

MASTER'S THESIS 2007:20

Modeling sound scattering from an audience
- an application to a model of the
Gothenburg Opera House

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Chalmers Room Acoustics Group
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2007

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Cover:

The cover picture is a screen shot from the CATT Acoustics 3D View Module, showing the interior of the Gothenburg Opera House.

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Abstract

A traditional approach when modeling sound scattering from an audience in concert halls is to model the audience by a plane and specify the mean absorption coefficient and mean diffusion coefficient of the plane in octave bands. A drawback with an approach like this is that the more detailed parts of the scattered sound field, such as the scattering by the heads of the audience, is not modeled. The work of this thesis has therefore revolved around making the simulation of the sound in the Gothenburg Opera House more realistic by adding the contribution of sound scattered by hemispheres (resembling the heads of the listeners) to the impulse response given by the commercial computer software CATT Acoustics.

When the CATT Acoustics impulse responses had been modified they were convolved with various anechoic recordings and listening tests could be performed. The results of these tests exposed that the introduction of hemisphere scattering results in a clear audible difference when the impulse responses are convolved with white noise. The difference is not as clear when convolving with other anechoic recordings but results indicate that expanded modeling could change that.

Keywords: Hemisphere Scattering, Fast Fourier Transform, Digital Signal Processing, Cubic Spline Interpolation, Discrete Convolution, Auralization, Separation of Variables

Modellering av ljudspridning från en publik - en tillämpning på en modell av Göteborgsoperan

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Sammanfattning

Ett traditionellt tillvägagångssätt när man modellerar ljudspridning från en publik i konserthallar är att modellera publiken med ett plan och specificera planets absorptionskoefficient och diffusionskoefficient i oktavband. En nackdel med detta tillvägagångssätt är att de mer detaljerade delarna av det spridda ljudfältet, såsom spridningen från åhörarnas huvuden, inte modelleras. Därför har detta examensarbete kretsat kring att göra simuleringen av ljudet i Göteborgsoperan mer verklighetstrogen genom att addera bidraget från ljud som spritts från halvsfärer (liknande åhörarnas huvuden) till det impulssvar som ges av det kommersiella datorprogrammet CATT Acoustics.

När impulssvaren från CATT Acoustics hade modifierats så faltades de med olika ekofria inspelningar och därefter kunde lyssningstest genomföras. Resultaten från dessa tester visade att införandet av halvsfärsspridning resulterar i en klart hörbar skillnad då impulssvaren faltas med vitt brus. Skillnaden är inte lika klar då man faltar med andra ekofria inspelningar men resultaten indikerar att utvidgad modellering skulle kunna ändra detta.

Nyckelord: Halvsfärsspridning, Fast Fourier Transform, digital signalbehandling, kubisk splineinterpolation, diskret faltning, auralisering, variabelseparation

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Finally I would like to thank my wonderful girlfriend Moa Berglund for inspiring me in so many ways and for turning the fall of 2006 into something bright and fun!

1 Introduction

This introductory chapter will give you a brief look into the scope of this thesis. The purpose of this project and its implementation is also described in brevity.

1.1 Background and Purpose

Predicting accurately how something will sound in large concert halls is a difficult and subtle problem. One needs the absorption and diffusion coefficients for all the elements building up the entire hall. This is of course not enough. Since many elements in a concert hall has complex geometrical properties one immediately runs into problems when it comes to computing the scattered sound field. There exists analytical solutions for the scattered sound field from a few simple bodies, but for more complicated structures one has to start approximating in order to get anywhere. Many commercial computer software are modeling the audience plane in concert halls by simply specifying the mean absorption coefficient and the mean diffusion coefficient of the plane in octave bands. A modeling approach like that will not resolve details such as scattering from the listeners heads. This was one of the key conclusions leading me into picking sound scattering as my field for the master thesis.

The purpose of this thesis is to investigate if one can obtain a more realistic modeling of the sound in the Gothenburg Opera House by using the Impulse Response given by CATT Acoustics and then add another impulse response corresponding to the scattering from hemispheres (resembling the heads of the listeners). The hemisphere impulse responses are computed with a number of in-house coded Matlab scripts using the data in the output files from CATT Acoustics as in parameters. Once the impulse response from CATT Acoustics and the hemisphere impulse response have been added one can perform listening tests investigating if there are any audible differences.

This master thesis is a natural continuation of my supervisor Georgios Natsiopoulos' master thesis, see [3].

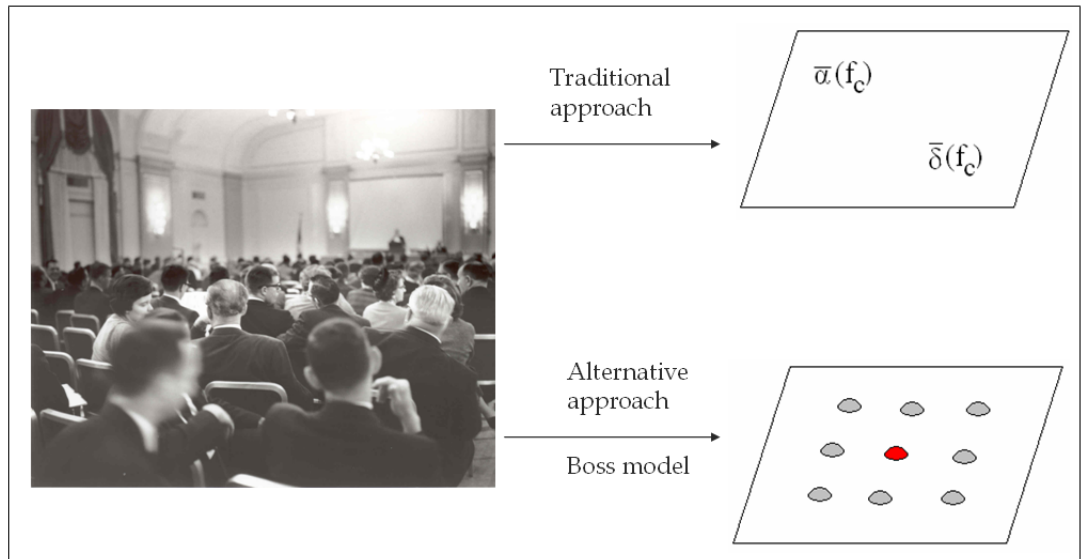


Figure 1.1: The two approaches.

2 Theory

This chapter will guide you through the theory forming the base for the equations used in the Matlab scripts. The largest part of this chapter considers scattering for Neumann Boundary Conditions. Furthermore the SumBank approximation and a way of modeling coupling are introduced.

2.1 Scattering for Neumann Boundary Conditions

The theoretical starting point of this thesis is scattering for Neumann Boundary Conditions. This means that the scattering surfaces are modeled as hard, impenetrable and immobile. As a consequence the particle velocity normal to the surface is zero which in turn implies that the gradient of the pressure also is zero.

In the coming subsections I will present a number of equations without derivation. For an in depth derivation of these equations, see [1].

2.1.1 Scattering from a hard sphere

Our first step will be looking into the scattering from a hard sphere. Let us use the following geometry

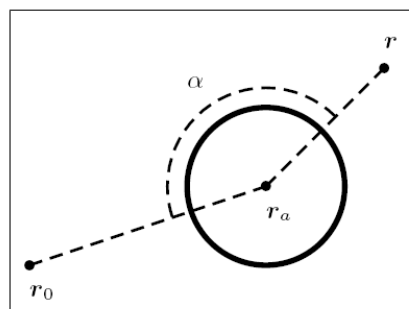


Figure 2.1: The geometry for scattering from a hard sphere.

We will assume that a point source is located at \mathbf{r}_0 , that a sphere of radius a has its center at \mathbf{r}_a and that the listening position is located at \mathbf{r} relative the origin $\mathbf{0}$. For convenience, let us also introduce the following vectors and distances:

$$\begin{aligned} \mathbf{R}_a &= \mathbf{r} - \mathbf{r}_a & R_a &= |\mathbf{R}_a| & \hat{\mathbf{R}}_a &= \frac{\mathbf{R}_a}{R_a} \\ \mathbf{R}_{a,0} &= \mathbf{r}_0 - \mathbf{r}_a & R_{a,0} &= |\mathbf{R}_{a,0}| & \hat{\mathbf{R}}_{a,0} &= \frac{\mathbf{R}_{a,0}}{R_{a,0}} \end{aligned}$$

The scattered pressure can then be written as

$$p_{sc}(\mathbf{r}, \omega) = -ik\hat{U}(\omega) \sum_{m=0}^{\infty} Q_m(ka)h_m(kR_a)h_m(kR_{a,0})P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}) \quad (2.1)$$

- i is the imaginary unit satisfying the equality $i^2 = -1$
- \mathbf{k} is the wave vector.
- $\hat{U}(\omega)$ is the source strength as function of frequency.
- Q_m is the function determining the strength of the scattered field as a function of frequency. Essentially it is a combination of spherical Bessel and Hankel functions.
- h_m is the spherical Hankel function of the first kind.
- P_m is the Legendre polynomial, mainly determining the directivity properties of the scattered field. Note that its argument $\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0} = \cos(\alpha)$.

2.1.2 Approximations

The following two relations are useful when approximating Eq. 2.1.

$$h_m^{(1)}(\eta) \rightarrow \frac{e^{i\eta}}{i\eta} i^{-m} \quad \text{when } \eta \rightarrow \infty \quad (2.2)$$

$$P_m(-x) = (-1)^m P_m(x) \quad (2.3)$$

Farzone

The farzone approximation is obtained by letting $kR_a \rightarrow \infty$. Practically this shall be interpreted as letting the listening position slide infinitely far away from the sphere along the line that leaves α unchanged. Using Eq. 2.2 the result is

$$p_{sc}(\mathbf{r}, \omega) \approx -\hat{U}(\omega) \frac{e^{ikR_a}}{R_a} \sum_{m=0}^{\infty} i^{-m} Q_m(ka) h_m(kR_{a,0}) P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}) \quad (2.4)$$

Farfield

The field approximation is obtained by letting $kR_{a,0} \rightarrow \infty$. Practically this shall be interpreted as letting the source slide infinitely far away from the sphere along the line that leaves α unchanged. Using Eq. 2.2 the result is

$$p_{sc}(\mathbf{r}, \omega) \approx -\hat{U}(\omega) \frac{e^{ikR_{a,0}}}{R_{a,0}} \sum_{m=0}^{\infty} i^{-m} Q_m(ka) h_m(kR_a) P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}) \quad (2.5)$$

Farzone and Farfield

The result when applying the farzone and farfield approximations simultaneously (using both Eq. 2.2 and Eq. 2.3) is

$$p_{sc}(\mathbf{r}, \omega) \approx \frac{i}{k} \hat{U}(\omega) \frac{e^{ik(R_a+R_{a,0})}}{R_a R_{a,0}} \sum_{m=0}^{\infty} Q_m(ka) P_m(-\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}) \quad (2.6)$$

It is worth mentioning that a factor e^{ikR} corresponds to a time delay of R/c seconds and that distance from source to receiver via the sphere center is $R_a + R_{a,0}$.

