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Influence of Delay on the Vicsek Model

Master's thesis in Complex Adaptive Systems Master Programme

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Abstract

The aim of this work is to show that sensorial delay influences the behaviour of self-propelling agents using self-aligning interactions. The model was based on the Vicsek model, which is a two-dimensional system of self-propelling particles that are able to detect and align with each other within a certain radius. We prove that the introduction of short delays favours cluster and swarm formation, while extending the delay to higher values or implementation of negative delays significantly harms this process. We introduce a global clustering parameter, which is based on the use of the Voronoi tessellation, which allows us to measure the emergence of clusters. The sensorial delay might play a crucial role in systems that exhibit swarming behaviours and its better understanding can result in the construction of key tools for the realisation and manipulation of complex networks of autonomous robots.

Keywords: swarming, clustering, Vicsek model, sensorial delay, autonomous robots.

Acknowledgements

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Simulations used in the research were carried out with use of MATLAB R2016a Software.

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Introduction

1.1 Collective motion in nature and science

In the worlds of nature, chemical reactions or human behaviours, we can often observe collective motion of various active particles [1]. Either if the particle is a bird in a large flock or a human in a dense crowd on busy street, the motion of each agent is controlled by many connected complex mechanisms [2]. To fully understand and simulate all of these relations is close to impossible, but crucial characteristics of such complex collective motion can be reconstructed with the use of rather simple rules. This allows us to push the boundary of understanding of collective behaviours present in swarms and apply gathered knowledge in various areas.
Examples of areas, where one can observe complex behaviours, are animal flocks [2, 3, 4, 5], chemically powered nanorods [6], actin networks driven by molecular motors [7], swarms of robots [8] and human crowds [9]. Despite occurring on vastly different scales, one can find behaviours and motion mechanisms that are universal for all mentioned environments and give satisfactory results while designing similar systems. An important aspect is that they are completely independent of the active agents forming the swarm [3, 4, 5]. For many years, it remained as a daunting task for theoretical physics to construct minimal dynamical models that capture completely these features with all the consequences and possibilities that they carry with them. First attempts, that are considered successful to certain extent, were carried out by C. W. Reynolds in 1987 [10]. He proposed the boids model in order to simulate the swarm behaviour of flock of birds in computer graphics. This model was operating with many, independent boids that followed basic swarming mechanics. A more complete model was introduced in 1995 by a team led by T. Vicsek [11]. They constructed what is known as the Vicsek model, which was the first successful attempt to study swarming behaviour in terms of a noise-induced phase transition. Combining all of its modifications, it has become one of the most widely used models describing collective motion [2, 12, 13, 15, 16, 26]. Thanks to that, it was a natural choice to use this model as a base for this research.

1.2 Principles of operation of Vicsek model

The Vicsek model operates with very basic rules, namely it describes the behaviour of volume-less particles in an empty arena. The arena has a rectangular shape and is supplied with a periodic boundary condition. This result in a simulation of an infinite area when observed from the perspective of a single particle. Particles themselves are supplied with short-range detection mechanism, which allows them to discover other particles and gain information about their direction of motion. At each timestep, particles carry out their motion with constant speed, which is the same for all agents in the system and is independent of carried actions. Once the particle checks its close surrounding and detects other particles, it performs mean direction evaluation of all particles in detected area, including itself. It is then supplied with obtained this way direction vector and continues its motion on a new path. It is important to notice, that this system is not supplied with any other mechanisms, thus swarming behaviour emerges from this rules only. In spite of the fact that each particle senses only its own surrounding and cannot communicate in any other way with the rest of particles present in the system, one can observe that eventually all particles create a single cluster. This tendency can be however significantly hindered or even completely stopped through introduction of noise in the orientation of each particle. Its influence was studied by Vicsek et al. themselves, but also by many others [11, 12, 13] and currently its influence on the system is rather well defined.

