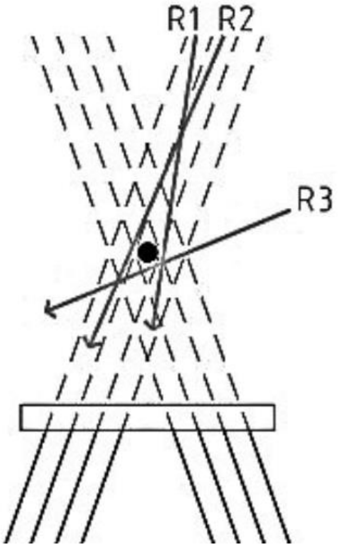

Pile Group Design

An Investigation and Development of a Software to Optimize the Design of Pile Groups

Master's thesis in Master Program Structural Engineering and Building Technology

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DEPARTMENT OF ARCHITECTURE AND CIVIL ENGINEERING

CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2021
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Master's thesis ACEX30

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Abstract

The design of pile groups is a very time consuming and iterative procedure. Piling is also very expensive, hence it is important to keep the number of piles to a minimum. Today, there are no established guidelines how to design a robust pile group and the design differs depending on the load effects and soil conditions.

This thesis investigates a method for designing a pile group based on the definitions of pile center and load center. A stand-alone software was developed to optimize double and single symmetric pile groups for bridges. The software optimizes the pile groups by minimizing the distance between the pile center and the load center while trying to achieve as high moment capacity in the pile group as possible. The software was evaluated by conducting 27 cases studies, where a vast majority of the generated pile groups were designed with either fewer or the same number of piles as the original pile groups. The pile groups were designed without collisions between the piles and the simulation time varied between 2 and 85 seconds.

The placement of the pile center is mainly limited by the dimensions of the pile cap and the inclination of the piles. The load center was sometimes shown to vary substantially between the different load cases, which made it difficult to match the position of the pile center with the load center. The thesis can conclude that designing a pile group based on the definitions of pile center and load center is a good method when the position of the load center is fairly constant. When this is not the case, the moment capacity of the pile group is of high importance.

Keywords: Pile Group, Optimization, Pile Center, Load Center, Objective Function.

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Martin Ekström, Gothenburg, June 2021

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1

Introduction

1.1 Background

Foundations using pile groups is often necessary when building large structures on weak soil. A piled foundation is used to transfer the loads from the superstructure down to stronger soil or rock. The piles are often made of steel or reinforced concrete and sometimes timber. Piles can be used individually or grouped together as a pile group for which the piles are linked together at their tops with a pile cap, often consisting of a reinforced concrete slab.

Today, there are no established guidelines when designing a pile group and the design differs depending on the soil conditions and load effects. Historically, the design of pile groups has been done under supervision by experienced engineers using their acquired knowledge [2]. The design of a pile group can be very time-consuming. The great number of input parameters such as length, location, angle of rotation and inclination of the pile makes the number of geometric configurations almost infinite, even for a pile group with few piles. Piling is also very expensive, hence it is desirable to find an optimal solution, usually meaning that the pile group is able to carry the desired load while keeping the number of piles to a minimum. Pile groups are designed to sustain vertical forces, horizontal forces and moments and the design process is ideally iterative. Deformations and sectional forces in the piles are calculated and the pile configuration is then adjusted until the results are satisfying.

There are no fully developed optimization software for pile groups accessible on the market today. This thesis aims to fill that gap by investigating and developing a software which calculates pile forces and optimize the design of pile groups, given the pile capacity and load effects. By studying which parameters that influence the pile groups the most and then systematically positioning the piles within the pile group, an optimal pile group can be generated within seconds.

1.2 Aim

The aim of this thesis is to deepen the understanding of the parameters influencing the design and how to optimize pile groups for bridges in an efficient way. A standalone software that optimizes the design of pile groups given the pile capacity and load effects will be developed.

1.3 Limitations

To be able to complete this thesis within the given time frame, some limitations are set:

- This thesis will only focus on pile groups for bridges.
- Only pile groups with rectangular/quadratic pile caps will be analyzed.
- The pile groups will be designed with respect to ultimate limit state.
- Pile type and pile length are kept as constant parameters and will not be optimized. The designer will however be able to define these parameters for every analysis.
- The pile cap is assumed to be infinitely stiff, as its deformations are considered to be negligible compared to the deformations of the piles.

1.4 Specific issues under investigation

- What is the most efficient pile placement to ensure that the pile group is able to carry the desired load while keeping the number of piles to a minimum?
- Which parameters governs an efficient pile placement?

1.5 Methodology

This project is divided into three phases, where the main focus is to develop an optimization software for pile groups. The first phase is consisting of a literature study covering the theory behind the design of pile groups, calculation methods and optimization routines. The parameters influencing the design of pile groups are related to the geometry of the piles and the pile cap as well as the soil type. The literature study will evaluate these parameters separately, trying to identify how they affect the design of a pile group. Middle supports and abutments are subjected to different types of load cases. These load cases will be studied to identify an efficient pile placement so that the pile groups can sustain the applied loads.

In the second phase, the optimization software will be developed for both double symmetric and single symmetric supports. The calculation time for a pile group can be very long due to the great number of geometric configurations a pile group can have. The optimization software will therefore try to limit the number of configurations based on the knowledge gained in phase one. The calculation time will also be reduced by making use of symmetry when designing the pile groups. The software will then generate a pile group that can sustain all applied load cases with the least number of piles. The software will be written in the programming language Python. The Python code will then be implemented in another software written in C# for a more user-friendly interface.

In the third and last phase of the thesis, case studies will be performed to verify the created software.

2

Theory

2.1 Soil types

When building a heavy structure on weak soil that is not capable of supporting the structure, a deep foundation is often necessary. A deep foundation transmits the load from the superstructure down to stronger soil or rock. A common type of deep foundation is piling. When describing the geotechnical properties of a soil type, a distinction is made between the concepts *friction soil* and *cohesive soil.* In friction soil, for example sand, the strength is mainly built up by friction between the soil grains. The strength can be defined with a friction angle that differs between different types of friction soils. In cohesive soils, typically clay, there are both friction forces and cohesion that characterizes the strength. Cohesion is a force that holds together the molecules in the soil and the strength can be described by the undrained shear strength of the soil [6].

According to the Commission on Pile Research [5], the subgrade modulus, *kd*, is used to relate the deformations in the soil with respect to pressure. For cohesive soil, the subgrade modulus can be assumed to be constant along the depth and is calculated as:

$$kd = \lambda \cdot c_u \tag{2.1}$$

where

c_u = Undrained shear strength $[\frac{N}{m^2}]$
λ = Constant depending on short or long term loading $[-]$

For friction soil, the subgrade modulus is linear and increasing in proportion to the depth as:

$$(kd)_z = n_h \cdot z \tag{2.2}$$

where

z = Depth $[m]$
n_h = Growth rate factor $[\frac{N}{m^3}]$

2.2 Piles and Pile Groups

Piles are mainly divided into two categories, *end bearing piles* and *friction piles* and are often made of either concrete, steel or sometimes timber. An end bearing pile transmits the load from the pile mainly at the pile end, usually standing on solid rock. A friction pile transmits the load from cohesion between the pile and the surrounding soil. The two pile types are illustrated in Figure 2.1.

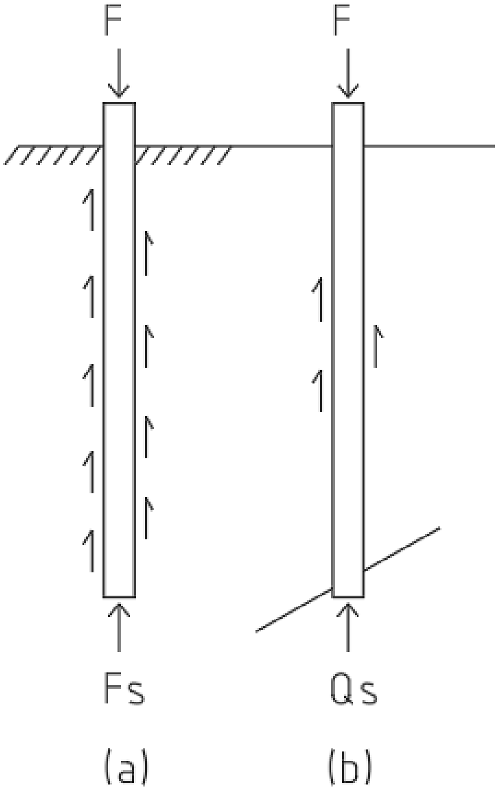


Figure 2.1: Friction pile (a) and end bearing pile (b)

As mentioned in Section 1.1, the number of piles in a pile group are linked together with a pile cap at their tops. Failure in a pile group can generally occur in two different ways. It can occur when the load capacity of a single pile is exceeded, or by a global failure of the pile group, also called block failure. When there is a small distance between the piles, the piles and the soil enclosed by the piles will act as a rigid body and a global failure will be governing. Single pile failure is most likely to occur for a pile group with larger spacing between the piles. Verification must be done for both failure modes [3].

2.3 Static Action of a Pile Group

A pile group should be designed to withstand vertical forces, horizontal forces, moments and a combination of these. This section will discuss the force flow in a pile group and how the forces and moments are resisted in a pile group.

2.3.1 Vertical Forces

Both vertical and inclined piles can be loaded with vertical forces. The inclined piles must however be placed with inclination in both directions for them to take vertical forces, due to fulfillment of horizontal equilibrium. As can be seen in Figure 2.2, the inclined piles do not take any vertical forces due to horizontal equilibrium. In Figure 2.3, the inclined piles are placed in both directions and the vertical force is distributed relatively equal between the four piles. Since the vertical stiffness differs between the piles, the piles with index b will take somewhat more vertical load.

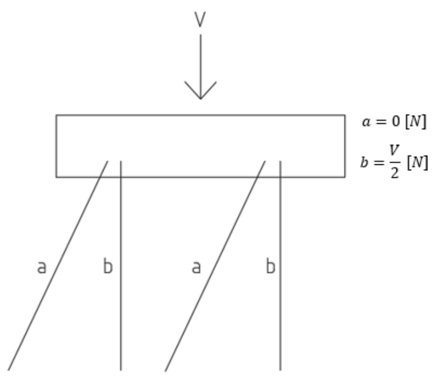


Figure 2.2: Example of inclined pile placement that can not take vertical forces.

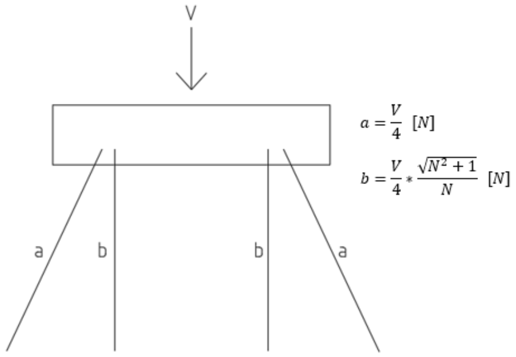


Figure 2.3: Example of inclined pile placement that can take vertical forces. N describing the N:1 inclination of the pile

2.3.2 Horizontal Forces

Horizontal forces acting in the plane of the pile cap can be taken by all piles that are inclined in the same direction as the horizontal force. Piles inclined in the x-direction can only be loaded with horizontal forces acting in the x-direction etc. The larger inclination of the pile, the larger capacity for horizontal forces. Equation 2.3 and equation 2.4 describes the contribution from horizontal forces in y-direction, $N_{y,h}$, and in x-direction, $N_{x,h}$.

$$N_{y,h} = \pm \frac{\sqrt{N^2+1} \cdot H_y}{n_y} \tag{2.3}$$

$$N_{x,h} = \pm \frac{\sqrt{N^2+1} \cdot H_x}{n_x} \tag{2.4}$$

where

n_y, n_x = Number of piles in the x, y direction
H_x, H_y = Applied horizontal forces in x and y direction [N]
N = Pile inclination $[\frac{m}{m}]$

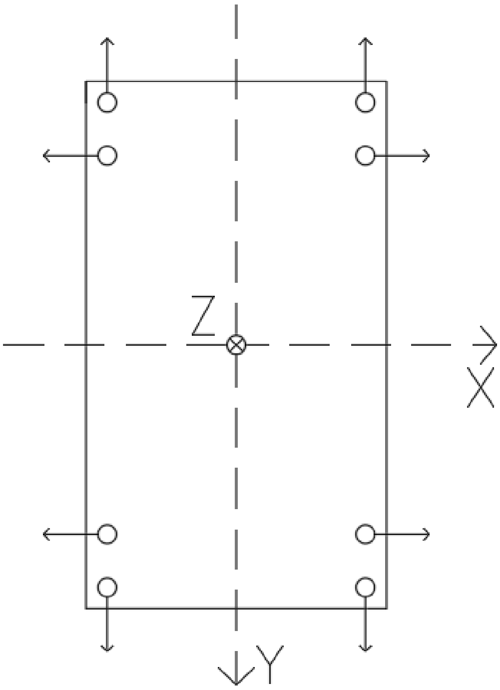


Figure 2.4: Visualization of the in-plane coordinate system with piles in the x, y direction

2.3.3 Moment

Moment arises from horizontal forces, eccentrically placed columns, friction forces from bearing etc. The moments can be eliminated by placing the pile center in the same position as the load center. Pile center and load center will be discussed further in 2.3.4. Moments must be taken up by force couples in the pile group. If not accounting for the lateral resistance of the soil, the piles can only take normal forces. Piles that are inclined in the same direction and with a certain distance between each other can form a force couple. See examples in Figure 2.5.

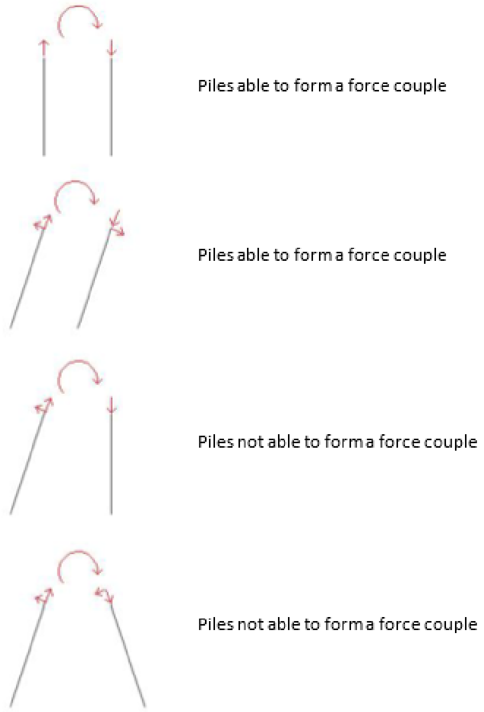


Figure 2.5: Example of how piles can form force couples

Forces in the piles caused by moments can be calculated according to:

$$\mathrm{N}_{x,m} = \frac{M_x \cdot a_i}{I_x} \tag{2.5}$$

$$\mathrm{N}_{y,m} = \frac{M_y \cdot a_i}{I_y} \tag{2.6}$$

where

a_i = A piles distance to the pile center $[m]$
M_x, M_y = Applied moment around the x and y axis [Nm]
I_x, I_y = Stiffness of the pile cap around the pile center in x, y direction, i.e. the sum of the squared distance from the piles to the pile center in x, y direction $[m^2]$

2.3.4 Pile Center and Load Center

The difficulty in designing a pile group is not so much the calculation of actual forces in the piles, but to find the number of piles and the optimal placement. A common notion when working with pile groups is *pile center*. It is the point where a force or a moment acts which only causes a translation of the pile cap in the force direction or a rotation around the axis of the moment. The pile center for pile groups is similar to the center of mass for a cross section. This means that eccentricities from the force resultants, the resisting moment and moment of inertia is calculated with respect to the pile center, i.e. the relationship between the pile center and the load center affect the magnitude of the generated forces in the piles [9]. The pile center and load center is shown illustratively in Figure 2.6, where the pile center is located in the smallest part of the '*broom*', formed by the extension lines of the piles. This is true when the resistance of the soil is not considered and the piles have equal axial stiffness. The location of the pile center when taking the soil resistance into account and how to calculate load center is explained in Section 2.8.

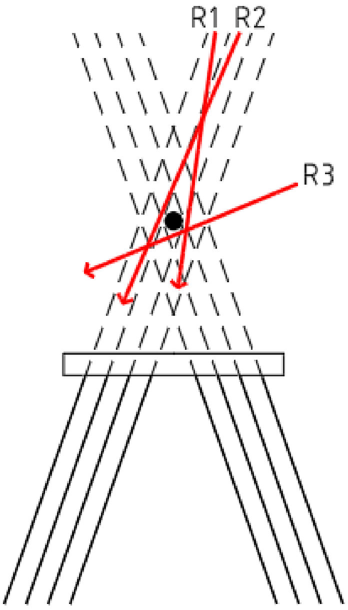


Figure 2.6: Load center and pile center coincides. Pile center represented by a broom-shaped figure formed by the extension lines of the piles. R representing force resultants. Figure redrawn from [9].

To decrease the pile forces coming from the moments, the pile center should be placed as close as possible to the load center of the force resultants. At the same time, a pile groups sensitivity against applied moment increases if the lines of extension of the piles form a small 'neck', see Figure 2.6, where the pile center is situated. To increase the area moment of inertia of a pile group, the extensions of the piles in level with the pile center should be placed as far away from the pile center as possible [9].

2.4 Geometric Restrictions

The following section will focus on the geometrical restrictions to consider when designing a pile group.

2.4.1 Center- and Edge Distance

When installing piles in friction soil, there is a risk that the soil will get very compact and the last piles will be difficult to install. Compact blocks in the soil could also lead to the pile breaking during the installation and the pile has to be replaced with a new one. Due to these reasons, the piles in a pile group must be installed with relatively large center-to-center distance. The Swedish Transport Administration [12] recommends to have at least 0.8 meters between piles at the pile cap. This applies to piles with different direction and inclination, provided that collision does not occur. The center-to-center distance between piles depend on the pile diameter and pile length and increases when installing long, parallel piles. The distance between the outer part of the pile cap and the closest pile must be at least 200 mm [11].

2.4.2 Inclination

There are some recommendations on how large inclination a pile should have. A lower inclination results in higher risk for tension in the pile and a larger inclination results in higher resistance against horizontal forces. Due to limitations for the pile crane and safety issues in the work environment, the piles should not have a larger inclination than 4:1 [7].

2.4.3 Tolerances

Depending on the heterogeneity of the soil and type of installation, the piles will deviate more or less from its intended position. Unless specified otherwise, the maximum construction tolerances are specified in [8] as:

- Deviation in plane: ± 0.1 meter
- Inclination of pile: 4cm/m
- Direction of pile: 2°

2.5 General Design Assumptions

There are some common assumptions made when designing a pile group which will be discussed in this chapter.

2.5.1 Deformations of the Pile Cap

When designing a pile group, the deformations of the pile cap is often neglected. This is because the pile cap can be regarded as infinitely stiff compared to the piles. The displacements of the pile cap are instead calculated as for a whole rigid body. The displacements of the pile cap are then used to calculate the displacements at every pile top in the pile group [9]. The pile cap must however fulfill the conditions as infinitely stiff [10]. This assumption is valid for concentrated pile groups, which is treated in this thesis. If the pile cap is very wide, the flexibility of the pile cap must be considered [9].

2.5.2 Boundary Conditions for the Piles

The boundary condition for the pile top is assumed to be either hinged or fixed and the boundary condition for the pile end is assumed to be hinged. In older design codes, both pile ends have generally been seen as hinged and the calculations were also assuming no lateral resistance of the soil. This will only give axial forces in the piles and will give an inaccurate result, since there will be both bending and tensile stresses in the piles [9]. To account for this, the piles must be modeled with lateral soil resistance together with a hinged or a fixed connection at the pile top.

2.5.3 Lateral Resistance of the Soil

According to the Swedish Transport Administration [10], high and low characteristic values for the lateral resistance of the soil should be considered when dimensioning forces and moments in a pile group for bridges. A high value of the lateral resistance could be dimensioning for the piles with respect to higher moment and shear forces, which will reduce the pile capacity. A lower value could be dimensioning for eventual compression and tension forces in the piles. The influence that the lateral soil resistance has on the modeling of a pile group is presented in Section 2.7.3. For railway bridges and in the case with low value for the lateral resistance, a calculation should be done without any lateral resistance of the soil. The low value for the lateral resistance of the soil should be used as a safety against upcoming tension forces in the piles due to deviations during installation.

2.6 Optimization Method

There are several methods that can be used to optimize a structure. This thesis will make use of *topology optimization.* Topology optimization is a method which optimizes predefined objectives with associated constraints, aiming to maximize the performance of the structure in a certain aspect. The objectives to be optimized can for example be displacements, stresses or sectional forces. This is defined with an objective function. The objective function includes a design variable and a state variable. The design variable is a function or a vector that can for example represent the geometry of the pile group. The state variable represents the response of the pile group, i.e. displacements, stresses or sectional forces. The objective function is then subjected to behavioral constraints, design constraints and equilibrium constraints. The behavioral constraints are connected to the state variable and the design constraints are connected to the design variable. The equilibrium constraints specify the stability of the structure [1]. The optimization method is explained more in detail in Section 3.2.

2.7 Evaluation of Pile Group Model

The Commission on Pile Research published a report in 1983, presenting matrix calculations to determine pile forces and displacements in a pile group [5]. The calculations in that report are reproduced in this chapter and will form the basis of the upcoming optimization software.

2.7.1 Pile Forces and Displacements

The pile cap is seen as the boundary of the pile group with a global coordinate system inserted at the position of the applied loads, e.g. at the origin of the pile cap. A local coordinate system is placed at the top of each pile q, where one pile can have six load components at the pile top, see Figure 2.7.

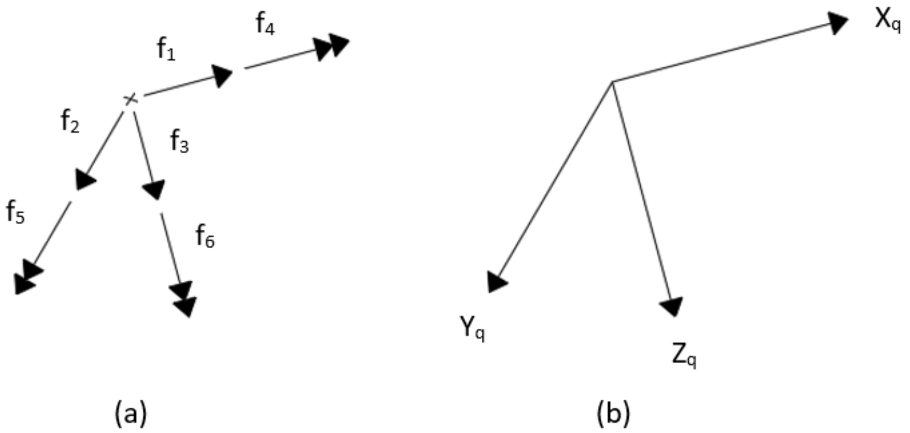


Figure 2.7: Pile forces and moments (a) and local coordinate system for a pile (b)

The relationship between the forces F_q and the displacements X_q can be written, with the element stiffness matrix K_q, as:

$$F_q = K_q X_q \tag{2.7}$$

where,

$$F_q = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix} \qquad K_q = \begin{pmatrix} k_{11} & 0 & 0 & 0 & k_{15} & 0 \\ 0 & k_{22} & 0 & k_{24} & 0 & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 \\ 0 & k_{42} & 0 & k_{44} & 0 & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} \end{pmatrix} \qquad X_q = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}$$

Another local coordinate system (X', Y', Z') is placed parallel to the global system of the pile cap, to which the forces F_q are transformed. The relationship between the forces in the local coordinate system F_q and the transformed forces F'_q can be written as:

$$F'_q = A_q F_q = A_q K_q X_q \tag{2.8}$$

where A_q is the transformation matrix described as:

$$A_q = \begin{pmatrix} A' & 0 \\ 0 & A' \end{pmatrix} \qquad A' = \begin{pmatrix} \cos\beta\cos\alpha & -\sin\alpha & \sin\beta\cos\alpha \\ \cos\beta\sin\alpha & \cos\alpha & \sin\beta\sin\alpha \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \tag{2.9}$$

where

α = in-plane direction of the pile
β = pile inclination

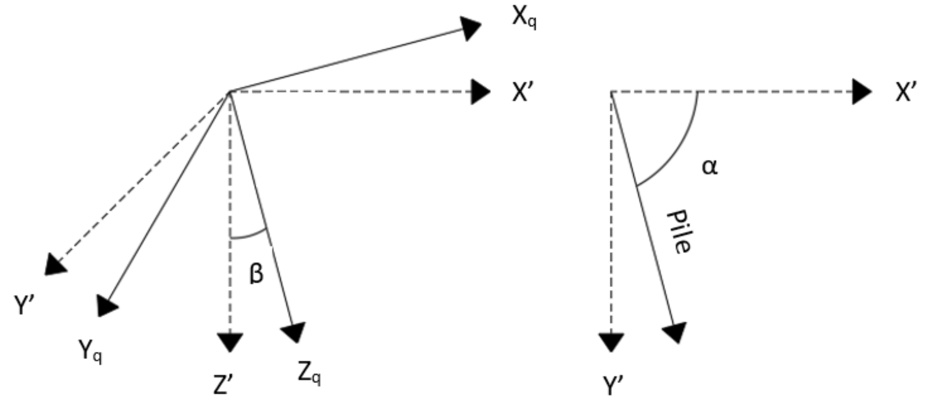


Figure 2.8: Coordinate system of the pile transformed to another coordinate system parallel to the pile cap

The correlation between displacements in the two coordinate systems is given by:

$$X_q = A_q^T X'_q \tag{2.10}$$

2.7.2 Group Effect

The forces F'_q at every pile top must be in equilibrium with the force vector P_q, acting at the origin of the pile cap. By extension, this means that all external forces acting in the origin of the pile cap, R, must be in equilibrium with the pile forces in all piles.

$$P_q = C_q F'_q \tag{2.11}$$

$$R = \sum_{q=1}^{n_{piles}} P_q \tag{2.12}$$

C_q includes the global coordinates of the pile (x, y, z) together with its support condition as:

$$C_q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z & y & m & 0 & 0 \\ z & 0 & -x & 0 & m & 0 \\ -y & x & 0 & 0 & 0 & m \end{pmatrix} \tag{2.13}$$

where

$m = 0$ describes a hinged connection
$m = 1$ describes a fixed connection

The resulting displacements at the origin of the pile cap caused by the external forces is summed up in a displacement vector U. With the assumption that the pile cap is stiff, the correlation between the displacements at the pile top and the displacements at the origin of the pile cap can be written as:

$$X'_q = C_q^T U \tag{2.14}$$

Furthermore, the relation between the loads acting on the pile cap, R, and its displacement, U, is described with a global stiffness matrix S. The stiffness matrix is symmetric and each element S_{ij} is representing the force coming from each pile in the direction i which gives the pile cap a displacement in the direction j equal to one. The contribution from each pile to S_{ij} is given by a combination of equations 2.8, 2.10 and 2.14 inserted into 2.11:

$$P_q = C_q A_q K_q A_q^T C_q^T U \tag{2.15}$$

Put $D_q = C_q A_q$ and use $A^T C^T = (CA)^T$ which gives:

$$P_q = S'_q U \quad \text{where} \quad S'_q = D_q K_q D_q^T$$

Each pile will contribute with its stiffness to the global stiffness matrix of the pile cap as:

$$S = \sum_{q=1}^{n_{piles}} S'_q \tag{2.16}$$

Given the external loads acting on the pile cap, R, the displacement of the pile cap is calculated as:

$$U = S^{-1} R \tag{2.17}$$

With the calculated displacement at the pile cap, the displacements at every pile top are calculated with equation 2.18. Then, the pile forces are calculated using equation 2.19.

$$X_q = A_q X'_q = D_q^T U \tag{2.18}$$

$$F_q = K_q X_q \tag{2.19}$$

2.7.3 Lateral Soil Resistance and Pile Constants

The matrix element k_{ij} required to calculate the pile element stiffness matrix K_q, presented in Section 2.7.1, are dependent on the soil properties and the pile stiffness. It is calculated differently depending on if the piles are installed in friction soil or cohesive soil.