2.1.3 Scattering from a hard hemisphere on a hard infinite plane

For the hemisphere scattering, let us use the following geometry

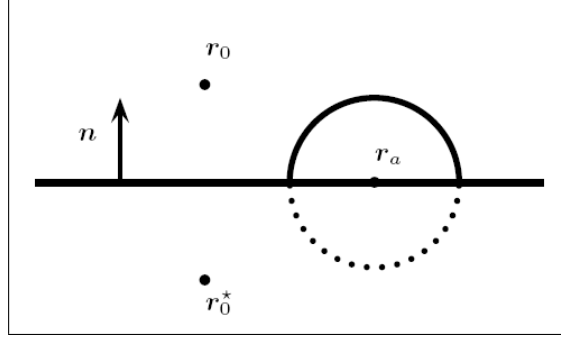


Figure 2.2: The geometry for scattering from a hard hemisphere.

When calculating the scattered sound field from a hard hemisphere on an hard infinite plane one can make use of the solution for the hard sphere. By replacing the plane with a mirror source (at the specific coordinate \mathbf{r}_0^*) one obtains an equivalent scenario - at least mathematically equivalent - above the plane. The new vectors and distances are

$$\mathbf{r}_0^* = \mathbf{r}_0 - 2|\mathbf{R}_{a,0} \cdot \hat{\mathbf{n}}|\hat{\mathbf{n}}$$

$$\mathbf{R}_{a,0}^* \equiv \mathbf{r}_0^* - \mathbf{r}_a = \mathbf{R}_{a,0} - 2|\mathbf{R}_{a,0} \cdot \hat{\mathbf{n}}|\hat{\mathbf{n}}$$

$$R_{a,0}^* = R_{a,0}$$

We can now formulate an expression for the scattered field from a hemisphere on a plane. It is done by adding what is scattered when the source at \mathbf{r}_0 radiates on a sphere at \mathbf{r}_a to what is scattered when the mirror source at \mathbf{r}_0^* radiates on the same sphere. In mathematical terms this is formulated as

$$p_{sc}(\mathbf{r}, \omega) + p_{sc}^*(\mathbf{r}, \omega) = -ik\hat{U}(\omega) \sum_{m=0}^{\infty} Q_m(ka)h_m(kR_a)h_m(kR_{a,0})[P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}) + P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0}^*)] \quad (2.7)$$

The farzone and farfield approximations are applicable in this case too, due to the principle of superposition.

2.2 Modeling finite impedance

What we have dealt with so far is when Neumann Boundary Conditions apply. If we want to model the more realistic case - the case of a hemisphere with finite impedance - we will have to take a closer look at $Q_m(ka)$ in Eq. 2.1. The general expression is:

$$Q_m(ka) = (2m + 1) \frac{j'_m(ka) + i\beta j_m(ka)}{h'_m(ka) + i\beta h_m(ka)} \quad (2.8)$$

where β is the specific admittance. See [4] page 425 for details.

When Neumann Boundary Conditions apply we have $\beta = 0$, so the expression for $Q_m(ka)$ can be simplified into

$$Q_m(ka) = (2m + 1) \frac{j'_m(ka)}{h'_m(ka)} = (2m + 1) \frac{mj_m(ka) - kaj_{m+1}(ka)}{mh_m(ka) - kah_{m+1}(ka)} \quad (2.9)$$

For finite impedances β will be none-zero. A suitable admittance to make use of in our case could be the admittance of a thick, porous and locally reacting absorber.

$$\beta(k) = \frac{1}{\sqrt{1 - \frac{i\Xi}{\rho_0 c k}}} \quad (2.10)$$

Ξ is the flow resistivity of the porous absorber. See [5] page 169 for details.

Going all the way with finite impedance modeling would mean taking into account that the plane which the hemispheres are resting on is not infinitely hard. This has not been done in this project. A consequence of this is that the part of the scattered sound field originating from the mirror source at \mathbf{r}_0^* (see Figure 2.2) will be slightly overestimated, especially for high frequencies.

2.3 The SumBank approximation

If we take a close look at Eq. 2.1 we can see that some cumbersome calculations will be required if you want to compute the scattered pressure exact. We will have to start approximating to get anywhere. A first step is to truncate the infinite sum. This is commonly done by breaking the summation when the ratio between a partial sum and the previous partial sum is of a certain order. The next step might be to apply the farzone

and farfield approximations. In spite of the fact that these approximations would reduce the number of parameters needed for the summation from four ($Q_m(ka)$, $h_m(kR_a)$, $h_m(kR_{a,0})$ and $P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0})$) to two ($Q_m(ka)$ and $P_m(\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0})$) the computation time might have to be cut even more. This is when the *SumBank* approximation comes into the picture.

As you can see in equation Eq. 2.6 we need a wave vector k and the angle α (remember that $\hat{\mathbf{R}}_a \cdot \hat{\mathbf{R}}_{a,0} = \cos(\alpha)$) in order to compute the sums value. If we in advance knew the frequency range of interest and the angle α associated with each wave hitting the scattering object we could easily compute the sums value, store it in a clever way and later call for it with instant access. The computation itself would be done. Based on this idea I built up the so called *SumBank*. As the name hints it is a bank of sums. The picture below will expose its structure.

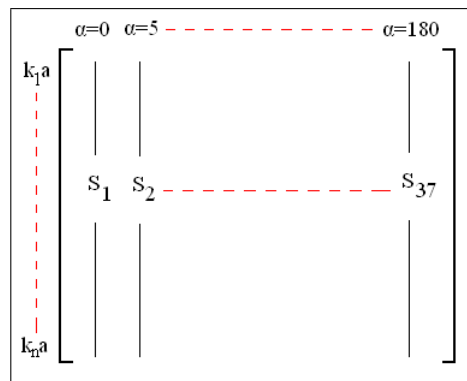


Figure 2.3: Structure of the SumBank.

In Figure 2.3 you can see the matrix structure of the SumBank. Each column vector constitutes the sums value for a specific value of the angle α given the wave vector range k_1, k_2, \dots, k_n . Obviously α is now a discrete parameter. Therefore another approximation needs to be introduced. We will need to round α to the α_d ($\alpha_d = d \cdot 5^\circ$, $d = 0, 1, 2, \dots, 36$) that minimizes $|\alpha - \alpha_d|$ in order to pick the most suitable column vector in the SumBank matrix.

2.4 Modeling coupling

Since the modeling I have described so far has not included coupling we will need to incorporate this somehow. The reason for this is that without coupling each scattering object is viewed as existing alone. A sound wave scattered from one scatterer can not be impeded by another scatterer, which is unphysical. To include full coupling in the analytical equations is very hard but one can use a simplified model which at least will give a more accurate result than if we modeled no coupling at all.

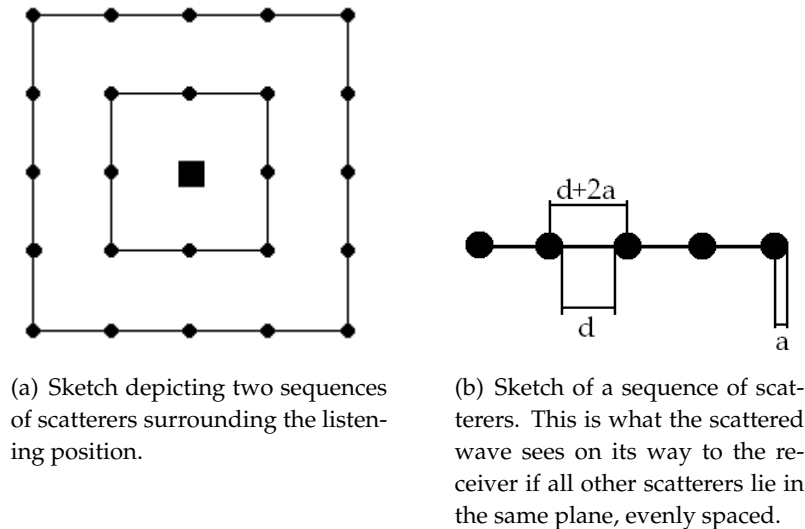


Figure 2.4: Figures describing the simplified way of modeling coupling.

Imagine a scattered sound wave propagating through a sequence of scatterers (as in Figure 2.4(b)) on its way to the listening position. Let us then define the fraction of opening, R

$$R \equiv \frac{d}{d + 2a} \quad (2.11)$$

We can now let R^n be the factor scaling the amplitude of the scattered pressure wave. n is an integer representing the number of scattering sequences there are on the way towards the listening position.

3 Implementation

This chapter will walk you through the implementation of my project. The idea is to follow the project chronologically, starting with the modeling in CATT Acoustics, then moving on through the Matlab scripts and finally the listening tests.

Before we get into specifics I would like to show a schematic sketch of this project's implementation. I will divide this chapter into sections agreeing with Figure 3.1

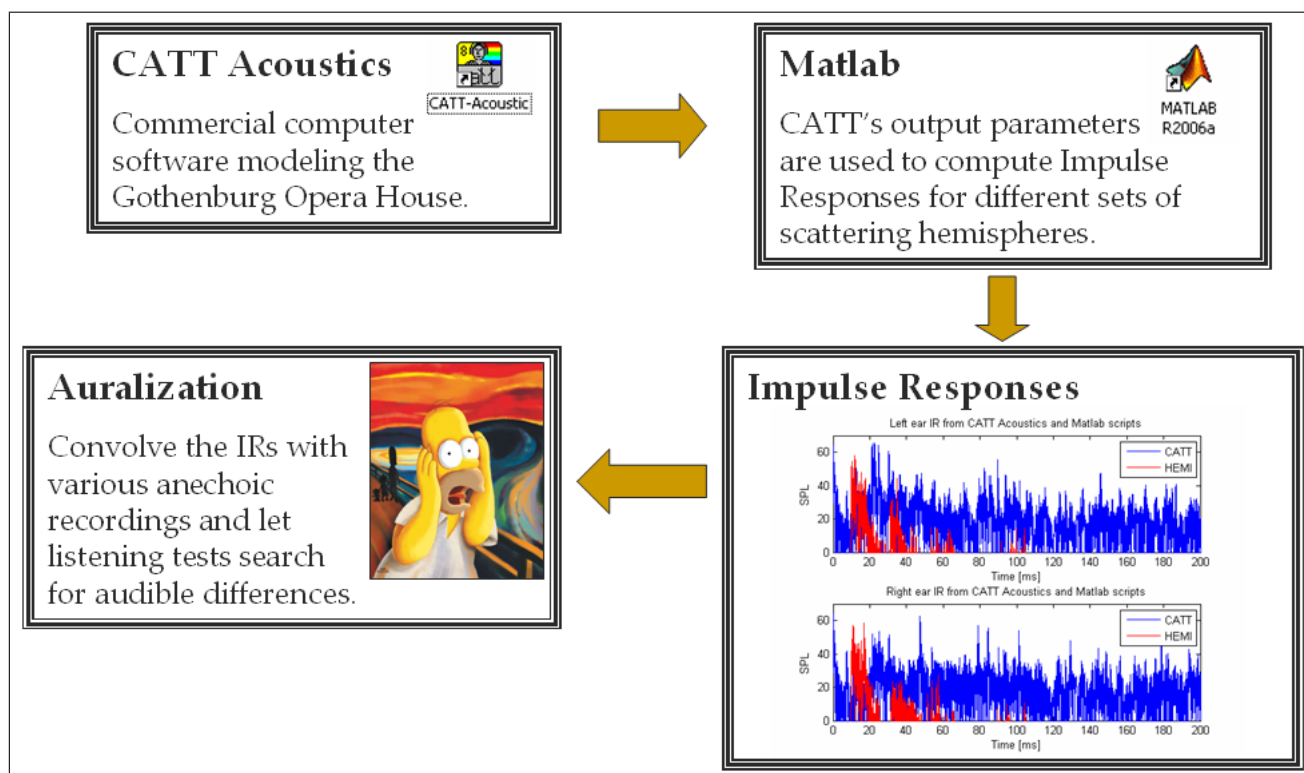


Figure 3.1: Schematic sketch of the implementation.