1.3 Model used in the research

Even though system used in the simulations is based on the Vicsek model, it was necessary to supply it with parameters that will give clear and reliable results with respect to desired area of research. Orientational noise is represented in the system through random number drawn from uniform distribution $[\frac{-\eta}{2}, \frac{\eta}{2}]$ where its magnitude, basing on reconstruction of results obtained by Vicsek, was chosen to be equal $\eta = 0.4$. Such amount of noise prevents formation of permanent clusters, but at the same time is small enough, so particle aligning remains a main mechanism controlling behaviour of particles. Due to hardware limitations connected with performing the simulations, system size was limited to 400 independent agents which during initialisation of the model were randomly scattered over the square arena with the size of 2000 by 2000 units, which resulted in density of $M = 0.0001 \cdot A$, where $A = \pi \cdot R^2$ is the area of the detection circle. Such size of the system is already big enough to study it successfully and eliminate flaws connected with small system sizes, like for example density waves, which occur in the systems with significantly higher densities [13, 27, 28]. Speed of the particles was designated to be equal $\nu = 3$, what results in achieving the steady-state after maximally 5000 steps of the simulation, while time limit was chosen to be 10000 steps.

1.4 Introduction of measurement tools

In order to investigate the behaviour of the particles and groups of them in clusters, it is necessary to introduce proper measuring tools. Order parameter was introduced already by Vicsek et al. in their original paper [11] and is described as absolute value of the average normalised velocity, which denotes what part of the overall number of particles follow a particular direction. This parameter is a widely used tool to describe such systems, however it is far from being a perfect measuring tool, as it does not generate sufficient information about the formation of clusters. The value of this parameter is $\psi \approx 0$ in the case of full system randomisation, while coherent direction of motion of most of the particles results in parameter value reaching values $\psi \approx 1$. Parameter is calculated with use of the modulus of normalised velocity given by equation 1.1, where N denotes the number of particles in the system, ν is the absolute velocity of the particles and v_i is the velocity of the i-th agent.

$$\psi = \frac{1}{N \cdot \nu} | \sum_{i=1}^{N} v_i | \tag{1.1}$$

Second parameter describing the model is the cluster parameter - it approximates what percentage of the total amount of particles is currently a part of clusters. It was constructed with use of Voronoi tessellation [14], which partitions the arena into Voronoi cells corresponding to the agents: Each agent is associated to a Voronoi cell constituted by all those points of the plane that are closer to it than to any other agent. Size of each such cell is measured and proportion of Voronoi cells whose size is smaller than a detection area denotes the parameter, as it is represented in equation 1.2. Such approach counts particles in all clusters present in the system, independently from the size and number of clusters, however such measuring system does not count the particles that are forming the edge of a cluster. Those particles are often treated as outside of the cluster, as their cells are much bigger than the detection area. Due to such limitation cluster parameter cannot reach value of 1, thus achieving range of $0.8 - 0.9$ is considered to be a very good result and proof of reaching the steady-state.

$$c = \frac{count(A_i < \pi R^2)}{N} \tag{1.2}$$

Mentioned measurement parameters are applied to gauge the influence of wide range of delay levels on the behaviour of the clusters.

1.5 Outline

Starting from *introduction (Chapter 1)*, where the background of the research was presented, together with the basic model description this report proceeds to *Introduction of the delay (Chapter 2)* in which one can find a description of the delay mechanism implementation, justification of its use in the system and discussion of obtained simulation results describing influence of the delay on the collective motion of the particles. This is followed by *Particle velocity investigation (Chapter 3)*, where investigation of effects caused by change in velocity of particles in the model is carried out and after that, report moves to *Influence of extrapolated delay (Chapter 4)*, which treats about the results of application of extrapolated data as the basis for the delay. Last, the report is going to end with *Conclusions (Chapter 5)* and *Bibliography.*

2

Introduction of the delay

2.1 Delay application

The initial task that was put forward was the recreation of the model proposed by Vicsek et al. and the implementation of the time delay between the information gathering process and its application mechanism. Delay mechanism enhances model through bringing it closer to the real-life counterparts, as there does not exist a real system without a delay. If one considers a flock of birds [3], swarm of robots [17, 18] or human crowds [9], in all these cases we observe existence of the delay. It spans over many different scales, but nevertheless it is present in all system, thus its simulation can bring us valuable information about behaviour of these systems.
In order to simulate delay, it was necessary to create a memory matrix for each particle present in the system. Particles gather information about their close surrounding, evaluate the average direction of motion of all particles (including themselves) and store obtained this way information in the supplied matrix. Data in the matrix is stored for desired amount of time, which depends on the chosen delay of the system, and then is transferred back to the particles. Particles then apply obtained information, while saving new set of data, which completes the circle of operation. This procedure is repeated at every step and is carried through the whole simulation. One can notice that through first few steps, there is no information stored yet that particles could be supplied with, therefore particles are kept in a dummy-state, in which they all follow the same direction and they do not react to presence of any other particles. Once the necessary amount of steps is carried out and information is ready to be supplied, all the principles of particle motion are applied, together with initial random direction of motion. This procedure ensures that simulation is based on the delay mechanics from its very beginning and is not biased by lack of information present through initial steps of the simulation.