For friction soil, the piles are seen as beams which are either hinged or fixed in the pile cap and fixed at the depth L_i. The pile length L_i is calculated as:

$$L_i = 1.8 \sqrt[5]{\frac{EI}{n_h}} \quad \text{for} \quad L > 4L_i \tag{2.20}$$

where

L = Real pile length $[m]$
EI = Bending stiffness of the pile $[N \cdot m^2]$
n_h = Growth rate factor for the linear subgrade modulus for friction soil $[\frac{N}{m^3}]$

The stiffness matrix for a pile in friction soil then yields:

$$K_q = \begin{pmatrix} \frac{(1+3m)3EI}{L_i^3} & 0 & 0 & 0 & \frac{6mEI}{L_i^2} & 0 \\ 0 & \frac{(1+3m)3EI}{L_i^3} & 0 & -\frac{6mEI}{L_i^2} & 0 & 0 \\ 0 & 0 & \frac{AE}{L} & 0 & 0 & 0 \\ 0 & -\frac{6mEI}{L_i^2} & 0 & \frac{4mEI}{L_i} & 0 & 0 \\ \frac{6mEI}{L_i^2} & 0 & 0 & 0 & \frac{4mEI}{L_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{mJG}{L} \end{pmatrix} \tag{2.21}$$

where

$m = 0$ for hinged connection and m = 1 for fixed connection at the pile cap
$A =$ Cross-section area of the pile [m]
$L_i =$ Pile length [m]
$JG =$ Torsional stiffness for the cross-section of the pile $[N \cdot m^2]$

In cohesion material, the piles can be seen as semi-infinite long beams on an elastic foundation. The stiffness is modeled as horizontal elastic springs with a fictitious pile length L_e. The subgrade modulus kd is assumed to be constant along the depth.

$$L_e = \sqrt[4]{\frac{4EI}{kd}} \quad \text{for} \quad L > 3L_e \tag{2.22}$$

The stiffness matrix for a pile in cohesion soil becomes:

$$K_q = \begin{pmatrix} \frac{(1+m)2EI}{L_e^3} & 0 & 0 & 0 & \frac{m2EI}{L_e^2} & 0 \\ 0 & \frac{(1+m)2EI}{L_e^3} & 0 & -\frac{m2EI}{L_e^2} & 0 & 0 \\ 0 & 0 & \frac{AE}{L} & 0 & 0 & 0 \\ 0 & -\frac{m2EI}{L_e^2} & 0 & \frac{m2EI}{L_e} & 0 & 0 \\ \frac{m2EI}{L_e^2} & 0 & 0 & 0 & \frac{m2EI}{L_e} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{mJG}{L} \end{pmatrix} \tag{2.23}$$

If the lateral soil resistance is not considered, the pile constants in the stiffness matrix will be the same as for a beam with a fixed connection at the pile cap and a hinged connection at the pile end.

$$K_q = \begin{pmatrix} \frac{3EI}{L^3} & 0 & 0 & 0 & \frac{6EI}{L^2} & 0 \\ 0 & \frac{3EI}{L^3} & 0 & \frac{6EI}{L^2} & 0 & 0 \\ 0 & 0 & \frac{AE}{L} & 0 & 0 & 0 \\ 0 & \frac{6EI}{L^2} & 0 & \frac{3EI}{L} & 0 & 0 \\ \frac{6EI}{L^2} & 0 & 0 & 0 & \frac{3EI}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{JG}{L} \end{pmatrix} \tag{2.24}$$

If the pile is hinged in the pile cap and no lateral soil resistance is considered, the piles will only take normal forces and the only remaining matrix element is k_{33} while the rest are zero.

2.7.4 Design Forces

The next step is to calculate maximum moment and shear force in each pile. The axial force can be seen as constant along the pile length where maximum axial force occur.

Maximum shear force occur at the pile cap and is calculated as:

$$f_T = \sqrt{f_1^2 - f_2^2} \tag{2.25}$$

Maximum bending moment is calculated depending on the support condition at the pile cap. For a hinged connection, the moment is calculated accounting for the soil condition as:

$$\begin{aligned} &\text{Friction Soil: } f_M = 0.43 f_T L_i \quad \text{at} \quad 0.8L_i \quad \text{below ground surface} \\ &\text{Cohesion Soil: } f_M = 0.32 f_T L_e \quad \text{at} \quad 0.8L_e \quad \text{below ground surface} \end{aligned} \tag{2.26}$$

For a fixed connection at the pile cap, the equation yields:

$$f_M = \sqrt{f_4^2 - f_5^2} \tag{2.27}$$

The resulting stresses are then compared to the permitted values for each pile type.

2.8 Calculation of Pile Center and Load Center

This section derives the mathematical expression for the pile center when taking the soil resistance into account and how to calculate the load center for a pile group.

2.8.1 Pile Center

Pile center without accounting for lateral soil resistance was written about in Section 2.3.4. When accounting for the lateral soil resistance, the pile center depends on both the stiffness of the piles and the stiffness of the soil. The definition of the pile center is that it is the point where a force acts and only causes a translation of the pile cap in the force direction or where a moment only causes a rotation around the axis of the moment. The pile center is calculated for a two-dimensional plane and it is made correspondingly for each plane. The mathematical expression for the pile center is derived for the XZ-plane below.

The displacement of the pile cap due to an external load is calculated with Equation 2.17 as:

$$U = S^{-1}R$$

If applying a force in x-direction to create a unit displacement in x, the expanded equation is written as:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} S^{-1}_{1,1} & S^{-1}_{1,2} & S^{-1}_{1,3} & S^{-1}_{1,4} & S^{-1}_{1,5} & S^{-1}_{1,6} \\ S^{-1}_{2,1} & S^{-1}_{2,2} & S^{-1}_{2,3} & S^{-1}_{2,4} & S^{-1}_{2,5} & S^{-1}_{2,6} \\ S^{-1}_{3,1} & S^{-1}_{3,2} & S^{-1}_{3,3} & S^{-1}_{3,4} & S^{-1}_{3,5} & S^{-1}_{3,6} \\ S^{-1}_{4,1} & S^{-1}_{4,2} & S^{-1}_{4,3} & S^{-1}_{4,4} & S^{-1}_{4,5} & S^{-1}_{4,6} \\ S^{-1}_{5,1} & S^{-1}_{5,2} & S^{-1}_{5,3} & S^{-1}_{5,4} & S^{-1}_{5,5} & S^{-1}_{5,6} \\ S^{-1}_{6,1} & S^{-1}_{6,2} & S^{-1}_{6,3} & S^{-1}_{6,4} & S^{-1}_{6,5} & S^{-1}_{6,6} \end{pmatrix} \begin{pmatrix} f_1 \\ 0 \\ 0 \\ 0 \\ f_1 \cdot -PC_{xz} \\ f_1 \cdot PC_{xy} \end{pmatrix} \tag{2.28}$$

where

$-PC_{xz}$ = Pile center in the XZ-plane for negative Z-direction [m]
PC_{xy} = Pile center in the XY-plane [m]

Using the definition of pile center with no rotations, the pile center can be seen as the eccentricity between the cut-off level of the piles and the position where the moment is zero. Solving the equation system for PC_{xz} with row five and six gives:

$$PC_{xz} = \frac{S^{-1}_{6,6} \cdot S^{-1}_{5,1} - S^{-1}_{5,6} \cdot S^{-1}_{6,1}}{S^{-1}_{6,6} \cdot S^{-1}_{5,5} - S^{-1}_{6,5} \cdot S^{-1}_{5,6}} \tag{2.29}$$

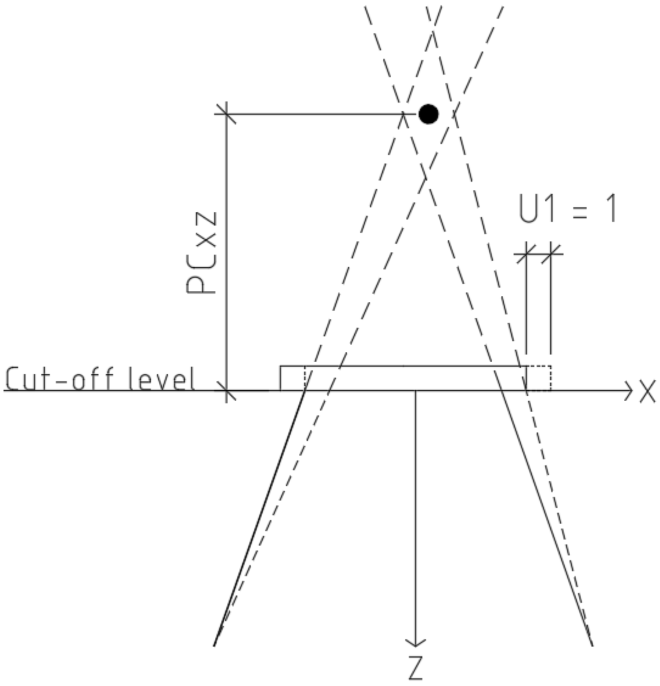


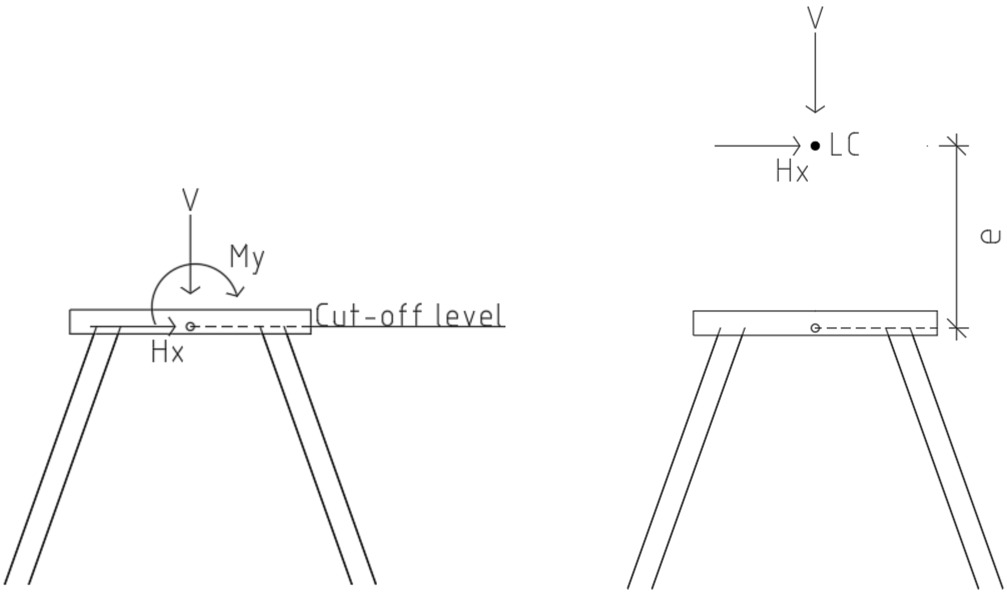
Figure 2.9: Pile center in XZ-plane with a unit displacement U_1

2.8.2 Load Center

The external forces are applied at the cut-off level of the piles. The load center is calculated using eccentricities and they are calculated for both the XZ-plane and the YZ-plane. The load center is found at the distance where the horizontal force applied at the cut-off level times an eccentricity is equally large as the moment applied at the cut-off level. This is illustrated for the load center in the XZ-plane in Figure 2.10 and mathematically for both the XZ-plane and the YZ-plane as:

$$e_{xz} = LC_{xz} = \frac{M_y}{H_x} \tag{2.30}$$

$$e_{yz} = LC_{yz} = \frac{-M_x}{H_y} \tag{2.31}$$



(a) Applied forces

(b) Position of load center in the XZ-plane

Figure 2.10: Visualization of load center

The mathematical expressions for the load center above is valid for double symmetric supports. If this is not the case and the pile center is shifted in the xy-plane, the moment contribution from the vertical force has to be accounted for as well.

3

Method

The optimization software was developed using topology optimization, as mentioned in Section 2.6. The objective function, design and state variables were formulated together with their constraints and necessary input data. This is presented in Section 3.2.2 and Section 3.1. Case studies were performed to evaluate the software. The case studies consists of different types of pile groups for bridges with both 'typical' and 'atypical' load cases, to ensure that the software could handle several different load situations. A verification of the software was performed in terms of comparing sectional forces in the piles and pile center with the existing software Rymdpålgrupp [4].

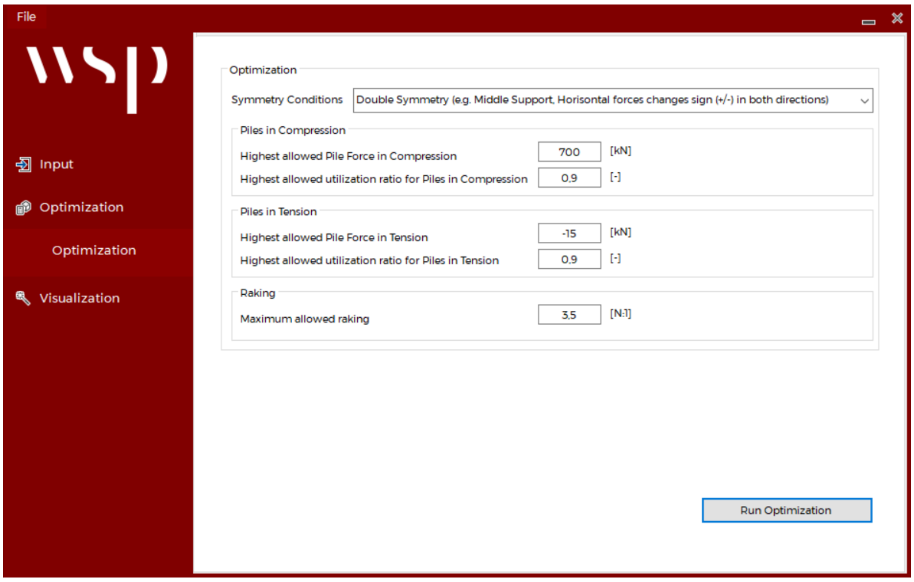
3.1 Input Data

The designer needs to specify the following input data for the software:

- *Load cases* - All load cases consisting of three forces and three moments acting at the cut-off level of the piles
- *Inclination of the piles*
- *Pile capacity* - Pile capacity for ultimate limit state
- *Utilization* - Maximum utilization ratio for the piles
- *Pile cap* - Geometry of the pile cap as well as the center-to-center distance between the piles and the minimum distance from the pile to the edge of the pile cap
- *Pile parameters* - Material parameters, length and dimensions of the cross-section
- *Soil parameters* - Type of soil and its subgrade modulus
- *Support condition* - If the pile is hinged or fixed at the pile cap

3.2 Optimization Software

The optimization software was developed in Python. In order to get a better visualization and a user-friendly interface, the optimization code was implemented in another software written in C#. This software makes it easier for the user to enter all input data in various menu tabs. After the simulation has been run, a plot of the generated pile group is displayed together with the pile coordinates and pile forces. The software also gives the user the opportunity to save the pile group and the file can then be opened in the software Rymdpålgrupp [4]. Here, the user can print a report or make changes in the pile group if desired. The interface software written in C# was developed by the author's supervisor at WSP, Daniel Josefsson. Figure 3.1 shows the layout of the interface software for one of the menu bars.



Optimization

Symmetry Conditions: Double Symmetry (e.g. Middle Support, Horisontal forces changes sign (+/-) in both directions)

Piles in Compression

Highest allowed Pile Force in Compression: 700 [kN]
Highest allowed utilization ratio for Piles in Compression: 0,9 [-]

Piles in Tension

Highest allowed Pile Force in Tension: -15 [kN]
Highest allowed utilization ratio for Piles in Tension: 0,9 [-]

Raking

Maximum allowed raking: 3,5 [N:1]

Figure 3.1: Layout of the interface for the optimization software

The Python code begins with calculating an estimated number of piles in x and y direction for every load case. This estimation is based on that the pile center and load center coincide and that moments associated with the load case are eliminated, as discussed in Section 2.3.4. The estimation is calculated by combining equation 2.3 and 2.4 and also taking the vertical load into account. The total number of piles n_p can thereby be estimated as:

$$n_p = \frac{(H_x + H_y) \cdot \frac{\sqrt{N^2+1}}{1} + V_z \cdot \frac{\sqrt{N^2+1}}{N}}{N_{Rd} \cdot \eta} \tag{3.1}$$

where
H_x, H_y = Applied horizontal forces in x and y direction [N]
V_z = Applied vertical load [N]
N = Inclination of the pile $[\frac{m}{m}]$
N_{Rd} = Pile capacity [N]
η = Utilization ratio for the piles [-]

Two vectors are created based on the estimation of number of piles with $\pm\Delta n$ number of piles in x and y direction. Adding $\pm\Delta n$ to these two vectors is done to account for all extremes of every load case and because Equation 3.1 is only an estimation. This gives one matrix for the number of piles in x direction, n_{px}, and one matrix for the number of piles in y direction, n_{py}, with k rows equal to number of load cases as:

$$n_{px} = \begin{bmatrix} n_{px_i} \\ . \\ . \\ . \\ n_{px_k} \end{bmatrix} \qquad n_{py} = \begin{bmatrix} n_{py_i} \\ . \\ . \\ . \\ n_{py_k} \end{bmatrix} \qquad i = \begin{bmatrix} 1, 2, \ldots, k \end{bmatrix}$$

$$n_{px_i} = \begin{bmatrix} x_{p_i} - \Delta n_i & \cdots & x_{p_i} & \cdots & x_{p_i} + \Delta n_i \end{bmatrix}$$

$$n_{py_i} = \begin{bmatrix} y_{p_i} - \Delta n_i \\ . \\ . \\ . \\ y_{p_i} \\ . \\ . \\ . \\ y_{p_i} + \Delta n_i \end{bmatrix}$$

where

x_p, y_p = Estimated number of piles in x, y direction for one load case
i = Number of load cases

Every combination of these vectors for each load case respectively are used for to generate a pile group. Each pile group is then analyzed for every load case. The pile group with the least number of piles and with the lowest utilization ratio for maximum and minimum normal forces is identified as the most optimal pile group. A flow chart for the software is presented in Figure 3.2.

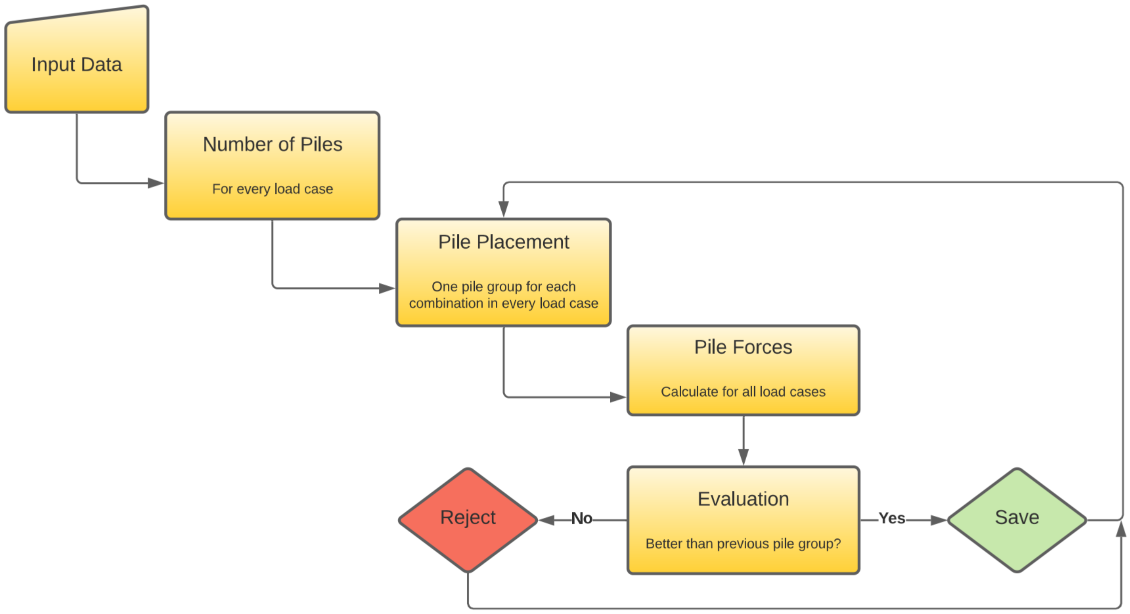


Figure 3.2: Flow chart of optimization software

3.2.1 Pile Placement

The pile cap is divided into a grid in x and y direction. The size of the grid depends on the distance between the piles and the minimum distance to the edge of the pile cap. Each intersection gives a free pile coordinate. This is shown in Figure 3.3.

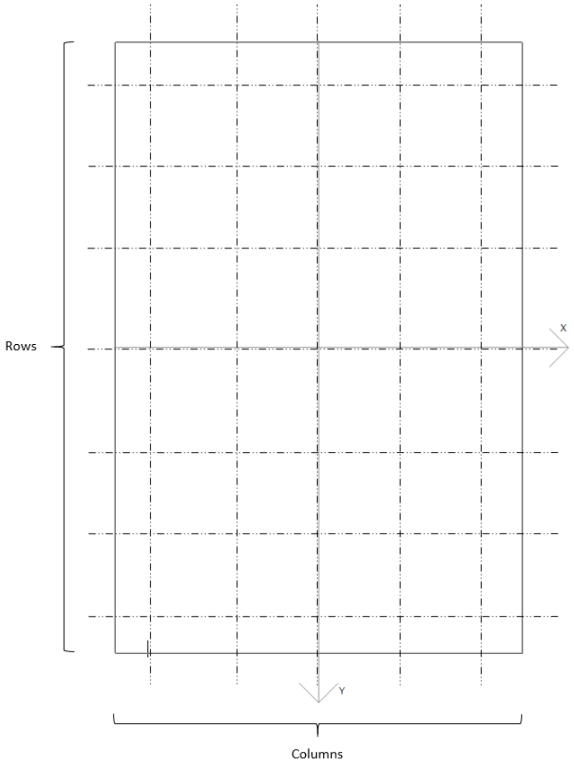
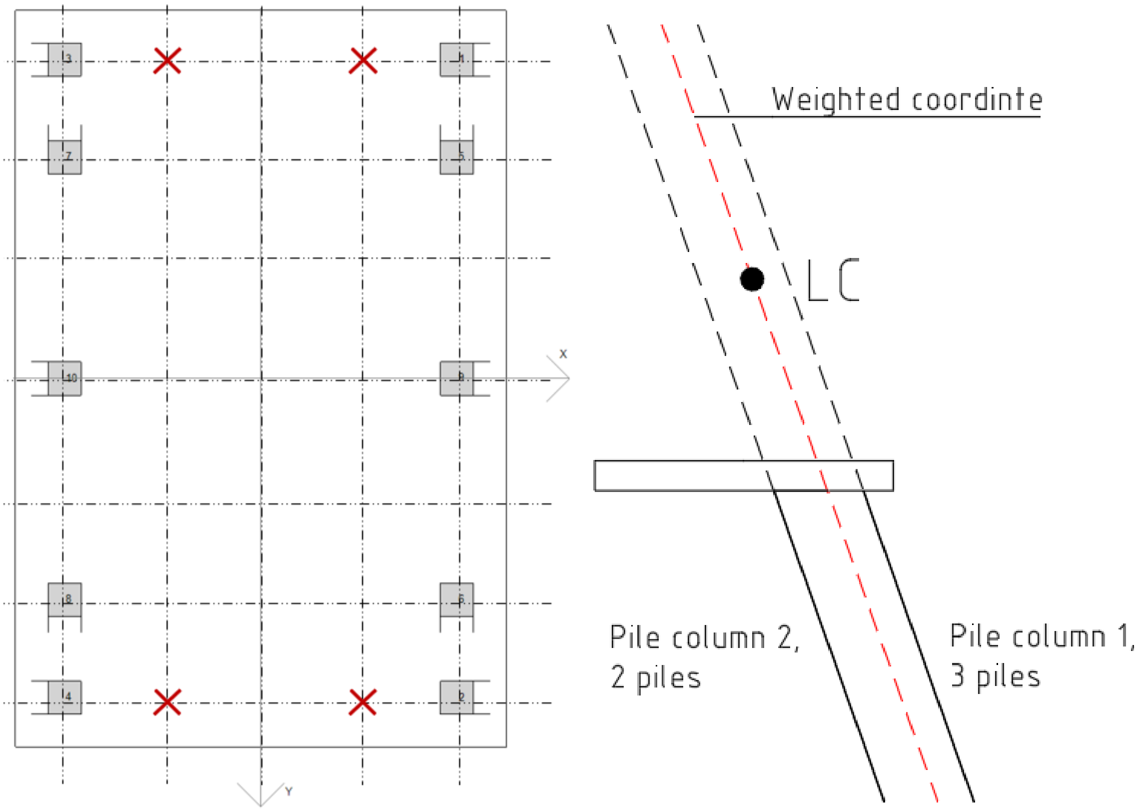


Figure 3.3: Available coordinates for a pile group

In general, the piles with the largest number of piles leaning in the same direction are placed first. The pile placement is always trying to match the pile center with the load center. The placement is therefor always made column/row wise. This means that if there are piles inclined in the same direction divided into two columns/rows, one row of piles is placed out first. The position for the second row of piles has to be adapted based on the coordinate for the first row, to place the pile center in the same position as the load center. This is exemplified in Figure 3.4. This pile group is double symmetric with 5 piles in the x direction and 2 piles in the y direction on each side of the symmetry lines. The piles in x direction are divided into two columns, where three piles are placed in the first column and two piles are placed in the second column.

Figure 3.4a shows the pile group when the piles in the x direction for the first column are placed out first and the piles in y direction are placed second. The piles in y direction are placed out before the second column of piles in x direction to also give the piles in y direction as many free coordinates as possible for them to position the pile center within the load center. The red **x**'s in Figure 3.4a illustrates 'occupied' coordinates for piles leaning in the x direction to avoid collision.



(a) Pile placement and grid of pile coordinates

(b) Pile coordinate for the second column of piles in x direction

Figure 3.4: Approach for pile placement

The principle of placing the second column of piles in the x direction is illustrated in Figure 3.4b. The coordinate of the second pile column is based on the weighted coordinate of both columns. The weighted coordinate is positioned so that its extended line intersects through the load center. Pile column 2 is then placed in the nearest available coordinate from the grid of coordinates so that this can match. The final pile group is shown in Figure 3.5.

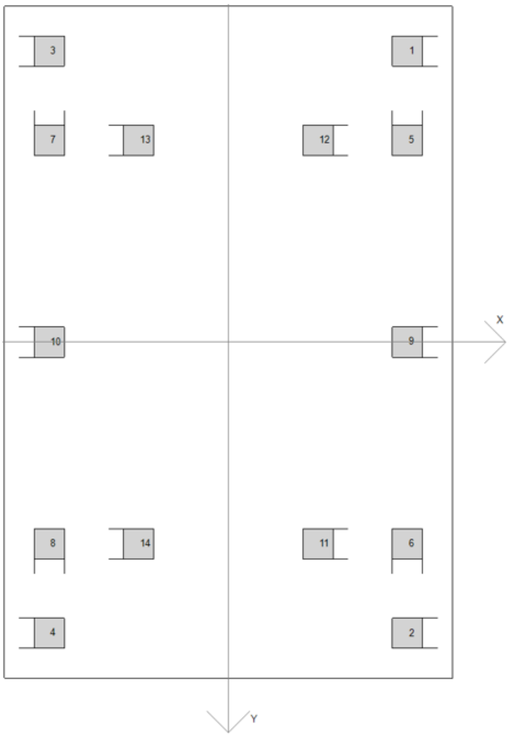


Figure 3.5: Pile group where the pile center and load center coincide

For some load cases it will not be possible to match the pile center with the load center. The position of the load center can sometimes be too high or too low for the pile center to be placed in the same position, due to the limitation of the dimensions of the pile cap and/or the inclination of the piles. For these cases, the piles are placed to create as much moment resistance in the pile group as possible. Consider the pile group presented in Figure 3.5. If the pile center can not be placed in the same position as the load center in the XZ-plane, the piles will instead be placed to create high moment resistance around the x-axis. This pile arrangement is shown in Figure 3.6. The piles in x-direction are now placed such that the distance L1, i.e. the lever arm for these piles, is increased as much as possible. The piles in y-direction have the same y-coordinate as before but had to be positioned closer to the middle of the pile group to avoid collision. This results in a decrease in moment capacity around the y-axis since the distance L3 decreased. The column wise placement of the piles in x-direction gives however an increase of the moment resistance around the y-axis since they form a force couple with the lever arm L2.