3.1 CATT Acoustics

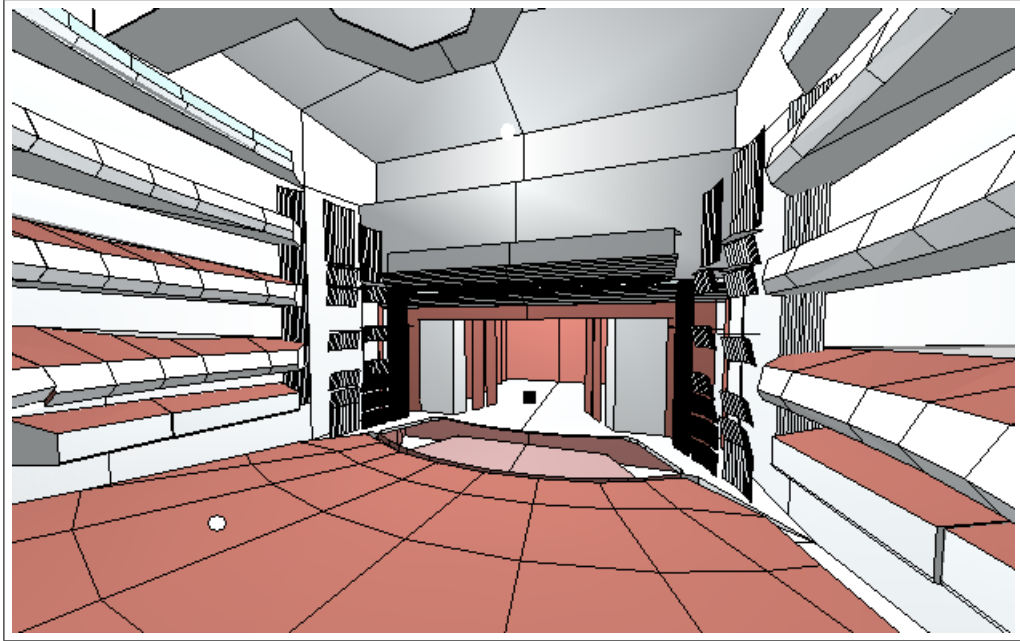


Figure 3.2: Screen shot from CATT Acoustics showing source (black square) and listening position (white circle).

The implementation starts in a commercial computer software called CATT Acoustics. This program has a wide range of features but I have mainly used it for two purposes: firstly to obtain the full detailed calculated impulse response of the Gothenburg Opera House and secondly to extract data such as arrival times, angles of incidence and amplitude for a number of user defined receiver points.

As mentioned earlier, the main task of this project is to investigate the effect of introducing hemisphere scattering into the present acoustical modeling. Therefore step one is to - as detailed as possible - calculate the impulse response without scattering hemispheres present. This is exactly what CATT Acoustics does. Step two would then be to refine an impulse response corresponding to the hemisphere scattering exclusively, enabling us to start comparing. How step two is implemented will be discussed in the next section, however one shall note that the raw data used for the analysis of hemisphere scattering is created in the first step by CATT Acoustics.

3.1.1 Absorption and diffusion of the audience plane in CATT Acoustics

It is important to emphasize that CATT Acoustics has its own way of modeling absorption and diffusion for the audience plane. The following table will give you an idea of how.

Octave band f_c	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
α (%)	40	60	75	88	88	85
δ (%)	30	40	45	70	80	90

Table 3.1: Parameters defining the absorption and the diffusion of the audience plane in CATT Acoustics.

The octave band averages of the absorption coefficient, seen in Table 3.1, served as guidelines when the value of the flow resistivity, Ξ in Eq. 2.10 was set. Ξ was chosen so that the tabulated α values from CATT Acoustics agreed fairly well with values given by Eq. 3.1.

$$\alpha(\beta) = \frac{4\Re\{\beta^{-1}\}}{|\beta^{-1}|^2 + 2\Re\{\beta^{-1}\} + 1} \quad (3.1)$$

\Re denotes the real part of a complex quantity.

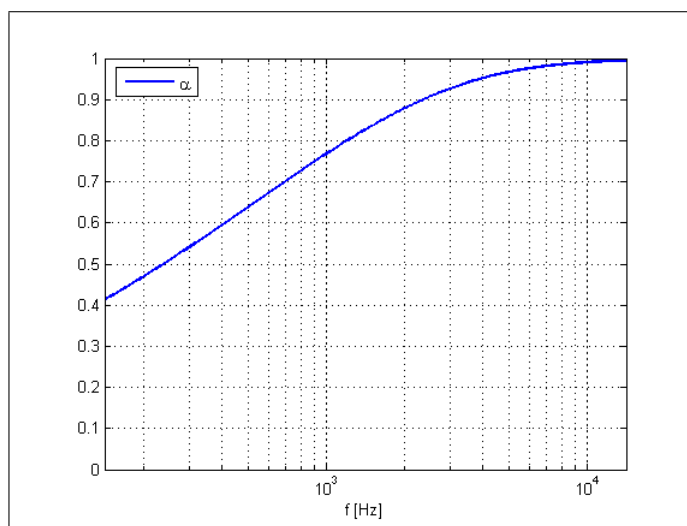


Figure 3.3: Absorption coefficient as a function of frequency according to Eq. 3.1.

3.1.2 Full detailed calculation of impulse response

CATT Acoustics computes the impulse response in a rather complex manner. The early part of the impulse response (typically 300-500 ms in concert halls) consists of first order reflections, second order specular reflections and the direct sound. This part is handled deterministically. In the late part of the impulse response a method handling diffuse reflections is needed. The method utilized by CATT Acoustics is called Randomized Tail-corrected Cone-tracing (RTC). This is a method combining features of both specular cone-tracing, standard ray-tracing and the image source modeling.

3.1.3 Gathering data for analysis of hemisphere scattering

Since there is no way to model hemisphere scattering exact in CATT Acoustics, receivers were placed around the listening position at the coordinates where hemispheres were wanted. After an early part detailed image source modeling in CATT Acoustics (where the highest reflection order was set to 2) a separate text file is generated for each receiver. The parameters needed for the calculation of hemisphere scattering are all in these text files. Before we move on to how the Matlab scripts extract and use these parameters, let me show some typical receiver configurations.

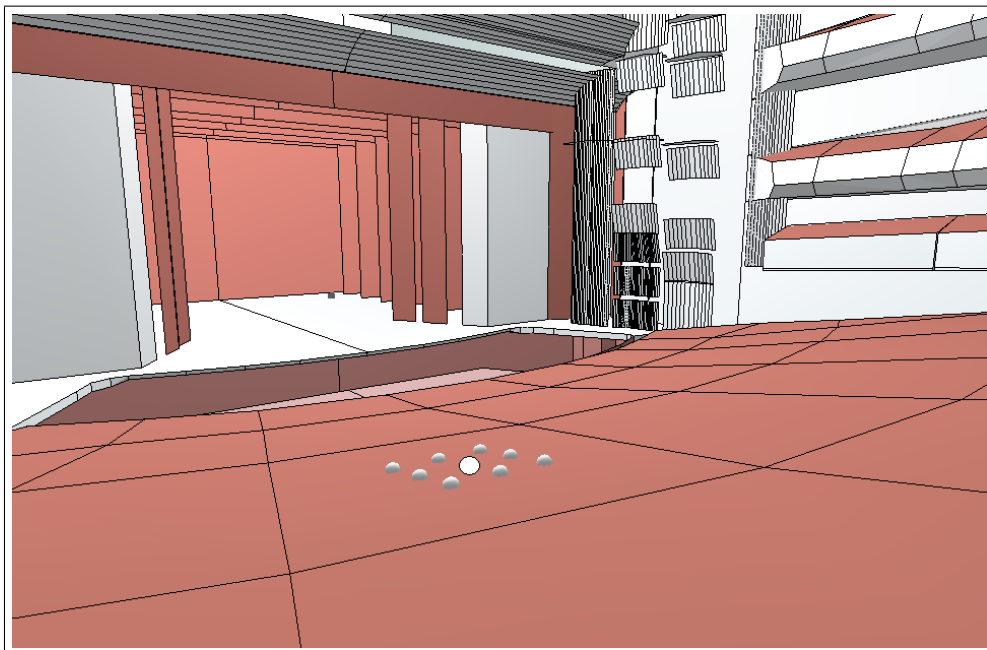


Figure 3.4: Listening position (white circle) surrounded by one receiver cluster.

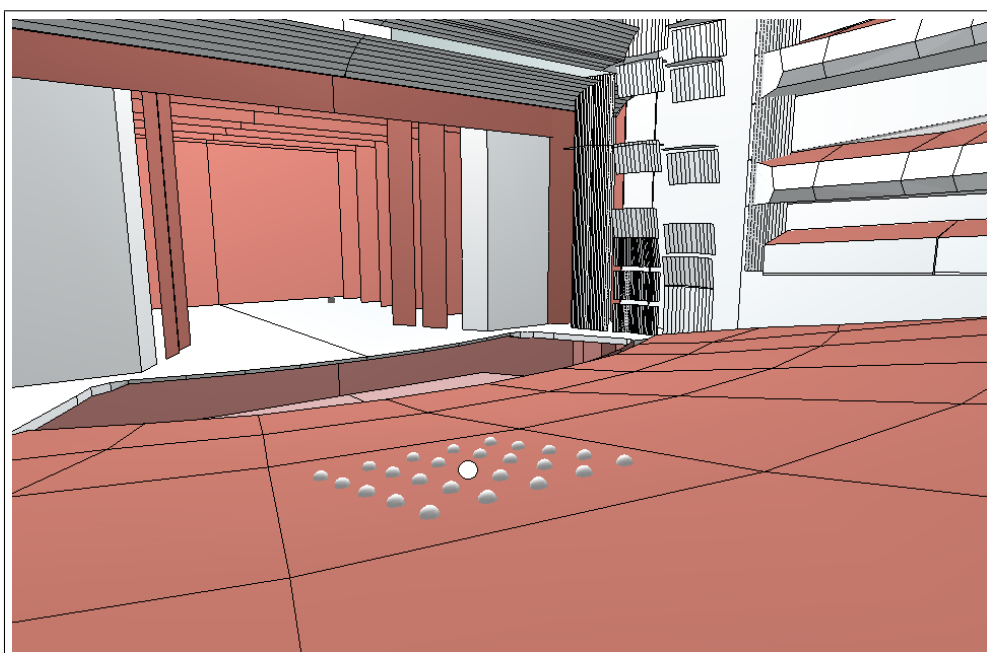


Figure 3.5: Listening position (white circle) surrounded by two receiver clusters.

