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Characterization of challenging tasks intended for gifted students and what is needed for teachers to provide them

Master's thesis in Learning and Leadership

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Abstract

Enrichment tasks are often used to stimulate gifted students in school, who most of the time receive too little challenge in their classrooms. These challenging tasks could possibly encourage students to show gifted behavior. To investigate if this is the case, a framework was constructed to determine what stimulates gifted behavior, to analyze if challenging tasks successfully stimulate gifted behavior. In addition, it is known that the reason students often do not receive enough challenge in school is due to teachers not having sufficient time to spare. Therefore, the needs of teachers are also investigated in relation to supporting gifted students.

Both perspectives are investigated through semi-structured interviews with upper secondary school teachers in Sweden, all of which had tested at least one challenging task in their classroom prior to the interview. Content analysis was used to analyze these interviews, and was performed twice: Once to analyze if the tasks stimulate gifted behavior, and once to analyze the needs of teachers.

It was found that challenging tasks can stimulate gifted behavior, and all tested tasks do so in different ways, however they do not always stimulate all aspects of gifted behavior at the same time. Teachers confirm that they are short on time, and therefore wish for ways to quickly find tasks that are appropriate for these students. The most important thing was being able to find them based on course and subject area. In addition, teachers value student hints being coupled together with the tasks, as well as clear proposed solutions. Lastly, it may be important to have a variety of difficulties of tasks to be able to meet the needs of different students, such as underperforming and high achieving gifted students.

Keywords: giftedness, enrichment, mathematics, tasks.

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Emilia Ekstrand, May 2023

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1. Introduction

1.1 Background

Giftedness (*Swe: Särskild begåvning*) as a term is ill-defined, and several different definitions of it exist (Mattsson, 2013; Mattsson & Pettersson, 2015). Furthermore, how many percent of the student population are considered as gifted varies between different definitions of giftedness. The Swedish National Agency for Education (*Swe: Skolverket)* has published support material for teachers, where they decided on using 5% as a general estimate of the proportion of students who are considered gifted (Mattsson & Pettersson, 2015). In addition, there is a distinction between high-achieving students and gifted students. Being high achieving does not necessarily mean the student is gifted. However, it is possible that the student is both (Mattsson & Pettersson, 2015). Sometimes gifted students underperform, which can be due to an unsupportive environment leading to boredom, or to better fit in with other students (Persson, 2010; Mattsson & Pettersson, 2015).

A common model for giftedness is the Multifactor model, which says that giftedness is a combination of the personality traits creativity, exceptional abilities, and motivation (Mönks & Ypenburg, 2009). However, since humans are social beings Mönks and Ypenburg (2009) argue that support from the school, family, and peers is necessary for the personality traits to develop.

A study by the Swedish Schools Inspectorate (*Swe: Skolinspektionen*) in 2018 showed that one-third of lessons are not sufficiently adapted for high-achieving students, and that they sometimes have no tasks to work on. According to the report, mathematics lessons suffer the most from this issue. The structure of the lesson is often the same and there is little room for variation. Furthermore, if students are given extra tasks, they often do not receive sufficient support from their teachers when solving them (Swedish Schools Inspectorate, 2018). A reason commonly given by teachers of several subjects for not prioritizing high-achieving students is that the teachers don't have enough time (Swedish Schools Inspectorate, 2018; Pettersson, 2011). In conclusion, schools are often unable to properly support high-achieving students, which can include gifted students. Furthermore, a lack of support can lead to boredom for gifted students, and thus lower achievements (Persson, 2010; Mattsson & Pettersson, 2015). Swedish schools can therefore, in many cases, not provide the support gifted students need to properly develop the personality traits in the Multifactor model.

A possible way to support these students is by providing enrichment opportunities (Pettersson, 2011), which could be through challenging tasks. These tasks can, for example, come from resource banks that teachers can use, to, in turn, give to their students. However, it is important that these challenging tasks properly stimulate gifted behavior in the students. Furthermore, it is important that such resources function well within teachers' work processes. This will therefore be investigated in this study.

1.2 Purpose

The purpose of the study is to examine if challenging tasks benefit gifted students by investigating if they stimulate gifted behavior. In addition, it is to investigate teachers' needs from challenging tasks, and the resources they belong to, to effectively integrate them into their work process and be able to provide them to students.

1.3 Research questions

This study aims to answer the following three research questions:

1. Do teachers interpret the selected tasks as challenging for students that have reached far in their mathematical development?
2. Which aspects of gifted behavior do teachers think challenging tasks stimulate students to exhibit?
3. What needs and wishes do teachers have from resources with challenging tasks, and their tasks, to function well in their work process?

Note that the first research question is not the primary objective of this study. It is instead a pre-requisite to be able to answer the second question.

1.4 Limitations

- This study will not focus on identifying gifted students during the testing of assignments.
- This study does not directly investigate the students' experience with the tasks and resources, and instead focuses on the teacher's observations, opinions, and experiences.

2. Theory
In this section critical background theory regarding mathematical giftedness will be presented. Furthermore, a framework for analyzing if tasks promote gifted behavior will be presented.

2.1 Mathematical giftedness
Mathematical giftedness is a concept with no clear definition, and instead there are many different definitions that are in use (Mattsson, 2013). Furthermore, these various definitions estimate that different percentages of the student population are gifted. The Swedish National Agency for Education, when constructing their supplementary material for teachers about gifted students, consider 5% of the student population as gifted (Mattsson & Pettersson, 2015). Furthermore, they describe a practical, but unscientific definition of gifted students: Gifted students often surprise by their abilities in different ways, both in school, and outside of school (Mattsson & Pettersson, 2015, p. 9).

A common model to describe giftedness is Renzulli's three-ring conception of giftedness, which says that giftedness is a combination of the factors creativity, above-average ability, and task commitment (Renzulli, 2011). The model was later modified by Mönks (Mönks & Katzko, 2005), who added three external factors – Peers, School, and Family - that influence the personal traits. In addition to that the names of the personal traits are changed to Exceptional Abilities, Creativity and Motivation. This extended model is called the Multifactor model of giftedness. Furthermore, Mönks and Katzko define giftedness as "Giftedness is an individual potential for exceptional or outstanding achievements in one or more domains." (Mönks & Katzko, 2005, p. 191). They also describe how these outstanding achievements can be reached by a positive interaction between the three personality traits, and the three external factors (Mönks & Katzko, 2005). Notably, Renzulli (2000) prefers to talk about the development of gifted behavior rather than giftedness. In this case, gifted behavior means an interaction between the three personality traits in the three-ring conception of giftedness (Renzulli, 2000). In this study, when discussing gifted behavior, the three personality traits in the Multifactor Model will be used.

Of interest is that gifted students and high-achieving students are not necessarily the same students. High-achieving students do not have to be gifted, which is highlighted by 15-20% of students being high-achieving, and only 5% being considered gifted, according to the Swedish National Agency for Education. In the same way, gifted students do not have to be high achieving (Mattsson & Pettersson, 2015). If met by an unsupportive environment, gifted students can become bored and underperform, or hide their giftedness. This was found to be the case in Swedish primary schools, but it was found that the trend often continues into upper secondary school (Persson, 2010).

One method, by which you can support gifted students in various ways, is differentiated instruction. One of those ways is acceleration, which means that students are allowed to work at a faster pace with exercises within a course or to complete the courses faster. Another method is enrichment, which means providing students with additional learning opportunities within courses (Pettersson, 2011). Providing students with additional challenging material, like the material that will be tested in this study, could be considered a form of enrichment.

2.2 Swedish Agency for Education's abilities
An essential set of abilities used in Sweden are the abilities used by the Swedish Agency for Education in relation to Mathematics courses (Juter, 2014; Swedish National Agency for Education, 2022). These abilities are a central part of the grading of students in mathematics courses in upper secondary school in Sweden. The constructed framework, which will be presented in the next section, will make connections to some of these abilities.

The abilities are the following:

- *The problem-solving ability* refers to the ability to solve problems where the method is not known in advance. To solve problems like this different strategies can be employed.
- *The procedure ability* is to perform procedures, such as algorithms and methods, and decide which one is suitable.

- *The reasoning ability* is to explain and argue for mathematical conclusions, such as when performing a proof.
- *The conceptual ability* is the ability to describe concepts and understand their relationships to other concepts.
- *The communicative ability* is to communicate mathematics efficiently, which can be through different mathematical representations such as figures, graphs or symbols.
- *The modelling ability* is to create a mathematical model describing the relationships within a certain phenomenon.

2.3. Framework

To help answer the second research question and to evaluate how well the chosen tasks stimulate students to exhibit creativity, exceptional abilities, and motivation a framework for what encourages these traits, or are signs of these traits, is needed. The starting point for this framework will be the Multifactor model, as described by Mönks and Ypenburg (2009), where the focus is on mathematical tasks providing support for students to develop the three personality traits from the Multifactor model. Since the selected tasks can be seen as a resource intended to help teachers, they can be considered as a part of the external factor school. In the following sections research will be presented for how creativity, exceptional abilities and motivation can be supported.

2.3.1 Creativity

There are several frameworks that describe what makes a task creative, as well as general theory about things that promote creativity. In this section, some of them will be described, and finally some key points will be summarized. These key points will be included in this framework for evaluating if a task promotes creativity.

Most tasks in schools are routine tasks, which are characterized by having a known solution method, and a fixed answer. These tasks do not allow students to explore mathematics, and do not allow them to work creatively (Pettersson, 2011). Having a known method for solving a task is akin to using imitative reasoning for solving them, or more specifically algorithmic reasoning, where a known algorithm is applied to the task to solve them. This type of task does generally not require any creative reasoning (Boesen et al., 2010). Tasks that are best solved with creative reasoning are most often different from typical tasks found in textbooks (Boesen et al., 2010). In Swedish education the ability to solve these routine tasks is referred to as the ability to use procedures (Juter, 2014), and thus it is reasonable to investigate if the tasks are of a procedural character or not. Notably, what is considered routine depends on the student. Yeo (2007) explains how a task may be routine for someone who knows the procedure to solve it. If a student does not know the procedure, it is instead a problem, and not routine. This matches well with the Swedish National Agency for Education's description of what a problem in relation to problem solving is (Juter, 2014). Therefore, if a task requires problem-solving it can be an indication of the task being non-routine.

Creativity can also be promoted using open-ended tasks (Kandemir et al., 2019), which can be characterized in several ways according to Yeo (2017). A few of these ways are:

> Open method - Tasks can be open in the method that is used to solve them, in which case several solution methods are viable (Yeo, 2017).
> Open answer – Tasks can have an open answer, which means that the answers are not determined. A determinate answer is something where all answers can be found, for example a quadratic equation has multiple answers, but is determinate because all of them can be found. A question with an open answer can, for example as explained by Yeo (2017), be a task to design a playground.
> Open goal – Tasks can also have an open goal, which means that the objective of the problem is open. For example, a task to investigate a sequence of numbers has an open goal (Yeo, 2017).

This matches what Hadar and Tirosh (2019) describe as being positive for creativity in their framework for evaluating if a task is positive for creativity. According to them solving a task with more than one solution promotes creativity (open answer), as well as trying to find alternative solutions to a task by using different pathways (open method). Open-ended tasks will therefore be regarded as something positive for creativity in the framework constructed in this thesis and can be identified by descriptions matching Yeo's (2017) different characterization. Furthermore, tasks where students pose their own mathematical problems is positive for creativity (Hadar and Tirosh, 2019). Lastly, mathematical modelling activities are also positive for promoting mathematical creativity (Kandemir et al., 2019).