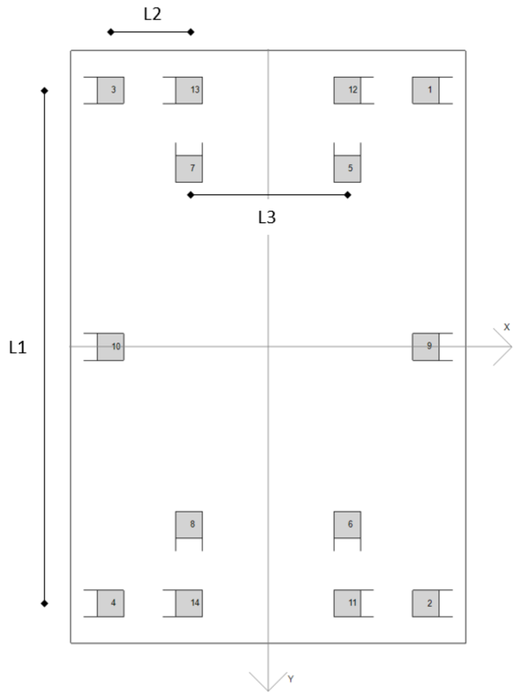


Figure 3.6: Pile group with high moment resistance in x-direction

3.2.2 Topology Optimization Method

An overview of the optimization method was presented in Section 2.6. This section will describe the implementation of the method used in the software. According to Christensen et al. [1] the general structural optimization problem is formulated as:

$$\begin{cases} \text{minimize } f(m,n) \text{ with respect to m and n} \\ \text{subject to } \begin{cases} \text{behavioral constraints on n} \\ \text{design constraints on m} \\ \text{equilibrium constraints} \end{cases} \end{cases}$$

The pile group is optimized using an objective function $f(m,n)$. This includes the design variable m and the state variable n, which are explained more in detail later in this chapter. The objective function is set to represent the distance between the load center and the pile center as:

$$f = LC - PC \tag{3.2}$$

where

LC = Load center for one load case [m]
PC = Pile center of the pile group [m]

Since the position of the load center changes for every load case, the pile group is only optimized against one load case at the time. The topology optimization is thereby only used when calculating the placement of the piles, which is done according to the flow chart in Figure 3.2. To account for the change of position of the load center for all load cases, a fictitious load case is created. This is based on weighted forces and moments from all load cases, aiming to find a pile group with a pile center positioned equally good/bad for all load cases. This weighted load case is only used for the pile placement and not considered when calculating pile forces.

The design variable m is represented by the geometry of the pile group and placement of the piles. It consists of a matrix with k rows equal to the number of piles in the pile group. Every row consists of a vector m_i representing one pile in the pile group.

$$m = \begin{bmatrix} m_i \\ \cdot \\ \cdot \\ \cdot \\ m_k \end{bmatrix} \qquad i = \left[1, 2, \ldots, k\right]$$

$$m_i = \begin{bmatrix} x_i & y_i & z_i & \alpha_i & N & L \end{bmatrix}$$

where

x_i, y_i, z_i = Global coordinates of the pile within the pile cap dimensions [m]
α_i = In-plane direction of the pile [°]
N = Inclination of the pile $[\frac{m}{m}]$
L = Length of the pile [m]

The state variable n represents the response of the pile group in terms of section forces, n_i, for each pile and displacements, U, for the whole pile group. The sectional forces and displacements are calculated according to Section 2.7. The sectional forces consist of three forces and three moments at every pile top and the displacements consists of three translations and three rotations at the origin of the pile cap.

$$n_i = \begin{bmatrix} H_{x_i} & H_{y_i} & V_{z_i} & M_{x_i} & M_{y_i} & M_{z_i} \end{bmatrix} \qquad U = \begin{bmatrix} u_x & u_y & u_z & \theta_x & \theta_y & \theta_z \end{bmatrix}$$

The behavioral constraints apply to the sectional forces and displacements. The constraints regarding sectional forces are user defined and refers to maximum and minimum pile capacity. The behavioral constraints applied to the displacements are built into the software and limits the translations to exceed ± 1 meter in x and y direction and 20‰ for the rotations. No limit has been set for the translations in z direction. The behavioral constraints relating to displacements are given high values due to the permissible displacements will differ depending on the type of project.

These constraints will however make sure that the software sort out the most unstable pile groups.

The design constraints limit the position of the piles to the area of the pile cap. The piles center-to-center distance and the distance between the edge of the pile cap to the closest pile, as mentioned in Section 2.4.1, are also constraints but user defined. The in-plane direction of the piles, α_i, is free to vary with 90 degrees intervals. The restriction on 90 degrees intervals is to make it easier to prevent the piles to collide.

The equilibrium constraint is implemented to ensure that the pile group is stable. This is achieved by checking that the determinant of the stiffness matrix S is not zero.

3.2.3 Assumptions and Limitations

The pile placement is always made without considering lateral soil resistance, where it is first taken into account when calculating pile forces. This is because the placement of the piles is based on the pile center matching the load center and an exact position of the pile center, when considering the lateral soil resistance, is not possible to compute without having the final pile coordinates. If the lateral soil resistance would be considered, it would lead to a slight change of pile center when calculating pile forces compared to the pile center calculated when the pile placement was made. This means that the pile center and load center will no longer coincide and moments will arise in the pile group. The distance between the new pile center and load center will however be small if the pile center and load center coincided when not accounting for lateral soil resistance, hence resulting in small moments.

For double symmetric pile groups that require very few piles, it can be troublesome to achieve moment resistance and a robust pile group. This is accounted for by not generating a pile group with less than eight piles in total. With the selected variation of the in-plane direction of the piles, this is the least number of piles required to create moment resistance and a robust pile group.

The pile parameters such as length and cross-section area are given as an input and will be the same for all piles. This applies to the inclination of the piles as well.

The subgrade modulus has to be reduced due to group effect if the piles are standing closer than three times the pile diameter, according to [5]. This is not accounted for in the software and is left for the designer to verify.

As mentioned earlier, the in-plane direction of the piles is only free to vary with 90 degrees intervals to make it easier to prevent the piles to collide. Allowing this parameter to vary more would make control of collision almost impossible and the calculation time would increase severely. One downside with this restriction is that it could generate more piles than necessary. For the worst-case scenario, with a double symmetric support, the generated pile group could give up to two more piles compared to if the direction of the piles was allowed to vary. One example of this is shown in Figure 3.7, where the desired pile group would have eight piles in x-direction and two piles in y-direction. The optimization software would generate a pile group with 12 piles in total to create moment resistance around both the x-axis and y-axis, whereas it would be possible with 10 piles only if four piles were allowed to be rotated with 45 degrees intervals instead of 90 degrees intervals. The designer can however remove piles and change the direction afterwards. This is considered to be a much easier modification than to control pile collision, especially for pile groups with large number of piles.

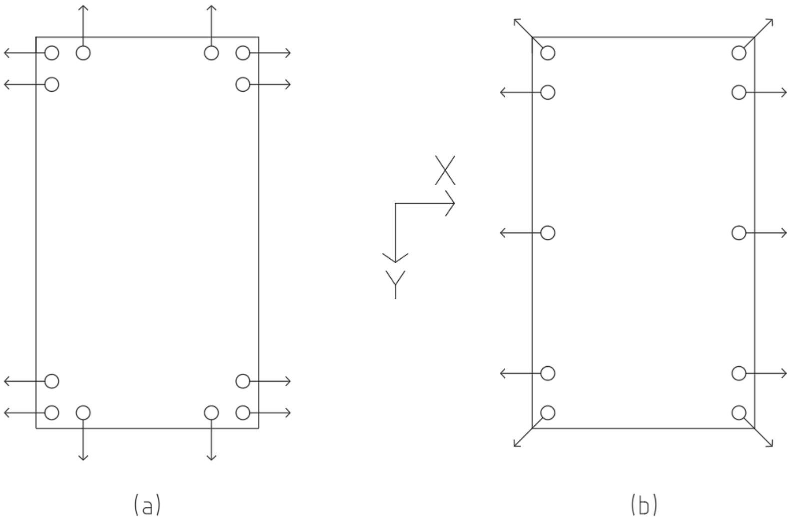


Figure 3.7: Pile group generated by optimization program with 90 degrees intervals (a). Possible pile group with 45 degrees intervals (b).

3.3 Case Studies

The case studies were performed for 27 different pile groups, where four of them are presented in detail in the following chapter and the rest can be found in Appendix B. These four cases were selected to show pile groups for both middle supports and abutments where the software managed to generate less number of piles compared to the existing pile groups, but also where the software generated pile groups with more number of piles than the existing pile groups. The input data used in the optimization software is the same as for the constructed pile groups. There is however common that no information about the pile capacity and the utilization ratio of the piles is given the existing calculations and that the piles in the existing pile group have different inclinations. If none of the above are specified and if several inclinations are used, the following assumptions are made:

- If there is tension in the piles in the original calculations, the value for the worst loaded pile in tension is set as minimum allowable tension force. If no tension exists, this value is set to 0 kN.
- The utilization ratio for the piles is assumed to be 90 %.
- The pile capacity with respect to compression is calculated by dividing the highest normal force in one pile, $N_{Ed,max}$, from the existing calculations with the utilization ratio, η, as:

$$N_{Rd,max} = \frac{N_{Ed,max}}{\eta}$$

- If several inclinations are used in a pile group, the largest allowable inclination is used for all piles in the optimization software. This has to do with the optimization method where the pile center and load center are intended to coincide. Using a large inclination (low value on N) in Equation 3.1 will result in fewer piles than for a small inclination, which is also the aim for the optimization.

3.3.1 Case I - Railway Bridge Mid Support

This bridge is a single span railway bridge with retaining walls where support one and two, according to Figure 3.8, are identical. Although the supports are abutments, they are modeled as middle supports due to the fact that the loads applied are double symmetrical, which is characteristic of a middle support. A selection of the input data is presented in Table 3.1 where the only difference is that the optimization software uses an inclination of 3.5:1 for all piles. The utilization ratio for the pile capacity is set to 100 % since the pile capacity was specified in the existing calculations for the pile group.

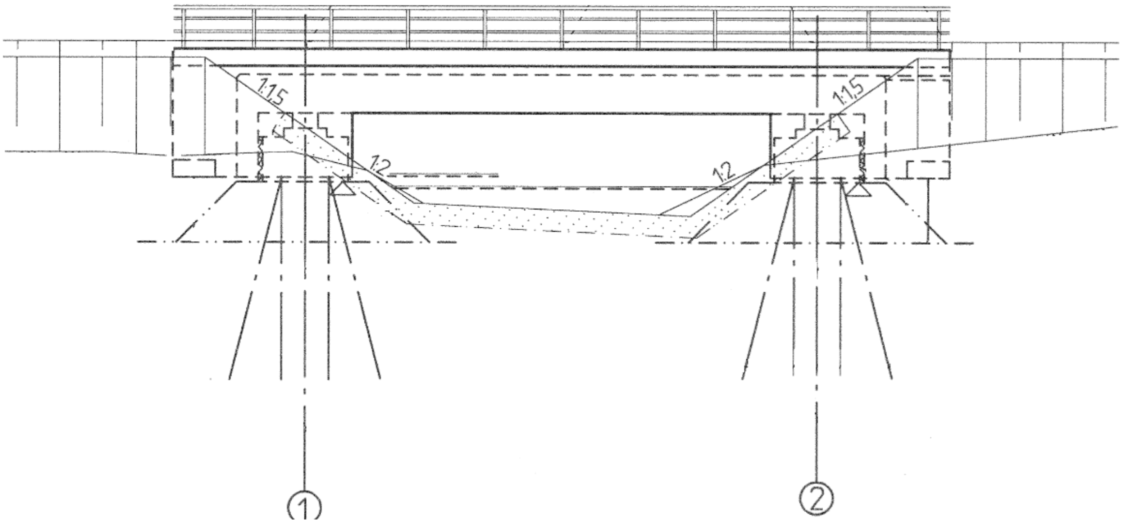
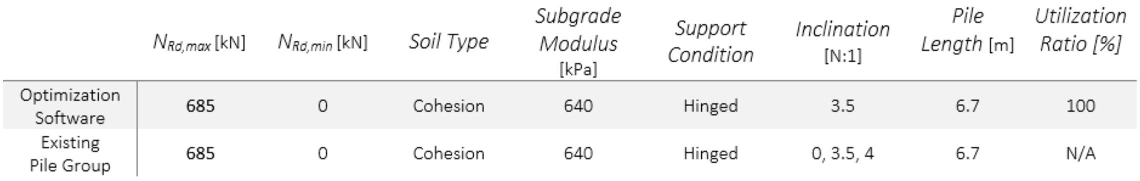


Figure 3.8: Elevation of the bridge for Case I



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Software	**685**	0	Cohesion	640	Hinged	3.5	6.7	100
Existing Pile Group	**685**	0	Cohesion	640	Hinged	0, 3.5, 4	6.7	N/A

Table 3.1: Selection of the input data for the optimization software and existing bridge for Case I

3.3.2 Case II - Road Bridge Mid Support

The second case study is performed for the middle support of a road bridge. The support simulated is support 11, shown in Figure 3.9. The pile capacity for this support is not given in the existing calculations and there are piles loaded in tension. The pile capacity is therefor assumed according to Section 3.3. An overview of the input data is shown in Table 3.2.

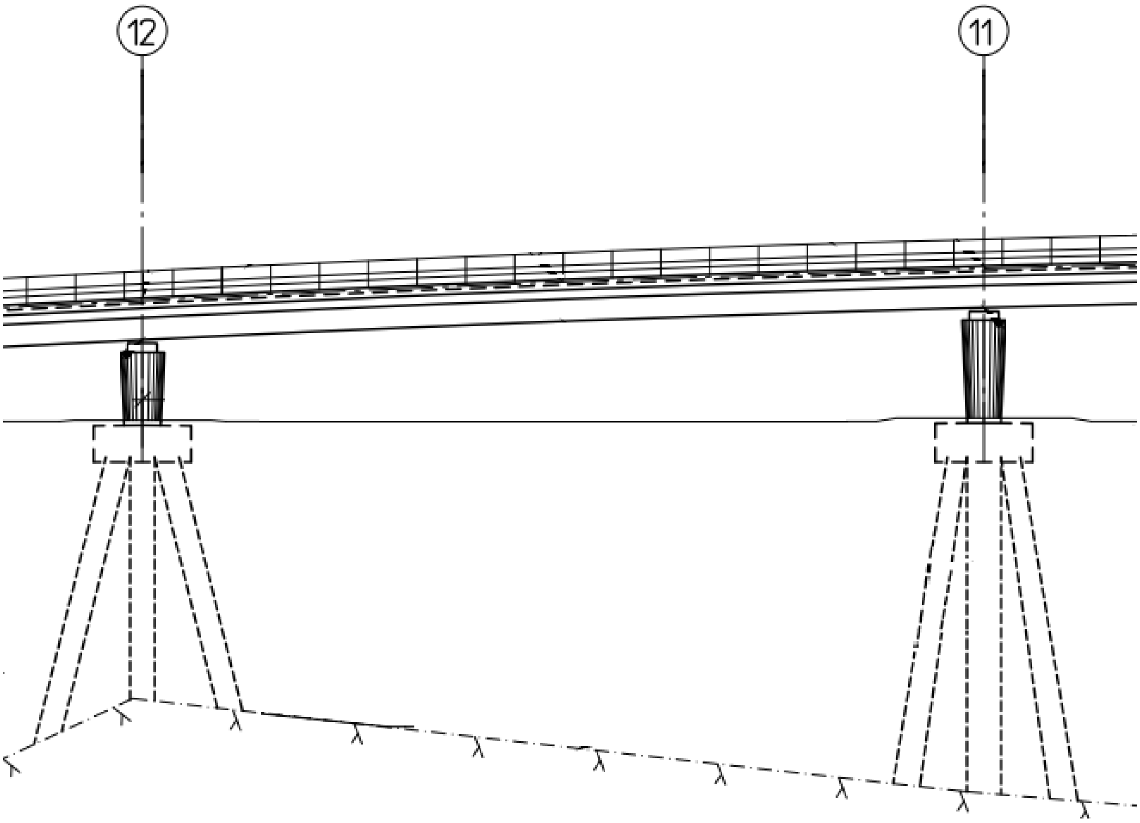


Figure 3.9: Elevation of the bridge for Case II

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Software	$\frac{1352}{0,9}$	-118	Cohesion	250	Hinged	4	15	90
Existing Pile Group	$\frac{1352}{0,9}$	-118	Cohesion	250	Hinged	4, 6, 7	15	N/A

Table 3.2: Selection of the input data for the optimization software and existing bridge for Case II

3.3.3 Case III - Road Bridge Abutment

The third case study is made for an abutment for a road bridge. The bridge is 73 meters long with three supports, where the simulation for support one is presented here. The piles are made out of concrete, with an original inclination of 4:1 and 5:1. The pile group is modeled without any lateral soil resistance.

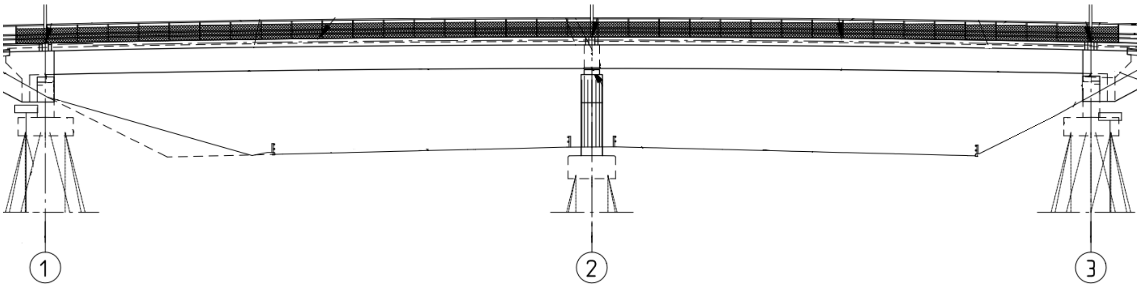
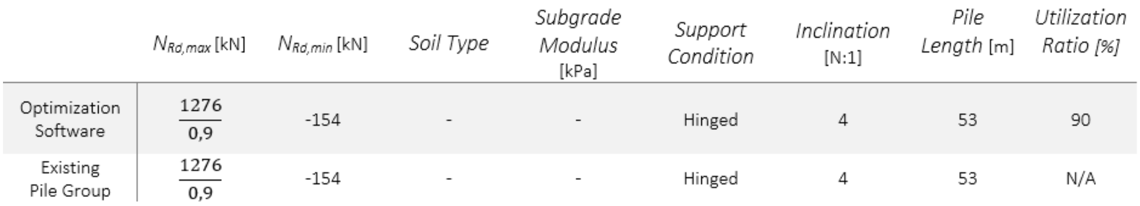


Figure 3.10: Elevation of the bridge for Case III



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Software	$\frac{1276}{0,9}$	-154	-	-	Hinged	4	53	90
Existing Pile Group	$\frac{1276}{0,9}$	-154	-	-	Hinged	4	53	N/A

Table 3.3: Selection of the input data for the optimization software and existing bridge for Case III

3.3.4 Case IV - Road Bridge Abutment

The last case study presented is also for a road bridge abutment, but for a much shorter bridge than in Case III. The bridge is 13,6 meters long with two identical supports, piled with concrete piles which are allowed to take tension forces. The pile group is modeled without lateral soil resistance.

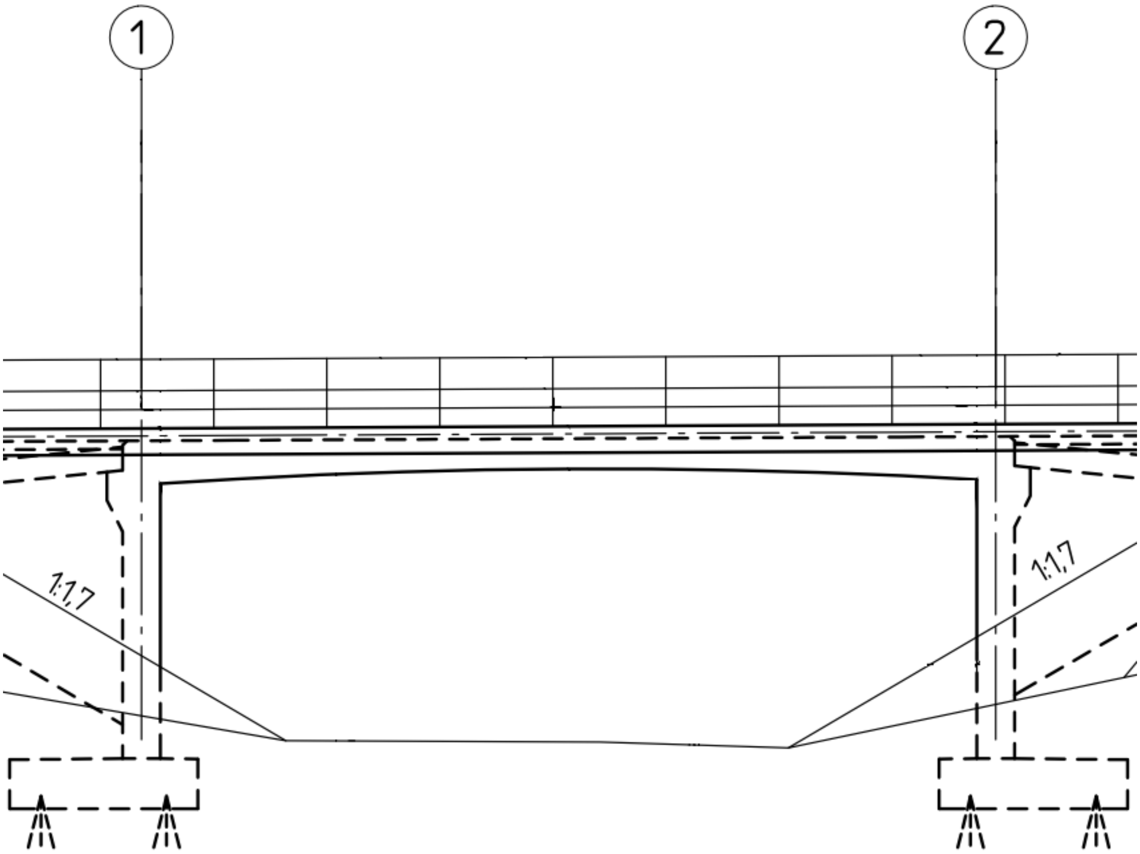
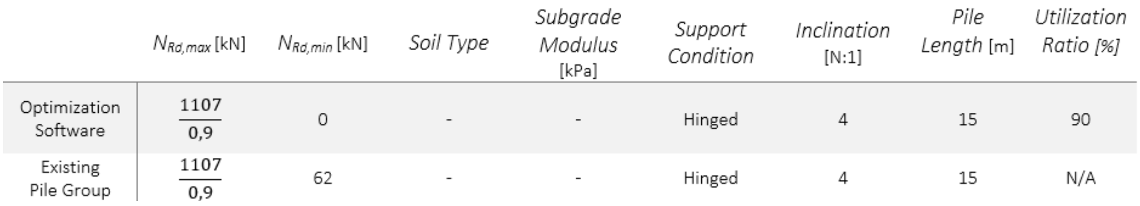


Figure 3.11: Elevation of the bridge for Case IV



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Software	$\frac{1107}{0,9}$	0	-	-	Hinged	4	15	90
Existing Pile Group	$\frac{1107}{0,9}$	62	-	-	Hinged	4	15	N/A

Table 3.4: Selection of the input data for the optimization software and existing bridge for Case IV

3.3.5 Verification of Optimization Software

The verification of the software is made with respect to calculation of pile forces, displacements and pile center for a given pile group. The pile forces and displacements generated by the optimization software were compared with the results from the software Rymdpålgrupp [4]. During the development of the software, the software was verified for several cases but the verification presented is made for the pile group design in Case I. The calculations are compared for the actual input data in Case I but also for calculations with friction soil, with no lateral soil resistance and for different types of cross-sections for the piles. The verification is made in Appendix A.

4

Results

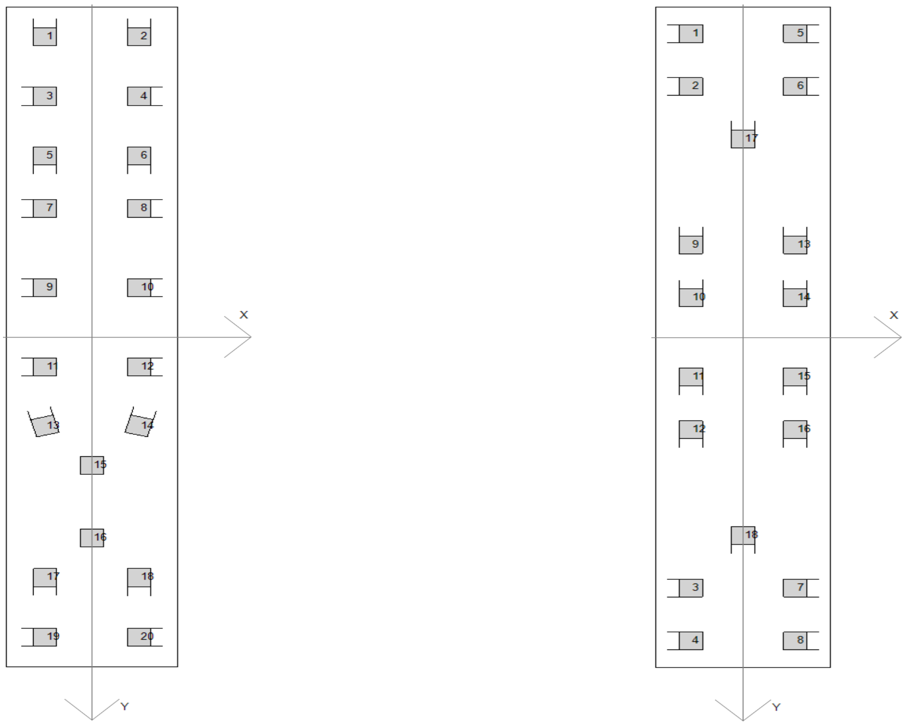
This section will present the results from the case studies. The software was tested on 18 double symmetric middle supports and 9 single symmetric abutments. The software modeled all pile groups successfully without any collision between the piles. The software generated pile groups with the same number of piles or fewer for 23 out of 27 supports. The calculation time varied between two seconds to 1 minute and 25 seconds.

4.1 Case Studies

Out of the 27 simulated supports, four of them are presented more in detail. Two of them are double symmetric middle supports and two are single symmetric abutments, which are described in Section 3.3. The software generates sectional forces for all piles, deformations and rotations for the pile cap, pile coordinates, pile center and load center for all load cases. The data presented for the case studies are however limited to a plot of the pile group, maximum and minimum normal force and a comparison between the load centers and the pile center.

4.1.1 Case I

The existing pile group was constructed as a single symmetric support with 20 piles. The optimized pile group was simulated in three seconds, testing 566 different pile groups where the best pile group had 18 piles.

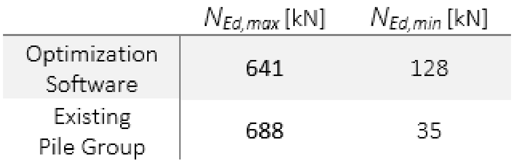


(a) Original pile group, 20 piles

(b) Optimized pile group, 18 piles

Figure 4.1: Pile groups Case I

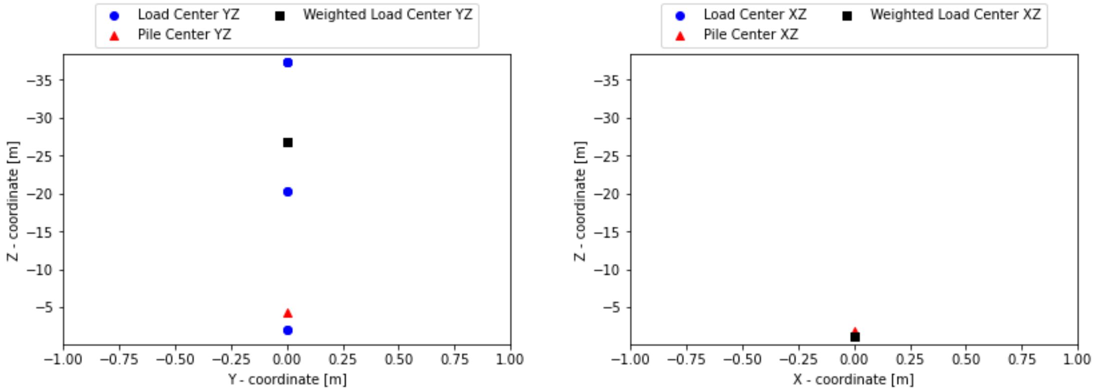
Maximum and minimum normal force for both pile groups can be seen in Table 4.1. The optimized pile group has lower maximum normal force and higher minimum normal force. This results in higher redundancy for the pile group with respect to normal force, even though the optimized pile group has less piles than the existing one.