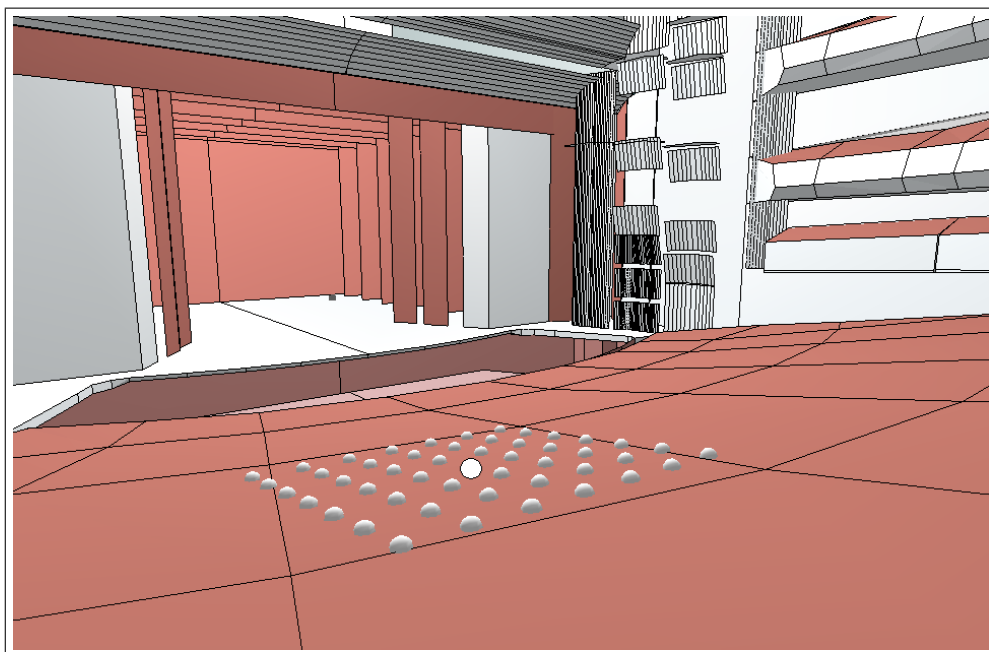


Figure 3.6: Listening position (white circle) surrounded by three receiver clusters.

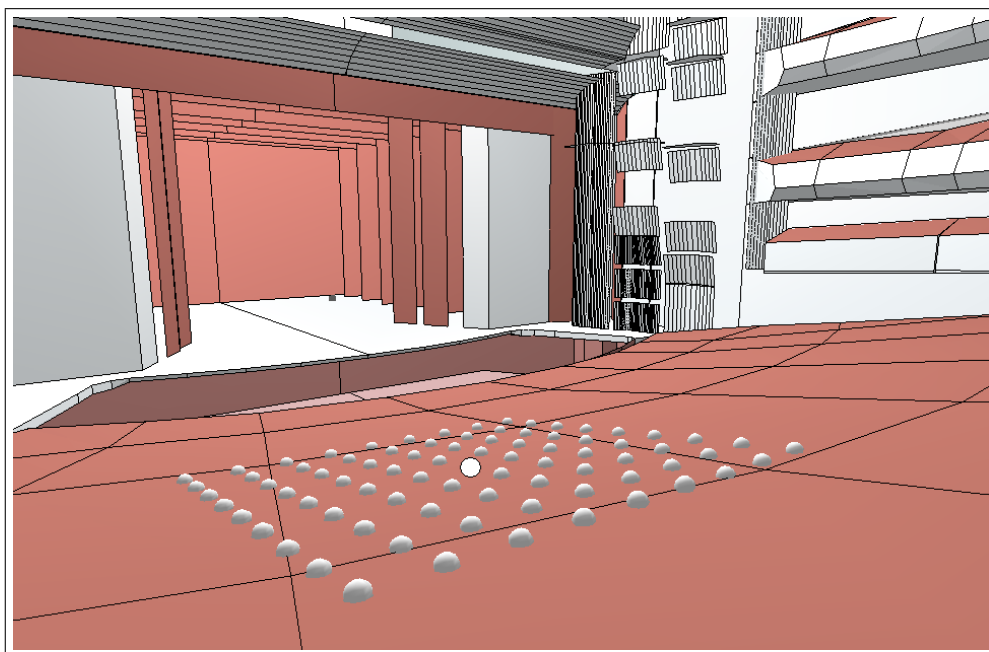
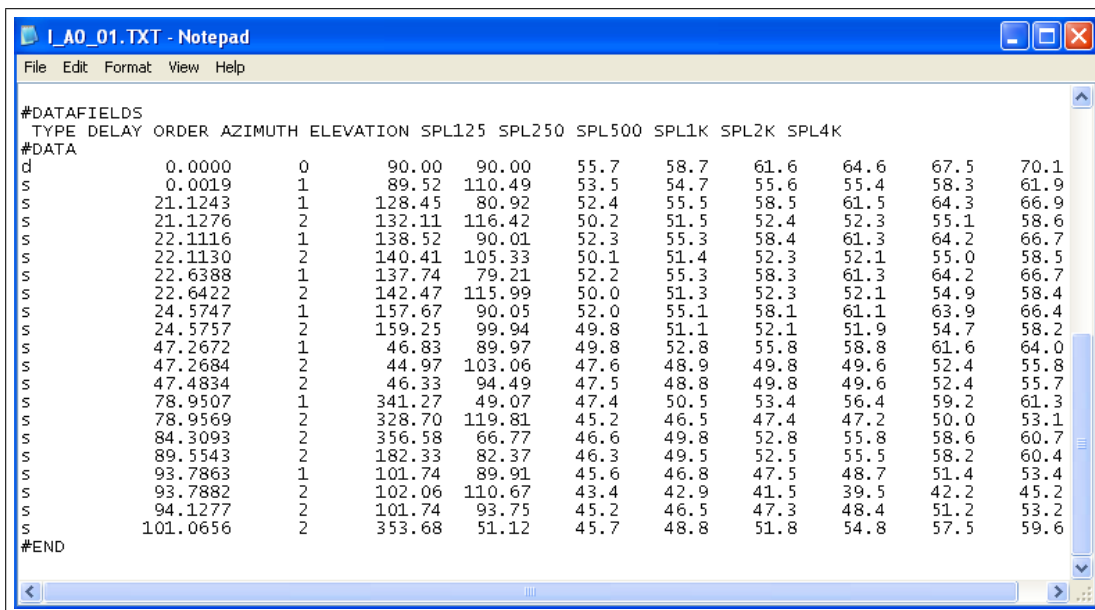


Figure 3.7: Listening position (white circle) surrounded by four receiver clusters.

3.2 Matlab scripts



```
#DATAFIELDS
TYPE DELAY ORDER AZIMUTH ELEVATION SPL125 SPL250 SPL500 SPL1K SPL2K SPL4K
#DATA
d 0.0000 0 90.00 90.00 55.7 58.7 61.6 64.6 67.5 70.1
s 0.0019 1 89.52 110.49 53.5 54.7 55.6 55.4 58.3 61.9
s 21.1243 1 128.45 80.92 52.4 55.5 58.5 61.5 64.3 66.9
s 21.1276 2 132.11 116.42 50.2 51.5 52.4 52.3 55.1 58.6
s 22.1116 1 138.52 90.01 52.3 55.3 58.4 61.3 64.2 66.7
s 22.1130 2 140.41 105.33 50.1 51.4 52.3 52.1 55.0 58.5
s 22.6388 1 137.74 79.21 52.2 55.3 58.3 61.3 64.2 66.7
s 22.6422 2 142.47 115.99 50.0 51.3 52.3 52.1 54.9 58.4
s 24.5747 1 157.67 90.05 52.0 55.1 58.1 61.1 63.9 66.4
s 24.5757 2 159.25 99.94 49.8 51.1 52.1 51.9 54.7 58.2
s 47.2672 1 46.83 89.97 49.8 52.8 55.8 58.8 61.6 64.0
s 47.2684 2 44.97 103.06 47.6 48.9 49.8 49.6 52.4 55.8
s 47.4834 2 46.33 94.49 47.5 48.8 49.8 49.6 52.4 55.7
s 78.9507 1 341.27 49.07 47.4 50.5 53.4 56.4 59.2 61.3
s 78.9569 2 328.70 119.81 45.2 46.5 47.4 47.2 50.0 53.1
s 84.3093 2 356.58 66.77 46.6 49.8 52.8 55.8 58.6 60.7
s 89.5543 2 182.33 82.37 46.3 49.5 52.5 55.5 58.2 60.4
s 93.7863 1 101.74 89.91 45.6 46.8 47.5 48.7 51.4 53.4
s 93.7882 2 102.06 110.67 43.4 42.9 41.5 39.5 42.2 45.2
s 94.1277 2 101.74 93.75 45.2 46.5 47.3 48.4 51.2 53.2
s 101.0656 2 353.68 51.12 45.7 48.8 51.8 54.8 57.5 59.6
#END
```

Figure 3.8: A typical output text file from CATT Acoustics used to extract in parameters for the Matlab scripts.

Above in Figure 3.8 you can see how one of the output text files from CATT Acoustic looks. Each row contains data about a specific reflection. The first column called *TYPE* states if the reflection is direct (*d*), specular (*s*) or diffuse (*). Since we are only concerned with the deterministic scattering from hemispheres the diffuse reflection order is set to zero, hence no (*) in the first column. The second column called *DELAY* simply states the delay time of the reflection in *ms* relative to the direct sound. The third column called *ORDER* states the reflection order. The column called *AZIMUTH* contains the incident angle in the receiver's horizontal plane. The column called *ELEVATION* contains the incident angle in the receiver's vertical plane. Finally the last six columns specify the sound pressure level as octave band averages.

The first task for the Matlab scripts is to extract the parameters needed to calculate the pressure scattered by the hemispheres out of files such as the one in Figure 3.8. Since every output text file has the same structure it is easy to code a Matlab routine going through the files and saving the data needed in a clever way.

3.2.1 Shaping a spectrum from six octave band averages

As you can see in Figure 3.8 there are six parameters describing the spectrum of a reflection. Using all these parameters in conjunction with a cubic spline interpolation gave a more fine-tuned modeling of the spectrum compared to previous modeling. Since the values given from CATT Acoustics were octave band averages, each value was normalized with the number of frequency components in the actual octave band. Let me show a typical example using this approach.

	SPL125	SPL250	SPL500	SPL1k	SPL2k	SPL4k
<i>Oct. band av.</i>	53.5	54.7	55.6	55.4	58.3	61.9

Table 3.2: Sequence describing the spectrum for a typical reflection.

