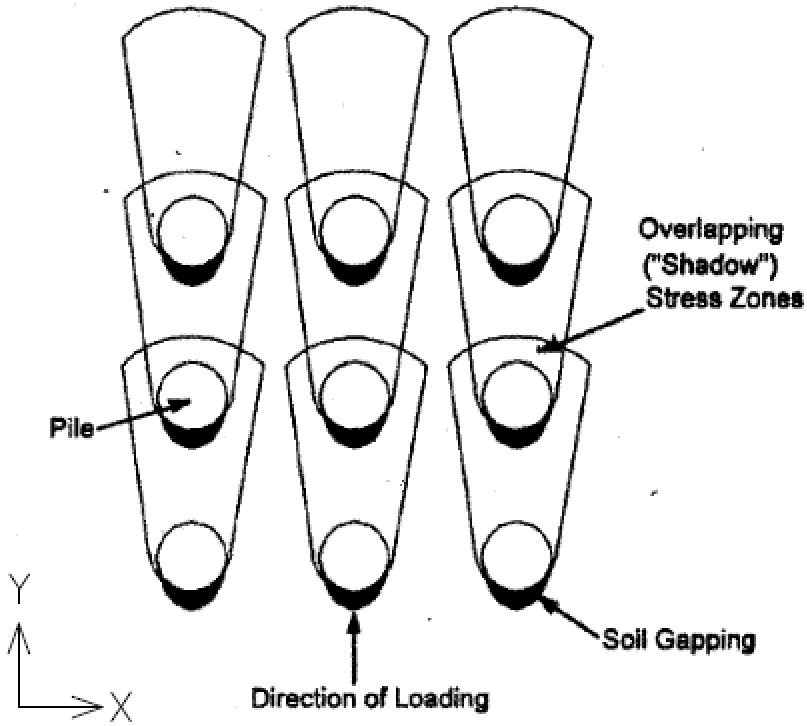

Lateral stability of pile groups

Old timber piles in clay deposit

Master of Science Thesis in the Master's Programme Structural Engineering and Building Technology

ERIK SKANSEBO
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Department of Civil and Environmental Engineering
Division of GeoEngineering
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CHALMERS UNIVERSITY OF TECHNOLOGY
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Master's Thesis 2013:77

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Cover:
Plan view of pile group displaying the so called *shadow effect* in pile groups (adopted from (Rollins, et al., 1998)).

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ABSTRACT

In Sweden, an often overlooked property of pile foundations is the capacity to withstand lateral forces. Regarding new constructions, the engineer can use inclined piles to counter horizontal forces and thus relying fully on the piles axial capacity. This greatly simplifies the calculations and is also the recommended standard procedure in Sweden.

However, in a situation when optimization is important or when the lateral forces are pronounced, the capacity might be needed. The background for this study is concerning old foundations that need verification. In Gothenburg, and many other cities, many buildings and structures are founded on timber piles. In many cases these were installed tightly spaced and more or less vertical, which standard calculation procedures deems unsafe.

On the other hand, the structures have endured for centuries and it is obvious that a lateral load bearing capacity is present. The study investigates the problem of lateral capacity of piles and the pile-soil-pile interaction in a pile group. A tightly spaced pile group often have less horizontal load bearing capacity than the sum of the individual piles. This is due to the so called *shadowing* effect which is explained further in report.

This study compares two methods for analysing the lateral capacity of piles; hand calculation using beam on elastic foundation as presented in *Commission on Pile Research, report 101*, and a 3D finite element analysis. This study focuses on timber piles in cohesive soil and exemplifies the methods by applying them to the foundation of Fontänbron in central Gothenburg. Parametric studies are performed to highlight difficulties with the different methods.

In general, both methods rely on the interpretation of the soil behaviour and the choice of material parameters. Finding reliable general recommendations for many parameters have been proven difficult, but could be overcome by proper field testing.

Keywords:

Pile group, lateral capacity, clay, Winkler, Finite element method, Abaqus

Horisontalstabilitet för pålgrupper
Gamla träpålar i lera
Examensarbete inom Structural engineering and building technology
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SAMMANFATTNING

En i Sverige ofta förbisedd egenskap hos pålgrundläggningar är dess kapacitet att motstå horisontalbelastning. För nya konstruktioner kan ingenjören använda sig utav lutande pålar för att handskas med horisontallaster och då endast behöva utnyttja axialkapaciteten. Detta underlättar beräkningsproceduren och är även praxis i Sverige.

I situationer när optimering är viktigt eller när horisontalkrafterna är extra uttalade, kan det vara nödvändigt kontrollera kapaciteten för horisontalbelastning. Bakgrunden för denna studie rör sig om gamla konstruktioner i behov av verifiering. I Göteborg och i andra städer är många gamla konstruktioner grundlagda på träpålar. I flera fall är pålgrupperna väldigt tätt konstruerade med endast vertikala pålar. Det vanligen förekommande beräkningssättet klassar dem då som ostabila.

Dock har grundläggningarna överlevt i århundraden och det är uppenbart att den omgivande jorden ger någon form av stabilitet i horisontalled. Denna studie undersöker problemet med horisontalbelastade pålar och interaktionen mellan pålar och jord i en pålgrupp. En tätt konstruerad pålgrupp har ofta lägre horisontalkapacitet än summan av de enskilda pålarna. Detta är på grund av *skuggningseffekten*, vilken förklaras ytterligare i rapporten.

Denna studie jämför två metoder för analys av horisontalbelastade pålar; handberäkningsmodell grundad på teorin om balk på fjädrande bädd, presenterad i Pålkommissionen rapport 101 samt en 3D finita element analys. Studien fokuserar på träpålar i kohesionsjord och exemplifieras genom att närmare studera grundläggningen till Fontänbron i centrala Göteborg. Parametriska studier för att lyfta fram svårigheter med de båda metoderna visas i rapporten.

Generellt beror båda beräkningsmetoderna på tolkningen av jordens egenskaper och valet av materialparametrar. Det har visat sig svårt att hitta generella rekommendationer för många parametrar och fältundersökningar har visat sig nödvändiga.

Nyckelord:

Pålgrupp, horisontalkapacitet, Winkler, Finita element metoden, Abaqus

Contents

Preface

This master´s thesis has been written at the division of GeoEngineering at Chalmers university of Technology as a part of the master's program Structural Engineering and Building technology. The study was initially requested by ÅF Infrastructure AB, which had an on-going project regarding overall stability of Fontänbron. The work was mainly carried out at ÅF, the department of Bridge and Plant Design.

Claes Alén, division of GeoEngineering, has been representing Chalmers as the examiner and supervisor for the project. Many thanks for his helpful input and enthusiasm for our project.

We also would like to thank Ludwig Lundberg who has been our supervisor at ÅF Infrastructure. He provided much insight and motivation to work harder and dig deeper.

Finally we would like to send our regards to the whole department of Bridge and Plant design with manager Mattias Hansson, for a positive spirit and for always taking time to answer questions and making us feel welcome at ÅF.

Göteborg, June 2013

Erik Skansebo & Yordan Venev

Notations

Roman upper case letters

A	Area [m^2]
E	Young's Modulus [Pa]
G	Shear modulus [Pa]
G_0	Initial shear modulus [Pa]
I	Moment of inertia [m^4]
K_u	Subgrade reaction coefficient [Pa]
L	Length [m]
M	Moment [Nm]
N_c	Load bearing factor [-]
P	Point load [N]
S	Spacing between pile centres [m]
U	Soil resistance [Pa]
U_y	Ultimate soil resistance [Pa]

Roman lower case letters

c_u	Undrained shear strength [Pa]
f_m	P-multiplier (reduction factor) [-]
k_1	Amount of increasing shear strength [Pa/m]
k_0	Parameter used for determining the subgrade reaction coefficient [-]
p	Pressure [Pa]
y	Deflection [m]
z	Depth from ground level [m]

Greek lower case letters

α	Pile-clay interaction factor [-]
δ	Deflection [m]
γ	Shear strain [-] or the elastic slip [m]
μ	Friction coefficient [-]
σ	Stress [Pa]
τ	Shear force [Pa]

1 Introduction to laterally loaded piles

In modern design procedure of piles and pile groups in Sweden, the lateral load capacity of piles is not a big concern. In fact, it is recommended to only utilize the axial capacity and install inclined piles to counter horizontal forces (Holm, 1993) (Rankka, 1991). This is of a great practical value for the engineer since the design of axially loaded piles is straight forward and simple without too much uncertainty. The simplicity comes at the expense of a possibly over dimensioned structure.

Sometimes, however, situations appear when the lateral capacity of piles is of interest. It could concern optimization of an ordinary structure or a pile foundation exposed to a more pronounced lateral force such as for example a pier or a bridge foundation. Furthermore, inclined, or battered, piles could in case of ongoing settlements be subjected to extensive lateral loading from the surrounding soil.

It could also, which is sometimes the case in Gothenburg, concern old foundations where there are simply no or few inclined piles. The subgrade of the city is full of poorly documented, old and unverified pile foundations dating as far back as the 17^{th} century (Almquist, 1929). When the city is evolving it is of great interest to know if the old bridges, streets and retaining walls will endure the changing loading conditions.

Ingvar Larsson[1], senior structural engineer at ÅF-Infrastructure AB, has described several cases in Gothenburg when the lateral load bearing capacity of old timber pile groups could not be properly estimated. He mentioned one example in particular where the responsible engineering company admitted to not have access or knowledge about any reliable methods for calculating the capacity. Considering that the structure had already endured for a very long time it was considered to *probably* hold for a yet some time. The owner, in this case the city of Gothenburg, was given no choice but to take over the responsibility of structural integrity regarding lateral load for the construction. A possible solution could of course be to replace the old constructions. As very often, the economic situation of the city was referred to as incapable of handling such an operation and that option was out of question for the time being. In this thesis the case of Fontänbron in central Gothenburg is examined further and used as an example for when comparing different methods of calculations of lateral stability.

In Sweden there is a lack of research on this matter, although some recommendations exist. The Swedish Pile commission has published a report (Svahn & Alén, 2006) concerning how to assess the lateral capacity of piles in a rather simplified manner. The authors, Per-Ola Svahn and Claes Alén present equations for different special cases and how to combine them to fit different conditions. The procedure is based on the most internationally commonly used method of analyzing lateral capacity of piles, and consequently it is also one of the most internationally debated procedure.

In other countries, for example in the United States, it is more common to utilize the lateral capacity of the piles and thus more research is produced there. Generally it can be said that although many have tried, it has been proven hard to account for the complex nature of the soil in a hand calculation procedure. Methods like that often rely heavily on field test results and site specific data which is problematic to apply in a general way.

[1] Structural engineer at ÅF Infrastructure AB, interviewed January 2013.

A common approach is to utilize for example the finite element method to be able to add more parameters and complexity to the calculations. Modern FE software, such as Ansys and Abaqus, also makes it accessible for engineers to go even further and model the pile-soil interaction with the help of 3D solid continuum elements. The latter method has the possibility to overcome many of the assumptions necessary for the simplified methods but in return demands more time, computer power and knowledge about FE design.

1.1 Group effect

The capacity to withstand lateral force for a group of piles is often less than the sum of the piles individual response. When the piles are spaced too dense their stressed zones in the soil overlap each other as showed in Figure 1. This is commonly referred to as the shadowing effect and is acknowledged fact by the research society (Brown, et al., 1987) (Poulos, 1971)(Rollins, et al., 2006). The amount of reduction in general depends on the spacing between the piles, the type of soil and deflection of the piles.

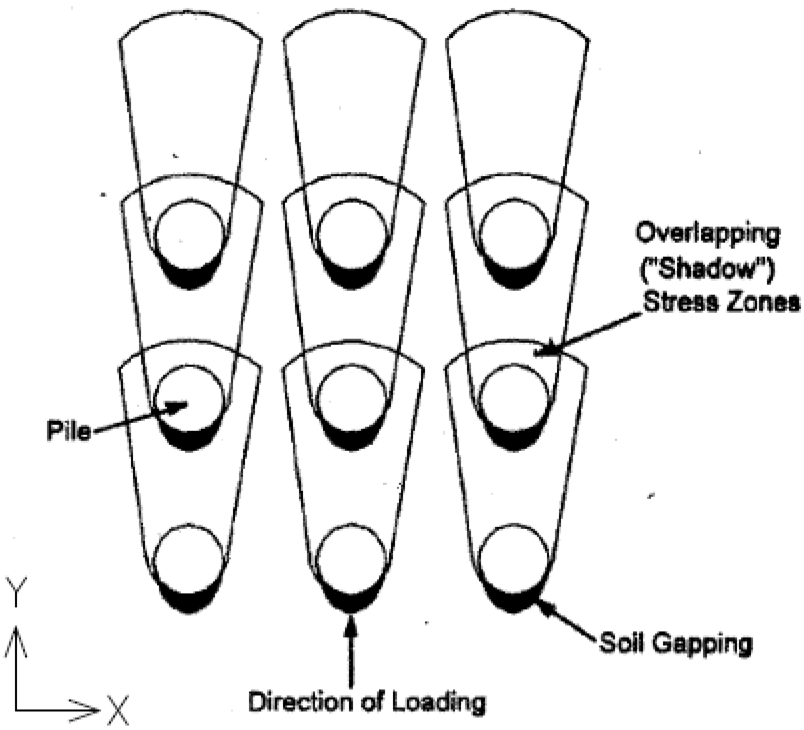


Figure 1. Shadow effect (adopted from Rollins et. al 1998)

1.2 Aim of the study

The purpose of this thesis is to give an overview of the phenomenon of pile-soil-pile interaction within a group of piles. Also to investigate the current state of the art of how to assess the lateral load bearing capacity in a pile group analysis. The focus is on finding a practical method with reasonable accuracy. To exemplify the analysed methods, the calculation procedures are applied in a case study of Fontänbron.

Investigations in this report:

- Literature study of the current state of the art, regarding laterally loaded pile groups.
- Examination of the method for hand calculation presented by the Commission on Pile Research (Svahn & Alén, 2006).
- Adding the p-multiplier to the calculations, a popular reduction factor accounting for the group effect.
- Creation of a 3D Finite element model of a single pile in clay using Abaqus/CAE, including a parametric study of the impact of design choices in the FE-software.
- Using the knowledge gained from the single pile 3D model to create a simple pile group of 3x1 piles.
- Comparing the results of the different methods.

1.3 Methods

Literature study is preformed to gather information on regarding up-to-date research of available analysis methods. The methods which are deemed most common and practical for the investigated case, timber piles in clay, are chosen for further investigation. They are further applied to a realistic scenario to test the different procedures with the aim to recommend a good methodology for engineers in practice.

1.4 Limitations

The thesis is limited to analyse the case of timber piles in clay. The methods for pile group analysis deemed the most common are identified, with focus on which are best suited for practical use. Thus, not all available methods are examined and compared.

Regarding the interpretation of the complex material behaviour of the clay, this study is limited to the use of bi-linear, elasto-plastic, stress-displacement relationships. This simplification is discussed further in the report.

The phenomenon of laterally loaded piles and pile groups is examined in general and exemplified at the end of the report. The example is brief and the clay strength assumptions made are based on the undrained shear strength and no long term effects are accounted for.

2 Common methods for a pile group analysis

Different methods have been developed to analyse the interaction between the pile and the soil. In general they are based on either idealisation of the soil as a series of springs or modelling the earth as a solid continuum. The first method is a structural approach which is used by many engineers and is relatively straight forward and simple, but highly dependent on the estimations made of material properties. Utilising the more extravagant continuum model provides the possibility to overcome many of these estimations but is on the other hand more time consuming to execute.

2.1 Material behaviour of clay

The material behaviour of soils is in general a complicated, non-linear matter. In their publication *Jords egenskaper* (Larsson, 2008), the Swedish institute of geology, SGI, describes, among other things, the strength of clays. A general relationship between shear force and deformation is presented in Figure 2. For undrained conditions with $\upsilon=0.5$, the *E-modulus* is stated to depend linearly on the shear modulus by $E=3G$. As shown in the picture, the stiffness of the clay varies with the deformation, with higher stiffness for small deformations (*strain softening*). G_0 is the small strain modulus and is normally applicable in vertical loading situations, while it is too high for horizontal loading where greater strains are expected (Flemming, et al., 2009).

A common estimation for the *E-modulus* of clay in practical calculations regarding undrained conditions is 150 multiplied by the undrained shear strength. This is a rough estimation that is used for instant settlement calculations . This assumption is adopted for this study, but for a real design situation the *E-modulus* should be estimated using for example a triaxial test, as suggested by SGI (Larsson, 2008).

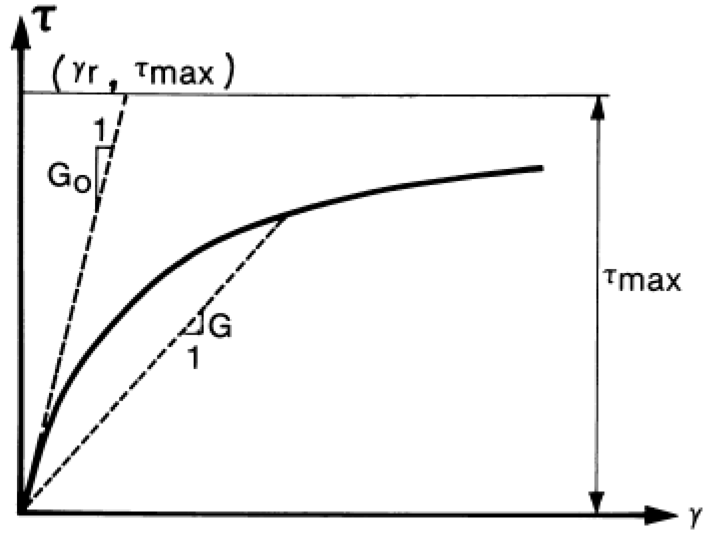


Figure 2. Shear force versus deformation, adopted from (Larsson, 2008)

The pressure, *p*, of the clay on a laterally deflecting, *y*, pile has been thoroughly investigated. In coherence with previously described material behaviour of clay, the response is concluded highly non-linear. How to estimate and construct p-y curves of the response for different soils is presented in (Matlock, 1970) and also in (Reese &

Impe, 2001) with some difference. Examples of different p-y curves are shown in Figure 3, from which it is clear that the relation is highly non-linear and also depends on the soil strength.

The idea of a linear or bi-linear p-y relation is intriguing from an engineer's point of view, since it will make the calculation procedure more practical. A suggestion of how this could be done is presented in (Baugelin, et al., 1977). This recommendation is however a bit rough, and do not properly consider the relation to the undrain shear strength of the clay. For softer clays the elasto-plastic model estimates a higher relative strength than for stiffer clays.