	$N_{Ed,max}$ [kN]	$N_{Ed,min}$ [kN]
Optimization Software	641	128
Existing Pile Group	688	35

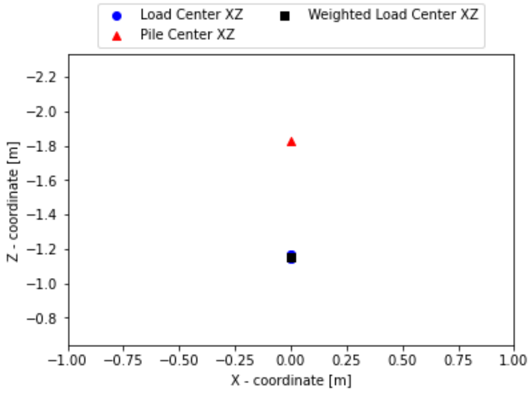
Table 4.1: Maximum and minimum normal force for the pile groups for Case I

Figure 4.2 shows the placement of the pile center for the optimized pile group and the placement of the load center for all load cases. In the XZ plane, the pile center and load center are positioned close to each other for every load case and the load center is almost in the same position for every load case. There is however a big difference in the YZ plane. The pile cap is very narrow in the x-direction, resulting in that there is only room for one column of piles on each side of the y-symmetry line. This makes it more difficult to place the pile center in the location of the load center for both the XZ and YZ plane. There is also a big difference between the location of the load center in the YZ plane for the different load cases, making it hard to construct a pile group where the pile center and load center is close to each other for every load case.



(a) Load center and pile center in YZ plane

(b) Load center and pile center in XZ plane

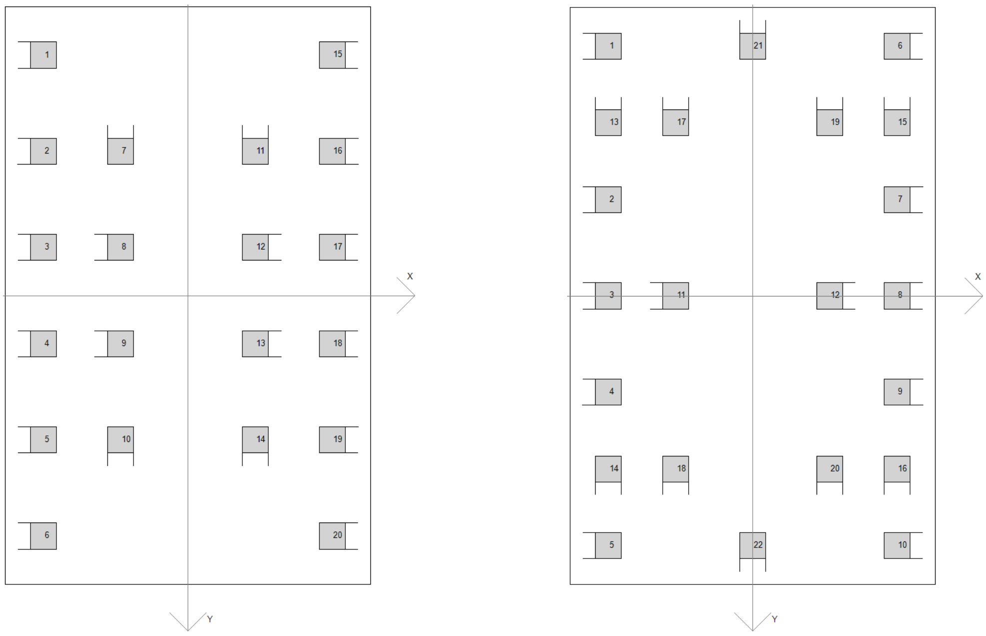


(c) Load center and pile center in XZ plane zoomed in

Figure 4.2: Comparison between load center and pile center for the optimized pile group for Case I

4.1.2 Case II

The original pile group is constructed as double symmetric with 20 piles. The optimization software generated a pile group with 22 piles in 21 seconds and tested 972 different pile group combinations. A possible explanation for this increase in number of piles can be the restriction in pile capacity and utilization ratio. Since the pile capacity and utilization ratio are unknown, a fairly strict limitation is set for those two parameters in the optimization software. A higher pile capacity and/or higher utilization ratio can have a large impact on the number of piles. A test simulation was made where the utilization ratio was changed to 91% and the pile capacity was left unchanged, resulting in a pile group with the same number of piles as the original pile group. This pile group is however not presented. The restriction of pile capacity and utilization ratio is reflected more upon in Section 5.3. The two pile groups can be seen in Figure 4.3 and the design normal forces in Table 4.2.



(a) Original pile group, 20 piles

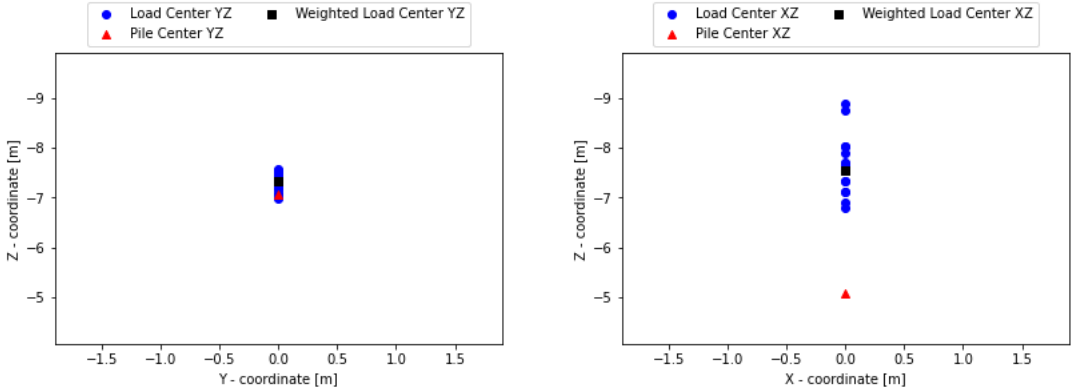
(b) Optimized pile group, 22 piles

Figure 4.3: Pile groups Case II

	$N_{Ed,max}$ [kN]	$N_{Ed,min}$ [kN]
Optimization Software	1246	-101
Existing Pile Group	1352	-118

Table 4.2: Maximum and minimum normal force for the pile groups for Case II

The optimized pile group has less compression and tension in the worst loaded piles. Since the load center in YZ-plane is almost in the same position for all load cases, the pile center will fit close to all of them. The pile cap is however to narrow in the x-direction to increase the height of the pile center and place it closer to the load centers in the XZ-plane.



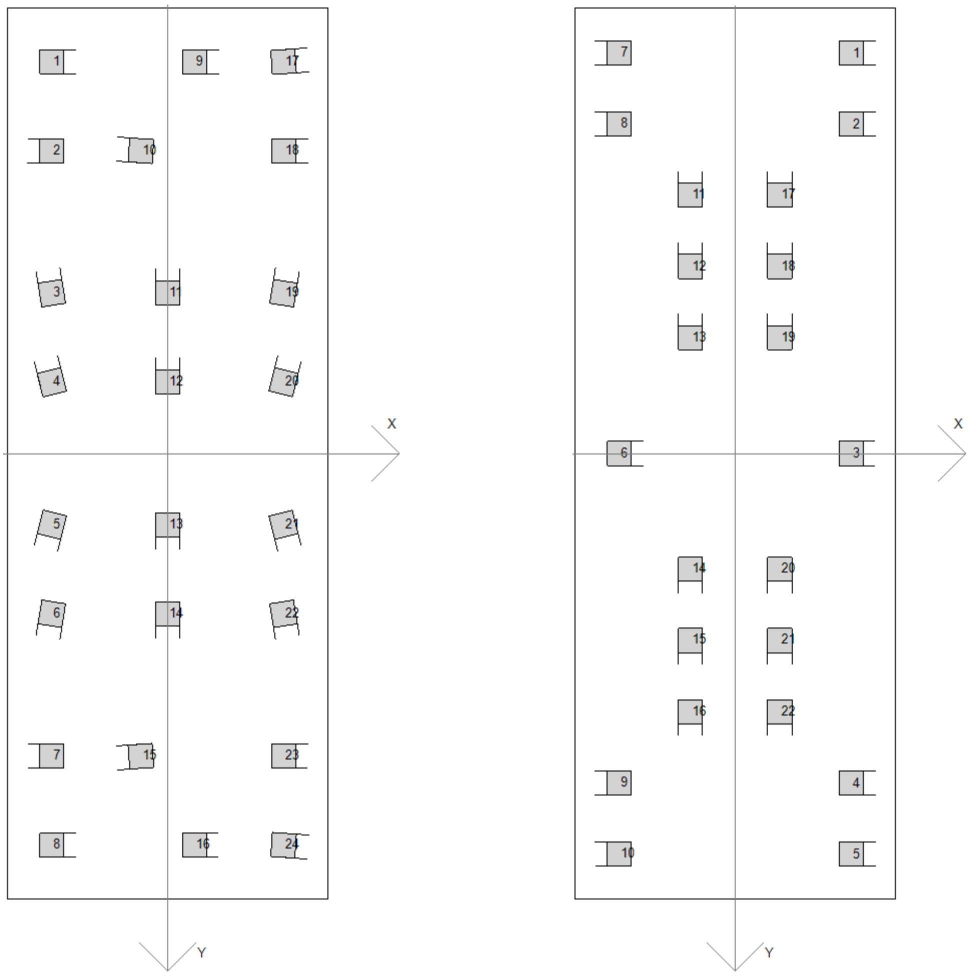
(a) Load center and pile center in YZ plane

(b) Load center and pile center in XZ plane

Figure 4.4: Comparison between load center and pile center for the optimized pile group for Case II

4.1.3 Case III

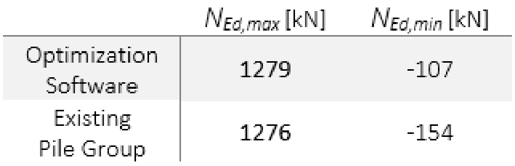
The optimization software generated a single symmetric pile group with 22 piles. It was simulated in 41 seconds and tested 5845 different pile group combinations. The original pile group is constructed with 24 piles. Both pile groups are shown in Figure 4.5. Maximum and minimum normal force in the pile groups can be seen in Table 4.3. The optimized pile group exceeds the maximum normal force with 3 kN which is considered to be negligible. The highest tension force for the optimized pile group is however considerably lower.



(a) Original pile group, 24 piles

(b) Optimized pile group, 22 piles

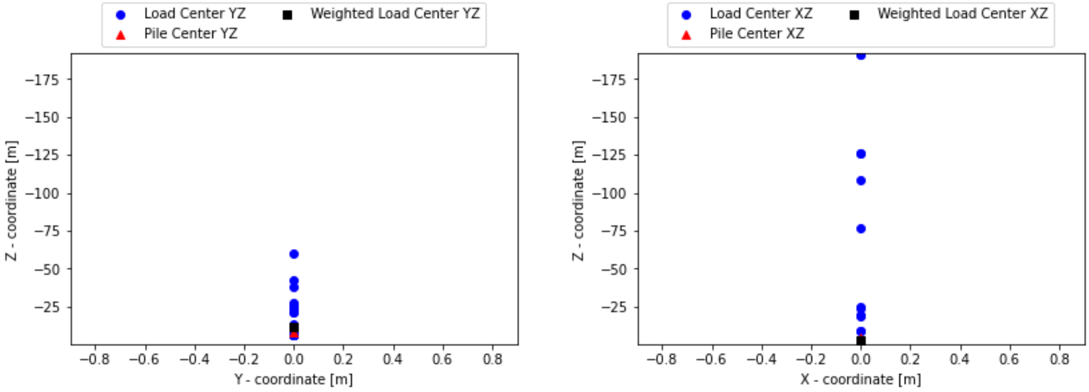
Figure 4.5: Pile groups Case III



	$N_{Ed,max}$ [kN]	$N_{Ed,min}$ [kN]
Optimization Software	1279	-107
Existing Pile Group	1276	-154

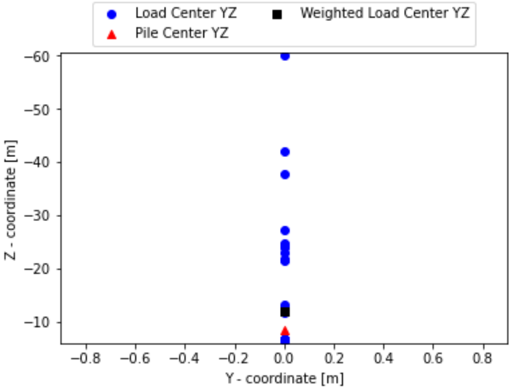
Table 4.3: Maximum and minimum normal force for the pile groups for Case III

Figure 4.6 shows the comparison between the pile center and load center for the optimized pile group. There are large differences between the location of the load centers for both the YZ-plane and the XZ-plane, although the differences are much larger in the XZ-plane. The biggest difference between the load centers in the XZ-direction is around 190 meters. Even if the pile center would be positioned in the middle between the two load cases with the biggest difference, it would still lead to very large moments due to the large lever arm. The piles in x-direction are instead positioned to achieve high moment resistance for the pile group around the x-axis. The row-wise placement of the piles in y-direction is also contributing to the moment resistance around the x-axis.



(a) Load center and pile center in YZ plane

(b) Load center and pile center in XZ plane

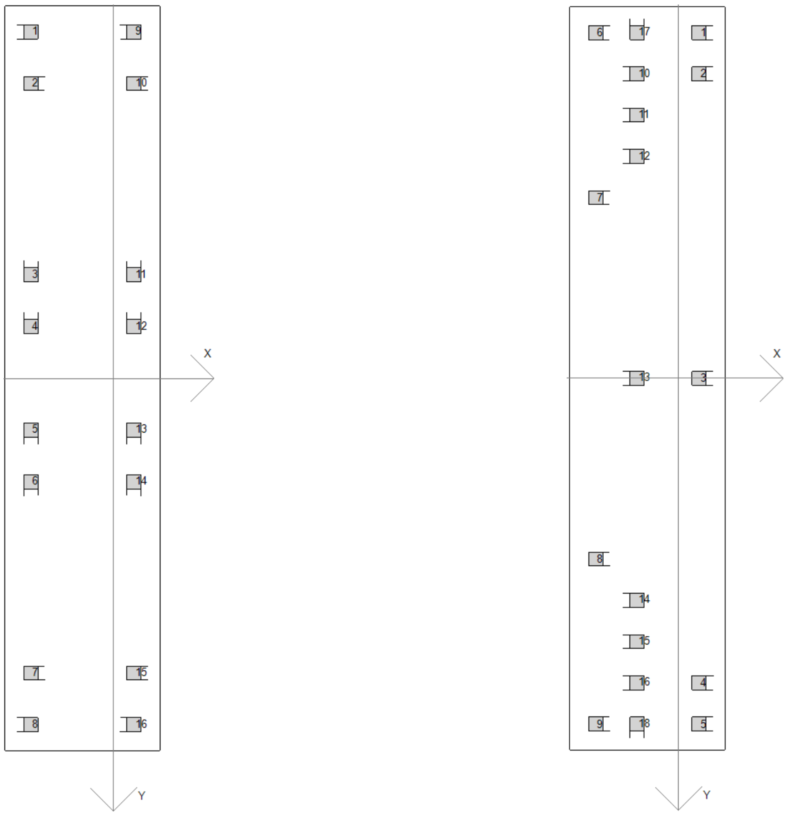


(c) Load center and pile center in YZ plane zoomed in

Figure 4.6: Comparison between load center and pile center for the optimized pile group for Case III

4.1.4 Case IV

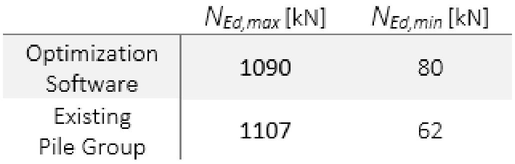
The original pile group is single symmetric with 16 piles. The optimization software simulated 4550 different pile group combinations in 14 seconds. This resulted in a pile group with 18 piles. Maximum and minimum normal forces for the pile groups, shown in Table 4.4, are similar for both pile groups. The design of the pile groups differs a lot. The original pile group achieves moment resistance in both directions by placing piles inclined in the same direction in two columns. The piles inclined in the same direction in the x direction are also placed as far away as possible from each other, which also increases the moment resistance as written about in Section 3.2.1. The optimized pile group only gets moment resistance from the piles in x direction. The piles in y-direction are very few and are only taking vertical forces and horizontal forces in y-direction.



(a) Original pile group, 16 piles

(b) Optimized pile group, 18 piles

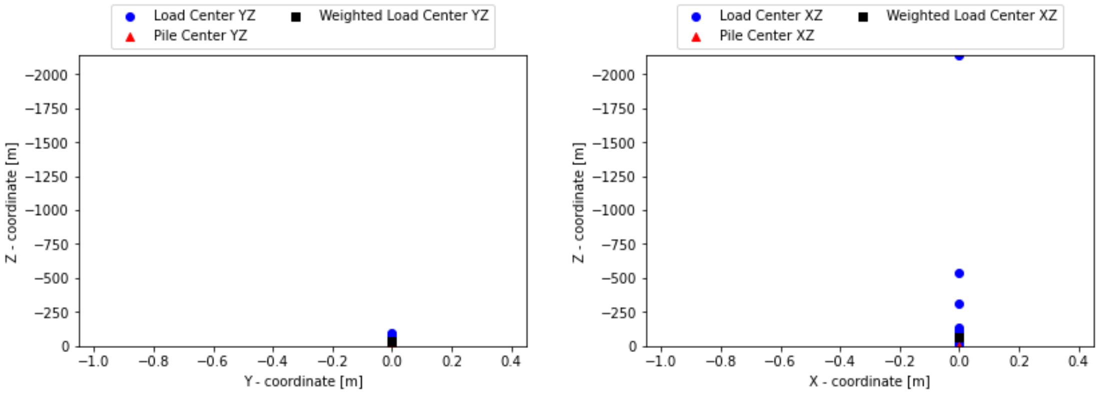
Figure 4.7: Pile groups Case IV



	$N_{Ed,max}$ [kN]	$N_{Ed,min}$ [kN]
Optimization Software	1090	80
Existing Pile Group	1107	62

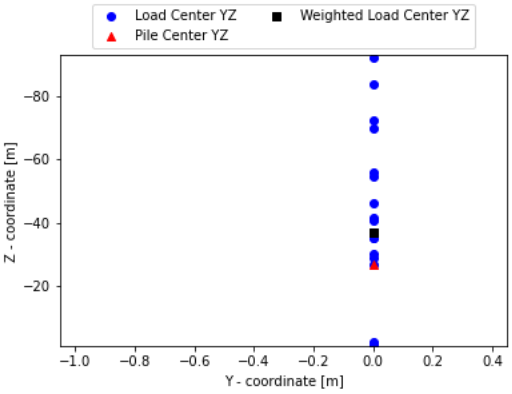
Table 4.4: Maximum and minimum normal force for the pile groups for Case IV

This pile group has similar issues as for the pile group in Case III, where the difference between the load centers are very large, see Figure 4.8. The largest difference between the load centers occurs in the XZ-plane, where it is as much as 2118 meters. Once again, this makes it very difficult to design the pile group based on that the pile center and the load center should coincide.



(a) Load center and pile center in YZ plane

(b) Load center and pile center in XZ plane



(c) Load center and pile center in YZ plane zoomed in

Figure 4.8: Comparison between load center and pile center for the optimized pile group for Case IV

4.1.5 Summary of All Case Studies

A summary of all the case studies is presented in Table 4.5. There are only four cases where the optimization software generates more piles than the original pile groups. The software also generates pile groups where the lowest normal force is higher for all cases except five. Worth noting is also the calculation time for the pile groups, where most pile groups are generated in less than 20 seconds. The design of the pile groups can be seen in Appendix B.

	Optimization Software				*Existing Pile Group*		
Case	*Number of Piles*	*$N_{Ed,max}$ [kN]*	*$N_{Ed,min}$ [kN]*	*Calculation Time [s]*	*Number of Piles*	*$N_{Ed,max}$ [kN]*	*$N_{Ed,min}$ [kN]*
🟩 I	18	641	128	3	20	688	35
🟥 II	22	1246	-101	21	20	1352	-118
🟩 III	22	1279	-107	41	24	1276	-154
🟥 IV	18	1090	80	14	16	1107	62
🟩 V	30	1223	178	13	32	1222	124
🟩 VI	24	1261	158	14	24	1281	120
🟩 VII	24	1263	142	10	24	1288	116
🟩 VIII	24	1095	49	11	24	1175	13
🟩 IX	26	1203	239	9	26	1259	169
🟩 X	24	1149	101	8	26	1159	102
🟩 XI	32	1085	177	18	32	1154	163
🟩 XII	26	1271	171	15	26	1330	120
🟩 XIII	26	1308	63	16	26	1326	94
🟩 XIV	28	1201	98	15	28	1256	61
🟩 XV	26	1061	63	11	26	1118	58
🟩 XVI	30	1389	18	16	32	1399	-53
🟩 XVII	12	981	-61	6	12	1021	-152
🟥 XVIII	28	940	97	14	27	1070	52
🟩 XIX	26	882	105	2	28	873	174
🟩 XX	12	1055	179	3	12	1115	28
🟩 XXI	22	1199	32	37	22	1256	-8
🟩 XXII	12	792	-23	25	12	783	-354
🟩 XXIII	10	896	3	17	12	898	-191
🟩 XXIV	17	1228	137	43	18	1274	2
🟥 XXV	18	930	-127	74	16	932	-133
🟩 XXVI	24	1188	-79	85	24	1240	-167
🟩 XXVII	10	1027	-46	2	10	1043	-36

Table 4.5: Summary of all case studies. Green color represents fewer or the same number of piles generated by the software compared to the existing pile group and red color represents more.

5

Discussion

This chapter will discuss the results of the thesis and the restrictions of the software/method. The aim of this thesis was to develop a software that optimize the design of pile groups and to deepen the understanding of the parameters influencing the design. 27 different supports were simulated where the software successfully generated a pile group for every support within the limits of the pile capacity. The software generated pile groups with the same number of piles or fewer for 23 out of 27 supports.

5.1 Objective Function

The objective function of the software was designed to minimize the distance between the load center and the pile center, thereby decreasing the forces coming from the moments. This was proven to be harder to achieve in practice than in theory, especially for abutments. One reason for this was the limitation of the size of the pile cap. The load center was often positioned high above the piles cut-off plane, which made it impossible to place the pile center in the same position as the load center. For several of the supports, this would have been difficult to achieve even if the size of the pile cap was changeable, since it would have required a massive pile cap in some cases. Another difficulty with this approach was the large difference in the position of the load center between the different load cases. The biggest difference was for Case IV, where it was as much as 2118 meters. This is a difficulty since every simulated pile group is optimized for one load case and then evaluated against every other load case. It is thereby hard to find a pile center positioned close to all different load centers. As mentioned, this was primarily a problem for the abutments and especially abutments with heavy superstructures and high horizontal forces in the longitudinal direction of the bridge. When studying the existing calculations for the bridges, these horizontal forces often arose from braking and acceleration forces. These forces can either interact or counteract the earth pressure acting on the abutment wall. This leads to load cases with very small or very large horizontal forces in the longitudinal direction of the bridge together with a high vertical force. The difference in the position of the load center between these load cases can therefore be very large. The same type of problem can occur in the transverse direction of the bridge if the bridge is constructed with a radius. Then, an eccentric unidirectional centrifugal force arises which creates a moment around the longitudinal axis of the bridge which is difficult to counteract.

For the cases where the load centers varied severely, it was proven to be difficult to design a pile group based on positioning the pile center within the load center. For these cases, the pile group was instead designed with the focus on creating as much moment resistance as possible in the pile group, as mentioned in Section 3.2.1. This situation can clearly be seen for the pile groups in Case II and Case IV. Figure 4.4 shows that the pile center is close to the load center in the YZ-plane but not in the XZ-plane for Case II. This also corresponds to the pile placement in Figure 4.3. The piles in y-direction are positioned to coincide with the load center whereas the piles in x-direction are placed to create a high moment resistance. For the pile group in Case IV, neither of the pile centers are close to the load centers but the biggest difference occurs in the XZ-plane. The piles are therefor placed to create as much moment resistance around the x-axis as possible. The piles in y-direction are not generating any moment resistance but the column wise placement of the piles in x-direction is contributing to the moment resistance around the y-axis as well.

5.2 Restrictions of the Optimization Software

The optimization software comes with some restrictions. One of the most challenging parts of the programming was to avoid collision between the piles. To reduce this problem, the in-plane direction of the piles was limited to only vary with 90 degrees intervals. The software was able to generate pile groups for all simulated supports without any collisions between the piles. More case studies must however be performed to be able to guarantee this for all cases. A drawback with this restriction is that it can result in more piles than a completely optimized pile group would require since the software only generates pile groups that are either single or double symmetric. This means that the total number of piles in a direction that uses symmetry can only be an even number. Hence it would be easier to reduce the number of piles if the direction of the piles were allowed to vary more freely. This would however make the control for collision of piles much more complicated but is something that would be interesting to develop further.

The inclination of the piles are user-specified but it applies to all the piles in the pile group, i.e. different inclinations in one pile group is not possible. This is also a restriction when it comes to optimizing a pile group. If for example the pile group is subjected to a large vertical load, it could be advantageous to place some vertical piles and the other piles could be inclined. Less inclined piles also contribute to a more moment resistant pile group which could be preferable sometimes. With the possibility of varying the inclination of the piles, it would also increase the possibilities of placing the pile center in more positions. This would mean that the position of the pile center is not as limited by the dimensions of the pile cap as it is when only one inclination for all piles must be used. The pile group optimization could thereby be improved if the piles were allowed to have different inclinations.

The placement of the piles is, as mentioned, based on that the pile center should coincide with the load center. One challenging part with this was which piles to place first. The piles placed first can influence the position of the pile center a good deal since all coordinates within the pile cap are available. The last piles placed in the pile group can then naturally not affect the position of the pile center as much since the available coordinates are now much less. The software is now basing the order of the placement for a 'typical' load case. In general, the piles with the largest number of piles leaning in the same direction are placed first. However, this may not be the best solution for every load case, which means that this part should be investigated further. The method of how the pile placement is made is explained in Section 3.2.1.

5.3 General Discussion

The calculations for evaluating the pile groups in this thesis are based on the calculations presented by the Commission on Pile Research [5] which are reproduced in Section 2.7. This calculation method contains some simplifications. The pile can either be seen as hinged or fixed in the pile cap and fixed at a fictitious length at the pile end. This is not always the case in reality where a degree of fixation might be more accurate. The pile cap is seen as infinitely stiff but a more precise calculation would be to account for the displacements of the pile cap as for a beam on elastic supports. Another simplification of this method is that it can only be calculated with one homogeneous soil layer. All of these simplifications are however reasonable for the types of pile groups investigated in this thesis.

In order to obtain comparable result in the case studies, the input data for the optimization software were chosen as far as possible to be the same as in the existing calculations. The only differences that appeared in the case studies were the choice of inclination, as discussed above, and/or the assumption about the pile capacity and its degree of utilization. The pile capacity was rarely specified in the existing calculations. The requirement for the optimization software regarding the pile capacity was then set so that the worst loaded pile in the pile group was never allowed to exceed the maximum normal force or fall below the lowest normal force found in the existing calculations. This is a fairly strict requirement because a small difference in the pile capacity can make big differences for the design of the pile group. As mentioned in Section 4.1.2 for Case II, the allowed utilization ratio for the pile capacity was changed to 91 % instead of 90 %, i.e. the allowed pile capacity increased by just under 15 kN. This small increase led to a reduction of two piles. This gives an indication that if the pile capacity had been known for all case studies performed, the total reduction in the number of piles would probably have increased.

As mentioned earlier, the design of pile groups is an iterative process, which also applies to the calculation of the loads acting on a pile group. A stiffer structure attracts more load than a less stiff structure. This means that a pile group with more piles attracts more load than a pile group with fewer piles since it is stiffer. The loads used in all case studies are based on the existing configuration of the pile groups. If the optimization software reduced the number of piles for a pile group, some of the loads originally acting on the pile group would probably be taken up by the nearby supports. The opposite is true if the optimization program designed a pile group with more piles than the original pile group, i.e. the pile group would probably attract more load. This is worth mentioning for the evaluation of the results from the case studies.