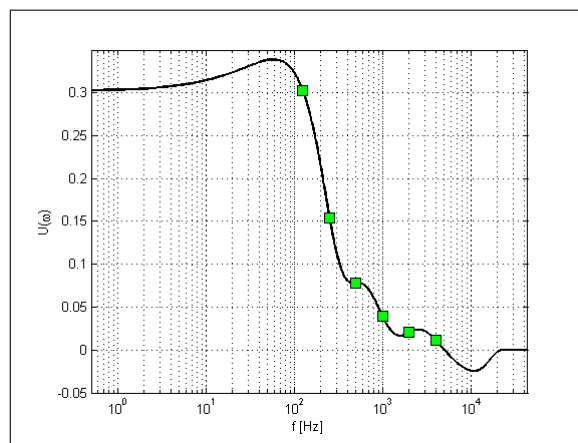


Figure 3.9: Modeling the spectrum for the reflection in Table 3.2, using cubic spline interpolation and normalizing each octave band value with its number of frequency components.

3.2.2 The SumBank routine

Once all the needed parameters have been calculated, the Matlab routine *H_SumBank.m* (attached in the Appendix) can be executed. What this routine does in essence is that it uses the in parameters and the *SumBank* (described in Chapter 2) to compute the frequency response of the hemispheres. If the number of hemispheres is greater than one the output parameter of this routine will be a matrix with dimension (*number of frequency components* \times *number of hemispheres*). Consequently each column in this output matrix constitutes the frequency response of one specific hemisphere. This comes

in handy later if one wants to look at the impulse response for a specific set of hemispheres.

Some important parameter values involved when using the SumBank should also be stated. The radius of the hemisphere was set to approximately 8.9 cm. Since I wanted the sampling frequency to agree with sampling frequency of the CATT Acoustics impulse responses I set $N = 88200$ ($N =$ number of frequency components) and $df = 0.5$ ($df =$ the frequency interval between two adjacent frequency components). These parameter values will generate a rather large SumBank. I will be the first to admit that the size of the SumBank could have been reduced drastically by saving the value of the sum with a much larger value of df and then interpolate. Nevertheless, the parameter values stated above were the ones I used during this work.

Since our goal is to find out whether the introduction of hemispherical scattering results in an audible change or not we need the impulse response for the scattering hemispheres. This impulse response is obtained by taking the inverse Fast Fourier Transform (*FFT*) of the frequency response computed by *H_SumBank.m*. Since the *FFT* algorithm assumes the time signal to be periodic, the level in the end of the impulse response will in some cases rise in a very un-physical way, contradicting causality. To minimize this effect all the impulse responses were translated 10 ms forward in time. When the inverse *FFT* has been done it is also important to keep in mind that various errors such as interpolation errors can affect the levels of the impulse response. It is crucial for the validity of this work that x dB in CATT Acoustics correspond to exactly x dB in the Matlab scripts. A bullet-proof way of making sure that this is really the case is to have a look at the specular impulse responses. By summing the squared pressure values of the impulse response from CATT Acoustics and the Matlab scripts in the time domain one can make sure that the *total* energy content of the signals is the same, although some variations will occur if one compares octave band per octave band.

The specular impulse response calculated by the Matlab scripts should ideally be an identical copy of the specular impulse response calculated by CATT Acoustics. However this is very hard to put into practice since there is no phase information at all (we only have the six octave band values at disposal). Knowing this we can still try to compensate a possible mismatch between the two impulse responses by scaling the impulse response calculated by the matlab scripts with a factor. This factor is the factor which will make the energy content of the two impulse responses equal. Once this is done we know that we have comparability and the possibility of combining impulse responses calculated by CATT Acoustics and the Matlab scripts. At this point we are ready to start the final part of the analysis. That is to search for audible differences through a listening test.

3.3 Listening tests

Two parts formed this listening test. The purpose of the first part was to investigate if there is an audible difference between modeling with or without hemispherical scattering. Part number two deals with the comparison between the exact and the approximate solution for the scattered sound field.

The test was performed in the VR lab at the Division of Applied Acoustics, Chalmers. In order to keep the background level as low as possible the computer was placed outside the room where the test was performed. This resulted in a background level of 35 dBA. The sound pressure level at the left ear when playing the reference signal was 73 dBA.

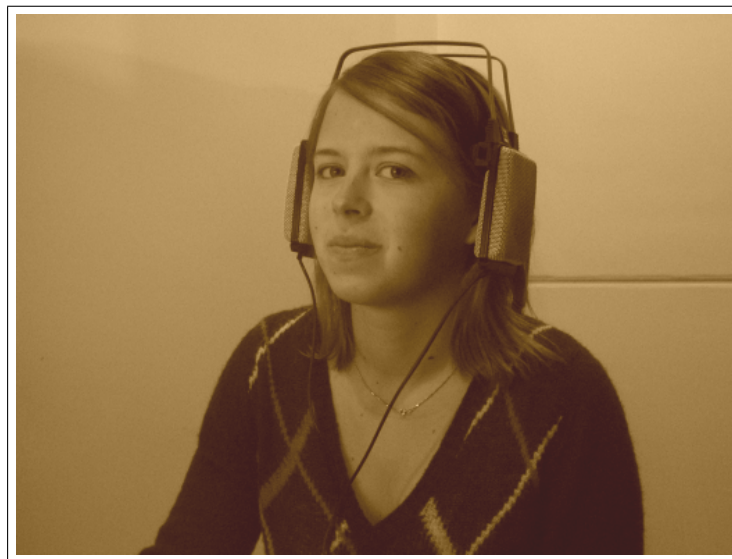


Figure 3.10: Moa during the listening test.

3.3.1 Part 1 - The effect of hemisphere scattering

This part had five subparts. Each one of them had the same structure. The main idea is to compare a reference signal with four other samples. The listener was asked to answer if he or she could hear an audible difference between the reference signal and any of the other four samples. If the answer was yes, then the listener was asked to state the number of the samples that differed and if more than one sample differed from the reference signal they were also asked to state the one differing the most. Let me explain the the content of the various signals.

- **Reference:** Impulse response from a full detailed calculation in CATT Acoustics convoluted with digital recording.
- **Sample 1:** The sum of the impulse response given from CATT Acoustics and the impulse response for one set of scattering hemispheres (see Figure 3.4) convoluted with digital recording.
- **Sample 2:** The sum of the impulse response given from CATT Acoustics and the impulse response for two sets of scattering hemispheres (see Figure 3.5) convoluted with digital recording.
- **Sample 3:** The sum of the impulse response given from CATT Acoustics and the impulse response for three sets of scattering hemispheres (see Figure 3.6) convoluted with digital recording.
- **Sample 4:** The sum of the impulse response given from CATT Acoustics and the impulse response for four sets of scattering hemispheres (see Figure 3.7) convoluted with digital recording.

The digital recordings used for convolution in the five different subparts were

- **Part 1.1:** White Noise
- **Part 1.2:** No convolution (i.e. pure impulse responses)
- **Part 1.3:** Anechoic recording of an acoustic guitar
- **Part 1.4:** Anechoic recording of a female talker
- **Part 1.5:** Anechoic recording of a trumpet

3.3.2 Part 2 - Exact vs. approximative solution

The other main question with the listening test was: Is there an audible difference between the exact and the approximative solution? A simple situation aiming to answer that question was therefore refined.

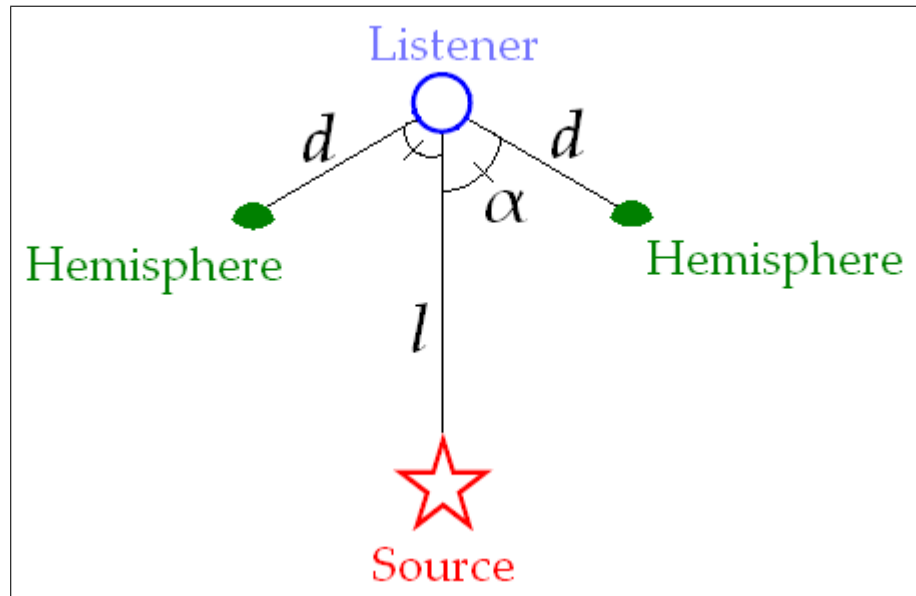


Figure 3.11: Situation refined for comparing the two solutions.

The situation in part 2 can be seen in Figure 3.11. The distance between the source and the listening position (l) was set to 16 m. Furthermore the source is not actually in the same plane as the listening position and the hemispheres, it is elevated approximately 3 m above the plane. The angle α was set to $[0^\circ, 30^\circ, 60^\circ, 120^\circ, 180^\circ]$ and the distance between the listening position and the hemispheres (d) was for each value of α set to $[d, 2d, 4d]$. For each combination of α and d the impulse response for the two hemispheres was computed, exact and approximately. The two impulse responses were then convolved with white noise. When this was done for all the configurations the pairwise comparisons between the exact and the approximative solution could begin.

3.3.3 Identical samples

In order to judge the reliability of results accurately it is important to study the case when identical samples are compared by the listeners. In the test there were three different pairs of samples which were a bit special. The two samples forming each of these pairs were identical. The outcome of this test will serve as a reference when analyzing the listening test.

4 Results

The aim of this chapter is to give the reader a short and precise presentation of the results from the analysis in Matlab and the listening tests. In the end of the chapter there is also a discussion about the results.

4.1 Analysis in Matlab

Many hours have been spent on making different kinds of investigations with the Matlab scripts. The plots could go on and on for pages. However, the most important once can be seen here.

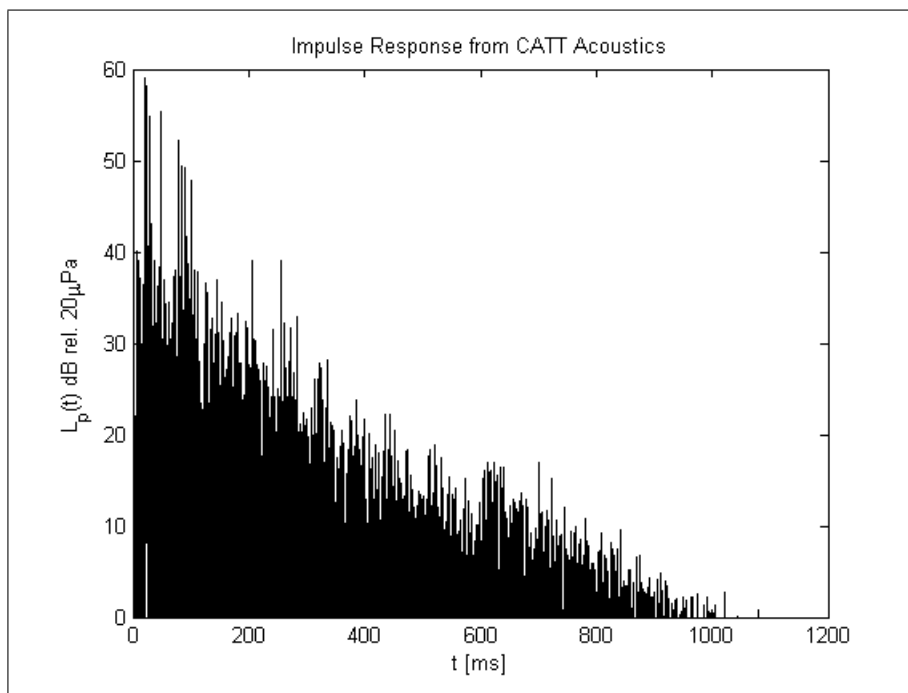


Figure 4.1: Full detailed calculation of the impulse response made by CATT Acoustics.

