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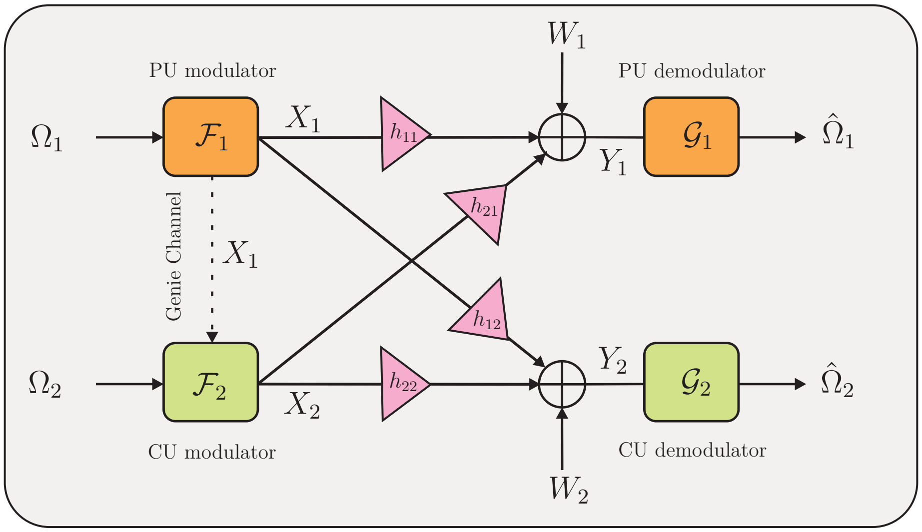

UNCODED COGNITIVE TRANSMISSION OVER AWGN AND FADING CHANNELS

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This thesis has been prepared using LaTeX.

Front Cover: The figure on the front cover is the two-user cognitive radio channel.

To the memory of my mother and father

ABSTRACT

In this thesis a new modulation method is studied for an uncoded cognitive transmission (secondary user transmission) in presence of a Primary User (PU) for AWGN and time-varying flat-fading channels. Interference symbol of the PU is assumed to be known at the transmitter of the Cognitive User (CU) beforehand. Based on this knowledge and using a symbol by symbol approach, we design a CU modulation which can fulfill the coexistence conditions of the CU and the PU. In this scheme, the modulator and demodulator of CU are designed jointly by solving an optimization problem to mitigate the interference of the PU and maximize the performance of the CU communication link without increasing the symbol error probability (P_e) of the PU. The proposed method is a low-complexity modulation approach in a single (complex-valued) dimension rather than a high dimensional coding scheme, but still it achieves good performance. The robustness of the method is also investigated in case of having an imperfect knowledge about the PU transmitted symbols. An implementation algorithm for our modulation method is presented and its performance is evaluated by experiments.

Keywords: Cognitive Radio, Modulation, Uncoded Communication, AWGN Channel, Fading Channel, Interference Channel, Interference Avoidance, Imperfect Side Information, Costa Precoding, Dirty Paper Coding, Relay.

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Doing the master thesis is like fishing in Delsjön Lake! Not only you need to have the proper knowledge beforehand, but also you need persistence and should spend a lot of time. At the end if you are lucky, you will get a big Gädda!

In research you do not know what may happen at the end unless you have superior supervisor and colleagues as well as enough interest and motivation.

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My sincerest gratitude goes to my family. My master studies would have not been possible without your never-ending support. Thank you for everything.

Last but not least, I would like to thank my girlfriend, Fatemeh, for her love, support and patience. The one from the management world who at first had a lot of questions about basic communication concepts and now has a lot to say about cognitive radio!

Mohammad Reza Khanzadi
Göteborg, September 2010

CONTENTS

NOTATION

Abbreviations

AWGN	additive white Gaussian noise
BPSK	binary phase shift keying
CDF	cumulative distribution function
CR	cognitive radio
CU	cognitive user
DPC	dirty paper coding
MAP	maximum a posterior probability
PAM	pulse amplitude modulation
PDF	probability density function
PMF	probability mass function
PU	primary user
SNR	signal to noise ratio

Capital Letters

$\mathcal{B}$	demodulator decision regions
$\mathbb{C}$	complex value
E	expectation
$\mathcal{F}$	modulator function
$\mathcal{G}$	demodulator function
$I(;)$	mutual information
K	number of channel gain quantization levels
M	number of transmitted information messages
P_e	average symbol error probability
P_c	average symbol correctness probability
P_{CU}	maximum transmitted power of the cognitive user
P_{PU}	maximum transmitted power of the primary user
$\mathbb{R}^+$	positive real value
W	additive white Gaussian noise as a random variable
X	transmitted signal as a random variable
Y	received signal as a random variable

Small Letters

f	probability density function
h	channel coefficient
p	probability mass function
x	transmitted signal
y	received signal

Greek Letters

α	cross-talk channel from the CU transmitter to the PU receiver
α^i	probability of being in the quantization region i
β	cross-talk channel from the PU transmitter to the CU receiver
γ	channel gain
λ	Lagrange multiplier
Ω	transmitted information message as a random variable
ω	transmitted information message
$\hat{\Omega}$	demodulated received signal as a random variable
$\hat{\omega}$	demodulated received signal
σ^2	additive white Gaussian noise power

Subscripts

CU	cognitive user
PU	primary user

Part I

Introduction

1 Introduction

The concept of cognitive radio–a wireless device that can sense and adapt to the spectrum–was first introduced by J. Mitola [1]. It is recommended as an option for dynamic and secondary spectrum licensing to overcome the problem of overcrowded and insufficient licensed spectrum [2], [3].

In the previous studies, different general techniques for cognitive transmission in presence of the primary (licensed) users are introduced (e.g., the interweave and overlay techniques [4]). In the interweave technique, the cognitive user (CU) takes advantage of the vacant frequency holes in the spectrum of the primary user (PU) for its own transmission. The CU exploits different spectrum sensing methods to find these unoccupied segments of the licensed spectrum of the PU and adapt its transmission to these free frequency bands [5]. On the contrary, in the overlay technique, the CU transmits its information in the same time and frequency as the PU. Having a pre-knowledge about the PU transmitted signals, the CU adapts its transmission to mitigate the interference introduced by the PU transmission while it does not degrade the performance of the PU communication link which is the owner of the licensed frequency band.

In this thesis our focus is on the overlay technique. In several information-theoretical studies on the cognitive transmission using the overlay technique (e.g., [6] and [7]), a proper combination of the selfish [8] (dirty paper coding [9]) and selfless [8] scenarios (relay) is suggested in order to fulfill the coexistence conditions [7] of the CU and PU. The coexistence conditions of the cognitive transmission are as follow. Firstly, the PU is not aware of the presence of the CU. It has a fixed transmitter and receiver and is not capable of adapting to the CU's transmission. Secondly, the CU should not degrade the performance of the PU's link by introducing any harmful interference.

Although these information-theoretical schemes introduce acceptable rates for coded cognitive radio channels, the infinite length of the codewords (infinite time intervals) and high dimensional coding make them complex for a practical implementation. The aim of this project is finding a new method of overlay cognitive transmission, closer to the real case applications with less implementation complexity. To reduce the complexity, a practical method for the overlay cognitive transmission in one dimension (a complex-valued dimension) is proposed. In other words, to produce the transmitted symbol of the CU in each channel use, a single transmitted symbol of the PU is exploited instead of using the whole sequence of its known transmitted codeword (interference). Although the performance of the introduced method is not as good as the case in which the whole sequence of the interference is used, it is shown that this is a very low-complexity method for an uncoded cognitive transmission which still has a remarkable performance.

In the first part of the project, the method of design of the CU optimal modulator and demodulator is presented for the uncoded cognitive transmission in the AWGN case. Next, this method is modified for the fading environment. Finally, the assumption of having the perfect knowledge about the PU transmitted symbols at the CU transmitter is relaxed and the performance of the method is restudied for this case.

2 The Case of AWGN Channel

In our problem, the cognitive radio transmission is modeled as a two-user channel (Fig. 1) with a fixed and given PU modulator and demodulator. In each channel use, the CU transmitter is aware of the PU transmitted symbol by means of a genie aided channel [8]. Using this prior information, an optimization is formed to design the optimal CU modulator. Solving this optimization, the CU performance is maximized without introducing any harmful interference to the PU link.

First, the average symbol error probability is used as the optimization criterion. Moreover, to guarantee the performance of the PU link, a constraint must be added to the optimization. This constraint can be formed by comparing the performance of the PU in two cases of the absence and presence of the CU. The performance measure for the PU communication link is also its average symbol error probability. As the CU is limited by its transmission power, another constraint is added to the optimization, concerning this limitation.

To solve the optimization, a combination of the Newton-Raphson and a fixed point iterative method is used. The average symbol error probability of the CU is calculated in each iteration. Therefore, the optimal demodulator must be designed in each round of the optimization. The CU receiver has a posterior probability

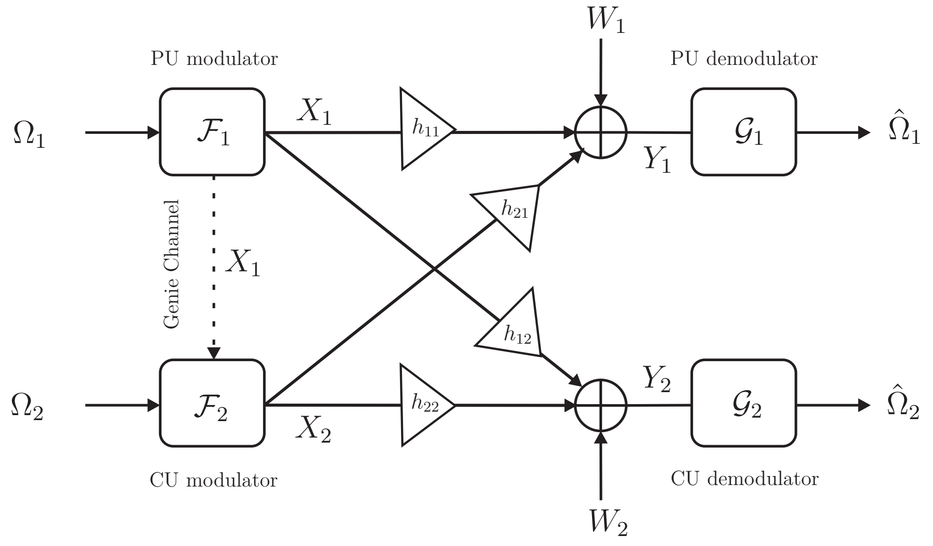

Figure 1: System Model.

mass function (pmf) of the PU modulation. Based on this prior information and using the Maximum a Posterior probability (MAP), the demodulator of the CU is designed.

Next, the mutual information between the CU transmitted and received signals is used as the optimization criterion. As this mutual information is not dependent on the demodulation procedure, the demodulator is designed once, after completing the design of the modulator, which it makes the optimization less complex.

The performance of this method is evaluated for the antipodal binary modulation (BPSK) case. Both the PU and CU have two information messages. The optimal modulator and demodulator of the PU are designed based on the mentioned optimization method. The result is compared with the interference and the optimal cancellation cases ([10] and [11]). Results show that using the cancellation method in [10] and [11] degrades the performance of the PU which is against the mentioned co-existence conditions. But using our method of cognitive transmission, in presence of the CU the PU performance is the same as its single user case. Although, the performance of the CU link decreases in this case compared to the interference cancellation case, it is still notably better than the interference case.