The placement of the piles is only limited by the pile cap dimensions, the center-to-center distance between the piles and the distance to the edge of the pile cap. The latter two are only requirements for minimum distances and there are no set requirements for these maximum distances. Too large a distance between the piles can affect the dimensioning of the pile cap. This is however left for the designer to check since the dimensioning of the pile cap is not accounted for in this thesis.

Decreasing the tension forces and minimizing the difference between the highest and lowest normal force is important for the robustness of a pile group. The summary of all case studies in Table 4.5 shows that the optimization software generates pile groups where the tension forces are decreased or eliminated and the difference between the normal forces are much smaller for a majority of the pile groups compared to the original pile groups. An example of this is for Case XXIII, where the software generated a pile group with two less piles and a lowest normal force of 3 kN compared to the original pile group where the lowest normal force is -191 kN. So even if the software could not reduce the number of piles for some of the cases compared to the existing pile groups, the robustness of the pile group is also an important aspect to analyze.

For Case III, Figure 4.5 shows that the pile placement for the optimized pile group is done to create a high moment resistance around the x-axis and the piles in the y-direction are seemed to be placed to match one of the load centers. To create higher moment resistance around the y-axis, the piles inclined in the y-direction should be placed with a larger distance between each other in the x-direction, as discussed in Section 3.2.1. One explanation for why this is not done may be that the moment acting around the y-axis is favorable for the compressive forces acting on the piles leaning in the positive x-direction. If the piles in y-direction are then placed with a greater distance between each other in the x-direction, this means that these piles will now take up a larger part of the moment around the y-axis. The favorable moment for the piles in the positive x-direction will be smaller, which then leads to the compressive forces increasing in these piles. To achieve horizontal equilibrium in the piles, the tensile forces in the piles leaning in the negative x-direction will then also increase.

The minimum normal force for Case III is higher for the optimized pile group compared to the existing pile group, as seen in Table 4.3. The existing pile group has placed the tensioned piles, i.e. the piles leaning in negative x-direction, column wise which allows them to take up moment. That placement is increasing the moment resistance around the y-axis but it will also lead to higher tension forces in the piles due to the increased force from the moment. The optimized pile group has not placed the tensioned piles in columns why they are unable to take up moment acting around the y-axis. This could be a reason for the difference in tension forces between the pile groups.

6

Conclusion

One aim of this thesis was to develop an optimization software for designing pile groups. This was achieved successfully and the thesis can conclude that an optimization software is a powerful tool to use when designing pile groups. The design of pile groups is an iterative process and it is very time consuming to find a suitable pile group for several load cases, where the help from a computer comes in handy. Even if the software has certain restrictions and there remain some controls that the designer is left to check, the software is suitable for a preliminary design of a pile group. Especially since it is compatible with the software Rymdpålgrupp, where the designer can make any desired changes afterwards.

The thesis also investigated the most efficient pile placement for a pile group and which parameters that governs an efficient pile placement. Eliminating the moments in the pile group by adjusting the pile center to fit the load center had a great effect on the design of the pile group. This was however proven to be difficult in many cases, since the position of the pile center is much dependent on the pile cap dimensions and the inclination of the piles. The in-plane direction of the piles is also an important parameter since this limits the position of the piles due to the risk of collision. When the pile center could not be placed in the same position as the load center, the moment resistance was of great importance for the stability of the pile group. A high moment resistance was achieved by increasing the lever arm between the piles forming force couples in the pile group. This is also limited by the dimensions of the pile cap which determines the maximum distance between the piles. The thesis can thereby conclude that a pile group can be optimized by minimizing the distance between the pile center and the load center at the same time as the piles are positioned in a way such that high moment capacity in the pile group is created.

6.1 Suggestions for Further Work

The following subjects are suggestions for further work to refine the optimization software:

- To further optimize the pile group and increase the flexibility of the position of the pile center, each pile should have its unique inclination and length and the in-plane direction should be able to vary more freely.

- The generated pile groups for all case studies was made without any collision between the piles. However, this can not be fully guaranteed for other cases, hence pile collision should be investigated more carefully.

- The allowed deformations for the pile group are now given high permissible values and are hard-coded into the software. Allowed deformations should instead be implemented as an input data.

- The piles will always deviate from their intended position when installed. A redundancy calculation could be performed by the optimization software that calculates pile forces and deformations for the pile group based on the maximum permissible deviation values.

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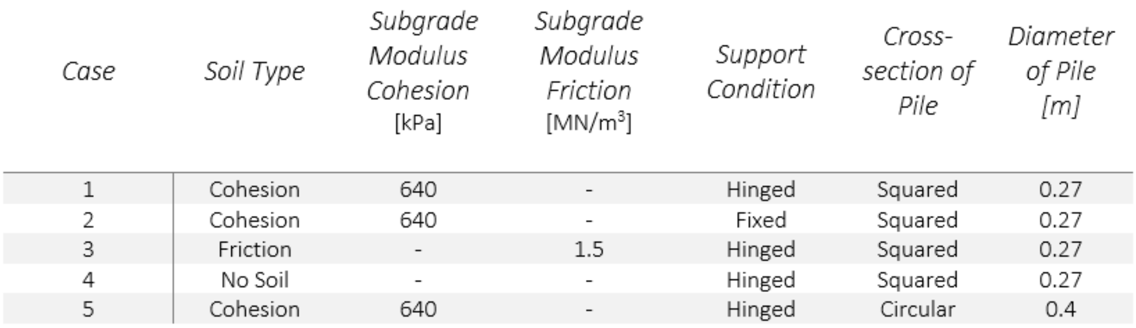
A

Verification of Optimization Software

The verification of the software is made for the pile group in Case I. The pile forces, deformations and pile centers generated by the optimization software is compared with the results from the program Rymdpålgrupp [4]. To ensure that the software calculates correctly regardless of the choice of input data, the verification is done for five different choices of input data. The soil condition is changed for three cases, the support condition is set to be fixed in the pile cap for one case and the cross-section of the pile is changed for the last comparison. The input data for all comparisons can be seen in Table A.1. For the cases with friction soil and different cross-section of the piles, arbitrary values are used. The load cases are the same for all comparisons and the input data that is left unchanged for all cases can be seen in Table A.2. The pile forces are compared for one load case only.

The comparisons are presented in Sections A.1 to A.5. The results of the calculations from the optimization software correspond to the results from the program Rymdpålgrupp for all cases.



Case	*Soil Type*	*Subgrade Modulus Cohesion* [kPa]	*Subgrade Modulus Friction* $[MN/m^3]$	*Support Condition*	*Cross-section of Pile*	*Diameter of Pile [m]*
1	Cohesion	640	-	Hinged	Squared	0.27
2	Cohesion	640	-	Fixed	Squared	0.27
3	Friction	-	1.5	Hinged	Squared	0.27
4	No Soil	-	-	Hinged	Squared	0.27
5	Cohesion	640	-	Hinged	Circular	0.4

Table A.1: Change of input data for the different comparisons

$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Pile Length [m]*	*Inclination* [N:1]	*Utilization Ratio [%]*
685	0	6.7	4	100

Table A.2: Constant input data

A.1 Verification 1

This comparison is made for a calculation with the input data as stated for the original pile group in Section 3.3.1.

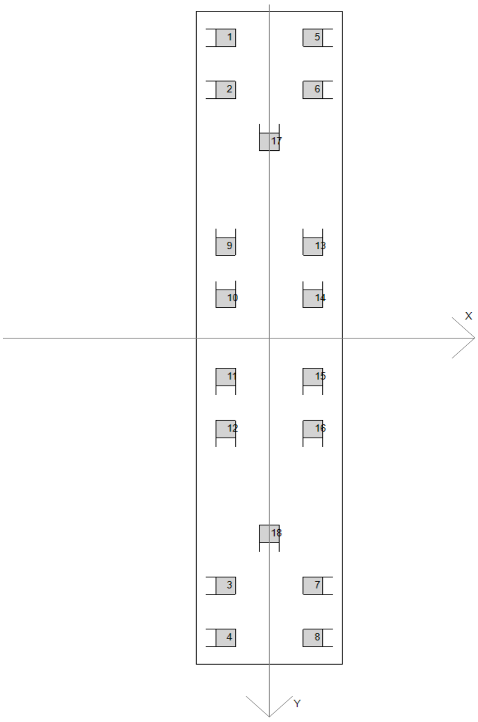
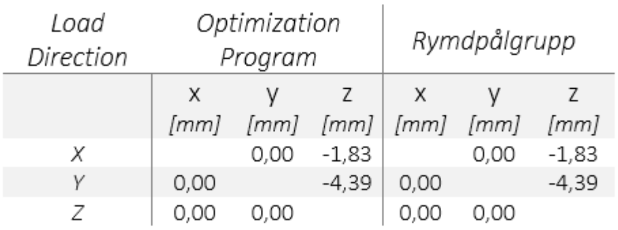
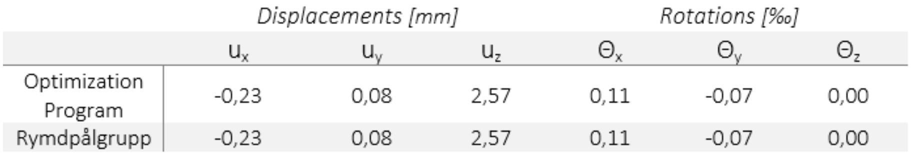


Figure A.1: Optimized pile group Case 1



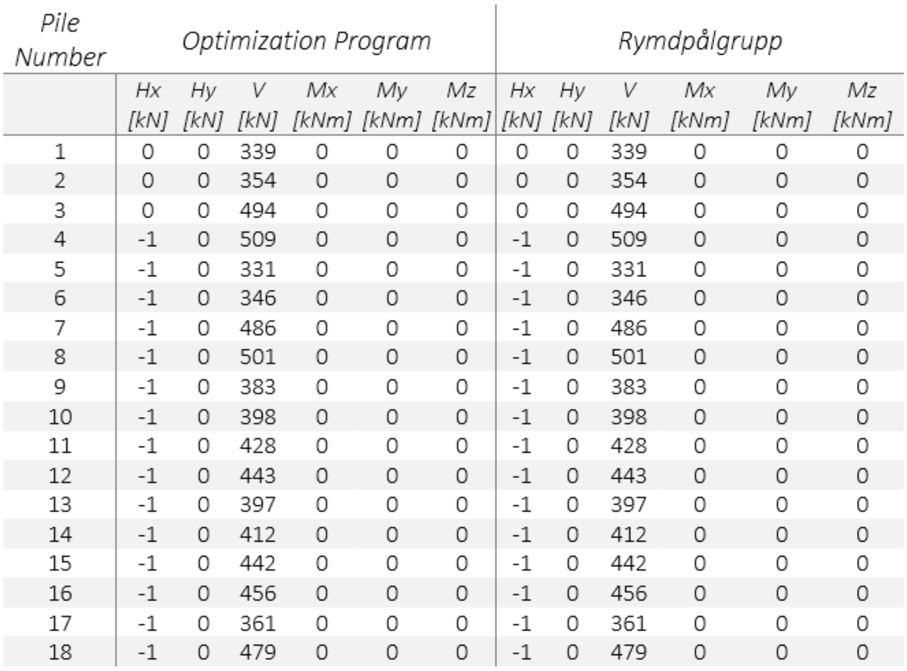
Load Direction	*Optimization Program*			*Rymdpålgrupp*		
	x *[mm]*	y *[mm]*	z *[mm]*	x *[mm]*	y *[mm]*	z *[mm]*
X		0,00	-1,83		0,00	-1,83
Y	0,00		-4,39	0,00		-4,39
Z	0,00	0,00		0,00	0,00	

Table A.3: Comparison of pile centers for Case 1



	Displacements [mm]			*Rotations [‰]*		
	u_x	u_y	u_z	Θ_x	Θ_y	Θ_z
Optimization Program	-0,23	0,08	2,57	0,11	-0,07	0,00
Rymdpålgrupp	-0,23	0,08	2,57	0,11	-0,07	0,00

Table A.4: Comparison of deformations for one load case for Case 1



Pile Number	Optimization Program						Rymdpålgrupp					
	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
1	0	0	339	0	0	0	0	0	339	0	0	0
2	0	0	354	0	0	0	0	0	354	0	0	0
3	0	0	494	0	0	0	0	0	494	0	0	0
4	-1	0	509	0	0	0	-1	0	509	0	0	0
5	-1	0	331	0	0	0	-1	0	331	0	0	0
6	-1	0	346	0	0	0	-1	0	346	0	0	0
7	-1	0	486	0	0	0	-1	0	486	0	0	0
8	-1	0	501	0	0	0	-1	0	501	0	0	0
9	-1	0	383	0	0	0	-1	0	383	0	0	0
10	-1	0	398	0	0	0	-1	0	398	0	0	0
11	-1	0	428	0	0	0	-1	0	428	0	0	0
12	-1	0	443	0	0	0	-1	0	443	0	0	0
13	-1	0	397	0	0	0	-1	0	397	0	0	0
14	-1	0	412	0	0	0	-1	0	412	0	0	0
15	-1	0	442	0	0	0	-1	0	442	0	0	0
16	-1	0	456	0	0	0	-1	0	456	0	0	0
17	-1	0	361	0	0	0	-1	0	361	0	0	0
18	-1	0	479	0	0	0	-1	0	479	0	0	0

Table A.5: Comparison of pile forces for one load case for Case 1

A.2 Verification 2

This comparison is made for a calculation with a fixed connection at the pile cap.

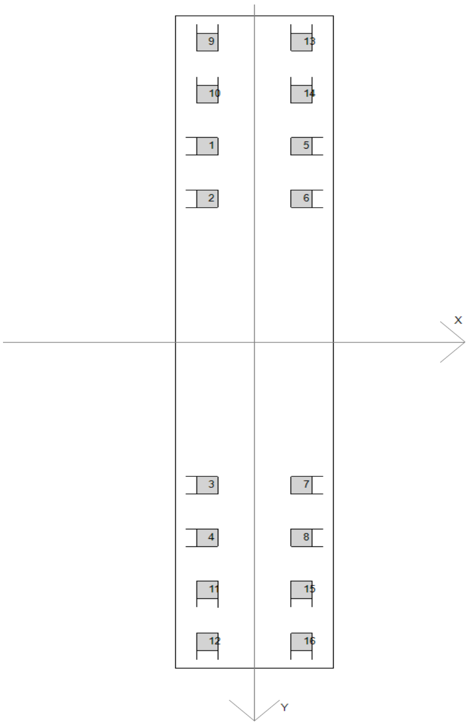
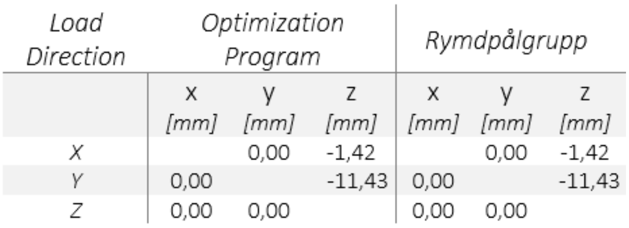
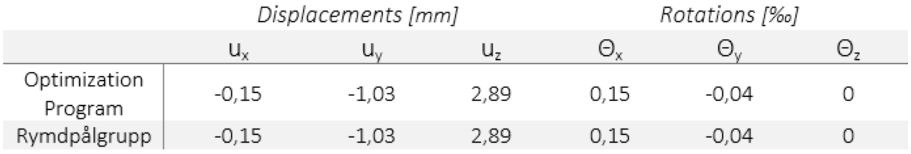


Figure A.2: Optimized pile group Case 2



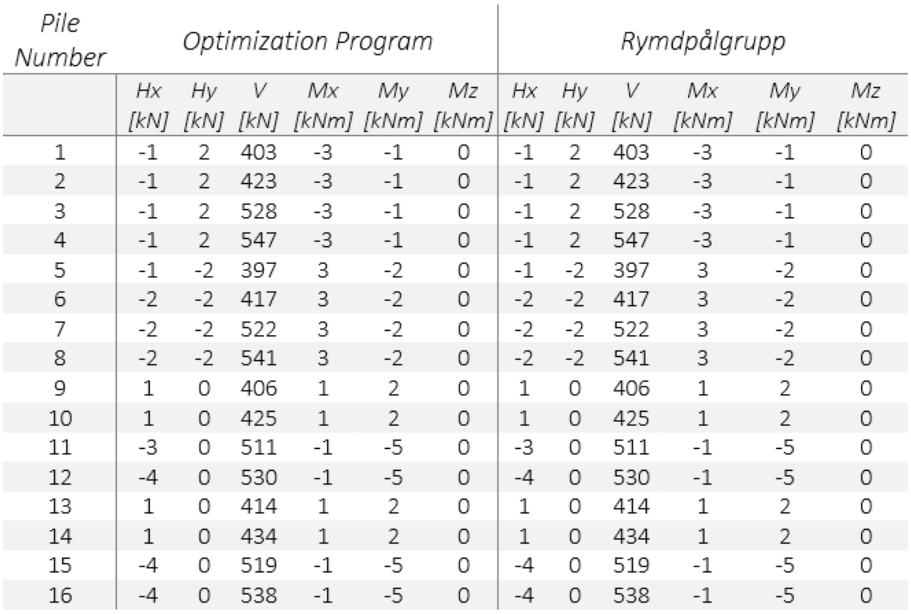
Load Direction	*Optimization Program*			*Rymdpålgrupp*		
	x *[mm]*	y *[mm]*	z *[mm]*	x *[mm]*	y *[mm]*	z *[mm]*
X		0,00	-1,42		0,00	-1,42
Y	0,00		-11,43	0,00		-11,43
Z	0,00	0,00		0,00	0,00	

Table A.6: Comparison of pile centers for Case 2



	Displacements [mm]			*Rotations [‰]*		
	u_x	u_y	u_z	Θ_x	Θ_y	Θ_z
Optimization Program	-0,15	-1,03	2,89	0,15	-0,04	0
Rymdpålgrupp	-0,15	-1,03	2,89	0,15	-0,04	0

Table A.7: Comparison of deformations for one load case for Case 2



Pile Number	Optimization Program						Rymdpålgrupp					
	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
1	-1	2	403	-3	-1	0	-1	2	403	-3	-1	0
2	-1	2	423	-3	-1	0	-1	2	423	-3	-1	0
3	-1	2	528	-3	-1	0	-1	2	528	-3	-1	0
4	-1	2	547	-3	-1	0	-1	2	547	-3	-1	0
5	-1	-2	397	3	-2	0	-1	-2	397	3	-2	0
6	-2	-2	417	3	-2	0	-2	-2	417	3	-2	0
7	-2	-2	522	3	-2	0	-2	-2	522	3	-2	0
8	-2	-2	541	3	-2	0	-2	-2	541	3	-2	0
9	1	0	406	1	2	0	1	0	406	1	2	0
10	1	0	425	1	2	0	1	0	425	1	2	0
11	-3	0	511	-1	-5	0	-3	0	511	-1	-5	0
12	-4	0	530	-1	-5	0	-4	0	530	-1	-5	0
13	1	0	414	1	2	0	1	0	414	1	2	0
14	1	0	434	1	2	0	1	0	434	1	2	0
15	-4	0	519	-1	-5	0	-4	0	519	-1	-5	0
16	-4	0	538	-1	-5	0	-4	0	538	-1	-5	0

Table A.8: Comparison of pile forces for one load case for Case 2

A.3 Verification 3

This comparison is made for a calculation without friction soil.

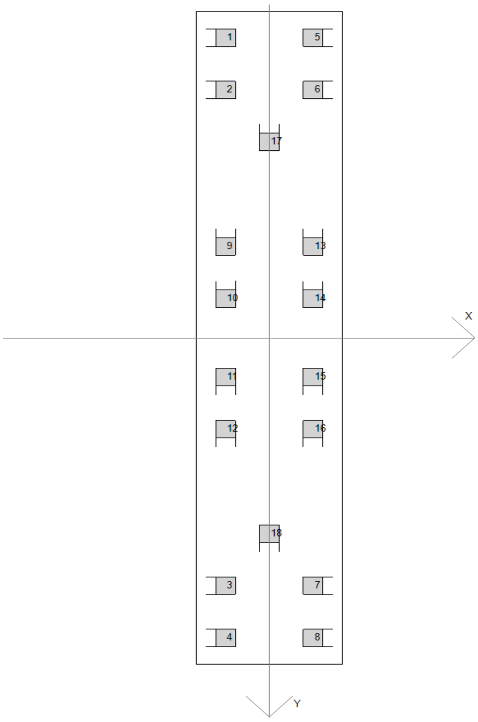
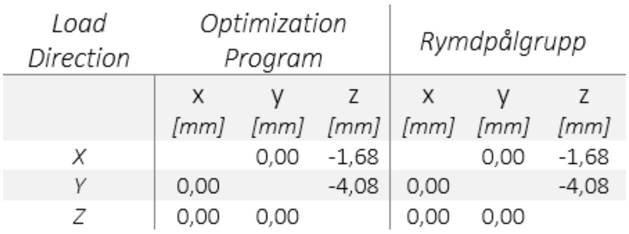
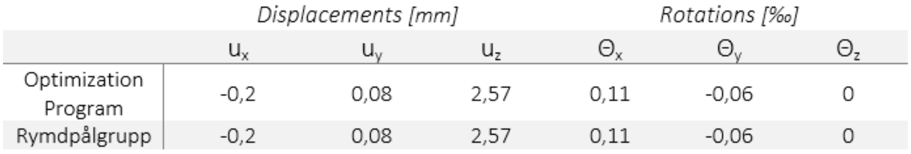


Figure A.3: Optimized pile group Case 3



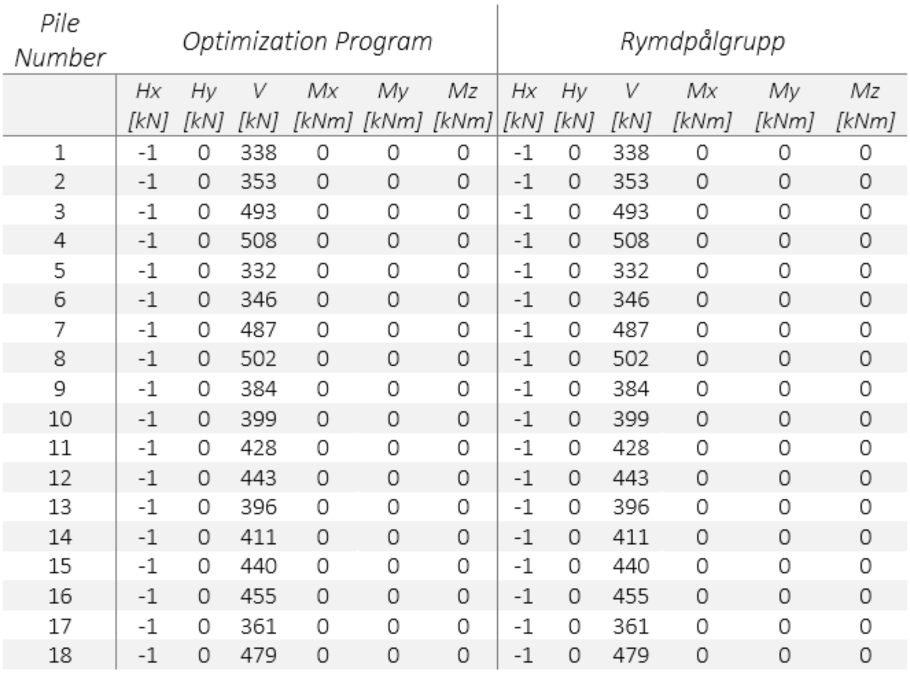
Load Direction	*Optimization Program*			*Rymdpålgrupp*		
	x *[mm]*	y *[mm]*	z *[mm]*	x *[mm]*	y *[mm]*	z *[mm]*
X		0,00	-1,68		0,00	-1,68
Y	0,00		-4,08	0,00		-4,08
Z	0,00	0,00		0,00	0,00	

Table A.9: Comparison of pile centers for Case 3



	Displacements [mm]			*Rotations [‰]*		
	u_x	u_y	u_z	Θ_x	Θ_y	Θ_z
Optimization Program	-0,2	0,08	2,57	0,11	-0,06	0
Rymdpålgrupp	-0,2	0,08	2,57	0,11	-0,06	0

Table A.10: Comparison of deformations for one load case for Case 3



Pile Number	Optimization Program						Rymdpålgrupp					
	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
1	-1	0	338	0	0	0	-1	0	338	0	0	0
2	-1	0	353	0	0	0	-1	0	353	0	0	0
3	-1	0	493	0	0	0	-1	0	493	0	0	0
4	-1	0	508	0	0	0	-1	0	508	0	0	0
5	-1	0	332	0	0	0	-1	0	332	0	0	0
6	-1	0	346	0	0	0	-1	0	346	0	0	0
7	-1	0	487	0	0	0	-1	0	487	0	0	0
8	-1	0	502	0	0	0	-1	0	502	0	0	0
9	-1	0	384	0	0	0	-1	0	384	0	0	0
10	-1	0	399	0	0	0	-1	0	399	0	0	0
11	-1	0	428	0	0	0	-1	0	428	0	0	0
12	-1	0	443	0	0	0	-1	0	443	0	0	0
13	-1	0	396	0	0	0	-1	0	396	0	0	0
14	-1	0	411	0	0	0	-1	0	411	0	0	0
15	-1	0	440	0	0	0	-1	0	440	0	0	0
16	-1	0	455	0	0	0	-1	0	455	0	0	0
17	-1	0	361	0	0	0	-1	0	361	0	0	0
18	-1	0	479	0	0	0	-1	0	479	0	0	0

Table A.11: Comparison of pile forces for one load case for Case 3

A.4 Verification 4

This comparison is made for a calculation with no lateral soil resistance.

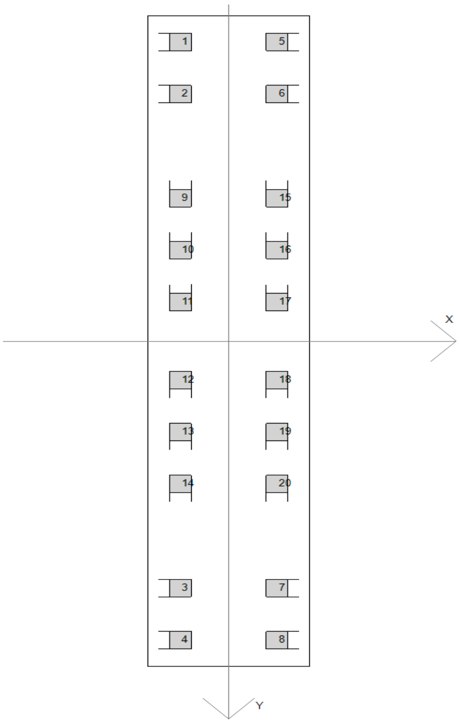


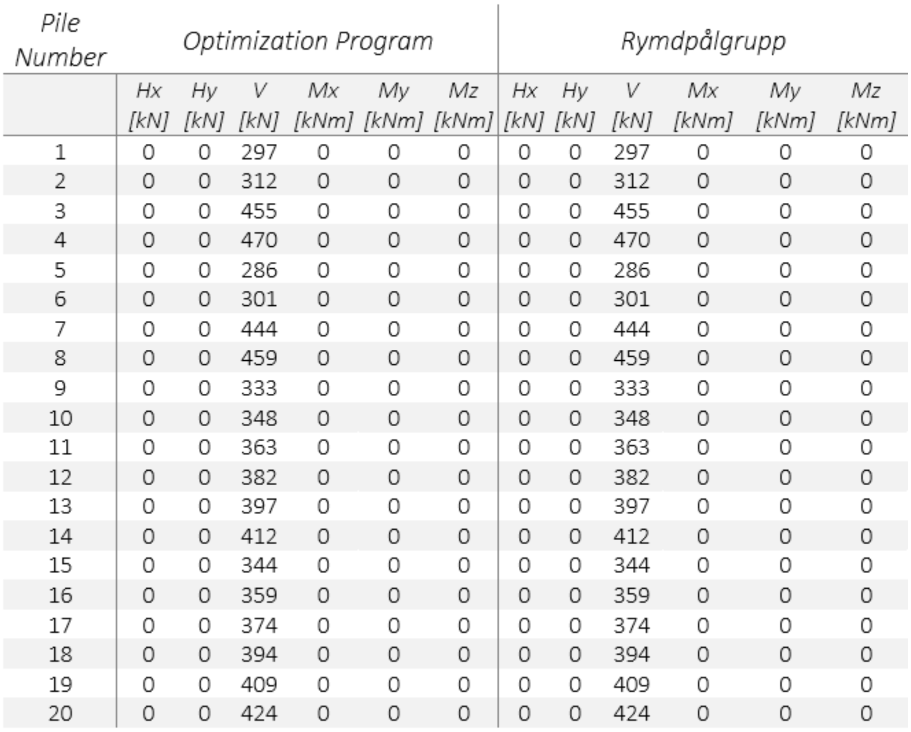
Figure A.4: Optimized pile group Case 4

Load Direction	*Optimization Program*			*Rymdpålgrupp*		
	x *[mm]*	y *[mm]*	z *[mm]*	x *[mm]*	y *[mm]*	z *[mm]*
X		0,00	-2.1		0,00	-2.1
Y	0,00		-4,9	0,00		-4,9
Z	0,00	0,00		0,00	0,00	

Table A.12: Comparison of pile centers for Case 4

	Displacements [mm]			*Rotations [‰]*		
	u_x	u_y	u_z	Θ_x	Θ_y	Θ_z
Optimization Program	-0,24	-0,03	2,32	0,11	-0,06	0,00
Rymdpålgrupp	-0,24	-0,03	2,32	0,11	-0,06	0,00

Table A.13: Comparison of deformations for one load case for Case 4



Pile Number	Optimization Program						Rymdpålgrupp					
	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
1	0	0	297	0	0	0	0	0	297	0	0	0
2	0	0	312	0	0	0	0	0	312	0	0	0
3	0	0	455	0	0	0	0	0	455	0	0	0
4	0	0	470	0	0	0	0	0	470	0	0	0
5	0	0	286	0	0	0	0	0	286	0	0	0
6	0	0	301	0	0	0	0	0	301	0	0	0
7	0	0	444	0	0	0	0	0	444	0	0	0
8	0	0	459	0	0	0	0	0	459	0	0	0
9	0	0	333	0	0	0	0	0	333	0	0	0
10	0	0	348	0	0	0	0	0	348	0	0	0
11	0	0	363	0	0	0	0	0	363	0	0	0
12	0	0	382	0	0	0	0	0	382	0	0	0
13	0	0	397	0	0	0	0	0	397	0	0	0
14	0	0	412	0	0	0	0	0	412	0	0	0
15	0	0	344	0	0	0	0	0	344	0	0	0
16	0	0	359	0	0	0	0	0	359	0	0	0
17	0	0	374	0	0	0	0	0	374	0	0	0
18	0	0	394	0	0	0	0	0	394	0	0	0
19	0	0	409	0	0	0	0	0	409	0	0	0
20	0	0	424	0	0	0	0	0	424	0	0	0

Table A.14: Comparison of pile forces for one load case for Case 4

A.5 Verification 5

This comparison is made for a calculation with 400 mm circular piles.