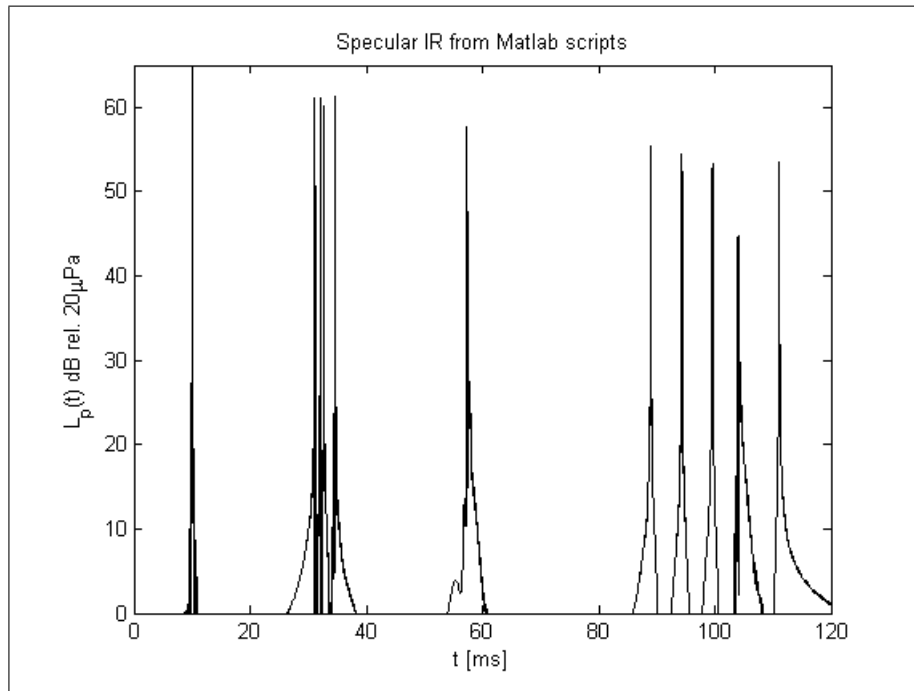


Figure 4.2: Specular impulse response calculated with the Matlab scripts using out parameters from CATT Acoustics.

The following sequence of impulse responses correspond to the four receiver configurations shown in Figure 3.4 - Figure 3.7. Note that the plots concern the hemisphere scattered sound exclusively.

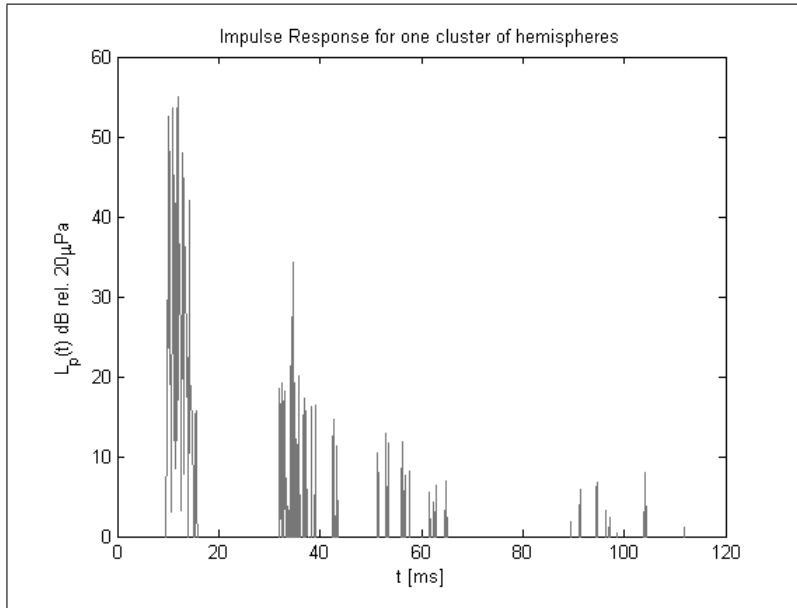


Figure 4.3: Impulse response for one cluster of hemispheres.

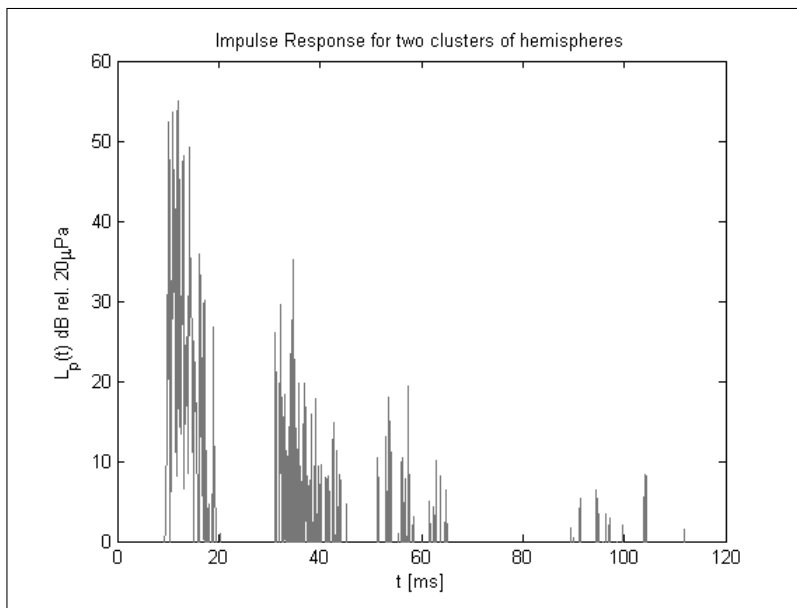


Figure 4.4: Impulse response for two clusters of hemispheres.

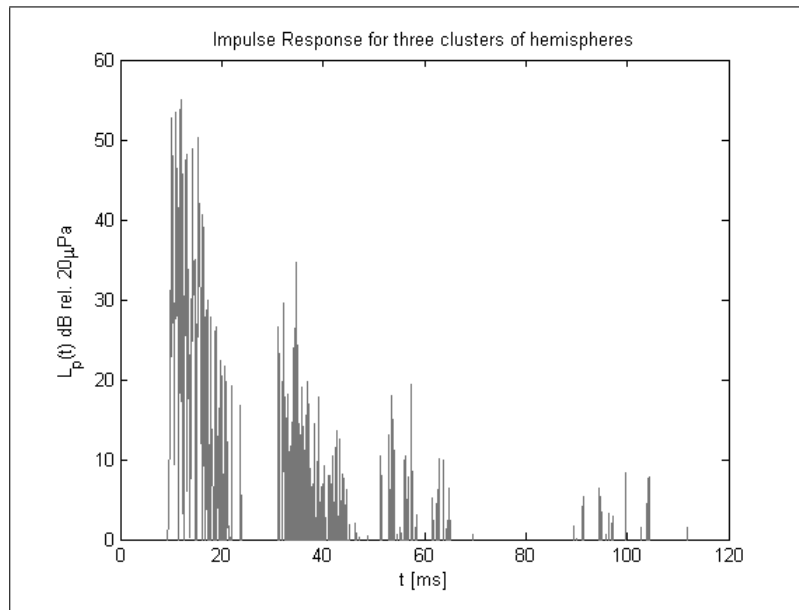


Figure 4.5: Impulse response for three clusters of hemispheres.

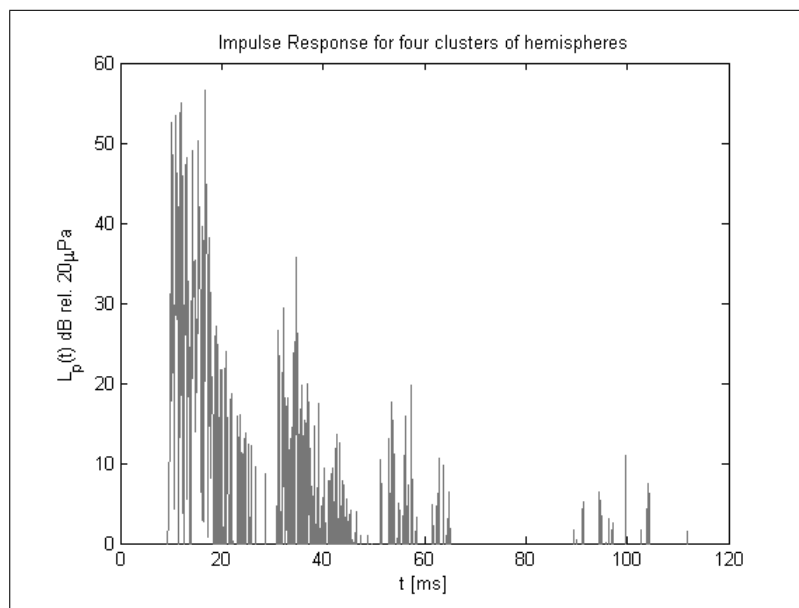


Figure 4.6: Impulse response for four clusters of hemispheres.

Let us also have a look at the plots showing the impulse response from CATT Acoustics in the same figure.

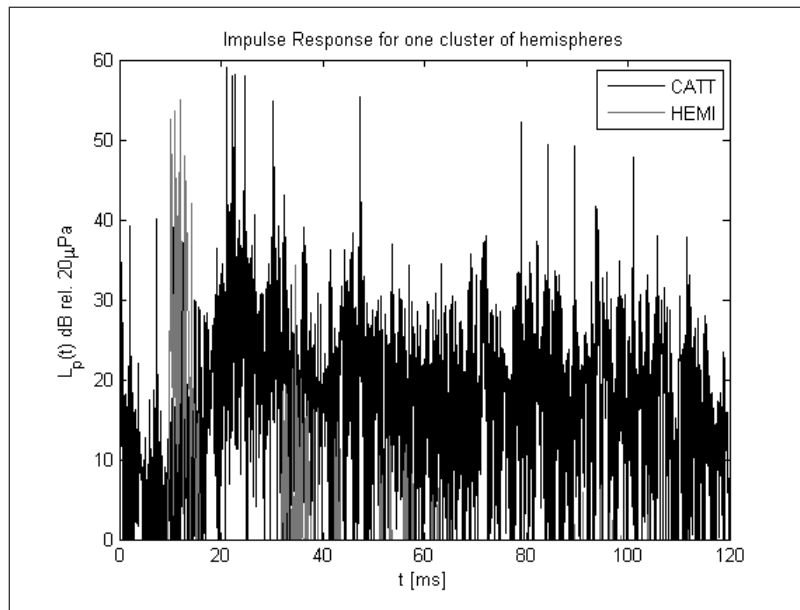


Figure 4.7: IR from CATT Acoustics together with IR for one cluster of hemispheres.