2.2 Influence of delay application

To fully understand the consequences tied to the existence of the delay in the mechanism steering the particle, it is best to compare it to delay-free case of the system. In the case of the delay-free model, which is characterised by $d = 0$, one can observe that clusters are quickly dissolving due to presence of the noise. Such behaviour is caused by the fact that particles align and engage in clusters in the very moment they detect each other, thus distances between them are relatively large and connections weak, as even minimal amount of noise can result in cluster decomposition.

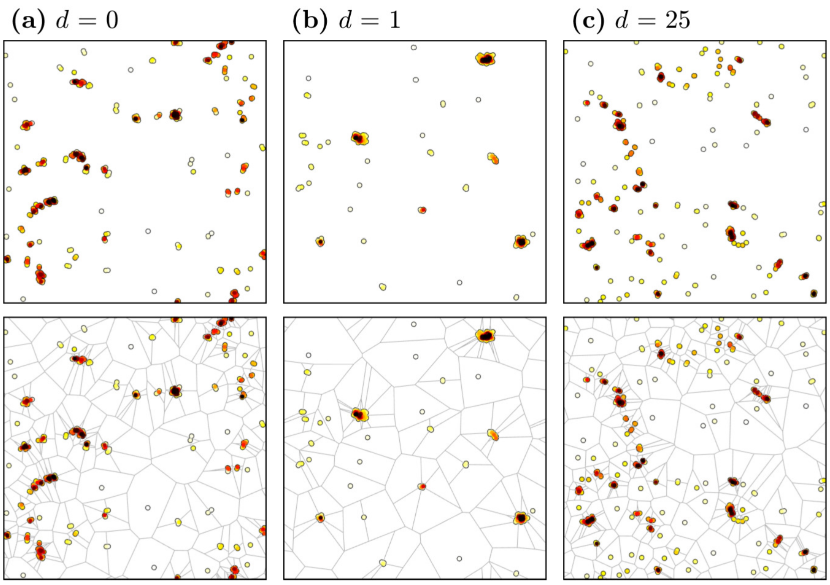

Figure 2.1: System graphical representation. Sample frame from the system (top) and corresponding Voronoi system of cells around particles (bottom). The Colour of each particle denotes the area of a cell surrounding it, with black colour identifying cells with smallest areas and white colour denoting those with largest areas.

Delay happens to deal very well with this problem, as particles have a chance for deeper penetration of each others' detection zones. Such mechanism leads to drastic reduction of the space between the particles in a cluster and that makes the newly formed clusters much more resistant to erosive influence of the noise. Effects of the delay application are visible on the figure 2.1b, where system is much more stabilised than corresponding delay-free system visible on figure 2.1a.
Introduction of small amounts of noise (i.e. $d \cdot \nu < R$) turned out to be very beneficial for the system, as clusters were forming much more often and were able to survive much longer periods of time. Some of the clusters can be considered as candidates for permanent clusters, as they do not exhibit tendency to dissolve after long simulation time due to the influence of noise, however to ensure that infinitely long simulation would be required or some solid theoretical arguments.
Addition of the delay can also lead to system further destabilisation, when compared to delay-free case. Such behaviour can be observed in the situation when too much time passes between particle detection and application of the gathered data. Particle is supplied with the data about its surrounding when particles that were detected are no longer in its region of detection. This can lead to collapse of the clustering mechanism and significantly hinder complex collective motion. High destabilisation of the system is clearly visible on figure 2.1c, especially when compared to partial steady-state achieved in corresponding figure 2.1b.