For the framework used in this study, this is summarized into the following key points which are positive for promoting creativity:

- Non-routine tasks
- Open-ended tasks
- Creation of mathematical problems
- Mathematical modelling activities

2.3.2 Exceptional Abilities

Exceptional abilities in the multifactor model is similar to above average abilities in Renzulli's three ring conception, with the difference being that the exceptional ability requirement in the multifactor model is higher than above-average ability requirement in the three-ring conception of giftedness (Mattsson, 2013). Renzulli describes how the above average abilities consist of general abilities, but also specific abilities in a certain domain in the three-ring conception of giftedness (Renzulli, 2005). In this case the domain would be mathematics. This should reasonably apply to the multifactor model as well, however with the requirement that the abilities are stronger. To evaluate these specific abilities the framework of mathematical abilities in gifted students by Krutetskii (1976) will be used (Krutetskii, 1976). He defined eight different mathematical abilities, which are the following:

- Ability for formalized perception of mathematic material
- Logical ability
- Ability to generalize
- Ability to curtail a mathematical process
- Ability to be flexible with mental processes
- Striving for clarity and rationality of solutions
- Ability to reverse a thought process
- Mathematical memory for proofs, schemes, etc.

These abilities are all related, and together make up what Krutetskii calls a mathematical cast of mind (Krutetskii, 1976). For this framework some of these will be omitted, due to tasks not being able to directly require or support them on their own. These are the ability to curtail a mathematical process and striving for clarity and rationality of solutions. Furthermore, the ability to reverse a thought process will be omitted due to it being deemed too specific, and not general enough to be widely identifiable. A more detailed description of the remaining abilities will now be given.

The ability for formalized perception of mathematical material means that a student can understand and interpret a mathematical problem's structure. Krutetskii (1976) describes it as if all the specifics of the material have been removed, and only the structure, or correlations, remain. Krutetskii also refers to this as grasping the formal structure. When using the framework to identify if this is necessary for a task attention will be paid to if interpreting and understanding is of special importance for the task.

The logical ability pertains to both logical connections within mathematical notation, as well as spatial relationships. It is also indicated by sequential and systematic thoughts. An example of this given by Krutetskii (1976) is how a student easily can see implications, and that said student is good at proofs.

When using the framework identifying if a task requires proofs is therefore appropriate, as well as general statements about logic.

The ability to generalize is to take knowledge from a specific case and apply it in a general case. For example, Krutetskii (1976) let students learn the formula for the square of a binomial. They were then given a problem where they had to use the formula on a trinomial, $(C+D+E)^2$, and had to generalize the information they knew about the square of a binomial to be able to solve it. When using the framework to evaluate if a task promotes use of generalization descriptions of generalization, such as the one in the example above, will be considered.

Ability to be flexible with mental processes means that a student, when solving a problem, can switch their mental processes between different operations. Furthermore, the mental processes are not constrained to traditional solving methods. Interestingly this ability is also commonly connected with creativity, which means that this ability does not exclusively fit into the category of exceptional abilities (Eriksson & Petersson, 2015). When using the framework to identify this if tasks encourage this ability, the tasks can, for example, require the students to try different solving methods that are unconventional to them.

Mathematical memory is a memory for previous proofs and schemes. This type of memory is not, however, about remembering specific numbers or details about a task. Krutetskii (1976) gives an example where students are first given a task, and then given a different task of the same type months later. Students would then feel familiar with the task and feel like they have done the task before. The conclusion by Krutetskii is that students remember the type of task, but not the specific details for each individual task. When using the framework to identify this ability tasks which require students to remember previous knowledge will be searched for.

This can be summarized as the following:

- Ability to understand formal structures
- Generalization ability
- Logical ability
- Flexibility
- Mathematical memory

2.3.3 Motivation

There are several theories about motivation as well as much research about what motivates students. In this section one such model, self-determination theory, will be presented as well as some general research on what motivates students.

Self-determination theory (SDT) is a motivational theory which claims that people have different types of motivation: intrinsic and extrinsic. Intrinsic motivation is doing something because the activity itself is interesting or enjoyable (Ryan & Deci, 2000). Extrinsic motivation is instead doing something not because it is fun, but because of external reasons (Hornstra et al., 2020). Extrinsic motivation can, however, vary in how self-determined the motivation is according to Ryan and Deci (2000). The least self-determined kind of extrinsic motivation is characterized by doing activities due to external demands or rewards. The most self-determined extrinsic motivation is instead when a person has internalized the reasons for doing a task with their own values and needs. An example given by Ryan and Deci (2000) is how a student might do their homework to avoid consequences from their parents, whereas another student might do the homework because they recognize its value for their future. This is still, however, distinct from intrinsic motivation which means to do something because the activity itself is enjoyable (Ryan & Deci, 2000).

According to SDT, intrinsic motivation can be promoted by supporting the needs of autonomy, competence, and relatedness (Schukajlow et al., 2017). Furthermore, supporting these three needs can help facilitate internalization in extrinsic motivation, and thus make their motivation more self-determined (Ryan & Deci, 2000). These three needs will be some of the factors included in the framework to help identify if tasks support motivation. An explanation of these three needs will follow, as well as how these factors may be used to identify motivation supporting tasks.

Autonomy is a person's need to act out of their own volition (Schukajlow et al., 2017; Hornstra et al., 2020), or in other words their behavior is self-determined (Ryan & Doci, 2000). Supporting this need in students can be done with what Hornstra et al. (2020) calls autonomy-supportive teaching. This type of teaching is characterized by "providing choice, explaining the relevance of learning tasks, acknowledging negative feelings, and nurturing students' inner motivational resources" (Hornstra et al, 2017, p. 2). This is similar the Swedish National Agency for Education's supportive material for gifted students, where they describe how gifted students often want to create their own interest, and as such want to be part of the problem posing process (Eriksson & Petersson, 2015). Reasonably, this cannot be provided by a task alone, and instead concerns the support the teacher provides. However, it is possible that the tasks can be a part of autonomy-supporting teaching. When using the framework, it is therefore important to look out for if tasks can be used in this way. Furthermore, the assumption that open tasks allow for more autonomy within students will be made, since they allow students to make more of their own choices, and thus have some control over their own learning.

Competence is instead the need to "feel effective and in control, and to be able to stretch one's capabilities" (Hornstra et al., 2020, p. 2). To support this a teacher can, among other things, give students feedback and guidance, and provide material suitable for the student's ability level (Hornstra et al., 2020). Providing suitable material so the students can stretch their abilities can reasonably be compared to providing mathematical challenges. A key point in these challenges is that they can be overcome (Leikin, 2010), highlighting the importance the material being well selected for the student's level. When using the framework to identify tasks supporting competence, challenging tasks that students can overcome will be looked for, as well as material that can help provide guidance.

Lastly, when it comes to relatedness, students need to feel a sense of belonging according to Hornstra et al. (2020). This can both be done by positive connections with the teacher, or with classmates. Notably, Eriksson and Petersson (2015) describe how it is important that gifted students can interact with other gifted students and not just their teacher, and that the students are less likely to feel alone this way. When using the framework to identify if tasks can support relatedness, looking for tasks that are possible to work with in pairs or groups will be looked for. The tasks themselves cannot create the connection with peers, but if they are fitting for working with others, they may be able to support it indirectly.

In addition to SDT there is other research about motivation within students, and how it can be facilitated. Interest is a topic that has been researched in relation to motivation and will be included as a factor in the framework. According to Hidi (2006) interest is a motivational variable which "occurs during interactions between persons and their objects of interest, and is characterized by increased attention, concentration and affect" (Hidi, 2006, p. 70.). Furthermore, there are two types of interest: situational and personal interest. Situational interest refers to an interest triggered by the environment, for example seeing a newspaper in a waiting room (Hidi & Renninger, 2006). It can also be triggered by already having an interest in a subject, and something different allowing them to engage with it. Hidi and Renninger (2006) give the example of someone becoming interested in technology because of an already existing interest in mathematics, which can be further explored with technology. An individual interest is instead something that develops over time (Hidi, 2006) and refers to a person's tendency to reengage with a certain content (Hidi & Renninger, 2006; Hidi, 2006). Notably situational interest can lead to a personal interest over time. Another interesting point is that the three needs in SDT have a relation to interest. This relationship is reciprocal, which means that the three needs can support interest, and interest can support the three needs (Hidi & Renninger, 2006). Lastly, when students start to develop an individual interest for a subject, they begin asking curiosity questions about the subject (Hidi & Renninger, 2006). Interest in mathematics may be difficult to evaluate, since it depends on the students themselves, and their already existing individual interest. However, it is possible for the tasks to be situationally interesting and provide further material for students that already have an individual interest. When using the framework to identify tasks that support interest students asking curiosity questions will be used as an identifier. Furthermore, general discussion about tasks being interesting is worth looking out for.

This can be summarized into the following points:

- Autonomy
- Competence
- Relatedness
- Interest

3. Method

To evaluate the two research questions, upper secondary school mathematics teachers were contacted with a request to test an enrichment task in their regular classroom as well as participate in an interview afterwards. The purpose of the interview was to collect data to answer both research questions, and the testing of the task helps facilitate this for both questions. The intent was that the testing of the task allowed the teachers to familiarize themselves with the task, as well as observe how such a task works in practice, which can help with answering questions related to the first research question. Furthermore, experiencing working with gifted students with tasks like this lets the teachers discover what their needs are, if they did not already know. Notably, the first research question will be analyzed in relation to the framework constructed in section 2.3, whereas the method for analyzing the second research question takes a more inductive approach.

3.1 Selection of respondents

The respondents were selected by convenience sampling, which means the teachers who were the most easily accessible were selected (Berg, 2009). In addition, one teacher was contacted based on a recommendation from another respondent. Convenience sampling is known to have risks involved with it, for example due to the respondents not belonging to the group that is to be studied (Berg, 2009). The choice to use this method was still made due to the relatively small size of the project, as well as the expected difficulty finding respondents willing to test tasks. Furthermore, the respondents were still deemed to be fit for the current study, due to them all being teachers, and as such belonging to the group which the study aims to research.

All respondents were contacted through e-mail, where the purpose of the study was explained to them, and they were asked if they wanted to participate. In total 6 teachers participated in the study. All of them were teaching at an upper secondary school and at the technology program or natural sciences program. The six teachers will be referred to as T1-T6.

3.2 Selection of tasks

The tasks used in this study were selected from the resource MatteSherpa, which is a resource with challenging tasks for gifted students in upper secondary school in Sweden. This resource is run by the Department of Mathematical Sciences, which is a joint department between Chalmers University of Technology and the University of Gothenburg. Each task has a corresponding piece of supportive material for teachers, which includes a solution to the task, and sometimes hints that can be given to the students.

All tasks were selected by the researcher, after consulting with the teachers about what would currently be a suitable topic for their mathematics course. After figuring out the topic for the tasks, they were selected subjectively, prioritizing tasks that seemed well formulated and easy to understand. This did not, however, mean that the difficulty was low, merely that the formulations were clear. Furthermore, the selected tasks were checked for any potential errors, to ensure that the students did not spend time on a task that was incorrectly formulated or had errors in the solution example. Two tasks were edited due to incorrect mathematical formulations or errors in the suggested solution. Furthermore, the formatting of most tasks was updated to be more easily read, as well as some terminology to match the current Swedish school curriculum. All teachers tested one task each, except for one teacher who tested two. In that case two tasks were sent during the process of trying to find a suitable task, and the teacher ended up using both.

In total seven tasks were selected, which will be summarized below. The full tasks can be seen in Appendix C. T1-T5 tested the tasks corresponding to their number, and T6 tested tasks 6 and 7.

Task 1: Determine how to optimally view a screen hanging on a wall, with some initial conditions such as the size of the screen, and how high up it is hanging.

Task 2: Given some properties of the Bernoulli polynomials, determine the first three polynomials, as well as prove a property for them. In addition, search for information about the Bernoulli polynomials.

Task 3: Three curves are plotted in a graph, as well as two equal areas between the curves, also determined by a rectangle. Given the function for two of the curves, determine the function of the third.

Task 4: First, a short introduction to the parametrization of curves is given. Using this knowledge, determine the shortest distance between two moving points in the plane, described by two given parametrizations. In addition, determine the equations of the curves that the moving points create.

Task 5: A definition of a Pythagorean triple is given, as well as a few examples. The task is to prove two statements about the divisibility of numbers in a Pythagorean triple, as well as to search for further divisors than the two proven ones.