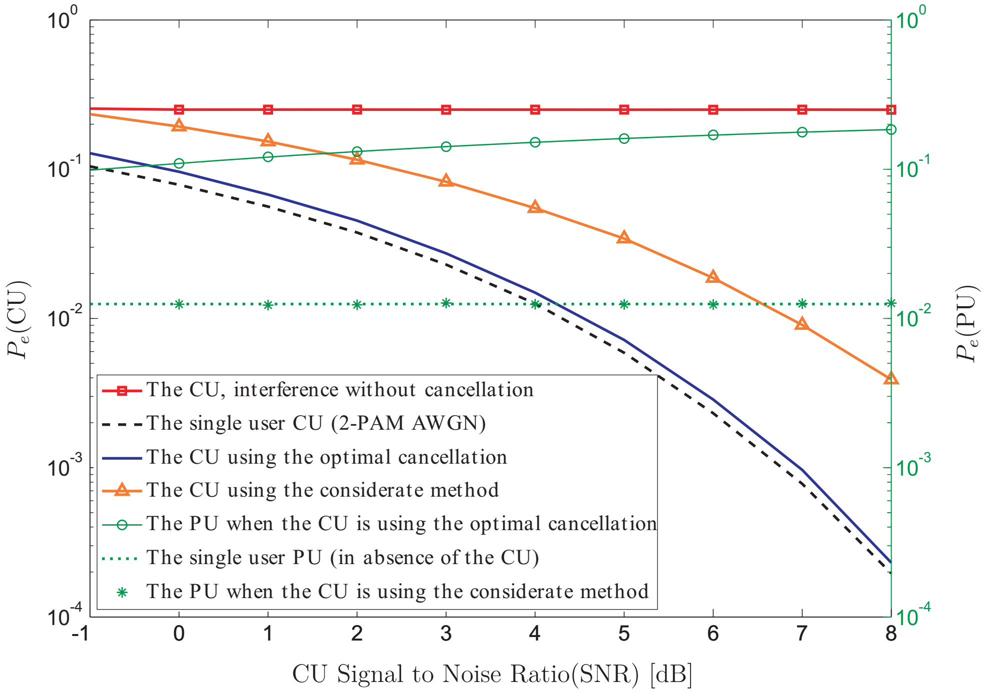

Figure 2: Performance of the PU and CU in different AWGN scenarios vs. CU SNR.

3 Fading Environment

In this section, we propose a fairly general framework to design the modulator and demodulator of the CU in time-varying flat-fading channels. Similar to the AWGN case, we maximize the performance of the CU link while the coexistence conditions are fulfilled. We assume that the distribution of each channel (direct links and interference links in our two user model) is known for the CU. In addition, the CU is completely aware of the states of the channels during each channel use. These assumptions are provided by the CU capability to listen and observe the channel states.

To design the CU modulator and demodulator, the channel gain distributions of all four independent channels are quantized. Next, for each combination of these quantized channel gain values, a CU modulator and demodulator is designed by the AWGN design method of the previous section.

Two different transmission power policies are studied in the fading case namely, the short-term and long-term average power constraints [12]. By the short-term average power policy, the transmission power of each combination is limited to the maximum acceptable transmission power. On the contrary, in the long-term aver-

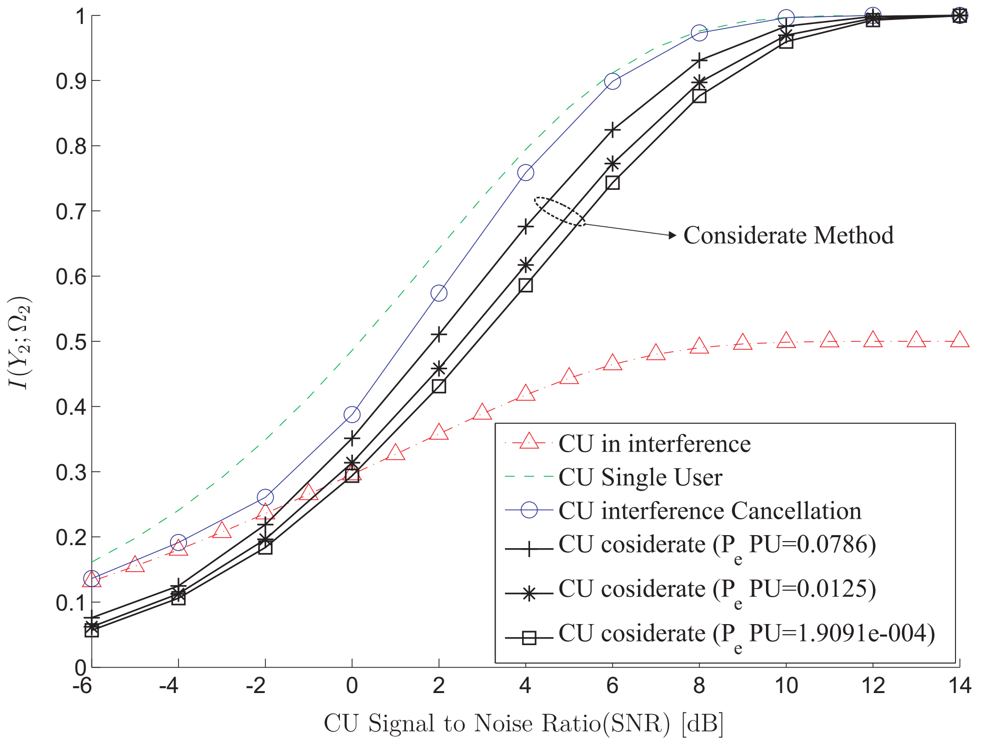

Figure 3: The CU performance in terms of mutual information vs. the CU SNR in different AWGN scenarios.

age power policy, the average power over all combination of the quantized channel gain values is limited. Since the channel values are time-varying, maximizing the CU performance using the long-term policy is a sort of “water-filling” power adaptation strategy [13]. This results in spending the power in those combinations of the quantized channel gain values for which the CU link has a better performance.

The proposed method is designed and simulated for the BPSK modulation over a Rayleigh fading environment and the results of using the two mentioned power policies are compared.

4 Imperfect knowledge of the PU Transmitted Symbols

In the previous sections, the CU was assumed to have a perfect knowledge about the PU transmitted symbols beforehand by means of a genie aided channel. Practically, it means that, we assumed an instantaneous ideal channel between the PU and CU transmitters. Due to the imperfections, a more realistic assumption is that the CU must detect the PU transmitted symbols through an AWGN channel, and

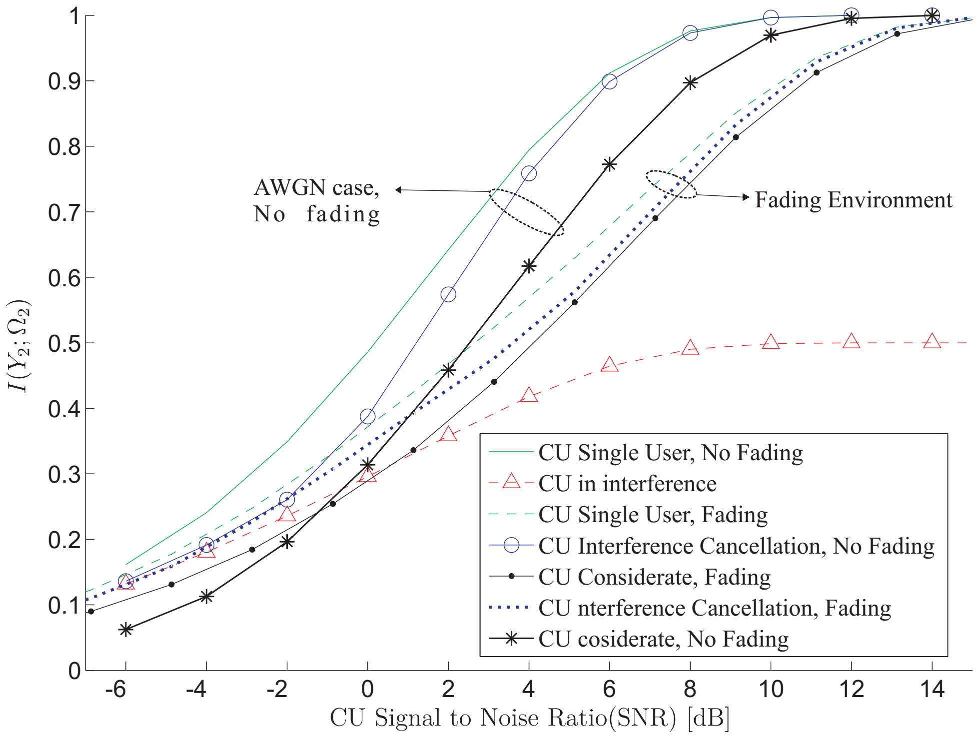

Figure 4: Comparing the performance of the CU in fading (short-term power policy) and AWGN cases.

transmit the proper signal based on this knowledge and using the designed modulator. Since this AWGN genie channel is erroneous, the CU acquires imperfect knowledge of the PU transmitted symbols. Any error in detecting the PU symbols results a wrong choice of CU transmitted signal. This will cause the performance of the CU to decrease as well as introducing harmful interference into the PU link, which is against the co-existence conditions. The simulations for the BPSK case show that generally, if we have an imperfect genie channel with the SNR about 4dB higher than the direct PU link, the performance of the method is close to the case in which the CU has the perfect knowledge of the PU transmitted symbols. Broadly speaking, if we assume the path loss as the only factor that decreases the received power, it can be concluded that the distance between the PU and CU transmitters must be less than the distance between the PU transmitter and receiver.

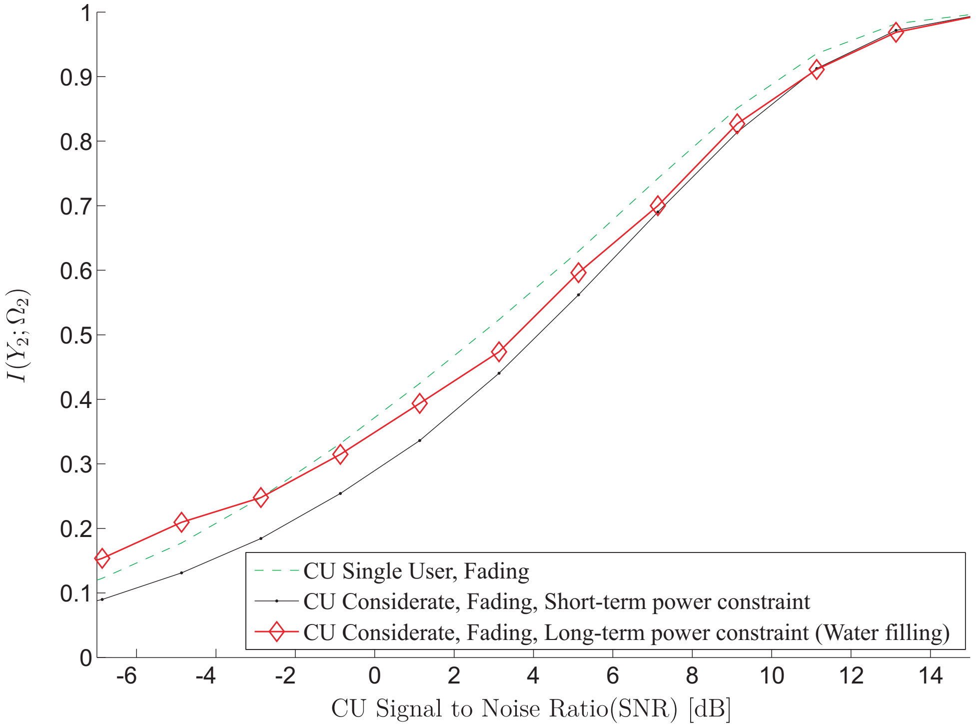

Figure 5: Comparing the performance of CU using short-term and long-term power policies in a fading environment.

5 Conclusion and future work

In this project, methods of designing the optimal modulator and demodulator is proposed for the uncoded cognitive transmission in the AWGN and fading channels. These methods can be classified as an overlay technique of cognitive transmission. Our numerical results show that the cognitive user in this method achieves a notable performance without introducing any detrimental effect on the performance of the licensed user. In the fading environment, the channel gain quantization is used to design the optimal CU modulator and demodulator. The long-term average transmitted power policy (water-filling) yields a better performance compared to the short-term strategy.

In the case of the imperfect knowledge of the PU symbols, we neglect the important fact of delay in detecting the PU symbols in the transmitter of the CU. The methods to compensate the effects of such a delay can be investigated in future studies. In the fading case, the performance can be improved by generalizing the method, for example, by taking the quantization regions as unknown parameters into account to be found inside the optimization. This method also has the potential to be extended to the higher number of symbols (higher dimensions) instead of the symbol by symbol strategy which can improve the performance of the CU link.

The results and findings of this thesis are presented with more details in two conference papers [14] and [15] which are included in two upcoming chapters. In [14], we concentrate on design of the considerate method in the AWGN case. A general method for solving the optimization to design the CU modulator and demodulator is also presented and the performance of the method is evaluated. A general technique to design the modulator and demodulator of the CU in time varying flat-fading channels is proposed in [15]. The effect of imperfect knowledge about the PU symbols on the performance of the method is also investigated in this paper.