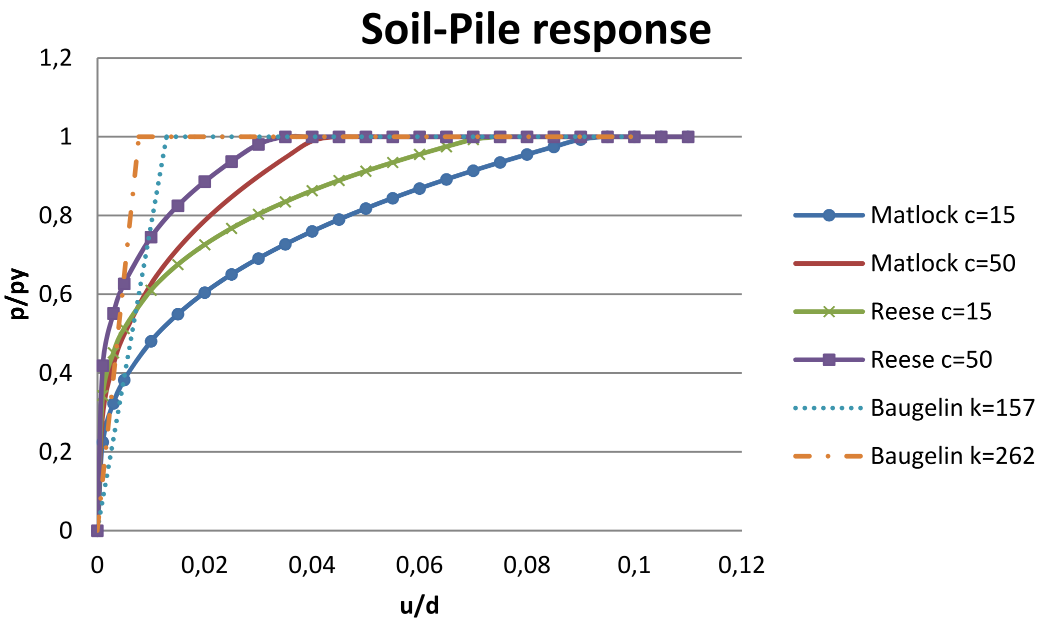


Figure 3. p-y curves as proposed by (Matlock, 1970) and (Reese & Impe, 2001), also linearization from (Baugelin, et al., 1977).

Considering a soft clay, comparing the non-linear curve suggested by (Reese & Impe, 2001) to a bi-linear curve, a value of k=100 appears more reasonable. The value, as explained before, is strain dependent and even if yielding is reached in the top of the clay, the majority of the depth might only experience small strains. To establish a good and reasonable value for k, the calculations should preferably be verified by testing. Figure 4 shows a suggestion for lowered k-value.

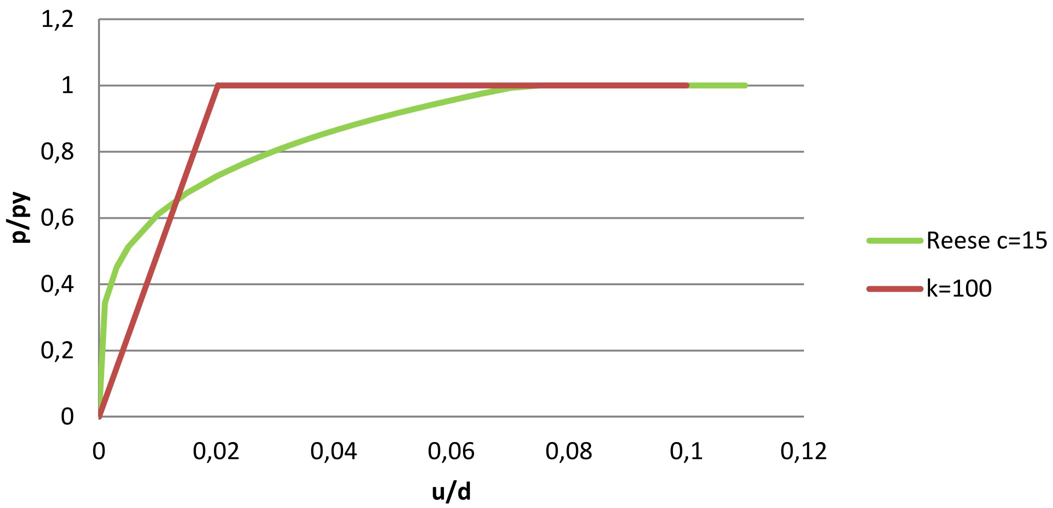


Figure 4. Suggestion for lowered k-value

2.2 Analytical models

Talking about analytical methods for analysing the lateral load bearing capacity of a pile group generally refers to the idea of a beam on elastic foundation i.e. discretisation of the soil as springs. This method of analysing beams is often referred to as the Winkler method, named after the Prague-based Dr E. Winkler who first proposed the idea in the late 19th century (Winkler, 1867).

When representing the soil with springs a series of assumptions has to be made. This method gives an approximation of the reality which is not always accurate and should be used conservatively (Kwangkuk, 2003). The main assumption for this model is that the reaction force of the soil is proportional to the deflection of the pile in the same point. This assumption disregards the more complex reality of the soil, for example the effects of the Poisson ratio. The major advantage with the Winkler model over a more detailed analysis is the possibility to perform hand calculations and faster computer calculations.

The Winkler method has been thoroughly investigated over the years and many variations and improvements have been proposed. In general they can be divided into two categories; one-parameter and two-parameter models. As the name suggests, for one parameter models the relationship between the pile deflection y and the soil pressure p on the pile is depending on one parameter, namely the spring constant K_u. This relation is shown in equation (1). The spring constant is often referred to as a *subgrade reaction coefficient*.

$$p = \frac{K_u}{d} \cdot y \quad [kPa] \tag{1}$$

The factor d is the pile diameter or width. The relation is also commonly expressed as force per unit length of the pile

$$U = K_u \cdot y \quad \left[\frac{N}{m}\right] \tag{2}$$

The subgrade reaction coefficient represents the slope of the p-y curve (see Figure 3) and is most conveniently used as a constant, producing a linear relation. The limiting pressure p_y for when the clay starts to yield is commonly defined as

$$p_y = N_c \cdot c_u \quad [kPa] \tag{3}$$

The relationship presented in eq. (3) was first introduced by (Broms, 1964). The yield pressure depends on the diameter (or width if rectangular) of the pile, the undrained shear strength of the clay and a load bearing parameter. Broms recommended that N_c varies between 8-12 for normal consolidated clays, and lowered to 6 to consider long term effects. The value depends on the interaction between pile and soil where the lower value goes for a perfect smooth pile surface and the higher for a rough surface. The parameter regulates the calculated strength of the clay, which was proven in reality to be very low at the surface and gradually increase until it converges at a certain depth. Broms recommended a value of 0 at the surface which then gradually increases until it reaches the full capacity,9 ,at a depth of *3d*.

Another recommendation is presented by (Randolph & Houlsby, 1984) who derived equation (4) for how to calculate the parameter along the depth with respect to the shaft friction μ. A perfectly smooth pile corresponds to $\mu=0$ and a rough pile thus corresponds to $\mu=1$. This proposal was less conservative than previous and assumed a value of $N_c=2$ at surface level, which their investigation proved to correspond well to their tests.

$$N_c(z) = N_{pl} - \left(N_{pl} - 2\right)e^{\frac{-\xi \cdot z}{d}} \quad [-] \tag{4}$$

$$N_{pl} = \pi + 2\Delta + 4\cos\left(\frac{\pi - \Delta}{4}\right) \cdot \left(\sqrt{2} + sin\left(\frac{\pi - \Delta}{4}\right)\right) \tag{5}$$

$$\Delta = \text{asin}\,(\mu) \tag{6}$$

$$\xi = 0.25 + 0.05 \frac{c_u(0)}{k_1 \cdot d} \quad [-] \tag{7}$$

The different suggestions are visualized in Figure 5, from which it is clear that Broms suggestion is the most conservative one.

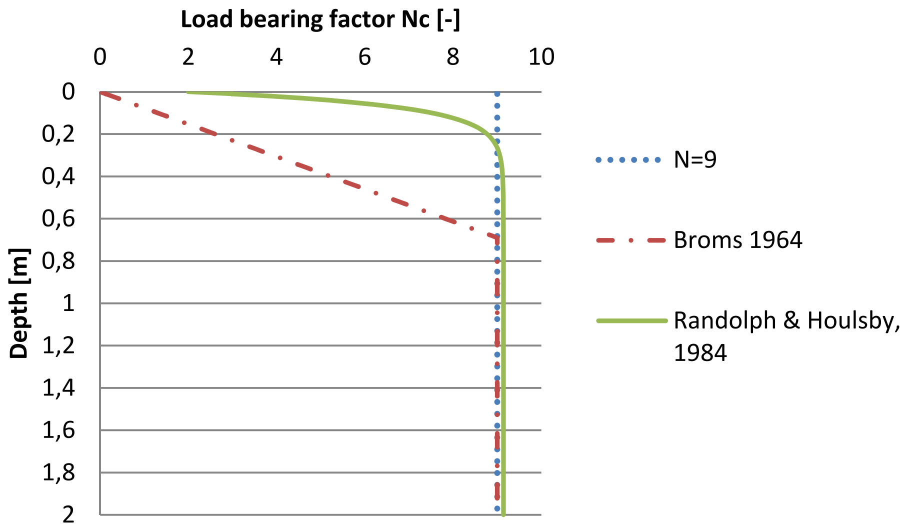
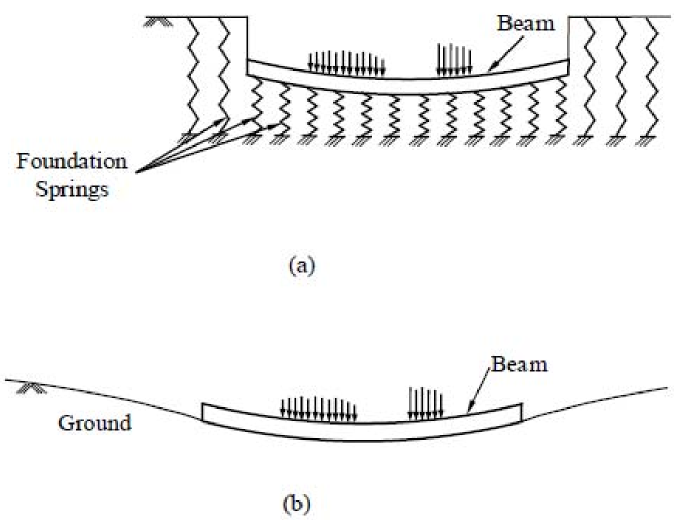


Figure 5. Different suggestions for the load bearing factor Nc

For the two-parameter approach one extra dimension is added in the p-y relation in form of a shear parameter *t* which accounts for the transfer of shear forces in the soil. This method is not as commonly used as the one parameter method since the extra parameter extends the calculations and the advantage of the hand calculation is reduced. It will not be treated further in this investigation. The principal difference between using individual springs and the expected behaviour of the soil is shown in Figure 6a-b. The purpose of the two parameters is to get a response closer to Figure



6b.

Figure 6. a) Deflection predicted by one-parameter model b) Expected soil behaviour (adopted from Basu et al, 2008)

The Winkler model is relatively straight forward to implement and is used either in one of many commercially available computer programs or as a method for hand calculations. A thorough guide of how to use the Winkler model for hand calculations is presented in Pålkommissionen rapport 101 (Svahn & Alén, 2006). Their one-parameter approach utilizes the simplifications of a constant strength of the soil and also a constant stiffness of the pile to make hand calculations feasible. If for example a linear variation of the pile stiffness and the strength of the soil were to be included in the calculations it would be necessary to use for example *finite element method*, FEM technique to solve the problem.

2.2.1 Group effect: The p-multiplier

The Winkler model functions for calculations of single piles and has to be adjusted to incorporate the group effect. A suggestion was made by (Davison, 1970) to reduce the modulus of subgrade reaction linearly with respect to the spacing between the piles. A reduction to $0.25K_u$ for 3d spacing (three times the diameter) up to $1.0\ K_u$ for 8d, is recommended.

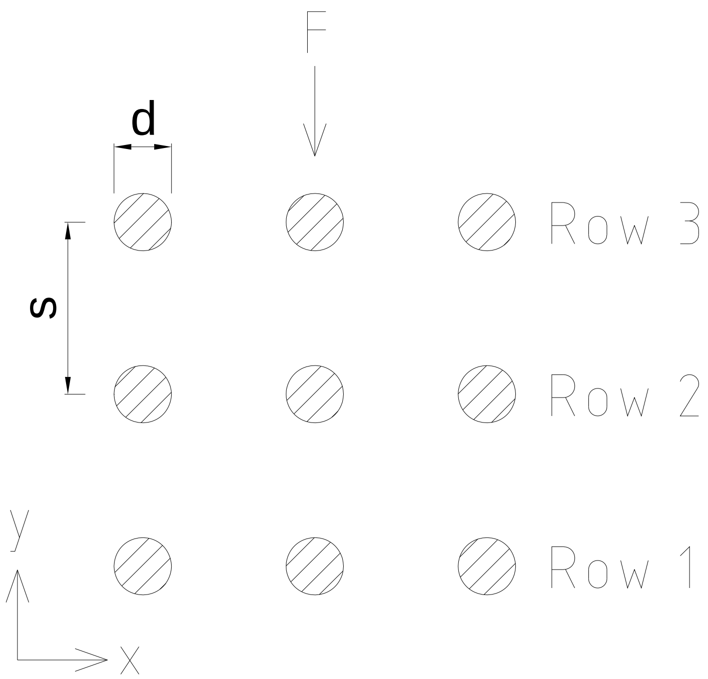


Figure 7. Plan view of a pile group

A more refined, and less conservative, method is the p-multiplier method introduced by (Brown, et al., 1987). It is one of the most adopted methods of accounting for the group effect in a calculation using the Winkler method. Brown noticed in his research how the leading row in a pile group (row 1 in Figure 7) was subjected to a higher pressure than subsequent rows, thus recommending different reduction factors for different rows. As the name suggest the reduction applies to the pressure *p* in the *p-y* relation, as shown in Figure 8.

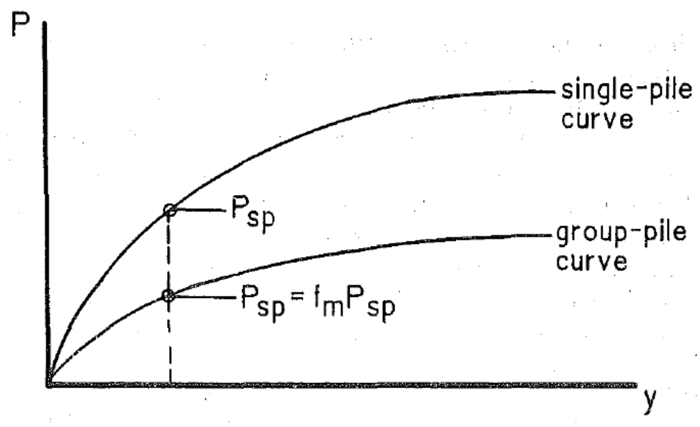


Figure 8. P-multiplier concept (adopted from (Brown, et al., 1987))

The method relies on experimental data and the recommendations for p-multipliers vary between tests in different soils. Table 1 shows a summary of recommendations for p-multipliers with respect to soil properties. In their research, Mohamed Ashour and Hamed Ardalan(Ashour & Ardalan, 2011) explain how the p-multiplier also vary greatly due to the level of loading. The group effect develops from nothing in a group where small deflections are present, and increases gradually with the deflection. The action in a laterally loaded pile is concentrated to the top part of the pile. After a certain characteristic pile length, increasing the length does not influence the results (Randolph, 1981) This means that the group effect will vary with the depth of the pile.

2.2.2 Group effect: Brom’s method

As explained in the last section the p-multiplier approach is purely empirical. An alternative approach is to use Brom´s method which is an analytical method also suited for hand calculations. As explained in the book Piling Engineering a special failure mode was identified and assumed to be dominant for pile groups loaded laterally. Figure 9 shows the failure mode in which the system fails in blocks parallel to the direction of loading. This is assumed to occur when the soil resistance to shear between piles is less than the soil resistance to pressure for a single pile (eq. (8)).

$$p_{y.group} < p_{y.single} \tag{8}$$

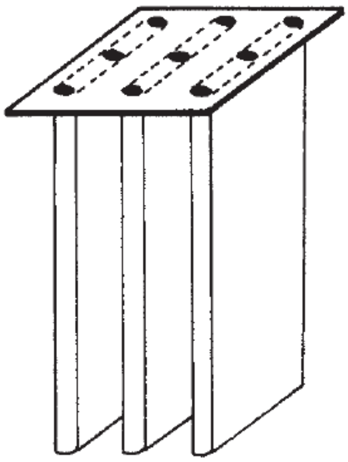


Figure 9. Failure mode for pile groups under lateral loading

(adopted from (Flemming, et al., 2009))

The resistance of a single pile is calculated according to eq. (3). The shear resistance of a row of piles is calculated as the undrained shear strength τ_u of the soil multiplied the pile spacing *S,* see Figure 10.

$$p_{y.group} = 2 \cdot S \cdot \tau_u \tag{9}$$

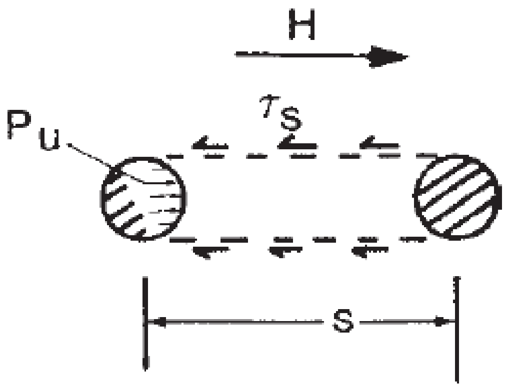


Figure 10. Plan view of block failure under lateral loading

(adopted from (Flemming, et al., 2009))

2.3 Numerical methods (Solid continuum)

When better precision is needed the soil can be represented by a solid continuum instead of springs. The soil is then assumed to completely fill the space of the concerned object, in the contrary to the discrete springs. This assumption does not concern the distance between atoms but is applicable to problems on a scale where the molecular structure can be ignored, which is true for the vast majority of civil engineering applications.

Solid mechanics is a branch of continuum mechanics in which the Euler-Bernoulli beam equation can be found, which is one of the most common practical applications of an elastic solid continuum.(Batra, 2006). The accuracy provided by this method is of course accompanied by a more complex mathematical problem. The general system of equations for an object modelled as an elastic continuum is highly indeterminate. Apart from equilibrium and compatibility conditions like the conservation of energy and the conservation of mass, also constitutive relationships governing the behaviour of the unique material has to be defined. The constitutive relation in the previously mentioned case of a linear elastic beam is Hooke´s law. (Teodoru, 2009)

Concerning the soil a different constitutive relationship has to be used to describe the material behaviour. The soil has a highly non-linear behaviour which means it is not often sufficient to assume only an elastic response of the soil. Only in some cases, when analysing the response in the service state, it can be sufficient with the assumption of linear elastic reaction in the soil.

3 Method application

The purpose of the following chapter is to describe the particular implementation of the calculation methods presented in chapter 2 and how to use the methods in practice. In the first section the approach derived in Commission on Pile research report 101(Svahn & Alén, 2006) is applied as a method for hand calculations. The following section describes the use of the elastic continuum in the FEM software Abaqus. It should also be noted that the hand calculations are most conveniently performed with computer aid, such as MathCad. Involving yielding of the clay, which was necessary in this case, calls for iterative calculations which can easily be programmed with a computer.