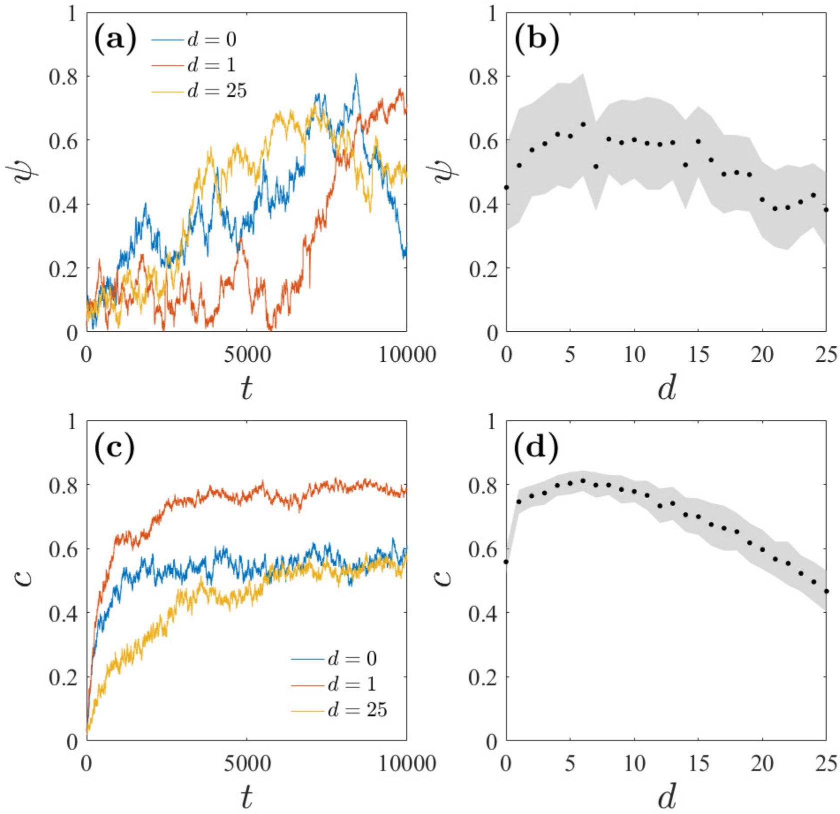

Figure 2.2: Order and cluster parameters. (a) Sample order parameter measured for the system with 3 delay settings (b) Average order parameter and corresponding standard deviation measured over 10 independent runs. (c) Sample cluster parameter measured for the same system. (d) Average cluster parameter and corresponding standard deviation measured over 10 independent runs.

Figure 2.2b shows that order parameter turned out to be rather unreliable tool in measuring the influence of the delay, as standard deviation spans over rather wide range. Obtained average values exhibit a trend of raise till the delay of $d = 6$, followed by a drop for higher values of the delay, but deviation is too high to draw meaningful conclusions. Due to these complications, attention was directed to cluster parameter, which proved to be more reliable tool. Figure 2.2d provides a proof our previous observations, as cluster parameter value visibly drops with increase in delay. Key values to system stabilisation, seem to be connected with a distance passed by a particle during the delay time and detection radius size - the best clustering results are obtained when these values are close to equal. From this observation, relation was drawn in the form

$$\frac{d \cdot \nu}{R} \approx 1 \tag{2.1}$$

Velocity ν is a crucial factor in this equation, thus in the next chapter system was tested with various velocity settings.

3

Particle velocity investigation

3.1 Particle velocity

In the simulations carried out so far, the velocity of the particles was equal to $\nu = 3$. This velocity was chosen to make simulations fast enough, so one can observe transformation of the system under the influence of noise and delay in relatively short time. Simulations are computationally heavy and due to that decision was made to limit them to 10000 steps. Velocity $nu = 3$ proved to be very good choice, as in such amount of time it was possible for the system to achieve the steady-state. Such speed allowed also observation of the influence of noise application in the system. However, even though velocity $\nu = 3$ was found to be fitting the simulation very well, it was necessary to test also different scenarios of velocity.

Results obtained through addition of the delay to the system clearly show, how big influence on the behaviour of the system has a velocity of the particle itself. It is strictly related to application of the delay and manipulation of this variable can change dramatically the outcome of the simulation. Due to such importance of this parameter, it was necessary to test configurations with particles travelling slower and faster than the initial case. In order to achieve that two velocity parameter settings were chosen: $\nu = 1$ and $\nu = 7$. Due to these changes particles travel much smaller distance (for $\nu = 1$) or much larger distance (for $\nu = 7$) during a single step of the simulation, than in the default case of $\nu = 3$. Prediction was formed that this will dramatically influence the cluster parameter with respect to applied delay.