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Part II

Included papers

Paper A

A Novel Cognitive Modulation Method Considering the Performance of Primary User

Mohammad Reza Khanzadi, Kasra Haghighi, Ashkan Panahi and Thomas Eriksson

Published in
Proceedings of 6th IEEE Wireless Advanced (WiAD) Conference
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Abstract

This paper proposes a new modulation method for an uncoded cognitive transmission (secondary user transmission) in presence of a Primary User (PU) for the AWGN channel. Interference of the PU is assumed to be known at the transmitter of Cognitive User (CU) non-causally. Based on this knowledge, for the design of the modulator and demodulator of the CU, a symbol by symbol approach is studied which can fulfill the coexistence conditions of the CU and the PU of the band. In this scheme, the modulator and demodulator of CU are designed jointly by solving an optimization problem to mitigate the interference of the PU and minimize the symbol error probability (P_e) in CU's communication link without increasing the symbol error probability (P_e) of the PU. The proposed method is a modulation approach in a single (complex-valued) dimension rather than a high dimensional coding scheme. Although this one-dimensional method is not capacity achieving, we show it still has a remarkable performance with low amount of complexity. An implementation algorithm for our modulation method is also presented and the performance of this method is evaluated by experimental results.

KEYWORDS: Cognitive Radio, Costa Precoding, Dirty Paper Coding, Relay, Interference Channel, Modulation, Uncoded Communication, Interference Avoidance.

1 Introduction

According to recent studies of Federal Communication Commission (FCC), the licensed spectrum is severely underutilized [1]. Therefore, *Cognitive Radio* is recommended for dynamic and secondary spectrum licensing by FCC as an option to reduce the amount of unused spectrum [2]. The concept of cognitive radio–a wireless device that can sense and adapt to the spectrum–was first introduced by J. Mitola [3]. There have been several information-theoretical studies on achievable rates and modeling of cognitive radio networks during recent years (e.g., [4] and [5]). In [4], both links of Primary User (PU) and Cognitive User (CU) are error free with infinite length codewords. In addition, PU and CU cooperate and jointly design their encoder and decoder pairs. In reality, the problem is often different. The PUs are radio devices which have fixed and non-adaptive designs, and they cannot change their encoding and decoding procedure jointly with the CUs. A more realistic study of cognitive radio for the additive Gaussian case is done in [5], where the cognitive transmission is studied based on two coexistence conditions:

1. The PU is not aware of the presence of the CU. It has a fixed transmitter-receiver and is not capable of adapting to the CU's transmission.

2. The CU should not degrade performance of the PU's link by introducing the harmful interference.

The problem of cognitive transmission is an extension of designing the transmitter and the receiver for cancellation of the known interference at the transmitter. For this interference cancellation case, dirty paper coding (DPC) or Costa precoding has been suggested in [6]. The main difference of DPC compared to the cognitive scenario is that the effect of the interfered user (cognitive user) on the performance of the interferer's (non-cognitive user) link is neglected in DPC. This method is denoted as *selfish*, since the CU does not care about the non-cognitive user [7]. On the other hand, another case can be studied in which the CU can act as a relay based on the knowledge of the non-cognitive user's transmitted signals. In this case, the CU disregards performance of its own link and fully relays the non-cognitive user's messages; This method is called *selfless* [7].

In several previous studies on cognitive transmission (e.g., [5]) a proper combination of selfish and selfless scenarios (DPC and Relay) is suggested in order to fulfill the mentioned coexistence conditions. Although these information-theoretical schemes introduce acceptable achievable rates for coded cognitive radio channels, the infinite length of the codewords (infinite time intervals) and high dimensional coding make them complex for practical implementations.

To reduce the complexity, we propose a practical method for the cognitive transmission in one dimension (a complex-valued dimension). It means that, instead of using the whole sequence of the known PU codeword (PU interference), a single transmitted symbol of the PU in each channel use is exploited to produce the transmitted symbol of the CU. Although the performance of this method is worse than the case in which the whole sequence of interference is used, we will show this low complexity method still has a remarkable performance.

The design of the optimal modulator-demodulator pair for cancellation of known interference in one dimension based on a symbol by symbol method is recently studied in [8]. In [8], unlike our proposed method, the interferer is not necessarily a user and its performance is not analyzed in presence of the interfered user (cognitive user). Therefore, we first reintroduce the method of [8] but for the case in which the interferer is also a user. For convenience, the term *optimal cancellation* is used here to refer to this method. Then, a new scheme for designing the modulator and demodulator of the CU for an uncoded relay channel is presented. We use the term *full relay* for referring to this method. Finally, a practical combination of these two methods for designing the modulator and demodulator of the CU is presented, which can fulfill the coexistence conditions of our uncoded cognitive transmission.

Here, the primary and cognitive transmissions are considered erroneous in the same way as real communication links which is another difference of our case and the information-theoretical studies (e.g., [4] and [5]). As it is a one dimensional method, instead of using the information-theoretical rates, the performance of the primary and cognitive user's links for different scenarios are evaluated by calculation of the symbol error probability (P_e) of each link.

2 Mathematical Formulation of The Model

Information messages of the PU, Ω_1, is a discrete random variable uniformly distributed over the set $\{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$. During each channel use, one of the realizations of the Ω_1 is transmitted. This message is modulated by the modulator function $\mathcal{F}_1 : \{\omega_{1,1}, \ldots, \omega_{1,M_1}\} \to X_1 \in \mathbb{C}$ of the PU. The output of $\mathcal{F}_1$ is the complex-valued transmitted signal of X_1. At the receiver, a complex Gaussian noise W_1, zero mean with variance equal to σ_1^2 is added to the X_1. The received signal $Y_1 = \mathcal{F}_1(\Omega_1) + W_1 = X_1 + W_1$ is demodulated by the demodulation function $\mathcal{G}_1 : Y_1 \in \mathbb{C} \to \{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$.

Due to our model, in which the PU has a fixed and non-adapting design, $\mathcal{F}_1$ and $\mathcal{G}_1$ are two fixed functions and cannot be adapted in presence of the CU. For the given demodulator of the PU, decision regions $\mathcal{B}_{\omega_{1,i}}$ are also fixed and can be

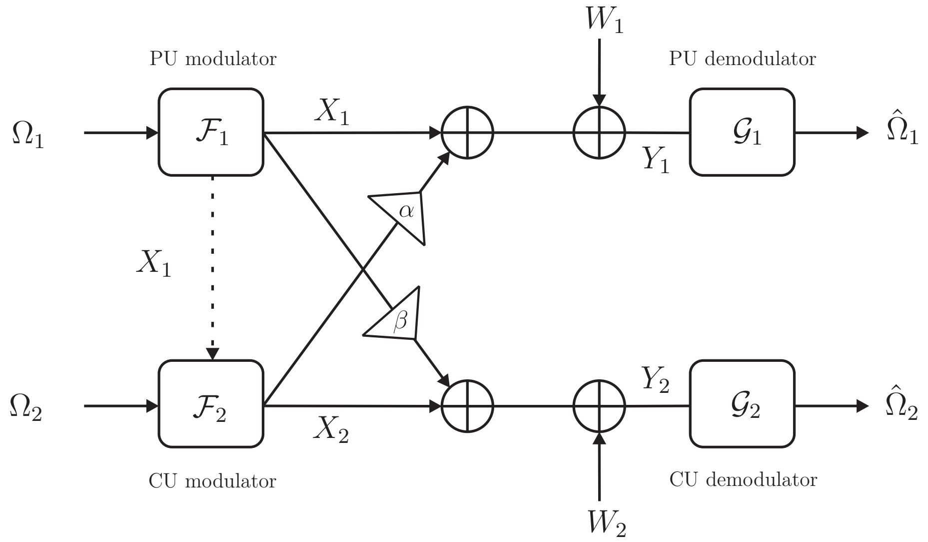

Figure 1: System Model.

defined as

$$\mathcal{B}_{\omega_{1,i}} = \{y_1 | \mathcal{G}_1(y_1) = \omega_{1,i}\}, \quad i = \{1, \ldots, M_1\} \tag{1}$$

which is the set of received signals y_1 that results in the output $\omega_{1,i}$ of the demodulator function.

Following [5], we assume the *Standard Form* for the cognitive radio channel, where the direct channel gain between the transmitter and receiver of the PU is equal to one. The gain of the cross talk channel (interference) between the transmitter of the CU and the receiver of the PU is equal to α. In this case the received signal of the PU is $Y_1 = \mathcal{F}_1(\Omega_1) + W_1 + \alpha X_2 = X_1 + W_1 + \alpha X_2$ where X_2 is the complex-valued transmitted signal of the CU that will be introduced in more detail later.

In the single PU case where the CU is not present (or $\alpha = 0$), the average symbol error probability of the PU using the demodulation function $\mathcal{G}_1(Y_1) = \hat{\Omega}_1$ is equal to

$$P_e(\text{Single PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is not transmitted}). \tag{2}$$

In the presence of the cognitive user, the symbol error probability is

$$P_e(\text{PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is transmitted}). \tag{3}$$

Given the decision regions $\mathcal{B}_{\omega_{1,i}}$ of the PU's demodulator, the average symbol error probability can be calculated as

$$\begin{aligned} P_e(\mathrm{PU}) &= 1 - \frac{1}{M_1}\sum_{i=1}^{M_1}\int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1}(y_1|\omega_{1,i})\mathrm{d}y_1 \\ &= 1 - \underbrace{\frac{1}{M_1 M_2}\sum_{i=1,j=1}^{M_1,M_2}\int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i},x_{2,ij})\mathrm{d}y_1}_{P_c(\mathrm{PU})} \end{aligned} \tag{4}$$

where according to the complex Gaussian noise and additive channel

$$f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i},x_{2,ij}) = \frac{1}{2\pi\sigma_1^2}\exp\Big(-\frac{1}{2\sigma_1^2}|y_1 - x_{1,i} - \alpha x_{2,ij}|^2\Big). \tag{5}$$

We assume that the transmitter of the CU is aware of the transmitted symbol of the PU in each channel use. The receiver of the CU, however, is not aware of this message but only a posterior Probability Mass Function (pmf) of the PU's modulation. The discrete random variable Ω_2 represents information messages of the CU and is defined uniformly over the set $\{\omega_{2,1},\dots,\omega_{2,M_2}\}$. The modulator of the CU $\mathcal{F}_2 : \{\omega_{2,1},\dots,\omega_{2,M_2}\} \times \mathbb{C} \to X_2 \in \mathbb{C}$ maps Ω_2 and the known transmitted signal from the PU (X_1) to the proper complex-valued signal X_2 which will be transmitted later. At the receiver of the CU, a complex Gaussian noise W_2 with mean zero and variance σ_2^2 is added to this signal. The received signal Y_2 is demodulated by demodulator function $\mathcal{G}_2 : Y_2 \in \mathbb{C} \to \{\omega_{2,1},\dots,\omega_{2,M_2}\}$.

Using the *Standard Form* of cognitive radio channel [5], the direct channel gain between the transmitter and receiver of the CU is assumed to be one, and β is gain of the cross talk channel from the transmitter of PU to the CU's receiver. Thus, the received signal of the CU is $Y_2 = \mathcal{F}_2(\Omega_2, X_1) + W_2 + \beta X_1 = X_2 + W_2 + \beta X_1$. Based on the demodulation function $\mathcal{G}_2(Y_2) = \hat{\Omega}_2$, the average symbol error probability for the CU is $P_e(\mathrm{CU}) = \Pr(\hat{\Omega}_2 \neq \Omega_2)$. For the given demodulator of the CU, decision regions $\mathcal{B}_{\omega_{2,j}}$ can be defined as

$$\mathcal{B}_{\omega_{2,j}} = \{y_2|\mathcal{G}_2(y_2) = \omega_{2,j}\}, \quad j = \{1,\dots,M_2\}. \tag{6}$$

$\mathcal{B}_{\omega_{2,j}}$ is a set of received signals y_2 which $\omega_{2,j}$ is the result of the CU's demodulator. The decision regions of the CU's demodulator are not fixed and can be changed adaptively according to the requirements. Based on these decision regions

$$\begin{aligned} P_e(\mathrm{CU}) &= 1 - \frac{1}{M_2}\sum_{j=1}^{M_2}\int_{\mathcal{B}_{\omega_{2,j}}} f_{Y_2|\Omega_2}(y_2|\omega_{2,j})\mathrm{d}y_2 \\ &= 1 - \underbrace{\frac{1}{M_1 M_2}\sum_{i=1,j=1}^{M_1,M_2}\int_{\mathcal{B}_{\omega_{2,j}}} f_{Y_2|\Omega_2,X_1}(y_2|\omega_{2,j},x_{1,i})\mathrm{d}y_2}_{P_c(\mathrm{CU})}\,. \end{aligned} \tag{7}$$

Where

$$f_{Y_2|\Omega_2,X_1}(y_2|\omega_{2,j},x_{1,i}) = \frac{1}{2\pi\sigma_2^2}\exp\Big(-\frac{1}{2\sigma_2^2}|y_2-\beta x_{1,i}-x_{2,ij}|^2\Big). \tag{8}$$

Along with the definition of the cognitive radio as a wireless device which can sense and adapt its transmission to the environment [3], $\mathcal{F}_2$ and $\mathcal{G}_2$ (and decision regions $\mathcal{B}_{\omega_{2,j}}$) can be designed based on different scenarios. As the CU is limited by its transmission power, we have a constraint on the power of its transmitted signal X_2.

$$\begin{aligned} E|X_2|^2 =&\frac{1}{M_1M_2}\sum_{i=1,j=1}^{M_1,M_2}|\mathcal{F}_2(\omega_{2,j},x_{1,i})|^2 \\ =&\frac{1}{M_1M_2}\sum_{i=1,j=1}^{M_1,M_2}|x_{2,ij}|^2 \le P_{\mathrm{CU}} \end{aligned} \tag{9}$$

where P_{CU} is the maximum acceptable power for the CU's transmission.