Task 6: Given three angles in a triangle, and a relationship between the angles, prove that the triangle is an isosceles triangle.

Task 7: Given an arbitrary third-degree polynomial, and an arbitrary straight line, that cross each other three times, where one crossing is in the inflection point of the third-degree polynomial. Prove that the two areas created by their intersection are equal.

3.3 Interviews

Semi-structured interviews were selected as a method of data collection. This method is found suitable when trying to understand the respondent's own worldview (Kvale & Brinkmann, 2014). The third research question aims to investigate what needs teachers have when supporting gifted students, and as such understanding their worldview is relevant to answering the question, which motivates why the method was selected for this question. Furthermore, Kvale & Brinkmann (2014) describe how an interview also can be used to test a hypothesis, which fits the first and second research questions well. Notably, these two different purposes result in differences in how the interview is carried out, which will be described below.

To begin with, semi-structured interviews are characterized by not being completely closed or open. A semi-structured interview does not follow a strict interview guide; it instead contains themes that are to be discussed, and suggestions for questions to be asked. The questions do not have to be asked in a pre-set order. If a specific question order is followed, and if the phrasing of the questions is determined in advance, depends on the specific study in question. For example, interviews that focus on testing a hypothesis are generally more structured than those that do not (Kvale & Brinkmann, 2014).

The interviews that were conducted in this study were a mix of testing a hypothesis and exploring a worldview. Therefore, different parts of the interview were constructed differently. The part investigating the second research question still had open and general questions to begin with, which also did not have to adhere to a strict formulation. However, this part of the interview was characterized by more targeted questions pretraining to the constructed framework, which is especially relevant for the analysis for this research question[1]. These questions were generally phrased as direct questions, with a varying degree of openness. For example, "Do you think this task allows students to explore mathematics?", or "How would you describe the degree of openness in this task?"

Of particular note for the questions regarding the first research question is that during one question respondents are shown a piece of paper with examples of "special abilities" (Swe: *Särskilda*

[1] When performing a deductive content analysis, after asking open-ended questions it is reasonable to use targeted questions that are related to the categories in the content analysis (Hsieh & Shannon, 2005). This was done in some cases for questions relating to research question 1, but not exclusively.

förmågor), which in Swedish has a meaning closer to "gifted abilities". These abilities were the selection of abilities from Krutetskii's (1976) framework that are in turn used in the constructed framework for this thesis. They were in some cases slightly reformulated to be easier to understand intuitively. This decision was made to be able to investigate this selection of abilities without explicitly asking about each one. Since the abilities are specific, they were deemed unlikely encountered in an open question, which motivated the direct approach.

The remaining sections of the interview focused on the third research question and general background of how the tasks were tested, which can be relevant for all research questions. These parts primarily had open questions; however, some direct follow-up questions were still suggested in the interview guide. These direct questions should generally be left towards the end of the interview according to Kvale and Brinkmann (2014) so the respondent can give their spontaneous view first. In this case a decision was made to start with open questions, and later follow up with direct questions that were only relevant to this specific part of the interview. The only exception to this rule is the questions concerning the second research question, which took a more direct and structured approach.

The questions and themes for the interview guide were constructed with the help of a preliminary version of the framework in section 2.3 for the second research question. The framework changed in some minor ways since the creation of the interview guide, which explains the mischaracterization of a few suggested questions. A more detailed explanation can be found together with the interview guide in Appendix A. Note that the version presented in section 2.3 was used for the analysis. For the third research question the interview questions were primarily open and focused on the teachers' experiences. Some questions were, however, constructed with the help of literature, such as teachers experiencing time constraints. The complete interview guide can be seen translated and untranslated in Appendixes A and B respectively.

3.4 Analysis of interviews

3.4.1 Transcription of interviews and quotations

All interviews were recorded and manually transcribed while following the recommendations of Kvale and Brinkmann (2014). They describe how transcribing is a translation from an oral to written speech. This is a transformative process, and several choices can be made during the transcription process depending on what kind of analysis is performed.

In this study the meaning of what the respondents answered was analyzed, not the specific wording. Therefore, the transcriptions were translated to written speech to the extent it was possible, except when the intent of the language was unclear. The transcriptions also included words such as repetitions or filler words the respondents used while answering questions. However, some other details were omitted. For example, intonation, people speaking simultaneously, and filler words intending to signal agreement while someone else was talking were not transcribed due to a lack of relevance. Exceptions to these rules were made when it was deemed necessary for the meaning of the words, for example when respondents started answering a question before it was fully formulated. Since one person conducted this study, it was not possible to create two transcriptions of each interview to increase the reliability, as suggested by Kvale and Brinkmann (2014). When discussing validity Kvale and Brinkmann (2014) suggest asking which type of transcription is most suited for the current research, giving the example of literal transcripts being necessary for analyzing speech patterns. In this case, the meaning was relevant, which led to the decisions described above.

Lastly, when presenting quotes from the interviews in the results, repetitions, filler words and unclear words that were previously included in the transcription were omitted to increase the clarity of the quotes. Exceptions to this was made when it was necessary to preserve the meaning. In some cases, connecting words were added to increase clarity, if it did not change the meaning. Furthermore, these quotes were translated from Swedish to English, with a focus on preserving the meaning.

3.4.2 Content analysis

To answer the three research questions two separate content analyses were conducted as described by Berg (2009). The general idea of the method is to identify categories into which different sections of the text can be coded, and then analyze the material to find patterns within the categories. This can be done both deductively and inductively. Deductively means to identify categories from existing theory or research question, and inductively means to identify categories from the data itself (Berg, 2009). The first content analysis used a primarily deductive approach to answer the first and second research question. This was due to a framework being constructed to investigate gifted behavior, as well as question one investigating something very specific. The second content analysis used a primarily inductive approach to answer the third research question. This was done due to the intent being to openly investigate teachers' needs that are for the most part not known beforehand. However, most of the steps in the method remained the same.

Unit of analysis

Before a content analysis can start the unit of analysis has to be selected according to Berg (2009). This is the unit that will be analyzed from the data and can be something as small as a specific word occurring, or entire paragraphs. The unit of analysis for both content analyses in this study was selected to be passages of words describing a specific theme (Berg, 2009; Bryman, 2016).

Content analysis

The content analysis followed the steps outlined by Berg (2009, p. 362-364), which is a combined approach where categories are identified both deductively and inductively. In the outlined steps categories identified by these approaches are called analytical and grounded categories respectively. The first step was to identify research questions, which was already complete. Secondly, analytic categories were to be defined deductively. In addition to being identified from existing theory, these categories can also be connected to interview or research questions.

For the first content analysis taking a primarily deductive approach these analytical categories were identified from the framework presented in section 2.3. In addition, two extra categories *challenging tasks* and *students* were added, corresponding to the first research question. In the second content analysis taking a primarily indicative approach, some analytic categories were created from the interview questions, and some general theories about the difficulties teachers face. However, the focus was not placed on these analytical categories.

After analytic categories were defined grounded categories were identified inductively. This was done by reading a large part of the gathered data and writing down categories relevant to the research questions that could be observed on a surface level of the data. This was done during both content analyses that were performed. However, in the first content analysis that was primarily deductive, the grounded categories were found to overlap with the analytical ones and were omitted. For example, the category *Threshold* was a grounded category, and referred to there being a threshold to start solving the exercise that was difficult to overcome. This was determined to overlap with the analytical category *Competence,* due to the category concerning the difficulty of the task, and thus whether students can feel competent while working with it.

In the second content analysis for the third research question, most categories were grounded categories. The idea was to explore what teachers experienced, with only some previous knowledge of the difficulties that exist with supporting gifted students.

After categories were identified criteria for sorting data into these categories were defined. For the analysis related to the first and second research question, these can be seen in Appendix D, and are defined with the help of the framework described in section 2.3. For the second content analysis for research question three, the criteria were less reliant on theory, and the focus was instead placed on creating consistency within the inductively identified grounded categories. The criteria can be seen in Appendix E.

Notably, both data supporting the existence of the category, as well as data supporting the absence of it, were coded into the different categories. For example, in the category *Open Tasks*, both data indicating that the task was open, as well as data indicating that the task was not open, were included in the category. This was marked by a + or – sign next to supportive and unsupportive data, respectively, as described by Kvale and Brinkmann (2014). This was done to allow for contrary points to be coded without potentially doubling the number of categories. In some cases, a theme included both supportive and unsupportive descriptions, in which case it was marked with both a + and – sign. This method was only used for categories corresponding to the framework in section 2.3, as well as the category challenging tasks for the first research question, since other categories did not seek to confirm the existence or absence of a theme.

The data was then sorted into categories with the help of the defined criteria. In some cases, categories and criteria had to be revised, which was more common in the content analysis related to the third research question due to the inductive approach. Afterwards, the amount of data entries supporting, and not supporting, each category was counted, and the data was analyzed to identify patterns within the categories. Lastly, these patterns were analyzed with the theory presented in section 2 in mind.

3.4.3 Determining the character of the tested tasks

After the content analysis was finished, which investigated patterns emerging from all tasks, the results were also used to determine if the tested tasks individually stimulate creativity, exceptional abilities and motivation. This was done by taking one task at a time, and checking if there was coded data from the content analysis supporting of that task being stimulating for creativity, exceptional abilities or motivation.

A task was determined to stimulate a personality trait (category in content analysis), if it had data supporting one of its subcategories. However, sometimes a task could have supportive data for one subcategory, while there was data saying it did not support some others. For example, a task could be determined to stimulate motivation if it had data supporting competence, but data saying it was not supportive of relatedness. In some cases, tasks had data both supportive and not supportive of a specific subcategory, in which case one positive piece of data was enough to categorize the task as supportive. This is motivated by the task, in that case, stimulating a personality trait in some regard. It should not be taken as an indication that the task cannot stimulate the personality trait better.

Note that this method did not build upon any specific theory. There was also a limited amount of data for each individual task, such as data being missing for certain sub-categories for a given task. The intent was to create an overview of which tasks stimulated which personality traits in Mönks' Multifactor model (Mönks & Ypenburg, 2009), but not for it to be the primary result answering the second research question.

3.5 Ethics

A key consideration regarding ethics for this study was to not cause any harm to any teachers or students due to the material that was tested. To help ensure no harm occurred all tasks provided to teachers were checked for errors, and had their formatting updated, as described in section 3.2.

Furthermore, all teachers were informed of the purpose of the study and gave their consent to participate through a signed form. In this form teachers were also informed about how they can retract their consent, as well as how the data gathered for the study will be used. All teachers were given a copy of the form to ensure they had the necessary contact information if they wanted to retract their consent.

3.6 Limitations of the method

During the process of conducting the study, changes occurred on the way. As mentioned in section 3.3, the interview guide was constructed based on a preliminary version of the framework. This meant that some questions proved to not be relevant for the finished framework. This, however, did not cause any issues. It merely resulted in more information to sift through during the analysis process.

However, there were also some questions that could have been better formulated. For example, interest turned out to be a key concept for the finished framework, whereas it was a smaller part in the preliminary one. Due to this there were no direct questions about interest in the interview guide. However, the question about curiosity questions could be used as an indicator of interest. In addition, many teachers ended up talking about interest spontaneously. Therefore, there was still enough data for this category to be properly analyzed but having it in mind from the start could have improved the interview process. Another example of questions that could have been better formulated is autonomy. The question about autonomy included working independently, which may have led to less useful answers since working autonomously and independently are not the same thing.

Furthermore, during the interview process, some of the direct questions could have been better formulated. These types of questions are not entirely problematic for this type of theory testing studies, but I still retain that they could have been formulated in a more neutral and less leading way. For example, asking “Can you describe the degree of openness of this task?” would be a less leading question than “Do you think this task is open?” However, it is worthy of note that teachers would answer no to questions such as the second example, but often a bit more hesitant. This was considered in the analysis, since it focused on the meaning of what was being said, rather than the exact words.

4. Results
In this section the result of the content analyses will be presented separately for each research question. All identified categories will be listed, and their patterns will be explained. Furthermore, the results will be exemplified with quotes from the interviews.