Fig. 3.1a represents comparison between the obtained data - increase in speed results in rapid deterioration of clustering accompanying the raise in the delay coefficient. This tendency is most likely connected with the lesser accuracy of the aligning mechanism - particle travelling with velocity $\nu = 7$ travels for maximally only 3 simulation steps inside detection area of another particle during a head-on pass, thus even small amounts of delay will have a significant influence on the particles' behaviour. Such increase in speed of the particle also significantly increases the power of noise in the evaluation of the average direction - even the slightest change in direction will cause the particle to drift far away from desired position. One can observe the opposite situation for the results of slowly moving particle with velocity $\nu = 1$. Particles travel in each others detection areas through 20 simulation steps during the head on pass, thus only very high values of delay will cause significant changes in particle's behaviour. Influence of noise is also much smaller in this case, as the direction of the particle is corrected relatively often in comparison to other settings.

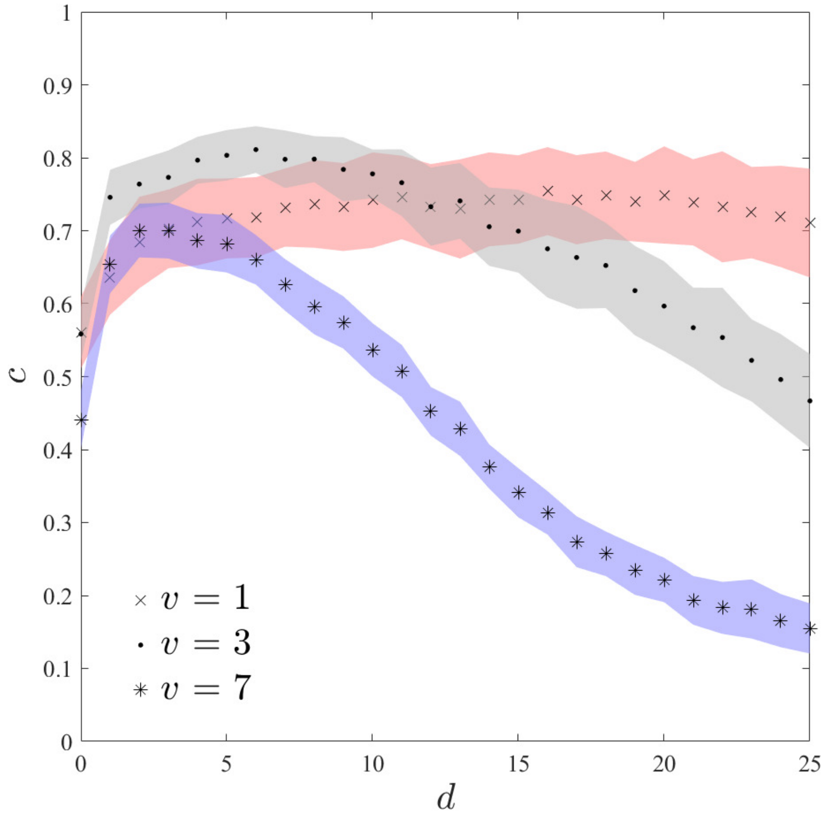

Figure 3.1: Particle velocity. Average cluster parameter and corresponding standard deviation measured for various amounts of delay and changing particle velocities.

3.2 Delay - velocity relation and density waves

Relation described in Eq. 2.1 holds for all 3 tested particle velocity settings and proof of it is visible on the Fig. 3.1b. Maximum for all 3 plots is placed approximately around $\frac{d \cdot \nu}{R} \approx 1$. This relation could be used as a guidance for tuning autonomous robots' control systems and one can predict relatively promising results of its application taking into consideration its performance in the case of the system investigated in this research. This relation points us also to the conclusion that delay is strictly connected with the velocity of the particles. Combination of these parameters might be responsible for a very interesting behaviour that can be observed in the environment of high particle velocity and high delay application. In this region the system exhibits a tendency to create waves very similar to waves caused by a significant increase in density of particles in the arena [19, 27, 28], however in this case the density remained the same throughout all simulations. By the term "wave", we understand the particular behaviour of the system, in which almost all particles form a giant cluster spanning throughout most of the arena, with rather big distances between the agents. An example of such wave can be seen on Fig 3.2. A shape of the cluster is connected with the shape of the arena and is usually resembling a "C-shaped" wave, with a high density of particles on its front edge followed by a gradual decrease in density up to the region of relatively empty space. A possible explanation for this phenomenon is complex synergy between the particle-velocity, the delay and the density. Previous research in the area showed that high density and/or high velocity of particles results in creation of density waves, but delay was never taken into account. In the case of this research delay is proven to be a part of this mechanism, as the waves are created only in the environment of high delay and nonexistent in the presence of small amounts of the delay, while all the other parameters are kept constant.