3.1 The Winkler method for hand calculations

The calculation procedures described in Commission on Pile research report 101(Svahn & Alén, 2006) is derived for a series of special conditions. These special cases can be combined to represent several different problems. The method was aimed to be simplified and, as mentioned in section 2.2, some assumptions where made to shorten the equations;

- Linear subgrade reaction coefficient

As mentioned in section 2 the soil has non-linear strength properties, but is approximated as bi-linear, see Figure 3 in section 2.1. The yielding strength of the soil p_y beyond which the response is considered as plastic is calculated by equation (3), as explained in section 2.2.

- Constant shear strength of the clay along the pile

As mentioned above, for the procedure used in the particular hand calculations for modelling the soil and its interaction with the piles, the subgrade reaction coefficient K_u is used. K_u is assumed to be a function of c_u, the undrained shear strength of the cohesive soil, according to equation (10).

$$K_u = k_0 c_{ud} \tag{10}$$

In equation (10) k_0 is an empirical value that can be adjusted to account for either short term or long term loading including creep. The shear strength of the soil most often depends on the depth and thus a representative value has to be chosen. The design value of c_{ud} is calculated according to eq. (11), where z is chosen to a depth which is assumed to give reasonable, constant properties.

$$c_{ud} \approx c_u(z) \tag{11}$$

- Constant diameter of the pile

Timber piles do usually not have a uniform diameter, due to the nature of the raw material. The diameter normally varies somewhat linearly with depth which affects the stiffness of the pile. The thickest and stiffest end is usually located at the ground surface.

- Constant yield pressure of clay

The yield pressure of the clay is calculated according to eq. (3), and as discussed in section 2.2 the yield pressure should be considerably lower in the top part of the clay. This consideration is added to the calculations proposed in the report, since not originally included. A comparison of the pile deflection for a lateral load of 20kN with and without varying yield pressure is displayed in Figure 11, where a noticeable difference is shown. For the case of a timber pile in soft clay, the top deflection can be 25% larger for varying U_y under the same load. The difference depends on the level of loading and the stiffness of the clay. For a lower load or stiffer clay where no yielding takes place, the difference is zero. Concerning the suggested value from (Randolph & Houlsby, 1984), the difference to constant Nc is insignificant since almost no yielding occurs for this particular load.

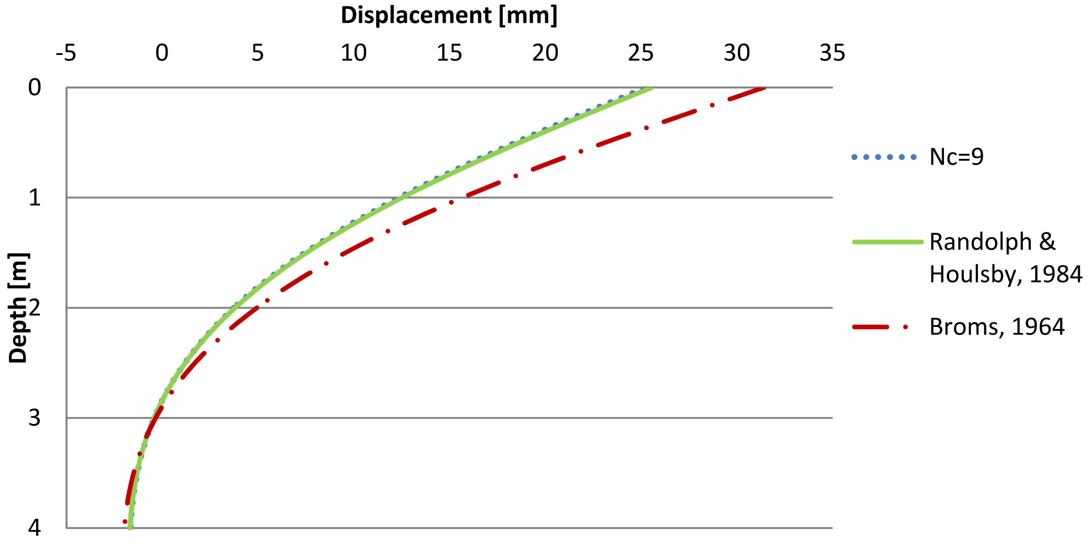


Figure 11. Difference in deflection between constant and varying load factor Nc

The yield pressure U_y changes from constant to non-linear when adding a varying load bearing factor N_c. This was implemented in the calculating procedure by replacing the constant with the integral of the function $U_y(z)$. Examples of how this was integrated are shown in appendix A.

3.1.1 Group reduction factors

Svahn and Alén suggest a group reduction factor in their report (Svahn & Alén, 2006), which was originally proposed by (Davison, 1970) and based on test of pile groups in sand. This method recommends a linear reduction factor to all piles in the group, and should be chosen as explained in section 2.2.1. This produces a single factor which is used to reduce the undrained shear strength in a single-pile calculation. This is rather easy to implement and very convenient for hand calculations.

The previously mentioned reduction factor was proven too conservative in many cases, which led to the introduction of the p-multiplier (Brown, et al., 1987). The reduction factor here evolves to also incorporate the difference in soil resistance between different rows, normal to the loading direction, of piles in a group.

Table 1 shows a collection of different recommendations of p-multipliers. It is clear that variation exists between the suggestions. The multiplier has to be chosen carefully with respect to the individual situation. Although the values of the reduction factor vary, a general conclusion from researchers performing lateral load tests on pile groups has been made. The difference in the deflection between the rows (see Figure 7) decreases from the first row and back. For the third and subsequent rows the reduction factor can be assumed the same. The group effect, as mentioned in section 1.1, varies with the spacing between the piles, increasing the spacing to a certain distance the piles will not affect each other. This study is focused on very dense pile groups, which can be found for example underneath Fontänbron, as explained further in chapter 4. Therefore Table 1 only refers to tests made on similarly spaced pile groups.

Table 1. Collection of suggested p multipliers

Reference	Normalized spacing S/d	Soil	P multipliers, f_m			Type of test
			Row 1	Row 2	Row 3 and trailing	
(Ilyas, et al., 2004)	3	Soft clay (c_u=20 kPa)	0.65	0.5	0.48	Centrifuge
(Rollins, et al., 1998)	2.82	Med. Stiff (c_u =48kPa)	0.6	0.4	0.4	Full Scale
(Brown, et al., 1987)	3	Stiff clay (c_u =72kPa)	0.7	0.6	0.5	Full scale

3.1.2 Iterative calculations in MathCad

For this study, the mathematical software MathCad has been used extensively. In the report on laterally loaded piles from the Commission on Pile Research (Svahn & Alén, 2006), a manual procedure is suggested. This was proven as very time-consuming during the making of this study and instead, an automated procedure was implemented.

To exemplify, an iterative procedure is needed in the calculation of moment equilibrium in the pile between a plastic part and an elastic part. In equation (12) the investigated parameter is the depth z for which the equilibrium exists.

$$M_{plastic}(z) - M_{elastic}(z) = 0 \tag{12}$$

This can be automated using the *root*-function in MathCad as shown in (13). The program returns the first root found in z_y with respect to the chosen tolerance.

The root function should, although convenient, be used with caution. In a published user's guide (PTC, 2007) by the company behind MathCad, the problem is discussed and different solutions to deal with eventual problems are presented.

$$z_y = root(M_{plastic}(z) - M_{elastic}(z), z) \tag{13}$$

3.2 Finite Element Method

The software of choice for this problem became Abaqus/CAE v6.11, which provides numerous possibilities to model the soil and the pile-soil interaction. The software is very general and the computational options come with a responsibility to choose suitable ones for the problem of interest. This section aims to provide some insight in how to construct an accurate pile-soil model in Abaqus. The manual provided together with the software contains extensive and good formulated information about both theory behind the calculations and practical information on how certain options function. It is highly recommended to consult the manual if uncertainty occurs.

3.2.1 Constitutive models

There are many options in Abaqus for how to model the soil, including cam-clay, modified cap, extended Drucker-Prager and Mohr-Coulomb plasticity models. They work in conjunction with an appropriate elastic model which can be either elastic or porous-elastic. Together they describe how the material behaves under stress and formulates a constitutive model.

For this investigation the Mohr-Coulomb plasticity model was used together with an elastic model. This setup made it possible to model short term, undrained conditions in the soil. The stress-strain relationship is then represented by a bi-linear curve as discussed in section 2.2. Table 2 describes the necessary input data that is required for the calculations, and the corresponding values used throughout this study.

Table 2. Input for elasticity and Mohr-Coulomb plasticity model

Elasticity		**Mohr-Coulomb**	
E-modulus	150c_u	Friction angle	0
Poisson ratio	0.495	Dilatation angle	0
		Cohesion yield strength	c_u
		Abs. plastic strain	0

The cohesion yield strength c and the internal friction angle Φ is used by Abaqus to calculate the failure envelope using the equation

$$\tau_u = c - \sigma \tan(\phi) \tag{14}$$

During the analysis the principal stresses are calculated and if they together reach the failure envelope according to the Mohr-Coulomb model, yielding occurs.

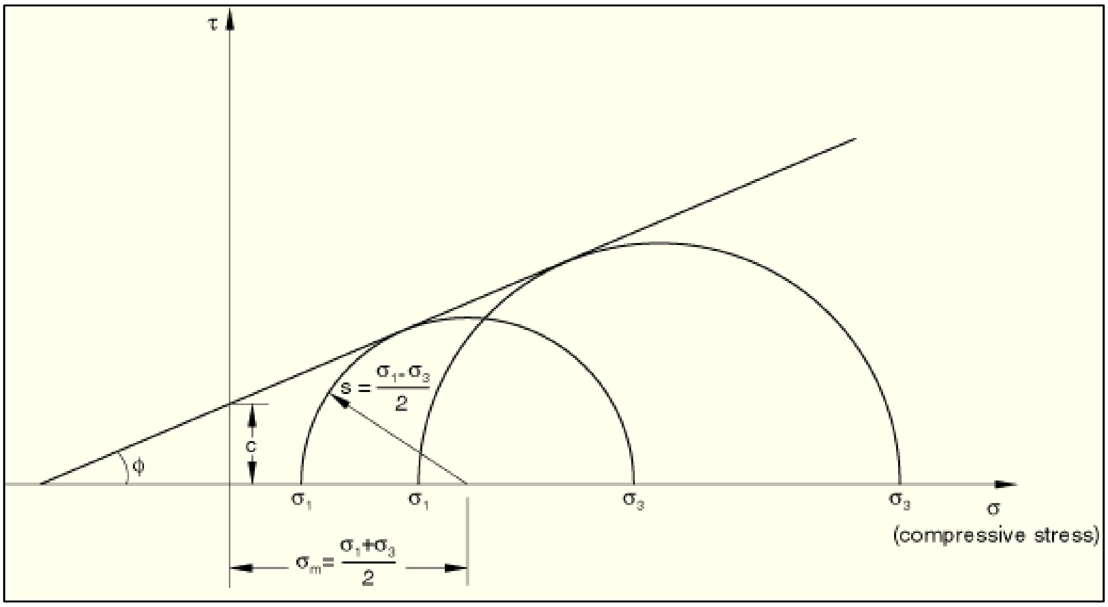


Figure 12. Mohr-Coulomb plasticity model (adopted from (Simulia Inc., 2013))

3.2.2 Element types

As mentioned section 2.3**,** the soil can be represented by a 3D solid continuum. In Abaqus this corresponds to their selection of solid elements. It is most straight forward to also model the pile with solid elements, which makes it easy to assign interaction properties between the two parts. Maryam Mardfekri with partners compared FE-models with shell and beam elements to hand calculations based on the Winkler model (Mardfekri, et al., 2012). They concluded that using beam elements to model a hollow-core steel pile together with 3D solid elements for soil, and thus neglecting the diameter of the pile, was inappropriate and gave unreliable results. Shell elements on the other hand were better and gave results closer to their analytical results.

Apart from the elements, it is provided a great deal of additional options and the elements has to be chosen carefully depending on the type of problem they are meant to describe. The process of choosing appropriate configuration to a specific problem most often call for experimentation.

Concerning the piles it is a known issue that solid elements have problem catching bending behaviour (Broo, et al., 2008) (Simulia inc., 2013). One way to address the problem can be to change the element type to a structural element such as beam or shell. Other possibilities are available by changing element properties to correspond better to the specific problem. This section describes the effects of elements type, elements properties and the size of the mesh. The results are compared to analytical results in order to determine the best configuration. Parametric studies are performed on a cantilever beam as shown in Figure 13, where the bending stress is obtained by Navier´s formula (15) at the outermost fibre in the cross section. The cantilever is used as a good enough representation of the pile for when accurate hand calculation results can be obtained.

$$\sigma(x) = \frac{M(x)}{I} z_{max} \tag{15}$$

The deflection is calculated by equation (16), based on the elementary case of a cantilever beam. The small and therefore often neglected deformation due to shear is also included.

$$\delta = \frac{PL^3}{3EI} + \frac{PL}{AG} \tag{16}$$

Three similar studies were performed, two for a cantilever with solid elements and one for a cantilever with shell elements. The two models with solid elements were performed on piles with different diameters, and also using different mesh variations. The different elements used in this study is presented and described in Table 3.

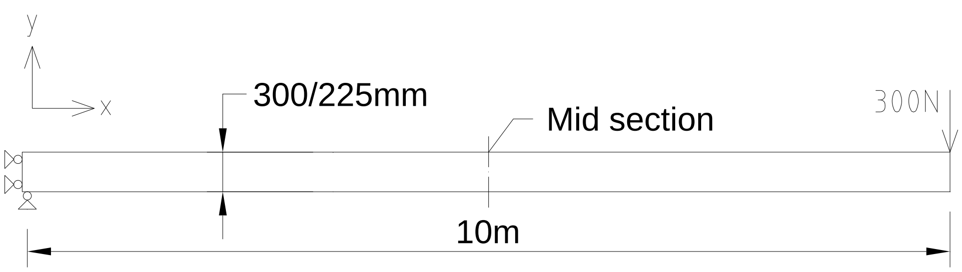


Figure 13. Cantilever studied to determine element properties

Table 3. Descriptions of element types used in this study

Element type	Description
C3D8	Linear 8-node brick element
C3D8R	Linear 8-node brick element with reduced integration
C3D8I	Linear 8-node brick element with incompatible modes
C3D20	Quadratic 20-node brick element
C3D20R	Quadratic 20-node brick element with reduced integration
CIN3D8	Linear 8-node infinite elements
S4	Liner 4-node quadrilateral element
S4R	Liner 4-node quadrilateral element with reduced integration
S8R	Quadratic 8-node quadrilateral element with reduced integration

3.2.2.1 Solid elements

In general the Abaqus manual recommends Quadrilaterals and Hexahedral elements, which have good convergence rates. Triangular and tetrahedral elements should according to the manual only be used in non-critical areas since they do not usually provide acceptable accuracy (Simulia Inc., 2013). The shape and proportions of the elements are also of a great importance. A test comparing the different meshing techniques *sweep* and *structured* is presented in Table 4. The Sweep algorithm creates a radial mesh around the mid axis and the structured algorithm creates elements with better proportions. The test results show that a structured mesh provides better results regarding the deflection than a slightly more dense radial mesh.

Table 4. Different meshing techniques

	Element size						
Element type	***x-direction***	***y-direction***	u_{max}	Δu	σ_{max} ***at center***	$\Delta\sigma$	***Mesh algorithm***
	[mm]	**[-]**	**[mm]**	**[%]**	**[MPa]**	**[%]**	**-**
Hand calculation			**28,0**		**0,57**		
C3D8R	40	8	29,1	3,9%	0,51	10%	Structured
C3D8R	20	8	30,0	7,3%	0,51	10%	Sweep

Choosing between first order and second order elements, it can be said that second order elements generally provide a better accuracy and is more effective in bending-dominated problems. However they can have trouble converging if the problem involves complicated interactions between different parts in the model. A second order, or quadratic, element has more nodes than a first order element and thus requires more computational time. First-order elements with triangular and tetrahedral geometry should be avoided since they can be too stiff and do not converge fast with decreased mesh size (Simulia inc., 2013). A test displayed in Table 5 shows that 2nd order elements gives better results, as expected, compared to their linear version with regard to bending.

Table 5. Comparison between 1st and 2nd order elements

Element type	Element size: x-direction	Element size: y-direction	u_{max}	Δu	σ_{max} at center	Δσ
	[mm]	[-]	[mm]	[%]	[MPa]	[%]
Hand calculation			**28,0**		**0,57**	
C3D8R	20	12	28,9	3,2%	0,53	7%
C3D20	20	12	27,9	-0,3%	0,57	-1%

The option to use a lower order integration of the element stiffness, i.e. lower the number of integration points in each element is called reduced integration. This is available for both first and second order element. Guidelines from the Abaqus Analysis User´s Manual states that reduced integration generally provides more accurate results for the 2nd order elements. For the first order elements the option can provide distortion problems, which often can be avoided using the hourglass control option. It is recommended to use reduced integration if the 1st order elements are used in bending problems (Simulia Inc., 2013). Table 6 shows a comparison between linear and quadratic elements with reduced integration.

Table 6. Reduced integration on 1st and 2nd order elements

Element type	Element size x-direction	Element size y-direction	u_{max}	Δu	σ_{max} at center	Δσ
	[mm]	[-]	[mm]	[%]	[MPa]	[%]
Hand calculation			**28,0**		**0,57**	
C3D8R	100	4	37,8	35,6%	0,52	22%
C3D20R	100	4	27,9	-0,2%	0,66	-17%
C3D20	100	4	27,9	-0,2%	0,60	-6%

To a higher computational cost the option to use incompatible modes can also be used to improve the results in bending. This adds extra degrees of freedom inside the elements which purpose are to eliminate effects of so called “parasitic shear stress” and “artificial stiffening” which are explained further in the Abaqus Analysis User´s manual(Simulia Inc., 2013). A comparison between solid brick elements together with options reduced integration and incompatible modes are shown in Table 7. The option to include incompatible modes improves the results and for the finest mesh it shows very small differences to the hand calculations.

Table 7. Adding Incompatible modes to a brick element

Element type	Element size x-direction	Element size y-direction	u_{max}	Δu	σ_{max} at center	Δσ
	[mm]	[-]	[mm]	[%]	[MPa]	[%]
Hand calculation			**28,0**		**0,57**	
C3D8R	20	12	28,9	3,2%	0,53	7%
C3D8R	20	16	28,2	0,8%	0,53	6%
C3D8I	20	12	28,6	2,3%	0,57	0%
C3D8I	20	16	28,1	0,5%	0,57	0%

Figure 14 a-d shows the full results from the performed cantilever study regarding the solid elements. The diagrams show that the density of the mesh has a considerable influence on the results in this problem. To accurately calculate the deflection and stresses in the cantilevers, a very dense mesh is needed. From the results presented in Figure 14 b-c it is clear that the stresses are on the safe side if incompatible mode is used, in contrast to ordinary and reduced integration elements. Regarding the displacement displayed in Figure 14 a-b, all elements overestimate the results. To get close to zero error, element C3D8I with a very dense mesh should be used.

