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Considering Wake Effects in a Mixed Integer Linear Programming Model for Optimizing Wind Farm Layout

Master's Thesis in Sustainable Energy Systems

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Abstract

In this thesis is an already existing mixed integer linear programming (MILP) model for designing wind power farms developed further. The development is focused on the wake effects, since those are not linear and most of the variables in the problem are due to wakes. The new models were tested on a potential site Moskogen, nearby Järpen in Jämtland, Sweden.

Mathematical optimization is a powerful tool, which unlike most used methods for designing wind power farms, controls that it actually have found the best solution for the stated problem. The largest drawback is that only some groups of mathematical optimization problems can be solved, and those mostly require a lot of time. MILP do have methods to find the best solution.

When designing wind power farms it is important to maximize the power extraction, at the same time as environmental and other restrictions are respected. In the MILP model is the maximum number of turbines restricted due to budget reasons and there are sound limits on some nearby positions.

Wind power turbines extract energy from and are obstacles for the wind, which means that they decrease the wind speed and increase the turbulence downwind. This change of speed and turbulence is called a wake. Measurements from existing farms and simulations show that for turbines standing close to other turbines the stress on the structure is substantial and the losses in productivity are large. Therefore a minimum distance is applied, which is calculated to 5 rotor diameters. From 5 to 12 rotor diameters the wake causes an extraction loss in power. Further away the extraction losses due to a wake are assumed negligible.

The first development of the MILP-model is that no wakes are accounted for if there is more than 12 rotor diameters between two turbines. Then the model were developed into three different modifications: i) The wake losses are not calculated, instead it has to be at least 12 rotor diameters between the turbines in dominant wind directions, ii) The wakes are only in constraints limiting the wake effect and the wakes are excluded from the objective function and iii) The wake effects are scaled so that the sum of all wakes might be more correct, but single wakes will be less correct.

Due to time limits on the project no tests were ran more than 24 hours. The standard model without wake losses gave the highest extraction, 16% higher than previous results, but also had a lower capacity factor. Model i) without wake losses and model ii) with the wake losses constrained to 5% of the extraction gave 9 and 3% higher extraction, respectively. The other models gave lower extraction than previous results, probably due to the time limit.

Sammanfattning

The following text is an extensive Swedish summary.

I examensarbetet utvecklades en redan existerande blandad heltals- och linjärprogrammeringsmodell som kan användas för att designa vindkraftsparker. Utvecklingen fokuserar främst på hur vakarna ska beräknas och beskrivas matematiskt. De nya modellerna testades på ett potentiellt område för vindkraft i Moskogen, som ligger utanför Järpen i Jämtland, Sverige. Modellerna testades med tre olika turbiner: Nordex N117 3,0MW, Siemens SWT-3,0-113 och Vestas V112 3,075MW.

När vindkraftsparker designas är det viktigt att utvinna så mycket el som möjligt för att maximera inkomsterna och samhällsnyttan, samtidigt som det måste tas hänsyn till miljön, budget, regler och människor i närheten. I det här avsnittet gås det igenom några av de viktigaste sakerna att ta hänsyn till samt om de ingår i den matematiska modellerna eller inte.

För några platser i närområdet sattes en ljudgräns på 35 dB, vilket är gränsvärdet för så kallade ljudkänsliga områden. Tidigare har gränsen 40 dB använts. Den möjliga påverkan på natur och kultur har studerats i miljökonsekvensbeskrivningen, men då vissa av faktorerna är väldigt subjektiva, t.ex. vindkraftparkens utseende, är det svårt att bestämma ett hårt värde av dem som implementeras i modellen. Av hänsyn till naturupplevelsen för boende och turister som vandrar i fjällen lades Hottögsfjället till bland de ljudkänsliga platserna. Andra saker, t.ex. buller under byggfasen och snöröjning, ligger utanför ramarna för examensarbetet.

Som ett enkelt mått på lönsamheten hos parken används kapacitetsfaktorn, som är den genomsnittliga årliga effekten dividerat med den installerade kapaciteten. Den totala kapaciteten ska vara under 130 MW, vilket motsvarar maximalt 43 eller 42 turbiner, beroende på turbinleverantör.

När vinden går igenom en turbin, kommer en del av den kinetiska energin i vinden omvandlas till mekanisk energi i rotorn och turbinen är också ett hinder som vinden måste ta sig runt. Därför kommer turbinen att minska vindhastigheten och öka turbulensen, vilket kallas för turbinens vak.

Mätningarna från existerande parker har visat att belastningen på turbiner som står i en annan turbins vak kan vara betydligt högre än belastningen på turbiner som inte står i en vak. Den förhöjda belastningen kan förkorta livslängden på turbinen. Dessutom är förlusterna i produktion stora när turbinerna står nära varandra. Därför bestäms ett minimiavstånd för att minska både belastningar och stora förluster i produktion. Minimiavståndet beräknades till 4,96 rotordiametrar, vilket avrundades till 5 rotordiametrar.

Om turbiner står längre från varandra än minimiavståndet, kommer de att orsaka varandra produktionsförluster på grund av fartreduceringen av vinden i vaken. Efter 12 rotordiametrar är fartreduceringen från en vak försumbar enligt mätningar från vindkraftsparker. Det finns flera modeller för att beräkna fartreduceringen i vaken, här användes Jensens modell, som är snabb och välprövad. Den totala fartreduceringen av flera vakar fås fram genom att dra kvadratroten ur summan av kvadraten på de enskilda vakarnas fartreducering, vilket skapar problem i optimeringsmodellerna.

Matematisk optimering är ett kraftfullt verktyg som, till skillnad från de vanligaste metoderna idag för att designa vindkraftsparker, verifierar att det är den bästa lösningen som har hittats. Nackdelarna är att bara vissa grupper av matematisk optimering har lösningsmetoder och i regel kräver dessa mycket tid för att hitta den bästa lösningen. I det här projektet användes blandad heltals- och linjärprogrammering (MILP), vilket är en gren inom matematisk optimering som har lösningsmetoder.

Alla optimeringsproblem har formen att en målfunktion ska maximeras eller minimeras under särskilda bivillkor. Målfunktionen berättar vad som är syftet med optimeringen, t.ex. kan det vara att hitta kortaste vägen eller den mest lönsamma investeringen. Bivillkoren säger vad som måste vara uppfyllt eller vad som inte är tillåtet i lösningen.

I de matematiska modellerna i det här projektet måste målfunktionen och alla bivillkor vara linjära, annars slutar problemet att vara ett MILP-problem och därmed troligen inte tillhöra en grupp med matematiska optimeringsproblem som har lösningsmetoder. Som det stod tidigare i sammanfattningen, fås den totala fartreducering genom att dra kvadratroten ur summan av kvadraten på de enskilda vakarnas fartreducering, vilket inte är ett linjär samband. Modellerna kommer därmed inte beskriva vakförlusterna korrekt, när flera vakar samverkar.

Eftersom lösningsmetoderna för MILP-problem är relativt långsamma, körs de i regel inte tills det är helt säkert att den bästa lösningen är hittad. Istället används ett mått kallat det relativa gapet (mätt i %) för att uppskatta hur nära det är till den bästa lösningen. När det relativa gapet är 0% så har garanterat den bästa lösningen hittats. Den första utvecklingen som görs av modellen är att inte ta med förluster i produktionen när det är mer än 12 rotordiametrar mellan turbinerna. Sen skapas tre olika modifieringar av denna modell. Alla modeller syftar till att ha olika tillvägagångssätt på vakarna.

Målfunktionen i standardmodellen ska maximera den totala produktionen minus alla vakförluster. Första bivillkoret är att det maximala antalet turbiner är 42 respektive 43 turbiner. Andra bivillkoret är att ljudet inte får överstiga 35 dB på bestämda ljudkänsliga positioner. Tredje bivillkoret är att turbinerna inte får stå närmare varandra än 5 rotordiametrar. Fjärde bivillkoret får vakarna att existera mellan de turbiner som står varandra närmare än 12 rotordiametrar.

Det blåser olika mycket i olika riktningar i Moskogen. Över en längre period, t.ex. ett år, kommer den totala vakförlusten påverkas starkt av hur mycket det blåser i en viss riktning. I den här modellen sätts minimiavståndet till 12 rotor diametrar i de sex mest dominanta vindriktningarna, utav totalt tolv. I övriga riktningar behålls minimiavståndet 5 rotordiametrar. Målfunktionen i vindrosmodellen ska maximera den totala produktionen. Första bivillkoret är att det maximala antalet turbiner är 42 respektive 43 turbiner. Andra bivillkoret är att ljudet inte får överstiga 35 dB på bestämda ljudkänsliga positioner. Tredje bivillkoret är att turbinerna inte får stå närmare varandra än 12 rotordiametrar i dominanta vindriktningar, annars 5 rotordiametrar. Den här modellen testas både utan och med vakförluster i målfunktionen.

För att få ner antalet variabler och få en modell som automatiskt undviker stora vakförluster, omformulerades bivillkoren om vakarna så att vakförlusterna kunde tas bort från målfunktionen. Målfunktionen i modellen med vakförluster bara i bivillkor ska maximera den totala produktionen. Första bivillkoret är att det maximala antalet turbiner är 42 respektive 43 turbiner. Andra bivillkoret är att ljudet inte får överstiga 35 dB på bestämda ljudkänsliga positioner. Tredje bivillkoret är att turbinerna inte får stå närmare varandra än 5 rotordiametrar. Fjärde bivillkoret är att vakförlusterna som en turbin har inte får överstiga en viss andel av dess produktion. Femte bivillkoret är att vakförlusterna som en turbin skapar inte får överstiga en viss andel av dess produktion.

I standardmodellen blir vakförlusterna överskattade. För att få de totala valförlusterna att bli mer korrekta, multipliceras varje enskild vakförlust med en koefficient mindre än 1. Koefficienten bestäms till $1/\sqrt{2}$. Målfunktionen i modellen med skalade vakförluster ska maximera den totala produktionen minus alla vakförluster, där vakförlusterna är multiplicerade med $1/\sqrt{2}$. Första bivillkoret är att det maximala antalet turbiner är 42 respektive 43 turbiner. Andra bivillkoret är att ljudet inte får överstiga 35 dB på bestämda ljudkänsliga positioner. Tredje bivillkoret är att turbinerna inte får stå närmare varandra än 5 rotordiametrar. Fjärde bivillkoret får vakarna att existera mellan de turbiner som står varandra närmare än 12 rotordiametrar. (Bivillkoren i den här modellen är samma som för standardmodellen.)

Den av modellerna som gav högst produktion var standardmodellen när inga vakförluster togs med i målfunktionen. Den modellen gav cirka 16 % mer el än tidigare designer, på bekostnad av en sänkt kapacitetsfaktor. Den av modellerna som gav näst högst produktion var vindrosmodellen när inga vakförluster togs med i målfunktionen. Den modellen gav cirka 9 % mer el än tidigare designer, vilket delvis kan förklaras med att den ställde ut fler turbiner. Utöver den gav även modellen med vakförlusterna bara i bivillkor högre produktion när vakförlusterna fick vara max 5% av produktion, cirka 3% mer än tidigare designer. För både de fallen var kapacitetsfaktorn ungefär densamma som för tidigare designer. Övriga modeller gav lägre produktion, men hade också ett väldigt stort relativt gap när körningarna var klara.

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Contents

Acronyms

CF	Capacity factor
LP	Linear Programming
MCP	Model-Predict-Correlate
MILP	Mixed Integer Linear Programming
SL	Sound Level
SP	Sound Pressure

Nomenclature

Latin Symbols

ΔCF	Relative difference in capacity factor between previous designs and the result from the MILP-model
ΔP	Relative difference in power extraction between previous designs and the result from the MILP-model
Δu	Velocity deficit due to one wake
ΔU	Velocity deficit due to several wakes
a	Sound pressure
a_{max}	Maximum allowed sound pressure
A	The sweep area of the rotor blades
CF	Capacity factor
C_P	Power coefficient
C_T	Thrust coefficient
d	Direction of the wind
E	The entire set of edges, in the graph representing the site
E_m	The set of all edges which are shorter than αR
$E_{m,i}$	The set of all edges connected to i which are shorter than αR
E_w	The set of all edges which are longer than αR and shorter than $12R$
f	Weibull probability density function
g	Objective value of an optimization problem
I	Total installed capacity
k	Wake decay constant
L_n	Approximate length of the near wake
m	Total number of non-negligible wakes
$\dot{m}_{wind}$	Mass speed of the wind
n	Number of wind power turbines
N	Maximum number of wind power turbines
p_i	Maximum power extraction in position i
P	Power extraction from the wind power farm
P_0	Power extraction when the wakes are unaccounted for
q_{ij}	Wake losses in position i due to a wake from a turbine in j
Q	Total wake losses in a wind power farm
R	Rotor diameter

R_w	Width of a wake
s	Sound sensitive location
S	Set of all sound sensitive locations
T	Thrust force
u	Wind speed in a wake
U	Ambient wind speed
$\overline{U}$	Average wind speed
V	Set of all nodes
w_{ij}	Is 1 if there is a wake between node i and j, otherwise 0
x	Variables in a MILP-problem
X	Total number of possible positions on site

Greek Symbols

α	Minimum distance between two turbines expressed as number of wind turbines
γ	Maximum share of the power extraction that may be wake losses
Γ	The gamma function
θ	Probability that the wind blows in a certain direction
κ	The shape parameter in a Weibull distribution
λ	The scale parameter in a Weibull distribution
ρ	Density (of the wind)
τ	Term of diffused turbulence
χ	Distance downstream the turbine divided by the rotor diameter of the turbine
ψ	The wind speed on site which is unaffected by the turbines divided by the wind speed in a wake

Subscripts

b	Bad wind conditions; lower average wind speed than an average year
d	Direction of the wind
i	Position where it is possible to put a wind turbine
j	Position where it is possible to put a wind turbine
LP	Optimal solution of a linear programming problem
s	Sound sensitive location

1

Introduction

Wind power is a renewable source of electricity. The basic idea is to extract kinetic energy from the wind and convert it into useful energy. Historically it has been mechanical work, e.g. running mills, but today it is mostly to produce electricity [1]. Wind power has grown markedly in Sweden in the last years. 724 MW was installed in Sweden during 2013, the fourth largest annual installations in Europe, giving a total capacity of 4 470 MW in the end of 2013 [2]. There are several advantages with wind power: it is renewable, it does not rely on imported or expensive fuel and it does not have any emissions except when it is built.