3 Different Secondary Transmission Scenarios

Based on our definitions, three general cases can be assumed for uncoded secondary transmission in the AWGN channel: optimal cancellation, full relay and *Considerate* (combination of optimal cancellation and full relay methods). These cases are described as follows:

3.1 Optimal Cancellation

In this scenario, the CU is employing the optimal cancellation method introduced in [8] for cancelling the interference produced by the PU. Here, the focus is on maximization of the performance of the CU's link, and no concern is given to the possibly detrimental effects on the PU's performance. As mentioned before, our interferer is a user, and comparing to [8] which uses a continuous random variable for modeling the interference, we model it using a discrete random variable.

For design of the optimal modulator and demodulator pair, first it is assumed that the optimal modulator $\mathcal{F}_2$ is given and the decision regions for correct demodulation are defined based on the maximum likelihood rule.

$$\begin{aligned} \hat{\omega}_{2,j} =&\mathcal{G}_2(y_2) \\ =&\underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\operatorname{argmax}} f_{Y_2|\Omega_2}(y_2|\omega_{2,j}) \\ =&\underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\operatorname{argmax}} \sum_{i=1}^{M_1}\Big\{\exp\Big(-\frac{1}{2\sigma_2^2}|y_2-\beta x_{1,i}-\mathcal{F}_2(\omega_{2,j},x_{1,i})|^2\Big)\Big\}. \end{aligned} \tag{10}$$

Now we assume the demodulator $\mathcal{G}_2$ is given and optimal modulator must be designed. Design of the modulator can be reformulated as an optimization problem. The aim of this optimization is maximization of the performance of CU's link with respect to the power constraint (7).

$$\text{Optimal Cancellation:} \begin{cases} \underset{x_{2,ij} \in \mathbb{C}}{\text{minimize}} \ P_e(\text{CU}) \\ \text{subject to } E|X_2|^2 \leq P_{\text{CU}} \end{cases} \tag{11}$$

For solving this optimization problem, the same as [8] a proper objective function is found using (7) and (7). Then, it is differentiated with respect to $x_{2,ij}$ and is set equal to zero. Using an iterative method, a nonlinear system of equations consisting of $M_1 \times M_2 + 1$ equations is solved for finding the transmitted signals of secondary user (cognitive user). For jointly designing of the optimal modulator and demodulator pair, after each iteration the decision regions are updated based on (11). Due to the space constraints we refer to [8] for more details on this iterative optimization method.

3.2 Full Relay

In this case, the CU is not concerned about its own transmission, and just helps the PU's transmission by relaying its messages. From another point of view, this is an optimization problem in which the proper transmission signals of the CU ($x_{2,ij}$) must be found to minimize the symbol error probability of the PU's link. Still the power constraint (7) must be considered.

$$\text{Full Relay:} \begin{cases} \underset{x_{2,ij} \in \mathbb{C}}{\text{minimize}} \ P_e(\text{PU}) \\ \text{subject to } E|X_2|^2 \leq P_{\text{CU}} \end{cases} \tag{12}$$

Minimization of $P_e(\text{PU})$ is the same as maximization of $P_c(\text{PU})$ defined in (10). Using (10), power constraint (7) and Lagrange multiplier λ_1, the objective function for finding a proper $x_{2,ij}$ can be written as

$$\frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} \left\{ \int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i}, x_{2,ij}) \mathrm{d}y_1 - \lambda_1 |x_{2,ij}|^2 \right\}. \tag{13}$$

Now to find the values of $x_{2,ij}$ which maximize the objecting function (13), (2) is used, derivatives are taken with respect to $x_{2,ij}$ and the result is set equal to zero.

$$\frac{1}{2\pi\sigma_1^2} \frac{\alpha}{\sigma_1^2} \int_{\mathcal{B}_{\omega_{1,i}}} \left\{ (y_1 - x_{1,i} - \alpha x_{2,ij}) \exp\left(- \frac{1}{2\sigma_1^2} |y_1 - x_{1,i} - \alpha x_{2,ij}|^2 \right) \right\} \mathrm{d}y_1 = 2\lambda_1 |x_{2,ij}|. \tag{14}$$

Using (14) with the power constraint (7), we have a nonlinear system of equations with $M_1 \times M_2 + 1$ equations and the same number of unknown variables (λ_1 and $x_{2,ij}$). We suggest a fixed point iteration method for solving the system. Using an initial value for $x_{2,ij}$ we calculate the left hand side of (14). Current value of λ_1 is found using the power constraint (7) and current values of $x_{2,ij}$. Left hand side of (14) is divided by $2\lambda_1$ and current value for $x_{2,ij}$ is found. This algorithm is repeated until it converges. In general, the information messages of the CU (Ω_2) is independent of the PU messages (Ω_1). Thus, the transmitted signals of CU (X_2) in this scenario are only functions of PU's transmitted signals (X_1). CU in this scenario is selfless and designing a demodulator for it is meaningless. The symbol error probability of the PU in this case is a lower bound for any other case (one-dimensional case) where the CU is also available.

3.3 Considerate

None of the two previous scenarios can fulfill the coexistence conditions. Thus, *a proper combination of the Optimal Cancellation and Full Relay* must be used. Similar to the selfish scenario, in order to design the optimal modulator and demodulator jointly we split the procedure in two steps of designing the demodulator for a given modulator and vice versa. In this case, the performance of the CU should be maximized (minimizing the symbol probability of error). In addition to the power constraint for CU's transmission, another constraint must be added to the optimization to guarantee the performance of the PU's link. This new constraint can be formed by comparing the performance of the PU in absence of the CU with the case where the CU is also available. Therefore, the optimization can be written as

$$\begin{aligned} &\underset{x_{2,ij}\in\mathbb{C}}{\text{minimize}} \; P_e(\text{CU}) \\ &\text{subject to} \begin{cases} P_e(\text{PU}) = P_e(\text{Single PU}) \\ E|X_2|^2 \le P_{\text{CU}} \end{cases} \end{aligned} \tag{15}$$

The objective function which must be maximized is written using equations (10), (7) and two Lagrange multipliers λ_1 and λ_2 for including the CU's power constraint (7), and PU's performance constraint as

$$\begin{aligned} \frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} \Big\{ &\int_{\mathcal{B}_{\omega_{2,j}}} f_{Y_2|\Omega_2,X_1}(y_2|\omega_{2,j}, x_{1,i})\mathrm{d}y_2 \\ &- \lambda_1 \int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i}, x_{2,ij})\mathrm{d}y_1 - \lambda_2 |x_{2,ij}|^2 \Big\}. \end{aligned} \tag{16}$$

By taking derivatives of (16) in respect to $x_{2,ij}$ we have

$$\begin{aligned}
&\underbrace{\frac{\partial P_c(\mathrm{CU})}{\partial x_{2,ij}}}_{K_{ij}} - \lambda_1 \underbrace{\frac{\partial P_c(\mathrm{PU})}{\partial x_{2,ij}}}_{L_{ij}} - \lambda_2 \underbrace{\frac{\partial P_{\mathrm{CU}}}{\partial x_{2,ij}}}_{2x_{2,ij}} \qquad (17)\\
&= \frac{1}{2\pi\sigma_2^2}\frac{1}{\sigma_2^2}\int_{\mathcal{B}_{\omega_{2,j}}} \Big\{(y_2 - \beta x_{1,i} - x_{2,ij}) \exp\Big(-\frac{1}{2\sigma_2^2}|y_2 - \beta x_{1,i} - x_{2,ij}|^2\Big)\Big\} \mathrm{d}y_2\\
&- \lambda_1 \frac{1}{2\pi\sigma_1^2}\frac{\alpha}{\sigma_1^2}\int_{\mathcal{B}_{\omega_{1,i}}} \Big\{(y_1 - x_{1,i} - \alpha x_{2,ij}) \exp\Big(-\frac{1}{2\sigma_1^2}|y_1 - x_{1,i} - \alpha x_{2,ij}|^2\Big)\Big\} \mathrm{d}y_1\\
&- 2\lambda_2 |x_{2,ij}|.
\end{aligned}$$

Setting (17) equal to zero and using two discussed constraints, we have a system of $M_1 \times M_2 + 2$ nonlinear equations and the same number of unknown variables ($x_{2,ij}$, λ_1 and λ_2). The method of solving this nonlinear system of equations and designing the modulator and demodulator pair jointly is discussed in the next section. Exploiting the *considerate method*, the coexistence conditions of our uncoded cognitive radio channel can be fulfilled.

4 Implementation And Numerical Results

4.1 Implementation Of the Considerate Method

For the joint optimization of the modulator and demodulator of the CU, we have used a variation of the iterative method used in [8]. Setting (17) equal to zero, dividing both sides by $2\lambda_2$, and renaming $\frac{1}{2\lambda_2} \to \lambda_3$ and $\frac{-\lambda_1}{2\lambda_2} \to \lambda_4$ we have

$$\lambda_3 K_{ij} + \lambda_4 L_{ij} = x_{2,ij}. \qquad (18)$$

Solving (18) along with the constraints in (8) leads to the proper solution for this scenario. The two constraints can be written as

$$\begin{aligned}
\frac{1}{M_1 M_2}\sum_{i=1,j=1}^{M_1,M_2}\int_{\mathcal{B}_{\omega_{1,i}}} \Big\{\exp\Big(-\frac{1}{2\sigma_1^2}|y_1 - x_{1,i} - \alpha(\lambda_3 K_{ij} + \lambda_4 L_{ij})|^2\Big)\Big\}\mathrm{d}y_1 \qquad (19a)\\
= P_e(\text{Single PU}),
\end{aligned}$$

$$\frac{1}{M_1 M_2}\sum_{i=1,j=1}^{M_1,M_2} |\lambda_3 K_{ij} + \lambda_4 L_{ij}|^2 \le P_{\mathrm{CU}}. \qquad (19b)$$

Using the fixed point iteration and the definitions above we propose the following steps:

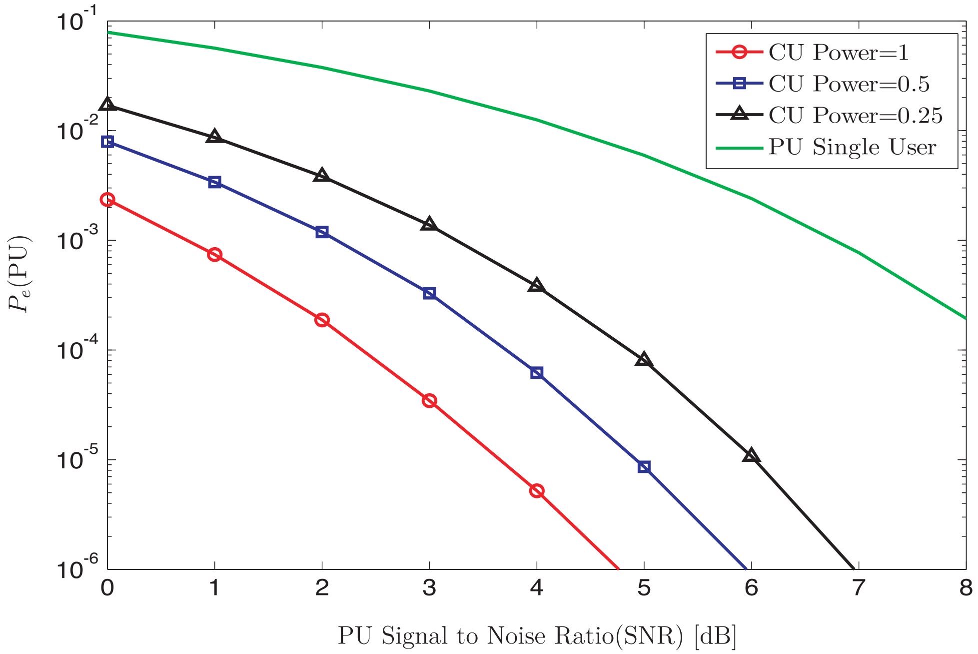

Figure 2: Performance of PU using the full relay method vs. PU SNR. $\alpha = 1$, $\beta = 1$ and Power of PU=1.