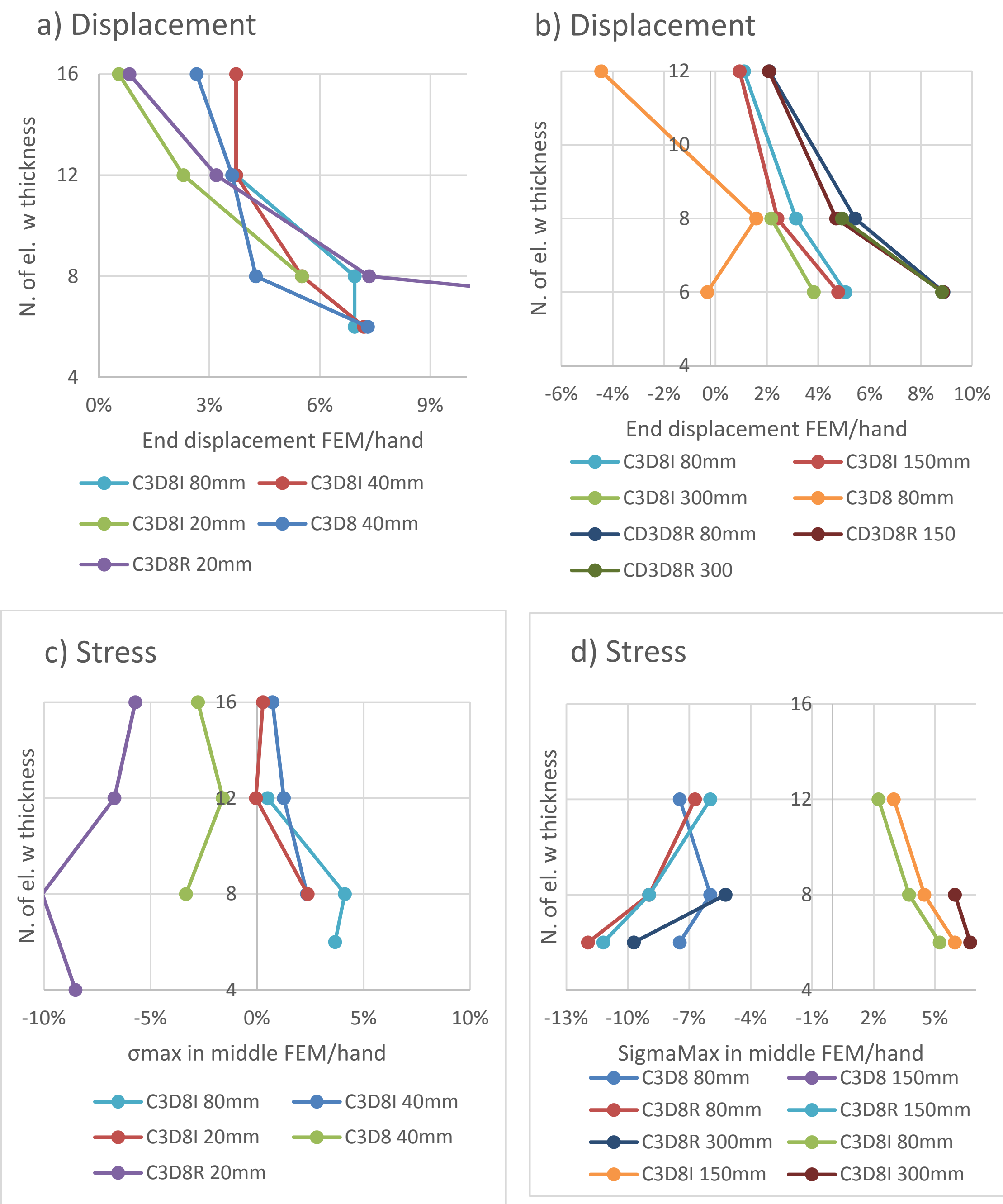


Figure 14. a) Displacement convergence for d=30cm b) Displacement convergence for d=22.5cm c) Stress convergence for d=30cm d) Stress convergence for d=22.5cm

3.2.2.2 Shell elements

Concerning the shell elements it can be said that they are applicable for structural members subjected to loading effects where bending is dominant. This is typical for thin walled sections where the bending resistance of the material is governing and the shear stresses magnitude is slightly pronounced. For instance, different thin wall steel profiles, considered as rather slender structural members, are supposed to be successfully modelled by using shell elements.

However, modelling the geometry of a pile with a solid circular cross section can be a quite challenging task because the shell elements are appropriate for members with rather small thickness in comparison to the other dimensions. Therefore the shell elements are meant to describe a surface which is to be assigned a certain thickness representing the rigidity of the structural member. In order to calculate the cross-section behaviour it has to be decided the thickness integration rule and the number of the integration points within the thickness. Based on the Abaqus User´s manual for the particular parametric study with the shell elements it is decided to use *Gauss quadrature* with two integration points recommended as appropriate for linear problems. By using the following rule Abaqus automatically also provides a proper *fraction* which gives an *offset* of 0,5, see Figure 15. The fraction defines the distance between the shell's midsurface and the reference surface which contains the elements' nodes. For offset = 0,5 the reference surface is the top surface which makes it able to read the stresses at the top location regarding the section points. The offset is an option available in the *Field output variable for Probe* when one would like to read the stresses from a FEM model (Simulia inc., 2013).

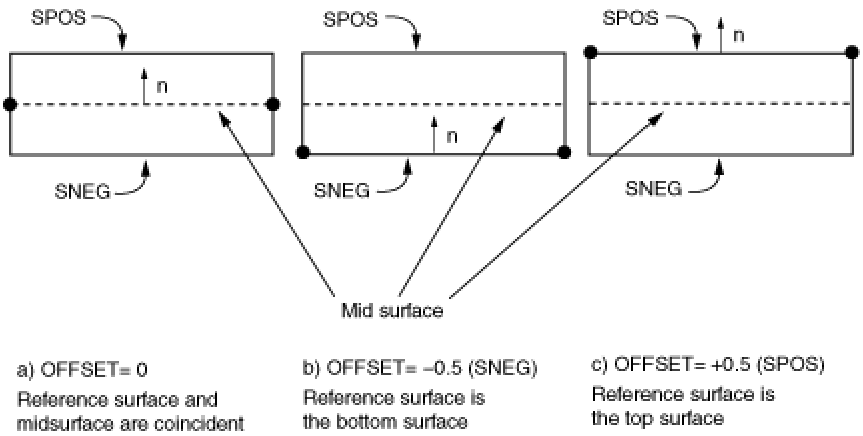


Figure 15. Shell offset and the position of the reference surface (adopted from (Simulia Inc., 2013))

In Table 8 is presented a comparison between the results from the analytical solution and a cantilever with the small wall thickness, Figure 16a, and different mesh sizes applied. Apparently, relatively small differences are noticed when using mesh size of 10mm and 50mm in x-direction. The mesh size of 10mm is found to be the most appropriate especially considering the smallest difference in the deflection Δu. On the contrary, the difference in the stresses $\Delta\sigma$ for the first mesh used, size 250mm, is the smallest. This little difference is not expected for such a coarse mesh, however it is disregarded since the stresses close to the support of the cantilever deviate significantly and seem not to be a stable index for making comparisons.

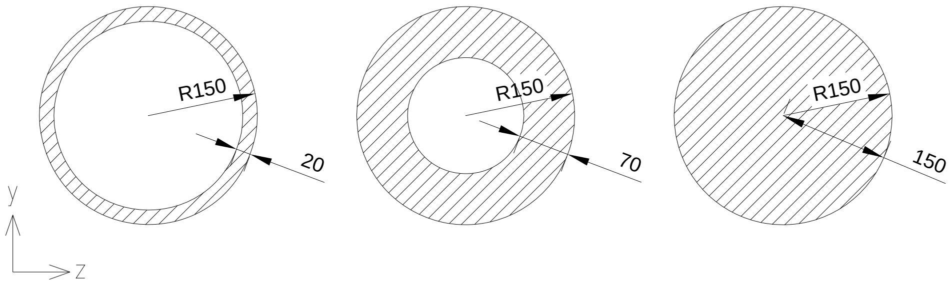


Figure 16. Cross sections with different wall thicknesses: a) 20mm b) 70mm c) 150mm

Table 8. Comparison between the hand calculation results and the models with different mesh sizes

Element type	Element size x-direction	Wall thickness	u_{max}	Δu	$σ_{max}$ middle	$σ_{max}$ support	Δσ center	Δσ support
	[mm]	[mm]	[mm]	[%]	[MPa]	[MPa]	[%]	[%]
hand calculation			**64,2**		**1,30**	**2,60**		
S4R	250	20	77,4	-20,6%	1,27	2,49	1,9%	4,1%
S4R	50	20	66,8	-4,1%	1,26	2,43	2,9%	6,4%
S4R	10	20	64,6	-0,6%	1,27	2,44	2,6%	6,0%

To summarise, the results about the deflection *Δu* and the stresses *Δσ* for the test model presented in Table 8 show rather small difference but still do not converge to zero. However for the case with such a thin wall shall elements a convergence is expected and demanded. This proves that the cantilever alternative with shell elements is not fully accurate even though using thin walls.

Additionally, the option *reduced integration* is used as it is described for the solid elements in section 3.2.2.1. Conversely to the model with the solid elements, the option for shell elements is only available when they are linear i.e. first order elements. In Table 9 is shown the difference in the results between the models with the option switched on and off and also the case with quadratic elements when it is not available. Apparently, little advantage occurs for the model with the quadratic elements but the differences in the deflection and the stresses still exist and do not converge to zero. The demanded accuracy with the shell elements is not reached.

Table 9. Reduced integration on 1st order elements

Element type	Element size x-direction	Wall thickness	u_{max}	Δu	$σ_{max}$ middle	$σ_{max}$ support	Δσ center	Δσ support
	[mm]	[mm]	[mm]	[%]	[MPa]	[MPa]	[%]	[%]
hand calculation			**64,2**		**1,30**	**2,60**		
S4R	10	20	64,6	-0,6%	1,27	2,44	2,6%	6,0%
S4	10	20	64,6	-0,6%	1,27	2,45	2,5%	5,8%
S8R	10	20	64,5	-0,5%	1,27	2,45	2,5%	5,5%

Nevertheless the parametric study over the shell elements and the possibility to use them to model a pile with a solid circular section is studied in further. Mainly two approaches are employed to make the thin wall pipe analogical to a circular pile with a solid cross section:

- Increase the thickness of the shell elements until it is equal to the radius of the pile so that the cross section becomes practically solid, see Figure 16c.
- Use higher *E-modulus* for the material of the pipe cross section so that the stiffness *EI* is kept the same as for a solid cross section. The new *E-modulus* is calculated as follows:

$$E_{new} = E \frac{EI_{solid}}{EI_{hollow\ core}} \tag{17}$$

Where *E* is the modulus as decided for the timber material used for the pile.

With respect to the first approach a parametric study on the same cantilever beam as discussed is carried out by changing the thickness assigned to the shell elements as showed in Figure 16a,b. Following the results, observe Table 10, a tendency occurs: the more increasing the thickness of the shell, the bigger difference is found in the maximum deflection compared to the analytical results. The stresses read for the middle section of the cantilever beam seem to be wrong as well. A possible reason for the difference in the deflection could be the size of the radius for the shell's middle section surface. The radius was chosen smaller in such a way that the middle surface of the shell, Figure 17, provides the cross section to match the outer size of the real circular cross section for a certain thickness assigned.

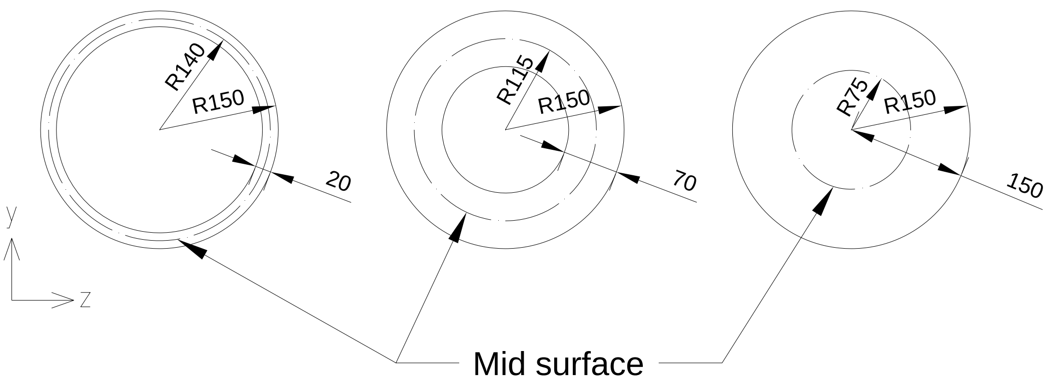


Figure 17. Cross sections with mid surface shown

However this technique gives wrong results for the deflection and for the stresses in the mid length which makes it not applicable. Moreover, such a tendency is expected since the accuracy in the results is not satisfying even for the shell models with thin walls as discussed above in the same section.

Table 10. Results with different shell thicknesses used

Element type	Element size x-direction	Wall thickness	u_{max}	Δu	σ_{max} middle	σ_{max} support	Δσ center	Δσ support
	[mm]	[mm]	[mm]	[%]	[MPa]	[MPa]	[%]	[%]
hand calculation			**30,4**		0,62	1,23		
S8R	10mm	70mm	32,3	-6,1%	0,59	1,04	4,5%	15,2%
hand calculation			28,0		0,57	1,13		
S8R	10mm	150mm	42,3	-51,3%	0,67	1,14	-18,2%	-0,6%

Considering the second approach by changing the *E-modulus* for the material, the shell elements are still not working in a preferable way. To keep the same stiffness but using a pipe cross section is estimated as a good correlation regarding the maximum deflection. It was compared with the corresponding deflection from a model with a solid cross section, see Table 11. However, that is not enough since the stresses differ and do not provide satisfying convergence in the results. The results from the table prove it and show impossibility to compare stresses between different cross sections with the same *EI* parameters.

Table 11. Results for increased E-modulus and stiffness EI correlated to a solid cross section

Element type	Element size x-direction	Wall thickness	u_{max}	Δu	σ_{max} middle	σ_{max} support	Δσ center	Δσ support
	[mm]	[mm]	[mm]	[%]	[MPa]	[MPa]	[%]	[%]
Hand calculation for the equivalent solid section			**28,0**		0,57	1,13		
Hand calculation for the same pipe section			**28,0**		1,30	2,60		
S8R, E modulus transformed	10mm	20mm	28,1	-0,5%	1,26	2,45	-123,4%	-116,9%
according to formula (15)				-0,5%			2,7%	5,5%

Based on the observations stated in the following section the shell elements were regarded as an inappropriate choice for modelling a solid timber pile with FE software. As follows the FE models presented later in the thesis were created without considering the usage of the shell elements.

3.2.3 Geometry

The geometry used in the FEM software to describe a certain structural system is an important part of the modelling and strongly depends on the specific conditions in the analysis. A three dimensional geometry is preferable in order to catch the complicated behaviour of a pile-soil system with surface interaction included. However, when modelling a pile in soil the geometry might be simplified to a two-dimensional system as well. That could be possible with certain assumptions which may not give as good accuracy as the 3D model. The most significant reason behind is in the horizontal forces which have to be transferred from the pile surface to the soil. The soil resistance may not be fully considered meaning that part of its contribution is disregarded and the stressed state in the 2D model is not as in the reality, see Figure 18.

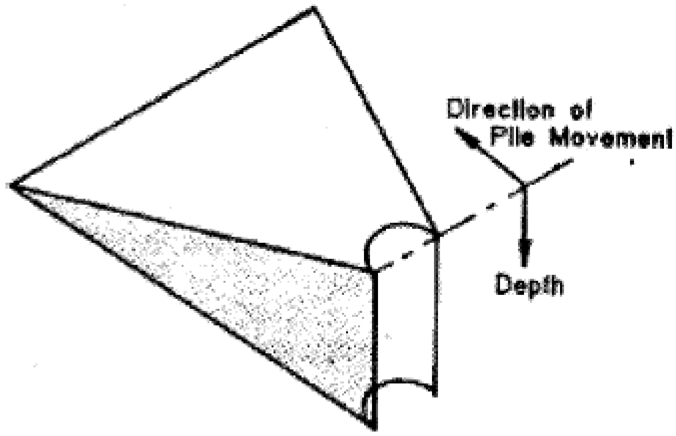


Figure 18. Shear zone in the soil for a laterally loaded pile (adopted from (Kwangkuk, 2003))

Furthermore the geometry has to be constructed accounting for the type of loading applied. For instance, when the loads are applied at the pile's head a higher concentration of stresses appears in the soil regions close to the ground surface level. That requires fine and well arranged mesh for these regions. Therefore the geometrical shape of the model has to be suitable to allow for creating a dense mesh in the critical regions, for example, observe Figure 25a in section 4.4.4.

A possible measure to reduce the computational time, which affects the geometry, is to use the symmetry of the real structure which is to be modelled. For instance, if the original structure has a cylindrical shape as could be the case for the pile-soil model, just half of the cylinder has to be modelled. That is done by means of a symmetry plane which goes through the middle of the pile-soil system, parallel to the direction of loading.

Another aspect about the choice of the geometry for a pile-soil model is the global size of the soil. The model in Abaqus is supposed to represent, as much as possible, a real soil medium. In order to simulate the infinity in the horizontal direction it is required to use a relatively big soil model. To decide a reasonable size the most relevant is to test different geometry alternatives and study the influence on the results. Then a minimal geometrical size could be determined for a precision which is demanded. However, such a pile-soil model simulating the infinity of the soil medium in an accurate way, is expected to be rather heavy and slow to operate with. In order to overcome this problem a special attention has to be given to the mesh arrangement and to the mesh elements as well. A practical approach is to divide the soil into two main parts: inner and outer field regions where a mesh with different density is used. The inner field regions are considered as critical since they are the most stressed, thus finer mesh is required. The point for the far field regions is instead to be meshed with bigger mesh elements in order to reduce the computation time and the complexity of the model.

A possible alternative technique for the far field regions is to use *infinite elements* instead. They are typical for analysing problems where the area of interest is rather small compared to the surrounding media. The infinite elements can be successfully combined with finite elements providing stiffness and linear behaviour of the materials used in static analyses (Simulia Inc., 2013). The main advantage considered is saving computational time and resources. Other advantage could be regarding the geometry size of the soil. Once it is decided as big enough to represent the infinity in the horizontal direction, it may allow to be reduced if using the infinite elements. However that is to be applied after detailed study over the problem.

3.2.4 Interaction

In Abaqus there are several approaches for modelling interaction between two bodies. For a certain type of interaction, appropriate interaction properties have to be assigned in order to account for the nature of the materials used. The purpose is to simulate the real contact in the most accurate way.

To assign a *tangential behaviour* is one of the available options for modelling mechanical contact in Abaqus. It is also possible to combine it with another contact property - *normal behaviour.* That is found as especially relevant in the case of a laterally loaded pile in soil. Since horizontal forces are applied on the pile they have to be transferred to the soil by friction and pressure which represent the mechanical contact in the interface. Therefore using tangential and normal behaviour allows simulating both friction and pressure respectively.

Particularly in Abaqus, when the tangential behaviour property is assigned a friction formulation has to be specified. The different formulations provide choices for no friction, friction based on the Mohr-Coulomb theory (Figure 19a), and a stiff contact with no slip allowed. The methods where the Mohr-Coulomb theory is employed require defining a friction coefficient μ which is the ratio between the equivalent shear stress and the contact pressure in the interface. By means of μ, the critical shear stress is calculated that governs the maximum friction force that can be reached between the two contact surfaces.