Howver, one of the far most important reasons for investing in wind power is the profitability. The choice of site for wind power farms is important to increase the power extraction and to reduce the costs for building the wind power farm, but it is also important to find a good design for the wind power farm to make as much use as possible of the selected site. At the same time the environmental and societal impacts should be limited.

In 2010 Patrik Fagerfjäll [3] wrote a master thesis where he developed a Mixed Integer Linear Programming (MILP) model to get a layout of a wind power farm that was optimal with respect to maximal power extraction. It considered budget limitations as well as sound limits. Fagerfjäll's model resulted in 5-10% more power than methods in commercial software and in an extreme case up to 40% more. One of the issues that Fagerfjäll pointed out with the MILP-model is that linear superposition of several wakes overestimates the total wake effect, which might lead to that the design found actually is not the optimal one [3]. A wake is an area downwind the turbine with decreased wind speed and increased turbulence compared to ambient wind conditions.

Historically MILP models have not been widely used since they typically require a lot of computional time and space. Instead so called heuristic methods is often used, which gives a good but maybe not the best solution and requires much less computional effort [4]. The capacity of computers is growing, making MILP models more appealing for large applications.

1.1 Aim of the Thesis

The aim of this thesis is to develop Fagerfjäll's MILP-model, mainly with respect to the wakes. The developed model shall be tested on a site in Moskogen and compared to previous designs.

1.2 The Test Site

The company JP Vind AB has a possible site for wind power outside Järpen called Moskogen, see figure 1.1. There already exists one possible layout with about 40 wind power turbines for the park [5]. JP Vind wishes an alternative suggestion, which will be provided within this master thesis.

Figure 1.1: The location of Moskogen in the region Jämtland in Sweden. Figure from [5].

1.3 Problem Description

There are two main concerns when using mixed integer linear programming for designing wind power farms. One is that MILP problems require lots of computional time and space, and a reduction of this might make the method more attractive for industrial applications. The other one is that considering the total effect of several wakes cannot be both accurate and linear. If the wakes are described accurately, the problem will not be linear anymore, which might mean that it is not even possible to find an optimal solution. Linearity will instead overestimate the total wake effects.

1.4 Project Boundaries

This project has a general perspective on an entire wind power farm. It does not look into individual turbines, e.g. if they can be yawed to reduce losses. Neither does this project look into how the farm might affect the electrical grid in entire Jämtland, Sweden or Europe.

The perspective is also limited to when the wind power farm is running. In an entire project it is necassary to also consider the restoration and the impacts during the construction period.

1.5 Structure of the Report

The report begins with facts about wind power turbines and farms, including wake effects and continues with describing optimization and other mathematical tools. Then are the MILP models described before the results are presented, compared and discussed. Chapter by chapter is the structure of the report as follows:

Chapter 2 introduces the planning of wind power farms; what has to be considered when the turbines positions are decided. The information in this and the next chapters is the basis for the MILP-model.

Chapter 3 outlines the theory and models for wake effects in wind power farms. The chapter starts closest to the turbine with the near wake, continues with the far wake and ends with how several wakes in a wind power farm interact.

Chapter 4 explains the mathematical tools used when designing wind power farms: wind analysis, mixed integer linear optimization (MILP) and graph representation. Heuristic methods used is also explained. To explain optmization a simple example is given.

Chapter 5 presents the MILP-models used within this project and possible alterations to make the model easier to use. There is one standard model and three models with modifications.

Chapter 6 presents the results from testing the models presented in chapter 5.

Chapter 7 discusses and compares the results. There are also recommendations on future work.

2

Wind Farms

Wind power is a renewable and low-emitting source of electricity, nevertheless it has other impacts on nature. When wind power turbines are positioned these impacts have to be respected at the same time as the companies building them want a profit and the societal benefits shall be maximized.

In this chapter some of the more important factors will be presented. A Swedish perspective will be applied, since the location of the possible site is in Sweden.

2.1 Properties of the Wind

Since the wind has a mass and a velocity, it has kinetic energy. The power in the wind can be expressed as

$$P_{wind} = \frac{\dot{m}_{wind} U^2}{2}, \tag{2.1}$$

where $\dot{m}_{wind}$ is the mass flow of the wind and U is the speed of the wind. The mass flow of the wind going through a wind turbine with the sweep area A can be expressed as $\dot{m}_{wind} = \rho A U$, and then is

$$P_{wind} = \frac{\rho A U^3}{2}. \tag{2.2}$$

The share of the power in the wind that the turbine can extract is denoted the power coefficient C_P so the power extracted by a turbine is [6], [1]

$$P_{turbine} = C_P \frac{\rho A U^3}{2}. \tag{2.3}$$

The power that is extracted by the wind power turbine is thus proportional to U^3. This is relevant, since it means that when calculating the average power extraction of a site the mean wind speed cannot be used due to $\sum U^3 \neq (\sum U)^3$. $P_{turbine}$ needs to be calculated through summing up over an interval of relevant wind speeds. Many sites have a Weibull distribution of the wind speed in a long term perspective [7],[1]. The Weibull probability distribution is defined by its its scale parameter λ and shape parameter κ; two examples can be seen in figure 2.1. The power extracted from a wind power farm is thus highly dependent on the wind conditions.

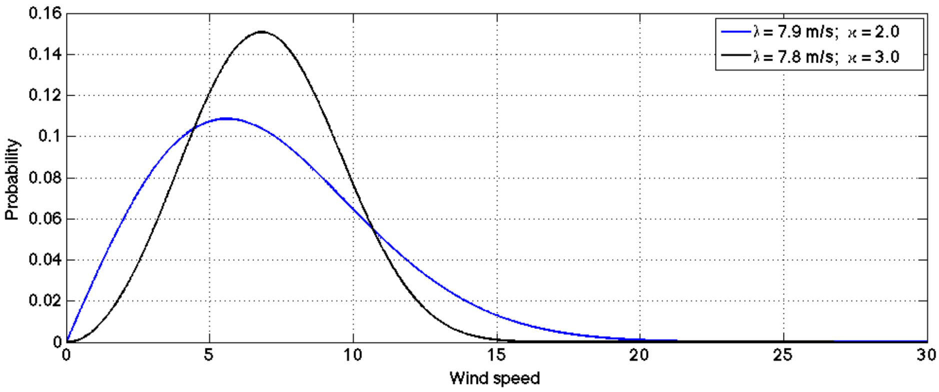

Figure 2.1: Two different Weibull distributions, both with the mean wind speed 7.0 m/s.

Most wind farms have an expected life time of about 20 years, which makes it interesting to approximately know the wind conditions as many years into the future when building a wind power farm [8]. To estimate the wind conditions the wind speed is measured with an anemometer to find the distrubution of the wind speeds. The recommended time periods for wind meauserments varies from 1 to 30 years [7]. The measurements are then going through long term correction, to be more reliable for forecasting. More about long term correction can be found in chapter 4.

For the turbine to start, there needs to be enough kinetic energy in the wind. The lowest wind speed when a wind turbine runs is denoted cut-in speed. When the wind speed is too high there is a risk that the wind damages the wind turbine. Therefore wind turbines are stopped at the so called cut-off speed. [1] The MILP models were tested with three different turbines: Nordex N117 3.0MW [9], Siemens SWT-3.0-113 [10],[11],[12] and Vestas V112 3.075MW [13],[14]. All of these turbines have a cut-in speed at 3 m/s and a cut-off speed at 25 m/s.

2.2 Budget of Wind Power Projects

It is approximated that between 70 and 80 % of the costs for a wind power farm are the turbines [15]. The maximum number of turbines is therefore largely an economical question, but can also be a request from the landowners, the municipality or the operator of the electrical grid. For Moskogen the limit for maximum installed capacity is

130 MW, because a higher capacity would require large investment in the electrical grid. The suggested turbines have a capacity of 3.0 or 3.075 MW, so the maximum number of turbines is 43 and 42 turbines, respectively.

The income from the park will be from the sold electricity while the turbines are a large part of the costs, so a common measurement for the profability of an investment in a power plant or farm is the **capacity factor**, defined as:

$$CF = \frac{P}{I} \tag{2.4}$$

where P is the total annual average power output from the farm (in Watt) and I is the total installed capacity (in Watt).

2.3 Sound Levels

The sound from wind power has the same frequency as wind that goes through leaves. It can still create stress, since this sound has a rhythm. Modern wind turbines have a sound emission of approximately 100 dB and it decreases the further it comes from its source. Formulas for how the sound changes depending on the distance to one or several wind power turbines are provided by the Swedish Naturvårdsverket [16]. In living areas the sound immission must not exceed 40 dB and in sound sensitive areas it must not exceed 35 dB. What a sound sensitive area is, is not exactly defined. As a comparison, refrigators normally have a sound emission of 35-40 dB. [16], [1]

Since the site lies close to areas where nature is important both for the community and local businesses [5], the limit 35 dB is choosen for nearby houses. When the calculations are compared to measurements, there might be an error of up to 1 dB [16]. Therefore 34 dB is implemented as constraint in the model. In previous design 40 dB has been used as a limit.

None of the above includes the noise when a wind farm eventually is built, which is considered to be outside of the scope of this thesis.

2.4 Nature and Culture

A well-known part of the impacts from wind power on nature, is that wind turbines have effects on bats and birds. When planning a wind power farm, there have to be searches for bats and birds and analysis of what they do on the site. Previously the distance between the site and identified bats and birds nest has been found sufficient [5].

In north and middle of Sweden reindeer herding is a very important part of the nature and culture, especially for the Sami people. This has been taken into account during the work with this site, e.g. agreements on which time of the year the farm should be built, on snow removal and communication between JP Vind and Sami. [5] None of this concerns can be directly implemented in the mathematical models in this thesis.

The site is in the same municipality as several popular winter- and ski-resorts as well as popular places for hiking. The ski-industry can be neglected, since the wind farm could only been seen from the largest ski-hill during certain wheather conditions, and is then very hard to spot, see figure 2.2. There are still tourist destinations from where the wind turbines are easily spotted. [5] It is hard to say if and how the wind turbines will affect the tourism, and therefore it is not included as a constraint or otherwise in the mathematical model in this thesis.

There is one popular hill for hiking, Hottögsfjället, about 5.5 km from the edge of the site [5]. The distance is so far that even a cluster of turbines is unlikely to cause sound immission above 34 dB, but to be safe the hill is included in the sound-sensitive locations and the sound constraints. Figure 2.3 shows a photomontage, where the photo is taken on Hottögsfjället.

2.5 Turbine Interference

The turbines have to be so far away from each other that their rotor blads do not collide with each other, so there has to be more than one rotor diameter between the towers of the turbines. The turbines do however also affect each other through the wind.

Wind power turbines extract energy from and are obstacles for the wind, which means that they decrease the wind speed and increase the turbulence downwind the turbine [15], [17]. This change of speed and turbulence is called a wake. If turbines are closer than 10-12 rotor diameters to each other they may affect each other through the wakes. If the turbines interfer each other depends on which direction the wind blows. Therefore when wind measurements are done, see chapter 2.1, is not only the speed but also the direction of the wind measured. [17],[1]

Calculating and taking the wake effects into considerations is one of the largest challenges of this project. Therefore is the entire next chapter dedicated to turbine interference and wake effects.

Figure 2.2: A photomontage of the wind farm from a very popular mountain for skiing. The turbines can be spotted inside the red circle. Figure from [5].

Figure 2.3: A photomontage of the wind farm from the nearby mountain Hottögsfjället, where hiking is popular. Hottögsfjället is about 5.5 km from the edge of the site. Figure from [5].

3

Wake Effects

When the wind goes through the wind power turbine, a part of the kinetic energy is converted into mechanical energy, which reduces the wind speed downwind. The turbine is also a roughness element in the terrain, so it increases the turbulence. This reduced velocity and increased turbulence nearby the turbine is called the wake of the turbine.

For wind power turbines, the wake is often divided into the **near wake** and the **far wake**. How far behind the turbine the near wake reaches is dependent on several factors. The transition region, i.e. when it goes from near wake to far wake, is often approximated to 2-5 rotor diameters behind the turbine [18], [19], [20], [17]. After 10-12 rotor diameters the velocity deficits of one wake are negligible [17], [1].