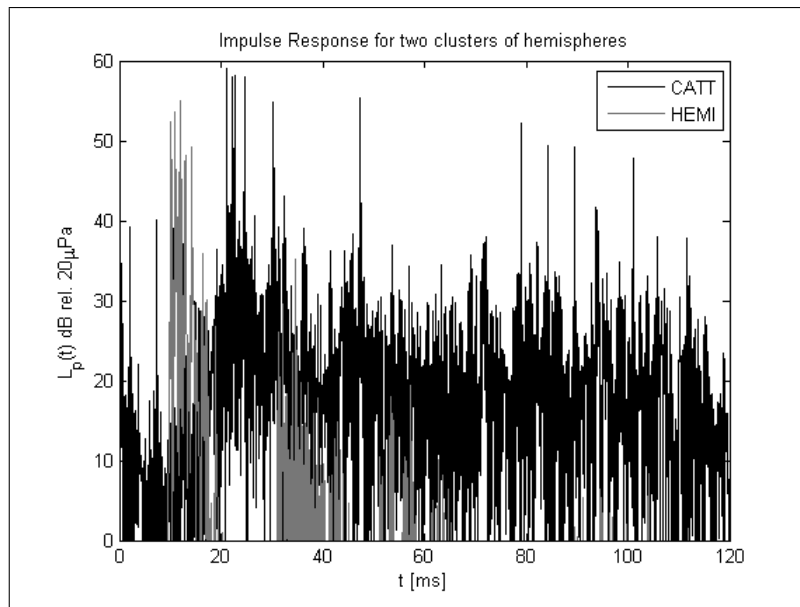


Figure 4.8: IR from CATT Acoustics together with IR for two clusters of hemispheres.

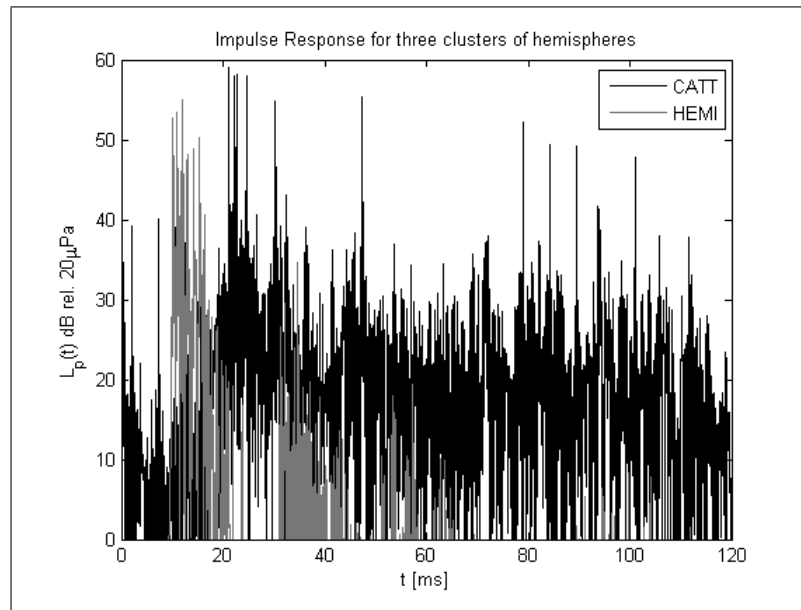


Figure 4.9: IR from CATT Acoustics together with IR for three clusters of hemispheres.

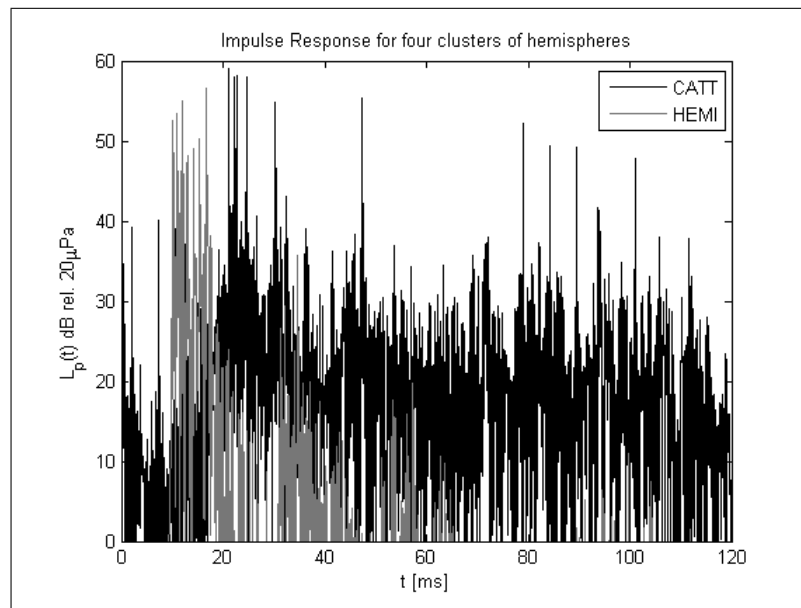


Figure 4.10: IR from CATT Acoustics together with IR for four clusters of hemispheres.

4.2 Listening test

4.2.1 Part 1 - The effect of hemisphere scattering

Here follows the compiled results of part 1 of the listening test.

Part	Can distinguish ref. from any of the samples	Can distinguish more than one sample from ref.	Can distinguish ref. and				Largest difference between ref. and			
			1	2	3	4	1	2	3	4
1.1	15/15	11/15	6/15	6/15	9/15	15/15	-	-	1/15	14/15
1.2	3/15	2/3	-	1/15	1/15	3/15	1/3	-	1/3	1/3
1.3	7/15	1/7	-	1/15	2/15	5/15	-	1/7	2/7	4/7
1.4	8/15	4/8	5/15	1/15	3/15	3/15	3/8	-	2/8	3/8
1.5	9/15	7/9	5/15	3/15	3/15	8/15	1/9	1/9	1/9	6/9

Table 4.1: Results from part 1.

4.2.2 Part 2 - Exact vs. approximative solution

Here follows the compiled results of part 2 of the listening test.

α	$d = 0.6m$	$d = 1.2m$	$d = 2.4m$
	Do hear a difference	Do hear a difference	Do hear a difference
0°	10/15	5/15	5/15
30°	10/15	4/15	8/15
60°	15/15	4/15	8/15
120°	12/15	7/15	4/15
180°	12/15	6/15	3/15

Table 4.2: Results from part 2.

4.2.3 Identical samples

The comparisons of the pairwise identical samples came out as follows

	Could hear a difference
Identical pair 1	3/15
Identical pair 2	2/15
Identical pair 3	3/15

Table 4.3: Results from from comparisons of identical samples.

Another way of presenting the results from the study of identical samples is the following

- **9/15** listeners gave 0 erroneous answers (i.e. claimed that there *was* a difference) when comparing the samples forming these three pairs.
- **4/15** listeners gave 1 erroneous answers (i.e. claimed that there *was* a difference) when comparing the samples forming these three pairs.
- **2/15** listeners gave 2 erroneous answers (i.e. claimed that there *was* a difference) when comparing the samples forming these three pairs.
- **0/15** listeners gave 3 erroneous answers (i.e. claimed that there *was* a difference) when comparing the samples forming these three pairs.

4.3 Discussion

It is hard to foresee the influence of hemisphere scattering by just visually inspecting the plots in Figure 4.3 - Figure 4.10. As one can see in Figure 4.3 - Figure 4.6 the impulse response grows more and more dense as the number of scattering hemispheres is increased. This makes sense since you are increasing the number of objects scattering sound to the listening position. But the important question still remains: Will the hemisphere scattering be audible when its impulse response is added up will the impulse response from CATT Acoustics? The next four plots in Figure 4.7 - Figure 4.10 can give us a first idea to the answer. We can see that there are time regions where the CATT and HEMI levels are of the same order. Nevertheless, one could still argue that these time regions are too short. So, in order to continue a meaningful discussion we also need the results from the subjective listening tests.

By looking at Table 4.1 we can conclude that 100% (15 out of 15 test subjects) could tell a difference between the reference and any of the four samples in the case of white noise convolution. 14/15 claimed that sample number four produced the greatest difference. This is good news but also expected news. It is no secret that differences in pitch and level are easily exposed when convolving the different impulse responses with white noise. If we take a look in the other end of the spectra we are studying Part 1.2 where the impulse responses are not convolved at all, we are just listening to pure impulse responses. In that case only 3/15 could tell the difference between the reference and any of the four samples. If you take a look in Table 4.3 you can see that the fraction 3/15 constitutes some kind of truncation criterion. This is because three out of fifteen subjects claimed that there *was* a difference when comparing completely identical signals. Consequently, we can with highest likelihood state that hemisphere scattering - modeled in a small scale like here - do not change the audible impression of the pure impulse responses. However, if we proceed with parts 1.3-1.5 there is no doubt that people do hear a difference. In the last part where the impulse responses have been convolved with an anechoic recording of a trumpet 9/15 subjects heard a difference and 6/9 claimed that sample number four produced the greatest difference.

In Table 4.2 the results of the listening test's part 2 are compiled. Since the important approximations done are the farzone and farfield approximations you would hope that few people could tell a difference between exact and approximated solutions as you increase the distance d . This is the case when $\alpha > 90^\circ$ but not when $\alpha < 90^\circ$. Perhaps this is not too surprising after all since increasing d when $\alpha < 90^\circ$ means moving the scattering hemisphere closer to the source (consequently the farfield approximation breaks down). It sure seems like that is the case because when you look at the results from the last two angles ($\alpha = 120^\circ$ and $\alpha = 180^\circ$) the trend looks more like what one expects. The number of test subjects being able to distinguish between the two solutions is decreasing as the distance d is increased. It would have been desirable to perform this experiment for a greater number of angles though. More information about the angular dependence can be found in [1], pages 17-18.

It is also important to remember that this expanded way of modeling sound scattering - in this scale - only concerns the early part of the impulse response. If one had the computational capacity and time to fill the entire audience plane with receivers the results might have looked different. However, doing that would in turn require a better way of modeling coupling.

5 Conclusions

Introducing hemisphere scattering into the present acoustical modeling makes a difference. To exactly quantify the differences between modeling with and without hemisphere scattering is a tough task. To analyze the differences I have essentially dealt with two kinds of impulse responses.

- **A:** The impulse response from a full detailed calculation in CATT Acoustics.
- **B:** The impulse response from a full detailed calculation in CATT Acoustics with an impulse response corresponding to hemisphere scattering added to it.

My studies show that there is a substantial audible difference when comparing the white noise convolution of A with the white noise convolution of B. When convolving A and B with other anechoic recordings (such as musical instruments and speech) the differences are not as clear but can nevertheless be pointed out through listening tests.