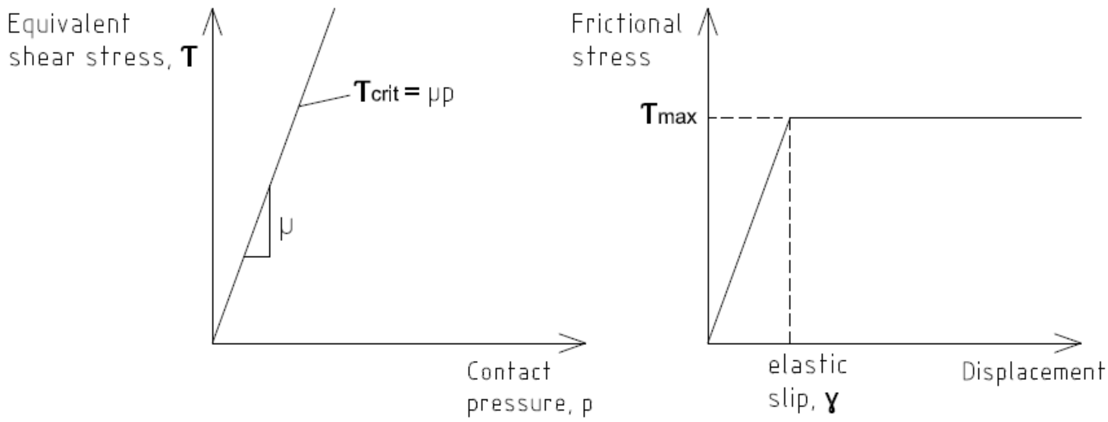


Figure 19. a) Mohr-Coulomb friction b) Penalty friction formulation in Abaqus (Simulia Inc., 2013)

The penalty method available in Abaqus is a friction formulation which enforces constrains in the friction behaviour of two bodies in contact, see Figure 19b. The constrains to be introduced as input data are the elastic slip γ, the friction coefficient μ and the shear stress limit τ_{max}. Depending on how big the elastic slip is defined it is allowed to occur a certain relative motion provided that two of the bodies are still 'sticking'. The choice for the elastic slip and the friction coefficient might be hard without the presence on any real testing data. In such cases a sensitivity analysis is suggested in order to study the influence of those parameters over the interaction.

The shear stress limit τ_{max} is an additional parameter available in the penalty formulation in Abaqus. It is a limit on the magnitude of the equivalent stresses in the interface. The stress limit provides an option to make sure that the material strength is not surpassed. When either the stress limit or the critical stress is reached, slipping occurs. (Simulia Inc., 2013). Normally it is calculated in accordance to the shear strength of the materials. If the problem concerns soil material and especially clay used for pile foundations then is possible to determine the stress limit through the soil properties. In the third edition of Piling Engineering (Flemming, et al., 2009), a formula is recommended for calculating the shaft friction which can serve as the shear stress limit. By means of the undrained shear strength for clay it is estimated:

$$\tau_s = \alpha \cdot c_u \tag{18}$$

The value for the empirical factor α has been determined based on pile load tests. For clay material with low shear strength a value of 1 can be assumed.

4 Case study – Fontänbron

In 1990 the bridge Fontänbron in central Gothenburg had its superstructure replaced and during that work the load capacity of the support constructions were evaluated. The assessment was ordered by Gatukontoret AB and performed by ProjektTeamet AB. There are two major issues with those calculations:

- Lateral stability not checked
- Uplift force in one of the piles, which it is unable to carry

This chapter implements the previously discussed methods, FE analysis and Winkler method, on this foundation and explains the difficulties involved with the two models. The choice of parameters and how they affect the results of the two chosen design approaches is presented.

4.1 Site conditions

The bridge is located in Brunsparken, central Gothenburg and is daily passed by many trams & busses. Originally the bridge was much smaller but after the canal at Östra Hamngatan between Brunsparken and Lilla Bommen was filled in the 1920's it was extended to the west and became significantly wider. Due to this and other changes at the site during the history of the bridge it is now supported by several different foundations. The documentation of the structure is poor and this case study focuses on the section previously investigated by ProjektTeamet AB as mentioned before. The analysed section is presented in Figure 20, which displays the piles and the different loads coming from the super structure, soil and traffic. For more details about the calculation procedure refer to (ProjektTeamet AB, 1989).

Concerning the soil, a geotechnical study of the nearby surroundings has been performed (Norconsult AB, 2012) and values for the undrained shear strength of the clay were taken from the test performed closest to the bridge. The clay is considered to be soft, slightly over consolidated with undrained shear strength varying according to

$$c_u(z) = 14.5kPa + 1kPa \cdot z \tag{19}$$

There are, at the time of writing, no data on the strength and stiffness of the timber material in the piles. Instead properties according to timber strength class C18 was assumed in the calculations. There are indications pointing towards this being a conservative assumption, but since no data is available the choice is a bit arbitrary. The timber pile properties used are presented in Table 12.

Table 12. Timber pile properties

Length	15m
diameter	30-15cm
$E_{0,mean}$ parallel to grain	9Gpa

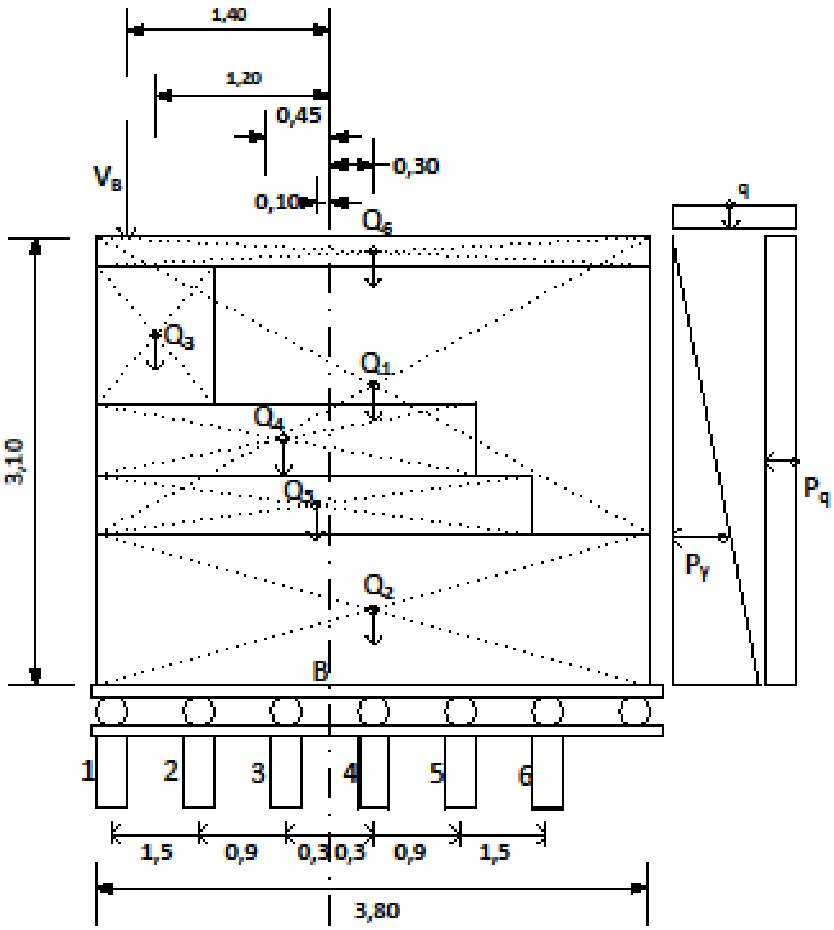


Figure 20. Section over a bridge support at Fontänbron

4.2 Previous investigation of the foundation

The calculations performed by ProjektTeamet AB (ProjektTeamet AB, 1989) was examined and as previously mentioned the lateral stability of the piles were not treated. They based their calculations on geometrical assumptions presented in Figure 20, with the conclusion that the 7th pile from left was in tension. Some different changes to the structure were presented to overcome the tension force, but the original structure was decided by the owner to remain and thus no changes were made.

For this report a new stability analysis was performed with the same assumptions as before, but only including the 6 piles in compression. The resulting pile forces are presented in Table 13. The forces are presented per pile. The spacing is assumed to 60cm centre-to-centre, and divided by the diameter of the pile the normalized spacing becomes S/D=2 at the top and 4 at the bottom of the pile.

Table 13. Pile reaction forces

Pile (from left)	1	2	3	4	5	6
Normal force [kN]	92.3	75.3	58.3	41.3	24.3	7.4
Horizontal force [kN]	3.7	3.7	3.7	3.7	3.7	3.7
Total horizontal force [kN]	22.3					

4.3 Hand calculations – Winkler method

As presented in previous chapters, to calculate the lateral capacity the Winkler method is implemented in general according to Pålkommissionen rapport 101 (Svahn & Alén, 2006). This section explains the assumptions and input data for those calculations.

4.3.1 Assumptions for soil strength and pile geometry

In the chosen hand calculation procedure, the diameter of the pile along with the strength of the clay are treated as constants. In reality, both values vary with depth and thus representative values for both parameters have to be chosen. For this report the values for the pile diameter and the undrained shear strength of clay were picked from a reference depth, z_c. The influence on the result for different depths is presented in Figure 21. The investigation was done for a representative load case and the difference did not show any tendency to depend on the applied load. For the first three meters the difference is negligible, and for greater depths the displacement increases more noticeably. Since the deflection of the pile in general is concentrated to the top 2-4m, a reference depth of $z_c=3m$ is considered as representative.

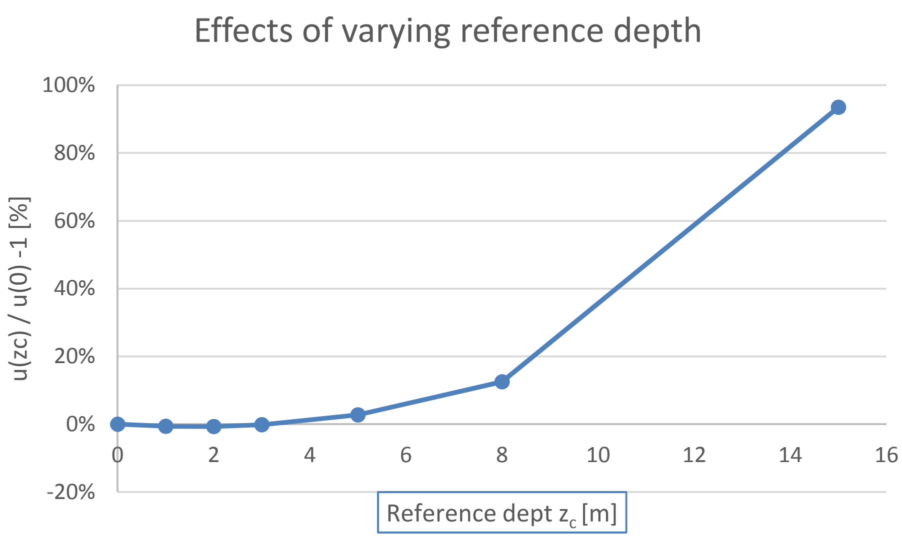


Figure 21. Difference in the pile head displacement for different reference depths.

4.3.2 Determination of the failure mode for the pile-soil system

In case of transversally loaded piles it is important to distinguish the dominant failure mode that is possible to happen. The following calculation method analyzes a single pile out of the group of piles from the case study. The purpose is to determine the structural behaviour of the piles in the soil when loaded transversally. According to the type of the pile *short* or *long,* the failure mode which dominates for ultimate loading is determined: it is either failure in the soil, for short piles, or also structural failure in the pile for long piles.

In order to perform the calculations, the bending capacity of the piles is to be considered together with the yielding strength of the soil.

The timber pile properties and the pile geometry are as presented for Model 1, see section 4.4.

The parameters used for calculating the soil response are the undrained shear strength $c_u(3m)=17.5kPa$ and the load bearing factor $N_c = 9$.

From equilibrium conditions the maximum bending moment which is possible to appear in the pile is found to be $M_{max}=684kN/m$ which rather exceeds the resistant moment of the piles of $M_R=8.6\ kN/m$. The result clearly determines the piles as long which means that both the failure modes of the soil and failure in the piles could be expected and accounted in case of lateral loading. Regarding the detailed calculation procedure, refer to Commission on Pile Research report 101, p.18 (Svahn & Alén, 2006).

4.4 FEM – Model 1, Single pile

The model represents a laterally loaded timber pile in soil medium created by using the FEM software Abaqus/CAE v6.11. It is a simple test alternative with general assumptions which facilitate the comparison with the hand calculation model. Especially to study how the pile-soil interaction works two main alternatives of Model 1 were created:

1. Single pile in clay with contact surface interaction
2. Single pile in clay merged without interaction

For the aim of the comparison between Model 1 and the hand calculations the corresponding parameters were adjusted to be identical. Then it was possible to provide understanding on how the extra parameters used in Abaqus influence the results from Model 1 provided the same assumptions as in the hand calculations. Furthermore the conclusions from the results were considered in the analysis for Model 2.

4.4.1 Geometry

Model 1 was constructed with a 3D geometrical shape with dimensions as shown in

Figure 22. The advantage of the symmetry was taken by considering only a half of the cylindrical shape of both the pile and the soil around. The pile section's diameter was chosen to be constant in depth meaning that the pile is assumed to be straight. Calculated as 0,225m the section diameter is the mean value between the top and the bottom one along the length of the pile, see Table 12 in section 4.1. Regarding the soil medium radius, it was assumed to be 4m providing a proportional shape. In that way the distribution of the stresses in the horizontal direction was considered not to be

influenced from the boundary conditions on the outer sides of the half cylinder. Defined with such geometry, the model was estimated to require reasonable computational time.

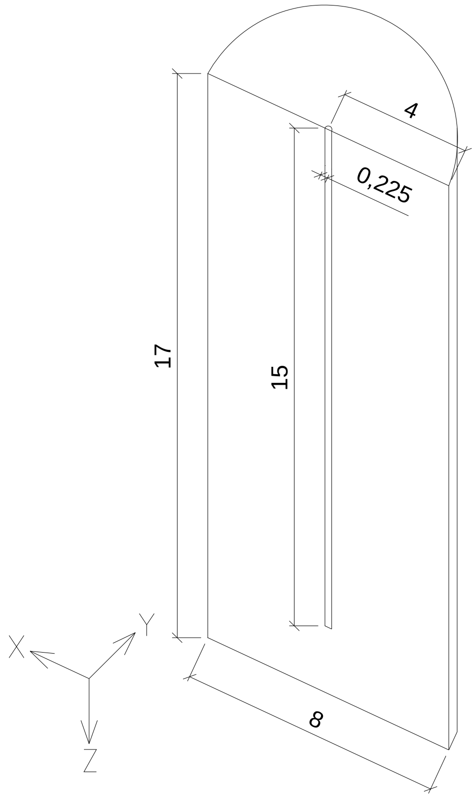


Figure 22. Geometry of Model 1 [m]

4.4.2 E-modulus

In Model 1 the *E-modulus* used for the pile is 9GPa as presented in Table 12, section 4.1. The corresponding timber material which is represented by means of the modulus was introduced in Abaqus as linear elastic.

The *E-modulus* for the soil was chosen based on the formula stated in section 2.1 where $E = 150c_u$. This estimation was believed to be rather conservative meaning that for clay with $c_u = 17{,}5kPa$ the stiffness is calculated as $E = 2{,}625MPa$. Therefore to examine how much the result from Model 1 is influenced it was introduced one more value for the $E = 200c_u$. The results from the comparison between two of the interaction models show 11.9% less displacement for the model with $E = 200c_u$. The comparison was carried out with the rest of the parameters being equal.

4.4.3 Loads and Boundary conditions

The loads used in Model 1 are as follows:

- 0.5*30kN=15kN lateral load applied as a point load on the top of the pile
- Gravity load for the pile and the clay

The lateral load was obtained in relation to the symmetrical shape of Model 1 and thus, calculated as half of the full magnitude which corresponds to the real geometry of the pile. The load is assumed as a reasonable, allowing for the pile-soil system to be analyzed in the ultimate limit state or close to it. Higher magnitudes were considered as not relevant. Furthermore, couple of different steps were defined in the model so that it was possible to apply the load in portions. In that way the computer analysis was facilitated and the likelihood of errors was reduced, especially for the more complex models discussed further in the thesis.

Apart from the lateral load a gravity load was introduced as well. The magnitude was automatically calculated from Abaqus by the geometry of the model and the density of the materials used. In order to prove the effect of the gravity load it was tested two different interaction models with and without the gravity load assigned. In the case without gravity load a 30% higher magnitude of the head displacement of the pile was observed that is significant and should not be underestimated.

With respect to the boundary conditions, as a first, has to be mentioned that all the restrictions except those for the symmetry plane are applied to the soil block. The pile being a deep foundation is supposed to be restricted only from the soil and therefore the interaction behaviour was introduced, see section 4.4.6.

Regarding the symmetry plane in the model, it is restricted the displacements in the normal direction to the plane, as well as the rotations in the plane of the symmetry. The outer side surface of the soil medium and the bottom are fully locked excepting the vertical displacements for the sides. Contrary, the alternative without the vertical restrictions for the side surface is considered as not accurate since additional tension forces might be created in the soil after the gravity load is introduced.

4.4.4 Mesh technique

The mesh created for the two alternatives of Model 1 can be seen at Figure 23. Two types of mesh techniques were used in the FE analysis for both the interaction and the merged alternatives. The purpose of the coarse mesh was to allow the model quick computational time and check whether it was assembled correctly.

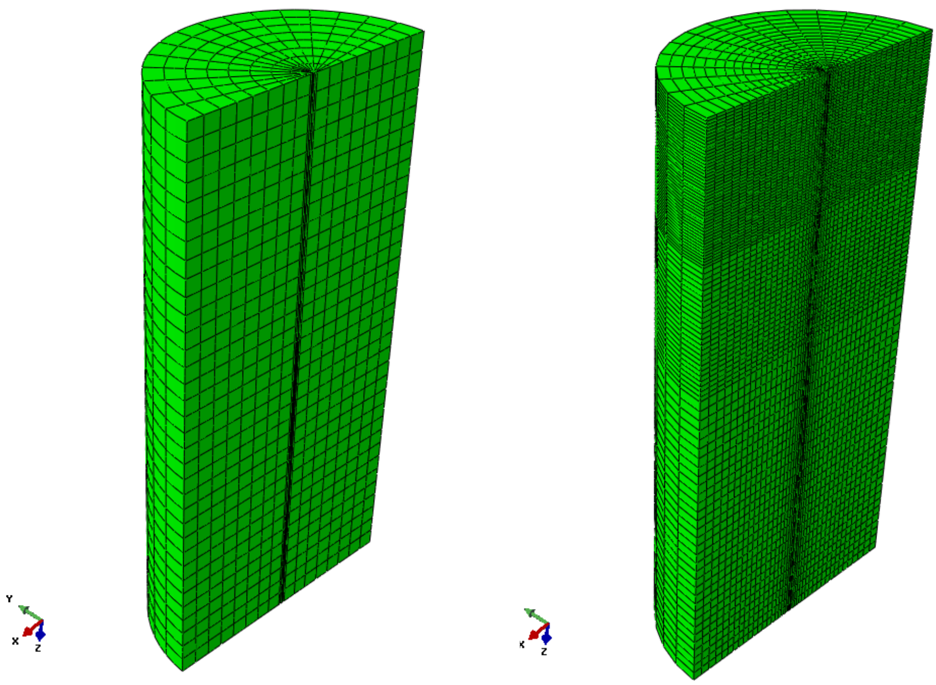


Figure 23. Mesh size types: a) coarse mesh b) refined mesh

Indeed, when using the coarse mesh size the model was able to run without distorted elements.