This chapter starts with some theoretical background about wakes, then continues with going through charachteristics and consequences both of the near wake region and the far wake region. Some calculations are also done for the conditions in Moskogen and with the turbines used in the mathematical model. At the end is it about how several wakes interact in a wind power farm.

3.1 Theoretical Background

When the wind reaches the rotor blade the wind will act both as a lift force and as a drag force on the rotor blade, where the lift force is perpendicular to the ambient velocity and directed upward while the drag force is parallell to the ambient velocity and directed downwind. As a reactant force to the drag force, will there be a thrust force, resulting from the pressure drop over the rotor. The thrust force has an upwind direction, see figure 3.1. [6] The thrust force T is

$$T = \frac{1}{2} C_T \rho U^2 A, \tag{3.1}$$

where C_T is the thrust coefficient, ρ the density of the wind, U the ambient wind speed and A is the sweep area of the rotor blades. Theoritically, the wind speed u after the wind turbine is [15]

$$\frac{u}{U} = \sqrt{1 - C_T}, \tag{3.2}$$

where u is the wind speed in the wake.

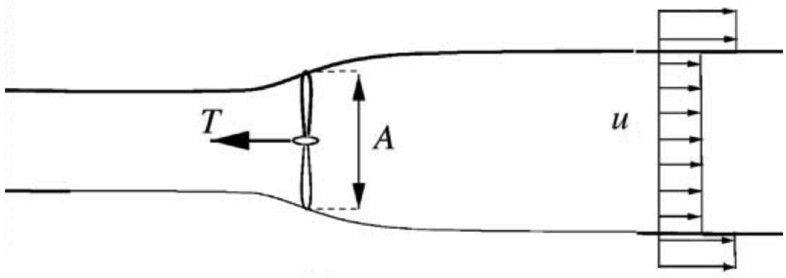

Figure 3.1: Illustration of a wake. T is the thrust force, A is the sweep area and u is the speed in the wake. Figure from [6].

3.2 Near Wake

Close to the wind turbine the wind velocity field is strongly dependent on the rotor blades. There is a lot of turbulence and the velocity field is relatively difficult to calculate. Measurements from some existing wind power farms shows that up to 8 rotor diameters behind the turbine can the increase in fatigue load on the wind turbine behind be 10-80%, while others can see no such increase if the distance is larger than 7 rotor diameters [20]. A too high load on the rotors can shorten their life expectancy [6]. Simulations made by [21] concludes that increasing the distance between two turbines from 3 to 5 rotor diameters gives an significant increase in power extraction, while the increase from 5 to 7 rotor diameters gives a much smaller increase in power extraction. [21] suggests a minimum distance of 5 rotor diameters.

3.2.1 The Abramovich Model

The length of the near wake is dependent on the velocity, turbulence and the rotor. A model developed by Abramovich can be used to calculate an approximate length of the near wake. Here are the formulas from Kiranoudis et.al. [22], where the near wake is denoted as the length of ”the initial constant potential core of conical shape region”, see region (1) in figure 3.2. The velocity u directly behind the wind turbine is calculated with the formula

$$\left(\frac{u}{U}\right)^3 + \left(\frac{u}{U}\right)^2 - \frac{u}{U} + (2C_P - 1) = 0, \tag{3.3}$$

where C_P is the power coefficient. If $\psi = U/u$ then is the length of the near wake

$$L_n = \Big(\frac{\sqrt{35}}{3} - 1\Big)\frac{R}{2\sqrt{\big(\frac{\tau}{0.51}\big)^2 + \big(\frac{0.22(\psi-1)^2}{\psi^2-4\psi+1}\big)^2}}, \tag{3.4}$$

where τ is the term of diffused turbulence, which is set to $\tau = 0.05$ [22].

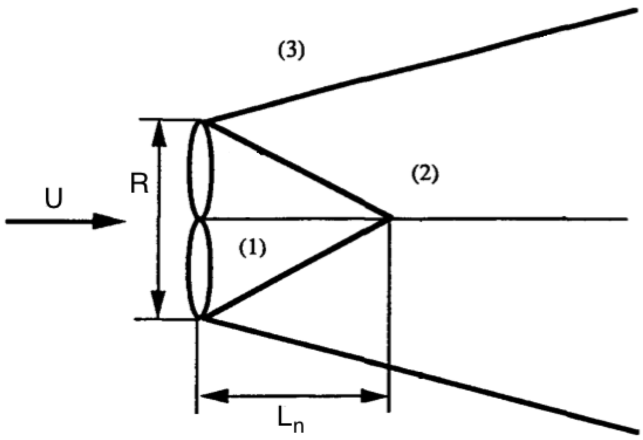

Figure 3.2: The wake region divided into sections according to Abramovich's wake model. (1) is the initial core of the wake. (2) is the region where the velocity is gradually approaches the ambient velocity. (3) is the ambient region. Modified version of figure from [22].

3.3 Far Wake

In the far wake the turbines affect each other through reduced wind speed, which reduces the power extraction. The optimal design of a wind power farm migh still include wind power turbines standing in the wake region of other turbines.

Wake models for far wakes are often divided into explicit models (also known as kinematic models) and implicit models (also known as field models). Explicit models assume a self-similar velocity deficit in the wake, while implicit models calculate (simplified) velocity fields in the wake, either two- or three-dimensional. Implicit models requires substiantally more computer capacity than explicit models. [20], [17]

In the implementation of the mathematical models only one far wake model will be used, due to time limits on this project. The Jensen model is chosen for its simplicity, speed and good accuracy. Examples of other well-spread wake models are the Larsen model and the Ainslie model [23].

3.3.1 The Jensen Model

The Jensen model is an explicit model and also one of the oldest models for wakes downwind wind turbines. The Jensen model was developed with the purpuse of being used for planning wind farms. It assumes a linear expansion of the wake. The width of the wake according to the Jensen model is [24]

$$R_w = R + 2kx, \tag{3.5}$$

where R is the rotor diameter, k is the wake decay constant and x is the distance downwind the turbine. The wake decay constant is dependent on turbulence, both ambient and turbine induced, and athmospheric stability [24]. It is often chosen to 0.075 for on-shore wind farms [15], [23].

The velocity in the wake is assumed to depend on the distance from the turbine and that directly behind the turbine the velocity deficit is according to equation 3.2 [15]. It does not consider elevation differences or if the turbine is close to the edge of the wake or in the middle of it. The velocity in the wake according to the Jensen model is

$$\frac{u}{U} = 1 - \frac{1 - \sqrt{1 - C_T}}{(1 + 2k\chi)^2}, \tag{3.6}$$

where $\chi = x/R$. For this formula to work, it is necessary that $C_T \leq 1$, which is the case for all turbines used within this project, regardless of wind speed. Despite the simplicity for the model and the limitation for C_T it is proven to correspond well with reality, both in wind tunnels and in wind farms [24], [23]. In figure 3.3 the velocity deficit for three different and relevant thrust coefficient is shown.

3.4 Wake Modelling in Wind Power Farms

In models which calculates or optimizes power extraction in wind farms the interaction between several wakes is highly relevant. Calculating the interaction between several wakes is denoted far wake modelling or wind farm modelling.

The best known approximation on how the velocity deficit from several wakes interact, is the square root of a linear superposition of the wakes squared [20], [24], [17]

$$\Delta U = \sqrt{\sum_i \Delta u_i^2}, \tag{3.7}$$

where ΔU is the velocity deficit due to wake effects in one direction and Δu_i is the individual velocity deficit due to one wake which lays in the same direction. However,

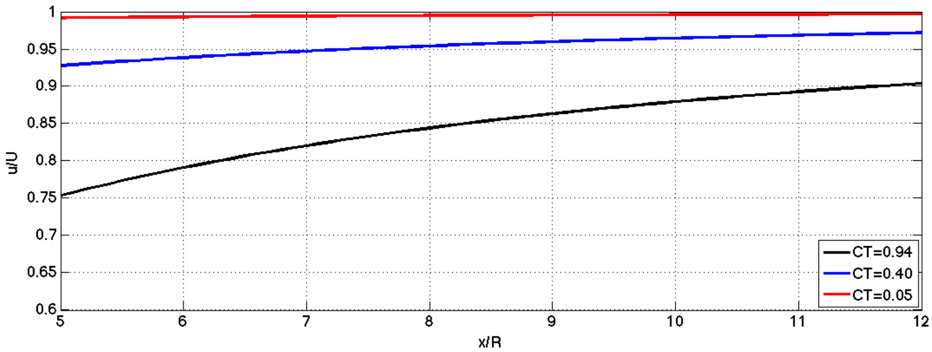

Figure 3.3: Velocity deficit according to the Jensen model [24] for relevant but different thrust coefficient. $C_T = 0.94$ is the highest thrust coefficient for all turbines and wind speeds, and is for Nordex right above cut-in wind speed. $C_T = 0.40$ is the average thrust coefficient for the turbines used in this thesis and wind speeds between 3 and 25 m/s. $C_T = 0.05$ is the thrust coefficient for the turbines used in this thesis right before cut-off speed.

in this project a linear model is used whereas linear superposition is used to summarize the wakes

$$\Delta U = \sum_i \Delta u_i. \tag{3.8}$$

This will overestimate the wake effects if there is more than two turbines in a row [17]. In extreme cases the total velocity deficit may become unrealistically large, e.g. larger than the ambient wind speed U. According to Vermeer et.al. [17] is there no (known) physical reason why equation 3.7 gives a good approximation, but according to Katic et.al. [24] it is derived from the conservation of the kinetic energy of the wind.

An example of the square-relationsship can be seen in figure 3.4, which shows how the wind speed changes in a row of turbines. If the velocity deficit of several wakes could be calculated through linear superposition, as equation 3.8, the lines would have been straight. Now equation 3.7 describes the situation better, which gives the lines their curved shape. The velocity deficit reaches an equilibrium after 3-4 turbines [24], which also can be seen in figure 3.4.

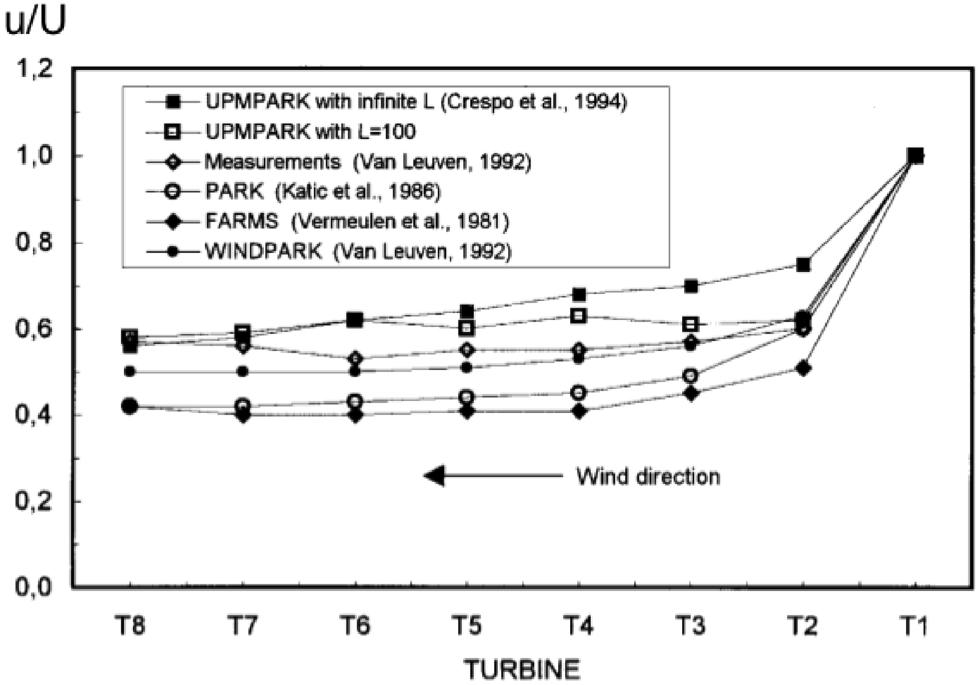

Figure 3.4: The velocity deficit in a row of turbines. The wind comes from the right side of the figure, passes through the first turbine T1, which reduces the wind velocity, then the wind continues through turbine T2 where the velocity is reduced further and so on. The different lines are for different models and measurements. Figure from [20].

4

The Mathematical Tools

Several mathematical tools are used in this thesis, whereof the most central ones are explained in this chapter. The chapter begins with wind analysis, an important part of the pre-processing of the optimization models, and then continues with graph representation, which is used to build the basic representation of the models. After that comes the main theme: mathematical optimization, focusing on mixed integer and linear optimization. Mathematical optimization is a powerful tool that sometimes is difficult to implement. Heuristics and the widely used methods today for designing wind farms are also explained.

4.1 Wind Analysis

The power extracted from a wind power farm is highly dependent on the wind conditions. Most wind farms have an expected life time of about 20 years, which makes it interesting to know the wind conditions many years into the future when building a farm [8]. The methods for predicting the wind conditions on site are often divided into physical or statistical methods, but can be used in combination [7],[1].