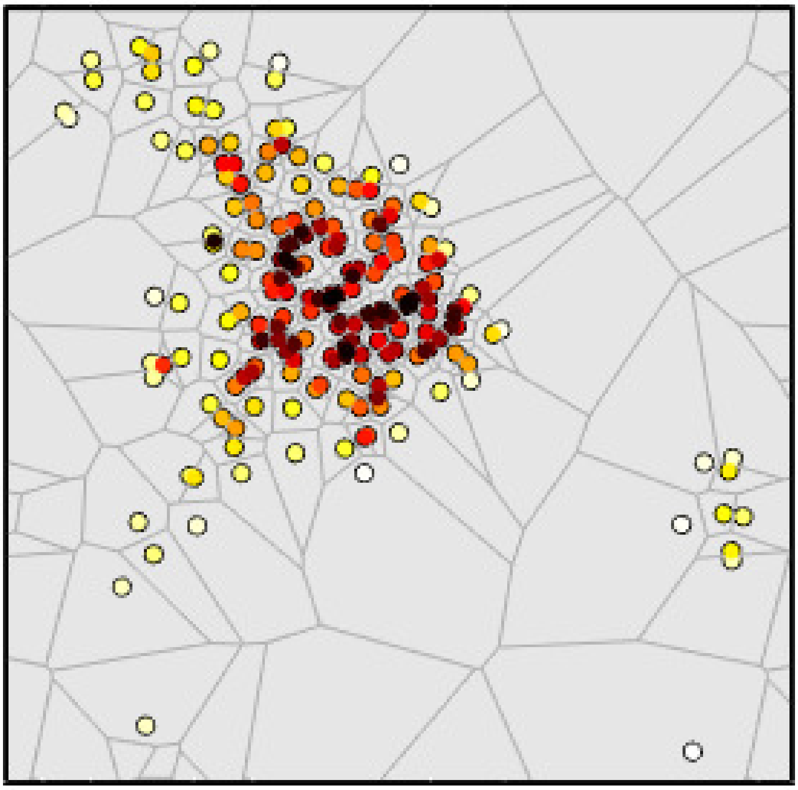

Figure 3.2: Density wave. System supplied with 200 particles scattered randomly over the square-shaped area of 200 x 200 units.

4

Influence of extrapolated delay

4.1 Application of linear extrapolation

A Natural step in further investigation of delay influence was re-adaptation of memory matrix used by particles to obtain direction information through extrapolation of data. In the delay mechanism used so far, data about the neighbourhood of the particle is stored for certain amount of time and then used in unchanged form to guide the particle. This approach seems to be lacking in accuracy, as in the moment of application neighbourhood of the particle is often completely different, than neighbourhood registered by delayed data. In order to achieve more smooth adaptation to the changing environment the system was supplied with an extrapolation mechanism operating with the memory of 5 steps for each particle separately. Data are collected in the same manner as in the standard delay model, but they are linearly extrapolated to desired delay position, which is denoted by delay parameter d. This allows the particle to adapt to sudden changes of environment, like approaching cluster, through smooth aligning. Such approach better simulates real-life systems, when often one can observe slower aligning to desired direction, rather than rapid changes in direction of motion. Good example might be behaviour of the human crowd, where people try to find balance in active avoiding each other (in order to not collide) and to remain on their way to the target (avoiding unnecessary changes in direction).

4.2 Future prediction mechanism

Thanks to application of linear extrapolation, particles gained ability not only to gradually adapt to changing environment, but also to predict their future to certain extent. This is due to the fact that gathered data can be linearly extrapolated to the future, giving a semi-accurate picture of upcoming events in particles behaviour. Future prediction performed by particles sounds very promising in terms of ability to achieve the steady state and permanent cluster creation, however this system is far from perfect. Main obstacle lies in the fact that particles cannot communicate with each other, thus future prediction has to be constantly updated by inclusion of the influence generated by particles entering the detection zone for the first time or exclusion of the influence of particles that escaped the detection zone. This problem forcibly deteriorates quality of the predictions, as during the stabilisation process during the early stages of the simulation, encountering new particles is a very common event. Due to such problems achieving steady state is significantly slowed down or even completely stopped.