1. Start from a proper initial point $x_{2,ij}$ and its corresponding decision region $\mathcal{B}_{\omega_{2,j}}$. This can be, for example, the original constellation points and the decision regions of a single user case.

2. K_{ij} and L_{ij} are calculated using the current $x_{2,ij}$. Substituting these values in (19a) and (19b), a system of two nonlinear equations is constructed. In this system λ_3 and λ_4 are the unknown variables to be found. Another iterative method such as Newton's method is suggested for solving this system.

3. After solving the system (19a) and (19b), the left hand side of (18) is calculated using the current values of λ_3, λ_4, K_{ij} and L_{ij}. The result is the updated value of $x_{2,ij}$.

4. The decision regions $\mathcal{B}_{\omega_{2,j}}$ are updated using the new value of $x_{2,ij}$ and the likelihood function (11). If the difference of the current and the previous value of $x_{2,ij}$ is larger than a threshold we go to Step 2 and start another iteration with the current values. Otherwise, the algorithm is converged.

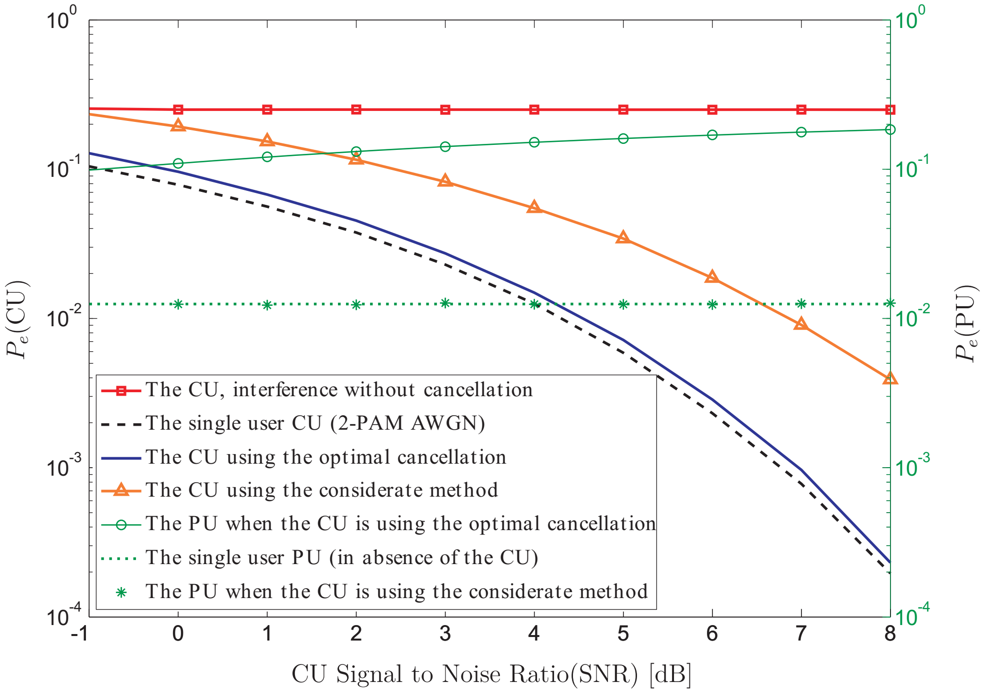

Figure 3: Performance of PU and CU in different scenarios vs. CU SNR. In all cases $\alpha = 1$, $\beta = 1$, Transmission Power of CU=1, Transmission Power of PU=1 and SNR of PU= 4 dB.

4.2 Numerical Results

The simulation setup and the results presented here are based on the system model discussed in Section II (Fig. 1). In our simulations, both PU and CU have two information messages ($M_1 = 2, M_2 = 2$). The PU uses binary Pulse Amplitude Modulation (2-PAM). In the full relay scenario, the CU also uses a two-point constellation corresponding to the PU's transmitted signals, regardless of its own information messages Ω_2. In the two other scenarios, the CU needs to use a four-point constellation corresponding to each combination of PU's transmitted signals X_1 and its own information messages Ω_2. The designed modulator and demodulator pairs of the discussed scenarios are evaluated for different values of signal and noise power in the PU and CU's links. The Monte Carlo simulation method is used to compute the performance of each case.

The performance evaluation results of the PU's link corresponding to the full relay scenario are illustrated in Fig. 2. The CU behaves as a relay and spends all of its transmission power to help the PU's link. It can be seen that the more power the CU is allowed to use; the better performance is achievable in the PU's link.

Fig. 3 compares the performance of CU in different scenarios. In addition,

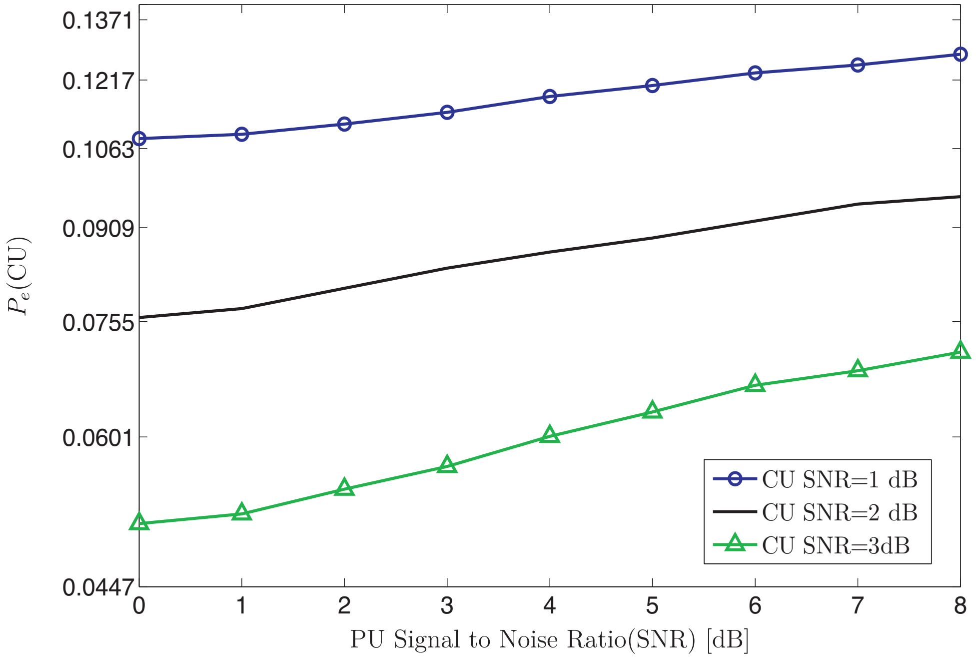

Figure 4: Performance of CU using the considerate method vs. PU SNR. $\alpha = 1$, $\beta = 1$, Transmission Power of CU=1 and Transmission Power of PU=1.

the effects of using the optimal cancellation method and considerate case on the performance of the PU link are shown in this figure. Here, the PU link has a constant SNR and consequently a certain symbol error probability. Using the optimal cancellation method, CU cancels out a large portion of interference and its symbol error probability is close to the case in which there is no interference. But as it is mentioned before, the performance of PU link is degraded and its probability of error is increased. It can be seen that the CU in *considerate* scenario performs much better than the interference case (interference without cancellation). On the other hand, the performance of the CU's link is degraded compared to the optimal cancellation case. However, this degradation is the result of the same symbol error probability for the PU's link before and after presence of the CU.

Fig. 4 depicts results of exploiting the *considerate* method for different SNRs of the PU's link (different P_e(Single PU)). Generally, all three curves in this figure show that increasing the SNR of the PU's link decreases the performance of the CU's link. Increasing the SNR of the PU is the same as improving its performance (decreasing the P_e(Single PU)). Therefore, the CU must care more about the PU's link compared to its own link. Thus, the selfless side of the method is dominant compared to the selfishness. Another effect that can be seen in Fig. 4 is the improved performance of the CU's link with increased SNR. This result was expected, and is the same in any other communication link.

5 Further Discussion

Observing the information of the primary user messages beforehand is an important issue. There are some practical solutions for this problem. For example, it can be assumed that the transmitters of the primary and cognitive user are two base stations which have a high capacity and instantaneous link between. As a result, the transmitted sequences of the primary user can be available for the cognitive user's transmitter in advance. Another scenario is assuming that the two transmitters are closer to each other physically compared to the distance between the transmitter and receiver of the primary user. In this case, generally the SNR of the wireless channel between the transmitters is more than the SNR of the link between the transmitter of the primary user and its receiver. Thus, the transmitter of cognitive user can decode the transmitted messages of the primary user in fewer channel uses, compared to what the primary user receiver needs for decoding. Therefore, cognitive user can listen to the primary user's link and after decoding a part of transmitted sequence acquires the upcoming part of it beforehand.

6 Conclusion

Three different scenarios for designing the modulator and demodulator of the cognitive user for an uncoded cognitive transmission (secondary user transmission) and their implementation methods have been studied in this paper. The *considerate method* is the most appropriate scheme which can fulfill the requirements of the real cognitive radio channels. Using this method, the cognitive user improves the performance of its own link as much as possible on the promise of no degradation on the quality of the primary user's link. Comparing the symbol error probability, it can be seen that the performance of the cognitive user is much better than the interference case. However, the cognitive user's performance is degraded compared to the optimal cancellation method. But as its presence is not harmful for primary user's communication, it can communicate in the same frequency band as the primary (licensed) user of the band. Note that this system is an uncoded cognitive radio channel. Therefore, without changing the method, it can be connected to an outer channel coding for increasing the performance of the cognitive user's link.

The approaches used in the considerate method -the symbol by symbol strategy for an uncoded channel and the constraint of symbol error probability of the primary user link- can be used as a low complexity practical solution for the secondary spectrum licensing and increase the spectral efficiency.

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Paper B

Uncoded Cognitive Transmission Over AWGN and Fading Channels

Mohammad Reza Khanzadi, Kasra Haghighi and Thomas Eriksson

Ready to submit.

Abstract

This paper proposes a new modulation method for an uncoded cognitive transmission (secondary user transmission) in presence of a Primary User (PU) for AWGN and time-varying flat-fading channels. Interference symbol of the PU is assumed to be known at the transmitter of the Cognitive User (CU) beforehand. Based on this knowledge and using a symbol by symbol approach, we design a CU modulation which can fulfill the coexistence conditions of the CU and the PU. The proposed method is a low-complexity modulation approach in a single (complex-valued) dimension rather than a high dimensional coding scheme, but still it achieves good performance. The robustness of the method is also investigated in case of having an imperfect knowledge about the PU transmitted symbols. An implementation algorithm for our modulation method is presented and its performance is evaluated by experiments.

Keywords: Cognitive Radio, Fading Channel, Interference Channel, Modulation, Uncoded Communication, Interference Avoidance, Imperfect Side Information.

1 Introduction

Cognitive radio [1] is recommended as an option for dynamic and secondary spectrum licensing to overcome the problem of overcrowded and insufficient licensed spectrum [2], [3].

In previous studies, different general techniques for cognitive transmission in presence of the primary (licensed) users have been introduced (e.g., the interweave and overlay techniques [4]). In the interweave technique, the cognitive user (CU) takes advantage of the vacant frequency holes in the spectrum of the primary user (PU). The CU exploits different spectrum sensing methods to find these unoccupied segments of the licensed spectrum of the PU and adapt its transmission to these free frequency bands [5]. On the contrary, in the overlay technique, the CU transmits its information in the same time and frequency as the PU. Having a pre-knowledge about the PU transmitted signals, the CU adapts its transmission to mitigate the interference introduced by the PU transmission while it does not degrade the performance of the PU communication link which is the owner of the licensed frequency band. In this paper our focus is on the overlay technique.

In several information-theoretical studies on the cognitive transmission using the overlay technique (e.g., [6] and [7]), a proper combination of the selfish [8] (dirty paper coding [9]) and selfless [8] scenarios (relay) is suggested in order to fulfill the coexistence conditions [7] of the CU and PU. The coexistence conditions of the cognitive transmission are as follow: Firstly, the PU is not aware of the presence of the CU. It has a fixed transmitter and receiver and is not capable of adapting to the CU's transmission. Secondly, the CU should not degrade the performance of the PU's link by introducing any harmful interference. Although these information-theoretical schemes introduce acceptable rates for coded cognitive radio channels, the infinite length of the codewords (infinite time intervals) and high dimensional coding make them complex for a practical implementation.