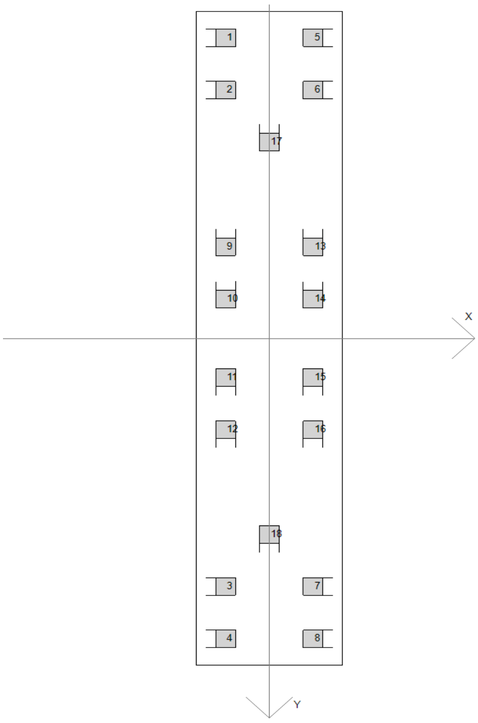
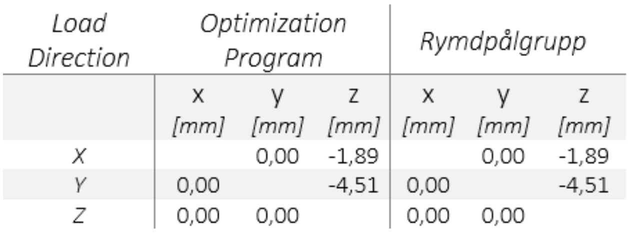
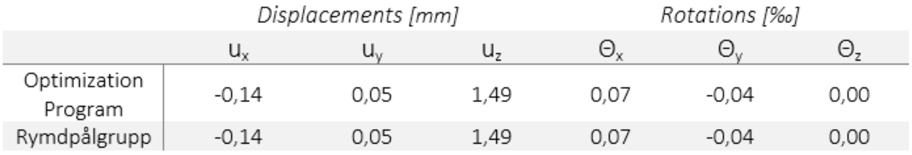


Figure A.5: Optimized pile group Case 5



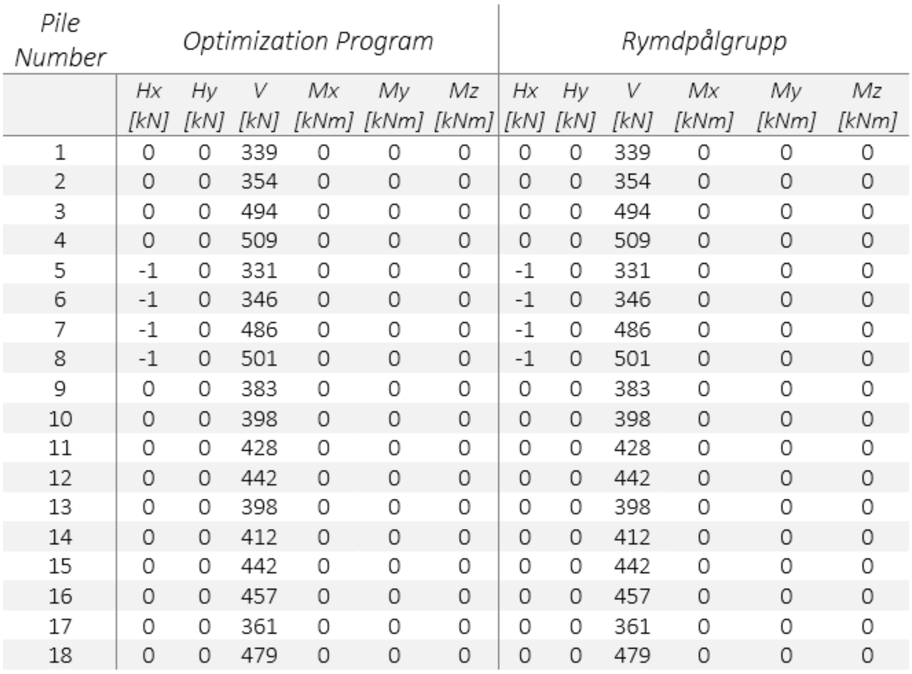
Load Direction	*Optimization Program*			*Rymdpålgrupp*		
	x *[mm]*	y *[mm]*	z *[mm]*	x *[mm]*	y *[mm]*	z *[mm]*
X		0,00	-1,89		0,00	-1,89
Y	0,00		-4,51	0,00		-4,51
Z	0,00	0,00		0,00	0,00	

Table A.15: Comparison of pile centers for Case 5



	Displacements [mm]			*Rotations [‰]*		
	u_x	u_y	u_z	Θ_x	Θ_y	Θ_z
Optimization Program	-0,14	0,05	1,49	0,07	-0,04	0,00
Rymdpålgrupp	-0,14	0,05	1,49	0,07	-0,04	0,00

Table A.16: Comparison of deformations for one load case for Case 5



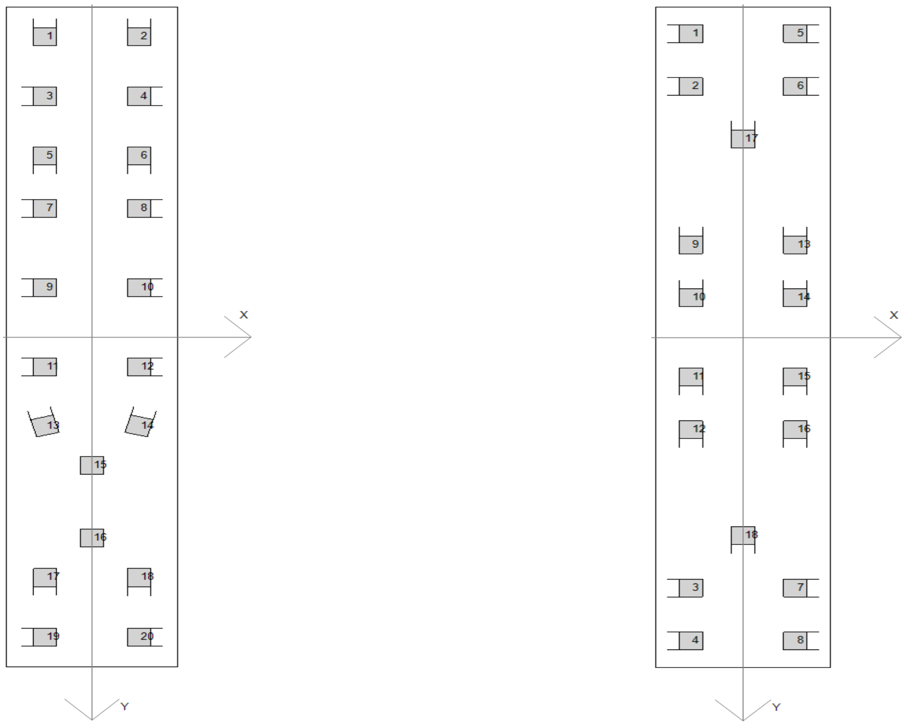
Pile Number	Optimization Program						Rymdpålgrupp					
	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]	Hx [kN]	Hy [kN]	V [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
1	0	0	339	0	0	0	0	0	339	0	0	0
2	0	0	354	0	0	0	0	0	354	0	0	0
3	0	0	494	0	0	0	0	0	494	0	0	0
4	0	0	509	0	0	0	0	0	509	0	0	0
5	-1	0	331	0	0	0	-1	0	331	0	0	0
6	-1	0	346	0	0	0	-1	0	346	0	0	0
7	-1	0	486	0	0	0	-1	0	486	0	0	0
8	-1	0	501	0	0	0	-1	0	501	0	0	0
9	0	0	383	0	0	0	0	0	383	0	0	0
10	0	0	398	0	0	0	0	0	398	0	0	0
11	0	0	428	0	0	0	0	0	428	0	0	0
12	0	0	442	0	0	0	0	0	442	0	0	0
13	0	0	398	0	0	0	0	0	398	0	0	0
14	0	0	412	0	0	0	0	0	412	0	0	0
15	0	0	442	0	0	0	0	0	442	0	0	0
16	0	0	457	0	0	0	0	0	457	0	0	0
17	0	0	361	0	0	0	0	0	361	0	0	0
18	0	0	479	0	0	0	0	0	479	0	0	0

Table A.17: Comparison of pile forces for one load case for Case 5

B

Summary of All Case Studies

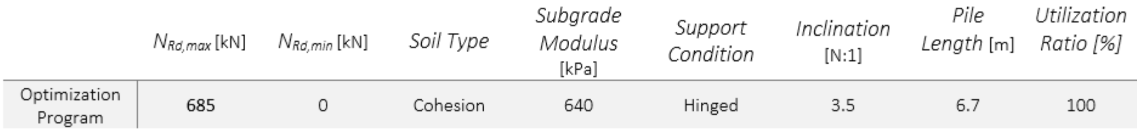
B.1 Case I



(a) Original pile group, 20 piles

(b) Optimized pile group, 18 piles

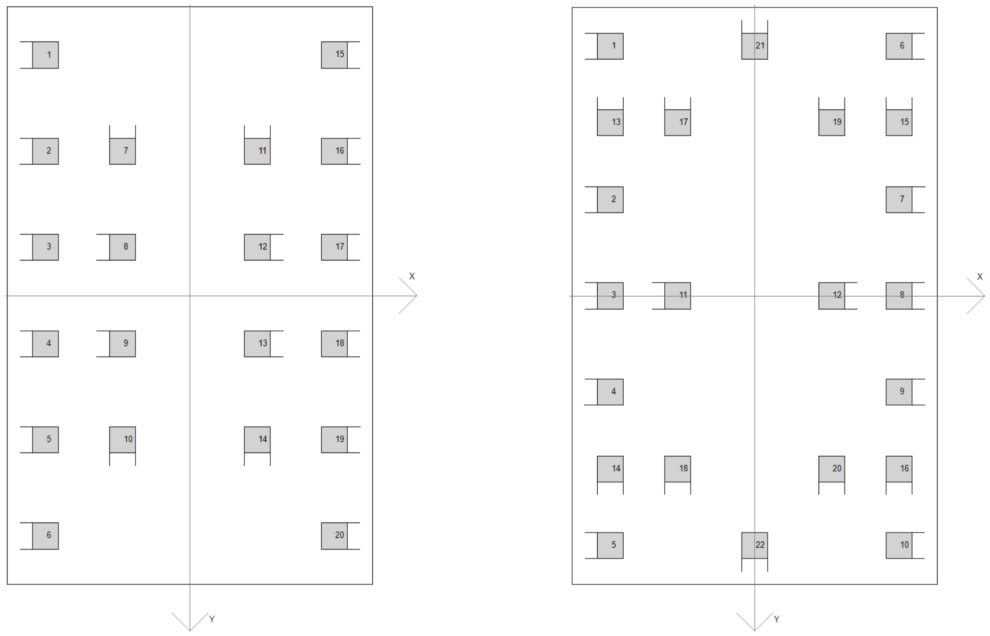
Figure B.1: Pile groups Case I



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	685	0	Cohesion	640	Hinged	3.5	6.7	100

Table B.1: Input data for the optimization software for Case I

B.2 Case II



(a) Original pile group, 20 piles

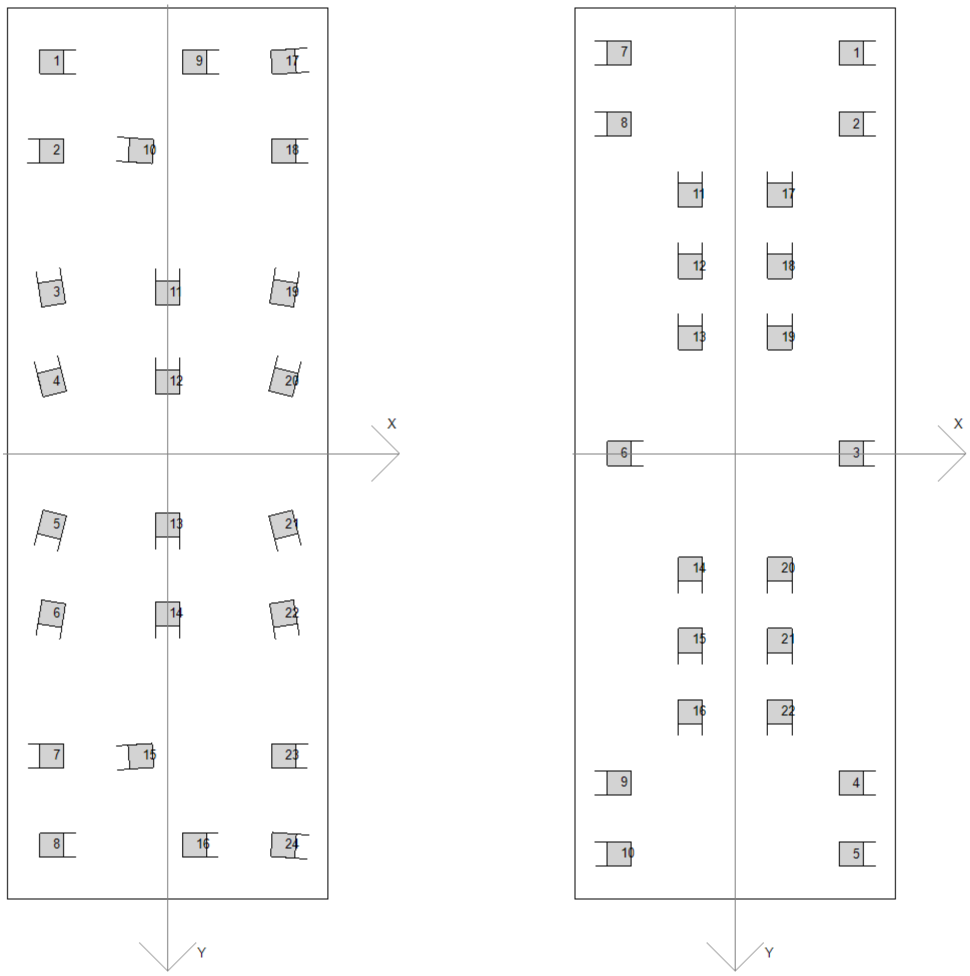
(b) Optimized pile group, 22 piles

Figure B.2: Pile groups Case II

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1352}{0,9}$	-118	Cohesion	250	Hinged	4	15	90

Table B.2: Input data for the optimization software for Case II

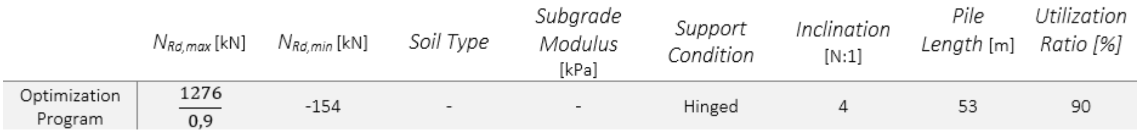
B.3 Case III



(a) Original pile group, 24 piles

(b) Optimized pile group, 22 piles

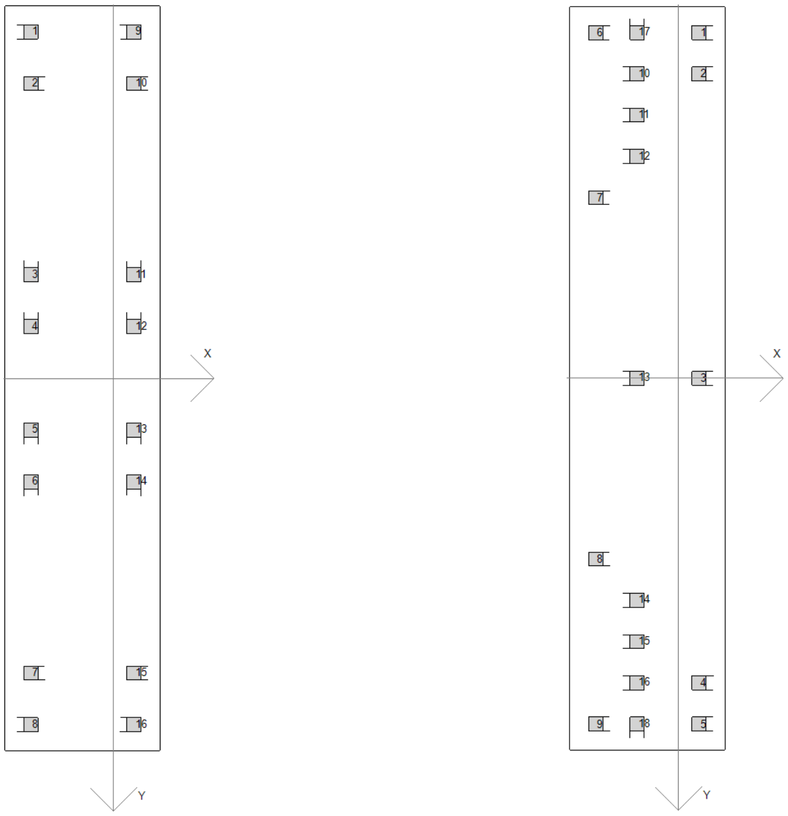
Figure B.3: Pile groups Case III



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1276}{0,9}$	-154	-	-	Hinged	4	53	90

Table B.3: Input data for the optimization software for Case III

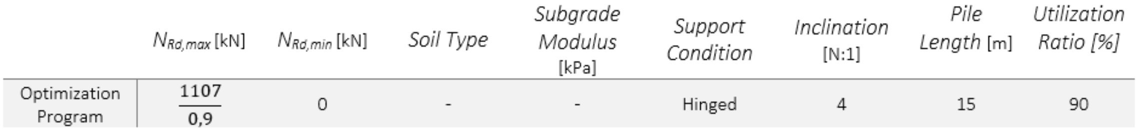
B.4 Case IV



(a) Original pile group, 16 piles

(b) Optimized pile group, 18 piles

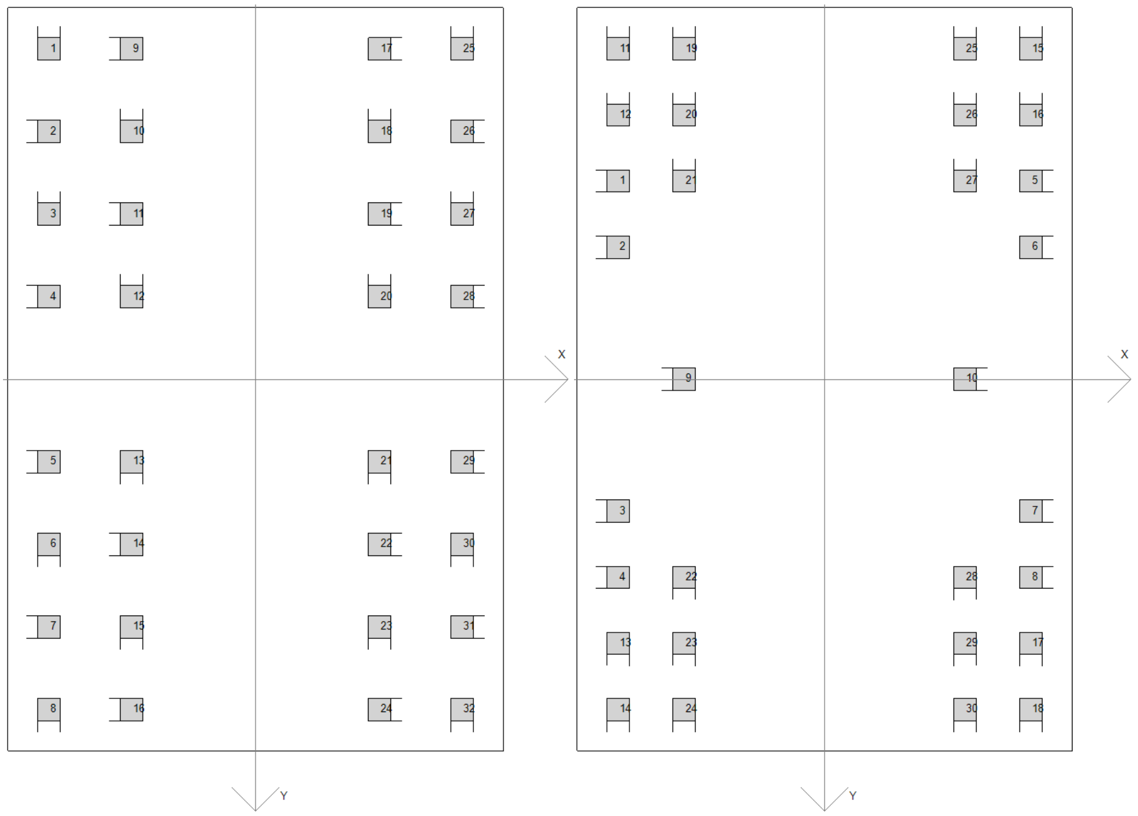
Figure B.4: Pile groups Case IV



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1107}{0,9}$	0	-	-	Hinged	4	15	90

Table B.4: Input data for the optimization software for Case IV

B.5 Case V



(a) Original pile group, 32 piles **(b)** Optimized pile group, 30 piles

Figure B.5: Pile groups Case V

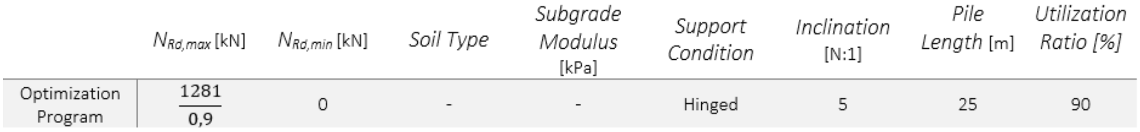
	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1223}{0,9}$	0	-	-	Hinged	4	6	90

Table B.5: Input data for the optimization software for Case V

B.6 Case VI

(a) Original pile group, 24 piles **(b)** Optimized pile group, 24 piles

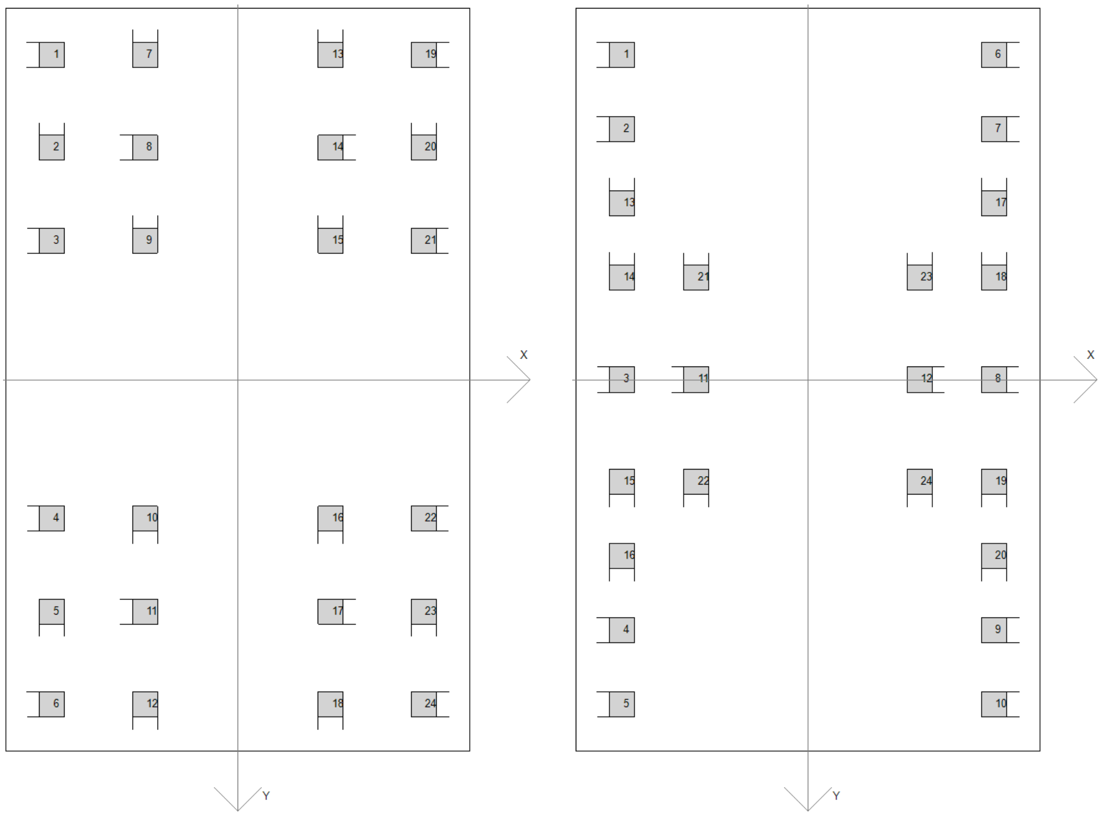
Figure B.6: Pile groups Case VI



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1281}{0{,}9}$	0	-	-	Hinged	5	25	90

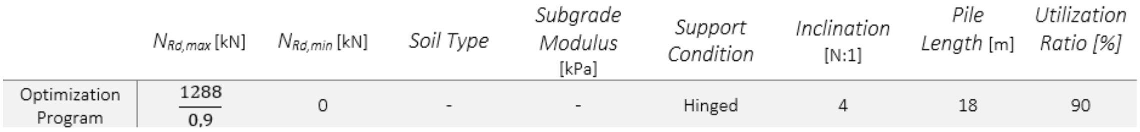
Table B.6: Input data for the optimization software for Case VI

