020; Ogbonnaya, 2010; Thambi and Eu, 2013).

The specific subject matter of the fourth lesson was writing linear functions in its general form, i.e. $y = ax + b$ where $a, b \subseteq \mathbb{R}$. The reasons behind the deviating result of the fourth lesson could be attributed to a variety of things. Some of it could be situational, e.g. teacher performance or students' attentiveness while some may be attributed to the execution of the lesson itself; it may have lacked in various aspects, not utilizing the full potential of ICT. The specific subject matter is heavily focused on procedural calculations. When creating the fourth lesson there was an attempt to make visual representations of the manually calculated algebraic expressions, yet this may have left students in the experimental group with less time to practice their procedural abilities compared to the control group.

Students in both classes were instructed on how to express a linear function in its general form depending on what information is available: (1) the value of a and b; (2) the slope's value a and a given point on the line P_1: (x_1, y_1); (3) the line's intersection with the y-axis b and a given point on the line P_1: (x_1, y_1); and (4) two given points on the line, P_1: (x_1, y_1) and P_2: (x_2, y_2). These tasks, particularly the last three, are heavily procedural. If students in the experimental group solved these exercises by e.g. putting two points on GeoGebra and drawing a line between them, their problem-solving process was less challenging than that of the control group. The phenomenological concept of 'struggle' is vital in mathematics (Hiebert & Grouws, 2007). If the pupils in the control group struggled to a greater extent with the exercises it may have strengthened their procedural understanding of how to solve such problems, making them more equipped to repeat the procedure during the quiz.

Theory of probability

The results for theory of probability were insignificant between the experimental group, i.e. class 21C, and the control group, i.e. 21B. The experimental group did, however, perform slightly better on all three lessons, indicating that ICT may have aided their learning but the differences are most likely negligible.

There exists a wide range of explanations as to why the results between the two groups were not as distinguishable as for linear functions. For instance, there is a plethora of scientific literature on linear functions and ICT, while it is scarce for theory of probability. This made the preparation in constructing lessons in probability more difficult as we were more reliant on self-reflection and personal pedagogical ability, as opposed to applying previously tested and thought-out methods of teaching.

5.1.3 Long term knowledge

The results from both groups' exam were close to identical for the two mathematical topics. The lack of discrepancy between the groups shows that the visible increase in understanding during the lessons did not translate into long-term understanding. This infers that this attempted method of implementing ICT does not have an effect on long-term understanding.

In order to create a more fruitful learning environment the students might need continuous work with digital tools e.g. GeoGebra. This would make ICT a more integrated part of their problem-solving strategies. They would become increasingly proficient in the digital landscape through regular use, benefiting from the full potential of ICT.

Both groups had relatively few students that performed above the passing limit. This indicates that the groups likely spent an insufficient time studying for this exam. The context in which the students wrote this test might also have impacted their performances. The test was scheduled close to the end of the semester; a period in which students generally experience a higher work load. The exam had more variables that could affect the results. Factoring in different components such as study technique or work continuity increase the difficulty of isolating the specific effect of ICT on knowledge retention.

5.2 Incorporating ICT

This report showed that ICT can have positive impact on students' understanding of linear functions. There are, however, challenges in successfully incorporating ICT in education. The issues are multifaceted, involving teacher readiness and acceptance of using new and unfamiliar technology as well as training and teaching educators to become effective facilitators of technological landscapes.

There is a worldwide ambition to enhance the quality of education (Todd, 2010). With educational reforms, *what* and *how* teachers teach needs to change (Bautista et al., 2016). Educators play a key part in enforcing such comprehensive reforms (Guskey, 2002). There is widespread consensus around teachers' essential part in meeting the demands put on the educational system (Desimone, 2009). Barber and Mourshed (2007) additionally conclude that a school's educational quality is limited to the quality of the teachers, why continuously improving educators is considered necessary if we are to facilitate academic progress.

Traditional methods of developing teachers' pedagogical abilities pertaining the use of ICT, e.g. seminars, conferences and workshops (Gersten et al., 2010), have created little to no effect on educational quality (Garet et al., 2008; Garet et al., 2011; O'Dwyer et al., 2010). These events tend to occur sporadically and as isolated events, making teachers passive recipients of information, leaving little room for meaningful and engaging learning environments. Employing these traditional strategies, all though well-intended, generate inadequate results and is what Darling-Hammond (2010) calls "the spray and pray approach" (p. 22) or what Ball (1995) labels as "style-shows" (p. 39). In order to develop means to successfully integrate ICT there are different acts of Teacher Professional Development (TPD) needed. TPD is defined as being:

> "[...] about teachers learning, learning how to learn, and transforming their knowledge into practice for the benefit of their students' growth. Teacher professional learning is a complex process, which requires cognitive and emotional involvement of teachers individually and collectively, the capacity and willingness to examine where each one stands in terms of convictions and beliefs and the perusal and enactment of appropriate alternatives for improvement or change." (Avalos, 2011, p. 10)

There might exist some value in increasing the amount of minor studies conducted on a local level. Teachers may find studies organized locally more relevant, hence increasing the likelihood of developing and attaining new technological skills. The studies should, according to Davis' (1989) Technology Acceptance Model (TAM), focus on a technological system's ease of use and/or it's educational usefulness in order to influence the intention to use it.

There indeed seems to exist a reluctance among teachers to attain relevant know-how in areas of ICT (Twining et al., 2013). However, all though the findings of this study suggests that ICT may improve understanding in some branches of mathematics, the logistics of both training educators and providing infrastructural prerequisites for implementing ICT can be costly and might not generate substantial results in regards to students' knowledge attainment.

Activities that educators seem to appreciate the most are those that put great emphasis on the teacher's specific subject(s) (Bautista et al., 2015; Borko, 2004). Strategizing towards this type of further training for teachers might lead to a qualitative change in teachers' ability to teach. By exploring and learning how to use digital tools, e.g. GeoGebra, in a proficient manner with the intent to promote learning, students will likely experience a progression in educational achievements, making them better equipped to pursue future academic endeavors.

6

Conclusion

This study found no significant effect on ICT-based teaching on the understanding of linear functions and theory of probability. The results on linear functions showed some promise as the experimental group slightly outperformed the control group. However, the results weren't conclusive enough to make definitive statements on the efficacy of GeoGebra's effect on understanding linear functions. Based on the results of this report we draw the conclusion that the use of ICT in teaching, GeoGebra in particular, had no effect on the learning of issues related to linear functions or theory of probability. Further research in order to improve upon the conducted study is needed in order to draw definitive conclusions on GeoGebra's potential usefulness in teaching both mathematical topics. These results, however, contradict much of the established literature on this subject.

We also found no support for ICT improving students' long-term understanding of linear functions or probability as the results from the exam were inconclusive. We instead choose to evoke Vygotsky's (1934) citation of Francis Bacon (1600), translated by Trouché (2004):

> "Human hand and intelligence, alone, are powerless: what gives them power are tools and assistants provided by culture" (p. 283).

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