To reduce the complexity, a practical method of cognitive transmission in one dimension (a complex-valued dimension) for additive white gaussian noise (AWGN) channel is introduced in [10]. In this work, instead of using the whole sequence of known PU codeword (PU interference), a single transmitted symbol of the PU in each channel use is exploited to produce the transmitted signal of the CU. It is shown that this low complexity method for the uncoded cognitive transmission has a remarkable performance. In [10], the average symbol error probability is used as a measure for evaluating the performance of the CU link. Thus, to design the optimal modulator and demodulator pair of the CU, the demodulator must be redesigned in each round of the modulator optimization. In contrast, in this paper the mutual information [11] between the CU transmitted and received signals is used as the optimization criterion. As this mutual information is not dependent on the demodulation procedure, the demodulator is designed

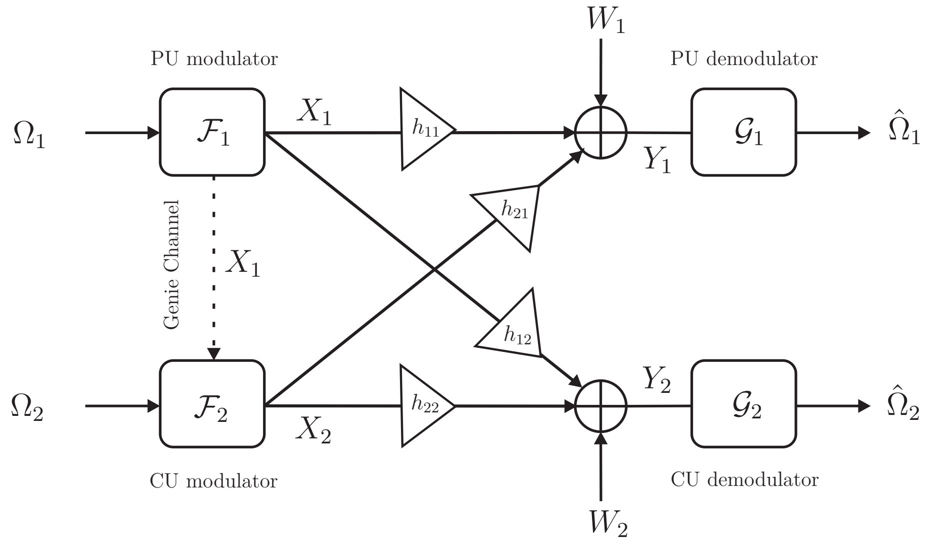

Figure 1: System Model.

once, after completing the design of the modulator, which makes the optimization less complex.

In addition, we propose a general framework to design the modulator and demodulator of the CU in time-varying flat-fading channels. The method of uncoded cognitive transmission in AWGN case is modified for the fading environment by means of a channel gain distribution quantization technique. The effect of using different power allocation policies on the performance of our method is also investigated in the fading case. Finally, the assumption of having the perfect knowledge about the PU transmitted symbols at the CU transmitter is relaxed and the performance of the method is restudied for this case.

2 Mathematical Formulation of The Model

Information messages of the PU are represented as a discrete random variable Ω_1, uniformly distributed over the set $\{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$. During each channel use, one of the realizations of the Ω_1 is transmitted. This message is modulated by the modulator function $\mathcal{F}_1 : \{\omega_{1,1}, \ldots, \omega_{1,M_1}\} \to X_1 \in \mathbb{C}$ of the PU. The output of $\mathcal{F}_1$ is the complex-valued transmitted signal of X_1. At the receiver, a complex Gaussian noise W_1, zero mean with variance equal to σ_1^2 is added to the X_1. The received signal $Y_1 = h_{11}\mathcal{F}_1(\Omega_1) + W_1 = h_{11}X_1 + W_1$ is demodulated by the demodulation function $\mathcal{G}_1 : Y_1 \in \mathbb{C} \to \{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$.

In our model, in which the PU has a fixed and non-adapting design, $\mathcal{F}_1$ and $\mathcal{G}_1$

are two fixed functions and cannot be adapted in presence of the CU. For the given demodulator of the PU, decision regions $\mathcal{B}_{\omega_{1,i}}$ are also fixed and can be defined as

$$\mathcal{B}_{\omega_{1,i}} = \{y_1 | \mathcal{G}_1(y_1) = \omega_{1,i}\}, \quad i = \{1, \dots, M_1\} \tag{1}$$

which is the set of received signals y_1 that results in the output $\omega_{1,i}$ of the demodulator function.

In presence of the CU, the received signal of the PU is $Y_1 = h_{11}\mathcal{F}_1(\Omega_1) + W_1 + h_{21}X_2 = h_{11}X_1 + W_1 + h_{21}X_2$ where X_2 is the complex-valued transmitted signal of the CU that will be introduced in more detail later. Assuming the complex Gaussian noise and additive channel, the conditional probability density function (pdf) of the received signal Y_1 given the Ω_1 and X_2 can be written as

$$f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i}, x_{2,ij}) = \frac{1}{2\pi\sigma_1^2} \exp\Big(- \frac{1}{2\sigma_1^2} |y_1 - h_{11}x_{1,i} - h_{21}x_{2,ij}|^2 \Big). \tag{2}$$

In the single PU case where the CU is not present, the average symbol error probability of the PU using the demodulation function $\mathcal{G}_1(Y_1) = \hat{\Omega}_1$ is equal to

$$P_e(\text{Single PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is not transmitted}). \tag{3}$$

In the presence of the cognitive user, the average symbol error probability is

$$P_e(\text{PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is transmitted}). \tag{4}$$

We assume that the transmitter of the CU is aware of the transmitted symbol of the PU in each channel use by means of a genie aided channel [8]. The receiver of the CU, however, is not aware of this message but only a posterior probability mass function (pmf) of the PU's modulation. The discrete random variable Ω_2 represents information messages of the CU and is defined uniformly over the set $\{\omega_{2,1}, \dots, \omega_{2,M_2}\}$. The modulator of the CU $\mathcal{F}_2 : \{\omega_{2,1}, \dots, \omega_{2,M_2}\} \times \mathbb{C} \to X_2 \in \mathbb{C}$ maps Ω_2 and the known transmitted signal from the PU (X_1) to the proper complex-valued signal X_2 which will be transmitted later. At the receiver of the CU, a complex Gaussian noise W_2 with mean zero and variance σ_2^2 is added to this signal. The received signal Y_2 is demodulated by demodulator function $\mathcal{G}_2 : Y_2 \in \mathbb{C} \to \{\omega_{2,1}, \dots, \omega_{2,M_2}\}$.

The received signal of the CU is $Y_2 = h_{22}\mathcal{F}_2(\Omega_2, X_1) + W_2 + h_{12}X_1 = h_{22}X_2 + W_2 + h_{12}X_1$. Based on the demodulation function $\mathcal{G}_2(Y_2) = \hat{\Omega}_2$, the average symbol error probability for the CU is $P_e(\text{CU}) = \Pr(\hat{\Omega}_2 \neq \Omega_2)$. The conditional pdf of the received signal Y_2 given the Ω_2 and X_1 is written as

$$f_{Y_2|\Omega_2,X_1}(y_2|\omega_{2,j}, x_{1,i}) = \frac{1}{2\pi\sigma_2^2} \exp\Big(- \frac{1}{2\sigma_2^2} |y_2 - h_{12}x_{1,i} - h_{22}x_{2,ij}|^2 \Big). \tag{5}$$

For the given demodulator of the CU, decision regions $\mathcal{B}_{\omega_{2,j}}$ can be defined as

$$\mathcal{B}_{\omega_{2,j}} = \{y_2 | \mathcal{G}_2(y_2) = \omega_{2,j}\}\,, \quad j = \{1, \ldots, M_2\}. \tag{6}$$

$\mathcal{B}_{\omega_{2,j}}$ is a set of received signals y_2, where $\omega_{2,j}$ is the result of the CU's demodulator.

Along with the definition of the cognitive radio as a wireless device which can sense and adapt its transmission to the environment [1], $\mathcal{F}_2$ and $\mathcal{G}_2$ (and decision regions $\mathcal{B}_{\omega_{2,j}}$) are not fixed and can be designed adaptively according to the requirements of the different scenarios.

3 Considerate Method

We want to design the optimal modulator and demodulator of the CU for the uncoded cognitive transmission to fulfill the coexistence conditions. The problem is formulated as an optimization in which the focus is on maximization of the performance of the CU link as well as avoiding the possible detrimental effects on the PU performance.

First, we design the modulator. The mutual information between the transmitted information message Ω_2 and the received signal Y_2 is used as a criterion for the CU link performance in the optimization. This mutual information $I(Y_2; \Omega_2)$ is used for a special case of symbol by symbol cancellation of the known interference in [12]. It is easy to show this mutual information is equal to the communication rate of the CU in this case.

As the CU is limited by its transmission power, we have a constraint on the power of its transmitted signal X_2.

$$\begin{aligned} E|X_2|^2 =& \frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} |\mathcal{F}_2(\omega_{2,j}, x_{1,i})|^2 \\ =& \frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} |x_{2,ij}|^2 \leq P_{\mathrm{CU}} \end{aligned} \tag{7}$$

where P_{CU} is the maximum acceptable CU transmission power.

In addition to the power constraint, another constraint must be added to the optimization in order to guarantee the performance of the PU link. This new constraint can be formed by comparing the performance of the PU in two cases of the absence and presence of the CU. The performance measure which we suggest for the PU communication link is its average symbol error probability.

Based on these definitions, the optimization for design of the modulator $\mathcal{F}_2$ can be written as

$$\begin{aligned}&\underset{x_{2,ij}\in\mathbb{C}}{\text{maximize}}\ I(Y_2;\Omega_2)\\&\text{subject to}\ \begin{cases}P_e(\text{PU})\leq P_e(\text{Single PU})\\E|X_2|^2\leq P_{\text{CU}}\end{cases}\end{aligned}\tag{8}$$

The $I(Y_2;\Omega_2)$ is calculated in (9) and the PU average symbol error probability in presence of the CU is also computed as (10).

$$\begin{aligned}&I(Y_2;\Omega_2)=H(\Omega_2)-H(\Omega_2|Y_2)\\&=\sum_{j=1}^{M_2}\int_{-\infty}^{\infty}f(y_2,\omega_{2,j})\text{log}p(\omega_{2,j}|y_2)\text{d}y_2-p(\omega_{2,j})\text{log}p(\omega_{2,j})\\&=\sum_{j=1}^{M_2}p(\omega_{2,j})\int_{-\infty}^{\infty}f(y_2|\omega_{2,j})\text{log}\frac{f(y_2|\omega_{2,j})}{f(y_2)}\text{d}y_2\\&=\sum_{i=1,j=1}^{M_1,M_2}\Big\{p(\omega_{1,i})p(\omega_{2,j})\int_{-\infty}^{\infty}f(y_2|\omega_{1,i},\omega_{2,j})\text{log}\frac{\sum_{k=1}^{M_1}f(y_2|\omega_{1,k},\omega_{2,j})p(\omega_{1,k})}{f(y_2)}\text{d}y_2\Big\}\\&=\frac{1}{M_1M_2}\sum_{i=1,j=1}^{M_1,M_2}\Big\{\int_{-\infty}^{\infty}f(y_2|\omega_{2,j},x_{1,i})\text{log}\frac{\frac{1}{M_1}\sum_{k=1}^{M_1}f(y_2|\omega_{2,j},x_{1,k})}{f(y_2)}\text{d}y_2\Big\}.\end{aligned}\tag{9}$$

$$\begin{aligned}P_e(\text{PU})&=1-\frac{1}{M_1}\sum_{i=1}^{M_1}\int_{\mathcal{B}_{\omega_{1,i}}}f_{Y_1|\Omega_1}(y_1|\omega_{1,i})\text{d}y_1\\&=1-\frac{1}{M_1M_2}\sum_{i=1,j=1}^{M_1,M_2}\int_{\mathcal{B}_{\omega_{1,i}}}f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i},x_{2,ij})\text{d}y_1.\end{aligned}\tag{10}$$

After solving this optimization, the optimal modulator $\mathcal{F}_2$ is given and the decision regions for the correct demodulation are defined based on the maximum likelihood rule (11).

$$\begin{aligned}\hat{\omega}_{2,j}=&\mathcal{G}_2(y_2)\\=&\underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\text{argmax}}f_{Y_2|\Omega_2}(y_2|\omega_{2,j})\\=&\underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\text{argmax}}\sum_{i=1}^{M_1}\Big\{\\&\exp\Big(-\frac{1}{2\sigma_2^2}|y_2-h_{12}x_{1,i}-h_{22}\mathcal{F}_2(\omega_{2,j},x_{1,i})|^2\Big)\Big\}.\end{aligned}\tag{11}$$

Now, the secondary transmission is just a lookup table. Based on the PU transmitted signal (message) and the CU transmitted information message, we look inside the designed modulator table and find the proper CU transmitted signal.