Physical models use knowledge about how the wind behave around rough surface, slopes and obstacles to determine the wind conditions on site. Firstly they need wind measurements, which preferably should have been processed with a long term correction, i.e. a statistical model. [1]

The statistical methods in use are often so called Model-Correlate-Predict (MCP) models. MCP requires wind data from the site, measured during a short period of time, which is then correlated to a nearby long term measurement (e.g. from a wheater station) and then is the wind data from the site adjusted so it can give predictions into the future, see figure 4.1. MCP models requires no or very little information about the terrain, which physical models need. It is however important that the terrain and surface roughness are similar at the site and the reference station, when using MCP. Despite these simplifications, MCP models tend to give slightly better predictions than physical models, if they are used seperately [7]. From year to year will the wind still change, which makes it hard to predict how much power that can be extracted.

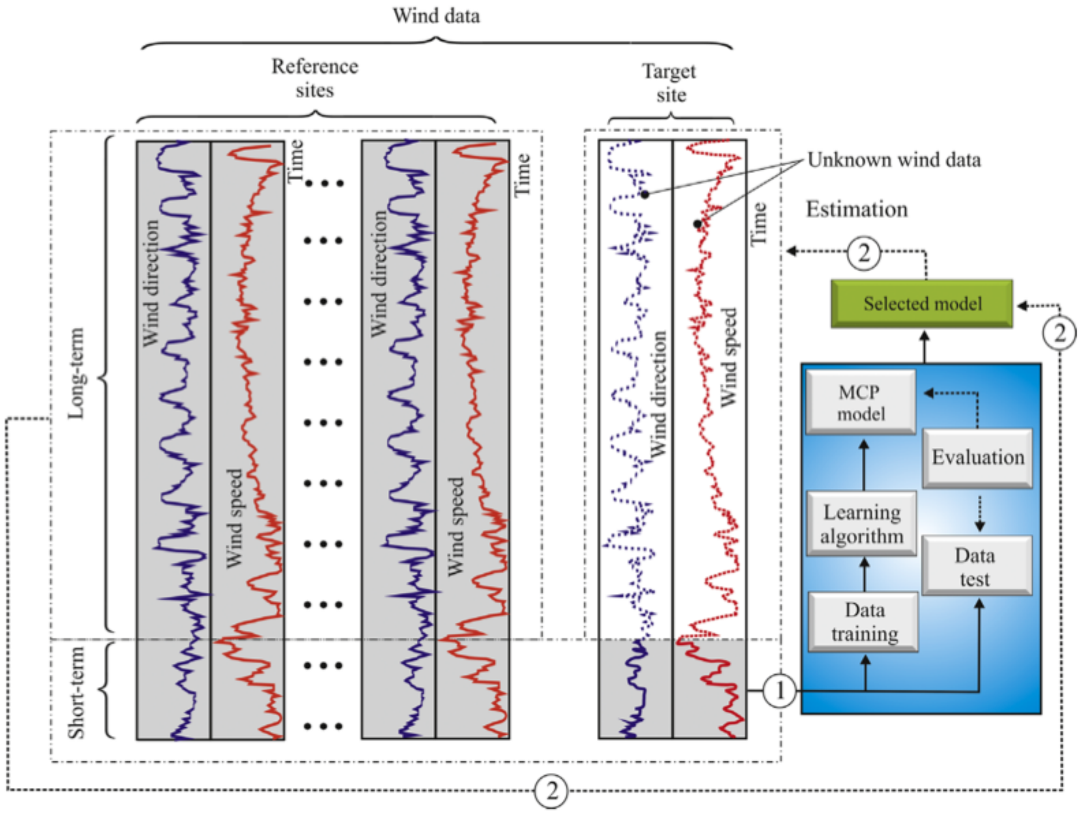

Figure 4.1: A graphic explanation on how MCP (Model-Correlate-Predict) models work. Figure from [7].

4.2 Graph Representation

A graph $G = (V, E)$ is a set of nodes or vertices V, that are linked together through directed or undirected edges E. A simple example is train stations linked together through rail tracks. The train stations can be represented through nodes and the rail tracks through edges. If train station i and j are nodes in the graph and are connected, then $i \in V$, $j \in V$ and $(i,j) \in E$. The edges can have weights representing ie.g. the time or the cost to travel that edge. If the graphs have undirected edges, then $(i,j) = (j,i) \forall i,j$ and the eventual weights of (i,j) are the same as for (j,i). Otherwise the graph has directed edges. Graph representation when designing wind power farms has previous been used by e.g. [25] and [3].

4.3 Mathematical Optimization

Mathematical optimization is when an objective function is minimized or maxmimized, sometimes with constraints. The constraints tell what is allowed or what has to be fullfilled in the solution. Almost everything can be formulated as an optimization problem, but only a few classes of optimization problems have methods to find a solution and guaranteeing it is the best one. In this thesis mixed integer and linear programming (MILP) is used, where there exists methods to find the maximum or minimum of the objective function. In industrial applications for designing wind power farms heuristic methods are often used, which is to find a good solution without verification that it is the best one. [4]

An Example

Here a very simple example of optimization is presented. The example is first solved as a linear programming problem and then as an integer linear programming problem.

Kim wants to sell homemade gloves and caps in a nearby town market and earn as much money as possible. Information about the gloves and caps can be found in the table 4.1. Kim has a bag that carries up to 8 kg and holds up to 12 litre.

Table 4.1: Information about the optimization example.

	Gloves	Caps
Profit per piece (SEK)	50	40
Weight per piece (kg)	0.25	0.15
Volume per piece (litre)	0.3	0.4

4.3.1 Linear Programming

The objective is to maximize the profit with the constraints that it is possible to carry up to 8 kg and 12 litre, respectively. Thus this is a maxmization problem and not a minimization problem. Mathematically this can be written as:

$$\begin{aligned} &\text{maximize } g = 50x_1 + 40x_2 \\ &\text{subject to} \\ &\quad 0.25x_1 + 0.15x_2 \leq 8, \\ &\quad 0.3x_1 + 0.4x_2 \leq 12, \\ &\quad x_1 \geq 0, x_2 \geq 0, \end{aligned} \tag{4.1}$$

where g is the total profit, x_1 is the number of gloves and x_2 is the number of caps. The first constraint $0.25x_1 + 0.15x_2 \leq 8$ says that the total weight may not exceed 8 kg. The second constraint $0.3x_1 + 0.4x_2 \leq 12$ says that the total volume may not exceed 12 litre. The last constraint says that the gloves and caps cannot be negative. This is a linear optimization problem, denoted **linear programming**, since both the objective function and all the constraints are linear. The problem can be drawn as in figure 4.2.

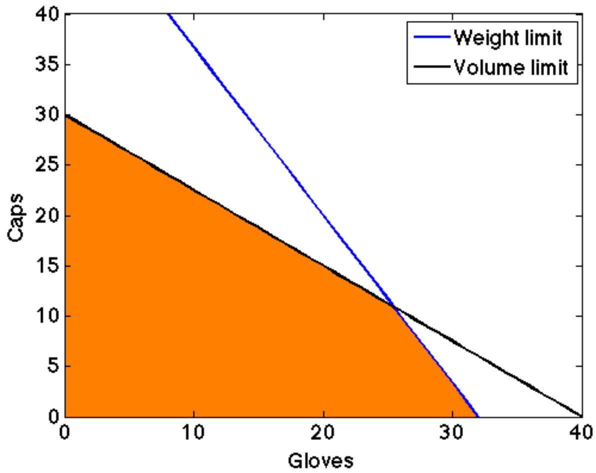

Figure 4.2: A picture illustrating the LP-problem. The orange area are all the feasible solutions.

It is known from the fundamental theorem of linear programming [4] that for this kind of problem the best solution is in an extreme point. In figure 4.2 the corners are the extreme points. Extreme points in two dimensional problems are limited by at least two constraints, e.g. at the extreme point at the top of the picture is both $x_2 = 0$ and $0.3x_1 + 0.4x_2 = 12$. For such a small problem as this, the easiest method to solve the problem can be to calculate the value of g in each corner and then compare the values to see when g is at its maximum.

For larger problems one extreme point is found, then the neighbouring extreme point is searched which potentially can have the largest increase (or decrease for a minimization problem). If there is no neighbouring corners where g will increase (or decrease) has the optimal solution been found. This is called the simplex method. For this problem the optimal solution is $x_1 = 25\frac{5}{11}$, $x_2 = 10\frac{10}{11}$, which gives $g = 1709\frac{1}{11}$.

4.3.2 Integer Linear Programming

Kim cannot sell only a part of a glove or a cap. Therefore a constraint is added which says that x_1 and x_2 have to be positive integers:

$$\begin{aligned}
&\text{maximize } g = 50x_1 + 40x_2 \\
&\text{subject to} \\
&\qquad 0.25x_1 + 0.15x_2 \leq 8, \\
&\qquad 0.3x_1 + 0.4x_2 \leq 12, \\
&\qquad x_1 \in \mathbb{N}, x_2 \in \mathbb{N}.
\end{aligned} \tag{4.2}$$

Due to the added constraints, this is now a Integer Linear Programming (ILP) problem. The problem can be drawn as in figure 4.3. If some variable would have been integer and some continuous it would have been a Mixed Integer Linear Programming (MILP) problem.

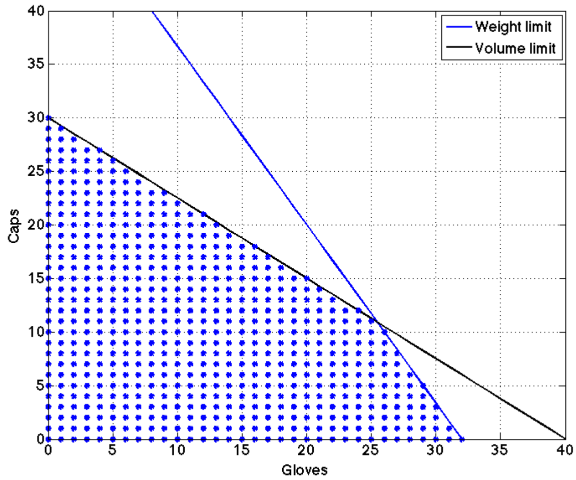

Figure 4.3: A picture illustrating the ILP-problem. The dots are the feasible solutions.

There are different methods for solving MILP-problems. The method used later is Branch-and-Bound with LP-relaxation. Step by step the following is done:

1. Treat all variables as continuous and find the solution of the LP-problem.
2. Choose one of the variables x_i that is not an integer in the LP-solution.
3. Branch the problem into two problems:

   i) in one of the branches add the constraint $x_i \leq \lfloor x_{LP,i} \rfloor$,

   ii) in the other branch add the constraint $x_i \geq \lceil x_{LP,i} \rceil$.

4. Find the solution of the LP-problem in each branch.

5. Keep on branching until all variables are integers.

From chapter 4.3.1 is it known that the solution of the LP-problem is $x_1 = 25\frac{5}{11}$ and $x_2 = 10\frac{10}{11}$. From this solution an integer solution can be found.

The search tree for this problem can be seen in figure 4.4. In the first step is x_2 chosen as the branching variable. In the branch to the left the constraint $x_2 \leq 10$ is added. This gives the solution $x_1 = 26$ and $x_2 = 10$. Since all variables are integers no further branching is needed from that branch. In the right branch the constraint $x_2 \geq 11$ is added. The solution to that LP-problem is $x_2 = 11$ and $x_1 = 25\frac{1}{3}$. Since x_1 is not an integer, further branching is needed. The added constraints $x_2 \geq 11$ and $x_1 \geq 26$ cannot be fullfilled without violating the weight- and volume constraints. This branch is infeasible. However, the branch with the added constraint $x_1 \leq 25$ is feasible, and the branching continues there. In the end three integer solutions are found, and the best one of those is $x_1 = 26$ and $x_2 = 10$, which gives $g = 1700$. That is the solution of the MILP-problem.

How difficult a MILP problem is depends on the number of integer variables and the structure of the problem. The structure is e.g. how close the extreme points of the LP-relaxed problem is to the feasible integer solutions.

4.3.3 The Relative Gap

To know approximately how close a MILP problem is to being solved the pessimistic value and the optimistic value can be used. The pessimistic value is usually the value of the objective function for the best integer solution so far. The optimistic value is the value of the objective function when some constraints might be violated, either the integer constrations or some other. [26] These two can be used together to get the gap or relative gap of the problem. **The relative gap** used within this project is defined as

$$\text{Relative Gap } = \frac{\text{Optimistic value } - \text{ Pessimistic value}}{\text{Optimistic value } + 1}, \tag{4.3}$$

+1 in the denominater is in case the optimistic value is zero. When the relative gap is zero the optimistic and the pessimistic values are the same and the optimal solution has been found. The relative gap can be considered of as the highest potential improvement of the best know integer solution at the moment.