The second type of mesh used in Model 1, as shown at Figure 25 b), was refined for both parts of the model: the pile and the soil medium. Based on the results from the cantilever study, the refined mesh for the pile consists of 8 elements in the horizontal direction instead of 6 as for the coarse mesh, Figure 24. Thus the error in the results due to the mesh size was approximately between 2-6% depending on the type of elements used, see Figure 14b.

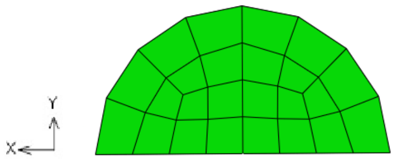


Figure 24. Refined mesh for the pile cross section

The mesh size for the soil, Figure 25 b), was refined with the purpose of obtaining well proportioned mesh elements in the areas close to the pile head where the stresses are the highest.

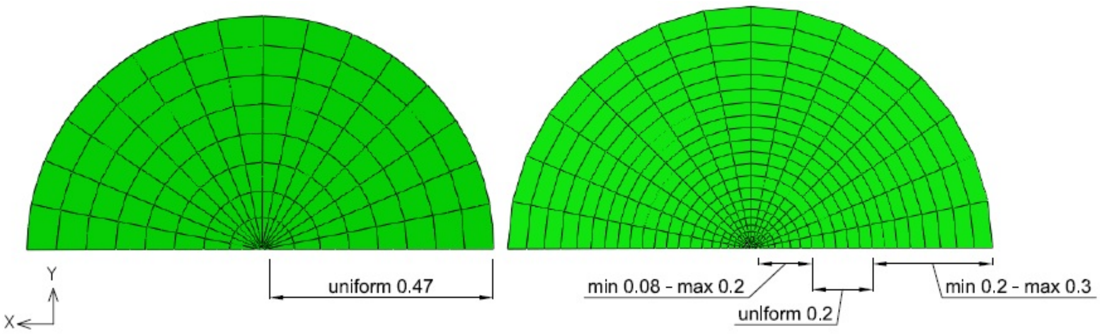


Figure 25. Mesh size in plan: a) coarse mesh b) refined mesh by using Bias elements [m]

Accordingly, the results from a comparison between the two different dense meshes show a noticeable difference in the displacement of the pile head for both the merged and the interaction alternatives, see Table 14. The difference was expected following the tendency from the cantilever study, Figure 14 in section 3.2.2.1. It can be concluded that the coarse mesh presented at this stage is not suitable for Model 1 and gives results that one should not rely on. However the refined mesh could be more trustful but still needs to be calibrated with a finer mesh even and draw a final conclusion if the results converge.

Table 14. Displacement results, coarse vs. fine mesh

		"Merged" alt.		"Interaction" alt.	
Mesh and elements	Direction	"Coarse" mesh	"Fine" mesh	"Coarse" mesh	"Fine" mesh
Pile elements	-	C3D20	C3D20	C3D8	C3D8
Clay elements	-	C3D20	C3D20	C3D8	C3D8
Mesh size pile [m]	horizontal	6 elements	8 elements	6 elements	8 elements
	vertical	0.5	0.08/ 0.15/ 0.30	0.5	0.08/ 0.15/ 0.30
Mesh size clay [m]	horizontal (radial)	0.47	0.08-0.2/ 0.2/ 0.2-0.3	0.47	0.08-0.2/ 0.2/ 0.2-0.3
	vertical	0.5	0.08/ 0.15/ 0.30	0.5	0.08/ 0.15/ 0.30
Pile head displacement [mm]	horizontal	13	19	14	43

4.4.5 Mesh elements type

The mesh elements of interest for Model 1 were two main groups – first and second order elements, or linear and quadratic. For the merged alternative of Model 1, the 1st order elements were used in combinations with additional options as reduced integration and incompatible modes. The interaction alternative of Model 1 was tested with the same set of elements as well. As mentioned in section 3.2.2, the short names of the elements used in Abaqus are as follows: C3D8, C3D8R, C3D8I and C3D20.

Regarding the results delivered, for the merged model it was observed that the horizontal displacement of the pile is slightly affected from the type of the elements for the mesh, see Table 15. Apparently the elements are not the most important choice considering the results. Thus using C3D8R could be accurate enough and save computational time even though C3D20 were recommended as a better alternative from the cantilever study. In particular, C3D20 was only tested for the merged alternative of Model 1 since it is not recommended for interaction problems with modelling of contact between two materials, see section 3.2.2.1.

The results from the interaction model provide a little bit bigger margin in the pile displacement when different elements are used for the mesh. According to the values from Table 15 it is observed that C3D8R elements provide approximately 7% bigger displacement compared to C3D8 elements under equal conditions for two of the model alternatives. A possible reason for this tendency could be due to the specific usage of the reduced integration option for the linear elements, C3D8R. According to the Abaqus manual they are recommended in particular for bending problems, (Simulia Inc., 2013). However the stressed state in the pile is assessed as not pure bending due to its solid cross section subjected to lateral loading. Therefore using linear elements with reduced integration (C3D8R) may not provide more accurate results than C3D8 for the analysis of Model 1.

The last element type C3D8I was more complex and problematic to be employed in the analysis. The incompatible mode option was not able to be used in Model 1 when gravity load was assigned. Since the gravity load was considered as an important action it was decided not to perform analysis without it, thus disregard C3D8I for Model 1.

Table 15. Displacement results for different mesh elements

	Test alternatives				
Mesh elements	**"Merged" alt.**			**"Interaction" alt.**	
Pile elements	C3D8R	C3D8	C3D20	C3D8	C3D8R
Clay elements	C3D8R	C3D8	C3D20	C3D8	C3D8R
Horizontal pile head displacement [mm]	20	19	19	43	46

4.4.6 Interaction

The interaction used for the interface between the timber pile and the clay was chosen as a mechanical contact *surface to surface*. The surface of the pile was defined to act as a *master* surface and the one for the soil as a *slave* surface respectively. Hence, when the load is applied the pile acts as a rigid deformable body which provides the clay to deform accordingly, (Simulia inc., 2013).

Another important option to be discussed regarding the interaction is the sliding formulation. The choice has to be decided under the assumption that the friction contact between the pile surface and the clay allows relatively small sliding. Thus, for Model 1 *small sliding* is considered as more relevant and accurate to use instead of *finite sliding* formulation, (Simulia Inc., 2013).

Following the analysis, the next step after assigning the interaction is to determine the corresponding parameters, see Table 16. *Tangential* behaviour and *normal* behaviour were chosen as relevant properties describing the interaction problem in the most proper way. Moreover, it has been carried out a sensitivity analysis to provide understanding on how the most important parameters affect the final results. Model 1 was used as a test alternative to test some of the different options listed in Table 16.

Table 16. Interaction parameters

	Properties chosen for Model 1	**Alternative options**
Interaction	Mechanical contact "surface to surface"	"Node to surface"
Theory model	Frictinal behaviour by Mohr-Coulomb theory	Various
Sliding Formulation	Small sliding	Finite sliding
Interaction properties		
Tangential behaviour		
Friction formulation	Penalty friction	Various
Friction coeff [-]	0.7	0.5
Shear stress limit	NO	Optional limit can be set
Elastic slip, absolute distance [m]	0.001	0.0005
Normal behaviour, hard contact with penalty constraint method		
Separation after contact	Allowed	Not allowed
Tie contact for the tip of the pile	Assigned	-

First the friction coefficient was tested with two different values of 0.5 and 0.7 under the other parameters in the model being equal. Apparently the coefficient influences slightly the result for this particular case of a pile in clay medium. The value of 0.7 provided just 2% bigger displacement of the pile head which is considered as insignificant.

As second the influence of the elastic slip was studied. The parameter was defined as an absolute distance and the values tested are 0.001m and 0.0005m. No difference was observed in the results of the head displacement with a precision of 1mm.

The third tested option was *separation after contact* which is to be defined in the *normal behaviour* property of the interaction. As a consequence it was noticed a quite significant difference of more than 100% in the results depending on whether the separation is allowed or not. The displacement observed for the head of the pile is much higher when the separation is allowed and this option was the matter of choice for Model 1 as well. The reason for that is the capacity of the clay as a soil material which does not bear any tension forces. Therefore on the tension side of the pile there is practically no lateral support from the soil which could lead to gap formation, see Figure 26.

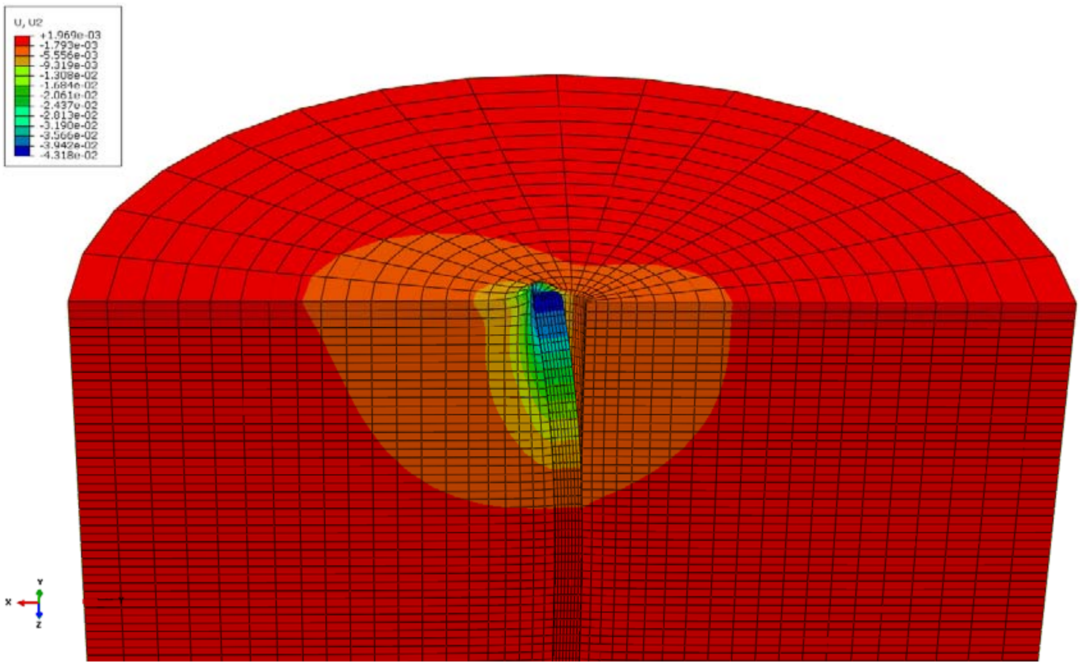


Figure 26. Deformed shape of model 1 with the option *separation allowed*

To summarise it turns out that the interaction parameters, the friction coefficient and the elastic slip are not highly sensitive regarding the displacement of the pile. However the separation option influences much more and significantly determines the behaviour of the pile related to the soil. For Model 1 allowing the separation was considered as closest to the real scenario.

4.5 FEM – Model 2, Single pile

This model is an improvement of Model 1, to also include more correct pile geometry and varying soil properties with depth. Properties are in general based on experience from Model 1, with some additional tests. The interaction parameters used for the model are presented in Table 17. In early testing of the model, convergence problems arise using the sliding formulation small sliding. This led to the usage of *finite sliding* for further analysis of Model 2 with no additional comparison made for the option.

Table 17. Interaction properties for Model 2

Friction coefficient	0.7
Interaction discretization	Surface to surface
Sliding formulation	Finite sliding
Allow separation	Y

4.5.1 Geometry

For the geometry, the same dimensions as for Model 1 were used as a starting point, see Table 18. For a model like this, where a part of something big (the clay) is taken out of its context and analysed separately, the size is very important. It has to be concluded that the model is sufficiently large so that the boundary doesn't affect the result. Two different radius of the clay was tested with and without the use of infinite elements, see Figure 28.

Table 18. Geometry of Model 2

Near-field radius	2m
Far-field radius	3-4m
Infinite elements	Y/N
Pile diameter	0.3-0.15m
Soil layers	8

When changing the outer radius of the model a small difference was noticed in the results. Less displacement in the head of the pile as a result of the smaller radius indicates that the boundaries affect the results as expected. Further, if using infinite elements in the far field region a small decrease in the displacement for the 4m radius is detected. This indicates that also the larger outer radius of 4m affects the results, since it would be expected to have, for a sufficiently large model, the same results for regular and infinite elements in the far field region. However, the differences were quite small and the 4m outer radius together with infinite elements was concluded as a good enough representation of the soil, see Figure 27.

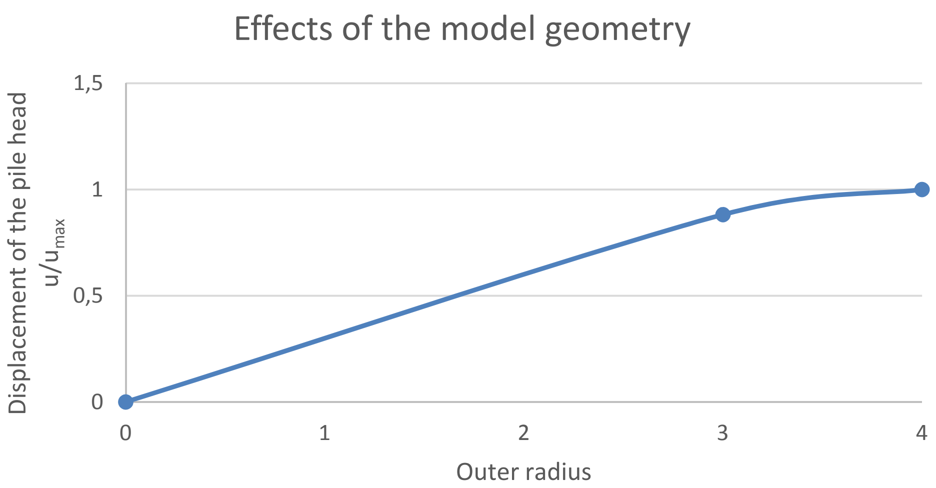


Figure 27. The effects of the model geometry on the pile head deflection on a model with infinite elements in the far field region

4.5.2 E-modulus

Young's modulus was chosen, as explained in section 3.2.1 to $E = 150c_u$. To include the varying soil properties with depth, the soil was divided into 8 layers, with constant properties for each layer as presented in Table 19.

Table 19. Soil layers

Layer	*Thickness of layers [m]*	*c_u [kPa]*	*Young's modulus [Mpa]*
1	1	15.0	2.25
2	1	16.0	2.40
3	1	17.0	2.55
4	1	18.0	2.70
5	1	19.0	2.85
6	2	20.5	3.08
7	8	25.5	3.82
8	2	30.5	4.58

4.5.3 Loads and Boundary conditions

Boundary conditions were chosen according to results from Model 1, with symmetry direction, radial directions on outer surface and bottom surface locked. The model was loaded with gravity and a lateral load of *0.5*20kN=10kN*. The lateral load was halved in the same manner as for Model 1, accounting for the symmetrical geometry. Additionally, the load was divided in increments in order to make it possible to extract stresses for different magnitudes of the load.

4.5.4 Mesh and elements

In general, the mesh was created denser in the regions of interest. From previously made research it is established that the most action in the pile occurs in the top 2-4m, which was also confirmed in this analysis. In depth, three regions with different mesh sizes was created. In radial direction, bias elements were used with a linear increase of the element size away from the pile centre. The mesh in the clay was created to match the pile mesh, for which different sizes was tested. 6, 8 and 10 elements with the thickness of the pile were tested in the same conditions. Mesh properties are displayed in Figure 28. Consistent with the cantilever study it was concluded that a very dense mesh was needed to get a converging solution. The cantilever study suggested at least 12 element with the thickness of the pile. To limit the computational time 10 was chosen for this model, for which the solution started to converge. Based on the cantilever study, the displacement is in this case expected to be overestimated by up to 5%. In the cantilever study the incompatible mode element, C3D8I, converged best to the hand calculations and is therefore the element of choice for the pile. For the soil, the brick element C3D8R with reduced integration is used.

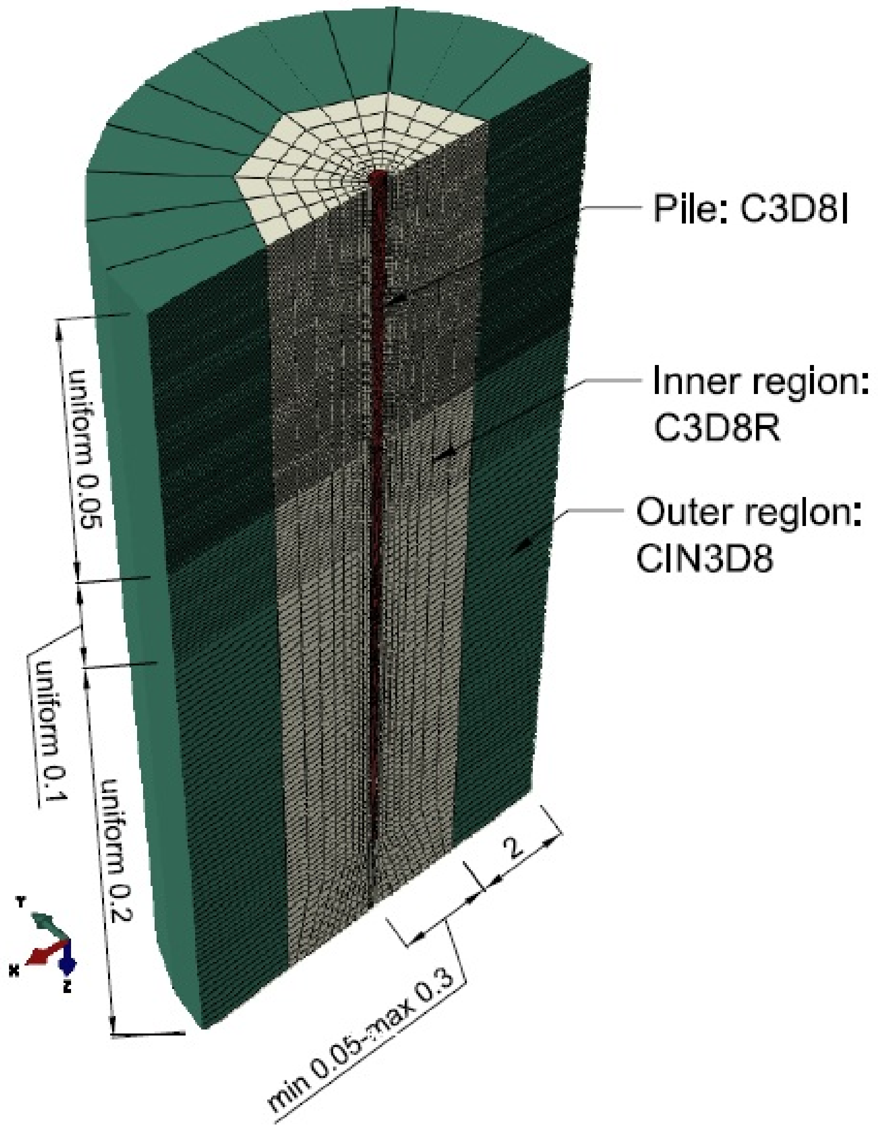


Figure 28. Different Element types

The infinite elements used are not directly supported in Abaqus/CAE. To apply the elements, the input file created when starting a job has to be altered. A region assigned with a mesh type of same geometry as the infinite elements can easily be transformed by editing the input file in a text editor. To be able to find the correct region in the input file, it is suitable to assign a different element for the infinite region than for the rest of the model. For this study the infinite element CIN3D8 was tested, which has the same geometry as the acoustic element AC3D8. To apply the changes, a new job has to be created in Abaqus using the input file.