4.1 Research question 1: Challenging tasks
To answer if the tasks were challenging for students that have reached far in their mathematical development it must be confirmed that the students who the teachers gave the tasks to match this description. Furthermore, it must be confirmed that teachers thought the tasks were challenging for them. The categories *students* and *challenging tasks* from the first content analysis will be presented here to answer that question.

Students
The teachers who participated in this study gave their task or tasks to students who were either interested in mathematics, high achieving in mathematics, or needed additional stimulation in class and could benefit from these tasks. Notably, in two cases the task was given as an extra assignment to students who were done with their regular material. These students have reasonably gotten far in their mathematical development. However, it was not confirmed whether the students were gifted, but the groups the tasks were given to can also be gifted, so it is possible that they were. At least 15 students in total attempted to solve the tasks, but the exact number is not possible to confirm from the collected data.

Challenging tasks
Most teachers described the tasks as challenging, in some cases even slightly too challenging for the selected students. This did however depend on the individual students solving the tasks. Some students managed to solve the tasks on their own, and if students did not manage to solve it on their own, they often managed with guidance from their teacher. Furthermore, several teachers said that their students enjoyed the tasks. Since the students who attempted these tasks generally had gotten far in their mathematical development, and teachers think the tasks were challenging for them, the answer to the first research question is yes. Note that all teachers gave all their students the same task or tasks. T1-T5 only had one task each, which was given to all their students. T6 had two tasks, both of which were given to all their students. In addition, all teachers tested different tasks, as described in section 3.2.

4.2 Research question 2: Stimulation of gifted behavior
To answer the question if challenging tasks stimulate gifted behavior the remaining categories from the first content analysis will be presented, with descriptions of if they can be considered present in the tasks. These categories correspond to the summaries in sections 2.3.1-2.3.3. Furthermore, every task has individually been characterized by if they stimulate each of the three personality traits in the Multifactor model, which can be seen in table 1.

Task	Creativity	Exceptional abilities	Motivation
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x

Table 1: A table showing which of the three personality traits in the Multifactor model each individual task stimulates. A task stimulating a personality trait is indicated by an x.

4.2.1 Creativity

Non-routine tasks

Overall, teachers seemed to think that their tested tasks are not of a procedural character, at least not primarily. This is an indication of the tasks also being non-routine, due to procedure tasks reasonably having been seen by students before. Especially since this information came from their teacher, who should have a grasp of what type of procedures the students have worked with. For example, when asked if they think the task is a procedure task, T2 said:

> *"No, only partially. There is procedure included in it. There is, because you were forced to calculate the primitive […], but there was a lot of interpretation and understanding. So, it is on a higher level. I think."*

Furthermore, a link was found between the tested tasks and problem-solving. Almost every teacher described the tasks are problem-solving tasks. These are, as described by Juter (2014), tasks that are new to students and pose a problem and are hence non-routine as described in section 2.3.1. For example, when asked to describe the character of the task, T1 said:

> *"[…] And of course, there is a huge amount of reasoning and communication, so it is a very rich task in that way. It covers all abilities. Maybe procedure the least. Which I appreciate a lot, that the calculation itself, or how you say it. Sure, there is some Pythagorean theorem usage and such, but it is relatively basic compared to the difficulty of reasoning and problem-solving. […]"*

Lastly, when asked why they think their tested task is creative, T4 gave an interesting description. This description matches well with what Boesen et al. (2010) describe as creative reasoning, which is something that is best used to solve tasks that are new to the students and not commonly encountered. They said:

> *"Since it is not something they have done before, and then need to find these ways, and think in a new way. And for me that is creativity. To think in a new way, it is, well, you have to stretch yourself, your way of seeing things."*

Open-ended tasks

Teachers expressed a variety of opinions regarding the openness of the tasks. The most prevalent opinion was that the tasks have an open solution method, however, one T1's description of their tested task matched an open goal and an open answer. In addition, T4 described how the openness of the task can be varied depending on how it is used.

To begin with, an open solution method was for example described by T3:

> *"It's not that open when it comes to what you're supposed to do. It's very clearly defined. But the method, they had a lot of different ways to try to solve it."*

A similar description was given by T2, when asked to describe the openness of the task:

> *"What was supposed to be done was described, after all. […] So, you were supposed to follow and see something, so it wasn't very open in that way. Even if it was open in the sense that you had to figure out that you need to start integrating and develop it step by step and build it up."*

The teachers described some sort of open solution method; however, it is unclear if multiple solutions were actually viable. According to Yeo (2017) an open solution method means that there are multiple ways to solve it, and not just that the students have to figure out a way without knowing it before. It is therefore possible that the solution method is open, but it is difficult to confirm. For example, in T3's case, several different methods were tested to solve the problem, but in the end, students singled in on the proposed solution method in the supportive material to be able to solve it. But it is still possible that there were other viable solution methods.

Secondly, T1 described an open goal and an open answer, which looks as follows:

> *"[…] It is a problem-solving task, I'm thinking. And an open task, because the answer itself somehow builds on assumptions to some extent. And that makes it so the focus is more on strategies instead of procedures, if one is supposed to make the distinction between those two."*

The goal statement is reasonably open, since assumptions can play a part of the task. In addition, since the answer depends on the goal statement, there are multiple viable answers, and as such the answer is also open. Furthermore, this teacher placed a heavy emphasis on strategies being important for solving this task. This could be considered the method, and therefore the method could possibly be open as well.

Lastly, T4 described how the openness of their tested task depends on how the task is used. They said:

> *"[…] Everything there is related to creativity and open questions. And so, it's clear that it is fairly open. […] I could have done a little walkthrough, or such about it, and then we would have closed it more."*

It seems like they considered their tested task to be open if it is used as it is. But if students are provided with additional help, through a walkthrough, then it would be more closed. Perhaps this is because the students would have a better idea of a suitable method after such a walkthrough, and not explore different alternative solution methods.

Creation of mathematical problems
Two teachers talked about the creation of own problems. It seems like the tasks can inspire students to create their own problems, however, no teacher has described that the tasks explicitly instruct students to create their own problems. For example, when asked if they think the tasks motivate students to create their own tasks, T4 said:

> *"Absolutely. I think so. I definitely think so. And I think that it opens up that mathematics is different things. And I think it's very important that they get that."*

Furthermore, when asked the same question, T6 said:

> *" They could definitely be used in that way, at least."*

When prompted to explain more, T6 said they think the tasks can evoke thoughts, depending on what kind of problem it is, such as how the task would be if it was applied to another polynomial. Furthermore, it seems like simply reading the task, and thinking about it, can prompt students to create their own variations of the problem according to this teacher. When asked to clarify if they thought it was the problem itself that evoked the thoughts, or the students working solving it, T6 answered:

> *"To just read the formulation and task. […] Because it builds on them plotting up, in this case, a third-degree polynomial, what an inflection point is, and that they need to prove different areas. And then they might think 'If the graphs looked like this instead […] what would have happened then?'"*

Mathematical modelling
It seems like mathematical modelling can show up in the tasks, but the extent of it depends on the specific task. Two teachers talked about this topic. T2 spontaneously described that the task includes mathematical modelling when asked to describe the character of the task:

> *"There is a decent amount of interpreting what it is about. To understand what it is. These B:s, and all that meant, and how to think about it. So, some sort of modelling, problem-solving, from that."*

T6 was instead directly asked how connected to mathematical modelling their two tested tasks were. They described how both were more related to mathematical modelling than many tasks in their textbook, but that task 7 had a stronger connection to it than task 6. They said:

> *"This one very much [Referring to task 7]. […] They needed to create a model for third degree polynomials and think how they would set it up. This one not as much [Referring to task 6]. But yeah. Draw a triangle, create a model and so yeah. Depending on the task, more or less. But absolutely. Perhaps more than many tasks in in the book that often are very procedure focused […]."*

4.2.2 Exceptional abilities

Ability to understand formal structures

The ability to understand and interpret formal structures seems to be a key ability needed to solve the tested tasks, according to teachers. Therefore, it is reasonably also stimulated during the solving process, since it needs to be used. To begin with, several teachers voiced how this ability is important after being shown the paper with abilities. For example, T3 said:

> *"Yes, this with interpreting the task and setting up a mathematical relationship is brilliant for this task. Because that is really what you need to do. […] To set up an integral where you don't have the function is fairly difficult for them, and to understand that you need to all with the integral expressions and work with them, as what do you call it? Terms. They're not really used to, either."*

In addition, T6 said:

> *"Yes, absolutely, all of them I would say. […] To understand and interpret tasks and set up a mathematical relationship. Exactly. This is what both of the tasks were about. And that is where I experienced that they got stuck. And that is what they need to practice, as well."*

These teachers express that understanding and interpreting the task is an important part of the task, but also a difficult one. T3 expressed that the integral expressions had to be used in ways they are not used to, and T6 expressed that the interpretation of the task was where they got stuck. The third teacher had expressed earlier in the interview that the students specifically got stuck with variable names not being familiar to them and confirmed that notation was what they were referring to with the cited statement. This ability was therefore important and necessary for solving these tasks.

However, T1 had a different opinion. They thought that it was the least important ability out of the examples. Notably, the description focused on the mathematical relationship of the ability. However, they still thought it was a necessary ability:

> *"I would probably say that which is challenged least is understand and interpreting the task and setting up a mathematical relationship. Mostly because, I've thought about the last part about that. It is done of course. But the focus, for solving, for finding the strategy, I'm thinking that that part follows for the students. The mathematical relationship. […] interpreting and understanding is of course needed. But the others […] have some sort of stronger connection to the ability of formulating a reasonable strategy."*

Generalization ability

The ability to generalize was commonly described, and it seems like this ability is stimulated by the tasks. T1 asked if they should rank the different abilities provided on the paper, and upon doing so they said:

> *"[…] It is difficult, actually, I realize now. It is difficult to separate, because all of them tangent each other. But maybe number one is to 'perform generalizations and see patterns'. Which is an A level ability, if you translate it. So that one I would say is number one, even if the others are also accessible."*

Furthermore, when T5 was asked if the example abilities on the paper were stimulated, they described how different abilities are interlinked, and how generalization is a part of creating a certain table suggested by the supportive material. They said:

> *"Absolutely. It feels like it would cover most of them. Because you will need flexibility between solution methods. Because I looked at the supportive material, and then it's a good tool to create a table for example. And you have to, based on that, perform generalizations and see patterns. And I would say that requires a logical ability, to do that."*

Logical ability

Logical ability was commonly expressed as something that these tasks support. Often these descriptions were expressed as a need for logical abilities, or an ability to perform proofs, or both. For example, after being showed the examples of special abilities, T4 said:

> *"Absolutely. Above all, perhaps, both logical ability and flexibility between solution methods. In that one that we worked with, that is."*

Several similar descriptions were given by other teachers. Furthermore, many teachers mentioned that the tasks require a strong ability for performing proofs. Performing proofs is something that often requires logical abilities, such as in the example given by Krutetskii (1976), which can be seen in section 2.3.2. For example, T5 said:

> *"[…] It feels advanced. It requires, after all, a pretty strong ability to perform proofs."*

Mathematical memory

A memory of previous mathematical knowledge was brought up by several teachers in the interviews. However, it usually does not concern specific schemes or proofs, which is what the mathematical memory is about according to Krutetskii (1976). For example, T3 said:

> *"[…] Here they were forced to recall. When they eventually understood that it was necessary to use the inverse functions […], they have to think about what it is, and how to draw them."*

T3 described students having to remember a concept they have previously learned to be able to solve the task. T5 also said it was necessary for students to recall knowledge about remainder classes to solve the problem. These examples are indicative of retrieving conceptual knowledge, and not necessarily knowledge of a certain method or solution scheme.