Since MILP problems can take a long time to solve, it can be implemented that when the relative gap is small enough the pessimistic value will be presented as the solution. Once again looking at figure 4.4: Assume that it would have been implemented that the algorithm should stop when the relative gap is less than 1%. After the relaxations $x \leq 10$ and $x \geq 11$, $g = 1707$ could be considered an optmistic value and $g = 1700$ the

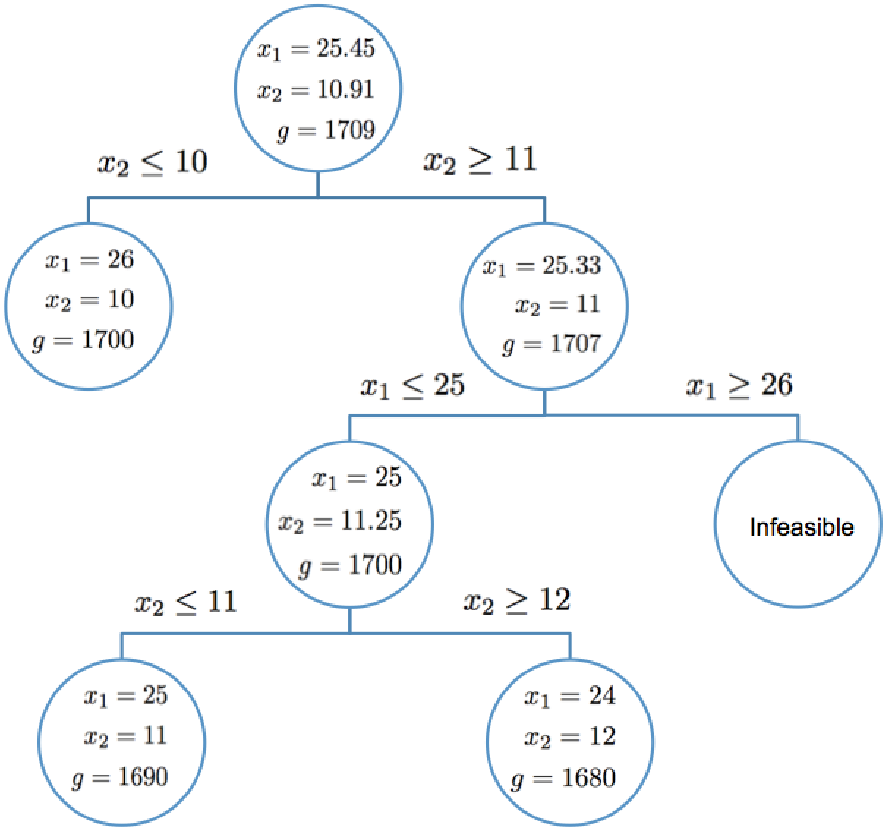

Figure 4.4: A picture illustrating the branch and bound for the example MILP problem.

pessimistic value. Then the relative gap is (see equation 4.3)

$$\frac{1707 - 1700}{1700 + 1} = 0.41\%, \tag{4.4}$$

which is less than 1% and the algorithm would have stopped. There is no guarantee that this is the best solution, but this drawback may outweigh the reduction in computional time.

4.3.4 Heuristics

Since all optimization problems cannot be solved and those which can usually require a lot of time to be solved, heuristic methods are used instead. **Heuristic methods** find a good solution, but does not verify that it is the optimal solution.

For the example above an heuristic method could have been to fill the bag with gloves, which has a larger profit per piece than caps, until no more of them fit. Then eventual additional space or weight could be used for caps. This is a so called greedy method, which does what seems to be the best choice at the moment. This method will result in 32 gloves and no caps, and the objective value 1600. Note that all integer solutions found in figure 4.4 have a better value of the objective function.

4.4 Comparison between a Greedy Method and MILP when Designing Wind Power Farms

Figure 4.5 and 4.6, originally from [3], illustrates a problem that might occur with a greedy heuristic method. The greedy method used in the pictures is to put the turbine in a position with the highest possible power extraction and then excludes positions which are too close to the turbine. This is done until there are no positions left.

In figure 4.5 the heuristic method generates that only one turbine fit with the extraction 5, while the MILP solution has two turbines and the extraction 8. In figure 4.6 both methods give solutions with two turbines. The greedy method takes the position with the highest possible extraction, but then only the position with the lowest possible extraction is left. The MILP-solution decreases the production of one of the turbines with 1, but instead increases the other one with 2, which in total gives additional extraction of 1.

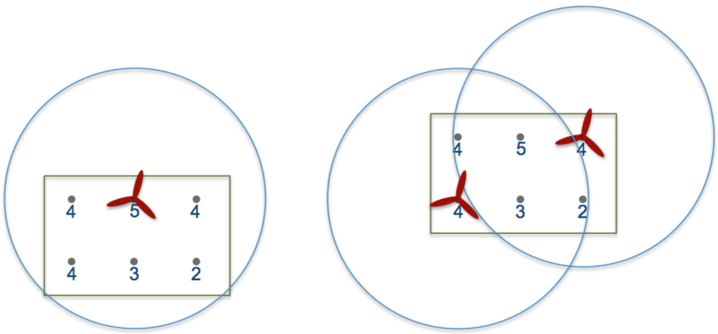

Figure 4.5: An illustration of what can happen when no verification is done. The left solution is a greedy method, while the right one is the MILP-solution. The number is the possible power extraction on that spot and the circle is the minium distance to the turbine. Figure from [3].

4.5 Tools in Commercial Software

WindPro is one of the most wide spread software for calculations on wind power. In WindPro's software package there is moduls where it is possible both to build your own wind power farms and for the software to decide the layout of the farm. The resolution

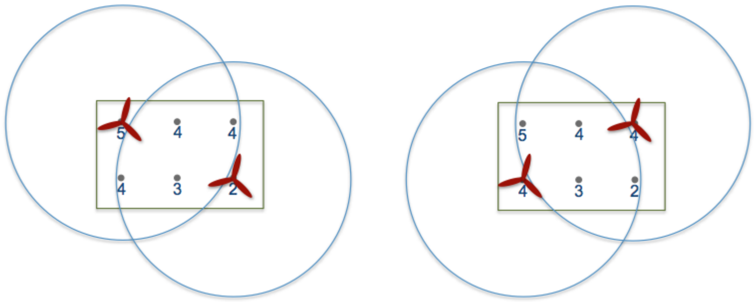

Figure 4.6: An illustration of what can happen when no verification is done. The left solution is a greedy method, while the right one is the MILP-solution. The number is the possible power extraction on that spot and the circle is the minium distance to the turbine. Figure from [3].

of the discretization of the site is then down to 10 metres. Four different options on how the farm should be designed can be chosen:

- The layout is optimized with respect to that the turbines should be in parallell lines or other geometrical requirement.
- One turbine at the time is added to the position which is the most beneficial regarding maximizing power extraction.
- The number of wind turbines on the site is maximized.
- Minimize the production losses while taking noise limits into consideration.

The second alternative is not optimization in a mathematical sense, but a (greedy) heuristic method with no verification that the result is the optimal solution. Whether the other options are mathematical optimization or heuristics method cannot be determined from the information on WindPro's homepage. [27]

5

Formulating the Models

The mathematical models tested within this project are presented in this chapter. It starts out with the original model, where one change has been done compared to previous work. After that, three different suggestions are made on how to modify the MILP-model. In the first one the wind rose used is so that fewer or no wakes needs to be calculated. In the second one a limit for the wakes are chosen, and then the wakes are excluded from the goal function and instead considered in additional constraints. In the third one the wakes losses from single wakes will be scaled down, which might make the linear superposition of several wakes more accurate. But first the wind data are discretized and the graph constructed.

5.1 Pre-processing the Problem

5.1.1 Discretization of the Wind Conditions

The wind analysis is discretized to make it computable. The processing of wind data has previously been done by Kjeller Vindteknikk for JP Vind. They have used wind data from about 4.3 years, which have been processed both with physical and MCP models. There was no reason to question their result, which is therefore implemented when calculating possible extraction and extraction losses due to wakes, herefrom denoted wake losses. In the wind analysis the wind directions are divided into 12 sectors.

The first step in the discretization is to make a square grid of the site, each sqaure has a side of 100 metres. Squares which are located in water are excluded. Each square is assumed to have the same wind rose, divided into 12 sectors. The wind is assumed to have Weibull distribution and is considered as integers between 3 and 25 m/s, which is the lowest cut-in speed and highest cut-off speed, respectively. Then the potential power extraction p for each square i can be calculated as

$$p_i = \sum_{d=1}^{12} \sum_{U=3}^{25} P(U)\theta(d) f_{i,d}(U), \tag{5.1}$$

where d is the direction, U is the wind speed, θ is the probability that the wind blows in a certain direction, $f_{i,d}$ is the Weibull probability density function in direction d and square i and P is the power extraction from the turbine, which is given for each U from the turbine company.

The positions interact through the wakes. The wake losses between sqaure i and j is

$$q_{ij} = \theta(d)\Big(\sum_{d=1}^{12} \sum_{U=3}^{25} P(U) f_{i,d}(U) - \sum_{d=1}^{12} \sum_{U=3}^{25} P(u) f_{i,d}(u) \Big), \tag{5.2}$$

where u is the velocity in the wake. u is calulcated with the Jensen model, presented in section 3.3.1, and depends on the ambient wind speed U and the thrust coefficient C_T. The Jensen model assumes that the velocity in the entire wake is constant, which makes it irrelevant where in the wake the position affected is.

5.1.2 Constructing the Graph

All possible positions on the site are represented through nodes. The entire set of possible positions or nodes are denoted V. The entire set of edges is denoted E. In the complete graph every node has one edge to all other nodes except itself. Each edge contains information about its length and direction. Since the directions are different for edge (i,j) and edge (j,i), the edges are directed.

When calculating the minimum distance between the nodes, a subset of the edges E_m is used. E_m only contains edges that are shorter then the minimum allowed distance between two turbines. $E_{m,i}$ is the subset of edges connected to i that are shorter than the minimum allowed distance between two turbines. E_m and $E_{m,i}$ are altered in the model where the wind rose is used.

When calculating the wakes a subset of the edges E_w is used. E_w only contains edges that are longer than or equal to minimum allowed distance between two turbines. Turbines will not be closer than the minimum allowed distance, therefore there will be no wake effects between nodes closer than this distance. Edges that are so long that the wake losses can be neglected are also excluded, which is set to 12 rotor diameters, which is when the velocity deficits are negligible. This has the consequence that q_{ij} does not need to be calculated for each combination of (i,j), only for $(i,j) \in E_w$. In the model where the wind rose is used a subset of E_w will be used, more about that in section 5.3.

5.1.3 Verifaction of Extraction Calculations

To verify that the results in this thesis can be compared with previous designs, the coordinates from previous results is put together with the graph here. Therefrom the total production is calculated from the $p_i \forall i \in V$ and compared to the total produciton according to previous designs. If the errors are small, $p_i \forall i \in V$ can be considered reliable.

5.2 Standard MILP Model

Here are several of the aspects presented in chapter 2 formulated as parts of the mathematical problem. At the end of this section all parts are summarized to the standard model.

5.2.1 Possible Extraction

In position $i \in V$ where V is the set with all possible positions for the wind turbine, the possible power extraction is p_i. The total possible power extraction in the wind power farm is

$$\sum_{i \in V} p_i x_i. \tag{5.3}$$

This is the first part of the objective function, which is maximized.

5.2.2 Maximum Number of Wind Turbines

The maximum installed capacity of the park is limited to 130 MW. In order to have a better structure of the problem this number is recalculated to an integer number of turbines. In this thesis turbines have a capacity 3 or 3.075 MW, which gives a maximum number of 43 and 42 turbines, respectively. The constraint is

$$\sum_{i \in V} x_i \leq N, \tag{5.4}$$

where N is the maximum number of wind turbines. The stricter constraint [28]

$$\sum_{i \in V} x_i = n \tag{5.5}$$

could be used, where n is the exact number of turbines. However, the exakt number of turbines does not have to be known in advance, which is the case in this project.

5.2.3 Maximum Sound Level

The sound level from each turbine is calculated to its responding sound pressure with the formula $SL = 10 \log_{10} SP$. Formulas for sound immission from wind power turbines are provided by the Swedish Naturvårdsverket [16]. The sound pressure from a turbine in position i to a sound sensitive location s is denoted a_{is}. The sound pressure from each turbine can be added together to get the total sound pressure at a sound sensitive location s where the sound from the wind power farm may not exceed the limit a_{max}. Mathematically, this constraint can be written as

$$\sum_{i \in V} a_{is} x_i \leq a_{max}, \qquad \forall s \in S, \tag{5.6}$$

where S is the set of all sound sensitive locations. The sound level is restricted to 35 dB for nearby houses and the closest mountain top. The model for calculating the sounds [16] might have an error of up to 1 dB, so 34 dB is implemented.

5.2.4 Minimum Distance between Wind Turbines

To reduce the increase in fatigue loading and avoid large reductions in power extraction a minimum distance is implemented in the model. The maximum length of the near wake is approximated to find a suitable minimum distance. Calculations from Abramovich's wake model showed that for the turbines used in this project the near wake exceeds 4.96 rotor diameters, see figure 5.1. The minimum distance implemented is 5 rotor diameters. The constraint for this is

$$\sum_{j \in E_{m,i}} x_j \leq 1, \qquad i \in V, \tag{5.7}$$

where $E_{m,i}$ is the set of edges from i that are shorter than αR, see section 5.1.2. As a test sometimes $\alpha = 12$ will be used, and therefore in the report it will be written α and not the number 5.