B.7 Case VII



(a) Original pile group, 24 piles **(b)** Optimized pile group, 24 piles

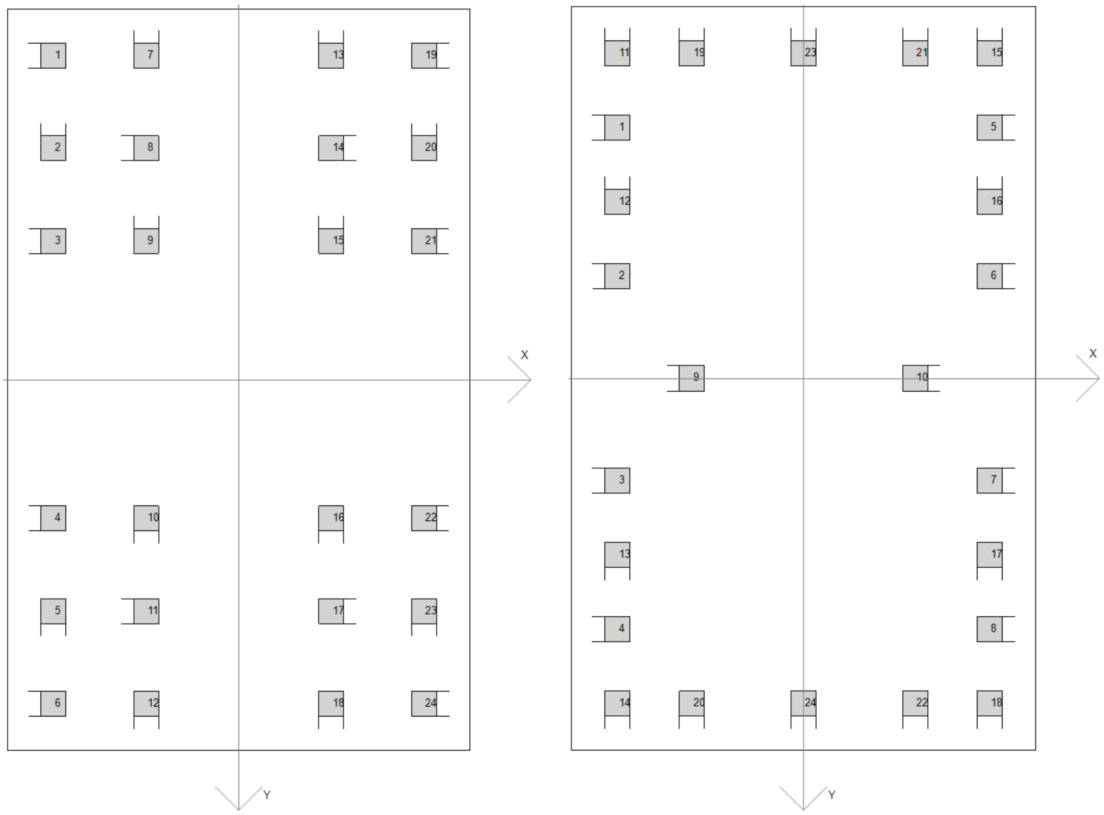
Figure B.7: Pile groups Case VII



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1288}{0.9}$	0	-	-	Hinged	4	18	90

Table B.7: Input data for the optimization software for Case VII

B.8 Case VIII



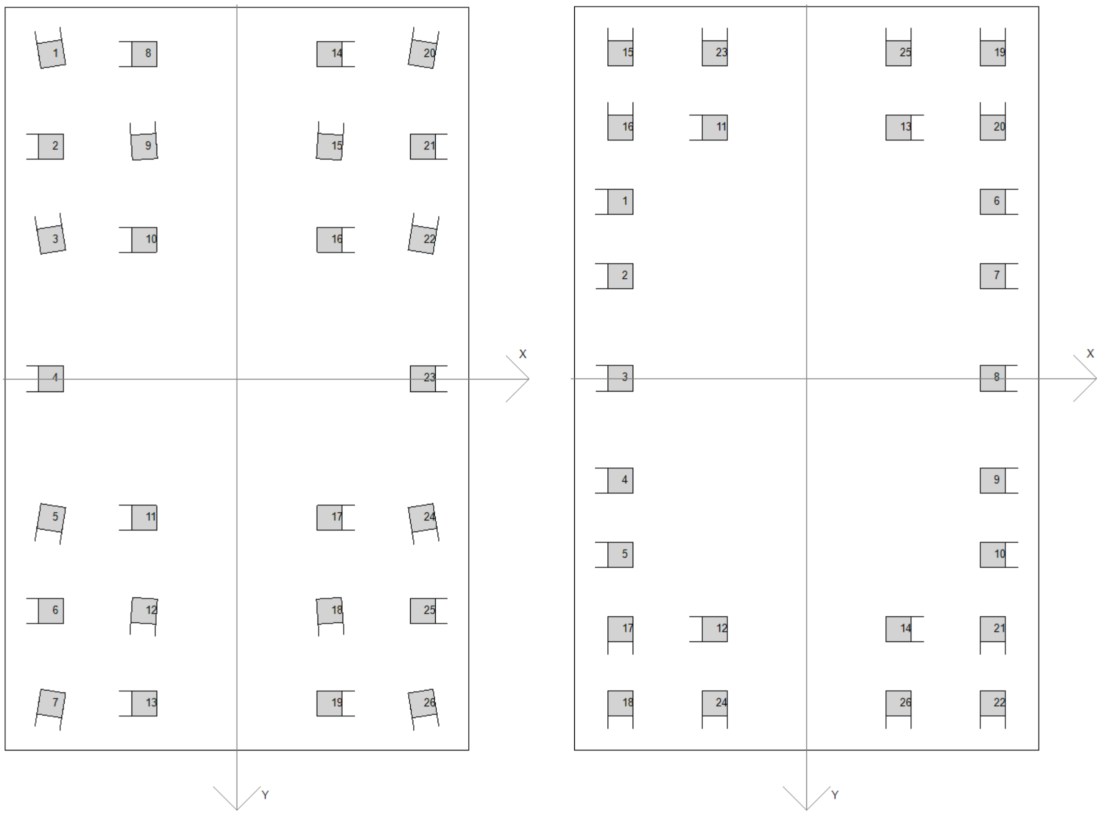
(a) Original pile group, 24 piles **(b)** Optimized pile group, 24 piles

Figure B.8: Pile groups Case VIII

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1175}{0,9}$	0	-	-	Hinged	4	18	90

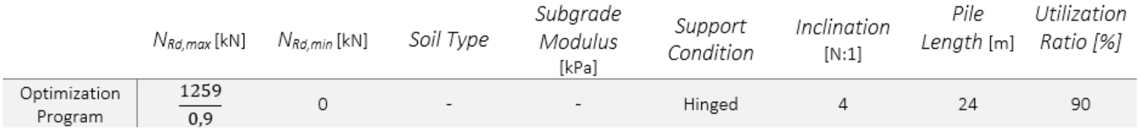
Table B.8: Input data for the optimization software for Case VIII

B.9 Case IX



(a) Original pile group, 26 piles **(b)** Optimized pile group, 26 piles

Figure B.9: Pile groups Case IX



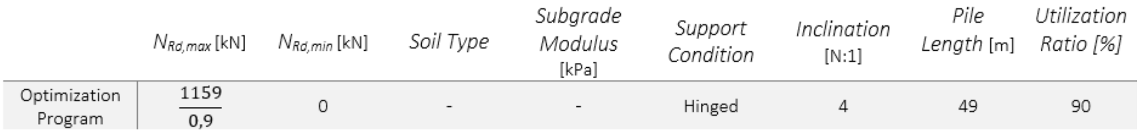
	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1259}{0,9}$	0	-	-	Hinged	4	24	90

Table B.9: Input data for the optimization software for Case IX

B.10 Case X

(a) Original pile group, 26 piles **(b)** Optimized pile group, 24 piles

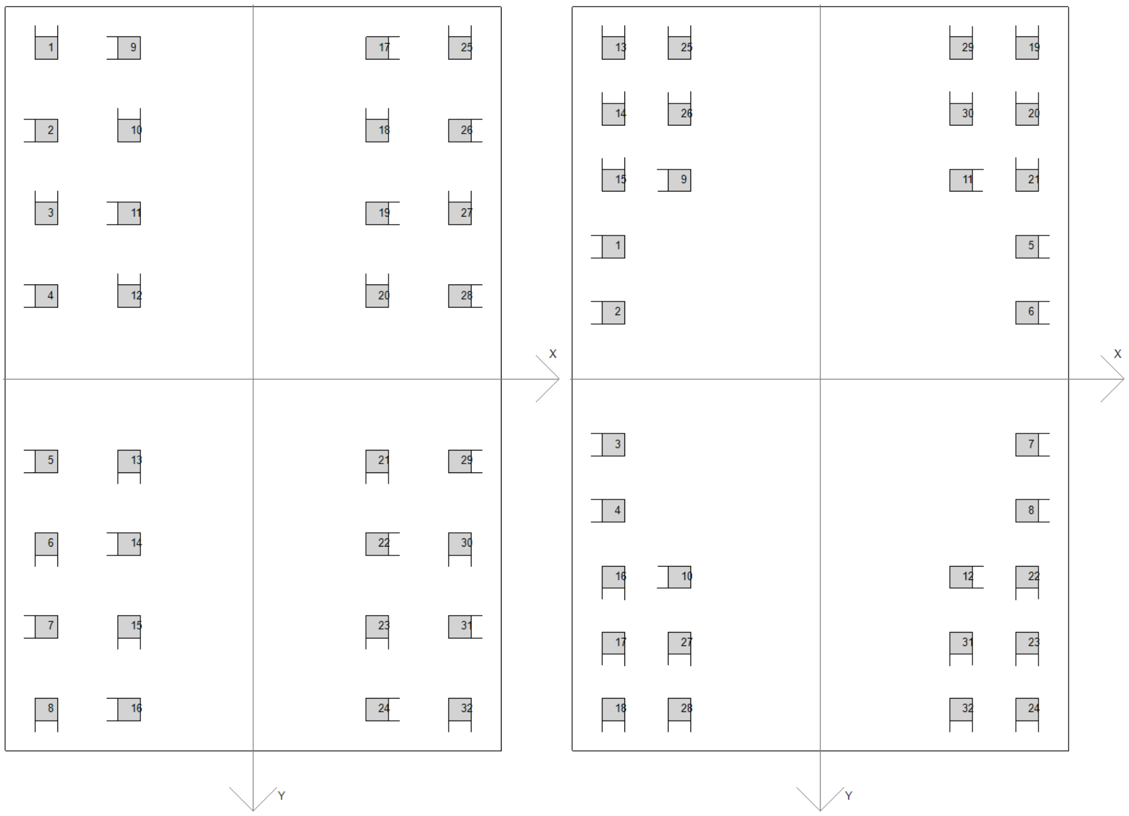
Figure B.10: Pile groups Case X



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1159}{0,9}$	0	-	-	Hinged	4	49	90

Table B.10: Input data for the optimization software for Case X

B.11 Case XI



(a) Original pile group, 32 piles

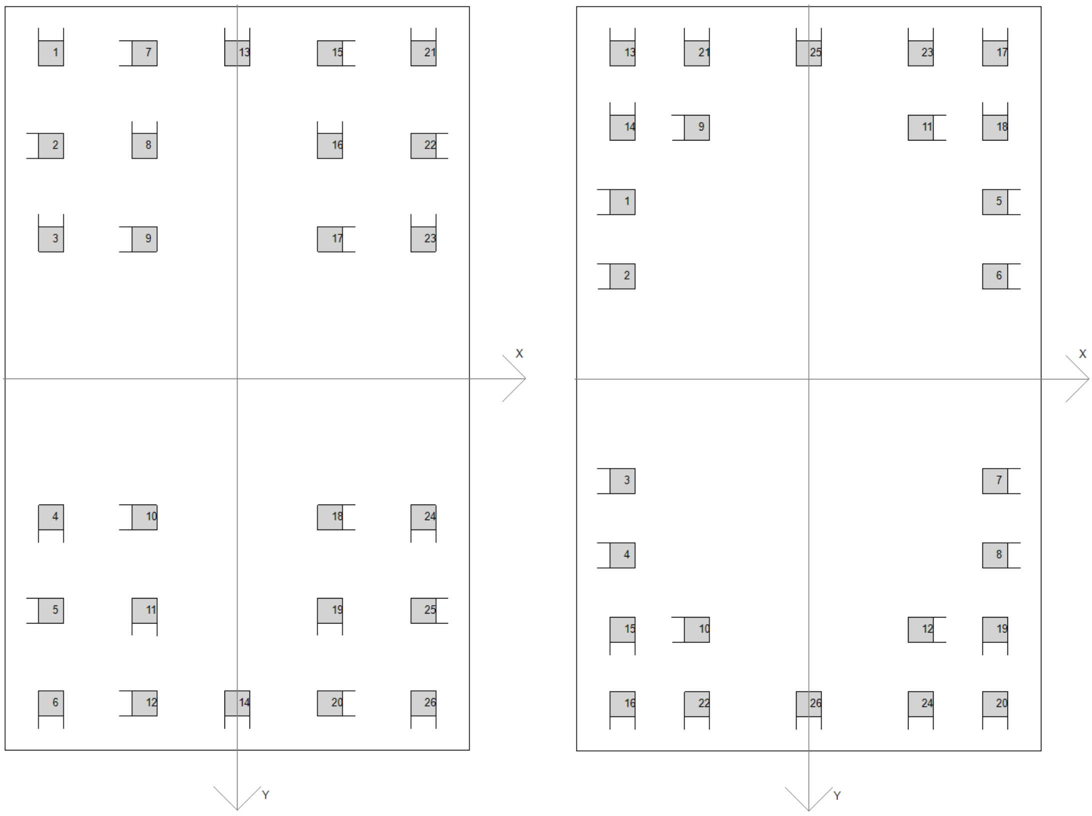
(b) Optimized pile group, 32 piles

Figure B.11: Pile groups Case XI

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1154}{0,9}$	0	-	-	Hinged	4	12	90

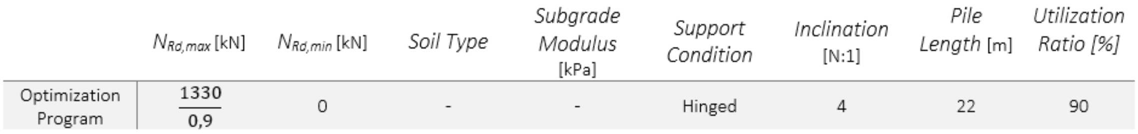
Table B.11: Input data for the optimization software for Case XI

B.12 Case XII



(a) Original pile group, 26 piles **(b)** Optimized pile group, 26 piles

Figure B.12: Pile groups Case XII



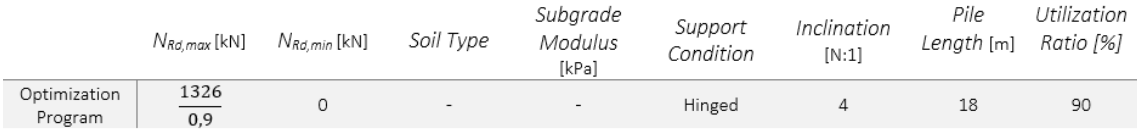
	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1330}{0,9}$	0	-	-	Hinged	4	22	90

Table B.12: Input data for the optimization software for Case XII

B.13 Case XIII

(a) Original pile group, 26 piles **(b)** Optimized pile group, 26 piles

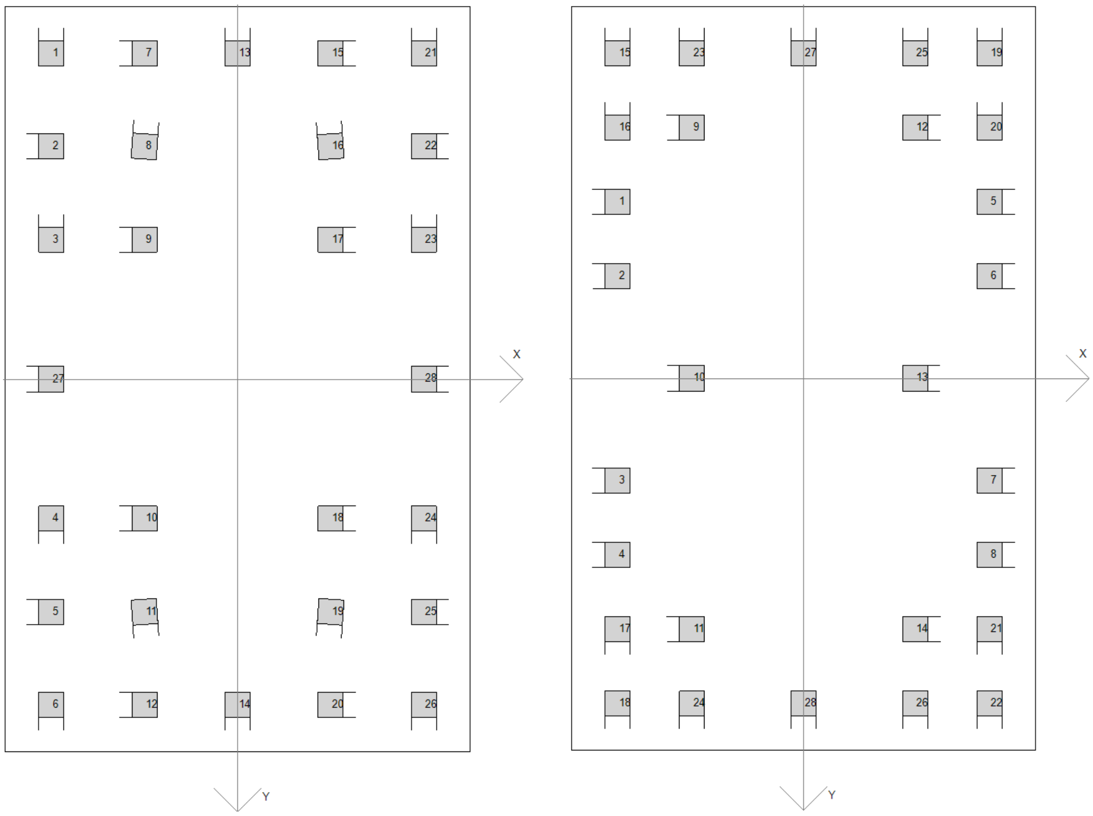
Figure B.13: Pile groups Case XIII



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1326}{0,9}$	0	-	-	Hinged	4	18	90

Table B.13: Input data for the optimization software for Case XIII

B.14 Case XIV



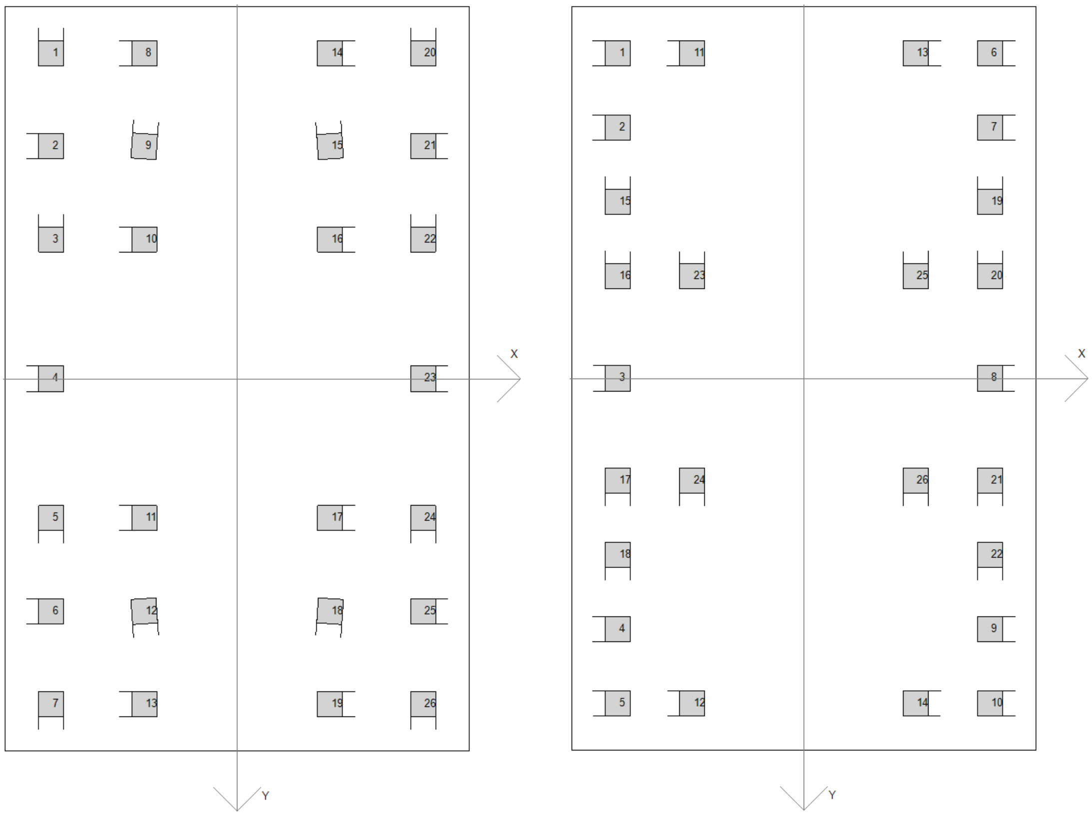
(a) Original pile group, 28 piles (b) Optimized pile group, 28 piles

Figure B.14: Pile groups Case XIV

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1256}{0,9}$	0	-	-	Hinged	4	28	90

Table B.14: Input data for the optimization software for Case XIV

B.15 Case XV



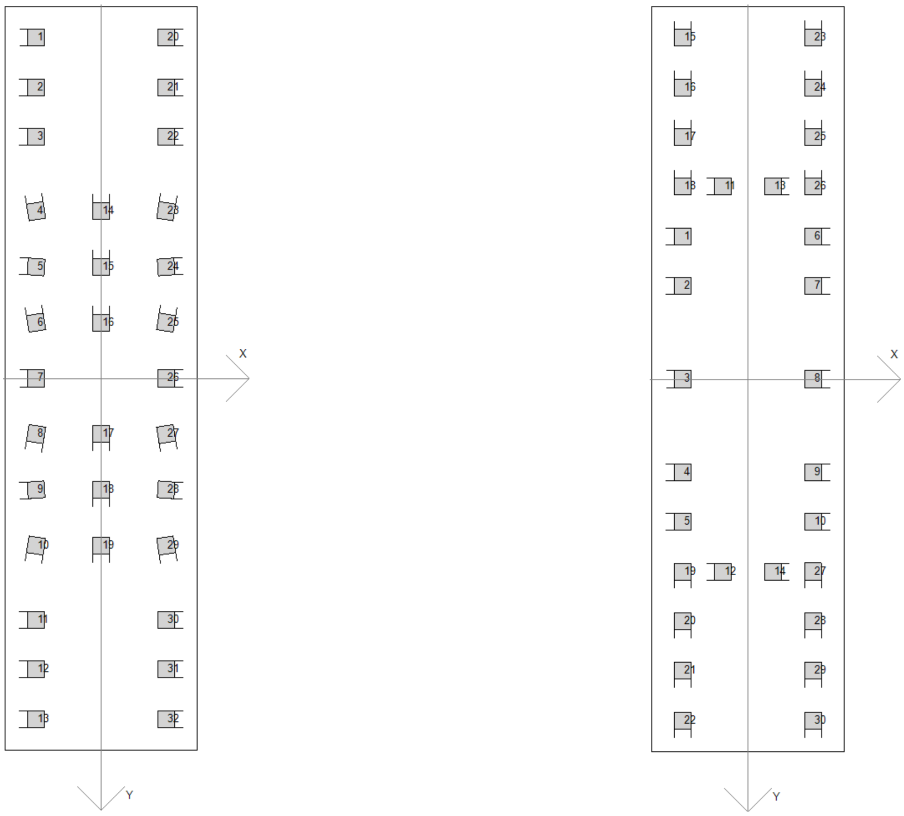
(a) Original pile group, 26 piles **(b)** Optimized pile group, 26 piles

Figure B.15: Pile groups Case XV

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1118}{0{,}9}$	0	-	-	Hinged	4	42	90

Table B.15: Input data for the optimization software for Case XV

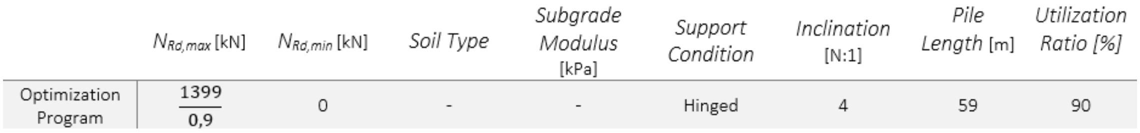
B.16 Case XVI



(a) Original pile group, 32 piles

(b) Optimized pile group, 30 piles

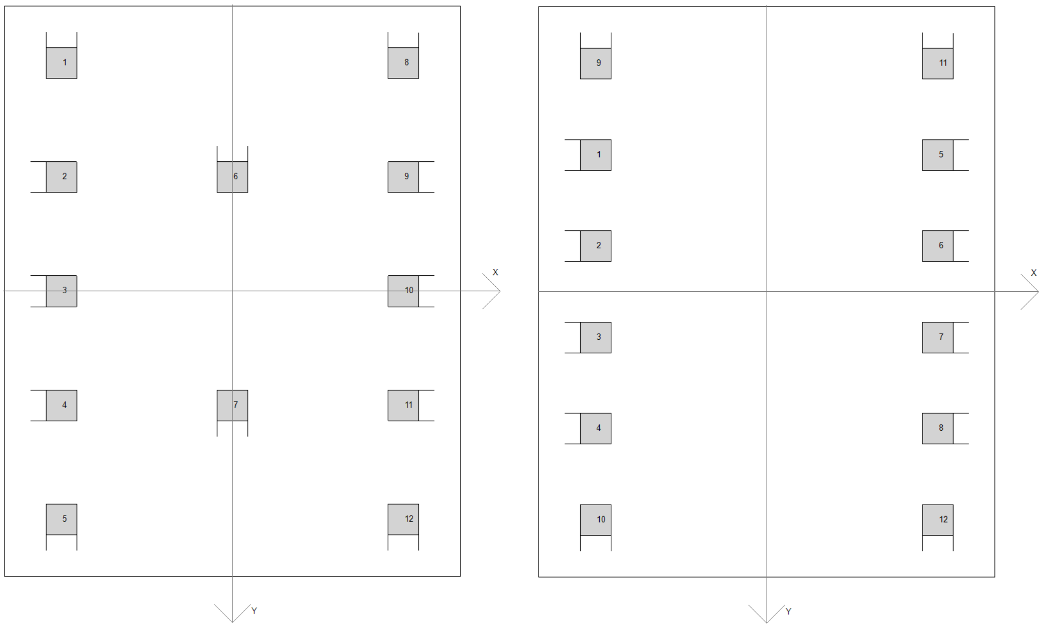
Figure B.16: Pile groups Case XVI



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1399}{0,9}$	0	-	-	Hinged	4	59	90

Table B.16: Input data for the optimization software for Case XVI

B.17 Case XVII



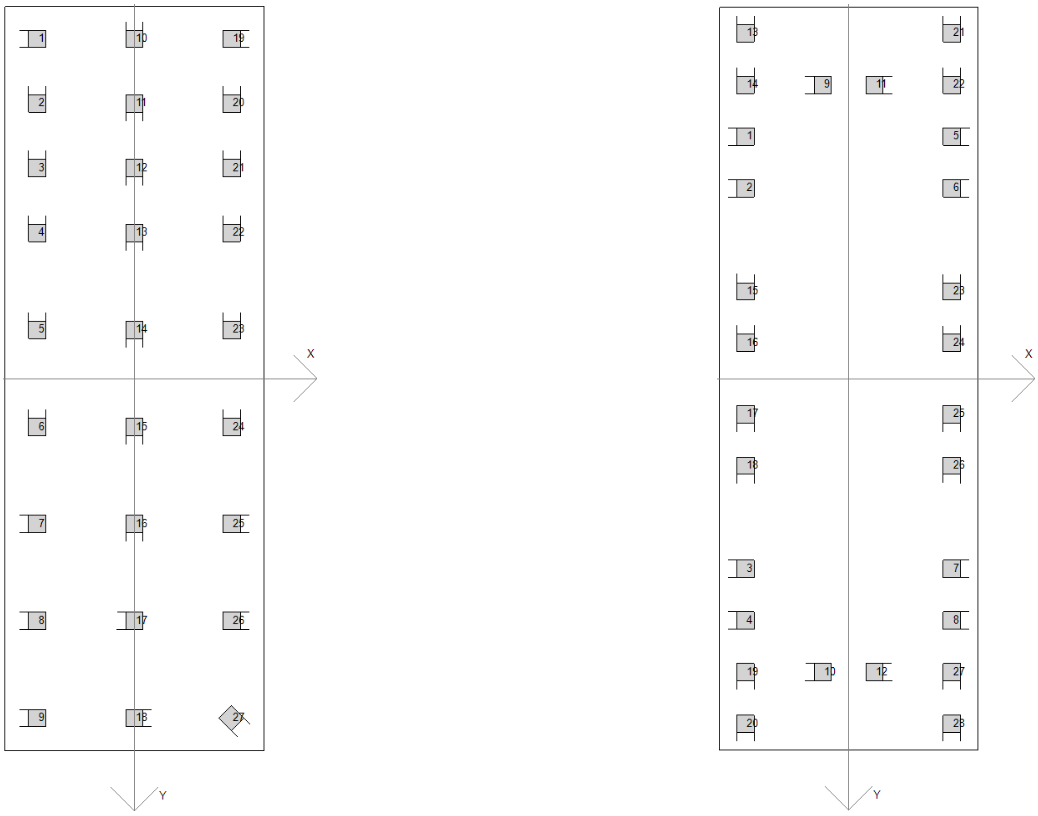
(a) Original pile group, 12 piles **(b)** Optimized pile group, 12 piles