When asked about if the example special abilities on the piece of paper are stimulated, T6 said:

> *"Yes, absolutely, all of them, I would say. […] Associating tasks with previous knowledge. Absolutely. And that was also what they had difficulties with. Like, 'How do I do it?', 'What should I do?' […]. 'Which theorems shall I use here?'"*

When later asked if a certain method had to be remembered for this task, T6 said:

> *"It was probably methods they recognized. Because it's a lot of setting up, well, in this case it was the sine theorem and the cosine theorem, and throwing it around with formulas. And they are used to that."*

This teacher also described recalling concepts, such as the sine theorem, but also possibly the method of algebraically manipulating the expression, as indicated by "throwing it around with formulas". However, algebraic manipulation remains a very general solution method which is used in many tasks. This reasonably also cannot be considered the mathematical memory Krutetskii (1976) speaks of, which instead is a specific method or scheme for a specific type of task.

Flexibility

Flexibility between different methods when trying to solve a task was described by several teachers. Often it has to do with switching from a non-working solution method to one that is working. Alternatively, it could be stimulated through a discussion about different solutions when working with tasks that have multiple viable solutions. For example, after being showed the piece of paper with examples of special abilities, T3 said:

> *"[...] And flexibility between solution methods, that one I think they absolutely practice, but is also something they have a hard time with. This thing with switching strategy if what they are used to doesn't, and what they thought of doesn't work. It is difficult to come up with different ones."*

T3 had earlier mentioned that the students attempted to solve the task in many different ways, and when prompted to talk about that, they said:

> *"Exactly. But maybe also very similar ways. So, very similarly, that they maybe test the same method twice. It doesn't work, and they say 'Okay, what do I do instead?' But I experience that a lot of students think it's difficult to switch strategy entirely. And you really needed to do that here actually."*

T3 describes how students needed to switch their strategy when their initial one did not work. This proved important, because the task in question required a very specific strategy to be solved. Another example of how flexibility could have been used for task 6 was given by T6:

> *"Flexibility between solution methods. [Pause]. Yes. Here it would really have been possible to do that, with math 4 and such. Such a discussion didn't occur between us, but it would really have been possible. If it had been done with an entire class and they had gotten time and you really worked with it, then maybe different students had solved it in different ways. If it had been given to a math 4 class, for example. [...]"*

Task 6 could be solved with knowledge from the courses Mathematics 3 or Mathematics 4, with different solution methods. T6 described how they did not discuss different solution methods, but it would have been possible to do such a thing with this task. Notably, the way Krutetskii (1976) describes flexibility is not about the possibility of multiple solutions existing, but instead the ability to switch the mental thought process between such methods. However, this could possibly have been stimulated in the structure the teacher described.

Several other teachers also agreed that their tested task stimulates flexibility between solution methods. However, T2 expressed that it was not being very prevalent in their tested task.

4.2.3 Motivation

Autonomy

Several teachers described different ways the tasks can help support autonomy, for example due to them feeling meaningful, or how the tasks are used as a way to provide choice. T4 voiced that they think tasks outside of the classroom increase the feeling of autonomy for students. When asked if students can work autonomously with these types of tasks, where autonomy among other things refers to making their own decisions, T4 said:

> *"Yes, I think so. Absolutely. [...] When we go into the classroom, we have decided that we are going to work with the textbook, and to have this and this with us, and that you are supposed to listen to me when I talk a certain time and such. So, everything that happens outside of that, I think, increases the feeling of autonomy for students. And that they, especially if they have the opportunity to decide. 'Do I want to do this task or do I not?'. Definitely then, I think."*

T4 is saying that providing students with extra tasks, and the ability to decide if they want to do them, supports autonomy. This is in line with autonomy in SDT, where providing choices for students supports autonomy (Hornstra et al., 2017).

In addition, some teachers explained how students felt motivated because of the type of task it was. T1 said that a student was especially motivated due to how well it matched how mathematics can be used in the real world, with descriptions about how open the task was, and how it allowed for many interpretations and factors to be considered. For this student it seems like the task felt meaningful. Furthermore, T6 teacher expressed that students were motivated by the task because it came from Chalmers[2], and as such presumably felt more meaningful. Hornstra et al. (2020) says that explaining the relevance of tasks for students is supportive of autonomy. Students feeling like the task is meaningful is reasonably similar to this, with the difference that it did not require an explanation.

Competence
Several teachers expressed varying opinions when it came to how challenging the test tasks were, and as such how appropriate they are for supporting competence within students.

When asked about the difficulty of their tested task, and if it was solvable but still challenging, T3 answered:

> *"[…] It was on the border to being a bit difficult, I think. I had a student that solved it themselves without looking at the solution. The other two who did it and managed it needed support from me. And so I really had to sit down with them and explain this with, with why we needed to switch the axis and such. So it's definitely difficult, but I think. And now everyone didn't do it either. I am certain that if it had been done with more that more would have managed it. Then I think it was a good task for when it is supposed to be challenging. So good difficulty level."*

T3 thought the difficulty level was fitting, and that several students would have been able to solve it if more attempted it. Later in the interview they elaborated and said that while the task is of a difficulty higher than most students in a classroom should work with, they think many students can manage more than they think. Furthermore, the students that attempted the task received guidance from the teacher did manage to solve it in the end. Providing such guidance is positive for supporting competence, as well as having tasks that are challenging but solvable. This specific task may require such guidance for it to be solvable, and therefore to be positive for competence. Notably, the students attempting this task were students who had finished their tasks for the current lesson, and who the teacher described as good at mathematics.

T1 had a differing account. When asked the same question as before about the difficulty, they expressed that it might be a bit too difficult, and said:

> *"Maybe the difficulty is on the high side. Just because it was difficult to see. But with a hint I think it's perfect."*

They further explained that the difficulty was reaching the key insight that the problem required for solving it. This insight was a conceptual insight about how angles work, and no student reached it on their own. T1 therefore thinks a hint about that would make the task of an appropriate level.

Teachers also expressed students getting stuck on a certain step (T1, T3, T4, T6), and several teachers expressed that they think this demotivates students (T1, T4, T6). It seems that proper guidance being provided is of vital importance, so students are able to solve the task. This can either be from the teacher, or perhaps through a hint being included to lower the difficulty of the task, as described by T1. It is worth noting that some of the tested tasks had hints, while some did not.

Furthermore, task 4 and task 7 included concepts that are not part of the standard Swedish curriculum. This proved to be a challenge in task 7 and became an obstacle for the students. Notably, in this case

[2] Note that the tasks are from the resource MatteSherpa, ran by the Department of Mathematical Sciences, which is a joint department between Chalmers University of Technology and the University of Gothenburg. The students had presumably only heard that it was from Chalmers, due to the researcher mentioning that they are a Chalmer student in their introduction.

the task provided no guidance for what the concept was about. In task 4 guidance was provided, which explained what the concept was about, even if it was new. In that case the teacher did not think it was an issue at all. When discussing their wishes for the resource in general, T4 said:

> *"[…] Like with how it was with the parameterization of a line. It does not matter that they don't know it. It's only good that they don't know it. But they need to connect, and that you can receive a little text. They get to learn something, use it, and connect it to the course."*

One interpretation from these two accounts is that it is important to provide students with an explanation of new concepts, and not leave it to them to figure it out on their own. The conceptual explanation does not have to be an explanation of the method to solve the task, however.

In conclusion, the tested tasks seem to be of a high difficulty that is not fitting for most students in a classroom. But gifted students, or high performing students, will be able to solve these tasks assuming they receive proper guidance, which can either be from their teacher, or hints provided with the task. These hints currently exist for some tasks, but several teachers expressed a further need for these hints. Notably, it is also important to provide explanations of new concepts, which the tasks do in some cases.

Relatedness
Supporting relatedness was primarily investigated through finding out if tasks can be used in work forms including multiple people, for example by working in pairs or groups, and as such encouraging interactions with peers. This was found to be possible, but often teachers thought it requires the groups to be arranged so the students in the group are of a similar level. Furthermore, several teachers wanted to use the tasks with the entire class, and thought it possible, but in one case with some modifications. To begin with, when asked if they think the task would be suitable in other work forms, and being given the example of pairs or groups, T2 said:

> *"[…] I definitely think so, that it would have been possible at least in pairs. […] It's not a proper discussion task to sit and reason about with too many. But to sit in pairs and help each other, and learn to interpret, that would have definitely worked. With two students that are on an equal level."*

T1 expressed a similar sentiment:

> *"One thing that I think is tricky in education in general is to create activities that everyone has access to. Not physical access, but more like, for example: A task like this the rest of the class, or at least two thirds of the class, won't get anywhere with. Which makes it so if they were to sit in groups and work with this, and the groups were made so every group has someone who could manage it, it happens so easily that that student becomes in the center. And the other sit and don't really learn anything."*

It seems like it is of importance that the groups are well made for this type of task to work in a group, or that it is two people of equal level if it is to work in a pair. T5 echoed this, saying that they think small groups are good for these types of tasks, where everyone belongs to the group that the tasks are intended for.

Notably, several teachers express a want to adapt the task to be fitting for the entire class, or simply wanting to perform it with the entire class. T3 expressed wanting to dedicate an entire lesson to the task. They said:

> *"Actually, I would have liked to dedicate an entire lesson to this. […] That the task is provided to everyone, and they get to sit and work with it in a group. I would have liked it, I think. Or that they got to sit alone first for a bit and made a draft of the problem. Drawing it and looking at what it is. I think most people would have found an expression for area A then. The first little area. And then they would have needed to discuss their way what to do with the*

rest. And then it would have been possible to provide the hints successively. It would have been more fun, and more students would also have done it."

This teacher does not discuss the groups having to be arranged in a certain way like the other teachers, possibly due to the support provided by spaced-out hints. Furthermore, T1 said the following when asked if they thought it would work in a group or pairs:

"[…] I definitely think so. […] And I'm thinking like this. A whiteboard, someone who is responsible for taking notes, someone who directs the conversation. That roles are delegated so everyone is active."

T1 described how they think there is a possibility of students working individually with the task, despite them being in a group. They mentioned roles to solve this and voiced a wish for the task to be reformulated for it to be used as a group-task. Furthermore, T1 also described a possibility of the task being modified to be simpler, such as by making it more of an open discussion task, instead of one that actively seeks an answer. This way it can be used with the entire class. They also proposed having different versions of the task; one for everyone, and one for gifted students, and perhaps one for some other group.

Interest
The tested tasks seem to be interesting and motivating for students already interested in mathematics. But whether they are sufficiently situationally interesting for students who do not have a pre-existing interest in mathematics is not confirmed.

For example, when asked if they think tasks like their tested task stimulate students to ask follow-up questions, answered T3:

"I absolutely think so. And it probably depends a lot on what student it is. But the students that try these tasks are also those that often are interested in math. […] Then maybe it will be even a little easier for them to ask questions if it is connected to reality. Because in this case I think it was a little difficult for them... Because it is math at such a high level that I think it is difficult for them to ask questions about it. To go even further. But I absolutely think that those who understand the math would have done it."

This teacher thought that students would ask follow-up questions if they were able to understand the mathematics. They also described how students who try these tasks often have a pre-existing individual interest for mathematics. This may mean that if such follow-up questions are asked, they are asked by students with an already existing interest.

T4 also voiced how they think students might ask questions about how the tasks would have been if something was slightly different, when asked about if they think it motivates students to create their own problems. They were asked if they thought this was out of curiosity, which they confirmed that they think. This also corresponds well to the idea of asking curiosity questions, which is an indication of interest. Notably, neither of these teachers directly observed such questions being asked, and T5 also said they did not receive any such questions, but they think it could happen more often than with standard textbook tasks.

Lastly, when asked if they think tasks like their tested task is motivating for the students, T2 said:

"Yes, I think so. It is after all a challenge to try to understand 'what is it I'm supposed to do', and to see if you remember it. I think so, for such a student that is interested in math, it is a little motivating."

This teacher further expressed that it would not be a suitable task for most of their students. It must be students that are interested in mathematics, and not necessarily ones that want the highest grade. One interpretation that can be made from this is that the tasks are fitting for stimulating an already existing interest in mathematics. However, whether the tasks are situationally interesting for students without an already existing interest is not confirmed.