4.3 Results of linear extrapolation application

The above observations are supported by many simulations of various system settings. Combined results of the order parameter are presented on Fig. 4.1a. One can observe that order parameter picks up at approximately the same area as it is suggested by the relation in Eq. 2.1, what can serve as its further confirmation, however standard deviation, gathered through 10 iterations of the simulation, is still very big, which limits its reliability. Values achieved by the system using prediction mechanism are comparable to the default delay system in the mentioned region of $d = 5$, but for higher values of the delay system is significantly worse. This tendency is most likely caused by overestimation of gathered data - point of estimation significantly overshoots detected values. Prediction of the future i.e. data extrapolation to values of negative delay is afflicted by the same problem, but also by lack of information about particles approaching outside of the detection zone. These two elements significantly hinder efficiency of the prediction system, thus results of its operation are worse than ones obtained through application of regular delay.
Corresponding situation for the cluster parameter readings is presented on Fig. 4.1b. Systems achieves very similar high cluster parameter results in the same region as the ones achieved for the regular delay. For higher values of delay d, the system with prediction applied deteriorates much more rapidly, which suggests that linear extrapolation is not suitable in application of high delay in the system.

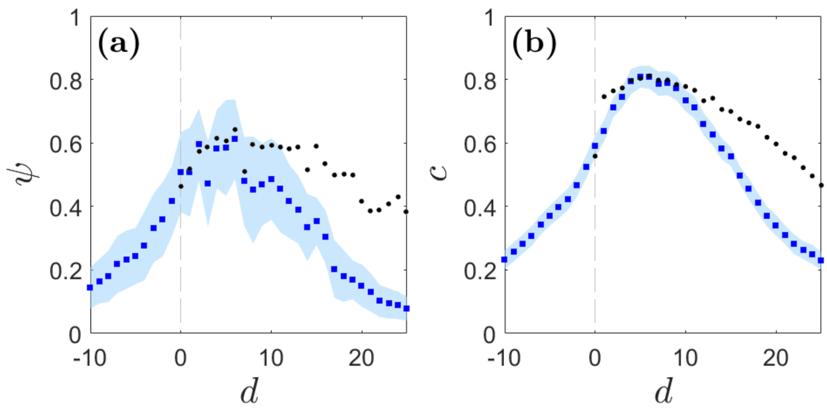

Figure 4.1: Order and cluster parameters. (a) Order parameter and corresponding standard deviation (blue) measured for various amounts of delays with application of data prediction through extrapolation of 5 last steps. Record of order parameter in the case of delayed model without prediction mechanism (black) (b) Corresponding cluster parameter record (blue) with comparison to the record of the system not using prediction system (black).

5

Conclusions

In conclusion, Vicsek model was recreated and expanded by the delay and linear extrapolation delay mechanisms. In order to better evaluate the system order and cluster parameters were constructed and applied in each simulation. It has been shown with numerical simulations that the introduction of a sensorial delay in the Vicsek model can improve its clustering behaviour. Specifically, short positive delays lead to a more ordered and coherent motion of the ensemble of active agents, while larger positive delays and negative delays lead to a disruption of the order of the system, preventing the emergence of clustering and swarming. Some living entities, such as bacteria, are known to respond to the temporal evolution of stimuli [20, 21] and the presence of a delay might be already at play in natural systems due to the time it takes to acquire, process and react to environmental information. Furthermore, engineering of sensorial delay can be used to control and tune the clustering and swarming behaviour of large collectives of active agents in applications, such as swarm robotics [8], environmental monitoring [22, 23], and self-assembly [24, 25].

Bibliography

[1] C. Bechinger, R. Di Leonardo, H. Lowen, C. Reichhardt, G. Volpe, and G. Volpe, Active particles in complex and crowded environments, Rev. Mod. Phys. 88, 045006 (2016).

[2] T. Vicsek and A. Zafeiris, Collective motion, Phys. Rep. 517, 71 (2012).

[3] J. Toner, Y. Tu, and S. Ramaswamy, Hydrodynamics and phases of flocks , Annals Phys. 318, 170 (2005).

[4] Y.-X. Li, R. Lukeman, and L. Edelstein-Keshet, Minimal mechanisms for school formation in self-propelled particles , Physica D 237, 699 (2008).

[5] J. Buhl, D. J. Sumpter, I. D. Couzin, J. J. Hale, E. Despland, E. R. Miller, and S. J. Simpson, From Disorder to Order in Marching Locusts , Science 312, 1402 (2006).