A very central part of this project has been coding a set of Matlab scripts solving for the sound field scattered by hemispheres. This is a very time consuming procedure (even for a modern computer). Therefore the so called *SumBank* was built. By combining the farzone and farfield approximations the SumBank greatly reduces the computation time.

Important to note is that what I present in this report is a first attempt to investigate the influence of introducing hemisphere scattering. The largest set of hemispheres I did calculations for was a set containing four clusters (80 hemispheres). Everyone knows that there are more than eighty spectators in a large opera house such as the Gothenburg Opera House. Still, even with incomplete audience modeling the results from the listening tests show that hemisphere scattering does make a difference and should not be neglected.

Future work would be to expand and improve the modeling. A larger number of hemispheres would make the modeling more realistic and would probably also increase the audible difference between A and B. However, a new way of modeling coupling would then be desirable.

Another area worth improving if continuing this work is the modeling of finite impedance for the plane on which the hemispheres are resting. The starting point for this would be [7] and [8].

References

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- [8] Thomasson, S.I.: *Reflection of waves from a point source by an impedance boundary*, JASA 59(4), 780-785, 1976

A Matlab Scripts

A.1 Main program: *ConvertCATTtoMATLAB*

```
000: clear all, clc, close all
001:
002: % Parameters and directory paths for CATT-import etc
003:
004: commonpath = 'C:\jhw\Xjobbet\';
005: datadir = [ commonpath 'CATT\GOP\GOP_UT\'];
006: hrtfdir = [ commonpath 'HRTFs\'];
007: anechstr = [ commonpath 'WAV\WhiteNoise.wav'];
008:
009: srcid = 'a0';
010: recid = '01';
011:
012: bossid_nbr = [ 11:18 21:92 ];
013: Nboss = length(bossid_nbr);
014: a_i = 2.54*3.5/100 .* ones(1,Nboss);
015:
016: % Creating a matrix where each column vector represents the plane normal
017: % associated with a certain boss
018: pn = [-0.0356 -0.1782 0.9834]';
019: plane_normal = repmat(pn,1,Nboss);
020:
021: % Due to numerical rounding in CATT we need to normalize the plane normals
022: for j = 1:size(plane_normal,2)
023:
024:     plane_normal(:,j) = plane_normal(:,j)/sqrt( sum( plane_normal(:,j).^2 ) );
025:
026: end
027:
028: Df = 0.5;
029: N = 88200;
030:
031: disp('Read CATT-file')
032:
033: [ c, Rbar_0k, theta_dk, phi_dk, A_k, ...
034:   R_i, R_ik, cos_ik, cos_ik_star, A_ik, ...
035:   theta_i, phi_i, nx_ni_si, ny_ni_si, nz_ni_si ] = ...
036: coordinates_CATT(datadir, srcid, recid, bossid_nbr, plane_normal);
037:
038:
039: % Ger data fr kontrollutkrift av ankomsttiderna i lyssnarpositionen
040: [ SpecularArrivals_ms DiffuseArrivals_ms ] = ArrivalTimes( Rbar_0k, R_ik, R_i, c);
041:
042:
```

```

043: disp('Calculate sound field')
044:
045: k = 2*pi/c*(1:N)*Df;
046:
047: % Creating the SumBank
048: % SumBank2 = CreateSumBank2( a_i(1)*k, 1e-6 );
049: % [ SumBank , NOI ] = CreateSumBank( a_i(1)*k, 1e-6 );
050:
051: % Translating both spectras td seconds
052: td = 0.01;
053:
054: FR_hemi = H.SumBank(k, a_i, R_i, R_ik, cos_ik, cos_ik_star, A_ik, ...
055: nx_ni_si, ny_ni_si, nz_ni_si, plane_normal, N, Df, td);
056:
057: FR_spec = H.spec_CATT(k, Rbar_0k, A_k, N, Df, td);
058:
059: % Fraction of opening, R
060:
061: R = 0.6/(0.6 + 2*a_i(1));
062:
063: FR1 = FR_hemi(:,1:8);
064: FR2 = FR_hemi(:,9:24).*R;
065: FR3 = FR_hemi(:,25:48).*R^2;
066: FR4 = FR_hemi(:,49:80).*R^3;
067:
068: FR_hemi = [FR1 FR2 FR3 FR4];
069:
070: disp('Plotting')
071:
072: % Plotting of frequency response (FR) and impulse response (IR)
073: FR_spec_sum = sum(FR_spec,2);
074: FR_hemi_sum = sum(FR_hemi,2);
075:
076:
077: plotting( 2, Df, N, [ FR_spec_sum FR_hemi_sum ], ...
078: strvcac( 'Specular part', 'Scattered part' ))
079:
080: plotting( 3, Df, N, [ FR_spec_sum FR_spec_sum+FR_hemi_sum ], ...
081: strvcac( 'Specular part', 'Total field' ))
082:
083: Boss_IR.Plotting( 4, Df, N, FR_hemi, FR_spec )
084:
085:
086: disp('Calculate BRIR')
087:
088: [ BRIR_L_spec, BRIR_R_spec ] = brir( hrtfdir, FR_spec, phi_dk, theta_dk, Df );
089: [ BRIR_L_hemi, BRIR_R_hemi ] = brir( hrtfdir, FR_hemi, phi_i, theta_i, Df );
090: [ BRIR_L_hemi_1, BRIR_R_hemi_1 ] = brir( hrtfdir, [FR1], phi_i(1:8), theta_i(1:8),
Df );
091: [ BRIR_L_hemi_2, BRIR_R_hemi_2 ] = brir( hrtfdir, [FR1 FR2], phi_i(1:24), theta_i(1:24),
Df );
092: [ BRIR_L_hemi_3, BRIR_R_hemi_3 ] = brir( hrtfdir, [FR1 FR2 FR3], phi_i(1:48), theta_i(1:48),
Df );
093: [ BRIR_L_hemi_4, BRIR_R_hemi_4 ] = brir( hrtfdir, [FR1 FR2 FR3 FR4], phi_i(1:80),
theta_i(1:80), Df );
094:

```

```

095: BRIR_L_hemi = BRIR_L_hemi_4;
096: BRIR_R_hemi = BRIR_R_hemi_4;
097:
098: BRIR_L = BRIR_L_spec + BRIR_L_hemi;
099: BRIR_R = BRIR_R_spec + BRIR_R_hemi;
100:
101: BRIR_hemi = [ BRIR_L_hemi, BRIR_R_hemi ];
102: BRIR_hemi_1 = [ BRIR_L_hemi_1, BRIR_R_hemi_1 ];
103: BRIR_hemi_2 = [ BRIR_L_hemi_2, BRIR_R_hemi_2 ];
104: BRIR_hemi_3 = [ BRIR_L_hemi_3, BRIR_R_hemi_3 ];
105: BRIR_hemi_4 = [ BRIR_L_hemi_4, BRIR_R_hemi_4 ];
106: BRIR = [ BRIR_L, BRIR_R ];
107:
108:
109: disp('Convolution with anechoic recording')
110:
111: % The source signal and its spectrum
112:
113: [ AnechoicSignal, fs ] = wavread( anechstr );
114: NormalizedAnechoicSignal = AnechoicSignal / max( abs(AnechoicSignal) );
115:
116: BRR_spec = brr( BRIR_L_spec, BRIR_R_spec, NormalizedAnechoicSignal );
117: BRR_hemi = brr( BRIR_L_hemi, BRIR_R_hemi, NormalizedAnechoicSignal );
118:
119: BRR_hemi_1 = brr( BRIR_L_hemi_1, BRIR_R_hemi_1, NormalizedAnechoicSignal );
120: BRR_hemi_2 = brr( BRIR_L_hemi_2, BRIR_R_hemi_2, NormalizedAnechoicSignal );
121: BRR_hemi_3 = brr( BRIR_L_hemi_3, BRIR_R_hemi_3, NormalizedAnechoicSignal );
122: BRR_hemi_4 = brr( BRIR_L_hemi_4, BRIR_R_hemi_4, NormalizedAnechoicSignal );
123:
124: BRR = brr( BRIR_L, BRIR_R, NormalizedAnechoicSignal );
125:
126:
127: % Auralization
128:
129: disp('Auralization')
130:
131: disp('Only Specular Reflections')
132: wavplay( BRR_spec/max(max(abs(BRR_spec))), fs, 'sync' )
133:
134: disp('Specular and Hemisphere Reflections')
135: wavplay( BRR/max(max(abs(BRR))), fs, 'sync' )
136:
137: disp('Only Hemisphere Reflections')
138: wavplay( BRR_hemi/max(max(abs(BRR_hemi))), fs, 'sync' )

```

A.2 Important function: $H_SumBank$

```
00: function H_sc_SumBank = H_SumBank(k, a_i, R_si, R_ni_si, cos_ni_si, cos_ni_si_star, A_ni_si, N, df,td)
01:
02:     load SumBank_GRANDE.mat
03:
04:     f = (1:N)*df;
05:     sf = 680;
06:
07:     Nboss = length(a_i);
08:     H_sc_SumBank = zeros( length(k),Nboss );
09:
10:     k = k';
11:     boss = 0;
12:
13:     % Loop for bosses
14:     for ii = 1:Nboss
15:
16:         boss = boss + 1
17:         NOR = length(R_ni_siii);
18:
19:         BossSpectra = zeros( length(k),1 );
20:
21:         kr = k*R_si(ii);
22:
23:         % Loop for reflections associated with boss ii
24:         for nn = 1:NOR
25:
26:             alpha = 180/pi*acos(cos_ni_siii(nn));
27:             alpha_star = 180/pi*acos(cos_ni_si_starii(nn));
28:
29:             kr_0 = k*R_ni_siii(nn);
30:             A = A_ni_siii(nn,:);
31:
32:             kol = floor(alpha/5) + 1;
33:             kol_star = floor(alpha_star/5) + 1;
34:
35:             p_sc = SumBank(:,kol) + SumBank(:,kol_star);
36:
37:             BossSpectra = BossSpectra + ...
38:             i.*k .* Interpolate(A, N, df)' .* exp(i.*kr_0)./kr_0 .*exp(i.*kr)./kr .*p_sc;
39:
40:         end
41:
42:         H_sc_SumBank(:,ii) = BossSpectra.*exp(i*2*pi*f'*td)/sf;
43:
44:     end
```


A.3 Important function: *Interpolate*

```
00: function yy = Interpolate( YY, N, df )
01:
02:     fs = N*df;
03:
04:     % xx - the frequency vector or if one wants points of evaluation
05:     f = (1:N)*df;
06:
07:     % XX - discrete X values of the intrepolant
08:     XX = [ 125 250 500 1000 2000 4000 ];
09:
10:     % YY - discrete Y values of the intrepolant
11:     YY = YY.^2;
12:
13:
14:     for i=1:length(XX)
15:
16:         nofc(i) = length( find( f>=XX(i)/sqrt(2) & f<=XX(i)*sqrt(2) ) );
17:
18:     end
19:
20:     YYpfc = YY./nofc;
21:
22:     fc = find(f == fs/2);
23:
24:     yy2 = spline( [0 XX fs/2 f(fc+1:end)], [YYpfc(1) YYpfc 0 zeros(size(f(fc+1:end)))] , f);
25:
26:     yy = sqrt(yy2);
```