4.3 Research question 3: Teachers' needs and wishes

For the third research question regarding teachers' needs five categories were inductively identified. Some of these overlapped with the analytically identified categories, which were *time aspect* and *usefulness of supportive material,* and have since been combined. The final categories were: *Finding tasks, preparation, lesson, material quality,* and *student hints.* They will be presented in the following sections.

4.3.1 Finding tasks

Several teachers voiced that constructing challenging tasks, or finding challenging tasks, is difficult and takes a lot of time. Therefore, it is important that finding the tasks is quick and easy. A common request regarding this is for the tasks to be sorted based on course and subject area.

To begin with, when asked if it was easy or difficult to find the time for this type of task, T3 said:

> *"This time it was really easy because I got a task from you for free. [...] So that was really good. But I absolutely think that if you know there is a page with these type of problems, then it's also really easy to find them. The tough part is if you need to, either make them yourself, or search the internet for tasks, because that takes a lot of time. So if there is a specific page with a lot of good material you are really happy about that as a teacher."*

Furthermore, several teachers emphasized that it is important for them that the process of finding the tasks is easy and quick. Of special note is that finding tasks belonging to a specific course, or a specific subject area, should be easy to do. For example, when asked what they wish for in a resource with challenging tasks, T6 said:

> *"[...] That it is easy to use and you kind of want it to be divided. Very clearly divided depending on course and content, that still fits, well, the school curriculum and what the content is."*

In addition, T4 had a wish to connect the tasks to chapters, or their titles, in mathematics course books. They think this would make it easier for them to quickly find tasks. Furthermore, T4 also wished for a high number of tasks to be found, so students can be given tasks often. Another teacher specifically wished for there to be a lot of tasks related to problem-solving, reasoning, communication as well as open tasks.

Lastly, T4 described how these tasks may be useful for students who may not be gifted, but who are aiming for high grades. According to them education is often too simple to properly support the students who need support to reach a high grade and think challenging tasks like these may be of assistance. But they think it would be preferable if the tasks were slightly easier than the one they tested.

4.3.2 Preparation

Familiarizing oneself with the task is something that teachers mentioned as a necessary preparation for using the tasks. This is something that can be difficult, but overall teachers thought the amount of time it takes to be reasonable. A reason for this was due to the task and supportive material being well developed and clear.

When asked if they found it difficult to use tasks like this spontaneously, without preparation, T5 said:

> *"Yes. I feel like I need to be familiar with this to be able to give proper guidance."*

Furthermore, T1 voiced that there is a challenge to familiarize themself with this type of task, since it was created by someone else. When asked about what their experience with using the material during their lesson was, they said:

> *"This was very rewarding and I thought it was appropriate for them. The challenge was that I, in other words, that when you inherit a task it's always difficult when it hasn't been created by oneself. And that I was not in on the core idea with the task, and have to familiarize myself with it."*

However, this teacher still found the amount of time they needed to spend on preparation and the lesson to be a reasonable amount. They said that the task was so well developed that it was easy to use. This is also echoed by other teachers, and most teachers found that it was a reasonable amount of time that was necessary to use the task.

Of particular note is that T3 felt fine using the task during the lesson without having solved it before. They said:

> *"I thought it was really good! Absolutely. Super clear and easy to follow. So it was also very easy to use the task because there was good supportive material. Because I can imagine that it is also a resistance. If you first have to sit down and solve it and understand it yourself. Then it becomes more work."*

One important takeaway from this is therefore that the tasks and supportive material need to be well developed to allow for the preparation time to be low.

4.3.3 Lesson

Several teachers expressed that it is difficult to find the time in their course to include these types of tasks. However, it also seems to depend on the course. T4 said that it depends on the course they have, giving the example of Mathematics 2 having more free room than Mathematics 1. Furthermore, they voiced that how easy it is to fit in depends on the current topic they are discussing. When asked if they had enough time to help the students, while still having time to help students working with the regular classroom activities, T4 said:

> *"No, but it felt good. It did. […] Most of the class understood what they were doing. And then it worked well. If it was a subject like logarithms, for example, then it would've probably been a bit worse. […] But with minima and maxima values it was a bit like 'Yes, yes. We'll figure this out'."*

T6 also voiced that there are time constraints in the course, and that they to some extent may be at fault for not prioritizing these types of tasks. But at the same time, they have to balance it with having enough time for other students that need help to pass the course to begin with. Furthermore, they voiced that a task like this would probably fit better into their course In a few weeks during a repetition lesson. T2 voiced a similar thought, which is that how well the tasks fit in depends on the lesson, and that during a lesson with a lot of working individually it is manageable to use these tasks.

4.3.4 Supportive material

Something important about the material for teachers is that the tasks have clear supportive material, with a proposal solution. When asked about what a teacher wishes from resources containing tasks, T3 said:

> *"[…] And a clear supportive material, I think. That is, a proposed solution in the supportive material. So that in the worst case it's possible to post it and let the students look themselves. […]"*

This teacher used the supportive material this way, and found that it worked well for them, even though they later also needed to give guidance themselves. T4, however, thinks that it would not work to give students the supportive material directly. When asked if the supportive material was clear enough, T4 said:

> *"Yes, I think so. Absolutely. But I don't think it would have worked to give to students. But to teachers, absolutely."*

There seems to be a variation in how clear the supportive material is, and if it is possible to let students see it, according to teachers. But they seem clear enough for teachers to make use of them.

In addition, T2 wished for the material to clearly indicate how the tasks relate to the abilities defined by the Swedish National Agency for Education (Juter, 2014). Such as if it is related to problem-solving, or mathematical concepts. The reason being that it helps to be able to explain to students that while doing these tasks they are also practicing the abilities needed for their current course.

Lastly, T5 mentioned that there is a need for quality control of the tasks, so they know they are reasonable to give to students. The teacher did, however, not express that their tested task lacked in this respect, and instead thinks of it as a broad thing necessary for a resource with tasks.

In summary, teachers wanted clear supportive material, that can ideally be given directly to students, should it be necessary. All tested tasks do not have supportive material that is fitting for this application. However, most teachers expressed that the material was clear enough for them as teachers. Furthermore, there was a wish for a clear connection to the abilities defined by the Swedish National Agency for Education, as well as a quality control of tasks contained in a resource.

4.3.5 Student hints

A common topic brought up was that of the student hints included in some of the supportive material for the tasks. T3, who had student hints in the supportive material they were given, said the following when describing how they used the material:

> *"[…] And then I showed them a bit about how they can think, and these student hints in the supportive material were really good. […]"*

T6, who also had student hints in the supportive material for task 7, said the following when asked if having student hints help with the time aspect during lessons:

> *"Yes, it was nice. But I thought they were a bit slyly formulated. […] concepts show up that they aren't familiar with. And then I still need to be the one to explain it. So, for example, to translate a curve. Some might understand it. But then alpha shows up here. Then things stop for them as well."*

This teacher had to come up with their own tip instead, to make up for the hints provided being too difficult to understand. T4 also made up their own hints, but that teacher explained that it was out of preference and having sufficient time to prepare such hints. Furthermore, several teachers thought that when time is short these hints can be helpful, so they do not have to spend extra time making their own hints.

Additionally, the teachers that did not have any hints in the supportive material they received voiced that they wanted such hints. T1 said it would have helped them assist their students, and T5 said it would have saved them time, which is already short. T5 also found it confusing that the material said, "proposed solution, including student hints", when there were no student hints included in their supportive material.

In conclusion, teachers think student hints are a good idea that can help them save time, and better assist students. However, some of the student hints are currently formulated in a way that is too difficult to understand for students, for example by introducing new concepts they are not familiar with. In addition, several tasks are missing these hints. Reasonably the hints should be simplified, and added to the supportive material that is missing them, to properly assist teachers in using these tasks in their work process.

5. Discussion

In this section the results will be discussed and compared to each other, as well as relevant literature.

5.1 Challenging tasks

Challenging tasks were selected to identify if the tasks were suitable for stimulating these students. It is after all challenge that is lacking in their education according to the Swedish Schools Inspectorate (Swedish Schools Inspectorate, 2018). In general, this was found to be the case. However, often the tasks were considered to be slightly too challenging by teachers and requiring guidance for the students to be able to solve them. It is unclear if this depends on the student group, or if it depends on the tasks. The student groups were not confirmed to all be gifted, and instead teachers subjectively selected students they thought could benefit from these tasks. Often this meant they were good at mathematics, however, some teachers questioned whether they were in the target group for these tasks. This is not unreasonable. The Swedish National Agency for Education considers 5% of students to be gifted (Mattsson & Pettersson, 2015), or 1/20 students. This is approximately one student per class, and often teachers tested the tasks with several students. As such it might be reasonable that the tasks are slightly too difficult for these students, but it is hard to safely conclude due to this study not being quantitative. It is not possible to conclude that all of them were not gifted. Regardless, the tasks can be considered challenging, and especially in comparison to standard teaching material, which makes them suitable for students who have gotten far in their mathematical development. This could be gifted students or high-achieving students. But, as mentioned by T4, it seems like these tasks can also be useful for students aiming for high grades that do not perform to that level yet. Therefore, these tasks may not be restricted to being used just by gifted students but can also assist other students in their mathematical development.

5.2 Stimulation of gifted behavior

Creativity

Regarding creativity, it seems that most of the tasks are non-routine tasks, which require creative reasoning. Furthermore, often teachers voiced that the tasks were open, at least in some regard, most commonly through an open solution method. However, it is interesting to note that the other factors in the framework were not nearly as prevalent. Mathematical modelling was rarely mentioned, as was the creation of own tasks. Notably, the creation of own tasks was investigated by asking teachers if they thought the tasks encouraged students to create such problems of their own. The tasks themselves never explicitly told students to create their own problem (see Appendix C or section 3.2). This is therefore lacking in the tested tasks, together with mathematical modelling.

However, these factors not being present does not necessarily mean that the tasks are not creative. Teachers have voiced that they think of their tested task as creative, even with these factors lacking. Furthermore, the literature says that the first two factors should already promote creativity. But it does show that the tasks are not as diverse as they could be. If some tasks, for example, included more mathematical modelling students would get to practice more aspects of creativity than they otherwise would.

In addition, it is important to note that there is the possibility of the factors being brought up less often due to direct questions about them being less frequent. They were less frequent than, for example, direct questions about open tasks. However, it is also the case that teachers more rarely brought up these two factors out of their own accord. Only one teacher spontaneously mentioned mathematical modelling when asked an open question to describe the task, and no teacher mentioned creating their own problem. Open tasks were, instead, often mentioned spontaneously. I therefore do not consider it to be an issue that there were fewer direct questions about the creation of problems or mathematical modelling. The lack of direct questions could result in them not being mentioned, but if they were not mentioned spontaneously, they were reasonably not the most prevalent factors within the tested tasks.

Exceptional abilities
Exceptional abilities seem to be stimulated, since several of Krutetskii's (1976) abilities were described to be stimulated, or necessary, for solving the tasks. All abilities were commonly described by teachers, however two of them are of particular interest to discuss. To begin with, the mathematical memory ability was often described by teachers, but their descriptions did not match how Krutetskii (1976) describes the ability. Krutetskii describes the ability as a memory for the problem itself, and the solution scheme for that type of problem. Teachers instead often describe how students need to recall previous conceptual knowledge, such as the inverse function. This could be because the ability was described as "associating to previous knowledge" on the paper with abilities. It does not clearly specify that it is solution schemes that is the ability, which may have led to teachers not describing it. Another interpretation is that the mathematical memory Krutetskii (1976) describes was not present while solving these tasks, and therefore teachers did not describe it. If this is the case, it is not necessarily a bad thing. The idea is for the tasks to be challenging, and if students recall the solution scheme from previous tasks, they are arguably not challenging, and instead variations of a task they have already seen. This is akin to them being routine tasks for these students, even though they may not be for most students, due to their difficulty. Therefore, not finding evidence for the mathematical memory being used or stimulated may mean that the tasks are non-routine, which instead means the tasks stimulate creativity.