4.5.5 Non-linear geometry

When the option to use large displacement theory, *Nlgeom*, is switched on, the displacement of the pile increases noticeably. For a load of 10kN the difference was 5.6mm at the head of the pile.

4.6 FEM – Model 3, Pile group

The group model consists of 3x1 piles and its main purpose is to confirm the group effect explained in previous chapters. This model incorporates the findings from the single pile models, and is aimed to provide understanding of how the foundation under Fontänbron, and pile groups in general, behaves. Interaction was assumed according to Table 17 and material layers according to Table 19 with the exception of the last layer which is not used, see also Figure 29.

One noteworthy finding from the single pile analysis performed is that introducing friction to the model, greatly increases the computational time. In order to limit the models and the calculation process some simplifications were made. To put this in context, a single pile analysis took about 1-2h to complete while a group analysis took between 6-9h, including the simplifications.

4.6.1 Model geometry

The geometry is chosen according to Figure 29. The model is extended a bit less than the single pile models. The length from pile to outer border in the direction of loading is 2.5m, compared to 3 and 4 from the single pile analysis. The difference between the two tested single pile models was small but noticeable, and for this particular case 4m would be preferable as discussed in section 4.5.1.

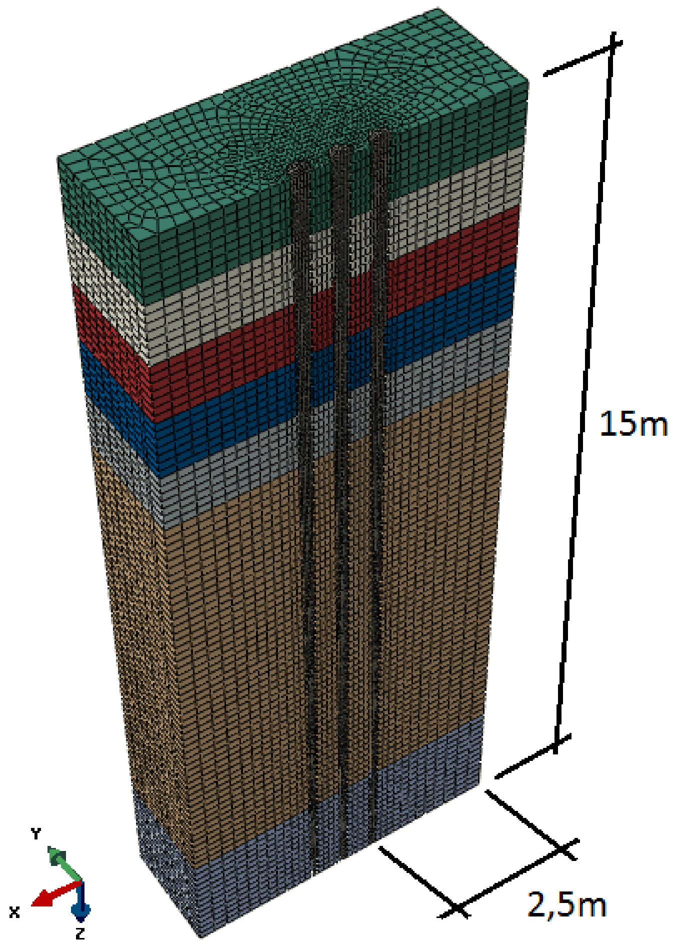


Figure 29. Geometry of the group model. Colours indicate soil layers.

For the single pile models subjected to lateral loading, only very small movements in the bottom of the piles were detected. Therefore, the model was not extended in depth beyond the piles. Instead boundary conditions were placed directly on the piles' tip.

4.6.2 Applied loads

The piles were subjected to, apart from gravity, individual horizontal loads of *0.5*30kN=15kN* each. The symmetry was considered by applying reduced magnitude of the loading in the same way as for Model 1 and Model 2. The loads were applied at individual nodes in the centre of the piles at level with the clay surface. Different steps were introduced where to assign the loads as follows: first the gravity and than the horizontal loads.

After applying the gravity load in the single pile models, a tendency for the clay to "hang" on the piles was noticed. Since the clay is a softer and denser material than the timber, it settles more due to the gravity load. The friction between the materials then prohibits the clay next to the piles to go down and it appears to be "hanging" on the piles. To escape from this phenomenon, the friction was deactivated when applying the gravity load and the clay was settled evenly, see Figure 30. This also had a positive effect on the model performance, for which the convergence rate became much better.

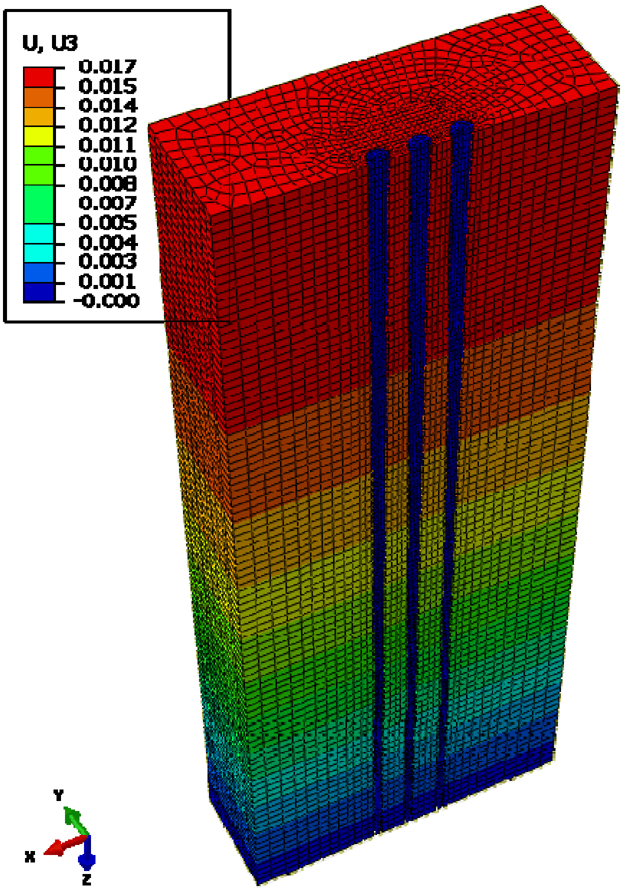


Figure 30. Settlement in z-direction due to gravity load [m]

4.6.3 Applied mesh

The mesh, which is visible in Figure 29, was slightly coarser than suggested by the study of the single piles. Most noticeably, the number of elements with the thickness of the piles was set to 6, compared to suggested 10-12. Worth mentioning is although the number of elements is less, the overall element proportions are better compared to the single pile models.

4.6.4 Element type

The suggested incompatible mode elements, C3D8I, included more internal integration points compared to full- or reduced integration elements, C3D8 & C3D8R. The reduced integration option was therefore chosen to limit the needed calculations as much as possible. In the cantilever study, C3D8R was the relatively worst element in the comparison. As expected, the head deflections were around 10-20% higher for C3D8R than C3D8I. Therefore the results using this element should only be taken as an indication of what the result might be. The deflection is expected to be overestimated and the stress, according to the cantilever study, is expected to be underestimated.

4.7 Comparison and results

4.7.1 Model 1

Model 1 is the single pile model which corresponds best to the hand calculation procedure, with constant shear strength and a pile diameter. First the model is analysed without any friction between pile and soil, i.e. the model is fully merged. Using the recommended input values presented in Table 20, the Winkler method is unable to match the stresses and the displacements from the FE-analysis. Both the head displacement and the maximum bending stress are overestimated in the hand calculations by 72% and 33% respectively. Figure 31a-b shows the displacement and the stresses calculated by Abaqus and hand calculations.

Table 20. Parameter configurations for the Winkler method

		Configurations				
Parameters	**Recommended values from the literature**	a	b	c	d	e
N_c	0-9, according to Broms	0-9	0-9	0-9	0-9	0-9
k	204.5, mid value according to Baugelin	1020	114	168	200	155
z_c	-	-	-	3m	7.5m	3m

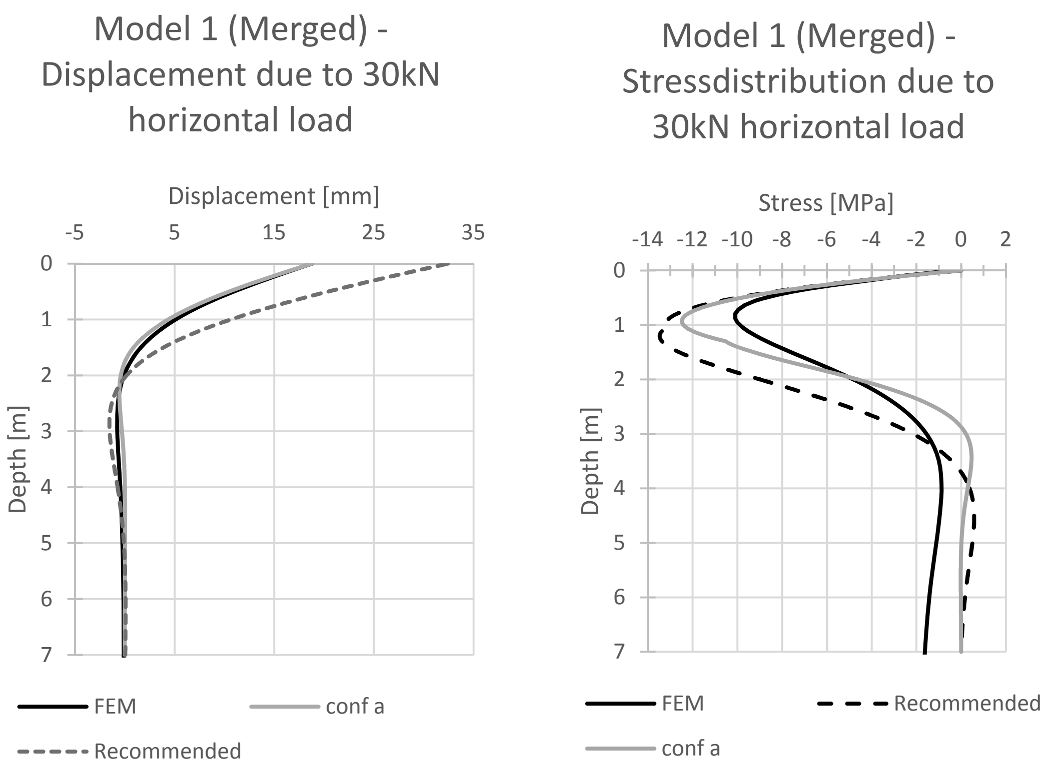


Figure 31. Model 1, merged a) Displacement b) Bending stresses

To match the FE-results better, the parameters presented in Table 20, which are non-related to the FE-input, are altered. When disregarding the recommendations, a better correlation between the displacement in FEM and in hand calculations is possible. As Figure 31a-b shows, it is not possible to match both displacement and stresses for the same configuration of the two parameters. Configuration a match the displacement curve well, but the maximum stress is on the other hand overestimated by 24%.

When interaction properties are included, the Winkler method corresponds much better to the FE-results. Using the recommended settings, the head displacement and the maximum stress are instead underestimated by 25% and 11% respectively. Applying configuration b, the Winkler method is able to predict both the maximum stress and the maximum displacement from Model 1, see Figure 32.

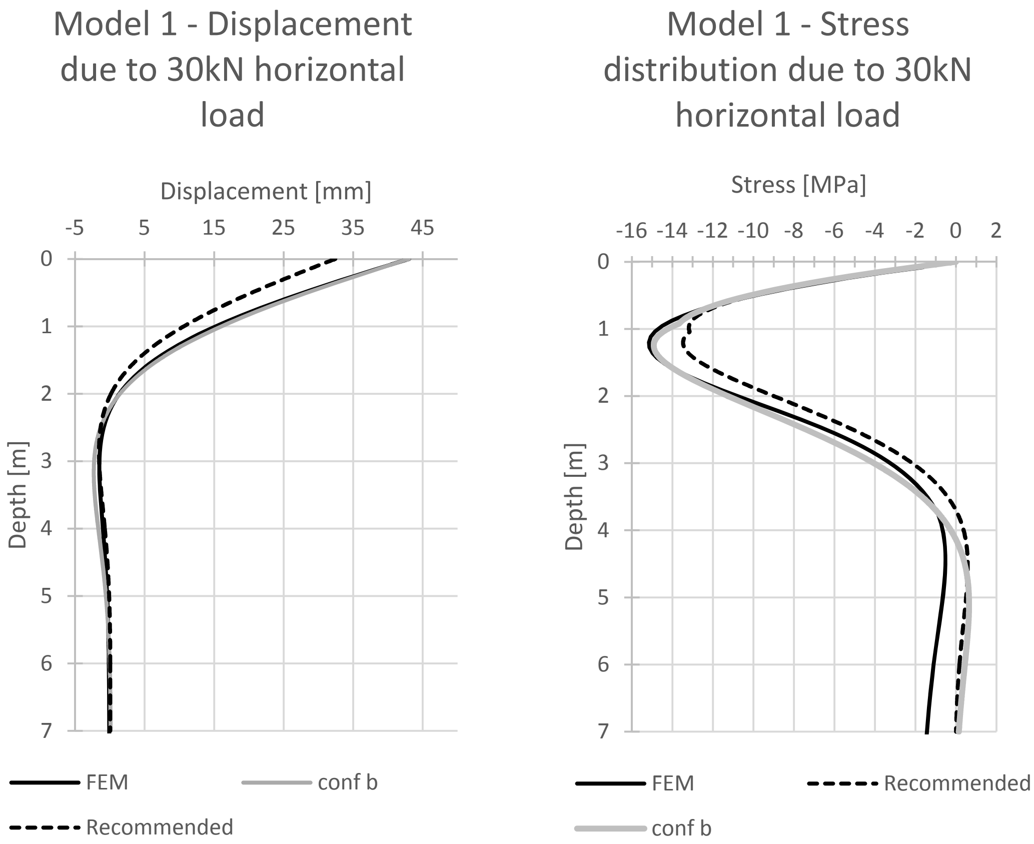


Figure 32. Model 1, interaction a) Displacement b) Bending stresses

4.7.2 Model 2

When comparing Model 2 to the hand calculations, one extra variable parameter appears; the reference depth, which is discussed in section 4.3.1. When applying recommended values according to Table 20 at a reasonable reference depth of $z_c=3m$ the displacement is underestimated as for Model 1, but only by 11%. The Winkler model is on the other hand able to predict the maxim stress by only 2% error. Altering the parameters according to configuration c, a better correlation is established for the displacement. The maximum stress is then overestimated by a moderate 5%, see Figure 33.

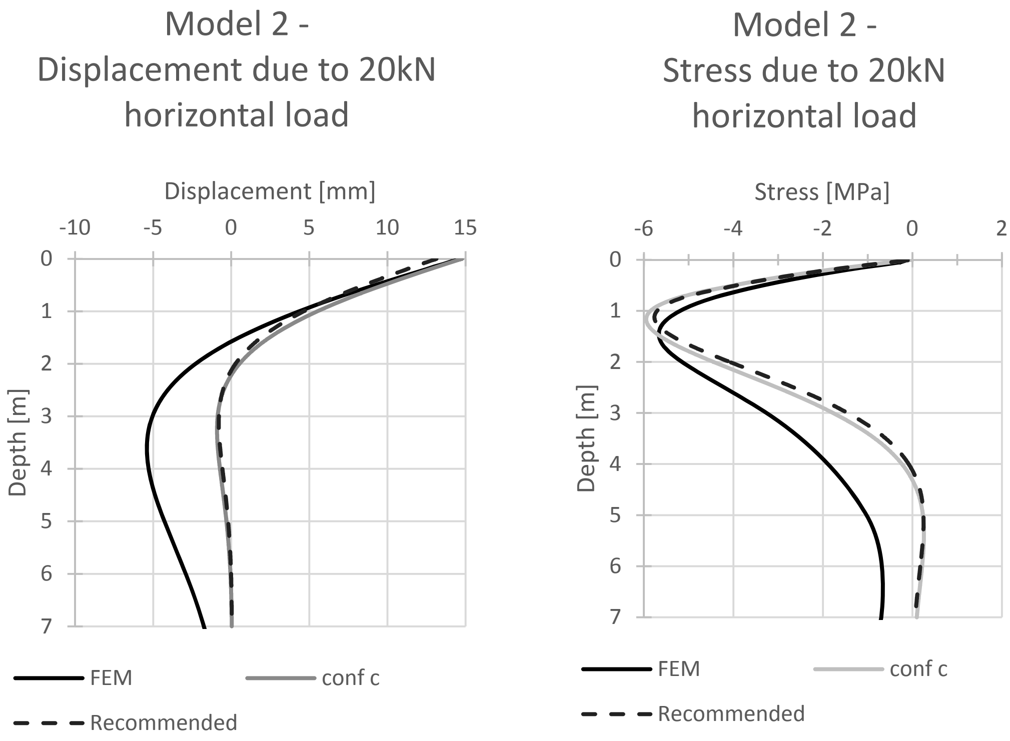


Figure 33. Model 2 a) Displacement b) Bending stresses

Regarding the reference depth, no better correllation can be found if further altered. When aplying a reference depth of 7.5m (half the pile length) according to configuration d, a greater distorsion of the results appear, see Figure 34a-b. The head displacement is matched, but the maximum stress is greatly overestimated. Altering the load bearing factor Nc does not influence the relation much. Appling the higher Nc(z) recommended by (Randolph & Houlsby, 1984), underestimates the stresses compared to FE-resulus. Using a lower Nc instead overestimates the stresses when displacement are matched. The recommended value from (Broms, 1964), as discussed in section 2.2, seems most resonable int his case.