4 Considerate Method in The Fading Case

In this section, we propose a fairly general framework to design the modulator and demodulator of the CU in the time-varying flat-fading channels. Similar to the AWGN case, we maximize the performance of the CU link while the coexistence conditions are fulfilled. We assume that the distribution of each channel (h_{11}, h_{12}, h_{21} and h_{22}) is known for the CU. In addition, the CU is completely aware of the states of the channels during each channel use. These assumptions are provided by the CU capability to listen and observe the channel states. To design the CU modulator and demodulator, the channel gain distributions of all four independent channels are quantized. For example, the continuous channel gain $\gamma_{11} = |h_{11}|$ is quantized to K_{11} discrete samples γ_{11}^{i} using the quantization regions $[\gamma_{11}^{a}, \gamma_{11}^{b})_{i=1,\ldots,K_{11}}$. $\gamma_{11,i}^{a}$ and $\gamma_{11,i}^{b}$ are the boundaries of each quantization region and the probability of being in each region is defined as

$$\alpha_{11}^{i} = F(\gamma_{11,i}^{b}) - F(\gamma_{11,i}^{a}) \tag{12}$$

where $F(\gamma_{11})$ is the cumulative distribution function (cdf) of the h_{11} channel gain.

All four independent channel gain distributions ($\gamma_{11}, \gamma_{12}, \gamma_{21}$ and γ_{22}) are quantized that results in $K_{11} \times K_{12} \times K_{21} \times K_{22}$ independent combination of the quantized channel gains. The CU modulator and demodulator can be designed for each combination similar to the AWGN case. The constraint for respecting the PU link is its average symbol error probability over all of the quantized channel gain combinations. The CU performance criterion is the average mutual information $I(Y_2; \Omega_2)$ over all different combinations. In order to limit the transmission power of the CU, two different power constraints, namely short-term and long -term average power constraints [13] are used as follow:

4.1 Short-Term Average Power Constraint

Using the short-term average power constraint [13], there is a constant power limit (P_{CU}) on the transmission power of each combination ($P_{k,l,r,s}$) independently. In order to design the CU modulator in this case, the optimization problem is rewritten as (13). The designed modulator is again a lookup table. The CU can find a suitable transmitted signal from this table in each channel use, knowing the PU transmitted signal and the instantaneous channel gain values.

To design the demodulator, the CU forms the likelihood function (11) for each combination of the channel gain quantized values. The proper likelihood function in each channel use can be found based on the instantaneous channel gain values and the demodulation is done using the received signal.

$$
\begin{aligned}
&\underset{x_{2,ij}\in\mathbb{C}}{\text{maximize}} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (13)\\
&\sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^{k}\alpha_{12}^{l}\alpha_{21}^{r}\alpha_{22}^{s} I(Y_2;\Omega_2|\{|h_{11}|=\gamma_{11}^{k},|h_{12}|=\gamma_{12}^{l},|h_{21}|=\gamma_{21}^{r},|h_{22}|=\gamma_{22}^{s}\})\\
&\text{subject to } \{\\
&\qquad \sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^{k}\alpha_{12}^{l}\alpha_{21}^{r}\alpha_{22}^{s} P_e(\text{PU})|_{|h_{11}|=\gamma_{11}^{k},|h_{12}|=\gamma_{12}^{l},|h_{21}|=\gamma_{21}^{r},|h_{22}|=\gamma_{22}^{s}}\\
&\qquad\qquad\qquad\qquad\qquad\qquad \le \sum_{k=1}^{K_{11}} P_e(\text{Single PU})|_{|h_{11}|=\gamma_{11}^{k}},\\
&\qquad \text{and} \;\; P_{k,l,r,s} = E|X_2|^2 \le P_{\text{CU}}\}.
\end{aligned}
$$

4.2 Long-Term Average Power Constraint

Here, a long-term average power constraint strategy [13] is employed. In other words, instead of limiting the average power of each combination independently, the average transmission power over all combination of the quantized channel gain values is limited to P_{CU}. This power constraint can be written as

$$
\sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^{k}\alpha_{12}^{l}\alpha_{21}^{r}\alpha_{22}^{s} P_{k,l,r,s} \le P_{\text{CU}} \qquad (14)
$$

where $P_{k,l,r,s} \in \mathbb{R}^{+}$ is the proper transmission power of each combination which also must be found inside the optimization problem.

Hence, the CU can adapt its transmission power based on the channels condition. For instance, assume the h_{22} channel has a small value because of the fading. In this case, the CU transmits with small amount of power. On the other hand, when the value of the interference channel from the CU transmitter to the PU receiver (h_{21}) is small, the CU can transmit with more power without degrading the performance of the PU link. Since the channel values are time-varying, maximizing the CU performance using the long-term policy is a sort of "water-filling" power adaptation strategy [14]. This strategy results in spending the power in those combinations of the quantized channel gain values for which the CU link has a better performance. The modulator optimization is similar to (13) with a difference in the power constraint as (14). The demodulator is designed in the same way as the short-term average power case.

5 Implementation and Numerical Results

In this section, the implementation method of our uncoded cognitive transmission for the antipodal binary modulation (BPSK) case is investigated.

In this case, the PU has two information messages $\omega_{1,1} = 0$ and $\omega_{1,2} = 1$ ($M_1 = 2$) with the same probability of transmission. The PU transmission power is P_{PU} and its transmitted signals are $x_{1,1} = -\sqrt{P_{\text{PU}}}$ and $x_{1,2} = \sqrt{P_{\text{PU}}}$.

The CU also has two equal probable information messages $\omega_{2,1} = 0$ and $\omega_{2,2} = 1$ ($M_2 = 2$). The transmitted signals $x_{2,ij}$ must be found for different scenarios.

5.1 AWGN Case

Our method of implementation is stimulated by the optimal cancellation method of known interference in [12]. Under our assumptions of the BPSK case, there are four ($M_1 \times M_2 = 4$) different choices of $x_{2,ij}$ transmitted signals as below.

	$x_{1,1} = -\sqrt{P_{\text{PU}}}$	$x_{1,2} = \sqrt{P_{\text{PU}}}$
$\omega_{2,1} = 0$	$x_{2,11}$	$x_{2,21}$
$\omega_{2,2} = 1$	$x_{2,12}$	$x_{2,22}$

The probability densities of the $x_{1,i}$, $\omega_{2,j}$ and the white Gaussian noise are symmetric. Therefore, we have $x_{2,ij} \in \{-a, -b, a, b\}$ where a and b are positive real constants. First, a and b must be found. Then $x_{2,ij}$ must be mapped to the set $\{-a, -b, a, b\}$. As a and b are not ordered, there will be $\frac{4!}{2!} = 12$ possibilities for this mapping set.

For implementation of the optimization (8), first the real values between 0 to $\sqrt{P_{\text{CU}}}$ is quantized uniformly and a grid of possible values for the a and b is made. Then the optimization is done as follows:

- **Step 1:** Find all of the combinations of the grid points for a and b that can fulfil the power constraint (7) which can be rewritten as $\frac{a^2+b^2}{2} \leq P_{\text{CU}}$.
- **Step 2:** For the set of a and b found in Step 1, form the 12 possibilities of the set $\{-a, -b, a, b\}$.
- **Step 3:** Find all of the combinations from the result of Step 2 which can fulfil the constraint of average symbol error probability of the PU by calculating the $P_e(\text{PU})$ using (10) and comparing the result with the $P_e(\text{Single PU})$.
- **Step 4:** For the result set of the Step 3, the $I(Y_2; \Omega_2)$ is calculated using (9) and the set which can maximize this value is chosen as the proper transmitted signal of the CU ($x_{2,ij}$). The infinite integration inside (9) is computed numerically exploiting the Simpson's rule.

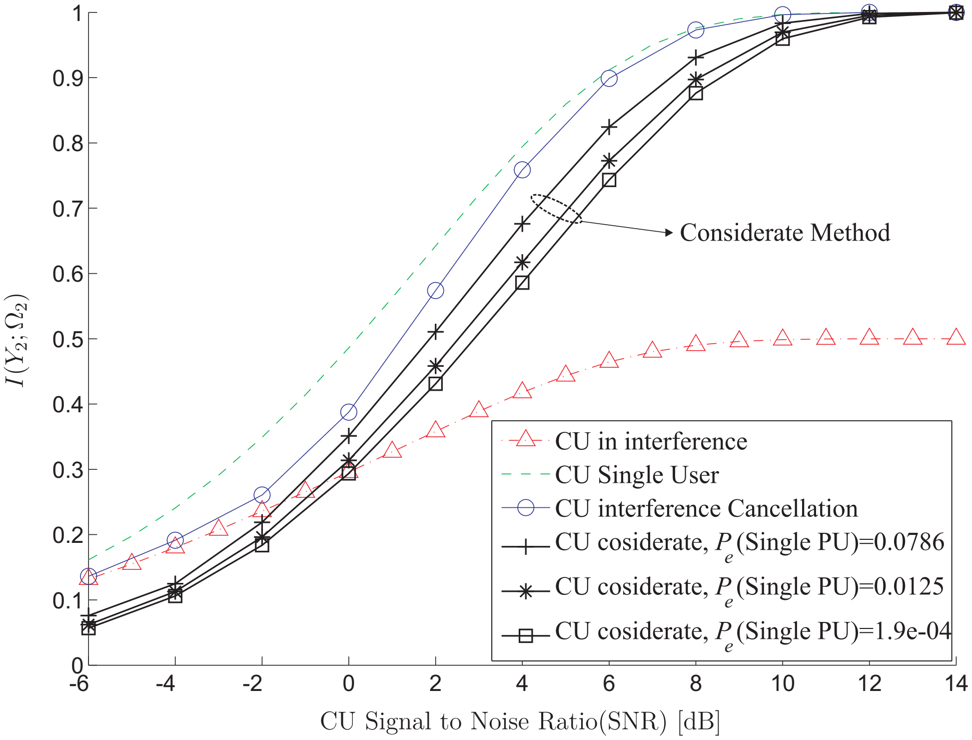

Figure 2: The CU performance in terms of mutual information vs. the CU SNR in different AWGN scenarios. Transmission power of the CU = 1, Transmission power of the PU = 1.

Result of using the CU considerate method is compared with the single user, the optimal interference cancellation [12] and the interference cases in Fig. 2. Fig. 3 shows the PU link performance for the different scenarios of the Fig. 2. The CU cancels out a large portion of the interference by using the optimal cancellation method [12] and its $I(Y_2;\Omega_2)$ is close to the no-interference (single user) case. But as it can be seen in Fig. 3, the PU link performance is degraded and its P_e is increased. The CU in considerate scenario performs much better than the interference case (interference without cancellation). On the other hand, its performance is degraded compared to the optimal cancellation case. However, this degradation is the result of the same symbol error probability for the PU link before and after presence of the CU (Fig. 3). Fig. 2 also depicts the effect of changing the PU link performance in the single user case on the CU performance in the considerate method. Improving the performance of the PU link (decreasing the P_e), the CU must care more about the PU link compared to its own link. As a result, the selfless side of the method is dominant compared to the selfishness and performance of the CU link is decreased.