Furthermore, it must be noted that the current study is not ideal for observing this ability. To begin with, teachers' perspective and observations are what is investigated. The focus is not on the students and their experiences, which reasonably is preferred when wanting to investigate an internal process that is highly individual, and that depends on what each student has seen before. Teachers can only provide their perspective about what they think the task would stimulate, as well as the observations they have done during their lesson. These observations are not guaranteed to contain what students' experiences were, since discussion time is limited due to them holding a lesson at the same time. In addition, this type of test is not ideal for observing this ability because of the tasks only being tested once. When investigating the mathematical memory specifically, a test more similar to Krutetskii's (1976) where students got to solve two similar exercises with a delay between them, would be preferred.

Therefore, it may be reasonable that the mathematical memory ability was not accurately described, and hence not found to be stimulated or used. In an ideal situation, it simply has to do with the tasks being entirely new for the students, and the ability not being used. But it is also possible that this study is not well fit for observing the ability.

The other ability of note is flexibility. This ability has connections to creativity (Eriksson & Petersson, 2015), and it is not certain if it truly belongs in this category. However, it was commonly found to be stimulated by the tasks, which is a good thing regardless of which category it belongs to. Either it stimulates exceptional abilities, creativity, or both. Due to it being described as a creative ability, it may be reasonable to categorize it that way. If it is also an exceptional ability is difficult to determine, and was an initial categorization made during the construction of the framework which may have been flawed. The idea was to use Krutetskii's abilities to model exceptional abilities, but only later it was discovered that this ability is related to creativity.

Motivation
One of the most interesting results from this study is that of whether competence is stimulated by tasks or not. As mentioned in 5.2 and 4.2.2, students often needed guidance to be able to solve their tasks. However, this is not necessarily an issue. Competence can, as discussed by Hornstra et al. (2020) be supported by providing tasks of a suitable level, but also by providing guidance to students. This guidance being provided is therefore not bad, it is instead helpful for supporting competence.

A way to analyze this would be with Vygotsky's zone of proximal development (Phillips & Soltis, 2014). These tasks would be within students' zone of proximal development since they are able to solve them when they receive guidance. This is not a bad thing, and instead allows students to stretch their abilities, since the proximal zone of development represents the level of competence the students can reach. This is also how Hornstra et al. (2020) described how competence should be supported. Tasks that are so challenging that guidance is required are therefore not necessarily a bad thing. But they could be if their teacher does not have the time to give this guidance. Time is a common issue for teachers, which was both found in this study, as well as by the Swedish Schools Inspectorate (2018). In this study teacher did generally not have trouble with finding time to give guidance, but they also voiced that this is very reliant on what type of lesson it is due to them also having a responsibility to help the students that are not high achieving. This could be a potential issue for the usability of these tasks and requires teachers to carefully plan when they use them. In addition, although it is not mentioned by teachers, there is the possibility of a lot of students needing more help than expected by teachers, resulting in them having less time to support the high achieving students.

A way to assist with this is student hints. They could reasonably provide some form of support, and thus enable students to be within their proximal zone of development. Although less effectively than a teacher would be able to, due to the hints being pre-defined, whereas a teacher can provide individualized guidance. This is something teachers voiced that they want available. For example, T1 said they believe the difficulty level would be just right with such hints, which matches well with the idea of hints allowing students to enter their zone of proximal development. Therefore, they would be able to save teachers time, which can be especially important during some lessons, even though they are no replacement for teacher given guidance.

However, some students solved the tasks without guidance, and at least in one case without student hints. For these students the difficulty would be easy enough to accomplish alone without guidance. These students would therefore not be in their zone of proximal development, which may not be ideal for stretching one’s capabilities. Nevertheless, the tasks remain challenging, and at an appropriate level for the students, which is reasonably still supportive of competence. But for these students the ideal situation would be for the tasks to be even more challenging. Similarly to how T4 wished for tasks of a slightly lower difficulty for students in need of stimulation to reach higher grades, it may be beneficial to have tasks that are even more difficult for these students that did not need guidance.

This relates well to a common wish expressed by teachers categorized as relatedness. Teachers often mention how they want to use the tasks with the entire class, and not just the students they consider gifted, or perhaps high achieving. This opens the possibility of underperforming gifted students feeling engaged. As mentioned by Persson (2010), students can hide their abilities, or get bored and underperform. Only giving the tasks to students that are performing will therefore not assist gifted students that underperform. But by adapting the tasks so they can be used in the entire class, it opens the possibility for these students to become engaged in mathematics, and perhaps to start performing to their potential. A possible development for the tested tasks, and similar tasks, is therefore to make them easier to use with the entire class. T1 described how there can be different versions of the tasks, and perhaps that way students can work their way up to the harder versions if they become sufficiently engaged by the easier versions. It is reasonably in these situations that the tasks being situationally interesting is especially important, which is unconfirmed in this study.

This adaptation with various versions, or difficulties, could support a variety of student groups. Not just gifted students for which these tasks are in their proximal zone of development. But also gifted students who are underperforming, students who are aiming for higher grades and need additional support, and students that are able to solve the tasks on their own. One important thing to consider in this case is that a core need teachers have is that the process of finding the tasks is quick and easy. To ensure that this is the case similar categorization as teachers mentioned for courses and subject areas should be used for difficulty levels in a resource providing these tasks. In addition, asking the question of how broad the resource should be is necessary. Is the focus on all these groups of students, or is it only on one of them?

As the tasks are right now, it can be concluded that they are within the zone of proximal development for many of the students who attempted the tasks. However, teachers also voiced that the tasks are a bit too difficult in general. A slight reduction of the difficulty, perhaps through new student hints as T1 suggested, or improved ones in the case of them existing, may be suitable.

Lastly, regarding another factor stimulating motivation, it seems like autonomy can be supported through these tasks, but not directly. Autonomy cannot be forced, as per definition. Therefore, it is reasonable that the tasks only indirectly stimulate autonomy, by being something teachers can use to provide students with choice. Here it is not the tasks themselves that support autonomy, but the action of the teacher to provide students with multiple options. This is what is called autonomy-supportive teaching by Hornstra et al. (2020). This matches well with what T4 described about providing choice for students, with activities outside of the classroom. However, it can be argued activities inside the classroom can also be autonomy supportive. If these opportunities for choice become commonplace, they are reasonably inside of the classroom, but would still be considered autonomy supportive.

5.3 Teachers' needs

Time seems to be a central theme for teachers when it comes to their needs and wishes. This time aspect being so prevalent could be due to the existence of questions focusing on the time teachers have. However, some teachers talked about the time in response to open questions. Furthermore, it is not entirely unreasonable to focus on the time aspect, given that it is known that teachers do not have enough time (Swedish Schools Inspectorate, 2018), so finding what can help assist them in this regard is of value. In addition, there are also some needs that are not related to the time aspect. For example, T2 wanted a connection to the abilities used by the Swedish school curriculum to help explain to students why it is a worthwhile task to do. However, T2 also explained that students care about going beyond those abilities. A good solution to meet this need may be to include descriptions of the abilities in the Swedish school curriculum, as well as which of the personality traits in the Multifactor model (Mönks & Katzko, 2005) are stimulated.

One common thing brought up by teachers as an opportunity for improvement for these tasks are the student hints. This would, as mentioned before, make the difficulty slightly lower, which is preferable. However, it would also save teachers time, for example when preparing for the usage of the tasks. Currently, most teachers described the time needed to use the tasks are reasonable, but due to how important the time factor seems to be, if any improvements can be made, they should be made. These tasks do not seem to be in danger of not being used due to time constraints, but it is still important to be wary of time requirements. For the tasks to provide a challenge to the students, they need to enter the classroom, and if teachers lack the time to use them, the challenge cannot be provided. If proper hints are provided, perhaps these tasks can enter slightly more lessons than they currently would, due it taking less time for the teachers. This is therefore an especially important factor when it comes to these tasks being used and helping improve the situation described by the Swedish School Inspectorate (2018), and creating opportunities to stimulate gifted behavior.

5.4 Future research

The framework used in this study could be applied to other tasks to perform a similar analysis regarding if they stimulate gifted behavior. This could be tasks belonging to existing resources, regardless of if they are intended for gifted students or not. Similarly, it could be applied to tasks in textbooks to analyze how well existing material in schools stimulate gifted behavior, and perhaps to compare that to tasks like the ones used in this study.

Notably, this framework could also be improved. Flexibility between solution methods is, as explained by Krutetskii (1976), related to creativity. Perhaps this should be part of the creativity category instead to increase the validity of the framework. Furthermore, other parts of the framework proved difficult to evaluate in tasks. Autonomy and relatedness are often factors that the tasks themselves could not stimulate, and instead only indirectly stimulated due to the possibility of using the tasks in a way that is favorable for these two factors. Further research regarding if these factors are fitting for a framework analyzing tasks could be warranted, as well as if there are other options for fitting motivational factors to include in the framework.

In addition, the situational interest of the tasks could not be evaluated in this study. This could be because of a lack of focus on it during the interviews, as well as a lack of methods to identify it described in the framework. Further research regarding if the tested tasks are situationally interesting, as well as if the interest factor of the framework can be developed, could therefore be valuable.

6. Conclusions

This study has created a framework to classify if tasks stimulate gifted behavior, by using Mönks' Multifactor model as a starting point. The created framework goes into detail about how task specific criteria can help stimulate gifted behavior. Similar classifications of tasks have been done before, for example by Hadar and Tirosh (2019), which focused on creativity. Szabo (2017) also performed similar studies using a framework based on Krutetskii's (1976) abilities, investigating which abilities students use. This study instead looks at gifted behavior overall, with the help of a framework based on the Multifactor model (Mönks & Katzko, 2005), which in turn includes creativity, Krutetskii's abilities to investigate exceptional abilities, as well as motivation.

This study used a selection of tasks to investigate with the help of this framework, which were confirmed to be challenging, and as such appropriate for testing if they can stimulate gifted behavior. It was confirmed that all tasks stimulate gifted behavior, but the tasks varied in which personality traits they stimulated from the Multifactor model. Notably, not all tasks could be considered to support all three personality traits, but this could also be due to the limited scope of the interviews. However, looking at the tasks all together as a group, gifted behavior seems to be well stimulated.

What can be concluded, however, is that the tasks did vary in what they supported most. For example, multiple tasks were creative, but only the task tested by T1 was considered to have an open goal and open answer. Other tasks were creative in other regards, such as by being non-routine. Notably, some factors in the model were less prevalent than others, such as mathematical modelling, the mathematical memory and autonomy, but tasks could still stimulate gifted behavior in other ways.

Furthermore, when it comes to teachers' needs from resources providing these types of tasks, and the tasks themselves, they primarily concern ways to save the teachers' time. This can be time necessary to find the tasks, as well as preparation time, or potentially time during the lesson. This could be influenced by the choice of questions, but if it is the case, it is not deemed unreasonable. Time is, after all, confirmed to be an issue for teachers when it comes to providing challenges in Swedish upper secondary schools (Swedish Schools Inspectorate, 2018). And as such, ways to save time for teachers are essential for these tasks to be provided to the students who need them. This can be achieved by providing hints for students, as well as by resources properly categorizing their tasks.

It is also important to note that teachers wish for a variety of difficulties for different student groups, and to be able to use tasks with the entire class. This could benefit gifted students who are underperforming by them getting to engage with challenging material they may have previously lacked. Furthermore, some students were able to solve the tasks without guidance and may be able to solve tasks even more difficult, especially if guidance is provided. A variety of difficulties, both easier and more difficult than the tested tasks, can therefore be positive for resources containing such tasks to properly stimulate gifted students, and meet the needs of teachers.

Regarding future research, development of the framework could be valuable. Several factors could benefit from further development, or switching categories they belong to. The situational interest of the tasks used in this study could also benefit from further studying. In addition, the framework could be used to characterize other tasks, such as tasks from textbooks, to identify whether they also stimulate gifted behavior.

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Appendix A: Translated interview guide

The following translated interview guide was used for all interviews. The original Swedish interview guide is presented in Appendix B. Furthermore, the list of special abilities that was shown to the teachers during the interview will be presented after the interview guide.