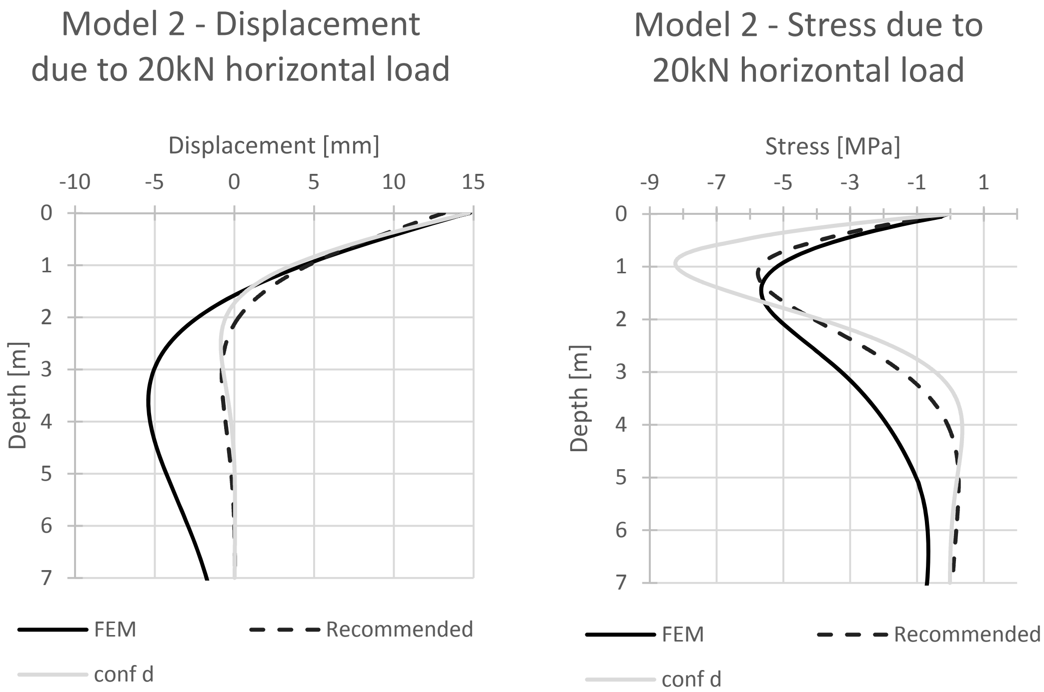


Figure 34. Model 2, altered zc a) Displacement b) Bending stresses

4.7.3 Model 3

The properties assigned to Model 3 are similar to those of Model 2. The differences include the clay geometry and the element types, which are both determined with respect to computational time. Model 3 of the pile group is analysed twice. First with lateral load applied to only one pile, and second with lateral loads on all three piles. The applied lateral load is $\frac{30}{2} = 15kN$ per pile.

The single pile experiences a similar deflection curve as Model 1, where the negative deflection at z=3m is less pronounced than for Model 2, see Figure 35. Probably either the passive piles influence the result, or the difference in geometry and element settings is responsible for the difference to Model 2. It could also be due to the load, which it is somewhat larger for Model 1 and 3.

When applying recommended values according to Table 20 at a reference depth of $z_c=3m$, the displacement is underestimated as for Model 2. The bending stress is on the other hand overestimated by 25%. Matching the deflection by applying configuration e only enlarges the stress even further, see Figure 35. The hand calculations would probably match the FE-results better with a higher N_c value, as discussed earlier in this section. The results from this analysis are although deemed best as comparison to the group analysis, since they are from the exact same model.

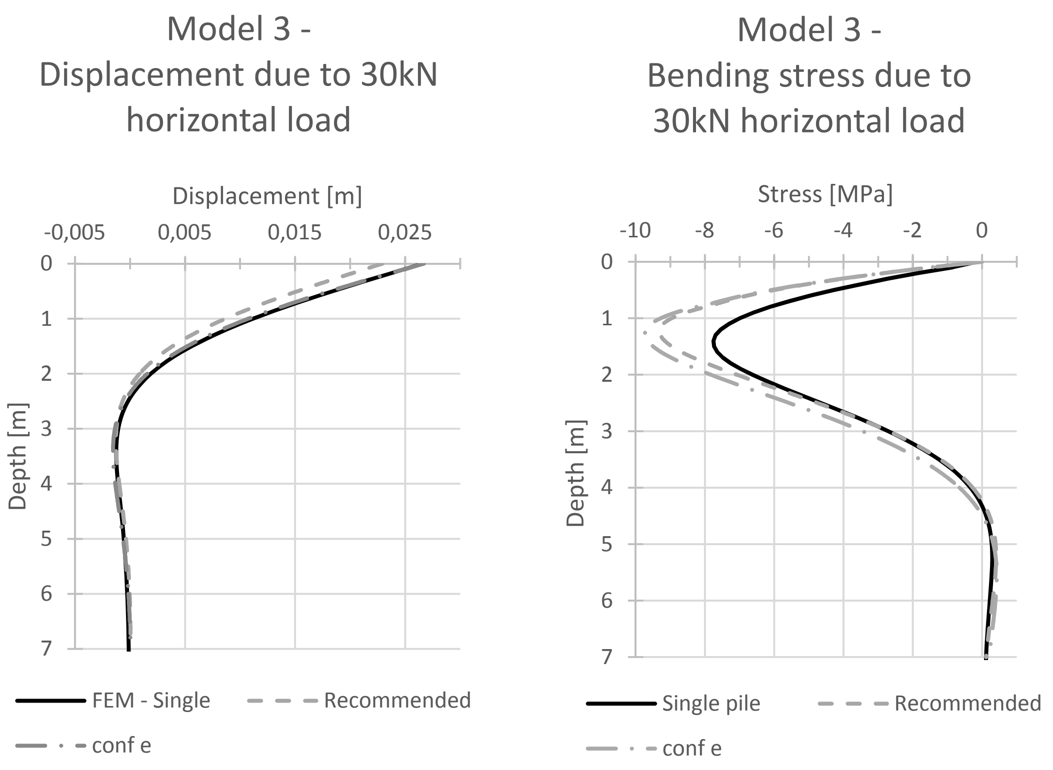


Figure 35. Model 3, single pile a) Displacement b) Bending stresses

Figure 36 shows the displacement of the pile group when the load is applied to all the piles. The displacements confirm the expected behaviour discussed in section 2.2.1. When the first row moves laterally it reduces the soil resistance for the trailing rows, i.e. the first row has the most support from the soil and thus deflects the least. The difference between deflections of the different rows is the largest between the first and the second, and very small between the second and the third. This is in coherence with the findings of (Ilyas, et al., 2004) and (Rollins, et al., 2006), which proposed reduction factors for each row to apply on single pile calculations, as presented in Table 1. Both research teams concluded that the difference in the reduction between the different rows can be considered zero between the third and the trailing.

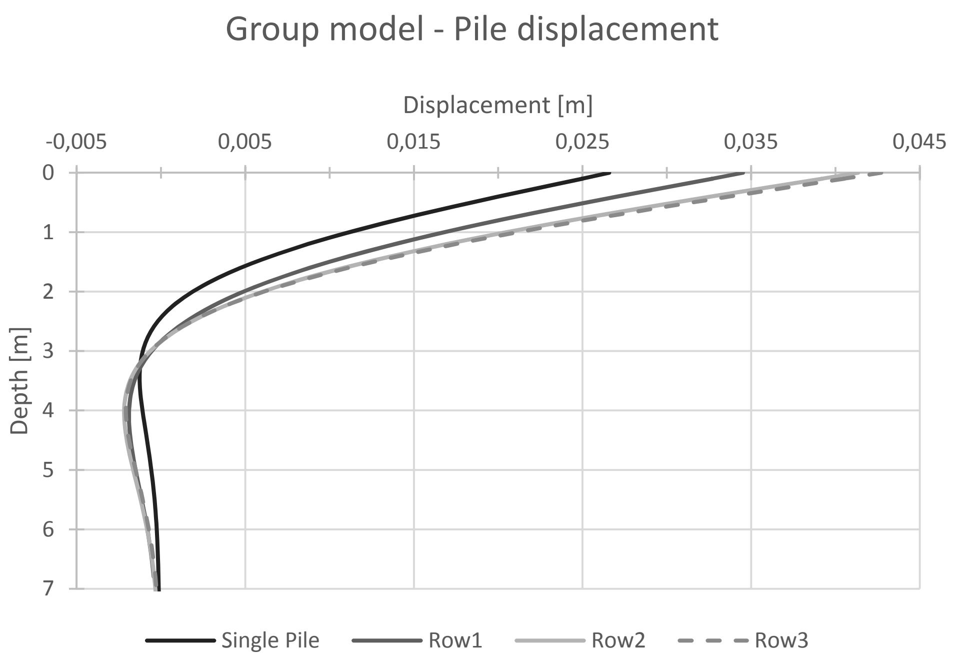


Figure 36. Model 3, displacement of different rows in pile group

When matching the displacement to the FE-results of the group model, the stresses are overestimated as for the single pile analysis of Model 3. Focusing on the displacement, reduction factors for each row is back calculated and presented in Table 21. The stresses for row 1 and 2 with the reduction factors are compared to FE-results in Figure 37.

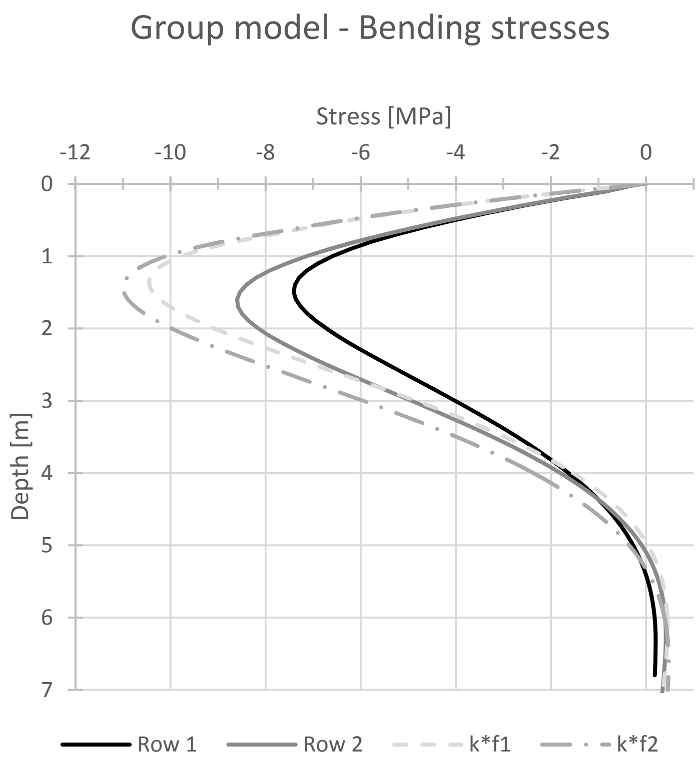


Figure 37. Model 3, bending stresses of different rows in pile group

Table 21. Back-calculated reduction factors

	Row 1	*Row 2*	*Row 3*
Reduction factors $(z_c = 3m\ N_c = 0\text{-}9)$	f_1=0.64	f_2=0.55	f_3=0.52

5 Conclusions

- Although some difference occurred between the results of the different models, the hand calculation procedure described in Commission on Pile Research (Svahn & Alén, 2006) appears to have acceptable correspondence with the FE-analysis.

- The making of a soil-pile FE-model in Abaqus/CAE is a delicate process with many options to consider. Parameters that greatly affected the results were; Element type, Mesh size, interaction properties, global geometry and E-modulus.

- The recommendation for the load bearing factor, N_c from (Broms, 1964) appears to correspond best to FE-results, however further analysis is needed.

- The linear interpretation of the more accurate, strain dependent, *E-modulus* $E(\varepsilon)$ is problematic and a big responsibility lays on the engineer in charge for defining this parameter.

- In analogy with the FE-analysis, the linearization of the p-y curve with the subgrade reaction coefficient in the hand calculations have a similar big impact on the result and should be chosen carefully.

- The FE-models with soil-pile interaction, as discussed in the report, correspond best to the Winkler method, rather than the merged FE alternative.

- Both the FE and the hand calculation procedure need to be verified against field tests. Both tested methods rely on simplified material relations, where it is hard to find good recommendations for necessary input data.

6 Discussion

6.1 One- and two-parameter Winkler models

The limitation of the one-parameter Winkler model regarding the inability to account for shear transfer was investigated in (Eriksson & Caselunghe, 2012). Their work considered spread foundations and the authors explained how the main problem with the individual springs arise at the boundary regions as seen in Figure 6 in section 2.2. They proposed a solution where the soil stiffness is lowered by a factor of 4 at corner regions to better describe the reality. This idea does not however seem to fit the case of piles since there is no presence of soil above the pile. Also the loading conditions are different and the action in a pile subjected to lateral force is concentrated to the top, as shown principally in Figure 38, which makes a reduction of the soil stiffness at the lower boundary insignificant. Although, the presented one-parameter seemed unable to correctly match the results from FEM, and perhaps it is necessary to incorporate the shear transfer in the analytical method.

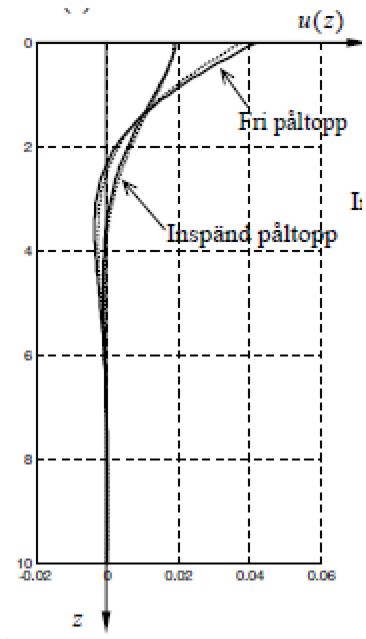


Figure 38. Principal deflection of pile subjected to lateral force

6.2 Material models in FE-analysis

As mentioned in section 2.1, the linearization of the material behaviour is problematic. A comparison to a more detailed material model is needed to be confident with the behaviour of the elasto-plastic model. The value of $E=150c_u$ used in the report is believed to be a very conservative value, and a higher capacity could possibly be reached using another material model (or higher *E-modulus*).

6.3 Mesh proportions in FE-analysis

In the making of this report, available computer resources were only of average capacity at best. The Abaqus software is a very comprehensive program and the analysis process can take a lot of time for large models. The single pile models analysed for this report took between 1h and 2h to complete, and would take even longer for a more refined mesh. Due to this fact the mesh for each model was created more dense in regions of interest and coarse in other regions. The effects of this choice are not fully investigated in this work. Since the mesh size was concluded important both in the cantilever study and in the later models, an even better mesh should probably be tested to verify the results.

One effect of changing mesh size in the model is that the proportions of the elements also vary in the model. When creating the mesh for different models in this study the priority was to have good proportions in the top part close to the pile. How the less good proportions of some elements affected the result was not concluded in this study. Figure 39 shows element proportions for different mesh sizes in horizontal and vertical direction. In the top part close to the pile, the elements were about 0.8 times longer in z-direction for the most refined mesh used for the single pile analysis.

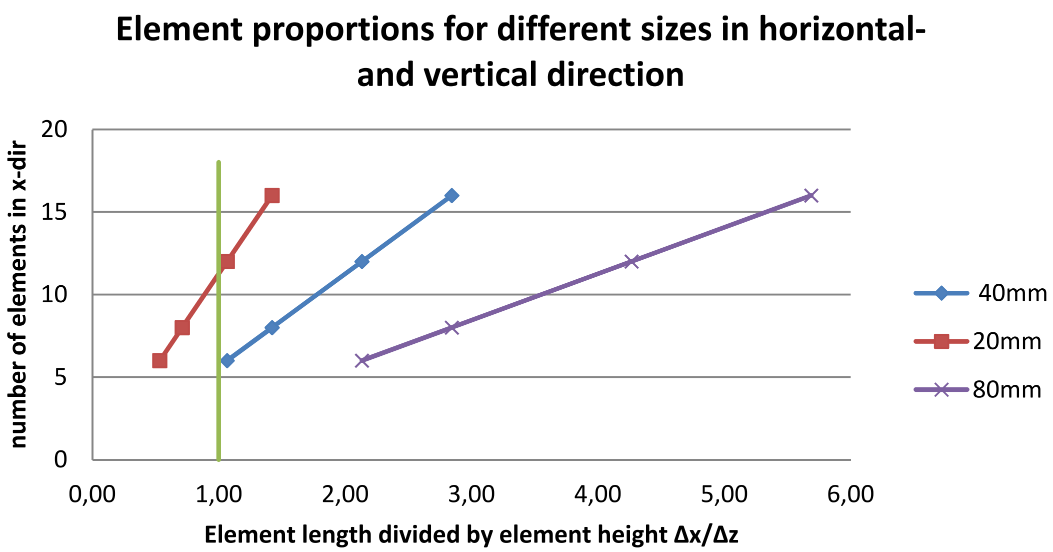


Figure 39. Element proportions

6.4 Load application in FE-analysis

Due to the gravity load applied, the clay settles about 17mm relatively to the piles in the group model. This adds an eccentricity of 17mm to the horizontal loads, which then gives a slightly higher displacement than if the loads were applied in ground level. This could be considered when optimizing a model in Abaqus.

7 Recommendations for further studies

The Winkler model has been thoroughly examined and further development of the model is not generally requested. Although, a comparison to a two-, or multi-parameter Winkler method could be interesting. The dominant disadvantage of the model is instead uncertainties of the input data which are highly site specific. Values for pile behaviour from a local field test would increase the accuracy of the Winkler model substantially.

A representative site with old timber piles, as explained in the report, could be chosen as a test site. The piles could be tested for horizontal loading until failure and then be replaced by a new construction. This could generate data which allowed for a more rational decision regarding other similar structures in the city.

Regarding long-term loading situations – could a lowering of the strength properties similar to the hand calculation be useful for FE analysis? Or is a more comprehensive material model incorporating pore pressure needed to more accurately model long term loads? In Abaqus for example it is possible to divide the load into a long-term and short-term part where the long term load is applied during long time and the short term instantaneously.

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Appendix A – The Load bearing factor N_c

Implementation of varying Load bearing factor Nc(z) into equations derived in Pålkommissionen rapport 101. Example equations chosen to illustrate the alteration is taken from example 3 p.35.

The load bearing factor Nc(z) is used when calculating the yielding pressure of the soil. In the procedure proposed in Pålkommissionen rapport 101 the value is constant due to constant shear strength and diameter. When implementing varying Nc(z) also Uy(z) becomes non-linear.

$$U_y(z) := N_c(z) \cdot c_u \cdot d$$

For example, when calculating the depth zy which divides the upper plastic and the lower elastic zone in the soil a shear force equilibrium is used. Calculating the contribution from the plastic part, the external load plus the resisting earth pressure is added togheter. The original equation was as follows:

$$V_{pl} := -F_x + U_y \cdot z_y$$

With varying earth pressure instead, it had to be integrated over the lenght

$$V_{pl}(z_y) := -F_x + \int_0^{z_y} U_y(z)\, dz$$

Calculating the moment along the pile in the plastic zone the original equation for a pile subjected to horizontal force, normal force and fixed at the head is as follows

$$M_{pl}(z, z_y) := \left[\left(F_x + m_y(z_y) \cdot N\right) \cdot (z_y) \cdot C_{F\psi N}(z, z_y) \ldots + \frac{-U_y \cdot z_y^{\,2}}{2} \cdot C_{U1\psi N}(z, z_y) + M_y(z_y) \cdot C_{M\psi N}(z, z_y) \right]$$

And with varying yield pressure the moment becomes

$$M_{pl}(z, z_y) := \left[\left(F_x + m_y(z_y) \cdot N\right) \cdot (z_y) \cdot C_{F\psi N}(z, z_y) \ldots + \int_0^{z_y} -U_y(z) \cdot z\, dz \cdot C_{U1\psi N}(z, z_y) + M_y(z_y) \cdot C_{M\psi N}(z, z_y) \right]$$