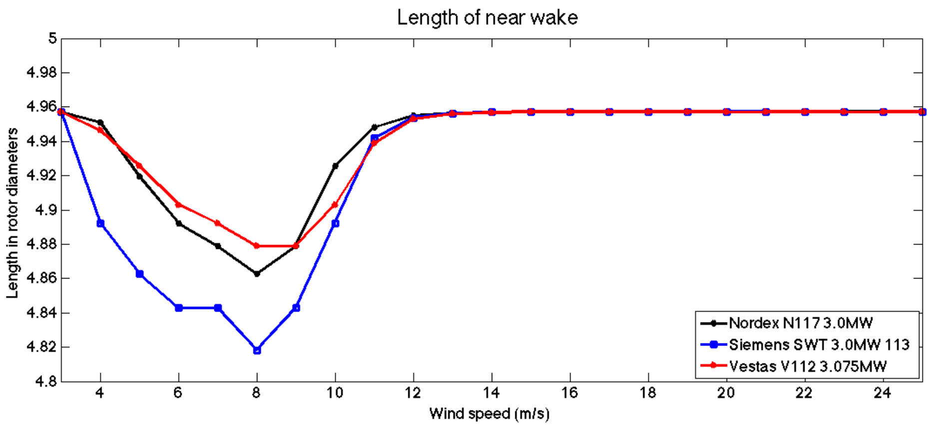

Figure 5.1: The length of the near wake, according to the formulas from [22], for the turbines used in the simulations.

5.2.5 Wake Losses

If there are several wind turbines, these will affect each other and reduce the power extraction with q_{ij}. In this report q_{ij} is the potential wake losses a turbine in position j

can cause on a turbine in position i. The total wake losses in the farm is

$$\sum_{(i,j)\in E_w} q_{ij}w_{ij}. \tag{5.8}$$

If there are wind turbines in position i and j the corresponding wake w_{ij} has to be set to 1. This is done with the constraint

$$x_i + x_j - w_{ij} \leq 1, \qquad \forall (i,j) \in E_w, \tag{5.9}$$

where E_w is the subset of edges that are longer than αR and shorter than $12R$. w_{ij} does not have to be set to binary; due to the objective function w_{ij} will be binary if both x_i and x_j are binary and the constraint above is fullfilled [25], so w_{ij} has to fullfill

$$0 \leq w_{ij} \leq 1 \tag{5.10}$$

5.2.6 The Complete Standard Model

$$
\begin{aligned}
&\text{maximize} \sum_{i\in V} p_i x_i - \sum_{(i,j)\in E_w} q_{ij}w_{ij}\\
&\text{subject to}\\
&\qquad \sum_{i\in V} x_i \leq N,\\
&\qquad \sum_{i\in V} a_{is}x_i \leq a_{max}, \qquad \forall s \in S,\\
&\qquad \sum_{j\in E_{m,i}} x_j \leq 1, \qquad \forall i \in V,\\
&\qquad x_i + x_j - w_{ij} \leq 1, \qquad \forall (i,j) \in E_w,\\
&\qquad x_i \in \{0,1\}, \qquad \forall i \in V,\\
&\qquad w_{ij} \in [0,1], \qquad \forall (i,j) \in E_w.
\end{aligned}
$$

Here all the parts that will be taken into account in the standard model are put together. The objective function says that the power extraction minus wakes losses shall be maximized. The first constraint limits the number of turbines. The second constraint limits the sound in nearby sound sensitive positions. The third constraint says that the minimum distance between the turbines is αR, since $E_{m,i}$ is the subset of edges connected to node i that are shorter than αR, where $\alpha = 5$ and $\alpha = 12$ will be tested. The fourth constraint says that if there is a turbine in position i and a turbines in position j and the edge (i,j) is in E_w, i.e. i and j are further away than αR and closer than $12R$ to each other, there will be a wake between i and j. The two remaining constraints makes sure that the variables will be binary.

5.3 MILP Model where the Wind Rose Is Used

Measurements from existing wind power farms show that when there is more than 12 rotor diameters between the turbines, the velocity deficit due to a wake can be neglected compared to other influences. For the site Moskogen the wind rose is quite uneven, see figure 5.2. The figure shows, that if the wind directions are divided into twelve sectors, in the most dominant one in Moskogen it blows almost a fourth of the time, while the least likely wind direction is so small in the picture it is difficult to see. Over longer time periodes, the wake losses will be larger in dominant wind directions, see equation 5.2. Therefore the minimum distance is set to 12 rotor diameters in the dominant wind directions and to α rotor diameters in the other wind directions.

In the calculations are 12 different wind directions used and the limit chosen for when a wind direction is dominant is 8.33% ($\approx 1/12$). For the implementation the subsets $\widehat{E_{m,i}} \forall i$ are created, which are the edges connected to position i that are shorter than $12R$ in dominant wind directions and αR otherwise. This is also tested for when the wake losses are considered in the not dominant wind directions.

This approach requires an unevenly distributed wind rose. On this site the wind rose is not as uneven as some other wind roses, for example figure 5.3 from [15]. There it is easy to determine in which direction to have 12 rotor diameters as minimum distance, and in which ones it can be shorter. Donovan [25] makes an easier version of this model, which assumes a wind roses similar to the one in figure 5.3.

5.3.1 The Complete Model where the Wind Rose Is Used

$$
\begin{aligned}
&\text{maximize} \sum_{i \in V} p_i x_i \\
&\text{subject to} \\
&\qquad \sum_{i \in V} x_i \leq N, \\
&\qquad \sum_{i \in V} a_{is} x_i \leq a_{max}, \qquad \forall s \in S, \\
&\qquad \sum_{j \in \widehat{E_{m,i}}} x_j \leq 1, \qquad \forall i \in V, \\
&\qquad x_i \in \{0,1\}, \qquad \forall i \in V.
\end{aligned}
$$

Here all the parts that will be taken into account in the first modified model are put together. The objective function says that the extraction shall be maximized; wake losses are excluded. The first constraint limits the number of turbines. The second constraint limits the sound in nearby sound sensitive locations. The third constraint says that the minimum distance between the turbines is $12R$ in dominant wind directions and αR

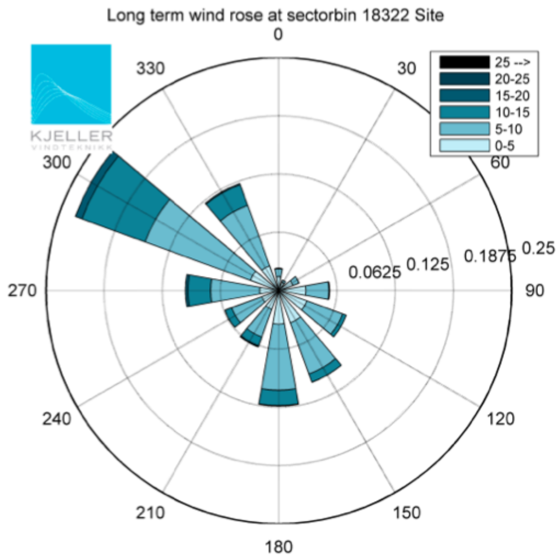

Figure 5.2: The wind rose for the site. The length of each sector shows the probability that the wind blows in that direction, which corresponds to θ in the equations, e.g. equation 5.2. Made by Kjeller Vindteknikk for JP Vind. Published with permission from JP Vind and Jämtkraft.

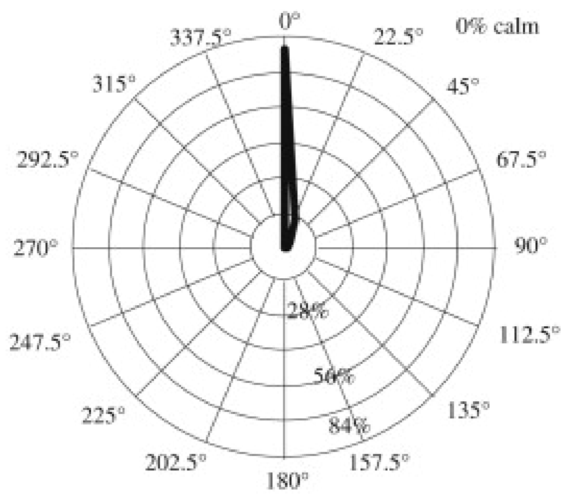

Figure 5.3: A very unevenly distributed wind rose from [15].

otherwise, since $\widehat{E_{m,i}}$ is the subset of edges connected to node i that are shorter than $12R$ in dominant wind directions and αR otherwise, where $\alpha = 5$ The remaining constraint states that the variables will be binary.

5.3.2 The Complete Model where the Wind Rose and the Wake Losses Are Used

$$
\begin{aligned}
&\text{maximize} \sum_{i \in V} p_i x_i - \sum_{(i,j) \in E_w \setminus \widehat{E_{m,i}}} q_{ij} w_{ij} \\
&\text{subject to} \\
&\quad \sum_{i \in V} x_i \leq N, \\
&\quad \sum_{i \in V} a_{is} x_i \leq a_{max}, \qquad \forall s \in S, \\
&\quad \sum_{j \in E_{m,i}} x_j \leq 1, \qquad \forall i \in V, \\
&\quad x_i + x_j - w_{ij} \leq 1, \qquad \forall (i,j) \in E_w \setminus \widehat{E_{m,i}}, \\
&\quad x_i \in \{0,1\}, \qquad \forall i \in V, \\
&\quad w_{ij} \in [0,1], \qquad \forall (i,j) \in E_w \setminus \widehat{E_{m,i}}.
\end{aligned}
$$

This is also combined with the standard model, so that the wakes are calculated in the not dominant directions and in the dominant directions it should be at least 12 rotor diameters between the turbines.

The objective function says that the extraction minus wake losses shall be maximized. The first constraint limits the number of turbines. The second constraint limits the sound in nearby sound sensitive locations. The third constraint says that the minimum distance between the turbines is $12R$ in dominant wind directions and αR otherwise, since $\widehat{E_{m,i}}$ is the subset of edges connected to node i that are shorter than $12R$ in dominant wind directions and αR otherwise. The fourth constraint says that if there is a turbine in position i and a turbines in position j and the edge (i,j) is in E_w but not in $\widehat{E_{m,i}}$ there will be a wake between i and j. An edge is in E_w but not in $\widehat{E_{m,i}}$ if it is longer than αR, shorter than $12R$ and is not in a dominant wind direction. The two remaining constraints make sure that the variables will be binary.

5.4 MILP Model where the Wake Losses are Only in Constraints

In the standard model the number of variables is proportional to the number of positions squared. Reducing the number of variables could speed up the calculations significantly, therefore the wake losses were taken away from the objective function and instead put into added constraints.

Starting with the constraint saying there should be a wake w_{ij} if there are turbines in both position i and j:

$$x_i + x_j - 1 \leq w_{ij}. \tag{5.11}$$

Multiplying the entire inequality with the wake losses q_{ij}:

$$q_{ij}(x_i + x_j - 1) \leq q_{ij}w_{ij}. \tag{5.12}$$

Summing this over all $j \in V$:

$$\sum_{j\in V} q_{ij}(x_i + x_j - 1) \leq \sum_{j\in V} q_{ij}w_{ij} \Leftrightarrow$$
$$\sum_{j\in V} q_{ij}x_i + \sum_{j\in V} q_{ij}x_j - \sum_{j\in V} q_{ij} \leq \sum_{j\in V} q_{ij}w_{ij}.$$

Then the total wake losses on turbine in position i is constrained to be less than γ share of the potential power extraction in position i. This gives

$$\sum_{j\in V} q_{ij}w_{ij} \leq \gamma p_i x_i \Leftrightarrow \tag{5.13}$$
$$\sum_{j\in V} q_{ij}x_i + \sum_{j\in V} q_{ij}x_j - \sum_{j\in V} q_{ij} \leq \gamma p_i x_i \Leftrightarrow$$
$$(-\gamma p_i + \sum_{j\in V} q_{ij})x_i + \sum_{j\in V} q_{ij}x_j \leq \sum_{j\in V} q_{ij}. \tag{5.14}$$

The above constraints make sure a turbine will not be in position i if γ of its power extraction is exceeded by the wake losses on it. In the same way it is possible to get the constraint

$$\big(- \gamma p_i + \sum_{j\in V} q_{ji}\big)x_i + \sum_{j\in V} q_{ji}x_j \leq \sum_{j\in V} q_{ji}. \tag{5.15}$$

Keeping in mind that $w_{ij} = w_{ji}$ but q_{ij} is usually not equal to q_{ji}. This constraint says that a turbine will not be in position i if the wake losses it creates exceeds γ of the power extraction in position i. The model is tested for $\gamma = 2.5\%$ and $\gamma = 5\%$, which corresponds to very low and normal, respectively, wake losses.