Note that some questions were incorrectly categorized into motivation, exceptional abilities and creativity. This was, as described in 3.3, resolved for the analysis. These were the following:

"Does the task allow the students to explore mathematics?"

> The topic of exploratory tasks was removed from the framework due to a limited amount of literature describing what it is.

"Is the task connected to reality?"

> Reality connection was originally thought as something that could promote relatedness in SDT, but upon further research, was not found to correspond well with the descriptions of relatedness in the literature.

"Does the task allow students to work independently and autonomously?"

> Working independently does not necessarily match autonomy in SDT. However, the second part of the question remains relevant.

Interview guide

Theme 1: Implementation
How did you use the material in the classroom?
Did a few students get to work with the task, or the entire class?

Theme 2: Experiences with the material – Task perspective (15 min)
How would you describe the character of the task you tested?
Creativity
Do you think students are encouraged to be creative by the task?

- How open was the task?
    - Were there several solution methods, several answers, or similar?
- Was the task of modelling character?
- Does the task allow the students to explore mathematics?
- Were students encouraged to create own problems?
- Was the task of procedure character?

Motivation
Do you think the task motivates the students?

- Is the task connected to reality?
- Do you think the tasks is of a reasonable level that students can solve, but still is challenging?
- Do you think the task stimulates the students to ask follow-up questions?

Does the task encourage students to use special abilities?
[Teacher is shown a paper with examples of special abilities]
Exceptional abilities

- Does the task challenge the students logical ability?
- Do the students need to generalize previous knowledge to solve the task?
- Do the students need to use previously learned solution methods to solve the task?
- Does the task demand that the students practice understanding and interpreting the task as well as setting up a mathematical relationship?
- Do the students need to practice using several different solution methods?

Which forms of work can the task be used for?

Motivation

- Is the task suitable for working both in pairs and alone?
- Does the task allow students to work independently and autonomously?

Theme 3: Experiences with the material – Teacher perspective (5 min)
How did you experience using the material as a teacher?

- Do you as a teacher receive sufficient support from the material to yourself be able to support the students?
- Is the supportive material clear?
- Did you have a dialogue with the students about the task?
  - Did you use the student hints during the dialogue?
  - Were they helpful?
- How did you experience the time consumption the task demanded?
  - Was it worth the time consumption?
  - How difficult/easy was it to make time for the usage of the task?

Theme 4: Needs and wishes from the resource MatteSherpa (5 min)
Which needs do you as teachers have of resources to support gifted students in mathematics?

- How can MatteSherpa accomplish this?
-

Examples of special abilities

Logical ability
Perform generalizations and see patterns
Understand and interpret the task as well as setting up a mathematical relationship
Flexibility between solution methods
Associate the task to previous knowledge
...

Appendix B: Untranslated interview guide

The following interview guide was used for all interviews. Note that some questions were incorrectly categorized into motivation, exceptional abilities and creativity. This was, as described in 3.3, resolved for the analysis. See Appendix A for a list of these questions.
Furthermore, the list of special abilities that was shown to the teachers during the interview will be presented after the interview guide.

Interview guide

Tema 1: Genomförande
Hur använde du dig av materialet i klassrummet?
Fick enstaka elever arbeta med uppgiften, eller hela klassen?

Tema 2: Upplevelser med materialet – Uppgiftsperspektiv (15 min)
Hur skulle du beskriva karaktären av uppgiften du testade?
Kreativitet
Tror du att elever uppmuntras att vara kreativa av uppgiften?

- Hur öppen var uppgiften?
    - Fanns det flera lösningsmetoder, eller flera svar, eller liknande?
- Var uppgiften av modelleringskaraktär?
- Tillåter uppgiften eleverna att utforska matematik?
- Uppmuntrades elever att skapa egna problem?
- Var uppgiften av procedurkaraktär?

Motivation
Tror du uppgiften motiverar eleverna?

- Är uppgiften kopplad till verkligheten?
- Tror du uppgiften av en rimlig nivå som elever kan lösa, men fortfarande är utmanande?
- Tror du uppgiften stimulerar eleverna till att ställa följdfrågor?

Uppmuntrar uppgiften eleverna att använda sig av särskilda förmågor?
[Lärare visas en blankett med exempel på särskilda förmågor]
Exceptionella förmågor

- Utmanar uppgiften elevernas logiska förmåga?
- Behöver eleverna generalisera tidigare kunskap för att lösa uppgiften?
- Behöver eleverna använda sig av tidigare lärda lösningsmetoder för att lösa uppgiften?
- Kräver uppgiften att eleverna tränar på att förstå och tolka uppgiften samt ställa upp ett matematiskt samband?
- Behöver eleverna träna på att använda flera olika lösningsmetoder?

Vilka arbetsformer kan uppgiften användas för?
Motivation

- Är uppgiften passande för att arbeta både i par och ensam?
- Tillåter uppgiften eleverna att arbeta självständigt och autonomt?

Tema 3: Upplevelser med materialet - Lärarperspektiv (5 min)
Hur upplevde du att det var att använda materialet som lärare?

- Får du som lärare tillräckligt stöd från materialet för att själv kunna stödja eleverna?
- Är handledningen tydlig?
- Hade du en dialog med eleverna om uppgiften?
    - Använde du elevtipsen under dialogen?
    - Var de hjälpsamma?
- Hur upplevde du tidsåtgången uppgiften krävde?

  - Var det värt tidsåtgången?
  - Hur svårt/enkelt var det att ta dig tiden till användandet av uppgiften?

Tema 4: Behov och önskemål från resursen Mattesherpa (5 min)
Vilka behov har ni som lärare av resurser för att bemöta elever med särskild begåvning för matematik?

- Hur kan MatteSherpa uppfylla detta?

Examples of special abilities

Logisk förmåga
Utföra generaliseringar och se mönster
Förstå och tolka uppgiften samt ställa upp ett matematiskt samband
Flexibilitet mellan lösningsmetoder
Associera uppgiften till tidigare kunskap
...

Appendix C: Tasks

The tasks used by the teachers in this study are the following:

Task number	Task name	Task description	Supportive material
1	Bästa Platsen På Torget	http://mattesherpa.se/basta-platsen-pa-torget/	https://mattesherpa.se/handledning-basta-platsen-pa-torget/
2	Bernouli-Polynom	http://mattesherpa.se/bernoulli-polynom/	http://mattesherpa.se/handledning-bernoulli-polynom/
3	Kurvan som delar lika	https://mattesherpa.se/kurvan-som-delar-lika/	https://mattesherpa.se/handledning-kurvan-som-delar-lika/
4	Parameterpunkter	https://mattesherpa.se/parameterpunkter/	http://mattesherpa.se/handledning-parameterpunkter/
5	Pytagoreiska Tripplar	http://mattesherpa.se/pytagoreiska-tripplar/	https://mattesherpa.se/handledning-pytagoreiska-tripplar/
6	Triangelsatsen	http://mattesherpa.se/triangelsatsen/	http://mattesherpa.se/handledning-triangelsatsen/
7	Lika Areor Tredjegradspolynom	https://mattesherpa.se/lika-areor-tredjegradspolynom/	https://mattesherpa.se/handledning-lika-areor-tredjegradspolynom/

Appendix D: Content analysis criteria for research question 1 & 2

Students & challenging tasks

Students:
Themes related to what students the teachers gave their task or tasks to, and how they were given the tasks. For example, if the students were interested in mathematics, and if the task was their primary assignment for the lesson, or if it was an extra assignment if they had time for it.

Challenging tasks:
Themes related to if the tasks were challenging. This can for example be teachers describing the tasks as challenging in general or describing that their students found them challenging. Furthermore, it can include themes related to the tasks not being challenging.

Creativity

Open-ended tasks:
Themes related to open-ended tasks, or the openness of a task. It does not have to specifically use these words and can instead describe it with other words. For example, the presence of several solution methods, or several answers. The themes can describe either the presence or absence of openness in a task, as well as discuss open-ended tasks generally.

Modelling tasks:
Themes related to mathematical modelling. It does not have to specifically use these words and can instead describe it with other words. For example, describing a real phenomenon with mathematics could correspond to mathematical modelling. The themes can describe either the presence or absence of mathematical modelling in a task, as well as discuss mathematical modelling generally.

Creating own problems:
Themes related to students creating their own problems. It does not have to specifically use these words and can instead describe it with other words. The themes can describe either the presence or absence of students creating their own problem, as well as discuss the creation of own problems generally.

Non-routine tasks:
Themes related to non-routine tasks. It does not have to specifically use these words and can instead describe it with other words. For example, descriptions of how the tasks are not of a procedural character, or how problem-solving is required, can correspond to this category. The themes can describe either the presence or absence of non-routine tasks, as well as discuss non-routine tasks generally.

Exceptional abilities

Ability to understand formal structures:
Themes related to the ability to understand formal structures. It does not have to specifically use these words and can instead describe it with other words. For example, the words interpret, or understand, in connection to the task. The themes can describe either the presence or absence of the promotion and stimulation of this ability, as well as discuss the ability in general.

Generalization ability:
Themes related to the ability to generalize. It does not have to specifically use these words and can instead describe it with other words. For example, applying knowledge in new situations. The themes can describe either the presence or absence of the promotion and stimulation of this ability, as well as discuss the ability in general.

Logical ability:
Themes related to logical ability. It does not have to specifically use these words and can instead describe it with other words. For example, this ability is important for mathematical proofs as well as

general logic in mathematical notation and spatial relationships. The themes can describe either the presence or absence of the promotion and stimulation of this ability, as well as discuss the ability in general.

Mathematical memory ability:
Themes related to mathematical memory. It does not have to specifically use these words and can instead describe it with other words. For example, being able to retain information of previous solution methods, and to be able to apply them again. The themes can describe either the presence or absence of the promotion and stimulation of mathematical memory, as well as discuss it in general.

Flexibility:
Themes related to the ability to be flexible when solving problems. It does not have to specifically use these words and can instead describe it with other words. For example, being able to switch between solution methods. The themes can describe either the presence or absence of the promotion and stimulation of this ability, as well as discuss the ability in general.

Motivation

Relatedness
Themes related to the dimension of relatedness in SDT. It does not have to specifically use these words and can instead describe it with other words. For example, working with peers and feeling part of a group. The themes can describe either the presence or absence of relatedness, as well as discuss relatedness in general.

Autonomy
Themes related to the dimension of autonomy in SDT. It does not have to specifically use these words and can instead describe it with other words. For example, freedom of choice in how to work, and the learning tasks feeling relevant. The themes can describe either the presence or absence of autonomy, as well as discuss autonomy in general.

Competence
Themes related to the dimension of competence in SDT. It does not have to specifically use these words and can instead describe it with other words. For example, solving tasks at a suitable level for the students, and providing support and guidance. The themes can describe either the presence or absence of competence, as well as discuss competence in general.

Interest
Themes related to students' interest in mathematics. It does not have to specifically use these words and can instead describe it with other words. For example, students showing interest in a topic can be indicated by asking curious questions about it. The themes can describe either the presence or absence of interest, as well as discuss interest in general.

Appending E: Content analysis criteria for research question 3

Finding tasks

Themes related to how teachers find tasks. This can for example include difficulties teachers face when searching for tasks. It can also be wishes or needs they that can improve the process of finding tasks, as well as general descriptions of the process.

Preparation

Themes related to the preparation teachers need to do when using tasks. This can difficulties they face when preparing, or need teachers have that may assist in the preparation process. General descriptions of the preparation process can also be included.

Lesson

Themes related to using tasks during a lesson. This can relate things during the lesson itself, but it can also include things related to picking the lesson, such as the course schedule. Difficulties, needs, wishes, general descriptions, and things that assist in this process can be included.

Supportive material

Themes related to the supportive material that is provided together with the tested tasks. This can include wishes and needs from the material, and descriptions of how it is to use. For example, if it is clear and easy to use, or if there are experienced difficulties.

Student hints

Themes related to the student hints in the supportive material. This can include wishes or needs from the hints, how they can improve, or difficulties faced with the hints as they are. It can also include general descriptions of the hints, such as how they can be used, or what is positive about them.

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