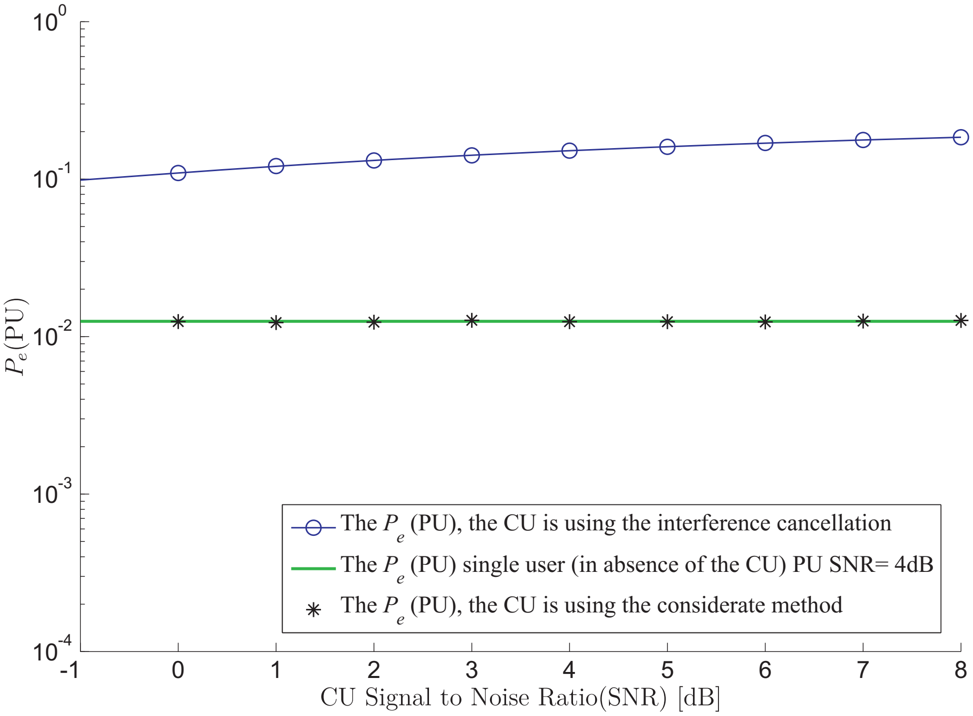

Figure 3: The PU performance in AWGN scenarios vs. CU SNR. PU SNR = 4 dB.

5.2 Fading Case with Short-Term Average Power Constraint

We assumed the Rayleigh distribution for each independent channel gain. To implement the optimization (13), first the channel gain distributions are quantized using two levels of quantization ($K_{11} = 2, K_{12} = 2, K_{21} = 2, K_{22} = 2$). The four step optimization method of the AWGN case is used independently for each of the sixteen combinations to fulfill the optimization criteria. The final performance measure is the average of $I(Y_2; \Omega_2)$ over all of the combinations. Fig. 4 compares the results of considerate method in the Rayleigh fading environment ($\sigma^2 = 0.1$) using the short-term average power constraint and the AWGN case. We also extend the results of the optimal cancellation method [12] to the fading case by means of our channel gain quantization method. The performance of the considerate method with fading is generally less than the AWGN case. But as it can be seen in this figure, the considerate method performance in the fading case is closer to the single user result compared to the AWGN case. As it is discussed before, this improvement is the result of fading in the interference channels (h_{12} and h_{21}) and the PU direct link (h_{11}).

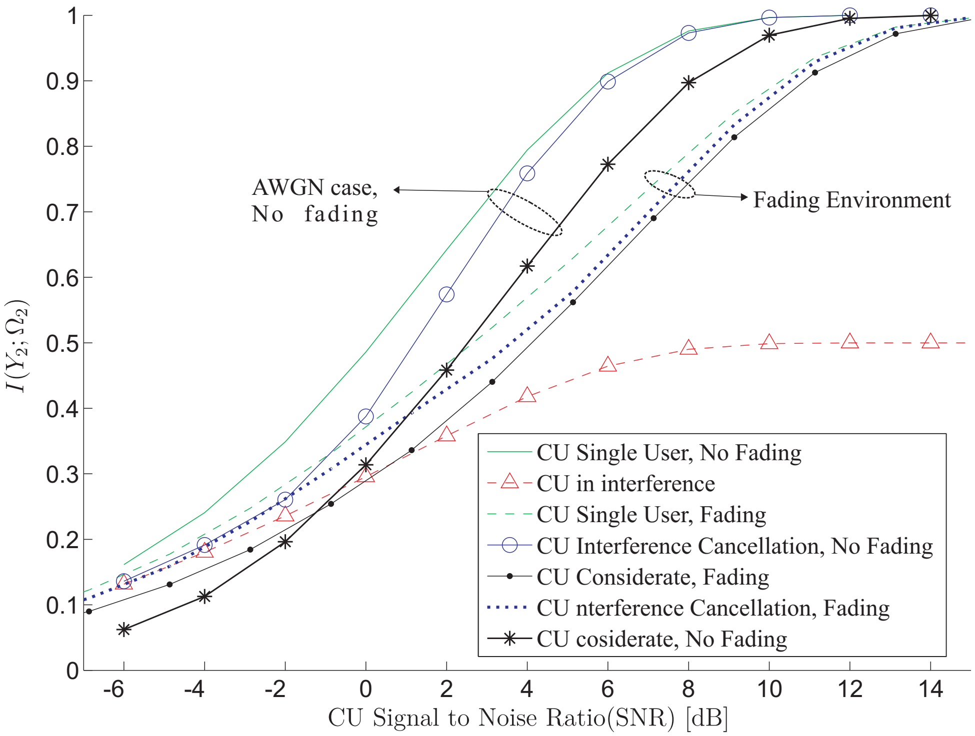

Figure 4: Comparing the performance of the CU in fading (short-term power policy) and AWGN cases. Transmission power of the CU = 1, transmission power of the PU = 1 and average SNR of the PU = 4 dB.

5.3 Fading Case with Long-Term Average Power Constraint

The transmission power of each combination is not limited to a constant value in the long-term average power policy and it must be optimized during the optimization. A vector of dynamic power constraints, each elements corresponds to one of the sixteen possible combinations is defined with the initial value of P_{CU}. A numerical gradient decent method with constraint over average power of all combinations is exploited to assign the optimal power to each combination. This power allocation method (water-filling) besides the procedure used in the short-term case implements the optimal CU modulator. Fig. 5 compares the results of the considerate method using the short-term and long-term average power policies in the fading environment. By using the long-term method (water-filling), the CU link performance is improved due to the wiser power allocation technique.

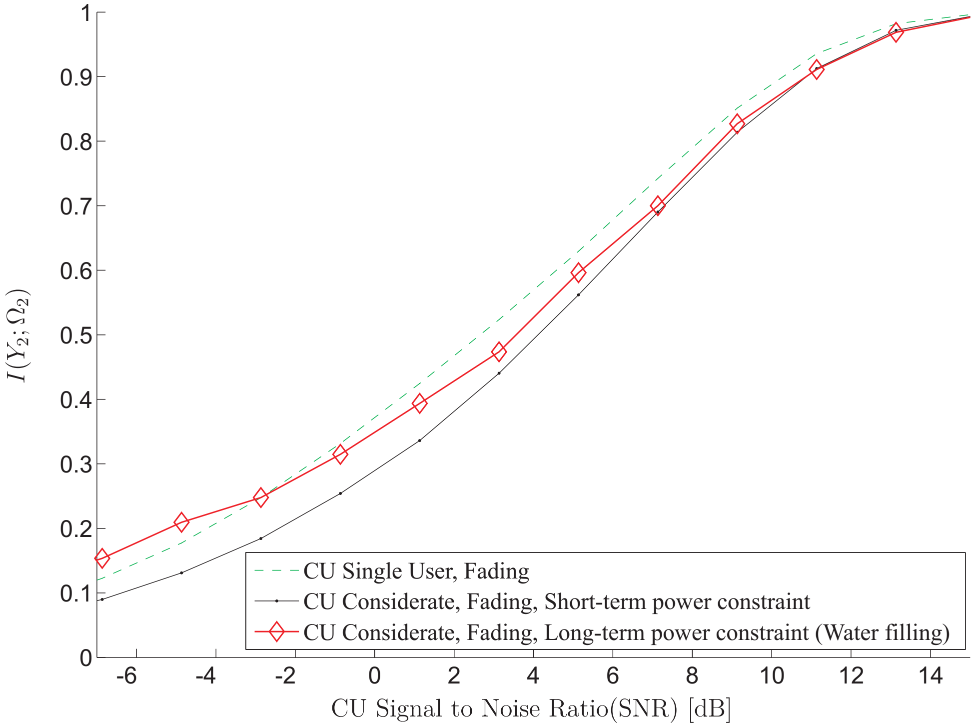

Figure 5: Comparing the performance of CU using short-term and long-term power policies in a fading environment. Transmission power of the CU = 1, Transmission power of the PU = 1.

6 Imperfect Knowledge of the PU Transmitted Symbols

In the previous sections, the CU was assumed to have a perfect knowledge about the PU transmitted symbols beforehand by means of a genie aided channel. Practically, it means that, we assumed an instantaneous ideal channel between the PU and CU transmitters. Due to the imperfections, a more realistic assumption is that the CU must detect the PU transmitted symbols through an AWGN channel, and transmit the proper signal based on this knowledge and using the designed modulator. Since this AWGN genie channel is noisy, the CU acquires imperfect knowledge of the PU transmitted symbols. Any error in detecting the PU symbols results in a wrong choice of CU transmitted signal. This will cause the performance of the CU to decrease as well as introducing harmful interference into the PU link, which is against the co-existence conditions. The PU link performance in the considerate method for the AWGN channel is evaluated vs. the quality of the genie channel in Fig. 6. There is a SNR value of the genie channel in which

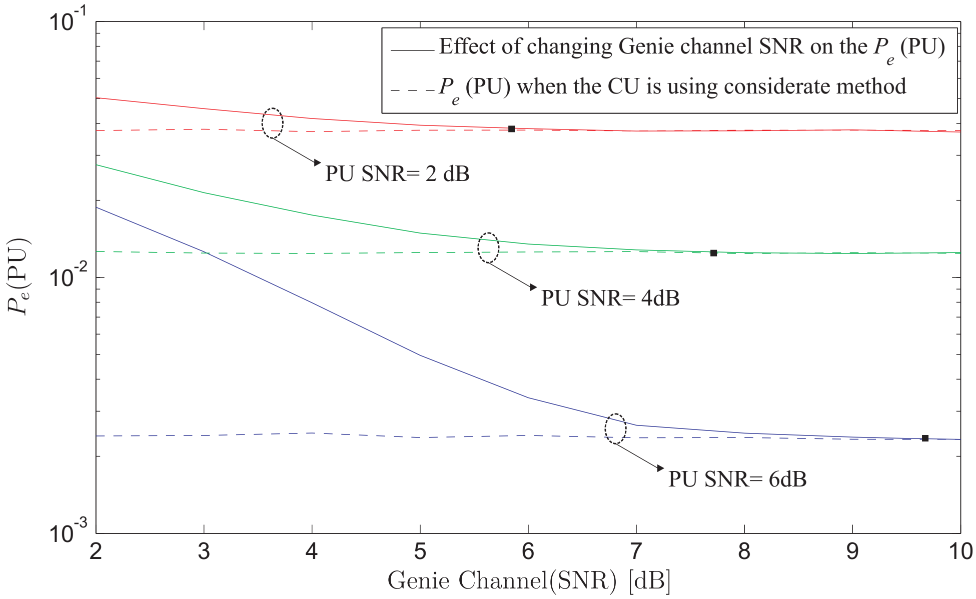

Figure 6: Effect of changing the genie channel quality (SNR) on the PU performance in AWGN case.

the quality of the PU link decreases from the perfect knowledge case. This genie channel SNR value is not the same for the PU links with different qualities. To be more specific, the quality of the PU link is a function of the difference between the quality of the PU link and the genie channel. In general, the simulations for the BPSK case show that if we have an imperfect genie channel with the SNR about 4dB higher than the direct PU link, the performance of the method is close to the case in which the CU has the perfect knowledge of the PU transmitted symbols. Broadly speaking, if we assume the path loss [14] as the only factor that decreases the received power, it can be concluded that the distance between the PU and CU transmitters must be less than 0.6 of the distance between the PU transmitter and receiver using the free-space path loss model [14].

7 Conclusion and Future Work

In this paper, methods of designing the optimal modulator and demodulator are proposed for the uncoded cognitive transmission in the AWGN and fading channels. Our numerical results show that the CU in this method achieves a notable performance without introducing any detrimental effect on the performance of the licensed user. Hence, it can communicate in the same frequency band as the primary (licensed) user. In the fading environment, the channel gain quantization is used to design the optimal CU modulator and demodulator. The long-term average transmitted power policy (water-filling) yields a better performance compared to the short-term strategy. The effect of having imperfect knowledge about the PU transmitted symbols on the performance of the method is also investigated.

In the case of the imperfect knowledge of the PU symbols, we neglect the important fact of delay in detecting the PU symbols in the transmitter of the CU. The methods to compensate the effects of such a delay can be investigated in the future studies. In the fading case, the performance can be improved by generalizing the method, for example, by taking the quantization regions as unknown parameters into account to be found inside the optimization. This method also has the potential to be extended to the higher number of symbols (higher dimensions) instead of the symbol by symbol strategy which can improve the performance of the CU link.

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