Figure B.17: Pile groups Case XVII

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{\mathbf{1021}}{\mathbf{0,9}}$	-152	Cohesion	850	Hinged	4	25	90

Table B.17: Input data for the optimization software for Case XVII

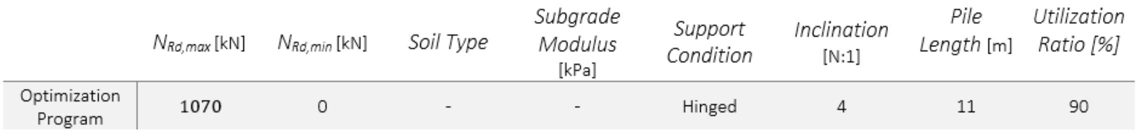
B.18 Case XVIII



(a) Original pile group, 27 piles

(b) Optimized pile group, 28 piles

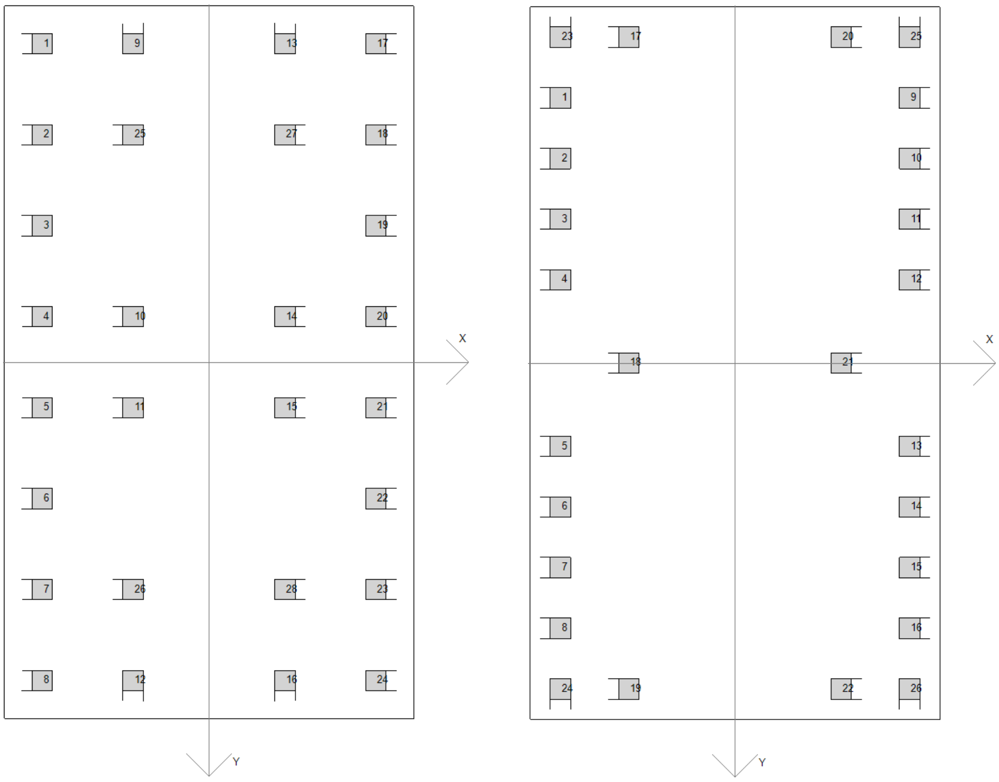
Figure B.18: Pile groups Case XVIII



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	**1070**	0	-	-	Hinged	4	11	90

Table B.18: Input data for the optimization software for Case XVIII

B.19 Case XIX



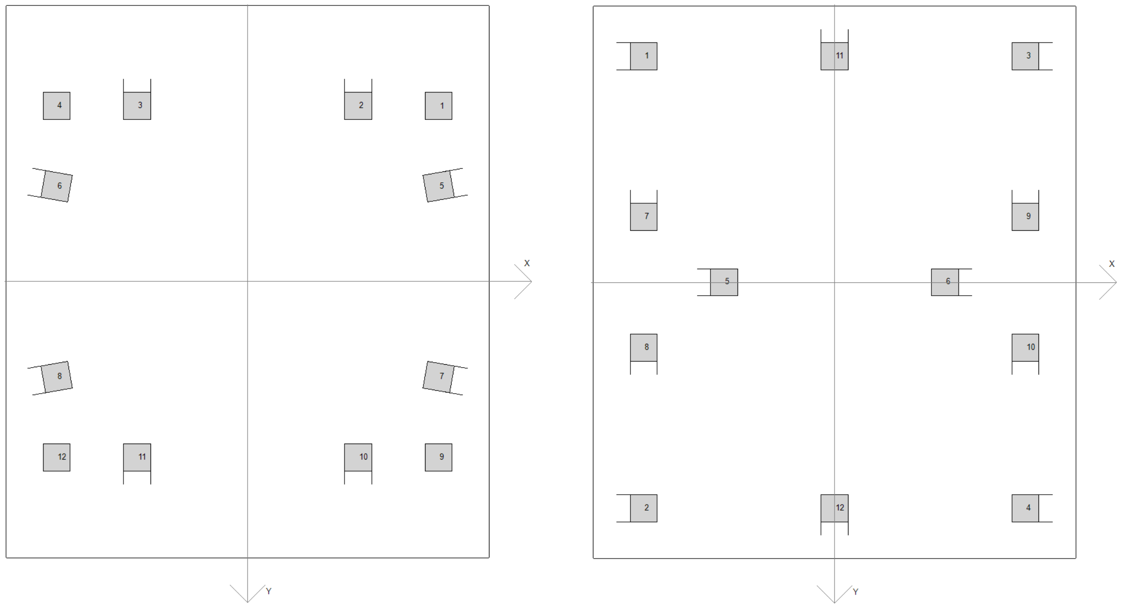
(a) Original pile group, 28 piles **(b)** Optimized pile group, 26 piles

Figure B.19: Pile groups Case XIX

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{873}{0,9}$	0	-	-	Hinged	4.5	9.5	90

Table B.19: Input data for the optimization software for Case XIX

B.20 Case XX



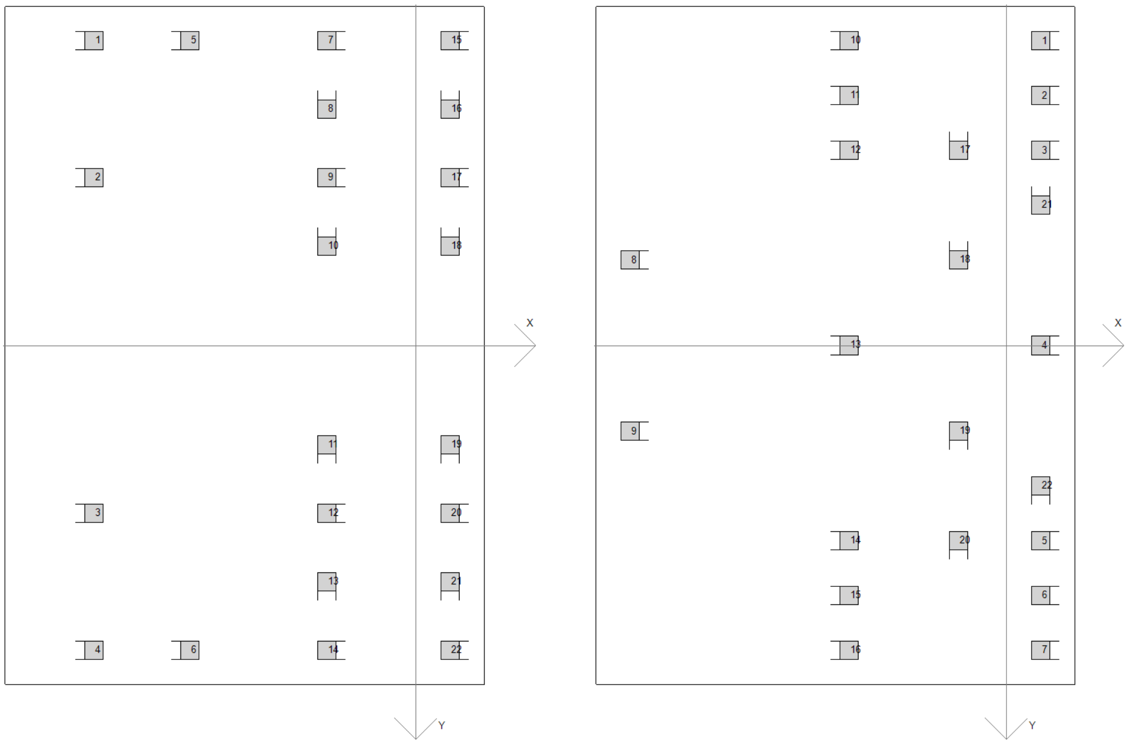
(a) Original pile group, 12 piles **(b)** Optimized pile group, 12 piles

Figure B.20: Pile groups Case XX

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1115}{0,9}$	0	Cohesion	640	Hinged	4	20	90

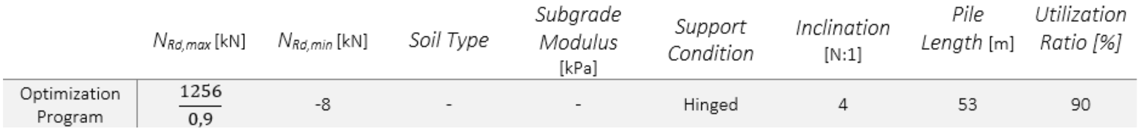
Table B.20: Input data for the optimization software for Case XX

B.21 Case XXI



(a) Original pile group, 22 piles **(b)** Optimized pile group, 22 piles

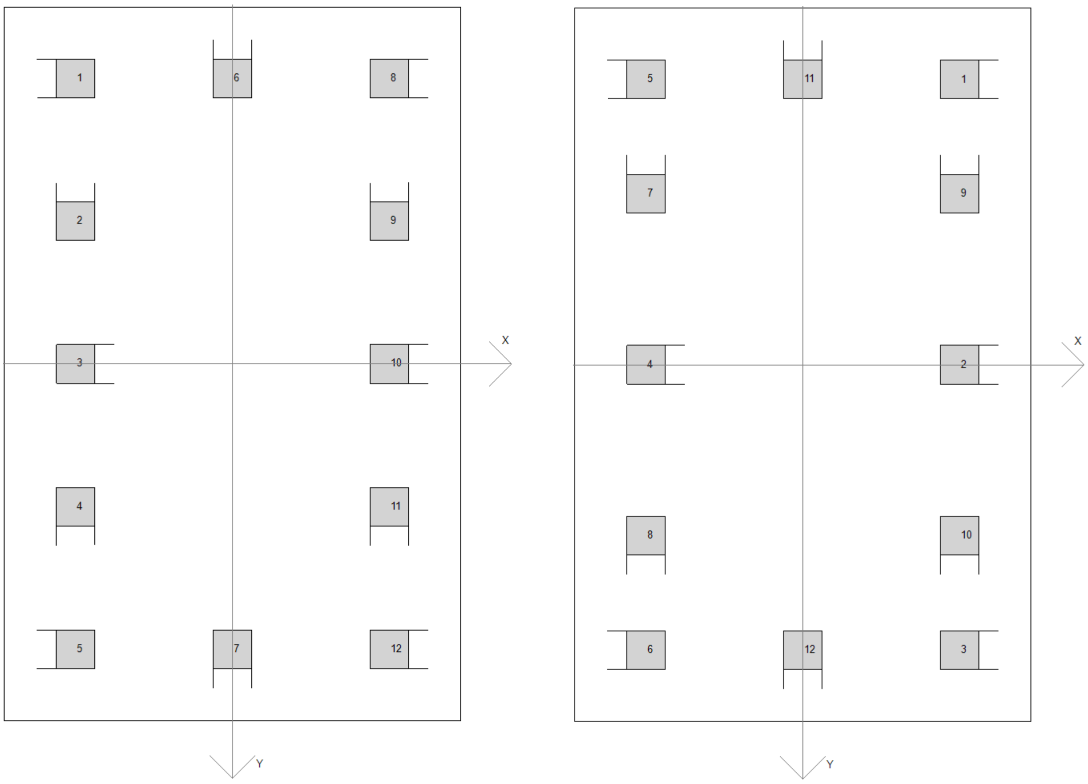
Figure B.21: Pile groups Case XXI



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1256}{0,9}$	-8	-	-	Hinged	4	53	90

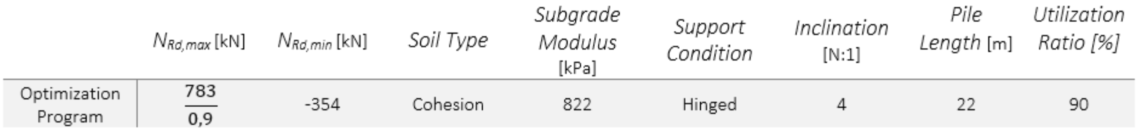
Table B.21: Input data for the optimization software for Case XXI

B.22 Case XXII



(a) Original pile group, 12 piles **(b)** Optimized pile group, 12 piles

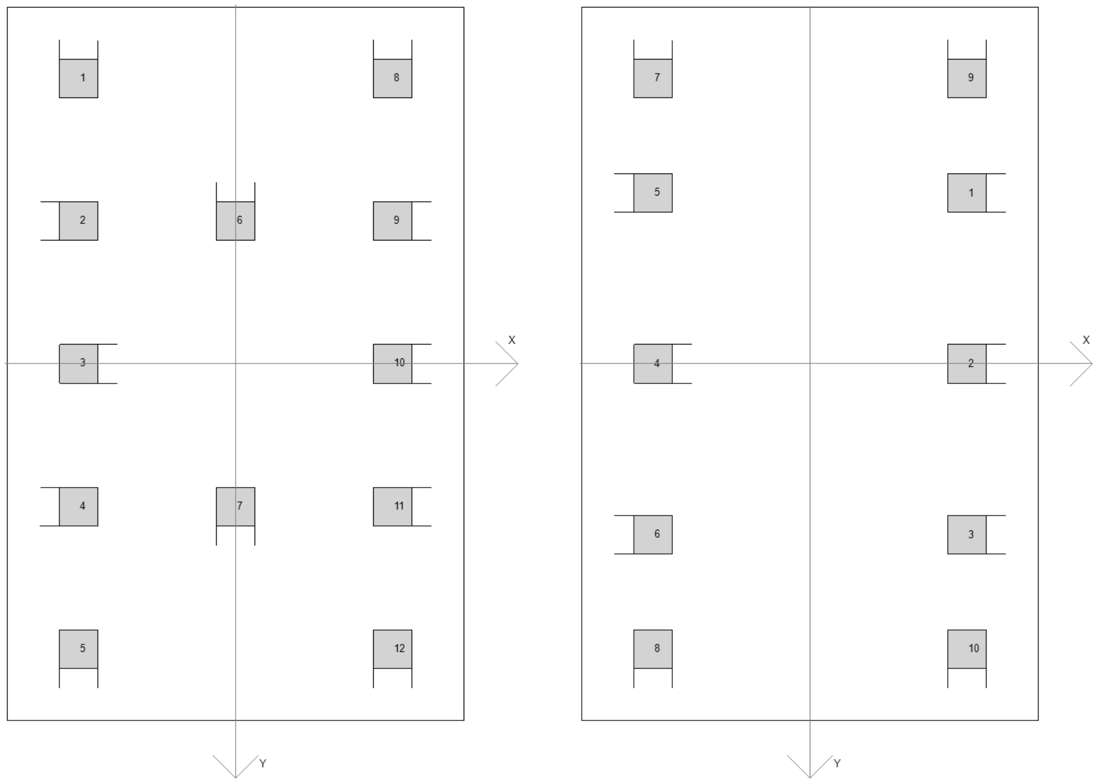
Figure B.22: Pile groups Case XXII



	$N_{Rd,max}$ [kN]	*$N_{Rd,min}$* [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{783}{0{,}9}$	-354	Cohesion	822	Hinged	4	22	90

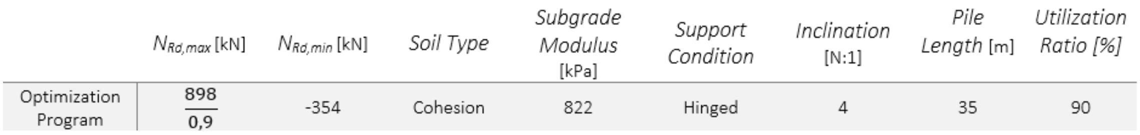
Table B.22: Input data for the optimization software for Case XXII

B.23 Case XXIII



(a) Original pile group, 12 piles **(b)** Optimized pile group, 10 piles

Figure B.23: Pile groups Case XXIII



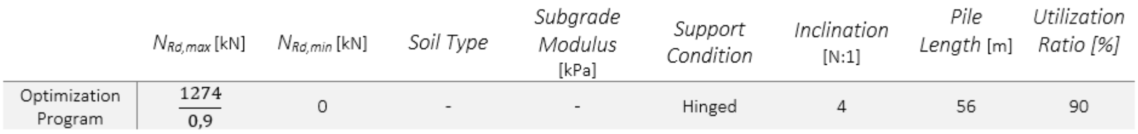
	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{898}{0,9}$	-354	Cohesion	822	Hinged	4	35	90

Table B.23: Input data for the optimization software for Case XXIII

B.24 Case XXIV

(a) Original pile group, 18 piles **(b)** Optimized pile group, 17 piles

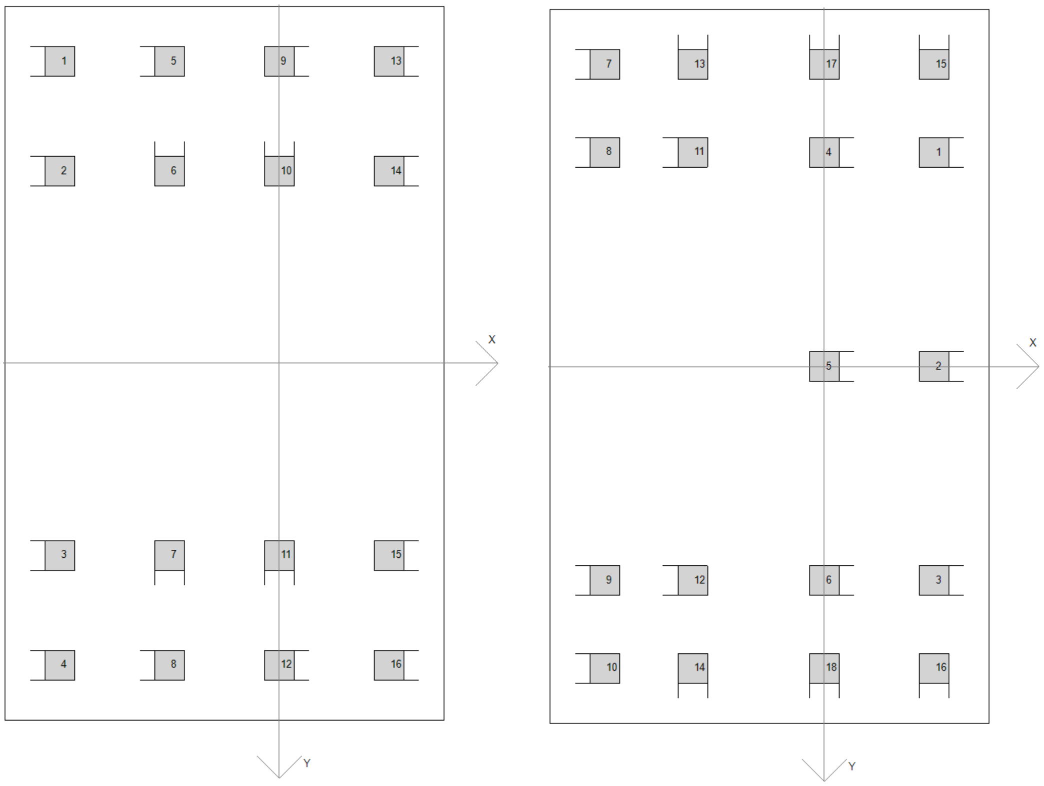
Figure B.24: Pile groups Case XXIV



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1274}{0,9}$	0	-	-	Hinged	4	56	90

Table B.24: Input data for the optimization software for Case XXIV

B.25 Case XXV



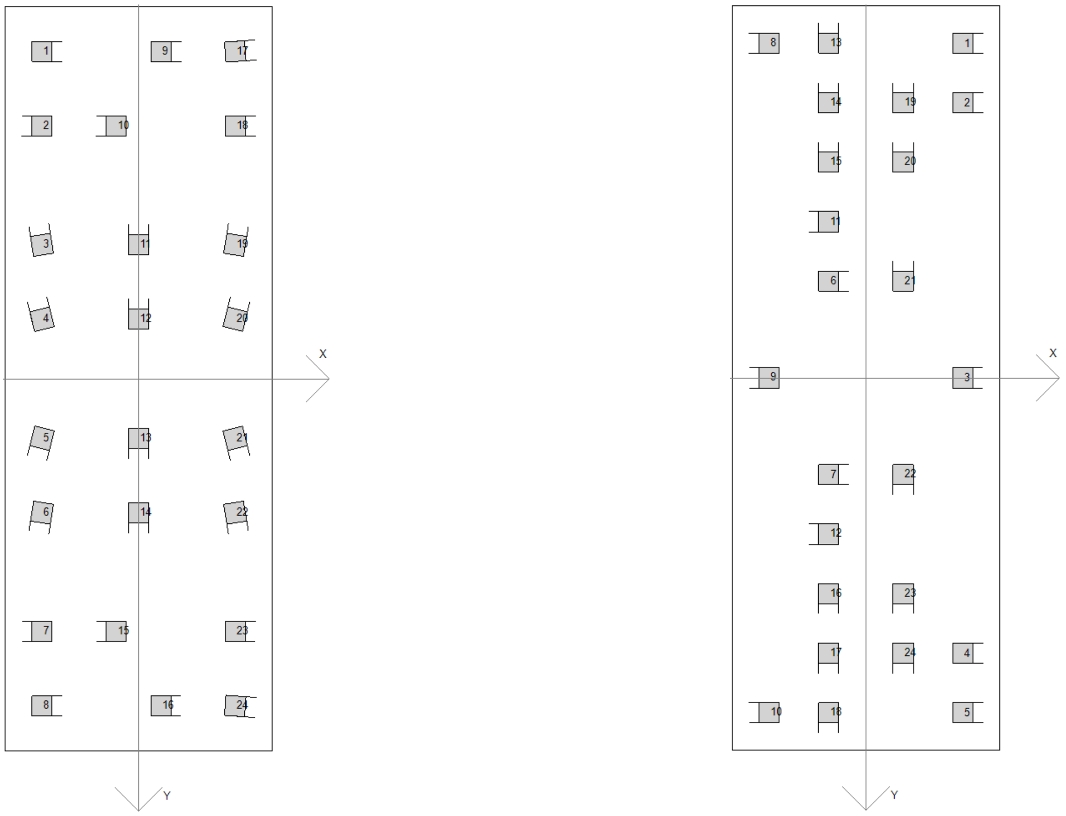
(a) Original pile group, 16 piles **(b)** Optimized pile group, 18 piles

Figure B.25: Pile groups Case XXV

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{932}{0,9}$	-133	Cohesion	240	Hinged	4	23	90

Table B.25: Input data for the optimization software for Case XXV

B.26 Case XXVI



(a) Original pile group, 24 piles

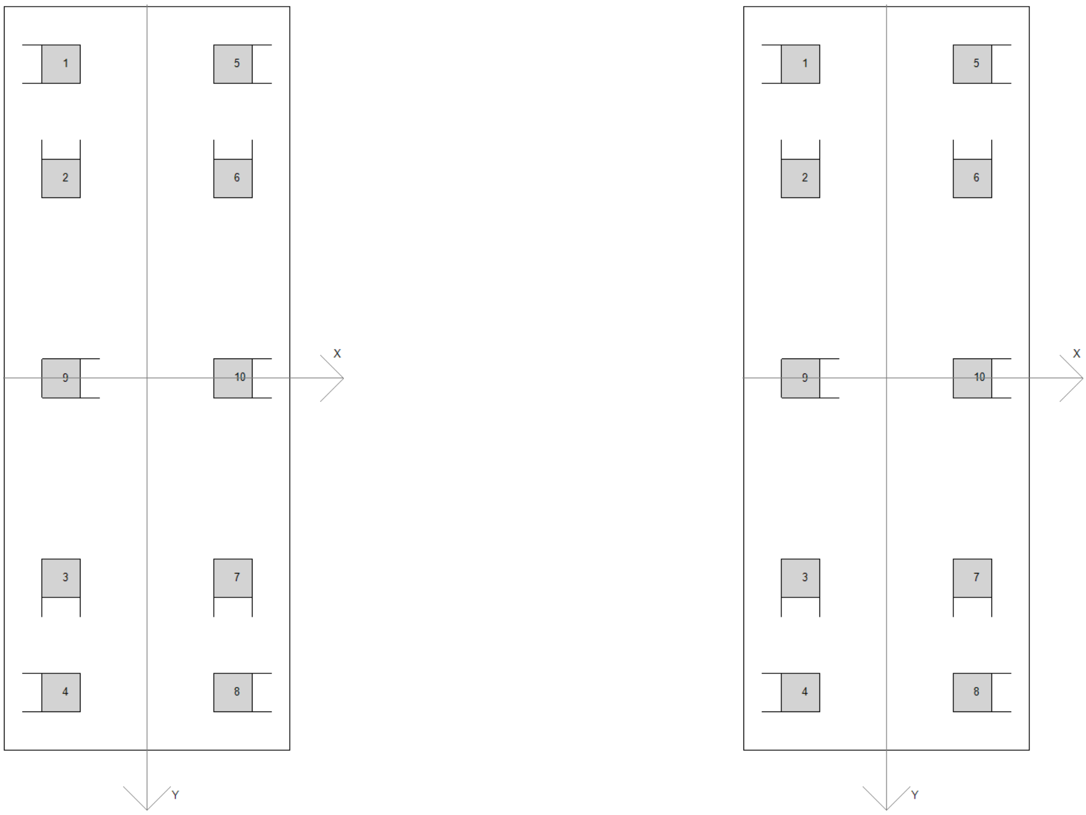
(b) Optimized pile group, 24 piles

Figure B.26: Pile groups Case XXVI

	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1240}{0,9}$	-167	-	-	Hinged	4	65	90

Table B.26: Input data for the optimization software for Case XXVI

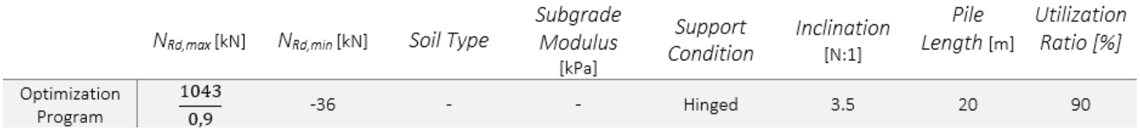
B.27 Case XXVII



(a) Original pile group, 10 piles

(b) Optimized pile group, 10 piles

Figure B.27: Pile groups Case XXVII



	$N_{Rd,max}$ [kN]	$N_{Rd,min}$ [kN]	*Soil Type*	*Subgrade Modulus* [kPa]	*Support Condition*	*Inclination* [N:1]	*Pile Length* [m]	*Utilization Ratio [%]*
Optimization Program	$\frac{1043}{0,9}$	-36	-	-	Hinged	3.5	20	90

Table B.27: Input data for the optimization software for Case XXVII

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