[6] T. Xu, F. Soto, W. Gao, R. Dong, V. Garcia-Gradilla, E. Magaa, X. Zhang, and J. Wang, Reversible Swarming and Separation of Self-Propelled Chemically Powered Nanomotors under Acoustic Fields, J. Am. Chem. Soc. 137, 2163 (2015).

[7] T. Sanchez, D. T. Chen, S. J. DeCamp, M. Heymann, and Z. Dogic, Spontaneous motion in hierarchically assembled active matter, Nature 491, 431 (2012).

[8] M. Brambilla, E. Ferrante, M. Birattari, and M. Dorigo, Swarm robotics: a review from the swarm engineering perspective, Swarm Intelligence 7, 1 (2013).

[9] M. Moussaïd, N. Perozo, S. Garnier, D. Helbing, and G. Theraulaz, The Walking Behaviour of Pedestrian Social Groups and Its Impact on Crowd Dynamics, PloS ONE 5, e10047 (2010).

[10] C. W. Reynolds, Flocks, herds and schools: A distributed behavioral model, in ACM SIGGRAPH Computer Graphics, Vol. 21 (1987) pp. 25-34.

[11] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Novel Type of Phase Transition in a System of Self-Driven Particles, Phys. Rev. Lett. 75, 1226 (1995).

[12] G. Chen, Small Noise May Diversify Collective Motion in Vicsek Model, IEEE Transactions on Automatic Control, Volume 62, p. 636-651, 2017.

[13] M. Nagy, I. Daruka and T. Vicsek, New aspects of the continuous phase transition in the scalar noise model (SNM) of collective motion, Physica A. 373, p. 445-454, 2007.

[14] Q. Du, V. Faber and M. Gunzburger, Centroidal Voronoi Tessellations: Applications and Algorithms, SIAM Review, Vol. 41, No. 4, p. 637-676, December 1999.

[15] A. Czirók and T. Vicsek, Collective behavior of interacting self-propelled particles, Physica A 281, 17 (2000).

[16] H. Chaté, F. Ginelli, G. Gréegoire, F. Peruani, and F. Raynaud, Modeling collective motion: variations on the Vicsek model, Eur. Phys. J. B 64, 451 (2008).

[17] M. Mijalkov, A. McDaniel, J. Wehr, and G. Volpe, Engineering Sensorial Delay to Control Phototaxis and Emergent Collective Behaviors, Phys. Rev. X 6, 011008 (2016).

[18] G. Volpe and J. Wehr, Effective drifts in dynamical systems with multiplicative noise: a review of recent progress, Rep. Prog. Phys. 79, 053901 (2016).

[19] M. Nagy, I. Daruka, and T. Vicsek, New aspects of the continuous phase transition in the scalar noise model (SNM) of collective motion, Physica A 373, 445 (2007).

[20] R. M. Macnab and D. Koshland, The Gradient-Sensing Mechanism in Bacterial Chemotaxis, Proc. Natl. Acad. Sci. U.S.A. 69, 2509 (1972).

[21] J. E. Segall, S. M. Block, and H. C. Berg, Temporal comparisons in bacterial chemotaxis, Proc. Natl. Acad. Sci. U.S.A. 83, 8987 (1986).

[22] A. Dhariwal, G. S. Sukhatme, and A. A. Requicha, Bacterium-inspired robots for environmental monitoring, in Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on, Vol. 2 (2004) pp. 1436-1443.

[23] M. Dunbabin and L. Marques, Robots for Environmental Monitoring: Significant Advancements and Applications, IEEE Robotics Automation Magazine 19, 24 (2012).

[24] M. Rubenstein, A. Cornejo, and R. Nagpal, Programmable self-assembly in a thousand-robot swarm, Science 345, 795 (2014).

[25] J. Werfel, K. Petersen, and R. Nagpal, Designing Collective Behavior in a Termite-Inspired Robot Construction Team, Science 343, 754 (2014).

[26] H. Chaté, F. Ginelli, G. Grégoire and F. Raynaud, Collective motion of self-propelled particles interacting without cohesion, Phys. Rev. E 77, 046113 – April 2008

[27] T. Ihle, Kinetic theory of flocking: Derivation of hydrodynamic equations, Phys. Rev. E 83, 030901(R) – March 2011

[28] E. Bertin, M. Droz, and G. Grégoire, Boltzmann and hydrodynamic description for self-propelled particles, Phys. Rev. E 74, 022101 – August 2006