What Does Not Work

It would be even more efficient if this constraint could be exactly one, instead of proportional to the number of possible positions on the site. However, if equation 5.15 would be summed over all i:

$$\sum_{i\in V}\big(-\gamma p_i+\sum_{j\in V} q_{ji}\big)x_i+\sum_{i\in V}\sum_{j\in V} q_{ij}x_j = -\gamma\sum_{i\in V} p_i x_i+\sum_{i\in V}\sum_{j\in V} q_{ji}x_i+\sum_{i\in V}\sum_{j\in V} q_{ij}x_i \leq \sum_{i\in V}\sum_{j\in V} q_{ji}. \tag{5.16}$$

Reordered:

$$\sum_{i\in V}\sum_{j\in V} q_{ij}x_i \leq \sum_{i\in V}\sum_{j\in V}(1-x_i)q_{ji} + \gamma\sum_{i\in V} p_i x_i. \tag{5.17}$$

Both the constraint about maximum number of turbine and the constratin about minimum distance will make most of the x_i zero. Therefore will

$$\sum_{i\in V}\sum_{j\in V} q_{ij}x_i << \sum_{i\in V}\sum_{j\in V}(1-x_i)q_{ji}. \tag{5.18}$$

So the constraint becomes too weak, i.e. it is unlikely it will be a limiting constraint, if it is written as one constraint instead of one or two constraints per position.

5.4.1 The Complete Model where the Wake Losses are Only in Constraints

$$\begin{aligned}
&\text{maximize} \sum_{i\in V} p_i x_i \\
&\text{subject to} \\
&\quad \sum_{i\in V} x_i \leq N, \\
&\quad \sum_{i\in V} a_{is}x_i \leq a_{max}, \qquad \forall s \in S, \\
&\quad \sum_{j\in E_{m,i}} x_j \leq 1, \qquad \forall i \in V, \\
&\quad \big(-\gamma p_i + \sum_{j\in V} q_{ji}\big)x_i + \sum_{j\in V} q_{ji}x_j \leq \sum_{j\in V} q_{ji}, \qquad \forall i \in V, \\
&\quad (-\gamma p_i + \sum_{j\in V} q_{ij})x_i + \sum_{j\in V} q_{ij}x_j \leq \sum_{j\in V} q_{ij}, \qquad \forall i \in V, \\
&\quad x_i \in \{0,1\}, \qquad \forall i \in V.
\end{aligned}$$

Here all the parts that will be taken into account in the second modified model are put together. The objective function says that the extraction shall be maximized; wake losses are excluded. The first constraint limits the number of turbines. The second constraint limits the sound in nearby sound sensitive locations. The third constraint says that the minimum distance between the turbines is αR, since $E_{m,i}$ is the subset of edges connected to node i that are shorter than αR. The fourth constraint says that if there is a turbine in position i the relative wake losses on i may not exceed γ. The fifth constraint says that if there is a turbine in position i the relative wake losses due to i may not exceed γ. The remaining constraint makes the variables binary.

5.5 MILP Model with Scaled Wake Losses

Since a linear mathematical tool is used, the wake losses also have to be calculated as linear. This will overestimate the wake losses. This model is an attempt to make the total wake losses more accurate.

The power extraction P for a turbine standing in a wake, caused be one or several turbines, is

$$P \propto (U - \Delta U)^3, \tag{5.19}$$

where U is the ambient wind speed and ΔU is the speed deficit due to several wakes. Without the wakes $P_0 \propto U^3$, where P_0 is the power extraction when the turbine is unaffected by wakes, so the wake losses Q' are

$$Q' = P - P_0 \propto (U - \Delta U)^3 - U^3 = 3U^2\Delta U - 3U(\Delta U)^2 + (\Delta U)^3, \tag{5.20}$$

but when linear superposition is used:

$$Q \propto U^3 - (\Delta U)^3. \tag{5.21}$$

Then

$$\frac{Q'}{Q} \propto \frac{3U^2\Delta U - 3U(\Delta U)^2 + (\Delta U)^3}{U^3 - (\Delta U)^3}. \tag{5.22}$$

Assuming $U >> \Delta U$

$$\frac{Q'}{Q} \propto \frac{3U^2\Delta U - 3U(\Delta U)^2 + (\Delta U)^3}{U^3 - (\Delta U)^3} \approx \frac{3U^2\Delta U}{U^3} = \frac{3\Delta U}{U}, \tag{5.23}$$

and writing it as a function of several wakes:

$$\frac{Q'}{Q} \propto 3\frac{\Delta U}{U} = 3\left(\frac{\sqrt{\sum_{i\in V}(\Delta u_i)^2}}{U}\right) = 3\sqrt{\sum_{i\in V}\left(\frac{\Delta u_i}{U}\right)^2}, \tag{5.24}$$

where Δu_i is the velocity deficit due to one wake. If one turbine is influenced by m wakes then

$$\frac{Q'}{Q} \propto 3\sqrt{\sum_{j\in V}\left(\frac{\Delta u_i}{U}\right)^2} = 3\sqrt{m}\sqrt{\overline{\left(\frac{\Delta u_i}{U}\right)^2}}, \tag{5.25}$$

where a bar above a symbol is to show that it is the average. To make a modification of the standard model as easy as possible, $3\sqrt{\overline{\left(\frac{\Delta u_i}{U}\right)^2}}$ is assumed to be constant. Then $\frac{Q'}{Q}$ is proportional to a constant times $\sqrt{m}$, i.e.

$$\frac{Q'}{Q} \propto \sqrt{m}. \tag{5.26}$$

Then also $\frac{Q'}{Q} \propto 1$ when there is only one turbine causing the wake effects, which is consistent with reality. This is implemented in the objective function such that

$$\text{maximize} \sum_{i \in V} p_i x_i - \frac{1}{\sqrt{m}} \sum_{(i,j) \in E_w} q_{ij} w_{ij}. \tag{5.27}$$

Further, there can be at the most two turbines in the same direction which are closer than twelve rotor diameters to the affected turbine, when the minimum distance between turbines is five rotor diameters, see figure 5.4. Therefore m can be approximated to two. This model can also be viewed upon as a sensitivity analysis on how much the wakes influence where the standard model positiones the turbines.

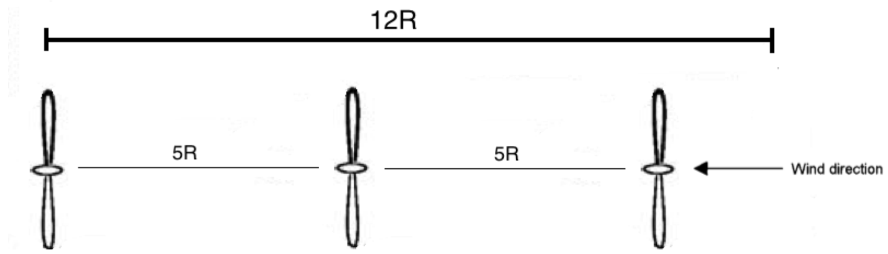

Figure 5.4: Three turbines in a row, illustrating how many turbines that can affect one turbine through wakes. The turbine furthest to the left can only be affected by two other turbines, other eventual turbines are too far away. This is under the assumption that all wakes further away than 12 rotor diameters can be neglected. R in the figure stands for rotor diameter.

5.5.1 The Complete Model with Scaled Wake Losses

$$\text{maximize} \sum_{i \in V} p_i x_i - \frac{1}{\sqrt{2}} \sum_{(i,j) \in E_w} q_{ij} w_{ij}$$

subject to

$$\begin{aligned}
&\sum_{i \in V} x_i \leq n, \\
&\sum_{i \in V} a_{is} x_i \leq a_{max}, \qquad \forall s \in S, \\
&\sum_{j \in E_{m,i}} x_j \leq 1, \qquad \forall i \in V, \\
&x_i + x_j - w_{ij} \leq 1, \qquad \forall (i,j) \in E_w, \\
&x_i \in \{0,1\}, \qquad \forall i \in V, \\
&w_{ij} \in [0,1], \qquad \forall (i,j) \in E_w.
\end{aligned}$$

Here all the parts that will be taken into account in the third modified model are put together. The objective function says that the power extraction minus wake losses shall be maximized. The first constraint limits the number of turbines. The second constraint limits the sound in nearby sound sensitive locations. The third constraint says that the minimum distance between the turbines is αR, since $E_{m,i}$ is the subset of edges connected to node i that are shorter than αR, where $\alpha = 5$. The fourth constraint says that if there is a turbine in position i and a turbines in position j and the edge (i,j) is in E_w, i.e. i and j are further away than αR and closer than $12R$ to each other, there will be a wake between i and j. The two remaining constraints make sure that the variables will be binary. The model is almost the same as the standard model, the only change is that some of the variables in the objective function are divided with $\sqrt{2}$.

5.6 Sensitivity Analysis

The hardest thing to predict is how the wind is going to blow. On the site wind measurements have been done for years, which then have been processed both through long term correction and physical models. From year to year the wind will still change, which makes it hard to predict how profitable the farm will be.

Since the wind is hard to predict, but plays a very large role on the objective function, some of the simulations will also be done for a wind conditions with lower average wind speed than the estimated long term average. According to Liléo et.al. [8] is the interannual variability of wind is 3-7% and long term correction error for 3-4 years of wind measurements is 1.7-2.8% in Scandinavia and the Baltics. The uncertainity of the wind measurements has previously been determined to 3.7%. The summarized uncertainity used in the sensitivity analysis in this thesis is therefore set to

$$1 - (1 - 0.07)(1 - 0.028)(1 - 0.037) = 12.95\%. \tag{5.28}$$

This is implemented by reducing the average wind speed by 12.95%, using the Weibull Scale Method [7], [8], [1]. The Weibull Scale Method is usually used to determine the scale and shape parameters for a target site, using the known parameters for a reference site. In this thesis it is used to scale the parameters from average wind conditions to a year with bad wind conditions.

It is known that the average wind speed an average year is [29]

$$\overline{U} = \lambda\Gamma(1 + 1/\kappa), \tag{5.29}$$

where Γ is the gamma function, λ is the scale factor for the Weibull distribution and κ is the shape factor. For bad wind conditions the annual average is

$$\overline{U}_b = \lambda_b\Gamma(1 + 1/\kappa_b), \tag{5.30}$$

where index b is for a bad wind year. The Weibull Scale Method assumes

$$\frac{\kappa_b}{\lambda_b} = \frac{\kappa}{\lambda}, \tag{5.31}$$

where $\overline{U}$, λ and κ are known, $\overline{U}_b$ is set to $(1-0.1295)\overline{U}$ and from that and the equations above can λ_b and κ_b be calculated for each position and direction.

6

Results

The results of the different models are presented in this chapter. Model by model the power extraction, approximate wake losses, number of turbines and capacity factor are presented, in order to determine how well the models performed. The relative gap and running time will also be presented to determine their computational performance and possible industrial applicalibility. When it says no wake model in the presented results, it means no wake model used in the MILP-model, but afterwards the wakes are calculated with the Jensen model and through linear superposition.

The models were run with three different turbines: Nordex N117 3.0MW [9], Siemens SWT-3.0-113 [10],[11],[12] and Vestas V112 3.075MW [13],[14]. For clarity, only the average results for all three turbines are presented here. The models are run with MATLAB R2014a and the MILP-problem is solved through the function *intlinprog* [30]. *intlinprog* was run on a MacBook Pro with a 2.3 GHz Intel Core i5 processer and a 4 GB 1333 MHz DDR3 RAM.

This thesis is a quantitive study. Several approaches on considering the wakes have been adopted, with the drawback that the testing had to be cut shorter. This is clear in some results, where the relative gap is large.

6.1 Verifaction of Extraction Calculations

The coordinates from previous designs are put into the MILP-model to see what production it gives according to the standard model and the model with scaled wake losses. Firstly no wake losses are accounted for, and then the standard model gave between 0.10% and 0.70% higher extraction compared to previous calculations. The standard model (with wake losses) gave between 1.87% and 2.08% lower production compared to previous calculations. The model with scaled wake losses gave between 0.22% and 0.71% higher production compared to previous calculations.

6.2 Standard MILP Model

The results are summarized in table 6.1. ΔP and ΔCF is expressed as the relative difference to previous results. Q is expressed as percentage of P. Q is calculated after the MILP model is done when it says $\alpha = 5$ and no wake model in table 6.1. The calculations are done with linear superpositions, to be comparable with the MILP model when the wake losses are in the objective function. When $\alpha = 12$ is it assumed that $Q = 0$.

When the wakes are considered the model has a very large relative gap after the time is up. When they are not considered, the model is done within one or a few hours, but to exclude the wake effects when the minimum distance is five rotor diameters means great wake losses that the MILP model does not consider. It does however give a higher power extraction than previous models, but with the drawback that the capacity factor is reduced slightly.

To increase the number of turbines and force more of the variables w_{ij} to become one, the constraint $\sum_i x_i > 30$ was added. After 12 hours no integer solution at all had been found, and the attempt was terminated. When the constraint $\sum_i x_i > 30$ was not in the model, it took between 4.5 and 5 hours, depending on which turbine, until the first integer solution was found.

To investigate what constraints that were limiting the problem, the constraint $\sum_{i \in V} x_i \leq N$ was removed, no wakes were taken into account and $\alpha = 5$. It resulted at the most in 47 turbines. After 1 hour the gap was 12.29%, so this version of the standard model was slower than the results presented for no wake model and $\alpha = 5$ in table 6.4, with a relative gap of 3.2% after 1 hour for the smallest turbine.

6.3 MILP Model where the Wind Rose Is Used

The results are summarized in table 6.2. ΔP and ΔCF is expressed as the relative difference to previous results. Q is expressed as percentage of P. As in previous section Q is calculated after the MILP model is done when it says no wake model and $\alpha = 5$ in table 6.2. The calculations are done with linear superpositions, to be comparable with the MILP model when the wake losses are in the objective function.

This model takes the wakes into account through a minimum distance depending on the probability for the wind that blows in a certain direction. This reduces the variables significantly, which in turn reduces the required running time to get an acceptable relative gap. The model is slow when the wake losses in not dominant wind directions are

Table 6.1: Results for the standard MILP-model.

Wake model	Jensen	No wakes	No wakes
α	5	5	12
n	23.33	42.33	10.33
ΔP	-31.05%	+16.00%	-68.85%
ΔCF	+6.46%	-1.36%	+2.95%
Q	0.00%	5.07%	0%
Relative Gap	78.27%	1.27%	0 %
Running time	24 hours	1 hour	35 minutes

included in the objective function. Then is also the relative gap large after the maximum running time is over.

Table 6.2: Results for the MILP-model where the wind rose is used.

Wake model	Jensen	No wakes
α	5	5
n	24.33	39.0
ΔP	-28.06%	+9.19%
ΔCF	+6.59	+0.78%
Q	0.00%	2.15%
Relative Gap	69.71%	9.09%
Running time	24 hours	4 hours

6.4 MILP Model where the Wake Losses are Only in Constraints

The results are summarized in table 6.3. ΔP and ΔCF is expressed as the relative difference to previous results. Q is expressed as percentage of P. After the MILP model was run was the wake losses calculated through linear superposition, that is the result shown as Q in table 6.3. Q was lower than 2.5 respecitively 5%, which can be expected since γ sets a limit on each indivdual turbine and not the whole farm.

To move the wake losses from the objective function to constraints speeded up the process significantly, compared to the standard model. When $\gamma = 2.5\%$ was the model much slower compared to when $\gamma = 5.0\%$.

Table 6.3: Results for the MILP-model which only have the wake losses in constraints.

γ	2.5%	5%
α	5	5
n	27.67	36.67
ΔP	-18.59	+2.75%
ΔCF	+5.05%	+0.98%
Q	1.61%	3.61%
Relative Gap	52.14%	16.18%
Running time	8 hours	8 hours

6.5 MILP Model with Scaled Wake Losses

The results are summarized in table 6.4. ΔP and ΔCF is expressed as the relative difference to previous results. Q is expressed as percentage of P. Q in table 6.4 is calculated after the MILP model was run through linear superposition and with the same values of q_{ij} as in the standard model.

This model gave exactly the same results as the standard model. But it does say that changing the values of q_{ij} by dividing them with $\sqrt{2}$ does not change the results, at least not with this time limit.

Table 6.4: Results for the MILP-model with scaled wake losses.

Wake model	Jensen
α	5
n	23.33
ΔP	-31.05%
ΔCF	+6.46%
Q	0.00%
Relative Gap	78.27%
Running time	24 hours

6.6 Sensitivity Analysis

The sensitivity analysis is done for the standard model and for the model where the wind rose is used, since it gave the best results after a limited time. ΔP, $\Delta \sum_i p_i x_{b,i}$

and ΔCF is expressed as difference to previous results, where $\sum_i p_i x_{b,i}$ is the power extraction under average wind conditions, with the design from the model under bad wind conditions. Q is expressed as percentage of P and is calculated with linear superposition; in the case of the model where the wind rose is used are these calculations done after the model was run.

The sensitivity analysis is similar for both models. The possible power extraction for the entire farm is reduced during bad wind years, but when those positions are put in the original objective function the result is similar to the result from when the average values are used in the MILP model.

Table 6.5: Results for sensitivity analysis.

MILP model	Standard	Wind Rose
n	24.33	38.67
$\Delta \sum_i p_i x_{b,i}$	-28.00%	+8.66%
ΔP_b	-43.56%	-19.77%
$\frac{P_b - P}{P}$	-18.14%	-26.16%
ΔCF	-16.50%	-25.99%
Q	0.00%	7.75%
Relative Gap	70.57%	9.63%
Running time	24 hours	4 hours

6.7 Computional Size of the Problem

To see how the computional size of the problems varies a subset of $G = (V,E)$ is used, i.e. only a part of the site is used, for the standard model and the model with scaled wake losses. This is mostly due to the time needed when the resolution is 50 meter.

Making a polynomial fit to the subproblem gives that the number of variables is proportional to approximately $0.45X^2$ where X is the total number of possible positions. A doubled resolution (e.g. going from 100 metres resolution to 50 metres) makes X approximately four time as big. The number of constraints is proportional to the number of variables for all models except when the wake losses are only in the constraints. Then the number of constraints is proportional to the number of possible wakes.

How much RAM that is used shall be seen as a guideline, and not an exact number, since this depends on what else is running on the computer. Table 6.6 shows still that it increases fast with higher resolution.

Table 6.6: Computional size of the standard MILP-model as well as the MILP-model with scaled wake losses for a part of the site. The numbers are rounded and show at the most four significant digits.

Resolution	RAM	Possible positions	Variables	Constraints
500 m	1.0	45	707	741
100 m	1.2	753	258 300	258 300
50 m	2.3	3016	4 116 000	4 175 000

7

Conclusions and Discussion

IN THIS CHAPTER some concluding remarks are given concerning the models and possible further developments as well as the suitability of the models in industrial applications are discussed.

7.1 General Remarks

7.1.1 Verifaction of Extraction Calculations

The size of the errors and uncertainties considered in the sensitivity analysis, see section 5.6, are all approximately the same or larger than the errors that occured in the verification. As could be expected, the largest error occurs when the total wake losses are calculated through linear superposition of the single wake losses.

Wind Sensitivity Analysis

When the wind conditions are worse but proportional to average conditions, the extraction went down, but in general the results were similar to results from average conditions. Since there is differences in the results, it might always be a good habit to run the MILP model or models for both average and bad wind conditions.

The standard model was insensitive to changes in the size of q_{ij}. The mere existence of variables that worsened the objective function made the method uneager to put any of the variables w_{ij} equal to one.

Budget and Sound Limits

For this site the sound limit and the limit on total installed capacity seem to constrain the maximum number of turbines to approximately the same level, but it was better for the performance of the standard model with a constraint on maximum number of turbines, since it speeded up the model.

An interesting side note is that the capacity factor was the same or higher for most of the results; the exception was the standard model with $\alpha = 5$. So even if the power extraction is lower, some of these design might still be more profitable due to lower investment costs in comparison with the income.

The Non-linearity of Wakes

Since all wakes between turbines that are further away than twelve rotor diameters are neglected, a turbine can at most have wake losses due to two turbines. Assuming a linear extraction loss due to wakes would still overestimate the total effect, but looking at figure 3.4 it seems like the use of E_w instead of E can avoid unrealistic results like higher wake losses on a turbine than possible power extraction from it. The reasoning behind the model with scaled wake losses shows that the error might be approximately $1 - 1/\sqrt{2} \approx 29.3\%$, which is large but doubtless between zero and one. The comparison to previous calculations showed that linear superposition can overestimate the wake losses between between 38.16% and 45.24%.

If all edges E are used to consider the wake losses, then is the total number of possible wakes in the model approximately $X(X-1)$, where X is the total number of possible positions. When the size of the subproblem was considered the total number of possible wakes was approximately $0.45X^2$, which means that less than half of the wakes can be considered. The reduction is most likely larger for the entire problem, since then there are more edges which are longer then twelve rotor diameters.

7.2 Standard MILP model

The standard MILP model is slow, and does have a very large gap even after 24 hours. For it to be suitable for industrial applications, the site would need to be smaller or it would require a lot more time to run. The result was however good considering the power extraction, when no wake losses were considered in the wake model.

The model seemed uneager to position turbines so close that any w_{ij} were one, which might explain the low power extraction and the few turbines. A method that might circumvent this problem is to find some constraint that forces more w_{ij} to become one, but still respects that the best solution might be that there is at least 12 rotor diameters between the turbines.

7.3 MILP Model where the Wind Rose Is Used

When Donovan [25] implemented a similar metod, the author called the method ”blunt”. Donovan’s version also assumed that the wind blew from one direction, but here it seems like a fast method with good results. Since it makes big simplifications on how to regard the wake losses it cannot make claims on finding the best solution, but for Moskogen it found better solutions than previous used methods.

When half of the wakes were accounted for the method became much slower and after 24 hours the gap was still large. It was not a big improvement in time compared with the standard model.

7.4 MILP Model where the Wake Losses are Only in Constraints

$\gamma = 2.5\%$ seemed to be a too limiting constraint and it did not only seem to give a worse result, but also to significantly slow the process down. The MIP-gap was around 50% after 8 hours when $\gamma = 2.5\%$, compared to 16.18% after the same amount of time when $\gamma = 5\%$. When $\gamma = 5\%$ the power extraction increased and the capacity factor increased slightly.

7.5 MILP Model with Scaled Wake Losses

Most of all the MILP model with scaled wake losses made it clear that the standard model is fairly insensitive to the size of the wake losses. q_{ij} was divided by $\sqrt{2}$ but the model still positioned turbines so as many as possible of the variables w_{ij} were zero. If there are many closely positioned turbines this model might be more accurate than the standard model.

Scaling the wake losses can also be combined with the model where the wind rose and wake losses accounted for or the model where the wake losses are only in the constraints. This could, if an appriopiate factor is used, make the calculations of several wakes in these models more accurate or speed them up.

7.6 Comparison of all MILP models

Undoubtly the fastest methods were those when no wakes were considered: the standard model without the wake effects, the model where the wind rose was used and the model where the wake losses were only in constraints. For implementation in industrial applications, these models would be recommendable, at least until there has been a break-through in solving MILP problems faster or the capacity of computers has increased significantly.

The probable reason for the bad result for some of the MILP models is the time limit. The gap between the best integer solution and the optimistic solution (partly LP-relaxed problem) was significant, especially in the standard model, the wind rose model when wake losses were accounted for and the model where the wake losses were scaled. These models are presently maybe only suitable for smaller wind power farms, since the size of the problem makes them computionally slow.

7.7 Suggestion on Further Developments

In this section recommendations on future work to develope and improve the model further are given. A general suggestion is that many of the approximated number here, the limit for when a direction is dominant, γ and what q_{ij}, can have much more effort put into them to get more accurate and well-adapted numbers than the ones used within this thesis. Within this thesis the models which do not have wake losses in the objective function have the best result, and therefore is it recommended to prioritize developments of these methods.

Turbulence Modelling

Both in the calculations of the possible power extraction and in the Jensen wake model very little consideration was taken of the turbulence. However, turbulence does affect both turbines in wakes and turbines only affected by natural wind. The turbulence may also differ from different locations on site, which may make some locations more or less suitable for wind turbines, either due to losses in power extraction or the fatigue load.

Additional Fatigue Load due to Wakes

The information found about additional load on the turbine due to wakes was often done on only one farm, if not only one turbine. Since this is one of the factors deciding the minimum distance between the turbines, a large scale investigations on the additional load in both off- and on-shore farms which is then connected to a minimum distance between turbines would be recommended. This could be put in data base, so different values of α could be used for different terrains, wind directions etc. This could help improving the life expectancy of the wind power farms. For the model where wake losses are only in constraints, it would be interesting to find a value of γ so additional fatigue load is reduced.

At What Distance Should the Wakes Be Neglected?

In this thesis it is assumed that the velocity deficit due to one wake can be neglected after 12 rotor diameters, which is supported by measurements, experiments and modelling by several others. This is however a general rule and for uneven windroses the distance can vary for different directions. An statistical approach on measurements and modelling from existing farms might give knowledge when the velocity deficit due to one wake can be neglected. This can improve the models by reducing the number of possible wakes that need to be accounted for.

It would also be interesting to find an approach, where the wake losses in each direction are only calculated for the closest turbine in that direction. In a row of wind turbines it is the first one that reduces the wind speed the most, and therefore the most interesting turbine, when calculating wake effects, is the closest one.

In Which Directions Should the Wakes be Neglected?

What is an appropiate limit for when there should be at least 12 rotor diameters between the turbines and when there should be less can be examined much further. Here the average has been used, but there could be other more appriopiate limits. This can also be done with more dynamic choices of minimum distance, instead of only 12 and 5 rotor diameters. In [7] a MCP model is presented where sectors of different sizes of the directions is used, instead of 12 sectors with 30^o each. That might be interesting here to find appropiate distances.

How Much Shall q_{ij} be Scaled?

This can be done in a more thoroughly and complemented with simulations in e.g. WindPro. Additionally, experiments in wind tunnels and measurements from existing wind power farms can also be used. When the power extraction for previous designs were calculated with the standard model and the model with scaled wake losses, multiplying the wake losses with $1/\sqrt{2}$ gave a much better approximation of the wake losses than the standard model did.

It also has to be questioned, if q_{ij} shall be scaled for an accurate description of the wake losses or to make a MILP model more open to position turbines in other turbines wakes. Maybe is it better to find a scaling factor that reduces some of q_{ij} enough for the model to think of the wakes losses as sufficiently small.

Speeding Up the Process

Speeding up the process is not mostly important because the investors needs the designs faster, but in order to be able to have better resolutions of the site. WindPro has a resolution only a tenth of what is used in this thesis. Removing the wakes from the objective function seemed as an efficient way of speeding up the process. Fagerfjäll [3] does have interesting suggestions on reducing the number of variables, but most of them fall when all wakes further away then twelve rotor diameters are neglected.

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